

Computer algebra independent integration tests

4-Trig-functions/4.5-Secant/4.5.3.1-a+b-sec-^m-d-sec-ⁿ-A+B-sec-

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3.175	$\int \frac{A+A \sec(c+dx)}{(a-a \sec(c+dx))^{5/2}} dx$	1021
3.176	$\int \frac{\cos(c+dx)(A+A \sec(c+dx))}{(a-a \sec(c+dx))^{5/2}} dx$	1027
3.177	$\int \frac{\cos^2(c+dx)(A+A \sec(c+dx))}{(a-a \sec(c+dx))^{5/2}} dx$	1033
3.178	$\int \frac{\cos^3(c+dx)(A+A \sec(c+dx))}{(a-a \sec(c+dx))^{5/2}} dx$	1040
3.179	$\int \sec^{\frac{5}{2}}(c+dx)(a+a \sec(c+dx))(A+B \sec(c+dx)) dx$	1047
3.180	$\int \sec^{\frac{3}{2}}(c+dx)(a+a \sec(c+dx))(A+B \sec(c+dx)) dx$	1052
3.181	$\int \sqrt{\sec(c+dx)}(a+a \sec(c+dx))(A+B \sec(c+dx)) dx$	1057
3.182	$\int \frac{(a+a \sec(c+dx))(A+B \sec(c+dx))}{\sqrt{\sec(c+dx)}} dx$	1061
3.183	$\int \frac{(a+a \sec(c+dx))(A+B \sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)} dx$	1065

- 3.184 $\int \frac{(a+a \sec(c+dx))(A+B \sec(c+dx))}{\sec^{\frac{5}{2}}(c+dx)} dx \dots\dots\dots .1069$
- 3.185 $\int \frac{(a+a \sec(c+dx))(A+B \sec(c+dx))}{\sec^{\frac{7}{2}}(c+dx)} dx \dots\dots\dots .1074$
- 3.186 $\int \sec^{\frac{3}{2}}(c+dx)(a+a \sec(c+dx))^2(A+B \sec(c+dx)) dx \dots\dots\dots .1079$
- 3.187 $\int \sqrt{\sec(c+dx)}(a+a \sec(c+dx))^2(A+B \sec(c+dx)) dx \dots\dots\dots .1084$
- 3.188 $\int \frac{(a+a \sec(c+dx))^2(A+B \sec(c+dx))}{\sqrt{\sec(c+dx)}} dx \dots\dots\dots .1089$
- 3.189 $\int \frac{(a+a \sec(c+dx))^2(A+B \sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)} dx \dots\dots\dots .1094$
- 3.190 $\int \frac{(a+a \sec(c+dx))^2(A+B \sec(c+dx))}{\sec^{\frac{5}{2}}(c+dx)} dx \dots\dots\dots .1099$
- 3.191 $\int \frac{(a+a \sec(c+dx))^2(A+B \sec(c+dx))}{\sec^{\frac{7}{2}}(c+dx)} dx \dots\dots\dots .1104$
- 3.192 $\int \frac{(a+a \sec(c+dx))^2(A+B \sec(c+dx))}{\sec^{\frac{9}{2}}(c+dx)} dx \dots\dots\dots .1109$
- 3.193 $\int \sec^{\frac{3}{2}}(c+dx)(a+a \sec(c+dx))^3(A+B \sec(c+dx)) dx \dots\dots\dots .1114$
- 3.194 $\int \sqrt{\sec(c+dx)}(a+a \sec(c+dx))^3(A+B \sec(c+dx)) dx \dots\dots\dots .1120$
- 3.195 $\int \frac{(a+a \sec(c+dx))^3(A+B \sec(c+dx))}{\sqrt{\sec(c+dx)}} dx \dots\dots\dots .1126$
- 3.196 $\int \frac{(a+a \sec(c+dx))^3(A+B \sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)} dx \dots\dots\dots .1131$
- 3.197 $\int \frac{(a+a \sec(c+dx))^3(A+B \sec(c+dx))}{\sec^{\frac{5}{2}}(c+dx)} dx \dots\dots\dots .1136$
- 3.198 $\int \frac{(a+a \sec(c+dx))^3(A+B \sec(c+dx))}{\sec^{\frac{7}{2}}(c+dx)} dx \dots\dots\dots .1141$
- 3.199 $\int \frac{(a+a \sec(c+dx))^3(A+B \sec(c+dx))}{\sec^{\frac{9}{2}}(c+dx)} dx \dots\dots\dots .1146$
- 3.200 $\int \frac{(a+a \sec(c+dx))^3(A+B \sec(c+dx))}{\sec^{\frac{11}{2}}(c+dx)} dx \dots\dots\dots .1151$
- 3.201 $\int \frac{\sec^{\frac{7}{2}}(c+dx)(A+B \sec(c+dx))}{a+a \sec(c+dx)} dx \dots\dots\dots .1156$
- 3.202 $\int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B \sec(c+dx))}{a+a \sec(c+dx)} dx \dots\dots\dots .1162$
- 3.203 $\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B \sec(c+dx))}{a+a \sec(c+dx)} dx \dots\dots\dots .1167$
- 3.204 $\int \frac{\sqrt{\sec(c+dx)}(A+B \sec(c+dx))}{a+a \sec(c+dx)} dx \dots\dots\dots .1172$
- 3.205 $\int \frac{A+B \sec(c+dx)}{\sqrt{\sec(c+dx)}(a+a \sec(c+dx))} dx \dots\dots\dots .1176$
- 3.206 $\int \frac{A+B \sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a \sec(c+dx))} dx \dots\dots\dots .1180$

3.207	$\int \frac{A+B \sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+a \sec(c+dx))} dx$1185
3.208	$\int \frac{A+B \sec(c+dx)}{\sec^{\frac{7}{2}}(c+dx)(a+a \sec(c+dx))} dx$1190
3.209	$\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B \sec(c+dx))}{(a+a \sec(c+dx))^2} dx$1195
3.210	$\int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B \sec(c+dx))}{(a+a \sec(c+dx))^2} dx$1201
3.211	$\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B \sec(c+dx))}{(a+a \sec(c+dx))^2} dx$1206
3.212	$\int \frac{\sqrt{\sec(c+dx)}(A+B \sec(c+dx))}{(a+a \sec(c+dx))^2} dx$1211
3.213	$\int \frac{A+B \sec(c+dx)}{\sqrt{\sec(c+dx)}(a+a \sec(c+dx))^2} dx$1216
3.214	$\int \frac{A+B \sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a \sec(c+dx))^2} dx$1221
3.215	$\int \frac{A+B \sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+a \sec(c+dx))^2} dx$1227
3.216	$\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B \sec(c+dx))}{(a+a \sec(c+dx))^3} dx$1233
3.217	$\int \frac{\sec^{\frac{7}{2}}(c+dx)(A+B \sec(c+dx))}{(a+a \sec(c+dx))^3} dx$1239
3.218	$\int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B \sec(c+dx))}{(a+a \sec(c+dx))^3} dx$1245
3.219	$\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B \sec(c+dx))}{(a+a \sec(c+dx))^3} dx$1251
3.220	$\int \frac{\sqrt{\sec(c+dx)}(A+B \sec(c+dx))}{(a+a \sec(c+dx))^3} dx$1257
3.221	$\int \frac{A+B \sec(c+dx)}{\sqrt{\sec(c+dx)}(a+a \sec(c+dx))^3} dx$1263
3.222	$\int \frac{A+B \sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a \sec(c+dx))^3} dx$1269
3.223	$\int \frac{A+B \sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+a \sec(c+dx))^3} dx$1275
3.224	$\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+a \sec(c+dx)}(A+B \sec(c+dx)) dx$1281
3.225	$\int \sec^{\frac{5}{2}}(c+dx)\sqrt{a+a \sec(c+dx)}(A+B \sec(c+dx)) dx$1288
3.226	$\int \sqrt{\sec(c+dx)}\sqrt{a+a \sec(c+dx)}(A+B \sec(c+dx)) dx$1294
3.227	$\int \frac{\sqrt{a+a \sec(c+dx)}(A+B \sec(c+dx))}{\sqrt{\sec(c+dx)}} dx$1299
3.228	$\int \frac{\sqrt{a+a \sec(c+dx)}(A+B \sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)} dx$1303
3.229	$\int \frac{\sqrt{a+a \sec(c+dx)}(A+B \sec(c+dx))}{\sec^{\frac{5}{2}}(c+dx)} dx$1307

- 3.230 $\int \frac{\sqrt{a+a \sec(c+dx)}(A+B \sec(c+dx))}{\sec^{\frac{7}{2}}(c+dx)} dx \dots\dots\dots .1311$
- 3.231 $\int \sec^{\frac{5}{2}}(c+dx)(a+a \sec(c+dx))^{3/2}(A+B \sec(c+dx)) dx \dots\dots\dots .1315$
- 3.232 $\int \sec^{\frac{3}{2}}(c+dx)(a+a \sec(c+dx))^{3/2}(A+B \sec(c+dx)) dx \dots\dots\dots .1324$
- 3.233 $\int \sqrt{\sec(c+dx)}(a+a \sec(c+dx))^{3/2}(A+B \sec(c+dx)) dx \dots\dots\dots .1333$
- 3.234 $\int \frac{(a+a \sec(c+dx))^{3/2}(A+B \sec(c+dx))}{\sqrt{\sec(c+dx)}} dx \dots\dots\dots .1340$
- 3.235 $\int \frac{(a+a \sec(c+dx))^{3/2}(A+B \sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)} dx \dots\dots\dots .1346$
- 3.236 $\int \frac{(a+a \sec(c+dx))^{3/2}(A+B \sec(c+dx))}{\sec^{\frac{5}{2}}(c+dx)} dx \dots\dots\dots .1351$
- 3.237 $\int \frac{(a+a \sec(c+dx))^{3/2}(A+B \sec(c+dx))}{\sec^{\frac{7}{2}}(c+dx)} dx \dots\dots\dots .1355$
- 3.238 $\int \frac{(a+a \sec(c+dx))^{3/2}(A+B \sec(c+dx))}{\sec^{\frac{9}{2}}(c+dx)} dx \dots\dots\dots .1360$
- 3.239 $\int \sec^{\frac{5}{2}}(c+dx)(a+a \sec(c+dx))^{5/2}(A+B \sec(c+dx)) dx \dots\dots\dots .1365$
- 3.240 $\int \sec^{\frac{3}{2}}(c+dx)(a+a \sec(c+dx))^{5/2}(A+B \sec(c+dx)) dx \dots\dots\dots .1376$
- 3.241 $\int \sqrt{\sec(c+dx)}(a+a \sec(c+dx))^{5/2}(A+B \sec(c+dx)) dx \dots\dots\dots .1386$
- 3.242 $\int \frac{(a+a \sec(c+dx))^{5/2}(A+B \sec(c+dx))}{\sqrt{\sec(c+dx)}} dx \dots\dots\dots .1395$
- 3.243 $\int \frac{(a+a \sec(c+dx))^{5/2}(A+B \sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)} dx \dots\dots\dots .1400$
- 3.244 $\int \frac{(a+a \sec(c+dx))^{5/2}(A+B \sec(c+dx))}{\sec^{\frac{5}{2}}(c+dx)} dx \dots\dots\dots .1405$
- 3.245 $\int \frac{(a+a \sec(c+dx))^{5/2}(A+B \sec(c+dx))}{\sec^{\frac{7}{2}}(c+dx)} dx \dots\dots\dots .1410$
- 3.246 $\int \frac{(a+a \sec(c+dx))^{5/2}(A+B \sec(c+dx))}{\sec^{\frac{9}{2}}(c+dx)} dx \dots\dots\dots .1414$
- 3.247 $\int \frac{(a+a \sec(c+dx))^{5/2}(A+B \sec(c+dx))}{\sec^{\frac{11}{2}}(c+dx)} dx \dots\dots\dots .1419$
- 3.248 $\int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B \sec(c+dx))}{\sqrt{a+a \sec(c+dx)}} dx \dots\dots\dots .1425$
- 3.249 $\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B \sec(c+dx))}{\sqrt{a+a \sec(c+dx)}} dx \dots\dots\dots .1433$
- 3.250 $\int \frac{\sqrt{\sec(c+dx)}(A+B \sec(c+dx))}{\sqrt{a+a \sec(c+dx)}} dx \dots\dots\dots .1439$
- 3.251 $\int \frac{A+B \sec(c+dx)}{\sqrt{\sec(c+dx)} \sqrt{a+a \sec(c+dx)}} dx \dots\dots\dots .1444$
- 3.252 $\int \frac{A+B \sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx) \sqrt{a+a \sec(c+dx)}} dx \dots\dots\dots .1448$

3.253	$\int \frac{A+B \sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a \sec(c+dx)}} dx$.1453
3.254	$\int \frac{A+B \sec(c+dx)}{\sec^{\frac{7}{2}}(c+dx)\sqrt{a+a \sec(c+dx)}} dx$.1458
3.255	$\int \frac{\sec^{\frac{7}{2}}(c+dx)(A+B \sec(c+dx))}{(a+a \sec(c+dx))^{\frac{3}{2}}} dx$.1464
3.256	$\int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B \sec(c+dx))}{(a+a \sec(c+dx))^{\frac{3}{2}}} dx$.1470
3.257	$\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B \sec(c+dx))}{(a+a \sec(c+dx))^{\frac{3}{2}}} dx$.1480
3.258	$\int \frac{\sqrt{\sec(c+dx)}(A+B \sec(c+dx))}{(a+a \sec(c+dx))^{\frac{3}{2}}} dx$.1485
3.259	$\int \frac{A+B \sec(c+dx)}{\sqrt{\sec(c+dx)}(a+a \sec(c+dx))^{\frac{3}{2}}} dx$.1489
3.260	$\int \frac{A+B \sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a \sec(c+dx))^{\frac{3}{2}}} dx$.1499
3.261	$\int \frac{A+B \sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+a \sec(c+dx))^{\frac{3}{2}}} dx$.1504
3.262	$\int \frac{\sec^{\frac{7}{2}}(c+dx)(A+B \sec(c+dx))}{(a+a \sec(c+dx))^{\frac{5}{2}}} dx$.1510
3.263	$\int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B \sec(c+dx))}{(a+a \sec(c+dx))^{\frac{5}{2}}} dx$.1516
3.264	$\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B \sec(c+dx))}{(a+a \sec(c+dx))^{\frac{5}{2}}} dx$.1522
3.265	$\int \frac{\sqrt{\sec(c+dx)}(A+B \sec(c+dx))}{(a+a \sec(c+dx))^{\frac{5}{2}}} dx$.1527
3.266	$\int \frac{A+B \sec(c+dx)}{\sqrt{\sec(c+dx)}(a+a \sec(c+dx))^{\frac{5}{2}}} dx$.1536
3.267	$\int \frac{A+B \sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a \sec(c+dx))^{\frac{5}{2}}} dx$.1541
3.268	$\int \frac{A+B \sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+a \sec(c+dx))^{\frac{5}{2}}} dx$.1547
3.269	$\int (a+a \sec(c+dx))^{\frac{2}{3}}(A+B \sec(c+dx)) dx$.1553
3.270	$\int \frac{A+B \sec(c+dx)}{\sqrt[3]{a+a \sec(c+dx)}} dx$.1561
3.271	$\int \frac{A+B \sec(c+dx)}{(a+a \sec(c+dx))^{\frac{4}{3}}} dx$.1567
3.272	$\int (a+a \sec(c+dx))^{\frac{4}{3}}(A+B \sec(c+dx)) dx$.1574
3.273	$\int \sqrt[3]{a+a \sec(c+dx)}(A+B \sec(c+dx)) dx$.1583
3.274	$\int \frac{A+B \sec(c+dx)}{(a+a \sec(c+dx))^{\frac{2}{3}}} dx$.1589
3.275	$\int (c \sec(e+fx))^n (a+a \sec(e+fx))^m (A+B \sec(e+fx)) dx$.1598
3.276	$\int \sec^{-1-n}(c+dx)(a+a \sec(c+dx))^n (A+B \sec(c+dx)) dx$.1604
3.277	$\int \sec^3(c+dx)(a+b \sec(c+dx))(A+B \sec(c+dx)) dx$.1608
3.278	$\int \sec^2(c+dx)(a+b \sec(c+dx))(A+B \sec(c+dx)) dx$.1612

3.279	$\int \sec(c + dx)(a + b \sec(c + dx))(A + B \sec(c + dx)) dx$.1616
3.280	$\int (a + b \sec(c + dx))(A + B \sec(c + dx)) dx$.1620
3.281	$\int \cos(c + dx)(a + b \sec(c + dx))(A + B \sec(c + dx)) dx$.1624
3.282	$\int \cos^2(c + dx)(a + b \sec(c + dx))(A + B \sec(c + dx)) dx$.1627
3.283	$\int \cos^3(c + dx)(a + b \sec(c + dx))(A + B \sec(c + dx)) dx$.1631
3.284	$\int \cos^4(c + dx)(a + b \sec(c + dx))(A + B \sec(c + dx)) dx$.1635
3.285	$\int \sec^3(c + dx)(a + b \sec(c + dx))^2(A + B \sec(c + dx)) dx$.1639
3.286	$\int \sec^2(c + dx)(a + b \sec(c + dx))^2(A + B \sec(c + dx)) dx$.1644
3.287	$\int \sec(c + dx)(a + b \sec(c + dx))^2(A + B \sec(c + dx)) dx$.1649
3.288	$\int (a + b \sec(c + dx))^2(A + B \sec(c + dx)) dx$.1654
3.289	$\int \cos(c + dx)(a + b \sec(c + dx))^2(A + B \sec(c + dx)) dx$.1658
3.290	$\int \cos^2(c + dx)(a + b \sec(c + dx))^2(A + B \sec(c + dx)) dx$.1662
3.291	$\int \cos^3(c + dx)(a + b \sec(c + dx))^2(A + B \sec(c + dx)) dx$.1666
3.292	$\int \cos^4(c + dx)(a + b \sec(c + dx))^2(A + B \sec(c + dx)) dx$.1670
3.293	$\int \cos^5(c + dx)(a + b \sec(c + dx))^2(A + B \sec(c + dx)) dx$.1675
3.294	$\int \sec^2(c + dx)(a + b \sec(c + dx))^3(A + B \sec(c + dx)) dx$.1680
3.295	$\int \sec(c + dx)(a + b \sec(c + dx))^3(A + B \sec(c + dx)) dx$.1686
3.296	$\int (a + b \sec(c + dx))^3(A + B \sec(c + dx)) dx$.1691
3.297	$\int \cos(c + dx)(a + b \sec(c + dx))^3(A + B \sec(c + dx)) dx$.1696
3.298	$\int \cos^2(c + dx)(a + b \sec(c + dx))^3(A + B \sec(c + dx)) dx$.1701
3.299	$\int \cos^3(c + dx)(a + b \sec(c + dx))^3(A + B \sec(c + dx)) dx$.1706
3.300	$\int \cos^4(c + dx)(a + b \sec(c + dx))^3(A + B \sec(c + dx)) dx$.1712
3.301	$\int \cos^5(c + dx)(a + b \sec(c + dx))^3(A + B \sec(c + dx)) dx$.1717
3.302	$\int \sec^2(c + dx)(a + b \sec(c + dx))^4(A + B \sec(c + dx)) dx$.1723
3.303	$\int \sec(c + dx)(a + b \sec(c + dx))^4(A + B \sec(c + dx)) dx$.1730
3.304	$\int (a + b \sec(c + dx))^4(A + B \sec(c + dx)) dx$.1736
3.305	$\int \cos(c + dx)(a + b \sec(c + dx))^4(A + B \sec(c + dx)) dx$.1742
3.306	$\int \cos^2(c + dx)(a + b \sec(c + dx))^4(A + B \sec(c + dx)) dx$.1748
3.307	$\int \cos^3(c + dx)(a + b \sec(c + dx))^4(A + B \sec(c + dx)) dx$.1754
3.308	$\int \cos^4(c + dx)(a + b \sec(c + dx))^4(A + B \sec(c + dx)) dx$.1761
3.309	$\int \cos^5(c + dx)(a + b \sec(c + dx))^4(A + B \sec(c + dx)) dx$.1767
3.310	$\int \cos^6(c + dx)(a + b \sec(c + dx))^4(A + B \sec(c + dx)) dx$.1773
3.311	$\int \frac{\sec^4(c+dx)(A+B \sec(c+dx))}{a+b \sec(c+dx)} dx$.1779
3.312	$\int \frac{\sec^3(c+dx)(A+B \sec(c+dx))}{a+b \sec(c+dx)} dx$.1787
3.313	$\int \frac{\sec^2(c+dx)(A+B \sec(c+dx))}{a+b \sec(c+dx)} dx$.1795
3.314	$\int \frac{\sec(c+dx)(A+B \sec(c+dx))}{a+b \sec(c+dx)} dx$.1801

3.315	$\int \frac{A+B \sec(c+dx)}{a+b \sec(c+dx)} dx$1806
3.316	$\int \frac{\cos(c+dx)(A+B \sec(c+dx))}{a+b \sec(c+dx)} dx$1810
3.317	$\int \frac{\cos^2(c+dx)(A+B \sec(c+dx))}{a+b \sec(c+dx)} dx$1815
3.318	$\int \frac{\cos^3(c+dx)(A+B \sec(c+dx))}{a+b \sec(c+dx)} dx$1822
3.319	$\int \frac{\cos^4(c+dx)(A+B \sec(c+dx))}{a+b \sec(c+dx)} dx$1830
3.320	$\int \frac{\sec^4(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^2} dx$1839
3.321	$\int \frac{\sec^3(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^2} dx$1849
3.322	$\int \frac{\sec^2(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^2} dx$1857
3.323	$\int \frac{\sec(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^2} dx$1864
3.324	$\int \frac{A+B \sec(c+dx)}{(a+b \sec(c+dx))^2} dx$1869
3.325	$\int \frac{\cos(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^2} dx$1875
3.326	$\int \frac{\cos^2(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^2} dx$1883
3.327	$\int \frac{\cos^3(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^2} dx$1892
3.328	$\int \frac{\sec^5(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^3} dx$1902
3.329	$\int \frac{\sec^4(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^3} dx$1916
3.330	$\int \frac{\sec^3(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^3} dx$1928
3.331	$\int \frac{\sec^2(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^3} dx$1938
3.332	$\int \frac{\sec(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^3} dx$1944
3.333	$\int \frac{A+B \sec(c+dx)}{(a+b \sec(c+dx))^3} dx$1950
3.334	$\int \frac{\cos(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^3} dx$1959
3.335	$\int \frac{\cos^2(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^3} dx$1969
3.336	$\int \frac{\sec^5(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^4} dx$1983
3.337	$\int \frac{\sec^4(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^4} dx$1999
3.338	$\int \frac{\sec^3(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^4} dx$2012
3.339	$\int \frac{\sec^2(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^4} dx$2019
3.340	$\int \frac{\sec(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^4} dx$2026
3.341	$\int \frac{A+B \sec(c+dx)}{(a+b \sec(c+dx))^4} dx$2033

3.342	$\int \frac{\cos(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^4} dx$.2045
3.343	$\int \frac{\cos^2(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^4} dx$.2058
3.344	$\int \frac{\frac{bB}{a} + B \sec(c+dx)}{a+b \sec(c+dx)} dx$.2076
3.345	$\int \frac{\frac{aB}{b} + B \sec(c+dx)}{a+b \sec(c+dx)} dx$.2080
3.346	$\int \frac{a+b \sec(c+dx)}{(b+a \sec(c+dx))^2} dx$.2083
3.347	$\int \frac{3+\sec(c+dx)}{2-\sec(c+dx)} dx$.2088
3.348	$\int \sec^4(c+dx)\sqrt{a+b \sec(c+dx)}(A+B \sec(c+dx)) dx$.2092
3.349	$\int \sec^3(c+dx)\sqrt{a+b \sec(c+dx)}(A+B \sec(c+dx)) dx$.2101
3.350	$\int \sec^2(c+dx)\sqrt{a+b \sec(c+dx)}(A+B \sec(c+dx)) dx$.2109
3.351	$\int \sec(c+dx)\sqrt{a+b \sec(c+dx)}(A+B \sec(c+dx)) dx$.2116
3.352	$\int \sqrt{a+b \sec(c+dx)}(A+B \sec(c+dx)) dx$.2121
3.353	$\int \cos(c+dx)\sqrt{a+b \sec(c+dx)}(A+B \sec(c+dx)) dx$.2126
3.354	$\int \cos^2(c+dx)\sqrt{a+b \sec(c+dx)}(A+B \sec(c+dx)) dx$.2132
3.355	$\int \cos^3(c+dx)\sqrt{a+b \sec(c+dx)}(A+B \sec(c+dx)) dx$.2138
3.356	$\int \sec^3(c+dx)(a+b \sec(c+dx))^{3/2}(A+B \sec(c+dx)) dx$.2145
3.357	$\int \sec^2(c+dx)(a+b \sec(c+dx))^{3/2}(A+B \sec(c+dx)) dx$.2154
3.358	$\int \sec(c+dx)(a+b \sec(c+dx))^{3/2}(A+B \sec(c+dx)) dx$.2162
3.359	$\int (a+b \sec(c+dx))^{3/2}(A+B \sec(c+dx)) dx$.2168
3.360	$\int \cos(c+dx)(a+b \sec(c+dx))^{3/2}(A+B \sec(c+dx)) dx$.2174
3.361	$\int \cos^2(c+dx)(a+b \sec(c+dx))^{3/2}(A+B \sec(c+dx)) dx$.2180
3.362	$\int \cos^3(c+dx)(a+b \sec(c+dx))^{3/2}(A+B \sec(c+dx)) dx$.2187
3.363	$\int \sec^3(c+dx)(a+b \sec(c+dx))^{5/2}(A+B \sec(c+dx)) dx$.2194
3.364	$\int \sec^2(c+dx)(a+b \sec(c+dx))^{5/2}(A+B \sec(c+dx)) dx$.2201
3.365	$\int \sec(c+dx)(a+b \sec(c+dx))^{5/2}(A+B \sec(c+dx)) dx$.2210
3.366	$\int (a+b \sec(c+dx))^{5/2}(A+B \sec(c+dx)) dx$.2218
3.367	$\int \cos(c+dx)(a+b \sec(c+dx))^{5/2}(A+B \sec(c+dx)) dx$.2224
3.368	$\int \cos^2(c+dx)(a+b \sec(c+dx))^{5/2}(A+B \sec(c+dx)) dx$.2230
3.369	$\int \cos^3(c+dx)(a+b \sec(c+dx))^{5/2}(A+B \sec(c+dx)) dx$.2237
3.370	$\int \cos^4(c+dx)(a+b \sec(c+dx))^{5/2}(A+B \sec(c+dx)) dx$.2245
3.371	$\int \frac{\sec^3(c+dx)(A+B \sec(c+dx))}{\sqrt{a+b \sec(c+dx)}} dx$.2253
3.372	$\int \frac{\sec^2(c+dx)(A+B \sec(c+dx))}{\sqrt{a+b \sec(c+dx)}} dx$.2260
3.373	$\int \frac{\sec(c+dx)(A+B \sec(c+dx))}{\sqrt{a+b \sec(c+dx)}} dx$.2265
3.374	$\int \frac{A+B \sec(c+dx)}{\sqrt{a+b \sec(c+dx)}} dx$.2269

3.375	$\int \frac{\cos(c+dx)(A+B \sec(c+dx))}{\sqrt{a+b \sec(c+dx)}} dx$2273
3.376	$\int \frac{\cos^2(c+dx)(A+B \sec(c+dx))}{\sqrt{a+b \sec(c+dx)}} dx$2279
3.377	$\int \frac{\cos^3(c+dx)(A+B \sec(c+dx))}{\sqrt{a+b \sec(c+dx)}} dx$2286
3.378	$\int \frac{\sec^3(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^{3/2}} dx$2293
3.379	$\int \frac{\sec^2(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^{3/2}} dx$2301
3.380	$\int \frac{\sec(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^{3/2}} dx$2307
3.381	$\int \frac{A+B \sec(c+dx)}{(a+b \sec(c+dx))^{3/2}} dx$2312
3.382	$\int \frac{\cos(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^{3/2}} dx$2318
3.383	$\int \frac{\cos^2(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^{3/2}} dx$2325
3.384	$\int \frac{\cos^3(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^{3/2}} dx$2334
3.385	$\int \frac{\sec^4(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^{5/2}} dx$2341
3.386	$\int \frac{\sec^3(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^{5/2}} dx$2348
3.387	$\int \frac{\sec^2(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^{5/2}} dx$2355
3.388	$\int \frac{\sec(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^{5/2}} dx$2361
3.389	$\int \frac{A+B \sec(c+dx)}{(a+b \sec(c+dx))^{5/2}} dx$2368
3.390	$\int \frac{\cos(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^{5/2}} dx$2374
3.391	$\int \frac{\cos^2(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^{5/2}} dx$2381
3.392	$\int \frac{\sec(e+fx)(A+A \sec(e+fx))}{\sqrt{a+b \sec(e+fx)}} dx$2387
3.393	$\int \frac{\sec(e+fx)(A-A \sec(e+fx))}{\sqrt{a+b \sec(e+fx)}} dx$2391
3.394	$\int \sec^2(c+dx)(a+b \sec(c+dx))(A+B \sec(c+dx)) dx$2395
3.395	$\int \sqrt{\sec(c+dx)}(a+b \sec(c+dx))(A+B \sec(c+dx)) dx$2399
3.396	$\int \frac{(a+b \sec(c+dx))(A+B \sec(c+dx))}{\sqrt{\sec(c+dx)}} dx$2403
3.397	$\int \frac{(a+b \sec(c+dx))(A+B \sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)} dx$2407
3.398	$\int \frac{(a+b \sec(c+dx))(A+B \sec(c+dx))}{\sec^{\frac{5}{2}}(c+dx)} dx$2411
3.399	$\int \frac{(a+b \sec(c+dx))(A+B \sec(c+dx))}{\sec^{\frac{7}{2}}(c+dx)} dx$2416
3.400	$\int \sec^{\frac{3}{2}}(c+dx)(a+b \sec(c+dx))^2(A+B \sec(c+dx)) dx$2421
3.401	$\int \sqrt{\sec(c+dx)}(a+b \sec(c+dx))^2(A+B \sec(c+dx)) dx$2426

3.402	$\int \frac{(a+b \sec(c+dx))^2(A+B \sec(c+dx))}{\sqrt{\sec(c+dx)}} dx \dots\dots\dots$.2431
3.403	$\int \frac{(a+b \sec(c+dx))^2(A+B \sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)} dx \dots\dots\dots$.2436
3.404	$\int \frac{(a+b \sec(c+dx))^2(A+B \sec(c+dx))}{\sec^{\frac{5}{2}}(c+dx)} dx \dots\dots\dots$.2441
3.405	$\int \frac{(a+b \sec(c+dx))^2(A+B \sec(c+dx))}{\sec^{\frac{7}{2}}(c+dx)} dx \dots\dots\dots$.2446
3.406	$\int \frac{(a+b \sec(c+dx))^2(A+B \sec(c+dx))}{\sec^{\frac{9}{2}}(c+dx)} dx \dots\dots\dots$.2451
3.407	$\int \sec^{\frac{3}{2}}(c+dx)(a+b \sec(c+dx))^3(A+B \sec(c+dx)) dx \dots\dots\dots$.2456
3.408	$\int \sqrt{\sec(c+dx)}(a+b \sec(c+dx))^3(A+B \sec(c+dx)) dx \dots\dots\dots$.2462
3.409	$\int \frac{(a+b \sec(c+dx))^3(A+B \sec(c+dx))}{\sqrt{\sec(c+dx)}} dx \dots\dots\dots$.2467
3.410	$\int \frac{(a+b \sec(c+dx))^3(A+B \sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)} dx \dots\dots\dots$.2472
3.411	$\int \frac{(a+b \sec(c+dx))^3(A+B \sec(c+dx))}{\sec^{\frac{5}{2}}(c+dx)} dx \dots\dots\dots$.2478
3.412	$\int \frac{(a+b \sec(c+dx))^3(A+B \sec(c+dx))}{\sec^{\frac{7}{2}}(c+dx)} dx \dots\dots\dots$.2483
3.413	$\int \frac{(a+b \sec(c+dx))^3(A+B \sec(c+dx))}{\sec^{\frac{9}{2}}(c+dx)} dx \dots\dots\dots$.2488
3.414	$\int \frac{(a+b \sec(c+dx))^3(A+B \sec(c+dx))}{\sec^{\frac{11}{2}}(c+dx)} dx \dots\dots\dots$.2493
3.415	$\int \frac{\sec^{\frac{7}{2}}(c+dx)(A+B \sec(c+dx))}{a+b \sec(c+dx)} dx \dots\dots\dots$.2499
3.416	$\int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B \sec(c+dx))}{a+b \sec(c+dx)} dx \dots\dots\dots$.2505
3.417	$\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B \sec(c+dx))}{a+b \sec(c+dx)} dx \dots\dots\dots$.2510
3.418	$\int \frac{\sqrt{\sec(c+dx)}(A+B \sec(c+dx))}{a+b \sec(c+dx)} dx \dots\dots\dots$.2515
3.419	$\int \frac{A+B \sec(c+dx)}{\sqrt{\sec(c+dx)}(a+b \sec(c+dx))} dx \dots\dots\dots$.2519
3.420	$\int \frac{A+B \sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+b \sec(c+dx))} dx \dots\dots\dots$.2524
3.421	$\int \frac{A+B \sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+b \sec(c+dx))} dx \dots\dots\dots$.2530
3.422	$\int \frac{\sec^{\frac{7}{2}}(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^2} dx \dots\dots\dots$.2536
3.423	$\int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^2} dx \dots\dots\dots$.2543

- 3.424 $\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^2} dx \dots \dots \dots .2549$
- 3.425 $\int \frac{\sqrt{\sec(c+dx)}(A+B \sec(c+dx))}{(a+b \sec(c+dx))^2} dx \dots \dots \dots .2555$
- 3.426 $\int \frac{A+B \sec(c+dx)}{\sqrt{\sec(c+dx)}(a+b \sec(c+dx))^2} dx \dots \dots \dots .2561$
- 3.427 $\int \frac{A+B \sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+b \sec(c+dx))^2} dx \dots \dots \dots .2567$
- 3.428 $\int \frac{\sec^{\frac{2}{9}}(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^3} dx \dots \dots \dots .2573$
- 3.429 $\int \frac{\sec^{\frac{7}{2}}(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^3} dx \dots \dots \dots .2581$
- 3.430 $\int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^3} dx \dots \dots \dots .2588$
- 3.431 $\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^3} dx \dots \dots \dots .2595$
- 3.432 $\int \frac{\sqrt{\sec(c+dx)}(A+B \sec(c+dx))}{(a+b \sec(c+dx))^3} dx \dots \dots \dots .2602$
- 3.433 $\int \frac{A+B \sec(c+dx)}{\sqrt{\sec(c+dx)}(a+b \sec(c+dx))^3} dx \dots \dots \dots .2609$
- 3.434 $\int \frac{A+B \sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+b \sec(c+dx))^3} dx \dots \dots \dots .2616$
- 3.435 $\int \sec^{\frac{2}{3}}(c+dx)\sqrt{a+b \sec(c+dx)}(A+B \sec(c+dx)) dx \dots \dots \dots .2624$
- 3.436 $\int \sqrt{\sec(c+dx)}\sqrt{a+b \sec(c+dx)}(A+B \sec(c+dx)) dx \dots \dots \dots .2631$
- 3.437 $\int \frac{\sqrt{a+b \sec(c+dx)}(A+B \sec(c+dx))}{\sqrt{\sec(c+dx)}} dx \dots \dots \dots .2638$
- 3.438 $\int \frac{\sqrt{a+b \sec(c+dx)}(A+B \sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)} dx \dots \dots \dots .2645$
- 3.439 $\int \frac{\sqrt{a+b \sec(c+dx)}(A+B \sec(c+dx))}{\sec^{\frac{5}{2}}(c+dx)} dx \dots \dots \dots .2651$
- 3.440 $\int \frac{\sqrt{a+b \sec(c+dx)}(A+B \sec(c+dx))}{\sec^{\frac{7}{2}}(c+dx)} dx \dots \dots \dots .2658$
- 3.441 $\int \sec^{\frac{3}{2}}(c+dx)(a+b \sec(c+dx))^{\frac{3}{2}}(A+B \sec(c+dx)) dx \dots \dots \dots .2666$
- 3.442 $\int \sqrt{\sec(c+dx)}(a+b \sec(c+dx))^{\frac{3}{2}}(A+B \sec(c+dx)) dx \dots \dots \dots .2675$
- 3.443 $\int \frac{(a+b \sec(c+dx))^{\frac{3}{2}}(A+B \sec(c+dx))}{\sqrt{\sec(c+dx)}} dx \dots \dots \dots .2683$
- 3.444 $\int \frac{(a+b \sec(c+dx))^{\frac{3}{2}}(A+B \sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)} dx \dots \dots \dots .2690$
- 3.445 $\int \frac{(a+b \sec(c+dx))^{\frac{3}{2}}(A+B \sec(c+dx))}{\sec^{\frac{5}{2}}(c+dx)} dx \dots \dots \dots .2697$
- 3.446 $\int \frac{(a+b \sec(c+dx))^{\frac{3}{2}}(A+B \sec(c+dx))}{\sec^{\frac{7}{2}}(c+dx)} dx \dots \dots \dots .2704$

- 3.447 $\int \frac{(a+b \sec(c+dx))^{3/2}(A+B \sec(c+dx))}{\sec^2(c+dx)} dx \dots\dots\dots .2712$
- 3.448 $\int \sec^3(c+dx)(a+b \sec(c+dx))^{5/2}(A+B \sec(c+dx)) dx \dots\dots\dots .2721$
- 3.449 $\int \sqrt{\sec(c+dx)}(a+b \sec(c+dx))^{5/2}(A+B \sec(c+dx)) dx \dots\dots\dots .2728$
- 3.450 $\int \frac{(a+b \sec(c+dx))^{5/2}(A+B \sec(c+dx))}{\sqrt{\sec(c+dx)}} dx \dots\dots\dots .2737$
- 3.451 $\int \frac{(a+b \sec(c+dx))^{5/2}(A+B \sec(c+dx))}{\sec^3(c+dx)} dx \dots\dots\dots .2746$
- 3.452 $\int \frac{(a+b \sec(c+dx))^{5/2}(A+B \sec(c+dx))}{\sec^5(c+dx)} dx \dots\dots\dots .2754$
- 3.453 $\int \frac{(a+b \sec(c+dx))^{5/2}(A+B \sec(c+dx))}{\sec^7(c+dx)} dx \dots\dots\dots .2762$
- 3.454 $\int \frac{(a+b \sec(c+dx))^{5/2}(A+B \sec(c+dx))}{\sec^9(c+dx)} dx \dots\dots\dots .2770$
- 3.455 $\int \frac{(a+b \sec(c+dx))^{5/2}(A+B \sec(c+dx))}{\sec^{11}(c+dx)} dx \dots\dots\dots .2779$
- 3.456 $\int \frac{\sec^5(c+dx)(A+B \sec(c+dx))}{\sqrt{a+b \sec(c+dx)}} dx \dots\dots\dots .2785$
- 3.457 $\int \frac{\sec^3(c+dx)(A+B \sec(c+dx))}{\sqrt{a+b \sec(c+dx)}} dx \dots\dots\dots .2793$
- 3.458 $\int \frac{\sqrt{\sec(c+dx)}(A+B \sec(c+dx))}{\sqrt{a+b \sec(c+dx)}} dx \dots\dots\dots .2800$
- 3.459 $\int \frac{A+B \sec(c+dx)}{\sqrt{\sec(c+dx)} \sqrt{a+b \sec(c+dx)}} dx \dots\dots\dots .2805$
- 3.460 $\int \frac{\sec^3(c+dx) \sqrt{a+b \sec(c+dx)}}{A+B \sec(c+dx)} dx \dots\dots\dots .2810$
- 3.461 $\int \frac{\sec^5(c+dx) \sqrt{a+b \sec(c+dx)}}{A+B \sec(c+dx)} dx \dots\dots\dots .2816$
- 3.462 $\int \frac{\sec^5(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^{3/2}} dx \dots\dots\dots .2823$
- 3.463 $\int \frac{\sec^3(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^{3/2}} dx \dots\dots\dots .2831$
- 3.464 $\int \frac{\sqrt{\sec(c+dx)}(A+B \sec(c+dx))}{(a+b \sec(c+dx))^{3/2}} dx \dots\dots\dots .2837$
- 3.465 $\int \frac{A+B \sec(c+dx)}{\sqrt{\sec(c+dx)}(a+b \sec(c+dx))^{3/2}} dx \dots\dots\dots .2843$
- 3.466 $\int \frac{A+B \sec(c+dx)}{\sec^3(c+dx)(a+b \sec(c+dx))^{3/2}} dx \dots\dots\dots .2849$
- 3.467 $\int \frac{A+B \sec(c+dx)}{\sec^5(c+dx)(a+b \sec(c+dx))^{3/2}} dx \dots\dots\dots .2856$
- 3.468 $\int \frac{\sec^5(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^{5/2}} dx \dots\dots\dots .2864$

3.469	$\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^{5/2}} dx$.2871
3.470	$\int \frac{\sqrt{\sec(c+dx)}(A+B \sec(c+dx))}{(a+b \sec(c+dx))^{5/2}} dx$.2879
3.471	$\int \frac{A+B \sec(c+dx)}{\sqrt{\sec(c+dx)}(a+b \sec(c+dx))^{5/2}} dx$.2887
3.472	$\int \frac{A+B \sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+b \sec(c+dx))^{5/2}} dx$.2893
3.473	$\int \frac{A+B \sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+b \sec(c+dx))^{5/2}} dx$.2899
3.474	$\int (a+b \sec(c+dx))^{2/3}(A+B \sec(c+dx)) dx$.2905
3.475	$\int \sqrt[3]{a+b \sec(c+dx)}(A+B \sec(c+dx)) dx$.2908
3.476	$\int \frac{A+B \sec(c+dx)}{\sqrt[3]{a+b \sec(c+dx)}} dx$.2911
3.477	$\int \frac{A+B \sec(c+dx)}{(a+b \sec(c+dx))^{2/3}} dx$.2914
3.478	$\int (c \sec(e+fx))^n(a+b \sec(e+fx))^m(A+B \sec(e+fx)) dx$.2917
3.479	$\int \sec^m(c+dx)(a+b \sec(c+dx))^4(A+B \sec(c+dx)) dx$.2920
3.480	$\int \sec^m(c+dx)(a+b \sec(c+dx))^3(A+B \sec(c+dx)) dx$.2925
3.481	$\int \sec^m(c+dx)(a+b \sec(c+dx))^2(A+B \sec(c+dx)) dx$.2930
3.482	$\int \sec^m(c+dx)(a+b \sec(c+dx))(A+B \sec(c+dx)) dx$.2935
3.483	$\int \cos^{\frac{7}{2}}(c+dx)(a+a \sec(c+dx))(A+B \sec(c+dx)) dx$.2939
3.484	$\int \cos^{\frac{5}{2}}(c+dx)(a+a \sec(c+dx))(A+B \sec(c+dx)) dx$.2945
3.485	$\int \cos^{\frac{3}{2}}(c+dx)(a+a \sec(c+dx))(A+B \sec(c+dx)) dx$.2951
3.486	$\int \sqrt{\cos(c+dx)}(a+a \sec(c+dx))(A+B \sec(c+dx)) dx$.2956
3.487	$\int \frac{(a+a \sec(c+dx))(A+B \sec(c+dx))}{\sqrt{\cos(c+dx)}} dx$.2961
3.488	$\int \frac{(a+a \sec(c+dx))(A+B \sec(c+dx))}{\cos^{\frac{3}{2}}(c+dx)} dx$.2967
3.489	$\int \cos^{\frac{9}{2}}(c+dx)(a+a \sec(c+dx))^2(A+B \sec(c+dx)) dx$.2973
3.490	$\int \cos^{\frac{7}{2}}(c+dx)(a+a \sec(c+dx))^2(A+B \sec(c+dx)) dx$.2979
3.491	$\int \cos^{\frac{5}{2}}(c+dx)(a+a \sec(c+dx))^2(A+B \sec(c+dx)) dx$.2985
3.492	$\int \cos^{\frac{3}{2}}(c+dx)(a+a \sec(c+dx))^2(A+B \sec(c+dx)) dx$.2991
3.493	$\int \sqrt{\cos(c+dx)}(a+a \sec(c+dx))^2(A+B \sec(c+dx)) dx$.2997
3.494	$\int \frac{(a+a \sec(c+dx))^2(A+B \sec(c+dx))}{\sqrt{\cos(c+dx)}} dx$.3003
3.495	$\int \frac{(a+a \sec(c+dx))^2(A+B \sec(c+dx))}{\cos^{\frac{3}{2}}(c+dx)} dx$.3009
3.496	$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B \sec(c+dx))}{a+a \sec(c+dx)} dx$.3015

3.497	$\int \frac{\cos^3(c+dx)(A+B \sec(c+dx))}{a+a \sec(c+dx)} dx$3020
3.498	$\int \frac{\sqrt{\cos(c+dx)}(A+B \sec(c+dx))}{a+a \sec(c+dx)} dx$3025
3.499	$\int \frac{A+B \sec(c+dx)}{\sqrt{\cos(c+dx)}(a+a \sec(c+dx))} dx$3030
3.500	$\int \frac{A+B \sec(c+dx)}{\cos^3(c+dx)(a+a \sec(c+dx))} dx$3035
3.501	$\int \frac{\cos^5(c+dx)(A+B \sec(c+dx))}{A+B \sec(c+dx)} dx$3041
3.502	$\int \frac{\cos^2(c+dx)(A+B \sec(c+dx))}{(a+a \sec(c+dx))^2} dx$3047
3.503	$\int \frac{\cos^3(c+dx)(A+B \sec(c+dx))}{(a+a \sec(c+dx))^2} dx$3053
3.504	$\int \frac{\sqrt{\cos(c+dx)}(A+B \sec(c+dx))}{(a+a \sec(c+dx))^2} dx$3059
3.505	$\int \frac{A+B \sec(c+dx)}{\sqrt{\cos(c+dx)}(a+a \sec(c+dx))^2} dx$3064
3.506	$\int \frac{A+B \sec(c+dx)}{\cos^3(c+dx)(a+a \sec(c+dx))^2} dx$3070
3.507	$\int \frac{A+B \sec(c+dx)}{\cos^5(c+dx)(a+a \sec(c+dx))^2} dx$3075
3.508	$\int \frac{A+B \sec(c+dx)}{\cos^7(c+dx)(a+a \sec(c+dx))^2} dx$3081
3.509	$\int \frac{\cos^3(c+dx)(A+B \sec(c+dx))}{(a+a \sec(c+dx))^3} dx$3087
3.510	$\int \frac{\sqrt{\cos(c+dx)}(A+B \sec(c+dx))}{(a+a \sec(c+dx))^3} dx$3093
3.511	$\int \frac{A+B \sec(c+dx)}{\sqrt{\cos(c+dx)}(a+a \sec(c+dx))^3} dx$3099
3.512	$\int \frac{A+B \sec(c+dx)}{\cos^3(c+dx)(a+a \sec(c+dx))^3} dx$3105
3.513	$\int \frac{A+B \sec(c+dx)}{\cos^5(c+dx)(a+a \sec(c+dx))^3} dx$3111
3.514	$\int \frac{A+B \sec(c+dx)}{\cos^7(c+dx)(a+a \sec(c+dx))^3} dx$3117
3.515	$\int \cos^{\frac{9}{7}}(c+dx) \sqrt{a+a \sec(c+dx)} (A+B \sec(c+dx)) dx$3123
3.516	$\int \cos^{\frac{7}{5}}(c+dx) \sqrt{a+a \sec(c+dx)} (A+B \sec(c+dx)) dx$3128
3.517	$\int \cos^{\frac{5}{3}}(c+dx) \sqrt{a+a \sec(c+dx)} (A+B \sec(c+dx)) dx$3133
3.518	$\int \cos^{\frac{3}{2}}(c+dx) \sqrt{a+a \sec(c+dx)} (A+B \sec(c+dx)) dx$3137
3.519	$\int \sqrt{\cos(c+dx)} \sqrt{a+a \sec(c+dx)} (A+B \sec(c+dx)) dx$3141
3.520	$\int \frac{\sqrt{a+a \sec(c+dx)}(A+B \sec(c+dx))}{\sqrt{\cos(c+dx)}} dx$3145

- 3.521 $\int \frac{\sqrt{a+a \sec(c+dx)} (A+B \sec(c+dx))}{\cos^{\frac{3}{2}}(c+dx)} dx \dots \dots \dots .3150$
- 3.522 $\int \frac{\sqrt{a+a \sec(c+dx)} (A+B \sec(c+dx))}{\cos^{\frac{5}{2}}(c+dx)} dx \dots \dots \dots .3157$
- 3.523 $\int \cos^{\frac{11}{2}}(c+dx)(a+a \sec(c+dx))^{3/2}(A+B \sec(c+dx)) dx \dots \dots \dots .3165$
- 3.524 $\int \cos^{\frac{9}{2}}(c+dx)(a+a \sec(c+dx))^{3/2}(A+B \sec(c+dx)) dx \dots \dots \dots .3170$
- 3.525 $\int \cos^{\frac{7}{2}}(c+dx)(a+a \sec(c+dx))^{3/2}(A+B \sec(c+dx)) dx \dots \dots \dots .3175$
- 3.526 $\int \cos^{\frac{5}{2}}(c+dx)(a+a \sec(c+dx))^{3/2}(A+B \sec(c+dx)) dx \dots \dots \dots .3180$
- 3.527 $\int \cos^{\frac{3}{2}}(c+dx)(a+a \sec(c+dx))^{3/2}(A+B \sec(c+dx)) dx \dots \dots \dots .3184$
- 3.528 $\int \sqrt{\cos(c+dx)}(a+a \sec(c+dx))^{3/2}(A+B \sec(c+dx)) dx \dots \dots \dots .3189$
- 3.529 $\int \frac{(a+a \sec(c+dx))^{3/2}(A+B \sec(c+dx))}{\sqrt{\cos(c+dx)}} dx \dots \dots \dots .3195$
- 3.530 $\int \frac{(a+a \sec(c+dx))^{3/2}(A+B \sec(c+dx))}{\cos^{\frac{3}{2}}(c+dx)} dx \dots \dots \dots .3203$
- 3.531 $\int \frac{(a+a \sec(c+dx))^{3/2}(A+B \sec(c+dx))}{\cos^{\frac{5}{2}}(c+dx)} dx \dots \dots \dots .3212$
- 3.532 $\int \cos^{\frac{11}{2}}(c+dx)(a+a \sec(c+dx))^{5/2}(A+B \sec(c+dx)) dx \dots \dots \dots .3221$
- 3.533 $\int \cos^{\frac{9}{2}}(c+dx)(a+a \sec(c+dx))^{5/2}(A+B \sec(c+dx)) dx \dots \dots \dots .3227$
- 3.534 $\int \cos^{\frac{7}{2}}(c+dx)(a+a \sec(c+dx))^{5/2}(A+B \sec(c+dx)) dx \dots \dots \dots .3232$
- 3.535 $\int \cos^{\frac{5}{2}}(c+dx)(a+a \sec(c+dx))^{5/2}(A+B \sec(c+dx)) dx \dots \dots \dots .3237$
- 3.536 $\int \cos^{\frac{3}{2}}(c+dx)(a+a \sec(c+dx))^{5/2}(A+B \sec(c+dx)) dx \dots \dots \dots .3242$
- 3.537 $\int \sqrt{\cos(c+dx)}(a+a \sec(c+dx))^{5/2}(A+B \sec(c+dx)) dx \dots \dots \dots .3249$
- 3.538 $\int \frac{(a+a \sec(c+dx))^{5/2}(A+B \sec(c+dx))}{\sqrt{\cos(c+dx)}} dx \dots \dots \dots .3254$
- 3.539 $\int \frac{(a+a \sec(c+dx))^{5/2}(A+B \sec(c+dx))}{\cos^{\frac{3}{2}}(c+dx)} dx \dots \dots \dots .3264$
- 3.540 $\int \frac{(a+a \sec(c+dx))^{5/2}(A+B \sec(c+dx))}{\cos^{\frac{5}{2}}(c+dx)} dx \dots \dots \dots .3274$
- 3.541 $\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B \sec(c+dx))}{\sqrt{a+a \sec(c+dx)}} dx \dots \dots \dots .3285$
- 3.542 $\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B \sec(c+dx))}{\sqrt{a+a \sec(c+dx)}} dx \dots \dots \dots .3291$
- 3.543 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B \sec(c+dx))}{\sqrt{a+a \sec(c+dx)}} dx \dots \dots \dots .3297$
- 3.544 $\int \frac{\sqrt{\cos(c+dx)}(A+B \sec(c+dx))}{\sqrt{a+a \sec(c+dx)}} dx \dots \dots \dots .3303$
- 3.545 $\int \frac{A+B \sec(c+dx)}{\sqrt{\cos(c+dx)} \sqrt{a+a \sec(c+dx)}} dx \dots \dots \dots .3308$

3.546	$\int \frac{A+B \sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx) \sqrt{a+a \sec(c+dx)}} dx$3313
3.547	$\int \frac{A+B \sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx) \sqrt{a+a \sec(c+dx)}} dx$3320
3.548	$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B \sec(c+dx))}{(a+a \sec(c+dx))^{\frac{3}{2}}} dx$3328
3.549	$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B \sec(c+dx))}{(a+a \sec(c+dx))^{\frac{3}{2}}} dx$3334
3.550	$\int \frac{\sqrt{\cos(c+dx)}(A+B \sec(c+dx))}{(a+a \sec(c+dx))^{\frac{3}{2}}} dx$3339
3.551	$\int \frac{A+B \sec(c+dx)}{\sqrt{\cos(c+dx)}(a+a \sec(c+dx))^{\frac{3}{2}}} dx$3350
3.552	$\int \frac{A+B \sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a \sec(c+dx))^{\frac{3}{2}}} dx$3356
3.553	$\int \frac{A+B \sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a \sec(c+dx))^{\frac{3}{2}}} dx$3361
3.554	$\int \frac{A+B \sec(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+a \sec(c+dx))^{\frac{3}{2}}} dx$3372
3.555	$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B \sec(c+dx))}{(a+a \sec(c+dx))^{\frac{5}{2}}} dx$3378
3.556	$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B \sec(c+dx))}{(a+a \sec(c+dx))^{\frac{5}{2}}} dx$3384
3.557	$\int \frac{\sqrt{\cos(c+dx)}(A+B \sec(c+dx))}{(a+a \sec(c+dx))^{\frac{5}{2}}} dx$3390
3.558	$\int \frac{A+B \sec(c+dx)}{\sqrt{\cos(c+dx)}(a+a \sec(c+dx))^{\frac{5}{2}}} dx$3396
3.559	$\int \frac{A+B \sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a \sec(c+dx))^{\frac{5}{2}}} dx$3405
3.560	$\int \frac{A+B \sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a \sec(c+dx))^{\frac{5}{2}}} dx$3414
3.561	$\int \frac{A+B \sec(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+a \sec(c+dx))^{\frac{5}{2}}} dx$3420
3.562	$\int \cos^{\frac{7}{2}}(c+dx)(a+b \sec(c+dx))(A+B \sec(c+dx)) dx$3427
3.563	$\int \cos^{\frac{5}{2}}(c+dx)(a+b \sec(c+dx))(A+B \sec(c+dx)) dx$3432
3.564	$\int \cos^{\frac{3}{2}}(c+dx)(a+b \sec(c+dx))(A+B \sec(c+dx)) dx$3437
3.565	$\int \sqrt{\cos(c+dx)}(a+b \sec(c+dx))(A+B \sec(c+dx)) dx$3441
3.566	$\int \frac{(a+b \sec(c+dx))(A+B \sec(c+dx))}{\sqrt{\cos(c+dx)}} dx$3445
3.567	$\int \frac{(a+b \sec(c+dx))(A+B \sec(c+dx))}{\cos^{\frac{3}{2}}(c+dx)} dx$3450
3.568	$\int \cos^{\frac{7}{2}}(c+dx)(a+b \sec(c+dx))^2(A+B \sec(c+dx)) dx$3455
3.569	$\int \cos^{\frac{5}{2}}(c+dx)(a+b \sec(c+dx))^2(A+B \sec(c+dx)) dx$3460

3.570	$\int \cos^{\frac{3}{2}}(c+dx)(a+b \sec(c+dx))^2(A+B \sec(c+dx)) dx$3465
3.571	$\int \sqrt{\cos(c+dx)}(a+b \sec(c+dx))^2(A+B \sec(c+dx)) dx$3470
3.572	$\int \frac{(a+b \sec(c+dx))^2(A+B \sec(c+dx))}{\sqrt{\cos(c+dx)}} dx$3475
3.573	$\int \frac{(a+b \sec(c+dx))^2(A+B \sec(c+dx))}{\cos^{\frac{3}{2}}(c+dx)} dx$3480
3.574	$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B \sec(c+dx))}{a+b \sec(c+dx)} dx$3486
3.575	$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B \sec(c+dx))}{a+b \sec(c+dx)} dx$3492
3.576	$\int \frac{\sqrt{\cos(c+dx)}(A+B \sec(c+dx))}{a+b \sec(c+dx)} dx$3497
3.577	$\int \frac{A+B \sec(c+dx)}{\sqrt{\cos(c+dx)}(a+b \sec(c+dx))} dx$3501
3.578	$\int \frac{A+B \sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b \sec(c+dx))} dx$3505
3.579	$\int \frac{\cos^{\frac{5}{2}}(c+dx)(a+b \sec(c+dx))}{A+B \sec(c+dx)} dx$3510
3.580	$\int \frac{\cos^{\frac{7}{2}}(c+dx)(a+b \sec(c+dx))}{A+B \sec(c+dx)} dx$3516
3.581	$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^2} dx$3522
3.582	$\int \frac{\sqrt{\cos(c+dx)}(A+B \sec(c+dx))}{(a+b \sec(c+dx))^2} dx$3528
3.583	$\int \frac{A+B \sec(c+dx)}{\sqrt{\cos(c+dx)}(a+b \sec(c+dx))^2} dx$3533
3.584	$\int \frac{A+B \sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b \sec(c+dx))^2} dx$3538
3.585	$\int \frac{\cos^{\frac{5}{2}}(c+dx)(a+b \sec(c+dx))^2}{A+B \sec(c+dx)} dx$3543
3.586	$\int \frac{\cos^{\frac{7}{2}}(c+dx)(a+b \sec(c+dx))^2}{A+B \sec(c+dx)} dx$3549
3.587	$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^3} dx$3556
3.588	$\int \frac{\sqrt{\cos(c+dx)}(A+B \sec(c+dx))}{(a+b \sec(c+dx))^3} dx$3563
3.589	$\int \frac{A+B \sec(c+dx)}{\sqrt{\cos(c+dx)}(a+b \sec(c+dx))^3} dx$3570
3.590	$\int \frac{A+B \sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b \sec(c+dx))^3} dx$3577
3.591	$\int \frac{\cos^{\frac{5}{2}}(c+dx)(a+b \sec(c+dx))^3}{A+B \sec(c+dx)} dx$3584
3.592	$\int \frac{\cos^{\frac{7}{2}}(c+dx)(a+b \sec(c+dx))^3}{A+B \sec(c+dx)} dx$3591

- 3.593 $\int \frac{A+B \sec(c+dx)}{\cos^{\frac{9}{2}}(c+dx)(a+b \sec(c+dx))^3} dx \dots \dots \dots .3598$
- 3.594 $\int \cos^{\frac{7}{2}}(c+dx) \sqrt{a+b \sec(c+dx)} (A+B \sec(c+dx)) dx \dots \dots \dots .3605$
- 3.595 $\int \cos^{\frac{5}{2}}(c+dx) \sqrt{a+b \sec(c+dx)} (A+B \sec(c+dx)) dx \dots \dots \dots .3612$
- 3.596 $\int \cos^{\frac{3}{2}}(c+dx) \sqrt{a+b \sec(c+dx)} (A+B \sec(c+dx)) dx \dots \dots \dots .3619$
- 3.597 $\int \sqrt{\cos(c+dx)} \sqrt{a+b \sec(c+dx)} (A+B \sec(c+dx)) dx \dots \dots \dots .3625$
- 3.598 $\int \frac{\sqrt{a+b \sec(c+dx)} (A+B \sec(c+dx))}{\sqrt{\cos(c+dx)}} dx \dots \dots \dots .3631$
- 3.599 $\int \frac{\sqrt{a+b \sec(c+dx)} (A+B \sec(c+dx))}{\cos^{\frac{3}{2}}(c+dx)} dx \dots \dots \dots .3637$
- 3.600 $\int \cos^{\frac{9}{2}}(c+dx) (a+b \sec(c+dx))^{\frac{3}{2}} (A+B \sec(c+dx)) dx \dots \dots \dots .3645$
- 3.601 $\int \cos^{\frac{7}{2}}(c+dx) (a+b \sec(c+dx))^{\frac{3}{2}} (A+B \sec(c+dx)) dx \dots \dots \dots .3653$
- 3.602 $\int \cos^{\frac{5}{2}}(c+dx) (a+b \sec(c+dx))^{\frac{3}{2}} (A+B \sec(c+dx)) dx \dots \dots \dots .3660$
- 3.603 $\int \cos^{\frac{3}{2}}(c+dx) (a+b \sec(c+dx))^{\frac{3}{2}} (A+B \sec(c+dx)) dx \dots \dots \dots .3667$
- 3.604 $\int \sqrt{\cos(c+dx)} (a+b \sec(c+dx))^{\frac{3}{2}} (A+B \sec(c+dx)) dx \dots \dots \dots .3674$
- 3.605 $\int \frac{(a+b \sec(c+dx))^{\frac{3}{2}} (A+B \sec(c+dx))}{\sqrt{\cos(c+dx)}} dx \dots \dots \dots .3681$
- 3.606 $\int \frac{(a+b \sec(c+dx))^{\frac{3}{2}} (A+B \sec(c+dx))}{\cos^{\frac{3}{2}}(c+dx)} dx \dots \dots \dots .3689$
- 3.607 $\int \cos^{\frac{11}{2}}(c+dx) (a+b \sec(c+dx))^{\frac{5}{2}} (A+B \sec(c+dx)) dx \dots \dots \dots .3697$
- 3.608 $\int \cos^{\frac{9}{2}}(c+dx) (a+b \sec(c+dx))^{\frac{5}{2}} (A+B \sec(c+dx)) dx \dots \dots \dots .3706$
- 3.609 $\int \cos^{\frac{7}{2}}(c+dx) (a+b \sec(c+dx))^{\frac{5}{2}} (A+B \sec(c+dx)) dx \dots \dots \dots .3714$
- 3.610 $\int \cos^{\frac{5}{2}}(c+dx) (a+b \sec(c+dx))^{\frac{5}{2}} (A+B \sec(c+dx)) dx \dots \dots \dots .3721$
- 3.611 $\int \cos^{\frac{3}{2}}(c+dx) (a+b \sec(c+dx))^{\frac{5}{2}} (A+B \sec(c+dx)) dx \dots \dots \dots .3729$
- 3.612 $\int \sqrt{\cos(c+dx)} (a+b \sec(c+dx))^{\frac{5}{2}} (A+B \sec(c+dx)) dx \dots \dots \dots .3737$
- 3.613 $\int \frac{(a+b \sec(c+dx))^{\frac{5}{2}} (A+B \sec(c+dx))}{\sqrt{\cos(c+dx)}} dx \dots \dots \dots .3745$
- 3.614 $\int \frac{(a+b \sec(c+dx))^{\frac{5}{2}} (A+B \sec(c+dx))}{\cos^{\frac{3}{2}}(c+dx)} dx \dots \dots \dots .3753$
- 3.615 $\int \frac{\cos^{\frac{5}{2}}(c+dx) (A+B \sec(c+dx))}{\sqrt{a+b \sec(c+dx)}} dx \dots \dots \dots .3762$
- 3.616 $\int \frac{\cos^{\frac{3}{2}}(c+dx) (A+B \sec(c+dx))}{\sqrt{a+b \sec(c+dx)}} dx \dots \dots \dots .3769$
- 3.617 $\int \frac{\sqrt{\cos(c+dx)} (A+B \sec(c+dx))}{\sqrt{a+b \sec(c+dx)}} dx \dots \dots \dots .3775$
- 3.618 $\int \frac{A+B \sec(c+dx)}{\sqrt{\cos(c+dx)} \sqrt{a+b \sec(c+dx)}} dx \dots \dots \dots .3780$

3.619	$\int \frac{A+B \sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx) \sqrt{a+b \sec(c+dx)}} dx$3785
3.620	$\int \frac{A+B \sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx) \sqrt{a+b \sec(c+dx)}} dx$3791
3.621	$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^{\frac{3}{2}}} dx$3799
3.622	$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^{\frac{3}{2}}} dx$3806
3.623	$\int \frac{\sqrt{\cos(c+dx)} (A+B \sec(c+dx))}{(a+b \sec(c+dx))^{\frac{3}{2}}} dx$3813
3.624	$\int \frac{A+B \sec(c+dx)}{\sqrt{\cos(c+dx)} (a+b \sec(c+dx))^{\frac{3}{2}}} dx$3819
3.625	$\int \frac{A+B \sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b \sec(c+dx))^{\frac{3}{2}}} dx$3825
3.626	$\int \frac{A+B \sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b \sec(c+dx))^{\frac{3}{2}}} dx$3831
3.627	$\int \frac{A+B \sec(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+b \sec(c+dx))^{\frac{3}{2}}} dx$3839
3.628	$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^{\frac{5}{2}}} dx$3847
3.629	$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^{\frac{5}{2}}} dx$3856
3.630	$\int \frac{\sqrt{\cos(c+dx)} (A+B \sec(c+dx))}{(a+b \sec(c+dx))^{\frac{5}{2}}} dx$3866
3.631	$\int \frac{A+B \sec(c+dx)}{\sqrt{\cos(c+dx)} (a+b \sec(c+dx))^{\frac{5}{2}}} dx$3874
3.632	$\int \frac{A+B \sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b \sec(c+dx))^{\frac{5}{2}}} dx$3881
3.633	$\int \frac{A+B \sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b \sec(c+dx))^{\frac{5}{2}}} dx$3888
3.634	$\int \frac{A+B \sec(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+b \sec(c+dx))^{\frac{5}{2}}} dx$3896

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Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from <https://rulebasedintegration.org>

The number of integrals in this report is [634]. This is test number [123].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or Hypergeometric2F1 functions. RootSum and RootOf are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100.00 (634)	% 0.00 (0)
Mathematica	% 100.00 (634)	% 0.00 (0)
Maple	% 92.43 (586)	% 7.57 (48)
Maxima	% 30.44 (193)	% 69.56 (441)
Fricas	% 47.32 (300)	% 52.68 (334)
Sympy	% 1.26 (8)	% 98.74 (626)
Giac	% 32.97 (209)	% 67.03 (425)
Mupad	% 30.76 (195)	% 69.24 (439)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

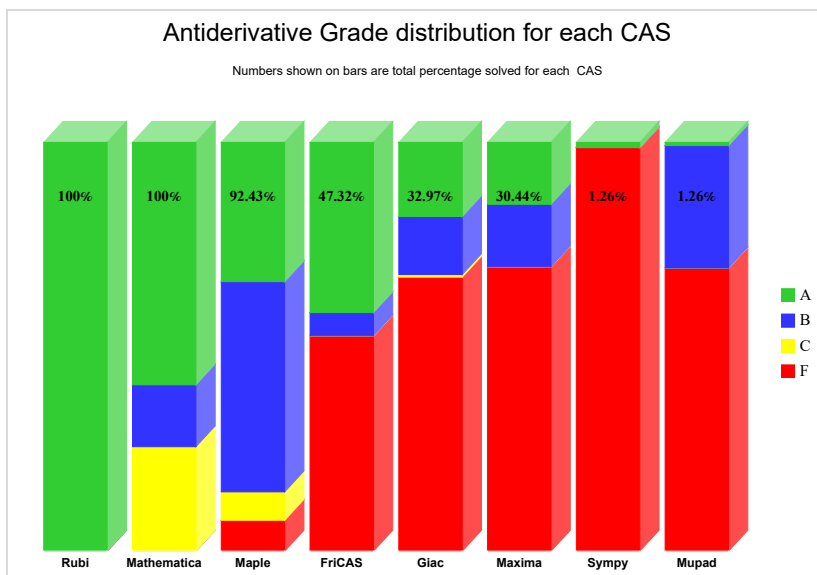
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

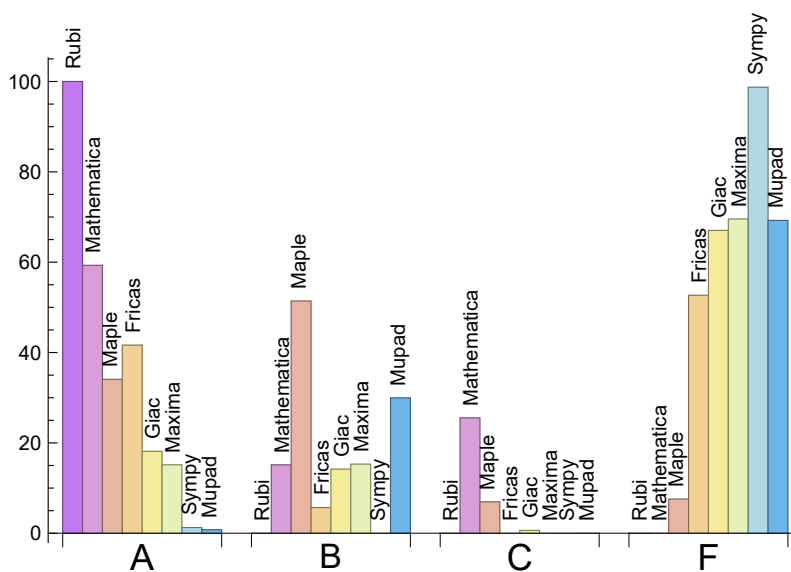
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	59.31	15.14	25.55	0.00
Maple	34.07	51.42	6.94	7.57
Maxima	15.14	15.30	0.00	69.56
Fricas	41.64	5.68	0.00	52.68
Sympy	1.26	0.00	0.00	98.74
Giac	18.14	14.20	0.63	67.03
Mupad	0.79	29.97	0.00	69.24

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This is the typical normal failure **F**.

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	48	100.00 %	0.00 %	0.00 %
Maxima	441	61.90 %	28.57 %	9.52 %
Fricas	334	76.05 %	23.95 %	0.00 %
Sympy	626	56.23 %	43.77 %	0.00 %
Giac	425	99.29 %	0.00 %	0.71 %
Mupad	439	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

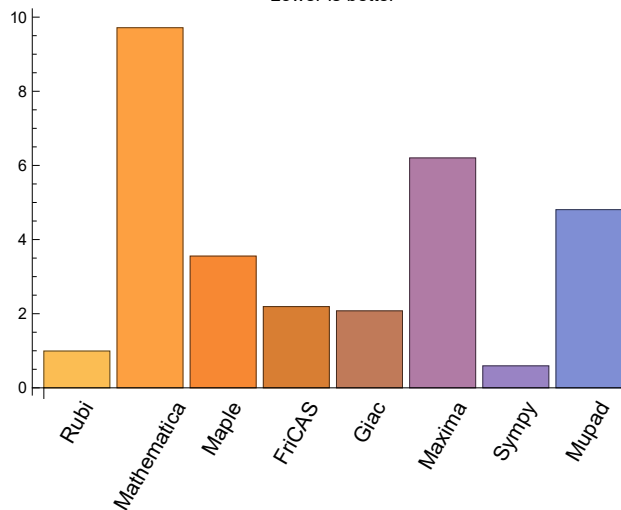
System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.56	217.71	0.99	187.00	1.00
Mathematica	5.64	3150.77	9.71	222.50	1.16
Maple	3.81	1002.57	3.55	435.00	2.62
Maxima	0.89	1093.94	6.20	278.00	1.84
Fricas	1.87	394.73	2.19	306.50	1.99
Sympy	2.64	18.13	0.59	0.00	0.00
Giac	1.47	345.75	2.08	233.00	1.72
Mupad	4.32	1066.04	4.81	198.00	1.42

Table 1.5: Time and leaf size performance for each CAS

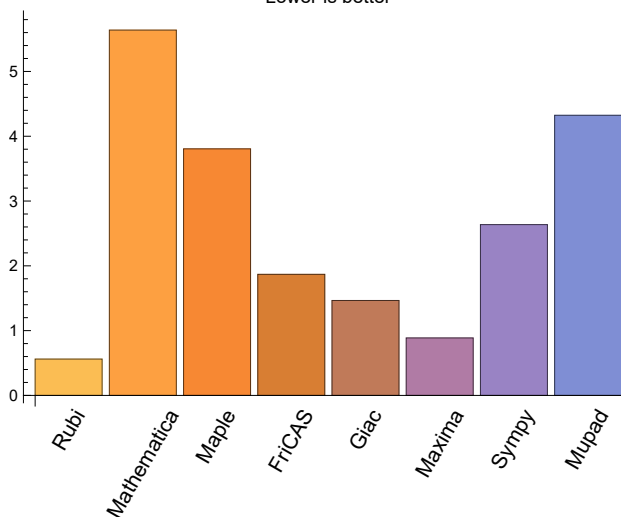
The following are bar charts for the normalized leafsize and time used columns from the above table.

Normalized mean size of antiderivative

Lower is better

**Mean time used (seconds)**

Lower is better



1.4 list of integrals that has no closed form antiderivative

{474, 475, 476, 477, 478}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {149, 156, 157, 164, 165, 193, 220, 222, 223, 234, 235, 242, 243, 244, 251, 252, 259, 260, 266, 267, 269, 270, 271, 272, 273, 274, 275, 320, 329, 348, 349, 350, 351, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 375, 377, 378, 379, 380, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 415, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 441, 443, 444, 448, 449, 450, 451, 452, 456, 468, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 527, 535, 536, 548, 549, 550, 553, 554, 555, 556, 557, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using `sagemath` (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate if the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

1.9.2 Important note about FriCAS and Giac/X-CAS results

There are few integrals which failed due to SageMath not being able to translate the result back to SageMath syntax and not because these CAS systems were not able to do the integrations.

These will fail with error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the builtin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user slelievre at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))

except Exception as ee:
    leafCount = 1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

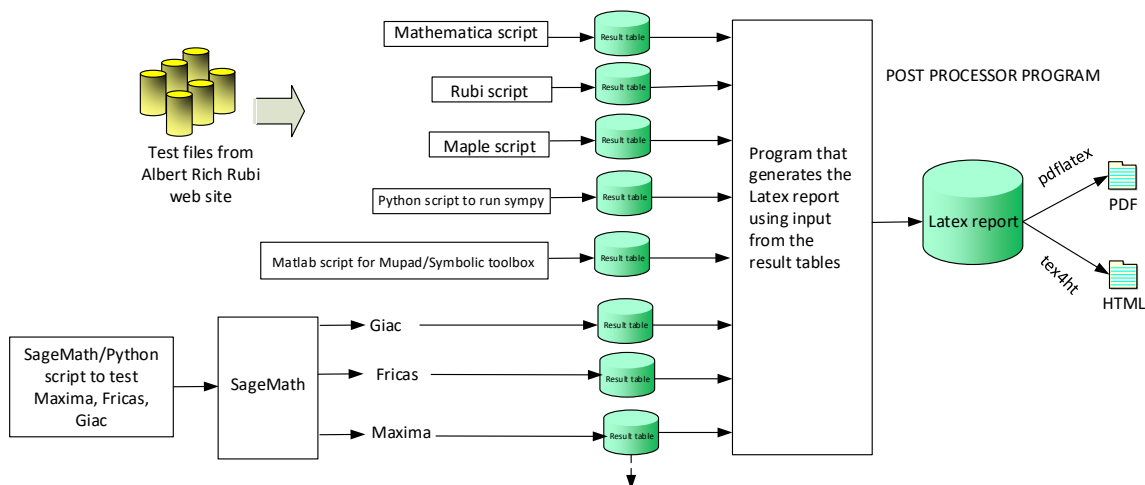
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine, 'cos(x)*sin(x)')
the_variable = evalin(symengine, 'x')
anti = int(integrand, the_variable)
```

Which gives $\sin(x)^2/2$

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
 2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
 3. integer. Leaf size of result.
 4. integer. Leaf size of the optimal antiderivative.
 5. number. CPU time used to solve this integral. 0 if failed.
 6. string. The integral in Latex format
 7. string. The input used in CAS own syntax.
 8. string. The result (antiderivative) produced by CAS in Latex format
 9. string. The optimal antiderivative in Latex format.
 10. integer. 0 or 1. Indicates if problem has known antiderivative or not
 11. String. The result (antiderivative) in CAS own syntax.
 12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
- The following field present only in Rubi and Mathematica Tables*
13. integer. 1 if result was verified or 0 if not verified.
- The following fields present only in Rubi Tables*
14. integer. Number of rules used.
 15. integer. Integrand leaf size.
 16. real number. Ratio of field 14 over field 15
 17. integer. 1 if result was verified or 0 if not verified.
 18. String of form "{n,n,..}" which is list of the rules used by Rubi

**High level overview of the CAS
independent integration test
build system**

Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549,

550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 58, 59, 60, 61, 62, 63, 68, 69, 70, 71, 72, 73, 78, 79, 80, 81, 87, 94, 101, 102, 103, 110, 111, 112, 113, 118, 119, 120, 121, 122, 123, 127, 128, 129, 130, 131, 132, 133, 134, 136, 137, 138, 139, 140, 144, 145, 146, 147, 148, 150, 151, 152, 153, 154, 155, 160, 161, 162, 179, 180, 181, 182, 183, 184, 185, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 256, 257, 258, 259, 260, 261, 264, 265, 266, 267, 268, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 298, 299, 300, 301, 302, 303, 304, 306, 307, 308, 309, 310, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 344, 345, 346, 347, 351, 358, 372, 373, 374, 379, 380, 388, 391, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 416, 417, 418, 419, 422, 427, 428, 429, 430, 433, 434, 437, 438, 439, 440, 445, 446, 447, 453, 454, 455, 458, 459, 460, 461, 464, 465, 466, 467, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593 }

B grade: { 55, 56, 57, 64, 65, 66, 67, 74, 75, 76, 77, 82, 83, 84, 85, 86, 88, 89, 90, 91, 92, 93, 95, 96, 97, 98, 99, 100, 104, 105, 106, 107, 108, 109, 114, 115, 116, 117, 135, 255, 262, 263, 269, 270, 271, 272, 273, 274, 275, 297, 305, 311, 312, 341, 342, 343, 348, 349, 350, 354, 355, 356, 357, 359, 360, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 377, 378, 382, 384, 385, 386, 387, 390, 392, 415, 420, 421, 423, 424, 425, 426, 431, 432, 560, 561, 578 }

C grade: { 124, 125, 126, 141, 142, 143, 149, 156, 157, 158, 159, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 352, 353, 361, 375, 376, 381, 383, 389, 435, 436, 441, 442, 443, 444, 448, 449, 450, 451, 452, 456, 457, 462, 463, 468, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616,

617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634 }

F grade: { }

2.1.3 Maple

A grade: { 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 85, 86, 87, 92, 93, 94, 95, 96, 97, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 127, 128, 129, 135, 136, 137, 167, 168, 182, 185, 190, 191, 192, 198, 199, 200, 203, 204, 205, 206, 207, 208, 211, 212, 213, 214, 215, 218, 219, 220, 221, 222, 223, 228, 229, 230, 235, 236, 237, 238, 244, 245, 246, 247, 251, 252, 253, 254, 260, 261, 268, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 314, 315, 323, 331, 332, 338, 339, 340, 345, 347, 374, 396, 399, 416, 417, 418, 419, 474, 475, 476, 477, 478, 489, 490, 496, 497, 498, 499, 500, 502, 509, 515, 516, 517, 518, 523, 524, 525, 526, 527, 532, 533, 534, 535, 541, 542, 543, 544, 545, 548, 549, 550, 551, 552, 555, 556, 557, 558, 576, 577 }

B grade: { 82, 83, 84, 88, 89, 90, 91, 98, 122, 123, 124, 125, 126, 130, 131, 132, 133, 134, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 183, 184, 186, 187, 188, 189, 193, 194, 195, 196, 197, 201, 202, 209, 210, 216, 217, 224, 225, 226, 227, 231, 232, 233, 234, 239, 240, 241, 242, 243, 248, 249, 250, 255, 256, 257, 258, 259, 262, 263, 264, 265, 266, 267, 311, 312, 313, 316, 317, 318, 319, 320, 321, 322, 324, 325, 326, 327, 328, 329, 330, 333, 334, 335, 336, 337, 341, 342, 343, 344, 346, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 397, 398, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 438, 439, 440, 445, 446, 447, 453, 454, 455, 459, 460, 461, 464, 465, 466, 467, 469, 470, 471, 472, 473, 483, 484, 485, 486, 487, 488, 491, 492, 493, 494, 495, 501, 503, 504, 505, 506, 507, 508, 510, 511, 512, 513, 514, 519, 520, 521, 522, 528, 529, 530, 531, 536, 537, 538, 539, 540, 546, 547, 553, 554, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 600, 601, 602, 607, 608, 609, 615, 616, 617, 621, 622, 623, 624, 628, 629, 630, 631, 632 }

C grade: { 1, 2, 3, 4, 5, 6, 435, 436, 437, 441, 442, 443, 444, 448, 449, 450, 451, 452, 456, 457, 458, 462, 463, 468, 597, 598, 599, 603, 604, 605, 606, 610, 611, 612, 613, 614, 618, 619, 620, 625, 626, 627, 633, 634 }

F grade: { 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 269, 270, 271, 272, 273, 274, 275, 276, 479, 480, 481, 482 }

2.1.4 Maxima

A grade: { 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 65, 66, 67, 68, 69, 70, 71, 75, 76, 77, 78, 79, 80, 81, 93, 94, 95, 99, 100, 101, 102, 103, 104, 105, 106, 108, 109, 110, 111,

112, 113, 114, 115, 116, 117, 228, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 347, 474, 475, 476, 477, 478, 544 }

B grade: { 62, 63, 64, 72, 73, 74, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 96, 97, 98, 107, 122, 123, 124, 125, 126, 130, 131, 138, 139, 224, 225, 226, 227, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 256, 259, 265, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 538, 539, 540, 541, 542, 543, 545, 546, 547, 550, 551, 553, 558, 559 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 118, 119, 120, 121, 127, 128, 129, 132, 133, 134, 135, 136, 137, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 242, 243, 255, 257, 258, 260, 261, 262, 263, 264, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 537, 548, 549, 552, 554, 555, 556, 557, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634 }

2.1.5 FriCAS

A grade: { 43, 44, 45, 46, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 157, 158, 159, 160, 161, 162, 163, 165, 166, 167, 168, 169, 170, 172, 173, 174, 176, 177, 178, 224, 225, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 258, 259, 260, 261, 262, 264, 265, 266, 267, 268, 277, 278, 279, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307,

308, 309, 310, 314, 315, 316, 317, 318, 319, 323, 326, 327, 344, 345, 346, 347, 478, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561 }

B grade: { 47, 84, 155, 156, 164, 171, 175, 226, 227, 257, 263, 280, 311, 312, 313, 320, 321, 322, 324, 325, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 269, 270, 271, 272, 273, 274, 275, 276, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634 }

2.1.6 Sympy

A grade: { 47, 280, 345, 474, 475, 476, 477, 478 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277,

278, 279, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634 }

2.1.7 Giac

A grade: { 43, 44, 45, 50, 51, 52, 53, 54, 55, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 127, 128, 129, 135, 136, 137, 144, 145, 146, 152, 153, 154, 160, 161, 162, 163, 171, 173, 174, 175, 176, 177, 178, 298, 307, 313, 314, 316, 317, 320, 322, 323, 324, 326, 327, 334, 346, 347, 474, 475, 476, 477, 478 }

B grade: { 46, 47, 48, 49, 56, 57, 121, 122, 123, 124, 125, 126, 130, 131, 132, 133, 134, 138, 139, 140, 141, 142, 143, 147, 149, 150, 151, 155, 157, 158, 159, 165, 166, 172, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 299, 300, 301, 302, 303, 304, 305, 306, 308, 309, 310, 311, 312, 315, 318, 319, 321, 325, 328, 329, 330, 331, 332, 333, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345 }

C grade: { 167, 168, 169, 170 }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 148, 156, 164, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424,

425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634 }

2.1.8 Mupad

A grade: { 474, 475, 476, 477, 478 }

B grade: { 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 127, 128, 129, 135, 136, 137, 228, 229, 230, 236, 237, 238, 245, 246, 247, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 122, 123, 124, 125, 126, 130, 131, 132, 133, 134, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 231, 232, 233, 234, 235, 239, 240, 241, 242, 243, 244, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 479, 480, 481, 482, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583,

584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604,
605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625,
626, 627, 628, 629, 630, 631, 632, 633, 634 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table, the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	99	518	0	0	0	0	-1
normalized size	1	1.00	0.58	3.03	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.119	0.478	2.027	0.000	0.954	0.000	0.000	0.000
Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	87	499	0	0	0	0	-1
normalized size	1	1.00	0.64	3.67	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.101	0.273	1.414	0.000	0.528	0.000	0.000	0.000
Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	73	453	0	0	0	0	-1
normalized size	1	1.00	0.70	4.36	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.082	0.122	1.521	0.000	0.433	0.000	0.000	0.000

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	54	445	0	0	0	0	-1
normalized size	1	1.00	0.66	5.43	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.068	0.090	1.520	0.000	0.460	0.000	0.000	0.000

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	86	470	0	0	0	0	-1
normalized size	1	1.00	0.74	4.05	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.091	0.185	1.324	0.000	0.440	0.000	0.000	0.000

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	88	482	0	0	0	0	-1
normalized size	1	1.00	0.60	3.28	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.104	0.511	1.327	0.000	0.445	0.000	0.000	0.000

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	90	0	0	0	0	0	-1
normalized size	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.098	0.262	0.765	0.000	0.443	0.000	0.000	0.000

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	91	0	0	0	0	0	-1
normalized size	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.098	0.137	0.712	0.000	0.454	0.000	0.000	0.000

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	88	0	0	0	0	0	-1
normalized size	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.086	0.088	0.971	0.000	0.441	0.000	0.000	0.000

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	88	0	0	0	0	0	-1
normalized size	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.104	0.096	1.938	0.000	0.466	0.000	0.000	0.000

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	88	0	0	0	0	0	-1
normalized size	1	1.00	0.74	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.121	0.141	2.657	0.000	0.464	0.000	0.000	0.000

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	90	0	0	0	0	0	-1
normalized size	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.101	0.304	0.766	0.000	0.454	0.000	0.000	0.000

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	91	0	0	0	0	0	-1
normalized size	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.099	0.151	0.752	0.000	0.442	0.000	0.000	0.000

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	88	0	0	0	0	0	-1
normalized size	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.088	0.112	0.985	0.000	0.465	0.000	0.000	0.000

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	87	0	0	0	0	0	-1
normalized size	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.105	0.116	2.164	0.000	0.443	0.000	0.000	0.000

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	88	0	0	0	0	0	-1
normalized size	1	1.00	0.74	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.123	0.096	3.102	0.000	0.465	0.000	0.000	0.000

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	90	0	0	0	0	0	-1
normalized size	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.097	0.255	0.740	0.000	0.457	0.000	0.000	0.000

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	90	0	0	0	0	0	-1
normalized size	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.093	0.103	0.716	0.000	0.487	0.000	0.000	0.000

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	87	0	0	0	0	0	-1
normalized size	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.087	0.090	0.951	0.000	0.454	0.000	0.000	0.000

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	90	0	0	0	0	0	-1
normalized size	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.093	0.025	0.046	0.000	0.449	0.000	0.000	0.000

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	90	0	0	0	0	0	-1
normalized size	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.098	0.024	0.038	0.000	0.455	0.000	0.000	0.000

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	91	0	0	0	0	0	-1
normalized size	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.097	0.232	0.698	0.000	0.438	0.000	0.000	0.000

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	91	0	0	0	0	0	-1
normalized size	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.093	0.101	0.753	0.000	0.493	0.000	0.000	0.000

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	87	0	0	0	0	0	-1
normalized size	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.089	0.143	0.800	0.000	0.469	0.000	0.000	0.000

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	91	0	0	0	0	0	-1
normalized size	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.093	0.085	0.040	0.000	0.491	0.000	0.000	0.000

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	91	0	0	0	0	0	-1
normalized size	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.097	0.188	0.034	0.000	0.433	0.000	0.000	0.000

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	140	0	0	0	0	0	-1
normalized size	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.120	0.385	1.736	0.000	0.460	0.000	0.000	0.000

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	140	0	0	0	0	0	-1
normalized size	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.117	0.222	1.869	0.000	0.441	0.000	0.000	0.000

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	140	0	0	0	0	0	-1
normalized size	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.112	0.275	1.530	0.000	0.433	0.000	0.000	0.000

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	140	0	0	0	0	0	-1
normalized size	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.109	0.250	1.244	0.000	0.459	0.000	0.000	0.000

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	140	0	0	0	0	0	-1
normalized size	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.115	0.236	1.236	0.000	0.475	0.000	0.000	0.000

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	140	0	0	0	0	0	-1
normalized size	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.120	0.340	1.219	0.000	0.489	0.000	0.000	0.000

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	126	0	0	0	0	0	-1
normalized size	1	1.00	0.73	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.111	0.223	4.106	0.000	0.432	0.000	0.000	0.000

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	119	0	0	0	0	0	-1
normalized size	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.127	0.228	3.651	0.000	0.422	0.000	0.000	0.000

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	119	0	0	0	0	0	-1
normalized size	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.115	0.257	2.947	0.000	0.435	0.000	0.000	0.000

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	107	0	0	0	0	0	-1
normalized size	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.096	0.156	3.007	0.000	0.426	0.000	0.000	0.000

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	107	0	0	0	0	0	-1
normalized size	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.127	0.161	4.005	0.000	0.413	0.000	0.000	0.000

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	114	0	0	0	0	0	-1
normalized size	1	1.00	0.75	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.142	0.311	4.487	0.000	0.418	0.000	0.000	0.000

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	140	0	0	0	0	0	-1
normalized size	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.116	0.329	1.769	0.000	0.429	0.000	0.000	0.000

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	140	0	0	0	0	0	-1
normalized size	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.111	0.244	1.570	0.000	0.449	0.000	0.000	0.000

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	135	0	0	0	0	0	-1
normalized size	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.113	0.364	1.639	0.000	0.457	0.000	0.000	0.000

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	140	0	0	0	0	0	-1
normalized size	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.113	0.327	1.702	0.000	0.454	0.000	0.000	0.000

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	87	213	200	137	0	214	198
normalized size	1	1.00	0.65	1.59	1.49	1.02	0.00	1.60	1.48
time (sec)	N/A	0.141	0.777	1.204	0.333	0.442	0.000	1.114	4.751

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	77	171	163	127	0	188	166
normalized size	1	1.00	0.73	1.61	1.54	1.20	0.00	1.77	1.57
time (sec)	N/A	0.123	0.406	1.125	0.336	0.459	0.000	0.258	4.617

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	56	128	127	105	0	154	126
normalized size	1	1.00	0.65	1.49	1.48	1.22	0.00	1.79	1.47
time (sec)	N/A	0.115	0.345	1.146	0.335	0.432	0.000	0.990	3.988

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	75	86	88	89	0	124	94
normalized size	1	1.00	1.34	1.54	1.57	1.59	0.00	2.21	1.68
time (sec)	N/A	0.067	0.027	0.941	0.322	0.425	0.000	0.278	2.732

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	43	65	56	79	71	84	100
normalized size	1	1.00	1.34	2.03	1.75	2.47	2.22	2.62	3.12
time (sec)	N/A	0.033	0.016	0.707	0.319	0.436	7.785	1.411	2.234

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	46	56	58	51	0	79	100
normalized size	1	1.00	1.44	1.75	1.81	1.59	0.00	2.47	3.12
time (sec)	N/A	0.047	0.027	0.622	0.326	0.442	0.000	2.347	2.149

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	44	57	55	38	0	93	50
normalized size	1	1.00	0.94	1.21	1.17	0.81	0.00	1.98	1.06
time (sec)	N/A	0.086	0.100	0.755	0.324	0.417	0.000	0.226	2.087

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	65	85	79	56	0	124	84
normalized size	1	1.00	0.84	1.10	1.03	0.73	0.00	1.61	1.09
time (sec)	N/A	0.108	0.181	1.134	0.323	0.452	0.000	0.308	2.102

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	75	107	101	74	0	156	184
normalized size	1	1.00	0.77	1.10	1.04	0.76	0.00	1.61	1.90
time (sec)	N/A	0.119	0.242	1.287	0.336	0.502	0.000	0.475	4.668

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	77	128	124	88	0	184	212
normalized size	1	1.00	0.62	1.02	0.99	0.70	0.00	1.47	1.70
time (sec)	N/A	0.134	0.248	1.601	0.341	0.427	0.000	0.260	4.808

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	280	235	278	165	0	246	224
normalized size	1	1.00	1.66	1.39	1.64	0.98	0.00	1.46	1.33
time (sec)	N/A	0.244	1.399	1.400	0.346	0.471	0.000	0.729	4.606

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	262	187	230	145	0	212	183
normalized size	1	1.00	1.90	1.36	1.67	1.05	0.00	1.54	1.33
time (sec)	N/A	0.228	1.328	1.347	0.346	0.456	0.000	0.321	4.465

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	481	141	167	125	0	178	145
normalized size	1	1.00	4.67	1.37	1.62	1.21	0.00	1.73	1.41
time (sec)	N/A	0.113	6.193	1.149	0.341	0.580	0.000	0.627	3.809

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	307	113	128	119	0	154	162
normalized size	1	1.00	3.74	1.38	1.56	1.45	0.00	1.88	1.98
time (sec)	N/A	0.084	1.323	0.897	0.326	0.467	0.000	0.491	2.005

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	258	107	105	108	0	157	161
normalized size	1	1.00	3.53	1.47	1.44	1.48	0.00	2.15	2.21
time (sec)	N/A	0.130	1.662	0.928	0.338	0.480	0.000	0.962	2.006

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	96	108	101	79	0	145	141
normalized size	1	1.00	1.09	1.23	1.15	0.90	0.00	1.65	1.60
time (sec)	N/A	0.145	0.158	0.820	0.340	0.464	0.000	0.283	2.049

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	61	116	110	70	0	142	98
normalized size	1	1.00	0.60	1.14	1.08	0.69	0.00	1.39	0.96
time (sec)	N/A	0.153	0.174	1.051	0.331	0.435	0.000	0.547	1.890

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	86	154	144	90	0	176	134
normalized size	1	1.00	0.64	1.14	1.07	0.67	0.00	1.30	0.99
time (sec)	N/A	0.231	0.354	1.285	0.339	0.462	0.000	0.293	1.933

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	108	186	178	110	0	210	247
normalized size	1	1.00	0.68	1.16	1.11	0.69	0.00	1.31	1.54
time (sec)	N/A	0.252	0.446	1.624	0.341	0.449	0.000	0.330	4.680

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	346	281	405	185	0	280	262
normalized size	1	1.00	1.65	1.34	1.93	0.88	0.00	1.33	1.25
time (sec)	N/A	0.399	1.885	1.632	0.355	0.450	0.000	1.455	4.626

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	294	234	337	165	0	246	224
normalized size	1	1.00	1.80	1.44	2.07	1.01	0.00	1.51	1.37
time (sec)	N/A	0.268	1.457	1.702	0.365	0.472	0.000	0.767	4.596

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	273	188	262	145	0	212	185
normalized size	1	1.00	2.18	1.50	2.10	1.16	0.00	1.70	1.48
time (sec)	N/A	0.143	1.296	1.385	0.367	0.460	0.000	0.372	4.474

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	1056	158	198	141	0	189	209
normalized size	1	1.00	9.51	1.42	1.78	1.27	0.00	1.70	1.88
time (sec)	N/A	0.144	6.406	1.164	0.359	0.467	0.000	0.767	2.078

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	335	144	165	137	0	192	207
normalized size	1	1.00	3.10	1.33	1.53	1.27	0.00	1.78	1.92
time (sec)	N/A	0.238	2.709	1.114	0.356	0.466	0.000	0.505	2.123

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	302	145	140	127	0	192	197
normalized size	1	1.00	2.58	1.24	1.20	1.09	0.00	1.64	1.68
time (sec)	N/A	0.264	5.095	0.885	0.355	0.479	0.000	0.337	2.076

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	113	153	148	102	0	180	178
normalized size	1	1.00	0.90	1.22	1.18	0.82	0.00	1.44	1.42
time (sec)	N/A	0.271	0.241	1.073	0.359	0.490	0.000	1.865	2.152

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	86	176	167	90	0	176	134
normalized size	1	1.00	0.69	1.42	1.35	0.73	0.00	1.42	1.08
time (sec)	N/A	0.170	0.270	1.324	0.361	0.453	0.000	0.322	2.005

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	108	223	213	110	0	210	247
normalized size	1	1.00	0.61	1.27	1.21	0.62	0.00	1.19	1.40
time (sec)	N/A	0.372	0.431	1.798	0.361	0.450	0.000	0.626	4.710

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	134	266	262	130	0	244	285
normalized size	1	1.00	0.67	1.32	1.30	0.65	0.00	1.21	1.42
time (sec)	N/A	0.410	0.556	1.981	0.352	0.437	0.000	0.400	4.681

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	358	280	464	185	0	280	262
normalized size	1	1.00	1.85	1.44	2.39	0.95	0.00	1.44	1.35
time (sec)	N/A	0.318	2.380	1.798	0.352	0.436	0.000	0.782	4.606

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	306	234	369	165	0	246	224
normalized size	1	1.00	1.92	1.47	2.32	1.04	0.00	1.55	1.41
time (sec)	N/A	0.179	1.747	1.704	0.349	0.460	0.000	2.219	4.542

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	326	204	293	157	0	223	255
normalized size	1	1.00	2.16	1.35	1.94	1.04	0.00	1.48	1.69
time (sec)	N/A	0.214	2.081	1.422	0.355	0.448	0.000	0.634	2.097

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	1202	189	235	159	0	227	254
normalized size	1	1.00	7.96	1.25	1.56	1.05	0.00	1.50	1.68
time (sec)	N/A	0.368	6.479	1.327	0.354	0.460	0.000	0.804	2.100

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	373	182	199	156	0	230	243
normalized size	1	1.00	2.33	1.14	1.24	0.98	0.00	1.44	1.52
time (sec)	N/A	0.390	4.969	1.055	0.354	0.448	0.000	0.319	2.172

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	342	190	187	150	0	226	242
normalized size	1	1.00	2.07	1.15	1.13	0.91	0.00	1.37	1.47
time (sec)	N/A	0.410	2.012	1.145	0.353	0.437	0.000	0.580	2.230

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	138	199	205	118	0	214	188
normalized size	1	1.00	0.80	1.15	1.18	0.68	0.00	1.24	1.09
time (sec)	N/A	0.403	0.387	1.213	0.351	0.447	0.000	0.334	2.450

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	108	248	236	110	0	210	248
normalized size	1	1.00	0.68	1.57	1.49	0.70	0.00	1.33	1.57
time (sec)	N/A	0.202	0.359	1.497	0.354	0.490	0.000	0.643	4.699

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	134	306	297	130	0	244	286
normalized size	1	1.00	0.61	1.39	1.35	0.59	0.00	1.11	1.30
time (sec)	N/A	0.532	0.611	1.995	0.353	0.450	0.000	0.931	4.624

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	156	358	356	150	0	278	323
normalized size	1	1.00	0.65	1.49	1.48	0.62	0.00	1.15	1.34
time (sec)	N/A	0.567	0.744	2.154	0.350	0.440	0.000	0.417	4.128

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	635	340	368	170	0	182	152
normalized size	1	1.00	4.85	2.60	2.81	1.30	0.00	1.39	1.16
time (sec)	N/A	0.171	6.301	0.572	0.345	0.450	0.000	0.799	2.437

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	311	252	282	156	0	156	119
normalized size	1	1.00	2.88	2.33	2.61	1.44	0.00	1.44	1.10
time (sec)	N/A	0.163	3.876	0.580	0.344	0.476	0.000	0.299	2.114

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	224	163	196	127	0	109	79
normalized size	1	1.00	3.61	2.63	3.16	2.05	0.00	1.76	1.27
time (sec)	N/A	0.117	1.406	0.563	0.345	0.435	0.000	0.790	2.033

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	109	78	99	74	0	70	41
normalized size	1	1.00	2.53	1.81	2.30	1.72	0.00	1.63	0.95
time (sec)	N/A	0.082	0.272	0.679	0.347	0.428	0.000	0.242	1.931

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	72	56	73	44	0	44	32
normalized size	1	1.00	2.06	1.60	2.09	1.26	0.00	1.26	0.91
time (sec)	N/A	0.059	0.159	0.734	0.425	0.413	0.000	0.290	1.900

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	76	108	143	63	0	79	65
normalized size	1	1.00	1.27	1.80	2.38	1.05	0.00	1.32	1.08
time (sec)	N/A	0.109	0.378	1.089	0.434	0.435	0.000	0.231	2.002

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	197	211	225	81	0	123	107
normalized size	1	1.00	2.01	2.15	2.30	0.83	0.00	1.26	1.09
time (sec)	N/A	0.150	0.462	1.066	0.446	0.445	0.000	0.216	2.213

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	249	281	310	97	0	151	138
normalized size	1	1.00	2.04	2.30	2.54	0.80	0.00	1.24	1.13
time (sec)	N/A	0.159	0.748	1.415	0.448	0.445	0.000	2.816	3.024

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	764	382	425	245	0	226	202
normalized size	1	1.00	4.27	2.13	2.37	1.37	0.00	1.26	1.13
time (sec)	N/A	0.321	6.434	1.050	0.356	0.449	0.000	1.573	1.980

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	496	294	336	228	0	198	166
normalized size	1	1.00	3.18	1.88	2.15	1.46	0.00	1.27	1.06
time (sec)	N/A	0.306	4.187	0.680	0.356	0.450	0.000	0.287	1.929

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	292	205	244	195	0	151	120
normalized size	1	1.00	2.70	1.90	2.26	1.81	0.00	1.40	1.11
time (sec)	N/A	0.257	1.940	0.696	0.344	0.473	0.000	0.294	1.929

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	169	119	145	129	0	112	74
normalized size	1	1.00	2.14	1.51	1.84	1.63	0.00	1.42	0.94
time (sec)	N/A	0.187	0.564	0.849	0.351	0.438	0.000	0.496	1.918

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	76	60	93	58	0	60	45
normalized size	1	1.00	1.17	0.92	1.43	0.89	0.00	0.92	0.69
time (sec)	N/A	0.080	0.212	0.845	0.344	0.409	0.000	0.263	1.892

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	153	97	120	94	0	85	65
normalized size	1	1.00	2.19	1.39	1.71	1.34	0.00	1.21	0.93
time (sec)	N/A	0.112	0.385	0.920	0.436	0.437	0.000	0.216	1.926

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	245	149	191	123	0	121	109
normalized size	1	1.00	2.50	1.52	1.95	1.26	0.00	1.23	1.11
time (sec)	N/A	0.230	0.637	1.212	0.436	0.443	0.000	0.252	2.040

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	315	252	283	138	0	164	154
normalized size	1	1.00	2.20	1.76	1.98	0.97	0.00	1.15	1.08
time (sec)	N/A	0.300	0.777	1.112	0.445	0.431	0.000	0.258	2.065

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	369	322	372	157	0	192	187
normalized size	1	1.00	2.17	1.89	2.19	0.92	0.00	1.13	1.10
time (sec)	N/A	0.319	0.771	1.169	0.450	0.446	0.000	0.263	2.082

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	768	334	377	295	0	233	216
normalized size	1	1.00	3.80	1.65	1.87	1.46	0.00	1.15	1.07
time (sec)	N/A	0.475	6.402	0.697	0.359	0.449	0.000	0.348	2.026

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	480	245	286	256	0	186	168
normalized size	1	1.00	3.08	1.57	1.83	1.64	0.00	1.19	1.08
time (sec)	N/A	0.429	4.279	0.602	0.350	0.443	0.000	0.306	2.038

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	197	159	187	183	0	147	124
normalized size	1	1.00	1.58	1.27	1.50	1.46	0.00	1.18	0.99
time (sec)	N/A	0.315	0.928	0.749	0.340	0.459	0.000	0.673	1.977

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	96	64	115	93	0	75	66
normalized size	1	1.00	0.94	0.63	1.13	0.91	0.00	0.74	0.65
time (sec)	N/A	0.203	0.307	0.730	0.332	0.397	0.000	1.010	1.918

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	135	64	115	93	0	75	66
normalized size	1	1.00	1.32	0.63	1.13	0.91	0.00	0.74	0.65
time (sec)	N/A	0.114	0.346	0.752	0.329	0.427	0.000	1.908	1.923

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	241	137	160	138	0	121	133
normalized size	1	1.00	2.23	1.27	1.48	1.28	0.00	1.12	1.23
time (sec)	N/A	0.186	0.596	0.796	0.421	0.431	0.000	0.648	2.127

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	365	189	231	173	0	157	155
normalized size	1	1.00	2.68	1.39	1.70	1.27	0.00	1.15	1.14
time (sec)	N/A	0.367	1.084	1.043	0.437	0.424	0.000	1.525	1.982

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	435	292	322	190	0	200	204
normalized size	1	1.00	2.33	1.56	1.72	1.02	0.00	1.07	1.09
time (sec)	N/A	0.470	0.830	1.221	0.435	0.465	0.000	0.472	1.997

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	491	362	412	205	0	228	237
normalized size	1	1.00	2.25	1.66	1.89	0.94	0.00	1.05	1.09
time (sec)	N/A	0.495	1.232	1.284	0.450	0.500	0.000	0.306	2.050

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	238	880	374	419	358	0	267	272
normalized size	1	1.00	3.70	1.57	1.76	1.50	0.00	1.12	1.14
time (sec)	N/A	0.656	6.516	0.628	0.357	0.493	0.000	0.866	2.055

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	754	285	326	317	0	220	237
normalized size	1	1.00	3.89	1.47	1.68	1.63	0.00	1.13	1.22
time (sec)	N/A	0.616	6.446	0.686	0.357	0.441	0.000	1.331	2.031

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	239	199	228	236	0	181	198
normalized size	1	1.00	1.47	1.22	1.40	1.45	0.00	1.11	1.21
time (sec)	N/A	0.475	1.570	0.714	0.361	0.453	0.000	0.578	2.128

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	109	88	175	125	0	117	85
normalized size	1	1.00	0.75	0.60	1.20	0.86	0.00	0.80	0.58
time (sec)	N/A	0.229	0.376	0.740	0.357	0.421	0.000	0.318	2.033

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	163	88	174	124	0	117	84
normalized size	1	1.00	1.18	0.64	1.26	0.90	0.00	0.85	0.61
time (sec)	N/A	0.265	0.416	0.716	0.357	0.428	0.000	0.303	1.984

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	193	90	175	124	0	117	88
normalized size	1	1.00	1.40	0.65	1.27	0.90	0.00	0.85	0.64
time (sec)	N/A	0.151	0.476	0.739	0.352	0.407	0.000	0.281	1.988

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	329	177	201	181	0	154	163
normalized size	1	1.00	2.38	1.28	1.46	1.31	0.00	1.12	1.18
time (sec)	N/A	0.268	0.793	0.799	0.715	0.436	0.000	0.288	2.034

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	485	229	271	223	0	190	202
normalized size	1	1.00	2.92	1.38	1.63	1.34	0.00	1.14	1.22
time (sec)	N/A	0.573	1.108	1.228	0.708	0.448	0.000	0.698	2.056

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	555	332	364	240	0	233	179
normalized size	1	1.00	2.49	1.49	1.63	1.08	0.00	1.04	0.80
time (sec)	N/A	0.649	1.205	1.165	0.711	0.445	0.000	0.282	1.998

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	256	256	611	402	452	257	0	261	300
normalized size	1	1.00	2.39	1.57	1.77	1.00	0.00	1.02	1.17
time (sec)	N/A	0.705	1.801	1.257	0.458	0.457	0.000	0.340	2.050

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	98	138	0	122	0	268	512
normalized size	1	1.00	0.52	0.74	0.00	0.65	0.00	1.43	2.74
time (sec)	N/A	0.338	0.601	1.637	0.000	0.420	0.000	2.279	10.047

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	81	116	0	105	0	222	407
normalized size	1	1.00	0.56	0.81	0.00	0.73	0.00	1.54	2.83
time (sec)	N/A	0.277	0.290	1.513	0.000	0.437	0.000	14.355	6.163

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	80	94	0	87	0	176	212
normalized size	1	1.00	0.79	0.93	0.00	0.86	0.00	1.74	2.10
time (sec)	N/A	0.228	0.335	1.454	0.000	0.488	0.000	1.164	6.158

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	53	70	0	66	0	129	159
normalized size	1	1.00	0.85	1.13	0.00	1.06	0.00	2.08	2.56
time (sec)	N/A	0.094	0.171	1.762	0.000	0.437	0.000	0.988	1.966

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	76	118	147	235	0	193	-1
normalized size	1	1.00	1.15	1.79	2.23	3.56	0.00	2.92	-0.02
time (sec)	N/A	0.088	0.322	1.581	0.502	0.453	0.000	1.447	0.000

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	93	198	939	261	0	336	-1
normalized size	1	1.00	1.37	2.91	13.81	3.84	0.00	4.94	-0.01
time (sec)	N/A	0.106	0.252	1.517	0.644	0.540	0.000	5.765	0.000

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	117	398	1851	308	0	630	-1
normalized size	1	1.00	1.00	3.40	15.82	2.63	0.00	5.38	-0.01
time (sec)	N/A	0.177	0.412	1.701	0.772	0.539	0.000	1.577	0.000

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	70	580	2981	346	0	889	-1
normalized size	1	1.00	0.44	3.62	18.63	2.16	0.00	5.56	-0.01
time (sec)	N/A	0.242	0.187	1.887	1.004	0.508	0.000	4.966	0.000

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	70	762	8561	380	0	1080	-1
normalized size	1	1.00	0.34	3.75	42.17	1.87	0.00	5.32	-0.00
time (sec)	N/A	0.298	0.182	1.677	1.424	0.534	0.000	1.811	0.000

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	100	139	0	127	0	258	596
normalized size	1	1.00	0.53	0.74	0.00	0.67	0.00	1.37	3.15
time (sec)	N/A	0.461	0.787	1.603	0.000	0.422	0.000	2.094	9.604

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	82	117	0	108	0	215	479
normalized size	1	1.00	0.59	0.85	0.00	0.78	0.00	1.56	3.47
time (sec)	N/A	0.297	0.421	1.431	0.000	0.418	0.000	1.552	6.889

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	70	95	0	89	0	170	213
normalized size	1	1.00	0.69	0.94	0.00	0.88	0.00	1.68	2.11
time (sec)	N/A	0.140	0.333	1.412	0.000	0.417	0.000	2.069	5.877

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	102	237	998	314	0	263	-1
normalized size	1	1.00	0.97	2.26	9.50	2.99	0.00	2.50	-0.01
time (sec)	N/A	0.146	0.622	1.484	0.642	0.459	0.000	6.971	0.000

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	97	212	1801	292	0	406	-1
normalized size	1	1.00	0.94	2.06	17.49	2.83	0.00	3.94	-0.01
time (sec)	N/A	0.241	0.462	1.472	0.835	0.514	0.000	1.597	0.000

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	111	399	0	320	0	639	-1
normalized size	1	1.00	0.93	3.35	0.00	2.69	0.00	5.37	-0.01
time (sec)	N/A	0.273	0.813	1.576	0.000	0.498	0.000	1.868	0.000

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	137	581	0	360	0	897	-1
normalized size	1	1.00	0.84	3.54	0.00	2.20	0.00	5.47	-0.01
time (sec)	N/A	0.365	1.013	1.807	0.000	0.515	0.000	1.930	0.000

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	154	763	0	396	0	1088	-1
normalized size	1	1.00	0.74	3.65	0.00	1.89	0.00	5.21	-0.00
time (sec)	N/A	0.449	1.478	1.624	0.000	0.558	0.000	8.925	0.000

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-1)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	487	163	0	157	0	306	856
normalized size	1	1.00	2.05	0.69	0.00	0.66	0.00	1.29	3.61
time (sec)	N/A	0.657	6.190	1.640	0.000	0.435	0.000	2.273	13.506

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	96	141	0	136	0	261	723
normalized size	1	1.00	0.55	0.81	0.00	0.78	0.00	1.49	4.13
time (sec)	N/A	0.353	0.635	1.515	0.000	0.432	0.000	2.064	10.815

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	89	119	0	115	0	216	590
normalized size	1	1.00	0.64	0.86	0.00	0.83	0.00	1.57	4.28
time (sec)	N/A	0.184	0.514	1.327	0.000	0.431	0.000	1.776	6.348

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	128	341	1396	378	0	309	-1
normalized size	1	1.00	0.90	2.40	9.83	2.66	0.00	2.18	-0.01
time (sec)	N/A	0.223	0.943	1.441	1.007	0.491	0.000	1.961	0.000

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	126	256	2780	386	0	480	-1
normalized size	1	1.00	0.88	1.79	19.44	2.70	0.00	3.36	-0.01
time (sec)	N/A	0.411	0.828	1.447	1.298	0.513	0.000	1.962	0.000

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	116	410	0	348	0	709	-1
normalized size	1	1.00	0.75	2.66	0.00	2.26	0.00	4.60	-0.01
time (sec)	N/A	0.419	0.965	1.685	0.000	0.538	0.000	2.069	0.000

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	312	583	0	380	0	905	-1
normalized size	1	1.00	1.90	3.55	0.00	2.32	0.00	5.52	-0.01
time (sec)	N/A	0.456	1.039	1.708	0.000	0.522	0.000	2.482	0.000

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	366	765	0	420	0	1096	-1
normalized size	1	1.00	1.75	3.66	0.00	2.01	0.00	5.24	-0.00
time (sec)	N/A	0.580	1.296	1.495	0.000	0.561	0.000	2.864	0.000

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	416	947	0	460	0	1377	-1
normalized size	1	1.00	1.64	3.73	0.00	1.81	0.00	5.42	-0.00
time (sec)	N/A	0.654	1.817	1.611	0.000	0.741	0.000	4.183	0.000

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	140	785	0	432	0	287	-1
normalized size	1	1.00	0.69	3.89	0.00	2.14	0.00	1.42	-0.00
time (sec)	N/A	0.606	0.537	1.876	0.000	0.513	0.000	2.350	0.000

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	123	595	0	397	0	271	-1
normalized size	1	1.00	0.77	3.74	0.00	2.50	0.00	1.70	-0.01
time (sec)	N/A	0.420	0.410	1.696	0.000	0.494	0.000	2.684	0.000

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	106	405	0	352	0	186	-1
normalized size	1	1.00	0.90	3.43	0.00	2.98	0.00	1.58	-0.01
time (sec)	N/A	0.257	0.302	1.664	0.000	0.514	0.000	2.137	0.000

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	88	200	0	287	0	144	-1
normalized size	1	1.00	1.13	2.56	0.00	3.68	0.00	1.85	-0.01
time (sec)	N/A	0.107	0.174	1.587	0.000	0.483	0.000	1.985	0.000

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	92	194	0	307	0	0	-1
normalized size	1	1.00	1.01	2.13	0.00	3.37	0.00	0.00	-0.01
time (sec)	N/A	0.107	0.292	1.593	0.000	1.210	0.000	0.000	0.000

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	11162	353	0	458	0	393	-1
normalized size	1	1.00	93.80	2.97	0.00	3.85	0.00	3.30	-0.01
time (sec)	N/A	0.229	26.700	1.686	0.000	1.478	0.000	6.625	0.000

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	135	717	0	506	0	649	-1
normalized size	1	1.00	0.82	4.35	0.00	3.07	0.00	3.93	-0.01
time (sec)	N/A	0.369	0.448	1.864	0.000	2.450	0.000	6.152	0.000

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	150	1067	0	539	0	846	-1
normalized size	1	1.00	0.73	5.18	0.00	2.62	0.00	4.11	-0.00
time (sec)	N/A	0.555	0.707	1.776	0.000	2.445	0.000	2.248	0.000

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	160	793	0	504	0	312	-1
normalized size	1	1.00	0.74	3.67	0.00	2.33	0.00	1.44	-0.00
time (sec)	N/A	0.633	2.396	2.013	0.000	0.498	0.000	6.953	0.000

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	141	603	0	459	0	296	-1
normalized size	1	1.00	0.82	3.53	0.00	2.68	0.00	1.73	-0.01
time (sec)	N/A	0.461	1.356	2.135	0.000	0.515	0.000	2.366	0.000

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	125	405	0	386	0	190	-1
normalized size	1	1.00	1.06	3.43	0.00	3.27	0.00	1.61	-0.01
time (sec)	N/A	0.259	0.872	1.773	0.000	0.500	0.000	2.501	0.000

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	127	404	0	367	0	154	-1
normalized size	1	1.00	1.46	4.64	0.00	4.22	0.00	1.77	-0.01
time (sec)	N/A	0.122	0.817	1.416	0.000	0.514	0.000	1.942	0.000

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	11183	554	0	548	0	0	-1
normalized size	1	1.00	88.06	4.36	0.00	4.31	0.00	0.00	-0.01
time (sec)	N/A	0.181	26.787	1.395	0.000	3.221	0.000	0.000	0.000

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	11954	713	0	609	0	453	-1
normalized size	1	1.00	70.32	4.19	0.00	3.58	0.00	2.66	-0.01
time (sec)	N/A	0.405	27.271	1.616	0.000	4.047	0.000	13.188	0.000

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	395	1075	0	644	0	673	-1
normalized size	1	1.00	1.79	4.86	0.00	2.91	0.00	3.05	-0.00
time (sec)	N/A	0.583	2.444	1.796	0.000	5.839	0.000	3.180	0.000

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	502	1425	0	675	0	851	-1
normalized size	1	1.00	1.87	5.32	0.00	2.52	0.00	3.18	-0.00
time (sec)	N/A	0.780	6.153	1.682	0.000	5.776	0.000	3.343	0.000

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	161	795	0	557	0	311	-1
normalized size	1	1.00	0.75	3.68	0.00	2.58	0.00	1.44	-0.00
time (sec)	N/A	0.655	2.708	1.750	0.000	0.495	0.000	3.395	0.000

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	144	597	0	484	0	289	-1
normalized size	1	1.00	0.85	3.53	0.00	2.86	0.00	1.71	-0.01
time (sec)	N/A	0.455	1.577	1.632	0.000	0.477	0.000	6.394	0.000

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	131	602	0	475	0	191	-1
normalized size	1	1.00	1.04	4.78	0.00	3.77	0.00	1.52	-0.01
time (sec)	N/A	0.276	1.640	1.674	0.000	0.478	0.000	13.492	0.000

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	206	594	0	475	0	191	-1
normalized size	1	1.00	1.63	4.71	0.00	3.77	0.00	1.52	-0.01
time (sec)	N/A	0.165	1.593	1.487	0.000	0.522	0.000	7.466	0.000

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	11243	824	0	670	0	0	-1
normalized size	1	1.00	68.55	5.02	0.00	4.09	0.00	0.00	-0.01
time (sec)	N/A	0.254	27.040	1.433	0.000	6.312	0.000	0.000	0.000

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F(-1)	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	12012	1065	0	739	0	499	-1
normalized size	1	1.00	58.03	5.14	0.00	3.57	0.00	2.41	-0.00
time (sec)	N/A	0.558	27.310	1.627	0.000	8.200	0.000	4.674	0.000

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	512	1427	0	776	0	720	-1
normalized size	1	1.00	1.94	5.41	0.00	2.94	0.00	2.73	-0.00
time (sec)	N/A	0.790	6.169	1.901	0.000	10.814	0.000	4.667	0.000

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	140	120	0	305	0	166	-1
normalized size	1	1.00	1.57	1.35	0.00	3.43	0.00	1.87	-0.01
time (sec)	N/A	0.146	0.644	1.541	0.000	0.466	0.000	1.126	0.000

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	269	154	0	435	0	238	-1
normalized size	1	1.00	2.34	1.34	0.00	3.78	0.00	2.07	-0.01
time (sec)	N/A	0.221	1.495	1.715	0.000	0.455	0.000	1.147	0.000

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	297	367	0	462	0	265	-1
normalized size	1	1.00	1.92	2.37	0.00	2.98	0.00	1.71	-0.01
time (sec)	N/A	0.362	1.789	1.945	0.000	0.463	0.000	4.071	0.000

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	330	625	0	484	0	290	-1
normalized size	1	1.00	1.72	3.26	0.00	2.52	0.00	1.51	-0.01
time (sec)	N/A	0.524	1.950	2.118	0.000	0.466	0.000	4.361	0.000

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	133	322	298	0	502	0	196	-1
normalized size	1	1.15	2.78	2.57	0.00	4.33	0.00	1.69	-0.01
time (sec)	N/A	0.200	6.693	1.543	0.000	0.488	0.000	1.148	0.000

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	361	462	0	526	0	261	-1
normalized size	1	1.00	2.47	3.16	0.00	3.60	0.00	1.79	-0.01
time (sec)	N/A	0.355	6.628	1.643	0.000	0.475	0.000	2.212	0.000

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	408	883	0	550	0	295	-1
normalized size	1	1.00	2.10	4.55	0.00	2.84	0.00	1.52	-0.01
time (sec)	N/A	0.529	6.766	1.926	0.000	0.467	0.000	1.454	0.000

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	452	1104	0	572	0	320	-1
normalized size	1	1.00	1.92	4.68	0.00	2.42	0.00	1.36	-0.00
time (sec)	N/A	0.701	6.733	1.856	0.000	0.492	0.000	1.529	0.000

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	185	387	695	0	590	0	222	-1
normalized size	1	1.22	2.55	4.57	0.00	3.88	0.00	1.46	-0.01
time (sec)	N/A	0.206	6.814	1.554	0.000	0.469	0.000	1.365	0.000

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	423	788	0	612	0	295	-1
normalized size	1	1.00	2.30	4.28	0.00	3.33	0.00	1.60	-0.01
time (sec)	N/A	0.505	6.801	1.772	0.000	0.477	0.000	1.469	0.000

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	458	1475	0	634	0	308	-1
normalized size	1	1.00	1.94	6.25	0.00	2.69	0.00	1.31	-0.00
time (sec)	N/A	0.731	6.821	1.886	0.000	0.493	0.000	2.918	0.000

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	280	514	1964	0	656	0	346	-1
normalized size	1	1.00	1.84	7.01	0.00	2.34	0.00	1.24	-0.00
time (sec)	N/A	0.906	6.841	2.066	0.000	0.519	0.000	1.936	0.000

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	200	691	0	0	0	0	-1
normalized size	1	1.00	1.01	3.47	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.179	0.780	12.410	0.000	0.441	0.000	0.000	0.000

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	168	662	0	0	0	0	-1
normalized size	1	1.00	0.98	3.85	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.165	0.674	12.301	0.000	0.416	0.000	0.000	0.000

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	94	427	0	0	0	0	-1
normalized size	1	1.00	0.70	3.16	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.144	0.530	10.069	0.000	0.449	0.000	0.000	0.000

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	77	240	0	0	0	0	-1
normalized size	1	1.00	0.73	2.26	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.134	0.307	4.863	0.000	0.423	0.000	0.000	0.000

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	83	321	0	0	0	0	-1
normalized size	1	1.00	0.75	2.92	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.130	0.318	4.983	0.000	0.450	0.000	0.000	0.000

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	99	355	0	0	0	0	-1
normalized size	1	1.00	0.70	2.52	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.153	0.588	4.372	0.000	0.458	0.000	0.000	0.000

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	113	383	0	0	0	0	-1
normalized size	1	1.00	0.66	2.23	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.161	0.987	4.197	0.000	0.465	0.000	0.000	0.000

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	463	852	0	0	0	0	-1
normalized size	1	1.00	1.98	3.64	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.342	4.666	15.561	0.000	0.438	0.000	0.000	0.000

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	321	743	0	0	0	0	-1
normalized size	1	1.00	1.61	3.73	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.299	6.570	13.120	0.000	0.422	0.000	0.000	0.000

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	295	513	0	0	0	0	-1
normalized size	1	1.00	1.84	3.21	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.278	3.211	5.141	0.000	0.465	0.000	0.000	0.000

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	299	388	0	0	0	0	-1
normalized size	1	1.00	1.89	2.46	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.256	3.011	4.635	0.000	0.445	0.000	0.000	0.000

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	153	357	0	0	0	0	-1
normalized size	1	1.00	0.92	2.15	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.260	1.797	4.533	0.000	0.455	0.000	0.000	0.000

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	193	385	0	0	0	0	-1
normalized size	1	1.00	0.96	1.92	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.290	2.382	4.427	0.000	0.453	0.000	0.000	0.000

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	217	413	0	0	0	0	-1
normalized size	1	1.00	0.93	1.76	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.320	2.997	4.720	0.000	0.468	0.000	0.000	0.000

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	793	1180	0	0	0	0	-1
normalized size	1	1.00	2.86	4.26	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.541	6.956	18.522	0.000	0.450	0.000	0.000	0.000

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	465	931	0	0	0	0	-1
normalized size	1	1.00	1.91	3.82	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.439	5.231	15.402	0.000	0.424	0.000	0.000	0.000

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	244	916	0	0	0	0	-1
normalized size	1	1.00	1.16	4.34	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.416	2.410	12.877	0.000	0.470	0.000	0.000	0.000

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	202	654	0	0	0	0	-1
normalized size	1	1.00	1.02	3.29	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.409	2.049	5.722	0.000	0.486	0.000	0.000	0.000

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	207	519	0	0	0	0	-1
normalized size	1	1.00	0.98	2.46	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.413	1.774	4.783	0.000	0.449	0.000	0.000	0.000

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	194	385	0	0	0	0	-1
normalized size	1	1.00	0.92	1.82	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.437	2.697	5.016	0.000	0.432	0.000	0.000	0.000

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	196	413	0	0	0	0	-1
normalized size	1	1.00	0.80	1.69	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.477	3.169	4.459	0.000	0.458	0.000	0.000	0.000

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	239	441	0	0	0	0	-1
normalized size	1	1.00	0.86	1.59	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.510	3.737	4.476	0.000	0.468	0.000	0.000	0.000

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	814	806	0	0	0	0	-1
normalized size	1	1.00	3.55	3.52	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.248	7.698	14.321	0.000	0.444	0.000	0.000	0.000

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	372	493	0	0	0	0	-1
normalized size	1	1.00	1.94	2.57	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.227	3.530	11.618	0.000	0.466	0.000	0.000	0.000

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	420	318	0	0	0	0	-1
normalized size	1	1.00	2.75	2.08	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.186	4.829	8.625	0.000	0.459	0.000	0.000	0.000

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	200	243	0	0	0	0	-1
normalized size	1	1.00	1.63	1.98	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.171	1.175	4.813	0.000	0.429	0.000	0.000	0.000

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	445	244	0	0	0	0	-1
normalized size	1	1.00	3.48	1.91	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.178	2.784	4.412	0.000	0.427	0.000	0.000	0.000

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	232	262	0	0	0	0	-1
normalized size	1	1.00	1.41	1.60	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.194	2.425	4.997	0.000	0.485	0.000	0.000	0.000

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	540	282	0	0	0	0	-1
normalized size	1	1.00	2.74	1.43	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.213	3.978	4.901	0.000	0.455	0.000	0.000	0.000

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	568	300	0	0	0	0	-1
normalized size	1	1.00	2.47	1.30	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.230	4.158	5.003	0.000	0.474	0.000	0.000	0.000

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	865	750	0	0	0	0	-1
normalized size	1	1.00	3.65	3.16	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.371	8.035	15.795	0.000	0.446	0.000	0.000	0.000

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	455	492	0	0	0	0	-1
normalized size	1	1.00	2.23	2.41	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.345	7.134	5.877	0.000	0.439	0.000	0.000	0.000

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	256	350	0	0	0	0	-1
normalized size	1	1.00	1.59	2.17	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.303	2.947	5.030	0.000	0.448	0.000	0.000	0.000

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	256	350	0	0	0	0	-1
normalized size	1	1.00	1.52	2.08	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.310	3.649	5.660	0.000	0.462	0.000	0.000	0.000

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	854	421	0	0	0	0	-1
normalized size	1	1.00	4.82	2.38	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.326	6.879	5.823	0.000	0.471	0.000	0.000	0.000

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	899	435	0	0	0	0	-1
normalized size	1	1.00	4.26	2.06	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.357	6.912	6.279	0.000	0.448	0.000	0.000	0.000

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	946	465	0	0	0	0	-1
normalized size	1	1.00	3.88	1.91	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.382	7.154	5.617	0.000	0.490	0.000	0.000	0.000

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	292	953	876	0	0	0	0	-1
normalized size	1	1.00	3.26	3.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.560	8.498	6.971	0.000	0.456	0.000	0.000	0.000

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	924	685	0	0	0	0	-1
normalized size	1	1.00	3.54	2.62	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.536	7.518	6.015	0.000	0.443	0.000	0.000	0.000

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	919	451	0	0	0	0	-1
normalized size	1	1.00	4.18	2.05	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.490	7.106	5.149	0.000	0.435	0.000	0.000	0.000

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	918	451	0	0	0	0	-1
normalized size	1	1.00	4.25	2.09	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.484	6.941	4.830	0.000	0.426	0.000	0.000	0.000

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	919	451	0	0	0	0	-1
normalized size	1	1.00	4.14	2.03	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.489	7.097	6.000	0.000	0.440	0.000	0.000	0.000

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	943	451	0	0	0	0	-1
normalized size	1	1.00	4.14	1.98	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.499	7.157	5.188	0.000	0.463	0.000	0.000	0.000

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	988	465	0	0	0	0	-1
normalized size	1	1.00	3.79	1.78	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.553	7.276	5.295	0.000	0.465	0.000	0.000	0.000

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	294	294	1032	493	0	0	0	0	-1
normalized size	1	1.00	3.51	1.68	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.570	7.586	5.900	0.000	0.472	0.000	0.000	0.000

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	131	408	3342	448	0	0	-1
normalized size	1	1.00	0.74	2.32	18.99	2.55	0.00	0.00	-0.01
time (sec)	N/A	0.288	1.553	2.428	2.227	0.563	0.000	0.000	0.000

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	106	344	1927	402	0	0	-1
normalized size	1	1.00	0.81	2.63	14.71	3.07	0.00	0.00	-0.01
time (sec)	N/A	0.237	0.523	2.481	1.713	0.564	0.000	0.000	0.000

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	89	277	905	322	0	0	-1
normalized size	1	1.00	1.14	3.55	11.60	4.13	0.00	0.00	-0.01
time (sec)	N/A	0.160	0.286	2.190	1.593	0.749	0.000	0.000	0.000

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	83	178	262	307	0	0	-1
normalized size	1	1.00	1.09	2.34	3.45	4.04	0.00	0.00	-0.01
time (sec)	N/A	0.157	0.471	2.516	1.202	0.472	0.000	0.000	0.000

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	56	75	134	74	0	0	81
normalized size	1	1.00	0.68	0.91	1.63	0.90	0.00	0.00	0.99
time (sec)	N/A	0.158	0.237	2.598	1.268	0.424	0.000	0.000	2.785

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	71	96	317	92	0	0	106
normalized size	1	1.00	0.55	0.74	2.44	0.71	0.00	0.00	0.82
time (sec)	N/A	0.222	0.320	2.484	1.260	0.406	0.000	0.000	3.380

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	91	118	498	110	0	0	130
normalized size	1	1.00	0.52	0.67	2.85	0.63	0.00	0.00	0.74
time (sec)	N/A	0.288	0.354	2.954	1.320	0.406	0.000	0.000	4.188

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	153	479	5879	494	0	0	-1
normalized size	1	1.00	0.67	2.11	25.90	2.18	0.00	0.00	-0.00
time (sec)	N/A	0.547	1.504	2.421	2.615	0.685	0.000	0.000	0.000

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	134	415	4606	458	0	0	-1
normalized size	1	1.00	0.74	2.31	25.59	2.54	0.00	0.00	-0.01
time (sec)	N/A	0.419	1.548	2.417	1.953	0.589	0.000	0.000	0.000

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	107	353	3389	410	0	0	-1
normalized size	1	1.00	0.80	2.65	25.48	3.08	0.00	0.00	-0.01
time (sec)	N/A	0.336	0.731	2.112	1.522	0.614	0.000	0.000	0.000

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	107	346	1417	364	0	0	-1
normalized size	1	1.00	0.86	2.79	11.43	2.94	0.00	0.00	-0.01
time (sec)	N/A	0.314	1.737	2.543	1.304	0.540	0.000	0.000	0.000

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	109	211	314	368	0	0	-1
normalized size	1	1.00	0.87	1.69	2.51	2.94	0.00	0.00	-0.01
time (sec)	N/A	0.339	0.656	2.606	1.528	0.471	0.000	0.000	0.000

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	73	97	250	94	0	0	107
normalized size	1	1.00	0.56	0.74	1.91	0.72	0.00	0.00	0.82
time (sec)	N/A	0.256	0.506	2.617	1.499	0.419	0.000	0.000	3.405

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	92	119	514	113	0	0	131
normalized size	1	1.00	0.51	0.66	2.84	0.62	0.00	0.00	0.72
time (sec)	N/A	0.438	0.494	2.595	1.273	0.421	0.000	0.000	4.338

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	110	141	700	132	0	0	155
normalized size	1	1.00	0.48	0.62	3.07	0.58	0.00	0.00	0.68
time (sec)	N/A	0.508	0.672	3.123	1.345	0.428	0.000	0.000	5.299

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	178	543	9242	558	0	0	-1
normalized size	1	1.00	0.65	1.98	33.73	2.04	0.00	0.00	-0.00
time (sec)	N/A	0.693	2.277	2.463	5.201	0.680	0.000	0.000	0.000

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	154	479	7331	518	0	0	-1
normalized size	1	1.00	0.68	2.11	32.30	2.28	0.00	0.00	-0.00
time (sec)	N/A	0.595	1.595	2.457	3.092	0.696	0.000	0.000	0.000

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	133	419	6297	478	0	0	-1
normalized size	1	1.00	0.74	2.33	34.98	2.66	0.00	0.00	-0.01
time (sec)	N/A	0.513	1.449	2.162	8.721	0.592	0.000	0.000	0.000

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	137	386	0	454	0	0	-1
normalized size	1	1.00	0.76	2.14	0.00	2.52	0.00	0.00	-0.01
time (sec)	N/A	0.504	2.266	2.988	0.000	0.557	0.000	0.000	0.000

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	133	376	0	424	0	0	-1
normalized size	1	1.00	0.75	2.12	0.00	2.40	0.00	0.00	-0.01
time (sec)	N/A	0.505	1.052	3.287	0.000	0.605	0.000	0.000	0.000

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	127	235	655	424	0	0	-1
normalized size	1	1.00	0.74	1.37	3.81	2.47	0.00	0.00	-0.01
time (sec)	N/A	0.489	1.957	2.718	1.658	0.485	0.000	0.000	0.000

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	91	121	385	120	0	0	133
normalized size	1	1.00	0.51	0.68	2.16	0.67	0.00	0.00	0.75
time (sec)	N/A	0.317	0.569	2.612	1.395	0.441	0.000	0.000	4.347

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	108	143	746	141	0	0	157
normalized size	1	1.00	0.47	0.63	3.27	0.62	0.00	0.00	0.69
time (sec)	N/A	0.632	0.708	2.684	1.492	0.452	0.000	0.000	5.549

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	127	165	945	162	0	0	392
normalized size	1	1.00	0.46	0.60	3.44	0.59	0.00	0.00	1.43
time (sec)	N/A	0.699	4.250	2.788	1.507	0.478	0.000	0.000	8.768

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	125	423	2524	617	0	0	-1
normalized size	1	1.00	0.66	2.23	13.28	3.25	0.00	0.00	-0.01
time (sec)	N/A	0.573	0.889	2.641	1.639	0.648	0.000	0.000	0.000

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	106	352	1353	531	0	0	-1
normalized size	1	1.00	0.75	2.50	9.60	3.77	0.00	0.00	-0.01
time (sec)	N/A	0.387	0.422	2.587	1.555	0.598	0.000	0.000	0.000

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	95	211	567	366	0	0	-1
normalized size	1	1.00	0.95	2.11	5.67	3.66	0.00	0.00	-0.01
time (sec)	N/A	0.232	0.198	2.347	1.576	0.516	0.000	0.000	0.000

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	114	150	195	306	0	0	-1
normalized size	1	1.00	1.15	1.52	1.97	3.09	0.00	0.00	-0.01
time (sec)	N/A	0.185	0.272	2.372	1.359	0.467	0.000	0.000	0.000

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	132	183	387	354	0	0	-1
normalized size	1	1.00	0.93	1.29	2.73	2.49	0.00	0.00	-0.01
time (sec)	N/A	0.331	0.389	2.524	1.229	0.458	0.000	0.000	0.000

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	133	205	640	388	0	0	-1
normalized size	1	1.00	0.71	1.10	3.42	2.07	0.00	0.00	-0.01
time (sec)	N/A	0.507	1.192	2.671	1.390	0.460	0.000	0.000	0.000

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	152	227	805	422	0	0	-1
normalized size	1	1.00	0.66	0.99	3.50	1.83	0.00	0.00	-0.00
time (sec)	N/A	0.689	1.640	2.839	2.129	0.483	0.000	0.000	0.000

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-1)	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	497	541	0	761	0	0	-1
normalized size	1	1.00	2.01	2.19	0.00	3.08	0.00	0.00	-0.00
time (sec)	N/A	0.783	4.636	2.365	0.000	0.666	0.000	0.000	0.000

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	132	479	7057	669	0	0	-1
normalized size	1	1.00	0.67	2.43	35.82	3.40	0.00	0.00	-0.01
time (sec)	N/A	0.601	1.941	2.378	4.027	0.663	0.000	0.000	0.000

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	113	313	0	601	0	0	-1
normalized size	1	1.00	0.78	2.16	0.00	4.14	0.00	0.00	-0.01
time (sec)	N/A	0.395	0.857	2.560	0.000	0.535	0.000	0.000	0.000

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	84	219	0	376	0	0	-1
normalized size	1	1.00	0.79	2.05	0.00	3.51	0.00	0.00	-0.01
time (sec)	N/A	0.195	0.287	2.180	0.000	0.465	0.000	0.000	0.000

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	174	287	8208	430	0	0	-1
normalized size	1	1.00	1.12	1.84	52.62	2.76	0.00	0.00	-0.01
time (sec)	N/A	0.362	1.523	2.457	1.255	0.461	0.000	0.000	0.000

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	173	317	0	464	0	0	-1
normalized size	1	1.00	0.85	1.56	0.00	2.29	0.00	0.00	-0.00
time (sec)	N/A	0.553	1.795	2.501	0.000	0.464	0.000	0.000	0.000

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	171	339	0	500	0	0	-1
normalized size	1	1.00	0.68	1.36	0.00	2.00	0.00	0.00	-0.00
time (sec)	N/A	0.734	1.496	2.803	0.000	0.488	0.000	0.000	0.000

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-1)	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	941	831	0	803	0	0	-1
normalized size	1	1.00	3.83	3.38	0.00	3.26	0.00	0.00	-0.00
time (sec)	N/A	0.820	6.171	2.640	0.000	0.721	0.000	0.000	0.000

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-1)	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	845	550	0	749	0	0	-1
normalized size	1	1.00	4.36	2.84	0.00	3.86	0.00	0.00	-0.01
time (sec)	N/A	0.588	6.162	2.824	0.000	0.569	0.000	0.000	0.000

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	106	350	0	498	0	0	-1
normalized size	1	1.00	0.68	2.24	0.00	3.19	0.00	0.00	-0.01
time (sec)	N/A	0.272	0.812	2.633	0.000	0.469	0.000	0.000	0.000

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	203	103	347	5924	502	0	0	-1
normalized size	1	1.30	0.66	2.22	37.97	3.22	0.00	0.00	-0.01
time (sec)	N/A	0.572	1.156	2.380	2.729	0.709	0.000	0.000	0.000

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	206	419	0	524	0	0	-1
normalized size	1	1.00	1.01	2.06	0.00	2.58	0.00	0.00	-0.00
time (sec)	N/A	0.571	2.607	2.742	0.000	0.477	0.000	0.000	0.000

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	193	449	0	564	0	0	-1
normalized size	1	1.00	0.77	1.80	0.00	2.26	0.00	0.00	-0.00
time (sec)	N/A	0.761	1.889	2.876	0.000	0.485	0.000	0.000	0.000

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	196	471	0	592	0	0	-1
normalized size	1	1.00	0.66	1.59	0.00	1.99	0.00	0.00	-0.00
time (sec)	N/A	0.956	2.395	3.024	0.000	0.468	0.000	0.000	0.000

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	406	406	4445	0	0	0	0	0	-1
normalized size	1	1.00	10.95	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.632	20.717	1.253	0.000	0.000	0.000	0.000	0.000

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	354	354	2709	0	0	0	0	0	-1
normalized size	1	1.00	7.65	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.366	19.155	1.324	0.000	0.000	0.000	0.000	0.000

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	415	415	2901	0	0	0	0	0	-1
normalized size	1	1.00	6.99	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.433	19.561	1.520	0.000	0.000	0.000	0.000	0.000

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	787	787	4110	0	0	0	0	0	-1
normalized size	1	1.00	5.22	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.839	19.609	1.731	0.000	0.000	0.000	0.000	0.000

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	739	739	5094	0	0	0	0	0	-1
normalized size	1	1.00	6.89	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.702	21.338	1.461	0.000	0.000	0.000	0.000	0.000

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	764	764	4066	0	0	0	0	0	-1
normalized size	1	1.00	5.32	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.733	19.363	1.322	0.000	0.000	0.000	0.000	0.000

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	4897	0	0	0	0	0	-1
normalized size	1	1.00	24.86	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.361	23.075	5.063	0.000	0.952	0.000	0.000	0.000

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	111	0	0	0	0	0	-1
normalized size	1	1.00	0.68	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.255	1.127	4.560	0.000	0.867	0.000	0.000	0.000

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	85	171	163	136	0	304	194
normalized size	1	1.00	0.75	1.50	1.43	1.19	0.00	2.67	1.70
time (sec)	N/A	0.145	0.634	1.247	0.880	0.968	0.000	0.300	5.727

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	67	128	127	115	0	210	145
normalized size	1	1.00	0.72	1.38	1.37	1.24	0.00	2.26	1.56
time (sec)	N/A	0.133	0.291	1.221	0.657	0.454	0.000	0.272	4.385

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	75	86	88	96	0	153	104
normalized size	1	1.00	1.23	1.41	1.44	1.57	0.00	2.51	1.70
time (sec)	N/A	0.078	0.026	0.965	0.613	0.442	0.000	0.235	3.143

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	43	65	56	85	71	84	114
normalized size	1	1.00	1.23	1.86	1.60	2.43	2.03	2.40	3.26
time (sec)	N/A	0.035	0.011	0.729	0.836	0.479	7.956	0.258	2.244

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	46	56	58	54	0	79	100
normalized size	1	1.00	1.31	1.60	1.66	1.54	0.00	2.26	2.86
time (sec)	N/A	0.055	0.027	0.805	0.445	0.462	0.000	0.467	2.191

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	51	57	55	42	0	121	50
normalized size	1	1.00	0.98	1.10	1.06	0.81	0.00	2.33	0.96
time (sec)	N/A	0.096	0.086	0.783	0.781	0.462	0.000	0.223	2.034

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	75	85	79	60	0	180	84
normalized size	1	1.00	0.89	1.01	0.94	0.71	0.00	2.14	1.00
time (sec)	N/A	0.125	0.162	1.313	0.905	0.428	0.000	0.786	2.067

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	91	107	101	81	0	272	117
normalized size	1	1.00	0.87	1.02	0.96	0.77	0.00	2.59	1.11
time (sec)	N/A	0.139	0.236	1.517	2.051	0.440	0.000	0.237	2.126

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	150	312	276	208	0	528	359
normalized size	1	1.00	0.76	1.58	1.39	1.05	0.00	2.67	1.81
time (sec)	N/A	0.291	1.559	1.523	0.667	0.535	0.000	0.331	5.707

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	120	241	228	180	0	478	317
normalized size	1	1.00	0.67	1.35	1.27	1.01	0.00	2.67	1.77
time (sec)	N/A	0.322	0.754	1.428	0.958	0.448	0.000	0.289	5.687

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	92	174	165	150	0	294	227
normalized size	1	1.00	0.79	1.50	1.42	1.29	0.00	2.53	1.96
time (sec)	N/A	0.180	0.487	1.182	1.657	0.451	0.000	0.435	5.444

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	67	133	126	136	0	192	176
normalized size	1	1.00	0.78	1.55	1.47	1.58	0.00	2.23	2.05
time (sec)	N/A	0.081	0.281	0.945	1.390	0.458	0.000	0.292	2.737

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	109	104	103	117	0	154	163
normalized size	1	1.00	1.82	1.73	1.72	1.95	0.00	2.57	2.72
time (sec)	N/A	0.102	0.495	0.867	0.908	0.485	0.000	0.276	2.547

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	120	120	99	87	0	178	169
normalized size	1	1.00	1.50	1.50	1.24	1.09	0.00	2.22	2.11
time (sec)	N/A	0.174	0.229	0.713	0.883	0.467	0.000	0.454	2.381

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	90	114	108	85	0	254	115
normalized size	1	1.00	0.84	1.07	1.01	0.79	0.00	2.37	1.07
time (sec)	N/A	0.216	0.238	1.064	1.171	0.466	0.000	0.493	2.101

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	118	152	142	114	0	437	169
normalized size	1	1.00	0.87	1.12	1.04	0.84	0.00	3.21	1.24
time (sec)	N/A	0.260	0.461	1.409	1.674	0.448	0.000	0.255	2.183

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	146	184	176	142	0	487	307
normalized size	1	1.00	0.81	1.02	0.98	0.79	0.00	2.71	1.71
time (sec)	N/A	0.268	0.546	1.576	0.960	0.480	0.000	0.640	5.808

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	181	382	341	249	0	722	470
normalized size	1	1.00	0.72	1.52	1.35	0.99	0.00	2.87	1.87
time (sec)	N/A	0.479	3.496	1.708	0.827	0.538	0.000	0.817	5.787

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	140	290	266	211	0	586	395
normalized size	1	1.00	0.78	1.61	1.48	1.17	0.00	3.26	2.19
time (sec)	N/A	0.333	0.949	1.379	0.741	0.480	0.000	0.702	6.015

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	108	223	202	189	0	336	526
normalized size	1	1.00	0.79	1.63	1.47	1.38	0.00	2.45	3.84
time (sec)	N/A	0.190	0.591	1.181	0.325	0.471	0.000	0.653	4.026

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	131	399	172	169	167	0	241	249
normalized size	1	1.10	3.35	1.45	1.42	1.40	0.00	2.03	2.09
time (sec)	N/A	0.223	0.979	1.171	0.766	0.473	0.000	0.383	3.598

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	217	168	144	152	0	234	236
normalized size	1	1.00	1.75	1.35	1.16	1.23	0.00	1.89	1.90
time (sec)	N/A	0.333	0.708	0.951	0.810	0.473	0.000	0.350	3.335

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	159	207	152	131	0	314	1924
normalized size	1	1.00	1.10	1.43	1.05	0.90	0.00	2.17	13.27
time (sec)	N/A	0.347	0.375	2.004	0.696	0.507	0.000	0.674	3.902

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	140	180	171	136	0	536	202
normalized size	1	1.00	0.78	1.01	0.96	0.76	0.00	2.99	1.13
time (sec)	N/A	0.423	0.427	1.622	0.670	0.464	0.000	0.316	2.490

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	176	227	217	174	0	672	277
normalized size	1	1.00	0.80	1.03	0.98	0.79	0.00	3.04	1.25
time (sec)	N/A	0.494	0.727	2.108	0.682	0.450	0.000	0.342	2.728

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	334	334	244	550	474	327	0	1186	709
normalized size	1	1.00	0.73	1.65	1.42	0.98	0.00	3.55	2.12
time (sec)	N/A	0.711	2.854	1.850	0.764	0.490	0.000	2.277	5.737

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	198	431	379	281	0	850	555
normalized size	1	1.00	0.79	1.72	1.52	1.12	0.00	3.40	2.22
time (sec)	N/A	0.520	3.940	1.671	1.019	0.476	0.000	1.082	6.014

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	160	338	303	250	0	635	1969
normalized size	1	1.00	0.80	1.69	1.52	1.25	0.00	3.18	9.84
time (sec)	N/A	0.327	1.032	1.378	0.839	0.494	0.000	2.389	5.006

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	1051	262	245	219	0	387	636
normalized size	1	1.00	5.39	1.34	1.26	1.12	0.00	1.98	3.26
time (sec)	N/A	0.368	6.294	1.480	0.627	0.484	0.000	0.372	4.902

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	310	236	209	202	0	528	330
normalized size	1	1.00	1.48	1.13	1.00	0.97	0.00	2.53	1.58
time (sec)	N/A	0.463	2.010	1.072	0.719	0.488	0.000	2.107	4.390

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	257	255	197	196	0	371	2523
normalized size	1	1.00	1.30	1.29	0.99	0.99	0.00	1.87	12.74
time (sec)	N/A	0.591	1.119	1.158	0.723	0.487	0.000	1.859	4.308

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	210	319	215	183	0	603	369
normalized size	1	1.00	0.97	1.48	1.00	0.85	0.00	2.79	1.71
time (sec)	N/A	0.609	0.616	1.184	0.703	0.485	0.000	0.421	3.384

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	258	263	258	246	197	0	791	307
normalized size	1	1.00	1.02	1.00	0.95	0.76	0.00	3.07	1.19
time (sec)	N/A	0.691	0.649	1.563	0.746	0.465	0.000	0.324	2.713

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	309	309	333	316	307	243	0	1127	403
normalized size	1	1.00	1.08	1.02	0.99	0.79	0.00	3.65	1.30
time (sec)	N/A	0.820	1.242	1.850	0.867	0.470	0.000	3.048	3.183

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	422	688	0	743	0	412	4667
normalized size	1	1.00	2.26	3.68	0.00	3.97	0.00	2.20	24.96
time (sec)	N/A	0.676	3.008	0.583	0.000	1.180	0.000	0.282	6.931

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	300	410	0	609	0	269	4047
normalized size	1	1.00	2.10	2.87	0.00	4.26	0.00	1.88	28.30
time (sec)	N/A	0.395	1.917	0.653	0.000	4.835	0.000	0.619	6.083

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	130	228	0	472	0	176	719
normalized size	1	1.00	1.33	2.33	0.00	4.82	0.00	1.80	7.34
time (sec)	N/A	0.229	0.711	0.533	0.000	0.620	0.000	0.628	2.971

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	112	135	0	316	0	127	573
normalized size	1	1.00	1.47	1.78	0.00	4.16	0.00	1.67	7.54
time (sec)	N/A	0.126	0.188	0.752	0.000	1.041	0.000	1.331	3.085

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	68	113	0	250	0	274	573
normalized size	1	1.00	1.01	1.69	0.00	3.73	0.00	4.09	8.55
time (sec)	N/A	0.099	0.133	0.800	0.000	0.467	0.000	0.434	3.196

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	85	172	0	328	0	141	740
normalized size	1	1.00	0.94	1.91	0.00	3.64	0.00	1.57	8.22
time (sec)	N/A	0.149	0.226	1.247	0.000	0.484	0.000	0.923	3.359

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	121	367	0	427	0	227	3740
normalized size	1	1.00	0.90	2.74	0.00	3.19	0.00	1.69	27.91
time (sec)	N/A	0.403	0.360	1.139	0.000	0.498	0.000	0.273	6.019

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	152	641	0	547	0	360	4572
normalized size	1	1.00	0.85	3.60	0.00	3.07	0.00	2.02	25.69
time (sec)	N/A	0.642	0.537	1.218	0.000	0.559	0.000	0.305	6.904

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	202	1212	0	685	0	642	5903
normalized size	1	1.00	0.84	5.05	0.00	2.85	0.00	2.68	24.60
time (sec)	N/A	0.982	0.671	1.177	0.000	0.546	0.000	0.301	8.636

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	272	272	438	698	0	1343	0	384	6678
normalized size	1	1.00	1.61	2.57	0.00	4.94	0.00	1.41	24.55
time (sec)	N/A	0.866	6.297	0.572	0.000	20.609	0.000	0.339	11.171

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	240	510	0	1114	0	404	5436
normalized size	1	1.00	1.46	3.11	0.00	6.79	0.00	2.46	33.15
time (sec)	N/A	0.578	2.231	0.661	0.000	13.157	0.000	0.363	10.257

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	191	350	0	694	0	231	3751
normalized size	1	1.00	1.46	2.67	0.00	5.30	0.00	1.76	28.63
time (sec)	N/A	0.301	0.715	0.749	0.000	4.056	0.000	0.307	9.654

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	97	132	0	389	0	172	106
normalized size	1	1.00	0.97	1.32	0.00	3.89	0.00	1.72	1.06
time (sec)	N/A	0.134	0.370	0.718	0.000	0.482	0.000	0.326	2.422

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	155	328	0	561	0	201	3763
normalized size	1	1.00	1.25	2.65	0.00	4.52	0.00	1.62	30.35
time (sec)	N/A	0.207	0.669	0.789	0.000	0.537	0.000	0.300	9.656

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	221	453	0	788	0	1107	3264
normalized size	1	1.00	1.23	2.52	0.00	4.38	0.00	6.15	18.13
time (sec)	N/A	0.569	1.131	1.149	0.000	0.558	0.000	0.489	7.073

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	184	651	0	970	0	340	6731
normalized size	1	1.00	0.70	2.49	0.00	3.72	0.00	1.30	25.79
time (sec)	N/A	0.891	1.138	1.099	0.000	0.599	0.000	1.159	11.106

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	346	346	224	926	0	1167	0	473	7763
normalized size	1	1.00	0.65	2.68	0.00	3.37	0.00	1.37	22.44
time (sec)	N/A	1.275	1.363	1.403	0.000	0.618	0.000	3.262	11.663

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	407	407	507	1599	0	2444	0	1391	10533
normalized size	1	1.00	1.25	3.93	0.00	6.00	0.00	3.42	25.88
time (sec)	N/A	1.959	3.081	0.883	0.000	72.774	0.000	0.498	14.229

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	289	289	418	1406	0	2111	0	581	9286
normalized size	1	1.00	1.45	4.87	0.00	7.30	0.00	2.01	32.13
time (sec)	N/A	1.424	6.515	0.687	0.000	48.002	0.000	0.459	14.543

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	270	1085	0	1419	0	486	6899
normalized size	1	1.00	1.23	4.93	0.00	6.45	0.00	2.21	31.36
time (sec)	N/A	0.686	2.054	0.803	0.000	18.230	0.000	0.480	11.530

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	157	238	0	750	0	400	251
normalized size	1	1.00	0.87	1.32	0.00	4.17	0.00	2.22	1.39
time (sec)	N/A	0.336	0.731	0.717	0.000	0.519	0.000	3.443	5.417

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	172	236	0	752	0	399	251
normalized size	1	1.00	1.05	1.44	0.00	4.59	0.00	2.43	1.53
time (sec)	N/A	0.264	0.919	0.718	0.000	0.512	0.000	0.377	5.352

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	267	1063	0	1152	0	457	6909
normalized size	1	1.00	1.30	5.19	0.00	5.62	0.00	2.23	33.70
time (sec)	N/A	0.536	1.466	0.864	0.000	0.548	0.000	0.374	11.785

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	290	290	306	1349	0	1568	0	546	5530
normalized size	1	1.00	1.06	4.65	0.00	5.41	0.00	1.88	19.07
time (sec)	N/A	1.535	2.072	1.179	0.000	0.623	0.000	0.399	9.730
Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	393	393	734	1552	0	1811	0	2700	10586
normalized size	1	1.00	1.87	3.95	0.00	4.61	0.00	6.87	26.94
time (sec)	N/A	1.999	4.692	1.319	0.000	0.701	0.000	0.787	14.170
Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	418	418	548	2948	0	3434	0	1005	13092
normalized size	1	1.00	1.31	7.05	0.00	8.22	0.00	2.40	31.32
time (sec)	N/A	5.273	3.247	0.651	0.000	137.705	0.000	0.470	19.961
Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	369	2264	0	2278	0	844	9713
normalized size	1	1.00	1.19	7.30	0.00	7.35	0.00	2.72	31.33
time (sec)	N/A	1.366	1.992	0.738	0.000	48.762	0.000	0.456	14.145
Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	226	375	0	1230	0	693	439
normalized size	1	1.00	0.82	1.37	0.00	4.49	0.00	2.53	1.60
time (sec)	N/A	0.700	2.829	0.807	0.000	0.604	0.000	0.421	6.799

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	252	388	0	1242	0	726	451
normalized size	1	1.00	0.96	1.48	0.00	4.72	0.00	2.76	1.71
time (sec)	N/A	0.615	1.267	0.792	0.000	0.584	0.000	1.987	6.646

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	404	376	0	1238	0	693	439
normalized size	1	1.00	1.70	1.59	0.00	5.22	0.00	2.92	1.85
time (sec)	N/A	0.510	1.106	0.728	0.000	0.588	0.000	0.404	6.673

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	292	769	2242	0	1867	0	814	9721
normalized size	1	1.00	2.63	7.68	0.00	6.39	0.00	2.79	33.29
time (sec)	N/A	1.068	3.506	0.890	0.000	0.655	0.000	0.414	14.414

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	411	411	1205	2891	0	2560	0	966	7863
normalized size	1	1.00	2.93	7.03	0.00	6.23	0.00	2.35	19.13
time (sec)	N/A	5.595	6.357	1.413	0.000	0.814	0.000	1.964	14.203

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	538	538	1452	3099	0	2890	0	1052	14438
normalized size	1	1.00	2.70	5.76	0.00	5.37	0.00	1.96	26.84
time (sec)	N/A	6.844	6.120	1.152	0.000	0.939	0.000	0.413	15.817

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	61	116	0	197	0	187	91
normalized size	1	1.00	1.00	1.90	0.00	3.23	0.00	3.07	1.49
time (sec)	N/A	0.115	0.166	1.029	0.000	0.488	0.000	0.708	2.559

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	0	6	3	13	6
normalized size	1	1.00	1.00	1.17	0.00	1.00	0.50	2.17	1.00
time (sec)	N/A	0.001	0.001	0.043	0.000	0.400	5.347	0.241	2.231

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	97	163	0	279	0	139	444
normalized size	1	1.00	1.13	1.90	0.00	3.24	0.00	1.62	5.16
time (sec)	N/A	0.180	0.376	0.800	0.000	0.478	0.000	0.773	2.574

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	39	39	80	84	0	58	26
normalized size	1	1.00	0.45	0.45	0.92	0.97	0.00	0.67	0.30
time (sec)	N/A	0.074	0.096	0.836	1.109	0.455	0.000	0.318	2.298

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-1)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	485	485	3734	4394	0	0	0	0	-1
normalized size	1	1.00	7.70	9.06	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.437	26.140	3.220	0.000	0.482	0.000	0.000	0.000

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-1)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	397	397	3330	3438	0	0	0	0	-1
normalized size	1	1.00	8.39	8.66	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.933	25.016	2.740	0.000	0.466	0.000	0.000	0.000

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	314	314	2905	2498	0	0	0	0	-1
normalized size	1	1.00	9.25	7.96	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.598	22.018	2.293	0.000	0.477	0.000	0.000	0.000

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	256	256	408	1752	0	0	0	0	-1
normalized size	1	1.00	1.59	6.84	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.340	14.652	2.007	0.000	0.473	0.000	0.000	0.000

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	320	913	1372	0	0	0	0	-1
normalized size	1	1.00	2.85	4.29	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.290	17.961	2.048	0.000	0.828	0.000	0.000	0.000

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	344	344	1107	1389	0	0	0	0	-1
normalized size	1	1.00	3.22	4.04	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.369	18.307	1.983	0.000	49.588	0.000	0.000	0.000

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	429	429	1149	2065	0	0	0	0	-1
normalized size	1	1.00	2.68	4.81	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.733	18.937	1.962	0.000	1.590	0.000	0.000	0.000

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	509	509	1548	2954	0	0	0	0	-1
normalized size	1	1.00	3.04	5.80	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.129	20.040	2.493	0.000	59.425	0.000	0.000	0.000

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-1)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	475	475	3766	4395	0	0	0	0	-1
normalized size	1	1.00	7.93	9.25	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.225	26.439	4.003	0.000	0.473	0.000	0.000	0.000

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-1)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	388	388	3342	3424	0	0	0	0	-1
normalized size	1	1.00	8.61	8.82	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.829	25.264	2.918	0.000	0.475	0.000	0.000	0.000

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	312	312	502	2683	0	0	0	0	-1
normalized size	1	1.00	1.61	8.60	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.570	19.259	2.184	0.000	0.448	0.000	0.000	0.000

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	381	381	6063	2337	0	0	0	0	-1
normalized size	1	1.00	15.91	6.13	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.465	24.621	2.053	0.000	23.525	0.000	0.000	0.000

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	361	361	971	2196	0	0	0	0	-1
normalized size	1	1.00	2.69	6.08	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.451	16.716	2.042	0.000	48.202	0.000	0.000	0.000

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	428	428	1598	2439	0	0	0	0	-1
normalized size	1	1.00	3.73	5.70	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.792	19.571	1.910	0.000	1.598	0.000	0.000	0.000

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	520	520	1535	3142	0	0	0	0	-1
normalized size	1	1.00	2.95	6.04	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.275	19.460	2.146	0.000	2.105	0.000	0.000	0.000

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	566	566	4227	5368	0	0	0	0	-1
normalized size	1	1.00	7.47	9.48	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.784	27.342	4.067	0.000	0.504	0.000	0.000	0.000

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	469	469	3781	4395	0	0	0	0	-1
normalized size	1	1.00	8.06	9.37	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.183	26.630	3.132	0.000	0.479	0.000	0.000	0.000

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-1)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	384	384	2957	3637	0	0	0	0	-1
normalized size	1	1.00	7.70	9.47	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.807	23.167	2.689	0.000	0.460	0.000	0.000	0.000

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	442	442	7138	3285	0	0	0	0	-1
normalized size	1	1.00	16.15	7.43	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.656	25.559	2.421	0.000	22.713	0.000	0.000	0.000

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	433	433	7745	3215	0	0	0	0	-1
normalized size	1	1.00	17.89	7.42	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.703	26.077	2.285	0.000	51.700	0.000	0.000	0.000

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	450	450	1326	3271	0	0	0	0	-1
normalized size	1	1.00	2.95	7.27	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.835	19.789	2.420	0.000	56.181	0.000	0.000	0.000

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	518	518	1551	3511	0	0	0	0	-1
normalized size	1	1.00	2.99	6.78	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.365	19.535	2.237	0.000	63.484	0.000	0.000	0.000

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	617	617	5172	4231	0	0	0	0	-1
normalized size	1	1.00	8.38	6.86	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.832	24.271	2.285	0.000	65.086	0.000	0.000	0.000

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-1)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	329	329	3000	2499	0	0	0	0	-1
normalized size	1	1.00	9.12	7.60	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.618	22.582	2.569	0.000	0.460	0.000	0.000	0.000

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	372	1563	0	0	0	0	-1
normalized size	1	1.00	1.43	5.99	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.397	15.835	2.301	0.000	0.438	0.000	0.000	0.000

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	356	829	0	0	0	0	-1
normalized size	1	1.00	1.70	3.95	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.206	14.402	2.137	0.000	0.473	0.000	0.000	0.000

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	145	215	0	0	0	0	-1
normalized size	1	1.00	0.70	1.03	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.124	2.336	2.010	0.000	0.831	0.000	0.000	0.000

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	348	348	1027	1025	0	0	0	0	-1
normalized size	1	1.00	2.95	2.95	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.405	17.044	2.170	0.000	49.793	0.000	0.000	0.000

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	435	435	1639	1886	0	0	0	0	-1
normalized size	1	1.00	3.77	4.34	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.724	16.235	2.164	0.000	0.000	0.000	0.000	0.000

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	525	525	1569	2954	0	0	0	0	-1
normalized size	1	1.00	2.99	5.63	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.168	19.789	2.311	0.000	4.124	0.000	0.000	0.000

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-1)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	329	329	3460	3333	0	0	0	0	-1
normalized size	1	1.00	10.52	10.13	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.720	24.644	2.549	0.000	1.081	0.000	0.000	0.000

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	467	2276	0	0	0	0	-1
normalized size	1	1.00	1.70	8.28	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.464	18.076	2.309	0.000	1.248	0.000	0.000	0.000

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	468	1633	0	0	0	0	-1
normalized size	1	1.00	1.84	6.43	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.348	15.246	2.108	0.000	0.998	0.000	0.000	0.000

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	376	376	1491	2010	0	0	0	0	-1
normalized size	1	1.00	3.97	5.35	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.433	14.581	2.098	0.000	0.000	0.000	0.000	0.000

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	427	427	1597	2871	0	0	0	0	-1
normalized size	1	1.00	3.74	6.72	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.701	19.609	2.116	0.000	2.060	0.000	0.000	0.000

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	531	531	2667	3980	0	0	0	0	-1
normalized size	1	1.00	5.02	7.50	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.139	18.299	2.872	0.000	2.646	0.000	0.000	0.000

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	630	630	2319	5086	0	0	0	0	-1
normalized size	1	1.00	3.68	8.07	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.669	22.292	2.925	0.000	62.086	0.000	0.000	0.000

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-1)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	510	510	4342	8044	0	0	0	0	-1
normalized size	1	1.00	8.51	15.77	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.588	27.420	3.432	0.000	0.600	0.000	0.000	0.000

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-1)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	417	417	3920	6455	0	0	0	0	-1
normalized size	1	1.00	9.40	15.48	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.016	26.771	2.676	0.000	0.558	0.000	0.000	0.000

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-1)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	387	387	3514	5170	0	0	0	0	-1
normalized size	1	1.00	9.08	13.36	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.694	24.459	2.139	0.000	0.533	0.000	0.000	0.000

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	353	353	603	4213	0	0	0	0	-1
normalized size	1	1.00	1.71	11.93	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.598	18.873	1.977	0.000	0.541	0.000	0.000	0.000

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	495	495	2083	5712	0	0	0	0	-1
normalized size	1	1.00	4.21	11.54	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.767	16.806	2.034	0.000	22.365	0.000	0.000	0.000

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	582	582	2366	8545	0	0	0	0	-1
normalized size	1	1.00	4.07	14.68	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.214	21.987	2.010	0.000	1.427	0.000	0.000	0.000

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	686	686	821	10322	0	0	0	0	-1
normalized size	1	1.00	1.20	15.05	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.050	15.267	2.398	0.000	57.036	0.000	0.000	0.000

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	248	642	0	0	0	0	-1
normalized size	1	1.00	2.36	6.11	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.081	10.692	2.011	0.000	0.461	0.000	0.000	0.000

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	211	457	0	0	0	0	-1
normalized size	1	1.00	1.97	4.27	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.085	7.940	1.923	0.000	0.452	0.000	0.000	0.000

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	132	663	0	0	0	0	-1
normalized size	1	1.00	0.73	3.68	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.183	1.857	12.436	0.000	0.462	0.000	0.000	0.000

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	104	428	0	0	0	0	-1
normalized size	1	1.00	0.73	2.99	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.147	0.886	9.666	0.000	0.438	0.000	0.000	0.000

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	84	244	0	0	0	0	-1
normalized size	1	1.00	0.76	2.20	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.139	0.294	4.593	0.000	0.445	0.000	0.000	0.000

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	90	326	0	0	0	0	-1
normalized size	1	1.00	0.78	2.83	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.147	0.262	4.167	0.000	0.470	0.000	0.000	0.000

Problem 398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	108	371	0	0	0	0	-1
normalized size	1	1.00	0.73	2.51	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.163	0.694	4.342	0.000	0.456	0.000	0.000	0.000

Problem 399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	125	413	0	0	0	0	-1
normalized size	1	1.00	0.69	2.29	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.181	1.081	4.535	0.000	0.468	0.000	0.000	0.000

Problem 400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	221	859	0	0	0	0	-1
normalized size	1	1.00	0.84	3.27	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.373	4.486	16.161	0.000	0.449	0.000	0.000	0.000

Problem 401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	171	750	0	0	0	0	-1
normalized size	1	1.00	0.77	3.39	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.315	2.721	13.532	0.000	0.452	0.000	0.000	0.000

Problem 402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	125	677	0	0	0	0	-1
normalized size	1	1.00	0.71	3.82	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.270	1.281	10.251	0.000	0.452	0.000	0.000	0.000

Problem 403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	124	404	0	0	0	0	-1
normalized size	1	1.00	0.77	2.51	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.248	0.758	4.633	0.000	0.464	0.000	0.000	0.000

Problem 404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	128	487	0	0	0	0	-1
normalized size	1	1.00	0.75	2.85	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.260	0.960	4.781	0.000	0.476	0.000	0.000	0.000

Problem 405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	161	548	0	0	0	0	-1
normalized size	1	1.00	0.76	2.57	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.289	1.584	4.632	0.000	0.482	0.000	0.000	0.000

Problem 406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	189	610	0	0	0	0	-1
normalized size	1	1.00	0.74	2.40	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.337	1.870	4.659	0.000	0.503	0.000	0.000	0.000

Problem 407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	345	345	452	1193	0	0	0	0	-1
normalized size	1	1.00	1.31	3.46	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.572	6.565	20.417	0.000	0.498	0.000	0.000	0.000

Problem 408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	225	944	0	0	0	0	-1
normalized size	1	1.00	0.76	3.20	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.504	3.876	16.359	0.000	0.458	0.000	0.000	0.000

Problem 409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	190	997	0	0	0	0	-1
normalized size	1	1.00	0.78	4.09	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.483	2.548	13.163	0.000	0.471	0.000	0.000	0.000

Problem 410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	166	1212	0	0	0	0	-1
normalized size	1	1.00	0.69	5.07	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.510	1.953	13.405	0.000	0.479	0.000	0.000	0.000

Problem 411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	172	867	0	0	0	0	-1
normalized size	1	1.00	0.73	3.67	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.461	1.689	5.214	0.000	0.475	0.000	0.000	0.000

Problem 412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	180	664	0	0	0	0	-1
normalized size	1	1.00	0.73	2.71	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.467	1.357	4.501	0.000	0.475	0.000	0.000	0.000

Problem 413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	219	745	0	0	0	0	-1
normalized size	1	1.00	0.74	2.53	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.538	2.035	4.755	0.000	0.467	0.000	0.000	0.000

Problem 414	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	345	345	256	825	0	0	0	0	-1
normalized size	1	1.00	0.74	2.39	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.574	3.232	4.811	0.000	0.509	0.000	0.000	0.000

Problem 415	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	664	785	0	0	0	0	-1
normalized size	1	1.00	2.40	2.83	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.014	6.938	15.566	0.000	0.000	0.000	0.000	0.000

Problem 416	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	225	466	0	0	0	0	-1
normalized size	1	1.00	1.07	2.22	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.713	3.660	11.188	0.000	0.000	0.000	0.000	0.000

Problem 417	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	123	325	0	0	0	0	-1
normalized size	1	1.00	0.98	2.58	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.401	1.368	8.388	0.000	0.000	0.000	0.000	0.000

Problem 418	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	76	217	0	0	0	0	-1
normalized size	1	1.00	0.75	2.15	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.198	0.582	4.437	0.000	0.000	0.000	0.000	0.000

Problem 419	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	220	295	0	0	0	0	-1
normalized size	1	1.00	1.48	1.98	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.257	7.023	4.851	0.000	144.225	0.000	0.000	0.000

Problem 420	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	540	786	0	0	0	0	-1
normalized size	1	1.00	2.76	4.01	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.467	6.740	4.870	0.000	0.000	0.000	0.000	0.000

Problem 421	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	612	1074	0	0	0	0	-1
normalized size	1	1.00	2.53	4.44	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.755	6.913	5.680	0.000	0.000	0.000	0.000	0.000

Problem 422	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	406	406	733	1024	0	0	0	0	-1
normalized size	1	1.00	1.81	2.52	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.162	7.318	19.500	0.000	0.000	0.000	0.000	0.000

Problem 423	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	315	315	680	877	0	0	0	0	-1
normalized size	1	1.00	2.16	2.78	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.837	6.998	12.400	0.000	0.000	0.000	0.000	0.000

Problem 424	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-1)	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	638	715	0	0	0	0	-1
normalized size	1	1.00	2.48	2.78	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.527	7.003	10.083	0.000	0.000	0.000	0.000	0.000

Problem 425	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-1)	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	722	802	0	0	0	0	-1
normalized size	1	1.00	2.75	3.05	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.506	6.955	10.880	0.000	0.000	0.000	0.000	0.000

Problem 426	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	652	843	0	0	0	0	-1
normalized size	1	1.00	2.30	2.98	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.569	6.926	13.447	0.000	0.000	0.000	0.000	0.000

Problem 427	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	365	365	699	1059	0	0	0	0	-1
normalized size	1	1.00	1.92	2.90	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.859	7.021	15.594	0.000	120.487	0.000	0.000	0.000

Problem 428	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	583	583	897	2178	0	0	0	0	-1
normalized size	1	1.00	1.54	3.74	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.779	7.502	30.140	0.000	0.000	0.000	0.000	0.000

Problem 429	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	480	480	842	2024	0	0	0	0	-1
normalized size	1	1.00	1.75	4.22	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.378	7.250	19.725	0.000	0.000	0.000	0.000	0.000

Problem 430	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	402	402	795	1768	0	0	0	0	-1
normalized size	1	1.00	1.98	4.40	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.914	6.964	16.344	0.000	0.000	0.000	0.000	0.000

Problem 431	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	402	402	882	1872	0	0	0	0	-1
normalized size	1	1.00	2.19	4.66	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.914	6.986	16.602	0.000	0.000	0.000	0.000	0.000

Problem 432	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-1)	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	402	402	885	1959	0	0	0	0	-1
normalized size	1	1.00	2.20	4.87	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.865	6.983	18.139	0.000	0.000	0.000	0.000	0.000

Problem 433	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	427	427	818	2000	0	0	0	0	-1
normalized size	1	1.00	1.92	4.68	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.998	7.226	20.238	0.000	0.000	0.000	0.000	0.000

Problem 434	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	521	521	863	2216	0	0	0	0	-1
normalized size	1	1.00	1.66	4.25	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.444	7.527	23.622	0.000	0.000	0.000	0.000	0.000

Problem 435	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	336	336	422	2521	0	0	0	0	-1
normalized size	1	1.00	1.26	7.50	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.108	5.521	2.245	0.000	0.000	0.000	0.000	0.000

Problem 436	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	377	1431	0	0	0	0	-1
normalized size	1	1.00	1.49	5.66	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.783	6.366	2.292	0.000	0.000	0.000	0.000	0.000

Problem 437	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	122	1549	0	0	0	0	-1
normalized size	1	1.00	0.59	7.45	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.543	2.635	2.984	0.000	3.474	0.000	0.000	0.000

Problem 438	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	165	1926	0	0	0	0	-1
normalized size	1	1.00	0.82	9.58	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.479	0.815	2.368	0.000	0.577	0.000	0.000	0.000

Problem 439	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	200	2737	0	0	0	0	-1
normalized size	1	1.00	0.75	10.25	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.748	1.206	2.242	0.000	0.501	0.000	0.000	0.000

Problem 440	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	343	343	208	3778	0	0	0	0	-1
normalized size	1	1.00	0.61	11.01	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.035	1.364	2.576	0.000	0.519	0.000	0.000	0.000

Problem 441	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	421	421	673	4051	0	0	0	0	-1
normalized size	1	1.00	1.60	9.62	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.597	6.880	2.289	0.000	0.000	0.000	0.000	0.000

Problem 442	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	339	339	595	2947	0	0	0	0	-1
normalized size	1	1.00	1.76	8.69	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.207	6.883	1.900	0.000	0.000	0.000	0.000	0.000

Problem 443	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	272	272	554	2595	0	0	0	0	-1
normalized size	1	1.00	2.04	9.54	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.868	6.706	2.311	0.000	0.000	0.000	0.000	0.000

Problem 444	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	276	276	437	2552	0	0	0	0	-1
normalized size	1	1.00	1.58	9.25	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.918	4.532	2.218	0.000	0.000	0.000	0.000	0.000

Problem 445	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	201	2915	0	0	0	0	-1
normalized size	1	1.00	0.76	10.96	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.792	1.838	2.194	0.000	1.191	0.000	0.000	0.000

Problem 446	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	342	342	255	3752	0	0	0	0	-1
normalized size	1	1.00	0.75	10.97	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.129	2.015	2.447	0.000	0.854	0.000	0.000	0.000

Problem 447	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	427	427	313	4846	0	0	0	0	-1
normalized size	1	1.00	0.73	11.35	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.495	2.310	3.002	0.000	0.828	0.000	0.000	0.000

Problem 448	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	513	513	768	5392	0	0	0	0	-1
normalized size	1	1.00	1.50	10.51	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.998	6.962	2.626	0.000	0.000	0.000	0.000	0.000

Problem 449	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	422	422	678	4258	0	0	0	0	-1
normalized size	1	1.00	1.61	10.09	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.594	7.220	2.156	0.000	0.000	0.000	0.000	0.000

Problem 450	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	359	359	628	3939	0	0	0	0	-1
normalized size	1	1.00	1.75	10.97	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.249	6.873	2.266	0.000	0.000	0.000	0.000	0.000

Problem 451	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	349	349	599	3663	0	0	0	0	-1
normalized size	1	1.00	1.72	10.50	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.250	7.024	2.247	0.000	0.000	0.000	0.000	0.000

Problem 452	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	342	342	616	3564	0	0	0	0	-1
normalized size	1	1.00	1.80	10.42	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.223	7.002	2.380	0.000	0.000	0.000	0.000	0.000

Problem 453	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	340	340	257	3980	0	0	0	0	-1
normalized size	1	1.00	0.76	11.71	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.155	1.833	2.673	0.000	1.060	0.000	0.000	0.000

Problem 454	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	425	425	313	4847	0	0	0	0	-1
normalized size	1	1.00	0.74	11.40	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.518	2.705	2.892	0.000	0.563	0.000	0.000	0.000

Problem 455	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	519	519	380	5946	0	0	0	0	-1
normalized size	1	1.00	0.73	11.46	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.960	3.804	3.030	0.000	0.570	0.000	0.000	0.000

Problem 456	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	344	344	451	2738	0	0	0	0	-1
normalized size	1	1.00	1.31	7.96	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.111	4.153	2.330	0.000	0.000	0.000	0.000	0.000

Problem 457	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	256	256	339	1440	0	0	0	0	-1
normalized size	1	1.00	1.32	5.62	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.731	7.185	2.568	0.000	0.000	0.000	0.000	0.000

Problem 458	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	91	283	0	0	0	0	-1
normalized size	1	1.00	0.66	2.05	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.392	0.272	2.095	0.000	0.000	0.000	0.000	0.000

Problem 459	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	103	940	0	0	0	0	-1
normalized size	1	1.00	0.69	6.27	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.310	3.891	2.506	0.000	1.439	0.000	0.000	0.000

Problem 460	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	161	1731	0	0	0	0	-1
normalized size	1	1.00	0.76	8.17	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.480	0.883	2.331	0.000	0.632	0.000	0.000	0.000

Problem 461	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	280	198	2738	0	0	0	0	-1
normalized size	1	1.00	0.71	9.78	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.750	1.388	2.478	0.000	0.522	0.000	0.000	0.000

Problem 462	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	371	371	518	2656	0	0	0	0	-1
normalized size	1	1.00	1.40	7.16	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.269	5.971	2.556	0.000	0.000	0.000	0.000	0.000

Problem 463	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	464	1585	0	0	0	0	-1
normalized size	1	1.00	2.11	7.20	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.625	4.892	3.116	0.000	0.000	0.000	0.000	0.000

Problem 464	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	161	941	0	0	0	0	-1
normalized size	1	1.00	0.75	4.38	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.572	0.818	2.447	0.000	1.716	0.000	0.000	0.000

Problem 465	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	178	1448	0	0	0	0	-1
normalized size	1	1.00	0.76	6.16	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.579	1.065	2.360	0.000	0.715	0.000	0.000	0.000

Problem 466	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	326	326	252	2285	0	0	0	0	-1
normalized size	1	1.00	0.77	7.01	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.836	1.608	2.311	0.000	0.576	0.000	0.000	0.000

Problem 467	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	423	423	316	3156	0	0	0	0	-1
normalized size	1	1.00	0.75	7.46	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.222	2.403	2.499	0.000	0.952	0.000	0.000	0.000

Problem 468	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	399	399	726	5195	0	0	0	0	-1
normalized size	1	1.00	1.82	13.02	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.374	6.861	2.365	0.000	0.000	0.000	0.000	0.000

Problem 469	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	329	329	217	3138	0	0	0	0	-1
normalized size	1	1.00	0.66	9.54	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.842	2.305	2.405	0.000	1.041	0.000	0.000	0.000

Problem 470	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	346	346	245	3857	0	0	0	0	-1
normalized size	1	1.00	0.71	11.15	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.822	2.215	2.250	0.000	0.770	0.000	0.000	0.000

Problem 471	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	368	368	297	5169	0	0	0	0	-1
normalized size	1	1.00	0.81	14.05	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.937	2.691	2.444	0.000	0.667	0.000	0.000	0.000

Problem 472	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	472	472	353	6746	0	0	0	0	-1
normalized size	1	1.00	0.75	14.29	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.409	3.113	2.630	0.000	0.940	0.000	0.000	0.000

Problem 473	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	588	588	392	8251	0	0	0	0	-1
normalized size	1	1.00	0.67	14.03	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.879	4.298	2.940	0.000	0.992	0.000	0.000	0.000

Problem 474	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	126	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.158	22.712	1.079	0.000	0.000	0.000	0.000	0.000

Problem 475	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	126	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.146	19.323	1.084	0.000	0.000	0.000	0.000	0.000

Problem 476	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	126	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.176	3.533	1.116	0.000	0.000	0.000	0.000	0.000

Problem 477	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	126	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.162	3.460	1.209	0.000	0.000	0.000	0.000	0.000

Problem 478	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	36	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.093	4.778	3.327	0.000	0.916	0.000	0.000	0.000

Problem 479	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-1)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	544	544	365	0	0	0	0	0	-1
normalized size	1	1.00	0.67	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.630	4.925	2.477	0.000	0.993	0.000	0.000	0.000

Problem 480	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	366	366	307	0	0	0	0	0	-1
normalized size	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.786	2.558	1.993	0.000	0.800	0.000	0.000	0.000

Problem 481	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	239	0	0	0	0	0	-1
normalized size	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.406	1.076	3.596	0.000	0.658	0.000	0.000	0.000

Problem 482	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	168	0	0	0	0	0	-1
normalized size	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.201	0.462	2.180	0.000	0.967	0.000	0.000	0.000

Problem 483	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	872	383	0	0	0	0	166
normalized size	1	1.00	6.61	2.90	0.00	0.00	0.00	0.00	1.26
time (sec)	N/A	0.229	6.295	4.663	0.000	1.073	0.000	0.000	0.891

Problem 484	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	830	355	0	0	0	0	128
normalized size	1	1.00	8.22	3.51	0.00	0.00	0.00	0.00	1.27
time (sec)	N/A	0.210	6.221	4.074	0.000	0.681	0.000	0.000	0.358

Problem 485	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	309	321	0	0	0	0	85
normalized size	1	1.00	4.41	4.59	0.00	0.00	0.00	0.00	1.21
time (sec)	N/A	0.186	6.240	4.404	0.000	0.470	0.000	0.000	2.661

Problem 486	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	252	240	0	0	0	0	96
normalized size	1	1.00	3.82	3.64	0.00	0.00	0.00	0.00	1.45
time (sec)	N/A	0.194	6.055	4.920	0.000	0.459	0.000	0.000	3.059

Problem 487	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	813	426	0	0	0	0	150
normalized size	1	1.00	8.56	4.48	0.00	0.00	0.00	0.00	1.58
time (sec)	N/A	0.217	6.373	11.025	0.000	0.478	0.000	0.000	3.293

Problem 488	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	865	661	0	0	0	0	177
normalized size	1	1.00	6.55	5.01	0.00	0.00	0.00	0.00	1.34
time (sec)	N/A	0.232	6.427	12.333	0.000	0.444	0.000	0.000	3.553

Problem 489	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	F	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	1086	413	0	0	0	0	266
normalized size	1	1.00	5.60	2.13	0.00	0.00	0.00	0.00	1.37
time (sec)	N/A	0.404	6.307	4.868	0.000	0.492	0.000	0.000	3.189

Problem 490	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	1040	385	0	0	0	0	231
normalized size	1	1.00	6.46	2.39	0.00	0.00	0.00	0.00	1.43
time (sec)	N/A	0.362	6.267	5.046	0.000	0.463	0.000	0.000	3.012

Problem 491	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	994	357	0	0	0	0	153
normalized size	1	1.00	7.89	2.83	0.00	0.00	0.00	0.00	1.21
time (sec)	N/A	0.347	6.310	4.205	0.000	0.462	0.000	0.000	2.915

Problem 492	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	735	388	0	0	0	0	134
normalized size	1	1.00	6.34	3.34	0.00	0.00	0.00	0.00	1.16
time (sec)	N/A	0.337	6.392	4.592	0.000	0.464	0.000	0.000	3.162

Problem 493	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	736	513	0	0	0	0	196
normalized size	1	1.00	6.13	4.28	0.00	0.00	0.00	0.00	1.63
time (sec)	N/A	0.350	6.480	4.806	0.000	0.470	0.000	0.000	3.938

Problem 494	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	1025	741	0	0	0	0	229
normalized size	1	1.00	6.45	4.66	0.00	0.00	0.00	0.00	1.44
time (sec)	N/A	0.381	6.585	12.757	0.000	0.470	0.000	0.000	4.153

Problem 495	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	1067	851	0	0	0	0	235
normalized size	1	1.00	5.50	4.39	0.00	0.00	0.00	0.00	1.21
time (sec)	N/A	0.415	6.641	15.458	0.000	0.450	0.000	0.000	4.692

Problem 496	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	1292	282	0	0	0	0	-1
normalized size	1	1.00	8.23	1.80	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.265	6.693	5.240	0.000	0.455	0.000	0.000	0.000

Problem 497	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	1239	262	0	0	0	0	-1
normalized size	1	1.00	9.99	2.11	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.244	6.597	4.322	0.000	0.442	0.000	0.000	0.000

Problem 498	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	1208	244	0	0	0	0	-1
normalized size	1	1.00	13.73	2.77	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.220	6.473	4.794	0.000	0.458	0.000	0.000	0.000

Problem 499	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	1204	243	0	0	0	0	-1
normalized size	1	1.00	14.51	2.93	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.222	6.509	4.669	0.000	0.440	0.000	0.000	0.000

Problem 500	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	1240	318	0	0	0	0	-1
normalized size	1	1.00	10.97	2.81	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.240	6.705	8.911	0.000	0.473	0.000	0.000	0.000

Problem 501	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	1277	493	0	0	0	0	-1
normalized size	1	1.00	8.40	3.24	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.260	7.173	11.491	0.000	0.438	0.000	0.000	0.000

Problem 502	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	1396	465	0	0	0	0	-1
normalized size	1	1.00	6.84	2.28	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.419	7.078	5.006	0.000	0.449	0.000	0.000	0.000

Problem 503	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	1352	435	0	0	0	0	-1
normalized size	1	1.00	7.91	2.54	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.401	6.806	5.539	0.000	0.467	0.000	0.000	0.000

Problem 504	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	1318	421	0	0	0	0	-1
normalized size	1	1.00	9.62	3.07	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.375	6.700	5.153	0.000	0.456	0.000	0.000	0.000

Problem 505	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	921	350	0	0	0	0	-1
normalized size	1	1.00	7.61	2.89	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.346	6.570	5.228	0.000	0.461	0.000	0.000	0.000

Problem 506	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	921	350	0	0	0	0	-1
normalized size	1	1.00	7.61	2.89	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.355	6.586	4.748	0.000	0.453	0.000	0.000	0.000

Problem 507	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	1351	492	0	0	0	0	-1
normalized size	1	1.00	8.24	3.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.397	6.900	5.756	0.000	0.469	0.000	0.000	0.000

Problem 508	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	1392	750	0	0	0	0	-1
normalized size	1	1.00	7.07	3.81	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.428	7.500	15.142	0.000	0.490	0.000	0.000	0.000

Problem 509	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	1448	465	0	0	0	0	-1
normalized size	1	1.00	6.55	2.10	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.582	7.079	5.704	0.000	0.480	0.000	0.000	0.000

Problem 510	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	1415	451	0	0	0	0	-1
normalized size	1	1.00	7.53	2.40	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.548	6.932	5.371	0.000	0.460	0.000	0.000	0.000

Problem 511	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	1407	451	0	0	0	0	-1
normalized size	1	1.00	7.73	2.48	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.539	6.957	5.613	0.000	0.470	0.000	0.000	0.000

Problem 512	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	1406	451	0	0	0	0	-1
normalized size	1	1.00	7.90	2.53	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.522	6.862	5.324	0.000	0.448	0.000	0.000	0.000

Problem 513	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	1407	451	0	0	0	0	-1
normalized size	1	1.00	7.82	2.51	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.533	6.839	5.569	0.000	0.492	0.000	0.000	0.000

Problem 514	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	1447	685	0	0	0	0	-1
normalized size	1	1.00	6.55	3.10	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.577	7.194	6.056	0.000	0.460	0.000	0.000	0.000

Problem 515	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	119	130	547	116	0	0	-1
normalized size	1	1.00	0.54	0.59	2.49	0.53	0.00	0.00	-0.00
time (sec)	N/A	0.476	0.545	1.955	0.755	0.433	0.000	0.000	0.000

Problem 516	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	96	108	418	99	0	0	-1
normalized size	1	1.00	0.55	0.62	2.39	0.57	0.00	0.00	-0.01
time (sec)	N/A	0.403	0.367	1.798	0.791	0.441	0.000	0.000	0.000

Problem 517	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	79	86	296	81	0	0	-1
normalized size	1	1.00	0.61	0.66	2.28	0.62	0.00	0.00	-0.01
time (sec)	N/A	0.332	0.160	1.786	0.838	0.429	0.000	0.000	0.000

Problem 518	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	56	65	141	64	0	0	-1
normalized size	1	1.00	0.68	0.79	1.72	0.78	0.00	0.00	-0.01
time (sec)	N/A	0.262	0.197	1.739	0.663	0.439	0.000	0.000	0.000

Problem 519	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	94	169	262	298	0	0	-1
normalized size	1	1.00	0.98	1.76	2.73	3.10	0.00	0.00	-0.01
time (sec)	N/A	0.259	0.323	1.664	0.640	0.484	0.000	0.000	0.000

Problem 520	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	89	275	905	351	0	0	-1
normalized size	1	1.00	0.91	2.81	9.23	3.58	0.00	0.00	-0.01
time (sec)	N/A	0.257	0.399	2.408	0.864	0.537	0.000	0.000	0.000

Problem 521	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	106	342	1927	401	0	0	-1
normalized size	1	1.00	0.70	2.26	12.76	2.66	0.00	0.00	-0.01
time (sec)	N/A	0.331	0.655	2.238	0.829	0.517	0.000	0.000	0.000

Problem 522	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	131	404	3342	439	0	0	-1
normalized size	1	1.00	0.67	2.06	17.05	2.24	0.00	0.00	-0.01
time (sec)	N/A	0.396	1.224	2.294	1.232	0.550	0.000	0.000	0.000

Problem 523	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	131	153	703	142	0	0	-1
normalized size	1	1.00	0.48	0.56	2.56	0.52	0.00	0.00	-0.00
time (sec)	N/A	0.714	0.540	2.130	0.763	0.425	0.000	0.000	0.000

Problem 524	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	118	131	558	124	0	0	-1
normalized size	1	1.00	0.52	0.57	2.45	0.54	0.00	0.00	-0.00
time (sec)	N/A	0.686	0.423	2.574	0.698	0.433	0.000	0.000	0.000

Problem 525	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	100	109	451	105	0	0	-1
normalized size	1	1.00	0.55	0.60	2.49	0.58	0.00	0.00	-0.01
time (sec)	N/A	0.617	0.324	1.952	0.722	0.473	0.000	0.000	0.000

Problem 526	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	80	87	276	86	0	0	-1
normalized size	1	1.00	0.61	0.66	2.11	0.66	0.00	0.00	-0.01
time (sec)	N/A	0.383	0.288	1.880	0.682	0.415	0.000	0.000	0.000

Problem 527	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	101	201	583	343	0	0	-1
normalized size	1	1.00	0.70	1.39	4.02	2.37	0.00	0.00	-0.01
time (sec)	N/A	0.444	0.440	1.797	0.864	0.491	0.000	0.000	0.000

Problem 528	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	133	306	1417	389	0	0	-1
normalized size	1	1.00	0.92	2.12	9.84	2.70	0.00	0.00	-0.01
time (sec)	N/A	0.433	0.855	1.875	0.802	0.513	0.000	0.000	0.000

Problem 529	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	107	343	3389	409	0	0	-1
normalized size	1	1.00	0.70	2.24	22.15	2.67	0.00	0.00	-0.01
time (sec)	N/A	0.454	0.845	2.072	0.896	0.525	0.000	0.000	0.000

Problem 530	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	134	405	4606	449	0	0	-1
normalized size	1	1.00	0.67	2.02	23.03	2.24	0.00	0.00	-0.00
time (sec)	N/A	0.540	1.278	1.818	1.020	0.578	0.000	0.000	0.000

Problem 531	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	153	467	5879	485	0	0	-1
normalized size	1	1.00	0.62	1.89	23.80	1.96	0.00	0.00	-0.00
time (sec)	N/A	0.634	1.964	1.818	1.462	0.630	0.000	0.000	0.000

Problem 532	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	137	155	754	154	0	0	-1
normalized size	1	1.00	0.50	0.56	2.74	0.56	0.00	0.00	-0.00
time (sec)	N/A	0.831	0.598	2.081	0.748	0.451	0.000	0.000	0.000

Problem 533	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	116	133	596	133	0	0	-1
normalized size	1	1.00	0.51	0.58	2.61	0.58	0.00	0.00	-0.00
time (sec)	N/A	0.758	0.550	1.914	0.921	0.456	0.000	0.000	0.000

Problem 534	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	99	111	482	112	0	0	-1
normalized size	1	1.00	0.56	0.62	2.71	0.63	0.00	0.00	-0.01
time (sec)	N/A	0.458	0.414	1.954	0.694	0.447	0.000	0.000	0.000

Problem 535	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	118	225	352	399	0	0	-1
normalized size	1	1.00	0.61	1.17	1.83	2.08	0.00	0.00	-0.01
time (sec)	N/A	0.619	0.678	1.942	0.683	0.489	0.000	0.000	0.000

Problem 536	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	117	368	2589	449	0	0	-1
normalized size	1	1.00	0.59	1.87	13.14	2.28	0.00	0.00	-0.01
time (sec)	N/A	0.626	0.682	2.054	0.840	0.557	0.000	0.000	0.000

Problem 537	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	173	376	0	453	0	0	-1
normalized size	1	1.00	0.86	1.88	0.00	2.26	0.00	0.00	-0.00
time (sec)	N/A	0.627	0.951	2.250	0.000	0.549	0.000	0.000	0.000

Problem 538	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	133	407	6297	469	0	0	-1
normalized size	1	1.00	0.66	2.04	31.48	2.34	0.00	0.00	-0.00
time (sec)	N/A	0.651	1.344	2.509	4.677	0.566	0.000	0.000	0.000

Problem 539	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	154	469	7331	509	0	0	-1
normalized size	1	1.00	0.62	1.90	29.68	2.06	0.00	0.00	-0.00
time (sec)	N/A	0.766	1.983	2.169	1.411	0.608	0.000	0.000	0.000

Problem 540	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	294	178	531	9242	549	0	0	-1
normalized size	1	1.00	0.61	1.81	31.44	1.87	0.00	0.00	-0.00
time (sec)	N/A	0.857	2.883	2.122	2.280	0.594	0.000	0.000	0.000

Problem 541	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	170	217	749	400	0	0	-1
normalized size	1	1.00	0.68	0.87	3.00	1.60	0.00	0.00	-0.00
time (sec)	N/A	0.843	1.378	2.071	0.812	0.492	0.000	0.000	0.000

Problem 542	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	154	195	581	368	0	0	-1
normalized size	1	1.00	0.74	0.94	2.81	1.78	0.00	0.00	-0.00
time (sec)	N/A	0.632	0.796	2.351	0.751	0.472	0.000	0.000	0.000

Problem 543	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	124	173	478	336	0	0	-1
normalized size	1	1.00	0.77	1.07	2.95	2.07	0.00	0.00	-0.01
time (sec)	N/A	0.447	0.351	2.548	0.703	0.469	0.000	0.000	0.000

Problem 544	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	140	142	195	306	0	0	-1
normalized size	1	1.00	1.18	1.19	1.64	2.57	0.00	0.00	-0.01
time (sec)	N/A	0.286	0.308	2.268	0.648	0.460	0.000	0.000	0.000

Problem 545	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	115	201	699	357	0	0	-1
normalized size	1	1.00	0.82	1.44	4.99	2.55	0.00	0.00	-0.01
time (sec)	N/A	0.343	0.227	2.401	0.716	0.488	0.000	0.000	0.000

Problem 546	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	114	342	1509	575	0	0	-1
normalized size	1	1.00	0.63	1.89	8.34	3.18	0.00	0.00	-0.01
time (sec)	N/A	0.503	0.668	2.337	0.829	0.560	0.000	0.000	0.000

Problem 547	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	137	413	2704	621	0	0	-1
normalized size	1	1.00	0.60	1.80	11.76	2.70	0.00	0.00	-0.00
time (sec)	N/A	0.700	1.046	2.314	0.917	0.562	0.000	0.000	0.000

Problem 548	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	178	329	0	480	0	0	-1
normalized size	1	1.00	0.66	1.22	0.00	1.78	0.00	0.00	-0.00
time (sec)	N/A	0.866	1.346	2.372	0.000	0.477	0.000	0.000	0.000

Problem 549	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	155	307	0	442	0	0	-1
normalized size	1	1.00	0.70	1.38	0.00	1.98	0.00	0.00	-0.00
time (sec)	N/A	0.687	1.237	2.242	0.000	0.506	0.000	0.000	0.000

Problem 550	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	198	235	8208	410	0	0	-1
normalized size	1	1.00	1.12	1.34	46.64	2.33	0.00	0.00	-0.01
time (sec)	N/A	0.490	1.943	2.323	0.940	0.451	0.000	0.000	0.000

Problem 551	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	86	209	2166	376	0	0	-1
normalized size	1	1.00	0.68	1.65	17.06	2.96	0.00	0.00	-0.01
time (sec)	N/A	0.317	0.523	2.112	0.826	0.473	0.000	0.000	0.000

Problem 552	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	113	303	0	592	0	0	-1
normalized size	1	1.00	0.61	1.64	0.00	3.20	0.00	0.00	-0.01
time (sec)	N/A	0.531	1.137	2.219	0.000	0.526	0.000	0.000	0.000

Problem 553	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	288	468	7057	716	0	0	-1
normalized size	1	1.00	1.22	1.97	29.78	3.02	0.00	0.00	-0.00
time (sec)	N/A	0.744	2.282	2.243	1.435	0.581	0.000	0.000	0.000

Problem 554	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	328	531	0	764	0	0	-1
normalized size	1	1.00	1.14	1.85	0.00	2.66	0.00	0.00	-0.00
time (sec)	N/A	0.947	3.519	2.036	0.000	0.598	0.000	0.000	0.000

Problem 555	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	317	317	207	461	0	572	0	0	-1
normalized size	1	1.00	0.65	1.45	0.00	1.80	0.00	0.00	-0.00
time (sec)	N/A	1.123	2.344	2.297	0.000	0.530	0.000	0.000	0.000

Problem 556	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	183	439	0	542	0	0	-1
normalized size	1	1.00	0.68	1.63	0.00	2.01	0.00	0.00	-0.00
time (sec)	N/A	0.910	1.698	2.286	0.000	0.471	0.000	0.000	0.000

Problem 557	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	228	365	0	504	0	0	-1
normalized size	1	1.00	1.02	1.64	0.00	2.26	0.00	0.00	-0.00
time (sec)	N/A	0.700	2.694	2.088	0.000	0.460	0.000	0.000	0.000

Problem 558	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	108	339	5924	482	0	0	-1
normalized size	1	1.00	0.48	1.52	26.57	2.16	0.00	0.00	-0.00
time (sec)	N/A	0.712	1.016	2.241	1.549	0.472	0.000	0.000	0.000

Problem 559	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	108	340	5356	478	0	0	-1
normalized size	1	1.00	0.61	1.93	30.43	2.72	0.00	0.00	-0.01
time (sec)	N/A	0.403	0.834	2.226	1.240	0.492	0.000	0.000	0.000

Problem 560	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-1)	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	965	540	0	720	0	0	-1
normalized size	1	1.00	4.12	2.31	0.00	3.08	0.00	0.00	-0.00
time (sec)	N/A	0.714	6.170	2.270	0.000	0.565	0.000	0.000	0.000

Problem 561	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-1)	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	286	286	1061	821	0	850	0	0	-1
normalized size	1	1.00	3.71	2.87	0.00	2.97	0.00	0.00	-0.00
time (sec)	N/A	0.958	6.203	2.744	0.000	0.613	0.000	0.000	0.000

Problem 562	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	103	413	0	0	0	0	166
normalized size	1	1.00	0.74	2.95	0.00	0.00	0.00	0.00	1.19
time (sec)	N/A	0.231	0.919	5.284	0.000	0.472	0.000	0.000	0.765

Problem 563	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	86	371	0	0	0	0	128
normalized size	1	1.00	0.80	3.44	0.00	0.00	0.00	0.00	1.19
time (sec)	N/A	0.214	0.441	4.406	0.000	0.451	0.000	0.000	0.622

Problem 564	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	67	326	0	0	0	0	85
normalized size	1	1.00	0.89	4.35	0.00	0.00	0.00	0.00	1.13
time (sec)	N/A	0.193	0.273	4.675	0.000	0.463	0.000	0.000	0.624

Problem 565	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	64	244	0	0	0	0	96
normalized size	1	1.00	0.90	3.44	0.00	0.00	0.00	0.00	1.35
time (sec)	N/A	0.193	0.372	5.326	0.000	0.478	0.000	0.000	3.341

Problem 566	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	107	428	0	0	0	0	150
normalized size	1	1.00	1.04	4.16	0.00	0.00	0.00	0.00	1.46
time (sec)	N/A	0.212	0.599	9.894	0.000	0.442	0.000	0.000	3.858

Problem 567	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	134	663	0	0	0	0	177
normalized size	1	1.00	0.96	4.74	0.00	0.00	0.00	0.00	1.26
time (sec)	N/A	0.231	1.018	13.225	0.000	0.450	0.000	0.000	4.301

Problem 568	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	139	548	0	0	0	0	229
normalized size	1	1.00	0.76	3.01	0.00	0.00	0.00	0.00	1.26
time (sec)	N/A	0.368	1.275	4.833	0.000	0.464	0.000	0.000	3.170

Problem 569	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	106	487	0	0	0	0	177
normalized size	1	1.00	0.76	3.48	0.00	0.00	0.00	0.00	1.26
time (sec)	N/A	0.332	0.673	5.316	0.000	0.482	0.000	0.000	3.037

Problem 570	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	102	404	0	0	0	0	158
normalized size	1	1.00	0.84	3.34	0.00	0.00	0.00	0.00	1.31
time (sec)	N/A	0.318	0.664	5.301	0.000	0.459	0.000	0.000	3.314

Problem 571	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	105	677	0	0	0	0	194
normalized size	1	1.00	0.83	5.37	0.00	0.00	0.00	0.00	1.54
time (sec)	N/A	0.335	1.225	10.267	0.000	0.475	0.000	0.000	4.274

Problem 572	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	175	750	0	0	0	0	227
normalized size	1	1.00	1.02	4.36	0.00	0.00	0.00	0.00	1.32
time (sec)	N/A	0.375	1.280	14.557	0.000	0.451	0.000	0.000	4.635

Problem 573	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	191	859	0	0	0	0	233
normalized size	1	1.00	0.89	4.01	0.00	0.00	0.00	0.00	1.09
time (sec)	N/A	0.396	5.260	16.099	0.000	0.477	0.000	0.000	4.976

Problem 574	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	260	1074	0	0	0	0	-1
normalized size	1	1.00	1.43	5.90	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.855	2.804	5.943	0.000	0.000	0.000	0.000	0.000

Problem 575	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	207	786	0	0	0	0	-1
normalized size	1	1.00	1.52	5.78	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.586	1.472	5.595	0.000	0.000	0.000	0.000	0.000

Problem 576	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	128	295	0	0	0	0	-1
normalized size	1	1.00	1.44	3.31	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.276	0.986	5.162	0.000	146.311	0.000	0.000	0.000

Problem 577	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	58	217	0	0	0	0	-1
normalized size	1	1.00	0.95	3.56	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.219	0.233	5.049	0.000	0.000	0.000	0.000	0.000

Problem 578	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	206	325	0	0	0	0	-1
normalized size	1	1.00	2.40	3.78	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.393	2.800	8.678	0.000	0.000	0.000	0.000	0.000

Problem 579	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	260	466	0	0	0	0	-1
normalized size	1	1.00	1.73	3.11	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.836	2.305	11.585	0.000	0.000	0.000	0.000	0.000

Problem 580	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	326	785	0	0	0	0	-1
normalized size	1	1.00	1.50	3.62	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.188	4.861	15.598	0.000	0.000	0.000	0.000	0.000

Problem 581	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	305	305	318	1059	0	0	0	0	-1
normalized size	1	1.00	1.04	3.47	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.018	3.782	15.997	0.000	170.433	0.000	0.000	0.000

Problem 582	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	281	843	0	0	0	0	-1
normalized size	1	1.00	1.26	3.78	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.702	2.851	13.830	0.000	123.898	0.000	0.000	0.000

Problem 583	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	260	802	0	0	0	0	-1
normalized size	1	1.00	1.28	3.95	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.614	2.711	11.822	0.000	0.000	0.000	0.000	0.000

Problem 584	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	273	715	0	0	0	0	-1
normalized size	1	1.00	1.39	3.63	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.667	2.797	11.371	0.000	0.000	0.000	0.000	0.000

Problem 585	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	317	877	0	0	0	0	-1
normalized size	1	1.00	1.24	3.44	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.944	4.731	14.906	0.000	0.000	0.000	0.000	0.000

Problem 586	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	346	346	427	1024	0	0	0	0	-1
normalized size	1	1.00	1.23	2.96	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.311	6.941	20.449	0.000	0.000	0.000	0.000	0.000

Problem 587	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	461	461	461	2216	0	0	0	0	-1
normalized size	1	1.00	1.00	4.81	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.585	6.140	22.695	0.000	0.000	0.000	0.000	0.000

Problem 588	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	367	367	390	2000	0	0	0	0	-1
normalized size	1	1.00	1.06	5.45	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.108	5.127	21.886	0.000	0.000	0.000	0.000	0.000

Problem 589	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	346	346	361	1959	0	0	0	0	-1
normalized size	1	1.00	1.04	5.66	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.102	4.852	19.949	0.000	0.000	0.000	0.000	0.000

Problem 590	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	338	338	364	1872	0	0	0	0	-1
normalized size	1	1.00	1.08	5.54	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.006	4.932	20.359	0.000	0.000	0.000	0.000	0.000

Problem 591	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	342	342	383	1768	0	0	0	0	-1
normalized size	1	1.00	1.12	5.17	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.096	5.136	19.436	0.000	0.000	0.000	0.000	0.000

Problem 592	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	420	420	458	2024	0	0	0	0	-1
normalized size	1	1.00	1.09	4.82	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.478	6.185	23.806	0.000	0.000	0.000	0.000	0.000

Problem 593	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	523	523	570	2178	0	0	0	0	-1
normalized size	1	1.00	1.09	4.16	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.980	7.355	35.259	0.000	0.000	0.000	0.000	0.000

Problem 594	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	343	343	455	2364	0	0	0	0	-1
normalized size	1	1.00	1.33	6.89	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.219	18.081	2.872	0.000	0.558	0.000	0.000	0.000

Problem 595	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	353	1699	0	0	0	0	-1
normalized size	1	1.00	1.32	6.36	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.914	14.923	2.102	0.000	0.756	0.000	0.000	0.000

Problem 596	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	305	1162	0	0	0	0	-1
normalized size	1	1.00	1.52	5.78	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.620	8.949	2.603	0.000	0.938	0.000	0.000	0.000

Problem 597	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	25347	822	0	0	0	0	-1
normalized size	1	1.00	121.86	3.95	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.683	29.934	2.232	0.000	2.296	0.000	0.000	0.000

Problem 598	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	52603	789	0	0	0	0	-1
normalized size	1	1.00	207.92	3.12	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.935	32.452	2.863	0.000	0.000	0.000	0.000	0.000

Problem 599	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	336	336	77879	1475	0	0	0	0	-1
normalized size	1	1.00	231.78	4.39	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.262	33.019	1.966	0.000	0.000	0.000	0.000	0.000

Problem 600	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	427	427	540	3069	0	0	0	0	-1
normalized size	1	1.00	1.26	7.19	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.708	18.652	3.375	0.000	1.680	0.000	0.000	0.000

Problem 601	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	342	342	466	2326	0	0	0	0	-1
normalized size	1	1.00	1.36	6.80	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.304	17.254	2.148	0.000	0.634	0.000	0.000	0.000

Problem 602	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	369	1749	0	0	0	0	-1
normalized size	1	1.00	1.39	6.58	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.970	14.539	2.793	0.000	0.811	0.000	0.000	0.000

Problem 603	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	276	276	45958	1429	0	0	0	0	-1
normalized size	1	1.00	166.51	5.18	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.093	34.220	2.198	0.000	0.000	0.000	0.000	0.000

Problem 604	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	272	272	66581	1410	0	0	0	0	-1
normalized size	1	1.00	244.78	5.18	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.023	33.180	2.834	0.000	6.500	0.000	0.000	0.000

Problem 605	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	339	339	79375	1659	0	0	0	0	-1
normalized size	1	1.00	234.14	4.89	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.418	33.617	2.555	0.000	0.000	0.000	0.000	0.000

Problem 606	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	421	421	104716	2351	0	0	0	0	-1
normalized size	1	1.00	248.73	5.58	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.802	33.903	3.862	0.000	0.000	0.000	0.000	0.000

Problem 607	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	519	519	626	3816	0	0	0	0	-1
normalized size	1	1.00	1.21	7.35	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.174	20.645	4.773	0.000	1.328	0.000	0.000	0.000

Problem 608	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	425	425	542	3069	0	0	0	0	-1
normalized size	1	1.00	1.28	7.22	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.716	19.410	2.774	0.000	0.558	0.000	0.000	0.000

Problem 609	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	340	340	470	2450	0	0	0	0	-1
normalized size	1	1.00	1.38	7.21	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.322	19.412	3.026	0.000	1.095	0.000	0.000	0.000

Problem 610	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	342	342	49609	2052	0	0	0	0	-1
normalized size	1	1.00	145.06	6.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.393	35.489	2.177	0.000	0.000	0.000	0.000	0.000

Problem 611	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	349	349	73332	2073	0	0	0	0	-1
normalized size	1	1.00	210.12	5.94	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.426	34.152	2.847	0.000	6.065	0.000	0.000	0.000

Problem 612	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	359	359	97208	2216	0	0	0	0	-1
normalized size	1	1.00	270.77	6.17	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.429	34.735	2.317	0.000	9.723	0.000	0.000	0.000

Problem 613	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	422	422	106199	2441	0	0	0	0	-1
normalized size	1	1.00	251.66	5.78	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.795	34.619	2.870	0.000	0.000	0.000	0.000	0.000

Problem 614	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	513	513	131553	3175	0	0	0	0	-1
normalized size	1	1.00	256.44	6.19	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.248	35.403	2.327	0.000	0.000	0.000	0.000	0.000

Problem 615	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	280	280	363	1700	0	0	0	0	-1
normalized size	1	1.00	1.30	6.07	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.916	16.471	3.356	0.000	1.021	0.000	0.000	0.000

Problem 616	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	311	1080	0	0	0	0	-1
normalized size	1	1.00	1.47	5.09	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.632	9.233	2.414	0.000	0.886	0.000	0.000	0.000

Problem 617	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	260	564	0	0	0	0	-1
normalized size	1	1.00	1.73	3.76	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.435	7.051	3.277	0.000	0.961	0.000	0.000	0.000

Problem 618	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	16611	257	0	0	0	0	-1
normalized size	1	1.00	120.37	1.86	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.526	28.809	2.373	0.000	0.000	0.000	0.000	0.000

Problem 619	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	256	256	51168	776	0	0	0	0	-1
normalized size	1	1.00	199.88	3.03	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.889	32.954	3.306	0.000	0.000	0.000	0.000	0.000

Problem 620	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	344	344	77909	1569	0	0	0	0	-1
normalized size	1	1.00	226.48	4.56	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.281	32.916	2.357	0.000	0.000	0.000	0.000	0.000

Problem 621	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	423	423	533	2084	0	0	0	0	-1
normalized size	1	1.00	1.26	4.93	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.404	20.985	3.183	0.000	1.000	0.000	0.000	0.000

Problem 622	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	326	326	469	1460	0	0	0	0	-1
normalized size	1	1.00	1.44	4.48	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.032	18.030	2.369	0.000	0.656	0.000	0.000	0.000

Problem 623	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	445	889	0	0	0	0	-1
normalized size	1	1.00	1.89	3.78	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.720	16.516	3.136	0.000	0.513	0.000	0.000	0.000

Problem 624	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	328	564	0	0	0	0	-1
normalized size	1	1.00	1.53	2.62	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.665	10.925	2.437	0.000	0.467	0.000	0.000	0.000

Problem 625	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	50122	840	0	0	0	0	-1
normalized size	1	1.00	227.83	3.82	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.788	32.369	2.954	0.000	0.000	0.000	0.000	0.000

Problem 626	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	371	371	95694	1441	0	0	0	0	-1
normalized size	1	1.00	257.94	3.88	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.446	33.841	2.283	0.000	0.000	0.000	0.000	0.000

Problem 627	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	487	487	140027	2295	0	0	0	0	-1
normalized size	1	1.00	287.53	4.71	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.859	35.276	2.924	0.000	0.000	0.000	0.000	0.000

Problem 628	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	588	588	4179	5675	0	0	0	0	-1
normalized size	1	1.00	7.11	9.65	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.067	25.412	2.651	0.000	0.983	0.000	0.000	0.000

Problem 629	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	472	472	3758	4480	0	0	0	0	-1
normalized size	1	1.00	7.96	9.49	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.540	24.261	3.245	0.000	1.389	0.000	0.000	0.000

Problem 630	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	368	368	621	3337	0	0	0	0	-1
normalized size	1	1.00	1.69	9.07	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.105	19.783	2.689	0.000	0.890	0.000	0.000	0.000

Problem 631	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	346	346	463	2416	0	0	0	0	-1
normalized size	1	1.00	1.34	6.98	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.008	18.028	3.169	0.000	0.970	0.000	0.000	0.000

Problem 632	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	329	329	487	1925	0	0	0	0	-1
normalized size	1	1.00	1.48	5.85	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.043	16.796	2.292	0.000	0.743	0.000	0.000	0.000

Problem 633	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	399	399	97528	3159	0	0	0	0	-1
normalized size	1	1.00	244.43	7.92	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.500	34.308	3.038	0.000	0.000	0.000	0.000	0.000

Problem 634	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	526	526	184379	5358	0	0	0	0	-1
normalized size	1	1.00	350.53	10.19	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.990	37.095	2.481	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [272] had the largest ratio of [.4400]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	8	5	1.00	23	0.217
2	A	7	5	1.00	23	0.217
3	A	6	5	1.00	23	0.217
4	A	5	4	1.00	23	0.174
5	A	6	5	1.00	23	0.217
6	A	7	5	1.00	23	0.217
7	A	6	4	1.00	31	0.129
8	A	6	4	1.00	29	0.138
9	A	5	3	1.00	23	0.130
10	A	6	4	1.00	29	0.138

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
11	A	6	4	1.00	31	0.129
12	A	6	4	1.00	31	0.129
13	A	6	4	1.00	29	0.138
14	A	5	3	1.00	23	0.130
15	A	6	4	1.00	29	0.138
16	A	6	4	1.00	31	0.129
17	A	6	4	1.00	31	0.129
18	A	6	4	1.00	29	0.138
19	A	5	3	1.00	23	0.130
20	A	6	4	1.00	29	0.138
21	A	6	4	1.00	31	0.129
22	A	6	4	1.00	31	0.129
23	A	6	4	1.00	29	0.138
24	A	5	3	1.00	23	0.130
25	A	6	4	1.00	29	0.138
26	A	6	4	1.00	31	0.129
27	A	6	4	1.00	31	0.129
28	A	6	4	1.00	31	0.129
29	A	6	4	1.00	31	0.129
30	A	6	4	1.00	31	0.129
31	A	6	4	1.00	31	0.129
32	A	6	4	1.00	31	0.129
33	A	6	4	1.00	29	0.138
34	A	6	4	1.00	29	0.138
35	A	6	4	1.00	27	0.148
36	A	5	3	1.00	21	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
37	A	6	4	1.00	27	0.148
38	A	6	4	1.00	29	0.138
39	A	6	4	1.00	31	0.129
40	A	6	4	1.00	31	0.129
41	A	6	4	1.00	31	0.129
42	A	6	4	1.00	31	0.129
43	A	7	5	1.00	29	0.172
44	A	6	5	1.00	29	0.172
45	A	6	6	1.00	29	0.207
46	A	5	5	1.00	27	0.185
47	A	4	4	1.00	21	0.190
48	A	3	2	1.00	27	0.074
49	A	4	4	1.00	29	0.138
50	A	5	5	1.00	29	0.172
51	A	6	5	1.00	29	0.172
52	A	7	5	1.00	29	0.172
53	A	7	6	1.00	31	0.194
54	A	7	7	1.00	31	0.226
55	A	6	6	1.00	29	0.207
56	A	5	5	1.00	23	0.217
57	A	4	3	1.00	29	0.103
58	A	4	3	1.00	31	0.097
59	A	5	5	1.00	31	0.161
60	A	6	6	1.00	31	0.194
61	A	7	6	1.00	31	0.194
62	A	8	6	1.00	31	0.194

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
63	A	11	7	1.00	31	0.226
64	A	10	6	1.00	29	0.207
65	A	6	5	1.00	23	0.217
66	A	5	3	1.00	29	0.103
67	A	5	4	1.00	31	0.129
68	A	5	3	1.00	31	0.097
69	A	8	6	1.00	31	0.194
70	A	7	6	1.00	31	0.194
71	A	8	6	1.00	31	0.194
72	A	14	7	1.00	31	0.226
73	A	13	6	1.00	29	0.207
74	A	7	5	1.00	23	0.217
75	A	6	3	1.00	29	0.103
76	A	6	4	1.00	31	0.129
77	A	6	4	1.00	31	0.129
78	A	6	3	1.00	31	0.097
79	A	11	6	1.00	31	0.194
80	A	8	6	1.00	31	0.194
81	A	9	6	1.00	31	0.194
82	A	6	5	1.00	31	0.161
83	A	6	6	1.00	31	0.194
84	A	5	5	1.00	31	0.161
85	A	3	3	1.00	29	0.103
86	A	2	2	1.00	23	0.087
87	A	4	4	1.00	29	0.138
88	A	5	5	1.00	31	0.161

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
89	A	6	5	1.00	31	0.161
90	A	7	5	1.00	31	0.161
91	A	7	6	1.00	31	0.194
92	A	6	6	1.00	31	0.194
93	A	4	4	1.00	31	0.129
94	A	2	2	1.00	29	0.069
95	A	3	3	1.00	23	0.130
96	A	5	4	1.00	29	0.138
97	A	6	5	1.00	31	0.161
98	A	7	5	1.00	31	0.161
99	A	8	6	1.00	31	0.194
100	A	7	6	1.00	31	0.194
101	A	5	5	1.00	31	0.161
102	A	3	3	1.00	31	0.097
103	A	3	3	1.00	29	0.103
104	A	4	3	1.00	23	0.130
105	A	6	4	1.00	29	0.138
106	A	7	5	1.00	31	0.161
107	A	8	5	1.00	31	0.161
108	A	9	6	1.00	31	0.194
109	A	8	6	1.00	31	0.194
110	A	6	5	1.00	31	0.161
111	A	4	4	1.00	31	0.129
112	A	4	4	1.00	31	0.129
113	A	4	3	1.00	29	0.103
114	A	5	3	1.00	23	0.130

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
115	A	7	4	1.00	29	0.138
116	A	8	5	1.00	31	0.161
117	A	9	5	1.00	31	0.161
118	A	5	5	1.00	33	0.152
119	A	4	4	1.00	33	0.121
120	A	3	3	1.00	33	0.091
121	A	2	2	1.00	31	0.065
122	A	4	4	1.00	25	0.160
123	A	3	3	1.00	31	0.097
124	A	4	4	1.00	33	0.121
125	A	5	4	1.00	33	0.121
126	A	6	4	1.00	33	0.121
127	A	5	5	1.00	33	0.152
128	A	4	4	1.00	33	0.121
129	A	3	3	1.00	31	0.097
130	A	5	5	1.00	25	0.200
131	A	4	4	1.00	31	0.129
132	A	4	4	1.00	33	0.121
133	A	5	5	1.00	33	0.152
134	A	6	5	1.00	33	0.152
135	A	6	5	1.00	33	0.152
136	A	5	4	1.00	33	0.121
137	A	4	3	1.00	31	0.097
138	A	6	5	1.00	25	0.200
139	A	5	4	1.00	31	0.129
140	A	5	5	1.00	33	0.152

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
141	A	5	4	1.00	33	0.121
142	A	6	5	1.00	33	0.152
143	A	7	5	1.00	33	0.152
144	A	6	5	1.00	33	0.152
145	A	5	5	1.00	33	0.152
146	A	4	4	1.00	33	0.121
147	A	3	3	1.00	31	0.097
148	A	5	4	1.00	25	0.160
149	A	6	5	1.00	31	0.161
150	A	7	5	1.00	33	0.152
151	A	8	5	1.00	33	0.152
152	A	6	6	1.00	33	0.182
153	A	5	5	1.00	33	0.152
154	A	4	4	1.00	33	0.121
155	A	3	3	1.00	31	0.097
156	A	6	5	1.00	25	0.200
157	A	7	6	1.00	31	0.194
158	A	8	6	1.00	33	0.182
159	A	9	6	1.00	33	0.182
160	A	6	5	1.00	33	0.152
161	A	5	5	1.00	33	0.152
162	A	4	4	1.00	33	0.121
163	A	4	4	1.00	31	0.129
164	A	7	5	1.00	25	0.200
165	A	8	6	1.00	31	0.194
166	A	9	6	1.00	33	0.182
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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
167	A	5	4	1.00	26	0.154
168	A	6	5	1.00	32	0.156
169	A	7	5	1.00	34	0.147
170	A	8	5	1.00	34	0.147
171	A	6	5	1.15	26	0.192
172	A	7	6	1.00	32	0.188
173	A	8	6	1.00	34	0.176
174	A	9	6	1.00	34	0.176
175	A	7	6	1.22	26	0.231
176	A	8	6	1.00	32	0.188
177	A	9	6	1.00	34	0.176
178	A	10	6	1.00	34	0.176
179	A	9	6	1.00	31	0.194
180	A	8	6	1.00	31	0.194
181	A	7	6	1.00	31	0.194
182	A	6	5	1.00	31	0.161
183	A	6	5	1.00	31	0.161
184	A	7	6	1.00	31	0.194
185	A	8	6	1.00	31	0.194
186	A	9	7	1.00	33	0.212
187	A	8	7	1.00	33	0.212
188	A	7	6	1.00	33	0.182
189	A	7	6	1.00	33	0.182
190	A	7	6	1.00	33	0.182
191	A	8	7	1.00	33	0.212
192	A	9	7	1.00	33	0.212

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
193	A	10	7	1.00	33	0.212
194	A	9	7	1.00	33	0.212
195	A	8	6	1.00	33	0.182
196	A	8	7	1.00	33	0.212
197	A	8	6	1.00	33	0.182
198	A	8	6	1.00	33	0.182
199	A	9	7	1.00	33	0.212
200	A	10	7	1.00	33	0.212
201	A	9	6	1.00	33	0.182
202	A	8	6	1.00	33	0.182
203	A	7	6	1.00	33	0.182
204	A	6	5	1.00	33	0.152
205	A	6	5	1.00	33	0.152
206	A	7	6	1.00	33	0.182
207	A	8	6	1.00	33	0.182
208	A	9	6	1.00	33	0.182
209	A	9	6	1.00	33	0.182
210	A	8	6	1.00	33	0.182
211	A	7	5	1.00	33	0.152
212	A	7	6	1.00	33	0.182
213	A	7	5	1.00	33	0.152
214	A	8	6	1.00	33	0.182
215	A	9	6	1.00	33	0.182
216	A	10	6	1.00	33	0.182
217	A	9	6	1.00	33	0.182
218	A	8	5	1.00	33	0.152

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
219	A	8	6	1.00	33	0.182
220	A	8	6	1.00	33	0.182
221	A	8	5	1.00	33	0.152
222	A	9	6	1.00	33	0.182
223	A	10	6	1.00	33	0.182
224	A	5	4	1.00	35	0.114
225	A	4	4	1.00	35	0.114
226	A	3	3	1.00	35	0.086
227	A	3	3	1.00	35	0.086
228	A	2	2	1.00	35	0.057
229	A	3	3	1.00	35	0.086
230	A	4	3	1.00	35	0.086
231	A	6	5	1.00	35	0.143
232	A	5	5	1.00	35	0.143
233	A	4	4	1.00	35	0.114
234	A	4	4	1.00	35	0.114
235	A	4	4	1.00	35	0.114
236	A	3	3	1.00	35	0.086
237	A	4	4	1.00	35	0.114
238	A	5	4	1.00	35	0.114
239	A	7	5	1.00	35	0.143
240	A	6	5	1.00	35	0.143
241	A	5	4	1.00	35	0.114
242	A	5	4	1.00	35	0.114
243	A	5	5	1.00	35	0.143
244	A	5	4	1.00	35	0.114

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
245	A	4	3	1.00	35	0.086
246	A	5	4	1.00	35	0.114
247	A	6	4	1.00	35	0.114
248	A	7	6	1.00	35	0.171
249	A	6	6	1.00	35	0.171
250	A	5	5	1.00	35	0.143
251	A	3	3	1.00	35	0.086
252	A	4	4	1.00	35	0.114
253	A	5	4	1.00	35	0.114
254	A	6	4	1.00	35	0.114
255	A	8	7	1.00	35	0.200
256	A	7	7	1.00	35	0.200
257	A	6	6	1.00	35	0.171
258	A	3	3	1.00	35	0.086
259	A	4	4	1.00	35	0.114
260	A	5	5	1.00	35	0.143
261	A	6	5	1.00	35	0.143
262	A	8	7	1.00	35	0.200
263	A	7	6	1.00	35	0.171
264	A	4	4	1.00	35	0.114
265	A	5	5	1.30	35	0.143
266	A	5	4	1.00	35	0.114
267	A	6	5	1.00	35	0.143
268	A	7	5	1.00	35	0.143
269	A	9	9	1.00	25	0.360
270	A	8	8	1.00	25	0.320

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
271	A	9	9	1.00	25	0.360
272	A	11	11	1.00	25	0.440
273	A	10	10	1.00	25	0.400
274	A	11	11	1.00	25	0.440
275	A	7	4	1.00	33	0.121
276	A	4	4	1.00	35	0.114
277	A	6	5	1.00	29	0.172
278	A	6	6	1.00	29	0.207
279	A	5	5	1.00	27	0.185
280	A	4	4	1.00	21	0.190
281	A	3	2	1.00	27	0.074
282	A	4	4	1.00	29	0.138
283	A	5	5	1.00	29	0.172
284	A	6	5	1.00	29	0.172
285	A	7	6	1.00	31	0.194
286	A	7	7	1.00	31	0.226
287	A	6	6	1.00	29	0.207
288	A	5	4	1.00	23	0.174
289	A	5	4	1.00	29	0.138
290	A	5	5	1.00	31	0.161
291	A	5	5	1.00	31	0.161
292	A	7	6	1.00	31	0.194
293	A	7	6	1.00	31	0.194
294	A	8	7	1.00	31	0.226
295	A	7	6	1.00	29	0.207
296	A	6	5	1.00	23	0.217

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
297	A	6	5	1.10	29	0.172
298	A	6	6	1.00	31	0.194
299	A	6	6	1.00	31	0.194
300	A	6	6	1.00	31	0.194
301	A	8	7	1.00	31	0.226
302	A	9	7	1.00	31	0.226
303	A	8	6	1.00	29	0.207
304	A	7	6	1.00	23	0.261
305	A	7	6	1.00	29	0.207
306	A	7	6	1.00	31	0.194
307	A	7	7	1.00	31	0.226
308	A	7	7	1.00	31	0.226
309	A	7	7	1.00	31	0.226
310	A	9	8	1.00	31	0.258
311	A	8	8	1.00	31	0.258
312	A	7	7	1.00	31	0.226
313	A	7	7	1.00	31	0.226
314	A	5	5	1.00	29	0.172
315	A	4	4	1.00	23	0.174
316	A	5	5	1.00	29	0.172
317	A	6	6	1.00	31	0.194
318	A	7	6	1.00	31	0.194
319	A	8	6	1.00	31	0.194
320	A	8	8	1.00	31	0.258
321	A	7	7	1.00	31	0.226
322	A	6	6	1.00	31	0.194

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
323	A	5	5	1.00	29	0.172
324	A	5	5	1.00	23	0.217
325	A	6	6	1.00	29	0.207
326	A	7	6	1.00	31	0.194
327	A	8	6	1.00	31	0.194
328	A	9	9	1.00	31	0.290
329	A	8	8	1.00	31	0.258
330	A	7	7	1.00	31	0.226
331	A	6	6	1.00	31	0.194
332	A	6	5	1.00	29	0.172
333	A	6	6	1.00	23	0.261
334	A	7	7	1.00	29	0.241
335	A	8	7	1.00	31	0.226
336	A	9	9	1.00	31	0.290
337	A	8	8	1.00	31	0.258
338	A	7	7	1.00	31	0.226
339	A	7	6	1.00	31	0.194
340	A	7	5	1.00	29	0.172
341	A	7	6	1.00	23	0.261
342	A	8	7	1.00	29	0.241
343	A	9	7	1.00	31	0.226
344	A	4	4	1.00	28	0.143
345	A	2	2	1.00	28	0.071
346	A	5	5	1.00	23	0.217
347	A	4	4	1.00	21	0.190
348	A	7	7	1.00	33	0.212

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
349	A	6	6	1.00	33	0.182
350	A	5	5	1.00	33	0.152
351	A	4	4	1.00	31	0.129
352	A	5	5	1.00	25	0.200
353	A	6	6	1.00	31	0.194
354	A	7	7	1.00	33	0.212
355	A	8	7	1.00	33	0.212
356	A	7	6	1.00	33	0.182
357	A	6	5	1.00	33	0.152
358	A	5	4	1.00	31	0.129
359	A	6	6	1.00	25	0.240
360	A	6	6	1.00	31	0.194
361	A	7	7	1.00	33	0.212
362	A	8	7	1.00	33	0.212
363	A	8	6	1.00	33	0.182
364	A	7	5	1.00	33	0.152
365	A	6	4	1.00	31	0.129
366	A	7	7	1.00	25	0.280
367	A	7	7	1.00	31	0.226
368	A	7	7	1.00	33	0.212
369	A	8	8	1.00	33	0.242
370	A	9	8	1.00	33	0.242
371	A	5	5	1.00	33	0.152
372	A	4	4	1.00	33	0.121
373	A	3	3	1.00	31	0.097
374	A	3	3	1.00	25	0.120

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
375	A	6	6	1.00	31	0.194
376	A	7	7	1.00	33	0.212
377	A	8	7	1.00	33	0.212
378	A	5	5	1.00	33	0.152
379	A	4	4	1.00	33	0.121
380	A	4	4	1.00	31	0.129
381	A	6	6	1.00	25	0.240
382	A	7	7	1.00	31	0.226
383	A	8	8	1.00	33	0.242
384	A	9	8	1.00	33	0.242
385	A	6	6	1.00	33	0.182
386	A	5	5	1.00	33	0.152
387	A	5	5	1.00	33	0.152
388	A	5	4	1.00	31	0.129
389	A	7	7	1.00	25	0.280
390	A	8	8	1.00	31	0.258
391	A	9	8	1.00	33	0.242
392	A	1	1	1.00	31	0.032
393	A	1	1	1.00	32	0.031
394	A	8	6	1.00	31	0.194
395	A	7	6	1.00	31	0.194
396	A	6	5	1.00	31	0.161
397	A	6	5	1.00	31	0.161
398	A	7	6	1.00	31	0.194
399	A	8	6	1.00	31	0.194
400	A	9	7	1.00	33	0.212

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
401	A	8	7	1.00	33	0.212
402	A	7	6	1.00	33	0.182
403	A	7	6	1.00	33	0.182
404	A	7	6	1.00	33	0.182
405	A	8	7	1.00	33	0.212
406	A	9	7	1.00	33	0.212
407	A	10	8	1.00	33	0.242
408	A	9	8	1.00	33	0.242
409	A	8	7	1.00	33	0.212
410	A	8	7	1.00	33	0.212
411	A	8	7	1.00	33	0.212
412	A	8	7	1.00	33	0.212
413	A	9	8	1.00	33	0.242
414	A	10	8	1.00	33	0.242
415	A	11	9	1.00	33	0.273
416	A	10	9	1.00	33	0.273
417	A	7	7	1.00	33	0.212
418	A	5	5	1.00	33	0.152
419	A	7	7	1.00	33	0.212
420	A	9	8	1.00	33	0.242
421	A	10	9	1.00	33	0.273
422	A	11	9	1.00	33	0.273
423	A	10	9	1.00	33	0.273
424	A	9	8	1.00	33	0.242
425	A	9	8	1.00	33	0.242
426	A	9	8	1.00	33	0.242

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
427	A	10	9	1.00	33	0.273
428	A	12	10	1.00	33	0.303
429	A	11	10	1.00	33	0.303
430	A	10	9	1.00	33	0.273
431	A	10	9	1.00	33	0.273
432	A	10	9	1.00	33	0.273
433	A	10	9	1.00	33	0.273
434	A	11	10	1.00	33	0.303
435	A	13	13	1.00	35	0.371
436	A	12	12	1.00	35	0.343
437	A	11	11	1.00	35	0.314
438	A	8	8	1.00	35	0.229
439	A	9	9	1.00	35	0.257
440	A	10	9	1.00	35	0.257
441	A	14	13	1.00	35	0.371
442	A	13	13	1.00	35	0.371
443	A	12	12	1.00	35	0.343
444	A	12	12	1.00	35	0.343
445	A	9	9	1.00	35	0.257
446	A	10	9	1.00	35	0.257
447	A	11	9	1.00	35	0.257
448	A	15	14	1.00	35	0.400
449	A	14	14	1.00	35	0.400
450	A	13	13	1.00	35	0.371
451	A	13	13	1.00	35	0.371
452	A	13	13	1.00	35	0.371

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
453	A	10	10	1.00	35	0.286
454	A	11	10	1.00	35	0.286
455	A	12	10	1.00	35	0.286
456	A	13	13	1.00	35	0.371
457	A	12	12	1.00	35	0.343
458	A	7	7	1.00	35	0.200
459	A	7	7	1.00	35	0.200
460	A	8	8	1.00	35	0.229
461	A	9	9	1.00	35	0.257
462	A	13	13	1.00	35	0.371
463	A	9	9	1.00	35	0.257
464	A	8	8	1.00	35	0.229
465	A	8	8	1.00	35	0.229
466	A	9	9	1.00	35	0.257
467	A	10	9	1.00	35	0.257
468	A	13	13	1.00	35	0.371
469	A	9	9	1.00	35	0.257
470	A	9	9	1.00	35	0.257
471	A	9	9	1.00	35	0.257
472	A	10	10	1.00	35	0.286
473	A	11	10	1.00	35	0.286
474	A	0	0	0.00	0	0.000
475	A	0	0	0.00	0	0.000
476	A	0	0	0.00	0	0.000
477	A	0	0	0.00	0	0.000
478	A	0	0	0.00	0	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
479	A	9	7	1.00	31	0.226
480	A	8	6	1.00	31	0.194
481	A	7	5	1.00	31	0.161
482	A	6	4	1.00	29	0.138
483	A	8	7	1.00	31	0.226
484	A	7	7	1.00	31	0.226
485	A	6	6	1.00	31	0.194
486	A	6	6	1.00	31	0.194
487	A	7	7	1.00	31	0.226
488	A	8	7	1.00	31	0.226
489	A	9	8	1.00	33	0.242
490	A	8	8	1.00	33	0.242
491	A	7	7	1.00	33	0.212
492	A	7	7	1.00	33	0.212
493	A	7	7	1.00	33	0.212
494	A	8	8	1.00	33	0.242
495	A	9	8	1.00	33	0.242
496	A	7	6	1.00	33	0.182
497	A	6	6	1.00	33	0.182
498	A	5	5	1.00	33	0.152
499	A	5	5	1.00	33	0.152
500	A	6	6	1.00	33	0.182
501	A	7	6	1.00	33	0.182
502	A	8	6	1.00	33	0.182
503	A	7	6	1.00	33	0.182
504	A	6	5	1.00	33	0.152

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
505	A	6	6	1.00	33	0.182
506	A	6	5	1.00	33	0.152
507	A	7	6	1.00	33	0.182
508	A	8	6	1.00	33	0.182
509	A	8	6	1.00	33	0.182
510	A	7	5	1.00	33	0.152
511	A	7	6	1.00	33	0.182
512	A	7	6	1.00	33	0.182
513	A	7	5	1.00	33	0.152
514	A	8	6	1.00	33	0.182
515	A	6	4	1.00	35	0.114
516	A	5	4	1.00	35	0.114
517	A	4	4	1.00	35	0.114
518	A	3	3	1.00	35	0.086
519	A	4	4	1.00	35	0.114
520	A	4	4	1.00	35	0.114
521	A	5	5	1.00	35	0.143
522	A	6	5	1.00	35	0.143
523	A	7	5	1.00	35	0.143
524	A	6	5	1.00	35	0.143
525	A	5	5	1.00	35	0.143
526	A	4	4	1.00	35	0.114
527	A	5	5	1.00	35	0.143
528	A	5	5	1.00	35	0.143
529	A	5	5	1.00	35	0.143
530	A	6	6	1.00	35	0.171

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
531	A	7	6	1.00	35	0.171
532	A	7	5	1.00	35	0.143
533	A	6	5	1.00	35	0.143
534	A	5	4	1.00	35	0.114
535	A	6	5	1.00	35	0.143
536	A	6	6	1.00	35	0.171
537	A	6	5	1.00	35	0.143
538	A	6	5	1.00	35	0.143
539	A	7	6	1.00	35	0.171
540	A	8	6	1.00	35	0.171
541	A	7	5	1.00	35	0.143
542	A	6	5	1.00	35	0.143
543	A	5	5	1.00	35	0.143
544	A	4	4	1.00	35	0.114
545	A	6	6	1.00	35	0.171
546	A	7	7	1.00	35	0.200
547	A	8	7	1.00	35	0.200
548	A	7	6	1.00	35	0.171
549	A	6	6	1.00	35	0.171
550	A	5	5	1.00	35	0.143
551	A	4	4	1.00	35	0.114
552	A	7	7	1.00	35	0.200
553	A	8	8	1.00	35	0.229
554	A	9	8	1.00	35	0.229
555	A	8	6	1.00	35	0.171
556	A	7	6	1.00	35	0.171

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
557	A	6	5	1.00	35	0.143
558	A	6	6	1.00	35	0.171
559	A	5	5	1.00	35	0.143
560	A	8	7	1.00	35	0.200
561	A	9	8	1.00	35	0.229
562	A	8	7	1.00	31	0.226
563	A	7	7	1.00	31	0.226
564	A	6	6	1.00	31	0.194
565	A	6	6	1.00	31	0.194
566	A	7	7	1.00	31	0.226
567	A	8	7	1.00	31	0.226
568	A	7	7	1.00	33	0.212
569	A	6	6	1.00	33	0.182
570	A	6	6	1.00	33	0.182
571	A	6	6	1.00	33	0.182
572	A	7	7	1.00	33	0.212
573	A	8	7	1.00	33	0.212
574	A	8	8	1.00	33	0.242
575	A	7	7	1.00	33	0.212
576	A	6	6	1.00	33	0.182
577	A	4	4	1.00	33	0.121
578	A	6	6	1.00	33	0.182
579	A	8	8	1.00	33	0.242
580	A	9	8	1.00	33	0.242
581	A	8	8	1.00	33	0.242
582	A	7	7	1.00	33	0.212

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
583	A	7	7	1.00	33	0.212
584	A	7	7	1.00	33	0.212
585	A	8	8	1.00	33	0.242
586	A	9	8	1.00	33	0.242
587	A	9	9	1.00	33	0.273
588	A	8	8	1.00	33	0.242
589	A	8	8	1.00	33	0.242
590	A	8	8	1.00	33	0.242
591	A	8	8	1.00	33	0.242
592	A	9	8	1.00	33	0.242
593	A	10	8	1.00	33	0.242
594	A	11	10	1.00	35	0.286
595	A	10	10	1.00	35	0.286
596	A	9	9	1.00	35	0.257
597	A	12	12	1.00	35	0.343
598	A	13	13	1.00	35	0.371
599	A	14	14	1.00	35	0.400
600	A	12	10	1.00	35	0.286
601	A	11	10	1.00	35	0.286
602	A	10	10	1.00	35	0.286
603	A	13	13	1.00	35	0.371
604	A	13	13	1.00	35	0.371
605	A	14	14	1.00	35	0.400
606	A	15	14	1.00	35	0.400
607	A	13	11	1.00	35	0.314
608	A	12	11	1.00	35	0.314

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
609	A	11	11	1.00	35	0.314
610	A	14	14	1.00	35	0.400
611	A	14	14	1.00	35	0.400
612	A	14	14	1.00	35	0.400
613	A	15	15	1.00	35	0.429
614	A	16	15	1.00	35	0.429
615	A	10	10	1.00	35	0.286
616	A	9	9	1.00	35	0.257
617	A	8	8	1.00	35	0.229
618	A	8	8	1.00	35	0.229
619	A	13	13	1.00	35	0.371
620	A	14	14	1.00	35	0.400
621	A	11	10	1.00	35	0.286
622	A	10	10	1.00	35	0.286
623	A	9	9	1.00	35	0.257
624	A	9	9	1.00	35	0.257
625	A	10	10	1.00	35	0.286
626	A	14	14	1.00	35	0.400
627	A	15	14	1.00	35	0.400
628	A	12	11	1.00	35	0.314
629	A	11	11	1.00	35	0.314
630	A	10	10	1.00	35	0.286
631	A	10	10	1.00	35	0.286
632	A	10	10	1.00	35	0.286
633	A	14	14	1.00	35	0.400
634	A	15	15	1.00	35	0.429

Chapter 3

Listing of integrals

3.1 $\int (b \sec(c + dx))^{5/2} (A + B \sec(c + dx)) dx$

Optimal. Leaf size=171

$$\frac{2Ab^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{3d} + \frac{2Ab\sin(c+dx)(b\sec(c+dx))^{3/2}}{3d} - \frac{6b^3BE\left(\frac{1}{2}(c+dx)\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

[Out] $2/3*A*b*(b*\sec(d*x+c))^{(3/2)}*\sin(d*x+c)/d+2/5*B*(b*\sec(d*x+c))^{(5/2)}*\sin(d*x+c)/d-6/5*b^3*B*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c),2^{(1/2)})/d/\cos(d*x+c)^{(1/2)}/(b*\sec(d*x+c))^{(1/2)}+6/5*b^2*B*\sin(d*x+c)*(b*\sec(d*x+c))^{(1/2)}/d+2/3*A*b^2*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c),2^{(1/2)})*\cos(d*x+c)^{(1/2)}*(b*\sec(d*x+c))^{(1/2)}/d$

Rubi [A] time = 0.12, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3787, 3768, 3771, 2641, 2639}

$$\frac{2Ab^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{3d} + \frac{2Ab\sin(c+dx)(b\sec(c+dx))^{3/2}}{3d} + \frac{6b^2B\sin(c+dx)\sqrt{b\sec(c+dx)}}{5d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(b*\text{Sec}[c + d*x])^{(5/2)}*(A + B*\text{Sec}[c + d*x]),x]$

[Out] $(-6*b^3*B*\text{EllipticE}[(c + d*x)/2, 2])/((5*d*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sqrt}[b*\text{Sec}[c + d*x]]) + (2*A*b^2*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticF}[(c + d*x)/2, 2]*\text{Sqrt}[b*\text{Sec}[c + d*x]])/(3*d) + (6*b^2*B*\text{Sqrt}[b*\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/((5*d) + (2*A$

$$*b*(b*\text{Sec}[c + d*x])^{(3/2)}*\text{Sin}[c + d*x]/(3*d) + (2*B*(b*\text{Sec}[c + d*x])^{(5/2)}* \text{Sin}[c + d*x])/ (5*d)$$

Rule 2639

$$\text{Int}[\text{Sqrt}[\text{sin}[(c_.) + (d_.)*(x_.)]], x_Symbol] \text{ :> } \text{Simp}[(2*\text{EllipticE}[(1*(c - \text{Pi}/2 + d*x))/2, 2])/d, x] \text{ /; } \text{FreeQ}\{c, d\}, x]$$

Rule 2641

$$\text{Int}[1/\text{Sqrt}[\text{sin}[(c_.) + (d_.)*(x_.)]], x_Symbol] \text{ :> } \text{Simp}[(2*\text{EllipticF}[(1*(c - \text{Pi}/2 + d*x))/2, 2])/d, x] \text{ /; } \text{FreeQ}\{c, d\}, x]$$

Rule 3768

$$\text{Int}[(\text{csc}[(c_.) + (d_.)*(x_.)]*(b_.))^{(n_.)}, x_Symbol] \text{ :> } -\text{Simp}[(b*\text{Cos}[c + d*x] * (b*\text{Csc}[c + d*x])^{(n-1)})/(d*(n-1)), x] + \text{Dist}[(b^2*(n-2))/(n-1), \text{Int}[(b*\text{Csc}[c + d*x])^{(n-2)}, x], x] \text{ /; } \text{FreeQ}\{b, c, d\}, x \ \&\& \ \text{GtQ}[n, 1] \ \&\& \ \text{IntegerQ}[2*n]$$

Rule 3771

$$\text{Int}[(\text{csc}[(c_.) + (d_.)*(x_.)]*(b_.))^{(n_.)}, x_Symbol] \text{ :> } \text{Dist}[(b*\text{Csc}[c + d*x])^{(n-1)}*\text{Sin}[c + d*x]^n, \text{Int}[1/\text{Sin}[c + d*x]^n, x], x] \text{ /; } \text{FreeQ}\{b, c, d\}, x \ \&\& \ \text{EqQ}[n^2, 1/4]$$

Rule 3787

$$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(d_.))^{(n_.)}*(\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] \text{ :> } \text{Dist}[a, \text{Int}[(d*\text{Csc}[e + f*x])^n, x], x] + \text{Dist}[b/d, \text{Int}[(d*\text{Csc}[e + f*x])^{(n+1)}, x], x] \text{ /; } \text{FreeQ}\{a, b, d, e, f, n\}, x]$$

Rubi steps

$$\begin{aligned}
\int (b \sec(c + dx))^{5/2} (A + B \sec(c + dx)) dx &= A \int (b \sec(c + dx))^{5/2} dx + \frac{B \int (b \sec(c + dx))^{7/2} dx}{b} \\
&= \frac{2Ab(b \sec(c + dx))^{3/2} \sin(c + dx)}{3d} + \frac{2B(b \sec(c + dx))^{5/2} \sin(c + dx)}{5d} \\
&= \frac{6b^2 B \sqrt{b \sec(c + dx)} \sin(c + dx)}{5d} + \frac{2Ab(b \sec(c + dx))^{3/2} \sin(c + dx)}{3d} \\
&= \frac{2Ab^2 \sqrt{\cos(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{b \sec(c + dx)}}{3d} + \frac{6b^2 B \sqrt{b \sec(c + dx)}}{3d} \\
&= -\frac{6b^3 B E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{5d \sqrt{\cos(c + dx)} \sqrt{b \sec(c + dx)}} + \frac{2Ab^2 \sqrt{\cos(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3d}
\end{aligned}$$

Mathematica [A] time = 0.48, size = 99, normalized size = 0.58

$$\frac{(b \sec(c + dx))^{5/2} \left(10A \sin(2(c + dx)) + 20A \cos^2(c + dx) F\left(\frac{1}{2}(c + dx) \middle| 2\right) + 21B \sin(c + dx) + 9B \sin(3(c + dx))\right)}{30d}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]

[Out] ((b*Sec[c + d*x])^(5/2)*(-36*B*Cos[c + d*x]^(5/2)*EllipticE[(c + d*x)/2, 2] + 20*A*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] + 21*B*Sin[c + d*x] + 10*A*Sin[2*(c + d*x)] + 9*B*Sin[3*(c + d*x)])/(30*d)

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb^2 \sec(dx + c)^3 + Ab^2 \sec(dx + c)^2\right) \sqrt{b \sec(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] integral((B*b^2*sec(d*x + c)^3 + A*b^2*sec(d*x + c)^2)*sqrt(b*sec(d*x + c)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A) (b \sec(dx + c))^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(5/2), x)

maple [C] time = 2.03, size = 518, normalized size = 3.03

$$2(1 + \cos(dx + c))^2 (-1 + \cos(dx + c))^2 \left(5iA \sqrt{\frac{1}{1 + \cos(dx + c)}} \sqrt{\frac{\cos(dx + c)}{1 + \cos(dx + c)}} \operatorname{EllipticF}\left(\frac{i(-1 + \cos(dx + c))}{\sin(dx + c)}, i\right) \sin(dx + c) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x)

[Out] $\frac{2}{15}d*(1+\cos(d*x+c))^2*(-1+\cos(d*x+c))^2*(5*I*A*(1/(1+\cos(d*x+c)))^{1/2}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*\operatorname{EllipticF}(I*(-1+\cos(d*x+c))/\sin(d*x+c),I)*\sin(d*x+c)*\cos(d*x+c)^3-9*I*B*\cos(d*x+c)^3*(1/(1+\cos(d*x+c)))^{1/2}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*\operatorname{EllipticF}(I*(-1+\cos(d*x+c))/\sin(d*x+c),I)*\sin(d*x+c)+9*I*B*\cos(d*x+c)^3*(1/(1+\cos(d*x+c)))^{1/2}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*\operatorname{EllipticE}(I*(-1+\cos(d*x+c))/\sin(d*x+c),I)*\sin(d*x+c)+5*I*A*\cos(d*x+c)^2*(1/(1+\cos(d*x+c)))^{1/2}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*\operatorname{EllipticF}(I*(-1+\cos(d*x+c))/\sin(d*x+c),I)*\sin(d*x+c)-9*I*B*\cos(d*x+c)^2*(1/(1+\cos(d*x+c)))^{1/2}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*\operatorname{EllipticF}(I*(-1+\cos(d*x+c))/\sin(d*x+c),I)*\sin(d*x+c)+9*I*B*\cos(d*x+c)^2*(1/(1+\cos(d*x+c)))^{1/2}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*\operatorname{EllipticE}(I*(-1+\cos(d*x+c))/\sin(d*x+c),I)*\sin(d*x+c)-5*A*\cos(d*x+c)^3-9*B*\cos(d*x+c)^3+6*B*\cos(d*x+c)^2+5*A*\cos(d*x+c)+3*B)*(b/\cos(d*x+c))^{5/2}/\sin(d*x+c)^5$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A) (b \sec(dx + c))^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(5/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A + \frac{B}{\cos(c + dx)} \right) \left(\frac{b}{\cos(c + dx)} \right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B/cos(c + d*x))*(b/cos(c + d*x))^(5/2), x)`

[Out] `int((A + B/cos(c + d*x))*(b/cos(c + d*x))^(5/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \sec(c + dx))^{\frac{5}{2}} (A + B \sec(c + dx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)), x)`

[Out] `Integral((b*sec(c + d*x))**(5/2)*(A + B*sec(c + d*x)), x)`

3.2 $\int (b \sec(c + dx))^{3/2} (A + B \sec(c + dx)) dx$

Optimal. Leaf size=136

$$-\frac{2Ab^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{2Ab\sin(c+dx)\sqrt{b\sec(c+dx)}}{d} + \frac{2B\sin(c+dx)(b\sec(c+dx))^{3/2}}{3d} + \frac{2bB\sqrt{\cos(c+dx)}}{3d}$$

[Out] $2/3*B*(b*\sec(d*x+c))^{(3/2)}*\sin(d*x+c)/d-2*A*b^2*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c),2^{(1/2)})/d/\cos(d*x+c)^{(1/2)}/(b*\sec(d*x+c))^{(1/2)}+2*A*b*\sin(d*x+c)*(b*\sec(d*x+c))^{(1/2)}/d+2/3*b*B*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c),2^{(1/2)})*\cos(d*x+c)^{(1/2)}*(b*\sec(d*x+c))^{(1/2)}/d$

Rubi [A] time = 0.10, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3787, 3768, 3771, 2639, 2641}

$$-\frac{2Ab^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{2Ab\sin(c+dx)\sqrt{b\sec(c+dx)}}{d} + \frac{2B\sin(c+dx)(b\sec(c+dx))^{3/2}}{3d} + \frac{2bB\sqrt{\cos(c+dx)}}{3d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(b*\text{Sec}[c + d*x])^{(3/2)}*(A + B*\text{Sec}[c + d*x]),x]$

[Out] $(-2*A*b^2*\text{EllipticE}[(c + d*x)/2, 2])/(d*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sqrt}[b*\text{Sec}[c + d*x]]) + (2*b*B*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticF}[(c + d*x)/2, 2]*\text{Sqrt}[b*\text{Sec}[c + d*x]])/(3*d) + (2*A*b*\text{Sqrt}[b*\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/d + (2*B*(b*\text{Sec}[c + d*x])^{(3/2)}*\text{Sin}[c + d*x])/(3*d)$

Rule 2639

$\text{Int}[\text{Sqrt}[\sin[(c_.) + (d_.)*(x_.)]], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticE}[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 2641

$\text{Int}[1/\text{Sqrt}[\sin[(c_.) + (d_.)*(x_.)]], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticF}[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 3768

$\text{Int}[(\text{csc}[(c_.) + (d_.)*(x_.)]*(b_.))^{(n_.)}, x_Symbol] \rightarrow -\text{Simp}[(b*\text{Cos}[c + d*x]*\text{Csc}[c + d*x]^{(n-1)})/(d*(n-1)), x] + \text{Dist}[(b^2*(n-2))/(n-1), \text{Int}[(b*\text{Csc}[c + d*x])^{(n-2)}, x], x] /; \text{FreeQ}\{b, c, d\}, x] \&\& \text{GtQ}[n, 1] \&\&$

IntegerQ[2*n]

Rule 3771

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rubi steps

$$\begin{aligned}
 \int (b \sec(c + dx))^{3/2} (A + B \sec(c + dx)) dx &= A \int (b \sec(c + dx))^{3/2} dx + \frac{B \int (b \sec(c + dx))^{5/2} dx}{b} \\
 &= \frac{2Ab\sqrt{b \sec(c + dx)} \sin(c + dx)}{d} + \frac{2B(b \sec(c + dx))^{3/2} \sin(c + dx)}{3d} \\
 &= \frac{2Ab\sqrt{b \sec(c + dx)} \sin(c + dx)}{d} + \frac{2B(b \sec(c + dx))^{3/2} \sin(c + dx)}{3d} \\
 &= -\frac{2Ab^2 E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{d\sqrt{\cos(c + dx)}\sqrt{b \sec(c + dx)}} + \frac{2bB\sqrt{\cos(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3d}
 \end{aligned}$$

Mathematica [A] time = 0.27, size = 87, normalized size = 0.64

$$\frac{(b \sec(c + dx))^{3/2} \left(2 \sin(c + dx) (3A \cos(c + dx) + B) - 6A \cos^3(c + dx) E\left(\frac{1}{2}(c + dx) \middle| 2\right) + 2B \cos^3(c + dx) F\left(\frac{1}{2}(c + dx) \middle| 2\right) \right)}{3d}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]

[Out] ((b*Sec[c + d*x])^(3/2)*(-6*A*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2] + 2*B*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + 2*(B + 3*A*Cos[c + d*x])*Sin[c + d*x]))/(3*d)

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb \sec(dx + c)^2 + Ab \sec(dx + c)\right)\sqrt{b \sec(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] integral((B*b*sec(d*x + c)^2 + A*b*sec(d*x + c))*sqrt(b*sec(d*x + c)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A) (b \sec(dx + c))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(3/2), x)

maple [C] time = 1.41, size = 499, normalized size = 3.67

$$2(1 + \cos(dx + c))^2 (-1 + \cos(dx + c))^2 \left(3iA (\cos^2(dx + c)) \sqrt{\frac{1}{1 + \cos(dx + c)}} \sqrt{\frac{\cos(dx + c)}{1 + \cos(dx + c)}} \operatorname{EllipticE}\left(\frac{i(-1 + \cos(dx + c))}{\sin(dx + c)}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x)

[Out] 2/3/d*(1+cos(d*x+c))^2*(-1+cos(d*x+c))^2*(3*I*A*cos(d*x+c)^2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)-3*I*A*cos(d*x+c)^2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)+I*B*cos(d*x+c)^2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)+3*I*A*cos(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)-3*I*A*cos(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)+I*B*cos(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)-3*A*cos(d*x+c)^2-B*cos(d*x+c)^2+3*A*cos(d*x+c)+B)*(b/cos(d*x+c))^(3/2)/sin(d*x+c)^5

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A) (b \sec(dx + c))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A + \frac{B}{\cos(c + dx)} \right) \left(\frac{b}{\cos(c + dx)} \right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))*(b/cos(c + d*x))^(3/2), x)

[Out] int((A + B/cos(c + d*x))*(b/cos(c + d*x))^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \sec(c + dx))^{\frac{3}{2}} (A + B \sec(c + dx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)), x)

[Out] Integral((b*sec(c + d*x))**(3/2)*(A + B*sec(c + d*x)), x)

3.3 $\int \sqrt{b \sec(c + dx)} (A + B \sec(c + dx)) dx$

Optimal. Leaf size=104

$$\frac{2A\sqrt{\cos(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{b \sec(c + dx)}}{d} + \frac{2B \sin(c + dx) \sqrt{b \sec(c + dx)}}{d} - \frac{2bBE\left(\frac{1}{2}(c + dx) \middle| 2\right)}{d\sqrt{\cos(c + dx)} \sqrt{b \sec(c + dx)}}$$

[Out] $-2*b*B*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/d/\cos(d*x+c)^{(1/2)}/(b*\sec(d*x+c))^{(1/2)}+2*B*\sin(d*x+c)*(b*\sec(d*x+c))^{(1/2)}/d+2*A*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*(b*\sec(d*x+c))^{(1/2)}/d$

Rubi [A] time = 0.08, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3787, 3771, 2641, 3768, 2639}

$$\frac{2A\sqrt{\cos(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{b \sec(c + dx)}}{d} + \frac{2B \sin(c + dx) \sqrt{b \sec(c + dx)}}{d} - \frac{2bBE\left(\frac{1}{2}(c + dx) \middle| 2\right)}{d\sqrt{\cos(c + dx)} \sqrt{b \sec(c + dx)}}$$

Antiderivative was successfully verified.

[In] `Int[Sqrt[b*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]`

[Out] $(-2*b*B*\text{EllipticE}[(c + d*x)/2, 2])/(d*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sqrt}[b*\text{Sec}[c + d*x]]) + (2*A*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticF}[(c + d*x)/2, 2]*\text{Sqrt}[b*\text{Sec}[c + d*x]])/d + (2*B*\text{Sqrt}[b*\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/d$

Rule 2639

`Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]`

Rule 2641

`Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]`

Rule 3768

`Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := -Simp[(b*Csc[c + d*x])*(b*Csc[c + d*x])^(n - 1)/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]`

Rule 3771

Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.))^(n_), x_Symbol] :=> Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] :=> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rubi steps

$$\begin{aligned}
 \int \sqrt{b \sec(c + dx)} (A + B \sec(c + dx)) dx &= A \int \sqrt{b \sec(c + dx)} dx + \frac{B \int (b \sec(c + dx))^{3/2} dx}{b} \\
 &= \frac{2B\sqrt{b \sec(c + dx)} \sin(c + dx)}{d} - (bB) \int \frac{1}{\sqrt{b \sec(c + dx)}} dx + (A\sqrt{c}) \\
 &= \frac{2A\sqrt{\cos(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{b \sec(c + dx)}}{d} + \frac{2B\sqrt{b \sec(c + dx)}}{d} \\
 &= -\frac{2bBE\left(\frac{1}{2}(c + dx) \middle| 2\right)}{d\sqrt{\cos(c + dx)} \sqrt{b \sec(c + dx)}} + \frac{2A\sqrt{\cos(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{d}
 \end{aligned}$$

Mathematica [A] time = 0.12, size = 73, normalized size = 0.70

$$\frac{2\sqrt{b \sec(c + dx)} \left(A\sqrt{\cos(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right) + B \sin(c + dx) - B\sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) \right)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[b*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]

[Out] (2*Sqrt[b*Sec[c + d*x]]*(-(B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) + A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + B*Sin[c + d*x]))/d

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left((B \sec(dx + c) + A)\sqrt{b \sec(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] integral((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A) \sqrt{b \sec(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c)), x)

maple [C] time = 1.52, size = 453, normalized size = 4.36

$$2\sqrt{\frac{b}{\cos(dx+c)}} (1 + \cos(dx + c))^2 (-1 + \cos(dx + c))^2 \left(iA \cos(dx + c) \sqrt{\frac{1}{1+\cos(dx+c)}} \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} \operatorname{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x)

[Out] 2/d*(b/cos(d*x+c))^(1/2)*(1+cos(d*x+c))^2*(-1+cos(d*x+c))^2*(I*A*sin(d*x+c)*cos(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)-I*B*cos(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)+I*B*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)*cos(d*x+c)+I*A*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-I*B*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)+I*B*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-B*cos(d*x+c)+B)/sin(d*x+c)^5

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A) \sqrt{b \sec(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A + \frac{B}{\cos(c + dx)} \right) \sqrt{\frac{b}{\cos(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))*(b/cos(c + d*x))^(1/2), x)

[Out] int((A + B/cos(c + d*x))*(b/cos(c + d*x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sec(c + dx)} (A + B \sec(c + dx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*sec(d*x+c))**(1/2)*(A+B*sec(d*x+c)), x)

[Out] Integral(sqrt(b*sec(c + d*x))*(A + B*sec(c + d*x)), x)

3.4 $\int \frac{A+B \sec(c+dx)}{\sqrt{b \sec(c+dx)}} dx$

Optimal. Leaf size=82

$$\frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{2B\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{bd}$$

[Out] $2*A*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/d/\cos(d*x+c)^{(1/2)}/(b*\sec(d*x+c))^{(1/2)}+2*B*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*(b*\sec(d*x+c))^{(1/2)}/b/d$

Rubi [A] time = 0.07, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3787, 3771, 2639, 2641}

$$\frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{2B\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{bd}$$

Antiderivative was successfully verified.

[In] `Int[(A + B*Sec[c + d*x])/Sqrt[b*Sec[c + d*x]], x]`

[Out] $(2*A*\text{EllipticE}[(c + d*x)/2, 2])/(d*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sqrt}[b*\text{Sec}[c + d*x]]) + (2*B*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticF}[(c + d*x)/2, 2]*\text{Sqrt}[b*\text{Sec}[c + d*x]])/(b*d)$

Rule 2639

`Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]`

Rule 2641

`Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]`

Rule 3771

`Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]`

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rubi steps

$$\begin{aligned} \int \frac{A + B \sec(c + dx)}{\sqrt{b \sec(c + dx)}} dx &= A \int \frac{1}{\sqrt{b \sec(c + dx)}} dx + \frac{B \int \sqrt{b \sec(c + dx)} dx}{b} \\ &= \frac{A \int \sqrt{\cos(c + dx)} dx}{\sqrt{\cos(c + dx)} \sqrt{b \sec(c + dx)}} + \frac{(B \sqrt{\cos(c + dx)} \sqrt{b \sec(c + dx)}) \int \frac{1}{\sqrt{\cos(c + dx)}} dx}{b} \\ &= \frac{2AE \left(\frac{1}{2}(c + dx) \middle| 2 \right)}{d \sqrt{\cos(c + dx)} \sqrt{b \sec(c + dx)}} + \frac{2B \sqrt{\cos(c + dx)} F \left(\frac{1}{2}(c + dx) \middle| 2 \right) \sqrt{b \sec(c + dx)}}{bd} \end{aligned}$$

Mathematica [A] time = 0.09, size = 54, normalized size = 0.66

$$\frac{2 \left(AE \left(\frac{1}{2}(c + dx) \middle| 2 \right) + BF \left(\frac{1}{2}(c + dx) \middle| 2 \right) \right)}{d \sqrt{\cos(c + dx)} \sqrt{b \sec(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sec[c + d*x])/Sqrt[b*Sec[c + d*x]],x]

[Out] (2*(A*EllipticE[(c + d*x)/2, 2] + B*EllipticF[(c + d*x)/2, 2]))/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]])

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{(B \sec(dx + c) + A) \sqrt{b \sec(dx + c)}}{b \sec(dx + c)}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(b*sec(d*x+c))^(1/2),x, algorithm="fricas")

[Out] integral((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c))/(b*sec(d*x + c)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{\sqrt{b \sec(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(b*sec(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)/sqrt(b*sec(d*x + c)), x)

maple [C] time = 1.52, size = 445, normalized size = 5.43

$$2 \left(iA \cos(dx + c) \sqrt{\frac{1}{1+\cos(dx+c)}} \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} \operatorname{EllipticF} \left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)}, i \right) \sin(dx + c) - iA \cos(dx + c) \sqrt{\frac{1}{1+\cos(dx+c)}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))/(b*sec(d*x+c))^(1/2),x)

[Out] 2/d*(I*A*sin(d*x+c)*cos(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)-I*A*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)*cos(d*x+c)*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+I*B*cos(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)+I*A*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-I*A*sin(d*x+c)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+I*B*sin(d*x+c)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-A*cos(d*x+c)^2+A*cos(d*x+c))*(b/cos(d*x+c))^(1/2)/sin(d*x+c)/b

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{\sqrt{b \sec(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(b*sec(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)/sqrt(b*sec(d*x + c)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\sqrt{\frac{b}{\cos(c+dx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B/cos(c + d*x))/(b/cos(c + d*x))^(1/2), x)`

[Out] `int((A + B/cos(c + d*x))/(b/cos(c + d*x))^(1/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \sec(c + dx)}{\sqrt{b \sec(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*sec(d*x+c))/(b*sec(d*x+c))**(1/2), x)`

[Out] `Integral((A + B*sec(c + d*x))/sqrt(b*sec(c + d*x)), x)`

$$3.5 \quad \int \frac{A+B \sec(c+dx)}{(b \sec(c+dx))^{3/2}} dx$$

Optimal. Leaf size=116

$$\frac{2A\sqrt{\cos(c+dx)} F\left(\frac{1}{2}(c+dx) \middle| 2\right) \sqrt{b \sec(c+dx)}}{3b^2d} + \frac{2A \sin(c+dx)}{3bd\sqrt{b \sec(c+dx)}} + \frac{2BE\left(\frac{1}{2}(c+dx) \middle| 2\right)}{bd\sqrt{\cos(c+dx)} \sqrt{b \sec(c+dx)}}$$

[Out] $2/3*A*\sin(d*x+c)/b/d/(b*\sec(d*x+c))^{(1/2)}+2*B*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/b/d/\cos(d*x+c)^{(1/2)}/(b*\sec(d*x+c))^{(1/2)}+2/3*A*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*(b*\sec(d*x+c))^{(1/2)}/b^{2/d}$

Rubi [A] time = 0.09, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3787, 3769, 3771, 2641, 2639}

$$\frac{2A\sqrt{\cos(c+dx)} F\left(\frac{1}{2}(c+dx) \middle| 2\right) \sqrt{b \sec(c+dx)}}{3b^2d} + \frac{2A \sin(c+dx)}{3bd\sqrt{b \sec(c+dx)}} + \frac{2BE\left(\frac{1}{2}(c+dx) \middle| 2\right)}{bd\sqrt{\cos(c+dx)} \sqrt{b \sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sec[c + d*x])/(b*Sec[c + d*x])^(3/2), x]

[Out] $(2*B*\text{EllipticE}[(c+d*x)/2, 2])/(b*d*\text{Sqrt}[\text{Cos}[c+d*x]]*\text{Sqrt}[b*\text{Sec}[c+d*x]]) + (2*A*\text{Sqrt}[\text{Cos}[c+d*x]]*\text{EllipticF}[(c+d*x)/2, 2]*\text{Sqrt}[b*\text{Sec}[c+d*x]])/(3*b^2*d) + (2*A*\text{Sin}[c+d*x])/(3*b*d*\text{Sqrt}[b*\text{Sec}[c+d*x]])$

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 3769

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

]

Rule 3771

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rubi steps

$$\begin{aligned}
 \int \frac{A + B \sec(c + dx)}{(b \sec(c + dx))^{3/2}} dx &= A \int \frac{1}{(b \sec(c + dx))^{3/2}} dx + \frac{B \int \frac{1}{\sqrt{b \sec(c + dx)}} dx}{b} \\
 &= \frac{2A \sin(c + dx)}{3bd \sqrt{b \sec(c + dx)}} + \frac{A \int \sqrt{b \sec(c + dx)} dx}{3b^2} + \frac{B \int \sqrt{\cos(c + dx)} dx}{b \sqrt{\cos(c + dx)} \sqrt{b \sec(c + dx)}} \\
 &= \frac{2BE \left(\frac{1}{2}(c + dx) \Big| 2 \right)}{bd \sqrt{\cos(c + dx)} \sqrt{b \sec(c + dx)}} + \frac{2A \sin(c + dx)}{3bd \sqrt{b \sec(c + dx)}} + \frac{(A \sqrt{\cos(c + dx)} \sqrt{b \sec(c + dx)})}{3b^2} \\
 &= \frac{2BE \left(\frac{1}{2}(c + dx) \Big| 2 \right)}{bd \sqrt{\cos(c + dx)} \sqrt{b \sec(c + dx)}} + \frac{2A \sqrt{\cos(c + dx)} F \left(\frac{1}{2}(c + dx) \Big| 2 \right) \sqrt{b \sec(c + dx)}}{3b^2 d}
 \end{aligned}$$

Mathematica [A] time = 0.18, size = 86, normalized size = 0.74

$$\frac{\sec^2(c + dx) \left(A \left(\sin(2(c + dx)) + 2\sqrt{\cos(c + dx)} F \left(\frac{1}{2}(c + dx) \Big| 2 \right) \right) + 6B \sqrt{\cos(c + dx)} E \left(\frac{1}{2}(c + dx) \Big| 2 \right) \right)}{3d(b \sec(c + dx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sec[c + d*x])/(b*Sec[c + d*x])^(3/2), x]

[Out] (Sec[c + d*x]^2*(6*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + A*(2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + Sin[2*(c + d*x)])))/(3*d*(b*Sec[c + d*x])^(3/2))

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B \sec(dx+c) + A)\sqrt{b \sec(dx+c)}}{b^2 \sec(dx+c)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(b*sec(d*x+c))^(3/2),x, algorithm="fricas")

[Out] integral((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c))/(b^2*sec(d*x + c)^2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx+c) + A}{(b \sec(dx+c))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(b*sec(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)/(b*sec(d*x + c))^(3/2), x)

maple [C] time = 1.32, size = 470, normalized size = 4.05

$$\frac{2iA \cos(dx+c) \sqrt{\frac{1}{1+\cos(dx+c)}} \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} \text{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)}, i\right) \sin(dx+c)}{3} - 2iB \sqrt{\frac{1}{1+\cos(dx+c)}} \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} \sin(dx+c) \text{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)}, i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))/(b*sec(d*x+c))^(3/2),x)

[Out] 2/3/d*(I*A*sin(d*x+c)*cos(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)-3*I*B*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)*cos(d*x+c)+3*I*B*cos(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)+I*A*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-3*I*B*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+3*I*B*sin(d*x+c)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-A*cos(d*x+c)^3-3*B*cos(d*x+c)^2+A*cos(d*x+c)+3*B*cos(d*x+c))/sin(d*x+c)/cos(d*x+c)^2/(b/cos(d*x+c))^(3/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(b \sec(dx + c))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(b*sec(d*x+c))^(3/2),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)/(b*sec(d*x + c))^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\left(\frac{b}{\cos(c+dx)}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/(b/cos(c + d*x))^(3/2), x)

[Out] int((A + B/cos(c + d*x))/(b/cos(c + d*x))^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \sec(c + dx)}{(b \sec(c + dx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(b*sec(d*x+c))**(3/2), x)

[Out] Integral((A + B*sec(c + d*x))/(b*sec(c + d*x))**(3/2), x)

$$3.6 \quad \int \frac{A+B \sec(c+dx)}{(b \sec(c+dx))^{5/2}} dx$$

Optimal. Leaf size=147

$$\frac{6AE \left(\frac{1}{2}(c+dx) \middle| 2 \right)}{5b^2 d \sqrt{\cos(c+dx)} \sqrt{b \sec(c+dx)}} + \frac{2A \sin(c+dx)}{5bd(b \sec(c+dx))^{3/2}} + \frac{2B \sqrt{\cos(c+dx)} F \left(\frac{1}{2}(c+dx) \middle| 2 \right) \sqrt{b \sec(c+dx)}}{3b^3 d} + \frac{2}{3b^2 d}$$

[Out] $2/5*A*\sin(d*x+c)/b/d/(b*\sec(d*x+c))^{(3/2)}+2/3*B*\sin(d*x+c)/b^2/d/(b*\sec(d*x+c))^{(1/2)}+6/5*A*(\cos(1/2*d*x+1/2*c))^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/b^2/d/\cos(d*x+c)^{(1/2)}/(b*\sec(d*x+c))^{(1/2)}+2/3*B*(\cos(1/2*d*x+1/2*c))^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*(b*\sec(d*x+c))^{(1/2)}/b^3/d$

Rubi [A] time = 0.10, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3787, 3769, 3771, 2639, 2641}

$$\frac{6AE \left(\frac{1}{2}(c+dx) \middle| 2 \right)}{5b^2 d \sqrt{\cos(c+dx)} \sqrt{b \sec(c+dx)}} + \frac{2A \sin(c+dx)}{5bd(b \sec(c+dx))^{3/2}} + \frac{2B \sin(c+dx)}{3b^2 d \sqrt{b \sec(c+dx)}} + \frac{2B \sqrt{\cos(c+dx)} F \left(\frac{1}{2}(c+dx) \middle| 2 \right)}{3b^3 d}$$

Antiderivative was successfully verified.

[In] `Int[(A + B*Sec[c + d*x])/(b*Sec[c + d*x])^(5/2), x]`

[Out] $(6*A*\text{EllipticE}[(c+d*x)/2, 2])/(5*b^2*d*\text{Sqrt}[\text{Cos}[c+d*x]]*\text{Sqrt}[b*\text{Sec}[c+d*x]]) + (2*B*\text{Sqrt}[\text{Cos}[c+d*x]]*\text{EllipticF}[(c+d*x)/2, 2]*\text{Sqrt}[b*\text{Sec}[c+d*x]])/(3*b^3*d) + (2*A*\text{Sin}[c+d*x])/(5*b*d*(b*\text{Sec}[c+d*x])^{(3/2)}) + (2*B*\text{Sin}[c+d*x])/(3*b^2*d*\text{Sqrt}[b*\text{Sec}[c+d*x]])$

Rule 2639

`Int[Sqrt[sin[(c_.) + (d_.)*(x_.)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]`

Rule 2641

`Int[1/Sqrt[sin[(c_.) + (d_.)*(x_.)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]`

Rule 3769

`Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.))^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +`

$d*x])^{(n + 2)}, x], x] /; \text{FreeQ}[\{b, c, d\}, x] \ \&\& \ \text{LtQ}[n, -1] \ \&\& \ \text{IntegerQ}[2*n]$

Rule 3771

$\text{Int}[(\text{csc}[c_.] + (d_.)*(x_)]*(b_.)^{(n_.)}, x_Symbol] \text{:>} \text{Dist}[(b*\text{Csc}[c + d*x])^{n*} \text{Sin}[c + d*x]^n, \text{Int}[1/\text{Sin}[c + d*x]^n, x], x] /; \text{FreeQ}[\{b, c, d\}, x] \ \&\& \ \text{EqQ}[n^2, 1/4]$

Rule 3787

$\text{Int}[(\text{csc}[e_.] + (f_.)*(x_)]*(d_.)^{(n_.)}*(\text{csc}[e_.] + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] \text{:>} \text{Dist}[a, \text{Int}[(d*\text{Csc}[e + f*x])^n, x], x] + \text{Dist}[b/d, \text{Int}[(d*\text{Csc}[e + f*x])^{(n + 1)}, x], x] /; \text{FreeQ}[\{a, b, d, e, f, n\}, x]$

Rubi steps

$$\begin{aligned} \int \frac{A + B \sec(c + dx)}{(b \sec(c + dx))^{5/2}} dx &= A \int \frac{1}{(b \sec(c + dx))^{5/2}} dx + \frac{B \int \frac{1}{(b \sec(c + dx))^{3/2}} dx}{b} \\ &= \frac{2A \sin(c + dx)}{5bd(b \sec(c + dx))^{3/2}} + \frac{2B \sin(c + dx)}{3b^2 d \sqrt{b \sec(c + dx)}} + \frac{(3A) \int \frac{1}{\sqrt{b \sec(c + dx)}} dx}{5b^2} + \frac{B \int \sqrt{b \sec(c + dx)} dx}{3b^3} \\ &= \frac{2A \sin(c + dx)}{5bd(b \sec(c + dx))^{3/2}} + \frac{2B \sin(c + dx)}{3b^2 d \sqrt{b \sec(c + dx)}} + \frac{(3A) \int \sqrt{\cos(c + dx)} dx}{5b^2 \sqrt{\cos(c + dx)} \sqrt{b \sec(c + dx)}} + \frac{B \int \sqrt{b \sec(c + dx)} dx}{3b^3} \\ &= \frac{6AE \left(\frac{1}{2}(c + dx) \Big| 2 \right)}{5b^2 d \sqrt{\cos(c + dx)} \sqrt{b \sec(c + dx)}} + \frac{2B \sqrt{\cos(c + dx)} F \left(\frac{1}{2}(c + dx) \Big| 2 \right) \sqrt{b \sec(c + dx)}}{3b^3 d} \end{aligned}$$

Mathematica [A] time = 0.51, size = 88, normalized size = 0.60

$$\frac{2 \left(\sin(c + dx) \sqrt{\cos(c + dx)} (3A \cos(c + dx) + 5B) + 9AE \left(\frac{1}{2}(c + dx) \Big| 2 \right) + 5BF \left(\frac{1}{2}(c + dx) \Big| 2 \right) \right)}{15d \cos^{\frac{5}{2}}(c + dx) (b \sec(c + dx))^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sec[c + d*x])/(b*Sec[c + d*x])^(5/2), x]

[Out] (2*(9*A*EllipticE[(c + d*x)/2, 2] + 5*B*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(5*B + 3*A*Cos[c + d*x])*Sin[c + d*x]))/(15*d*Cos[c + d*x]^(5/2)*(b*Sec[c + d*x])^(5/2))

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B \sec(dx+c) + A)\sqrt{b \sec(dx+c)}}{b^3 \sec(dx+c)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(b*sec(d*x+c))^(5/2),x, algorithm="fricas")

[Out] integral((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c))/(b^3*sec(d*x + c)^3), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx+c) + A}{(b \sec(dx+c))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(b*sec(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)/(b*sec(d*x + c))^(5/2), x)

maple [C] time = 1.33, size = 482, normalized size = 3.28

$$\frac{6iA \cos(dx+c) \sqrt{\frac{1}{1+\cos(dx+c)}} \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} \text{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)}, i\right) \sin(dx+c)}{5} - \frac{6iA \cos(dx+c) \sqrt{\frac{1}{1+\cos(dx+c)}} \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} \text{EllipticE}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))/(b*sec(d*x+c))^(5/2),x)

[Out] 2/15/d*(9*I*A*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*cos(d*x+c)-9*I*A*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)+5*I*B*cos(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)+9*I*A*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-9*I*A*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+5*I*B*sin(d*x+c)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-3*A*cos(d*x+c)^4-5*B*cos(d*x+c)^3-6*A*cos(d*x+c)^2+9*A*cos(d*x+c)+5*B*cos(d*x+c))/cos(d*x+c)^3/(b/cos(d*x+c))^(5/2)/sin(d*x+c)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(b \sec(dx + c))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(b*sec(d*x+c))^(5/2),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)/(b*sec(d*x + c))^(5/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\left(\frac{b}{\cos(c+dx)}\right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/(b/cos(c + d*x))^(5/2), x)

[Out] int((A + B/cos(c + d*x))/(b/cos(c + d*x))^(5/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \sec(c + dx)}{(b \sec(c + dx))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(b*sec(d*x+c))**(5/2), x)

[Out] Integral((A + B*sec(c + d*x))/(b*sec(c + d*x))**(5/2), x)

3.7 $\int \sec^2(c + dx)(b \sec(c + dx))^{2/3}(A + B \sec(c + dx)) dx$

Optimal. Leaf size=119

$$\frac{3A \sin(c + dx)(b \sec(c + dx))^{5/3} {}_2F_1\left(-\frac{5}{6}, \frac{1}{2}; \frac{1}{6}; \cos^2(c + dx)\right)}{5bd\sqrt{\sin^2(c + dx)}} + \frac{3B \sin(c + dx)(b \sec(c + dx))^{8/3} {}_2F_1\left(-\frac{4}{3}, \frac{1}{2}; -\frac{1}{3}; \cos^2(c + dx)\right)}{8b^2d\sqrt{\sin^2(c + dx)}}$$

[Out] 3/5*A*hypergeom([-5/6, 1/2], [1/6], cos(d*x+c)^2)*(b*sec(d*x+c))^(5/3)*sin(d*x+c)/b/d/(sin(d*x+c)^2)^(1/2)+3/8*B*hypergeom([-4/3, 1/2], [-1/3], cos(d*x+c)^2)*(b*sec(d*x+c))^(8/3)*sin(d*x+c)/b^2/d/(sin(d*x+c)^2)^(1/2)

Rubi [A] time = 0.10, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {16, 3787, 3772, 2643}

$$\frac{3A \sin(c + dx)(b \sec(c + dx))^{5/3} {}_2F_1\left(-\frac{5}{6}, \frac{1}{2}; \frac{1}{6}; \cos^2(c + dx)\right)}{5bd\sqrt{\sin^2(c + dx)}} + \frac{3B \sin(c + dx)(b \sec(c + dx))^{8/3} {}_2F_1\left(-\frac{4}{3}, \frac{1}{2}; -\frac{1}{3}; \cos^2(c + dx)\right)}{8b^2d\sqrt{\sin^2(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d*x]^2*(b*Sec[c + d*x])^(2/3)*(A + B*Sec[c + d*x]), x]

[Out] (3*A*Hypergeometric2F1[-5/6, 1/2, 1/6, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(5/3)*Sin[c + d*x])/(5*b*d*Sqrt[Sin[c + d*x]^2]) + (3*B*Hypergeometric2F1[-4/3, 1/2, -1/3, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(8/3)*Sin[c + d*x])/(8*b^2*d*Sqrt[Sin[c + d*x]^2])

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 2643

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rule 3772

Int[(csc[(c_) + (d_)*(x_)]*(b_))^(n_), x_Symbol] := Simp[(b*Csc[c + d*x])^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr

eeQ[{b, c, d, n}, x] && !IntegerQ[n]

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rubi steps

$$\begin{aligned} \int \sec^2(c + dx)(b \sec(c + dx))^{2/3}(A + B \sec(c + dx)) dx &= \frac{\int (b \sec(c + dx))^{8/3}(A + B \sec(c + dx)) dx}{b^2} \\ &= \frac{A \int (b \sec(c + dx))^{8/3} dx}{b^2} + \frac{B \int (b \sec(c + dx))^{11/3} dx}{b^3} \\ &= \frac{\left(A \left(\frac{\cos(c+dx)}{b} \right)^{2/3} (b \sec(c + dx))^{2/3} \right) \int \frac{1}{\left(\frac{\cos(c+dx)}{b} \right)^{8/3}} dx}{b^2} + \dots \\ &= \frac{3A {}_2F_1\left(-\frac{5}{6}, \frac{1}{2}; \frac{1}{6}; \cos^2(c + dx)\right) (b \sec(c + dx))^{5/3} \sin(c + dx)}{5bd\sqrt{\sin^2(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.26, size = 90, normalized size = 0.76

$$\frac{3(-\tan^2(c + dx))^{3/2} \csc^3(c + dx)(b \sec(c + dx))^{2/3} \left(11A \cos(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{4}{3}; \frac{7}{3}; \sec^2(c + dx)\right) + 8B {}_2F_1\left(\frac{1}{2}, \frac{11}{6}; \frac{17}{6}; \sec^2(c + dx)\right) \right)}{88d}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[c + d*x]^2*(b*Sec[c + d*x])^(2/3)*(A + B*Sec[c + d*x]),x]

[Out] (-3*Csc[c + d*x]^3*(11*A*Cos[c + d*x]*Hypergeometric2F1[1/2, 4/3, 7/3, Sec[c + d*x]^2] + 8*B*Hypergeometric2F1[1/2, 11/6, 17/6, Sec[c + d*x]^2])*(b*Sec[c + d*x])^(2/3)*(-Tan[c + d*x]^2)^(3/2))/(88*d)

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(B \sec(dx + c)^3 + A \sec(dx + c)^2\right) (b \sec(dx + c))^{2/3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] integral((B*sec(d*x + c)^3 + A*sec(d*x + c)^2)*(b*sec(d*x + c))^(2/3), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A) (b \sec(dx + c))^{\frac{2}{3}} \sec(dx + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(2/3)*sec(d*x + c)^2, x)

maple [F] time = 0.76, size = 0, normalized size = 0.00

$$\int (\sec^2(dx + c)) (b \sec(dx + c))^{\frac{2}{3}} (A + B \sec(dx + c)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^2*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x)

[Out] int(sec(d*x+c)^2*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A) (b \sec(dx + c))^{\frac{2}{3}} \sec(dx + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(2/3)*sec(d*x + c)^2, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(\frac{b}{\cos(c+dx)}\right)^{2/3}}{\cos(c+dx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(b/cos(c + d*x))^(2/3))/cos(c + d*x)^2,x)

[Out] `int(((A + B/cos(c + d*x))*(b/cos(c + d*x))^(2/3))/cos(c + d*x)^2, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \sec(c + dx))^{\frac{2}{3}} (A + B \sec(c + dx)) \sec^2(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)**2*(b*sec(d*x+c))**(2/3)*(A+B*sec(d*x+c)),x)`

[Out] `Integral((b*sec(c + d*x))**(2/3)*(A + B*sec(c + d*x))*sec(c + d*x)**2, x)`

3.8 $\int \sec(c+dx)(b \sec(c+dx))^{2/3}(A+B \sec(c+dx)) dx$

Optimal. Leaf size=116

$$\frac{3A \sin(c+dx)(b \sec(c+dx))^{2/3} {}_2F_1\left(-\frac{1}{3}, \frac{1}{2}; \frac{2}{3}; \cos^2(c+dx)\right)}{2d\sqrt{\sin^2(c+dx)}} + \frac{3B \sin(c+dx)(b \sec(c+dx))^{5/3} {}_2F_1\left(-\frac{5}{6}, \frac{1}{2}; \frac{1}{6}; \cos^2(c+dx)\right)}{5bd\sqrt{\sin^2(c+dx)}}$$

[Out] $3/2*A*\text{hypergeom}([-1/3, 1/2], [2/3], \cos(d*x+c)^2)*(b*\sec(d*x+c))^{(2/3)}*\sin(d*x+c)/d/(\sin(d*x+c)^2)^{(1/2)}+3/5*B*\text{hypergeom}([-5/6, 1/2], [1/6], \cos(d*x+c)^2)*(b*\sec(d*x+c))^{(5/3)}*\sin(d*x+c)/b/d/(\sin(d*x+c)^2)^{(1/2)}$

Rubi [A] time = 0.10, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {16, 3787, 3772, 2643}

$$\frac{3A \sin(c+dx)(b \sec(c+dx))^{2/3} {}_2F_1\left(-\frac{1}{3}, \frac{1}{2}; \frac{2}{3}; \cos^2(c+dx)\right)}{2d\sqrt{\sin^2(c+dx)}} + \frac{3B \sin(c+dx)(b \sec(c+dx))^{5/3} {}_2F_1\left(-\frac{5}{6}, \frac{1}{2}; \frac{1}{6}; \cos^2(c+dx)\right)}{5bd\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sec}[c+d*x]*(b*\text{Sec}[c+d*x])^{(2/3)}*(A+B*\text{Sec}[c+d*x]),x]$

[Out] $(3*A*\text{Hypergeometric2F1}[-1/3, 1/2, 2/3, \text{Cos}[c+d*x]^2]*(b*\text{Sec}[c+d*x])^{(2/3)}*\text{Sin}[c+d*x])/(2*d*\text{Sqrt}[\text{Sin}[c+d*x]^2]) + (3*B*\text{Hypergeometric2F1}[-5/6, 1/2, 1/6, \text{Cos}[c+d*x]^2]*(b*\text{Sec}[c+d*x])^{(5/3)}*\text{Sin}[c+d*x])/(5*b*d*\text{Sqrt}[\text{Sin}[c+d*x]^2])$

Rule 16

$\text{Int}[(u_*)^{(v_*)^{(m_*)}}*((b_*)^{(v_*)})^{(n_*)}, x_Symbol] \rightarrow \text{Dist}[1/b^m, \text{Int}[u*(b*v)^{(m+n)}, x], x] /; \text{FreeQ}[\{b, n\}, x] \ \&\amp; \ \text{IntegerQ}[m]$

Rule 2643

$\text{Int}[(b_*)*\sin[(c_*) + (d_*)*(x_*)]^{(n_*)}, x_Symbol] \rightarrow \text{Simp}[(\text{Cos}[c+d*x]*(b*\text{Sin}[c+d*x])^{(n+1)}*\text{Hypergeometric2F1}[1/2, (n+1)/2, (n+3)/2, \text{Sin}[c+d*x]^2])/(b*d*(n+1)*\text{Sqrt}[\text{Cos}[c+d*x]^2]), x] /; \text{FreeQ}[\{b, c, d, n\}, x] \ \&\amp; \ !\text{IntegerQ}[2*n]$

Rule 3772

$\text{Int}[(\text{csc}[(c_*) + (d_*)*(x_*)]*(b_*)^{(n_*)}, x_Symbol] \rightarrow \text{Simp}[(b*\text{Csc}[c+d*x])^{(n-1)}*((\text{Sin}[c+d*x]/b)^{(n-1)}*\text{Int}[1/(\text{Sin}[c+d*x]/b)^n, x]), x] /; \text{Fr}$

eeQ[{b, c, d, n}, x] && !IntegerQ[n]

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rubi steps

$$\begin{aligned} \int \sec(c + dx)(b \sec(c + dx))^{2/3}(A + B \sec(c + dx)) dx &= \frac{\int (b \sec(c + dx))^{5/3}(A + B \sec(c + dx)) dx}{b} \\ &= \frac{A \int (b \sec(c + dx))^{5/3} dx}{b} + \frac{B \int (b \sec(c + dx))^{8/3} dx}{b^2} \\ &= \frac{\left(A \left(\frac{\cos(c+dx)}{b} \right)^{2/3} (b \sec(c + dx))^{2/3} \right) \int \frac{1}{\left(\frac{\cos(c+dx)}{b} \right)^{5/3}} dx}{b} + \dots \\ &= \frac{3A {}_2F_1\left(-\frac{1}{3}, \frac{1}{2}; \frac{2}{3}; \cos^2(c + dx)\right) (b \sec(c + dx))^{2/3} \sin(c + dx)}{2d\sqrt{\sin^2(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.14, size = 91, normalized size = 0.78

$$\frac{3\sqrt{-\tan^2(c + dx)} \csc(c + dx)(b \sec(c + dx))^{5/3} \left(8A \cos(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{5}{6}; \frac{11}{6}; \sec^2(c + dx)\right) + 5B {}_2F_1\left(\frac{1}{2}, \frac{4}{3}; \frac{7}{3}; \sec^2(c + dx)\right) \right)}{40bd}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[c + d*x]*(b*Sec[c + d*x])^(2/3)*(A + B*Sec[c + d*x]),x]

[Out] (3*Csc[c + d*x]*(8*A*Cos[c + d*x]*Hypergeometric2F1[1/2, 5/6, 11/6, Sec[c + d*x]^2] + 5*B*Hypergeometric2F1[1/2, 4/3, 7/3, Sec[c + d*x]^2])*(b*Sec[c + d*x])^(5/3)*Sqrt[-Tan[c + d*x]^2])/(40*b*d)

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(B \sec(dx + c)^2 + A \sec(dx + c)\right) (b \sec(dx + c))^{2/3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] integral((B*sec(d*x + c)^2 + A*sec(d*x + c))*(b*sec(d*x + c))^(2/3), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A) (b \sec(dx + c))^{\frac{2}{3}} \sec(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(2/3)*sec(d*x + c), x)

maple [F] time = 0.71, size = 0, normalized size = 0.00

$$\int \sec(dx + c) (b \sec(dx + c))^{\frac{2}{3}} (A + B \sec(dx + c)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x)

[Out] int(sec(d*x+c)*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A) (b \sec(dx + c))^{\frac{2}{3}} \sec(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(2/3)*sec(d*x + c), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(\frac{b}{\cos(c+dx)}\right)^{2/3}}{\cos(c+dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(b/cos(c + d*x))^(2/3))/cos(c + d*x),x)

[Out] `int(((A + B/cos(c + d*x))*(b/cos(c + d*x))^(2/3))/cos(c + d*x), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \sec(c + dx))^{\frac{2}{3}} (A + B \sec(c + dx)) \sec(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)*(b*sec(d*x+c))**(2/3)*(A+B*sec(d*x+c)),x)`

[Out] `Integral((b*sec(c + d*x))**(2/3)*(A + B*sec(c + d*x))*sec(c + d*x), x)`

3.9 $\int (b \sec(c + dx))^{2/3} (A + B \sec(c + dx)) dx$

Optimal. Leaf size=112

$$\frac{3B \sin(c + dx)(b \sec(c + dx))^{2/3} {}_2F_1\left(-\frac{1}{3}, \frac{1}{2}; \frac{2}{3}; \cos^2(c + dx)\right)}{2d\sqrt{\sin^2(c + dx)}} - \frac{3Ab \sin(c + dx) {}_2F_1\left(\frac{1}{6}, \frac{1}{2}; \frac{7}{6}; \cos^2(c + dx)\right)}{d\sqrt{\sin^2(c + dx)} \sqrt[3]{b \sec(c + dx)}}$$

[Out] $-3*A*b*\text{hypergeom}([1/6, 1/2], [7/6], \cos(d*x+c)^2)*\sin(d*x+c)/d/(b*\sec(d*x+c))^{1/3}/(\sin(d*x+c)^2)^{1/2}+3/2*B*\text{hypergeom}([-1/3, 1/2], [2/3], \cos(d*x+c)^2)*(b*\sec(d*x+c))^{2/3}*\sin(d*x+c)/d/(\sin(d*x+c)^2)^{1/2}$

Rubi [A] time = 0.09, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3787, 3772, 2643}

$$\frac{3B \sin(c + dx)(b \sec(c + dx))^{2/3} {}_2F_1\left(-\frac{1}{3}, \frac{1}{2}; \frac{2}{3}; \cos^2(c + dx)\right)}{2d\sqrt{\sin^2(c + dx)}} - \frac{3Ab \sin(c + dx) {}_2F_1\left(\frac{1}{6}, \frac{1}{2}; \frac{7}{6}; \cos^2(c + dx)\right)}{d\sqrt{\sin^2(c + dx)} \sqrt[3]{b \sec(c + dx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(b*\text{Sec}[c + d*x])^{2/3}*(A + B*\text{Sec}[c + d*x]), x]$

[Out] $(-3*A*b*\text{Hypergeometric2F1}[1/6, 1/2, 7/6, \text{Cos}[c + d*x]^2]*\text{Sin}[c + d*x])/d*(b*\text{Sec}[c + d*x])^{1/3}*\text{Sqrt}[\text{Sin}[c + d*x]^2]) + (3*B*\text{Hypergeometric2F1}[-1/3, 1/2, 2/3, \text{Cos}[c + d*x]^2]*(b*\text{Sec}[c + d*x])^{2/3}*\text{Sin}[c + d*x])/(2*d*\text{Sqrt}[\text{Sin}[c + d*x]^2])$

Rule 2643

$\text{Int}[(b_*)*\sin[(c_*) + (d_*)*(x_*)]^{(n_*)}, x_Symbol] \rightarrow \text{Simp}[(\text{Cos}[c + d*x]*(b*\text{Sin}[c + d*x])^{(n + 1)}*\text{Hypergeometric2F1}[1/2, (n + 1)/2, (n + 3)/2, \text{Sin}[c + d*x]^2])/(b*d*(n + 1)*\text{Sqrt}[\text{Cos}[c + d*x]^2]), x] /;$ FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rule 3772

$\text{Int}[(\text{csc}[(c_*) + (d_*)*(x_*)]*(b_*))^{(n_*)}, x_Symbol] \rightarrow \text{Simp}[(b*\text{Csc}[c + d*x])^{(n - 1)}*((\text{Sin}[c + d*x]/b)^{(n - 1)}*\text{Int}[1/(\text{Sin}[c + d*x]/b)^n, x]), x] /;$ FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

Rule 3787

$\text{Int}[(\text{csc}[(e_*) + (f_*)*(x_*)]*(d_*))^{(n_*)}*(\text{csc}[(e_*) + (f_*)*(x_*)]*(b_*) + (a_*)), x_Symbol] \rightarrow \text{Dist}[a, \text{Int}[(d*\text{Csc}[e + f*x])^n, x], x] + \text{Dist}[b/d, \text{Int}[\text{csc}[(e_*) + (f_*)*(x_*)], x], x]$

$(d \cdot \text{Csc}[e + f \cdot x])^{(n + 1)}, x], x] /; \text{FreeQ}[\{a, b, d, e, f, n\}, x]$

Rubi steps

$$\begin{aligned} \int (b \sec(c + dx))^{2/3} (A + B \sec(c + dx)) dx &= A \int (b \sec(c + dx))^{2/3} dx + \frac{B \int (b \sec(c + dx))^{5/3} dx}{b} \\ &= \left(A \left(\frac{\cos(c + dx)}{b} \right)^{2/3} (b \sec(c + dx))^{2/3} \right) \int \frac{1}{\left(\frac{\cos(c + dx)}{b} \right)^{2/3}} dx + \frac{B \int \left(\frac{\cos(c + dx)}{b} \right)^{-5/3} dx}{b} \\ &= \frac{3B {}_2F_1\left(-\frac{1}{3}, \frac{1}{2}; \frac{2}{3}; \cos^2(c + dx)\right) (b \sec(c + dx))^{2/3} \sin(c + dx)}{2d \sqrt{\sin^2(c + dx)}} - \frac{3A}{b} \end{aligned}$$

Mathematica [A] time = 0.09, size = 88, normalized size = 0.79

$$\frac{3\sqrt{-\tan^2(c + dx)} \csc(c + dx) (b \sec(c + dx))^{2/3} \left(5A \cos(c + dx) {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; \sec^2(c + dx)\right) + 2B {}_2F_1\left(\frac{1}{2}, \frac{5}{6}; \frac{11}{6}; \sec^2(c + dx)\right) \right)}{10d}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Sec[c + d*x])^(2/3)*(A + B*Sec[c + d*x]),x]

[Out] (3*Csc[c + d*x]*(5*A*Cos[c + d*x]*Hypergeometric2F1[1/3, 1/2, 4/3, Sec[c + d*x]^2] + 2*B*Hypergeometric2F1[1/2, 5/6, 11/6, Sec[c + d*x]^2])*(b*Sec[c + d*x])^(2/3)*Sqrt[-Tan[c + d*x]^2])/(10*d)

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left((B \sec(dx + c) + A) (b \sec(dx + c))^{2/3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] integral((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(2/3), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A) (b \sec(dx + c))^{2/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(2/3), x)

maple [F] time = 0.97, size = 0, normalized size = 0.00

$$\int (b \sec(dx + c))^{\frac{2}{3}} (A + B \sec(dx + c)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x)

[Out] int((b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A) (b \sec(dx + c))^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(2/3), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A + \frac{B}{\cos(c + dx)} \right) \left(\frac{b}{\cos(c + dx)} \right)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))*(b/cos(c + d*x))^(2/3),x)

[Out] int((A + B/cos(c + d*x))*(b/cos(c + d*x))^(2/3), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \sec(c + dx))^{\frac{2}{3}} (A + B \sec(c + dx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*sec(d*x+c))**(2/3)*(A+B*sec(d*x+c)),x)

[Out] Integral((b*sec(c + d*x))**(2/3)*(A + B*sec(c + d*x)), x)

3.10 $\int \cos(c + dx)(b \sec(c + dx))^{2/3}(A + B \sec(c + dx)) dx$

Optimal. Leaf size=115

$$-\frac{3Ab^2 \sin(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; \cos^2(c + dx)\right)}{4d\sqrt{\sin^2(c + dx)}(b \sec(c + dx))^{4/3}} - \frac{3bB \sin(c + dx) {}_2F_1\left(\frac{1}{6}, \frac{1}{2}; \frac{7}{6}; \cos^2(c + dx)\right)}{d\sqrt{\sin^2(c + dx)}\sqrt[3]{b \sec(c + dx)}}$$

[Out] $-3/4*A*b^2*\text{hypergeom}([1/2, 2/3], [5/3], \cos(d*x+c)^2)*\sin(d*x+c)/d/(b*\sec(d*x+c))^{(4/3)}/(\sin(d*x+c)^2)^{(1/2)}-3*b*B*\text{hypergeom}([1/6, 1/2], [7/6], \cos(d*x+c)^2)*\sin(d*x+c)/d/(b*\sec(d*x+c))^{(1/3)}/(\sin(d*x+c)^2)^{(1/2)}$

Rubi [A] time = 0.10, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {16, 3787, 3772, 2643}

$$-\frac{3Ab^2 \sin(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; \cos^2(c + dx)\right)}{4d\sqrt{\sin^2(c + dx)}(b \sec(c + dx))^{4/3}} - \frac{3bB \sin(c + dx) {}_2F_1\left(\frac{1}{6}, \frac{1}{2}; \frac{7}{6}; \cos^2(c + dx)\right)}{d\sqrt{\sin^2(c + dx)}\sqrt[3]{b \sec(c + dx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Cos}[c + d*x]*(b*\text{Sec}[c + d*x])^{(2/3)}*(A + B*\text{Sec}[c + d*x]), x]$

[Out] $(-3*A*b^2*\text{Hypergeometric2F1}[1/2, 2/3, 5/3, \text{Cos}[c + d*x]^2]*\text{Sin}[c + d*x])/(4*d*(b*\text{Sec}[c + d*x])^{(4/3)}*\text{Sqrt}[\text{Sin}[c + d*x]^2]) - (3*b*B*\text{Hypergeometric2F1}[1/6, 1/2, 7/6, \text{Cos}[c + d*x]^2]*\text{Sin}[c + d*x])/(d*(b*\text{Sec}[c + d*x])^{(1/3)}*\text{Sqrt}[\text{Sin}[c + d*x]^2])$

Rule 16

$\text{Int}[(u_*)^{(v_*)^{(m_*)}}*((b_*)^{(v_*)})^{(n_*)}, x_Symbol] \rightarrow \text{Dist}[1/b^m, \text{Int}[u*(b*v)^{(m+n)}, x], x] /; \text{FreeQ}[\{b, n\}, x] \ \&\& \ \text{IntegerQ}[m]$

Rule 2643

$\text{Int}[(b_*)^{\sin[(c_*) + (d_*)*(x_)]}]^{(n_*)}, x_Symbol] \rightarrow \text{Simp}[(\text{Cos}[c + d*x]*(b*\text{Sin}[c + d*x])^{(n+1)}*\text{Hypergeometric2F1}[1/2, (n+1)/2, (n+3)/2, \text{Sin}[c + d*x]^2])/(b*d*(n+1)*\text{Sqrt}[\text{Cos}[c + d*x]^2]), x] /; \text{FreeQ}[\{b, c, d, n\}, x] \ \&\& \ !\text{IntegerQ}[2*n]$

Rule 3772

$\text{Int}[(\text{csc}[(c_*) + (d_*)*(x_)]*(b_*)^{(n_*)}, x_Symbol] \rightarrow \text{Simp}[(b*\text{Csc}[c + d*x])^{(n-1)}*((\text{Sin}[c + d*x]/b)^{(n-1)}*\text{Int}[1/(\text{Sin}[c + d*x]/b)^n, x]), x] /; \text{Fr}$

eeQ[{b, c, d, n}, x] && !IntegerQ[n]

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rubi steps

$$\begin{aligned} \int \cos(c + dx)(b \sec(c + dx))^{2/3}(A + B \sec(c + dx)) dx &= b \int \frac{A + B \sec(c + dx)}{\sqrt[3]{b \sec(c + dx)}} dx \\ &= (Ab) \int \frac{1}{\sqrt[3]{b \sec(c + dx)}} dx + B \int (b \sec(c + dx))^{2/3} dx \\ &= \left(Ab \left(\frac{\cos(c + dx)}{b} \right)^{2/3} (b \sec(c + dx))^{2/3} \right) \int \sqrt[3]{\frac{\cos(c + dx)}{b}} dx \\ &= -\frac{3B \cos(c + dx) {}_2F_1\left(\frac{1}{6}, \frac{1}{2}; \frac{7}{6}; \cos^2(c + dx)\right) (b \sec(c + dx))^{2/3}}{d \sqrt{\sin^2(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.10, size = 88, normalized size = 0.77

$$\frac{3\sqrt{-\tan^2(c + dx)} \cot(c + dx)(b \sec(c + dx))^{2/3} \left(2A \cos(c + dx) {}_2F_1\left(-\frac{1}{6}, \frac{1}{2}; \frac{5}{6}; \sec^2(c + dx)\right) - B {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; \sec^2(c + dx)\right) \right)}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]*(b*Sec[c + d*x])^(2/3)*(A + B*Sec[c + d*x]), x]

[Out] (-3*Cot[c + d*x]*(2*A*Cos[c + d*x]*Hypergeometric2F1[-1/6, 1/2, 5/6, Sec[c + d*x]^2] - B*Hypergeometric2F1[1/3, 1/2, 4/3, Sec[c + d*x]^2])*(b*Sec[c + d*x])^(2/3)*Sqrt[-Tan[c + d*x]^2])/(2*d)

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(B \cos(dx + c) \sec(dx + c) + A \cos(dx + c)\right) (b \sec(dx + c))^{2/3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] integral((B*cos(d*x + c)*sec(d*x + c) + A*cos(d*x + c))*(b*sec(d*x + c))^(2/3), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A) (b \sec(dx + c))^{\frac{2}{3}} \cos(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(2/3)*cos(d*x + c), x)

maple [F] time = 1.94, size = 0, normalized size = 0.00

$$\int \cos(dx + c) (b \sec(dx + c))^{\frac{2}{3}} (A + B \sec(dx + c)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x)

[Out] int(cos(d*x+c)*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A) (b \sec(dx + c))^{\frac{2}{3}} \cos(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(2/3)*cos(d*x + c), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c + dx) \left(A + \frac{B}{\cos(c + dx)} \right) \left(\frac{b}{\cos(c + dx)} \right)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)*(A + B/cos(c + d*x))*(b/cos(c + d*x))^(2/3),x)

```
[Out] int(cos(c + d*x)*(A + B/cos(c + d*x))*(b/cos(c + d*x))^(2/3), x)
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))**(2/3)*(A+B*sec(d*x+c)), x)
```

```
[Out] Timed out
```


3.11 $\int \cos^2(c + dx)(b \sec(c + dx))^{2/3}(A + B \sec(c + dx)) dx$

Optimal. Leaf size=119

$$\frac{3Ab^3 \sin(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{7}{6}; \frac{13}{6}; \cos^2(c + dx)\right)}{7d\sqrt{\sin^2(c + dx)}(b \sec(c + dx))^{7/3}} - \frac{3b^2B \sin(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; \cos^2(c + dx)\right)}{4d\sqrt{\sin^2(c + dx)}(b \sec(c + dx))^{4/3}}$$

[Out] $-3/7*A*b^3*\text{hypergeom}([1/2, 7/6], [13/6], \cos(d*x+c)^2)*\sin(d*x+c)/d/(b*\sec(d*x+c))^{(7/3)}/(\sin(d*x+c)^2)^{(1/2)}-3/4*b^2*B*\text{hypergeom}([1/2, 2/3], [5/3], \cos(d*x+c)^2)*\sin(d*x+c)/d/(b*\sec(d*x+c))^{(4/3)}/(\sin(d*x+c)^2)^{(1/2)}$

Rubi [A] time = 0.12, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {16, 3787, 3772, 2643}

$$\frac{3Ab^3 \sin(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{7}{6}; \frac{13}{6}; \cos^2(c + dx)\right)}{7d\sqrt{\sin^2(c + dx)}(b \sec(c + dx))^{7/3}} - \frac{3b^2B \sin(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; \cos^2(c + dx)\right)}{4d\sqrt{\sin^2(c + dx)}(b \sec(c + dx))^{4/3}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Cos}[c + d*x]^2*(b*\text{Sec}[c + d*x])^{(2/3)}*(A + B*\text{Sec}[c + d*x]), x]$

[Out] $(-3*A*b^3*\text{Hypergeometric2F1}[1/2, 7/6, 13/6, \text{Cos}[c + d*x]^2]*\text{Sin}[c + d*x])/((7*d*(b*\text{Sec}[c + d*x])^{(7/3)}*\text{Sqrt}[\text{Sin}[c + d*x]^2]) - (3*b^2*B*\text{Hypergeometric2F1}[1/2, 2/3, 5/3, \text{Cos}[c + d*x]^2]*\text{Sin}[c + d*x])/(4*d*(b*\text{Sec}[c + d*x])^{(4/3)}*\text{Sqrt}[\text{Sin}[c + d*x]^2])$

Rule 16

$\text{Int}[(u_*)^{(v_*)^{(m_*)}}*((b_*)^{(v_*)})^{(n_*)}, x_Symbol] \rightarrow \text{Dist}[1/b^m, \text{Int}[u*(b*v)^{(m+n)}, x], x] /; \text{FreeQ}[\{b, n\}, x] \ \&\& \ \text{IntegerQ}[m]$

Rule 2643

$\text{Int}[(b_*)*\sin[(c_*) + (d_*)*(x_*)]^{(n_*)}, x_Symbol] \rightarrow \text{Simp}[(\text{Cos}[c + d*x]*(b*\text{Sin}[c + d*x])^{(n+1)}*\text{Hypergeometric2F1}[1/2, (n+1)/2, (n+3)/2, \text{Sin}[c + d*x]^2])/(b*d*(n+1)*\text{Sqrt}[\text{Cos}[c + d*x]^2]), x] /; \text{FreeQ}[\{b, c, d, n\}, x] \ \&\& \ !\text{IntegerQ}[2*n]$

Rule 3772

$\text{Int}[(\text{csc}[(c_*) + (d_*)*(x_*)]*(b_*)^{(n_*)}, x_Symbol] \rightarrow \text{Simp}[(b*\text{Csc}[c + d*x])^{(n-1)}*((\text{Sin}[c + d*x]/b)^{(n-1)}*\text{Int}[1/(\text{Sin}[c + d*x]/b)^n, x]), x] /; \text{Fr}$

eeQ[{b, c, d, n}, x] && !IntegerQ[n]

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n_*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rubi steps

$$\begin{aligned} \int \cos^2(c + dx)(b \sec(c + dx))^{2/3}(A + B \sec(c + dx)) dx &= b^2 \int \frac{A + B \sec(c + dx)}{(b \sec(c + dx))^{4/3}} dx \\ &= (Ab^2) \int \frac{1}{(b \sec(c + dx))^{4/3}} dx + (bB) \int \frac{1}{\sqrt[3]{b \sec(c + dx)}} dx \\ &= \left(Ab^2 \left(\frac{\cos(c + dx)}{b} \right)^{2/3} (b \sec(c + dx))^{2/3} \right) \int \left(\frac{\cos(c + dx)}{b} \right) dx \\ &= -\frac{3B \cos^2(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; \cos^2(c + dx)\right) (b \sec(c + dx))^{2/3}}{4d\sqrt{\sin^2(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.14, size = 88, normalized size = 0.74

$$\frac{3b\sqrt{-\tan^2(c + dx)} \cot(c + dx) \left(A \cos(c + dx) {}_2F_1\left(-\frac{2}{3}, \frac{1}{2}; \frac{1}{3}; \sec^2(c + dx)\right) + 4B {}_2F_1\left(-\frac{1}{6}, \frac{1}{2}; \frac{5}{6}; \sec^2(c + dx)\right) \right)}{4d\sqrt[3]{b \sec(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]^2*(b*Sec[c + d*x])^(2/3)*(A + B*Sec[c + d*x]),x]

[Out] (-3*b*Cot[c + d*x]*(A*Cos[c + d*x]*Hypergeometric2F1[-2/3, 1/2, 1/3, Sec[c + d*x]^2] + 4*B*Hypergeometric2F1[-1/6, 1/2, 5/6, Sec[c + d*x]^2])*Sqrt[-Tan[c + d*x]^2])/(4*d*(b*Sec[c + d*x])^(1/3))

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(B \cos(dx + c)^2 \sec(dx + c) + A \cos(dx + c)^2\right) (b \sec(dx + c))^{2/3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] integral((B*cos(d*x + c)^2*sec(d*x + c) + A*cos(d*x + c)^2)*(b*sec(d*x + c))^(2/3), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A) (b \sec(dx + c))^{\frac{2}{3}} \cos(dx + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(2/3)*cos(d*x + c)^2, x)

maple [F] time = 2.66, size = 0, normalized size = 0.00

$$\int (\cos^2(dx + c)) (b \sec(dx + c))^{\frac{2}{3}} (A + B \sec(dx + c)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^2*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x)

[Out] int(cos(d*x+c)^2*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A) (b \sec(dx + c))^{\frac{2}{3}} \cos(dx + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(2/3)*cos(d*x + c)^2, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c + dx)^2 \left(A + \frac{B}{\cos(c + dx)} \right) \left(\frac{b}{\cos(c + dx)} \right)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^2*(A + B/cos(c + d*x))*(b/cos(c + d*x))^(2/3),x)

```
[Out] int(cos(c + d*x)^2*(A + B/cos(c + d*x))*(b/cos(c + d*x))^(2/3), x)
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(b*sec(d*x+c))**(2/3)*(A+B*sec(d*x+c)), x)
```

```
[Out] Timed out
```

3.12 $\int \sec^2(c + dx)(b \sec(c + dx))^{4/3}(A + B \sec(c + dx)) dx$

Optimal. Leaf size=119

$$\frac{3A \sin(c + dx)(b \sec(c + dx))^{7/3} {}_2F_1\left(-\frac{7}{6}, \frac{1}{2}; -\frac{1}{6}; \cos^2(c + dx)\right)}{7bd\sqrt{\sin^2(c + dx)}} + \frac{3B \sin(c + dx)(b \sec(c + dx))^{10/3} {}_2F_1\left(-\frac{5}{3}, \frac{1}{2}; -\frac{2}{3}; \cos^2(c + dx)\right)}{10b^2d\sqrt{\sin^2(c + dx)}}$$

[Out] $3/7*A*\text{hypergeom}([-7/6, 1/2], [-1/6], \cos(d*x+c)^2)*(b*\sec(d*x+c))^{(7/3)*\sin(d*x+c)/b/d/(\sin(d*x+c)^2)^{(1/2)}+3/10*B*\text{hypergeom}([-5/3, 1/2], [-2/3], \cos(d*x+c)^2)*(b*\sec(d*x+c))^{(10/3)*\sin(d*x+c)/b^2/d/(\sin(d*x+c)^2)^{(1/2)}$

Rubi [A] time = 0.10, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {16, 3787, 3772, 2643}

$$\frac{3A \sin(c + dx)(b \sec(c + dx))^{7/3} {}_2F_1\left(-\frac{7}{6}, \frac{1}{2}; -\frac{1}{6}; \cos^2(c + dx)\right)}{7bd\sqrt{\sin^2(c + dx)}} + \frac{3B \sin(c + dx)(b \sec(c + dx))^{10/3} {}_2F_1\left(-\frac{5}{3}, \frac{1}{2}; -\frac{2}{3}; \cos^2(c + dx)\right)}{10b^2d\sqrt{\sin^2(c + dx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sec}[c + d*x]^2*(b*\text{Sec}[c + d*x])^{(4/3)}*(A + B*\text{Sec}[c + d*x]), x]$

[Out] $(3*A*\text{Hypergeometric2F1}[-7/6, 1/2, -1/6, \text{Cos}[c + d*x]^2]*(b*\text{Sec}[c + d*x])^{(7/3)*\text{Sin}[c + d*x]})/(7*b*d*\text{Sqrt}[\text{Sin}[c + d*x]^2]) + (3*B*\text{Hypergeometric2F1}[-5/3, 1/2, -2/3, \text{Cos}[c + d*x]^2]*(b*\text{Sec}[c + d*x])^{(10/3)*\text{Sin}[c + d*x]})/(10*b^2*d*\text{Sqrt}[\text{Sin}[c + d*x]^2])$

Rule 16

$\text{Int}[(u_*)^{(v_*)^{(m_*)}}*((b_*)^{(v_*)})^{(n_*)}, x_Symbol] \rightarrow \text{Dist}[1/b^m, \text{Int}[u*(b*v)^{(m+n)}, x], x] /; \text{FreeQ}\{b, n, x\} \ \&\& \ \text{IntegerQ}[m]$

Rule 2643

$\text{Int}[(b_*)^{\sin(c_*) + (d_*)*(x_*)}^{(n_*)}, x_Symbol] \rightarrow \text{Simp}[(\text{Cos}[c + d*x]*(b*\text{Sin}[c + d*x])^{(n+1)}*\text{Hypergeometric2F1}[1/2, (n+1)/2, (n+3)/2, \text{Sin}[c + d*x]^2])/(b*d*(n+1)*\text{Sqrt}[\text{Cos}[c + d*x]^2]), x] /; \text{FreeQ}\{b, c, d, n, x\} \ \&\& \ !\text{IntegerQ}[2*n]$

Rule 3772

$\text{Int}[(\text{csc}[(c_*) + (d_*)*(x_*)]*(b_*)^{(n_*)}, x_Symbol] \rightarrow \text{Simp}[(b*\text{Csc}[c + d*x])^{(n-1)}*((\text{Sin}[c + d*x]/b)^{(n-1)}*\text{Int}[1/(\text{Sin}[c + d*x]/b)^n, x]), x] /; \text{Fr}$

eeQ[{b, c, d, n}, x] && !IntegerQ[n]

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n_.*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rubi steps

$$\begin{aligned} \int \sec^2(c + dx)(b \sec(c + dx))^{4/3}(A + B \sec(c + dx)) dx &= \frac{\int (b \sec(c + dx))^{10/3}(A + B \sec(c + dx)) dx}{b^2} \\ &= \frac{A \int (b \sec(c + dx))^{10/3} dx}{b^2} + \frac{B \int (b \sec(c + dx))^{13/3} dx}{b^3} \\ &= \frac{\left(A \sqrt[3]{\frac{\cos(c+dx)}{b}} \sqrt[3]{b \sec(c + dx)}\right) \int \frac{1}{\left(\frac{\cos(c+dx)}{b}\right)^{10/3}} dx}{b^2} + \frac{\left(B \sqrt[3]{\frac{\cos(c+dx)}{b}} \sqrt[3]{b \sec(c + dx)}\right) \int \frac{1}{\left(\frac{\cos(c+dx)}{b}\right)^{13/3}} dx}{b^3} \\ &= \frac{3Ab {}_2F_1\left(-\frac{7}{6}, \frac{1}{2}; -\frac{1}{6}; \cos^2(c + dx)\right) \sec(c + dx) \sqrt[3]{b \sec(c + dx)}}{7d \sqrt{\sin^2(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.30, size = 90, normalized size = 0.76

$$\frac{3(-\tan^2(c + dx))^{3/2} \csc^3(c + dx)(b \sec(c + dx))^{4/3} \left(13A \cos(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{5}{3}, \frac{8}{3}; \sec^2(c + dx)\right) + 10B {}_2F_1\left(\frac{1}{2}, \frac{13}{6}, \frac{19}{6}; \sec^2(c + dx)\right)\right) (b \sec(c + dx))^{4/3} (-\tan^2(c + dx))^{3/2}}{130d}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[c + d*x]^2*(b*Sec[c + d*x])^(4/3)*(A + B*Sec[c + d*x]),x]

[Out] (-3*Csc[c + d*x]^3*(13*A*Cos[c + d*x]*Hypergeometric2F1[1/2, 5/3, 8/3, Sec[c + d*x]^2] + 10*B*Hypergeometric2F1[1/2, 13/6, 19/6, Sec[c + d*x]^2])*(b*Sec[c + d*x])^(4/3)*(-Tan[c + d*x]^2)^(3/2))/(130*d)

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb \sec(dx + c)^4 + Ab \sec(dx + c)^3\right) (b \sec(dx + c))^{1/3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] integral((B*b*sec(d*x + c)^4 + A*b*sec(d*x + c)^3)*(b*sec(d*x + c))^(1/3), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A) (b \sec(dx + c))^{\frac{4}{3}} \sec(dx + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(4/3)*sec(d*x + c)^2, x)

maple [F] time = 0.77, size = 0, normalized size = 0.00

$$\int (\sec^2(dx + c)) (b \sec(dx + c))^{\frac{4}{3}} (A + B \sec(dx + c)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^2*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)), x)

[Out] int(sec(d*x+c)^2*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)), x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A) (b \sec(dx + c))^{\frac{4}{3}} \sec(dx + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(4/3)*sec(d*x + c)^2, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(\frac{b}{\cos(c+dx)}\right)^{4/3}}{\cos(c+dx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(b/cos(c + d*x))^(4/3))/cos(c + d*x)^2,x)
```

```
[Out] int(((A + B/cos(c + d*x))*(b/cos(c + d*x))^(4/3))/cos(c + d*x)^2, x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(b*sec(d*x+c))**(4/3)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```


3.13 $\int \sec(c + dx)(b \sec(c + dx))^{4/3}(A + B \sec(c + dx)) dx$

Optimal. Leaf size=116

$$\frac{3A \sin(c + dx)(b \sec(c + dx))^{4/3} {}_2F_1\left(-\frac{2}{3}, \frac{1}{2}; \frac{1}{3}; \cos^2(c + dx)\right)}{4d\sqrt{\sin^2(c + dx)}} + \frac{3B \sin(c + dx)(b \sec(c + dx))^{7/3} {}_2F_1\left(-\frac{7}{6}, \frac{1}{2}; -\frac{1}{6}; \cos^2(c + dx)\right)}{7bd\sqrt{\sin^2(c + dx)}}$$

[Out] $3/4*A*\text{hypergeom}([-2/3, 1/2], [1/3], \cos(d*x+c)^2)*(b*\sec(d*x+c))^{(4/3)}*\sin(d*x+c)/d/(\sin(d*x+c)^2)^{(1/2)}+3/7*B*\text{hypergeom}([-7/6, 1/2], [-1/6], \cos(d*x+c)^2)*(b*\sec(d*x+c))^{(7/3)}*\sin(d*x+c)/b/d/(\sin(d*x+c)^2)^{(1/2)}$

Rubi [A] time = 0.10, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {16, 3787, 3772, 2643}

$$\frac{3A \sin(c + dx)(b \sec(c + dx))^{4/3} {}_2F_1\left(-\frac{2}{3}, \frac{1}{2}; \frac{1}{3}; \cos^2(c + dx)\right)}{4d\sqrt{\sin^2(c + dx)}} + \frac{3B \sin(c + dx)(b \sec(c + dx))^{7/3} {}_2F_1\left(-\frac{7}{6}, \frac{1}{2}; -\frac{1}{6}; \cos^2(c + dx)\right)}{7bd\sqrt{\sin^2(c + dx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sec}[c + d*x]*(b*\text{Sec}[c + d*x])^{(4/3)}*(A + B*\text{Sec}[c + d*x]), x]$

[Out] $(3*A*\text{Hypergeometric2F1}[-2/3, 1/2, 1/3, \text{Cos}[c + d*x]^2]*(b*\text{Sec}[c + d*x])^{(4/3)}*\text{Sin}[c + d*x])/(4*d*\text{Sqrt}[\text{Sin}[c + d*x]^2]) + (3*B*\text{Hypergeometric2F1}[-7/6, 1/2, -1/6, \text{Cos}[c + d*x]^2]*(b*\text{Sec}[c + d*x])^{(7/3)}*\text{Sin}[c + d*x])/(7*b*d*\text{Sqrt}[\text{Sin}[c + d*x]^2])$

Rule 16

$\text{Int}[(u_*)^{(v_*)}*(b_*)^{(v_*)}*(v_*)^{(n_*)}, x_Symbol] \rightarrow \text{Dist}[1/b^m, \text{Int}[u*(b*v)^{(m+n)}, x], x] /; \text{FreeQ}\{b, n, x\} \ \&\& \ \text{IntegerQ}[m]$

Rule 2643

$\text{Int}[(b_*)*\sin[(c_*) + (d_*)*(x_*)]^{(n_*)}, x_Symbol] \rightarrow \text{Simp}[(\text{Cos}[c + d*x]*(b*\text{Sin}[c + d*x])^{(n+1)}*\text{Hypergeometric2F1}[1/2, (n+1)/2, (n+3)/2, \text{Sin}[c + d*x]^2])/(b*d*(n+1)*\text{Sqrt}[\text{Cos}[c + d*x]^2]), x] /; \text{FreeQ}\{b, c, d, n, x\} \ \&\& \ !\text{IntegerQ}[2*n]$

Rule 3772

$\text{Int}[(\text{csc}[(c_*) + (d_*)*(x_*)]*(b_*)^{(n_*)}, x_Symbol] \rightarrow \text{Simp}[(b*\text{Csc}[c + d*x])^{(n-1)}*((\text{Sin}[c + d*x]/b)^{(n-1)}*\text{Int}[1/(\text{Sin}[c + d*x]/b)^n, x]), x] /; \text{FreeQ}\{b, c, d, n, x\}$

eeQ[{b, c, d, n}, x] && !IntegerQ[n]

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rubi steps

$$\begin{aligned} \int \sec(c + dx)(b \sec(c + dx))^{4/3}(A + B \sec(c + dx)) dx &= \frac{\int (b \sec(c + dx))^{7/3}(A + B \sec(c + dx)) dx}{b} \\ &= \frac{A \int (b \sec(c + dx))^{7/3} dx}{b} + \frac{B \int (b \sec(c + dx))^{10/3} dx}{b^2} \\ &= \frac{\left(A \sqrt[3]{\frac{\cos(c+dx)}{b}} \sqrt[3]{b \sec(c + dx)} \right) \int \frac{1}{\left(\frac{\cos(c+dx)}{b} \right)^{7/3}} dx}{b} + \frac{\left(B \sqrt[3]{\frac{\cos(c+dx)}{b}} \sqrt[3]{b \sec(c + dx)} \right) \int \frac{1}{\left(\frac{\cos(c+dx)}{b} \right)^{10/3}} dx}{b} \\ &= \frac{3A {}_2F_1\left(-\frac{2}{3}, \frac{1}{2}; \frac{1}{3}; \cos^2(c + dx)\right) (b \sec(c + dx))^{4/3} \sin(c + dx)}{4d \sqrt{\sin^2(c + dx)}} + \frac{7B {}_2F_1\left(\frac{1}{2}, \frac{5}{3}; \frac{8}{3}; \sec^2(c + dx)\right) (b \sec(c + dx))^{7/3} \sin(c + dx)}{4d \sqrt{\sin^2(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.15, size = 91, normalized size = 0.78

$$\frac{3\sqrt{-\tan^2(c + dx)} \csc(c + dx)(b \sec(c + dx))^{7/3} \left(10A \cos(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{7}{6}; \frac{13}{6}; \sec^2(c + dx)\right) + 7B {}_2F_1\left(\frac{1}{2}, \frac{5}{3}; \frac{8}{3}; \sec^2(c + dx)\right) \right)}{70bd}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[c + d*x]*(b*Sec[c + d*x])^(4/3)*(A + B*Sec[c + d*x]), x]

[Out] (3*Csc[c + d*x]*(10*A*Cos[c + d*x]*Hypergeometric2F1[1/2, 7/6, 13/6, Sec[c + d*x]^2] + 7*B*Hypergeometric2F1[1/2, 5/3, 8/3, Sec[c + d*x]^2])*(b*Sec[c + d*x])^(7/3)*Sqrt[-Tan[c + d*x]^2])/(70*b*d)

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb \sec(dx + c)^3 + Ab \sec(dx + c)^2\right) (b \sec(dx + c))^{1/3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] integral((B*b*sec(d*x + c)^3 + A*b*sec(d*x + c)^2)*(b*sec(d*x + c))^(1/3), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A) (b \sec(dx + c))^{\frac{4}{3}} \sec(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(4/3)*sec(d*x + c), x)

maple [F] time = 0.75, size = 0, normalized size = 0.00

$$\int \sec(dx + c) (b \sec(dx + c))^{\frac{4}{3}} (A + B \sec(dx + c)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)), x)

[Out] int(sec(d*x+c)*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)), x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A) (b \sec(dx + c))^{\frac{4}{3}} \sec(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(4/3)*sec(d*x + c), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(\frac{b}{\cos(c+dx)}\right)^{4/3}}{\cos(c+dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(b/cos(c + d*x))^(4/3))/cos(c + d*x), x)
```

```
[Out] int(((A + B/cos(c + d*x))*(b/cos(c + d*x))^(4/3))/cos(c + d*x), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))**(4/3)*(A+B*sec(d*x+c)), x)
```

```
[Out] Timed out
```

3.14 $\int (b \sec(c + dx))^{4/3} (A + B \sec(c + dx)) dx$

Optimal. Leaf size=112

$$\frac{3Ab \sin(c + dx) \sqrt[3]{b \sec(c + dx)} {}_2F_1\left(-\frac{1}{6}, \frac{1}{2}; \frac{5}{6}; \cos^2(c + dx)\right)}{d\sqrt{\sin^2(c + dx)}} + \frac{3B \sin(c + dx) (b \sec(c + dx))^{4/3} {}_2F_1\left(-\frac{2}{3}, \frac{1}{2}; \frac{1}{3}; \cos^2(c + dx)\right)}{4d\sqrt{\sin^2(c + dx)}}$$

[Out] 3*A*b*hypergeom([-1/6, 1/2], [5/6], cos(d*x+c)^2)*(b*sec(d*x+c))^(1/3)*sin(d*x+c)/d/(sin(d*x+c)^2)^(1/2)+3/4*B*hypergeom([-2/3, 1/2], [1/3], cos(d*x+c)^2)*(b*sec(d*x+c))^(4/3)*sin(d*x+c)/d/(sin(d*x+c)^2)^(1/2)

Rubi [A] time = 0.09, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3787, 3772, 2643}

$$\frac{3Ab \sin(c + dx) \sqrt[3]{b \sec(c + dx)} {}_2F_1\left(-\frac{1}{6}, \frac{1}{2}; \frac{5}{6}; \cos^2(c + dx)\right)}{d\sqrt{\sin^2(c + dx)}} + \frac{3B \sin(c + dx) (b \sec(c + dx))^{4/3} {}_2F_1\left(-\frac{2}{3}, \frac{1}{2}; \frac{1}{3}; \cos^2(c + dx)\right)}{4d\sqrt{\sin^2(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[(b*Sec[c + d*x])^(4/3)*(A + B*Sec[c + d*x]),x]

[Out] (3*A*b*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(1/3)*Sin[c + d*x])/(d*Sqrt[Sin[c + d*x]^2]) + (3*B*Hypergeometric2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(4/3)*Sin[c + d*x])/(4*d*Sqrt[Sin[c + d*x]^2])

Rule 2643

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rule 3772

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x])^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[

$(d \cdot \text{Csc}[e + f \cdot x])^{(n + 1)}, x], x] /; \text{FreeQ}[\{a, b, d, e, f, n\}, x]$

Rubi steps

$$\begin{aligned} \int (b \sec(c + dx))^{4/3} (A + B \sec(c + dx)) dx &= A \int (b \sec(c + dx))^{4/3} dx + \frac{B \int (b \sec(c + dx))^{7/3} dx}{b} \\ &= \left(A \sqrt[3]{\frac{\cos(c + dx)}{b}} \sqrt[3]{b \sec(c + dx)} \right) \int \frac{1}{\left(\frac{\cos(c + dx)}{b}\right)^{4/3}} dx + \frac{\left(B \sqrt[3]{\frac{\cos(c + dx)}{b}} \right)}{\left(\frac{\cos(c + dx)}{b}\right)^{4/3}} \\ &= \frac{3Ab {}_2F_1\left(-\frac{1}{6}, \frac{1}{2}; \frac{5}{6}; \cos^2(c + dx)\right) \sqrt[3]{b \sec(c + dx)} \sin(c + dx)}{d \sqrt{\sin^2(c + dx)}} + \frac{3B {}_2F_1\left(\frac{1}{2}, \frac{7}{6}; \frac{13}{6}; \sec^2(c + dx)\right) \sqrt[3]{b \sec(c + dx)}}{d \sqrt{\sin^2(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.11, size = 88, normalized size = 0.79

$$\frac{3\sqrt{-\tan^2(c + dx)} \csc(c + dx) (b \sec(c + dx))^{4/3} \left(7A \cos(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; \sec^2(c + dx)\right) + 4B {}_2F_1\left(\frac{1}{2}, \frac{7}{6}; \frac{13}{6}; \sec^2(c + dx)\right) \right)}{28d}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Sec[c + d*x])^(4/3)*(A + B*Sec[c + d*x]),x]

[Out] (3*Csc[c + d*x]*(7*A*Cos[c + d*x]*Hypergeometric2F1[1/2, 2/3, 5/3, Sec[c + d*x]^2] + 4*B*Hypergeometric2F1[1/2, 7/6, 13/6, Sec[c + d*x]^2])*(b*Sec[c + d*x])^(4/3)*Sqrt[-Tan[c + d*x]^2])/(28*d)

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb \sec(dx + c)^2 + Ab \sec(dx + c)\right) (b \sec(dx + c))^{1/3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] integral((B*b*sec(d*x + c)^2 + A*b*sec(d*x + c))*(b*sec(d*x + c))^(1/3), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A) (b \sec(dx + c))^{4/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(4/3), x)

maple [F] time = 0.98, size = 0, normalized size = 0.00

$$\int (b \sec(dx + c))^{\frac{4}{3}} (A + B \sec(dx + c)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x)

[Out] int((b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A) (b \sec(dx + c))^{\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(4/3), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A + \frac{B}{\cos(c + dx)} \right) \left(\frac{b}{\cos(c + dx)} \right)^{\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))*(b/cos(c + d*x))^(4/3),x)

[Out] int((A + B/cos(c + d*x))*(b/cos(c + d*x))^(4/3), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \sec(c + dx))^{\frac{4}{3}} (A + B \sec(c + dx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*sec(d*x+c))**(4/3)*(A+B*sec(d*x+c)),x)

[Out] Integral((b*sec(c + d*x))**(4/3)*(A + B*sec(c + d*x)), x)

3.15 $\int \cos(c + dx)(b \sec(c + dx))^{4/3}(A + B \sec(c + dx)) dx$

Optimal. Leaf size=115

$$\frac{3bB \sin(c + dx) \sqrt[3]{b \sec(c + dx)} {}_2F_1\left(-\frac{1}{6}, \frac{1}{2}; \frac{5}{6}; \cos^2(c + dx)\right)}{d\sqrt{\sin^2(c + dx)}} - \frac{3Ab^2 \sin(c + dx) {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; \cos^2(c + dx)\right)}{2d\sqrt{\sin^2(c + dx)} (b \sec(c + dx))^{2/3}}$$

[Out] $-3/2*A*b^2*\text{hypergeom}([1/3, 1/2], [4/3], \cos(d*x+c)^2)*\sin(d*x+c)/d/(b*\sec(d*x+c))^{2/3}/(\sin(d*x+c)^2)^{1/2}+3*b*B*\text{hypergeom}([-1/6, 1/2], [5/6], \cos(d*x+c)^2)*(b*\sec(d*x+c))^{1/3}*\sin(d*x+c)/d/(\sin(d*x+c)^2)^{1/2}$

Rubi [A] time = 0.10, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {16, 3787, 3772, 2643}

$$\frac{3bB \sin(c + dx) \sqrt[3]{b \sec(c + dx)} {}_2F_1\left(-\frac{1}{6}, \frac{1}{2}; \frac{5}{6}; \cos^2(c + dx)\right)}{d\sqrt{\sin^2(c + dx)}} - \frac{3Ab^2 \sin(c + dx) {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; \cos^2(c + dx)\right)}{2d\sqrt{\sin^2(c + dx)} (b \sec(c + dx))^{2/3}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Cos}[c + d*x]*(b*\text{Sec}[c + d*x])^{4/3}*(A + B*\text{Sec}[c + d*x]), x]$

[Out] $(-3*A*b^2*\text{Hypergeometric2F1}[1/3, 1/2, 4/3, \text{Cos}[c + d*x]^2]*\text{Sin}[c + d*x])/(2*d*(b*\text{Sec}[c + d*x])^{2/3}*\text{Sqrt}[\text{Sin}[c + d*x]^2]) + (3*b*B*\text{Hypergeometric2F1}[-1/6, 1/2, 5/6, \text{Cos}[c + d*x]^2]*(b*\text{Sec}[c + d*x])^{1/3}*\text{Sin}[c + d*x])/(d*\text{Sqrt}[\text{Sin}[c + d*x]^2])$

Rule 16

$\text{Int}[(u_*)^{(v_*)^{(m_*)}}*((b_*)^{(v_*)})^{(n_*)}, x_Symbol] \rightarrow \text{Dist}[1/b^m, \text{Int}[u*(b*v)^{(m+n)}, x], x] /; \text{FreeQ}\{b, n\}, x \ \&\& \ \text{IntegerQ}[m]$

Rule 2643

$\text{Int}[(b_*)*\sin[(c_*) + (d_*)*(x_*)]^{(n_*)}, x_Symbol] \rightarrow \text{Simp}[(\text{Cos}[c + d*x]*(b*\text{Sin}[c + d*x])^{(n+1)}*\text{Hypergeometric2F1}[1/2, (n+1)/2, (n+3)/2, \text{Sin}[c + d*x]^2])/(b*d*(n+1)*\text{Sqrt}[\text{Cos}[c + d*x]^2]), x] /; \text{FreeQ}\{b, c, d, n\}, x \ \&\& \ !\text{IntegerQ}[2*n]$

Rule 3772

$\text{Int}[(\text{csc}[(c_*) + (d_*)*(x_*)]*(b_*)^{(n_*)}), x_Symbol] \rightarrow \text{Simp}[(b*\text{Csc}[c + d*x])^{(n-1)}*((\text{Sin}[c + d*x]/b)^{(n-1)}*\text{Int}[1/(\text{Sin}[c + d*x]/b)^n, x]), x] /; \text{Fr}$

eeQ[{b, c, d, n}, x] && !IntegerQ[n]

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rubi steps

$$\begin{aligned} \int \cos(c + dx)(b \sec(c + dx))^{4/3}(A + B \sec(c + dx)) dx &= b \int \sqrt[3]{b \sec(c + dx)} (A + B \sec(c + dx)) dx \\ &= (Ab) \int \sqrt[3]{b \sec(c + dx)} dx + B \int (b \sec(c + dx))^{4/3} dx \\ &= \left(Ab \sqrt[3]{\frac{\cos(c + dx)}{b}} \sqrt[3]{b \sec(c + dx)} \right) \int \frac{1}{\sqrt[3]{\frac{\cos(c + dx)}{b}}} dx + \\ &= \frac{3bB {}_2F_1\left(-\frac{1}{6}, \frac{1}{2}; \frac{5}{6}; \cos^2(c + dx)\right) \sqrt[3]{b \sec(c + dx)} \sin(c + dx)}{d \sqrt{\sin^2(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.12, size = 87, normalized size = 0.76

$$\frac{3\sqrt{-\tan^2(c + dx)} \cot(c + dx)(b \sec(c + dx))^{4/3} \left(4A \cos(c + dx) {}_2F_1\left(\frac{1}{6}, \frac{1}{2}; \frac{7}{6}; \sec^2(c + dx)\right) + B {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; \sec^2(c + dx)\right) \right)}{4d}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]*(b*Sec[c + d*x])^(4/3)*(A + B*Sec[c + d*x]),x]

[Out] (3*Cot[c + d*x]*(4*A*Cos[c + d*x]*Hypergeometric2F1[1/6, 1/2, 7/6, Sec[c + d*x]^2] + B*Hypergeometric2F1[1/2, 2/3, 5/3, Sec[c + d*x]^2])*(b*Sec[c + d*x])^(4/3)*Sqrt[-Tan[c + d*x]^2])/(4*d)

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb \cos(dx + c) \sec(dx + c)^2 + Ab \cos(dx + c) \sec(dx + c)\right) (b \sec(dx + c))^{1/3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] `integral((B*b*cos(d*x + c)*sec(d*x + c)^2 + A*b*cos(d*x + c)*sec(d*x + c))*(b*sec(d*x + c))^(1/3), x)`

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A) (b \sec(dx + c))^{\frac{4}{3}} \cos(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x, algorithm="giac")`

[Out] `integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(4/3)*cos(d*x + c), x)`

maple [F] time = 2.16, size = 0, normalized size = 0.00

$$\int \cos(dx + c) (b \sec(dx + c))^{\frac{4}{3}} (A + B \sec(dx + c)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(d*x+c)*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x)`

[Out] `int(cos(d*x+c)*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x)`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A) (b \sec(dx + c))^{\frac{4}{3}} \cos(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x, algorithm="maxima")`

[Out] `integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(4/3)*cos(d*x + c), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c + dx) \left(A + \frac{B}{\cos(c + dx)} \right) \left(\frac{b}{\cos(c + dx)} \right)^{\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(c + d*x)*(A + B/cos(c + d*x))*(b/cos(c + d*x))^(4/3),x)`

[Out] `int(cos(c + d*x)*(A + B/cos(c + d*x))*(b/cos(c + d*x))^(4/3), x)`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(b*sec(d*x+c))**(4/3)*(A+B*sec(d*x+c)),x)

[Out] Timed out

3.16 $\int \cos^2(c + dx)(b \sec(c + dx))^{4/3}(A + B \sec(c + dx)) dx$

Optimal. Leaf size=119

$$-\frac{3Ab^3 \sin(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{5}{6}; \frac{11}{6}; \cos^2(c + dx)\right)}{5d\sqrt{\sin^2(c + dx)}(b \sec(c + dx))^{5/3}} - \frac{3b^2B \sin(c + dx) {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; \cos^2(c + dx)\right)}{2d\sqrt{\sin^2(c + dx)}(b \sec(c + dx))^{2/3}}$$

[Out] $-3/5*A*b^3*\text{hypergeom}([1/2, 5/6], [11/6], \cos(d*x+c)^2)*\sin(d*x+c)/d/(b*\sec(d*x+c))^{5/3}/(\sin(d*x+c)^2)^{(1/2)} - 3/2*b^2*B*\text{hypergeom}([1/3, 1/2], [4/3], \cos(d*x+c)^2)*\sin(d*x+c)/d/(b*\sec(d*x+c))^{2/3}/(\sin(d*x+c)^2)^{(1/2)}$

Rubi [A] time = 0.12, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {16, 3787, 3772, 2643}

$$-\frac{3Ab^3 \sin(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{5}{6}; \frac{11}{6}; \cos^2(c + dx)\right)}{5d\sqrt{\sin^2(c + dx)}(b \sec(c + dx))^{5/3}} - \frac{3b^2B \sin(c + dx) {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; \cos^2(c + dx)\right)}{2d\sqrt{\sin^2(c + dx)}(b \sec(c + dx))^{2/3}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Cos}[c + d*x]^2*(b*\text{Sec}[c + d*x])^{4/3}*(A + B*\text{Sec}[c + d*x]), x]$

[Out] $(-3*A*b^3*\text{Hypergeometric2F1}[1/2, 5/6, 11/6, \text{Cos}[c + d*x]^2]*\text{Sin}[c + d*x])/(5*d*(b*\text{Sec}[c + d*x])^{5/3}*\text{Sqrt}[\text{Sin}[c + d*x]^2]) - (3*b^2*B*\text{Hypergeometric2F1}[1/3, 1/2, 4/3, \text{Cos}[c + d*x]^2]*\text{Sin}[c + d*x])/(2*d*(b*\text{Sec}[c + d*x])^{2/3}*\text{Sqrt}[\text{Sin}[c + d*x]^2])$

Rule 16

$\text{Int}[(u_*)*(v_*)^{(m_*)}*((b_*)*(v_*))^{(n_*)}, x_Symbol] \rightarrow \text{Dist}[1/b^m, \text{Int}[u*(b*v)^{(m+n)}, x], x] /; \text{FreeQ}\{b, n\}, x \&\& \text{IntegerQ}[m]$

Rule 2643

$\text{Int}[(b_*)*\sin[(c_*) + (d_*)*(x_*)]^{(n_*)}, x_Symbol] \rightarrow \text{Simp}[(\text{Cos}[c + d*x]*(b*\text{Sin}[c + d*x])^{(n+1)}*\text{Hypergeometric2F1}[1/2, (n+1)/2, (n+3)/2, \text{Sin}[c + d*x]^2])/(b*d*(n+1)*\text{Sqrt}[\text{Cos}[c + d*x]^2]), x] /; \text{FreeQ}\{b, c, d, n\}, x \&\& \text{IntegerQ}[2*n]$

Rule 3772

$\text{Int}[(\text{csc}[(c_*) + (d_*)*(x_*)]*(b_*))^{(n_*)}, x_Symbol] \rightarrow \text{Simp}[(b*\text{Csc}[c + d*x])^{(n-1)}*((\text{Sin}[c + d*x]/b)^{(n-1)}*\text{Int}[1/(\text{Sin}[c + d*x]/b)^n, x]), x] /; \text{Fr}$

eeQ[{b, c, d, n}, x] && !IntegerQ[n]

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rubi steps

$$\begin{aligned} \int \cos^2(c + dx)(b \sec(c + dx))^{4/3}(A + B \sec(c + dx)) dx &= b^2 \int \frac{A + B \sec(c + dx)}{(b \sec(c + dx))^{2/3}} dx \\ &= (Ab^2) \int \frac{1}{(b \sec(c + dx))^{2/3}} dx + (bB) \int \sqrt[3]{b \sec(c + dx)} dx \\ &= \left(Ab^2 \sqrt[3]{\frac{\cos(c + dx)}{b}} \sqrt[3]{b \sec(c + dx)} \right) \int \left(\frac{\cos(c + dx)}{b} \right) dx \\ &= \frac{3bB \cos(c + dx) {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; \cos^2(c + dx)\right) \sqrt[3]{b \sec(c + dx)}}{2d \sqrt{\sin^2(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.10, size = 88, normalized size = 0.74

$$\frac{3b \sqrt{-\tan^2(c + dx)} \cot(c + dx) \sqrt[3]{b \sec(c + dx)} \left(A \cos(c + dx) {}_2F_1\left(-\frac{1}{3}, \frac{1}{2}; \frac{2}{3}; \sec^2(c + dx)\right) - 2B {}_2F_1\left(\frac{1}{6}, \frac{1}{2}; \frac{7}{6}; \sec^2(c + dx)\right) \right)}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]^2*(b*Sec[c + d*x])^(4/3)*(A + B*Sec[c + d*x]),x]

[Out] (-3*b*Cot[c + d*x]*(A*Cos[c + d*x]*Hypergeometric2F1[-1/3, 1/2, 2/3, Sec[c + d*x]^2] - 2*B*Hypergeometric2F1[1/6, 1/2, 7/6, Sec[c + d*x]^2])*(b*Sec[c + d*x])^(1/3)*Sqrt[-Tan[c + d*x]^2])/(2*d)

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb \cos(dx + c)^2 \sec(dx + c)^2 + Ab \cos(dx + c)^2 \sec(dx + c)\right) (b \sec(dx + c))^{1/3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] integral((B*b*cos(d*x + c)^2*sec(d*x + c)^2 + A*b*cos(d*x + c)^2*sec(d*x + c))*(b*sec(d*x + c))^(1/3), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A) (b \sec(dx + c))^{\frac{4}{3}} \cos(dx + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(4/3)*cos(d*x + c)^2, x)

maple [F] time = 3.10, size = 0, normalized size = 0.00

$$\int (\cos^2(dx + c)) (b \sec(dx + c))^{\frac{4}{3}} (A + B \sec(dx + c)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^2*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x)

[Out] int(cos(d*x+c)^2*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A) (b \sec(dx + c))^{\frac{4}{3}} \cos(dx + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(4/3)*cos(d*x + c)^2, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c + dx)^2 \left(A + \frac{B}{\cos(c + dx)} \right) \left(\frac{b}{\cos(c + dx)} \right)^{\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^2*(A + B/cos(c + d*x))*(b/cos(c + d*x))^(4/3),x)

```
[Out] int(cos(c + d*x)^2*(A + B/cos(c + d*x))*(b/cos(c + d*x))^(4/3), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(b*sec(d*x+c))**(4/3)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

$$3.17 \quad \int \frac{\sec^2(c+dx)(A+B \sec(c+dx))}{(b \sec(c+dx))^{2/3}} dx$$

Optimal. Leaf size=117

$$\frac{3A \sin(c+dx) \sqrt[3]{b \sec(c+dx)} {}_2F_1\left(-\frac{1}{6}, \frac{1}{2}; \frac{5}{6}; \cos^2(c+dx)\right)}{bd \sqrt{\sin^2(c+dx)}} + \frac{3B \sin(c+dx) (b \sec(c+dx))^{4/3} {}_2F_1\left(-\frac{2}{3}, \frac{1}{2}; \frac{1}{3}; \cos^2(c+dx)\right)}{4b^2 d \sqrt{\sin^2(c+dx)}}$$

[Out] 3*A*hypergeom([-1/6, 1/2], [5/6], cos(d*x+c)^2)*(b*sec(d*x+c))^(1/3)*sin(d*x+c)/b/d/(sin(d*x+c)^2)^(1/2)+3/4*B*hypergeom([-2/3, 1/2], [1/3], cos(d*x+c)^2)*(b*sec(d*x+c))^(4/3)*sin(d*x+c)/b^2/d/(sin(d*x+c)^2)^(1/2)

Rubi [A] time = 0.10, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {16, 3787, 3772, 2643}

$$\frac{3A \sin(c+dx) \sqrt[3]{b \sec(c+dx)} {}_2F_1\left(-\frac{1}{6}, \frac{1}{2}; \frac{5}{6}; \cos^2(c+dx)\right)}{bd \sqrt{\sin^2(c+dx)}} + \frac{3B \sin(c+dx) (b \sec(c+dx))^{4/3} {}_2F_1\left(-\frac{2}{3}, \frac{1}{2}; \frac{1}{3}; \cos^2(c+dx)\right)}{4b^2 d \sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(b*Sec[c + d*x])^(2/3), x]

[Out] (3*A*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(1/3)*Sin[c + d*x])/(b*d*Sqrt[Sin[c + d*x]^2]) + (3*B*Hypergeometric2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(4/3)*Sin[c + d*x])/(4*b^2*d*Sqrt[Sin[c + d*x]^2])

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 2643

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n+1)*Hypergeometric2F1[1/2, (n+1)/2, (n+3)/2, Sin[c + d*x]^2])/(b*d*(n+1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rule 3772

Int[(csc[(c_) + (d_)*(x_)]*(b_))^(n_), x_Symbol] := Simp[(b*Csc[c + d*x])^(n-1)*((Sin[c + d*x]/b)^(n-1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr

eeQ[{b, c, d, n}, x] && !IntegerQ[n]

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rubi steps

$$\begin{aligned} \int \frac{\sec^2(c + dx)(A + B \sec(c + dx))}{(b \sec(c + dx))^{2/3}} dx &= \frac{\int (b \sec(c + dx))^{4/3} (A + B \sec(c + dx)) dx}{b^2} \\ &= \frac{A \int (b \sec(c + dx))^{4/3} dx}{b^2} + \frac{B \int (b \sec(c + dx))^{7/3} dx}{b^3} \\ &= \frac{\left(A \sqrt[3]{\frac{\cos(c+dx)}{b}} \sqrt[3]{b \sec(c + dx)} \right) \int \frac{1}{\left(\frac{\cos(c+dx)}{b} \right)^{4/3}} dx}{b^2} + \frac{\left(B \sqrt[3]{\frac{\cos(c+dx)}{b}} \sqrt[3]{b \sec(c + dx)} \right) \int \frac{1}{\left(\frac{\cos(c+dx)}{b} \right)^{4/3}} dx}{b^2} \\ &= \frac{3A {}_2F_1\left(-\frac{1}{6}, \frac{1}{2}; \frac{5}{6}; \cos^2(c + dx)\right) \sqrt[3]{b \sec(c + dx)} \sin(c + dx)}{bd \sqrt{\sin^2(c + dx)}} + \frac{3B {}_2F_1\left(-\frac{1}{6}, \frac{1}{2}; \frac{5}{6}; \cos^2(c + dx)\right) \sqrt[3]{b \sec(c + dx)} \sin(c + dx)}{bd \sqrt{\sin^2(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.26, size = 90, normalized size = 0.77

$$\frac{3 \left(-\tan^2(c + dx) \right)^{3/2} \csc^3(c + dx) \left(7A \cos(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; \sec^2(c + dx)\right) + 4B {}_2F_1\left(\frac{1}{2}, \frac{7}{6}; \frac{13}{6}; \sec^2(c + dx)\right) \right)}{28d(b \sec(c + dx))^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(b*Sec[c + d*x])^(2/3), x]

[Out] (-3*Csc[c + d*x]^3*(7*A*Cos[c + d*x]*Hypergeometric2F1[1/2, 2/3, 5/3, Sec[c + d*x]^2] + 4*B*Hypergeometric2F1[1/2, 7/6, 13/6, Sec[c + d*x]^2])*(-Tan[c + d*x]^2)^(3/2))/(28*d*(b*Sec[c + d*x])^(2/3))

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{(B \sec(dx + c)^2 + A \sec(dx + c)) (b \sec(dx + c))^{1/3}}{b}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(2/3),x, algorithm="fricas")

[Out] integral((B*sec(d*x + c)^2 + A*sec(d*x + c))*(b*sec(d*x + c))^(1/3)/b, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sec(dx + c)^2}{(b \sec(dx + c))^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(2/3),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^2/(b*sec(d*x + c))^(2/3), x)

maple [F] time = 0.74, size = 0, normalized size = 0.00

$$\int \frac{(\sec^2(dx + c))(A + B \sec(dx + c))}{(b \sec(dx + c))^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^2*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(2/3),x)

[Out] int(sec(d*x+c)^2*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(2/3),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sec(dx + c)^2}{(b \sec(dx + c))^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(2/3),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^2/(b*sec(d*x + c))^(2/3), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\cos(c+dx)^2 \left(\frac{b}{\cos(c+dx)}\right)^{2/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B/cos(c + d*x))/(cos(c + d*x)^2*(b/cos(c + d*x))^(2/3)),x)`

[Out] `int((A + B/cos(c + d*x))/(cos(c + d*x)^2*(b/cos(c + d*x))^(2/3)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \sec^2(c + dx)}{(b \sec(c + dx))^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)**2*(A+B*sec(d*x+c))/(b*sec(d*x+c))**(2/3),x)`

[Out] `Integral((A + B*sec(c + d*x))*sec(c + d*x)**2/(b*sec(c + d*x))**(2/3), x)`

$$3.18 \quad \int \frac{\sec(c+dx)(A+B \sec(c+dx))}{(b \sec(c+dx))^{2/3}} dx$$

Optimal. Leaf size=114

$$\frac{3B \sin(c+dx) \sqrt[3]{b \sec(c+dx)} {}_2F_1\left(-\frac{1}{6}, \frac{1}{2}; \frac{5}{6}; \cos^2(c+dx)\right)}{bd \sqrt{\sin^2(c+dx)}} - \frac{3A \sin(c+dx) {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; \cos^2(c+dx)\right)}{2d \sqrt{\sin^2(c+dx)} (b \sec(c+dx))^{2/3}}$$

[Out] $-3/2*A*\text{hypergeom}([1/3, 1/2], [4/3], \cos(d*x+c)^2)*\sin(d*x+c)/d/(b*\sec(d*x+c))^{2/3}/(\sin(d*x+c)^2)^{1/2}+3*B*\text{hypergeom}([-1/6, 1/2], [5/6], \cos(d*x+c)^2)*(b*\sec(d*x+c))^{1/3}*\sin(d*x+c)/b/d/(\sin(d*x+c)^2)^{1/2}$

Rubi [A] time = 0.09, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {16, 3787, 3772, 2643}

$$\frac{3B \sin(c+dx) \sqrt[3]{b \sec(c+dx)} {}_2F_1\left(-\frac{1}{6}, \frac{1}{2}; \frac{5}{6}; \cos^2(c+dx)\right)}{bd \sqrt{\sin^2(c+dx)}} - \frac{3A \sin(c+dx) {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; \cos^2(c+dx)\right)}{2d \sqrt{\sin^2(c+dx)} (b \sec(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(\text{Sec}[c + d*x]*(A + B*\text{Sec}[c + d*x]))/(b*\text{Sec}[c + d*x])^{2/3}, x]$

[Out] $(-3*A*\text{Hypergeometric2F1}[1/3, 1/2, 4/3, \text{Cos}[c + d*x]^2]*\text{Sin}[c + d*x])/(2*d*(b*\text{Sec}[c + d*x])^{2/3}*\text{Sqrt}[\text{Sin}[c + d*x]^2]) + (3*B*\text{Hypergeometric2F1}[-1/6, 1/2, 5/6, \text{Cos}[c + d*x]^2]*(b*\text{Sec}[c + d*x])^{1/3}*\text{Sin}[c + d*x])/(b*d*\text{Sqrt}[\text{Sin}[c + d*x]^2])$

Rule 16

$\text{Int}[(u_*)*(v_)^{(m_*)}*((b_)*(v_))^{(n_*)}, x_Symbol] \rightarrow \text{Dist}[1/b^m, \text{Int}[u*(b*v)^{(m+n)}, x], x] /; \text{FreeQ}\{b, n, x\} \&\& \text{IntegerQ}[m]$

Rule 2643

$\text{Int}[(b_*)*\sin[(c_*) + (d_*)*(x_*)]^{(n_*)}, x_Symbol] \rightarrow \text{Simp}[(\text{Cos}[c + d*x]*(b*\text{Sin}[c + d*x])^{(n+1)}*\text{Hypergeometric2F1}[1/2, (n+1)/2, (n+3)/2, \text{Sin}[c + d*x]^2])/(b*d*(n+1)*\text{Sqrt}[\text{Cos}[c + d*x]^2]), x] /; \text{FreeQ}\{b, c, d, n, x\} \&\& !\text{IntegerQ}[2*n]$

Rule 3772

$\text{Int}[(\text{csc}[(c_*) + (d_*)*(x_*)]*(b_*)^{(n_*)}, x_Symbol] \rightarrow \text{Simp}[(b*\text{Csc}[c + d*x])^{(n-1)}*((\text{Sin}[c + d*x]/b)^{(n-1)}*\text{Int}[1/(\text{Sin}[c + d*x]/b)^n, x]), x] /; \text{Fr}$

eeQ[{b, c, d, n}, x] && !IntegerQ[n]

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rubi steps

$$\begin{aligned} \int \frac{\sec(c + dx)(A + B \sec(c + dx))}{(b \sec(c + dx))^{2/3}} dx &= \frac{\int \sqrt[3]{b \sec(c + dx)} (A + B \sec(c + dx)) dx}{b} \\ &= \frac{A \int \sqrt[3]{b \sec(c + dx)} dx}{b} + \frac{B \int (b \sec(c + dx))^{4/3} dx}{b^2} \\ &= \frac{\left(A \sqrt[3]{\frac{\cos(c+dx)}{b}} \sqrt[3]{b \sec(c + dx)} \right) \int \frac{1}{\sqrt[3]{\frac{\cos(c+dx)}{b}}} dx}{b} + \frac{\left(B \sqrt[3]{\frac{\cos(c+dx)}{b}} \sqrt[3]{b \sec(c + dx)} \right) \int \frac{1}{\sqrt[3]{\frac{\cos(c+dx)}{b}}} dx}{b} \\ &= \frac{3B {}_2F_1\left(-\frac{1}{6}, \frac{1}{2}; \frac{5}{6}; \cos^2(c + dx)\right) \sqrt[3]{b \sec(c + dx)} \sin(c + dx)}{bd \sqrt{\sin^2(c + dx)}} - \frac{3A \cos(c + dx)}{bd} \end{aligned}$$

Mathematica [A] time = 0.10, size = 90, normalized size = 0.79

$$\frac{3\sqrt{-\tan^2(c + dx)} \csc(c + dx) \sqrt[3]{b \sec(c + dx)} \left(4A \cos(c + dx) {}_2F_1\left(\frac{1}{6}, \frac{1}{2}; \frac{7}{6}; \sec^2(c + dx)\right) + B {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; \sec^2(c + dx)\right) \right)}{4bd}$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/(b*Sec[c + d*x])^(2/3),x]

[Out] (3*Csc[c + d*x]*(4*A*Cos[c + d*x]*Hypergeometric2F1[1/6, 1/2, 7/6, Sec[c + d*x]^2] + B*Hypergeometric2F1[1/2, 2/3, 5/3, Sec[c + d*x]^2])*(b*Sec[c + d*x])^(1/3)*Sqrt[-Tan[c + d*x]^2])/(4*b*d)

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{(B \sec(dx + c) + A) (b \sec(dx + c))^{1/3}}{b}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(2/3),x, algorithm="fricas")

[Out] integral((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(1/3)/b, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sec(dx + c)}{(b \sec(dx + c))^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(2/3),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)/(b*sec(d*x + c))^(2/3), x)

maple [F] time = 0.72, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx + c) (A + B \sec(dx + c))}{(b \sec(dx + c))^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(2/3),x)

[Out] int(sec(d*x+c)*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(2/3),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sec(dx + c)}{(b \sec(dx + c))^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(2/3),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)/(b*sec(d*x + c))^(2/3), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\cos(c + dx) \left(\frac{b}{\cos(c+dx)}\right)^{2/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B/cos(c + d*x))/(cos(c + d*x)*(b/cos(c + d*x))^(2/3)),x)`

[Out] `int((A + B/cos(c + d*x))/(cos(c + d*x)*(b/cos(c + d*x))^(2/3)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \sec(c + dx)}{(b \sec(c + dx))^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(b*sec(d*x+c))**(2/3),x)`

[Out] `Integral((A + B*sec(c + d*x))*sec(c + d*x)/(b*sec(c + d*x))**(2/3), x)`

$$3.19 \quad \int \frac{A+B \sec(c+dx)}{(b \sec(c+dx))^{2/3}} dx$$

Optimal. Leaf size=114

$$\frac{3Ab \sin(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{5}{6}; \frac{11}{6}; \cos^2(c+dx)\right)}{5d\sqrt{\sin^2(c+dx)}(b \sec(c+dx))^{5/3}} - \frac{3B \sin(c+dx) {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; \cos^2(c+dx)\right)}{2d\sqrt{\sin^2(c+dx)}(b \sec(c+dx))^{2/3}}$$

[Out] $-3/5*A*b*\text{hypergeom}([1/2, 5/6], [11/6], \cos(d*x+c)^2)*\sin(d*x+c)/d/(b*\sec(d*x+c))^{(5/3)}/(\sin(d*x+c)^2)^{(1/2)}-3/2*B*\text{hypergeom}([1/3, 1/2], [4/3], \cos(d*x+c)^2)*\sin(d*x+c)/d/(b*\sec(d*x+c))^{(2/3)}/(\sin(d*x+c)^2)^{(1/2)}$

Rubi [A] time = 0.09, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3787, 3772, 2643}

$$\frac{3Ab \sin(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{5}{6}; \frac{11}{6}; \cos^2(c+dx)\right)}{5d\sqrt{\sin^2(c+dx)}(b \sec(c+dx))^{5/3}} - \frac{3B \sin(c+dx) {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; \cos^2(c+dx)\right)}{2d\sqrt{\sin^2(c+dx)}(b \sec(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(A + B*\text{Sec}[c + d*x])/(b*\text{Sec}[c + d*x])^{(2/3)}, x]$

[Out] $(-3*A*b*\text{Hypergeometric2F1}[1/2, 5/6, 11/6, \text{Cos}[c + d*x]^2]*\text{Sin}[c + d*x])/(5*d*(b*\text{Sec}[c + d*x])^{(5/3)}*\text{Sqrt}[\text{Sin}[c + d*x]^2]) - (3*B*\text{Hypergeometric2F1}[1/3, 1/2, 4/3, \text{Cos}[c + d*x]^2]*\text{Sin}[c + d*x])/(2*d*(b*\text{Sec}[c + d*x])^{(2/3)}*\text{Sqrt}[\text{Sin}[c + d*x]^2])$

Rule 2643

$\text{Int}[(b_*)*\sin[(c_*) + (d_*)*(x_*)]^{(n_*)}, x_Symbol] \rightarrow \text{Simp}[(\text{Cos}[c + d*x]*(b*\text{Sin}[c + d*x])^{(n + 1)}*\text{Hypergeometric2F1}[1/2, (n + 1)/2, (n + 3)/2, \text{Sin}[c + d*x]^2])/(b*d*(n + 1)*\text{Sqrt}[\text{Cos}[c + d*x]^2]), x] /;$ $\text{FreeQ}\{b, c, d, n, x\}$ && $! \text{IntegerQ}[2*n]$

Rule 3772

$\text{Int}[(\text{csc}[(c_*) + (d_*)*(x_*)]*(b_*))^{(n_*)}, x_Symbol] \rightarrow \text{Simp}[(b*\text{Csc}[c + d*x])^{(n - 1)}*((\text{Sin}[c + d*x]/b)^{(n - 1)}*\text{Int}[1/(\text{Sin}[c + d*x]/b)^n, x]), x] /;$ $\text{FreeQ}\{b, c, d, n, x\}$ && $! \text{IntegerQ}[n]$

Rule 3787

$\text{Int}[(\text{csc}[(e_*) + (f_*)*(x_*)]*(d_*))^{(n_*)}*(\text{csc}[(e_*) + (f_*)*(x_*)]*(b_*) + (a_*)), x_Symbol] \rightarrow \text{Dist}[a, \text{Int}[(d*\text{Csc}[e + f*x])^n, x], x] + \text{Dist}[b/d, \text{Int}[\text{csc}[(e_*) + (f_*)*(x_*)]^{(n_*)}, x], x]$

$(d \cdot \text{Csc}[e + f \cdot x])^{(n + 1)}, x], x] /; \text{FreeQ}[\{a, b, d, e, f, n\}, x]$

Rubi steps

$$\begin{aligned} \int \frac{A + B \sec(c + dx)}{(b \sec(c + dx))^{2/3}} dx &= A \int \frac{1}{(b \sec(c + dx))^{2/3}} dx + \frac{B \int \sqrt[3]{b \sec(c + dx)} dx}{b} \\ &= \left(A \sqrt[3]{\frac{\cos(c + dx)}{b}} \sqrt[3]{b \sec(c + dx)} \right) \int \left(\frac{\cos(c + dx)}{b} \right)^{2/3} dx + \frac{\left(B \sqrt[3]{\frac{\cos(c + dx)}{b}} \sqrt[3]{b \sec(c + dx)} \right)}{b} \\ &= -\frac{3B \cos(c + dx) {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; \cos^2(c + dx)\right) \sqrt[3]{b \sec(c + dx)} \sin(c + dx)}{2bd \sqrt{\sin^2(c + dx)}} - \frac{3A \cos^2(c + dx)}{2bd \sqrt{\sin^2(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.09, size = 87, normalized size = 0.76

$$\frac{3\sqrt{-\tan^2(c + dx)} \csc(c + dx) \left(A \cos(c + dx) {}_2F_1\left(-\frac{1}{3}, \frac{1}{2}; \frac{2}{3}; \sec^2(c + dx)\right) - 2B {}_2F_1\left(\frac{1}{6}, \frac{1}{2}; \frac{7}{6}; \sec^2(c + dx)\right) \right)}{2d(b \sec(c + dx))^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sec[c + d*x])/(b*Sec[c + d*x])^(2/3), x]

[Out] $(-3 \cdot \text{Csc}[c + d \cdot x] \cdot (A \cdot \text{Cos}[c + d \cdot x] \cdot \text{Hypergeometric2F1}[-1/3, 1/2, 2/3, \text{Sec}[c + d \cdot x]^2] - 2 \cdot B \cdot \text{Hypergeometric2F1}[1/6, 1/2, 7/6, \text{Sec}[c + d \cdot x]^2]) \cdot \text{Sqrt}[-\text{Tan}[c + d \cdot x]^2]) / (2 \cdot d \cdot (b \cdot \text{Sec}[c + d \cdot x])^{2/3})$

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B \sec(dx + c) + A)(b \sec(dx + c))^{1/3}}{b \sec(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(b*sec(d*x+c))^(2/3), x, algorithm="fricas")

[Out] integral((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(1/3)/(b*sec(d*x + c)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(b \sec(dx + c))^{2/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(b*sec(d*x+c))^(2/3),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)/(b*sec(d*x + c))^(2/3), x)

maple [F] time = 0.95, size = 0, normalized size = 0.00

$$\int \frac{A + B \sec(dx + c)}{(b \sec(dx + c))^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))/(b*sec(d*x+c))^(2/3),x)

[Out] int((A+B*sec(d*x+c))/(b*sec(d*x+c))^(2/3),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(b \sec(dx + c))^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(b*sec(d*x+c))^(2/3),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)/(b*sec(d*x + c))^(2/3), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\left(\frac{b}{\cos(c+dx)}\right)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/(b/cos(c + d*x))^(2/3),x)

[Out] int((A + B/cos(c + d*x))/(b/cos(c + d*x))^(2/3), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \sec(c + dx)}{(b \sec(c + dx))^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(b*sec(d*x+c))**(2/3),x)

[Out] Integral((A + B*sec(c + d*x))/(b*sec(c + d*x))**(2/3), x)

$$3.20 \quad \int \frac{\sec(c+dx)(A+B \sec(c+dx))}{(b \sec(c+dx))^{2/3}} dx$$

Optimal. Leaf size=114

$$\frac{3B \sin(c+dx) \sqrt[3]{b \sec(c+dx)} {}_2F_1\left(-\frac{1}{6}, \frac{1}{2}; \frac{5}{6}; \cos^2(c+dx)\right)}{bd \sqrt{\sin^2(c+dx)}} - \frac{3A \sin(c+dx) {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; \cos^2(c+dx)\right)}{2d \sqrt{\sin^2(c+dx)} (b \sec(c+dx))^{2/3}}$$

[Out] $-3/2*A*\text{hypergeom}([1/3, 1/2], [4/3], \cos(d*x+c)^2)*\sin(d*x+c)/d/(b*\sec(d*x+c))^{2/3}/(\sin(d*x+c)^2)^{1/2}+3*B*\text{hypergeom}([-1/6, 1/2], [5/6], \cos(d*x+c)^2)*(b*\sec(d*x+c))^{1/3}*\sin(d*x+c)/b/d/(\sin(d*x+c)^2)^{1/2}$

Rubi [A] time = 0.09, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {16, 3787, 3772, 2643}

$$\frac{3B \sin(c+dx) \sqrt[3]{b \sec(c+dx)} {}_2F_1\left(-\frac{1}{6}, \frac{1}{2}; \frac{5}{6}; \cos^2(c+dx)\right)}{bd \sqrt{\sin^2(c+dx)}} - \frac{3A \sin(c+dx) {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; \cos^2(c+dx)\right)}{2d \sqrt{\sin^2(c+dx)} (b \sec(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(\text{Sec}[c + d*x]*(A + B*\text{Sec}[c + d*x]))/(b*\text{Sec}[c + d*x])^{2/3}, x]$

[Out] $(-3*A*\text{Hypergeometric2F1}[1/3, 1/2, 4/3, \text{Cos}[c + d*x]^2]*\text{Sin}[c + d*x])/(2*d*(b*\text{Sec}[c + d*x])^{2/3}*\text{Sqrt}[\text{Sin}[c + d*x]^2]) + (3*B*\text{Hypergeometric2F1}[-1/6, 1/2, 5/6, \text{Cos}[c + d*x]^2]*(b*\text{Sec}[c + d*x])^{1/3}*\text{Sin}[c + d*x])/(b*d*\text{Sqrt}[\text{Sin}[c + d*x]^2])$

Rule 16

$\text{Int}[(u_*)*(v_)^{(m_*)}*((b_)*(v_))^{(n_*)}, x_Symbol] \rightarrow \text{Dist}[1/b^m, \text{Int}[u*(b*v)^{(m+n)}, x], x] /; \text{FreeQ}\{b, n, x\} \ \&\& \ \text{IntegerQ}[m]$

Rule 2643

$\text{Int}[(b_*)*\sin[(c_*) + (d_*)(x_*)]^{(n_*)}, x_Symbol] \rightarrow \text{Simp}[(\text{Cos}[c + d*x]*(b*\text{Sin}[c + d*x])^{(n+1)}*\text{Hypergeometric2F1}[1/2, (n+1)/2, (n+3)/2, \text{Sin}[c + d*x]^2])/(b*d*(n+1)*\text{Sqrt}[\text{Cos}[c + d*x]^2]), x] /; \text{FreeQ}\{b, c, d, n, x\} \ \&\& \ !\text{IntegerQ}[2*n]$

Rule 3772

$\text{Int}[(\text{csc}[(c_*) + (d_*)(x_*)]*(b_*)^{(n_*)}, x_Symbol] \rightarrow \text{Simp}[(b*\text{Csc}[c + d*x])^{(n-1)}*((\text{Sin}[c + d*x]/b)^{(n-1)}*\text{Int}[1/(\text{Sin}[c + d*x]/b)^n, x]), x] /; \text{Fr}$

eeQ[{b, c, d, n}, x] && !IntegerQ[n]

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rubi steps

$$\begin{aligned} \int \frac{\sec(c + dx)(A + B \sec(c + dx))}{(b \sec(c + dx))^{2/3}} dx &= \frac{\int \sqrt[3]{b \sec(c + dx)} (A + B \sec(c + dx)) dx}{b} \\ &= \frac{A \int \sqrt[3]{b \sec(c + dx)} dx}{b} + \frac{B \int (b \sec(c + dx))^{4/3} dx}{b^2} \\ &= \frac{\left(A \sqrt[3]{\frac{\cos(c+dx)}{b}} \sqrt[3]{b \sec(c + dx)} \right) \int \frac{1}{\sqrt[3]{\frac{\cos(c+dx)}{b}}} dx}{b} + \frac{\left(B \sqrt[3]{\frac{\cos(c+dx)}{b}} \sqrt[3]{b \sec(c + dx)} \right) \int \frac{1}{\sqrt[3]{\frac{\cos(c+dx)}{b}}} dx}{b^2} \\ &= \frac{3B {}_2F_1\left(-\frac{1}{6}, \frac{1}{2}; \frac{5}{6}; \cos^2(c + dx)\right) \sqrt[3]{b \sec(c + dx)} \sin(c + dx)}{bd \sqrt{\sin^2(c + dx)}} - \frac{3A \cos(c + dx)}{bd \sqrt{\sin^2(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.02, size = 90, normalized size = 0.79

$$\frac{3\sqrt{-\tan^2(c + dx)} \csc(c + dx) \sqrt[3]{b \sec(c + dx)} \left(4A \cos(c + dx) {}_2F_1\left(\frac{1}{6}, \frac{1}{2}; \frac{7}{6}; \sec^2(c + dx)\right) + B {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; \sec^2(c + dx)\right) \right)}{4bd}$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/(b*Sec[c + d*x])^(2/3),x]

[Out] (3*Csc[c + d*x]*(4*A*Cos[c + d*x]*Hypergeometric2F1[1/6, 1/2, 7/6, Sec[c + d*x]^2] + B*Hypergeometric2F1[1/2, 2/3, 5/3, Sec[c + d*x]^2])*(b*Sec[c + d*x])^(1/3)*Sqrt[-Tan[c + d*x]^2])/(4*b*d)

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{(B \sec(dx + c) + A) (b \sec(dx + c))^{1/3}}{b}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(2/3),x, algorithm="fricas")

[Out] integral((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(1/3)/b, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sec(dx + c)}{(b \sec(dx + c))^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(2/3),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)/(b*sec(d*x + c))^(2/3), x)

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx + c)(A + B \sec(dx + c))}{(b \sec(dx + c))^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(2/3),x)

[Out] int(sec(d*x+c)*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(2/3),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sec(dx + c)}{(b \sec(dx + c))^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(2/3),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)/(b*sec(d*x + c))^(2/3), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\cos(c + dx) \left(\frac{b}{\cos(c+dx)}\right)^{2/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B/cos(c + d*x))/(cos(c + d*x)*(b/cos(c + d*x))^(2/3)), x)`

[Out] `int((A + B/cos(c + d*x))/(cos(c + d*x)*(b/cos(c + d*x))^(2/3)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \sec(c + dx)}{(b \sec(c + dx))^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(b*sec(d*x+c))**(2/3), x)`

[Out] `Integral((A + B*sec(c + d*x))*sec(c + d*x)/(b*sec(c + d*x))**(2/3), x)`

$$3.21 \quad \int \frac{\sec^2(c+dx)(A+B \sec(c+dx))}{(b \sec(c+dx))^{2/3}} dx$$

Optimal. Leaf size=117

$$\frac{3A \sin(c+dx) \sqrt[3]{b \sec(c+dx)} {}_2F_1\left(-\frac{1}{6}, \frac{1}{2}; \frac{5}{6}; \cos^2(c+dx)\right)}{bd \sqrt{\sin^2(c+dx)}} + \frac{3B \sin(c+dx) (b \sec(c+dx))^{4/3} {}_2F_1\left(-\frac{2}{3}, \frac{1}{2}; \frac{1}{3}; \cos^2(c+dx)\right)}{4b^2 d \sqrt{\sin^2(c+dx)}}$$

[Out] 3*A*hypergeom([-1/6, 1/2], [5/6], cos(d*x+c)^2)*(b*sec(d*x+c))^(1/3)*sin(d*x+c)/b/d/(sin(d*x+c)^2)^(1/2)+3/4*B*hypergeom([-2/3, 1/2], [1/3], cos(d*x+c)^2)*(b*sec(d*x+c))^(4/3)*sin(d*x+c)/b^2/d/(sin(d*x+c)^2)^(1/2)

Rubi [A] time = 0.10, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {16, 3787, 3772, 2643}

$$\frac{3A \sin(c+dx) \sqrt[3]{b \sec(c+dx)} {}_2F_1\left(-\frac{1}{6}, \frac{1}{2}; \frac{5}{6}; \cos^2(c+dx)\right)}{bd \sqrt{\sin^2(c+dx)}} + \frac{3B \sin(c+dx) (b \sec(c+dx))^{4/3} {}_2F_1\left(-\frac{2}{3}, \frac{1}{2}; \frac{1}{3}; \cos^2(c+dx)\right)}{4b^2 d \sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(b*Sec[c + d*x])^(2/3), x]

[Out] (3*A*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(1/3)*Sin[c + d*x]/(b*d*Sqrt[Sin[c + d*x]^2]) + (3*B*Hypergeometric2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(4/3)*Sin[c + d*x])/(4*b^2*d*Sqrt[Sin[c + d*x]^2])

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 2643

Int[((b_)*sin[(c_.) + (d_)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n+1)*Hypergeometric2F1[1/2, (n+1)/2, (n+3)/2, Sin[c + d*x]^2])/(b*d*(n+1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rule 3772

Int[(csc[(c_.) + (d_)*(x_)])*(b_.)^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x])^(n-1)*((Sin[c + d*x]/b)^(n-1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr

eeQ[{b, c, d, n}, x] && !IntegerQ[n]

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n_.*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rubi steps

$$\begin{aligned} \int \frac{\sec^2(c + dx)(A + B \sec(c + dx))}{(b \sec(c + dx))^{2/3}} dx &= \frac{\int (b \sec(c + dx))^{4/3} (A + B \sec(c + dx)) dx}{b^2} \\ &= \frac{A \int (b \sec(c + dx))^{4/3} dx}{b^2} + \frac{B \int (b \sec(c + dx))^{7/3} dx}{b^3} \\ &= \frac{\left(A \sqrt[3]{\frac{\cos(c+dx)}{b}} \sqrt[3]{b \sec(c + dx)} \right) \int \frac{1}{\left(\frac{\cos(c+dx)}{b}\right)^{4/3}} dx}{b^2} + \frac{\left(B \sqrt[3]{\frac{\cos(c+dx)}{b}} \sqrt[3]{b \sec(c + dx)} \right) \int \frac{1}{\left(\frac{\cos(c+dx)}{b}\right)^{4/3}} dx}{b^2} \\ &= \frac{3A {}_2F_1\left(-\frac{1}{6}, \frac{1}{2}; \frac{5}{6}; \cos^2(c + dx)\right) \sqrt[3]{b \sec(c + dx)} \sin(c + dx)}{bd \sqrt{\sin^2(c + dx)}} + \frac{3B {}_2F_1\left(-\frac{1}{6}, \frac{1}{2}; \frac{5}{6}; \cos^2(c + dx)\right) \sqrt[3]{b \sec(c + dx)} \sin(c + dx)}{bd \sqrt{\sin^2(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.02, size = 90, normalized size = 0.77

$$\frac{3 \left(-\tan^2(c + dx)\right)^{3/2} \csc^3(c + dx) \left(7A \cos(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; \sec^2(c + dx)\right) + 4B {}_2F_1\left(\frac{1}{2}, \frac{7}{6}; \frac{13}{6}; \sec^2(c + dx)\right)\right)}{28d(b \sec(c + dx))^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(b*Sec[c + d*x])^(2/3), x]

[Out] (-3*Csc[c + d*x]^3*(7*A*Cos[c + d*x]*Hypergeometric2F1[1/2, 2/3, 5/3, Sec[c + d*x]^2] + 4*B*Hypergeometric2F1[1/2, 7/6, 13/6, Sec[c + d*x]^2])*(-Tan[c + d*x]^2)^(3/2))/(28*d*(b*Sec[c + d*x])^(2/3))

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{(B \sec(dx + c))^2 + A \sec(dx + c)}{b} (b \sec(dx + c))^{1/3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(2/3),x, algorithm="fricas")

[Out] integral((B*sec(d*x + c)^2 + A*sec(d*x + c))*(b*sec(d*x + c))^(1/3)/b, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sec(dx + c)^2}{(b \sec(dx + c))^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(2/3),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^2/(b*sec(d*x + c))^(2/3), x)

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(\sec^2(dx + c))(A + B \sec(dx + c))}{(b \sec(dx + c))^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^2*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(2/3), x)

[Out] int(sec(d*x+c)^2*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(2/3), x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sec(dx + c)^2}{(b \sec(dx + c))^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(2/3),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^2/(b*sec(d*x + c))^(2/3), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\cos(c+dx)^2 \left(\frac{b}{\cos(c+dx)}\right)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B/cos(c + d*x))/(cos(c + d*x)^2*(b/cos(c + d*x))^(2/3)), x)`

[Out] `int((A + B/cos(c + d*x))/(cos(c + d*x)^2*(b/cos(c + d*x))^(2/3)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \sec^2(c + dx)}{(b \sec(c + dx))^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)**2*(A+B*sec(d*x+c))/(b*sec(d*x+c))**(2/3), x)`

[Out] `Integral((A + B*sec(c + d*x))*sec(c + d*x)**2/(b*sec(c + d*x))**(2/3), x)`

$$3.22 \quad \int \frac{\sec^2(c+dx)(A+B \sec(c+dx))}{(b \sec(c+dx))^{4/3}} dx$$

Optimal. Leaf size=117

$$\frac{3B \sin(c+dx)(b \sec(c+dx))^{2/3} {}_2F_1\left(-\frac{1}{3}, \frac{1}{2}; \frac{2}{3}; \cos^2(c+dx)\right)}{2b^2 d \sqrt{\sin^2(c+dx)}} - \frac{3A \sin(c+dx) {}_2F_1\left(\frac{1}{6}, \frac{1}{2}; \frac{7}{6}; \cos^2(c+dx)\right)}{bd \sqrt{\sin^2(c+dx)} \sqrt[3]{b \sec(c+dx)}}$$

[Out] $-3A \operatorname{hypergeom}\left(\left[\frac{1}{6}, \frac{1}{2}\right], \left[\frac{7}{6}\right], \cos(d*x+c)^2\right) * \sin(d*x+c) / b / d / (b * \sec(d*x+c))^{1/3} / (\sin(d*x+c)^2)^{1/2} + 3/2 * B * \operatorname{hypergeom}\left(\left[-\frac{1}{3}, \frac{1}{2}\right], \left[\frac{2}{3}\right], \cos(d*x+c)^2\right) * (b * \sec(d*x+c))^{2/3} * \sin(d*x+c) / b^2 / d / (\sin(d*x+c)^2)^{1/2}$

Rubi [A] time = 0.10, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {16, 3787, 3772, 2643}

$$\frac{3B \sin(c+dx)(b \sec(c+dx))^{2/3} {}_2F_1\left(-\frac{1}{3}, \frac{1}{2}; \frac{2}{3}; \cos^2(c+dx)\right)}{2b^2 d \sqrt{\sin^2(c+dx)}} - \frac{3A \sin(c+dx) {}_2F_1\left(\frac{1}{6}, \frac{1}{2}; \frac{7}{6}; \cos^2(c+dx)\right)}{bd \sqrt{\sin^2(c+dx)} \sqrt[3]{b \sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(\operatorname{Sec}[c + d*x]^2 * (A + B * \operatorname{Sec}[c + d*x])) / (b * \operatorname{Sec}[c + d*x])^{4/3}, x]$

[Out] $(-3 * A * \operatorname{Hypergeometric2F1}[1/6, 1/2, 7/6, \operatorname{Cos}[c + d*x]^2] * \operatorname{Sin}[c + d*x]) / (b * d * (b * \operatorname{Sec}[c + d*x])^{1/3} * \operatorname{Sqrt}[\operatorname{Sin}[c + d*x]^2]) + (3 * B * \operatorname{Hypergeometric2F1}[-1/3, 1/2, 2/3, \operatorname{Cos}[c + d*x]^2] * (b * \operatorname{Sec}[c + d*x])^{2/3} * \operatorname{Sin}[c + d*x]) / (2 * b^2 * d * \operatorname{Sqrt}[\operatorname{Sin}[c + d*x]^2])$

Rule 16

$\operatorname{Int}[(u_*) * (v_*)^{(m_*)} * ((b_*) * (v_*))^{(n_*)}, x_Symbol] \rightarrow \operatorname{Dist}[1/b^m, \operatorname{Int}[u * (b*v)^{(m+n)}, x], x] /; \operatorname{FreeQ}[\{b, n\}, x] \ \&\& \ \operatorname{IntegerQ}[m]$

Rule 2643

$\operatorname{Int}[(b_*) * \sin[(c_*) + (d_*) * (x_*)]^{(n_*)}, x_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Cos}[c + d*x] * (b * \operatorname{Sin}[c + d*x])^{(n+1)} * \operatorname{Hypergeometric2F1}[1/2, (n+1)/2, (n+3)/2, \operatorname{Sin}[c + d*x]^2]) / (b * d * (n+1) * \operatorname{Sqrt}[\operatorname{Cos}[c + d*x]^2]), x] /; \operatorname{FreeQ}[\{b, c, d, n\}, x] \ \&\& \ !\operatorname{IntegerQ}[2*n]$

Rule 3772

$\operatorname{Int}[(\operatorname{csc}[(c_*) + (d_*) * (x_*)] * (b_*))^{(n_*)}, x_Symbol] \rightarrow \operatorname{Simp}[(b * \operatorname{Csc}[c + d*x])^{(n-1)} * ((\operatorname{Sin}[c + d*x] / b)^{(n-1)} * \operatorname{Int}[1 / (\operatorname{Sin}[c + d*x] / b)^n, x]), x] /; \operatorname{Fr}$

eeQ[{b, c, d, n}, x] && !IntegerQ[n]

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rubi steps

$$\begin{aligned} \int \frac{\sec^2(c + dx)(A + B \sec(c + dx))}{(b \sec(c + dx))^{4/3}} dx &= \frac{\int (b \sec(c + dx))^{2/3} (A + B \sec(c + dx)) dx}{b^2} \\ &= \frac{A \int (b \sec(c + dx))^{2/3} dx}{b^2} + \frac{B \int (b \sec(c + dx))^{5/3} dx}{b^3} \\ &= \frac{\left(A \left(\frac{\cos(c+dx)}{b} \right)^{2/3} (b \sec(c + dx))^{2/3} \right) \int \frac{1}{\left(\frac{\cos(c+dx)}{b} \right)^{2/3}} dx}{b^2} + \frac{\left(B \left(\frac{\cos(c+dx)}{b} \right)^{2/3} \right) \int \frac{1}{\left(\frac{\cos(c+dx)}{b} \right)^{2/3}} dx}{b^3} \\ &= \frac{3B {}_2F_1\left(-\frac{1}{3}, \frac{1}{2}; \frac{2}{3}; \cos^2(c + dx)\right) (b \sec(c + dx))^{2/3} \sin(c + dx)}{2b^2 d \sqrt{\sin^2(c + dx)}} - \frac{3A \cos(c + dx)}{2b^2 d \sqrt{\sin^2(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.23, size = 91, normalized size = 0.78

$$\frac{3\sqrt{-\tan^2(c + dx)} \csc(c + dx) (b \sec(c + dx))^{2/3} \left(5A \cos(c + dx) {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; \sec^2(c + dx)\right) + 2B {}_2F_1\left(\frac{1}{2}, \frac{5}{6}; \frac{11}{6}; \sec^2(c + dx)\right) \right)}{10b^2 d}$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(b*Sec[c + d*x])^(4/3),x]

[Out] (3*Csc[c + d*x]*(5*A*Cos[c + d*x]*Hypergeometric2F1[1/3, 1/2, 4/3, Sec[c + d*x]^2] + 2*B*Hypergeometric2F1[1/2, 5/6, 11/6, Sec[c + d*x]^2])*(b*Sec[c + d*x])^(2/3)*Sqrt[-Tan[c + d*x]^2])/(10*b^2*d)

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{(B \sec(dx + c) + A) (b \sec(dx + c))^{2/3}}{b^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(4/3),x, algorithm="fricas")

[Out] integral((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(2/3)/b^2, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sec(dx + c)^2}{(b \sec(dx + c))^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(4/3),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^2/(b*sec(d*x + c))^(4/3), x)

maple [F] time = 0.70, size = 0, normalized size = 0.00

$$\int \frac{(\sec^2(dx + c))(A + B \sec(dx + c))}{(b \sec(dx + c))^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^2*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(4/3),x)

[Out] int(sec(d*x+c)^2*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(4/3),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sec(dx + c)^2}{(b \sec(dx + c))^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(4/3),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^2/(b*sec(d*x + c))^(4/3), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\cos(c+dx)^2 \left(\frac{b}{\cos(c+dx)}\right)^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B/cos(c + d*x))/(cos(c + d*x)^2*(b/cos(c + d*x))^(4/3)), x)`

[Out] `int((A + B/cos(c + d*x))/(cos(c + d*x)^2*(b/cos(c + d*x))^(4/3)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \sec^2(c + dx)}{(b \sec(c + dx))^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)**2*(A+B*sec(d*x+c))/(b*sec(d*x+c))**(4/3), x)`

[Out] `Integral((A + B*sec(c + d*x))*sec(c + d*x)**2/(b*sec(c + d*x))**(4/3), x)`

$$3.23 \quad \int \frac{\sec(c+dx)(A+B \sec(c+dx))}{(b \sec(c+dx))^{4/3}} dx$$

Optimal. Leaf size=114

$$\frac{3A \sin(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; \cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}(b \sec(c+dx))^{4/3}} - \frac{3B \sin(c+dx) {}_2F_1\left(\frac{1}{6}, \frac{1}{2}; \frac{7}{6}; \cos^2(c+dx)\right)}{bd\sqrt{\sin^2(c+dx)}\sqrt[3]{b \sec(c+dx)}}$$

[Out] $-3/4*A*\text{hypergeom}([1/2, 2/3], [5/3], \cos(d*x+c)^2)*\sin(d*x+c)/d/(b*\sec(d*x+c))^{4/3}/(\sin(d*x+c)^2)^{1/2}-3*B*\text{hypergeom}([1/6, 1/2], [7/6], \cos(d*x+c)^2)*\sin(d*x+c)/b/d/(b*\sec(d*x+c))^{1/3}/(\sin(d*x+c)^2)^{1/2}$

Rubi [A] time = 0.09, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {16, 3787, 3772, 2643}

$$\frac{3A \sin(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; \cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}(b \sec(c+dx))^{4/3}} - \frac{3B \sin(c+dx) {}_2F_1\left(\frac{1}{6}, \frac{1}{2}; \frac{7}{6}; \cos^2(c+dx)\right)}{bd\sqrt{\sin^2(c+dx)}\sqrt[3]{b \sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(\text{Sec}[c + d*x]*(A + B*\text{Sec}[c + d*x]))/(b*\text{Sec}[c + d*x])^{4/3}, x]$

[Out] $(-3*A*\text{Hypergeometric2F1}[1/2, 2/3, 5/3, \text{Cos}[c + d*x]^2]*\text{Sin}[c + d*x])/(4*d*(b*\text{Sec}[c + d*x])^{4/3}*\text{Sqrt}[\text{Sin}[c + d*x]^2]) - (3*B*\text{Hypergeometric2F1}[1/6, 1/2, 7/6, \text{Cos}[c + d*x]^2]*\text{Sin}[c + d*x])/(b*d*(b*\text{Sec}[c + d*x])^{1/3}*\text{Sqrt}[\text{Sin}[c + d*x]^2])$

Rule 16

$\text{Int}[(u_*)*(v_)^{(m_*)}*((b_)*(v_))^{(n_)}, x_Symbol] := \text{Dist}[1/b^m, \text{Int}[u*(b*v)^{(m+n)}, x], x] /; \text{FreeQ}\{b, n\}, x \ \&\& \ \text{IntegerQ}[m]$

Rule 2643

$\text{Int}[(b_*)*\sin[(c_*) + (d_*)(x_)]^{(n_)}, x_Symbol] := \text{Simp}[(\text{Cos}[c + d*x]*(b*\text{Sin}[c + d*x])^{(n+1)}*\text{Hypergeometric2F1}[1/2, (n+1)/2, (n+3)/2, \text{Sin}[c + d*x]^2])/(b*d*(n+1)*\text{Sqrt}[\text{Cos}[c + d*x]^2]), x] /; \text{FreeQ}\{b, c, d, n\}, x \ \&\& \ !\text{IntegerQ}[2*n]$

Rule 3772

$\text{Int}[(\text{csc}[(c_*) + (d_*)(x_)]*(b_*)^{(n_)}, x_Symbol] := \text{Simp}[(b*\text{Csc}[c + d*x])^{(n-1)}*((\text{Sin}[c + d*x]/b)^{(n-1)}*\text{Int}[1/(\text{Sin}[c + d*x]/b)^n, x]), x] /; \text{Fr}$

eeQ[{b, c, d, n}, x] && !IntegerQ[n]

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rubi steps

$$\begin{aligned} \int \frac{\sec(c + dx)(A + B \sec(c + dx))}{(b \sec(c + dx))^{4/3}} dx &= \frac{\int \frac{A+B \sec(c+dx)}{\sqrt[3]{b \sec(c+dx)}} dx}{b} \\ &= \frac{A \int \frac{1}{\sqrt[3]{b \sec(c+dx)}} dx}{b} + \frac{B \int (b \sec(c + dx))^{2/3} dx}{b^2} \\ &= \frac{\left(A \left(\frac{\cos(c+dx)}{b}\right)^{2/3} (b \sec(c + dx))^{2/3}\right) \int \sqrt[3]{\frac{\cos(c+dx)}{b}} dx}{b} + \frac{\left(B \left(\frac{\cos(c+dx)}{b}\right)^{2/3} (b \sec(c + dx))^{2/3}\right) \int \sqrt[3]{\frac{\cos(c+dx)}{b}} dx}{b} \\ &= \frac{3B \cos(c + dx) {}_2F_1\left(\frac{1}{6}, \frac{1}{2}; \frac{7}{6}; \cos^2(c + dx)\right) (b \sec(c + dx))^{2/3} \sin(c + dx)}{b^2 d \sqrt{\sin^2(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.10, size = 91, normalized size = 0.80

$$\frac{3\sqrt{-\tan^2(c + dx)} \csc(c + dx) \left(2A \cos(c + dx) {}_2F_1\left(-\frac{1}{6}, \frac{1}{2}; \frac{5}{6}; \sec^2(c + dx)\right) - B {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; \sec^2(c + dx)\right)\right)}{2bd\sqrt[3]{b \sec(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/(b*Sec[c + d*x])^(4/3), x]

[Out] (-3*Csc[c + d*x]*(2*A*Cos[c + d*x]*Hypergeometric2F1[-1/6, 1/2, 5/6, Sec[c + d*x]^2] - B*Hypergeometric2F1[1/3, 1/2, 4/3, Sec[c + d*x]^2])*Sqrt[-Tan[c + d*x]^2])/(2*b*d*(b*Sec[c + d*x])^(1/3))

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{(B \sec(dx + c) + A) (b \sec(dx + c))^{2/3}}{b^2 \sec(dx + c)}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(4/3),x, algorithm="fricas")

[Out] integral((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(2/3)/(b^2*sec(d*x + c)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sec(dx + c)}{(b \sec(dx + c))^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(4/3),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)/(b*sec(d*x + c))^(4/3), x)

maple [F] time = 0.75, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx + c)(A + B \sec(dx + c))}{(b \sec(dx + c))^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(4/3),x)

[Out] int(sec(d*x+c)*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(4/3),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sec(dx + c)}{(b \sec(dx + c))^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(4/3),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)/(b*sec(d*x + c))^(4/3), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\cos(c + dx) \left(\frac{b}{\cos(c+dx)}\right)^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B/cos(c + d*x))/(cos(c + d*x)*(b/cos(c + d*x))^(4/3)), x)`

[Out] `int((A + B/cos(c + d*x))/(cos(c + d*x)*(b/cos(c + d*x))^(4/3)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \sec(c + dx)}{(b \sec(c + dx))^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(b*sec(d*x+c))**(4/3), x)`

[Out] `Integral((A + B*sec(c + d*x))*sec(c + d*x)/(b*sec(c + d*x))**(4/3), x)`

$$3.24 \quad \int \frac{A+B \sec(c+dx)}{(b \sec(c+dx))^{4/3}} dx$$

Optimal. Leaf size=114

$$\frac{3Ab \sin(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{7}{6}; \frac{13}{6}; \cos^2(c+dx)\right)}{7d\sqrt{\sin^2(c+dx)}(b \sec(c+dx))^{7/3}} - \frac{3B \sin(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; \cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}(b \sec(c+dx))^{4/3}}$$

[Out] $-3/7*A*b*\text{hypergeom}([1/2, 7/6], [13/6], \cos(d*x+c)^2)*\sin(d*x+c)/d/(b*\sec(d*x+c))^{(7/3)}/(\sin(d*x+c)^2)^{(1/2)}-3/4*B*\text{hypergeom}([1/2, 2/3], [5/3], \cos(d*x+c)^2)*\sin(d*x+c)/d/(b*\sec(d*x+c))^{(4/3)}/(\sin(d*x+c)^2)^{(1/2)}$

Rubi [A] time = 0.09, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3787, 3772, 2643}

$$\frac{3Ab \sin(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{7}{6}; \frac{13}{6}; \cos^2(c+dx)\right)}{7d\sqrt{\sin^2(c+dx)}(b \sec(c+dx))^{7/3}} - \frac{3B \sin(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; \cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}(b \sec(c+dx))^{4/3}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(A + B*\text{Sec}[c + d*x])/(b*\text{Sec}[c + d*x])^{(4/3)}, x]$

[Out] $(-3*A*b*\text{Hypergeometric2F1}[1/2, 7/6, 13/6, \text{Cos}[c + d*x]^2]*\text{Sin}[c + d*x])/(7*d*(b*\text{Sec}[c + d*x])^{(7/3)}*\text{Sqrt}[\text{Sin}[c + d*x]^2]) - (3*B*\text{Hypergeometric2F1}[1/2, 2/3, 5/3, \text{Cos}[c + d*x]^2]*\text{Sin}[c + d*x])/(4*d*(b*\text{Sec}[c + d*x])^{(4/3)}*\text{Sqrt}[\text{Sin}[c + d*x]^2])$

Rule 2643

$\text{Int}[(b_*)*\sin[(c_*) + (d_*)*(x_)]^{(n_)}, x_Symbol] \rightarrow \text{Simp}[(\text{Cos}[c + d*x]*(b*\text{Sin}[c + d*x])^{(n+1)}*\text{Hypergeometric2F1}[1/2, (n+1)/2, (n+3)/2, \text{Sin}[c + d*x]^2])/(b*d*(n+1)*\text{Sqrt}[\text{Cos}[c + d*x]^2]), x] /; \text{FreeQ}\{b, c, d, n\}, x \&\& \text{IntegerQ}[2*n]$

Rule 3772

$\text{Int}[(\text{csc}[(c_*) + (d_*)*(x_)]*(b_*))^{(n_)}, x_Symbol] \rightarrow \text{Simp}[(b*\text{Csc}[c + d*x])^{(n-1)}*((\text{Sin}[c + d*x]/b)^{(n-1)}*\text{Int}[1/(\text{Sin}[c + d*x]/b)^n, x]), x] /; \text{FreeQ}\{b, c, d, n\}, x \&\& \text{IntegerQ}[n]$

Rule 3787

$\text{Int}[(\text{csc}[(e_*) + (f_*)*(x_)]*(d_*))^{(n_*)}*(\text{csc}[(e_*) + (f_*)*(x_)]*(b_*) + (a_*)), x_Symbol] \rightarrow \text{Dist}[a, \text{Int}[(d*\text{Csc}[e + f*x])^n, x], x] + \text{Dist}[b/d, \text{Int}[\text{Csc}[e + f*x]^n, x], x]$

$(d \cdot \text{Csc}[e + f \cdot x])^{(n + 1)}, x], x] /; \text{FreeQ}[\{a, b, d, e, f, n\}, x]$

Rubi steps

$$\begin{aligned} \int \frac{A + B \sec(c + dx)}{(b \sec(c + dx))^{4/3}} dx &= A \int \frac{1}{(b \sec(c + dx))^{4/3}} dx + \frac{B \int \frac{1}{\sqrt[3]{b \sec(c + dx)}} dx}{b} \\ &= \left(A \left(\frac{\cos(c + dx)}{b} \right)^{2/3} (b \sec(c + dx))^{2/3} \right) \int \left(\frac{\cos(c + dx)}{b} \right)^{4/3} dx + \frac{\left(B \left(\frac{\cos(c + dx)}{b} \right)^{2/3} (b \sec(c + dx))^{2/3} \right) \int \left(\frac{\cos(c + dx)}{b} \right)^{4/3} dx}{b} \\ &= -\frac{3B \cos^2(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; \cos^2(c + dx)\right) (b \sec(c + dx))^{2/3} \sin(c + dx)}{4b^2 d \sqrt{\sin^2(c + dx)}} - \frac{3A \cos^3(c + dx)}{4b^2 d \sqrt{\sin^2(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.14, size = 87, normalized size = 0.76

$$\frac{3\sqrt{-\tan^2(c + dx)} \csc(c + dx) \left(A \cos(c + dx) {}_2F_1\left(-\frac{2}{3}, \frac{1}{2}; \frac{1}{3}; \sec^2(c + dx)\right) + 4B {}_2F_1\left(-\frac{1}{6}, \frac{1}{2}; \frac{5}{6}; \sec^2(c + dx)\right) \right)}{4d(b \sec(c + dx))^{4/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sec[c + d*x])/(b*Sec[c + d*x])^(4/3), x]

[Out] (-3*Csc[c + d*x]*(A*Cos[c + d*x]*Hypergeometric2F1[-2/3, 1/2, 1/3, Sec[c + d*x]^2] + 4*B*Hypergeometric2F1[-1/6, 1/2, 5/6, Sec[c + d*x]^2])*Sqrt[-Tan[c + d*x]^2])/(4*d*(b*Sec[c + d*x])^(4/3))

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B \sec(dx + c) + A)(b \sec(dx + c))^{2/3}}{b^2 \sec(dx + c)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(b*sec(d*x+c))^(4/3), x, algorithm="fricas")

[Out] integral((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(2/3)/(b^2*sec(d*x + c)^2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(b \sec(dx + c))^{4/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(b*sec(d*x+c))^(4/3),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)/(b*sec(d*x + c))^(4/3), x)

maple [F] time = 0.80, size = 0, normalized size = 0.00

$$\int \frac{A + B \sec(dx + c)}{(b \sec(dx + c))^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))/(b*sec(d*x+c))^(4/3),x)

[Out] int((A+B*sec(d*x+c))/(b*sec(d*x+c))^(4/3),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(b \sec(dx + c))^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(b*sec(d*x+c))^(4/3),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)/(b*sec(d*x + c))^(4/3), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\left(\frac{b}{\cos(c+dx)}\right)^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/(b/cos(c + d*x))^(4/3),x)

[Out] int((A + B/cos(c + d*x))/(b/cos(c + d*x))^(4/3), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \sec(c + dx)}{(b \sec(c + dx))^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(b*sec(d*x+c))**(4/3),x)

[Out] Integral((A + B*sec(c + d*x))/(b*sec(c + d*x))**(4/3), x)

$$3.25 \quad \int \frac{\sec(c+dx)(A+B \sec(c+dx))}{(b \sec(c+dx))^{4/3}} dx$$

Optimal. Leaf size=114

$$\frac{3A \sin(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; \cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}(b \sec(c+dx))^{4/3}} - \frac{3B \sin(c+dx) {}_2F_1\left(\frac{1}{6}, \frac{1}{2}; \frac{7}{6}; \cos^2(c+dx)\right)}{bd\sqrt{\sin^2(c+dx)}\sqrt[3]{b \sec(c+dx)}}$$

[Out] $-3/4*A*\text{hypergeom}([1/2, 2/3], [5/3], \cos(d*x+c)^2)*\sin(d*x+c)/d/(b*\sec(d*x+c))^{4/3}/(\sin(d*x+c)^2)^{(1/2)}-3*B*\text{hypergeom}([1/6, 1/2], [7/6], \cos(d*x+c)^2)*\sin(d*x+c)/b/d/(b*\sec(d*x+c))^{1/3}/(\sin(d*x+c)^2)^{(1/2)}$

Rubi [A] time = 0.09, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {16, 3787, 3772, 2643}

$$\frac{3A \sin(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; \cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}(b \sec(c+dx))^{4/3}} - \frac{3B \sin(c+dx) {}_2F_1\left(\frac{1}{6}, \frac{1}{2}; \frac{7}{6}; \cos^2(c+dx)\right)}{bd\sqrt{\sin^2(c+dx)}\sqrt[3]{b \sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(\text{Sec}[c + d*x]*(A + B*\text{Sec}[c + d*x]))/(b*\text{Sec}[c + d*x])^{4/3}, x]$

[Out] $(-3*A*\text{Hypergeometric2F1}[1/2, 2/3, 5/3, \text{Cos}[c + d*x]^2]*\text{Sin}[c + d*x])/(4*d*(b*\text{Sec}[c + d*x])^{4/3}*\text{Sqrt}[\text{Sin}[c + d*x]^2]) - (3*B*\text{Hypergeometric2F1}[1/6, 1/2, 7/6, \text{Cos}[c + d*x]^2]*\text{Sin}[c + d*x])/(b*d*(b*\text{Sec}[c + d*x])^{1/3}*\text{Sqrt}[\text{Sin}[c + d*x]^2])$

Rule 16

$\text{Int}[(u_*)*(v_*)^{(m_*)}*((b_*)*(v_*)^{(n_*)}), x_Symbol] \rightarrow \text{Dist}[1/b^m, \text{Int}[u*(b*v)^{(m+n)}, x], x] /; \text{FreeQ}\{b, n, x\} \ \&\amp; \ \text{IntegerQ}[m]$

Rule 2643

$\text{Int}[(b_*)*\sin[(c_*) + (d_*)*(x_*)]^{(n_*)}, x_Symbol] \rightarrow \text{Simp}[(\text{Cos}[c + d*x]*(b*\text{Sin}[c + d*x])^{(n+1)}*\text{Hypergeometric2F1}[1/2, (n+1)/2, (n+3)/2, \text{Sin}[c + d*x]^2])/(b*d*(n+1)*\text{Sqrt}[\text{Cos}[c + d*x]^2]), x] /; \text{FreeQ}\{b, c, d, n, x\} \ \&\amp; \ !\text{IntegerQ}[2*n]$

Rule 3772

$\text{Int}[(\text{csc}[(c_*) + (d_*)*(x_*)]*(b_*)^{(n_*)}), x_Symbol] \rightarrow \text{Simp}[(b*\text{Csc}[c + d*x])^{(n-1)}*((\text{Sin}[c + d*x]/b)^{(n-1)}*\text{Int}[1/(\text{Sin}[c + d*x]/b)^n, x]), x] /; \text{Fr}$

eeQ[{b, c, d, n}, x] && !IntegerQ[n]

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rubi steps

$$\begin{aligned} \int \frac{\sec(c + dx)(A + B \sec(c + dx))}{(b \sec(c + dx))^{4/3}} dx &= \frac{\int \frac{A+B \sec(c+dx)}{\sqrt[3]{b \sec(c+dx)}} dx}{b} \\ &= \frac{A \int \frac{1}{\sqrt[3]{b \sec(c+dx)}} dx}{b} + \frac{B \int (b \sec(c + dx))^{2/3} dx}{b^2} \\ &= \frac{\left(A \left(\frac{\cos(c+dx)}{b} \right)^{2/3} (b \sec(c + dx))^{2/3} \right) \int \sqrt[3]{\frac{\cos(c+dx)}{b}} dx}{b} + \frac{\left(B \left(\frac{\cos(c+dx)}{b} \right)^{2/3} \right) \int \sqrt[3]{\frac{\cos(c+dx)}{b}} dx}{b} \\ &= \frac{3B \cos(c + dx) {}_2F_1\left(\frac{1}{6}, \frac{1}{2}; \frac{7}{6}; \cos^2(c + dx)\right) (b \sec(c + dx))^{2/3} \sin(c + dx)}{b^2 d \sqrt{\sin^2(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.09, size = 91, normalized size = 0.80

$$\frac{3\sqrt{-\tan^2(c + dx)} \csc(c + dx) \left(2A \cos(c + dx) {}_2F_1\left(-\frac{1}{6}, \frac{1}{2}; \frac{5}{6}; \sec^2(c + dx)\right) - B {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; \sec^2(c + dx)\right) \right)}{2bd \sqrt[3]{b \sec(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/(b*Sec[c + d*x])^(4/3), x]

[Out] (-3*Csc[c + d*x]*(2*A*Cos[c + d*x]*Hypergeometric2F1[-1/6, 1/2, 5/6, Sec[c + d*x]^2] - B*Hypergeometric2F1[1/3, 1/2, 4/3, Sec[c + d*x]^2])*Sqrt[-Tan[c + d*x]^2])/(2*b*d*(b*Sec[c + d*x])^(1/3))

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{(B \sec(dx + c) + A) (b \sec(dx + c))^{2/3}}{b^2 \sec(dx + c)}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(4/3),x, algorithm="fricas")

[Out] integral((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(2/3)/(b^2*sec(d*x + c)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sec(dx + c)}{(b \sec(dx + c))^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(4/3),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)/(b*sec(d*x + c))^(4/3), x)

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx + c) (A + B \sec(dx + c))}{(b \sec(dx + c))^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(4/3),x)

[Out] int(sec(d*x+c)*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(4/3),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sec(dx + c)}{(b \sec(dx + c))^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(4/3),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)/(b*sec(d*x + c))^(4/3), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\cos(c+dx) \left(\frac{b}{\cos(c+dx)}\right)^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B/cos(c + d*x))/(cos(c + d*x)*(b/cos(c + d*x))^(4/3)),x)`

[Out] `int((A + B/cos(c + d*x))/(cos(c + d*x)*(b/cos(c + d*x))^(4/3)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \sec(c + dx)}{(b \sec(c + dx))^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(b*sec(d*x+c))**(4/3),x)`

[Out] `Integral((A + B*sec(c + d*x))*sec(c + d*x)/(b*sec(c + d*x))**(4/3), x)`

$$3.26 \quad \int \frac{\sec^2(c+dx)(A+B \sec(c+dx))}{(b \sec(c+dx))^{4/3}} dx$$

Optimal. Leaf size=117

$$\frac{3B \sin(c+dx)(b \sec(c+dx))^{2/3} {}_2F_1\left(-\frac{1}{3}, \frac{1}{2}; \frac{2}{3}; \cos^2(c+dx)\right)}{2b^2 d \sqrt{\sin^2(c+dx)}} - \frac{3A \sin(c+dx) {}_2F_1\left(\frac{1}{6}, \frac{1}{2}; \frac{7}{6}; \cos^2(c+dx)\right)}{bd \sqrt{\sin^2(c+dx)} \sqrt[3]{b \sec(c+dx)}}$$

[Out] $-3*A*\text{hypergeom}([1/6, 1/2], [7/6], \cos(d*x+c)^2)*\sin(d*x+c)/b/d/(b*\sec(d*x+c))^{1/3}/(\sin(d*x+c)^2)^{1/2}+3/2*B*\text{hypergeom}([-1/3, 1/2], [2/3], \cos(d*x+c)^2)*(b*\sec(d*x+c))^{2/3}*\sin(d*x+c)/b^2/d/(\sin(d*x+c)^2)^{1/2}$

Rubi [A] time = 0.10, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {16, 3787, 3772, 2643}

$$\frac{3B \sin(c+dx)(b \sec(c+dx))^{2/3} {}_2F_1\left(-\frac{1}{3}, \frac{1}{2}; \frac{2}{3}; \cos^2(c+dx)\right)}{2b^2 d \sqrt{\sin^2(c+dx)}} - \frac{3A \sin(c+dx) {}_2F_1\left(\frac{1}{6}, \frac{1}{2}; \frac{7}{6}; \cos^2(c+dx)\right)}{bd \sqrt{\sin^2(c+dx)} \sqrt[3]{b \sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(\text{Sec}[c + d*x]^2*(A + B*\text{Sec}[c + d*x]))/(b*\text{Sec}[c + d*x]^{4/3}), x]$

[Out] $(-3*A*\text{Hypergeometric2F1}[1/6, 1/2, 7/6, \text{Cos}[c + d*x]^2]*\text{Sin}[c + d*x])/(b*d*(b*\text{Sec}[c + d*x]^{1/3}*\text{Sqrt}[\text{Sin}[c + d*x]^2]) + (3*B*\text{Hypergeometric2F1}[-1/3, 1/2, 2/3, \text{Cos}[c + d*x]^2]*(b*\text{Sec}[c + d*x]^{2/3}*\text{Sin}[c + d*x])/(2*b^2*d*\text{Sqrt}[\text{Sin}[c + d*x]^2]))$

Rule 16

$\text{Int}[(u_*)*(v_)^{(m_*)}*((b_*)*(v_))^{(n_*)}, x_Symbol] \rightarrow \text{Dist}[1/b^m, \text{Int}[u*(b*v)^{(m+n)}, x], x] /; \text{FreeQ}\{b, n\}, x \ \&\& \ \text{IntegerQ}[m]$

Rule 2643

$\text{Int}[(b_*)*\sin[(c_*) + (d_*)*(x_)]^{(n_*)}, x_Symbol] \rightarrow \text{Simp}[(\text{Cos}[c + d*x]*(b*\text{Sin}[c + d*x])^{(n+1)}*\text{Hypergeometric2F1}[1/2, (n+1)/2, (n+3)/2, \text{Sin}[c + d*x]^2])/(b*d*(n+1)*\text{Sqrt}[\text{Cos}[c + d*x]^2]), x] /; \text{FreeQ}\{b, c, d, n\}, x \ \&\& \ !\text{IntegerQ}[2*n]$

Rule 3772

$\text{Int}[(\text{csc}[(c_*) + (d_*)*(x_)]*(b_*)^{(n_*)}, x_Symbol] \rightarrow \text{Simp}[(b*\text{Csc}[c + d*x])^{(n-1)}*((\text{Sin}[c + d*x]/b)^{(n-1)}*\text{Int}[1/(\text{Sin}[c + d*x]/b)^n, x]), x] /; \text{Fr}$

eeQ[{b, c, d, n}, x] && !IntegerQ[n]

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rubi steps

$$\begin{aligned} \int \frac{\sec^2(c + dx)(A + B \sec(c + dx))}{(b \sec(c + dx))^{4/3}} dx &= \frac{\int (b \sec(c + dx))^{2/3} (A + B \sec(c + dx)) dx}{b^2} \\ &= \frac{A \int (b \sec(c + dx))^{2/3} dx}{b^2} + \frac{B \int (b \sec(c + dx))^{5/3} dx}{b^3} \\ &= \frac{\left(A \left(\frac{\cos(c+dx)}{b} \right)^{2/3} (b \sec(c + dx))^{2/3} \right) \int \frac{1}{\left(\frac{\cos(c+dx)}{b} \right)^{2/3}} dx}{b^2} + \frac{\left(B \left(\frac{\cos(c+dx)}{b} \right)^{2/3} \right)}{b^2} \\ &= \frac{3B {}_2F_1\left(-\frac{1}{3}, \frac{1}{2}; \frac{2}{3}; \cos^2(c + dx)\right) (b \sec(c + dx))^{2/3} \sin(c + dx) - 3A \cos(c + dx)}{2b^2 d \sqrt{\sin^2(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.19, size = 91, normalized size = 0.78

$$\frac{3\sqrt{-\tan^2(c + dx)} \csc(c + dx)(b \sec(c + dx))^{2/3} \left(5A \cos(c + dx) {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; \sec^2(c + dx)\right) + 2B {}_2F_1\left(\frac{1}{2}, \frac{5}{6}; \frac{11}{6}; \sec^2(c + dx)\right) \right)}{10b^2 d}$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(b*Sec[c + d*x])^(4/3),x]

[Out] (3*Csc[c + d*x]*(5*A*Cos[c + d*x]*Hypergeometric2F1[1/3, 1/2, 4/3, Sec[c + d*x]^2] + 2*B*Hypergeometric2F1[1/2, 5/6, 11/6, Sec[c + d*x]^2])*(b*Sec[c + d*x])^(2/3)*Sqrt[-Tan[c + d*x]^2])/(10*b^2*d)

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{(B \sec(dx + c) + A) (b \sec(dx + c))^{2/3}}{b^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(4/3),x, algorithm="fricas")

[Out] integral((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(2/3)/b^2, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sec(dx + c)^2}{(b \sec(dx + c))^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(4/3),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^2/(b*sec(d*x + c))^(4/3), x)

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(\sec^2(dx + c))(A + B \sec(dx + c))}{(b \sec(dx + c))^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^2*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(4/3),x)

[Out] int(sec(d*x+c)^2*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(4/3),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sec(dx + c)^2}{(b \sec(dx + c))^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(4/3),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^2/(b*sec(d*x + c))^(4/3), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\cos(c+dx)^2 \left(\frac{b}{\cos(c+dx)}\right)^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B/cos(c + d*x))/(cos(c + d*x)^2*(b/cos(c + d*x))^(4/3)),x)`

[Out] `int((A + B/cos(c + d*x))/(cos(c + d*x)^2*(b/cos(c + d*x))^(4/3)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \sec^2(c + dx)}{(b \sec(c + dx))^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)**2*(A+B*sec(d*x+c))/(b*sec(d*x+c))**(4/3),x)`

[Out] `Integral((A + B*sec(c + d*x))*sec(c + d*x)**2/(b*sec(c + d*x))**(4/3), x)`

3.27 $\int \sec^m(c + dx)(b \sec(c + dx))^{4/3}(A + B \sec(c + dx)) dx$

Optimal. Leaf size=167

$$\frac{3Ab \sin(c + dx) \sqrt[3]{b \sec(c + dx)} \sec^m(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(-3m - 1); \frac{1}{6}(5 - 3m); \cos^2(c + dx)\right) + 3bB \sin(c + dx) \sqrt[3]{b \sec(c + dx)}}{d(3m + 1)\sqrt{\sin^2(c + dx)}}$$

[Out] 3*A*b*hypergeom([1/2, -1/6-1/2*m], [5/6-1/2*m], cos(d*x+c)^2)*sec(d*x+c)^m*(b*sec(d*x+c))^(1/3)*sin(d*x+c)/d/(1+3*m)/(sin(d*x+c)^2)^(1/2)+3*b*B*hypergeom([1/2, -2/3-1/2*m], [1/3-1/2*m], cos(d*x+c)^2)*sec(d*x+c)^(1+m)*(b*sec(d*x+c))^(1/3)*sin(d*x+c)/d/(4+3*m)/(sin(d*x+c)^2)^(1/2)

Rubi [A] time = 0.12, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {20, 3787, 3772, 2643}

$$\frac{3Ab \sin(c + dx) \sqrt[3]{b \sec(c + dx)} \sec^m(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(-3m - 1); \frac{1}{6}(5 - 3m); \cos^2(c + dx)\right) + 3bB \sin(c + dx) \sqrt[3]{b \sec(c + dx)}}{d(3m + 1)\sqrt{\sin^2(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d*x]^m*(b*Sec[c + d*x])^(4/3)*(A + B*Sec[c + d*x]), x]

[Out] (3*A*b*Hypergeometric2F1[1/2, (-1 - 3*m)/6, (5 - 3*m)/6, Cos[c + d*x]^2]*Sec[c + d*x]^m*(b*Sec[c + d*x])^(1/3)*Sin[c + d*x])/(d*(1 + 3*m)*Sqrt[Sin[c + d*x]^2]) + (3*b*B*Hypergeometric2F1[1/2, (-4 - 3*m)/6, (2 - 3*m)/6, Cos[c + d*x]^2]*Sec[c + d*x]^(1 + m)*(b*Sec[c + d*x])^(1/3)*Sin[c + d*x])/(d*(4 + 3*m)*Sqrt[Sin[c + d*x]^2])

Rule 20

Int[(u_)*((a_)*(v_))^(m_)*((b_)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m+n), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m+n]

Rule 2643

Int[((b_)*sin[(c_)+(d_)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(b*Ssin[c + d*x])^(n+1)*Hypergeometric2F1[1/2, (n+1)/2, (n+3)/2, Sin[c + d*x]^2])/(b*d*(n+1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rule 3772

Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x])^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rubi steps

$$\begin{aligned} \int \sec^m(c + dx)(b \sec(c + dx))^{4/3}(A + B \sec(c + dx)) dx &= \frac{(b^3 \sqrt[3]{b \sec(c + dx)}) \int \sec^{\frac{4}{3}+m}(c + dx)(A + B \sec(c + dx)) dx}{\sqrt[3]{\sec(c + dx)}} \\ &= \frac{(Ab^3 \sqrt[3]{b \sec(c + dx)}) \int \sec^{\frac{4}{3}+m}(c + dx) dx}{\sqrt[3]{\sec(c + dx)}} + \frac{(bB^3 \sqrt[3]{b \sec(c + dx)}) \int \sec^{\frac{4}{3}+m}(c + dx) dx}{\sqrt[3]{\sec(c + dx)}} \\ &= \left(Ab \cos^{\frac{1}{3}+m}(c + dx) \sec^m(c + dx) \sqrt[3]{b \sec(c + dx)} \right) \int \sec^{\frac{4}{3}+m}(c + dx) dx \\ &= \frac{3Ab {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(-1 - 3m); \frac{1}{6}(5 - 3m); \cos^2(c + dx)\right) \sec^{\frac{4}{3}+m}(c + dx)}{d(1 + 3m)\sqrt{\sin^2(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.39, size = 140, normalized size = 0.84

$$\frac{3\sqrt{-\tan^2(c + dx)} \csc(c + dx)(b \sec(c + dx))^{4/3} \sec^m(c + dx) \left(A(3m + 7) \cos(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(3m + 4); \frac{m}{2} + \frac{5}{3}; \sin^2(c + dx)\right) + B(4 + 3m) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(3m + 4); \frac{m}{2} + \frac{5}{3}; \sin^2(c + dx)\right) \right)}{d(3m + 4)(3m + 7)}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[c + d*x]^m*(b*Sec[c + d*x])^(4/3)*(A + B*Sec[c + d*x]),x]

[Out] (3*Csc[c + d*x]*(A*(7 + 3*m)*Cos[c + d*x]*Hypergeometric2F1[1/2, (4 + 3*m)/6, 5/3 + m/2, Sec[c + d*x]^2] + B*(4 + 3*m)*Hypergeometric2F1[1/2, (7 + 3*m)/6, (13 + 3*m)/6, Sec[c + d*x]^2])*Sec[c + d*x]^m*(b*Sec[c + d*x])^(4/3)*Sqrt[-Tan[c + d*x]^2]/(d*(4 + 3*m)*(7 + 3*m))

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb \sec(dx + c)^2 + Ab \sec(dx + c)\right) (b \sec(dx + c))^{\frac{1}{3}} \sec(dx + c)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^m*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] integral((B*b*sec(d*x + c)^2 + A*b*sec(d*x + c))*(b*sec(d*x + c))^(1/3)*sec(d*x + c)^m, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A) (b \sec(dx + c))^{\frac{4}{3}} \sec(dx + c)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^m*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(4/3)*sec(d*x + c)^m, x)

maple [F] time = 1.74, size = 0, normalized size = 0.00

$$\int (\sec^m(dx + c)) (b \sec(dx + c))^{\frac{4}{3}} (A + B \sec(dx + c)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^m*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x)

[Out] int(sec(d*x+c)^m*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A) (b \sec(dx + c))^{\frac{4}{3}} \sec(dx + c)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^m*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(4/3)*sec(d*x + c)^m, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A + \frac{B}{\cos(c + dx)} \right) \left(\frac{b}{\cos(c + dx)} \right)^{\frac{4}{3}} \left(\frac{1}{\cos(c + dx)} \right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.


```
[In] int((A + B/cos(c + d*x))*(b/cos(c + d*x))^(4/3)*(1/cos(c + d*x))^m,x)
```

```
[Out] int((A + B/cos(c + d*x))*(b/cos(c + d*x))^(4/3)*(1/cos(c + d*x))^m, x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**m*(b*sec(d*x+c))**(4/3)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

3.28 $\int \sec^m(c + dx)(b \sec(c + dx))^{2/3}(A + B \sec(c + dx)) dx$

Optimal. Leaf size=165

$$\frac{3B \sin(c + dx)(b \sec(c + dx))^{2/3} \sec^m(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(-3m - 2); \frac{1}{6}(4 - 3m); \cos^2(c + dx)\right) - 3A \sin(c + dx)(b \sec(c + dx))^{2/3} \sec^m(c + dx)}{d(3m + 2)\sqrt{\sin^2(c + dx)}}$$

[Out] $-3*A*\text{hypergeom}\left(\left[\frac{1}{2}, \frac{1}{6}-\frac{1}{2}*m\right], \left[\frac{7}{6}-\frac{1}{2}*m\right], \cos(d*x+c)^2\right)*\sec(d*x+c)^{-1+m}*(b*\sec(d*x+c))^{2/3}*\sin(d*x+c)/d/(1-3*m)/(\sin(d*x+c)^2)^{1/2}+3*B*\text{hypergeom}\left(\left[\frac{1}{2}, -\frac{1}{3}-\frac{1}{2}*m\right], \left[\frac{2}{3}-\frac{1}{2}*m\right], \cos(d*x+c)^2\right)*\sec(d*x+c)^m*(b*\sec(d*x+c))^{2/3}*\sin(d*x+c)/d/(2+3*m)/(\sin(d*x+c)^2)^{1/2}$

Rubi [A] time = 0.12, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {20, 3787, 3772, 2643}

$$\frac{3B \sin(c + dx)(b \sec(c + dx))^{2/3} \sec^m(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(-3m - 2); \frac{1}{6}(4 - 3m); \cos^2(c + dx)\right) - 3A \sin(c + dx)(b \sec(c + dx))^{2/3} \sec^m(c + dx)}{d(3m + 2)\sqrt{\sin^2(c + dx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sec}[c + d*x]^m*(b*\text{Sec}[c + d*x])^{2/3}*(A + B*\text{Sec}[c + d*x]), x]$

[Out] $(-3*A*\text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1 - 3*m}{6}, \frac{7 - 3*m}{6}, \text{Cos}[c + d*x]^2\right]*\text{Sec}[c + d*x]^{-1 + m}*(b*\text{Sec}[c + d*x])^{2/3}*\text{Sin}[c + d*x])/(d*(1 - 3*m)*\text{Sqrt}[\text{Sin}[c + d*x]^2]) + (3*B*\text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{-2 - 3*m}{6}, \frac{4 - 3*m}{6}, \text{Cos}[c + d*x]^2\right]*\text{Sec}[c + d*x]^m*(b*\text{Sec}[c + d*x])^{2/3}*\text{Sin}[c + d*x])/(d*(2 + 3*m)*\text{Sqrt}[\text{Sin}[c + d*x]^2])$

Rule 20

$\text{Int}[(u_*)*((a_*)*(v_))^{(m_*)}*((b_*)*(v_))^{(n_*)}, x_Symbol] \rightarrow \text{Dist}[(b^{\text{IntPart}[n]}*(b*v)^{\text{FracPart}[n]})/(a^{\text{IntPart}[n]}*(a*v)^{\text{FracPart}[n]}), \text{Int}[u*(a*v)^{(m+n)}, x], x] /;$ FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m + n]

Rule 2643

$\text{Int}[(b_*)*\sin[(c_*) + (d_*)*(x_)]^{(n_*)}, x_Symbol] \rightarrow \text{Simp}[(\text{Cos}[c + d*x]*(b*\text{Sin}[c + d*x])^{(n+1)}*\text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{(n+1)}{2}, \frac{(n+3)}{2}, \text{Sin}[c + d*x]^2\right])/(b*d*(n+1)*\text{Sqrt}[\text{Cos}[c + d*x]^2]), x] /;$ FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rule 3772

Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.))^(n_.), x_Symbol] :> Simp[(b*Csc[c + d*x])^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rubi steps

$$\begin{aligned} \int \sec^m(c + dx)(b \sec(c + dx))^{2/3}(A + B \sec(c + dx)) dx &= \frac{(b \sec(c + dx))^{2/3} \int \sec^{\frac{2}{3}+m}(c + dx)(A + B \sec(c + dx)) dx}{\sec^{\frac{2}{3}}(c + dx)} \\ &= \frac{(A(b \sec(c + dx))^{2/3}) \int \sec^{\frac{2}{3}+m}(c + dx) dx}{\sec^{\frac{2}{3}}(c + dx)} + \frac{(B(b \sec(c + dx))^{2/3}) \int \sec^{\frac{2}{3}+m}(c + dx) dx}{\sec^{\frac{2}{3}}(c + dx)} \\ &= \left(A \cos^{\frac{2}{3}+m}(c + dx) \sec^m(c + dx)(b \sec(c + dx))^{2/3} \right) \int \sec^{\frac{2}{3}+m}(c + dx) dx \\ &= \frac{3A {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(1 - 3m); \frac{1}{6}(7 - 3m); \cos^2(c + dx)\right) \sec^{\frac{2}{3}+m}(c + dx)}{d(1 - 3m)\sqrt{\sin^2(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.22, size = 140, normalized size = 0.85

$$\frac{3\sqrt{-\tan^2(c + dx)} \csc(c + dx)(b \sec(c + dx))^{2/3} \sec^m(c + dx) \left(A(3m + 5) \cos(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(3m + 2); \frac{1}{6}(3m + 5); \cos^2(c + dx)\right) + B(2 + 3m) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(3m + 2); \frac{1}{6}(3m + 5); \cos^2(c + dx)\right) \right)}{d(3m + 2)(3m + 5)}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[c + d*x]^m*(b*Sec[c + d*x])^(2/3)*(A + B*Sec[c + d*x]), x]

[Out] (3*Csc[c + d*x]*(A*(5 + 3*m)*Cos[c + d*x]*Hypergeometric2F1[1/2, (2 + 3*m)/6, (8 + 3*m)/6, Sec[c + d*x]^2] + B*(2 + 3*m)*Hypergeometric2F1[1/2, (5 + 3*m)/6, (11 + 3*m)/6, Sec[c + d*x]^2])*Sec[c + d*x]^m*(b*Sec[c + d*x])^(2/3)*Sqrt[-Tan[c + d*x]^2]/(d*(2 + 3*m)*(5 + 3*m))

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left((B \sec(dx + c) + A)(b \sec(dx + c))^{\frac{2}{3}} \sec(dx + c)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^m*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] integral((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(2/3)*sec(d*x + c)^m, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A) (b \sec(dx + c))^{\frac{2}{3}} \sec(dx + c)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^m*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(2/3)*sec(d*x + c)^m, x)

maple [F] time = 1.87, size = 0, normalized size = 0.00

$$\int (\sec^m(dx + c)) (b \sec(dx + c))^{\frac{2}{3}} (A + B \sec(dx + c)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^m*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x)

[Out] int(sec(d*x+c)^m*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A) (b \sec(dx + c))^{\frac{2}{3}} \sec(dx + c)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^m*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(2/3)*sec(d*x + c)^m, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A + \frac{B}{\cos(c + dx)} \right) \left(\frac{b}{\cos(c + dx)} \right)^{2/3} \left(\frac{1}{\cos(c + dx)} \right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B/cos(c + d*x))*(b/cos(c + d*x))^(2/3)*(1/cos(c + d*x))^m,x)`

[Out] `int((A + B/cos(c + d*x))*(b/cos(c + d*x))^(2/3)*(1/cos(c + d*x))^m, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \sec(c + dx))^{\frac{2}{3}} (A + B \sec(c + dx)) \sec^m(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)**m*(b*sec(d*x+c))**(2/3)*(A+B*sec(d*x+c)),x)`

[Out] `Integral((b*sec(c + d*x))**(2/3)*(A + B*sec(c + d*x))*sec(c + d*x)**m, x)`

3.29 $\int \sec^m(c+dx) \sqrt[3]{b \sec(c+dx)} (A+B \sec(c+dx)) dx$

Optimal. Leaf size=165

$$\frac{3B \sin(c+dx) \sqrt[3]{b \sec(c+dx)} \sec^m(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(-3m-1); \frac{1}{6}(5-3m); \cos^2(c+dx)\right) + 3A \sin(c+dx) \sqrt[3]{b \sec(c+dx)}}{d(3m+1)\sqrt{\sin^2(c+dx)}}$$

[Out] $-3*A*\text{hypergeom}([1/2, 1/3-1/2*m], [4/3-1/2*m], \cos(d*x+c)^2)*\sec(d*x+c)^{-1+m}*(b*\sec(d*x+c))^{1/3}*\sin(d*x+c)/d/(2-3*m)/(\sin(d*x+c)^2)^{1/2}+3*B*\text{hypergeom}([1/2, -1/6-1/2*m], [5/6-1/2*m], \cos(d*x+c)^2)*\sec(d*x+c)^m*(b*\sec(d*x+c))^{1/3}*\sin(d*x+c)/d/(1+3*m)/(\sin(d*x+c)^2)^{1/2}$

Rubi [A] time = 0.11, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {20, 3787, 3772, 2643}

$$\frac{3B \sin(c+dx) \sqrt[3]{b \sec(c+dx)} \sec^m(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(-3m-1); \frac{1}{6}(5-3m); \cos^2(c+dx)\right) + 3A \sin(c+dx) \sqrt[3]{b \sec(c+dx)}}{d(3m+1)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sec}[c+d*x]^m*(b*\text{Sec}[c+d*x])^{1/3}*(A+B*\text{Sec}[c+d*x]),x]$

[Out] $(-3*A*\text{Hypergeometric2F1}[1/2, (2-3*m)/6, (8-3*m)/6, \text{Cos}[c+d*x]^2]*\text{Sec}[c+d*x]^{-1+m}*(b*\text{Sec}[c+d*x])^{1/3}*\text{Sin}[c+d*x])/(d*(2-3*m)*\text{Sqrt}[\text{Sin}[c+d*x]^2])+(3*B*\text{Hypergeometric2F1}[1/2, (-1-3*m)/6, (5-3*m)/6, \text{Cos}[c+d*x]^2]*\text{Sec}[c+d*x]^m*(b*\text{Sec}[c+d*x])^{1/3}*\text{Sin}[c+d*x])/(d*(1+3*m)*\text{Sqrt}[\text{Sin}[c+d*x]^2])$

Rule 20

$\text{Int}[(u_*)*((a_*)*(v_))^{(m_)}*((b_*)*(v_))^{(n_)}, x_Symbol] \rightarrow \text{Dist}[(b^{\text{IntPart}[n]}*(b*v)^{\text{FracPart}[n]})/(a^{\text{IntPart}[n]}*(a*v)^{\text{FracPart}[n]}), \text{Int}[u*(a*v)^{(m+n)}, x], x] /;$ FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m+n]

Rule 2643

$\text{Int}[(b_*)*\sin[(c_*)+(d_*)*(x_)]^{(n_)}, x_Symbol] \rightarrow \text{Simp}[(\text{Cos}[c+d*x]*(b*\text{Sin}[c+d*x])^{(n+1)}*\text{Hypergeometric2F1}[1/2, (n+1)/2, (n+3)/2, \text{Sin}[c+d*x]^2])/(b*d*(n+1)*\text{Sqrt}[\text{Cos}[c+d*x]^2]), x] /;$ FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rule 3772

```
Int[(csc[(c_) + (d_)*(x_)]*(b_))^(n_), x_Symbol] := Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 3787

```
Int[(csc[(e_) + (f_)*(x_)]*(d_))^(n_)*(csc[(e_) + (f_)*(x_)]*(b_) +
(a_)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned} \int \sec^m(c + dx) \sqrt[3]{b \sec(c + dx)} (A + B \sec(c + dx)) dx &= \frac{\sqrt[3]{b \sec(c + dx)} \int \sec^{\frac{1}{3}+m}(c + dx) (A + B \sec(c + dx)) dx}{\sqrt[3]{\sec(c + dx)}} \\ &= \frac{(A \sqrt[3]{b \sec(c + dx)}) \int \sec^{\frac{1}{3}+m}(c + dx) dx}{\sqrt[3]{\sec(c + dx)}} + \frac{(B \sqrt[3]{b \sec(c + dx)}) \int \sec^{\frac{1}{3}+m}(c + dx) dx}{\sqrt[3]{\sec(c + dx)}} \\ &= \left(A \cos^{\frac{1}{3}+m}(c + dx) \sec^m(c + dx) \sqrt[3]{b \sec(c + dx)} \right) \int \cos^{-1-2m} \\ &= -\frac{3A {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(2 - 3m); \frac{1}{6}(8 - 3m); \cos^2(c + dx)\right) \sec^{-1-2m}}{d(2 - 3m)\sqrt{\sin^2(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.28, size = 140, normalized size = 0.85

$$\frac{3\sqrt{-\tan^2(c + dx)} \csc(c + dx) \sqrt[3]{b \sec(c + dx)} \sec^m(c + dx) \left(A(3m + 4) \cos(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(3m + 1); \frac{1}{6}(3m + 7); \cos^2(c + dx)\right) + B(3m + 1) \right)}{d(3m + 1)(3m + 4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^m*(b*Sec[c + d*x])^(1/3)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (3*Csc[c + d*x]*(A*(4 + 3*m)*Cos[c + d*x]*Hypergeometric2F1[1/2, (1 + 3*m)/
6, (7 + 3*m)/6, Sec[c + d*x]^2] + B*(1 + 3*m)*Hypergeometric2F1[1/2, (4 + 3
*m)/6, 5/3 + m/2, Sec[c + d*x]^2])*Sec[c + d*x]^m*(b*Sec[c + d*x])^(1/3)*Sq
rt[-Tan[c + d*x]^2])/(d*(1 + 3*m)*(4 + 3*m))
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left((B \sec(dx + c) + A)(b \sec(dx + c))^{\frac{1}{3}} \sec(dx + c)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^m*(b*sec(d*x+c))^(1/3)*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] integral((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(1/3)*sec(d*x + c)^m, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A) (b \sec(dx + c))^{\frac{1}{3}} \sec(dx + c)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^m*(b*sec(d*x+c))^(1/3)*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(1/3)*sec(d*x + c)^m, x)

maple [F] time = 1.53, size = 0, normalized size = 0.00

$$\int (\sec^m(dx + c)) (b \sec(dx + c))^{\frac{1}{3}} (A + B \sec(dx + c)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^m*(b*sec(d*x+c))^(1/3)*(A+B*sec(d*x+c)),x)

[Out] int(sec(d*x+c)^m*(b*sec(d*x+c))^(1/3)*(A+B*sec(d*x+c)),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A) (b \sec(dx + c))^{\frac{1}{3}} \sec(dx + c)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^m*(b*sec(d*x+c))^(1/3)*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(1/3)*sec(d*x + c)^m, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A + \frac{B}{\cos(c + dx)} \right) \left(\frac{b}{\cos(c + dx)} \right)^{1/3} \left(\frac{1}{\cos(c + dx)} \right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B/cos(c + d*x))*(b/cos(c + d*x))^(1/3)*(1/cos(c + d*x))^m,x)`

[Out] `int((A + B/cos(c + d*x))*(b/cos(c + d*x))^(1/3)*(1/cos(c + d*x))^m, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{b \sec(c + dx)} (A + B \sec(c + dx)) \sec^m(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)**m*(b*sec(d*x+c))**(1/3)*(A+B*sec(d*x+c)),x)`

[Out] `Integral((b*sec(c + d*x))**(1/3)*(A + B*sec(c + d*x))*sec(c + d*x)**m, x)`

$$3.30 \quad \int \frac{\sec^m(c+dx)(A+B \sec(c+dx))}{\sqrt[3]{b \sec(c+dx)}} dx$$

Optimal. Leaf size=165

$$\frac{3A \sin(c+dx) \sec^{m-1}(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(4-3m); \frac{1}{6}(10-3m); \cos^2(c+dx)\right)}{d(4-3m)\sqrt{\sin^2(c+dx)} \sqrt[3]{b \sec(c+dx)}} - \frac{3B \sin(c+dx) \sec^m(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(4-3m); \frac{1}{6}(10-3m); \cos^2(c+dx)\right)}{d(1-3m)\sqrt{\sin^2(c+dx)}}$$

[Out] -3*A*hypergeom([1/2, 2/3-1/2*m], [5/3-1/2*m], cos(d*x+c)^2)*sec(d*x+c)^(-1+m)*sin(d*x+c)/d/(4-3*m)/(b*sec(d*x+c))^(1/3)/(sin(d*x+c)^2)^(1/2)-3*B*hypergeom([1/2, 1/6-1/2*m], [7/6-1/2*m], cos(d*x+c)^2)*sec(d*x+c)^m*sin(d*x+c)/d/(1-3*m)/(b*sec(d*x+c))^(1/3)/(sin(d*x+c)^2)^(1/2)

Rubi [A] time = 0.11, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {20, 3787, 3772, 2643}

$$\frac{3A \sin(c+dx) \sec^{m-1}(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(4-3m); \frac{1}{6}(10-3m); \cos^2(c+dx)\right)}{d(4-3m)\sqrt{\sin^2(c+dx)} \sqrt[3]{b \sec(c+dx)}} - \frac{3B \sin(c+dx) \sec^m(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(4-3m); \frac{1}{6}(10-3m); \cos^2(c+dx)\right)}{d(1-3m)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(Sec[c + d*x]^m*(A + B*Sec[c + d*x]))/(b*Sec[c + d*x])^(1/3), x]

[Out] (-3*A*Hypergeometric2F1[1/2, (4 - 3*m)/6, (10 - 3*m)/6, Cos[c + d*x]^2]*Sec[c + d*x]^(-1 + m)*Sin[c + d*x])/(d*(4 - 3*m)*(b*Sec[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2]) - (3*B*Hypergeometric2F1[1/2, (1 - 3*m)/6, (7 - 3*m)/6, Cos[c + d*x]^2]*Sec[c + d*x]^m*Sin[c + d*x])/(d*(1 - 3*m)*(b*Sec[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2])

Rule 20

Int[(u_.)*((a_.)*(v_))^(m_.)*((b_.)*(v_))^(n_.), x_Symbol] :> Dist[(b^IntPart[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m+n), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m+n]

Rule 2643

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[(Cos[c + d*x]*(b*SIN[c + d*x])^(n+1)*Hypergeometric2F1[1/2, (n+1)/2, (n+3)/2, Sin[c + d*x]^2])/(b*d*(n+1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rule 3772

Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.))^(n_.), x_Symbol] :> Simp[(b*Csc[c + d*x])^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rubi steps

$$\begin{aligned} \int \frac{\sec^m(c + dx)(A + B \sec(c + dx))}{\sqrt[3]{b \sec(c + dx)}} dx &= \frac{\sqrt[3]{\sec(c + dx)} \int \sec^{-\frac{1}{3}+m}(c + dx)(A + B \sec(c + dx)) dx}{\sqrt[3]{b \sec(c + dx)}} \\ &= \frac{(A \sqrt[3]{\sec(c + dx)}) \int \sec^{-\frac{1}{3}+m}(c + dx) dx}{\sqrt[3]{b \sec(c + dx)}} + \frac{(B \sqrt[3]{\sec(c + dx)}) \int \sec^{\frac{2}{3}+m}(c + dx) dx}{\sqrt[3]{b \sec(c + dx)}} \\ &= \frac{(A \cos^{\frac{2}{3}+m}(c + dx) \sec^{1+m}(c + dx)) \int \cos^{\frac{1}{3}-m}(c + dx) dx}{\sqrt[3]{b \sec(c + dx)}} + \frac{(B \cos^{\frac{2}{3}+m}(c + dx) \sec^{1+m}(c + dx)) \int \cos^{\frac{1}{3}-m}(c + dx) dx}{\sqrt[3]{b \sec(c + dx)}} \\ &= -\frac{3A {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(4 - 3m); \frac{1}{6}(10 - 3m); \cos^2(c + dx)\right) \sec^{-1+m}(c + dx) \sin(c + dx)}{d(4 - 3m) \sqrt[3]{b \sec(c + dx)} \sqrt{\sin^2(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.25, size = 140, normalized size = 0.85

$$\frac{3\sqrt{-\tan^2(c + dx)} \csc(c + dx) \sec^m(c + dx) \left(A(3m + 2) \cos(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(3m - 1); \frac{1}{6}(3m + 5); \sec^2(c + dx)\right) + B(3m + 2) \sec(c + dx) \right)}{d(3m - 1)(3m + 2) \sqrt[3]{b \sec(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[c + d*x]^m*(A + B*Sec[c + d*x]))/(b*Sec[c + d*x])^(1/3),x]
 [Out] (3*Csc[c + d*x]*(A*(2 + 3*m)*Cos[c + d*x]*Hypergeometric2F1[1/2, (-1 + 3*m)/6, (5 + 3*m)/6, Sec[c + d*x]^2] + B*(-1 + 3*m)*Hypergeometric2F1[1/2, (2 + 3*m)/6, (8 + 3*m)/6, Sec[c + d*x]^2])*Sec[c + d*x]^m*sqrt[-Tan[c + d*x]^2]/(d*(-1 + 3*m)*(2 + 3*m)*(b*Sec[c + d*x])^(1/3))

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{(B \sec(dx + c) + A) (b \sec(dx + c))^{\frac{2}{3}} \sec(dx + c)^m}{b \sec(dx + c)}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^m*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(1/3),x, algorithm="fricas")

[Out] integral((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(2/3)*sec(d*x + c)^m/(b*sec(d*x + c)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sec(dx + c)^m}{(b \sec(dx + c))^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^m*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(1/3),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^m/(b*sec(d*x + c))^(1/3), x)

maple [F] time = 1.24, size = 0, normalized size = 0.00

$$\int \frac{(\sec^m(dx + c)) (A + B \sec(dx + c))}{(b \sec(dx + c))^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^m*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(1/3),x)

[Out] int(sec(d*x+c)^m*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(1/3),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sec(dx + c)^m}{(b \sec(dx + c))^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^m*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(1/3),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^m/(b*sec(d*x + c))^(1/3), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(\frac{1}{\cos(c+dx)}\right)^m}{\left(\frac{b}{\cos(c+dx)}\right)^{1/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^m)/(b/cos(c + d*x))^(1/3), x)

[Out] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^m)/(b/cos(c + d*x))^(1/3), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \sec^m(c + dx)}{\sqrt[3]{b \sec(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**m*(A+B*sec(d*x+c))/(b*sec(d*x+c))**(1/3), x)

[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**m/(b*sec(c + d*x))**(1/3), x)

$$3.31 \quad \int \frac{\sec^m(c+dx)(A+B \sec(c+dx))}{(b \sec(c+dx))^{2/3}} dx$$

Optimal. Leaf size=165

$$\frac{3A \sin(c+dx) \sec^{m-1}(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(5-3m); \frac{1}{6}(11-3m); \cos^2(c+dx)\right)}{d(5-3m)\sqrt{\sin^2(c+dx)}(b \sec(c+dx))^{2/3}} - \frac{3B \sin(c+dx) \sec^m(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(5-3m); \frac{1}{6}(11-3m); \cos^2(c+dx)\right)}{d(2-3m)\sqrt{\sin^2(c+dx)}}$$

[Out] $-3*A*\text{hypergeom}\left(\left[\frac{1}{2}, \frac{5}{6}-\frac{1}{2}*m\right], \left[\frac{11}{6}-\frac{1}{2}*m\right], \cos(d*x+c)^2\right)*\sec(d*x+c)^{-1+m}*\sin(d*x+c)/d/(5-3*m)/(b*\sec(d*x+c))^{2/3}/(\sin(d*x+c)^2)^{1/2}-3*B*\text{hypergeom}\left(\left[\frac{1}{2}, \frac{1}{3}-\frac{1}{2}*m\right], \left[\frac{4}{3}-\frac{1}{2}*m\right], \cos(d*x+c)^2\right)*\sec(d*x+c)^m*\sin(d*x+c)/d/(2-3*m)/(b*\sec(d*x+c))^{2/3}/(\sin(d*x+c)^2)^{1/2}$

Rubi [A] time = 0.12, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {20, 3787, 3772, 2643}

$$\frac{3A \sin(c+dx) \sec^{m-1}(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(5-3m); \frac{1}{6}(11-3m); \cos^2(c+dx)\right)}{d(5-3m)\sqrt{\sin^2(c+dx)}(b \sec(c+dx))^{2/3}} - \frac{3B \sin(c+dx) \sec^m(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(5-3m); \frac{1}{6}(11-3m); \cos^2(c+dx)\right)}{d(2-3m)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(\text{Sec}[c + d*x]^m*(A + B*\text{Sec}[c + d*x]))/(b*\text{Sec}[c + d*x]^{2/3}), x]$

[Out] $(-3*A*\text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{(5-3*m)}{6}, \frac{(11-3*m)}{6}, \text{Cos}[c + d*x]^2\right]*\text{Sec}[c + d*x]^{-1+m}*\text{Sin}[c + d*x])/(d*(5-3*m)*(b*\text{Sec}[c + d*x]^{2/3})*\text{Sqrt}[\text{Sin}[c + d*x]^2]) - (3*B*\text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{(2-3*m)}{6}, \frac{(8-3*m)}{6}, \text{Cos}[c + d*x]^2\right]*\text{Sec}[c + d*x]^m*\text{Sin}[c + d*x])/(d*(2-3*m)*(b*\text{Sec}[c + d*x]^{2/3})*\text{Sqrt}[\text{Sin}[c + d*x]^2])$

Rule 20

$\text{Int}[(u_*)*((a_*)*(v_))^{(m_)}*((b_*)*(v_))^{(n_)}, x_Symbol] \rightarrow \text{Dist}[(b^{\text{IntPart}[n]}*(b*v)^{\text{FracPart}[n]})/(a^{\text{IntPart}[n]}*(a*v)^{\text{FracPart}[n]}), \text{Int}[u*(a*v)^{(m+n)}, x], x] /;$ FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m+n]

Rule 2643

$\text{Int}[(b_*\sin[(c_*) + (d_*)(x_)])^{(n_)}, x_Symbol] \rightarrow \text{Simp}[(\text{Cos}[c + d*x]*(b*\text{Sin}[c + d*x])^{(n+1)}*\text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{(n+1)}{2}, \frac{(n+3)}{2}, \text{Sin}[c + d*x]^2\right])/(b*d*(n+1)*\text{Sqrt}[\text{Cos}[c + d*x]^2]), x] /;$ FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rule 3772

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x])^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rubi steps

$$\begin{aligned} \int \frac{\sec^m(c + dx)(A + B \sec(c + dx))}{(b \sec(c + dx))^{2/3}} dx &= \frac{\sec^{2/3}(c + dx) \int \sec^{-2/3+m}(c + dx)(A + B \sec(c + dx)) dx}{(b \sec(c + dx))^{2/3}} \\ &= \frac{\left(A \sec^{2/3}(c + dx)\right) \int \sec^{-2/3+m}(c + dx) dx}{(b \sec(c + dx))^{2/3}} + \frac{\left(B \sec^{2/3}(c + dx)\right) \int \sec^{1/3+m}(c + dx) dx}{(b \sec(c + dx))^{2/3}} \\ &= \frac{\left(A \cos^{1/3+m}(c + dx) \sec^{1+m}(c + dx)\right) \int \cos^{2/3-m}(c + dx) dx}{(b \sec(c + dx))^{2/3}} + \frac{\left(B \cos^{1/3+m}(c + dx) \sec^{1+m}(c + dx)\right) \int \cos^{2/3-m}(c + dx) dx}{(b \sec(c + dx))^{2/3}} \\ &= -\frac{3A {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(5 - 3m); \frac{1}{6}(11 - 3m); \cos^2(c + dx)\right) \sec^{-1+m}(c + dx) \sin(c + dx)}{d(5 - 3m)(b \sec(c + dx))^{2/3} \sqrt{\sin^2(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.24, size = 140, normalized size = 0.85

$$\frac{3\sqrt{-\tan^2(c + dx)} \csc(c + dx) \sec^m(c + dx) \left(A(3m + 1) \cos(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(3m - 2); \frac{1}{6}(3m + 4); \sec^2(c + dx)\right) + B(-2 + 3m) \sec(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(3m + 4); \sec^2(c + dx)\right)\right)}{d(3m - 2)(3m + 1)(b \sec(c + dx))^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[c + d*x]^m*(A + B*Sec[c + d*x]))/(b*Sec[c + d*x])^(2/3), x]
 [Out] (3*Csc[c + d*x]*(A*(1 + 3*m)*Cos[c + d*x]*Hypergeometric2F1[1/2, (-2 + 3*m)/6, (4 + 3*m)/6, Sec[c + d*x]^2] + B*(-2 + 3*m)*Hypergeometric2F1[1/2, (1 + 3*m)/6, (7 + 3*m)/6, Sec[c + d*x]^2])*Sec[c + d*x]^m*Sqrt[-Tan[c + d*x]^2]/(d*(-2 + 3*m)*(1 + 3*m)*(b*Sec[c + d*x])^(2/3))

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{(B \sec(dx + c) + A) (b \sec(dx + c))^{\frac{1}{3}} \sec(dx + c)^m}{b \sec(dx + c)}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^m*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(2/3),x, algorithm="fricas")

[Out] integral((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(1/3)*sec(d*x + c)^m/(b*sec(d*x + c)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sec(dx + c)^m}{(b \sec(dx + c))^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^m*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(2/3),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^m/(b*sec(d*x + c))^(2/3), x)

maple [F] time = 1.24, size = 0, normalized size = 0.00

$$\int \frac{(\sec^m(dx + c)) (A + B \sec(dx + c))}{(b \sec(dx + c))^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^m*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(2/3),x)

[Out] int(sec(d*x+c)^m*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(2/3),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sec(dx + c)^m}{(b \sec(dx + c))^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^m*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(2/3),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^m/(b*sec(d*x + c))^(2/3), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(\frac{1}{\cos(c+dx)}\right)^m}{\left(\frac{b}{\cos(c+dx)}\right)^{2/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^m)/(b/cos(c + d*x))^(2/3), x)

[Out] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^m)/(b/cos(c + d*x))^(2/3), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \sec^m(c + dx)}{(b \sec(c + dx))^{2/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**m*(A+B*sec(d*x+c))/(b*sec(d*x+c))**(2/3), x)

[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**m/(b*sec(c + d*x))**(2/3), x)

$$3.32 \quad \int \frac{\sec^m(c+dx)(A+B \sec(c+dx))}{(b \sec(c+dx))^{4/3}} dx$$

Optimal. Leaf size=173

$$\frac{3A \sin(c+dx) \sec^{m-2}(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(7-3m); \frac{1}{6}(13-3m); \cos^2(c+dx)\right)}{bd(7-3m)\sqrt{\sin^2(c+dx)} \sqrt[3]{b \sec(c+dx)}} - \frac{3B \sin(c+dx) \sec^{m-1}(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(7-3m); \frac{1}{6}(13-3m); \cos^2(c+dx)\right)}{bd(4-3m)\sqrt{\sin^2(c+dx)}}$$

[Out] $-3*A*\text{hypergeom}\left(\left[\frac{1}{2}, \frac{7}{6}-\frac{1}{2}*m\right], \left[\frac{13}{6}-\frac{1}{2}*m\right], \cos(d*x+c)^2\right)*\sec(d*x+c)^{-2+m}*\sin(d*x+c)/b/d/(7-3*m)/(b*\sec(d*x+c))^{1/3}/(\sin(d*x+c)^2)^{1/2}-3*B*\text{hypergeom}\left(\left[\frac{1}{2}, \frac{2}{3}-\frac{1}{2}*m\right], \left[\frac{5}{3}-\frac{1}{2}*m\right], \cos(d*x+c)^2\right)*\sec(d*x+c)^{-1+m}*\sin(d*x+c)/b/d/(4-3*m)/(b*\sec(d*x+c))^{1/3}/(\sin(d*x+c)^2)^{1/2}$

Rubi [A] time = 0.12, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {20, 3787, 3772, 2643}

$$\frac{3A \sin(c+dx) \sec^{m-2}(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(7-3m); \frac{1}{6}(13-3m); \cos^2(c+dx)\right)}{bd(7-3m)\sqrt{\sin^2(c+dx)} \sqrt[3]{b \sec(c+dx)}} - \frac{3B \sin(c+dx) \sec^{m-1}(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(7-3m); \frac{1}{6}(13-3m); \cos^2(c+dx)\right)}{bd(4-3m)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(\text{Sec}[c + d*x]^m*(A + B*\text{Sec}[c + d*x]))/(b*\text{Sec}[c + d*x]^{4/3}), x]$

[Out] $(-3*A*\text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{(7-3*m)}{6}, \frac{(13-3*m)}{6}, \text{Cos}[c + d*x]^2\right]*\text{Sec}[c + d*x]^{-2+m}*\text{Sin}[c + d*x])/(b*d*(7-3*m)*(b*\text{Sec}[c + d*x])^{1/3}*\text{Sqrt}[\text{Sin}[c + d*x]^2]) - (3*B*\text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{(4-3*m)}{6}, \frac{(10-3*m)}{6}, \text{Cos}[c + d*x]^2\right]*\text{Sec}[c + d*x]^{-1+m}*\text{Sin}[c + d*x])/(b*d*(4-3*m)*(b*\text{Sec}[c + d*x])^{1/3}*\text{Sqrt}[\text{Sin}[c + d*x]^2])$

Rule 20

$\text{Int}[(u_*)*((a_*)*(v_))^{(m_)}*((b_*)*(v_))^{(n_)}, x_Symbol] \rightarrow \text{Dist}[(b^{\text{IntPart}[n]}*(b*v)^{\text{FracPart}[n]})/(a^{\text{IntPart}[n]}*(a*v)^{\text{FracPart}[n]}), \text{Int}[u*(a*v)^{(m+n)}, x], x] /;$ FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m+n]

Rule 2643

$\text{Int}[(b_*\sin[(c_*) + (d_*)(x_)])^{(n_)}, x_Symbol] \rightarrow \text{Simp}[(\text{Cos}[c + d*x]*(b*\text{Sin}[c + d*x])^{(n+1)}*\text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{(n+1)}{2}, \frac{(n+3)}{2}, \text{Sin}[c + d*x]^2\right])/(b*d*(n+1)*\text{Sqrt}[\text{Cos}[c + d*x]^2]), x] /;$ FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rule 3772

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x])^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rubi steps

$$\begin{aligned} \int \frac{\sec^m(c + dx)(A + B \sec(c + dx))}{(b \sec(c + dx))^{4/3}} dx &= \frac{\sqrt[3]{\sec(c + dx)} \int \sec^{-\frac{4}{3}+m}(c + dx)(A + B \sec(c + dx)) dx}{b\sqrt[3]{b \sec(c + dx)}} \\ &= \frac{(A\sqrt[3]{\sec(c + dx)}) \int \sec^{-\frac{4}{3}+m}(c + dx) dx}{b\sqrt[3]{b \sec(c + dx)}} + \frac{(B\sqrt[3]{\sec(c + dx)}) \int \sec^{-\frac{1}{3}+m}(c + dx) dx}{b\sqrt[3]{b \sec(c + dx)}} \\ &= \frac{(A \cos^{\frac{2}{3}+m}(c + dx) \sec^{1+m}(c + dx)) \int \cos^{\frac{4}{3}-m}(c + dx) dx}{b\sqrt[3]{b \sec(c + dx)}} + \frac{(B \cos^{\frac{2}{3}+m}(c + dx) \sec^{1+m}(c + dx)) \int \cos^{\frac{4}{3}-m}(c + dx) dx}{b\sqrt[3]{b \sec(c + dx)}} \\ &= -\frac{3A {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(7 - 3m); \frac{1}{6}(13 - 3m); \cos^2(c + dx)\right) \sec^{-2+m}(c + dx) \sin(c + dx)}{bd(7 - 3m)\sqrt[3]{b \sec(c + dx)} \sqrt{\sin^2(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.34, size = 140, normalized size = 0.81

$$\frac{3\sqrt{-\tan^2(c + dx)} \csc(c + dx) \sec^m(c + dx) \left(A(3m - 1) \cos(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(3m - 4); \frac{1}{6}(3m + 2); \sec^2(c + dx)\right) + B(-4 + 3m) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(3m - 4); \frac{1}{6}(3m + 2); \sec^2(c + dx)\right) \right)}{d(3m - 4)(3m - 1)(b \sec(c + dx))^{4/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[c + d*x]^m*(A + B*Sec[c + d*x]))/(b*Sec[c + d*x])^(4/3), x]

[Out] (3*Csc[c + d*x]*(A*(-1 + 3*m)*Cos[c + d*x]*Hypergeometric2F1[1/2, (-4 + 3*m)/6, (2 + 3*m)/6, Sec[c + d*x]^2] + B*(-4 + 3*m)*Hypergeometric2F1[1/2, (-1 + 3*m)/6, (5 + 3*m)/6, Sec[c + d*x]^2])*Sec[c + d*x]^m*Sqrt[-Tan[c + d*x]^2]/(d*(-4 + 3*m)*(-1 + 3*m)*(b*Sec[c + d*x])^(4/3))

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{(B \sec(dx + c) + A) (b \sec(dx + c))^{\frac{2}{3}} \sec(dx + c)^m}{b^2 \sec(dx + c)^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^m*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(4/3),x, algorithm="fricas")

[Out] integral((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(2/3)*sec(d*x + c)^m/(b^2*sec(d*x + c)^2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sec(dx + c)^m}{(b \sec(dx + c))^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^m*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(4/3),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^m/(b*sec(d*x + c))^(4/3), x)

maple [F] time = 1.22, size = 0, normalized size = 0.00

$$\int \frac{(\sec^m(dx + c)) (A + B \sec(dx + c))}{(b \sec(dx + c))^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^m*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(4/3),x)

[Out] int(sec(d*x+c)^m*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(4/3),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sec(dx + c)^m}{(b \sec(dx + c))^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^m*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(4/3),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^m/(b*sec(d*x + c))^(4/3), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(\frac{1}{\cos(c+dx)}\right)^m}{\left(\frac{b}{\cos(c+dx)}\right)^{4/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^m)/(b/cos(c + d*x))^(4/3), x)

[Out] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^m)/(b/cos(c + d*x))^(4/3), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \sec^m(c + dx)}{(b \sec(c + dx))^{4/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**m*(A+B*sec(d*x+c))/(b*sec(d*x+c))**(4/3), x)

[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**m/(b*sec(c + d*x))**(4/3), x)

3.33 $\int \sec^m(c + dx)(b \sec(c + dx))^n(A + B \sec(c + dx)) dx$

Optimal. Leaf size=172

$$\frac{B \sin(c + dx) \sec^m(c + dx)(b \sec(c + dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{2}(-m - n); \frac{1}{2}(-m - n + 2); \cos^2(c + dx)\right)}{d(m + n)\sqrt{\sin^2(c + dx)}} - \frac{A \sin(c + dx) \sec^{m-1}(c + dx)}{d(m + n)\sqrt{\sin^2(c + dx)}}$$

[Out] -A*hypergeom([1/2, 1/2-1/2*m-1/2*n], [3/2-1/2*m-1/2*n], cos(d*x+c)^2)*sec(d*x+c)^(-1+m)*(b*sec(d*x+c))^n*sin(d*x+c)/d/(1-m-n)/(sin(d*x+c)^2)^(1/2)+B*hypergeom([1/2, -1/2*m-1/2*n], [1-1/2*m-1/2*n], cos(d*x+c)^2)*sec(d*x+c)^m*(b*sec(d*x+c))^n*sin(d*x+c)/d/(m+n)/(sin(d*x+c)^2)^(1/2)

Rubi [A] time = 0.11, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {20, 3787, 3772, 2643}

$$\frac{B \sin(c + dx) \sec^m(c + dx)(b \sec(c + dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{2}(-m - n); \frac{1}{2}(-m - n + 2); \cos^2(c + dx)\right)}{d(m + n)\sqrt{\sin^2(c + dx)}} - \frac{A \sin(c + dx) \sec^{m-1}(c + dx)}{d(m + n)\sqrt{\sin^2(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d*x]^m*(b*Sec[c + d*x])^n*(A + B*Sec[c + d*x]),x]

[Out] -((A*Hypergeometric2F1[1/2, (1 - m - n)/2, (3 - m - n)/2, Cos[c + d*x]^2]*Sec[c + d*x]^(-1 + m)*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(1 - m - n)*Sqrt[Sin[c + d*x]^2])) + (B*Hypergeometric2F1[1/2, (-m - n)/2, (2 - m - n)/2, Cos[c + d*x]^2]*Sec[c + d*x]^m*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(m + n)*Sqrt[Sin[c + d*x]^2])

Rule 20

Int[(u_)*((a_)*(v_))^(m_)*((b_)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m + n]

Rule 2643

Int[((b_)*sin[(c_.) + (d_)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rule 3772

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]^(n - 1))*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rubi steps

$$\begin{aligned} \int \sec^m(c + dx)(b \sec(c + dx))^n(A + B \sec(c + dx)) dx &= (\sec^{-n}(c + dx)(b \sec(c + dx))^n) \int \sec^{m+n}(c + dx)(A + B \sec(c + dx)) dx \\ &= (A \sec^{-n}(c + dx)(b \sec(c + dx))^n) \int \sec^{m+n}(c + dx) dx \\ &= (A \cos^{m+n}(c + dx) \sec^m(c + dx)(b \sec(c + dx))^n) \int \cos^{m+n}(c + dx) dx \\ &= \frac{A {}_2F_1\left(\frac{1}{2}, \frac{1}{2}(1 - m - n); \frac{1}{2}(3 - m - n); \cos^2(c + dx)\right) \sec^m(c + dx)(b \sec(c + dx))^n}{d(1 - m - n)\sqrt{\sin^2(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.22, size = 126, normalized size = 0.73

$$\frac{\sqrt{-\tan^2(c + dx)} \csc(c + dx) \sec^m(c + dx)(b \sec(c + dx))^n \left(A(m + n + 1) \cos(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{m+n}{2}; \frac{1}{2}(m + n + 2); \cos^2(c + dx)\right) + B(m + n) \sec(c + dx) \right)}{d(m + n)(m + n + 1)}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[c + d*x]^m*(b*Sec[c + d*x])^n*(A + B*Sec[c + d*x]), x]

[Out] (Csc[c + d*x]*(A*(1 + m + n)*Cos[c + d*x]*Hypergeometric2F1[1/2, (m + n)/2, (2 + m + n)/2, Sec[c + d*x]^2] + B*(m + n)*Hypergeometric2F1[1/2, (1 + m + n)/2, (3 + m + n)/2, Sec[c + d*x]^2])*Sec[c + d*x]^m*(b*Sec[c + d*x])^n*sqrt[-Tan[c + d*x]^2]/(d*(m + n)*(1 + m + n))

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left((B \sec(dx + c) + A)(b \sec(dx + c))^n \sec(dx + c)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^m*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] integral((B*sec(d*x + c) + A)*(b*sec(d*x + c))^n*sec(d*x + c)^m, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A) (b \sec(dx + c))^n \sec(dx + c)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^m*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^n*sec(d*x + c)^m, x)

maple [F] time = 4.11, size = 0, normalized size = 0.00

$$\int (\sec^m(dx + c)) (b \sec(dx + c))^n (A + B \sec(dx + c)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^m*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x)

[Out] int(sec(d*x+c)^m*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A) (b \sec(dx + c))^n \sec(dx + c)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^m*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^n*sec(d*x + c)^m, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A + \frac{B}{\cos(c + dx)} \right) \left(\frac{b}{\cos(c + dx)} \right)^n \left(\frac{1}{\cos(c + dx)} \right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))*(b/cos(c + d*x))^n*(1/cos(c + d*x))^m,x)

[Out] int((A + B/cos(c + d*x))*(b/cos(c + d*x))^n*(1/cos(c + d*x))^m, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \sec(c + dx))^n (A + B \sec(c + dx)) \sec^m(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)**m*(b*sec(d*x+c))**n*(A+B*sec(d*x+c)), x)`

[Out] `Integral((b*sec(c + d*x))**n*(A + B*sec(c + d*x))*sec(c + d*x)**m, x)`

3.34 $\int \sec^2(c + dx)(b \sec(c + dx))^n (A + B \sec(c + dx)) dx$

Optimal. Leaf size=143

$$\frac{A \sin(c + dx)(b \sec(c + dx))^{n+1} {}_2F_1\left(\frac{1}{2}, \frac{1}{2}(-n-1); \frac{1-n}{2}; \cos^2(c + dx)\right)}{bd(n+1)\sqrt{\sin^2(c + dx)}} + \frac{B \sin(c + dx)(b \sec(c + dx))^{n+2} {}_2F_1\left(\frac{1}{2}, \frac{1}{2}(-n-1); \frac{1-n}{2}; \cos^2(c + dx)\right)}{b^2d(n+2)\sqrt{\sin^2(c + dx)}}$$

[Out] A*hypergeom([1/2, -1/2-1/2*n], [1/2-1/2*n], cos(d*x+c)^2)*(b*sec(d*x+c))^(1+n)*sin(d*x+c)/b/d/(1+n)/(sin(d*x+c)^2)^(1/2)+B*hypergeom([1/2, -1-1/2*n], [-1/2*n], cos(d*x+c)^2)*(b*sec(d*x+c))^(2+n)*sin(d*x+c)/b^2/d/(2+n)/(sin(d*x+c)^2)^(1/2)

Rubi [A] time = 0.13, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {16, 3787, 3772, 2643}

$$\frac{A \sin(c + dx)(b \sec(c + dx))^{n+1} {}_2F_1\left(\frac{1}{2}, \frac{1}{2}(-n-1); \frac{1-n}{2}; \cos^2(c + dx)\right)}{bd(n+1)\sqrt{\sin^2(c + dx)}} + \frac{B \sin(c + dx)(b \sec(c + dx))^{n+2} {}_2F_1\left(\frac{1}{2}, \frac{1}{2}(-n-1); \frac{1-n}{2}; \cos^2(c + dx)\right)}{b^2d(n+2)\sqrt{\sin^2(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d*x]^2*(b*Sec[c + d*x])^n*(A + B*Sec[c + d*x]),x]

[Out] (A*Hypergeometric2F1[1/2, (-1 - n)/2, (1 - n)/2, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(1 + n)*Sin[c + d*x]/(b*d*(1 + n)*Sqrt[Sin[c + d*x]^2]) + (B*Hypergeometric2F1[1/2, (-2 - n)/2, -n/2, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(2 + n)*Sin[c + d*x]/(b^2*d*(2 + n)*Sqrt[Sin[c + d*x]^2]))

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 2643

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned} \int \sec^2(c + dx)(b \sec(c + dx))^n (A + B \sec(c + dx)) dx &= \frac{\int (b \sec(c + dx))^{2+n} (A + B \sec(c + dx)) dx}{b^2} \\ &= \frac{A \int (b \sec(c + dx))^{2+n} dx}{b^2} + \frac{B \int (b \sec(c + dx))^{3+n} dx}{b^3} \\ &= \frac{\left(A \left(\frac{\cos(c+dx)}{b} \right)^n (b \sec(c + dx))^n \int \left(\frac{\cos(c+dx)}{b} \right)^{-2-n} dx}{b^2} + \dots \\ &= \frac{A {}_2F_1\left(\frac{1}{2}, \frac{1}{2}(-1-n); \frac{1-n}{2}; \cos^2(c + dx)\right) (b \sec(c + dx))^{1+n}}{bd(1+n)\sqrt{\sin^2(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.23, size = 119, normalized size = 0.83

$$\frac{\sqrt{-\tan^2(c + dx)} \csc(c + dx) \sec(c + dx) (b \sec(c + dx))^n \left(A(n + 3) {}_2F_1\left(\frac{1}{2}, \frac{n+2}{2}; \frac{n+4}{2}; \sec^2(c + dx)\right) + B(n + 2) \sec(c + dx) \right)}{d(n + 2)(n + 3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(b*Sec[c + d*x])^n*(A + B*Sec[c + d*x]),x]
```

```
[Out] (Csc[c + d*x]*Sec[c + d*x]*(b*Sec[c + d*x])^n*(A*(3 + n)*Hypergeometric2F1[
1/2, (2 + n)/2, (4 + n)/2, Sec[c + d*x]^2] + B*(2 + n)*Hypergeometric2F1[1/
2, (3 + n)/2, (5 + n)/2, Sec[c + d*x]^2]*Sec[c + d*x]*Sqrt[-Tan[c + d*x]^2
])/ (d*(2 + n)*(3 + n))
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(B \sec(dx + c)^3 + A \sec(dx + c)^2\right) (b \sec(dx + c))^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] integral((B*sec(d*x + c)^3 + A*sec(d*x + c)^2)*(b*sec(d*x + c))^n, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A) (b \sec(dx + c))^n \sec(dx + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^n*sec(d*x + c)^2, x)

maple [F] time = 3.65, size = 0, normalized size = 0.00

$$\int (\sec^2(dx + c)) (b \sec(dx + c))^n (A + B \sec(dx + c)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^2*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x)

[Out] int(sec(d*x+c)^2*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A) (b \sec(dx + c))^n \sec(dx + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^n*sec(d*x + c)^2, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(\frac{b}{\cos(c+dx)}\right)^n}{\cos(c+dx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B/cos(c + d*x))*(b/cos(c + d*x))^n)/cos(c + d*x)^2,x)`

[Out] `int(((A + B/cos(c + d*x))*(b/cos(c + d*x))^n)/cos(c + d*x)^2, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \sec(c + dx))^n (A + B \sec(c + dx)) \sec^2(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)**2*(b*sec(d*x+c))**n*(A+B*sec(d*x+c)),x)`

[Out] `Integral((b*sec(c + d*x))**n*(A + B*sec(c + d*x))*sec(c + d*x)**2, x)`

3.35 $\int \sec(c+dx)(b \sec(c+dx))^n (A+B \sec(c+dx)) dx$

Optimal. Leaf size=136

$$\frac{A \sin(c+dx)(b \sec(c+dx))^n {}_2F_1\left(\frac{1}{2}, -\frac{n}{2}; \frac{2-n}{2}; \cos^2(c+dx)\right)}{dn\sqrt{\sin^2(c+dx)}} + \frac{B \sin(c+dx)(b \sec(c+dx))^{n+1} {}_2F_1\left(\frac{1}{2}, \frac{1}{2}(-n-1); \frac{1-n}{2}; \cos^2(c+dx)\right)}{bd(n+1)\sqrt{\sin^2(c+dx)}}$$

[Out] A*hypergeom([1/2, -1/2*n], [1-1/2*n], cos(d*x+c)^2)*(b*sec(d*x+c))^n*sin(d*x+c)/d/n/(sin(d*x+c)^2)^(1/2)+B*hypergeom([1/2, -1/2-1/2*n], [1/2-1/2*n], cos(d*x+c)^2)*(b*sec(d*x+c))^(1+n)*sin(d*x+c)/b/d/(1+n)/(sin(d*x+c)^2)^(1/2)

Rubi [A] time = 0.12, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {16, 3787, 3772, 2643}

$$\frac{A \sin(c+dx)(b \sec(c+dx))^n {}_2F_1\left(\frac{1}{2}, -\frac{n}{2}; \frac{2-n}{2}; \cos^2(c+dx)\right)}{dn\sqrt{\sin^2(c+dx)}} + \frac{B \sin(c+dx)(b \sec(c+dx))^{n+1} {}_2F_1\left(\frac{1}{2}, \frac{1}{2}(-n-1); \frac{1-n}{2}; \cos^2(c+dx)\right)}{bd(n+1)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d*x]*(b*Sec[c + d*x])^n*(A + B*Sec[c + d*x]), x]

[Out] (A*Hypergeometric2F1[1/2, -n/2, (2 - n)/2, Cos[c + d*x]^2]*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*n*Sqrt[Sin[c + d*x]^2]) + (B*Hypergeometric2F1[1/2, (-1 - n)/2, (1 - n)/2, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(1 + n)*Sin[c + d*x])/(b*d*(1 + n)*Sqrt[Sin[c + d*x]^2])

Rule 16

Int[(u_.)*(v_)^(m_.)*((b_.)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 2643

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rule 3772

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Simp[(b*Csc[c + d*x])^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr

eeQ[{b, c, d, n}, x] && !IntegerQ[n]

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rubi steps

$$\begin{aligned} \int \sec(c + dx)(b \sec(c + dx))^n (A + B \sec(c + dx)) dx &= \frac{\int (b \sec(c + dx))^{1+n} (A + B \sec(c + dx)) dx}{b} \\ &= \frac{A \int (b \sec(c + dx))^{1+n} dx}{b} + \frac{B \int (b \sec(c + dx))^{2+n} dx}{b^2} \\ &= \frac{\left(A \left(\frac{\cos(c+dx)}{b} \right)^n (b \sec(c + dx))^n \right) \int \left(\frac{\cos(c+dx)}{b} \right)^{-1-n} dx}{b} + \dots \\ &= \frac{A {}_2F_1\left(\frac{1}{2}, -\frac{n}{2}; \frac{2-n}{2}; \cos^2(c + dx)\right) (b \sec(c + dx))^n \sin(c + dx)}{dn \sqrt{\sin^2(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.26, size = 119, normalized size = 0.88

$$\frac{\sqrt{-\tan^2(c + dx)} \csc(c + dx) \sec(c + dx) (b \sec(c + dx))^n \left(A(n + 2) \cos(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{n+1}{2}; \frac{n+3}{2}; \sec^2(c + dx)\right) + \dots \right)}{d(n + 1)(n + 2)}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[c + d*x]*(b*Sec[c + d*x])^n*(A + B*Sec[c + d*x]),x]

[Out] (Csc[c + d*x]*(A*(2 + n)*Cos[c + d*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Sec[c + d*x]^2] + B*(1 + n)*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Sec[c + d*x]^2])*Sec[c + d*x]*(b*Sec[c + d*x])^n*Sqrt[-Tan[c + d*x]^2])/(d*(1 + n)*(2 + n))

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(B \sec(dx + c)^2 + A \sec(dx + c)\right) (b \sec(dx + c))^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] integral((B*sec(d*x + c)^2 + A*sec(d*x + c))*(b*sec(d*x + c))^n, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A) (b \sec(dx + c))^n \sec(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^n*sec(d*x + c), x)

maple [F] time = 2.95, size = 0, normalized size = 0.00

$$\int \sec(dx + c) (b \sec(dx + c))^n (A + B \sec(dx + c)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x)

[Out] int(sec(d*x+c)*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A) (b \sec(dx + c))^n \sec(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^n*sec(d*x + c), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(\frac{b}{\cos(c+dx)}\right)^n}{\cos(c+dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(b/cos(c + d*x))^n)/cos(c + d*x),x)

[Out] int(((A + B/cos(c + d*x))*(b/cos(c + d*x))^n)/cos(c + d*x), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \sec(c + dx))^n (A + B \sec(c + dx)) \sec(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))**n*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((b*sec(c + d*x))**n*(A + B*sec(c + d*x))*sec(c + d*x), x)
```

3.36 $\int (b \sec(c + dx))^n (A + B \sec(c + dx)) dx$

Optimal. Leaf size=137

$$\frac{B \sin(c + dx)(b \sec(c + dx))^n {}_2F_1\left(\frac{1}{2}, -\frac{n}{2}; \frac{2-n}{2}; \cos^2(c + dx)\right)}{dn\sqrt{\sin^2(c + dx)}} - \frac{Ab \sin(c + dx)(b \sec(c + dx))^{n-1} {}_2F_1\left(\frac{1}{2}, \frac{1-n}{2}; \frac{3-n}{2}; \cos^2(c + dx)\right)}{d(1-n)\sqrt{\sin^2(c + dx)}}$$

[Out] $-A*b*hypergeom([1/2, 1/2-1/2*n], [3/2-1/2*n], \cos(d*x+c)^2)*(b*\sec(d*x+c))^{(-1+n)}*\sin(d*x+c)/d/(1-n)/(\sin(d*x+c)^2)^{(1/2)}+B*hypergeom([1/2, -1/2*n], [1-1/2*n], \cos(d*x+c)^2)*(b*\sec(d*x+c))^n*\sin(d*x+c)/d/n/(\sin(d*x+c)^2)^{(1/2)}$

Rubi [A] time = 0.10, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3787, 3772, 2643}

$$\frac{B \sin(c + dx)(b \sec(c + dx))^n {}_2F_1\left(\frac{1}{2}, -\frac{n}{2}; \frac{2-n}{2}; \cos^2(c + dx)\right)}{dn\sqrt{\sin^2(c + dx)}} - \frac{Ab \sin(c + dx)(b \sec(c + dx))^{n-1} {}_2F_1\left(\frac{1}{2}, \frac{1-n}{2}; \frac{3-n}{2}; \cos^2(c + dx)\right)}{d(1-n)\sqrt{\sin^2(c + dx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(b*\text{Sec}[c + d*x])^n*(A + B*\text{Sec}[c + d*x]), x]$

[Out] $-\left(\frac{A*b*\text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{(1-n)}{2}, \frac{(3-n)}{2}, \text{Cos}[c + d*x]^2\right]*(b*\text{Sec}[c + d*x])^{(-1+n)}*\text{Sin}[c + d*x]}{d*(1-n)*\text{Sqrt}[\text{Sin}[c + d*x]^2]}\right) + \left(\frac{B*\text{Hypergeometric2F1}\left[\frac{1}{2}, -\frac{n}{2}, \frac{(2-n)}{2}, \text{Cos}[c + d*x]^2\right]*(b*\text{Sec}[c + d*x])^n*\text{Sin}[c + d*x]}{d*n*\text{Sqrt}[\text{Sin}[c + d*x]^2]}\right)$

Rule 2643

$\text{Int}[(b_*)*\sin[(c_*) + (d_*)*(x_*)]^{(n_*)}, x_Symbol] \rightarrow \text{Simp}[(\text{Cos}[c + d*x]*(b*\text{Sin}[c + d*x])^{(n+1)}*\text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{(n+1)}{2}, \frac{(n+3)}{2}, \text{Sin}[c + d*x]^2\right])/(b*d*(n+1)*\text{Sqrt}[\text{Cos}[c + d*x]^2]), x] /; \text{FreeQ}\{b, c, d, n\}, x] \&\& \text{!IntegerQ}[2*n]$

Rule 3772

$\text{Int}[(\text{csc}[(c_*) + (d_*)*(x_*)]*(b_*))^{(n_*)}, x_Symbol] \rightarrow \text{Simp}[(b*\text{Csc}[c + d*x])^{(n-1)}*((\text{Sin}[c + d*x]/b)^{(n-1)}*\text{Int}[1/(\text{Sin}[c + d*x]/b)^n, x]), x] /; \text{FreeQ}\{b, c, d, n\}, x] \&\& \text{!IntegerQ}[n]$

Rule 3787

$\text{Int}[(\text{csc}[(e_*) + (f_*)*(x_*)]*(d_*))^{(n_*)}*(\text{csc}[(e_*) + (f_*)*(x_*)]*(b_*) + (a_*)), x_Symbol] \rightarrow \text{Dist}[a, \text{Int}[(d*\text{Csc}[e + f*x])^n, x], x] + \text{Dist}[b/d, \text{Int}[\text{csc}[(e_*) + (f_*)*(x_*)]^{(n_*)}, x], x]$

$(d \cdot \text{Csc}[e + f \cdot x])^{(n + 1)}, x], x] /; \text{FreeQ}[\{a, b, d, e, f, n\}, x]$

Rubi steps

$$\begin{aligned} \int (b \sec(c + dx))^n (A + B \sec(c + dx)) dx &= A \int (b \sec(c + dx))^n dx + \frac{B \int (b \sec(c + dx))^{1+n} dx}{b} \\ &= \left(A \left(\frac{\cos(c + dx)}{b} \right)^n (b \sec(c + dx))^n \right) \int \left(\frac{\cos(c + dx)}{b} \right)^{-n} dx + \frac{B \left(\frac{\cos(c + dx)}{b} \right)^{n+1} \int \left(\frac{\cos(c + dx)}{b} \right)^{-n-1} dx}{b} \\ &= -\frac{A \cos(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{1-n}{2}; \frac{3-n}{2}; \cos^2(c + dx)\right) (b \sec(c + dx))^n \sin(c + dx)}{d(1-n)\sqrt{\sin^2(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.16, size = 107, normalized size = 0.78

$$\frac{\sqrt{-\tan^2(c + dx)} \csc(c + dx) (b \sec(c + dx))^n \left(A(n + 1) \cos(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{n}{2}; \frac{n+2}{2}; \sec^2(c + dx)\right) + Bn {}_2F_1\left(\frac{1}{2}, \frac{n+1}{2}; \frac{n+3}{2}; \sec^2(c + dx)\right) \right)}{dn(n + 1)}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Sec[c + d*x])^n*(A + B*Sec[c + d*x]),x]

[Out] (Csc[c + d*x]*(A*(1 + n)*Cos[c + d*x]*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Sec[c + d*x]^2] + B*n*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Sec[c + d*x]^2))*(b*Sec[c + d*x])^n*Sqrt[-Tan[c + d*x]^2]/(d*n*(1 + n))

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral} \left((B \sec(dx + c) + A) (b \sec(dx + c))^n, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] integral((B*sec(d*x + c) + A)*(b*sec(d*x + c))^n, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A) (b \sec(dx + c))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^n, x)

maple [F] time = 3.01, size = 0, normalized size = 0.00

$$\int (b \sec(dx + c))^n (A + B \sec(dx + c)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x)

[Out] int((b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A) (b \sec(dx + c))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^n, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A + \frac{B}{\cos(c + dx)} \right) \left(\frac{b}{\cos(c + dx)} \right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))*(b/cos(c + d*x))^n,x)

[Out] int((A + B/cos(c + d*x))*(b/cos(c + d*x))^n, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \sec(c + dx))^n (A + B \sec(c + dx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*sec(d*x+c))**n*(A+B*sec(d*x+c)),x)

[Out] Integral((b*sec(c + d*x))**n*(A + B*sec(c + d*x)), x)

3.37 $\int \cos(c+dx)(b \sec(c+dx))^n(A+B \sec(c+dx)) dx$

Optimal. Leaf size=151

$$\frac{Ab^2 \sin(c+dx)(b \sec(c+dx))^{n-2} {}_2F_1\left(\frac{1}{2}, \frac{2-n}{2}; \frac{4-n}{2}; \cos^2(c+dx)\right)}{d(2-n)\sqrt{\sin^2(c+dx)}} - \frac{bB \sin(c+dx)(b \sec(c+dx))^{n-1} {}_2F_1\left(\frac{1}{2}, \frac{1-n}{2}; \frac{3-n}{2}; \cos^2(c+dx)\right)}{d(1-n)\sqrt{\sin^2(c+dx)}}$$

[Out] $-A*b^2*\text{hypergeom}([1/2, 1-1/2*n], [2-1/2*n], \cos(d*x+c)^2)*(b*\sec(d*x+c))^{(-2+n)}*\sin(d*x+c)/d/(2-n)/(\sin(d*x+c)^2)^{(1/2)}-b*B*\text{hypergeom}([1/2, 1/2-1/2*n], [3/2-1/2*n], \cos(d*x+c)^2)*(b*\sec(d*x+c))^{(-1+n)}*\sin(d*x+c)/d/(1-n)/(\sin(d*x+c)^2)^{(1/2)}$

Rubi [A] time = 0.13, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {16, 3787, 3772, 2643}

$$\frac{Ab^2 \sin(c+dx)(b \sec(c+dx))^{n-2} {}_2F_1\left(\frac{1}{2}, \frac{2-n}{2}; \frac{4-n}{2}; \cos^2(c+dx)\right)}{d(2-n)\sqrt{\sin^2(c+dx)}} - \frac{bB \sin(c+dx)(b \sec(c+dx))^{n-1} {}_2F_1\left(\frac{1}{2}, \frac{1-n}{2}; \frac{3-n}{2}; \cos^2(c+dx)\right)}{d(1-n)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Cos}[c + d*x]*(b*\text{Sec}[c + d*x])^n*(A + B*\text{Sec}[c + d*x]), x]$

[Out] $-((A*b^2*\text{Hypergeometric2F1}[1/2, (2-n)/2, (4-n)/2, \text{Cos}[c + d*x]^2]*(b*\text{Sec}[c + d*x])^{(-2+n)}*\text{Sin}[c + d*x])/(d*(2-n)*\text{Sqrt}[\text{Sin}[c + d*x]^2])) - (b*B*\text{Hypergeometric2F1}[1/2, (1-n)/2, (3-n)/2, \text{Cos}[c + d*x]^2]*(b*\text{Sec}[c + d*x])^{(-1+n)}*\text{Sin}[c + d*x])/(d*(1-n)*\text{Sqrt}[\text{Sin}[c + d*x]^2])$

Rule 16

$\text{Int}[(u_*)^{(v_*)^{(m_*)}}*((b_*)^{(v_*)})^{(n_*)}, x_Symbol] \rightarrow \text{Dist}[1/b^m, \text{Int}[u*(b*v)^{(m+n)}, x], x] /; \text{FreeQ}\{b, n, x\} \ \&\& \ \text{IntegerQ}[m]$

Rule 2643

$\text{Int}[(b_*)*\sin[(c_*) + (d_*)*(x_*)]^{(n_*)}, x_Symbol] \rightarrow \text{Simp}[(\text{Cos}[c + d*x]*(b*\text{Sin}[c + d*x])^{(n+1)}*\text{Hypergeometric2F1}[1/2, (n+1)/2, (n+3)/2, \text{Sin}[c + d*x]^2])/(b*d*(n+1)*\text{Sqrt}[\text{Cos}[c + d*x]^2]), x] /; \text{FreeQ}\{b, c, d, n, x\} \ \&\& \ !\text{IntegerQ}[2*n]$

Rule 3772

$\text{Int}[(\text{csc}[(c_*) + (d_*)*(x_*)]*(b_*)^{(n_*)}, x_Symbol] \rightarrow \text{Simp}[(b*\text{Csc}[c + d*x])^{(n-1)}*((\text{Sin}[c + d*x]/b)^{(n-1)}*\text{Int}[1/(\text{Sin}[c + d*x]/b)^n, x]), x] /; \text{FreeQ}\{b, c, d, n, x\}$

eeQ[{b, c, d, n}, x] && !IntegerQ[n]

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rubi steps

$$\begin{aligned}
 \int \cos(c + dx)(b \sec(c + dx))^n(A + B \sec(c + dx)) dx &= b \int (b \sec(c + dx))^{-1+n}(A + B \sec(c + dx)) dx \\
 &= (Ab) \int (b \sec(c + dx))^{-1+n} dx + B \int (b \sec(c + dx))^n dx \\
 &= \left(Ab \left(\frac{\cos(c + dx)}{b} \right)^n (b \sec(c + dx))^n \right) \int \left(\frac{\cos(c + dx)}{b} \right)^{1-n} \\
 &\quad B \cos(c + dx) {}_2F_1 \left(\frac{1}{2}, \frac{1-n}{2}; \frac{3-n}{2}; \cos^2(c + dx) \right) (b \sec(c + dx)) \\
 &= -\frac{B \cos(c + dx) {}_2F_1 \left(\frac{1}{2}, \frac{1-n}{2}; \frac{3-n}{2}; \cos^2(c + dx) \right) (b \sec(c + dx))}{d(1-n)\sqrt{\sin^2(c + dx)}}
 \end{aligned}$$

Mathematica [A] time = 0.16, size = 107, normalized size = 0.71

$$\frac{\sqrt{-\tan^2(c + dx)} \cot(c + dx)(b \sec(c + dx))^n \left(An \cos(c + dx) {}_2F_1 \left(\frac{1}{2}, \frac{n-1}{2}; \frac{n+1}{2}; \sec^2(c + dx) \right) + B(n-1) {}_2F_1 \left(\frac{1}{2}, \frac{n}{2}; \right) \right)}{d(n-1)n}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]*(b*Sec[c + d*x])^n*(A + B*Sec[c + d*x]),x]

[Out] (Cot[c + d*x]*(A*n*Cos[c + d*x]*Hypergeometric2F1[1/2, (-1 + n)/2, (1 + n)/2, Sec[c + d*x]^2] + B*(-1 + n)*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Sec[c + d*x]^2])*(b*Sec[c + d*x])^n*Sqrt[-Tan[c + d*x]^2])/(d*(-1 + n)*n)

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral} \left((B \cos(dx + c) \sec(dx + c) + A \cos(dx + c)) (b \sec(dx + c))^n, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] `integral((B*cos(d*x + c)*sec(d*x + c) + A*cos(d*x + c))*(b*sec(d*x + c))^n, x)`

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A) (b \sec(dx + c))^n \cos(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x, algorithm="giac")`

[Out] `integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^n*cos(d*x + c), x)`

maple [F] time = 4.00, size = 0, normalized size = 0.00

$$\int \cos(dx + c) (b \sec(dx + c))^n (A + B \sec(dx + c)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(d*x+c)*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x)`

[Out] `int(cos(d*x+c)*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x)`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A) (b \sec(dx + c))^n \cos(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x, algorithm="maxima")`

[Out] `integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^n*cos(d*x + c), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c + dx) \left(A + \frac{B}{\cos(c + dx)} \right) \left(\frac{b}{\cos(c + dx)} \right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(c + d*x)*(A + B/cos(c + d*x))*(b/cos(c + d*x))^n,x)`

[Out] `int(cos(c + d*x)*(A + B/cos(c + d*x))*(b/cos(c + d*x))^n, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \sec(c + dx))^n (A + B \sec(c + dx)) \cos(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)*(b*sec(d*x+c))**n*(A+B*sec(d*x+c)), x)`

[Out] `Integral((b*sec(c + d*x))**n*(A + B*sec(c + d*x))*cos(c + d*x), x)`

3.38 $\int \cos^2(c + dx)(b \sec(c + dx))^n (A + B \sec(c + dx)) dx$

Optimal. Leaf size=153

$$\frac{Ab^3 \sin(c + dx)(b \sec(c + dx))^{n-3} {}_2F_1\left(\frac{1}{2}, \frac{3-n}{2}; \frac{5-n}{2}; \cos^2(c + dx)\right)}{d(3-n)\sqrt{\sin^2(c + dx)}} - \frac{b^2 B \sin(c + dx)(b \sec(c + dx))^{n-2} {}_2F_1\left(\frac{1}{2}, \frac{2-n}{2}\right)}{d(2-n)\sqrt{\sin^2(c + dx)}}$$

[Out] $-A*b^3*\text{hypergeom}([1/2, 3/2-1/2*n], [5/2-1/2*n], \cos(d*x+c)^2)*(b*\sec(d*x+c))^{(-3+n)*\sin(d*x+c)/d/(3-n)/(\sin(d*x+c)^2)^{(1/2)}-b^2*B*\text{hypergeom}([1/2, 1-1/2*n], [2-1/2*n], \cos(d*x+c)^2)*(b*\sec(d*x+c))^{(-2+n)*\sin(d*x+c)/d/(2-n)/(\sin(d*x+c)^2)^{(1/2)}}$

Rubi [A] time = 0.14, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {16, 3787, 3772, 2643}

$$\frac{Ab^3 \sin(c + dx)(b \sec(c + dx))^{n-3} {}_2F_1\left(\frac{1}{2}, \frac{3-n}{2}; \frac{5-n}{2}; \cos^2(c + dx)\right)}{d(3-n)\sqrt{\sin^2(c + dx)}} - \frac{b^2 B \sin(c + dx)(b \sec(c + dx))^{n-2} {}_2F_1\left(\frac{1}{2}, \frac{2-n}{2}\right)}{d(2-n)\sqrt{\sin^2(c + dx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Cos}[c + d*x]^2*(b*\text{Sec}[c + d*x])^n*(A + B*\text{Sec}[c + d*x]), x]$

[Out] $-((A*b^3*\text{Hypergeometric2F1}[1/2, (3 - n)/2, (5 - n)/2, \text{Cos}[c + d*x]^2]*(b*\text{Sec}[c + d*x])^{(-3 + n)*\text{Sin}[c + d*x]})/(d*(3 - n)*\text{Sqrt}[\text{Sin}[c + d*x]^2])) - (b^2*B*\text{Hypergeometric2F1}[1/2, (2 - n)/2, (4 - n)/2, \text{Cos}[c + d*x]^2]*(b*\text{Sec}[c + d*x])^{(-2 + n)*\text{Sin}[c + d*x]})/(d*(2 - n)*\text{Sqrt}[\text{Sin}[c + d*x]^2])$

Rule 16

$\text{Int}[(u_*)*(v_*)^{(m_*)}*((b_*)*(v_*))^{(n_*)}, x_Symbol] \rightarrow \text{Dist}[1/b^m, \text{Int}[u*(b*v)^{(m+n)}, x], x] /; \text{FreeQ}[\{b, n\}, x] \ \&\& \ \text{IntegerQ}[m]$

Rule 2643

$\text{Int}[(b_*)*\sin[(c_*) + (d_*)*(x_*)]^{(n_*)}, x_Symbol] \rightarrow \text{Simp}[(\text{Cos}[c + d*x]*(b*\text{Sin}[c + d*x])^{(n+1)}*\text{Hypergeometric2F1}[1/2, (n+1)/2, (n+3)/2, \text{Sin}[c + d*x]^2])/(b*d*(n+1)*\text{Sqrt}[\text{Cos}[c + d*x]^2]), x] /; \text{FreeQ}[\{b, c, d, n\}, x] \ \&\& \ !\text{IntegerQ}[2*n]$

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.))^(n_), x_Symbol] := Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) +
(a_.)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned} \int \cos^2(c + dx)(b \sec(c + dx))^n (A + B \sec(c + dx)) dx &= b^2 \int (b \sec(c + dx))^{-2+n} (A + B \sec(c + dx)) dx \\ &= (Ab^2) \int (b \sec(c + dx))^{-2+n} dx + (bB) \int (b \sec(c + dx))^{-1} dx \\ &= \left(Ab^2 \left(\frac{\cos(c + dx)}{b} \right)^n (b \sec(c + dx))^n \right) \int \left(\frac{\cos(c + dx)}{b} \right)^2 dx \\ &= \frac{B \cos^2(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{2-n}{2}; \frac{4-n}{2}; \cos^2(c + dx)\right) (b \sec(c + dx))^n}{d(2-n)\sqrt{\sin^2(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.31, size = 114, normalized size = 0.75

$$\frac{b\sqrt{-\tan^2(c + dx)} \cot(c + dx)(b \sec(c + dx))^{n-1} \left(A(n-1) \cos(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{n-2}{2}; \frac{n}{2}; \sec^2(c + dx)\right) + B(n-2) {}_2F_1\left(\frac{1}{2}, \frac{n-2}{2}; \frac{n}{2}; \sec^2(c + dx)\right) \right)}{d(n-2)(n-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(b*Sec[c + d*x])^n*(A + B*Sec[c + d*x]),x]
```

```
[Out] (b*Cot[c + d*x]*(A*(-1 + n)*Cos[c + d*x]*Hypergeometric2F1[1/2, (-2 + n)/2,
n/2, Sec[c + d*x]^2] + B*(-2 + n)*Hypergeometric2F1[1/2, (-1 + n)/2, (1 +
n)/2, Sec[c + d*x]^2])*(b*Sec[c + d*x])^(-1 + n)*Sqrt[-Tan[c + d*x]^2])/(d*
(-2 + n)*(-1 + n))
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(B \cos(dx + c)^2 \sec(dx + c) + A \cos(dx + c)^2\right) (b \sec(dx + c))^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] integral((B*cos(d*x + c)^2*sec(d*x + c) + A*cos(d*x + c)^2)*(b*sec(d*x + c))^n, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A) (b \sec(dx + c))^n \cos(dx + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^n*cos(d*x + c)^2, x)

maple [F] time = 4.49, size = 0, normalized size = 0.00

$$\int (\cos^2(dx + c)) (b \sec(dx + c))^n (A + B \sec(dx + c)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^2*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x)

[Out] int(cos(d*x+c)^2*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A) (b \sec(dx + c))^n \cos(dx + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^n*cos(d*x + c)^2, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c + dx)^2 \left(A + \frac{B}{\cos(c + dx)} \right) \left(\frac{b}{\cos(c + dx)} \right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^2*(A + B/cos(c + d*x))*(b/cos(c + d*x))^n,x)

[Out] `int(cos(c + d*x)^2*(A + B/cos(c + d*x))*(b/cos(c + d*x))^n, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \sec(c + dx))^n (A + B \sec(c + dx)) \cos^2(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)**2*(b*sec(d*x+c))**n*(A+B*sec(d*x+c)), x)`

[Out] `Integral((b*sec(c + d*x))**n*(A + B*sec(c + d*x))*cos(c + d*x)**2, x)`

$$3.39 \quad \int \sec^2(c + dx)(b \sec(c + dx))^n(A + B \sec(c + dx)) dx$$

Optimal. Leaf size=163

$$\frac{2A \sin(c + dx) \sqrt{\sec(c + dx)} (b \sec(c + dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(-2n - 1); \frac{1}{4}(3 - 2n); \cos^2(c + dx)\right)}{d(2n + 1) \sqrt{\sin^2(c + dx)}} + \frac{2B \sin(c + dx) \sec^2(c + dx)}{d(2n + 1) \sqrt{\sin^2(c + dx)}}$$

[Out] 2*B*hypergeom([1/2, -3/4-1/2*n], [1/4-1/2*n], cos(d*x+c)^2)*sec(d*x+c)^(3/2)*(b*sec(d*x+c))^n*sin(d*x+c)/d/(3+2*n)/(sin(d*x+c)^2)^(1/2)+2*A*hypergeom([1/2, -1/4-1/2*n], [3/4-1/2*n], cos(d*x+c)^2)*(b*sec(d*x+c))^n*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(1+2*n)/(sin(d*x+c)^2)^(1/2)

Rubi [A] time = 0.12, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {20, 3787, 3772, 2643}

$$\frac{2A \sin(c + dx) \sqrt{\sec(c + dx)} (b \sec(c + dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(-2n - 1); \frac{1}{4}(3 - 2n); \cos^2(c + dx)\right)}{d(2n + 1) \sqrt{\sin^2(c + dx)}} + \frac{2B \sin(c + dx) \sec^2(c + dx)}{d(2n + 1) \sqrt{\sin^2(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d*x]^(3/2)*(b*Sec[c + d*x])^n*(A + B*Sec[c + d*x]), x]

[Out] (2*A*Hypergeometric2F1[1/2, (-1 - 2*n)/4, (3 - 2*n)/4, Cos[c + d*x]^2]*Sqrt[Sec[c + d*x]]*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(1 + 2*n)*Sqrt[Sin[c + d*x]^2]) + (2*B*Hypergeometric2F1[1/2, (-3 - 2*n)/4, (1 - 2*n)/4, Cos[c + d*x]^2]*Sec[c + d*x]^(3/2)*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(3 + 2*n)*Sqrt[Sin[c + d*x]^2])

Rule 20

Int[(u_)*((a_)*(v_))^(m_)*((b_)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m + n]

Rule 2643

Int[((b_)*sin[(c_.) + (d_)*(x_)]^(n_)), x_Symbol] :> Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]

&& !IntegerQ[2*n]

Rule 3772

Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.))^(n_), x_Symbol] := Simp[(b*Csc[c + d*x])^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rubi steps

$$\begin{aligned} \int \sec^{\frac{3}{2}}(c + dx)(b \sec(c + dx))^n (A + B \sec(c + dx)) dx &= (\sec^{-n}(c + dx)(b \sec(c + dx))^n) \int \sec^{\frac{3}{2}+n}(c + dx)(A + B \sec(c + dx)) dx \\ &= (A \sec^{-n}(c + dx)(b \sec(c + dx))^n) \int \sec^{\frac{3}{2}+n}(c + dx) dx + (B \sec^{-n}(c + dx)(b \sec(c + dx))^n) \int \sec^{\frac{3}{2}+n}(c + dx) \sec(c + dx) dx \\ &= \left(A \cos^{\frac{1}{2}+n}(c + dx) \sqrt{\sec(c + dx)} (b \sec(c + dx))^n \right) \int \cos^{\frac{3}{2}+n}(c + dx) dx \\ &= \frac{2A {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(-1 - 2n); \frac{1}{4}(3 - 2n); \cos^2(c + dx)\right) \sqrt{\sec(c + dx)}}{d(1 + 2n)\sqrt{\sin^2(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.33, size = 140, normalized size = 0.86

$$\frac{2\sqrt{-\tan^2(c + dx)} \csc(c + dx) \sec^{\frac{3}{2}}(c + dx)(b \sec(c + dx))^n \left(A(2n + 5) \cos(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(2n + 3); \frac{1}{4}(2n + 7); \sin^2(c + dx)\right) + B(2n + 3) \right)}{d(2n + 3)(2n + 5)}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[c + d*x]^(3/2)*(b*Sec[c + d*x])^n*(A + B*Sec[c + d*x]),x]

[Out] (2*Csc[c + d*x]*(A*(5 + 2*n)*Cos[c + d*x]*Hypergeometric2F1[1/2, (3 + 2*n)/4, (7 + 2*n)/4, Sec[c + d*x]^2] + B*(3 + 2*n)*Hypergeometric2F1[1/2, (5 + 2*n)/4, (9 + 2*n)/4, Sec[c + d*x]^2])*Sec[c + d*x]^(3/2)*(b*Sec[c + d*x])^n*Sqrt[-Tan[c + d*x]^2]/(d*(3 + 2*n)*(5 + 2*n))

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(B \sec(dx + c)^2 + A \sec(dx + c)\right) (b \sec(dx + c))^n \sqrt{\sec(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] integral((B*sec(d*x + c)^2 + A*sec(d*x + c))*(b*sec(d*x + c))^n*sqrt(sec(d*x + c)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A) (b \sec(dx + c))^n \sec(dx + c)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^n*sec(d*x + c)^(3/2), x)

maple [F] time = 1.77, size = 0, normalized size = 0.00

$$\int \left(\sec^{\frac{3}{2}}(dx + c) \right) (b \sec(dx + c))^n (A + B \sec(dx + c)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x)

[Out] int(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A) (b \sec(dx + c))^n \sec(dx + c)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^n*sec(d*x + c)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A + \frac{B}{\cos(c + dx)} \right) \left(\frac{b}{\cos(c + dx)} \right)^n \left(\frac{1}{\cos(c + dx)} \right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))*(b/cos(c + d*x))^n*(1/cos(c + d*x))^(3/2),x)
```

```
[Out] int((A + B/cos(c + d*x))*(b/cos(c + d*x))^n*(1/cos(c + d*x))^(3/2), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```


3.40 $\int \sqrt{\sec(c + dx)} (b \sec(c + dx))^n (A + B \sec(c + dx)) dx$

Optimal. Leaf size=163

$$\frac{2B \sin(c + dx) \sqrt{\sec(c + dx)} (b \sec(c + dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(-2n - 1); \frac{1}{4}(3 - 2n); \cos^2(c + dx)\right)}{d(2n + 1) \sqrt{\sin^2(c + dx)}} - \frac{2A \sin(c + dx) (b \sec(c + dx))^n}{d(2n + 1) \sqrt{\sin^2(c + dx)}}$$

[Out] $-2*A*\text{hypergeom}([1/2, 1/4-1/2*n], [5/4-1/2*n], \cos(d*x+c)^2)*(b*\sec(d*x+c))^n*\sin(d*x+c)/d/(1-2*n)/\sec(d*x+c)^{(1/2)}/(\sin(d*x+c)^2)^{(1/2)}+2*B*\text{hypergeom}([1/2, -1/4-1/2*n], [3/4-1/2*n], \cos(d*x+c)^2)*(b*\sec(d*x+c))^n*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/d/(1+2*n)/(\sin(d*x+c)^2)^{(1/2)}$

Rubi [A] time = 0.11, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {20, 3787, 3772, 2643}

$$\frac{2B \sin(c + dx) \sqrt{\sec(c + dx)} (b \sec(c + dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(-2n - 1); \frac{1}{4}(3 - 2n); \cos^2(c + dx)\right)}{d(2n + 1) \sqrt{\sin^2(c + dx)}} - \frac{2A \sin(c + dx) (b \sec(c + dx))^n}{d(2n + 1) \sqrt{\sin^2(c + dx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sqrt}[\text{Sec}[c + d*x]]*(b*\text{Sec}[c + d*x])^n*(A + B*\text{Sec}[c + d*x]), x]$

[Out] $(-2*A*\text{Hypergeometric2F1}[1/2, (1 - 2*n)/4, (5 - 2*n)/4, \text{Cos}[c + d*x]^2]*(b*\text{Sec}[c + d*x])^n*\text{Sin}[c + d*x])/d*(1 - 2*n)*\text{Sqrt}[\text{Sec}[c + d*x]]*\text{Sqrt}[\text{Sin}[c + d*x]^2] + (2*B*\text{Hypergeometric2F1}[1/2, (-1 - 2*n)/4, (3 - 2*n)/4, \text{Cos}[c + d*x]^2]*\text{Sqrt}[\text{Sec}[c + d*x]]*(b*\text{Sec}[c + d*x])^n*\text{Sin}[c + d*x])/d*(1 + 2*n)*\text{Sqrt}[\text{Sin}[c + d*x]^2])$

Rule 20

$\text{Int}[(u_*)*((a_*)*(v_))^{(m_)}*((b_*)*(v_))^{(n_)}, x_Symbol] \rightarrow \text{Dist}[(b^{\text{IntPart}[n]}*(b*v)^{\text{FracPart}[n]})/(a^{\text{IntPart}[n]}*(a*v)^{\text{FracPart}[n]}), \text{Int}[u*(a*v)^{(m+n)}, x], x] /; \text{FreeQ}\{a, b, m, n\}, x \&\& \text{IntegerQ}[m] \&\& \text{IntegerQ}[n] \&\& \text{IntegerQ}[m+n]$

Rule 2643

$\text{Int}[(b_*)*\sin[(c_*) + (d_*)*(x_)]^{(n_)}, x_Symbol] \rightarrow \text{Simp}[(\text{Cos}[c + d*x]*(b*\text{Sin}[c + d*x])^{(n+1)}*\text{Hypergeometric2F1}[1/2, (n+1)/2, (n+3)/2, \text{Sin}[c + d*x]^2])/(b*d*(n+1)*\text{Sqrt}[\text{Cos}[c + d*x]^2]), x] /; \text{FreeQ}\{b, c, d, n\}, x]$

&& !IntegerQ[2*n]

Rule 3772

Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.))^(n_), x_Symbol] := Simp[(b*Csc[c + d*x])^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rubi steps

$$\begin{aligned} \int \sqrt{\sec(c + dx)} (b \sec(c + dx))^n (A + B \sec(c + dx)) dx &= (\sec^{-n}(c + dx)(b \sec(c + dx))^n) \int \sec^{\frac{1}{2}+n}(c + dx)(A + B \sec(c + dx)) dx \\ &= (A \sec^{-n}(c + dx)(b \sec(c + dx))^n) \int \sec^{\frac{1}{2}+n}(c + dx) dx \\ &= \left(A \cos^{\frac{1}{2}+n}(c + dx) \sqrt{\sec(c + dx)} (b \sec(c + dx))^n \right) \int \cos^{\frac{1}{2}+n}(c + dx) dx \\ &= -\frac{2A {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(1 - 2n); \frac{1}{4}(5 - 2n); \cos^2(c + dx)\right) (b \sec(c + dx))^n}{d(1 - 2n) \sqrt{\sec(c + dx)} \sqrt{\sin^2(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.24, size = 140, normalized size = 0.86

$$\frac{2\sqrt{-\tan^2(c + dx)} \csc(c + dx)(b \sec(c + dx))^n \left(A(2n + 3) {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(2n + 1); \frac{1}{4}(2n + 5); \sec^2(c + dx)\right) + B(2n + 1) \right)}{d(2n + 1)(2n + 3)\sqrt{\sec(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[Sec[c + d*x]]*(b*Sec[c + d*x])^n*(A + B*Sec[c + d*x]),x]

[Out] (2*Csc[c + d*x]*(b*Sec[c + d*x])^n*(A*(3 + 2*n)*Hypergeometric2F1[1/2, (1 + 2*n)/4, (5 + 2*n)/4, Sec[c + d*x]^2] + B*(1 + 2*n)*Hypergeometric2F1[1/2, (3 + 2*n)/4, (7 + 2*n)/4, Sec[c + d*x]^2]*Sec[c + d*x])*Sqrt[-Tan[c + d*x]^2]/(d*(1 + 2*n)*(3 + 2*n)*Sqrt[Sec[c + d*x]])

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral} \left((B \sec(dx + c) + A) (b \sec(dx + c))^n \sqrt{\sec(dx + c)}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*sec(d*x+c))^n*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x, algorithm="fricas")

[Out] integral((B*sec(d*x + c) + A)*(b*sec(d*x + c))^n*sqrt(sec(d*x + c)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A) (b \sec(dx + c))^n \sqrt{\sec(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*sec(d*x+c))^n*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^n*sqrt(sec(d*x + c)), x)

maple [F] time = 1.57, size = 0, normalized size = 0.00

$$\int (b \sec(dx + c))^n (A + B \sec(dx + c)) (\sqrt{\sec(dx + c)}) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*sec(d*x+c))^n*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x)

[Out] int((b*sec(d*x+c))^n*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A) (b \sec(dx + c))^n \sqrt{\sec(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*sec(d*x+c))^n*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^n*sqrt(sec(d*x + c)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A + \frac{B}{\cos(c + dx)} \right) \left(\frac{b}{\cos(c + dx)} \right)^n \sqrt{\frac{1}{\cos(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B/cos(c + d*x))*(b/cos(c + d*x))^n*(1/cos(c + d*x))^(1/2), x)`

[Out] `int((A + B/cos(c + d*x))*(b/cos(c + d*x))^n*(1/cos(c + d*x))^(1/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \sec(c + dx))^n (A + B \sec(c + dx)) \sqrt{\sec(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*sec(d*x+c))**n*(A+B*sec(d*x+c))*sec(d*x+c)**(1/2), x)`

[Out] `Integral((b*sec(c + d*x))**n*(A + B*sec(c + d*x))*sqrt(sec(c + d*x)), x)`

$$3.41 \quad \int \frac{(b \sec(c+dx))^n (A+B \sec(c+dx))}{\sqrt{\sec(c+dx)}} dx$$

Optimal. Leaf size=163

$$\frac{2A \sin(c+dx)(b \sec(c+dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(3-2n); \frac{1}{4}(7-2n); \cos^2(c+dx)\right)}{d(3-2n)\sqrt{\sin^2(c+dx)} \sec^{\frac{3}{2}}(c+dx)} - \frac{2B \sin(c+dx)(b \sec(c+dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(3-2n); \frac{1}{4}(7-2n); \cos^2(c+dx)\right)}{d(1-2n)\sqrt{\sin^2(c+dx)}}$$

[Out] $-2*A*\text{hypergeom}([1/2, 3/4-1/2*n], [7/4-1/2*n], \cos(d*x+c)^2)*(b*\sec(d*x+c))^n*\sin(d*x+c)/d/(3-2*n)/\sec(d*x+c)^{(3/2)}/(\sin(d*x+c)^2)^{(1/2)}-2*B*\text{hypergeom}([1/2, 1/4-1/2*n], [5/4-1/2*n], \cos(d*x+c)^2)*(b*\sec(d*x+c))^n*\sin(d*x+c)/d/(1-2*n)/\sec(d*x+c)^{(1/2)}/(\sin(d*x+c)^2)^{(1/2)}$

Rubi [A] time = 0.11, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {20, 3787, 3772, 2643}

$$\frac{2A \sin(c+dx)(b \sec(c+dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(3-2n); \frac{1}{4}(7-2n); \cos^2(c+dx)\right)}{d(3-2n)\sqrt{\sin^2(c+dx)} \sec^{\frac{3}{2}}(c+dx)} - \frac{2B \sin(c+dx)(b \sec(c+dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(3-2n); \frac{1}{4}(7-2n); \cos^2(c+dx)\right)}{d(1-2n)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\frac{(b*\text{Sec}[c+d*x])^n*(A+B*\text{Sec}[c+d*x])}{\text{Sqrt}[\text{Sec}[c+d*x]]}, x]$

[Out] $(-2*A*\text{Hypergeometric2F1}[1/2, (3-2*n)/4, (7-2*n)/4, \text{Cos}[c+d*x]^2]*(b*\text{Sec}[c+d*x])^n*\text{Sin}[c+d*x])/d*(3-2*n)*\text{Sec}[c+d*x]^{(3/2)}*\text{Sqrt}[\text{Sin}[c+d*x]^2]) - (2*B*\text{Hypergeometric2F1}[1/2, (1-2*n)/4, (5-2*n)/4, \text{Cos}[c+d*x]^2]*(b*\text{Sec}[c+d*x])^n*\text{Sin}[c+d*x])/d*(1-2*n)*\text{Sqrt}[\text{Sec}[c+d*x]]*\text{Sqrt}[\text{Sin}[c+d*x]^2])$

Rule 20

$\text{Int}[(u_*)*((a_*)*(v_))^{(m_)}*((b_*)*(v_))^{(n_)}, x_Symbol] \rightarrow \text{Dist}[(b^{\text{IntPart}[n]}*(b*v)^{\text{FracPart}[n]})/(a^{\text{IntPart}[n]}*(a*v)^{\text{FracPart}[n]}), \text{Int}[u*(a*v)^{(m+n)}, x], x] /;$ FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m+n]

Rule 2643

$\text{Int}[(b_*)*\sin[(c_*)+(d_*)(x_)]^{(n_)}, x_Symbol] \rightarrow \text{Simp}[(\text{Cos}[c+d*x]*(b*\text{Sin}[c+d*x])^{(n+1)}*\text{Hypergeometric2F1}[1/2, (n+1)/2, (n+3)/2, \text{Sin}[c+d*x]^2])/(b*d*(n+1)*\text{Sqrt}[\text{Cos}[c+d*x]^2]), x] /;$ FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rule 3772

Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.))^n], x_Symbol] := Simp[(b*Csc[c + d*x])^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rubi steps

$$\begin{aligned} \int \frac{(b \sec(c + dx))^n (A + B \sec(c + dx))}{\sqrt{\sec(c + dx)}} dx &= (\sec^{-n}(c + dx)(b \sec(c + dx))^n) \int \sec^{-\frac{1}{2}+n}(c + dx)(A + B \sec(c + dx)) dx \\ &= (A \sec^{-n}(c + dx)(b \sec(c + dx))^n) \int \sec^{-\frac{1}{2}+n}(c + dx) dx + (B \sec^{-n}(c + dx)(b \sec(c + dx))^n) \int \sec^{-\frac{1}{2}+n}(c + dx) \sec(c + dx) dx \\ &= \left(A \cos^{\frac{1}{2}+n}(c + dx) \sqrt{\sec(c + dx)} (b \sec(c + dx))^n \right) \int \cos^{\frac{1}{2}-n}(c + dx) dx \\ &= -\frac{2A {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(3 - 2n); \frac{1}{4}(7 - 2n); \cos^2(c + dx)\right) (b \sec(c + dx))^n \sin(c + dx)}{d(3 - 2n) \sec^{\frac{3}{2}}(c + dx) \sqrt{\sin^2(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.36, size = 135, normalized size = 0.83

$$\frac{2\sqrt{-\tan^2(c + dx)} \csc(c + dx)(b \sec(c + dx))^n \left(A(2n + 1) {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(2n - 1); \frac{1}{4}(2n + 3); \sec^2(c + dx)\right) + B(2n - 1) \sec(c + dx) \right)}{d(4n^2 - 1) \sec^{\frac{3}{2}}(c + dx)}$$

Antiderivative was successfully verified.

[In] Integrate[((b*Sec[c + d*x])^n*(A + B*Sec[c + d*x]))/Sqrt[Sec[c + d*x]], x]

[Out] (2*Csc[c + d*x]*(b*Sec[c + d*x])^n*(A*(1 + 2*n)*Hypergeometric2F1[1/2, (-1 + 2*n)/4, (3 + 2*n)/4, Sec[c + d*x]^2] + B*(-1 + 2*n)*Hypergeometric2F1[1/2, (1 + 2*n)/4, (5 + 2*n)/4, Sec[c + d*x]^2]*Sec[c + d*x])*Sqrt[-Tan[c + d*x]^2])/(d*(-1 + 4*n^2)*Sec[c + d*x]^(3/2))

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B \sec(dx + c) + A)(b \sec(dx + c))^n}{\sqrt{\sec(dx + c)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*sec(d*x+c))^n*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="fricas")

[Out] integral((B*sec(d*x + c) + A)*(b*sec(d*x + c))^n/sqrt(sec(d*x + c)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(b \sec(dx + c))^n}{\sqrt{\sec(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*sec(d*x+c))^n*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^n/sqrt(sec(d*x + c)), x)

maple [F] time = 1.64, size = 0, normalized size = 0.00

$$\int \frac{(b \sec(dx + c))^n (A + B \sec(dx + c))}{\sqrt{\sec(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*sec(d*x+c))^n*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x)

[Out] int((b*sec(d*x+c))^n*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(b \sec(dx + c))^n}{\sqrt{\sec(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*sec(d*x+c))^n*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^n/sqrt(sec(d*x + c)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(\frac{b}{\cos(c+dx)}\right)^n}{\sqrt{\frac{1}{\cos(c+dx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B/cos(c + d*x))*(b/cos(c + d*x))^n)/(1/cos(c + d*x))^(1/2), x)`

[Out] `int(((A + B/cos(c + d*x))*(b/cos(c + d*x))^n)/(1/cos(c + d*x))^(1/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \sec(c + dx))^n (A + B \sec(c + dx))}{\sqrt{\sec(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*sec(d*x+c))**n*(A+B*sec(d*x+c))/sec(d*x+c)**(1/2), x)`

[Out] `Integral((b*sec(c + d*x))**n*(A + B*sec(c + d*x))/sqrt(sec(c + d*x)), x)`

$$3.42 \quad \int \frac{(b \sec(c+dx))^n (A+B \sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)} dx$$

Optimal. Leaf size=163

$$\frac{2A \sin(c+dx)(b \sec(c+dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(5-2n); \frac{1}{4}(9-2n); \cos^2(c+dx)\right)}{d(5-2n)\sqrt{\sin^2(c+dx)} \sec^{\frac{5}{2}}(c+dx)} - \frac{2B \sin(c+dx)(b \sec(c+dx))^n {}_2F_1\left(\frac{1}{2}, \frac{3}{4}(5-2n); \frac{1}{4}(9-2n); \cos^2(c+dx)\right)}{d(3-2n)\sqrt{\sin^2(c+dx)}}$$

[Out] $-2A \operatorname{hypergeom}\left(\left[\frac{1}{2}, \frac{5}{4}-\frac{1}{2}n\right], \left[\frac{9}{4}-\frac{1}{2}n\right], \cos(d*x+c)^2\right) * (b*\sec(d*x+c))^n * \sin(d*x+c) / d / (5-2n) / \sec(d*x+c)^{(5/2)} / (\sin(d*x+c)^2)^{(1/2)} - 2B \operatorname{hypergeom}\left(\left[\frac{1}{2}, \frac{3}{4}-\frac{1}{2}n\right], \left[\frac{7}{4}-\frac{1}{2}n\right], \cos(d*x+c)^2\right) * (b*\sec(d*x+c))^n * \sin(d*x+c) / d / (3-2n) / \sec(d*x+c)^{(3/2)} / (\sin(d*x+c)^2)^{(1/2)}$

Rubi [A] time = 0.11, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {20, 3787, 3772, 2643}

$$\frac{2A \sin(c+dx)(b \sec(c+dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(5-2n); \frac{1}{4}(9-2n); \cos^2(c+dx)\right)}{d(5-2n)\sqrt{\sin^2(c+dx)} \sec^{\frac{5}{2}}(c+dx)} - \frac{2B \sin(c+dx)(b \sec(c+dx))^n {}_2F_1\left(\frac{1}{2}, \frac{3}{4}(5-2n); \frac{1}{4}(9-2n); \cos^2(c+dx)\right)}{d(3-2n)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(b*\operatorname{Sec}[c+d*x])^n*(A+B*\operatorname{Sec}[c+d*x])]/\operatorname{Sec}[c+d*x]^{(3/2)},x]$

[Out] $(-2*A*\operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{(5-2n)}{4}, \frac{(9-2n)}{4}, \operatorname{Cos}[c+d*x]^2\right]*(b*\operatorname{Sec}[c+d*x])^n*\operatorname{Sin}[c+d*x])/(d*(5-2n)*\operatorname{Sec}[c+d*x]^{(5/2)}*\operatorname{Sqrt}[\operatorname{Sin}[c+d*x]^2]) - (2*B*\operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{(3-2n)}{4}, \frac{(7-2n)}{4}, \operatorname{Cos}[c+d*x]^2\right]*(b*\operatorname{Sec}[c+d*x])^n*\operatorname{Sin}[c+d*x])/(d*(3-2n)*\operatorname{Sec}[c+d*x]^{(3/2)}*\operatorname{Sqrt}[\operatorname{Sin}[c+d*x]^2])$

Rule 20

$\operatorname{Int}[(u_*)*((a_*)*(v_))^{(m_)}*((b_*)*(v_))^{(n_)}, x_Symbol] \rightarrow \operatorname{Dist}[(b^{\operatorname{IntPart}[n]}*(b*v)^{\operatorname{FracPart}[n]})/(a^{\operatorname{IntPart}[n]}*(a*v)^{\operatorname{FracPart}[n]})], \operatorname{Int}[u*(a*v)^{(m+n)}, x], x] /;$ $\operatorname{FreeQ}\{a, b, m, n\}, x \&\& \operatorname{IntegerQ}[m] \&\& \operatorname{IntegerQ}[n] \&\& \operatorname{IntegerQ}[m+n]$

Rule 2643

$\operatorname{Int}[(b_*)*\sin[(c_*)+(d_*)(x_)]^{(n_)}, x_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Cos}[c+d*x]*(b*\operatorname{Sin}[c+d*x])^{(n+1)}*\operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{(n+1)}{2}, \frac{(n+3)}{2}, \operatorname{Sin}[c+d*x]^2\right])/(b*d*(n+1)*\operatorname{Sqrt}[\operatorname{Cos}[c+d*x]^2]), x] /;$ $\operatorname{FreeQ}\{b, c, d, n\}, x]$

&& !IntegerQ[2*n]

Rule 3772

Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.))^n], x_Symbol] := Simp[(b*Csc[c + d*x])^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rubi steps

$$\begin{aligned} \int \frac{(b \sec(c + dx))^n (A + B \sec(c + dx))}{\sec^{\frac{3}{2}}(c + dx)} dx &= (\sec^{-n}(c + dx)(b \sec(c + dx))^n) \int \sec^{-\frac{3}{2}+n}(c + dx)(A + B \sec(c + dx)) dx \\ &= (A \sec^{-n}(c + dx)(b \sec(c + dx))^n) \int \sec^{-\frac{3}{2}+n}(c + dx) dx + (B \sec^{-n}(c + dx)(b \sec(c + dx))^n) \int \sec^{-\frac{1}{2}+n}(c + dx) dx \\ &= \left(A \cos^{\frac{1}{2}+n}(c + dx) \sqrt{\sec(c + dx)} (b \sec(c + dx))^n \right) \int \cos^{\frac{3}{2}-n}(c + dx) dx + \left(B \cos^{\frac{1}{2}+n}(c + dx) \sqrt{\sec(c + dx)} (b \sec(c + dx))^n \right) \int \cos^{\frac{1}{2}-n}(c + dx) dx \\ &= \frac{2A {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(5 - 2n); \frac{1}{4}(9 - 2n); \cos^2(c + dx)\right) (b \sec(c + dx))^n \sin(c + dx) + 2B {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(5 - 2n); \frac{1}{4}(9 - 2n); \cos^2(c + dx)\right) (b \sec(c + dx))^n \sin(c + dx)}{d(5 - 2n) \sec^{\frac{5}{2}}(c + dx) \sqrt{\sin^2(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.33, size = 140, normalized size = 0.86

$$\frac{2\sqrt{-\tan^2(c + dx)} \csc(c + dx)(b \sec(c + dx))^n \left(A {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(2n - 3); \frac{1}{4}(2n + 1); \sec^2(c + dx)\right) + B {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(2n - 3); \frac{1}{4}(2n + 1); \sec^2(c + dx)\right) \right) + B(2n - 3) \sec(c + dx)}{d(2n - 3)(2n - 1) \sec^{\frac{5}{2}}(c + dx)}$$

Antiderivative was successfully verified.

[In] Integrate[((b*Sec[c + d*x])^n*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(3/2),x]

[Out] (2*Csc[c + d*x]*(b*Sec[c + d*x])^n*(A*(-1 + 2*n)*Hypergeometric2F1[1/2, (-3 + 2*n)/4, (1 + 2*n)/4, Sec[c + d*x]^2] + B*(-3 + 2*n)*Hypergeometric2F1[1/2, (-1 + 2*n)/4, (3 + 2*n)/4, Sec[c + d*x]^2]*Sec[c + d*x])*Sqrt[-Tan[c + d*x]^2]/(d*(-3 + 2*n)*(-1 + 2*n)*Sec[c + d*x]^(5/2))

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B \sec(dx + c) + A)(b \sec(dx + c))^n}{\sec(dx + c)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*sec(d*x+c))^n*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="fricas")

[Out] integral((B*sec(d*x + c) + A)*(b*sec(d*x + c))^n/sec(d*x + c)^(3/2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(b \sec(dx + c))^n}{\sec(dx + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*sec(d*x+c))^n*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^n/sec(d*x + c)^(3/2), x)

maple [F] time = 1.70, size = 0, normalized size = 0.00

$$\int \frac{(b \sec(dx + c))^n (A + B \sec(dx + c))}{\sec(dx + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*sec(d*x+c))^n*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x)

[Out] int((b*sec(d*x+c))^n*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(b \sec(dx + c))^n}{\sec(dx + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*sec(d*x+c))^n*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^n/sec(d*x + c)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(\frac{b}{\cos(c+dx)}\right)^n}{\left(\frac{1}{\cos(c+dx)}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B/cos(c + d*x))*(b/cos(c + d*x))^n)/(1/cos(c + d*x))^(3/2), x)`

[Out] `int(((A + B/cos(c + d*x))*(b/cos(c + d*x))^n)/(1/cos(c + d*x))^(3/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \sec(c + dx))^n (A + B \sec(c + dx))}{\sec^{\frac{3}{2}}(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*sec(d*x+c))**n*(A+B*sec(d*x+c))/sec(d*x+c)**(3/2), x)`

[Out] `Integral((b*sec(c + d*x))**n*(A + B*sec(c + d*x))/sec(c + d*x)**(3/2), x)`

3.43 $\int \sec^4(c + dx)(a + a \sec(c + dx))(A + B \sec(c + dx)) dx$

Optimal. Leaf size=134

$$\frac{a(5A + 4B) \tan^3(c + dx)}{15d} + \frac{a(5A + 4B) \tan(c + dx)}{5d} + \frac{3a(A + B) \tanh^{-1}(\sin(c + dx))}{8d} + \frac{a(A + B) \tan(c + dx) \sec^3(c + dx)}{4d}$$

[Out] $\frac{3}{8} a (A+B) \operatorname{arctanh}(\sin(dx+c)) / d + \frac{1}{5} a (5A+4B) \tan(dx+c) / d + \frac{3}{8} a (A+B) \sec(dx+c) \tan(dx+c) / d + \frac{1}{4} a (A+B) \sec(dx+c)^3 \tan(dx+c) / d + \frac{1}{5} a B \sec(dx+c)^4 \tan(dx+c) / d + \frac{1}{15} a (5A+4B) \tan(dx+c)^3 / d$

Rubi [A] time = 0.14, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {3997, 3787, 3767, 3768, 3770}

$$\frac{a(5A + 4B) \tan^3(c + dx)}{15d} + \frac{a(5A + 4B) \tan(c + dx)}{5d} + \frac{3a(A + B) \tanh^{-1}(\sin(c + dx))}{8d} + \frac{a(A + B) \tan(c + dx) \sec^3(c + dx)}{4d}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d*x]^4*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]

[Out] $(3*a*(A + B)*\operatorname{ArcTanh}[\operatorname{Sin}[c + d*x]])/(8*d) + (a*(5*A + 4*B)*\operatorname{Tan}[c + d*x])/(5*d) + (3*a*(A + B)*\operatorname{Sec}[c + d*x]*\operatorname{Tan}[c + d*x])/(8*d) + (a*(A + B)*\operatorname{Sec}[c + d*x]^3*\operatorname{Tan}[c + d*x])/(4*d) + (a*B*\operatorname{Sec}[c + d*x]^4*\operatorname{Tan}[c + d*x])/(5*d) + (a*(5*A + 4*B)*\operatorname{Tan}[c + d*x]^3)/(15*d)$

Rule 3767

Int[csc[(c_) + (d_)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 3768

Int[(csc[(c_) + (d_)*(x_)]*(b_))^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x])*(b*Csc[c + d*x])^(n - 1)/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3770

Int[csc[(c_) + (d_)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) +
(a_.)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) +
(a_.))*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> -Simp[(b*B*Cot[e
+ f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && !LeQ[n,
-1]
```

Rubi steps

$$\begin{aligned}
\int \sec^4(c + dx)(a + a \sec(c + dx))(A + B \sec(c + dx)) dx &= \frac{aB \sec^4(c + dx) \tan(c + dx)}{5d} + \frac{1}{5} \int \sec^4(c + dx)(a(5A + 4B) + 3B \sec(c + dx)) dx \\
&= \frac{aB \sec^4(c + dx) \tan(c + dx)}{5d} + (a(A + B)) \int \sec^5(c + dx) dx \\
&= \frac{a(A + B) \sec^3(c + dx) \tan(c + dx)}{4d} + \frac{aB \sec^4(c + dx) \tan(c + dx)}{5d} \\
&= \frac{a(5A + 4B) \tan(c + dx)}{5d} + \frac{3a(A + B) \sec(c + dx) \tan(c + dx)}{8d} \\
&= \frac{3a(A + B) \tanh^{-1}(\sin(c + dx))}{8d} + \frac{a(5A + 4B) \tan(c + dx)}{5d}
\end{aligned}$$

Mathematica [A] time = 0.78, size = 87, normalized size = 0.65

$$\frac{a \left(45(A + B) \tanh^{-1}(\sin(c + dx)) + \tan(c + dx) \left(8 \left(5(A + 2B) \tan^2(c + dx) + 15(A + B) + 3B \tan^4(c + dx) \right) + 30 \right) \right)}{120d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]), x]
```

```
[Out] (a*(45*(A + B)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(45*(A + B)*Sec[c + d*x]
] + 30*(A + B)*Sec[c + d*x]^3 + 8*(15*(A + B) + 5*(A + 2*B)*Tan[c + d*x]^2
+ 3*B*Tan[c + d*x]^4)))/(120*d)
```

fricas [A] time = 0.44, size = 137, normalized size = 1.02

$$\frac{45(A+B)a \cos(dx+c)^5 \log(\sin(dx+c)+1) - 45(A+B)a \cos(dx+c)^5 \log(-\sin(dx+c)+1) + 2(16(5A+4B)a \cos(dx+c)^4 + 45(A+B)a \cos(dx+c)^3 + 8(5A+4B)a \cos(dx+c)^2 + 30(A+B)a \cos(dx+c) + 24Ba) \sin(dx+c)}{(d \cos(dx+c))^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^4*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] 1/240*(45*(A+B)*a*cos(d*x+c)^5*log(sin(d*x+c)+1) - 45*(A+B)*a*cos(d*x+c)^5*log(-sin(d*x+c)+1) + 2*(16*(5*A+4*B)*a*cos(d*x+c)^4 + 45*(A+B)*a*cos(d*x+c)^3 + 8*(5*A+4*B)*a*cos(d*x+c)^2 + 30*(A+B)*a*cos(d*x+c) + 24*B*a)*sin(d*x+c)/(d*cos(d*x+c)^5)

giac [A] time = 1.11, size = 214, normalized size = 1.60

$$45(Aa + Ba) \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right|\right) - 45(Aa + Ba) \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1\right|\right) - \frac{2\left(45Aa \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^9 + 45Ba\right)}{(d \cos(dx+c))^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^4*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] 1/120*(45*(A*a + B*a)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 45*(A*a + B*a)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(45*A*a*tan(1/2*d*x + 1/2*c)^9 + 45*B*a*tan(1/2*d*x + 1/2*c)^9 - 290*A*a*tan(1/2*d*x + 1/2*c)^7 - 130*B*a*tan(1/2*d*x + 1/2*c)^7 + 400*A*a*tan(1/2*d*x + 1/2*c)^5 + 464*B*a*tan(1/2*d*x + 1/2*c)^5 - 350*A*a*tan(1/2*d*x + 1/2*c)^3 - 190*B*a*tan(1/2*d*x + 1/2*c)^3 + 195*A*a*tan(1/2*d*x + 1/2*c) + 195*B*a*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d

maple [A] time = 1.20, size = 213, normalized size = 1.59

$$\frac{2aA \tan(dx+c)}{3d} + \frac{aA (\sec^2(dx+c)) \tan(dx+c)}{3d} + \frac{aB (\sec^3(dx+c)) \tan(dx+c)}{4d} + \frac{3aB \sec(dx+c) \tan(dx+c)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^4*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x)

[Out] 2/3*a*A*tan(d*x+c)/d+1/3*a*A*sec(d*x+c)^2*tan(d*x+c)/d+1/4*a*B*sec(d*x+c)^3*tan(d*x+c)/d+3/8/d*a*B*sec(d*x+c)*tan(d*x+c)+3/8/d*a*B*ln(sec(d*x+c)+tan(d*x+c))

$*x+c))+1/4*a*A*\sec(d*x+c)^3*\tan(d*x+c)/d+3/8*a*A*\sec(d*x+c)*\tan(d*x+c)/d+3/8/d*a*A*\ln(\sec(d*x+c)+\tan(d*x+c))+8/15/d*a*B*\tan(d*x+c)+1/5*a*B*\sec(d*x+c)^4*\tan(d*x+c)/d+4/15/d*a*B*\tan(d*x+c)*\sec(d*x+c)^2$

maxima [A] time = 0.33, size = 200, normalized size = 1.49

$$80 \left(\tan(dx+c)^3 + 3 \tan(dx+c) \right) Aa + 16 \left(3 \tan(dx+c)^5 + 10 \tan(dx+c)^3 + 15 \tan(dx+c) \right) Ba - 15 Aa \left(\frac{2(3}{\sin} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^4*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] $1/240*(80*(\tan(dx+c)^3 + 3*\tan(dx+c))*A*a + 16*(3*\tan(dx+c)^5 + 10*\tan(dx+c)^3 + 15*\tan(dx+c))*B*a - 15*A*a*(2*(3*\sin(dx+c)^3 - 5*\sin(dx+c))/(\sin(dx+c)^4 - 2*\sin(dx+c)^2 + 1) - 3*\log(\sin(dx+c) + 1) + 3*\log(\sin(dx+c) - 1)) - 15*B*a*(2*(3*\sin(dx+c)^3 - 5*\sin(dx+c))/(\sin(dx+c)^4 - 2*\sin(dx+c)^2 + 1) - 3*\log(\sin(dx+c) + 1) + 3*\log(\sin(dx+c) - 1)))/d$

mupad [B] time = 4.75, size = 198, normalized size = 1.48

$$\frac{3 a \operatorname{atanh}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right) (A + B) \left(\frac{3 A a}{4} + \frac{3 B a}{4}\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^9 + \left(-\frac{29 A a}{6} - \frac{13 B a}{6}\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^7 + \left(\frac{20 A a}{3} + \frac{116 B a}{15}\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5}{4 d} - \frac{d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^{10} - 5 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^8 + 10 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^6 - 10 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 + 5 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 - 1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x)))/cos(c + d*x)^4,x)

[Out] $(3*a*\operatorname{atanh}(\tan(c/2 + (d*x)/2))*(A + B))/(4*d) - (\tan(c/2 + (d*x)/2))*((13*A*a)/4 + (13*B*a)/4) + \tan(c/2 + (d*x)/2)^9*((3*A*a)/4 + (3*B*a)/4) - \tan(c/2 + (d*x)/2)^7*((29*A*a)/6 + (13*B*a)/6) - \tan(c/2 + (d*x)/2)^5*((35*A*a)/6 + (19*B*a)/6) + \tan(c/2 + (d*x)/2)^3*((20*A*a)/3 + (116*B*a)/15)/(d*(5*\tan(c/2 + (d*x)/2)^2 - 10*\tan(c/2 + (d*x)/2)^4 + 10*\tan(c/2 + (d*x)/2)^6 - 5*\tan(c/2 + (d*x)/2)^8 + \tan(c/2 + (d*x)/2)^{10} - 1))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a \left(\int A \sec^4(c + dx) dx + \int A \sec^5(c + dx) dx + \int B \sec^5(c + dx) dx + \int B \sec^6(c + dx) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.


```
[In] integrate(sec(d*x+c)**4*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] a*(Integral(A*sec(c + d*x)**4, x) + Integral(A*sec(c + d*x)**5, x) + Integr  
al(B*sec(c + d*x)**5, x) + Integral(B*sec(c + d*x)**6, x))
```

3.44 $\int \sec^3(c + dx)(a + a \sec(c + dx))(A + B \sec(c + dx)) dx$

Optimal. Leaf size=106

$$\frac{a(A + B) \tan^3(c + dx)}{3d} + \frac{a(A + B) \tan(c + dx)}{d} + \frac{a(4A + 3B) \tanh^{-1}(\sin(c + dx))}{8d} + \frac{a(4A + 3B) \tan(c + dx) \sec(c + dx)}{8d}$$

[Out] $1/8*a*(4*A+3*B)*\operatorname{arctanh}(\sin(d*x+c))/d+a*(A+B)*\tan(d*x+c)/d+1/8*a*(4*A+3*B)*\sec(d*x+c)*\tan(d*x+c)/d+1/4*a*B*\sec(d*x+c)^3*\tan(d*x+c)/d+1/3*a*(A+B)*\tan(d*x+c)^3/d$

Rubi [A] time = 0.12, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {3997, 3787, 3768, 3770, 3767}

$$\frac{a(A + B) \tan^3(c + dx)}{3d} + \frac{a(A + B) \tan(c + dx)}{d} + \frac{a(4A + 3B) \tanh^{-1}(\sin(c + dx))}{8d} + \frac{a(4A + 3B) \tan(c + dx) \sec(c + dx)}{8d}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Sec}[c + d*x]^3*(a + a*\operatorname{Sec}[c + d*x])*(A + B*\operatorname{Sec}[c + d*x]), x]$

[Out] $(a*(4*A + 3*B)*\operatorname{ArcTanh}[\operatorname{Sin}[c + d*x]])/(8*d) + (a*(A + B)*\operatorname{Tan}[c + d*x])/d + (a*(4*A + 3*B)*\operatorname{Sec}[c + d*x]*\operatorname{Tan}[c + d*x])/(8*d) + (a*B*\operatorname{Sec}[c + d*x]^3*\operatorname{Tan}[c + d*x])/(4*d) + (a*(A + B)*\operatorname{Tan}[c + d*x]^3)/(3*d)$

Rule 3767

$\operatorname{Int}[\operatorname{csc}[(c_.) + (d_.)*(x_.)]^{(n_.)}, x_Symbol] \rightarrow -\operatorname{Dist}[d^{(-1)}, \operatorname{Subst}[\operatorname{Int}[\operatorname{ExpandIntegrand}[(1 + x^2)^{(n/2 - 1)}, x], x], x, \operatorname{Cot}[c + d*x]], x] /;$ FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 3768

$\operatorname{Int}[(\operatorname{csc}[(c_.) + (d_.)*(x_.)]*(b_.))^{(n_.)}, x_Symbol] \rightarrow -\operatorname{Simp}[(b*\operatorname{Cos}[c + d*x])*(b*\operatorname{Csc}[c + d*x])^{(n - 1)})/(d*(n - 1)), x] + \operatorname{Dist}[(b^2*(n - 2))/(n - 1), \operatorname{Int}[(b*\operatorname{Csc}[c + d*x])^{(n - 2)}, x], x] /;$ FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3770

$\operatorname{Int}[\operatorname{csc}[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow -\operatorname{Simp}[\operatorname{ArcTanh}[\operatorname{Cos}[c + d*x]]/d, x] /;$ FreeQ[{c, d}, x]

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
+ f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && !LeQ[n,
-1]
```

Rubi steps

$$\begin{aligned} \int \sec^3(c + dx)(a + a \sec(c + dx))(A + B \sec(c + dx)) dx &= \frac{aB \sec^3(c + dx) \tan(c + dx)}{4d} + \frac{1}{4} \int \sec^3(c + dx)(a(4A + 3B) + a \sec(c + dx)) dx \\ &= \frac{aB \sec^3(c + dx) \tan(c + dx)}{4d} + (a(A + B)) \int \sec^4(c + dx) dx \\ &= \frac{a(4A + 3B) \sec(c + dx) \tan(c + dx)}{8d} + \frac{aB \sec^3(c + dx) \tan(c + dx)}{4d} \\ &= \frac{a(4A + 3B) \tanh^{-1}(\sin(c + dx))}{8d} + \frac{a(A + B) \tan(c + dx) \sec^3(c + dx)}{d} \end{aligned}$$

Mathematica [A] time = 0.41, size = 77, normalized size = 0.73

$$\frac{a \left(3(4A + 3B) \tanh^{-1}(\sin(c + dx)) + \tan(c + dx) \sec(c + dx) \left(8(A + B)(\cos(2(c + dx)) + 2) \sec(c + dx) + 12A + 9B \right) \right)}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a*(3*(4*A + 3*B)*ArcTanh[Sin[c + d*x]] + Sec[c + d*x]*(12*A + 9*B + 8*(A +
B)*(2 + Cos[2*(c + d*x)])*Sec[c + d*x] + 6*B*Sec[c + d*x]^2)*Tan[c + d*x])
)/(24*d)
```

fricas [A] time = 0.46, size = 127, normalized size = 1.20

$$\frac{3(4A + 3B)a \cos(dx + c)^4 \log(\sin(dx + c) + 1) - 3(4A + 3B)a \cos(dx + c)^4 \log(-\sin(dx + c) + 1) + 2(16(A + B) \cos(dx + c) \sec^3(dx + c) \tan(dx + c) + 3a \sec^3(dx + c) \tan(dx + c))}{48d \cos(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] $\frac{1}{48}*(3*(4*A + 3*B)*a*\cos(d*x + c)^4*\log(\sin(d*x + c) + 1) - 3*(4*A + 3*B)*a*\cos(d*x + c)^4*\log(-\sin(d*x + c) + 1) + 2*(16*(A + B)*a*\cos(d*x + c)^3 + 3*(4*A + 3*B)*a*\cos(d*x + c)^2 + 8*(A + B)*a*\cos(d*x + c) + 6*B*a)*\sin(d*x + c))/(d*\cos(d*x + c)^4)$

giac [A] time = 0.26, size = 188, normalized size = 1.77

$$3(4Aa + 3Ba) \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right|\right) - 3(4Aa + 3Ba) \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1\right|\right) - \frac{2\left(12Aa \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right)^7 + 9B^2a^2}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] $\frac{1}{24}*(3*(4*A*a + 3*B*a)*\log(\text{abs}(\tan(1/2*d*x + 1/2*c) + 1)) - 3*(4*A*a + 3*B*a)*\log(\text{abs}(\tan(1/2*d*x + 1/2*c) - 1)) - 2*(12*A*a*\tan(1/2*d*x + 1/2*c)^7 + 9*B*a*\tan(1/2*d*x + 1/2*c)^7 - 28*A*a*\tan(1/2*d*x + 1/2*c)^5 - 49*B*a*\tan(1/2*d*x + 1/2*c)^5 + 52*A*a*\tan(1/2*d*x + 1/2*c)^3 + 31*B*a*\tan(1/2*d*x + 1/2*c)^3 - 36*A*a*\tan(1/2*d*x + 1/2*c) - 39*B*a*\tan(1/2*d*x + 1/2*c))/(\tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d$

maple [A] time = 1.12, size = 171, normalized size = 1.61

$$\frac{aA \sec(dx + c) \tan(dx + c)}{2d} + \frac{aA \ln(\sec(dx + c) + \tan(dx + c))}{2d} + \frac{2aB \tan(dx + c)}{3d} + \frac{aB \tan(dx + c) (\sec^2(dx + c) + \tan^2(dx + c))}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^3*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x)

[Out] $\frac{1}{2}*a*A*\sec(d*x+c)*\tan(d*x+c)/d + \frac{1}{2}*d*a*A*\ln(\sec(d*x+c)+\tan(d*x+c)) + \frac{2}{3}*d*a*B*\tan(d*x+c) + \frac{1}{3}*d*a*B*\tan(d*x+c)*\sec(d*x+c)^2 + \frac{2}{3}*a*A*\tan(d*x+c)/d + \frac{1}{3}*a*A*\sec(d*x+c)^2*\tan(d*x+c)/d + \frac{1}{4}*a*B*\sec(d*x+c)^3*\tan(d*x+c)/d + \frac{3}{8}*d*a*B*\sec(d*x+c)*\tan(d*x+c) + \frac{3}{8}*d*a*B*\ln(\sec(d*x+c)+\tan(d*x+c))$

maxima [A] time = 0.34, size = 163, normalized size = 1.54

$$16\left(\tan(dx + c)^3 + 3 \tan(dx + c)\right)Aa + 16\left(\tan(dx + c)^3 + 3 \tan(dx + c)\right)Ba - 3Ba \left(\frac{2\left(3 \sin(dx+c)^3 - 5 \sin(dx+c)\right)}{\sin(dx+c)^4 - 2 \sin(dx+c)^2 + 1} - 3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] $\frac{1}{48} \cdot (16 \cdot (\tan(dx + c))^3 + 3 \cdot \tan(dx + c)) \cdot A \cdot a + 16 \cdot (\tan(dx + c))^3 + 3 \cdot \tan(dx + c) \cdot B \cdot a - 3 \cdot B \cdot a \cdot (2 \cdot (3 \cdot \sin(dx + c))^3 - 5 \cdot \sin(dx + c)) / (\sin(dx + c)^4 - 2 \cdot \sin(dx + c)^2 + 1) - 3 \cdot \log(\sin(dx + c) + 1) + 3 \cdot \log(\sin(dx + c) - 1) - 12 \cdot A \cdot a \cdot (2 \cdot \sin(dx + c) / (\sin(dx + c)^2 - 1) - \log(\sin(dx + c) + 1) + \log(\sin(dx + c) - 1)) / d$

mupad [B] time = 4.62, size = 166, normalized size = 1.57

$$\frac{\left(-Aa - \frac{3Ba}{4}\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^7 + \left(\frac{7Aa}{3} + \frac{49Ba}{12}\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5 + \left(-\frac{13Aa}{3} - \frac{31Ba}{12}\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 + \left(3Aa + \frac{13Ba}{4}\right)}{d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^8 - 4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^6 + 6 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 - 4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x)))/cos(c + d*x)^3,x)

[Out] $(\tan(c/2 + (dx)/2) \cdot (3Aa + (13Ba)/4) - \tan(c/2 + (dx)/2)^7 \cdot (Aa + (3Ba)/4) - \tan(c/2 + (dx)/2)^3 \cdot ((13Aa)/3 + (31Ba)/12) + \tan(c/2 + (dx)/2)^5 \cdot ((7Aa)/3 + (49Ba)/12)) / (d \cdot (6 \cdot \tan(c/2 + (dx)/2)^4 - 4 \cdot \tan(c/2 + (dx)/2)^2 - 4 \cdot \tan(c/2 + (dx)/2)^6 + \tan(c/2 + (dx)/2)^8 + 1)) + (a \cdot \operatorname{atanh}(\tan(c/2 + (dx)/2)) \cdot (4A + 3B)) / (4d)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a \left(\int A \sec^3(c + dx) dx + \int A \sec^4(c + dx) dx + \int B \sec^4(c + dx) dx + \int B \sec^5(c + dx) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**3*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x)

[Out] $a \cdot (\operatorname{Integral}(A \cdot \sec(c + dx) ** 3, x) + \operatorname{Integral}(A \cdot \sec(c + dx) ** 4, x) + \operatorname{Integral}(B \cdot \sec(c + dx) ** 4, x) + \operatorname{Integral}(B \cdot \sec(c + dx) ** 5, x))$

3.45 $\int \sec^2(c + dx)(a + a \sec(c + dx))(A + B \sec(c + dx)) dx$

Optimal. Leaf size=86

$$\frac{a(3A + 2B) \tan(c + dx)}{3d} + \frac{a(A + B) \tanh^{-1}(\sin(c + dx))}{2d} + \frac{a(A + B) \tan(c + dx) \sec(c + dx)}{2d} + \frac{aB \tan(c + dx) \sec^2(c + dx)}{3d}$$

[Out] $1/2*a*(A+B)*\operatorname{arctanh}(\sin(d*x+c))/d+1/3*a*(3*A+2*B)*\tan(d*x+c)/d+1/2*a*(A+B)*\sec(d*x+c)*\tan(d*x+c)/d+1/3*a*B*\sec(d*x+c)^2*\tan(d*x+c)/d$

Rubi [A] time = 0.12, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {3997, 3787, 3767, 8, 3768, 3770}

$$\frac{a(3A + 2B) \tan(c + dx)}{3d} + \frac{a(A + B) \tanh^{-1}(\sin(c + dx))}{2d} + \frac{a(A + B) \tan(c + dx) \sec(c + dx)}{2d} + \frac{aB \tan(c + dx) \sec^2(c + dx)}{3d}$$

Antiderivative was successfully verified.

[In] `Int[Sec[c + d*x]^2*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]`

[Out] $(a*(A + B)*\operatorname{ArcTanh}[\operatorname{Sin}[c + d*x]])/(2*d) + (a*(3*A + 2*B)*\operatorname{Tan}[c + d*x])/(3*d) + (a*(A + B)*\operatorname{Sec}[c + d*x]*\operatorname{Tan}[c + d*x])/(2*d) + (a*B*\operatorname{Sec}[c + d*x]^2*\operatorname{Tan}[c + d*x])/(3*d)$

Rule 8

`Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

Rule 3767

`Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]`

Rule 3768

`Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]`

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x]
  /; FreeQ[{c, d}, x]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)])*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)])*(b_.) +
(a_), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)])*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)])*(b_.) +
(a_)*(csc[(e_.) + (f_.)*(x_)])*(B_.) + (A_), x_Symbol] := -Simp[(b*B*Cot[e
+ f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && !LeQ[n,
-1]
```

Rubi steps

$$\begin{aligned} \int \sec^2(c + dx)(a + a \sec(c + dx))(A + B \sec(c + dx)) dx &= \frac{aB \sec^2(c + dx) \tan(c + dx)}{3d} + \frac{1}{3} \int \sec^2(c + dx)(a(3 \\ &= \frac{aB \sec^2(c + dx) \tan(c + dx)}{3d} + (a(A + B)) \int \sec^3(c + dx) dx \\ &= \frac{a(A + B) \sec(c + dx) \tan(c + dx)}{2d} + \frac{aB \sec^2(c + dx)}{3d} \\ &= \frac{a(A + B) \tanh^{-1}(\sin(c + dx))}{2d} + \frac{a(3A + 2B) \tan(c + dx)}{3d} \end{aligned}$$

Mathematica [A] time = 0.34, size = 56, normalized size = 0.65

$$\frac{a \left(3(A + B) \tanh^{-1}(\sin(c + dx)) + \tan(c + dx) \left(3(A + B) \sec(c + dx) + 6(A + B) + 2B \tan^2(c + dx) \right) \right)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]), x]
```

```
[Out] (a*(3*(A + B)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(6*(A + B) + 3*(A + B)*S
ec[c + d*x] + 2*B*Tan[c + d*x]^2))/(6*d)
```

fricas [A] time = 0.43, size = 105, normalized size = 1.22

$$\frac{3(A+B)a \cos(dx+c)^3 \log(\sin(dx+c)+1) - 3(A+B)a \cos(dx+c)^3 \log(-\sin(dx+c)+1) + 2(2(3A+2B) + B)a \cos(dx+c)^2 + 3(A+B)a \cos(dx+c) + 2B^2 a \sin(dx+c)}{12d \cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] 1/12*(3*(A+B)*a*cos(d*x+c)^3*log(sin(d*x+c)+1) - 3*(A+B)*a*cos(d*x+c)^3*log(-sin(d*x+c)+1) + 2*(2*(3*A+2*B)*a*cos(d*x+c)^2 + 3*(A+B)*a*cos(d*x+c) + 2*B*a*sin(d*x+c))/(d*cos(d*x+c)^3)

giac [A] time = 0.99, size = 154, normalized size = 1.79

$$3(Aa + Ba) \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right|\right) - 3(Aa + Ba) \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1\right|\right) - \frac{2\left(3Aa \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5 + 3Ba \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right)}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] 1/6*(3*(A*a + B*a)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(A*a + B*a)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(3*A*a*tan(1/2*d*x + 1/2*c)^5 + 3*B*a*tan(1/2*d*x + 1/2*c)^5 - 12*A*a*tan(1/2*d*x + 1/2*c)^3 - 4*B*a*tan(1/2*d*x + 1/2*c)^3 + 9*A*a*tan(1/2*d*x + 1/2*c) + 9*B*a*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3/d

maple [A] time = 1.15, size = 128, normalized size = 1.49

$$\frac{aA \tan(dx+c)}{d} + \frac{aB \sec(dx+c) \tan(dx+c)}{2d} + \frac{aB \ln(\sec(dx+c) + \tan(dx+c))}{2d} + \frac{aA \sec(dx+c) \tan(dx+c)}{2d} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^2*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x)

[Out] a*A*tan(d*x+c)/d+1/2/d*a*B*sec(d*x+c)*tan(d*x+c)+1/2/d*a*B*ln(sec(d*x+c)+tan(d*x+c))+1/2*a*A*sec(d*x+c)*tan(d*x+c)/d+1/2/d*a*A*ln(sec(d*x+c)+tan(d*x+c))+2/3/d*a*B*tan(d*x+c)+1/3/d*a*B*tan(d*x+c)*sec(d*x+c)^2

maxima [A] time = 0.33, size = 127, normalized size = 1.48

$$\frac{4 \left(\tan(dx+c)^3 + 3 \tan(dx+c) \right) Ba - 3 Aa \left(\frac{2 \sin(dx+c)}{\sin(dx+c)^2-1} - \log(\sin(dx+c)+1) + \log(\sin(dx+c)-1) \right) - 3 Ba}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a - 3*A*a*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 3*B*a*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 12*A*a*tan(d*x + c))/d

mupad [B] time = 3.99, size = 126, normalized size = 1.47

$$\frac{a \operatorname{atanh}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right) (A+B) (Aa + Ba) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5 + \left(-4Aa - \frac{4Ba}{3}\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 + (3Aa + 3Ba) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^6 - 3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 + 3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 - 1 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x)))/cos(c + d*x)^2,x)

[Out] (a*atanh(tan(c/2 + (d*x)/2))*(A + B))/d - (tan(c/2 + (d*x)/2)*(3*A*a + 3*B*a) + tan(c/2 + (d*x)/2)^5*(A*a + B*a) - tan(c/2 + (d*x)/2)^3*(4*A*a + (4*B*a)/3))/(d*(3*tan(c/2 + (d*x)/2)^2 - 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a \left(\int A \sec^2(c + dx) dx + \int A \sec^3(c + dx) dx + \int B \sec^3(c + dx) dx + \int B \sec^4(c + dx) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**2*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x)

[Out] a*(Integral(A*sec(c + d*x)**2, x) + Integral(A*sec(c + d*x)**3, x) + Integral(B*sec(c + d*x)**3, x) + Integral(B*sec(c + d*x)**4, x))

3.46 $\int \sec(c + dx)(a + a \sec(c + dx))(A + B \sec(c + dx)) dx$

Optimal. Leaf size=56

$$\frac{a(A + B) \tan(c + dx)}{d} + \frac{a(2A + B) \tanh^{-1}(\sin(c + dx))}{2d} + \frac{aB \tan(c + dx) \sec(c + dx)}{2d}$$

[Out] $1/2*a*(2*A+B)*\operatorname{arctanh}(\sin(d*x+c))/d+a*(A+B)*\tan(d*x+c)/d+1/2*a*B*\sec(d*x+c)*\tan(d*x+c)/d$

Rubi [A] time = 0.07, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {3997, 3787, 3770, 3767, 8}

$$\frac{a(A + B) \tan(c + dx)}{d} + \frac{a(2A + B) \tanh^{-1}(\sin(c + dx))}{2d} + \frac{aB \tan(c + dx) \sec(c + dx)}{2d}$$

Antiderivative was successfully verified.

[In] `Int[Sec[c + d*x]*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]`

[Out] $(a*(2*A + B)*\operatorname{ArcTanh}[\operatorname{Sin}[c + d*x]])/(2*d) + (a*(A + B)*\operatorname{Tan}[c + d*x])/d + (a*B*\operatorname{Sec}[c + d*x]*\operatorname{Tan}[c + d*x])/(2*d)$

Rule 8

`Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

Rule 3767

`Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]`

Rule 3770

`Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]`

Rule 3787

`Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[`

$(d \cdot \text{Csc}[e + f \cdot x])^{(n + 1)}, x], x] /; \text{FreeQ}[\{a, b, d, e, f, n\}, x]$

Rule 3997

$\text{Int}[(\text{csc}[(e_.) + (f_.) \cdot (x_)] \cdot (d_.))^{(n_.)} \cdot (\text{csc}[(e_.) + (f_.) \cdot (x_)] \cdot (b_.) + (a_.) \cdot (\text{csc}[(e_.) + (f_.) \cdot (x_)] \cdot (B_.) + (A_.)), x_Symbol] :> -\text{Simp}[(b \cdot B \cdot \text{Cot}[e + f \cdot x] \cdot (d \cdot \text{Csc}[e + f \cdot x])^n) / (f \cdot (n + 1)), x] + \text{Dist}[1 / (n + 1), \text{Int}[(d \cdot \text{Csc}[e + f \cdot x])^n \cdot \text{Simp}[A \cdot a \cdot (n + 1) + B \cdot b \cdot n + (A \cdot b + B \cdot a) \cdot (n + 1) \cdot \text{Csc}[e + f \cdot x], x], x] /; \text{FreeQ}[\{a, b, d, e, f, A, B\}, x] \&\& \text{NeQ}[A \cdot b - a \cdot B, 0] \&\& !\text{LeQ}[n, -1]$

Rubi steps

$$\begin{aligned} \int \sec(c + dx)(a + a \sec(c + dx))(A + B \sec(c + dx)) dx &= \frac{aB \sec(c + dx) \tan(c + dx)}{2d} + \frac{1}{2} \int \sec(c + dx)(a(2A + B) + a \sec^2(c + dx)) dx \\ &= \frac{aB \sec(c + dx) \tan(c + dx)}{2d} + (a(A + B)) \int \sec^2(c + dx) dx \\ &= \frac{a(2A + B) \tanh^{-1}(\sin(c + dx))}{2d} + \frac{aB \sec(c + dx) \tan(c + dx)}{2d} \\ &= \frac{a(2A + B) \tanh^{-1}(\sin(c + dx))}{2d} + \frac{a(A + B) \tan(c + dx)}{d} \end{aligned}$$

Mathematica [A] time = 0.03, size = 75, normalized size = 1.34

$$\frac{aA \tan(c + dx)}{d} + \frac{aA \tanh^{-1}(\sin(c + dx))}{d} + \frac{aB \tan(c + dx)}{d} + \frac{aB \tanh^{-1}(\sin(c + dx))}{2d} + \frac{aB \tan(c + dx) \sec(c + dx)}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[c + d*x]*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]

[Out] (a*A*ArcTanh[Sin[c + d*x]])/d + (a*B*ArcTanh[Sin[c + d*x]])/(2*d) + (a*A*Tan[c + d*x])/d + (a*B*Tan[c + d*x])/d + (a*B*Sec[c + d*x]*Tan[c + d*x])/(2*d)

fricas [A] time = 0.42, size = 89, normalized size = 1.59

$$\frac{(2A + B)a \cos(dx + c)^2 \log(\sin(dx + c) + 1) - (2A + B)a \cos(dx + c)^2 \log(-\sin(dx + c) + 1) + 2(2(A + B)a \tan(dx + c) + a \sec^2(dx + c))}{4d \cos(dx + c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] $\frac{1}{4} * ((2 * A + B) * a * \cos(d * x + c) ^ 2 * \log(\sin(d * x + c) + 1) - (2 * A + B) * a * \cos(d * x + c) ^ 2 * \log(-\sin(d * x + c) + 1) + 2 * (2 * (A + B) * a * \cos(d * x + c) + B * a) * \sin(d * x + c)) / (d * \cos(d * x + c) ^ 2)$

giac [B] time = 0.28, size = 124, normalized size = 2.21

$$\frac{(2 A a + B a) \log\left(\left|\tan\left(\frac{1}{2} d x + \frac{1}{2} c\right) + 1\right|\right) - (2 A a + B a) \log\left(\left|\tan\left(\frac{1}{2} d x + \frac{1}{2} c\right) - 1\right|\right) - \frac{2\left(2 A a \tan\left(\frac{1}{2} d x + \frac{1}{2} c\right)^3 + B a \tan\left(\frac{1}{2} d x + \frac{1}{2} c\right)\right)}{2 d}}{2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] $\frac{1}{2} * ((2 * A * a + B * a) * \log(\operatorname{abs}(\tan(1/2 * d * x + 1/2 * c) + 1)) - (2 * A * a + B * a) * \log(\operatorname{abs}(\tan(1/2 * d * x + 1/2 * c) - 1))) - 2 * (2 * A * a * \tan(1/2 * d * x + 1/2 * c) ^ 3 + B * a * \tan(1/2 * d * x + 1/2 * c) ^ 3 - 2 * A * a * \tan(1/2 * d * x + 1/2 * c) - 3 * B * a * \tan(1/2 * d * x + 1/2 * c)) / (\tan(1/2 * d * x + 1/2 * c) ^ 2 - 1) ^ 2 / d$

maple [A] time = 0.94, size = 86, normalized size = 1.54

$$\frac{a A \ln(\sec(dx+c) + \tan(dx+c))}{d} + \frac{a B \tan(dx+c)}{d} + \frac{a A \tan(dx+c)}{d} + \frac{a B \sec(dx+c) \tan(dx+c)}{2 d} + \frac{a B \ln(\sec(dx+c) + \tan(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x)

[Out] $\frac{1}{d} * a * A * \ln(\sec(d * x + c) + \tan(d * x + c)) + \frac{1}{d} * a * B * \tan(d * x + c) + a * A * \tan(d * x + c) / d + \frac{1}{2} / d * a * B * \sec(d * x + c) * \tan(d * x + c) + \frac{1}{2} / d * a * B * \ln(\sec(d * x + c) + \tan(d * x + c))$

maxima [A] time = 0.32, size = 88, normalized size = 1.57

$$\frac{B a \left(\frac{2 \sin(dx+c)}{\sin(dx+c)^2-1} - \log(\sin(dx+c) + 1) + \log(\sin(dx+c) - 1) \right) - 4 A a \log(\sec(dx+c) + \tan(dx+c)) - 4 A a \tan(dx+c)}{4 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] $-\frac{1}{4} * (B * a * (2 * \sin(d * x + c) / (\sin(d * x + c) ^ 2 - 1) - \log(\sin(d * x + c) + 1) + \log(\sin(d * x + c) - 1)) - 4 * A * a * \log(\sec(d * x + c) + \tan(d * x + c)) - 4 * A * a * \tan(d * x + c) - 4 * B * a * \tan(d * x + c)) / d$

mupad [B] time = 2.73, size = 94, normalized size = 1.68

$$\frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right) (2Aa + 3Ba) - \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 (2Aa + Ba) + a \operatorname{atanh}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right) (2A + B)}{d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 - 2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + 1 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x)))/cos(c + d*x), x)`

[Out] `(tan(c/2 + (d*x)/2)*(2*A*a + 3*B*a) - tan(c/2 + (d*x)/2)^3*(2*A*a + B*a))/(d*(tan(c/2 + (d*x)/2)^4 - 2*tan(c/2 + (d*x)/2)^2 + 1) + (a*atanh(tan(c/2 + (d*x)/2))*(2*A + B))/d`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a \left(\int A \sec(c + dx) dx + \int A \sec^2(c + dx) dx + \int B \sec^2(c + dx) dx + \int B \sec^3(c + dx) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)), x)`

[Out] `a*(Integral(A*sec(c + d*x), x) + Integral(A*sec(c + d*x)**2, x) + Integral(B*sec(c + d*x)**2, x) + Integral(B*sec(c + d*x)**3, x))`

3.47 $\int (a + a \sec(c + dx))(A + B \sec(c + dx)) dx$

Optimal. Leaf size=32

$$\frac{a(A + B) \tanh^{-1}(\sin(c + dx))}{d} + aAx + \frac{aB \tan(c + dx)}{d}$$

[Out] a*A*x+a*(A+B)*arctanh(sin(d*x+c))/d+a*B*tan(d*x+c)/d

Rubi [A] time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3914, 3767, 8, 3770}

$$\frac{a(A + B) \tanh^{-1}(\sin(c + dx))}{d} + aAx + \frac{aB \tan(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]

[Out] a*A*x + (a*(A + B)*ArcTanh[Sin[c + d*x]])/d + (a*B*Tan[c + d*x])/d

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 3767

Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3914

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.)), x_Symbol] := Simp[a*c*x, x] + (Dist[b*d, Int[Csc[e + f*x]^2, x], x] + Dist[b*c + a*d, Int[Csc[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]

Rubi steps

$$\begin{aligned}
\int (a + a \sec(c + dx))(A + B \sec(c + dx)) dx &= aAx + (aB) \int \sec^2(c + dx) dx + (a(A + B)) \int \sec(c + dx) dx \\
&= aAx + \frac{a(A + B) \tanh^{-1}(\sin(c + dx))}{d} - \frac{(aB) \operatorname{Subst}(\int 1 dx, x, -\tan(c + dx))}{d} \\
&= aAx + \frac{a(A + B) \tanh^{-1}(\sin(c + dx))}{d} + \frac{aB \tan(c + dx)}{d}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 43, normalized size = 1.34

$$\frac{aA \tanh^{-1}(\sin(c + dx))}{d} + aAx + \frac{aB \tan(c + dx)}{d} + \frac{aB \tanh^{-1}(\sin(c + dx))}{d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]

[Out] a*A*x + (a*A*ArcTanh[Sin[c + d*x]])/d + (a*B*ArcTanh[Sin[c + d*x]])/d + (a*B*Tan[c + d*x])/d

fricas [B] time = 0.44, size = 79, normalized size = 2.47

$$\frac{2 A a d x \cos(dx + c) + (A + B) a \cos(dx + c) \log(\sin(dx + c) + 1) - (A + B) a \cos(dx + c) \log(-\sin(dx + c) + 1)}{2 d \cos(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] 1/2*(2*A*a*d*x*cos(d*x + c) + (A + B)*a*cos(d*x + c)*log(sin(d*x + c) + 1) - (A + B)*a*cos(d*x + c)*log(-sin(d*x + c) + 1) + 2*B*a*sin(d*x + c))/(d*cos(d*x + c))

giac [B] time = 1.41, size = 84, normalized size = 2.62

$$\frac{(dx + c)Aa + (Aa + Ba) \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 1\right|\right) - (Aa + Ba) \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 1\right|\right) - \frac{2Ba \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] $((d*x + c)*A*a + (A*a + B*a)*\log(\text{abs}(\tan(1/2*d*x + 1/2*c) + 1)) - (A*a + B*a)*\log(\text{abs}(\tan(1/2*d*x + 1/2*c) - 1)) - 2*B*a*\tan(1/2*d*x + 1/2*c)/(\tan(1/2*d*x + 1/2*c)^2 - 1))/d$

maple [A] time = 0.71, size = 65, normalized size = 2.03

$$aAx + \frac{aA \ln(\sec(dx + c) + \tan(dx + c))}{d} + \frac{Aac}{d} + \frac{aB \ln(\sec(dx + c) + \tan(dx + c))}{d} + \frac{aB \tan(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x)`

[Out] $a*A*x + 1/d*a*A*\ln(\sec(d*x+c) + \tan(d*x+c)) + 1/d*A*a*c + 1/d*a*B*\ln(\sec(d*x+c) + \tan(d*x+c)) + 1/d*a*B*\tan(d*x+c)$

maxima [A] time = 0.32, size = 56, normalized size = 1.75

$$\frac{(dx + c)Aa + Aa \log(\sec(dx + c) + \tan(dx + c)) + Ba \log(\sec(dx + c) + \tan(dx + c)) + Ba \tan(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="maxima")`

[Out] $((d*x + c)*A*a + A*a*\log(\sec(d*x + c) + \tan(d*x + c)) + B*a*\log(\sec(d*x + c) + \tan(d*x + c)) + B*a*\tan(d*x + c))/d$

mupad [B] time = 2.23, size = 100, normalized size = 3.12

$$\frac{Ba \tan(c + dx)}{d} + \frac{2Aa \operatorname{atan}\left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)}\right)}{d} + \frac{2Aa \operatorname{atanh}\left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)}\right)}{d} + \frac{2Ba \operatorname{atanh}\left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B/cos(c + d*x))*(a + a/cos(c + d*x)),x)`

[Out] $(B*a*\tan(c + d*x))/d + (2*A*a*\operatorname{atan}(\sin(c/2 + (d*x)/2)/\cos(c/2 + (d*x)/2)))/d + (2*A*a*\operatorname{atanh}(\sin(c/2 + (d*x)/2)/\cos(c/2 + (d*x)/2)))/d + (2*B*a*\operatorname{atanh}(\sin(c/2 + (d*x)/2)/\cos(c/2 + (d*x)/2)))/d$

sympy [A] time = 7.79, size = 71, normalized size = 2.22

$$\begin{cases} \frac{Aa(c+dx) + Aa \log(\tan(c+dx) + \sec(c+dx)) + Ba \log(\tan(c+dx) + \sec(c+dx)) + Ba \tan(c+dx)}{d} & \text{for } d \neq 0 \\ x(A + B \sec(c))(a \sec(c) + a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] Piecewise(((A*a*(c + d*x) + A*a*log(tan(c + d*x) + sec(c + d*x)) + B*a*log(tan(c + d*x) + sec(c + d*x)) + B*a*tan(c + d*x))/d, Ne(d, 0)), (x*(A + B*sec(c))*(a*sec(c) + a), True))
```

$$3.48 \quad \int \cos(c + dx)(a + a \sec(c + dx))(A + B \sec(c + dx)) dx$$

Optimal. Leaf size=32

$$ax(A + B) + \frac{aA \sin(c + dx)}{d} + \frac{aB \tanh^{-1}(\sin(c + dx))}{d}$$

[Out] a*(A+B)*x+a*B*arctanh(sin(d*x+c))/d+a*A*sin(d*x+c)/d

Rubi [A] time = 0.05, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {3996, 3770}

$$ax(A + B) + \frac{aA \sin(c + dx)}{d} + \frac{aB \tanh^{-1}(\sin(c + dx))}{d}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]

[Out] a*(A + B)*x + (a*B*ArcTanh[Sin[c + d*x]])/d + (a*A*Sin[c + d*x])/d

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3996

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] := Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]

Rubi steps

$$\begin{aligned} \int \cos(c + dx)(a + a \sec(c + dx))(A + B \sec(c + dx)) dx &= \frac{aA \sin(c + dx)}{d} - \int (-a(A + B) - aB \sec(c + dx)) dx \\ &= a(A + B)x + \frac{aA \sin(c + dx)}{d} + (aB) \int \sec(c + dx) dx \\ &= a(A + B)x + \frac{aB \tanh^{-1}(\sin(c + dx))}{d} + \frac{aA \sin(c + dx)}{d} \end{aligned}$$

Mathematica [A] time = 0.03, size = 46, normalized size = 1.44

$$\frac{aA \sin(c) \cos(dx)}{d} + \frac{aA \cos(c) \sin(dx)}{d} + aAx + \frac{aB \tanh^{-1}(\sin(c + dx))}{d} + aBx$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]

[Out] a*A*x + a*B*x + (a*B*ArcTanh[Sin[c + d*x]])/d + (a*A*Cos[d*x]*Sin[c])/d + (a*A*Cos[c]*Sin[d*x])/d

fricas [A] time = 0.44, size = 51, normalized size = 1.59

$$\frac{2(A+B)adx + Ba \log(\sin(dx+c)+1) - Ba \log(-\sin(dx+c)+1) + 2Aa \sin(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] 1/2*(2*(A + B)*a*d*x + B*a*log(sin(d*x + c) + 1) - B*a*log(-sin(d*x + c) + 1) + 2*A*a*sin(d*x + c))/d

giac [B] time = 2.35, size = 79, normalized size = 2.47

$$\frac{Ba \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right|\right) - Ba \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1\right|\right) + (Aa + Ba)(dx + c) + \frac{2Aa \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)}{\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + 1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] (B*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - B*a*log(abs(tan(1/2*d*x + 1/2*c) - 1))) + (A*a + B*a)*(d*x + c) + 2*A*a*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 + 1))/d

maple [A] time = 0.62, size = 56, normalized size = 1.75

$$aAx + Bxa + \frac{aA \sin(dx+c)}{d} + \frac{Aac}{d} + \frac{aB \ln(\sec(dx+c) + \tan(dx+c))}{d} + \frac{Bac}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x)

[Out] $a \cdot A \cdot x + B \cdot x \cdot a + a \cdot A \cdot \sin(dx + c) / d + 1/d \cdot A \cdot a \cdot c + 1/d \cdot a \cdot B \cdot \ln(\sec(dx + c) + \tan(dx + c)) + 1/d \cdot B \cdot a \cdot c$

maxima [A] time = 0.33, size = 58, normalized size = 1.81

$$\frac{2(dx + c)Aa + 2(dx + c)Ba + Ba(\log(\sin(dx + c) + 1) - \log(\sin(dx + c) - 1)) + 2Aa \sin(dx + c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="maxima")`

[Out] $1/2 \cdot (2 \cdot (dx + c) \cdot A \cdot a + 2 \cdot (dx + c) \cdot B \cdot a + B \cdot a \cdot (\log(\sin(dx + c) + 1) - \log(\sin(dx + c) - 1)) + 2 \cdot A \cdot a \cdot \sin(dx + c)) / d$

mupad [B] time = 2.15, size = 100, normalized size = 3.12

$$\frac{Aa \sin(c + dx)}{d} + \frac{2Aa \operatorname{atan}\left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)}\right)}{d} + \frac{2Ba \operatorname{atan}\left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)}\right)}{d} + \frac{2Ba \operatorname{atanh}\left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(c + d*x)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x)),x)`

[Out] $(A \cdot a \cdot \sin(c + dx)) / d + (2 \cdot A \cdot a \cdot \operatorname{atan}(\sin(c/2 + (dx)/2) / \cos(c/2 + (dx)/2))) / d + (2 \cdot B \cdot a \cdot \operatorname{atan}(\sin(c/2 + (dx)/2) / \cos(c/2 + (dx)/2))) / d + (2 \cdot B \cdot a \cdot \operatorname{atanh}(\sin(c/2 + (dx)/2) / \cos(c/2 + (dx)/2))) / d$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a \left(\int A \cos(c + dx) dx + \int A \cos(c + dx) \sec(c + dx) dx + \int B \cos(c + dx) \sec(c + dx) dx + \int B \cos(c + dx) \sec(c + dx) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x)`

[Out] $a \cdot (\operatorname{Integral}(A \cdot \cos(c + dx), x) + \operatorname{Integral}(A \cdot \cos(c + dx) \cdot \sec(c + dx), x) + \operatorname{Integral}(B \cdot \cos(c + dx) \cdot \sec(c + dx), x) + \operatorname{Integral}(B \cdot \cos(c + dx) \cdot \sec(c + dx) \cdot \sec(c + dx), x))$

$$3.49 \quad \int \cos^2(c + dx)(a + a \sec(c + dx))(A + B \sec(c + dx)) dx$$

Optimal. Leaf size=47

$$\frac{a(A + B) \sin(c + dx)}{d} + \frac{1}{2}ax(A + 2B) + \frac{aA \sin(c + dx) \cos(c + dx)}{2d}$$

[Out] 1/2*a*(A+2*B)*x+a*(A+B)*sin(d*x+c)/d+1/2*a*A*cos(d*x+c)*sin(d*x+c)/d

Rubi [A] time = 0.09, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {3996, 3787, 2637, 8}

$$\frac{a(A + B) \sin(c + dx)}{d} + \frac{1}{2}ax(A + 2B) + \frac{aA \sin(c + dx) \cos(c + dx)}{2d}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^2*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]

[Out] (a*(A + 2*B)*x)/2 + (a*(A + B)*Sin[c + d*x])/d + (a*A*Cos[c + d*x]*Sin[c + d*x])/(2*d)

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_)])*(d_.)^(n_.)*(csc[(e_.) + (f_.)*(x_)])*(b_.) + (a_.), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 3996

Int[(csc[(e_.) + (f_.)*(x_)])*(d_.)^(n_.)*(csc[(e_.) + (f_.)*(x_)])*(b_.) + (a_.)*(csc[(e_.) + (f_.)*(x_)])*(B_.) + (A_.), x_Symbol] := Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x] /

; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]

Rubi steps

$$\begin{aligned} \int \cos^2(c + dx)(a + a \sec(c + dx))(A + B \sec(c + dx)) dx &= \frac{aA \cos(c + dx) \sin(c + dx)}{2d} - \frac{1}{2} \int \cos(c + dx)(-2a(A + B \sec(c + dx))) dx \\ &= \frac{aA \cos(c + dx) \sin(c + dx)}{2d} + (a(A + B)) \int \cos(c + dx) dx \\ &= \frac{1}{2}a(A + 2B)x + \frac{a(A + B) \sin(c + dx)}{d} + \frac{aA \cos(c + dx)}{2d} \end{aligned}$$

Mathematica [A] time = 0.10, size = 44, normalized size = 0.94

$$\frac{a(4(A + B) \sin(c + dx) + A \sin(2(c + dx)) + 2Ac + 2Adx + 4Bdx)}{4d}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]^2*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]

[Out] (a*(2*A*c + 2*A*d*x + 4*B*d*x + 4*(A + B)*Sin[c + d*x] + A*Ssin[2*(c + d*x)]))/(4*d)

fricas [A] time = 0.42, size = 38, normalized size = 0.81

$$\frac{(A + 2B)adx + (Aa \cos(dx + c) + 2(A + B)a) \sin(dx + c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] 1/2*((A + 2*B)*a*d*x + (A*a*cos(d*x + c) + 2*(A + B)*a)*sin(d*x + c))/d

giac [B] time = 0.23, size = 93, normalized size = 1.98

$$\frac{(Aa + 2Ba)(dx + c) + \frac{2\left(Aa \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 + 2Ba \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 + 3Aa \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 2Ba \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] $\frac{1}{2} * ((A*a + 2*B*a) * (d*x + c) + 2 * (A*a * \tan(1/2*d*x + 1/2*c)^3 + 2*B*a * \tan(1/2*d*x + 1/2*c)^3 + 3*A*a * \tan(1/2*d*x + 1/2*c) + 2*B*a * \tan(1/2*d*x + 1/2*c))) / (\tan(1/2*d*x + 1/2*c)^2 + 1)^2 / d$

maple [A] time = 0.76, size = 57, normalized size = 1.21

$$\frac{aA \left(\frac{\cos(dx+c) \sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2} \right) + aA \sin(dx+c) + aB \sin(dx+c) + B(dx+c)a}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^2*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x)

[Out] $\frac{1}{d} * (a*A*(1/2*\cos(d*x+c)*\sin(d*x+c)+1/2*d*x+1/2*c)+a*A*\sin(d*x+c)+a*B*\sin(d*x+c)+B*(d*x+c)*a)$

maxima [A] time = 0.32, size = 55, normalized size = 1.17

$$\frac{(2 dx + 2 c + \sin(2 dx + 2 c))Aa + 4 (dx + c)Ba + 4 Aa \sin(dx + c) + 4 Ba \sin(dx + c)}{4 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] $\frac{1}{4} * ((2*d*x + 2*c + \sin(2*d*x + 2*c)) * A*a + 4 * (d*x + c) * B*a + 4 * A*a * \sin(d*x + c) + 4 * B*a * \sin(d*x + c)) / d$

mupad [B] time = 2.09, size = 50, normalized size = 1.06

$$\frac{A a x}{2} + B a x + \frac{A a \sin(c + d x)}{d} + \frac{B a \sin(c + d x)}{d} + \frac{A a \sin(2 c + 2 d x)}{4 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^2*(A + B/cos(c + d*x))*(a + a/cos(c + d*x)),x)

[Out] $(A*a*x)/2 + B*a*x + (A*a*\sin(c + d*x))/d + (B*a*\sin(c + d*x))/d + (A*a*\sin(2*c + 2*d*x))/(4*d)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a \left(\int A \cos^2(c + dx) dx + \int A \cos^2(c + dx) \sec(c + dx) dx + \int B \cos^2(c + dx) \sec(c + dx) dx + \int B \cos^2(c + dx) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] a*(Integral(A*cos(c + d*x)**2, x) + Integral(A*cos(c + d*x)**2*sec(c + d*x), x) + Integral(B*cos(c + d*x)**2*sec(c + d*x), x) + Integral(B*cos(c + d*x)**2*sec(c + d*x)**2, x))
```


3.50 $\int \cos^3(c + dx)(a + a \sec(c + dx))(A + B \sec(c + dx)) dx$

Optimal. Leaf size=77

$$\frac{a(2A + 3B) \sin(c + dx)}{3d} + \frac{a(A + B) \sin(c + dx) \cos(c + dx)}{2d} + \frac{1}{2}ax(A+B) + \frac{aA \sin(c + dx) \cos^2(c + dx)}{3d}$$

[Out] $\frac{1}{2}a*(A+B)*x + \frac{1}{3}a*(2*A+3*B)*\sin(d*x+c)/d + \frac{1}{2}a*(A+B)*\cos(d*x+c)*\sin(d*x+c)/d + \frac{1}{3}a*A*\cos(d*x+c)^2*\sin(d*x+c)/d$

Rubi [A] time = 0.11, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {3996, 3787, 2635, 8, 2637}

$$\frac{a(2A + 3B) \sin(c + dx)}{3d} + \frac{a(A + B) \sin(c + dx) \cos(c + dx)}{2d} + \frac{1}{2}ax(A+B) + \frac{aA \sin(c + dx) \cos^2(c + dx)}{3d}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^3*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]

[Out] $(a*(A + B)*x)/2 + (a*(2*A + 3*B)*\sin[c + d*x])/(3*d) + (a*(A + B)*\cos[c + d*x]*\sin[c + d*x])/(2*d) + (a*A*\cos[c + d*x]^2*\sin[c + d*x])/(3*d)$

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x])*(b*SIN[c + d*x])^(n - 1)/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*SIN[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[SIN[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_)])*(d_.)^(n_.)*(csc[(e_.) + (f_.)*(x_)])*(b_.) + (a_.), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[

$(d \cdot \text{Csc}[e + f \cdot x])^{(n + 1)}, x], x] /; \text{FreeQ}[\{a, b, d, e, f, n\}, x]$

Rule 3996

$\text{Int}[(\text{csc}[(e_.) + (f_.) \cdot (x_.)] \cdot (d_.))^{(n_.)} \cdot (\text{csc}[(e_.) + (f_.) \cdot (x_.)] \cdot (b_.) + (a_.)) \cdot (\text{csc}[(e_.) + (f_.) \cdot (x_.)] \cdot (B_.) + (A_.)), x_Symbol] :> \text{Simp}[(A \cdot a \cdot \text{Cot}[e + f \cdot x] \cdot (d \cdot \text{Csc}[e + f \cdot x])^n) / (f \cdot n), x] + \text{Dist}[1 / (d \cdot n), \text{Int}[(d \cdot \text{Csc}[e + f \cdot x])^{(n + 1)} \cdot \text{Simp}[n \cdot (B \cdot a + A \cdot b) + (B \cdot b \cdot n + A \cdot a \cdot (n + 1)) \cdot \text{Csc}[e + f \cdot x], x], x], x] /; \text{FreeQ}[\{a, b, d, e, f, A, B\}, x] \&\& \text{NeQ}[A \cdot b - a \cdot B, 0] \&\& \text{LeQ}[n, -1]$

Rubi steps

$$\begin{aligned} \int \cos^3(c + dx)(a + a \sec(c + dx))(A + B \sec(c + dx)) dx &= \frac{aA \cos^2(c + dx) \sin(c + dx)}{3d} - \frac{1}{3} \int \cos^2(c + dx)(-3a + a \sec(c + dx)) dx \\ &= \frac{aA \cos^2(c + dx) \sin(c + dx)}{3d} + (a(A + B)) \int \cos^2(c + dx) dx \\ &= \frac{a(2A + 3B) \sin(c + dx)}{3d} + \frac{a(A + B) \cos(c + dx) \sin(c + dx)}{2d} \\ &= \frac{1}{2} a(A + B)x + \frac{a(2A + 3B) \sin(c + dx)}{3d} + \frac{a(A + B) \cos(c + dx) \sin(c + dx)}{2d} \end{aligned}$$

Mathematica [A] time = 0.18, size = 65, normalized size = 0.84

$$\frac{a(3(3A + 4B) \sin(c + dx) + 3(A + B) \sin(2(c + dx)) + A \sin(3(c + dx)) + 6Ac + 6Adx + 6Bc + 6Bdx)}{12d}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]^3*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]

[Out] (a*(6*A*c + 6*B*c + 6*A*d*x + 6*B*d*x + 3*(3*A + 4*B)*Sin[c + d*x] + 3*(A + B)*Sin[2*(c + d*x)] + A*Ssin[3*(c + d*x)]))/(12*d)

fricas [A] time = 0.45, size = 56, normalized size = 0.73

$$\frac{3(A + B)adx + (2Aa \cos(dx + c)^2 + 3(A + B)a \cos(dx + c) + 2(2A + 3B)a) \sin(dx + c)}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] $1/6*(3*(A + B)*a*d*x + (2*A*a*cos(d*x + c)^2 + 3*(A + B)*a*cos(d*x + c) + 2*(2*A + 3*B)*a)*sin(d*x + c))/d$

giac [A] time = 0.31, size = 124, normalized size = 1.61

$$3(Aa + Ba)(dx + c) + \frac{2\left(3Aa \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5 + 3Ba \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5 + 4Aa \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 + 12Ba \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 + 9Aa \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 9Ba \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + 1\right)^3}$$

$$6d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="giac")`

[Out] $1/6*(3*(A*a + B*a)*(d*x + c) + 2*(3*A*a*tan(1/2*d*x + 1/2*c)^5 + 3*B*a*tan(1/2*d*x + 1/2*c)^5 + 4*A*a*tan(1/2*d*x + 1/2*c)^3 + 12*B*a*tan(1/2*d*x + 1/2*c)^3 + 9*A*a*tan(1/2*d*x + 1/2*c) + 9*B*a*tan(1/2*d*x + 1/2*c)))/(tan(1/2*d*x + 1/2*c)^2 + 1)^3/d$

maple [A] time = 1.13, size = 85, normalized size = 1.10

$$\frac{aA(2+\cos^2(dx+c))\sin(dx+c)}{3} + aA\left(\frac{\cos(dx+c)\sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2}\right) + aB\left(\frac{\cos(dx+c)\sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2}\right) + aB\sin(dx+c)$$

$$d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(d*x+c)^3*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x)`

[Out] $1/d*(1/3*a*A*(2+\cos(d*x+c)^2)*\sin(d*x+c)+a*A*(1/2*\cos(d*x+c)*\sin(d*x+c)+1/2*d*x+1/2*c)+a*B*(1/2*\cos(d*x+c)*\sin(d*x+c)+1/2*d*x+1/2*c)+a*B*\sin(d*x+c))$

maxima [A] time = 0.32, size = 79, normalized size = 1.03

$$\frac{4(\sin(dx+c)^3 - 3\sin(dx+c))Aa - 3(2dx+2c+\sin(2dx+2c))Aa - 3(2dx+2c+\sin(2dx+2c))Ba}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="maxima")`

[Out] $-1/12*(4*(\sin(d*x + c)^3 - 3*\sin(d*x + c))*A*a - 3*(2*d*x + 2*c + \sin(2*d*x + 2*c))*A*a - 3*(2*d*x + 2*c + \sin(2*d*x + 2*c))*B*a - 12*B*a*\sin(d*x + c))/d$

mupad [B] time = 2.10, size = 84, normalized size = 1.09

$$\frac{Aax}{2} + \frac{Bax}{2} + \frac{3Aa \sin(c+dx)}{4d} + \frac{Ba \sin(c+dx)}{d} + \frac{Aa \sin(2c+2dx)}{4d} + \frac{Aa \sin(3c+3dx)}{12d} + \frac{Ba \sin(2c+2dx)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(c + d*x)^3*(A + B/cos(c + d*x))*(a + a/cos(c + d*x)),x)`

[Out] $(A*a*x)/2 + (B*a*x)/2 + (3*A*a*\sin(c + d*x))/(4*d) + (B*a*\sin(c + d*x))/d + (A*a*\sin(2*c + 2*d*x))/(4*d) + (A*a*\sin(3*c + 3*d*x))/(12*d) + (B*a*\sin(2*c + 2*d*x))/(4*d)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a \left(\int A \cos^3(c+dx) dx + \int A \cos^3(c+dx) \sec(c+dx) dx + \int B \cos^3(c+dx) \sec(c+dx) dx + \int B \cos^3(c+dx) \sec^2(c+dx) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)**3*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x)`

[Out] `a*(Integral(A*cos(c + d*x)**3, x) + Integral(A*cos(c + d*x)**3*sec(c + d*x), x) + Integral(B*cos(c + d*x)**3*sec(c + d*x), x) + Integral(B*cos(c + d*x)**3*sec(c + d*x)**2, x))`

3.51 $\int \cos^4(c + dx)(a + a \sec(c + dx))(A + B \sec(c + dx)) dx$

Optimal. Leaf size=97

$$-\frac{a(A+B)\sin^3(c+dx)}{3d} + \frac{a(A+B)\sin(c+dx)}{d} + \frac{a(3A+4B)\sin(c+dx)\cos(c+dx)}{8d} + \frac{1}{8}ax(3A+4B) + \frac{aA\sin(c+dx)}{d}$$

[Out] $1/8*a*(3*A+4*B)*x+a*(A+B)*\sin(d*x+c)/d+1/8*a*(3*A+4*B)*\cos(d*x+c)*\sin(d*x+c)/d+1/4*a*A*\cos(d*x+c)^3*\sin(d*x+c)/d-1/3*a*(A+B)*\sin(d*x+c)^3/d$

Rubi [A] time = 0.12, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {3996, 3787, 2633, 2635, 8}

$$-\frac{a(A+B)\sin^3(c+dx)}{3d} + \frac{a(A+B)\sin(c+dx)}{d} + \frac{a(3A+4B)\sin(c+dx)\cos(c+dx)}{8d} + \frac{1}{8}ax(3A+4B) + \frac{aA\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

[In] `Int[Cos[c + d*x]^4*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]`

[Out] $(a*(3*A + 4*B)*x)/8 + (a*(A + B)*\sin[c + d*x])/d + (a*(3*A + 4*B)*\cos[c + d*x]*\sin[c + d*x])/(8*d) + (a*A*\cos[c + d*x]^3*\sin[c + d*x])/(4*d) - (a*(A + B)*\sin[c + d*x]^3)/(3*d)$

Rule 8

`Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

Rule 2633

`Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]`

Rule 2635

`Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[(b*cos[c + d*x])*(b*sin[c + d*x])^(n - 1)/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]`

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_.)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_.))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] :> Simp[(A*a*Cot[e +
f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
+ 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rubi steps

$$\begin{aligned} \int \cos^4(c + dx)(a + a \sec(c + dx))(A + B \sec(c + dx)) dx &= \frac{aA \cos^3(c + dx) \sin(c + dx)}{4d} - \frac{1}{4} \int \cos^3(c + dx)(-4a + B \sec(c + dx)) dx \\ &= \frac{aA \cos^3(c + dx) \sin(c + dx)}{4d} + (a(A + B)) \int \cos^3(c + dx) dx \\ &= \frac{a(3A + 4B) \cos(c + dx) \sin(c + dx)}{8d} + \frac{aA \cos^3(c + dx)}{4d} \\ &= \frac{1}{8} a(3A + 4B)x + \frac{a(A + B) \sin(c + dx)}{d} + \frac{a(3A + 4B)}{8d} \end{aligned}$$

Mathematica [A] time = 0.24, size = 75, normalized size = 0.77

$$\frac{a(-32(A + B) \sin^3(c + dx) + 96(A + B) \sin(c + dx) + 24(A + B) \sin(2(c + dx)) + 3A \sin(4(c + dx)) + 36Ac + 36Bc)}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a*(36*A*c + 48*B*c + 36*A*d*x + 48*B*d*x + 96*(A + B)*Sin[c + d*x] - 32*(A
+ B)*Sin[c + d*x]^3 + 24*(A + B)*Sin[2*(c + d*x)] + 3*A*Ssin[4*(c + d*x)]))
/(96*d)
```

fricas [A] time = 0.50, size = 74, normalized size = 0.76

$$\frac{3(3A + 4B)adx + (6Aa \cos(dx + c)^3 + 8(A + B)a \cos(dx + c)^2 + 3(3A + 4B)a \cos(dx + c) + 16(A + B)a) \sin(dx + c)}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] $\frac{1}{24}*(3*(3*A + 4*B)*a*d*x + (6*A*a*\cos(d*x + c)^3 + 8*(A + B)*a*\cos(d*x + c)^2 + 3*(3*A + 4*B)*a*\cos(d*x + c) + 16*(A + B)*a)*\sin(d*x + c))/d$

giac [A] time = 0.47, size = 156, normalized size = 1.61

$$\frac{3(3Aa + 4Ba)(dx + c) + \frac{2\left(9Aa \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^7 + 12Ba \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^7 + 49Aa \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5 + 28Ba \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5 + 31Aa \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 + 16Ba \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 + 9Aa \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 16Ba \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + 1\right)^4}}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] $\frac{1}{24}*(3*(3*A*a + 4*B*a)*(d*x + c) + 2*(9*A*a*\tan(1/2*d*x + 1/2*c)^7 + 12*B*a*\tan(1/2*d*x + 1/2*c)^7 + 49*A*a*\tan(1/2*d*x + 1/2*c)^5 + 28*B*a*\tan(1/2*d*x + 1/2*c)^5 + 31*A*a*\tan(1/2*d*x + 1/2*c)^3 + 52*B*a*\tan(1/2*d*x + 1/2*c)^3 + 39*A*a*\tan(1/2*d*x + 1/2*c) + 36*B*a*\tan(1/2*d*x + 1/2*c)))/(\tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d$

maple [A] time = 1.29, size = 107, normalized size = 1.10

$$\frac{aA\left(\frac{\left(\cos^3(dx+c) + \frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4} + \frac{3dx}{8} + \frac{3c}{8}\right) + \frac{aA(2+\cos^2(dx+c))\sin(dx+c)}{3} + \frac{aB(2+\cos^2(dx+c))\sin(dx+c)}{3} + aB\left(\frac{\cos(dx+c)\sin(dx+c)}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^4*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x)

[Out] $\frac{1}{d}*(a*A*(1/4*(\cos(d*x+c)^3 + 3/2*\cos(d*x+c))*\sin(d*x+c) + 3/8*d*x + 3/8*c) + 1/3*a*A*(2+\cos(d*x+c)^2)*\sin(d*x+c) + 1/3*a*B*(2+\cos(d*x+c)^2)*\sin(d*x+c) + a*B*(1/2*\cos(d*x+c)*\sin(d*x+c) + 1/2*d*x + 1/2*c))$

maxima [A] time = 0.34, size = 101, normalized size = 1.04

$$\frac{32\left(\sin(dx+c)^3 - 3\sin(dx+c)\right)Aa - 3(12dx + 12c + \sin(4dx + 4c) + 8\sin(2dx + 2c))Aa + 32\left(\sin(dx+c)\cos(dx+c) + \frac{1}{2}dx + \frac{1}{2}c\right)aB}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] $-1/96*(32*(\sin(d*x + c))^3 - 3*\sin(d*x + c))*A*a - 3*(12*d*x + 12*c + \sin(4*d*x + 4*c) + 8*\sin(2*d*x + 2*c))*A*a + 32*(\sin(d*x + c))^3 - 3*\sin(d*x + c))$
 $*B*a - 24*(2*d*x + 2*c + \sin(2*d*x + 2*c))*B*a)/d$

mupad [B] time = 4.67, size = 184, normalized size = 1.90

$$\frac{\left(\frac{3Aa}{4} + Ba\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^7 + \left(\frac{49Aa}{12} + \frac{7Ba}{3}\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5 + \left(\frac{31Aa}{12} + \frac{13Ba}{3}\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 + \left(\frac{13Aa}{4} + 3Ba\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^8 + 4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^6 + 6 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 + 4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + 1 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(c + d*x)^4*(A + B/cos(c + d*x))*(a + a/cos(c + d*x)),x)`

[Out] $(\tan(c/2 + (d*x)/2)*((13*A*a)/4 + 3*B*a) + \tan(c/2 + (d*x)/2)^7*((3*A*a)/4 + B*a) + \tan(c/2 + (d*x)/2)^3*((31*A*a)/12 + (13*B*a)/3) + \tan(c/2 + (d*x)/2)^5*((49*A*a)/12 + (7*B*a)/3))/(d*(4*\tan(c/2 + (d*x)/2)^2 + 6*\tan(c/2 + (d*x)/2)^4 + 4*\tan(c/2 + (d*x)/2)^6 + \tan(c/2 + (d*x)/2)^8 + 1)) + (a*\operatorname{atan}((a*\tan(c/2 + (d*x)/2)*(3*A + 4*B))/(4*((3*A*a)/4 + B*a)))*(3*A + 4*B))/(4*d)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a \left(\int A \cos^4(c + dx) dx + \int A \cos^4(c + dx) \sec(c + dx) dx + \int B \cos^4(c + dx) \sec(c + dx) dx + \int B \cos^4(c + dx) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)**4*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x)`

[Out] $a*(\operatorname{Integral}(A*\cos(c + d*x)**4, x) + \operatorname{Integral}(A*\cos(c + d*x)**4*\sec(c + d*x), x) + \operatorname{Integral}(B*\cos(c + d*x)**4*\sec(c + d*x), x) + \operatorname{Integral}(B*\cos(c + d*x)**4*\sec(c + d*x)**2, x))$

3.52 $\int \cos^5(c + dx)(a + a \sec(c + dx))(A + B \sec(c + dx)) dx$

Optimal. Leaf size=125

$$-\frac{a(4A + 5B) \sin^3(c + dx)}{15d} + \frac{a(4A + 5B) \sin(c + dx)}{5d} + \frac{a(A + B) \sin(c + dx) \cos^3(c + dx)}{4d} + \frac{3a(A + B) \sin(c + dx)}{8d}$$

[Out] $3/8*a*(A+B)*x+1/5*a*(4*A+5*B)*\sin(d*x+c)/d+3/8*a*(A+B)*\cos(d*x+c)*\sin(d*x+c)/d+1/4*a*(A+B)*\cos(d*x+c)^3*\sin(d*x+c)/d+1/5*a*A*\cos(d*x+c)^4*\sin(d*x+c)/d-1/15*a*(4*A+5*B)*\sin(d*x+c)^3/d$

Rubi [A] time = 0.13, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {3996, 3787, 2635, 8, 2633}

$$-\frac{a(4A + 5B) \sin^3(c + dx)}{15d} + \frac{a(4A + 5B) \sin(c + dx)}{5d} + \frac{a(A + B) \sin(c + dx) \cos^3(c + dx)}{4d} + \frac{3a(A + B) \sin(c + dx)}{8d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Cos}[c + d*x]^5*(a + a*\text{Sec}[c + d*x])*(A + B*\text{Sec}[c + d*x]), x]$

[Out] $(3*a*(A + B)*x)/8 + (a*(4*A + 5*B)*\text{Sin}[c + d*x])/(5*d) + (3*a*(A + B)*\text{Cos}[c + d*x]*\text{Sin}[c + d*x])/(8*d) + (a*(A + B)*\text{Cos}[c + d*x]^3*\text{Sin}[c + d*x])/(4*d) + (a*A*\text{Cos}[c + d*x]^4*\text{Sin}[c + d*x])/(5*d) - (a*(4*A + 5*B)*\text{Sin}[c + d*x]^3)/(15*d)$

Rule 8

$\text{Int}[a_, x_Symbol] \rightarrow \text{Simp}[a*x, x] /; \text{FreeQ}[a, x]$

Rule 2633

$\text{Int}[\sin[(c_.) + (d_.)*(x_.)]^{(n_.)}, x_Symbol] \rightarrow -\text{Dist}[d^{(-1)}, \text{Subst}[\text{Int}[\text{Expand}[(1 - x^2)^{((n - 1)/2)}, x], x], x, \text{Cos}[c + d*x]], x] /; \text{FreeQ}\{c, d\}, x \&\& \text{IGtQ}[(n - 1)/2, 0]$

Rule 2635

$\text{Int}[(b_.)*\sin[(c_.) + (d_.)*(x_.)]^{(n_.)}, x_Symbol] \rightarrow -\text{Simp}[(b*\text{Cos}[c + d*x])*(b*\text{Sin}[c + d*x])^{(n - 1)}]/(d*n), x] + \text{Dist}[(b^2*(n - 1))/n, \text{Int}[(b*\text{Sin}[c + d*x])^{(n - 2)}, x], x] /; \text{FreeQ}\{b, c, d\}, x \&\& \text{GtQ}[n, 1] \&\& \text{IntegerQ}[2*n]$

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) +
(a_.)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) +
(a_.))*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> Simp[(A*a*Cot[e +
f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
+ 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \cos^5(c + dx)(a + a \sec(c + dx))(A + B \sec(c + dx)) dx &= \frac{aA \cos^4(c + dx) \sin(c + dx)}{5d} - \frac{1}{5} \int \cos^4(c + dx)(-5a + a \sec(c + dx)) dx \\
&= \frac{aA \cos^4(c + dx) \sin(c + dx)}{5d} + (a(A + B)) \int \cos^4(c + dx) dx \\
&= \frac{a(A + B) \cos^3(c + dx) \sin(c + dx)}{4d} + \frac{aA \cos^4(c + dx)}{5d} \\
&= \frac{a(4A + 5B) \sin(c + dx)}{5d} + \frac{3a(A + B) \cos(c + dx) \sin(c + dx)}{8d} \\
&= \frac{3}{8} a(A + B)x + \frac{a(4A + 5B) \sin(c + dx)}{5d} + \frac{3a(A + B) \cos(c + dx) \sin(c + dx)}{8d}
\end{aligned}$$

Mathematica [A] time = 0.25, size = 77, normalized size = 0.62

$$\frac{a(-160(2A + B) \sin^3(c + dx) + 480(A + B) \sin(c + dx) + 15(A + B)(12(c + dx) + 8 \sin(2(c + dx))) + \sin(4(c + dx)))}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]), x]
```

```
[Out] (a*(480*(A + B)*Sin[c + d*x] - 160*(2*A + B)*Sin[c + d*x]^3 + 96*A*SIN[c +
d*x]^5 + 15*(A + B)*(12*(c + d*x) + 8*SIN[2*(c + d*x)]) + SIN[4*(c + d*x)])
)/(480*d)
```

fricas [A] time = 0.43, size = 88, normalized size = 0.70

$$\frac{45(A + B)adx + (24Aa \cos(dx + c)^4 + 30(A + B)a \cos(dx + c)^3 + 8(4A + 5B)a \cos(dx + c)^2 + 45(A + B)a \cos(dx + c))}{120d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] 1/120*(45*(A + B)*a*d*x + (24*A*a*cos(d*x + c)^4 + 30*(A + B)*a*cos(d*x + c)^3 + 8*(4*A + 5*B)*a*cos(d*x + c)^2 + 45*(A + B)*a*cos(d*x + c) + 16*(4*A + 5*B)*a)*sin(d*x + c))/d

giac [A] time = 0.26, size = 184, normalized size = 1.47

$$45(Aa + Ba)(dx + c) + \frac{2\left(45Aa \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^9 + 45Ba \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^9 + 130Aa \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^7 + 290Ba \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^7 + 464Aa \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5 + 464Ba \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5 + 190Aa \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 + 350Ba \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 + 195Aa \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 195Ba \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right)}{120d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] 1/120*(45*(A*a + B*a)*(d*x + c) + 2*(45*A*a*tan(1/2*d*x + 1/2*c)^9 + 45*B*a*tan(1/2*d*x + 1/2*c)^9 + 130*A*a*tan(1/2*d*x + 1/2*c)^7 + 290*B*a*tan(1/2*d*x + 1/2*c)^7 + 464*A*a*tan(1/2*d*x + 1/2*c)^5 + 400*B*a*tan(1/2*d*x + 1/2*c)^5 + 190*A*a*tan(1/2*d*x + 1/2*c)^3 + 350*B*a*tan(1/2*d*x + 1/2*c)^3 + 195*A*a*tan(1/2*d*x + 1/2*c) + 195*B*a*tan(1/2*d*x + 1/2*c)))/(tan(1/2*d*x + 1/2*c)^2 + 1)^5)/d

maple [A] time = 1.60, size = 128, normalized size = 1.02

$$\frac{aA\left(\frac{8}{3} + \cos^4(dx+c) + \frac{4(\cos^2(dx+c))}{3}\right)\sin(dx+c)}{5} + aA\left(\frac{(\cos^3(dx+c) + \frac{3\cos(dx+c)}{2})\sin(dx+c)}{4} + \frac{3dx}{8} + \frac{3c}{8}\right) + aB\left(\frac{(\cos^3(dx+c) + \frac{3\cos(dx+c)}{2})\sin(dx+c)}{4}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^5*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x)

[Out] 1/d*(1/5*a*A*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+a*A*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+a*B*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3*a*B*(2+cos(d*x+c)^2)*sin(d*x+c))

maxima [A] time = 0.34, size = 124, normalized size = 0.99

$$\frac{32\left(3\sin(dx+c)^5 - 10\sin(dx+c)^3 + 15\sin(dx+c)\right)Aa + 15(12dx + 12c + \sin(4dx + 4c) + 8\sin(2dx + 2c))Ba}{480d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="maxima")`

[Out] $\frac{1}{480}*(32*(3*\sin(d*x + c)^5 - 10*\sin(d*x + c)^3 + 15*\sin(d*x + c))*A*a + 15*(12*d*x + 12*c + \sin(4*d*x + 4*c) + 8*\sin(2*d*x + 2*c))*A*a - 160*(\sin(d*x + c)^3 - 3*\sin(d*x + c))*B*a + 15*(12*d*x + 12*c + \sin(4*d*x + 4*c) + 8*\sin(2*d*x + 2*c))*B*a)/d$

mupad [B] time = 4.81, size = 212, normalized size = 1.70

$$\frac{\left(\frac{3Aa}{4} + \frac{3Ba}{4}\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^9 + \left(\frac{13Aa}{6} + \frac{29Ba}{6}\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^7 + \left(\frac{116Aa}{15} + \frac{20Ba}{3}\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5 + \left(\frac{19Aa}{6} + \frac{35Ba}{6}\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 + \left(\frac{3Aa}{4} + \frac{3Ba}{4}\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^{10} + 5 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^8 + 10 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^6 + 10 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 + 5 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + 1 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(c + d*x)^5*(A + B/cos(c + d*x))*(a + a/cos(c + d*x)),x)`

[Out] $(\tan(c/2 + (d*x)/2)*((13*A*a)/4 + (13*B*a)/4) + \tan(c/2 + (d*x)/2)^9*((3*A*a)/4 + (3*B*a)/4) + \tan(c/2 + (d*x)/2)^7*((13*A*a)/6 + (29*B*a)/6) + \tan(c/2 + (d*x)/2)^5*((19*A*a)/6 + (35*B*a)/6) + \tan(c/2 + (d*x)/2)^3*((116*A*a)/15 + (20*B*a)/3))/(d*(5*\tan(c/2 + (d*x)/2)^2 + 10*\tan(c/2 + (d*x)/2)^4 + 10*\tan(c/2 + (d*x)/2)^6 + 5*\tan(c/2 + (d*x)/2)^8 + \tan(c/2 + (d*x)/2)^{10} + 1)) + (3*a*atan((3*a*\tan(c/2 + (d*x)/2)*(A + B))/(4*((3*A*a)/4 + (3*B*a)/4)))*(A + B))/(4*d)$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)**5*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x)`

[Out] Timed out

3.53 $\int \sec^3(c + dx)(a + a \sec(c + dx))^2(A + B \sec(c + dx)) dx$

Optimal. Leaf size=169

$$\frac{a^2(10A + 9B) \tan^3(c + dx)}{15d} + \frac{a^2(10A + 9B) \tan(c + dx)}{5d} + \frac{a^2(7A + 6B) \tanh^{-1}(\sin(c + dx))}{8d} + \frac{a^2(5A + 6B) \tan(c + dx)}{20d}$$

[Out] $1/8*a^2*(7*A+6*B)*\operatorname{arctanh}(\sin(d*x+c))/d+1/5*a^2*(10*A+9*B)*\tan(d*x+c)/d+1/8*a^2*(7*A+6*B)*\sec(d*x+c)*\tan(d*x+c)/d+1/20*a^2*(5*A+6*B)*\sec(d*x+c)^3*\tan(d*x+c)/d+1/5*B*\sec(d*x+c)^3*(a^2+a^2*\sec(d*x+c))*\tan(d*x+c)/d+1/15*a^2*(10*A+9*B)*\tan(d*x+c)^3/d$

Rubi [A] time = 0.24, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4018, 3997, 3787, 3768, 3770, 3767}

$$\frac{a^2(10A + 9B) \tan^3(c + dx)}{15d} + \frac{a^2(10A + 9B) \tan(c + dx)}{5d} + \frac{a^2(7A + 6B) \tanh^{-1}(\sin(c + dx))}{8d} + \frac{a^2(5A + 6B) \tan(c + dx)}{20d}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Sec}[c + d*x]^3*(a + a*\operatorname{Sec}[c + d*x])^2*(A + B*\operatorname{Sec}[c + d*x]), x]$

[Out] $(a^2*(7*A + 6*B)*\operatorname{ArcTanh}[\operatorname{Sin}[c + d*x]])/(8*d) + (a^2*(10*A + 9*B)*\operatorname{Tan}[c + d*x])/(5*d) + (a^2*(7*A + 6*B)*\operatorname{Sec}[c + d*x]*\operatorname{Tan}[c + d*x])/(8*d) + (a^2*(5*A + 6*B)*\operatorname{Sec}[c + d*x]^3*\operatorname{Tan}[c + d*x])/(20*d) + (B*\operatorname{Sec}[c + d*x]^3*(a^2 + a^2*\operatorname{Sec}[c + d*x])*\operatorname{Tan}[c + d*x])/(5*d) + (a^2*(10*A + 9*B)*\operatorname{Tan}[c + d*x]^3)/(15*d)$

Rule 3767

$\operatorname{Int}[\operatorname{csc}[(c_.) + (d_.)*(x_.)]^{(n_.)}, x_Symbol] \rightarrow -\operatorname{Dist}[d^{(-1)}, \operatorname{Subst}[\operatorname{Int}[\operatorname{ExpandIntegrand}[(1 + x^2)^{(n/2 - 1)}, x], x], x, \operatorname{Cot}[c + d*x]], x] /;$ $\operatorname{FreeQ}\{c, d\}, x \&\& \operatorname{IGtQ}[n/2, 0]$

Rule 3768

$\operatorname{Int}[(\operatorname{csc}[(c_.) + (d_.)*(x_.)]*(b_.))^{(n_.)}, x_Symbol] \rightarrow -\operatorname{Simp}[(b*\operatorname{Cos}[c + d*x])*(b*\operatorname{Csc}[c + d*x])^{(n - 1)})/(d*(n - 1)), x] + \operatorname{Dist}[(b^2*(n - 2))/(n - 1), \operatorname{Int}[(b*\operatorname{Csc}[c + d*x])^{(n - 2)}, x], x] /;$ $\operatorname{FreeQ}\{b, c, d\}, x \&\& \operatorname{GtQ}[n, 1] \&\& \operatorname{IntegerQ}[2*n]$

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_)])*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)])*(b_.) +
 (a_), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
 (d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 3997

Int[(csc[(e_.) + (f_.)*(x_)])*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)])*(b_.) +
 (a_))*(csc[(e_.) + (f_.)*(x_)])*(B_.) + (A_)), x_Symbol] := -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
 + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
 x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && !LeQ[n,
 -1]

Rule 4018

Int[(csc[(e_.) + (f_.)*(x_)])*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)])*(b_.) +
 (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)])*(B_.) + (A_)), x_Symbol] := -Simp[(b*B*C
 ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
 *Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
 [e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
 B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && !LtQ[n, -1]

Rubi steps

$$\begin{aligned}
 \int \sec^3(c + dx)(a + a \sec(c + dx))^2(A + B \sec(c + dx)) dx &= \frac{B \sec^3(c + dx) (a^2 + a^2 \sec(c + dx)) \tan(c + dx)}{5d} + \frac{1}{5} \\
 &= \frac{a^2(5A + 6B) \sec^3(c + dx) \tan(c + dx)}{20d} + \frac{B \sec^3(c + dx)}{5d} \\
 &= \frac{a^2(5A + 6B) \sec^3(c + dx) \tan(c + dx)}{20d} + \frac{B \sec^3(c + dx)}{5d} \\
 &= \frac{a^2(7A + 6B) \sec(c + dx) \tan(c + dx)}{8d} + \frac{a^2(5A + 6B)}{5d} \\
 &= \frac{a^2(7A + 6B) \tanh^{-1}(\sin(c + dx))}{8d} + \frac{a^2(10A + 9B) \tan(c + dx)}{5d}
 \end{aligned}$$

Mathematica [A] time = 1.40, size = 280, normalized size = 1.66

$$a^2(\cos(c + dx) + 1)^2 \sec^4\left(\frac{1}{2}(c + dx)\right) \sec^5(c + dx) \left(240(7A + 6B) \cos^5(c + dx) \left(\log\left(\cos\left(\frac{1}{2}(c + dx)\right) - \sin\left(\frac{1}{2}(c + dx)\right)\right)\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]

[Out] $-1/7680*(a^2*(1 + \text{Cos}[c + d*x])^2*\text{Sec}[(c + d*x)/2]^4*\text{Sec}[c + d*x]^5*(240*(7*A + 6*B)*\text{Cos}[c + d*x]^5*(\text{Log}[\text{Cos}[(c + d*x)/2] - \text{Sin}[(c + d*x)/2]] - \text{Log}[\text{Cos}[(c + d*x)/2] + \text{Sin}[(c + d*x)/2]]) - \text{Sec}[c]*(80*(14*A + 15*B)*\text{Sin}[d*x] - 240*(2*A + B)*\text{Sin}[2*c + d*x] + 330*A*\text{Sin}[c + 2*d*x] + 420*B*\text{Sin}[c + 2*d*x] + 330*A*\text{Sin}[3*c + 2*d*x] + 420*B*\text{Sin}[3*c + 2*d*x] + 800*A*\text{Sin}[2*c + 3*d*x] + 720*B*\text{Sin}[2*c + 3*d*x] + 105*A*\text{Sin}[3*c + 4*d*x] + 90*B*\text{Sin}[3*c + 4*d*x] + 105*A*\text{Sin}[5*c + 4*d*x] + 90*B*\text{Sin}[5*c + 4*d*x] + 160*A*\text{Sin}[4*c + 5*d*x] + 144*B*\text{Sin}[4*c + 5*d*x])))/d$

fricas [A] time = 0.47, size = 165, normalized size = 0.98

$$15(7A + 6B)a^2 \cos(dx + c)^5 \log(\sin(dx + c) + 1) - 15(7A + 6B)a^2 \cos(dx + c)^5 \log(-\sin(dx + c) + 1) + 2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] $1/240*(15*(7*A + 6*B)*a^2*\cos(d*x + c)^5*\log(\sin(d*x + c) + 1) - 15*(7*A + 6*B)*a^2*\cos(d*x + c)^5*\log(-\sin(d*x + c) + 1) + 2*(16*(10*A + 9*B)*a^2*\cos(d*x + c)^4 + 15*(7*A + 6*B)*a^2*\cos(d*x + c)^3 + 8*(10*A + 9*B)*a^2*\cos(d*x + c)^2 + 30*(A + 2*B)*a^2*\cos(d*x + c) + 24*B*a^2)*\sin(d*x + c))/(d*\cos(d*x + c)^5)$

giac [A] time = 0.73, size = 246, normalized size = 1.46

$$15(7Aa^2 + 6Ba^2) \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right|\right) - 15(7Aa^2 + 6Ba^2) \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1\right|\right) - \frac{2\left(105Aa^2 \tan\left(\frac{1}{2}c\right)\right)}{\cos^2\left(\frac{1}{2}c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] $\frac{1}{120} \cdot (15 \cdot (7 \cdot A \cdot a^2 + 6 \cdot B \cdot a^2) \cdot \log(\tan(\frac{1}{2}dx + \frac{1}{2}c) + 1)) - 15 \cdot (7 \cdot A \cdot a^2 + 6 \cdot B \cdot a^2) \cdot \log(\tan(\frac{1}{2}dx + \frac{1}{2}c) - 1) - 2 \cdot (105 \cdot A \cdot a^2 \cdot \tan(\frac{1}{2}dx + \frac{1}{2}c)^9 + 90 \cdot B \cdot a^2 \cdot \tan(\frac{1}{2}dx + \frac{1}{2}c)^9 - 490 \cdot A \cdot a^2 \cdot \tan(\frac{1}{2}dx + \frac{1}{2}c)^7 - 420 \cdot B \cdot a^2 \cdot \tan(\frac{1}{2}dx + \frac{1}{2}c)^7 + 800 \cdot A \cdot a^2 \cdot \tan(\frac{1}{2}dx + \frac{1}{2}c)^5 + 864 \cdot B \cdot a^2 \cdot \tan(\frac{1}{2}dx + \frac{1}{2}c)^5 - 790 \cdot A \cdot a^2 \cdot \tan(\frac{1}{2}dx + \frac{1}{2}c)^3 - 540 \cdot B \cdot a^2 \cdot \tan(\frac{1}{2}dx + \frac{1}{2}c)^3 + 375 \cdot A \cdot a^2 \cdot \tan(\frac{1}{2}dx + \frac{1}{2}c) + 390 \cdot B \cdot a^2 \cdot \tan(\frac{1}{2}dx + \frac{1}{2}c)) / (\tan(\frac{1}{2}dx + \frac{1}{2}c)^2 - 1)^5 / d$

maple [A] time = 1.40, size = 235, normalized size = 1.39

$$\frac{7a^2 A \sec(dx+c) \tan(dx+c)}{8d} + \frac{7a^2 A \ln(\sec(dx+c) + \tan(dx+c))}{8d} + \frac{6a^2 B \tan(dx+c)}{5d} + \frac{3a^2 B (\sec^2(dx+c)) \tan(dx+c)}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sec(d*x+c)^3*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x)`

[Out] $\frac{7}{8} \cdot \frac{1}{d \cdot a^2} \cdot A \cdot \sec(dx+c) \cdot \tan(dx+c) + \frac{7}{8} \cdot \frac{1}{d \cdot a^2} \cdot A \cdot \ln(\sec(dx+c) + \tan(dx+c)) + \frac{6}{5} \cdot \frac{1}{a^2} \cdot B \cdot \frac{\tan(dx+c)}{d} + \frac{3}{5} \cdot \frac{1}{a^2} \cdot B \cdot \sec(dx+c)^2 \cdot \frac{\tan(dx+c)}{d} + \frac{4}{3} \cdot \frac{1}{a^2} \cdot A \cdot \frac{\tan(dx+c)}{d} + \frac{2}{3} \cdot \frac{1}{d \cdot a^2} \cdot A \cdot \tan(dx+c) \cdot \sec(dx+c)^2 + \frac{1}{2} \cdot \frac{1}{a^2} \cdot B \cdot \sec(dx+c)^3 \cdot \frac{\tan(dx+c)}{d} + \frac{3}{4} \cdot \frac{1}{a^2} \cdot B \cdot \sec(dx+c) \cdot \frac{\tan(dx+c)}{d} + \frac{3}{4} \cdot \frac{1}{d \cdot B \cdot a^2} \cdot \ln(\sec(dx+c) + \tan(dx+c)) + \frac{1}{4} \cdot \frac{1}{d \cdot a^2} \cdot A \cdot \tan(dx+c) \cdot \sec(dx+c)^3 + \frac{1}{5} \cdot \frac{1}{d \cdot a^2} \cdot B \cdot \frac{\tan(dx+c)}{d} \cdot \sec(dx+c)^4$

maxima [A] time = 0.35, size = 278, normalized size = 1.64

$$160 \left(\tan(dx+c)^3 + 3 \tan(dx+c) \right) A a^2 + 16 \left(3 \tan(dx+c)^5 + 10 \tan(dx+c)^3 + 15 \tan(dx+c) \right) B a^2 + 80 \left(\tan(dx+c)^3 + 3 \tan(dx+c) \right) A$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="maxima")`

[Out] $\frac{1}{240} \cdot (160 \cdot (\tan(dx+c)^3 + 3 \cdot \tan(dx+c)) \cdot A \cdot a^2 + 16 \cdot (3 \cdot \tan(dx+c)^5 + 10 \cdot \tan(dx+c)^3 + 15 \cdot \tan(dx+c)) \cdot B \cdot a^2 - 15 \cdot A \cdot a^2 \cdot (2 \cdot (3 \cdot \sin(dx+c)^3 - 5 \cdot \sin(dx+c)) / (\sin(dx+c)^4 - 2 \cdot \sin(dx+c)^2 + 1) - 3 \cdot \log(\sin(dx+c) + 1) + 3 \cdot \log(\sin(dx+c) - 1)) - 30 \cdot B \cdot a^2 \cdot (2 \cdot (3 \cdot \sin(dx+c)^3 - 5 \cdot \sin(dx+c)) / (\sin(dx+c)^4 - 2 \cdot \sin(dx+c)^2 + 1) - 3 \cdot \log(\sin(dx+c) + 1) + 3 \cdot \log(\sin(dx+c) - 1)) - 60 \cdot A \cdot a^2 \cdot (2 \cdot \sin(dx+c) / (\sin(dx+c)^2 - 1) - \log(\sin(dx+c) + 1) + \log(\sin(dx+c) - 1))) / d$

mupad [B] time = 4.61, size = 224, normalized size = 1.33

$$\frac{a^2 \operatorname{atanh}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right) (7A + 6B) \left(\frac{7Aa^2}{4} + \frac{3Ba^2}{2}\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^9 + \left(-\frac{49Aa^2}{6} - 7Ba^2\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^7 + \left(\frac{40A}{3}\right)}{4d} - \frac{d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^{10} - 5 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^8 + \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^6 - 1\right)}{d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^{10} - 5 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^8 + \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^6 - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^2)/cos(c + d*x)^3,x)

[Out] (a^2*atanh(tan(c/2 + (d*x)/2))*(7*A + 6*B))/(4*d) - (tan(c/2 + (d*x)/2)*((2*5*A*a^2)/4 + (13*B*a^2)/2) + tan(c/2 + (d*x)/2)^9*((7*A*a^2)/4 + (3*B*a^2)/2) - tan(c/2 + (d*x)/2)^7*((49*A*a^2)/6 + 7*B*a^2) - tan(c/2 + (d*x)/2)^3*((79*A*a^2)/6 + 9*B*a^2) + tan(c/2 + (d*x)/2)^5*((40*A*a^2)/3 + (72*B*a^2)/5))/d*(5*tan(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 - 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2 \left(\int A \sec^3(c + dx) dx + \int 2A \sec^4(c + dx) dx + \int A \sec^5(c + dx) dx + \int B \sec^4(c + dx) dx + \int 2B \sec^5(c + dx) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**3*(a+a*sec(d*x+c))**2*(A+B*sec(d*x+c)),x)

[Out] a**2*(Integral(A*sec(c + d*x)**3, x) + Integral(2*A*sec(c + d*x)**4, x) + Integral(A*sec(c + d*x)**5, x) + Integral(B*sec(c + d*x)**4, x) + Integral(2*B*sec(c + d*x)**5, x) + Integral(B*sec(c + d*x)**6, x))

3.54 $\int \sec^2(c + dx)(a + a \sec(c + dx))^2(A + B \sec(c + dx)) dx$

Optimal. Leaf size=138

$$\frac{a^2(8A + 7B) \tan(c + dx)}{6d} + \frac{a^2(8A + 7B) \tanh^{-1}(\sin(c + dx))}{8d} + \frac{a^2(8A + 7B) \tan(c + dx) \sec(c + dx)}{24d} + \frac{(4A - B) \tan(c + dx)}{4d}$$

[Out] 1/8*a^2*(8*A+7*B)*arctanh(sin(d*x+c))/d+1/6*a^2*(8*A+7*B)*tan(d*x+c)/d+1/24*a^2*(8*A+7*B)*sec(d*x+c)*tan(d*x+c)/d+1/12*(4*A-B)*(a+a*sec(d*x+c))^2*tan(d*x+c)/d+1/4*B*(a+a*sec(d*x+c))^3*tan(d*x+c)/a/d

Rubi [A] time = 0.23, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {4010, 4001, 3788, 3767, 8, 4046, 3770}

$$\frac{a^2(8A + 7B) \tan(c + dx)}{6d} + \frac{a^2(8A + 7B) \tanh^{-1}(\sin(c + dx))}{8d} + \frac{a^2(8A + 7B) \tan(c + dx) \sec(c + dx)}{24d} + \frac{(4A - B) \tan(c + dx)}{4d}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]), x]

[Out] (a^2*(8*A + 7*B)*ArcTanh[Sin[c + d*x]])/(8*d) + (a^2*(8*A + 7*B)*Tan[c + d*x])/(6*d) + (a^2*(8*A + 7*B)*Sec[c + d*x]*Tan[c + d*x])/(24*d) + ((4*A - B)*(a + a*Sec[c + d*x])^2*Tan[c + d*x])/(12*d) + (B*(a + a*Sec[c + d*x])^3*Tan[c + d*x])/(4*a*d)

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 3767

Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_.))^2, x_Symbol] := Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_.)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] := -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
+ 1), 0] && !LtQ[m, -2^(-1)]
```

Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_.)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] := -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Cs
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
0] && !LtQ[m, -1]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_.)), x_Symbol] := -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \sec^2(c + dx)(a + a \sec(c + dx))^2(A + B \sec(c + dx)) dx &= \frac{B(a + a \sec(c + dx))^3 \tan(c + dx)}{4ad} + \frac{\int \sec(c + dx)(a + a \sec(c + dx))^2(A + B \sec(c + dx)) dx}{4ad} \\
&= \frac{(4A - B)(a + a \sec(c + dx))^2 \tan(c + dx)}{12d} + \frac{B(a + a \sec(c + dx))^3 \tan(c + dx)}{4ad} \\
&= \frac{(4A - B)(a + a \sec(c + dx))^2 \tan(c + dx)}{12d} + \frac{B(a + a \sec(c + dx))^3 \tan(c + dx)}{4ad} \\
&= \frac{a^2(8A + 7B) \sec(c + dx) \tan(c + dx)}{24d} + \frac{(4A - B)(a + a \sec(c + dx))^2 \tan(c + dx)}{12d} \\
&= \frac{a^2(8A + 7B) \tanh^{-1}(\sin(c + dx))}{8d} + \frac{a^2(8A + 7B) \tan(c + dx)}{6d}
\end{aligned}$$

Mathematica [A] time = 1.33, size = 262, normalized size = 1.90

$$\frac{a^2(\cos(c + dx) + 1)^2 \sec^4\left(\frac{1}{2}(c + dx)\right) \sec^4(c + dx) \left(24(8A + 7B) \cos^4(c + dx) \left(\log\left(\cos\left(\frac{1}{2}(c + dx)\right)\right) - \sin\left(\frac{1}{2}(c + dx)\right)\right)\right)}{48d \cos(dx + c)}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]

[Out] -1/768*(a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*Sec[c + d*x]^4*(24*(8*A + 7*B)*Cos[c + d*x]^4*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - Sec[c]*(-24*(5*A + 4*B)*Sin[c] + 3*(8*A + 15*B)*Sin[d*x] + 24*A*Sin[2*c + d*x] + 45*B*Sin[2*c + d*x] + 136*A*Sin[c + 2*d*x] + 128*B*Sin[c + 2*d*x] - 24*A*Sin[3*c + 2*d*x] + 24*A*Sin[2*c + 3*d*x] + 21*B*Sin[2*c + 3*d*x] + 24*A*Sin[4*c + 3*d*x] + 21*B*Sin[4*c + 3*d*x] + 40*A*Sin[3*c + 4*d*x] + 32*B*Sin[3*c + 4*d*x]))/d

fricas [A] time = 0.46, size = 145, normalized size = 1.05

$$\frac{3(8A + 7B)a^2 \cos(dx + c)^4 \log(\sin(dx + c) + 1) - 3(8A + 7B)a^2 \cos(dx + c)^4 \log(-\sin(dx + c) + 1) + 2(8A + 7B)a^2 \cos(dx + c)^4 \log(\cos(dx + c) + 1)}{48d \cos(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] 1/48*(3*(8*A + 7*B)*a^2*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(8*A + 7*B)*a^2*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(8*(5*A + 4*B)*a^2*cos(d*x + c)^4*log(cos(d*x + c) + 1))

$$+ c)^3 + 3(8A + 7B)a^2 \cos(dx + c)^2 + 8(A + 2B)a^2 \cos(dx + c) + 6Ba^2 \sin(dx + c) / (d \cos(dx + c)^4)$$

giac [A] time = 0.32, size = 212, normalized size = 1.54

$$3(8Aa^2 + 7Ba^2) \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right|\right) - 3(8Aa^2 + 7Ba^2) \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1\right|\right) - \frac{2\left(24Aa^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(dx+c)^2*(a+a*sec(dx+c))^2*(A+B*sec(dx+c)),x, algorithm="giac")

[Out] 1/24*(3*(8*A*a^2 + 7*B*a^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(8*A*a^2 + 7*B*a^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(24*A*a^2*tan(1/2*d*x + 1/2*c)^7 + 21*B*a^2*tan(1/2*d*x + 1/2*c)^7 - 88*A*a^2*tan(1/2*d*x + 1/2*c)^5 - 77*B*a^2*tan(1/2*d*x + 1/2*c)^5 + 136*A*a^2*tan(1/2*d*x + 1/2*c)^3 + 83*B*a^2*tan(1/2*d*x + 1/2*c)^3 - 72*A*a^2*tan(1/2*d*x + 1/2*c) - 75*B*a^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4/d

maple [A] time = 1.35, size = 187, normalized size = 1.36

$$\frac{5a^2A \tan(dx + c)}{3d} + \frac{7a^2B \sec(dx + c) \tan(dx + c)}{8d} + \frac{7B a^2 \ln(\sec(dx + c) + \tan(dx + c))}{8d} + \frac{a^2A \sec(dx + c) \tan(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(dx+c)^2*(a+a*sec(dx+c))^2*(A+B*sec(dx+c)),x)

[Out] 5/3*a^2*A*tan(dx+c)/d+7/8*a^2*B*sec(dx+c)*tan(dx+c)/d+7/8/d*B*a^2*ln(sec(dx+c)+tan(dx+c))+1/d*a^2*A*sec(dx+c)*tan(dx+c)+1/d*a^2*A*ln(sec(dx+c)+tan(dx+c))+4/3*a^2*B*tan(dx+c)/d+2/3*a^2*B*sec(dx+c)^2*tan(dx+c)/d+1/3/d*a^2*A*tan(dx+c)*sec(dx+c)^2+1/4*a^2*B*sec(dx+c)^3*tan(dx+c)/d

maxima [A] time = 0.35, size = 230, normalized size = 1.67

$$16\left(\tan(dx + c)^3 + 3 \tan(dx + c)\right)Aa^2 + 32\left(\tan(dx + c)^3 + 3 \tan(dx + c)\right)Ba^2 - 3Ba^2 \left(\frac{2(3 \sin(dx+c)^3 - 5 \sin(dx+c))}{\sin(dx+c)^4 - 2 \sin(dx+c)^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(dx+c)^2*(a+a*sec(dx+c))^2*(A+B*sec(dx+c)),x, algorithm="maxima")

[Out] $\frac{1}{48} \cdot (16 \cdot (\tan(dx + c))^3 + 3 \cdot \tan(dx + c)) \cdot A \cdot a^2 + 32 \cdot (\tan(dx + c))^3 + 3 \cdot \tan(dx + c) \cdot B \cdot a^2 - 3 \cdot B \cdot a^2 \cdot (2 \cdot (3 \cdot \sin(dx + c))^3 - 5 \cdot \sin(dx + c)) / (\sin(dx + c)^4 - 2 \cdot \sin(dx + c)^2 + 1) - 3 \cdot \log(\sin(dx + c) + 1) + 3 \cdot \log(\sin(dx + c) - 1)) - 24 \cdot A \cdot a^2 \cdot (2 \cdot \sin(dx + c)) / (\sin(dx + c)^2 - 1) - \log(\sin(dx + c) + 1) + \log(\sin(dx + c) - 1)) - 12 \cdot B \cdot a^2 \cdot (2 \cdot \sin(dx + c)) / (\sin(dx + c)^2 - 1) - \log(\sin(dx + c) + 1) + \log(\sin(dx + c) - 1)) + 48 \cdot A \cdot a^2 \cdot \tan(dx + c) / d$

mupad [B] time = 4.47, size = 183, normalized size = 1.33

$$\frac{\left(-2 A a^2 - \frac{7 B a^2}{4}\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^7 + \left(\frac{22 A a^2}{3} + \frac{77 B a^2}{12}\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5 + \left(-\frac{34 A a^2}{3} - \frac{83 B a^2}{12}\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 + \left(6 A a^2 - \frac{7 B a^2}{4}\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^8 - 4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^6 + 6 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 - 4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^2)/cos(c + d*x)^2,x)`

[Out] $(\tan(c/2 + (dx)/2) \cdot (6 \cdot A \cdot a^2 + (25 \cdot B \cdot a^2)/4) - \tan(c/2 + (dx)/2)^7 \cdot (2 \cdot A \cdot a^2 + (7 \cdot B \cdot a^2)/4) + \tan(c/2 + (dx)/2)^5 \cdot ((22 \cdot A \cdot a^2)/3 + (77 \cdot B \cdot a^2)/12) - \tan(c/2 + (dx)/2)^3 \cdot ((34 \cdot A \cdot a^2)/3 + (83 \cdot B \cdot a^2)/12)) / (d \cdot (6 \cdot \tan(c/2 + (dx)/2)^4 - 4 \cdot \tan(c/2 + (dx)/2)^2 - 4 \cdot \tan(c/2 + (dx)/2)^6 + \tan(c/2 + (dx)/2)^8 + 1)) + (2 \cdot a^2 \cdot \operatorname{atanh}(\tan(c/2 + (dx)/2)) \cdot (A + (7 \cdot B)/8)) / d$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2 \left(\int A \sec^2(c + dx) dx + \int 2A \sec^3(c + dx) dx + \int A \sec^4(c + dx) dx + \int B \sec^3(c + dx) dx + \int 2B \sec^4(c + dx) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)**2*(a+a*sec(d*x+c))**2*(A+B*sec(d*x+c)),x)`

[Out] $a^2 \cdot (\operatorname{Integral}(A \cdot \sec(c + dx)^2, x) + \operatorname{Integral}(2 \cdot A \cdot \sec(c + dx)^3, x) + \operatorname{Integral}(A \cdot \sec(c + dx)^4, x) + \operatorname{Integral}(B \cdot \sec(c + dx)^3, x) + \operatorname{Integral}(2 \cdot B \cdot \sec(c + dx)^4, x) + \operatorname{Integral}(B \cdot \sec(c + dx)^5, x))$

3.55 $\int \sec(c + dx)(a + a \sec(c + dx))^2(A + B \sec(c + dx)) dx$

Optimal. Leaf size=103

$$\frac{2a^2(3A + 2B) \tan(c + dx)}{3d} + \frac{a^2(3A + 2B) \tanh^{-1}(\sin(c + dx))}{2d} + \frac{a^2(3A + 2B) \tan(c + dx) \sec(c + dx)}{6d} + \frac{B \tan(c + dx)}{d}$$

[Out] $1/2*a^2*(3*A+2*B)*\operatorname{arctanh}(\sin(d*x+c))/d+2/3*a^2*(3*A+2*B)*\tan(d*x+c)/d+1/6*a^2*(3*A+2*B)*\sec(d*x+c)*\tan(d*x+c)/d+1/3*B*(a+a*\sec(d*x+c))^2*\tan(d*x+c)/d$

Rubi [A] time = 0.11, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {4001, 3788, 3767, 8, 4046, 3770}

$$\frac{2a^2(3A + 2B) \tan(c + dx)}{3d} + \frac{a^2(3A + 2B) \tanh^{-1}(\sin(c + dx))}{2d} + \frac{a^2(3A + 2B) \tan(c + dx) \sec(c + dx)}{6d} + \frac{B \tan(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Sec}[c + d*x]*(a + a*\operatorname{Sec}[c + d*x])^2*(A + B*\operatorname{Sec}[c + d*x]), x]$

[Out] $(a^2*(3*A + 2*B)*\operatorname{ArcTanh}[\operatorname{Sin}[c + d*x]])/(2*d) + (2*a^2*(3*A + 2*B)*\operatorname{Tan}[c + d*x])/(3*d) + (a^2*(3*A + 2*B)*\operatorname{Sec}[c + d*x]*\operatorname{Tan}[c + d*x])/(6*d) + (B*(a + a*\operatorname{Sec}[c + d*x])^2*\operatorname{Tan}[c + d*x])/(3*d)$

Rule 8

$\operatorname{Int}[a_, x_Symbol] \rightarrow \operatorname{Simp}[a*x, x] /; \operatorname{FreeQ}[a, x]$

Rule 3767

$\operatorname{Int}[\operatorname{csc}[(c_) + (d_)*(x_)]^{(n_)}, x_Symbol] \rightarrow -\operatorname{Dist}[d^{(-1)}, \operatorname{Subst}[\operatorname{Int}[\operatorname{ExpandIntegrand}[(1 + x^2)^{(n/2 - 1)}, x], x], x, \operatorname{Cot}[c + d*x]], x] /; \operatorname{FreeQ}[\{c, d\}, x] \ \&\& \ \operatorname{IGtQ}[n/2, 0]$

Rule 3770

$\operatorname{Int}[\operatorname{csc}[(c_) + (d_)*(x_)], x_Symbol] \rightarrow -\operatorname{Simp}[\operatorname{ArcTanh}[\operatorname{Cos}[c + d*x]]/d, x] /; \operatorname{FreeQ}[\{c, d\}, x]$

Rule 3788

$\operatorname{Int}[(\operatorname{csc}[(e_) + (f_)*(x_)]*(d_.))^{(n_.)}*(\operatorname{csc}[(e_) + (f_)*(x_)]*(b_.) + (a_.))^2, x_Symbol] \rightarrow \operatorname{Dist}[(2*a*b)/d, \operatorname{Int}[(d*\operatorname{Csc}[e + f*x])^{(n + 1)}, x], x]$

+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 4001

Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^m*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m + 1), 0] && !LtQ[m, -2^(-1)]

Rule 4046

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^m*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_.)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]

Rubi steps

$$\begin{aligned} \int \sec(c + dx)(a + a \sec(c + dx))^2(A + B \sec(c + dx)) dx &= \frac{B(a + a \sec(c + dx))^2 \tan(c + dx)}{3d} + \frac{1}{3}(3A + 2B) \int \sec(c + dx) dx \\ &= \frac{B(a + a \sec(c + dx))^2 \tan(c + dx)}{3d} + \frac{1}{3}(3A + 2B) \int \sec(c + dx) dx \\ &= \frac{a^2(3A + 2B) \sec(c + dx) \tan(c + dx)}{6d} + \frac{B(a + a \sec(c + dx))^2 \tan(c + dx)}{3d} \\ &= \frac{a^2(3A + 2B) \tanh^{-1}(\sin(c + dx))}{2d} + \frac{2a^2(3A + 2B) \tan(c + dx)}{3d} \end{aligned}$$

Mathematica [B] time = 6.19, size = 481, normalized size = 4.67

$$a^2 \cos^3(c + dx) \sec^4\left(\frac{1}{2}(c + dx)\right) (\sec(c + dx) + 1)^2 (A + B \sec(c + dx)) \left(\frac{4(6A+5B) \sin\left(\frac{dx}{2}\right)}{\left(\cos\left(\frac{c}{2}\right) - \sin\left(\frac{c}{2}\right)\right) \left(\cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+dx)\right)\right)} + \frac{1}{\sin\left(\frac{1}{2}(c+dx)\right)} \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sec[c + d*x]*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]), x]


```
[Out] (a^2*cos[c + d*x]^3*Sec[(c + d*x)/2]^4*(1 + Sec[c + d*x])^2*(A + B*Sec[c +
d*x])*(-6*(3*A + 2*B)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 6*(3*A + 2
*B)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (2*B*Sin[(d*x)/2]))/((Cos[c/2
] - Sin[c/2])*Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3 + ((3*A + 7*B)*Cos[c
/2] - (3*A + 5*B)*Sin[c/2])/((Cos[c/2] - Sin[c/2])*Cos[(c + d*x)/2] - Sin[
(c + d*x)/2])^2 + (4*(6*A + 5*B)*Sin[(d*x)/2])/((Cos[c/2] - Sin[c/2])*Cos
[(c + d*x)/2] - Sin[(c + d*x)/2])) + (2*B*Sin[(d*x)/2])/((Cos[c/2] + Sin[c/
2])*Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3 - ((3*A + 7*B)*Cos[c/2] + (3*A
+ 5*B)*Sin[c/2])/((Cos[c/2] + Sin[c/2])*Cos[(c + d*x)/2] + Sin[(c + d*x)/
2])^2 + (4*(6*A + 5*B)*Sin[(d*x)/2])/((Cos[c/2] + Sin[c/2])*Cos[(c + d*x)
/2] + Sin[(c + d*x)/2])))/(48*d*(B + A*cos[c + d*x]))
```

fricas [A] time = 0.58, size = 125, normalized size = 1.21

$$\frac{3(3A + 2B)a^2 \cos(dx + c)^3 \log(\sin(dx + c) + 1) - 3(3A + 2B)a^2 \cos(dx + c)^3 \log(-\sin(dx + c) + 1) + 2(2B \sin(dx/2) \cos((c + dx)/2) - \sin((c + dx)/2))^3 + (3A + 7B) \cos(c/2) - (3A + 5B) \sin(c/2)}{12d \cos(dx + c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/12*(3*(3*A + 2*B)*a^2*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*(3*A + 2*B
)*a^2*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + 2*(2*(6*A + 5*B)*a^2*cos(d*x
+ c)^2 + 3*(A + 2*B)*a^2*cos(d*x + c) + 2*B*a^2)*sin(d*x + c))/(d*cos(d*x +
c)^3)
```

giac [A] time = 0.63, size = 178, normalized size = 1.73

$$3(3Aa^2 + 2Ba^2) \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right|\right) - 3(3Aa^2 + 2Ba^2) \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1\right|\right) - \frac{2(9Aa^2 \tan(\frac{1}{2}dx + \frac{1}{2}c) - 9Aa^2 \tan(\frac{1}{2}dx + \frac{1}{2}c) + 6Aa^2 \tan^3(\frac{1}{2}dx + \frac{1}{2}c) - 6Aa^2 \tan^5(\frac{1}{2}dx + \frac{1}{2}c) + 6Ba^2 \tan^3(\frac{1}{2}dx + \frac{1}{2}c) - 6Ba^2 \tan^5(\frac{1}{2}dx + \frac{1}{2}c) - 24Aa^2 \tan^3(\frac{1}{2}dx + \frac{1}{2}c) - 16Ba^2 \tan^3(\frac{1}{2}dx + \frac{1}{2}c) + 15Aa^2 \tan^5(\frac{1}{2}dx + \frac{1}{2}c) + 18Ba^2 \tan^5(\frac{1}{2}dx + \frac{1}{2}c))}{(\tan(\frac{1}{2}dx + \frac{1}{2}c)^2 - 1)^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/6*(3*(3*A*a^2 + 2*B*a^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(3*A*a^2
+ 2*B*a^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(9*A*a^2*tan(1/2*d*x + 1/
2*c)^5 + 6*B*a^2*tan(1/2*d*x + 1/2*c)^5 - 24*A*a^2*tan(1/2*d*x + 1/2*c)^3 -
16*B*a^2*tan(1/2*d*x + 1/2*c)^3 + 15*A*a^2*tan(1/2*d*x + 1/2*c) + 18*B*a^2
*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

maple [A] time = 1.15, size = 141, normalized size = 1.37

$$\frac{3a^2 A \ln(\sec(dx+c) + \tan(dx+c))}{2d} + \frac{5a^2 B \tan(dx+c)}{3d} + \frac{2a^2 A \tan(dx+c)}{d} + \frac{a^2 B \sec(dx+c) \tan(dx+c)}{d} + \frac{B a^2}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x)

[Out] 3/2/d*a^2*A*ln(sec(d*x+c)+tan(d*x+c))+5/3*a^2*B*tan(d*x+c)/d+2*a^2*A*tan(d*x+c)/d+a^2*B*sec(d*x+c)*tan(d*x+c)/d+1/d*B*a^2*ln(sec(d*x+c)+tan(d*x+c))+1/2/d*a^2*A*sec(d*x+c)*tan(d*x+c)+1/3*a^2*B*sec(d*x+c)^2*tan(d*x+c)/d

maxima [A] time = 0.34, size = 167, normalized size = 1.62

$$\frac{4 \left(\tan(dx+c)^3 + 3 \tan(dx+c) \right) B a^2 - 3 A a^2 \left(\frac{2 \sin(dx+c)}{\sin(dx+c)^2-1} - \log(\sin(dx+c)+1) + \log(\sin(dx+c)-1) \right) - 6 B a^2}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] 1/12*(4*(tan(d*x+c)^3+3*tan(d*x+c))*B*a^2-3*A*a^2*(2*sin(d*x+c)/(sin(d*x+c)^2-1)-log(sin(d*x+c)+1)+log(sin(d*x+c)-1))-6*B*a^2*(2*sin(d*x+c)/(sin(d*x+c)^2-1)-log(sin(d*x+c)+1)+log(sin(d*x+c)-1))+12*A*a^2*log(sec(d*x+c)+tan(d*x+c))+24*A*a^2*tan(d*x+c)+12*B*a^2*tan(d*x+c))/d

mupad [B] time = 3.81, size = 145, normalized size = 1.41

$$\frac{2 a^2 \operatorname{atanh}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right) \left(\frac{3A}{2} + B\right) \left(3 A a^2 + 2 B a^2\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5 + \left(-8 A a^2 - \frac{16 B a^2}{3}\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 + \left(5 A a^2 + 6 B a^2\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^6 - 3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 + 3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A+B/cos(c+d*x))*(a+a/cos(c+d*x))^2)/cos(c+d*x),x)

[Out] (2*a^2*atanh(tan(c/2+(d*x)/2))*((3*A)/2+B))/d-(tan(c/2+(d*x)/2)*(5*A*a^2+6*B*a^2)+tan(c/2+(d*x)/2)^5*(3*A*a^2+2*B*a^2)-tan(c/2+(d*x)/2)^3*(8*A*a^2+(16*B*a^2)/3))/(d*(3*tan(c/2+(d*x)/2)^2-3*tan(c/2+(d*x)/2)^4+tan(c/2+(d*x)/2)^6-1))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2 \left(\int A \sec(c + dx) dx + \int 2A \sec^2(c + dx) dx + \int A \sec^3(c + dx) dx + \int B \sec^2(c + dx) dx + \int 2B \sec^3(c + dx) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))**2*(A+B*sec(d*x+c)),x)

[Out] a**2*(Integral(A*sec(c + d*x), x) + Integral(2*A*sec(c + d*x)**2, x) + Integral(A*sec(c + d*x)**3, x) + Integral(B*sec(c + d*x)**2, x) + Integral(2*B*sec(c + d*x)**3, x) + Integral(B*sec(c + d*x)**4, x))

3.56 $\int (a + a \sec(c + dx))^2 (A + B \sec(c + dx)) dx$

Optimal. Leaf size=82

$$\frac{a^2(2A + 3B) \tan(c + dx)}{2d} + \frac{a^2(4A + 3B) \tanh^{-1}(\sin(c + dx))}{2d} + a^2Ax + \frac{B \tan(c + dx) (a^2 \sec(c + dx) + a^2)}{2d}$$

[Out] $a^2Ax + \frac{1}{2}a^2(4A+3B)\arctanh(\sin(dx+c))/d + \frac{1}{2}a^2(2A+3B)\tan(dx+c)/d + \frac{1}{2}B(a^2+a^2\sec(dx+c))\tan(dx+c)/d$

Rubi [A] time = 0.08, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3917, 3914, 3767, 8, 3770}

$$\frac{a^2(2A + 3B) \tan(c + dx)}{2d} + \frac{a^2(4A + 3B) \tanh^{-1}(\sin(c + dx))}{2d} + a^2Ax + \frac{B \tan(c + dx) (a^2 \sec(c + dx) + a^2)}{2d}$$

Antiderivative was successfully verified.

[In] `Int[(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]`

[Out] $a^2Ax + (a^2(4A + 3B)\text{ArcTanh}[\text{Sin}[c + d*x]])/(2*d) + (a^2(2A + 3B)\text{Tan}[c + d*x])/(2*d) + (B(a^2 + a^2\text{Sec}[c + d*x])\text{Tan}[c + d*x])/(2*d)$

Rule 8

`Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

Rule 3767

`Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]`

Rule 3770

`Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]`

Rule 3914

`Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.)), x_Symbol] := Simp[a*c*x, x] + (Dist[b*d, Int[Csc[e + f*x]^2, x], x] + Dist[b*c + a*d, Int[Csc[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]`

Rule 3917

Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.)), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1))/(f*m), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m - 1)*Simp[a*c*m + (b*c*m + a*d*(2*m - 1))*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && GtQ[m, 1] && EqQ[a^2 - b^2, 0] && IntegerQ[2*m]

Rubi steps

$$\begin{aligned} \int (a + a \sec(c + dx))^2 (A + B \sec(c + dx)) dx &= \frac{B(a^2 + a^2 \sec(c + dx)) \tan(c + dx)}{2d} + \frac{1}{2} \int (a + a \sec(c + dx))(2a + a \sec(c + dx)) dx \\ &= a^2 Ax + \frac{B(a^2 + a^2 \sec(c + dx)) \tan(c + dx)}{2d} + \frac{1}{2} (a^2(2A + 3B)) \\ &= a^2 Ax + \frac{a^2(4A + 3B) \tanh^{-1}(\sin(c + dx))}{2d} + \frac{B(a^2 + a^2 \sec(c + dx))}{2d} \\ &= a^2 Ax + \frac{a^2(4A + 3B) \tanh^{-1}(\sin(c + dx))}{2d} + \frac{a^2(2A + 3B) \tan(c + dx)}{2d} \end{aligned}$$

Mathematica [B] time = 1.32, size = 307, normalized size = 3.74

$$a^2 \cos^3(c + dx) \sec^4\left(\frac{1}{2}(c + dx)\right) (\sec(c + dx) + 1)^2 (A + B \sec(c + dx)) \left(\frac{4(A+2B) \sin\left(\frac{dx}{2}\right)}{d(\cos\left(\frac{c}{2}\right) - \sin\left(\frac{c}{2}\right))(\cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+dx)\right))} + \dots \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]

[Out] (a^2*Cos[c + d*x]^3*Sec[(c + d*x)/2]^4*(1 + Sec[c + d*x])^2*(A + B*Sec[c + d*x])*(4*A*x - (2*(4*A + 3*B)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/d + (2*(4*A + 3*B)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/d + B/(d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) + (4*(A + 2*B)*Sin[(d*x)/2])/(d*(Cos[c/2] - Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) - B/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (4*(A + 2*B)*Sin[(d*x)/2])/(d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])))/(16*(B + A*Cos[c + d*x]))

fricas [A] time = 0.47, size = 119, normalized size = 1.45

$$\frac{4 A a^2 dx \cos(dx + c)^2 + (4 A + 3 B) a^2 \cos(dx + c)^2 \log(\sin(dx + c) + 1) - (4 A + 3 B) a^2 \cos(dx + c)^2 \log(-\sin(dx + c) + 1)}{4 d \cos(dx + c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] $\frac{1}{4}*(4*A*a^2*d*x*\cos(d*x + c)^2 + (4*A + 3*B)*a^2*\cos(d*x + c)^2*\log(\sin(d*x + c) + 1) - (4*A + 3*B)*a^2*\cos(d*x + c)^2*\log(-\sin(d*x + c) + 1) + 2*(2*(A + 2*B)*a^2*\cos(d*x + c) + B*a^2)*\sin(d*x + c))/(d*\cos(d*x + c)^2)$

giac [B] time = 0.49, size = 154, normalized size = 1.88

$$\frac{2(dx+c)Aa^2 + (4Aa^2 + 3Ba^2)\log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right|\right) - (4Aa^2 + 3Ba^2)\log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1\right|\right) - \frac{2(2Aa^2 + 3Ba^2)\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)}{d}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] $\frac{1}{2}*(2*(d*x + c)*A*a^2 + (4*A*a^2 + 3*B*a^2)*\log(\text{abs}(\tan(1/2*d*x + 1/2*c) + 1)) - (4*A*a^2 + 3*B*a^2)*\log(\text{abs}(\tan(1/2*d*x + 1/2*c) - 1)) - 2*(2*A*a^2*\tan(1/2*d*x + 1/2*c)^3 + 3*B*a^2*\tan(1/2*d*x + 1/2*c)^3 - 2*A*a^2*\tan(1/2*d*x + 1/2*c) - 5*B*a^2*\tan(1/2*d*x + 1/2*c)))/(\tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d$

maple [A] time = 0.90, size = 113, normalized size = 1.38

$$a^2Ax + \frac{Aa^2c}{d} + \frac{3Ba^2 \ln(\sec(dx+c) + \tan(dx+c))}{2d} + \frac{2a^2A \ln(\sec(dx+c) + \tan(dx+c))}{d} + \frac{2a^2B \tan(dx+c)}{d} + \frac{a^2}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x)

[Out] $a^2*A*x + 1/d*A*a^2*c + 3/2/d*B*a^2*\ln(\sec(d*x+c) + \tan(d*x+c)) + 2/d*a^2*A*\ln(\sec(d*x+c) + \tan(d*x+c)) + 2*a^2*B*\tan(d*x+c)/d + a^2*A*\tan(d*x+c)/d + 1/2*a^2*B*\sec(d*x+c)*\tan(d*x+c)/d$

maxima [A] time = 0.33, size = 128, normalized size = 1.56

$$\frac{4(dx+c)Aa^2 - Ba^2\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1} - \log(\sin(dx+c)+1) + \log(\sin(dx+c)-1)\right) + 8Aa^2\log(\sec(dx+c) + \tan(dx+c))}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] $\frac{1}{4} \cdot (4 \cdot (d \cdot x + c) \cdot A \cdot a^2 - B \cdot a^2 \cdot (2 \cdot \sin(d \cdot x + c) / (\sin(d \cdot x + c)^2 - 1) - \log(\sin(d \cdot x + c) + 1) + \log(\sin(d \cdot x + c) - 1)) + 8 \cdot A \cdot a^2 \cdot \log(\sec(d \cdot x + c) + \tan(d \cdot x + c)) + 4 \cdot B \cdot a^2 \cdot \log(\sec(d \cdot x + c) + \tan(d \cdot x + c)) + 4 \cdot A \cdot a^2 \cdot \tan(d \cdot x + c) + 8 \cdot B \cdot a^2 \cdot \tan(d \cdot x + c)) / d$

mupad [B] time = 2.01, size = 162, normalized size = 1.98

$$\frac{2 A a^2 \operatorname{atan}\left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)}\right)}{d} + \frac{4 A a^2 \operatorname{atanh}\left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)}\right)}{d} + \frac{3 B a^2 \operatorname{atanh}\left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)}\right)}{d} + \frac{A a^2 \sin(c + dx)}{d \cos(c + dx)} + \frac{2 B a^2 \sin(c + dx)}{d \cos(c + dx)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^2,x)`

[Out] $(2 \cdot A \cdot a^2 \cdot \operatorname{atan}\left(\frac{\sin(c/2 + (d \cdot x)/2)}{\cos(c/2 + (d \cdot x)/2)}\right)) / d + (4 \cdot A \cdot a^2 \cdot \operatorname{atanh}\left(\frac{\sin(c/2 + (d \cdot x)/2)}{\cos(c/2 + (d \cdot x)/2)}\right)) / d + (3 \cdot B \cdot a^2 \cdot \operatorname{atanh}\left(\frac{\sin(c/2 + (d \cdot x)/2)}{\cos(c/2 + (d \cdot x)/2)}\right)) / d + (A \cdot a^2 \cdot \sin(c + d \cdot x)) / (d \cdot \cos(c + d \cdot x)) + (2 \cdot B \cdot a^2 \cdot \sin(c + d \cdot x)) / (d \cdot \cos(c + d \cdot x)) + (B \cdot a^2 \cdot \sin(c + d \cdot x)) / (2 \cdot d \cdot \cos(c + d \cdot x)^2)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2 \left(\int A dx + \int 2A \sec(c + dx) dx + \int A \sec^2(c + dx) dx + \int B \sec(c + dx) dx + \int 2B \sec^2(c + dx) dx + \int B \sec^3(c + dx) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sec(d*x+c))*2*(A+B*sec(d*x+c)),x)`

[Out] `a**2*(Integral(A, x) + Integral(2*A*sec(c + d*x), x) + Integral(A*sec(c + d*x)**2, x) + Integral(B*sec(c + d*x), x) + Integral(2*B*sec(c + d*x)**2, x) + Integral(B*sec(c + d*x)**3, x))`

$$3.57 \quad \int \cos(c + dx)(a + a \sec(c + dx))^2(A + B \sec(c + dx)) dx$$

Optimal. Leaf size=73

$$\frac{a^2(A - B) \sin(c + dx)}{d} + \frac{a^2(A + 2B) \tanh^{-1}(\sin(c + dx))}{d} + a^2x(2A+B) + \frac{B \sin(c + dx)(a^2 \sec(c + dx) + a^2)}{d}$$

[Out] $a^2*(2*A+B)*x+a^2*(A+2*B)*\operatorname{arctanh}(\sin(d*x+c))/d+a^2*(A-B)*\sin(d*x+c)/d+B*(a^2+a^2*\sec(d*x+c))*\sin(d*x+c)/d$

Rubi [A] time = 0.13, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {4018, 3996, 3770}

$$\frac{a^2(A - B) \sin(c + dx)}{d} + \frac{a^2(A + 2B) \tanh^{-1}(\sin(c + dx))}{d} + a^2x(2A+B) + \frac{B \sin(c + dx)(a^2 \sec(c + dx) + a^2)}{d}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Cos}[c + d*x]*(a + a*\operatorname{Sec}[c + d*x])^2*(A + B*\operatorname{Sec}[c + d*x]), x]$

[Out] $a^2*(2*A + B)*x + (a^2*(A + 2*B)*\operatorname{ArcTanh}[\operatorname{Sin}[c + d*x]])/d + (a^2*(A - B)*\operatorname{Sin}[c + d*x])/d + (B*(a^2 + a^2*\operatorname{Sec}[c + d*x])*\operatorname{Sin}[c + d*x])/d$

Rule 3770

$\operatorname{Int}[\operatorname{csc}[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow -\operatorname{Simp}[\operatorname{ArcTanh}[\operatorname{Cos}[c + d*x]]/d, x] /;$ $\operatorname{FreeQ}\{c, d\}, x]$

Rule 3996

$\operatorname{Int}[(\operatorname{csc}[(e_.) + (f_.)*(x_.)]*(d_.))^n*(\operatorname{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))*(\operatorname{csc}[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] \rightarrow \operatorname{Simp}[(A*a*\operatorname{Cot}[e + f*x]*(d*\operatorname{Csc}[e + f*x])^n)/(f*n), x] + \operatorname{Dist}[1/(d*n), \operatorname{Int}[(d*\operatorname{Csc}[e + f*x])^{n+1}*\operatorname{Simp}[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*\operatorname{Csc}[e + f*x], x], x] /;$ $\operatorname{FreeQ}\{a, b, d, e, f, A, B\}, x] \&\& \operatorname{NeQ}[A*b - a*B, 0] \&\& \operatorname{LeQ}[n, -1]$

Rule 4018

$\operatorname{Int}[(\operatorname{csc}[(e_.) + (f_.)*(x_.)]*(d_.))^n*(\operatorname{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m*(\operatorname{csc}[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] \rightarrow -\operatorname{Simp}[(b*B*\operatorname{Cot}[e + f*x]*(a + b*\operatorname{Csc}[e + f*x])^{m-1}*(d*\operatorname{Csc}[e + f*x])^n)/(f*(m + n)), x] + \operatorname{Dist}[1/(d*(m + n)), \operatorname{Int}[(a + b*\operatorname{Csc}[e + f*x])^{m-1}*(d*\operatorname{Csc}[e + f*x])^n*\operatorname{Simp}[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*\operatorname{Csc}$

$[e + f*x], x], x], x] /; \text{FreeQ}[\{a, b, d, e, f, A, B, n\}, x] \ \&\& \ \text{NeQ}[A*b - a*B, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{GtQ}[m, 1/2] \ \&\& \ \text{!LtQ}[n, -1]$

Rubi steps

$$\begin{aligned} \int \cos(c + dx)(a + a \sec(c + dx))^2(A + B \sec(c + dx)) dx &= \frac{B(a^2 + a^2 \sec(c + dx)) \sin(c + dx)}{d} + \int \cos(c + dx) \\ &= \frac{a^2(A - B) \sin(c + dx)}{d} + \frac{B(a^2 + a^2 \sec(c + dx)) \sin(c + dx)}{d} \\ &= a^2(2A + B)x + \frac{a^2(A - B) \sin(c + dx)}{d} + \frac{B(a^2 + a^2 \sec(c + dx)) \sin(c + dx)}{d} \\ &= a^2(2A + B)x + \frac{a^2(A + 2B) \tanh^{-1}(\sin(c + dx))}{d} + \frac{a^2 B \sec(c + dx) \sin(c + dx)}{d} \end{aligned}$$

Mathematica [B] time = 1.66, size = 258, normalized size = 3.53

$$a^2 \cos^3(c + dx) \sec^4\left(\frac{1}{2}(c + dx)\right) (\sec(c + dx) + 1)^2 (A + B \sec(c + dx)) \left(-\frac{(A+2B) \log\left(\cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+dx)\right)\right)}{d} + \frac{(A+2B) \log\left(\cos\left(\frac{1}{2}(c+dx)\right) + \sin\left(\frac{1}{2}(c+dx)\right)\right)}{d} \right)$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]

[Out] (a^2*Cos[c + d*x]^3*Sec[(c + d*x)/2]^4*(1 + Sec[c + d*x])^2*(A + B*Sec[c + d*x])*((2*A + B)*x - ((A + 2*B)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/d + ((A + 2*B)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/d + (A*Cos[d*x]*Sin[c])/d + (A*Cos[c]*Sin[d*x])/d + (B*Sin[(d*x)/2])/(d*(Cos[c/2] - Sin[c/2]))*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) + (B*Sin[(d*x)/2])/(d*(Cos[c/2] + Sin[c/2]))*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])))/(4*(B + A*Cos[c + d*x]))

fricas [A] time = 0.48, size = 108, normalized size = 1.48

$$\frac{2(2A + B)a^2 dx \cos(dx + c) + (A + 2B)a^2 \cos(dx + c) \log(\sin(dx + c) + 1) - (A + 2B)a^2 \cos(dx + c) \log(-\sin(dx + c) + 1)}{2d \cos(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] $\frac{1}{2}*(2*(2*A + B)*a^2*d*x*\cos(d*x + c) + (A + 2*B)*a^2*\cos(d*x + c)*\log(\sin(d*x + c) + 1) - (A + 2*B)*a^2*\cos(d*x + c)*\log(-\sin(d*x + c) + 1) + 2*(A*a^2*\cos(d*x + c) + B*a^2)*\sin(d*x + c))/(d*\cos(d*x + c))$

giac [B] time = 0.96, size = 157, normalized size = 2.15

$$\frac{(2 A a^2 + B a^2)(d x + c) + (A a^2 + 2 B a^2) \log \left(\left| \tan \left(\frac{1}{2} d x + \frac{1}{2} c \right) + 1 \right| \right) - (A a^2 + 2 B a^2) \log \left(\left| \tan \left(\frac{1}{2} d x + \frac{1}{2} c \right) - 1 \right| \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] $((2*A*a^2 + B*a^2)*(d*x + c) + (A*a^2 + 2*B*a^2)*\log(\text{abs}(\tan(1/2*d*x + 1/2*c) + 1)) - (A*a^2 + 2*B*a^2)*\log(\text{abs}(\tan(1/2*d*x + 1/2*c) - 1)) + 2*(A*a^2*\tan(1/2*d*x + 1/2*c)^3 - B*a^2*\tan(1/2*d*x + 1/2*c)^3 - A*a^2*\tan(1/2*d*x + 1/2*c) - B*a^2*\tan(1/2*d*x + 1/2*c)))/(\tan(1/2*d*x + 1/2*c)^4 - 1))/d$

maple [A] time = 0.93, size = 107, normalized size = 1.47

$$2a^2Ax + a^2Bx + \frac{a^2A \sin(dx + c)}{d} + \frac{a^2A \ln(\sec(dx + c) + \tan(dx + c))}{d} + \frac{2Aa^2c}{d} + \frac{a^2B \tan(dx + c)}{d} + \frac{2Ba^2 \ln(\sec(dx + c) + \tan(dx + c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x)

[Out] $2*a^2*A*x + a^2*B*x + 1/d*a^2*A*\sin(d*x+c) + 1/d*a^2*A*\ln(\sec(d*x+c) + \tan(d*x+c)) + 2/d*A*a^2*c + a^2*B*\tan(d*x+c)/d + 2/d*B*a^2*\ln(\sec(d*x+c) + \tan(d*x+c)) + 1/d*B*a^2*c$

maxima [A] time = 0.34, size = 105, normalized size = 1.44

$$\frac{4(dx + c)Aa^2 + 2(dx + c)Ba^2 + Aa^2(\log(\sin(dx + c) + 1) - \log(\sin(dx + c) - 1)) + 2Ba^2(\log(\sin(dx + c) + 1) - \log(\sin(dx + c) - 1))}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] $\frac{1}{2}*(4*(d*x + c)*A*a^2 + 2*(d*x + c)*B*a^2 + A*a^2*(\log(\sin(d*x + c) + 1) - \log(\sin(d*x + c) - 1)) + 2*B*a^2*(\log(\sin(d*x + c) + 1) - \log(\sin(d*x + c) - 1)) + 2*A*a^2*\sin(d*x + c) + 2*B*a^2*\tan(d*x + c))/d$

mupad [B] time = 2.01, size = 161, normalized size = 2.21

$$\frac{A a^2 \sin(c + dx)}{d} + \frac{4 A a^2 \operatorname{atan}\left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)}\right)}{d} + \frac{2 A a^2 \operatorname{atanh}\left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)}\right)}{d} + \frac{2 B a^2 \operatorname{atan}\left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)}\right)}{d} + \frac{4 B a^2 \operatorname{atanh}\left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(c + d*x)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^2,x)`

[Out] `(A*a^2*sin(c + d*x))/d + (4*A*a^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (2*A*a^2*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (2*B*a^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (4*B*a^2*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (B*a^2*sin(c + d*x))/(d*cos(c + d*x))`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2 \left(\int A \cos(c + dx) dx + \int 2A \cos(c + dx) \sec(c + dx) dx + \int A \cos(c + dx) \sec^2(c + dx) dx + \int B \cos(c + dx) \sec(c + dx) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)*(a+a*sec(d*x+c))**2*(A+B*sec(d*x+c)),x)`

[Out] `a**2*(Integral(A*cos(c + d*x), x) + Integral(2*A*cos(c + d*x)*sec(c + d*x), x) + Integral(A*cos(c + d*x)*sec(c + d*x)**2, x) + Integral(B*cos(c + d*x)*sec(c + d*x), x) + Integral(2*B*cos(c + d*x)*sec(c + d*x)**2, x) + Integral(B*cos(c + d*x)*sec(c + d*x)**3, x))`

3.58 $\int \cos^2(c + dx)(a + a \sec(c + dx))^2(A + B \sec(c + dx)) dx$

Optimal. Leaf size=88

$$\frac{a^2(3A + 2B) \sin(c + dx)}{2d} + \frac{1}{2}a^2x(3A+4B) + \frac{A \sin(c + dx) \cos(c + dx) (a^2 \sec(c + dx) + a^2)}{2d} + \frac{a^2B \tanh^{-1}(\sin(c + dx))}{d}$$

[Out] $1/2*a^2*(3*A+4*B)*x+a^2*B*\operatorname{arctanh}(\sin(d*x+c))/d+1/2*a^2*(3*A+2*B)*\sin(d*x+c)/d+1/2*A*\cos(d*x+c)*(a^2+a^2*\sec(d*x+c))*\sin(d*x+c)/d$

Rubi [A] time = 0.14, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {4017, 3996, 3770}

$$\frac{a^2(3A + 2B) \sin(c + dx)}{2d} + \frac{1}{2}a^2x(3A+4B) + \frac{A \sin(c + dx) \cos(c + dx) (a^2 \sec(c + dx) + a^2)}{2d} + \frac{a^2B \tanh^{-1}(\sin(c + dx))}{d}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Cos}[c + d*x]^2*(a + a*\operatorname{Sec}[c + d*x])^2*(A + B*\operatorname{Sec}[c + d*x]), x]$

[Out] $(a^2*(3*A + 4*B)*x)/2 + (a^2*B*\operatorname{ArcTanh}[\operatorname{Sin}[c + d*x]])/d + (a^2*(3*A + 2*B)*\operatorname{Sin}[c + d*x])/(2*d) + (A*\operatorname{Cos}[c + d*x]*(a^2 + a^2*\operatorname{Sec}[c + d*x])*\operatorname{Sin}[c + d*x])/(2*d)$

Rule 3770

$\operatorname{Int}[\operatorname{csc}[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow -\operatorname{Simp}[\operatorname{ArcTanh}[\operatorname{Cos}[c + d*x]]/d, x] /;$ $\operatorname{FreeQ}[\{c, d\}, x]$

Rule 3996

$\operatorname{Int}[(\operatorname{csc}[(e_.) + (f_.)*(x_.)]*(d_.))^n*(\operatorname{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))*(\operatorname{csc}[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] \rightarrow \operatorname{Simp}[(A*a*\operatorname{Cot}[e + f*x]*(d*\operatorname{Csc}[e + f*x])^n)/(f*n), x] + \operatorname{Dist}[1/(d*n), \operatorname{Int}[(d*\operatorname{Csc}[e + f*x])^{n+1}]*\operatorname{Simp}[n*(B*a + A*b) + (B*b*n + A*a*(n+1))*\operatorname{Csc}[e + f*x], x], x] /;$ $\operatorname{FreeQ}[\{a, b, d, e, f, A, B\}, x] \ \&\& \operatorname{NeQ}[A*b - a*B, 0] \ \&\& \operatorname{LeQ}[n, -1]$

Rule 4017

$\operatorname{Int}[(\operatorname{csc}[(e_.) + (f_.)*(x_.)]*(d_.))^n*(\operatorname{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m*(\operatorname{csc}[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] \rightarrow \operatorname{Simp}[(a*A*\operatorname{Cot}[e + f*x]*(a + b*\operatorname{Csc}[e + f*x])^{m-1}*(d*\operatorname{Csc}[e + f*x])^n)/(f*n), x] - \operatorname{Dist}[b/(a*d*n), \operatorname{Int}[(a + b*\operatorname{Csc}[e + f*x])^{m-1}*(d*\operatorname{Csc}[e + f*x])^{n+1}]*\operatorname{Simp}[\operatorname{Csc}[e + f*x], x], x] /;$

$[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /$
 $; FreeQ[\{a, b, d, e, f, A, B\}, x] \&\& NeQ[A*b - a*B, 0] \&\& EqQ[a^2 - b^2, 0]$
 $\&\& GtQ[m, 1/2] \&\& LtQ[n, -1]$

Rubi steps

$$\begin{aligned} \int \cos^2(c + dx)(a + a \sec(c + dx))^2(A + B \sec(c + dx)) dx &= \frac{A \cos(c + dx) (a^2 + a^2 \sec(c + dx)) \sin(c + dx)}{2d} + \frac{1}{2} \\ &= \frac{a^2(3A + 2B) \sin(c + dx)}{2d} + \frac{A \cos(c + dx) (a^2 + a^2 \sec(c + dx)) \sin(c + dx)}{2d} \\ &= \frac{1}{2} a^2(3A + 4B)x + \frac{a^2(3A + 2B) \sin(c + dx)}{2d} + \frac{A \cos(c + dx) (a^2 + a^2 \sec(c + dx)) \sin(c + dx)}{2d} \\ &= \frac{1}{2} a^2(3A + 4B)x + \frac{a^2 B \tanh^{-1}(\sin(c + dx))}{d} + \frac{a^2(3A + 2B) \sin(c + dx)}{2d} \end{aligned}$$

Mathematica [A] time = 0.16, size = 96, normalized size = 1.09

$$\frac{a^2 \left(4(2A + B) \sin(c + dx) + A \sin(2(c + dx)) + 6Adx - 4B \log \left(\cos \left(\frac{1}{2}(c + dx) \right) - \sin \left(\frac{1}{2}(c + dx) \right) \right) + 4B \log \left(\sin \left(\frac{1}{2}(c + dx) \right) + \cos \left(\frac{1}{2}(c + dx) \right) \right) \right)}{4d}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]

[Out] (a^2*(6*A*d*x + 8*B*d*x - 4*B*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 4*B*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 4*(2*A + B)*Sin[c + d*x] + A*Sin[2*(c + d*x)])/(4*d)

fricas [A] time = 0.46, size = 79, normalized size = 0.90

$$\frac{(3A + 4B)a^2 dx + Ba^2 \log(\sin(dx + c) + 1) - Ba^2 \log(-\sin(dx + c) + 1) + (Aa^2 \cos(dx + c) + 2(2A + B)a^2) \sin(dx + c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] 1/2*((3*A + 4*B)*a^2*d*x + B*a^2*log(sin(d*x + c) + 1) - B*a^2*log(-sin(d*x + c) + 1) + (A*a^2*cos(d*x + c) + 2*(2*A + B)*a^2)*sin(d*x + c))/d

giac [A] time = 0.28, size = 145, normalized size = 1.65

$$\frac{2 B a^2 \log \left(\left| \tan \left(\frac{1}{2} d x + \frac{1}{2} c \right) + 1 \right| \right) - 2 B a^2 \log \left(\left| \tan \left(\frac{1}{2} d x + \frac{1}{2} c \right) - 1 \right| \right) + (3 A a^2 + 4 B a^2)(d x + c) + \frac{2 \left(3 A a^2 \tan \left(\frac{1}{2} d x + \frac{1}{2} c \right) \right)}{2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] 1/2*(2*B*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 2*B*a^2*log(abs(tan(1/2*d*x + 1/2*c) - 1))) + (3*A*a^2 + 4*B*a^2)*(d*x + c) + 2*(3*A*a^2*tan(1/2*d*x + 1/2*c)^3 + 2*B*a^2*tan(1/2*d*x + 1/2*c)^3 + 5*A*a^2*tan(1/2*d*x + 1/2*c) + 2*B*a^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2/d

maple [A] time = 0.82, size = 108, normalized size = 1.23

$$\frac{a^2 A \cos(dx + c) \sin(dx + c)}{2d} + \frac{3a^2 Ax}{2} + \frac{3A a^2 c}{2d} + \frac{B a^2 \sin(dx + c)}{d} + \frac{2a^2 A \sin(dx + c)}{d} + 2a^2 Bx + \frac{2B a^2 c}{d} + \frac{B a^2 \ln(\sec(dx + c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^2*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x)

[Out] 1/2*a^2*A*cos(d*x+c)*sin(d*x+c)/d+3/2*a^2*A*x+3/2/d*A*a^2*c+1/d*B*a^2*sin(d*x+c)+2/d*a^2*A*sin(d*x+c)+2*a^2*B*x+2/d*B*a^2*c+1/d*B*a^2*ln(sec(d*x+c)+tan(d*x+c))

maxima [A] time = 0.34, size = 101, normalized size = 1.15

$$\frac{(2 d x + 2 c + \sin(2 d x + 2 c)) A a^2 + 4 (d x + c) A a^2 + 8 (d x + c) B a^2 + 2 B a^2 (\log(\sin(dx + c) + 1) - \log(\sin(dx + c) - 1))}{4 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*A*a^2 + 4*(d*x + c)*A*a^2 + 8*(d*x + c)*B*a^2 + 2*B*a^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 8*A*a^2*sin(d*x + c) + 4*B*a^2*sin(d*x + c))/d

mupad [B] time = 2.05, size = 141, normalized size = 1.60

$$\frac{2 A a^2 \sin(c + d x)}{d} + \frac{B a^2 \sin(c + d x)}{d} + \frac{3 A a^2 \operatorname{atan} \left(\frac{\sin \left(\frac{c}{2} + \frac{d x}{2} \right)}{\cos \left(\frac{c}{2} + \frac{d x}{2} \right)} \right)}{d} + \frac{4 B a^2 \operatorname{atan} \left(\frac{\sin \left(\frac{c}{2} + \frac{d x}{2} \right)}{\cos \left(\frac{c}{2} + \frac{d x}{2} \right)} \right)}{d} + \frac{2 B a^2 \operatorname{atanh} \left(\frac{\sin \left(\frac{c}{2} + \frac{d x}{2} \right)}{\cos \left(\frac{c}{2} + \frac{d x}{2} \right)} \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^2,x)
```

```
[Out] (2*A*a^2*sin(c + d*x))/d + (B*a^2*sin(c + d*x))/d + (3*A*a^2*atan(sin(c/2 +
(d*x)/2)/cos(c/2 + (d*x)/2)))/d + (4*B*a^2*atan(sin(c/2 + (d*x)/2)/cos(c/2
+ (d*x)/2)))/d + (2*B*a^2*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d
+ (A*a^2*sin(2*c + 2*d*x))/(4*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2 \left(\int A \cos^2(c + dx) dx + \int 2A \cos^2(c + dx) \sec(c + dx) dx + \int A \cos^2(c + dx) \sec^2(c + dx) dx + \int B \cos^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sec(d*x+c))**2*(A+B*sec(d*x+c)),x)
```

```
[Out] a**2*(Integral(A*cos(c + d*x)**2, x) + Integral(2*A*cos(c + d*x)**2*sec(c +
d*x), x) + Integral(A*cos(c + d*x)**2*sec(c + d*x)**2, x) + Integral(B*cos
(c + d*x)**2*sec(c + d*x), x) + Integral(2*B*cos(c + d*x)**2*sec(c + d*x)**
2, x) + Integral(B*cos(c + d*x)**2*sec(c + d*x)**3, x))
```

$$3.59 \quad \int \cos^3(c + dx)(a + a \sec(c + dx))^2(A + B \sec(c + dx)) dx$$

Optimal. Leaf size=102

$$\frac{2a^2(2A + 3B) \sin(c + dx)}{3d} + \frac{a^2(2A + 3B) \sin(c + dx) \cos(c + dx)}{6d} + \frac{1}{2}a^2x(2A+3B) + \frac{A \sin(c + dx) \cos^2(c + dx)(a \sec(c + dx))}{3d}$$

[Out] $1/2*a^2*(2*A+3*B)*x+2/3*a^2*(2*A+3*B)*\sin(d*x+c)/d+1/6*a^2*(2*A+3*B)*\cos(d*x+c)*\sin(d*x+c)/d+1/3*A*\cos(d*x+c)^2*(a+a*\sec(d*x+c))^2*\sin(d*x+c)/d$

Rubi [A] time = 0.15, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {4013, 3788, 2637, 4045, 8}

$$\frac{2a^2(2A + 3B) \sin(c + dx)}{3d} + \frac{a^2(2A + 3B) \sin(c + dx) \cos(c + dx)}{6d} + \frac{1}{2}a^2x(2A+3B) + \frac{A \sin(c + dx) \cos^2(c + dx)(a \sec(c + dx))}{3d}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]

[Out] $(a^2*(2*A + 3*B)*x)/2 + (2*a^2*(2*A + 3*B)*\text{Sin}[c + d*x])/(3*d) + (a^2*(2*A + 3*B)*\text{Cos}[c + d*x]*\text{Sin}[c + d*x])/(6*d) + (A*\text{Cos}[c + d*x]^2*(a + a*\text{Sec}[c + d*x])^2*\text{Sin}[c + d*x])/(3*d)$

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3788

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^2, x_Symbol] := Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 4013

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^m*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] := Simp[(A*Cot[

$e + f*x](a + b*\text{Csc}[e + f*x])^m*(d*\text{Csc}[e + f*x])^n/(f*n), x] - \text{Dist}[(a*A*m - b*B*n)/(b*d*n), \text{Int}[(a + b*\text{Csc}[e + f*x])^m*(d*\text{Csc}[e + f*x])^{n+1}, x], x] /; \text{FreeQ}\{a, b, d, e, f, A, B, m, n\}, x] \&\& \text{NeQ}[A*b - a*B, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{EqQ}[m + n + 1, 0] \&\& !\text{LeQ}[m, -1]$

Rule 4045

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.))^{(m_.)}*(\text{csc}[(e_.) + (f_.)*(x_.)]^{2*(C_.)} + (A_.)), x_Symbol] :> \text{Simp}[(A*\text{Cot}[e + f*x]*(b*\text{Csc}[e + f*x])^m)/(f*m), x] + \text{Dist}[(C*m + A*(m + 1))/(b^2*m), \text{Int}[(b*\text{Csc}[e + f*x])^{m+2}, x], x] /; \text{FreeEq}\{b, e, f, A, C\}, x] \&\& \text{NeQ}[C*m + A*(m + 1), 0] \&\& \text{LeQ}[m, -1]$

Rubi steps

$$\begin{aligned} \int \cos^3(c + dx)(a + a \sec(c + dx))^2(A + B \sec(c + dx)) dx &= \frac{A \cos^2(c + dx)(a + a \sec(c + dx))^2 \sin(c + dx)}{3d} + \frac{1}{3} \\ &= \frac{A \cos^2(c + dx)(a + a \sec(c + dx))^2 \sin(c + dx)}{3d} + \frac{1}{3} \\ &= \frac{2a^2(2A + 3B) \sin(c + dx)}{3d} + \frac{a^2(2A + 3B) \cos(c + dx)}{6d} \\ &= \frac{1}{2}a^2(2A + 3B)x + \frac{2a^2(2A + 3B) \sin(c + dx)}{3d} + \frac{a^2(2A + 3B) \cos(c + dx)}{6d} \end{aligned}$$

Mathematica [A] time = 0.17, size = 61, normalized size = 0.60

$$\frac{a^2(3(7A + 8B) \sin(c + dx) + 3(2A + B) \sin(2(c + dx)) + A \sin(3(c + dx))) + 12Adx + 18Bdx}{12d}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]), x]

[Out] (a^2*(12*A*d*x + 18*B*d*x + 3*(7*A + 8*B)*Sin[c + d*x] + 3*(2*A + B)*Sin[2*(c + d*x)] + A*Ssin[3*(c + d*x)]))/(12*d)

fricas [A] time = 0.44, size = 70, normalized size = 0.69

$$\frac{3(2A + 3B)a^2 dx + (2Aa^2 \cos(dx + c))^2 + 3(2A + B)a^2 \cos(dx + c) + 2(5A + 6B)a^2 \sin(dx + c)}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] 1/6*(3*(2*A + 3*B)*a^2*d*x + (2*A*a^2*cos(d*x + c)^2 + 3*(2*A + B)*a^2*cos(d*x + c) + 2*(5*A + 6*B)*a^2)*sin(d*x + c))/d

giac [A] time = 0.55, size = 142, normalized size = 1.39

$$3(2Aa^2 + 3Ba^2)(dx + c) + \frac{2\left(6Aa^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5 + 9Ba^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5 + 16Aa^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 + 24Ba^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 + 18Aa^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3\right)}{\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + 1\right)^3}$$

$6d$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] 1/6*(3*(2*A*a^2 + 3*B*a^2)*(d*x + c) + 2*(6*A*a^2*tan(1/2*d*x + 1/2*c)^5 + 9*B*a^2*tan(1/2*d*x + 1/2*c)^5 + 16*A*a^2*tan(1/2*d*x + 1/2*c)^3 + 24*B*a^2*tan(1/2*d*x + 1/2*c)^3 + 18*A*a^2*tan(1/2*d*x + 1/2*c) + 15*B*a^2*tan(1/2*d*x + 1/2*c)))/(tan(1/2*d*x + 1/2*c)^2 + 1)^3/d

maple [A] time = 1.05, size = 116, normalized size = 1.14

$$\frac{a^2 A (2 + \cos^2(dx+c)) \sin(dx+c)}{3} + 2a^2 A \left(\frac{\cos(dx+c) \sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2} \right) + B a^2 \left(\frac{\cos(dx+c) \sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2} \right) + a^2 A \sin(dx+c)$$

d

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^3*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x)

[Out] 1/d*(1/3*a^2*A*(2+cos(d*x+c)^2)*sin(d*x+c)+2*a^2*A*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+B*a^2*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+a^2*A*sin(d*x+c)+2*B*a^2*sin(d*x+c)+B*a^2*(d*x+c))

maxima [A] time = 0.33, size = 110, normalized size = 1.08

$$\frac{4(\sin(dx+c)^3 - 3\sin(dx+c))Aa^2 - 6(2dx+2c+\sin(2dx+2c))Aa^2 - 3(2dx+2c+\sin(2dx+2c))Ba^2}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] $-1/12*(4*(\sin(dx + c))^3 - 3*\sin(dx + c))*A*a^2 - 6*(2*dx + 2*c + \sin(2*d*x + 2*c))*A*a^2 - 3*(2*d*x + 2*c + \sin(2*d*x + 2*c))*B*a^2 - 12*(dx + c)*B*a^2 - 12*A*a^2*\sin(dx + c) - 24*B*a^2*\sin(dx + c))/d$

mupad [B] time = 1.89, size = 98, normalized size = 0.96

$$A a^2 x + \frac{3 B a^2 x}{2} + \frac{7 A a^2 \sin(c + dx)}{4 d} + \frac{2 B a^2 \sin(c + dx)}{d} + \frac{A a^2 \sin(2 c + 2 d x)}{2 d} + \frac{A a^2 \sin(3 c + 3 d x)}{12 d} + \frac{B a^2 \sin(2 c + 2 d x)}{4 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(c + d*x)^3*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^2,x)`

[Out] $A*a^2*x + (3*B*a^2*x)/2 + (7*A*a^2*\sin(c + d*x))/(4*d) + (2*B*a^2*\sin(c + d*x))/d + (A*a^2*\sin(2*c + 2*d*x))/(2*d) + (A*a^2*\sin(3*c + 3*d*x))/(12*d) + (B*a^2*\sin(2*c + 2*d*x))/(4*d)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2 \left(\int A \cos^3(c + dx) dx + \int 2A \cos^3(c + dx) \sec(c + dx) dx + \int A \cos^3(c + dx) \sec^2(c + dx) dx + \int B \cos^3(c + dx) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)**3*(a+a*sec(d*x+c))**2*(A+B*sec(d*x+c)),x)`

[Out] $a**2*(Integral(A*cos(c + d*x)**3, x) + Integral(2*A*cos(c + d*x)**3*sec(c + d*x), x) + Integral(A*cos(c + d*x)**3*sec(c + d*x)**2, x) + Integral(B*cos(c + d*x)**3*sec(c + d*x), x) + Integral(2*B*cos(c + d*x)**3*sec(c + d*x)**2, x) + Integral(B*cos(c + d*x)**3*sec(c + d*x)**3, x))$

$$3.60 \quad \int \cos^4(c + dx)(a + a \sec(c + dx))^2(A + B \sec(c + dx)) dx$$

Optimal. Leaf size=135

$$\frac{a^2(4A + 5B) \sin(c + dx)}{3d} + \frac{a^2(5A + 4B) \sin(c + dx) \cos^2(c + dx)}{12d} + \frac{a^2(7A + 8B) \sin(c + dx) \cos(c + dx)}{8d} + \frac{1}{8}a^2x(7A$$

[Out] $1/8*a^2*(7*A+8*B)*x+1/3*a^2*(4*A+5*B)*\sin(d*x+c)/d+1/8*a^2*(7*A+8*B)*\cos(d*x+c)*\sin(d*x+c)/d+1/12*a^2*(5*A+4*B)*\cos(d*x+c)^2*\sin(d*x+c)/d+1/4*A*\cos(d*x+c)^3*(a^2+a^2*\sec(d*x+c))*\sin(d*x+c)/d$

Rubi [A] time = 0.23, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4017, 3996, 3787, 2635, 8, 2637}

$$\frac{a^2(4A + 5B) \sin(c + dx)}{3d} + \frac{a^2(5A + 4B) \sin(c + dx) \cos^2(c + dx)}{12d} + \frac{a^2(7A + 8B) \sin(c + dx) \cos(c + dx)}{8d} + \frac{1}{8}a^2x(7A$$

Antiderivative was successfully verified.

[In] `Int[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]`

[Out] $(a^2*(7*A + 8*B)*x)/8 + (a^2*(4*A + 5*B)*\sin[c + d*x])/(3*d) + (a^2*(7*A + 8*B)*\cos[c + d*x]*\sin[c + d*x])/(8*d) + (a^2*(5*A + 4*B)*\cos[c + d*x]^2*\sin[c + d*x])/(12*d) + (A*\cos[c + d*x]^3*(a^2 + a^2*\sec[c + d*x])*\sin[c + d*x])/(4*d)$

Rule 8

`Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

Rule 2635

`Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x])*(b*Ssin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Ssin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]`

Rule 2637

`Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]`

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 3996

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]

Rule 4017

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> Simp[(a*A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && LtQ[n, -1]

Rubi steps

$$\begin{aligned}
 \int \cos^4(c + dx)(a + a \sec(c + dx))^2(A + B \sec(c + dx)) dx &= \frac{A \cos^3(c + dx) (a^2 + a^2 \sec(c + dx)) \sin(c + dx)}{4d} + \\
 &= \frac{a^2(5A + 4B) \cos^2(c + dx) \sin(c + dx)}{12d} + \frac{A \cos^3(c + dx)}{4d} + \\
 &= \frac{a^2(5A + 4B) \cos^2(c + dx) \sin(c + dx)}{12d} + \frac{A \cos^3(c + dx)}{4d} + \\
 &= \frac{a^2(4A + 5B) \sin(c + dx)}{3d} + \frac{a^2(7A + 8B) \cos(c + dx)}{8d} + \\
 &= \frac{1}{8} a^2(7A + 8B)x + \frac{a^2(4A + 5B) \sin(c + dx)}{3d} + \frac{a^2(7A + 8B) \cos(c + dx)}{8d}
 \end{aligned}$$

Mathematica [A] time = 0.35, size = 86, normalized size = 0.64

$$\frac{a^2(24(6A + 7B) \sin(c + dx) + 48(A + B) \sin(2(c + dx)) + 16A \sin(3(c + dx)) + 3A \sin(4(c + dx)) + 84Ac + 84A^2)}{96d}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]

[Out] (a^2*(84*A*c + 84*A*d*x + 96*B*d*x + 24*(6*A + 7*B)*Sin[c + d*x] + 48*(A + B)*Sin[2*(c + d*x)] + 16*A*Sin[3*(c + d*x)] + 8*B*Sin[3*(c + d*x)] + 3*A*Sin[4*(c + d*x)]))/(96*d)

fricas [A] time = 0.46, size = 90, normalized size = 0.67

$$\frac{3(7A + 8B)a^2 dx + (6Aa^2 \cos(dx + c)^3 + 8(2A + B)a^2 \cos(dx + c)^2 + 3(7A + 8B)a^2 \cos(dx + c) + 8(4A + 5B)a^2 \sin(dx + c))}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] 1/24*(3*(7*A + 8*B)*a^2*d*x + (6*A*a^2*cos(d*x + c)^3 + 8*(2*A + B)*a^2*cos(d*x + c)^2 + 3*(7*A + 8*B)*a^2*cos(d*x + c) + 8*(4*A + 5*B)*a^2)*sin(d*x + c))/d

giac [A] time = 0.29, size = 176, normalized size = 1.30

$$3(7Aa^2 + 8Ba^2)(dx + c) + \frac{2\left(21Aa^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^7 + 24Ba^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^7 + 77Aa^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5 + 88Ba^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5 + 83Aa^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 + 136Ba^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 + 75Aa^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 72Ba^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + 1\right)^4} \cdot \frac{1}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] 1/24*(3*(7*A*a^2 + 8*B*a^2)*(d*x + c) + 2*(21*A*a^2*tan(1/2*d*x + 1/2*c)^7 + 24*B*a^2*tan(1/2*d*x + 1/2*c)^7 + 77*A*a^2*tan(1/2*d*x + 1/2*c)^5 + 88*B*a^2*tan(1/2*d*x + 1/2*c)^5 + 83*A*a^2*tan(1/2*d*x + 1/2*c)^3 + 136*B*a^2*tan(1/2*d*x + 1/2*c)^3 + 75*A*a^2*tan(1/2*d*x + 1/2*c) + 72*B*a^2*tan(1/2*d*x + 1/2*c)))/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d

maple [A] time = 1.28, size = 154, normalized size = 1.14

$$\frac{a^2 A \left(\frac{\left(\cos^3(dx+c) + \frac{3 \cos(dx+c)}{2} \right) \sin(dx+c)}{4} + \frac{3dx}{8} + \frac{3c}{8} \right) + \frac{B a^2 (2 + \cos^2(dx+c)) \sin(dx+c)}{3} + \frac{2a^2 A (2 + \cos^2(dx+c)) \sin(dx+c)}{3} + 2B a^2 \left(\frac{\cos(dx+c)}{2} \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(d*x+c)^4*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x)`

[Out] $\frac{1}{d} \left(a^2 A \left(\frac{1}{4} (\cos(d*x+c)^3 + \frac{3}{2} \cos(d*x+c)) \sin(d*x+c) + \frac{3}{8} d*x + \frac{3}{8} c \right) + \frac{1}{3} B a^2 (2 + \cos(d*x+c)^2) \sin(d*x+c) + \frac{2}{3} a^2 A (2 + \cos(d*x+c)^2) \sin(d*x+c) + 2 B a^2 \left(\frac{1}{2} \cos(d*x+c) \sin(d*x+c) + \frac{1}{2} d*x + \frac{1}{2} c \right) + a^2 A \left(\frac{1}{2} \cos(d*x+c) \sin(d*x+c) + \frac{1}{2} d*x + \frac{1}{2} c \right) + B a^2 \sin(d*x+c) \right)$

maxima [A] time = 0.34, size = 144, normalized size = 1.07

$$\frac{64 (\sin(dx+c)^3 - 3 \sin(dx+c)) A a^2 - 3 (12 dx + 12 c + \sin(4 dx + 4 c) + 8 \sin(2 dx + 2 c)) A a^2 - 24 (2 dx + 2 c + \sin(2 dx + 2 c)) B a^2 + 32 (\sin(dx+c)^3 - 3 \sin(dx+c)) B a^2 - 48 (2 dx + 2 c + \sin(2 dx + 2 c)) B a^2 - 96 B a^2 \sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="maxima")`

[Out] $-1/96 * (64 * (\sin(dx+c)^3 - 3 * \sin(dx+c)) * A * a^2 - 3 * (12 * dx + 12 * c + \sin(4 * dx + 4 * c) + 8 * \sin(2 * dx + 2 * c)) * A * a^2 - 24 * (2 * dx + 2 * c + \sin(2 * dx + 2 * c)) * B * a^2 + 32 * (\sin(dx+c)^3 - 3 * \sin(dx+c)) * B * a^2 - 48 * (2 * dx + 2 * c + \sin(2 * dx + 2 * c)) * B * a^2 - 96 * B * a^2 * \sin(dx+c)) / d$

mupad [B] time = 1.93, size = 134, normalized size = 0.99

$$\frac{7 A a^2 x}{8} + B a^2 x + \frac{3 A a^2 \sin(c+dx)}{2d} + \frac{7 B a^2 \sin(c+dx)}{4d} + \frac{A a^2 \sin(2c+2dx)}{2d} + \frac{A a^2 \sin(3c+3dx)}{6d} + \frac{A a^2 \sin(4c+4dx)}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(c+d*x)^4*(A+B/cos(c+d*x))*(a+a/cos(c+d*x))^2,x)`

[Out] $(7 * A * a^2 * x) / 8 + B * a^2 * x + (3 * A * a^2 * \sin(c+dx)) / (2 * d) + (7 * B * a^2 * \sin(c+dx)) / (4 * d) + (A * a^2 * \sin(2 * c + 2 * dx)) / (2 * d) + (A * a^2 * \sin(3 * c + 3 * dx)) / (6 * d) + (A * a^2 * \sin(4 * c + 4 * dx)) / (12 * d) + (B * a^2 * \sin(2 * c + 2 * dx)) / (2 * d) + (B * a^2 * \sin(3 * c + 3 * dx)) / (6 * d)$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)**4*(a+a*sec(d*x+c))**2*(A+B*sec(d*x+c)),x)`

[Out] Timed out

3.61 $\int \cos^5(c + dx)(a + a \sec(c + dx))^2(A + B \sec(c + dx)) dx$

Optimal. Leaf size=160

$$-\frac{a^2(9A + 10B) \sin^3(c + dx)}{15d} + \frac{a^2(9A + 10B) \sin(c + dx)}{5d} + \frac{a^2(6A + 5B) \sin(c + dx) \cos^3(c + dx)}{20d} + \frac{a^2(6A + 7B) \sin(c + dx)}{15d}$$

[Out] $1/8*a^2*(6*A+7*B)*x+1/5*a^2*(9*A+10*B)*\sin(d*x+c)/d+1/8*a^2*(6*A+7*B)*\cos(d*x+c)*\sin(d*x+c)/d+1/20*a^2*(6*A+5*B)*\cos(d*x+c)^3*\sin(d*x+c)/d+1/5*A*\cos(d*x+c)^4*(a^2+a^2*\sec(d*x+c))*\sin(d*x+c)/d-1/15*a^2*(9*A+10*B)*\sin(d*x+c)^3/d$

Rubi [A] time = 0.25, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4017, 3996, 3787, 2633, 2635, 8}

$$-\frac{a^2(9A + 10B) \sin^3(c + dx)}{15d} + \frac{a^2(9A + 10B) \sin(c + dx)}{5d} + \frac{a^2(6A + 5B) \sin(c + dx) \cos^3(c + dx)}{20d} + \frac{a^2(6A + 7B) \sin(c + dx)}{15d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Cos}[c + d*x]^5*(a + a*\text{Sec}[c + d*x])^2*(A + B*\text{Sec}[c + d*x]), x]$

[Out] $(a^2*(6*A + 7*B)*x)/8 + (a^2*(9*A + 10*B)*\text{Sin}[c + d*x])/(5*d) + (a^2*(6*A + 7*B)*\text{Cos}[c + d*x]*\text{Sin}[c + d*x])/(8*d) + (a^2*(6*A + 5*B)*\text{Cos}[c + d*x]^3*\text{Sin}[c + d*x])/(20*d) + (A*\text{Cos}[c + d*x]^4*(a^2 + a^2*\text{Sec}[c + d*x])*\text{Sin}[c + d*x])/(5*d) - (a^2*(9*A + 10*B)*\text{Sin}[c + d*x]^3)/(15*d)$

Rule 8

$\text{Int}[a_, x_Symbol] := \text{Simp}[a*x, x] /; \text{FreeQ}[a, x]$

Rule 2633

$\text{Int}[\sin[(c_.) + (d_.)*(x_)]^{(n_)}, x_Symbol] := -\text{Dist}[d^{(-1)}, \text{Subst}[\text{Int}[\text{Expand}[(1 - x^2)^{((n - 1)/2)}, x], x], x, \text{Cos}[c + d*x]], x] /; \text{FreeQ}\{c, d\}, x] \&\& \text{IGtQ}[(n - 1)/2, 0]$

Rule 2635

$\text{Int}[(b_.)*\sin[(c_.) + (d_.)*(x_)]^{(n_)}, x_Symbol] := -\text{Simp}[(b*\text{Cos}[c + d*x])*(b*\text{Sin}[c + d*x])^{(n - 1)}]/(d*n), x] + \text{Dist}[(b^2*(n - 1))/n, \text{Int}[(b*\text{Sin}[c + d*x])^{(n - 2)}, x], x] /; \text{FreeQ}\{b, c, d\}, x] \&\& \text{GtQ}[n, 1] \&\& \text{IntegerQ}[2*n]$

]

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
+ 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (
a_)^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \cos^5(c + dx)(a + a \sec(c + dx))^2(A + B \sec(c + dx)) dx &= \frac{A \cos^4(c + dx) (a^2 + a^2 \sec(c + dx)) \sin(c + dx)}{5d} + \\
&= \frac{a^2(6A + 5B) \cos^3(c + dx) \sin(c + dx)}{20d} + \frac{A \cos^4(c + dx)}{5d} + \\
&= \frac{a^2(6A + 5B) \cos^3(c + dx) \sin(c + dx)}{20d} + \frac{A \cos^4(c + dx)}{5d} + \\
&= \frac{a^2(6A + 7B) \cos(c + dx) \sin(c + dx)}{8d} + \frac{a^2(6A + 5B)}{5d} + \\
&= \frac{1}{8} a^2(6A + 7B)x + \frac{a^2(9A + 10B) \sin(c + dx)}{5d} + \frac{a^2(6A + 5B)}{5d}
\end{aligned}$$

Mathematica [A] time = 0.45, size = 108, normalized size = 0.68

$$\frac{a^2(60(11A + 12B) \sin(c + dx) + 240(A + B) \sin(2(c + dx)) + 90A \sin(3(c + dx)) + 30A \sin(4(c + dx)) + 6A \sin(5(c + dx)))}{480d}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]^5*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]

[Out] (a^2*(360*A*c + 360*A*d*x + 420*B*d*x + 60*(11*A + 12*B)*Sin[c + d*x] + 240*(A + B)*Sin[2*(c + d*x)] + 90*A*Sin[3*(c + d*x)] + 80*B*Sin[3*(c + d*x)] + 30*A*Sin[4*(c + d*x)] + 15*B*Sin[4*(c + d*x)] + 6*A*Sin[5*(c + d*x)])/(480*d)

fricas [A] time = 0.45, size = 110, normalized size = 0.69

$$\frac{15(6A + 7B)a^2 dx + (24Aa^2 \cos(dx + c)^4 + 30(2A + B)a^2 \cos(dx + c)^3 + 8(9A + 10B)a^2 \cos(dx + c)^2 + 15(6A + 7B)a^2 \cos(dx + c))}{120d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] 1/120*(15*(6*A + 7*B)*a^2*d*x + (24*A*a^2*cos(d*x + c)^4 + 30*(2*A + B)*a^2*cos(d*x + c)^3 + 8*(9*A + 10*B)*a^2*cos(d*x + c)^2 + 15*(6*A + 7*B)*a^2*cos(d*x + c) + 16*(9*A + 10*B)*a^2*sin(d*x + c))/d

giac [A] time = 0.33, size = 210, normalized size = 1.31

$$15(6Aa^2 + 7Ba^2)(dx + c) + \frac{2\left(90Aa^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^9 + 105Ba^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^9 + 420Aa^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^7 + 490Ba^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^7 + 864Aa^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5 + 800Ba^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5 + 540Aa^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 + 790Ba^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 + 390Aa^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 375Ba^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right)}{(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + 1)^5}/d$$

120

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] 1/120*(15*(6*A*a^2 + 7*B*a^2)*(d*x + c) + 2*(90*A*a^2*tan(1/2*d*x + 1/2*c)^9 + 105*B*a^2*tan(1/2*d*x + 1/2*c)^9 + 420*A*a^2*tan(1/2*d*x + 1/2*c)^7 + 490*B*a^2*tan(1/2*d*x + 1/2*c)^7 + 864*A*a^2*tan(1/2*d*x + 1/2*c)^5 + 800*B*a^2*tan(1/2*d*x + 1/2*c)^5 + 540*A*a^2*tan(1/2*d*x + 1/2*c)^3 + 790*B*a^2*tan(1/2*d*x + 1/2*c)^3 + 390*A*a^2*tan(1/2*d*x + 1/2*c) + 375*B*a^2*tan(1/2*d*x + 1/2*c)))/(tan(1/2*d*x + 1/2*c)^2 + 1)^5)/d

maple [A] time = 1.62, size = 186, normalized size = 1.16

$$\frac{a^2 A \left(\frac{8}{3} + \cos^4(dx+c) + \frac{4(\cos^2(dx+c))}{3} \right) \sin(dx+c)}{5} + B a^2 \left(\frac{(\cos^3(dx+c) + \frac{3\cos(dx+c)}{2}) \sin(dx+c)}{4} + \frac{3dx}{8} + \frac{3c}{8} \right) + 2a^2 A \left(\frac{(\cos^3(dx+c) + \frac{3\cos(dx+c)}{2}) \sin(dx+c)}{4} \right)$$

d

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^5*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x)

[Out] 1/d*(1/5*a^2*A*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+B*a^2*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+2*a^2*A*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+2/3*B*a^2*(2+cos(d*x+c)^2)*sin(d*x+c)+1/3*a^2*A*(2+cos(d*x+c)^2)*sin(d*x+c)+B*a^2*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))

maxima [A] time = 0.34, size = 178, normalized size = 1.11

$$32 \left(3 \sin(dx+c)^5 - 10 \sin(dx+c)^3 + 15 \sin(dx+c) \right) A a^2 - 160 \left(\sin(dx+c)^3 - 3 \sin(dx+c) \right) A a^2 + 30 (12 dx + 12c + \sin(4dx + 4c) + 8 \sin(2dx + 2c)) B a^2 - 320 \left(\sin(dx+c)^3 - 3 \sin(dx+c) \right) B a^2 + 120 (2dx + 2c + \sin(2dx + 2c)) B a^2 / d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] 1/480*(32*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*A*a^2 - 160*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^2 + 30*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^2 - 320*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^2 + 15*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^2 + 120*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^2)/d

mupad [B] time = 4.68, size = 247, normalized size = 1.54

$$\frac{\left(\frac{3Aa^2}{2} + \frac{7Ba^2}{4} \right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^9 + \left(7Aa^2 + \frac{49Ba^2}{6} \right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^7 + \left(\frac{72Aa^2}{5} + \frac{40Ba^2}{3} \right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5 + \left(9Aa^2 + \frac{27Ba^2}{2} \right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 + \left(\frac{3Aa^2}{2} + \frac{7Ba^2}{4} \right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^{10} + 5 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^8 + 10 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^6 + 10 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 + 5 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + 1 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^5*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^2,x)

[Out] (tan(c/2 + (d*x)/2)*((13*A*a^2)/2 + (25*B*a^2)/4) + tan(c/2 + (d*x)/2)^9*((3*A*a^2)/2 + (7*B*a^2)/4) + tan(c/2 + (d*x)/2)^7*(7*A*a^2 + (49*B*a^2)/6) + tan(c/2 + (d*x)/2)^5*(9*A*a^2 + (27*B*a^2)/2) + tan(c/2 + (d*x)/2)^3*(3*A*a^2 + (9*B*a^2)/2) + tan(c/2 + (d*x)/2)*((3*A*a^2)/2 + (7*B*a^2)/4)) / (tan(c/2 + (d*x)/2)^10 + 5*tan(c/2 + (d*x)/2)^8 + 10*tan(c/2 + (d*x)/2)^6 + 10*tan(c/2 + (d*x)/2)^4 + 5*tan(c/2 + (d*x)/2)^2 + 1)

$$\tan(c/2 + (d*x)/2)^3*(9*A*a^2 + (79*B*a^2)/6) + \tan(c/2 + (d*x)/2)^5*((72*A*a^2)/5 + (40*B*a^2)/3)/(d*(5*\tan(c/2 + (d*x)/2)^2 + 10*\tan(c/2 + (d*x)/2)^4 + 10*\tan(c/2 + (d*x)/2)^6 + 5*\tan(c/2 + (d*x)/2)^8 + \tan(c/2 + (d*x)/2)^{10} + 1)) + (a^2*\operatorname{atan}((a^2*\tan(c/2 + (d*x)/2)*(6*A + 7*B))/(4*((3*A*a^2)/2 + (7*B*a^2)/4)))*(6*A + 7*B))/(4*d)$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**5*(a+a*sec(d*x+c))**2*(A+B*sec(d*x+c)),x)

[Out] Timed out

3.62 $\int \sec^3(c + dx)(a + a \sec(c + dx))^3(A + B \sec(c + dx)) dx$

Optimal. Leaf size=210

$$\frac{a^3(19A + 17B) \tan^3(c + dx)}{15d} + \frac{a^3(19A + 17B) \tan(c + dx)}{5d} + \frac{a^3(26A + 23B) \tanh^{-1}(\sin(c + dx))}{16d} + \frac{a^3(22A + 21B)}{16d}$$

[Out] $1/16*a^3*(26*A+23*B)*\arctanh(\sin(d*x+c))/d+1/5*a^3*(19*A+17*B)*\tan(d*x+c)/d+1/16*a^3*(26*A+23*B)*\sec(d*x+c)*\tan(d*x+c)/d+1/40*a^3*(22*A+21*B)*\sec(d*x+c)^3*\tan(d*x+c)/d+1/6*a*B*\sec(d*x+c)^3*(a+a*\sec(d*x+c))^2*\tan(d*x+c)/d+1/15*(3*A+4*B)*\sec(d*x+c)^3*(a^3+a^3*\sec(d*x+c))*\tan(d*x+c)/d+1/15*a^3*(19*A+17*B)*\tan(d*x+c)^3/d$

Rubi [A] time = 0.40, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4018, 3997, 3787, 3768, 3770, 3767}

$$\frac{a^3(19A + 17B) \tan^3(c + dx)}{15d} + \frac{a^3(19A + 17B) \tan(c + dx)}{5d} + \frac{a^3(26A + 23B) \tanh^{-1}(\sin(c + dx))}{16d} + \frac{a^3(22A + 21B)}{16d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sec}[c + d*x]^3*(a + a*\text{Sec}[c + d*x])^3*(A + B*\text{Sec}[c + d*x]), x]$

[Out] $(a^3*(26*A + 23*B)*\text{ArcTanh}[\text{Sin}[c + d*x]])/(16*d) + (a^3*(19*A + 17*B)*\text{Tan}[c + d*x])/(5*d) + (a^3*(26*A + 23*B)*\text{Sec}[c + d*x]*\text{Tan}[c + d*x])/(16*d) + (a^3*(22*A + 21*B)*\text{Sec}[c + d*x]^3*\text{Tan}[c + d*x])/(40*d) + (a*B*\text{Sec}[c + d*x]^3*(a + a*\text{Sec}[c + d*x])^2*\text{Tan}[c + d*x])/(6*d) + ((3*A + 4*B)*\text{Sec}[c + d*x]^3*(a^3 + a^3*\text{Sec}[c + d*x])*\text{Tan}[c + d*x])/(15*d) + (a^3*(19*A + 17*B)*\text{Tan}[c + d*x]^3)/(15*d)$

Rule 3767

$\text{Int}[\text{csc}[(c_.) + (d_.)*(x_)]^{(n_)}, x_Symbol] \rightarrow -\text{Dist}[d^{(-1)}, \text{Subst}[\text{Int}[\text{ExpandIntegrand}[(1 + x^2)^{(n/2 - 1)}, x], x], x, \text{Cot}[c + d*x]], x] /;$ $\text{FreeQ}\{c, d\}, x \ \&\& \ \text{IGtQ}[n/2, 0]$

Rule 3768

$\text{Int}[(\text{csc}[(c_.) + (d_.)*(x_)]*(b_.)^{(n_)}), x_Symbol] \rightarrow -\text{Simp}[(b*\text{Cos}[c + d*x]*(b*\text{Csc}[c + d*x])^{(n - 1)})/(d*(n - 1)), x] + \text{Dist}[(b^2*(n - 2))/(n - 1), \text{Int}[(b*\text{Csc}[c + d*x])^{(n - 2)}, x], x] /;$ $\text{FreeQ}\{b, c, d\}, x \ \&\& \ \text{GtQ}[n, 1] \ \&\& \ \text{IntegerQ}[2*n]$

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x]
  /; FreeQ[{c, d}, x]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)])*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)])*(b_.) +
(a_), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)])*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)])*(b_.) +
(a_)*(csc[(e_.) + (f_.)*(x_)])*(B_.) + (A_)), x_Symbol] := -Simp[(b*B*Cot[
e + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && !LeQ[n,
-1]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)])*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)])*(b_.) +
(a_)^(m_)*(csc[(e_.) + (f_.)*(x_)])*(B_.) + (A_)), x_Symbol] := -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && !LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \sec^3(c + dx)(a + a \sec(c + dx))^3(A + B \sec(c + dx)) dx &= \frac{aB \sec^3(c + dx)(a + a \sec(c + dx))^2 \tan(c + dx)}{6d} + \frac{1}{6} \\
&= \frac{aB \sec^3(c + dx)(a + a \sec(c + dx))^2 \tan(c + dx)}{6d} + \frac{1}{6} \\
&= \frac{a^3(22A + 21B) \sec^3(c + dx) \tan(c + dx)}{40d} + \frac{aB \sec^3(c + dx)}{6} \\
&= \frac{a^3(22A + 21B) \sec^3(c + dx) \tan(c + dx)}{40d} + \frac{aB \sec^3(c + dx)}{6} \\
&= \frac{a^3(26A + 23B) \sec(c + dx) \tan(c + dx)}{16d} + \frac{a^3(22A + 21B)}{40d} \\
&= \frac{a^3(26A + 23B) \tanh^{-1}(\sin(c + dx))}{16d} + \frac{a^3(19A + 17B)}{40d}
\end{aligned}$$

Mathematica [A] time = 1.89, size = 346, normalized size = 1.65

$$\frac{a^3(\cos(c + dx) + 1)^3 \sec^6\left(\frac{1}{2}(c + dx)\right) \sec^6(c + dx) \left(480(26A + 23B) \cos^6(c + dx) \left(\log\left(\cos\left(\frac{1}{2}(c + dx)\right) - \sin\left(\frac{1}{2}(c + dx)\right)\right)\right) - \sin\left(\frac{1}{2}(c + dx)\right)\right)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]

[Out] -1/61440*(a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*Sec[c + d*x]^6*(480*(26*A + 23*B)*Cos[c + d*x]^6*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - Sec[c]*(-320*(19*A + 17*B)*Sin[c] + 750*(2*A + 3*B)*Sin[d*x] + 1500*A*Sin[2*c + d*x] + 2250*B*Sin[2*c + d*x] + 7680*A*Sin[c + 2*d*x] + 7680*B*Sin[c + 2*d*x] - 1440*A*Sin[3*c + 2*d*x] - 480*B*Sin[3*c + 2*d*x] + 1890*A*Sin[2*c + 3*d*x] + 1955*B*Sin[2*c + 3*d*x] + 1890*A*Sin[4*c + 3*d*x] + 1955*B*Sin[4*c + 3*d*x] + 3648*A*Sin[3*c + 4*d*x] + 3264*B*Sin[3*c + 4*d*x] + 390*A*Sin[4*c + 5*d*x] + 345*B*Sin[4*c + 5*d*x] + 390*A*Sin[6*c + 5*d*x] + 345*B*Sin[6*c + 5*d*x] + 608*A*Sin[5*c + 6*d*x] + 544*B*Sin[5*c + 6*d*x])))/d

fricas [A] time = 0.45, size = 185, normalized size = 0.88

$$\frac{15(26A + 23B)a^3 \cos(dx + c)^6 \log(\sin(dx + c) + 1) - 15(26A + 23B)a^3 \cos(dx + c)^6 \log(-\sin(dx + c) + 1)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] $\frac{1}{480}*(15*(26*A + 23*B)*a^3*\cos(d*x + c)^6*\log(\sin(d*x + c) + 1) - 15*(26*A + 23*B)*a^3*\cos(d*x + c)^6*\log(-\sin(d*x + c) + 1) + 2*(32*(19*A + 17*B)*a^3*\cos(d*x + c)^5 + 15*(26*A + 23*B)*a^3*\cos(d*x + c)^4 + 16*(19*A + 17*B)*a^3*\cos(d*x + c)^3 + 10*(18*A + 23*B)*a^3*\cos(d*x + c)^2 + 48*(A + 3*B)*a^3*\cos(d*x + c) + 40*B*a^3)*\sin(d*x + c))/(d*\cos(d*x + c)^6)$

giac [A] time = 1.45, size = 280, normalized size = 1.33

$$15 \left(26 A a^3 + 23 B a^3 \right) \log \left(\left| \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) + 1 \right| \right) - 15 \left(26 A a^3 + 23 B a^3 \right) \log \left(\left| \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) - 1 \right| \right) - \frac{2 \left(390 A a^3 \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) \right)}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] $\frac{1}{240}*(15*(26*A*a^3 + 23*B*a^3)*\log(\text{abs}(\tan(1/2*d*x + 1/2*c) + 1)) - 15*(26*A*a^3 + 23*B*a^3)*\log(\text{abs}(\tan(1/2*d*x + 1/2*c) - 1)) - 2*(390*A*a^3*\tan(1/2*d*x + 1/2*c)^{11} + 345*B*a^3*\tan(1/2*d*x + 1/2*c)^{11} - 2210*A*a^3*\tan(1/2*d*x + 1/2*c)^9 - 1955*B*a^3*\tan(1/2*d*x + 1/2*c)^9 + 5148*A*a^3*\tan(1/2*d*x + 1/2*c)^7 + 4554*B*a^3*\tan(1/2*d*x + 1/2*c)^7 - 5988*A*a^3*\tan(1/2*d*x + 1/2*c)^5 - 5814*B*a^3*\tan(1/2*d*x + 1/2*c)^5 + 4190*A*a^3*\tan(1/2*d*x + 1/2*c)^3 + 3165*B*a^3*\tan(1/2*d*x + 1/2*c)^3 - 1530*A*a^3*\tan(1/2*d*x + 1/2*c) - 1575*B*a^3*\tan(1/2*d*x + 1/2*c)))/(\tan(1/2*d*x + 1/2*c)^2 - 1)^6)/d$

maple [A] time = 1.63, size = 281, normalized size = 1.34

$$\frac{13A a^3 \sec(dx + c) \tan(dx + c)}{8d} + \frac{13A a^3 \ln(\sec(dx + c) + \tan(dx + c))}{8d} + \frac{34a^3 B \tan(dx + c)}{15d} + \frac{17a^3 B \tan(dx + c)}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^3*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x)

[Out] $\frac{13}{8}/d*A*a^3*\sec(d*x+c)*\tan(d*x+c)+\frac{13}{8}/d*A*a^3*\ln(\sec(d*x+c)+\tan(d*x+c))+\frac{3}{4}/d*a^3*B*\tan(d*x+c)+\frac{17}{15}/d*a^3*B*\tan(d*x+c)*\sec(d*x+c)^2+\frac{38}{15}/d*A*a^3*\tan(d*x+c)+\frac{19}{15}/d*A*a^3*\tan(d*x+c)*\sec(d*x+c)^2+\frac{23}{24}/d*a^3*B*\tan(d*x+c)*\sec(d*x+c)^3+\frac{23}{16}/d*a^3*B*\sec(d*x+c)*\tan(d*x+c)+\frac{23}{16}/d*a^3*B*\ln(\sec(d*x+c)+\tan(d*x+c))+\frac{3}{4}/d*A*a^3*\tan(d*x+c)*\sec(d*x+c)^3+\frac{3}{5}/d*a^3*B*\tan(d*x+c)*\sec(d*x+c)^4+\frac{1}{5}/d*A*a^3*\tan(d*x+c)*\sec(d*x+c)^4+\frac{1}{6}/d*a^3*B*\tan(d*x+c)*\sec(d*x+c)^5$

maxima [B] time = 0.36, size = 405, normalized size = 1.93

$$32 \left(3 \tan(dx + c)^5 + 10 \tan(dx + c)^3 + 15 \tan(dx + c) \right) Aa^3 + 480 \left(\tan(dx + c)^3 + 3 \tan(dx + c) \right) Aa^3 + 96 \left(3 \tan(dx + c)^5 + 10 \tan(dx + c)^3 + 15 \tan(dx + c) \right) Ba^3 + 160 \left(\tan(dx + c)^3 + 3 \tan(dx + c) \right) Ba^3 - 5Ba^3 \left(2 \left(15 \sin(dx + c)^5 - 40 \sin(dx + c)^3 + 33 \sin(dx + c) \right) / \left(\sin(dx + c)^6 - 3 \sin(dx + c)^4 + 3 \sin(dx + c)^2 - 1 \right) - 15 \log(\sin(dx + c) + 1) + 15 \log(\sin(dx + c) - 1) \right) - 90Aa^3 \left(2 \left(3 \sin(dx + c)^3 - 5 \sin(dx + c) \right) / \left(\sin(dx + c)^4 - 2 \sin(dx + c)^2 + 1 \right) - 3 \log(\sin(dx + c) + 1) + 3 \log(\sin(dx + c) - 1) \right) - 90Ba^3 \left(2 \left(3 \sin(dx + c)^3 - 5 \sin(dx + c) \right) / \left(\sin(dx + c)^4 - 2 \sin(dx + c)^2 + 1 \right) - 3 \log(\sin(dx + c) + 1) + 3 \log(\sin(dx + c) - 1) \right) - 120Aa^3 \left(2 \sin(dx + c) / \left(\sin(dx + c)^2 - 1 \right) - \log(\sin(dx + c) + 1) + \log(\sin(dx + c) - 1) \right) / d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] 1/480*(32*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*A*a^3 + 480*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^3 + 96*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*B*a^3 + 160*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a^3 - 5*B*a^3*(2*(15*sin(d*x + c)^5 - 40*sin(d*x + c)^3 + 33*sin(d*x + c))/((sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) - 90*A*a^3*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/((sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 90*B*a^3*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/((sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 120*A*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)))/d

mupad [B] time = 4.63, size = 262, normalized size = 1.25

$$\frac{\left(-\frac{13Aa^3}{4} - \frac{23Ba^3}{8} \right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^{11} + \left(\frac{221Aa^3}{12} + \frac{391Ba^3}{24} \right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^9 + \left(-\frac{429Aa^3}{10} - \frac{759Ba^3}{20} \right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^7 + \left(\frac{499Aa^3}{10} + \frac{969Ba^3}{20} \right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5 + \left(\frac{13Aa^3}{4} + \frac{23Ba^3}{8} \right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 + \left(\frac{221Aa^3}{12} + \frac{391Ba^3}{24} \right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right) + \left(-\frac{429Aa^3}{10} - \frac{759Ba^3}{20} \right) \frac{1}{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)} + \left(\frac{499Aa^3}{10} + \frac{969Ba^3}{20} \right) \frac{1}{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3} + \left(-\frac{13Aa^3}{4} - \frac{23Ba^3}{8} \right) \frac{1}{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5} + \left(\frac{221Aa^3}{12} + \frac{391Ba^3}{24} \right) \frac{1}{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^7} + \left(-\frac{429Aa^3}{10} - \frac{759Ba^3}{20} \right) \frac{1}{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^9} + \left(\frac{499Aa^3}{10} + \frac{969Ba^3}{20} \right) \frac{1}{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^{11}}}{d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^{12} - 6 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^{10} + 15 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^8 - 20 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^6 + 15 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 - 6 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + 1 \right) + (a^3 \operatorname{atanh}(\tan(\frac{c}{2} + \frac{dx}{2}))) * (26A + 23B)) / (8*d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^3)/cos(c + d*x)^3,x)

[Out] (tan(c/2 + (d*x)/2)*((51*A*a^3)/4 + (105*B*a^3)/8) - tan(c/2 + (d*x)/2)^11*((13*A*a^3)/4 + (23*B*a^3)/8) - tan(c/2 + (d*x)/2)^3*((419*A*a^3)/12 + (211*B*a^3)/8) + tan(c/2 + (d*x)/2)^9*((221*A*a^3)/12 + (391*B*a^3)/24) - tan(c/2 + (d*x)/2)^7*((429*A*a^3)/10 + (759*B*a^3)/20) + tan(c/2 + (d*x)/2)^5*((499*A*a^3)/10 + (969*B*a^3)/20))/((d*(15*tan(c/2 + (d*x)/2)^4 - 6*tan(c/2 + (d*x)/2)^2 - 20*tan(c/2 + (d*x)/2)^6 + 15*tan(c/2 + (d*x)/2)^8 - 6*tan(c/2 + (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 + 1)) + (a^3*atanh(tan(c/2 + (d*x)/2)))*(26*A + 23*B))/(8*d)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3 \left(\int A \sec^3(c + dx) dx + \int 3A \sec^4(c + dx) dx + \int 3A \sec^5(c + dx) dx + \int A \sec^6(c + dx) dx + \int B \sec^4(c + dx) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+a*sec(d*x+c))**3*(A+B*sec(d*x+c)),x)
```

```
[Out] a**3*(Integral(A*sec(c + d*x)**3, x) + Integral(3*A*sec(c + d*x)**4, x) + I
ntegral(3*A*sec(c + d*x)**5, x) + Integral(A*sec(c + d*x)**6, x) + Integral
(B*sec(c + d*x)**4, x) + Integral(3*B*sec(c + d*x)**5, x) + Integral(3*B*se
c(c + d*x)**6, x) + Integral(B*sec(c + d*x)**7, x))
```

3.63 $\int \sec^2(c + dx)(a + a \sec(c + dx))^3(A + B \sec(c + dx)) dx$

Optimal. Leaf size=163

$$\frac{a^3(15A + 13B) \tan^3(c + dx)}{60d} + \frac{a^3(15A + 13B) \tan(c + dx)}{5d} + \frac{a^3(15A + 13B) \tanh^{-1}(\sin(c + dx))}{8d} + \frac{3a^3(15A + 13B)}{d}$$

[Out] $1/8*a^3*(15*A+13*B)*\operatorname{arctanh}(\sin(d*x+c))/d+1/5*a^3*(15*A+13*B)*\tan(d*x+c)/d+3/40*a^3*(15*A+13*B)*\sec(d*x+c)*\tan(d*x+c)/d+1/20*(5*A-B)*(a+a*\sec(d*x+c))^3*\tan(d*x+c)/d+1/5*B*(a+a*\sec(d*x+c))^4*\tan(d*x+c)/a/d+1/60*a^3*(15*A+13*B)*\tan(d*x+c)^3/d$

Rubi [A] time = 0.27, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {4010, 4001, 3791, 3770, 3767, 8, 3768}

$$\frac{a^3(15A + 13B) \tan^3(c + dx)}{60d} + \frac{a^3(15A + 13B) \tan(c + dx)}{5d} + \frac{a^3(15A + 13B) \tanh^{-1}(\sin(c + dx))}{8d} + \frac{3a^3(15A + 13B)}{d}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Sec}[c + d*x]^2*(a + a*\operatorname{Sec}[c + d*x])^3*(A + B*\operatorname{Sec}[c + d*x]), x]$

[Out] $(a^3*(15*A + 13*B)*\operatorname{ArcTanh}[\operatorname{Sin}[c + d*x]])/(8*d) + (a^3*(15*A + 13*B)*\operatorname{Tan}[c + d*x])/(5*d) + (3*a^3*(15*A + 13*B)*\operatorname{Sec}[c + d*x]*\operatorname{Tan}[c + d*x])/(40*d) + ((5*A - B)*(a + a*\operatorname{Sec}[c + d*x])^3*\operatorname{Tan}[c + d*x])/(20*d) + (B*(a + a*\operatorname{Sec}[c + d*x])^4*\operatorname{Tan}[c + d*x])/(5*a*d) + (a^3*(15*A + 13*B)*\operatorname{Tan}[c + d*x]^3)/(60*d)$

Rule 8

$\operatorname{Int}[a_, x_Symbol] \rightarrow \operatorname{Simp}[a*x, x] /; \operatorname{FreeQ}[a, x]$

Rule 3767

$\operatorname{Int}[\operatorname{csc}[(c_.) + (d_.)*(x_.)]^{(n_.)}, x_Symbol] \rightarrow -\operatorname{Dist}[d^{(-1)}, \operatorname{Subst}[\operatorname{Int}[\operatorname{ExpandIntegrand}[(1 + x^2)^{(n/2 - 1)}, x], x], x], \operatorname{Cot}[c + d*x]], x] /; \operatorname{FreeQ}[\{c, d\}, x] \ \&\& \operatorname{IGtQ}[n/2, 0]$

Rule 3768

$\operatorname{Int}[(\operatorname{csc}[(c_.) + (d_.)*(x_.)]*(b_.))^{(n_.)}, x_Symbol] \rightarrow -\operatorname{Simp}[(b*\operatorname{Cos}[c + d*x])*(b*\operatorname{Csc}[c + d*x])^{(n - 1)}]/(d*(n - 1)), x] + \operatorname{Dist}[(b^2*(n - 2))/(n - 1), \operatorname{Int}[(b*\operatorname{Csc}[c + d*x])^{(n - 2)}, x], x] /; \operatorname{FreeQ}[\{b, c, d\}, x] \ \&\& \operatorname{GtQ}[n, 1] \ \&\&$

IntegerQ[2*n]

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3791

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_), x_Symbol] := Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && !GtQ[m, 0] && RationalQ[n]

Rule 4001

Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] := -Simp[(B*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m + 1), 0] && !LtQ[m, -2^(-1)]

Rule 4010

Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] := -Simp[(B*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && !LtQ[m, -1]

Rubi steps

$$\begin{aligned}
\int \sec^2(c + dx)(a + a \sec(c + dx))^3(A + B \sec(c + dx)) dx &= \frac{B(a + a \sec(c + dx))^4 \tan(c + dx)}{5ad} + \frac{\int \sec(c + dx)(a + a \sec(c + dx))^3(A + B \sec(c + dx)) dx}{5ad} \\
&= \frac{(5A - B)(a + a \sec(c + dx))^3 \tan(c + dx)}{20d} + \frac{B(a + a \sec(c + dx))^4 \tan(c + dx)}{5ad} \\
&= \frac{(5A - B)(a + a \sec(c + dx))^3 \tan(c + dx)}{20d} + \frac{B(a + a \sec(c + dx))^4 \tan(c + dx)}{5ad} \\
&= \frac{(5A - B)(a + a \sec(c + dx))^3 \tan(c + dx)}{20d} + \frac{B(a + a \sec(c + dx))^4 \tan(c + dx)}{5ad} \\
&= \frac{a^3(15A + 13B) \tanh^{-1}(\sin(c + dx))}{20d} + \frac{3a^3(15A + 13B) \tanh^{-1}(\sin(c + dx))}{20d} \\
&= \frac{a^3(15A + 13B) \tanh^{-1}(\sin(c + dx))}{8d} + \frac{a^3(15A + 13B) \tanh^{-1}(\sin(c + dx))}{8d}
\end{aligned}$$

Mathematica [A] time = 1.46, size = 294, normalized size = 1.80

$$\frac{a^3(\cos(c + dx) + 1)^3 \sec^6\left(\frac{1}{2}(c + dx)\right) \sec^5(c + dx) \left(240(15A + 13B) \cos^5(c + dx) \left(\log\left(\cos\left(\frac{1}{2}(c + dx)\right) - \sin\left(\frac{1}{2}(c + dx)\right)\right)\right) - \log\left(\cos\left(\frac{1}{2}(c + dx)\right) + \sin\left(\frac{1}{2}(c + dx)\right)\right) - \sec[c] \left(80(30A + 29B) \sin[dx] - 240(5A + 3B) \sin[2c + dx] + 570A \sin[c + 2dx] + 750B \sin[c + 2dx] + 570A \sin[3c + 2dx] + 750B \sin[3c + 2dx] + 1680A \sin[2c + 3dx] + 1520B \sin[2c + 3dx] - 120A \sin[4c + 3dx] + 225A \sin[3c + 4dx] + 195B \sin[3c + 4dx] + 225A \sin[5c + 4dx] + 195B \sin[5c + 4dx] + 360A \sin[4c + 5dx] + 304B \sin[4c + 5dx]\right)\right)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]

[Out] -1/15360*(a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*Sec[c + d*x]^5*(240*(15*A + 13*B)*Cos[c + d*x]^5*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - Sec[c]*(80*(30*A + 29*B)*Sin[d*x] - 240*(5*A + 3*B)*Sin[2*c + d*x] + 570*A*Sin[c + 2*d*x] + 750*B*Sin[c + 2*d*x] + 570*A*Sin[3*c + 2*d*x] + 750*B*Sin[3*c + 2*d*x] + 1680*A*Sin[2*c + 3*d*x] + 1520*B*Sin[2*c + 3*d*x] - 120*A*Sin[4*c + 3*d*x] + 225*A*Sin[3*c + 4*d*x] + 195*B*Sin[3*c + 4*d*x] + 225*A*Sin[5*c + 4*d*x] + 195*B*Sin[5*c + 4*d*x] + 360*A*Sin[4*c + 5*d*x] + 304*B*Sin[4*c + 5*d*x])))/d

fricas [A] time = 0.47, size = 165, normalized size = 1.01

$$\frac{15(15A + 13B)a^3 \cos(dx + c)^5 \log(\sin(dx + c) + 1) - 15(15A + 13B)a^3 \cos(dx + c)^5 \log(-\sin(dx + c) + 1)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] $\frac{1}{240} \cdot (15 \cdot (15A + 13B) \cdot a^3 \cdot \cos(dx + c)^5 \cdot \log(\sin(dx + c) + 1) - 15 \cdot (15A + 13B) \cdot a^3 \cdot \cos(dx + c)^5 \cdot \log(-\sin(dx + c) + 1) + 2 \cdot (8 \cdot (45A + 38B) \cdot a^3 \cdot \cos(dx + c)^4 + 15 \cdot (15A + 13B) \cdot a^3 \cdot \cos(dx + c)^3 + 8 \cdot (15A + 19B) \cdot a^3 \cdot \cos(dx + c)^2 + 30 \cdot (A + 3B) \cdot a^3 \cdot \cos(dx + c) + 24 \cdot B \cdot a^3) \cdot \sin(dx + c)) / (\cos(dx + c)^5)$

giac [A] time = 0.77, size = 246, normalized size = 1.51

$$15 (15 A a^3 + 13 B a^3) \log \left(\left| \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) + 1 \right| \right) - 15 (15 A a^3 + 13 B a^3) \log \left(\left| \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) - 1 \right| \right) - \frac{2 \left(225 A a^3 \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) \right)}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(dx+c)^2*(a+a*sec(dx+c))^3*(A+B*sec(dx+c)),x, algorithm="giac")

[Out] $\frac{1}{120} \cdot (15 \cdot (15A \cdot a^3 + 13B \cdot a^3) \cdot \log(\tan(1/2 \cdot dx + 1/2 \cdot c) + 1) - 15 \cdot (15A \cdot a^3 + 13B \cdot a^3) \cdot \log(\tan(1/2 \cdot dx + 1/2 \cdot c) - 1) - 2 \cdot (225 \cdot A \cdot a^3 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c)^9 + 195 \cdot B \cdot a^3 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c)^9 - 1050 \cdot A \cdot a^3 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c)^7 - 910 \cdot B \cdot a^3 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c)^7 + 1920 \cdot A \cdot a^3 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c)^5 + 1664 \cdot B \cdot a^3 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c)^5 - 1830 \cdot A \cdot a^3 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c)^3 - 1330 \cdot B \cdot a^3 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c)^3 + 735 \cdot A \cdot a^3 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c) + 765 \cdot B \cdot a^3 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c)) / (\tan(1/2 \cdot dx + 1/2 \cdot c)^2 - 1)^5) / d$

maple [A] time = 1.70, size = 234, normalized size = 1.44

$$\frac{3A a^3 \tan(dx + c)}{d} + \frac{13a^3 B \sec(dx + c) \tan(dx + c)}{8d} + \frac{13a^3 B \ln(\sec(dx + c) + \tan(dx + c))}{8d} + \frac{15A a^3 \sec(dx + c) \tan(dx + c)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(dx+c)^2*(a+a*sec(dx+c))^3*(A+B*sec(dx+c)),x)

[Out] $\frac{3}{d} \cdot A \cdot a^3 \cdot \tan(dx + c) + \frac{13}{8} \cdot \frac{1}{d} \cdot a^3 \cdot B \cdot \sec(dx + c) \cdot \tan(dx + c) + \frac{13}{8} \cdot \frac{1}{d} \cdot a^3 \cdot B \cdot \ln(\sec(dx + c) + \tan(dx + c)) + \frac{15}{8} \cdot \frac{1}{d} \cdot A \cdot a^3 \cdot \sec(dx + c) \cdot \tan(dx + c) + \frac{15}{8} \cdot \frac{1}{d} \cdot A \cdot a^3 \cdot \ln(\sec(dx + c) + \tan(dx + c)) + \frac{38}{15} \cdot \frac{1}{d} \cdot a^3 \cdot B \cdot \tan(dx + c) + \frac{19}{15} \cdot \frac{1}{d} \cdot a^3 \cdot B \cdot \tan(dx + c) \cdot \sec(dx + c)^2 + \frac{1}{d} \cdot A \cdot a^3 \cdot \tan(dx + c) \cdot \sec(dx + c)^2 + \frac{3}{4} \cdot \frac{1}{d} \cdot a^3 \cdot B \cdot \tan(dx + c) \cdot \sec(dx + c)^3 + \frac{1}{4} \cdot \frac{1}{d} \cdot A \cdot a^3 \cdot \tan(dx + c) \cdot \sec(dx + c)^3 + \frac{1}{5} \cdot \frac{1}{d} \cdot a^3 \cdot B \cdot \tan(dx + c) \cdot \sec(dx + c)^4$

maxima [B] time = 0.37, size = 337, normalized size = 2.07

$$240 \left(\tan(dx + c)^3 + 3 \tan(dx + c) \right) A a^3 + 16 \left(3 \tan(dx + c)^5 + 10 \tan(dx + c)^3 + 15 \tan(dx + c) \right) B a^3 + 240 \left(\tan(dx + c)^3 + 3 \tan(dx + c) \right) A a^3 + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] 1/240*(240*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^3 + 16*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*B*a^3 + 240*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a^3 - 15*A*a^3*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 45*B*a^3*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 180*A*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 60*B*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 240*A*a^3*tan(d*x + c))/d

mupad [B] time = 4.60, size = 224, normalized size = 1.37

$$\frac{a^3 \operatorname{atanh}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right) (15A + 13B) \left(\frac{15Aa^3}{4} + \frac{13Ba^3}{4}\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^9 + \left(-\frac{35Aa^3}{2} - \frac{91Ba^3}{6}\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^7 + \left(\frac{61Aa^3}{2} + \frac{133Ba^3}{6}\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5 + \left(\frac{32Aa^3}{15} + \frac{416Ba^3}{15}\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 + \left(\frac{10Aa^3}{15} + \frac{10Ba^3}{15}\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right) + \left(\frac{10Aa^3}{15} + \frac{10Ba^3}{15}\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^{-1}}{4d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^{10} - 5 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^8 + \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^6 - 5 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 + \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^3)/cos(c + d*x)^2,x)

[Out] (a^3*atanh(tan(c/2 + (d*x)/2))*(15*A + 13*B))/(4*d) - (tan(c/2 + (d*x)/2)*((49*A*a^3)/4 + (51*B*a^3)/4) + tan(c/2 + (d*x)/2)^9*((15*A*a^3)/4 + (13*B*a^3)/4) - tan(c/2 + (d*x)/2)^7*((35*A*a^3)/2 + (91*B*a^3)/6) - tan(c/2 + (d*x)/2)^5*((61*A*a^3)/2 + (133*B*a^3)/6) + tan(c/2 + (d*x)/2)^3*((32*A*a^3 + (416*B*a^3)/15))/(d*(5*tan(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 - 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3 \left(\int A \sec^2(c + dx) dx + \int 3A \sec^3(c + dx) dx + \int 3A \sec^4(c + dx) dx + \int A \sec^5(c + dx) dx + \int B \sec^3(c + dx) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**2*(a+a*sec(d*x+c))**3*(A+B*sec(d*x+c)),x)

[Out] a**3*(Integral(A*sec(c + d*x)**2, x) + Integral(3*A*sec(c + d*x)**3, x) + Integral(3*A*sec(c + d*x)**4, x) + Integral(A*sec(c + d*x)**5, x) + Integral(B*sec(c + d*x)**3, x) + Integral(3*B*sec(c + d*x)**4, x) + Integral(3*B*sec(c + d*x)**5, x) + Integral(B*sec(c + d*x)**6, x))

3.64 $\int \sec(c + dx)(a + a \sec(c + dx))^3(A + B \sec(c + dx)) dx$

Optimal. Leaf size=125

$$\frac{a^3(4A + 3B) \tan^3(c + dx)}{12d} + \frac{a^3(4A + 3B) \tan(c + dx)}{d} + \frac{5a^3(4A + 3B) \tanh^{-1}(\sin(c + dx))}{8d} + \frac{3a^3(4A + 3B) \tan(c + dx)}{8d}$$

[Out] $5/8*a^3*(4*A+3*B)*\operatorname{arctanh}(\sin(d*x+c))/d+a^3*(4*A+3*B)*\tan(d*x+c)/d+3/8*a^3*(4*A+3*B)*\sec(d*x+c)*\tan(d*x+c)/d+1/4*B*(a+a*\sec(d*x+c))^3*\tan(d*x+c)/d+1/12*a^3*(4*A+3*B)*\tan(d*x+c)^3/d$

Rubi [A] time = 0.14, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {4001, 3791, 3770, 3767, 8, 3768}

$$\frac{a^3(4A + 3B) \tan^3(c + dx)}{12d} + \frac{a^3(4A + 3B) \tan(c + dx)}{d} + \frac{5a^3(4A + 3B) \tanh^{-1}(\sin(c + dx))}{8d} + \frac{3a^3(4A + 3B) \tan(c + dx)}{8d}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Sec}[c + d*x]*(a + a*\operatorname{Sec}[c + d*x])^3*(A + B*\operatorname{Sec}[c + d*x]), x]$

[Out] $(5*a^3*(4*A + 3*B)*\operatorname{ArcTanh}[\operatorname{Sin}[c + d*x]])/(8*d) + (a^3*(4*A + 3*B)*\operatorname{Tan}[c + d*x])/d + (3*a^3*(4*A + 3*B)*\operatorname{Sec}[c + d*x]*\operatorname{Tan}[c + d*x])/(8*d) + (B*(a + a*\operatorname{Sec}[c + d*x])^3*\operatorname{Tan}[c + d*x])/(4*d) + (a^3*(4*A + 3*B)*\operatorname{Tan}[c + d*x]^3)/(12*d)$

Rule 8

$\operatorname{Int}[a_, x_Symbol] := \operatorname{Simp}[a*x, x] /; \operatorname{FreeQ}[a, x]$

Rule 3767

$\operatorname{Int}[\operatorname{csc}[(c_.) + (d_.)*(x_)]^{(n_)}, x_Symbol] := -\operatorname{Dist}[d^{(-1)}, \operatorname{Subst}[\operatorname{Int}[\operatorname{ExpandIntegrand}[(1 + x^2)^{(n/2 - 1)}, x], x], x, \operatorname{Cot}[c + d*x]], x] /; \operatorname{FreeQ}[\{c, d\}, x] \ \&\& \operatorname{IGtQ}[n/2, 0]$

Rule 3768

$\operatorname{Int}[(\operatorname{csc}[(c_.) + (d_.)*(x_)]*(b_.))^{(n_)}, x_Symbol] := -\operatorname{Simp}[(b*\operatorname{Cos}[c + d*x])*(b*\operatorname{Csc}[c + d*x])^{(n - 1)})/(d*(n - 1)), x] + \operatorname{Dist}[(b^2*(n - 2))/(n - 1), \operatorname{Int}[(b*\operatorname{Csc}[c + d*x])^{(n - 2)}, x], x] /; \operatorname{FreeQ}[\{b, c, d\}, x] \ \&\& \operatorname{GtQ}[n, 1] \ \&\& \operatorname{IntegerQ}[2*n]$

Rule 3770

`Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]`

Rule 3791

`Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_), x_Symbol] := Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && !GtQ[m, 0] && RationalQ[n]`

Rule 4001

`Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] := -Simp[(B*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m + 1), 0] && !LtQ[m, -2^(-1)]`

Rubi steps

$$\begin{aligned}
 \int \sec(c + dx)(a + a \sec(c + dx))^3(A + B \sec(c + dx)) dx &= \frac{B(a + a \sec(c + dx))^3 \tan(c + dx)}{4d} + \frac{1}{4}(4A + 3B) \int \sec(c + dx)(a + a \sec(c + dx))^3 dx \\
 &= \frac{B(a + a \sec(c + dx))^3 \tan(c + dx)}{4d} + \frac{1}{4}(4A + 3B) \int \sec(c + dx)(a + a \sec(c + dx))^3 dx \\
 &= \frac{B(a + a \sec(c + dx))^3 \tan(c + dx)}{4d} + \frac{1}{4}(a^3(4A + 3B) \int \sec(c + dx) dx) \\
 &= \frac{a^3(4A + 3B) \tanh^{-1}(\sin(c + dx))}{4d} + \frac{3a^3(4A + 3B) \sec(c + dx)}{4d} \\
 &= \frac{5a^3(4A + 3B) \tanh^{-1}(\sin(c + dx))}{8d} + \frac{a^3(4A + 3B) \sec(c + dx)}{d}
 \end{aligned}$$

Mathematica [B] time = 1.30, size = 273, normalized size = 2.18

$$a^3(\cos(c + dx) + 1)^3 \sec^6\left(\frac{1}{2}(c + dx)\right) \sec^4(c + dx) \left(120(4A + 3B) \cos^4(c + dx) \left(\log\left(\cos\left(\frac{1}{2}(c + dx)\right)\right) - \sin\left(\frac{1}{2}(c + dx)\right)\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[Sec[c + d*x]*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]

[Out]
$$-1/1536*(a^3*(1 + \cos[c + d*x])^3*\sec[(c + d*x)/2]^6*\sec[c + d*x]^4*(120*(4*A + 3*B)*\cos[c + d*x]^4*(\log[\cos[(c + d*x)/2] - \sin[(c + d*x)/2]] - \log[\cos[(c + d*x)/2] + \sin[(c + d*x)/2]]) - \sec[c]*(-24*(11*A + 9*B)*\sin[c] + (36*A + 69*B)*\sin[d*x] + 36*A*\sin[2*c + d*x] + 69*B*\sin[2*c + d*x] + 280*A*\sin[c + 2*d*x] + 264*B*\sin[c + 2*d*x] - 72*A*\sin[3*c + 2*d*x] - 24*B*\sin[3*c + 2*d*x] + 36*A*\sin[2*c + 3*d*x] + 45*B*\sin[2*c + 3*d*x] + 36*A*\sin[4*c + 3*d*x] + 45*B*\sin[4*c + 3*d*x] + 88*A*\sin[3*c + 4*d*x] + 72*B*\sin[3*c + 4*d*x])))/d$$

fricas [A] time = 0.46, size = 145, normalized size = 1.16

$$\frac{15(4A + 3B)a^3 \cos(dx + c)^4 \log(\sin(dx + c) + 1) - 15(4A + 3B)a^3 \cos(dx + c)^4 \log(-\sin(dx + c) + 1) + 2 \left(8 \cos(dx + c) \right)}{48 d \cos(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out]
$$1/48*(15*(4*A + 3*B)*a^3*\cos(d*x + c)^4*\log(\sin(d*x + c) + 1) - 15*(4*A + 3*B)*a^3*\cos(d*x + c)^4*\log(-\sin(d*x + c) + 1) + 2*(8*(11*A + 9*B)*a^3*\cos(d*x + c)^3 + 9*(4*A + 5*B)*a^3*\cos(d*x + c)^2 + 8*(A + 3*B)*a^3*\cos(d*x + c) + 6*B*a^3)*\sin(d*x + c))/(d*\cos(d*x + c)^4)$$

giac [A] time = 0.37, size = 212, normalized size = 1.70

$$15(4Aa^3 + 3Ba^3) \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right|\right) - 15(4Aa^3 + 3Ba^3) \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1\right|\right) - \frac{2\left(60Aa^3 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right)}{\cos(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out]
$$1/24*(15*(4*A*a^3 + 3*B*a^3)*\log(\operatorname{abs}(\tan(1/2*d*x + 1/2*c) + 1)) - 15*(4*A*a^3 + 3*B*a^3)*\log(\operatorname{abs}(\tan(1/2*d*x + 1/2*c) - 1)) - 2*(60*A*a^3*\tan(1/2*d*x + 1/2*c)^7 + 45*B*a^3*\tan(1/2*d*x + 1/2*c)^7 - 220*A*a^3*\tan(1/2*d*x + 1/2*c)^5 - 165*B*a^3*\tan(1/2*d*x + 1/2*c)^5 + 292*A*a^3*\tan(1/2*d*x + 1/2*c)^3 + 219*B*a^3*\tan(1/2*d*x + 1/2*c)^3 - 132*A*a^3*\tan(1/2*d*x + 1/2*c) - 147*B*a^3*\tan(1/2*d*x + 1/2*c))/(\tan(1/2*d*x + 1/2*c)^2 - 1)^4/d$$

maple [A] time = 1.38, size = 188, normalized size = 1.50

$$\frac{5Aa^3 \ln(\sec(dx+c) + \tan(dx+c))}{2d} + \frac{3a^3B \tan(dx+c)}{d} + \frac{11Aa^3 \tan(dx+c)}{3d} + \frac{15a^3B \sec(dx+c) \tan(dx+c)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x)

[Out] 5/2/d*A*a^3*ln(sec(d*x+c)+tan(d*x+c))+3/d*a^3*B*tan(d*x+c)+11/3/d*A*a^3*tan(d*x+c)+15/8/d*a^3*B*sec(d*x+c)*tan(d*x+c)+15/8/d*a^3*B*ln(sec(d*x+c)+tan(d*x+c))+3/2/d*A*a^3*sec(d*x+c)*tan(d*x+c)+1/d*a^3*B*tan(d*x+c)*sec(d*x+c)^2+1/3/d*A*a^3*tan(d*x+c)*sec(d*x+c)^2+1/4/d*a^3*B*tan(d*x+c)*sec(d*x+c)^3

maxima [B] time = 0.37, size = 262, normalized size = 2.10

$$16 \left(\tan(dx+c)^3 + 3 \tan(dx+c) \right) Aa^3 + 48 \left(\tan(dx+c)^3 + 3 \tan(dx+c) \right) Ba^3 - 3Ba^3 \left(\frac{2(3 \sin(dx+c)^3 - 5 \sin(dx+c))}{\sin(dx+c)^4 - 2 \sin(dx+c)^2 + 1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] 1/48*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^3 + 48*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a^3 - 3*B*a^3*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 36*A*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 36*B*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 48*A*a^3*log(sec(d*x + c) + tan(d*x + c)) + 144*A*a^3*tan(d*x + c) + 48*B*a^3*tan(d*x + c))/d

mupad [B] time = 4.47, size = 185, normalized size = 1.48

$$\frac{\left(-5Aa^3 - \frac{15Ba^3}{4}\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^7 + \left(\frac{55Aa^3}{3} + \frac{55Ba^3}{4}\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5 + \left(-\frac{73Aa^3}{3} - \frac{73Ba^3}{4}\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 + \left(11Aa^3 + \frac{15Ba^3}{4}\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^8 - 4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^6 + 6 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 - 4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + 1 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^3)/cos(c + d*x),x)

[Out] (tan(c/2 + (d*x)/2)*(11*A*a^3 + (49*B*a^3)/4) - tan(c/2 + (d*x)/2)^7*(5*A*a^3 + (15*B*a^3)/4) + tan(c/2 + (d*x)/2)^5*((55*A*a^3)/3 + (55*B*a^3)/4) - t

$$\frac{\tan(c/2 + (d*x)/2)^3 * ((73*A*a^3)/3 + (73*B*a^3)/4)}{(d*(6*\tan(c/2 + (d*x)/2)^4 - 4*\tan(c/2 + (d*x)/2)^2 - 4*\tan(c/2 + (d*x)/2)^6 + \tan(c/2 + (d*x)/2)^8 + 1)) + (5*a^3*\operatorname{atanh}(\tan(c/2 + (d*x)/2)))*(4*A + 3*B))/(4*d)}$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3 \left(\int A \sec(c + dx) dx + \int 3A \sec^2(c + dx) dx + \int 3A \sec^3(c + dx) dx + \int A \sec^4(c + dx) dx + \int B \sec^2(c + dx) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))**3*(A+B*sec(d*x+c)),x)

[Out] a**3*(Integral(A*sec(c + d*x), x) + Integral(3*A*sec(c + d*x)**2, x) + Integral(3*A*sec(c + d*x)**3, x) + Integral(A*sec(c + d*x)**4, x) + Integral(B*sec(c + d*x)**2, x) + Integral(3*B*sec(c + d*x)**3, x) + Integral(3*B*sec(c + d*x)**4, x) + Integral(B*sec(c + d*x)**5, x))

3.65 $\int (a + a \sec(c + dx))^3 (A + B \sec(c + dx)) dx$

Optimal. Leaf size=111

$$\frac{5a^3(A+B)\tan(c+dx)}{2d} + \frac{a^3(7A+5B)\tanh^{-1}(\sin(c+dx))}{2d} + \frac{(3A+5B)\tan(c+dx)(a^3\sec(c+dx)+a^3)}{6d} + a^3Ax$$

[Out] $a^3A*x + 1/2*a^3*(7*A+5*B)*\text{arctanh}(\sin(d*x+c))/d + 5/2*a^3*(A+B)*\tan(d*x+c)/d + 1/3*a*B*(a+a*\sec(d*x+c))^2*\tan(d*x+c)/d + 1/6*(3*A+5*B)*(a^3+a^3*\sec(d*x+c))*\tan(d*x+c)/d$

Rubi [A] time = 0.14, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3917, 3914, 3767, 8, 3770}

$$\frac{5a^3(A+B)\tan(c+dx)}{2d} + \frac{a^3(7A+5B)\tanh^{-1}(\sin(c+dx))}{2d} + \frac{(3A+5B)\tan(c+dx)(a^3\sec(c+dx)+a^3)}{6d} + a^3Ax$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]

[Out] $a^3A*x + (a^3*(7*A + 5*B)*\text{ArcTanh}[\text{Sin}[c + d*x]])/(2*d) + (5*a^3*(A + B)*\text{Tan}[c + d*x])/(2*d) + (a*B*(a + a*\text{Sec}[c + d*x])^2*\text{Tan}[c + d*x])/(3*d) + ((3*A + 5*B)*(a^3 + a^3*\text{Sec}[c + d*x])*\text{Tan}[c + d*x])/(6*d)$

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 3767

Int[csc[(c_.) + (d_.)*(x_.)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 3770

Int[csc[(c_.) + (d_.)*(x_.)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3914

Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))*(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.)), x_Symbol] := Simp[a*c*x, x] + (Dist[b*d, Int[Csc[e + f*x]^2, x], x] + Dist[b*c + a*d, Int[Csc[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x]

] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]

Rule 3917

Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.)), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1))/(f*m), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m - 1)*Simp[a*c*m + (b*c*m + a*d*(2*m - 1))*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && GtQ[m, 1] && EqQ[a^2 - b^2, 0] && IntegerQ[2*m]

Rubi steps

$$\begin{aligned}
 \int (a + a \sec(c + dx))^3 (A + B \sec(c + dx)) dx &= \frac{aB(a + a \sec(c + dx))^2 \tan(c + dx)}{3d} + \frac{1}{3} \int (a + a \sec(c + dx))^2 (3A + B \sec(c + dx)) dx \\
 &= \frac{aB(a + a \sec(c + dx))^2 \tan(c + dx)}{3d} + \frac{(3A + 5B)(a^3 + a^3 \sec(c + dx))}{6d} \\
 &= a^3 Ax + \frac{aB(a + a \sec(c + dx))^2 \tan(c + dx)}{3d} + \frac{(3A + 5B)(a^3 + a^3 \sec(c + dx))}{6d} \\
 &= a^3 Ax + \frac{a^3(7A + 5B) \tanh^{-1}(\sin(c + dx))}{2d} + \frac{aB(a + a \sec(c + dx))}{3d} \\
 &= a^3 Ax + \frac{a^3(7A + 5B) \tanh^{-1}(\sin(c + dx))}{2d} + \frac{5a^3(A + B) \tan(c + dx)}{2d}
 \end{aligned}$$

Mathematica [B] time = 6.41, size = 1056, normalized size = 9.51

result too large to display

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]

[Out] (A*x*Cos[c + d*x]^4*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]))/(8*(B + A*Cos[c + d*x])) + ((-7*A - 5*B)*Cos[c + d*x]^4*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]))/(16*d*(B + A*Cos[c + d*x])) + ((7*A + 5*B)*Cos[c + d*x]^4*Log[Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]))/(16*d*(B + A*Cos[c + d*x])) + (B*Cos[c + d*x]^4*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x])*Sin[(d*x)/2])/(48*d*(B + A*Cos[c + d*x])*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^3) + (Cos[c + d*x]^4*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]))/(8*(B + A*Cos[c + d*x]))

) / 2 ^ 6 * (a + a * Sec[c + d * x]) ^ 3 * (A + B * Sec[c + d * x]) * (3 * A * Cos[c / 2] + 10 * B * Cos[c / 2] - 3 * A * Sin[c / 2] - 8 * B * Sin[c / 2]) / (96 * d * (B + A * Cos[c + d * x]) * (Cos[c / 2] - Sin[c / 2]) * (Cos[c / 2 + (d * x) / 2] - Sin[c / 2 + (d * x) / 2]) ^ 2) + (Cos[c + d * x] ^ 4 * Sec[c / 2 + (d * x) / 2] ^ 6 * (a + a * Sec[c + d * x]) ^ 3 * (A + B * Sec[c + d * x]) * (9 * A * Sin[(d * x) / 2] + 11 * B * Sin[(d * x) / 2])) / (24 * d * (B + A * Cos[c + d * x]) * (Cos[c / 2] - Sin[c / 2]) * (Cos[c / 2 + (d * x) / 2] - Sin[c / 2 + (d * x) / 2])) + (B * Cos[c + d * x] ^ 4 * Sec[c / 2 + (d * x) / 2] ^ 6 * (a + a * Sec[c + d * x]) ^ 3 * (A + B * Sec[c + d * x]) * Sin[(d * x) / 2]) / (48 * d * (B + A * Cos[c + d * x]) * (Cos[c / 2] + Sin[c / 2]) * (Cos[c / 2 + (d * x) / 2] + Sin[c / 2 + (d * x) / 2]) ^ 3) + (Cos[c + d * x] ^ 4 * Sec[c / 2 + (d * x) / 2] ^ 6 * (a + a * Sec[c + d * x]) ^ 3 * (A + B * Sec[c + d * x]) * (-3 * A * Cos[c / 2] - 10 * B * Cos[c / 2] - 3 * A * Sin[c / 2] - 8 * B * Sin[c / 2])) / (96 * d * (B + A * Cos[c + d * x]) * (Cos[c / 2] + Sin[c / 2]) * (Cos[c / 2 + (d * x) / 2] + Sin[c / 2 + (d * x) / 2]) ^ 2) + (Cos[c + d * x] ^ 4 * Sec[c / 2 + (d * x) / 2] ^ 6 * (a + a * Sec[c + d * x]) ^ 3 * (A + B * Sec[c + d * x]) * (9 * A * Sin[(d * x) / 2] + 11 * B * Sin[(d * x) / 2])) / (24 * d * (B + A * Cos[c + d * x]) * (Cos[c / 2] + Sin[c / 2]) * (Cos[c / 2 + (d * x) / 2] + Sin[c / 2 + (d * x) / 2]))

fricas [A] time = 0.47, size = 141, normalized size = 1.27

$$\frac{12 A a^3 dx \cos(dx + c)^3 + 3(7 A + 5 B) a^3 \cos(dx + c)^3 \log(\sin(dx + c) + 1) - 3(7 A + 5 B) a^3 \cos(dx + c)^3 \log(\sin(dx + c) - 1) + 2(2(9 A + 11 B) a^3 \cos(dx + c)^2 + 3(A + 3 B) a^3 \cos(dx + c) + 2 B a^3) \sin(dx + c)}{12 d \cos(dx + c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] 1/12*(12*A*a^3*d*x*cos(d*x + c)^3 + 3*(7*A + 5*B)*a^3*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*(7*A + 5*B)*a^3*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + 2*(2*(9*A + 11*B)*a^3*cos(d*x + c)^2 + 3*(A + 3*B)*a^3*cos(d*x + c) + 2*B*a^3)*sin(d*x + c))/(d*cos(d*x + c)^3)

giac [A] time = 0.77, size = 189, normalized size = 1.70

$$6(dx + c)Aa^3 + 3(7Aa^3 + 5Ba^3) \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right|\right) - 3(7Aa^3 + 5Ba^3) \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1\right|\right) -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] 1/6*(6*(d*x + c)*A*a^3 + 3*(7*A*a^3 + 5*B*a^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(7*A*a^3 + 5*B*a^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(15*A*a^3*tan(1/2*d*x + 1/2*c)^5 + 15*B*a^3*tan(1/2*d*x + 1/2*c)^5 - 36*A*a^3*tan(1/2*d*x + 1/2*c)^3 - 40*B*a^3*tan(1/2*d*x + 1/2*c)^3 + 21*A*a^3*tan(1/2*d*x + 1/2*c)^3)

$$\frac{(dx + 1/2*c) + 33*B*a^3*\tan(1/2*d*x + 1/2*c)}{(\tan(1/2*d*x + 1/2*c)^2 - 1)^3} / d$$

maple [A] time = 1.16, size = 158, normalized size = 1.42

$$a^3 A x + \frac{A a^3 c}{d} + \frac{5 a^3 B \ln(\sec(dx + c) + \tan(dx + c))}{2d} + \frac{7 A a^3 \ln(\sec(dx + c) + \tan(dx + c))}{2d} + \frac{11 a^3 B \tan(dx + c)}{3d} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x)

[Out] $a^3 A x + 1/d A a^3 c + 5/2/d a^3 B \ln(\sec(d*x+c) + \tan(d*x+c)) + 7/2/d A a^3 \ln(\sec(d*x+c) + \tan(d*x+c)) + 11/3/d a^3 B \tan(d*x+c) + 3/d A a^3 \tan(d*x+c) + 3/2/d a^3 B \sec(d*x+c) * \tan(d*x+c) + 1/2/d A a^3 \sec(d*x+c) * \tan(d*x+c) + 1/3/d a^3 B \tan(d*x+c) * \sec(d*x+c)^2$

maxima [A] time = 0.36, size = 198, normalized size = 1.78

$$\frac{12(dx+c)Aa^3 + 4(\tan(dx+c)^3 + 3\tan(dx+c))Ba^3 - 3Aa^3 \left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1} - \log(\sin(dx+c)+1) + \log(\sin(dx+c)-1) \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] $1/12*(12*(d*x+c)*A*a^3 + 4*(\tan(d*x+c)^3 + 3*\tan(d*x+c))*B*a^3 - 3*A*a^3*(2*\sin(d*x+c)/(\sin(d*x+c)^2-1) - \log(\sin(d*x+c)+1) + \log(\sin(d*x+c)-1)) - 9*B*a^3*(2*\sin(d*x+c)/(\sin(d*x+c)^2-1) - \log(\sin(d*x+c)+1) + \log(\sin(d*x+c)-1)) + 36*A*a^3*\log(\sec(d*x+c) + \tan(d*x+c)) + 12*B*a^3*\log(\sec(d*x+c) + \tan(d*x+c)) + 36*A*a^3*\tan(d*x+c) + 36*B*a^3*\tan(d*x+c))/d$

mupad [B] time = 2.08, size = 209, normalized size = 1.88

$$\frac{2 A a^3 \operatorname{atan}\left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)}\right)}{d} + \frac{7 A a^3 \operatorname{atanh}\left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)}\right)}{d} + \frac{5 B a^3 \operatorname{atanh}\left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)}\right)}{d} + \frac{3 A a^3 \sin(c+dx)}{d \cos(c+dx)} + \frac{A a^3 \sin(c+dx)}{2 d \cos(c+dx)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^3,x)

[Out] $(2*A*a^3*\operatorname{atan}\left(\frac{\sin(c/2 + (d*x)/2)}{\cos(c/2 + (d*x)/2)}\right))/d + (7*A*a^3*\operatorname{atanh}\left(\frac{\sin(c/2 + (d*x)/2)}{\cos(c/2 + (d*x)/2)}\right))/d + (5*B*a^3*\operatorname{atanh}\left(\frac{\sin(c/2 + (d*x)/2)}{\cos(c/2 + (d*x)/2)}\right))/d$

$$\frac{1}{\cos(c/2 + (d*x)/2)} \Big/ d + (3*A*a^3*\sin(c + d*x))/(d*\cos(c + d*x)) + (A*a^3*\sin(c + d*x))/(2*d*\cos(c + d*x)^2) + (11*B*a^3*\sin(c + d*x))/(3*d*\cos(c + d*x)) + (3*B*a^3*\sin(c + d*x))/(2*d*\cos(c + d*x)^2) + (B*a^3*\sin(c + d*x))/(3*d*\cos(c + d*x)^3)$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3 \left(\int A dx + \int 3A \sec(c + dx) dx + \int 3A \sec^2(c + dx) dx + \int A \sec^3(c + dx) dx + \int B \sec(c + dx) dx + \int \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))**3*(A+B*sec(d*x+c)),x)

[Out] a**3*(Integral(A, x) + Integral(3*A*sec(c + d*x), x) + Integral(3*A*sec(c + d*x)**2, x) + Integral(A*sec(c + d*x)**3, x) + Integral(B*sec(c + d*x), x) + Integral(3*B*sec(c + d*x)**2, x) + Integral(3*B*sec(c + d*x)**3, x) + Integral(B*sec(c + d*x)**4, x))

3.66 $\int \cos(c + dx)(a + a \sec(c + dx))^3(A + B \sec(c + dx)) dx$

Optimal. Leaf size=108

$$\frac{a^3(6A + 7B) \tanh^{-1}(\sin(c + dx))}{2d} + \frac{(A + 2B) \sin(c + dx) (a^3 \sec(c + dx) + a^3)}{d} + a^3 x(3A + B) - \frac{5a^3 B \sin(c + dx)}{2d} + \dots$$

[Out] $a^3(3A+B)*x+1/2*a^3*(6A+7B)*\operatorname{arctanh}(\sin(dx+c))/d-5/2*a^3*B*\sin(dx+c)/d+1/2*a*B*(a+a*\sec(dx+c))^2*\sin(dx+c)/d+(A+2*B)*(a^3+a^3*\sec(dx+c))*\sin(dx+c)/d$

Rubi [A] time = 0.24, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {4018, 3996, 3770}

$$\frac{a^3(6A + 7B) \tanh^{-1}(\sin(c + dx))}{2d} + \frac{(A + 2B) \sin(c + dx) (a^3 \sec(c + dx) + a^3)}{d} + a^3 x(3A + B) - \frac{5a^3 B \sin(c + dx)}{2d} + \dots$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Cos}[c + d*x]*(a + a*\operatorname{Sec}[c + d*x])^3*(A + B*\operatorname{Sec}[c + d*x]), x]$

[Out] $a^3(3A + B)*x + (a^3(6A + 7B)*\operatorname{ArcTanh}[\operatorname{Sin}[c + d*x]])/(2*d) - (5*a^3*B*\operatorname{Sin}[c + d*x])/(2*d) + (a*B*(a + a*\operatorname{Sec}[c + d*x])^2*\operatorname{Sin}[c + d*x])/(2*d) + ((A + 2*B)*(a^3 + a^3*\operatorname{Sec}[c + d*x])*\operatorname{Sin}[c + d*x])/d$

Rule 3770

$\operatorname{Int}[\operatorname{csc}[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow -\operatorname{Simp}[\operatorname{ArcTanh}[\operatorname{Cos}[c + d*x]]/d, x] /; \operatorname{FreeQ}\{c, d\}, x]$

Rule 3996

$\operatorname{Int}[(\operatorname{csc}[(e_.) + (f_.)*(x_.)]*(d_.))^n*(\operatorname{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))*(\operatorname{csc}[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] \rightarrow \operatorname{Simp}[A*a*\operatorname{Cot}[e + f*x]*(d*\operatorname{Csc}[e + f*x])^n]/(f*n), x] + \operatorname{Dist}[1/(d*n), \operatorname{Int}[(d*\operatorname{Csc}[e + f*x])^{n+1}]*\operatorname{Simp}[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*\operatorname{Csc}[e + f*x], x], x] /; \operatorname{FreeQ}\{a, b, d, e, f, A, B\}, x] \&\& \operatorname{NeQ}[A*b - a*B, 0] \&\& \operatorname{LeQ}[n, -1]$

Rule 4018

$\operatorname{Int}[(\operatorname{csc}[(e_.) + (f_.)*(x_.)]*(d_.))^n*(\operatorname{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m*(\operatorname{csc}[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] \rightarrow -\operatorname{Simp}[(b*B*\operatorname{Cot}[e + f*x]*(a + b*\operatorname{Csc}[e + f*x])^{m-1}*(d*\operatorname{Csc}[e + f*x])^n)/(f*(m + n)), x]$

] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n *Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc[e + f*x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && !LtQ[n, -1]

Rubi steps

$$\begin{aligned} \int \cos(c + dx)(a + a \sec(c + dx))^3(A + B \sec(c + dx)) dx &= \frac{aB(a + a \sec(c + dx))^2 \sin(c + dx)}{2d} + \frac{1}{2} \int \cos(c + dx) dx \\ &= \frac{aB(a + a \sec(c + dx))^2 \sin(c + dx)}{2d} + \frac{(A + 2B)(a^3 + a^2 \sec(c + dx))}{2d} \\ &= -\frac{5a^3 B \sin(c + dx)}{2d} + \frac{aB(a + a \sec(c + dx))^2 \sin(c + dx)}{2d} \\ &= a^3(3A + B)x - \frac{5a^3 B \sin(c + dx)}{2d} + \frac{aB(a + a \sec(c + dx))^2 \sin(c + dx)}{2d} \\ &= a^3(3A + B)x + \frac{a^3(6A + 7B) \tanh^{-1}(\sin(c + dx))}{2d} - \frac{5a^3 B \sin(c + dx)}{2d} \end{aligned}$$

Mathematica [B] time = 2.71, size = 335, normalized size = 3.10

$$a^3 \cos^4(c + dx) \sec^6\left(\frac{1}{2}(c + dx)\right) (\sec(c + dx) + 1)^3 (A + B \sec(c + dx)) \left(\frac{4(A+3B) \sin\left(\frac{dx}{2}\right)}{d(\cos\left(\frac{c}{2}\right) - \sin\left(\frac{c}{2}\right))(\cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+dx)\right))} + \dots \right)$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]

[Out] (a^3*Cos[c + d*x]^4*Sec[(c + d*x)/2]^6*(1 + Sec[c + d*x])^3*(A + B*Sec[c + d*x])*(4*(3*A + B)*x - (2*(6*A + 7*B)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/d + (2*(6*A + 7*B)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/d + (4*A*Cos[d*x]*Sin[c])/d + (4*A*Cos[c]*Sin[d*x])/d + B/(d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) + (4*(A + 3*B)*Sin[(d*x)/2])/(d*(Cos[c/2] - Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) - B/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (4*(A + 3*B)*Sin[(d*x)/2])/(d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])))/(32*(B + A*Cos[c + d*x]))

fricas [A] time = 0.47, size = 137, normalized size = 1.27

$$\frac{4(3A + B)a^3 dx \cos(dx + c)^2 + (6A + 7B)a^3 \cos(dx + c)^2 \log(\sin(dx + c) + 1) - (6A + 7B)a^3 \cos(dx + c)^2 \log(\sin(dx + c) - 1)}{4d \cos(dx + c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] $\frac{1}{4}*(4*(3*A + B)*a^3*d*x*\cos(d*x + c)^2 + (6*A + 7*B)*a^3*\cos(d*x + c)^2*\log(\sin(d*x + c) + 1) - (6*A + 7*B)*a^3*\cos(d*x + c)^2*\log(-\sin(d*x + c) + 1) + 2*(2*A*a^3*\cos(d*x + c)^2 + 2*(A + 3*B)*a^3*\cos(d*x + c) + B*a^3)*\sin(d*x + c))/(d*\cos(d*x + c)^2)$

giac [A] time = 0.51, size = 192, normalized size = 1.78

$$\frac{4 A a^3 \tan\left(\frac{1}{2} d x + \frac{1}{2} c\right)}{\tan\left(\frac{1}{2} d x + \frac{1}{2} c\right)^2 + 1} + 2 \left(3 A a^3 + B a^3\right)(d x + c) + \left(6 A a^3 + 7 B a^3\right) \log\left(\left|\tan\left(\frac{1}{2} d x + \frac{1}{2} c\right) + 1\right|\right) - \left(6 A a^3 + 7 B a^3\right) \log\left(\left|\tan\left(\frac{1}{2} d x + \frac{1}{2} c\right) - 1\right|\right)$$

$$2 d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] $\frac{1}{2}*(4*A*a^3*\tan(1/2*d*x + 1/2*c)/(\tan(1/2*d*x + 1/2*c)^2 + 1) + 2*(3*A*a^3 + B*a^3)*(d*x + c) + (6*A*a^3 + 7*B*a^3)*\log(\tan(1/2*d*x + 1/2*c) + 1) - (6*A*a^3 + 7*B*a^3)*\log(\tan(1/2*d*x + 1/2*c) - 1) - 2*(2*A*a^3*\tan(1/2*d*x + 1/2*c)^3 + 5*B*a^3*\tan(1/2*d*x + 1/2*c)^3 - 2*A*a^3*\tan(1/2*d*x + 1/2*c) - 7*B*a^3*\tan(1/2*d*x + 1/2*c))/(\tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d$

maple [A] time = 1.11, size = 144, normalized size = 1.33

$$\frac{a^3 A \sin(dx + c)}{d} + a^3 B x + \frac{a^3 B c}{d} + 3 a^3 A x + \frac{3 A a^3 c}{d} + \frac{7 a^3 B \ln(\sec(dx + c) + \tan(dx + c))}{2 d} + \frac{3 A a^3 \ln(\sec(dx + c) + \tan(dx + c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x)

[Out] $a^3*A*\sin(d*x+c)/d+a^3*B*x+1/d*a^3*B*c+3*a^3*A*x+3/d*A*a^3*c+7/2/d*a^3*B*\ln(\sec(d*x+c)+\tan(d*x+c))+3/d*A*a^3*\ln(\sec(d*x+c)+\tan(d*x+c))+3/d*a^3*B*\tan(d*x+c)+1/d*A*a^3*\tan(d*x+c)+1/2/d*a^3*B*\sec(d*x+c)*\tan(d*x+c)$

maxima [A] time = 0.36, size = 165, normalized size = 1.53

$$12(dx + c)Aa^3 + 4(dx + c)Ba^3 - Ba^3\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1} - \log(\sin(dx + c) + 1) + \log(\sin(dx + c) - 1)\right) + 6Aa^3(\log(\sec(dx + c) + \tan(dx + c)))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] $\frac{1}{4}*(12*(d*x + c)*A*a^3 + 4*(d*x + c)*B*a^3 - B*a^3*(2*\sin(d*x + c)/(\sin(d*x + c)^2 - 1) - \log(\sin(d*x + c) + 1) + \log(\sin(d*x + c) - 1)) + 6*A*a^3*(\log(\sin(d*x + c) + 1) - \log(\sin(d*x + c) - 1)) + 6*B*a^3*(\log(\sin(d*x + c) + 1) - \log(\sin(d*x + c) - 1)) + 4*A*a^3*\sin(d*x + c) + 4*A*a^3*\tan(d*x + c) + 12*B*a^3*\tan(d*x + c))/d$

mupad [B] time = 2.12, size = 207, normalized size = 1.92

$$\frac{A a^3 \sin(c + d x)}{d} + \frac{6 A a^3 \operatorname{atan}\left(\frac{\sin\left(\frac{c}{2} + \frac{d x}{2}\right)}{\cos\left(\frac{c}{2} + \frac{d x}{2}\right)}\right)}{d} + \frac{6 A a^3 \operatorname{atanh}\left(\frac{\sin\left(\frac{c}{2} + \frac{d x}{2}\right)}{\cos\left(\frac{c}{2} + \frac{d x}{2}\right)}\right)}{d} + \frac{2 B a^3 \operatorname{atan}\left(\frac{\sin\left(\frac{c}{2} + \frac{d x}{2}\right)}{\cos\left(\frac{c}{2} + \frac{d x}{2}\right)}\right)}{d} + \frac{7 B a^3 \operatorname{atanh}\left(\frac{\sin\left(\frac{c}{2} + \frac{d x}{2}\right)}{\cos\left(\frac{c}{2} + \frac{d x}{2}\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^3,x)

[Out] $(A*a^3*\sin(c + d*x))/d + (6*A*a^3*\operatorname{atan}(\sin(c/2 + (d*x)/2)/\cos(c/2 + (d*x)/2)))/d + (6*A*a^3*\operatorname{atanh}(\sin(c/2 + (d*x)/2)/\cos(c/2 + (d*x)/2)))/d + (2*B*a^3*\operatorname{atan}(\sin(c/2 + (d*x)/2)/\cos(c/2 + (d*x)/2)))/d + (7*B*a^3*\operatorname{atanh}(\sin(c/2 + (d*x)/2)/\cos(c/2 + (d*x)/2)))/d + (A*a^3*\sin(c + d*x))/(d*\cos(c + d*x)) + (3*B*a^3*\sin(c + d*x))/(d*\cos(c + d*x)) + (B*a^3*\sin(c + d*x))/(2*d*\cos(c + d*x)^2)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3 \left(\int A \cos(c + dx) dx + \int 3A \cos(c + dx) \sec(c + dx) dx + \int 3A \cos(c + dx) \sec^2(c + dx) dx + \int A \cos(c + dx) \sec^3(c + dx) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))**3*(A+B*sec(d*x+c)),x)

[Out] $a**3*(\operatorname{Integral}(A*\cos(c + d*x), x) + \operatorname{Integral}(3*A*\cos(c + d*x)*\sec(c + d*x), x) + \operatorname{Integral}(3*A*\cos(c + d*x)*\sec(c + d*x)**2, x) + \operatorname{Integral}(A*\cos(c + d*x)*\sec(c + d*x)**3, x) + \operatorname{Integral}(B*\cos(c + d*x)*\sec(c + d*x), x) + \operatorname{Integral}(3*B*\cos(c + d*x)*\sec(c + d*x)**2, x) + \operatorname{Integral}(3*B*\cos(c + d*x)*\sec(c + d*x)**3, x) + \operatorname{Integral}(B*\cos(c + d*x)*\sec(c + d*x)**4, x))$

3.67 $\int \cos^2(c + dx)(a + a \sec(c + dx))^3(A + B \sec(c + dx)) dx$

Optimal. Leaf size=117

$$\frac{a^3(A + 3B) \tanh^{-1}(\sin(c + dx))}{d} - \frac{(A - 2B) \sin(c + dx) (a^3 \sec(c + dx) + a^3)}{2d} + \frac{1}{2} a^3 x (7A + 6B) + \frac{5a^3 A \sin(c + dx)}{2d} +$$

[Out] $1/2*a^3*(7*A+6*B)*x+a^3*(A+3*B)*\arctanh(\sin(d*x+c))/d+5/2*a^3*A*\sin(d*x+c)/d+1/2*a*A*\cos(d*x+c)*(a+a*\sec(d*x+c))^2*\sin(d*x+c)/d-1/2*(A-2*B)*(a^3+a^3*\sec(d*x+c))*\sin(d*x+c)/d$

Rubi [A] time = 0.26, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {4017, 4018, 3996, 3770}

$$\frac{a^3(A + 3B) \tanh^{-1}(\sin(c + dx))}{d} - \frac{(A - 2B) \sin(c + dx) (a^3 \sec(c + dx) + a^3)}{2d} + \frac{1}{2} a^3 x (7A + 6B) + \frac{5a^3 A \sin(c + dx)}{2d} +$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Cos}[c + d*x]^2*(a + a*\text{Sec}[c + d*x])^3*(A + B*\text{Sec}[c + d*x]), x]$

[Out] $(a^3*(7*A + 6*B)*x)/2 + (a^3*(A + 3*B)*\text{ArcTanh}[\text{Sin}[c + d*x]])/d + (5*a^3*A*\text{Sin}[c + d*x])/(2*d) + (a*A*\text{Cos}[c + d*x]*(a + a*\text{Sec}[c + d*x])^2*\text{Sin}[c + d*x])/(2*d) - ((A - 2*B)*(a^3 + a^3*\text{Sec}[c + d*x])*\text{Sin}[c + d*x])/(2*d)$

Rule 3770

$\text{Int}[\text{csc}[(c_.) + (d_.)*(x_.)], x_Symbol] \text{ :> } -\text{Simp}[\text{ArcTanh}[\text{Cos}[c + d*x]]/d, x] /;$ FreeQ[{c, d}, x]

Rule 3996

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(d_.))^n*(\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))*(\text{csc}[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] \text{ :> } \text{Simp}[(A*a*\text{Cot}[e + f*x]*(d*\text{Csc}[e + f*x])^n)/(f*n), x] + \text{Dist}[1/(d*n), \text{Int}[(d*\text{Csc}[e + f*x])^{n+1}]*\text{Simp}[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*\text{Csc}[e + f*x], x], x] /;$ FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]

Rule 4017

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(d_.))^n*(\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m*(\text{csc}[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] \text{ :> } \text{Simp}[(a*A*\text{Cot}[e + f*x]*(a + b*\text{Csc}[e + f*x])^{m-1}*(d*\text{Csc}[e + f*x])^n)/(f*n), x] - \text{Dis}$

```
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (
a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && !LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
 \int \cos^2(c + dx)(a + a \sec(c + dx))^3(A + B \sec(c + dx)) dx &= \frac{aA \cos(c + dx)(a + a \sec(c + dx))^2 \sin(c + dx)}{2d} + \frac{1}{2} \\
 &= \frac{aA \cos(c + dx)(a + a \sec(c + dx))^2 \sin(c + dx)}{2d} - \frac{1}{2} \\
 &= \frac{5a^3 A \sin(c + dx)}{2d} + \frac{aA \cos(c + dx)(a + a \sec(c + dx))}{2d} \\
 &= \frac{1}{2} a^3 (7A + 6B)x + \frac{5a^3 A \sin(c + dx)}{2d} + \frac{aA \cos(c + dx)}{d} \\
 &= \frac{1}{2} a^3 (7A + 6B)x + \frac{a^3 (A + 3B) \tanh^{-1}(\sin(c + dx))}{d}
 \end{aligned}$$

Mathematica [B] time = 5.09, size = 302, normalized size = 2.58

$$a^3 \cos^4(c + dx) \sec^6\left(\frac{1}{2}(c + dx)\right) (\sec(c + dx) + 1)^3 (A + B \sec(c + dx)) \left(\frac{4(3A+B) \sin(c) \cos(dx)}{d} + \frac{4(3A+B) \cos(c) \sin(dx)}{d} \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^3*Cos[c + d*x]^4*Sec[(c + d*x)/2]^6*(1 + Sec[c + d*x])^3*(A + B*Sec[c +
d*x]))*(2*(7*A + 6*B)*x - (4*(A + 3*B)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/
```

2]])/d + (4*(A + 3*B)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/d + (4*(3*A + B)*Cos[d*x]*Sin[c])/d + (A*Cos[2*d*x]*Sin[2*c])/d + (4*(3*A + B)*Cos[c]*Sin[d*x])/d + (A*Cos[2*c]*Sin[2*d*x])/d + (4*B*Sin[(d*x)/2])/(d*(Cos[c/2] - Sin[c/2]))*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) + (4*B*Sin[(d*x)/2])/(d*(Cos[c/2] + Sin[c/2]))*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])))/(32*(B + A*Cos[c + d*x]))

fricas [A] time = 0.48, size = 127, normalized size = 1.09

$$\frac{(7A + 6B)a^3 dx \cos(dx + c) + (A + 3B)a^3 \cos(dx + c) \log(\sin(dx + c) + 1) - (A + 3B)a^3 \cos(dx + c) \log(-\sin(dx + c) + 1)}{2d \cos(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] 1/2*((7*A + 6*B)*a^3*d*x*cos(d*x + c) + (A + 3*B)*a^3*cos(d*x + c)*log(sin(d*x + c) + 1) - (A + 3*B)*a^3*cos(d*x + c)*log(-sin(d*x + c) + 1) + (A*a^3*cos(d*x + c)^2 + 2*(3*A + B)*a^3*cos(d*x + c) + 2*B*a^3)*sin(d*x + c))/(d*cos(d*x + c))

giac [A] time = 0.34, size = 192, normalized size = 1.64

$$\frac{4Ba^3 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)}{\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - 1} - (7Aa^3 + 6Ba^3)(dx + c) - 2(Aa^3 + 3Ba^3) \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right|\right) + 2(Aa^3 + 3Ba^3) \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1\right|\right)$$

2d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] -1/2*(4*B*a^3*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1) - (7*A*a^3 + 6*B*a^3)*(d*x + c) - 2*(A*a^3 + 3*B*a^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) + 2*(A*a^3 + 3*B*a^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(5*A*a^3*tan(1/2*d*x + 1/2*c)^3 + 2*B*a^3*tan(1/2*d*x + 1/2*c)^3 + 7*A*a^3*tan(1/2*d*x + 1/2*c) + 2*B*a^3*tan(1/2*d*x + 1/2*c)))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2/d

maple [A] time = 0.88, size = 145, normalized size = 1.24

$$\frac{Aa^3 \cos(dx + c) \sin(dx + c)}{2d} + \frac{7a^3 Ax}{2} + \frac{7Aa^3 c}{2d} + \frac{a^3 B \sin(dx + c)}{d} + \frac{3a^3 A \sin(dx + c)}{d} + 3a^3 Bx + \frac{3a^3 Bc}{d} + \frac{3a^3 B \ln(\sec(dx + c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(d*x+c)^2*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x)`

[Out] $\frac{1}{2}dAa^3\cos(d*x+c)\sin(d*x+c)+\frac{7}{2}a^3Ax+\frac{7}{2}dAa^3c+a^3B\sin(d*x+c)$
 $\frac{1}{d+3a^3A\sin(d*x+c)}+\frac{1}{d+3a^3Bx+3/d*a^3B*c+3/d*a^3B*\ln(\sec(d*x+c))+\tan(d*x+c))+\frac{1}{dAa^3\ln(\sec(d*x+c))+\tan(d*x+c))+\frac{1}{da^3B*\tan(d*x+c)}$

maxima [A] time = 0.36, size = 140, normalized size = 1.20

$$\frac{(2dx + 2c + \sin(2dx + 2c))Aa^3 + 12(dx + c)Aa^3 + 12(dx + c)Ba^3 + 2Aa^3(\log(\sin(dx + c) + 1) - \log(\sin(dx + c) - 1)) + 6B*a^3(\log(\sin(dx + c) + 1) - \log(\sin(dx + c) - 1)) + 12Aa^3\sin(dx + c) + 4B*a^3\sin(dx + c) + 4B*a^3\tan(dx + c))/d}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="maxima")`

[Out] $\frac{1}{4}((2dx + 2c + \sin(2dx + 2c))Aa^3 + 12(dx + c)Aa^3 + 12(dx + c)Ba^3 + 2Aa^3(\log(\sin(dx + c) + 1) - \log(\sin(dx + c) - 1)) + 6B*a^3(\log(\sin(dx + c) + 1) - \log(\sin(dx + c) - 1)) + 12Aa^3\sin(dx + c) + 4B*a^3\sin(dx + c) + 4B*a^3\tan(dx + c))/d$

mupad [B] time = 2.08, size = 197, normalized size = 1.68

$$\frac{3Aa^3\sin(c+dx)}{d} + \frac{Ba^3\sin(c+dx)}{d} + \frac{7Aa^3\operatorname{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{d} + \frac{2Aa^3\operatorname{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{d} + \frac{6Ba^3\operatorname{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(c+d*x)^2*(A+B/cos(c+d*x))*(a+a/cos(c+d*x))^3,x)`

[Out] $\frac{3Aa^3\sin(c+dx)}{d} + \frac{Ba^3\sin(c+dx)}{d} + \frac{7Aa^3\operatorname{atan}\left(\frac{\sin(c/2+(dx)/2)}{\cos(c/2+(dx)/2)}\right)}{d} + \frac{2Aa^3\operatorname{atanh}\left(\frac{\sin(c/2+(dx)/2)}{\cos(c/2+(dx)/2)}\right)}{d} + \frac{6B*a^3\operatorname{atan}\left(\frac{\sin(c/2+(dx)/2)}{\cos(c/2+(dx)/2)}\right)}{d} + \frac{6B*a^3\operatorname{atanh}\left(\frac{\sin(c/2+(dx)/2)}{\cos(c/2+(dx)/2)}\right)}{d} + \frac{Ba^3\sin(c+dx)}{d\cos(c+dx)} + \frac{Aa^3\cos(c+dx)\sin(c+dx)}{2d}$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int A\cos^2(c+dx)dx + \int 3A\cos^2(c+dx)\sec(c+dx)dx + \int 3A\cos^2(c+dx)\sec^2(c+dx)dx + \int A\cos^2(c+dx)\sec^3(c+dx)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)**2*(a+a*sec(d*x+c))**3*(A+B*sec(d*x+c)),x)`

[Out] $a^3\left(\operatorname{Integral}(A\cos(c+dx)**2,x) + \operatorname{Integral}(3A\cos(c+dx)**2*\sec(c+dx),x) + \operatorname{Integral}(3A\cos(c+dx)**2*\sec(c+dx)**2,x) + \operatorname{Integral}(A\cos(c+dx)**2*\sec(c+dx)**3,x)\right)$

$\cos(c + d*x)**2*\sec(c + d*x)**3, x) + \text{Integral}(B*\cos(c + d*x)**2*\sec(c + d*x), x) + \text{Integral}(3*B*\cos(c + d*x)**2*\sec(c + d*x)**2, x) + \text{Integral}(3*B*\cos(c + d*x)**2*\sec(c + d*x)**3, x) + \text{Integral}(B*\cos(c + d*x)**2*\sec(c + d*x)**4, x))$

3.68 $\int \cos^3(c + dx)(a + a \sec(c + dx))^3(A + B \sec(c + dx)) dx$

Optimal. Leaf size=125

$$\frac{5a^3(A + B) \sin(c + dx)}{2d} + \frac{(5A + 3B) \sin(c + dx) \cos(c + dx) (a^3 \sec(c + dx) + a^3)}{6d} + \frac{1}{2}a^3x(5A+7B) + \frac{a^3B \tanh^{-1}(\sin(dx+c))}{d}$$

[Out] $1/2*a^3*(5*A+7*B)*x+a^3*B*\arctanh(\sin(d*x+c))/d+5/2*a^3*(A+B)*\sin(d*x+c)/d+1/3*a*A*\cos(d*x+c)^2*(a+a*\sec(d*x+c))^2*\sin(d*x+c)/d+1/6*(5*A+3*B)*\cos(d*x+c)*(a^3+a^3*\sec(d*x+c))*\sin(d*x+c)/d$

Rubi [A] time = 0.27, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {4017, 3996, 3770}

$$\frac{5a^3(A + B) \sin(c + dx)}{2d} + \frac{(5A + 3B) \sin(c + dx) \cos(c + dx) (a^3 \sec(c + dx) + a^3)}{6d} + \frac{1}{2}a^3x(5A+7B) + \frac{a^3B \tanh^{-1}(\sin(dx+c))}{d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Cos}[c + d*x]^3*(a + a*\text{Sec}[c + d*x])^3*(A + B*\text{Sec}[c + d*x]), x]$

[Out] $(a^3*(5*A + 7*B)*x)/2 + (a^3*B*\text{ArcTanh}[\text{Sin}[c + d*x]])/d + (5*a^3*(A + B)*\text{Sin}[c + d*x])/(2*d) + (a*A*\text{Cos}[c + d*x]^2*(a + a*\text{Sec}[c + d*x])^2*\text{Sin}[c + d*x])/(3*d) + ((5*A + 3*B)*\text{Cos}[c + d*x]*(a^3 + a^3*\text{Sec}[c + d*x])*\text{Sin}[c + d*x])/(6*d)$

Rule 3770

$\text{Int}[\text{csc}[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow -\text{Simp}[\text{ArcTanh}[\text{Cos}[c + d*x]]/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 3996

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(d_.))^{(n_.)}*(\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))*(\text{csc}[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] \rightarrow \text{Simp}[(A*a*\text{Cot}[e + f*x]*(d*\text{Csc}[e + f*x])^n)/(f*n), x] + \text{Dist}[1/(d*n), \text{Int}[(d*\text{Csc}[e + f*x])^{(n + 1)}*\text{Simp}[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*\text{Csc}[e + f*x], x], x], x] /; \text{FreeQ}\{a, b, d, e, f, A, B\}, x] \&\& \text{NeQ}[A*b - a*B, 0] \&\& \text{LeQ}[n, -1]$

Rule 4017

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(d_.))^{(n_.)}*(\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^{(m_.)}*(\text{csc}[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] \rightarrow \text{Simp}[(a*A*\text{Co}$

```
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned} \int \cos^3(c + dx)(a + a \sec(c + dx))^3(A + B \sec(c + dx)) dx &= \frac{aA \cos^2(c + dx)(a + a \sec(c + dx))^2 \sin(c + dx)}{3d} + \frac{1}{3} \\ &= \frac{aA \cos^2(c + dx)(a + a \sec(c + dx))^2 \sin(c + dx)}{3d} + \frac{5a^3(A + B) \sin(c + dx)}{2d} + \frac{aA \cos^2(c + dx)(a + a \sec(c + dx))^2 \sin(c + dx)}{3d} \\ &= \frac{5a^3(A + B) \sin(c + dx)}{2d} + \frac{aA \cos^2(c + dx)(a + a \sec(c + dx))^2 \sin(c + dx)}{3d} \\ &= \frac{1}{2}a^3(5A + 7B)x + \frac{5a^3(A + B) \sin(c + dx)}{2d} + \frac{aA \cos^2(c + dx)(a + a \sec(c + dx))^2 \sin(c + dx)}{3d} \\ &= \frac{1}{2}a^3(5A + 7B)x + \frac{a^3B \tanh^{-1}(\sin(c + dx))}{d} + \frac{5a^3(A + B) \sin(c + dx)}{2d} \end{aligned}$$

Mathematica [A] time = 0.24, size = 113, normalized size = 0.90

$$\frac{a^3 \left(9(5A + 4B) \sin(c + dx) + 3(3A + B) \sin(2(c + dx)) + A \sin(3(c + dx)) + 30Adx - 12B \log \left(\cos \left(\frac{1}{2}(c + dx) \right) \right) \right)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^3*(30*A*d*x + 42*B*d*x - 12*B*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] +
12*B*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 9*(5*A + 4*B)*Sin[c + d*x]
+ 3*(3*A + B)*Sin[2*(c + d*x)] + A*Ssin[3*(c + d*x)])/(12*d)
```

fricas [A] time = 0.49, size = 102, normalized size = 0.82

$$\frac{3(5A + 7B)a^3 dx + 3Ba^3 \log(\sin(dx + c) + 1) - 3Ba^3 \log(-\sin(dx + c) + 1) + (2Aa^3 \cos(dx + c)^2 + 3(3A + B)a^3) \sin(dx + c)}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="fricas")
```

[Out] $\frac{1}{6}*(3*(5*A + 7*B)*a^3*d*x + 3*B*a^3*\log(\sin(d*x + c) + 1) - 3*B*a^3*\log(-\sin(d*x + c) + 1) + (2*A*a^3*\cos(d*x + c)^2 + 3*(3*A + B)*a^3*\cos(d*x + c) + 2*(11*A + 9*B)*a^3)*\sin(d*x + c))/d$

giac [A] time = 1.87, size = 180, normalized size = 1.44

$$6Ba^3 \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right|\right) - 6Ba^3 \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1\right|\right) + 3(5Aa^3 + 7Ba^3)(dx + c) + \frac{2(15Aa^3 \tan(\frac{1}{2}dx + \frac{1}{2}c) + 15Aa^3 \sec(\frac{1}{2}dx + \frac{1}{2}c) + 15Aa^3 \csc(\frac{1}{2}dx + \frac{1}{2}c) + 15Aa^3 \cot(\frac{1}{2}dx + \frac{1}{2}c) + 15Aa^3 \operatorname{arctan}(\frac{1}{2}dx + \frac{1}{2}c) + 15Aa^3 \operatorname{arccot}(\frac{1}{2}dx + \frac{1}{2}c) + 15Aa^3 \operatorname{arcsin}(\frac{1}{2}dx + \frac{1}{2}c) + 15Aa^3 \operatorname{arccos}(\frac{1}{2}dx + \frac{1}{2}c) + 15Aa^3 \operatorname{arctan}(\frac{1}{2}dx + \frac{1}{2}c) + 15Aa^3 \operatorname{arccot}(\frac{1}{2}dx + \frac{1}{2}c) + 15Aa^3 \operatorname{arcsin}(\frac{1}{2}dx + \frac{1}{2}c) + 15Aa^3 \operatorname{arccos}(\frac{1}{2}dx + \frac{1}{2}c))}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="giac")`

[Out] $\frac{1}{6}*(6*B*a^3*\log(\operatorname{abs}(\tan(1/2*d*x + 1/2*c) + 1)) - 6*B*a^3*\log(\operatorname{abs}(\tan(1/2*d*x + 1/2*c) - 1)) + 3*(5*A*a^3 + 7*B*a^3)*(d*x + c) + 2*(15*A*a^3*\tan(1/2*d*x + 1/2*c)^5 + 15*B*a^3*\tan(1/2*d*x + 1/2*c)^5 + 40*A*a^3*\tan(1/2*d*x + 1/2*c)^3 + 36*B*a^3*\tan(1/2*d*x + 1/2*c)^3 + 33*A*a^3*\tan(1/2*d*x + 1/2*c) + 21*B*a^3*\tan(1/2*d*x + 1/2*c)))/(\tan(1/2*d*x + 1/2*c)^2 + 1)^3/d$

maple [A] time = 1.07, size = 153, normalized size = 1.22

$$\frac{A \sin(dx + c) (\cos^2(dx + c)) a^3}{3d} + \frac{11a^3 A \sin(dx + c)}{3d} + \frac{a^3 B \cos(dx + c) \sin(dx + c)}{2d} + \frac{7a^3 Bx}{2} + \frac{7a^3 Bc}{2d} + \frac{3Aa^3 \cos(dx + c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(d*x+c)^3*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x)`

[Out] $\frac{1}{3}/d*A*\sin(d*x+c)*\cos(d*x+c)^2*a^3+11/3*a^3*A*\sin(d*x+c)/d+1/2/d*a^3*B*\cos(d*x+c)*\sin(d*x+c)+7/2*a^3*B*x+7/2/d*a^3*B*c+3/2/d*A*a^3*\cos(d*x+c)*\sin(d*x+c)+5/2*a^3*A*x+5/2/d*A*a^3*c+3*a^3*B*\sin(d*x+c)/d+1/d*a^3*B*\ln(\sec(d*x+c)+\tan(d*x+c))$

maxima [A] time = 0.36, size = 148, normalized size = 1.18

$$\frac{4(\sin(dx + c)^3 - 3 \sin(dx + c))Aa^3 - 9(2dx + 2c + \sin(2dx + 2c))Aa^3 - 12(dx + c)Aa^3 - 3(2dx + 2c + \sin(2dx + 2c))Aa^3}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="maxima")`

[Out] $-1/12*(4*(\sin(dx + c))^3 - 3*\sin(dx + c))*A*a^3 - 9*(2*dx + 2*c + \sin(2*d*x + 2*c))*A*a^3 - 12*(dx + c)*A*a^3 - 3*(2*d*x + 2*c + \sin(2*d*x + 2*c))*B*a^3 - 36*(dx + c)*B*a^3 - 6*B*a^3*(\log(\sin(dx + c) + 1) - \log(\sin(dx + c) - 1)) - 36*A*a^3*\sin(dx + c) - 36*B*a^3*\sin(dx + c))/d$

mupad [B] time = 2.15, size = 178, normalized size = 1.42

$$\frac{15 A a^3 \sin(c + dx)}{4d} + \frac{3 B a^3 \sin(c + dx)}{d} + \frac{5 A a^3 \operatorname{atan}\left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)}\right)}{d} + \frac{7 B a^3 \operatorname{atan}\left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)}\right)}{d} + \frac{2 B a^3 \operatorname{atanh}\left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(c + d*x)^3*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^3,x)`

[Out] $(15*A*a^3*\sin(c + d*x))/(4*d) + (3*B*a^3*\sin(c + d*x))/d + (5*A*a^3*\operatorname{atan}(\sin(c/2 + (d*x)/2)/\cos(c/2 + (d*x)/2)))/d + (7*B*a^3*\operatorname{atan}(\sin(c/2 + (d*x)/2)/\cos(c/2 + (d*x)/2)))/d + (2*B*a^3*\operatorname{atanh}(\sin(c/2 + (d*x)/2)/\cos(c/2 + (d*x)/2)))/d + (3*A*a^3*\sin(2*c + 2*d*x))/(4*d) + (A*a^3*\sin(3*c + 3*d*x))/(12*d) + (B*a^3*\sin(2*c + 2*d*x))/(4*d)$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)**3*(a+a*sec(d*x+c))**3*(A+B*sec(d*x+c)),x)`

[Out] Timed out

$$3.69 \quad \int \cos^4(c + dx)(a + a \sec(c + dx))^3(A + B \sec(c + dx)) dx$$

Optimal. Leaf size=124

$$\frac{a^3(3A + 4B) \sin^3(c + dx)}{12d} + \frac{a^3(3A + 4B) \sin(c + dx)}{d} + \frac{3a^3(3A + 4B) \sin(c + dx) \cos(c + dx)}{8d} + \frac{5}{8}a^3x(3A+4B) + \dots$$

[Out] $5/8*a^3*(3*A+4*B)*x+a^3*(3*A+4*B)*\sin(d*x+c)/d+3/8*a^3*(3*A+4*B)*\cos(d*x+c)*\sin(d*x+c)/d+1/4*A*\cos(d*x+c)^3*(a+a*\sec(d*x+c))^3*\sin(d*x+c)/d-1/12*a^3*(3*A+4*B)*\sin(d*x+c)^3/d$

Rubi [A] time = 0.17, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4013, 3791, 2637, 2635, 8, 2633}

$$\frac{a^3(3A + 4B) \sin^3(c + dx)}{12d} + \frac{a^3(3A + 4B) \sin(c + dx)}{d} + \frac{3a^3(3A + 4B) \sin(c + dx) \cos(c + dx)}{8d} + \frac{5}{8}a^3x(3A+4B) + \dots$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Cos}[c + d*x]^4*(a + a*\text{Sec}[c + d*x])^3*(A + B*\text{Sec}[c + d*x]), x]$

[Out] $(5*a^3*(3*A + 4*B)*x)/8 + (a^3*(3*A + 4*B)*\text{Sin}[c + d*x])/d + (3*a^3*(3*A + 4*B)*\text{Cos}[c + d*x]*\text{Sin}[c + d*x])/(8*d) + (A*\text{Cos}[c + d*x]^3*(a + a*\text{Sec}[c + d*x])^3*\text{Sin}[c + d*x])/(4*d) - (a^3*(3*A + 4*B)*\text{Sin}[c + d*x]^3)/(12*d)$

Rule 8

$\text{Int}[a_, x_Symbol] \text{ :> } \text{Simp}[a*x, x] \text{ /; } \text{FreeQ}[a, x]$

Rule 2633

$\text{Int}[\sin[(c_.) + (d_.)*(x_.)]^{(n_.)}, x_Symbol] \text{ :> } -\text{Dist}[d^{(-1)}, \text{Subst}[\text{Int}[\text{Expand}[(1 - x^2)^{((n - 1)/2)}, x], x], x, \text{Cos}[c + d*x]], x] \text{ /; } \text{FreeQ}[\{c, d\}, x] \text{ \&\& } \text{IGtQ}[(n - 1)/2, 0]$

Rule 2635

$\text{Int}[(b_.)*\sin[(c_.) + (d_.)*(x_.)]^{(n_.)}, x_Symbol] \text{ :> } -\text{Simp}[(b*\text{Cos}[c + d*x])*(b*\text{Sin}[c + d*x])^{(n - 1)}]/(d*n), x] + \text{Dist}[(b^2*(n - 1))/n, \text{Int}[(b*\text{Sin}[c + d*x])^{(n - 2)}, x], x] \text{ /; } \text{FreeQ}[\{b, c, d\}, x] \text{ \&\& } \text{GtQ}[n, 1] \text{ \&\& } \text{IntegerQ}[2*n]$

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] := Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] := Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
- b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] && !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
 \int \cos^4(c + dx)(a + a \sec(c + dx))^3(A + B \sec(c + dx)) dx &= \frac{A \cos^3(c + dx)(a + a \sec(c + dx))^3 \sin(c + dx)}{4d} + \frac{1}{4} \int \cos^3(c + dx)(a + a \sec(c + dx))^3(A + B \sec(c + dx)) dx \\
 &= \frac{A \cos^3(c + dx)(a + a \sec(c + dx))^3 \sin(c + dx)}{4d} + \frac{1}{4} \int \cos^2(c + dx)(a + a \sec(c + dx))^3(A + B \sec(c + dx)) dx \\
 &= \frac{1}{4} a^3(3A + 4B)x + \frac{A \cos^3(c + dx)(a + a \sec(c + dx))^3 \sin(c + dx)}{4d} \\
 &= \frac{1}{4} a^3(3A + 4B)x + \frac{3a^3(3A + 4B) \sin(c + dx)}{4d} + \frac{3a^3(3A + 4B) \sin^3(c + dx)}{4d} \\
 &= \frac{5}{8} a^3(3A + 4B)x + \frac{a^3(3A + 4B) \sin(c + dx)}{d} + \frac{3a^3(3A + 4B) \sin^3(c + dx)}{4d}
 \end{aligned}$$

Mathematica [A] time = 0.27, size = 86, normalized size = 0.69

$$\frac{a^3(24(13A + 15B) \sin(c + dx) + 24(4A + 3B) \sin(2(c + dx)) + 24A \sin(3(c + dx)) + 3A \sin(4(c + dx)) + 180Adx)}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```


[Out] $(a^3(180A dx + 240B dx + 24(13A + 15B)\sin[c + dx] + 24(4A + 3B)\sin[2(c + dx)] + 24A\sin[3(c + dx)] + 8B\sin[3(c + dx)] + 3A\sin[4(c + dx)]))/96d$

fricas [A] time = 0.45, size = 90, normalized size = 0.73

$$\frac{15(3A + 4B)a^3 dx + (6Aa^3 \cos(dx + c)^3 + 8(3A + B)a^3 \cos(dx + c)^2 + 9(5A + 4B)a^3 \cos(dx + c) + 8(9A + 11B)a^3 \sin(dx + c))}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="fricas")`

[Out] $1/24*(15*(3A + 4B)*a^3 dx + (6A*a^3 \cos(dx + c)^3 + 8*(3A + B)*a^3 \cos(dx + c)^2 + 9*(5A + 4B)*a^3 \cos(dx + c) + 8*(9A + 11B)*a^3 \sin(dx + c)))/d$

giac [A] time = 0.32, size = 176, normalized size = 1.42

$$15(3Aa^3 + 4Ba^3)(dx + c) + \frac{2(45Aa^3 \tan(\frac{1}{2}dx + \frac{1}{2}c)^7 + 60Ba^3 \tan(\frac{1}{2}dx + \frac{1}{2}c)^7 + 165Aa^3 \tan(\frac{1}{2}dx + \frac{1}{2}c)^5 + 220Ba^3 \tan(\frac{1}{2}dx + \frac{1}{2}c)^5 + 219Aa^3 \tan(\frac{1}{2}dx + \frac{1}{2}c)^3 + 292Ba^3 \tan(\frac{1}{2}dx + \frac{1}{2}c)^3 + 147Aa^3 \tan(\frac{1}{2}dx + \frac{1}{2}c) + 132Ba^3 \tan(\frac{1}{2}dx + \frac{1}{2}c))}{(\tan(\frac{1}{2}dx + \frac{1}{2}c)^2 + 1)^4} \cdot \frac{1}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="giac")`

[Out] $1/24*(15*(3A*a^3 + 4B*a^3)*(dx + c) + 2*(45*A*a^3 \tan(1/2 dx + 1/2 c)^7 + 60*B*a^3 \tan(1/2 dx + 1/2 c)^7 + 165*A*a^3 \tan(1/2 dx + 1/2 c)^5 + 220*B*a^3 \tan(1/2 dx + 1/2 c)^5 + 219*A*a^3 \tan(1/2 dx + 1/2 c)^3 + 292*B*a^3 \tan(1/2 dx + 1/2 c)^3 + 147*A*a^3 \tan(1/2 dx + 1/2 c) + 132*B*a^3 \tan(1/2 dx + 1/2 c)))/(\tan(1/2 dx + 1/2 c)^2 + 1)^4/d$

maple [A] time = 1.32, size = 176, normalized size = 1.42

$$Aa^3 \left(\frac{(\cos^3(dx+c) + \frac{3\cos(dx+c)}{2}) \sin(dx+c)}{4} + \frac{3dx}{8} + \frac{3c}{8} \right) + Aa^3 (2 + \cos^2(dx+c)) \sin(dx+c) + \frac{a^3 B (2 + \cos^2(dx+c)) \sin(dx+c)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(d*x+c)^4*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x)`

```
[Out] 1/d*(A*a^3*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+A*a^3*(2+cos(d*x+c)^2)*sin(d*x+c)+1/3*a^3*B*(2+cos(d*x+c)^2)*sin(d*x+c)+3*A*a^3*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+3*a^3*B*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+A*a^3*sin(d*x+c)+3*a^3*B*sin(d*x+c)+B*(d*x+c)*a^3)
```

maxima [A] time = 0.36, size = 167, normalized size = 1.35

$$\frac{96(\sin(dx+c)^3 - 3\sin(dx+c))Aa^3 - 3(12dx + 12c + \sin(4dx + 4c) + 8\sin(2dx + 2c))Aa^3 - 72(2dx + 2c)Aa^3 + 32(\sin(dx+c)^3 - 3\sin(dx+c))Ba^3 - 72(2dx + 2c)Ba^3 - 96(dx+c)Ba^3 - 96Aa^3\sin(dx+c) - 288Ba^3\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/96*(96*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^3 - 3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^3 - 72*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a^3 + 32*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^3 - 72*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^3 - 96*(d*x + c)*B*a^3 - 96*A*a^3*sin(d*x + c) - 288*B*a^3*sin(d*x + c))/d
```

mupad [B] time = 2.01, size = 134, normalized size = 1.08

$$\frac{15 A a^3 x}{8} + \frac{5 B a^3 x}{2} + \frac{13 A a^3 \sin(c + dx)}{4 d} + \frac{15 B a^3 \sin(c + dx)}{4 d} + \frac{A a^3 \sin(2c + 2dx)}{d} + \frac{A a^3 \sin(3c + 3dx)}{4 d} + \frac{A a^3 \sin(4c + 4dx)}{32 d} + \frac{3 B a^3 \sin(2c + 2dx)}{4 d} + \frac{B a^3 \sin(3c + 3dx)}{12 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^3,x)
```

```
[Out] (15*A*a^3*x)/8 + (5*B*a^3*x)/2 + (13*A*a^3*sin(c + d*x))/(4*d) + (15*B*a^3*sin(c + d*x))/(4*d) + (A*a^3*sin(2*c + 2*d*x))/d + (A*a^3*sin(3*c + 3*d*x))/(4*d) + (A*a^3*sin(4*c + 4*d*x))/(32*d) + (3*B*a^3*sin(2*c + 2*d*x))/(4*d) + (B*a^3*sin(3*c + 3*d*x))/(12*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*sec(d*x+c))**3*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

3.70 $\int \cos^5(c + dx)(a + a \sec(c + dx))^3(A + B \sec(c + dx)) dx$

Optimal. Leaf size=176

$$\frac{a^3(38A + 45B) \sin(c + dx)}{15d} + \frac{a^3(43A + 45B) \sin(c + dx) \cos^2(c + dx)}{60d} + \frac{a^3(13A + 15B) \sin(c + dx) \cos(c + dx)}{8d}$$

```
[Out] 1/8*a^3*(13*A+15*B)*x+1/15*a^3*(38*A+45*B)*sin(d*x+c)/d+1/8*a^3*(13*A+15*B)
*cos(d*x+c)*sin(d*x+c)/d+1/60*a^3*(43*A+45*B)*cos(d*x+c)^2*sin(d*x+c)/d+1/5
*a*A*cos(d*x+c)^4*(a+a*sec(d*x+c))^2*sin(d*x+c)/d+1/20*(7*A+5*B)*cos(d*x+c)
^3*(a^3+a^3*sec(d*x+c))*sin(d*x+c)/d
```

Rubi [A] time = 0.37, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4017, 3996, 3787, 2635, 8, 2637}

$$\frac{a^3(38A + 45B) \sin(c + dx)}{15d} + \frac{a^3(43A + 45B) \sin(c + dx) \cos^2(c + dx)}{60d} + \frac{a^3(13A + 15B) \sin(c + dx) \cos(c + dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^3*(13*A + 15*B)*x)/8 + (a^3*(38*A + 45*B)*Sin[c + d*x])/(15*d) + (a^3*(13*A + 15*B)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a^3*(43*A + 45*B)*Cos[c + d*x]^2*Sin[c + d*x])/(60*d) + (a*A*Cos[c + d*x]^4*(a + a*Sec[c + d*x])^2*Sin[c + d*x])/(5*d) + ((7*A + 5*B)*Cos[c + d*x]^3*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x])/(20*d)
```

Rule 8

```
Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x])*(b*SIN[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*SIN[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[SIN[c + d*x]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 3996

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]

Rule 4017

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> Simp[(a*A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && LtQ[n, -1]

Rubi steps

$$\begin{aligned}
 \int \cos^5(c + dx)(a + a \sec(c + dx))^3(A + B \sec(c + dx)) dx &= \frac{aA \cos^4(c + dx)(a + a \sec(c + dx))^2 \sin(c + dx)}{5d} + \frac{1}{5} \\
 &= \frac{aA \cos^4(c + dx)(a + a \sec(c + dx))^2 \sin(c + dx)}{5d} + \frac{7}{5} \\
 &= \frac{a^3(43A + 45B) \cos^2(c + dx) \sin(c + dx)}{60d} + \frac{aA \cos^4(c + dx)}{5d} \\
 &= \frac{a^3(43A + 45B) \cos^2(c + dx) \sin(c + dx)}{60d} + \frac{aA \cos^4(c + dx)}{5d} \\
 &= \frac{a^3(38A + 45B) \sin(c + dx)}{15d} + \frac{a^3(13A + 15B) \cos(c + dx)}{8d} \\
 &= \frac{1}{8}a^3(13A + 15B)x + \frac{a^3(38A + 45B) \sin(c + dx)}{15d} + \frac{a^3}{8}
 \end{aligned}$$

Mathematica [A] time = 0.43, size = 108, normalized size = 0.61

$$\frac{a^3(60(23A + 26B) \sin(c + dx) + 480(A + B) \sin(2(c + dx)) + 170A \sin(3(c + dx)) + 45A \sin(4(c + dx)) + 6A \sin(5(c + dx)))}{480d}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]^5*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]

[Out] (a^3*(780*A*c + 780*A*d*x + 900*B*d*x + 60*(23*A + 26*B)*Sin[c + d*x] + 480*(A + B)*Sin[2*(c + d*x)] + 170*A*Ssin[3*(c + d*x)] + 120*B*Ssin[3*(c + d*x)] + 45*A*Ssin[4*(c + d*x)] + 15*B*Ssin[4*(c + d*x)] + 6*A*Ssin[5*(c + d*x)]))/(480*d)

fricas [A] time = 0.45, size = 110, normalized size = 0.62

$$\frac{15(13A + 15B)a^3 dx + (24Aa^3 \cos(dx + c)^4 + 30(3A + B)a^3 \cos(dx + c)^3 + 8(19A + 15B)a^3 \cos(dx + c)^2 + 15(13A + 15B)a^3 \cos(dx + c) + 8(38A + 45B)a^3 \sin(dx + c))}{120d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] 1/120*(15*(13*A + 15*B)*a^3*d*x + (24*A*a^3*cos(d*x + c)^4 + 30*(3*A + B)*a^3*cos(d*x + c)^3 + 8*(19*A + 15*B)*a^3*cos(d*x + c)^2 + 15*(13*A + 15*B)*a^3*cos(d*x + c) + 8*(38*A + 45*B)*a^3*sin(d*x + c))/d

giac [A] time = 0.63, size = 210, normalized size = 1.19

$$15(13Aa^3 + 15Ba^3)(dx + c) + \frac{2\left(195Aa^3 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^9 + 225Ba^3 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^9 + 910Aa^3 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^7 + 1050Ba^3 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^7\right)}{\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + 1} / d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] 1/120*(15*(13*A*a^3 + 15*B*a^3)*(d*x + c) + 2*(195*A*a^3*tan(1/2*d*x + 1/2*c)^9 + 225*B*a^3*tan(1/2*d*x + 1/2*c)^9 + 910*A*a^3*tan(1/2*d*x + 1/2*c)^7 + 1050*B*a^3*tan(1/2*d*x + 1/2*c)^7 + 1664*A*a^3*tan(1/2*d*x + 1/2*c)^5 + 1920*B*a^3*tan(1/2*d*x + 1/2*c)^5 + 1330*A*a^3*tan(1/2*d*x + 1/2*c)^3 + 1830*B*a^3*tan(1/2*d*x + 1/2*c)^3 + 765*A*a^3*tan(1/2*d*x + 1/2*c) + 735*B*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^5/d

maple [A] time = 1.80, size = 223, normalized size = 1.27

$$\frac{Aa^3 \left(\frac{8}{3} + \cos^4(dx+c) + \frac{4(\cos^2(dx+c))}{3} \right) \sin(dx+c)}{5} + a^3 B \left(\frac{(\cos^3(dx+c) + \frac{3\cos(dx+c)}{2}) \sin(dx+c)}{4} + \frac{3dx}{8} + \frac{3c}{8} \right) + 3Aa^3 \left(\frac{(\cos^3(dx+c) + \frac{3\cos(dx+c)}{2}) \sin(dx+c)}{4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^5*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x)

[Out] 1/d*(1/5*A*a^3*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+a^3*B*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+3*A*a^3*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+a^3*B*(2+cos(d*x+c)^2)*sin(d*x+c)+A*a^3*(2+cos(d*x+c)^2)*sin(d*x+c)+3*a^3*B*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+A*a^3*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+a^3*B*sin(d*x+c))

maxima [A] time = 0.36, size = 213, normalized size = 1.21

$$\frac{32 \left(3 \sin(dx+c)^5 - 10 \sin(dx+c)^3 + 15 \sin(dx+c) \right) Aa^3 - 480 \left(\sin(dx+c)^3 - 3 \sin(dx+c) \right) Aa^3 + 45 (12 dx + 12c) Aa^3}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] 1/480*(32*(3*sin(d*x+c)^5 - 10*sin(d*x+c)^3 + 15*sin(d*x+c))*A*a^3 - 480*(sin(d*x+c)^3 - 3*sin(d*x+c))*A*a^3 + 45*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^3 + 120*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a^3 - 480*(sin(d*x+c)^3 - 3*sin(d*x+c))*B*a^3 + 15*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^3 + 360*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^3 + 480*B*a^3*sin(d*x+c))/d

mupad [B] time = 4.71, size = 247, normalized size = 1.40

$$\frac{\left(\frac{13Aa^3}{4} + \frac{15Ba^3}{4} \right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^9 + \left(\frac{91Aa^3}{6} + \frac{35Ba^3}{2} \right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^7 + \left(\frac{416Aa^3}{15} + 32Ba^3 \right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5 + \left(\frac{133Aa^3}{6} + 15Ba^3 \right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 + \left(\frac{13Aa^3}{4} + \frac{15Ba^3}{4} \right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^{10} + 5 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^8 + 10 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^6 + 10 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 + 5 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + 1 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c+d*x)^5*(A+B/cos(c+d*x))*(a+a/cos(c+d*x))^3,x)

```
[Out] (tan(c/2 + (d*x)/2)*((51*A*a^3)/4 + (49*B*a^3)/4) + tan(c/2 + (d*x)/2)^9*((
13*A*a^3)/4 + (15*B*a^3)/4) + tan(c/2 + (d*x)/2)^7*((91*A*a^3)/6 + (35*B*a^
3)/2) + tan(c/2 + (d*x)/2)^3*((133*A*a^3)/6 + (61*B*a^3)/2) + tan(c/2 + (d*
x)/2)^5*((416*A*a^3)/15 + 32*B*a^3))/(d*(5*tan(c/2 + (d*x)/2)^2 + 10*tan(c/
2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 + 5*tan(c/2 + (d*x)/2)^8 + tan(c/2
+ (d*x)/2)^10 + 1)) + (a^3*atan((a^3*tan(c/2 + (d*x)/2)*(13*A + 15*B))/(4*
((13*A*a^3)/4 + (15*B*a^3)/4)))*(13*A + 15*B))/(4*d)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+a*sec(d*x+c))**3*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

3.71 $\int \cos^6(c + dx)(a + a \sec(c + dx))^3(A + B \sec(c + dx)) dx$

Optimal. Leaf size=201

$$-\frac{a^3(17A + 19B) \sin^3(c + dx)}{15d} + \frac{a^3(17A + 19B) \sin(c + dx)}{5d} + \frac{a^3(21A + 22B) \sin(c + dx) \cos^3(c + dx)}{40d} + \frac{a^3(23A + 26B)x}{16d}$$

[Out] $1/16*a^3*(23*A+26*B)*x+1/5*a^3*(17*A+19*B)*\sin(d*x+c)/d+1/16*a^3*(23*A+26*B)*\cos(d*x+c)*\sin(d*x+c)/d+1/40*a^3*(21*A+22*B)*\cos(d*x+c)^3*\sin(d*x+c)/d+1/6*a*A*\cos(d*x+c)^5*(a+a*\sec(d*x+c))^2*\sin(d*x+c)/d+1/15*(4*A+3*B)*\cos(d*x+c)^4*(a^3+a^3*\sec(d*x+c))*\sin(d*x+c)/d-1/15*a^3*(17*A+19*B)*\sin(d*x+c)^3/d$

Rubi [A] time = 0.41, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4017, 3996, 3787, 2633, 2635, 8}

$$-\frac{a^3(17A + 19B) \sin^3(c + dx)}{15d} + \frac{a^3(17A + 19B) \sin(c + dx)}{5d} + \frac{a^3(21A + 22B) \sin(c + dx) \cos^3(c + dx)}{40d} + \frac{a^3(23A + 26B)x}{16d}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^6*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]), x]

[Out] $(a^3*(23*A + 26*B)*x)/16 + (a^3*(17*A + 19*B)*\sin[c + d*x])/(5*d) + (a^3*(23*A + 26*B)*\cos[c + d*x]*\sin[c + d*x])/(16*d) + (a^3*(21*A + 22*B)*\cos[c + d*x]^3*\sin[c + d*x])/(40*d) + (a*A*\cos[c + d*x]^5*(a + a*\sec[c + d*x])^2*\sin[c + d*x])/(6*d) + ((4*A + 3*B)*\cos[c + d*x]^4*(a^3 + a^3*\sec[c + d*x])*\sin[c + d*x])/(15*d) - (a^3*(17*A + 19*B)*\sin[c + d*x]^3)/(15*d)$

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2633

Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Simp[(b*cos[c + d*x])*(b*sin[c + d*x])^(n - 1)]/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

]

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
+ 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \cos^6(c + dx)(a + a \sec(c + dx))^3(A + B \sec(c + dx)) dx &= \frac{aA \cos^5(c + dx)(a + a \sec(c + dx))^2 \sin(c + dx)}{6d} + \dots \\
&= \frac{aA \cos^5(c + dx)(a + a \sec(c + dx))^2 \sin(c + dx)}{6d} + \dots \\
&= \frac{a^3(21A + 22B) \cos^3(c + dx) \sin(c + dx)}{40d} + \frac{aA \cos^5}{40d} \\
&= \frac{a^3(21A + 22B) \cos^3(c + dx) \sin(c + dx)}{40d} + \frac{aA \cos^5}{40d} \\
&= \frac{a^3(23A + 26B) \cos(c + dx) \sin(c + dx)}{16d} + \frac{a^3(21A + \dots)}{16d} \\
&= \frac{1}{16} a^3(23A + 26B)x + \frac{a^3(17A + 19B) \sin(c + dx)}{5d} + \dots
\end{aligned}$$

Mathematica [A] time = 0.56, size = 134, normalized size = 0.67

$$\frac{a^3(120(21A + 23B) \sin(c + dx) + 15(63A + 64B) \sin(2(c + dx)) + 380A \sin(3(c + dx)) + 135A \sin(4(c + dx)) + 30A \sin(5(c + dx)) + 12B \sin(6(c + dx)))}{960d}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]^6*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]

[Out] (a^3*(1380*A*c + 1380*A*d*x + 1560*B*d*x + 120*(21*A + 23*B)*Sin[c + d*x] + 15*(63*A + 64*B)*Sin[2*(c + d*x)] + 380*A*Ssin[3*(c + d*x)] + 340*B*Ssin[3*(c + d*x)] + 135*A*Ssin[4*(c + d*x)] + 90*B*Ssin[4*(c + d*x)] + 36*A*Ssin[5*(c + d*x)] + 12*B*Ssin[5*(c + d*x)] + 5*A*Ssin[6*(c + d*x)]))/(960*d)

fricas [A] time = 0.44, size = 130, normalized size = 0.65

$$\frac{15(23A + 26B)a^3 dx + (40Aa^3 \cos(dx + c)^5 + 48(3A + B)a^3 \cos(dx + c)^4 + 10(23A + 18B)a^3 \cos(dx + c)^3 + 16(17A + 19B)a^3 \cos(dx + c)^2 + 15(23A + 26B)a^3 \cos(dx + c) + 32(17A + 19B)a^3 \sin(dx + c))/d}{240d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^6*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] 1/240*(15*(23*A + 26*B)*a^3*d*x + (40*A*a^3*cos(d*x + c)^5 + 48*(3*A + B)*a^3*cos(d*x + c)^4 + 10*(23*A + 18*B)*a^3*cos(d*x + c)^3 + 16*(17*A + 19*B)*a^3*cos(d*x + c)^2 + 15*(23*A + 26*B)*a^3*cos(d*x + c) + 32*(17*A + 19*B)*a^3*sin(d*x + c))/d

giac [A] time = 0.40, size = 244, normalized size = 1.21

$$15(23Aa^3 + 26Ba^3)(dx + c) + \frac{2\left(345Aa^3 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^{11} + 390Ba^3 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^{11} + 1955Aa^3 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^9 + 2210Ba^3 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^9\right)}{240d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^6*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] 1/240*(15*(23*A*a^3 + 26*B*a^3)*(d*x + c) + 2*(345*A*a^3*tan(1/2*d*x + 1/2*c)^11 + 390*B*a^3*tan(1/2*d*x + 1/2*c)^11 + 1955*A*a^3*tan(1/2*d*x + 1/2*c)^9 + 2210*B*a^3*tan(1/2*d*x + 1/2*c)^9 + 4554*A*a^3*tan(1/2*d*x + 1/2*c)^7 + 5148*B*a^3*tan(1/2*d*x + 1/2*c)^7 + 5814*A*a^3*tan(1/2*d*x + 1/2*c)^5 + 5988*B*a^3*tan(1/2*d*x + 1/2*c)^5 + 3165*A*a^3*tan(1/2*d*x + 1/2*c)^3 + 4190

$$*B*a^3*\tan(1/2*d*x + 1/2*c)^3 + 1575*A*a^3*\tan(1/2*d*x + 1/2*c) + 1530*B*a^3*\tan(1/2*d*x + 1/2*c))/(\tan(1/2*d*x + 1/2*c)^2 + 1)^6)/d$$

maple [A] time = 1.98, size = 266, normalized size = 1.32

$$A a^3 \left(\frac{\left(\cos^5(dx+c) + \frac{5(\cos^3(dx+c))}{4} + \frac{15\cos(dx+c)}{8} \right) \sin(dx+c)}{6} + \frac{5dx}{16} + \frac{5c}{16} \right) + \frac{a^3 B \left(\frac{8}{3} + \cos^4(dx+c) + \frac{4(\cos^2(dx+c))}{3} \right) \sin(dx+c)}{5} + \frac{3A a^3 \left(\frac{8}{3} + \cos^4(dx+c) \right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^6*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x)

[Out] 1/d*(A*a^3*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/16*d*x+5/16*c)+1/5*a^3*B*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+3/5*A*a^3*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+3*a^3*B*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+3*A*a^3*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+a^3*B*(2+cos(d*x+c)^2)*sin(d*x+c)+1/3*A*a^3*(2+cos(d*x+c)^2)*sin(d*x+c)+a^3*B*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))

maxima [A] time = 0.35, size = 262, normalized size = 1.30

$$192 \left(3 \sin(dx+c)^5 - 10 \sin(dx+c)^3 + 15 \sin(dx+c) \right) A a^3 - 5 \left(4 \sin(2dx+2c)^3 - 60 dx - 60 c - 9 \sin(4dx+4c) \right) B a^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^6*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] 1/960*(192*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*A*a^3 - 5*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*B*a^3 - 320*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^3 + 90*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^3 + 64*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*B*a^3 - 960*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^3 + 90*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^3 + 240*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^3)/d

mupad [B] time = 4.68, size = 285, normalized size = 1.42

$$\frac{\left(\frac{23 A a^3}{8} + \frac{13 B a^3}{4} \right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^{11} + \left(\frac{391 A a^3}{24} + \frac{221 B a^3}{12} \right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^9 + \left(\frac{759 A a^3}{20} + \frac{429 B a^3}{10} \right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^7 + \left(\frac{969 A a^3}{8} + \frac{579 B a^3}{4} \right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5 + \left(\frac{23 A a^3}{8} + \frac{13 B a^3}{4} \right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 + \left(\frac{391 A a^3}{24} + \frac{221 B a^3}{12} \right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right) + \left(\frac{759 A a^3}{20} + \frac{429 B a^3}{10} \right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right) + \left(\frac{969 A a^3}{8} + \frac{579 B a^3}{4} \right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right) + 20 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^{12} + 6 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^{10} + 15 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^8 + 20 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^6 + 15 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 + 6 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + 1 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^3,x)
```

```
[Out] (tan(c/2 + (d*x)/2)*((105*A*a^3)/8 + (51*B*a^3)/4) + tan(c/2 + (d*x)/2)^11*
((23*A*a^3)/8 + (13*B*a^3)/4) + tan(c/2 + (d*x)/2)^3*((211*A*a^3)/8 + (419*
B*a^3)/12) + tan(c/2 + (d*x)/2)^9*((391*A*a^3)/24 + (221*B*a^3)/12) + tan(c
/2 + (d*x)/2)^7*((759*A*a^3)/20 + (429*B*a^3)/10) + tan(c/2 + (d*x)/2)^5*((
969*A*a^3)/20 + (499*B*a^3)/10))/(d*(6*tan(c/2 + (d*x)/2)^2 + 15*tan(c/2 +
(d*x)/2)^4 + 20*tan(c/2 + (d*x)/2)^6 + 15*tan(c/2 + (d*x)/2)^8 + 6*tan(c/2
+ (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 + 1)) + (a^3*atan((a^3*tan(c/2 + (d*x
)/2)*(23*A + 26*B))/(8*((23*A*a^3)/8 + (13*B*a^3)/4)))*(23*A + 26*B))/(8*d)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+a*sec(d*x+c))**3*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

3.72 $\int \sec^2(c + dx)(a + a \sec(c + dx))^4(A + B \sec(c + dx)) dx$

Optimal. Leaf size=194

$$\frac{2a^4(8A + 7B) \tan^3(c + dx)}{15d} + \frac{4a^4(8A + 7B) \tan(c + dx)}{5d} + \frac{7a^4(8A + 7B) \tanh^{-1}(\sin(c + dx))}{16d} + \frac{a^4(8A + 7B) \tan(c + dx)}{4d}$$

[Out] $7/16*a^4*(8*A+7*B)*\operatorname{arctanh}(\sin(d*x+c))/d+4/5*a^4*(8*A+7*B)*\tan(d*x+c)/d+27/80*a^4*(8*A+7*B)*\sec(d*x+c)*\tan(d*x+c)/d+1/40*a^4*(8*A+7*B)*\sec(d*x+c)^3*\tan(d*x+c)/d+1/30*(6*A-B)*(a+a*\sec(d*x+c))^4*\tan(d*x+c)/d+1/6*B*(a+a*\sec(d*x+c))^5*\tan(d*x+c)/a/d+2/15*a^4*(8*A+7*B)*\tan(d*x+c)^3/d$

Rubi [A] time = 0.32, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {4010, 4001, 3791, 3770, 3767, 8, 3768}

$$\frac{2a^4(8A + 7B) \tan^3(c + dx)}{15d} + \frac{4a^4(8A + 7B) \tan(c + dx)}{5d} + \frac{7a^4(8A + 7B) \tanh^{-1}(\sin(c + dx))}{16d} + \frac{a^4(8A + 7B) \tan(c + dx)}{4d}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Sec}[c + d*x]^2*(a + a*\operatorname{Sec}[c + d*x])^4*(A + B*\operatorname{Sec}[c + d*x]), x]$

[Out] $(7*a^4*(8*A + 7*B)*\operatorname{ArcTanh}[\operatorname{Sin}[c + d*x]])/(16*d) + (4*a^4*(8*A + 7*B)*\operatorname{Tan}[c + d*x])/(5*d) + (27*a^4*(8*A + 7*B)*\operatorname{Sec}[c + d*x]*\operatorname{Tan}[c + d*x])/(80*d) + (a^4*(8*A + 7*B)*\operatorname{Sec}[c + d*x]^3*\operatorname{Tan}[c + d*x])/(40*d) + ((6*A - B)*(a + a*\operatorname{Sec}[c + d*x])^4*\operatorname{Tan}[c + d*x])/(30*d) + (B*(a + a*\operatorname{Sec}[c + d*x])^5*\operatorname{Tan}[c + d*x])/(6*a*d) + (2*a^4*(8*A + 7*B)*\operatorname{Tan}[c + d*x]^3)/(15*d)$

Rule 8

$\operatorname{Int}[a_, x_Symbol] := \operatorname{Simp}[a*x, x] /; \operatorname{FreeQ}[a, x]$

Rule 3767

$\operatorname{Int}[\operatorname{csc}[(c_.) + (d_.)*(x_.)]^{(n_.)}, x_Symbol] := -\operatorname{Dist}[d^{(-1)}, \operatorname{Subst}[\operatorname{Int}[\operatorname{ExpandIntegrand}[(1 + x^2)^{(n/2 - 1)}, x], x], x, \operatorname{Cot}[c + d*x]], x] /; \operatorname{FreeQ}[\{c, d\}, x] \ \&\& \operatorname{IGtQ}[n/2, 0]$

Rule 3768

$\operatorname{Int}[(\operatorname{csc}[(c_.) + (d_.)*(x_.)]*(b_.))^{(n_.)}, x_Symbol] := -\operatorname{Simp}[(b*\operatorname{Cos}[c + d*x])*(b*\operatorname{Csc}[c + d*x])^{(n - 1)}]/(d*(n - 1)), x] + \operatorname{Dist}[(b^2*(n - 2))/(n - 1), \operatorname{Int}[(b*\operatorname{Csc}[c + d*x])^{(n - 2)}, x], x] /; \operatorname{FreeQ}[\{b, c, d\}, x] \ \&\& \operatorname{GtQ}[n, 1] \ \&\&$

IntegerQ[2*n]

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3791

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_), x_Symbol] := Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && !GtQ[m, 0] && RationalQ[n]

Rule 4001

Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] := -Simp[(B*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m + 1), 0] && !LtQ[m, -2^(-1)]

Rule 4010

Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] := -Simp[(B*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && !LtQ[m, -1]

Rubi steps

$$\begin{aligned}
\int \sec^2(c + dx)(a + a \sec(c + dx))^4(A + B \sec(c + dx)) dx &= \frac{B(a + a \sec(c + dx))^5 \tan(c + dx)}{6ad} + \frac{\int \sec(c + dx)(a + a \sec(c + dx))^4(A + B \sec(c + dx)) dx}{6ad} \\
&= \frac{(6A - B)(a + a \sec(c + dx))^4 \tan(c + dx)}{30d} + \frac{B(a + a \sec(c + dx))^5 \tan(c + dx)}{6ad} \\
&= \frac{(6A - B)(a + a \sec(c + dx))^4 \tan(c + dx)}{30d} + \frac{B(a + a \sec(c + dx))^5 \tan(c + dx)}{6ad} \\
&= \frac{(6A - B)(a + a \sec(c + dx))^4 \tan(c + dx)}{30d} + \frac{B(a + a \sec(c + dx))^5 \tan(c + dx)}{6ad} \\
&= \frac{a^4(8A + 7B) \tanh^{-1}(\sin(c + dx))}{10d} + \frac{3a^4(8A + 7B) \tanh^{-1}(\sin(c + dx))}{10d} \\
&= \frac{2a^4(8A + 7B) \tanh^{-1}(\sin(c + dx))}{5d} + \frac{4a^4(8A + 7B) \tanh^{-1}(\sin(c + dx))}{5d} \\
&= \frac{7a^4(8A + 7B) \tanh^{-1}(\sin(c + dx))}{16d} + \frac{4a^4(8A + 7B) \tanh^{-1}(\sin(c + dx))}{5d}
\end{aligned}$$

Mathematica [A] time = 2.38, size = 358, normalized size = 1.85

$$a^4(\cos(c + dx) + 1)^4 \sec^8\left(\frac{1}{2}(c + dx)\right) \sec^6(c + dx) \left(3360(8A + 7B) \cos^6(c + dx) \left(\log\left(\cos\left(\frac{1}{2}(c + dx)\right) - \sin\left(\frac{1}{2}(c + dx)\right)\right)\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]

[Out] -1/122880*(a^4*(1 + Cos[c + d*x])^4*Sec[(c + d*x)/2]^8*Sec[c + d*x]^6*(3360*(8*A + 7*B)*Cos[c + d*x]^6*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])) - Sec[c]*(-160*(83*A + 72*B)*Sin[c] + 30*(88*A + 125*B)*Sin[d*x] + 2640*A*Sin[2*c + d*x] + 3750*B*Sin[2*c + d*x] + 15840*A*Sin[c + 2*d*x] + 15360*B*Sin[c + 2*d*x] - 4080*A*Sin[3*c + 2*d*x] - 1920*B*Sin[3*c + 2*d*x] + 3480*A*Sin[2*c + 3*d*x] + 3845*B*Sin[2*c + 3*d*x] + 3480*A*Sin[4*c + 3*d*x] + 3845*B*Sin[4*c + 3*d*x] + 7728*A*Sin[3*c + 4*d*x] + 6912*B*Sin[3*c + 4*d*x] - 240*A*Sin[5*c + 4*d*x] + 840*A*Sin[4*c + 5*d*x] + 735*B*Sin[4*c + 5*d*x] + 840*A*Sin[6*c + 5*d*x] + 735*B*Sin[6*c + 5*d*x] + 1328*A*Sin[5*c + 6*d*x] + 1152*B*Sin[5*c + 6*d*x]))/d

fricas [A] time = 0.44, size = 185, normalized size = 0.95

$$105(8A + 7B)a^4 \cos(dx + c)^6 \log(\sin(dx + c) + 1) - 105(8A + 7B)a^4 \cos(dx + c)^6 \log(-\sin(dx + c) + 1) +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] $\frac{1}{480} \cdot (105 \cdot (8A + 7B) \cdot a^4 \cdot \cos(dx + c)^6 \cdot \log(\sin(dx + c) + 1) - 105 \cdot (8A + 7B) \cdot a^4 \cdot \cos(dx + c)^6 \cdot \log(-\sin(dx + c) + 1) + 2 \cdot (16 \cdot (83A + 72B) \cdot a^4 \cdot \cos(dx + c)^5 + 105 \cdot (8A + 7B) \cdot a^4 \cdot \cos(dx + c)^4 + 32 \cdot (17A + 18B) \cdot a^4 \cdot \cos(dx + c)^3 + 10 \cdot (24A + 41B) \cdot a^4 \cdot \cos(dx + c)^2 + 48 \cdot (A + 4B) \cdot a^4 \cdot \cos(dx + c) + 40 \cdot B \cdot a^4) \cdot \sin(dx + c)) / (d \cdot \cos(dx + c)^6)$

giac [A] time = 0.78, size = 280, normalized size = 1.44

$$105 (8 A a^4 + 7 B a^4) \log \left(\left| \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) + 1 \right| \right) - 105 (8 A a^4 + 7 B a^4) \log \left(\left| \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) - 1 \right| \right) - \frac{2 \left(840 A a^4 \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) \right)}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] $\frac{1}{240} \cdot (105 \cdot (8A \cdot a^4 + 7B \cdot a^4) \cdot \log(\text{abs}(\tan(1/2 \cdot dx + 1/2 \cdot c) + 1)) - 105 \cdot (8A \cdot a^4 + 7B \cdot a^4) \cdot \log(\text{abs}(\tan(1/2 \cdot dx + 1/2 \cdot c) - 1)) - 2 \cdot (840 \cdot A \cdot a^4 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c)^{11} + 735 \cdot B \cdot a^4 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c)^{11} - 4760 \cdot A \cdot a^4 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c)^9 - 4165 \cdot B \cdot a^4 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c)^9 + 11088 \cdot A \cdot a^4 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c)^7 + 9702 \cdot B \cdot a^4 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c)^7 - 13488 \cdot A \cdot a^4 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c)^5 - 11802 \cdot B \cdot a^4 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c)^5 + 9320 \cdot A \cdot a^4 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c)^3 + 7355 \cdot B \cdot a^4 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c)^3 - 3000 \cdot A \cdot a^4 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c) - 3105 \cdot B \cdot a^4 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c)) / (\tan(1/2 \cdot dx + 1/2 \cdot c)^2 - 1)^6 / d$

maple [A] time = 1.80, size = 280, normalized size = 1.44

$$\frac{83A a^4 \tan(dx + c)}{15d} + \frac{49a^4 B \sec(dx + c) \tan(dx + c)}{16d} + \frac{49a^4 B \ln(\sec(dx + c) + \tan(dx + c))}{16d} + \frac{7A a^4 \sec(dx + c) \tan(dx + c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^2*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x)

[Out] $\frac{83}{15} \cdot \frac{1}{d} \cdot A \cdot a^4 \cdot \tan(dx + c) + \frac{49}{16} \cdot \frac{1}{d} \cdot a^4 \cdot B \cdot \sec(dx + c) \cdot \tan(dx + c) + \frac{49}{16} \cdot \frac{1}{d} \cdot a^4 \cdot B \cdot \ln(\sec(dx + c) + \tan(dx + c)) + \frac{7}{2} \cdot \frac{1}{d} \cdot A \cdot a^4 \cdot \sec(dx + c) \cdot \tan(dx + c) + \frac{7}{2} \cdot \frac{1}{d} \cdot A \cdot a^4 \cdot \ln(\sec(dx + c) + \tan(dx + c)) + \frac{24}{5} \cdot \frac{1}{d} \cdot a^4 \cdot B \cdot \tan(dx + c) + \frac{12}{5} \cdot \frac{1}{d} \cdot a^4 \cdot B \cdot \tan(dx + c) \cdot \sec(dx + c)^2 + \frac{34}{15} \cdot \frac{1}{d} \cdot A \cdot a^4 \cdot \tan(dx + c) \cdot \sec(dx + c)^2 + \frac{41}{24} \cdot \frac{1}{d} \cdot a^4 \cdot B \cdot \tan(dx + c) \cdot \sec(dx + c)^3 + \frac{1}{d} \cdot A \cdot a^4 \cdot \tan(dx + c) \cdot \sec(dx + c)^3 + \frac{4}{5} \cdot \frac{1}{d} \cdot a^4 \cdot B \cdot \tan(dx + c) \cdot \sec(dx + c)^4 + \frac{1}{5} \cdot \frac{1}{d} \cdot A \cdot a^4 \cdot \tan(dx + c) \cdot \sec(dx + c)^4 + \frac{1}{6} \cdot \frac{1}{d} \cdot a^4 \cdot B \cdot \tan(dx + c) \cdot \sec(dx + c)^5$

maxima [B] time = 0.35, size = 464, normalized size = 2.39

$$32 \left(3 \tan(dx + c)^5 + 10 \tan(dx + c)^3 + 15 \tan(dx + c) \right) Aa^4 + 960 \left(\tan(dx + c)^3 + 3 \tan(dx + c) \right) Aa^4 + 128 \left(\tan(dx + c)^5 + 10 \tan(dx + c)^3 + 15 \tan(dx + c) \right) Ba^4 + 640 \left(\tan(dx + c)^3 + 3 \tan(dx + c) \right) Ba^4 - 5Ba^4 \left(2 \left(15 \sin(dx + c)^5 - 40 \sin(dx + c)^3 + 33 \sin(dx + c) \right) / \left(\sin(dx + c)^6 - 3 \sin(dx + c)^4 + 3 \sin(dx + c)^2 - 1 \right) - 15 \log(\sin(dx + c) + 1) + 15 \log(\sin(dx + c) - 1) \right) - 120Aa^4 \left(2 \left(3 \sin(dx + c)^3 - 5 \sin(dx + c) \right) / \left(\sin(dx + c)^4 - 2 \sin(dx + c)^2 + 1 \right) - 3 \log(\sin(dx + c) + 1) + 3 \log(\sin(dx + c) - 1) \right) - 180Ba^4 \left(2 \left(3 \sin(dx + c)^3 - 5 \sin(dx + c) \right) / \left(\sin(dx + c)^4 - 2 \sin(dx + c)^2 + 1 \right) - 3 \log(\sin(dx + c) + 1) + 3 \log(\sin(dx + c) - 1) \right) - 480Aa^4 \left(2 \sin(dx + c) / \left(\sin(dx + c)^2 - 1 \right) - \log(\sin(dx + c) + 1) + \log(\sin(dx + c) - 1) \right) - 120Ba^4 \left(2 \sin(dx + c) / \left(\sin(dx + c)^2 - 1 \right) - \log(\sin(dx + c) + 1) + \log(\sin(dx + c) - 1) \right) + 480Aa^4 \tan(dx + c) / d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] 1/480*(32*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*A*a^4 + 960*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^4 + 128*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*B*a^4 + 640*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a^4 - 5*B*a^4*(2*(15*sin(d*x + c)^5 - 40*sin(d*x + c)^3 + 33*sin(d*x + c))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) - 120*A*a^4*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 180*B*a^4*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 480*A*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 120*B*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 480*A*a^4*tan(d*x + c))/d

mupad [B] time = 4.61, size = 262, normalized size = 1.35

$$\frac{\left(-7Aa^4 - \frac{49Ba^4}{8}\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^{11} + \left(\frac{119Aa^4}{3} + \frac{833Ba^4}{24}\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^9 + \left(-\frac{462Aa^4}{5} - \frac{1617Ba^4}{20}\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^7 + \left(\frac{562Aa^4}{5} + \frac{1967Ba^4}{20}\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5 + \left(\frac{562Aa^4}{5} + \frac{1967Ba^4}{20}\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 + \left(\frac{562Aa^4}{5} + \frac{1967Ba^4}{20}\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^{12} - 6 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^{10} + 15 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^8 - 20 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^6 + 15 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 - 6 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + 1 \right) + (7a^4 \operatorname{atanh}(\tan(\frac{c}{2} + \frac{dx}{2})) * (8a + 7B)) / (8*d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^4)/cos(c + d*x)^2,x)

[Out] (tan(c/2 + (d*x)/2)*(25*A*a^4 + (207*B*a^4)/8) - tan(c/2 + (d*x)/2)^11*(7*A*a^4 + (49*B*a^4)/8) + tan(c/2 + (d*x)/2)^9*((119*A*a^4)/3 + (833*B*a^4)/24) - tan(c/2 + (d*x)/2)^3*((233*A*a^4)/3 + (1471*B*a^4)/24) - tan(c/2 + (d*x)/2)^7*((462*A*a^4)/5 + (1617*B*a^4)/20) + tan(c/2 + (d*x)/2)^5*((562*A*a^4)/5 + (1967*B*a^4)/20))/d*(15*tan(c/2 + (d*x)/2)^4 - 6*tan(c/2 + (d*x)/2)^2 - 20*tan(c/2 + (d*x)/2)^6 + 15*tan(c/2 + (d*x)/2)^8 - 6*tan(c/2 + (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 + 1)) + (7*a^4*atanh(tan(c/2 + (d*x)/2))*(8*a + 7*B))/(8*d)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^4 \left(\int A \sec^2(c + dx) dx + \int 4A \sec^3(c + dx) dx + \int 6A \sec^4(c + dx) dx + \int 4A \sec^5(c + dx) dx + \int A \sec^6(c + dx) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**2*(a+a*sec(d*x+c))**4*(A+B*sec(d*x+c)),x)

[Out] a**4*(Integral(A*sec(c + d*x)**2, x) + Integral(4*A*sec(c + d*x)**3, x) + Integral(6*A*sec(c + d*x)**4, x) + Integral(4*A*sec(c + d*x)**5, x) + Integral(A*sec(c + d*x)**6, x) + Integral(B*sec(c + d*x)**3, x) + Integral(4*B*sec(c + d*x)**4, x) + Integral(6*B*sec(c + d*x)**5, x) + Integral(4*B*sec(c + d*x)**6, x) + Integral(B*sec(c + d*x)**7, x))

3.73 $\int \sec(c + dx)(a + a \sec(c + dx))^4(A + B \sec(c + dx)) dx$

Optimal. Leaf size=159

$$\frac{4a^4(5A + 4B) \tan^3(c + dx)}{15d} + \frac{8a^4(5A + 4B) \tan(c + dx)}{5d} + \frac{7a^4(5A + 4B) \tanh^{-1}(\sin(c + dx))}{8d} + \frac{a^4(5A + 4B) \tan(c + dx)}{2d}$$

[Out] $7/8*a^4*(5*A+4*B)*\operatorname{arctanh}(\sin(d*x+c))/d+8/5*a^4*(5*A+4*B)*\tan(d*x+c)/d+27/40*a^4*(5*A+4*B)*\sec(d*x+c)*\tan(d*x+c)/d+1/20*a^4*(5*A+4*B)*\sec(d*x+c)^3*\tan(d*x+c)/d+1/5*B*(a+a*\sec(d*x+c))^4*\tan(d*x+c)/d+4/15*a^4*(5*A+4*B)*\tan(d*x+c)^3/d$

Rubi [A] time = 0.18, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {4001, 3791, 3770, 3767, 8, 3768}

$$\frac{4a^4(5A + 4B) \tan^3(c + dx)}{15d} + \frac{8a^4(5A + 4B) \tan(c + dx)}{5d} + \frac{7a^4(5A + 4B) \tanh^{-1}(\sin(c + dx))}{8d} + \frac{a^4(5A + 4B) \tan(c + dx)}{2d}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Sec}[c + d*x]*(a + a*\operatorname{Sec}[c + d*x])^4*(A + B*\operatorname{Sec}[c + d*x]), x]$

[Out] $(7*a^4*(5*A + 4*B)*\operatorname{ArcTanh}[\operatorname{Sin}[c + d*x]])/(8*d) + (8*a^4*(5*A + 4*B)*\operatorname{Tan}[c + d*x])/(5*d) + (27*a^4*(5*A + 4*B)*\operatorname{Sec}[c + d*x]*\operatorname{Tan}[c + d*x])/(40*d) + (a^4*(5*A + 4*B)*\operatorname{Sec}[c + d*x]^3*\operatorname{Tan}[c + d*x])/(20*d) + (B*(a + a*\operatorname{Sec}[c + d*x])^4*\operatorname{Tan}[c + d*x])/(5*d) + (4*a^4*(5*A + 4*B)*\operatorname{Tan}[c + d*x]^3)/(15*d)$

Rule 8

$\operatorname{Int}[a_, x_Symbol] \rightarrow \operatorname{Simp}[a*x, x] /; \operatorname{FreeQ}[a, x]$

Rule 3767

$\operatorname{Int}[\operatorname{csc}[(c_.) + (d_.)*(x_.)]^{(n_.)}, x_Symbol] \rightarrow -\operatorname{Dist}[d^{(-1)}, \operatorname{Subst}[\operatorname{Int}[\operatorname{ExpandIntegrand}[(1 + x^2)^{(n/2 - 1)}, x], x], x, \operatorname{Cot}[c + d*x]], x] /; \operatorname{FreeQ}[\{c, d\}, x] \ \&\& \ \operatorname{IGtQ}[n/2, 0]$

Rule 3768

$\operatorname{Int}[(\operatorname{csc}[(c_.) + (d_.)*(x_.)]*(b_.))^{(n_.)}, x_Symbol] \rightarrow -\operatorname{Simp}[(b*\operatorname{Cos}[c + d*x])*(b*\operatorname{Csc}[c + d*x])^{(n - 1)}]/(d*(n - 1)), x] + \operatorname{Dist}[(b^2*(n - 2))/(n - 1), \operatorname{Int}[(b*\operatorname{Csc}[c + d*x])^{(n - 2)}, x], x] /; \operatorname{FreeQ}[\{b, c, d\}, x] \ \&\& \ \operatorname{GtQ}[n, 1] \ \&\&$

IntegerQ[2*n]

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x]
  /; FreeQ[{c, d}, x]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_.)^(m_.), x_Symbol] := Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)^(m_.)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] := -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
+ 1), 0] && !LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}
\int \sec(c + dx)(a + a \sec(c + dx))^4(A + B \sec(c + dx)) dx &= \frac{B(a + a \sec(c + dx))^4 \tan(c + dx)}{5d} + \frac{1}{5}(5A + 4B) \int \sec(c + dx)(a + a \sec(c + dx))^4 dx \\
&= \frac{B(a + a \sec(c + dx))^4 \tan(c + dx)}{5d} + \frac{1}{5}(5A + 4B) \int (a + a \sec(c + dx))^4 dx \\
&= \frac{B(a + a \sec(c + dx))^4 \tan(c + dx)}{5d} + \frac{1}{5}(a^4(5A + 4B)) \int \sec(c + dx) dx \\
&= \frac{a^4(5A + 4B) \tanh^{-1}(\sin(c + dx))}{5d} + \frac{3a^4(5A + 4B) \sec(c + dx)}{5d} \\
&= \frac{4a^4(5A + 4B) \tanh^{-1}(\sin(c + dx))}{5d} + \frac{8a^4(5A + 4B) \tan(c + dx)}{5d} \\
&= \frac{7a^4(5A + 4B) \tanh^{-1}(\sin(c + dx))}{8d} + \frac{8a^4(5A + 4B) \tan(c + dx)}{5d}
\end{aligned}$$

Mathematica [A] time = 1.75, size = 306, normalized size = 1.92

$$\frac{a^4(\cos(c + dx) + 1)^4 \sec^8\left(\frac{1}{2}(c + dx)\right) \sec^5(c + dx) \left(1680(5A + 4B) \cos^5(c + dx) \left(\log\left(\cos\left(\frac{1}{2}(c + dx)\right)\right) - \sin\left(\frac{1}{2}(c + dx)\right)\right)\right)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[c + d*x]*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]

[Out]
$$-1/30720*(a^4*(1 + \cos[c + d*x])^4*\sec[(c + d*x)/2]^8*\sec[c + d*x]^5*(1680*(5*A + 4*B)*\cos[c + d*x]^5*(\log[\cos[(c + d*x)/2] - \sin[(c + d*x)/2]] - \log[\cos[(c + d*x)/2] + \sin[(c + d*x)/2]]) - \sec[c]*(80*(64*A + 59*B)*\sin[d*x] - 960*(3*A + 2*B)*\sin[2*c + d*x] + 930*A*\sin[c + 2*d*x] + 1320*B*\sin[c + 2*d*x] + 930*A*\sin[3*c + 2*d*x] + 1320*B*\sin[3*c + 2*d*x] + 3520*A*\sin[2*c + 3*d*x] + 3200*B*\sin[2*c + 3*d*x] - 480*A*\sin[4*c + 3*d*x] - 120*B*\sin[4*c + 3*d*x] + 405*A*\sin[3*c + 4*d*x] + 420*B*\sin[3*c + 4*d*x] + 405*A*\sin[5*c + 4*d*x] + 420*B*\sin[5*c + 4*d*x] + 800*A*\sin[4*c + 5*d*x] + 664*B*\sin[4*c + 5*d*x])))/d$$

fricas [A] time = 0.46, size = 165, normalized size = 1.04

$$\frac{105(5A + 4B)a^4 \cos(dx + c)^5 \log(\sin(dx + c) + 1) - 105(5A + 4B)a^4 \cos(dx + c)^5 \log(-\sin(dx + c) + 1)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out]
$$1/240*(105*(5*A + 4*B)*a^4*\cos(d*x + c)^5*\log(\sin(d*x + c) + 1) - 105*(5*A + 4*B)*a^4*\cos(d*x + c)^5*\log(-\sin(d*x + c) + 1) + 2*(8*(100*A + 83*B)*a^4*\cos(d*x + c)^4 + 15*(27*A + 28*B)*a^4*\cos(d*x + c)^3 + 16*(10*A + 17*B)*a^4*\cos(d*x + c)^2 + 30*(A + 4*B)*a^4*\cos(d*x + c) + 24*B*a^4)*\sin(d*x + c))/(d*\cos(d*x + c)^5)$$

giac [A] time = 2.22, size = 246, normalized size = 1.55

$$\frac{105(5Aa^4 + 4Ba^4) \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right|\right) - 105(5Aa^4 + 4Ba^4) \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1\right|\right) - \frac{2(525Aa^4 \tan^2(\frac{1}{2}dx + \frac{1}{2}c) + 1050Aa^4 \tan(\frac{1}{2}dx + \frac{1}{2}c) + 525Aa^4)}{d}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] $1/120*(105*(5*A*a^4 + 4*B*a^4)*\log(\abs{\tan(1/2*d*x + 1/2*c) + 1}) - 105*(5*A*a^4 + 4*B*a^4)*\log(\abs{\tan(1/2*d*x + 1/2*c) - 1}) - 2*(525*A*a^4*\tan(1/2*d*x + 1/2*c)^9 + 420*B*a^4*\tan(1/2*d*x + 1/2*c)^9 - 2450*A*a^4*\tan(1/2*d*x + 1/2*c)^7 - 1960*B*a^4*\tan(1/2*d*x + 1/2*c)^7 + 4480*A*a^4*\tan(1/2*d*x + 1/2*c)^5 + 3584*B*a^4*\tan(1/2*d*x + 1/2*c)^5 - 3950*A*a^4*\tan(1/2*d*x + 1/2*c)^3 - 3160*B*a^4*\tan(1/2*d*x + 1/2*c)^3 + 1395*A*a^4*\tan(1/2*d*x + 1/2*c) + 1500*B*a^4*\tan(1/2*d*x + 1/2*c))/(\tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d$

maple [A] time = 1.70, size = 234, normalized size = 1.47

$$\frac{35Aa^4 \ln(\sec(dx+c) + \tan(dx+c))}{8d} + \frac{83a^4B \tan(dx+c)}{15d} + \frac{20Aa^4 \tan(dx+c)}{3d} + \frac{7a^4B \sec(dx+c) \tan(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sec(d*x+c)*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x)`

[Out] $35/8/d*A*a^4*\ln(\sec(d*x+c)+\tan(d*x+c))+83/15/d*a^4*B*\tan(d*x+c)+20/3/d*A*a^4*\tan(d*x+c)+7/2/d*a^4*B*\sec(d*x+c)*\tan(d*x+c)+7/2/d*a^4*B*\ln(\sec(d*x+c)+\tan(d*x+c))+27/8/d*A*a^4*\sec(d*x+c)*\tan(d*x+c)+34/15/d*a^4*B*\tan(d*x+c)*\sec(d*x+c)^2+4/3/d*A*a^4*\tan(d*x+c)*\sec(d*x+c)^2+1/d*a^4*B*\tan(d*x+c)*\sec(d*x+c)^3+1/4/d*A*a^4*\tan(d*x+c)*\sec(d*x+c)^3+1/5/d*a^4*B*\tan(d*x+c)*\sec(d*x+c)^4$

maxima [B] time = 0.35, size = 369, normalized size = 2.32

$$320\left(\tan(dx+c)^3 + 3 \tan(dx+c)\right)Aa^4 + 16\left(3 \tan(dx+c)^5 + 10 \tan(dx+c)^3 + 15 \tan(dx+c)\right)Ba^4 + 480\left(\tan(dx+c)^3 + 3 \tan(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="maxima")`

[Out] $1/240*(320*(\tan(d*x + c)^3 + 3*\tan(d*x + c))*A*a^4 + 16*(3*\tan(d*x + c)^5 + 10*\tan(d*x + c)^3 + 15*\tan(d*x + c))*B*a^4 + 480*(\tan(d*x + c)^3 + 3*\tan(d*x + c))*B*a^4 - 15*A*a^4*(2*(3*\sin(d*x + c)^3 - 5*\sin(d*x + c)))/(\sin(d*x + c)^4 - 2*\sin(d*x + c)^2 + 1) - 3*\log(\sin(d*x + c) + 1) + 3*\log(\sin(d*x + c) - 1)) - 60*B*a^4*(2*(3*\sin(d*x + c)^3 - 5*\sin(d*x + c)))/(\sin(d*x + c)^4 - 2*\sin(d*x + c)^2 + 1) - 3*\log(\sin(d*x + c) + 1) + 3*\log(\sin(d*x + c) - 1)) - 360*A*a^4*(2*\sin(d*x + c))/(\sin(d*x + c)^2 - 1) - \log(\sin(d*x + c) + 1) + \log(\sin(d*x + c) - 1)) - 240*B*a^4*(2*\sin(d*x + c))/(\sin(d*x + c)^2 - 1) - \log(\sin(d*x + c) + 1) + \log(\sin(d*x + c) - 1)) + 240*A*a^4*\log(\sec(d*x + c) + \tan(d*x + c)) + 960*A*a^4*\tan(d*x + c) + 240*B*a^4*\tan(d*x + c))/d$

mupad [B] time = 4.54, size = 224, normalized size = 1.41

$$\frac{7a^4 \operatorname{atanh}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right) (5A + 4B) \left(\frac{35Aa^4}{4} + 7Ba^4\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^9 + \left(-\frac{245Aa^4}{6} - \frac{98Ba^4}{3}\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^7 + \dots}{4d} - \frac{d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right)^{10} - 5 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^7 + \dots}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^4)/cos(c + d*x), x)

[Out] (7*a^4*atanh(tan(c/2 + (d*x)/2))*(5*A + 4*B))/(4*d) - (tan(c/2 + (d*x)/2)*((93*A*a^4)/4 + 25*B*a^4) + tan(c/2 + (d*x)/2)^9*((35*A*a^4)/4 + 7*B*a^4) - tan(c/2 + (d*x)/2)^7*((245*A*a^4)/6 + (98*B*a^4)/3) - tan(c/2 + (d*x)/2)^3*((395*A*a^4)/6 + (158*B*a^4)/3) + tan(c/2 + (d*x)/2)^5*((224*A*a^4)/3 + (896*B*a^4)/15))/(d*(5*tan(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 - 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^4 \left(\int A \sec(c + dx) dx + \int 4A \sec^2(c + dx) dx + \int 6A \sec^3(c + dx) dx + \int 4A \sec^4(c + dx) dx + \int A \sec^5(c + dx) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))**4*(A+B*sec(d*x+c)), x)

[Out] a**4*(Integral(A*sec(c + d*x), x) + Integral(4*A*sec(c + d*x)**2, x) + Integral(6*A*sec(c + d*x)**3, x) + Integral(4*A*sec(c + d*x)**4, x) + Integral(A*sec(c + d*x)**5, x) + Integral(B*sec(c + d*x)**2, x) + Integral(4*B*sec(c + d*x)**3, x) + Integral(6*B*sec(c + d*x)**4, x) + Integral(4*B*sec(c + d*x)**5, x) + Integral(B*sec(c + d*x)**6, x))

3.74 $\int (a + a \sec(c + dx))^4 (A + B \sec(c + dx)) dx$

Optimal. Leaf size=151

$$\frac{5a^4(8A + 7B) \tan(c + dx)}{8d} + \frac{a^4(48A + 35B) \tanh^{-1}(\sin(c + dx))}{8d} + \frac{(32A + 35B) \tan(c + dx) (a^4 \sec(c + dx) + a^4)}{24d}$$

[Out] $a^4 A x + 1/8 a^4 (48 A + 35 B) \operatorname{arctanh}(\sin(d x + c)) / d + 5/8 a^4 (8 A + 7 B) \tan(d x + c) / d + 1/4 a^4 B (a + a \sec(d x + c))^3 \tan(d x + c) / d + 1/12 (4 A + 7 B) (a^2 + a^2 \sec(d x + c))^2 \tan(d x + c) / d + 1/24 (32 A + 35 B) (a^4 + a^4 \sec(d x + c)) \tan(d x + c) / d$

Rubi [A] time = 0.21, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3917, 3914, 3767, 8, 3770}

$$\frac{5a^4(8A + 7B) \tan(c + dx)}{8d} + \frac{a^4(48A + 35B) \tanh^{-1}(\sin(c + dx))}{8d} + \frac{(4A + 7B) \tan(c + dx) (a^2 \sec(c + dx) + a^2)^2}{12d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + a \operatorname{Sec}[c + d x])^4 (A + B \operatorname{Sec}[c + d x]), x]$

[Out] $a^4 A x + (a^4 (48 A + 35 B) \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]) / (8 d) + (5 a^4 (8 A + 7 B) \operatorname{Tan}[c + d x]) / (8 d) + (a^4 B (a + a \operatorname{Sec}[c + d x])^3 \operatorname{Tan}[c + d x]) / (4 d) + ((4 A + 7 B) (a^2 + a^2 \operatorname{Sec}[c + d x])^2 \operatorname{Tan}[c + d x]) / (12 d) + ((32 A + 35 B) (a^4 + a^4 \operatorname{Sec}[c + d x]) \operatorname{Tan}[c + d x]) / (24 d)$

Rule 8

$\text{Int}[a_, x_Symbol] \rightarrow \text{Simp}[a x, x] /; \text{FreeQ}[a, x]$

Rule 3767

$\text{Int}[\operatorname{csc}[(c_) + (d_)(x_)]^{(n_)}, x_Symbol] \rightarrow -\text{Dist}[d^{(-1)}, \text{Subst}[\text{Int}[\text{ExpandIntegrand}[(1 + x^2)^{(n/2 - 1)}, x], x], x, \operatorname{Cot}[c + d x]], x] /; \text{FreeQ}[\{c, d\}, x] \ \&\& \ \text{IGtQ}[n/2, 0]$

Rule 3770

$\text{Int}[\operatorname{csc}[(c_) + (d_)(x_)], x_Symbol] \rightarrow -\text{Simp}[\operatorname{ArcTanh}[\operatorname{Cos}[c + d x]] / d, x] /; \text{FreeQ}[\{c, d\}, x]$

Rule 3914

$\text{Int}[(\operatorname{csc}[(e_) + (f_)(x_)](b_) + (a_))(\operatorname{csc}[(e_) + (f_)(x_)](d_) + (c_)), x_Symbol] \rightarrow \text{Simp}[a c x, x] + (\text{Dist}[b d, \text{Int}[\operatorname{Csc}[e + f x]^2, x], x]$

+ Dist[b*c + a*d, Int[Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]

Rule 3917

Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.)), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1))/(f*m), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m - 1)*Simp[a*c*m + (b*c*m + a*d*(2*m - 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && GtQ[m, 1] && EqQ[a^2 - b^2, 0] && IntegerQ[2*m]

Rubi steps

$$\begin{aligned}
 \int (a + a \sec(c + dx))^4 (A + B \sec(c + dx)) dx &= \frac{aB(a + a \sec(c + dx))^3 \tan(c + dx)}{4d} + \frac{1}{4} \int (a + a \sec(c + dx))^3 (4A + B \sec(c + dx)) dx \\
 &= \frac{aB(a + a \sec(c + dx))^3 \tan(c + dx)}{4d} + \frac{(4A + 7B)(a^2 + a^2 \sec(c + dx))}{12d} \\
 &= \frac{aB(a + a \sec(c + dx))^3 \tan(c + dx)}{4d} + \frac{(4A + 7B)(a^2 + a^2 \sec(c + dx))}{12d} \\
 &= a^4 Ax + \frac{aB(a + a \sec(c + dx))^3 \tan(c + dx)}{4d} + \frac{(4A + 7B)(a^2 + a^2 \sec(c + dx))}{12d} \\
 &= a^4 Ax + \frac{a^4(48A + 35B) \tanh^{-1}(\sin(c + dx))}{8d} + \frac{aB(a + a \sec(c + dx))^3 \tan(c + dx)}{4d} \\
 &= a^4 Ax + \frac{a^4(48A + 35B) \tanh^{-1}(\sin(c + dx))}{8d} + \frac{5a^4(8A + 7B) \tan(c + dx)}{8d}
 \end{aligned}$$

Mathematica [B] time = 2.08, size = 326, normalized size = 2.16

$$\frac{a^4 \sec^8\left(\frac{1}{2}(c + dx)\right) (\sec(c + dx) + 1)^4 \left(\sec(c)(48A \sin(2c + dx) + 496A \sin(c + 2dx) - 144A \sin(3c + 2dx) + 48A \sin(4c + 2dx))\right)}{8d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]

[Out] (a^4*Sec[(c + d*x)/2]^8*(1 + Sec[c + d*x])^4*(-24*(48*A + 35*B)*Cos[c + d*x] + (Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Sec[c]*(72*A*d*x*Cos[c] + 48*A*d*x*Cos[c + 2*d*x] + 48*A*d

$$\begin{aligned} & *x*\text{Cos}[3*c + 2*d*x] + 12*A*d*x*\text{Cos}[3*c + 4*d*x] + 12*A*d*x*\text{Cos}[5*c + 4*d*x] \\ & - 480*A*\text{Sin}[c] - 480*B*\text{Sin}[c] + 48*A*\text{Sin}[d*x] + 105*B*\text{Sin}[d*x] + 48*A*\text{Sin}[\\ & 2*c + d*x] + 105*B*\text{Sin}[2*c + d*x] + 496*A*\text{Sin}[c + 2*d*x] + 544*B*\text{Sin}[c + 2* \\ & d*x] - 144*A*\text{Sin}[3*c + 2*d*x] - 96*B*\text{Sin}[3*c + 2*d*x] + 48*A*\text{Sin}[2*c + 3*d* \\ & x] + 81*B*\text{Sin}[2*c + 3*d*x] + 48*A*\text{Sin}[4*c + 3*d*x] + 81*B*\text{Sin}[4*c + 3*d*x] \\ & + 160*A*\text{Sin}[3*c + 4*d*x] + 160*B*\text{Sin}[3*c + 4*d*x] \end{aligned} \Big) \Big) / (3072*d)$$

fricas [A] time = 0.45, size = 157, normalized size = 1.04

$$48 A a^4 dx \cos(dx + c)^4 + 3(48 A + 35 B) a^4 \cos(dx + c)^4 \log(\sin(dx + c) + 1) - 3(48 A + 35 B) a^4 \cos(dx + c)^4 \log(\sin(dx + c) - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] $\frac{1}{48}*(48*A*a^4*d*x*\cos(d*x + c)^4 + 3*(48*A + 35*B)*a^4*\cos(d*x + c)^4*\log(\sin(d*x + c) + 1) - 3*(48*A + 35*B)*a^4*\cos(d*x + c)^4*\log(-\sin(d*x + c) + 1) + 2*(160*(A + B)*a^4*\cos(d*x + c)^3 + 3*(16*A + 27*B)*a^4*\cos(d*x + c)^2 + 8*(A + 4*B)*a^4*\cos(d*x + c) + 6*B*a^4)*\sin(d*x + c))/(d*\cos(d*x + c)^4)$

giac [A] time = 0.63, size = 223, normalized size = 1.48

$$24(dx + c)Aa^4 + 3(48Aa^4 + 35Ba^4) \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right|\right) - 3(48Aa^4 + 35Ba^4) \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] $\frac{1}{24}*(24*(d*x + c)*A*a^4 + 3*(48*A*a^4 + 35*B*a^4)*\log(\text{abs}(\tan(1/2*d*x + 1/2*c) + 1)) - 3*(48*A*a^4 + 35*B*a^4)*\log(\text{abs}(\tan(1/2*d*x + 1/2*c) - 1)) - 2*(120*A*a^4*\tan(1/2*d*x + 1/2*c)^7 + 105*B*a^4*\tan(1/2*d*x + 1/2*c)^7 - 424*A*a^4*\tan(1/2*d*x + 1/2*c)^5 - 385*B*a^4*\tan(1/2*d*x + 1/2*c)^5 + 520*A*a^4*\tan(1/2*d*x + 1/2*c)^3 + 511*B*a^4*\tan(1/2*d*x + 1/2*c)^3 - 216*A*a^4*\tan(1/2*d*x + 1/2*c) - 279*B*a^4*\tan(1/2*d*x + 1/2*c))/(\tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d$

maple [A] time = 1.42, size = 204, normalized size = 1.35

$$A a^4 x + \frac{A a^4 c}{d} + \frac{35 a^4 B \ln(\sec(dx + c) + \tan(dx + c))}{8d} + \frac{6 A a^4 \ln(\sec(dx + c) + \tan(dx + c))}{d} + \frac{20 a^4 B \tan(dx + c)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x)`

[Out] $Aa^4x + 1/dAa^4c + 35/8/dA^4B \ln(\sec(dx+c) + \tan(dx+c)) + 6/dA^4a^4 \ln(\sec(dx+c) + \tan(dx+c)) + 20/3/dA^4B \tan(dx+c) + 20/3/dA^4a^4 \tan(dx+c) + 27/8/dA^4B \sec(dx+c) \tan(dx+c) + 2/dA^4a^4 \sec(dx+c) \tan(dx+c) + 4/3/dA^4B \tan(dx+c) \sec(dx+c)^2 + 1/3/dA^4a^4 \tan(dx+c) \sec(dx+c)^2 + 1/4/dA^4B \tan(dx+c) \sec(dx+c)^3$

maxima [B] time = 0.36, size = 293, normalized size = 1.94

$$16 \left(\tan(dx+c)^3 + 3 \tan(dx+c) \right) Aa^4 + 48(dx+c)Aa^4 + 64 \left(\tan(dx+c)^3 + 3 \tan(dx+c) \right) Ba^4 - 3Ba^4 \left(\frac{2(3 \sin(dx+c)^3 - 5 \sin(dx+c))}{\sin(dx+c)^4 - 2 \sin(dx+c)^2 + 1} - 3 \log(\sin(dx+c) + 1) + 3 \log(\sin(dx+c) - 1) - 48Aa^4 \frac{2 \sin(dx+c)}{\sin(dx+c)^2 - 1} - \log(\sin(dx+c) + 1) + \log(\sin(dx+c) - 1) - 72Ba^4 \frac{2 \sin(dx+c)}{\sin(dx+c)^2 - 1} - \log(\sin(dx+c) + 1) + \log(\sin(dx+c) - 1) + 192Aa^4 \log(\sec(dx+c) + \tan(dx+c)) + 48Ba^4 \log(\sec(dx+c) + \tan(dx+c)) + 288Aa^4 \tan(dx+c) + 192Ba^4 \tan(dx+c) \right) / d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="maxima")`

[Out] $1/48(16(\tan(dx+c)^3 + 3 \tan(dx+c))Aa^4 + 48(dx+c)Aa^4 + 64(\tan(dx+c)^3 + 3 \tan(dx+c))Ba^4 - 3Ba^4(2(3 \sin(dx+c)^3 - 5 \sin(dx+c))/(\sin(dx+c)^4 - 2 \sin(dx+c)^2 + 1) - 3 \log(\sin(dx+c) + 1) + 3 \log(\sin(dx+c) - 1) - 48Aa^4(2 \sin(dx+c)/(\sin(dx+c)^2 - 1) - \log(\sin(dx+c) + 1) + \log(\sin(dx+c) - 1) - 72Ba^4(2 \sin(dx+c)/(\sin(dx+c)^2 - 1) - \log(\sin(dx+c) + 1) + \log(\sin(dx+c) - 1)) + 192Aa^4 \log(\sec(dx+c) + \tan(dx+c)) + 48Ba^4 \log(\sec(dx+c) + \tan(dx+c)) + 288Aa^4 \tan(dx+c) + 192Ba^4 \tan(dx+c)) / d$

mupad [B] time = 2.10, size = 255, normalized size = 1.69

$$\frac{2Aa^4 \operatorname{atan}\left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)}\right)}{d} + \frac{12Aa^4 \operatorname{atanh}\left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)}\right)}{d} + \frac{35Ba^4 \operatorname{atanh}\left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)}\right)}{4d} + \frac{20Aa^4 \sin(c+dx)}{3d \cos(c+dx)} + \frac{2Aa^4 \sin(c+dx)}{d \cos(c+dx)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^4,x)`

[Out] $(2Aa^4 \operatorname{atan}(\sin(c/2 + (dx)/2)/\cos(c/2 + (dx)/2)))/d + (12Aa^4 \operatorname{atanh}(\sin(c/2 + (dx)/2)/\cos(c/2 + (dx)/2)))/d + (35Ba^4 \operatorname{atanh}(\sin(c/2 + (dx)/2)/\cos(c/2 + (dx)/2)))/(4d) + (20Aa^4 \sin(c+dx))/(3d \cos(c+dx)) + (2Aa^4 \sin(c+dx))/(d \cos(c+dx)^2) + (Aa^4 \sin(c+dx))/(3d \cos(c+dx)^3) + (20Ba^4 \sin(c+dx))/(3d \cos(c+dx)) + (27Ba^4 \sin(c+dx))/(8d \cos(c+dx)^2) + (4Ba^4 \sin(c+dx))/(3d \cos(c+dx)^3) + (Ba^4 \sin(c+dx))/(4d \cos(c+dx)^4)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^4 \left(\int A dx + \int 4A \sec(c+dx) dx + \int 6A \sec^2(c+dx) dx + \int 4A \sec^3(c+dx) dx + \int A \sec^4(c+dx) dx + \dots \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**4*(A+B*sec(d*x+c)),x)
```

```
[Out] a**4*(Integral(A, x) + Integral(4*A*sec(c + d*x), x) + Integral(6*A*sec(c +  
d*x)**2, x) + Integral(4*A*sec(c + d*x)**3, x) + Integral(A*sec(c + d*x)**  
4, x) + Integral(B*sec(c + d*x), x) + Integral(4*B*sec(c + d*x)**2, x) + In  
tegral(6*B*sec(c + d*x)**3, x) + Integral(4*B*sec(c + d*x)**4, x) + Integra  
l(B*sec(c + d*x)**5, x))
```

3.75 $\int \cos(c + dx)(a + a \sec(c + dx))^4(A + B \sec(c + dx)) dx$

Optimal. Leaf size=151

$$\frac{5a^4(A + 2B) \sin(c + dx)}{2d} + \frac{a^4(13A + 12B) \tanh^{-1}(\sin(c + dx))}{2d} + \frac{(9A + 11B) \sin(c + dx) (a^4 \sec(c + dx) + a^4)}{3d} +$$

[Out] $a^4(4A+B)x + \frac{1}{2}a^4(13A+12B)\operatorname{arctanh}(\sin(dx+c))/d - \frac{5}{2}a^4(A+2B)\sin(dx+c)/d + \frac{1}{3}a^4B(a+a\sec(dx+c))^3\sin(dx+c)/d + \frac{1}{2}(A+2B)(a^2+a^2\sec(dx+c))^2\sin(dx+c)/d + \frac{1}{3}(9A+11B)(a^4+a^4\sec(dx+c))\sin(dx+c)/d$

Rubi [A] time = 0.37, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {4018, 3996, 3770}

$$\frac{5a^4(A + 2B) \sin(c + dx)}{2d} + \frac{a^4(13A + 12B) \tanh^{-1}(\sin(c + dx))}{2d} + \frac{(A + 2B) \sin(c + dx) (a^2 \sec(c + dx) + a^2)^2}{2d} +$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Cos}[c + d*x]*(a + a*\operatorname{Sec}[c + d*x])^4*(A + B*\operatorname{Sec}[c + d*x]), x]$

[Out] $a^4(4A + B)x + (a^4(13A + 12B)\operatorname{ArcTanh}[\operatorname{Sin}[c + d*x]])/(2*d) - (5*a^4*(A + 2*B)*\operatorname{Sin}[c + d*x])/(2*d) + (a*B*(a + a*\operatorname{Sec}[c + d*x])^3*\operatorname{Sin}[c + d*x])/(3*d) + ((A + 2*B)*(a^2 + a^2*\operatorname{Sec}[c + d*x])^2*\operatorname{Sin}[c + d*x])/(2*d) + ((9*A + 11*B)*(a^4 + a^4*\operatorname{Sec}[c + d*x])*\operatorname{Sin}[c + d*x])/(3*d)$

Rule 3770

$\operatorname{Int}[\operatorname{csc}[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow -\operatorname{Simp}[\operatorname{ArcTanh}[\operatorname{Cos}[c + d*x]]/d, x] /;$ $\operatorname{FreeQ}\{c, d\}, x]$

Rule 3996

$\operatorname{Int}[(\operatorname{csc}[(e_.) + (f_.)*(x_.)]*(d_.))^{(n_.)}*(\operatorname{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))*(\operatorname{csc}[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] \rightarrow \operatorname{Simp}[(A*a*\operatorname{Cot}[e + f*x]*(d*\operatorname{Csc}[e + f*x])^n)/(f*n), x] + \operatorname{Dist}[1/(d*n), \operatorname{Int}[(d*\operatorname{Csc}[e + f*x])^{(n + 1)}*\operatorname{Simp}[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*\operatorname{Csc}[e + f*x], x], x], x] /;$ $\operatorname{FreeQ}\{a, b, d, e, f, A, B\}, x] \&\& \operatorname{NeQ}[A*b - a*B, 0] \&\& \operatorname{LeQ}[n, -1]$

Rule 4018

$\operatorname{Int}[(\operatorname{csc}[(e_.) + (f_.)*(x_.)]*(d_.))^{(n_.)}*(\operatorname{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^{(m_.)}*(\operatorname{csc}[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] \rightarrow -\operatorname{Simp}[(b*B*\operatorname{C}$

```

ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && !LtQ[n, -1]

```

Rubi steps

$$\begin{aligned}
\int \cos(c + dx)(a + a \sec(c + dx))^4(A + B \sec(c + dx)) dx &= \frac{aB(a + a \sec(c + dx))^3 \sin(c + dx)}{3d} + \frac{1}{3} \int \cos(c + dx) \\
&= \frac{aB(a + a \sec(c + dx))^3 \sin(c + dx)}{3d} + \frac{(A + 2B)(a^2 + a \sec(c + dx))^3 \sin(c + dx)}{3d} \\
&= \frac{aB(a + a \sec(c + dx))^3 \sin(c + dx)}{3d} + \frac{(A + 2B)(a^2 + a \sec(c + dx))^3 \sin(c + dx)}{3d} \\
&= -\frac{5a^4(A + 2B) \sin(c + dx)}{2d} + \frac{aB(a + a \sec(c + dx))^3 \sin(c + dx)}{3d} \\
&= a^4(4A + B)x - \frac{5a^4(A + 2B) \sin(c + dx)}{2d} + \frac{aB(a + a \sec(c + dx))^3 \sin(c + dx)}{3d} \\
&= a^4(4A + B)x + \frac{a^4(13A + 12B) \tanh^{-1}(\sin(c + dx))}{2d} - \frac{aB(a + a \sec(c + dx))^3 \sin(c + dx)}{3d}
\end{aligned}$$

Mathematica [B] time = 6.48, size = 1202, normalized size = 7.96

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]
```

```
[Out] ((4*A + B)*x*Cos[c + d*x]^5*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4*(A
+ B*Sec[c + d*x]))/(16*(B + A*Cos[c + d*x])) + ((-13*A - 12*B)*Cos[c + d*x]
^5*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sec[c/2 + (d*x)/2]^8*(a + a
*Sec[c + d*x])^4*(A + B*Sec[c + d*x]))/(32*d*(B + A*Cos[c + d*x])) + ((13*A
+ 12*B)*Cos[c + d*x]^5*Log[Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]]*Sec[c/
2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x]))/(32*d*(B + A*Co
s[c + d*x])) + (A*Cos[d*x]*Cos[c + d*x]^5*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c
+ d*x])^4*(A + B*Sec[c + d*x])*Sin[c])/((16*d*(B + A*Cos[c + d*x])) + (A*Co
s[c]*Cos[c + d*x]^5*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4*(A + B*Sec[
c + d*x])*Sin[d*x]))/(16*d*(B + A*Cos[c + d*x])) + (B*Cos[c + d*x]^5*Sec[c/2
```

$$\begin{aligned} & + (dx/2)^8(a + a\sec[c + dx])^4(A + B\sec[c + dx])\sin[(dx)/2]/(96 \\ & *d*(B + A\cos[c + dx])*(\cos[c/2] - \sin[c/2])*(\cos[c/2 + (dx)/2] - \sin[c/2 \\ & + (dx)/2])^3 + (\cos[c + dx]^5\sec[c/2 + (dx)/2]^8(a + a\sec[c + dx]) \\ & ^4*(A + B\sec[c + dx])*(3A\cos[c/2] + 13B\cos[c/2] - 3A\sin[c/2] - 11B \\ & * \sin[c/2]))/(192*d*(B + A\cos[c + dx])*(\cos[c/2] - \sin[c/2])*(\cos[c/2 + (d \\ & *x)/2] - \sin[c/2 + (dx)/2])^2) + (\cos[c + dx]^5\sec[c/2 + (dx)/2]^8(a + \\ & a\sec[c + dx])^4*(A + B\sec[c + dx])*(3A\sin[(dx)/2] + 5B\sin[(dx)/2 \\ &])))/(12*d*(B + A\cos[c + dx])*(\cos[c/2] - \sin[c/2])*(\cos[c/2 + (dx)/2] - \\ & \sin[c/2 + (dx)/2])) + (B\cos[c + dx]^5\sec[c/2 + (dx)/2]^8(a + a\sec[c \\ & + dx])^4*(A + B\sec[c + dx])\sin[(dx)/2])/ (96*d*(B + A\cos[c + dx])*(\cos[c/2] + \sin[c/2])*(\cos[c/2 + (dx)/2] + \sin[c/2 + (dx)/2])^3) + (\cos[c + dx]^5\sec[c/2 + (dx)/2]^8(a + a\sec[c + dx])^4*(A + B\sec[c + dx])*(-3A\cos[c/2] - 13B\cos[c/2] - 3A\sin[c/2] - 11B\sin[c/2]))/(192*d*(B + A\cos[c + dx])*(\cos[c/2] + \sin[c/2])*(\cos[c/2 + (dx)/2] + \sin[c/2 + (dx)/2])^2) + (\cos[c + dx]^5\sec[c/2 + (dx)/2]^8(a + a\sec[c + dx])^4*(A + B\sec[c + dx])*(3A\sin[(dx)/2] + 5B\sin[(dx)/2]))/(12*d*(B + A\cos[c + dx])*(\cos[c/2] + \sin[c/2])*(\cos[c/2 + (dx)/2] + \sin[c/2 + (dx)/2])) \end{aligned}$$

fricas [A] time = 0.46, size = 159, normalized size = 1.05

$$\frac{12(4A + B)a^4 dx \cos(dx + c)^3 + 3(13A + 12B)a^4 \cos(dx + c)^3 \log(\sin(dx + c) + 1) - 3(13A + 12B)a^4 \cos(dx + c)^3 \log(-\sin(dx + c) + 1) + 2(6Aa^4 \cos(dx + c)^3 + 8(3A + 5B)a^4 \cos(dx + c)^2 + 3(A + 4B)a^4 \cos(dx + c) + 2Ba^4) \sin(dx + c)}{(d \cos(dx + c))^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(dx+c)*(a+a*sec(dx+c))^4*(A+B*sec(dx+c)),x, algorithm="fricas")

[Out] 1/12*(12*(4*A + B)*a^4*d*x*cos(dx + c)^3 + 3*(13*A + 12*B)*a^4*cos(dx + c)^3*log(sin(dx + c) + 1) - 3*(13*A + 12*B)*a^4*cos(dx + c)^3*log(-sin(dx + c) + 1) + 2*(6*A*a^4*cos(dx + c)^3 + 8*(3*A + 5*B)*a^4*cos(dx + c)^2 + 3*(A + 4*B)*a^4*cos(dx + c) + 2*B*a^4)*sin(dx + c))/(d*cos(dx + c)^3)

giac [A] time = 0.80, size = 227, normalized size = 1.50

$$\frac{12Aa^4 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)}{\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + 1} + 6(4Aa^4 + Ba^4)(dx + c) + 3(13Aa^4 + 12Ba^4) \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right|\right) - 3(13Aa^4 + 12Ba^4) \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(dx+c)*(a+a*sec(dx+c))^4*(A+B*sec(dx+c)),x, algorithm="giac")

[Out] $\frac{1}{6} * (12 * A * a^4 * \tan(1/2 * d * x + 1/2 * c) / (\tan(1/2 * d * x + 1/2 * c)^2 + 1) + 6 * (4 * A * a^4 + B * a^4) * (d * x + c) + 3 * (13 * A * a^4 + 12 * B * a^4) * \log(\text{abs}(\tan(1/2 * d * x + 1/2 * c) + 1)) - 3 * (13 * A * a^4 + 12 * B * a^4) * \log(\text{abs}(\tan(1/2 * d * x + 1/2 * c) - 1)) - 2 * (21 * A * a^4 * \tan(1/2 * d * x + 1/2 * c)^5 + 30 * B * a^4 * \tan(1/2 * d * x + 1/2 * c)^5 - 48 * A * a^4 * \tan(1/2 * d * x + 1/2 * c)^3 - 76 * B * a^4 * \tan(1/2 * d * x + 1/2 * c)^3 + 27 * A * a^4 * \tan(1/2 * d * x + 1/2 * c) + 54 * B * a^4 * \tan(1/2 * d * x + 1/2 * c)) / (\tan(1/2 * d * x + 1/2 * c)^2 - 1)^3) / d$

maple [A] time = 1.33, size = 189, normalized size = 1.25

$$\frac{A a^4 \sin(dx + c)}{d} + a^4 B x + \frac{a^4 B c}{d} + 4 A a^4 x + \frac{4 A a^4 c}{d} + \frac{6 a^4 B \ln(\sec(dx + c) + \tan(dx + c))}{d} + \frac{13 A a^4 \ln(\sec(dx + c) + \tan(dx + c))}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(d*x+c)*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x)`

[Out] $\frac{1}{d} * A * a^4 * \sin(d * x + c) + a^4 * B * x + \frac{1}{d} * a^4 * B * c + 4 * A * a^4 * x + \frac{4}{d} * A * a^4 * c + \frac{6}{d} * a^4 * B * \ln(\sec(d * x + c) + \tan(d * x + c)) + \frac{13}{2} * \frac{1}{d} * A * a^4 * \ln(\sec(d * x + c) + \tan(d * x + c)) + \frac{20}{3} * \frac{1}{d} * a^4 * B * \tan(d * x + c) + \frac{4}{d} * A * a^4 * \tan(d * x + c) + \frac{2}{d} * a^4 * B * \sec(d * x + c) * \tan(d * x + c) + \frac{1}{2} * \frac{1}{d} * A * a^4 * \sec(d * x + c) * \tan(d * x + c) + \frac{1}{3} * \frac{1}{d} * a^4 * B * \tan(d * x + c) * \sec(d * x + c)^2$

maxima [A] time = 0.35, size = 235, normalized size = 1.56

$$\frac{48(dx + c)Aa^4 + 4(\tan(dx + c)^3 + 3 \tan(dx + c))Ba^4 + 12(dx + c)Ba^4 - 3Aa^4 \left(\frac{2 \sin(dx + c)}{\sin(dx + c)^2 - 1} - \log(\sin(dx + c) + 1) + \log(\sin(dx + c) - 1) \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="maxima")`

[Out] $\frac{1}{12} * (48 * (d * x + c) * A * a^4 + 4 * (\tan(d * x + c)^3 + 3 * \tan(d * x + c)) * B * a^4 + 12 * (d * x + c) * B * a^4 - 3 * A * a^4 * (2 * \sin(d * x + c) / (\sin(d * x + c)^2 - 1) - \log(\sin(d * x + c) + 1) + \log(\sin(d * x + c) - 1)) - 12 * B * a^4 * (2 * \sin(d * x + c) / (\sin(d * x + c)^2 - 1) - \log(\sin(d * x + c) + 1) + \log(\sin(d * x + c) - 1)) + 36 * A * a^4 * (\log(\sin(d * x + c) + 1) - \log(\sin(d * x + c) - 1)) + 24 * B * a^4 * (\log(\sin(d * x + c) + 1) - \log(\sin(d * x + c) - 1)) + 12 * A * a^4 * \sin(d * x + c) + 48 * A * a^4 * \tan(d * x + c) + 72 * B * a^4 * \tan(d * x + c)) / d$

mupad [B] time = 2.10, size = 254, normalized size = 1.68

$$\frac{A a^4 \sin(c + dx)}{d} + \frac{8 A a^4 \operatorname{atan}\left(\frac{\sin\left(\frac{c + dx}{2}\right)}{\cos\left(\frac{c + dx}{2}\right)}\right)}{d} + \frac{13 A a^4 \operatorname{atanh}\left(\frac{\sin\left(\frac{c + dx}{2}\right)}{\cos\left(\frac{c + dx}{2}\right)}\right)}{d} + \frac{2 B a^4 \operatorname{atan}\left(\frac{\sin\left(\frac{c + dx}{2}\right)}{\cos\left(\frac{c + dx}{2}\right)}\right)}{d} + \frac{12 B a^4 \operatorname{atanh}\left(\frac{\sin\left(\frac{c + dx}{2}\right)}{\cos\left(\frac{c + dx}{2}\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^4,x)
```

```
[Out] (A*a^4*sin(c + d*x))/d + (8*A*a^4*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (13*A*a^4*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (2*B*a^4*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (12*B*a^4*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (4*A*a^4*sin(c + d*x))/(d*cos(c + d*x)) + (A*a^4*sin(c + d*x))/(2*d*cos(c + d*x)^2) + (20*B*a^4*sin(c + d*x))/(3*d*cos(c + d*x)) + (2*B*a^4*sin(c + d*x))/(d*cos(c + d*x)^2) + (B*a^4*sin(c + d*x))/(3*d*cos(c + d*x)^3)
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$a^4 \left(\int A \cos(c + dx) dx + \int 4A \cos(c + dx) \sec(c + dx) dx + \int 6A \cos(c + dx) \sec^2(c + dx) dx + \int 4A \cos(c + dx) \sec^3(c + dx) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))**4*(A+B*sec(d*x+c)),x)
```

```
[Out] a**4*(Integral(A*cos(c + d*x), x) + Integral(4*A*cos(c + d*x)*sec(c + d*x), x) + Integral(6*A*cos(c + d*x)*sec(c + d*x)**2, x) + Integral(4*A*cos(c + d*x)*sec(c + d*x)**3, x) + Integral(A*cos(c + d*x)*sec(c + d*x)**4, x) + Integral(B*cos(c + d*x)*sec(c + d*x), x) + Integral(4*B*cos(c + d*x)*sec(c + d*x)**2, x) + Integral(6*B*cos(c + d*x)*sec(c + d*x)**3, x) + Integral(4*B*cos(c + d*x)*sec(c + d*x)**4, x) + Integral(B*cos(c + d*x)*sec(c + d*x)**5, x))
```

3.76 $\int \cos^2(c + dx)(a + a \sec(c + dx))^4(A + B \sec(c + dx)) dx$

Optimal. Leaf size=160

$$\frac{5a^4(A - B) \sin(c + dx)}{2d} + \frac{a^4(8A + 13B) \tanh^{-1}(\sin(c + dx))}{2d} + \frac{(A + 6B) \sin(c + dx) (a^4 \sec(c + dx) + a^4)}{2d} + \frac{1}{2}a^4x(13$$

[Out] $1/2*a^4*(13*A+8*B)*x+1/2*a^4*(8*A+13*B)*\operatorname{arctanh}(\sin(d*x+c))/d+5/2*a^4*(A-B)*\sin(d*x+c)/d+1/2*a^4*A*\cos(d*x+c)*(a+a*\sec(d*x+c))^3*\sin(d*x+c)/d-1/2*(A-B)*(a^2+a^2*\sec(d*x+c))^2*\sin(d*x+c)/d+1/2*(A+6*B)*(a^4+a^4*\sec(d*x+c))*\sin(d*x+c)/d$

Rubi [A] time = 0.39, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {4017, 4018, 3996, 3770}

$$\frac{5a^4(A - B) \sin(c + dx)}{2d} + \frac{a^4(8A + 13B) \tanh^{-1}(\sin(c + dx))}{2d} - \frac{(A - B) \sin(c + dx) (a^2 \sec(c + dx) + a^2)^2}{2d} + \frac{(A + 6B) \sin(c + dx) (a^4 \sec(c + dx) + a^4)}{2d}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Cos}[c + d*x]^2*(a + a*\operatorname{Sec}[c + d*x])^4*(A + B*\operatorname{Sec}[c + d*x]), x]$

[Out] $(a^4*(13*A + 8*B)*x)/2 + (a^4*(8*A + 13*B)*\operatorname{ArcTanh}[\operatorname{Sin}[c + d*x]])/(2*d) + (5*a^4*(A - B)*\operatorname{Sin}[c + d*x])/(2*d) + (a*A*\operatorname{Cos}[c + d*x]*(a + a*\operatorname{Sec}[c + d*x])^3*\operatorname{Sin}[c + d*x])/(2*d) - ((A - B)*(a^2 + a^2*\operatorname{Sec}[c + d*x])^2*\operatorname{Sin}[c + d*x])/(2*d) + ((A + 6*B)*(a^4 + a^4*\operatorname{Sec}[c + d*x])*\operatorname{Sin}[c + d*x])/(2*d)$

Rule 3770

$\operatorname{Int}[\operatorname{csc}[(c_.) + (d_.)*(x_.)], x_Symbol] := -\operatorname{Simp}[\operatorname{ArcTanh}[\operatorname{Cos}[c + d*x]]/d, x] /; \operatorname{FreeQ}\{c, d\}, x]$

Rule 3996

$\operatorname{Int}[(\operatorname{csc}[(e_.) + (f_.)*(x_.)]*(d_.))^n*(\operatorname{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))*(\operatorname{csc}[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := \operatorname{Simp}[(A*a*\operatorname{Cot}[e + f*x]*(d*\operatorname{Csc}[e + f*x])^n)/(f*n), x] + \operatorname{Dist}[1/(d*n), \operatorname{Int}[(d*\operatorname{Csc}[e + f*x])^{n+1}*\operatorname{Simp}[n*(B*a + A*b) + (B*b*n + A*a*(n+1))*\operatorname{Csc}[e + f*x], x], x] /; \operatorname{FreeQ}\{a, b, d, e, f, A, B\}, x] \&\& \operatorname{NeQ}[A*b - a*B, 0] \&\& \operatorname{LeQ}[n, -1]$

Rule 4017

```

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (
a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1]

```

Rule 4018

```

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (
a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && !LtQ[n, -1]

```

Rubi steps

$$\begin{aligned}
\int \cos^2(c + dx)(a + a \sec(c + dx))^4(A + B \sec(c + dx)) dx &= \frac{aA \cos(c + dx)(a + a \sec(c + dx))^3 \sin(c + dx)}{2d} + \frac{1}{2} \\
&= \frac{aA \cos(c + dx)(a + a \sec(c + dx))^3 \sin(c + dx)}{2d} - \frac{1}{2} \\
&= \frac{aA \cos(c + dx)(a + a \sec(c + dx))^3 \sin(c + dx)}{2d} - \frac{1}{2} \\
&= \frac{5a^4(A - B) \sin(c + dx)}{2d} + \frac{aA \cos(c + dx)(a + a \sec(c + dx))^3 \sin(c + dx)}{2d} \\
&= \frac{1}{2}a^4(13A + 8B)x + \frac{5a^4(A - B) \sin(c + dx)}{2d} + \frac{aA \cos(c + dx)(a + a \sec(c + dx))^3 \sin(c + dx)}{2d} \\
&= \frac{1}{2}a^4(13A + 8B)x + \frac{a^4(8A + 13B) \tanh^{-1}(\sin(c + dx))}{2d}
\end{aligned}$$

Mathematica [B] time = 4.97, size = 373, normalized size = 2.33

$$a^4 \cos^5(c + dx) \sec^8\left(\frac{1}{2}(c + dx)\right) (\sec(c + dx) + 1)^4 (A + B \sec(c + dx)) \left(\frac{4(4A+B) \sin(c) \cos(dx)}{d} + \frac{4(4A+B) \cos(c) \sin(dx)}{d} + \dots \right)$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]

[Out] (a^4*cos[c + d*x]^5*Sec[(c + d*x)/2]^8*(1 + Sec[c + d*x])^4*(A + B*Sec[c + d*x])*(2*(13*A + 8*B)*x - (2*(8*A + 13*B)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]))/d + (2*(8*A + 13*B)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/d + (4*(4*A + B)*Cos[d*x]*Sin[c])/d + (A*cos[2*d*x]*Sin[2*c])/d + (4*(4*A + B)*Cos[c]*Sin[d*x])/d + (A*cos[2*c]*Sin[2*d*x])/d + B/(d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) + (4*(A + 4*B)*Sin[(d*x)/2])/(d*(Cos[c/2] - Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) - B/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (4*(A + 4*B)*Sin[(d*x)/2])/(d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])))/(64*(B + A*cos[c + d*x]))

fricas [A] time = 0.45, size = 156, normalized size = 0.98

$$\frac{2(13A + 8B)a^4 dx \cos(dx + c)^2 + (8A + 13B)a^4 \cos(dx + c)^2 \log(\sin(dx + c) + 1) - (8A + 13B)a^4 \cos(dx + c)}{64(B + A \cos[c + d*x])}$$

4

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] 1/4*(2*(13*A + 8*B)*a^4*d*x*cos(d*x + c)^2 + (8*A + 13*B)*a^4*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (8*A + 13*B)*a^4*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(A*a^4*cos(d*x + c)^3 + 2*(4*A + B)*a^4*cos(d*x + c)^2 + 2*(A + 4*B)*a^4*cos(d*x + c) + B*a^4)*sin(d*x + c))/(d*cos(d*x + c)^2)

giac [A] time = 0.32, size = 230, normalized size = 1.44

$$(13Aa^4 + 8Ba^4)(dx + c) + (8Aa^4 + 13Ba^4) \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right|\right) - (8Aa^4 + 13Ba^4) \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] 1/2*((13*A*a^4 + 8*B*a^4)*(d*x + c) + (8*A*a^4 + 13*B*a^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - (8*A*a^4 + 13*B*a^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(5*A*a^4*tan(1/2*d*x + 1/2*c)^7 - 5*B*a^4*tan(1/2*d*x + 1/2*c)^7 - 7*A*a^4*tan(1/2*d*x + 1/2*c)^5 - 7*B*a^4*tan(1/2*d*x + 1/2*c)^5 - 9*A*a^4*tan(1/2*d*x + 1/2*c)^3 + 9*B*a^4*tan(1/2*d*x + 1/2*c)^3 + 11*A*a^4*tan(1/2*d*x + 1/2*c))

$$+ 1/2*c) + 11*B*a^4*\tan(1/2*d*x + 1/2*c))/(\tan(1/2*d*x + 1/2*c)^4 - 1)^2)/d$$

maple [A] time = 1.06, size = 182, normalized size = 1.14

$$\frac{A a^4 \cos(dx + c) \sin(dx + c)}{2d} + \frac{13A a^4 x}{2} + \frac{13A a^4 c}{2d} + \frac{a^4 B \sin(dx + c)}{d} + \frac{4A a^4 \sin(dx + c)}{d} + 4a^4 Bx + \frac{4a^4 Bc}{d} + \frac{13a^4 B}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^2*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x)

[Out] 1/2/d*A*a^4*cos(d*x+c)*sin(d*x+c)+13/2*A*a^4*x+13/2/d*A*a^4*c+1/d*a^4*B*sin(d*x+c)+4/d*A*a^4*sin(d*x+c)+4*a^4*B*x+4/d*a^4*B*c+13/2/d*a^4*B*ln(sec(d*x+c)+tan(d*x+c))+4/d*A*a^4*ln(sec(d*x+c)+tan(d*x+c))+4/d*a^4*B*tan(d*x+c)+1/d*A*a^4*tan(d*x+c)+1/2/d*a^4*B*sec(d*x+c)*tan(d*x+c)

maxima [A] time = 0.35, size = 199, normalized size = 1.24

$$(2 dx + 2 c + \sin(2 dx + 2 c))Aa^4 + 24 (dx + c)Aa^4 + 16 (dx + c)Ba^4 - Ba^4 \left(\frac{2 \sin(dx+c)}{\sin(dx+c)^2-1} - \log(\sin(dx+c)+1) - \log(\sin(dx+c)-1) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*A*a^4 + 24*(d*x + c)*A*a^4 + 16*(d*x + c)*B*a^4 - B*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 8*A*a^4*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 12*B*a^4*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 16*A*a^4*sin(d*x + c) + 4*B*a^4*sin(d*x + c) + 4*A*a^4*tan(d*x + c) + 16*B*a^4*tan(d*x + c))/d

mupad [B] time = 2.17, size = 243, normalized size = 1.52

$$\frac{4 A a^4 \sin(c + dx)}{d} + \frac{B a^4 \sin(c + dx)}{d} + \frac{13 A a^4 \operatorname{atan}\left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)}\right)}{d} + \frac{8 A a^4 \operatorname{atanh}\left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)}\right)}{d} + \frac{8 B a^4 \operatorname{atan}\left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^2*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^4,x)

[Out] (4*A*a^4*sin(c + d*x))/d + (B*a^4*sin(c + d*x))/d + (13*A*a^4*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (8*A*a^4*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (8*B*a^4*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d

$$\begin{aligned} & /2 + (d*x)/2)))/d + (8*B*a^4*\operatorname{atan}(\sin(c/2 + (d*x)/2)/\cos(c/2 + (d*x)/2)))/d \\ & + (13*B*a^4*\operatorname{atanh}(\sin(c/2 + (d*x)/2)/\cos(c/2 + (d*x)/2)))/d + (A*a^4*\sin(c \\ & + d*x))/(\cos(c + d*x)) + (4*B*a^4*\sin(c + d*x))/(\cos(c + d*x)) + (B*a^4 \\ & * \sin(c + d*x))/(2*\cos(c + d*x)^2) + (A*a^4*\cos(c + d*x)*\sin(c + d*x))/(2 \\ & *d) \end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**2*(a+a*sec(d*x+c))**4*(A+B*sec(d*x+c)),x)

[Out] Timed out

$$3.77 \quad \int \cos^3(c + dx)(a + a \sec(c + dx))^4(A + B \sec(c + dx)) dx$$

Optimal. Leaf size=165

$$\frac{5a^4(2A + B) \sin(c + dx)}{2d} + \frac{a^4(A + 4B) \tanh^{-1}(\sin(c + dx))}{d} - \frac{(8A - 3B) \sin(c + dx)(a^4 \sec(c + dx) + a^4)}{6d} + \frac{1}{2}a^4x$$

[Out] $\frac{1}{2}a^4(12A+13B)x + a^4(A+4B)\operatorname{arctanh}(\sin(dx+c))/d + 5/2a^4(2A+B)\sin(dx+c)/d + 1/3a^4A\cos(dx+c)^2(a+a\sec(dx+c))^3\sin(dx+c)/d + 1/2(2A+B)\cos(dx+c)(a^2+a^2\sec(dx+c))^2\sin(dx+c)/d - 1/6(8A-3B)(a^4+a^4\sec(dx+c))\sin(dx+c)/d$

Rubi [A] time = 0.41, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {4017, 4018, 3996, 3770}

$$\frac{5a^4(2A + B) \sin(c + dx)}{2d} + \frac{a^4(A + 4B) \tanh^{-1}(\sin(c + dx))}{d} - \frac{(8A - 3B) \sin(c + dx)(a^4 \sec(c + dx) + a^4)}{6d} + \frac{(2A + B) \sin(c + dx)}{2d}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Cos}[c + d*x]^3(a + a*\operatorname{Sec}[c + d*x])^4(A + B*\operatorname{Sec}[c + d*x]), x]$

[Out] $(a^4(12A + 13B)x)/2 + (a^4(A + 4B)*\operatorname{ArcTanh}[\operatorname{Sin}[c + d*x]])/d + (5a^4(2A + B)*\operatorname{Sin}[c + d*x])/(2*d) + (a^4A*\operatorname{Cos}[c + d*x]^2(a + a*\operatorname{Sec}[c + d*x])^3*\operatorname{Sin}[c + d*x])/(3*d) + ((2A + B)*\operatorname{Cos}[c + d*x]*(a^2 + a^2*\operatorname{Sec}[c + d*x])^2*\operatorname{Sin}[c + d*x])/(2*d) - ((8A - 3B)*(a^4 + a^4*\operatorname{Sec}[c + d*x])*\operatorname{Sin}[c + d*x])/(6*d)$

Rule 3770

$\operatorname{Int}[\operatorname{csc}[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow -\operatorname{Simp}[\operatorname{ArcTanh}[\operatorname{Cos}[c + d*x]]/d, x] /; \operatorname{FreeQ}\{c, d\}, x]$

Rule 3996

$\operatorname{Int}[(\operatorname{csc}[(e_.) + (f_.)*(x_.)]*(d_.))^n*(\operatorname{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))*(\operatorname{csc}[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] \rightarrow \operatorname{Simp}[(A*a*\operatorname{Cot}[e + f*x]*(d*\operatorname{Csc}[e + f*x])^n)/(f*n), x] + \operatorname{Dist}[1/(d*n), \operatorname{Int}[(d*\operatorname{Csc}[e + f*x])^{n+1}*\operatorname{Simp}[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*\operatorname{Csc}[e + f*x], x], x] /; \operatorname{FreeQ}\{a, b, d, e, f, A, B\}, x] \&\amp; \operatorname{NeQ}[A*b - a*B, 0] \&\amp; \operatorname{LeQ}[n, -1]$

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && !LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \cos^3(c + dx)(a + a \sec(c + dx))^4(A + B \sec(c + dx)) dx &= \frac{aA \cos^2(c + dx)(a + a \sec(c + dx))^3 \sin(c + dx)}{3d} + \frac{1}{3} \\
&= \frac{aA \cos^2(c + dx)(a + a \sec(c + dx))^3 \sin(c + dx)}{3d} + \frac{(2)}{3} \\
&= \frac{aA \cos^2(c + dx)(a + a \sec(c + dx))^3 \sin(c + dx)}{3d} + \frac{(2)}{3} \\
&= \frac{5a^4(2A + B) \sin(c + dx)}{2d} + \frac{aA \cos^2(c + dx)(a + a \sec(c + dx))^3 \sin(c + dx)}{3d} \\
&= \frac{1}{2}a^4(12A + 13B)x + \frac{5a^4(2A + B) \sin(c + dx)}{2d} + \frac{aA \cos^2(c + dx)(a + a \sec(c + dx))^3 \sin(c + dx)}{3d} \\
&= \frac{1}{2}a^4(12A + 13B)x + \frac{a^4(A + 4B) \tanh^{-1}(\sin(c + dx))}{d}
\end{aligned}$$

Mathematica [B] time = 2.01, size = 342, normalized size = 2.07

$$a^4 \cos^5(c + dx) \sec^8\left(\frac{1}{2}(c + dx)\right) (\sec(c + dx) + 1)^4 (A + B \sec(c + dx)) \left(\frac{3(27A + 16B) \sin(c) \cos(dx)}{d} + \frac{3(4A + B) \sin(2c) \cos(2dx)}{d} \right)$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]

[Out] (a^4*cos[c + d*x]^5*Sec[(c + d*x)/2]^8*(1 + Sec[c + d*x])^4*(A + B*Sec[c + d*x])*(72*A*x + 78*B*x - (12*(A + 4*B)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/d + (12*(A + 4*B)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/d + (3*(27*A + 16*B)*Cos[d*x]*Sin[c])/d + (3*(4*A + B)*Cos[2*d*x]*Sin[2*c])/d + (A*Cos[3*d*x]*Sin[3*c])/d + (3*(27*A + 16*B)*Cos[c]*Sin[d*x])/d + (3*(4*A + B)*Cos[2*c]*Sin[2*d*x])/d + (A*Cos[3*c]*Sin[3*d*x])/d + (12*B*Sin[(d*x)/2])/(d*(Cos[c/2] - Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) + (12*B*Sin[(d*x)/2])/(d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])))/(192*(B + A*cos[c + d*x]))

fricas [A] time = 0.44, size = 150, normalized size = 0.91

$$\frac{3(12A + 13B)a^4 dx \cos(dx + c) + 3(A + 4B)a^4 \cos(dx + c) \log(\sin(dx + c) + 1) - 3(A + 4B)a^4 \cos(dx + c) \log(\sin(dx + c) - 1)}{192(B + A \cos(c + dx))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] 1/6*(3*(12*A + 13*B)*a^4*d*x*cos(d*x + c) + 3*(A + 4*B)*a^4*cos(d*x + c)*log(sin(d*x + c) + 1) - 3*(A + 4*B)*a^4*cos(d*x + c)*log(-sin(d*x + c) + 1) + (2*A*a^4*cos(d*x + c)^3 + 3*(4*A + B)*a^4*cos(d*x + c)^2 + 8*(5*A + 3*B)*a^4*cos(d*x + c) + 6*B*a^4)*sin(d*x + c))/(d*cos(d*x + c))

giac [A] time = 0.58, size = 226, normalized size = 1.37

$$\frac{12Ba^4 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)}{\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - 1} - 3(12Aa^4 + 13Ba^4)(dx + c) - 6(Aa^4 + 4Ba^4) \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right|\right) + 6(Aa^4 + 4Ba^4) \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] -1/6*(12*B*a^4*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1) - 3*(12*A*a^4 + 13*B*a^4)*(d*x + c) - 6*(A*a^4 + 4*B*a^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) + 6*(A*a^4 + 4*B*a^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(30*A*a^4*tan(1/2*d*x + 1/2*c)^5 + 21*B*a^4*tan(1/2*d*x + 1/2*c)^5 + 76*A*a^4*tan(1/2*d*x + 1/2*c)^3 + 48*B*a^4*tan(1/2*d*x + 1/2*c)^3 + 54*A*a^4*tan(1/2*d*x + 1/2*c)))/(d*cos(d*x + c))

$x + 1/2*c) + 27*B*a^4*\tan(1/2*d*x + 1/2*c))/(\tan(1/2*d*x + 1/2*c)^2 + 1)^3$
/d

maple [A] time = 1.14, size = 190, normalized size = 1.15

$$\frac{A \sin(dx + c) (\cos^2(dx + c)) a^4}{3d} + \frac{20A a^4 \sin(dx + c)}{3d} + \frac{a^4 B \cos(dx + c) \sin(dx + c)}{2d} + \frac{13a^4 Bx}{2} + \frac{13a^4 Bc}{2d} + \frac{2A a^4 \cos(dx + c)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(d*x+c)^3*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x)`

[Out] $1/3/d*A*\sin(d*x+c)*\cos(d*x+c)^2*a^4+20/3/d*A*a^4*\sin(d*x+c)+1/2/d*a^4*B*\cos(d*x+c)*\sin(d*x+c)+13/2*a^4*B*x+13/2/d*a^4*B*c+2/d*A*a^4*\cos(d*x+c)*\sin(d*x+c)+6*A*a^4*x+6/d*A*a^4*c+4/d*a^4*B*\sin(d*x+c)+4/d*a^4*B*\ln(\sec(d*x+c)+\tan(d*x+c))+1/d*A*a^4*\ln(\sec(d*x+c)+\tan(d*x+c))+1/d*a^4*B*\tan(d*x+c)$

maxima [A] time = 0.35, size = 187, normalized size = 1.13

$$\frac{4(\sin(dx + c)^3 - 3 \sin(dx + c))Aa^4 - 12(2dx + 2c + \sin(2dx + 2c))Aa^4 - 48(dx + c)Aa^4 - 3(2dx + 2c + \sin(2dx + 2c))Ba^4 - 72(dx + c)Ba^4 - 6Aa^4(\log(\sin(dx + c) + 1) - \log(\sin(dx + c) - 1)) - 24Ba^4(\log(\sin(dx + c) + 1) - \log(\sin(dx + c) - 1)) - 72Aa^4\sin(dx + c) - 48Ba^4\sin(dx + c) - 12Ba^4\tan(dx + c))/d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="maxima")`

[Out] $-1/12*(4*(\sin(dx + c)^3 - 3*\sin(dx + c))*A*a^4 - 12*(2*d*x + 2*c + \sin(2*d*x + 2*c))*A*a^4 - 48*(d*x + c)*A*a^4 - 3*(2*d*x + 2*c + \sin(2*d*x + 2*c))*B*a^4 - 72*(d*x + c)*B*a^4 - 6*A*a^4*(\log(\sin(dx + c) + 1) - \log(\sin(dx + c) - 1)) - 24*B*a^4*(\log(\sin(dx + c) + 1) - \log(\sin(dx + c) - 1)) - 72*A*a^4*\sin(dx + c) - 48*B*a^4*\sin(dx + c) - 12*B*a^4*\tan(dx + c))/d$

mupad [B] time = 2.23, size = 242, normalized size = 1.47

$$\frac{20 A a^4 \sin(c + dx)}{3 d} + \frac{4 B a^4 \sin(c + dx)}{d} + \frac{12 A a^4 \operatorname{atan}\left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)}\right)}{d} + \frac{2 A a^4 \operatorname{atanh}\left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)}\right)}{d} + \frac{13 B a^4 \operatorname{atan}\left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(c + d*x)^3*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^4,x)`

[Out] $(20*A*a^4*\sin(c + d*x))/(3*d) + (4*B*a^4*\sin(c + d*x))/d + (12*A*a^4*\operatorname{atan}(\sin(c/2 + (d*x)/2)/\cos(c/2 + (d*x)/2)))/d + (2*A*a^4*\operatorname{atanh}(\sin(c/2 + (d*x)/2)/\cos(c/2 + (d*x)/2)))/d + (13*B*a^4*\operatorname{atan}(\sin(c/2 + (d*x)/2)/\cos(c/2 + (d*x)/2)))/d$

```
)/2))/d + (8*B*a^4*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (A*a^
4*cos(c + d*x)^2*sin(c + d*x))/(3*d) + (B*a^4*sin(c + d*x))/(d*cos(c + d*x)
) + (2*A*a^4*cos(c + d*x)*sin(c + d*x))/d + (B*a^4*cos(c + d*x)*sin(c + d*x
))/(2*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sec(d*x+c))**4*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

$$3.78 \quad \int \cos^4(c + dx)(a + a \sec(c + dx))^4(A + B \sec(c + dx)) dx$$

Optimal. Leaf size=173

$$\frac{5a^4(7A + 8B) \sin(c + dx)}{8d} + \frac{(35A + 32B) \sin(c + dx) \cos(c + dx) (a^4 \sec(c + dx) + a^4)}{24d} + \frac{1}{8} a^4 x (35A + 48B) + \frac{a^4 B \tan(c + dx)}{d}$$

[Out] $1/8*a^4*(35*A+48*B)*x+a^4*B*\operatorname{arctanh}(\sin(d*x+c))/d+5/8*a^4*(7*A+8*B)*\sin(d*x+c)/d+1/4*a*A*\cos(d*x+c)^3*(a+a*\sec(d*x+c))^3*\sin(d*x+c)/d+1/12*(7*A+4*B)*\cos(d*x+c)^2*(a^2+a^2*\sec(d*x+c))^2*\sin(d*x+c)/d+1/24*(35*A+32*B)*\cos(d*x+c)*(a^4+a^4*\sec(d*x+c))*\sin(d*x+c)/d$

Rubi [A] time = 0.40, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {4017, 3996, 3770}

$$\frac{5a^4(7A + 8B) \sin(c + dx)}{8d} + \frac{(7A + 4B) \sin(c + dx) \cos^2(c + dx) (a^2 \sec(c + dx) + a^2)^2}{12d} + \frac{(35A + 32B) \sin(c + dx) \cos(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] `Int[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x]), x]`

[Out] $(a^4*(35*A + 48*B)*x)/8 + (a^4*B*\operatorname{ArcTanh}[\sin[c + d*x]])/d + (5*a^4*(7*A + 8*B)*\sin[c + d*x])/(8*d) + (a*A*\cos[c + d*x]^3*(a + a*\sec[c + d*x])^3*\sin[c + d*x])/(4*d) + ((7*A + 4*B)*\cos[c + d*x]^2*(a^2 + a^2*\sec[c + d*x])^2*\sin[c + d*x])/(12*d) + ((35*A + 32*B)*\cos[c + d*x]*(a^4 + a^4*\sec[c + d*x])*\sin[c + d*x])/(24*d)$

Rule 3770

`Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]`

Rule 3996

`Int[(csc[(e_.) + (f_.)*(x_)])*(d_.)^n*(csc[(e_.) + (f_.)*(x_)])*(b_.) + (a_.)*(csc[(e_.) + (f_.)*(x_)])*(B_.) + (A_.), x_Symbol] := Simp[(A*a*Cot[e + f*x])*(d*Csc[e + f*x])^n/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]`

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (
a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \cos^4(c + dx)(a + a \sec(c + dx))^4(A + B \sec(c + dx)) dx &= \frac{aA \cos^3(c + dx)(a + a \sec(c + dx))^3 \sin(c + dx)}{4d} + \dots \\
&= \frac{aA \cos^3(c + dx)(a + a \sec(c + dx))^3 \sin(c + dx)}{4d} + \dots \\
&= \frac{aA \cos^3(c + dx)(a + a \sec(c + dx))^3 \sin(c + dx)}{4d} + \dots \\
&= \frac{5a^4(7A + 8B) \sin(c + dx)}{8d} + \frac{aA \cos^3(c + dx)(a + a \sec(c + dx))^3 \sin(c + dx)}{4d} + \dots \\
&= \frac{1}{8}a^4(35A + 48B)x + \frac{5a^4(7A + 8B) \sin(c + dx)}{8d} + \dots \\
&= \frac{1}{8}a^4(35A + 48B)x + \frac{a^4B \tanh^{-1}(\sin(c + dx))}{d} + \dots
\end{aligned}$$

Mathematica [A] time = 0.39, size = 138, normalized size = 0.80

$$a^4 \left(24(28A + 27B) \sin(c + dx) + 24(7A + 4B) \sin(2(c + dx)) + 32A \sin(3(c + dx)) + 3A \sin(4(c + dx)) + 420A \right)$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]

[Out] (a^4*(420*A*d*x + 576*B*d*x - 96*B*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 96*B*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 24*(28*A + 27*B)*Sin[c + d*x] + 24*(7*A + 4*B)*Sin[2*(c + d*x)] + 32*A*Sin[3*(c + d*x)] + 8*B*Sin[3*(c + d*x)] + 3*A*Sin[4*(c + d*x)]))/(96*d)

fricas [A] time = 0.45, size = 118, normalized size = 0.68

$$3(35A + 48B)a^4 dx + 12Ba^4 \log(\sin(dx + c) + 1) - 12Ba^4 \log(-\sin(dx + c) + 1) + (6Aa^4 \cos(dx + c)^3 + 8Aa^4 \sin(dx + c)^3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] 1/24*(3*(35*A + 48*B)*a^4*d*x + 12*B*a^4*log(sin(d*x + c) + 1) - 12*B*a^4*log(-sin(d*x + c) + 1) + (6*A*a^4*cos(d*x + c)^3 + 8*(4*A + B)*a^4*cos(d*x + c)^2 + 3*(27*A + 16*B)*a^4*cos(d*x + c) + 160*(A + B)*a^4)*sin(d*x + c))/d

giac [A] time = 0.33, size = 214, normalized size = 1.24

$$24 Ba^4 \log \left(\left| \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) + 1 \right| \right) - 24 Ba^4 \log \left(\left| \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) - 1 \right| \right) + 3 (35 Aa^4 + 48 Ba^4)(dx + c) + \frac{2 (105 Aa^4}{$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] 1/24*(24*B*a^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 24*B*a^4*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 3*(35*A*a^4 + 48*B*a^4)*(d*x + c) + 2*(105*A*a^4*tan(1/2*d*x + 1/2*c)^7 + 120*B*a^4*tan(1/2*d*x + 1/2*c)^7 + 385*A*a^4*tan(1/2*d*x + 1/2*c)^5 + 424*B*a^4*tan(1/2*d*x + 1/2*c)^5 + 511*A*a^4*tan(1/2*d*x + 1/2*c)^3 + 520*B*a^4*tan(1/2*d*x + 1/2*c)^3 + 279*A*a^4*tan(1/2*d*x + 1/2*c) + 216*B*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d

maple [A] time = 1.21, size = 199, normalized size = 1.15

$$\frac{A a^4 \sin(dx + c) (\cos^3(dx + c))}{4d} + \frac{27 A a^4 \cos(dx + c) \sin(dx + c)}{8d} + \frac{35 A a^4 x}{8} + \frac{35 A a^4 c}{8d} + \frac{B \sin(dx + c) (\cos^2(dx + c))}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^4*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x)

[Out] 1/4/d*A*a^4*sin(d*x+c)*cos(d*x+c)^3+27/8/d*A*a^4*cos(d*x+c)*sin(d*x+c)+35/8*A*a^4*x+35/8/d*A*a^4*c+1/3/d*B*sin(d*x+c)*cos(d*x+c)^2*a^4+20/3/d*a^4*B*sin(d*x+c)+4/3/d*A*sin(d*x+c)*cos(d*x+c)^2*a^4+20/3/d*A*a^4*sin(d*x+c)+2/d*a^4*B*cos(d*x+c)*sin(d*x+c)+6*a^4*B*x+6/d*a^4*B*c+1/d*a^4*B*ln(sec(d*x+c)+tan(d*x+c))

maxima [A] time = 0.35, size = 205, normalized size = 1.18

$$128 (\sin(dx + c)^3 - 3 \sin(dx + c)) Aa^4 - 3 (12 dx + 12 c + \sin(4 dx + 4 c) + 8 \sin(2 dx + 2 c)) Aa^4 - 144 (2 dx +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out]
$$-1/96*(128*(\sin(dx + c)^3 - 3*\sin(dx + c))*A*a^4 - 3*(12*dx + 12*c + \sin(4*dx + 4*c) + 8*\sin(2*dx + 2*c))*A*a^4 - 144*(2*dx + 2*c + \sin(2*dx + 2*c))*A*a^4 - 96*(dx + c)*A*a^4 + 32*(\sin(dx + c)^3 - 3*\sin(dx + c))*B*a^4 - 96*(2*dx + 2*c + \sin(2*dx + 2*c))*B*a^4 - 384*(dx + c)*B*a^4 - 48*B*a^4*(\log(\sin(dx + c) + 1) - \log(\sin(dx + c) - 1)) - 384*A*a^4*\sin(dx + c) - 576*B*a^4*\sin(dx + c))/d$$

mupad [B] time = 2.45, size = 188, normalized size = 1.09

$$105 A a^4 \operatorname{atan}\left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)}\right) + 144 B a^4 \operatorname{atan}\left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)}\right) + 24 B a^4 \operatorname{atanh}\left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)}\right) + 21 A a^4 \sin(2c + 2dx) + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^4*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^4,x)

[Out]
$$(105*A*a^4*\operatorname{atan}(\sin(c/2 + (d*x)/2)/\cos(c/2 + (d*x)/2)) + 144*B*a^4*\operatorname{atan}(\sin(c/2 + (d*x)/2)/\cos(c/2 + (d*x)/2)) + 24*B*a^4*\operatorname{atanh}(\sin(c/2 + (d*x)/2)/\cos(c/2 + (d*x)/2)) + 21*A*a^4*\sin(2*c + 2*d*x) + 4*A*a^4*\sin(3*c + 3*d*x) + (3*A*a^4*\sin(4*c + 4*d*x))/8 + 12*B*a^4*\sin(2*c + 2*d*x) + B*a^4*\sin(3*c + 3*d*x) + 84*A*a^4*\sin(c + d*x) + 81*B*a^4*\sin(c + d*x))/(12*d)$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**4*(a+a*sec(d*x+c))**4*(A+B*sec(d*x+c)),x)

[Out] Timed out

$$3.79 \quad \int \cos^5(c + dx)(a + a \sec(c + dx))^4(A + B \sec(c + dx)) dx$$

Optimal. Leaf size=158

$$-\frac{4a^4(4A + 5B) \sin^3(c + dx)}{15d} + \frac{8a^4(4A + 5B) \sin(c + dx)}{5d} + \frac{a^4(4A + 5B) \sin(c + dx) \cos^3(c + dx)}{20d} + \frac{27a^4(4A + 5B)}{40d}$$

[Out] 7/8*a^4*(4*A+5*B)*x+8/5*a^4*(4*A+5*B)*sin(d*x+c)/d+27/40*a^4*(4*A+5*B)*cos(d*x+c)*sin(d*x+c)/d+1/20*a^4*(4*A+5*B)*cos(d*x+c)^3*sin(d*x+c)/d+1/5*A*cos(d*x+c)^4*(a+a*sec(d*x+c))^4*sin(d*x+c)/d-4/15*a^4*(4*A+5*B)*sin(d*x+c)^3/d

Rubi [A] time = 0.20, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4013, 3791, 2637, 2635, 8, 2633}

$$-\frac{4a^4(4A + 5B) \sin^3(c + dx)}{15d} + \frac{8a^4(4A + 5B) \sin(c + dx)}{5d} + \frac{a^4(4A + 5B) \sin(c + dx) \cos^3(c + dx)}{20d} + \frac{27a^4(4A + 5B)}{40d}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^5*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x]), x]

[Out] (7*a^4*(4*A + 5*B)*x)/8 + (8*a^4*(4*A + 5*B)*Sin[c + d*x])/(5*d) + (27*a^4*(4*A + 5*B)*Cos[c + d*x]*Sin[c + d*x])/(40*d) + (a^4*(4*A + 5*B)*Cos[c + d*x]^3*SIN[c + d*x])/(20*d) + (A*cos[c + d*x]^4*(a + a*Sec[c + d*x])^4*SIN[c + d*x])/(5*d) - (4*a^4*(4*A + 5*B)*Sin[c + d*x]^3)/(15*d)

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2633

Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Simp[(b*cos[c + d*x])*(b*sin[c + d*x])^(n - 1)/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] := Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] := Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
- b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] && !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \cos^5(c + dx)(a + a \sec(c + dx))^4(A + B \sec(c + dx)) dx &= \frac{A \cos^4(c + dx)(a + a \sec(c + dx))^4 \sin(c + dx)}{5d} + \frac{1}{5} \\
&= \frac{A \cos^4(c + dx)(a + a \sec(c + dx))^4 \sin(c + dx)}{5d} + \frac{1}{5} \\
&= \frac{1}{5} a^4(4A + 5B)x + \frac{A \cos^4(c + dx)(a + a \sec(c + dx))^4 \sin(c + dx)}{5d} \\
&= \frac{1}{5} a^4(4A + 5B)x + \frac{4a^4(4A + 5B) \sin(c + dx)}{5d} + \frac{3a^4}{5} \\
&= \frac{4}{5} a^4(4A + 5B)x + \frac{8a^4(4A + 5B) \sin(c + dx)}{5d} + \frac{27a^4}{5} \\
&= \frac{7}{8} a^4(4A + 5B)x + \frac{8a^4(4A + 5B) \sin(c + dx)}{5d} + \frac{27a^4}{5}
\end{aligned}$$

Mathematica [A] time = 0.36, size = 108, normalized size = 0.68

$$\frac{a^4(420(7A + 8B) \sin(c + dx) + 120(8A + 7B) \sin(2(c + dx)) + 290A \sin(3(c + dx)) + 60A \sin(4(c + dx)) + 6A \sin(5(c + dx)))}{480d}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]^5*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]

[Out] (a^4*(1680*A*d*x + 2100*B*d*x + 420*(7*A + 8*B)*Sin[c + d*x] + 120*(8*A + 7*B)*Sin[2*(c + d*x)] + 290*A*Ssin[3*(c + d*x)] + 160*B*Ssin[3*(c + d*x)] + 60*A*Ssin[4*(c + d*x)] + 15*B*Ssin[4*(c + d*x)] + 6*A*Ssin[5*(c + d*x)])/(480*d)

fricas [A] time = 0.49, size = 110, normalized size = 0.70

$$\frac{105(4A + 5B)a^4 dx + (24Aa^4 \cos(dx + c)^4 + 30(4A + B)a^4 \cos(dx + c)^3 + 16(17A + 10B)a^4 \cos(dx + c)^2 + 15(28A + 27B)a^4 \cos(dx + c) + 8(83A + 100B)a^4 \sin(dx + c))}{120d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] 1/120*(105*(4*A + 5*B)*a^4*d*x + (24*A*a^4*cos(d*x + c)^4 + 30*(4*A + B)*a^4*cos(d*x + c)^3 + 16*(17*A + 10*B)*a^4*cos(d*x + c)^2 + 15*(28*A + 27*B)*a^4*cos(d*x + c) + 8*(83*A + 100*B)*a^4*sin(d*x + c))/d

giac [A] time = 0.64, size = 210, normalized size = 1.33

$$105(4Aa^4 + 5Ba^4)(dx + c) + \frac{2\left(420Aa^4 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^9 + 525Ba^4 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^9 + 1960Aa^4 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^7 + 2450Ba^4 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^7 + 3584Aa^4 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5 + 4480Ba^4 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5 + 3160Aa^4 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 + 3950Ba^4 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 + 1500Aa^4 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1395Ba^4 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right)}{(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + 1)^5}/d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] 1/120*(105*(4*A*a^4 + 5*B*a^4)*(d*x + c) + 2*(420*A*a^4*tan(1/2*d*x + 1/2*c)^9 + 525*B*a^4*tan(1/2*d*x + 1/2*c)^9 + 1960*A*a^4*tan(1/2*d*x + 1/2*c)^7 + 2450*B*a^4*tan(1/2*d*x + 1/2*c)^7 + 3584*A*a^4*tan(1/2*d*x + 1/2*c)^5 + 4480*B*a^4*tan(1/2*d*x + 1/2*c)^5 + 3160*A*a^4*tan(1/2*d*x + 1/2*c)^3 + 3950*B*a^4*tan(1/2*d*x + 1/2*c)^3 + 1500*A*a^4*tan(1/2*d*x + 1/2*c) + 1395*B*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^5)/d

maple [A] time = 1.50, size = 248, normalized size = 1.57

$$\frac{Aa^4\left(\frac{8}{3} + \cos^4(dx+c) + \frac{4(\cos^2(dx+c))}{3}\right)\sin(dx+c)}{5} + 4Aa^4\left(\frac{\left(\cos^3(dx+c) + \frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4} + \frac{3dx}{8} + \frac{3c}{8}\right) + a^4B\left(\frac{\left(\cos^3(dx+c) + \frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(d*x+c)^5*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x)`

[Out] $\frac{1}{d} \left(\frac{1}{5} A a^4 (8/3 + \cos(d*x+c)^4 + 4/3 \cos(d*x+c)^2) \sin(d*x+c) + 4 A a^4 (1/4 (\cos(d*x+c)^3 + 3/2 \cos(d*x+c)) \sin(d*x+c) + 3/8 d*x + 3/8 c) + a^4 B (1/4 (\cos(d*x+c)^3 + 3/2 \cos(d*x+c)) \sin(d*x+c) + 3/8 d*x + 3/8 c) + 2 A a^4 (2 + \cos(d*x+c)^2) \sin(d*x+c) + 4/3 a^4 B (2 + \cos(d*x+c)^2) \sin(d*x+c) + 4 A a^4 (1/2 \cos(d*x+c) \sin(d*x+c) + 1/2 d*x + 1/2 c) + 6 a^4 B (1/2 \cos(d*x+c) \sin(d*x+c) + 1/2 d*x + 1/2 c) + A a^4 \sin(d*x+c) + 4 a^4 B \sin(d*x+c) + a^4 B (d*x+c) \right)$

maxima [A] time = 0.35, size = 236, normalized size = 1.49

$$\frac{32 \left(3 \sin(dx+c)^5 - 10 \sin(dx+c)^3 + 15 \sin(dx+c) \right) A a^4 - 960 \left(\sin(dx+c)^3 - 3 \sin(dx+c) \right) A a^4 + 60 (12 d x + 12 c + \sin(4 d x + 4 c) + 8 \sin(2 d x + 2 c)) B a^4 + 480 (2 d x + 2 c + \sin(2 d x + 2 c)) A a^4 - 640 (\sin(dx+c)^3 - 3 \sin(dx+c)) B a^4 + 15 (12 d x + 12 c + \sin(4 d x + 4 c) + 8 \sin(2 d x + 2 c)) B a^4 + 720 (2 d x + 2 c + \sin(2 d x + 2 c)) B a^4 + 480 (d x + c) B a^4 + 480 A a^4 \sin(dx+c) + 1920 B a^4 \sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="maxima")`

[Out] $\frac{1}{480} \left(32 \left(3 \sin(dx+c)^5 - 10 \sin(dx+c)^3 + 15 \sin(dx+c) \right) A a^4 - 960 \left(\sin(dx+c)^3 - 3 \sin(dx+c) \right) A a^4 + 60 (12 d x + 12 c + \sin(4 d x + 4 c) + 8 \sin(2 d x + 2 c)) B a^4 + 480 (2 d x + 2 c + \sin(2 d x + 2 c)) A a^4 - 640 (\sin(dx+c)^3 - 3 \sin(dx+c)) B a^4 + 15 (12 d x + 12 c + \sin(4 d x + 4 c) + 8 \sin(2 d x + 2 c)) B a^4 + 720 (2 d x + 2 c + \sin(2 d x + 2 c)) B a^4 + 480 (d x + c) B a^4 + 480 A a^4 \sin(dx+c) + 1920 B a^4 \sin(dx+c) \right) / d$

mupad [B] time = 4.70, size = 248, normalized size = 1.57

$$\frac{\left(7 A a^4 + \frac{35 B a^4}{4} \right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^9 + \left(\frac{98 A a^4}{3} + \frac{245 B a^4}{6} \right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^7 + \left(\frac{896 A a^4}{15} + \frac{224 B a^4}{3} \right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5 + \left(\frac{158 A a^4}{3} \right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 + \left(\frac{395 B a^4}{6} \right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^{10} + 5 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^8 + 10 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^6 + 10 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 + 5 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + \tan\left(\frac{c}{2} + \frac{dx}{2}\right) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(c+d*x)^5*(A+B/cos(c+d*x))*(a+a/cos(c+d*x))^4,x)`

[Out] $\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right) \left(25 A a^4 + \frac{93 B a^4}{4} \right) + \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^9 \left(7 A a^4 + \frac{35 B a^4}{4} \right) + \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^7 \left(\frac{98 A a^4}{3} + \frac{245 B a^4}{6} \right) + \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5 \left(\frac{158 A a^4}{3} + \frac{395 B a^4}{6} \right) + \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 \left(\frac{158 A a^4}{3} + \frac{395 B a^4}{6} \right) + \tan\left(\frac{c}{2} + \frac{dx}{2}\right) \left(\frac{896 A a^4}{15} + \frac{224 B a^4}{3} \right) \right) / \left(d \left(5 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + 10 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 + 10 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^6 + 5 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^8 + \tan\left(\frac{c}{2} + \frac{dx}{2}\right) + 1 \right) \right)$

$$\left(\frac{d*x}{2}\right)^{10} + 1)) + (7*a^4*atan((7*a^4*tan(c/2 + (d*x)/2)*(4*A + 5*B))/(4*(7*A*a^4 + (35*B*a^4)/4)))*(4*A + 5*B))/(4*d)$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**5*(a+a*sec(d*x+c))**4*(A+B*sec(d*x+c)),x)

[Out] Timed out

$$3.80 \quad \int \cos^6(c + dx)(a + a \sec(c + dx))^4(A + B \sec(c + dx)) dx$$

Optimal. Leaf size=220

$$\frac{a^4(72A + 83B) \sin(c + dx)}{15d} + \frac{a^4(159A + 176B) \sin(c + dx) \cos^2(c + dx)}{120d} + \frac{7a^4(7A + 8B) \sin(c + dx) \cos(c + dx)}{16d}$$

[Out] 7/16*a^4*(7*A+8*B)*x+1/15*a^4*(72*A+83*B)*sin(d*x+c)/d+7/16*a^4*(7*A+8*B)*cos(d*x+c)*sin(d*x+c)/d+1/120*a^4*(159*A+176*B)*cos(d*x+c)^2*sin(d*x+c)/d+1/6*a^4*A*cos(d*x+c)^5*(a+a*sec(d*x+c))^3*sin(d*x+c)/d+1/10*(3*A+2*B)*cos(d*x+c)^4*(a^2+a^2*sec(d*x+c))^2*sin(d*x+c)/d+1/120*(73*A+72*B)*cos(d*x+c)^3*(a^4+a^4*sec(d*x+c))*sin(d*x+c)/d

Rubi [A] time = 0.53, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4017, 3996, 3787, 2635, 8, 2637}

$$\frac{a^4(72A + 83B) \sin(c + dx)}{15d} + \frac{a^4(159A + 176B) \sin(c + dx) \cos^2(c + dx)}{120d} + \frac{7a^4(7A + 8B) \sin(c + dx) \cos(c + dx)}{16d}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^6*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]

[Out] (7*a^4*(7*A + 8*B)*x)/16 + (a^4*(72*A + 83*B)*Sin[c + d*x])/(15*d) + (7*a^4*(7*A + 8*B)*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (a^4*(159*A + 176*B)*Cos[c + d*x]^2*Sin[c + d*x])/(120*d) + (a^4*A*Cos[c + d*x]^5*(a + a*Sec[c + d*x])^3*Sin[c + d*x])/(6*d) + ((3*A + 2*B)*Cos[c + d*x]^4*(a^2 + a^2*Sec[c + d*x])^2*Sin[c + d*x])/(10*d) + ((73*A + 72*B)*Cos[c + d*x]^3*(a^4 + a^4*Sec[c + d*x])*Sin[c + d*x])/(120*d)

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x])*(b*SIN[c + d*x])^(n - 1)/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*SIN[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)])*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)])*(b_.) +
(a_), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)])*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)])*(b_.) + (
a_)*(csc[(e_.) + (f_.)*(x_)])*(B_.) + (A_)), x_Symbol] := Simp[(A*a*Cot[e +
f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
+ 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)])*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)])*(b_.) + (
a_)^(m_.)*(csc[(e_.) + (f_.)*(x_)])*(B_.) + (A_)), x_Symbol] := Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \cos^6(c + dx)(a + a \sec(c + dx))^4(A + B \sec(c + dx)) dx &= \frac{aA \cos^5(c + dx)(a + a \sec(c + dx))^3 \sin(c + dx)}{6d} + \dots \\
&= \frac{aA \cos^5(c + dx)(a + a \sec(c + dx))^3 \sin(c + dx)}{6d} + \dots \\
&= \frac{aA \cos^5(c + dx)(a + a \sec(c + dx))^3 \sin(c + dx)}{6d} + \dots \\
&= \frac{a^4(159A + 176B) \cos^2(c + dx) \sin(c + dx)}{120d} + \frac{aA \cos^5(c + dx)}{6d} \\
&= \frac{a^4(159A + 176B) \cos^2(c + dx) \sin(c + dx)}{120d} + \frac{aA \cos^5(c + dx)}{6d} \\
&= \frac{a^4(72A + 83B) \sin(c + dx)}{15d} + \frac{7a^4(7A + 8B) \cos(c + dx)}{16d} \\
&= \frac{7}{16} a^4(7A + 8B)x + \frac{a^4(72A + 83B) \sin(c + dx)}{15d} + \dots
\end{aligned}$$

Mathematica [A] time = 0.61, size = 134, normalized size = 0.61

$$\frac{a^4(120(44A + 49B) \sin(c + dx) + 15(127A + 128B) \sin(2(c + dx)) + 720A \sin(3(c + dx)) + 225A \sin(4(c + dx)) + \dots}{960d}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]^6*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]

[Out] (a^4*(2940*A*c + 2940*A*d*x + 3360*B*d*x + 120*(44*A + 49*B)*Sin[c + d*x] + 15*(127*A + 128*B)*Sin[2*(c + d*x)] + 720*A*Ssin[3*(c + d*x)] + 580*B*Ssin[3*(c + d*x)] + 225*A*Ssin[4*(c + d*x)] + 120*B*Ssin[4*(c + d*x)] + 48*A*Ssin[5*(c + d*x)] + 12*B*Ssin[5*(c + d*x)] + 5*A*Ssin[6*(c + d*x)]))/(960*d)

fricas [A] time = 0.45, size = 130, normalized size = 0.59

$$\frac{105(7A + 8B)a^4 dx + (40Aa^4 \cos(dx + c)^5 + 48(4A + B)a^4 \cos(dx + c)^4 + 10(41A + 24B)a^4 \cos(dx + c)^3 + \dots}{240d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^6*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] $\frac{1}{240} \cdot (105 \cdot (7A + 8B) \cdot a^4 \cdot dx + (40A \cdot a^4 \cdot \cos(dx + c)^5 + 48 \cdot (4A + B) \cdot a^4 \cdot \cos(dx + c)^4 + 10 \cdot (41A + 24B) \cdot a^4 \cdot \cos(dx + c)^3 + 32 \cdot (18A + 17B) \cdot a^4 \cdot \cos(dx + c)^2 + 105 \cdot (7A + 8B) \cdot a^4 \cdot \cos(dx + c) + 16 \cdot (72A + 83B) \cdot a^4 \cdot \sin(dx + c)) / d$

giac [A] time = 0.93, size = 244, normalized size = 1.11

$$105 (7 A a^4 + 8 B a^4) (dx + c) + \frac{2 \left(735 A a^4 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^{11} + 840 B a^4 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^{11} + 4165 A a^4 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^9 + 4760 B a^4 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^9 \right)}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(dx+c)^6*(a+a*sec(dx+c))^4*(A+B*sec(dx+c)),x, algorithm="giac")`

[Out] $\frac{1}{240} \cdot (105 \cdot (7A \cdot a^4 + 8B \cdot a^4) \cdot (dx + c) + 2 \cdot (735 \cdot A \cdot a^4 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c)^{11} + 840 \cdot B \cdot a^4 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c)^{11} + 4165 \cdot A \cdot a^4 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c)^9 + 4760 \cdot B \cdot a^4 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c)^9 + 9702 \cdot A \cdot a^4 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c)^7 + 11088 \cdot B \cdot a^4 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c)^7 + 11802 \cdot A \cdot a^4 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c)^5 + 13488 \cdot B \cdot a^4 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c)^5 + 7355 \cdot A \cdot a^4 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c)^3 + 9320 \cdot B \cdot a^4 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c)^3 + 3105 \cdot A \cdot a^4 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c) + 3000 \cdot B \cdot a^4 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c)) / (\tan(1/2 \cdot dx + 1/2 \cdot c)^2 + 1)^6 / d$

maple [A] time = 2.00, size = 306, normalized size = 1.39

$$A a^4 \left(\frac{\left(\cos^5(dx+c) + \frac{5 \cos^3(dx+c)}{4} + \frac{15 \cos(dx+c)}{8} \right) \sin(dx+c)}{6} + \frac{5 dx}{16} + \frac{5c}{16} \right) + \frac{a^4 B \left(\frac{8}{3} + \cos^4(dx+c) + \frac{4 \cos^2(dx+c)}{3} \right) \sin(dx+c)}{5} + \frac{4 A a^4 \left(\frac{8}{3} + \cos^4(dx+c) \right)}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(dx+c)^6*(a+a*sec(dx+c))^4*(A+B*sec(dx+c)),x)`

[Out] $\frac{1}{d} \cdot (A \cdot a^4 \cdot (1/6 \cdot \cos(dx+c)^5 + 5/4 \cdot \cos(dx+c)^3 + 15/8 \cdot \cos(dx+c)) \cdot \sin(dx+c) + 5/16 \cdot dx + 5/16 \cdot c) + 1/5 \cdot a^4 \cdot B \cdot (8/3 + \cos(dx+c)^4 + 4/3 \cdot \cos(dx+c)^2) \cdot \sin(dx+c) + 4/5 \cdot A \cdot a^4 \cdot (8/3 + \cos(dx+c)^4 + 4/3 \cdot \cos(dx+c)^2) \cdot \sin(dx+c) + 4 \cdot a^4 \cdot B \cdot (1/4 \cdot (\cos(dx+c)^3 + 3/2 \cdot \cos(dx+c)) \cdot \sin(dx+c) + 3/8 \cdot dx + 3/8 \cdot c) + 6 \cdot A \cdot a^4 \cdot (1/4 \cdot (\cos(dx+c)^3 + 3/2 \cdot \cos(dx+c)) \cdot \sin(dx+c) + 3/8 \cdot dx + 3/8 \cdot c) + 2 \cdot a^4 \cdot B \cdot (2 + \cos(dx+c)^2) \cdot \sin(dx+c) + 4/3 \cdot A \cdot a^4 \cdot (2 + \cos(dx+c)^2) \cdot \sin(dx+c) + 4 \cdot a^4 \cdot B \cdot (1/2 \cdot \cos(dx+c) \cdot \sin(dx+c) + 1/2 \cdot dx + 1/2 \cdot c) + A \cdot a^4 \cdot (1/2 \cdot \cos(dx+c) \cdot \sin(dx+c) + 1/2 \cdot dx + 1/2 \cdot c) + a^4 \cdot B \cdot \sin(dx+c))$

maxima [A] time = 0.35, size = 297, normalized size = 1.35

$$\frac{256 \left(3 \sin(dx + c)^5 - 10 \sin(dx + c)^3 + 15 \sin(dx + c) \right) A a^4 - 5 \left(4 \sin(2dx + 2c)^3 - 60 dx - 60 c - 9 \sin(4dx + 4c) \right) A a^4 - 1280 \left(\sin(dx + c)^3 - 3 \sin(dx + c) \right) A a^4 + 180 \left(12 dx + 12 c + \sin(4dx + 4c) + 8 \sin(2dx + 2c) \right) A a^4 + 240 \left(2 dx + 2c + \sin(2dx + 2c) \right) A a^4 + 64 \left(3 \sin(dx + c)^5 - 10 \sin(dx + c)^3 + 15 \sin(dx + c) \right) B a^4 - 1920 \left(\sin(dx + c)^3 - 3 \sin(dx + c) \right) B a^4 + 120 \left(12 dx + 12 c + \sin(4dx + 4c) + 8 \sin(2dx + 2c) \right) B a^4 + 960 \left(2 dx + 2c + \sin(2dx + 2c) \right) B a^4 + 960 B a^4 \sin(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^6*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] 1/960*(256*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*A*a^4 - 5*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*A*a^4 - 1280*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^4 + 180*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^4 + 240*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a^4 + 64*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*B*a^4 - 1920*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^4 + 120*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^4 + 960*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^4 + 960*B*a^4*sin(d*x + c))/d

mupad [B] time = 4.62, size = 286, normalized size = 1.30

$$\frac{\left(\frac{49 A a^4}{8} + 7 B a^4 \right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^{11} + \left(\frac{833 A a^4}{24} + \frac{119 B a^4}{3} \right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^9 + \left(\frac{1617 A a^4}{20} + \frac{462 B a^4}{5} \right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^7 + \left(\frac{1967 A a^4}{20} + \frac{562 B a^4}{5} \right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5 + \left(\frac{127 A a^4}{20} + \frac{127 B a^4}{5} \right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 + \left(\frac{127 A a^4}{20} + \frac{127 B a^4}{5} \right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^{12} + 6 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^{10} + 15 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^8 + 20 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^6 + 15 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 + 6 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + 1 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^6*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^4,x)

[Out] (tan(c/2 + (d*x)/2)*((207*A*a^4)/8 + 25*B*a^4) + tan(c/2 + (d*x)/2)^11*((49*A*a^4)/8 + 7*B*a^4) + tan(c/2 + (d*x)/2)^9*((833*A*a^4)/24 + (119*B*a^4)/3) + tan(c/2 + (d*x)/2)^7*((1617*A*a^4)/20 + (462*B*a^4)/5) + tan(c/2 + (d*x)/2)^5*((1967*A*a^4)/20 + (562*B*a^4)/5))/(d*(6*tan(c/2 + (d*x)/2)^2 + 15*tan(c/2 + (d*x)/2)^4 + 20*tan(c/2 + (d*x)/2)^6 + 15*tan(c/2 + (d*x)/2)^8 + 6*tan(c/2 + (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 + 1)) + (7*a^4*atan((7*a^4*tan(c/2 + (d*x)/2)*(7*A + 8*B))/(8*((49*A*a^4)/8 + 7*B*a^4)))*(7*A + 8*B))/(8*d)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+a*sec(d*x+c))**4*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

3.81 $\int \cos^7(c + dx)(a + a \sec(c + dx))^4(A + B \sec(c + dx)) dx$

Optimal. Leaf size=241

$$-\frac{a^4(227A + 252B) \sin^3(c + dx)}{105d} + \frac{a^4(227A + 252B) \sin(c + dx)}{35d} + \frac{a^4(276A + 301B) \sin(c + dx) \cos^3(c + dx)}{280d} + \frac{a^4}{d}$$

[Out] $1/16*a^4*(44*A+49*B)*x+1/35*a^4*(227*A+252*B)*\sin(d*x+c)/d+1/16*a^4*(44*A+49*B)*\cos(d*x+c)*\sin(d*x+c)/d+1/280*a^4*(276*A+301*B)*\cos(d*x+c)^3*\sin(d*x+c)/d+1/7*a^4*A*\cos(d*x+c)^6*(a+a*\sec(d*x+c))^3*\sin(d*x+c)/d+1/42*(10*A+7*B)*\cos(d*x+c)^5*(a^2+a^2*\sec(d*x+c))^2*\sin(d*x+c)/d+7/15*(A+B)*\cos(d*x+c)^4*(a^4+a^4*\sec(d*x+c))*\sin(d*x+c)/d-1/105*a^4*(227*A+252*B)*\sin(d*x+c)^3/d$

Rubi [A] time = 0.57, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4017, 3996, 3787, 2633, 2635, 8}

$$-\frac{a^4(227A + 252B) \sin^3(c + dx)}{105d} + \frac{a^4(227A + 252B) \sin(c + dx)}{35d} + \frac{a^4(276A + 301B) \sin(c + dx) \cos^3(c + dx)}{280d} + \frac{a^4}{d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Cos}[c + d*x]^7*(a + a*\text{Sec}[c + d*x])^4*(A + B*\text{Sec}[c + d*x]), x]$

[Out] $(a^4*(44*A + 49*B)*x)/16 + (a^4*(227*A + 252*B)*\text{Sin}[c + d*x])/(35*d) + (a^4*(44*A + 49*B)*\text{Cos}[c + d*x]*\text{Sin}[c + d*x])/(16*d) + (a^4*(276*A + 301*B)*\text{Cos}[c + d*x]^3*\text{Sin}[c + d*x])/(280*d) + (a^4*A*\text{Cos}[c + d*x]^6*(a + a*\text{Sec}[c + d*x])^3*\text{Sin}[c + d*x])/(7*d) + ((10*A + 7*B)*\text{Cos}[c + d*x]^5*(a^2 + a^2*\text{Sec}[c + d*x])^2*\text{Sin}[c + d*x])/(42*d) + (7*(A + B)*\text{Cos}[c + d*x]^4*(a^4 + a^4*\text{Sec}[c + d*x])*\text{Sin}[c + d*x])/(15*d) - (a^4*(227*A + 252*B)*\text{Sin}[c + d*x]^3)/(105*d)$

Rule 8

$\text{Int}[a_, x_Symbol] \rightarrow \text{Simp}[a*x, x] /; \text{FreeQ}[a, x]$

Rule 2633

$\text{Int}[\sin[(c_.) + (d_.)*(x_)]^{(n_.)}, x_Symbol] \rightarrow -\text{Dist}[d^{(-1)}, \text{Subst}[\text{Int}[\text{Expand}[(1 - x^2)^{((n - 1)/2)}, x], x], x, \text{Cos}[c + d*x]], x] /; \text{FreeQ}\{c, d\}, x \&\& \text{IGtQ}[(n - 1)/2, 0]$

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]
)*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] := Simp[(A*a*Cot[e +
f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
+ 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] := Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \cos^7(c + dx)(a + a \sec(c + dx))^4(A + B \sec(c + dx)) dx &= \frac{aA \cos^6(c + dx)(a + a \sec(c + dx))^3 \sin(c + dx)}{7d} + \dots \\
&= \frac{aA \cos^6(c + dx)(a + a \sec(c + dx))^3 \sin(c + dx)}{7d} + \dots \\
&= \frac{aA \cos^6(c + dx)(a + a \sec(c + dx))^3 \sin(c + dx)}{7d} + \dots \\
&= \frac{a^4(276A + 301B) \cos^3(c + dx) \sin(c + dx)}{280d} + \frac{aA \cos^6(c + dx)}{7d} + \dots \\
&= \frac{a^4(276A + 301B) \cos^3(c + dx) \sin(c + dx)}{280d} + \frac{aA \cos^6(c + dx)}{7d} + \dots \\
&= \frac{a^4(44A + 49B) \cos(c + dx) \sin(c + dx)}{16d} + \frac{a^4(276A + 301B)}{35d} + \dots \\
&= \frac{1}{16} a^4(44A + 49B)x + \frac{a^4(227A + 252B) \sin(c + dx)}{35d} + \dots
\end{aligned}$$

Mathematica [A] time = 0.74, size = 156, normalized size = 0.65

$$\frac{a^4(105(323A + 352B) \sin(c + dx) + 105(124A + 127B) \sin(2(c + dx)) + 5495A \sin(3(c + dx)) + 2100A \sin(4(c + dx)) + 1575B \sin(4(c + dx)) + 5040B \sin(3(c + dx)) + 2100A \sin(4(c + dx)) + 1575B \sin(4(c + dx)) + 651A \sin(5(c + dx)) + 336B \sin(5(c + dx)) + 140A \sin(6(c + dx)) + 35B \sin(6(c + dx)) + 15A \sin(7(c + dx)))}{(6720*d)}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]^7*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]

[Out] (a^4*(18480*A*c + 18480*A*d*x + 20580*B*d*x + 105*(323*A + 352*B)*Sin[c + d*x] + 105*(124*A + 127*B)*Sin[2*(c + d*x)] + 5495*A*Ssin[3*(c + d*x)] + 5040*B*Ssin[3*(c + d*x)] + 2100*A*Ssin[4*(c + d*x)] + 1575*B*Ssin[4*(c + d*x)] + 651*A*Ssin[5*(c + d*x)] + 336*B*Ssin[5*(c + d*x)] + 140*A*Ssin[6*(c + d*x)] + 35*B*Ssin[6*(c + d*x)] + 15*A*Ssin[7*(c + d*x)]))/(6720*d)

fricas [A] time = 0.44, size = 150, normalized size = 0.62

$$\frac{105(44A + 49B)a^4 dx + (240Aa^4 \cos(dx + c)^6 + 280(4A + B)a^4 \cos(dx + c)^5 + 192(12A + 7B)a^4 \cos(dx + c)^4 + 105(323A + 352B)a^4 \sin(dx + c) + 105(124A + 127B)a^4 \sin(2(dx + c)) + 5495Aa^4 \sin(3(dx + c)) + 2100Aa^4 \sin(4(dx + c)) + 1575Ba^4 \sin(4(dx + c)) + 5040Ba^4 \sin(3(dx + c)) + 2100Aa^4 \sin(4(dx + c)) + 1575Ba^4 \sin(4(dx + c)) + 651Aa^4 \sin(5(dx + c)) + 336Ba^4 \sin(5(dx + c)) + 140Aa^4 \sin(6(dx + c)) + 35Ba^4 \sin(6(dx + c)) + 15Aa^4 \sin(7(dx + c)))}{6720d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^7*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] $\frac{1}{1680} \cdot (105 \cdot (44 \cdot A + 49 \cdot B) \cdot a^4 \cdot dx + (240 \cdot A \cdot a^4 \cdot \cos(dx + c)^6 + 280 \cdot (4 \cdot A + B) \cdot a^4 \cdot \cos(dx + c)^5 + 192 \cdot (12 \cdot A + 7 \cdot B) \cdot a^4 \cdot \cos(dx + c)^4 + 70 \cdot (44 \cdot A + 41 \cdot B) \cdot a^4 \cdot \cos(dx + c)^3 + 16 \cdot (227 \cdot A + 252 \cdot B) \cdot a^4 \cdot \cos(dx + c)^2 + 105 \cdot (44 \cdot A + 49 \cdot B) \cdot a^4 \cdot \cos(dx + c) + 32 \cdot (227 \cdot A + 252 \cdot B) \cdot a^4) \cdot \sin(dx + c)) / d$

giac [A] time = 0.42, size = 278, normalized size = 1.15

$$105 \left(44 A a^4 + 49 B a^4 \right) (dx + c) + \frac{2 \left(4620 A a^4 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^{13} + 5145 B a^4 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^{13} + 30800 A a^4 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^{11} + 34300 B a^4 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^{11} \right)}{\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(dx+c)^7*(a+a*sec(dx+c))^4*(A+B*sec(dx+c)),x, algorithm="giac")`

[Out] $\frac{1}{1680} \cdot (105 \cdot (44 \cdot A \cdot a^4 + 49 \cdot B \cdot a^4) \cdot (dx + c) + 2 \cdot (4620 \cdot A \cdot a^4 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c)^{13} + 5145 \cdot B \cdot a^4 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c)^{13} + 30800 \cdot A \cdot a^4 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c)^{11} + 34300 \cdot B \cdot a^4 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c)^{11} + 87164 \cdot A \cdot a^4 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c)^9 + 97069 \cdot B \cdot a^4 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c)^9 + 135168 \cdot A \cdot a^4 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c)^7 + 150528 \cdot B \cdot a^4 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c)^7 + 126084 \cdot A \cdot a^4 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c)^5 + 134099 \cdot B \cdot a^4 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c)^5 + 58800 \cdot A \cdot a^4 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c)^3 + 73220 \cdot B \cdot a^4 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c)^3 + 22260 \cdot A \cdot a^4 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c) + 21735 \cdot B \cdot a^4 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c)) / (\tan(1/2 \cdot dx + 1/2 \cdot c)^2 + 1)^7) / d$

maple [A] time = 2.15, size = 358, normalized size = 1.49

$$\frac{A a^4 \left(\frac{16}{5} + \cos^6(dx+c) + \frac{6 \cos^4(dx+c)}{5} + \frac{8 \cos^2(dx+c)}{5} \right) \sin(dx+c)}{7} + a^4 B \left(\frac{\left(\cos^5(dx+c) + \frac{5 \cos^3(dx+c)}{4} + \frac{15 \cos(dx+c)}{8} \right) \sin(dx+c)}{6} + \frac{5 dx}{16} + \frac{5 c}{16} \right) + 4 A a^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(dx+c)^7*(a+a*sec(dx+c))^4*(A+B*sec(dx+c)),x)`

[Out] $\frac{1}{d} \cdot (1/7 \cdot A \cdot a^4 \cdot (16/5 + \cos(dx+c)^6 + 6/5 \cdot \cos(dx+c)^4 + 8/5 \cdot \cos(dx+c)^2) \cdot \sin(dx+c) + a^4 \cdot B \cdot (1/6 \cdot (\cos(dx+c)^5 + 5/4 \cdot \cos(dx+c)^3 + 15/8 \cdot \cos(dx+c)) \cdot \sin(dx+c) + 5/16 \cdot dx + 5/16 \cdot c) + 4 \cdot A \cdot a^4 \cdot (1/6 \cdot (\cos(dx+c)^5 + 5/4 \cdot \cos(dx+c)^3 + 15/8 \cdot \cos(dx+c)) \cdot \sin(dx+c) + 5/16 \cdot dx + 5/16 \cdot c) + 4/5 \cdot a^4 \cdot B \cdot (8/3 + \cos(dx+c)^4 + 4/3 \cdot \cos(dx+c)^2) \cdot \sin(dx+c) + 6/5 \cdot A \cdot a^4 \cdot (8/3 + \cos(dx+c)^4 + 4/3 \cdot \cos(dx+c)^2) \cdot \sin(dx+c) + 6 \cdot a^4 \cdot B \cdot (1/4 \cdot (\cos(dx+c)^3 + 3/2 \cdot \cos(dx+c)) \cdot \sin(dx+c) + 3/8 \cdot dx + 3/8 \cdot c) + 4 \cdot A \cdot a^4 \cdot (1/4 \cdot (\cos(dx+c)^3 + 3/2 \cdot \cos(dx+c)) \cdot \sin(dx+c) + 3/8 \cdot dx + 3/8 \cdot c) + 4/3 \cdot a^4 \cdot B \cdot (2 + \cos(dx+c)^2) \cdot \sin(dx+c) + 2/3 \cdot A \cdot a^4 \cdot (2 + \cos(dx+c)^2) \cdot \sin(dx+c) + 2/3 \cdot a^4 \cdot B \cdot (2 + \cos(dx+c)^2) \cdot \sin(dx+c)) / d$

$(d*x+c)^2)*\sin(d*x+c)+1/3*A*a^4*(2+\cos(d*x+c)^2)*\sin(d*x+c)+a^4*B*(1/2*\cos(d*x+c)*\sin(d*x+c)+1/2*d*x+1/2*c))$

maxima [A] time = 0.35, size = 356, normalized size = 1.48

$$\frac{192(5 \sin(dx + c)^7 - 21 \sin(dx + c)^5 + 35 \sin(dx + c)^3 - 35 \sin(dx + c))Aa^4 - 2688(3 \sin(dx + c)^5 - 10 \sin(dx + c)^3 - 10 \sin(dx + c))Ba^4}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^7*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] $-1/6720*(192*(5*\sin(d*x + c)^7 - 21*\sin(d*x + c)^5 + 35*\sin(d*x + c)^3 - 35*\sin(d*x + c))*A*a^4 - 2688*(3*\sin(d*x + c)^5 - 10*\sin(d*x + c)^3 + 15*\sin(d*x + c))*A*a^4 + 140*(4*\sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*\sin(4*d*x + 4*c) - 48*\sin(2*d*x + 2*c))*A*a^4 + 2240*(\sin(d*x + c)^3 - 3*\sin(d*x + c))*A*a^4 - 840*(12*d*x + 12*c + \sin(4*d*x + 4*c) + 8*\sin(2*d*x + 2*c))*A*a^4 - 1792*(3*\sin(d*x + c)^5 - 10*\sin(d*x + c)^3 + 15*\sin(d*x + c))*B*a^4 + 35*(4*\sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*\sin(4*d*x + 4*c) - 48*\sin(2*d*x + 2*c))*B*a^4 + 8960*(\sin(d*x + c)^3 - 3*\sin(d*x + c))*B*a^4 - 1260*(12*d*x + 12*c + \sin(4*d*x + 4*c) + 8*\sin(2*d*x + 2*c))*B*a^4 - 1680*(2*d*x + 2*c + \sin(2*d*x + 2*c))*B*a^4)/d$

mupad [B] time = 4.13, size = 323, normalized size = 1.34

$$\frac{\left(\frac{11Aa^4}{2} + \frac{49Ba^4}{8}\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^{13} + \left(\frac{110Aa^4}{3} + \frac{245Ba^4}{6}\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^{11} + \left(\frac{3113Aa^4}{30} + \frac{13867Ba^4}{120}\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^9 + \left(\frac{1501Aa^4}{10} + \frac{19157Ba^4}{120}\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^7 + \left(\frac{5632Aa^4}{35} + \frac{896Ba^4}{5}\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5 + \left(\frac{110Aa^4}{3} + \frac{245Ba^4}{6}\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 + \left(\frac{11Aa^4}{2} + \frac{49Ba^4}{8}\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right) + 1}{d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^{14} + 7 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^{12} + 21 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^{10} + 35 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^8 + 21 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^6 + 7 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 + \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + 1 \right) + (a^4 \operatorname{atan}\left(\frac{a^4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right) + 1}{(8*((11Aa^4)/2 + (49Ba^4)/8))}\right) * (44A + 49B)) / (8*d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^7*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^4,x)

[Out] $(\tan(c/2 + (d*x)/2)*((53*A*a^4)/2 + (207*B*a^4)/8) + \tan(c/2 + (d*x)/2)^{13}*((11*A*a^4)/2 + (49*B*a^4)/8) + \tan(c/2 + (d*x)/2)^{11}*((110*A*a^4)/3 + (245*B*a^4)/6) + \tan(c/2 + (d*x)/2)^9*((3113*A*a^4)/30 + (13867*B*a^4)/120) + \tan(c/2 + (d*x)/2)^7*((1501*A*a^4)/10 + (19157*B*a^4)/120) + \tan(c/2 + (d*x)/2)^5*((5632*A*a^4)/35 + (896*B*a^4)/5) + \tan(c/2 + (d*x)/2)^3*((110*A*a^4)/3 + (245*B*a^4)/6) + \tan(c/2 + (d*x)/2) * ((11*A*a^4)/2 + (49*B*a^4)/8) + 1) + (a^4 \operatorname{atan}\left(\frac{a^4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right) + 1}{(8*((11Aa^4)/2 + (49Ba^4)/8))}\right) * (44A + 49B)) / (8*d)$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**7*(a+a*sec(d*x+c))**4*(A+B*sec(d*x+c)),x)

[Out] Timed out

$$3.82 \quad \int \frac{\sec^4(c+dx)(A+B \sec(c+dx))}{a+a \sec(c+dx)} dx$$

Optimal. Leaf size=131

$$\frac{(3A-4B) \tan^3(c+dx)}{3ad} - \frac{(3A-4B) \tan(c+dx)}{ad} + \frac{3(A-B) \tanh^{-1}(\sin(c+dx))}{2ad} + \frac{(A-B) \tan(c+dx) \sec^3(c+dx)}{d(a \sec(c+dx) + a)}$$

[Out] 3/2*(A-B)*arctanh(sin(d*x+c))/a/d-(3*A-4*B)*tan(d*x+c)/a/d+3/2*(A-B)*sec(d*x+c)*tan(d*x+c)/a/d+(A-B)*sec(d*x+c)^3*tan(d*x+c)/d/(a+a*sec(d*x+c))-1/3*(3*A-4*B)*tan(d*x+c)^3/a/d

Rubi [A] time = 0.17, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {4019, 3787, 3768, 3770, 3767}

$$\frac{(3A-4B) \tan^3(c+dx)}{3ad} - \frac{(3A-4B) \tan(c+dx)}{ad} + \frac{3(A-B) \tanh^{-1}(\sin(c+dx))}{2ad} + \frac{(A-B) \tan(c+dx) \sec^3(c+dx)}{d(a \sec(c+dx) + a)}$$

Antiderivative was successfully verified.

[In] Int[(Sec[c + d*x]^4*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x]),x]

[Out] (3*(A - B)*ArcTanh[Sin[c + d*x]])/(2*a*d) - ((3*A - 4*B)*Tan[c + d*x])/(a*d) + (3*(A - B)*Sec[c + d*x]*Tan[c + d*x])/(2*a*d) + ((A - B)*Sec[c + d*x]^3*Tan[c + d*x])/(d*(a + a*Sec[c + d*x])) - ((3*A - 4*B)*Tan[c + d*x]^3)/(3*a*d)

Rule 3767

Int[csc[(c_) + (d_)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 3768

Int[(csc[(c_) + (d_)*(x_)]*(b_.))^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3770

Int[csc[(c_) + (d_)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 4019

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> Simp[(d*(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{\sec^4(c + dx)(A + B \sec(c + dx))}{a + a \sec(c + dx)} dx &= \frac{(A - B) \sec^3(c + dx) \tan(c + dx)}{d(a + a \sec(c + dx))} + \frac{\int \sec^3(c + dx)(3a(A - B) - a(3A - B)) dx}{a^2} \\
 &= \frac{(A - B) \sec^3(c + dx) \tan(c + dx)}{d(a + a \sec(c + dx))} - \frac{(3A - 4B) \int \sec^4(c + dx) dx}{a} + \frac{3(A - B) \int \sec^2(c + dx) dx}{a} \\
 &= \frac{3(A - B) \sec(c + dx) \tan(c + dx)}{2ad} + \frac{(A - B) \sec^3(c + dx) \tan(c + dx)}{d(a + a \sec(c + dx))} + \frac{3(A - B) \tan(c + dx)}{a} \\
 &= \frac{3(A - B) \tanh^{-1}(\sin(c + dx))}{2ad} - \frac{(3A - 4B) \tan(c + dx)}{ad} + \frac{3(A - B) \sec(c + dx)}{a}
 \end{aligned}$$

Mathematica [B] time = 6.30, size = 635, normalized size = 4.85

$$\frac{\sec\left(\frac{c}{2}\right) \sec(c) \cos\left(\frac{c}{2} + \frac{dx}{2}\right) \sec^3(c + dx) \left(12A \sin\left(c - \frac{dx}{2}\right) + 6A \sin\left(c + \frac{dx}{2}\right) + 24A \sin\left(2c + \frac{dx}{2}\right) - 9A \sin\left(c + \frac{3a}{2}\right)\right)}{a^2}$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[c + d*x]^4*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x]),x]

[Out] (3*(-A + B)*Cos[c/2 + (d*x)/2]^2*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])*(A + B*Sec[c + d*x])/(d*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])) - (3*(-A + B)*Cos[c/2 + (d*x)/2]^2*Log[Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])

$$\frac{(A + B \sec[c + dx])}{(d(B + A \cos[c + dx])(a + a \sec[c + dx]))} + (\cos[c/2 + (dx)/2] \sec[c/2] \sec[c] \sec[c + dx]^3 (A + B \sec[c + dx]) (6A \sin[(dx)/2] + 6B \sin[(dx)/2] - 27A \sin[(3dx)/2] + 39B \sin[(3dx)/2] + 12A \sin[c - (dx)/2] - 24B \sin[c - (dx)/2] + 6A \sin[c + (dx)/2] - 6B \sin[c + (dx)/2] + 24A \sin[2c + (dx)/2] - 24B \sin[2c + (dx)/2] - 9A \sin[c + (3dx)/2] + 21B \sin[c + (3dx)/2] - 9A \sin[2c + (3dx)/2] + 9B \sin[2c + (3dx)/2] + 9A \sin[3c + (3dx)/2] - 9B \sin[3c + (3dx)/2] - 3A \sin[c + (5dx)/2] + 7B \sin[c + (5dx)/2] + 3A \sin[2c + (5dx)/2] + B \sin[2c + (5dx)/2] + 3A \sin[3c + (5dx)/2] - 3B \sin[3c + (5dx)/2] + 9A \sin[4c + (5dx)/2] - 9B \sin[4c + (5dx)/2] - 12A \sin[2c + (7dx)/2] + 16B \sin[2c + (7dx)/2] - 6A \sin[3c + (7dx)/2] + 10B \sin[3c + (7dx)/2] - 6A \sin[4c + (7dx)/2] + 6B \sin[4c + (7dx)/2])}{(48d(B + A \cos[c + dx])(a + a \sec[c + dx]))}$$

fricas [A] time = 0.45, size = 170, normalized size = 1.30

$$\frac{9((A - B) \cos(dx + c)^4 + (A - B) \cos(dx + c)^3) \log(\sin(dx + c) + 1) - 9((A - B) \cos(dx + c)^4 + (A - B) \cos(dx + c)^3) \log(-\sin(dx + c) + 1) - 2(4(3A - 4B) \cos(dx + c)^3 + (3A - 7B) \cos(dx + c)^2 - (3A - B) \cos(dx + c) - 2B) \sin(dx + c)}{12(ad \cos(dx + c)^4 + a^2 \cos(dx + c)^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(dx+c)^4*(A+B*sec(dx+c))/(a+a*sec(dx+c)),x, algorithm="fricas")

[Out] 1/12*(9*((A - B)*cos(dx + c)^4 + (A - B)*cos(dx + c)^3)*log(sin(dx + c) + 1) - 9*((A - B)*cos(dx + c)^4 + (A - B)*cos(dx + c)^3)*log(-sin(dx + c) + 1) - 2*(4*(3*A - 4*B)*cos(dx + c)^3 + (3*A - 7*B)*cos(dx + c)^2 - (3*A - B)*cos(dx + c) - 2*B)*sin(dx + c))/(a*d*cos(dx + c)^4 + a*d*cos(dx + c)^3)

giac [A] time = 0.80, size = 182, normalized size = 1.39

$$\frac{9(A-B) \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 1\right|\right)}{a} - \frac{9(A-B) \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 1\right|\right)}{a} - \frac{6\left(A \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - B \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right)}{a} + \frac{2\left(9A \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 - 15B \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 - 12A \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^4 + 12B \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^4\right)}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(dx+c)^4*(A+B*sec(dx+c))/(a+a*sec(dx+c)),x, algorithm="giac")

[Out] 1/6*(9*(A - B)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - 9*(A - B)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a - 6*(A*tan(1/2*d*x + 1/2*c) - B*tan(1/2*d*x + 1/2*c))/a + 2*(9*A*tan(1/2*d*x + 1/2*c)^5 - 15*B*tan(1/2*d*x + 1/2*c)^5 - 12*A*tan(1/2*d*x + 1/2*c)^4 + 12*B*tan(1/2*d*x + 1/2*c)^4))/6d

$\tan(1/2dx + 1/2c)^3 + 16B\tan(1/2dx + 1/2c)^3 + 3A\tan(1/2dx + 1/2c) - 9B\tan(1/2dx + 1/2c) / ((\tan(1/2dx + 1/2c)^2 - 1)^3 a) / d$

maple [B] time = 0.57, size = 340, normalized size = 2.60

$$\frac{A \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{ad} + \frac{B \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{ad} - \frac{B}{3ad \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)^3} - \frac{B}{ad \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)^2} + \frac{A}{2ad \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)^2} + \frac{3}{ad \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x)`

[Out] $-1/a/d*A*\tan(1/2*d*x+1/2*c)+1/a/d*B*\tan(1/2*d*x+1/2*c)-1/3/a/d*B/(\tan(1/2*d*x+1/2*c)-1)^3-1/a/d/(\tan(1/2*d*x+1/2*c)-1)^2*B+1/2/a/d*A/(\tan(1/2*d*x+1/2*c)-1)^2+3/2/a/d*\ln(\tan(1/2*d*x+1/2*c)-1)*B-3/2/a/d*A*\ln(\tan(1/2*d*x+1/2*c)-1)-5/2/a/d/(\tan(1/2*d*x+1/2*c)-1)*B+3/2/a/d*A/(\tan(1/2*d*x+1/2*c)-1)-1/3/a/d*B/(\tan(1/2*d*x+1/2*c)+1)^3-1/2/a/d*A/(\tan(1/2*d*x+1/2*c)+1)^2+1/a/d/(\tan(1/2*d*x+1/2*c)+1)^2*B-5/2/a/d/(\tan(1/2*d*x+1/2*c)+1)*B+3/2/a/d*A/(\tan(1/2*d*x+1/2*c)+1)-3/2/a/d*\ln(\tan(1/2*d*x+1/2*c)+1)*B+3/2/a/d*A*\ln(\tan(1/2*d*x+1/2*c)+1)$

maxima [B] time = 0.34, size = 368, normalized size = 2.81

$$\frac{B \left(\frac{2 \left(\frac{9 \sin(dx+c)}{\cos(dx+c)+1} - \frac{16 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{15 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} \right)}{a - \frac{3a \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{3a \sin(dx+c)^4}{(\cos(dx+c)+1)^4} - \frac{a \sin(dx+c)^6}{(\cos(dx+c)+1)^6}} - \frac{9 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1} + 1\right)}{a} + \frac{9 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1} - 1\right)}{a} + \frac{6 \sin(dx+c)}{a(\cos(dx+c)+1)} \right) - 3A \left(\frac{2 \left(\frac{\sin(dx+c)}{\cos(dx+c)+1} - \frac{2a \sin(dx+c)^2}{(\cos(dx+c)+1)^2} \right)}{a - \frac{2a \sin(dx+c)^2}{(\cos(dx+c)+1)^2}} \right)}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="maxima")`

[Out] $1/6*(B*(2*(9*\sin(d*x + c)/(\cos(d*x + c) + 1) - 16*\sin(d*x + c)^3/(\cos(d*x + c) + 1)^3 + 15*\sin(d*x + c)^5/(\cos(d*x + c) + 1)^5)/(a - 3*a*\sin(d*x + c)^2/(\cos(d*x + c) + 1)^2 + 3*a*\sin(d*x + c)^4/(\cos(d*x + c) + 1)^4 - a*\sin(d*x + c)^6/(\cos(d*x + c) + 1)^6) - 9*\log(\sin(d*x + c)/(\cos(d*x + c) + 1) + 1)/a + 9*\log(\sin(d*x + c)/(\cos(d*x + c) + 1) - 1)/a + 6*\sin(d*x + c)/(a*(\cos(d*x + c) + 1))) - 3*A*(2*(\sin(d*x + c)/(\cos(d*x + c) + 1) - 3*\sin(d*x + c)^3/(\cos(d*x + c) + 1)^3)/(a - 2*a*\sin(d*x + c)^2/(\cos(d*x + c) + 1)^2 + a*\sin(d*x + c)^4/(\cos(d*x + c) + 1)^4) - 3*\log(\sin(d*x + c)/(\cos(d*x + c) + 1) + 1)/a + 3*\log(\sin(d*x + c)/(\cos(d*x + c) + 1) - 1)/a + 2*\sin(d*x + c)/(a*(\cos(d*x + c) + 1))))/d$

mupad [B] time = 2.44, size = 152, normalized size = 1.16

$$\frac{3 \operatorname{atanh}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right) (A - B) (3A - 5B) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5 + \left(\frac{16B}{3} - 4A\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 + (A - 3B) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{a d} - \frac{d \left(-a \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^6 + 3a \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 - 3a \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + a \right)}{a d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B/cos(c + d*x))/(cos(c + d*x)^4*(a + a/cos(c + d*x))),x)`

[Out] `(3*atanh(tan(c/2 + (d*x)/2))*(A - B))/(a*d) - (tan(c/2 + (d*x)/2)^5*(3*A - 5*B) - tan(c/2 + (d*x)/2)^3*(4*A - (16*B)/3) + tan(c/2 + (d*x)/2)*(A - 3*B))/(d*(a - 3*a*tan(c/2 + (d*x)/2)^2 + 3*a*tan(c/2 + (d*x)/2)^4 - a*tan(c/2 + (d*x)/2)^6) - (tan(c/2 + (d*x)/2)*(A - B))/(a*d)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A \sec^4(c+dx)}{\sec(c+dx)+1} dx + \int \frac{B \sec^5(c+dx)}{\sec(c+dx)+1} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)**4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x)`

[Out] `(Integral(A*sec(c + d*x)**4/(sec(c + d*x) + 1), x) + Integral(B*sec(c + d*x)**5/(sec(c + d*x) + 1), x))/a`

$$3.83 \quad \int \frac{\sec^3(c+dx)(A+B \sec(c+dx))}{a+a \sec(c+dx)} dx$$

Optimal. Leaf size=108

$$\frac{2(A-B) \tan(c+dx)}{ad} - \frac{(2A-3B) \tanh^{-1}(\sin(c+dx))}{2ad} + \frac{(A-B) \tan(c+dx) \sec^2(c+dx)}{d(a \sec(c+dx) + a)} - \frac{(2A-3B) \tan(c+dx)}{2ad}$$

[Out] $-1/2*(2*A-3*B)*\operatorname{arctanh}(\sin(d*x+c))/a/d+2*(A-B)*\tan(d*x+c)/a/d-1/2*(2*A-3*B)*\sec(d*x+c)*\tan(d*x+c)/a/d+(A-B)*\sec(d*x+c)^2*\tan(d*x+c)/d/(a+a*\sec(d*x+c))$

Rubi [A] time = 0.16, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4019, 3787, 3767, 8, 3768, 3770}

$$\frac{2(A-B) \tan(c+dx)}{ad} - \frac{(2A-3B) \tanh^{-1}(\sin(c+dx))}{2ad} + \frac{(A-B) \tan(c+dx) \sec^2(c+dx)}{d(a \sec(c+dx) + a)} - \frac{(2A-3B) \tan(c+dx)}{2ad}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(\operatorname{Sec}[c+d*x]^3*(A+B*\operatorname{Sec}[c+d*x]))/(a+a*\operatorname{Sec}[c+d*x]),x]$

[Out] $-((2*A-3*B)*\operatorname{ArcTanh}[\operatorname{Sin}[c+d*x]])/(2*a*d) + (2*(A-B)*\operatorname{Tan}[c+d*x])/(a*d) - ((2*A-3*B)*\operatorname{Sec}[c+d*x]*\operatorname{Tan}[c+d*x])/(2*a*d) + ((A-B)*\operatorname{Sec}[c+d*x]^2*\operatorname{Tan}[c+d*x])/(d*(a+a*\operatorname{Sec}[c+d*x]))$

Rule 8

$\operatorname{Int}[a_, x_Symbol] \rightarrow \operatorname{Simp}[a*x, x] /; \operatorname{FreeQ}[a, x]$

Rule 3767

$\operatorname{Int}[\operatorname{csc}[(c_.) + (d_.)*(x_)]^{(n_)}, x_Symbol] \rightarrow -\operatorname{Dist}[d^{(-1)}, \operatorname{Subst}[\operatorname{Int}[\operatorname{ExpandIntegrand}[(1+x^2)^{(n/2-1)}, x], x], x, \operatorname{Cot}[c+d*x]], x] /; \operatorname{FreeQ}\{c, d\}, x \ \&\& \operatorname{IGtQ}[n/2, 0]$

Rule 3768

$\operatorname{Int}[(\operatorname{csc}[(c_.) + (d_.)*(x_)]*(b_.)^{(n_)}), x_Symbol] \rightarrow -\operatorname{Simp}[(b*\operatorname{Cos}[c+d*x]*(b*\operatorname{Csc}[c+d*x])^{(n-1)})/(d*(n-1)), x] + \operatorname{Dist}[(b^2*(n-2))/(n-1), \operatorname{Int}[(b*\operatorname{Csc}[c+d*x])^{(n-2)}, x], x] /; \operatorname{FreeQ}\{b, c, d\}, x \ \&\& \operatorname{GtQ}[n, 1] \ \&\& \operatorname{IntegerQ}[2*n]$

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x]
  /; FreeQ[{c, d}, x]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_.)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] := Simp[(d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rubi steps

$$\begin{aligned}
 \int \frac{\sec^3(c + dx)(A + B \sec(c + dx))}{a + a \sec(c + dx)} dx &= \frac{(A - B) \sec^2(c + dx) \tan(c + dx)}{d(a + a \sec(c + dx))} + \frac{\int \sec^2(c + dx)(2a(A - B) - a(2A - B) \sec(c + dx))}{a^2} \\
 &= \frac{(A - B) \sec^2(c + dx) \tan(c + dx)}{d(a + a \sec(c + dx))} - \frac{(2A - 3B) \int \sec^3(c + dx) dx}{a} + \frac{(2A - 3B) \int \sec^2(c + dx) dx}{a} \\
 &= -\frac{(2A - 3B) \sec(c + dx) \tan(c + dx)}{2ad} + \frac{(A - B) \sec^2(c + dx) \tan(c + dx)}{d(a + a \sec(c + dx))} \\
 &= -\frac{(2A - 3B) \tanh^{-1}(\sin(c + dx))}{2ad} + \frac{2(A - B) \tan(c + dx)}{ad} - \frac{(2A - 3B)}{ad}
 \end{aligned}$$

Mathematica [B] time = 3.88, size = 311, normalized size = 2.88

$$\cos\left(\frac{1}{2}(c + dx)\right) (A + B \sec(c + dx)) \left(4(A - B) \sec\left(\frac{c}{2}\right) \sin\left(\frac{dx}{2}\right) + \cos\left(\frac{1}{2}(c + dx)\right) \right) \left(\frac{1}{(\cos(\frac{c}{2}) - \sin(\frac{c}{2}))(\sin(\frac{c}{2}) + \cos(\frac{c}{2}))(\cos(\frac{c}{2}) + \sin(\frac{c}{2}))} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x]),x]

[Out] (Cos[(c + d*x)/2]*(A + B*Sec[c + d*x])*(4*(A - B)*Sec[c/2]*Sin[(d*x)/2] + Cos[(c + d*x)/2]*((4*A - 6*B)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 4*A*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 6*B*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + B/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 - B/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + (4*(A - B)*Sin[d*x])/((Cos[c/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])))))/(2*a*d*(B + A*Cos[c + d*x])*(1 + Sec[c + d*x]))

fricas [A] time = 0.48, size = 156, normalized size = 1.44

$$\frac{\left((2A - 3B) \cos(dx + c)^3 + (2A - 3B) \cos(dx + c)^2\right) \log(\sin(dx + c) + 1) - \left((2A - 3B) \cos(dx + c)^3 + (2A - 3B) \cos(dx + c)^2\right) \log(-\sin(dx + c) + 1) - 2*(4*(A - B)*\cos(dx + c)^2 + (2*A - B)*\cos(dx + c) + B)*\sin(dx + c)}{4(ad \cos(dx + c))^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="fricas")

[Out] -1/4*((2*A - 3*B)*cos(d*x + c)^3 + (2*A - 3*B)*cos(d*x + c)^2)*log(sin(d*x + c) + 1) - ((2*A - 3*B)*cos(d*x + c)^3 + (2*A - 3*B)*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) - 2*(4*(A - B)*cos(d*x + c)^2 + (2*A - B)*cos(d*x + c) + B)*sin(d*x + c)/(a*d*cos(d*x + c)^3 + a*d*cos(d*x + c)^2)

giac [A] time = 0.30, size = 156, normalized size = 1.44

$$\frac{(2A-3B) \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 1\right|\right)}{a} - \frac{(2A-3B) \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 1\right|\right)}{a} - \frac{2\left(A \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - B \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right)}{a} + \frac{2\left(2A \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right)^3 - 3B \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{a}$$

2d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="giac")

[Out] -1/2*((2*A - 3*B)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - (2*A - 3*B)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a - 2*(A*tan(1/2*d*x + 1/2*c) - B*tan(1/2*d*x + 1/2*c))/a + 2*(2*A*tan(1/2*d*x + 1/2*c)^3 - 3*B*tan(1/2*d*x + 1/2*c)^3 - 2*A*tan(1/2*d*x + 1/2*c) + B*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a))/d

maple [B] time = 0.58, size = 252, normalized size = 2.33

$$\frac{A \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{ad} - \frac{B \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{ad} + \frac{B}{2ad \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)^2} + \frac{3B}{2ad \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)} - \frac{A}{ad \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)} - \frac{3 \ln}{ad \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x)

[Out] 1/a/d*A*tan(1/2*d*x+1/2*c)-1/a/d*B*tan(1/2*d*x+1/2*c)+1/2/a/d/(tan(1/2*d*x+1/2*c)-1)^2*B+3/2/a/d/(tan(1/2*d*x+1/2*c)-1)*B-1/a/d*A/(tan(1/2*d*x+1/2*c)-1)-3/2/a/d*ln(tan(1/2*d*x+1/2*c)-1)*B+1/a/d*A*ln(tan(1/2*d*x+1/2*c)-1)-1/2/a/d/(tan(1/2*d*x+1/2*c)+1)^2*B+3/2/a/d*ln(tan(1/2*d*x+1/2*c)+1)*B-1/a/d*A*ln(tan(1/2*d*x+1/2*c)+1)+3/2/a/d/(tan(1/2*d*x+1/2*c)+1)*B-1/a/d*A/(tan(1/2*d*x+1/2*c)+1)

maxima [B] time = 0.34, size = 282, normalized size = 2.61

$$\frac{B \left(2 \left(\frac{\sin(dx+c)}{\cos(dx+c)+1} - \frac{3 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} \right) - \frac{3 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1} + 1\right)}{a} + \frac{3 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1} - 1\right)}{a} + \frac{2 \sin(dx+c)}{a(\cos(dx+c)+1)} \right) + 2A \left(\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1} + 1\right)}{a} - \frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1} - 1\right)}{a} \right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="maxima")

[Out] -1/2*(B*(2*(sin(d*x + c)/(cos(d*x + c) + 1) - 3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/(a - 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) - 3*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a + 3*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a + 2*sin(d*x + c)/(a*(cos(d*x + c) + 1))) + 2*A*(log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a - log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a - 2*sin(d*x + c)/((a - a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)) - sin(d*x + c)/(a*(cos(d*x + c) + 1))))/d

mupad [B] time = 2.11, size = 119, normalized size = 1.10

$$\frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right) (A - B)}{ad} - \frac{2 \operatorname{atanh}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right) \left(A - \frac{3B}{2}\right)}{ad} - \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 (2A - 3B) - \tan\left(\frac{c}{2} + \frac{dx}{2}\right) (2A - B)}{d \left(a \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 - 2a \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + a \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B/cos(c + d*x))/(cos(c + d*x)^3*(a + a/cos(c + d*x))),x)`

[Out] $(\tan(c/2 + (d*x)/2)*(A - B))/(a*d) - (2*\operatorname{atanh}(\tan(c/2 + (d*x)/2))*(A - (3*B)/2))/(a*d) - (\tan(c/2 + (d*x)/2)^3*(2*A - 3*B) - \tan(c/2 + (d*x)/2)*(2*A - B))/(d*(a - 2*a*\tan(c/2 + (d*x)/2)^2 + a*\tan(c/2 + (d*x)/2)^4)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A \sec^3(c+dx)}{\sec(c+dx)+1} dx + \int \frac{B \sec^4(c+dx)}{\sec(c+dx)+1} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)**3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x)`

[Out] $(\operatorname{Integral}(A*\sec(c + d*x)**3/(\sec(c + d*x) + 1), x) + \operatorname{Integral}(B*\sec(c + d*x)**4/(\sec(c + d*x) + 1), x))/a$

$$3.84 \quad \int \frac{\sec^2(c+dx)(A+B \sec(c+dx))}{a+a \sec(c+dx)} dx$$

Optimal. Leaf size=62

$$\frac{(A-B) \tanh^{-1}(\sin(c+dx))}{ad} - \frac{(A-B) \tan(c+dx)}{d(a \sec(c+dx)+a)} + \frac{B \tan(c+dx)}{ad}$$

[Out] (A-B)*arctanh(sin(d*x+c))/a/d+B*tan(d*x+c)/a/d-(A-B)*tan(d*x+c)/d/(a+a*sec(d*x+c))

Rubi [A] time = 0.12, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {4008, 3787, 3770, 3767, 8}

$$\frac{(A-B) \tanh^{-1}(\sin(c+dx))}{ad} - \frac{(A-B) \tan(c+dx)}{d(a \sec(c+dx)+a)} + \frac{B \tan(c+dx)}{ad}$$

Antiderivative was successfully verified.

[In] Int[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x]),x]

[Out] ((A - B)*ArcTanh[Sin[c + d*x]]/(a*d) + (B*Tan[c + d*x])/(a*d) - ((A - B)*Tan[c + d*x])/(d*(a + a*Sec[c + d*x])))

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 3767

Int[csc[(c_.) + (d_.)*(x_.)]^(n_.), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 3770

Int[csc[(c_.) + (d_.)*(x_.)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 4008

```
Int[csc[(e_.) + (f_.)*(x_.)]^2*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_)*(
csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := -Simp[((A*b - a*B)*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(b^2*(2*m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*B*
(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A
*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^2(c + dx)(A + B \sec(c + dx))}{a + a \sec(c + dx)} dx &= -\frac{(A - B) \tan(c + dx)}{d(a + a \sec(c + dx))} - \frac{\int \sec(c + dx)(-a(A - B) - aB \sec(c + dx)) dx}{a^2} \\
&= -\frac{(A - B) \tan(c + dx)}{d(a + a \sec(c + dx))} + \frac{(A - B) \int \sec(c + dx) dx}{a} + \frac{B \int \sec^2(c + dx) dx}{a} \\
&= \frac{(A - B) \tanh^{-1}(\sin(c + dx))}{ad} - \frac{(A - B) \tan(c + dx)}{d(a + a \sec(c + dx))} - \frac{B \operatorname{Subst}(\int 1 dx, x)}{ad} \\
&= \frac{(A - B) \tanh^{-1}(\sin(c + dx))}{ad} + \frac{B \tan(c + dx)}{ad} - \frac{(A - B) \tan(c + dx)}{d(a + a \sec(c + dx))}
\end{aligned}$$

Mathematica [B] time = 1.41, size = 224, normalized size = 3.61

$$\frac{2 \cos\left(\frac{1}{2}(c + dx)\right) (A + B \sec(c + dx)) \left((B - A) \sec\left(\frac{c}{2}\right) \sin\left(\frac{dx}{2}\right) + \cos\left(\frac{1}{2}(c + dx)\right) \right)}{\left(\cos\left(\frac{c}{2}\right) - \sin\left(\frac{c}{2}\right) \right) \left(\sin\left(\frac{c}{2}\right) + \cos\left(\frac{c}{2}\right) \right) \left(\cos\left(\frac{c}{2}\right) + \sin\left(\frac{c}{2}\right) \right)} ad(\sec(c + dx))$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x]),x]
```

```
[Out] (2*Cos[(c + d*x)/2]*(A + B*Sec[c + d*x])*((-A + B)*Sec[c/2]*Sin[(d*x)/2] +
Cos[(c + d*x)/2]*(-((A - B)*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log
[Cos[(c + d*x)/2] + Sin[(c + d*x)/2])) + (B*Sin[d*x])/((Cos[c/2] - Sin[c/2
])*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d
x)/2] + Sin[(c + d*x)/2]))))/(a*d*(B + A*Cos[c + d*x])*(1 + Sec[c + d*x]))
```

fricas [B] time = 0.43, size = 127, normalized size = 2.05

$$\frac{\left((A - B) \cos(dx + c)^2 + (A - B) \cos(dx + c) \right) \log(\sin(dx + c) + 1) - \left((A - B) \cos(dx + c)^2 + (A - B) \cos(dx + c) \right)}{2 \left(ad \cos(dx + c)^2 + ad \cos(dx + c) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="fricas")

[Out] $\frac{1}{2} * ((A - B) * \cos(d*x + c)^2 + (A - B) * \cos(d*x + c)) * \log(\sin(d*x + c) + 1) - ((A - B) * \cos(d*x + c)^2 + (A - B) * \cos(d*x + c)) * \log(-\sin(d*x + c) + 1) - 2 * ((A - 2*B) * \cos(d*x + c) - B) * \sin(d*x + c) / (a*d*\cos(d*x + c)^2 + a*d*\cos(d*x + c))$

giac [A] time = 0.79, size = 109, normalized size = 1.76

$$\frac{\frac{(A-B)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a} - \frac{(A-B)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a} - \frac{A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a} - \frac{2B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)^2-1}}{d}a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="giac")

[Out] $((A - B) * \log(\text{abs}(\tan(1/2*d*x + 1/2*c) + 1)) / a - (A - B) * \log(\text{abs}(\tan(1/2*d*x + 1/2*c) - 1)) / a - (A * \tan(1/2*d*x + 1/2*c) - B * \tan(1/2*d*x + 1/2*c)) / a - 2 * B * \tan(1/2*d*x + 1/2*c) / ((\tan(1/2*d*x + 1/2*c)^2 - 1) * a)) / d$

maple [B] time = 0.56, size = 163, normalized size = 2.63

$$\frac{A \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{ad} + \frac{B \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{ad} - \frac{B}{ad \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)} - \frac{A \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}{ad} + \frac{\ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right) B}{ad} - \frac{B}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x)

[Out] $-1/a/d*A*\tan(1/2*d*x+1/2*c)+1/a/d*B*\tan(1/2*d*x+1/2*c)-1/a/d/(\tan(1/2*d*x+1/2*c)-1)*B-1/a/d*A*\ln(\tan(1/2*d*x+1/2*c)-1)+1/a/d*\ln(\tan(1/2*d*x+1/2*c)-1)*B-1/a/d/(\tan(1/2*d*x+1/2*c)+1)*B+1/a/d*A*\ln(\tan(1/2*d*x+1/2*c)+1)-1/a/d*\ln(\tan(1/2*d*x+1/2*c)+1)*B$

maxima [B] time = 0.35, size = 196, normalized size = 3.16

$$\frac{B \left(\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a} - \frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a} - \frac{2 \sin(dx+c)}{\left(a - \frac{a \sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)} - \frac{\sin(dx+c)}{a(\cos(dx+c)+1)} \right) - A \left(\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a} - \frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a} \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="maxima")

[Out] $-(B*(\log(\sin(d*x + c)/(\cos(d*x + c) + 1) + 1)/a - \log(\sin(d*x + c)/(\cos(d*x + c) + 1) - 1)/a - 2*\sin(d*x + c)/((a - a*\sin(d*x + c)^2/(\cos(d*x + c) + 1)^2)*(\cos(d*x + c) + 1)) - \sin(d*x + c)/(a*(\cos(d*x + c) + 1))) - A*(\log(\sin(d*x + c)/(\cos(d*x + c) + 1) + 1)/a - \log(\sin(d*x + c)/(\cos(d*x + c) + 1) - 1)/a - \sin(d*x + c)/(a*(\cos(d*x + c) + 1))))/d$

mupad [B] time = 2.03, size = 79, normalized size = 1.27

$$\frac{2B \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{d \left(a - a \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2\right)} + \frac{2 \operatorname{atanh}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right) (A - B)}{a d} - \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right) (A - B)}{a d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^2*(a + a/cos(c + d*x))),x)

[Out] $(2*B*\tan(c/2 + (d*x)/2))/(d*(a - a*\tan(c/2 + (d*x)/2)^2)) + (2*\operatorname{atanh}(\tan(c/2 + (d*x)/2))*(A - B))/(a*d) - (\tan(c/2 + (d*x)/2)*(A - B))/(a*d)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A \sec^2(c+dx)}{\sec(c+dx)+1} dx + \int \frac{B \sec^3(c+dx)}{\sec(c+dx)+1} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x)

[Out] $(\operatorname{Integral}(A*\sec(c + d*x)**2/(\sec(c + d*x) + 1), x) + \operatorname{Integral}(B*\sec(c + d*x)**3/(\sec(c + d*x) + 1), x))/a$

$$3.85 \quad \int \frac{\sec(c+dx)(A+B \sec(c+dx))}{a+a \sec(c+dx)} dx$$

Optimal. Leaf size=43

$$\frac{(A-B) \tan(c+dx)}{d(a \sec(c+dx)+a)} + \frac{B \tanh^{-1}(\sin(c+dx))}{ad}$$

[Out] B*arctanh(sin(d*x+c))/a/d+(A-B)*tan(d*x+c)/d/(a+a*sec(d*x+c))

Rubi [A] time = 0.08, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {3998, 3770, 3794}

$$\frac{(A-B) \tan(c+dx)}{d(a \sec(c+dx)+a)} + \frac{B \tanh^{-1}(\sin(c+dx))}{ad}$$

Antiderivative was successfully verified.

[In] Int[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x]),x]

[Out] (B*ArcTanh[Sin[c + d*x]]/(a*d) + ((A - B)*Tan[c + d*x])/(d*(a + a*Sec[c + d*x])))

Rule 3770

Int[csc[(c_.) + (d_.)*(x_.)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3794

Int[csc[(e_.) + (f_.)*(x_.)]/(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] := -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 3998

Int[(csc[(e_.) + (f_.)*(x_.)]*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)))/(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] := Dist[B/b, Int[Csc[e + f*x], x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]

Rubi steps

$$\int \frac{\sec(c+dx)(A+B\sec(c+dx))}{a+a\sec(c+dx)} dx = (A-B) \int \frac{\sec(c+dx)}{a+a\sec(c+dx)} dx + \frac{B \int \sec(c+dx) dx}{a}$$

$$= \frac{B \tanh^{-1}(\sin(c+dx))}{ad} + \frac{(A-B) \tan(c+dx)}{d(a+a\sec(c+dx))}$$

Mathematica [B] time = 0.27, size = 109, normalized size = 2.53

$$\frac{2 \cos\left(\frac{1}{2}(c+dx)\right) \left((A-B) \sec\left(\frac{c}{2}\right) \sin\left(\frac{dx}{2}\right) + B \cos\left(\frac{1}{2}(c+dx)\right) \left(\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) + \cos\left(\frac{1}{2}(c+dx)\right) \right) \right) - \log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{ad(\cos(c+dx)+1)}$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x]),x]

[Out] (2*Cos[(c + d*x)/2]*(B*Cos[(c + d*x)/2]*(-Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])) + (A - B)*Sec[c/2]*Sin[(d*x)/2])/(a*d*(1 + Cos[c + d*x]))

fricas [A] time = 0.43, size = 74, normalized size = 1.72

$$\frac{(B \cos(dx+c) + B) \log(\sin(dx+c) + 1) - (B \cos(dx+c) + B) \log(-\sin(dx+c) + 1) + 2(A-B) \sin(dx+c)}{2(ad \cos(dx+c) + ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="fricas")

[Out] 1/2*((B*cos(d*x + c) + B)*log(sin(d*x + c) + 1) - (B*cos(d*x + c) + B)*log(-sin(d*x + c) + 1) + 2*(A - B)*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)

giac [A] time = 0.24, size = 70, normalized size = 1.63

$$\frac{\frac{B \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right|\right)}{a} - \frac{B \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1\right|\right)}{a} + \frac{A \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - B \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="giac")

[Out] (B*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - B*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a + (A*tan(1/2*d*x + 1/2*c) - B*tan(1/2*d*x + 1/2*c))/a)/d

maple [A] time = 0.68, size = 78, normalized size = 1.81

$$\frac{A \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{ad} + \frac{\ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)B}{ad} - \frac{B \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{ad} - \frac{\ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)B}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x)`

[Out] `1/a/d*A*tan(1/2*d*x+1/2*c)+1/a/d*ln(tan(1/2*d*x+1/2*c)+1)*B-1/a/d*B*tan(1/2*d*x+1/2*c)-1/a/d*ln(tan(1/2*d*x+1/2*c)-1)*B`

maxima [B] time = 0.35, size = 99, normalized size = 2.30

$$\frac{B\left(\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a} - \frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a} - \frac{\sin(dx+c)}{a(\cos(dx+c)+1)}\right) + \frac{A \sin(dx+c)}{a(\cos(dx+c)+1)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="maxima")`

[Out] `(B*(log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a - log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a - sin(d*x + c)/(a*(cos(d*x + c) + 1)))) + A*sin(d*x + c)/(a*(cos(d*x + c) + 1)))/d`

mupad [B] time = 1.93, size = 41, normalized size = 0.95

$$\frac{2B \operatorname{atanh}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{ad} + \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)(A - B)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B/cos(c + d*x))/(cos(c + d*x)*(a + a/cos(c + d*x))),x)`

[Out] `(2*B*atanh(tan(c/2 + (d*x)/2)))/(a*d) + (tan(c/2 + (d*x)/2)*(A - B))/(a*d)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A \sec(c+dx)}{\sec(c+dx)+1} dx + \int \frac{B \sec^2(c+dx)}{\sec(c+dx)+1} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x)
```

```
[Out] (Integral(A*sec(c + d*x)/(sec(c + d*x) + 1), x) + Integral(B*sec(c + d*x)**  
2/(sec(c + d*x) + 1), x))/a
```

$$3.86 \quad \int \frac{A+B \sec(c+dx)}{a+a \sec(c+dx)} dx$$

Optimal. Leaf size=35

$$\frac{Ax}{a} - \frac{(A-B) \tan(c+dx)}{d(a \sec(c+dx) + a)}$$

[Out] $A*x/a - (A-B)*\tan(d*x+c)/d/(a+a*\sec(d*x+c))$

Rubi [A] time = 0.06, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3919, 3794}

$$\frac{Ax}{a} - \frac{(A-B) \tan(c+dx)}{d(a \sec(c+dx) + a)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(A + B*\text{Sec}[c + d*x])/(a + a*\text{Sec}[c + d*x]), x]$

[Out] $(A*x)/a - ((A - B)*\text{Tan}[c + d*x])/(d*(a + a*\text{Sec}[c + d*x]))$

Rule 3794

$\text{Int}[\text{csc}[(e_.) + (f_.)*(x_.)]/(\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol]$
 $:= -\text{Simp}[\text{Cot}[e + f*x]/(f*(b + a*\text{Csc}[e + f*x])), x] /;$ $\text{FreeQ}\{a, b, e, f\}, x]$ && $\text{EqQ}[a^2 - b^2, 0]$

Rule 3919

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))/(\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol]$
 $:= \text{Simp}[(c*x)/a, x] - \text{Dist}[(b*c - a*d)/a, \text{Int}[\text{Csc}[e + f*x]/(a + b*\text{Csc}[e + f*x]), x], x] /;$ $\text{FreeQ}\{a, b, c, d, e, f\}, x]$ && $\text{NeQ}[b*c - a*d, 0]$

Rubi steps

$$\begin{aligned} \int \frac{A+B \sec(c+dx)}{a+a \sec(c+dx)} dx &= \frac{Ax}{a} - (A-B) \int \frac{\sec(c+dx)}{a+a \sec(c+dx)} dx \\ &= \frac{Ax}{a} - \frac{(A-B) \tan(c+dx)}{d(a+a \sec(c+dx))} \end{aligned}$$

Mathematica [B] time = 0.16, size = 72, normalized size = 2.06

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(2(B-A)\sin\left(\frac{dx}{2}\right)+Adx\cos\left(c+\frac{dx}{2}\right)+Adx\cos\left(\frac{dx}{2}\right)\right)}{ad(\cos(c+dx)+1)}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sec[c + d*x])/(a + a*Sec[c + d*x]), x]

[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(A*d*x*Cos[(d*x)/2] + A*d*x*Cos[c + (d*x)/2] + 2*(-A + B)*Sin[(d*x)/2]))/(a*d*(1 + Cos[c + d*x]))

fricas [A] time = 0.41, size = 44, normalized size = 1.26

$$\frac{Adx\cos(dx+c)+Adx-(A-B)\sin(dx+c)}{ad\cos(dx+c)+ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c)), x, algorithm="fricas")

[Out] (A*d*x*cos(d*x + c) + A*d*x - (A - B)*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)

giac [A] time = 0.29, size = 44, normalized size = 1.26

$$\frac{\frac{(dx+c)A}{a} - \frac{A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c)), x, algorithm="giac")

[Out] ((d*x + c)*A/a - (A*tan(1/2*d*x + 1/2*c) - B*tan(1/2*d*x + 1/2*c))/a)/d

maple [A] time = 0.73, size = 56, normalized size = 1.60

$$-\frac{A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad} + \frac{2A\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad} + \frac{B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))/(a+a*sec(d*x+c)), x)

[Out] -1/a/d*A*tan(1/2*d*x+1/2*c)+2/a/d*A*arctan(tan(1/2*d*x+1/2*c))+1/a/d*B*tan(1/2*d*x+1/2*c)

maxima [B] time = 0.43, size = 73, normalized size = 2.09

$$\frac{A \left(\frac{2 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a} - \frac{\sin(dx+c)}{a(\cos(dx+c)+1)} \right) + \frac{B \sin(dx+c)}{a(\cos(dx+c)+1)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="maxima")

[Out] (A*(2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a - sin(d*x + c)/(a*(cos(d*x + c) + 1))) + B*sin(d*x + c)/(a*(cos(d*x + c) + 1)))/d

mupad [B] time = 1.90, size = 32, normalized size = 0.91

$$-\frac{\frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)(A-B)}{a} - \frac{A dx}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/(a + a/cos(c + d*x)),x)

[Out] -((tan(c/2 + (d*x)/2)*(A - B))/a - (A*d*x)/a)/d

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A}{\sec(c+dx)+1} dx + \int \frac{B \sec(c+dx)}{\sec(c+dx)+1} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x)

[Out] (Integral(A/(sec(c + d*x) + 1), x) + Integral(B*sec(c + d*x)/(sec(c + d*x) + 1), x))/a

$$3.87 \quad \int \frac{\cos(c+dx)(A+B \sec(c+dx))}{a+a \sec(c+dx)} dx$$

Optimal. Leaf size=60

$$\frac{(2A - B) \sin(c + dx)}{ad} - \frac{(A - B) \sin(c + dx)}{d(a \sec(c + dx) + a)} - \frac{x(A - B)}{a}$$

[Out] $-(A-B)*x/a+(2*A-B)*\sin(d*x+c)/a/d-(A-B)*\sin(d*x+c)/d/(a+a*\sec(d*x+c))$

Rubi [A] time = 0.11, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {4020, 3787, 2637, 8}

$$\frac{(2A - B) \sin(c + dx)}{ad} - \frac{(A - B) \sin(c + dx)}{d(a \sec(c + dx) + a)} - \frac{x(A - B)}{a}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(\text{Cos}[c + d*x]*(A + B*\text{Sec}[c + d*x]))/(a + a*\text{Sec}[c + d*x]),x]$

[Out] $-\left(\frac{(A - B)*x}{a}\right) + \left(\frac{(2*A - B)*\text{Sin}[c + d*x]}{a*d} - \frac{(A - B)*\text{Sin}[c + d*x]}{d*(a + a*\text{Sec}[c + d*x])}\right)$

Rule 8

$\text{Int}[a_, x_Symbol] \rightarrow \text{Simp}[a*x, x] /; \text{FreeQ}[a, x]$

Rule 2637

$\text{Int}[\sin[\text{Pi}/2 + (c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow \text{Simp}[\text{Sin}[c + d*x]/d, x] /; \text{FreeQ}[\{c, d\}, x]$

Rule 3787

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(d_.))^{\text{(n_.)}}*(\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] \rightarrow \text{Dist}[a, \text{Int}[(d*\text{Csc}[e + f*x])^n, x], x] + \text{Dist}[b/d, \text{Int}[(d*\text{Csc}[e + f*x])^{n+1}, x], x] /; \text{FreeQ}[\{a, b, d, e, f, n\}, x]$

Rule 4020

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(d_.))^{\text{(n_.)}}*(\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^{\text{(m_.)}}*(\text{csc}[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] \rightarrow -\text{Simp}[\left(\frac{(A*b - a*B)*\text{Cot}[e + f*x]*(a + b*\text{Csc}[e + f*x])^m*(d*\text{Csc}[e + f*x])^n}{(b*f*(2*m + 1))}\right), x] - \text{Dist}\left[\frac{1}{a^2*(2*m + 1)}, \text{Int}[(a + b*\text{Csc}[e + f*x])^{m+1}*(d*\text{Csc}[e + f*x])^n*\text{Simp}[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*\text{Csc}[e + f*x], x]\right]$

$f*x], x], x], x] /; \text{FreeQ}[\{a, b, d, e, f, A, B, n\}, x] \ \&\& \ \text{NeQ}[A*b - a*B, 0]$
 $] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{LtQ}[m, -2^{(-1)}] \ \&\& \ \text{!GtQ}[n, 0]$

Rubi steps

$$\begin{aligned} \int \frac{\cos(c+dx)(A+B \sec(c+dx))}{a+a \sec(c+dx)} dx &= -\frac{(A-B) \sin(c+dx)}{d(a+a \sec(c+dx))} + \frac{\int \cos(c+dx)(a(2A-B) - a(A-B) \sec(c+dx)) dx}{a^2} \\ &= -\frac{(A-B) \sin(c+dx)}{d(a+a \sec(c+dx))} - \frac{(A-B) \int 1 dx}{a} + \frac{(2A-B) \int \cos(c+dx) dx}{a} \\ &= -\frac{(A-B)x}{a} + \frac{(2A-B) \sin(c+dx)}{ad} - \frac{(A-B) \sin(c+dx)}{d(a+a \sec(c+dx))} \end{aligned}$$

Mathematica [A] time = 0.38, size = 76, normalized size = 1.27

$$\frac{2 \cos\left(\frac{1}{2}(c+dx)\right) \left(\cos\left(\frac{1}{2}(c+dx)\right) (dx(B-A) + A \sin(c+dx)) + (A-B) \sec\left(\frac{c}{2}\right) \sin\left(\frac{dx}{2}\right) \right)}{ad(\cos(c+dx) + 1)}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x]),x]

[Out] (2*Cos[(c + d*x)/2]*((A - B)*Sec[c/2]*Sin[(d*x)/2] + Cos[(c + d*x)/2]*((-A + B)*d*x + A*Sin[c + d*x]))/(a*d*(1 + Cos[c + d*x]))

fricas [A] time = 0.44, size = 63, normalized size = 1.05

$$\frac{(A-B)dx \cos(dx+c) + (A-B)dx - (A \cos(dx+c) + 2A - B) \sin(dx+c)}{ad \cos(dx+c) + ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="fricas")

[Out] -((A - B)*d*x*cos(d*x + c) + (A - B)*d*x - (A*cos(d*x + c) + 2*A - B)*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)

giac [A] time = 0.23, size = 79, normalized size = 1.32

$$\frac{\frac{(dx+c)(A-B)}{a} - \frac{A \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - B \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{a}}{\left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right)^2 + 1} \frac{2A \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{a}$$

d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="giac")

[Out] -((d*x + c)*(A - B)/a - (A*tan(1/2*d*x + 1/2*c) - B*tan(1/2*d*x + 1/2*c))/a - 2*A*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 + 1)*a))/d

maple [A] time = 1.09, size = 108, normalized size = 1.80

$$\frac{A \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{ad} - \frac{B \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{ad} + \frac{2A \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{da \left(1 + \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} - \frac{2A \arctan\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{ad} + \frac{2 \arctan\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right)\right) B}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x)

[Out] 1/a/d*A*tan(1/2*d*x+1/2*c)-1/a/d*B*tan(1/2*d*x+1/2*c)+2/d/a*A*tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)-2/a/d*A*arctan(tan(1/2*d*x+1/2*c))+2/a/d*arctan(tan(1/2*d*x+1/2*c))*B

maxima [B] time = 0.43, size = 143, normalized size = 2.38

$$\frac{A \left(\frac{2 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a} - \frac{2 \sin(dx+c)}{\left(a + \frac{a \sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)} - \frac{\sin(dx+c)}{a(\cos(dx+c)+1)} \right) - B \left(\frac{2 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a} - \frac{\sin(dx+c)}{a(\cos(dx+c)+1)} \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="maxima")

[Out] -(A*(2*arctan(sin(d*x + c)/(cos(d*x + c) + 1)))/a - 2*sin(d*x + c)/((a + a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)) - sin(d*x + c)/(a*(cos(d*x + c) + 1))) - B*(2*arctan(sin(d*x + c)/(cos(d*x + c) + 1)))/a - sin(d*x + c)/(a*(cos(d*x + c) + 1)))/d

mupad [B] time = 2.00, size = 65, normalized size = 1.08

$$\frac{2A \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{d \left(a \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + a\right)} - \frac{x(A-B)}{a} + \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)(A-B)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((cos(c + d*x)*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x)),x)`

[Out] $(2*A*\tan(c/2 + (d*x)/2))/(d*(a + a*\tan(c/2 + (d*x)/2)^2) - (x*(A - B))/a + (\tan(c/2 + (d*x)/2)*(A - B))/(a*d)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A \cos(c+dx)}{\sec(c+dx)+1} dx + \int \frac{B \cos(c+dx) \sec(c+dx)}{\sec(c+dx)+1} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x)`

[Out] $(\text{Integral}(A*\cos(c + d*x)/(\sec(c + d*x) + 1), x) + \text{Integral}(B*\cos(c + d*x)*\sec(c + d*x)/(\sec(c + d*x) + 1), x))/a$

$$3.88 \quad \int \frac{\cos^2(c+dx)(A+B \sec(c+dx))}{a+a \sec(c+dx)} dx$$

Optimal. Leaf size=98

$$-\frac{2(A-B) \sin(c+dx)}{ad} + \frac{(3A-2B) \sin(c+dx) \cos(c+dx)}{2ad} - \frac{(A-B) \sin(c+dx) \cos(c+dx)}{d(a \sec(c+dx) + a)} + \frac{x(3A-2B)}{2a}$$

[Out] 1/2*(3*A-2*B)*x/a-2*(A-B)*sin(d*x+c)/a/d+1/2*(3*A-2*B)*cos(d*x+c)*sin(d*x+c)/a/d-(A-B)*cos(d*x+c)*sin(d*x+c)/d/(a+a*sec(d*x+c))

Rubi [A] time = 0.15, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {4020, 3787, 2635, 8, 2637}

$$-\frac{2(A-B) \sin(c+dx)}{ad} + \frac{(3A-2B) \sin(c+dx) \cos(c+dx)}{2ad} - \frac{(A-B) \sin(c+dx) \cos(c+dx)}{d(a \sec(c+dx) + a)} + \frac{x(3A-2B)}{2a}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x]),x]

[Out] ((3*A - 2*B)*x)/(2*a) - (2*(A - B)*Sin[c + d*x])/(a*d) + ((3*A - 2*B)*Cos[c + d*x]*Sin[c + d*x])/(2*a*d) - ((A - B)*Cos[c + d*x]*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]))

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x])* (b*SIN[c + d*x])^(n - 1)]/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*SIN[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[SIN[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[

$(d \cdot \text{Csc}[e + f \cdot x])^{(n + 1)}, x], x] /; \text{FreeQ}[\{a, b, d, e, f, n\}, x]$

Rule 4020

$\text{Int}[(\text{csc}[(e_.) + (f_.) \cdot (x_)] \cdot (d_.)^{(n_)} \cdot (\text{csc}[(e_.) + (f_.) \cdot (x_)] \cdot (b_.) + (a_))^{(m_)} \cdot (\text{csc}[(e_.) + (f_.) \cdot (x_)] \cdot (B_.) + (A_))], x_Symbol] :> -\text{Simp}[(A \cdot b - a \cdot B) \cdot \text{Cot}[e + f \cdot x] \cdot (a + b \cdot \text{Csc}[e + f \cdot x])^m \cdot (d \cdot \text{Csc}[e + f \cdot x])^n] / (b \cdot f \cdot (2 \cdot m + 1)), x] - \text{Dist}[1 / (a^2 \cdot (2 \cdot m + 1)), \text{Int}[(a + b \cdot \text{Csc}[e + f \cdot x])^{(m + 1)} \cdot (d \cdot \text{Csc}[e + f \cdot x])^n \cdot \text{Simp}[b \cdot B \cdot n - a \cdot A \cdot (2 \cdot m + n + 1) + (A \cdot b - a \cdot B) \cdot (m + n + 1) \cdot \text{Csc}[e + f \cdot x], x], x], x] /; \text{FreeQ}[\{a, b, d, e, f, A, B, n\}, x] \&\& \text{NeQ}[A \cdot b - a \cdot B, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{LtQ}[m, -2^{(-1)}] \&\& !\text{GtQ}[n, 0]$

Rubi steps

$$\begin{aligned} \int \frac{\cos^2(c + dx)(A + B \sec(c + dx))}{a + a \sec(c + dx)} dx &= -\frac{(A - B) \cos(c + dx) \sin(c + dx)}{d(a + a \sec(c + dx))} + \frac{\int \cos^2(c + dx)(a(3A - 2B) - 2a(A - B) \sec(c + dx))}{a^2} \\ &= -\frac{(A - B) \cos(c + dx) \sin(c + dx)}{d(a + a \sec(c + dx))} + \frac{(3A - 2B) \int \cos^2(c + dx) dx}{a} - \frac{2(A - B) \int \cos(c + dx) \sec(c + dx) dx}{a} \\ &= -\frac{2(A - B) \sin(c + dx)}{ad} + \frac{(3A - 2B) \cos(c + dx) \sin(c + dx)}{2ad} - \frac{(A - B) \sin(c + dx)}{d} \\ &= \frac{(3A - 2B)x}{2a} - \frac{2(A - B) \sin(c + dx)}{ad} + \frac{(3A - 2B) \cos(c + dx) \sin(c + dx)}{2ad} \end{aligned}$$

Mathematica [B] time = 0.46, size = 197, normalized size = 2.01

$$\frac{\sec\left(\frac{c}{2}\right) \cos\left(\frac{1}{2}(c + dx)\right) \left(4dx(3A - 2B) \cos\left(c + \frac{dx}{2}\right) + 4dx(3A - 2B) \cos\left(\frac{dx}{2}\right) - 4A \sin\left(c + \frac{dx}{2}\right) - 3A \sin\left(c + \frac{3dx}{2}\right) + 4B \sin\left(c + \frac{dx}{2}\right) - 3B \sin\left(c + \frac{3dx}{2}\right) + 4B \sin\left(c + \frac{5dx}{2}\right) + A \sin\left[2c + \frac{3dx}{2}\right] + A \sin\left[3c + \frac{5dx}{2}\right])}{(8ad \cos(c + dx) + ad)}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x]),x]

[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(4*(3*A - 2*B)*d*x*Cos[(d*x)/2] + 4*(3*A - 2*B)*d*x*Cos[c + (d*x)/2] - 20*A*Sin[(d*x)/2] + 20*B*Sin[(d*x)/2] - 4*A*Sin[c + (d*x)/2] + 4*B*Sin[c + (d*x)/2] - 3*A*Sin[c + (3*d*x)/2] + 4*B*Sin[c + (3*d*x)/2] - 3*A*Sin[2*c + (3*d*x)/2] + 4*B*Sin[2*c + (3*d*x)/2] + A*Sin[2*c + (5*d*x)/2] + A*Sin[3*c + (5*d*x)/2]))/(8*a*d*(1 + Cos[c + d*x]))

fricas [A] time = 0.45, size = 81, normalized size = 0.83

$$\frac{(3A - 2B)dx \cos(dx + c) + (3A - 2B)dx + (A \cos(dx + c))^2 - (A - 2B) \cos(dx + c) - 4A + 4B \sin(dx + c)}{2(ad \cos(dx + c) + ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="fricas")

[Out] 1/2*((3*A - 2*B)*d*x*cos(d*x + c) + (3*A - 2*B)*d*x + (A*cos(d*x + c)^2 - (A - 2*B)*cos(d*x + c) - 4*A + 4*B)*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)

giac [A] time = 0.22, size = 123, normalized size = 1.26

$$\frac{\frac{(dx+c)(3A-2B)}{a} - \frac{2\left(A \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - B \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right)}{a} - \frac{2\left(3A \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 - 2B \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 + A \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 2B \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + 1\right)^2 a}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="giac")

[Out] 1/2*((d*x + c)*(3*A - 2*B)/a - 2*(A*tan(1/2*d*x + 1/2*c) - B*tan(1/2*d*x + 1/2*c))/a - 2*(3*A*tan(1/2*d*x + 1/2*c)^3 - 2*B*tan(1/2*d*x + 1/2*c)^3 + A*tan(1/2*d*x + 1/2*c) - 2*B*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a))/d

maple [B] time = 1.07, size = 211, normalized size = 2.15

$$-\frac{A \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{ad} + \frac{B \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{ad} - \frac{3\left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)A}{ad\left(1 + \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^2} + \frac{2\left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)B}{ad\left(1 + \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^2} - \frac{A \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{ad\left(1 + \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^2} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x)

[Out] -1/a/d*A*tan(1/2*d*x+1/2*c)+1/a/d*B*tan(1/2*d*x+1/2*c)-3/a/d/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3*A+2/a/d/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3*B-1/a/d/(1+tan(1/2*d*x+1/2*c)^2)^2*A*tan(1/2*d*x+1/2*c)+2/a/d/(1+tan(1/2*d*x+1/2*c)^2)^2*B*tan(1/2*d*x+1/2*c)+3/a/d*A*arctan(tan(1/2*d*x+1/2*c))-2/a/d*arctan(tan(1/2*d*x+1/2*c))*B

maxima [B] time = 0.45, size = 225, normalized size = 2.30

$$\frac{A \left(\frac{\frac{\sin(dx+c)}{\cos(dx+c)+1} + \frac{3 \sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a + \frac{2a \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{a \sin(dx+c)^4}{(\cos(dx+c)+1)^4}} - \frac{3 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a} + \frac{\sin(dx+c)}{a(\cos(dx+c)+1)} \right) + B \left(\frac{2 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a} - \frac{2 \sin(dx+c)}{\left(a + \frac{a \sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)} \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="maxima")`

[Out]
$$-(A*((\sin(dx + c)/(\cos(dx + c) + 1) + 3*\sin(dx + c)^3/(\cos(dx + c) + 1)^3)/(a + 2*a*\sin(dx + c)^2/(\cos(dx + c) + 1)^2 + a*\sin(dx + c)^4/(\cos(dx + c) + 1)^4) - 3*\arctan(\sin(dx + c)/(\cos(dx + c) + 1))/a + \sin(dx + c)/(a*(\cos(dx + c) + 1))) + B*(2*\arctan(\sin(dx + c)/(\cos(dx + c) + 1))/a - 2*\sin(dx + c)/((a + a*\sin(dx + c)^2/(\cos(dx + c) + 1)^2)*(\cos(dx + c) + 1)) - \sin(dx + c)/(a*(\cos(dx + c) + 1))))/d$$

mupad [B] time = 2.21, size = 107, normalized size = 1.09

$$\frac{x(3A - 2B)}{2a} - \frac{(3A - 2B)\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 + (A - 2B)\tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{d\left(a\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 + 2a\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + a\right)} - \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)(A - B)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((cos(c + d*x)^2*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x)),x)`

[Out]
$$(x*(3*A - 2*B))/(2*a) - (\tan(c/2 + (d*x)/2)^3*(3*A - 2*B) + \tan(c/2 + (d*x)/2)*(A - 2*B))/(d*(a + 2*a*\tan(c/2 + (d*x)/2)^2 + a*\tan(c/2 + (d*x)/2)^4) - (\tan(c/2 + (d*x)/2)*(A - B))/(a*d)$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A \cos^2(c+dx)}{\sec(c+dx)+1} dx + \int \frac{B \cos^2(c+dx) \sec(c+dx)}{\sec(c+dx)+1} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)**2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x)`

[Out]
$$(\text{Integral}(A*\cos(c + d*x)**2/(\sec(c + d*x) + 1), x) + \text{Integral}(B*\cos(c + d*x)**2*\sec(c + d*x)/(\sec(c + d*x) + 1), x))/a$$

$$3.89 \quad \int \frac{\cos^3(c+dx)(A+B \sec(c+dx))}{a+a \sec(c+dx)} dx$$

Optimal. Leaf size=122

$$-\frac{(4A-3B)\sin^3(c+dx)}{3ad} + \frac{(4A-3B)\sin(c+dx)}{ad} - \frac{3(A-B)\sin(c+dx)\cos(c+dx)}{2ad} - \frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{d(a\sec(c+dx)+a)}$$

[Out] $-3/2*(A-B)*x/a+(4*A-3*B)*\sin(d*x+c)/a/d-3/2*(A-B)*\cos(d*x+c)*\sin(d*x+c)/a/d$
 $-(A-B)*\cos(d*x+c)^2*\sin(d*x+c)/d/(a+a*\sec(d*x+c))-1/3*(4*A-3*B)*\sin(d*x+c)^3/a/d$

Rubi [A] time = 0.16, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {4020, 3787, 2633, 2635, 8}

$$-\frac{(4A-3B)\sin^3(c+dx)}{3ad} + \frac{(4A-3B)\sin(c+dx)}{ad} - \frac{3(A-B)\sin(c+dx)\cos(c+dx)}{2ad} - \frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{d(a\sec(c+dx)+a)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(\text{Cos}[c + d*x]^3*(A + B*\text{Sec}[c + d*x]))/(a + a*\text{Sec}[c + d*x]), x]$

[Out] $(-3*(A - B)*x)/(2*a) + ((4*A - 3*B)*\text{Sin}[c + d*x])/(a*d) - (3*(A - B)*\text{Cos}[c + d*x]*\text{Sin}[c + d*x])/(2*a*d) - ((A - B)*\text{Cos}[c + d*x]^2*\text{Sin}[c + d*x])/(d*(a + a*\text{Sec}[c + d*x])) - ((4*A - 3*B)*\text{Sin}[c + d*x]^3)/(3*a*d)$

Rule 8

$\text{Int}[a_, x_Symbol] \rightarrow \text{Simp}[a*x, x] /; \text{FreeQ}[a, x]$

Rule 2633

$\text{Int}[\sin[(c_.) + (d_.)*(x_)]^{(n_)}, x_Symbol] \rightarrow -\text{Dist}[d^{(-1)}, \text{Subst}[\text{Int}[\text{Expand}[(1 - x^2)^{((n - 1)/2)}, x], x], x, \text{Cos}[c + d*x]], x] /; \text{FreeQ}\{c, d\}, x] \&\& \text{IGtQ}[(n - 1)/2, 0]$

Rule 2635

$\text{Int}[(b_.)*\sin[(c_.) + (d_.)*(x_)]^{(n_)}, x_Symbol] \rightarrow -\text{Simp}[(b*\text{Cos}[c + d*x])*(b*\text{Sin}[c + d*x])^{(n - 1)})/(d*n), x] + \text{Dist}[(b^2*(n - 1))/n, \text{Int}[(b*\text{Sin}[c + d*x])^{(n - 2)}, x], x] /; \text{FreeQ}\{b, c, d\}, x] \&\& \text{GtQ}[n, 1] \&\& \text{IntegerQ}[2*n]$

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) +
(a_.)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (
a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
+ f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{\cos^3(c + dx)(A + B \sec(c + dx))}{a + a \sec(c + dx)} dx &= -\frac{(A - B) \cos^2(c + dx) \sin(c + dx)}{d(a + a \sec(c + dx))} + \frac{\int \cos^3(c + dx)(a(4A - 3B) - 3a)}{a^2} \\ &= -\frac{(A - B) \cos^2(c + dx) \sin(c + dx)}{d(a + a \sec(c + dx))} + \frac{(4A - 3B) \int \cos^3(c + dx) dx}{a} - \frac{3(A - B)}{a} \\ &= -\frac{3(A - B) \cos(c + dx) \sin(c + dx)}{2ad} - \frac{(A - B) \cos^2(c + dx) \sin(c + dx)}{d(a + a \sec(c + dx))} \\ &= -\frac{3(A - B)x}{2a} + \frac{(4A - 3B) \sin(c + dx)}{ad} - \frac{3(A - B) \cos(c + dx) \sin(c + dx)}{2ad} \end{aligned}$$

Mathematica [B] time = 0.75, size = 249, normalized size = 2.04

$$\sec\left(\frac{c}{2}\right) \cos\left(\frac{1}{2}(c + dx)\right) \left(-36dx(A - B) \cos\left(c + \frac{dx}{2}\right) - 36dx(A - B) \cos\left(\frac{dx}{2}\right) + 21A \sin\left(c + \frac{dx}{2}\right) + 18A \sin\left(c + \frac{dx}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(-36*(A - B)*d*x*Cos[(d*x)/2] - 36*(A - B)*d*x*Cos[c + (d*x)/2] + 69*A*Sin[(d*x)/2] - 60*B*Sin[(d*x)/2] + 21*A*Sin[c + (d*x)/2] - 12*B*Sin[c + (d*x)/2] + 18*A*Sin[c + (3*d*x)/2] - 9*B*Sin[c + (3*d*x)/2] + 18*A*Sin[2*c + (3*d*x)/2] - 9*B*Sin[2*c + (3*d*x)/2] - 2*A*Sin[2*c + (5*d*x)/2] + 3*B*Sin[2*c + (5*d*x)/2] - 2*A*Sin[3*c + (5*d*x)/2] + 3*B*Sin
```

$[3*c + (5*d*x)/2] + A*\sin[3*c + (7*d*x)/2] + A*\sin[4*c + (7*d*x)/2])/(24*a*d*(1 + \cos[c + d*x]))$

fricas [A] time = 0.45, size = 97, normalized size = 0.80

$$\frac{9(A-B)dx \cos(dx+c) + 9(A-B)dx - (2A \cos(dx+c)^3 - (A-3B) \cos(dx+c)^2 + (7A-3B) \cos(dx+c))}{6(ad \cos(dx+c) + ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="fricas")

[Out] $-1/6*(9*(A-B)*d*x*\cos(d*x+c) + 9*(A-B)*d*x - (2*A*\cos(d*x+c)^3 - (A-3*B)*\cos(d*x+c)^2 + (7*A-3*B)*\cos(d*x+c) + 16*A - 12*B)*\sin(d*x+c))/(a*d*\cos(d*x+c) + a*d)$

giac [A] time = 2.82, size = 151, normalized size = 1.24

$$\frac{\frac{9(dx+c)(A-B)}{a} - \frac{6\left(A \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - B \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right)}{a} - \frac{2\left(15A \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5 - 9B \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5 + 16A \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 - 12B \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + 1\right)^3 a}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="giac")

[Out] $-1/6*(9*(d*x+c)*(A-B)/a - 6*(A*\tan(1/2*d*x + 1/2*c) - B*\tan(1/2*d*x + 1/2*c))/a - 2*(15*A*\tan(1/2*d*x + 1/2*c)^5 - 9*B*\tan(1/2*d*x + 1/2*c)^5 + 16*A*\tan(1/2*d*x + 1/2*c)^3 - 12*B*\tan(1/2*d*x + 1/2*c)^3 + 9*A*\tan(1/2*d*x + 1/2*c) - 3*B*\tan(1/2*d*x + 1/2*c))/((\tan(1/2*d*x + 1/2*c)^2 + 1)^3*a))/d$

maple [B] time = 1.42, size = 281, normalized size = 2.30

$$\frac{A \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{ad} - \frac{B \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{ad} - \frac{3\left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right)B}{ad\left(1 + \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^3} + \frac{5\left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right)A}{ad\left(1 + \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^3} - \frac{4\left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)B}{ad\left(1 + \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^3} + \frac{1}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x)

[Out] $1/a/d*A*\tan(1/2*d*x+1/2*c) - 1/a/d*B*\tan(1/2*d*x+1/2*c) - 3/a/d/(1+\tan(1/2*d*x+1/2*c)^2)^3*\tan(1/2*d*x+1/2*c)^5*B + 5/a/d/(1+\tan(1/2*d*x+1/2*c)^2)^3*\tan(1/2$

$*d*x+1/2*c)^5*A-4/a/d/(1+\tan(1/2*d*x+1/2*c))^2)^3*\tan(1/2*d*x+1/2*c)^3*B+16/3/a/d/(1+\tan(1/2*d*x+1/2*c))^2)^3*\tan(1/2*d*x+1/2*c)^3*A-1/a/d/(1+\tan(1/2*d*x+1/2*c))^2)^3*B*\tan(1/2*d*x+1/2*c)+3/a/d/(1+\tan(1/2*d*x+1/2*c))^2)^3*A*\tan(1/2*d*x+1/2*c)-3/a/d*A*\arctan(\tan(1/2*d*x+1/2*c))+3/a/d*\arctan(\tan(1/2*d*x+1/2*c))*B$

maxima [B] time = 0.45, size = 310, normalized size = 2.54

$$\frac{A \left(\frac{9 \sin(dx+c)}{\cos(dx+c)+1} + \frac{16 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{15 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{9 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a} + \frac{3 \sin(dx+c)}{a(\cos(dx+c)+1)} \right) - 3B \left(\frac{\frac{\sin(dx+c)}{\cos(dx+c)+1} + \frac{3 \sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a + \frac{2a \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{a \sin(dx+c)^4}{(\cos(dx+c)+1)^4}} - \frac{3a}{a d} \right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="maxima")

[Out] $\frac{1}{3} * (A * ((9 * \sin(d * x + c) / (\cos(d * x + c) + 1) + 16 * \sin(d * x + c)^3 / (\cos(d * x + c) + 1)^3 + 15 * \sin(d * x + c)^5 / (\cos(d * x + c) + 1)^5) / (a + 3 * a * \sin(d * x + c)^2 / (\cos(d * x + c) + 1)^2 + 3 * a * \sin(d * x + c)^4 / (\cos(d * x + c) + 1)^4 + a * \sin(d * x + c)^6 / (\cos(d * x + c) + 1)^6) - 9 * \arctan(\sin(d * x + c) / (\cos(d * x + c) + 1)) / a + 3 * \sin(d * x + c) / (a * (\cos(d * x + c) + 1))) - 3 * B * ((\sin(d * x + c) / (\cos(d * x + c) + 1) + 3 * \sin(d * x + c)^3 / (\cos(d * x + c) + 1)^3) / (a + 2 * a * \sin(d * x + c)^2 / (\cos(d * x + c) + 1)^2 + a * \sin(d * x + c)^4 / (\cos(d * x + c) + 1)^4) - 3 * \arctan(\sin(d * x + c) / (\cos(d * x + c) + 1)) / a + \sin(d * x + c) / (a * (\cos(d * x + c) + 1)))) / d$

mupad [B] time = 3.02, size = 138, normalized size = 1.13

$$\frac{(5A - 3B) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5 + \left(\frac{16A}{3} - 4B\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 + (3A - B) \tan\left(\frac{c}{2} + \frac{dx}{2}\right) - \frac{3x(A - B)}{2a} + \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{ad}}{d \left(a \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^6 + 3a \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 + 3a \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + a \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)^3*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x)),x)

[Out] $(\tan(c/2 + (d*x)/2)^5 * (5*A - 3*B) + \tan(c/2 + (d*x)/2)^3 * ((16*A)/3 - 4*B) + \tan(c/2 + (d*x)/2) * (3*A - B)) / (d * (a + 3*a*\tan(c/2 + (d*x)/2)^2 + 3*a*\tan(c/2 + (d*x)/2)^4 + a*\tan(c/2 + (d*x)/2)^6)) - (3*x*(A - B)) / (2*a) + (\tan(c/2 + (d*x)/2) * (A - B)) / (a*d)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A \cos^3(c+dx)}{\sec(c+dx)+1} dx + \int \frac{B \cos^3(c+dx) \sec(c+dx)}{\sec(c+dx)+1} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x)
```

```
[Out] (Integral(A*cos(c + d*x)**3/(sec(c + d*x) + 1), x) + Integral(B*cos(c + d*x)**3*sec(c + d*x)/(sec(c + d*x) + 1), x))/a
```

$$3.90 \quad \int \frac{\sec^5(c+dx)(A+B \sec(c+dx))}{(a+a \sec(c+dx))^2} dx$$

Optimal. Leaf size=179

$$\frac{4(2A-3B) \tan^3(c+dx)}{3a^2d} - \frac{4(2A-3B) \tan(c+dx)}{a^2d} + \frac{(7A-10B) \tanh^{-1}(\sin(c+dx))}{2a^2d} + \frac{(7A-10B) \tan(c+dx)}{3a^2d(\sec(c+dx))}$$

[Out] 1/2*(7*A-10*B)*arctanh(sin(d*x+c))/a^2/d-4*(2*A-3*B)*tan(d*x+c)/a^2/d+1/2*(7*A-10*B)*sec(d*x+c)*tan(d*x+c)/a^2/d+1/3*(7*A-10*B)*sec(d*x+c)^3*tan(d*x+c)/a^2/d/(1+sec(d*x+c))+1/3*(A-B)*sec(d*x+c)^4*tan(d*x+c)/d/(a+a*sec(d*x+c))^2-4/3*(2*A-3*B)*tan(d*x+c)^3/a^2/d

Rubi [A] time = 0.32, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {4019, 3787, 3768, 3770, 3767}

$$\frac{4(2A-3B) \tan^3(c+dx)}{3a^2d} - \frac{4(2A-3B) \tan(c+dx)}{a^2d} + \frac{(7A-10B) \tanh^{-1}(\sin(c+dx))}{2a^2d} + \frac{(7A-10B) \tan(c+dx)}{3a^2d(\sec(c+dx))}$$

Antiderivative was successfully verified.

[In] Int[(Sec[c + d*x]^5*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^2,x]

[Out] ((7*A - 10*B)*ArcTanh[Sin[c + d*x]]/(2*a^2*d) - (4*(2*A - 3*B)*Tan[c + d*x])/(a^2*d) + ((7*A - 10*B)*Sec[c + d*x]*Tan[c + d*x])/(2*a^2*d) + ((7*A - 10*B)*Sec[c + d*x]^3*Tan[c + d*x])/(3*a^2*d*(1 + Sec[c + d*x])) + ((A - B)*Sec[c + d*x]^4*Tan[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2) - (4*(2*A - 3*B)*Tan[c + d*x]^3)/(3*a^2*d)

Rule 3767

Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 3768

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := -Simp[(b*Csc[c + d*x]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3770

`Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x]
/; FreeQ[{c, d}, x]`

Rule 3787

`Int[(csc[(e_.) + (f_.)*(x_)])*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)])*(b_.) +
(a_), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]`

Rule 4019

`Int[(csc[(e_.) + (f_.)*(x_)])*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)])*(b_.) + (a_)^(m_)
(csc[(e_.) + (f_.)*(x_)])*(B_.) + (A_.)), x_Symbol] := Simp[(d*(A*b - a*B)*Cot[e + f*x]*
(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) -
d*(a*B*(m - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] &&
NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0]`

Rubi steps

$$\begin{aligned} \int \frac{\sec^5(c + dx)(A + B \sec(c + dx))}{(a + a \sec(c + dx))^2} dx &= \frac{(A - B) \sec^4(c + dx) \tan(c + dx)}{3d(a + a \sec(c + dx))^2} + \frac{\int \frac{\sec^4(c + dx)(4a(A - B) - 3a(A - 2B) \sec(c + dx))}{a + a \sec(c + dx)} dx}{3a^2} \\ &= \frac{(7A - 10B) \sec^3(c + dx) \tan(c + dx)}{3a^2 d(1 + \sec(c + dx))} + \frac{(A - B) \sec^4(c + dx) \tan(c + dx)}{3d(a + a \sec(c + dx))^2} \\ &= \frac{(7A - 10B) \sec^3(c + dx) \tan(c + dx)}{3a^2 d(1 + \sec(c + dx))} + \frac{(A - B) \sec^4(c + dx) \tan(c + dx)}{3d(a + a \sec(c + dx))^2} \\ &= \frac{(7A - 10B) \sec(c + dx) \tan(c + dx)}{2a^2 d} + \frac{(7A - 10B) \sec^3(c + dx) \tan(c + dx)}{3a^2 d(1 + \sec(c + dx))} \\ &= \frac{(7A - 10B) \tanh^{-1}(\sin(c + dx))}{2a^2 d} - \frac{4(2A - 3B) \tan(c + dx)}{a^2 d} + \frac{(7A - 10B) \sec^3(c + dx) \tan(c + dx)}{3a^2 d(1 + \sec(c + dx))} \end{aligned}$$

Mathematica [B] time = 6.43, size = 764, normalized size = 4.27

$$\frac{\sec\left(\frac{c}{2}\right) \sec(c) \cos\left(\frac{c}{2} + \frac{dx}{2}\right) \sec^4(c + dx) \left(195A \sin\left(c - \frac{dx}{2}\right) - 51A \sin\left(c + \frac{dx}{2}\right) + 189A \sin\left(2c + \frac{dx}{2}\right) - A \sin\left(c + \frac{dx}{2}\right)\right)}{3a^2 d(1 + \sec(c + dx))^2}$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[c + d*x]^5*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^2,x]

[Out] $(2*(-7*A + 10*B)*\cos[c/2 + (d*x)/2]^4*\log[\cos[c/2 + (d*x)/2] - \sin[c/2 + (d*x)/2]]*\sec[c + d*x]*(A + B*\sec[c + d*x]))/(d*(B + A*\cos[c + d*x])*(a + a*\sec[c + d*x])^2) - (2*(-7*A + 10*B)*\cos[c/2 + (d*x)/2]^4*\log[\cos[c/2 + (d*x)/2] + \sin[c/2 + (d*x)/2]]*\sec[c + d*x]*(A + B*\sec[c + d*x]))/(d*(B + A*\cos[c + d*x])*(a + a*\sec[c + d*x])^2) + (\cos[c/2 + (d*x)/2]*\sec[c/2]*\sec[c]*\sec[c + d*x]^4*(A + B*\sec[c + d*x]))*(45*A*\sin[(d*x)/2] - 6*B*\sin[(d*x)/2] - 201*A*\sin[(3*d*x)/2] + 310*B*\sin[(3*d*x)/2] + 195*A*\sin[c - (d*x)/2] - 306*B*\sin[c - (d*x)/2] - 51*A*\sin[c + (d*x)/2] + 42*B*\sin[c + (d*x)/2] + 189*A*\sin[2*c + (d*x)/2] - 270*B*\sin[2*c + (d*x)/2] - A*\sin[c + (3*d*x)/2] + 50*B*\sin[c + (3*d*x)/2] - 81*A*\sin[2*c + (3*d*x)/2] + 90*B*\sin[2*c + (3*d*x)/2] + 119*A*\sin[3*c + (3*d*x)/2] - 170*B*\sin[3*c + (3*d*x)/2] - 129*A*\sin[c + (5*d*x)/2] + 198*B*\sin[c + (5*d*x)/2] - 9*A*\sin[2*c + (5*d*x)/2] + 42*B*\sin[2*c + (5*d*x)/2] - 57*A*\sin[3*c + (5*d*x)/2] + 66*B*\sin[3*c + (5*d*x)/2] + 63*A*\sin[4*c + (5*d*x)/2] - 90*B*\sin[4*c + (5*d*x)/2] - 75*A*\sin[2*c + (7*d*x)/2] + 114*B*\sin[2*c + (7*d*x)/2] - 15*A*\sin[3*c + (7*d*x)/2] + 36*B*\sin[3*c + (7*d*x)/2] - 39*A*\sin[4*c + (7*d*x)/2] + 48*B*\sin[4*c + (7*d*x)/2] + 21*A*\sin[5*c + (7*d*x)/2] - 30*B*\sin[5*c + (7*d*x)/2] - 32*A*\sin[3*c + (9*d*x)/2] + 48*B*\sin[3*c + (9*d*x)/2] - 12*A*\sin[4*c + (9*d*x)/2] + 22*B*\sin[4*c + (9*d*x)/2] - 20*A*\sin[5*c + (9*d*x)/2] + 26*B*\sin[5*c + (9*d*x)/2]))/(96*d*(B + A*\cos[c + d*x])*(a + a*\sec[c + d*x])^2)$

fricas [A] time = 0.45, size = 245, normalized size = 1.37

$$\frac{3((7A - 10B)\cos(dx + c)^5 + 2(7A - 10B)\cos(dx + c)^4 + (7A - 10B)\cos(dx + c)^3)\log(\sin(dx + c) + 1) - \dots}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^5*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="fricas")

[Out] $1/12*(3*((7*A - 10*B)*\cos(d*x + c)^5 + 2*(7*A - 10*B)*\cos(d*x + c)^4 + (7*A - 10*B)*\cos(d*x + c)^3)*\log(\sin(d*x + c) + 1) - 3*((7*A - 10*B)*\cos(d*x + c)^5 + 2*(7*A - 10*B)*\cos(d*x + c)^4 + (7*A - 10*B)*\cos(d*x + c)^3)*\log(-\sin(d*x + c) + 1) - 2*(16*(2*A - 3*B)*\cos(d*x + c)^4 + (43*A - 66*B)*\cos(d*x + c)^3 + 6*(A - 2*B)*\cos(d*x + c)^2 - (3*A - 2*B)*\cos(d*x + c) - 2*B)*\sin(d*x + c))/(a^2*d*\cos(d*x + c)^5 + 2*a^2*d*\cos(d*x + c)^4 + a^2*d*\cos(d*x + c)^3)$

giac [A] time = 1.57, size = 226, normalized size = 1.26

$$\frac{3(7A-10B)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^2} - \frac{3(7A-10B)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^2} + \frac{2\left(15A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-30B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-24A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^5*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="giac")

[Out] $\frac{1}{6}*(3*(7*A - 10*B)*\log(\tan(\frac{1}{2}*d*x + \frac{1}{2}*c) + 1))/a^2 - 3*(7*A - 10*B)*\log(\tan(\frac{1}{2}*d*x + \frac{1}{2}*c) - 1))/a^2 + 2*(15*A*\tan(\frac{1}{2}*d*x + \frac{1}{2}*c)^5 - 30*B*\tan(\frac{1}{2}*d*x + \frac{1}{2}*c)^5 - 24*A*\tan(\frac{1}{2}*d*x + \frac{1}{2}*c)^3 + 40*B*\tan(\frac{1}{2}*d*x + \frac{1}{2}*c)^3 + 9*A*\tan(\frac{1}{2}*d*x + \frac{1}{2}*c) - 18*B*\tan(\frac{1}{2}*d*x + \frac{1}{2}*c))/((\tan(\frac{1}{2}*d*x + \frac{1}{2}*c)^2 - 1)^3*a^2) - (A*a^4*\tan(\frac{1}{2}*d*x + \frac{1}{2}*c)^3 - B*a^4*\tan(\frac{1}{2}*d*x + \frac{1}{2}*c)^3 + 21*A*a^4*\tan(\frac{1}{2}*d*x + \frac{1}{2}*c) - 27*B*a^4*\tan(\frac{1}{2}*d*x + \frac{1}{2}*c))/a^6)/d$

maple [B] time = 1.05, size = 382, normalized size = 2.13

$$-\frac{\left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right) A}{6d a^2} + \frac{B\left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{6d a^2} - \frac{7A \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{2d a^2} + \frac{9B \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{2d a^2} + \frac{A}{2d a^2 \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)^2} - \frac{B}{2d a^2 \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^5*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x)

[Out] $-\frac{1}{6}/d/a^2*\tan(\frac{1}{2}*d*x+\frac{1}{2}*c)^3*A+\frac{1}{6}/d/a^2*B*\tan(\frac{1}{2}*d*x+\frac{1}{2}*c)^3-\frac{7}{2}/d/a^2*A*\tan(\frac{1}{2}*d*x+\frac{1}{2}*c)+\frac{9}{2}/d/a^2*B*\tan(\frac{1}{2}*d*x+\frac{1}{2}*c)+\frac{1}{2}/d/a^2*A/(\tan(\frac{1}{2}*d*x+\frac{1}{2}*c)-1)^2-\frac{3}{2}/d/a^2/(\tan(\frac{1}{2}*d*x+\frac{1}{2}*c)-1)^2*B-\frac{7}{2}/d/a^2*A*\ln(\tan(\frac{1}{2}*d*x+\frac{1}{2}*c)-1)+\frac{5}{2}/d/a^2*\ln(\tan(\frac{1}{2}*d*x+\frac{1}{2}*c)-1)*B-\frac{5}{2}/d/a^2/(\tan(\frac{1}{2}*d*x+\frac{1}{2}*c)-1)*B+\frac{5}{2}/d/a^2*A/(\tan(\frac{1}{2}*d*x+\frac{1}{2}*c)-1)-\frac{1}{3}/d/a^2*B/(\tan(\frac{1}{2}*d*x+\frac{1}{2}*c)-1)^3+\frac{7}{2}/d/a^2*A*\ln(\tan(\frac{1}{2}*d*x+\frac{1}{2}*c)+1)-\frac{5}{2}/d/a^2*\ln(\tan(\frac{1}{2}*d*x+\frac{1}{2}*c)+1)*B+\frac{3}{2}/d/a^2/(\tan(\frac{1}{2}*d*x+\frac{1}{2}*c)+1)^2*B-\frac{1}{2}/d/a^2*A/(\tan(\frac{1}{2}*d*x+\frac{1}{2}*c)+1)^2-\frac{5}{2}/d/a^2/(\tan(\frac{1}{2}*d*x+\frac{1}{2}*c)+1)*B+\frac{5}{2}/d/a^2*A/(\tan(\frac{1}{2}*d*x+\frac{1}{2}*c)+1)-\frac{1}{3}/d/a^2*B/(\tan(\frac{1}{2}*d*x+\frac{1}{2}*c)+1)^3$

maxima [B] time = 0.36, size = 425, normalized size = 2.37

$$B \left(\frac{4 \left(\frac{9 \sin(dx+c)}{\cos(dx+c)+1} - \frac{20 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{15 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} \right)}{a^2 - \frac{3 a^2 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{3 a^2 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} - \frac{a^2 \sin(dx+c)^6}{(\cos(dx+c)+1)^6}} + \frac{\frac{27 \sin(dx+c)}{\cos(dx+c)+1} + \frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2} - \frac{30 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1} + 1\right)}{a^2} + \frac{30 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1} - 1\right)}{a^2} \right) - A \left(\frac{6}{a^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^5*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="maxima")

[Out] $\frac{1}{6}*(B*(4*(9*\sin(d*x + c))/(\cos(d*x + c) + 1) - 20*\sin(d*x + c)^3/(\cos(d*x + c) + 1)^3 + 15*\sin(d*x + c)^5/(\cos(d*x + c) + 1)^5)/(a^2 - 3*a^2*\sin(d*x +$

$$\begin{aligned} & c)^2/(\cos(dx + c) + 1)^2 + 3a^2\sin(dx + c)^4/(\cos(dx + c) + 1)^4 - a^2\sin(dx + c)^6/(\cos(dx + c) + 1)^6 + (27\sin(dx + c)/(\cos(dx + c) + 1) \\ &) + \sin(dx + c)^3/(\cos(dx + c) + 1)^3/a^2 - 30\log(\sin(dx + c)/(\cos(dx + c) + 1) + 1)/a^2 + 30\log(\sin(dx + c)/(\cos(dx + c) + 1) - 1)/a^2) - A* \\ & (6*(3\sin(dx + c)/(\cos(dx + c) + 1) - 5\sin(dx + c)^3/(\cos(dx + c) + 1)^3)/(a^2 - 2a^2\sin(dx + c)^2/(\cos(dx + c) + 1)^2 + a^2\sin(dx + c)^4/(\cos(dx + c) + 1)^4) \\ & + (21\sin(dx + c)/(\cos(dx + c) + 1) + \sin(dx + c)^3/(\cos(dx + c) + 1)^3)/a^2 - 21\log(\sin(dx + c)/(\cos(dx + c) + 1) + 1)/a^2 + 21\log(\sin(dx + c)/(\cos(dx + c) + 1) - 1)/a^2))/d \end{aligned}$$

mupad [B] time = 1.98, size = 202, normalized size = 1.13

$$\frac{(5A - 10B) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5 + \left(\frac{40B}{3} - 8A\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 + (3A - 6B) \tan\left(\frac{c}{2} + \frac{dx}{2}\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right) \left(\frac{2(A-B)}{a^2} + \frac{3}{d}\right)}{d \left(a^2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^6 - 3a^2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 + 3a^2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 - a^2 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^5*(a + a/cos(c + d*x))^2), x)

[Out] $(\tan(c/2 + (d*x)/2)^5*(5*A - 10*B) - \tan(c/2 + (d*x)/2)^3*(8*A - (40*B)/3) + \tan(c/2 + (d*x)/2)*(3*A - 6*B))/(d*(3*a^2*\tan(c/2 + (d*x)/2)^2 - 3*a^2*\tan(c/2 + (d*x)/2)^4 + a^2*\tan(c/2 + (d*x)/2)^6 - a^2)) - (\tan(c/2 + (d*x)/2)*((2*(A - B))/a^2 + (3*A - 5*B)/(2*a^2)))/d - (\tan(c/2 + (d*x)/2)^3*(A - B))/(6*a^2*d) + (\operatorname{atanh}(\tan(c/2 + (d*x)/2))*(7*A - 10*B))/(a^2*d)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A \sec^5(c+dx)}{\sec^2(c+dx)+2 \sec(c+dx)+1} dx + \int \frac{B \sec^6(c+dx)}{\sec^2(c+dx)+2 \sec(c+dx)+1} dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**5*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**2, x)

[Out] $(\operatorname{Integral}(A*\sec(c + d*x)**5/(\sec(c + d*x)**2 + 2*\sec(c + d*x) + 1), x) + \operatorname{Integral}(B*\sec(c + d*x)**6/(\sec(c + d*x)**2 + 2*\sec(c + d*x) + 1), x))/a**2$

$$3.91 \quad \int \frac{\sec^4(c+dx)(A+B \sec(c+dx))}{(a+a \sec(c+dx))^2} dx$$

Optimal. Leaf size=156

$$\frac{2(5A-8B) \tan(c+dx)}{3a^2d} - \frac{(4A-7B) \tanh^{-1}(\sin(c+dx))}{2a^2d} + \frac{(5A-8B) \tan(c+dx) \sec^2(c+dx)}{3a^2d(\sec(c+dx)+1)} - \frac{(4A-7B) \tan(c+dx)}{2a^2d}$$

[Out] $-1/2*(4*A-7*B)*\operatorname{arctanh}(\sin(d*x+c))/a^2/d+2/3*(5*A-8*B)*\tan(d*x+c)/a^2/d-1/2*(4*A-7*B)*\sec(d*x+c)*\tan(d*x+c)/a^2/d+1/3*(5*A-8*B)*\sec(d*x+c)^2*\tan(d*x+c)/a^2/d/(1+\sec(d*x+c))+1/3*(A-B)*\sec(d*x+c)^3*\tan(d*x+c)/d/(a+a*\sec(d*x+c))^2$

Rubi [A] time = 0.31, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4019, 3787, 3767, 8, 3768, 3770}

$$\frac{2(5A-8B) \tan(c+dx)}{3a^2d} - \frac{(4A-7B) \tanh^{-1}(\sin(c+dx))}{2a^2d} + \frac{(5A-8B) \tan(c+dx) \sec^2(c+dx)}{3a^2d(\sec(c+dx)+1)} - \frac{(4A-7B) \tan(c+dx)}{2a^2d}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(\operatorname{Sec}[c+d*x]^4*(A+B*\operatorname{Sec}[c+d*x]))/(a+a*\operatorname{Sec}[c+d*x])^2,x]$

[Out] $-((4*A-7*B)*\operatorname{ArcTanh}[\operatorname{Sin}[c+d*x]])/(2*a^2*d) + (2*(5*A-8*B)*\operatorname{Tan}[c+d*x])/((3*a^2*d) - ((4*A-7*B)*\operatorname{Sec}[c+d*x]*\operatorname{Tan}[c+d*x])/(2*a^2*d) + ((5*A-8*B)*\operatorname{Sec}[c+d*x]^2*\operatorname{Tan}[c+d*x])/(3*a^2*d*(1+\operatorname{Sec}[c+d*x])) + ((A-B)*\operatorname{Sec}[c+d*x]^3*\operatorname{Tan}[c+d*x])/(3*d*(a+a*\operatorname{Sec}[c+d*x])^2)$

Rule 8

$\operatorname{Int}[a_, x_Symbol] := \operatorname{Simp}[a*x, x] /; \operatorname{FreeQ}[a, x]$

Rule 3767

$\operatorname{Int}[\operatorname{csc}[(c_.) + (d_.)*(x_)]^{(n_.)}, x_Symbol] := -\operatorname{Dist}[d^{(-1)}, \operatorname{Subst}[\operatorname{Int}[\operatorname{ExpandIntegrand}[(1+x^2)^{(n/2-1)}, x], x], x, \operatorname{Cot}[c+d*x]], x] /; \operatorname{FreeQ}[\{c, d\}, x] \&\& \operatorname{IGtQ}[n/2, 0]$

Rule 3768

$\operatorname{Int}[(\operatorname{csc}[(c_.) + (d_.)*(x_)]*(b_.))^{(n_.)}, x_Symbol] := -\operatorname{Simp}[(b*\operatorname{Cos}[c+d*x])*(b*\operatorname{Csc}[c+d*x])^{(n-1)})/(d*(n-1)), x] + \operatorname{Dist}[(b^2*(n-2))/(n-1), \operatorname{Int}[(b*\operatorname{Csc}[c+d*x])^{(n-2)}, x], x] /; \operatorname{FreeQ}[\{b, c, d\}, x] \&\& \operatorname{GtQ}[n, 1] \&\& \operatorname{IntegerQ}[2*n]$

Rule 3770

Int[csc[(c_.) + (d_.)*(x_.)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 4019

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := Simp[(d*(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{\sec^4(c + dx)(A + B \sec(c + dx))}{(a + a \sec(c + dx))^2} dx &= \frac{(A - B) \sec^3(c + dx) \tan(c + dx)}{3d(a + a \sec(c + dx))^2} + \frac{\int \frac{\sec^3(c + dx)(3a(A - B) - a(2A - 5B) \sec(c + dx))}{a + a \sec(c + dx)} dx}{3a^2} \\
 &= \frac{(5A - 8B) \sec^2(c + dx) \tan(c + dx)}{3a^2 d(1 + \sec(c + dx))} + \frac{(A - B) \sec^3(c + dx) \tan(c + dx)}{3d(a + a \sec(c + dx))^2} \\
 &= \frac{(5A - 8B) \sec^2(c + dx) \tan(c + dx)}{3a^2 d(1 + \sec(c + dx))} + \frac{(A - B) \sec^3(c + dx) \tan(c + dx)}{3d(a + a \sec(c + dx))^2} \\
 &= -\frac{(4A - 7B) \sec(c + dx) \tan(c + dx)}{2a^2 d} + \frac{(5A - 8B) \sec^2(c + dx) \tan(c + dx)}{3a^2 d(1 + \sec(c + dx))} \\
 &= -\frac{(4A - 7B) \tanh^{-1}(\sin(c + dx))}{2a^2 d} + \frac{2(5A - 8B) \tan(c + dx)}{3a^2 d} - \frac{(4A - 7B)}{3a^2 d}
 \end{aligned}$$

Mathematica [B] time = 4.19, size = 496, normalized size = 3.18

$$96(4A - 7B) \cos^4\left(\frac{1}{2}(c + dx)\right) \left(\log\left(\cos\left(\frac{1}{2}(c + dx)\right) - \sin\left(\frac{1}{2}(c + dx)\right)\right) - \log\left(\sin\left(\frac{1}{2}(c + dx)\right) + \cos\left(\frac{1}{2}(c + dx)\right)\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[c + d*x]^4*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^2,x]

[Out] (96*(4*A - 7*B)*Cos[(c + d*x)/2]^4*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Cos[(c + d*x)/2]*Sec[c/2]*Sec[c]*Sec[c + d*x]^2*(-14*(A - B)*Sin[(d*x)/2] + (64*A - 97*B)*Sin[(3*d*x)/2] - 84*A*Sin[c - (d*x)/2] + 126*B*Sin[c - (d*x)/2] + 42*A*Sin[c + (d*x)/2] - 42*B*Sin[c + (d*x)/2] - 56*A*Sin[2*c + (d*x)/2] + 98*B*Sin[2*c + (d*x)/2] - 6*A*Sin[c + (3*d*x)/2] + 3*B*Sin[c + (3*d*x)/2] + 34*A*Sin[2*c + (3*d*x)/2] - 37*B*Sin[2*c + (3*d*x)/2] - 36*A*Sin[3*c + (3*d*x)/2] + 63*B*Sin[3*c + (3*d*x)/2] + 48*A*Sin[c + (5*d*x)/2] - 75*B*Sin[c + (5*d*x)/2] + 6*A*Sin[2*c + (5*d*x)/2] - 15*B*Sin[2*c + (5*d*x)/2] + 30*A*Sin[3*c + (5*d*x)/2] - 39*B*Sin[3*c + (5*d*x)/2] - 12*A*Sin[4*c + (5*d*x)/2] + 21*B*Sin[4*c + (5*d*x)/2] + 20*A*Sin[2*c + (7*d*x)/2] - 32*B*Sin[2*c + (7*d*x)/2] + 6*A*Sin[3*c + (7*d*x)/2] - 12*B*Sin[3*c + (7*d*x)/2] + 14*A*Sin[4*c + (7*d*x)/2] - 20*B*Sin[4*c + (7*d*x)/2]))/(48*a^2*d*(1 + Cos[c + d*x])^2)

fricas [A] time = 0.45, size = 228, normalized size = 1.46

$$\frac{3\left((4A - 7B)\cos(dx + c)^4 + 2(4A - 7B)\cos(dx + c)^3 + (4A - 7B)\cos(dx + c)^2\right)\log(\sin(dx + c) + 1) - 3\left(\dots\right)}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="fricas")

[Out] -1/12*(3*((4*A - 7*B)*cos(d*x + c)^4 + 2*(4*A - 7*B)*cos(d*x + c)^3 + (4*A - 7*B)*cos(d*x + c)^2)*log(sin(d*x + c) + 1) - 3*((4*A - 7*B)*cos(d*x + c)^4 + 2*(4*A - 7*B)*cos(d*x + c)^3 + (4*A - 7*B)*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) - 2*(4*(5*A - 8*B)*cos(d*x + c)^3 + (28*A - 43*B)*cos(d*x + c)^2 + 6*(A - B)*cos(d*x + c) + 3*B)*sin(d*x + c))/(a^2*d*cos(d*x + c)^4 + 2*a^2*d*cos(d*x + c)^3 + a^2*d*cos(d*x + c)^2)

giac [A] time = 0.29, size = 198, normalized size = 1.27

$$\frac{3(4A-7B)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^2} - \frac{3(4A-7B)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^2} + \frac{6\left(2A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-5B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-2A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+3B\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2 a^2}$$

6d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="giac")

[Out]
$$-1/6*(3*(4*A - 7*B)*\log(\text{abs}(\tan(1/2*d*x + 1/2*c) + 1))/a^2 - 3*(4*A - 7*B)*\log(\text{abs}(\tan(1/2*d*x + 1/2*c) - 1))/a^2 + 6*(2*A*\tan(1/2*d*x + 1/2*c)^3 - 5*B*\tan(1/2*d*x + 1/2*c)^3 - 2*A*\tan(1/2*d*x + 1/2*c) + 3*B*\tan(1/2*d*x + 1/2*c))/((\tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^2) - (A*a^4*\tan(1/2*d*x + 1/2*c)^3 - B*a^4*\tan(1/2*d*x + 1/2*c)^3 + 15*A*a^4*\tan(1/2*d*x + 1/2*c) - 21*B*a^4*\tan(1/2*d*x + 1/2*c))/a^6)/d$$

maple [B] time = 0.68, size = 294, normalized size = 1.88

$$\frac{\left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right) A}{6d a^2} - \frac{B \left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{6d a^2} + \frac{5A \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{2d a^2} - \frac{7B \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{2d a^2} - \frac{A}{d a^2 \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)} + \frac{1}{2d a^2 \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(\sec(dx+c)^4*(A+B*\sec(dx+c))/(a+a*\sec(dx+c))^2, x)$

[Out]
$$1/6/d/a^2*\tan(1/2*d*x+1/2*c)^3*A-1/6/d/a^2*B*\tan(1/2*d*x+1/2*c)^3+5/2/d/a^2*A*\tan(1/2*d*x+1/2*c)-7/2/d/a^2*B*\tan(1/2*d*x+1/2*c)-1/d/a^2*A/(\tan(1/2*d*x+1/2*c)-1)+5/2/d/a^2/(\tan(1/2*d*x+1/2*c)-1)*B+2/d/a^2*A*\ln(\tan(1/2*d*x+1/2*c)-1)-7/2/d/a^2*\ln(\tan(1/2*d*x+1/2*c)-1)*B+1/2/d/a^2/(\tan(1/2*d*x+1/2*c)-1)^2*B-1/d/a^2*A/(\tan(1/2*d*x+1/2*c)+1)+5/2/d/a^2/(\tan(1/2*d*x+1/2*c)+1)*B-2/d/a^2*A*\ln(\tan(1/2*d*x+1/2*c)+1)+7/2/d/a^2*\ln(\tan(1/2*d*x+1/2*c)+1)*B-1/2/d/a^2/(\tan(1/2*d*x+1/2*c)+1)^2*B$$

maxima [B] time = 0.36, size = 336, normalized size = 2.15

$$\frac{B \left(\frac{6 \left(\frac{3 \sin(dx+c)}{\cos(dx+c)+1} - \frac{5 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} \right)}{a^2 - \frac{2a^2 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{a^2 \sin(dx+c)^4}{(\cos(dx+c)+1)^4}} + \frac{21 \sin(dx+c)}{\cos(dx+c)+1} + \frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3} - \frac{21 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1} + 1\right)}{a^2} + \frac{21 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1} - 1\right)}{a^2} \right) - A \left(\frac{15 \sin(dx+c)}{\cos(dx+c)+1} + \frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3} \right)}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(\sec(dx+c)^4*(A+B*\sec(dx+c))/(a+a*\sec(dx+c))^2, x, \text{algorithm}="maxima")$

[Out]
$$-1/6*(B*(6*(3*\sin(dx + c)/(\cos(dx + c) + 1) - 5*\sin(dx + c)^3/(\cos(dx + c) + 1)^3)/(a^2 - 2*a^2*\sin(dx + c)^2/(\cos(dx + c) + 1)^2 + a^2*\sin(dx + c)^4/(\cos(dx + c) + 1)^4) + (21*\sin(dx + c)/(\cos(dx + c) + 1) + \sin(dx + c)^3/(\cos(dx + c) + 1)^3)/a^2 - 21*\log(\sin(dx + c)/(\cos(dx + c) + 1) + 1)/a^2 + 21*\log(\sin(dx + c)/(\cos(dx + c) + 1) - 1)/a^2) - A*((15*\sin(dx + c)/(\cos(dx + c) + 1) + \sin(dx + c)^3/(\cos(dx + c) + 1)^3)/a^2 - 12*\log(\sin(dx + c)/(\cos(dx + c) + 1) + 1)/a^2 + 12*\log(\sin(dx + c)/(\cos(dx + c) + 1) - 1)/a^2 + 12*\sin(dx + c)/((a^2 - a^2*\sin(dx + c)^2/(\cos(dx + c) + 1)^2)*(cos(dx + c) + 1))))/d$$

mupad [B] time = 1.93, size = 166, normalized size = 1.06

$$\frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right) \left(\frac{3(A-B)}{2a^2} + \frac{2A-4B}{2a^2}\right)}{d} - \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 (2A - 5B) - \tan\left(\frac{c}{2} + \frac{dx}{2}\right) (2A - 3B)}{d \left(a^2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 - 2a^2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + a^2\right)} + \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 (A - B)}{6a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^4*(a + a/cos(c + d*x))^2), x)

[Out] (tan(c/2 + (d*x)/2)*((3*(A - B))/(2*a^2) + (2*A - 4*B)/(2*a^2)))/d - (tan(c/2 + (d*x)/2)^3*(2*A - 5*B) - tan(c/2 + (d*x)/2)*(2*A - 3*B))/(d*(a^2*tan(c/2 + (d*x)/2)^4 - 2*a^2*tan(c/2 + (d*x)/2)^2 + a^2)) + (tan(c/2 + (d*x)/2)^3*(A - B))/(6*a^2*d) - (atanh(tan(c/2 + (d*x)/2))*(4*A - 7*B))/(a^2*d)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A \sec^4(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1} dx + \int \frac{B \sec^5(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1} dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**2, x)

[Out] (Integral(A*sec(c + d*x)**4/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x) + Integral(B*sec(c + d*x)**5/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x))/a**2

$$3.92 \quad \int \frac{\sec^3(c+dx)(A+B \sec(c+dx))}{(a+a \sec(c+dx))^2} dx$$

Optimal. Leaf size=108

$$\frac{(A-4B) \tan(c+dx)}{3a^2d} + \frac{(A-2B) \tanh^{-1}(\sin(c+dx))}{a^2d} - \frac{(A-2B) \tan(c+dx)}{a^2d(\sec(c+dx)+1)} + \frac{(A-B) \tan(c+dx) \sec^2(c+dx)}{3d(a \sec(c+dx)+a)^2}$$

[Out] (A-2*B)*arctanh(sin(d*x+c))/a^2/d-1/3*(A-4*B)*tan(d*x+c)/a^2/d-(A-2*B)*tan(d*x+c)/a^2/d/(1+sec(d*x+c))+1/3*(A-B)*sec(d*x+c)^2*tan(d*x+c)/d/(a+a*sec(d*x+c))^2

Rubi [A] time = 0.26, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4019, 4008, 3787, 3770, 3767, 8}

$$\frac{(A-4B) \tan(c+dx)}{3a^2d} + \frac{(A-2B) \tanh^{-1}(\sin(c+dx))}{a^2d} - \frac{(A-2B) \tan(c+dx)}{a^2d(\sec(c+dx)+1)} + \frac{(A-B) \tan(c+dx) \sec^2(c+dx)}{3d(a \sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

[In] Int[(Sec[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^2,x]

[Out] ((A - 2*B)*ArcTanh[Sin[c + d*x]]/(a^2*d) - ((A - 4*B)*Tan[c + d*x])/(3*a^2*d) - ((A - 2*B)*Tan[c + d*x])/(a^2*d*(1 + Sec[c + d*x])) + ((A - B)*Sec[c + d*x]^2*Tan[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2)

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 3767

Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_)])*(d_.)^(n_.)*(csc[(e_.) + (f_.)*(x_)])*(b_.) + (a_.), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[

$(d*\text{Csc}[e + f*x])^{(n + 1)}, x], x] /; \text{FreeQ}[\{a, b, d, e, f, n\}, x]$

Rule 4008

$\text{Int}[\text{csc}[(e_.) + (f_.)*(x_.)]^2*(\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^{(m_.)}*(\text{csc}[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> -\text{Simp}[\frac{(A*b - a*B)*\text{Cot}[e + f*x]*(a + b*\text{Csc}[e + f*x])^m}{b*f*(2*m + 1)}, x] + \text{Dist}[1/(b^2*(2*m + 1)), \text{Int}[\text{Csc}[e + f*x]*(a + b*\text{Csc}[e + f*x])^{(m + 1)}*\text{Simp}[A*b*m - a*B*m + b*B*(2*m + 1)*\text{Csc}[e + f*x], x], x], x] /; \text{FreeQ}[\{a, b, e, f, A, B\}, x] \&\& \text{NeQ}[A*b - a*B, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{LtQ}[m, -2^{(-1)}]$

Rule 4019

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(d_.))^{(n_.)}*(\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^{(m_.)}*(\text{csc}[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> \text{Simp}[\frac{(d*(A*b - a*B)*\text{Cot}[e + f*x]*(a + b*\text{Csc}[e + f*x])^m*(d*\text{Csc}[e + f*x])^{(n - 1)})}{a*f*(2*m + 1)}, x] - \text{Dist}[1/(a*b*(2*m + 1)), \text{Int}[(a + b*\text{Csc}[e + f*x])^{(m + 1)}*(d*\text{Csc}[e + f*x])^{(n - 1)}*\text{Simp}[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m - n + 1) + A*b*(m + n))*\text{Csc}[e + f*x], x], x], x] /; \text{FreeQ}[\{a, b, d, e, f, A, B\}, x] \&\& \text{NeQ}[A*b - a*B, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{LtQ}[m, -2^{(-1)}] \&\& \text{GtQ}[n, 0]$

Rubi steps

$$\begin{aligned} \int \frac{\sec^3(c + dx)(A + B \sec(c + dx))}{(a + a \sec(c + dx))^2} dx &= \frac{(A - B) \sec^2(c + dx) \tan(c + dx)}{3d(a + a \sec(c + dx))^2} + \frac{\int \frac{\sec^2(c + dx)(2a(A - B) - a(A - 4B) \sec(c + dx))}{a + a \sec(c + dx)} dx}{3a^2} \\ &= -\frac{(A - 2B) \tan(c + dx)}{a^2 d (1 + \sec(c + dx))} + \frac{(A - B) \sec^2(c + dx) \tan(c + dx)}{3d(a + a \sec(c + dx))^2} - \frac{\int \sec(c + dx)}{3a^2} \\ &= -\frac{(A - 2B) \tan(c + dx)}{a^2 d (1 + \sec(c + dx))} + \frac{(A - B) \sec^2(c + dx) \tan(c + dx)}{3d(a + a \sec(c + dx))^2} - \frac{(A - 4B) \int \sec(c + dx)}{3a^2} \\ &= \frac{(A - 2B) \tanh^{-1}(\sin(c + dx))}{a^2 d} - \frac{(A - 2B) \tan(c + dx)}{a^2 d (1 + \sec(c + dx))} + \frac{(A - B) \sec^2(c + dx) \tan(c + dx)}{3d(a + a \sec(c + dx))^2} \\ &= \frac{(A - 2B) \tanh^{-1}(\sin(c + dx))}{a^2 d} - \frac{(A - 4B) \tan(c + dx)}{3a^2 d} - \frac{(A - 2B) \tan(c + dx)}{a^2 d (1 + \sec(c + dx))} \end{aligned}$$

Mathematica [B] time = 1.94, size = 292, normalized size = 2.70

$$2 \cos\left(\frac{1}{2}(c + dx)\right) \sec(c + dx)(A + B \sec(c + dx)) \left(-(A - B) \tan\left(\frac{c}{2}\right) \cos\left(\frac{1}{2}(c + dx)\right) + (B - A) \sec\left(\frac{c}{2}\right) \sin\left(\frac{dx}{2}\right) \right) +$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^2,x]

[Out] (2*Cos[(c + d*x)/2]*Sec[c + d*x]*(A + B*Sec[c + d*x])*((-A + B)*Sec[c/2]*Sin[(d*x)/2] - 2*(4*A - 7*B)*Cos[(c + d*x)/2]^2*Sec[c/2]*Sin[(d*x)/2] + Cos[(c + d*x)/2]^3*(-6*(A - 2*B)*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])) + (6*B*Sin[d*x])/((Cos[c/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])) - (A - B)*Cos[(c + d*x)/2]*Tan[c/2]))/(3*a^2*d*(B + A*Cos[c + d*x])*(1 + Sec[c + d*x])^2)

fricas [A] time = 0.47, size = 195, normalized size = 1.81

$$3 \left((A - 2B) \cos(dx + c)^3 + 2(A - 2B) \cos(dx + c)^2 + (A - 2B) \cos(dx + c) \right) \log(\sin(dx + c) + 1) - 3 \left((A - 2B) \cos(dx + c)^3 + 2(A - 2B) \cos(dx + c)^2 + (A - 2B) \cos(dx + c) \right) \log(-\sin(dx + c) + 1) - 2 \left((2A - 5B) \cos(dx + c)^2 + (5A - 14B) \cos(dx + c) - 3B \sin(dx + c) \right) / (a^2 d \cos(dx + c)^3 + 2 a^2 d \cos(dx + c)^2 + a^2 d \cos(dx + c))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="fricas")

[Out] 1/6*(3*((A - 2*B)*cos(d*x + c)^3 + 2*(A - 2*B)*cos(d*x + c)^2 + (A - 2*B)*cos(d*x + c))*log(sin(d*x + c) + 1) - 3*((A - 2*B)*cos(d*x + c)^3 + 2*(A - 2*B)*cos(d*x + c)^2 + (A - 2*B)*cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(2*(2*A - 5*B)*cos(d*x + c)^2 + (5*A - 14*B)*cos(d*x + c) - 3*B*sin(d*x + c)) / (a^2*d*cos(d*x + c)^3 + 2*a^2*d*cos(d*x + c)^2 + a^2*d*cos(d*x + c))

giac [A] time = 0.29, size = 151, normalized size = 1.40

$$\frac{6(A-2B) \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right|\right)}{a^2} - \frac{6(A-2B) \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1\right|\right)}{a^2} - \frac{12B \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1\right)a^2} - \frac{Aa^4 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 - Ba^4 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)}{a^2} + \frac{6d}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="giac")

[Out] $\frac{1}{6} * (6 * (A - 2 * B) * \log(\text{abs}(\tan(1/2 * d * x + 1/2 * c) + 1)) / a^2 - 6 * (A - 2 * B) * \log(\text{abs}(\tan(1/2 * d * x + 1/2 * c) - 1)) / a^2 - 12 * B * \tan(1/2 * d * x + 1/2 * c) / ((\tan(1/2 * d * x + 1/2 * c)^2 - 1) * a^2) - (A * a^4 * \tan(1/2 * d * x + 1/2 * c)^3 - B * a^4 * \tan(1/2 * d * x + 1/2 * c)^3 + 9 * A * a^4 * \tan(1/2 * d * x + 1/2 * c) - 15 * B * a^4 * \tan(1/2 * d * x + 1/2 * c)) / a^6) / d$

maple [A] time = 0.70, size = 205, normalized size = 1.90

$$-\frac{\left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right) A}{6d a^2} + \frac{B \left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{6d a^2} - \frac{3A \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{2d a^2} + \frac{5B \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{2d a^2} - \frac{A \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}{d a^2} + \frac{2 \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}{d a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x)`

[Out] $-1/6/d/a^2 * \tan(1/2 * d * x + 1/2 * c)^3 * A + 1/6/d/a^2 * B * \tan(1/2 * d * x + 1/2 * c)^3 - 3/2/d/a^2 * A * \tan(1/2 * d * x + 1/2 * c) + 5/2/d/a^2 * B * \tan(1/2 * d * x + 1/2 * c) - 1/d/a^2 * A * \ln(\tan(1/2 * d * x + 1/2 * c) - 1) + 2/d/a^2 * \ln(\tan(1/2 * d * x + 1/2 * c) - 1) * B - 1/d/a^2 * (\tan(1/2 * d * x + 1/2 * c) - 1) * B + 1/d/a^2 * A * \ln(\tan(1/2 * d * x + 1/2 * c) + 1) - 2/d/a^2 * \ln(\tan(1/2 * d * x + 1/2 * c) + 1) * B - 1/d/a^2 * (\tan(1/2 * d * x + 1/2 * c) + 1) * B$

maxima [B] time = 0.34, size = 244, normalized size = 2.26

$$B \left(\frac{\frac{15 \sin(dx+c)}{\cos(dx+c)+1} + \frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2} - \frac{12 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1} + 1\right)}{a^2} + \frac{12 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1} - 1\right)}{a^2} + \frac{12 \sin(dx+c)}{\left(a^2 - \frac{a^2 \sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right) (\cos(dx+c)+1)} \right) - A \left(\frac{\frac{9 \sin(dx+c)}{\cos(dx+c)+1} + \frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2} - \frac{12 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1} + 1\right)}{a^2} + \frac{12 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1} - 1\right)}{a^2} + \frac{12 \sin(dx+c)}{\left(a^2 - \frac{a^2 \sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right) (\cos(dx+c)+1)} \right) \frac{1}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="maxima")`

[Out] $\frac{1}{6} * (B * ((15 * \sin(d * x + c) / (\cos(d * x + c) + 1) + \sin(d * x + c)^3 / (\cos(d * x + c) + 1)^3) / a^2 - 12 * \log(\sin(d * x + c) / (\cos(d * x + c) + 1) + 1) / a^2 + 12 * \log(\sin(d * x + c) / (\cos(d * x + c) + 1) - 1) / a^2 + 12 * \sin(d * x + c) / ((a^2 - a^2 * \sin(d * x + c)^2 / (\cos(d * x + c) + 1)^2) * (\cos(d * x + c) + 1))) - A * ((9 * \sin(d * x + c) / (\cos(d * x + c) + 1) + \sin(d * x + c)^3 / (\cos(d * x + c) + 1)^3) / a^2 - 6 * \log(\sin(d * x + c) / (\cos(d * x + c) + 1) + 1) / a^2 + 6 * \log(\sin(d * x + c) / (\cos(d * x + c) + 1) - 1) / a^2)) / d$

mupad [B] time = 1.93, size = 120, normalized size = 1.11

$$\frac{2 \operatorname{atanh}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right) (A - 2B)}{a^2 d} - \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 (A - B)}{6 a^2 d} - \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right) \left(\frac{A-B}{a^2} + \frac{A-3B}{2a^2}\right)}{d} - \frac{2 B \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{d \left(a^2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 - a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B/cos(c + d*x))/(cos(c + d*x)^3*(a + a/cos(c + d*x))^2),x)`

[Out] $(2*\operatorname{atanh}(\tan(c/2 + (d*x)/2))*(A - 2*B))/(a^2*d) - (\tan(c/2 + (d*x)/2)^3*(A - B))/(6*a^2*d) - (\tan(c/2 + (d*x)/2)*((A - B)/a^2 + (A - 3*B)/(2*a^2)))/d - (2*B*\tan(c/2 + (d*x)/2))/(d*(a^2*\tan(c/2 + (d*x)/2)^2 - a^2))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A \sec^3(c+dx)}{\sec^2(c+dx)+2 \sec(c+dx)+1} dx + \int \frac{B \sec^4(c+dx)}{\sec^2(c+dx)+2 \sec(c+dx)+1} dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)**3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**2,x)`

[Out] $(\operatorname{Integral}(A*\sec(c + d*x)**3/(\sec(c + d*x)**2 + 2*\sec(c + d*x) + 1), x) + \operatorname{Integral}(B*\sec(c + d*x)**4/(\sec(c + d*x)**2 + 2*\sec(c + d*x) + 1), x))/a**2$

$$3.93 \quad \int \frac{\sec^2(c+dx)(A+B \sec(c+dx))}{(a+a \sec(c+dx))^2} dx$$

Optimal. Leaf size=79

$$\frac{(2A-5B) \tan(c+dx)}{3a^2d(\sec(c+dx)+1)} + \frac{B \tanh^{-1}(\sin(c+dx))}{a^2d} - \frac{(A-B) \tan(c+dx)}{3d(a \sec(c+dx)+a)^2}$$

[Out] B*arctanh(sin(d*x+c))/a^2/d+1/3*(2*A-5*B)*tan(d*x+c)/a^2/d/(1+sec(d*x+c))-1/3*(A-B)*tan(d*x+c)/d/(a+a*sec(d*x+c))^2

Rubi [A] time = 0.19, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {4008, 3998, 3770, 3794}

$$\frac{(2A-5B) \tan(c+dx)}{3a^2d(\sec(c+dx)+1)} + \frac{B \tanh^{-1}(\sin(c+dx))}{a^2d} - \frac{(A-B) \tan(c+dx)}{3d(a \sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

[In] Int[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^2,x]

[Out] (B*ArcTanh[Sin[c + d*x]]/(a^2*d) + ((2*A - 5*B)*Tan[c + d*x])/(3*a^2*d*(1 + Sec[c + d*x])) - ((A - B)*Tan[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2)

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3794

Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] := -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 3998

Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] := Dist[B/b, Int[Csc[e + f*x], x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]

Rule 4008

```
Int[csc[(e_.) + (f_.)*(x_.)]^2*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(
csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := -Simp[((A*b - a*B)*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(b^2*(2*m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*B*
(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A
*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned} \int \frac{\sec^2(c + dx)(A + B \sec(c + dx))}{(a + a \sec(c + dx))^2} dx &= -\frac{(A - B) \tan(c + dx)}{3d(a + a \sec(c + dx))^2} - \frac{\int \frac{\sec(c+dx)(-2a(A-B)-3aB \sec(c+dx))}{a+a \sec(c+dx)} dx}{3a^2} \\ &= -\frac{(A - B) \tan(c + dx)}{3d(a + a \sec(c + dx))^2} + \frac{(2A - 5B) \int \frac{\sec(c+dx)}{a+a \sec(c+dx)} dx}{3a} + \frac{B \int \sec(c + dx)}{a^2} \\ &= \frac{B \tanh^{-1}(\sin(c + dx))}{a^2 d} - \frac{(A - B) \tan(c + dx)}{3d(a + a \sec(c + dx))^2} + \frac{(2A - 5B) \tan(c + dx)}{3d(a^2 + a^2 \sec(c + dx))} \end{aligned}$$

Mathematica [B] time = 0.56, size = 169, normalized size = 2.14

$$\frac{2 \cos\left(\frac{1}{2}(c + dx)\right) \left(-(A - B) \tan\left(\frac{c}{2}\right) \cos\left(\frac{1}{2}(c + dx)\right) + (B - A) \sec\left(\frac{c}{2}\right) \sin\left(\frac{dx}{2}\right) - 2(A - 4B) \sec\left(\frac{c}{2}\right) \sin\left(\frac{dx}{2}\right) \cos\left(\frac{1}{2}(c + dx)\right) \right)}{3a^2 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^2,x]
[Out] (-2*Cos[(c + d*x)/2]*(6*B*Cos[(c + d*x)/2]^3*(Log[Cos[(c + d*x)/2] - Sin[(c
+ d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + (-A + B)*Sec[c/2]
*Sin[(d*x)/2] - 2*(A - 4*B)*Cos[(c + d*x)/2]^2*Sec[c/2]*Sin[(d*x)/2] - (A -
B)*Cos[(c + d*x)/2]*Tan[c/2))/(3*a^2*d*(1 + Cos[c + d*x])^2)
```

fricas [A] time = 0.44, size = 129, normalized size = 1.63

$$\frac{3 \left(B \cos(dx + c)^2 + 2 B \cos(dx + c) + B \right) \log(\sin(dx + c) + 1) - 3 \left(B \cos(dx + c)^2 + 2 B \cos(dx + c) + B \right) \log(\sin(dx + c) - 1)}{6 \left(a^2 d \cos(dx + c)^2 + 2 a^2 d \cos(dx + c) + a^2 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="fr
icas")
```

[Out] $1/6*(3*(B*\cos(dx + c)^2 + 2*B*\cos(dx + c) + B)*\log(\sin(dx + c) + 1) - 3*(B*\cos(dx + c)^2 + 2*B*\cos(dx + c) + B)*\log(-\sin(dx + c) + 1) + 2*((A - 4*B)*\cos(dx + c) + 2*A - 5*B)*\sin(dx + c))/(a^2*d*\cos(dx + c)^2 + 2*a^2*d*\cos(dx + c) + a^2*d)$

giac [A] time = 0.50, size = 112, normalized size = 1.42

$$\frac{6B \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right|\right)}{a^2} - \frac{6B \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1\right|\right)}{a^2} + \frac{Aa^4 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 - Ba^4 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 + 3Aa^4 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 9Ba^4 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)}{6d a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(dx+c)^2*(A+B*sec(dx+c))/(a+a*sec(dx+c))^2,x, algorithm="giac")`

[Out] $1/6*(6*B*\log(\text{abs}(\tan(1/2*d*x + 1/2*c) + 1))/a^2 - 6*B*\log(\text{abs}(\tan(1/2*d*x + 1/2*c) - 1))/a^2 + (A*a^4*\tan(1/2*d*x + 1/2*c)^3 - B*a^4*\tan(1/2*d*x + 1/2*c)^3 + 3*A*a^4*\tan(1/2*d*x + 1/2*c) - 9*B*a^4*\tan(1/2*d*x + 1/2*c))/a^6)/d$

maple [A] time = 0.85, size = 119, normalized size = 1.51

$$\frac{\left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)A}{6d a^2} - \frac{B\left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{6d a^2} + \frac{A \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{2d a^2} - \frac{3B \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{2d a^2} - \frac{\ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)B}{d a^2} + \frac{\ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)B}{d a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sec(dx+c)^2*(A+B*sec(dx+c))/(a+a*sec(dx+c))^2,x)`

[Out] $1/6/d/a^2*\tan(1/2*d*x+1/2*c)^3*A - 1/6/d/a^2*B*\tan(1/2*d*x+1/2*c)^3 + 1/2/d/a^2*A*\tan(1/2*d*x+1/2*c) - 3/2/d/a^2*B*\tan(1/2*d*x+1/2*c) - 1/d/a^2*\ln(\tan(1/2*d*x+1/2*c)-1)*B + 1/d/a^2*\ln(\tan(1/2*d*x+1/2*c)+1)*B$

maxima [A] time = 0.35, size = 145, normalized size = 1.84

$$\frac{B\left(\frac{9 \sin(dx+c)}{\cos(dx+c)+1} + \frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3} - \frac{6 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1} + 1\right)}{a^2} + \frac{6 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1} - 1\right)}{a^2}\right)}{6d} - \frac{A\left(\frac{3 \sin(dx+c)}{\cos(dx+c)+1} + \frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(dx+c)^2*(A+B*sec(dx+c))/(a+a*sec(dx+c))^2,x, algorithm="maxima")`

[Out] $-1/6*(B*((9*\sin(dx + c))/(\cos(dx + c) + 1) + \sin(dx + c)^3/(\cos(dx + c) + 1)^3)/a^2 - 6*\log(\sin(dx + c)/(\cos(dx + c) + 1) + 1)/a^2 + 6*\log(\sin(dx + c)/(\cos(dx + c) + 1) - 1)/a^2) + A*(3*\sin(dx + c)/(\cos(dx + c) + 1) + \sin(dx + c)^3/(\cos(dx + c) + 1)^3)/a^2$

$x + c)/(\cos(dx + c) + 1) - 1)/a^2) - A*(3*\sin(dx + c)/(\cos(dx + c) + 1) + \sin(dx + c)^3/(\cos(dx + c) + 1)^3)/a^2)/d$

mupad [B] time = 1.92, size = 74, normalized size = 0.94

$$\frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right) \left(\frac{A-B}{2a^2} - \frac{B}{a^2}\right)}{d} + \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 (A-B)}{6a^2 d} + \frac{2B \operatorname{atanh}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B/cos(c + d*x))/(cos(c + d*x)^2*(a + a/cos(c + d*x))^2), x)`

[Out] $(\tan(c/2 + (d*x)/2)*((A - B)/(2*a^2) - B/a^2))/d + (\tan(c/2 + (d*x)/2)^3*(A - B))/(6*a^2*d) + (2*B*atanh(\tan(c/2 + (d*x)/2)))/(a^2*d)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A \sec^2(c+dx)}{\sec^2(c+dx)+2 \sec(c+dx)+1} dx + \int \frac{B \sec^3(c+dx)}{\sec^2(c+dx)+2 \sec(c+dx)+1} dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)**2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**2, x)`

[Out] $(\operatorname{Integral}(A*\sec(c + d*x)**2/(\sec(c + d*x)**2 + 2*\sec(c + d*x) + 1), x) + \operatorname{Integral}(B*\sec(c + d*x)**3/(\sec(c + d*x)**2 + 2*\sec(c + d*x) + 1), x))/a**2$

$$3.94 \quad \int \frac{\sec(c+dx)(A+B \sec(c+dx))}{(a+a \sec(c+dx))^2} dx$$

Optimal. Leaf size=65

$$\frac{(A+2B) \tan(c+dx)}{3d(a^2 \sec(c+dx) + a^2)} + \frac{(A-B) \tan(c+dx)}{3d(a \sec(c+dx) + a)^2}$$

[Out] 1/3*(A-B)*tan(d*x+c)/d/(a+a*sec(d*x+c))^2+1/3*(A+2*B)*tan(d*x+c)/d/(a^2+a^2*sec(d*x+c))

Rubi [A] time = 0.08, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {4000, 3794}

$$\frac{(A+2B) \tan(c+dx)}{3d(a^2 \sec(c+dx) + a^2)} + \frac{(A-B) \tan(c+dx)}{3d(a \sec(c+dx) + a)^2}$$

Antiderivative was successfully verified.

[In] Int[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^2,x]

[Out] ((A - B)*Tan[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2) + ((A + 2*B)*Tan[c + d*x])/(3*d*(a^2 + a^2*Sec[c + d*x]))

Rule 3794

Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 4000

Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[((A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]

Rubi steps

$$\int \frac{\sec(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^2} dx = \frac{(A-B)\tan(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{(A+2B)\int \frac{\sec(c+dx)}{a+a\sec(c+dx)} dx}{3a}$$

$$= \frac{(A-B)\tan(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{(A+2B)\tan(c+dx)}{3d(a^2+a^2\sec(c+dx))}$$

Mathematica [A] time = 0.21, size = 76, normalized size = 1.17

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left((2A+B)\sin\left(c+\frac{3dx}{2}\right)+3(A+B)\sin\left(\frac{dx}{2}\right)-3A\sin\left(c+\frac{dx}{2}\right)\right)}{3a^2d(\cos(c+dx)+1)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^2,x]

[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(3*(A + B)*Sin[(d*x)/2] - 3*A*Sin[c + (d*x)/2] + (2*A + B)*Sin[c + (3*d*x)/2]))/(3*a^2*d*(1 + Cos[c + d*x])^2)

fricas [A] time = 0.41, size = 58, normalized size = 0.89

$$\frac{((2A+B)\cos(dx+c)+A+2B)\sin(dx+c)}{3(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="fricas")

[Out] 1/3*((2*A + B)*cos(d*x + c) + A + 2*B)*sin(d*x + c)/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)

giac [A] time = 0.26, size = 60, normalized size = 0.92

$$\frac{A \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 - B \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 - 3A \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 3B \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{6a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="giac")

[Out] $-1/6*(A*\tan(1/2*d*x + 1/2*c)^3 - B*\tan(1/2*d*x + 1/2*c)^3 - 3*A*\tan(1/2*d*x + 1/2*c) - 3*B*\tan(1/2*d*x + 1/2*c))/(a^2*d)$

maple [A] time = 0.84, size = 60, normalized size = 0.92

$$\frac{-\frac{\left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)A}{3} + \frac{B\left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{3} + A \tan\left(\frac{dx}{2} + \frac{c}{2}\right) + B \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{2d a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x)`

[Out] $1/2/d/a^2*(-1/3*\tan(1/2*d*x+1/2*c)^3*A+1/3*B*\tan(1/2*d*x+1/2*c)^3+A*\tan(1/2*d*x+1/2*c)+B*\tan(1/2*d*x+1/2*c))$

maxima [A] time = 0.34, size = 93, normalized size = 1.43

$$\frac{\frac{B\left(\frac{3 \sin(dx+c)}{\cos(dx+c)+1} + \frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^2} + \frac{A\left(\frac{3 \sin(dx+c)}{\cos(dx+c)+1} - \frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^2}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="maxima")`

[Out] $1/6*(B*(3*\sin(d*x + c)/(\cos(d*x + c) + 1) + \sin(d*x + c)^3/(\cos(d*x + c) + 1)^3)/a^2 + A*(3*\sin(d*x + c)/(\cos(d*x + c) + 1) - \sin(d*x + c)^3/(\cos(d*x + c) + 1)^3)/a^2)/d$

mupad [B] time = 1.89, size = 45, normalized size = 0.69

$$\frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right) (A + B)}{2 a^2 d} - \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 (A - B)}{6 a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B/cos(c + d*x))/(cos(c + d*x)*(a + a/cos(c + d*x))^2),x)`

[Out] $(\tan(c/2 + (d*x)/2)*(A + B))/(2*a^2*d) - (\tan(c/2 + (d*x)/2)^3*(A - B))/(6*a^2*d)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A \sec(c+dx)}{\sec^2(c+dx)+2 \sec(c+dx)+1} dx + \int \frac{B \sec^2(c+dx)}{\sec^2(c+dx)+2 \sec(c+dx)+1} dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**2,x)
```

```
[Out] (Integral(A*sec(c + d*x)/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x) + Integral(B*sec(c + d*x)**2/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x))/a**2
```

$$3.95 \quad \int \frac{A+B \sec(c+dx)}{(a+a \sec(c+dx))^2} dx$$

Optimal. Leaf size=70

$$-\frac{(4A-B) \tan(c+dx)}{3a^2 d(\sec(c+dx)+1)} + \frac{Ax}{a^2} - \frac{(A-B) \tan(c+dx)}{3d(a \sec(c+dx)+a)^2}$$

[Out] A*x/a^2-1/3*(4*A-B)*tan(d*x+c)/a^2/d/(1+sec(d*x+c))-1/3*(A-B)*tan(d*x+c)/d/(a+a*sec(d*x+c))^2

Rubi [A] time = 0.11, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3922, 3919, 3794}

$$-\frac{(4A-B) \tan(c+dx)}{3a^2 d(\sec(c+dx)+1)} + \frac{Ax}{a^2} - \frac{(A-B) \tan(c+dx)}{3d(a \sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sec[c + d*x])/(a + a*Sec[c + d*x])^2,x]

[Out] (A*x)/a^2 - ((4*A - B)*Tan[c + d*x])/(3*a^2*d*(1 + Sec[c + d*x])) - ((A - B)*Tan[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2)

Rule 3794

Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 3919

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 3922

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> -Simp[((b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && EqQ[a^2 - b^2, 0] && IntegerQ[2*m]

Rubi steps

$$\begin{aligned} \int \frac{A + B \sec(c + dx)}{(a + a \sec(c + dx))^2} dx &= -\frac{(A - B) \tan(c + dx)}{3d(a + a \sec(c + dx))^2} - \frac{\int \frac{-3aA + a(A - B) \sec(c + dx)}{a + a \sec(c + dx)} dx}{3a^2} \\ &= \frac{Ax}{a^2} - \frac{(A - B) \tan(c + dx)}{3d(a + a \sec(c + dx))^2} - \frac{(4A - B) \int \frac{\sec(c + dx)}{a + a \sec(c + dx)} dx}{3a} \\ &= \frac{Ax}{a^2} - \frac{(A - B) \tan(c + dx)}{3d(a + a \sec(c + dx))^2} - \frac{(4A - B) \tan(c + dx)}{3d(a^2 + a^2 \sec(c + dx))} \end{aligned}$$

Mathematica [B] time = 0.38, size = 153, normalized size = 2.19

$$\frac{\sec\left(\frac{c}{2}\right) \sec^3\left(\frac{1}{2}(c + dx)\right) \left(12A \sin\left(c + \frac{dx}{2}\right) - 10A \sin\left(c + \frac{3dx}{2}\right) + 9Adx \cos\left(c + \frac{dx}{2}\right) + 3Adx \cos\left(c + \frac{3dx}{2}\right) + 3A\right)}{24a^2d}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sec[c + d*x])/(a + a*Sec[c + d*x])^2,x]

[Out] (Sec[c/2]*Sec[(c + d*x)/2]^3*(9*A*d*x*Cos[(d*x)/2] + 9*A*d*x*Cos[c + (d*x)/2] + 3*A*d*x*Cos[c + (3*d*x)/2] + 3*A*d*x*Cos[2*c + (3*d*x)/2] - 18*A*Sin[(d*x)/2] + 6*B*Sin[(d*x)/2] + 12*A*Sin[c + (d*x)/2] - 6*B*Sin[c + (d*x)/2] - 10*A*Sin[c + (3*d*x)/2] + 4*B*Sin[c + (3*d*x)/2]))/(24*a^2*d)

fricas [A] time = 0.44, size = 94, normalized size = 1.34

$$\frac{3 Adx \cos(dx + c)^2 + 6 Adx \cos(dx + c) + 3 Adx - ((5 A - 2 B) \cos(dx + c) + 4 A - B) \sin(dx + c)}{3(a^2d \cos(dx + c)^2 + 2a^2d \cos(dx + c) + a^2d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="fricas")

[Out] 1/3*(3*A*d*x*cos(d*x + c)^2 + 6*A*d*x*cos(d*x + c) + 3*A*d*x - ((5*A - 2*B)*cos(d*x + c) + 4*A - B)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)

giac [A] time = 0.22, size = 85, normalized size = 1.21

$$\frac{\frac{6(dx+c)A}{a^2} + \frac{Aa^4 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 - Ba^4 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 - 9Aa^4 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 3Ba^4 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)}{a^6}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="giac")

[Out] $\frac{1}{6}*(6*(d*x + c)*A/a^2 + (A*a^4*\tan(1/2*d*x + 1/2*c))^3 - B*a^4*\tan(1/2*d*x + 1/2*c)^3 - 9*A*a^4*\tan(1/2*d*x + 1/2*c) + 3*B*a^4*\tan(1/2*d*x + 1/2*c))/a^6/d$

maple [A] time = 0.92, size = 97, normalized size = 1.39

$$\frac{\left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right) A}{6d a^2} - \frac{B \left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{6d a^2} - \frac{3A \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{2d a^2} + \frac{B \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{2d a^2} + \frac{2 \arctan\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right)\right) A}{d a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x)

[Out] $\frac{1}{6}/d/a^2*\tan(1/2*d*x+1/2*c)^3*A - \frac{1}{6}/d/a^2*B*\tan(1/2*d*x+1/2*c)^3 - \frac{3}{2}/d/a^2*A*\tan(1/2*d*x+1/2*c) + \frac{1}{2}/d/a^2*B*\tan(1/2*d*x+1/2*c) + \frac{2}{d}/a^2*\arctan(\tan(1/2*d*x+1/2*c))*A$

maxima [A] time = 0.44, size = 120, normalized size = 1.71

$$\frac{A \left(\frac{9 \sin(dx+c) - \frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2} - \frac{12 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2} \right) - \frac{B \left(\frac{3 \sin(dx+c) - \frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2} \right)}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="maxima")

[Out] $\frac{-1}{6}*(A*((9*\sin(d*x + c)/(\cos(d*x + c) + 1) - \sin(d*x + c)^3/(\cos(d*x + c) + 1)^3)/a^2 - 12*\arctan(\sin(d*x + c)/(\cos(d*x + c) + 1))/a^2) - B*(3*\sin(d*x + c)/(\cos(d*x + c) + 1) - \sin(d*x + c)^3/(\cos(d*x + c) + 1)^3)/a^2)/d$

mupad [B] time = 1.93, size = 65, normalized size = 0.93

$$\frac{3B \tan\left(\frac{c}{2} + \frac{dx}{2}\right) - 9A \tan\left(\frac{c}{2} + \frac{dx}{2}\right) + A \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 - B \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 + 6A dx}{6a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/(a + a/cos(c + d*x))^2,x)

[Out] $(3*B*\tan(c/2 + (d*x)/2) - 9*A*\tan(c/2 + (d*x)/2) + A*\tan(c/2 + (d*x)/2)^3 - B*\tan(c/2 + (d*x)/2)^3 + 6*A*d*x)/(6*a^2*d)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A}{\sec^2(c+dx)+2\sec(c+dx)+1} dx + \int \frac{B \sec(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1} dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))**2,x)

[Out] (Integral(A/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x) + Integral(B*sec(c + d*x)/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x))/a**2

$$3.96 \quad \int \frac{\cos(c+dx)(A+B \sec(c+dx))}{(a+a \sec(c+dx))^2} dx$$

Optimal. Leaf size=98

$$\frac{2(5A-2B)\sin(c+dx)}{3a^2d} - \frac{(2A-B)\sin(c+dx)}{a^2d(\sec(c+dx)+1)} - \frac{x(2A-B)}{a^2} - \frac{(A-B)\sin(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

[Out] $-(2*A-B)*x/a^2+2/3*(5*A-2*B)*\sin(d*x+c)/a^2/d-(2*A-B)*\sin(d*x+c)/a^2/d/(1+\sec(d*x+c))-1/3*(A-B)*\sin(d*x+c)/d/(a+a*\sec(d*x+c))^2$

Rubi [A] time = 0.23, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {4020, 3787, 2637, 8}

$$\frac{2(5A-2B)\sin(c+dx)}{3a^2d} - \frac{(2A-B)\sin(c+dx)}{a^2d(\sec(c+dx)+1)} - \frac{x(2A-B)}{a^2} - \frac{(A-B)\sin(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(\text{Cos}[c+d*x]*(A+B*\text{Sec}[c+d*x]))/(a+a*\text{Sec}[c+d*x])^2,x]$

[Out] $-\left(\frac{(2A-B)x}{a^2}\right) + \frac{2(5A-2B)\text{Sin}[c+d*x]}{(3a^2d)} - \frac{(2A-B)\text{Sin}[c+d*x]}{(a^2d(1+\text{Sec}[c+d*x]))} - \frac{(A-B)\text{Sin}[c+d*x]}{(3d(a+a*\text{Sec}[c+d*x])^2)}$

Rule 8

$\text{Int}[a_, x_Symbol] \rightarrow \text{Simp}[a*x, x] /; \text{FreeQ}[a, x]$

Rule 2637

$\text{Int}[\sin[\text{Pi}/2 + (c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow \text{Simp}[\text{Sin}[c+d*x]/d, x] /; \text{FreeQ}[\{c, d\}, x]$

Rule 3787

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(d_.))^{(n_.)}*(\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] \rightarrow \text{Dist}[a, \text{Int}[(d*\text{Csc}[e+f*x])^n, x], x] + \text{Dist}[b/d, \text{Int}[(d*\text{Csc}[e+f*x])^{(n+1)}, x], x] /; \text{FreeQ}[\{a, b, d, e, f, n\}, x]$

Rule 4020

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(d_.))^{(n_.)}*(\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^{(m_.)}*(\text{csc}[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] \rightarrow -\text{Simp}[(A*b - a*B)*\text{Cot}[e+f*x]*(a+b*\text{Csc}[e+f*x])^m*(d*\text{Csc}[e+f*x])^n]/(b*f*(2*m +$

1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e + f*x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0]

Rubi steps

$$\begin{aligned} \int \frac{\cos(c + dx)(A + B \sec(c + dx))}{(a + a \sec(c + dx))^2} dx &= -\frac{(A - B) \sin(c + dx)}{3d(a + a \sec(c + dx))^2} + \frac{\int \frac{\cos(c+dx)(a(4A-B)-2a(A-B)\sec(c+dx))}{a+a \sec(c+dx)} dx}{3a^2} \\ &= -\frac{(2A - B) \sin(c + dx)}{a^2d(1 + \sec(c + dx))} - \frac{(A - B) \sin(c + dx)}{3d(a + a \sec(c + dx))^2} + \frac{\int \cos(c + dx) (2a^2(5A - 2B) \sec(c + dx) + 3a^2)}{3a^2} \\ &= -\frac{(2A - B) \sin(c + dx)}{a^2d(1 + \sec(c + dx))} - \frac{(A - B) \sin(c + dx)}{3d(a + a \sec(c + dx))^2} + \frac{(2(5A - 2B)) \int \cos(c + dx)}{3a^2} \\ &= -\frac{(2A - B)x}{a^2} + \frac{2(5A - 2B) \sin(c + dx)}{3a^2d} - \frac{(A - B) \sin(c + dx)}{a^2d(1 + \sec(c + dx))} - \frac{(A - B) \sin(c + dx)}{3d(a + a \sec(c + dx))^2} \end{aligned}$$

Mathematica [B] time = 0.64, size = 245, normalized size = 2.50

$$\sec\left(\frac{c}{2}\right) \cos\left(\frac{1}{2}(c + dx)\right) \left(-18dx(2A - B) \cos\left(c + \frac{dx}{2}\right) - 18dx(2A - B) \cos\left(\frac{dx}{2}\right) - 30A \sin\left(c + \frac{dx}{2}\right) + 41A \sin\left(c - \frac{dx}{2}\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^2,x]

[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(-18*(2*A - B)*d*x*Cos[(d*x)/2] - 18*(2*A - B)*d*x*Cos[c + (d*x)/2] - 12*A*d*x*Cos[c + (3*d*x)/2] + 6*B*d*x*Cos[c + (3*d*x)/2] - 12*A*d*x*Cos[2*c + (3*d*x)/2] + 6*B*d*x*Cos[2*c + (3*d*x)/2] + 66*A*Sin[(d*x)/2] - 36*B*Sin[(d*x)/2] - 30*A*Sin[c + (d*x)/2] + 24*B*Sin[c + (d*x)/2] + 41*A*Sin[c + (3*d*x)/2] - 20*B*Sin[c + (3*d*x)/2] + 9*A*Sin[2*c + (3*d*x)/2] + 3*A*Sin[2*c + (5*d*x)/2] + 3*A*Sin[3*c + (5*d*x)/2]))/(12*a^2*d*(1 + Cos[c + d*x])^2)

fricas [A] time = 0.44, size = 123, normalized size = 1.26

$$\frac{3(2A - B)dx \cos(dx + c)^2 + 6(2A - B)dx \cos(dx + c) + 3(2A - B)dx - (3A \cos(dx + c)^2 + (14A - 5B) \cos(dx + c))}{3(a^2d \cos(dx + c)^2 + 2a^2d \cos(dx + c) + a^2d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="fricas")

[Out]
$$-1/3*(3*(2*A - B)*d*x*cos(d*x + c)^2 + 6*(2*A - B)*d*x*cos(d*x + c) + 3*(2*A - B)*d*x - (3*A*cos(d*x + c)^2 + (14*A - 5*B)*cos(d*x + c) + 10*A - 4*B)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)$$

giac [A] time = 0.25, size = 121, normalized size = 1.23

$$\frac{\frac{6(dx+c)(2A-B)}{a^2} - \frac{12A \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right)a^2} + \frac{Aa^4 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 - Ba^4 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 - 15Aa^4 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 9Ba^4 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)}{a^6}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="giac")

[Out]
$$-1/6*(6*(d*x + c)*(2*A - B)/a^2 - 12*A*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 + 1)*a^2) + (A*a^4*tan(1/2*d*x + 1/2*c)^3 - B*a^4*tan(1/2*d*x + 1/2*c)^3 - 15*A*a^4*tan(1/2*d*x + 1/2*c) + 9*B*a^4*tan(1/2*d*x + 1/2*c))/a^6)/d$$

maple [A] time = 1.21, size = 149, normalized size = 1.52

$$-\frac{\left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)A}{6da^2} + \frac{B\left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{6da^2} + \frac{5A \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{2da^2} - \frac{3B \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{2da^2} + \frac{2A \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{da^2\left(1 + \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} - \frac{4 \arctan\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{da^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x)

[Out]
$$-1/6/d/a^2*tan(1/2*d*x+1/2*c)^3*A+1/6/d/a^2*B*tan(1/2*d*x+1/2*c)^3+5/2/d/a^2*A*tan(1/2*d*x+1/2*c)-3/2/d/a^2*B*tan(1/2*d*x+1/2*c)+2/d/a^2*A*tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)-4/d/a^2*arctan(tan(1/2*d*x+1/2*c))*A+2/d/a^2*arctan(tan(1/2*d*x+1/2*c))*B$$

maxima [B] time = 0.44, size = 191, normalized size = 1.95

$$A \left(\frac{\frac{15 \sin(dx+c)}{\cos(dx+c)+1} - \frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2} - \frac{24 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2} + \frac{12 \sin(dx+c)}{\left(a^2 + \frac{a^2 \sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)} \right) - B \left(\frac{\frac{9 \sin(dx+c)}{\cos(dx+c)+1} - \frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2} - \frac{12 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2} \right)$$

$6d$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="maxima")

[Out] $\frac{1}{6} \cdot \frac{A \left(\frac{15 \sin(d*x+c)}{\cos(d*x+c)+1} - \frac{\sin(d*x+c)^3}{(\cos(d*x+c)+1)^3} \right)}{a^2} - \frac{24 \arctan\left(\frac{\sin(d*x+c)}{\cos(d*x+c)+1}\right)}{a^2} + \frac{12 \sin(d*x+c)}{((a^2 + a^2 \sin(d*x+c)^2 / (\cos(d*x+c)+1)^2) * (\cos(d*x+c)+1))} - \frac{B \left(\frac{9 \sin(d*x+c)}{\cos(d*x+c)+1} - \frac{\sin(d*x+c)^3}{(\cos(d*x+c)+1)^3} \right)}{a^2} - \frac{12 \arctan\left(\frac{\sin(d*x+c)}{\cos(d*x+c)+1}\right)}{a^2} / d$

mupad [B] time = 2.04, size = 109, normalized size = 1.11

$$\frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right) \left(\frac{A-B}{a^2} + \frac{3A-B}{2a^2}\right)}{d} - \frac{x(2A-B)}{a^2} + \frac{2A \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{d \left(a^2 \tan^2\left(\frac{c}{2} + \frac{dx}{2}\right) + a^2\right)} - \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 (A-B)}{6a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^2,x)

[Out] $\frac{\tan(c/2 + (d*x)/2) * ((A - B)/a^2 + (3*A - B)/(2*a^2))}{d} - \frac{x * (2*A - B)}{a^2} + \frac{2*A * \tan(c/2 + (d*x)/2)}{d * (a^2 * \tan^2(c/2 + (d*x)/2) + a^2)} - \frac{\tan(c/2 + (d*x)/2)^3 * (A - B)}{6*a^2*d}$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A \cos(c+dx)}{\sec^2(c+dx)+2 \sec(c+dx)+1} dx + \int \frac{B \cos(c+dx) \sec(c+dx)}{\sec^2(c+dx)+2 \sec(c+dx)+1} dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**2,x)

[Out] $\left(\text{Integral}\left(\frac{A \cos(c + d*x)}{(\sec(c + d*x))^{**2} + 2 * \sec(c + d*x) + 1}, x\right) + \text{Integral}\left(\frac{B * \cos(c + d*x) * \sec(c + d*x)}{(\sec(c + d*x))^{**2} + 2 * \sec(c + d*x) + 1}, x\right)\right) / a^{**2}$

$$3.97 \quad \int \frac{\cos^2(c+dx)(A+B \sec(c+dx))}{(a+a \sec(c+dx))^2} dx$$

Optimal. Leaf size=143

$$-\frac{2(8A-5B)\sin(c+dx)}{3a^2d} + \frac{(7A-4B)\sin(c+dx)\cos(c+dx)}{2a^2d} - \frac{(8A-5B)\sin(c+dx)\cos(c+dx)}{3a^2d(\sec(c+dx)+1)} + \frac{x(7A-4B)}{2a^2} - \frac{(8A-5B)\sin(c+dx)\cos(c+dx)}{3a^2d(\sec(c+dx)+1)}$$

[Out] 1/2*(7*A-4*B)*x/a^2-2/3*(8*A-5*B)*sin(d*x+c)/a^2/d+1/2*(7*A-4*B)*cos(d*x+c)*sin(d*x+c)/a^2/d-1/3*(8*A-5*B)*cos(d*x+c)*sin(d*x+c)/a^2/d/(1+sec(d*x+c))-1/3*(A-B)*cos(d*x+c)*sin(d*x+c)/d/(a+a*sec(d*x+c))^2

Rubi [A] time = 0.30, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {4020, 3787, 2635, 8, 2637}

$$-\frac{2(8A-5B)\sin(c+dx)}{3a^2d} + \frac{(7A-4B)\sin(c+dx)\cos(c+dx)}{2a^2d} - \frac{(8A-5B)\sin(c+dx)\cos(c+dx)}{3a^2d(\sec(c+dx)+1)} + \frac{x(7A-4B)}{2a^2} - \frac{(8A-5B)\sin(c+dx)\cos(c+dx)}{3a^2d(\sec(c+dx)+1)}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^2,x]

[Out] ((7*A - 4*B)*x)/(2*a^2) - (2*(8*A - 5*B)*Sin[c + d*x])/(3*a^2*d) + ((7*A - 4*B)*Cos[c + d*x]*Sin[c + d*x])/(2*a^2*d) - ((8*A - 5*B)*Cos[c + d*x]*Sin[c + d*x])/(3*a^2*d*(1 + Sec[c + d*x])) - ((A - B)*Cos[c + d*x]*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2)

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x])*(b*Ssin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Ssin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) +
(a_.)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (
a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
+ f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{\cos^2(c+dx)(A+B \sec(c+dx))}{(a+a \sec(c+dx))^2} dx &= -\frac{(A-B) \cos(c+dx) \sin(c+dx)}{3d(a+a \sec(c+dx))^2} + \frac{\int \frac{\cos^2(c+dx)(a(5A-2B)-3a(A-B) \sec(c+dx))}{a+a \sec(c+dx)} dx}{3a^2} \\ &= -\frac{(8A-5B) \cos(c+dx) \sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{(A-B) \cos(c+dx) \sin(c+dx)}{3d(a+a \sec(c+dx))^2} \\ &= -\frac{(8A-5B) \cos(c+dx) \sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{(A-B) \cos(c+dx) \sin(c+dx)}{3d(a+a \sec(c+dx))^2} \\ &= -\frac{2(8A-5B) \sin(c+dx)}{3a^2d} + \frac{(7A-4B) \cos(c+dx) \sin(c+dx)}{2a^2d} - \frac{(8A-5B) \cos(c+dx) \sin(c+dx)}{3d(a+a \sec(c+dx))^2} \\ &= \frac{(7A-4B)x}{2a^2} - \frac{2(8A-5B) \sin(c+dx)}{3a^2d} + \frac{(7A-4B) \cos(c+dx) \sin(c+dx)}{2a^2d} \end{aligned}$$

Mathematica [B] time = 0.78, size = 315, normalized size = 2.20

$$\frac{\sec\left(\frac{c}{2}\right) \cos\left(\frac{1}{2}(c+dx)\right) \left(36dx(7A-4B) \cos\left(c+\frac{dx}{2}\right) + 36dx(7A-4B) \cos\left(\frac{dx}{2}\right) + 147A \sin\left(c+\frac{dx}{2}\right) - 239A \sin\left(\frac{dx}{2}\right)\right)}{3a^2d(1+\sec(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^2, x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(36*(7*A - 4*B)*d*x*Cos[(d*x)/2] + 36*(7*A - 4*B)
)*d*x*Cos[c + (d*x)/2] + 84*A*d*x*Cos[c + (3*d*x)/2] - 48*B*d*x*Cos[c + (3*
d*x)/2] + 84*A*d*x*Cos[2*c + (3*d*x)/2] - 48*B*d*x*Cos[2*c + (3*d*x)/2] - 3
```

$$81*A*\sin[(d*x)/2] + 264*B*\sin[(d*x)/2] + 147*A*\sin[c + (d*x)/2] - 120*B*\sin[c + (d*x)/2] - 239*A*\sin[c + (3*d*x)/2] + 164*B*\sin[c + (3*d*x)/2] - 63*A*\sin[2*c + (3*d*x)/2] + 36*B*\sin[2*c + (3*d*x)/2] - 15*A*\sin[2*c + (5*d*x)/2] + 12*B*\sin[2*c + (5*d*x)/2] - 15*A*\sin[3*c + (5*d*x)/2] + 12*B*\sin[3*c + (5*d*x)/2] + 3*A*\sin[3*c + (7*d*x)/2] + 3*A*\sin[4*c + (7*d*x)/2]))/(48*a^2*d*(1 + \cos[c + d*x])^2)$$

fricas [A] time = 0.43, size = 138, normalized size = 0.97

$$\frac{3(7A - 4B)dx \cos(dx + c)^2 + 6(7A - 4B)dx \cos(dx + c) + 3(7A - 4B)dx + (3A \cos(dx + c)^3 - 6(A - B) \cos(dx + c))}{6(a^2d \cos(dx + c)^2 + 2a^2d \cos(dx + c) + a^2d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="fricas")

[Out] 1/6*(3*(7*A - 4*B)*d*x*cos(d*x + c)^2 + 6*(7*A - 4*B)*d*x*cos(d*x + c) + 3*(7*A - 4*B)*d*x + (3*A*cos(d*x + c)^3 - 6*(A - B)*cos(d*x + c)^2 - (43*A - 28*B)*cos(d*x + c) - 32*A + 20*B)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)

giac [A] time = 0.26, size = 164, normalized size = 1.15

$$\frac{\frac{3(dx+c)(7A-4B)}{a^2} - \frac{6\left(5A \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 - 2B \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 + 3A \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 2B \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + 1\right)^2 a^2} + \frac{Aa^4 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 - Ba^4 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)}{a^2}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="giac")

[Out] 1/6*(3*(d*x + c)*(7*A - 4*B)/a^2 - 6*(5*A*tan(1/2*d*x + 1/2*c)^3 - 2*B*tan(1/2*d*x + 1/2*c)^3 + 3*A*tan(1/2*d*x + 1/2*c) - 2*B*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^2 + (A*a^4*tan(1/2*d*x + 1/2*c)^3 - B*a^4*tan(1/2*d*x + 1/2*c)^3 - 21*A*a^4*tan(1/2*d*x + 1/2*c) + 15*B*a^4*tan(1/2*d*x + 1/2*c))/a^6)/d

maple [A] time = 1.11, size = 252, normalized size = 1.76

$$\frac{\left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)A}{6da^2} - \frac{B\left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{6da^2} - \frac{7A \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{2da^2} + \frac{5B \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{2da^2} - \frac{5\left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)A}{da^2\left(1 + \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^2} + \frac{2B\left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{da^2\left(1 + \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(\cos(dx+c)^2*(A+B*\sec(dx+c))/(a+a*\sec(dx+c))^2, x)$

[Out] $\frac{1}{6} \frac{d}{a^2} \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 A - \frac{1}{6} \frac{d}{a^2} B \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 - \frac{7}{2} \frac{d}{a^2} A \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + \frac{5}{2} \frac{d}{a^2} B \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - \frac{5}{d} \frac{d}{a^2} \frac{1}{(1 + \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2)^2} \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 A + \frac{2}{d} \frac{d}{a^2} \frac{1}{(1 + \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2)^2} B \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 - \frac{3}{d} \frac{d}{a^2} \frac{1}{(1 + \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2)^2} A \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + \frac{2}{d} \frac{d}{a^2} \frac{1}{(1 + \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2)^2} B \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + \frac{7}{d} \frac{d}{a^2} \arctan\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right) A - \frac{4}{d} \frac{d}{a^2} \arctan\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right) B$

maxima [B] time = 0.44, size = 283, normalized size = 1.98

$$\frac{A \left(\frac{6 \left(\frac{3 \sin(dx+c)}{\cos(dx+c)+1} + \frac{5 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} \right)}{a^2 + \frac{2a^2 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{a^2 \sin(dx+c)^4}{(\cos(dx+c)+1)^4}} + \frac{21 \sin(dx+c)}{\cos(dx+c)+1} - \frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3} - \frac{42 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2} \right) - B \left(\frac{15 \sin(dx+c)}{\cos(dx+c)+1} - \frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3} - \frac{24 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2} \right)}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(\cos(dx+c)^2*(A+B*\sec(dx+c))/(a+a*\sec(dx+c))^2, x, \text{algorithm}="maxima")$

[Out] $- \frac{1}{6} * (A * (6 * (3 * \sin(dx + c) / (\cos(dx + c) + 1) + 5 * \sin(dx + c)^3 / (\cos(dx + c) + 1)^3) / (a^2 + 2 * a^2 * \sin(dx + c)^2 / (\cos(dx + c) + 1)^2 + a^2 * \sin(dx + c)^4 / (\cos(dx + c) + 1)^4) + (21 * \sin(dx + c) / (\cos(dx + c) + 1) - \sin(dx + c)^3 / (\cos(dx + c) + 1)^3) / a^2 - 42 * \arctan(\sin(dx + c) / (\cos(dx + c) + 1)) / a^2 - B * ((15 * \sin(dx + c) / (\cos(dx + c) + 1) - \sin(dx + c)^3 / (\cos(dx + c) + 1)^3) / a^2 - 24 * \arctan(\sin(dx + c) / (\cos(dx + c) + 1)) / a^2 + 12 * \sin(dx + c) / ((a^2 + a^2 * \sin(dx + c)^2 / (\cos(dx + c) + 1)^2) * (\cos(dx + c) + 1)))) / d$

mupad [B] time = 2.07, size = 154, normalized size = 1.08

$$\frac{x(7A - 4B)}{2a^2} - \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right) \left(\frac{3(A-B)}{2a^2} + \frac{4A-2B}{2a^2}\right)}{d} - \frac{(5A - 2B) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 + (3A - 2B) \tan\left(\frac{c}{2} + \frac{dx}{2}\right) \tan\left(\frac{c}{2}\right)}{d \left(a^2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 + 2a^2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + a^2\right)} + \frac{\tan\left(\frac{c}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((\cos(c + dx))^2*(A + B/\cos(c + dx)))/(a + a/\cos(c + dx))^2, x)$

[Out] $\frac{(x*(7*A - 4*B))/(2*a^2) - (\tan(c/2 + (dx)/2)*((3*(A - B))/(2*a^2) + (4*A - 2*B)/(2*a^2)))/d - (\tan(c/2 + (dx)/2)^3*(5*A - 2*B) + \tan(c/2 + (dx)/2)*(3*A - 2*B))/(d*(2*a^2*\tan(c/2 + (dx)/2)^2 + a^2*\tan(c/2 + (dx)/2)^4 + a^2)) + (\tan(c/2 + (dx)/2)^3*(A - B))/(6*a^2*d)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A \cos^2(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1} dx + \int \frac{B \cos^2(c+dx) \sec(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1} dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**2,x)

[Out] (Integral(A*cos(c + d*x)**2/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x) + Integral(B*cos(c + d*x)**2*sec(c + d*x)/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x))/a**2

$$3.98 \quad \int \frac{\cos^3(c+dx)(A+B \sec(c+dx))}{(a+a \sec(c+dx))^2} dx$$

Optimal. Leaf size=170

$$\frac{4(3A-2B)\sin^3(c+dx)}{3a^2d} + \frac{4(3A-2B)\sin(c+dx)}{a^2d} - \frac{(10A-7B)\sin(c+dx)\cos(c+dx)}{2a^2d} - \frac{(10A-7B)\sin(c+dx)}{3a^2d(\sec(c+dx))}$$

[Out] $-1/2*(10*A-7*B)*x/a^2+4*(3*A-2*B)*\sin(d*x+c)/a^2/d-1/2*(10*A-7*B)*\cos(d*x+c)*\sin(d*x+c)/a^2/d-1/3*(10*A-7*B)*\cos(d*x+c)^2*\sin(d*x+c)/a^2/d/(1+\sec(d*x+c))-1/3*(A-B)*\cos(d*x+c)^2*\sin(d*x+c)/d/(a+a*\sec(d*x+c))^2-4/3*(3*A-2*B)*\sin(d*x+c)^3/a^2/d$

Rubi [A] time = 0.32, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {4020, 3787, 2633, 2635, 8}

$$\frac{4(3A-2B)\sin^3(c+dx)}{3a^2d} + \frac{4(3A-2B)\sin(c+dx)}{a^2d} - \frac{(10A-7B)\sin(c+dx)\cos(c+dx)}{2a^2d} - \frac{(10A-7B)\sin(c+dx)}{3a^2d(\sec(c+dx))}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^2,x]

[Out] $-((10*A-7*B)*x)/(2*a^2)+ (4*(3*A-2*B)*Sin[c+d*x])/(a^2*d) - ((10*A-7*B)*Cos[c+d*x]*Sin[c+d*x])/(2*a^2*d) - ((10*A-7*B)*Cos[c+d*x]^2*Sin[c+d*x])/(3*a^2*d*(1+Sec[c+d*x])) - ((A-B)*Cos[c+d*x]^2*Sin[c+d*x])/(3*d*(a+a*Sec[c+d*x])^2) - (4*(3*A-2*B)*Sin[c+d*x]^3)/(3*a^2*d)$

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2633

Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[Expand[(1 - x^2)^(n-1)/2], x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n-1)/2, 0]

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x])*(b*SIN[c + d*x])^(n-1)/(d*n), x] + Dist[(b^2*(n-1))/n, Int[(b*SIN[c + d*x])^(n-2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

]

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) +
(a_.)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) +
(a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
+ f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^3(c+dx)(A+B \sec(c+dx))}{(a+a \sec(c+dx))^2} dx &= -\frac{(A-B) \cos^2(c+dx) \sin(c+dx)}{3d(a+a \sec(c+dx))^2} + \int \frac{\cos^3(c+dx)(3a(2A-B)-4a(A-B) \sec(c+dx))}{a+a \sec(c+dx)} dx \\
&= -\frac{(10A-7B) \cos^2(c+dx) \sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{(A-B) \cos^2(c+dx) \sin(c+dx)}{3d(a+a \sec(c+dx))^2} \\
&= -\frac{(10A-7B) \cos^2(c+dx) \sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{(A-B) \cos^2(c+dx) \sin(c+dx)}{3d(a+a \sec(c+dx))^2} \\
&= -\frac{(10A-7B) \cos(c+dx) \sin(c+dx)}{2a^2d} - \frac{(10A-7B) \cos^2(c+dx) \sin(c+dx)}{3a^2d(1+\sec(c+dx))} \\
&= -\frac{(10A-7B)x}{2a^2} + \frac{4(3A-2B) \sin(c+dx)}{a^2d} - \frac{(10A-7B) \cos(c+dx) \sin(c+dx)}{2a^2d}
\end{aligned}$$

Mathematica [B] time = 0.77, size = 369, normalized size = 2.17

$$\frac{\sec\left(\frac{c}{2}\right) \cos\left(\frac{1}{2}(c+dx)\right) \left(-36dx(10A-7B) \cos\left(c+\frac{dx}{2}\right) - 36dx(10A-7B) \cos\left(\frac{dx}{2}\right) - 156A \sin\left(c+\frac{dx}{2}\right) + 342As\right)}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^2, x]
```



```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(-36*(10*A - 7*B)*d*x*Cos[(d*x)/2] - 36*(10*A - 7*B)*d*x*Cos[c + (d*x)/2] - 120*A*d*x*Cos[c + (3*d*x)/2] + 84*B*d*x*Cos[c + (3*d*x)/2] - 120*A*d*x*Cos[2*c + (3*d*x)/2] + 84*B*d*x*Cos[2*c + (3*d*x)/2] + 516*A*Sin[(d*x)/2] - 381*B*Sin[(d*x)/2] - 156*A*Sin[c + (d*x)/2] + 147*B*Sin[c + (d*x)/2] + 342*A*Sin[c + (3*d*x)/2] - 239*B*Sin[c + (3*d*x)/2] + 118*A*Sin[2*c + (3*d*x)/2] - 63*B*Sin[2*c + (3*d*x)/2] + 30*A*Sin[2*c + (5*d*x)/2] - 15*B*Sin[2*c + (5*d*x)/2] + 30*A*Sin[3*c + (5*d*x)/2] - 15*B*Sin[3*c + (5*d*x)/2] - 3*A*Sin[3*c + (7*d*x)/2] + 3*B*Sin[3*c + (7*d*x)/2] - 3*A*Sin[4*c + (7*d*x)/2] + 3*B*Sin[4*c + (7*d*x)/2] + A*Sin[4*c + (9*d*x)/2] + A*Sin[5*c + (9*d*x)/2]))/(48*a^2*d*(1 + Cos[c + d*x])^2)
```

fricas [A] time = 0.45, size = 157, normalized size = 0.92

$$\frac{3(10A - 7B)dx \cos(dx + c)^2 + 6(10A - 7B)dx \cos(dx + c) + 3(10A - 7B)dx - (2A \cos(dx + c)^4 - (2A - 3B) \cos(dx + c)^3 + 6(2A - B) \cos(dx + c)^2 + (66A - 43B) \cos(dx + c) + 48A - 32B) \sin(dx + c)}{6(a^2d \cos(dx + c))^2 + 2a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/6*(3*(10*A - 7*B)*d*x*cos(d*x + c)^2 + 6*(10*A - 7*B)*d*x*cos(d*x + c) + 3*(10*A - 7*B)*d*x - (2*A*cos(d*x + c)^4 - (2*A - 3*B)*cos(d*x + c)^3 + 6*(2*A - B)*cos(d*x + c)^2 + (66*A - 43*B)*cos(d*x + c) + 48*A - 32*B)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

giac [A] time = 0.26, size = 192, normalized size = 1.13

$$\frac{3(dx+c)(10A-7B)}{a^2} - \frac{2\left(30A \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5 - 15B \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5 + 40A \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 - 24B \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 + 18A \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 9B \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right)^3 a^2}$$

6d

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/6*(3*(d*x + c)*(10*A - 7*B)/a^2 - 2*(30*A*tan(1/2*d*x + 1/2*c)^5 - 15*B*tan(1/2*d*x + 1/2*c)^5 + 40*A*tan(1/2*d*x + 1/2*c)^3 - 24*B*tan(1/2*d*x + 1/2*c)^3 + 18*A*tan(1/2*d*x + 1/2*c) - 9*B*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^3*a^2) + (A*a^4*tan(1/2*d*x + 1/2*c)^3 - B*a^4*tan(1/2*d*x + 1/2*c)^3 - 27*A*a^4*tan(1/2*d*x + 1/2*c) + 21*B*a^4*tan(1/2*d*x + 1/2*c))/a^6)/d
```

maple [B] time = 1.17, size = 322, normalized size = 1.89

$$-\frac{\left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)A}{6d a^2} + \frac{B\left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{6d a^2} + \frac{9A \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{2d a^2} - \frac{7B \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{2d a^2} + \frac{10\left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right)A}{d a^2 \left(1 + \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^3} - \frac{5\left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right)B}{d a^2 \left(1 + \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x)

[Out] $-1/6/d/a^2*\tan(1/2*d*x+1/2*c)^3*A+1/6/d/a^2*B*\tan(1/2*d*x+1/2*c)^3+9/2/d/a^2*A*\tan(1/2*d*x+1/2*c)-7/2/d/a^2*B*\tan(1/2*d*x+1/2*c)+10/d/a^2/(1+\tan(1/2*d*x+1/2*c)^2)^3*\tan(1/2*d*x+1/2*c)^5*A-5/d/a^2/(1+\tan(1/2*d*x+1/2*c)^2)^3*\tan(1/2*d*x+1/2*c)^5*B+40/3/d/a^2/(1+\tan(1/2*d*x+1/2*c)^2)^3*A*\tan(1/2*d*x+1/2*c)^3-8/d/a^2/(1+\tan(1/2*d*x+1/2*c)^2)^3*B*\tan(1/2*d*x+1/2*c)^3+6/d/a^2/(1+\tan(1/2*d*x+1/2*c)^2)^3*A*\tan(1/2*d*x+1/2*c)-3/d/a^2/(1+\tan(1/2*d*x+1/2*c)^2)^3*B*\tan(1/2*d*x+1/2*c)-10/d/a^2*\arctan(\tan(1/2*d*x+1/2*c))*A+7/d/a^2*\arctan(\tan(1/2*d*x+1/2*c))*B$

maxima [B] time = 0.45, size = 372, normalized size = 2.19

$$A \left(\frac{4 \left(\frac{9 \sin(dx+c)}{\cos(dx+c)+1} + \frac{20 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{15 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} \right)}{a^2 + \frac{3a^2 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{3a^2 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{a^2 \sin(dx+c)^6}{(\cos(dx+c)+1)^6}} + \frac{27 \sin(dx+c)}{\cos(dx+c)+1} - \frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3} - \frac{60 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2} \right) - B \left(\frac{6 \left(\frac{3 \sin(dx+c)}{\cos(dx+c)+1} + \frac{5 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} \right)}{a^2 + \frac{2a^2 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{a^2 \sin(dx+c)^4}{(\cos(dx+c)+1)^4}} \right) \frac{1}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="maxima")

[Out] $1/6*(A*(4*(9*\sin(d*x + c)/(\cos(d*x + c) + 1) + 20*\sin(d*x + c)^3/(\cos(d*x + c) + 1)^3 + 15*\sin(d*x + c)^5/(\cos(d*x + c) + 1)^5)/(a^2 + 3*a^2*\sin(d*x + c)^2/(\cos(d*x + c) + 1)^2 + 3*a^2*\sin(d*x + c)^4/(\cos(d*x + c) + 1)^4 + a^2*\sin(d*x + c)^6/(\cos(d*x + c) + 1)^6) + (27*\sin(d*x + c)/(\cos(d*x + c) + 1) - \sin(d*x + c)^3/(\cos(d*x + c) + 1)^3)/a^2 - 60*\arctan(\sin(d*x + c)/(\cos(d*x + c) + 1))/a^2 - B*(6*(3*\sin(d*x + c)/(\cos(d*x + c) + 1) + 5*\sin(d*x + c)^3/(\cos(d*x + c) + 1)^3)/(a^2 + 2*a^2*\sin(d*x + c)^2/(\cos(d*x + c) + 1)^2 + a^2*\sin(d*x + c)^4/(\cos(d*x + c) + 1)^4) + (21*\sin(d*x + c)/(\cos(d*x + c) + 1) - \sin(d*x + c)^3/(\cos(d*x + c) + 1)^3)/a^2 - 42*\arctan(\sin(d*x + c)/(\cos(d*x + c) + 1))/a^2)/d$

mupad [B] time = 2.08, size = 187, normalized size = 1.10

$$\frac{(10A - 5B) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5 + \left(\frac{40A}{3} - 8B\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 + (6A - 3B) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{d \left(a^2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^6 + 3a^2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 + 3a^2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + a^2 \right)} \cdot \frac{x(10A - 7B) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{2a^2} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)^3*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^2,x)

[Out] (tan(c/2 + (d*x)/2)^5*(10*A - 5*B) + tan(c/2 + (d*x)/2)^3*((40*A)/3 - 8*B) + tan(c/2 + (d*x)/2)*(6*A - 3*B))/(d*(3*a^2*tan(c/2 + (d*x)/2)^2 + 3*a^2*tan(c/2 + (d*x)/2)^4 + a^2*tan(c/2 + (d*x)/2)^6 + a^2)) - (x*(10*A - 7*B))/(2*a^2) + (tan(c/2 + (d*x)/2)*((2*(A - B))/a^2 + (5*A - 3*B)/(2*a^2)))/d - (tan(c/2 + (d*x)/2)^3*(A - B))/(6*a^2*d)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A \cos^3(c+dx)}{\sec^2(c+dx)+2 \sec(c+dx)+1} dx + \int \frac{B \cos^3(c+dx) \sec(c+dx)}{\sec^2(c+dx)+2 \sec(c+dx)+1} dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**2,x)

[Out] (Integral(A*cos(c + d*x)**3/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x) + Integral(B*cos(c + d*x)**3*sec(c + d*x)/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x))/a**2

$$3.99 \quad \int \frac{\sec^5(c+dx)(A+B \sec(c+dx))}{(a+a \sec(c+dx))^3} dx$$

Optimal. Leaf size=202

$$\frac{8(9A - 19B) \tan(c + dx)}{15a^3d} - \frac{(6A - 13B) \tanh^{-1}(\sin(c + dx))}{2a^3d} + \frac{4(9A - 19B) \tan(c + dx) \sec^2(c + dx)}{15d(a^3 \sec(c + dx) + a^3)} - \frac{(6A - 13B) \tan(c + dx)}{15d(a^3 \sec(c + dx) + a^3)}$$

[Out] $-1/2*(6*A-13*B)*\operatorname{arctanh}(\sin(d*x+c))/a^3/d+8/15*(9*A-19*B)*\tan(d*x+c)/a^3/d-1/2*(6*A-13*B)*\sec(d*x+c)*\tan(d*x+c)/a^3/d+1/5*(A-B)*\sec(d*x+c)^4*\tan(d*x+c)/d/(a+a*\sec(d*x+c))^3+1/15*(6*A-11*B)*\sec(d*x+c)^3*\tan(d*x+c)/a/d/(a+a*\sec(d*x+c))^2+4/15*(9*A-19*B)*\sec(d*x+c)^2*\tan(d*x+c)/d/(a^3+a^3*\sec(d*x+c))$

Rubi [A] time = 0.47, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4019, 3787, 3767, 8, 3768, 3770}

$$\frac{8(9A - 19B) \tan(c + dx)}{15a^3d} - \frac{(6A - 13B) \tanh^{-1}(\sin(c + dx))}{2a^3d} + \frac{4(9A - 19B) \tan(c + dx) \sec^2(c + dx)}{15d(a^3 \sec(c + dx) + a^3)} - \frac{(6A - 13B) \tan(c + dx)}{15d(a^3 \sec(c + dx) + a^3)}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(\operatorname{Sec}[c + d*x]^5*(A + B*\operatorname{Sec}[c + d*x]))/(a + a*\operatorname{Sec}[c + d*x])^3, x]$

[Out] $-((6*A - 13*B)*\operatorname{ArcTanh}[\operatorname{Sin}[c + d*x]])/(2*a^3*d) + (8*(9*A - 19*B)*\operatorname{Tan}[c + d*x])/(15*a^3*d) - ((6*A - 13*B)*\operatorname{Sec}[c + d*x]*\operatorname{Tan}[c + d*x])/(2*a^3*d) + ((A - B)*\operatorname{Sec}[c + d*x]^4*\operatorname{Tan}[c + d*x])/(5*d*(a + a*\operatorname{Sec}[c + d*x])^3) + ((6*A - 11*B)*\operatorname{Sec}[c + d*x]^3*\operatorname{Tan}[c + d*x])/(15*a*d*(a + a*\operatorname{Sec}[c + d*x])^2) + (4*(9*A - 19*B)*\operatorname{Sec}[c + d*x]^2*\operatorname{Tan}[c + d*x])/(15*d*(a^3 + a^3*\operatorname{Sec}[c + d*x]))$

Rule 8

$\operatorname{Int}[a_, x_Symbol] \rightarrow \operatorname{Simp}[a*x, x] /; \operatorname{FreeQ}[a, x]$

Rule 3767

$\operatorname{Int}[\operatorname{csc}[(c_.) + (d_.)*(x_.)]^{(n_.)}, x_Symbol] \rightarrow -\operatorname{Dist}[d^{(-1)}, \operatorname{Subst}[\operatorname{Int}[\operatorname{ExpandIntegrand}[(1 + x^2)^{(n/2 - 1)}, x], x], x], \operatorname{Cot}[c + d*x]], x] /; \operatorname{FreeQ}[\{c, d\}, x] \ \&\& \ \operatorname{IGtQ}[n/2, 0]$

Rule 3768

$\operatorname{Int}[(\operatorname{csc}[(c_.) + (d_.)*(x_.)]*(b_.))^{(n_.)}, x_Symbol] \rightarrow -\operatorname{Simp}[(b*\operatorname{Cos}[c + d*x])*(b*\operatorname{Csc}[c + d*x])^{(n - 1)})/(d*(n - 1)), x] + \operatorname{Dist}[(b^2*(n - 2))/(n - 1), \operatorname{Int}[(b*\operatorname{Csc}[c + d*x])^{(n - 2)}, x], x] /; \operatorname{FreeQ}[\{b, c, d\}, x] \ \&\& \ \operatorname{GtQ}[n, 1] \ \&\&$

IntegerQ[2*n]

Rule 3770

Int[csc[(c_.) + (d_.)*(x_.)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 4019

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := Simp[(d*(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0]

Rubi steps

$$\begin{aligned}
\int \frac{\sec^5(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^3} dx &= \frac{(A-B)\sec^4(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{\int \frac{\sec^4(c+dx)(4a(A-B)-a(2A-7B)\sec(c+dx))}{(a+a\sec(c+dx))^2}}{5a^2} \\
&= \frac{(A-B)\sec^4(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(6A-11B)\sec^3(c+dx)\tan(c+dx)}{15ad(a+a\sec(c+dx))^2} \\
&= \frac{(A-B)\sec^4(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(6A-11B)\sec^3(c+dx)\tan(c+dx)}{15ad(a+a\sec(c+dx))^2} \\
&= \frac{(A-B)\sec^4(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(6A-11B)\sec^3(c+dx)\tan(c+dx)}{15ad(a+a\sec(c+dx))^2} \\
&= -\frac{(6A-13B)\sec(c+dx)\tan(c+dx)}{2a^3d} + \frac{(A-B)\sec^4(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} \\
&= -\frac{(6A-13B)\tanh^{-1}(\sin(c+dx))}{2a^3d} + \frac{8(9A-19B)\tan(c+dx)}{15a^3d} - \frac{(6A-11B)\sec^3(c+dx)\tan(c+dx)}{15ad(a+a\sec(c+dx))^2}
\end{aligned}$$

Mathematica [B] time = 6.40, size = 768, normalized size = 3.80

$$\frac{\sec\left(\frac{c}{2}\right)\sec(c)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^4(c+dx)\left(-2094A\sin\left(c-\frac{dx}{2}\right)+1314A\sin\left(c+\frac{dx}{2}\right)-1650A\sin\left(2c+\frac{dx}{2}\right)-450A\sin\left(3c+\frac{dx}{2}\right)\right)}{(a+a\sec(c+dx))^3}$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[c + d*x]^5*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^3,x]

[Out] (-4*(-6*A + 13*B)*Cos[c/2 + (d*x)/2]^6*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])*Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(d*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^3) + (4*(-6*A + 13*B)*Cos[c/2 + (d*x)/2]^6*Log[Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])*Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(d*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^3) + (Cos[c/2 + (d*x)/2]*Sec[c/2]*Sec[c]*Sec[c + d*x]^4*(A + B*Sec[c + d*x]))*(-870*A*Sin[(d*x)/2] + 1235*B*Sin[(d*x)/2] + 1830*A*Sin[(3*d*x)/2] - 3805*B*Sin[(3*d*x)/2] - 2094*A*Sin[c - (d*x)/2] + 4329*B*Sin[c - (d*x)/2] + 1314*A*Sin[c + (d*x)/2] - 1989*B*Sin[c + (d*x)/2] - 1650*A*Sin[2*c + (d*x)/2] + 3575*B*Sin[2*c + (d*x)/2] - 450*A*Sin[c + (3*d*x)/2] + 475*B*Sin[c + (3*d*x)/2] + 1230*A*Sin[2*c + (3*d*x)/2] - 2005*B*Sin[2*c + (3*d*x)/2] - 1050*A*Sin[3*c + (3*d*x)/2] + 2275*B*Sin[3*c + (3*d*x)/2] + 1278*A*Sin[c + (5*d*x)/2] - 2673*B*Sin[c + (5*d*x)/2] - 90*A*Sin[2*c + (5*d*x)/2] - 105*B*Sin[2*c + (5*d*x)/2] + 918*A*Sin[3*c + (5*d*x)/2] - 1593*B*Sin[3*c + (5*d*x)/2] - 450*A*Sin[4*c + (5*d*x)/2] + 975*B*Sin[4*c + (5*d*x)/2])

$$\begin{aligned} & [4*c + (5*d*x)/2] + 630*A*\sin[2*c + (7*d*x)/2] - 1325*B*\sin[2*c + (7*d*x)/2] \\ &] + 60*A*\sin[3*c + (7*d*x)/2] - 255*B*\sin[3*c + (7*d*x)/2] + 480*A*\sin[4*c \\ & + (7*d*x)/2] - 875*B*\sin[4*c + (7*d*x)/2] - 90*A*\sin[5*c + (7*d*x)/2] + 195 \\ & *B*\sin[5*c + (7*d*x)/2] + 144*A*\sin[3*c + (9*d*x)/2] - 304*B*\sin[3*c + (9*d \\ & *x)/2] + 30*A*\sin[4*c + (9*d*x)/2] - 90*B*\sin[4*c + (9*d*x)/2] + 114*A*\sin[\\ & 5*c + (9*d*x)/2] - 214*B*\sin[5*c + (9*d*x)/2]))/(480*d*(B + A*\cos[c + d*x]) \\ & *(a + a*\sec[c + d*x])^3) \end{aligned}$$

fricas [A] time = 0.45, size = 295, normalized size = 1.46

$$\frac{15((6A - 13B)\cos(dx + c)^5 + 3(6A - 13B)\cos(dx + c)^4 + 3(6A - 13B)\cos(dx + c)^3 + (6A - 13B)\cos(dx + c))}{(a + a\sec(c + dx))^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^5*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="fricas")

[Out]
$$\begin{aligned} & -1/60*(15*((6*A - 13*B)*\cos(d*x + c)^5 + 3*(6*A - 13*B)*\cos(d*x + c)^4 + 3* \\ & (6*A - 13*B)*\cos(d*x + c)^3 + (6*A - 13*B)*\cos(d*x + c)^2)*\log(\sin(d*x + c) \\ & + 1) - 15*((6*A - 13*B)*\cos(d*x + c)^5 + 3*(6*A - 13*B)*\cos(d*x + c)^4 + 3 \\ & *(6*A - 13*B)*\cos(d*x + c)^3 + (6*A - 13*B)*\cos(d*x + c)^2)*\log(-\sin(d*x + \\ & c) + 1) - 2*(16*(9*A - 19*B)*\cos(d*x + c)^4 + 3*(114*A - 239*B)*\cos(d*x + c \\ &)^3 + (234*A - 479*B)*\cos(d*x + c)^2 + 15*(2*A - 3*B)*\cos(d*x + c) + 15*B)* \\ & \sin(d*x + c))/(a^3*d*\cos(d*x + c)^5 + 3*a^3*d*\cos(d*x + c)^4 + 3*a^3*d*\cos(\\ & d*x + c)^3 + a^3*d*\cos(d*x + c)^2) \end{aligned}$$

giac [A] time = 0.35, size = 233, normalized size = 1.15

$$\frac{30(6A-13B)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^3} - \frac{30(6A-13B)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^3} + \frac{60\left(2A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-7B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-2A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2 a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^5*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="giac")

[Out]
$$\begin{aligned} & -1/60*(30*(6*A - 13*B)*\log(\operatorname{abs}(\tan(1/2*d*x + 1/2*c) + 1))/a^3 - 30*(6*A - 1 \\ & 3*B)*\log(\operatorname{abs}(\tan(1/2*d*x + 1/2*c) - 1))/a^3 + 60*(2*A*\tan(1/2*d*x + 1/2*c)^ \\ & 3 - 7*B*\tan(1/2*d*x + 1/2*c)^3 - 2*A*\tan(1/2*d*x + 1/2*c) + 5*B*\tan(1/2*d*x \\ & + 1/2*c))/((\tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^3) - (3*A*a^12*\tan(1/2*d*x + 1 \\ & /2*c)^5 - 3*B*a^12*\tan(1/2*d*x + 1/2*c)^5 + 30*A*a^12*\tan(1/2*d*x + 1/2*c)^ \\ & 3 - 40*B*a^12*\tan(1/2*d*x + 1/2*c)^3 + 255*A*a^12*\tan(1/2*d*x + 1/2*c) - 46 \\ & 5*B*a^12*\tan(1/2*d*x + 1/2*c))/a^15)/d \end{aligned}$$

maple [A] time = 0.70, size = 334, normalized size = 1.65

$$\frac{A \left(\tan^5 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{20d a^3} - \frac{B \left(\tan^5 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{20d a^3} + \frac{\left(\tan^3 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) A}{2d a^3} - \frac{2B \left(\tan^3 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{3d a^3} + \frac{17A \tan \left(\frac{dx}{2} + \frac{c}{2} \right)}{4d a^3} - \frac{31B \tan \left(\frac{dx}{2} + \frac{c}{2} \right)}{4d a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^5*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x)

[Out] $\frac{1}{20} \frac{d}{a^3} A \tan^5 \left(\frac{1}{2} d x + \frac{1}{2} c \right) - \frac{1}{20} \frac{d}{a^3} B \tan^5 \left(\frac{1}{2} d x + \frac{1}{2} c \right) + \frac{1}{2} \frac{d}{a^3} A \tan^3 \left(\frac{1}{2} d x + \frac{1}{2} c \right) - \frac{2}{3} \frac{d}{a^3} B \tan^3 \left(\frac{1}{2} d x + \frac{1}{2} c \right) + \frac{17}{4} \frac{d}{a^3} A \tan \left(\frac{1}{2} d x + \frac{1}{2} c \right) - \frac{31}{4} \frac{d}{a^3} B \tan \left(\frac{1}{2} d x + \frac{1}{2} c \right) + \frac{3}{d a^3} \ln \left(\tan \left(\frac{1}{2} d x + \frac{1}{2} c \right) - 1 \right) A - \frac{13}{2} \frac{d}{a^3} \ln \left(\tan \left(\frac{1}{2} d x + \frac{1}{2} c \right) - 1 \right) B + \frac{7}{2} \frac{d}{a^3} \frac{1}{\left(\tan \left(\frac{1}{2} d x + \frac{1}{2} c \right) - 1 \right)} A - \frac{1}{d a^3} \frac{1}{\left(\tan \left(\frac{1}{2} d x + \frac{1}{2} c \right) - 1 \right)} B + \frac{1}{2} \frac{d}{a^3} \frac{1}{\left(\tan \left(\frac{1}{2} d x + \frac{1}{2} c \right) - 1 \right)^2} B + \frac{7}{2} \frac{d}{a^3} \frac{1}{\left(\tan \left(\frac{1}{2} d x + \frac{1}{2} c \right) + 1 \right)} B - \frac{1}{d a^3} \frac{1}{\left(\tan \left(\frac{1}{2} d x + \frac{1}{2} c \right) + 1 \right)} A - \frac{3}{d a^3} \ln \left(\tan \left(\frac{1}{2} d x + \frac{1}{2} c \right) + 1 \right) A + \frac{13}{2} \frac{d}{a^3} \ln \left(\tan \left(\frac{1}{2} d x + \frac{1}{2} c \right) + 1 \right) B - \frac{1}{2} \frac{d}{a^3} \frac{1}{\left(\tan \left(\frac{1}{2} d x + \frac{1}{2} c \right) + 1 \right)^2} B$

maxima [A] time = 0.36, size = 377, normalized size = 1.87

$$\frac{B \left(\frac{60 \left(\frac{5 \sin(dx+c)}{\cos(dx+c)+1} - \frac{7 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} \right)}{a^3 - \frac{2a^3 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{a^3 \sin(dx+c)^4}{(\cos(dx+c)+1)^4}} + \frac{\frac{465 \sin(dx+c)}{\cos(dx+c)+1} + \frac{40 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{3 \sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3} - \frac{390 \log \left(\frac{\sin(dx+c)}{\cos(dx+c)+1} + 1 \right)}{a^3} + \frac{390 \log \left(\frac{\sin(dx+c)}{\cos(dx+c)+1} - 1 \right)}{a^3} \right)}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^5*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="maxima")

[Out] $-\frac{1}{60} \left(B \left(60 \frac{5 \sin(dx+c)}{\cos(dx+c)+1} - \frac{7 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} \right) / \left(a^3 - \frac{2a^3 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{a^3 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} \right) + \frac{465 \sin(dx+c)}{\cos(dx+c)+1} + \frac{40 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{3 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} \right) / a^3 - \frac{390 \log \left(\frac{\sin(dx+c)}{\cos(dx+c)+1} + 1 \right)}{a^3} + \frac{390 \log \left(\frac{\sin(dx+c)}{\cos(dx+c)+1} - 1 \right)}{a^3} - 3A \frac{40 \sin(dx+c)}{\left(a^3 - \frac{2a^3 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{a^3 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} \right)} + \frac{85 \sin(dx+c)}{\cos(dx+c)+1} + \frac{10 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{\sin(dx+c)^5}{\cos(dx+c)+1} \right) / a^3 - \frac{60 \log \left(\frac{\sin(dx+c)}{\cos(dx+c)+1} + 1 \right)}{a^3} + \frac{60 \log \left(\frac{\sin(dx+c)}{\cos(dx+c)+1} - 1 \right)}{a^3} \right) / d$

mupad [B] time = 2.03, size = 216, normalized size = 1.07

$$\frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right) \left(\frac{3(A-B)}{2a^3} + \frac{3(3A-5B)}{4a^3} + \frac{2A-10B}{4a^3}\right)}{d} - \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 (2A-7B) - \tan\left(\frac{c}{2} + \frac{dx}{2}\right) (2A-5B)}{d \left(a^3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 - 2a^3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + a^3\right)} + \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^5*(a + a/cos(c + d*x))^3), x)

[Out] (tan(c/2 + (d*x)/2)*((3*(A - B))/(2*a^3) + (3*(3*A - 5*B))/(4*a^3) + (2*A - 10*B)/(4*a^3))/d - (tan(c/2 + (d*x)/2)^3*(2*A - 7*B) - tan(c/2 + (d*x)/2)*(2*A - 5*B))/(d*(a^3*tan(c/2 + (d*x)/2)^4 - 2*a^3*tan(c/2 + (d*x)/2)^2 + a^3)) + (tan(c/2 + (d*x)/2)^3*((A - B)/(4*a^3) + (3*A - 5*B)/(12*a^3)))/d + (tan(c/2 + (d*x)/2)^5*(A - B))/(20*a^3*d) - (atanh(tan(c/2 + (d*x)/2))*(6*A - 13*B))/(a^3*d)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A \sec^5(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1} dx + \int \frac{B \sec^6(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1} dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**5*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**3, x)

[Out] (Integral(A*sec(c + d*x)**5/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d*x) + 1), x) + Integral(B*sec(c + d*x)**6/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d*x) + 1), x))/a**3

$$3.100 \quad \int \frac{\sec^4(c+dx)(A+B \sec(c+dx))}{(a+a \sec(c+dx))^3} dx$$

Optimal. Leaf size=156

$$-\frac{(7A-27B)\tan(c+dx)}{15a^3d} + \frac{(A-3B)\tanh^{-1}(\sin(c+dx))}{a^3d} - \frac{(A-3B)\tan(c+dx)}{d(a^3\sec(c+dx)+a^3)} + \frac{(A-B)\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

[Out] (A-3*B)*arctanh(sin(d*x+c))/a^3/d-1/15*(7*A-27*B)*tan(d*x+c)/a^3/d+1/5*(A-B)*sec(d*x+c)^3*tan(d*x+c)/d/(a+a*sec(d*x+c))^3+1/15*(4*A-9*B)*sec(d*x+c)^2*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^2-(A-3*B)*tan(d*x+c)/d/(a^3+a^3*sec(d*x+c))

Rubi [A] time = 0.43, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4019, 4008, 3787, 3770, 3767, 8}

$$-\frac{(7A-27B)\tan(c+dx)}{15a^3d} + \frac{(A-3B)\tanh^{-1}(\sin(c+dx))}{a^3d} - \frac{(A-3B)\tan(c+dx)}{d(a^3\sec(c+dx)+a^3)} + \frac{(A-B)\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

Antiderivative was successfully verified.

[In] Int[(Sec[c + d*x]^4*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^3,x]

[Out] ((A - 3*B)*ArcTanh[Sin[c + d*x]])/(a^3*d) - ((7*A - 27*B)*Tan[c + d*x])/(15*a^3*d) + ((A - B)*Sec[c + d*x]^3*Tan[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) + ((4*A - 9*B)*Sec[c + d*x]^2*Tan[c + d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) - ((A - 3*B)*Tan[c + d*x])/(d*(a^3 + a^3*Sec[c + d*x]))

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 3767

Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) +
(a_.)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4008

```
Int[csc[(e_.) + (f_.)*(x_.)]^2*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(
csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(b^2*(2*m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*B*
(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A
*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (
a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> Simp[(d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^4(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^3} dx &= \frac{(A-B)\sec^3(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{\int \frac{\sec^3(c+dx)(3a(A-B)-a(A-6B)\sec(c+dx))}{(a+a\sec(c+dx))^2} dx}{5a^2} \\
&= \frac{(A-B)\sec^3(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(4A-9B)\sec^2(c+dx)\tan(c+dx)}{15ad(a+a\sec(c+dx))^2} \\
&= \frac{(A-B)\sec^3(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(4A-9B)\sec^2(c+dx)\tan(c+dx)}{15ad(a+a\sec(c+dx))^2} \\
&= \frac{(A-B)\sec^3(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(4A-9B)\sec^2(c+dx)\tan(c+dx)}{15ad(a+a\sec(c+dx))^2} \\
&= \frac{(A-3B)\tanh^{-1}(\sin(c+dx))}{a^3d} + \frac{(A-B)\sec^3(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(4A-9B)\sec^2(c+dx)\tan(c+dx)}{15ad(a+a\sec(c+dx))^2} \\
&= \frac{(A-3B)\tanh^{-1}(\sin(c+dx))}{a^3d} - \frac{(7A-27B)\tan(c+dx)}{15a^3d} + \frac{(A-B)\sec^3(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3}
\end{aligned}$$

Mathematica [B] time = 4.28, size = 480, normalized size = 3.08

$$\frac{\sec\left(\frac{c}{2}\right)\sec(c)\cos\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(5(32A-51B)\sin\left(\frac{dx}{2}\right)+(567B-167A)\sin\left(\frac{3dx}{2}\right)+170A\sin\left(c-\frac{dx}{2}\right)\right)}{(a+a\sec(c+dx))^3}$$

Antiderivative was successfully verified.

```

[In] Integrate[(Sec[c + d*x]^4*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^3,x]
[Out] (-960*(A - 3*B)*Cos[(c + d*x)/2]^6*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Cos[(c + d*x)/2]*Sec[c/2]*Sec[c]*Sec[c + d*x]*(5*(32*A - 51*B)*Sin[(d*x)/2] + (-167*A + 567*B)*Sin[(3*d*x)/2] + 170*A*Sin[c - (d*x)/2] - 600*B*Sin[c - (d*x)/2] - 170*A*Sin[c + (d*x)/2] + 375*B*Sin[c + (d*x)/2] + 160*A*Sin[2*c + (d*x)/2] - 480*B*Sin[2*c + (d*x)/2] + 75*A*Sin[c + (3*d*x)/2] - 60*B*Sin[c + (3*d*x)/2] - 167*A*Sin[2*c + (3*d*x)/2] + 402*B*Sin[2*c + (3*d*x)/2] + 75*A*Sin[3*c + (3*d*x)/2] - 225*B*Sin[3*c + (3*d*x)/2] - 95*A*Sin[c + (5*d*x)/2] + 315*B*Sin[c + (5*d*x)/2] + 15*A*Sin[2*c + (5*d*x)/2] + 30*B*Sin[2*c + (5*d*x)/2] - 95*A*Sin[3*c + (5*d*x)/2] + 240*B*Sin[3*c + (5*d*x)/2] + 15*A*Sin[4*c + (5*d*x)/2] - 45*B*Sin[4*c + (5*d*x)/2] - 22*A*Sin[2*c + (7*d*x)/2] + 72*B*Sin[2*c + (7*d*x)/2] + 15*B*Sin[3*c + (7*d*x)/2] - 22*A*Sin[4*c + (7*d*x)/2] + 57*B*Sin[4*c + (7*d*x)/2))/(120*a^3*d*(1 + Cos[c + d*x])^3)

```

fricas [A] time = 0.44, size = 256, normalized size = 1.64

$$\frac{15 \left((A - 3B) \cos(dx + c)^4 + 3(A - 3B) \cos(dx + c)^3 + 3(A - 3B) \cos(dx + c)^2 + (A - 3B) \cos(dx + c) \right) \log(\sin(dx + c) + 1) - 15 \left((A - 3B) \cos(dx + c)^4 + 3(A - 3B) \cos(dx + c)^3 + 3(A - 3B) \cos(dx + c)^2 + (A - 3B) \cos(dx + c) \right) \log(-\sin(dx + c) + 1) - 2 \left((11A - 36B) \cos(dx + c)^3 + 3(17A - 57B) \cos(dx + c)^2 + (32A - 117B) \cos(dx + c) - 15B \sin(dx + c) \right)}{a^3 d \cos(dx + c)^4 + 3a^3 d \cos(dx + c)^3 + 3a^3 d \cos(dx + c)^2 + a^3 d \cos(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="fricas")

[Out] 1/30*(15*((A - 3*B)*cos(d*x + c)^4 + 3*(A - 3*B)*cos(d*x + c)^3 + 3*(A - 3*B)*cos(d*x + c)^2 + (A - 3*B)*cos(d*x + c))*log(sin(d*x + c) + 1) - 15*((A - 3*B)*cos(d*x + c)^4 + 3*(A - 3*B)*cos(d*x + c)^3 + 3*(A - 3*B)*cos(d*x + c)^2 + (A - 3*B)*cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(2*(11*A - 36*B)*cos(d*x + c)^3 + 3*(17*A - 57*B)*cos(d*x + c)^2 + (32*A - 117*B)*cos(d*x + c) - 15*B*sin(d*x + c))/(a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + a^3*d*cos(d*x + c))

giac [A] time = 0.31, size = 186, normalized size = 1.19

$$\frac{60(A-3B) \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right|\right)}{a^3} - \frac{60(A-3B) \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1\right|\right)}{a^3} - \frac{120B \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right)^2 - 1} a^3 - \frac{3Aa^{12} \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5 - 3Ba^{12} \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="giac")

[Out] 1/60*(60*(A - 3*B)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 60*(A - 3*B)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^3 - 120*B*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 - 1)*a^3) - (3*A*a^12*tan(1/2*d*x + 1/2*c)^5 - 3*B*a^12*tan(1/2*d*x + 1/2*c)^5 + 20*A*a^12*tan(1/2*d*x + 1/2*c)^3 - 30*B*a^12*tan(1/2*d*x + 1/2*c)^3 + 105*A*a^12*tan(1/2*d*x + 1/2*c) - 255*B*a^12*tan(1/2*d*x + 1/2*c))/a^15)/d

maple [A] time = 0.60, size = 245, normalized size = 1.57

$$\frac{A \left(\tan^5 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{20d a^3} + \frac{B \left(\tan^5 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{20d a^3} - \frac{\left(\tan^3 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) A}{3d a^3} + \frac{B \left(\tan^3 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{2d a^3} - \frac{7A \tan \left(\frac{dx}{2} + \frac{c}{2} \right)}{4d a^3} + \frac{17B \tan \left(\frac{dx}{2} + \frac{c}{2} \right)}{4d a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x)`

[Out]
$$-1/20/d/a^3*A*\tan(1/2*d*x+1/2*c)^5+1/20/d/a^3*B*\tan(1/2*d*x+1/2*c)^5-1/3/d/a^3*\tan(1/2*d*x+1/2*c)^3*A+1/2/d/a^3*B*\tan(1/2*d*x+1/2*c)^3-7/4/d/a^3*A*\tan(1/2*d*x+1/2*c)+17/4/d/a^3*B*\tan(1/2*d*x+1/2*c)-1/d/a^3*\ln(\tan(1/2*d*x+1/2*c)-1)*A+3/d/a^3*\ln(\tan(1/2*d*x+1/2*c)-1)*B-1/d/a^3/(\tan(1/2*d*x+1/2*c)-1)*B+1/d/a^3*\ln(\tan(1/2*d*x+1/2*c)+1)*A-3/d/a^3*\ln(\tan(1/2*d*x+1/2*c)+1)*B-1/d/a^3/(\tan(1/2*d*x+1/2*c)+1)*B$$

maxima [A] time = 0.35, size = 286, normalized size = 1.83

$$3B \left(\frac{40 \sin(dx+c)}{\left(a^3 - \frac{a^3 \sin^2(dx+c)}{(\cos(dx+c)+1)^2} \right) (\cos(dx+c)+1)} + \frac{\frac{85 \sin(dx+c)}{\cos(dx+c)+1} + \frac{10 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3} - \frac{60 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1} + 1\right)}{a^3} + \frac{60 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1} - 1\right)}{a^3} \right) - A \left(\frac{\dots}{60d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="maxima")`

[Out]
$$\frac{1}{60} * (3*B*(40*\sin(d*x + c)/((a^3 - a^3*\sin(d*x + c)^2/(\cos(d*x + c) + 1)^2) * (\cos(d*x + c) + 1)) + (85*\sin(d*x + c)/(\cos(d*x + c) + 1) + 10*\sin(d*x + c)^3/(\cos(d*x + c) + 1)^3 + \sin(d*x + c)^5/(\cos(d*x + c) + 1)^5)/a^3 - 60*\log(\sin(d*x + c)/(\cos(d*x + c) + 1) + 1)/a^3 + 60*\log(\sin(d*x + c)/(\cos(d*x + c) + 1) - 1)/a^3 - A*((105*\sin(d*x + c)/(\cos(d*x + c) + 1) + 20*\sin(d*x + c)^3/(\cos(d*x + c) + 1)^3 + 3*\sin(d*x + c)^5/(\cos(d*x + c) + 1)^5)/a^3 - 60*\log(\sin(d*x + c)/(\cos(d*x + c) + 1) + 1)/a^3 + 60*\log(\sin(d*x + c)/(\cos(d*x + c) + 1) - 1)/a^3))/d$$

mupad [B] time = 2.04, size = 168, normalized size = 1.08

$$\frac{2 \operatorname{atanh}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right) (A - 3B)}{a^3 d} - \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right) \left(\frac{3(A-B)}{4a^3} - \frac{3B}{2a^3} + \frac{2A-4B}{2a^3}\right)}{d} - \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5 (A - B)}{20 a^3 d} - \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{20 a^3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B/cos(c + d*x))/(cos(c + d*x)^4*(a + a/cos(c + d*x))^3),x)`

[Out]
$$(2*\operatorname{atanh}(\tan(c/2 + (d*x)/2))*(A - 3*B))/(a^3*d) - (\tan(c/2 + (d*x)/2))*((3*(A - B))/(4*a^3) - (3*B)/(2*a^3) + (2*A - 4*B)/(2*a^3))/d - (\tan(c/2 + (d*x)/2)^5*(A - B))/(20*a^3*d) - (\tan(c/2 + (d*x)/2)^3*((A - B)/(6*a^3) + (2*A - 4*B)/(12*a^3)))/d - (2*B*\tan(c/2 + (d*x)/2))/(d*(a^3*\tan(c/2 + (d*x)/2)^2 - a^3))$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A \sec^4(c+dx)}{\sec^3(c+dx)+3 \sec^2(c+dx)+3 \sec(c+dx)+1} dx + \int \frac{B \sec^5(c+dx)}{\sec^3(c+dx)+3 \sec^2(c+dx)+3 \sec(c+dx)+1} dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**3,x)

[Out] (Integral(A*sec(c + d*x)**4/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d*x) + 1), x) + Integral(B*sec(c + d*x)**5/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d*x) + 1), x))/a**3

$$3.101 \quad \int \frac{\sec^3(c+dx)(A+B \sec(c+dx))}{(a+a \sec(c+dx))^3} dx$$

Optimal. Leaf size=125

$$\frac{(4A - 29B) \tan(c + dx)}{15d(a^3 \sec(c + dx) + a^3)} + \frac{B \tanh^{-1}(\sin(c + dx))}{a^3 d} + \frac{(A - B) \tan(c + dx) \sec^2(c + dx)}{5d(a \sec(c + dx) + a)^3} - \frac{(2A - 7B) \tan(c + dx)}{15ad(a \sec(c + dx) + a)^2}$$

[Out] B*arctanh(sin(d*x+c))/a^3/d+1/5*(A-B)*sec(d*x+c)^2*tan(d*x+c)/d/(a+a*sec(d*x+c))^3-1/15*(2*A-7*B)*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^2+1/15*(4*A-29*B)*tan(d*x+c)/d/(a^3+a^3*sec(d*x+c))

Rubi [A] time = 0.32, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {4019, 4008, 3998, 3770, 3794}

$$\frac{(4A - 29B) \tan(c + dx)}{15d(a^3 \sec(c + dx) + a^3)} + \frac{B \tanh^{-1}(\sin(c + dx))}{a^3 d} + \frac{(A - B) \tan(c + dx) \sec^2(c + dx)}{5d(a \sec(c + dx) + a)^3} - \frac{(2A - 7B) \tan(c + dx)}{15ad(a \sec(c + dx) + a)^2}$$

Antiderivative was successfully verified.

[In] Int[(Sec[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^3,x]

[Out] (B*ArcTanh[Sin[c + d*x]]/(a^3*d) + ((A - B)*Sec[c + d*x]^2*Tan[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) - ((2*A - 7*B)*Tan[c + d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) + ((4*A - 29*B)*Tan[c + d*x])/(15*d*(a^3 + a^3*Sec[c + d*x]))

Rule 3770

Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3794

Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 3998

Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)))/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := Dist[B/b, Int[Csc[e + f*x], x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]

Rule 4008

```
Int[csc[(e_.) + (f_.)*(x_.)]^2*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*
csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.), x_Symbol] :> -Simp[((A*b - a*B)*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(b^2*(2*m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*B*
(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A
*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (
a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.), x_Symbol] :> Simp[(d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{\sec^3(c + dx)(A + B \sec(c + dx))}{(a + a \sec(c + dx))^3} dx &= \frac{(A - B) \sec^2(c + dx) \tan(c + dx)}{5d(a + a \sec(c + dx))^3} + \frac{\int \frac{\sec^2(c + dx)(2a(A - B) + 5aB \sec(c + dx))}{(a + a \sec(c + dx))^2} dx}{5a^2} \\ &= \frac{(A - B) \sec^2(c + dx) \tan(c + dx)}{5d(a + a \sec(c + dx))^3} - \frac{(2A - 7B) \tan(c + dx)}{15ad(a + a \sec(c + dx))^2} - \frac{\int \frac{\sec(c + dx)}{a + a \sec(c + dx)} dx}{15ad(a + a \sec(c + dx))^2} \\ &= \frac{(A - B) \sec^2(c + dx) \tan(c + dx)}{5d(a + a \sec(c + dx))^3} - \frac{(2A - 7B) \tan(c + dx)}{15ad(a + a \sec(c + dx))^2} + \frac{(4A - 7B) \tan(c + dx)}{15ad(a + a \sec(c + dx))^2} \\ &= \frac{B \tanh^{-1}(\sin(c + dx))}{a^3 d} + \frac{(A - B) \sec^2(c + dx) \tan(c + dx)}{5d(a + a \sec(c + dx))^3} - \frac{(2A - 7B) \tan(c + dx)}{15ad(a + a \sec(c + dx))^2} \end{aligned}$$

Mathematica [A] time = 0.93, size = 197, normalized size = 1.58

$$\sec\left(\frac{c}{2}\right) \cos\left(\frac{1}{2}(c + dx)\right) \left(5(4A - 29B) \sin\left(\frac{dx}{2}\right) + 10A \sin\left(c + \frac{3dx}{2}\right) + 2A \sin\left(2c + \frac{5dx}{2}\right) + 75B \sin\left(c + \frac{dx}{2}\right) - 95B \sin\left(2c + \frac{3dx}{2}\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^3,x]

[Out] $(-240*B*\cos[(c + d*x)/2]^6*(\log[\cos[(c + d*x)/2] - \sin[(c + d*x)/2]] - \log[\cos[(c + d*x)/2] + \sin[(c + d*x)/2]]) + \cos[(c + d*x)/2]*\sec[c/2]*(5*(4*A - 29*B)*\sin[(d*x)/2] + 75*B*\sin[c + (d*x)/2] + 10*A*\sin[c + (3*d*x)/2] - 95*B*\sin[c + (3*d*x)/2] + 15*B*\sin[2*c + (3*d*x)/2] + 2*A*\sin[2*c + (5*d*x)/2] - 22*B*\sin[2*c + (5*d*x)/2]))/(30*a^3*d*(1 + \cos[c + d*x])^3)$

fricas [A] time = 0.46, size = 183, normalized size = 1.46

$$\frac{15 \left(B \cos(dx + c)^3 + 3 B \cos(dx + c)^2 + 3 B \cos(dx + c) + B \right) \log(\sin(dx + c) + 1) - 15 \left(B \cos(dx + c)^3 + 3 B \cos(dx + c)^2 + 3 B \cos(dx + c) + B \right) \log(-\sin(dx + c) + 1) + 2 * (2 * (A - 11 * B) * \cos(dx + c)^2 + 3 * (2 * A - 17 * B) * \cos(dx + c) + 7 * A - 32 * B) * \sin(dx + c)}{30 \left(a^3 d \cos(dx + c) + a^3 d \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="fricas")

[Out] $\frac{1}{30} * (15 * (B * \cos(dx + c)^3 + 3 * B * \cos(dx + c)^2 + 3 * B * \cos(dx + c) + B) * \log(\sin(dx + c) + 1) - 15 * (B * \cos(dx + c)^3 + 3 * B * \cos(dx + c)^2 + 3 * B * \cos(dx + c) + B) * \log(-\sin(dx + c) + 1) + 2 * (2 * (A - 11 * B) * \cos(dx + c)^2 + 3 * (2 * A - 17 * B) * \cos(dx + c) + 7 * A - 32 * B) * \sin(dx + c)) / (a^3 * d * \cos(dx + c)^3 + 3 * a^3 * d * \cos(dx + c)^2 + 3 * a^3 * d * \cos(dx + c) + a^3 * d)$

giac [A] time = 0.67, size = 147, normalized size = 1.18

$$\frac{60 B \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 1\right|\right)}{a^3} - \frac{60 B \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 1\right|\right)}{a^3} + \frac{3 A a^{12} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 - 3 B a^{12} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 + 10 A a^{12} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 - 20 B a^{12} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3}{a^{15}}}{60 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="giac")

[Out] $\frac{1}{60} * (60 * B * \log(\text{abs}(\tan(1/2 * d * x + 1/2 * c) + 1)) / a^3 - 60 * B * \log(\text{abs}(\tan(1/2 * d * x + 1/2 * c) - 1)) / a^3 + (3 * A * a^{12} * \tan(1/2 * d * x + 1/2 * c)^5 - 3 * B * a^{12} * \tan(1/2 * d * x + 1/2 * c)^5 + 10 * A * a^{12} * \tan(1/2 * d * x + 1/2 * c)^3 - 20 * B * a^{12} * \tan(1/2 * d * x + 1/2 * c)^3 + 15 * A * a^{12} * \tan(1/2 * d * x + 1/2 * c) - 105 * B * a^{12} * \tan(1/2 * d * x + 1/2 * c)) / a^{15}) / d$

maple [A] time = 0.75, size = 159, normalized size = 1.27

$$\frac{A \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{4d a^3} - \frac{\ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right) B}{d a^3} - \frac{7B \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{4d a^3} + \frac{\ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right) B}{d a^3} + \frac{A \left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{20d a^3} - \frac{B \left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{20d a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(\sec(dx+c)^3*(A+B*\sec(dx+c))/(a+a*\sec(dx+c))^3, x)$

[Out] $\frac{1}{4} \frac{1}{d} \frac{1}{a^3} A \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - \frac{1}{d} \frac{1}{a^3} \ln\left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 1\right) * B - \frac{7}{4} \frac{1}{d} \frac{1}{a^3} * B \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + \frac{1}{d} \frac{1}{a^3} \ln\left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 1\right) * B + \frac{1}{20} \frac{1}{d} \frac{1}{a^3} A \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 - \frac{1}{20} \frac{1}{d} \frac{1}{a^3} * B \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 + \frac{1}{6} \frac{1}{d} \frac{1}{a^3} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 * A - \frac{1}{3} \frac{1}{d} \frac{1}{a^3} * B \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3$

maxima [A] time = 0.34, size = 187, normalized size = 1.50

$$\frac{B \left(\frac{105 \sin(dx+c)}{\cos(dx+c)+1} + \frac{20 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{3 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{60 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1} + 1\right)}{a^3} + \frac{60 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1} - 1\right)}{a^3} \right) - \frac{A \left(\frac{15 \sin(dx+c)}{\cos(dx+c)+1} + \frac{10 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{3 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} \right)}{60 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(\sec(dx+c)^3*(A+B*\sec(dx+c))/(a+a*\sec(dx+c))^3, x, \text{algorithm}="maxima")$

[Out] $- \frac{1}{60} * (B * ((105 * \sin(dx+c) / (\cos(dx+c)+1) + 20 * \sin(dx+c)^3 / (\cos(dx+c)+1)^3 + 3 * \sin(dx+c)^5 / (\cos(dx+c)+1)^5) / a^3 - 60 * \log(\sin(dx+c) / (\cos(dx+c)+1) + 1) / a^3 + 60 * \log(\sin(dx+c) / (\cos(dx+c)+1) - 1) / a^3) - A * (15 * \sin(dx+c) / (\cos(dx+c)+1) + 10 * \sin(dx+c)^3 / (\cos(dx+c)+1)^3 + 3 * \sin(dx+c)^5 / (\cos(dx+c)+1)^5) / a^3) / d$

mupad [B] time = 1.98, size = 124, normalized size = 0.99

$$\frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 \left(\frac{A-B}{12a^3} + \frac{A-3B}{12a^3}\right)}{d} + \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right) \left(\frac{A-B}{4a^3} + \frac{A-3B}{4a^3} - \frac{A+3B}{4a^3}\right)}{d} + \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5 (A-B)}{20a^3 d} + \frac{2B \operatorname{atanh}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{a^3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((A + B/\cos(c + dx))/(\cos(c + dx)^3*(a + a/\cos(c + dx))^3), x)$

[Out] $\frac{(\tan(c/2 + (dx)/2))^3 * ((A - B)/(12*a^3) + (A - 3*B)/(12*a^3))}{d} + \frac{(\tan(c/2 + (dx)/2) * ((A - B)/(4*a^3) + (A - 3*B)/(4*a^3) - (A + 3*B)/(4*a^3)))}{d} + \frac{(\tan(c/2 + (dx)/2))^5 * (A - B)}{(20*a^3*d)} + \frac{(2*B*\operatorname{atanh}(\tan(c/2 + (dx)/2)))}{(a^3*d)}$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A \sec^3(c+dx)}{\sec^3(c+dx)+3 \sec^2(c+dx)+3 \sec(c+dx)+1} dx + \int \frac{B \sec^4(c+dx)}{\sec^3(c+dx)+3 \sec^2(c+dx)+3 \sec(c+dx)+1} dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**3,x)
```

```
[Out] (Integral(A*sec(c + d*x)**3/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d*x) + 1), x) + Integral(B*sec(c + d*x)**4/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d*x) + 1), x))/a**3
```

$$3.102 \quad \int \frac{\sec^2(c+dx)(A+B \sec(c+dx))}{(a+a \sec(c+dx))^3} dx$$

Optimal. Leaf size=102

$$\frac{(3A+7B) \tan(c+dx)}{15d(a^3 \sec(c+dx)+a^3)} + \frac{(3A-8B) \tan(c+dx)}{15ad(a \sec(c+dx)+a)^2} - \frac{(A-B) \tan(c+dx)}{5d(a \sec(c+dx)+a)^3}$$

[Out] $-1/5*(A-B)*\tan(d*x+c)/d/(a+a*\sec(d*x+c))^3+1/15*(3*A-8*B)*\tan(d*x+c)/a/d/(a+a*\sec(d*x+c))^2+1/15*(3*A+7*B)*\tan(d*x+c)/d/(a^3+a^3*\sec(d*x+c))$

Rubi [A] time = 0.20, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {4008, 4000, 3794}

$$\frac{(3A+7B) \tan(c+dx)}{15d(a^3 \sec(c+dx)+a^3)} + \frac{(3A-8B) \tan(c+dx)}{15ad(a \sec(c+dx)+a)^2} - \frac{(A-B) \tan(c+dx)}{5d(a \sec(c+dx)+a)^3}$$

Antiderivative was successfully verified.

[In] Int[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^3,x]

[Out] $-((A-B)*\text{Tan}[c+d*x])/(5*d*(a+a*\text{Sec}[c+d*x])^3) + ((3*A-8*B)*\text{Tan}[c+d*x])/(15*a*d*(a+a*\text{Sec}[c+d*x])^2) + ((3*A+7*B)*\text{Tan}[c+d*x])/(15*d*(a^3+a^3*\text{Sec}[c+d*x]))$

Rule 3794

Int[csc[(e_.) + (f_.)*(x_.)]/(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] := -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 4000

Int[csc[(e_.) + (f_.)*(x_.)]*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := Simp[((A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]

Rule 4008

Int[csc[(e_.) + (f_.)*(x_.)]^2*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := -Simp[((A*b - a*B)*Cot[

```
e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(b^2*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*B*(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned} \int \frac{\sec^2(c + dx)(A + B \sec(c + dx))}{(a + a \sec(c + dx))^3} dx &= -\frac{(A - B) \tan(c + dx)}{5d(a + a \sec(c + dx))^3} - \frac{\int \frac{\sec(c+dx)(-3a(A-B)-5aB \sec(c+dx))}{(a+a \sec(c+dx))^2} dx}{5a^2} \\ &= -\frac{(A - B) \tan(c + dx)}{5d(a + a \sec(c + dx))^3} + \frac{(3A - 8B) \tan(c + dx)}{15ad(a + a \sec(c + dx))^2} + \frac{(3A + 7B) \int \frac{\sec}{a+a \sec}}{15a^2} \\ &= -\frac{(A - B) \tan(c + dx)}{5d(a + a \sec(c + dx))^3} + \frac{(3A - 8B) \tan(c + dx)}{15ad(a + a \sec(c + dx))^2} + \frac{(3A + 7B) \tan(c + dx)}{15d(a^3 + a^3 \sec(c + dx))} \end{aligned}$$

Mathematica [A] time = 0.31, size = 96, normalized size = 0.94

$$\frac{\sec\left(\frac{c}{2}\right) \cos\left(\frac{1}{2}(c + dx)\right) \left((3A + 2B) \left(5 \sin\left(c + \frac{3dx}{2}\right) + \sin\left(2c + \frac{5dx}{2}\right) \right) + 5(3A + 4B) \sin\left(\frac{dx}{2}\right) - 15A \sin\left(c + \frac{dx}{2}\right) \right)}{30a^3d(\cos(c + dx) + 1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(5*(3*A + 4*B)*Sin[(d*x)/2] - 15*A*Sin[c + (d*x)/2] + (3*A + 2*B)*(5*Sin[c + (3*d*x)/2] + Sin[2*c + (5*d*x)/2]))) / (30*a^3*d*(1 + Cos[c + d*x])^3)
```

fricas [A] time = 0.40, size = 93, normalized size = 0.91

$$\frac{\left((3A + 2B) \cos(dx + c)^2 + 3(3A + 2B) \cos(dx + c) + 3A + 7B \right) \sin(dx + c)}{15(a^3d \cos(dx + c)^3 + 3a^3d \cos(dx + c)^2 + 3a^3d \cos(dx + c) + a^3d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/15*((3*A + 2*B)*cos(d*x + c)^2 + 3*(3*A + 2*B)*cos(d*x + c) + 3*A + 7*B)*sin(d*x + c)/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

giac [A] time = 1.01, size = 75, normalized size = 0.74

$$\frac{3A \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5 - 3B \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5 - 10B \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 - 15A \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 15B \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)}{60a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="giac")

[Out] -1/60*(3*A*tan(1/2*d*x + 1/2*c)^5 - 3*B*tan(1/2*d*x + 1/2*c)^5 - 10*B*tan(1/2*d*x + 1/2*c)^3 - 15*A*tan(1/2*d*x + 1/2*c) - 15*B*tan(1/2*d*x + 1/2*c))/ (a^3*d)

maple [A] time = 0.73, size = 64, normalized size = 0.63

$$\frac{\frac{(-A+B)\left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{5} + \frac{2B\left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{3} + A \tan\left(\frac{dx}{2} + \frac{c}{2}\right) + B \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{4d a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x)

[Out] 1/4/d/a^3*(1/5*(-A+B)*tan(1/2*d*x+1/2*c)^5+2/3*B*tan(1/2*d*x+1/2*c)^3+A*tan(1/2*d*x+1/2*c)+B*tan(1/2*d*x+1/2*c))

maxima [A] time = 0.33, size = 115, normalized size = 1.13

$$\frac{B\left(\frac{15 \sin(dx+c)}{\cos(dx+c)+1} + \frac{10 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{3 \sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^3} + \frac{3A\left(\frac{5 \sin(dx+c)}{\cos(dx+c)+1} - \frac{\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^3}$$

$$60d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="maxima")

[Out] 1/60*(B*(15*sin(d*x + c)/(cos(d*x + c) + 1) + 10*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 + 3*A*(5*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3)/d

mupad [B] time = 1.92, size = 66, normalized size = 0.65

$$\frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right) \left(15A + 15B - 3A \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 + 10B \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + 3B \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4\right)}{60a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B/cos(c + d*x))/(cos(c + d*x)^2*(a + a/cos(c + d*x))^3),x)`

[Out] $(\tan(c/2 + (d*x)/2)*(15*A + 15*B - 3*A*\tan(c/2 + (d*x)/2)^4 + 10*B*\tan(c/2 + (d*x)/2)^2 + 3*B*\tan(c/2 + (d*x)/2)^4))/(60*a^3*d)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A \sec^2(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1} dx + \int \frac{B \sec^3(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1} dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)**2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**3,x)`

[Out] $(\text{Integral}(A*\sec(c + d*x)**2/(\sec(c + d*x)**3 + 3*\sec(c + d*x)**2 + 3*\sec(c + d*x) + 1), x) + \text{Integral}(B*\sec(c + d*x)**3/(\sec(c + d*x)**3 + 3*\sec(c + d*x)**2 + 3*\sec(c + d*x) + 1), x))/a**3$

$$3.103 \quad \int \frac{\sec(c+dx)(A+B \sec(c+dx))}{(a+a \sec(c+dx))^3} dx$$

Optimal. Leaf size=102

$$\frac{(2A+3B) \tan(c+dx)}{15d(a^3 \sec(c+dx)+a^3)} + \frac{(2A+3B) \tan(c+dx)}{15ad(a \sec(c+dx)+a)^2} + \frac{(A-B) \tan(c+dx)}{5d(a \sec(c+dx)+a)^3}$$

[Out] 1/5*(A-B)*tan(d*x+c)/d/(a+a*sec(d*x+c))^3+1/15*(2*A+3*B)*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^2+1/15*(2*A+3*B)*tan(d*x+c)/d/(a^3+a^3*sec(d*x+c))

Rubi [A] time = 0.11, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {4000, 3796, 3794}

$$\frac{(2A+3B) \tan(c+dx)}{15d(a^3 \sec(c+dx)+a^3)} + \frac{(2A+3B) \tan(c+dx)}{15ad(a \sec(c+dx)+a)^2} + \frac{(A-B) \tan(c+dx)}{5d(a \sec(c+dx)+a)^3}$$

Antiderivative was successfully verified.

[In] Int[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^3,x]

[Out] ((A - B)*Tan[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) + ((2*A + 3*B)*Tan[c + d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) + ((2*A + 3*B)*Tan[c + d*x])/(15*d*(a^3 + a^3*Sec[c + d*x]))

Rule 3794

Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 3796

Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_), x_Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && IntegerQ[2*m]

Rule 4000

Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] :> Simp[((A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]

/; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]

Rubi steps

$$\begin{aligned} \int \frac{\sec(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^3} dx &= \frac{(A-B)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(2A+3B)\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^2} dx}{5a} \\ &= \frac{(A-B)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(2A+3B)\tan(c+dx)}{15ad(a+a\sec(c+dx))^2} + \frac{(2A+3B)\int \frac{\sec(c+dx)}{a+a\sec(c+dx)} dx}{15a^2} \\ &= \frac{(A-B)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(2A+3B)\tan(c+dx)}{15ad(a+a\sec(c+dx))^2} + \frac{(2A+3B)\tan(c+dx)}{15d(a^3+a^3\sec(c+dx))} \end{aligned}$$

Mathematica [A] time = 0.35, size = 135, normalized size = 1.32

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(-15(2A+B)\sin\left(c+\frac{dx}{2}\right)+5(8A+3B)\sin\left(\frac{dx}{2}\right)+20A\sin\left(c+\frac{3dx}{2}\right)-15A\sin\left(2c+\frac{3dx}{2}\right)\right)}{30a^3d(\cos(c+dx)+1)^3}$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^3,x]

[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(5*(8*A + 3*B)*Sin[(d*x)/2] - 15*(2*A + B)*Sin[c + (d*x)/2] + 20*A*Sin[c + (3*d*x)/2] + 15*B*Sin[c + (3*d*x)/2] - 15*A*Sin[2*c + (3*d*x)/2] + 7*A*Sin[2*c + (5*d*x)/2] + 3*B*Sin[2*c + (5*d*x)/2]))/(3*0*a^3*d*(1 + Cos[c + d*x])^3)

fricas [A] time = 0.43, size = 93, normalized size = 0.91

$$\frac{((7A + 3B)\cos(dx + c)^2 + 3(2A + 3B)\cos(dx + c) + 2A + 3B)\sin(dx + c)}{15(a^3d\cos(dx + c)^3 + 3a^3d\cos(dx + c)^2 + 3a^3d\cos(dx + c) + a^3d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="fricas")

[Out] 1/15*((7*A + 3*B)*cos(d*x + c)^2 + 3*(2*A + 3*B)*cos(d*x + c) + 2*A + 3*B)*sin(d*x + c)/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)

giac [A] time = 1.91, size = 75, normalized size = 0.74

$$\frac{3A \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5 - 3B \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5 - 10A \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 + 15A \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 15B \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)}{60a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="giac")

[Out] 1/60*(3*A*tan(1/2*d*x + 1/2*c)^5 - 3*B*tan(1/2*d*x + 1/2*c)^5 - 10*A*tan(1/2*d*x + 1/2*c)^3 + 15*A*tan(1/2*d*x + 1/2*c) + 15*B*tan(1/2*d*x + 1/2*c))/(a^3*d)

maple [A] time = 0.75, size = 64, normalized size = 0.63

$$\frac{\frac{(A-B)\left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{5} - \frac{2\left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)A}{3} + A \tan\left(\frac{dx}{2} + \frac{c}{2}\right) + B \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{4d a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x)

[Out] 1/4/d/a^3*(1/5*(A-B)*tan(1/2*d*x+1/2*c)^5-2/3*tan(1/2*d*x+1/2*c)^3*A+A*tan(1/2*d*x+1/2*c)+B*tan(1/2*d*x+1/2*c))

maxima [A] time = 0.33, size = 115, normalized size = 1.13

$$\frac{A\left(\frac{15 \sin(dx+c)}{\cos(dx+c)+1} - \frac{10 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{3 \sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right) + 3B\left(\frac{5 \sin(dx+c)}{\cos(dx+c)+1} - \frac{\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="maxima")

[Out] 1/60*(A*(15*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 + 3*B*(5*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3)/d

mupad [B] time = 1.92, size = 66, normalized size = 0.65

$$\frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right) \left(15A + 15B - 10A \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + 3A \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 - 3B \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4\right)}{60a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B/cos(c + d*x))/(cos(c + d*x)*(a + a/cos(c + d*x))^3), x)`

[Out] $(\tan(c/2 + (d*x)/2)*(15*A + 15*B - 10*A*\tan(c/2 + (d*x)/2)^2 + 3*A*\tan(c/2 + (d*x)/2)^4 - 3*B*\tan(c/2 + (d*x)/2)^4))/(60*a^3*d)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A \sec(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1} dx + \int \frac{B \sec^2(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1} dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**3, x)`

[Out] $(\text{Integral}(A*\sec(c + d*x)/(\sec(c + d*x)**3 + 3*\sec(c + d*x)**2 + 3*\sec(c + d*x) + 1), x) + \text{Integral}(B*\sec(c + d*x)**2/(\sec(c + d*x)**3 + 3*\sec(c + d*x)**2 + 3*\sec(c + d*x) + 1), x))/a**3$

$$3.104 \quad \int \frac{A+B \sec(c+dx)}{(a+a \sec(c+dx))^3} dx$$

Optimal. Leaf size=108

$$\frac{2(11A - B) \tan(c + dx)}{15d(a^3 \sec(c + dx) + a^3)} + \frac{Ax}{a^3} - \frac{(7A - 2B) \tan(c + dx)}{15ad(a \sec(c + dx) + a)^2} - \frac{(A - B) \tan(c + dx)}{5d(a \sec(c + dx) + a)^3}$$

[Out] $A*x/a^3 - 1/5*(A-B)*\tan(d*x+c)/d/(a+a*\sec(d*x+c))^3 - 1/15*(7*A-2*B)*\tan(d*x+c)/a/d/(a+a*\sec(d*x+c))^2 - 2/15*(11*A-B)*\tan(d*x+c)/d/(a^3+a^3*\sec(d*x+c))$

Rubi [A] time = 0.19, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3922, 3919, 3794}

$$\frac{2(11A - B) \tan(c + dx)}{15d(a^3 \sec(c + dx) + a^3)} + \frac{Ax}{a^3} - \frac{(7A - 2B) \tan(c + dx)}{15ad(a \sec(c + dx) + a)^2} - \frac{(A - B) \tan(c + dx)}{5d(a \sec(c + dx) + a)^3}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sec[c + d*x])/(a + a*Sec[c + d*x])^3, x]

[Out] $(A*x)/a^3 - ((A - B)*\tan[c + d*x])/(5*d*(a + a*\sec[c + d*x])^3) - ((7*A - 2*B)*\tan[c + d*x])/(15*a*d*(a + a*\sec[c + d*x])^2) - (2*(11*A - B)*\tan[c + d*x])/(15*d*(a^3 + a^3*\sec[c + d*x]))$

Rule 3794

Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 3919

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 3922

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.)), x_Symbol] := -Simp[((b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && EqQ

[a^2 - b^2, 0] && IntegerQ[2*m]

Rubi steps

$$\begin{aligned}
 \int \frac{A + B \sec(c + dx)}{(a + a \sec(c + dx))^3} dx &= -\frac{(A - B) \tan(c + dx)}{5d(a + a \sec(c + dx))^3} - \frac{\int \frac{-5aA + 2a(A - B) \sec(c + dx)}{(a + a \sec(c + dx))^2} dx}{5a^2} \\
 &= -\frac{(A - B) \tan(c + dx)}{5d(a + a \sec(c + dx))^3} - \frac{(7A - 2B) \tan(c + dx)}{15ad(a + a \sec(c + dx))^2} + \frac{\int \frac{15a^2A - a^2(7A - 2B) \sec(c + dx)}{a + a \sec(c + dx)} dx}{15a^4} \\
 &= \frac{Ax}{a^3} - \frac{(A - B) \tan(c + dx)}{5d(a + a \sec(c + dx))^3} - \frac{(7A - 2B) \tan(c + dx)}{15ad(a + a \sec(c + dx))^2} - \frac{(2(11A - B)) \int \frac{\sec(c + dx)}{a + a \sec(c + dx)} dx}{15a^2} \\
 &= \frac{Ax}{a^3} - \frac{(A - B) \tan(c + dx)}{5d(a + a \sec(c + dx))^3} - \frac{(7A - 2B) \tan(c + dx)}{15ad(a + a \sec(c + dx))^2} - \frac{2(11A - B) \tan(c + dx)}{15d(a^3 + a^3 \sec(c + dx))}
 \end{aligned}$$

Mathematica [B] time = 0.60, size = 241, normalized size = 2.23

$$\frac{\sec\left(\frac{c}{2}\right) \sec^5\left(\frac{1}{2}(c + dx)\right) \left(270A \sin\left(c + \frac{dx}{2}\right) - 230A \sin\left(c + \frac{3dx}{2}\right) + 90A \sin\left(2c + \frac{3dx}{2}\right) - 64A \sin\left(2c + \frac{5dx}{2}\right) + 15A \sin\left(2c + \frac{7dx}{2}\right)\right)}{480a^3d}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sec[c + d*x])/(a + a*Sec[c + d*x])^3,x]

[Out] (Sec[c/2]*Sec[(c + d*x)/2]^5*(150*A*d*x*Cos[(d*x)/2] + 150*A*d*x*Cos[c + (d*x)/2] + 75*A*d*x*Cos[c + (3*d*x)/2] + 75*A*d*x*Cos[2*c + (3*d*x)/2] + 15*A*d*x*Cos[2*c + (5*d*x)/2] + 15*A*d*x*Cos[3*c + (5*d*x)/2] - 370*A*Sin[(d*x)/2] + 80*B*Sin[(d*x)/2] + 270*A*Sin[c + (d*x)/2] - 60*B*Sin[c + (d*x)/2] - 230*A*Sin[c + (3*d*x)/2] + 40*B*Sin[c + (3*d*x)/2] + 90*A*Sin[2*c + (3*d*x)/2] - 30*B*Sin[2*c + (3*d*x)/2] - 64*A*Sin[2*c + (5*d*x)/2] + 14*B*Sin[2*c + (5*d*x)/2]))/(480*a^3*d)

fricas [A] time = 0.43, size = 138, normalized size = 1.28

$$\frac{15 A dx \cos(dx + c)^3 + 45 A dx \cos(dx + c)^2 + 45 A dx \cos(dx + c) + 15 A dx - \left((32 A - 7 B) \cos(dx + c)^2 + 3(17 A - 7 B) \cos(dx + c) + 3 B\right)}{15 \left(a^3 d \cos(dx + c)^3 + 3 a^3 d \cos(dx + c)^2 + 3 a^3 d \cos(dx + c) + a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="fricas")

[Out] $1/15*(15*A*d*x*\cos(d*x + c)^3 + 45*A*d*x*\cos(d*x + c)^2 + 45*A*d*x*\cos(d*x + c) + 15*A*d*x - ((32*A - 7*B)*\cos(d*x + c)^2 + 3*(17*A - 2*B)*\cos(d*x + c) + 22*A - 2*B)*\sin(d*x + c))/(a^3*d*\cos(d*x + c)^3 + 3*a^3*d*\cos(d*x + c)^2 + 3*a^3*d*\cos(d*x + c) + a^3*d)$

giac [A] time = 0.65, size = 121, normalized size = 1.12

$$\frac{60(dx+c)A}{a^3} - \frac{3Aa^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5 - 3Ba^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5 - 20Aa^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 + 10Ba^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 + 105Aa^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) - 15Ba^{12}}{a^{15}}$$

$$60d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="giac")`

[Out] $1/60*(60*(d*x + c)*A/a^3 - (3*A*a^{12}*\tan(1/2*d*x + 1/2*c)^5 - 3*B*a^{12}*\tan(1/2*d*x + 1/2*c)^5 - 20*A*a^{12}*\tan(1/2*d*x + 1/2*c)^3 + 10*B*a^{12}*\tan(1/2*d*x + 1/2*c)^3 + 105*A*a^{12}*\tan(1/2*d*x + 1/2*c) - 15*B*a^{12}*\tan(1/2*d*x + 1/2*c))/a^{15})/d$

maple [A] time = 0.80, size = 137, normalized size = 1.27

$$\frac{A\left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{20d a^3} + \frac{B\left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{20d a^3} + \frac{\left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)A}{3d a^3} - \frac{B\left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{6d a^3} - \frac{7A \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{4d a^3} + \frac{B \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{4d a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x)`

[Out] $-1/20/d/a^3*A*\tan(1/2*d*x+1/2*c)^5+1/20/d/a^3*B*\tan(1/2*d*x+1/2*c)^5+1/3/d/a^3*\tan(1/2*d*x+1/2*c)^3*A-1/6/d/a^3*B*\tan(1/2*d*x+1/2*c)^3-7/4/d/a^3*A*\tan(1/2*d*x+1/2*c)+1/4/d/a^3*B*\tan(1/2*d*x+1/2*c)+2/d/a^3*\arctan(\tan(1/2*d*x+1/2*c))*A$

maxima [A] time = 0.42, size = 160, normalized size = 1.48

$$\frac{A\left(\frac{105 \sin(dx+c)}{\cos(dx+c)+1} - \frac{20 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{3 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{120 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}\right)}{a^3} - \frac{B\left(\frac{15 \sin(dx+c)}{\cos(dx+c)+1} - \frac{10 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{3 \sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^3}$$

$$60d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="maxima")`

[Out] $-1/60*(A*((105*\sin(d*x + c))/(\cos(d*x + c) + 1) - 20*\sin(d*x + c)^3/(\cos(d*x + c) + 1)^3 + 3*\sin(d*x + c)^5/(\cos(d*x + c) + 1)^5)/a^3 - 120*\arctan(\sin($

$d*x + c)/(\cos(d*x + c) + 1)/a^3) - B*(15*\sin(d*x + c)/(\cos(d*x + c) + 1) - 10*\sin(d*x + c)^3/(\cos(d*x + c) + 1)^3 + 3*\sin(d*x + c)^5/(\cos(d*x + c) + 1)^5)/a^3)/d$

mupad [B] time = 2.13, size = 133, normalized size = 1.23

$$\frac{Ax}{a^3} + \frac{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)^2 \left(\frac{A \sin\left(\frac{c}{2} + \frac{dx}{2}\right)^3}{3} - \frac{B \sin\left(\frac{c}{2} + \frac{dx}{2}\right)^3}{6} \right) - \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^4 \left(\frac{7A \sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{4} - \frac{B \sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{4} \right) - \frac{A \sin\left(\frac{c}{2} + \frac{dx}{2}\right)^5}{20} + \frac{B \sin\left(\frac{c}{2} + \frac{dx}{2}\right)^5}{20}}{a^3 d \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B/cos(c + d*x))/(a + a/cos(c + d*x))^3,x)`

[Out] $(A*x)/a^3 + (\cos(c/2 + (d*x)/2)^2*((A*\sin(c/2 + (d*x)/2)^3)/3 - (B*\sin(c/2 + (d*x)/2)^3)/6) - \cos(c/2 + (d*x)/2)^4*((7*A*\sin(c/2 + (d*x)/2))/4 - (B*\sin(c/2 + (d*x)/2))/4) - (A*\sin(c/2 + (d*x)/2)^5)/20 + (B*\sin(c/2 + (d*x)/2)^5)/20)/(a^3*d*\cos(c/2 + (d*x)/2)^5)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1} dx + \int \frac{B \sec(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1} dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))**3,x)`

[Out] $(\text{Integral}(A/(\sec(c + d*x)**3 + 3*\sec(c + d*x)**2 + 3*\sec(c + d*x) + 1), x) + \text{Integral}(B*\sec(c + d*x)/(\sec(c + d*x)**3 + 3*\sec(c + d*x)**2 + 3*\sec(c + d*x) + 1), x))/a**3$

$$3.105 \quad \int \frac{\cos(c+dx)(A+B \sec(c+dx))}{(a+a \sec(c+dx))^3} dx$$

Optimal. Leaf size=136

$$\frac{2(36A - 11B) \sin(c + dx)}{15a^3d} - \frac{(3A - B) \sin(c + dx)}{d(a^3 \sec(c + dx) + a^3)} - \frac{x(3A - B)}{a^3} - \frac{(9A - 4B) \sin(c + dx)}{15ad(a \sec(c + dx) + a)^2} - \frac{(A - B) \sin(c + dx)}{5d(a \sec(c + dx) + a)}$$

[Out] $-(3A-B)*x/a^3+2/15*(36A-11*B)*\sin(d*x+c)/a^3/d-1/5*(A-B)*\sin(d*x+c)/d/(a+a*\sec(d*x+c))^3-1/15*(9A-4*B)*\sin(d*x+c)/a/d/(a+a*\sec(d*x+c))^2-(3A-B)*\sin(d*x+c)/d/(a^3+a^3*\sec(d*x+c))$

Rubi [A] time = 0.37, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {4020, 3787, 2637, 8}

$$\frac{2(36A - 11B) \sin(c + dx)}{15a^3d} - \frac{(3A - B) \sin(c + dx)}{d(a^3 \sec(c + dx) + a^3)} - \frac{x(3A - B)}{a^3} - \frac{(9A - 4B) \sin(c + dx)}{15ad(a \sec(c + dx) + a)^2} - \frac{(A - B) \sin(c + dx)}{5d(a \sec(c + dx) + a)}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^3,x]

[Out] $-(((3A - B)*x)/a^3) + (2*(36A - 11*B)*Sin[c + d*x])/(15*a^3*d) - ((A - B)*Sin[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) - ((9*A - 4*B)*Sin[c + d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) - ((3A - B)*Sin[c + d*x])/(d*(a^3 + a^3*Sec[c + d*x]))$

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_)])*(d_.)^(n_.)*(csc[(e_.) + (f_.)*(x_)])*(b_.) + (a_.), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m + 1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{\cos(c+dx)(A+B \sec(c+dx))}{(a+a \sec(c+dx))^3} dx &= -\frac{(A-B) \sin(c+dx)}{5d(a+a \sec(c+dx))^3} + \frac{\int \frac{\cos(c+dx)(a(6A-B)-3a(A-B) \sec(c+dx))}{(a+a \sec(c+dx))^2} dx}{5a^2} \\ &= -\frac{(A-B) \sin(c+dx)}{5d(a+a \sec(c+dx))^3} - \frac{(9A-4B) \sin(c+dx)}{15ad(a+a \sec(c+dx))^2} + \frac{\int \frac{\cos(c+dx)(a^2(27A-7B)-3a^2(A-B) \sec(c+dx))}{a+a \sec(c+dx)} dx}{15a^2d} \\ &= -\frac{(A-B) \sin(c+dx)}{5d(a+a \sec(c+dx))^3} - \frac{(9A-4B) \sin(c+dx)}{15ad(a+a \sec(c+dx))^2} - \frac{(3A-B) \sin(c+dx)}{d(a^3+a^3 \sec(c+dx))} \\ &= -\frac{(A-B) \sin(c+dx)}{5d(a+a \sec(c+dx))^3} - \frac{(9A-4B) \sin(c+dx)}{15ad(a+a \sec(c+dx))^2} - \frac{(3A-B) \sin(c+dx)}{d(a^3+a^3 \sec(c+dx))} \\ &= -\frac{(3A-B)x}{a^3} + \frac{2(36A-11B) \sin(c+dx)}{15a^3d} - \frac{(A-B) \sin(c+dx)}{5d(a+a \sec(c+dx))^3} - \frac{(9A-4B) \sin(c+dx)}{15ad(a+a \sec(c+dx))^2} \end{aligned}$$

Mathematica [B] time = 1.08, size = 365, normalized size = 2.68

$$\sec\left(\frac{c}{2}\right) \cos\left(\frac{1}{2}(c+dx)\right) \left(-300dx(3A-B) \cos\left(c+\frac{dx}{2}\right) - 300dx(3A-B) \cos\left(\frac{dx}{2}\right) - 1125A \sin\left(c+\frac{dx}{2}\right) + 1215A \sin\left(\frac{dx}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^3, x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(-300*(3*A - B)*d*x*Cos[(d*x)/2] - 300*(3*A - B)*d*x*Cos[c + (d*x)/2] - 450*A*d*x*Cos[c + (3*d*x)/2] + 150*B*d*x*Cos[c + (3*d*x)/2] - 450*A*d*x*Cos[2*c + (3*d*x)/2] + 150*B*d*x*Cos[2*c + (3*d*x)/2] - 90*A*d*x*Cos[2*c + (5*d*x)/2] + 30*B*d*x*Cos[2*c + (5*d*x)/2] - 90*A*d*x*Cos[3*c + (5*d*x)/2] + 30*B*d*x*Cos[3*c + (5*d*x)/2] + 1755*A*Sin[(d*x)/2] - 740*B*Sin[(d*x)/2] - 1125*A*Sin[c + (d*x)/2] + 540*B*Sin[c + (d*x)/2] + 1215*A*Sin[c + (3*d*x)/2] - 460*B*Sin[c + (3*d*x)/2] - 225*A*Sin[2*c + (3*d*x)/2])
```

$$\frac{x)/2] + 180*B*\sin[2*c + (3*d*x)/2] + 363*A*\sin[2*c + (5*d*x)/2] - 128*B*\sin[2*c + (5*d*x)/2] + 75*A*\sin[3*c + (5*d*x)/2] + 15*A*\sin[3*c + (7*d*x)/2] + 15*A*\sin[4*c + (7*d*x)/2])/(120*a^3*d*(1 + \cos[c + d*x])^3}$$

fricas [A] time = 0.42, size = 173, normalized size = 1.27

$$\frac{15(3A - B)dx \cos(dx + c)^3 + 45(3A - B)dx \cos(dx + c)^2 + 45(3A - B)dx \cos(dx + c) + 15(3A - B)dx - (15(a^3d \cos(dx + c))^3 + 3a^3d \cos(dx + c))}{15(a^3d \cos(dx + c))^3 + 3a^3d \cos(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="fricas")

[Out]
$$-1/15*(15*(3*A - B)*d*x*\cos(d*x + c)^3 + 45*(3*A - B)*d*x*\cos(d*x + c)^2 + 45*(3*A - B)*d*x*\cos(d*x + c) + 15*(3*A - B)*d*x - (15*A*\cos(d*x + c)^3 + (117*A - 32*B)*\cos(d*x + c)^2 + 3*(57*A - 17*B)*\cos(d*x + c) + 72*A - 22*B)*\sin(d*x + c))/(a^3*d*\cos(d*x + c)^3 + 3*a^3*d*\cos(d*x + c)^2 + 3*a^3*d*\cos(d*x + c) + a^3*d)$$

giac [A] time = 1.52, size = 157, normalized size = 1.15

$$\frac{\frac{60(dx+c)(3A-B)}{a^3} - \frac{120A \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + 1\right)a^3} - \frac{3Aa^{12} \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5 - 3Ba^{12} \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5 - 30Aa^{12} \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 + 20Ba^{12} \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)}{a^{15}}}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="giac")

[Out]
$$-1/60*(60*(d*x + c)*(3*A - B)/a^3 - 120*A*\tan(1/2*d*x + 1/2*c)/((\tan(1/2*d*x + 1/2*c)^2 + 1)*a^3) - (3*A*a^{12}*\tan(1/2*d*x + 1/2*c)^5 - 3*B*a^{12}*\tan(1/2*d*x + 1/2*c)^5 - 30*A*a^{12}*\tan(1/2*d*x + 1/2*c)^3 + 20*B*a^{12}*\tan(1/2*d*x + 1/2*c)^3 + 255*A*a^{12}*\tan(1/2*d*x + 1/2*c) - 105*B*a^{12}*\tan(1/2*d*x + 1/2*c))/a^{15})/d$$

maple [A] time = 1.04, size = 189, normalized size = 1.39

$$\frac{A \left(\tan^5 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{20d a^3} - \frac{B \left(\tan^5 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{20d a^3} - \frac{\left(\tan^3 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) A}{2d a^3} + \frac{B \left(\tan^3 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{3d a^3} + \frac{17A \tan \left(\frac{dx}{2} + \frac{c}{2} \right)}{4d a^3} - \frac{7B \tan \left(\frac{dx}{2} + \frac{c}{2} \right)}{4d a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x)`

[Out] $\frac{1}{20} \frac{d}{a^3} A \tan\left(\frac{1}{2} d x + \frac{1}{2} c\right)^5 - \frac{1}{20} \frac{d}{a^3} B \tan\left(\frac{1}{2} d x + \frac{1}{2} c\right)^5 - \frac{1}{2} \frac{d}{a^3} A \tan\left(\frac{1}{2} d x + \frac{1}{2} c\right)^3 + \frac{1}{3} \frac{d}{a^3} B \tan\left(\frac{1}{2} d x + \frac{1}{2} c\right)^3 + \frac{17}{4} \frac{d}{a^3} A \tan\left(\frac{1}{2} d x + \frac{1}{2} c\right) - \frac{7}{4} \frac{d}{a^3} B \tan\left(\frac{1}{2} d x + \frac{1}{2} c\right) + \frac{2}{d} \frac{d}{a^3} A \tan\left(\frac{1}{2} d x + \frac{1}{2} c\right) / \left(1 + \tan\left(\frac{1}{2} d x + \frac{1}{2} c\right)^2\right) - \frac{6}{d} \frac{d}{a^3} \arctan\left(\tan\left(\frac{1}{2} d x + \frac{1}{2} c\right)\right) * A + \frac{2}{d} \frac{d}{a^3} \arctan\left(\tan\left(\frac{1}{2} d x + \frac{1}{2} c\right)\right) * B$

maxima [A] time = 0.44, size = 231, normalized size = 1.70

$$\frac{3A \left(\frac{40 \sin(dx+c)}{\left(a^3 + \frac{a^3 \sin^2(dx+c)}{(\cos(dx+c)+1)^2}\right) (\cos(dx+c)+1)} + \frac{\frac{85 \sin(dx+c)}{\cos(dx+c)+1} - \frac{10 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3} - \frac{120 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3} \right) - B \left(\frac{\frac{105 \sin(dx+c)}{\cos(dx+c)+1} - \frac{20 \sin(dx+c)}{(\cos(dx+c)+1)^3}}{a^3} \right)}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="maxima")`

[Out] $\frac{1}{60} * (3A * (40 * \sin(dx + c) / ((a^3 + a^3 * \sin(dx + c)^2 / (\cos(dx + c) + 1)^2) * (\cos(dx + c) + 1)) + (85 * \sin(dx + c) / (\cos(dx + c) + 1) - 10 * \sin(dx + c)^3 / (\cos(dx + c) + 1)^3 + \sin(dx + c)^5 / (\cos(dx + c) + 1)^5) / a^3 - 120 * \arctan(\sin(dx + c) / (\cos(dx + c) + 1)) / a^3) - B * ((105 * \sin(dx + c) / (\cos(dx + c) + 1) - 20 * \sin(dx + c)^3 / (\cos(dx + c) + 1)^3 + 3 * \sin(dx + c)^5 / (\cos(dx + c) + 1)^5) / a^3 - 120 * \arctan(\sin(dx + c) / (\cos(dx + c) + 1)) / a^3) / d$

mupad [B] time = 1.98, size = 155, normalized size = 1.14

$$\frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right) \left(\frac{3A}{2a^3} + \frac{3(A-B)}{4a^3} + \frac{4A-2B}{2a^3}\right)}{d} - \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 \left(\frac{A-B}{6a^3} + \frac{4A-2B}{12a^3}\right)}{d} - \frac{x(3A-B)}{a^3} + \frac{2A \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{d \left(a^3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((cos(c+d*x)*(A+B/cos(c+d*x)))/(a+a/cos(c+d*x))^3,x)`

[Out] $\left(\tan\left(\frac{c}{2} + \frac{d x}{2}\right) * \left(\frac{3 A}{2 a^3} + \frac{3(A-B)}{4 a^3} + \frac{4 A-2 B}{2 a^3}\right)\right) / d - \left(\tan\left(\frac{c}{2} + \frac{d x}{2}\right)^3 * \left(\frac{A-B}{6 a^3} + \frac{4 A-2 B}{12 a^3}\right)\right) / d - \left(x * (3 A-B)\right) / a^3 + \left(2 A * \tan\left(\frac{c}{2} + \frac{d x}{2}\right)\right) / \left(d * \left(a^3 * \tan\left(\frac{c}{2} + \frac{d x}{2}\right)^2 + a^3\right)\right) + \left(\tan\left(\frac{c}{2} + \frac{d x}{2}\right)^5 * (A-B)\right) / \left(20 * a^3 * d\right)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A \cos(c+dx)}{\sec^3(c+dx)+3 \sec^2(c+dx)+3 \sec(c+dx)+1} dx + \int \frac{B \cos(c+dx) \sec(c+dx)}{\sec^3(c+dx)+3 \sec^2(c+dx)+3 \sec(c+dx)+1} dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**3,x)
```

```
[Out] (Integral(A*cos(c + d*x)/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d*x) + 1), x) + Integral(B*cos(c + d*x)*sec(c + d*x)/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d*x) + 1), x))/a**3
```

$$3.106 \quad \int \frac{\cos^2(c+dx)(A+B \sec(c+dx))}{(a+a \sec(c+dx))^3} dx$$

Optimal. Leaf size=187

$$-\frac{8(19A-9B)\sin(c+dx)}{15a^3d} + \frac{(13A-6B)\sin(c+dx)\cos(c+dx)}{2a^3d} - \frac{4(19A-9B)\sin(c+dx)\cos(c+dx)}{15d(a^3\sec(c+dx)+a^3)} + \frac{x(13A-6B)}{2a^3}$$

[Out] 1/2*(13*A-6*B)*x/a^3-8/15*(19*A-9*B)*sin(d*x+c)/a^3/d+1/2*(13*A-6*B)*cos(d*x+c)*sin(d*x+c)/a^3/d-1/5*(A-B)*cos(d*x+c)*sin(d*x+c)/d/(a+a*sec(d*x+c))^3-1/15*(11*A-6*B)*cos(d*x+c)*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^2-4/15*(19*A-9*B)*cos(d*x+c)*sin(d*x+c)/d/(a^3+a^3*sec(d*x+c))

Rubi [A] time = 0.47, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {4020, 3787, 2635, 8, 2637}

$$-\frac{8(19A-9B)\sin(c+dx)}{15a^3d} + \frac{(13A-6B)\sin(c+dx)\cos(c+dx)}{2a^3d} - \frac{4(19A-9B)\sin(c+dx)\cos(c+dx)}{15d(a^3\sec(c+dx)+a^3)} + \frac{x(13A-6B)}{2a^3}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^3,x]

[Out] ((13*A - 6*B)*x)/(2*a^3) - (8*(19*A - 9*B)*Sin[c + d*x])/(15*a^3*d) + ((13*A - 6*B)*Cos[c + d*x]*Sin[c + d*x])/(2*a^3*d) - ((A - B)*Cos[c + d*x]*Sin[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) - ((11*A - 6*B)*Cos[c + d*x]*Sin[c + d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) - (4*(19*A - 9*B)*Cos[c + d*x]*Sin[c + d*x])/(15*d*(a^3 + a^3*Sec[c + d*x]))

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x])*(b*Ssin[c + d*x])^(n-1))/(d*n), x] + Dist[(b^2*(n-1))/n, Int[(b*Ssin[c + d*x])^(n-2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 4020

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m + 1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{\cos^2(c + dx)(A + B \sec(c + dx))}{(a + a \sec(c + dx))^3} dx &= -\frac{(A - B) \cos(c + dx) \sin(c + dx)}{5d(a + a \sec(c + dx))^3} + \frac{\int \frac{\cos^2(c + dx)(a(7A - 2B) - 4a(A - B) \sec(c + dx))}{(a + a \sec(c + dx))^2} dx}{5a^2} \\
 &= -\frac{(A - B) \cos(c + dx) \sin(c + dx)}{5d(a + a \sec(c + dx))^3} - \frac{(11A - 6B) \cos(c + dx) \sin(c + dx)}{15ad(a + a \sec(c + dx))^2} \\
 &= -\frac{(A - B) \cos(c + dx) \sin(c + dx)}{5d(a + a \sec(c + dx))^3} - \frac{(11A - 6B) \cos(c + dx) \sin(c + dx)}{15ad(a + a \sec(c + dx))^2} \\
 &= -\frac{(A - B) \cos(c + dx) \sin(c + dx)}{5d(a + a \sec(c + dx))^3} - \frac{(11A - 6B) \cos(c + dx) \sin(c + dx)}{15ad(a + a \sec(c + dx))^2} \\
 &= -\frac{8(19A - 9B) \sin(c + dx)}{15a^3d} + \frac{(13A - 6B) \cos(c + dx) \sin(c + dx)}{2a^3d} - \frac{(A - B) \cos(c + dx) \sin(c + dx)}{5d(a + a \sec(c + dx))^3} \\
 &= \frac{(13A - 6B)x}{2a^3} - \frac{8(19A - 9B) \sin(c + dx)}{15a^3d} + \frac{(13A - 6B) \cos(c + dx) \sin(c + dx)}{2a^3d}
 \end{aligned}$$

Mathematica [B] time = 0.83, size = 435, normalized size = 2.33

$$\frac{\sec\left(\frac{c}{2}\right) \cos\left(\frac{1}{2}(c + dx)\right) \left(600dx(13A - 6B) \cos\left(c + \frac{dx}{2}\right) + 600dx(13A - 6B) \cos\left(\frac{dx}{2}\right) + 7560A \sin\left(c + \frac{dx}{2}\right) - 92\right)}{15a^3d}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^3,x]

[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(600*(13*A - 6*B)*d*x*cos[(d*x)/2] + 600*(13*A - 6*B)*d*x*cos[c + (d*x)/2] + 3900*A*d*x*cos[c + (3*d*x)/2] - 1800*B*d*x*cos[c + (3*d*x)/2] + 3900*A*d*x*cos[2*c + (3*d*x)/2] - 1800*B*d*x*cos[2*c + (3*d*x)/2] + 780*A*d*x*cos[2*c + (5*d*x)/2] - 360*B*d*x*cos[2*c + (5*d*x)/2] + 780*A*d*x*cos[3*c + (5*d*x)/2] - 360*B*d*x*cos[3*c + (5*d*x)/2] - 12760*A*sin[(d*x)/2] + 7020*B*sin[(d*x)/2] + 7560*A*sin[c + (d*x)/2] - 4500*B*sin[c + (d*x)/2] - 9230*A*sin[c + (3*d*x)/2] + 4860*B*sin[c + (3*d*x)/2] + 930*A*sin[2*c + (3*d*x)/2] - 900*B*sin[2*c + (3*d*x)/2] - 2782*A*sin[2*c + (5*d*x)/2] + 1452*B*sin[2*c + (5*d*x)/2] - 750*A*sin[3*c + (5*d*x)/2] + 300*B*sin[3*c + (5*d*x)/2] - 105*A*sin[3*c + (7*d*x)/2] + 60*B*sin[3*c + (7*d*x)/2] - 105*A*sin[4*c + (7*d*x)/2] + 60*B*sin[4*c + (7*d*x)/2] + 15*A*sin[4*c + (9*d*x)/2] + 15*A*sin[5*c + (9*d*x)/2]))/(480*a^3*d*(1 + Cos[c + d*x])^3)

fricas [A] time = 0.46, size = 190, normalized size = 1.02

$$\frac{15(13A - 6B)dx \cos(dx + c)^3 + 45(13A - 6B)dx \cos(dx + c)^2 + 45(13A - 6B)dx \cos(dx + c) + 15(13A - 6B)}{30(a^3 d \cos(dx + c))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="fricas")

[Out] 1/30*(15*(13*A - 6*B)*d*x*cos(d*x + c)^3 + 45*(13*A - 6*B)*d*x*cos(d*x + c)^2 + 45*(13*A - 6*B)*d*x*cos(d*x + c) + 15*(13*A - 6*B)*d*x + (15*A*cos(d*x + c)^4 - 15*(3*A - 2*B)*cos(d*x + c)^3 - (479*A - 234*B)*cos(d*x + c)^2 - 3*(239*A - 114*B)*cos(d*x + c) - 304*A + 144*B)*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)

giac [A] time = 0.47, size = 200, normalized size = 1.07

$$\frac{30(dx+c)(13A-6B)}{a^3} - \frac{60\left(7A \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - 2B \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 + 5A \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) - 2B \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 + 1\right)^2 a^3} - \frac{3Aa^{12} \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5 - 3Ba^{12} \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="giac")

[Out] 1/60*(30*(d*x + c)*(13*A - 6*B)/a^3 - 60*(7*A*tan(1/2*d*x + 1/2*c)^3 - 2*B*tan(1/2*d*x + 1/2*c)^3 + 5*A*tan(1/2*d*x + 1/2*c) - 2*B*tan(1/2*d*x + 1/2*c)))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^3) - (3*A*a^12*tan(1/2*d*x + 1/2*c)^5 - 3*B*a^12*tan(1/2*d*x + 1/2*c))/60d

$$- 3B a^{12} \tan(1/2 dx + 1/2 c)^5 - 40A a^{12} \tan(1/2 dx + 1/2 c)^3 + 30B a^{12} \tan(1/2 dx + 1/2 c)^3 + 465A a^{12} \tan(1/2 dx + 1/2 c) - 255B a^{12} \tan(1/2 dx + 1/2 c) / a^{15} / d$$

maple [A] time = 1.22, size = 292, normalized size = 1.56

$$\frac{A \left(\tan^5 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{20d a^3} + \frac{B \left(\tan^5 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{20d a^3} + \frac{2 \left(\tan^3 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) A}{3d a^3} - \frac{B \left(\tan^3 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{2d a^3} - \frac{31A \tan \left(\frac{dx}{2} + \frac{c}{2} \right)}{4d a^3} + \frac{17B \tan \left(\frac{dx}{2} + \frac{c}{2} \right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(dx+c)^2*(A+B*sec(dx+c))/(a+a*sec(dx+c))^3,x)

[Out] $-1/20/d/a^3 A \tan(1/2 dx + 1/2 c)^5 + 1/20/d/a^3 B \tan(1/2 dx + 1/2 c)^5 + 2/3/d/a^3 \tan(1/2 dx + 1/2 c)^3 A - 1/2/d/a^3 B \tan(1/2 dx + 1/2 c)^3 - 31/4/d/a^3 A \tan(1/2 dx + 1/2 c) + 17/4/d/a^3 B \tan(1/2 dx + 1/2 c) - 7/d/a^3 / (1 + \tan(1/2 dx + 1/2 c)^2)^2 \tan(1/2 dx + 1/2 c)^3 A + 2/d/a^3 / (1 + \tan(1/2 dx + 1/2 c)^2)^2 B \tan(1/2 dx + 1/2 c)^3 - 5/d/a^3 / (1 + \tan(1/2 dx + 1/2 c)^2)^2 A \tan(1/2 dx + 1/2 c) + 2/d/a^3 / (1 + \tan(1/2 dx + 1/2 c)^2)^2 B \tan(1/2 dx + 1/2 c) + 13/d/a^3 \arctan(\tan(1/2 dx + 1/2 c)) A - 6/d/a^3 \arctan(\tan(1/2 dx + 1/2 c)) B$

maxima [A] time = 0.43, size = 322, normalized size = 1.72

$$\frac{A \left(\frac{60 \left(\frac{5 \sin(dx+c)}{\cos(dx+c)+1} + \frac{7 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} \right)}{a^3 + \frac{2a^3 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{a^3 \sin(dx+c)^4}{(\cos(dx+c)+1)^4}} + \frac{\frac{465 \sin(dx+c)}{\cos(dx+c)+1} - \frac{40 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{3 \sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3} - \frac{780 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3} \right) - 3B \left(\frac{40 \sin(dx+c)}{a^3 + \frac{a^3 \sin(dx+c)^2}{(\cos(dx+c)+1)^2}} \right)}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(dx+c)^2*(A+B*sec(dx+c))/(a+a*sec(dx+c))^3,x, algorithm="maxima")

[Out] $-1/60*(A*(60*(5*\sin(dx + c)/(\cos(dx + c) + 1) + 7*\sin(dx + c)^3/(\cos(dx + c) + 1)^3)/(a^3 + 2*a^3*\sin(dx + c)^2/(\cos(dx + c) + 1)^2 + a^3*\sin(dx + c)^4/(\cos(dx + c) + 1)^4) + (465*\sin(dx + c)/(\cos(dx + c) + 1) - 40*\sin(dx + c)^3/(\cos(dx + c) + 1)^3 + 3*\sin(dx + c)^5/(\cos(dx + c) + 1)^5)/a^3 - 780*\arctan(\sin(dx + c)/(\cos(dx + c) + 1))/a^3) - 3*B*(40*\sin(dx + c)/((a^3 + a^3*\sin(dx + c)^2/(\cos(dx + c) + 1)^2)*(\cos(dx + c) + 1)) + (85*\sin(dx + c)/(\cos(dx + c) + 1) - 10*\sin(dx + c)^3/(\cos(dx + c) + 1)^3 + \sin(dx + c)^5/(\cos(dx + c) + 1)^5)/a^3 - 120*\arctan(\sin(dx + c)/(\cos(dx + c) + 1))/a^3)/d$

mupad [B] time = 2.00, size = 204, normalized size = 1.09

$$\frac{x(13A - 6B)}{2a^3} - \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right) \left(\frac{3(A-B)}{2a^3} + \frac{3(5A-3B)}{4a^3} + \frac{10A-2B}{4a^3}\right)}{d} - \frac{(7A - 2B) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 + (5A - 2B) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{d \left(a^3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 + 2a^3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)^2*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^3,x)

[Out] (x*(13*A - 6*B))/(2*a^3) - (tan(c/2 + (d*x)/2)*((3*(A - B))/(2*a^3) + (3*(5*A - 3*B))/(4*a^3) + (10*A - 2*B)/(4*a^3)))/d - (tan(c/2 + (d*x)/2)^3*(7*A - 2*B) + tan(c/2 + (d*x)/2)*(5*A - 2*B))/(d*(2*a^3*tan(c/2 + (d*x)/2)^2 + a^3*tan(c/2 + (d*x)/2)^4 + a^3)) + (tan(c/2 + (d*x)/2)^3*((A - B)/(4*a^3) + (5*A - 3*B)/(12*a^3)))/d - (tan(c/2 + (d*x)/2)^5*(A - B))/(20*a^3*d)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A \cos^2(c+dx)}{\sec^3(c+dx)+3 \sec^2(c+dx)+3 \sec(c+dx)+1} dx + \int \frac{B \cos^2(c+dx) \sec(c+dx)}{\sec^3(c+dx)+3 \sec^2(c+dx)+3 \sec(c+dx)+1} dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**3,x)

[Out] (Integral(A*cos(c + d*x)**2/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d*x) + 1), x) + Integral(B*cos(c + d*x)**2*sec(c + d*x)/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d*x) + 1), x))/a**3

$$3.107 \quad \int \frac{\cos^3(c+dx)(A+B \sec(c+dx))}{(a+a \sec(c+dx))^3} dx$$

Optimal. Leaf size=218

$$\frac{4(34A - 19B) \sin^3(c + dx)}{15a^3d} + \frac{4(34A - 19B) \sin(c + dx)}{5a^3d} - \frac{(23A - 13B) \sin(c + dx) \cos(c + dx)}{2a^3d} - \frac{(23A - 13B) \sin(c + dx)}{3d(a^3 \sec(c + dx))}$$

[Out] $-1/2*(23*A-13*B)*x/a^3+4/5*(34*A-19*B)*\sin(d*x+c)/a^3/d-1/2*(23*A-13*B)*\cos(d*x+c)*\sin(d*x+c)/a^3/d-1/5*(A-B)*\cos(d*x+c)^2*\sin(d*x+c)/d/(a+a*\sec(d*x+c))^3-1/15*(13*A-8*B)*\cos(d*x+c)^2*\sin(d*x+c)/a/d/(a+a*\sec(d*x+c))^2-1/3*(23*A-13*B)*\cos(d*x+c)^2*\sin(d*x+c)/d/(a^3+a^3*\sec(d*x+c))-4/15*(34*A-19*B)*\sin(d*x+c)^3/a^3/d$

Rubi [A] time = 0.49, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {4020, 3787, 2633, 2635, 8}

$$\frac{4(34A - 19B) \sin^3(c + dx)}{15a^3d} + \frac{4(34A - 19B) \sin(c + dx)}{5a^3d} - \frac{(23A - 13B) \sin(c + dx) \cos(c + dx)}{2a^3d} - \frac{(23A - 13B) \sin(c + dx)}{3d(a^3 \sec(c + dx))}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^3,x]

[Out] $-((23*A - 13*B)*x)/(2*a^3) + (4*(34*A - 19*B)*\sin[c + d*x])/(5*a^3*d) - ((23*A - 13*B)*\cos[c + d*x]*\sin[c + d*x])/(2*a^3*d) - ((A - B)*\cos[c + d*x]^2*\sin[c + d*x])/(5*d*(a + a*\sec[c + d*x])^3) - ((13*A - 8*B)*\cos[c + d*x]^2*\sin[c + d*x])/(15*a*d*(a + a*\sec[c + d*x])^2) - ((23*A - 13*B)*\cos[c + d*x]^2*\sin[c + d*x])/(3*d*(a^3 + a^3*\sec[c + d*x])) - (4*(34*A - 19*B)*\sin[c + d*x]^3)/(15*a^3*d)$

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2633

Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[Expand[(1 - x^2)^(n - 1)/2], x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[(b*cos[c + d*x])*(b*sin[c + d*x])^(n - 1)/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*sin[c + d*x])^(n - 1), x], x]

+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 4020

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m + 1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{\cos^3(c + dx)(A + B \sec(c + dx))}{(a + a \sec(c + dx))^3} dx &= -\frac{(A - B) \cos^2(c + dx) \sin(c + dx)}{5d(a + a \sec(c + dx))^3} + \frac{\int \frac{\cos^3(c + dx)(a(8A - 3B) - 5a(A - B) \sec(c + dx))}{(a + a \sec(c + dx))^2}}{5a^2} \\
 &= -\frac{(A - B) \cos^2(c + dx) \sin(c + dx)}{5d(a + a \sec(c + dx))^3} - \frac{(13A - 8B) \cos^2(c + dx) \sin(c + dx)}{15ad(a + a \sec(c + dx))^2} \\
 &= -\frac{(A - B) \cos^2(c + dx) \sin(c + dx)}{5d(a + a \sec(c + dx))^3} - \frac{(13A - 8B) \cos^2(c + dx) \sin(c + dx)}{15ad(a + a \sec(c + dx))^2} \\
 &= -\frac{(A - B) \cos^2(c + dx) \sin(c + dx)}{5d(a + a \sec(c + dx))^3} - \frac{(13A - 8B) \cos^2(c + dx) \sin(c + dx)}{15ad(a + a \sec(c + dx))^2} \\
 &= -\frac{(23A - 13B) \cos(c + dx) \sin(c + dx)}{2a^3d} - \frac{(A - B) \cos^2(c + dx) \sin(c + dx)}{5d(a + a \sec(c + dx))^3} \\
 &= -\frac{(23A - 13B)x}{2a^3} + \frac{4(34A - 19B) \sin(c + dx)}{5a^3d} - \frac{(23A - 13B) \cos(c + dx)}{2a^3d}
 \end{aligned}$$

Mathematica [B] time = 1.23, size = 491, normalized size = 2.25

$$\sec\left(\frac{c}{2}\right) \cos\left(\frac{1}{2}(c + dx)\right) \left(-600dx(23A - 13B) \cos\left(c + \frac{dx}{2}\right) - 600dx(23A - 13B) \cos\left(\frac{dx}{2}\right) - 11110A \sin\left(c + \frac{dx}{2}\right) - \dots\right)$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^3,x]

[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(-600*(23*A - 13*B)*d*x*Cos[(d*x)/2] - 600*(23*A - 13*B)*d*x*Cos[c + (d*x)/2] - 6900*A*d*x*Cos[c + (3*d*x)/2] + 3900*B*d*x*Cos[c + (3*d*x)/2] - 6900*A*d*x*Cos[2*c + (3*d*x)/2] + 3900*B*d*x*Cos[2*c + (3*d*x)/2] - 1380*A*d*x*Cos[2*c + (5*d*x)/2] + 780*B*d*x*Cos[2*c + (5*d*x)/2] - 1380*A*d*x*Cos[3*c + (5*d*x)/2] + 780*B*d*x*Cos[3*c + (5*d*x)/2] + 20410*A*Sin[(d*x)/2] - 12760*B*Sin[(d*x)/2] - 11110*A*Sin[c + (d*x)/2] + 7560*B*Sin[c + (d*x)/2] + 15380*A*Sin[c + (3*d*x)/2] - 9230*B*Sin[c + (3*d*x)/2] - 380*A*Sin[2*c + (3*d*x)/2] + 930*B*Sin[2*c + (3*d*x)/2] + 4777*A*Sin[2*c + (5*d*x)/2] - 2782*B*Sin[2*c + (5*d*x)/2] + 1625*A*Sin[3*c + (5*d*x)/2] - 750*B*Sin[3*c + (5*d*x)/2] + 230*A*Sin[3*c + (7*d*x)/2] - 105*B*Sin[3*c + (7*d*x)/2] + 230*A*Sin[4*c + (7*d*x)/2] - 105*B*Sin[4*c + (7*d*x)/2] - 20*A*Sin[4*c + (9*d*x)/2] + 15*B*Sin[4*c + (9*d*x)/2] - 20*A*Sin[5*c + (9*d*x)/2] + 15*B*Sin[5*c + (9*d*x)/2] + 5*A*Sin[5*c + (11*d*x)/2] + 5*A*Sin[6*c + (11*d*x)/2]))/(480*a^3*d*(1 + Cos[c + d*x])^3)

fricas [A] time = 0.50, size = 205, normalized size = 0.94

$$15(23A - 13B)dx \cos(dx + c)^3 + 45(23A - 13B)dx \cos(dx + c)^2 + 45(23A - 13B)dx \cos(dx + c) + 15(23A - 13B)dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="fricas")

[Out] -1/30*(15*(23*A - 13*B)*d*x*cos(d*x + c)^3 + 45*(23*A - 13*B)*d*x*cos(d*x + c)^2 + 45*(23*A - 13*B)*d*x*cos(d*x + c) + 15*(23*A - 13*B)*d*x - (10*A*cos(d*x + c)^5 - 15*(A - B)*cos(d*x + c)^4 + 5*(19*A - 9*B)*cos(d*x + c)^3 + (869*A - 479*B)*cos(d*x + c)^2 + 3*(429*A - 239*B)*cos(d*x + c) + 544*A - 304*B)*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)

giac [A] time = 0.31, size = 228, normalized size = 1.05

$$\frac{30(dx+c)(23A-13B)}{a^3} - \frac{20\left(51A \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5 - 21B \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5 + 76A \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 - 36B \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 + 33A \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 15B \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right)^3 a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="giac")

[Out]
$$\frac{-1/60*(30*(d*x + c)*(23*A - 13*B)/a^3 - 20*(51*A*\tan(1/2*d*x + 1/2*c)^5 - 21*B*\tan(1/2*d*x + 1/2*c)^5 + 76*A*\tan(1/2*d*x + 1/2*c)^3 - 36*B*\tan(1/2*d*x + 1/2*c)^3 + 33*A*\tan(1/2*d*x + 1/2*c) - 15*B*\tan(1/2*d*x + 1/2*c))/((\tan(1/2*d*x + 1/2*c)^2 + 1)^3*a^3) - (3*A*a^{12}*\tan(1/2*d*x + 1/2*c)^5 - 3*B*a^{12}*\tan(1/2*d*x + 1/2*c)^5 - 50*A*a^{12}*\tan(1/2*d*x + 1/2*c)^3 + 40*B*a^{12}*\tan(1/2*d*x + 1/2*c)^3 + 735*A*a^{12}*\tan(1/2*d*x + 1/2*c) - 465*B*a^{12}*\tan(1/2*d*x + 1/2*c))/a^{15}}{d}$$

maple [A] time = 1.28, size = 362, normalized size = 1.66

$$\frac{A \left(\tan^5 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{20d a^3} - \frac{B \left(\tan^5 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{20d a^3} - \frac{5 \left(\tan^3 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) A}{6d a^3} + \frac{2B \left(\tan^3 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{3d a^3} + \frac{49A \tan \left(\frac{dx}{2} + \frac{c}{2} \right)}{4d a^3} - \frac{31B \tan \left(\frac{dx}{2} + \frac{c}{2} \right)}{4d a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x)

[Out]
$$\frac{1}{20} \frac{1}{d} \frac{1}{a^3} A \tan^5 \left(\frac{1}{2} d x + \frac{1}{2} c \right) - \frac{1}{20} \frac{1}{d} \frac{1}{a^3} B \tan^5 \left(\frac{1}{2} d x + \frac{1}{2} c \right) - \frac{5}{6} \frac{1}{d} \frac{1}{a^3} \tan^3 \left(\frac{1}{2} d x + \frac{1}{2} c \right)^3 A + \frac{2}{3} \frac{1}{d} \frac{1}{a^3} B \tan^3 \left(\frac{1}{2} d x + \frac{1}{2} c \right)^3 + \frac{49}{4} \frac{1}{d} \frac{1}{a^3} A \tan \left(\frac{1}{2} d x + \frac{1}{2} c \right) - \frac{31}{4} \frac{1}{d} \frac{1}{a^3} B \tan \left(\frac{1}{2} d x + \frac{1}{2} c \right) + \frac{17}{d} \frac{1}{a^3} \frac{1}{(1 + \tan^2 \left(\frac{1}{2} d x + \frac{1}{2} c \right))^{3/2}} \tan^5 \left(\frac{1}{2} d x + \frac{1}{2} c \right) A - \frac{7}{d} \frac{1}{a^3} \frac{1}{(1 + \tan^2 \left(\frac{1}{2} d x + \frac{1}{2} c \right))^{3/2}} \tan^5 \left(\frac{1}{2} d x + \frac{1}{2} c \right) B + \frac{76}{3} \frac{1}{d} \frac{1}{a^3} \frac{1}{(1 + \tan^2 \left(\frac{1}{2} d x + \frac{1}{2} c \right))^{3/2}} \tan^3 \left(\frac{1}{2} d x + \frac{1}{2} c \right) A - \frac{12}{d} \frac{1}{a^3} \frac{1}{(1 + \tan^2 \left(\frac{1}{2} d x + \frac{1}{2} c \right))^{3/2}} \tan^3 \left(\frac{1}{2} d x + \frac{1}{2} c \right) B + \frac{11}{d} \frac{1}{a^3} \frac{1}{(1 + \tan^2 \left(\frac{1}{2} d x + \frac{1}{2} c \right))^{3/2}} \tan \left(\frac{1}{2} d x + \frac{1}{2} c \right) A - \frac{5}{d} \frac{1}{a^3} \frac{1}{(1 + \tan^2 \left(\frac{1}{2} d x + \frac{1}{2} c \right))^{3/2}} \tan \left(\frac{1}{2} d x + \frac{1}{2} c \right) B - \frac{23}{d} \frac{1}{a^3} \arctan \left(\tan \left(\frac{1}{2} d x + \frac{1}{2} c \right) \right) A + \frac{13}{d} \frac{1}{a^3} \arctan \left(\tan \left(\frac{1}{2} d x + \frac{1}{2} c \right) \right) B$$

maxima [B] time = 0.45, size = 412, normalized size = 1.89

$$A \left(\frac{20 \left(\frac{33 \sin(dx+c)}{\cos(dx+c)+1} + \frac{76 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{51 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} \right)}{a^3 + \frac{3a^3 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{3a^3 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{a^3 \sin(dx+c)^6}{(\cos(dx+c)+1)^6}} + \frac{\frac{735 \sin(dx+c)}{\cos(dx+c)+1} - \frac{50 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{3 \sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3} - \frac{1380 \arctan \left(\frac{\sin(dx+c)}{\cos(dx+c)+1} \right)}{a^3} \right) - B \left(\frac{60 \left(\frac{5 \sin(dx+c)}{\cos(dx+c)+1} + \frac{2a^3 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} \right)}{a^3} \right)$$

$60d$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="maxima")

[Out]
$$\frac{1}{60} \frac{1}{a^3} \left(A \left(20 \frac{33 \sin(dx+c)}{\cos(dx+c)+1} + 76 \frac{\sin^3(dx+c)}{(\cos(dx+c)+1)^3} + 51 \frac{\sin^5(dx+c)}{(\cos(dx+c)+1)^5} \right) + B \left(60 \frac{5 \sin(dx+c)}{\cos(dx+c)+1} + 2a^3 \frac{\sin^3(dx+c)}{(\cos(dx+c)+1)^3} \right) \right)$$

$x + c)^2 / (\cos(dx + c) + 1)^2 + 3a^3 \sin(dx + c)^4 / (\cos(dx + c) + 1)^4 + a^3 \sin(dx + c)^6 / (\cos(dx + c) + 1)^6 + (735 \sin(dx + c) / (\cos(dx + c) + 1) - 50 \sin(dx + c)^3 / (\cos(dx + c) + 1)^3 + 3 \sin(dx + c)^5 / (\cos(dx + c) + 1)^5) / a^3 - 1380 \arctan(\sin(dx + c) / (\cos(dx + c) + 1)) / a^3 - B(60(5 \sin(dx + c) / (\cos(dx + c) + 1) + 7 \sin(dx + c)^3 / (\cos(dx + c) + 1)^3) / (a^3 + 2a^3 \sin(dx + c)^2 / (\cos(dx + c) + 1)^2 + a^3 \sin(dx + c)^4 / (\cos(dx + c) + 1)^4) + (465 \sin(dx + c) / (\cos(dx + c) + 1) - 40 \sin(dx + c)^3 / (\cos(dx + c) + 1)^3 + 3 \sin(dx + c)^5 / (\cos(dx + c) + 1)^5) / a^3 - 780 \arctan(\sin(dx + c) / (\cos(dx + c) + 1)) / a^3) / d$

mupad [B] time = 2.05, size = 237, normalized size = 1.09

$$\frac{(17A - 7B) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5 + \left(\frac{76A}{3} - 12B\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 + (11A - 5B) \tan\left(\frac{c}{2} + \frac{dx}{2}\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right) \left(\frac{5(A-B)}{2a^3} + \frac{5(A-B)}{2a^3}\right)}{d \left(a^3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^6 + 3a^3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 + 3a^3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + a^3 \right)} + \frac{5(A-B)}{2a^3} \frac{1}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)^3*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^3,x)

[Out] $(\tan(c/2 + (dx)/2)^5(17A - 7B) + \tan(c/2 + (dx)/2)^3((76A)/3 - 12B) + \tan(c/2 + (dx)/2)(11A - 5B)) / (d(3a^3 \tan(c/2 + (dx)/2)^2 + 3a^3 \tan(c/2 + (dx)/2)^4 + a^3 \tan(c/2 + (dx)/2)^6 + a^3) + (\tan(c/2 + (dx)/2) * ((5(A - B)) / (2a^3) + (6A - 4B) / a^3 + (15A - 5B) / (4a^3))) / d - (x * (23A - 13B) / (2a^3) - (\tan(c/2 + (dx)/2)^3 * ((A - B) / (3a^3) + (6A - 4B) / (12a^3))) / d + (\tan(c/2 + (dx)/2)^5 * (A - B)) / (20a^3 * d)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A \cos^3(c+dx)}{\sec^3(c+dx)+3 \sec^2(c+dx)+3 \sec(c+dx)+1} dx + \int \frac{B \cos^3(c+dx) \sec(c+dx)}{\sec^3(c+dx)+3 \sec^2(c+dx)+3 \sec(c+dx)+1} dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**3,x)

[Out] $(\text{Integral}(A \cos(c + d*x)**3 / (\sec(c + d*x)**3 + 3 \sec(c + d*x)**2 + 3 \sec(c + d*x) + 1), x) + \text{Integral}(B \cos(c + d*x)**3 \sec(c + d*x) / (\sec(c + d*x)**3 + 3 \sec(c + d*x)**2 + 3 \sec(c + d*x) + 1), x)) / a**3$

$$3.108 \quad \int \frac{\sec^6(c+dx)(A+B \sec(c+dx))}{(a+a \sec(c+dx))^4} dx$$

Optimal. Leaf size=238

$$\frac{8(83A - 216B) \tan(c + dx)}{105a^4d} - \frac{(8A - 21B) \tanh^{-1}(\sin(c + dx))}{2a^4d} + \frac{(52A - 129B) \tan(c + dx) \sec^3(c + dx)}{105a^4d(\sec(c + dx) + 1)^2} + \frac{4(83A - 216B) \tan(c + dx)}{105a^4d}$$

[Out] $-1/2*(8*A-21*B)*\operatorname{arctanh}(\sin(d*x+c))/a^4/d+8/105*(83*A-216*B)*\tan(d*x+c)/a^4/d-1/2*(8*A-21*B)*\sec(d*x+c)*\tan(d*x+c)/a^4/d+1/105*(52*A-129*B)*\sec(d*x+c)^3*\tan(d*x+c)/a^4/d/(1+\sec(d*x+c))^2+4/105*(83*A-216*B)*\sec(d*x+c)^2*\tan(d*x+c)/a^4/d/(1+\sec(d*x+c))+1/7*(A-B)*\sec(d*x+c)^5*\tan(d*x+c)/d/(a+a*\sec(d*x+c))^4+1/5*(A-2*B)*\sec(d*x+c)^4*\tan(d*x+c)/a/d/(a+a*\sec(d*x+c))^3$

Rubi [A] time = 0.66, antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4019, 3787, 3767, 8, 3768, 3770}

$$\frac{8(83A - 216B) \tan(c + dx)}{105a^4d} - \frac{(8A - 21B) \tanh^{-1}(\sin(c + dx))}{2a^4d} + \frac{(52A - 129B) \tan(c + dx) \sec^3(c + dx)}{105a^4d(\sec(c + dx) + 1)^2} + \frac{4(83A - 216B) \tan(c + dx)}{105a^4d}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(\operatorname{Sec}[c + d*x]^6*(A + B*\operatorname{Sec}[c + d*x]))/(a + a*\operatorname{Sec}[c + d*x])^4, x]$

[Out] $-((8*A - 21*B)*\operatorname{ArcTanh}[\operatorname{Sin}[c + d*x]])/(2*a^4*d) + (8*(83*A - 216*B)*\operatorname{Tan}[c + d*x])/(105*a^4*d) - ((8*A - 21*B)*\operatorname{Sec}[c + d*x]*\operatorname{Tan}[c + d*x])/(2*a^4*d) + ((52*A - 129*B)*\operatorname{Sec}[c + d*x]^3*\operatorname{Tan}[c + d*x])/(105*a^4*d*(1 + \operatorname{Sec}[c + d*x])^2) + (4*(83*A - 216*B)*\operatorname{Sec}[c + d*x]^2*\operatorname{Tan}[c + d*x])/(105*a^4*d*(1 + \operatorname{Sec}[c + d*x])) + ((A - B)*\operatorname{Sec}[c + d*x]^5*\operatorname{Tan}[c + d*x])/(7*d*(a + a*\operatorname{Sec}[c + d*x])^4) + ((A - 2*B)*\operatorname{Sec}[c + d*x]^4*\operatorname{Tan}[c + d*x])/(5*a*d*(a + a*\operatorname{Sec}[c + d*x])^3)$

Rule 8

$\operatorname{Int}[a_, x_Symbol] \rightarrow \operatorname{Simp}[a*x, x] /; \operatorname{FreeQ}[a, x]$

Rule 3767

$\operatorname{Int}[\operatorname{csc}[(c_.) + (d_.)*(x_.)]^{(n_.)}, x_Symbol] \rightarrow -\operatorname{Dist}[d^{(-1)}, \operatorname{Subst}[\operatorname{Int}[\operatorname{ExpandIntegrand}[(1 + x^2)^{(n/2 - 1)}, x], x], x, \operatorname{Cot}[c + d*x]], x] /; \operatorname{FreeQ}[\{c, d\}, x] \ \&\& \ \operatorname{IGtQ}[n/2, 0]$

Rule 3768

$\operatorname{Int}[(\operatorname{csc}[(c_.) + (d_.)*(x_.)]*(b_.))^{(n_.)}, x_Symbol] \rightarrow -\operatorname{Simp}[(b*\operatorname{Cos}[c + d*x])*(b*\operatorname{Csc}[c + d*x])^{(n - 1)}]/(d*(n - 1)), x] + \operatorname{Dist}[(b^2*(n - 2))/(n - 1), I$


```
Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_.)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x]
/; FreeQ[{c, d}, x]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) +
(a_.), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (
a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.), x_Symbol] := Simp[(d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^6(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^4} dx &= \frac{(A-B)\sec^5(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{\int \frac{\sec^5(c+dx)(5a(A-B)-a(2A-9B)\sec(c+dx))}{(a+a\sec(c+dx))^3}}{7a^2} \\
&= \frac{(A-B)\sec^5(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{(A-2B)\sec^4(c+dx)\tan(c+dx)}{5ad(a+a\sec(c+dx))^3} + \\
&= \frac{(52A-129B)\sec^3(c+dx)\tan(c+dx)}{105a^4d(1+\sec(c+dx))^2} + \frac{(A-B)\sec^5(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} \\
&= \frac{(52A-129B)\sec^3(c+dx)\tan(c+dx)}{105a^4d(1+\sec(c+dx))^2} + \frac{(A-B)\sec^5(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} \\
&= \frac{(52A-129B)\sec^3(c+dx)\tan(c+dx)}{105a^4d(1+\sec(c+dx))^2} + \frac{(A-B)\sec^5(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} \\
&= -\frac{(8A-21B)\sec(c+dx)\tan(c+dx)}{2a^4d} + \frac{(52A-129B)\sec^3(c+dx)\tan(c+dx)}{105a^4d(1+\sec(c+dx))^2} \\
&= -\frac{(8A-21B)\tanh^{-1}(\sin(c+dx))}{2a^4d} + \frac{8(83A-216B)\tan(c+dx)}{105a^4d} - \frac{(8A-21B)\tan(c+dx)}{105a^4d}
\end{aligned}$$

Mathematica [B] time = 6.52, size = 880, normalized size = 3.70

$$\frac{8(21B-8A)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\sec^3(c+dx)(A+B\sec(c+dx))\cos^8\left(\frac{c}{2}+\frac{dx}{2}\right)}{d(B+A\cos(c+dx))(\sec(c+dx)a+a)^4} + \frac{8(21B-8A)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d(B+A\cos(c+dx))(\sec(c+dx)a+a)^4}$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[c + d*x]^6*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^4, x]

[Out] (-8*(-8*A + 21*B)*Cos[c/2 + (d*x)/2]^8*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])*Sec[c + d*x]^3*(A + B*Sec[c + d*x))/(d*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^4) + (8*(-8*A + 21*B)*Cos[c/2 + (d*x)/2]^8*Log[Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])*Sec[c + d*x]^3*(A + B*Sec[c + d*x))/(d*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^4) + (Cos[c/2 + (d*x)/2]*Sec[c/2]*Sec[c]*Sec[c + d*x]^5*(A + B*Sec[c + d*x])*(-38668*A*Sin[(d*x)/2] + 73206*B*Sin[(d*x)/2] + 64384*A*Sin[(3*d*x)/2] - 166668*B*Sin[(3*d*x)/2] - 70896*A*Sin[c - (d*x)/2] + 183162*B*Sin[c - (d*x)/2] + 50316*A*Sin[c + (d*x)/2] - 100842*B*Sin[c + (d*x)/2] - 59248*A*Sin[2*c + (d*x)/2] + 155526*B*Sin[2*c + (d*x)/2] - 22820*A*Sin[c + (3*d*x)/2] + 37380*B*Sin[c + (3*d*x)/2] + 48004*A*Sin[2*c + (3*d*x)/2] - 101148*B*Sin[2*c + (3*d*x)/2] - 39200*A*Sin[3*c + (3*d*x)/2] - 39200*B*Sin[3*c + (3*d*x)/2])

$x)/2] + 102900*B*\sin[3*c + (3*d*x)/2] + 46032*A*\sin[c + (5*d*x)/2] - 119364$
 $*B*\sin[c + (5*d*x)/2] - 8750*A*\sin[2*c + (5*d*x)/2] + 8820*B*\sin[2*c + (5*d$
 $*x)/2] + 35742*A*\sin[3*c + (5*d*x)/2] - 78204*B*\sin[3*c + (5*d*x)/2] - 1904$
 $0*A*\sin[4*c + (5*d*x)/2] + 49980*B*\sin[4*c + (5*d*x)/2] + 24664*A*\sin[2*c +$
 $(7*d*x)/2] - 64053*B*\sin[2*c + (7*d*x)/2] - 1050*A*\sin[3*c + (7*d*x)/2] -$
 $3885*B*\sin[3*c + (7*d*x)/2] + 19834*A*\sin[4*c + (7*d*x)/2] - 44733*B*\sin[4*$
 $c + (7*d*x)/2] - 5880*A*\sin[5*c + (7*d*x)/2] + 15435*B*\sin[5*c + (7*d*x)/2]$
 $+ 8456*A*\sin[3*c + (9*d*x)/2] - 21987*B*\sin[3*c + (9*d*x)/2] + 630*A*\sin[4$
 $*c + (9*d*x)/2] - 3675*B*\sin[4*c + (9*d*x)/2] + 6986*A*\sin[5*c + (9*d*x)/2]$
 $- 16107*B*\sin[5*c + (9*d*x)/2] - 840*A*\sin[6*c + (9*d*x)/2] + 2205*B*\sin[6$
 $*c + (9*d*x)/2] + 1328*A*\sin[4*c + (11*d*x)/2] - 3456*B*\sin[4*c + (11*d*x)/$
 $2] + 210*A*\sin[5*c + (11*d*x)/2] - 840*B*\sin[5*c + (11*d*x)/2] + 1118*A*\sin$
 $[6*c + (11*d*x)/2] - 2616*B*\sin[6*c + (11*d*x)/2]))/(6720*d*(B + A*\cos[c +$
 $d*x]))*(a + a*\sec[c + d*x])^4$

fricas [A] time = 0.49, size = 358, normalized size = 1.50

$$\frac{105 \left((8A - 21B) \cos(dx + c)^6 + 4(8A - 21B) \cos(dx + c)^5 + 6(8A - 21B) \cos(dx + c)^4 + 4(8A - 21B) \cos(dx + c)^3 + (8A - 21B) \cos(dx + c)^2 \right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^6*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x, algorithm="fricas")

[Out] $-1/420*(105*((8*A - 21*B)*\cos(d*x + c)^6 + 4*(8*A - 21*B)*\cos(d*x + c)^5 + 6*(8*A - 21*B)*\cos(d*x + c)^4 + 4*(8*A - 21*B)*\cos(d*x + c)^3 + (8*A - 21*B)*\cos(d*x + c)^2)*\log(\sin(d*x + c) + 1) - 105*((8*A - 21*B)*\cos(d*x + c)^6 + 4*(8*A - 21*B)*\cos(d*x + c)^5 + 6*(8*A - 21*B)*\cos(d*x + c)^4 + 4*(8*A - 21*B)*\cos(d*x + c)^3 + (8*A - 21*B)*\cos(d*x + c)^2)*\log(-\sin(d*x + c) + 1) - 2*(16*(83*A - 216*B)*\cos(d*x + c)^5 + (4472*A - 11619*B)*\cos(d*x + c)^4 + 4*(1318*A - 3411*B)*\cos(d*x + c)^3 + 4*(592*A - 1509*B)*\cos(d*x + c)^2 + 210*(A - 2*B)*\cos(d*x + c) + 105*B*\sin(d*x + c))/(a^4*d*\cos(d*x + c)^6 + 4*a^4*d*\cos(d*x + c)^5 + 6*a^4*d*\cos(d*x + c)^4 + 4*a^4*d*\cos(d*x + c)^3 + a^4*d*\cos(d*x + c)^2)$

giac [A] time = 0.87, size = 267, normalized size = 1.12

$$\frac{420(8A-21B)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^4} - \frac{420(8A-21B)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^4} + \frac{840\left(2A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-9B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-2A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)^2 a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^6*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x, algorithm="giac")

[Out]
$$-1/840*(420*(8*A - 21*B)*\log(\text{abs}(\tan(1/2*d*x + 1/2*c) + 1))/a^4 - 420*(8*A - 21*B)*\log(\text{abs}(\tan(1/2*d*x + 1/2*c) - 1))/a^4 + 840*(2*A*\tan(1/2*d*x + 1/2*c)^3 - 9*B*\tan(1/2*d*x + 1/2*c)^3 - 2*A*\tan(1/2*d*x + 1/2*c) + 7*B*\tan(1/2*d*x + 1/2*c))/((\tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^4) - (15*A*a^{24}*\tan(1/2*d*x + 1/2*c)^7 - 15*B*a^{24}*\tan(1/2*d*x + 1/2*c)^7 + 147*A*a^{24}*\tan(1/2*d*x + 1/2*c)^5 - 189*B*a^{24}*\tan(1/2*d*x + 1/2*c)^5 + 805*A*a^{24}*\tan(1/2*d*x + 1/2*c)^3 - 1365*B*a^{24}*\tan(1/2*d*x + 1/2*c)^3 + 5145*A*a^{24}*\tan(1/2*d*x + 1/2*c) - 11655*B*a^{24}*\tan(1/2*d*x + 1/2*c))/a^{28}/d$$

maple [A] time = 0.63, size = 374, normalized size = 1.57

$$\frac{\left(\tan^7\left(\frac{dx}{2} + \frac{c}{2}\right)\right) A}{56d a^4} - \frac{B \left(\tan^7\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{56d a^4} + \frac{7A \left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{40d a^4} - \frac{9B \left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{40d a^4} + \frac{23 \left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right) A}{24d a^4} - \frac{13B \left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{24d a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^6*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x)

[Out]
$$1/56/d/a^4*\tan(1/2*d*x+1/2*c)^7*A-1/56/d/a^4*B*\tan(1/2*d*x+1/2*c)^7+7/40/d/a^4*A*\tan(1/2*d*x+1/2*c)^5-9/40/d/a^4*B*\tan(1/2*d*x+1/2*c)^5+23/24/d/a^4*\tan(1/2*d*x+1/2*c)^3*A-13/8/d/a^4*B*\tan(1/2*d*x+1/2*c)^3+49/8/d/a^4*A*\tan(1/2*d*x+1/2*c)-111/8/d/a^4*B*\tan(1/2*d*x+1/2*c)+4/d/a^4*\ln(\tan(1/2*d*x+1/2*c)-1)*A-21/2/d/a^4*\ln(\tan(1/2*d*x+1/2*c)-1)*B+9/2/d/a^4/(\tan(1/2*d*x+1/2*c)-1)*B-1/d/a^4/(\tan(1/2*d*x+1/2*c)-1)*A+1/2/d/a^4*B/(\tan(1/2*d*x+1/2*c)-1)^2+9/2/d/a^4/(\tan(1/2*d*x+1/2*c)+1)*B-1/d/a^4/(\tan(1/2*d*x+1/2*c)+1)*A-4/d/a^4*\ln(\tan(1/2*d*x+1/2*c)+1)*A+21/2/d/a^4*\ln(\tan(1/2*d*x+1/2*c)+1)*B-1/2/d/a^4*B/(\tan(1/2*d*x+1/2*c)+1)^2$$

maxima [A] time = 0.36, size = 419, normalized size = 1.76

$$3B \frac{\left(\frac{280 \left(\frac{7 \sin(dx+c)}{\cos(dx+c)+1} - \frac{9 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} \right)}{a^4 - \frac{2a^4 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{a^4 \sin(dx+c)^4}{(\cos(dx+c)+1)^4}} \right) + \frac{\frac{3885 \sin(dx+c)}{\cos(dx+c)+1} + \frac{455 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{63 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} + \frac{5 \sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4} - \frac{2940 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1} + 1\right)}{a^4} + \frac{2940 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1} - 1\right)}{a^4}}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^6*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x, algorithm="maxima")

[Out]
$$-1/840*(3*B*(280*(7*\sin(d*x + c))/(\cos(d*x + c) + 1) - 9*\sin(d*x + c)^3/(\cos(d*x + c) + 1)^3)/(a^4 - 2*a^4*\sin(d*x + c)^2/(\cos(d*x + c) + 1)^2 + a^4*\sin(d*x + c)^4/(\cos(d*x + c) + 1)^4) + 9/40*(2*A*\tan(1/2*d*x + 1/2*c)^3 - 9*B*\tan(1/2*d*x + 1/2*c)^3 - 2*A*\tan(1/2*d*x + 1/2*c) + 7*B*\tan(1/2*d*x + 1/2*c))/((\tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^4) - (15*A*a^{24}*\tan(1/2*d*x + 1/2*c)^7 - 15*B*a^{24}*\tan(1/2*d*x + 1/2*c)^7 + 147*A*a^{24}*\tan(1/2*d*x + 1/2*c)^5 - 189*B*a^{24}*\tan(1/2*d*x + 1/2*c)^5 + 805*A*a^{24}*\tan(1/2*d*x + 1/2*c)^3 - 1365*B*a^{24}*\tan(1/2*d*x + 1/2*c)^3 + 5145*A*a^{24}*\tan(1/2*d*x + 1/2*c) - 11655*B*a^{24}*\tan(1/2*d*x + 1/2*c))/a^{28}/d$$

$$\frac{n(dx + c)^4/(\cos(dx + c) + 1)^4 + (3885\sin(dx + c)/(\cos(dx + c) + 1) + 455\sin(dx + c)^3/(\cos(dx + c) + 1)^3 + 63\sin(dx + c)^5/(\cos(dx + c) + 1)^5 + 5\sin(dx + c)^7/(\cos(dx + c) + 1)^7)/a^4 - 2940\log(\sin(dx + c)/(\cos(dx + c) + 1) + 1)/a^4 + 2940\log(\sin(dx + c)/(\cos(dx + c) + 1) - 1)/a^4 - A(1680\sin(dx + c)/((a^4 - a^4\sin(dx + c)^2/(\cos(dx + c) + 1)^2)(\cos(dx + c) + 1)) + (5145\sin(dx + c)/(\cos(dx + c) + 1) + 805\sin(dx + c)^3/(\cos(dx + c) + 1)^3 + 147\sin(dx + c)^5/(\cos(dx + c) + 1)^5 + 15\sin(dx + c)^7/(\cos(dx + c) + 1)^7)/a^4 - 3360\log(\sin(dx + c)/(\cos(dx + c) + 1) + 1)/a^4 + 3360\log(\sin(dx + c)/(\cos(dx + c) + 1) - 1)/a^4)}{d}$$

mupad [B] time = 2.05, size = 272, normalized size = 1.14

$$\frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 \left(\frac{A-B}{4a^4} + \frac{4A-6B}{8a^4} + \frac{5A-15B}{24a^4}\right)}{d} + \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right) \left(\frac{5(A-B)}{4a^4} - \frac{5B}{2a^4} + \frac{3(4A-6B)}{4a^4} + \frac{3(5A-15B)}{8a^4}\right)}{d} - \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3}{d \left(a^4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^6*(a + a/cos(c + d*x))^4), x)

[Out] (tan(c/2 + (d*x)/2)^3*((A - B)/(4*a^4) + (4*A - 6*B)/(8*a^4) + (5*A - 15*B)/(24*a^4)))/d + (tan(c/2 + (d*x)/2)*((5*(A - B))/(4*a^4) - (5*B)/(2*a^4) + (3*(4*A - 6*B))/(4*a^4) + (3*(5*A - 15*B))/(8*a^4)))/d - (tan(c/2 + (d*x)/2)^3*(2*A - 9*B) - tan(c/2 + (d*x)/2)*(2*A - 7*B))/(d*(a^4*tan(c/2 + (d*x)/2)^4 - 2*a^4*tan(c/2 + (d*x)/2)^2 + a^4)) + (tan(c/2 + (d*x)/2)^5*((3*(A - B))/(40*a^4) + (4*A - 6*B)/(40*a^4)))/d + (tan(c/2 + (d*x)/2)^7*(A - B))/(56*a^4*d) - (atanh(tan(c/2 + (d*x)/2))*(8*A - 21*B))/(a^4*d)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A \sec^6(c+dx)}{\sec^4(c+dx)+4\sec^3(c+dx)+6\sec^2(c+dx)+4\sec(c+dx)+1} dx + \int \frac{B \sec^7(c+dx)}{\sec^4(c+dx)+4\sec^3(c+dx)+6\sec^2(c+dx)+4\sec(c+dx)+1} dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**6*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**4, x)

[Out] (Integral(A*sec(c + d*x)**6/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c + d*x)**2 + 4*sec(c + d*x) + 1), x) + Integral(B*sec(c + d*x)**7/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c + d*x)**2 + 4*sec(c + d*x) + 1), x))/a**4

$$3.109 \quad \int \frac{\sec^5(c+dx)(A+B \sec(c+dx))}{(a+a \sec(c+dx))^4} dx$$

Optimal. Leaf size=194

$$-\frac{(55A-244B) \tan(c+dx)}{105a^4d} + \frac{(A-4B) \tanh^{-1}(\sin(c+dx))}{a^4d} + \frac{(25A-88B) \tan(c+dx) \sec^2(c+dx)}{105a^4d(\sec(c+dx)+1)^2} - \frac{(A-4B) \tan(c+dx)}{a^4d(\sec(c+dx)+1)}$$

[Out] (A-4*B)*arctanh(sin(d*x+c))/a^4/d-1/105*(55*A-244*B)*tan(d*x+c)/a^4/d+1/105*(25*A-88*B)*sec(d*x+c)^2*tan(d*x+c)/a^4/d/(1+sec(d*x+c))^2-(A-4*B)*tan(d*x+c)/a^4/d/(1+sec(d*x+c))+1/7*(A-B)*sec(d*x+c)^4*tan(d*x+c)/d/(a+a*sec(d*x+c))^4+1/35*(5*A-12*B)*sec(d*x+c)^3*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^3

Rubi [A] time = 0.62, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4019, 4008, 3787, 3770, 3767, 8}

$$-\frac{(55A-244B) \tan(c+dx)}{105a^4d} + \frac{(A-4B) \tanh^{-1}(\sin(c+dx))}{a^4d} + \frac{(25A-88B) \tan(c+dx) \sec^2(c+dx)}{105a^4d(\sec(c+dx)+1)^2} - \frac{(A-4B) \tan(c+dx)}{a^4d(\sec(c+dx)+1)}$$

Antiderivative was successfully verified.

[In] Int[(Sec[c + d*x]^5*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^4, x]

[Out] ((A - 4*B)*ArcTanh[Sin[c + d*x]]/(a^4*d) - ((55*A - 244*B)*Tan[c + d*x])/(105*a^4*d) + ((25*A - 88*B)*Sec[c + d*x]^2*Tan[c + d*x])/(105*a^4*d*(1 + Sec[c + d*x])^2) - ((A - 4*B)*Tan[c + d*x])/(a^4*d*(1 + Sec[c + d*x])) + ((A - B)*Sec[c + d*x]^4*Tan[c + d*x])/(7*d*(a + a*Sec[c + d*x])^4) + ((5*A - 12*B)*Sec[c + d*x]^3*Tan[c + d*x])/(35*a*d*(a + a*Sec[c + d*x])^3)

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 3767

Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) +
(a_.), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4008

```
Int[csc[(e_.) + (f_.)*(x_.)]^2*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(
csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.), x_Symbol] :> -Simp[((A*b - a*B)*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(b^2*(2*m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*B*
(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A
*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (
a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.), x_Symbol] :> Simp[(d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^5(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^4} dx &= \frac{(A-B)\sec^4(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{\int \frac{\sec^4(c+dx)(4a(A-B)-a(A-8B)\sec(c+dx))}{(a+a\sec(c+dx))^3} dx}{7a^2} \\
&= \frac{(A-B)\sec^4(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{(5A-12B)\sec^3(c+dx)\tan(c+dx)}{35ad(a+a\sec(c+dx))^3} \\
&= \frac{(25A-88B)\sec^2(c+dx)\tan(c+dx)}{105a^4d(1+\sec(c+dx))^2} + \frac{(A-B)\sec^4(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} \\
&= \frac{(25A-88B)\sec^2(c+dx)\tan(c+dx)}{105a^4d(1+\sec(c+dx))^2} + \frac{(A-B)\sec^4(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} \\
&= \frac{(25A-88B)\sec^2(c+dx)\tan(c+dx)}{105a^4d(1+\sec(c+dx))^2} + \frac{(A-B)\sec^4(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} \\
&= \frac{(A-4B)\tanh^{-1}(\sin(c+dx))}{a^4d} + \frac{(25A-88B)\sec^2(c+dx)\tan(c+dx)}{105a^4d(1+\sec(c+dx))^2} \\
&= \frac{(A-4B)\tanh^{-1}(\sin(c+dx))}{a^4d} - \frac{(55A-244B)\tan(c+dx)}{105a^4d} + \frac{(25A-88B)\sec^2(c+dx)\tan(c+dx)}{105a^4d(1+\sec(c+dx))^2}
\end{aligned}$$

Mathematica [B] time = 6.45, size = 754, normalized size = 3.89

$$\frac{\sec\left(\frac{c}{2}\right)\sec(c)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^4(c+dx)\left(4795A\sin\left(c-\frac{dx}{2}\right)-4795A\sin\left(c+\frac{dx}{2}\right)+4165A\sin\left(2c+\frac{dx}{2}\right)+2275A\sin\left(3c+\frac{dx}{2}\right)-4690B\sin\left(c+\frac{dx}{2}\right)-4445A\sin\left(2c+\frac{dx}{2}\right)+14378B\sin\left(2c+\frac{dx}{2}\right)+2275A\sin\left(3c+\frac{dx}{2}\right)-9100B\sin\left(3c+\frac{dx}{2}\right)\right)}{105a^4d(1+\sec(c+dx))^2}$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[c + d*x]^5*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^4,x]

[Out] (16*(-A + 4*B)*Cos[c/2 + (d*x)/2]^8*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])*Sec[c + d*x]^3*(A + B*Sec[c + d*x])/(d*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^4) - (16*(-A + 4*B)*Cos[c/2 + (d*x)/2]^8*Log[Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])*Sec[c + d*x]^3*(A + B*Sec[c + d*x])/(d*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^4) + (Cos[c/2 + (d*x)/2]*Sec[c/2]*Sec[c]*Sec[c + d*x]^4*(A + B*Sec[c + d*x])*(4165*A*Sin[(d*x)/2] - 10780*B*Sin[(d*x)/2] - 4445*A*Sin[(3*d*x)/2] + 18788*B*Sin[(3*d*x)/2] + 4795*A*Sin[c - (d*x)/2] - 20524*B*Sin[c - (d*x)/2] - 4795*A*Sin[c + (d*x)/2] + 14644*B*Sin[c + (d*x)/2] + 4165*A*Sin[2*c + (d*x)/2] - 16660*B*Sin[2*c + (d*x)/2] + 2275*A*Sin[c + (3*d*x)/2] - 4690*B*Sin[c + (3*d*x)/2] - 4445*A*Sin[2*c + (3*d*x)/2] + 14378*B*Sin[2*c + (3*d*x)/2] + 2275*A*Sin[3*c + (3*d*x)/2] - 9100*B*Sin[3*c + (3*d*x)/2])

*c + (3*d*x)/2] - 2785*A*Sin[c + (5*d*x)/2] + 11668*B*Sin[c + (5*d*x)/2] + 735*A*Sin[2*c + (5*d*x)/2] - 630*B*Sin[2*c + (5*d*x)/2] - 2785*A*Sin[3*c + (5*d*x)/2] + 9358*B*Sin[3*c + (5*d*x)/2] + 735*A*Sin[4*c + (5*d*x)/2] - 2940*B*Sin[4*c + (5*d*x)/2] - 1015*A*Sin[2*c + (7*d*x)/2] + 4228*B*Sin[2*c + (7*d*x)/2] + 105*A*Sin[3*c + (7*d*x)/2] + 315*B*Sin[3*c + (7*d*x)/2] - 1015*A*Sin[4*c + (7*d*x)/2] + 3493*B*Sin[4*c + (7*d*x)/2] + 105*A*Sin[5*c + (7*d*x)/2] - 420*B*Sin[5*c + (7*d*x)/2] - 160*A*Sin[3*c + (9*d*x)/2] + 664*B*Sin[3*c + (9*d*x)/2] + 105*B*Sin[4*c + (9*d*x)/2] - 160*A*Sin[5*c + (9*d*x)/2] + 559*B*Sin[5*c + (9*d*x)/2]))/(1680*d*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^4)

fricas [A] time = 0.44, size = 317, normalized size = 1.63

$$\frac{105 \left((A - 4B) \cos(dx + c)^5 + 4(A - 4B) \cos(dx + c)^4 + 6(A - 4B) \cos(dx + c)^3 + 4(A - 4B) \cos(dx + c)^2 + \dots \right)}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^5*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x, algorithm="fricas")

[Out] 1/210*(105*((A - 4*B)*cos(d*x + c)^5 + 4*(A - 4*B)*cos(d*x + c)^4 + 6*(A - 4*B)*cos(d*x + c)^3 + 4*(A - 4*B)*cos(d*x + c)^2 + (A - 4*B)*cos(d*x + c))*log(sin(d*x + c) + 1) - 105*((A - 4*B)*cos(d*x + c)^5 + 4*(A - 4*B)*cos(d*x + c)^4 + 6*(A - 4*B)*cos(d*x + c)^3 + 4*(A - 4*B)*cos(d*x + c)^2 + (A - 4*B)*cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(8*(20*A - 83*B)*cos(d*x + c)^4 + (535*A - 2236*B)*cos(d*x + c)^3 + 4*(155*A - 659*B)*cos(d*x + c)^2 + 4*(65*A - 296*B)*cos(d*x + c) - 105*B)*sin(d*x + c))/(a^4*d*cos(d*x + c)^5 + 4*a^4*d*cos(d*x + c)^4 + 6*a^4*d*cos(d*x + c)^3 + 4*a^4*d*cos(d*x + c)^2 + a^4*d*cos(d*x + c))

giac [A] time = 1.33, size = 220, normalized size = 1.13

$$\frac{840(A-4B)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^4} - \frac{840(A-4B)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^4} - \frac{1680B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)a^4} - \frac{15Aa^{24}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-15Ba^{24}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^5*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x, algorithm="giac")

[Out] 1/840*(840*(A - 4*B)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - 840*(A - 4*B)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^4 - 1680*B*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 - 1)*a^4) - (15*A*a^24*tan(1/2*d*x + 1/2*c)^7 - 15*B*a^24*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)*a^4))

$24*\tan(1/2*d*x + 1/2*c)^7 + 105*A*a^{24}*\tan(1/2*d*x + 1/2*c)^5 - 147*B*a^{24}*\tan(1/2*d*x + 1/2*c)^5 + 385*A*a^{24}*\tan(1/2*d*x + 1/2*c)^3 - 805*B*a^{24}*\tan(1/2*d*x + 1/2*c)^3 + 1575*A*a^{24}*\tan(1/2*d*x + 1/2*c) - 5145*B*a^{24}*\tan(1/2*d*x + 1/2*c))/a^{28}/d$

maple [A] time = 0.69, size = 285, normalized size = 1.47

$$-\frac{\left(\tan^7\left(\frac{dx}{2} + \frac{c}{2}\right)\right)A}{56da^4} + \frac{B\left(\tan^7\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{56da^4} - \frac{A\left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{8da^4} + \frac{7B\left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{40da^4} - \frac{11\left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)A}{24da^4} + \frac{23B\left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{24da^4} + \frac{1575A - 5145B}{a^{28}d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sec(d*x+c)^5*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x)`

[Out] $-1/56/d/a^4*\tan(1/2*d*x+1/2*c)^7*A+1/56/d/a^4*B*\tan(1/2*d*x+1/2*c)^7-1/8/d/a^4*A*\tan(1/2*d*x+1/2*c)^5+7/40/d/a^4*B*\tan(1/2*d*x+1/2*c)^5-11/24/d/a^4*\tan(1/2*d*x+1/2*c)^3*A+23/24/d/a^4*B*\tan(1/2*d*x+1/2*c)^3-15/8/d/a^4*A*\tan(1/2*d*x+1/2*c)+49/8/d/a^4*B*\tan(1/2*d*x+1/2*c)-1/d/a^4*\ln(\tan(1/2*d*x+1/2*c)-1)*A+4/d/a^4*\ln(\tan(1/2*d*x+1/2*c)-1)*B-1/d/a^4/(\tan(1/2*d*x+1/2*c)-1)*B+1/d/a^4*\ln(\tan(1/2*d*x+1/2*c)+1)*A-4/d/a^4*\ln(\tan(1/2*d*x+1/2*c)+1)*B-1/d/a^4/(\tan(1/2*d*x+1/2*c)+1)*B$

maxima [A] time = 0.36, size = 326, normalized size = 1.68

$$B \left(\frac{1680 \sin(dx+c)}{\left(a^4 - \frac{a^4 \sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)} + \frac{\frac{5145 \sin(dx+c)}{\cos(dx+c)+1} + \frac{805 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{147 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} + \frac{15 \sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4} - \frac{3360 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1} + 1\right)}{a^4} + \frac{3360 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1} - 1\right)}{a^4} \right) / 840d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^5*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x, algorithm="maxima")`

[Out] $1/840*(B*(1680*\sin(d*x + c)/((a^4 - a^4*\sin(d*x + c)^2/(\cos(d*x + c) + 1)^2))*(\cos(d*x + c) + 1)) + (5145*\sin(d*x + c)/(\cos(d*x + c) + 1) + 805*\sin(d*x + c)^3/(\cos(d*x + c) + 1)^3 + 147*\sin(d*x + c)^5/(\cos(d*x + c) + 1)^5 + 15*\sin(d*x + c)^7/(\cos(d*x + c) + 1)^7)/a^4 - 3360*\log(\sin(d*x + c)/(\cos(d*x + c) + 1) + 1)/a^4 + 3360*\log(\sin(d*x + c)/(\cos(d*x + c) + 1) - 1)/a^4 - 5*A*((315*\sin(d*x + c)/(\cos(d*x + c) + 1) + 77*\sin(d*x + c)^3/(\cos(d*x + c) + 1)^3 + 21*\sin(d*x + c)^5/(\cos(d*x + c) + 1)^5 + 3*\sin(d*x + c)^7/(\cos(d*x + c) + 1)^7)/a^4 - 168*\log(\sin(d*x + c)/(\cos(d*x + c) + 1) + 1)/a^4 + 168*\log(\sin(d*x + c)/(\cos(d*x + c) + 1) - 1)/a^4))/d$

mupad [B] time = 2.03, size = 237, normalized size = 1.22

$$\frac{2 \operatorname{atanh}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right) (A - 4B)}{a^4 d} - \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5 \left(\frac{A-B}{20a^4} + \frac{3A-5B}{40a^4}\right)}{d} - \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right) \left(\frac{A-B}{2a^4} + \frac{3(3A-5B)}{8a^4} + \frac{2A-10B}{4a^4}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B/cos(c + d*x))/(cos(c + d*x)^5*(a + a/cos(c + d*x))^4), x)`

[Out] $(2*\operatorname{atanh}(\tan(c/2 + (d*x)/2))*(A - 4*B))/(a^4*d) - (\tan(c/2 + (d*x)/2)^5*((A - B)/(20*a^4) + (3*A - 5*B)/(40*a^4)))/d - (\tan(c/2 + (d*x)/2)*((A - B)/(2*a^4) + (3*(3*A - 5*B))/(8*a^4) + (2*A - 10*B)/(4*a^4) - (2*A + 10*B)/(8*a^4)))/d - (\tan(c/2 + (d*x)/2)^7*(A - B))/(56*a^4*d) - (\tan(c/2 + (d*x)/2)^3*((A - B)/(8*a^4) + (3*A - 5*B)/(12*a^4) + (2*A - 10*B)/(24*a^4)))/d - (2*B*\tan(c/2 + (d*x)/2))/(d*(a^4*\tan(c/2 + (d*x)/2)^2 - a^4))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A \sec^5(c+dx)}{\sec^4(c+dx)+4\sec^3(c+dx)+6\sec^2(c+dx)+4\sec(c+dx)+1} dx + \int \frac{B \sec^6(c+dx)}{\sec^4(c+dx)+4\sec^3(c+dx)+6\sec^2(c+dx)+4\sec(c+dx)+1} dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)**5*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**4, x)`

[Out] $(\operatorname{Integral}(A*\sec(c + d*x)**5/(\sec(c + d*x)**4 + 4*\sec(c + d*x)**3 + 6*\sec(c + d*x)**2 + 4*\sec(c + d*x) + 1), x) + \operatorname{Integral}(B*\sec(c + d*x)**6/(\sec(c + d*x)**4 + 4*\sec(c + d*x)**3 + 6*\sec(c + d*x)**2 + 4*\sec(c + d*x) + 1), x))/a**4$

$$3.110 \quad \int \frac{\sec^4(c+dx)(A+B \sec(c+dx))}{(a+a \sec(c+dx))^4} dx$$

Optimal. Leaf size=163

$$\frac{(12A - 215B) \tan(c + dx)}{105a^4d(\sec(c + dx) + 1)} - \frac{(6A - 55B) \tan(c + dx)}{105a^4d(\sec(c + dx) + 1)^2} + \frac{B \tanh^{-1}(\sin(c + dx))}{a^4d} + \frac{(A - B) \tan(c + dx) \sec^3(c + dx)}{7d(a \sec(c + dx) + a)^4} +$$

[Out] B*arctanh(sin(d*x+c))/a^4/d-1/105*(6*A-55*B)*tan(d*x+c)/a^4/d/(1+sec(d*x+c))^2+1/105*(12*A-215*B)*tan(d*x+c)/a^4/d/(1+sec(d*x+c))+1/7*(A-B)*sec(d*x+c)^3*tan(d*x+c)/d/(a+a*sec(d*x+c))^4+1/35*(3*A-10*B)*sec(d*x+c)^2*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^3

Rubi [A] time = 0.48, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {4019, 4008, 3998, 3770, 3794}

$$\frac{(12A - 215B) \tan(c + dx)}{105a^4d(\sec(c + dx) + 1)} - \frac{(6A - 55B) \tan(c + dx)}{105a^4d(\sec(c + dx) + 1)^2} + \frac{B \tanh^{-1}(\sin(c + dx))}{a^4d} + \frac{(A - B) \tan(c + dx) \sec^3(c + dx)}{7d(a \sec(c + dx) + a)^4} +$$

Antiderivative was successfully verified.

[In] Int[(Sec[c + d*x]^4*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^4, x]

[Out] (B*ArcTanh[Sin[c + d*x]])/(a^4*d) - ((6*A - 55*B)*Tan[c + d*x])/(105*a^4*d*(1 + Sec[c + d*x])^2) + ((12*A - 215*B)*Tan[c + d*x])/(105*a^4*d*(1 + Sec[c + d*x])) + ((A - B)*Sec[c + d*x]^3*Tan[c + d*x])/(7*d*(a + a*Sec[c + d*x])^4) + ((3*A - 10*B)*Sec[c + d*x]^2*Tan[c + d*x])/(35*a*d*(a + a*Sec[c + d*x])^3)

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3794

Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] := -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 3998

Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] := Dist[B/b, Int[Csc[e + f*x], x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]

;/ FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]

Rule 4008

Int[csc[(e_.) + (f_.)*(x_.)]^2*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(b^2*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*B*(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]

Rule 4019

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> Simp[(d*(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0]

Rubi steps

$$\begin{aligned} \int \frac{\sec^4(c + dx)(A + B \sec(c + dx))}{(a + a \sec(c + dx))^4} dx &= \frac{(A - B) \sec^3(c + dx) \tan(c + dx)}{7d(a + a \sec(c + dx))^4} + \frac{\int \frac{\sec^3(c + dx)(3a(A - B) + 7aB \sec(c + dx))}{(a + a \sec(c + dx))^3} dx}{7a^2} \\ &= \frac{(A - B) \sec^3(c + dx) \tan(c + dx)}{7d(a + a \sec(c + dx))^4} + \frac{(3A - 10B) \sec^2(c + dx) \tan(c + dx)}{35ad(a + a \sec(c + dx))^3} \\ &= -\frac{(6A - 55B) \tan(c + dx)}{105a^4d(1 + \sec(c + dx))^2} + \frac{(A - B) \sec^3(c + dx) \tan(c + dx)}{7d(a + a \sec(c + dx))^4} + \frac{(3A - 10B) \sec^2(c + dx) \tan(c + dx)}{35ad(a + a \sec(c + dx))^3} \\ &= -\frac{(6A - 55B) \tan(c + dx)}{105a^4d(1 + \sec(c + dx))^2} + \frac{(A - B) \sec^3(c + dx) \tan(c + dx)}{7d(a + a \sec(c + dx))^4} + \frac{(3A - 10B) \sec^2(c + dx) \tan(c + dx)}{35ad(a + a \sec(c + dx))^3} \\ &= \frac{B \tanh^{-1}(\sin(c + dx))}{a^4d} - \frac{(6A - 55B) \tan(c + dx)}{105a^4d(1 + \sec(c + dx))^2} + \frac{(A - B) \sec^3(c + dx) \tan(c + dx)}{7d(a + a \sec(c + dx))^4} \end{aligned}$$

Mathematica [A] time = 1.57, size = 239, normalized size = 1.47

$$\sec\left(\frac{c}{2}\right) \cos\left(\frac{1}{2}(c + dx)\right) \left(70(3A - 49B) \sin\left(\frac{dx}{2}\right) + 126A \sin\left(c + \frac{3dx}{2}\right) + 42A \sin\left(2c + \frac{5dx}{2}\right) + 6A \sin\left(3c + \frac{7dx}{2}\right) + \dots\right)$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[c + d*x]^4*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^4,x]

[Out] (-6720*B*Cos[(c + d*x)/2]^8*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Cos[(c + d*x)/2]*Sec[c/2]*(70*(3*A - 49*B)*Sin[(d*x)/2] + 2170*B*Sin[c + (d*x)/2] + 126*A*Sin[c + (3*d*x)/2] - 2625*B*Sin[c + (3*d*x)/2] + 735*B*Sin[2*c + (3*d*x)/2] + 42*A*Sin[2*c + (5*d*x)/2] - 1015*B*Sin[2*c + (5*d*x)/2] + 105*B*Sin[3*c + (5*d*x)/2] + 6*A*Sin[3*c + (7*d*x)/2] - 160*B*Sin[3*c + (7*d*x)/2]))/(420*a^4*d*(1 + Cos[c + d*x])^4)

fricas [A] time = 0.45, size = 236, normalized size = 1.45

$$105 \left(B \cos(dx + c)^4 + 4B \cos(dx + c)^3 + 6B \cos(dx + c)^2 + 4B \cos(dx + c) + B \right) \log(\sin(dx + c) + 1) - 105 \left(B \cos(dx + c)^4 + 4B \cos(dx + c)^3 + 6B \cos(dx + c)^2 + 4B \cos(dx + c) + B \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x, algorithm="fricas")

[Out] 1/210*(105*(B*cos(d*x + c)^4 + 4*B*cos(d*x + c)^3 + 6*B*cos(d*x + c)^2 + 4*B*cos(d*x + c) + B)*log(sin(d*x + c) + 1) - 105*(B*cos(d*x + c)^4 + 4*B*cos(d*x + c)^3 + 6*B*cos(d*x + c)^2 + 4*B*cos(d*x + c) + B)*log(-sin(d*x + c) + 1) + 2*(2*(3*A - 80*B)*cos(d*x + c)^3 + (24*A - 535*B)*cos(d*x + c)^2 + (39*A - 620*B)*cos(d*x + c) + 36*A - 260*B)*sin(d*x + c))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)

giac [A] time = 0.58, size = 181, normalized size = 1.11

$$\frac{840 B \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 1\right|\right)}{a^4} - \frac{840 B \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 1\right|\right)}{a^4} + \frac{15 A a^{24} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^7 - 15 B a^{24} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^7 + 63 A a^{24} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 - 105 B a^{24} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5}{840 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x, algorithm="giac")

[Out] $\frac{1}{840} \cdot (840 \cdot B \cdot \log(\tan(\frac{1}{2} \cdot d \cdot x + \frac{1}{2} \cdot c) + 1)) / a^4 - 840 \cdot B \cdot \log(\tan(\frac{1}{2} \cdot d \cdot x + \frac{1}{2} \cdot c) - 1)) / a^4 + (15 \cdot A \cdot a^{24} \cdot \tan(\frac{1}{2} \cdot d \cdot x + \frac{1}{2} \cdot c)^7 - 15 \cdot B \cdot a^{24} \cdot \tan(\frac{1}{2} \cdot d \cdot x + \frac{1}{2} \cdot c)^7 + 63 \cdot A \cdot a^{24} \cdot \tan(\frac{1}{2} \cdot d \cdot x + \frac{1}{2} \cdot c)^5 - 105 \cdot B \cdot a^{24} \cdot \tan(\frac{1}{2} \cdot d \cdot x + \frac{1}{2} \cdot c)^5 + 105 \cdot A \cdot a^{24} \cdot \tan(\frac{1}{2} \cdot d \cdot x + \frac{1}{2} \cdot c)^3 - 385 \cdot B \cdot a^{24} \cdot \tan(\frac{1}{2} \cdot d \cdot x + \frac{1}{2} \cdot c)^3 + 105 \cdot A \cdot a^{24} \cdot \tan(\frac{1}{2} \cdot d \cdot x + \frac{1}{2} \cdot c) - 1575 \cdot B \cdot a^{24} \cdot \tan(\frac{1}{2} \cdot d \cdot x + \frac{1}{2} \cdot c)) / a^{28} / d$

maple [A] time = 0.71, size = 199, normalized size = 1.22

$$\frac{A \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{8d a^4} + \frac{\left(\tan^7\left(\frac{dx}{2} + \frac{c}{2}\right)\right) A}{56d a^4} - \frac{B \left(\tan^7\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{56d a^4} - \frac{\ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right) B}{d a^4} - \frac{15B \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{8d a^4} + \frac{\ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{d a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(\sec(dx+c)^4 \cdot (A+B \cdot \sec(dx+c)) / (a+a \cdot \sec(dx+c))^4, x)$

[Out] $\frac{1}{8} \cdot \frac{1}{d} \cdot \frac{1}{a^4} \cdot A \cdot \tan(\frac{1}{2} \cdot d \cdot x + \frac{1}{2} \cdot c) + \frac{1}{56} \cdot \frac{1}{d} \cdot \frac{1}{a^4} \cdot \tan(\frac{1}{2} \cdot d \cdot x + \frac{1}{2} \cdot c)^7 \cdot A - \frac{1}{56} \cdot \frac{1}{d} \cdot \frac{1}{a^4} \cdot B \cdot \tan(\frac{1}{2} \cdot d \cdot x + \frac{1}{2} \cdot c)^7 - \frac{1}{d} \cdot \frac{1}{a^4} \cdot \ln(\tan(\frac{1}{2} \cdot d \cdot x + \frac{1}{2} \cdot c) - 1) \cdot B - \frac{15}{8} \cdot \frac{1}{d} \cdot \frac{1}{a^4} \cdot B \cdot \tan(\frac{1}{2} \cdot d \cdot x + \frac{1}{2} \cdot c) + \frac{1}{d} \cdot \frac{1}{a^4} \cdot \ln(\tan(\frac{1}{2} \cdot d \cdot x + \frac{1}{2} \cdot c) + 1) \cdot B + \frac{3}{40} \cdot \frac{1}{d} \cdot \frac{1}{a^4} \cdot A \cdot \tan(\frac{1}{2} \cdot d \cdot x + \frac{1}{2} \cdot c)^5 - \frac{1}{8} \cdot \frac{1}{d} \cdot \frac{1}{a^4} \cdot B \cdot \tan(\frac{1}{2} \cdot d \cdot x + \frac{1}{2} \cdot c)^5 + \frac{1}{8} \cdot \frac{1}{d} \cdot \frac{1}{a^4} \cdot \tan(\frac{1}{2} \cdot d \cdot x + \frac{1}{2} \cdot c)^3 \cdot A - \frac{11}{24} \cdot \frac{1}{d} \cdot \frac{1}{a^4} \cdot B \cdot \tan(\frac{1}{2} \cdot d \cdot x + \frac{1}{2} \cdot c)^3$

maxima [A] time = 0.36, size = 228, normalized size = 1.40

$$\frac{5B \left(\frac{\frac{315 \sin(dx+c)}{\cos(dx+c)+1} + \frac{77 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{21 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} + \frac{3 \sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4} - \frac{168 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1} + 1\right)}{a^4} + \frac{168 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1} - 1\right)}{a^4} \right) - 3A \left(\frac{35 \sin(dx+c)}{\cos(dx+c)+1} + \frac{35 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{21 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} + \frac{5 \sin(dx+c)^7}{(\cos(dx+c)+1)^7} \right)}{840d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(\sec(dx+c)^4 \cdot (A+B \cdot \sec(dx+c)) / (a+a \cdot \sec(dx+c))^4, x, \text{algorithm}="maxima")$

[Out] $-\frac{1}{840} \cdot (5 \cdot B \cdot ((315 \cdot \sin(dx+c) / (\cos(dx+c)+1) + 77 \cdot \sin(dx+c)^3 / (\cos(dx+c)+1)^3 + 21 \cdot \sin(dx+c)^5 / (\cos(dx+c)+1)^5 + 3 \cdot \sin(dx+c)^7 / (\cos(dx+c)+1)^7) / a^4 - 168 \cdot \log(\sin(dx+c) / (\cos(dx+c)+1) + 1) / a^4 + 168 \cdot \log(\sin(dx+c) / (\cos(dx+c)+1) - 1) / a^4 - 3 \cdot A \cdot (35 \cdot \sin(dx+c) / (\cos(dx+c)+1) + 35 \cdot \sin(dx+c)^3 / (\cos(dx+c)+1)^3 + 21 \cdot \sin(dx+c)^5 / (\cos(dx+c)+1)^5 + 5 \cdot \sin(dx+c)^7 / (\cos(dx+c)+1)^7) / a^4) / d$

mupad [B] time = 2.13, size = 198, normalized size = 1.21

$$\frac{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)^4 \left(\frac{A \sin\left(\frac{c}{2} + \frac{dx}{2}\right)^3}{8} - \frac{11 B \sin\left(\frac{c}{2} + \frac{dx}{2}\right)^3}{24} \right) + \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^2 \left(\frac{3 A \sin\left(\frac{c}{2} + \frac{dx}{2}\right)^5}{40} - \frac{B \sin\left(\frac{c}{2} + \frac{dx}{2}\right)^5}{8} \right) + \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^6 \left(\frac{A \sin\left(\frac{c}{2} + \frac{dx}{2}\right)^7}{56} - \frac{15 B \sin\left(\frac{c}{2} + \frac{dx}{2}\right)^7}{8} \right)}{a^4 d \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^4*(a + a/cos(c + d*x))^4), x)

[Out] (cos(c/2 + (d*x)/2)^4*((A*sin(c/2 + (d*x)/2)^3)/8 - (11*B*sin(c/2 + (d*x)/2)^3)/24) + cos(c/2 + (d*x)/2)^2*((3*A*sin(c/2 + (d*x)/2)^5)/40 - (B*sin(c/2 + (d*x)/2)^5)/8) + cos(c/2 + (d*x)/2)^6*((A*sin(c/2 + (d*x)/2))^7)/56 - (15*B*sin(c/2 + (d*x)/2))^7)/56)/(a^4*d*cos(c/2 + (d*x)/2)^7) + (2*B*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(a^4*d)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A \sec^4(c+dx)}{\sec^4(c+dx)+4 \sec^3(c+dx)+6 \sec^2(c+dx)+4 \sec(c+dx)+1} dx + \int \frac{B \sec^5(c+dx)}{\sec^4(c+dx)+4 \sec^3(c+dx)+6 \sec^2(c+dx)+4 \sec(c+dx)+1} dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**4, x)

[Out] (Integral(A*sec(c + d*x)**4/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c + d*x)**2 + 4*sec(c + d*x) + 1), x) + Integral(B*sec(c + d*x)**5/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c + d*x)**2 + 4*sec(c + d*x) + 1), x))/a**4

$$3.111 \quad \int \frac{\sec^3(c+dx)(A+B \sec(c+dx))}{(a+a \sec(c+dx))^4} dx$$

Optimal. Leaf size=146

$$\frac{(4A+3B) \tan(c+dx)}{15d(a^4 \sec(c+dx) + a^4)} - \frac{8(4A+3B) \tan(c+dx)}{105d(a^2 \sec(c+dx) + a^2)^2} - \frac{(A-B) \tan(c+dx) \sec^3(c+dx)}{7d(a \sec(c+dx) + a)^4} + \frac{(4A+3B) \tan(c+dx)}{35ad(a \sec(c+dx) + a)}$$

[Out] $-1/7*(A-B)*\sec(d*x+c)^3*\tan(d*x+c)/d/(a+a*\sec(d*x+c))^4+1/35*(4*A+3*B)*\tan(d*x+c)/a/d/(a+a*\sec(d*x+c))^3-8/105*(4*A+3*B)*\tan(d*x+c)/d/(a^2+a^2*\sec(d*x+c))^2+1/15*(4*A+3*B)*\tan(d*x+c)/d/(a^4+a^4*\sec(d*x+c))$

Rubi [A] time = 0.23, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {4012, 3799, 4000, 3794}

$$\frac{(4A+3B) \tan(c+dx)}{15d(a^4 \sec(c+dx) + a^4)} - \frac{8(4A+3B) \tan(c+dx)}{105d(a^2 \sec(c+dx) + a^2)^2} - \frac{(A-B) \tan(c+dx) \sec^3(c+dx)}{7d(a \sec(c+dx) + a)^4} + \frac{(4A+3B) \tan(c+dx)}{35ad(a \sec(c+dx) + a)}$$

Antiderivative was successfully verified.

[In] Int[(Sec[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^4,x]

[Out] $-((A-B)*\text{Sec}[c+d*x]^3*\text{Tan}[c+d*x])/(7*d*(a+a*\text{Sec}[c+d*x])^4) + ((4*A+3*B)*\text{Tan}[c+d*x])/(35*a*d*(a+a*\text{Sec}[c+d*x])^3) - (8*(4*A+3*B)*\text{Tan}[c+d*x])/(105*d*(a^2+a^2*\text{Sec}[c+d*x])^2) + ((4*A+3*B)*\text{Tan}[c+d*x])/(15*d*(a^4+a^4*\text{Sec}[c+d*x]))$

Rule 3794

Int[csc[(e_.) + (f_.)*(x_.)]/(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 3799

Int[csc[(e_.) + (f_.)*(x_.)]^3*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.), x_Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] - Dist[1/(a^2*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(a*m - b*(2*m + 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]

Rule 4000

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] := Simp[((A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Rule 4012

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] := -Simp[((A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m + 1)), x] + Dist[(a*A*m + b*B*(m + 1))/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && LeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{\sec^3(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^4} dx &= -\frac{(A-B)\sec^3(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{(4A+3B)\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^3} dx}{7a} \\ &= -\frac{(A-B)\sec^3(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{(4A+3B)\tan(c+dx)}{35ad(a+a\sec(c+dx))^3} + \frac{(4A+3B)\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^3} dx}{7a} \\ &= -\frac{(A-B)\sec^3(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{(4A+3B)\tan(c+dx)}{35ad(a+a\sec(c+dx))^3} - \frac{8(4A+3B)\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^3} dx}{105d} \\ &= -\frac{(A-B)\sec^3(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{(4A+3B)\tan(c+dx)}{35ad(a+a\sec(c+dx))^3} - \frac{8(4A+3B)\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^3} dx}{105d} \end{aligned}$$

Mathematica [A] time = 0.38, size = 109, normalized size = 0.75

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left((4A+3B)\left(21\sin\left(c+\frac{3dx}{2}\right)+7\sin\left(2c+\frac{5dx}{2}\right)+\sin\left(3c+\frac{7dx}{2}\right)\right)+35(2A+3B)\sin\left(\frac{dx}{2}\right)\right)}{210a^4d(\cos(c+dx)+1)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^4, x]
```

[Out] $(\cos[(c + dx)/2] \sec[c/2] (35(2A + 3B) \sin[(dx)/2] - 70A \sin[c + (dx)/2]) + (4A + 3B)(21 \sin[c + (3dx)/2] + 7 \sin[2c + (5dx)/2] + \sin[3c + (7dx)/2])) / (210a^4d(1 + \cos[c + dx])^4)$

fricas [A] time = 0.42, size = 125, normalized size = 0.86

$$\frac{(2(4A + 3B) \cos(dx + c)^3 + 8(4A + 3B) \cos(dx + c)^2 + 13(4A + 3B) \cos(dx + c) + 13A + 36B) \sin(dx + c)}{105(a^4d \cos(dx + c)^4 + 4a^4d \cos(dx + c)^3 + 6a^4d \cos(dx + c)^2 + 4a^4d \cos(dx + c) + a^4d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(dx+c)^3*(A+B*sec(dx+c))/(a+a*sec(dx+c))^4,x, algorithm="fricas")`

[Out] $1/105*(2*(4A + 3B)*\cos(dx + c)^3 + 8*(4A + 3B)*\cos(dx + c)^2 + 13*(4A + 3B)*\cos(dx + c) + 13*A + 36*B)*\sin(dx + c)/(a^4*d*\cos(dx + c)^4 + 4*a^4*d*\cos(dx + c)^3 + 6*a^4*d*\cos(dx + c)^2 + 4*a^4*d*\cos(dx + c) + a^4*d)$

giac [A] time = 0.32, size = 117, normalized size = 0.80

$$\frac{15A \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^7 - 15B \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^7 + 21A \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5 - 63B \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5 - 35A \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 - 105B \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 - 105A \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 105B \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)}{840a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(dx+c)^3*(A+B*sec(dx+c))/(a+a*sec(dx+c))^4,x, algorithm="giac")`

[Out] $-1/840*(15*A*\tan(1/2*d*x + 1/2*c)^7 - 15*B*\tan(1/2*d*x + 1/2*c)^7 + 21*A*\tan(1/2*d*x + 1/2*c)^5 - 63*B*\tan(1/2*d*x + 1/2*c)^5 - 35*A*\tan(1/2*d*x + 1/2*c)^3 - 105*B*\tan(1/2*d*x + 1/2*c)^3 - 105*A*\tan(1/2*d*x + 1/2*c) - 105*B*\tan(1/2*d*x + 1/2*c))/(a^4*d)$

maple [A] time = 0.74, size = 88, normalized size = 0.60

$$\frac{\frac{(-A+B)\left(\tan^7\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{7} + \frac{(-A+3B)\left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{5} + \frac{(A+3B)\left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{3} + A \tan\left(\frac{dx}{2} + \frac{c}{2}\right) + B \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{8da^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sec(dx+c)^3*(A+B*sec(dx+c))/(a+a*sec(dx+c))^4,x)`

[Out] $1/8/d/a^4*(1/7*(-A+B)*\tan(1/2*d*x+1/2*c)^7+1/5*(-A+3*B)*\tan(1/2*d*x+1/2*c)^5+1/3*(A+3*B)*\tan(1/2*d*x+1/2*c)^3+A*\tan(1/2*d*x+1/2*c)+B*\tan(1/2*d*x+1/2*c))$

maxima [A] time = 0.36, size = 175, normalized size = 1.20

$$\frac{A \left(\frac{105 \sin(dx+c)}{\cos(dx+c)+1} + \frac{35 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} - \frac{21 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{15 \sin(dx+c)^7}{(\cos(dx+c)+1)^7} \right)}{a^4} + \frac{3B \left(\frac{35 \sin(dx+c)}{\cos(dx+c)+1} + \frac{35 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{21 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} + \frac{5 \sin(dx+c)^7}{(\cos(dx+c)+1)^7} \right)}{a^4}}{840d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x, algorithm="maxima")

[Out] 1/840*(A*(105*sin(d*x + c)/(cos(d*x + c) + 1) + 35*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 15*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 + 3*B*(35*sin(d*x + c)/(cos(d*x + c) + 1) + 35*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 5*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4/d

mupad [B] time = 2.03, size = 85, normalized size = 0.58

$$\frac{\frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 (A+3B)}{24a^4} - \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5 (A-3B)}{40a^4} - \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^7 (A-B)}{56a^4} + \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right) (A+B)}{8a^4}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^3*(a + a/cos(c + d*x))^4),x)

[Out] ((tan(c/2 + (d*x)/2)^3*(A + 3*B))/(24*a^4) - (tan(c/2 + (d*x)/2)^5*(A - 3*B))/(40*a^4) - (tan(c/2 + (d*x)/2)^7*(A - B))/(56*a^4) + (tan(c/2 + (d*x)/2)*(A + B))/(8*a^4))/d

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A \sec^3(c+dx)}{\sec^4(c+dx)+4 \sec^3(c+dx)+6 \sec^2(c+dx)+4 \sec(c+dx)+1} dx + \int \frac{B \sec^4(c+dx)}{\sec^4(c+dx)+4 \sec^3(c+dx)+6 \sec^2(c+dx)+4 \sec(c+dx)+1} dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**4,x)

[Out] (Integral(A*sec(c + d*x)**3/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c + d*x)**2 + 4*sec(c + d*x) + 1), x) + Integral(B*sec(c + d*x)**4/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c + d*x)**2 + 4*sec(c + d*x) + 1), x))/a**4

$$3.112 \quad \int \frac{\sec^2(c+dx)(A+B \sec(c+dx))}{(a+a \sec(c+dx))^4} dx$$

Optimal. Leaf size=138

$$\frac{(8A + 13B) \tan(c + dx)}{105d (a^4 \sec(c + dx) + a^4)} + \frac{(8A + 13B) \tan(c + dx)}{105d (a^2 \sec(c + dx) + a^2)^2} + \frac{(4A - 11B) \tan(c + dx)}{35ad (a \sec(c + dx) + a)^3} - \frac{(A - B) \tan(c + dx)}{7d (a \sec(c + dx) + a)^4}$$

[Out] $-1/7*(A-B)*\tan(d*x+c)/d/(a+a*\sec(d*x+c))^4+1/35*(4*A-11*B)*\tan(d*x+c)/a/d/(a+a*\sec(d*x+c))^3+1/105*(8*A+13*B)*\tan(d*x+c)/d/(a^2+a^2*\sec(d*x+c))^2+1/105*(8*A+13*B)*\tan(d*x+c)/d/(a^4+a^4*\sec(d*x+c))$

Rubi [A] time = 0.26, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {4008, 4000, 3796, 3794}

$$\frac{(8A + 13B) \tan(c + dx)}{105d (a^4 \sec(c + dx) + a^4)} + \frac{(8A + 13B) \tan(c + dx)}{105d (a^2 \sec(c + dx) + a^2)^2} + \frac{(4A - 11B) \tan(c + dx)}{35ad (a \sec(c + dx) + a)^3} - \frac{(A - B) \tan(c + dx)}{7d (a \sec(c + dx) + a)^4}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(\text{Sec}[c + d*x]^2*(A + B*\text{Sec}[c + d*x]))/(a + a*\text{Sec}[c + d*x])^4, x]$

[Out] $-((A - B)*\text{Tan}[c + d*x])/(7*d*(a + a*\text{Sec}[c + d*x])^4) + ((4*A - 11*B)*\text{Tan}[c + d*x])/(35*a*d*(a + a*\text{Sec}[c + d*x])^3) + ((8*A + 13*B)*\text{Tan}[c + d*x])/(105*d*(a^2 + a^2*\text{Sec}[c + d*x])^2) + ((8*A + 13*B)*\text{Tan}[c + d*x])/(105*d*(a^4 + a^4*\text{Sec}[c + d*x]))$

Rule 3794

$\text{Int}[\text{csc}[(e_.) + (f_.)*(x_.)]/(\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol]$ $\rightarrow -\text{Simp}[\text{Cot}[e + f*x]/(f*(b + a*\text{Csc}[e + f*x])), x] /;$ $\text{FreeQ}[\{a, b, e, f\}, x]$ $\&\& \text{EqQ}[a^2 - b^2, 0]$

Rule 3796

$\text{Int}[\text{csc}[(e_.) + (f_.)*(x_.)]*(\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^{(m_.)}, x_Symbol]$ $\rightarrow \text{Simp}[(b*\text{Cot}[e + f*x]*(a + b*\text{Csc}[e + f*x])^m)/(a*f*(2*m + 1)), x] + \text{Dist}[(m + 1)/(a*(2*m + 1)), \text{Int}[\text{Csc}[e + f*x]*(a + b*\text{Csc}[e + f*x])^{(m + 1)}, x], x] /;$ $\text{FreeQ}[\{a, b, e, f\}, x]$ $\&\& \text{EqQ}[a^2 - b^2, 0]$ $\&\& \text{LtQ}[m, -2^{(-1)}]$ $\&\& \text{IntegerQ}[2*m]$

Rule 4000

$\text{Int}[\text{csc}[(e_.) + (f_.)*(x_.)]*(\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^{(m_.)}*(\text{csc}[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol]$ $\rightarrow \text{Simp}[(A*b - a*B)*\text{Cot}[e +$

```
f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Rule 4008

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] := -Simp[((A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(b^2*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*B*(2*m + 1)*Csc[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned} \int \frac{\sec^2(c + dx)(A + B \sec(c + dx))}{(a + a \sec(c + dx))^4} dx &= -\frac{(A - B) \tan(c + dx)}{7d(a + a \sec(c + dx))^4} - \frac{\int \frac{\sec(c + dx)(-4a(A - B) - 7aB \sec(c + dx))}{(a + a \sec(c + dx))^3} dx}{7a^2} \\ &= -\frac{(A - B) \tan(c + dx)}{7d(a + a \sec(c + dx))^4} + \frac{(4A - 11B) \tan(c + dx)}{35ad(a + a \sec(c + dx))^3} + \frac{(8A + 13B) \int \frac{1}{(a + a \sec(c + dx))^2} dx}{35a^2} \\ &= -\frac{(A - B) \tan(c + dx)}{7d(a + a \sec(c + dx))^4} + \frac{(4A - 11B) \tan(c + dx)}{35ad(a + a \sec(c + dx))^3} + \frac{(8A + 13B) \tan(c + dx)}{105d(a^2 + a^2 \sec^2(c + dx))} \\ &= -\frac{(A - B) \tan(c + dx)}{7d(a + a \sec(c + dx))^4} + \frac{(4A - 11B) \tan(c + dx)}{35ad(a + a \sec(c + dx))^3} + \frac{(8A + 13B) \tan(c + dx)}{105d(a^2 + a^2 \sec^2(c + dx))} \end{aligned}$$

Mathematica [A] time = 0.42, size = 163, normalized size = 1.18

$$\frac{\sec\left(\frac{c}{2}\right) \cos\left(\frac{1}{2}(c + dx)\right) \left(-35(5A + 4B) \sin\left(c + \frac{dx}{2}\right) + 140(2A + B) \sin\left(\frac{dx}{2}\right) + 168A \sin\left(c + \frac{3dx}{2}\right) - 105A \sin\left(2c + \frac{3dx}{2}\right)\right)}{420a^4 d (\cos(c + dx) + 1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^4, x]
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(140*(2*A + B)*Sin[(d*x)/2] - 35*(5*A + 4*B)*Sin[c + (d*x)/2] + 168*A*Sin[c + (3*d*x)/2] + 168*B*Sin[c + (3*d*x)/2] - 105*A*Sin[2*c + (3*d*x)/2] + 91*A*Sin[2*c + (5*d*x)/2] + 56*B*Sin[2*c + (5*d*x)/2])/(420*a^4*d*(Cos[c + dx] + 1))
```

$$2] + 13*A*\sin[3*c + (7*d*x)/2] + 8*B*\sin[3*c + (7*d*x)/2]))/(420*a^4*d*(1 + \cos[c + d*x])^4)$$

fricas [A] time = 0.43, size = 124, normalized size = 0.90

$$\frac{((13A + 8B)\cos(dx + c)^3 + 4(13A + 8B)\cos(dx + c)^2 + 4(8A + 13B)\cos(dx + c) + 8A + 13B)\sin(dx + c)}{105(a^4d\cos(dx + c)^4 + 4a^4d\cos(dx + c)^3 + 6a^4d\cos(dx + c)^2 + 4a^4d\cos(dx + c) + a^4d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x, algorithm="fricas")

[Out] 1/105*((13*A + 8*B)*cos(d*x + c)^3 + 4*(13*A + 8*B)*cos(d*x + c)^2 + 4*(8*A + 13*B)*cos(d*x + c) + 8*A + 13*B)*sin(d*x + c)/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)

giac [A] time = 0.30, size = 117, normalized size = 0.85

$$\frac{15A \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^7 - 15B \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^7 - 21A \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5 - 21B \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5 - 35A \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 + 35B \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 + 105A \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 105B \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)}{840a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x, algorithm="giac")

[Out] 1/840*(15*A*tan(1/2*d*x + 1/2*c)^7 - 15*B*tan(1/2*d*x + 1/2*c)^7 - 21*A*tan(1/2*d*x + 1/2*c)^5 - 21*B*tan(1/2*d*x + 1/2*c)^5 - 35*A*tan(1/2*d*x + 1/2*c)^3 + 35*B*tan(1/2*d*x + 1/2*c)^3 + 105*A*tan(1/2*d*x + 1/2*c) + 105*B*tan(1/2*d*x + 1/2*c))/(a^4*d)

maple [A] time = 0.72, size = 88, normalized size = 0.64

$$\frac{\frac{(A-B)\left(\tan^7\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{7} + \frac{(-A-B)\left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{5} + \frac{(-A+B)\left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{3} + A \tan\left(\frac{dx}{2} + \frac{c}{2}\right) + B \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{8da^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x)

[Out] 1/8/d/a^4*(1/7*(A-B)*tan(1/2*d*x+1/2*c)^7+1/5*(-A-B)*tan(1/2*d*x+1/2*c)^5+1/3*(-A+B)*tan(1/2*d*x+1/2*c)^3+A*tan(1/2*d*x+1/2*c)+B*tan(1/2*d*x+1/2*c))

maxima [A] time = 0.36, size = 174, normalized size = 1.26

$$\frac{B \left(\frac{105 \sin(dx+c)}{\cos(dx+c)+1} + \frac{35 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} - \frac{21 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{15 \sin(dx+c)^7}{(\cos(dx+c)+1)^7} \right)}{a^4} + \frac{A \left(\frac{105 \sin(dx+c)}{\cos(dx+c)+1} - \frac{35 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} - \frac{21 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} + \frac{15 \sin(dx+c)^7}{(\cos(dx+c)+1)^7} \right)}{a^4}}{840 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x, algorithm="maxima")

[Out] 1/840*(B*(105*sin(d*x + c)/(cos(d*x + c) + 1) + 35*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 15*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 + A*(105*sin(d*x + c)/(cos(d*x + c) + 1) - 35*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 15*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4/d

mupad [B] time = 1.98, size = 84, normalized size = 0.61

$$\frac{\frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5 (A+B)}{40 a^4} + \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 (A-B)}{24 a^4} - \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^7 (A-B)}{56 a^4} - \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right) (A+B)}{8 a^4}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^2*(a + a/cos(c + d*x))^4),x)

[Out] -((tan(c/2 + (d*x)/2)^5*(A + B))/(40*a^4) + (tan(c/2 + (d*x)/2)^3*(A - B))/(24*a^4) - (tan(c/2 + (d*x)/2)^7*(A - B))/(56*a^4) - (tan(c/2 + (d*x)/2)*(A + B))/(8*a^4))/d

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A \sec^2(c+dx)}{\sec^4(c+dx)+4 \sec^3(c+dx)+6 \sec^2(c+dx)+4 \sec(c+dx)+1} dx + \int \frac{B \sec^3(c+dx)}{\sec^4(c+dx)+4 \sec^3(c+dx)+6 \sec^2(c+dx)+4 \sec(c+dx)+1} dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**4,x)

[Out] (Integral(A*sec(c + d*x)**2/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c + d*x)**2 + 4*sec(c + d*x) + 1), x) + Integral(B*sec(c + d*x)**3/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c + d*x)**2 + 4*sec(c + d*x) + 1), x))/a**4

$$3.113 \quad \int \frac{\sec(c+dx)(A+B \sec(c+dx))}{(a+a \sec(c+dx))^4} dx$$

Optimal. Leaf size=138

$$\frac{2(3A+4B) \tan(c+dx)}{105d(a^4 \sec(c+dx)+a^4)} + \frac{2(3A+4B) \tan(c+dx)}{105d(a^2 \sec(c+dx)+a^2)^2} + \frac{(3A+4B) \tan(c+dx)}{35ad(a \sec(c+dx)+a)^3} + \frac{(A-B) \tan(c+dx)}{7d(a \sec(c+dx)+a)^4}$$

[Out] 1/7*(A-B)*tan(d*x+c)/d/(a+a*sec(d*x+c))^4+1/35*(3*A+4*B)*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^3+2/105*(3*A+4*B)*tan(d*x+c)/d/(a^2+a^2*sec(d*x+c))^2+2/105*(3*A+4*B)*tan(d*x+c)/d/(a^4+a^4*sec(d*x+c))

Rubi [A] time = 0.15, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {4000, 3796, 3794}

$$\frac{2(3A+4B) \tan(c+dx)}{105d(a^4 \sec(c+dx)+a^4)} + \frac{2(3A+4B) \tan(c+dx)}{105d(a^2 \sec(c+dx)+a^2)^2} + \frac{(3A+4B) \tan(c+dx)}{35ad(a \sec(c+dx)+a)^3} + \frac{(A-B) \tan(c+dx)}{7d(a \sec(c+dx)+a)^4}$$

Antiderivative was successfully verified.

[In] Int[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^4, x]

[Out] ((A - B)*Tan[c + d*x])/(7*d*(a + a*Sec[c + d*x])^4) + ((3*A + 4*B)*Tan[c + d*x])/(35*a*d*(a + a*Sec[c + d*x])^3) + (2*(3*A + 4*B)*Tan[c + d*x])/(105*d*(a^2 + a^2*Sec[c + d*x])^2) + (2*(3*A + 4*B)*Tan[c + d*x])/(105*d*(a^4 + a^4*Sec[c + d*x]))

Rule 3794

Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 3796

Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_), x_Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && IntegerQ[2*m]

Rule 4000

Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] :> Simp[((A*b - a*B)*Cot[e +

```
f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
/; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned} \int \frac{\sec(c + dx)(A + B \sec(c + dx))}{(a + a \sec(c + dx))^4} dx &= \frac{(A - B) \tan(c + dx)}{7d(a + a \sec(c + dx))^4} + \frac{(3A + 4B) \int \frac{\sec(c + dx)}{(a + a \sec(c + dx))^3} dx}{7a} \\ &= \frac{(A - B) \tan(c + dx)}{7d(a + a \sec(c + dx))^4} + \frac{(3A + 4B) \tan(c + dx)}{35ad(a + a \sec(c + dx))^3} + \frac{(2(3A + 4B)) \int \frac{\sec(c + dx)}{(a + a \sec(c + dx))^2} dx}{35a^2} \\ &= \frac{(A - B) \tan(c + dx)}{7d(a + a \sec(c + dx))^4} + \frac{(3A + 4B) \tan(c + dx)}{35ad(a + a \sec(c + dx))^3} + \frac{2(3A + 4B) \tan(c + dx)}{105d(a^2 + a^2 \sec(c + dx))} \\ &= \frac{(A - B) \tan(c + dx)}{7d(a + a \sec(c + dx))^4} + \frac{(3A + 4B) \tan(c + dx)}{35ad(a + a \sec(c + dx))^3} + \frac{2(3A + 4B) \tan(c + dx)}{105d(a^2 + a^2 \sec(c + dx))} \end{aligned}$$

Mathematica [A] time = 0.48, size = 193, normalized size = 1.40

$$\frac{\sec\left(\frac{c}{2}\right) \cos\left(\frac{1}{2}(c + dx)\right) \left(-35(18A + 5B) \sin\left(c + \frac{dx}{2}\right) + 70(9A + 4B) \sin\left(\frac{dx}{2}\right) + 441A \sin\left(c + \frac{3dx}{2}\right) - 315A \sin\left(2c + \frac{3dx}{2}\right)\right)}{(a + a \sec(c + dx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(70*(9*A + 4*B)*Sin[(d*x)/2] - 35*(18*A + 5*B)*Sin[c + (d*x)/2] + 441*A*Sin[c + (3*d*x)/2] + 168*B*Sin[c + (3*d*x)/2] - 315*A*Sin[2*c + (3*d*x)/2] - 105*B*Sin[2*c + (3*d*x)/2] + 147*A*Sin[2*c + (5*d*x)/2] + 91*B*Sin[2*c + (5*d*x)/2] - 105*A*Sin[3*c + (5*d*x)/2] + 36*A*Sin[3*c + (7*d*x)/2] + 13*B*Sin[3*c + (7*d*x)/2]))/(420*a^4*d*(1 + Cos[c + d*x])^4)
```

fricas [A] time = 0.41, size = 124, normalized size = 0.90

$$\frac{((36A + 13B) \cos(dx + c)^3 + 13(3A + 4B) \cos(dx + c)^2 + 8(3A + 4B) \cos(dx + c) + 6A + 8B) \sin(dx + c)}{105(a^4d \cos(dx + c)^4 + 4a^4d \cos(dx + c)^3 + 6a^4d \cos(dx + c)^2 + 4a^4d \cos(dx + c) + a^4d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x, algorithm="fricas")

[Out] 1/105*((36*A + 13*B)*cos(d*x + c)^3 + 13*(3*A + 4*B)*cos(d*x + c)^2 + 8*(3*A + 4*B)*cos(d*x + c) + 6*A + 8*B)*sin(d*x + c)/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)

giac [A] time = 0.28, size = 117, normalized size = 0.85

$$\frac{15 A \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^7 - 15 B \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^7 - 63 A \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 + 21 B \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 + 105 A \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 + 35 B \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 - 105 A \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 105 B \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{840 a^4 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x, algorithm="giac")

[Out] -1/840*(15*A*tan(1/2*d*x + 1/2*c)^7 - 15*B*tan(1/2*d*x + 1/2*c)^7 - 63*A*tan(1/2*d*x + 1/2*c)^5 + 21*B*tan(1/2*d*x + 1/2*c)^5 + 105*A*tan(1/2*d*x + 1/2*c)^3 + 35*B*tan(1/2*d*x + 1/2*c)^3 - 105*A*tan(1/2*d*x + 1/2*c) - 105*B*tan(1/2*d*x + 1/2*c))/(a^4*d)

maple [A] time = 0.74, size = 90, normalized size = 0.65

$$\frac{\frac{(-A+B)\left(\tan^7\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{7} + \frac{(3A-B)\left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{5} + \frac{(-3A-B)\left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{3} + A \tan\left(\frac{dx}{2} + \frac{c}{2}\right) + B \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{8d a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x)

[Out] 1/8/d/a^4*(1/7*(-A+B)*tan(1/2*d*x+1/2*c)^7+1/5*(3*A-B)*tan(1/2*d*x+1/2*c)^5+1/3*(-3*A-B)*tan(1/2*d*x+1/2*c)^3+A*tan(1/2*d*x+1/2*c)+B*tan(1/2*d*x+1/2*c))

maxima [A] time = 0.35, size = 175, normalized size = 1.27

$$\frac{B\left(\frac{105 \sin(dx+c)}{\cos(dx+c)+1} - \frac{35 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} - \frac{21 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} + \frac{15 \sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)}{a^4} + \frac{3A\left(\frac{35 \sin(dx+c)}{\cos(dx+c)+1} - \frac{35 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{21 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{5 \sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)}{a^4}}{840 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x, algorithm="maxima")

[Out] $\frac{1}{840} * (B * (105 * \sin(dx + c) / (\cos(dx + c) + 1) - 35 * \sin(dx + c)^3 / (\cos(dx + c) + 1)^3 - 21 * \sin(dx + c)^5 / (\cos(dx + c) + 1)^5 + 15 * \sin(dx + c)^7 / (\cos(dx + c) + 1)^7) / a^4 + 3 * A * (35 * \sin(dx + c) / (\cos(dx + c) + 1) - 35 * \sin(dx + c)^3 / (\cos(dx + c) + 1)^3 + 21 * \sin(dx + c)^5 / (\cos(dx + c) + 1)^5 - 5 * \sin(dx + c)^7 / (\cos(dx + c) + 1)^7) / a^4) / d$

mupad [B] time = 1.99, size = 88, normalized size = 0.64

$$\frac{\frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 (3A+B)}{24a^4} + \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^7 (A-B)}{56a^4} - \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right) (A+B)}{8a^4} - \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5 (3A-B)}{40a^4}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B/cos(c + d*x))/(cos(c + d*x)*(a + a/cos(c + d*x))^4), x)`

[Out] $-\left(\frac{\tan(c/2 + (d*x)/2)^3 * (3*A + B)}{(24*a^4)} + \frac{\tan(c/2 + (d*x)/2)^7 * (A - B)}{(56*a^4)} - \frac{\tan(c/2 + (d*x)/2) * (A + B)}{(8*a^4)} - \frac{\tan(c/2 + (d*x)/2)^5 * (3*A - B)}{(40*a^4)}\right) / d$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A \sec(c+dx)}{\sec^4(c+dx)+4 \sec^3(c+dx)+6 \sec^2(c+dx)+4 \sec(c+dx)+1} dx + \int \frac{B \sec^2(c+dx)}{\sec^4(c+dx)+4 \sec^3(c+dx)+6 \sec^2(c+dx)+4 \sec(c+dx)+1} dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**4, x)`

[Out] $(\text{Integral}(A * \sec(c + d*x) / (\sec(c + d*x)**4 + 4 * \sec(c + d*x)**3 + 6 * \sec(c + d*x)**2 + 4 * \sec(c + d*x) + 1), x) + \text{Integral}(B * \sec(c + d*x)**2 / (\sec(c + d*x)**4 + 4 * \sec(c + d*x)**3 + 6 * \sec(c + d*x)**2 + 4 * \sec(c + d*x) + 1), x)) / a**4$

$$3.114 \quad \int \frac{A+B \sec(c+dx)}{(a+a \sec(c+dx))^4} dx$$

Optimal. Leaf size=138

$$\frac{2(80A-3B)\tan(c+dx)}{105a^4d(\sec(c+dx)+1)} - \frac{(55A-6B)\tan(c+dx)}{105a^4d(\sec(c+dx)+1)^2} + \frac{Ax}{a^4} - \frac{(10A-3B)\tan(c+dx)}{35ad(a\sec(c+dx)+a)^3} - \frac{(A-B)\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

[Out] A*x/a^4-1/105*(55*A-6*B)*tan(d*x+c)/a^4/d/(1+sec(d*x+c))^2-2/105*(80*A-3*B)*tan(d*x+c)/a^4/d/(1+sec(d*x+c))-1/7*(A-B)*tan(d*x+c)/d/(a+a*sec(d*x+c))^4-1/35*(10*A-3*B)*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^3

Rubi [A] time = 0.27, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3922, 3919, 3794}

$$\frac{2(80A-3B)\tan(c+dx)}{105a^4d(\sec(c+dx)+1)} - \frac{(55A-6B)\tan(c+dx)}{105a^4d(\sec(c+dx)+1)^2} + \frac{Ax}{a^4} - \frac{(10A-3B)\tan(c+dx)}{35ad(a\sec(c+dx)+a)^3} - \frac{(A-B)\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sec[c + d*x])/(a + a*Sec[c + d*x])^4, x]

[Out] (A*x)/a^4 - ((55*A - 6*B)*Tan[c + d*x])/(105*a^4*d*(1 + Sec[c + d*x])^2) - (2*(80*A - 3*B)*Tan[c + d*x])/(105*a^4*d*(1 + Sec[c + d*x])) - ((A - B)*Tan[c + d*x])/(7*d*(a + a*Sec[c + d*x])^4) - ((10*A - 3*B)*Tan[c + d*x])/(35*a*d*(a + a*Sec[c + d*x])^3)

Rule 3794

Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 3919

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 3922

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.)), x_Symbol] := -Simp[((b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x]

])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && EqQ
 [a^2 - b^2, 0] && IntegerQ[2*m]

Rubi steps

$$\begin{aligned} \int \frac{A + B \sec(c + dx)}{(a + a \sec(c + dx))^4} dx &= -\frac{(A - B) \tan(c + dx)}{7d(a + a \sec(c + dx))^4} - \frac{\int \frac{-7aA + 3a(A - B) \sec(c + dx)}{(a + a \sec(c + dx))^3} dx}{7a^2} \\ &= -\frac{(A - B) \tan(c + dx)}{7d(a + a \sec(c + dx))^4} - \frac{(10A - 3B) \tan(c + dx)}{35ad(a + a \sec(c + dx))^3} + \frac{\int \frac{35a^2A - 2a^2(10A - 3B) \sec(c + dx)}{(a + a \sec(c + dx))^2} dx}{35a^4} \\ &= -\frac{(55A - 6B) \tan(c + dx)}{105a^4d(1 + \sec(c + dx))^2} - \frac{(A - B) \tan(c + dx)}{7d(a + a \sec(c + dx))^4} - \frac{(10A - 3B) \tan(c + dx)}{35ad(a + a \sec(c + dx))^3} - \int \frac{35a^2A - 2a^2(10A - 3B) \sec(c + dx)}{(a + a \sec(c + dx))^2} dx \\ &= \frac{Ax}{a^4} - \frac{(55A - 6B) \tan(c + dx)}{105a^4d(1 + \sec(c + dx))^2} - \frac{(A - B) \tan(c + dx)}{7d(a + a \sec(c + dx))^4} - \frac{(10A - 3B) \tan(c + dx)}{35ad(a + a \sec(c + dx))^3} \\ &= \frac{Ax}{a^4} - \frac{(55A - 6B) \tan(c + dx)}{105a^4d(1 + \sec(c + dx))^2} - \frac{(A - B) \tan(c + dx)}{7d(a + a \sec(c + dx))^4} - \frac{(10A - 3B) \tan(c + dx)}{35ad(a + a \sec(c + dx))^3} \end{aligned}$$

Mathematica [B] time = 0.79, size = 329, normalized size = 2.38

$$\sec\left(\frac{c}{2}\right) \sec^7\left(\frac{1}{2}(c + dx)\right) \left(8260A \sin\left(c + \frac{dx}{2}\right) - 7140A \sin\left(c + \frac{3dx}{2}\right) + 3780A \sin\left(2c + \frac{3dx}{2}\right) - 2800A \sin\left(2c + \frac{5dx}{2}\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sec[c + d*x])/(a + a*Sec[c + d*x])^4,x]

[Out] (Sec[c/2]*Sec[(c + d*x)/2]^7*(3675*A*d*x*Cos[(d*x)/2] + 3675*A*d*x*Cos[c + (d*x)/2] + 2205*A*d*x*Cos[c + (3*d*x)/2] + 2205*A*d*x*Cos[2*c + (3*d*x)/2] + 735*A*d*x*Cos[2*c + (5*d*x)/2] + 735*A*d*x*Cos[3*c + (5*d*x)/2] + 105*A*d*x*Cos[3*c + (7*d*x)/2] + 105*A*d*x*Cos[4*c + (7*d*x)/2] - 9940*A*Sin[(d*x)/2] + 1260*B*Sin[(d*x)/2] + 8260*A*Sin[c + (d*x)/2] - 1260*B*Sin[c + (d*x)/2] - 7140*A*Sin[c + (3*d*x)/2] + 882*B*Sin[c + (3*d*x)/2] + 3780*A*Sin[2*c + (3*d*x)/2] - 630*B*Sin[2*c + (3*d*x)/2] - 2800*A*Sin[2*c + (5*d*x)/2] + 294*B*Sin[2*c + (5*d*x)/2] + 840*A*Sin[3*c + (5*d*x)/2] - 210*B*Sin[3*c + (5*d*x)/2] - 520*A*Sin[3*c + (7*d*x)/2] + 72*B*Sin[3*c + (7*d*x)/2]))/(13440*a^4*d)

fricas [A] time = 0.44, size = 181, normalized size = 1.31

$$\frac{105 A d x \cos (d x + c)^4 + 420 A d x \cos (d x + c)^3 + 630 A d x \cos (d x + c)^2 + 420 A d x \cos (d x + c) + 105 A d x - (420 A d x \cos (d x + c)^3 + 630 A d x \cos (d x + c)^2 + 420 A d x \cos (d x + c) + 105 A d x)}{105 \left(a^4 d \cos (d x + c)^4 + 4 a^4 d \cos (d x + c)^3 + 6 a^4 d \cos (d x + c)^2 + 4 a^4 d \cos (d x + c) + a^4 d \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x, algorithm="fricas")

[Out] 1/105*(105*A*d*x*cos(d*x + c)^4 + 420*A*d*x*cos(d*x + c)^3 + 630*A*d*x*cos(d*x + c)^2 + 420*A*d*x*cos(d*x + c) + 105*A*d*x - (4*(65*A - 9*B)*cos(d*x + c)^3 + (620*A - 39*B)*cos(d*x + c)^2 + (535*A - 24*B)*cos(d*x + c) + 160*A - 6*B)*sin(d*x + c))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)

giac [A] time = 0.29, size = 154, normalized size = 1.12

$$\frac{\frac{840(dx+c)A}{a^4} + \frac{15 A a^{24} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^7 - 15 B a^{24} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^7 - 105 A a^{24} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 + 63 B a^{24} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 + 385 A a^{24} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 - 105 B a^{24} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 - 1575 A a^{24} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 105 B a^{24} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{a^{28}}}{840 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x, algorithm="giac")

[Out] 1/840*(840*(d*x + c)*A/a^4 + (15*A*a^24*tan(1/2*d*x + 1/2*c)^7 - 15*B*a^24*tan(1/2*d*x + 1/2*c)^7 - 105*A*a^24*tan(1/2*d*x + 1/2*c)^5 + 63*B*a^24*tan(1/2*d*x + 1/2*c)^5 + 385*A*a^24*tan(1/2*d*x + 1/2*c)^3 - 105*B*a^24*tan(1/2*d*x + 1/2*c)^3 - 1575*A*a^24*tan(1/2*d*x + 1/2*c) + 105*B*a^24*tan(1/2*d*x + 1/2*c))/a^28)/d

maple [A] time = 0.80, size = 177, normalized size = 1.28

$$\frac{\left(\tan^7\left(\frac{dx}{2} + \frac{c}{2}\right)\right) A}{56d a^4} - \frac{B \left(\tan^7\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{56d a^4} - \frac{A \left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{8d a^4} + \frac{3B \left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{40d a^4} + \frac{11 \left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right) A}{24d a^4} - \frac{B \left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{24d a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x)

[Out] 1/56/d/a^4*tan(1/2*d*x+1/2*c)^7*A-1/56/d/a^4*B*tan(1/2*d*x+1/2*c)^7-1/8/d/a^4*A*tan(1/2*d*x+1/2*c)^5+3/40/d/a^4*B*tan(1/2*d*x+1/2*c)^5+11/24/d/a^4*tan(1/2*d*x+1/2*c)^3*A-1/8/d/a^4*B*tan(1/2*d*x+1/2*c)^3-15/8/d/a^4*A*tan(1/2*d*x+1/2*c)+1/8/d/a^4*B*tan(1/2*d*x+1/2*c)+2/d/a^4*A*arctan(tan(1/2*d*x+1/2*c))

maxima [A] time = 0.72, size = 201, normalized size = 1.46

$$\frac{5A \left(\frac{\frac{315 \sin(dx+c)}{\cos(dx+c)+1} - \frac{77 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{21 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{3 \sin(dx+c)^7}{(\cos(dx+c)+1)^7} - \frac{336 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4} \right) - 3B \left(\frac{35 \sin(dx+c)}{\cos(dx+c)+1} - \frac{35 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{21 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{5 \sin(dx+c)^7}{(\cos(dx+c)+1)^7} \right)}{840d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x, algorithm="maxima")

[Out] -1/840*(5*A*((315*sin(d*x + c)/(cos(d*x + c) + 1) - 77*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 336*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^4) - 3*B*(35*sin(d*x + c)/(cos(d*x + c) + 1) - 35*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 5*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4)/d

mupad [B] time = 2.03, size = 163, normalized size = 1.18

$$\frac{Ax \left(\frac{52A \sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{21} - \frac{12B \sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{35} \right) \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^6 + \left(\frac{23B \sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{70} - \frac{16A \sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{21} \right) \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^4 + \left(\frac{5A \sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{28} \right) \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^2}{a^4} - \frac{a^4 d \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^7}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/(a + a/cos(c + d*x))^4,x)

[Out] (A*x)/a^4 - ((B*sin(c/2 + (d*x)/2))/56 - (A*sin(c/2 + (d*x)/2))/56 + cos(c/2 + (d*x)/2)^2*((5*A*sin(c/2 + (d*x)/2))/28 - (9*B*sin(c/2 + (d*x)/2))/70) + cos(c/2 + (d*x)/2)^6*((52*A*sin(c/2 + (d*x)/2))/21 - (12*B*sin(c/2 + (d*x)/2))/35) - cos(c/2 + (d*x)/2)^4*((16*A*sin(c/2 + (d*x)/2))/21 - (23*B*sin(c/2 + (d*x)/2))/70))/(a^4*d*cos(c/2 + (d*x)/2)^7)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A}{\sec^4(c+dx)+4\sec^3(c+dx)+6\sec^2(c+dx)+4\sec(c+dx)+1} dx + \int \frac{B \sec(c+dx)}{\sec^4(c+dx)+4\sec^3(c+dx)+6\sec^2(c+dx)+4\sec(c+dx)+1} dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))**4,x)

[Out] (Integral(A/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c + d*x)**2 + 4*sec(c + d*x) + 1), x) + Integral(B*sec(c + d*x)/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c + d*x)**2 + 4*sec(c + d*x) + 1), x))/a**4

$$3.115 \quad \int \frac{\cos(c+dx)(A+B \sec(c+dx))}{(a+a \sec(c+dx))^4} dx$$

Optimal. Leaf size=166

$$\frac{8(83A - 20B) \sin(c + dx)}{105a^4d} - \frac{(4A - B) \sin(c + dx)}{a^4d(\sec(c + dx) + 1)} - \frac{(88A - 25B) \sin(c + dx)}{105a^4d(\sec(c + dx) + 1)^2} - \frac{x(4A - B)}{a^4} - \frac{(12A - 5B) \sin(c + dx)}{35ad(a \sec(c + dx) + 1)}$$

[Out] $-(4*A-B)*x/a^4+8/105*(83*A-20*B)*\sin(d*x+c)/a^4/d-1/105*(88*A-25*B)*\sin(d*x+c)/a^4/d/(1+\sec(d*x+c))^2-(4*A-B)*\sin(d*x+c)/a^4/d/(1+\sec(d*x+c))-1/7*(A-B)*\sin(d*x+c)/d/(a+a*\sec(d*x+c))^4-1/35*(12*A-5*B)*\sin(d*x+c)/a/d/(a+a*\sec(d*x+c))^3$

Rubi [A] time = 0.57, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {4020, 3787, 2637, 8}

$$\frac{8(83A - 20B) \sin(c + dx)}{105a^4d} - \frac{(4A - B) \sin(c + dx)}{a^4d(\sec(c + dx) + 1)} - \frac{(88A - 25B) \sin(c + dx)}{105a^4d(\sec(c + dx) + 1)^2} - \frac{x(4A - B)}{a^4} - \frac{(12A - 5B) \sin(c + dx)}{35ad(a \sec(c + dx) + 1)}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^4,x]

[Out] $-(((4*A - B)*x)/a^4) + (8*(83*A - 20*B)*\text{Sin}[c + d*x])/(105*a^4*d) - ((88*A - 25*B)*\text{Sin}[c + d*x])/(105*a^4*d*(1 + \text{Sec}[c + d*x])^2) - ((4*A - B)*\text{Sin}[c + d*x])/(a^4*d*(1 + \text{Sec}[c + d*x])) - ((A - B)*\text{Sin}[c + d*x])/(7*d*(a + a*\text{Sec}[c + d*x])^4) - ((12*A - 5*B)*\text{Sin}[c + d*x])/(35*a*d*(a + a*\text{Sec}[c + d*x])^3)$

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_)])*(d_.)^(n_.)*(csc[(e_.) + (f_.)*(x_)])*(b_.) + (a_.), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m + 1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{\cos(c + dx)(A + B \sec(c + dx))}{(a + a \sec(c + dx))^4} dx &= -\frac{(A - B) \sin(c + dx)}{7d(a + a \sec(c + dx))^4} + \frac{\int \frac{\cos(c + dx)(a(8A - B) - 4a(A - B) \sec(c + dx))}{(a + a \sec(c + dx))^3} dx}{7a^2} \\ &= -\frac{(A - B) \sin(c + dx)}{7d(a + a \sec(c + dx))^4} - \frac{(12A - 5B) \sin(c + dx)}{35ad(a + a \sec(c + dx))^3} + \frac{\int \frac{\cos(c + dx)(2a^2(26A - 5B) - 4a(A - B) \sec(c + dx))}{(a + a \sec(c + dx))^2} dx}{7a^2} \\ &= -\frac{(88A - 25B) \sin(c + dx)}{105a^4d(1 + \sec(c + dx))^2} - \frac{(A - B) \sin(c + dx)}{7d(a + a \sec(c + dx))^4} - \frac{(12A - 5B) \sin(c + dx)}{35ad(a + a \sec(c + dx))^3} \\ &= -\frac{(88A - 25B) \sin(c + dx)}{105a^4d(1 + \sec(c + dx))^2} - \frac{(A - B) \sin(c + dx)}{7d(a + a \sec(c + dx))^4} - \frac{(12A - 5B) \sin(c + dx)}{35ad(a + a \sec(c + dx))^3} \\ &= -\frac{(88A - 25B) \sin(c + dx)}{105a^4d(1 + \sec(c + dx))^2} - \frac{(A - B) \sin(c + dx)}{7d(a + a \sec(c + dx))^4} - \frac{(12A - 5B) \sin(c + dx)}{35ad(a + a \sec(c + dx))^3} \\ &= -\frac{(4A - B)x}{a^4} + \frac{8(83A - 20B) \sin(c + dx)}{105a^4d} - \frac{(88A - 25B) \sin(c + dx)}{105a^4d(1 + \sec(c + dx))^2} - \frac{(12A - 5B) \sin(c + dx)}{35ad(a + a \sec(c + dx))^3} \end{aligned}$$

Mathematica [B] time = 1.11, size = 485, normalized size = 2.92

$$\frac{\sec\left(\frac{c}{2}\right) \cos\left(\frac{1}{2}(c + dx)\right) \left(-7350dx(4A - B) \cos\left(c + \frac{dx}{2}\right) - 7350dx(4A - B) \cos\left(\frac{dx}{2}\right) - 46130A \sin\left(c + \frac{dx}{2}\right) + 46130A \sin\left(\frac{dx}{2}\right)\right)}{7350dx(4A - B) \cos\left(c + \frac{dx}{2}\right) + 7350dx(4A - B) \cos\left(\frac{dx}{2}\right) + 46130A \sin\left(c + \frac{dx}{2}\right) - 46130A \sin\left(\frac{dx}{2}\right)}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^4,x]

[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(-7350*(4*A - B)*d*x*Cos[(d*x)/2] - 7350*(4*A - B)*d*x*Cos[c + (d*x)/2] - 17640*A*d*x*Cos[c + (3*d*x)/2] + 4410*B*d*x*Cos[c + (3*d*x)/2] - 17640*A*d*x*Cos[2*c + (3*d*x)/2] + 4410*B*d*x*Cos[2*c + (3*d*x)/2] - 5880*A*d*x*Cos[2*c + (5*d*x)/2] + 1470*B*d*x*Cos[2*c + (5*d*x)/2])

$$\begin{aligned}
& - 5880*A*d*x*\text{Cos}[3*c + (5*d*x)/2] + 1470*B*d*x*\text{Cos}[3*c + (5*d*x)/2] - 840* \\
& A*d*x*\text{Cos}[3*c + (7*d*x)/2] + 210*B*d*x*\text{Cos}[3*c + (7*d*x)/2] - 840*A*d*x*\text{Cos} \\
& [4*c + (7*d*x)/2] + 210*B*d*x*\text{Cos}[4*c + (7*d*x)/2] + 60830*A*\text{Sin}[(d*x)/2] - \\
& 19880*B*\text{Sin}[(d*x)/2] - 46130*A*\text{Sin}[c + (d*x)/2] + 16520*B*\text{Sin}[c + (d*x)/2] \\
& + 46116*A*\text{Sin}[c + (3*d*x)/2] - 14280*B*\text{Sin}[c + (3*d*x)/2] - 18060*A*\text{Sin}[2* \\
& c + (3*d*x)/2] + 7560*B*\text{Sin}[2*c + (3*d*x)/2] + 19292*A*\text{Sin}[2*c + (5*d*x)/2] \\
& - 5600*B*\text{Sin}[2*c + (5*d*x)/2] - 2100*A*\text{Sin}[3*c + (5*d*x)/2] + 1680*B*\text{Sin}[3 \\
& *c + (5*d*x)/2] + 3791*A*\text{Sin}[3*c + (7*d*x)/2] - 1040*B*\text{Sin}[3*c + (7*d*x)/2] \\
& + 735*A*\text{Sin}[4*c + (7*d*x)/2] + 105*A*\text{Sin}[4*c + (9*d*x)/2] + 105*A*\text{Sin}[5*c \\
& + (9*d*x)/2]) / (1680*a^4*d*(1 + \text{Cos}[c + d*x])^4)
\end{aligned}$$

fricas [A] time = 0.45, size = 223, normalized size = 1.34

$$\frac{105(4A - B)dx \cos(dx + c)^4 + 420(4A - B)dx \cos(dx + c)^3 + 630(4A - B)dx \cos(dx + c)^2 + 420(4A - B)dx \cos(dx + c) + 105(4A - B)dx - (105A \cos(dx + c)^4 + 4*(296A - 65B) \cos(dx + c)^3 + 4*(659A - 155B) \cos(dx + c)^2 + (2236A - 535B) \cos(dx + c) + 664A - 160B) \sin(dx + c)}{a^4 d (\tan(\frac{1}{2} dx + \frac{1}{2} c)^2 + 1) a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x, algorithm="fricas")

[Out]
$$\begin{aligned}
& -1/105*(105*(4*A - B)*d*x*\text{cos}(d*x + c)^4 + 420*(4*A - B)*d*x*\text{cos}(d*x + c)^3 \\
& + 630*(4*A - B)*d*x*\text{cos}(d*x + c)^2 + 420*(4*A - B)*d*x*\text{cos}(d*x + c) + 105* \\
& (4*A - B)*d*x - (105*A*\text{cos}(d*x + c)^4 + 4*(296*A - 65*B)*\text{cos}(d*x + c)^3 + 4 \\
& *(659*A - 155*B)*\text{cos}(d*x + c)^2 + (2236*A - 535*B)*\text{cos}(d*x + c) + 664*A - 1 \\
& 60*B)*\text{sin}(d*x + c)) / (a^4*d*\text{cos}(d*x + c)^4 + 4*a^4*d*\text{cos}(d*x + c)^3 + 6*a^4* \\
& d*\text{cos}(d*x + c)^2 + 4*a^4*d*\text{cos}(d*x + c) + a^4*d)
\end{aligned}$$

giac [A] time = 0.70, size = 190, normalized size = 1.14

$$\frac{840(dx+c)(4A-B)}{a^4} - \frac{1680A \tan(\frac{1}{2} dx + \frac{1}{2} c)}{(\tan(\frac{1}{2} dx + \frac{1}{2} c)^2 + 1) a^4} + \frac{15Aa^{24} \tan(\frac{1}{2} dx + \frac{1}{2} c)^7 - 15Ba^{24} \tan(\frac{1}{2} dx + \frac{1}{2} c)^7 - 147Aa^{24} \tan(\frac{1}{2} dx + \frac{1}{2} c)^5 + 105Ba^{24} \tan(\frac{1}{2} dx + \frac{1}{2} c)^3 - 5145Aa^{24} \tan(\frac{1}{2} dx + \frac{1}{2} c) + 1575Ba^{24} \tan(\frac{1}{2} dx + \frac{1}{2} c)}}{a^{28} d}$$

840 d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x, algorithm="giac")

[Out]
$$\begin{aligned}
& -1/840*(840*(d*x + c)*(4*A - B)/a^4 - 1680*A*\text{tan}(1/2*d*x + 1/2*c) / ((\text{tan}(1/2 \\
& *d*x + 1/2*c)^2 + 1)*a^4) + (15*A*a^{24}*\text{tan}(1/2*d*x + 1/2*c)^7 - 15*B*a^{24}* \\
& \text{an}(1/2*d*x + 1/2*c)^7 - 147*A*a^{24}*\text{tan}(1/2*d*x + 1/2*c)^5 + 105*B*a^{24}* \\
& \text{tan}(1/2*d*x + 1/2*c)^5 + 805*A*a^{24}*\text{tan}(1/2*d*x + 1/2*c)^3 - 385*B*a^{24}* \\
& \text{tan}(1/2*d*x + 1/2*c)^3 - 5145*A*a^{24}*\text{tan}(1/2*d*x + 1/2*c) + 1575*B*a^{24}* \\
& \text{tan}(1/2*d*x + 1/2*c)) / a^{28} / d
\end{aligned}$$

maple [A] time = 1.23, size = 229, normalized size = 1.38

$$\frac{\left(\tan^7\left(\frac{dx}{2} + \frac{c}{2}\right)\right) A}{56d a^4} + \frac{B\left(\tan^7\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{56d a^4} + \frac{7A\left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{40d a^4} + \frac{B\left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{8d a^4} + \frac{23\left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right) A}{24d a^4} + \frac{11B\left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{24d a^4} + \frac{7A\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{4d a^4} + \frac{B\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{4d a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x)`

[Out]
$$-1/56/d/a^4*\tan(1/2*d*x+1/2*c)^7*A+1/56/d/a^4*B*\tan(1/2*d*x+1/2*c)^7+7/40/d/a^4*A*\tan(1/2*d*x+1/2*c)^5-1/8/d/a^4*B*\tan(1/2*d*x+1/2*c)^5-23/24/d/a^4*\tan(1/2*d*x+1/2*c)^3*A+11/24/d/a^4*B*\tan(1/2*d*x+1/2*c)^3+49/8/d/a^4*A*\tan(1/2*d*x+1/2*c)-15/8/d/a^4*B*\tan(1/2*d*x+1/2*c)+2/d/a^4*A*\tan(1/2*d*x+1/2*c)/(1+\tan(1/2*d*x+1/2*c)^2)-8/d/a^4*A*\arctan(\tan(1/2*d*x+1/2*c))+2/d/a^4*\arctan(\tan(1/2*d*x+1/2*c))*B$$

maxima [A] time = 0.71, size = 271, normalized size = 1.63

$$A \left(\frac{1680 \sin(dx+c)}{\left(a^4 + \frac{a^4 \sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right) (\cos(dx+c)+1)} + \frac{\frac{5145 \sin(dx+c)}{\cos(dx+c)+1} - \frac{805 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{147 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{15 \sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4} - \frac{6720 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4} \right) - 5 B \left(\frac{315 \sin(dx+c)}{\cos(dx+c)+1} - \frac{77 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{21 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{3 \sin(dx+c)^7}{(\cos(dx+c)+1)^7} - \frac{336 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4} \right) / 840 d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x, algorithm="maxima")`

[Out]
$$1/840*(A*(1680*\sin(d*x + c)/((a^4 + a^4*\sin(d*x + c)^2/(\cos(d*x + c) + 1)^2))*(\cos(d*x + c) + 1)) + (5145*\sin(d*x + c)/(\cos(d*x + c) + 1) - 805*\sin(d*x + c)^3/(\cos(d*x + c) + 1)^3 + 147*\sin(d*x + c)^5/(\cos(d*x + c) + 1)^5 - 15*\sin(d*x + c)^7/(\cos(d*x + c) + 1)^7)/a^4 - 6720*\arctan(\sin(d*x + c)/(\cos(d*x + c) + 1))/a^4 - 5*B*((315*\sin(d*x + c)/(\cos(d*x + c) + 1) - 77*\sin(d*x + c)^3/(\cos(d*x + c) + 1)^3 + 21*\sin(d*x + c)^5/(\cos(d*x + c) + 1)^5 - 3*\sin(d*x + c)^7/(\cos(d*x + c) + 1)^7)/a^4 - 336*\arctan(\sin(d*x + c)/(\cos(d*x + c) + 1))/a^4)/d$$

mupad [B] time = 2.06, size = 202, normalized size = 1.22

$$\frac{\left(\frac{764 A \sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{105} - \frac{52 B \sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{21}\right) \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^6 + \left(\frac{16 B \sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{21} - \frac{143 A \sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{105}\right) \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^4 + \left(\frac{8 A \sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{35}\right) \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^2}{a^4 d \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((cos(c + d*x)*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^4,x)`

[Out] $((B*\sin(c/2 + (d*x)/2))/56 - (A*\sin(c/2 + (d*x)/2))/56 + \cos(c/2 + (d*x)/2)^2*((8*A*\sin(c/2 + (d*x)/2))/35 - (5*B*\sin(c/2 + (d*x)/2))/28) - \cos(c/2 + (d*x)/2)^4*((143*A*\sin(c/2 + (d*x)/2))/105 - (16*B*\sin(c/2 + (d*x)/2))/21) + \cos(c/2 + (d*x)/2)^6*((764*A*\sin(c/2 + (d*x)/2))/105 - (52*B*\sin(c/2 + (d*x)/2))/21))/(a^4*d*\cos(c/2 + (d*x)/2)^7) - (4*A*d*x - B*d*x)/(a^4*d) + (2*A*\cos(c/2 + (d*x)/2)*\sin(c/2 + (d*x)/2))/(a^4*d)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A \cos(c+dx)}{\sec^4(c+dx)+4\sec^3(c+dx)+6\sec^2(c+dx)+4\sec(c+dx)+1} dx + \int \frac{B \cos(c+dx) \sec(c+dx)}{\sec^4(c+dx)+4\sec^3(c+dx)+6\sec^2(c+dx)+4\sec(c+dx)+1} dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**4,x)`

[Out] $(\text{Integral}(A*\cos(c + d*x)/(\sec(c + d*x)**4 + 4*\sec(c + d*x)**3 + 6*\sec(c + d*x)**2 + 4*\sec(c + d*x) + 1), x) + \text{Integral}(B*\cos(c + d*x)*\sec(c + d*x)/(\sec(c + d*x)**4 + 4*\sec(c + d*x)**3 + 6*\sec(c + d*x)**2 + 4*\sec(c + d*x) + 1), x))/a**4$

$$3.116 \quad \int \frac{\cos^2(c+dx)(A+B \sec(c+dx))}{(a+a \sec(c+dx))^4} dx$$

Optimal. Leaf size=223

$$-\frac{8(216A - 83B) \sin(c + dx)}{105a^4d} + \frac{(21A - 8B) \sin(c + dx) \cos(c + dx)}{2a^4d} - \frac{4(216A - 83B) \sin(c + dx) \cos(c + dx)}{105a^4d(\sec(c + dx) + 1)} - \frac{(129A - 52B) \cos(c + dx) \sin(c + dx)}{105a^4d(1 + \sec(c + dx))^2}$$

[Out] 1/2*(21*A-8*B)*x/a^4-8/105*(216*A-83*B)*sin(d*x+c)/a^4/d+1/2*(21*A-8*B)*cos(d*x+c)*sin(d*x+c)/a^4/d-1/105*(129*A-52*B)*cos(d*x+c)*sin(d*x+c)/a^4/d/(1+sec(d*x+c))^2-4/105*(216*A-83*B)*cos(d*x+c)*sin(d*x+c)/a^4/d/(1+sec(d*x+c))-1/7*(A-B)*cos(d*x+c)*sin(d*x+c)/d/(a+a*sec(d*x+c))^4-1/5*(2*A-B)*cos(d*x+c)*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^3

Rubi [A] time = 0.65, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {4020, 3787, 2635, 8, 2637}

$$-\frac{8(216A - 83B) \sin(c + dx)}{105a^4d} + \frac{(21A - 8B) \sin(c + dx) \cos(c + dx)}{2a^4d} - \frac{4(216A - 83B) \sin(c + dx) \cos(c + dx)}{105a^4d(\sec(c + dx) + 1)} - \frac{(129A - 52B) \cos(c + dx) \sin(c + dx)}{105a^4d(1 + \sec(c + dx))^2}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^4,x]

[Out] ((21*A - 8*B)*x)/(2*a^4) - (8*(216*A - 83*B)*Sin[c + d*x])/(105*a^4*d) + ((21*A - 8*B)*Cos[c + d*x]*Sin[c + d*x])/(2*a^4*d) - ((129*A - 52*B)*Cos[c + d*x]*Sin[c + d*x])/(105*a^4*d*(1 + Sec[c + d*x])^2) - (4*(216*A - 83*B)*Cos[c + d*x]*Sin[c + d*x])/(105*a^4*d*(1 + Sec[c + d*x])) - ((A - B)*Cos[c + d*x]*Sin[c + d*x])/(7*d*(a + a*Sec[c + d*x])^4) - ((2*A - B)*Cos[c + d*x]*Sin[c + d*x])/(5*a*d*(a + a*Sec[c + d*x])^3)

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x])*(b*Ssin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Ssin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 4020

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] := -Simp[((A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m + 1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{\cos^2(c + dx)(A + B \sec(c + dx))}{(a + a \sec(c + dx))^4} dx &= -\frac{(A - B) \cos(c + dx) \sin(c + dx)}{7d(a + a \sec(c + dx))^4} + \frac{\int \frac{\cos^2(c + dx)(a(9A - 2B) - 5a(A - B) \sec(c + dx))}{(a + a \sec(c + dx))^3} dx}{7a^2} \\
 &= -\frac{(A - B) \cos(c + dx) \sin(c + dx)}{7d(a + a \sec(c + dx))^4} - \frac{(2A - B) \cos(c + dx) \sin(c + dx)}{5ad(a + a \sec(c + dx))^3} \\
 &= -\frac{(129A - 52B) \cos(c + dx) \sin(c + dx)}{105a^4d(1 + \sec(c + dx))^2} - \frac{(A - B) \cos(c + dx) \sin(c + dx)}{7d(a + a \sec(c + dx))^4} \\
 &= -\frac{(129A - 52B) \cos(c + dx) \sin(c + dx)}{105a^4d(1 + \sec(c + dx))^2} - \frac{(A - B) \cos(c + dx) \sin(c + dx)}{7d(a + a \sec(c + dx))^4} \\
 &= -\frac{(129A - 52B) \cos(c + dx) \sin(c + dx)}{105a^4d(1 + \sec(c + dx))^2} - \frac{(A - B) \cos(c + dx) \sin(c + dx)}{7d(a + a \sec(c + dx))^4} \\
 &= -\frac{8(216A - 83B) \sin(c + dx)}{105a^4d} + \frac{(21A - 8B) \cos(c + dx) \sin(c + dx)}{2a^4d} \\
 &= \frac{(21A - 8B)x}{2a^4} - \frac{8(216A - 83B) \sin(c + dx)}{105a^4d} + \frac{(21A - 8B) \cos(c + dx) \sin(c + dx)}{2a^4d}
 \end{aligned}$$

Mathematica [B] time = 1.20, size = 555, normalized size = 2.49

$$\sec\left(\frac{c}{2}\right) \cos\left(\frac{1}{2}(c + dx)\right) \left(14700dx(21A - 8B) \cos\left(c + \frac{dx}{2}\right) + 14700dx(21A - 8B) \cos\left(\frac{dx}{2}\right) + 386190A \sin\left(c + \frac{dx}{2}\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^4,x]

[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(14700*(21*A - 8*B)*d*x*Cos[(d*x)/2] + 14700*(21*A - 8*B)*d*x*Cos[c + (d*x)/2] + 185220*A*d*x*Cos[c + (3*d*x)/2] - 70560*B*d*x*Cos[c + (3*d*x)/2] + 185220*A*d*x*Cos[2*c + (3*d*x)/2] - 70560*B*d*x*Cos[2*c + (3*d*x)/2] + 61740*A*d*x*Cos[2*c + (5*d*x)/2] - 23520*B*d*x*Cos[2*c + (5*d*x)/2] + 61740*A*d*x*Cos[3*c + (5*d*x)/2] - 23520*B*d*x*Cos[3*c + (5*d*x)/2] + 8820*A*d*x*Cos[3*c + (7*d*x)/2] - 3360*B*d*x*Cos[3*c + (7*d*x)/2] + 8820*A*d*x*Cos[4*c + (7*d*x)/2] - 3360*B*d*x*Cos[4*c + (7*d*x)/2] - 539490*A*Sin[(d*x)/2] + 243320*B*Sin[(d*x)/2] + 386190*A*Sin[c + (d*x)/2] - 184520*B*Sin[c + (d*x)/2] - 422478*A*Sin[c + (3*d*x)/2] + 184464*B*Sin[c + (3*d*x)/2] + 132930*A*Sin[2*c + (3*d*x)/2] - 72240*B*Sin[2*c + (3*d*x)/2] - 181461*A*Sin[2*c + (5*d*x)/2] + 77168*B*Sin[2*c + (5*d*x)/2] + 3675*A*Sin[3*c + (5*d*x)/2] - 8400*B*Sin[3*c + (5*d*x)/2] - 36003*A*Sin[3*c + (7*d*x)/2] + 15164*B*Sin[3*c + (7*d*x)/2] - 9555*A*Sin[4*c + (7*d*x)/2] + 2940*B*Sin[4*c + (7*d*x)/2] - 945*A*Sin[4*c + (9*d*x)/2] + 420*B*Sin[4*c + (9*d*x)/2] - 945*A*Sin[5*c + (9*d*x)/2] + 420*B*Sin[5*c + (9*d*x)/2] + 105*A*Sin[5*c + (11*d*x)/2] + 105*A*Sin[6*c + (11*d*x)/2]))/(6720*a^4*d*(1 + Cos[c + d*x])^4)

fricas [A] time = 0.45, size = 240, normalized size = 1.08

$$105(21A - 8B)dx \cos(dx + c)^4 + 420(21A - 8B)dx \cos(dx + c)^3 + 630(21A - 8B)dx \cos(dx + c)^2 + 420(21A - 8B)dx \cos(dx + c) + 105(21A - 8B)d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x, algorithm="fricas")

[Out] 1/210*(105*(21*A - 8*B)*d*x*cos(d*x + c)^4 + 420*(21*A - 8*B)*d*x*cos(d*x + c)^3 + 630*(21*A - 8*B)*d*x*cos(d*x + c)^2 + 420*(21*A - 8*B)*d*x*cos(d*x + c) + 105*(21*A - 8*B)*d*x + (105*A*cos(d*x + c)^5 - 210*(2*A - B)*cos(d*x + c)^4 - 4*(1509*A - 592*B)*cos(d*x + c)^3 - 4*(3411*A - 1318*B)*cos(d*x + c)^2 - (11619*A - 4472*B)*cos(d*x + c) - 3456*A + 1328*B)*sin(d*x + c))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)

giac [A] time = 0.28, size = 233, normalized size = 1.04

$$\frac{420(dx+c)(21A-8B)}{a^4} - \frac{840\left(9A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - 2B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 + 7A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) - 2B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 + 1\right)^2 a^4} + \frac{15Aa^{24}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7 - 15Ba^{24}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7}{a^{28}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x, algorithm="giac")

[Out] 1/840*(420*(d*x + c)*(21*A - 8*B)/a^4 - 840*(9*A*tan(1/2*d*x + 1/2*c)^3 - 2*B*tan(1/2*d*x + 1/2*c)^3 + 7*A*tan(1/2*d*x + 1/2*c) - 2*B*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^4) + (15*A*a^24*tan(1/2*d*x + 1/2*c)^7 - 15*B*a^24*tan(1/2*d*x + 1/2*c)^7 - 189*A*a^24*tan(1/2*d*x + 1/2*c)^5 + 147*B*a^24*tan(1/2*d*x + 1/2*c)^5 + 1365*A*a^24*tan(1/2*d*x + 1/2*c)^3 - 805*B*a^24*tan(1/2*d*x + 1/2*c)^3 - 11655*A*a^24*tan(1/2*d*x + 1/2*c) + 5145*B*a^24*tan(1/2*d*x + 1/2*c))/a^28)/d

maple [A] time = 1.16, size = 332, normalized size = 1.49

$$\frac{\left(\tan^7\left(\frac{dx}{2} + \frac{c}{2}\right)\right)A}{56d a^4} - \frac{B\left(\tan^7\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{56d a^4} - \frac{9A\left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{40d a^4} + \frac{7B\left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{40d a^4} + \frac{13\left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)A}{8d a^4} - \frac{23B\left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{8d a^4} - \frac{13A\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{8d a^4} + \frac{23B\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{8d a^4} - \frac{13A}{8d a^4} + \frac{23B}{8d a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x)

[Out] 1/56/d/a^4*tan(1/2*d*x+1/2*c)^7*A-1/56/d/a^4*B*tan(1/2*d*x+1/2*c)^7-9/40/d/a^4*A*tan(1/2*d*x+1/2*c)^5+7/40/d/a^4*B*tan(1/2*d*x+1/2*c)^5+13/8/d/a^4*tan(1/2*d*x+1/2*c)^3*A-23/24/d/a^4*B*tan(1/2*d*x+1/2*c)^3-111/8/d/a^4*A*tan(1/2*d*x+1/2*c)+49/8/d/a^4*B*tan(1/2*d*x+1/2*c)-9/d/a^4/(1+tan(1/2*d*x+1/2*c))^2)^2*A*tan(1/2*d*x+1/2*c)^3+2/d/a^4/(1+tan(1/2*d*x+1/2*c))^2)^2*B*tan(1/2*d*x+1/2*c)^3-7/d/a^4/(1+tan(1/2*d*x+1/2*c))^2)^2*A*tan(1/2*d*x+1/2*c)+2/d/a^4/(1+tan(1/2*d*x+1/2*c))^2)^2*B*tan(1/2*d*x+1/2*c)+21/d/a^4*A*arctan(tan(1/2*d*x+1/2*c))-8/d/a^4*arctan(tan(1/2*d*x+1/2*c))*B

maxima [A] time = 0.71, size = 364, normalized size = 1.63

$$3A\left(\frac{280\left(\frac{7\sin(dx+c)}{\cos(dx+c)+1} + \frac{9\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^4 + \frac{2a^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{a^4\sin(dx+c)^4}{(\cos(dx+c)+1)^4}} + \frac{\frac{3885\sin(dx+c)}{\cos(dx+c)+1} - \frac{455\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{63\sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{5\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4} - \frac{5880\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4}\right) - B\left(\frac{13\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right)\right)A}{8d a^4} - \frac{23B\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{8d a^4} - \frac{13A}{8d a^4} + \frac{23B}{8d a^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x, algorithm="maxima")

[Out]
$$-1/840*(3*A*(280*(7*\sin(d*x + c)/(\cos(d*x + c) + 1) + 9*\sin(d*x + c)^3/(\cos(d*x + c) + 1)^3)/(a^4 + 2*a^4*\sin(d*x + c)^2/(\cos(d*x + c) + 1)^2 + a^4*\sin(d*x + c)^4/(\cos(d*x + c) + 1)^4) + (3885*\sin(d*x + c)/(\cos(d*x + c) + 1) - 455*\sin(d*x + c)^3/(\cos(d*x + c) + 1)^3 + 63*\sin(d*x + c)^5/(\cos(d*x + c) + 1)^5 - 5*\sin(d*x + c)^7/(\cos(d*x + c) + 1)^7)/a^4 - 5880*\arctan(\sin(d*x + c)/(\cos(d*x + c) + 1))/a^4 - B*(1680*\sin(d*x + c)/((a^4 + a^4*\sin(d*x + c)^2/(\cos(d*x + c) + 1)^2)*(\cos(d*x + c) + 1)) + (5145*\sin(d*x + c)/(\cos(d*x + c) + 1) - 805*\sin(d*x + c)^3/(\cos(d*x + c) + 1)^3 + 147*\sin(d*x + c)^5/(\cos(d*x + c) + 1)^5 - 15*\sin(d*x + c)^7/(\cos(d*x + c) + 1)^7)/a^4 - 6720*\arctan(\sin(d*x + c)/(\cos(d*x + c) + 1))/a^4)/d$$

mupad [B] time = 2.00, size = 179, normalized size = 0.80

$$\frac{\frac{21A dx}{2} - 4B dx}{a^4 d} - \frac{(9A - 2B) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 + (7A - 2B) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{a^4 d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + 1\right)^2} - \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right) \left(\frac{111A}{8} - \frac{49B}{8}\right)}{a^4 d} + \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{a^4 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)^2*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^4,x)

[Out]
$$\left(\frac{21*A*d*x}{2} - 4*B*d*x\right)/(a^4*d) - \left(\tan\left(\frac{c}{2} + \frac{d*x}{2}\right)\right)^3*(9*A - 2*B) + \tan\left(\frac{c}{2} + \frac{d*x}{2}\right)*(7*A - 2*B)/(a^4*d*(\tan\left(\frac{c}{2} + \frac{d*x}{2}\right)^2 + 1)^2) - \left(\tan\left(\frac{c}{2} + \frac{d*x}{2}\right)*\left(\frac{111*A}{8} - \frac{49*B}{8}\right)\right)/(a^4*d) + \left(\tan\left(\frac{c}{2} + \frac{d*x}{2}\right)\right)^3*\left(\frac{13*A}{8} - \frac{23*B}{24}\right)/(a^4*d) - \left(\tan\left(\frac{c}{2} + \frac{d*x}{2}\right)\right)^5*\left(\frac{9*A}{40} - \frac{7*B}{40}\right)/(a^4*d) + \left(\tan\left(\frac{c}{2} + \frac{d*x}{2}\right)\right)^7*(A/56 - B/56)/(a^4*d)$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A \cos^2(c+dx)}{\sec^4(c+dx)+4\sec^3(c+dx)+6\sec^2(c+dx)+4\sec(c+dx)+1} dx + \int \frac{B \cos^2(c+dx) \sec(c+dx)}{\sec^4(c+dx)+4\sec^3(c+dx)+6\sec^2(c+dx)+4\sec(c+dx)+1} dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**4,x)

[Out]
$$\left(\text{Integral}\left(A*\cos(c + d*x)**2/(\sec(c + d*x)**4 + 4*\sec(c + d*x)**3 + 6*\sec(c + d*x)**2 + 4*\sec(c + d*x) + 1)\right), x\right) + \text{Integral}\left(B*\cos(c + d*x)**2*\sec(c + d*x)/(\sec(c + d*x)**4 + 4*\sec(c + d*x)**3 + 6*\sec(c + d*x)**2 + 4*\sec(c + d*x) + 1)\right), x)/a**4$$

$$3.117 \quad \int \frac{\cos^3(c+dx)(A+B \sec(c+dx))}{(a+a \sec(c+dx))^4} dx$$

Optimal. Leaf size=256

$$\frac{8(227A - 108B) \sin^3(c + dx)}{105a^4d} + \frac{8(227A - 108B) \sin(c + dx)}{35a^4d} - \frac{(44A - 21B) \sin(c + dx) \cos(c + dx)}{2a^4d} - \frac{(44A - 21B)}{3a^4d}$$

[Out] $-1/2*(44*A-21*B)*x/a^4+8/35*(227*A-108*B)*\sin(d*x+c)/a^4/d-1/2*(44*A-21*B)*\cos(d*x+c)*\sin(d*x+c)/a^4/d-1/105*(178*A-87*B)*\cos(d*x+c)^2*\sin(d*x+c)/a^4/d/(1+\sec(d*x+c))^2-1/3*(44*A-21*B)*\cos(d*x+c)^2*\sin(d*x+c)/a^4/d/(1+\sec(d*x+c))-1/7*(A-B)*\cos(d*x+c)^2*\sin(d*x+c)/d/(a+a*\sec(d*x+c))^4-1/35*(16*A-9*B)*\cos(d*x+c)^2*\sin(d*x+c)/a/d/(a+a*\sec(d*x+c))^3-8/105*(227*A-108*B)*\sin(d*x+c)^3/a^4/d$

Rubi [A] time = 0.71, antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {4020, 3787, 2633, 2635, 8}

$$\frac{8(227A - 108B) \sin^3(c + dx)}{105a^4d} + \frac{8(227A - 108B) \sin(c + dx)}{35a^4d} - \frac{(44A - 21B) \sin(c + dx) \cos(c + dx)}{2a^4d} - \frac{(44A - 21B)}{3a^4d}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^4,x]

[Out] $-((44*A - 21*B)*x)/(2*a^4) + (8*(227*A - 108*B)*\text{Sin}[c + d*x])/(35*a^4*d) - ((44*A - 21*B)*\text{Cos}[c + d*x]*\text{Sin}[c + d*x])/(2*a^4*d) - ((178*A - 87*B)*\text{Cos}[c + d*x]^2*\text{Sin}[c + d*x])/(105*a^4*d*(1 + \text{Sec}[c + d*x])^2) - ((44*A - 21*B)*\text{Cos}[c + d*x]^2*\text{Sin}[c + d*x])/(3*a^4*d*(1 + \text{Sec}[c + d*x])) - ((A - B)*\text{Cos}[c + d*x]^2*\text{Sin}[c + d*x])/(7*d*(a + a*\text{Sec}[c + d*x])^4) - ((16*A - 9*B)*\text{Cos}[c + d*x]^2*\text{Sin}[c + d*x])/(35*a*d*(a + a*\text{Sec}[c + d*x])^3) - (8*(227*A - 108*B)*\text{Sin}[c + d*x]^3)/(105*a^4*d)$

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2633

Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]
)*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] := -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
+ f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^3(c+dx)(A+B \sec(c+dx))}{(a+a \sec(c+dx))^4} dx &= -\frac{(A-B) \cos^2(c+dx) \sin(c+dx)}{7d(a+a \sec(c+dx))^4} + \frac{\int \frac{\cos^3(c+dx)(a(10A-3B)-6a(A-B) \sec(c+dx))}{(a+a \sec(c+dx))^3} dx}{7a^2} \\
&= -\frac{(A-B) \cos^2(c+dx) \sin(c+dx)}{7d(a+a \sec(c+dx))^4} - \frac{(16A-9B) \cos^2(c+dx) \sin(c+dx)}{35ad(a+a \sec(c+dx))^3} \\
&= -\frac{(178A-87B) \cos^2(c+dx) \sin(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{(A-B) \cos^2(c+dx) \sin(c+dx)}{7d(a+a \sec(c+dx))^4} \\
&= -\frac{(178A-87B) \cos^2(c+dx) \sin(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{(A-B) \cos^2(c+dx) \sin(c+dx)}{7d(a+a \sec(c+dx))^4} \\
&= -\frac{(178A-87B) \cos^2(c+dx) \sin(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{(A-B) \cos^2(c+dx) \sin(c+dx)}{7d(a+a \sec(c+dx))^4} \\
&= -\frac{(44A-21B) \cos(c+dx) \sin(c+dx)}{2a^4d} - \frac{(178A-87B) \cos^2(c+dx) \sin(c+dx)}{105a^4d(1+\sec(c+dx))^2} \\
&= -\frac{(44A-21B)x}{2a^4} + \frac{8(227A-108B) \sin(c+dx)}{35a^4d} - \frac{(44A-21B) \cos(c+dx)}{2a^4d}
\end{aligned}$$

Mathematica [B] time = 1.80, size = 611, normalized size = 2.39

$$\frac{\sec\left(\frac{c}{2}\right) \cos\left(\frac{1}{2}(c+dx)\right) \left(-14700dx(44A-21B) \cos\left(c+\frac{dx}{2}\right) - 14700dx(44A-21B) \cos\left(\frac{dx}{2}\right) - 687260A \sin\left(c+\frac{dx}{2}\right)\right)}{2a^4d}$$

Antiderivative was successfully verified.

```

[In] Integrate[(Cos[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^4,x]
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(-14700*(44*A - 21*B)*d*x*Cos[(d*x)/2] - 14700*(44*A - 21*B)*d*x*Cos[c + (d*x)/2] - 388080*A*d*x*Cos[c + (3*d*x)/2] + 185220*B*d*x*Cos[c + (3*d*x)/2] - 388080*A*d*x*Cos[2*c + (3*d*x)/2] + 185220*B*d*x*Cos[2*c + (3*d*x)/2] - 129360*A*d*x*Cos[2*c + (5*d*x)/2] + 61740*B*d*x*Cos[2*c + (5*d*x)/2] - 129360*A*d*x*Cos[3*c + (5*d*x)/2] + 61740*B*d*x*Cos[3*c + (5*d*x)/2] - 18480*A*d*x*Cos[3*c + (7*d*x)/2] + 8820*B*d*x*Cos[3*c + (7*d*x)/2] - 18480*A*d*x*Cos[4*c + (7*d*x)/2] + 8820*B*d*x*Cos[4*c + (7*d*x)/2] + 1010660*A*Sin[(d*x)/2] - 539490*B*Sin[(d*x)/2] - 687260*A*Sin[c + (d*x)/2] + 386190*B*Sin[c + (d*x)/2] + 814107*A*Sin[c + (3*d*x)/2] - 422478*B*Sin[c + (3*d*x)/2] - 204645*A*Sin[2*c + (3*d*x)/2] + 132930*B*Sin[2*c + (3*d*x)/2] + 357609*A*Sin[2*c + (5*d*x)/2] - 181461*B*Sin[2*c + (5*d*x)/2] + 1

```

$$\begin{aligned} & 8025*A*\sin[3*c + (5*d*x)/2] + 3675*B*\sin[3*c + (5*d*x)/2] + 72522*A*\sin[3*c \\ & + (7*d*x)/2] - 36003*B*\sin[3*c + (7*d*x)/2] + 24010*A*\sin[4*c + (7*d*x)/2] \\ & - 9555*B*\sin[4*c + (7*d*x)/2] + 2310*A*\sin[4*c + (9*d*x)/2] - 945*B*\sin[4*c \\ & + (9*d*x)/2] + 2310*A*\sin[5*c + (9*d*x)/2] - 945*B*\sin[5*c + (9*d*x)/2] - \\ & 175*A*\sin[5*c + (11*d*x)/2] + 105*B*\sin[5*c + (11*d*x)/2] - 175*A*\sin[6*c \\ & + (11*d*x)/2] + 105*B*\sin[6*c + (11*d*x)/2] + 35*A*\sin[6*c + (13*d*x)/2] + \\ & 35*A*\sin[7*c + (13*d*x)/2]) / (6720*a^4*d*(1 + \cos[c + d*x])^4) \end{aligned}$$

fricas [A] time = 0.46, size = 257, normalized size = 1.00

$$\frac{105(44A - 21B)dx \cos(dx + c)^4 + 420(44A - 21B)dx \cos(dx + c)^3 + 630(44A - 21B)dx \cos(dx + c)^2 + 420(44A - 21B)dx \cos(dx + c) + 105(44A - 21B)dx - (70A \cos(dx + c)^6 - 35(4A - 3B) \cos(dx + c)^5 + 140(7A - 3B) \cos(dx + c)^4 + 4(3196A - 1509B) \cos(dx + c)^3 + 4(7184A - 3411B) \cos(dx + c)^2 + (24436A - 11619B) \cos(dx + c) + 7264A - 3456B) \sin(dx + c)}{(a^4 d \cos(dx + c)^4 + 4a^4 d \cos(dx + c)^3 + 6a^4 d \cos(dx + c)^2 + 4a^4 d \cos(dx + c) + a^4 d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x, algorithm="fricas")

[Out] -1/210*(105*(44*A - 21*B)*d*x*cos(d*x + c)^4 + 420*(44*A - 21*B)*d*x*cos(d*x + c)^3 + 630*(44*A - 21*B)*d*x*cos(d*x + c)^2 + 420*(44*A - 21*B)*d*x*cos(d*x + c) + 105*(44*A - 21*B)*d*x - (70*A*cos(d*x + c)^6 - 35*(4*A - 3*B)*cos(d*x + c)^5 + 140*(7*A - 3*B)*cos(d*x + c)^4 + 4*(3196*A - 1509*B)*cos(d*x + c)^3 + 4*(7184*A - 3411*B)*cos(d*x + c)^2 + (24436*A - 11619*B)*cos(d*x + c) + 7264*A - 3456*B)*sin(d*x + c))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)

giac [A] time = 0.34, size = 261, normalized size = 1.02

$$\frac{420(dx+c)(44A-21B)}{a^4} - \frac{280\left(78A \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5 - 27B \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5 + 124A \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 - 48B \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 + 54A \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 21B\right)}{\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + 1\right)^3 a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x, algorithm="giac")

[Out] -1/840*(420*(d*x + c)*(44*A - 21*B)/a^4 - 280*(78*A*tan(1/2*d*x + 1/2*c)^5 - 27*B*tan(1/2*d*x + 1/2*c)^5 + 124*A*tan(1/2*d*x + 1/2*c)^3 - 48*B*tan(1/2*d*x + 1/2*c)^3 + 54*A*tan(1/2*d*x + 1/2*c) - 21*B*tan(1/2*d*x + 1/2*c)) / ((tan(1/2*d*x + 1/2*c)^2 + 1)^3*a^4) + (15*A*a^24*tan(1/2*d*x + 1/2*c)^7 - 15*B*a^24*tan(1/2*d*x + 1/2*c)^7 - 231*A*a^24*tan(1/2*d*x + 1/2*c)^5 + 189*B*a^24*tan(1/2*d*x + 1/2*c)^5 + 2065*A*a^24*tan(1/2*d*x + 1/2*c)^3 - 1365*B*a^24*tan(1/2*d*x + 1/2*c)^3 - 21945*A*a^24*tan(1/2*d*x + 1/2*c) + 11655*B*a^24*tan(1/2*d*x + 1/2*c))/a^28)/d

maple [A] time = 1.26, size = 402, normalized size = 1.57

$$\frac{\left(\tan^7\left(\frac{dx}{2} + \frac{c}{2}\right)\right) A}{56d a^4} + \frac{B \left(\tan^7\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{56d a^4} + \frac{11A \left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{40d a^4} - \frac{9B \left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{40d a^4} - \frac{59 \left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right) A}{24d a^4} + \frac{13B \left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{24d a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x)`

[Out]
$$-1/56/d/a^4*\tan(1/2*d*x+1/2*c)^7*A+1/56/d/a^4*B*\tan(1/2*d*x+1/2*c)^7+11/40/d/a^4*A*\tan(1/2*d*x+1/2*c)^5-9/40/d/a^4*B*\tan(1/2*d*x+1/2*c)^5-59/24/d/a^4*\tan(1/2*d*x+1/2*c)^3*A+13/8/d/a^4*B*\tan(1/2*d*x+1/2*c)^3+209/8/d/a^4*A*\tan(1/2*d*x+1/2*c)-111/8/d/a^4*B*\tan(1/2*d*x+1/2*c)+26/d/a^4/(1+\tan(1/2*d*x+1/2*c)^2)^3*\tan(1/2*d*x+1/2*c)^5*A-9/d/a^4/(1+\tan(1/2*d*x+1/2*c)^2)^3*\tan(1/2*d*x+1/2*c)^5*B+124/3/d/a^4/(1+\tan(1/2*d*x+1/2*c)^2)^3*A*\tan(1/2*d*x+1/2*c)^3-16/d/a^4/(1+\tan(1/2*d*x+1/2*c)^2)^3*B*\tan(1/2*d*x+1/2*c)^3+18/d/a^4/(1+\tan(1/2*d*x+1/2*c)^2)^3*A*\tan(1/2*d*x+1/2*c)-7/d/a^4/(1+\tan(1/2*d*x+1/2*c)^2)^3*B*\tan(1/2*d*x+1/2*c)-44/d/a^4*A*\arctan(\tan(1/2*d*x+1/2*c))+21/d/a^4*\arctan(\tan(1/2*d*x+1/2*c))*B$$

maxima [A] time = 0.46, size = 452, normalized size = 1.77

$$A \left(\frac{560 \left(\frac{27 \sin(dx+c)}{\cos(dx+c)+1} + \frac{62 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{39 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} \right)}{a^4 + \frac{3a^4 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{3a^4 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{a^4 \sin(dx+c)^6}{(\cos(dx+c)+1)^6}} + \frac{\frac{21945 \sin(dx+c)}{\cos(dx+c)+1} - \frac{2065 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{231 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{15 \sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4} - \frac{36960 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4} \right)$$

840 d

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x, algorithm="maxima")`

[Out]
$$1/840*(A*(560*(27*\sin(d*x + c)/(\cos(d*x + c) + 1) + 62*\sin(d*x + c)^3/(\cos(d*x + c) + 1)^3 + 39*\sin(d*x + c)^5/(\cos(d*x + c) + 1)^5)/(a^4 + 3*a^4*\sin(d*x + c)^2/(\cos(d*x + c) + 1)^2 + 3*a^4*\sin(d*x + c)^4/(\cos(d*x + c) + 1)^4 + a^4*\sin(d*x + c)^6/(\cos(d*x + c) + 1)^6) + (21945*\sin(d*x + c)/(\cos(d*x + c) + 1) - 2065*\sin(d*x + c)^3/(\cos(d*x + c) + 1)^3 + 231*\sin(d*x + c)^5/(\cos(d*x + c) + 1)^5 - 15*\sin(d*x + c)^7/(\cos(d*x + c) + 1)^7)/a^4 - 36960*a*\arctan(\sin(d*x + c)/(\cos(d*x + c) + 1))/a^4 - 3*B*(280*(7*\sin(d*x + c)/(\cos(d*x + c) + 1) + 9*\sin(d*x + c)^3/(\cos(d*x + c) + 1)^3)/(a^4 + 2*a^4*\sin(d*x + c)^2/(\cos(d*x + c) + 1)^2 + a^4*\sin(d*x + c)^4/(\cos(d*x + c) + 1)^4) + (3885*\sin(d*x + c)/(\cos(d*x + c) + 1) - 455*\sin(d*x + c)^3/(\cos(d*x + c) + 1)^3 + 63*\sin(d*x + c)^5/(\cos(d*x + c) + 1)^5 - 5*\sin(d*x + c)^7/(\cos(d*x + c) + 1)^7)/a^4 - 5880*\arctan(\sin(d*x + c)/(\cos(d*x + c) + 1))/a^4)/d$$

mupad [B] time = 2.05, size = 300, normalized size = 1.17

$$\frac{(26A - 9B) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5 + \left(\frac{124A}{3} - 16B\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 + (18A - 7B) \tan\left(\frac{c}{2} + \frac{dx}{2}\right) + x(44A - 21B) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{d \left(a^4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^6 + 3a^4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 + 3a^4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + a^4 \right) - \frac{x(44A - 21B) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{2a^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)^3*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^4,x)

[Out] (tan(c/2 + (d*x)/2)^5*(26*A - 9*B) + tan(c/2 + (d*x)/2)^3*((124*A)/3 - 16*B) + tan(c/2 + (d*x)/2)*(18*A - 7*B))/(d*(3*a^4*tan(c/2 + (d*x)/2)^2 + 3*a^4*tan(c/2 + (d*x)/2)^4 + a^4*tan(c/2 + (d*x)/2)^6 + a^4)) - (x*(44*A - 21*B))/(2*a^4) - (tan(c/2 + (d*x)/2)^3*((5*(A - B))/(12*a^4) + (7*A - 5*B)/(6*a^4) + (21*A - 9*B)/(24*a^4)))/d + (tan(c/2 + (d*x)/2)^5*((A - B)/(10*a^4) + (7*A - 5*B)/(40*a^4)))/d + (tan(c/2 + (d*x)/2)*((5*(A - B))/(2*a^4) + (5*(7*A - 5*B))/(4*a^4) + (21*A - 9*B)/(2*a^4) + (35*A - 5*B)/(8*a^4)))/d - (tan(c/2 + (d*x)/2)^7*(A - B))/(56*a^4*d)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**4,x)

[Out] Timed out

$$3.118 \quad \int \sec^4(c+dx) \sqrt{a + a \sec(c + dx)} (A+B \sec(c+dx)) dx$$

Optimal. Leaf size=187

$$\frac{2a(9A + 8B) \tan(c + dx) \sec^3(c + dx)}{63d\sqrt{a \sec(c + dx) + a}} + \frac{4(9A + 8B) \tan(c + dx)(a \sec(c + dx) + a)^{3/2}}{105ad} - \frac{8(9A + 8B) \tan(c + dx)\sqrt{a \sec(c + dx) + a}}{315d}$$

[Out] 4/105*(9*A+8*B)*(a+a*sec(d*x+c))^(3/2)*tan(d*x+c)/a/d+4/45*a*(9*A+8*B)*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+2/63*a*(9*A+8*B)*sec(d*x+c)^3*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+2/9*a*B*sec(d*x+c)^4*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)-8/315*(9*A+8*B)*(a+a*sec(d*x+c))^(1/2)*tan(d*x+c)/d

Rubi [A] time = 0.34, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {4016, 3803, 3800, 4001, 3792}

$$\frac{2a(9A + 8B) \tan(c + dx) \sec^3(c + dx)}{63d\sqrt{a \sec(c + dx) + a}} + \frac{4(9A + 8B) \tan(c + dx)(a \sec(c + dx) + a)^{3/2}}{105ad} - \frac{8(9A + 8B) \tan(c + dx)\sqrt{a \sec(c + dx) + a}}{315d}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d*x]^4*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]

[Out] (4*a*(9*A + 8*B)*Tan[c + d*x])/(45*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*(9*A + 8*B)*Sec[c + d*x]^3*Tan[c + d*x])/(63*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*B*Sec[c + d*x]^4*Tan[c + d*x])/(9*d*Sqrt[a + a*Sec[c + d*x]]) - (8*(9*A + 8*B)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(315*d) + (4*(9*A + 8*B)*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(105*a*d)

Rule 3792

Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 3800

Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m + 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)]

Rule 3803

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.)
+ (a_.)], x_Symbol] := Simp[(-2*b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/
(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(2*a*d*(n - 1))/(b*(2*n -
1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_.)]*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_)*(cs
c[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
+ 1), 0] && !LtQ[m, -2^(-1)]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.)
+ (a_.)]*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
+ Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && !
LtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \sec^4(c + dx) \sqrt{a + a \sec(c + dx)} (A + B \sec(c + dx)) dx &= \frac{2aB \sec^4(c + dx) \tan(c + dx)}{9d\sqrt{a + a \sec(c + dx)}} + \frac{1}{9}(9A + 8B) \int \sec^4(c + dx) \sqrt{a + a \sec(c + dx)} dx \\
&= \frac{2a(9A + 8B) \sec^3(c + dx) \tan(c + dx)}{63d\sqrt{a + a \sec(c + dx)}} + \frac{2aB \sec^4(c + dx)}{9d\sqrt{a + a \sec(c + dx)}} \\
&= \frac{2a(9A + 8B) \sec^3(c + dx) \tan(c + dx)}{63d\sqrt{a + a \sec(c + dx)}} + \frac{2aB \sec^4(c + dx)}{9d\sqrt{a + a \sec(c + dx)}} \\
&= \frac{2a(9A + 8B) \sec^3(c + dx) \tan(c + dx)}{63d\sqrt{a + a \sec(c + dx)}} + \frac{2aB \sec^4(c + dx)}{9d\sqrt{a + a \sec(c + dx)}} \\
&= \frac{4a(9A + 8B) \tan(c + dx)}{45d\sqrt{a + a \sec(c + dx)}} + \frac{2a(9A + 8B) \sec^3(c + dx)}{63d\sqrt{a + a \sec(c + dx)}}
\end{aligned}$$

Mathematica [A] time = 0.60, size = 98, normalized size = 0.52

$$\frac{2a \tan(c + dx) \left(5(9A + 8B) \sec^3(c + dx) + 6(9A + 8B) \sec^2(c + dx) + 8(9A + 8B) \sec(c + dx) + 16(9A + 8B) + 315d\sqrt{a(\sec(c + dx) + 1)} \right)}{315d\sqrt{a(\sec(c + dx) + 1)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[c + d*x]^4*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]

[Out] (2*a*(16*(9*A + 8*B) + 8*(9*A + 8*B)*Sec[c + d*x] + 6*(9*A + 8*B)*Sec[c + d*x]^2 + 5*(9*A + 8*B)*Sec[c + d*x]^3 + 35*B*Sec[c + d*x]^4)*Tan[c + d*x])/(315*d*Sqrt[a*(1 + Sec[c + d*x])])

fricas [A] time = 0.42, size = 122, normalized size = 0.65

$$\frac{2 \left(16(9A + 8B) \cos(dx + c)^4 + 8(9A + 8B) \cos(dx + c)^3 + 6(9A + 8B) \cos(dx + c)^2 + 5(9A + 8B) \cos(dx + c) + 315 \left(d \cos(dx + c)^5 + d \cos(dx + c)^4 \right) \right)}{315 \left(d \cos(dx + c)^5 + d \cos(dx + c)^4 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^4*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] 2/315*(16*(9*A + 8*B)*cos(d*x + c)^4 + 8*(9*A + 8*B)*cos(d*x + c)^3 + 6*(9*A + 8*B)*cos(d*x + c)^2 + 5*(9*A + 8*B)*cos(d*x + c) + 35*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^5 + d*cos(d*x + c)^4)

giac [A] time = 2.28, size = 268, normalized size = 1.43

$$2 \left(315 \sqrt{2} A a^5 \operatorname{sgn}(\cos(dx + c)) + 315 \sqrt{2} B a^5 \operatorname{sgn}(\cos(dx + c)) - \left(630 \sqrt{2} A a^5 \operatorname{sgn}(\cos(dx + c)) + 420 \sqrt{2} B a^5 \operatorname{sgn}(\cos(dx + c)) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^4*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] 2/315*(315*sqrt(2)*A*a^5*sgn(cos(d*x + c)) + 315*sqrt(2)*B*a^5*sgn(cos(d*x + c)) - (630*sqrt(2)*A*a^5*sgn(cos(d*x + c)) + 420*sqrt(2)*B*a^5*sgn(cos(d*x + c)) - (756*sqrt(2)*A*a^5*sgn(cos(d*x + c)) + 882*sqrt(2)*B*a^5*sgn(cos(d*x + c)) - (522*sqrt(2)*A*a^5*sgn(cos(d*x + c)) + 324*sqrt(2)*B*a^5*sgn(cos(d*x + c))))

$s(dx + c) - (81\sqrt{2})Aa^5\text{sgn}(\cos(dx + c)) + 107\sqrt{2}Ba^5\text{sgn}(\cos(dx + c))\tan(1/2dx + 1/2c)^2 \tan(1/2dx + 1/2c)^2 \tan(1/2dx + 1/2c)^2 \tan(1/2dx + 1/2c)^2 \tan(1/2dx + 1/2c) / ((a\tan(1/2dx + 1/2c)^2 - a)^4 \sqrt{-a\tan(1/2dx + 1/2c)^2 + a})d$

maple [A] time = 1.64, size = 138, normalized size = 0.74

$$\frac{2(-1 + \cos(dx + c))(144A(\cos^4(dx + c)) + 128B(\cos^4(dx + c)) + 72A(\cos^3(dx + c)) + 64B(\cos^3(dx + c)))}{315d \cos(dx + c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sec(dx+c)^4*(a+a*sec(dx+c))^(1/2)*(A+B*sec(dx+c)),x)`

[Out] $-2/315/d*(-1+\cos(dx+c))*(144*A*\cos(dx+c)^4+128*B*\cos(dx+c)^4+72*A*\cos(dx+c)^3+64*B*\cos(dx+c)^3+54*A*\cos(dx+c)^2+48*B*\cos(dx+c)^2+45*A*\cos(dx+c)+40*B*\cos(dx+c)+35*B)*(a*(1+\cos(dx+c))/\cos(dx+c))^(1/2)/\cos(dx+c)^4/\sin(dx+c)$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(dx+c)^4*(a+a*sec(dx+c))^(1/2)*(A+B*sec(dx+c)),x, algorithm="maxima")`

[Out] Timed out

mupad [B] time = 10.05, size = 512, normalized size = 2.74

$$\frac{\sqrt{a + \frac{a}{\frac{e^{-c1i-dx1i}}{2} + \frac{e^{c1i+dx1i}}{2}}}}{\left(e^{c1i+dx1i} + 1\right) \left(e^{c2i+dx2i} + 1\right)^2} \left(e^{c1i+dx1i} \left(\frac{A16i}{5d} + \frac{(48A-32B)1i}{105d}\right) + \frac{(336A+672B)1i}{105d}\right) \sqrt{a + \frac{a}{\frac{e^{-c1i-dx1i}}{2} + \frac{e^{c1i+dx1i}}{2}}}}{\left(e^{c1i+dx1i} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(1/2))/cos(c + d*x)^4,x)`

[Out] $((a + a/(\exp(-c*1i - d*x*1i)/2 + \exp(c*1i + d*x*1i)/2))^(1/2)*(exp(c*1i + d*x*1i)*((A*16i)/(5*d) + ((48*A - 32*B)*1i)/(105*d)) + ((336*A + 672*B)*1i)/(105*d)))/((\exp(c*1i + d*x*1i) + 1)*(\exp(c*2i + d*x*2i) + 1)^2) - ((a + a/(\exp(-c*1i - d*x*1i)/2 + \exp(c*1i + d*x*1i)/2))^(1/2)*(exp(c*1i + d*x*1i)*((A*16i)/(7*d) - (B*320i)/(63*d)) + (B*32i)/(7*d) + ((144*A + 288*B)*1i)/(6$

```

3*d)))/((exp(c*1i + d*x*1i) + 1)*(exp(c*2i + d*x*2i) + 1)^3) + ((a + a/(exp
(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(exp(c*1i + d*x*1i)*((A*
16i)/(9*d) - ((16*A + 32*B)*1i)/(9*d)) - (A*16i)/(9*d) + ((16*A + 32*B)*1i)
/(9*d)))/((exp(c*1i + d*x*1i) + 1)*(exp(c*2i + d*x*2i) + 1)^4) - (exp(c*1i
+ d*x*1i)*(a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(28
8*A + 256*B)*1i)/(315*d*(exp(c*1i + d*x*1i) + 1)) - (exp(c*1i + d*x*1i)*(a
+ a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(144*A + 128*B)*
1i)/(315*d*(exp(c*1i + d*x*1i) + 1)*(exp(c*2i + d*x*2i) + 1))

```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a(\sec(c + dx) + 1)} (A + B \sec(c + dx)) \sec^4(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+a*sec(d*x+c))**(1/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*(A + B*sec(c + d*x))*sec(c + d*x)**4, x
)
```

$$3.119 \quad \int \sec^3(c+dx) \sqrt{a + a \sec(c + dx)} (A+B \sec(c+dx)) dx$$

Optimal. Leaf size=144

$$\frac{2(7A + 6B) \tan(c + dx)(a \sec(c + dx) + a)^{3/2}}{35ad} - \frac{4(7A + 6B) \tan(c + dx) \sqrt{a \sec(c + dx) + a}}{105d} + \frac{2a(7A + 6B) \tan(c + dx)}{15d \sqrt{a \sec(c + dx) + a}}$$

[Out] 2/35*(7*A+6*B)*(a+a*sec(d*x+c))^(3/2)*tan(d*x+c)/a/d+2/15*a*(7*A+6*B)*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+2/7*a*B*sec(d*x+c)^3*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)-4/105*(7*A+6*B)*(a+a*sec(d*x+c))^(1/2)*tan(d*x+c)/d

Rubi [A] time = 0.28, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {4016, 3800, 4001, 3792}

$$\frac{2(7A + 6B) \tan(c + dx)(a \sec(c + dx) + a)^{3/2}}{35ad} - \frac{4(7A + 6B) \tan(c + dx) \sqrt{a \sec(c + dx) + a}}{105d} + \frac{2a(7A + 6B) \tan(c + dx)}{15d \sqrt{a \sec(c + dx) + a}}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d*x]^3*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]

[Out] (2*a*(7*A + 6*B)*Tan[c + d*x])/(15*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*B*Sec[c + d*x]^3*Tan[c + d*x])/(7*d*Sqrt[a + a*Sec[c + d*x]]) - (4*(7*A + 6*B)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(105*d) + (2*(7*A + 6*B)*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(35*a*d)

Rule 3792

Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 3800

Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m + 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)]

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_.)]*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := -Simp[(B*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m + 1), 0] && !LtQ[m, -2^(-1)]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := Simp[(-2*b*B*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && !LtQ[n, 0]
```

Rubi steps

$$\begin{aligned} \int \sec^3(c + dx)\sqrt{a + a \sec(c + dx)}(A + B \sec(c + dx)) dx &= \frac{2aB \sec^3(c + dx) \tan(c + dx)}{7d\sqrt{a + a \sec(c + dx)}} + \frac{1}{7}(7A + 6B) \int \sec^3(c + dx) \sqrt{a + a \sec(c + dx)} dx \\ &= \frac{2aB \sec^3(c + dx) \tan(c + dx)}{7d\sqrt{a + a \sec(c + dx)}} + \frac{2(7A + 6B)(a + a \sec(c + dx))}{7d\sqrt{a + a \sec(c + dx)}} \\ &= \frac{2aB \sec^3(c + dx) \tan(c + dx)}{7d\sqrt{a + a \sec(c + dx)}} - \frac{4(7A + 6B)\sqrt{a + a \sec(c + dx)}}{7d\sqrt{a + a \sec(c + dx)}} \\ &= \frac{2a(7A + 6B) \tan(c + dx)}{15d\sqrt{a + a \sec(c + dx)}} + \frac{2aB \sec^3(c + dx) \tan(c + dx)}{7d\sqrt{a + a \sec(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.29, size = 81, normalized size = 0.56

$$\frac{2a \tan(c + dx) (3(7A + 6B) \sec^2(c + dx) + 4(7A + 6B) \sec(c + dx) + 8(7A + 6B) + 15B \sec^3(c + dx))}{105d\sqrt{a(\sec(c + dx) + 1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
[Out] (2*a*(8*(7*A + 6*B) + 4*(7*A + 6*B)*Sec[c + d*x] + 3*(7*A + 6*B)*Sec[c + d*x]^2 + 15*B*Sec[c + d*x]^3)*Tan[c + d*x])/(105*d*Sqrt[a*(1 + Sec[c + d*x])])
```

fricas [A] time = 0.44, size = 105, normalized size = 0.73

$$\frac{2 \left(8 (7 A + 6 B) \cos (d x + c)^3 + 4 (7 A + 6 B) \cos (d x + c)^2 + 3 (7 A + 6 B) \cos (d x + c) + 15 B \right) \sqrt{\frac{a \cos (d x + c) + a}{\cos (d x + c)}} \sin (d x + c)}{105 \left(d \cos (d x + c)^4 + d \cos (d x + c)^3 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] 2/105*(8*(7*A + 6*B)*cos(d*x + c)^3 + 4*(7*A + 6*B)*cos(d*x + c)^2 + 3*(7*A + 6*B)*cos(d*x + c) + 15*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3)

giac [A] time = 14.36, size = 222, normalized size = 1.54

$$2 \left(105 \sqrt{2} A a^4 \operatorname{sgn}(\cos (d x + c)) + 105 \sqrt{2} B a^4 \operatorname{sgn}(\cos (d x + c)) - \left(175 \sqrt{2} A a^4 \operatorname{sgn}(\cos (d x + c)) + 105 \sqrt{2} B a^4 \operatorname{sgn}(\cos (d x + c)) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] -2/105*(105*sqrt(2)*A*a^4*sgn(cos(d*x + c)) + 105*sqrt(2)*B*a^4*sgn(cos(d*x + c)) - (175*sqrt(2)*A*a^4*sgn(cos(d*x + c)) + 105*sqrt(2)*B*a^4*sgn(cos(d*x + c)) - (119*sqrt(2)*A*a^4*sgn(cos(d*x + c)) + 147*sqrt(2)*B*a^4*sgn(cos(d*x + c)) - (49*sqrt(2)*A*a^4*sgn(cos(d*x + c)) + 27*sqrt(2)*B*a^4*sgn(cos(d*x + c))))*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^3*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*d)

maple [A] time = 1.51, size = 116, normalized size = 0.81

$$\frac{2(-1 + \cos (d x + c)) \left(56 A \left(\cos ^3 (d x + c) \right) + 48 B \left(\cos ^3 (d x + c) \right) + 28 A \left(\cos ^2 (d x + c) \right) + 24 B \left(\cos ^2 (d x + c) \right) + \right)}{105 d \cos (d x + c)^3 \sin (d x + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^3*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x)

[Out]
$$-2/105/d*(-1+\cos(d*x+c))*(56*A*\cos(d*x+c)^3+48*B*\cos(d*x+c)^3+28*A*\cos(d*x+c)^2+24*B*\cos(d*x+c)^2+21*A*\cos(d*x+c)+18*B*\cos(d*x+c)+15*B)*(a*(1+\cos(d*x+c)))/\cos(d*x+c)^{(1/2)}/\cos(d*x+c)^3/\sin(d*x+c)$$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x, algorithm="maxima")`

[Out] Timed out

mupad [B] time = 6.16, size = 407, normalized size = 2.83

$$\frac{\sqrt{a + \frac{a}{\frac{e^{-c1i-dx1i}}{2} + \frac{e^{c1i+dx1i}}{2}}} \left(-\frac{A8i}{5d} + \frac{B16i}{5d} + e^{c1i+dx1i} \left(\frac{B16i}{35d} + \frac{(56A+112B)1i}{35d} \right) \right)}{(e^{c1i+dx1i} + 1) (e^{c2i+dx2i} + 1)^2} + \frac{\sqrt{a + \frac{a}{\frac{e^{-c1i-dx1i}}{2} + \frac{e^{c1i+dx1i}}{2}}} \left(\frac{A8i}{7d} + e^{c1i+dx1i} \right)}{(e^{c1i+dx1i} + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(1/2))/cos(c + d*x)^3,x)`

[Out]
$$\left((a + a/(\exp(-c*1i - d*x*1i)/2 + \exp(c*1i + d*x*1i)/2))^{1/2} * ((B*16i)/(5*d) - (A*8i)/(5*d) + \exp(c*1i + d*x*1i) * ((B*16i)/(35*d) + ((56*A + 112*B)*1i)/(35*d))) / ((\exp(c*1i + d*x*1i) + 1) * (\exp(c*2i + d*x*2i) + 1)^2) + ((a + a/(\exp(-c*1i - d*x*1i)/2 + \exp(c*1i + d*x*1i)/2))^{1/2} * ((A*8i)/(7*d) + \exp(c*1i + d*x*1i) * ((A*8i)/(7*d) - ((8*A + 16*B)*1i)/(7*d) - ((8*A + 16*B)*1i)/(7*d))) / ((\exp(c*1i + d*x*1i) + 1) * (\exp(c*2i + d*x*2i) + 1)^3) + (((A*8i)/(3*d) - (\exp(c*1i + d*x*1i) * (56*A + 48*B) * 1i) / (105*d)) * (a + a/(\exp(-c*1i - d*x*1i)/2 + \exp(c*1i + d*x*1i)/2))^{1/2}) / ((\exp(c*1i + d*x*1i) + 1) * (\exp(c*2i + d*x*2i) + 1)) - (\exp(c*1i + d*x*1i) * (a + a/(\exp(-c*1i - d*x*1i)/2 + \exp(c*1i + d*x*1i)/2))^{1/2} * (112*A + 96*B) * 1i) / (105*d * (\exp(c*1i + d*x*1i) + 1)) \right)$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a(\sec(c + dx) + 1)} (A + B \sec(c + dx)) \sec^3(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)**3*(a+a*sec(d*x+c))**(1/2)*(A+B*sec(d*x+c)),x)`

[Out] `Integral(sqrt(a*(sec(c + d*x) + 1))*(A + B*sec(c + d*x))*sec(c + d*x)**3, x)`

$$3.120 \quad \int \sec^2(c+dx) \sqrt{a + a \sec(c + dx)} (A+B \sec(c+dx)) dx$$

Optimal. Leaf size=101

$$\frac{2(5A - 2B) \tan(c + dx) \sqrt{a \sec(c + dx) + a}}{15d} + \frac{2a(5A + 7B) \tan(c + dx)}{15d \sqrt{a \sec(c + dx) + a}} + \frac{2B \tan(c + dx) (a \sec(c + dx) + a)^{3/2}}{5ad}$$

[Out] $2/5*B*(a+a*\sec(d*x+c))^{(3/2)}*\tan(d*x+c)/a/d+2/15*a*(5*A+7*B)*\tan(d*x+c)/d/(a+a*\sec(d*x+c))^{(1/2)}+2/15*(5*A-2*B)*(a+a*\sec(d*x+c))^{(1/2)}*\tan(d*x+c)/d$

Rubi [A] time = 0.23, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4010, 4001, 3792}

$$\frac{2(5A - 2B) \tan(c + dx) \sqrt{a \sec(c + dx) + a}}{15d} + \frac{2a(5A + 7B) \tan(c + dx)}{15d \sqrt{a \sec(c + dx) + a}} + \frac{2B \tan(c + dx) (a \sec(c + dx) + a)^{3/2}}{5ad}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sec}[c + d*x]^2*\text{Sqrt}[a + a*\text{Sec}[c + d*x]]*(A + B*\text{Sec}[c + d*x]), x]$

[Out] $(2*a*(5*A + 7*B)*\text{Tan}[c + d*x])/(15*d*\text{Sqrt}[a + a*\text{Sec}[c + d*x]]) + (2*(5*A - 2*B)*\text{Sqrt}[a + a*\text{Sec}[c + d*x]]*\text{Tan}[c + d*x])/(15*d) + (2*B*(a + a*\text{Sec}[c + d*x])^{(3/2)}*\text{Tan}[c + d*x])/(5*a*d)$

Rule 3792

$\text{Int}[\text{csc}[(e_.) + (f_.)*(x_.)]*\text{Sqrt}[\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] \rightarrow \text{Simp}[(-2*b*\text{Cot}[e + f*x])/(f*\text{Sqrt}[a + b*\text{Csc}[e + f*x]]), x] /;$ Free Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 4001

$\text{Int}[\text{csc}[(e_.) + (f_.)*(x_.)]*(\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^{(m)}*(\text{csc}[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] \rightarrow -\text{Simp}[(B*\text{Cot}[e + f*x]*(a + b*\text{Csc}[e + f*x])^m)/(f*(m + 1)), x] + \text{Dist}[(a*B*m + A*b*(m + 1))/(b*(m + 1)), \text{Int}[\text{Csc}[e + f*x]*(a + b*\text{Csc}[e + f*x])^m, x], x] /;$ FreeQ[{a, b, A, B, e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m + 1), 0] && !LtQ[m, -2^(-1)]

Rule 4010

$\text{Int}[\text{csc}[(e_.) + (f_.)*(x_.)]^2*(\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^{(m)}*(\text{csc}[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] \rightarrow -\text{Simp}[(B*\text{Cot}[e + f*x]*($

```
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Csc[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \sec^2(c + dx) \sqrt{a + a \sec(c + dx)} (A + B \sec(c + dx)) dx &= \frac{2B(a + a \sec(c + dx))^{3/2} \tan(c + dx)}{5ad} + \frac{2 \int \sec(c + dx) \sqrt{a + a \sec(c + dx)} dx}{5ad} \\ &= \frac{2(5A - 2B) \sqrt{a + a \sec(c + dx)} \tan(c + dx)}{15d} + \frac{2B(a + a \sec(c + dx))^{3/2} \tan(c + dx)}{15d} \\ &= \frac{2a(5A + 7B) \tan(c + dx)}{15d \sqrt{a + a \sec(c + dx)}} + \frac{2(5A - 2B) \sqrt{a + a \sec(c + dx)} \tan(c + dx)}{15d} \end{aligned}$$

Mathematica [A] time = 0.34, size = 80, normalized size = 0.79

$$\frac{2 \tan(c + dx) \sec(c + dx) \sqrt{a(\sec(c + dx) + 1)} ((5A + 4B) \cos(c + dx) + (5A + 4B) \cos(2(c + dx)) + 5A + 7B)}{15d(\cos(c + dx) + 1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
[Out] (2*(5*A + 7*B + (5*A + 4*B)*Cos[c + d*x] + (5*A + 4*B)*Cos[2*(c + d*x)])*Sec[c + d*x]*Sqrt[a*(1 + Sec[c + d*x])]*Tan[c + d*x])/(15*d*(1 + Cos[c + d*x]))
```

fricas [A] time = 0.49, size = 87, normalized size = 0.86

$$\frac{2 \left(2(5A + 4B) \cos(dx + c)^2 + (5A + 4B) \cos(dx + c) + 3B \right) \sqrt{\frac{a \cos(dx + c) + a}{\cos(dx + c)}} \sin(dx + c)}{15 \left(d \cos(dx + c)^3 + d \cos(dx + c)^2 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 2/15*(2*(5*A + 4*B)*cos(d*x + c)^2 + (5*A + 4*B)*cos(d*x + c) + 3*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2)
```

giac [A] time = 1.16, size = 176, normalized size = 1.74

$$\frac{2 \left(15 \sqrt{2} A a^3 \operatorname{sgn}(\cos(dx+c)) + 15 \sqrt{2} B a^3 \operatorname{sgn}(\cos(dx+c)) - \left(20 \sqrt{2} A a^3 \operatorname{sgn}(\cos(dx+c)) + 10 \sqrt{2} B a^3 \operatorname{sgn}(\cos(dx+c)) \right) \right)}{15 \left(a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] $\frac{2}{15} \left(15 \sqrt{2} A a^3 \operatorname{sgn}(\cos(dx+c)) + 15 \sqrt{2} B a^3 \operatorname{sgn}(\cos(dx+c)) - \left(20 \sqrt{2} A a^3 \operatorname{sgn}(\cos(dx+c)) + 10 \sqrt{2} B a^3 \operatorname{sgn}(\cos(dx+c)) \right) - \left(5 \sqrt{2} A a^3 \operatorname{sgn}(\cos(dx+c)) + 7 \sqrt{2} B a^3 \operatorname{sgn}(\cos(dx+c)) \right) \right) \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) / \left(\left(a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - a \right)^2 \sqrt{-a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + a} \right) d$

maple [A] time = 1.45, size = 94, normalized size = 0.93

$$\frac{2(-1 + \cos(dx+c)) \left(10A \left(\cos^2(dx+c) \right) + 8B \left(\cos^2(dx+c) \right) + 5A \cos(dx+c) + 4B \cos(dx+c) + 3B \right) \sqrt{\frac{a(1-\cos(dx+c))}{c}}}{15d \cos(dx+c)^2 \sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^2*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x)

[Out] $\frac{-2}{15} \frac{1}{d} (-1 + \cos(dx+c)) \left(10A \cos^2(dx+c) + 8B \cos^2(dx+c) + 5A \cos(dx+c) + 4B \cos(dx+c) + 3B \right) \frac{a(1 + \cos(dx+c))}{\cos(dx+c)} \frac{1}{\cos(dx+c)^2} \frac{1}{\sin(dx+c)}$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] Timed out

mupad [B] time = 6.16, size = 212, normalized size = 2.10

$$\frac{4 \left(e^{c li+dx li} - 1 \right) \sqrt{a + \frac{a}{\frac{e^{-c li-dx li}}{2} + \frac{e^{c li+dx li}}{2}}} \left(A 5i + B 4i + A e^{c li+dx li} 5i + A e^{c 2i+dx 2i} 10i + A e^{c 3i+dx 3i} 5i + A e^{c 4i+dx 4i} 10i + A e^{c 5i+dx 5i} 5i \right)}{15d \left(e^{c li+dx li} + 1 \right) \left(e^{c 2i+dx 2i} + 1 \right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(1/2))/cos(c + d*x)^2,x)
```

```
[Out] -(4*(exp(c*1i + d*x*1i) - 1)*(a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*
x*1i)/2))^(1/2)*(A*5i + B*4i + A*exp(c*1i + d*x*1i)*5i + A*exp(c*2i + d*x*2
i)*10i + A*exp(c*3i + d*x*3i)*5i + A*exp(c*4i + d*x*4i)*5i + B*exp(c*1i + d
*x*1i)*4i + B*exp(c*2i + d*x*2i)*14i + B*exp(c*3i + d*x*3i)*4i + B*exp(c*4i
+ d*x*4i)*4i))/(15*d*(exp(c*1i + d*x*1i) + 1)*(exp(c*2i + d*x*2i) + 1)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a(\sec(c + dx) + 1)} (A + B \sec(c + dx)) \sec^2(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sec(d*x+c))**(1/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*(A + B*sec(c + d*x))*sec(c + d*x)**2, x
)
```

$$3.121 \quad \int \sec(c+dx) \sqrt{a + a \sec(c+dx)} (A+B \sec(c+dx)) dx$$

Optimal. Leaf size=62

$$\frac{2a(3A+B) \tan(c+dx)}{3d\sqrt{a \sec(c+dx)+a}} + \frac{2B \tan(c+dx) \sqrt{a \sec(c+dx)+a}}{3d}$$

[Out] $2/3*a*(3*A+B)*\tan(d*x+c)/d/(a+a*\sec(d*x+c))^{(1/2)}+2/3*B*(a+a*\sec(d*x+c))^{(1/2)}*\tan(d*x+c)/d$

Rubi [A] time = 0.09, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {4001, 3792}

$$\frac{2a(3A+B) \tan(c+dx)}{3d\sqrt{a \sec(c+dx)+a}} + \frac{2B \tan(c+dx) \sqrt{a \sec(c+dx)+a}}{3d}$$

Antiderivative was successfully verified.

[In] `Int[Sec[c + d*x]*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]`

[Out] $(2*a*(3*A + B)*\tan[c + d*x])/(3*d*\sqrt{a + a*\sec[c + d*x]}) + (2*B*\sqrt{a + a*\sec[c + d*x]}*\tan[c + d*x])/(3*d)$

Rule 3792

`Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] := Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]`

Rule 4001

`Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] := -Simp[(B*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m + 1), 0] && !LtQ[m, -2^(-1)]`

Rubi steps

$$\int \sec(c + dx) \sqrt{a + a \sec(c + dx)} (A + B \sec(c + dx)) dx = \frac{2B \sqrt{a + a \sec(c + dx)} \tan(c + dx)}{3d} + \frac{1}{3}(3A + B) \int \sec(c + dx) dx$$

$$= \frac{2a(3A + B) \tan(c + dx)}{3d \sqrt{a + a \sec(c + dx)}} + \frac{2B \sqrt{a + a \sec(c + dx)} \tan(c + dx)}{3d}$$

Mathematica [A] time = 0.17, size = 53, normalized size = 0.85

$$\frac{2 \tan(c + dx) \sqrt{a(\sec(c + dx) + 1)} ((3A + 2B) \cos(c + dx) + B)}{3d(\cos(c + dx) + 1)}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[c + d*x]*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]

[Out] (2*(B + (3*A + 2*B)*Cos[c + d*x])*Sqrt[a*(1 + Sec[c + d*x])]*Tan[c + d*x])/(3*d*(1 + Cos[c + d*x]))

fricas [A] time = 0.44, size = 66, normalized size = 1.06

$$\frac{2((3A + 2B) \cos(dx + c) + B) \sqrt{\frac{a \cos(dx+c)+a}{\cos(dx+c)}} \sin(dx + c)}{3(d \cos(dx + c)^2 + d \cos(dx + c))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] 2/3*((3*A + 2*B)*cos(d*x + c) + B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^2 + d*cos(d*x + c))

giac [B] time = 0.99, size = 129, normalized size = 2.08

$$\frac{2 \left(3 \sqrt{2} A a^2 \operatorname{sgn}(\cos(dx + c)) + 3 \sqrt{2} B a^2 \operatorname{sgn}(\cos(dx + c)) - (3 \sqrt{2} A a^2 \operatorname{sgn}(\cos(dx + c)) + \sqrt{2} B a^2 \operatorname{sgn}(\cos(dx + c))) \right)}{3 \left(a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - a \right) \sqrt{-a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + a d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] $-2/3*(3*\sqrt{2}*A*a^2*\text{sgn}(\cos(dx + c)) + 3*\sqrt{2}*B*a^2*\text{sgn}(\cos(dx + c)) - (3*\sqrt{2}*A*a^2*\text{sgn}(\cos(dx + c)) + \sqrt{2}*B*a^2*\text{sgn}(\cos(dx + c)))*\tan(1/2*dx + 1/2*c)^2)*\tan(1/2*dx + 1/2*c)/((a*\tan(1/2*dx + 1/2*c)^2 - a)*\sqrt{-a*\tan(1/2*dx + 1/2*c)^2 + a}*d)$

maple [A] time = 1.76, size = 70, normalized size = 1.13

$$\frac{2(-1 + \cos(dx + c))(3A \cos(dx + c) + 2B \cos(dx + c) + B) \sqrt{\frac{a(1 + \cos(dx + c))}{\cos(dx + c)}}}{3d \sin(dx + c) \cos(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sec(dx+c)*(a+a*sec(dx+c))^(1/2)*(A+B*sec(dx+c)),x)`

[Out] $-2/3/d*(-1+\cos(dx+c))*(3*A*\cos(dx+c)+2*B*\cos(dx+c)+B)*(a*(1+\cos(dx+c)))/\cos(dx+c)^(1/2)/\sin(dx+c)/\cos(dx+c)$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(dx+c)*(a+a*sec(dx+c))^(1/2)*(A+B*sec(dx+c)),x, algorithm="maxima")`

[Out] Timed out

mupad [B] time = 1.97, size = 159, normalized size = 2.56

$$\frac{2 \sqrt{\frac{a(\cos(c+dx)+1)}{\cos(c+dx)}} (6A \sin(c+dx) + 6B \sin(c+dx) + 6A \sin(2c+2dx) + 6A \sin(3c+3dx) + 3A \sin(4c+4dx))}{3d (12 \cos(c+dx) + 8 \cos(2c+2dx) + 4 \cos(3c+3dx) + \cos(4c+4dx) + 7)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B/cos(c + dx))*(a + a/cos(c + dx))^(1/2))/cos(c + dx),x)`

[Out] $(2*((a*(\cos(c + dx) + 1))/\cos(c + dx))^(1/2)*(6*A*\sin(c + dx) + 6*B*\sin(c + dx) + 6*A*\sin(2*c + 2*d*x) + 6*A*\sin(3*c + 3*d*x) + 3*A*\sin(4*c + 4*d*x) + 8*B*\sin(2*c + 2*d*x) + 6*B*\sin(3*c + 3*d*x) + 2*B*\sin(4*c + 4*d*x)))/(3*d*(12*\cos(c + d*x) + 8*\cos(2*c + 2*d*x) + 4*\cos(3*c + 3*d*x) + \cos(4*c + 4*d*x) + 7))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a(\sec(c + dx) + 1)} (A + B \sec(c + dx)) \sec(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))**(1/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*(A + B*sec(c + d*x))*sec(c + d*x), x)
```

3.122 $\int \sqrt{a + a \sec(c + dx)} (A + B \sec(c + dx)) dx$

Optimal. Leaf size=66

$$\frac{2\sqrt{a} A \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}}\right)}{d} + \frac{2aB \tan(c+dx)}{d\sqrt{a \sec(c+dx)+a}}$$

[Out] $2*A*\arctan(a^{(1/2)}*\tan(d*x+c)/(a+a*\sec(d*x+c))^{(1/2)})*a^{(1/2)}/d+2*a*B*\tan(d*x+c)/d/(a+a*\sec(d*x+c))^{(1/2)}$

Rubi [A] time = 0.09, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3915, 3774, 203, 3792}

$$\frac{2\sqrt{a} A \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}}\right)}{d} + \frac{2aB \tan(c+dx)}{d\sqrt{a \sec(c+dx)+a}}$$

Antiderivative was successfully verified.

[In] `Int[Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]`

[Out] $(2*\text{Sqrt}[a]*A*\text{ArcTan}[(\text{Sqrt}[a]*\text{Tan}[c + d*x])/\text{Sqrt}[a + a*\text{Sec}[c + d*x]])/d + (2*a*B*\text{Tan}[c + d*x])/(d*\text{Sqrt}[a + a*\text{Sec}[c + d*x]])$

Rule 203

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])`

Rule 3774

`Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] := Dist[(-2*b)/d, Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]`

Rule 3792

`Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] := Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]`

Rule 3915

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.)), x_Symbol] := Dist[c, Int[Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[d, Int[Sqrt[a + b*Csc[e + f*x]]*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{a + a \sec(c + dx)} (A + B \sec(c + dx)) dx = A \int \sqrt{a + a \sec(c + dx)} dx + B \int \sec(c + dx) \sqrt{a + a \sec(c + dx)} dx$$

$$= \frac{2aB \tan(c + dx)}{d\sqrt{a + a \sec(c + dx)}} - \frac{(2aA) \text{Subst}\left(\int \frac{1}{a+x^2} dx, x, -\frac{a \tan(c+dx)}{\sqrt{a+a \sec(c+dx)}}\right)}{d}$$

$$= \frac{2\sqrt{a} A \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a+a \sec(c+dx)}}\right)}{d} + \frac{2aB \tan(c + dx)}{d\sqrt{a + a \sec(c + dx)}}$$

Mathematica [A] time = 0.32, size = 76, normalized size = 1.15

$$\frac{\sec\left(\frac{1}{2}(c + dx)\right) \sqrt{a(\sec(c + dx) + 1)} \left(\sqrt{2} A \sin^{-1}\left(\sqrt{2} \sin\left(\frac{1}{2}(c + dx)\right)\right) \sqrt{\cos(c + dx)} + 2B \sin\left(\frac{1}{2}(c + dx)\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```

```
[Out] (Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(Sqrt[2]*A*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]] + 2*B*Sin[(c + d*x)/2]))/d
```

fricas [A] time = 0.45, size = 235, normalized size = 3.56

$$\left[\frac{(A \cos(dx + c) + A)\sqrt{-a} \log\left(\frac{2a \cos(dx+c)^2 - 2\sqrt{-a} \sqrt{\frac{a \cos(dx+c)+a}{\cos(dx+c)}} \cos(dx+c) \sin(dx+c) + a \cos(dx+c) - a}{\cos(dx+c)+1}\right) + 2B \sqrt{\frac{a \cos(dx+c)+a}{\cos(dx+c)}}}{d \cos(dx + c) + d} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] [((A*cos(d*x + c) + A)*sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*B*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d), -2*((A*cos(d*x + c) + A)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - B*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d)]
```

giac [B] time = 1.45, size = 193, normalized size = 2.92

$$\frac{2\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}B\operatorname{sgn}(\cos(dx+c))\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a} + \frac{A\sqrt{-a}a\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-4\sqrt{2}|a|-6a}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+4\sqrt{2}|a|-6a}\right)\right)\operatorname{sgn}(\cos(dx+c))}{|a|}$$

d

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -(2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*B*a*sgn(cos(d*x + c))*tan(1/2*d*x + 1/2*c)/(a*tan(1/2*d*x + 1/2*c)^2 - a) + A*sqrt(-a)*a*log(abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sqrt(2)*abs(a) - 6*a))*sgn(cos(d*x + c))/abs(a)/d
```

maple [B] time = 1.58, size = 118, normalized size = 1.79

$$\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(A\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\sqrt{\frac{-2\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)+2B\cos(dx+c)-2B\right)}{d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] -1/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(A*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+2*B*cos(d*x+c)-2*B)/sin(d*x+c)
```

maxima [B] time = 0.50, size = 147, normalized size = 2.23

$$A\sqrt{a} \arctan\left(\left(\cos(2dx + 2c)^2 + \sin(2dx + 2c)^2 + 2\cos(2dx + 2c) + 1\right)^{\frac{1}{4}} \sin\left(\frac{1}{2} \arctan(\sin(2dx + 2c), \cos(2dx + 2c) + 1)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] A*sqrt(a)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + cos(d*x + c))/d

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \left(A + \frac{B}{\cos(c + dx)} \right) \sqrt{a + \frac{a}{\cos(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(1/2), x)

[Out] int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a(\sec(c + dx) + 1)} (A + B \sec(c + dx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))**(1/2)*(A+B*sec(d*x+c)),x)

[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*(A + B*sec(c + d*x)), x)

$$3.123 \quad \int \cos(c+dx) \sqrt{a + a \sec(c + dx)} (A + B \sec(c + dx)) dx$$

Optimal. Leaf size=68

$$\frac{\sqrt{a}(A + 2B) \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}}\right)}{d} + \frac{aA \sin(c + dx)}{d\sqrt{a \sec(c + dx) + a}}$$

[Out] (A+2*B)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*a^(1/2)/d+a*A*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)

Rubi [A] time = 0.11, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {4015, 3774, 203}

$$\frac{\sqrt{a}(A + 2B) \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}}\right)}{d} + \frac{aA \sin(c + dx)}{d\sqrt{a \sec(c + dx) + a}}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]), x]

[Out] (Sqrt[a]*(A + 2*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]])/d + (a*A*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 3774

Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_.)], x_Symbol] := Dist[(-2*b)/d, Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 4015

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^n*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] := Simp[(A*b^2*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e

$+ f*x])^{(n + 1), x], x] /; \text{FreeQ}[\{a, b, d, e, f, A, B\}, x] \ \&\& \ \text{NeQ}[A*b - a*B, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{NeQ}[A*b*(2*n + 1) + 2*a*B*n, 0] \ \&\& \ \text{LtQ}[n, 0]$

Rubi steps

$$\begin{aligned} \int \cos(c + dx) \sqrt{a + a \sec(c + dx)} (A + B \sec(c + dx)) dx &= \frac{aA \sin(c + dx)}{d \sqrt{a + a \sec(c + dx)}} + \frac{1}{2} (A + 2B) \int \sqrt{a + a \sec(c + dx)} dx \\ &= \frac{aA \sin(c + dx)}{d \sqrt{a + a \sec(c + dx)}} - \frac{(a(A + 2B)) \text{Subst}\left(\int \frac{1}{a+x^2} dx\right)}{d} \\ &= \frac{\sqrt{a} (A + 2B) \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a+a \sec(c+dx)}}\right)}{d} + \frac{aA \sin(c + dx)}{d \sqrt{a + a \sec(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.25, size = 93, normalized size = 1.37

$$\frac{\sqrt{\cos(c + dx)} \sec\left(\frac{1}{2}(c + dx)\right) \sqrt{a(\sec(c + dx) + 1)} \left(\sqrt{2} (A + 2B) \sin^{-1}\left(\sqrt{2} \sin\left(\frac{1}{2}(c + dx)\right)\right)\right) + 2A \sin\left(\frac{1}{2}(c + dx)\right)}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]

[Out] (Sqrt[Cos[c + d*x]]*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(Sqrt[2]*(A + 2*B)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*A*Sqrt[Cos[c + d*x]]*Sin[(c + d*x)/2]))/(2*d)

fricas [A] time = 0.54, size = 261, normalized size = 3.84

$$\left[\frac{2A \sqrt{\frac{a \cos(dx+c)+a}{\cos(dx+c)}} \cos(dx+c) \sin(dx+c) + ((A + 2B) \cos(dx+c) + A + 2B) \sqrt{-a} \log\left(\frac{2a \cos(dx+c)^2 - 2\sqrt{-a} \sqrt{\frac{a \cos(dx+c)+a}{\cos(dx+c)}}}{2(d \cos(dx+c) + d)}\right)}{2(d \cos(dx+c) + d)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] $\left[\frac{1}{2} * (2 * A * \sqrt{(a * \cos(dx + c) + a) / \cos(dx + c)}) * \cos(dx + c) * \sin(dx + c) + ((A + 2 * B) * \cos(dx + c) + A + 2 * B) * \sqrt{-a} * \log((2 * a * \cos(dx + c))^2 - 2 * \sqrt{-a} * \sqrt{(a * \cos(dx + c) + a) / \cos(dx + c)}) * \cos(dx + c) * \sin(dx + c) + a * \cos(dx + c) - a) / (\cos(dx + c) + 1)) / (d * \cos(dx + c) + d), (A * \sqrt{(a * \cos(dx + c) + a) / \cos(dx + c)}) * \cos(dx + c) * \sin(dx + c) - ((A + 2 * B) * \cos(dx + c) + A + 2 * B) * \sqrt{a} * \arctan(\sqrt{(a * \cos(dx + c) + a) / \cos(dx + c)}) * \cos(dx + c) / (\sqrt{a} * \sin(dx + c))) / (d * \cos(dx + c) + d) \right]$

giac [B] time = 5.77, size = 336, normalized size = 4.94

$$\left(A \sqrt{-a} \operatorname{sgn}(\cos(dx + c)) + 2B \sqrt{-a} \operatorname{sgn}(\cos(dx + c)) \right) \log \left(\left(\sqrt{-a} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - \sqrt{-a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + a} \right)^2 + a \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x, algorithm="giac")`

[Out] $-1/2 * ((A * \sqrt{-a} * \operatorname{sgn}(\cos(dx + c)) + 2 * B * \sqrt{-a} * \operatorname{sgn}(\cos(dx + c))) * \log(\operatorname{abs}(\sqrt{-a} * \tan(1/2 * dx + 1/2 * c) - \sqrt{-a * \tan(1/2 * dx + 1/2 * c)^2 + a}))^2 - a * (2 * \sqrt{2} + 3)) - (A * \sqrt{-a} * \operatorname{sgn}(\cos(dx + c)) + 2 * B * \sqrt{-a} * \operatorname{sgn}(\cos(dx + c))) * \log(\operatorname{abs}(\sqrt{-a} * \tan(1/2 * dx + 1/2 * c) - \sqrt{-a * \tan(1/2 * dx + 1/2 * c)^2 + a}))^2 + a * (2 * \sqrt{2} - 3)) + 4 * (3 * \sqrt{2}) * (\sqrt{-a} * \tan(1/2 * dx + 1/2 * c) - \sqrt{-a * \tan(1/2 * dx + 1/2 * c)^2 + a})^2 * A * \sqrt{-a} * a * \operatorname{sgn}(\cos(dx + c)) - \sqrt{2} * A * \sqrt{-a} * a^2 * \operatorname{sgn}(\cos(dx + c))) / ((\sqrt{-a} * \tan(1/2 * dx + 1/2 * c) - \sqrt{-a * \tan(1/2 * dx + 1/2 * c)^2 + a})^4 - 6 * (\sqrt{-a} * \tan(1/2 * dx + 1/2 * c) - \sqrt{-a * \tan(1/2 * dx + 1/2 * c)^2 + a})^2 * a + a^2) / d$

maple [B] time = 1.52, size = 198, normalized size = 2.91

$$\left(A \sqrt{2} \operatorname{arctanh} \left(\frac{\sqrt{-\frac{2 \cos(dx+c)}{1+\cos(dx+c)}} \sin(dx+c) \sqrt{2}}{2 \cos(dx+c)} \right) \sqrt{-\frac{2 \cos(dx+c)}{1+\cos(dx+c)}} \sin(dx+c) + 2B \sqrt{-\frac{2 \cos(dx+c)}{1+\cos(dx+c)}} \operatorname{arctanh} \left(\frac{\sqrt{-\frac{2 \cos(dx+c)}{1+\cos(dx+c)}}}{2 \cos(dx+c)} \right) \right) / (2d \sin(dx+c))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(d*x+c)*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x)`

[Out] $-1/2/d * (A^2)^{1/2} * \operatorname{arctanh}(1/2 * (-2 * \cos(dx+c) / (1 + \cos(dx+c))))^{1/2} * \sin(dx+c) / \cos(dx+c) * 2^{1/2} * (-2 * \cos(dx+c) / (1 + \cos(dx+c)))^{1/2} * \sin(dx+c) + 2 * B * (-2 * \cos(dx+c) / (1 + \cos(dx+c)))^{1/2} * \operatorname{arctanh}(1/2 * (-2 * \cos(dx+c) / (1 + \cos(dx+c))))^{1/2} * \sin(dx+c) / \cos(dx+c)$

c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*2^(1/2)*sin(d*x+c)+2*A*cos(d*x+c)
 ^2-2*A*cos(d*x+c))*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)

maxima [B] time = 0.64, size = 939, normalized size = 13.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] 1/4*(4*B*sqrt(a)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + cos(d*x + c)) + (2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sin(d*x + c) - (cos(d*x + c) - 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))))*sqrt(a) + sqrt(a)*(arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))) + 1) - arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))) - 1) - arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) + arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1)))A)/d

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c + dx) \left(A + \frac{B}{\cos(c + dx)} \right) \sqrt{a + \frac{a}{\cos(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(c + d*x)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(1/2), x)`

[Out] `int(cos(c + d*x)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(1/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a(\sec(c + dx) + 1)} (A + B \sec(c + dx)) \cos(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)*(a+a*sec(d*x+c))**(1/2)*(A+B*sec(d*x+c)), x)`

[Out] `Integral(sqrt(a*(sec(c + d*x) + 1))*(A + B*sec(c + d*x))*cos(c + d*x), x)`

3.124 $\int \cos^2(c+dx) \sqrt{a + a \sec(c + dx)} (A+B \sec(c+dx)) dx$

Optimal. Leaf size=117

$$\frac{a(3A + 4B) \sin(c + dx)}{4d\sqrt{a \sec(c + dx) + a}} + \frac{\sqrt{a} (3A + 4B) \tan^{-1} \left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}} \right)}{4d} + \frac{aA \sin(c + dx) \cos(c + dx)}{2d\sqrt{a \sec(c + dx) + a}}$$

[Out] $1/4*(3*A+4*B)*\arctan(a^{(1/2)}*\tan(d*x+c)/(a+a*\sec(d*x+c))^{(1/2)})*a^{(1/2)}/d+1/4*a*(3*A+4*B)*\sin(d*x+c)/d/(a+a*\sec(d*x+c))^{(1/2)}+1/2*a*A*\cos(d*x+c)*\sin(d*x+c)/d/(a+a*\sec(d*x+c))^{(1/2)}$

Rubi [A] time = 0.18, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {4015, 3805, 3774, 203}

$$\frac{a(3A + 4B) \sin(c + dx)}{4d\sqrt{a \sec(c + dx) + a}} + \frac{\sqrt{a} (3A + 4B) \tan^{-1} \left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}} \right)}{4d} + \frac{aA \sin(c + dx) \cos(c + dx)}{2d\sqrt{a \sec(c + dx) + a}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Cos}[c + d*x]^2*\text{Sqrt}[a + a*\text{Sec}[c + d*x]]*(A + B*\text{Sec}[c + d*x]),x]$

[Out] $(\text{Sqrt}[a]*(3*A + 4*B)*\text{ArcTan}[(\text{Sqrt}[a]*\text{Tan}[c + d*x])/(\text{Sqrt}[a + a*\text{Sec}[c + d*x]])]/(4*d) + (a*(3*A + 4*B)*\text{Sin}[c + d*x])/(4*d*\text{Sqrt}[a + a*\text{Sec}[c + d*x]]) + (a*A*\text{Cos}[c + d*x]*\text{Sin}[c + d*x])/(2*d*\text{Sqrt}[a + a*\text{Sec}[c + d*x]]))$

Rule 203

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1*\text{ArcTan}[(\text{Rt}[b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[b, 2]), x] /; \text{FreeQ}\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{GtQ}[b, 0])$

Rule 3774

$\text{Int}[\text{Sqrt}[\text{csc}[c_ + (d_)*(x_)]*(b_ + (a_))], x_Symbol] \rightarrow \text{Dist}[(-2*b)/d, \text{Subst}[\text{Int}[1/(a + x^2), x], x, (b*\text{Cot}[c + d*x])/(\text{Sqrt}[a + b*\text{Csc}[c + d*x]])], x] /; \text{FreeQ}\{a, b, c, d\}, x] \ \&\& \ \text{EqQ}[a^2 - b^2, 0]$

Rule 3805

$\text{Int}[(\text{csc}[e_ + (f_)*(x_)]*(d_))^{(n_)}*\text{Sqrt}[\text{csc}[e_ + (f_)*(x_)]*(b_ + (a_))], x_Symbol] \rightarrow \text{Simp}[(a*\text{Cot}[e + f*x]*(d*\text{Csc}[e + f*x])^n)/(f*n*\text{Sqrt}[a$

+ b*Csc[e + f*x]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]

Rule 4015

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> Simp[(A*b^2*Cos[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]

Rubi steps

$$\begin{aligned} \int \cos^2(c + dx) \sqrt{a + a \sec(c + dx)} (A + B \sec(c + dx)) dx &= \frac{aA \cos(c + dx) \sin(c + dx)}{2d\sqrt{a + a \sec(c + dx)}} + \frac{1}{4}(3A + 4B) \int \cos(c + dx) \sqrt{a + a \sec(c + dx)} dx \\ &= \frac{a(3A + 4B) \sin(c + dx)}{4d\sqrt{a + a \sec(c + dx)}} + \frac{aA \cos(c + dx) \sin(c + dx)}{2d\sqrt{a + a \sec(c + dx)}} \\ &= \frac{a(3A + 4B) \sin(c + dx)}{4d\sqrt{a + a \sec(c + dx)}} + \frac{aA \cos(c + dx) \sin(c + dx)}{2d\sqrt{a + a \sec(c + dx)}} \\ &= \frac{\sqrt{a} (3A + 4B) \tan^{-1}\left(\frac{\sqrt{a} \tan(c + dx)}{\sqrt{a + a \sec(c + dx)}}\right)}{4d} + \frac{a(3A + 4B) \sin(c + dx)}{4d\sqrt{a + a \sec(c + dx)}} \end{aligned}$$

Mathematica [C] time = 0.41, size = 117, normalized size = 1.00

$$\frac{\tan\left(\frac{1}{2}(c + dx)\right) \sqrt{a(\sec(c + dx) + 1)} \left(2A\sqrt{1 - \sec(c + dx)} {}_2F_1\left(\frac{1}{2}, 3; \frac{3}{2}; 1 - \sec(c + dx)\right) + B(\cos(c + dx)\sqrt{1 - \sec(c + dx)} + \sin(c + dx))\right)}{d\sqrt{1 - \sec(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]^2*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]), x]
 [Out] ((B*(ArcTanh[Sqrt[1 - Sec[c + d*x]]] + Cos[c + d*x]*Sqrt[1 - Sec[c + d*x]]) + 2*A*Hypergeometric2F1[1/2, 3, 3/2, 1 - Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]])*Sqrt[a*(1 + Sec[c + d*x]]*Tan[(c + d*x)/2])/(d*Sqrt[1 - Sec[c + d*x]])

fricas [A] time = 0.54, size = 308, normalized size = 2.63

$$\left[\frac{((3A + 4B) \cos(dx + c) + 3A + 4B) \sqrt{-a} \log \left(\frac{2a \cos(dx+c)^2 - 2\sqrt{-a} \sqrt{\frac{a \cos(dx+c)+a}{\cos(dx+c)}} \cos(dx+c) \sin(dx+c) + a \cos(dx+c) - a}{\cos(dx+c)+1} \right)}{8(d \cos(dx + c) + d)} + 2 \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] [1/8*(((3*A + 4*B)*cos(d*x + c) + 3*A + 4*B)*sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(2*A*cos(d*x + c)^2 + (3*A + 4*B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d), -1/4*(((3*A + 4*B)*cos(d*x + c) + 3*A + 4*B)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - (2*A*cos(d*x + c)^2 + (3*A + 4*B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d)]

giac [B] time = 1.58, size = 630, normalized size = 5.38

$$(3A\sqrt{-a} \operatorname{sgn}(\cos(dx + c)) + 4B\sqrt{-a} \operatorname{sgn}(\cos(dx + c))) \log \left(\left| \sqrt{-a} \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - \sqrt{-a \tan^2\left(\frac{1}{2}dx + \frac{1}{2}c\right) + a} \right| \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] -1/8*(((3*A*sqrt(-a)*sgn(cos(d*x + c)) + 4*B*sqrt(-a)*sgn(cos(d*x + c))))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3))) - (3*A*sqrt(-a)*sgn(cos(d*x + c)) + 4*B*sqrt(-a)*sgn(cos(d*x + c))) * log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))) - 4*sqrt(2)*(5*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*A*sqrt(-a)*a*sgn(cos(d*x + c)) - 12*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*B*sqrt(-a)*a*sgn(cos(d*x + c)) + 19*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*A*sqrt(-a)*a*sgn(cos(d*x + c)) + 19*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*B*sqrt(-a)*a*sgn(cos(d*x + c)))

$$\begin{aligned} & /2*c) - \sqrt{-a*\tan(1/2*d*x + 1/2*c)^2 + a})^4*A*\sqrt{-a}*a^2*\text{sgn}(\cos(d*x + \\ & c)) + 76*(\sqrt{-a}*\tan(1/2*d*x + 1/2*c) - \sqrt{-a*\tan(1/2*d*x + 1/2*c)^2 + \\ & a})^4*B*\sqrt{-a}*a^2*\text{sgn}(\cos(d*x + c)) - 17*(\sqrt{-a}*\tan(1/2*d*x + 1/2*c) \\ & - \sqrt{-a*\tan(1/2*d*x + 1/2*c)^2 + a})^2*A*\sqrt{-a}*a^3*\text{sgn}(\cos(d*x + c)) \\ & - 36*(\sqrt{-a}*\tan(1/2*d*x + 1/2*c) - \sqrt{-a*\tan(1/2*d*x + 1/2*c)^2 + a})^2 \\ & *B*\sqrt{-a}*a^3*\text{sgn}(\cos(d*x + c)) + A*\sqrt{-a}*a^4*\text{sgn}(\cos(d*x + c)) + 4*B \\ & *\sqrt{-a}*a^4*\text{sgn}(\cos(d*x + c)))/((\sqrt{-a}*\tan(1/2*d*x + 1/2*c) - \sqrt{-a* \\ & \tan(1/2*d*x + 1/2*c)^2 + a})^4 - 6*(\sqrt{-a}*\tan(1/2*d*x + 1/2*c) - \sqrt{-a* \\ & \tan(1/2*d*x + 1/2*c)^2 + a})^2*a + a^2)^2)/d \end{aligned}$$

maple [B] time = 1.70, size = 398, normalized size = 3.40

$$\left(-3A \cos(dx + c) \operatorname{arctanh} \left(\frac{\sqrt{-\frac{2 \cos(dx+c)}{1+\cos(dx+c)}} \sin(dx+c) \sqrt{2}}{2 \cos(dx+c)} \right) \left(-\frac{2 \cos(dx+c)}{1+\cos(dx+c)} \right)^{\frac{3}{2}} \sin(dx + c) \sqrt{2} - 4B \cos(dx + c) \operatorname{arctanh} \left(\frac{\sqrt{-\frac{2 \cos(dx+c)}{1+\cos(dx+c)}} \sin(dx+c) \sqrt{2}}{2 \cos(dx+c)} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(d*x+c)^2*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x)`

[Out]
$$\begin{aligned} & -1/16/d*(-3*A*\cos(d*x+c)*\operatorname{arctanh}(1/2*(-2*\cos(d*x+c)/(1+\cos(d*x+c))))^{(1/2)*\sin(d*x+c)/\cos(d*x+c)*2^{(1/2)}}*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{(3/2)*\sin(d*x+c)*2^{(1/2)}}-4*B*\cos(d*x+c)*\operatorname{arctanh}(1/2*(-2*\cos(d*x+c)/(1+\cos(d*x+c))))^{(1/2)*\sin(d*x+c)/\cos(d*x+c)*2^{(1/2)}}*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{(3/2)*\sin(d*x+c)*2^{(1/2)}}-3*A*2^{(1/2)*\operatorname{arctanh}(1/2*(-2*\cos(d*x+c)/(1+\cos(d*x+c))))^{(1/2)*\sin(d*x+c)/\cos(d*x+c)*2^{(1/2)}}*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{(3/2)*\sin(d*x+c)-4*B*2^{(1/2)*\operatorname{arctanh}(1/2*(-2*\cos(d*x+c)/(1+\cos(d*x+c))))^{(1/2)*\sin(d*x+c)/\cos(d*x+c)*2^{(1/2)}}*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{(3/2)*\sin(d*x+c)+8*A*\cos(d*x+c)^4+4*A*\cos(d*x+c)^3+16*B*\cos(d*x+c)^3-12*A*\cos(d*x+c)^2-16*B*\cos(d*x+c)^2)*(a*(1+\cos(d*x+c))/\cos(d*x+c))^{(1/2)}/\cos(d*x+c)/\sin(d*x+c)} \end{aligned}$$

maxima [B] time = 0.77, size = 1851, normalized size = 15.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x, algorithm="maxima")`

[Out]
$$\begin{aligned} & 1/16*((2*(\cos(2*d*x + 2*c))^2 + \sin(2*d*x + 2*c))^2 + 2*\cos(2*d*x + 2*c) + 1) \\ & ^{(1/4)*((\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))*\sin(2*d*x + 2 \\ & *c) - (\cos(2*d*x + 2*c) - 2)*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2 \\ & *c)))) + \sin(2*d*x + 2*c))*\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2 \\ & *c) + 1)) + ((\cos(2*d*x + 2*c) - 2)*\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2* \end{aligned}$$

$$\begin{aligned}
& d*x + 2*c))) + \sin(2*d*x + 2*c)*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x \\
& + 2*c))) - \cos(2*d*x + 2*c) + 2)*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d \\
& *x + 2*c) + 1)))*\sqrt{a} + 3*\sqrt{a}*(\arctan2((\cos(2*d*x + 2*c)^2 + \sin(2*d \\
& *x + 2*c)^2 + 2*\cos(2*d*x + 2*c) + 1)^{1/4}*(\cos(1/2*\arctan2(\sin(2*d*x + 2* \\
& c), \cos(2*d*x + 2*c))))*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) + \\
& 1)) - \cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) + 1))*\sin(1/2*\ar \\
& \tan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))))), (\cos(2*d*x + 2*c)^2 + \sin(2*d*x \\
& + 2*c)^2 + 2*\cos(2*d*x + 2*c) + 1)^{1/4}*(\cos(1/2*\arctan2(\sin(2*d*x + 2*c) \\
& , \cos(2*d*x + 2*c) + 1))*\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) \\
&)) + \sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) + 1))*\sin(1/2*\arcta \\
& n2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))) + 1) - \arctan2((\cos(2*d*x + 2*c)^2 \\
& + \sin(2*d*x + 2*c)^2 + 2*\cos(2*d*x + 2*c) + 1)^{1/4}*(\cos(1/2*\arctan2(\sin(\\
& 2*d*x + 2*c), \cos(2*d*x + 2*c))))*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d* \\
& x + 2*c) + 1)) - \cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) + 1))*s \\
& in(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))))), (\cos(2*d*x + 2*c)^2 + \\
& \sin(2*d*x + 2*c)^2 + 2*\cos(2*d*x + 2*c) + 1)^{1/4}*(\cos(1/2*\arctan2(\sin(2* \\
& d*x + 2*c), \cos(2*d*x + 2*c) + 1))*\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2* \\
& d*x + 2*c))) + \sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) + 1))*\sin \\
& (1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))) - 1) - \arctan2((\cos(2*d* \\
& x + 2*c)^2 + \sin(2*d*x + 2*c)^2 + 2*\cos(2*d*x + 2*c) + 1)^{1/4}*\sin(1/2*\arc \\
& \tan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) + 1)), (\cos(2*d*x + 2*c)^2 + \sin(2* \\
& d*x + 2*c)^2 + 2*\cos(2*d*x + 2*c) + 1)^{1/4}*\cos(1/2*\arctan2(\sin(2*d*x + 2* \\
& c), \cos(2*d*x + 2*c) + 1)) + 1) + \arctan2((\cos(2*d*x + 2*c)^2 + \sin(2*d*x + \\
& 2*c)^2 + 2*\cos(2*d*x + 2*c) + 1)^{1/4}*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), c \\
& os(2*d*x + 2*c) + 1)), (\cos(2*d*x + 2*c)^2 + \sin(2*d*x + 2*c)^2 + 2*\cos(2*d \\
& *x + 2*c) + 1)^{1/4}*\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) + 1 \\
&)) - 1))) * A + 4*(2*(\cos(2*d*x + 2*c)^2 + \sin(2*d*x + 2*c)^2 + 2*\cos(2*d*x + \\
& 2*c) + 1)^{1/4}*(\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) + 1))* \\
& \sin(d*x + c) - (\cos(d*x + c) - 1)*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d \\
& *x + 2*c) + 1)))*\sqrt{a} + \sqrt{a}*(\arctan2(-(\cos(2*d*x + 2*c)^2 + \sin(2*d* \\
& x + 2*c)^2 + 2*\cos(2*d*x + 2*c) + 1)^{1/4}*(\cos(1/2*\arctan2(\sin(2*d*x + 2*c) \\
&), \cos(2*d*x + 2*c) + 1))*\sin(d*x + c) - \cos(d*x + c)*\sin(1/2*\arctan2(\sin(2 \\
& *d*x + 2*c), \cos(2*d*x + 2*c) + 1))), (\cos(2*d*x + 2*c)^2 + \sin(2*d*x + 2*c) \\
&)^2 + 2*\cos(2*d*x + 2*c) + 1)^{1/4}*(\cos(d*x + c)*\cos(1/2*\arctan2(\sin(2*d*x \\
& + 2*c), \cos(2*d*x + 2*c) + 1)) + \sin(d*x + c)*\sin(1/2*\arctan2(\sin(2*d*x + \\
& 2*c), \cos(2*d*x + 2*c) + 1)))) + 1) - \arctan2(-(\cos(2*d*x + 2*c)^2 + \sin(2*d \\
& *x + 2*c)^2 + 2*\cos(2*d*x + 2*c) + 1)^{1/4}*(\cos(1/2*\arctan2(\sin(2*d*x + 2* \\
& c), \cos(2*d*x + 2*c) + 1))*\sin(d*x + c) - \cos(d*x + c)*\sin(1/2*\arctan2(\sin(\\
& 2*d*x + 2*c), \cos(2*d*x + 2*c) + 1))), (\cos(2*d*x + 2*c)^2 + \sin(2*d*x + 2* \\
& c)^2 + 2*\cos(2*d*x + 2*c) + 1)^{1/4}*(\cos(d*x + c)*\cos(1/2*\arctan2(\sin(2*d* \\
& x + 2*c), \cos(2*d*x + 2*c) + 1)) + \sin(d*x + c)*\sin(1/2*\arctan2(\sin(2*d*x + \\
& 2*c), \cos(2*d*x + 2*c) + 1)))) - 1) - \arctan2((\cos(2*d*x + 2*c)^2 + \sin(2*d \\
& *x + 2*c)^2 + 2*\cos(2*d*x + 2*c) + 1)^{1/4}*\sin(1/2*\arctan2(\sin(2*d*x + 2*c) \\
&), \cos(2*d*x + 2*c) + 1)), (\cos(2*d*x + 2*c)^2 + \sin(2*d*x + 2*c)^2 + 2*\cos \\
& (2*d*x + 2*c) + 1)^{1/4}*\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)
\end{aligned}$$

+ 1)) + 1) + arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))) * B) / d

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c + dx)^2 \left(A + \frac{B}{\cos(c + dx)} \right) \sqrt{a + \frac{a}{\cos(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^2*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(1/2), x)

[Out] int(cos(c + d*x)^2*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a(\sec(c + dx) + 1)} (A + B \sec(c + dx)) \cos^2(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**2*(a+a*sec(d*x+c))**(1/2)*(A+B*sec(d*x+c)), x)

[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*(A + B*sec(c + d*x))*cos(c + d*x)**2, x)

$$3.125 \quad \int \cos^3(c+dx) \sqrt{a + a \sec(c + dx)} (A+B \sec(c+dx)) dx$$

Optimal. Leaf size=160

$$\frac{a(5A + 6B) \sin(c + dx)}{8d\sqrt{a \sec(c + dx) + a}} + \frac{\sqrt{a} (5A + 6B) \tan^{-1} \left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}} \right)}{8d} + \frac{a(5A + 6B) \sin(c + dx) \cos(c + dx)}{12d\sqrt{a \sec(c + dx) + a}} + \frac{aA \sin(c + dx)}{3d\sqrt{a \sec(c + dx) + a}}$$

[Out] 1/8*(5*A+6*B)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*a^(1/2)/d+1/8*a*(5*A+6*B)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/12*a*(5*A+6*B)*cos(d*x+c)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/3*a*A*cos(d*x+c)^2*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)

Rubi [A] time = 0.24, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {4015, 3805, 3774, 203}

$$\frac{a(5A + 6B) \sin(c + dx)}{8d\sqrt{a \sec(c + dx) + a}} + \frac{\sqrt{a} (5A + 6B) \tan^{-1} \left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}} \right)}{8d} + \frac{a(5A + 6B) \sin(c + dx) \cos(c + dx)}{12d\sqrt{a \sec(c + dx) + a}} + \frac{aA \sin(c + dx)}{3d\sqrt{a \sec(c + dx) + a}}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^3*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]

[Out] (Sqrt[a]*(5*A + 6*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]])/(8*d) + (a*(5*A + 6*B)*Sin[c + d*x])/(8*d*Sqrt[a + a*Sec[c + d*x]]) + (a*(5*A + 6*B)*Cos[c + d*x]*Sin[c + d*x])/(12*d*Sqrt[a + a*Sec[c + d*x]]) + (a*A*Cos[c + d*x]^2*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]])

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 3774

Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] := Dist[(-2*b)/d, Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n)*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.)
+ (a_.)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
+ b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n)*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.)
+ (a_.)]*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[
e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rubi steps

$$\begin{aligned} \int \cos^3(c + dx) \sqrt{a + a \sec(c + dx)} (A + B \sec(c + dx)) dx &= \frac{aA \cos^2(c + dx) \sin(c + dx)}{3d\sqrt{a + a \sec(c + dx)}} + \frac{1}{6}(5A + 6B) \int \cos^2(c + dx) \sqrt{a + a \sec(c + dx)} dx \\ &= \frac{a(5A + 6B) \cos(c + dx) \sin(c + dx)}{12d\sqrt{a + a \sec(c + dx)}} + \frac{aA \cos^2(c + dx) \sin(c + dx)}{3d\sqrt{a + a \sec(c + dx)}} \\ &= \frac{a(5A + 6B) \sin(c + dx)}{8d\sqrt{a + a \sec(c + dx)}} + \frac{a(5A + 6B) \cos(c + dx) \sin(c + dx)}{12d\sqrt{a + a \sec(c + dx)}} \\ &= \frac{a(5A + 6B) \sin(c + dx)}{8d\sqrt{a + a \sec(c + dx)}} + \frac{a(5A + 6B) \cos(c + dx) \sin(c + dx)}{12d\sqrt{a + a \sec(c + dx)}} \\ &= \frac{\sqrt{a} (5A + 6B) \tan^{-1} \left(\frac{\sqrt{a} \tan(c + dx)}{\sqrt{a + a \sec(c + dx)}} \right)}{8d} + \frac{a(5A + 6B) \sin(c + dx)}{8d\sqrt{a + a \sec(c + dx)}} \end{aligned}$$

Mathematica [C] time = 0.19, size = 70, normalized size = 0.44

$$\frac{2 \tan \left(\frac{1}{2}(c + dx) \right) \sqrt{a(\sec(c + dx) + 1)} \left(A {}_2F_1 \left(\frac{1}{2}, 4; \frac{3}{2}; 1 - \sec(c + dx) \right) + B {}_2F_1 \left(\frac{1}{2}, 3; \frac{3}{2}; 1 - \sec(c + dx) \right) \right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```

[Out] $(2*(B*\text{Hypergeometric2F1}[1/2, 3, 3/2, 1 - \text{Sec}[c + d*x]] + A*\text{Hypergeometric2F1}[1/2, 4, 3/2, 1 - \text{Sec}[c + d*x]])*\text{Sqrt}[a*(1 + \text{Sec}[c + d*x])]*\text{Tan}[(c + d*x)/2])/d$

fricas [A] time = 0.51, size = 346, normalized size = 2.16

$$\frac{3((5A + 6B)\cos(dx + c) + 5A + 6B)\sqrt{-a} \log\left(\frac{2a\cos(dx+c)^2 - 2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right) + 2}{48(d\cos(dx + c) + d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x, algorithm="fricas")`

[Out] $[1/48*(3*((5*A + 6*B)*\cos(d*x + c) + 5*A + 6*B)*\text{sqrt}(-a)*\log((2*a*\cos(d*x + c)^2 - 2*\text{sqrt}(-a)*\text{sqrt}((a*\cos(d*x + c) + a)/\cos(d*x + c))*\cos(d*x + c)*\sin(d*x + c) + a*\cos(d*x + c) - a)/(\cos(d*x + c) + 1)) + 2*(8*A*\cos(d*x + c)^3 + 2*(5*A + 6*B)*\cos(d*x + c)^2 + 3*(5*A + 6*B)*\cos(d*x + c))*\text{sqrt}((a*\cos(d*x + c) + a)/\cos(d*x + c))*\sin(d*x + c))/(d*\cos(d*x + c) + d), -1/24*(3*((5*A + 6*B)*\cos(d*x + c) + 5*A + 6*B)*\text{sqrt}(a)*\arctan(\text{sqrt}((a*\cos(d*x + c) + a)/\cos(d*x + c))*\cos(d*x + c)/(\text{sqrt}(a)*\sin(d*x + c))) - (8*A*\cos(d*x + c)^3 + 2*(5*A + 6*B)*\cos(d*x + c)^2 + 3*(5*A + 6*B)*\cos(d*x + c))*\text{sqrt}((a*\cos(d*x + c) + a)/\cos(d*x + c))*\sin(d*x + c))/(d*\cos(d*x + c) + d)]$

giac [B] time = 4.97, size = 889, normalized size = 5.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x, algorithm="giac")`

[Out] $-1/48*(3*(5*A*\text{sqrt}(-a)*\text{sgn}(\cos(d*x + c)) + 6*B*\text{sqrt}(-a)*\text{sgn}(\cos(d*x + c))))*\log(\text{abs}((\text{sqrt}(-a)*\tan(1/2*d*x + 1/2*c) - \text{sqrt}(-a*\tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*\text{sqrt}(2) + 3))) - 3*(5*A*\text{sqrt}(-a)*\text{sgn}(\cos(d*x + c)) + 6*B*\text{sqrt}(-a)*\text{sgn}(\cos(d*x + c)))*\log(\text{abs}((\text{sqrt}(-a)*\tan(1/2*d*x + 1/2*c) - \text{sqrt}(-a*\tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*\text{sqrt}(2) - 3))) + 4*(63*\text{sqrt}(2)*(\text{sqrt}(-a)*\tan(1/2*d*x + 1/2*c) - \text{sqrt}(-a*\tan(1/2*d*x + 1/2*c)^2 + a))^10*A*\text{sqrt}(-a)*a*\text{sgn}(\cos(d*x + c)) - 30*\text{sqrt}(2)*(\text{sqrt}(-a)*\tan(1/2*d*x + 1/2*c) - \text{sqrt}(-a*\tan(1/2*d*x + 1/2*c)^2 + a))^10*B*\text{sqrt}(-a)*a*\text{sgn}(\cos(d*x + c)) - 369*\text{sqrt}(2)*(\text{sqrt}(-a)*\tan(1/2*d*x + 1/2*c) - \text{sqrt}(-a*\tan(1/2*d*x + 1/2*c)^2 + a))^8*A*s$

```

qrt(-a)*a^2*sgn(cos(d*x + c)) + 66*sqrt(2)*(sqrt(-a)*tan(1/2*d*x + 1/2*c) -
sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^8*B*sqrt(-a)*a^2*sgn(cos(d*x + c)) +
1638*sqrt(2)*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^
2 + a))^6*A*sqrt(-a)*a^3*sgn(cos(d*x + c)) + 756*sqrt(2)*(sqrt(-a)*tan(1/2*
d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*B*sqrt(-a)*a^3*sgn(co
s(d*x + c)) - 1074*sqrt(2)*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2
*d*x + 1/2*c)^2 + a))^4*A*sqrt(-a)*a^4*sgn(cos(d*x + c)) - 732*sqrt(2)*(sqr
t(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*B*sqrt(
-a)*a^4*sgn(cos(d*x + c)) + 171*sqrt(2)*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sq
rt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*sqrt(-a)*a^5*sgn(cos(d*x + c)) + 138
*sqrt(2)*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 +
a))^2*B*sqrt(-a)*a^5*sgn(cos(d*x + c)) - 13*sqrt(2)*A*sqrt(-a)*a^6*sgn(cos(
d*x + c)) - 6*sqrt(2)*B*sqrt(-a)*a^6*sgn(cos(d*x + c)))/((sqrt(-a)*tan(1/2*
d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2
*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^3)/d

```

maple [B] time = 1.89, size = 580, normalized size = 3.62

$$\left(15A \sin(dx + c) \left(-\frac{2 \cos(dx+c)}{1+\cos(dx+c)} \right)^{\frac{5}{2}} \operatorname{arctanh} \left(\frac{\sqrt{-\frac{2 \cos(dx+c)}{1+\cos(dx+c)}} \sin(dx+c) \sqrt{2}}{2 \cos(dx+c)} \right) \sqrt{2} \left(\cos^2(dx + c) \right) + 18B \sin(dx + c) \left(-\frac{2 \cos(dx+c)}{1+\cos(dx+c)} \right)^{\frac{5}{2}} \operatorname{arctanh} \left(\frac{\sqrt{-\frac{2 \cos(dx+c)}{1+\cos(dx+c)}} \sin(dx+c) \sqrt{2}}{2 \cos(dx+c)} \right) \sqrt{2} \left(\cos^2(dx + c) \right) \right) / \left(\left(\sqrt{-a} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - \sqrt{-a \tan^2\left(\frac{1}{2} dx + \frac{1}{2} c\right) + a} \right)^4 - 6 \left(\sqrt{-a} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - \sqrt{-a \tan^2\left(\frac{1}{2} dx + \frac{1}{2} c\right) + a} \right)^2 a + a^2 \right)^3 \right) / d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^3*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x)

```

[Out] -1/192/d*(15*A*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*arctanh(1/2*
(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*2^(1/2)
*cos(d*x+c)^2+18*B*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*arctanh(
1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*2^(
1/2)*cos(d*x+c)^2+30*A*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*arct
anh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))
*2^(1/2)*cos(d*x+c)+36*B*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*ar
ctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2
))*2^(1/2)*cos(d*x+c)+15*A*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*2^(1/2)*arc
tanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2)
)*sin(d*x+c)+18*B*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*2^(1/2)*arctanh(1/2*
(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*sin(d*x
+c)+64*A*cos(d*x+c)^6+16*A*cos(d*x+c)^5+96*B*cos(d*x+c)^5+40*A*cos(d*x+c)^4
+48*B*cos(d*x+c)^4-120*A*cos(d*x+c)^3-144*B*cos(d*x+c)^3)*(a*(1+cos(d*x+c))
/cos(d*x+c))^(1/2)/sin(d*x+c)/cos(d*x+c)^2

```

maxima [B] time = 1.00, size = 2981, normalized size = 18.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] 1/96*((4*(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*
arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))^2 + 2*cos(2/3*arctan2(sin(3*d*
x + 3*c), cos(3*d*x + 3*c))) + 1)^(3/4)*(cos(3/2*arctan2(sin(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3
*d*x + 3*c))) + 1))*sin(3*d*x + 3*c) - (cos(3*d*x + 3*c) - 1)*sin(3/2*arcta
n2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))*sqrt(a) + 6*(cos(2/3*arctan2(sin(
3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(
3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) +
1)^(1/4)*((sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 5*sin(1/
3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*cos(1/2*arctan2(sin(2/3*arc
tan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c)
, cos(3*d*x + 3*c))) + 1)) - (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
3*c))) + 3*cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) - 4)*sin(1
/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*ar
ctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))*sqrt(a) + 15*sqrt(a)*(arc
tan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arc
tan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x +
3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*arctan2(sin(3
*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*
x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) - cos
(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) * sin(1/2*arctan2(sin(2/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*
c), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
+ 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*co
s(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) * cos(1/2*arctan2(sin(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3
*d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) *
sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2
/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))) + 1) - arctan2(-(cos
(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3
*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos
(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c)
), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
+ 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) - cos(1/3*arcta
n2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) * sin(1/2*arctan2(sin(2/3*arctan2(sin
(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*
d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^
2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arct
```

$$\begin{aligned} & \text{an2}(\sin(3*d*x + 3*c), \cos(3*d*x + 3*c))) + 1)^{(1/4)} * (\cos(1/3*\arctan2(\sin(3*d*x \\ & + 3*c), \cos(3*d*x + 3*c))) * \cos(1/2*\arctan2(\sin(2/3*\arctan2(\sin(3*d*x + \\ & 3*c), \cos(3*d*x + 3*c))), \cos(2/3*\arctan2(\sin(3*d*x + 3*c), \cos(3*d*x + 3*c) \\ &))) + 1)) + \sin(1/3*\arctan2(\sin(3*d*x + 3*c), \cos(3*d*x + 3*c))) * \sin(1/2*\ar \\ & \text{ctan2}(\sin(2/3*\arctan2(\sin(3*d*x + 3*c), \cos(3*d*x + 3*c))), \cos(2/3*\arctan2 \\ & (\sin(3*d*x + 3*c), \cos(3*d*x + 3*c))) + 1))) - 1) - \arctan2((\cos(2/3*\arctan \\ & 2(\sin(3*d*x + 3*c), \cos(3*d*x + 3*c)))^2 + \sin(2/3*\arctan2(\sin(3*d*x + 3*c) \\ & , \cos(3*d*x + 3*c)))^2 + 2*\cos(2/3*\arctan2(\sin(3*d*x + 3*c), \cos(3*d*x + 3* \\ & c))) + 1)^{(1/4)} * \sin(1/2*\arctan2(\sin(2/3*\arctan2(\sin(3*d*x + 3*c), \cos(3*d*x \\ & + 3*c))), \cos(2/3*\arctan2(\sin(3*d*x + 3*c), \cos(3*d*x + 3*c))) + 1)), (\cos \\ & (2/3*\arctan2(\sin(3*d*x + 3*c), \cos(3*d*x + 3*c)))^2 + \sin(2/3*\arctan2(\sin(3 \\ & *d*x + 3*c), \cos(3*d*x + 3*c)))^2 + 2*\cos(2/3*\arctan2(\sin(3*d*x + 3*c), \cos \\ & (3*d*x + 3*c))) + 1)^{(1/4)} * \cos(1/2*\arctan2(\sin(2/3*\arctan2(\sin(3*d*x + 3*c) \\ & , \cos(3*d*x + 3*c))), \cos(2/3*\arctan2(\sin(3*d*x + 3*c), \cos(3*d*x + 3*c))) \\ & + 1)) + 1) + \arctan2((\cos(2/3*\arctan2(\sin(3*d*x + 3*c), \cos(3*d*x + 3*c)))^2 \\ & + \sin(2/3*\arctan2(\sin(3*d*x + 3*c), \cos(3*d*x + 3*c)))^2 + 2*\cos(2/3*\arct \\ & \text{an2}(\sin(3*d*x + 3*c), \cos(3*d*x + 3*c))) + 1)^{(1/4)} * \sin(1/2*\arctan2(\sin(2/3 \\ & *\arctan2(\sin(3*d*x + 3*c), \cos(3*d*x + 3*c))), \cos(2/3*\arctan2(\sin(3*d*x + \\ & 3*c), \cos(3*d*x + 3*c))) + 1)), (\cos(2/3*\arctan2(\sin(3*d*x + 3*c), \cos(3*d* \\ & x + 3*c)))^2 + \sin(2/3*\arctan2(\sin(3*d*x + 3*c), \cos(3*d*x + 3*c)))^2 + 2*c \\ & \text{os}(2/3*\arctan2(\sin(3*d*x + 3*c), \cos(3*d*x + 3*c))) + 1)^{(1/4)} * \cos(1/2*\arct \\ & \text{an2}(\sin(2/3*\arctan2(\sin(3*d*x + 3*c), \cos(3*d*x + 3*c))), \cos(2/3*\arctan2(s \\ & \text{in}(3*d*x + 3*c), \cos(3*d*x + 3*c))) + 1)) - 1))) * A + 6*(2*(\cos(2*d*x + 2*c) \\ & ^2 + \sin(2*d*x + 2*c)^2 + 2*\cos(2*d*x + 2*c) + 1)^{(1/4)} * ((\cos(1/2*\arctan2(s \\ & \text{in}(2*d*x + 2*c), \cos(2*d*x + 2*c))) * \sin(2*d*x + 2*c) - (\cos(2*d*x + 2*c) - \\ & 2)*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + \sin(2*d*x + 2*c)) \\ & *\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) + 1)) + ((\cos(2*d*x + 2 \\ & *c) - 2)*\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + \sin(2*d*x + \\ & 2*c)*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) - \cos(2*d*x + 2* \\ & c) + 2)*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) + 1))) * \text{sqrt}(a) + \\ & 3*\text{sqrt}(a) * (\arctan2((\cos(2*d*x + 2*c)^2 + \sin(2*d*x + 2*c)^2 + 2*\cos(2*d*x \\ & + 2*c) + 1)^{(1/4)} * (\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) * \sin \\ & (1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) + 1)) - \cos(1/2*\arctan2(\sin \\ & (2*d*x + 2*c), \cos(2*d*x + 2*c) + 1)) * \sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos \\ & (2*d*x + 2*c))))), (\cos(2*d*x + 2*c)^2 + \sin(2*d*x + 2*c)^2 + 2*\cos(2*d*x + \\ & 2*c) + 1)^{(1/4)} * (\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) + 1)) * \c \\ & \text{os}(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + \sin(1/2*\arctan2(\sin(2 \\ & *d*x + 2*c), \cos(2*d*x + 2*c) + 1)) * \sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2 \\ & *d*x + 2*c)))) + 1) - \arctan2((\cos(2*d*x + 2*c)^2 + \sin(2*d*x + 2*c)^2 + 2* \\ & \text{cos}(2*d*x + 2*c) + 1)^{(1/4)} * (\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + \\ & 2*c))) * \sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) + 1)) - \cos(1/2*a \\ & \text{rctan2}(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) + 1)) * \sin(1/2*\arctan2(\sin(2*d*x + \\ & 2*c), \cos(2*d*x + 2*c))))), (\cos(2*d*x + 2*c)^2 + \sin(2*d*x + 2*c)^2 + 2*c \\ & \text{os}(2*d*x + 2*c) + 1)^{(1/4)} * (\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2* \\ & c) + 1)) * \cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + \sin(1/2*\arct \end{aligned}$$

```
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c)))) - 1) - arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2
*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
+ 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
+ 1) + arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*
c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (co
s(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))) * B) / d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c + dx)^3 \left(A + \frac{B}{\cos(c + dx)} \right) \sqrt{a + \frac{a}{\cos(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(1/2), x)
```

```
[Out] int(cos(c + d*x)^3*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a(\sec(c + dx) + 1)} (A + B \sec(c + dx)) \cos^3(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sec(d*x+c))**(1/2)*(A+B*sec(d*x+c)), x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*(A + B*sec(c + d*x))*cos(c + d*x)**3, x
)
```

$$3.126 \quad \int \cos^4(c+dx) \sqrt{a + a \sec(c + dx)} (A+B \sec(c+dx)) dx$$

Optimal. Leaf size=203

$$\frac{5a(7A + 8B) \sin(c + dx)}{64d\sqrt{a \sec(c + dx) + a}} + \frac{5\sqrt{a} (7A + 8B) \tan^{-1} \left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}} \right)}{64d} + \frac{a(7A + 8B) \sin(c + dx) \cos^2(c + dx)}{24d\sqrt{a \sec(c + dx) + a}} + \frac{5a(7A + 8B) \sin(c + dx)}{96d\sqrt{a \sec(c + dx) + a}}$$

[Out] $5/64*(7*A+8*B)*\arctan(a^{(1/2)}*\tan(d*x+c)/(a+a*\sec(d*x+c))^{(1/2)})*a^{(1/2)}/d+5/64*a*(7*A+8*B)*\sin(d*x+c)/d/(a+a*\sec(d*x+c))^{(1/2)}+5/96*a*(7*A+8*B)*\cos(d*x+c)*\sin(d*x+c)/d/(a+a*\sec(d*x+c))^{(1/2)}+1/24*a*(7*A+8*B)*\cos(d*x+c)^2*\sin(d*x+c)/d/(a+a*\sec(d*x+c))^{(1/2)}+1/4*a*A*\cos(d*x+c)^3*\sin(d*x+c)/d/(a+a*\sec(d*x+c))^{(1/2)}$

Rubi [A] time = 0.30, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {4015, 3805, 3774, 203}

$$\frac{5a(7A + 8B) \sin(c + dx)}{64d\sqrt{a \sec(c + dx) + a}} + \frac{5\sqrt{a} (7A + 8B) \tan^{-1} \left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}} \right)}{64d} + \frac{a(7A + 8B) \sin(c + dx) \cos^2(c + dx)}{24d\sqrt{a \sec(c + dx) + a}} + \frac{5a(7A + 8B) \sin(c + dx)}{96d\sqrt{a \sec(c + dx) + a}}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^4*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]

[Out] $(5*\text{Sqrt}[a]*(7*A + 8*B)*\text{ArcTan}[(\text{Sqrt}[a]*\text{Tan}[c + d*x])/(\text{Sqrt}[a + a*\text{Sec}[c + d*x]])]/(64*d) + (5*a*(7*A + 8*B)*\text{Sin}[c + d*x])/((64*d*\text{Sqrt}[a + a*\text{Sec}[c + d*x]]) + (5*a*(7*A + 8*B)*\text{Cos}[c + d*x]*\text{Sin}[c + d*x])/((96*d*\text{Sqrt}[a + a*\text{Sec}[c + d*x]]) + (a*(7*A + 8*B)*\text{Cos}[c + d*x]^2*\text{Sin}[c + d*x])/((24*d*\text{Sqrt}[a + a*\text{Sec}[c + d*x]]) + (a*A*\text{Cos}[c + d*x]^3*\text{Sin}[c + d*x])/((4*d*\text{Sqrt}[a + a*\text{Sec}[c + d*x]])$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 3774

Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_.)], x_Symbol] := Dist[(-2*b)/d, Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.)
+ (a_.)], x_Symbol] := Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
+ b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.)
+ (a_.)]*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
+ f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \cos^4(c + dx) \sqrt{a + a \sec(c + dx)} (A + B \sec(c + dx)) dx &= \frac{aA \cos^3(c + dx) \sin(c + dx)}{4d\sqrt{a + a \sec(c + dx)}} + \frac{1}{8}(7A + 8B) \int \cos^3(c + dx) \sqrt{a + a \sec(c + dx)} dx \\
&= \frac{a(7A + 8B) \cos^2(c + dx) \sin(c + dx)}{24d\sqrt{a + a \sec(c + dx)}} + \frac{aA \cos^3(c + dx) \sin(c + dx)}{4d\sqrt{a + a \sec(c + dx)}} \\
&= \frac{5a(7A + 8B) \cos(c + dx) \sin(c + dx)}{96d\sqrt{a + a \sec(c + dx)}} + \frac{a(7A + 8B) \cos^3(c + dx) \sin(c + dx)}{24d\sqrt{a + a \sec(c + dx)}} \\
&= \frac{5a(7A + 8B) \sin(c + dx)}{64d\sqrt{a + a \sec(c + dx)}} + \frac{5a(7A + 8B) \cos(c + dx)}{96d\sqrt{a + a \sec(c + dx)}} \\
&= \frac{5a(7A + 8B) \sin(c + dx)}{64d\sqrt{a + a \sec(c + dx)}} + \frac{5a(7A + 8B) \cos(c + dx)}{96d\sqrt{a + a \sec(c + dx)}} \\
&= \frac{5\sqrt{a} (7A + 8B) \tan^{-1}\left(\frac{\sqrt{a} \tan(c + dx)}{\sqrt{a + a \sec(c + dx)}}\right)}{64d} + \frac{5a(7A + 8B) \cos(c + dx)}{64d\sqrt{a + a \sec(c + dx)}}
\end{aligned}$$

Mathematica [C] time = 0.18, size = 70, normalized size = 0.34

$$\frac{2 \tan\left(\frac{1}{2}(c + dx)\right) \sqrt{a(\sec(c + dx) + 1)} \left(A {}_2F_1\left(\frac{1}{2}, 5; \frac{3}{2}; 1 - \sec(c + dx)\right) + B {}_2F_1\left(\frac{1}{2}, 4; \frac{3}{2}; 1 - \sec(c + dx)\right) \right)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]^4*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]

[Out] (2*(B*Hypergeometric2F1[1/2, 4, 3/2, 1 - Sec[c + d*x]] + A*Hypergeometric2F1[1/2, 5, 3/2, 1 - Sec[c + d*x]])*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/d

fricas [A] time = 0.53, size = 380, normalized size = 1.87

$$\left[\frac{15((7A + 8B)\cos(dx + c) + 7A + 8B)\sqrt{-a} \log\left(\frac{2a\cos(dx+c)^2 - 2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c) + a\cos(dx+c)-a}{\cos(dx+c)+1}\right)}{\dots} \right] + 2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] [1/384*(15*((7*A + 8*B)*cos(d*x + c) + 7*A + 8*B)*sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(48*A*cos(d*x + c)^4 + 8*(7*A + 8*B)*cos(d*x + c)^3 + 10*(7*A + 8*B)*cos(d*x + c)^2 + 15*(7*A + 8*B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d), -1/192*(15*((7*A + 8*B)*cos(d*x + c) + 7*A + 8*B)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - (48*A*cos(d*x + c)^4 + 8*(7*A + 8*B)*cos(d*x + c)^3 + 10*(7*A + 8*B)*cos(d*x + c)^2 + 15*(7*A + 8*B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d)]

giac [B] time = 1.81, size = 1080, normalized size = 5.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] -1/384*(15*(7*A*sqrt(-a)*sgn(cos(d*x + c)) + 8*B*sqrt(-a)*sgn(cos(d*x + c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3))) - 15*(7*A*sqrt(-a)*sgn(cos(d*x + c)) + 8*B*sqrt(-a)*sgn(cos(d*x + c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))) - 4*sqrt(2)*(279*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^14*A*sqrt(-

$$\begin{aligned}
& a) * a * \operatorname{sgn}(\cos(dx + c)) - 504 * (\sqrt{-a}) * \tan(1/2 * dx + 1/2 * c) - \sqrt{-a * \tan(1/2 * dx + 1/2 * c)^2 + a} \\
& ^{14} * B * \sqrt{-a} * a * \operatorname{sgn}(\cos(dx + c)) + 285 * (\sqrt{-a}) * \tan(1/2 * dx + 1/2 * c) - \sqrt{-a * \tan(1/2 * dx + 1/2 * c)^2 + a} \\
& ^{12} * A * \sqrt{-a} * a^2 * \operatorname{sgn}(\cos(dx + c)) + 5976 * (\sqrt{-a}) * \tan(1/2 * dx + 1/2 * c) - \sqrt{-a * \tan(1/2 * dx + 1/2 * c)^2 + a} \\
& ^{12} * B * \sqrt{-a} * a^2 * \operatorname{sgn}(\cos(dx + c)) - 4605 * (\sqrt{-a}) * \tan(1/2 * dx + 1/2 * c) - \sqrt{-a * \tan(1/2 * dx + 1/2 * c)^2 + a} \\
& ^{10} * A * \sqrt{-a} * a^3 * \operatorname{sgn}(\cos(dx + c)) - 31320 * (\sqrt{-a}) * \tan(1/2 * dx + 1/2 * c) - \sqrt{-a * \tan(1/2 * dx + 1/2 * c)^2 + a} \\
& ^{10} * B * \sqrt{-a} * a^3 * \operatorname{sgn}(\cos(dx + c)) + 37281 * (\sqrt{-a}) * \tan(1/2 * dx + 1/2 * c) - \sqrt{-a * \tan(1/2 * dx + 1/2 * c)^2 + a} \\
& ^8 * A * \sqrt{-a} * a^4 * \operatorname{sgn}(\cos(dx + c)) + 90168 * (\sqrt{-a}) * \tan(1/2 * dx + 1/2 * c) - \sqrt{-a * \tan(1/2 * dx + 1/2 * c)^2 + a} \\
& ^8 * B * \sqrt{-a} * a^4 * \operatorname{sgn}(\cos(dx + c)) - 35643 * (\sqrt{-a}) * \tan(1/2 * dx + 1/2 * c) - \sqrt{-a * \tan(1/2 * dx + 1/2 * c)^2 + a} \\
& ^6 * A * \sqrt{-a} * a^5 * \operatorname{sgn}(\cos(dx + c)) - 66024 * (\sqrt{-a}) * \tan(1/2 * dx + 1/2 * c) - \sqrt{-a * \tan(1/2 * dx + 1/2 * c)^2 + a} \\
& ^6 * B * \sqrt{-a} * a^5 * \operatorname{sgn}(\cos(dx + c)) + 9175 * (\sqrt{-a}) * \tan(1/2 * dx + 1/2 * c) - \sqrt{-a * \tan(1/2 * dx + 1/2 * c)^2 + a} \\
& ^4 * A * \sqrt{-a} * a^6 * \operatorname{sgn}(\cos(dx + c)) + 16904 * (\sqrt{-a}) * \tan(1/2 * dx + 1/2 * c) - \sqrt{-a * \tan(1/2 * dx + 1/2 * c)^2 + a} \\
& ^4 * B * \sqrt{-a} * a^6 * \operatorname{sgn}(\cos(dx + c)) - 1311 * (\sqrt{-a}) * \tan(1/2 * dx + 1/2 * c) - \sqrt{-a * \tan(1/2 * dx + 1/2 * c)^2 + a} \\
& ^2 * A * \sqrt{-a} * a^7 * \operatorname{sgn}(\cos(dx + c)) - 1992 * (\sqrt{-a}) * \tan(1/2 * dx + 1/2 * c) - \sqrt{-a * \tan(1/2 * dx + 1/2 * c)^2 + a} \\
& ^2 * B * \sqrt{-a} * a^7 * \operatorname{sgn}(\cos(dx + c)) + 43 * A * \sqrt{-a} * a^8 * \operatorname{sgn}(\cos(dx + c)) + 104 * B * \sqrt{-a} * a^8 * \operatorname{sgn}(\cos(dx + c)) \\
&) / ((\sqrt{-a}) * \tan(1/2 * dx + 1/2 * c) - \sqrt{-a * \tan(1/2 * dx + 1/2 * c)^2 + a})^4 - 6 * (\sqrt{-a}) * \tan(1/2 * dx + 1/2 * c) - \sqrt{-a * \tan(1/2 * dx + 1/2 * c)^2 + a} \\
& ^2 * a + a^2)^4) / d
\end{aligned}$$

maple [B] time = 1.68, size = 762, normalized size = 3.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\int (\cos(dx+c))^4 (a+a*\sec(dx+c))^{(1/2)} (A+B*\sec(dx+c)), x$

[Out] $\frac{1}{3072} \frac{1}{d} (105 A^2)^{(1/2)} (-2 \cos(dx+c) / (1 + \cos(dx+c)))^{(7/2)} \operatorname{arctanh}(1/2 * (-2 \cos(dx+c) / (1 + \cos(dx+c)))^{(1/2)} \sin(dx+c) / \cos(dx+c) * 2^{(1/2)}) \sin(dx+c) \cos(dx+c)^3 + 120 B * 2^{(1/2)} (-2 \cos(dx+c) / (1 + \cos(dx+c)))^{(7/2)} \operatorname{arctanh}(1/2 * (-2 \cos(dx+c) / (1 + \cos(dx+c)))^{(1/2)} \sin(dx+c) / \cos(dx+c) * 2^{(1/2)}) \sin(dx+c) \cos(dx+c)^3 + 315 A^2)^{(1/2)} (-2 \cos(dx+c) / (1 + \cos(dx+c)))^{(7/2)} \operatorname{arctanh}(1/2 * (-2 \cos(dx+c) / (1 + \cos(dx+c)))^{(1/2)} \sin(dx+c) / \cos(dx+c) * 2^{(1/2)}) \sin(dx+c) \cos(dx+c)^2 + 360 B * 2^{(1/2)} (-2 \cos(dx+c) / (1 + \cos(dx+c)))^{(7/2)} \operatorname{arctanh}(1/2 * (-2 \cos(dx+c) / (1 + \cos(dx+c)))^{(1/2)} \sin(dx+c) / \cos(dx+c) * 2^{(1/2)}) \sin(dx+c) \cos(dx+c)^2 + 315 A^2)^{(1/2)} (-2 \cos(dx+c) / (1 + \cos(dx+c)))^{(7/2)} \operatorname{arctanh}(1/2 * (-2 \cos(dx+c) / (1 + \cos(dx+c)))^{(1/2)} \sin(dx+c) / \cos(dx+c) * 2^{(1/2)}) \sin(dx+c) \cos(dx+c) + 360 B * 2^{(1/2)} (-2 \cos(dx+c) / (1 + \cos(dx+c)))^{(7/2)} \operatorname{arctanh}(1/2 * (-2 \cos(dx+c) / (1 + \cos(dx+c)))^{(1/2)} \sin(dx+c) / \cos(dx+c) * 2^{(1/2)}) \sin(dx+c) \cos(dx+c) + 105 A * (-2 \cos(dx+c) / (1 + \cos(dx+c)))^{(7/2)}$

$$\begin{aligned} & /2) * 2^{(1/2)} * \operatorname{arctanh}(1/2 * (-2 * \cos(d*x+c) / (1 + \cos(d*x+c)))^{(1/2)} * \sin(d*x+c) / \cos \\ & (d*x+c) * 2^{(1/2)}) * \sin(d*x+c) + 120 * B * (-2 * \cos(d*x+c) / (1 + \cos(d*x+c)))^{(7/2)} * 2^{(1 \\ & /2)} * \operatorname{arctanh}(1/2 * (-2 * \cos(d*x+c) / (1 + \cos(d*x+c)))^{(1/2)} * \sin(d*x+c) / \cos(d*x+c) * \\ & 2^{(1/2)}) * \sin(d*x+c) - 768 * A * \cos(d*x+c)^8 - 128 * A * \cos(d*x+c)^7 - 1024 * B * \cos(d*x+c) \\ & ^7 - 224 * A * \cos(d*x+c)^6 - 256 * B * \cos(d*x+c)^6 - 560 * A * \cos(d*x+c)^5 - 640 * B * \cos(d*x+c) \\ & ^5 + 1680 * A * \cos(d*x+c)^4 + 1920 * B * \cos(d*x+c)^4 * (a * (1 + \cos(d*x+c)) / \cos(d*x+c))^{(1/2)} / \sin(d*x+c) / \cos(d*x+c)^3 \end{aligned}$$

maxima [B] time = 1.42, size = 8561, normalized size = 42.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out]
$$\begin{aligned} & 1/768 * (8 * (4 * (\cos(2/3 * \operatorname{arctan2}(\sin(3*d*x + 3*c), \cos(3*d*x + 3*c)))^2 + \sin(2 \\ & /3 * \operatorname{arctan2}(\sin(3*d*x + 3*c), \cos(3*d*x + 3*c)))^2 + 2 * \cos(2/3 * \operatorname{arctan2}(\sin(3 \\ & *d*x + 3*c), \cos(3*d*x + 3*c))) + 1)^{(3/4)} * (\cos(3/2 * \operatorname{arctan2}(\sin(2/3 * \operatorname{arctan2} \\ & (\sin(3*d*x + 3*c), \cos(3*d*x + 3*c))), \cos(2/3 * \operatorname{arctan2}(\sin(3*d*x + 3*c), \cos \\ & (3*d*x + 3*c))) + 1)) * \sin(3*d*x + 3*c) - (\cos(3*d*x + 3*c) - 1) * \sin(3/2 * \operatorname{ar} \\ & \operatorname{ctan2}(\sin(2/3 * \operatorname{arctan2}(\sin(3*d*x + 3*c), \cos(3*d*x + 3*c))), \cos(2/3 * \operatorname{arctan2} \\ & (\sin(3*d*x + 3*c), \cos(3*d*x + 3*c))) + 1))) * \sqrt{a} + 6 * (\cos(2/3 * \operatorname{arctan2}(s \\ & \operatorname{in}(3*d*x + 3*c), \cos(3*d*x + 3*c)))^2 + \sin(2/3 * \operatorname{arctan2}(\sin(3*d*x + 3*c), \cos \\ & (3*d*x + 3*c)))^2 + 2 * \cos(2/3 * \operatorname{arctan2}(\sin(3*d*x + 3*c), \cos(3*d*x + 3*c))) \\ &) + 1)^{(1/4)} * ((\sin(2/3 * \operatorname{arctan2}(\sin(3*d*x + 3*c), \cos(3*d*x + 3*c))) + 5 * \sin \\ & (1/3 * \operatorname{arctan2}(\sin(3*d*x + 3*c), \cos(3*d*x + 3*c)))) * \cos(1/2 * \operatorname{arctan2}(\sin(2/3 * \\ & \operatorname{arctan2}(\sin(3*d*x + 3*c), \cos(3*d*x + 3*c))), \cos(2/3 * \operatorname{arctan2}(\sin(3*d*x + 3 \\ & *c), \cos(3*d*x + 3*c))) + 1)) - (\cos(2/3 * \operatorname{arctan2}(\sin(3*d*x + 3*c), \cos(3*d* \\ & x + 3*c))) + 3 * \cos(1/3 * \operatorname{arctan2}(\sin(3*d*x + 3*c), \cos(3*d*x + 3*c))) - 4) * \sin \\ & (1/2 * \operatorname{arctan2}(\sin(2/3 * \operatorname{arctan2}(\sin(3*d*x + 3*c), \cos(3*d*x + 3*c))), \cos(2/3 \\ & * \operatorname{arctan2}(\sin(3*d*x + 3*c), \cos(3*d*x + 3*c))) + 1))) * \sqrt{a} + 15 * \sqrt{a} * (\\ & \operatorname{arctan2}(-(\cos(2/3 * \operatorname{arctan2}(\sin(3*d*x + 3*c), \cos(3*d*x + 3*c)))^2 + \sin(2/3 * \\ & \operatorname{arctan2}(\sin(3*d*x + 3*c), \cos(3*d*x + 3*c)))^2 + 2 * \cos(2/3 * \operatorname{arctan2}(\sin(3*d* \\ & x + 3*c), \cos(3*d*x + 3*c))) + 1)^{(1/4)} * (\cos(1/2 * \operatorname{arctan2}(\sin(2/3 * \operatorname{arctan2}(\sin \\ & (3*d*x + 3*c), \cos(3*d*x + 3*c))), \cos(2/3 * \operatorname{arctan2}(\sin(3*d*x + 3*c), \cos(3 \\ & *d*x + 3*c))) + 1)) * \sin(1/3 * \operatorname{arctan2}(\sin(3*d*x + 3*c), \cos(3*d*x + 3*c))) - \\ & \cos(1/3 * \operatorname{arctan2}(\sin(3*d*x + 3*c), \cos(3*d*x + 3*c))) * \sin(1/2 * \operatorname{arctan2}(\sin(2/ \\ & 3 * \operatorname{arctan2}(\sin(3*d*x + 3*c), \cos(3*d*x + 3*c))), \cos(2/3 * \operatorname{arctan2}(\sin(3*d*x + \\ & 3*c), \cos(3*d*x + 3*c))) + 1))), (\cos(2/3 * \operatorname{arctan2}(\sin(3*d*x + 3*c), \cos(3* \\ & d*x + 3*c)))^2 + \sin(2/3 * \operatorname{arctan2}(\sin(3*d*x + 3*c), \cos(3*d*x + 3*c)))^2 + 2 \\ & * \cos(2/3 * \operatorname{arctan2}(\sin(3*d*x + 3*c), \cos(3*d*x + 3*c))) + 1)^{(1/4)} * (\cos(1/3 * \operatorname{ar} \\ & \operatorname{ctan2}(\sin(3*d*x + 3*c), \cos(3*d*x + 3*c))) * \cos(1/2 * \operatorname{arctan2}(\sin(2/3 * \operatorname{arctan2} \\ & (\sin(3*d*x + 3*c), \cos(3*d*x + 3*c))), \cos(2/3 * \operatorname{arctan2}(\sin(3*d*x + 3*c), \cos \\ & (3*d*x + 3*c))) + 1)) + \sin(1/3 * \operatorname{arctan2}(\sin(3*d*x + 3*c), \cos(3*d*x + 3*c) \end{aligned}$$

$$\begin{aligned}
& x + 4*c)) * \cos(1/2 * \arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c))) - (32 * (\cos(4*d*x + 4*c)^2 + \sin(4*d*x + 4*c)^2 - 2 * \cos(4*d*x + 4*c) + 1) * \cos(1/2 * \arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c)))^2 + 32 * (\cos(4*d*x + 4*c)^2 + \sin(4*d*x + 4*c)^2 + 2 * \cos(4*d*x + 4*c) + 1) * \sin(1/2 * \arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c)))^2 + 8 * \cos(4*d*x + 4*c)^2 + 2 * (16 * \cos(4*d*x + 4*c)^2 + 16 * \sin(4*d*x + 4*c)^2 - 7 * \cos(4*d*x + 4*c) - 9) * \cos(1/2 * \arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c))) + 8 * \sin(4*d*x + 4*c)^2 - 2 * (64 * \cos(1/2 * \arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c))) * \sin(4*d*x + 4*c) + 7 * \sin(4*d*x + 4*c)) * \sin(1/2 * \arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c))) + 9 * \cos(4*d*x + 4*c)) * \sin(3/4 * \arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c))) - 36 * (4 * \cos(1/2 * \arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c))) * \sin(4*d*x + 4*c)^2 + \sin(4*d*x + 4*c)^2 * \sin(1/2 * \arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c)))) * \cos(3/2 * \arctan2(\sin(1/2 * \arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c))), \cos(1/2 * \arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c)))) + 1) - (9 * \cos(4*d*x + 4*c)^3 + 4 * (9 * \cos(4*d*x + 4*c)^3 + (9 * \cos(4*d*x + 4*c) + 8) * \sin(4*d*x + 4*c)^2 - 10 * \cos(4*d*x + 4*c)^2 - 7 * \cos(4*d*x + 4*c) + 8) * \cos(1/2 * \arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c)))^2 + (9 * \cos(4*d*x + 4*c) + 8) * \sin(4*d*x + 4*c)^2 + 4 * (9 * \cos(4*d*x + 4*c)^3 + (9 * \cos(4*d*x + 4*c) + 8) * \sin(4*d*x + 4*c)^2 + 26 * \cos(4*d*x + 4*c)^2 + 25 * \cos(4*d*x + 4*c) + 8) * \sin(1/2 * \arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c)))^2 + 8 * \cos(4*d*x + 4*c)^2 - (32 * (\cos(4*d*x + 4*c)^2 + \sin(4*d*x + 4*c)^2 - 2 * \cos(4*d*x + 4*c) + 1) * \cos(1/2 * \arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c)))^2 + 32 * (\cos(4*d*x + 4*c)^2 + \sin(4*d*x + 4*c)^2 + 2 * \cos(4*d*x + 4*c) + 1) * \sin(1/2 * \arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c)))^2 + 8 * \cos(4*d*x + 4*c)^2 + 2 * (16 * \cos(4*d*x + 4*c)^2 + 16 * \sin(4*d*x + 4*c)^2 - 7 * \cos(4*d*x + 4*c) - 9) * \cos(1/2 * \arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c))) + 8 * \sin(4*d*x + 4*c)^2 - 2 * (64 * \cos(1/2 * \arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c))) * \sin(4*d*x + 4*c) + 7 * \sin(4*d*x + 4*c)) * \sin(1/2 * \arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c))) + 9 * \cos(4*d*x + 4*c)) * \cos(3/4 * \arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c))) + 4 * (9 * \cos(4*d*x + 4*c)^3 + (9 * \cos(4*d*x + 4*c) + 8) * \sin(4*d*x + 4*c)^2 - \cos(4*d*x + 4*c)^2 - 8 * \cos(4*d*x + 4*c)) * \cos(1/2 * \arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c))) - 9 * (2 * \cos(1/2 * \arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c))) * \sin(4*d*x + 4*c) - 2 * (\cos(4*d*x + 4*c) + 1) * \sin(1/2 * \arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c))) + \sin(4*d*x + 4*c)) * \sin(3/4 * \arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c))) - 4 * (4 * (9 * \cos(4*d*x + 4*c) + 8) * \cos(1/2 * \arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c))) * \sin(4*d*x + 4*c) + (9 * \cos(4*d*x + 4*c) + 8) * \sin(4*d*x + 4*c)) * \sin(1/2 * \arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c)))) * \sin(3/2 * \arctan2(\sin(1/2 * \arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c))), \cos(1/2 * \arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c)))) + 1) * \sqrt{a} - 6 * (\cos(1/2 * \arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c)))^2 + \sin(1/2 * \arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c)))^2 + 2 * \cos(1/2 * \arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c))) + 1)^(1/4) * ((64 * (\cos(4*d*x + 4*c)^2 + \sin(4*d*x + 4*c)^2 + 2 * \cos(4*d*x + 4*c) + 1) * \sin(1/2 * \arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c)))^3 + 20 * (\sin(4*d*x + 4*c)^3 + (\cos(4*d*x + 4*c)^2 - 2 * \cos(4*d*x + 4*c) + 1) * \sin(4*d*x + 4*c) + 8 * (\cos(4*d*x + 4*c)^2 + \sin(4*d*x + 4*c)^2 - 2 * \cos(4*d*x + 4*c) + 1) * \sin(1/4 * \arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c)))
\end{aligned}$$

$$\begin{aligned}
&)) * \cos(1/2 * \arctan2(\sin(4 * d * x + 4 * c), \cos(4 * d * x + 4 * c)))^2 + 5 * \cos(4 * d * x + 4 * c)^2 * \sin(4 * d * x + 4 * c) + 5 * \sin(4 * d * x + 4 * c)^3 + 4 * (5 * \sin(4 * d * x + 4 * c)^3 + (\\
&5 * \cos(4 * d * x + 4 * c)^2 + 10 * \cos(4 * d * x + 4 * c) - 11) * \sin(4 * d * x + 4 * c) - 64 * \cos(\\
&1/2 * \arctan2(\sin(4 * d * x + 4 * c), \cos(4 * d * x + 4 * c))) * \sin(4 * d * x + 4 * c) + 40 * (\cos \\
&(4 * d * x + 4 * c)^2 + \sin(4 * d * x + 4 * c)^2 + 2 * \cos(4 * d * x + 4 * c) + 1) * \sin(1/4 * \arctan2(\sin(4 * d * x + 4 * c), \\
&\cos(4 * d * x + 4 * c)))^2 + 10 * (2 * \sin(4 * d * x + 4 * c)^3 + 2 * (\cos(4 * d * x + 4 * c)^2 - \\
&\cos(4 * d * x + 4 * c)) * \sin(4 * d * x + 4 * c) + \cos(1/4 * \arctan2(\sin(4 * d * x + 4 * c), \cos(\\
&4 * d * x + 4 * c))) * \sin(4 * d * x + 4 * c) + (16 * \cos(4 * d * x + 4 * c)^2 + 16 * \sin(4 * d * x + 4 * c)^2 - \\
&17 * \cos(4 * d * x + 4 * c) + 1) * \sin(1/4 * \arctan2(\sin(4 * d * x + 4 * c), \cos(4 * d * x + 4 * c))) * \cos(1/2 * \arctan2(\sin(4 * d * x + 4 * c), \cos(4 * d * x + 4 * c))) + 5 * \cos(1/ \\
&4 * \arctan2(\sin(4 * d * x + 4 * c), \cos(4 * d * x + 4 * c))) * \sin(4 * d * x + 4 * c) + 2 * (32 * (\cos \\
&(4 * d * x + 4 * c)^2 + \sin(4 * d * x + 4 * c)^2 - 2 * \cos(4 * d * x + 4 * c) + 1) * \cos(1/2 * \arctan2(\sin(4 * d * x + 4 * c), \cos(4 * d * x + 4 * c)))^2 + 8 * \cos(4 * d * x + 4 * c)^2 + 8 * (4 * c \\
&\cos(4 * d * x + 4 * c)^2 - \sin(4 * d * x + 4 * c)^2 - 40 * \sin(4 * d * x + 4 * c) * \sin(1/4 * \arctan2(\sin(4 * d * x + 4 * c), \cos(4 * d * x + 4 * c))) - 4 * \cos(4 * d * x + 4 * c)) * \cos(1/2 * \arctan2(\sin(4 * d * x + 4 * c), \cos(4 * d * x + 4 * c))) - 5 * (\cos(4 * d * x + 4 * c) + 1) * \cos(1/4 * \arctan2(\sin(4 * d * x + 4 * c), \cos(4 * d * x + 4 * c))) - 2 * \sin(4 * d * x + 4 * c)^2 - 85 * \sin(4 * d * x + 4 * c) * \sin(1/4 * \arctan2(\sin(4 * d * x + 4 * c), \cos(4 * d * x + 4 * c))) * \sin(1/2 * \arctan2(\sin(4 * d * x + 4 * c), \cos(4 * d * x + 4 * c))) + 5 * (8 * \cos(4 * d * x + 4 * c)^2 + 8 * \sin(4 * d * x + 4 * c)^2 - \cos(4 * d * x + 4 * c)) * \sin(1/4 * \arctan2(\sin(4 * d * x + 4 * c), \cos(4 * d * x + 4 * c))) * \cos(1/2 * \arctan2(\sin(1/2 * \arctan2(\sin(4 * d * x + 4 * c), \cos(4 * d * x + 4 * c))), \cos(1/2 * \arctan2(\sin(4 * d * x + 4 * c), \cos(4 * d * x + 4 * c))) + 1)) - (64 * (\cos(4 * d * x + 4 * c)^2 + \sin(4 * d * x + 4 * c)^2 - 2 * \cos(4 * d * x + 4 * c) + 1) * \cos(1/2 * \arctan2(\sin(4 * d * x + 4 * c), \cos(4 * d * x + 4 * c)))^3 + 5 * \cos(4 * d * x + 4 * c)^3 + 4 * (5 * \cos(4 * d * x + 4 * c)^3 + (5 * \cos(4 * d * x + 4 * c) - 8) * \sin(4 * d * x + 4 * c)^2 - 18 * \cos(4 * d * x + 4 * c)^2 + 8 * (\cos(4 * d * x + 4 * c)^2 + \sin(4 * d * x + 4 * c)^2 - 2 * \cos(4 * d * x + 4 * c) + 1) * \cos(1/4 * \arctan2(\sin(4 * d * x + 4 * c), \cos(4 * d * x + 4 * c))) + 37 * \cos(4 * d * x + 4 * c) - 24) * \cos(1/2 * \arctan2(\sin(4 * d * x + 4 * c), \cos(4 * d * x + 4 * c)))^2 + (5 * \cos(4 * d * x + 4 * c) - 24) * \sin(4 * d * x + 4 * c)^2 + 4 * (5 * \cos(4 * d * x + 4 * c)^3 + (5 * \cos(4 * d * x + 4 * c) - 24) * \sin(4 * d * x + 4 * c)^2 - 14 * \cos(4 * d * x + 4 * c)^2 + 16 * (\cos(4 * d * x + 4 * c)^2 + \sin(4 * d * x + 4 * c)^2 + 2 * \cos(4 * d * x + 4 * c) + 1) * \cos(1/2 * \arctan2(\sin(4 * d * x + 4 * c), \cos(4 * d * x + 4 * c))) + 8 * (\cos(4 * d * x + 4 * c)^2 + \sin(4 * d * x + 4 * c)^2 + 2 * \cos(4 * d * x + 4 * c) + 1) * \cos(1/4 * \arctan2(\sin(4 * d * x + 4 * c), \cos(4 * d * x + 4 * c))) - 43 * \cos(4 * d * x + 4 * c) - 24) * \sin(1/2 * \arctan2(\sin(4 * d * x + 4 * c), \cos(4 * d * x + 4 * c)))^2 - 24 * \cos(4 * d * x + 4 * c)^2 + 2 * (10 * \cos(4 * d * x + 4 * c)^3 + 10 * (\cos(4 * d * x + 4 * c) - 4) * \sin(4 * d * x + 4 * c)^2 - 50 * \cos(4 * d * x + 4 * c)^2 + (16 * \cos(4 * d * x + 4 * c)^2 + 16 * \sin(4 * d * x + 4 * c)^2 - 21 * \cos(4 * d * x + 4 * c) + 5) * \cos(1/4 * \arctan2(\sin(4 * d * x + 4 * c), \cos(4 * d * x + 4 * c))) - 5 * \sin(4 * d * x + 4 * c) * \sin(1/4 * \arctan2(\sin(4 * d * x + 4 * c), \cos(4 * d * x + 4 * c))) + 48 * \cos(4 * d * x + 4 * c) * \cos(1/2 * \arctan2(\sin(4 * d * x + 4 * c), \cos(4 * d * x + 4 * c))) + (8 * \cos(4 * d * x + 4 * c)^2 + 8 * \sin(4 * d * x + 4 * c)^2 - 5 * \cos(4 * d * x + 4 * c)) * \cos(1/4 * \arctan2(\sin(4 * d * x + 4 * c), \cos(4 * d * x + 4 * c))) - 2 * (128 * \cos(1/2 * \arctan2(\sin(4 * d * x + 4 * c), \cos(4 * d * x + 4 * c)))^2 * \sin(4 * d * x + 4 * c) + 8 * (5 * (\cos(4 * d * x + 4 * c) - 4) * \sin(4 * d * x + 4 * c) + 8 * \cos(1/4 * \arctan2(\sin(4 * d * x + 4 * c), \cos(4 * d * x + 4 * c))) * \sin(4 * d * x + 4 * c)
\end{aligned}$$


```
*d*x + 4*c))) * sin(4*d*x + 4*c) + sin(4*d*x + 4*c)) * sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c + dx)^4 \left(A + \frac{B}{\cos(c + dx)} \right) \sqrt{a + \frac{a}{\cos(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(1/2), x)
```

```
[Out] int(cos(c + d*x)^4*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*sec(d*x+c))**(1/2)*(A+B*sec(d*x+c)), x)
```

```
[Out] Timed out
```

$$3.127 \quad \int \sec^3(c+dx)(a+a \sec(c+dx))^{3/2}(A+B \sec(c+dx)) dx$$

Optimal. Leaf size=189

$$\frac{2a^2(9A+10B) \tan(c+dx) \sec^3(c+dx)}{63d\sqrt{a \sec(c+dx)+a}} + \frac{2a^2(39A+34B) \tan(c+dx)}{45d\sqrt{a \sec(c+dx)+a}} + \frac{2(39A+34B) \tan(c+dx)(a \sec(c+dx))^{3/2}}{105d}$$

[Out] $2/105*(39*A+34*B)*(a+a*\sec(d*x+c))^{(3/2)}*\tan(d*x+c)/d+2/45*a^2*(39*A+34*B)*\tan(d*x+c)/d/(a+a*\sec(d*x+c))^{(1/2)}+2/63*a^2*(9*A+10*B)*\sec(d*x+c)^3*\tan(d*x+c)/d/(a+a*\sec(d*x+c))^{(1/2)}-4/315*a*(39*A+34*B)*(a+a*\sec(d*x+c))^{(1/2)}*\tan(d*x+c)/d+2/9*a*B*\sec(d*x+c)^3*(a+a*\sec(d*x+c))^{(1/2)}*\tan(d*x+c)/d$

Rubi [A] time = 0.46, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {4018, 4016, 3800, 4001, 3792}

$$\frac{2a^2(9A+10B) \tan(c+dx) \sec^3(c+dx)}{63d\sqrt{a \sec(c+dx)+a}} + \frac{2a^2(39A+34B) \tan(c+dx)}{45d\sqrt{a \sec(c+dx)+a}} + \frac{2(39A+34B) \tan(c+dx)(a \sec(c+dx))^{3/2}}{105d}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]), x]

[Out] $(2*a^2*(39*A+34*B)*\tan[c+d*x])/(45*d*\sqrt{a+a*\sec[c+d*x]}) + (2*a^2*(9*A+10*B)*\sec[c+d*x]^3*\tan[c+d*x])/(63*d*\sqrt{a+a*\sec[c+d*x]}) - (4*a*(39*A+34*B)*\sqrt{a+a*\sec[c+d*x]}*\tan[c+d*x])/(315*d) + (2*a*B*\sec[c+d*x]^3*\sqrt{a+a*\sec[c+d*x]}*\tan[c+d*x])/(9*d) + (2*(39*A+34*B)*(a+a*\sec[c+d*x])^{(3/2)}*\tan[c+d*x])/(105*d)$

Rule 3792

Int[csc[(e_.) + (f_.)*(x_.)]*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 3800

Int[csc[(e_.) + (f_.)*(x_.)]^3*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m + 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)]

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] := -Simp[(B*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m + 1), 0] && !LtQ[m, -2^(-1)]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] := Simp[(-2*b*B*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && !LtQ[n, 0]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] := -Simp[(b*B*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && !LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \sec^3(c + dx)(a + a \sec(c + dx))^{3/2}(A + B \sec(c + dx)) dx &= \frac{2aB \sec^3(c + dx)\sqrt{a + a \sec(c + dx)} \tan(c + dx)}{9d} \\
&= \frac{2a^2(9A + 10B) \sec^3(c + dx) \tan(c + dx)}{63d\sqrt{a + a \sec(c + dx)}} + \frac{2aB \sec^3(c + dx) \tan(c + dx)}{63d\sqrt{a + a \sec(c + dx)}} \\
&= \frac{2a^2(9A + 10B) \sec^3(c + dx) \tan(c + dx)}{63d\sqrt{a + a \sec(c + dx)}} + \frac{2aB \sec^3(c + dx) \tan(c + dx)}{63d\sqrt{a + a \sec(c + dx)}} \\
&= \frac{2a^2(9A + 10B) \sec^3(c + dx) \tan(c + dx)}{63d\sqrt{a + a \sec(c + dx)}} - \frac{4a(39A + 34B) \sec^2(c + dx) \tan(c + dx)}{63d\sqrt{a + a \sec(c + dx)}} \\
&= \frac{2a^2(39A + 34B) \tan(c + dx)}{45d\sqrt{a + a \sec(c + dx)}} + \frac{2a^2(9A + 10B) \sec^3(c + dx) \tan(c + dx)}{63d\sqrt{a + a \sec(c + dx)}}
\end{aligned}$$

Mathematica [A] time = 0.79, size = 100, normalized size = 0.53

$$\frac{2a^2 \tan(c + dx) (5(9A + 17B) \sec^3(c + dx) + 3(39A + 34B) \sec^2(c + dx) + 4(39A + 34B) \sec(c + dx) + 8(39A + 34B))}{315d\sqrt{a(\sec(c + dx) + 1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
[Out] (2*a^2*(8*(39*A + 34*B) + 4*(39*A + 34*B)*Sec[c + d*x] + 3*(39*A + 34*B)*Sec[c + d*x]^2 + 5*(9*A + 17*B)*Sec[c + d*x]^3 + 35*B*Sec[c + d*x]^4)*Tan[c + d*x])/(315*d*Sqrt[a*(1 + Sec[c + d*x])])
```

fricas [A] time = 0.42, size = 127, normalized size = 0.67

$$\frac{2(8(39A + 34B)a \cos(dx + c)^4 + 4(39A + 34B)a \cos(dx + c)^3 + 3(39A + 34B)a \cos(dx + c)^2 + 5(9A + 17B)a \cos(dx + c) + 35B)}{315(d \cos(dx + c)^5 + d \cos(dx + c)^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 2/315*(8*(39*A + 34*B)*a*cos(d*x + c)^4 + 4*(39*A + 34*B)*a*cos(d*x + c)^3 + 3*(39*A + 34*B)*a*cos(d*x + c)^2 + 5*(9*A + 17*B)*a*cos(d*x + c) + 35*B*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^5 + d*cos(d*x + c)^4)
```

giac [A] time = 2.09, size = 258, normalized size = 1.37

$$4 \left(\left(\left(\left(2 \sqrt{2} (57 A a^6 \operatorname{sgn}(\cos(dx+c)) + 47 B a^6 \operatorname{sgn}(\cos(dx+c))) \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 9 \sqrt{2} (57 A a^6 \operatorname{sgn}(\cos(dx+c)) + 47 B a^6 \operatorname{sgn}(\cos(dx+c))) \right)^2 - 819 \sqrt{2} (A a^6 \operatorname{sgn}(\cos(dx+c)) + B a^6 \operatorname{sgn}(\cos(dx+c))) \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 105 \sqrt{2} (7 A a^6 \operatorname{sgn}(\cos(dx+c)) + 5 B a^6 \operatorname{sgn}(\cos(dx+c))) \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 315 \sqrt{2} (A a^6 \operatorname{sgn}(\cos(dx+c)) + B a^6 \operatorname{sgn}(\cos(dx+c))) \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) \right) / ((a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right))^2 - a)^4 \sqrt{-a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + a} \right) \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] 4/315*(((2*sqrt(2)*(57*A*a^6*sgn(cos(d*x + c)) + 47*B*a^6*sgn(cos(d*x + c))) * tan(1/2*d*x + 1/2*c)^2 - 9*sqrt(2)*(57*A*a^6*sgn(cos(d*x + c)) + 47*B*a^6*sgn(cos(d*x + c)))) * tan(1/2*d*x + 1/2*c)^2 + 819*sqrt(2)*(A*a^6*sgn(cos(d*x + c)) + B*a^6*sgn(cos(d*x + c)))) * tan(1/2*d*x + 1/2*c)^2 - 105*sqrt(2)*(7*A*a^6*sgn(cos(d*x + c)) + 5*B*a^6*sgn(cos(d*x + c)))) * tan(1/2*d*x + 1/2*c)^2 + 315*sqrt(2)*(A*a^6*sgn(cos(d*x + c)) + B*a^6*sgn(cos(d*x + c)))) * tan(1/2*d*x + 1/2*c) / ((a*tan(1/2*d*x + 1/2*c)^2 - a)^4*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*d)

maple [A] time = 1.60, size = 139, normalized size = 0.74

$$2(-1 + \cos(dx+c)) \left(312A (\cos^4(dx+c)) + 272B (\cos^4(dx+c)) + 156A (\cos^3(dx+c)) + 136B (\cos^3(dx+c)) \right) / (315d \cos(dx+c))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^3*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x)

[Out] -2/315/d*(-1+cos(d*x+c))*(312*A*cos(d*x+c)^4+272*B*cos(d*x+c)^4+156*A*cos(d*x+c)^3+136*B*cos(d*x+c)^3+117*A*cos(d*x+c)^2+102*B*cos(d*x+c)^2+45*A*cos(d*x+c)+85*B*cos(d*x+c)+35*B)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^4/sin(d*x+c)*a

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] Timed out

mupad [B] time = 9.60, size = 596, normalized size = 3.15

$$\frac{\sqrt{a + \frac{a}{\frac{e^{-c1i-dx1i}}{2} + \frac{e^{c1i+dx1i}}{2}}}}{\left(e^{c1i+dx1i} \left(-\frac{a(3A+2B)8i}{7d} + \frac{a(A+4B)8i}{7d} + \frac{Ba32i}{63d} \right) + \frac{Aa8i}{7d} - \frac{a(A+2B)24i}{7d} - \frac{Ba32i}{7d} \right) \left(e^{c1i+dx1i} + 1 \right) \left(e^{c2i+dx2i} + 1 \right)^3} + \sqrt{a + \frac{a}{\frac{e^{-c1i-dx1i}}{2} + \frac{e^{c1i+dx1i}}{2}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(3/2))/cos(c + d*x)^3,x)

[Out] ((a + a/(exp(-c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(exp(c*1i + d*x*1i)*((a*(A + 4*B)*8i)/(7*d) - (a*(3*A + 2*B)*8i)/(7*d) + (B*a*32i)/(63*d)) + (A*a*8i)/(7*d) - (a*(A + 2*B)*24i)/(7*d) - (B*a*32i)/(7*d)))/((exp(c*1i + d*x*1i) + 1)*(exp(c*2i + d*x*2i) + 1)^3) + ((a + a/(exp(-c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(exp(c*1i + d*x*1i)*((a*(3*A + 2*B)*8i)/(9*d) - (a*(2*A + 3*B)*16i)/(9*d) + (A*a*8i)/(9*d) + (a*(2*A + 3*B)*16i)/(9*d) - (a*(3*A + 2*B)*8i)/(9*d) - (A*a*8i)/(9*d)))/((exp(c*1i + d*x*1i) + 1)*(exp(c*2i + d*x*2i) + 1)^4) + (((A*a*8i)/(3*d) - (a*exp(c*1i + d*x*1i)*(39*A + 34*B)*8i)/(315*d))*(a + a/(exp(-c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2))/((exp(c*1i + d*x*1i) + 1)*(exp(c*2i + d*x*2i) + 1)) + ((a + a/(exp(-c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(exp(c*1i + d*x*1i)*((a*(3*A + 2*B)*8i)/(5*d) + (a*(3*A + B)*16i)/(105*d)) - (A*a*8i)/(5*d) + (a*(A + 3*B)*16i)/(5*d)))/((exp(c*1i + d*x*1i) + 1)*(exp(c*2i + d*x*2i) + 1)^2) - (a*exp(c*1i + d*x*1i)*(a + a/(exp(-c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(39*A + 34*B)*16i)/(315*d*(exp(c*1i + d*x*1i) + 1))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sec(c + dx) + 1))^{\frac{3}{2}} (A + B \sec(c + dx)) \sec^3(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**3*(a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)),x)

[Out] Integral((a*(sec(c + d*x) + 1))**(3/2)*(A + B*sec(c + d*x))*sec(c + d*x)**3, x)

$$3.128 \quad \int \sec^2(c+dx)(a+a \sec(c+dx))^{3/2}(A+B \sec(c+dx)) dx$$

Optimal. Leaf size=138

$$\frac{8a^2(21A+19B) \tan(c+dx)}{105d\sqrt{a \sec(c+dx)+a}} + \frac{2(7A-2B) \tan(c+dx)(a \sec(c+dx)+a)^{3/2}}{35d} + \frac{2a(21A+19B) \tan(c+dx)\sqrt{a \sec(c+dx)}}{105d}$$

[Out] $2/35*(7*A-2*B)*(a+a*\sec(d*x+c))^{(3/2)}*\tan(d*x+c)/d+2/7*B*(a+a*\sec(d*x+c))^{(5/2)}*\tan(d*x+c)/a/d+8/105*a^2*(21*A+19*B)*\tan(d*x+c)/d/(a+a*\sec(d*x+c))^{(1/2)}+2/105*a*(21*A+19*B)*(a+a*\sec(d*x+c))^{(1/2)}*\tan(d*x+c)/d$

Rubi [A] time = 0.30, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {4010, 4001, 3793, 3792}

$$\frac{8a^2(21A+19B) \tan(c+dx)}{105d\sqrt{a \sec(c+dx)+a}} + \frac{2(7A-2B) \tan(c+dx)(a \sec(c+dx)+a)^{3/2}}{35d} + \frac{2a(21A+19B) \tan(c+dx)\sqrt{a \sec(c+dx)}}{105d}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]), x]

[Out] $(8*a^2*(21*A+19*B)*Tan[c+d*x])/(105*d*Sqrt[a+a*Sec[c+d*x]]) + (2*a*(21*A+19*B)*Sqrt[a+a*Sec[c+d*x]]*Tan[c+d*x])/(105*d) + (2*(7*A-2*B)*(a+a*Sec[c+d*x])^{(3/2)}*Tan[c+d*x])/(35*d) + (2*B*(a+a*Sec[c+d*x])^{(5/2)}*Tan[c+d*x])/(7*a*d)$

Rule 3792

Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 3793

Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m-1))/(f*m), x] + Dist[(a*(2*m-1))/m, Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m-1), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && IntegerQ[2*m]

Rule 4001


```
Int[csc[(e_.) + (f_.)*(x_.)]*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m + 1), 0] && !LtQ[m, -2^(-1)]
```

Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_.)]^2*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \sec^2(c + dx)(a + a \sec(c + dx))^{3/2}(A + B \sec(c + dx)) dx &= \frac{2B(a + a \sec(c + dx))^{5/2} \tan(c + dx)}{7ad} + \frac{2 \int \sec(c + dx)(a + a \sec(c + dx))^{3/2}(A + B \sec(c + dx)) dx}{7ad} \\ &= \frac{2(7A - 2B)(a + a \sec(c + dx))^{3/2} \tan(c + dx)}{35d} + \frac{2 \int \sec(c + dx)(a + a \sec(c + dx))^{3/2}(A + B \sec(c + dx)) dx}{35d} \\ &= \frac{2a(21A + 19B)\sqrt{a + a \sec(c + dx)} \tan(c + dx)}{105d} + \frac{2 \int \sec(c + dx)(a + a \sec(c + dx))^{3/2}(A + B \sec(c + dx)) dx}{105d} \\ &= \frac{8a^2(21A + 19B) \tan(c + dx)}{105d\sqrt{a + a \sec(c + dx)}} + \frac{2a(21A + 19B)\sqrt{a + a \sec(c + dx)}}{105d} \end{aligned}$$

Mathematica [A] time = 0.42, size = 82, normalized size = 0.59

$$\frac{2a^2 \tan(c + dx) (3(7A + 13B) \sec^2(c + dx) + (63A + 52B) \sec(c + dx) + 2(63A + 52B) + 15B \sec^3(c + dx))}{105d\sqrt{a(\sec(c + dx) + 1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
[Out] (2*a^2*(2*(63*A + 52*B) + (63*A + 52*B)*Sec[c + d*x] + 3*(7*A + 13*B)*Sec[c + d*x]^2 + 15*B*Sec[c + d*x]^3)*Tan[c + d*x])/(105*d*Sqrt[a*(1 + Sec[c + d*x])])
```

fricas [A] time = 0.42, size = 108, normalized size = 0.78

$$\frac{2 \left(2 (63 A + 52 B) a \cos(dx + c)^3 + (63 A + 52 B) a \cos(dx + c)^2 + 3 (7 A + 13 B) a \cos(dx + c) + 15 B a \right) \sqrt{\frac{a \cos(dx + c) + a}{\cos(dx + c)}}}{105 \left(d \cos(dx + c)^4 + d \cos(dx + c)^3 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] 2/105*(2*(63*A + 52*B)*a*cos(d*x + c)^3 + (63*A + 52*B)*a*cos(d*x + c)^2 + 3*(7*A + 13*B)*a*cos(d*x + c) + 15*B*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3)

giac [A] time = 1.55, size = 215, normalized size = 1.56

$$4 \left(\left(\left(2 \sqrt{2} \left(21 A a^5 \operatorname{sgn}(\cos(dx + c)) + 19 B a^5 \operatorname{sgn}(\cos(dx + c)) \right) \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) \right)^2 - 7 \sqrt{2} \left(21 A a^5 \operatorname{sgn}(\cos(dx + c)) + 19 B a^5 \operatorname{sgn}(\cos(dx + c)) \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] 4/105*(((2*sqrt(2)*(21*A*a^5*sgn(cos(d*x + c)) + 19*B*a^5*sgn(cos(d*x + c))))*tan(1/2*d*x + 1/2*c)^2 - 7*sqrt(2)*(21*A*a^5*sgn(cos(d*x + c)) + 19*B*a^5*sgn(cos(d*x + c))))*tan(1/2*d*x + 1/2*c)^2 + 70*sqrt(2)*(3*A*a^5*sgn(cos(d*x + c)) + 2*B*a^5*sgn(cos(d*x + c))))*tan(1/2*d*x + 1/2*c)^2 - 105*sqrt(2)*(A*a^5*sgn(cos(d*x + c)) + B*a^5*sgn(cos(d*x + c))))*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^3*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*d)

maple [A] time = 1.43, size = 117, normalized size = 0.85

$$\frac{2(-1 + \cos(dx + c)) \left(126A \left(\cos^3(dx + c) \right) + 104B \left(\cos^3(dx + c) \right) + 63A \left(\cos^2(dx + c) \right) + 52B \left(\cos^2(dx + c) \right) \right)}{105d \cos(dx + c)^3 \sin(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^2*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x)

[Out] $-2/105/d*(-1+\cos(dx+c))*(126*A*\cos(dx+c)^3+104*B*\cos(dx+c)^3+63*A*\cos(dx+c)^2+52*B*\cos(dx+c)^2+21*A*\cos(dx+c)+39*B*\cos(dx+c)+15*B)*(a*(1+\cos(dx+c))/\cos(dx+c))^{1/2}/\cos(dx+c)^3/\sin(dx+c)*a$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(dx+c)^2*(a+a*sec(dx+c))^(3/2)*(A+B*sec(dx+c)),x, algorithm="maxima")`

[Out] Timed out

mupad [B] time = 6.89, size = 479, normalized size = 3.47

$$\frac{\sqrt{a + \frac{a}{\frac{e^{-c1i-dx1i}}{2} + \frac{e^{c1i+dx1i}}{2}}} \left(e^{c1i+dx1i} \left(\frac{a(7A+13B)8i}{105d} - \frac{Aa4i}{3d} \right) - \frac{a(3A+2B)4i}{3d} \right)}{(e^{c1i+dx1i} + 1) (e^{2i+dx2i} + 1)} + \frac{\sqrt{a + \frac{a}{\frac{e^{-c1i-dx1i}}{2} + \frac{e^{c1i+dx1i}}{2}}} \left(e^{c1i+dx1i} \right)}{(e^{c1i+dx1i} + 1) (e^{2i+dx2i} + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B/cos(c + dx))*(a + a/cos(c + dx))^(3/2))/cos(c + dx)^2,x)`

[Out] $((a + a/(\exp(-c1i - dx*1i)/2 + \exp(c1i + dx*1i)/2))^{1/2}*(\exp(c1i + dx*1i)*((a*(3*A + 2*B)*4i)/(7*d) - (a*(2*A + 3*B)*8i)/(7*d) + (A*a*4i)/(7*d)) - (a*(2*A + 3*B)*8i)/(7*d) + (a*(3*A + 2*B)*4i)/(7*d) + (A*a*4i)/(7*d)))/((\exp(c1i + dx*1i) + 1)*(\exp(c2i + dx*2i) + 1)^3 - ((a + a/(\exp(-c1i - dx*1i)/2 + \exp(c1i + dx*1i)/2))^{1/2}*(\exp(c1i + dx*1i)*((a*(7*A + 13*B)*8i)/(105*d) - (A*a*4i)/(3*d)) - (a*(3*A + 2*B)*4i)/(3*d)))/((\exp(c1i + dx*1i) + 1)*(\exp(c2i + dx*2i) + 1)) + ((a + a/(\exp(-c1i - dx*1i)/2 + \exp(c1i + dx*1i)/2))^{1/2}*(\exp(c1i + dx*1i)*((a*(A + 2*B)*12i)/(5*d) - (A*a*4i)/(5*d) + (B*a*16i)/(35*d)) - (a*(3*A + 2*B)*4i)/(5*d) + (a*(A + 4*B)*4i)/(5*d)))/((\exp(c1i + dx*1i) + 1)*(\exp(c2i + dx*2i) + 1)^2 - (a*\exp(c1i + dx*1i)*(a + a/(\exp(-c1i - dx*1i)/2 + \exp(c1i + dx*1i)/2))^{1/2}*(63*A + 52*B)*4i)/(105*d*(\exp(c1i + dx*1i) + 1)))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sec(c + dx) + 1))^{\frac{3}{2}} (A + B \sec(c + dx)) \sec^2(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(dx+c)**2*(a+a*sec(dx+c))**(3/2)*(A+B*sec(dx+c)),x)`

[Out] `Integral((a*(sec(c + dx) + 1))**(3/2)*(A + B*sec(c + dx))*sec(c + dx)**2, x)`

$$3.129 \quad \int \sec(c+dx)(a+a \sec(c+dx))^{3/2}(A+B \sec(c+dx)) dx$$

Optimal. Leaf size=101

$$\frac{8a^2(5A+3B) \tan(c+dx)}{15d\sqrt{a \sec(c+dx)+a}} + \frac{2a(5A+3B) \tan(c+dx)\sqrt{a \sec(c+dx)+a}}{15d} + \frac{2B \tan(c+dx)(a \sec(c+dx)+a)^{3/2}}{5d}$$

[Out] $2/5*B*(a+a*\sec(d*x+c))^{3/2}*tan(d*x+c)/d+8/15*a^2*(5*A+3*B)*tan(d*x+c)/d/(a+a*\sec(d*x+c))^{1/2}+2/15*a*(5*A+3*B)*(a+a*\sec(d*x+c))^{1/2}*tan(d*x+c)/d$

Rubi [A] time = 0.14, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {4001, 3793, 3792}

$$\frac{8a^2(5A+3B) \tan(c+dx)}{15d\sqrt{a \sec(c+dx)+a}} + \frac{2a(5A+3B) \tan(c+dx)\sqrt{a \sec(c+dx)+a}}{15d} + \frac{2B \tan(c+dx)(a \sec(c+dx)+a)^{3/2}}{5d}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d*x]*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]), x]

[Out] $(8*a^2*(5*A+3*B)*Tan[c+d*x])/(15*d*Sqrt[a+a*Sec[c+d*x]])+(2*a*(5*A+3*B)*Sqrt[a+a*Sec[c+d*x]]*Tan[c+d*x])/(15*d)+(2*B*(a+a*Sec[c+d*x])^{3/2}*Tan[c+d*x])/(5*d)$

Rule 3792

Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 3793

Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_), x_Symbol] :> -Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m-1))/(f*m), x] + Dist[(a*(2*m-1))/m, Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m-1), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && IntegerQ[2*m]

Rule 4001

Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m+1)), x] + Dist[(a*B*m + A*b*(m+1))/(b*(m+1))

)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m + 1), 0] && !LtQ[m, -2^(-1)]

Rubi steps

$$\begin{aligned} \int \sec(c + dx)(a + a \sec(c + dx))^{3/2}(A + B \sec(c + dx)) dx &= \frac{2B(a + a \sec(c + dx))^{3/2} \tan(c + dx)}{5d} + \frac{1}{5}(5A + 3B) \\ &= \frac{2a(5A + 3B)\sqrt{a + a \sec(c + dx)} \tan(c + dx)}{15d} + \frac{2B}{15d} \\ &= \frac{8a^2(5A + 3B) \tan(c + dx)}{15d\sqrt{a + a \sec(c + dx)}} + \frac{2a(5A + 3B)\sqrt{a + a \sec(c + dx)}}{15d} \end{aligned}$$

Mathematica [A] time = 0.33, size = 70, normalized size = 0.69

$$\frac{2a\sqrt{a(\sec(c + dx) + 1)}((25A + 18B) \sin(c + dx) + \tan(c + dx)(5A + 3B \sec(c + dx) + 9B))}{15d(\cos(c + dx) + 1)}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[c + d*x]*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]

[Out] (2*a*Sqrt[a*(1 + Sec[c + d*x])]*((25*A + 18*B)*Sin[c + d*x] + (5*A + 9*B + 3*B*Sec[c + d*x])*Tan[c + d*x]))/(15*d*(1 + Cos[c + d*x]))

fricas [A] time = 0.42, size = 89, normalized size = 0.88

$$\frac{2 \left((25A + 18B)a \cos(dx + c)^2 + (5A + 9B)a \cos(dx + c) + 3Ba \right) \sqrt{\frac{a \cos(dx + c) + a}{\cos(dx + c)}} \sin(dx + c)}{15 \left(d \cos(dx + c)^3 + d \cos(dx + c)^2 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] 2/15*((25*A + 18*B)*a*cos(d*x + c)^2 + (5*A + 9*B)*a*cos(d*x + c) + 3*B*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2)

giac [A] time = 2.07, size = 170, normalized size = 1.68

$$\frac{4 \left(\left(2 \sqrt{2} \left(5 A a^4 \operatorname{sgn}(\cos(dx+c)) + 3 B a^4 \operatorname{sgn}(\cos(dx+c)) \right) \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 5 \sqrt{2} \left(5 A a^4 \operatorname{sgn}(\cos(dx+c)) + 3 B a^4 \operatorname{sgn}(\cos(dx+c)) \right) \right) \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 5 \sqrt{2} \left(5 A a^4 \operatorname{sgn}(\cos(dx+c)) + 3 B a^4 \operatorname{sgn}(\cos(dx+c)) \right) \right)}{15 \left(a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - a \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] 4/15*((2*sqrt(2)*(5*A*a^4*sgn(cos(d*x + c)) + 3*B*a^4*sgn(cos(d*x + c))))*tan(1/2*d*x + 1/2*c)^2 - 5*sqrt(2)*(5*A*a^4*sgn(cos(d*x + c)) + 3*B*a^4*sgn(cos(d*x + c))))*tan(1/2*d*x + 1/2*c)^2 + 15*sqrt(2)*(A*a^4*sgn(cos(d*x + c)) + B*a^4*sgn(cos(d*x + c))))*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^2*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*d)

maple [A] time = 1.41, size = 95, normalized size = 0.94

$$\frac{2(-1 + \cos(dx+c)) \left(25A(\cos^2(dx+c)) + 18B(\cos^2(dx+c)) + 5A \cos(dx+c) + 9B \cos(dx+c) + 3B \right) \sqrt{\frac{a}{\cos(dx+c)}}}{15d \cos(dx+c)^2 \sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x)

[Out] -2/15/d*(-1+cos(d*x+c))*(25*A*cos(d*x+c)^2+18*B*cos(d*x+c)^2+5*A*cos(d*x+c)+9*B*cos(d*x+c)+3*B)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^2/sin(d*x+c)*a

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] Timed out

mupad [B] time = 5.88, size = 213, normalized size = 2.11

$$\frac{2a \left(e^{c1i+dx1i} - 1 \right) \sqrt{a + \frac{a}{\frac{e^{-c1i-dx1i}}{2} + \frac{e^{c1i+dx1i}}{2}}} \left(A25i + B18i + A e^{c1i+dx1i} 10i + A e^{c2i+dx2i} 50i + A e^{c3i+dx3i} 10i \right)}{15d \left(e^{c1i+dx1i} + 1 \right) \left(e^{c2i+dx2i} + 1 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(3/2))/cos(c + d*x),x)
```

```
[Out] -(2*a*(exp(c*1i + d*x*1i) - 1)*(a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(A*25i + B*18i + A*exp(c*1i + d*x*1i)*10i + A*exp(c*2i + d*x*2i)*50i + A*exp(c*3i + d*x*3i)*10i + A*exp(c*4i + d*x*4i)*25i + B*exp(c*1i + d*x*1i)*18i + B*exp(c*2i + d*x*2i)*48i + B*exp(c*3i + d*x*3i)*18i + B*exp(c*4i + d*x*4i)*18i))/(15*d*(exp(c*1i + d*x*1i) + 1)*(exp(c*2i + d*x*2i) + 1)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sec(c + dx) + 1))^{\frac{3}{2}} (A + B \sec(c + dx)) \sec(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**(3/2)*(A + B*sec(c + d*x))*sec(c + d*x), x)
```

3.130 $\int (a + a \sec(c + dx))^{3/2} (A + B \sec(c + dx)) dx$

Optimal. Leaf size=105

$$\frac{2a^{3/2} A \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}}\right)}{d} + \frac{2a^2(3A + 4B) \tan(c + dx)}{3d\sqrt{a \sec(c + dx) + a}} + \frac{2aB \tan(c + dx)\sqrt{a \sec(c + dx) + a}}{3d}$$

[Out] $2*a^{(3/2)}*A*\arctan(a^{(1/2)}*\tan(d*x+c)/(a+a*\sec(d*x+c))^{(1/2)})/d+2/3*a^2*(3*A+4*B)*\tan(d*x+c)/d/(a+a*\sec(d*x+c))^{(1/2)}+2/3*a*B*(a+a*\sec(d*x+c))^{(1/2)}*\tan(d*x+c)/d$

Rubi [A] time = 0.15, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3917, 3915, 3774, 203, 3792}

$$\frac{2a^2(3A + 4B) \tan(c + dx)}{3d\sqrt{a \sec(c + dx) + a}} + \frac{2a^{3/2} A \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}}\right)}{d} + \frac{2aB \tan(c + dx)\sqrt{a \sec(c + dx) + a}}{3d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + a*\text{Sec}[c + d*x])^{(3/2)}*(A + B*\text{Sec}[c + d*x]), x]$

[Out] $(2*a^{(3/2)}*A*\text{ArcTan}[(\text{Sqrt}[a]*\text{Tan}[c + d*x])/(\text{Sqrt}[a + a*\text{Sec}[c + d*x]])]/d + (2*a^2*(3*A + 4*B)*\text{Tan}[c + d*x])/((3*d*\text{Sqrt}[a + a*\text{Sec}[c + d*x]]) + (2*a*B*\text{Sqrt}[a + a*\text{Sec}[c + d*x]]*\text{Tan}[c + d*x]))/(3*d)$

Rule 203

$\text{Int}[(a_.) + (b_.)*(x_)^2]^{-1}, x_Symbol] \rightarrow \text{Simp}[(1*\text{ArcTan}[(\text{Rt}[b, 2]*x)/(\text{Rt}[a, 2])])]/(\text{Rt}[a, 2]*\text{Rt}[b, 2]), x] /; \text{FreeQ}\{a, b, x\} \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{GtQ}[b, 0])$

Rule 3774

$\text{Int}[\text{Sqrt}[\text{csc}[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] \rightarrow \text{Dist}[(-2*b)/d, \text{Subst}[\text{Int}[1/(a + x^2), x], x, (b*\text{Cot}[c + d*x])/(\text{Sqrt}[a + b*\text{Csc}[c + d*x]])], x] /; \text{FreeQ}\{a, b, c, d, x\} \ \&\& \ \text{EqQ}[a^2 - b^2, 0]$

Rule 3792

$\text{Int}[\text{csc}[(e_.) + (f_.)*(x_)]*\text{Sqrt}[\text{csc}[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] \rightarrow \text{Simp}[(-2*b*\text{Cot}[e + f*x])/(f*\text{Sqrt}[a + b*\text{Csc}[e + f*x]]), x] /; \text{FreeQ}\{a, b, e, f, x\} \ \&\& \ \text{EqQ}[a^2 - b^2, 0]$

Rule 3915

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.)), x_Symbol] := Dist[c, Int[Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[d, Int[Sqrt[a + b*Csc[e + f*x]]*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3917

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.)), x_Symbol] := -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1))/(f*m), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m - 1)*Simp[a*c*m + (b*c*m + a*d*(2*m - 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && GtQ[m, 1] && EqQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned} \int (a + a \sec(c + dx))^{3/2} (A + B \sec(c + dx)) dx &= \frac{2aB\sqrt{a + a \sec(c + dx)} \tan(c + dx)}{3d} + \frac{2}{3} \int \sqrt{a + a \sec(c + dx)} dx \\ &= \frac{2aB\sqrt{a + a \sec(c + dx)} \tan(c + dx)}{3d} + (aA) \int \sqrt{a + a \sec(c + dx)} dx \\ &= \frac{2a^2(3A + 4B) \tan(c + dx)}{3d\sqrt{a + a \sec(c + dx)}} + \frac{2aB\sqrt{a + a \sec(c + dx)} \tan(c + dx)}{3d} \\ &= \frac{2a^{3/2} A \tan^{-1}\left(\frac{\sqrt{a} \tan(c + dx)}{\sqrt{a + a \sec(c + dx)}}\right)}{d} + \frac{2a^2(3A + 4B) \tan(c + dx)}{3d\sqrt{a + a \sec(c + dx)}} + \frac{2}{3} \int \sqrt{a + a \sec(c + dx)} dx \end{aligned}$$

Mathematica [A] time = 0.62, size = 102, normalized size = 0.97

$$\frac{a \sec\left(\frac{1}{2}(c + dx)\right) \sec(c + dx) \sqrt{a(\sec(c + dx) + 1)} \left(2 \sin\left(\frac{1}{2}(c + dx)\right) ((3A + 5B) \cos(c + dx) + B) + 3\sqrt{2} A \sin^{-1}\left(\frac{\sqrt{a} \tan(c + dx)}{\sqrt{a + a \sec(c + dx)}}\right)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a*Sec[(c + d*x)/2]*Sec[c + d*x]*Sqrt[a*(1 + Sec[c + d*x])]*(3*Sqrt[2]*A*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(3/2) + 2*(B + (3*A + 5*B)*Cos[c + d*x])*Sin[(c + d*x)/2]))/(3*d)
```

fricas [A] time = 0.46, size = 314, normalized size = 2.99

$$\frac{3 \left(Aa \cos(dx+c)^2 + Aa \cos(dx+c) \right) \sqrt{-a} \log \left(\frac{2a \cos(dx+c)^2 - 2\sqrt{-a} \sqrt{\frac{a \cos(dx+c)+a}{\cos(dx+c)}} \cos(dx+c) \sin(dx+c) + a \cos(dx+c) - a}{\cos(dx+c)+1} \right) + 2 \left(\dots \right)}{3 \left(d \cos(dx+c)^2 + d \cos(dx+c) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] [1/3*(3*(A*a*cos(d*x + c)^2 + A*a*cos(d*x + c))*sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*((3*A + 5*B)*a*cos(d*x + c) + B*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2 + d*cos(d*x + c)), -2/3*(3*(A*a*cos(d*x + c)^2 + A*a*cos(d*x + c))*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - ((3*A + 5*B)*a*cos(d*x + c) + B*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2 + d*cos(d*x + c))]

giac [B] time = 6.97, size = 263, normalized size = 2.50

$$\frac{3A\sqrt{-a}a^2 \log \left(\frac{\left| 2 \left(\sqrt{-a} \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - \sqrt{-a \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + a} \right)^2 - 4\sqrt{2}|a| - 6a}{\left| 2 \left(\sqrt{-a} \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - \sqrt{-a \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + a} \right)^2 + 4\sqrt{2}|a| - 6a} \right| \operatorname{sgn}(\cos(dx+c))}{|a|} \right) + \frac{2 \left(3\sqrt{2}Aa^3 \operatorname{sgn}(\cos(dx+c)) + 6\sqrt{2}Ba^3 \operatorname{sgn}(\cos(dx+c)) - \left(a \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) \right)^2 \right)}{3d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] -1/3*(3*A*sqrt(-a)*a^2*log(abs(2*(sqrt(-a))*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-a))*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sqrt(2)*abs(a) - 6*a))*sgn(cos(d*x + c))/abs(a) + 2*(3*sqrt(2)*A*a^3*sgn(cos(d*x + c)) + 6*sqrt(2)*B*a^3*sgn(cos(d*x + c)) - (3*sqrt(2)*A*a^3*sgn(cos(d*x + c)) + 4*sqrt(2)*B*a^3*sgn(cos(d*x + c)))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/d

maple [B] time = 1.48, size = 237, normalized size = 2.26

$$\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}} \left(3A \cos(dx+c) \operatorname{arctanh} \left(\frac{\sqrt{\frac{-2\cos(dx+c)}{1+\cos(dx+c)}} \sin(dx+c) \sqrt{2}}{2\cos(dx+c)} \right) \left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)} \right)^{\frac{3}{2}} \sin(dx+c) \sqrt{2} + 3A\sqrt{2} \operatorname{arctanh} \left(\frac{\sqrt{\frac{-2\cos(dx+c)}{1+\cos(dx+c)}} \sin(dx+c) \sqrt{2}}{2\cos(dx+c)} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x)`

[Out] `1/6/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(3*A*cos(d*x+c)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*sin(d*x+c)*2^(1/2)+3*A*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*sin(d*x+c)-12*A*cos(d*x+c)^2-20*B*cos(d*x+c)^2+12*A*cos(d*x+c)+16*B*cos(d*x+c)+4*B)/cos(d*x+c)/sin(d*x+c)*a`

maxima [B] time = 0.64, size = 998, normalized size = 9.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="maxima")`

[Out] `1/2*((a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))) + 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))) - 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))`

+ 1) + a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sqrt(a) + 4*(a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) - (a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) - a)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a))*A/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*d)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A + \frac{B}{\cos(c + dx)} \right) \left(a + \frac{a}{\cos(c + dx)} \right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(3/2), x)

[Out] int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sec(c + dx) + 1))^{\frac{3}{2}} (A + B \sec(c + dx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)), x)

[Out] Integral((a*(sec(c + d*x) + 1))**(3/2)*(A + B*sec(c + d*x)), x)

$$3.131 \quad \int \cos(c+dx)(a+a \sec(c+dx))^{3/2}(A+B \sec(c+dx)) dx$$

Optimal. Leaf size=103

$$\frac{a^{3/2}(3A+2B) \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}}\right)}{d} + \frac{a^2(A-2B) \sin(c+dx)}{d\sqrt{a \sec(c+dx)+a}} + \frac{2aB \sin(c+dx)\sqrt{a \sec(c+dx)+a}}{d}$$

[Out] a^(3/2)*(3*A+2*B)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+a^2*(A-2*B)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+2*a*B*sin(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d

Rubi [A] time = 0.24, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {4018, 4015, 3774, 203}

$$\frac{a^2(A-2B) \sin(c+dx)}{d\sqrt{a \sec(c+dx)+a}} + \frac{a^{3/2}(3A+2B) \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}}\right)}{d} + \frac{2aB \sin(c+dx)\sqrt{a \sec(c+dx)+a}}{d}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]

[Out] (a^(3/2)*(3*A + 2*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]])/d + (a^2*(A - 2*B)*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*B*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/d

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 3774

Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_.)], x_Symbol] := Dist[(-2*b)/d, Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 4015

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] := Simp[(A*b^2*C

ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
 [(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
 B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]

Rule 4018

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> -Simp[(b*B*C
 ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
 *Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
 [e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
 B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && !LtQ[n, -1]

Rubi steps

$$\begin{aligned} \int \cos(c + dx)(a + a \sec(c + dx))^{3/2}(A + B \sec(c + dx)) dx &= \frac{2aB\sqrt{a + a \sec(c + dx)} \sin(c + dx)}{d} + 2 \int \cos(c + dx) \sqrt{a + a \sec(c + dx)} dx \\ &= \frac{a^2(A - 2B) \sin(c + dx)}{d\sqrt{a + a \sec(c + dx)}} + \frac{2aB\sqrt{a + a \sec(c + dx)} \sin(c + dx)}{d} \\ &= \frac{a^2(A - 2B) \sin(c + dx)}{d\sqrt{a + a \sec(c + dx)}} + \frac{2aB\sqrt{a + a \sec(c + dx)} \sin(c + dx)}{d} \\ &= \frac{a^{3/2}(3A + 2B) \tan^{-1}\left(\frac{\sqrt{a} \tan(c + dx)}{\sqrt{a + a \sec(c + dx)}}\right)}{d} + \frac{a^2(A - 2B) \sin(c + dx)}{d\sqrt{a + a \sec(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.46, size = 97, normalized size = 0.94

$$\frac{a \sec\left(\frac{1}{2}(c + dx)\right) \sqrt{a(\sec(c + dx) + 1)} \left(\sqrt{2}(3A + 2B) \sin^{-1}\left(\sqrt{2} \sin\left(\frac{1}{2}(c + dx)\right)\right)\right) \sqrt{\cos(c + dx)} + 2 \sin\left(\frac{1}{2}(c + dx)\right)}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]

[Out] (a*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(Sqrt[2]*(3*A + 2*B)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]] + 2*(2*B + A*Cos[c + d*x])*Sin[(c + d*x)/2]))/(2*d)

fricas [A] time = 0.51, size = 292, normalized size = 2.83

$$\left[\frac{((3A + 2B)a \cos(dx + c) + (3A + 2B)a)\sqrt{-a} \log\left(\frac{2a \cos(dx+c)^2 - 2\sqrt{-a} \sqrt{\frac{a \cos(dx+c)+a}{\cos(dx+c)}} \cos(dx+c) \sin(dx+c) + a \cos(dx+c) - a}{\cos(dx+c)+1}\right)}{2(d \cos(dx + c) + d)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] [1/2*(((3*A + 2*B)*a*cos(d*x + c) + (3*A + 2*B)*a)*sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(A*a*cos(d*x + c) + 2*B*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d), -(((3*A + 2*B)*a*cos(d*x + c) + (3*A + 2*B)*a)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - (A*a*cos(d*x + c) + 2*B*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d)]

giac [B] time = 1.60, size = 406, normalized size = 3.94

$$\frac{4\sqrt{2}\sqrt{-a \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + a} B a^2 \operatorname{sgn}(\cos(dx+c)) \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)}{a \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - a} + (3A\sqrt{-a} \operatorname{sgn}(\cos(dx+c)) + 2B\sqrt{-a} \operatorname{sgn}(\cos(dx+c)))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] -1/2*(4*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*B*a^2*sgn(cos(d*x + c))*tan(1/2*d*x + 1/2*c)/(a*tan(1/2*d*x + 1/2*c)^2 - a) + (3*A*sqrt(-a)*a*sgn(cos(d*x + c)) + 2*B*sqrt(-a)*a*sgn(cos(d*x + c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3))) - (3*A*sqrt(-a)*a*sgn(cos(d*x + c)) + 2*B*sqrt(-a)*a*sgn(cos(d*x + c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))) + 4*(3*sqrt(2)*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*sqrt(-a)*a^2*sgn(cos(d*x + c)) - sqrt(2)*A*sqrt(-a)*a^3*sgn(cos(d*x + c)))/((sqrt(-a)*tan(1/2*d*x + 1/2*c) - s

$\frac{\sqrt{-a \tan(1/2 dx + 1/2 c)^2 + a} - 6 \sqrt{-a} \tan(1/2 dx + 1/2 c) - \sqrt{-a \tan(1/2 dx + 1/2 c)^2 + a}^{2a + a^2}}{d}$

maple [B] time = 1.47, size = 212, normalized size = 2.06

$$\frac{\left(3A\sqrt{2} \operatorname{arctanh} \left(\frac{\sqrt{-\frac{2 \cos(dx+c)}{1+\cos(dx+c)}} \sin(dx+c) \sqrt{2}}{2 \cos(dx+c)} \right) \sqrt{\frac{2 \cos(dx+c)}{1+\cos(dx+c)}} \sin(dx+c) + 2B \sqrt{\frac{2 \cos(dx+c)}{1+\cos(dx+c)}} \operatorname{arctanh} \left(\frac{\sqrt{-\frac{2 \cos(dx+c)}{1+\cos(dx+c)}} \sin(dx+c) \sqrt{2}}{2 \cos(dx+c)} \right) \right)}{2d \sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x)

[Out] $-\frac{1}{2} \frac{d}{\cos(dx+c)} \left(3A^2 \arctanh \left(\frac{-2 \cos(dx+c)}{1+\cos(dx+c)} \right) \right)^{1/2} \sin(dx+c) + 2B \left(-2 \cos(dx+c) \right)^{1/2} \arctanh \left(\frac{-2 \cos(dx+c)}{1+\cos(dx+c)} \right) \sin(dx+c) + 2A \cos(dx+c) \sqrt{-2 \cos(dx+c)} + 4B \cos(dx+c) - 4B \left(\frac{a(1+\cos(dx+c))}{\cos(dx+c)} \right)^{1/2} \sin(dx+c) \cdot a$

maxima [B] time = 0.84, size = 1801, normalized size = 17.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] $\frac{1}{4} \left((2(a \cos(1/2 \arctan^2(\sin(2dx+2c), \cos(2dx+2c)+1)) \sin(dx+c) - (a \cos(dx+c) - a) \sin(1/2 \arctan^2(\sin(2dx+2c), \cos(2dx+2c)+1))) \sqrt{\cos(2dx+2c)^2 + \sin(2dx+2c)^2 + 2 \cos(2dx+2c) + 1}^{1/4} + 3(a \arctan^2(-(\cos(2dx+2c)^2 + \sin(2dx+2c)^2 + 2 \cos(2dx+2c) + 1))^{1/4} \cos(1/2 \arctan^2(\sin(2dx+2c), \cos(2dx+2c)+1)) \sin(dx+c) - \cos(dx+c) \sin(1/2 \arctan^2(\sin(2dx+2c), \cos(2dx+2c)+1))) \sqrt{\cos(2dx+2c)^2 + \sin(2dx+2c)^2 + 2 \cos(2dx+2c) + 1}^{1/4} + (\cos(dx+c) \cos(1/2 \arctan^2(\sin(2dx+2c), \cos(2dx+2c)+1)) + \sin(dx+c) \sin(1/2 \arctan^2(\sin(2dx+2c), \cos(2dx+2c)+1))) \sqrt{\cos(2dx+2c)^2 + \sin(2dx+2c)^2 + 2 \cos(2dx+2c) + 1}^{1/4} - a \arctan^2(-(\cos(2dx+2c)^2 + \sin(2dx+2c)^2 + 2 \cos(2dx+2c) + 1))^{1/4} \cos(1/2 \arctan^2(\sin(2dx+2c), \cos(2dx+2c)+1)) \sin(dx+c) - \cos(dx+c) \sin(1/2 \arctan^2(\sin(2dx+2c), \cos(2dx+2c)+1))) \sqrt{\cos(2dx+2c)^2 + \sin(2dx+2c)^2 + 2 \cos(2dx+2c) + 1}^{1/4} + (\cos(dx+c) \cos(1/2 \arctan^2(\sin(2dx+2c), \cos(2dx+2c)+1)) + \sin(dx+c) \sin(1/2 \arctan^2(\sin(2dx+2c), \cos(2dx+2c)+1))) \sqrt{\cos(2dx+2c)^2 + \sin(2dx+2c)^2 + 2 \cos(2dx+2c) + 1}^{1/4} - 1 - a \arctan^2(\cos(2dx+2c)^2 + \sin(2dx+2c)^2 + 2 \cos(2dx+2c) + 1) \sqrt{\cos(2dx+2c)^2 + \sin(2dx+2c)^2 + 2 \cos(2dx+2c) + 1}^{1/4} \right)$


```

c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
+ 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
+ 1) + a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2
*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (c
os(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*sqrt(a))*A + 2*(
(a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + sin(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c)))) + 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d
*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*
sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
+ 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)
)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + sin(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c)))) - 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*
c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1)
+ a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
+ 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2
*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*(cos(2*d*x + 2*c)^2
+ sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sqrt(a) + 4*(a*cos(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) - (a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c)))) - a)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*s
qrt(a))*B/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1
)^(1/4))/d

```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c + dx) \left(A + \frac{B}{\cos(c + dx)} \right) \left(a + \frac{a}{\cos(c + dx)} \right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(3/2), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

$$3.132 \quad \int \cos^2(c+dx)(a+a \sec(c+dx))^{3/2}(A+B \sec(c+dx)) dx$$

Optimal. Leaf size=119

$$\frac{a^{3/2}(7A+12B) \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}}\right)}{4d} + \frac{a^2(5A+4B) \sin(c+dx)}{4d\sqrt{a \sec(c+dx)+a}} + \frac{aA \sin(c+dx) \cos(c+dx) \sqrt{a \sec(c+dx)+a}}{2d}$$

[Out] 1/4*a^(3/2)*(7*A+12*B)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+1/4*a^2*(5*A+4*B)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/2*a*A*cos(d*x+c)*sin(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d

Rubi [A] time = 0.27, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {4017, 4015, 3774, 203}

$$\frac{a^2(5A+4B) \sin(c+dx)}{4d\sqrt{a \sec(c+dx)+a}} + \frac{a^{3/2}(7A+12B) \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}}\right)}{4d} + \frac{aA \sin(c+dx) \cos(c+dx) \sqrt{a \sec(c+dx)+a}}{2d}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]

[Out] (a^(3/2)*(7*A + 12*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]])/(4*d) + (a^2*(5*A + 4*B)*Sin[c + d*x])/(4*d*Sqrt[a + a*Sec[c + d*x]]) + (a*A*Cos[c + d*x]*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(2*d)

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 3774

Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_.)], x_Symbol] := Dist[(-2*b)/d, Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 4015

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] := Simp[(A*b^2*C

ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
 [(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
 B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]

Rule 4017

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> Simp[(a*A*Co
 t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
 t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
 [a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x] /
 ; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]

Rubi steps

$$\begin{aligned} \int \cos^2(c + dx)(a + a \sec(c + dx))^{3/2}(A + B \sec(c + dx)) dx &= \frac{aA \cos(c + dx)\sqrt{a + a \sec(c + dx)} \sin(c + dx)}{2d} + \frac{1}{2} \\ &= \frac{a^2(5A + 4B) \sin(c + dx)}{4d\sqrt{a + a \sec(c + dx)}} + \frac{aA \cos(c + dx)\sqrt{a + a \sec(c + dx)}}{2d} \\ &= \frac{a^2(5A + 4B) \sin(c + dx)}{4d\sqrt{a + a \sec(c + dx)}} + \frac{aA \cos(c + dx)\sqrt{a + a \sec(c + dx)}}{2d} \\ &= \frac{a^{3/2}(7A + 12B) \tan^{-1}\left(\frac{\sqrt{a} \tan(c + dx)}{\sqrt{a + a \sec(c + dx)}}\right)}{4d} + \frac{a^2(5A + 4B) \sin(c + dx)}{4d\sqrt{a + a \sec(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.81, size = 111, normalized size = 0.93

$$\frac{a\sqrt{\cos(c + dx)} \sec\left(\frac{1}{2}(c + dx)\right) \sqrt{a(\sec(c + dx) + 1)} \left(\sqrt{2}(7A + 12B) \sin^{-1}\left(\sqrt{2} \sin\left(\frac{1}{2}(c + dx)\right)\right)\right) + 2 \sin\left(\frac{1}{2}(c + dx)\right)}{8d}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
 [Out] (a*Sqrt[Cos[c + d*x]]*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(Sqrt[2]*
 (7*A + 12*B)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*(7*A +
 4*B + 2*A*Cos[c + d*x])*Sin[(c + d*x)/2]))/(8*d)

fricas [A] time = 0.50, size = 320, normalized size = 2.69

$$\left[\frac{((7A + 12B)a \cos(dx + c) + (7A + 12B)a)\sqrt{-a} \log \left(\frac{2a \cos(dx+c)^2 - 2\sqrt{-a} \sqrt{\frac{a \cos(dx+c)+a}{\cos(dx+c)}} \cos(dx+c) \sin(dx+c) + a \cos(dx+c) - a}{\cos(dx+c)+1} \right)}{8(d \cos(dx + c) + d)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] [1/8*(((7*A + 12*B)*a*cos(d*x + c) + (7*A + 12*B)*a)*sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(2*A*a*cos(d*x + c)^2 + (7*A + 4*B)*a*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d), -1/4*(((7*A + 12*B)*a*cos(d*x + c) + (7*A + 12*B)*a)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - (2*A*a*cos(d*x + c)^2 + (7*A + 4*B)*a*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d)]

giac [B] time = 1.87, size = 639, normalized size = 5.37

$$(7A\sqrt{-a} \operatorname{asgn}(\cos(dx + c)) + 12B\sqrt{-a} \operatorname{asgn}(\cos(dx + c))) \log \left(\left| \sqrt{-a} \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - \sqrt{-a} \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) \right| \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] -1/8*(((7*A*sqrt(-a)*a*sgn(cos(d*x + c)) + 12*B*sqrt(-a)*a*sgn(cos(d*x + c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3))) - (7*A*sqrt(-a)*a*sgn(cos(d*x + c)) + 12*B*sqrt(-a)*a*sgn(cos(d*x + c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))) + 4*sqrt(2)*(7*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*A*sqrt(-a)*a^2*sgn(cos(d*x + c)) + 12*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*B*sqrt(-a)*a^2*sgn(cos(d*x + c))

$$\begin{aligned} & /2*d*x + 1/2*c)^2 + a))^6*B*\sqrt{-a}*a^2*\text{sgn}(\cos(dx + c)) - 95*(\sqrt{-a})*\tan(1/2*d*x + 1/2*c) - \sqrt{-a*\tan(1/2*d*x + 1/2*c)^2 + a})^4*A*\sqrt{-a}*a^3 \\ & *\text{sgn}(\cos(dx + c)) - 76*(\sqrt{-a})*\tan(1/2*d*x + 1/2*c) - \sqrt{-a*\tan(1/2*d*x + 1/2*c)^2 + a})^4*B*\sqrt{-a}*a^3*\text{sgn}(\cos(dx + c)) + 53*(\sqrt{-a})*\tan(1/2*d*x + 1/2*c) - \sqrt{-a*\tan(1/2*d*x + 1/2*c)^2 + a})^2*A*\sqrt{-a}*a^4*\text{sgn}(\cos(dx + c)) + 36*(\sqrt{-a})*\tan(1/2*d*x + 1/2*c) - \sqrt{-a*\tan(1/2*d*x + 1/2*c)^2 + a})^2*B*\sqrt{-a}*a^4*\text{sgn}(\cos(dx + c)) - 5*A*\sqrt{-a}*a^5*\text{sgn}(\cos(dx + c)) - 4*B*\sqrt{-a}*a^5*\text{sgn}(\cos(dx + c)))/((\sqrt{-a})*\tan(1/2*d*x + 1/2*c) - \sqrt{-a*\tan(1/2*d*x + 1/2*c)^2 + a})^4 - 6*(\sqrt{-a})*\tan(1/2*d*x + 1/2*c) - \sqrt{-a*\tan(1/2*d*x + 1/2*c)^2 + a})^2*a + a^2)^2/d \end{aligned}$$

maple [B] time = 1.58, size = 399, normalized size = 3.35

$$\left(-7A \cos(dx + c) \operatorname{arctanh} \left(\frac{\sqrt{\frac{2 \cos(dx+c)}{1+\cos(dx+c)}} \sin(dx+c) \sqrt{2}}{2 \cos(dx+c)} \right) \left(\frac{2 \cos(dx+c)}{1+\cos(dx+c)} \right)^{\frac{3}{2}} \sin(dx + c) \sqrt{2} - 12B \cos(dx + c) \operatorname{arctanh} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(d*x+c)^2*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x)`

[Out] $-1/16/d*(-7*A*\cos(dx+c)*\operatorname{arctanh}(1/2*(-2*\cos(dx+c)/(1+\cos(dx+c))))^{(1/2)}*\sin(dx+c)/\cos(dx+c)*2^{(1/2)}*(-2*\cos(dx+c)/(1+\cos(dx+c)))^{(3/2)}*\sin(dx+c)*2^{(1/2)}-12*B*\cos(dx+c)*\operatorname{arctanh}(1/2*(-2*\cos(dx+c)/(1+\cos(dx+c))))^{(1/2)}*\sin(dx+c)/\cos(dx+c)*2^{(1/2)}*(-2*\cos(dx+c)/(1+\cos(dx+c)))^{(3/2)}*\sin(dx+c)*2^{(1/2)}-7*A*2^{(1/2)}*\operatorname{arctanh}(1/2*(-2*\cos(dx+c)/(1+\cos(dx+c))))^{(1/2)}*\sin(dx+c)/\cos(dx+c)*2^{(1/2)}*(-2*\cos(dx+c)/(1+\cos(dx+c)))^{(3/2)}*\sin(dx+c)-12*B*2^{(1/2)}*\operatorname{arctanh}(1/2*(-2*\cos(dx+c)/(1+\cos(dx+c))))^{(1/2)}*\sin(dx+c)/\cos(dx+c)*2^{(1/2)}*(-2*\cos(dx+c)/(1+\cos(dx+c)))^{(3/2)}*\sin(dx+c)+8*A*\cos(dx+c)^4+20*A*\cos(dx+c)^3+16*B*\cos(dx+c)^3-28*A*\cos(dx+c)^2-16*B*\cos(dx+c)^2)*(a*(1+\cos(dx+c))/\cos(dx+c))^{(1/2)}/\cos(dx+c)/\sin(dx+c)*a$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="maxima")`

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c + dx)^2 \left(A + \frac{B}{\cos(c + dx)} \right) \left(a + \frac{a}{\cos(c + dx)} \right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^2*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(3/2), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

$$3.133 \quad \int \cos^3(c+dx)(a+a \sec(c+dx))^{3/2}(A+B \sec(c+dx)) dx$$

Optimal. Leaf size=164

$$\frac{a^{3/2}(11A + 14B) \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}}\right)}{8d} + \frac{a^2(11A + 14B) \sin(c + dx)}{8d\sqrt{a \sec(c + dx) + a}} + \frac{a^2(7A + 6B) \sin(c + dx) \cos(c + dx)}{12d\sqrt{a \sec(c + dx) + a}} + \frac{aA \sin(c + dx)}{12d\sqrt{a \sec(c + dx) + a}}$$

[Out] 1/8*a^(3/2)*(11*A+14*B)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d + 1/8*a^2*(11*A+14*B)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/12*a^2*(7*A+6*B)*cos(d*x+c)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/3*a*A*cos(d*x+c)^2*sin(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d

Rubi [A] time = 0.37, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {4017, 4015, 3805, 3774, 203}

$$\frac{a^2(11A + 14B) \sin(c + dx)}{8d\sqrt{a \sec(c + dx) + a}} + \frac{a^{3/2}(11A + 14B) \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}}\right)}{8d} + \frac{a^2(7A + 6B) \sin(c + dx) \cos(c + dx)}{12d\sqrt{a \sec(c + dx) + a}} + \frac{aA \sin(c + dx)}{12d\sqrt{a \sec(c + dx) + a}}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]), x]

[Out] (a^(3/2)*(11*A + 14*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]])/(8*d) + (a^2*(11*A + 14*B)*Sin[c + d*x])/(8*d*Sqrt[a + a*Sec[c + d*x]]) + (a^2*(7*A + 6*B)*Cos[c + d*x]*Sin[c + d*x])/(12*d*Sqrt[a + a*Sec[c + d*x]]) + (a*A*Cos[c + d*x]^2*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(3*d)

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 3774

Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] := Dist[(-2*b)/d, Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 3805


```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.)
+ (a_.)], x_Symbol] := Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
+ b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.)
+ (a_.)]*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
+ f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (
a_.))^(m_)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \cos^3(c + dx)(a + a \sec(c + dx))^{3/2}(A + B \sec(c + dx)) dx &= \frac{aA \cos^2(c + dx)\sqrt{a + a \sec(c + dx)} \sin(c + dx)}{3d} + \\
&= \frac{a^2(7A + 6B) \cos(c + dx) \sin(c + dx)}{12d\sqrt{a + a \sec(c + dx)}} + \frac{aA \cos^2(c + dx)}{8d\sqrt{a + a \sec(c + dx)}} \\
&= \frac{a^2(11A + 14B) \sin(c + dx)}{8d\sqrt{a + a \sec(c + dx)}} + \frac{a^2(7A + 6B) \cos(c + dx)}{12d\sqrt{a + a \sec(c + dx)}} \\
&= \frac{a^2(11A + 14B) \sin(c + dx)}{8d\sqrt{a + a \sec(c + dx)}} + \frac{a^2(7A + 6B) \cos(c + dx)}{12d\sqrt{a + a \sec(c + dx)}} \\
&= \frac{a^{3/2}(11A + 14B) \tan^{-1}\left(\frac{\sqrt{a} \tan(c + dx)}{\sqrt{a + a \sec(c + dx)}}\right)}{8d} + \frac{a^2(11A + 14B)}{8d\sqrt{a}}
\end{aligned}$$

Mathematica [A] time = 1.01, size = 137, normalized size = 0.84

$$\frac{a \cos(c + dx) \sqrt{a(\sec(c + dx) + 1)} \left(\sin(c + dx) \sqrt{1 - \sec(c + dx)} (2(11A + 6B) \cos(c + dx) + 4A \cos(2(c + dx))) + 3 \right)}{24d(\cos(c + dx) + 1) \sqrt{1 - \sec(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]

[Out] (a*Cos[c + d*x]*Sqrt[a*(1 + Sec[c + d*x])]*((37*A + 42*B + 2*(11*A + 6*B))*Cos[c + d*x] + 4*A*Cos[2*(c + d*x)])*Sqrt[1 - Sec[c + d*x]]*Sin[c + d*x] + 3*(11*A + 14*B)*ArcTanh[Sqrt[1 - Sec[c + d*x]]]*Tan[c + d*x])/(24*d*(1 + Cos[c + d*x])*Sqrt[1 - Sec[c + d*x]])

fricas [A] time = 0.52, size = 360, normalized size = 2.20

$$\left[\frac{3((11A + 14B)a \cos(dx + c) + (11A + 14B)a) \sqrt{-a} \log \left(\frac{2a \cos(dx+c)^2 - 2\sqrt{-a} \sqrt{\frac{a \cos(dx+c)+a}{\cos(dx+c)}} \cos(dx+c) \sin(dx+c) + a \cos(dx+c)}{\cos(dx+c)+1} \right)}{48(d \cos(dx+c) + d)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] [1/48*(3*((11*A + 14*B)*a*cos(d*x + c) + (11*A + 14*B)*a)*sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(8*A*a*cos(d*x + c)^3 + 2*(11*A + 6*B)*a*cos(d*x + c)^2 + 3*(11*A + 14*B)*a*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d), -1/24*(3*((11*A + 14*B)*a*cos(d*x + c) + (11*A + 14*B)*a)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - (8*A*a*cos(d*x + c)^3 + 2*(11*A + 6*B)*a*cos(d*x + c)^2 + 3*(11*A + 14*B)*a*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d)]

giac [B] time = 1.93, size = 897, normalized size = 5.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out]
$$\begin{aligned} & -1/48*(3*(11*A*\sqrt{-a})*a*\operatorname{sgn}(\cos(dx+c)) + 14*B*\sqrt{-a})*a*\operatorname{sgn}(\cos(dx+c)) \\ & * \log(\operatorname{abs}(\sqrt{-a}*\tan(1/2*dx+1/2*c) - \sqrt{-a*\tan(1/2*dx+1/2*c)^2+a})^2 - a*(2*\sqrt{2}+3))) - 3*(11*A*\sqrt{-a})*a*\operatorname{sgn}(\cos(dx+c)) + 14*B*\sqrt{-a})*a*\operatorname{sgn}(\cos(dx+c)) \\ & * \log(\operatorname{abs}(\sqrt{-a}*\tan(1/2*dx+1/2*c) - \sqrt{-a*\tan(1/2*dx+1/2*c)^2+a})^2 + a*(2*\sqrt{2}-3))) + 4*(33*\sqrt{2})*(\sqrt{-a}*\tan(1/2*dx+1/2*c) - \sqrt{-a*\tan(1/2*dx+1/2*c)^2+a})^{10} \\ & * A*\sqrt{-a})*a^2*\operatorname{sgn}(\cos(dx+c)) + 42*\sqrt{2}*(\sqrt{-a}*\tan(1/2*dx+1/2*c) - \sqrt{-a*\tan(1/2*dx+1/2*c)^2+a})^{10} \\ & * B*\sqrt{-a})*a^2*\operatorname{sgn}(\cos(dx+c)) - 303*\sqrt{2}*(\sqrt{-a}*\tan(1/2*dx+1/2*c) - \sqrt{-a*\tan(1/2*dx+1/2*c)^2+a})^8 \\ & * A*\sqrt{-a})*a^3*\operatorname{sgn}(\cos(dx+c)) - 822*\sqrt{2}*(\sqrt{-a}*\tan(1/2*dx+1/2*c) - \sqrt{-a*\tan(1/2*dx+1/2*c)^2+a})^8 \\ & * B*\sqrt{-a})*a^3*\operatorname{sgn}(\cos(dx+c)) + 2394*\sqrt{2}*(\sqrt{-a}*\tan(1/2*dx+1/2*c) - \sqrt{-a*\tan(1/2*dx+1/2*c)^2+a})^6 \\ & * A*\sqrt{-a})*a^4*\operatorname{sgn}(\cos(dx+c)) + 3780*\sqrt{2}*(\sqrt{-a}*\tan(1/2*dx+1/2*c) - \sqrt{-a*\tan(1/2*dx+1/2*c)^2+a})^6 \\ & * B*\sqrt{-a})*a^4*\operatorname{sgn}(\cos(dx+c)) - 1806*\sqrt{2}*(\sqrt{-a}*\tan(1/2*dx+1/2*c) - \sqrt{-a*\tan(1/2*dx+1/2*c)^2+a})^4 \\ & * A*\sqrt{-a})*a^5*\operatorname{sgn}(\cos(dx+c)) - 2508*\sqrt{2}*(\sqrt{-a}*\tan(1/2*dx+1/2*c) - \sqrt{-a*\tan(1/2*dx+1/2*c)^2+a})^4 \\ & * B*\sqrt{-a})*a^5*\operatorname{sgn}(\cos(dx+c)) + 309*\sqrt{2}*(\sqrt{-a}*\tan(1/2*dx+1/2*c) - \sqrt{-a*\tan(1/2*dx+1/2*c)^2+a})^2 \\ & * A*\sqrt{-a})*a^6*\operatorname{sgn}(\cos(dx+c)) + 498*\sqrt{2}*(\sqrt{-a}*\tan(1/2*dx+1/2*c) - \sqrt{-a*\tan(1/2*dx+1/2*c)^2+a})^2 \\ & * B*\sqrt{-a})*a^6*\operatorname{sgn}(\cos(dx+c)) - 19*\sqrt{2}*(\sqrt{-a}*\tan(1/2*dx+1/2*c) - \sqrt{-a*\tan(1/2*dx+1/2*c)^2+a}) \\ & * A*\sqrt{-a})*a^7*\operatorname{sgn}(\cos(dx+c)) - 30*\sqrt{2}*(\sqrt{-a}*\tan(1/2*dx+1/2*c) - \sqrt{-a*\tan(1/2*dx+1/2*c)^2+a}) \\ & * B*\sqrt{-a})*a^7*\operatorname{sgn}(\cos(dx+c)))/((\sqrt{-a}*\tan(1/2*dx+1/2*c) - \sqrt{-a*\tan(1/2*dx+1/2*c)^2+a})^4 - 6*(\sqrt{-a}*\tan(1/2*dx+1/2*c) - \sqrt{-a*\tan(1/2*dx+1/2*c)^2+a})^2*a + a^2)^3)/d \end{aligned}$$

maple [B] time = 1.81, size = 581, normalized size = 3.54

$$\left(33A \sin(dx+c) \left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)} \right)^{\frac{5}{2}} \operatorname{arctanh} \left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}} \sin(dx+c) \sqrt{2}}{2\cos(dx+c)} \right) \sqrt{2} (\cos^2(dx+c)) + 42B \sin(dx+c) \left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)} \right)^{\frac{5}{2}} \operatorname{arctanh} \left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}} \sin(dx+c) \sqrt{2}}{2\cos(dx+c)} \right) \sqrt{2} (\cos^2(dx+c)) \right) / \left((\sqrt{-a}*\tan(1/2*dx+1/2*c) - \sqrt{-a*\tan(1/2*dx+1/2*c)^2+a})^4 - 6*(\sqrt{-a}*\tan(1/2*dx+1/2*c) - \sqrt{-a*\tan(1/2*dx+1/2*c)^2+a})^2*a + a^2 \right)^3 / d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^3*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x)

[Out]
$$\begin{aligned} & -1/192/d*(33*A*\sin(dx+c)*(-2*\cos(dx+c)/(1+\cos(dx+c)))^{5/2}*\operatorname{arctanh}(1/2*(-2*\cos(dx+c)/(1+\cos(dx+c)))^{1/2}*\sin(dx+c)/\cos(dx+c)*2^{1/2})*2^{1/2} \\ & * \cos(dx+c)^2+42*B*\sin(dx+c)*(-2*\cos(dx+c)/(1+\cos(dx+c)))^{5/2}*\operatorname{arctanh}(1/2*(-2*\cos(dx+c)/(1+\cos(dx+c)))^{1/2}*\sin(dx+c)/\cos(dx+c)*2^{1/2})*2^{1/2} \\ & * \cos(dx+c)^2+66*A*\sin(dx+c)*(-2*\cos(dx+c)/(1+\cos(dx+c)))^{5/2}*\operatorname{arctanh}(1/2*(-2*\cos(dx+c)/(1+\cos(dx+c)))^{1/2}*\sin(dx+c)/\cos(dx+c)*2^{1/2})*2^{1/2} \\ & * \cos(dx+c)^2) \end{aligned}$$

```
*2^(1/2)*cos(d*x+c)+84*B*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*ar
ctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2
))*2^(1/2)*cos(d*x+c)+33*A*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*2^(1/2)*arc
tanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2
))*sin(d*x+c)+42*B*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*2^(1/2)*arctanh(1/2*
(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*sin(d*x
+c)+64*A*cos(d*x+c)^6+112*A*cos(d*x+c)^5+96*B*cos(d*x+c)^5+88*A*cos(d*x+c)^
4+240*B*cos(d*x+c)^4-264*A*cos(d*x+c)^3-336*B*cos(d*x+c)^3)*(a*(1+cos(d*x+c
))/cos(d*x+c))^(1/2)/cos(d*x+c)^2/sin(d*x+c)*a
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm
="maxima")
```

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c + dx)^3 \left(A + \frac{B}{\cos(c + dx)} \right) \left(a + \frac{a}{\cos(c + dx)} \right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^3*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)),x)
```

[Out] Timed out

$$3.134 \quad \int \cos^4(c+dx)(a+a \sec(c+dx))^{3/2}(A+B \sec(c+dx)) dx$$

Optimal. Leaf size=209

$$\frac{a^{3/2}(75A + 88B) \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}}\right)}{64d} + \frac{a^2(75A + 88B) \sin(c + dx)}{64d\sqrt{a \sec(c + dx) + a}} + \frac{a^2(9A + 8B) \sin(c + dx) \cos^2(c + dx)}{24d\sqrt{a \sec(c + dx) + a}} + \frac{a^2}{d}$$

[Out] 1/64*a^(3/2)*(75*A+88*B)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+1/64*a^2*(75*A+88*B)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/96*a^2*(75*A+88*B)*cos(d*x+c)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/24*a^2*(9*A+8*B)*cos(d*x+c)^2*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/4*a*A*cos(d*x+c)^3*sin(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d

Rubi [A] time = 0.45, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {4017, 4015, 3805, 3774, 203}

$$\frac{a^2(75A + 88B) \sin(c + dx)}{64d\sqrt{a \sec(c + dx) + a}} + \frac{a^{3/2}(75A + 88B) \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}}\right)}{64d} + \frac{a^2(9A + 8B) \sin(c + dx) \cos^2(c + dx)}{24d\sqrt{a \sec(c + dx) + a}} + \frac{a^2}{d}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]

[Out] (a^(3/2)*(75*A + 88*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(64*d) + (a^2*(75*A + 88*B)*Sin[c + d*x])/(64*d*Sqrt[a + a*Sec[c + d*x]]) + (a^2*(75*A + 88*B)*Cos[c + d*x]*Sin[c + d*x])/(96*d*Sqrt[a + a*Sec[c + d*x]]) + (a^2*(9*A + 8*B)*Cos[c + d*x]^2*Sin[c + d*x])/(24*d*Sqrt[a + a*Sec[c + d*x]]) + (a*A*Cos[c + d*x]^3*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(4*d)

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 3774

Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] := Dist[(-2*b)/d, Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.)
+ (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
+ b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.)
+ (a_)]*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
+ f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \cos^4(c + dx)(a + a \sec(c + dx))^{3/2}(A + B \sec(c + dx)) dx &= \frac{aA \cos^3(c + dx) \sqrt{a + a \sec(c + dx)} \sin(c + dx)}{4d} + \\
&= \frac{a^2(9A + 8B) \cos^2(c + dx) \sin(c + dx)}{24d \sqrt{a + a \sec(c + dx)}} + \frac{aA \cos^3(c + dx)}{24d \sqrt{a + a \sec(c + dx)}} \\
&= \frac{a^2(75A + 88B) \cos(c + dx) \sin(c + dx)}{96d \sqrt{a + a \sec(c + dx)}} + \frac{a^2(9A + 8B) \cos^3(c + dx)}{96d \sqrt{a + a \sec(c + dx)}} \\
&= \frac{a^2(75A + 88B) \sin(c + dx)}{64d \sqrt{a + a \sec(c + dx)}} + \frac{a^2(75A + 88B) \cos(c + dx)}{96d \sqrt{a + a \sec(c + dx)}} \\
&= \frac{a^2(75A + 88B) \sin(c + dx)}{64d \sqrt{a + a \sec(c + dx)}} + \frac{a^2(75A + 88B) \cos(c + dx)}{96d \sqrt{a + a \sec(c + dx)}} \\
&= \frac{a^{3/2}(75A + 88B) \tan^{-1}\left(\frac{\sqrt{a} \tan(c + dx)}{\sqrt{a + a \sec(c + dx)}}\right)}{64d} + \frac{a^2(75A + 88B)}{64d \sqrt{a + a \sec(c + dx)}}
\end{aligned}$$

Mathematica [A] time = 1.48, size = 154, normalized size = 0.74

$$\frac{a \cos(c + dx) \sqrt{a(\sec(c + dx) + 1)} \left(\sin(c + dx) \sqrt{1 - \sec(c + dx)} (2(93A + 88B) \cos(c + dx) + 4(15A + 8B) \cos(2c + 2dx)) + 192d(\cos(c + dx) + 1) \right)}{192d(\cos(c + dx) + 1) \sqrt{a(\sec(c + dx) + 1)}}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]

[Out] (a*Cos[c + d*x]*Sqrt[a*(1 + Sec[c + d*x])]*((285*A + 296*B + 2*(93*A + 88*B)*Cos[c + d*x] + 4*(15*A + 8*B)*Cos[2*(c + d*x)] + 12*A*Cos[3*(c + d*x)]))*Sqrt[1 - Sec[c + d*x]]*Sin[c + d*x] + 3*(75*A + 88*B)*ArcTanh[Sqrt[1 - Sec[c + d*x]]*Tan[c + d*x]])/(192*d*(1 + Cos[c + d*x])*Sqrt[1 - Sec[c + d*x]])

fricas [A] time = 0.56, size = 396, normalized size = 1.89

$$\left[\frac{3((75A + 88B)a \cos(dx + c) + (75A + 88B)a) \sqrt{-a} \log \left(\frac{2a \cos(dx + c)^2 - 2\sqrt{-a} \sqrt{\frac{a \cos(dx + c) + a}{\cos(dx + c)}} \cos(dx + c) \sin(dx + c) + a \cos(dx + c)}{\cos(dx + c) + 1} \right)}{\dots} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] [1/384*(3*((75*A + 88*B)*a*cos(d*x + c) + (75*A + 88*B)*a)*sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(48*A*a*cos(d*x + c)^4 + 8*(15*A + 8*B)*a*cos(d*x + c)^3 + 2*(75*A + 88*B)*a*cos(d*x + c)^2 + 3*(75*A + 88*B)*a*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d), -1/192*(3*((75*A + 88*B)*a*cos(d*x + c) + (75*A + 88*B)*a)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - (48*A*a*cos(d*x + c)^4 + 8*(15*A + 8*B)*a*cos(d*x + c)^3 + 2*(75*A + 88*B)*a*cos(d*x + c)^2 + 3*(75*A + 88*B)*a*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d)]

giac [B] time = 8.93, size = 1088, normalized size = 5.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] -1/384*(3*(75*A*sqrt(-a)*a*sgn(cos(d*x + c)) + 88*B*sqrt(-a)*a*sgn(cos(d*x + c))) * log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3))) - 3*(75*A*sqrt(-a)*a*sgn(cos(d*x + c)) + 88*B*sqrt(-a)*a*sgn(cos(d*x + c))) * log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))) + 4*sqrt(2)*(225*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^14*A*sqrt(-a)*a^2*sgn(cos(d*x + c)) + 264*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^14*B*sqrt(-a)*a^2*sgn(cos(d*x + c)) - 6261*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^12*A*sqrt(-a)*a^3*sgn(cos(d*x + c)) - 4008*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^12*B*sqrt(-a)*a^3*sgn(cos(d*x + c)) + 35925*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^10*A*sqrt(-a)*a^4*sgn(cos(d*x + c)) + 33960*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^10*B*sqrt(-a)*a^4*sgn(cos(d*x + c)) - 127449*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^8*A*sqrt(-a)*a^5*sgn(cos(d*x + c)) - 131784*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^8*B*sqrt(-a)*a^5*sgn(cos(d*x + c)) + 101667*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*A*sqrt(-a)*a^6*sgn(cos(d*x + c)) + 108312*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*B*sqrt(-a)*a^6*sgn(cos(d*x + c)) - 26079*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*A*sqrt(-a)*a^7*sgn(cos(d*x + c)) - 29432*(sqrt(-a)*tan(1/2*d*x

$$\frac{\begin{aligned} &+ 1/2*c) - \sqrt{-a*\tan(1/2*d*x + 1/2*c)^2 + a})^4*B*\sqrt{-a}*a^7*\text{sgn}(\cos(d*x + c)) + 3303*(\sqrt{-a}*\tan(1/2*d*x + 1/2*c) - \sqrt{-a*\tan(1/2*d*x + 1/2*c)^2 + a})^2*A*\sqrt{-a}*a^8*\text{sgn}(\cos(d*x + c)) + 3384*(\sqrt{-a}*\tan(1/2*d*x + 1/2*c) - \sqrt{-a*\tan(1/2*d*x + 1/2*c)^2 + a})^2*B*\sqrt{-a}*a^8*\text{sgn}(\cos(d*x + c)) - 147*A*\sqrt{-a}*a^9*\text{sgn}(\cos(d*x + c)) - 152*B*\sqrt{-a}*a^9*\text{sgn}(\cos(d*x + c)) \end{aligned}}{((\sqrt{-a}*\tan(1/2*d*x + 1/2*c) - \sqrt{-a*\tan(1/2*d*x + 1/2*c)^2 + a})^4 - 6*(\sqrt{-a}*\tan(1/2*d*x + 1/2*c) - \sqrt{-a*\tan(1/2*d*x + 1/2*c)^2 + a})^2*a + a^2)^4}/d$$

maple [B] time = 1.62, size = 763, normalized size = 3.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(d*x+c)^4*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x)`

[Out] $\frac{1}{3072} \frac{1}{d} \left(225 A^2 \sqrt{\frac{-2 \cos(d*x+c)}{1+\cos(d*x+c)}} \arctanh\left(\frac{1}{2} \sqrt{\frac{-2 \cos(d*x+c)}{1+\cos(d*x+c)}} \sin(d*x+c)\right) \cos(d*x+c)^3 + 264 B^2 \sqrt{\frac{-2 \cos(d*x+c)}{1+\cos(d*x+c)}} \arctanh\left(\frac{1}{2} \sqrt{\frac{-2 \cos(d*x+c)}{1+\cos(d*x+c)}} \sin(d*x+c)\right) \cos(d*x+c)^2 + 675 A^2 \sqrt{\frac{-2 \cos(d*x+c)}{1+\cos(d*x+c)}} \arctanh\left(\frac{1}{2} \sqrt{\frac{-2 \cos(d*x+c)}{1+\cos(d*x+c)}} \sin(d*x+c)\right) \cos(d*x+c)^2 + 792 B^2 \sqrt{\frac{-2 \cos(d*x+c)}{1+\cos(d*x+c)}} \arctanh\left(\frac{1}{2} \sqrt{\frac{-2 \cos(d*x+c)}{1+\cos(d*x+c)}} \sin(d*x+c)\right) \cos(d*x+c)^2 + 675 A^2 \sqrt{\frac{-2 \cos(d*x+c)}{1+\cos(d*x+c)}} \arctanh\left(\frac{1}{2} \sqrt{\frac{-2 \cos(d*x+c)}{1+\cos(d*x+c)}} \sin(d*x+c)\right) \cos(d*x+c)^2 + 792 B^2 \sqrt{\frac{-2 \cos(d*x+c)}{1+\cos(d*x+c)}} \arctanh\left(\frac{1}{2} \sqrt{\frac{-2 \cos(d*x+c)}{1+\cos(d*x+c)}} \sin(d*x+c)\right) \cos(d*x+c)^2 + 225 A \sqrt{\frac{-2 \cos(d*x+c)}{1+\cos(d*x+c)}} \arctanh\left(\frac{1}{2} \sqrt{\frac{-2 \cos(d*x+c)}{1+\cos(d*x+c)}} \sin(d*x+c)\right) \cos(d*x+c)^2 + 264 B \sqrt{\frac{-2 \cos(d*x+c)}{1+\cos(d*x+c)}} \arctanh\left(\frac{1}{2} \sqrt{\frac{-2 \cos(d*x+c)}{1+\cos(d*x+c)}} \sin(d*x+c)\right) \cos(d*x+c)^2 - 768 A \cos(d*x+c)^8 - 1152 A \cos(d*x+c)^7 - 1024 B \cos(d*x+c)^7 - 480 A \cos(d*x+c)^6 - 1792 B \cos(d*x+c)^6 - 1200 A \cos(d*x+c)^5 - 1408 B \cos(d*x+c)^5 + 3600 A \cos(d*x+c)^4 + 4224 B \cos(d*x+c)^4 \right) \frac{a(1+\cos(d*x+c))}{\cos(d*x+c)^{3/2} \sin(d*x+c)}$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="maxima")`

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c + dx)^4 \left(A + \frac{B}{\cos(c + dx)} \right) \left(a + \frac{a}{\cos(c + dx)} \right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(c + d*x)^4*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(3/2), x)`

[Out] `int(cos(c + d*x)^4*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(3/2), x)`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)**4*(a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)), x)`

[Out] Timed out

3.135 $\int \sec^3(c+dx)(a+a \sec(c+dx))^{5/2}(A+B \sec(c+dx)) dx$

Optimal. Leaf size=237

$$\frac{2a^3(209A + 194B) \tan(c + dx) \sec^3(c + dx)}{693d\sqrt{a \sec(c + dx) + a}} + \frac{2a^3(803A + 710B) \tan(c + dx)}{495d\sqrt{a \sec(c + dx) + a}} + \frac{2a^2(11A + 14B) \tan(c + dx) \sec^3(c + dx)}{99d}$$

[Out] $2/1155*a*(803*A+710*B)*(a+a*\sec(d*x+c))^{(3/2)*\tan(d*x+c)/d+2/11*a*B*\sec(d*x+c)^3*(a+a*\sec(d*x+c))^{(3/2)*\tan(d*x+c)/d+2/495*a^3*(803*A+710*B)*\tan(d*x+c)/d/(a+a*\sec(d*x+c))^{(1/2)+2/693*a^3*(209*A+194*B)*\sec(d*x+c)^3*\tan(d*x+c)/d/(a+a*\sec(d*x+c))^{(1/2)-4/3465*a^2*(803*A+710*B)*(a+a*\sec(d*x+c))^{(1/2)*\tan(d*x+c)/d+2/99*a^2*(11*A+14*B)*\sec(d*x+c)^3*(a+a*\sec(d*x+c))^{(1/2)*\tan(d*x+c)/d}}$

Rubi [A] time = 0.66, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {4018, 4016, 3800, 4001, 3792}

$$\frac{2a^3(209A + 194B) \tan(c + dx) \sec^3(c + dx)}{693d\sqrt{a \sec(c + dx) + a}} + \frac{2a^2(11A + 14B) \tan(c + dx) \sec^3(c + dx)\sqrt{a \sec(c + dx) + a}}{99d} + \frac{2a^3}{4}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sec}[c + d*x]^3*(a + a*\text{Sec}[c + d*x])^{(5/2)*(A + B*\text{Sec}[c + d*x])}, x]$

[Out] $(2*a^3*(803*A + 710*B)*\text{Tan}[c + d*x])/(495*d*\text{Sqrt}[a + a*\text{Sec}[c + d*x]]) + (2*a^3*(209*A + 194*B)*\text{Sec}[c + d*x]^3*\text{Tan}[c + d*x])/(693*d*\text{Sqrt}[a + a*\text{Sec}[c + d*x]]) - (4*a^2*(803*A + 710*B)*\text{Sqrt}[a + a*\text{Sec}[c + d*x]]*\text{Tan}[c + d*x])/(3465*d) + (2*a^2*(11*A + 14*B)*\text{Sec}[c + d*x]^3*\text{Sqrt}[a + a*\text{Sec}[c + d*x]]*\text{Tan}[c + d*x])/(99*d) + (2*a*(803*A + 710*B)*(a + a*\text{Sec}[c + d*x])^{(3/2)*\text{Tan}[c + d*x]})/(1155*d) + (2*a*B*\text{Sec}[c + d*x]^3*(a + a*\text{Sec}[c + d*x])^{(3/2)*\text{Tan}[c + d*x]})/(11*d)$

Rule 3792

$\text{Int}[\text{csc}[(e_.) + (f_.)*(x_.)]*\text{Sqrt}[\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] \rightarrow \text{Simp}[(-2*b*\text{Cot}[e + f*x])/(f*\text{Sqrt}[a + b*\text{Csc}[e + f*x]]), x] /; \text{FreeQ}\{a, b, e, f, x\} \ \&\& \ \text{EqQ}[a^2 - b^2, 0]$

Rule 3800

$\text{Int}[\text{csc}[(e_.) + (f_.)*(x_.)]^3*(\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^{(m_.)}, x_Symbol] \rightarrow -\text{Simp}[(\text{Cot}[e + f*x]*(a + b*\text{Csc}[e + f*x])^{(m+1)})/(b*f*(m+2)), x] + \text{Dist}[1/(b*(m+2)), \text{Int}[\text{Csc}[e + f*x]*(a + b*\text{Csc}[e + f*x])^m*(b*(m+2)), x]$

1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)]

Rule 4001

Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m + 1), 0] && !LtQ[m, -2^(-1)]

Rule 4016

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] :> Simp[(-2*b*B*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && !LtQ[n, 0]

Rule 4018

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] :> -Simp[(b*B*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && !LtQ[n, -1]

Rubi steps

$$\begin{aligned}
\int \sec^3(c + dx)(a + a \sec(c + dx))^{5/2}(A + B \sec(c + dx)) dx &= \frac{2aB \sec^3(c + dx)(a + a \sec(c + dx))^{3/2} \tan(c + dx)}{11d} \\
&= \frac{2a^2(11A + 14B) \sec^3(c + dx) \sqrt{a + a \sec(c + dx)} \tan(c + dx)}{99d} \\
&= \frac{2a^3(209A + 194B) \sec^3(c + dx) \tan(c + dx)}{693d \sqrt{a + a \sec(c + dx)}} + \frac{2a^2}{693d \sqrt{a + a \sec(c + dx)}} \\
&= \frac{2a^3(209A + 194B) \sec^3(c + dx) \tan(c + dx)}{693d \sqrt{a + a \sec(c + dx)}} + \frac{2a^2}{693d \sqrt{a + a \sec(c + dx)}} \\
&= \frac{2a^3(209A + 194B) \sec^3(c + dx) \tan(c + dx)}{693d \sqrt{a + a \sec(c + dx)}} - \frac{4a^2}{693d \sqrt{a + a \sec(c + dx)}} \\
&= \frac{2a^3(803A + 710B) \tan(c + dx)}{495d \sqrt{a + a \sec(c + dx)}} + \frac{2a^3(209A + 194B)}{693d \sqrt{a + a \sec(c + dx)}}
\end{aligned}$$

Mathematica [B] time = 6.19, size = 487, normalized size = 2.05

$$\frac{2A \tan(c + dx) \sec^3(c + dx)(a(\sec(c + dx) + 1))^{5/2}}{9d(\sec(c + dx) + 1)^2} + \frac{38A \tan(c + dx) \sec^3(c + dx)(a(\sec(c + dx) + 1))^{5/2}}{63d(\sec(c + dx) + 1)^3} + \frac{146A}{693d \sqrt{a + a \sec(c + dx)}}$$

Antiderivative was successfully verified.

```

[In] Integrate[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
[Out] (1168*A*(a*(1 + Sec[c + d*x]))^(5/2)*Tan[c + d*x]/(315*d*(1 + Sec[c + d*x])^3) + (2272*B*(a*(1 + Sec[c + d*x]))^(5/2)*Tan[c + d*x]/(693*d*(1 + Sec[c + d*x])^3) + (584*A*Sec[c + d*x]*(a*(1 + Sec[c + d*x]))^(5/2)*Tan[c + d*x]/(315*d*(1 + Sec[c + d*x])^3) + (1136*B*Sec[c + d*x]*(a*(1 + Sec[c + d*x]))^(5/2)*Tan[c + d*x]/(693*d*(1 + Sec[c + d*x])^3) + (146*A*Sec[c + d*x]^2*(a*(1 + Sec[c + d*x]))^(5/2)*Tan[c + d*x]/(105*d*(1 + Sec[c + d*x])^3) + (284*B*Sec[c + d*x]^2*(a*(1 + Sec[c + d*x]))^(5/2)*Tan[c + d*x]/(231*d*(1 + Sec[c + d*x])^3) + (38*A*Sec[c + d*x]^3*(a*(1 + Sec[c + d*x]))^(5/2)*Tan[c + d*x]/(63*d*(1 + Sec[c + d*x])^3) + (710*B*Sec[c + d*x]^3*(a*(1 + Sec[c + d*x]))^(5/2)*Tan[c + d*x]/(693*d*(1 + Sec[c + d*x])^3) + (46*B*Sec[c + d*x]^4*(a*(1 + Sec[c + d*x]))^(5/2)*Tan[c + d*x]/(99*d*(1 + Sec[c + d*x])^3) + (2*A*Sec[c + d*x]^3*(a*(1 + Sec[c + d*x]))^(5/2)*Tan[c + d*x]/(9*d*(1 + Sec[c + d*x])^2) + (2*B*Sec[c + d*x]^4*(a*(1 + Sec[c + d*x]))^(5/2)*Tan[c + d*x]/(11*d*(1 + Sec[c + d*x])^2)

```

fricas [A] time = 0.43, size = 157, normalized size = 0.66

$$\frac{2 \left(8 (803 A + 710 B) a^2 \cos(dx + c)^5 + 4 (803 A + 710 B) a^2 \cos(dx + c)^4 + 3 (803 A + 710 B) a^2 \cos(dx + c)^3 + 5 (803 A + 710 B) a^2 \cos(dx + c)^2 + 35 (11 A + 32 B) a^2 \cos(dx + c) + 315 B a^2 \right) \sqrt{a \cos(dx + c) + a} \sin(dx + c)}{3465 \left(d \cos(dx + c) \right)^6 + d \cos(dx + c)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] 2/3465*(8*(803*A + 710*B)*a^2*cos(d*x + c)^5 + 4*(803*A + 710*B)*a^2*cos(d*x + c)^4 + 3*(803*A + 710*B)*a^2*cos(d*x + c)^3 + 5*(286*A + 355*B)*a^2*cos(d*x + c)^2 + 35*(11*A + 32*B)*a^2*cos(d*x + c) + 315*B*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^6 + d*cos(d*x + c)^5)

giac [A] time = 2.27, size = 306, normalized size = 1.29

$$8 \left(\left(\left(\left(4 \left(2 \sqrt{2} \left(143 A a^8 \operatorname{sgn}(\cos(dx + c)) + 125 B a^8 \operatorname{sgn}(\cos(dx + c)) \right) \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) \right)^2 - 11 \sqrt{2} \left(143 A a^8 \operatorname{sgn}(\cos(dx + c)) + 125 B a^8 \operatorname{sgn}(\cos(dx + c)) \right) \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] 8/3465*(((4*(2*sqrt(2)*(143*A*a^8*sgn(cos(d*x + c)) + 125*B*a^8*sgn(cos(d*x + c))))*tan(1/2*d*x + 1/2*c)^2 - 11*sqrt(2)*(143*A*a^8*sgn(cos(d*x + c)) + 125*B*a^8*sgn(cos(d*x + c))))*tan(1/2*d*x + 1/2*c)^2 + 99*sqrt(2)*(143*A*a^8*sgn(cos(d*x + c)) + 125*B*a^8*sgn(cos(d*x + c))))*tan(1/2*d*x + 1/2*c)^2 - 231*sqrt(2)*(69*A*a^8*sgn(cos(d*x + c)) + 65*B*a^8*sgn(cos(d*x + c))))*tan(1/2*d*x + 1/2*c)^2 + 1155*sqrt(2)*(9*A*a^8*sgn(cos(d*x + c)) + 7*B*a^8*sgn(cos(d*x + c))))*tan(1/2*d*x + 1/2*c)^2 - 3465*sqrt(2)*(A*a^8*sgn(cos(d*x + c)) + B*a^8*sgn(cos(d*x + c))))*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^5*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*d)

maple [A] time = 1.64, size = 163, normalized size = 0.69

$$2(-1 + \cos(dx + c)) \left(6424A \left(\cos^5(dx + c) \right) + 5680B \left(\cos^5(dx + c) \right) + 3212A \left(\cos^4(dx + c) \right) + 2840B \left(\cos^4(dx + c) \right) + \dots \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(\sec(dx+c)^3(a+a\sec(dx+c))^{5/2}(A+B\sec(dx+c)), x)$

[Out] $-2/3465/d*(-1+\cos(dx+c))*(6424*A*\cos(dx+c)^5+5680*B*\cos(dx+c)^5+3212*A*\cos(dx+c)^4+2840*B*\cos(dx+c)^4+2409*A*\cos(dx+c)^3+2130*B*\cos(dx+c)^3+1430*A*\cos(dx+c)^2+1775*B*\cos(dx+c)^2+385*A*\cos(dx+c)+1120*B*\cos(dx+c)+315*B)*(a*(1+\cos(dx+c))/\cos(dx+c))^{1/2}/\cos(dx+c)^5/\sin(dx+c)*a^2$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(\sec(dx+c)^3(a+a\sec(dx+c))^{5/2}(A+B\sec(dx+c)), x, \text{algorithm}="maxima")$

[Out] Timed out

mupad [B] time = 13.51, size = 856, normalized size = 3.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(((A + B/\cos(c + dx))*(a + a/\cos(c + dx))^{5/2})/\cos(c + dx)^3, x)$

[Out] $((a + a/(\exp(-c*1i - d*x*1i)/2 + \exp(c*1i + d*x*1i)/2))^{1/2}*((A*a^2*8i)/(3*d) - (a^2*\exp(c*1i + d*x*1i)*(803*A + 710*B)*8i)/(3465*d)))/((\exp(c*1i + d*x*1i) + 1)*(\exp(c*2i + d*x*2i) + 1)) - ((a + a/(\exp(-c*1i - d*x*1i)/2 + \exp(c*1i + d*x*1i)/2))^{1/2}*((A*a^2*24i)/(7*d) - \exp(c*1i + d*x*1i)*((a^2*(5*A + 16*B)*8i)/(7*d) - (a^2*(5*A + 2*B)*8i)/(7*d) + (a^2*(11*A + 50*B)*3*2i)/(693*d) + (a^2*(9*A + 10*B)*8i)/(7*d)))/((\exp(c*1i + d*x*1i) + 1)*(\exp(c*2i + d*x*2i) + 1)^3) + ((a + a/(\exp(-c*1i - d*x*1i)/2 + \exp(c*1i + d*x*1i)/2))^{1/2}*(\exp(c*1i + d*x*1i)*((A*a^2*8i)/(11*d) + (a^2*(3*A + 4*B)*40i)/(11*d) - (a^2*(5*A + 2*B)*8i)/(11*d) - (a^2*(11*A + 10*B)*8i)/(11*d)) + (A*a^2*8i)/(11*d) + (a^2*(3*A + 4*B)*40i)/(11*d) - (a^2*(5*A + 2*B)*8i)/(11*d) - (a^2*(11*A + 10*B)*8i)/(11*d)))/((\exp(c*1i + d*x*1i) + 1)*(\exp(c*2i + d*x*2i) + 1)^5) - ((a + a/(\exp(-c*1i - d*x*1i)/2 + \exp(c*1i + d*x*1i)/2))^{1/2}*((A*a^2*8i)/(9*d) - \exp(c*1i + d*x*1i)*((a^2*(A - 8*B)*8i)/(9*d) - (B*a^2*64i)/(99*d) + (a^2*(5*A + 2*B)*8i)/(9*d) - (a^2*(5*A + 9*B)*16i)/(9*d) + (a^2*(A + 2*B)*40i)/(9*d) + (B*a^2*64i)/(9*d) - (a^2*(A + B)*80i)/(9*d)))/((\exp(c*1i + d*x*1i) + 1)*(\exp(c*2i + d*x*2i) + 1)^4) + ((a + a/(\exp(-c*1i - d*x*1i)/2 + \exp(c*1i + d*x*1i)/2))^{1/2}*(\exp(c*1i + d*x*1i)*((a^2*(5*A + 2*B)*8i)/(5*d) + (a^2*(44*A - 31*B)*16i)/(1155*d) - (A*a^2*8i)/(5*d) + (a^2*(4*A + 5*B)*16i)/(5*d)))/((\exp(c*1i + d*x*1i) + 1)*(\exp(c*2i + d*x*2i) + 1)^3))$

$i) + 1)^2) - (a^2 \exp(c + dx) (a + a/(\exp(-c - dx)/2 + \exp(c + dx)/2))^{1/2} (803A + 710B) \cdot 16i) / (3465d (\exp(c + dx) + 1))$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**3*(a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)),x)

[Out] Timed out

3.136 $\int \sec^2(c+dx)(a+a \sec(c+dx))^{5/2}(A+B \sec(c+dx)) dx$

Optimal. Leaf size=175

$$\frac{64a^3(15A+13B)\tan(c+dx)}{315d\sqrt{a\sec(c+dx)+a}} + \frac{16a^2(15A+13B)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{315d} + \frac{2(9A-2B)\tan(c+dx)(a\sec(c+dx))^{5/2}}{63d}$$

[Out] $2/105*a*(15*A+13*B)*(a+a*\sec(d*x+c))^{3/2}*\tan(d*x+c)/d+2/63*(9*A-2*B)*(a+a*\sec(d*x+c))^{5/2}*\tan(d*x+c)/d+2/9*B*(a+a*\sec(d*x+c))^{7/2}*\tan(d*x+c)/a/d+64/315*a^3*(15*A+13*B)*\tan(d*x+c)/d/(a+a*\sec(d*x+c))^{1/2}+16/315*a^2*(15*A+13*B)*(a+a*\sec(d*x+c))^{1/2}*\tan(d*x+c)/d$

Rubi [A] time = 0.35, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {4010, 4001, 3793, 3792}

$$\frac{16a^2(15A+13B)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{315d} + \frac{64a^3(15A+13B)\tan(c+dx)}{315d\sqrt{a\sec(c+dx)+a}} + \frac{2(9A-2B)\tan(c+dx)(a\sec(c+dx))^{5/2}}{63d}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]), x]

[Out] $(64*a^3*(15*A + 13*B)*Tan[c + d*x])/(315*d*Sqrt[a + a*Sec[c + d*x]]) + (16*a^2*(15*A + 13*B)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(315*d) + (2*a*(15*A + 13*B)*(a + a*Sec[c + d*x])^{3/2}*Tan[c + d*x])/(105*d) + (2*(9*A - 2*B)*(a + a*Sec[c + d*x])^{5/2}*Tan[c + d*x])/(63*d) + (2*B*(a + a*Sec[c + d*x])^{7/2}*Tan[c + d*x])/(9*a*d)$

Rule 3792

Int[csc[(e_.) + (f_.)*(x_.)]*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 3793

Int[csc[(e_.) + (f_.)*(x_.)]*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.), x_Symbol] :> -Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1))/(f*m), x] + Dist[(a*(2*m - 1))/m, Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && IntegerQ[2*m]

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_.)]*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := -Simp[(B*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m + 1), 0] && !LtQ[m, -2^(-1)]
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Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_.)]^2*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := -Simp[(B*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \sec^2(c + dx)(a + a \sec(c + dx))^{5/2}(A + B \sec(c + dx)) dx &= \frac{2B(a + a \sec(c + dx))^{7/2} \tan(c + dx)}{9ad} + \frac{2 \int \sec(c + dx) (a + a \sec(c + dx))^{5/2} (A + B \sec(c + dx)) dx}{9ad} \\ &= \frac{2(9A - 2B)(a + a \sec(c + dx))^{5/2} \tan(c + dx)}{63d} + \frac{2B \int \sec(c + dx) (a + a \sec(c + dx))^{5/2} dx}{63d} \\ &= \frac{2a(15A + 13B)(a + a \sec(c + dx))^{3/2} \tan(c + dx)}{105d} + \frac{2B \int \sec(c + dx) (a + a \sec(c + dx))^{5/2} dx}{105d} \\ &= \frac{16a^2(15A + 13B)\sqrt{a + a \sec(c + dx)} \tan(c + dx)}{315d} + \frac{2B \int \sec(c + dx) (a + a \sec(c + dx))^{5/2} dx}{315d} \\ &= \frac{64a^3(15A + 13B) \tan(c + dx)}{315d\sqrt{a + a \sec(c + dx)}} + \frac{16a^2(15A + 13B)\sqrt{a + a \sec(c + dx)}}{315d} \end{aligned}$$

Mathematica [A] time = 0.64, size = 96, normalized size = 0.55

$$\frac{2a^3 \tan(c + dx) (5(9A + 26B) \sec^3(c + dx) + 3(60A + 73B) \sec^2(c + dx) + (345A + 292B) \sec(c + dx) + 690A + 690B)}{315d\sqrt{a(\sec(c + dx) + 1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]), x]
```

[Out] $(2a^3(690A + 584B + (345A + 292B)\text{Sec}[c + dx] + 3(60A + 73B)\text{Sec}[c + dx]^2 + 5(9A + 26B)\text{Sec}[c + dx]^3 + 35B\text{Sec}[c + dx]^4)\text{Tan}[c + dx]) / (315d\sqrt{a(1 + \text{Sec}[c + dx])})$

fricas [A] time = 0.43, size = 136, normalized size = 0.78

$$\frac{2(2(345A + 292B)a^2 \cos(dx + c)^4 + (345A + 292B)a^2 \cos(dx + c)^3 + 3(60A + 73B)a^2 \cos(dx + c)^2 + 5(9A + 26B)a^2 \cos(dx + c) + 35B a^2) \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)}{315(d \cos(dx + c)^5 + d \cos(dx + c)^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(dx+c)^2*(a+a*sec(dx+c))^(5/2)*(A+B*sec(dx+c)),x, algorithm="fricas")`

[Out] $2/315*(2*(345A + 292B)*a^2*\cos(dx + c)^4 + (345A + 292B)*a^2*\cos(dx + c)^3 + 3*(60A + 73B)*a^2*\cos(dx + c)^2 + 5*(9A + 26B)*a^2*\cos(dx + c) + 35*B*a^2)*\sqrt{(a*\cos(dx + c) + a)/\cos(dx + c)}*\sin(dx + c)/(d*\cos(dx + c)^5 + d*\cos(dx + c)^4)$

giac [A] time = 2.06, size = 261, normalized size = 1.49

$$8 \left(\left(\left(4 \left(2\sqrt{2} (15 A a^7 \text{sgn}(\cos(dx + c)) + 13 B a^7 \text{sgn}(\cos(dx + c))) \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 9\sqrt{2} (15 A a^7 \text{sgn}(\cos(dx + c)) + 13 B a^7 \text{sgn}(\cos(dx + c))) \right) \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(dx+c)^2*(a+a*sec(dx+c))^(5/2)*(A+B*sec(dx+c)),x, algorithm="giac")`

[Out] $8/315*(((4*(2*\sqrt{2}*(15*A*a^7*\text{sgn}(\cos(dx + c)) + 13*B*a^7*\text{sgn}(\cos(dx + c))))*\tan(1/2*d*x + 1/2*c)^2 - 9*\sqrt{2}*(15*A*a^7*\text{sgn}(\cos(dx + c)) + 13*B*a^7*\text{sgn}(\cos(dx + c))))*\tan(1/2*d*x + 1/2*c)^2 + 63*\sqrt{2}*(15*A*a^7*\text{sgn}(\cos(dx + c)) + 13*B*a^7*\text{sgn}(\cos(dx + c))))*\tan(1/2*d*x + 1/2*c)^2 - 210*\sqrt{2}*(4*A*a^7*\text{sgn}(\cos(dx + c)) + 3*B*a^7*\text{sgn}(\cos(dx + c))))*\tan(1/2*d*x + 1/2*c)^2 + 315*\sqrt{2}*(A*a^7*\text{sgn}(\cos(dx + c)) + B*a^7*\text{sgn}(\cos(dx + c))))*\tan(1/2*d*x + 1/2*c)/((a*\tan(1/2*d*x + 1/2*c)^2 - a)^4*\sqrt{-a*\tan(1/2*d*x + 1/2*c)^2 + a}*d)$

maple [A] time = 1.52, size = 141, normalized size = 0.81

$$\frac{2(-1 + \cos(dx + c)) (690A (\cos^4(dx + c)) + 584B (\cos^4(dx + c)) + 345A (\cos^3(dx + c)) + 292B (\cos^3(dx + c)) + 9A (\cos^2(dx + c)) + 26B (\cos^2(dx + c)) + 35B \cos(dx + c) + 35B)}{315d \cos(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sec(d*x+c)^2*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x)`

[Out]
$$-2/315/d*(-1+\cos(d*x+c))*(690*A*\cos(d*x+c)^4+584*B*\cos(d*x+c)^4+345*A*\cos(d*x+c)^3+292*B*\cos(d*x+c)^3+180*A*\cos(d*x+c)^2+219*B*\cos(d*x+c)^2+45*A*\cos(d*x+c)+130*B*\cos(d*x+c)+35*B)*(a*(1+\cos(d*x+c))/\cos(d*x+c))^{1/2}/\cos(d*x+c)^4/\sin(d*x+c)*a^2$$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="maxima")`

[Out] Timed out

mupad [B] time = 10.81, size = 723, normalized size = 4.13

$$\frac{\left(e^{c1i+d x1i} \left(\frac{Aa^2 4i}{3d} - \frac{a^2(60A+73B)8i}{315d}\right) + \frac{a^2(5A+2B)4i}{3d}\right) \sqrt{a + \frac{a}{\frac{e^{-c1i-d x1i}}{2} + \frac{e^{c1i+d x1i}}{2}}}}{\left(e^{c1i+d x1i} + 1\right) \left(e^{c2i+d x2i} + 1\right)} + \frac{\sqrt{a + \frac{a}{\frac{e^{-c1i-d x1i}}{2} + \frac{e^{c1i+d x1i}}{2}}}}{\left(e^{c1i+d x1i} + 1\right) \left(e^{c2i+d x2i} + 1\right)} \left(e^{c1i+d x1i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2))/cos(c + d*x)^2,x)`

[Out]
$$\left(\frac{\exp(c*1i + d*x*1i)*((A*a^2*4i)/(3*d) - (a^2*(60*A + 73*B)*8i)/(315*d)) + (a^2*(5*A + 2*B)*4i)/(3*d)}{\left(\frac{\exp(-c*1i - d*x*1i)}{2} + \frac{\exp(c*1i + d*x*1i)}{2}\right)^{1/2}}\right) / \left(\left(\frac{\exp(c*1i + d*x*1i)}{2} + \frac{\exp(c*2i + d*x*2i)}{2} + 1\right) + \left(\frac{\exp(-c*1i - d*x*1i)}{2} + \frac{\exp(c*1i + d*x*1i)}{2}\right)^{1/2} * \left(\frac{\exp(c*1i + d*x*1i)}{2} + \frac{\exp(c*2i + d*x*2i)}{2} + 1\right)\right) + \left(\frac{a + a/\left(\frac{\exp(-c*1i - d*x*1i)}{2} + \frac{\exp(c*1i + d*x*1i)}{2}\right)^{1/2}}{\left(\frac{\exp(c*1i + d*x*1i)}{2} + \frac{\exp(c*2i + d*x*2i)}{2} + 1\right)^2} + \left(\frac{\exp(-c*1i - d*x*1i)}{2} + \frac{\exp(c*1i + d*x*1i)}{2}\right)^{1/2} * \left(\frac{\exp(c*1i + d*x*1i)}{2} + \frac{\exp(c*2i + d*x*2i)}{2} + 1\right)\right) * \left(\frac{A*a^2*4i}{(7*d)} + \frac{a^2*(A + 2*B)*20i}{(7*d)} + \frac{B*a^2*32i}{(63*d)} - \frac{a^2*(A + B)*40i}{(7*d)} + \frac{a^2*(A - 8*B)*4i}{(7*d)} + \frac{a^2*(5*A + 2*B)*4i}{(7*d)} - \frac{a^2*(5*A + 9*B)*8i}{(7*d)}\right) / \left(\left(\frac{\exp(c*1i + d*x*1i)}{2} + \frac{\exp(c*2i + d*x*2i)}{2} + 1\right)^3 - \left(\frac{\exp(-c*1i - d*x*1i)}{2} + \frac{\exp(c*1i + d*x*1i)}{2}\right)^{1/2} * \left(\frac{\exp(c*1i + d*x*1i)}{2} + \frac{\exp(c*2i + d*x*2i)}{2} + 1\right)\right) - \left(\frac{a + a/\left(\frac{\exp(-c*1i - d*x*1i)}{2} + \frac{\exp(c*1i + d*x*1i)}{2}\right)^{1/2}}{\left(\frac{\exp(c*1i + d*x*1i)}{2} + \frac{\exp(c*2i + d*x*2i)}{2} + 1\right)^4} - \frac{a^2*\exp(c*1i + d*x*1i)}{\left(\frac{\exp(c*1i + d*x*1i)}{2} + \frac{\exp(c*2i + d*x*2i)}{2} + 1\right)^4}\right)$$

1i)*(a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(345*A + 292*B)*4i)/(315*d*(exp(c*1i + d*x*1i) + 1))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**2*(a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)),x)

[Out] Timed out

$$3.137 \quad \int \sec(c+dx)(a+a \sec(c+dx))^{5/2}(A+B \sec(c+dx)) dx$$

Optimal. Leaf size=138

$$\frac{64a^3(7A+5B) \tan(c+dx)}{105d\sqrt{a \sec(c+dx)+a}} + \frac{16a^2(7A+5B) \tan(c+dx)\sqrt{a \sec(c+dx)+a}}{105d} + \frac{2a(7A+5B) \tan(c+dx)(a \sec(c+dx))^{5/2}}{35d}$$

[Out] $2/35*a*(7*A+5*B)*(a+a*\sec(d*x+c))^{(3/2)}*\tan(d*x+c)/d+2/7*B*(a+a*\sec(d*x+c))^{(5/2)}*\tan(d*x+c)/d+64/105*a^3*(7*A+5*B)*\tan(d*x+c)/d/(a+a*\sec(d*x+c))^{(1/2)}+16/105*a^2*(7*A+5*B)*(a+a*\sec(d*x+c))^{(1/2)}*\tan(d*x+c)/d$

Rubi [A] time = 0.18, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {4001, 3793, 3792}

$$\frac{64a^3(7A+5B) \tan(c+dx)}{105d\sqrt{a \sec(c+dx)+a}} + \frac{16a^2(7A+5B) \tan(c+dx)\sqrt{a \sec(c+dx)+a}}{105d} + \frac{2a(7A+5B) \tan(c+dx)(a \sec(c+dx))^{5/2}}{35d}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d*x]*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]

[Out] $(64*a^3*(7*A+5*B)*\text{Tan}[c+d*x])/(105*d*\text{Sqrt}[a+a*\text{Sec}[c+d*x]]) + (16*a^2*(7*A+5*B)*\text{Sqrt}[a+a*\text{Sec}[c+d*x]]*\text{Tan}[c+d*x])/(105*d) + (2*a*(7*A+5*B)*(a+a*\text{Sec}[c+d*x])^{(3/2)}*\text{Tan}[c+d*x])/(35*d) + (2*B*(a+a*\text{Sec}[c+d*x])^{(5/2)}*\text{Tan}[c+d*x])/(7*d)$

Rule 3792

Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 3793

Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m-1))/(f*m), x] + Dist[(a*(2*m-1))/m, Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m-1), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && IntegerQ[2*m]

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_.)]*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := -Simp[(B*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m + 1), 0] && !LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned} \int \sec(c + dx)(a + a \sec(c + dx))^{5/2}(A + B \sec(c + dx)) dx &= \frac{2B(a + a \sec(c + dx))^{5/2} \tan(c + dx)}{7d} + \frac{1}{7}(7A + 5B) \\ &= \frac{2a(7A + 5B)(a + a \sec(c + dx))^{3/2} \tan(c + dx)}{35d} + \frac{2}{35} \\ &= \frac{16a^2(7A + 5B)\sqrt{a + a \sec(c + dx)} \tan(c + dx)}{105d} + \frac{2}{105} \\ &= \frac{64a^3(7A + 5B) \tan(c + dx)}{105d\sqrt{a + a \sec(c + dx)}} + \frac{16a^2(7A + 5B)\sqrt{a + a \sec(c + dx)}}{105d} \end{aligned}$$

Mathematica [A] time = 0.51, size = 89, normalized size = 0.64

$$\frac{2a^2\sqrt{a(\sec(c + dx) + 1)} \left((301A + 230B) \sin(c + dx) + \tan(c + dx) \left(3(7A + 20B) \sec(c + dx) + 98A + 15B \sec^2(c + dx) \right) \right)}{105d(\cos(c + dx) + 1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]), x]
```

```
[Out] (2*a^2*Sqrt[a*(1 + Sec[c + d*x])]*((301*A + 230*B)*Sin[c + d*x] + (98*A + 15*B + 3*(7*A + 20*B)*Sec[c + d*x] + 15*B*Sec[c + d*x]^2)*Tan[c + d*x]))/(105*d*(1 + Cos[c + d*x]))
```

fricas [A] time = 0.43, size = 115, normalized size = 0.83

$$\frac{2 \left((301A + 230B)a^2 \cos(dx + c)^3 + (98A + 115B)a^2 \cos(dx + c)^2 + 3(7A + 20B)a^2 \cos(dx + c) + 15Ba^2 \right) \sqrt{a + a \sec(dx + c)}}{105 \left(d \cos(dx + c)^4 + d \cos(dx + c)^3 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)), x, algorithm="fricas")
```

[Out] $\frac{2}{105} * ((301 * A + 230 * B) * a^2 * \cos(dx + c)^3 + (98 * A + 115 * B) * a^2 * \cos(dx + c)^2 + 3 * (7 * A + 20 * B) * a^2 * \cos(dx + c) + 15 * B * a^2) * \sqrt{(a * \cos(dx + c) + a) / \cos(dx + c)} * \sin(dx + c) / (d * \cos(dx + c)^4 + d * \cos(dx + c)^3)$

giac [A] time = 1.78, size = 216, normalized size = 1.57

$$8 \left(\left(4 \left(2 \sqrt{2} (7 A a^6 \operatorname{sgn}(\cos(dx + c)) + 5 B a^6 \operatorname{sgn}(\cos(dx + c))) \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 7 \sqrt{2} (7 A a^6 \operatorname{sgn}(\cos(dx + c)) \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="giac")`

[Out] $\frac{8}{105} * ((4 * (2 * \sqrt{2}) * (7 * A * a^6 * \operatorname{sgn}(\cos(dx + c)) + 5 * B * a^6 * \operatorname{sgn}(\cos(dx + c))) * \tan(1/2 * dx + 1/2 * c)^2 - 7 * \sqrt{2} * (7 * A * a^6 * \operatorname{sgn}(\cos(dx + c)) + 5 * B * a^6 * \operatorname{sgn}(\cos(dx + c)))) * \tan(1/2 * dx + 1/2 * c)^2 + 35 * \sqrt{2} * (7 * A * a^6 * \operatorname{sgn}(\cos(dx + c)) + 5 * B * a^6 * \operatorname{sgn}(\cos(dx + c)))) * \tan(1/2 * dx + 1/2 * c)^2 - 105 * \sqrt{2} * (A * a^6 * \operatorname{sgn}(\cos(dx + c)) + B * a^6 * \operatorname{sgn}(\cos(dx + c)))) * \tan(1/2 * dx + 1/2 * c) / ((a * \tan(1/2 * dx + 1/2 * c)^2 - a)^3 * \sqrt{-a * \tan(1/2 * dx + 1/2 * c)^2 + a} * d)$

maple [A] time = 1.33, size = 119, normalized size = 0.86

$$\frac{2(-1 + \cos(dx + c)) \left(301A \left(\cos^3(dx + c) \right) + 230B \left(\cos^3(dx + c) \right) + 98A \left(\cos^2(dx + c) \right) + 115B \left(\cos^2(dx + c) \right) \right)}{105d \cos(dx + c)^3 \sin(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sec(d*x+c)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x)`

[Out] $-\frac{2}{105} / d * (-1 + \cos(dx + c)) * (301 * A * \cos(dx + c)^3 + 230 * B * \cos(dx + c)^3 + 98 * A * \cos(dx + c)^2 + 115 * B * \cos(dx + c)^2 + 21 * A * \cos(dx + c) + 60 * B * \cos(dx + c) + 15 * B) * (a * (1 + \cos(dx + c))) / \cos(dx + c)^{(1/2)} / \cos(dx + c)^3 / \sin(dx + c) * a^2$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="maxima")`

[Out] Timed out

mupad [B] time = 6.35, size = 590, normalized size = 4.28

$$\frac{\sqrt{a + \frac{a}{\frac{e^{-c1i-dx1i}}{2} + \frac{e^{c1i+dx1i}}{2}}}}{e^{c1i+dx1i} + 1} \left(\frac{Aa^2 2i}{d} - \frac{a^2 e^{c1i+dx1i} (301A+230B) 2i}{105d} \right) - \sqrt{a + \frac{a}{\frac{e^{-c1i-dx1i}}{2} + \frac{e^{c1i+dx1i}}{2}}}}{e^{c1i+dx1i} + 1} \left(e^{c1i+dx1i} \left(\frac{Aa^2 2i}{7d} + \frac{a^2 (3A}{7d} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2))/cos(c + d*x),x)

[Out] ((a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*((A*a^2*2i)/d - (a^2*exp(c*1i + d*x*1i)*(301*A + 230*B)*2i)/(105*d)))/(exp(c*1i + d*x*1i) + 1) - ((a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(exp(c*1i + d*x*1i)*((A*a^2*2i)/(7*d) + (a^2*(3*A + 4*B)*10i)/(7*d) - (a^2*(5*A + 2*B)*2i)/(7*d) - (a^2*(11*A + 10*B)*2i)/(7*d)) + (A*a^2*2i)/(7*d) + (a^2*(3*A + 4*B)*10i)/(7*d) - (a^2*(5*A + 2*B)*2i)/(7*d) - (a^2*(11*A + 10*B)*2i)/(7*d)))/((exp(c*1i + d*x*1i) + 1)*(exp(c*2i + d*x*2i) + 1)^3) - ((a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(exp(c*1i + d*x*1i))*((a^2*(5*A + 2*B)*2i)/(5*d) - (a^2*(5*A + 9*B)*4i)/(5*d) + (a^2*(7*A - 8*B)*2i)/(35*d) - (A*a^2*2i)/(5*d) - (a^2*(A + 2*B)*2i)/d + (a^2*(A + B)*4i)/d))/((exp(c*1i + d*x*1i) + 1)*(exp(c*2i + d*x*2i) + 1)^2) + ((a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(exp(c*1i + d*x*1i))*((a^2*(5*A + 2*B)*2i)/(3*d) - (a^2*(63*A + 80*B)*2i)/(105*d) - (A*a^2*2i)/(3*d) + (a^2*(9*A + 10*B)*2i)/(3*d)))/((exp(c*1i + d*x*1i) + 1)*(exp(c*2i + d*x*2i) + 1)))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a (\sec(c + dx) + 1))^{\frac{5}{2}} (A + B \sec(c + dx)) \sec(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)),x)

[Out] Integral((a*(sec(c + d*x) + 1))**(5/2)*(A + B*sec(c + d*x))*sec(c + d*x), x)

3.138 $\int (a + a \sec(c + dx))^{5/2} (A + B \sec(c + dx)) dx$

Optimal. Leaf size=142

$$\frac{2a^{5/2} A \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}}\right)}{d} + \frac{2a^3(35A + 32B) \tan(c + dx)}{15d\sqrt{a \sec(c + dx) + a}} + \frac{2a^2(5A + 8B) \tan(c + dx)\sqrt{a \sec(c + dx) + a}}{15d} + \frac{2aB}{d}$$

[Out] $2*a^{(5/2)}*A*\arctan(a^{(1/2)}*\tan(d*x+c)/(a+a*\sec(d*x+c))^{(1/2)})/d+2/5*a*B*(a+a*\sec(d*x+c))^{(3/2)}*\tan(d*x+c)/d+2/15*a^3*(35*A+32*B)*\tan(d*x+c)/d/(a+a*\sec(d*x+c))^{(1/2)}+2/15*a^2*(5*A+8*B)*(a+a*\sec(d*x+c))^{(1/2)}*\tan(d*x+c)/d$

Rubi [A] time = 0.22, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3917, 3915, 3774, 203, 3792}

$$\frac{2a^3(35A + 32B) \tan(c + dx)}{15d\sqrt{a \sec(c + dx) + a}} + \frac{2a^2(5A + 8B) \tan(c + dx)\sqrt{a \sec(c + dx) + a}}{15d} + \frac{2a^{5/2} A \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}}\right)}{d} + \frac{2aB}{d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + a*\text{Sec}[c + d*x])^{(5/2)}*(A + B*\text{Sec}[c + d*x]), x]$

[Out] $(2*a^{(5/2)}*A*\text{ArcTan}[(\text{Sqrt}[a]*\text{Tan}[c + d*x])/(\text{Sqrt}[a + a*\text{Sec}[c + d*x]])]/d + (2*a^3*(35*A + 32*B)*\text{Tan}[c + d*x])/(15*d*\text{Sqrt}[a + a*\text{Sec}[c + d*x]]) + (2*a^2*(5*A + 8*B)*\text{Sqrt}[a + a*\text{Sec}[c + d*x]*\text{Tan}[c + d*x])/(15*d) + (2*a*B*(a + a*\text{Sec}[c + d*x])^{(3/2)}*\text{Tan}[c + d*x])/(5*d)$

Rule 203

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1*\text{ArcTan}[(\text{Rt}[b, 2]*x)/(\text{Rt}[a, 2])]/(\text{Rt}[a, 2]*\text{Rt}[b, 2]), x] /; \text{FreeQ}\{a, b, x\} \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{GtQ}[b, 0])$

Rule 3774

$\text{Int}[\text{Sqrt}[\text{csc}[(c_.) + (d_)*(x_)]*(b_.) + (a_)], x_Symbol] \rightarrow \text{Dist}[(-2*b)/d, \text{Subst}[\text{Int}[1/(a + x^2), x], x, (b*\text{Cot}[c + d*x])/(\text{Sqrt}[a + b*\text{Csc}[c + d*x])], x] /; \text{FreeQ}\{a, b, c, d, x\} \ \&\& \ \text{EqQ}[a^2 - b^2, 0]$

Rule 3792

$\text{Int}[\text{csc}[(e_.) + (f_)*(x_)]*\text{Sqrt}[\text{csc}[(e_.) + (f_)*(x_)]*(b_.) + (a_)], x_Symbol] \rightarrow \text{Simp}[(-2*b*\text{Cot}[e + f*x])/(f*\text{Sqrt}[a + b*\text{Csc}[e + f*x]]), x] /; \text{FreeQ}\{a, b, c, d, x\}$

$Q[\{a, b, e, f\}, x] \ \&\& \ \text{EqQ}[a^2 - b^2, 0]$

Rule 3915

$\text{Int}[\text{Sqrt}[\text{csc}[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(\text{csc}[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] \rightarrow \text{Dist}[c, \text{Int}[\text{Sqrt}[a + b*\text{Csc}[e + f*x]], x], x] + \text{Dist}[d, \text{Int}[\text{Sqrt}[a + b*\text{Csc}[e + f*x]]*\text{Csc}[e + f*x], x], x] \ /; \ \text{FreeQ}[\{a, b, c, d, e, f\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0]$

Rule 3917

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^{(m_)}*(\text{csc}[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] \rightarrow -\text{Simp}[(b*d*\text{Cot}[e + f*x]*(a + b*\text{Csc}[e + f*x])^{(m-1)})/(f*m), x] + \text{Dist}[1/m, \text{Int}[(a + b*\text{Csc}[e + f*x])^{(m-1)}*\text{Simp}[a*c*m + (b*c*m + a*d*(2*m-1))*\text{Csc}[e + f*x], x], x], x] \ /; \ \text{FreeQ}[\{a, b, c, d, e, f\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{GtQ}[m, 1] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{IntegerQ}[2*m]$

Rubi steps

$$\begin{aligned} \int (a + a \sec(c + dx))^{5/2} (A + B \sec(c + dx)) dx &= \frac{2aB(a + a \sec(c + dx))^{3/2} \tan(c + dx)}{5d} + \frac{2}{5} \int (a + a \sec(c + dx)) \\ &= \frac{2a^2(5A + 8B)\sqrt{a + a \sec(c + dx)} \tan(c + dx)}{15d} + \frac{2aB(a + a \sec(c + dx))^{3/2} \tan(c + dx)}{5d} \\ &= \frac{2a^2(5A + 8B)\sqrt{a + a \sec(c + dx)} \tan(c + dx)}{15d} + \frac{2aB(a + a \sec(c + dx))^{3/2} \tan(c + dx)}{5d} \\ &= \frac{2a^3(35A + 32B) \tan(c + dx)}{15d\sqrt{a + a \sec(c + dx)}} + \frac{2a^2(5A + 8B)\sqrt{a + a \sec(c + dx)} \tan(c + dx)}{15d} \\ &= \frac{2a^{5/2} A \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a+a \sec(c+dx)}}\right)}{d} + \frac{2a^3(35A + 32B) \tan(c + dx)}{15d\sqrt{a + a \sec(c + dx)}} + \end{aligned}$$

Mathematica [A] time = 0.94, size = 128, normalized size = 0.90

$$\frac{a^2 \sec\left(\frac{1}{2}(c + dx)\right) \sec^2(c + dx) \sqrt{a(\sec(c + dx) + 1)} \left(2 \sin\left(\frac{1}{2}(c + dx)\right) (2(5A + 14B) \cos(c + dx) + (40A + 43B))\right)}{30d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]

[Out] $(a^2 \operatorname{Sec}[(c + dx)/2] \operatorname{Sec}[c + dx]^2 \operatorname{Sqrt}[a(1 + \operatorname{Sec}[c + dx])]) \cdot (30 \operatorname{Sqrt}[2] \cdot A \operatorname{ArcSin}[\operatorname{Sqrt}[2] \operatorname{Sin}[(c + dx)/2]] \operatorname{Cos}[c + dx]^{5/2} + 2(40A + 49B + 2(5A + 14B)) \operatorname{Cos}[c + dx] + (40A + 43B) \operatorname{Cos}[2(c + dx)]) \operatorname{Sin}[(c + dx)/2]) / (30d)$

fricas [A] time = 0.49, size = 378, normalized size = 2.66

$$\frac{15 \left(Aa^2 \cos(dx + c)^3 + Aa^2 \cos(dx + c)^2 \right) \sqrt{-a} \log \left(\frac{2a \cos(dx+c)^2 - 2\sqrt{-a} \sqrt{\frac{a \cos(dx+c)+a}{\cos(dx+c)}} \cos(dx+c) \sin(dx+c) + a \cos(dx+c) - a}{\cos(dx+c)+1} \right)}{15 \left(d \cos(dx + c)^3 + d \cos(dx + c)^2 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="fricas")`

[Out] $[1/15 * (15 * (A * a^2 * \cos(dx + c)^3 + A * a^2 * \cos(dx + c)^2) * \operatorname{sqrt}(-a) * \log((2 * a * \cos(dx + c)^2 - 2 * \operatorname{sqrt}(-a) * \operatorname{sqrt}((a * \cos(dx + c) + a) / \cos(dx + c)) * \cos(dx + c) * \sin(dx + c) + a * \cos(dx + c) - a) / (\cos(dx + c) + 1)) + 2 * ((40 * A + 43 * B) * a^2 * \cos(dx + c)^2 + (5 * A + 14 * B) * a^2 * \cos(dx + c) + 3 * B * a^2) * \operatorname{sqrt}((a * \cos(dx + c) + a) / \cos(dx + c)) * \sin(dx + c)) / (d * \cos(dx + c)^3 + d * \cos(dx + c)^2), -2/15 * (15 * (A * a^2 * \cos(dx + c)^3 + A * a^2 * \cos(dx + c)^2) * \operatorname{sqrt}(a) * \operatorname{arctan}(\operatorname{sqrt}((a * \cos(dx + c) + a) / \cos(dx + c)) * \cos(dx + c) / (\operatorname{sqrt}(a) * \sin(dx + c)))) - ((40 * A + 43 * B) * a^2 * \cos(dx + c)^2 + (5 * A + 14 * B) * a^2 * \cos(dx + c) + 3 * B * a^2) * \operatorname{sqrt}((a * \cos(dx + c) + a) / \cos(dx + c)) * \sin(dx + c)) / (d * \cos(dx + c)^3 + d * \cos(dx + c)^2)]$

giac [B] time = 1.96, size = 309, normalized size = 2.18

$$\frac{15 A \sqrt{-a} a^3 \log \left(\frac{\left| 2 \left(\sqrt{-a} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - \sqrt{-a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + a} \right)^2 - 4 \sqrt{2} |a| - 6 a}{\left| 2 \left(\sqrt{-a} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - \sqrt{-a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + a} \right)^2 + 4 \sqrt{2} |a| - 6 a} \right| \operatorname{sgn}(\cos(dx+c))}{|a|} \right)}{2 \left(45 \sqrt{2} A a^5 \operatorname{sgn}(\cos(dx+c)) + 60 \sqrt{2} B a^5 \operatorname{sgn}(\cos(dx+c)) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="giac")`

[Out] $-1/15 * (15 * A * \operatorname{sqrt}(-a) * a^3 * \log(\operatorname{abs}(2 * (\operatorname{sqrt}(-a) * \tan(1/2 * dx + 1/2 * c) - \operatorname{sqrt}(-a * \tan(1/2 * dx + 1/2 * c)^2 + a))^2 - 4 * \operatorname{sqrt}(2) * \operatorname{abs}(a) - 6 * a) / \operatorname{abs}(2 * (\operatorname{sqrt}(-a) * \tan(1/2 * dx + 1/2 * c) - \operatorname{sqrt}(-a * \tan(1/2 * dx + 1/2 * c)^2 + a))^2 + 4 * \operatorname{sqrt}(2) * \operatorname{abs}(a) - 6 * a)) - 2 * (45 * \sqrt{2} * A * a^5 * \operatorname{sgn}(\cos(dx+c)) + 60 * \sqrt{2} * B * a^5 * \operatorname{sgn}(\cos(dx+c))) / (d * \cos(dx + c)^3 + d * \cos(dx + c)^2)$

$$\frac{\operatorname{arctan}\left(\frac{1}{2}d*x + \frac{1}{2}c\right) - \sqrt{-a*\operatorname{tan}\left(\frac{1}{2}d*x + \frac{1}{2}c\right)^2 + a}}{\sqrt{-a*\operatorname{tan}\left(\frac{1}{2}d*x + \frac{1}{2}c\right)^2 + a}} + 4*\sqrt{2}*ab$$

$$\frac{\operatorname{sgn}(\cos(d*x + c))}{\operatorname{abs}(a) - 2*(45*\sqrt{2}*A*a^5*\operatorname{sgn}(\cos(d*x + c))) + 60*\sqrt{2}*B*a^5*\operatorname{sgn}(\cos(d*x + c)) - (80*\sqrt{2}*A*a^5*\operatorname{sgn}(\cos(d*x + c))) + 80*\sqrt{2}*B*a^5*\operatorname{sgn}(\cos(d*x + c)) - (35*\sqrt{2}*A*a^5*\operatorname{sgn}(\cos(d*x + c))) + 32*\sqrt{2}*B*a^5*\operatorname{sgn}(\cos(d*x + c)))}*\operatorname{tan}\left(\frac{1}{2}d*x + \frac{1}{2}c\right)^2*\operatorname{tan}\left(\frac{1}{2}d*x + \frac{1}{2}c\right)^2*\operatorname{tan}\left(\frac{1}{2}d*x + \frac{1}{2}c\right)}{\left(\left(a*\operatorname{tan}\left(\frac{1}{2}d*x + \frac{1}{2}c\right)^2 - a\right)^2*\sqrt{-a*\operatorname{tan}\left(\frac{1}{2}d*x + \frac{1}{2}c\right)^2 + a}\right)} / d$$

maple [B] time = 1.44, size = 341, normalized size = 2.40

$$\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}} \left(15A \sin(dx+c) \left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)} \right)^{\frac{5}{2}} \operatorname{arctanh} \left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}} \sin(dx+c)\sqrt{2}}{2\cos(dx+c)} \right) \sqrt{2} (\cos^2(dx+c)) + 30A \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\operatorname{int}((a+a*\sec(d*x+c))^{5/2}*(A+B*\sec(d*x+c)), x)$

[Out] $-1/60/d*(a*(1+\cos(d*x+c))/\cos(d*x+c))^{1/2}*(15*A*\sin(d*x+c)*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{5/2}*\operatorname{arctanh}(1/2*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*\sin(d*x+c)/\cos(d*x+c)*2^{1/2})*2^{1/2}*\cos(d*x+c)^2+30*A*\sin(d*x+c)*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{5/2}*\operatorname{arctanh}(1/2*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2})*\sin(d*x+c)/\cos(d*x+c)*2^{1/2})*2^{1/2}*\cos(d*x+c)+15*A*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{5/2}*2^{1/2}*\operatorname{arctanh}(1/2*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2})*\sin(d*x+c)/\cos(d*x+c)*2^{1/2})*\sin(d*x+c)+320*A*\cos(d*x+c)^3+344*B*\cos(d*x+c)^3-280*A*\cos(d*x+c)^2-232*B*\cos(d*x+c)^2-40*A*\cos(d*x+c)-88*B*\cos(d*x+c)-24*B)/\sin(d*x+c)/\cos(d*x+c)^2*a^2$

maxima [B] time = 1.01, size = 1396, normalized size = 9.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\operatorname{integrate}((a+a*\sec(d*x+c))^{5/2}*(A+B*\sec(d*x+c)), x, \operatorname{algorithm}="maxima")$

[Out] $1/6*(30*(\cos(2*d*x + 2*c))^2 + \sin(2*d*x + 2*c))^2 + 2*\cos(2*d*x + 2*c) + 1)^{3/4}*a^{5/2}*\sin(1/2*\operatorname{arctan}2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) + 1)) - 2*(\cos(2*d*x + 2*c))^2 + \sin(2*d*x + 2*c))^2 + 2*\cos(2*d*x + 2*c) + 1)^{1/4}*((12*a^2*\cos(3/2*\operatorname{arctan}2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))))*\sin(2*d*x + 2*c) - 3*a^2*\sin(2*d*x + 2*c) - 4*(3*a^2*\cos(2*d*x + 2*c) + 4*a^2)*\sin(3/2*\operatorname{arctan}2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))))*\cos(3/2*\operatorname{arctan}2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) + 1)) + (12*a^2*\sin(2*d*x + 2*c)*\sin(3/2*\operatorname{arctan}2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))) + 3*a^2*\cos(2*d*x + 2*c) - a^2 + 4*(3*a^2*\cos(2*d*x + 2*c) + 4*a^2)*\cos(3/2*\operatorname{arctan}2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))))$

) $\sin(3/2\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) + 1))\sqrt{a} + 3*((a^2\cos(2*d*x + 2*c)^2 + a^2\sin(2*d*x + 2*c)^2 + 2*a^2\cos(2*d*x + 2*c) + a^2)\arctan2((\cos(2*d*x + 2*c)^2 + \sin(2*d*x + 2*c)^2 + 2*\cos(2*d*x + 2*c) + 1)^{1/4}*(\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))))*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) + 1)) - \cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) + 1))*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))))), (\cos(2*d*x + 2*c)^2 + \sin(2*d*x + 2*c)^2 + 2*\cos(2*d*x + 2*c) + 1)^{1/4}*(\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) + 1))*\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))) + \sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) + 1))*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))) + 1) - (a^2\cos(2*d*x + 2*c)^2 + a^2\sin(2*d*x + 2*c)^2 + 2*a^2\cos(2*d*x + 2*c) + a^2)\arctan2((\cos(2*d*x + 2*c)^2 + \sin(2*d*x + 2*c)^2 + 2*\cos(2*d*x + 2*c) + 1)^{1/4}*(\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))))*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) + 1)) - \cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) + 1))*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))))), (\cos(2*d*x + 2*c)^2 + \sin(2*d*x + 2*c)^2 + 2*\cos(2*d*x + 2*c) + 1)^{1/4}*(\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) + 1))*\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))) + \sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) + 1))*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))) - 1) - (a^2\cos(2*d*x + 2*c)^2 + a^2\sin(2*d*x + 2*c)^2 + 2*a^2\cos(2*d*x + 2*c) + a^2)\arctan2((\cos(2*d*x + 2*c)^2 + \sin(2*d*x + 2*c)^2 + 2*\cos(2*d*x + 2*c) + 1)^{1/4}*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) + 1))), (\cos(2*d*x + 2*c)^2 + \sin(2*d*x + 2*c)^2 + 2*\cos(2*d*x + 2*c) + 1)^{1/4}*\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) + 1)) + 1) + (a^2\cos(2*d*x + 2*c)^2 + a^2\sin(2*d*x + 2*c)^2 + 2*a^2\cos(2*d*x + 2*c) + a^2)\arctan2((\cos(2*d*x + 2*c)^2 + \sin(2*d*x + 2*c)^2 + 2*\cos(2*d*x + 2*c) + 1)^{1/4}*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) + 1))), (\cos(2*d*x + 2*c)^2 + \sin(2*d*x + 2*c)^2 + 2*\cos(2*d*x + 2*c) + 1)^{1/4}*\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) + 1)) - 1))*\sqrt{a})*A/((\cos(2*d*x + 2*c)^2 + \sin(2*d*x + 2*c)^2 + 2*\cos(2*d*x + 2*c) + 1)*d)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A + \frac{B}{\cos(c + dx)} \right) \left(a + \frac{a}{\cos(c + dx)} \right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2), x)

[Out] int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sec(c + dx) + 1))^{\frac{5}{2}} (A + B \sec(c + dx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**(5/2)*(A + B*sec(c + d*x)), x)
```

$$3.139 \quad \int \cos(c+dx)(a+a \sec(c+dx))^{5/2}(A+B \sec(c+dx)) dx$$

Optimal. Leaf size=143

$$\frac{a^{5/2}(5A+2B) \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}}\right)}{d} - \frac{a^3(3A+14B) \sin(c+dx)}{3d\sqrt{a \sec(c+dx)+a}} + \frac{2a^2(A+2B) \sin(c+dx)\sqrt{a \sec(c+dx)+a}}{d} + \frac{2a^2(A+2B) \sin(c+dx)\sqrt{a \sec(c+dx)+a}}{d}$$

[Out] $a^{(5/2)}*(5*A+2*B)*\arctan(a^{(1/2)}*\tan(d*x+c)/(a+a*\sec(d*x+c))^{(1/2)})/d+2/3*a*B*(a+a*\sec(d*x+c))^{(3/2)}*\sin(d*x+c)/d-1/3*a^3*(3*A+14*B)*\sin(d*x+c)/d/(a+a*\sec(d*x+c))^{(1/2)}+2*a^2*(A+2*B)*\sin(d*x+c)*(a+a*\sec(d*x+c))^{(1/2)}/d$

Rubi [A] time = 0.41, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {4018, 4015, 3774, 203}

$$-\frac{a^3(3A+14B) \sin(c+dx)}{3d\sqrt{a \sec(c+dx)+a}} + \frac{2a^2(A+2B) \sin(c+dx)\sqrt{a \sec(c+dx)+a}}{d} + \frac{a^{5/2}(5A+2B) \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}}\right)}{d} + \frac{2a^2(A+2B) \sin(c+dx)\sqrt{a \sec(c+dx)+a}}{d}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]), x]

[Out] $(a^{(5/2)}*(5*A+2*B)*\text{ArcTan}[(\text{Sqrt}[a]*\text{Tan}[c+d*x])/\text{Sqrt}[a+a*\text{Sec}[c+d*x]])]/d - (a^3*(3*A+14*B)*\text{Sin}[c+d*x])/(3*d*\text{Sqrt}[a+a*\text{Sec}[c+d*x]]) + (2*a^2*(A+2*B)*\text{Sqrt}[a+a*\text{Sec}[c+d*x])*\text{Sin}[c+d*x])/d + (2*a*B*(a+a*\text{Sec}[c+d*x])^{(3/2)}*\text{Sin}[c+d*x])/(3*d)$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 3774

Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] := Dist[(-2*b)/d, Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 4015

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_) *(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] := Simp[(A*b^2*c


```
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
+ f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (
a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && !LtQ[n, -1]
```

Rubi steps

$$\begin{aligned} \int \cos(c + dx)(a + a \sec(c + dx))^{5/2}(A + B \sec(c + dx)) dx &= \frac{2aB(a + a \sec(c + dx))^{3/2} \sin(c + dx)}{3d} + \frac{2}{3} \int \cos(c + dx)(a + a \sec(c + dx))^{5/2} dx \\ &= \frac{2a^2(A + 2B)\sqrt{a + a \sec(c + dx)} \sin(c + dx)}{d} + \frac{2aB}{3} \int \cos(c + dx)(a + a \sec(c + dx))^{5/2} dx \\ &= -\frac{a^3(3A + 14B) \sin(c + dx)}{3d\sqrt{a + a \sec(c + dx)}} + \frac{2a^2(A + 2B)\sqrt{a + a \sec(c + dx)}}{3d} \\ &= -\frac{a^3(3A + 14B) \sin(c + dx)}{3d\sqrt{a + a \sec(c + dx)}} + \frac{2a^2(A + 2B)\sqrt{a + a \sec(c + dx)}}{3d} \\ &= \frac{a^{5/2}(5A + 2B) \tan^{-1}\left(\frac{\sqrt{a} \tan(c + dx)}{\sqrt{a + a \sec(c + dx)}}\right)}{d} - \frac{a^3(3A + 14B) \sin(c + dx)}{3d\sqrt{a + a \sec(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.83, size = 126, normalized size = 0.88

$$\frac{a^2 \sec\left(\frac{1}{2}(c + dx)\right) \sec(c + dx) \sqrt{a(\sec(c + dx) + 1)} \left(3\sqrt{2}(5A + 2B) \sin^{-1}\left(\sqrt{2} \sin\left(\frac{1}{2}(c + dx)\right)\right)\right) \cos^{\frac{3}{2}}(c + dx) + s}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^2*Sec[(c + d*x)/2]*Sec[c + d*x]*Sqrt[a*(1 + Sec[c + d*x])]*(3*Sqrt[2]*(5
*A + 2*B)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(3/2) + (3*A + 4*B
```

+ 4*(3*A + 8*B)*Cos[c + d*x] + 3*A*Cos[2*(c + d*x)]*Sin[(c + d*x)/2]))/(6*d)

fricas [A] time = 0.51, size = 386, normalized size = 2.70

$$\frac{3 \left((5A + 2B)a^2 \cos(dx + c)^2 + (5A + 2B)a^2 \cos(dx + c) \right) \sqrt{-a} \log \left(\frac{2a \cos(dx+c)^2 - 2\sqrt{-a} \sqrt{\frac{a \cos(dx+c)+a}{\cos(dx+c)}} \cos(dx+c) \sin(dx+c)}{\cos(dx+c)+1} \right)}{6 \left(d \cos(dx + c) \right)^2 + a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] [1/6*(3*((5*A + 2*B)*a^2*cos(d*x + c)^2 + (5*A + 2*B)*a^2*cos(d*x + c))*sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(3*A*a^2*cos(d*x + c)^2 + 2*(3*A + 8*B)*a^2*cos(d*x + c) + 2*B*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2 + d*cos(d*x + c)), -1/3*(3*((5*A + 2*B)*a^2*cos(d*x + c)^2 + (5*A + 2*B)*a^2*cos(d*x + c))*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - (3*A*a^2*cos(d*x + c)^2 + 2*(3*A + 8*B)*a^2*cos(d*x + c) + 2*B*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2 + d*cos(d*x + c))]

giac [B] time = 1.96, size = 480, normalized size = 3.36

$$3 \left(5A\sqrt{-a}a^2 \operatorname{sgn}(\cos(dx + c)) + 2B\sqrt{-a}a^2 \operatorname{sgn}(\cos(dx + c)) \right) \log \left(\left| \sqrt{-a} \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - \sqrt{-a} \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) \right| \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] -1/6*(3*(5*A*sqrt(-a)*a^2*sgn(cos(d*x + c)) + 2*B*sqrt(-a)*a^2*sgn(cos(d*x + c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c))^2 + a))^2 - a*(2*sqrt(2) + 3))) - 3*(5*A*sqrt(-a)*a^2*sgn(cos(d*x + c)) + 2*B*sqrt(-a)*a^2*sgn(cos(d*x + c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c)

- sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))) + 4*(3*sqrt(2)*A*a^4*sgn(cos(d*x + c)) + 9*sqrt(2)*B*a^4*sgn(cos(d*x + c)) - (3*sqrt(2)*A*a^4*sgn(cos(d*x + c)) + 7*sqrt(2)*B*a^4*sgn(cos(d*x + c)))*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)) + 12*(3*sqrt(2)*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*sqrt(-a)*a^3*sgn(cos(d*x + c)) - sqrt(2)*A*sqrt(-a)*a^4*sgn(cos(d*x + c)))/((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2))/d

maple [B] time = 1.45, size = 256, normalized size = 1.79

$$\left(15A \sin(dx + c) \cos(dx + c) \sqrt{-\frac{2 \cos(dx+c)}{1+\cos(dx+c)}} \operatorname{arctanh} \left(\frac{\sqrt{-\frac{2 \cos(dx+c)}{1+\cos(dx+c)}} \sin(dx+c) \sqrt{2}}{2 \cos(dx+c)} \right) \sqrt{2} + 6B \sin(dx + c) \cos(dx + c) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)), x)

[Out] -1/6/d*(15*A*sin(d*x+c)*cos(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*2^(1/2)+6*B*sin(d*x+c)*cos(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*2^(1/2)+6*A*cos(d*x+c)^3+6*A*cos(d*x+c)^2+32*B*cos(d*x+c)^2-12*A*cos(d*x+c)-28*B*cos(d*x+c)-4*B)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)/sin(d*x+c)*a^2

maxima [B] time = 1.30, size = 2780, normalized size = 19.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)), x, algorithm="maxima")

[Out] 1/12*(3*(18*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(3/4)*a^(5/2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4))*((4*a^2*sin(3*d*x + 3*c) + 5*a^2*sin(2*d*x + 2*c) + 4*a^2*sin(d*x + c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + (a^2*cos(2*d*x + 2*c)^2*sin(d*x + c) + a^2*sin(2*d*x + 2*c)^2*sin(d*x + c) + 2*a^2*cos(2*d*x + 2*c)*sin(d*x + c) + a^2*sin(d*x + c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - (4*a^2*cos(3*d*x + 3*c) + 5*a^2*cos(2*d*x + 2*c) +

$$1)^{(1/4)} * (\cos(1/2 * \arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) + 1)), \cos(2*d*x + 2*c) + 1)) * \cos(1/2 * \arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + \sin(1/2 * \arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) + 1)) * \sin(1/2 * \arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 1) - (a^2 * \cos(2*d*x + 2*c)^2 + a^2 * \sin(2*d*x + 2*c)^2 + 2 * a^2 * \cos(2*d*x + 2*c) + a^2) * \arctan2((\cos(2*d*x + 2*c)^2 + \sin(2*d*x + 2*c)^2 + 2 * \cos(2*d*x + 2*c) + 1)^{(1/4)} * (\cos(1/2 * \arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) * \sin(1/2 * \arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) + 1)) - \cos(1/2 * \arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) + 1)) * \sin(1/2 * \arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))))), (\cos(2*d*x + 2*c)^2 + \sin(2*d*x + 2*c)^2 + 2 * \cos(2*d*x + 2*c) + 1)^{(1/4)} * (\cos(1/2 * \arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) + 1)) * \cos(1/2 * \arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + \sin(1/2 * \arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) + 1)) * \sin(1/2 * \arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) + 1)) * \sin(1/2 * \arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) + 1)) - 1) - (a^2 * \cos(2*d*x + 2*c)^2 + a^2 * \sin(2*d*x + 2*c)^2 + 2 * a^2 * \cos(2*d*x + 2*c) + a^2) * \arctan2((\cos(2*d*x + 2*c)^2 + \sin(2*d*x + 2*c)^2 + 2 * \cos(2*d*x + 2*c) + 1)^{(1/4)} * \sin(1/2 * \arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) + 1))), (\cos(2*d*x + 2*c)^2 + \sin(2*d*x + 2*c)^2 + 2 * \cos(2*d*x + 2*c) + 1)^{(1/4)} * \cos(1/2 * \arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) + 1)) + 1) + (a^2 * \cos(2*d*x + 2*c)^2 + a^2 * \sin(2*d*x + 2*c)^2 + 2 * a^2 * \cos(2*d*x + 2*c) + a^2) * \arctan2((\cos(2*d*x + 2*c)^2 + \sin(2*d*x + 2*c)^2 + 2 * \cos(2*d*x + 2*c) + 1)^{(1/4)} * \sin(1/2 * \arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) + 1))), (\cos(2*d*x + 2*c)^2 + \sin(2*d*x + 2*c)^2 + 2 * \cos(2*d*x + 2*c) + 1)^{(1/4)} * \cos(1/2 * \arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) + 1)) - 1)) * \sqrt{a}) * B / (\cos(2*d*x + 2*c)^2 + \sin(2*d*x + 2*c)^2 + 2 * \cos(2*d*x + 2*c) + 1)) / d$$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c + dx) \left(A + \frac{B}{\cos(c + dx)} \right) \left(a + \frac{a}{\cos(c + dx)} \right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2),x)

[Out] int(cos(c + d*x)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)),x)

[Out] Timed out

$$3.140 \quad \int \cos^2(c+dx)(a+a \sec(c+dx))^{5/2}(A+B \sec(c+dx)) dx$$

Optimal. Leaf size=154

$$\frac{a^{5/2}(19A + 20B) \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}}\right)}{4d} + \frac{a^3(9A - 4B) \sin(c + dx)}{4d\sqrt{a \sec(c + dx) + a}} - \frac{a^2(A - 4B) \sin(c + dx)\sqrt{a \sec(c + dx) + a}}{2d} + \frac{a^2(A - 4B) \sin(c + dx)\sqrt{a \sec(c + dx) + a}}{2d}$$

[Out] 1/4*a^(5/2)*(19*A+20*B)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d + 1/2*a*A*cos(d*x+c)*(a+a*sec(d*x+c))^(3/2)*sin(d*x+c)/d+1/4*a^3*(9*A-4*B)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)-1/2*a^2*(A-4*B)*sin(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d

Rubi [A] time = 0.42, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {4017, 4018, 4015, 3774, 203}

$$\frac{a^3(9A - 4B) \sin(c + dx)}{4d\sqrt{a \sec(c + dx) + a}} - \frac{a^2(A - 4B) \sin(c + dx)\sqrt{a \sec(c + dx) + a}}{2d} + \frac{a^{5/2}(19A + 20B) \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}}\right)}{4d} + \frac{a^2(A - 4B) \sin(c + dx)\sqrt{a \sec(c + dx) + a}}{2d}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]

[Out] (a^(5/2)*(19*A + 20*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(4*d) + (a^3*(9*A - 4*B)*Sin[c + d*x])/(4*d*Sqrt[a + a*Sec[c + d*x]]) - (a^2*(A - 4*B)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(2*d) + (a*A*Cos[c + d*x]*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(2*d)

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 3774

Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] := Dist[(-2*b)/d, Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.)
+ (a_.)]*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
+ f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (
a_.))^(m_)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (
a_.))^(m_)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && !LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
 \int \cos^2(c + dx)(a + a \sec(c + dx))^{5/2}(A + B \sec(c + dx)) dx &= \frac{aA \cos(c + dx)(a + a \sec(c + dx))^{3/2} \sin(c + dx)}{2d} \\
 &= -\frac{a^2(A - 4B)\sqrt{a + a \sec(c + dx)} \sin(c + dx)}{2d} + \frac{aA}{2d} \\
 &= \frac{a^3(9A - 4B) \sin(c + dx)}{4d\sqrt{a + a \sec(c + dx)}} - \frac{a^2(A - 4B)\sqrt{a + a \sec(c + dx)}}{2d} \\
 &= \frac{a^3(9A - 4B) \sin(c + dx)}{4d\sqrt{a + a \sec(c + dx)}} - \frac{a^2(A - 4B)\sqrt{a + a \sec(c + dx)}}{2d} \\
 &= \frac{a^{5/2}(19A + 20B) \tan^{-1}\left(\frac{\sqrt{a} \tan(c + dx)}{\sqrt{a + a \sec(c + dx)}}\right)}{4d} + \frac{a^3(9A - 4B)}{4d\sqrt{a - a \sec(c + dx)}}
 \end{aligned}$$

Mathematica [A] time = 0.97, size = 116, normalized size = 0.75

$$\frac{a^2 \sec\left(\frac{1}{2}(c + dx)\right) \sqrt{a(\sec(c + dx) + 1)} \left(\sqrt{2}(19A + 20B) \sin^{-1}\left(\sqrt{2} \sin\left(\frac{1}{2}(c + dx)\right)\right)\right) \sqrt{\cos(c + dx)} + 2 \sin\left(\frac{1}{2}(c + dx)\right)}{8d}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]

[Out] (a^2*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(Sqrt[2]*(19*A + 20*B)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]] + 2*(A + 8*B + (11*A + 4*B)*Cos[c + d*x] + A*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]))/(8*d)

fricas [A] time = 0.54, size = 348, normalized size = 2.26

$$\frac{\left((19A + 20B)a^2 \cos(dx + c) + (19A + 20B)a^2 \right) \sqrt{-a} \log\left(\frac{2a \cos(dx+c)^2 - 2\sqrt{-a} \sqrt{\frac{a \cos(dx+c)+a}{\cos(dx+c)}} \cos(dx+c) \sin(dx+c) + a \cos(dx+c)}{\cos(dx+c)+1} \right)}{8(d \cos(dx + c) + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] [1/8*(((19*A + 20*B)*a^2*cos(d*x + c) + (19*A + 20*B)*a^2)*sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(2*A*a^2*cos(d*x + c)^2 + (11*A + 4*B)*a^2*cos(d*x + c) + 8*B*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d), -1/4*(((19*A + 20*B)*a^2*cos(d*x + c) + (19*A + 20*B)*a^2)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - (2*A*a^2*cos(d*x + c)^2 + (11*A + 4*B)*a^2*cos(d*x + c) + 8*B*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d)]

giac [B] time = 2.07, size = 709, normalized size = 4.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="giac")


```
[Out] -1/8*(16*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*B*a^3*sgn(cos(d*x + c))
)*tan(1/2*d*x + 1/2*c)/(a*tan(1/2*d*x + 1/2*c)^2 - a) + (19*A*sqrt(-a)*a^2*
sgn(cos(d*x + c)) + 20*B*sqrt(-a)*a^2*sgn(cos(d*x + c)))*log(abs((sqrt(-a)*
tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2)
) + 3))) - (19*A*sqrt(-a)*a^2*sgn(cos(d*x + c)) + 20*B*sqrt(-a)*a^2*sgn(cos
(d*x + c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x +
1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))) + 4*sqrt(2)*(19*(sqrt(-a)*tan(1/2*d*
x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*A*sqrt(-a)*a^3*sgn(cos(
d*x + c)) + 12*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)
)^2 + a))^6*B*sqrt(-a)*a^3*sgn(cos(d*x + c)) - 171*(sqrt(-a)*tan(1/2*d*x +
1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*A*sqrt(-a)*a^4*sgn(cos(d*x
+ c)) - 76*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2
+ a))^4*B*sqrt(-a)*a^4*sgn(cos(d*x + c)) + 89*(sqrt(-a)*tan(1/2*d*x + 1/2*c)
) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*sqrt(-a)*a^5*sgn(cos(d*x + c))
+ 36*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))
^2*B*sqrt(-a)*a^5*sgn(cos(d*x + c)) - 9*A*sqrt(-a)*a^6*sgn(cos(d*x + c)) -
4*B*sqrt(-a)*a^6*sgn(cos(d*x + c)))/((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(
-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt
(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^2)/d
```

maple [B] time = 1.68, size = 410, normalized size = 2.66

$$\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}} \left(19A \cos(dx+c) \operatorname{arctanh} \left(\frac{\sqrt{\frac{-2\cos(dx+c)}{1+\cos(dx+c)}} \sin(dx+c) \sqrt{2}}{2\cos(dx+c)} \right) \left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)} \right)^{\frac{3}{2}} \sin(dx+c) \sqrt{2} + 20B \cos(dx+c) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] 1/16/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(19*A*cos(d*x+c)*arctanh(1/2*(-2
*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*
x+c)/(1+cos(d*x+c)))^(3/2)*sin(d*x+c)*2^(1/2)+20*B*cos(d*x+c)*arctanh(1/2*(-
2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(
d*x+c)/(1+cos(d*x+c)))^(3/2)*sin(d*x+c)*2^(1/2)+19*A*2^(1/2)*arctanh(1/2*(-
2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d
*x+c)/(1+cos(d*x+c)))^(3/2)*sin(d*x+c)+20*B*2^(1/2)*arctanh(1/2*(-2*cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+
cos(d*x+c)))^(3/2)*sin(d*x+c)-8*A*cos(d*x+c)^4-36*A*cos(d*x+c)^3-16*B*cos(d
*x+c)^3+44*A*cos(d*x+c)^2-16*B*cos(d*x+c)^2+32*B*cos(d*x+c))/sin(d*x+c)/cos
(d*x+c)*a^2
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c + dx)^2 \left(A + \frac{B}{\cos(c + dx)} \right) \left(a + \frac{a}{\cos(c + dx)} \right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^2*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2),x)

[Out] int(cos(c + d*x)^2*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**2*(a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)),x)

[Out] Timed out

$$3.141 \quad \int \cos^3(c+dx)(a+a \sec(c+dx))^{5/2}(A+B \sec(c+dx)) dx$$

Optimal. Leaf size=164

$$\frac{a^{5/2}(25A + 38B) \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}}\right)}{8d} + \frac{a^3(49A + 54B) \sin(c + dx)}{24d\sqrt{a \sec(c + dx) + a}} + \frac{a^2(3A + 2B) \sin(c + dx) \cos(c + dx)\sqrt{a \sec(c + dx) + a}}{4d}$$

[Out] 1/8*a^(5/2)*(25*A+38*B)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d + 1/3*a*A*cos(d*x+c)^2*(a+a*sec(d*x+c))^(3/2)*sin(d*x+c)/d + 1/24*a^3*(49*A+54*B)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2) + 1/4*a^2*(3*A+2*B)*cos(d*x+c)*sin(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d

Rubi [A] time = 0.46, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {4017, 4015, 3774, 203}

$$\frac{a^3(49A + 54B) \sin(c + dx)}{24d\sqrt{a \sec(c + dx) + a}} + \frac{a^{5/2}(25A + 38B) \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}}\right)}{8d} + \frac{a^2(3A + 2B) \sin(c + dx) \cos(c + dx)\sqrt{a \sec(c + dx) + a}}{4d}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]

[Out] (a^(5/2)*(25*A + 38*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(8*d) + (a^3*(49*A + 54*B)*Sin[c + d*x])/(24*d*Sqrt[a + a*Sec[c + d*x]]) + (a^2*(3*A + 2*B)*Cos[c + d*x]*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(4*d) + (a*A*cos[c + d*x]^2*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3*d)

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 3774

Int[Sqrt[csc[(c_.) + (d_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] := Dist[(-2*b)/d, Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.)
+ (a_)]*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*Co
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
+ f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned} \int \cos^3(c + dx)(a + a \sec(c + dx))^{5/2}(A + B \sec(c + dx)) dx &= \frac{aA \cos^2(c + dx)(a + a \sec(c + dx))^{3/2} \sin(c + dx)}{3d} + \\ &= \frac{a^2(3A + 2B) \cos(c + dx) \sqrt{a + a \sec(c + dx)} \sin(c + dx)}{4d} \\ &= \frac{a^3(49A + 54B) \sin(c + dx)}{24d \sqrt{a + a \sec(c + dx)}} + \frac{a^2(3A + 2B) \cos(c + dx)}{24d \sqrt{a + a \sec(c + dx)}} \\ &= \frac{a^3(49A + 54B) \sin(c + dx)}{24d \sqrt{a + a \sec(c + dx)}} + \frac{a^2(3A + 2B) \cos(c + dx)}{24d \sqrt{a + a \sec(c + dx)}} \\ &= \frac{a^{5/2}(25A + 38B) \tan^{-1}\left(\frac{\sqrt{a} \tan(c + dx)}{\sqrt{a + a \sec(c + dx)}}\right)}{8d} + \frac{a^3(49A + 54B) \sin(c + dx)}{24d \sqrt{a + a \sec(c + dx)}} \end{aligned}$$

Mathematica [C] time = 1.04, size = 312, normalized size = 1.90

$$\frac{a^2 \cos(c + dx) \sqrt{a(\sec(c + dx) + 1)} \left(-192A \tan(c + dx) \sqrt{1 - \sec(c + dx)} {}_2F_1\left(\frac{1}{2}, 4; \frac{3}{2}; 1 - \sec(c + dx)\right) - 165A \sin(c + dx) \right)}{24d \sqrt{a + a \sec(c + dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] -1/72*(a^2*Cos[c + d*x]*Sqrt[a*(1 + Sec[c + d*x])]*(-165*A*Sqrt[1 - Sec[c +
d*x]]*Sin[c + d*x] + 18*B*Sqrt[1 - Sec[c + d*x]]*Sin[c + d*x] + 8*A*Cos[c
+ d*x]^2*Sqrt[1 - Sec[c + d*x]]*Sin[c + d*x] - 31*A*Sqrt[1 - Sec[c + d*x]]*
Sin[2*(c + d*x)] + 54*B*Sqrt[1 - Sec[c + d*x]]*Sin[2*(c + d*x)] - 165*A*Arc
Tanh[Sqrt[1 - Sec[c + d*x]]]*Tan[c + d*x] - 126*B*ArcTanh[Sqrt[1 - Sec[c +
d*x]]]*Tan[c + d*x] - 576*B*Hypergeometric2F1[1/2, 3, 3/2, 1 - Sec[c + d*x]
]*Sqrt[1 - Sec[c + d*x]]*Tan[c + d*x] - 192*A*Hypergeometric2F1[1/2, 4, 3/2
, 1 - Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]]*Tan[c + d*x]))/(d*(1 + Cos[c + d
*x])*Sqrt[1 - Sec[c + d*x]])
```

fricas [A] time = 0.52, size = 380, normalized size = 2.32

$$\frac{3 \left((25A + 38B)a^2 \cos(dx + c) + (25A + 38B)a^2 \right) \sqrt{-a} \log \left(\frac{2a \cos(dx+c)^2 - 2\sqrt{-a} \sqrt{\frac{a \cos(dx+c)+a}{\cos(dx+c)}} \cos(dx+c) \sin(dx+c) + a \cos(dx+c)}{\cos(dx+c)+1} \right)}{48}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] [1/48*(3*((25*A + 38*B)*a^2*cos(d*x + c) + (25*A + 38*B)*a^2)*sqrt(-a)*log(
(2*a*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*co
s(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(8*A*
a^2*cos(d*x + c)^3 + 2*(17*A + 6*B)*a^2*cos(d*x + c)^2 + 3*(25*A + 22*B)*a^
2*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*co
s(d*x + c) + d), -1/24*(3*((25*A + 38*B)*a^2*cos(d*x + c) + (25*A + 38*B)*a
^2)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sq
rt(a)*sin(d*x + c))) - (8*A*a^2*cos(d*x + c)^3 + 2*(17*A + 6*B)*a^2*cos(d*x
+ c)^2 + 3*(25*A + 22*B)*a^2*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d
*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d)]
```

giac [B] time = 2.48, size = 905, normalized size = 5.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm
="giac")
```

```
[Out] -1/48*(3*(25*A*sqrt(-a)*a^2*sgn(cos(d*x + c)) + 38*B*sqrt(-a)*a^2*sgn(cos(d
*x + c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/
```

$$\begin{aligned}
& 2*c)^2 + a))^2 - a*(2*\sqrt{2} + 3))) - 3*(25*A*\sqrt{-a}*a^2*\text{sgn}(\cos(dx + c)) \\
&) + 38*B*\sqrt{-a}*a^2*\text{sgn}(\cos(dx + c)))*\log(\text{abs}((\sqrt{-a})*\tan(1/2*dx + 1 \\
& /2*c) - \sqrt{-a*\tan(1/2*dx + 1/2*c)^2 + a}))^2 + a*(2*\sqrt{2} - 3))) + 4*(7 \\
& 5*\sqrt{2}*(\sqrt{-a}*\tan(1/2*dx + 1/2*c) - \sqrt{-a*\tan(1/2*dx + 1/2*c)^2 + \\
& a}))^{10}*A*\sqrt{-a}*a^3*\text{sgn}(\cos(dx + c)) + 114*\sqrt{2}*(\sqrt{-a}*\tan(1/2*d*x \\
& x + 1/2*c) - \sqrt{-a*\tan(1/2*d*x + 1/2*c)^2 + a}))^{10}*B*\sqrt{-a}*a^3*\text{sgn}(\cos \\
& (d*x + c)) - 1125*\sqrt{2}*(\sqrt{-a}*\tan(1/2*d*x + 1/2*c) - \sqrt{-a*\tan(1/2* \\
& d*x + 1/2*c)^2 + a}))^8*A*\sqrt{-a}*a^4*\text{sgn}(\cos(d*x + c)) - 1710*\sqrt{2}*(\sqrt{ \\
& t(-a)*\tan(1/2*d*x + 1/2*c) - \sqrt{-a*\tan(1/2*d*x + 1/2*c)^2 + a}))^8*B*\sqrt{ \\
& (-a)*a^4*\text{sgn}(\cos(d*x + c)) + 6174*\sqrt{2}*(\sqrt{-a}*\tan(1/2*d*x + 1/2*c) - s \\
& \sqrt{-a*\tan(1/2*d*x + 1/2*c)^2 + a}))^6*A*\sqrt{-a}*a^5*\text{sgn}(\cos(d*x + c)) + 68 \\
& 04*\sqrt{2}*(\sqrt{-a}*\tan(1/2*d*x + 1/2*c) - \sqrt{-a*\tan(1/2*d*x + 1/2*c)^2 \\
& + a}))^6*B*\sqrt{-a}*a^5*\text{sgn}(\cos(d*x + c)) - 4314*\sqrt{2}*(\sqrt{-a}*\tan(1/2*d \\
& *x + 1/2*c) - \sqrt{-a*\tan(1/2*d*x + 1/2*c)^2 + a}))^4*A*\sqrt{-a}*a^6*\text{sgn}(\cos \\
& (d*x + c)) - 4284*\sqrt{2}*(\sqrt{-a}*\tan(1/2*d*x + 1/2*c) - \sqrt{-a*\tan(1/2* \\
& d*x + 1/2*c)^2 + a}))^4*B*\sqrt{-a}*a^6*\text{sgn}(\cos(d*x + c)) + 807*\sqrt{2}*(\sqrt{ \\
& (-a)*\tan(1/2*d*x + 1/2*c) - \sqrt{-a*\tan(1/2*d*x + 1/2*c)^2 + a}))^2*A*\sqrt{(- \\
& a)*a^7*\text{sgn}(\cos(d*x + c)) + 858*\sqrt{2}*(\sqrt{-a}*\tan(1/2*d*x + 1/2*c) - \sqrt{ \\
& t(-a*\tan(1/2*d*x + 1/2*c)^2 + a))^2*B*\sqrt{-a}*a^7*\text{sgn}(\cos(d*x + c)) - 49*s \\
& \sqrt{2}*A*\sqrt{-a}*a^8*\text{sgn}(\cos(d*x + c)) - 54*\sqrt{2}*B*\sqrt{-a}*a^8*\text{sgn}(\cos \\
& (d*x + c)))/((\sqrt{-a}*\tan(1/2*d*x + 1/2*c) - \sqrt{-a*\tan(1/2*d*x + 1/2*c)^ \\
& 2 + a}))^4 - 6*(\sqrt{-a}*\tan(1/2*d*x + 1/2*c) - \sqrt{-a*\tan(1/2*d*x + 1/2*c) \\
& ^2 + a}))^2*a + a^2)^3)/d
\end{aligned}$$

maple [B] time = 1.71, size = 583, normalized size = 3.55

$$\frac{\left(75A \sin(dx + c) \left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)} \right)^{\frac{5}{2}} \operatorname{arctanh} \left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}} \sin(dx+c) \sqrt{2}}{2\cos(dx+c)} \right) \sqrt{2} \left(\cos^2(dx + c) \right) + 114B \sin(dx + c) \left(-\frac{2}{1+\cos(dx+c)} \right)^{\frac{5}{2}} \operatorname{arctanh} \left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}} \sin(dx+c) \sqrt{2}}{2\cos(dx+c)} \right) \sqrt{2} \left(\cos^2(dx + c) \right) \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(d*x+c)^3*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x)`

[Out] `-1/192/d*(75*A*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*2^(1/2)*cos(d*x+c)^2+114*B*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*2^(1/2)*cos(d*x+c)^2+150*A*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*2^(1/2)*cos(d*x+c)+228*B*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*2^(1/2)*cos(d*x+c)+75*A*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))`

/2))*sin(d*x+c)+114*B*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*sin(d*x+c)+64*A*cos(d*x+c)^6+208*A*cos(d*x+c)^5+96*B*cos(d*x+c)^5+328*A*cos(d*x+c)^4+432*B*cos(d*x+c)^4-600*A*cos(d*x+c)^3-528*B*cos(d*x+c)^3)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^2/sin(d*x+c)*a^2

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c + dx)^3 \left(A + \frac{B}{\cos(c + dx)} \right) \left(a + \frac{a}{\cos(c + dx)} \right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^3*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2),x)

[Out] int(cos(c + d*x)^3*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**3*(a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)),x)

[Out] Timed out

$$3.142 \quad \int \cos^4(c+dx)(a+a \sec(c+dx))^{5/2}(A+B \sec(c+dx)) dx$$

Optimal. Leaf size=209

$$\frac{a^{5/2}(163A + 200B) \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}}\right)}{64d} + \frac{a^3(163A + 200B) \sin(c + dx)}{64d\sqrt{a \sec(c + dx) + a}} + \frac{a^3(95A + 104B) \sin(c + dx) \cos(c + dx)}{96d\sqrt{a \sec(c + dx) + a}}$$

[Out] 1/64*a^(5/2)*(163*A+200*B)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+1/4*a*A*cos(d*x+c)^3*(a+a*sec(d*x+c))^(3/2)*sin(d*x+c)/d+1/64*a^3*(163*A+200*B)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/96*a^3*(95*A+104*B)*cos(d*x+c)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/24*a^2*(11*A+8*B)*cos(d*x+c)^2*sin(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d

Rubi [A] time = 0.58, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {4017, 4015, 3805, 3774, 203}

$$\frac{a^3(163A + 200B) \sin(c + dx)}{64d\sqrt{a \sec(c + dx) + a}} + \frac{a^{5/2}(163A + 200B) \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}}\right)}{64d} + \frac{a^2(11A + 8B) \sin(c + dx) \cos^2(c + dx)}{24d}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]

[Out] (a^(5/2)*(163*A + 200*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]])/(64*d) + (a^3*(163*A + 200*B)*Sin[c + d*x])/(64*d*Sqrt[a + a*Sec[c + d*x]]) + (a^3*(95*A + 104*B)*Cos[c + d*x]*Sin[c + d*x])/(96*d*Sqrt[a + a*Sec[c + d*x]]) + (a^2*(11*A + 8*B)*Cos[c + d*x]^2*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(24*d) + (a*A*cos[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2)*sin[c + d*x])/(4*d)

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 3774

Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] := Dist[(-2*b)/d, Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_.)], x_Symbol] := Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
+ b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_.)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] := Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
+ f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] := Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \cos^4(c + dx)(a + a \sec(c + dx))^{5/2}(A + B \sec(c + dx)) dx &= \frac{aA \cos^3(c + dx)(a + a \sec(c + dx))^{3/2} \sin(c + dx)}{4d} + \\
&= \frac{a^2(11A + 8B) \cos^2(c + dx) \sqrt{a + a \sec(c + dx)} \sin(c + dx)}{24d} \\
&= \frac{a^3(95A + 104B) \cos(c + dx) \sin(c + dx)}{96d \sqrt{a + a \sec(c + dx)}} + \frac{a^2(11A + 8B) \cos^2(c + dx) \sin(c + dx)}{24d} \\
&= \frac{a^3(163A + 200B) \sin(c + dx)}{64d \sqrt{a + a \sec(c + dx)}} + \frac{a^3(95A + 104B) \cos(c + dx) \sin(c + dx)}{96d \sqrt{a + a \sec(c + dx)}} \\
&= \frac{a^3(163A + 200B) \sin(c + dx)}{64d \sqrt{a + a \sec(c + dx)}} + \frac{a^3(95A + 104B) \cos(c + dx) \sin(c + dx)}{96d \sqrt{a + a \sec(c + dx)}} \\
&= \frac{a^{5/2}(163A + 200B) \tan^{-1}\left(\frac{\sqrt{a} \tan(c + dx)}{\sqrt{a + a \sec(c + dx)}}\right)}{64d} + \frac{a^3(163A + 200B) \sin(c + dx)}{64d}
\end{aligned}$$

Mathematica [C] time = 1.30, size = 366, normalized size = 1.75

$$a^2 \sin(c + dx) \sqrt{a(\sec(c + dx) + 1)} \left(4608A \sqrt{1 - \sec(c + dx)} {}_2F_1\left(\frac{1}{2}, 5; \frac{3}{2}; 1 - \sec(c + dx)\right) + 2079A \sqrt{1 - \sec(c + dx)} \right)$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]

[Out] (a^2*(6075*A*ArcTanh[Sqrt[1 - Sec[c + d*x]]] + 6600*B*ArcTanh[Sqrt[1 - Sec[c + d*x]]] + 2079*A*Sqrt[1 - Sec[c + d*x]] + 1240*B*Sqrt[1 - Sec[c + d*x]] + 7641*A*Cos[c + d*x]*Sqrt[1 - Sec[c + d*x]] + 6360*B*Cos[c + d*x]*Sqrt[1 - Sec[c + d*x]] + 2097*A*Cos[2*(c + d*x)]*Sqrt[1 - Sec[c + d*x]] + 1240*B*Cos[2*(c + d*x)]*Sqrt[1 - Sec[c + d*x]] + 522*A*Cos[3*(c + d*x)]*Sqrt[1 - Sec[c + d*x]] - 80*B*Cos[3*(c + d*x)]*Sqrt[1 - Sec[c + d*x]] + 18*A*Cos[4*(c + d*x)]*Sqrt[1 - Sec[c + d*x]] + 7680*B*Hypergeometric2F1[1/2, 4, 3/2, 1 - Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]] + 4608*A*Hypergeometric2F1[1/2, 5, 3/2, 1 - Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]])*Sqrt[a*(1 + Sec[c + d*x])]*Sin[c + d*x])/(2880*d*(1 + Cos[c + d*x])*Sqrt[1 - Sec[c + d*x]])

fricas [A] time = 0.56, size = 420, normalized size = 2.01

$$\left[\frac{3 \left((163 A + 200 B) a^2 \cos(dx + c) + (163 A + 200 B) a^2 \right) \sqrt{-a} \log \left(\frac{2 a \cos(dx+c)^2 - 2 \sqrt{-a} \sqrt{\frac{a \cos(dx+c)+a}{\cos(dx+c)}} \cos(dx+c) \sin(dx+c)}{\cos(dx+c)+1} \right)}{\dots} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] [1/384*(3*((163*A + 200*B)*a^2*cos(d*x + c) + (163*A + 200*B)*a^2)*sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(48*A*a^2*cos(d*x + c)^4 + 8*(23*A + 8*B)*a^2*cos(d*x + c)^3 + 2*(163*A + 136*B)*a^2*cos(d*x + c)^2 + 3*(163*A + 200*B)*a^2*cos(d*x + c)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d), -1/192*(3*((163*A + 200*B)*a^2*cos(d*x + c) + (163*A + 200*B)*a^2)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - (48*A*a^2*cos(d*x + c)^4 + 8*(23*A + 8*B)*a^2*cos(d*x + c)^3 + 2*(163*A + 136*B)*a^2*cos(d*x + c)^2 + 3*(163*A + 200*B)*a^2*cos(d*x + c)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d)]

giac [B] time = 2.86, size = 1096, normalized size = 5.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] -1/384*(3*(163*A*sqrt(-a)*a^2*sgn(cos(d*x + c)) + 200*B*sqrt(-a)*a^2*sgn(cos(d*x + c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3))) - 3*(163*A*sqrt(-a)*a^2*sgn(cos(d*x + c)) + 200*B*sqrt(-a)*a^2*sgn(cos(d*x + c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))) + 4*sqrt(2)*(489*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^14*A*sqrt(-a)*a^3*sgn(cos(d*x + c)) + 600*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^14*B*sqrt(-a)*a^3*sgn(cos(d*x + c)) - 10269*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^12*A*sqrt(-a)*a^4*sgn(cos(d*x + c)) - 12600*(sqrt(-a)*tan(1/2*d

```

*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^12*B*sqrt(-a)*a^4*sgn(cos(
s(d*x + c)) + 69885*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x +
1/2*c)^2 + a))^10*A*sqrt(-a)*a^5*sgn(cos(d*x + c)) + 103992*(sqrt(-a)*tan(1
/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^10*B*sqrt(-a)*a^5*sg
n(cos(d*x + c)) - 259233*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d
*x + 1/2*c)^2 + a))^8*A*sqrt(-a)*a^6*sgn(cos(d*x + c)) - 339864*(sqrt(-a)*t
an(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^8*B*sqrt(-a)*a^6
*sgn(cos(d*x + c)) + 209979*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/
2*d*x + 1/2*c)^2 + a))^6*A*sqrt(-a)*a^7*sgn(cos(d*x + c)) + 262920*(sqrt(-a
)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*B*sqrt(-a)*
a^7*sgn(cos(d*x + c)) - 55511*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(
1/2*d*x + 1/2*c)^2 + a))^4*A*sqrt(-a)*a^8*sgn(cos(d*x + c)) - 73640*(sqrt(-
a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*B*sqrt(-a)
*a^8*sgn(cos(d*x + c)) + 6687*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(
1/2*d*x + 1/2*c)^2 + a))^2*A*sqrt(-a)*a^9*sgn(cos(d*x + c)) + 8808*(sqrt(-a
)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*B*sqrt(-a)*
a^9*sgn(cos(d*x + c)) - 299*A*sqrt(-a)*a^10*sgn(cos(d*x + c)) - 392*B*sqrt(
-a)*a^10*sgn(cos(d*x + c)))/((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1
/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(
1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^4)/d

```

maple [B] time = 1.50, size = 765, normalized size = 3.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x)
```

```

[Out] -1/3072/d*(-489*A*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*arctanh(1/2*
(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*sin(d*x
+c)*cos(d*x+c)^3-600*B*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*arctanh
(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*si
n(d*x+c)*cos(d*x+c)^3-1467*A*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*a
rctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/
2))*sin(d*x+c)*cos(d*x+c)^2-1800*B*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(
7/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)
*2^(1/2))*sin(d*x+c)*cos(d*x+c)^2-1467*A*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+
c)))^(7/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(
d*x+c)*2^(1/2))*sin(d*x+c)*cos(d*x+c)-1800*B*2^(1/2)*(-2*cos(d*x+c)/(1+cos(
d*x+c)))^(7/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/
cos(d*x+c)*2^(1/2))*sin(d*x+c)*cos(d*x+c)-489*A*(-2*cos(d*x+c)/(1+cos(d*x+c
)))^(7/2)*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+
c)/cos(d*x+c)*2^(1/2))*sin(d*x+c)-600*B*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2
)*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(
d*x+c)*2^(1/2))*sin(d*x+c)+768*A*cos(d*x+c)^8+2176*A*cos(d*x+c)^7+1024*B*cos

```

$(d*x+c)^7+2272*A*\cos(d*x+c)^6+3328*B*\cos(d*x+c)^6+2608*A*\cos(d*x+c)^5+5248*B*\cos(d*x+c)^5-7824*A*\cos(d*x+c)^4-9600*B*\cos(d*x+c)^4)*(a*(1+\cos(d*x+c)))/\cos(d*x+c)^{(1/2)}/\cos(d*x+c)^3/\sin(d*x+c)*a^2$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c + dx)^4 \left(A + \frac{B}{\cos(c + dx)} \right) \left(a + \frac{a}{\cos(c + dx)} \right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^4*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2),x)

[Out] int(cos(c + d*x)^4*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**4*(a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)),x)

[Out] Timed out

$$3.143 \quad \int \cos^5(c+dx)(a+a \sec(c+dx))^{5/2}(A+B \sec(c+dx)) dx$$

Optimal. Leaf size=254

$$\frac{a^{5/2}(283A + 326B) \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}}\right)}{128d} + \frac{a^3(283A + 326B) \sin(c+dx)}{128d\sqrt{a \sec(c+dx)+a}} + \frac{a^3(157A + 170B) \sin(c+dx) \cos^2(c+dx)}{240d\sqrt{a \sec(c+dx)+a}}$$

[Out] $1/128*a^{(5/2)}*(283*A+326*B)*\arctan(a^{(1/2)}*\tan(d*x+c)/(a+a*\sec(d*x+c))^{(1/2)})/d+1/5*a*A*\cos(d*x+c)^4*(a+a*\sec(d*x+c))^{(3/2)}*\sin(d*x+c)/d+1/128*a^3*(283*A+326*B)*\sin(d*x+c)/d/(a+a*\sec(d*x+c))^{(1/2)}+1/192*a^3*(283*A+326*B)*\cos(d*x+c)*\sin(d*x+c)/d/(a+a*\sec(d*x+c))^{(1/2)}+1/240*a^3*(157*A+170*B)*\cos(d*x+c)^2*\sin(d*x+c)/d/(a+a*\sec(d*x+c))^{(1/2)}+1/40*a^2*(13*A+10*B)*\cos(d*x+c)^3*\sin(d*x+c)*(a+a*\sec(d*x+c))^{(1/2)}/d$

Rubi [A] time = 0.65, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {4017, 4015, 3805, 3774, 203}

$$\frac{a^3(283A + 326B) \sin(c+dx)}{128d\sqrt{a \sec(c+dx)+a}} + \frac{a^{5/2}(283A + 326B) \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}}\right)}{128d} + \frac{a^2(13A + 10B) \sin(c+dx) \cos^3(c+dx)}{40d}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^5*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]

[Out] $(a^{(5/2)}*(283*A + 326*B)*\text{ArcTan}[(\text{Sqrt}[a]*\text{Tan}[c + d*x])/\text{Sqrt}[a + a*\text{Sec}[c + d*x]])/(128*d) + (a^3*(283*A + 326*B)*\text{Sin}[c + d*x])/(128*d*\text{Sqrt}[a + a*\text{Sec}[c + d*x]]) + (a^3*(283*A + 326*B)*\text{Cos}[c + d*x]*\text{Sin}[c + d*x])/(192*d*\text{Sqrt}[a + a*\text{Sec}[c + d*x]]) + (a^3*(157*A + 170*B)*\text{Cos}[c + d*x]^2*\text{Sin}[c + d*x])/(240*d*\text{Sqrt}[a + a*\text{Sec}[c + d*x]]) + (a^2*(13*A + 10*B)*\text{Cos}[c + d*x]^3*\text{Sqrt}[a + a*\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(40*d) + (a*A*\text{Cos}[c + d*x]^4*(a + a*\text{Sec}[c + d*x])^{(3/2)}*\text{Sin}[c + d*x])/(5*d)$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 3774

Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] := Dist[(-2*b)/d, Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],

$x] /; \text{FreeQ}\{a, b, c, d\}, x] \&\& \text{EqQ}[a^2 - b^2, 0]$

Rule 3805

$\text{Int}[(\text{csc}[(e_.) + (f_.)(x_.)]*(d_.))^n*\text{Sqrt}[\text{csc}[(e_.) + (f_.)(x_.)]*(b_.) + (a_.)], x_Symbol] :> \text{Simp}[(a*\text{Cot}[e + f*x]*(d*\text{Csc}[e + f*x])^n)/(f*n*\text{Sqrt}[a + b*\text{Csc}[e + f*x]]), x] + \text{Dist}[(a*(2*n + 1))/(2*b*d*n), \text{Int}[\text{Sqrt}[a + b*\text{Csc}[e + f*x]]*(d*\text{Csc}[e + f*x])^{n + 1}, x], x] /; \text{FreeQ}\{a, b, d, e, f\}, x] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{LtQ}[n, -2^{(-1)}] \&\& \text{IntegerQ}[2*n]$

Rule 4015

$\text{Int}[(\text{csc}[(e_.) + (f_.)(x_.)]*(d_.))^n*\text{Sqrt}[\text{csc}[(e_.) + (f_.)(x_.)]*(b_.) + (a_.)]*(\text{csc}[(e_.) + (f_.)(x_.)]*(B_.) + (A_.)), x_Symbol] :> \text{Simp}[(A*b^2*\text{Cot}[e + f*x]*(d*\text{Csc}[e + f*x])^n)/(a*f*n*\text{Sqrt}[a + b*\text{Csc}[e + f*x]]), x] + \text{Dist}[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), \text{Int}[\text{Sqrt}[a + b*\text{Csc}[e + f*x]]*(d*\text{Csc}[e + f*x])^{n + 1}, x], x] /; \text{FreeQ}\{a, b, d, e, f, A, B\}, x] \&\& \text{NeQ}[A*b - a*B, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{NeQ}[A*b*(2*n + 1) + 2*a*B*n, 0] \&\& \text{LtQ}[n, 0]$

Rule 4017

$\text{Int}[(\text{csc}[(e_.) + (f_.)(x_.)]*(d_.))^n*(\text{csc}[(e_.) + (f_.)(x_.)]*(b_.) + (a_.))^m*(\text{csc}[(e_.) + (f_.)(x_.)]*(B_.) + (A_.)), x_Symbol] :> \text{Simp}[(a*A*\text{Cot}[e + f*x]*(a + b*\text{Csc}[e + f*x])^{m - 1}*(d*\text{Csc}[e + f*x])^n)/(f*n), x] - \text{Dist}[b/(a*d*n), \text{Int}[(a + b*\text{Csc}[e + f*x])^{m - 1}*(d*\text{Csc}[e + f*x])^{n + 1}*\text{Simp}[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*\text{Csc}[e + f*x], x], x], x] /; \text{FreeQ}\{a, b, d, e, f, A, B\}, x] \&\& \text{NeQ}[A*b - a*B, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{GtQ}[m, 1/2] \&\& \text{LtQ}[n, -1]$

Rubi steps

$$\begin{aligned}
\int \cos^5(c + dx)(a + a \sec(c + dx))^{5/2}(A + B \sec(c + dx)) dx &= \frac{aA \cos^4(c + dx)(a + a \sec(c + dx))^{3/2} \sin(c + dx)}{5d} + \\
&= \frac{a^2(13A + 10B) \cos^3(c + dx) \sqrt{a + a \sec(c + dx)} \sin(c + dx)}{40d} \\
&= \frac{a^3(157A + 170B) \cos^2(c + dx) \sin(c + dx)}{240d \sqrt{a + a \sec(c + dx)}} + \frac{a^2(13A + 10B) \cos(c + dx) \sin(c + dx)}{240d \sqrt{a + a \sec(c + dx)}} \\
&= \frac{a^3(283A + 326B) \cos(c + dx) \sin(c + dx)}{192d \sqrt{a + a \sec(c + dx)}} + \frac{a^3(157A + 170B) \sin(c + dx)}{192d \sqrt{a + a \sec(c + dx)}} \\
&= \frac{a^3(283A + 326B) \sin(c + dx)}{128d \sqrt{a + a \sec(c + dx)}} + \frac{a^3(283A + 326B) \cos(c + dx)}{192d \sqrt{a + a \sec(c + dx)}} \\
&= \frac{a^3(283A + 326B) \sin(c + dx)}{128d \sqrt{a + a \sec(c + dx)}} + \frac{a^3(283A + 326B) \cos(c + dx)}{192d \sqrt{a + a \sec(c + dx)}} \\
&= \frac{a^{5/2}(283A + 326B) \tan^{-1}\left(\frac{\sqrt{a} \tan(c + dx)}{\sqrt{a + a \sec(c + dx)}}\right)}{128d} + \frac{a^3(283A + 326B)}{128d}
\end{aligned}$$

Mathematica [C] time = 1.82, size = 416, normalized size = 1.64

$$a^2 \sin(c + dx) \sqrt{a(\sec(c + dx) + 1)} \left(15360A \sqrt{1 - \sec(c + dx)} {}_2F_1\left(\frac{1}{2}, 6; \frac{3}{2}; 1 - \sec(c + dx)\right) + 11651A \sqrt{1 - \sec(c + dx)} \right)$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]^5*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]

[Out] (a^2*(25935*A*ArcTanh[Sqrt[1 - Sec[c + d*x]]] + 28350*B*ArcTanh[Sqrt[1 - Sec[c + d*x]]] + 11651*A*Sqrt[1 - Sec[c + d*x]] + 9702*B*Sqrt[1 - Sec[c + d*x]]) + 37029*A*Cos[c + d*x]*Sqrt[1 - Sec[c + d*x]] + 35658*B*Cos[c + d*x]*Sqrt[1 - Sec[c + d*x]] + 12653*A*Cos[2*(c + d*x)]*Sqrt[1 - Sec[c + d*x]] + 9786*B*Cos[2*(c + d*x)]*Sqrt[1 - Sec[c + d*x]] + 3818*A*Cos[3*(c + d*x)]*Sqrt[1 - Sec[c + d*x]] + 2436*B*Cos[3*(c + d*x)]*Sqrt[1 - Sec[c + d*x]] + 1002*A*Cos[4*(c + d*x)]*Sqrt[1 - Sec[c + d*x]] + 84*B*Cos[4*(c + d*x)]*Sqrt[1 - Sec[c + d*x]] + 72*A*Cos[5*(c + d*x)]*Sqrt[1 - Sec[c + d*x]] + 21504*B*Hypergeometric2F1[1/2, 5, 3/2, 1 - Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]] + 15360*A*Hypergeometric2F1[1/2, 6, 3/2, 1 - Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]])*Sqrt[a*(1 + Sec[c + d*x])*Sin[c + d*x]]/(13440*d*(1 + Cos[c + d*x])*Sqrt[1 - Sec[c + d*x]])

fricas [A] time = 0.74, size = 460, normalized size = 1.81

$$\left[\frac{15 \left((283A + 326B)a^2 \cos(dx + c) + (283A + 326B)a^2 \right) \sqrt{-a} \log \left(\frac{2a \cos(dx+c)^2 - 2\sqrt{-a} \sqrt{\frac{a \cos(dx+c)+a}{\cos(dx+c)}} \cos(dx+c) \sin(dx+c)}{\cos(dx+c)+1} \right)}{\dots} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] [1/3840*(15*((283*A + 326*B)*a^2*cos(d*x + c) + (283*A + 326*B)*a^2)*sqrt(-a)*log(((2*a*cos(d*x + c))^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(384*A*a^2*cos(d*x + c)^5 + 48*(29*A + 10*B)*a^2*cos(d*x + c)^4 + 8*(283*A + 230*B)*a^2*cos(d*x + c)^3 + 10*(283*A + 326*B)*a^2*cos(d*x + c)^2 + 15*(283*A + 326*B)*a^2*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c) + d), -1/1920*(15*((283*A + 326*B)*a^2*cos(d*x + c) + (283*A + 326*B)*a^2)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - (384*A*a^2*cos(d*x + c)^5 + 48*(29*A + 10*B)*a^2*cos(d*x + c)^4 + 8*(283*A + 230*B)*a^2*cos(d*x + c)^3 + 10*(283*A + 326*B)*a^2*cos(d*x + c)^2 + 15*(283*A + 326*B)*a^2*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c) + d)]

giac [B] time = 4.18, size = 1377, normalized size = 5.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] -1/3840*(15*(283*A*sqrt(-a)*a^2*sgn(cos(d*x + c)) + 326*B*sqrt(-a)*a^2*sgn(cos(d*x + c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3))) - 15*(283*A*sqrt(-a)*a^2*sgn(cos(d*x + c)) + 326*B*sqrt(-a)*a^2*sgn(cos(d*x + c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))) + 4*(4245*sqrt(2)*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^18*A*sqrt(-a)*a^3*sgn(cos(d*x + c)) + 4890*sqrt(2)*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^18*B*sqrt(-a)*

$$\begin{aligned}
& a^3 \operatorname{sgn}(\cos(dx + c)) - 114615 \sqrt{2} (\sqrt{-a} \tan(1/2 dx + 1/2 c) - \sqrt{-a \tan(1/2 dx + 1/2 c)^2 + a})^{16} A \sqrt{-a} a^4 \operatorname{sgn}(\cos(dx + c)) - 132030 \sqrt{2} (\sqrt{-a} \tan(1/2 dx + 1/2 c) - \sqrt{-a \tan(1/2 dx + 1/2 c)^2 + a})^{16} B \sqrt{-a} a^4 \operatorname{sgn}(\cos(dx + c)) + 1298820 \sqrt{2} (\sqrt{-a} \tan(1/2 dx + 1/2 c) - \sqrt{-a \tan(1/2 dx + 1/2 c)^2 + a})^{14} A \sqrt{-a} a^5 \operatorname{sgn}(\cos(dx + c)) + 1319880 \sqrt{2} (\sqrt{-a} \tan(1/2 dx + 1/2 c) - \sqrt{-a \tan(1/2 dx + 1/2 c)^2 + a})^{14} B \sqrt{-a} a^5 \operatorname{sgn}(\cos(dx + c)) - 6176700 \sqrt{2} (\sqrt{-a} \tan(1/2 dx + 1/2 c) - \sqrt{-a \tan(1/2 dx + 1/2 c)^2 + a})^{12} A \sqrt{-a} a^6 \operatorname{sgn}(\cos(dx + c)) - 6888120 \sqrt{2} (\sqrt{-a} \tan(1/2 dx + 1/2 c) - \sqrt{-a \tan(1/2 dx + 1/2 c)^2 + a})^{12} B \sqrt{-a} a^6 \operatorname{sgn}(\cos(dx + c)) + 16394598 \sqrt{2} (\sqrt{-a} \tan(1/2 dx + 1/2 c) - \sqrt{-a \tan(1/2 dx + 1/2 c)^2 + a})^{10} A \sqrt{-a} a^7 \operatorname{sgn}(\cos(dx + c)) + 18352620 \sqrt{2} (\sqrt{-a} \tan(1/2 dx + 1/2 c) - \sqrt{-a \tan(1/2 dx + 1/2 c)^2 + a})^{10} B \sqrt{-a} a^7 \operatorname{sgn}(\cos(dx + c)) - 14042770 \sqrt{2} (\sqrt{-a} \tan(1/2 dx + 1/2 c) - \sqrt{-a \tan(1/2 dx + 1/2 c)^2 + a})^8 A \sqrt{-a} a^8 \operatorname{sgn}(\cos(dx + c)) - 15746180 \sqrt{2} (\sqrt{-a} \tan(1/2 dx + 1/2 c) - \sqrt{-a \tan(1/2 dx + 1/2 c)^2 + a})^8 B \sqrt{-a} a^8 \operatorname{sgn}(\cos(dx + c)) + 4791060 \sqrt{2} (\sqrt{-a} \tan(1/2 dx + 1/2 c) - \sqrt{-a \tan(1/2 dx + 1/2 c)^2 + a})^6 A \sqrt{-a} a^9 \operatorname{sgn}(\cos(dx + c)) + 5497320 \sqrt{2} (\sqrt{-a} \tan(1/2 dx + 1/2 c) - \sqrt{-a \tan(1/2 dx + 1/2 c)^2 + a})^6 B \sqrt{-a} a^9 \operatorname{sgn}(\cos(dx + c)) - 860300 \sqrt{2} (\sqrt{-a} \tan(1/2 dx + 1/2 c) - \sqrt{-a \tan(1/2 dx + 1/2 c)^2 + a})^4 A \sqrt{-a} a^{10} \operatorname{sgn}(\cos(dx + c)) - 959320 \sqrt{2} (\sqrt{-a} \tan(1/2 dx + 1/2 c) - \sqrt{-a \tan(1/2 dx + 1/2 c)^2 + a})^4 B \sqrt{-a} a^{10} \operatorname{sgn}(\cos(dx + c)) + 75885 \sqrt{2} (\sqrt{-a} \tan(1/2 dx + 1/2 c) - \sqrt{-a \tan(1/2 dx + 1/2 c)^2 + a})^2 A \sqrt{-a} a^{11} \operatorname{sgn}(\cos(dx + c)) + 84810 \sqrt{2} (\sqrt{-a} \tan(1/2 dx + 1/2 c) - \sqrt{-a \tan(1/2 dx + 1/2 c)^2 + a})^2 B \sqrt{-a} a^{11} \operatorname{sgn}(\cos(dx + c)) - 2671 \sqrt{2} A \sqrt{-a} a^{12} \operatorname{sgn}(\cos(dx + c)) - 2990 \sqrt{2} B \sqrt{-a} a^{12} \operatorname{sgn}(\cos(dx + c)) / ((\sqrt{-a} \tan(1/2 dx + 1/2 c) - \sqrt{-a \tan(1/2 dx + 1/2 c)^2 + a})^4 - 6 (\sqrt{-a} \tan(1/2 dx + 1/2 c) - \sqrt{-a \tan(1/2 dx + 1/2 c)^2 + a})^2 a + a^2)^5) / d
\end{aligned}$$

maple [B] time = 1.61, size = 947, normalized size = 3.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\int (\cos(dx+c)^5 (a+a \sec(dx+c))^{5/2} (A+B \sec(dx+c)), x)$

[Out]
$$\begin{aligned}
& -1/61440/d * (4245 A \sin(dx+c) \cos(dx+c)^4 \operatorname{arctanh}(1/2 * (-2 \cos(dx+c) / (1 + \cos(dx+c))))^{(1/2)} \sin(dx+c) / \cos(dx+c) * 2^{(1/2)}) * (-2 \cos(dx+c) / (1 + \cos(dx+c)))^{(9/2)} * 2^{(1/2)} + 4890 B \sin(dx+c) \cos(dx+c)^4 \operatorname{arctanh}(1/2 * (-2 \cos(dx+c) / (1 + \cos(dx+c))))^{(1/2)} \sin(dx+c) / \cos(dx+c) * 2^{(1/2)}) * (-2 \cos(dx+c) / (1 + \cos(dx+c)))^{(9/2)} * 2^{(1/2)} + 16980 A \sin(dx+c) \cos(dx+c)^3 \operatorname{arctanh}(1/2 * (-2 \cos(dx+c) / (1 + \cos(dx+c))))^{(1/2)} \sin(dx+c) / \cos(dx+c) * 2^{(1/2)}) * (-2 \cos(dx+c) / (1 + \cos(dx+c)))^{(9/2)} * 2^{(1/2)} + 16980 B \sin(dx+c) \cos(dx+c)^3 \operatorname{arctanh}(1/2 * (-2 \cos(dx+c) / (1 + \cos(dx+c))))^{(1/2)} \sin(dx+c) / \cos(dx+c) * 2^{(1/2)}) * (-2 \cos(dx+c) / (1 + \cos(dx+c)))^{(9/2)} * 2^{(1/2)}
\end{aligned}$$

$$\begin{aligned} & /((1+\cos(dx+c))^{9/2} * 2^{1/2} + 19560 * B * \sin(dx+c) * \cos(dx+c)^3 * \operatorname{arctanh}(1/2 * \\ & (-2 * \cos(dx+c) / (1+\cos(dx+c)))^{1/2} * \sin(dx+c) / \cos(dx+c) * 2^{1/2})) * (-2 * \cos \\ & (dx+c) / (1+\cos(dx+c)))^{9/2} * 2^{1/2} + 25470 * A * \sin(dx+c) * \cos(dx+c)^2 * \operatorname{arctan} \\ & \operatorname{h}(1/2 * (-2 * \cos(dx+c) / (1+\cos(dx+c)))^{1/2} * \sin(dx+c) / \cos(dx+c) * 2^{1/2})) * \\ & (-2 * \cos(dx+c) / (1+\cos(dx+c)))^{9/2} * 2^{1/2} + 29340 * B * \sin(dx+c) * \cos(dx+c)^2 * \operatorname{arctan} \\ & \operatorname{h}(1/2 * (-2 * \cos(dx+c) / (1+\cos(dx+c)))^{1/2} * \sin(dx+c) / \cos(dx+c) * 2^{1/2})) * \\ & (-2 * \cos(dx+c) / (1+\cos(dx+c)))^{9/2} * 2^{1/2} + 16980 * A * \sin(dx+c) * \cos(dx+c) * \operatorname{arctan} \\ & \operatorname{h}(1/2 * (-2 * \cos(dx+c) / (1+\cos(dx+c)))^{1/2} * \sin(dx+c) / \cos(dx+c) * 2^{1/2})) * \\ & (-2 * \cos(dx+c) / (1+\cos(dx+c)))^{9/2} * 2^{1/2} + 19560 * B * \sin(dx+c) * \cos(dx+c) * \operatorname{arctan} \\ & \operatorname{h}(1/2 * (-2 * \cos(dx+c) / (1+\cos(dx+c)))^{1/2} * \sin(dx+c) / \cos(dx+c) * 2^{1/2})) * \\ & (-2 * \cos(dx+c) / (1+\cos(dx+c)))^{9/2} * 2^{1/2} + 4245 * A * 2^{1/2} * \operatorname{arctan} \\ & \operatorname{h}(1/2 * (-2 * \cos(dx+c) / (1+\cos(dx+c)))^{1/2} * \sin(dx+c) / \cos(dx+c) * 2^{1/2})) * \\ & (-2 * \cos(dx+c) / (1+\cos(dx+c)))^{9/2} * \sin(dx+c) + 4890 * B * 2^{1/2} * \operatorname{arctan} \\ & \operatorname{h}(1/2 * (-2 * \cos(dx+c) / (1+\cos(dx+c)))^{1/2} * \sin(dx+c) / \cos(dx+c) * 2^{1/2})) * \\ & (-2 * \cos(dx+c) / (1+\cos(dx+c)))^{9/2} * \sin(dx+c) + 12288 * A * \cos(dx+c)^{10} + 32256 \\ & * A * \cos(dx+c)^9 + 15360 * B * \cos(dx+c)^9 + 27904 * A * \cos(dx+c)^8 + 43520 * B * \cos(dx+c) \\ &)^8 + 18112 * A * \cos(dx+c)^7 + 45440 * B * \cos(dx+c)^7 + 45280 * A * \cos(dx+c)^6 + 52160 * B * \\ & \cos(dx+c)^6 - 135840 * A * \cos(dx+c)^5 - 156480 * B * \cos(dx+c)^5) * (a * (1+\cos(dx+c)) \\ & / \cos(dx+c))^{1/2} / \sin(dx+c) / \cos(dx+c)^4 * a^2 \end{aligned}$$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(dx+c)^5*(a+a*sec(dx+c))^(5/2)*(A+B*sec(dx+c)),x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c + dx)^5 \left(A + \frac{B}{\cos(c + dx)} \right) \left(a + \frac{a}{\cos(c + dx)} \right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^5*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2),x)

[Out] int(cos(c + d*x)^5*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

$$3.144 \quad \int \frac{\sec^4(c+dx)(A+B \sec(c+dx))}{\sqrt{a+a \sec(c+dx)}} dx$$

Optimal. Leaf size=202

$$\frac{2(7A - B) \tan(c + dx) \sec^2(c + dx)}{35d\sqrt{a \sec(c + dx) + a}} - \frac{\sqrt{2}(A - B) \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{2} \sqrt{a \sec(c+dx)+a}}\right)}{\sqrt{a} d} - \frac{2(7A - 31B) \tan(c + dx) \sqrt{a \sec(c + dx) + a}}{105ad}$$

[Out] $-(A-B) \cdot \arctan\left(\frac{1/2 \cdot a^{1/2} \cdot \tan(d \cdot x + c) \cdot 2^{1/2}}{(a + a \cdot \sec(d \cdot x + c))^{1/2}}\right) \cdot 2^{1/2} / d / a^{1/2} + 4/105 \cdot (49 \cdot A - 37 \cdot B) \cdot \tan(d \cdot x + c) / d / (a + a \cdot \sec(d \cdot x + c))^{1/2} + 2/35 \cdot (7 \cdot A - B) \cdot \sec(d \cdot x + c)^2 \cdot \tan(d \cdot x + c) / d / (a + a \cdot \sec(d \cdot x + c))^{1/2} + 2/7 \cdot B \cdot \sec(d \cdot x + c)^3 \cdot \tan(d \cdot x + c) / d / (a + a \cdot \sec(d \cdot x + c))^{1/2} - 2/105 \cdot (7 \cdot A - 31 \cdot B) \cdot (a + a \cdot \sec(d \cdot x + c))^{1/2} \cdot \tan(d \cdot x + c) / a / d$

Rubi [A] time = 0.61, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {4021, 4010, 4001, 3795, 203}

$$\frac{2(7A - B) \tan(c + dx) \sec^2(c + dx)}{35d\sqrt{a \sec(c + dx) + a}} - \frac{\sqrt{2}(A - B) \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{2} \sqrt{a \sec(c+dx)+a}}\right)}{\sqrt{a} d} - \frac{2(7A - 31B) \tan(c + dx) \sqrt{a \sec(c + dx) + a}}{105ad}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(\text{Sec}[c + d \cdot x]^4 \cdot (A + B \cdot \text{Sec}[c + d \cdot x])) / \text{Sqrt}[a + a \cdot \text{Sec}[c + d \cdot x]], x]$

[Out] $-\left(\frac{\text{Sqrt}[2] \cdot (A - B) \cdot \text{ArcTan}\left[\frac{\text{Sqrt}[a] \cdot \text{Tan}[c + d \cdot x]}{\text{Sqrt}[2] \cdot \text{Sqrt}[a + a \cdot \text{Sec}[c + d \cdot x]]}\right]}{\text{Sqrt}[a] \cdot d}\right) + \frac{4 \cdot (49 \cdot A - 37 \cdot B) \cdot \text{Tan}[c + d \cdot x]}{(105 \cdot d \cdot \text{Sqrt}[a + a \cdot \text{Sec}[c + d \cdot x]])} + \frac{2 \cdot (7 \cdot A - B) \cdot \text{Sec}[c + d \cdot x]^2 \cdot \text{Tan}[c + d \cdot x]}{(35 \cdot d \cdot \text{Sqrt}[a + a \cdot \text{Sec}[c + d \cdot x]])} + \frac{2 \cdot B \cdot \text{Sec}[c + d \cdot x]^3 \cdot \text{Tan}[c + d \cdot x]}{(7 \cdot d \cdot \text{Sqrt}[a + a \cdot \text{Sec}[c + d \cdot x]])} - \frac{2 \cdot (7 \cdot A - 31 \cdot B) \cdot \text{Sqrt}[a + a \cdot \text{Sec}[c + d \cdot x]] \cdot \text{Tan}[c + d \cdot x]}{(105 \cdot a \cdot d)}$

Rule 203

$\text{Int}[(a_) + (b_) \cdot (x_)^2]^{-1}, x_Symbol] \rightarrow \text{Simp}[(1 \cdot \text{ArcTan}[(\text{Rt}[b, 2] \cdot x) / \text{Rt}[a, 2]]) / (\text{Rt}[a, 2] \cdot \text{Rt}[b, 2]), x] /; \text{FreeQ}\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{GtQ}[b, 0])$

Rule 3795

$\text{Int}[\text{csc}[(e_) + (f_) \cdot (x_)] / \text{Sqrt}[\text{csc}[(e_) + (f_) \cdot (x_)] \cdot (b_) + (a_)], x_Symbol] \rightarrow \text{Dist}[-2/f, \text{Subst}[\text{Int}[1/(2 \cdot a + x^2), x], x, (b \cdot \text{Cot}[e + f \cdot x]) / \text{Sqrt}[a + b \cdot \text{Csc}[e + f \cdot x]]], x] /; \text{FreeQ}\{a, b, e, f\}, x] \ \&\& \ \text{EqQ}[a^2 - b^2, 0]$

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] := -Simp[(B*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m + 1), 0] && !LtQ[m, -2^(-1)]
```

Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] := -Simp[(B*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && !LtQ[m, -1]
```

Rule 4021

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] := -Simp[(B*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^4(c+dx)(A+B\sec(c+dx))}{\sqrt{a+a\sec(c+dx)}} dx &= \frac{2B\sec^3(c+dx)\tan(c+dx)}{7d\sqrt{a+a\sec(c+dx)}} + \frac{2\int \frac{\sec^3(c+dx)\left(3aB+\frac{1}{2}a(7A-B)\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}} dx}{7a} \\
&= \frac{2(7A-B)\sec^2(c+dx)\tan(c+dx)}{35d\sqrt{a+a\sec(c+dx)}} + \frac{2B\sec^3(c+dx)\tan(c+dx)}{7d\sqrt{a+a\sec(c+dx)}} + \frac{4}{7a} \\
&= \frac{2(7A-B)\sec^2(c+dx)\tan(c+dx)}{35d\sqrt{a+a\sec(c+dx)}} + \frac{2B\sec^3(c+dx)\tan(c+dx)}{7d\sqrt{a+a\sec(c+dx)}} - \frac{2}{7a} \\
&= \frac{4(49A-37B)\tan(c+dx)}{105d\sqrt{a+a\sec(c+dx)}} + \frac{2(7A-B)\sec^2(c+dx)\tan(c+dx)}{35d\sqrt{a+a\sec(c+dx)}} + \frac{2B}{7a} \\
&= \frac{4(49A-37B)\tan(c+dx)}{105d\sqrt{a+a\sec(c+dx)}} + \frac{2(7A-B)\sec^2(c+dx)\tan(c+dx)}{35d\sqrt{a+a\sec(c+dx)}} + \frac{2B}{7a} \\
&= -\frac{\sqrt{2}(A-B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d} + \frac{4(49A-37B)\tan(c+dx)}{105d\sqrt{a+a\sec(c+dx)}} + \frac{2B}{7a}
\end{aligned}$$

Mathematica [A] time = 0.54, size = 140, normalized size = 0.69

$$\frac{\tan(c+dx)\left(2\sqrt{1-\sec(c+dx)}\left(3(7A-B)\sec^2(c+dx)+(31B-7A)\sec(c+dx)+91A+15B\sec^3(c+dx)-4\right)\right)}{105d\sqrt{1-\sec(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[c + d*x]^4*(A + B*Sec[c + d*x]))/Sqrt[a + a*Sec[c + d*x]],x]

[Out] ((-105*Sqrt[2]*(A - B)*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]] + 2*Sqrt[1 - Sec[c + d*x]]*(91*A - 43*B + (-7*A + 31*B)*Sec[c + d*x] + 3*(7*A - B)*Sec[c + d*x]^2 + 15*B*Sec[c + d*x]^3))*Tan[c + d*x])/(105*d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])

fricas [A] time = 0.51, size = 432, normalized size = 2.14

$$\frac{105 \sqrt{2} \left((A - B) a \cos(dx + c)^4 + (A - B) a \cos(dx + c)^3 \right) \sqrt{-\frac{1}{a}} \log \left(\frac{2 \sqrt{2} \sqrt{\frac{a \cos(dx+c)+a}{\cos(dx+c)}} \sqrt{-\frac{1}{a}} \cos(dx+c) \sin(dx+c) - 3 \cos(dx+c)^2 + 2 \cos(dx+c) + 1}{\cos(dx+c)^2 + 2 \cos(dx+c) + 1} \right)}{210 \left((A - B) a \cos(dx + c)^4 + (A - B) a \cos(dx + c)^3 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")

[Out] [-1/210*(105*sqrt(2)*((A - B)*a*cos(d*x + c)^4 + (A - B)*a*cos(d*x + c)^3)*sqrt(-1/a)*log(-(2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*cos(d*x + c)*sin(d*x + c) - 3*cos(d*x + c)^2 - 2*cos(d*x + c) + 1)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*((91*A - 43*B)*cos(d*x + c)^3 - (7*A - 31*B)*cos(d*x + c)^2 + 3*(7*A - B)*cos(d*x + c) + 15*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c)^4 + a*d*cos(d*x + c)^3), 1/105*(2*((91*A - 43*B)*cos(d*x + c)^3 - (7*A - 31*B)*cos(d*x + c)^2 + 3*(7*A - B)*cos(d*x + c) + 15*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c) + 105*sqrt(2)*((A - B)*a*cos(d*x + c)^4 + (A - B)*a*cos(d*x + c)^3)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))/sqrt(a))/(a*d*cos(d*x + c)^4 + a*d*cos(d*x + c)^3)]

giac [A] time = 2.35, size = 287, normalized size = 1.42

$$\frac{105 \sqrt{2} (A-B) \log \left(\left| -\sqrt{-a} \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) + \sqrt{-a \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^2 + a} \right| \right)}{\sqrt{-a} \operatorname{sgn} \left(\tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^2 - 1 \right)} - \frac{2 \left(\frac{105 \sqrt{2} A a^3}{\operatorname{sgn} \left(\tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^2 - 1 \right)} - \left(\frac{\sqrt{2} (119 A a^3 - 92 B a^3) \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^2}{\operatorname{sgn} \left(\tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^2 - 1 \right)} - \frac{7 \sqrt{2} (37 A a^3)}{\operatorname{sgn} \left(\tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^2 - 1 \right)} \right)}{\left(a \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) \right)^2}$$

105d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")

[Out] -1/105*(105*sqrt(2)*(A - B)*log(abs(-sqrt(-a))*tan(1/2*d*x + 1/2*c) + sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) - 2*(105*sqrt(2)*A*a^3/sgn(tan(1/2*d*x + 1/2*c)^2 - 1) - ((sqrt(2)*(119*A*a^3 - 92*B*a^3)*tan(1/2*d*x + 1/2*c)^2/sgn(tan(1/2*d*x + 1/2*c)^2 - 1) - 7*s


```

qrt(2)*(37*A*a^3 - 16*B*a^3)/sgn(tan(1/2*d*x + 1/2*c)^2 - 1))*tan(1/2*d*x +
1/2*c)^2 + 35*sqrt(2)*(7*A*a^3 - 4*B*a^3)/sgn(tan(1/2*d*x + 1/2*c)^2 - 1))
*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 -
a)^3*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))/d

```

maple [B] time = 1.88, size = 785, normalized size = 3.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2), x)
```

```
[Out] 1/840/d*(105*A*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*
x+c)-1)/sin(d*x+c))*cos(d*x+c)^3*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^
(7/2)-105*B*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c
)-1)/sin(d*x+c))*cos(d*x+c)^3*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/
2)+315*A*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1
)/sin(d*x+c))*cos(d*x+c)^2*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)-
315*B*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/s
in(d*x+c))*cos(d*x+c)^2*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)+315
*A*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(
d*x+c))*cos(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)-315*B*ln
(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c
))*cos(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)+105*A*ln(-(-(-
2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-
2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*sin(d*x+c)-105*B*ln(-(-(-2*cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+
cos(d*x+c)))^(7/2)*sin(d*x+c)-1456*A*cos(d*x+c)^4+688*B*cos(d*x+c)^4+1568*A
*cos(d*x+c)^3-1184*B*cos(d*x+c)^3-448*A*cos(d*x+c)^2+544*B*cos(d*x+c)^2+336
*A*cos(d*x+c)-288*B*cos(d*x+c)+240*B)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/c
os(d*x+c)^3/sin(d*x+c)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sec(dx + c)^4}{\sqrt{a \sec(dx + c) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2), x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^4/sqrt(a*sec(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\cos(c+dx)^4 \sqrt{a + \frac{a}{\cos(c+dx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^4*(a + a/cos(c + d*x))^(1/2)),x)

[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^4*(a + a/cos(c + d*x))^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \sec^4(c + dx)}{\sqrt{a(\sec(c + dx) + 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(1/2),x)

[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**4/sqrt(a*(sec(c + d*x) + 1)), x)

$$3.145 \quad \int \frac{\sec^3(c+dx)(A+B \sec(c+dx))}{\sqrt{a+a \sec(c+dx)}} dx$$

Optimal. Leaf size=159

$$\frac{\sqrt{2}(A-B) \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{2} \sqrt{a \sec(c+dx)+a}}\right)}{\sqrt{a} d} + \frac{2(5A-B) \tan(c+dx) \sqrt{a \sec(c+dx)+a}}{15ad} - \frac{4(5A-7B) \tan(c+dx)}{15d \sqrt{a \sec(c+dx)+a}} + \frac{2B \tan(c+dx)}{5d \sqrt{a \sec(c+dx)+a}}$$

[Out] (A-B)*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2)/d/a^(1/2)-4/15*(5*A-7*B)*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+2/5*B*sec(d*x+c)^2*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+2/15*(5*A-B)*(a+a*sec(d*x+c))^(1/2)*tan(d*x+c)/a/d

Rubi [A] time = 0.42, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {4021, 4010, 4001, 3795, 203}

$$\frac{\sqrt{2}(A-B) \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{2} \sqrt{a \sec(c+dx)+a}}\right)}{\sqrt{a} d} + \frac{2(5A-B) \tan(c+dx) \sqrt{a \sec(c+dx)+a}}{15ad} - \frac{4(5A-7B) \tan(c+dx)}{15d \sqrt{a \sec(c+dx)+a}} + \frac{2B \tan(c+dx)}{5d \sqrt{a \sec(c+dx)+a}}$$

Antiderivative was successfully verified.

[In] Int[(Sec[c + d*x]^3*(A + B*Sec[c + d*x]))/Sqrt[a + a*Sec[c + d*x]],x]

[Out] (Sqrt[2]*(A - B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(Sqrt[a]*d) - (4*(5*A - 7*B)*Tan[c + d*x])/(15*d*Sqrt[a + a*Sec[c + d*x]]) + (2*B*Sec[c + d*x]^2*Tan[c + d*x])/(5*d*Sqrt[a + a*Sec[c + d*x]]) + (2*(5*A - B)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(15*a*d)

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 3795

Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] := Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 4001

Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] := -Simp[(B*Cot[e + f*x]*(a

+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m + 1), 0] && !LtQ[m, -2^(-1)]

Rule 4010

Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^m*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] := -Simp[(B*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Csc[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && !LtQ[m, -1]

Rule 4021

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^m*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] := -Simp[(B*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && GtQ[n, 1]

Rubi steps

$$\begin{aligned}
 \int \frac{\sec^3(c + dx)(A + B \sec(c + dx))}{\sqrt{a + a \sec(c + dx)}} dx &= \frac{2B \sec^2(c + dx) \tan(c + dx)}{5d\sqrt{a + a \sec(c + dx)}} + \frac{2 \int \frac{\sec^2(c + dx) \left(2aB + \frac{1}{2}a(5A - B) \sec(c + dx)\right)}{\sqrt{a + a \sec(c + dx)}} dx}{5a} \\
 &= \frac{2B \sec^2(c + dx) \tan(c + dx)}{5d\sqrt{a + a \sec(c + dx)}} + \frac{2(5A - B)\sqrt{a + a \sec(c + dx)} \tan(c + dx)}{15ad} \\
 &= -\frac{4(5A - 7B) \tan(c + dx)}{15d\sqrt{a + a \sec(c + dx)}} + \frac{2B \sec^2(c + dx) \tan(c + dx)}{5d\sqrt{a + a \sec(c + dx)}} + \frac{2(5A - B)\sqrt{a + a \sec(c + dx)} \tan(c + dx)}{15ad} \\
 &= -\frac{4(5A - 7B) \tan(c + dx)}{15d\sqrt{a + a \sec(c + dx)}} + \frac{2B \sec^2(c + dx) \tan(c + dx)}{5d\sqrt{a + a \sec(c + dx)}} + \frac{2(5A - B)\sqrt{a + a \sec(c + dx)} \tan(c + dx)}{15ad} \\
 &= \frac{\sqrt{2}(A - B) \tan^{-1}\left(\frac{\sqrt{a} \tan(c + dx)}{\sqrt{2}\sqrt{a + a \sec(c + dx)}}\right)}{\sqrt{a}d} - \frac{4(5A - 7B) \tan(c + dx)}{15d\sqrt{a + a \sec(c + dx)}} + \frac{2B \sec^2(c + dx) \tan(c + dx)}{5d\sqrt{a + a \sec(c + dx)}}
 \end{aligned}$$

Mathematica [A] time = 0.41, size = 123, normalized size = 0.77

$$\frac{\tan(c + dx) \left(2\sqrt{1 - \sec(c + dx)} \left((5A - B) \sec(c + dx) - 5A + 3B \sec^2(c + dx) + 13B \right) + 15\sqrt{2} (A - B) \tanh^{-1} \left(\frac{2\sqrt{1 - \sec(c + dx)}}{1 + \sec(c + dx)} \right) \right)}{15d\sqrt{1 - \sec(c + dx)} \sqrt{a(\sec(c + dx) + 1)}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[c + d*x]^3*(A + B*Sec[c + d*x]))/Sqrt[a + a*Sec[c + d*x]],x]

[Out] ((15*Sqrt[2]*(A - B)*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]] + 2*Sqrt[1 - Sec[c + d*x]]*(-5*A + 13*B + (5*A - B)*Sec[c + d*x] + 3*B*Sec[c + d*x]^2))*Tanh[c + d*x])/(15*d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])

fricas [A] time = 0.49, size = 397, normalized size = 2.50

$$\frac{15\sqrt{2} \left((A - B)a \cos(dx + c)^3 + (A - B)a \cos(dx + c)^2 \right) \sqrt{-\frac{1}{a}} \log \left(\frac{2\sqrt{2} \sqrt{\frac{a \cos(dx+c)+a}{\cos(dx+c)}} \sqrt{-\frac{1}{a}} \cos(dx+c) \sin(dx+c) + 3 \cos(dx+c)^2 + 2 \cos(dx+c) - 1}{\cos(dx+c)^2 + 2 \cos(dx+c) + 1} \right)}{30 \left(ad \cos(dx + c)^3 - \dots \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")

[Out] [-1/30*(15*sqrt(2)*((A - B)*a*cos(d*x + c)^3 + (A - B)*a*cos(d*x + c)^2)*sqrt(-1/a)*log((2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*cos(d*x + c)*sin(d*x + c) + 3*cos(d*x + c)^2 + 2*cos(d*x + c) - 1)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*((5*A - 13*B)*cos(d*x + c)^2 - (5*A - B)*cos(d*x + c) - 3*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)]/(a*d*cos(d*x + c)^3 + a*d*cos(d*x + c)^2), -1/15*(2*((5*A - 13*B)*cos(d*x + c)^2 - (5*A - B)*cos(d*x + c) - 3*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c) + 15*sqrt(2)*((A - B)*a*cos(d*x + c)^3 + (A - B)*a*cos(d*x + c)^2)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))/sqrt(a)]/(a*d*cos(d*x + c)^3 + a*d*cos(d*x + c)^2)]

giac [A] time = 2.68, size = 271, normalized size = 1.70

$$\frac{15(\sqrt{2}A - \sqrt{2}B) \log\left(-\sqrt{-a} \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + \sqrt{-a \tan^2\left(\frac{1}{2}dx + \frac{1}{2}c\right) + a}\right)}{\sqrt{-a} \operatorname{sgn}\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1\right)} - \frac{2\left(10\sqrt{2}Aa^2 \operatorname{sgn}\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - 1\right) - 20\sqrt{2}Ba^2 \operatorname{sgn}\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - 1\right)\right)}{\sqrt{-a} \operatorname{sgn}\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1\right)}$$

15

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")

[Out] 1/15*(15*(sqrt(2)*A - sqrt(2)*B)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) - 2*((10*sqrt(2)*A*a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1) - 20*sqrt(2)*B*a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1) - (10*sqrt(2)*A*a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1) - 17*sqrt(2)*B*a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1))*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2 + 15*sqrt(2)*B*a^2/sgn(tan(1/2*d*x + 1/2*c)^2 - 1))*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^2*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))/d

maple [B] time = 1.70, size = 595, normalized size = 3.74

$$\left(15A \left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{5}{2}} \ln\left(-\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}} \sin(dx+c) + \cos(dx+c) - 1}{\sin(dx+c)}\right)\right) \sin(dx+c) (\cos^2(dx+c)) - 15B \left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{5}{2}} \ln\left(-\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}} \sin(dx+c) + \cos(dx+c) - 1}{\sin(dx+c)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x)

[Out] 1/60/d*(15*A*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)^2-15*B*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)^2+30*A*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)-30*B*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)+15*A*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*sin(d*x+c)-15*B*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*sin(d*x+c))

$c)-1)/\sin(dx+c))*\sin(dx+c)+40*A*\cos(dx+c)^3-104*B*\cos(dx+c)^3-80*A*\cos(dx+c)^2+112*B*\cos(dx+c)^2+40*A*\cos(dx+c)-32*B*\cos(dx+c)+24*B)*(a*(1+\cos(dx+c))/\cos(dx+c))^{(1/2)}/\cos(dx+c)^2/\sin(dx+c)/a$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx+c) + A) \sec(dx+c)^3}{\sqrt{a \sec(dx+c) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(dx+c)^3*(A+B*sec(dx+c))/(a+a*sec(dx+c))^(1/2),x, algorithm="maxima")

[Out] integrate((B*sec(dx+c) + A)*sec(dx+c)^3/sqrt(a*sec(dx+c) + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\cos(c+dx)^3 \sqrt{a + \frac{a}{\cos(c+dx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + dx))/(cos(c + dx)^3*(a + a/cos(c + dx))^(1/2)),x)

[Out] int((A + B/cos(c + dx))/(cos(c + dx)^3*(a + a/cos(c + dx))^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \sec^3(c + dx)}{\sqrt{a (\sec(c + dx) + 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(dx+c)**3*(A+B*sec(dx+c))/(a+a*sec(dx+c))**(1/2),x)

[Out] Integral((A + B*sec(c + dx))*sec(c + dx)**3/sqrt(a*(sec(c + dx) + 1)), x)

$$3.146 \quad \int \frac{\sec^2(c+dx)(A+B \sec(c+dx))}{\sqrt{a+a \sec(c+dx)}} dx$$

Optimal. Leaf size=118

$$\frac{\sqrt{2}(A-B) \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{2} \sqrt{a \sec(c+dx)+a}}\right)}{\sqrt{a} d} + \frac{2(3A-2B) \tan(c+dx)}{3d \sqrt{a \sec(c+dx)+a}} + \frac{2B \tan(c+dx) \sqrt{a \sec(c+dx)+a}}{3ad}$$

[Out] $-(A-B) \cdot \arctan\left(\frac{1/2 \cdot a^{1/2} \cdot \tan(d \cdot x + c) \cdot 2^{1/2}}{(a + a \cdot \sec(d \cdot x + c))^{1/2}}\right) \cdot 2^{1/2} / d \cdot a^{1/2} + 2/3 \cdot (3A - 2B) \cdot \tan(d \cdot x + c) / d / (a + a \cdot \sec(d \cdot x + c))^{1/2} + 2/3 \cdot B \cdot (a + a \cdot \sec(d \cdot x + c))^{1/2} \cdot \tan(d \cdot x + c) / a / d$

Rubi [A] time = 0.26, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {4010, 4001, 3795, 203}

$$\frac{\sqrt{2}(A-B) \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{2} \sqrt{a \sec(c+dx)+a}}\right)}{\sqrt{a} d} + \frac{2(3A-2B) \tan(c+dx)}{3d \sqrt{a \sec(c+dx)+a}} + \frac{2B \tan(c+dx) \sqrt{a \sec(c+dx)+a}}{3ad}$$

Antiderivative was successfully verified.

[In] `Int[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/Sqrt[a + a*Sec[c + d*x]],x]`

[Out] $-\left(\left(\text{Sqrt}[2] \cdot (A - B) \cdot \text{ArcTan}\left[\frac{\text{Sqrt}[a] \cdot \text{Tan}[c + d \cdot x]}{\text{Sqrt}[2] \cdot \text{Sqrt}[a + a \cdot \text{Sec}[c + d \cdot x]]}\right]\right) / (\text{Sqrt}[a] \cdot d)\right) + \frac{2 \cdot (3A - 2B) \cdot \text{Tan}[c + d \cdot x]}{3 \cdot d \cdot \text{Sqrt}[a + a \cdot \text{Sec}[c + d \cdot x]]} + \frac{2 \cdot B \cdot \text{Sqrt}[a + a \cdot \text{Sec}[c + d \cdot x]] \cdot \text{Tan}[c + d \cdot x]}{3 \cdot a \cdot d}$

Rule 203

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])`

Rule 3795

`Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] := Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]`

Rule 4001

`Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] := -Simp[(B*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1))`

)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m + 1), 0] && !LtQ[m, -2^(-1)]

Rule 4010

Int[csc[(e_.) + (f_.)*(x_.)]^2*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && !LtQ[m, -1]

Rubi steps

$$\begin{aligned} \int \frac{\sec^2(c + dx)(A + B \sec(c + dx))}{\sqrt{a + a \sec(c + dx)}} dx &= \frac{2B\sqrt{a + a \sec(c + dx)} \tan(c + dx)}{3ad} + \frac{2 \int \frac{\sec(c+dx)\left(\frac{aB}{2} + \frac{1}{2}a(3A-2B)\sec(c+dx)\right)}{\sqrt{a+a \sec(c+dx)}}}{3a} \\ &= \frac{2(3A - 2B) \tan(c + dx)}{3d\sqrt{a + a \sec(c + dx)}} + \frac{2B\sqrt{a + a \sec(c + dx)} \tan(c + dx)}{3ad} + (-A + \\ &= \frac{2(3A - 2B) \tan(c + dx)}{3d\sqrt{a + a \sec(c + dx)}} + \frac{2B\sqrt{a + a \sec(c + dx)} \tan(c + dx)}{3ad} + \frac{(2(A - B) \tan(c + dx) - A)}{d} \\ &= -\frac{\sqrt{2}(A - B) \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{2}\sqrt{a+a \sec(c+dx)}}\right)}{\sqrt{a}d} + \frac{2(3A - 2B) \tan(c + dx)}{3d\sqrt{a + a \sec(c + dx)}} + \frac{2B}{d} \end{aligned}$$

Mathematica [A] time = 0.30, size = 106, normalized size = 0.90

$$\frac{\tan(c + dx) \left(2\sqrt{1 - \sec(c + dx)} (3A + B \sec(c + dx) - B) - 3\sqrt{2} (A - B) \tanh^{-1} \left(\frac{\sqrt{1 - \sec(c + dx)}}{\sqrt{2}} \right) \right)}{3d\sqrt{1 - \sec(c + dx)} \sqrt{a(\sec(c + dx) + 1)}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/Sqrt[a + a*Sec[c + d*x]], x]

[Out] ((-3*Sqrt[2]*(A - B)*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]] + 2*Sqrt[1 - Sec[c + d*x]]*(3*A - B + B*Sec[c + d*x]))*Tan[c + d*x]/(3*d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])))

fricas [A] time = 0.51, size = 352, normalized size = 2.98

$$\frac{3\sqrt{2}\left((A-B)a\cos(dx+c)^2+(A-B)a\cos(dx+c)\right)\sqrt{-\frac{1}{a}}\log\left(\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\cos(dx+c)\sin(dx+c)-3\cos(dx+c)}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)}{6\left(ad\cos(dx+c)^2+ad\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")

[Out] [-1/6*(3*sqrt(2)*((A - B)*a*cos(d*x + c)^2 + (A - B)*a*cos(d*x + c))*sqrt(-1/a)*log(-(2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*cos(d*x + c)*sin(d*x + c) - 3*cos(d*x + c)^2 - 2*cos(d*x + c) + 1)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*((3*A - B)*cos(d*x + c) + B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/(a*d*cos(d*x + c)^2 + a*d*cos(d*x + c)), 1/3*(2*((3*A - B)*cos(d*x + c) + B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c) + 3*sqrt(2)*((A - B)*a*cos(d*x + c)^2 + (A - B)*a*cos(d*x + c))*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))/sqrt(a))/(a*d*cos(d*x + c)^2 + a*d*cos(d*x + c))]

giac [A] time = 2.14, size = 186, normalized size = 1.58

$$\frac{3\sqrt{2}(A-B)\log\left(\left|-\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right|\right)}{\sqrt{-a}\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}+\frac{2\left(\frac{\sqrt{2}(3Aa-2Ba)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}-\frac{3\sqrt{2}Aa}{\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}$$

$3d$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")

[Out] -1/3*(3*sqrt(2)*(A - B)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) + 2*(sqrt(2)*(3*A*a - 2*B*a)*tan(1/2*d*x + 1/2*c)^2/sgn(tan(1/2*d*x + 1/2*c)^2 - 1) - 3*sqrt(2)*A*a/sgn(tan(1/2*d*x + 1/2*c)^2 - 1))*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))/d

maple [B] time = 1.66, size = 405, normalized size = 3.43

$$\left(-3A \cos(dx+c) \sin(dx+c) \ln \left(\frac{-\sqrt{\frac{2\cos(dx+c)}{1+\cos(dx+c)}} \sin(dx+c) + \cos(dx+c) - 1}{\sin(dx+c)} \right) \right) \left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)} \right)^{\frac{3}{2}} + 3B \cos(dx+c) \sin(dx+c)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x)`

[Out]
$$-1/6/d*(-3*A*\cos(d*x+c)*\sin(d*x+c)*\ln(-(-(-2*\cos(d*x+c)/(1+\cos(d*x+c))))^(1/2)*\sin(d*x+c)+\cos(d*x+c)-1)/\sin(d*x+c))*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^(3/2)+3*B*\cos(d*x+c)*\sin(d*x+c)*\ln(-(-(-2*\cos(d*x+c)/(1+\cos(d*x+c))))^(1/2)*\sin(d*x+c)+\cos(d*x+c)-1)/\sin(d*x+c))*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^(3/2)-3*A*\ln(-(-(-2*\cos(d*x+c)/(1+\cos(d*x+c))))^(1/2)*\sin(d*x+c)+\cos(d*x+c)-1)/\sin(d*x+c))*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^(3/2)*\sin(d*x+c)+3*B*\ln(-(-(-2*\cos(d*x+c)/(1+\cos(d*x+c))))^(1/2)*\sin(d*x+c)+\cos(d*x+c)-1)/\sin(d*x+c))*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^(3/2)*\sin(d*x+c)+12*A*\cos(d*x+c)^2-4*B*\cos(d*x+c)^2-12*A*\cos(d*x+c)+8*B*\cos(d*x+c)-4*B)*(a*(1+\cos(d*x+c))/\cos(d*x+c))^(1/2)/\sin(d*x+c)/\cos(d*x+c)/a$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx+c) + A) \sec(dx+c)^2}{\sqrt{a \sec(dx+c) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")`

[Out] `integrate((B*sec(d*x + c) + A)*sec(d*x + c)^2/sqrt(a*sec(d*x + c) + a), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\cos(c+dx)^2 \sqrt{a + \frac{a}{\cos(c+dx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B/cos(c + d*x))/(cos(c + d*x)^2*(a + a/cos(c + d*x))^(1/2)),x)`

[Out] `int((A + B/cos(c + d*x))/(cos(c + d*x)^2*(a + a/cos(c + d*x))^(1/2)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \sec^2(c + dx)}{\sqrt{a(\sec(c + dx) + 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(1/2),x)

[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**2/sqrt(a*(sec(c + d*x) + 1)), x
)

$$3.147 \quad \int \frac{\sec(c+dx)(A+B \sec(c+dx))}{\sqrt{a+a \sec(c+dx)}} dx$$

Optimal. Leaf size=78

$$\frac{\sqrt{2}(A-B) \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{2} \sqrt{a \sec(c+dx)+a}}\right)}{\sqrt{a} d} + \frac{2B \tan(c+dx)}{d \sqrt{a \sec(c+dx)+a}}$$

[Out] (A-B)*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2)/d/a^(1/2)+2*B*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)

Rubi [A] time = 0.11, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {4001, 3795, 203}

$$\frac{\sqrt{2}(A-B) \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{2} \sqrt{a \sec(c+dx)+a}}\right)}{\sqrt{a} d} + \frac{2B \tan(c+dx)}{d \sqrt{a \sec(c+dx)+a}}$$

Antiderivative was successfully verified.

[In] Int[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/Sqrt[a + a*Sec[c + d*x]], x]

[Out] (Sqrt[2]*(A - B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(Sqrt[a]*d) + (2*B*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 3795

Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] := Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 4001

Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] := -Simp[(B*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m

+ 1), 0] && !LtQ[m, -2^(-1)]

Rubi steps

$$\begin{aligned} \int \frac{\sec(c+dx)(A+B\sec(c+dx))}{\sqrt{a+a\sec(c+dx)}} dx &= \frac{2B \tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + (A-B) \int \frac{\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}} dx \\ &= \frac{2B \tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} - \frac{(2(A-B)) \text{Subst}\left(\int \frac{1}{2a+x^2} dx, x, -\frac{a \tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} \\ &= \frac{\sqrt{2}(A-B) \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d} + \frac{2B \tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} \end{aligned}$$

Mathematica [A] time = 0.17, size = 88, normalized size = 1.13

$$\frac{\tan(c+dx) \left(\sqrt{2}(A-B) \tanh^{-1}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right) + 2B\sqrt{1-\sec(c+dx)} \right)}{d\sqrt{1-\sec(c+dx)} \sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/Sqrt[a + a*Sec[c + d*x]], x]

[Out] ((Sqrt[2]*(A - B)*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]] + 2*B*Sqrt[1 - Sec[c + d*x]])*Tan[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])

fricas [A] time = 0.48, size = 287, normalized size = 3.68

$$\left[\frac{\sqrt{2}((A-B)a \cos(dx+c) + (A-B)a) \sqrt{-\frac{1}{a}} \log\left(\frac{2\sqrt{2} \sqrt{\frac{a \cos(dx+c)+a}{\cos(dx+c)}} \sqrt{-\frac{1}{a}} \cos(dx+c) \sin(dx+c) + 3 \cos(dx+c)^2 + 2 \cos(dx+c) - 1}{\cos(dx+c)^2 + 2 \cos(dx+c) + 1}\right)}{2(ad \cos(dx+c) + ad)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2), x, algorithm="fricas")

[Out] $[-1/2*(\sqrt{2}*((A - B)*a*\cos(dx + c) + (A - B)*a)*\sqrt{-1/a}*\log((2*\sqrt{2})*\sqrt{(a*\cos(dx + c) + a)/\cos(dx + c)})*\sqrt{-1/a}*\cos(dx + c)*\sin(dx + c) + 3*\cos(dx + c)^2 + 2*\cos(dx + c) - 1)/(\cos(dx + c)^2 + 2*\cos(dx + c) + 1)) - 4*B*\sqrt{(a*\cos(dx + c) + a)/\cos(dx + c)}*\sin(dx + c))/(a*d*\cos(dx + c) + a*d), (2*B*\sqrt{(a*\cos(dx + c) + a)/\cos(dx + c)}*\sin(dx + c) - \sqrt{2}*((A - B)*a*\cos(dx + c) + (A - B)*a)*\arctan(\sqrt{2}*\sqrt{(a*\cos(dx + c) + a)/\cos(dx + c)}*\cos(dx + c)/(\sqrt{a}*\sin(dx + c))))/\sqrt{a}]/(a*d*\cos(dx + c) + a*d)]$

giac [B] time = 1.98, size = 144, normalized size = 1.85

$$\frac{2\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)} + \frac{(\sqrt{2}A-\sqrt{2}B)\log\left(\left|-\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right|\right)}{\sqrt{-a}\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}$$

d

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(dx+c)*(A+B*sec(dx+c))/(a+a*sec(dx+c))^(1/2),x, algorithm="giac")`

[Out] $(2*\sqrt{2}*\sqrt{-a*\tan(1/2*d*x + 1/2*c)^2 + a}*B*\tan(1/2*d*x + 1/2*c)/((a*\tan(1/2*d*x + 1/2*c)^2 - a)*\operatorname{sgn}(\tan(1/2*d*x + 1/2*c)^2 - 1)) + (\sqrt{2}*A - \sqrt{2}*B)*\log(\operatorname{abs}(-\sqrt{-a}*\tan(1/2*d*x + 1/2*c) + \sqrt{-a*\tan(1/2*d*x + 1/2*c)^2 + a}))/(\sqrt{-a}*\operatorname{sgn}(\tan(1/2*d*x + 1/2*c)^2 - 1)))/d$

maple [B] time = 1.59, size = 200, normalized size = 2.56

$$\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}} \left(A \ln \left(\frac{-\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}} \sin(dx+c) + \cos(dx+c) - 1}{\sin(dx+c)} \right) \sqrt{\frac{2\cos(dx+c)}{1+\cos(dx+c)}} \sin(dx+c) - B \ln \left(\frac{-\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}} \sin(dx+c)}{\sin(dx+c)} \right) \right)}{d \sin(dx+c) a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sec(dx+c)*(A+B*sec(dx+c))/(a+a*sec(dx+c))^(1/2),x)`

[Out] $1/d*(a*(1+\cos(dx+c))/\cos(dx+c))^(1/2)*(A*\ln(-(-(-2*\cos(dx+c)/(1+\cos(dx+c))))^(1/2)*\sin(dx+c)+\cos(dx+c)-1)/\sin(dx+c))*(-2*\cos(dx+c)/(1+\cos(dx+c)))^(1/2)*\sin(dx+c)-B*\ln(-(-(-2*\cos(dx+c)/(1+\cos(dx+c))))^(1/2)*\sin(dx+c)+\cos(dx+c)-1)/\sin(dx+c))*(-2*\cos(dx+c)/(1+\cos(dx+c)))^(1/2)*\sin(dx+c)-2*B*\cos(dx+c)+2*B)/\sin(dx+c)/a$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx+c) + A) \sec(dx+c)}{\sqrt{a \sec(dx+c) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)/sqrt(a*sec(d*x + c) + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\cos(c+dx) \sqrt{a + \frac{a}{\cos(c+dx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/(cos(c + d*x)*(a + a/cos(c + d*x))^(1/2)),x)

[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)*(a + a/cos(c + d*x))^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \sec(c + dx)}{\sqrt{a(\sec(c + dx) + 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(1/2),x)

[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)/sqrt(a*(sec(c + d*x) + 1)), x)

$$3.148 \quad \int \frac{A+B \sec(c+dx)}{\sqrt{a+a \sec(c+dx)}} dx$$

Optimal. Leaf size=91

$$\frac{2A \tan^{-1} \left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}} \right)}{\sqrt{a} d} - \frac{\sqrt{2} (A-B) \tan^{-1} \left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{2} \sqrt{a \sec(c+dx)+a}} \right)}{\sqrt{a} d}$$

[Out] $2*A*\arctan(a^{(1/2)}*\tan(d*x+c)/(a+a*\sec(d*x+c))^{(1/2)})/d/a^{(1/2)}-(A-B)*\arctan(1/2*a^{(1/2)}*\tan(d*x+c)*2^{(1/2)}/(a+a*\sec(d*x+c))^{(1/2)})*2^{(1/2)}/d/a^{(1/2)}$

Rubi [A] time = 0.11, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3920, 3774, 203, 3795}

$$\frac{2A \tan^{-1} \left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}} \right)}{\sqrt{a} d} - \frac{\sqrt{2} (A-B) \tan^{-1} \left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{2} \sqrt{a \sec(c+dx)+a}} \right)}{\sqrt{a} d}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sec[c + d*x])/Sqrt[a + a*Sec[c + d*x]], x]

[Out] $(2*A*\text{ArcTan}[(\text{Sqrt}[a]*\text{Tan}[c + d*x])/(\text{Sqrt}[a + a*\text{Sec}[c + d*x]])]/(\text{Sqrt}[a]*d) - (\text{Sqrt}[2]*(A - B)*\text{ArcTan}[(\text{Sqrt}[a]*\text{Tan}[c + d*x])/(\text{Sqrt}[2]*\text{Sqrt}[a + a*\text{Sec}[c + d*x]])])/(\text{Sqrt}[a]*d)$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 3774

Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] := Dist[(-2*b)/d, Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 3795

Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] := Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 3920

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned} \int \frac{A + B \sec(c + dx)}{\sqrt{a + a \sec(c + dx)}} dx &= \frac{A \int \sqrt{a + a \sec(c + dx)} dx}{a} - (A - B) \int \frac{\sec(c + dx)}{\sqrt{a + a \sec(c + dx)}} dx \\ &= \frac{(2A) \text{Subst} \left(\int \frac{1}{a+x^2} dx, x, -\frac{a \tan(c+dx)}{\sqrt{a+a \sec(c+dx)}} \right)}{d} + \frac{(2(A-B)) \text{Subst} \left(\int \frac{1}{2a+x^2} dx, x, -\frac{a \tan(c+dx)}{\sqrt{a+a \sec(c+dx)}} \right)}{d} \\ &= \frac{2A \tan^{-1} \left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a+a \sec(c+dx)}} \right)}{\sqrt{a} d} - \frac{\sqrt{2} (A - B) \tan^{-1} \left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{2} \sqrt{a+a \sec(c+dx)}} \right)}{\sqrt{a} d} \end{aligned}$$

Mathematica [A] time = 0.29, size = 92, normalized size = 1.01

$$\frac{2 \cos \left(\frac{1}{2}(c + dx) \right) \left((B - A) \tan^{-1} \left(\frac{\sin \left(\frac{1}{2}(c + dx) \right)}{\sqrt{\cos(c + dx)}} \right) + \sqrt{2} A \sin^{-1} \left(\sqrt{2} \sin \left(\frac{1}{2}(c + dx) \right) \right) \right)}{d \sqrt{\cos(c + dx)} \sqrt{a(\sec(c + dx) + 1)}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sec[c + d*x])/Sqrt[a + a*Sec[c + d*x]],x]

[Out] (2*(Sqrt[2]*A*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + (-A + B)*ArcTan[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]])*Cos[(c + d*x)/2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])

fricas [A] time = 1.21, size = 307, normalized size = 3.37

$$\left[\frac{\sqrt{2} (A - B) a \sqrt{-\frac{1}{a}} \log \left(\frac{2 \sqrt{2} \sqrt{\frac{a \cos(dx+c)+a}{\cos(dx+c)}} \sqrt{-\frac{1}{a}} \cos(dx+c) \sin(dx+c) - 3 \cos(dx+c)^2 - 2 \cos(dx+c) + 1}{\cos(dx+c)^2 + 2 \cos(dx+c) + 1} \right)}{2 ad} + 2 A \sqrt{-a} \log \left(\frac{2 a \cos(dx+c)}{\dots} \right) \right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
[Out] [-1/2*(sqrt(2)*(A - B)*a*sqrt(-1/a)*log(-(2*sqrt(2)*sqrt((a*cos(d*x + c) +
a)/cos(d*x + c))*sqrt(-1/a)*cos(d*x + c)*sin(d*x + c) - 3*cos(d*x + c)^2 -
2*cos(d*x + c) + 1)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 2*A*sqrt(-a)*l
og((2*a*cos(d*x + c)^2 + 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))
*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)))/(a*d)
, (sqrt(2)*(A - B)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x
+ c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - 2*A*sqrt(a)*arctan(sqrt((a*co
s(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))))/(a*d)]
giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00
```

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Warning, integration of abs or sign assum
es constant sign by intervals (correct if the argument is real):Check [abs(
cos(d*t_nostep+c))]Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/
2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_no
step/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_
nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable t
o check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*
pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, assuming -2*a+a is positive. Hint
: run assume to make assumptions on a variableWarning, assuming -2*a+a is p
ositive. Hint: run assume to make assumptions on a variableWarning, assumin
g -2*a+a is positive. Hint: run assume to make assumptions on a variableWar
ning, assuming -2*a+a is positive. Hint: run assume to make assumptions on
a variableDiscontinuities at zeroes of cos(d*t_nostep+c) were not checkedWa
rning, integration of abs or sign assumes constant sign by intervals (corre
ct if the argument is real):Check [abs(t_nostep^2-1)]Evaluation time: 1.14i
ndex.cc index_m i_lex_is_greater Error: Bad Argument Value
```

maple [B] time = 1.59, size = 194, normalized size = 2.13

$$\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}} \sqrt{\frac{2\cos(dx+c)}{1+\cos(dx+c)}} \left(A\sqrt{2} \operatorname{arctanh} \left(\frac{\sqrt{\frac{2\cos(dx+c)}{1+\cos(dx+c)}} \sin(dx+c)\sqrt{2}}{2\cos(dx+c)} \right) + A \ln \left(-\frac{\sqrt{\frac{2\cos(dx+c)}{1+\cos(dx+c)}} \sin(dx+c)+\cos(dx+c)}{\sin(dx+c)} \right) \right)}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x)`

[Out]
$$-1/d*(a*(1+\cos(d*x+c))/\cos(d*x+c))^{(1/2)}*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*(A*2^{(1/2)}*\operatorname{arctanh}(1/2*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*\sin(d*x+c)/\cos(d*x+c)*2^{(1/2)})+A*\ln(-(-(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*\sin(d*x+c)+\cos(d*x+c)-1)/\sin(d*x+c))-B*\ln(-(-(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*\sin(d*x+c)+\cos(d*x+c)-1)/\sin(d*x+c)))/a$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: sign: argument cannot be imaginary; found %i

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\sqrt{a + \frac{a}{\cos(c+dx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B/cos(c + d*x))/(a + a/cos(c + d*x))^(1/2),x)`

[Out] `int((A + B/cos(c + d*x))/(a + a/cos(c + d*x))^(1/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \sec(c + dx)}{\sqrt{a(\sec(c + dx) + 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(1/2),x)`

[Out] `Integral((A + B*sec(c + d*x))/sqrt(a*(sec(c + d*x) + 1)), x)`

$$3.149 \quad \int \frac{\cos(c+dx)(A+B \sec(c+dx))}{\sqrt{a+a \sec(c+dx)}} dx$$

Optimal. Leaf size=119

$$-\frac{(A-2B) \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}}\right)}{\sqrt{a} d} + \frac{\sqrt{2}(A-B) \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{2} \sqrt{a \sec(c+dx)+a}}\right)}{\sqrt{a} d} + \frac{A \sin(c+dx)}{d \sqrt{a \sec(c+dx)+a}}$$

[Out] $-(A-2*B)*\arctan(a^{(1/2)}*\tan(d*x+c)/(a+a*\sec(d*x+c))^{(1/2)})/d/a^{(1/2)}+(A-B)*\arctan(1/2*a^{(1/2)}*\tan(d*x+c)*2^{(1/2)}/(a+a*\sec(d*x+c))^{(1/2)})*2^{(1/2)}/d/a^{(1/2)}+A*\sin(d*x+c)/d/(a+a*\sec(d*x+c))^{(1/2)}$

Rubi [A] time = 0.23, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {4022, 3920, 3774, 203, 3795}

$$-\frac{(A-2B) \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}}\right)}{\sqrt{a} d} + \frac{\sqrt{2}(A-B) \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{2} \sqrt{a \sec(c+dx)+a}}\right)}{\sqrt{a} d} + \frac{A \sin(c+dx)}{d \sqrt{a \sec(c+dx)+a}}$$

Antiderivative was successfully verified.

[In] `Int[(Cos[c + d*x]*(A + B*Sec[c + d*x]))/Sqrt[a + a*Sec[c + d*x]],x]`

[Out] `-(((A - 2*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]])]/(Sqrt[a]*d) + (Sqrt[2]*(A - B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]/(Sqrt[a]*d) + (A*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]))`

Rule 203

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])`

Rule 3774

`Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] := Dist[(-2*b)/d, Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]`

Rule 3795

`Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] := Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[`

$a + b \cdot \text{Csc}[e + f \cdot x]]], x] /; \text{FreeQ}[\{a, b, e, f\}, x] \ \&\& \ \text{EqQ}[a^2 - b^2, 0]$

Rule 3920

$\text{Int}[(\text{csc}[(e_.) + (f_.) \cdot (x_)] \cdot (d_.) + (c_.) / \text{Sqrt}[\text{csc}[(e_.) + (f_.) \cdot (x_)] \cdot (b_.) + (a_.)], x_Symbol] :> \text{Dist}[c/a, \text{Int}[\text{Sqrt}[a + b \cdot \text{Csc}[e + f \cdot x]], x], x] - \text{Dist}[(b \cdot c - a \cdot d)/a, \text{Int}[\text{Csc}[e + f \cdot x] / \text{Sqrt}[a + b \cdot \text{Csc}[e + f \cdot x]], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \ \&\& \ \text{NeQ}[b \cdot c - a \cdot d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0]$

Rule 4022

$\text{Int}[(\text{csc}[(e_.) + (f_.) \cdot (x_)] \cdot (d_.))^{(n_)} \cdot (\text{csc}[(e_.) + (f_.) \cdot (x_)] \cdot (b_.) + (a_.))^{(m_)} \cdot (\text{csc}[(e_.) + (f_.) \cdot (x_)] \cdot (B_.) + (A_.)], x_Symbol] :> \text{Simp}[(A \cdot \text{Cot}[e + f \cdot x] \cdot (a + b \cdot \text{Csc}[e + f \cdot x])^m \cdot (d \cdot \text{Csc}[e + f \cdot x])^n) / (f \cdot n), x] - \text{Dist}[1 / (b \cdot d \cdot n), \text{Int}[(a + b \cdot \text{Csc}[e + f \cdot x])^m \cdot (d \cdot \text{Csc}[e + f \cdot x])^{(n+1)} \cdot \text{Simp}[a \cdot A \cdot m - b \cdot B \cdot n - A \cdot b \cdot (m + n + 1) \cdot \text{Csc}[e + f \cdot x], x], x], x] /; \text{FreeQ}[\{a, b, d, e, f, A, B, m\}, x] \ \&\& \ \text{NeQ}[A \cdot b - a \cdot B, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{LtQ}[n, 0]$

Rubi steps

$$\begin{aligned} \int \frac{\cos(c + dx)(A + B \sec(c + dx))}{\sqrt{a + a \sec(c + dx)}} dx &= \frac{A \sin(c + dx)}{d \sqrt{a + a \sec(c + dx)}} + \frac{\int \frac{-\frac{1}{2}a(A-2B) + \frac{1}{2}aA \sec(c+dx)}{\sqrt{a+a \sec(c+dx)}} dx}{a} \\ &= \frac{A \sin(c + dx)}{d \sqrt{a + a \sec(c + dx)}} - \frac{(A - 2B) \int \sqrt{a + a \sec(c + dx)} dx}{2a} + (A - B) \int \frac{1}{\sqrt{a + a \sec(c + dx)}} dx \\ &= \frac{A \sin(c + dx)}{d \sqrt{a + a \sec(c + dx)}} + \frac{(A - 2B) \text{Subst}\left(\int \frac{1}{a+x^2} dx, x, -\frac{a \tan(c+dx)}{\sqrt{a+a \sec(c+dx)}}\right)}{d} - \frac{(A - B) \int \frac{1}{\sqrt{a + a \sec(c + dx)}} dx}{d} \\ &= -\frac{(A - 2B) \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a+a \sec(c+dx)}}\right)}{\sqrt{a} d} + \frac{\sqrt{2} (A - B) \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{2} \sqrt{a+a \sec(c+dx)}}\right)}{\sqrt{a} d} \end{aligned}$$

Mathematica [C] time = 26.70, size = 11162, normalized size = 93.80

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(Cos[c + d*x]*(A + B*Sec[c + d*x]))/Sqrt[a + a*Sec[c + d*x]],x]

[Out] Result too large to show

fricas [A] time = 1.48, size = 458, normalized size = 3.85

$$2 A \sqrt{\frac{a \cos(dx+c)+a}{\cos(dx+c)}} \cos(dx+c) \sin(dx+c) - \sqrt{2} ((A-B)a \cos(dx+c) + (A-B)a) \sqrt{-\frac{1}{a}} \log \left(\frac{2 \sqrt{2} \sqrt{\frac{a \cos(dx+c)+a}{\cos(dx+c)}}}{\dots} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")

[Out] [1/2*(2*A*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) - sqrt(2)*((A - B)*a*cos(d*x + c) + (A - B)*a)*sqrt(-1/a)*log((2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*cos(d*x + c)*sin(d*x + c) + 3*cos(d*x + c)^2 + 2*cos(d*x + c) - 1)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + ((A - 2*B)*cos(d*x + c) + A - 2*B)*sqrt(-a)*log((2*a*cos(d*x + c)^2 + 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)))/(a*d*cos(d*x + c) + a*d), (A*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + ((A - 2*B)*cos(d*x + c) + A - 2*B)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - sqrt(2)*((A - B)*a*cos(d*x + c) + (A - B)*a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))/sqrt(a))/(a*d*cos(d*x + c) + a*d)]

giac [B] time = 6.63, size = 393, normalized size = 3.30

$$\frac{\sqrt{2}(A-B) \log \left(\left(\sqrt{-a} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - \sqrt{-a \tan^2\left(\frac{1}{2} dx + \frac{1}{2} c\right) + a} \right)^2 \right)}{\sqrt{-a} \operatorname{sgn}\left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 1\right)} + \frac{(A-2B) \log \left(\left(\sqrt{-a} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - \sqrt{-a \tan^2\left(\frac{1}{2} dx + \frac{1}{2} c\right) + a} \right)^2 - a(2\sqrt{2}+3) \right)}{\sqrt{-a} \operatorname{sgn}\left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")

[Out] 1/2*(sqrt(2)*(A - B)*log((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2)/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) + (A - 2*

B)*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3)))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) - (A - 2*B)*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3)))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) + 4*sqrt(2)*(3*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*sqrt(-a) - A*sqrt(-a)*a)/(((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1))/d

maple [B] time = 1.69, size = 353, normalized size = 2.97

$$\left(A\sqrt{2} \operatorname{arctanh} \left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}} \sin(dx+c)\sqrt{2}}{2\cos(dx+c)} \right) \sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}} \sin(dx+c) - 2B\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}} \operatorname{arctanh} \left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}} \sin(dx+c)}{2\cos(dx+c)} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x)

[Out] 1/2/d*(A*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c))))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-2*B*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c))))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*2^(1/2)*sin(d*x+c)+2*A*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-2*B*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-2*A*cos(d*x+c)^2+2*A*cos(d*x+c))*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)/a

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx+c) + A) \cos(dx+c)}{\sqrt{a \sec(dx+c) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)/sqrt(a*sec(d*x + c) + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+dx) \left(A + \frac{B}{\cos(c+dx)} \right)}{\sqrt{a + \frac{a}{\cos(c+dx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((cos(c + d*x)*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^(1/2),x)`

[Out] `int((cos(c + d*x)*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^(1/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \cos(c + dx)}{\sqrt{a(\sec(c + dx) + 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(1/2),x)`

[Out] `Integral((A + B*sec(c + d*x))*cos(c + d*x)/sqrt(a*(sec(c + d*x) + 1)), x)`

$$3.150 \quad \int \frac{\cos^2(c+dx)(A+B \sec(c+dx))}{\sqrt{a+a \sec(c+dx)}} dx$$

Optimal. Leaf size=165

$$-\frac{(A-4B) \sin(c+dx)}{4d\sqrt{a \sec(c+dx)+a}} + \frac{(7A-4B) \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}}\right)}{4\sqrt{a}d} - \frac{\sqrt{2}(A-B) \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{2}\sqrt{a \sec(c+dx)+a}}\right)}{\sqrt{a}d} + \frac{A \sin(c+dx) \cos(c+dx)}{2d\sqrt{a \sec(c+dx)+a}}$$

[Out] 1/4*(7*A-4*B)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d/a^(1/2)-(A-B)*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2)/d/a^(1/2)-1/4*(A-4*B)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/2*A*cos(d*x+c)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)

Rubi [A] time = 0.37, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {4022, 3920, 3774, 203, 3795}

$$-\frac{(A-4B) \sin(c+dx)}{4d\sqrt{a \sec(c+dx)+a}} + \frac{(7A-4B) \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}}\right)}{4\sqrt{a}d} - \frac{\sqrt{2}(A-B) \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{2}\sqrt{a \sec(c+dx)+a}}\right)}{\sqrt{a}d} + \frac{A \sin(c+dx) \cos(c+dx)}{2d\sqrt{a \sec(c+dx)+a}}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^2*(A + B*Sec[c + d*x]))/Sqrt[a + a*Sec[c + d*x]],x]

[Out] ((7*A - 4*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]])/(4*Sqrt[a]*d) - (Sqrt[2]*(A - B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(Sqrt[a]*d) - ((A - 4*B)*Sin[c + d*x])/(4*d*Sqrt[a + a*Sec[c + d*x]]) + (A*Cos[c + d*x]*Sin[c + d*x])/(2*d*Sqrt[a + a*Sec[c + d*x]])

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 3774

Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_.)], x_Symbol] := Dist[(-2*b)/d, Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 3795

Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] := Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 3920

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] := Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]

Rule 4022

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^m*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] := Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{\cos^2(c + dx)(A + B \sec(c + dx))}{\sqrt{a + a \sec(c + dx)}} dx &= \frac{A \cos(c + dx) \sin(c + dx)}{2d\sqrt{a + a \sec(c + dx)}} + \frac{\int \frac{\cos(c + dx) \left(-\frac{1}{2}a(A - 4B) + \frac{3}{2}aA \sec(c + dx) \right)}{\sqrt{a + a \sec(c + dx)}} dx}{2a} \\
 &= -\frac{(A - 4B) \sin(c + dx)}{4d\sqrt{a + a \sec(c + dx)}} + \frac{A \cos(c + dx) \sin(c + dx)}{2d\sqrt{a + a \sec(c + dx)}} + \frac{\int \frac{\frac{1}{4}a^2(7A - 4B) - \frac{1}{4}}{\sqrt{a + a \sec(c + dx)}} dx}{2a} \\
 &= -\frac{(A - 4B) \sin(c + dx)}{4d\sqrt{a + a \sec(c + dx)}} + \frac{A \cos(c + dx) \sin(c + dx)}{2d\sqrt{a + a \sec(c + dx)}} + \frac{(7A - 4B) \int \sqrt{a + a \sec(c + dx)} dx}{2a} \\
 &= -\frac{(A - 4B) \sin(c + dx)}{4d\sqrt{a + a \sec(c + dx)}} + \frac{A \cos(c + dx) \sin(c + dx)}{2d\sqrt{a + a \sec(c + dx)}} - \frac{(7A - 4B) \operatorname{Subst}\left[\int \sqrt{a + a \sec(c + dx)} dx, x, \frac{\sqrt{a} \tan(c + dx)}{\sqrt{a + a \sec(c + dx)}}\right]}{2a} \\
 &= \frac{(7A - 4B) \tan^{-1}\left(\frac{\sqrt{a} \tan(c + dx)}{\sqrt{a + a \sec(c + dx)}}\right)}{4\sqrt{a}d} - \frac{\sqrt{2}(A - B) \tan^{-1}\left(\frac{\sqrt{a} \tan(c + dx)}{\sqrt{2}\sqrt{a + a \sec(c + dx)}}\right)}{\sqrt{a}d}
 \end{aligned}$$

Mathematica [A] time = 0.45, size = 135, normalized size = 0.82

$$\frac{\tan(c + dx) \left(\cos(c + dx) \sqrt{1 - \sec(c + dx)} (2A \cos(c + dx) - A + 4B) + (7A - 4B) \tanh^{-1} \left(\sqrt{1 - \sec(c + dx)} \right) - 4 \right)}{4d \sqrt{1 - \sec(c + dx)} \sqrt{a(\sec(c + dx) + 1)}}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]^2*(A + B*Sec[c + d*x]))/Sqrt[a + a*Sec[c + d*x]],x]

[Out] (((7*A - 4*B)*ArcTanh[Sqrt[1 - Sec[c + d*x]]] - 4*Sqrt[2]*(A - B)*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]] + Cos[c + d*x]*(-A + 4*B + 2*A*Cos[c + d*x])*Sqrt[1 - Sec[c + d*x]])*Tan[c + d*x])/(4*d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x]))]

fricas [A] time = 2.45, size = 506, normalized size = 3.07

$$\frac{4\sqrt{2}((A - B)a \cos(dx + c) + (A - B)a) \sqrt{-\frac{1}{a}} \log \left(-\frac{2\sqrt{2} \sqrt{\frac{a \cos(dx+c)+a}{\cos(dx+c)}} \sqrt{-\frac{1}{a}} \cos(dx+c) \sin(dx+c) - 3 \cos(dx+c)^2 - 2 \cos(dx+c) + 1}{\cos(dx+c)^2 + 2 \cos(dx+c) + 1} \right)}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")

[Out] [-1/8*(4*sqrt(2))*((A - B)*a*cos(d*x + c) + (A - B)*a)*sqrt(-1/a)*log(-(2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*cos(d*x + c)*sin(d*x + c) - 3*cos(d*x + c)^2 - 2*cos(d*x + c) + 1)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - ((7*A - 4*B)*cos(d*x + c) + 7*A - 4*B)*sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) - 2*(2*A*cos(d*x + c)^2 - (A - 4*B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/(a*d*cos(d*x + c) + a*d), -1/4*(((7*A - 4*B)*cos(d*x + c) + 7*A - 4*B)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - (2*A*cos(d*x + c)^2 - (A - 4*B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c) - 4*sqrt(2))*((A - B)*a*cos(d*x + c) + (A - B)*a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))/sqrt(a)/(a*d*cos(d*x + c) + a*d)]

giac [B] time = 6.15, size = 649, normalized size = 3.93

$$\frac{4\sqrt{2}(A-B)\log\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2\right)}{\sqrt{-a}\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)} + \frac{(7A-4B)\log\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-a(2\sqrt{2}+\dots)\right)}{\sqrt{-a}\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")

[Out]
$$\begin{aligned} & -1/8*(4*\sqrt{2}*(A - B)*\log((\sqrt{-a}*\tan(1/2*d*x + 1/2*c) - \sqrt{-a*\tan(1/2*d*x + 1/2*c)^2 + a})^2)/(\sqrt{-a}*\operatorname{sgn}(\tan(1/2*d*x + 1/2*c)^2 - 1)) + (7*A \\ & - 4*B)*\log(\operatorname{abs}((\sqrt{-a}*\tan(1/2*d*x + 1/2*c) - \sqrt{-a*\tan(1/2*d*x + 1/2*c)^2 + a})^2 - a*(2*\sqrt{2} + 3)))/(\sqrt{-a}*\operatorname{sgn}(\tan(1/2*d*x + 1/2*c)^2 - 1)) \\ & - (7*A - 4*B)*\log(\operatorname{abs}((\sqrt{-a}*\tan(1/2*d*x + 1/2*c) - \sqrt{-a*\tan(1/2*d*x + 1/2*c)^2 + a})^2 + a*(2*\sqrt{2} - 3)))/(\sqrt{-a}*\operatorname{sgn}(\tan(1/2*d*x + 1/2*c)^2 - 1)) \\ & + 4*\sqrt{2}*(17*(\sqrt{-a}*\tan(1/2*d*x + 1/2*c) - \sqrt{-a*\tan(1/2*d*x + 1/2*c)^2 + a})^6*A*\sqrt{-a} - 12*(\sqrt{-a}*\tan(1/2*d*x + 1/2*c) - \sqrt{-a*\tan(1/2*d*x + 1/2*c)^2 + a})^6*B*\sqrt{-a} \\ & - 57*(\sqrt{-a}*\tan(1/2*d*x + 1/2*c) - \sqrt{-a*\tan(1/2*d*x + 1/2*c)^2 + a})^4*A*\sqrt{-a}*a + 76*(\sqrt{-a}*\tan(1/2*d*x + 1/2*c) - \sqrt{-a*\tan(1/2*d*x + 1/2*c)^2 + a})^4*B*\sqrt{-a} \\ & *a + 19*(\sqrt{-a}*\tan(1/2*d*x + 1/2*c) - \sqrt{-a*\tan(1/2*d*x + 1/2*c)^2 + a})^2*A*\sqrt{-a}*a^2 - 36*(\sqrt{-a}*\tan(1/2*d*x + 1/2*c) - \sqrt{-a*\tan(1/2*d*x + 1/2*c)^2 + a})^2*B*\sqrt{-a}*a^2 \\ & - 3*A*\sqrt{-a}*a^3 + 4*B*\sqrt{-a}*a^3)/(((\sqrt{-a}*\tan(1/2*d*x + 1/2*c) - \sqrt{-a*\tan(1/2*d*x + 1/2*c)^2 + a})^4 - 6*(\sqrt{-a}*\tan(1/2*d*x + 1/2*c) - \sqrt{-a*\tan(1/2*d*x + 1/2*c)^2 + a})^2*a + a^2)^2*\operatorname{sgn}(\tan(1/2*d*x + 1/2*c)^2 - 1))/d \end{aligned}$$

maple [B] time = 1.86, size = 717, normalized size = 4.35

$$\left(7A \cos(dx + c) \operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}} \sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\right) \left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}} \sin(dx+c)\sqrt{2} - 4B \cos(dx+c) \operatorname{arctanh}\left(\dots\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x)

[Out]
$$\frac{1}{16/d}*(7*A*\cos(d*x+c)*\operatorname{arctanh}(1/2*(-2*\cos(d*x+c)/(1+\cos(d*x+c))))^(1/2)*\sin(d*x+c)/\cos(d*x+c)*2^(1/2))*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^(3/2)*\sin(d*x+c)$$

```

*2^(1/2)-4*B*cos(d*x+c)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin
n(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*sin(d*x+c
)*2^(1/2)+7*A*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(
d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*sin(d*x+c)+
8*A*cos(d*x+c)*sin(d*x+c)*ln(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*
x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-4*B*2^(
1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)
)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*sin(d*x+c)-8*B*cos(d*x+c)*si
n(d*x+c)*ln(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1
)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+8*A*ln(-(-2*cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/
(1+cos(d*x+c)))^(3/2)*sin(d*x+c)-8*B*ln(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3
/2)*sin(d*x+c)-8*A*cos(d*x+c)^4+12*A*cos(d*x+c)^3-16*B*cos(d*x+c)^3-4*A*cos
(d*x+c)^2+16*B*cos(d*x+c)^2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)
/sin(d*x+c)/a

```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \cos(dx + c)^2}{\sqrt{a \sec(dx + c) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^2/sqrt(a*sec(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c + dx)^2 \left(A + \frac{B}{\cos(c+dx)} \right)}{\sqrt{a + \frac{a}{\cos(c+dx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^(1/2),x)
```

```
[Out] int((cos(c + d*x)^2*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \cos^2(c + dx)}{\sqrt{a (\sec(c + dx) + 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*cos(c + d*x)**2/sqrt(a*(sec(c + d*x) + 1)), x  
)
```

$$3.151 \quad \int \frac{\cos^3(c+dx)(A+B \sec(c+dx))}{\sqrt{a+a \sec(c+dx)}} dx$$

Optimal. Leaf size=206

$$\frac{(7A-2B) \sin(c+dx)}{8d\sqrt{a \sec(c+dx)+a}} - \frac{(9A-14B) \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}}\right)}{8\sqrt{a}d} + \frac{\sqrt{2}(A-B) \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{2} \sqrt{a \sec(c+dx)+a}}\right)}{\sqrt{a}d} - \frac{(A-6B) \sin(c+dx)}{12d\sqrt{a \sec(c+dx)+a}}$$

[Out] $-1/8*(9*A-14*B)*\arctan(a^{(1/2)}*\tan(d*x+c)/(a+a*\sec(d*x+c))^{(1/2)})/d/a^{(1/2)} + (A-B)*\arctan(1/2*a^{(1/2)}*\tan(d*x+c)*2^{(1/2)}/(a+a*\sec(d*x+c))^{(1/2)})*2^{(1/2)}/d/a^{(1/2)} + 1/8*(7*A-2*B)*\sin(d*x+c)/d/(a+a*\sec(d*x+c))^{(1/2)} - 1/12*(A-6*B)*\cos(d*x+c)*\sin(d*x+c)/d/(a+a*\sec(d*x+c))^{(1/2)} + 1/3*A*\cos(d*x+c)^2*\sin(d*x+c)/d/(a+a*\sec(d*x+c))^{(1/2)}$

Rubi [A] time = 0.55, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {4022, 3920, 3774, 203, 3795}

$$\frac{(7A-2B) \sin(c+dx)}{8d\sqrt{a \sec(c+dx)+a}} - \frac{(9A-14B) \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}}\right)}{8\sqrt{a}d} + \frac{\sqrt{2}(A-B) \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{2} \sqrt{a \sec(c+dx)+a}}\right)}{\sqrt{a}d} - \frac{(A-6B) \sin(c+dx)}{12d\sqrt{a \sec(c+dx)+a}}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^3*(A + B*Sec[c + d*x]))/Sqrt[a + a*Sec[c + d*x]],x]

[Out] $-((9*A - 14*B)*\text{ArcTan}[(\text{Sqrt}[a]*\text{Tan}[c + d*x])/\text{Sqrt}[a + a*\text{Sec}[c + d*x]])/(8*\text{Sqrt}[a]*d) + (\text{Sqrt}[2]*(A - B)*\text{ArcTan}[(\text{Sqrt}[a]*\text{Tan}[c + d*x])/(\text{Sqrt}[2]*\text{Sqrt}[a + a*\text{Sec}[c + d*x]])]/(\text{Sqrt}[a]*d) + ((7*A - 2*B)*\text{Sin}[c + d*x])/(8*d*\text{Sqrt}[a + a*\text{Sec}[c + d*x]]) - ((A - 6*B)*\text{Cos}[c + d*x]*\text{Sin}[c + d*x])/(12*d*\text{Sqrt}[a + a*\text{Sec}[c + d*x]]) + (A*\text{Cos}[c + d*x]^2*\text{Sin}[c + d*x])/(3*d*\text{Sqrt}[a + a*\text{Sec}[c + d*x]])$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]]/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 3774

Int[Sqrt[csc[(c_.) + (d_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] := Dist[(-2*b)/d, Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_.)]/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol]
:> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x]
/; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol]
:> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol]
:> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n - A*b*(m + n + 1)*Csc[e + f*x], x], x], x]
/; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^3(c+dx)(A+B\sec(c+dx))}{\sqrt{a+a\sec(c+dx)}} dx &= \frac{A\cos^2(c+dx)\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{\int \frac{\cos^2(c+dx)\left(-\frac{1}{2}a(A-6B)+\frac{5}{2}aA\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}} dx}{3a} \\
&= -\frac{(A-6B)\cos(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}} + \frac{A\cos^2(c+dx)\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{\int \frac{\cos^2(c+dx)\left(-\frac{1}{2}a(A-6B)+\frac{5}{2}aA\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}} dx}{3a} \\
&= \frac{(7A-2B)\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}} - \frac{(A-6B)\cos(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}} + \frac{A\cos^2(c+dx)\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} \\
&= \frac{(7A-2B)\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}} - \frac{(A-6B)\cos(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}} + \frac{A\cos^2(c+dx)\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} \\
&= \frac{(7A-2B)\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}} - \frac{(A-6B)\cos(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}} + \frac{A\cos^2(c+dx)\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} \\
&= -\frac{(9A-14B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{8\sqrt{a}d} + \frac{\sqrt{2}(A-B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d}
\end{aligned}$$

Mathematica [A] time = 0.71, size = 150, normalized size = 0.73

$$\frac{\tan(c+dx)\left(\cos(c+dx)\sqrt{1-\sec(c+dx)}\left(-2(A-6B)\cos(c+dx)+8A\cos^2(c+dx)+21A-6B\right)+(42B-27A)\right)}{24d\sqrt{1-\sec(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]^3*(A + B*Sec[c + d*x]))/Sqrt[a + a*Sec[c + d*x]],x]

[Out] (((-27*A + 42*B)*ArcTanh[Sqrt[1 - Sec[c + d*x]]] + 24*Sqrt[2]*(A - B)*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]] + Cos[c + d*x]*(21*A - 6*B - 2*(A - 6*B)*Cos[c + d*x] + 8*A*Cos[c + d*x]^2)*Sqrt[1 - Sec[c + d*x]])*Tan[c + d*x])/(24*d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])

fricas [A] time = 2.44, size = 539, normalized size = 2.62

$$24\sqrt{2}((A-B)a\cos(dx+c) + (A-B)a)\sqrt{-\frac{1}{a}} \log \left(\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\cos(dx+c)\sin(dx+c)+3\cos(dx+c)^2+2\cos(dx+c)}{\cos(dx+c)^2+2\cos(dx+c)+1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")

[Out] [-1/48*(24*sqrt(2)*((A - B)*a*cos(d*x + c) + (A - B)*a)*sqrt(-1/a)*log((2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*cos(d*x + c)*sin(d*x + c) + 3*cos(d*x + c)^2 + 2*cos(d*x + c) - 1)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 3*((9*A - 14*B)*cos(d*x + c) + 9*A - 14*B)*sqrt(-a)*log((2*a*cos(d*x + c)^2 + 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) - 2*(8*A*cos(d*x + c)^3 - 2*(A - 6*B)*cos(d*x + c)^2 + 3*(7*A - 2*B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/(a*d*cos(d*x + c) + a*d), 1/24*(3*((9*A - 14*B)*cos(d*x + c) + 9*A - 14*B)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) + (8*A*cos(d*x + c)^3 - 2*(A - 6*B)*cos(d*x + c)^2 + 3*(7*A - 2*B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c) - 24*sqrt(2)*((A - B)*a*cos(d*x + c) + (A - B)*a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))/sqrt(a))/(a*d*cos(d*x + c) + a*d)]

giac [B] time = 2.25, size = 846, normalized size = 4.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")

[Out] 1/48*(24*sqrt(2)*(A - B)*log((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) + 3*(9*A - 14*B)*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3)))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c)^2

```

- 1)) - 3*(9*A - 14*B)*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a)*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3)))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) + 4*sqrt(2)*(165*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a)*tan(1/2*d*x + 1/2*c)^2 + a))^10*A*sqrt(-a) - 102*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a)*tan(1/2*d*x + 1/2*c)^2 + a))^10*B*sqrt(-a) - 1323*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a)*tan(1/2*d*x + 1/2*c)^2 + a))^8*A*sqrt(-a)*a + 954*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a)*tan(1/2*d*x + 1/2*c)^2 + a))^8*B*sqrt(-a)*a + 3906*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a)*tan(1/2*d*x + 1/2*c)^2 + a))^6*A*sqrt(-a)*a^2 - 2268*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a)*tan(1/2*d*x + 1/2*c)^2 + a))^6*B*sqrt(-a)*a^2 - 2118*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a)*tan(1/2*d*x + 1/2*c)^2 + a))^4*A*sqrt(-a)*a^3 + 1044*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a)*tan(1/2*d*x + 1/2*c)^2 + a))^4*B*sqrt(-a)*a^3 + 393*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a)*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*sqrt(-a)*a^4 - 222*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a)*tan(1/2*d*x + 1/2*c)^2 + a))^2*B*sqrt(-a)*a^4 - 31*A*sqrt(-a)*a^5 + 18*B*sqrt(-a)*a^5)/(((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a)*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a)*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^3*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))/d

```

maple [B] time = 1.78, size = 1067, normalized size = 5.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x)

```

[Out] -1/192/d*(-27*A*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*2^(1/2)*cos(d*x+c)^2+42*B*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*2^(1/2)*cos(d*x+c)^2-48*A*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*ln(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)^2-54*A*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*2^(1/2)*cos(d*x+c)+48*B*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*ln(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)^2+84*B*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*2^(1/2)*cos(d*x+c)-96*A*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*ln(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)-27*A*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*sin(d*x+c)+96*B*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*ln(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)+42*B*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+

```

$\cos(dx+c))^{1/2} \sin(dx+c) / \cos(dx+c) * 2^{1/2} \sin(dx+c) - 48A * (-2\cos(dx+c) / (1+\cos(dx+c)))^{5/2} \ln(-(-2\cos(dx+c) / (1+\cos(dx+c)))^{1/2} \sin(dx+c) + \cos(dx+c) - 1) / \sin(dx+c) * \sin(dx+c) + 48B * (-2\cos(dx+c) / (1+\cos(dx+c)))^{5/2} \ln(-(-2\cos(dx+c) / (1+\cos(dx+c)))^{1/2} \sin(dx+c) + \cos(dx+c) - 1) / \sin(dx+c) * \sin(dx+c) + 64A * \cos(dx+c)^6 - 80A * \cos(dx+c)^5 + 96B * \cos(dx+c)^5 + 184A * \cos(dx+c)^4 - 144B * \cos(dx+c)^4 - 168A * \cos(dx+c)^3 + 48B * \cos(dx+c)^3 * (a * (1+\cos(dx+c)) / \cos(dx+c))^{1/2} / \cos(dx+c)^2 / \sin(dx+c) / a$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx+c) + A) \cos(dx+c)^3}{\sqrt{a \sec(dx+c) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(dx+c)^3*(A+B*sec(dx+c))/(a+a*sec(dx+c))^(1/2),x, algorithm="maxima")

[Out] integrate((B*sec(dx+c) + A)*cos(dx+c)^3/sqrt(a*sec(dx+c) + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+dx)^3 \left(A + \frac{B}{\cos(c+dx)} \right)}{\sqrt{a + \frac{a}{\cos(c+dx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c+dx)^3*(A+B/cos(c+dx)))/(a+a/cos(c+dx))^(1/2),x)

[Out] int((cos(c+dx)^3*(A+B/cos(c+dx)))/(a+a/cos(c+dx))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c+dx)) \cos^3(c+dx)}{\sqrt{a(\sec(c+dx) + 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(dx+c)**3*(A+B*sec(dx+c))/(a+a*sec(dx+c))**(1/2),x)

[Out] Integral((A + B*sec(c+dx))*cos(c+dx)**3/sqrt(a*(sec(c+dx) + 1)), x)

$$3.152 \quad \int \frac{\sec^4(c+dx)(A+B \sec(c+dx))}{(a+a \sec(c+dx))^{3/2}} dx$$

Optimal. Leaf size=216

$$\frac{(11A - 15B) \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{2} \sqrt{a \sec(c+dx)+a}}\right)}{2\sqrt{2} a^{3/2} d} + \frac{(35A - 39B) \tan(c+dx) \sqrt{a \sec(c+dx)+a}}{30a^2 d} + \frac{(A - B) \tan(c+dx) \sec^3(c+dx)}{2d(a \sec(c+dx)+a)^{3/2}}$$

[Out] $1/4*(11*A-15*B)*\arctan(1/2*a^{(1/2)}*\tan(d*x+c)*2^{(1/2)}/(a+a*\sec(d*x+c))^{(1/2)})/a^{(3/2)}/d*2^{(1/2)}+1/2*(A-B)*\sec(d*x+c)^3*\tan(d*x+c)/d/(a+a*\sec(d*x+c))^{(3/2)}-1/15*(65*A-93*B)*\tan(d*x+c)/a/d/(a+a*\sec(d*x+c))^{(1/2)}-1/10*(5*A-9*B)*\sec(d*x+c)^2*\tan(d*x+c)/a/d/(a+a*\sec(d*x+c))^{(1/2)}+1/30*(35*A-39*B)*(a+a*\sec(d*x+c))^{(1/2)}*\tan(d*x+c)/a^2/d$

Rubi [A] time = 0.63, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4019, 4021, 4010, 4001, 3795, 203}

$$\frac{(11A - 15B) \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{2} \sqrt{a \sec(c+dx)+a}}\right)}{2\sqrt{2} a^{3/2} d} + \frac{(35A - 39B) \tan(c+dx) \sqrt{a \sec(c+dx)+a}}{30a^2 d} + \frac{(A - B) \tan(c+dx) \sec^3(c+dx)}{2d(a \sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(Sec[c + d*x]^4*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(3/2),x]

[Out] $((11*A - 15*B)*\text{ArcTan}[\frac{\sqrt{a}*\tan[c + d*x]}{\sqrt{2}*\sqrt{a + a*\sec[c + d*x]}}])/(2*\sqrt{2}*a^{(3/2)}*d) + ((A - B)*\sec[c + d*x]^3*\tan[c + d*x])/(2*d*(a + a*\sec[c + d*x])^{(3/2)}) - ((65*A - 93*B)*\tan[c + d*x])/(15*a*d*\sqrt{a + a*\sec[c + d*x]}) - ((5*A - 9*B)*\sec[c + d*x]^2*\tan[c + d*x])/(10*a*d*\sqrt{a + a*\sec[c + d*x]}) + ((35*A - 39*B)*\sqrt{a + a*\sec[c + d*x]}*\tan[c + d*x])/(30*a^2*d)$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 3795

Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_.)]*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m + 1), 0] && !LtQ[m, -2^(-1)]
```

Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_.)]^2*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Csc[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && !LtQ[m, -1]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> Simp[(d*(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0]
```

Rule 4021

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> -Simp[(B*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^4(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{3/2}} dx &= \frac{(A-B)\sec^3(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \int \frac{\sec^3(c+dx)\left(3a(A-B)-\frac{1}{2}a(5A-9B)\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}} \frac{1}{2a^2} \\
&= \frac{(A-B)\sec^3(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(5A-9B)\sec^2(c+dx)\tan(c+dx)}{10ad\sqrt{a+a\sec(c+dx)}} \\
&= \frac{(A-B)\sec^3(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(5A-9B)\sec^2(c+dx)\tan(c+dx)}{10ad\sqrt{a+a\sec(c+dx)}} \\
&= \frac{(A-B)\sec^3(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(65A-93B)\tan(c+dx)}{15ad\sqrt{a+a\sec(c+dx)}} - \frac{(5A-9B)\sec^2(c+dx)\tan(c+dx)}{10ad\sqrt{a+a\sec(c+dx)}} \\
&= \frac{(A-B)\sec^3(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(65A-93B)\tan(c+dx)}{15ad\sqrt{a+a\sec(c+dx)}} - \frac{(5A-9B)\sec^2(c+dx)\tan(c+dx)}{10ad\sqrt{a+a\sec(c+dx)}} \\
&= \frac{(11A-15B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} + \frac{(A-B)\sec^3(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}
\end{aligned}$$

Mathematica [A] time = 2.40, size = 160, normalized size = 0.74

$$\frac{\tan(c+dx)\left(\sqrt{1-\sec(c+dx)}\left(4(5A-3B)\sec^2(c+dx)-12(5A-9B)\sec(c+dx)-95A+12B\sec^3(c+dx)+1\right)\right)}{30d\sqrt{1-\sec(c+dx)}(a(\sec(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[c + d*x]^4*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(3/2), x]

[Out] ((15*sqrt[2]*(11*A - 15*B)*ArcTanh[Sqrt[1 - Sec[c + d*x]]/sqrt[2]]*Cos[(c + d*x)/2]^2*Sec[c + d*x] + Sqrt[1 - Sec[c + d*x]]*(-95*A + 147*B - 12*(5*A - 9*B)*Sec[c + d*x] + 4*(5*A - 3*B)*Sec[c + d*x]^2 + 12*B*Sec[c + d*x]^3))*Tan[c + d*x])/(30*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(3/2))

fricas [A] time = 0.50, size = 504, normalized size = 2.33

$$\left[\frac{15\sqrt{2} \left((11A - 15B) \cos(dx + c)^4 + 2(11A - 15B) \cos(dx + c)^3 + (11A - 15B) \cos(dx + c)^2 \right) \sqrt{-a} \log \left(-\frac{2\sqrt{2}}{\dots} \right)}{\dots} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")

[Out] [1/120*(15*sqrt(2)*((11*A - 15*B)*cos(d*x + c)^4 + 2*(11*A - 15*B)*cos(d*x + c)^3 + (11*A - 15*B)*cos(d*x + c)^2)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) - 3*a*cos(d*x + c)^2 - 2*a*cos(d*x + c) + a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*((95*A - 147*B)*cos(d*x + c)^3 + 12*(5*A - 9*B)*cos(d*x + c)^2 - 4*(5*A - 3*B)*cos(d*x + c) - 12*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^4 + 2*a^2*d*cos(d*x + c)^3 + a^2*d*cos(d*x + c)^2), -1/60*(15*sqrt(2)*((11*A - 15*B)*cos(d*x + c)^4 + 2*(11*A - 15*B)*cos(d*x + c)^3 + (11*A - 15*B)*cos(d*x + c)^2)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) + 2*((95*A - 147*B)*cos(d*x + c)^3 + 12*(5*A - 9*B)*cos(d*x + c)^2 - 4*(5*A - 3*B)*cos(d*x + c) - 12*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^4 + 2*a^2*d*cos(d*x + c)^3 + a^2*d*cos(d*x + c)^2)]

giac [A] time = 6.95, size = 312, normalized size = 1.44

$$\frac{15\sqrt{2}(11A-15B)\log\left(-\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)}{\sqrt{-a}\operatorname{asgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)} - \frac{\left(\left(\frac{15\sqrt{2}(Aa^3-Ba^3)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{a^2\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)} - \frac{\sqrt{2}(245Aa^3-381Ba^3)}{a^2\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}\right)}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")

[Out] 1/60*(15*sqrt(2)*(11*A - 15*B)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/(sqrt(-a)*a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) - (((15*sqrt(2)*(A*a^3 - B*a^3)*tan(1/2*d*x + 1/2*c)^2/(a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) - (sqrt(2)*(245*A*a^3 - 381*B*a^3)/(a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1))))*tan(1/2*d*x + 1/2*c))/(a*tan(1/2*d*x + 1/2*c))))

$$\frac{(2dx + 1/2c)^2 - 1) - \sqrt{2} \cdot (245Aa^3 - 381Ba^3) / (a^2 \operatorname{sgn}(\tan(1/2dx + 1/2c)^2 - 1)) \cdot \tan(1/2dx + 1/2c)^2 + 5\sqrt{2} \cdot (73Aa^3 - 105Ba^3) / (a^2 \operatorname{sgn}(\tan(1/2dx + 1/2c)^2 - 1)) \cdot \tan(1/2dx + 1/2c)^2 - 15\sqrt{2} \cdot (9Aa^3 - 17Ba^3) / (a^2 \operatorname{sgn}(\tan(1/2dx + 1/2c)^2 - 1)) \cdot \tan(1/2dx + 1/2c) / ((a \tan(1/2dx + 1/2c)^2 - a)^2 \sqrt{-a \tan(1/2dx + 1/2c)^2 + a}) / d$$

maple [B] time = 2.01, size = 793, normalized size = 3.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sec(dx+c)^4*(A+B*sec(dx+c))/(a+a*sec(dx+c))^(3/2),x)`

[Out]
$$\begin{aligned} & -1/240/d * (-1 + \cos(dx+c)) * (165A \cos(dx+c)^3 \sin(dx+c) * (-2 \cos(dx+c) / (1 + \cos(dx+c)))^{5/2} * \ln(-(-2 \cos(dx+c) / (1 + \cos(dx+c)))^{1/2} * \sin(dx+c) + \cos(dx+c) - 1) / \sin(dx+c)) - 225B \cos(dx+c)^3 \sin(dx+c) * (-2 \cos(dx+c) / (1 + \cos(dx+c)))^{5/2} * \ln(-(-2 \cos(dx+c) / (1 + \cos(dx+c)))^{1/2} * \sin(dx+c) + \cos(dx+c) - 1) / \sin(dx+c)) + 495A * (-2 \cos(dx+c) / (1 + \cos(dx+c)))^{5/2} * \ln(-(-2 \cos(dx+c) / (1 + \cos(dx+c)))^{1/2} * \sin(dx+c) + \cos(dx+c) - 1) / \sin(dx+c)) * \sin(dx+c) * \cos(dx+c)^2 - 675B * (-2 \cos(dx+c) / (1 + \cos(dx+c)))^{5/2} * \ln(-(-2 \cos(dx+c) / (1 + \cos(dx+c)))^{1/2} * \sin(dx+c) + \cos(dx+c) - 1) / \sin(dx+c)) * \sin(dx+c) * \cos(dx+c)^2 + 495A * (-2 \cos(dx+c) / (1 + \cos(dx+c)))^{5/2} * \ln(-(-2 \cos(dx+c) / (1 + \cos(dx+c)))^{1/2} * \sin(dx+c) + \cos(dx+c) - 1) / \sin(dx+c)) * \sin(dx+c) * \cos(dx+c) - 675B * (-2 \cos(dx+c) / (1 + \cos(dx+c)))^{5/2} * \ln(-(-2 \cos(dx+c) / (1 + \cos(dx+c)))^{1/2} * \sin(dx+c) + \cos(dx+c) - 1) / \sin(dx+c)) * \sin(dx+c) * \cos(dx+c) + 165A * (-2 \cos(dx+c) / (1 + \cos(dx+c)))^{5/2} * \ln(-(-2 \cos(dx+c) / (1 + \cos(dx+c)))^{1/2} * \sin(dx+c) + \cos(dx+c) - 1) / \sin(dx+c)) * \sin(dx+c) - 225B * (-2 \cos(dx+c) / (1 + \cos(dx+c)))^{5/2} * \ln(-(-2 \cos(dx+c) / (1 + \cos(dx+c)))^{1/2} * \sin(dx+c) + \cos(dx+c) - 1) / \sin(dx+c)) * \sin(dx+c) + 760A \cos(dx+c)^4 - 1176B \cos(dx+c)^4 - 280A \cos(dx+c)^3 + 312B \cos(dx+c)^3 - 640A \cos(dx+c)^2 + 960B \cos(dx+c)^2 + 160A \cos(dx+c) - 192B \cos(dx+c) + 96B * (a * (1 + \cos(dx+c)) / \cos(dx+c))^{1/2} / \cos(dx+c)^2 / \sin(dx+c)^3 / a^2 \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx+c) + A) \sec(dx+c)^4}{(a \sec(dx+c) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(dx+c)^4*(A+B*sec(dx+c))/(a+a*sec(dx+c))^(3/2),x, algorithm="maxima")`

[Out] `integrate((B*sec(dx+c) + A)*sec(dx+c)^4/(a*sec(dx+c) + a)^(3/2), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\cos(c+dx)^4 \left(a + \frac{a}{\cos(c+dx)}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^4*(a + a/cos(c + d*x))^(3/2)),x)

[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^4*(a + a/cos(c + d*x))^(3/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \sec^4(c + dx)}{(a(\sec(c + dx) + 1))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(3/2),x)

[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**4/(a*(sec(c + d*x) + 1))**(3/2), x)

$$3.153 \quad \int \frac{\sec^3(c+dx)(A+B \sec(c+dx))}{(a+a \sec(c+dx))^{3/2}} dx$$

Optimal. Leaf size=171

$$-\frac{(7A - 11B) \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{2} \sqrt{a \sec(c+dx)+a}}\right)}{2\sqrt{2} a^{3/2}d} - \frac{(3A - 7B) \tan(c+dx) \sqrt{a \sec(c+dx)+a}}{6a^2d} + \frac{(A - B) \tan(c+dx) \sec^2(c+dx)}{2d(a \sec(c+dx)+a)^{3/2}}$$

[Out] $-1/4*(7*A-11*B)*\arctan(1/2*a^{(1/2)}*\tan(d*x+c)*2^{(1/2)}/(a+a*\sec(d*x+c))^{(1/2)})/a^{(3/2)}/d*2^{(1/2)}+1/2*(A-B)*\sec(d*x+c)^2*\tan(d*x+c)/d/(a+a*\sec(d*x+c))^{(3/2)}+1/3*(9*A-13*B)*\tan(d*x+c)/a/d/(a+a*\sec(d*x+c))^{(1/2)}-1/6*(3*A-7*B)*(a+a*\sec(d*x+c))^{(1/2)}*\tan(d*x+c)/a^2/d$

Rubi [A] time = 0.46, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {4019, 4010, 4001, 3795, 203}

$$-\frac{(7A - 11B) \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{2} \sqrt{a \sec(c+dx)+a}}\right)}{2\sqrt{2} a^{3/2}d} - \frac{(3A - 7B) \tan(c+dx) \sqrt{a \sec(c+dx)+a}}{6a^2d} + \frac{(A - B) \tan(c+dx) \sec^2(c+dx)}{2d(a \sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(\text{Sec}[c + d*x]^3*(A + B*\text{Sec}[c + d*x]))/(a + a*\text{Sec}[c + d*x])^{(3/2)}, x]$

[Out] $-((7*A - 11*B)*\text{ArcTan}[(\text{Sqrt}[a]*\text{Tan}[c + d*x])/(\text{Sqrt}[2]*\text{Sqrt}[a + a*\text{Sec}[c + d*x]])])/(2*\text{Sqrt}[2]*a^{(3/2)}*d) + ((A - B)*\text{Sec}[c + d*x]^2*\text{Tan}[c + d*x])/(2*d*(a + a*\text{Sec}[c + d*x])^{(3/2)}) + ((9*A - 13*B)*\text{Tan}[c + d*x])/(3*a*d*\text{Sqrt}[a + a*\text{Sec}[c + d*x]]) - ((3*A - 7*B)*\text{Sqrt}[a + a*\text{Sec}[c + d*x]]*\text{Tan}[c + d*x])/(6*a^2*d)$

Rule 203

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] := \text{Simp}[(1*\text{ArcTan}[(\text{Rt}[b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[b, 2]), x] /; \text{FreeQ}\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{GtQ}[b, 0])$

Rule 3795

$\text{Int}[\text{csc}[(e_ + (f_)*(x_)]/\text{Sqrt}[\text{csc}[(e_ + (f_)*(x_)]*(b_ + (a_))], x_Symbol] := \text{Dist}[-2/f, \text{Subst}[\text{Int}[1/(2*a + x^2), x], x, (b*\text{Cot}[e + f*x])/\text{Sqrt}[a + b*\text{Csc}[e + f*x]]], x] /; \text{FreeQ}\{a, b, e, f\}, x] \ \&\& \ \text{EqQ}[a^2 - b^2, 0]$

Rule 4001

```

Int[csc[(e_.) + (f_.)*(x_.)]*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m + 1), 0] && !LtQ[m, -2^(-1)]

```

Rule 4010

```

Int[csc[(e_.) + (f_.)*(x_.)]^2*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && !LtQ[m, -1]

```

Rule 4019

```

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> Simp[(d*(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0]

```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^3(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{3/2}} dx &= \frac{(A-B)\sec^2(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \int \frac{\sec^2(c+dx)\left(2a(A-B)-\frac{1}{2}a(3A-7B)\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}} \frac{dx}{2a^2} \\
&= \frac{(A-B)\sec^2(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(3A-7B)\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{6a^2d} \\
&= \frac{(A-B)\sec^2(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(9A-13B)\tan(c+dx)}{3ad\sqrt{a+a\sec(c+dx)}} - \frac{(3A-7B)}{6a^2d} \\
&= \frac{(A-B)\sec^2(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(9A-13B)\tan(c+dx)}{3ad\sqrt{a+a\sec(c+dx)}} - \frac{(3A-7B)}{6a^2d} \\
&= -\frac{(7A-11B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} + \frac{(A-B)\sec^2(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}
\end{aligned}$$

Mathematica [A] time = 1.36, size = 141, normalized size = 0.82

$$\frac{\tan(c+dx)\left(\sqrt{1-\sec(c+dx)}\left(12(A-B)\sec(c+dx)+15A+4B\sec^2(c+dx)-19B\right)-3\sqrt{2}(7A-11B)\cos^2\left(\frac{1}{2}(c+dx)\right)\right)}{6d\sqrt{1-\sec(c+dx)}(a(\sec(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(3/2), x]

[Out] ((-3*Sqrt[2]*(7*A - 11*B)*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]*Cos[(c + d*x)/2]^2*Sec[c + d*x] + Sqrt[1 - Sec[c + d*x]]*(15*A - 19*B + 12*(A - B)*Sec[c + d*x] + 4*B*Sec[c + d*x]^2))*Tan[c + d*x]/(6*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(3/2))

fricas [A] time = 0.52, size = 459, normalized size = 2.68

$$\left[\frac{3\sqrt{2}\left((7A-11B)\cos(dx+c)^3+2(7A-11B)\cos(dx+c)^2+(7A-11B)\cos(dx+c)\right)\sqrt{-a}\log\left(\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)}{a+\sec(dx+c)}}}{24(a^2d\cos(dx+c)+\dots)}\right)}{24(a^2d\cos(dx+c)+\dots)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algorithm
="fricas")
```

```
[Out] [1/24*(3*sqrt(2)*((7*A - 11*B)*cos(d*x + c)^3 + 2*(7*A - 11*B)*cos(d*x + c)
^2 + (7*A - 11*B)*cos(d*x + c))*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + 3*a*cos(d*x + c)^
2 + 2*a*cos(d*x + c) - a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*((15*A
- 19*B)*cos(d*x + c)^2 + 12*(A - B)*cos(d*x + c) + 4*B)*sqrt((a*cos(d*x +
c) + a)/cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^3 + 2*a^2*d*cos(d*x
+ c)^2 + a^2*d*cos(d*x + c)), 1/12*(3*sqrt(2)*((7*A - 11*B)*cos(d*x + c)^3
+ 2*(7*A - 11*B)*cos(d*x + c)^2 + (7*A - 11*B)*cos(d*x + c))*sqrt(a)*arcta
n(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin
(d*x + c))) + 2*((15*A - 19*B)*cos(d*x + c)^2 + 12*(A - B)*cos(d*x + c) + 4
*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x +
c)^3 + 2*a^2*d*cos(d*x + c)^2 + a^2*d*cos(d*x + c))]
```

giac [A] time = 2.37, size = 296, normalized size = 1.73

$$\frac{\left(\frac{3 \left(\sqrt{2} A \operatorname{asgn} \left(\tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^2 - 1 \right) - \sqrt{2} B \operatorname{asgn} \left(\tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^2 - 1 \right) \right) \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^2}{a} - \frac{2 \left(15 \sqrt{2} A \operatorname{asgn} \left(\tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^2 - 1 \right) - 23 \sqrt{2} B \operatorname{asgn} \left(\tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^2 - 1 \right) \right)}{a} \right) \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)}{\left(a \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^2 - a \right) \sqrt{-a \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^2 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algorithm
="giac")
```

```
[Out] 1/12*(((3*(sqrt(2)*A*a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1) - sqrt(2)*B*a*sgn(ta
n(1/2*d*x + 1/2*c)^2 - 1))*tan(1/2*d*x + 1/2*c)^2/a - 2*(15*sqrt(2)*A*a*sgn
(tan(1/2*d*x + 1/2*c)^2 - 1) - 23*sqrt(2)*B*a*sgn(tan(1/2*d*x + 1/2*c)^2 -
1))/a)*tan(1/2*d*x + 1/2*c)^2 + 27*(sqrt(2)*A*a*sgn(tan(1/2*d*x + 1/2*c)^2
- 1) - sqrt(2)*B*a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1))/a)*tan(1/2*d*x + 1/2*c)
/((a*tan(1/2*d*x + 1/2*c)^2 - a)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)) - 3*(
7*sqrt(2)*A - 11*sqrt(2)*B)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-
a*tan(1/2*d*x + 1/2*c)^2 + a)))/(sqrt(-a)*a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1
))/d
```

maple [B] time = 2.14, size = 603, normalized size = 3.53

$$(-1 + \cos(dx + c)) \left(21A \ln \left(\frac{-\sqrt{\frac{2\cos(dx+c)}{1+\cos(dx+c)}} \sin(dx+c) + \cos(dx+c) - 1}{\sin(dx+c)} \right) \left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)} \right)^{\frac{3}{2}} \sin(dx+c) (\cos^2(dx+c)) - 33 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2), x)

[Out] -1/24/d*(-1+cos(d*x+c))*(21*A*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c))))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*sin(d*x+c)*cos(d*x+c)^2-33*B*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c))))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*sin(d*x+c)*cos(d*x+c)^2+42*A*cos(d*x+c)*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c))))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-66*B*cos(d*x+c)*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c))))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+21*A*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c))))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*sin(d*x+c)-33*B*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c))))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*sin(d*x+c)-60*A*cos(d*x+c)^3+76*B*cos(d*x+c)^3+12*A*cos(d*x+c)^2-28*B*cos(d*x+c)^2+48*A*cos(d*x+c)-64*B*cos(d*x+c)+16*B)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)^3/cos(d*x+c)/a^2

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sec(dx + c)^3}{(a \sec(dx + c) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2), x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^3/(a*sec(d*x + c) + a)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\cos(c+dx)^3 \left(a + \frac{a}{\cos(c+dx)} \right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B/cos(c + d*x))/(cos(c + d*x)^3*(a + a/cos(c + d*x))^(3/2)),x)`

[Out] `int((A + B/cos(c + d*x))/(cos(c + d*x)^3*(a + a/cos(c + d*x))^(3/2)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \sec^3(c + dx)}{(a(\sec(c + dx) + 1))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)**3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(3/2),x)`

[Out] `Integral((A + B*sec(c + d*x))*sec(c + d*x)**3/(a*(sec(c + d*x) + 1))**(3/2), x)`

$$3.154 \quad \int \frac{\sec^2(c+dx)(A+B \sec(c+dx))}{(a+a \sec(c+dx))^{3/2}} dx$$

Optimal. Leaf size=118

$$\frac{(3A - 7B) \tan^{-1} \left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{2} \sqrt{a \sec(c+dx)+a}} \right)}{2\sqrt{2} a^{3/2} d} - \frac{(A - B) \tan(c + dx)}{2d(a \sec(c + dx) + a)^{3/2}} + \frac{2B \tan(c + dx)}{ad\sqrt{a \sec(c + dx) + a}}$$

[Out] 1/4*(3*A-7*B)*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2)-1/2*(A-B)*tan(d*x+c)/d/(a+a*sec(d*x+c))^(3/2)+2*B*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^(1/2)

Rubi [A] time = 0.26, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {4008, 4001, 3795, 203}

$$\frac{(3A - 7B) \tan^{-1} \left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{2} \sqrt{a \sec(c+dx)+a}} \right)}{2\sqrt{2} a^{3/2} d} - \frac{(A - B) \tan(c + dx)}{2d(a \sec(c + dx) + a)^{3/2}} + \frac{2B \tan(c + dx)}{ad\sqrt{a \sec(c + dx) + a}}$$

Antiderivative was successfully verified.

[In] Int[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(3/2), x]

[Out] ((3*A - 7*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B)*Tan[c + d*x])/(2*d*(a + a*Sec[c + d*x])^(3/2)) + (2*B*Tan[c + d*x])/(a*d*Sqrt[a + a*Sec[c + d*x]])

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 3795

Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] := Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 4001

Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] := -Simp[(B*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1))

)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m + 1), 0] && !LtQ[m, -2^(-1)]

Rule 4008

Int[csc[(e_.) + (f_.)*(x_.)]^2*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(b^2*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*B*(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]

Rubi steps

$$\begin{aligned} \int \frac{\sec^2(c + dx)(A + B \sec(c + dx))}{(a + a \sec(c + dx))^{3/2}} dx &= -\frac{(A - B) \tan(c + dx)}{2d(a + a \sec(c + dx))^{3/2}} - \frac{\int \frac{\sec(c+dx)\left(-\frac{3}{2}a(A-B)-2aB \sec(c+dx)\right)}{\sqrt{a+a \sec(c+dx)}} dx}{2a^2} \\ &= -\frac{(A - B) \tan(c + dx)}{2d(a + a \sec(c + dx))^{3/2}} + \frac{2B \tan(c + dx)}{ad\sqrt{a + a \sec(c + dx)}} + \frac{(3A - 7B) \int \frac{\sec(c+dx)}{\sqrt{a+a \sec(c+dx)}} dx}{4a} \\ &= -\frac{(A - B) \tan(c + dx)}{2d(a + a \sec(c + dx))^{3/2}} + \frac{2B \tan(c + dx)}{ad\sqrt{a + a \sec(c + dx)}} - \frac{(3A - 7B) \operatorname{Subst} \int \frac{\sec(u)}{\sqrt{a+a \sec(u)}} du}{4a} \\ &= \frac{(3A - 7B) \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{2} \sqrt{a+a \sec(c+dx)}}\right)}{2\sqrt{2} a^{3/2} d} - \frac{(A - B) \tan(c + dx)}{2d(a + a \sec(c + dx))^{3/2}} + \frac{2B \tan(c + dx)}{ad\sqrt{a + a \sec(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.87, size = 125, normalized size = 1.06

$$\frac{\tan(c + dx) \left(\sqrt{1 - \sec(c + dx)} (-A + 4B \sec(c + dx) + 5B) + \sqrt{2} (3A - 7B) \cos^2\left(\frac{1}{2}(c + dx)\right) \sec(c + dx) \tanh^{-1}\left(\frac{\sqrt{2} \tan(c + dx)}{\sqrt{1 - \sec(c + dx)}}\right) \right)}{2d\sqrt{1 - \sec(c + dx)} (a(\sec(c + dx) + 1))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(3/2), x]

[Out] ((Sqrt[2]*(3*A - 7*B)*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]*Cos[(c + d*x)/2]^2*Sec[c + d*x] + Sqrt[1 - Sec[c + d*x]]*(-A + 5*B + 4*B*Sec[c + d*x]))*Tan[c + d*x])/(2*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(3/2))

fricas [A] time = 0.50, size = 386, normalized size = 3.27

$$\frac{\sqrt{2} \left((3A - 7B) \cos(dx + c)^2 + 2(3A - 7B) \cos(dx + c) + 3A - 7B \right) \sqrt{-a} \log \left(-\frac{2\sqrt{2}\sqrt{-a} \sqrt{\frac{a \cos(dx+c)+a}{\cos(dx+c)}} \cos(dx+c) \sin(dx+c)}{\cos(dx+c)^2 + 2a \cos(dx+c) + a^2} \right)}{8 \left(a^2 d \cos(dx + c)^2 + 2 a^2 d \cos(dx + c) + a^2 d \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")

[Out] [1/8*(sqrt(2)*((3*A - 7*B)*cos(d*x + c)^2 + 2*(3*A - 7*B)*cos(d*x + c) + 3*A - 7*B)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) - 3*a*cos(d*x + c)^2 - 2*a*cos(d*x + c) + a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*((A - 5*B)*cos(d*x + c) - 4*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), -1/4*(sqrt(2)*((3*A - 7*B)*cos(d*x + c)^2 + 2*(3*A - 7*B)*cos(d*x + c) + 3*A - 7*B)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) + 2*((A - 5*B)*cos(d*x + c) - 4*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]

giac [A] time = 2.50, size = 190, normalized size = 1.61

$$\frac{\left(\frac{\sqrt{2}(Aa^2 - Ba^2) \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2}{a^3 \operatorname{sgn}\left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 1\right)} - \frac{\sqrt{2}(Aa^2 - 9Ba^2)}{a^3 \operatorname{sgn}\left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 1\right)} \right) \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - \frac{\sqrt{2}(3A - 7B) \log\left(\left| -\sqrt{-a} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + \sqrt{-a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + a} \right|\right)}{\sqrt{-a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + a}}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")

[Out] -1/4*((sqrt(2)*(A*a^2 - B*a^2)*tan(1/2*d*x + 1/2*c)^2/(a^3*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) - sqrt(2)*(A*a^2 - 9*B*a^2)/(a^3*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))*tan(1/2*d*x + 1/2*c)/sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a) - sqrt(2)*(3*A - 7*B)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/(sqrt(-a)*a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1))/d

maple [B] time = 1.77, size = 405, normalized size = 3.43

$$\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}} (-1 + \cos(dx+c)) \left(3A \cos(dx+c) \sin(dx+c) \ln \left(\frac{-\sqrt{\frac{2\cos(dx+c)}{1+\cos(dx+c)}} \sin(dx+c) + \cos(dx+c) - 1}{\sin(dx+c)} \right) \right) \sqrt{\frac{2c}{1+}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2), x)

[Out]
$$-1/4/d*(a*(1+\cos(d*x+c))/\cos(d*x+c))^{(1/2)}*(-1+\cos(d*x+c))*(3*A*\cos(d*x+c)*\sin(d*x+c)*\ln(-(-(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*\sin(d*x+c)+\cos(d*x+c)-1)/\sin(d*x+c))*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}-7*B*\cos(d*x+c)*\sin(d*x+c)*\ln(-(-(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*\sin(d*x+c)+\cos(d*x+c)-1)/\sin(d*x+c))*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}+3*A*\ln(-(-(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*\sin(d*x+c)+\cos(d*x+c)-1)/\sin(d*x+c))*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*\sin(d*x+c)-7*B*\ln(-(-(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*\sin(d*x+c)+\cos(d*x+c)-1)/\sin(d*x+c))*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*\sin(d*x+c)+2*A*\cos(d*x+c)^2-10*B*\cos(d*x+c)^2-2*A*\cos(d*x+c)+2*B*\cos(d*x+c)+8*B)/\sin(d*x+c)^3/a^2$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx+c) + A) \sec(dx+c)^2}{(a \sec(dx+c) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2), x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^2/(a*sec(d*x + c) + a)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\cos(c+dx)^2 \left(a + \frac{a}{\cos(c+dx)} \right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^2*(a + a/cos(c + d*x))^(3/2)), x)

[Out] `int((A + B/cos(c + d*x))/(cos(c + d*x)^2*(a + a/cos(c + d*x))^(3/2)), x)`
sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \sec^2(c + dx)}{(a(\sec(c + dx) + 1))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)**2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(3/2), x)`

[Out] `Integral((A + B*sec(c + d*x))*sec(c + d*x)**2/(a*(sec(c + d*x) + 1))**(3/2), x)`

$$3.155 \quad \int \frac{\sec(c+dx)(A+B \sec(c+dx))}{(a+a \sec(c+dx))^{3/2}} dx$$

Optimal. Leaf size=87

$$\frac{(A+3B) \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{2} \sqrt{a \sec(c+dx)+a}}\right)}{2\sqrt{2} a^{3/2} d} + \frac{(A-B) \tan(c+dx)}{2d(a \sec(c+dx)+a)^{3/2}}$$

[Out] $1/4*(A+3*B)*\arctan(1/2*\sqrt{a}*\tan(d*x+c)*2^{(1/2)}/(a+a*\sec(d*x+c))^{(1/2)})/a^{(3/2)}/d*2^{(1/2)}+1/2*(A-B)*\tan(d*x+c)/d/(a+a*\sec(d*x+c))^{(3/2)}$

Rubi [A] time = 0.12, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {4000, 3795, 203}

$$\frac{(A+3B) \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{2} \sqrt{a \sec(c+dx)+a}}\right)}{2\sqrt{2} a^{3/2} d} + \frac{(A-B) \tan(c+dx)}{2d(a \sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(3/2), x]

[Out] ((A + 3*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]/(2*Sqrt[2]*a^(3/2)*d) + ((A - B)*Tan[c + d*x])/(2*d*(a + a*Sec[c + d*x])^(3/2))

Rule 203

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 3795

Int[csc[(e_) + (f_)*(x_)]/Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)], x_Symbol] := Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 4000

Int[csc[(e_) + (f_)*(x_)]*(csc[(e_) + (f_)*(x_)]*(b_) + (a_))^(m)*(csc[(e_) + (f_)*(x_)]*(B_) + (A_)), x_Symbol] := Simp[((A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]

/; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]

Rubi steps

$$\begin{aligned} \int \frac{\sec(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{3/2}} dx &= \frac{(A-B)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(A+3B) \int \frac{\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}} dx}{4a} \\ &= \frac{(A-B)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(A+3B) \operatorname{Subst}\left(\int \frac{1}{2a+x^2} dx, x, -\frac{a\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{2ad} \\ &= \frac{(A+3B) \tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} + \frac{(A-B)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} \end{aligned}$$

Mathematica [A] time = 0.82, size = 127, normalized size = 1.46

$$\frac{2(A-B)\sin(c+dx)\sqrt{1-\sec(c+dx)} + 2\sqrt{2}(A+3B)\cos^2\left(\frac{1}{2}(c+dx)\right)\tan(c+dx)\tanh^{-1}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)}{4ad(\cos(c+dx)+1)\sqrt{1-\sec(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(3/2), x]

[Out] (2*(A - B)*Sqrt[1 - Sec[c + d*x]]*Sin[c + d*x] + 2*Sqrt[2]*(A + 3*B)*ArcTan
h[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]*Cos[(c + d*x)/2]^2*Tan[c + d*x])/(4*a*d*(
1 + Cos[c + d*x])*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])

fricas [B] time = 0.51, size = 367, normalized size = 4.22

$$\left[\frac{4(A-B)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c) - \sqrt{2}\left((A+3B)\cos(dx+c)^2 + 2(A+3B)\cos(dx+c) + A\right)}{8\left(a^2d\cos(dx+c)^2 + 2a^2d\cos(dx+c) + \dots\right)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2), x, algorithm="fricas")


```
[Out] [1/8*(4*(A - B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) - sqrt(2)*((A + 3*B)*cos(d*x + c)^2 + 2*(A + 3*B)*cos(d*x + c) + A + 3*B)*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), 1/4*(2*(A - B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) - sqrt(2)*((A + 3*B)*cos(d*x + c)^2 + 2*(A + 3*B)*cos(d*x + c) + A + 3*B)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))]/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

giac [B] time = 1.94, size = 154, normalized size = 1.77

$$\frac{(\sqrt{2}A+3\sqrt{2}B)\log\left(-\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)}{\sqrt{-a}\operatorname{asgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)} - \frac{\left(\sqrt{2}A\operatorname{asgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)-\sqrt{2}B\operatorname{asgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)\right)\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{a^3}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*((sqrt(2)*A + 3*sqrt(2)*B)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/(sqrt(-a)*a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) - (sqrt(2)*A*a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1) - sqrt(2)*B*a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1))*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*tan(1/2*d*x + 1/2*c)/a^3)/d
```

maple [B] time = 1.42, size = 404, normalized size = 4.64

$$\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}} \left(-A \cos(dx+c) \sin(dx+c) \ln \left(-\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}} \sin(dx+c) + \cos(dx+c) - 1}{\sin(dx+c)} \right) \right) \sqrt{\frac{2\cos(dx+c)}{1+\cos(dx+c)}} - 3B \cos(dx+c)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/4/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-A*cos(d*x+c)*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-3*B*cos(d*x+c)*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-A*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*s
```

$\ln(d*x+c)+\cos(d*x+c)-1)/\sin(d*x+c))*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)*\sin(d*x+c)-3*B*\ln(-(-(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)*\sin(d*x+c)+\cos(d*x+c)-1)/\sin(d*x+c))*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)*\sin(d*x+c)+2*A*\cos(d*x+c)^2-2*B*\cos(d*x+c)^2-2*A*\cos(d*x+c)+2*B*\cos(d*x+c))/(1+\cos(d*x+c)))/\sin(d*x+c)/a^2$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sec(dx + c)}{(a \sec(dx + c) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)/(a*sec(d*x + c) + a)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\cos(c+dx) \left(a + \frac{a}{\cos(c+dx)}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/(cos(c + d*x)*(a + a/cos(c + d*x))^(3/2)),x)

[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)*(a + a/cos(c + d*x))^(3/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \sec(c + dx)}{(a (\sec(c + dx) + 1))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(3/2),x)

[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)/(a*(sec(c + d*x) + 1))**(3/2), x)

$$3.156 \quad \int \frac{A+B \sec(c+dx)}{(a+a \sec(c+dx))^{3/2}} dx$$

Optimal. Leaf size=127

$$-\frac{(5A-B) \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{2} \sqrt{a \sec(c+dx)+a}}\right)}{2\sqrt{2} a^{3/2}d} + \frac{2A \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}}\right)}{a^{3/2}d} - \frac{(A-B) \tan(c+dx)}{2d(a \sec(c+dx)+a)^{3/2}}$$

[Out] $2*A*\arctan(a^{(1/2)}*\tan(d*x+c)/(a+a*\sec(d*x+c))^{(1/2)})/a^{(3/2)}/d-1/4*(5*A-B)*\arctan(1/2*a^{(1/2)}*\tan(d*x+c)*2^{(1/2)}/(a+a*\sec(d*x+c))^{(1/2)})/a^{(3/2)}/d*2^{(1/2)}-1/2*(A-B)*\tan(d*x+c)/d/(a+a*\sec(d*x+c))^{(3/2)}$

Rubi [A] time = 0.18, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3922, 3920, 3774, 203, 3795}

$$-\frac{(5A-B) \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{2} \sqrt{a \sec(c+dx)+a}}\right)}{2\sqrt{2} a^{3/2}d} + \frac{2A \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}}\right)}{a^{3/2}d} - \frac{(A-B) \tan(c+dx)}{2d(a \sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

[In] `Int[(A + B*Sec[c + d*x])/(a + a*Sec[c + d*x])^(3/2), x]`

[Out] $(2*A*\text{ArcTan}[(\text{Sqrt}[a]*\text{Tan}[c + d*x])/\text{Sqrt}[a + a*\text{Sec}[c + d*x]])/(a^{(3/2)}*d) - ((5*A - B)*\text{ArcTan}[(\text{Sqrt}[a]*\text{Tan}[c + d*x])/(\text{Sqrt}[2]*\text{Sqrt}[a + a*\text{Sec}[c + d*x]])])/(2*\text{Sqrt}[2]*a^{(3/2)}*d) - ((A - B)*\text{Tan}[c + d*x])/(2*d*(a + a*\text{Sec}[c + d*x])^{(3/2)})$

Rule 203

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])`

Rule 3774

`Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] := Dist[(-2*b)/d, Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]`

Rule 3795

`Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] := Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[`

$a + b \cdot \text{Csc}[e + f \cdot x]]], x] /; \text{FreeQ}[\{a, b, e, f\}, x] \ \&\& \ \text{EqQ}[a^2 - b^2, 0]$

Rule 3920

$\text{Int}[(\text{csc}[(e_.) + (f_.) \cdot (x_)] \cdot (d_.) + (c_)) / \text{Sqrt}[\text{csc}[(e_.) + (f_.) \cdot (x_)] \cdot (b_.) + (a_)], x_Symbol] :> \text{Dist}[c/a, \text{Int}[\text{Sqrt}[a + b \cdot \text{Csc}[e + f \cdot x]], x], x] - \text{Dist}[(b \cdot c - a \cdot d)/a, \text{Int}[\text{Csc}[e + f \cdot x] / \text{Sqrt}[a + b \cdot \text{Csc}[e + f \cdot x]], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \ \&\& \ \text{NeQ}[b \cdot c - a \cdot d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0]$

Rule 3922

$\text{Int}[(\text{csc}[(e_.) + (f_.) \cdot (x_)] \cdot (b_.) + (a_))^{(m_)} \cdot (\text{csc}[(e_.) + (f_.) \cdot (x_)] \cdot (d_.) + (c_)), x_Symbol] :> -\text{Simp}[(b \cdot c - a \cdot d) \cdot \text{Cot}[e + f \cdot x] \cdot (a + b \cdot \text{Csc}[e + f \cdot x])^{(m)} / (b \cdot f \cdot (2 \cdot m + 1)), x] + \text{Dist}[1 / (a^2 \cdot (2 \cdot m + 1)), \text{Int}[(a + b \cdot \text{Csc}[e + f \cdot x])^{(m + 1)} \cdot \text{Simp}[a \cdot c \cdot (2 \cdot m + 1) - (b \cdot c - a \cdot d) \cdot (m + 1) \cdot \text{Csc}[e + f \cdot x], x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \ \&\& \ \text{NeQ}[b \cdot c - a \cdot d, 0] \ \&\& \ \text{LtQ}[m, -1] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{IntegerQ}[2 \cdot m]$

Rubi steps

$$\begin{aligned} \int \frac{A + B \sec(c + dx)}{(a + a \sec(c + dx))^{3/2}} dx &= -\frac{(A - B) \tan(c + dx)}{2d(a + a \sec(c + dx))^{3/2}} - \frac{\int \frac{-2aA + \frac{1}{2}a(A-B) \sec(c+dx)}{\sqrt{a+a \sec(c+dx)}} dx}{2a^2} \\ &= -\frac{(A - B) \tan(c + dx)}{2d(a + a \sec(c + dx))^{3/2}} + \frac{A \int \sqrt{a + a \sec(c + dx)} dx}{a^2} - \frac{(5A - B) \int \frac{\sec(c+dx)}{\sqrt{a+a \sec(c+dx)}} dx}{4a} \\ &= -\frac{(A - B) \tan(c + dx)}{2d(a + a \sec(c + dx))^{3/2}} - \frac{(2A) \text{Subst}\left(\int \frac{1}{a+x^2} dx, x, -\frac{a \tan(c+dx)}{\sqrt{a+a \sec(c+dx)}}\right)}{ad} + \frac{(5A - B) \int \frac{\sec(c+dx)}{\sqrt{a+a \sec(c+dx)}} dx}{4a} \\ &= \frac{2A \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a+a \sec(c+dx)}}\right)}{a^{3/2}d} - \frac{(5A - B) \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{2} \sqrt{a+a \sec(c+dx)}}\right)}{2\sqrt{2} a^{3/2}d} - \frac{(A - B) \tan(c + dx)}{2d(a + a \sec(c + dx))^{3/2}} \end{aligned}$$

Mathematica [C] time = 26.79, size = 11183, normalized size = 88.06

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(A + B*Sec[c + d*x])/(a + a*Sec[c + d*x])^(3/2),x]

[Out] Result too large to show

fricas [B] time = 3.22, size = 548, normalized size = 4.31

$$4(A - B) \sqrt{\frac{a \cos(dx+c)+a}{\cos(dx+c)}} \cos(dx+c) \sin(dx+c) - \sqrt{2} \left((5A - B) \cos(dx+c)^2 + 2(5A - B) \cos(dx+c) + 5 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")

[Out]
$$\begin{aligned} & [-1/8*(4*(A - B)*\sqrt{(a*\cos(d*x + c) + a)/\cos(d*x + c)}*\cos(d*x + c)*\sin(d \\ & *x + c) - \sqrt{2}*((5*A - B)*\cos(d*x + c)^2 + 2*(5*A - B)*\cos(d*x + c) + 5* \\ & A - B)*\sqrt{-a}*\log((2*\sqrt{2})*\sqrt{-a}*\sqrt{(a*\cos(d*x + c) + a)/\cos(d*x + \\ & c)}*\cos(d*x + c)*\sin(d*x + c) + 3*a*\cos(d*x + c)^2 + 2*a*\cos(d*x + c) - a) \\ & /(\cos(d*x + c)^2 + 2*\cos(d*x + c) + 1)) + 8*(A*\cos(d*x + c)^2 + 2*A*\cos(d*x \\ & + c) + A)*\sqrt{-a}*\log((2*a*\cos(d*x + c)^2 + 2*\sqrt{-a}*\sqrt{(a*\cos(d*x + \\ & c) + a)/\cos(d*x + c)}*\cos(d*x + c)*\sin(d*x + c) + a*\cos(d*x + c) - a)/(\cos(\\ & d*x + c) + 1)))/(a^2*d*\cos(d*x + c)^2 + 2*a^2*d*\cos(d*x + c) + a^2*d), -1/4 \\ & *(2*(A - B)*\sqrt{(a*\cos(d*x + c) + a)/\cos(d*x + c)}*\cos(d*x + c)*\sin(d*x + \\ & c) - \sqrt{2}*((5*A - B)*\cos(d*x + c)^2 + 2*(5*A - B)*\cos(d*x + c) + 5*A - B \\ &)*\sqrt{a}*\arctan(\sqrt{2}*\sqrt{(a*\cos(d*x + c) + a)/\cos(d*x + c)}*\cos(d*x + \\ & c)/(\sqrt{a}*\sin(d*x + c))) + 8*(A*\cos(d*x + c)^2 + 2*A*\cos(d*x + c) + A)*\sqrt{a} \\ & *\arctan(\sqrt{(a*\cos(d*x + c) + a)/\cos(d*x + c)}*\cos(d*x + c)/(\sqrt{a}*\sin(d*x + c))) \\ &))/(a^2*d*\cos(d*x + c)^2 + 2*a^2*d*\cos(d*x + c) + a^2*d)] \end{aligned}$$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
 UT:sage2:=int(sage0,x);;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
 Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
 gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Warni
 ng, integration of abs or sign assumes constant sign by intervals (correct
 if the argument is real):Check [abs(cos(d*t_nostep+c))]Unable to check sign
 : (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/
 2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_noste

p/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, assuming -2*a+a is positive. Hint: run assume to make assumptions on a variableWarning, assuming -2*a+a is positive. Hint: run assume to make assumptions on a variableWarning, assuming -2*a+a is positive. Hint: run assume to make assumptions on a variableDiscontinuities at zeroes of cos(d*t_nostep+c) were not checkedWarning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(t_nostep^2-1)]Evaluation time: 1.13index.cc index_m i_lex_is_greater Error: Bad Argument Value

maple [B] time = 1.40, size = 554, normalized size = 4.36

$$\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}} \left(4A \sin(dx+c) \cos(dx+c) \sqrt{\frac{-2\cos(dx+c)}{1+\cos(dx+c)}} \operatorname{arctanh}\left(\frac{\sqrt{\frac{-2\cos(dx+c)}{1+\cos(dx+c)}} \sin(dx+c) \sqrt{2}}{2\cos(dx+c)}}\right) \sqrt{2} + 5A \cos(dx+c) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2), x)

[Out]
$$-1/4/d*(a*(1+\cos(d*x+c))/\cos(d*x+c))^{1/2}*(4*A*\sin(d*x+c)*\cos(d*x+c)*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*\operatorname{arctanh}(1/2*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*\sin(d*x+c)/\cos(d*x+c)*2^{1/2})*2^{1/2}+5*A*\cos(d*x+c)*\sin(d*x+c)*\ln(-(-(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*\sin(d*x+c)+\cos(d*x+c)-1)/\sin(d*x+c))*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}+4*A*2^{1/2}*\operatorname{arctanh}(1/2*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*\sin(d*x+c)/\cos(d*x+c)*2^{1/2})*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*\sin(d*x+c)-B*\cos(d*x+c)*\sin(d*x+c)*\ln(-(-(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*\sin(d*x+c)+\cos(d*x+c)-1)/\sin(d*x+c))*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}+5*A*\ln(-(-(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*\sin(d*x+c)+\cos(d*x+c)-1)/\sin(d*x+c))*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*\sin(d*x+c)-B*\ln(-(-(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*\sin(d*x+c)+\cos(d*x+c)-1)/\sin(d*x+c))*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*\sin(d*x+c)-2*A*\cos(d*x+c)^2+2*B*\cos(d*x+c)^2+2*A*\cos(d*x+c)-2*B*\cos(d*x+c))/\sin(d*x+c)/a^2$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx+c) + A}{(a \sec(dx+c) + a)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)/(a*sec(d*x + c) + a)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\left(a + \frac{a}{\cos(c+dx)}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/(a + a/cos(c + d*x))^(3/2),x)

[Out] int((A + B/cos(c + d*x))/(a + a/cos(c + d*x))^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \sec(c + dx)}{(a(\sec(c + dx) + 1))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(3/2),x)

[Out] Integral((A + B*sec(c + d*x))/(a*(sec(c + d*x) + 1))**(3/2), x)

$$3.157 \quad \int \frac{\cos(c+dx)(A+B \sec(c+dx))}{(a+a \sec(c+dx))^{3/2}} dx$$

Optimal. Leaf size=170

$$-\frac{(3A-2B) \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}}\right)}{a^{3/2}d} + \frac{(9A-5B) \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{2} \sqrt{a \sec(c+dx)+a}}\right)}{2\sqrt{2} a^{3/2}d} + \frac{(3A-B) \sin(c+dx)}{2ad\sqrt{a \sec(c+dx)+a}} - \frac{(A-B) \sin(c+dx)}{2d(a \sec(c+dx)+a)}$$

[Out] $-(3A-2B) \arctan(a^{1/2} \tan(dx+c) / (a+a \sec(dx+c))^{1/2}) / a^{3/2} / d - 1/2 * (A-B) \sin(dx+c) / d / (a+a \sec(dx+c))^{3/2} + 1/4 * (9A-5B) \arctan(1/2 * a^{1/2} * \tan(dx+c) * 2^{1/2} / (a+a \sec(dx+c))^{1/2}) / a^{3/2} / d * 2^{1/2} + 1/2 * (3A-B) \sin(dx+c) / a / d / (a+a \sec(dx+c))^{1/2}$

Rubi [A] time = 0.41, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4020, 4022, 3920, 3774, 203, 3795}

$$-\frac{(3A-2B) \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}}\right)}{a^{3/2}d} + \frac{(9A-5B) \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{2} \sqrt{a \sec(c+dx)+a}}\right)}{2\sqrt{2} a^{3/2}d} + \frac{(3A-B) \sin(c+dx)}{2ad\sqrt{a \sec(c+dx)+a}} - \frac{(A-B) \sin(c+dx)}{2d(a \sec(c+dx)+a)}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(3/2), x]

[Out] $-(((3A-2B) \text{ArcTan}[\text{Sqrt}[a] \text{Tan}[c+d*x] / \text{Sqrt}[a+a \text{Sec}[c+d*x]]]) / (a^{3/2}d)) + ((9A-5B) \text{ArcTan}[\text{Sqrt}[a] \text{Tan}[c+d*x] / (\text{Sqrt}[2] \text{Sqrt}[a+a \text{Sec}[c+d*x]])]) / (2 \text{Sqrt}[2] a^{3/2}d) - ((A-B) \text{Sin}[c+d*x] / (2d(a+a \text{Sec}[c+d*x])^{3/2})) + ((3A-B) \text{Sin}[c+d*x] / (2a*d \text{Sqrt}[a+a \text{Sec}[c+d*x]]))$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 3774

Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_.)], x_Symbol] := Dist[(-2*b)/d, Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 3795


```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol]
:> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x]
/; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol]
:> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^m*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol]
:> -Simp[((A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m + 1)), x]
- Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e + f*x], x], x], x]
/; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0]
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^m*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol]
:> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n - A*b*(m + n + 1)*Csc[e + f*x], x], x], x]
/; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{3/2}} dx &= -\frac{(A-B)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{\int \frac{\cos(c+dx)\left(a(3A-B)-\frac{3}{2}a(A-B)\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}} dx}{2a^2} \\
&= -\frac{(A-B)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(3A-B)\sin(c+dx)}{2ad\sqrt{a+a\sec(c+dx)}} + \frac{\int \frac{-a^2(3A-2B)+\frac{1}{2}a^2(3A-2B)\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}} dx}{2a^3} \\
&= -\frac{(A-B)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(3A-B)\sin(c+dx)}{2ad\sqrt{a+a\sec(c+dx)}} + \frac{(9A-5B)\int \frac{\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}} dx}{4a} \\
&= -\frac{(A-B)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(3A-B)\sin(c+dx)}{2ad\sqrt{a+a\sec(c+dx)}} - \frac{(9A-5B)\text{Subst}\left(\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4a} \\
&= -\frac{(3A-2B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d} + \frac{(9A-5B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}
\end{aligned}$$

Mathematica [C] time = 27.27, size = 11954, normalized size = 70.32

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(Cos[c + d*x]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(3/2), x]

[Out] Result too large to show

fricas [A] time = 4.05, size = 609, normalized size = 3.58

$$\left[\sqrt{2} \left((9A - 5B) \cos(dx + c)^2 + 2(9A - 5B) \cos(dx + c) + 9A - 5B \right) \sqrt{-a} \log \left(-\frac{2\sqrt{2}\sqrt{-a} \sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}} \cos(dx+c) \sin(dx+c)}{\cos(dx+c)^2 + 2} \right) \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2), x, algorithm="fricas")

[Out] [1/8*(sqrt(2))*((9*A - 5*B)*cos(d*x + c)^2 + 2*(9*A - 5*B)*cos(d*x + c) + 9*A - 5*B)*sqrt(-a)*log(-(2*sqrt(2))*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))]

$$\begin{aligned}
& x + c)) \cdot \cos(dx + c) \cdot \sin(dx + c) - 3a \cdot \cos(dx + c)^2 - 2a \cdot \cos(dx + c) + \\
& a) / (\cos(dx + c)^2 + 2\cos(dx + c) + 1)) + 4 \cdot ((3A - 2B) \cdot \cos(dx + c)^2 \\
& + 2 \cdot (3A - 2B) \cdot \cos(dx + c) + 3A - 2B) \cdot \sqrt{-a} \cdot \log((2a \cdot \cos(dx + c)^2 \\
& + 2\sqrt{-a} \cdot \sqrt{(a \cdot \cos(dx + c) + a) / \cos(dx + c)}) \cdot \cos(dx + c) \cdot \sin(dx + \\
& c) + a \cdot \cos(dx + c) - a) / (\cos(dx + c) + 1)) + 4 \cdot (2A \cdot \cos(dx + c)^2 + (3A \\
& A - B) \cdot \cos(dx + c)) \cdot \sqrt{(a \cdot \cos(dx + c) + a) / \cos(dx + c)} \cdot \sin(dx + c)) / \\
& (a^2 \cdot d \cdot \cos(dx + c)^2 + 2a^2 \cdot d \cdot \cos(dx + c) + a^2 \cdot d), -1/4 \cdot (\sqrt{2}) \cdot ((9A \\
& - 5B) \cdot \cos(dx + c)^2 + 2 \cdot (9A - 5B) \cdot \cos(dx + c) + 9A - 5B) \cdot \sqrt{a} \cdot \arctan \\
& (\sqrt{2} \cdot \sqrt{(a \cdot \cos(dx + c) + a) / \cos(dx + c)}) \cdot \cos(dx + c) / (\sqrt{a} \cdot \sin \\
& (dx + c))) - 4 \cdot ((3A - 2B) \cdot \cos(dx + c)^2 + 2 \cdot (3A - 2B) \cdot \cos(dx + c) \\
& + 3A - 2B) \cdot \sqrt{a} \cdot \arctan(\sqrt{(a \cdot \cos(dx + c) + a) / \cos(dx + c)}) \cdot \cos(dx \\
& + c) / (\sqrt{a} \cdot \sin(dx + c))) - 2 \cdot (2A \cdot \cos(dx + c)^2 + (3A - B) \cdot \cos(dx + \\
& c)) \cdot \sqrt{(a \cdot \cos(dx + c) + a) / \cos(dx + c)} \cdot \sin(dx + c)) / (a^2 \cdot d \cdot \cos(dx + \\
& c)^2 + 2a^2 \cdot d \cdot \cos(dx + c) + a^2 \cdot d)]
\end{aligned}$$

giac [B] time = 13.19, size = 453, normalized size = 2.66

$$\frac{\sqrt{2} (9A - 5B) \log \left(\left(\sqrt{-a} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - \sqrt{-a \tan^2\left(\frac{1}{2} dx + \frac{1}{2} c\right) + a} \right)^2 \right)}{\sqrt{-a} \operatorname{sgn} \left(\tan^2\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 1 \right)} - \frac{4(3A - 2B) \log \left(\frac{\left(-17179869184 \left(\sqrt{-a} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - \sqrt{-a \tan^2\left(\frac{1}{2} dx + \frac{1}{2} c\right) + a} \right)^2 - 34359738368 \sqrt{2} \operatorname{abs}(a) + 51539607552 a \right)}{\left(-17179869184 \left(\sqrt{-a} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - \sqrt{-a \tan^2\left(\frac{1}{2} dx + \frac{1}{2} c\right) + a} \right)^2 + 34359738368 \sqrt{2} \operatorname{abs}(a) + 51539607552 a \right)} \right)}{\sqrt{-a} |\operatorname{sgn} \left(\tan^2\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 1 \right)|}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(dx+c)*(A+B*sec(dx+c))/(a+a*sec(dx+c))^(3/2),x, algorithm="giac")

[Out] 1/8*(sqrt(2)*(9A - 5*B)*log((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2)/(sqrt(-a)*a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) - 4*(3A - 2*B)*log(abs(-17179869184*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 34359738368*sqrt(2)*abs(a) + 51539607552*a)/abs(-17179869184*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 34359738368*sqrt(2)*abs(a) + 51539607552*a))/(sqrt(-a)*abs(a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) - 2*(sqrt(2)*A*a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1) - sqrt(2)*B*a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1))*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*tan(1/2*d*x + 1/2*c)/a^3 - 16*sqrt(2)*(3*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A - A*a)/(((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)*sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1))/d

maple [B] time = 1.62, size = 713, normalized size = 4.19

$$(-1 + \cos(dx + c)) \left(6A \sin(dx + c) \cos(dx + c) \sqrt{-\frac{2 \cos(dx+c)}{1+\cos(dx+c)}} \operatorname{arctanh} \left(\frac{\sqrt{-\frac{2 \cos(dx+c)}{1+\cos(dx+c)}} \sin(dx+c) \sqrt{2}}{2 \cos(dx+c)} \right) \sqrt{2} - 4B \sin \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2), x)`

[Out]
$$\begin{aligned} & -1/4/d*(-1+\cos(d*x+c))*(6*A*\sin(d*x+c)*\cos(d*x+c)*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*\operatorname{arctanh}(1/2*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*\sin(d*x+c)/\cos(d*x+c)*2^{(1/2)})*2^{(1/2)}-4*B*\sin(d*x+c)*\cos(d*x+c)*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*\operatorname{arctanh}(1/2*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*\sin(d*x+c)/\cos(d*x+c)*2^{(1/2)})*2^{(1/2)}+9*A*\cos(d*x+c)*\sin(d*x+c)*\ln(-(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*\sin(d*x+c)+\cos(d*x+c)-1)/\sin(d*x+c))*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}+6*A*2^{(1/2)}*\operatorname{arctanh}(1/2*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*\sin(d*x+c)/\cos(d*x+c)*2^{(1/2)})*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*\sin(d*x+c)-5*B*\cos(d*x+c)*\sin(d*x+c)*\ln(-(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*\sin(d*x+c)+\cos(d*x+c)-1)/\sin(d*x+c))*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}-4*B*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*\operatorname{arctanh}(1/2*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*\sin(d*x+c)/\cos(d*x+c)*2^{(1/2)})*2^{(1/2)}*\sin(d*x+c)+9*A*\ln(-(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*\sin(d*x+c)+\cos(d*x+c)-1)/\sin(d*x+c))*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*\sin(d*x+c)-4*A*\cos(d*x+c)^3-5*B*\ln(-(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*\sin(d*x+c)+\cos(d*x+c)-1)/\sin(d*x+c))*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*\sin(d*x+c)-2*A*\cos(d*x+c)^2+2*B*\cos(d*x+c)^2+6*A*\cos(d*x+c)-2*B*\cos(d*x+c))*(a*(1+\cos(d*x+c))/\cos(d*x+c))^{(1/2)}/\sin(d*x+c)^3/a^2 \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \cos(dx + c)}{(a \sec(dx + c) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2), x, algorithm="maxima")`

[Out] `integrate((B*sec(d*x + c) + A)*cos(d*x + c)/(a*sec(d*x + c) + a)^(3/2), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c + dx) \left(A + \frac{B}{\cos(c + dx)} \right)}{\left(a + \frac{a}{\cos(c + dx)} \right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((cos(c + d*x)*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^(3/2), x)`

[Out] `int((cos(c + d*x)*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^(3/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \cos(c + dx)}{(a (\sec(c + dx) + 1))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(3/2), x)`

[Out] `Integral((A + B*sec(c + d*x))*cos(c + d*x)/(a*(sec(c + d*x) + 1))**(3/2), x)`

$$3.158 \quad \int \frac{\cos^2(c+dx)(A+B \sec(c+dx))}{(a+a \sec(c+dx))^{3/2}} dx$$

Optimal. Leaf size=221

$$\frac{(19A - 12B) \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}}\right)}{4a^{3/2}d} - \frac{(13A - 9B) \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{2} \sqrt{a \sec(c+dx)+a}}\right)}{2\sqrt{2} a^{3/2}d} - \frac{(7A - 6B) \sin(c + dx)}{4ad\sqrt{a \sec(c + dx) + a}} + \frac{(2A - B) \sin(c + dx)}{2ad\sqrt{a \sec(c + dx) + a}}$$

[Out] $1/4*(19*A-12*B)*\arctan(a^{(1/2)}*\tan(d*x+c)/(a+a*\sec(d*x+c))^{(1/2)})/a^{(3/2)}/d$
 $-1/2*(A-B)*\cos(d*x+c)*\sin(d*x+c)/d/(a+a*\sec(d*x+c))^{(3/2)}-1/4*(13*A-9*B)*\ar$
 $\text{ctan}(1/2*a^{(1/2)}*\tan(d*x+c)*2^{(1/2)}/(a+a*\sec(d*x+c))^{(1/2)})/a^{(3/2)}/d*2^{(1/2)}$
 $-1/4*(7*A-6*B)*\sin(d*x+c)/a/d/(a+a*\sec(d*x+c))^{(1/2)}+1/2*(2*A-B)*\cos(d*x+c)*\sin(d*x+c)/a/d/(a+a*\sec(d*x+c))^{(1/2)}$

Rubi [A] time = 0.58, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4020, 4022, 3920, 3774, 203, 3795}

$$\frac{(19A - 12B) \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}}\right)}{4a^{3/2}d} - \frac{(13A - 9B) \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{2} \sqrt{a \sec(c+dx)+a}}\right)}{2\sqrt{2} a^{3/2}d} - \frac{(7A - 6B) \sin(c + dx)}{4ad\sqrt{a \sec(c + dx) + a}} + \frac{(2A - B) \sin(c + dx)}{2ad\sqrt{a \sec(c + dx) + a}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(\text{Cos}[c + d*x]^2*(A + B*\text{Sec}[c + d*x]))/(a + a*\text{Sec}[c + d*x])^{(3/2)}, x]$

[Out] $((19*A - 12*B)*\text{ArcTan}[(\text{Sqrt}[a]*\text{Tan}[c + d*x])/(\text{Sqrt}[a + a*\text{Sec}[c + d*x]])]/(4*a^{(3/2)*d}) - ((13*A - 9*B)*\text{ArcTan}[(\text{Sqrt}[a]*\text{Tan}[c + d*x])/(\text{Sqrt}[2]*\text{Sqrt}[a + a*\text{Sec}[c + d*x]])]/(2*\text{Sqrt}[2]*a^{(3/2)*d}) - ((A - B)*\text{Cos}[c + d*x]*\text{Sin}[c + d*x])/((2*d*(a + a*\text{Sec}[c + d*x])^{(3/2)}) - ((7*A - 6*B)*\text{Sin}[c + d*x])/((4*a*d*\text{Sqrt}[a + a*\text{Sec}[c + d*x]]) + ((2*A - B)*\text{Cos}[c + d*x]*\text{Sin}[c + d*x])/((2*a*d*\text{Sqrt}[a + a*\text{Sec}[c + d*x]])$

Rule 203

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1*\text{ArcTan}[(\text{Rt}[b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[b, 2]), x] /; \text{FreeQ}\{a, b, x\} \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{GtQ}[b, 0])$

Rule 3774

$\text{Int}[\text{Sqrt}[\text{csc}[(c_.) + (d_)*(x_)]*(b_.) + (a_)], x_Symbol] \rightarrow \text{Dist}[(-2*b)/d, \text{Subst}[\text{Int}[1/(a + x^2), x], x, (b*\text{Cot}[c + d*x])/(\text{Sqrt}[a + b*\text{Csc}[c + d*x]])], x] /; \text{FreeQ}\{a, b, c, d, x\} \ \&\& \ \text{EqQ}[a^2 - b^2, 0]$

Rule 3795

Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] := Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 3920

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] := Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]

Rule 4020

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^m*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] := -Simp[((A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m + 1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0]

Rule 4022

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^m*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] := Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{3/2}} dx &= -\frac{(A-B)\cos(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{\int \frac{\cos^2(c+dx)\left(2a(2A-B)-\frac{5}{2}a(A-B)\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}}{2a^2} \\
&= -\frac{(A-B)\cos(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(2A-B)\cos(c+dx)\sin(c+dx)}{2ad\sqrt{a+a\sec(c+dx)}} + \\
&= -\frac{(A-B)\cos(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(7A-6B)\sin(c+dx)}{4ad\sqrt{a+a\sec(c+dx)}} + \frac{(2A-B)}{2ad} \\
&= -\frac{(A-B)\cos(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(7A-6B)\sin(c+dx)}{4ad\sqrt{a+a\sec(c+dx)}} + \frac{(2A-B)}{2ad} \\
&= -\frac{(A-B)\cos(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(7A-6B)\sin(c+dx)}{4ad\sqrt{a+a\sec(c+dx)}} + \frac{(2A-B)}{2ad} \\
&= \frac{(19A-12B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4a^{3/2}d} - \frac{(13A-9B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}
\end{aligned}$$

Mathematica [C] time = 2.44, size = 395, normalized size = 1.79

$$\frac{\sec(c+dx)\left((91A-48B)(\sin(c+dx)+\tan(c+dx))\tanh^{-1}\left(\sqrt{1-\sec(c+dx)}\right)-40A\sqrt{1-\sec(c+dx)}\right)_2F_1\left(\frac{1}{2},3\right)}{16d\sqrt{1-\sec(c+dx)}(a(1+\sec(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(3/2), x]

[Out] (Sec[c + d*x]*(-52*sqrt[2]*A*ArcTanh[Sqrt[1 - Sec[c + d*x]]/sqrt[2]]*Sin[c + d*x] + 36*sqrt[2]*B*ArcTanh[Sqrt[1 - Sec[c + d*x]]/sqrt[2]]*Sin[c + d*x] - 13*A*sqrt[1 - Sec[c + d*x]]*Sin[c + d*x] + 24*B*sqrt[1 - Sec[c + d*x]]*Sin[c + d*x] + 18*A*cos[c + d*x]^2*sqrt[1 - Sec[c + d*x]]*Sin[c + d*x] + (13*A*sqrt[1 - Sec[c + d*x]]*Sin[2*(c + d*x)])/2 + 8*B*sqrt[1 - Sec[c + d*x]]*Sin[2*(c + d*x)] - 52*sqrt[2]*A*ArcTanh[Sqrt[1 - Sec[c + d*x]]/sqrt[2]]*Tan[c + d*x] + 36*sqrt[2]*B*ArcTanh[Sqrt[1 - Sec[c + d*x]]/sqrt[2]]*Tan[c + d*x] + (91*A - 48*B)*ArcTanh[Sqrt[1 - Sec[c + d*x]]]*(Sin[c + d*x] + Tan[c + d*x]) - 40*A*Hypergeometric2F1[1/2, 3, 3/2, 1 - Sec[c + d*x]]*sqrt[1 - Sec[c + d*x]]*(Sin[c + d*x] + Tan[c + d*x]))/(16*d*sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(3/2))

fricas [A] time = 5.84, size = 644, normalized size = 2.91

$$\left[\frac{\sqrt{2} \left((13A - 9B) \cos(dx + c)^2 + 2(13A - 9B) \cos(dx + c) + 13A - 9B \right) \sqrt{-a} \log \left(\frac{2\sqrt{2}\sqrt{-a} \sqrt{\frac{a \cos(dx+c)+a}{\cos(dx+c)}} \cos(dx+c)}{\cos(dx+c)} \right)}{\cos(dx+c)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")

[Out] [1/8*(sqrt(2)*((13*A - 9*B)*cos(d*x + c)^2 + 2*(13*A - 9*B)*cos(d*x + c) + 13*A - 9*B)*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + ((19*A - 12*B)*cos(d*x + c)^2 + 2*(19*A - 12*B)*cos(d*x + c) + 19*A - 12*B)*sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(2*A*cos(d*x + c)^3 - (3*A - 4*B)*cos(d*x + c)^2 - (7*A - 6*B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), 1/4*(sqrt(2)*((13*A - 9*B)*cos(d*x + c)^2 + 2*(13*A - 9*B)*cos(d*x + c) + 13*A - 9*B)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - ((19*A - 12*B)*cos(d*x + c)^2 + 2*(19*A - 12*B)*cos(d*x + c) + 19*A - 12*B)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) + (2*A*cos(d*x + c)^3 - (3*A - 4*B)*cos(d*x + c)^2 - (7*A - 6*B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]

giac [B] time = 3.18, size = 673, normalized size = 3.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")

[Out] -1/8*(sqrt(2)*(13*A - 9*B)*log((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2/(sqrt(-a)*a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) + (19*A - 12*B)*log(abs(147573952589676412928*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 295147905179352825856*sqrt(2)*a

```

bs(a) - 442721857769029238784*a)/abs(147573952589676412928*(sqrt(-a)*tan(1/
2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 29514790517935282
5856*sqrt(2)*abs(a) - 442721857769029238784*a))/(sqrt(-a)*abs(a)*sgn(tan(1/
2*d*x + 1/2*c)^2 - 1)) - 2*(sqrt(2)*A*a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1) - s
qrt(2)*B*a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1))*sqrt(-a*tan(1/2*d*x + 1/2*c)^2
+ a)*tan(1/2*d*x + 1/2*c)/a^3 - 4*sqrt(2)*(29*(sqrt(-a)*tan(1/2*d*x + 1/2*c
) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*A - 12*(sqrt(-a)*tan(1/2*d*x + 1
/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*B - 133*(sqrt(-a)*tan(1/2*d*
x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*A*a + 76*(sqrt(-a)*tan(
1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*B*a + 55*(sqrt(-a
)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*a^2 - 36*
(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*B*a
^2 - 7*A*a^3 + 4*B*a^3)/(((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*
d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2
*d*x + 1/2*c)^2 + a))^2*a + a^2)^2*sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1
))/d

```

maple [B] time = 1.80, size = 1075, normalized size = 4.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x)
```

```

[Out] -1/16/d*(-1+cos(d*x+c))*(19*A*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(
3/2)*cos(d*x+c)^2*sin(d*x+c)-12*B*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)
))^3/2*cos(d*x+c)^2*sin(d*x+c)+38*A*cos(d*x+c)*arctanh(1/2*(-2*cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos
(d*x+c)))^(3/2)*sin(d*x+c)*2^(1/2)+26*A*ln(-(-2*cos(d*x+c)/(1+cos(d*x+c))
))^1/2*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c))
)^(3/2)*sin(d*x+c)*cos(d*x+c)^2-24*B*cos(d*x+c)*arctanh(1/2*(-2*cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d
*x+c)))^(3/2)*sin(d*x+c)*2^(1/2)-18*B*ln(-(-2*cos(d*x+c)/(1+cos(d*x+c))
))^1/2*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c))
)^(3/2)*sin(d*x+c)*cos(d*x+c)^2+19*A*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)
))^3/2)*sin(d*x+c)+52*A*cos(d*x+c)*sin(d*x+c)*ln(-(-2*cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d
*x+c)))^(3/2)-12*B*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*sin(d*
x+c)-36*B*cos(d*x+c)*sin(d*x+c)*ln(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-8
*A*cos(d*x+c)^5+26*A*ln(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+

```

$\cos(dx+c)-1)/\sin(dx+c))*(-2*\cos(dx+c)/(1+\cos(dx+c)))^{(3/2)}*\sin(dx+c)-1$
 $8*B*\ln(-(-2*\cos(dx+c)/(1+\cos(dx+c)))^{(1/2)}*\sin(dx+c)+\cos(dx+c)-1)/\sin$
 $(dx+c))*(-2*\cos(dx+c)/(1+\cos(dx+c)))^{(3/2)}*\sin(dx+c)+20*A*\cos(dx+c)^4-$
 $16*B*\cos(dx+c)^4+16*A*\cos(dx+c)^3-8*B*\cos(dx+c)^3-28*A*\cos(dx+c)^2+24*B$
 $*\cos(dx+c)^2)*(a*(1+\cos(dx+c))/\cos(dx+c))^{(1/2)}/\cos(dx+c)/\sin(dx+c)^3/$
 a^2

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx+c) + A) \cos(dx+c)^2}{(a \sec(dx+c) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(dx+c)^2*(A+B*sec(dx+c))/(a+a*sec(dx+c))^(3/2),x, algorithm="maxima")

[Out] integrate((B*sec(dx+c) + A)*cos(dx+c)^2/(a*sec(dx+c) + a)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+dx)^2 \left(A + \frac{B}{\cos(c+dx)} \right)}{\left(a + \frac{a}{\cos(c+dx)} \right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c+dx)^2*(A+B/cos(c+dx)))/(a+a/cos(c+dx))^(3/2),x)

[Out] int((cos(c+dx)^2*(A+B/cos(c+dx)))/(a+a/cos(c+dx))^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c+dx)) \cos^2(c+dx)}{(a(\sec(c+dx) + 1))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(dx+c)**2*(A+B*sec(dx+c))/(a+a*sec(dx+c))**(3/2),x)

[Out] Integral((A + B*sec(c+dx))*cos(c+dx)**2/(a*(sec(c+dx) + 1))**(3/2), x)

$$3.159 \quad \int \frac{\cos^3(c+dx)(A+B \sec(c+dx))}{(a+a \sec(c+dx))^{3/2}} dx$$

Optimal. Leaf size=268

$$-\frac{(47A - 38B) \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}}\right)}{8a^{3/2}d} + \frac{(17A - 13B) \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{2} \sqrt{a \sec(c+dx)+a}}\right)}{2\sqrt{2} a^{3/2}d} + \frac{7(3A - 2B) \sin(c + dx)}{8ad\sqrt{a \sec(c + dx) + a}} + \frac{(5A - 3B)}{6ad}$$

[Out] $-1/8*(47*A-38*B)*\arctan(a^{(1/2)}*\tan(d*x+c)/(a+a*\sec(d*x+c))^{(1/2)})/a^{(3/2)}/d-1/2*(A-B)*\cos(d*x+c)^2*\sin(d*x+c)/d/(a+a*\sec(d*x+c))^{(3/2)}+1/4*(17*A-13*B)*\arctan(1/2*a^{(1/2)}*\tan(d*x+c)*2^{(1/2)}/(a+a*\sec(d*x+c))^{(1/2)})/a^{(3/2)}/d*2^{(1/2)}+7/8*(3*A-2*B)*\sin(d*x+c)/a/d/(a+a*\sec(d*x+c))^{(1/2)}-1/12*(13*A-12*B)*\cos(d*x+c)*\sin(d*x+c)/a/d/(a+a*\sec(d*x+c))^{(1/2)}+1/6*(5*A-3*B)*\cos(d*x+c)^2*\sin(d*x+c)/a/d/(a+a*\sec(d*x+c))^{(1/2)}$

Rubi [A] time = 0.78, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4020, 4022, 3920, 3774, 203, 3795}

$$-\frac{(47A - 38B) \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}}\right)}{8a^{3/2}d} + \frac{(17A - 13B) \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{2} \sqrt{a \sec(c+dx)+a}}\right)}{2\sqrt{2} a^{3/2}d} + \frac{7(3A - 2B) \sin(c + dx)}{8ad\sqrt{a \sec(c + dx) + a}} + \frac{(5A - 3B)}{6ad}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(3/2),x]

[Out] $-((47*A - 38*B)*\text{ArcTan}[(\text{Sqrt}[a]*\text{Tan}[c + d*x])/\text{Sqrt}[a + a*\text{Sec}[c + d*x]])/(8*a^{(3/2)*d}) + ((17*A - 13*B)*\text{ArcTan}[(\text{Sqrt}[a]*\text{Tan}[c + d*x])/(\text{Sqrt}[2]*\text{Sqrt}[a + a*\text{Sec}[c + d*x]])]/(2*\text{Sqrt}[2]*a^{(3/2)*d}) - ((A - B)*\text{Cos}[c + d*x]^2*\text{Sin}[c + d*x])/(2*d*(a + a*\text{Sec}[c + d*x])^{(3/2)}) + (7*(3*A - 2*B)*\text{Sin}[c + d*x])/(8*a*d*\text{Sqrt}[a + a*\text{Sec}[c + d*x]]) - ((13*A - 12*B)*\text{Cos}[c + d*x]*\text{Sin}[c + d*x])/(12*a*d*\text{Sqrt}[a + a*\text{Sec}[c + d*x]]) + ((5*A - 3*B)*\text{Cos}[c + d*x]^2*\text{Sin}[c + d*x])/(6*a*d*\text{Sqrt}[a + a*\text{Sec}[c + d*x]])$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 3774

Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] := Dist[(-2*b)/d, Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],

$x] /; \text{FreeQ}\{a, b, c, d\}, x\} \&\& \text{EqQ}[a^2 - b^2, 0]$

Rule 3795

$\text{Int}[\text{csc}[(e_.) + (f_.)(x_)]/\text{Sqrt}[\text{csc}[(e_.) + (f_.)(x_)]*(b_.) + (a_.)], x_Symbol] \rightarrow \text{Dist}[-2/f, \text{Subst}[\text{Int}[1/(2*a + x^2), x], x, (b*\text{Cot}[e + f*x])/\text{Sqrt}[a + b*\text{Csc}[e + f*x]]], x] /; \text{FreeQ}\{a, b, e, f\}, x\} \&\& \text{EqQ}[a^2 - b^2, 0]$

Rule 3920

$\text{Int}[(\text{csc}[(e_.) + (f_.)(x_)]*(d_.) + (c_.))/\text{Sqrt}[\text{csc}[(e_.) + (f_.)(x_)]*(b_.) + (a_.)], x_Symbol] \rightarrow \text{Dist}[c/a, \text{Int}[\text{Sqrt}[a + b*\text{Csc}[e + f*x]], x], x] - \text{Dist}[(b*c - a*d)/a, \text{Int}[\text{Csc}[e + f*x]/\text{Sqrt}[a + b*\text{Csc}[e + f*x]], x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x\} \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0]$

Rule 4020

$\text{Int}[(\text{csc}[(e_.) + (f_.)(x_)]*(d_.))^n*(\text{csc}[(e_.) + (f_.)(x_)]*(b_.) + (a_.))^m*(\text{csc}[(e_.) + (f_.)(x_)]*(B_.) + (A_.)), x_Symbol] \rightarrow -\text{Simp}[(A*b - a*B)*\text{Cot}[e + f*x]*(a + b*\text{Csc}[e + f*x])^m*(d*\text{Csc}[e + f*x])^n/(b*f*(2*m + 1)), x] - \text{Dist}[1/(a^2*(2*m + 1)), \text{Int}[(a + b*\text{Csc}[e + f*x])^{m+1}*(d*\text{Csc}[e + f*x])^n*\text{Simp}[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*\text{Csc}[e + f*x], x], x], x] /; \text{FreeQ}\{a, b, d, e, f, A, B, n\}, x\} \&\& \text{NeQ}[A*b - a*B, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{LtQ}[m, -2^{(-1)}] \&\& !\text{GtQ}[n, 0]$

Rule 4022

$\text{Int}[(\text{csc}[(e_.) + (f_.)(x_)]*(d_.))^n*(\text{csc}[(e_.) + (f_.)(x_)]*(b_.) + (a_.))^m*(\text{csc}[(e_.) + (f_.)(x_)]*(B_.) + (A_.)), x_Symbol] \rightarrow \text{Simp}[(A*\text{Cot}[e + f*x]*(a + b*\text{Csc}[e + f*x])^m*(d*\text{Csc}[e + f*x])^n)/(f*n), x] - \text{Dist}[1/(b*d*n), \text{Int}[(a + b*\text{Csc}[e + f*x])^m*(d*\text{Csc}[e + f*x])^{n+1}*\text{Simp}[a*A*m - b*B*n - A*b*(m + n + 1)*\text{Csc}[e + f*x], x], x], x] /; \text{FreeQ}\{a, b, d, e, f, A, B, m\}, x\} \&\& \text{NeQ}[A*b - a*B, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{LtQ}[n, 0]$

Rubi steps

$$\begin{aligned}
\int \frac{\cos^3(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{3/2}} dx &= -\frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \int \frac{\cos^3(c+dx)\left(a(5A-3B)-\frac{7}{2}a(A-B)\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)} \cdot 2a^2} \\
&= -\frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(5A-3B)\cos^2(c+dx)\sin(c+dx)}{6ad\sqrt{a+a\sec(c+dx)}} \\
&= -\frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(13A-12B)\cos(c+dx)\sin(c+dx)}{12ad\sqrt{a+a\sec(c+dx)}} \\
&= -\frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{7(3A-2B)\sin(c+dx)}{8ad\sqrt{a+a\sec(c+dx)}} - \frac{(13A-12B)\cos(c+dx)\sin(c+dx)}{12ad\sqrt{a+a\sec(c+dx)}} \\
&= -\frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{7(3A-2B)\sin(c+dx)}{8ad\sqrt{a+a\sec(c+dx)}} - \frac{(13A-12B)\cos(c+dx)\sin(c+dx)}{12ad\sqrt{a+a\sec(c+dx)}} \\
&= -\frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{7(3A-2B)\sin(c+dx)}{8ad\sqrt{a+a\sec(c+dx)}} - \frac{(13A-12B)\cos(c+dx)\sin(c+dx)}{12ad\sqrt{a+a\sec(c+dx)}} \\
&= -\frac{(47A-38B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{8a^{3/2}d} + \frac{(17A-13B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}
\end{aligned}$$

Mathematica [C] time = 6.15, size = 502, normalized size = 1.87

$$\frac{A(\sec(c+dx)+1)^{3/2} \left(\frac{336 \tan(c+dx) {}_2F_1\left(\frac{1}{2}, 4; \frac{3}{2}; 1-\sec(c+dx)\right)}{d\sqrt{\sec(c+dx)+1}} + \frac{17 \tan(c+dx) \left(-8 \cos^3(c+dx) \sqrt{1-\sec(c+dx)} + 2 \cos^2(c+dx) \sqrt{1-\sec(c+dx)} + \dots\right)}{d\sqrt{1-\sec(c+dx)}} \right)}{96(a(\sec(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(3/2), x]

[Out] -1/2*(B*Cos[c + d*x]*Sin[c + d*x])/(d*(a*(1 + Sec[c + d*x]))^(3/2)) - (A*Cos[c + d*x]^2*Sin[c + d*x])/(2*d*(a*(1 + Sec[c + d*x]))^(3/2)) - (B*(1 + Sec[c + d*x])^(3/2)*((40*Hypergeometric2F1[1/2, 3, 3/2, 1 - Sec[c + d*x]]*Tan[c + d*x])/(d*Sqrt[1 + Sec[c + d*x]]) - (13*(7*ArcTanh[Sqrt[1 - Sec[c + d*x]])] - 4*Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]] - Cos[c + d*x]*Sqrt[

$$\frac{1 - \operatorname{Sec}[c + d*x] + 2*\operatorname{Cos}[c + d*x]^2*\operatorname{Sqrt}[1 - \operatorname{Sec}[c + d*x]]*\operatorname{Tan}[c + d*x]}{(d*\operatorname{Sqrt}[1 - \operatorname{Sec}[c + d*x]]*\operatorname{Sqrt}[1 + \operatorname{Sec}[c + d*x]])}/(16*(a*(1 + \operatorname{Sec}[c + d*x]))^{(3/2)}) - (A*(1 + \operatorname{Sec}[c + d*x])^{(3/2)}*((336*\operatorname{Hypergeometric2F1}[1/2, 4, 3/2, 1 - \operatorname{Sec}[c + d*x]]*\operatorname{Tan}[c + d*x])/(d*\operatorname{Sqrt}[1 + \operatorname{Sec}[c + d*x]]) + (17*(3*(9*\operatorname{ArcTanh}[\operatorname{Sqrt}[1 - \operatorname{Sec}[c + d*x]])] - 8*\operatorname{Sqrt}[2]*\operatorname{ArcTanh}[\operatorname{Sqrt}[1 - \operatorname{Sec}[c + d*x]])/\operatorname{Sqrt}[2]] - 7*\operatorname{Cos}[c + d*x]*\operatorname{Sqrt}[1 - \operatorname{Sec}[c + d*x]]) + 2*\operatorname{Cos}[c + d*x]^2*\operatorname{Sqrt}[1 - \operatorname{Sec}[c + d*x]] - 8*\operatorname{Cos}[c + d*x]^3*\operatorname{Sqrt}[1 - \operatorname{Sec}[c + d*x]]*\operatorname{Tan}[c + d*x])/(d*\operatorname{Sqrt}[1 - \operatorname{Sec}[c + d*x]]*\operatorname{Sqrt}[1 + \operatorname{Sec}[c + d*x]])))/(96*(a*(1 + \operatorname{Sec}[c + d*x]))^{(3/2)})$$

fricas [A] time = 5.78, size = 675, normalized size = 2.52

$$\left[\frac{6\sqrt{2}\left((17A - 13B)\cos(dx + c)^2 + 2(17A - 13B)\cos(dx + c) + 17A - 13B\right)\sqrt{-a} \log\left(-\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}}{\dots}\right)}{\dots} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")

[Out] [1/48*(6*sqrt(2)*((17*A - 13*B)*cos(d*x + c)^2 + 2*(17*A - 13*B)*cos(d*x + c) + 17*A - 13*B)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) - 3*a*cos(d*x + c)^2 - 2*a*cos(d*x + c) + a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 3*((47*A - 38*B)*cos(d*x + c)^2 + 2*(47*A - 38*B)*cos(d*x + c) + 47*A - 38*B)*sqrt(-a)*log((2*a*cos(d*x + c)^2 + 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(8*A*cos(d*x + c)^4 - 6*(A - 2*B)*cos(d*x + c)^3 + (37*A - 18*B)*cos(d*x + c)^2 + 21*(3*A - 2*B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), -1/24*(6*sqrt(2)*((17*A - 13*B)*cos(d*x + c)^2 + 2*(17*A - 13*B)*cos(d*x + c) + 17*A - 13*B)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - 3*((47*A - 38*B)*cos(d*x + c)^2 + 2*(47*A - 38*B)*cos(d*x + c) + 47*A - 38*B)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - (8*A*cos(d*x + c)^4 - 6*(A - 2*B)*cos(d*x + c)^3 + (37*A - 18*B)*cos(d*x + c)^2 + 21*(3*A - 2*B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]

giac [B] time = 3.34, size = 851, normalized size = 3.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algorithm
="giac")
```

```
[Out] 1/48*(6*sqrt(2)*(17*A - 13*B)*log((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*
tan(1/2*d*x + 1/2*c)^2 + a))^2)/(sqrt(-a)*a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)
) - 3*(47*A - 38*B)*log(abs(-1947111321950560360698936123457536*(sqrt(-a)*t
an(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 389422264390
1120721397872246915072*sqrt(2)*abs(a) + 5841333965851681082096808370372608*
a)/abs(-1947111321950560360698936123457536*(sqrt(-a)*tan(1/2*d*x + 1/2*c) -
sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 389422264390112072139787224691507
2*sqrt(2)*abs(a) + 5841333965851681082096808370372608*a))/(sqrt(-a)*abs(a)*
sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) - 12*(sqrt(2)*A*a*sgn(tan(1/2*d*x + 1/2*c)
^2 - 1) - sqrt(2)*B*a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1))*sqrt(-a*tan(1/2*d*x
+ 1/2*c)^2 + a)*tan(1/2*d*x + 1/2*c)/a^3 - 4*sqrt(2)*(339*(sqrt(-a)*tan(1/2
*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^10*A - 174*(sqrt(-a)*t
an(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^10*B - 3165*(sqr
t(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^8*A*a + 1
842*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^8
*B*a + 9198*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2
+ a))^6*A*a^2 - 5292*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x
+ 1/2*c)^2 + a))^6*B*a^2 - 4938*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*ta
n(1/2*d*x + 1/2*c)^2 + a))^4*A*a^3 + 2820*(sqrt(-a)*tan(1/2*d*x + 1/2*c) -
sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*B*a^3 + 975*(sqrt(-a)*tan(1/2*d*x +
1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*a^4 - 582*(sqrt(-a)*tan(1
/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*B*a^4 - 73*A*a^5 +
42*B*a^5)/(((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^
2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)
^2 + a))^2*a + a^2)^3*sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1))/d
```

maple [B] time = 1.68, size = 1425, normalized size = 5.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/192/d*(-1+cos(d*x+c))*(423*A*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(
5/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)
*2^(1/2))*2^(1/2)*cos(d*x+c)-342*B*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)
))^5/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x
+c)*2^(1/2))*2^(1/2)*cos(d*x+c)+204*A*cos(d*x+c)^3*sin(d*x+c)*(-2*cos(d*x+c
)/(1+cos(d*x+c)))^(5/2)*ln(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+
c)+cos(d*x+c)-1)/sin(d*x+c))-156*B*cos(d*x+c)^3*sin(d*x+c)*(-2*cos(d*x+c)/(
```


$(1 + \cos(dx+c))^{5/2} \ln(-(-(-2\cos(dx+c)/(1+\cos(dx+c)))^{1/2} \sin(dx+c) + \cos(dx+c) - 1) / \sin(dx+c) - 208A\cos(dx+c)^4 + 192B\cos(dx+c)^4 + 141A(-2\cos(dx+c)/(1+\cos(dx+c)))^{5/2} \operatorname{arctanh}(1/2(-2\cos(dx+c)/(1+\cos(dx+c)))^{1/2} \sin(dx+c) / \cos(dx+c) * 2^{1/2}) * 2^{1/2} \cos(dx+c)^3 \sin(dx+c) - 342B\sin(dx+c) * (-2\cos(dx+c)/(1+\cos(dx+c)))^{5/2} \operatorname{arctanh}(1/2(-2\cos(dx+c)/(1+\cos(dx+c)))^{1/2} \sin(dx+c) / \cos(dx+c) * 2^{1/2}) * 2^{1/2} \cos(dx+c)^2 + 23A\sin(dx+c) * (-2\cos(dx+c)/(1+\cos(dx+c)))^{5/2} \operatorname{arctanh}(1/2(-2\cos(dx+c)/(1+\cos(dx+c)))^{1/2} \sin(dx+c) / \cos(dx+c) * 2^{1/2}) * 2^{1/2} \cos(dx+c)^2 + 612A(-2\cos(dx+c)/(1+\cos(dx+c)))^{5/2} \ln(-(-(-2\cos(dx+c)/(1+\cos(dx+c)))^{1/2} \sin(dx+c) + \cos(dx+c) - 1) / \sin(dx+c)) * \sin(dx+c) * \cos(dx+c)^2 - 468B(-2\cos(dx+c)/(1+\cos(dx+c)))^{5/2} \ln(-(-(-2\cos(dx+c)/(1+\cos(dx+c)))^{1/2} \sin(dx+c) + \cos(dx+c) - 1) / \sin(dx+c)) * \sin(dx+c) * \cos(dx+c)^2 + 612A(-2\cos(dx+c)/(1+\cos(dx+c)))^{5/2} \ln(-(-(-2\cos(dx+c)/(1+\cos(dx+c)))^{1/2} \sin(dx+c) + \cos(dx+c) - 1) / \sin(dx+c)) * \sin(dx+c) * \cos(dx+c) - 468B(-2\cos(dx+c)/(1+\cos(dx+c)))^{5/2} \ln(-(-(-2\cos(dx+c)/(1+\cos(dx+c)))^{1/2} \sin(dx+c) + \cos(dx+c) - 1) / \sin(dx+c)) * \sin(dx+c) * \cos(dx+c) + 504A\cos(dx+c)^3 - 336B\cos(dx+c)^3 - 344A\cos(dx+c)^5 + 240B\cos(dx+c)^5 + 204A(-2\cos(dx+c)/(1+\cos(dx+c)))^{5/2} \ln(-(-(-2\cos(dx+c)/(1+\cos(dx+c)))^{1/2} \sin(dx+c) + \cos(dx+c) - 1) / \sin(dx+c)) * \sin(dx+c) - 156B(-2\cos(dx+c)/(1+\cos(dx+c)))^{5/2} \ln(-(-(-2\cos(dx+c)/(1+\cos(dx+c)))^{1/2} \sin(dx+c) + \cos(dx+c) - 1) / \sin(dx+c)) * \sin(dx+c) - 114B(-2\cos(dx+c)/(1+\cos(dx+c)))^{5/2} \operatorname{arctanh}(1/2(-2\cos(dx+c)/(1+\cos(dx+c)))^{1/2} \sin(dx+c) / \cos(dx+c) * 2^{1/2}) * 2^{1/2} \cos(dx+c)^3 \sin(dx+c) + 112A\cos(dx+c)^6 - 64A\cos(dx+c)^7 + 141A(-2\cos(dx+c)/(1+\cos(dx+c)))^{5/2} * 2^{1/2} \operatorname{arctanh}(1/2(-2\cos(dx+c)/(1+\cos(dx+c)))^{1/2} \sin(dx+c) / \cos(dx+c) * 2^{1/2}) * \sin(dx+c) - 114B(-2\cos(dx+c)/(1+\cos(dx+c)))^{5/2} * 2^{1/2} \operatorname{arctanh}(1/2(-2\cos(dx+c)/(1+\cos(dx+c)))^{1/2} \sin(dx+c) / \cos(dx+c) * 2^{1/2}) * \sin(dx+c) - 96B\cos(dx+c)^6) * (a(1+\cos(dx+c)) / \cos(dx+c))^{1/2} / \cos(dx+c)^2 / \sin(dx+c)^3 / a^2$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx+c) + A) \cos(dx+c)^3}{(a \sec(dx+c) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(dx+c)^3*(A+B*sec(dx+c))/(a+a*sec(dx+c))^(3/2),x, algorithm="maxima")

[Out] integrate((B*sec(dx+c) + A)*cos(dx+c)^3/(a*sec(dx+c) + a)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c + dx)^3 \left(A + \frac{B}{\cos(c+dx)} \right)}{\left(a + \frac{a}{\cos(c+dx)} \right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)^3*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^(3/2), x)

[Out] int((cos(c + d*x)^3*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \cos^3(c + dx)}{(a(\sec(c + dx) + 1))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(3/2), x)

[Out] Integral((A + B*sec(c + d*x))*cos(c + d*x)**3/(a*(sec(c + d*x) + 1))**(3/2), x)

$$3.160 \quad \int \frac{\sec^4(c+dx)(A+B \sec(c+dx))}{(a+a \sec(c+dx))^{5/2}} dx$$

Optimal. Leaf size=216

$$\frac{(75A - 163B) \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{2} \sqrt{a \sec(c+dx)+a}}\right)}{16\sqrt{2} a^{5/2}d} - \frac{(39A - 95B) \tan(c+dx) \sqrt{a \sec(c+dx)+a}}{48a^3d} + \frac{(93A - 197B) \tan(c+dx)}{24a^2d \sqrt{a \sec(c+dx)+a}}$$

[Out] $-1/32*(75*A-163*B)*\arctan(1/2*a^{(1/2)}*\tan(d*x+c)*2^{(1/2)}/(a+a*\sec(d*x+c))^{(1/2)})/a^{(5/2)}/d*2^{(1/2)}+1/4*(A-B)*\sec(d*x+c)^3*\tan(d*x+c)/d/(a+a*\sec(d*x+c))^{(5/2)}+1/16*(9*A-17*B)*\sec(d*x+c)^2*\tan(d*x+c)/a/d/(a+a*\sec(d*x+c))^{(3/2)}+1/24*(93*A-197*B)*\tan(d*x+c)/a^2/d/(a+a*\sec(d*x+c))^{(1/2)}-1/48*(39*A-95*B)*(a+a*\sec(d*x+c))^{(1/2)}*\tan(d*x+c)/a^3/d$

Rubi [A] time = 0.65, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {4019, 4010, 4001, 3795, 203}

$$\frac{(75A - 163B) \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{2} \sqrt{a \sec(c+dx)+a}}\right)}{16\sqrt{2} a^{5/2}d} - \frac{(39A - 95B) \tan(c+dx) \sqrt{a \sec(c+dx)+a}}{48a^3d} + \frac{(93A - 197B) \tan(c+dx)}{24a^2d \sqrt{a \sec(c+dx)+a}}$$

Antiderivative was successfully verified.

[In] Int[(Sec[c + d*x]^4*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(5/2),x]

[Out] $-((75*A - 163*B)*\text{ArcTan}[(\text{Sqrt}[a]*\text{Tan}[c + d*x])/(\text{Sqrt}[2]*\text{Sqrt}[a + a*\text{Sec}[c + d*x]])])/((16*\text{Sqrt}[2]*a^{(5/2)}*d) + ((A - B)*\text{Sec}[c + d*x]^3*\text{Tan}[c + d*x])/((4*d*(a + a*\text{Sec}[c + d*x])^{(5/2)}) + ((9*A - 17*B)*\text{Sec}[c + d*x]^2*\text{Tan}[c + d*x])/((16*a*d*(a + a*\text{Sec}[c + d*x])^{(3/2)}) + ((93*A - 197*B)*\text{Tan}[c + d*x])/((24*a^2*d*\text{Sqrt}[a + a*\text{Sec}[c + d*x]) - ((39*A - 95*B)*\text{Sqrt}[a + a*\text{Sec}[c + d*x])*\text{Tan}[c + d*x])/((48*a^3*d)$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 3795

Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] := Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_.)]*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := -Simp[(B*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m + 1), 0] && !LtQ[m, -2^(-1)]
```

Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_.)]^2*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := -Simp[(B*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && !LtQ[m, -1]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := Simp[(d*(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^4(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{5/2}} dx &= \frac{(A-B)\sec^3(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \int \frac{\sec^3(c+dx)\left(3a(A-B)-\frac{1}{2}a(3A-11B)\sec(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}} \frac{1}{4a^2} dx \\
&= \frac{(A-B)\sec^3(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(9A-17B)\sec^2(c+dx)\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} \\
&= \frac{(A-B)\sec^3(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(9A-17B)\sec^2(c+dx)\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} \\
&= \frac{(A-B)\sec^3(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(9A-17B)\sec^2(c+dx)\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} \\
&= \frac{(A-B)\sec^3(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(9A-17B)\sec^2(c+dx)\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} \\
&= -\frac{(75A-163B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} + \frac{(A-B)\sec^3(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}}
\end{aligned}$$

Mathematica [A] time = 2.71, size = 161, normalized size = 0.75

$$\frac{\tan(c+dx)\left(\sqrt{1-\sec(c+dx)}\left(32(3A-5B)\sec^2(c+dx)+(255A-503B)\sec(c+dx)+147A+32B\sec^3(c+dx)\right)\right)}{48d\sqrt{1-\sec(c+dx)}(a(\sec(c+dx)))^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[c + d*x]^4*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(5/2), x]

[Out] ((-6*Sqrt[2]*(75*A - 163*B)*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]*Cos[(c + d*x)/2]^4*Sec[c + d*x]^2 + Sqrt[1 - Sec[c + d*x]]*(147*A - 299*B + (255*A - 503*B)*Sec[c + d*x] + 32*(3*A - 5*B)*Sec[c + d*x]^2 + 32*B*Sec[c + d*x]^3))*Tan[c + d*x])/(48*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(5/2))

fricas [A] time = 0.49, size = 557, normalized size = 2.58

$$3\sqrt{2}\left((75A - 163B)\cos(dx + c)^4 + 3(75A - 163B)\cos(dx + c)^3 + 3(75A - 163B)\cos(dx + c)^2 + (75A - 163B)\cos(dx + c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")

[Out] [1/192*(3*sqrt(2)*((75*A - 163*B)*cos(d*x + c)^4 + 3*(75*A - 163*B)*cos(d*x + c)^3 + 3*(75*A - 163*B)*cos(d*x + c)^2 + (75*A - 163*B)*cos(d*x + c))*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*((147*A - 299*B)*cos(d*x + c)^3 + (255*A - 503*B)*cos(d*x + c)^2 + 32*(3*A - 5*B)*cos(d*x + c) + 32*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + a^3*d*cos(d*x + c)), 1/96*(3*sqrt(2)*((75*A - 163*B)*cos(d*x + c)^4 + 3*(75*A - 163*B)*cos(d*x + c)^3 + 3*(75*A - 163*B)*cos(d*x + c)^2 + (75*A - 163*B)*cos(d*x + c))*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) + 2*((147*A - 299*B)*cos(d*x + c)^3 + (255*A - 503*B)*cos(d*x + c)^2 + 32*(3*A - 5*B)*cos(d*x + c) + 32*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + a^3*d*cos(d*x + c))]

giac [A] time = 3.40, size = 311, normalized size = 1.44

$$\frac{\left(\left(\left(\frac{2\sqrt{2}(Aa^5 - Ba^5)\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2}{a^6\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - 1\right)} + \frac{\sqrt{2}(15Aa^5 - 23Ba^5)}{a^6\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - 1\right)}\right)\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - \frac{4\sqrt{2}(75Aa^5 - 167Ba^5)}{a^6\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - 1\right)}\right)\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + \frac{3\sqrt{2}(83Aa^5 - 155Ba^5)}{a^6\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - 1\right)}\right)\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2}{\left(a\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - a\right)\sqrt{-a\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + a}}$$

96d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")

[Out] 1/96*(((3*(2*sqrt(2)*(A*a^5 - B*a^5))*tan(1/2*d*x + 1/2*c)^2/(a^6*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) + sqrt(2)*(15*A*a^5 - 23*B*a^5)/(a^6*sgn(tan(1/2*d*x

$$\begin{aligned} & + 1/2*c)^2 - 1))) * \tan(1/2*d*x + 1/2*c)^2 - 4*\sqrt{2}*(75*A*a^5 - 167*B*a^5 \\ &)/(a^6*\operatorname{sgn}(\tan(1/2*d*x + 1/2*c)^2 - 1))) * \tan(1/2*d*x + 1/2*c)^2 + 3*\sqrt{2} \\ & *(83*A*a^5 - 155*B*a^5)/(a^6*\operatorname{sgn}(\tan(1/2*d*x + 1/2*c)^2 - 1))) * \tan(1/2*d*x \\ & + 1/2*c)/((a*\tan(1/2*d*x + 1/2*c)^2 - a)*\sqrt{-a*\tan(1/2*d*x + 1/2*c)^2 + a \\ &)) - 3*\sqrt{2}*(75*A - 163*B)*\log(\operatorname{abs}(-\sqrt{-a})*\tan(1/2*d*x + 1/2*c) + \sqrt{ \\ & (-a*\tan(1/2*d*x + 1/2*c)^2 + a)))/(\sqrt{-a}*a^2*\operatorname{sgn}(\tan(1/2*d*x + 1/2*c)^2 \\ & - 1)))/d \end{aligned}$$

maple [B] time = 1.75, size = 795, normalized size = 3.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x)`

[Out]
$$\begin{aligned} & -1/192/d*(-1+\cos(d*x+c))^2*(-225*A*\cos(d*x+c)^3*\sin(d*x+c)*\ln(-(-(-2*\cos(d* \\ & x+c)/(1+\cos(d*x+c)))^{1/2}*\sin(d*x+c)+\cos(d*x+c)-1)/\sin(d*x+c))*(-2*\cos(d*x \\ & +c)/(1+\cos(d*x+c)))^{3/2}+489*B*\cos(d*x+c)^3*\sin(d*x+c)*\ln(-(-(-2*\cos(d*x+c) \\ &)/(1+\cos(d*x+c)))^{1/2}*\sin(d*x+c)+\cos(d*x+c)-1)/\sin(d*x+c))*(-2*\cos(d*x+c) \\ & /((1+\cos(d*x+c)))^{3/2}-675*A*\ln(-(-(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*\sin \\ & (d*x+c)+\cos(d*x+c)-1)/\sin(d*x+c))*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{3/2}* \sin \\ & (d*x+c)*\cos(d*x+c)^2+1467*B*\ln(-(-(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*\sin(d \\ & *x+c)+\cos(d*x+c)-1)/\sin(d*x+c))*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{3/2}* \sin(d* \\ & x+c)*\cos(d*x+c)^2-675*A*\cos(d*x+c)*\sin(d*x+c)*\ln(-(-(-2*\cos(d*x+c)/(1+\cos(d \\ & *x+c)))^{1/2}*\sin(d*x+c)+\cos(d*x+c)-1)/\sin(d*x+c))*(-2*\cos(d*x+c)/(1+\cos(d* \\ & x+c)))^{3/2}+1467*B*\cos(d*x+c)*\sin(d*x+c)*\ln(-(-(-2*\cos(d*x+c)/(1+\cos(d*x+c) \\ &)))^{1/2}*\sin(d*x+c)+\cos(d*x+c)-1)/\sin(d*x+c))*(-2*\cos(d*x+c)/(1+\cos(d*x+c) \\ &))^{3/2}-225*A*\ln(-(-(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*\sin(d*x+c)+\cos(d* \\ & x+c)-1)/\sin(d*x+c))*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{3/2}* \sin(d*x+c)+489*B* \ln \\ & (-(-(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*\sin(d*x+c)+\cos(d*x+c)-1)/\sin(d*x+ \\ & c))*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{3/2}* \sin(d*x+c)+588*A*\cos(d*x+c)^4-1196 \\ & *B*\cos(d*x+c)^4+432*A*\cos(d*x+c)^3-816*B*\cos(d*x+c)^3-636*A*\cos(d*x+c)^2+13 \\ & 72*B*\cos(d*x+c)^2-384*A*\cos(d*x+c)+768*B*\cos(d*x+c)-128*B)*(a*(1+\cos(d*x+c) \\ &)/\cos(d*x+c))^{1/2}/\sin(d*x+c)^5/\cos(d*x+c)/a^3 \end{aligned}$$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")`

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\cos(c+dx)^4 \left(a + \frac{a}{\cos(c+dx)}\right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^4*(a + a/cos(c + d*x))^(5/2)),x)

[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^4*(a + a/cos(c + d*x))^(5/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \sec^4(c + dx)}{(a(\sec(c + dx) + 1))^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(5/2),x)

[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**4/(a*(sec(c + d*x) + 1))**(5/2), x)

$$3.161 \quad \int \frac{\sec^3(c+dx)(A+B \sec(c+dx))}{(a+a \sec(c+dx))^{5/2}} dx$$

Optimal. Leaf size=169

$$\frac{(19A - 75B) \tan^{-1} \left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{2} \sqrt{a \sec(c+dx)+a}} \right)}{16\sqrt{2} a^{5/2} d} - \frac{(A - 9B) \tan(c + dx)}{4a^2 d \sqrt{a \sec(c + dx) + a}} + \frac{(A - B) \tan(c + dx) \sec^2(c + dx)}{4d(a \sec(c + dx) + a)^{5/2}} - \frac{(5A - 13B)}{16ad(a \sec(c + dx) + a)^{5/2}}$$

[Out] 1/32*(19*A-75*B)*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2)+1/4*(A-B)*sec(d*x+c)^2*tan(d*x+c)/d/(a+a*sec(d*x+c))^(5/2)-1/16*(5*A-13*B)*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^(3/2)-1/4*(A-9*B)*tan(d*x+c)/a^2/d/(a+a*sec(d*x+c))^(1/2)

Rubi [A] time = 0.45, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {4019, 4008, 4001, 3795, 203}

$$\frac{(19A - 75B) \tan^{-1} \left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{2} \sqrt{a \sec(c+dx)+a}} \right)}{16\sqrt{2} a^{5/2} d} - \frac{(A - 9B) \tan(c + dx)}{4a^2 d \sqrt{a \sec(c + dx) + a}} + \frac{(A - B) \tan(c + dx) \sec^2(c + dx)}{4d(a \sec(c + dx) + a)^{5/2}} - \frac{(5A - 13B)}{16ad(a \sec(c + dx) + a)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(Sec[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(5/2), x]

[Out] (((19*A - 75*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) + ((A - B)*Sec[c + d*x]^2*Tan[c + d*x])/(4*d*(a + a*Sec[c + d*x])^(5/2)) - ((5*A - 13*B)*Tan[c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/2)) - ((A - 9*B)*Tan[c + d*x])/(4*a^2*d*Sqrt[a + a*Sec[c + d*x]])

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 3795

Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] := Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] := -Simp[(B*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m + 1), 0] && !LtQ[m, -2^(-1)]
```

Rule 4008

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] := -Simp[((A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(b^2*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*B*(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] := Simp[(d*(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^3(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{5/2}} dx &= \frac{(A-B)\sec^2(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \int \frac{\sec^2(c+dx)\left(2a(A-B)-\frac{1}{2}a(A-9B)\sec(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}} \frac{1}{4a^2} \\
&= \frac{(A-B)\sec^2(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(5A-13B)\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \int \frac{\sec(c+dx)}{4a^2d} \\
&= \frac{(A-B)\sec^2(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(5A-13B)\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \frac{(A-B)\sec(c+dx)}{4a^2d} \\
&= \frac{(A-B)\sec^2(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(5A-13B)\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \frac{(A-B)\sec(c+dx)}{4a^2d} \\
&= \frac{(19A-75B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} + \frac{(A-B)\sec^2(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}}
\end{aligned}$$

Mathematica [A] time = 1.58, size = 144, normalized size = 0.85

$$\frac{\tan(c+dx)\left(\sqrt{1-\sec(c+dx)}\left((85B-13A)\sec(c+dx)-9A+32B\sec^2(c+dx)+49B\right)+2\sqrt{2}(19A-75B)\cos\left(\frac{c+dx}{2}\right)\right)}{16d\sqrt{1-\sec(c+dx)}(a(\sec(c+dx)+1))^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(5/2), x]

[Out] ((2*Sqrt[2]*(19*A - 75*B)*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]*Cos[(c + d*x)/2]^4*Sec[c + d*x]^2 + Sqrt[1 - Sec[c + d*x]]*(-9*A + 49*B + (-13*A + 85*B)*Sec[c + d*x] + 32*B*Sec[c + d*x]^2))*Tan[c + d*x])/(16*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(5/2))

fricas [A] time = 0.48, size = 484, normalized size = 2.86

$$\left[\frac{\sqrt{2}\left((19A-75B)\cos(dx+c)^3 + 3(19A-75B)\cos(dx+c)^2 + 3(19A-75B)\cos(dx+c) + 19A-75B\right)\sqrt{a+a\sec(c+dx)}}{64\left(a^3d\right)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")

[Out] [1/64*(sqrt(2)*((19*A - 75*B)*cos(d*x + c)^3 + 3*(19*A - 75*B)*cos(d*x + c)^2 + 3*(19*A - 75*B)*cos(d*x + c) + 19*A - 75*B)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) - 3*a*cos(d*x + c)^2 - 2*a*cos(d*x + c) + a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*((9*A - 49*B)*cos(d*x + c)^2 + (13*A - 85*B)*cos(d*x + c) - 32*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1/32*(sqrt(2)*((19*A - 75*B)*cos(d*x + c)^3 + 3*(19*A - 75*B)*cos(d*x + c)^2 + 3*(19*A - 75*B)*cos(d*x + c) + 19*A - 75*B)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) + 2*((9*A - 49*B)*cos(d*x + c)^2 + (13*A - 85*B)*cos(d*x + c) - 32*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]

giac [A] time = 6.39, size = 289, normalized size = 1.71

$$\frac{\left(\frac{2 \left(\sqrt{2} A a^6 \operatorname{sgn} \left(\tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^2 - 1 \right) - \sqrt{2} B a^6 \operatorname{sgn} \left(\tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^2 - 1 \right) \right) \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^2}{a^8} + \frac{9 \sqrt{2} A a^6 \operatorname{sgn} \left(\tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^2 - 1 \right) - 17 \sqrt{2} B a^6 \operatorname{sgn} \left(\tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^2 - 1 \right)}{a^8} \right) \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)}{\sqrt{-a \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^2 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")

[Out] -1/32*(((2*(sqrt(2)*A*a^6*sgn(tan(1/2*d*x + 1/2*c)^2 - 1) - sqrt(2)*B*a^6*sgn(tan(1/2*d*x + 1/2*c)^2 - 1))*tan(1/2*d*x + 1/2*c)^2/a^8 + (9*sqrt(2)*A*a^6*sgn(tan(1/2*d*x + 1/2*c)^2 - 1) - 17*sqrt(2)*B*a^6*sgn(tan(1/2*d*x + 1/2*c)^2 - 1))/a^8)*tan(1/2*d*x + 1/2*c)^2 - (11*sqrt(2)*A*a^6*sgn(tan(1/2*d*x + 1/2*c)^2 - 1) - 83*sqrt(2)*B*a^6*sgn(tan(1/2*d*x + 1/2*c)^2 - 1))/a^8)*tan(1/2*d*x + 1/2*c)/sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a) - (19*sqrt(2)*A - 75*sqrt(2)*B)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/(sqrt(-a)*a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1))/d

maple [B] time = 1.63, size = 597, normalized size = 3.53

$$\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}} (-1 + \cos(dx+c))^2 \left(19A \sin(dx+c) \ln \left(-\frac{\sqrt{\frac{2\cos(dx+c)}{1+\cos(dx+c)}} \sin(dx+c) + \cos(dx+c) - 1}{\sin(dx+c)} \right) \right) \sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}} \left(\cos(dx+c) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x)`

[Out] $\frac{1}{32}d \cdot (a \cdot (1 + \cos(dx+c)) / \cos(dx+c))^{1/2} \cdot (-1 + \cos(dx+c))^2 \cdot (19A \sin(dx+c) \cdot \ln(-(-(-2 \cos(dx+c) / (1 + \cos(dx+c)))^{1/2} \cdot \sin(dx+c) + \cos(dx+c) - 1) / \sin(dx+c)) \cdot (-2 \cos(dx+c) / (1 + \cos(dx+c)))^{1/2} \cdot \cos(dx+c)^2 - 75B \sin(dx+c) \cdot \ln(-(-(-2 \cos(dx+c) / (1 + \cos(dx+c)))^{1/2} \cdot \sin(dx+c) + \cos(dx+c) - 1) / \sin(dx+c)) \cdot (-2 \cos(dx+c) / (1 + \cos(dx+c)))^{1/2} \cdot \cos(dx+c)^2 + 38A \cos(dx+c) \cdot \sin(dx+c) \cdot \ln(-(-(-2 \cos(dx+c) / (1 + \cos(dx+c)))^{1/2} \cdot \sin(dx+c) + \cos(dx+c) - 1) / \sin(dx+c)) \cdot (-2 \cos(dx+c) / (1 + \cos(dx+c)))^{1/2} - 150B \cos(dx+c) \cdot \sin(dx+c) \cdot \ln(-(-(-2 \cos(dx+c) / (1 + \cos(dx+c)))^{1/2} \cdot \sin(dx+c) + \cos(dx+c) - 1) / \sin(dx+c)) \cdot (-2 \cos(dx+c) / (1 + \cos(dx+c)))^{1/2} + 19A \cdot \ln(-(-(-2 \cos(dx+c) / (1 + \cos(dx+c)))^{1/2} \cdot \sin(dx+c) + \cos(dx+c) - 1) / \sin(dx+c)) \cdot (-2 \cos(dx+c) / (1 + \cos(dx+c)))^{1/2} \cdot \sin(dx+c) + \cos(dx+c) - 1) / \sin(dx+c) \cdot (-2 \cos(dx+c) / (1 + \cos(dx+c)))^{1/2} \cdot \sin(dx+c) + 18A \cos(dx+c)^3 - 75B \cdot \ln(-(-(-2 \cos(dx+c) / (1 + \cos(dx+c)))^{1/2} \cdot \sin(dx+c) + \cos(dx+c) - 1) / \sin(dx+c)) \cdot (-2 \cos(dx+c) / (1 + \cos(dx+c)))^{1/2} \cdot \sin(dx+c) - 98B \cos(dx+c)^3 + 8A \cos(dx+c)^2 - 72B \cos(dx+c)^2 - 26A \cos(dx+c) + 106B \cos(dx+c) + 64B) / \sin(dx+c)^5 / a^3$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")`

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\cos(c+dx)^3 \left(a + \frac{a}{\cos(c+dx)} \right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B/cos(c + d*x))/(cos(c + d*x)^3*(a + a/cos(c + d*x))^(5/2)),x)`

[Out] `int((A + B/cos(c + d*x))/(cos(c + d*x)^3*(a + a/cos(c + d*x))^(5/2)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \sec^3(c + dx)}{(a(\sec(c + dx) + 1))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)**3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(5/2), x)`

[Out] `Integral((A + B*sec(c + d*x))*sec(c + d*x)**3/(a*(sec(c + d*x) + 1))**(5/2), x)`

$$3.162 \quad \int \frac{\sec^2(c+dx)(A+B \sec(c+dx))}{(a+a \sec(c+dx))^{5/2}} dx$$

Optimal. Leaf size=126

$$\frac{(5A + 19B) \tan^{-1} \left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{2} \sqrt{a \sec(c+dx)+a}} \right)}{16\sqrt{2} a^{5/2} d} + \frac{(5A - 13B) \tan(c + dx)}{16ad(a \sec(c + dx) + a)^{3/2}} - \frac{(A - B) \tan(c + dx)}{4d(a \sec(c + dx) + a)^{5/2}}$$

[Out] 1/32*(5*A+19*B)*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2)-1/4*(A-B)*tan(d*x+c)/d/(a+a*sec(d*x+c))^(5/2)+1/16*(5*A-13*B)*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^(3/2)

Rubi [A] time = 0.28, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {4008, 4000, 3795, 203}

$$\frac{(5A + 19B) \tan^{-1} \left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{2} \sqrt{a \sec(c+dx)+a}} \right)}{16\sqrt{2} a^{5/2} d} + \frac{(5A - 13B) \tan(c + dx)}{16ad(a \sec(c + dx) + a)^{3/2}} - \frac{(A - B) \tan(c + dx)}{4d(a \sec(c + dx) + a)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(5/2), x]

[Out] ((5*A + 19*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B)*Tan[c + d*x])/(4*d*(a + a*Sec[c + d*x])^(5/2)) + ((5*A - 13*B)*Tan[c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/2))

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[Rt[b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 3795

Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] := Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 4000

Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] := Simp[((A*b - a*B)*Cot[e +

```
f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Rule 4008

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^m*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] := -Simp[((A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(b^2*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*B*(2*m + 1)*Csc[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned} \int \frac{\sec^2(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{5/2}} dx &= -\frac{(A-B)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{\int \frac{\sec(c+dx)\left(-\frac{5}{2}a(A-B)-4aB\sec(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}} dx}{4a^2} \\ &= -\frac{(A-B)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(5A-13B)\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{(5A+19B)\int}{3} \\ &= -\frac{(A-B)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(5A-13B)\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \frac{(5A+19B)\text{Su}}{16\sqrt{2}a^{5/2}d} \\ &= \frac{(5A+19B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{(A-B)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(5A}{16ad} \end{aligned}$$

Mathematica [A] time = 1.64, size = 131, normalized size = 1.04

$$\frac{\tan(c+dx)\left(\sqrt{1-\sec(c+dx)}((5A-13B)\sec(c+dx)+A-9B)+2\sqrt{2}(5A+19B)\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\right)}{16d\sqrt{1-\sec(c+dx)}(a(\sec(c+dx)+1))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(5/2), x]
```

```
[Out] ((2*Sqrt[2]*(5*A + 19*B)*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]*Cos[(c + d*x)/2]^4*Sec[c + d*x]^2 + Sqrt[1 - Sec[c + d*x]]*(A - 9*B + (5*A - 13*B)*Se
```


$c[c + d*x]))*\text{Tan}[c + d*x]]/(16*d*\text{Sqrt}[1 - \text{Sec}[c + d*x]]*(a*(1 + \text{Sec}[c + d*x]))^{(5/2)})$

fricas [A] time = 0.48, size = 475, normalized size = 3.77

$$\frac{\sqrt{2} \left((5A + 19B) \cos(dx + c)^3 + 3(5A + 19B) \cos(dx + c)^2 + 3(5A + 19B) \cos(dx + c) + 5A + 19B \right) \sqrt{-a}}{64 \left(a^3 d \cos(dx + c) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")`

[Out] $[-1/64*(\text{sqrt}(2)*((5*A + 19*B)*\cos(d*x + c)^3 + 3*(5*A + 19*B)*\cos(d*x + c)^2 + 3*(5*A + 19*B)*\cos(d*x + c) + 5*A + 19*B)*\text{sqrt}(-a)*\log((2*\text{sqrt}(2)*\text{sqrt}(-a)*\text{sqrt}((a*\cos(d*x + c) + a)/\cos(d*x + c))*\cos(d*x + c)*\sin(d*x + c) + 3*a*\cos(d*x + c)^2 + 2*a*\cos(d*x + c) - a)/(\cos(d*x + c)^2 + 2*\cos(d*x + c) + 1)) - 4*((A - 9*B)*\cos(d*x + c)^2 + (5*A - 13*B)*\cos(d*x + c))*\text{sqrt}((a*\cos(d*x + c) + a)/\cos(d*x + c))*\sin(d*x + c))/(a^3*d*\cos(d*x + c)^3 + 3*a^3*d*\cos(d*x + c)^2 + 3*a^3*d*\cos(d*x + c) + a^3*d), -1/32*(\text{sqrt}(2)*((5*A + 19*B)*\cos(d*x + c)^3 + 3*(5*A + 19*B)*\cos(d*x + c)^2 + 3*(5*A + 19*B)*\cos(d*x + c) + 5*A + 19*B)*\text{sqrt}(a)*\arctan(\text{sqrt}(2)*\text{sqrt}((a*\cos(d*x + c) + a)/\cos(d*x + c))*\cos(d*x + c)/(\text{sqrt}(a)*\sin(d*x + c))) - 2*((A - 9*B)*\cos(d*x + c)^2 + (5*A - 13*B)*\cos(d*x + c))*\text{sqrt}((a*\cos(d*x + c) + a)/\cos(d*x + c))*\sin(d*x + c))/(a^3*d*\cos(d*x + c)^3 + 3*a^3*d*\cos(d*x + c)^2 + 3*a^3*d*\cos(d*x + c) + a^3*d)]$

giac [A] time = 13.49, size = 191, normalized size = 1.52

$$\frac{\sqrt{-a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + a} \left(\frac{2 \sqrt{2} (Aa^5 - Ba^5) \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2}{a^8 \text{sgn}\left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 1\right)} + \frac{\sqrt{2} (3Aa^5 - 11Ba^5)}{a^8 \text{sgn}\left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 1\right)} \right) \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - \frac{\sqrt{2} (5A + 19B) \log\left(\frac{\sqrt{2} (Aa^5 - Ba^5) \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + a}{a^8 \text{sgn}\left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 1\right)}\right)}{32 d}}{32 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")`

[Out] $-1/32*(\text{sqrt}(-a*\tan(1/2*d*x + 1/2*c)^2 + a))*(2*\text{sqrt}(2)*(A*a^5 - B*a^5)*\tan(1/2*d*x + 1/2*c)^2/(a^8*\text{sgn}(\tan(1/2*d*x + 1/2*c)^2 - 1)) + \text{sqrt}(2)*(3*A*a^5$

$- 11*B*a^5)/(a^8*\text{sgn}(\tan(1/2*d*x + 1/2*c)^2 - 1))*\tan(1/2*d*x + 1/2*c) - \text{sqrt}(2)*(5*A + 19*B)*\log(\text{abs}(-\text{sqrt}(-a)*\tan(1/2*d*x + 1/2*c) + \text{sqrt}(-a*\tan(1/2*d*x + 1/2*c)^2 + a)))/(\text{sqrt}(-a)*a^2*\text{sgn}(\tan(1/2*d*x + 1/2*c)^2 - 1))/d$

maple [B] time = 1.67, size = 602, normalized size = 4.78

$$\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}} (-1 + \cos(dx+c)) \left(-5A \sin(dx+c) \ln \left(-\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}} \sin(dx+c) + \cos(dx+c) - 1}{\sin(dx+c)}} \right) \sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}} (\cos^2(dx+c) - 1) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x)`

[Out] $1/32/d*(a*(1+\cos(d*x+c))/\cos(d*x+c))^{1/2}*(-1+\cos(d*x+c))*(-5*A*\sin(d*x+c)*\ln(-(-(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*\sin(d*x+c)+\cos(d*x+c)-1)/\sin(d*x+c))*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*\cos(d*x+c)^2-19*B*\sin(d*x+c)*\ln(-(-(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*\sin(d*x+c)+\cos(d*x+c)-1)/\sin(d*x+c))*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*\cos(d*x+c)^2-10*A*\cos(d*x+c)*\sin(d*x+c)*\ln(-(-(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*\sin(d*x+c)+\cos(d*x+c)-1)/\sin(d*x+c))*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}-38*B*\cos(d*x+c)*\sin(d*x+c)*\ln(-(-(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*\sin(d*x+c)+\cos(d*x+c)-1)/\sin(d*x+c))*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}+2*A*\cos(d*x+c)^3-5*A*\ln(-(-(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*\sin(d*x+c)+\cos(d*x+c)-1)/\sin(d*x+c))*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*\sin(d*x+c)-18*B*\cos(d*x+c)^3-19*B*\ln(-(-(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*\sin(d*x+c)+\cos(d*x+c)-1)/\sin(d*x+c))*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*\sin(d*x+c)+8*A*\cos(d*x+c)^2-8*B*\cos(d*x+c)^2-10*A*\cos(d*x+c)+26*B*\cos(d*x+c))/((1+\cos(d*x+c))/\sin(d*x+c))^3/a^3$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")`

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\cos(c+dx)^2 \left(a + \frac{a}{\cos(c+dx)} \right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B/cos(c + d*x))/(cos(c + d*x)^2*(a + a/cos(c + d*x))^(5/2)),x)`

[Out] `int((A + B/cos(c + d*x))/(cos(c + d*x)^2*(a + a/cos(c + d*x))^(5/2)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \sec^2(c + dx)}{(a(\sec(c + dx) + 1))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)**2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(5/2),x)`

[Out] `Integral((A + B*sec(c + d*x))*sec(c + d*x)**2/(a*(sec(c + d*x) + 1))**(5/2), x)`

$$3.163 \quad \int \frac{\sec(c+dx)(A+B \sec(c+dx))}{(a+a \sec(c+dx))^{5/2}} dx$$

Optimal. Leaf size=126

$$\frac{(3A + 5B) \tan^{-1} \left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{2} \sqrt{a \sec(c+dx)+a}} \right)}{16\sqrt{2} a^{5/2} d} + \frac{(3A + 5B) \tan(c + dx)}{16ad(a \sec(c + dx) + a)^{3/2}} + \frac{(A - B) \tan(c + dx)}{4d(a \sec(c + dx) + a)^{5/2}}$$

[Out] 1/32*(3*A+5*B)*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2)+1/4*(A-B)*tan(d*x+c)/d/(a+a*sec(d*x+c))^(5/2)+1/16*(3*A+5*B)*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^(3/2)

Rubi [A] time = 0.16, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {4000, 3796, 3795, 203}

$$\frac{(3A + 5B) \tan^{-1} \left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{2} \sqrt{a \sec(c+dx)+a}} \right)}{16\sqrt{2} a^{5/2} d} + \frac{(3A + 5B) \tan(c + dx)}{16ad(a \sec(c + dx) + a)^{3/2}} + \frac{(A - B) \tan(c + dx)}{4d(a \sec(c + dx) + a)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(5/2), x]

[Out] ((3*A + 5*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) + ((A - B)*Tan[c + d*x])/(4*d*(a + a*Sec[c + d*x])^(5/2)) + ((3*A + 5*B)*Tan[c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/2))

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[Rt[b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 3795

Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 3796

Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)

), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
&& IntegerQ[2*m]

Rule 4000

Int[csc[(e_.) + (f_.)*(x_.)]*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(cs
c[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> Simp[((A*b - a*B)*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
/; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]

Rubi steps

$$\begin{aligned} \int \frac{\sec(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{5/2}} dx &= \frac{(A-B)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(3A+5B)\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{3/2}} dx}{8a} \\ &= \frac{(A-B)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(3A+5B)\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{(3A+5B)\int \frac{1}{\sqrt{a+a\sec(c+dx)}} dx}{32a} \\ &= \frac{(A-B)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(3A+5B)\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \frac{(3A+5B)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+a\sec(c+dx)}} dx\right)}{32a} \\ &= \frac{(3A+5B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} + \frac{(A-B)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(3A+5B)\int \frac{1}{\sqrt{a+a\sec(c+dx)}} dx}{16ad(a+a\sec(c+dx))^{3/2}} \end{aligned}$$

Mathematica [C] time = 1.59, size = 206, normalized size = 1.63

$$\frac{64A \sin\left(\frac{1}{2}(c+dx)\right) \cos^5\left(\frac{1}{2}(c+dx)\right) \sqrt{1-\sec(c+dx)} \sec(c+dx) {}_2F_1\left(\frac{1}{2}, 3; \frac{3}{2}; \frac{1}{2}(1-\sec(c+dx))\right) + B(10 \sin(c+dx) \sqrt{1-\sec(c+dx)} + 10 \cos(c+dx) \sqrt{1-\sec(c+dx)})}{32a^2d(\cos(c+dx)+1)^2\sqrt{1-\sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(5/2), x]
[Out] (40*sqrt[2]*B*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]*Cos[(c + d*x)/2]^5*Sec[c + d*x]*Sin[(c + d*x)/2] + 64*A*Cos[(c + d*x)/2]^5*Hypergeometric2F1[1/2, 3, 3/2, (1 - Sec[c + d*x])/2]*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]*Sin[(c + d*x)/2] + B*Sqrt[1 - Sec[c + d*x]]*(10*Sin[c + d*x] + Sin[2*(c + d*x)]))/ (32*a^2*d*(1 + Cos[c + d*x])^2*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])

fricas [A] time = 0.52, size = 475, normalized size = 3.77

$$\frac{\sqrt{2} \left((3A + 5B) \cos(dx + c)^3 + 3(3A + 5B) \cos(dx + c)^2 + 3(3A + 5B) \cos(dx + c) + 3A + 5B \right) \sqrt{-a} \log \left(\frac{2 \sqrt{2} \sqrt{-a} \sqrt{(a \cos(dx + c) + a) / \cos(dx + c)} \cos(dx + c) \sin(dx + c) + 3a \cos(dx + c)^2 + 2a \cos(dx + c) - a}{(\cos(dx + c)^2 + 2 \cos(dx + c) + 1)} - 4 \frac{((7A + B) \cos(dx + c)^2 + (3A + 5B) \cos(dx + c)) \sqrt{(a \cos(dx + c) + a) / \cos(dx + c)} \sin(dx + c)}{a^3 d \cos(dx + c)^3 + 3a^3 d \cos(dx + c)^2 + 3a^3 d \cos(dx + c) + a^3 d} \right)}{64 (a^3 d \cos(dx + c)^3 + 3a^3 d \cos(dx + c)^2 + 3a^3 d \cos(dx + c) + a^3 d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")

[Out] [-1/64*(sqrt(2)*((3*A + 5*B)*cos(d*x + c)^3 + 3*(3*A + 5*B)*cos(d*x + c)^2 + 3*(3*A + 5*B)*cos(d*x + c) + 3*A + 5*B)*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*((7*A + B)*cos(d*x + c)^2 + (3*A + 5*B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1/32*(sqrt(2)*((3*A + 5*B)*cos(d*x + c)^3 + 3*(3*A + 5*B)*cos(d*x + c)^2 + 3*(3*A + 5*B)*cos(d*x + c) + 3*A + 5*B)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - 2*((7*A + B)*cos(d*x + c)^2 + (3*A + 5*B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]

giac [A] time = 7.47, size = 191, normalized size = 1.52

$$\frac{\sqrt{-a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + a} \left(\frac{2 \sqrt{2} (Aa^5 - Ba^5) \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2}{a^8 \operatorname{sgn}\left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 1\right)} - \frac{\sqrt{2} (5Aa^5 + 3Ba^5)}{a^8 \operatorname{sgn}\left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 1\right)} \right) \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + \frac{\sqrt{2} (3A + 5B) \log\left(\frac{2 \sqrt{2} \sqrt{-a} \sqrt{(a \cos(dx + c) + a) / \cos(dx + c)} \cos(dx + c) \sin(dx + c) + 3a \cos(dx + c)^2 + 2a \cos(dx + c) - a}{(\cos(dx + c)^2 + 2 \cos(dx + c) + 1)} - 4 \frac{((7A + B) \cos(dx + c)^2 + (3A + 5B) \cos(dx + c)) \sqrt{(a \cos(dx + c) + a) / \cos(dx + c)} \sin(dx + c)}{a^3 d \cos(dx + c)^3 + 3a^3 d \cos(dx + c)^2 + 3a^3 d \cos(dx + c) + a^3 d}\right)}}{32 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")

[Out] 1/32*(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*sqrt(2)*(A*a^5 - B*a^5)*tan(1/2*d*x + 1/2*c)^2/(a^8*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) - sqrt(2)*(5*A*a^5 + 3*B*a^5)/(a^8*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))*tan(1/2*d*x + 1/2*c) + sqrt(2)*(3*A + 5*B)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/(sqrt(-a)*a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1))/d

maple [B] time = 1.49, size = 594, normalized size = 4.71

$$\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}} \left(3A \sin(dx+c) \ln \left(-\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}} \sin(dx+c) + \cos(dx+c) - 1}{\sin(dx+c)} \right) \sqrt{\frac{2\cos(dx+c)}{1+\cos(dx+c)}} (\cos^2(dx+c)) + 5B \sin \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x)

[Out] 1/32/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(3*A*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2+5*B*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2+6*A*cos(d*x+c)*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+10*B*cos(d*x+c)*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+3*A*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-14*A*cos(d*x+c)^3+5*B*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-2*B*cos(d*x+c)^3+8*A*cos(d*x+c)^2-8*B*cos(d*x+c)^2+6*A*cos(d*x+c)+10*B*cos(d*x+c)/(1+cos(d*x+c))^2/sin(d*x+c)/a^3

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx+c) + A) \sec(dx+c)}{(a \sec(dx+c) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")

[Out] integrate((B*sec(d*x+c) + A)*sec(d*x+c)/(a*sec(d*x+c) + a)^(5/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\cos(c+dx) \left(a + \frac{a}{\cos(c+dx)} \right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B/cos(c + d*x))/(cos(c + d*x)*(a + a/cos(c + d*x))^(5/2)),x)`

[Out] `int((A + B/cos(c + d*x))/(cos(c + d*x)*(a + a/cos(c + d*x))^(5/2)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \sec(c + dx)}{(a(\sec(c + dx) + 1))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(5/2),x)`

[Out] `Integral((A + B*sec(c + d*x))*sec(c + d*x)/(a*(sec(c + d*x) + 1))**(5/2), x)`

$$3.164 \quad \int \frac{A+B \sec(c+dx)}{(a+a \sec(c+dx))^{5/2}} dx$$

Optimal. Leaf size=164

$$\frac{(43A - 3B) \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{2} \sqrt{a \sec(c+dx)+a}}\right)}{16\sqrt{2} a^{5/2}d} + \frac{2A \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}}\right)}{a^{5/2}d} - \frac{(11A - 3B) \tan(c+dx)}{16ad(a \sec(c+dx) + a)^{3/2}} - \frac{(A - B) \tan(c+dx)}{4d(a \sec(c+dx) + a)}$$

[Out] 2*A*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(5/2)/d-1/32*(43*A-3*B)*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2)-1/4*(A-B)*tan(d*x+c)/d/(a+a*sec(d*x+c))^(5/2)-1/16*(11*A-3*B)*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^(3/2)

Rubi [A] time = 0.25, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3922, 3920, 3774, 203, 3795}

$$\frac{(43A - 3B) \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{2} \sqrt{a \sec(c+dx)+a}}\right)}{16\sqrt{2} a^{5/2}d} + \frac{2A \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}}\right)}{a^{5/2}d} - \frac{(11A - 3B) \tan(c+dx)}{16ad(a \sec(c+dx) + a)^{3/2}} - \frac{(A - B) \tan(c+dx)}{4d(a \sec(c+dx) + a)}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sec[c + d*x])/(a + a*Sec[c + d*x])^(5/2), x]

[Out] (2*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]])/(a^(5/2)*d) - ((43*A - 3*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]/(16*Sqrt[2]*a^(5/2)*d) - ((A - B)*Tan[c + d*x]/(4*d*(a + a*Sec[c + d*x])^(5/2)) - ((11*A - 3*B)*Tan[c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/2)))

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 3774

Int[Sqrt[csc[(c_.) + (d_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] := Dist[(-2*b)/d, Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] := Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] := Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3922

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.)), x_Symbol] := -Simp[((b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && EqQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
 \int \frac{A + B \sec(c + dx)}{(a + a \sec(c + dx))^{5/2}} dx &= -\frac{(A - B) \tan(c + dx)}{4d(a + a \sec(c + dx))^{5/2}} - \frac{\int \frac{-4aA + \frac{3}{2}a(A-B) \sec(c+dx)}{(a+a \sec(c+dx))^{3/2}} dx}{4a^2} \\
 &= -\frac{(A - B) \tan(c + dx)}{4d(a + a \sec(c + dx))^{5/2}} - \frac{(11A - 3B) \tan(c + dx)}{16ad(a + a \sec(c + dx))^{3/2}} + \frac{\int \frac{8a^2A - \frac{1}{4}a^2(11A-3B) \sec(c+dx)}{\sqrt{a+a \sec(c+dx)}}}{8a^4} \\
 &= -\frac{(A - B) \tan(c + dx)}{4d(a + a \sec(c + dx))^{5/2}} - \frac{(11A - 3B) \tan(c + dx)}{16ad(a + a \sec(c + dx))^{3/2}} + \frac{A \int \sqrt{a + a \sec(c + dx)} dx}{a^3} \\
 &= -\frac{(A - B) \tan(c + dx)}{4d(a + a \sec(c + dx))^{5/2}} - \frac{(11A - 3B) \tan(c + dx)}{16ad(a + a \sec(c + dx))^{3/2}} - \frac{(2A) \text{Subst}\left(\int \frac{1}{a+x^2} dx, x, \sqrt{a + a \sec(c + dx)}\right)}{a^2d} \\
 &= \frac{2A \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a+a \sec(c+dx)}}\right)}{a^{5/2}d} - \frac{(43A - 3B) \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{2} \sqrt{a+a \sec(c+dx)}}\right)}{16\sqrt{2} a^{5/2}d} - \frac{(A - B) \tan(c + dx)}{4d(a + a \sec(c + dx))^{5/2}}
 \end{aligned}$$

Mathematica [C] time = 27.04, size = 11243, normalized size = 68.55

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x])/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] Result too large to show
```

```
fricas [B] time = 6.31, size = 670, normalized size = 4.09
```

$$\sqrt{2} \left((43A - 3B) \cos(dx + c)^3 + 3(43A - 3B) \cos(dx + c)^2 + 3(43A - 3B) \cos(dx + c) + 43A - 3B \right) \sqrt{-a} \ln \left| \frac{\dots}{\dots} \right|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/64*(sqrt(2)*((43*A - 3*B)*cos(d*x + c)^3 + 3*(43*A - 3*B)*cos(d*x + c)^2 + 3*(43*A - 3*B)*cos(d*x + c) + 43*A - 3*B)*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 64*(A*cos(d*x + c)^3 + 3*A*cos(d*x + c)^2 + 3*A*cos(d*x + c) + A)*sqrt(-a)*log((2*a*cos(d*x + c)^2 + 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) - 4*((15*A - 7*B)*cos(d*x + c)^2 + (11*A - 3*B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), 1/32*(sqrt(2)*((43*A - 3*B)*cos(d*x + c)^3 + 3*(43*A - 3*B)*cos(d*x + c)^2 + 3*(43*A - 3*B)*cos(d*x + c) + 43*A - 3*B)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - 64*(A*cos(d*x + c)^3 + 3*A*cos(d*x + c)^2 + 3*A*cos(d*x + c) + A)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - 2*((15*A - 7*B)*cos(d*x + c)^2 + (11*A - 3*B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

```
giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00
```

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Warni
ng, integration of abs or sign assumes constant sign by intervals (correct
if the argument is real):Check [abs(cos(d*t_nostep+c))]Unable to check sign
: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/
2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_noste
p/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to che
ck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_
nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/
t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warnin
g, assuming -2*a+a is positive. Hint: run assume to make assumptions on a v
ariableWarning, assuming -2*a+a is positive. Hint: run assume to make assum
ptions on a variableWarning, assuming -2*a+a is positive. Hint: run assume
to make assumptions on a variableWarning, assuming -2*a+a is positive. Hint
: run assume to make assumptions on a variableDiscontinuities at zeroes of
cos(d*t_nostep+c) were not checkedWarning, integration of abs or sign assum
es constant sign by intervals (correct if the argument is real):Check [abs(
t_nostep^2-1)]Evaluation time: 1.36index.cc index_m i_lex_is_greater Error:
Bad Argument Value
```

maple [B] time = 1.43, size = 824, normalized size = 5.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/32/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(32*A*sin(d*x+c)*cos(d*x+c)^2*(
-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)
)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*2^(1/2)+43*A*sin(d*x+c)*ln(-(-2*
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2+64*A*sin(d*x+c)*cos(d*x+c)*(-2
*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)
)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*2^(1/2)-3*B*sin(d*x+c)*ln(-(-(-2*cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2+86*A*cos(d*x+c)*sin(d*x+c)*ln(-(-
(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-
2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+32*A*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*sin(d*x+c)-6*B*cos(d*x+c)*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)+43*A*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x
```

+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-30*A*cos(d*x+c)^3-3*B*ln(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+14*B*cos(d*x+c)^3+8*A*cos(d*x+c)^2-8*B*cos(d*x+c)^2+22*A*cos(d*x+c)-6*B*cos(d*x+c))/(1+cos(d*x+c))^2/sin(d*x+c)/a^3

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(a \sec(dx + c) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)/(a*sec(d*x + c) + a)^(5/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\left(a + \frac{a}{\cos(c+dx)}\right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/(a + a/cos(c + d*x))^(5/2),x)

[Out] int((A + B/cos(c + d*x))/(a + a/cos(c + d*x))^(5/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \sec(c + dx)}{(a(\sec(c + dx) + 1))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(5/2),x)

[Out] Integral((A + B*sec(c + d*x))/(a*(sec(c + d*x) + 1))**(5/2), x)

$$3.165 \quad \int \frac{\cos(c+dx)(A+B \sec(c+dx))}{(a+a \sec(c+dx))^{5/2}} dx$$

Optimal. Leaf size=207

$$-\frac{(5A-2B) \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}}\right)}{a^{5/2}d} + \frac{(115A-43B) \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{2} \sqrt{a \sec(c+dx)+a}}\right)}{16\sqrt{2} a^{5/2}d} + \frac{(35A-11B) \sin(c+dx)}{16a^2d\sqrt{a \sec(c+dx)+a}} - \frac{(15A-7B) \sin(c+dx)}{16ad(a \sec(c+dx)+a)}$$

[Out] $-(5A-2B) \arctan(a^{1/2} \tan(dx+c) / (a+a \sec(dx+c))^{1/2}) / a^{5/2} / d - 1/4 * (A-B) \sin(dx+c) / d / (a+a \sec(dx+c))^{5/2} - 1/16 * (15A-7B) \sin(dx+c) / a / d / (a+a \sec(dx+c))^{3/2} + 1/32 * (115A-43B) \arctan(1/2 * a^{1/2} \tan(dx+c) * 2^{1/2}) / (a+a \sec(dx+c))^{1/2} / a^{5/2} / d * 2^{1/2} + 1/16 * (35A-11B) \sin(dx+c) / a^2 / d / (a+a \sec(dx+c))^{1/2}$

Rubi [A] time = 0.56, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4020, 4022, 3920, 3774, 203, 3795}

$$\frac{(35A-11B) \sin(c+dx)}{16a^2d\sqrt{a \sec(c+dx)+a}} - \frac{(5A-2B) \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}}\right)}{a^{5/2}d} + \frac{(115A-43B) \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{2} \sqrt{a \sec(c+dx)+a}}\right)}{16\sqrt{2} a^{5/2}d} - \frac{(15A-7B) \sin(c+dx)}{16ad(a \sec(c+dx)+a)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(\text{Cos}[c + d*x] * (A + B * \text{Sec}[c + d*x])) / (a + a * \text{Sec}[c + d*x])^{5/2}, x]$

[Out] $-(((5A-2B) * \text{ArcTan}[(\text{Sqrt}[a] * \text{Tan}[c + d*x]) / \text{Sqrt}[a + a * \text{Sec}[c + d*x]])]) / (a^{5/2} * d) + ((115A-43B) * \text{ArcTan}[(\text{Sqrt}[a] * \text{Tan}[c + d*x]) / (\text{Sqrt}[2] * \text{Sqrt}[a + a * \text{Sec}[c + d*x]])]) / (16 * \text{Sqrt}[2] * a^{5/2} * d) - ((A-B) * \text{Sin}[c + d*x]) / (4 * d * (a + a * \text{Sec}[c + d*x])^{5/2}) - ((15A-7B) * \text{Sin}[c + d*x]) / (16 * a * d * (a + a * \text{Sec}[c + d*x])^{3/2}) + ((35A-11B) * \text{Sin}[c + d*x]) / (16 * a^2 * d * \text{Sqrt}[a + a * \text{Sec}[c + d*x]])$

Rule 203

$\text{Int}[(a_ + (b_ * (x_)^2)^{-1}), x_Symbol] \rightarrow \text{Simp}[(1 * \text{ArcTan}[(\text{Rt}[b, 2] * x) / \text{Rt}[a, 2]]) / (\text{Rt}[a, 2] * \text{Rt}[b, 2]), x] /;$ FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 3774

$\text{Int}[\text{Sqrt}[\text{csc}[(c_ + (d_ * (x_))] * (b_ + (a_))], x_Symbol] \rightarrow \text{Dist}[(-2*b)/d, \text{Subst}[\text{Int}[1/(a + x^2), x], x, (b * \text{Cot}[c + d*x]) / \text{Sqrt}[a + b * \text{Csc}[c + d*x]]], x] /;$ FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol]
:> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x]
/; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol]
:> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^m*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol]
:> -Simp[((A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m + 1)), x]
- Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e + f*x], x], x], x]
/; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0]
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^m*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol]
:> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n - A*b*(m + n + 1)*Csc[e + f*x], x], x], x]
/; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{5/2}} dx &= -\frac{(A-B)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{\int \frac{\cos(c+dx)\left(a(5A-B)-\frac{5}{2}a(A-B)\sec(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}} dx}{4a^2} \\
&= -\frac{(A-B)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(15A-7B)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{\int \frac{\cos(c+dx)\left(\frac{1}{2}a^2(3A-2B)\sec(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}} dx}{16a^2d} \\
&= -\frac{(A-B)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(15A-7B)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{(35A-11B)\sin(c+dx)}{16a^2d\sqrt{a+a\sec(c+dx)}} \\
&= -\frac{(A-B)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(15A-7B)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{(35A-11B)\sin(c+dx)}{16a^2d\sqrt{a+a\sec(c+dx)}} \\
&= -\frac{(A-B)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(15A-7B)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{(35A-11B)\sin(c+dx)}{16a^2d\sqrt{a+a\sec(c+dx)}} \\
&= -\frac{(5A-2B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d} + \frac{(115A-43B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d}
\end{aligned}$$

Mathematica [C] time = 27.31, size = 12012, normalized size = 58.03

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(Cos[c + d*x]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(5/2), x]

[Out] Result too large to show

fricas [A] time = 8.20, size = 739, normalized size = 3.57

$$\left[\sqrt{2} \left((115A - 43B) \cos(dx + c)^3 + 3(115A - 43B) \cos(dx + c)^2 + 3(115A - 43B) \cos(dx + c) + 115A - 43B \right) \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")

[Out] [1/64*(sqrt(2)*((115*A - 43*B)*cos(d*x + c)^3 + 3*(115*A - 43*B)*cos(d*x + c)^2 + 3*(115*A - 43*B)*cos(d*x + c) + 115*A - 43*B)*sqrt(-a)*log(-(2*sqrt(2))*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) - 3*a*cos(d*x + c)^2 - 2*a*cos(d*x + c) + a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 32*((5*A - 2*B)*cos(d*x + c)^3 + 3*(5*A - 2*B)*cos(d*x + c)^2 + 3*(5*A - 2*B)*cos(d*x + c) + 5*A - 2*B)*sqrt(-a)*log((2*a*cos(d*x + c)^2 + 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 4*(16*A*cos(d*x + c)^3 + 5*(11*A - 3*B)*cos(d*x + c)^2 + (35*A - 11*B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1/32*(sqrt(2)*((115*A - 43*B)*cos(d*x + c)^3 + 3*(115*A - 43*B)*cos(d*x + c)^2 + 3*(115*A - 43*B)*cos(d*x + c) + 115*A - 43*B)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - 32*((5*A - 2*B)*cos(d*x + c)^3 + 3*(5*A - 2*B)*cos(d*x + c)^2 + 3*(5*A - 2*B)*cos(d*x + c) + 5*A - 2*B)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - 2*(16*A*cos(d*x + c)^3 + 5*(11*A - 3*B)*cos(d*x + c)^2 + (35*A - 11*B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]

giac [B] time = 4.67, size = 499, normalized size = 2.41

$$2\sqrt{-a \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + a} \left(\frac{2\sqrt{2}(Aa^5 - Ba^5) \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2}{a^8 \operatorname{sgn}\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - 1\right)} - \frac{\sqrt{2}(21Aa^5 - 13Ba^5)}{a^8 \operatorname{sgn}\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - 1\right)} \right) \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + \frac{\sqrt{2}(115A - 43B)}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")

[Out] 1/64*(2*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*sqrt(2)*(A*a^5 - B*a^5)*tan(1/2*d*x + 1/2*c)^2/(a^8*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) - sqrt(2)*(21*A*a^5 - 13*B*a^5)/(a^8*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))*tan(1/2*d*x + 1/2*c) + sqrt(2)*(115*A - 43*B)*log((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2)/(sqrt(-a)*a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) - 32*(5*A - 2*B)*log(abs(-562949953421312*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sq

$$\frac{\sqrt{-a \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + a}^2 - 1125899906842624 \sqrt{2} \operatorname{abs}(a) + 1688849860263936 a}{\operatorname{abs}\left(-562949953421312 \left(\sqrt{-a} \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - \sqrt{-a \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + a}\right)^2 + 1125899906842624 \sqrt{2} \operatorname{abs}(a) + 1688849860263936 a\right)} \frac{1}{\left(\sqrt{-a} a \operatorname{abs}(a) \operatorname{sgn}\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - 1\right) - 128 \sqrt{2} \left(3 \left(\sqrt{-a} \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - \sqrt{-a \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + a}\right)^2 A - A a\right) / \left(\left(\sqrt{-a} \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - \sqrt{-a \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + a}\right)^4 - 6 \left(\sqrt{-a} \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - \sqrt{-a \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + a}\right)^2 a + a^2\right) \sqrt{-a} a \operatorname{sgn}\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - 1\right)}\right)}{d}$$

maple [B] time = 1.63, size = 1065, normalized size = 5.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\int (\cos(dx+c) \cdot (A+B \sec(dx+c))) / (a+a \sec(dx+c))^{5/2}, x$

[Out] $\frac{1}{32} \frac{1}{d} \frac{(-1+\cos(dx+c))^2 (80A \sin(dx+c) \cos(dx+c)^2 (-2\cos(dx+c)/(1+\cos(dx+c)))^{1/2} \operatorname{arctanh}(1/2(-2\cos(dx+c)/(1+\cos(dx+c)))^{1/2} \sin(dx+c)/\cos(dx+c) \cdot 2^{1/2}) \cdot 2^{1/2} - 32B \sin(dx+c) \cos(dx+c)^2 (-2\cos(dx+c)/(1+\cos(dx+c)))^{1/2} \operatorname{arctanh}(1/2(-2\cos(dx+c)/(1+\cos(dx+c)))^{1/2} \sin(dx+c)/\cos(dx+c) \cdot 2^{1/2}) \cdot 2^{1/2} + 115A \sin(dx+c) \ln(-(-2\cos(dx+c)/(1+\cos(dx+c)))^{1/2} \sin(dx+c) + \cos(dx+c) - 1)/\sin(dx+c)) \cdot (-2\cos(dx+c)/(1+\cos(dx+c)))^{1/2} \cos(dx+c)^2 + 160A \sin(dx+c) \cos(dx+c) \cdot (-2\cos(dx+c)/(1+\cos(dx+c)))^{1/2} \operatorname{arctanh}(1/2(-2\cos(dx+c)/(1+\cos(dx+c)))^{1/2} \sin(dx+c)/\cos(dx+c) \cdot 2^{1/2}) \cdot 2^{1/2} - 43B \sin(dx+c) \ln(-(-2\cos(dx+c)/(1+\cos(dx+c)))^{1/2} \sin(dx+c) + \cos(dx+c) - 1)/\sin(dx+c)) \cdot (-2\cos(dx+c)/(1+\cos(dx+c)))^{1/2} \cos(dx+c)^2 - 64B \sin(dx+c) \cos(dx+c) \cdot (-2\cos(dx+c)/(1+\cos(dx+c)))^{1/2} \operatorname{arctanh}(1/2(-2\cos(dx+c)/(1+\cos(dx+c)))^{1/2} \sin(dx+c)/\cos(dx+c) \cdot 2^{1/2}) \cdot 2^{1/2} + 230A \cos(dx+c) \sin(dx+c) \ln(-(-2\cos(dx+c)/(1+\cos(dx+c)))^{1/2} \sin(dx+c) + \cos(dx+c) - 1)/\sin(dx+c)) \cdot (-2\cos(dx+c)/(1+\cos(dx+c)))^{1/2} + 80A \cdot 2^{1/2} \operatorname{arctanh}(1/2(-2\cos(dx+c)/(1+\cos(dx+c)))^{1/2} \sin(dx+c)/\cos(dx+c) \cdot 2^{1/2}) \cdot (-2\cos(dx+c)/(1+\cos(dx+c)))^{1/2} \sin(dx+c) - 32A \cos(dx+c)^4 - 86B \cos(dx+c) \sin(dx+c) \ln(-(-2\cos(dx+c)/(1+\cos(dx+c)))^{1/2} \sin(dx+c) + \cos(dx+c) - 1)/\sin(dx+c)) \cdot (-2\cos(dx+c)/(1+\cos(dx+c)))^{1/2} - 32B \cdot (-2\cos(dx+c)/(1+\cos(dx+c)))^{1/2} \operatorname{arctanh}(1/2(-2\cos(dx+c)/(1+\cos(dx+c)))^{1/2} \sin(dx+c)/\cos(dx+c) \cdot 2^{1/2}) \cdot 2^{1/2} \sin(dx+c) + 115A \ln(-(-2\cos(dx+c)/(1+\cos(dx+c)))^{1/2} \sin(dx+c) + \cos(dx+c) - 1)/\sin(dx+c)) \cdot (-2\cos(dx+c)/(1+\cos(dx+c)))^{1/2} \sin(dx+c) - 78A \cos(dx+c)^3 - 43B \ln(-(-2\cos(dx+c)/(1+\cos(dx+c)))^{1/2} \sin(dx+c) + \cos(dx+c) - 1)/\sin(dx+c)) \cdot (-2\cos(dx+c)/(1+\cos(dx+c)))^{1/2} \sin(dx+c) + 30B \cos(dx+c)^3 + 40A \cos(dx+c)^2 - 8B \cos(dx+c)^2 + 70A \cos(dx+c) - 22B \cos(dx+c)) \cdot (a(1+\cos(dx+c))/\cos(dx+c))^{1/2} / \sin(dx+c)^{5/a^3}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \cos(dx + c)}{(a \sec(dx + c) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)/(a*sec(d*x + c) + a)^(5/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c + dx) \left(A + \frac{B}{\cos(c+dx)} \right)}{\left(a + \frac{a}{\cos(c+dx)} \right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^(5/2),x)

[Out] int((cos(c + d*x)*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^(5/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(5/2),x)

[Out] Timed out

$$3.166 \quad \int \frac{\cos^2(c+dx)(A+B \sec(c+dx))}{(a+a \sec(c+dx))^{5/2}} dx$$

Optimal. Leaf size=264

$$\frac{(39A - 20B) \tan^{-1} \left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}} \right)}{4a^{5/2}d} - \frac{(219A - 115B) \tan^{-1} \left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{2} \sqrt{a \sec(c+dx)+a}} \right)}{16\sqrt{2} a^{5/2}d} - \frac{7(9A - 5B) \sin(c + dx)}{16a^2d\sqrt{a \sec(c + dx) + a}} + \frac{(31A - 15B) \cos(c + dx)}{16a^2d\sqrt{a \sec(c + dx) + a}}$$

[Out] 1/4*(39*A-20*B)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(5/2)/d - 1/4*(A-B)*cos(d*x+c)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(5/2)-1/16*(19*A-11*B)*cos(d*x+c)*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^(3/2)-1/32*(219*A-115*B)*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2)-7/16*(9*A-5*B)*sin(d*x+c)/a^2/d/(a+a*sec(d*x+c))^(1/2)+1/16*(31*A-15*B)*cos(d*x+c)*sin(d*x+c)/a^2/d/(a+a*sec(d*x+c))^(1/2)

Rubi [A] time = 0.79, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4020, 4022, 3920, 3774, 203, 3795}

$$-\frac{7(9A - 5B) \sin(c + dx)}{16a^2d\sqrt{a \sec(c + dx) + a}} + \frac{(39A - 20B) \tan^{-1} \left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}} \right)}{4a^{5/2}d} - \frac{(219A - 115B) \tan^{-1} \left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{2} \sqrt{a \sec(c+dx)+a}} \right)}{16\sqrt{2} a^{5/2}d} + \frac{(31A - 15B) \cos(c + dx)}{16a^2d\sqrt{a \sec(c + dx) + a}}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(5/2),x]

[Out] ((39*A - 20*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]])/(4*a^(5/2)*d) - ((219*A - 115*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B)*Cos[c + d*x]*Sin[c + d*x])/(4*d*(a + a*Sec[c + d*x])^(5/2)) - ((19*A - 11*B)*Cos[c + d*x]*Sin[c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/2)) - (7*(9*A - 5*B)*Sin[c + d*x])/(16*a^2*d*Sqrt[a + a*Sec[c + d*x]]) + ((31*A - 15*B)*Cos[c + d*x]*Sin[c + d*x])/(16*a^2*d*Sqrt[a + a*Sec[c + d*x]])

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 3774

Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] := Dist[(-2*b)/d, Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],

$x] /; \text{FreeQ}\{a, b, c, d\}, x\} \&\& \text{EqQ}[a^2 - b^2, 0]$

Rule 3795

$\text{Int}[\text{csc}[e_.] + (f_.)(x_.)]/\text{Sqrt}[\text{csc}[e_.] + (f_.)(x_.)]*(b_.) + (a_.)], x_Symbol] \rightarrow \text{Dist}[-2/f, \text{Subst}[\text{Int}[1/(2*a + x^2)], x], x, (b*\text{Cot}[e + f*x])/\text{Sqrt}[a + b*\text{Csc}[e + f*x]]], x] /; \text{FreeQ}\{a, b, e, f\}, x\} \&\& \text{EqQ}[a^2 - b^2, 0]$

Rule 3920

$\text{Int}[(\text{csc}[e_.] + (f_.)(x_.))*(d_.) + (c_.)]/\text{Sqrt}[\text{csc}[e_.] + (f_.)(x_.)]*(b_.) + (a_.)], x_Symbol] \rightarrow \text{Dist}[c/a, \text{Int}[\text{Sqrt}[a + b*\text{Csc}[e + f*x]], x], x] - \text{Dist}[(b*c - a*d)/a, \text{Int}[\text{Csc}[e + f*x]/\text{Sqrt}[a + b*\text{Csc}[e + f*x]], x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x\} \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0]$

Rule 4020

$\text{Int}[(\text{csc}[e_.] + (f_.)(x_.))*(d_.))^{(n_.)}*(\text{csc}[e_.] + (f_.)(x_.)]*(b_.) + (a_.))^{(m_.)}*(\text{csc}[e_.] + (f_.)(x_.)]*(B_.) + (A_.)], x_Symbol] \rightarrow -\text{Simp}[(A*b - a*B)*\text{Cot}[e + f*x]*(a + b*\text{Csc}[e + f*x])^m*(d*\text{Csc}[e + f*x])^n]/(b*f*(2*m + 1)), x] - \text{Dist}[1/(a^2*(2*m + 1)), \text{Int}[(a + b*\text{Csc}[e + f*x])^{(m + 1)}*(d*\text{Csc}[e + f*x])^n*\text{Simp}[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*\text{Csc}[e + f*x], x], x], x] /; \text{FreeQ}\{a, b, d, e, f, A, B, n\}, x\} \&\& \text{NeQ}[A*b - a*B, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{LtQ}[m, -2^{(-1)}] \&\& !\text{GtQ}[n, 0]$

Rule 4022

$\text{Int}[(\text{csc}[e_.] + (f_.)(x_.))*(d_.))^{(n_.)}*(\text{csc}[e_.] + (f_.)(x_.)]*(b_.) + (a_.))^{(m_.)}*(\text{csc}[e_.] + (f_.)(x_.)]*(B_.) + (A_.)], x_Symbol] \rightarrow \text{Simp}[(A*\text{Cot}[e + f*x]*(a + b*\text{Csc}[e + f*x])^m*(d*\text{Csc}[e + f*x])^n)/(f*n), x] - \text{Dist}[1/(b*d*n), \text{Int}[(a + b*\text{Csc}[e + f*x])^m*(d*\text{Csc}[e + f*x])^{(n + 1)}*\text{Simp}[a*A*m - b*B*n - A*b*(m + n + 1)*\text{Csc}[e + f*x], x], x], x] /; \text{FreeQ}\{a, b, d, e, f, A, B, m\}, x\} \&\& \text{NeQ}[A*b - a*B, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{LtQ}[n, 0]$

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{5/2}} dx &= -\frac{(A-B)\cos(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{\int \frac{\cos^2(c+dx)\left(2a(3A-B)-\frac{7}{2}a(A-B)\sec(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}}{4a^2} \\
&= -\frac{(A-B)\cos(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(19A-11B)\cos(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} \\
&= -\frac{(A-B)\cos(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(19A-11B)\cos(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} \\
&= -\frac{(A-B)\cos(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(19A-11B)\cos(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} \\
&= -\frac{(A-B)\cos(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(19A-11B)\cos(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} \\
&= -\frac{(A-B)\cos(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(19A-11B)\cos(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} \\
&= \frac{(39A-20B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4a^{5/2}d} - \frac{(219A-115B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d}
\end{aligned}$$

Mathematica [C] time = 6.17, size = 512, normalized size = 1.94

$$\frac{A(\sec(c+dx)+1)^{5/2} \left(\frac{760 \tan(c+dx) {}_2F_1\left(\frac{1}{2}, 3; \frac{3}{2}; 1-\sec(c+dx)\right)}{d\sqrt{\sec(c+dx)+1}} + \frac{152 \sin(c+dx)\cos(c+dx)}{d(\sec(c+dx)+1)^{3/2}} - \frac{219 \tan(c+dx)\left(2\cos^2(c+dx)\sqrt{1-\sec(c+dx)} - c\right)}{d(\sec(c+dx)+1)^{3/2}} \right)}{128(a(\sec(c+dx)+1))^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(5/2), x]

[Out] -1/4*(B*Sin[c + d*x])/(d*(a*(1 + Sec[c + d*x]))^(5/2)) - (A*Cos[c + d*x]*Sin[c + d*x])/(4*d*(a*(1 + Sec[c + d*x]))^(5/2)) - (5*B*(1 + Sec[c + d*x])^(5/2))*((6*Sin[c + d*x])/(d*(1 + Sec[c + d*x])^(3/2)) + (9*(Cos[c + d*x] + ArcTanh[Sqrt[1 - Sec[c + d*x]]])/Sqrt[1 - Sec[c + d*x]])*Tan[c + d*x])/(d*Sqrt[1 + Sec[c + d*x]]) + (23*(ArcTanh[Sqrt[1 - Sec[c + d*x]]] - Sqrt[2]*ArcTanh

$$\frac{(\sqrt{1 - \sec[c + dx]}/\sqrt{2}) - \cos[c + dx] \cdot \sqrt{1 - \sec[c + dx]}}{d \cdot \sqrt{1 - \sec[c + dx]} \cdot \sqrt{1 + \sec[c + dx]}} \cdot \tan[c + dx] \cdot \sqrt{1 - \sec[c + dx]}}{(32 \cdot (a \cdot (1 + \sec[c + dx]))^{5/2}) - (A \cdot (1 + \sec[c + dx])^{5/2} \cdot ((152 \cdot \cos[c + dx] \cdot \sin[c + dx]) / (d \cdot (1 + \sec[c + dx])^{3/2})) + (760 \cdot \text{Hypergeometric2F1}[1/2, 3, 3/2, 1 - \sec[c + dx]] \cdot \tan[c + dx]) / (d \cdot \sqrt{1 + \sec[c + dx]}) - (219 \cdot (7 \cdot \text{ArcTanh}[\sqrt{1 - \sec[c + dx]})] - 4 \cdot \sqrt{2} \cdot \text{ArcTanh}[\sqrt{1 - \sec[c + dx]}/\sqrt{2}] - \cos[c + dx] \cdot \sqrt{1 - \sec[c + dx]} + 2 \cdot \cos[c + dx]^2 \cdot \sqrt{1 - \sec[c + dx]}) \cdot \tan[c + dx]) / (d \cdot \sqrt{1 - \sec[c + dx]} \cdot \sqrt{1 + \sec[c + dx]})} \cdot \sqrt{1 - \sec[c + dx]}}{(128 \cdot (a \cdot (1 + \sec[c + dx]))^{5/2})}$$

fricas [A] time = 10.81, size = 776, normalized size = 2.94

$$\sqrt{2} \left((219A - 115B) \cos(dx + c)^3 + 3(219A - 115B) \cos(dx + c)^2 + 3(219A - 115B) \cos(dx + c) + 219A - 115B \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(dx+c)^2*(A+B*sec(dx+c))/(a+a*sec(dx+c))^(5/2),x, algorithm="fricas")

[Out] [1/64*(sqrt(2)*((219*A - 115*B)*cos(dx + c)^3 + 3*(219*A - 115*B)*cos(dx + c)^2 + 3*(219*A - 115*B)*cos(dx + c) + 219*A - 115*B)*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(dx + c) + a)/cos(dx + c))*cos(dx + c)*sin(dx + c) + 3*a*cos(dx + c)^2 + 2*a*cos(dx + c) - a)/(cos(dx + c)^2 + 2*cos(dx + c) + 1)) + 8*((39*A - 20*B)*cos(dx + c)^3 + 3*(39*A - 20*B)*cos(dx + c)^2 + 3*(39*A - 20*B)*cos(dx + c) + 39*A - 20*B)*sqrt(-a)*log((2*a*cos(dx + c)^2 - 2*sqrt(-a)*sqrt((a*cos(dx + c) + a)/cos(dx + c))*cos(dx + c)*sin(dx + c) + a*cos(dx + c) - a)/(cos(dx + c) + 1)) + 4*(8*A*cos(dx + c)^4 - 4*(5*A - 4*B)*cos(dx + c)^3 - 5*(19*A - 11*B)*cos(dx + c)^2 - 7*(9*A - 5*B)*cos(dx + c))*sqrt((a*cos(dx + c) + a)/cos(dx + c))*sin(dx + c))/(a^3*d*cos(dx + c)^3 + 3*a^3*d*cos(dx + c)^2 + 3*a^3*d*cos(dx + c) + a^3*d), 1/32*(sqrt(2)*((219*A - 115*B)*cos(dx + c)^3 + 3*(219*A - 115*B)*cos(dx + c)^2 + 3*(219*A - 115*B)*cos(dx + c) + 219*A - 115*B)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(dx + c) + a)/cos(dx + c))*cos(dx + c)/(sqrt(a)*sin(dx + c))) - 8*((39*A - 20*B)*cos(dx + c)^3 + 3*(39*A - 20*B)*cos(dx + c)^2 + 3*(39*A - 20*B)*cos(dx + c) + 39*A - 20*B)*sqrt(a)*arctan(sqrt((a*cos(dx + c) + a)/cos(dx + c))*cos(dx + c)/(sqrt(a)*sin(dx + c))) + 2*(8*A*cos(dx + c)^4 - 4*(5*A - 4*B)*cos(dx + c)^3 - 5*(19*A - 11*B)*cos(dx + c)^2 - 7*(9*A - 5*B)*cos(dx + c))*sqrt((a*cos(dx + c) + a)/cos(dx + c))*sin(dx + c))/(a^3*d*cos(dx + c)^3 + 3*a^3*d*cos(dx + c)^2 + 3*a^3*d*cos(dx + c) + a^3*d)]

giac [B] time = 4.67, size = 720, normalized size = 2.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")

[Out]
$$\begin{aligned} & -1/64*(2*\sqrt{-a*\tan(1/2*d*x + 1/2*c)^2 + a}*(2*\sqrt{2}*(A*a^5 - B*a^5)*\tan \\ & (1/2*d*x + 1/2*c)^2/(a^8*\operatorname{sgn}(\tan(1/2*d*x + 1/2*c)^2 - 1)) - \sqrt{2}*(29*A*a^5 \\ & - 21*B*a^5)/(a^8*\operatorname{sgn}(\tan(1/2*d*x + 1/2*c)^2 - 1)))*\tan(1/2*d*x + 1/2*c) \\ & + \sqrt{2}*(219*A - 115*B)*\log((\sqrt{-a}*\tan(1/2*d*x + 1/2*c) - \sqrt{-a*\tan(1/2*d*x \\ & + 1/2*c)^2 + a})^2)/(\sqrt{-a}*a^2*\operatorname{sgn}(\tan(1/2*d*x + 1/2*c)^2 - 1)) \\ & + 8*(39*A - 20*B)*\log(\operatorname{abs}(309485009821345068724781056*(\sqrt{-a}*\tan(1/2*d*x \\ & + 1/2*c) - \sqrt{-a*\tan(1/2*d*x + 1/2*c)^2 + a})^2 - 6189700196426901374495 \\ & 62112*\sqrt{2}*\operatorname{abs}(a) - 928455029464035206174343168*a)/\operatorname{abs}(30948500982134506 \\ & 8724781056*(\sqrt{-a}*\tan(1/2*d*x + 1/2*c) - \sqrt{-a*\tan(1/2*d*x + 1/2*c)^2 \\ & + a})^2 + 618970019642690137449562112*\sqrt{2}*\operatorname{abs}(a) - 92845502946403520617 \\ & 4343168*a))/(\sqrt{-a}*a*\operatorname{abs}(a)*\operatorname{sgn}(\tan(1/2*d*x + 1/2*c)^2 - 1)) - 32*\sqrt{2} \\ &)*(41*(\sqrt{-a}*\tan(1/2*d*x + 1/2*c) - \sqrt{-a*\tan(1/2*d*x + 1/2*c)^2 + a}) \\ & ^6*A - 12*(\sqrt{-a}*\tan(1/2*d*x + 1/2*c) - \sqrt{-a*\tan(1/2*d*x + 1/2*c)^2 + \\ & a})^6*B - 209*(\sqrt{-a}*\tan(1/2*d*x + 1/2*c) - \sqrt{-a*\tan(1/2*d*x + 1/2*c} \\ &)^2 + a)^4*A*a + 76*(\sqrt{-a}*\tan(1/2*d*x + 1/2*c) - \sqrt{-a*\tan(1/2*d*x + \\ & 1/2*c)^2 + a})^4*B*a + 91*(\sqrt{-a}*\tan(1/2*d*x + 1/2*c) - \sqrt{-a*\tan(1/2 \\ & *d*x + 1/2*c)^2 + a})^2*A*a^2 - 36*(\sqrt{-a}*\tan(1/2*d*x + 1/2*c) - \sqrt{-a \\ & *\tan(1/2*d*x + 1/2*c)^2 + a})^2*B*a^2 - 11*A*a^3 + 4*B*a^3)/(((\sqrt{-a}*\tan \\ & (1/2*d*x + 1/2*c) - \sqrt{-a*\tan(1/2*d*x + 1/2*c)^2 + a})^4 - 6*(\sqrt{-a}*\tan \\ & (1/2*d*x + 1/2*c) - \sqrt{-a*\tan(1/2*d*x + 1/2*c)^2 + a})^2*a + a^2)^2*\sqrt{2} \\ & (-a)*a*\operatorname{sgn}(\tan(1/2*d*x + 1/2*c)^2 - 1)))/d \end{aligned}$$

maple [B] time = 1.90, size = 1427, normalized size = 5.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x)

[Out]
$$\begin{aligned} & 1/64/d*(-1+\cos(d*x+c))^2*(140*B*\cos(d*x+c)^2-252*A*\cos(d*x+c)^2+300*A*\cos(d \\ & *x+c)^4-156*B*\cos(d*x+c)^4+468*A^2*(1/2)*\operatorname{arctanh}(1/2*(-2*\cos(d*x+c)/(1+\cos(\\ & d*x+c)))^(1/2)*\sin(d*x+c)/\cos(d*x+c)*2^(1/2))*(-2*\cos(d*x+c)/(1+\cos(d*x+c)) \\ &)^(3/2)*\cos(d*x+c)^2*\sin(d*x+c)+219*A*\ln(-(-(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^(\\ & 1/2)*\sin(d*x+c)+\cos(d*x+c)-1)/\sin(d*x+c))*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^(\\ & 3/2)*\sin(d*x+c)-115*B*\ln(-(-(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^(1/2)*\sin(d*x+c) \\ & +\cos(d*x+c)-1)/\sin(d*x+c))*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^(3/2)*\sin(d*x+c)- \end{aligned}$$

$$240*B*2^{(1/2)}*\operatorname{arctanh}(1/2*(-2*\cos(dx+c)/(1+\cos(dx+c)))^{(1/2)}*\sin(dx+c)/\cos(dx+c)*2^{(1/2)})*(-2*\cos(dx+c)/(1+\cos(dx+c)))^{(3/2)}*\cos(dx+c)^2*\sin(dx+c)-128*A*\cos(dx+c)^3+80*B*\cos(dx+c)^3-240*B*\cos(dx+c)*\operatorname{arctanh}(1/2*(-2*\cos(dx+c)/(1+\cos(dx+c)))^{(1/2)}*\sin(dx+c)/\cos(dx+c)*2^{(1/2)})*(-2*\cos(dx+c)/(1+\cos(dx+c)))^{(3/2)}*\sin(dx+c)*2^{(1/2)}+468*A*\cos(dx+c)*\operatorname{arctanh}(1/2*(-2*\cos(dx+c)/(1+\cos(dx+c)))^{(1/2)}*\sin(dx+c)/\cos(dx+c)*2^{(1/2)})*(-2*\cos(dx+c)/(1+\cos(dx+c)))^{(3/2)}*\sin(dx+c)*2^{(1/2)}+112*A*\cos(dx+c)^5-64*B*\cos(dx+c)^5-80*B*2^{(1/2)}*\operatorname{arctanh}(1/2*(-2*\cos(dx+c)/(1+\cos(dx+c)))^{(1/2)}*\sin(dx+c)/\cos(dx+c)*2^{(1/2)})*(-2*\cos(dx+c)/(1+\cos(dx+c)))^{(3/2)}*\sin(dx+c)+219*A*\cos(dx+c)^3*\sin(dx+c)*\ln(-(-2*\cos(dx+c)/(1+\cos(dx+c)))^{(1/2)}*\sin(dx+c)+\cos(dx+c)-1)/\sin(dx+c))*(-2*\cos(dx+c)/(1+\cos(dx+c)))^{(3/2)}-115*B*\cos(dx+c)^3*\sin(dx+c)*\ln(-(-2*\cos(dx+c)/(1+\cos(dx+c)))^{(1/2)}*\sin(dx+c)+\cos(dx+c)-1)/\sin(dx+c))*(-2*\cos(dx+c)/(1+\cos(dx+c)))^{(3/2)}-32*A*\cos(dx+c)^6+657*A*\ln(-(-2*\cos(dx+c)/(1+\cos(dx+c)))^{(1/2)}*\sin(dx+c)+\cos(dx+c)-1)/\sin(dx+c))*(-2*\cos(dx+c)/(1+\cos(dx+c)))^{(3/2)}*\sin(dx+c)*\cos(dx+c)^2-345*B*\ln(-(-2*\cos(dx+c)/(1+\cos(dx+c)))^{(1/2)}*\sin(dx+c)+\cos(dx+c)-1)/\sin(dx+c))*(-2*\cos(dx+c)/(1+\cos(dx+c)))^{(3/2)}*\sin(dx+c)*\cos(dx+c)^2-345*B*\cos(dx+c)*\sin(dx+c)*\ln(-(-2*\cos(dx+c)/(1+\cos(dx+c)))^{(1/2)}*\sin(dx+c)+\cos(dx+c)-1)/\sin(dx+c))*(-2*\cos(dx+c)/(1+\cos(dx+c)))^{(3/2)}+156*A*2^{(1/2)}*\operatorname{arctanh}(1/2*(-2*\cos(dx+c)/(1+\cos(dx+c)))^{(1/2)}*\sin(dx+c)/\cos(dx+c)*2^{(1/2)})*(-2*\cos(dx+c)/(1+\cos(dx+c)))^{(3/2)}*\sin(dx+c)+657*A*\cos(dx+c)*\sin(dx+c)*\ln(-(-2*\cos(dx+c)/(1+\cos(dx+c)))^{(1/2)}*\sin(dx+c)+\cos(dx+c)-1)/\sin(dx+c))*(-2*\cos(dx+c)/(1+\cos(dx+c)))^{(3/2)}+156*A*\operatorname{arctanh}(1/2*(-2*\cos(dx+c)/(1+\cos(dx+c)))^{(1/2)}*\sin(dx+c)/\cos(dx+c)*2^{(1/2)})*(-2*\cos(dx+c)/(1+\cos(dx+c)))^{(3/2)}*\sin(dx+c)*2^{(1/2)}*\cos(dx+c)^3-80*B*\operatorname{arctanh}(1/2*(-2*\cos(dx+c)/(1+\cos(dx+c)))^{(1/2)}*\sin(dx+c)/\cos(dx+c)*2^{(1/2)})*(-2*\cos(dx+c)/(1+\cos(dx+c)))^{(3/2)}*\sin(dx+c)*2^{(1/2)}*\cos(dx+c)^3*(a*(1+\cos(dx+c))/\cos(dx+c))^{(1/2)}/\sin(dx+c)^5/\cos(dx+c)/a^3$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx+c) + A) \cos(dx+c)^2}{(a \sec(dx+c) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(dx+c)^2*(A+B*sec(dx+c))/(a+a*sec(dx+c))^(5/2),x, algorithm="maxima")

[Out] integrate((B*sec(dx+c) + A)*cos(dx+c)^2/(a*sec(dx+c) + a)^(5/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c + dx)^2 \left(A + \frac{B}{\cos(c+dx)} \right)}{\left(a + \frac{a}{\cos(c+dx)} \right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)^2*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^(5/2), x)

[Out] int((cos(c + d*x)^2*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^(5/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \cos^2(c + dx)}{(a(\sec(c + dx) + 1))^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(5/2), x)

[Out] Integral((A + B*sec(c + d*x))*cos(c + d*x)**2/(a*(sec(c + d*x) + 1))**(5/2), x)

$$3.167 \quad \int \frac{A + A \sec(c+dx)}{\sqrt{a-a \sec(c+dx)}} dx$$

Optimal. Leaf size=89

$$\frac{2A \tan^{-1} \left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a-a \sec(c+dx)}} \right)}{\sqrt{a} d} - \frac{2\sqrt{2} A \tan^{-1} \left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{2} \sqrt{a-a \sec(c+dx)}} \right)}{\sqrt{a} d}$$

[Out] $2*A*\arctan(a^{(1/2)}*\tan(d*x+c)/(a-a*\sec(d*x+c))^{(1/2)})/d/a^{(1/2)}-2*A*\arctan(1/2*a^{(1/2)}*\tan(d*x+c)*2^{(1/2)}/(a-a*\sec(d*x+c))^{(1/2)})*2^{(1/2)}/d/a^{(1/2)}$

Rubi [A] time = 0.15, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3904, 3887, 481, 203}

$$\frac{2A \tan^{-1} \left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a-a \sec(c+dx)}} \right)}{\sqrt{a} d} - \frac{2\sqrt{2} A \tan^{-1} \left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{2} \sqrt{a-a \sec(c+dx)}} \right)}{\sqrt{a} d}$$

Antiderivative was successfully verified.

[In] Int[(A + A*Sec[c + d*x])/Sqrt[a - a*Sec[c + d*x]], x]

[Out] $(2*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a - a*Sec[c + d*x]])/(Sqrt[a]*d) - (2*Sqrt[2]*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[2]*Sqrt[a - a*Sec[c + d*x]])/(Sqrt[a]*d)$

Rule 203

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 481

Int[((e_)*(x_))^(m_)/(((a_) + (b_)*(x_)^(n_))*((c_) + (d_)*(x_)^(n_))), x_Symbol] := -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(a + b*x^n), x], x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m, 2*n - 1]

Rule 3887

Int[cot[(c_) + (d_)*(x_)]^(m_)*(csc[(c_) + (d_)*(x_)]*(b_) + (a_))^(n_), x_Symbol] := Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)

$(m/2 + n - 1/2)/(1 + a*x^2), x], x, \text{Cot}[c + d*x]/\text{Sqrt}[a + b*\text{Csc}[c + d*x]]$
 $], x] /;$ FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && IntegerQ[n - 1/2]

Rule 3904

Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))^(n_.), x_Symbol] :> Dist[(-a*c)^(m), Int[Cot[e + f*x]^(2*m)*(c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] && !IntegerQ[n] && GtQ[m - n, 0]

Rubi steps

$$\begin{aligned} \int \frac{A + A \sec(c + dx)}{\sqrt{a - a \sec(c + dx)}} dx &= - \left((aA) \int \frac{\tan^2(c + dx)}{(a - a \sec(c + dx))^{3/2}} dx \right) \\ &= \frac{(2aA) \text{Subst} \left(\int \frac{x^2}{(1+ax^2)(2+ax^2)} dx, x, -\frac{\tan(c+dx)}{\sqrt{a-a \sec(c+dx)}} \right)}{d} \\ &= -\frac{(2A) \text{Subst} \left(\int \frac{1}{1+ax^2} dx, x, -\frac{\tan(c+dx)}{\sqrt{a-a \sec(c+dx)}} \right)}{d} + \frac{(4A) \text{Subst} \left(\int \frac{1}{2+ax^2} dx, x, -\frac{\tan(c+dx)}{\sqrt{a-a \sec(c+dx)}} \right)}{d} \\ &= \frac{2A \tan^{-1} \left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a-a \sec(c+dx)}} \right)}{\sqrt{a} d} - \frac{2\sqrt{2} A \tan^{-1} \left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{2} \sqrt{a-a \sec(c+dx)}} \right)}{\sqrt{a} d} \end{aligned}$$

Mathematica [C] time = 0.64, size = 140, normalized size = 1.57

$$\frac{iA(-1 + e^{i(c+dx)}) \left(\sqrt{2} \sinh^{-1} \left(e^{i(c+dx)} \right) - 4 \tanh^{-1} \left(\frac{1 + e^{i(c+dx)}}{\sqrt{2} \sqrt{1 + e^{2i(c+dx)}}} \right) + \sqrt{2} \tanh^{-1} \left(\sqrt{1 + e^{2i(c+dx)}} \right) \right)}{\sqrt{2} d \sqrt{1 + e^{2i(c+dx)}} \sqrt{a - a \sec(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + A*Sec[c + d*x])/Sqrt[a - a*Sec[c + d*x]],x]

[Out] ((-I)*A*(-1 + E^(I*(c + d*x)))*(Sqrt[2]*ArcSinh[E^(I*(c + d*x))] - 4*ArcTanh[(1 + E^(I*(c + d*x))]/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))]]) + Sqrt[2]*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]])/(Sqrt[2]*d*Sqrt[1 + E^((2*I)*(c + d*x))])*Sqrt[a - a*Sec[c + d*x]])

fricas [A] time = 0.47, size = 305, normalized size = 3.43

$$\frac{\sqrt{2} A a \sqrt{-\frac{1}{a}} \log\left(\frac{2\sqrt{2}(\cos(dx+c)^2 + \cos(dx+c))\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}} - (3\cos(dx+c)+1)\sin(dx+c)}{(\cos(dx+c)-1)\sin(dx+c)}\right) - A\sqrt{-a} \log\left(\frac{2(\cos(dx+c)^2 + \cos(dx+c))\sqrt{-a}}{(\cos(dx+c)-1)\sin(dx+c)}\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(1/2),x, algorithm="fricas")

[Out] [(sqrt(2)*A*a*sqrt(-1/a)*log(-(2*sqrt(2)*(cos(d*x + c))^2 + cos(d*x + c))*sqrt((a*cos(d*x + c) - a)/cos(d*x + c))*sqrt(-1/a) - (3*cos(d*x + c) + 1)*sin(d*x + c))/((cos(d*x + c) - 1)*sin(d*x + c))) - A*sqrt(-a)*log((2*(cos(d*x + c))^2 + cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) - a)/cos(d*x + c)) - (2*a*cos(d*x + c) + a)*sin(d*x + c))/sin(d*x + c))/(a*d), 2*(sqrt(2)*A*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) - a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - A*sqrt(a)*arctan(sqrt((a*cos(d*x + c) - a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))))/(a*d)]

giac [C] time = 1.13, size = 166, normalized size = 1.87

$$\frac{2 \left(A \frac{\sqrt{2} \arctan\left(\frac{\sqrt{a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - a}}{\sqrt{a}}\right)}{\sqrt{a} \operatorname{sgn}\left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 1\right) \operatorname{sgn}\left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right)} - \frac{\arctan\left(\frac{\sqrt{2} \sqrt{a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - a}}{2 \sqrt{a}}\right)}{\sqrt{a} \operatorname{sgn}\left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 1\right) \operatorname{sgn}\left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right)} \right) + \frac{(\sqrt{2} A \sqrt{a} \arctan(-i) - A \sqrt{a} \arctan(-i))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(1/2),x, algorithm="giac")

[Out] -2*(A*(sqrt(2)*arctan(sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)/sqrt(a))/(sqrt(a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c))) - arctan(1/2*sqrt(2)*sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)/sqrt(a))/(sqrt(a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)))) + (sqrt(2)*A*sqrt(a)*arctan(-I) - A*sqrt(a)*arctan(-1/2*I*sqrt(2)))*sgn(tan(1/2*d*x + 1/2*c))/a/d

maple [A] time = 1.54, size = 120, normalized size = 1.35

$$A \left(\arctan \left(\frac{1}{\sqrt{\frac{2 \cos(dx+c)}{1+\cos(dx+c)}}} \right) \sqrt{2} + \arctan \left(\frac{\sqrt{\frac{2 \cos(dx+c)}{1+\cos(dx+c)}} \sqrt{2}}{2} \right) \right) \sqrt{\frac{a(-1+\cos(dx+c))}{\cos(dx+c)}} \sqrt{\frac{2 \cos(dx+c)}{1+\cos(dx+c)}} (1 + \cos(dx+c)) \sqrt{2}$$

$$d \sin(dx+c) a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(1/2),x)`

[Out] `A/d*(arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*2^(1/2)+arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)))*(a*(-1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c))/sin(d*x+c)/a*2^(1/2)`

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(1/2),x, algorithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: sign: argument cannot be imaginary; found %i

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + \frac{A}{\cos(c+dx)}}{\sqrt{a - \frac{a}{\cos(c+dx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + A/cos(c + d*x))/(a - a/cos(c + d*x))^(1/2),x)`

[Out] `int((A + A/cos(c + d*x))/(a - a/cos(c + d*x))^(1/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$A \left(\int \frac{\sec(c+dx)}{\sqrt{-a \sec(c+dx) + a}} dx + \int \frac{1}{\sqrt{-a \sec(c+dx) + a}} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+A*sec(d*x+c))/(a-a*sec(d*x+c))**(1/2),x)`

[Out] `A*(Integral(sec(c + d*x)/sqrt(-a*sec(c + d*x) + a), x) + Integral(1/sqrt(-a*sec(c + d*x) + a), x))`

$$3.168 \quad \int \frac{\cos(c+dx)(A+A \sec(c+dx))}{\sqrt{a-a \sec(c+dx)}} dx$$

Optimal. Leaf size=115

$$\frac{A \sin(c+dx)}{d\sqrt{a-a \sec(c+dx)}} + \frac{3A \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a-a \sec(c+dx)}}\right)}{\sqrt{a}d} - \frac{2\sqrt{2} A \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{2} \sqrt{a-a \sec(c+dx)}}\right)}{\sqrt{a}d}$$

[Out] $3*A*\arctan(a^{(1/2)}*\tan(d*x+c)/(a-a*\sec(d*x+c))^{(1/2)})/d/a^{(1/2)}-2*A*\arctan(1/2*a^{(1/2)}*\tan(d*x+c)*2^{(1/2)/(a-a*\sec(d*x+c))^{(1/2)}}*2^{(1/2)}/d/a^{(1/2)}+A*\sin(d*x+c)/d/(a-a*\sec(d*x+c))^{(1/2)}$

Rubi [A] time = 0.22, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {4022, 3920, 3774, 203, 3795}

$$\frac{A \sin(c+dx)}{d\sqrt{a-a \sec(c+dx)}} + \frac{3A \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a-a \sec(c+dx)}}\right)}{\sqrt{a}d} - \frac{2\sqrt{2} A \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{2} \sqrt{a-a \sec(c+dx)}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(\text{Cos}[c+d*x]*(A+A*\text{Sec}[c+d*x]))/\text{Sqrt}[a-a*\text{Sec}[c+d*x]],x]$

[Out] $(3*A*\text{ArcTan}[(\text{Sqrt}[a]*\text{Tan}[c+d*x])/\text{Sqrt}[a-a*\text{Sec}[c+d*x]]])/(\text{Sqrt}[a]*d) - (2*\text{Sqrt}[2]*A*\text{ArcTan}[(\text{Sqrt}[a]*\text{Tan}[c+d*x])/(\text{Sqrt}[2]*\text{Sqrt}[a-a*\text{Sec}[c+d*x]])])/(\text{Sqrt}[a]*d) + (A*\text{Sin}[c+d*x])/(\text{d}*\text{Sqrt}[a-a*\text{Sec}[c+d*x]])$

Rule 203

$\text{Int}[(a_+ + (b_+)*(x_+)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1*\text{ArcTan}[(\text{Rt}[b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[b, 2]), x] /; \text{FreeQ}\{a, b, x\} \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{GtQ}[b, 0])$

Rule 3774

$\text{Int}[\text{Sqrt}[\text{csc}[(c_+)+(d_+)*(x_+)]*(b_+)+(a_+)], x_Symbol] \rightarrow \text{Dist}[(-2*b)/d, \text{Subst}[\text{Int}[1/(a+x^2), x], x, (b*\text{Cot}[c+d*x])/\text{Sqrt}[a+b*\text{Csc}[c+d*x]]], x] /; \text{FreeQ}\{a, b, c, d, x\} \ \&\& \ \text{EqQ}[a^2-b^2, 0]$

Rule 3795

$\text{Int}[\text{csc}[(e_+)+(f_+)*(x_+)]/\text{Sqrt}[\text{csc}[(e_+)+(f_+)*(x_+)]*(b_+)+(a_+)], x_Symbol] \rightarrow \text{Dist}[-2/f, \text{Subst}[\text{Int}[1/(2*a+x^2), x], x, (b*\text{Cot}[e+f*x])/\text{Sqrt}[a+b*\text{Csc}[e+f*x]]], x] /; \text{FreeQ}\{a, b, e, f, x\} \ \&\& \ \text{EqQ}[a^2-b^2, 0]$

Rule 3920

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]

Rule 4022

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d*m), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]

Rubi steps

$$\begin{aligned} \int \frac{\cos(c + dx)(A + A \sec(c + dx))}{\sqrt{a - a \sec(c + dx)}} dx &= \frac{A \sin(c + dx)}{d\sqrt{a - a \sec(c + dx)}} - \int \frac{-\frac{3aA}{2} - \frac{1}{2}aA \sec(c + dx)}{\sqrt{a - a \sec(c + dx)}} dx \\ &= \frac{A \sin(c + dx)}{d\sqrt{a - a \sec(c + dx)}} + (2A) \int \frac{\sec(c + dx)}{\sqrt{a - a \sec(c + dx)}} dx + \frac{(3A) \int \sqrt{a - a \sec(c + dx)}}{2} \\ &= \frac{A \sin(c + dx)}{d\sqrt{a - a \sec(c + dx)}} + \frac{(3A) \text{Subst}\left(\int \frac{1}{a+x^2} dx, x, \frac{a \tan(c + dx)}{\sqrt{a - a \sec(c + dx)}}\right)}{d} \quad (4A) S \\ &= \frac{3A \tan^{-1}\left(\frac{\sqrt{a} \tan(c + dx)}{\sqrt{a - a \sec(c + dx)}}\right)}{\sqrt{a} d} - \frac{2\sqrt{2} A \tan^{-1}\left(\frac{\sqrt{a} \tan(c + dx)}{\sqrt{2} \sqrt{a - a \sec(c + dx)}}\right)}{\sqrt{a} d} + \frac{A \sin(c + dx)}{d\sqrt{a - a \sec(c + dx)}} \end{aligned}$$

Mathematica [C] time = 1.50, size = 269, normalized size = 2.34

$$A e^{-\frac{1}{2}i(c+dx)} \sin\left(\frac{1}{2}(c+dx)\right) \sec(c+dx) \left(\cos\left(\frac{1}{2}(c+dx)\right) + i \sin\left(\frac{1}{2}(c+dx)\right)\right) \left(3e^{-\frac{1}{2}i(c+dx)} \sqrt{1 + e^{2i(c+dx)}} \sinh^{-1}\left(e^{i(c+dx)}\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]*(A + A*Sec[c + d*x]))/Sqrt[a - a*Sec[c + d*x]],x]
 [Out] (A*((3*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcSinh[E^(I*(c + d*x))])/E^((I/2)*(c + d*x)) + (1 + E^((-I)*(c + d*x)) + E^(I*(c + d*x)) + E^((2*I)*(c + d*x)) -

$4\sqrt{2} \sqrt{1 + E^{(2I)(c + dx)}} \operatorname{ArcTanh}\left[\frac{1 + E^{I(c + dx)}}{\sqrt{1 + E^{(2I)(c + dx)}}}\right] + 3\sqrt{1 + E^{(2I)(c + dx)}} \operatorname{ArcTanh}\left[\sqrt{1 + E^{(2I)(c + dx)}}\right] / E^{(I/2)(c + dx)} \operatorname{Sec}[c + dx] (\cos[(c + dx)/2] + I \sin[(c + dx)/2]) \sin[(c + dx)/2] / (2d E^{(I/2)(c + dx)}) \sqrt{a - a \operatorname{Sec}[c + dx]}$

fricas [A] time = 0.45, size = 435, normalized size = 3.78

$$\left[\frac{2\sqrt{2} A a \sqrt{-\frac{1}{a}} \log\left(\frac{2\sqrt{2}(\cos(dx+c)^2 + \cos(dx+c)) \sqrt{\frac{a \cos(dx+c) - a}{\cos(dx+c)}} \sqrt{-\frac{1}{a}} - (3 \cos(dx+c) + 1) \sin(dx+c)}{(\cos(dx+c) - 1) \sin(dx+c)} \right) \sin(dx+c) - 3 A \sqrt{-a} \log\left(\frac{2\sqrt{2}(\cos(dx+c)^2 + \cos(dx+c)) \sqrt{\frac{a \cos(dx+c) - a}{\cos(dx+c)}} \sqrt{-\frac{1}{a}} - (3 \cos(dx+c) + 1) \sin(dx+c)}{(\cos(dx+c) - 1) \sin(dx+c)} \right)}{\dots} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(dx+c)*(A+A*sec(dx+c))/(a-a*sec(dx+c))^(1/2),x, algorithm="fricas")

[Out] $\frac{1}{2} (2\sqrt{2} A a \sqrt{-1/a} \log(-2\sqrt{2}(\cos(dx+c)^2 + \cos(dx+c)) \sqrt{(a \cos(dx+c) - a)/\cos(dx+c)} \sqrt{-1/a} - (3 \cos(dx+c) + 1) \sin(dx+c)) / ((\cos(dx+c) - 1) \sin(dx+c)) \sin(dx+c) - 3 A \sqrt{-a} \log((2(\cos(dx+c)^2 + \cos(dx+c)) \sqrt{-a} \sqrt{(a \cos(dx+c) - a)/\cos(dx+c)} - (2a \cos(dx+c) + a) \sin(dx+c)) / \sin(dx+c)) \sin(dx+c) - 2(A \cos(dx+c)^2 + A \cos(dx+c)) \sqrt{(a \cos(dx+c) - a)/\cos(dx+c)}) / (a d \sin(dx+c)), (2\sqrt{2} A \sqrt{a} \arctan(\sqrt{2} \sqrt{(a \cos(dx+c) - a)/\cos(dx+c)} \cos(dx+c) / (\sqrt{a} \sin(dx+c))) \sin(dx+c) - 3 A \sqrt{a} \arctan(\sqrt{(a \cos(dx+c) - a)/\cos(dx+c)} \cos(dx+c) / (\sqrt{a} \sin(dx+c))) \sin(dx+c) - (A \cos(dx+c)^2 + A \cos(dx+c)) \sqrt{(a \cos(dx+c) - a)/\cos(dx+c)}) / (a d \sin(dx+c)))$

giac [C] time = 1.15, size = 238, normalized size = 2.07

$$\frac{(-2i\sqrt{2} A \arctan(-i) + 3i A \arctan(-\frac{1}{2}i\sqrt{2}) + \sqrt{2} A) \operatorname{sgn}\left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right)}{\sqrt{-a}} + \frac{2\sqrt{2} A \arctan\left(\frac{\sqrt{a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - a}}{\sqrt{a}}\right)}{\sqrt{a} \operatorname{sgn}\left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 1\right) \operatorname{sgn}\left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right)} - \frac{3 A \arctan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{\sqrt{a} \operatorname{sgn}\left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right)} d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(dx+c)*(A+A*sec(dx+c))/(a-a*sec(dx+c))^(1/2),x, algorithm="giac")

```
[Out] -((-2*I*sqrt(2)*A*arctan(-I) + 3*I*A*arctan(-1/2*I*sqrt(2)) + sqrt(2)*A)*sgn(tan(1/2*d*x + 1/2*c))/sqrt(-a) + 2*sqrt(2)*A*arctan(sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)/sqrt(a))/(sqrt(a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c))) - 3*A*arctan(1/2*sqrt(2)*sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)/sqrt(a))/(sqrt(a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c))) - sqrt(2)*sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)*A/((a*tan(1/2*d*x + 1/2*c)^2 + a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)))/d
```

maple [A] time = 1.72, size = 154, normalized size = 1.34

$$A(1 + \cos(dx + c)) \left(-2\sqrt{2} \arctan \left(\frac{1}{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}} \right) \sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}} + \cos(dx + c) \sqrt{2} - 3 \arctan \left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}} \sqrt{2}}{2} \right) \right) / (2d \sin(dx + c) a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(1/2), x)
```

```
[Out] -1/2*A/d*(1+cos(d*x+c))*(-2*2^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c))))^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+cos(d*x+c)*2^(1/2)-3*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*(a*(-1+cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)/a*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A \sec(dx + c) + A) \cos(dx + c)}{\sqrt{-a \sec(dx + c) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(1/2), x, algorithm="maxima")
```

```
[Out] integrate((A*sec(d*x + c) + A)*cos(d*x + c)/sqrt(-a*sec(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c + dx) \left(A + \frac{A}{\cos(c+dx)} \right)}{\sqrt{a - \frac{a}{\cos(c+dx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(A + A/cos(c + d*x)))/(a - a/cos(c + d*x))^(1/2), x)
```

[Out] `int((cos(c + d*x)*(A + A/cos(c + d*x)))/(a - a/cos(c + d*x))^(1/2), x)`

sympy [F] `time = 0.00, size = 0, normalized size = 0.00`

$$A \left(\int \frac{\cos(c + dx)}{\sqrt{-a \sec(c + dx) + a}} dx + \int \frac{\cos(c + dx) \sec(c + dx)}{\sqrt{-a \sec(c + dx) + a}} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))**(1/2),x)`

[Out] `A*(Integral(cos(c + d*x)/sqrt(-a*sec(c + d*x) + a), x) + Integral(cos(c + d*x)*sec(c + d*x)/sqrt(-a*sec(c + d*x) + a), x))`

$$3.169 \quad \int \frac{\cos^2(c+dx)(A+A \sec(c+dx))}{\sqrt{a-a \sec(c+dx)}} dx$$

Optimal. Leaf size=155

$$\frac{5A \sin(c+dx)}{4d\sqrt{a-a \sec(c+dx)}} + \frac{11A \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a-a \sec(c+dx)}}\right)}{4\sqrt{a}d} - \frac{2\sqrt{2}A \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{2}\sqrt{a-a \sec(c+dx)}}\right)}{\sqrt{a}d} + \frac{A \sin(c+dx) \cos(c+dx)}{2d\sqrt{a-a \sec(c+dx)}}$$

[Out] $11/4*A*\arctan(a^{(1/2)}*\tan(d*x+c)/(a-a*\sec(d*x+c))^{(1/2)})/d/a^{(1/2)}-2*A*\arctan(1/2*a^{(1/2)}*\tan(d*x+c)*2^{(1/2)}/(a-a*\sec(d*x+c))^{(1/2)})*2^{(1/2)}/d/a^{(1/2)}+5/4*A*\sin(d*x+c)/d/(a-a*\sec(d*x+c))^{(1/2)}+1/2*A*\cos(d*x+c)*\sin(d*x+c)/d/(a-a*\sec(d*x+c))^{(1/2)}$

Rubi [A] time = 0.36, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {4022, 3920, 3774, 203, 3795}

$$\frac{5A \sin(c+dx)}{4d\sqrt{a-a \sec(c+dx)}} + \frac{11A \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a-a \sec(c+dx)}}\right)}{4\sqrt{a}d} - \frac{2\sqrt{2}A \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{2}\sqrt{a-a \sec(c+dx)}}\right)}{\sqrt{a}d} + \frac{A \sin(c+dx) \cos(c+dx)}{2d\sqrt{a-a \sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^2*(A + A*Sec[c + d*x]))/Sqrt[a - a*Sec[c + d*x]], x]

[Out] $(11*A*\text{ArcTan}[(\text{Sqrt}[a]*\text{Tan}[c + d*x])/(\text{Sqrt}[a - a*\text{Sec}[c + d*x]])]/(4*\text{Sqrt}[a]*d) - (2*\text{Sqrt}[2]*A*\text{ArcTan}[(\text{Sqrt}[a]*\text{Tan}[c + d*x])/(\text{Sqrt}[2]*\text{Sqrt}[a - a*\text{Sec}[c + d*x]])]/(\text{Sqrt}[a]*d) + (5*A*\text{Sin}[c + d*x])/((4*d*\text{Sqrt}[a - a*\text{Sec}[c + d*x]]) + (A*\text{Cos}[c + d*x]*\text{Sin}[c + d*x])/((2*d*\text{Sqrt}[a - a*\text{Sec}[c + d*x]])$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 3774

Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] := Dist[(-2*b)/d, Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 3795

Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] := Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[

$a + b*\text{Csc}[e + f*x]]], x] /; \text{FreeQ}[\{a, b, e, f\}, x] \ \&\& \ \text{EqQ}[a^2 - b^2, 0]$

Rule 3920

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/\text{Sqrt}[\text{csc}[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] \rightarrow \text{Dist}[c/a, \text{Int}[\text{Sqrt}[a + b*\text{Csc}[e + f*x]], x], x] - \text{Dist}[(b*c - a*d)/a, \text{Int}[\text{Csc}[e + f*x]/\text{Sqrt}[a + b*\text{Csc}[e + f*x]], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0]$

Rule 4022

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_)]*(d_.)^n*(\text{csc}[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^m*(\text{csc}[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] \rightarrow \text{Simp}[(A*\text{Cot}[e + f*x]*(a + b*\text{Csc}[e + f*x])^m*(d*\text{Csc}[e + f*x])^n)/(f*n), x] - \text{Dist}[1/(b*d*n), \text{Int}[(a + b*\text{Csc}[e + f*x])^m*(d*\text{Csc}[e + f*x])^{n+1}*\text{Simp}[a*A*m - b*B*n - A*b*(m + n + 1)*\text{Csc}[e + f*x], x], x], x] /; \text{FreeQ}[\{a, b, d, e, f, A, B, m\}, x] \ \&\& \ \text{NeQ}[A*b - a*B, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{LtQ}[n, 0]$

Rubi steps

$$\begin{aligned} \int \frac{\cos^2(c + dx)(A + A \sec(c + dx))}{\sqrt{a - a \sec(c + dx)}} dx &= \frac{A \cos(c + dx) \sin(c + dx)}{2d\sqrt{a - a \sec(c + dx)}} - \frac{\int \frac{\cos(c+dx)\left(-\frac{5aA}{2} - \frac{3}{2}aA \sec(c+dx)\right)}{\sqrt{a-a \sec(c+dx)}} dx}{2a} \\ &= \frac{5A \sin(c + dx)}{4d\sqrt{a - a \sec(c + dx)}} + \frac{A \cos(c + dx) \sin(c + dx)}{2d\sqrt{a - a \sec(c + dx)}} + \frac{\int \frac{\frac{11a^2A}{4} + \frac{5}{4}a^2A \sec(c+dx)}{\sqrt{a-a \sec(c+dx)}} dx}{2a^2} \\ &= \frac{5A \sin(c + dx)}{4d\sqrt{a - a \sec(c + dx)}} + \frac{A \cos(c + dx) \sin(c + dx)}{2d\sqrt{a - a \sec(c + dx)}} + (2A) \int \frac{\sec(c+dx)}{\sqrt{a-a \sec(c+dx)}} dx \\ &= \frac{5A \sin(c + dx)}{4d\sqrt{a - a \sec(c + dx)}} + \frac{A \cos(c + dx) \sin(c + dx)}{2d\sqrt{a - a \sec(c + dx)}} + \frac{(11A) \text{Subst}\left(\int \frac{\sec(u)}{\sqrt{a-a \sec(u)}} du\right)}{2a^2} \\ &= \frac{11A \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a-a \sec(c+dx)}}\right)}{4\sqrt{a} d} - \frac{2\sqrt{2} A \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{2} \sqrt{a-a \sec(c+dx)}}\right)}{\sqrt{a} d} + \frac{5A \sin(c + dx)}{4d\sqrt{a - a \sec(c + dx)}} \end{aligned}$$

Mathematica [C] time = 1.79, size = 297, normalized size = 1.92

$$Ae^{-\frac{1}{2}i(c+dx)} \sin\left(\frac{1}{2}(c + dx)\right) \sec(c + dx) \left(\cos\left(\frac{1}{2}(c + dx)\right) + i \sin\left(\frac{1}{2}(c + dx)\right)\right) \left(11e^{-\frac{1}{2}i(c+dx)} \sqrt{1 + e^{2i(c+dx)}} \sinh^{-1}\left(e^{i(c+dx)}\right) + \frac{5A \sin(c + dx)}{4d\sqrt{a - a \sec(c + dx)}}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]^2*(A + A*Sec[c + d*x]))/Sqrt[a - a*Sec[c + d*x]],x]

[Out] (A*((11*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcSinh[E^(I*(c + d*x))])/E^((I/2)*(c + d*x)) + (7 + 6/E^(I*(c + d*x)) + 7*E^(I*(c + d*x)) + E^((-2*I)*(c + d*x)) + 6*E^((2*I)*(c + d*x)) + E^((3*I)*(c + d*x)) - 16*Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[(1 + E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))]]) + 11*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]])/E^((I/2)*(c + d*x))*Sec[c + d*x]*(Cos[(c + d*x)/2] + I*Sin[(c + d*x)/2])*Sin[(c + d*x)/2])/(8*d*E^((I/2)*(c + d*x))*Sqrt[a - a*Sec[c + d*x]])

fricas [A] time = 0.46, size = 462, normalized size = 2.98

$$\left[\frac{8\sqrt{2} A a \sqrt{-\frac{1}{a}} \log\left(\frac{2\sqrt{2}(\cos(dx+c)^2 + \cos(dx+c)) \sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}} \sqrt{-\frac{1}{a}} - (3\cos(dx+c)+1)\sin(dx+c)}{(\cos(dx+c)-1)\sin(dx+c)} \right)}{\sin(dx+c) - 11A\sqrt{-a} \log\left(\dots \right)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(1/2),x, algorithm="fricas")

[Out] [1/8*(8*sqrt(2)*A*a*sqrt(-1/a)*log(-(2*sqrt(2))*(cos(d*x + c))^2 + cos(d*x + c))*sqrt((a*cos(d*x + c) - a)/cos(d*x + c))*sqrt(-1/a) - (3*cos(d*x + c) + 1)*sin(d*x + c))/((cos(d*x + c) - 1)*sin(d*x + c))*sin(d*x + c) - 11*A*sqrt(-a)*log((2*(cos(d*x + c))^2 + cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) - a)/cos(d*x + c)) - (2*a*cos(d*x + c) + a)*sin(d*x + c))/sin(d*x + c))*sin(d*x + c) - 2*(2*A*cos(d*x + c)^3 + 7*A*cos(d*x + c)^2 + 5*A*cos(d*x + c))*sqrt((a*cos(d*x + c) - a)/cos(d*x + c))/(a*d*sin(d*x + c)), 1/4*(8*sqrt(2)*A*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) - a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))*sin(d*x + c) - 11*A*sqrt(a)*arctan(sqrt((a*cos(d*x + c) - a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))*sin(d*x + c) - (2*A*cos(d*x + c)^3 + 7*A*cos(d*x + c)^2 + 5*A*cos(d*x + c))*sqrt((a*cos(d*x + c) - a)/cos(d*x + c))/(a*d*sin(d*x + c))]

giac [C] time = 4.07, size = 265, normalized size = 1.71

$$\frac{(-8i\sqrt{2}A\arctan(-i)+11iA\arctan(-\frac{1}{2}i\sqrt{2})+7\sqrt{2}A)\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\sqrt{-a}} + \frac{8\sqrt{2}A\arctan\left(\frac{\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a}}{\sqrt{a}}\right)}{\sqrt{a}\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)} - \frac{11A\arctan\left(\frac{\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a}}{\sqrt{a}}\right)}{\sqrt{a}\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}$$

4d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(1/2),x, algorithm="giac")

[Out] $-1/4*((-8*I*\sqrt{2})*A*\arctan(-I) + 11*I*A*\arctan(-1/2*I*\sqrt{2})) + 7*\sqrt{2})*A*\operatorname{sgn}(\tan(1/2*d*x + 1/2*c))/\sqrt{-a} + 8*\sqrt{2})*A*\arctan(\sqrt{a*\tan(1/2*d*x + 1/2*c)^2 - a}/\sqrt{a})/(\sqrt{a}*\operatorname{sgn}(\tan(1/2*d*x + 1/2*c)^2 - 1)*\operatorname{sgn}(\tan(1/2*d*x + 1/2*c))) - 11*A*\arctan(1/2*\sqrt{2})*\sqrt{a*\tan(1/2*d*x + 1/2*c)^2 - a}/\sqrt{a})/(\sqrt{a}*\operatorname{sgn}(\tan(1/2*d*x + 1/2*c)^2 - 1)*\operatorname{sgn}(\tan(1/2*d*x + 1/2*c))) - \sqrt{2}*(3*(a*\tan(1/2*d*x + 1/2*c)^2 - a)^(3/2)*A + 10*\sqrt{a*\tan(1/2*d*x + 1/2*c)^2 - a}*A*a)/((a*\tan(1/2*d*x + 1/2*c)^2 + a)^2*\operatorname{sgn}(\tan(1/2*d*x + 1/2*c)^2 - 1)*\operatorname{sgn}(\tan(1/2*d*x + 1/2*c)))/d$

maple [B] time = 1.94, size = 367, normalized size = 2.37

$$A(-1 + \cos(dx + c))^2 \left(-16 \left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)} \right)^{\frac{3}{2}} \cos(dx + c) \sqrt{2} + 6 \sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}} (\cos^3(dx + c)) \sqrt{2} - 16 \left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)} \right)^{\frac{3}{2}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^2*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(1/2),x)

[Out] $1/24*A/d*(-1+\cos(d*x+c))^2*(-16*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^(3/2)*\cos(d*x+c)*2^(1/2)+6*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^(1/2)*\cos(d*x+c)^3*2^(1/2)-16*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^(3/2)*2^(1/2)+27*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^(1/2)*\cos(d*x+c)^2*2^(1/2)+48*\arctan(1/(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^(1/2))*\cos(d*x+c)*2^(1/2)+4*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^(1/2)*\cos(d*x+c)*2^(1/2)+48*\arctan(1/(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^(1/2))*2^(1/2)+15*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^(1/2)*2^(1/2)+66*\arctan(1/2*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^(1/2)*2^(1/2))*\cos(d*x+c)+66*\arctan(1/2*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^(1/2)*2^(1/2)))/(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^(1/2)/(a*(-1+\cos(d*x+c)))/\cos(d*x+c))^(1/2)/\sin(d*x+c)^3*2^(1/2)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A \sec(dx + c) + A) \cos(dx + c)^2}{\sqrt{-a \sec(dx + c) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate((A*sec(d*x + c) + A)*cos(d*x + c)^2/sqrt(-a*sec(d*x + c) + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c + dx)^2 \left(A + \frac{A}{\cos(c+dx)} \right)}{\sqrt{a - \frac{a}{\cos(c+dx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)^2*(A + A/cos(c + d*x)))/(a - a/cos(c + d*x))^(1/2),x)

[Out] int((cos(c + d*x)^2*(A + A/cos(c + d*x)))/(a - a/cos(c + d*x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$A \left(\int \frac{\cos^2(c + dx)}{\sqrt{-a \sec(c + dx) + a}} dx + \int \frac{\cos^2(c + dx) \sec(c + dx)}{\sqrt{-a \sec(c + dx) + a}} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**2*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))**(1/2),x)

[Out] A*(Integral(cos(c + d*x)**2/sqrt(-a*sec(c + d*x) + a), x) + Integral(cos(c + d*x)**2*sec(c + d*x)/sqrt(-a*sec(c + d*x) + a), x))

$$3.170 \quad \int \frac{\cos^3(c+dx)(A+A \sec(c+dx))}{\sqrt{a-a \sec(c+dx)}} dx$$

Optimal. Leaf size=192

$$\frac{9A \sin(c+dx)}{8d\sqrt{a-a \sec(c+dx)}} + \frac{23A \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a-a \sec(c+dx)}}\right)}{8\sqrt{a}d} - \frac{2\sqrt{2}A \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{2}\sqrt{a-a \sec(c+dx)}}\right)}{\sqrt{a}d} + \frac{A \sin(c+dx) \cos^2(c+dx)}{3d\sqrt{a-a \sec(c+dx)}}$$

[Out] 23/8*A*arctan(a^(1/2)*tan(d*x+c)/(a-a*sec(d*x+c))^(1/2))/d/a^(1/2)-2*A*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a-a*sec(d*x+c))^(1/2))*2^(1/2)/d/a^(1/2)+9/8*A*sin(d*x+c)/d/(a-a*sec(d*x+c))^(1/2)+7/12*A*cos(d*x+c)*sin(d*x+c)/d/(a-a*sec(d*x+c))^(1/2)+1/3*A*cos(d*x+c)^2*sin(d*x+c)/d/(a-a*sec(d*x+c))^(1/2)

Rubi [A] time = 0.52, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {4022, 3920, 3774, 203, 3795}

$$\frac{9A \sin(c+dx)}{8d\sqrt{a-a \sec(c+dx)}} + \frac{23A \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a-a \sec(c+dx)}}\right)}{8\sqrt{a}d} - \frac{2\sqrt{2}A \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{2}\sqrt{a-a \sec(c+dx)}}\right)}{\sqrt{a}d} + \frac{A \sin(c+dx) \cos^2(c+dx)}{3d\sqrt{a-a \sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^3*(A + A*Sec[c + d*x]))/Sqrt[a - a*Sec[c + d*x]],x]

[Out] (23*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a - a*Sec[c + d*x]]])/(8*Sqrt[a]*d) - (2*Sqrt[2]*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a - a*Sec[c + d*x]])])/(Sqrt[a]*d) + (9*A*Sin[c + d*x])/(8*d*Sqrt[a - a*Sec[c + d*x]]) + (7*A*Cos[c + d*x]*Sin[c + d*x])/(12*d*Sqrt[a - a*Sec[c + d*x]]) + (A*Cos[c + d*x]^2*Sin[c + d*x])/(3*d*Sqrt[a - a*Sec[c + d*x]])

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 3774

Int[Sqrt[csc[(c_.) + (d_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] := Dist[(-2*b)/d, Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 3795

Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 3920

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]

Rule 4022

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^m*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{\cos^3(c + dx)(A + A \sec(c + dx))}{\sqrt{a - a \sec(c + dx)}} dx &= \frac{A \cos^2(c + dx) \sin(c + dx)}{3d\sqrt{a - a \sec(c + dx)}} - \frac{\int \frac{\cos^2(c + dx) \left(-\frac{7aA}{2} - \frac{5}{2}aA \sec(c + dx) \right)}{\sqrt{a - a \sec(c + dx)}} dx}{3a} \\
 &= \frac{7A \cos(c + dx) \sin(c + dx)}{12d\sqrt{a - a \sec(c + dx)}} + \frac{A \cos^2(c + dx) \sin(c + dx)}{3d\sqrt{a - a \sec(c + dx)}} + \frac{\int \frac{\cos(c + dx)}{\sqrt{a - a \sec(c + dx)}} dx}{3a} \\
 &= \frac{9A \sin(c + dx)}{8d\sqrt{a - a \sec(c + dx)}} + \frac{7A \cos(c + dx) \sin(c + dx)}{12d\sqrt{a - a \sec(c + dx)}} + \frac{A \cos^2(c + dx) \sin(c + dx)}{3d\sqrt{a - a \sec(c + dx)}} \\
 &= \frac{9A \sin(c + dx)}{8d\sqrt{a - a \sec(c + dx)}} + \frac{7A \cos(c + dx) \sin(c + dx)}{12d\sqrt{a - a \sec(c + dx)}} + \frac{A \cos^2(c + dx) \sin(c + dx)}{3d\sqrt{a - a \sec(c + dx)}} \\
 &= \frac{9A \sin(c + dx)}{8d\sqrt{a - a \sec(c + dx)}} + \frac{7A \cos(c + dx) \sin(c + dx)}{12d\sqrt{a - a \sec(c + dx)}} + \frac{A \cos^2(c + dx) \sin(c + dx)}{3d\sqrt{a - a \sec(c + dx)}} \\
 &= \frac{23A \tan^{-1} \left(\frac{\sqrt{a} \tan(c + dx)}{\sqrt{a - a \sec(c + dx)}} \right)}{8\sqrt{a} d} - \frac{2\sqrt{2} A \tan^{-1} \left(\frac{\sqrt{a} \tan(c + dx)}{\sqrt{2} \sqrt{a - a \sec(c + dx)}} \right)}{\sqrt{a} d} + \frac{9A \sin(c + dx)}{8d\sqrt{a - a \sec(c + dx)}}
 \end{aligned}$$

Mathematica [C] time = 1.95, size = 330, normalized size = 1.72

$$Ae^{-4i(c+dx)} \sin\left(\frac{1}{2}(c+dx)\right) \sec(c+dx) \left(\cos\left(\frac{1}{2}(c+dx)\right) + i \sin\left(\frac{1}{2}(c+dx)\right)\right) \left(9e^{i(c+dx)} + 40e^{2i(c+dx)} + 47e^{3i(c+dx)} + \dots\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + A*Sec[c + d*x])/Sqrt[a - a*Sec[c + d*x]],x]
[Out] (A*(2 + 9*E^(I*(c + d*x)) + 40*E^((2*I)*(c + d*x)) + 47*E^((3*I)*(c + d*x))
+ 47*E^((4*I)*(c + d*x)) + 40*E^((5*I)*(c + d*x)) + 9*E^((6*I)*(c + d*x))
+ 2*E^((7*I)*(c + d*x)) + 69*E^((3*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x)
)])*ArcSinh[E^(I*(c + d*x))] - 96*Sqrt[2]*E^((3*I)*(c + d*x))*Sqrt[1 + E^((
2*I)*(c + d*x))]*ArcTanh[(1 + E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(
c + d*x))])] + 69*E^((3*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh
[Sqrt[1 + E^((2*I)*(c + d*x))]])*Sec[c + d*x]*(Cos[(c + d*x)/2] + I*Sin[(c
+ d*x)/2])*Sin[(c + d*x)/2]/(48*d*E^((4*I)*(c + d*x))*Sqrt[a - a*Sec[c + d
*x]]))
```

fricas [A] time = 0.47, size = 484, normalized size = 2.52

$$\left[48 \sqrt{2} A a \sqrt{-\frac{1}{a}} \log \left(-\frac{2 \sqrt{2} (\cos(dx+c)^2 + \cos(dx+c)) \sqrt{\frac{a \cos(dx+c) - a}{\cos(dx+c)}} \sqrt{-\frac{1}{a}} - (3 \cos(dx+c) + 1) \sin(dx+c)}{(\cos(dx+c) - 1) \sin(dx+c)} \right) \sin(dx+c) - 69 A \sqrt{-a} \log \left(\dots \right) \right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] [1/48*(48*sqrt(2)*A*a*sqrt(-1/a)*log(-(2*sqrt(2))*(cos(d*x + c))^2 + cos(d*x
+ c))*sqrt((a*cos(d*x + c) - a)/cos(d*x + c))*sqrt(-1/a) - (3*cos(d*x + c)
+ 1)*sin(d*x + c))/((cos(d*x + c) - 1)*sin(d*x + c))*sin(d*x + c) - 69*A*s
qrt(-a)*log((2*(cos(d*x + c))^2 + cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c)
- a)/cos(d*x + c)) - (2*a*cos(d*x + c) + a)*sin(d*x + c))/sin(d*x + c))*s
in(d*x + c) - 2*(8*A*cos(d*x + c)^4 + 22*A*cos(d*x + c)^3 + 41*A*cos(d*x +
c)^2 + 27*A*cos(d*x + c))*sqrt((a*cos(d*x + c) - a)/cos(d*x + c))/(a*d*sin
(d*x + c)), 1/24*(48*sqrt(2)*A*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c)
- a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))*sin(d*x + c) - 69*A
*sqrt(a)*arctan(sqrt((a*cos(d*x + c) - a)/cos(d*x + c))*cos(d*x + c)/(sqrt(
a)*sin(d*x + c)))*sin(d*x + c) - (8*A*cos(d*x + c)^4 + 22*A*cos(d*x + c)^3
```

+ 41*A*cos(d*x + c)^2 + 27*A*cos(d*x + c))*sqrt((a*cos(d*x + c) - a)/cos(d*x + c)))/(a*d*sin(d*x + c))]

giac [C] time = 4.36, size = 290, normalized size = 1.51

$$\frac{\left(-48i\sqrt{2}A\arctan(-i)+69iA\arctan\left(-\frac{1}{2}i\sqrt{2}\right)+49\sqrt{2}A\right)\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\sqrt{-a}} + \frac{48\sqrt{2}A\arctan\left(\frac{\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a}}{\sqrt{a}}\right)}{\sqrt{a}\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)^2-1}\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right) - \frac{69A\arctan\left(\frac{1}{2}\sqrt{2}\right)}{\sqrt{a}\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)} - \frac{24d}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^3*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(1/2),x, algorithm="giac")

[Out] -1/24*((-48*I*sqrt(2)*A*arctan(-I) + 69*I*A*arctan(-1/2*I*sqrt(2)) + 49*sqrt(2)*A)*sgn(tan(1/2*d*x + 1/2*c))/sqrt(-a) + 48*sqrt(2)*A*arctan(sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)/sqrt(a))/(sqrt(a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c))) - 69*A*arctan(1/2*sqrt(2)*sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)/sqrt(a))/(sqrt(a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c))) - sqrt(2)*(21*(a*tan(1/2*d*x + 1/2*c)^2 - a)^(5/2)*A + 80*(a*tan(1/2*d*x + 1/2*c)^2 - a)^(3/2)*A*a + 108*sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)*A*a^2)/((a*tan(1/2*d*x + 1/2*c)^2 + a)^3*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)))/d

maple [B] time = 2.12, size = 625, normalized size = 3.26

$$A(-1 + \cos(dx + c))^3 \left(96 \left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)} \right)^{\frac{5}{2}} (\cos^2(dx + c)) \sqrt{2} + 192 \left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)} \right)^{\frac{5}{2}} \cos(dx + c) \sqrt{2} + 40 \sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^3*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(1/2),x)

[Out] -1/240*A/d*(-1+cos(d*x+c))^3*(96*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*cos(d*x+c)^2*2^(1/2)+192*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*cos(d*x+c)*2^(1/2)+40*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^5*2^(1/2)+96*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*2^(1/2)-160*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)^2*2^(1/2)+190*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^4*2^(1/2)-320*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)*2^(1/2)+465*(-

$2*\cos(d*x+c)/(1+\cos(d*x+c))^{(1/2)}*\cos(d*x+c)^3*2^{(1/2)}+480*\arctan(1/(-2*\cos(d*x+c)/(1+\cos(d*x+c))^{(1/2)})*\cos(d*x+c)^2*2^{(1/2)}-160*(-2*\cos(d*x+c)/(1+\cos(d*x+c))^{(3/2)}*2^{(1/2)}-49*(-2*\cos(d*x+c)/(1+\cos(d*x+c))^{(1/2)})*\cos(d*x+c)^2*2^{(1/2)}+960*\arctan(1/(-2*\cos(d*x+c)/(1+\cos(d*x+c))^{(1/2)}))*\cos(d*x+c)*2^{(1/2)}+155*(-2*\cos(d*x+c)/(1+\cos(d*x+c))^{(1/2)})*\cos(d*x+c)*2^{(1/2)}+690*\arctan(1/2*(-2*\cos(d*x+c)/(1+\cos(d*x+c))^{(1/2)})*2^{(1/2)})*\cos(d*x+c)^2+480*\arctan(1/(-2*\cos(d*x+c)/(1+\cos(d*x+c))^{(1/2)}))*2^{(1/2)}+135*(-2*\cos(d*x+c)/(1+\cos(d*x+c))^{(1/2)})*2^{(1/2)}+1380*\arctan(1/2*(-2*\cos(d*x+c)/(1+\cos(d*x+c))^{(1/2)})*2^{(1/2)}))*\cos(d*x+c)+690*\arctan(1/2*(-2*\cos(d*x+c)/(1+\cos(d*x+c))^{(1/2)})*2^{(1/2)}))/(-2*\cos(d*x+c)/(1+\cos(d*x+c))^{(1/2)})/(a*(-1+\cos(d*x+c))/\cos(d*x+c))^{(1/2)}/\sin(d*x+c)^5*2^{(1/2)}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A \sec(dx + c) + A) \cos(dx + c)^3}{\sqrt{-a \sec(dx + c) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^3*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate((A*sec(d*x + c) + A)*cos(d*x + c)^3/sqrt(-a*sec(d*x + c) + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c + dx)^3 \left(A + \frac{A}{\cos(c + dx)} \right)}{\sqrt{a - \frac{a}{\cos(c + dx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)^3*(A + A/cos(c + d*x)))/(a - a/cos(c + d*x))^(1/2),x)

[Out] int((cos(c + d*x)^3*(A + A/cos(c + d*x)))/(a - a/cos(c + d*x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$A \left(\int \frac{\cos^3(c + dx)}{\sqrt{-a \sec(c + dx) + a}} dx + \int \frac{\cos^3(c + dx) \sec(c + dx)}{\sqrt{-a \sec(c + dx) + a}} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**3*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))**(1/2),x)

[Out] A*(Integral(cos(c + d*x)**3/sqrt(-a*sec(c + d*x) + a), x) + Integral(cos(c + d*x)**3*sec(c + d*x)/sqrt(-a*sec(c + d*x) + a), x))

$$3.171 \quad \int \frac{A + A \sec(c+dx)}{(a - a \sec(c+dx))^{3/2}} dx$$

Optimal. Leaf size=116

$$\frac{2A \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a-a \sec(c+dx)}}\right)}{a^{3/2}d} - \frac{3A \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{2} \sqrt{a-a \sec(c+dx)}}\right)}{\sqrt{2} a^{3/2}d} - \frac{A \tan(c+dx)}{d(a - a \sec(c+dx))^{3/2}}$$

[Out] $2*A*\arctan(a^{(1/2)}*\tan(d*x+c)/(a-a*\sec(d*x+c))^{(1/2)})/a^{(3/2)}/d-3/2*A*\arctan(1/2*a^{(1/2)}*\tan(d*x+c)*2^{(1/2)}/(a-a*\sec(d*x+c))^{(1/2)})/a^{(3/2)}/d*2^{(1/2)}-A*\tan(d*x+c)/d/(a-a*\sec(d*x+c))^{(3/2)}$

Rubi [A] time = 0.20, antiderivative size = 133, normalized size of antiderivative = 1.15, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3904, 3887, 471, 522, 203}

$$\frac{2A \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a-a \sec(c+dx)}}\right)}{a^{3/2}d} - \frac{3A \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{2} \sqrt{a-a \sec(c+dx)}}\right)}{\sqrt{2} a^{3/2}d} + \frac{A \sin(c+dx) \csc^2\left(\frac{1}{2}(c+dx)\right)}{2ad\sqrt{a-a \sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(A + A*\text{Sec}[c + d*x])/(a - a*\text{Sec}[c + d*x])^{(3/2)}, x]$

[Out] $(2*A*\text{ArcTan}[(\text{Sqrt}[a]*\text{Tan}[c + d*x])/\text{Sqrt}[a - a*\text{Sec}[c + d*x]])/(a^{(3/2)*d}) - (3*A*\text{ArcTan}[(\text{Sqrt}[a]*\text{Tan}[c + d*x])/(\text{Sqrt}[2]*\text{Sqrt}[a - a*\text{Sec}[c + d*x]])]/(\text{Sqrt}[2]*a^{(3/2)*d}) + (A*\text{Csc}[(c + d*x)/2]^2*\text{Sin}[c + d*x])/(2*a*d*\text{Sqrt}[a - a*\text{Sec}[c + d*x]])$

Rule 203

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1*\text{ArcTan}[(\text{Rt}[b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[b, 2]), x] /; \text{FreeQ}\{a, b, x\} \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{GtQ}[b, 0])$

Rule 471

$\text{Int}[(e_)*(x_)^{(m_)}*((a_ + (b_)*(x_)^{(n_)})^{(p_)}*((c_ + (d_)*(x_)^{(n_)}))^{(q_)}), x_Symbol] \rightarrow \text{Simp}[(e^{(n-1)}*(e*x)^{(m-n+1)}*(a + b*x^n)^{(p+1)}*(c + d*x^n)^{(q+1)})/(n*(b*c - a*d)*(p+1)), x] - \text{Dist}[e^n/(n*(b*c - a*d)*(p+1)), \text{Int}[(e*x)^{(m-n)}*(a + b*x^n)^{(p+1)}*(c + d*x^n)^q*\text{Simp}[c*(m-n+1) + d*(m+n*(p+q+1)+1]*x^n, x], x] /; \text{FreeQ}\{a, b, c, d, e, q\}, x\} \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{LtQ}[p, -1] \ \&\& \ \text{GeQ}[n, m-n+1] \ \&\& \ \text{GtQ}[m-n+1, 0] \ \&\& \ \text{IntBinomialQ}[a, b, c, d, e, m, n, p, q, x]$

Rule 522

Int[((e_) + (f_)*(x_)^(n_))/(((a_) + (b_)*(x_)^(n_))*((c_) + (d_)*(x_)^(n_))), x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 3887

Int[cot[(c_) + (d_)*(x_)]^(m_)*(csc[(c_) + (d_)*(x_)]*(b_) + (a_))^(n_), x_Symbol] := Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && IntegerQ[n - 1/2]

Rule 3904

Int[(csc[(e_) + (f_)*(x_)]*(b_) + (a_))^(m_)*(csc[(e_) + (f_)*(x_)]*(d_) + (c_))^(n_), x_Symbol] := Dist[(-a*c)^m, Int[Cot[e + f*x]^(2*m)*(c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] && !IntegerQ[n] && GtQ[m - n, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{A + A \sec(c + dx)}{(a - a \sec(c + dx))^{3/2}} dx &= - \left((aA) \int \frac{\tan^2(c + dx)}{(a - a \sec(c + dx))^{5/2}} dx \right) \\
 &= \frac{(2A) \text{Subst} \left(\int \frac{x^2}{(1+ax^2)(2+ax^2)^2} dx, x, -\frac{\tan(c+dx)}{\sqrt{a-a \sec(c+dx)}} \right)}{d} \\
 &= \frac{A \csc^2 \left(\frac{1}{2}(c + dx) \right) \sin(c + dx)}{2ad\sqrt{a - a \sec(c + dx)}} - \frac{A \text{Subst} \left(\int \frac{1-ax^2}{(1+ax^2)(2+ax^2)} dx, x, -\frac{\tan(c+dx)}{\sqrt{a-a \sec(c+dx)}} \right)}{ad} \\
 &= \frac{A \csc^2 \left(\frac{1}{2}(c + dx) \right) \sin(c + dx)}{2ad\sqrt{a - a \sec(c + dx)}} - \frac{(2A) \text{Subst} \left(\int \frac{1}{1+ax^2} dx, x, -\frac{\tan(c+dx)}{\sqrt{a-a \sec(c+dx)}} \right)}{ad} + \dots \\
 &= \frac{2A \tan^{-1} \left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a-a \sec(c+dx)}} \right)}{a^{3/2}d} - \frac{3A \tan^{-1} \left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{2} \sqrt{a-a \sec(c+dx)}} \right)}{\sqrt{2} a^{3/2}d} + \frac{A \csc^2 \left(\frac{1}{2}(c + dx) \right) \sin(c + dx)}{2ad\sqrt{a - a \sec(c + dx)}}
 \end{aligned}$$

Mathematica [C] time = 6.69, size = 322, normalized size = 2.78

$$A \left(\frac{\sin^3\left(\frac{c}{2} + \frac{dx}{2}\right) \sec^2(c + dx) \left(-\frac{4 \sin\left(\frac{c}{2}\right) \sin\left(\frac{dx}{2}\right)}{d} + \frac{4 \cos\left(\frac{c}{2}\right) \cos\left(\frac{dx}{2}\right)}{d} - \frac{2 \cot\left(\frac{c}{2}\right) \csc\left(\frac{c}{2} + \frac{dx}{2}\right)}{d} + \frac{2 \csc\left(\frac{c}{2}\right) \sin\left(\frac{dx}{2}\right) \csc^2\left(\frac{c}{2} + \frac{dx}{2}\right)}{d} \right)}{(a - a \sec(c + dx))^{3/2}} \right) - 2\sqrt{a}$$

Antiderivative was successfully verified.

[In] Integrate[(A + A*Sec[c + d*x])/(a - a*Sec[c + d*x])^(3/2), x]

[Out] A*((-2*Sqrt[2]*Sqrt[E^(I*(c + d*x))]/(1 + E^((2*I)*(c + d*x))))*Sqrt[1 + E^((2*I)*(c + d*x))]*(ArcSinh[E^(I*(c + d*x))] - (3*ArcTanh[(1 + E^(I*(c + d*x))])/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))]])]/Sqrt[2] + ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]])*Sec[c + d*x]^(3/2)*Sin[c/2 + (d*x)/2]^3/(d*E^((I/2)*(c + d*x))*(a - a*Sec[c + d*x])^(3/2)) + (Sec[c + d*x]^2*((4*Cos[c/2]*Cos[(d*x)/2])/d - (2*Cot[c/2]*Csc[c/2 + (d*x)/2])/d + (2*Csc[c/2]*Csc[c/2 + (d*x)/2]^2*Sin[(d*x)/2])/d - (4*Sin[c/2]*Sin[(d*x)/2])/d)*Sin[c/2 + (d*x)/2]^3/(a - a*Sec[c + d*x])^(3/2))

fricas [B] time = 0.49, size = 502, normalized size = 4.33

$$\left(3\sqrt{2}(A \cos(dx + c) - A)\sqrt{-a} \log \left(\frac{2\sqrt{2}(\cos(dx+c)^2 + \cos(dx+c))\sqrt{-a} \sqrt{\frac{a \cos(dx+c) - a}{\cos(dx+c)} + (3a \cos(dx+c) + a) \sin(dx+c)}}{(\cos(dx+c) - 1) \sin(dx+c)} \right) \right) \sin(dx + c)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(3/2), x, algorithm="fricas")

[Out] [-1/4*(3*sqrt(2)*(A*cos(d*x + c) - A)*sqrt(-a)*log((2*sqrt(2)*(cos(d*x + c))^2 + cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) - a)/cos(d*x + c)) + (3*a*cos(d*x + c) + a)*sin(d*x + c))/((cos(d*x + c) - 1)*sin(d*x + c)))*sin(d*x + c) + 4*(A*cos(d*x + c) - A)*sqrt(-a)*log((2*(cos(d*x + c))^2 + cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) - a)/cos(d*x + c)) - (2*a*cos(d*x + c) + a)*sin(d*x + c))/sin(d*x + c)*sin(d*x + c) - 4*(A*cos(d*x + c)^2 + A*cos(d*x + c))*sqrt((a*cos(d*x + c) - a)/cos(d*x + c)))/((a^2*d*cos(d*x + c) - a^2*d)*sin(d*x + c)), 1/2*(3*sqrt(2)*(A*cos(d*x + c) - A)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) - a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))*sin(d*x + c) - 4*(A*cos(d*x + c) - A)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) - a)/cos(d*x + c))*sin(d*x + c)/(sqrt(a)*sin(d*x + c)))*sin(d*x + c)]

c) - a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))*sin(d*x + c) + 2*(A*cos(d*x + c)^2 + A*cos(d*x + c))*sqrt((a*cos(d*x + c) - a)/cos(d*x + c)))/((a^2*d*cos(d*x + c) - a^2*d)*sin(d*x + c))]

giac [A] time = 1.15, size = 196, normalized size = 1.69

$$\frac{3\sqrt{2}A \arctan\left(\frac{\sqrt{a \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}} \operatorname{sgn}\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - 1\right) \operatorname{sgn}\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right)} - \frac{4A \arctan\left(\frac{\sqrt{2} \sqrt{a \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - a}}{2\sqrt{a}}\right)}{a^{\frac{3}{2}} \operatorname{sgn}\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - 1\right) \operatorname{sgn}\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right)} - \frac{\sqrt{2} \sqrt{a \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - a}}{a^2 \operatorname{sgn}\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - 1\right) \operatorname{sgn}\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right)}$$

$2d$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(3/2),x, algorithm="giac")

[Out] -1/2*(3*sqrt(2)*A*arctan(sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)/sqrt(a))/(a^(3/2)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c))) - 4*A*arctan(1/2*sqrt(2)*sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)/sqrt(a))/(a^(3/2)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c))) - sqrt(2)*sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)*A/(a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c))*tan(1/2*d*x + 1/2*c)^2)/d

maple [B] time = 1.54, size = 298, normalized size = 2.57

$$A(-1 + \cos(dx + c))^2 \left(\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)} \right)^{\frac{3}{2}} \cos(dx + c) \sqrt{2} + \left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)} \right)^{\frac{3}{2}} \sqrt{2} + \sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}} \cos(dx + c) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(3/2),x)

[Out] -A/d*(-1+cos(d*x+c))^2*((-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)*2^(1/2)+(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*2^(1/2)+(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*2^(1/2)+3*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*cos(d*x+c)*2^(1/2)-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)-3*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*2^(1/2)+4*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*cos(d*x+c)-4*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)))/(a*(-1+cos(d*x+c))/cos(d*x+c))^(3/2)/sin(d*x+c)^3/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*2^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A \sec(dx + c) + A}{(-a \sec(dx + c) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(3/2),x, algorithm="maxima")

[Out] integrate((A*sec(d*x + c) + A)/(-a*sec(d*x + c) + a)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + \frac{A}{\cos(c+dx)}}{\left(a - \frac{a}{\cos(c+dx)}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + A/cos(c + d*x))/(a - a/cos(c + d*x))^(3/2),x)

[Out] int((A + A/cos(c + d*x))/(a - a/cos(c + d*x))^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$A \left(\int \frac{\sec(c + dx)}{-a\sqrt{-a \sec(c + dx) + a} \sec(c + dx) + a\sqrt{-a \sec(c + dx) + a}} dx + \int \frac{1}{-a\sqrt{-a \sec(c + dx) + a} \sec(c + dx)} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+A*sec(d*x+c))/(a-a*sec(d*x+c))**(3/2),x)

[Out] A*(Integral(sec(c + d*x)/(-a*sqrt(-a*sec(c + d*x) + a)*sec(c + d*x) + a*sqrt(-a*sec(c + d*x) + a)), x) + Integral(1/(-a*sqrt(-a*sec(c + d*x) + a)*sec(c + d*x) + a*sqrt(-a*sec(c + d*x) + a)), x))

$$3.172 \quad \int \frac{\cos(c+dx)(A+A \sec(c+dx))}{(a-a \sec(c+dx))^{3/2}} dx$$

Optimal. Leaf size=146

$$\frac{5A \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a-a \sec(c+dx)}}\right)}{a^{3/2}d} - \frac{7A \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{2} \sqrt{a-a \sec(c+dx)}}\right)}{\sqrt{2} a^{3/2}d} + \frac{2A \sin(c+dx)}{ad\sqrt{a-a \sec(c+dx)}} - \frac{A \sin(c+dx)}{d(a-a \sec(c+dx))^{3/2}}$$

[Out] $5*A*\arctan(a^{(1/2)}*\tan(d*x+c)/(a-a*\sec(d*x+c))^{(1/2)})/a^{(3/2)}/d-A*\sin(d*x+c)/d/(a-a*\sec(d*x+c))^{(3/2)}-7/2*A*\arctan(1/2*a^{(1/2)}*\tan(d*x+c)*2^{(1/2)}/(a-a*\sec(d*x+c))^{(1/2)})/a^{(3/2)}/d*2^{(1/2)}+2*A*\sin(d*x+c)/a/d/(a-a*\sec(d*x+c))^{(1/2)}$

Rubi [A] time = 0.35, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {4020, 4022, 3920, 3774, 203, 3795}

$$\frac{5A \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a-a \sec(c+dx)}}\right)}{a^{3/2}d} - \frac{7A \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{2} \sqrt{a-a \sec(c+dx)}}\right)}{\sqrt{2} a^{3/2}d} + \frac{2A \sin(c+dx)}{ad\sqrt{a-a \sec(c+dx)}} - \frac{A \sin(c+dx)}{d(a-a \sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]*(A + A*Sec[c + d*x]))/(a - a*Sec[c + d*x])^(3/2), x]

[Out] $(5*A*\text{ArcTan}[(\text{Sqrt}[a]*\text{Tan}[c + d*x])/(\text{Sqrt}[a - a*\text{Sec}[c + d*x]])]/(a^{(3/2)}*d) - (7*A*\text{ArcTan}[(\text{Sqrt}[a]*\text{Tan}[c + d*x])/(\text{Sqrt}[2]*\text{Sqrt}[a - a*\text{Sec}[c + d*x]])])/(\text{Sqrt}[2]*a^{(3/2)}*d) - (A*\text{Sin}[c + d*x])/(d*(a - a*\text{Sec}[c + d*x])^{(3/2)}) + (2*A*\text{Sin}[c + d*x])/(a*d*\text{Sqrt}[a - a*\text{Sec}[c + d*x]])$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 3774

Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] := Dist[(-2*b)/d, Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 3795

Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] := Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[

$a + b \cdot \text{Csc}[e + f \cdot x]$], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 3920

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]

Rule 4020

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> -Simp[(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n/(b*f*(2*m + 1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0]

Rule 4022

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]

Rubi steps

$$\begin{aligned}
\int \frac{\cos(c+dx)(A+A\sec(c+dx))}{(a-a\sec(c+dx))^{3/2}} dx &= -\frac{A\sin(c+dx)}{d(a-a\sec(c+dx))^{3/2}} + \frac{\int \frac{\cos(c+dx)(4aA+3aA\sec(c+dx))}{\sqrt{a-a\sec(c+dx)}} dx}{2a^2} \\
&= -\frac{A\sin(c+dx)}{d(a-a\sec(c+dx))^{3/2}} + \frac{2A\sin(c+dx)}{ad\sqrt{a-a\sec(c+dx)}} - \frac{\int \frac{-5a^2A-2a^2A\sec(c+dx)}{\sqrt{a-a\sec(c+dx)}}}{2a^3} \\
&= -\frac{A\sin(c+dx)}{d(a-a\sec(c+dx))^{3/2}} + \frac{2A\sin(c+dx)}{ad\sqrt{a-a\sec(c+dx)}} + \frac{(5A)\int \sqrt{a-a\sec(c+dx)}}{2a^2} \\
&= -\frac{A\sin(c+dx)}{d(a-a\sec(c+dx))^{3/2}} + \frac{2A\sin(c+dx)}{ad\sqrt{a-a\sec(c+dx)}} + \frac{(5A)\text{Subst}\left(\int \frac{1}{a+x^2}\right)}{2a^2} \\
&= \frac{5A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{a^{3/2}d} - \frac{7A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{2}a^{3/2}d} - \frac{A\sin(c+dx)}{d(a-a\sec(c+dx))^{3/2}}
\end{aligned}$$

Mathematica [C] time = 6.63, size = 361, normalized size = 2.47

$$A \left(\frac{\sin^3\left(\frac{c}{2} + \frac{dx}{2}\right) \sec^2(c+dx) \left(-\frac{2\sin\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)}{d} + \frac{2\sin\left(\frac{3c}{2}\right)\sin\left(\frac{3dx}{2}\right)}{d} + \frac{2\cos\left(\frac{c}{2}\right)\cos\left(\frac{dx}{2}\right)}{d} - \frac{2\cos\left(\frac{3c}{2}\right)\cos\left(\frac{3dx}{2}\right)}{d} - \frac{2\cot\left(\frac{c}{2}\right)\csc\left(\frac{c}{2} + \frac{dx}{2}\right)}{d} \right)}{(a-a\sec(c+dx))^{3/2}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]*(A + A*Sec[c + d*x]))/(a - a*Sec[c + d*x])^(3/2), x]

[Out] A*((Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))])*(-5*ArcSinh[E^(I*(c + d*x))] + 7*Sqrt[2]*ArcTanh[(1 + E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))]])] - 5*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]])*Sec[c + d*x]^(3/2)*Sin[c/2 + (d*x)/2]^3)/(d*E^((I/2)*(c + d*x))*(a - a*Sec[c + d*x])^(3/2)) + (Sec[c + d*x]^2*((2*Cos[c/2]*Cos[(d*x)/2])/d - (2*Cos[(3*c)/2]*Cos[(3*d*x)/2])/d - (2*Cot[c/2]*Csc[c/2 + (d*x)/2])/d + (2*Csc[c/2]*Csc[c/2 + (d*x)/2]^2*Sin[(d*x)/2])/d - (2*Sin[c/2]*Sin[(d*x)/2])/d + (2*Sin[(3*c)/2]*Sin[(3*d*x)/2])/d)*Sin[c/2 + (d*x)/2]^3)/(a - a*Sec[c + d*x])^(3/2))

fricas [A] time = 0.48, size = 526, normalized size = 3.60

$$\left[\frac{7\sqrt{2}(A\cos(dx+c) - A)\sqrt{-a} \log\left(\frac{2\sqrt{2}(\cos(dx+c)^2 + \cos(dx+c))\sqrt{-a} \sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}} + (3a\cos(dx+c)+a)\sin(dx+c)}{(\cos(dx+c)-1)\sin(dx+c)}\right)}{\sin(dx+c)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(3/2),x, algorithm="fricas")

[Out] [-1/4*(7*sqrt(2)*(A*cos(d*x + c) - A)*sqrt(-a)*log((2*sqrt(2)*(cos(d*x + c))^2 + cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) - a)/cos(d*x + c)) + (3*a*cos(d*x + c) + a)*sin(d*x + c))/((cos(d*x + c) - 1)*sin(d*x + c)))*sin(d*x + c) + 10*(A*cos(d*x + c) - A)*sqrt(-a)*log((2*(cos(d*x + c))^2 + cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) - a)/cos(d*x + c)) - (2*a*cos(d*x + c) + a)*sin(d*x + c))/sin(d*x + c))*sin(d*x + c) + 4*(A*cos(d*x + c)^3 - A*cos(d*x + c)^2 - 2*A*cos(d*x + c))*sqrt((a*cos(d*x + c) - a)/cos(d*x + c)))/((a^2*d*cos(d*x + c) - a^2*d)*sin(d*x + c)), 1/2*(7*sqrt(2)*(A*cos(d*x + c) - A)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) - a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))*sin(d*x + c) - 10*(A*cos(d*x + c) - A)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) - a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))*sin(d*x + c) - 2*(A*cos(d*x + c)^3 - A*cos(d*x + c)^2 - 2*A*cos(d*x + c))*sqrt((a*cos(d*x + c) - a)/cos(d*x + c)))/((a^2*d*cos(d*x + c) - a^2*d)*sin(d*x + c))]

giac [B] time = 2.21, size = 261, normalized size = 1.79

$$\frac{7\sqrt{2}A \arctan\left(\frac{\sqrt{a \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}} \operatorname{sgn}\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - 1\right) \operatorname{sgn}\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right)} - \frac{10A \arctan\left(\frac{\sqrt{2} \sqrt{a \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - a}}{2\sqrt{a}}\right)}{a^{\frac{3}{2}} \operatorname{sgn}\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - 1\right) \operatorname{sgn}\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right)} - \frac{3\sqrt{2} \left(a \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right)^2}{\left(\left(a \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right)^2 - a\right) + 3 \left(a \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right)}$$

$2d$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(3/2),x, algorithm="giac")

[Out] -1/2*(7*sqrt(2)*A*arctan(sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)/sqrt(a))/(a^(3/2)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c))) - 10*A*arctan

$$\frac{(1/2*\sqrt{2})*\sqrt{a*\tan(1/2*d*x + 1/2*c)^2 - a}/\sqrt{a}}{(a^{3/2}*\operatorname{sgn}(\tan(1/2*d*x + 1/2*c)^2 - 1)*\operatorname{sgn}(\tan(1/2*d*x + 1/2*c)))} - \frac{(3*\sqrt{2})*(a*\tan(1/2*d*x + 1/2*c)^2 - a)^{3/2}*A + 4*\sqrt{2}*\sqrt{a*\tan(1/2*d*x + 1/2*c)^2 - a}*A*a}{(((a*\tan(1/2*d*x + 1/2*c)^2 - a)^2 + 3*(a*\tan(1/2*d*x + 1/2*c)^2 - a)*a + 2*a^2)*a*\operatorname{sgn}(\tan(1/2*d*x + 1/2*c)^2 - 1)*\operatorname{sgn}(\tan(1/2*d*x + 1/2*c)))}/d$$

maple [B] time = 1.64, size = 462, normalized size = 3.16

$$A(-1 + \cos(dx + c))^3 \left(-3 \left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)} \right)^{\frac{5}{2}} (\cos^2(dx + c)) \sqrt{2} - 6 \left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)} \right)^{\frac{5}{2}} \cos(dx + c) \sqrt{2} - 7 \left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)} \right)^{\frac{5}{2}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(d*x+c)*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(3/2), x)`

[Out]
$$\frac{1}{3} \frac{A}{d} \frac{(-1 + \cos(dx + c))^{-3} (-3(-2\cos(dx + c)/(1 + \cos(dx + c)))^{5/2} \cos(dx + c)^2 2^{1/2} - 6(-2\cos(dx + c)/(1 + \cos(dx + c)))^{5/2} \cos(dx + c) 2^{1/2} - 7(-2\cos(dx + c)/(1 + \cos(dx + c)))^{5/2} \cos(dx + c)^2 2^{1/2} + 3(-2\cos(dx + c)/(1 + \cos(dx + c)))^{1/2} \cos(dx + c)^3 2^{1/2} + 21 \arctan(1/(-2\cos(dx + c)/(1 + \cos(dx + c)))^{1/2}) \cos(dx + c)^2 2^{1/2} - 2(-2\cos(dx + c)/(1 + \cos(dx + c)))^{1/2} \cos(dx + c)^2 2^{1/2} + 7(-2\cos(dx + c)/(1 + \cos(dx + c)))^{3/2} 2^{1/2} + 30 \arctan(1/2(-2\cos(dx + c)/(1 + \cos(dx + c)))^{1/2}) 2^{1/2}) \cos(dx + c)^2 + 5(-2\cos(dx + c)/(1 + \cos(dx + c)))^{1/2} 2^{1/2} \cos(dx + c)^2 - 21 \arctan(1/(-2\cos(dx + c)/(1 + \cos(dx + c)))^{1/2}) 2^{1/2} - 6(-2\cos(dx + c)/(1 + \cos(dx + c)))^{1/2} 2^{1/2} - 30 \arctan(1/2(-2\cos(dx + c)/(1 + \cos(dx + c)))^{1/2}) 2^{1/2})}{(-2\cos(dx + c)/(1 + \cos(dx + c)))^{3/2} (a(-1 + \cos(dx + c))/\cos(dx + c))^{3/2} \sin(dx + c)^5 2^{1/2}}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A \sec(dx + c) + A) \cos(dx + c)}{(-a \sec(dx + c) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(3/2), x, algorithm="maxima")`

[Out] `integrate((A*sec(d*x + c) + A)*cos(d*x + c)/(-a*sec(d*x + c) + a)^(3/2), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c + dx) \left(A + \frac{A}{\cos(c+dx)} \right)}{\left(a - \frac{a}{\cos(c+dx)} \right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)*(A + A/cos(c + d*x)))/(a - a/cos(c + d*x))^(3/2), x)

[Out] int((cos(c + d*x)*(A + A/cos(c + d*x)))/(a - a/cos(c + d*x))^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$A \left(\int \frac{\cos(c + dx)}{-a\sqrt{-a \sec(c + dx) + a} \sec(c + dx) + a\sqrt{-a \sec(c + dx) + a}} dx + \int \frac{\cos(c + dx) \sec(c + dx)}{-a\sqrt{-a \sec(c + dx) + a} \sec(c + dx)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))**(3/2), x)

[Out] A*(Integral(cos(c + d*x)/(-a*sqrt(-a*sec(c + d*x) + a)*sec(c + d*x) + a*sqrt(-a*sec(c + d*x) + a)), x) + Integral(cos(c + d*x)*sec(c + d*x)/(-a*sqrt(-a*sec(c + d*x) + a)*sec(c + d*x) + a*sqrt(-a*sec(c + d*x) + a)), x))

$$3.173 \quad \int \frac{\cos^2(c+dx)(A+A \sec(c+dx))}{(a-a \sec(c+dx))^{3/2}} dx$$

Optimal. Leaf size=194

$$\frac{31A \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a-a \sec(c+dx)}}\right)}{4a^{3/2}d} - \frac{11A \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{2} \sqrt{a-a \sec(c+dx)}}\right)}{\sqrt{2} a^{3/2}d} + \frac{13A \sin(c+dx)}{4ad\sqrt{a-a \sec(c+dx)}} + \frac{3A \sin(c+dx) \cos(c+dx)}{2ad\sqrt{a-a \sec(c+dx)}}$$

[Out] 31/4*A*arctan(a^(1/2)*tan(d*x+c)/(a-a*sec(d*x+c))^(1/2))/a^(3/2)/d-A*cos(d*x+c)*sin(d*x+c)/d/(a-a*sec(d*x+c))^(3/2)-11/2*A*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a-a*sec(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2)+13/4*A*sin(d*x+c)/a/d/(a-a*sec(d*x+c))^(1/2)+3/2*A*cos(d*x+c)*sin(d*x+c)/a/d/(a-a*sec(d*x+c))^(1/2)

Rubi [A] time = 0.53, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {4020, 4022, 3920, 3774, 203, 3795}

$$\frac{31A \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a-a \sec(c+dx)}}\right)}{4a^{3/2}d} - \frac{11A \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{2} \sqrt{a-a \sec(c+dx)}}\right)}{\sqrt{2} a^{3/2}d} + \frac{13A \sin(c+dx)}{4ad\sqrt{a-a \sec(c+dx)}} + \frac{3A \sin(c+dx) \cos(c+dx)}{2ad\sqrt{a-a \sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^2*(A + A*Sec[c + d*x]))/(a - a*Sec[c + d*x])^(3/2), x]

[Out] (31*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a - a*Sec[c + d*x]]]/(4*a^(3/2)*d) - (11*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/((Sqrt[2]*Sqrt[a - a*Sec[c + d*x]])])/(Sqrt[2]*a^(3/2)*d) - (A*Cos[c + d*x]*Sin[c + d*x])/(d*(a - a*Sec[c + d*x])^(3/2)) + (13*A*Sin[c + d*x])/(4*a*d*Sqrt[a - a*Sec[c + d*x]]) + (3*A*Cos[c + d*x]*Sin[c + d*x])/(2*a*d*Sqrt[a - a*Sec[c + d*x]])

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 3774

Int[Sqrt[csc[(c_.) + (d_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] := Dist[(-2*b)/d, Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol]
:> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x]
/; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol]
:> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^m*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol]
:> -Simp[((A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m + 1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e + f*x], x], x], x]
/; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0]
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^m*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol]
:> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n - A*b*(m + n + 1)*Csc[e + f*x], x], x], x]
/; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(c+dx)(A+A\sec(c+dx))}{(a-a\sec(c+dx))^{3/2}} dx &= -\frac{A\cos(c+dx)\sin(c+dx)}{d(a-a\sec(c+dx))^{3/2}} + \frac{\int \frac{\cos^2(c+dx)(6aA+5aA\sec(c+dx))}{\sqrt{a-a\sec(c+dx)}} dx}{2a^2} \\
&= -\frac{A\cos(c+dx)\sin(c+dx)}{d(a-a\sec(c+dx))^{3/2}} + \frac{3A\cos(c+dx)\sin(c+dx)}{2ad\sqrt{a-a\sec(c+dx)}} - \frac{\int \frac{\cos(c+dx)}{\sqrt{a-a\sec(c+dx)}} dx}{2a} \\
&= -\frac{A\cos(c+dx)\sin(c+dx)}{d(a-a\sec(c+dx))^{3/2}} + \frac{13A\sin(c+dx)}{4ad\sqrt{a-a\sec(c+dx)}} + \frac{3A\cos(c+dx)}{2ad\sqrt{a-a\sec(c+dx)}} \\
&= -\frac{A\cos(c+dx)\sin(c+dx)}{d(a-a\sec(c+dx))^{3/2}} + \frac{13A\sin(c+dx)}{4ad\sqrt{a-a\sec(c+dx)}} + \frac{3A\cos(c+dx)}{2ad\sqrt{a-a\sec(c+dx)}} \\
&= -\frac{A\cos(c+dx)\sin(c+dx)}{d(a-a\sec(c+dx))^{3/2}} + \frac{13A\sin(c+dx)}{4ad\sqrt{a-a\sec(c+dx)}} + \frac{3A\cos(c+dx)}{2ad\sqrt{a-a\sec(c+dx)}} \\
&= \frac{31A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{4a^{3/2}d} - \frac{11A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{2}a^{3/2}d} - \frac{A\cos(c+dx)}{d(a-a\sec(c+dx))^{3/2}}
\end{aligned}$$

Mathematica [C] time = 6.77, size = 408, normalized size = 2.10

$$A \left(\frac{\sin^3\left(\frac{c}{2} + \frac{dx}{2}\right) \sec^2(c+dx) \left(\frac{3\sin\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)}{2d} + \frac{5\sin\left(\frac{3c}{2}\right)\sin\left(\frac{3dx}{2}\right)}{d} + \frac{\sin\left(\frac{5c}{2}\right)\sin\left(\frac{5dx}{2}\right)}{2d} - \frac{3\cos\left(\frac{c}{2}\right)\cos\left(\frac{dx}{2}\right)}{2d} - \frac{5\cos\left(\frac{3c}{2}\right)\cos\left(\frac{3dx}{2}\right)}{d} \right)}{(a-a\sec(c+dx))^{3/2}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]^2*(A + A*Sec[c + d*x]))/(a - a*Sec[c + d*x])^(3/2), x]

[Out] A*(-1/2*(Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*(31*ArcSinh[E^(I*(c + d*x))] - 44*Sqrt[2]*ArcTanh[(1 + E^(I*(c + d*x))]/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))]]) + 31*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]])*Sec[c + d*x]^(3/2)*Sin[c/2 + (d*x)/2]^3)/(Sqrt[2]*d*E^(I/2*(c + d*x))*(a - a*Sec[c + d*x])^(3/2)) + (Sec[c + d*x]^2*((-3*Cos[c/2]*Cos[(d*x)/2])/(2*d) - (5*Cos[(3*c)/2]*Cos[(3*d*x)/2])/d - (Cos[(5*c)/2]*Cos[(5*d*x)/2])/(2*d) - (2*Cot[c/2]*Csc[c/2 + (d*x)/2])/d + (2*Csc[c/2]*Csc[c/2 + (d*x)/2]^2*Sin[(d*x)/2])/d + (3*Sin[c/2]*Sin[(d*x)/2])/(2*d) + (5*Sin

$$\frac{((3c)/2) \sin((3dx)/2)/d + (\sin((5c)/2) \sin((5dx)/2))/(2d) \sin(c/2 + (dx)/2)^3}{(a - a \sec(c + dx))^{3/2}}$$

fricas [A] time = 0.47, size = 550, normalized size = 2.84

$$\frac{22 \sqrt{2} (A \cos(dx + c) - A) \sqrt{-a} \log \left(\frac{2 \sqrt{2} (\cos(dx+c)^2 + \cos(dx+c)) \sqrt{-a} \sqrt{\frac{a \cos(dx+c) - a}{\cos(dx+c)}} + (3a \cos(dx+c) + a) \sin(dx+c)}{(\cos(dx+c) - 1) \sin(dx+c)} \right) \sin(dx + c)}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(dx+c)^2*(A+A*sec(dx+c))/(a-a*sec(dx+c))^(3/2),x, algorithm="fricas")

[Out]
$$\frac{-1/8 * (22 * \sqrt{2} * (A * \cos(dx + c) - A) * \sqrt{-a} * \log((2 * \sqrt{2} * (\cos(dx + c)^2 + \cos(dx + c)) * \sqrt{-a} * \sqrt{(a * \cos(dx + c) - a) / \cos(dx + c)} + (3 * a * \cos(dx + c) + a) * \sin(dx + c)) / ((\cos(dx + c) - 1) * \sin(dx + c))) * \sin(dx + c) + 31 * (A * \cos(dx + c) - A) * \sqrt{-a} * \log((2 * (\cos(dx + c)^2 + \cos(dx + c)) * \sqrt{-a} * \sqrt{(a * \cos(dx + c) - a) / \cos(dx + c)} - (2 * a * \cos(dx + c) + a) * \sin(dx + c)) / \sin(dx + c)) * \sin(dx + c) + 2 * (2 * A * \cos(dx + c)^4 + 9 * A * \cos(dx + c)^3 - 6 * A * \cos(dx + c)^2 - 13 * A * \cos(dx + c)) * \sqrt{(a * \cos(dx + c) - a) / \cos(dx + c)}) / ((a^2 * d * \cos(dx + c) - a^2 * d) * \sin(dx + c)), 1/4 * (22 * \sqrt{2} * (A * \cos(dx + c) - A) * \sqrt{a} * \arctan(\sqrt{2} * \sqrt{(a * \cos(dx + c) - a) / \cos(dx + c)}) * \cos(dx + c) / (\sqrt{a} * \sin(dx + c))) * \sin(dx + c) - 31 * (A * \cos(dx + c) - A) * \sqrt{a} * \arctan(\sqrt{(a * \cos(dx + c) - a) / \cos(dx + c)}) * \cos(dx + c) / (\sqrt{a} * \sin(dx + c))) * \sin(dx + c) - (2 * A * \cos(dx + c)^4 + 9 * A * \cos(dx + c)^3 - 6 * A * \cos(dx + c)^2 - 13 * A * \cos(dx + c)) * \sqrt{(a * \cos(dx + c) - a) / \cos(dx + c)}) / ((a^2 * d * \cos(dx + c) - a^2 * d) * \sin(dx + c))]}{\dots}$$

giac [A] time = 1.45, size = 295, normalized size = 1.52

$$\frac{22 \sqrt{2} A \arctan \left(\frac{\sqrt{a \tan(\frac{1}{2} dx + \frac{1}{2} c)^2 - a}}{\sqrt{a}} \right) + 31 A \arctan \left(\frac{\sqrt{2} \sqrt{a \tan(\frac{1}{2} dx + \frac{1}{2} c)^2 - a}}{2 \sqrt{a}} \right) + \sqrt{2} \left(7 \left(a \tan(\frac{1}{2} dx + \frac{1}{2} c)^2 - a \right)^{\frac{3}{2}} A + 18 \sqrt{a} \right)}{a^{\frac{3}{2}} \operatorname{sgn} \left(\tan(\frac{1}{2} dx + \frac{1}{2} c)^2 - 1 \right) \operatorname{sgn} \left(\tan(\frac{1}{2} dx + \frac{1}{2} c) \right) + a^{\frac{3}{2}} \operatorname{sgn} \left(\tan(\frac{1}{2} dx + \frac{1}{2} c)^2 - 1 \right) \operatorname{sgn} \left(\tan(\frac{1}{2} dx + \frac{1}{2} c) \right) + \left(a \tan(\frac{1}{2} dx + \frac{1}{2} c)^2 + a \right)^2 a \operatorname{sgn} \left(\tan(\frac{1}{2} dx + \frac{1}{2} c) \right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(dx+c)^2*(A+A*sec(dx+c))/(a-a*sec(dx+c))^(3/2),x, algorithm="giac")

```
[Out] -1/4*(22*sqrt(2)*A*arctan(sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)/sqrt(a))/(a^(3/2)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c))) - 31*A*arctan(1/2*sqrt(2)*sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)/sqrt(a))/(a^(3/2)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c))) - sqrt(2)*(7*(a*tan(1/2*d*x + 1/2*c)^2 - a)^(3/2)*A + 18*sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)*A*a)/((a*tan(1/2*d*x + 1/2*c)^2 + a)^2*a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c))) - 2*sqrt(2)*sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)*A/(a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c))*tan(1/2*d*x + 1/2*c)^2))/d
```

maple [B] time = 1.93, size = 883, normalized size = 4.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(3/2), x)
```

```
[Out] -1/60*A/d*(-1+cos(d*x+c))^4*(220*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)*2^(1/2)-930*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))+60*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*cos(d*x+c)^3+180*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*cos(d*x+c)^2+180*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*cos(d*x+c)+132*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*cos(d*x+c)^3-220*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)^3+660*2^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*cos(d*x+c)^3-195*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)+132*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*cos(d*x+c)^2*2^(1/2)-660*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*2^(1/2)-278*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^3*2^(1/2)+288*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*2^(1/2)-660*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*cos(d*x+c)*2^(1/2)-40*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*2^(1/2)-132*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*cos(d*x+c)*2^(1/2)+30*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^5*2^(1/2)-220*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)^2*2^(1/2)+195*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^4*2^(1/2)+660*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*cos(d*x+c)^2*2^(1/2)-930*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*cos(d*x+c)-132*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*2^(1/2)+930*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*cos(d*x+c)^2+220*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*2^(1/2)+930*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*cos(d*x+c)^3+60*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2))/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)/(a*(-1+cos(d*x+c))/cos(d*x+c))^(3/2)/sin(d*x+c)^7*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A \sec(dx + c) + A) \cos(dx + c)^2}{(-a \sec(dx + c) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(3/2),x, algorithm="maxima")

[Out] integrate((A*sec(d*x + c) + A)*cos(d*x + c)^2/(-a*sec(d*x + c) + a)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c + dx)^2 \left(A + \frac{A}{\cos(c+dx)} \right)}{\left(a - \frac{a}{\cos(c+dx)} \right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)^2*(A + A/cos(c + d*x)))/(a - a/cos(c + d*x))^(3/2),x)

[Out] int((cos(c + d*x)^2*(A + A/cos(c + d*x)))/(a - a/cos(c + d*x))^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$A \left(\int \frac{\cos^2(c + dx)}{-a\sqrt{-a \sec(c + dx) + a} \sec(c + dx) + a\sqrt{-a \sec(c + dx) + a}} dx + \int \frac{\cos^2(c + dx) \sec(c + dx)}{-a\sqrt{-a \sec(c + dx) + a} \sec(c + dx) + a\sqrt{-a \sec(c + dx) + a}} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**2*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))**(3/2),x)

[Out] A*(Integral(cos(c + d*x)**2/(-a*sqrt(-a*sec(c + d*x) + a)*sec(c + d*x) + a*sqrt(-a*sec(c + d*x) + a)), x) + Integral(cos(c + d*x)**2*sec(c + d*x)/(-a*sqrt(-a*sec(c + d*x) + a)*sec(c + d*x) + a*sqrt(-a*sec(c + d*x) + a)), x))

$$3.174 \quad \int \frac{\cos^3(c+dx)(A+A \sec(c+dx))}{(a-a \sec(c+dx))^{3/2}} dx$$

Optimal. Leaf size=236

$$\frac{85A \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a-a \sec(c+dx)}}\right)}{8a^{3/2}d} - \frac{15A \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{2} \sqrt{a-a \sec(c+dx)}}\right)}{\sqrt{2} a^{3/2}d} + \frac{35A \sin(c+dx)}{8ad\sqrt{a-a \sec(c+dx)}} + \frac{4A \sin(c+dx) \cos^2(c+dx)}{3ad\sqrt{a-a \sec(c+dx)}}$$

[Out] 85/8*A*arctan(a^(1/2)*tan(d*x+c)/(a-a*sec(d*x+c))^(1/2))/a^(3/2)/d-A*cos(d*x+c)^2*sin(d*x+c)/d/(a-a*sec(d*x+c))^(3/2)-15/2*A*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a-a*sec(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2)+35/8*A*sin(d*x+c)/a/d/(a-a*sec(d*x+c))^(1/2)+25/12*A*cos(d*x+c)*sin(d*x+c)/a/d/(a-a*sec(d*x+c))^(1/2)+4/3*A*cos(d*x+c)^2*sin(d*x+c)/a/d/(a-a*sec(d*x+c))^(1/2)

Rubi [A] time = 0.70, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {4020, 4022, 3920, 3774, 203, 3795}

$$\frac{85A \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a-a \sec(c+dx)}}\right)}{8a^{3/2}d} - \frac{15A \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{2} \sqrt{a-a \sec(c+dx)}}\right)}{\sqrt{2} a^{3/2}d} + \frac{35A \sin(c+dx)}{8ad\sqrt{a-a \sec(c+dx)}} + \frac{4A \sin(c+dx) \cos^2(c+dx)}{3ad\sqrt{a-a \sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^3*(A + A*Sec[c + d*x]))/(a - a*Sec[c + d*x])^(3/2), x]

[Out] (85*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a - a*Sec[c + d*x]]]/(8*a^(3/2)*d) - (15*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a - a*Sec[c + d*x]])]/(Sqrt[2]*a^(3/2)*d) - (A*Cos[c + d*x]^2*Sin[c + d*x])/(d*(a - a*Sec[c + d*x])^(3/2)) + (35*A*Sin[c + d*x])/(8*a*d*Sqrt[a - a*Sec[c + d*x]]) + (25*A*Cos[c + d*x]*Sin[c + d*x])/(12*a*d*Sqrt[a - a*Sec[c + d*x]]) + (4*A*Cos[c + d*x]^2*Sin[c + d*x])/(3*a*d*Sqrt[a - a*Sec[c + d*x]])

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 3774

Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] := Dist[(-2*b)/d, Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 3795

```
Int[(csc[(e_.) + (f_.)*(x_.)]/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol]
:> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x]
;/; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol]
:> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x]
;/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol]
:> -Simp[(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n/(b*f*(2*m + 1)), x]
- Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e + f*x], x], x], x]
;/; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0]
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol]
:> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n - A*b*(m + n + 1)*Csc[e + f*x], x], x], x]
;/; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^3(c+dx)(A+A\sec(c+dx))}{(a-a\sec(c+dx))^{3/2}} dx &= -\frac{A\cos^2(c+dx)\sin(c+dx)}{d(a-a\sec(c+dx))^{3/2}} + \frac{\int \frac{\cos^3(c+dx)(8aA+7aA\sec(c+dx))}{\sqrt{a-a\sec(c+dx)}} dx}{2a^2} \\
&= -\frac{A\cos^2(c+dx)\sin(c+dx)}{d(a-a\sec(c+dx))^{3/2}} + \frac{4A\cos^2(c+dx)\sin(c+dx)}{3ad\sqrt{a-a\sec(c+dx)}} - \frac{\int \frac{\cos^2(c+dx)}{\sqrt{a-a\sec(c+dx)}} dx}{3ad} \\
&= -\frac{A\cos^2(c+dx)\sin(c+dx)}{d(a-a\sec(c+dx))^{3/2}} + \frac{25A\cos(c+dx)\sin(c+dx)}{12ad\sqrt{a-a\sec(c+dx)}} + \frac{4A\cos(c+dx)}{3ad} \\
&= -\frac{A\cos^2(c+dx)\sin(c+dx)}{d(a-a\sec(c+dx))^{3/2}} + \frac{35A\sin(c+dx)}{8ad\sqrt{a-a\sec(c+dx)}} + \frac{25A\cos(c+dx)}{12ad\sqrt{a-a\sec(c+dx)}} \\
&= -\frac{A\cos^2(c+dx)\sin(c+dx)}{d(a-a\sec(c+dx))^{3/2}} + \frac{35A\sin(c+dx)}{8ad\sqrt{a-a\sec(c+dx)}} + \frac{25A\cos(c+dx)}{12ad\sqrt{a-a\sec(c+dx)}} \\
&= -\frac{A\cos^2(c+dx)\sin(c+dx)}{d(a-a\sec(c+dx))^{3/2}} + \frac{35A\sin(c+dx)}{8ad\sqrt{a-a\sec(c+dx)}} + \frac{25A\cos(c+dx)}{12ad\sqrt{a-a\sec(c+dx)}} \\
&= \frac{85A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{8a^{3/2}d} - \frac{15A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{2}a^{3/2}d} - \frac{A\cos^2(c+dx)}{d(a-a\sec(c+dx))^{3/2}}
\end{aligned}$$

Mathematica [C] time = 6.73, size = 452, normalized size = 1.92

$$A \left[\frac{\sin^3\left(\frac{c}{2} + \frac{dx}{2}\right) \sec^2(c+dx) \left(\frac{65\sin\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)}{12d} + \frac{25\sin\left(\frac{3c}{2}\right)\sin\left(\frac{3dx}{2}\right)}{3d} + \frac{5\sin\left(\frac{5c}{2}\right)\sin\left(\frac{5dx}{2}\right)}{4d} + \frac{\sin\left(\frac{7c}{2}\right)\sin\left(\frac{7dx}{2}\right)}{6d} - \frac{65\cos\left(\frac{c}{2}\right)\cos\left(\frac{dx}{2}\right)}{12d} \right)}{(a-a\sec(c+dx))^{3/2}} \right]$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]^3*(A + A*Sec[c + d*x]))/(a - a*Sec[c + d*x])^(3/2), x]

[Out] A*((-5*Sqrt[E^(I*(c + d*x))]/(1 + E^((2*I)*(c + d*x))))*Sqrt[1 + E^((2*I)*(c + d*x))])*(17*ArcSinh[E^(I*(c + d*x))] - 24*Sqrt[2]*ArcTanh[(1 + E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))]]) + 17*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]])*Sec[c + d*x]^(3/2)*Sin[c/2 + (d*x)/2]^3)/(4*Sqrt[2]*d*E^(I*(c + d*x)))

$$\begin{aligned} & \left(\frac{I}{2} \right) * (c + d*x) * (a - a*\text{Sec}[c + d*x])^{(3/2)} + (\text{Sec}[c + d*x]^2 * ((-65*\text{Cos}[c/2] * \text{Cos}[(d*x)/2]) / (12*d) - (25*\text{Cos}[(3*c)/2] * \text{Cos}[(3*d*x)/2]) / (3*d) - (5*\text{Cos}[(5*c)/2] * \text{Cos}[(5*d*x)/2]) / (4*d) - (\text{Cos}[(7*c)/2] * \text{Cos}[(7*d*x)/2]) / (6*d) - (2*\text{Cot}[c/2] * \text{Csc}[c/2 + (d*x)/2]) / d + (2*\text{Csc}[c/2] * \text{Csc}[c/2 + (d*x)/2]^2 * \text{Sin}[(d*x)/2]) / d + (65*\text{Sin}[c/2] * \text{Sin}[(d*x)/2]) / (12*d) + (25*\text{Sin}[(3*c)/2] * \text{Sin}[(3*d*x)/2]) / (3*d) + (5*\text{Sin}[(5*c)/2] * \text{Sin}[(5*d*x)/2]) / (4*d) + (\text{Sin}[(7*c)/2] * \text{Sin}[(7*d*x)/2]) / (6*d)) * \text{Sin}[c/2 + (d*x)/2]^3 / (a - a*\text{Sec}[c + d*x])^{(3/2)} \end{aligned}$$

fricas [A] time = 0.49, size = 572, normalized size = 2.42

$$\left[\frac{180 \sqrt{2} (A \cos(dx + c) - A) \sqrt{-a} \log \left(\frac{2 \sqrt{2} (\cos(dx+c)^2 + \cos(dx+c)) \sqrt{-a} \sqrt{\frac{a \cos(dx+c) - a}{\cos(dx+c)}} + (3a \cos(dx+c) + a) \sin(dx+c)}{(\cos(dx+c) - 1) \sin(dx+c)} \right)}{\sin(dx + c)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^3*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(3/2),x, algorithm="fricas")

[Out] [-1/48*(180*sqrt(2)*(A*cos(d*x + c) - A)*sqrt(-a)*log((2*sqrt(2)*(cos(d*x + c)^2 + cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) - a)/cos(d*x + c)) + (3*a*cos(d*x + c) + a)*sin(d*x + c))/((cos(d*x + c) - 1)*sin(d*x + c)))*sin(d*x + c) + 255*(A*cos(d*x + c) - A)*sqrt(-a)*log((2*(cos(d*x + c)^2 + cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) - a)/cos(d*x + c)) - (2*a*cos(d*x + c) + a)*sin(d*x + c))/sin(d*x + c))*sin(d*x + c) + 2*(8*A*cos(d*x + c)^5 + 26*A*cos(d*x + c)^4 + 73*A*cos(d*x + c)^3 - 50*A*cos(d*x + c)^2 - 105*A*cos(d*x + c))*sqrt((a*cos(d*x + c) - a)/cos(d*x + c))/((a^2*d*cos(d*x + c) - a^2*d)*sin(d*x + c)), 1/24*(180*sqrt(2)*(A*cos(d*x + c) - A)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) - a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))*sin(d*x + c) - 255*(A*cos(d*x + c) - A)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) - a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))*sin(d*x + c) - (8*A*cos(d*x + c)^5 + 26*A*cos(d*x + c)^4 + 73*A*cos(d*x + c)^3 - 50*A*cos(d*x + c)^2 - 105*A*cos(d*x + c))*sqrt((a*cos(d*x + c) - a)/cos(d*x + c))/((a^2*d*cos(d*x + c) - a^2*d)*sin(d*x + c))]

giac [A] time = 1.53, size = 320, normalized size = 1.36

$$\frac{180 \sqrt{2} A \arctan \left(\frac{\sqrt{a \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^2 - a}}{\sqrt{a}} \right)}{a^{\frac{3}{2}} \operatorname{sgn} \left(\tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^2 - 1 \right) \operatorname{sgn} \left(\tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) \right)} - \frac{255 A \arctan \left(\frac{\sqrt{2} \sqrt{a \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^2 - a}}{2 \sqrt{a}} \right)}{a^{\frac{3}{2}} \operatorname{sgn} \left(\tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^2 - 1 \right) \operatorname{sgn} \left(\tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) \right)} - \frac{12 \sqrt{2} \sqrt{a \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^2 - a}}{a^2 \operatorname{sgn} \left(\tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^2 - 1 \right) \operatorname{sgn} \left(\tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(3/2),x, algorithm
="giac")
```

```
[Out] -1/24*(180*sqrt(2)*A*arctan(sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)/sqrt(a))/(a^(
(3/2)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c))) - 255*A*ar
ctan(1/2*sqrt(2)*sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)/sqrt(a))/(a^(3/2)*sgn(t
an(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c))) - 12*sqrt(2)*sqrt(a*t
an(1/2*d*x + 1/2*c)^2 - a)*A/(a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1
/2*d*x + 1/2*c))*tan(1/2*d*x + 1/2*c)^2) - sqrt(2)*(63*(a*tan(1/2*d*x + 1/2
*c)^2 - a)^(5/2)*A + 272*(a*tan(1/2*d*x + 1/2*c)^2 - a)^(3/2)*A*a + 324*sqrt
(a*tan(1/2*d*x + 1/2*c)^2 - a)*A*a^2)/((a*tan(1/2*d*x + 1/2*c)^2 + a)^3*a*
sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c))))/d
```

maple [B] time = 1.86, size = 1104, normalized size = 4.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/168*A/d*(-1+cos(d*x+c))^5*(1680*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(
d*x+c)*2^(1/2)-360*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*cos(d*x+c)^
4-672*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(9/2)*cos(d*x+c)+504*2^(1/2)*(-
2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*cos(d*x+c)^4-840*2^(1/2)*(-2*cos(d*x+c)
/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)^4+2520*2^(1/2)*arctan(1/(-2*cos(d*x+c)/(1
+cos(d*x+c)))^(1/2))*cos(d*x+c)^4+56*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*cos(d*x+c)^7+350*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*
x+c)^6-3570*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))-720*2^(
1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*cos(d*x+c)^3+720*2^(1/2)*(-2*cos
(d*x+c)/(1+cos(d*x+c)))^(7/2)*cos(d*x+c)+1008*2^(1/2)*(-2*cos(d*x+c)/(1+cos
(d*x+c)))^(5/2)*cos(d*x+c)^3-1680*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3
/2)*cos(d*x+c)^3+5040*2^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
)*cos(d*x+c)^3-735*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)-2520*arctan
(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*2^(1/2)+1130*(-2*cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*cos(d*x+c)^3*2^(1/2)+952*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*cos(d*x+c)^2*2^(1/2)-5040*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))
*cos(d*x+c)*2^(1/2)-875*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*2^(
1/2)-168*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(9/2)*cos(d*x+c)^4-672*2^(1
/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(9/2)*cos(d*x+c)^3-1008*2^(1/2)*(-2*cos(
d*x+c)/(1+cos(d*x+c)))^(9/2)*cos(d*x+c)^2-1008*(-2*cos(d*x+c)/(1+cos(d*x+c)
))^(5/2)*cos(d*x+c)*2^(1/2)+1225*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d
*x+c)^5*2^(1/2)-2103*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^4*2^(1
/2)-168*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(9/2)+3570*arctan(1/2*(-2*co
```

$s(d*x+c)/(1+\cos(d*x+c))^{1/2}*2^{1/2})*\cos(d*x+c)^4-7140*\arctan(1/2*(-2*\cos(d*x+c)/(1+\cos(d*x+c))^{1/2}*2^{1/2})*\cos(d*x+c)-504*(-2*\cos(d*x+c)/(1+\cos(d*x+c))^{5/2}*2^{1/2}+840*(-2*\cos(d*x+c)/(1+\cos(d*x+c))^{3/2}*2^{1/2}+7140*\arctan(1/2*(-2*\cos(d*x+c)/(1+\cos(d*x+c))^{1/2}*2^{1/2})*\cos(d*x+c)^3+360*2^{1/2}*(-2*\cos(d*x+c)/(1+\cos(d*x+c))^{7/2}))/(-2*\cos(d*x+c)/(1+\cos(d*x+c))^{3/2})/(a*(-1+\cos(d*x+c))/\cos(d*x+c))^{3/2}/\sin(d*x+c)^9*2^{1/2}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A \sec(dx + c) + A) \cos(dx + c)^3}{(-a \sec(dx + c) + a)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^3*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(3/2),x, algorithm="maxima")

[Out] integrate((A*sec(d*x + c) + A)*cos(d*x + c)^3/(-a*sec(d*x + c) + a)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c + dx)^3 \left(A + \frac{A}{\cos(c + dx)} \right)}{\left(a - \frac{a}{\cos(c + dx)} \right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)^3*(A + A/cos(c + d*x)))/(a - a/cos(c + d*x))^(3/2),x)

[Out] int((cos(c + d*x)^3*(A + A/cos(c + d*x)))/(a - a/cos(c + d*x))^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$A \left(\int \frac{\cos^3(c + dx)}{-a\sqrt{-a \sec(c + dx) + a} \sec(c + dx) + a\sqrt{-a \sec(c + dx) + a}} dx + \int \frac{\cos^3(c + dx) \sec(c + dx)}{-a\sqrt{-a \sec(c + dx) + a} \sec(c + dx) + a\sqrt{-a \sec(c + dx) + a}} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**3*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))**(3/2),x)

[Out] A*(Integral(cos(c + d*x)**3/(-a*sqrt(-a*sec(c + d*x) + a)*sec(c + d*x) + a*sqrt(-a*sec(c + d*x) + a)), x) + Integral(cos(c + d*x)**3*sec(c + d*x)/(-a*sqrt(-a*sec(c + d*x) + a)*sec(c + d*x) + a*sqrt(-a*sec(c + d*x) + a)), x))

$$3.175 \quad \int \frac{A + A \sec(c+dx)}{(a - a \sec(c+dx))^{5/2}} dx$$

Optimal. Leaf size=152

$$\frac{2A \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a-a \sec(c+dx)}}\right)}{a^{5/2}d} - \frac{23A \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{2} \sqrt{a-a \sec(c+dx)}}\right)}{8\sqrt{2} a^{5/2}d} - \frac{7A \tan(c+dx)}{8ad(a-a \sec(c+dx))^{3/2}} - \frac{A \tan(c+dx)}{2d(a-a \sec(c+dx))^{5/2}}$$

[Out] $2*A*\arctan(a^{(1/2)}*\tan(d*x+c)/(a-a*\sec(d*x+c))^{(1/2)})/a^{(5/2)}/d-23/16*A*\arctan(1/2*a^{(1/2)}*\tan(d*x+c)*2^{(1/2)}/(a-a*\sec(d*x+c))^{(1/2)})/a^{(5/2)}/d*2^{(1/2)}-1/2*A*\tan(d*x+c)/d/(a-a*\sec(d*x+c))^{(5/2)}-7/8*A*\tan(d*x+c)/a/d/(a-a*\sec(d*x+c))^{(3/2)}$

Rubi [A] time = 0.21, antiderivative size = 185, normalized size of antiderivative = 1.22, number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3904, 3887, 471, 527, 522, 203}

$$\frac{2A \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a-a \sec(c+dx)}}\right)}{a^{5/2}d} - \frac{23A \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{2} \sqrt{a-a \sec(c+dx)}}\right)}{8\sqrt{2} a^{5/2}d} + \frac{7A \sin(c+dx) \csc^2\left(\frac{1}{2}(c+dx)\right)}{16a^2d\sqrt{a-a \sec(c+dx)}} - \frac{A \sin(c+dx) \cos(c+dx)}{8a^2d\sqrt{a-a \sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(A + A*Sec[c + d*x])/(a - a*Sec[c + d*x])^(5/2), x]

[Out] $(2*A*\text{ArcTan}[\text{Sqrt}[a]*\text{Tan}[c + d*x])/\text{Sqrt}[a - a*\text{Sec}[c + d*x]])/(a^{(5/2)*d} - (23*A*\text{ArcTan}[\text{Sqrt}[a]*\text{Tan}[c + d*x])/(\text{Sqrt}[2]*\text{Sqrt}[a - a*\text{Sec}[c + d*x]])))/(8*\text{Sqrt}[2]*a^{(5/2)*d} + (7*A*\text{Csc}[(c + d*x)/2]^2*\text{Sin}[c + d*x])/(16*a^2*d*\text{Sqrt}[a - a*\text{Sec}[c + d*x]]) - (A*\text{Cos}[c + d*x]*\text{Csc}[(c + d*x)/2]^4*\text{Sin}[c + d*x])/(8*a^2*d*\text{Sqrt}[a - a*\text{Sec}[c + d*x]]))$

Rule 203

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[Rt[b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 471

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := Simp[(e^(n-1)*(e*x)^(m-n+1)*(a+b*x^n)^(p+1)*(c+d*x^n)^(q+1))/(n*(b*c-a*d)*(p+1)), x] - Dist[e^n/(n*(b*c-a*d)*(p+1)), Int[(e*x)^(m-n)*(a+b*x^n)^(p+1)*(c+d*x^n)^q*Simp[c*(m-n+1)+d*(m+n*(p+q+1)+1]*x^n, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c-a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m-n+1]

1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 522

Int[((e_) + (f_)*(x_)^(n_))/(((a_) + (b_)*(x_)^(n_))*((c_) + (d_)*(x_)^(n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 527

Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_)*((e_) + (f_)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]

Rule 3887

Int[cot[(c_) + (d_)*(x_)]^(m_)*(csc[(c_) + (d_)*(x_)]*(b_) + (a_))^(n_), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && IntegerQ[n - 1/2]

Rule 3904

Int[(csc[(e_) + (f_)*(x_)]*(b_) + (a_))^(m_)*(csc[(e_) + (f_)*(x_)]*(d_) + (c_))^(n_), x_Symbol] :> Dist[(-a*c)^m, Int[Cot[e + f*x]^(2*m)*(c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] && !(IntegerQ[n] && GtQ[m - n, 0])

Rubi steps

$$\begin{aligned}
\int \frac{A + A \sec(c + dx)}{(a - a \sec(c + dx))^{5/2}} dx &= - \left((aA) \int \frac{\tan^2(c + dx)}{(a - a \sec(c + dx))^{7/2}} dx \right) \\
&= \frac{(2A) \text{Subst} \left(\int \frac{x^2}{(1+ax^2)(2+ax^2)^3} dx, x, -\frac{\tan(c+dx)}{\sqrt{a-a \sec(c+dx)}} \right)}{ad} \\
&= -\frac{A \cos(c + dx) \csc^4 \left(\frac{1}{2}(c + dx) \right) \sin(c + dx)}{8a^2 d \sqrt{a - a \sec(c + dx)}} - \frac{A \text{Subst} \left(\int \frac{1-3ax^2}{(1+ax^2)(2+ax^2)^2} dx, x, -\frac{\tan(c+dx)}{\sqrt{a-a \sec(c+dx)}} \right)}{2a^2 d} \\
&= \frac{7A \csc^2 \left(\frac{1}{2}(c + dx) \right) \sin(c + dx)}{16a^2 d \sqrt{a - a \sec(c + dx)}} - \frac{A \cos(c + dx) \csc^4 \left(\frac{1}{2}(c + dx) \right) \sin(c + dx)}{8a^2 d \sqrt{a - a \sec(c + dx)}} - \frac{A \text{Subst} \left(\int \frac{1-3ax^2}{(1+ax^2)(2+ax^2)^2} dx, x, -\frac{\tan(c+dx)}{\sqrt{a-a \sec(c+dx)}} \right)}{2a^2 d} \\
&= \frac{7A \csc^2 \left(\frac{1}{2}(c + dx) \right) \sin(c + dx)}{16a^2 d \sqrt{a - a \sec(c + dx)}} - \frac{A \cos(c + dx) \csc^4 \left(\frac{1}{2}(c + dx) \right) \sin(c + dx)}{8a^2 d \sqrt{a - a \sec(c + dx)}} - \frac{A \text{Subst} \left(\int \frac{1-3ax^2}{(1+ax^2)(2+ax^2)^2} dx, x, -\frac{\tan(c+dx)}{\sqrt{a-a \sec(c+dx)}} \right)}{2a^2 d} \\
&= \frac{2A \tan^{-1} \left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a-a \sec(c+dx)}} \right)}{a^{5/2} d} - \frac{23A \tan^{-1} \left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{2} \sqrt{a-a \sec(c+dx)}} \right)}{8\sqrt{2} a^{5/2} d} + \frac{7A \csc^2 \left(\frac{1}{2}(c + dx) \right) \sin(c + dx)}{16a^2 d \sqrt{a - a \sec(c + dx)}}
\end{aligned}$$

Mathematica [C] time = 6.81, size = 387, normalized size = 2.55

$$A \left(\frac{\sin^5 \left(\frac{c}{2} + \frac{dx}{2} \right) \sec^3(c + dx) \left(\frac{11 \sin \left(\frac{c}{2} \right) \sin \left(\frac{dx}{2} \right)}{d} - \frac{11 \cos \left(\frac{c}{2} \right) \cos \left(\frac{dx}{2} \right)}{d} - \frac{\cot \left(\frac{c}{2} \right) \csc^3 \left(\frac{c}{2} + \frac{dx}{2} \right)}{d} + \frac{15 \cot \left(\frac{c}{2} \right) \csc \left(\frac{c}{2} + \frac{dx}{2} \right)}{2d} + \frac{\csc \left(\frac{c}{2} \right) \sin \left(\frac{dx}{2} \right)}{d} \right)}{(a - a \sec(c + dx))^{5/2}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(A + A*Sec[c + d*x])/(a - a*Sec[c + d*x])^(5/2), x]

[Out] A*((Sqrt[E^(I*(c + d*x))]/(1 + E^((2*I)*(c + d*x))))*Sqrt[1 + E^((2*I)*(c + d*x))]*(8*ArcSinh[E^(I*(c + d*x))]) - (23*ArcTanh[(1 + E^(I*(c + d*x)))]/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))]]))/Sqrt[2] + 8*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]])*Sec[c + d*x]^(5/2)*Sin[c/2 + (d*x)/2]^5)/(Sqrt[2]*d*E^((I/2)*(c + d*x))*(a - a*Sec[c + d*x])^(5/2)) + (Sec[c + d*x]^3*((-11*Cos[c/2]*Cos[(d*x)/2])/d + (15*Cot[c/2]*Csc[c/2 + (d*x)/2])/(2*d) - (Cot[c/2]*Csc[c/2 + (d*x)/2]^3)/d - (15*Csc[c/2]*Csc[c/2 + (d*x)/2]^2*Sin[(d*x)/2])/(2*d) + (

$\text{Csc}[c/2] * \text{Csc}[c/2 + (d*x)/2]^4 * \text{Sin}[(d*x)/2]) / d + (11 * \text{Sin}[c/2] * \text{Sin}[(d*x)/2]) / d * \text{Sin}[c/2 + (d*x)/2]^5 / (a - a * \text{Sec}[c + d*x])^{(5/2)}$

fricas [B] time = 0.47, size = 590, normalized size = 3.88

$$\left[\frac{23 \sqrt{2} (A \cos(dx+c)^2 - 2A \cos(dx+c) + A) \sqrt{-a} \log \left(\frac{2 \sqrt{2} (\cos(dx+c)^2 + \cos(dx+c)) \sqrt{-a} \sqrt{\frac{a \cos(dx+c) - a}{\cos(dx+c)}} + (3a \cos(dx+c) + a)}{(\cos(dx+c) - 1) \sin(dx+c)} \right)}{\dots} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(5/2),x, algorithm="fricas")

[Out] $[-1/32 * (23 * \text{sqrt}(2) * (A * \cos(d*x + c)^2 - 2 * A * \cos(d*x + c) + A) * \text{sqrt}(-a) * \log((2 * \text{sqrt}(2) * (\cos(d*x + c)^2 + \cos(d*x + c)) * \text{sqrt}(-a) * \text{sqrt}((a * \cos(d*x + c) - a) / \cos(d*x + c)) + (3 * a * \cos(d*x + c) + a) * \sin(d*x + c)) / ((\cos(d*x + c) - 1) * \sin(d*x + c))) * \sin(d*x + c) + 32 * (A * \cos(d*x + c)^2 - 2 * A * \cos(d*x + c) + A) * \text{sqrt}(-a) * \log((2 * (\cos(d*x + c)^2 + \cos(d*x + c)) * \text{sqrt}(-a) * \text{sqrt}((a * \cos(d*x + c) - a) / \cos(d*x + c)) - (2 * a * \cos(d*x + c) + a) * \sin(d*x + c)) / \sin(d*x + c)) * \sin(d*x + c) - 4 * (11 * A * \cos(d*x + c)^3 + 4 * A * \cos(d*x + c)^2 - 7 * A * \cos(d*x + c)) * \text{sqrt}((a * \cos(d*x + c) - a) / \cos(d*x + c)) / ((a^3 * d * \cos(d*x + c)^2 - 2 * a^3 * d * \cos(d*x + c) + a^3 * d) * \sin(d*x + c)), 1/16 * (23 * \text{sqrt}(2) * (A * \cos(d*x + c)^2 - 2 * A * \cos(d*x + c) + A) * \text{sqrt}(a) * \arctan(\text{sqrt}(2) * \text{sqrt}((a * \cos(d*x + c) - a) / \cos(d*x + c)) * \cos(d*x + c) / (\text{sqrt}(a) * \sin(d*x + c))) * \sin(d*x + c) - 32 * (A * \cos(d*x + c)^2 - 2 * A * \cos(d*x + c) + A) * \text{sqrt}(a) * \arctan(\text{sqrt}((a * \cos(d*x + c) - a) / \cos(d*x + c)) * \cos(d*x + c) / (\text{sqrt}(a) * \sin(d*x + c))) * \sin(d*x + c) + 2 * (11 * A * \cos(d*x + c)^3 + 4 * A * \cos(d*x + c)^2 - 7 * A * \cos(d*x + c)) * \text{sqrt}((a * \cos(d*x + c) - a) / \cos(d*x + c)) / ((a^3 * d * \cos(d*x + c)^2 - 2 * a^3 * d * \cos(d*x + c) + a^3 * d) * \sin(d*x + c))]$

giac [A] time = 1.37, size = 222, normalized size = 1.46

$$\frac{23 \sqrt{2} A \arctan \left(\frac{\sqrt{a \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^2 - a}}{\sqrt{a}} \right)}{a^{\frac{5}{2}} \text{sgn} \left(\tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^2 - 1 \right) \text{sgn} \left(\tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) \right)} - \frac{32 A \arctan \left(\frac{\sqrt{2} \sqrt{a \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^2 - a}}{2 \sqrt{a}} \right)}{a^{\frac{5}{2}} \text{sgn} \left(\tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^2 - 1 \right) \text{sgn} \left(\tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) \right)} - \frac{\sqrt{2} \left(9 \left(a \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^2 - a \right)^{\frac{3}{2}} A + 7 \sqrt{a \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^2 - a} \right)}{a^4 \text{sgn} \left(\tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^2 - 1 \right) \text{sgn} \left(\tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) \right)}$$

16d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(5/2),x, algorithm="giac")

[Out]
$$-1/16*(23*\sqrt{2}*A*\arctan(\sqrt{a*\tan(1/2*d*x + 1/2*c)^2 - a}/\sqrt{a}))/a^{(5/2)}*\operatorname{sgn}(\tan(1/2*d*x + 1/2*c)^2 - 1)*\operatorname{sgn}(\tan(1/2*d*x + 1/2*c)) - 32*A*\arctan(1/2*\sqrt{2}*\sqrt{a*\tan(1/2*d*x + 1/2*c)^2 - a}/\sqrt{a}))/a^{(5/2)}*\operatorname{sgn}(\tan(1/2*d*x + 1/2*c)^2 - 1)*\operatorname{sgn}(\tan(1/2*d*x + 1/2*c)) - \sqrt{2}*(9*(a*\tan(1/2*d*x + 1/2*c)^2 - a)^{(3/2)}*A + 7*\sqrt{a*\tan(1/2*d*x + 1/2*c)^2 - a}*A*a)/(a^4*\operatorname{sgn}(\tan(1/2*d*x + 1/2*c)^2 - 1)*\operatorname{sgn}(\tan(1/2*d*x + 1/2*c))*\tan(1/2*d*x + 1/2*c)^4))/d$$

maple [B] time = 1.55, size = 695, normalized size = 4.57

$$A(-1 + \cos(dx + c))^4 \left(-21\sqrt{2} \left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)} \right)^{\frac{5}{2}} (\cos^3(dx + c)) - 33 \left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)} \right)^{\frac{5}{2}} (\cos^2(dx + c)) \sqrt{2} - 23\sqrt{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\operatorname{int}((A+A*\sec(dx+c))/(a-a*\sec(dx+c))^{(5/2)}, x)$

[Out]
$$\begin{aligned} &1/12*A/d*(-1+\cos(dx+c))^4*(-21*2^{(1/2)}*(-2*\cos(dx+c)/(1+\cos(dx+c)))^{(5/2)} \\ &)*\cos(dx+c)^3-33*(-2*\cos(dx+c)/(1+\cos(dx+c)))^{(5/2)}*\cos(dx+c)^2*2^{(1/2)} \\ &-23*2^{(1/2)}*(-2*\cos(dx+c)/(1+\cos(dx+c)))^{(3/2)}*\cos(dx+c)^3-3*(-2*\cos(dx+c)/(1+\cos(dx+c)))^{(5/2)} \\ &*\cos(dx+c)*2^{(1/2)}+23*(-2*\cos(dx+c)/(1+\cos(dx+c)))^{(3/2)}*\cos(dx+c)^2*2^{(1/2)}+9*(-2*\cos(dx+c)/(1+\cos(dx+c)))^{(5/2)} \\ &*2^{(1/2)}+5*(-2*\cos(dx+c)/(1+\cos(dx+c)))^{(1/2)}*\cos(dx+c)^3*2^{(1/2)}+69*2^{(1/2)}*a \\ &\operatorname{rctan}(1/(-2*\cos(dx+c)/(1+\cos(dx+c)))^{(1/2)})*\cos(dx+c)^3+23*(-2*\cos(dx+c)/(1+\cos(dx+c)))^{(3/2)} \\ &*\cos(dx+c)*2^{(1/2)}+96*\operatorname{arctan}(1/2*(-2*\cos(dx+c)/(1+\cos(dx+c)))^{(1/2)})*2^{(1/2)})*\cos(dx+c)^3+11 \\ &*(-2*\cos(dx+c)/(1+\cos(dx+c)))^{(1/2)}*\cos(dx+c)^2*2^{(1/2)}-69*\operatorname{arctan}(1/(-2*\cos(dx+c)/(1+\cos(dx+c)))^{(1/2)} \\ &)*\cos(dx+c)^2*2^{(1/2)}-23*(-2*\cos(dx+c)/(1+\cos(dx+c)))^{(3/2)}*2^{(1/2)}-96*a \\ &\operatorname{rctan}(1/2*(-2*\cos(dx+c)/(1+\cos(dx+c)))^{(1/2)})*2^{(1/2)})*\cos(dx+c)^2-37*(-2*\cos(dx+c)/(1+\cos(dx+c)))^{(1/2)} \\ &*\cos(dx+c)*2^{(1/2)}-69*\operatorname{arctan}(1/(-2*\cos(dx+c)/(1+\cos(dx+c)))^{(1/2)})*\cos(dx+c)*2^{(1/2)}-96*\operatorname{arctan}(1/2*(-2*\cos(dx+c)/(1+\cos(dx+c)))^{(1/2)} \\ &)*2^{(1/2)})*\cos(dx+c)+21*(-2*\cos(dx+c)/(1+\cos(dx+c)))^{(1/2)}*2^{(1/2)}+69*\operatorname{arctan}(1/(-2*\cos(dx+c)/(1+\cos(dx+c)))^{(1/2)} \\ &)*2^{(1/2)}+96*\operatorname{arctan}(1/2*(-2*\cos(dx+c)/(1+\cos(dx+c)))^{(1/2)})*2^{(1/2)}))/a*(-1+\cos(dx+c))/\cos(dx+c)^{(5/2)}/\sin(dx+c)^7/(-2*\cos(dx+c)/(1+\cos(dx+c)))^{(5/2)}*2^{(1/2)} \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A \sec(dx + c) + A}{(-a \sec(dx + c) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(5/2),x, algorithm="maxima")

[Out] integrate((A*sec(d*x + c) + A)/(-a*sec(d*x + c) + a)^(5/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + \frac{A}{\cos(c+dx)}}{\left(a - \frac{a}{\cos(c+dx)}\right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + A/cos(c + d*x))/(a - a/cos(c + d*x))^(5/2),x)

[Out] int((A + A/cos(c + d*x))/(a - a/cos(c + d*x))^(5/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$A \left(\int \frac{\sec(c + dx)}{a^2 \sqrt{-a \sec(c + dx) + a} \sec^2(c + dx) - 2a^2 \sqrt{-a \sec(c + dx) + a} \sec(c + dx) + a^2 \sqrt{-a \sec(c + dx) + a}} dx - \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+A*sec(d*x+c))/(a-a*sec(d*x+c))**(5/2),x)

[Out] A*(Integral(sec(c + d*x)/(a**2*sqrt(-a*sec(c + d*x) + a)*sec(c + d*x)**2 - 2*a**2*sqrt(-a*sec(c + d*x) + a)*sec(c + d*x) + a**2*sqrt(-a*sec(c + d*x) + a)), x) + Integral(1/(a**2*sqrt(-a*sec(c + d*x) + a)*sec(c + d*x)**2 - 2*a**2*sqrt(-a*sec(c + d*x) + a)*sec(c + d*x) + a**2*sqrt(-a*sec(c + d*x) + a)), x))

$$3.176 \quad \int \frac{\cos(c+dx)(A+A \sec(c+dx))}{(a-a \sec(c+dx))^{5/2}} dx$$

Optimal. Leaf size=184

$$\frac{7A \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a-a \sec(c+dx)}}\right)}{a^{5/2}d} - \frac{79A \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{2} \sqrt{a-a \sec(c+dx)}}\right)}{8\sqrt{2} a^{5/2}d} + \frac{23A \sin(c+dx)}{8a^2d\sqrt{a-a \sec(c+dx)}} - \frac{11A \sin(c+dx)}{8ad(a-a \sec(c+dx))^{3/2}} - \frac{23A \sin(c+dx)}{8a^2d\sqrt{a-a \sec(c+dx)}} + \frac{79A \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{2} \sqrt{a-a \sec(c+dx)}}\right)}{8\sqrt{2} a^{5/2}d} - \frac{11A \sin(c+dx)}{8ad(a-a \sec(c+dx))^{3/2}} - \frac{23A \sin(c+dx)}{8a^2d\sqrt{a-a \sec(c+dx)}}$$

[Out] 7*A*arctan(a^(1/2)*tan(d*x+c)/(a-a*sec(d*x+c))^(1/2))/a^(5/2)/d-1/2*A*sin(d*x+c)/d/(a-a*sec(d*x+c))^(5/2)-11/8*A*sin(d*x+c)/a/d/(a-a*sec(d*x+c))^(3/2)-79/16*A*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a-a*sec(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2)+23/8*A*sin(d*x+c)/a^2/d/(a-a*sec(d*x+c))^(1/2)

Rubi [A] time = 0.51, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {4020, 4022, 3920, 3774, 203, 3795}

$$\frac{23A \sin(c+dx)}{8a^2d\sqrt{a-a \sec(c+dx)}} + \frac{7A \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a-a \sec(c+dx)}}\right)}{a^{5/2}d} - \frac{79A \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{2} \sqrt{a-a \sec(c+dx)}}\right)}{8\sqrt{2} a^{5/2}d} - \frac{11A \sin(c+dx)}{8ad(a-a \sec(c+dx))^{3/2}} - \frac{23A \sin(c+dx)}{8a^2d\sqrt{a-a \sec(c+dx)}} + \frac{7A \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a-a \sec(c+dx)}}\right)}{a^{5/2}d} - \frac{79A \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{2} \sqrt{a-a \sec(c+dx)}}\right)}{8\sqrt{2} a^{5/2}d} - \frac{11A \sin(c+dx)}{8ad(a-a \sec(c+dx))^{3/2}} - \frac{23A \sin(c+dx)}{8a^2d\sqrt{a-a \sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]*(A + A*Sec[c + d*x]))/(a - a*Sec[c + d*x])^(5/2), x]

[Out] (7*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a - a*Sec[c + d*x]]])/(a^(5/2)*d) - (79*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a - a*Sec[c + d*x]])])/(8*Sqrt[2]*a^(5/2)*d) - (A*Sin[c + d*x])/(2*d*(a - a*Sec[c + d*x])^(5/2)) - (11*A*Sin[c + d*x])/(8*a*d*(a - a*Sec[c + d*x])^(3/2)) + (23*A*Sin[c + d*x])/(8*a^2*d*Sqrt[a - a*Sec[c + d*x]])

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 3774

Int[Sqrt[csc[(c_.) + (d_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] := Dist[(-2*b)/d, Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] := Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] := Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^m*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] := -Simp[((A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m + 1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0]
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^m*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] := Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(c+dx)(A+A\sec(c+dx))}{(a-a\sec(c+dx))^{5/2}} dx &= -\frac{A\sin(c+dx)}{2d(a-a\sec(c+dx))^{5/2}} + \frac{\int \frac{\cos(c+dx)(6aA+5aA\sec(c+dx))}{(a-a\sec(c+dx))^{3/2}} dx}{4a^2} \\
&= -\frac{A\sin(c+dx)}{2d(a-a\sec(c+dx))^{5/2}} - \frac{11A\sin(c+dx)}{8ad(a-a\sec(c+dx))^{3/2}} + \frac{\int \frac{\cos(c+dx)(23a^2)}{\sqrt{a-a\sec(c+dx)}} dx}{8a^2d\sqrt{a-a\sec(c+dx)}} \\
&= -\frac{A\sin(c+dx)}{2d(a-a\sec(c+dx))^{5/2}} - \frac{11A\sin(c+dx)}{8ad(a-a\sec(c+dx))^{3/2}} + \frac{23A\sin(c+dx)}{8a^2d\sqrt{a-a\sec(c+dx)}} \\
&= -\frac{A\sin(c+dx)}{2d(a-a\sec(c+dx))^{5/2}} - \frac{11A\sin(c+dx)}{8ad(a-a\sec(c+dx))^{3/2}} + \frac{23A\sin(c+dx)}{8a^2d\sqrt{a-a\sec(c+dx)}} \\
&= -\frac{A\sin(c+dx)}{2d(a-a\sec(c+dx))^{5/2}} - \frac{11A\sin(c+dx)}{8ad(a-a\sec(c+dx))^{3/2}} + \frac{23A\sin(c+dx)}{8a^2d\sqrt{a-a\sec(c+dx)}} \\
&= \frac{7A \tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{a^{5/2}d} - \frac{79A \tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a-a\sec(c+dx)}}\right)}{8\sqrt{2}a^{5/2}d} - \frac{A\sin(c+dx)}{2d(a-a\sec(c+dx))^{5/2}}
\end{aligned}$$

Mathematica [C] time = 6.80, size = 423, normalized size = 2.30

$$A \left(\frac{\sin^5\left(\frac{c}{2} + \frac{dx}{2}\right) \sec^3(c+dx) \left(\frac{15\sin\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)}{d} - \frac{4\sin\left(\frac{3c}{2}\right)\sin\left(\frac{3dx}{2}\right)}{d} - \frac{15\cos\left(\frac{c}{2}\right)\cos\left(\frac{dx}{2}\right)}{d} + \frac{4\cos\left(\frac{3c}{2}\right)\cos\left(\frac{3dx}{2}\right)}{d} - \frac{\cot\left(\frac{c}{2}\right)\csc^3\left(\frac{c}{2} + \frac{dx}{2}\right)}{d} \right)}{(a-a\sec(c+dx))^{5/2}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]*(A + A*Sec[c + d*x]))/(a - a*Sec[c + d*x])^(5/2), x]

[Out] A*((Sqrt[E^(I*(c + d*x))]/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*(28*ArcSinh[E^(I*(c + d*x))] - (79*ArcTanh[(1 + E^(I*(c + d*x)))]/Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))]))]/Sqrt[2] + 28*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]]*Sec[c + d*x]^(5/2)*Sin[c/2 + (d*x)/2]^5)/(Sqrt[2]*d*E^((I/2)*(c + d*x))*(a - a*Sec[c + d*x])^(5/2)) + (Sec[c + d*x]^3*(-15*Cos[c/2]*Cos[(d*x)/2])/d + (4*Cos[(3*c)/2]*Cos[(3*d*x)/2])/d + (23*Cot[c/2]*Csc[c/2 + (d*x)/2])/(2*d) - (Cot[c/2]*Csc[c/2 + (d*x)/2]^3)/d - (23*Csc[c/2]*Csc[c/2 + (d*x)/2]^2*Sin[(d*x)/2])/(2*d) + (Csc[c/2]*Csc[c/2 + (d*x)/2]^4*Sin[(d*x)/2])/(2*d))

$x)/2)/d + (15*\text{Sin}[c/2]*\text{Sin}[(d*x)/2])/d - (4*\text{Sin}[(3*c)/2]*\text{Sin}[(3*d*x)/2])/d$
 $)*\text{Sin}[c/2 + (d*x)/2]^5/(a - a*\text{Sec}[c + d*x])^{(5/2)}$

fricas [A] time = 0.48, size = 612, normalized size = 3.33

$$\left[\frac{79 \sqrt{2} \left(A \cos(dx+c)^2 - 2 A \cos(dx+c) + A \right) \sqrt{-a} \log \left(\frac{2 \sqrt{2} (\cos(dx+c)^2 + \cos(dx+c)) \sqrt{-a} \sqrt{\frac{a \cos(dx+c) - a}{\cos(dx+c)}} + (3 a \cos(dx+c) + a)}{(\cos(dx+c) - 1) \sin(dx+c)} \right)}{\dots} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(5/2),x, algorithm="fricas")

[Out] $[-1/32*(79*\text{sqrt}(2)*(A*\cos(d*x + c)^2 - 2*A*\cos(d*x + c) + A)*\text{sqrt}(-a)*\log((2*\text{sqrt}(2)*(\cos(d*x + c)^2 + \cos(d*x + c))*\text{sqrt}(-a)*\text{sqrt}((a*\cos(d*x + c) - a)/\cos(d*x + c)) + (3*a*\cos(d*x + c) + a)*\sin(d*x + c))/((\cos(d*x + c) - 1)*\sin(d*x + c)))*\sin(d*x + c) + 112*(A*\cos(d*x + c)^2 - 2*A*\cos(d*x + c) + A)*\text{sqrt}(-a)*\log((2*(\cos(d*x + c)^2 + \cos(d*x + c))*\text{sqrt}(-a)*\text{sqrt}((a*\cos(d*x + c) - a)/\cos(d*x + c)) - (2*a*\cos(d*x + c) + a)*\sin(d*x + c))/\sin(d*x + c))*\sin(d*x + c) + 4*(8*A*\cos(d*x + c)^4 - 27*A*\cos(d*x + c)^3 - 12*A*\cos(d*x + c)^2 + 23*A*\cos(d*x + c))*\text{sqrt}((a*\cos(d*x + c) - a)/\cos(d*x + c)))/((a^3*d*\cos(d*x + c)^2 - 2*a^3*d*\cos(d*x + c) + a^3*d)*\sin(d*x + c)), 1/16*(79*\text{sqrt}(2)*(A*\cos(d*x + c)^2 - 2*A*\cos(d*x + c) + A)*\text{sqrt}(a)*\arctan(\text{sqrt}(2)*\text{sqrt}((a*\cos(d*x + c) - a)/\cos(d*x + c))*\cos(d*x + c)/(\text{sqrt}(a)*\sin(d*x + c)))*\sin(d*x + c) - 112*(A*\cos(d*x + c)^2 - 2*A*\cos(d*x + c) + A)*\text{sqrt}(a)*\arctan(\text{sqrt}((a*\cos(d*x + c) - a)/\cos(d*x + c))*\cos(d*x + c)/(\text{sqrt}(a)*\sin(d*x + c)))*\sin(d*x + c) - 2*(8*A*\cos(d*x + c)^4 - 27*A*\cos(d*x + c)^3 - 12*A*\cos(d*x + c)^2 + 23*A*\cos(d*x + c))*\text{sqrt}((a*\cos(d*x + c) - a)/\cos(d*x + c)))/((a^3*d*\cos(d*x + c)^2 - 2*a^3*d*\cos(d*x + c) + a^3*d)*\sin(d*x + c))]$

giac [A] time = 1.47, size = 295, normalized size = 1.60

$$\frac{79 \sqrt{2} A \arctan \left(\frac{\sqrt{a \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^2 - a}}{\sqrt{a}} \right)}{a^{\frac{5}{2}} \text{sgn} \left(\tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^2 - 1 \right) \text{sgn} \left(\tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) \right)} - \frac{112 A \arctan \left(\frac{\sqrt{2} \sqrt{a \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^2 - a}}{2 \sqrt{a}} \right)}{a^{\frac{5}{2}} \text{sgn} \left(\tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^2 - 1 \right) \text{sgn} \left(\tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) \right)} - \frac{16 \sqrt{2} \sqrt{a \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^2}}{\left(a \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^2 + a \right) a^2 \text{sgn} \left(\tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) \right)} \cdot 16d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(5/2),x, algorithm="giac")

[Out]
$$-1/16*(79*\sqrt{2})*A*\arctan(\sqrt{a*\tan(1/2*d*x + 1/2*c)^2 - a}/\sqrt{a})/(a^{5/2}*\operatorname{sgn}(\tan(1/2*d*x + 1/2*c)^2 - 1)*\operatorname{sgn}(\tan(1/2*d*x + 1/2*c))) - 112*A*\arctan(1/2*\sqrt{2}*\sqrt{a*\tan(1/2*d*x + 1/2*c)^2 - a}/\sqrt{a})/(a^{5/2}*\operatorname{sgn}(\tan(1/2*d*x + 1/2*c)^2 - 1)*\operatorname{sgn}(\tan(1/2*d*x + 1/2*c))) - 16*\sqrt{2}*\sqrt{a*\tan(1/2*d*x + 1/2*c)^2 - a}*A/((a*\tan(1/2*d*x + 1/2*c)^2 + a)*a^2*\operatorname{sgn}(\tan(1/2*d*x + 1/2*c)^2 - 1)*\operatorname{sgn}(\tan(1/2*d*x + 1/2*c))) - \sqrt{2}*(17*(a*\tan(1/2*d*x + 1/2*c)^2 - a)^{(3/2)}*A + 15*\sqrt{a*\tan(1/2*d*x + 1/2*c)^2 - a}*A*a)/(a^4*\operatorname{sgn}(\tan(1/2*d*x + 1/2*c)^2 - 1)*\operatorname{sgn}(\tan(1/2*d*x + 1/2*c))*\tan(1/2*d*x + 1/2*c)^4)/d$$

maple [B] time = 1.77, size = 788, normalized size = 4.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(5/2),x)

[Out]
$$\begin{aligned} & -1/60*A/d*(-1+\cos(d*x+c))^5*(195*2^{(1/2)}*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{(7/2)}*\cos(d*x+c)^4+450*2^{(1/2)}*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{(7/2)}*\cos(d*x+c)^3+237*2^{(1/2)}*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{(5/2)}*\cos(d*x+c)^4+180*2^{(1/2)}*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{(7/2)}*\cos(d*x+c)^2-210*2^{(1/2)}*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{(7/2)}*\cos(d*x+c)-395*2^{(1/2)}*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{(3/2)}*\cos(d*x+c)^4-474*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{(5/2)}*\cos(d*x+c)^2*2^{(1/2)}-135*2^{(1/2)}*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{(7/2)}+120*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*\cos(d*x+c)^5*2^{(1/2)}-343*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*\cos(d*x+c)^4*2^{(1/2)}+1185*2^{(1/2)}*\arctan(1/(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)})*\cos(d*x+c)^4+790*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{(3/2)}*\cos(d*x+c)^2*2^{(1/2)}+237*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{(5/2)}*2^{(1/2)}+1680*\arctan(1/2*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*2^{(1/2)})*\cos(d*x+c)^4+736*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*\cos(d*x+c)^3*2^{(1/2)}-578*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*\cos(d*x+c)^2*2^{(1/2)}-2370*\arctan(1/(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)})*\cos(d*x+c)^2*2^{(1/2)}-395*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{(3/2)}*2^{(1/2)}-3360*\arctan(1/2*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*2^{(1/2)})*\cos(d*x+c)^2-280*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*\cos(d*x+c)*2^{(1/2)}+345*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*2^{(1/2)}+1185*\arctan(1/(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*2^{(1/2)})*\cos(d*x+c)^2+1680*\arctan(1/2*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*2^{(1/2)})/(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{(5/2)}/(a*(-1+\cos(d*x+c))/\cos(d*x+c))^{(5/2)}/\sin(d*x+c)^9*2^{(1/2)} \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A \sec(dx + c) + A) \cos(dx + c)}{(-a \sec(dx + c) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(5/2),x, algorithm="maxima")

[Out] integrate((A*sec(d*x + c) + A)*cos(d*x + c)/(-a*sec(d*x + c) + a)^(5/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c + dx) \left(A + \frac{A}{\cos(c+dx)} \right)}{\left(a - \frac{a}{\cos(c+dx)} \right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)*(A + A/cos(c + d*x)))/(a - a/cos(c + d*x))^(5/2),x)

[Out] int((cos(c + d*x)*(A + A/cos(c + d*x)))/(a - a/cos(c + d*x))^(5/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))**(5/2),x)

[Out] Timed out

$$3.177 \quad \int \frac{\cos^2(c+dx)(A+A \sec(c+dx))}{(a-a \sec(c+dx))^{5/2}} dx$$

Optimal. Leaf size=236

$$\frac{59A \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a-a \sec(c+dx)}}\right)}{4a^{5/2}d} - \frac{167A \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{2} \sqrt{a-a \sec(c+dx)}}\right)}{8\sqrt{2} a^{5/2}d} + \frac{49A \sin(c+dx)}{8a^2d\sqrt{a-a \sec(c+dx)}} + \frac{23A \sin(c+dx) \cos(c+dx)}{8a^2d\sqrt{a-a \sec(c+dx)}}$$

[Out] $59/4*A*\arctan(a^{(1/2)}*\tan(d*x+c)/(a-a*\sec(d*x+c))^{(1/2)})/a^{(5/2)}/d-1/2*A*\cos(d*x+c)*\sin(d*x+c)/d/(a-a*\sec(d*x+c))^{(5/2)}-15/8*A*\cos(d*x+c)*\sin(d*x+c)/a/d/(a-a*\sec(d*x+c))^{(3/2)}-167/16*A*\arctan(1/2*a^{(1/2)}*\tan(d*x+c)*2^{(1/2)})/(a-a*\sec(d*x+c))^{(1/2)}/a^{(5/2)}/d*2^{(1/2)}+49/8*A*\sin(d*x+c)/a^2/d/(a-a*\sec(d*x+c))^{(1/2)}+23/8*A*\cos(d*x+c)*\sin(d*x+c)/a^2/d/(a-a*\sec(d*x+c))^{(1/2)}$

Rubi [A] time = 0.73, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {4020, 4022, 3920, 3774, 203, 3795}

$$\frac{49A \sin(c+dx)}{8a^2d\sqrt{a-a \sec(c+dx)}} + \frac{59A \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a-a \sec(c+dx)}}\right)}{4a^{5/2}d} - \frac{167A \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{2} \sqrt{a-a \sec(c+dx)}}\right)}{8\sqrt{2} a^{5/2}d} + \frac{23A \sin(c+dx) \cos(c+dx)}{8a^2d\sqrt{a-a \sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(\text{Cos}[c+d*x]^2*(A+A*\text{Sec}[c+d*x]))/(a-a*\text{Sec}[c+d*x])^{(5/2)},x]$

[Out] $(59*A*\text{ArcTan}[(\text{Sqrt}[a]*\text{Tan}[c+d*x])/(\text{Sqrt}[a-a*\text{Sec}[c+d*x]])]/(4*a^{(5/2)*d}) - (167*A*\text{ArcTan}[(\text{Sqrt}[a]*\text{Tan}[c+d*x])/(\text{Sqrt}[2]*\text{Sqrt}[a-a*\text{Sec}[c+d*x]])])/(8*\text{Sqrt}[2]*a^{(5/2)*d}) - (A*\text{Cos}[c+d*x]*\text{Sin}[c+d*x])/(2*d*(a-a*\text{Sec}[c+d*x])^{(5/2)}) - (15*A*\text{Cos}[c+d*x]*\text{Sin}[c+d*x])/(8*a*d*(a-a*\text{Sec}[c+d*x])^{(3/2)}) + (49*A*\text{Sin}[c+d*x])/(8*a^2*d*\text{Sqrt}[a-a*\text{Sec}[c+d*x]]) + (23*A*\text{Cos}[c+d*x]*\text{Sin}[c+d*x])/(8*a^2*d*\text{Sqrt}[a-a*\text{Sec}[c+d*x]])$

Rule 203

$\text{Int}[(a_+ + (b_+)*(x_+)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1*\text{ArcTan}[(\text{Rt}[b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[b, 2]), x] /; \text{FreeQ}\{a, b, x\} \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{GtQ}[b, 0])$

Rule 3774

$\text{Int}[\text{Sqrt}[\text{csc}[(c_+ + (d_+)*(x_+)]*(b_+ + (a_+))], x_Symbol] \rightarrow \text{Dist}[(-2*b)/d, \text{Subst}[\text{Int}[1/(a+x^2), x], x, (b*\text{Cot}[c+d*x])/(\text{Sqrt}[a+b*\text{Csc}[c+d*x]])], x] /; \text{FreeQ}\{a, b, c, d, x\} \ \&\& \ \text{EqQ}[a^2 - b^2, 0]$

Rule 3795

```
Int[(csc[(e_.) + (f_.)*(x_)])/Sqrt[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))], x_Symbol]
:> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x]
/; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_)*(d_.) + (c_.)]/Sqrt[(csc[(e_.) + (f_.)*(x_)*(b_.) + (a_))], x_Symbol]
:> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)*(d_.)]^(n_)*(csc[(e_.) + (f_.)*(x_)*(b_.) + (a_)]^(m_)*(csc[(e_.) + (f_.)*(x_)*(B_.) + (A_.)]), x_Symbol]
:> -Simp[(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n/(b*f*(2*m + 1)), x]
- Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e + f*x], x], x], x]
/; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0]
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)*(d_.)]^(n_)*(csc[(e_.) + (f_.)*(x_)*(b_.) + (a_)]^(m_)*(csc[(e_.) + (f_.)*(x_)*(B_.) + (A_.)]), x_Symbol]
:> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n - A*b*(m + n + 1)*Csc[e + f*x], x], x], x]
/; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(c+dx)(A+A\sec(c+dx))}{(a-a\sec(c+dx))^{5/2}} dx &= -\frac{A\cos(c+dx)\sin(c+dx)}{2d(a-a\sec(c+dx))^{5/2}} + \frac{\int \frac{\cos^2(c+dx)(8aA+7aA\sec(c+dx))}{(a-a\sec(c+dx))^{3/2}} dx}{4a^2} \\
&= -\frac{A\cos(c+dx)\sin(c+dx)}{2d(a-a\sec(c+dx))^{5/2}} - \frac{15A\cos(c+dx)\sin(c+dx)}{8ad(a-a\sec(c+dx))^{3/2}} + \frac{\int \frac{\cos^2(c+dx)(8aA+7aA\sec(c+dx))}{(a-a\sec(c+dx))^{3/2}} dx}{8a^2d\sqrt{a}} \\
&= -\frac{A\cos(c+dx)\sin(c+dx)}{2d(a-a\sec(c+dx))^{5/2}} - \frac{15A\cos(c+dx)\sin(c+dx)}{8ad(a-a\sec(c+dx))^{3/2}} + \frac{23A\cos(c+dx)\sin(c+dx)}{8a^2d\sqrt{a}} \\
&= -\frac{A\cos(c+dx)\sin(c+dx)}{2d(a-a\sec(c+dx))^{5/2}} - \frac{15A\cos(c+dx)\sin(c+dx)}{8ad(a-a\sec(c+dx))^{3/2}} + \frac{49A\cos(c+dx)\sin(c+dx)}{8a^2d\sqrt{a}} \\
&= -\frac{A\cos(c+dx)\sin(c+dx)}{2d(a-a\sec(c+dx))^{5/2}} - \frac{15A\cos(c+dx)\sin(c+dx)}{8ad(a-a\sec(c+dx))^{3/2}} + \frac{49A\cos(c+dx)\sin(c+dx)}{8a^2d\sqrt{a}} \\
&= -\frac{A\cos(c+dx)\sin(c+dx)}{2d(a-a\sec(c+dx))^{5/2}} - \frac{15A\cos(c+dx)\sin(c+dx)}{8ad(a-a\sec(c+dx))^{3/2}} + \frac{49A\cos(c+dx)\sin(c+dx)}{8a^2d\sqrt{a}} \\
&= \frac{59A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{4a^{5/2}d} - \frac{167A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a-a\sec(c+dx)}}\right)}{8\sqrt{2}a^{5/2}d} - \frac{A\cos(c+dx)\sin(c+dx)}{2d(a-a\sec(c+dx))^{5/2}}
\end{aligned}$$

Mathematica [C] time = 6.82, size = 458, normalized size = 1.94

$$A \left(\frac{\sin^5\left(\frac{c}{2} + \frac{dx}{2}\right) \sec^3(c+dx) \left(\frac{12\sin\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)}{d} - \frac{14\sin\left(\frac{3c}{2}\right)\sin\left(\frac{3dx}{2}\right)}{d} - \frac{\sin\left(\frac{5c}{2}\right)\sin\left(\frac{5dx}{2}\right)}{d} - \frac{12\cos\left(\frac{c}{2}\right)\cos\left(\frac{dx}{2}\right)}{d} + \frac{14\cos\left(\frac{3c}{2}\right)\cos\left(\frac{3dx}{2}\right)}{d} \right)}{(a-a\sec(c+dx))^{5/2}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]^2*(A + A*Sec[c + d*x]))/(a - a*Sec[c + d*x])^(5/2), x]

[Out] A*((Sqrt[E^(I*(c + d*x))]/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))])*(59*ArcSinh[E^(I*(c + d*x))] - (167*ArcTanh[(1 + E^(I*(c + d*x)))]/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))]]))/Sqrt[2] + 59*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]])*Sec[c + d*x]^(5/2)*Sin[c/2 + (d*x)/2]^5)/(Sqrt[2]*d*E^(I*(c + d*x)))

$$\begin{aligned} & /2)*(c + d*x))*(a - a*\text{Sec}[c + d*x])^{(5/2)} + (\text{Sec}[c + d*x]^3*((-12*\text{Cos}[c/2] \\ & * \text{Cos}[(d*x)/2])/d + (14*\text{Cos}[(3*c)/2]*\text{Cos}[(3*d*x)/2])/d + (\text{Cos}[(5*c)/2]*\text{Cos}[(5*d*x)/2])/d \\ & + (31*\text{Cot}[c/2]*\text{Csc}[c/2 + (d*x)/2])/(2*d) - (\text{Cot}[c/2]*\text{Csc}[c/2 + (d*x)/2]^3)/d \\ & - (31*\text{Csc}[c/2]*\text{Csc}[c/2 + (d*x)/2]^2*\text{Sin}[(d*x)/2])/(2*d) + (\text{Csc}[c/2]*\text{Csc}[c/2 + (d*x)/2]^4*\text{Sin}[(d*x)/2])/d \\ & + (12*\text{Sin}[c/2]*\text{Sin}[(d*x)/2])/d - (14*\text{Sin}[(3*c)/2]*\text{Sin}[(3*d*x)/2])/d - (\text{Sin}[(5*c)/2]*\text{Sin}[(5*d*x)/2])/d \\ & * \text{Sin}[c/2 + (d*x)/2]^5)/(a - a*\text{Sec}[c + d*x])^{(5/2)} \end{aligned}$$

fricas [A] time = 0.49, size = 634, normalized size = 2.69

$$\left[\frac{167\sqrt{2}\left(A\cos(dx+c)^2 - 2A\cos(dx+c) + A\right)\sqrt{-a}\log\left(\frac{2\sqrt{2}\left(\cos(dx+c)^2 + \cos(dx+c)\right)\sqrt{-a}\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}} + (3a\cos(dx+c)+a)}}{(\cos(dx+c)-1)\sin(dx+c)}\right)}{\dots} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(5/2),x, algorithm="fricas")

[Out] [-1/32*(167*sqrt(2)*(A*cos(d*x + c)^2 - 2*A*cos(d*x + c) + A)*sqrt(-a)*log((2*sqrt(2)*(cos(d*x + c)^2 + cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) - a)/cos(d*x + c)) + (3*a*cos(d*x + c) + a)*sin(d*x + c))/((cos(d*x + c) - 1)*sin(d*x + c)))*sin(d*x + c) + 236*(A*cos(d*x + c)^2 - 2*A*cos(d*x + c) + A)*sqrt(-a)*log((2*(cos(d*x + c)^2 + cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) - a)/cos(d*x + c)) - (2*a*cos(d*x + c) + a)*sin(d*x + c))/sin(d*x + c))*sin(d*x + c) + 4*(4*A*cos(d*x + c)^5 + 22*A*cos(d*x + c)^4 - 57*A*cos(d*x + c)^3 - 26*A*cos(d*x + c)^2 + 49*A*cos(d*x + c))*sqrt((a*cos(d*x + c) - a)/cos(d*x + c)))/((a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) + a^3*d)*sin(d*x + c)), 1/16*(167*sqrt(2)*(A*cos(d*x + c)^2 - 2*A*cos(d*x + c) + A)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) - a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))*sin(d*x + c) - 236*(A*cos(d*x + c)^2 - 2*A*cos(d*x + c) + A)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) - a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))*sin(d*x + c) - 2*(4*A*cos(d*x + c)^5 + 22*A*cos(d*x + c)^4 - 57*A*cos(d*x + c)^3 - 26*A*cos(d*x + c)^2 + 49*A*cos(d*x + c))*sqrt((a*cos(d*x + c) - a)/cos(d*x + c)))/((a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) + a^3*d)*sin(d*x + c))]

giac [A] time = 2.92, size = 308, normalized size = 1.31

$$\frac{167\sqrt{2}A \arctan\left(\frac{\sqrt{a \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - a}}{\sqrt{a}}\right)}{a^{\frac{5}{2}} \operatorname{sgn}\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - 1\right) \operatorname{sgn}\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right)} - \frac{236A \arctan\left(\frac{\sqrt{2} \sqrt{a \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - a}}{2\sqrt{a}}\right)}{a^{\frac{5}{2}} \operatorname{sgn}\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - 1\right) \operatorname{sgn}\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right)} - \frac{\sqrt{2} \left(69 \left(a \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - a\right)^{\frac{7}{2}} A + 315 \left(a \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - a\right)^{\frac{5}{2}} A^2 + 444 \left(a \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - a\right)^{\frac{3}{2}} A^3 + 196 \sqrt{a \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - a} A^4 + 2a^2 \operatorname{sgn}\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - 1\right) \operatorname{sgn}\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right)\right)}{\left(\left(a \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - a\right)^2 + 3a \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right)^2} \cdot 16d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(5/2),x, algorithm="giac")

[Out]
$$-1/16*(167*\sqrt{2}*A*\arctan(\sqrt{a*\tan(1/2*d*x + 1/2*c)^2 - a}/\sqrt{a}))/a^{(5/2)}*\operatorname{sgn}(\tan(1/2*d*x + 1/2*c)^2 - 1)*\operatorname{sgn}(\tan(1/2*d*x + 1/2*c)) - 236*A*\arctan(1/2*\sqrt{2}*\sqrt{a*\tan(1/2*d*x + 1/2*c)^2 - a}/\sqrt{a}))/a^{(5/2)}*\operatorname{sgn}(\tan(1/2*d*x + 1/2*c)^2 - 1)*\operatorname{sgn}(\tan(1/2*d*x + 1/2*c)) - \sqrt{2}*(69*(a*\tan(1/2*d*x + 1/2*c)^2 - a)^{(7/2)}*A + 315*(a*\tan(1/2*d*x + 1/2*c)^2 - a)^{(5/2)}*A^2 + 444*(a*\tan(1/2*d*x + 1/2*c)^2 - a)^{(3/2)}*A^3 + 196*\sqrt{a*\tan(1/2*d*x + 1/2*c)^2 - a}*A^4 + 2*a^2*\operatorname{sgn}(\tan(1/2*d*x + 1/2*c)^2 - 1)*\operatorname{sgn}(\tan(1/2*d*x + 1/2*c)))/d$$

maple [B] time = 1.89, size = 1475, normalized size = 6.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^2*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(5/2),x)

[Out]
$$-1/420*A/d*(-1+\cos(d*x+c))^6*(5845*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{(3/2)}*\cos(d*x+c)^{2^{(1/2)}}-17535*2^{(1/2)}*\arctan(1/(-2*\cos(d*x+c)/(1+\cos(d*x+c))))^{(1/2)})*\cos(d*x+c)^5+1995*2^{(1/2)}*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{(9/2)}*\cos(d*x+c)^5+2505*2^{(1/2)}*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{(7/2)}*\cos(d*x+c)^5-3507*2^{(1/2)}*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{(5/2)}*\cos(d*x+c)^5+2505*2^{(1/2)}*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{(7/2)}*\cos(d*x+c)^4-4305*2^{(1/2)}*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{(9/2)}*\cos(d*x+c)-3507*2^{(1/2)}*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{(5/2)}*\cos(d*x+c)^4+5845*2^{(1/2)}*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{(3/2)}*\cos(d*x+c)^4-17535*2^{(1/2)}*\arctan(1/(-2*\cos(d*x+c)/(1+\cos(d*x+c))))^{(1/2)}*\cos(d*x+c)^4-420*2^{(1/2)}*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*\cos(d*x+c)^7-3570*2^{(1/2)}*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*\cos(d*x+c)^6-24780*\arctan(1/2*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)})*2^{(1/2)}-5010*2^{(1/2)}*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{(7/2)}*\cos(d*x+c)^3-5010*2^{(1/2)}*(-2*\cos(d*x+c)/(1+\cos(d*x+c)))^{(7/2)}$$

$$\begin{aligned} & /2) * \cos(dx+c)^2 + 2505 * 2^{1/2} * (-2 * \cos(dx+c) / (1 + \cos(dx+c)))^{7/2} * \cos(dx+c) \\ & + 7014 * 2^{1/2} * (-2 * \cos(dx+c) / (1 + \cos(dx+c)))^{5/2} * \cos(dx+c)^3 - 11690 * 2^{1/2} * \\ & (-2 * \cos(dx+c) / (1 + \cos(dx+c)))^{3/2} * \cos(dx+c)^3 + 35070 * 2^{1/2} * \arctan \\ & (1 / (-2 * \cos(dx+c) / (1 + \cos(dx+c))))^{1/2} * \cos(dx+c)^3 - 5145 * (-2 * \cos(dx+c) / \\ & (1 + \cos(dx+c)))^{1/2} * 2^{1/2} + 7014 * (-2 * \cos(dx+c) / (1 + \cos(dx+c)))^{5/2} * \cos \\ & (dx+c)^2 * 2^{1/2} - 17535 * \arctan(1 / (-2 * \cos(dx+c) / (1 + \cos(dx+c))))^{1/2} * 2^{1/2} \\ & + 5845 * 2^{1/2} * (-2 * \cos(dx+c) / (1 + \cos(dx+c)))^{3/2} * \cos(dx+c)^5 + 1322 * (-2 * \\ & \cos(dx+c) / (1 + \cos(dx+c)))^{1/2} * \cos(dx+c)^3 * 2^{1/2} + 12768 * (-2 * \cos(dx+c) / \\ & (1 + \cos(dx+c)))^{1/2} * \cos(dx+c)^2 * 2^{1/2} - 17535 * \arctan(1 / (-2 * \cos(dx+c) / (1 \\ & + \cos(dx+c)))^{1/2} * \cos(dx+c) * 2^{1/2} - 1015 * (-2 * \cos(dx+c) / (1 + \cos(dx+c))) \\ & ^{1/2} * \cos(dx+c) * 2^{1/2} - 24780 * \arctan(1/2 * (-2 * \cos(dx+c) / (1 + \cos(dx+c)))^{1/2} * \\ & 2^{1/2}) * \cos(dx+c)^5 + 6405 * 2^{1/2} * (-2 * \cos(dx+c) / (1 + \cos(dx+c)))^{9/2} \\ & * \cos(dx+c)^4 + 5670 * 2^{1/2} * (-2 * \cos(dx+c) / (1 + \cos(dx+c)))^{9/2} * \cos(dx+c) \\ & ^3 - 1470 * 2^{1/2} * (-2 * \cos(dx+c) / (1 + \cos(dx+c)))^{9/2} * \cos(dx+c)^2 - 3507 * (-2 * \\ & \cos(dx+c) / (1 + \cos(dx+c)))^{5/2} * \cos(dx+c) * 2^{1/2} + 11633 * (-2 * \cos(dx+c) / (1 \\ & + \cos(dx+c)))^{1/2} * \cos(dx+c)^5 * 2^{1/2} - 11690 * (-2 * \cos(dx+c) / (1 + \cos(dx+c) \\ &))^{3/2} * \cos(dx+c)^2 * 2^{1/2} - 15573 * (-2 * \cos(dx+c) / (1 + \cos(dx+c)))^{1/2} * \cos \\ & (dx+c)^4 * 2^{1/2} + 35070 * \arctan(1 / (-2 * \cos(dx+c) / (1 + \cos(dx+c)))^{1/2} * \cos \\ & (dx+c)^2 * 2^{1/2} - 1575 * 2^{1/2} * (-2 * \cos(dx+c) / (1 + \cos(dx+c)))^{9/2} - 24780 * \arctan \\ & (1/2 * (-2 * \cos(dx+c) / (1 + \cos(dx+c)))^{1/2} * 2^{1/2}) * \cos(dx+c)^4 - 24780 * \\ & \arctan(1/2 * (-2 * \cos(dx+c) / (1 + \cos(dx+c)))^{1/2} * 2^{1/2}) * \cos(dx+c) - 3507 * (- \\ & 2 * \cos(dx+c) / (1 + \cos(dx+c)))^{5/2} * 2^{1/2} + 49560 * \arctan(1/2 * (-2 * \cos(dx+c) / \\ & (1 + \cos(dx+c)))^{1/2} * 2^{1/2}) * \cos(dx+c)^2 + 5845 * (-2 * \cos(dx+c) / (1 + \cos(dx+c) \\ &))^{3/2} * 2^{1/2} + 49560 * \arctan(1/2 * (-2 * \cos(dx+c) / (1 + \cos(dx+c)))^{1/2} * 2^{1/2}) * \\ & \cos(dx+c)^3 + 2505 * 2^{1/2} * (-2 * \cos(dx+c) / (1 + \cos(dx+c)))^{7/2} / (-2 * \cos(dx+c) / \\ & (1 + \cos(dx+c)))^{5/2} / (a * (-1 + \cos(dx+c)) / \cos(dx+c))^{5/2} / \sin(dx+c)^{11} * 2^{1/2} \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A \sec(dx+c) + A) \cos(dx+c)^2}{(-a \sec(dx+c) + a)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(dx+c)^2*(A+A*sec(dx+c))/(a-a*sec(dx+c))^(5/2),x, algorithm="maxima")

[Out] integrate((A*sec(dx+c)+A)*cos(dx+c)^2/(-a*sec(dx+c)+a)^(5/2),x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+dx)^2 \left(A + \frac{A}{\cos(c+dx)} \right)}{\left(a - \frac{a}{\cos(c+dx)} \right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((cos(c + d*x)^2*(A + A/cos(c + d*x)))/(a - a/cos(c + d*x))^(5/2), x)`

[Out] `int((cos(c + d*x)^2*(A + A/cos(c + d*x)))/(a - a/cos(c + d*x))^(5/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$A \left(\int \frac{\cos^2(c + dx)}{a^2 \sqrt{-a \sec(c + dx) + a} \sec^2(c + dx) - 2a^2 \sqrt{-a \sec(c + dx) + a} \sec(c + dx) + a^2 \sqrt{-a \sec(c + dx) + a}} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)**2*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))**(5/2), x)`

[Out] `A*(Integral(cos(c + d*x)**2/(a**2*sqrt(-a*sec(c + d*x) + a)*sec(c + d*x)**2 - 2*a**2*sqrt(-a*sec(c + d*x) + a)*sec(c + d*x) + a**2*sqrt(-a*sec(c + d*x) + a)), x) + Integral(cos(c + d*x)**2*sec(c + d*x)/(a**2*sqrt(-a*sec(c + d*x) + a)*sec(c + d*x)**2 - 2*a**2*sqrt(-a*sec(c + d*x) + a)*sec(c + d*x) + a**2*sqrt(-a*sec(c + d*x) + a)), x))`

$$3.178 \quad \int \frac{\cos^3(c+dx)(A+A \sec(c+dx))}{(a-a \sec(c+dx))^{5/2}} dx$$

Optimal. Leaf size=280

$$\frac{203A \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a-a \sec(c+dx)}}\right)}{8a^{5/2}d} - \frac{287A \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{2} \sqrt{a-a \sec(c+dx)}}\right)}{8\sqrt{2} a^{5/2}d} + \frac{21A \sin(c+dx)}{2a^2d\sqrt{a-a \sec(c+dx)}} + \frac{77A \sin(c+dx) \cos^2(c+dx)}{24a^2d\sqrt{a-a \sec(c+dx)}}$$

[Out] 203/8*A*arctan(a^(1/2)*tan(d*x+c)/(a-a*sec(d*x+c))^(1/2))/a^(5/2)/d-1/2*A*cos(d*x+c)^2*sin(d*x+c)/d/(a-a*sec(d*x+c))^(5/2)-19/8*A*cos(d*x+c)^2*sin(d*x+c)/a/d/(a-a*sec(d*x+c))^(3/2)-287/16*A*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a-a*sec(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2)+21/2*A*sin(d*x+c)/a^2/d/(a-a*sec(d*x+c))^(1/2)+119/24*A*cos(d*x+c)*sin(d*x+c)/a^2/d/(a-a*sec(d*x+c))^(1/2)+77/24*A*cos(d*x+c)^2*sin(d*x+c)/a^2/d/(a-a*sec(d*x+c))^(1/2)

Rubi [A] time = 0.91, antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {4020, 4022, 3920, 3774, 203, 3795}

$$\frac{21A \sin(c+dx)}{2a^2d\sqrt{a-a \sec(c+dx)}} + \frac{203A \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a-a \sec(c+dx)}}\right)}{8a^{5/2}d} - \frac{287A \tan^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{2} \sqrt{a-a \sec(c+dx)}}\right)}{8\sqrt{2} a^{5/2}d} + \frac{77A \sin(c+dx) \cos^2(c+dx)}{24a^2d\sqrt{a-a \sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^3*(A + A*Sec[c + d*x]))/(a - a*Sec[c + d*x])^(5/2), x]

[Out] (203*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a - a*Sec[c + d*x]])/(8*a^(5/2)*d) - (287*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a - a*Sec[c + d*x]])])/(8*Sqrt[2]*a^(5/2)*d) - (A*Cos[c + d*x]^2*Sin[c + d*x])/(2*d*(a - a*Sec[c + d*x])^(5/2)) - (19*A*Cos[c + d*x]^2*Sin[c + d*x])/(8*a*d*(a - a*Sec[c + d*x])^(3/2)) + (21*A*Sin[c + d*x])/(2*a^2*d*Sqrt[a - a*Sec[c + d*x]]) + (119*A*Cos[c + d*x]*Sin[c + d*x])/(24*a^2*d*Sqrt[a - a*Sec[c + d*x]]) + (77*A*Cos[c + d*x]^2*Sin[c + d*x])/(24*a^2*d*Sqrt[a - a*Sec[c + d*x]])

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 3774

Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d, Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],

$x] /; \text{FreeQ}\{a, b, c, d\}, x\} \&\& \text{EqQ}[a^2 - b^2, 0]$

Rule 3795

$\text{Int}[\text{csc}[(e_.) + (f_.)(x_)]/\text{Sqrt}[\text{csc}[(e_.) + (f_.)(x_)]*(b_.) + (a_.)], x_Symbol] \rightarrow \text{Dist}[-2/f, \text{Subst}[\text{Int}[1/(2*a + x^2), x], x, (b*\text{Cot}[e + f*x])/\text{Sqrt}[a + b*\text{Csc}[e + f*x]]], x] /; \text{FreeQ}\{a, b, e, f\}, x\} \&\& \text{EqQ}[a^2 - b^2, 0]$

Rule 3920

$\text{Int}[(\text{csc}[(e_.) + (f_.)(x_)]*(d_.) + (c_.))/\text{Sqrt}[\text{csc}[(e_.) + (f_.)(x_)]*(b_.) + (a_.)], x_Symbol] \rightarrow \text{Dist}[c/a, \text{Int}[\text{Sqrt}[a + b*\text{Csc}[e + f*x]], x], x] - \text{Dist}[(b*c - a*d)/a, \text{Int}[\text{Csc}[e + f*x]/\text{Sqrt}[a + b*\text{Csc}[e + f*x]], x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x\} \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0]$

Rule 4020

$\text{Int}[(\text{csc}[(e_.) + (f_.)(x_)]*(d_.))^n*(\text{csc}[(e_.) + (f_.)(x_)]*(b_.) + (a_.))^m*(\text{csc}[(e_.) + (f_.)(x_)]*(B_.) + (A_.)), x_Symbol] \rightarrow -\text{Simp}[(A*b - a*B)*\text{Cot}[e + f*x]*(a + b*\text{Csc}[e + f*x])^m*(d*\text{Csc}[e + f*x])^n/(b*f*(2*m + 1)), x] - \text{Dist}[1/(a^2*(2*m + 1)), \text{Int}[(a + b*\text{Csc}[e + f*x])^{m+1}*(d*\text{Csc}[e + f*x])^n*\text{Simp}[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*\text{Csc}[e + f*x], x], x], x] /; \text{FreeQ}\{a, b, d, e, f, A, B, n\}, x\} \&\& \text{NeQ}[A*b - a*B, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{LtQ}[m, -2^{(-1)}] \&\& !\text{GtQ}[n, 0]$

Rule 4022

$\text{Int}[(\text{csc}[(e_.) + (f_.)(x_)]*(d_.))^n*(\text{csc}[(e_.) + (f_.)(x_)]*(b_.) + (a_.))^m*(\text{csc}[(e_.) + (f_.)(x_)]*(B_.) + (A_.)), x_Symbol] \rightarrow \text{Simp}[(A*\text{Cot}[e + f*x]*(a + b*\text{Csc}[e + f*x])^m*(d*\text{Csc}[e + f*x])^n)/(f*n), x] - \text{Dist}[1/(b*d*n), \text{Int}[(a + b*\text{Csc}[e + f*x])^m*(d*\text{Csc}[e + f*x])^{n+1}*\text{Simp}[a*A*m - b*B*n - A*b*(m + n + 1)*\text{Csc}[e + f*x], x], x], x] /; \text{FreeQ}\{a, b, d, e, f, A, B, m\}, x\} \&\& \text{NeQ}[A*b - a*B, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{LtQ}[n, 0]$

Rubi steps

$$\begin{aligned}
\int \frac{\cos^3(c+dx)(A+A\sec(c+dx))}{(a-a\sec(c+dx))^{5/2}} dx &= -\frac{A\cos^2(c+dx)\sin(c+dx)}{2d(a-a\sec(c+dx))^{5/2}} + \frac{\int \frac{\cos^3(c+dx)(10aA+9aA\sec(c+dx))}{(a-a\sec(c+dx))^{3/2}} dx}{4a^2} \\
&= -\frac{A\cos^2(c+dx)\sin(c+dx)}{2d(a-a\sec(c+dx))^{5/2}} - \frac{19A\cos^2(c+dx)\sin(c+dx)}{8ad(a-a\sec(c+dx))^{3/2}} + \frac{\int \frac{\cos^3(c+dx)(10aA+9aA\sec(c+dx))}{(a-a\sec(c+dx))^{3/2}} dx}{4a^2} \\
&= -\frac{A\cos^2(c+dx)\sin(c+dx)}{2d(a-a\sec(c+dx))^{5/2}} - \frac{19A\cos^2(c+dx)\sin(c+dx)}{8ad(a-a\sec(c+dx))^{3/2}} + \frac{77A\cos^2(c+dx)\sin(c+dx)}{24a^2d} \\
&= -\frac{A\cos^2(c+dx)\sin(c+dx)}{2d(a-a\sec(c+dx))^{5/2}} - \frac{19A\cos^2(c+dx)\sin(c+dx)}{8ad(a-a\sec(c+dx))^{3/2}} + \frac{119A\cos^2(c+dx)\sin(c+dx)}{24a^2d} \\
&= -\frac{A\cos^2(c+dx)\sin(c+dx)}{2d(a-a\sec(c+dx))^{5/2}} - \frac{19A\cos^2(c+dx)\sin(c+dx)}{8ad(a-a\sec(c+dx))^{3/2}} + \frac{21A\cos^2(c+dx)\sin(c+dx)}{2a^2d\sqrt{a}} \\
&= -\frac{A\cos^2(c+dx)\sin(c+dx)}{2d(a-a\sec(c+dx))^{5/2}} - \frac{19A\cos^2(c+dx)\sin(c+dx)}{8ad(a-a\sec(c+dx))^{3/2}} + \frac{21A\cos^2(c+dx)\sin(c+dx)}{2a^2d\sqrt{a}} \\
&= -\frac{A\cos^2(c+dx)\sin(c+dx)}{2d(a-a\sec(c+dx))^{5/2}} - \frac{19A\cos^2(c+dx)\sin(c+dx)}{8ad(a-a\sec(c+dx))^{3/2}} + \frac{21A\cos^2(c+dx)\sin(c+dx)}{2a^2d\sqrt{a}} \\
&= \frac{203A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{8a^{5/2}d} - \frac{287A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a-a\sec(c+dx)}}\right)}{8\sqrt{2}a^{5/2}d} - \frac{A\cos^2(c+dx)\sin(c+dx)}{2d(a-a\sec(c+dx))^{5/2}}
\end{aligned}$$

Mathematica [C] time = 6.84, size = 514, normalized size = 1.84

$$A \left[\frac{\sin^5\left(\frac{c}{2} + \frac{dx}{2}\right) \sec^3(c+dx) \left(\frac{7\sin\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)}{6d} - \frac{92\sin\left(\frac{3c}{2}\right)\sin\left(\frac{3dx}{2}\right)}{3d} - \frac{7\sin\left(\frac{5c}{2}\right)\sin\left(\frac{5dx}{2}\right)}{2d} - \frac{\sin\left(\frac{7c}{2}\right)\sin\left(\frac{7dx}{2}\right)}{3d} - \frac{7\cos\left(\frac{c}{2}\right)\cos\left(\frac{dx}{2}\right)}{6d} \right)}{\right]}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]^3*(A + A*Sec[c + d*x]))/(a - a*Sec[c + d*x])^(5/2), x]

[Out] A*((7*Sqrt[E^(I*(c + d*x))]/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))])*(29*ArcSinh[E^(I*(c + d*x))] - 41*Sqrt[2]*ArcTanh[(1 + E^(I*(c + d*x)))]

$x)))/(\text{Sqrt}[2]*\text{Sqrt}[1 + E^{((2*I)*(c + d*x))}]] + 29*\text{ArcTanh}[\text{Sqrt}[1 + E^{((2*I)*(c + d*x))}]]*\text{Sec}[c + d*x]^{(5/2)}*\text{Sin}[c/2 + (d*x)/2]^5)/(2*\text{Sqrt}[2]*d*E^{((I/2)*(c + d*x))}*(a - a*\text{Sec}[c + d*x])^{(5/2)}) + (\text{Sec}[c + d*x]^3*((-7*\text{Cos}[c/2]*\text{Cos}[(d*x)/2])/(6*d) + (92*\text{Cos}[(3*c)/2]*\text{Cos}[(3*d*x)/2])/(3*d) + (7*\text{Cos}[(5*c)/2]*\text{Cos}[(5*d*x)/2])/(2*d) + (\text{Cos}[(7*c)/2]*\text{Cos}[(7*d*x)/2])/(3*d) + (39*\text{Cot}[c/2]*\text{Csc}[c/2 + (d*x)/2])/(2*d) - (\text{Cot}[c/2]*\text{Csc}[c/2 + (d*x)/2]^3)/d - (39*\text{Csc}[c/2]*\text{Csc}[c/2 + (d*x)/2]^2*\text{Sin}[(d*x)/2])/(2*d) + (\text{Csc}[c/2]*\text{Csc}[c/2 + (d*x)/2]^4*\text{Sin}[(d*x)/2])/d + (7*\text{Sin}[c/2]*\text{Sin}[(d*x)/2])/(6*d) - (92*\text{Sin}[(3*c)/2]*\text{Sin}[(3*d*x)/2])/(3*d) - (7*\text{Sin}[(5*c)/2]*\text{Sin}[(5*d*x)/2])/(2*d) - (\text{Sin}[(7*c)/2]*\text{Sin}[(7*d*x)/2])/(3*d))*\text{Sin}[c/2 + (d*x)/2]^5)/(a - a*\text{Sec}[c + d*x])^{(5/2)}$

fricas [A] time = 0.52, size = 656, normalized size = 2.34

$$\frac{861 \sqrt{2} \left(A \cos(dx + c)^2 - 2 A \cos(dx + c) + A \right) \sqrt{-a} \log \left(\frac{2 \sqrt{2} (\cos(dx+c)^2 + \cos(dx+c)) \sqrt{-a} \sqrt{\frac{a \cos(dx+c) - a}{\cos(dx+c)}} + (3 a \cos(dx+c) - a) / \cos(dx+c)}{(\cos(dx+c) - 1) \sin(dx+c)} \right)}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^3*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(5/2),x, algorithm="fricas")

[Out] [-1/96*(861*sqrt(2)*(A*cos(d*x + c)^2 - 2*A*cos(d*x + c) + A)*sqrt(-a)*log((2*sqrt(2)*(cos(d*x + c)^2 + cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) - a)/cos(d*x + c)) + (3*a*cos(d*x + c) + a)*sin(d*x + c))/((cos(d*x + c) - 1)*sin(d*x + c)))*sin(d*x + c) + 1218*(A*cos(d*x + c)^2 - 2*A*cos(d*x + c) + A)*sqrt(-a)*log((2*(cos(d*x + c)^2 + cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) - a)/cos(d*x + c)) - (2*a*cos(d*x + c) + a)*sin(d*x + c))/sin(d*x + c))*sin(d*x + c) + 4*(8*A*cos(d*x + c)^6 + 30*A*cos(d*x + c)^5 + 113*A*cos(d*x + c)^4 - 294*A*cos(d*x + c)^3 - 133*A*cos(d*x + c)^2 + 252*A*cos(d*x + c))*sqrt((a*cos(d*x + c) - a)/cos(d*x + c)))/((a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) + a^3*d)*sin(d*x + c)), 1/48*(861*sqrt(2)*(A*cos(d*x + c)^2 - 2*A*cos(d*x + c) + A)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) - a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))*sin(d*x + c) - 1218*(A*cos(d*x + c)^2 - 2*A*cos(d*x + c) + A)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) - a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))*sin(d*x + c) - 2*(8*A*cos(d*x + c)^6 + 30*A*cos(d*x + c)^5 + 113*A*cos(d*x + c)^4 - 294*A*cos(d*x + c)^3 - 133*A*cos(d*x + c)^2 + 252*A*cos(d*x + c))*sqrt((a*cos(d*x + c) - a)/cos(d*x + c)))/((a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) + a^3*d)*sin(d*x + c)]]

giac [A] time = 1.94, size = 346, normalized size = 1.24

$$\frac{861 \sqrt{2} A \arctan\left(\frac{\sqrt{a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - a}}{\sqrt{a}}\right)}{a^{\frac{5}{2}} \operatorname{sgn}\left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 1\right) \operatorname{sgn}\left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right)} - \frac{1218 A \arctan\left(\frac{\sqrt{2} \sqrt{a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - a}}{2 \sqrt{a}}\right)}{a^{\frac{5}{2}} \operatorname{sgn}\left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 1\right) \operatorname{sgn}\left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right)} - \frac{2 \sqrt{2} \left(129 \left(a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - a\right)^{\frac{5}{2}} A + 560 \left(a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + a\right)^3 a^2 \operatorname{sgn}\left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right)\right)}{48 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^3*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(5/2),x, algorithm="giac")

[Out] -1/48*(861*sqrt(2)*A*arctan(sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)/sqrt(a))/(a^(5/2)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c))) - 1218*A*arctan(1/2*sqrt(2)*sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)/sqrt(a))/(a^(5/2)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c))) - 2*sqrt(2)*(129*(a*tan(1/2*d*x + 1/2*c)^2 - a)^(5/2)*A + 560*(a*tan(1/2*d*x + 1/2*c)^2 - a)^(3/2)*A*a + 636*sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)*A*a^2)/((a*tan(1/2*d*x + 1/2*c)^2 + a)^3*a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c))) - 3*sqrt(2)*(33*(a*tan(1/2*d*x + 1/2*c)^2 - a)^(3/2)*A + 31*sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)*A*a)/(a^4*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c))*tan(1/2*d*x + 1/2*c)^4)/d

maple [B] time = 2.07, size = 1964, normalized size = 7.01

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^3*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(5/2),x)

[Out] -1/180*A/d*(-1+cos(d*x+c))^7*(-8610*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)*2^(1/2)+2583*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*cos(d*x+c)^6+25830*2^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*cos(d*x+c)^5+2870*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(9/2)*cos(d*x+c)^5-3690*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*cos(d*x+c)^5+5166*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*cos(d*x+c)^5+1845*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*cos(d*x+c)^4+2870*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(9/2)*cos(d*x+c)-2583*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*cos(d*x+c)^4+4305*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)^4-12915*2^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*cos(d*x+c)^4+4215*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^7-15112*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^6+18270*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*2^(1/2))+7380*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))

$$\begin{aligned} & ^{(7/2)} \cos(dx+c)^3 + 1845 \cdot 2^{(1/2)} \cdot (-2 \cos(dx+c) / (1 + \cos(dx+c)))^{(7/2)} \cos(dx+c) \\ & - 10332 \cdot 2^{(1/2)} \cdot (-2 \cos(dx+c) / (1 + \cos(dx+c)))^{(5/2)} \cos(dx+c)^3 + 17220 \cdot 2^{(1/2)} \cdot (-2 \\ & \cdot \cos(dx+c) / (1 + \cos(dx+c)))^{(3/2)} \cos(dx+c)^3 - 51660 \cdot 2^{(1/2)} \cdot \arctan(1 / (-2 \cos(dx+c) / (1 + \cos(dx+c))))^{(1/2)} \\ & \cdot \cos(dx+c)^3 - 4305 \cdot 2^{(1/2)} \cdot \cos(dx+c)^6 \cdot (-2 \cos(dx+c) / (1 + \cos(dx+c)))^{(3/2)} + 12915 \cdot 2^{(1/2)} \cdot \arctan(1 / (-2 \cos(dx+c) / (1 + \cos(dx+c))))^{(1/2)} \\ & \cdot \cos(dx+c)^6 + 1125 \cdot 2^{(1/2)} \cdot (-2 \cos(dx+c) / (1 + \cos(dx+c)))^{(11/2)} \cdot \cos(dx+c)^6 + 4680 \cdot 2^{(1/2)} \cdot (-2 \cos(dx+c) / (1 + \cos(dx+c)))^{(11/2)} \cdot \cos(dx+c)^5 \\ & + 1435 \cdot 2^{(1/2)} \cdot (-2 \cos(dx+c) / (1 + \cos(dx+c)))^{(9/2)} \cdot \cos(dx+c)^6 + 6525 \cdot 2^{(1/2)} \cdot (-2 \cos(dx+c) / (1 + \cos(dx+c)))^{(11/2)} \cdot \cos(dx+c)^4 + 1800 \cdot 2^{(1/2)} \\ & \cdot (-2 \cos(dx+c) / (1 + \cos(dx+c)))^{(11/2)} \cdot \cos(dx+c)^3 - 1845 \cdot 2^{(1/2)} \cdot (-2 \cos(dx+c) / (1 + \cos(dx+c)))^{(7/2)} \cdot \cos(dx+c)^6 - 3825 \cdot 2^{(1/2)} \cdot (-2 \cos(dx+c) / (1 + \cos(dx+c)))^{(11/2)} \\ & \cdot \cos(dx+c)^2 - 3600 \cdot 2^{(1/2)} \cdot (-2 \cos(dx+c) / (1 + \cos(dx+c)))^{(11/2)} \cdot \cos(dx+c) + 120 \cdot 2^{(1/2)} \cdot \cos(dx+c)^9 \cdot (-2 \cos(dx+c) / (1 + \cos(dx+c)))^{(1/2)} \\ & + 930 \cdot 2^{(1/2)} \cdot \cos(dx+c)^8 \cdot (-2 \cos(dx+c) / (1 + \cos(dx+c)))^{(1/2)} + 3780 \cdot (-2 \cos(dx+c) / (1 + \cos(dx+c)))^{(1/2)} \cdot 2^{(1/2)} - 2583 \cdot (-2 \cos(dx+c) / (1 + \cos(dx+c)))^{(5/2)} \\ & \cdot \cos(dx+c)^2 \cdot 2^{(1/2)} + 12915 \cdot \arctan(1 / (-2 \cos(dx+c) / (1 + \cos(dx+c))))^{(1/2)} \cdot 2^{(1/2)} - 8610 \cdot 2^{(1/2)} \cdot (-2 \cos(dx+c) / (1 + \cos(dx+c)))^{(3/2)} \cdot \cos(dx+c)^5 \\ & - 10335 \cdot (-2 \cos(dx+c) / (1 + \cos(dx+c)))^{(1/2)} \cdot \cos(dx+c)^3 \cdot 2^{(1/2)} - 8652 \cdot (-2 \cos(dx+c) / (1 + \cos(dx+c)))^{(1/2)} \cdot \cos(dx+c)^2 \cdot 2^{(1/2)} + 25830 \cdot \arctan(1 / (-2 \cos(dx+c) / (1 + \cos(dx+c))))^{(1/2)} \\ & \cdot \cos(dx+c) \cdot 2^{(1/2)} + 4515 \cdot (-2 \cos(dx+c) / (1 + \cos(dx+c)))^{(1/2)} \cdot \cos(dx+c) \cdot 2^{(1/2)} + 36540 \cdot \arctan(1/2 \cdot (-2 \cos(dx+c) / (1 + \cos(dx+c))))^{(1/2)} \cdot 2^{(1/2)} \cdot \cos(dx+c)^5 \\ & - 1435 \cdot 2^{(1/2)} \cdot (-2 \cos(dx+c) / (1 + \cos(dx+c)))^{(9/2)} \cdot \cos(dx+c)^4 - 5740 \cdot 2^{(1/2)} \cdot (-2 \cos(dx+c) / (1 + \cos(dx+c)))^{(9/2)} \cdot \cos(dx+c)^3 - 1435 \cdot 2^{(1/2)} \cdot (-2 \cos(dx+c) / (1 + \cos(dx+c)))^{(9/2)} \\ & \cdot \cos(dx+c)^2 - 945 \cdot 2^{(1/2)} \cdot (-2 \cos(dx+c) / (1 + \cos(dx+c)))^{(11/2)} + 18270 \cdot \arctan(1/2 \cdot (-2 \cos(dx+c) / (1 + \cos(dx+c))))^{(1/2)} \cdot 2^{(1/2)} \cdot \cos(dx+c)^6 + 5166 \cdot (-2 \cos(dx+c) / (1 + \cos(dx+c)))^{(5/2)} \\ & \cdot \cos(dx+c) \cdot 2^{(1/2)} + 14285 \cdot (-2 \cos(dx+c) / (1 + \cos(dx+c)))^{(1/2)} \cdot \cos(dx+c)^5 \cdot 2^{(1/2)} + 4305 \cdot (-2 \cos(dx+c) / (1 + \cos(dx+c)))^{(3/2)} \cdot \cos(dx+c)^2 \cdot 2^{(1/2)} + 6254 \cdot (-2 \cos(dx+c) / (1 + \cos(dx+c)))^{(1/2)} \\ & \cdot \cos(dx+c)^4 \cdot 2^{(1/2)} - 12915 \cdot \arctan(1 / (-2 \cos(dx+c) / (1 + \cos(dx+c))))^{(1/2)} \cdot \cos(dx+c)^2 \cdot 2^{(1/2)} + 1435 \cdot 2^{(1/2)} \cdot (-2 \cos(dx+c) / (1 + \cos(dx+c)))^{(9/2)} - 18270 \cdot \arctan(1/2 \cdot (-2 \cos(dx+c) / (1 + \cos(dx+c))))^{(1/2)} \cdot 2^{(1/2)} \cdot \cos(dx+c)^4 \\ & + 36540 \cdot \arctan(1/2 \cdot (-2 \cos(dx+c) / (1 + \cos(dx+c))))^{(1/2)} \cdot 2^{(1/2)} \cdot \cos(dx+c) + 2583 \cdot (-2 \cos(dx+c) / (1 + \cos(dx+c)))^{(5/2)} \cdot 2^{(1/2)} - 18270 \cdot \arctan(1/2 \cdot (-2 \cos(dx+c) / (1 + \cos(dx+c))))^{(1/2)} \cdot 2^{(1/2)} \cdot \cos(dx+c)^2 - 4305 \cdot (-2 \cos(dx+c) / (1 + \cos(dx+c)))^{(3/2)} \cdot 2^{(1/2)} - 73080 \cdot \arctan(1/2 \cdot (-2 \cos(dx+c) / (1 + \cos(dx+c))))^{(1/2)} \cdot 2^{(1/2)} \cdot \cos(dx+c)^3 - 1845 \cdot 2^{(1/2)} \cdot (-2 \cos(dx+c) / (1 + \cos(dx+c)))^{(7/2)} / (-2 \cos(dx+c) / (1 + \cos(dx+c)))^{(5/2)} / (a \cdot (-1 + \cos(dx+c)) / \cos(dx+c))^{(5/2)} / \sin(dx+c)^{13} \cdot 2^{(1/2)} \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A \sec(dx+c) + A) \cos(dx+c)^3}{(-a \sec(dx+c) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((A*sec(d*x + c) + A)*cos(d*x + c)^3/(-a*sec(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c + dx)^3 \left(A + \frac{A}{\cos(c+dx)} \right)}{\left(a - \frac{a}{\cos(c+dx)} \right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*(A + A/cos(c + d*x)))/(a - a/cos(c + d*x))^(5/2),x)
```

```
[Out] int((cos(c + d*x)^3*(A + A/cos(c + d*x)))/(a - a/cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

$$3.179 \quad \int \sec^2(c + dx)(a + a \sec(c + dx))(A + B \sec(c + dx)) dx$$

Optimal. Leaf size=199

$$\frac{2a(A + B) \sin(c + dx) \sec^{\frac{5}{2}}(c + dx)}{5d} + \frac{2a(7A + 5B) \sin(c + dx) \sec^{\frac{3}{2}}(c + dx)}{21d} + \frac{6a(A + B) \sin(c + dx) \sqrt{\sec(c + dx)}}{5d}$$

[Out] $2/21*a*(7*A+5*B)*\sec(d*x+c)^{(3/2)}*\sin(d*x+c)/d+2/5*a*(A+B)*\sec(d*x+c)^{(5/2)}*\sin(d*x+c)/d+2/7*a*B*\sec(d*x+c)^{(7/2)}*\sin(d*x+c)/d+6/5*a*(A+B)*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/d-6/5*a*(A+B)*(cos(1/2*d*x+1/2*c)^2)^{(1/2)}/cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d+2/21*a*(7*A+5*B)*(cos(1/2*d*x+1/2*c)^2)^{(1/2)}/cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d$

Rubi [A] time = 0.18, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {3997, 3787, 3768, 3771, 2641, 2639}

$$\frac{2a(A + B) \sin(c + dx) \sec^{\frac{5}{2}}(c + dx)}{5d} + \frac{2a(7A + 5B) \sin(c + dx) \sec^{\frac{3}{2}}(c + dx)}{21d} + \frac{6a(A + B) \sin(c + dx) \sqrt{\sec(c + dx)}}{5d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sec}[c + d*x]^{(5/2)}*(a + a*\text{Sec}[c + d*x])*(A + B*\text{Sec}[c + d*x]), x]$

[Out] $(-6*a*(A + B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/(5*d) + (2*a*(7*A + 5*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticF}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/(21*d) + (6*a*(A + B)*\text{Sqrt}[\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(5*d) + (2*a*(7*A + 5*B)*\text{Sec}[c + d*x]^{(3/2)}*\text{Sin}[c + d*x])/(21*d) + (2*a*(A + B)*\text{Sec}[c + d*x]^{(5/2)}*\text{Sin}[c + d*x])/(5*d) + (2*a*B*\text{Sec}[c + d*x]^{(7/2)}*\text{Sin}[c + d*x])/(7*d)$

Rule 2639

$\text{Int}[\text{Sqrt}[\sin[(c_.) + (d_.)*(x_.)]], x_Symbol] := \text{Simp}[(2*\text{EllipticE}[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}[\{c, d\}, x]$

Rule 2641

$\text{Int}[1/\text{Sqrt}[\sin[(c_.) + (d_.)*(x_.)]], x_Symbol] := \text{Simp}[(2*\text{EllipticF}[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}[\{c, d\}, x]$

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]
)*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] := -Simp[(b*B*Cot[e
+ f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && !LeQ[n,
-1]
```

Rubi steps

$$\begin{aligned}
\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))(A+B\sec(c+dx))dx &= \frac{2aB\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{2}{7}\int \sec^{\frac{5}{2}}(c+dx)\left(\frac{2a}{7}\sec^{\frac{7}{2}}(c+dx)\right)dx \\
&= \frac{2aB\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d} + (a(A+B))\int \sec^{\frac{7}{2}}(c+dx)dx \\
&= \frac{2a(7A+5B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{21d} + \frac{2a(A+B)}{5d}\sqrt{\sec(c+dx)} \\
&= \frac{6a(A+B)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d} + \frac{2a(7A+5B)}{21d}\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)} \\
&= \frac{6a(A+B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}
\end{aligned}$$

Mathematica [A] time = 0.78, size = 200, normalized size = 1.01

$$a \sec^2\left(\frac{1}{2}(c+dx)\right)(\sec(c+dx)+1)(A+B\sec(c+dx))\left(5(7A+5B)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)-63(A+B)\sqrt{\sec(c+dx)}\right)$$

Antiderivative was successfully verified.

[In] Integrate[Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]

[Out] (a*Sec[(c + d*x)/2]^2*(1 + Sec[c + d*x])*(A + B*Sec[c + d*x])*(-63*(A + B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 5*(7*A + 5*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 63*A*Sin[c + d*x] + 63*B*Sin[c + d*x] + 35*A*Tan[c + d*x] + 25*B*Tan[c + d*x] + 21*A*Sec[c + d*x]*Tan[c + d*x] + 21*B*Sec[c + d*x]*Tan[c + d*x] + 15*B*Sec[c + d*x]^2*Tan[c + d*x]))/(105*d*(B + A*Cos[c + d*x])*Sec[c + d*x]^(3/2))

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Ba \sec(dx+c)^4 + (A+B)a \sec(dx+c)^3 + Aa \sec(dx+c)^2\right)\sqrt{\sec(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] integral((B*a*sec(d*x + c)^4 + (A + B)*a*sec(d*x + c)^3 + A*a*sec(d*x + c)^2)*sqrt(sec(d*x + c)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(a \sec(dx + c) + a) \sec(dx + c)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)*sec(d*x + c)^(5/2), x)

maple [B] time = 12.41, size = 691, normalized size = 3.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x)

[Out] -a*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*B*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-4/5*(1/2*A+1/2*B)/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*A*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(a \sec(dx + c) + a) \sec(dx + c)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)*sec(d*x + c)^(5/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A + \frac{B}{\cos(c + dx)} \right) \left(a + \frac{a}{\cos(c + dx)} \right) \left(\frac{1}{\cos(c + dx)} \right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))*(1/cos(c + d*x))^(5/2),x)

[Out] int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))*(1/cos(c + d*x))^(5/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**(5/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x)

[Out] Timed out

$$3.180 \quad \int \sec^{\frac{3}{2}}(c+dx)(a+a \sec(c+dx))(A+B \sec(c+dx)) dx$$

Optimal. Leaf size=172

$$\frac{2a(A+B) \sin(c+dx) \sec^{\frac{3}{2}}(c+dx)}{3d} + \frac{2a(5A+3B) \sin(c+dx) \sqrt{\sec(c+dx)}}{5d} + \frac{2a(A+B) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{3d}$$

[Out] $\frac{2}{3} a (A+B) \sec(d*x+c)^{(3/2)} \sin(d*x+c) / d + \frac{2}{5} a B \sec(d*x+c)^{(5/2)} \sin(d*x+c) / d + \frac{2}{5} a (5A+3B) \sin(d*x+c) \sec(d*x+c)^{(1/2)} / d - \frac{2}{5} a (5A+3B) (\cos(1/2*d*x+1/2*c)^2)^{(1/2)} / \cos(1/2*d*x+1/2*c) * \text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)}) * \cos(d*x+c)^{(1/2)} \sec(d*x+c)^{(1/2)} / d + \frac{2}{3} a (A+B) (\cos(1/2*d*x+1/2*c)^2)^{(1/2)} / \cos(1/2*d*x+1/2*c) * \text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)}) * \cos(d*x+c)^{(1/2)} \sec(d*x+c)^{(1/2)} / d$

Rubi [A] time = 0.16, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {3997, 3787, 3768, 3771, 2639, 2641}

$$\frac{2a(A+B) \sin(c+dx) \sec^{\frac{3}{2}}(c+dx)}{3d} + \frac{2a(5A+3B) \sin(c+dx) \sqrt{\sec(c+dx)}}{5d} + \frac{2a(A+B) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{3d}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]

[Out] $(-2*a*(5*A+3*B)*\text{Sqrt}[\text{Cos}[c+d*x]]*\text{EllipticE}[(c+d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c+d*x]])/(5*d) + (2*a*(A+B)*\text{Sqrt}[\text{Cos}[c+d*x]]*\text{EllipticF}[(c+d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c+d*x]])/(3*d) + (2*a*(5*A+3*B)*\text{Sqrt}[\text{Sec}[c+d*x]]*\text{Sin}[c+d*x])/(5*d) + (2*a*(A+B)*\text{Sec}[c+d*x]^{(3/2)}*\text{Sin}[c+d*x])/(3*d) + (2*a*B*\text{Sec}[c+d*x]^{(5/2)}*\text{Sin}[c+d*x])/(5*d)$

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 3768

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x] * (b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3771

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x])^n * Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 3997

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] := -Simp[(b*B*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e + f*x])^n * Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && !LeQ[n, -1]

Rubi steps

$$\begin{aligned}
 \int \sec^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))(A + B \sec(c + dx)) dx &= \frac{2aB \sec^{\frac{5}{2}}(c + dx) \sin(c + dx)}{5d} + \frac{2}{5} \int \sec^{\frac{3}{2}}(c + dx) \left(\frac{2aB \sec^{\frac{5}{2}}(c + dx) \sin(c + dx)}{5d} + (a(A + B)) \int \sec^{\frac{5}{2}}(c + dx) \right) dx \\
 &= \frac{2aB \sec^{\frac{5}{2}}(c + dx) \sin(c + dx)}{5d} + (a(A + B)) \int \sec^{\frac{5}{2}}(c + dx) dx \\
 &= \frac{2a(5A + 3B) \sqrt{\sec(c + dx)} \sin(c + dx)}{5d} + \frac{2a(A + B)}{5d} \int \sec^{\frac{5}{2}}(c + dx) dx \\
 &= \frac{2a(5A + 3B) \sqrt{\sec(c + dx)} \sin(c + dx)}{5d} + \frac{2a(A + B)}{5d} \int \sec^{\frac{5}{2}}(c + dx) dx \\
 &= \frac{2a(5A + 3B) \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{\sec(c + dx)}}{5d}
 \end{aligned}$$

Mathematica [A] time = 0.67, size = 168, normalized size = 0.98

$$\frac{a \sec^2\left(\frac{1}{2}(c + dx)\right) (\sec(c + dx) + 1)(A + B \sec(c + dx)) \left(5(A + B)\sqrt{\cos(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right) - 3(5A + 3B)\sqrt{\cos(c + dx)}\right)}{15d \sec^2(c + dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
[Out] (a*Sec[(c + d*x)/2]^2*(1 + Sec[c + d*x])*(A + B*Sec[c + d*x])*(-3*(5*A + 3*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 5*(A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 15*A*Sin[c + d*x] + 9*B*Sin[c + d*x] + 5*A*Tan[c + d*x] + 5*B*Tan[c + d*x] + 3*B*Sec[c + d*x]*Tan[c + d*x]))/(15*d*(B + A*Cos[c + d*x])*Sec[c + d*x]^(3/2))
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Ba \sec(dx + c)^3 + (A + B)a \sec(dx + c)^2 + Aa \sec(dx + c)\right)\sqrt{\sec(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="fricas")
[Out] integral((B*a*sec(d*x + c)^3 + (A + B)*a*sec(d*x + c)^2 + A*a*sec(d*x + c))*sqrt(sec(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(a \sec(dx + c) + a) \sec(dx + c)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="giac")
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)*sec(d*x + c)^(3/2), x)
```

maple [B] time = 12.30, size = 662, normalized size = 3.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] -a*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*(1/2*A+1/2*B)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*A*(-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)-2/5*B/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(a \sec(dx + c) + a) \sec(dx + c)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)*sec(d*x + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A + \frac{B}{\cos(c + dx)} \right) \left(a + \frac{a}{\cos(c + dx)} \right) \left(\frac{1}{\cos(c + dx)} \right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))*(1/cos(c + d*x))^(3/2), x)
```

```
[Out] int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))*(1/cos(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```


$$3.181 \quad \int \sqrt{\sec(c + dx)} (a + a \sec(c + dx))(A + B \sec(c + dx)) dx$$

Optimal. Leaf size=135

$$\frac{2a(A + B) \sin(c + dx) \sqrt{\sec(c + dx)}}{d} + \frac{2a(3A + B) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3d} - \frac{2a(A + B) \sqrt{\cos(c + dx)}}{3d}$$

[Out] $2/3*a*B*\sec(d*x+c)^{(3/2)}*\sin(d*x+c)/d+2*a*(A+B)*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/d-2*a*(A+B)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d+2/3*a*(3*A+B)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d$

Rubi [A] time = 0.14, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {3997, 3787, 3771, 2641, 3768, 2639}

$$\frac{2a(A + B) \sin(c + dx) \sqrt{\sec(c + dx)}}{d} + \frac{2a(3A + B) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3d} - \frac{2a(A + B) \sqrt{\cos(c + dx)}}{3d}$$

Antiderivative was successfully verified.

[In] `Int[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]), x]`

[Out] $(-2*a*(A + B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/d + (2*a*(3*A + B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticF}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/(3*d) + (2*a*(A + B)*\text{Sqrt}[\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/d + (2*a*B*\text{Sec}[c + d*x]^{(3/2)}*\text{Sin}[c + d*x])/(3*d)$

Rule 2639

`Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]`

Rule 2641

`Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]`

Rule 3768

`Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x])* (b*Csc[c + d*x])^(n - 1)/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I`

Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3771

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 3997

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] := -Simp[(b*B*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && !LeQ[n, -1]

Rubi steps

$$\begin{aligned}
 \int \sqrt{\sec(c + dx)} (a + a \sec(c + dx))(A + B \sec(c + dx)) dx &= \frac{2aB \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{3d} + \frac{2}{3} \int \sqrt{\sec(c + dx)} \left(\right. \\
 &= \frac{2aB \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{3d} + (a(A + B)) \int \sec^{\frac{3}{2}}(c + dx) dx \\
 &= \frac{2a(A + B) \sqrt{\sec(c + dx)} \sin(c + dx)}{d} + \frac{2aB \sec^{\frac{3}{2}}(c + dx)}{3d} \\
 &= \frac{2a(3A + B) \sqrt{\cos(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{\sec(c + dx)}}{3d} \\
 &= -\frac{2a(A + B) \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{\sec(c + dx)}}{d}
 \end{aligned}$$

Mathematica [A] time = 0.53, size = 94, normalized size = 0.70

$$\frac{a \sec^2(c + dx) \left(2(3A + B) \cos^2(c + dx) F\left(\frac{1}{2}(c + dx) \middle| 2\right) - 6(A + B) \cos^2(c + dx) E\left(\frac{1}{2}(c + dx) \middle| 2\right) + 2 \sin(c + dx) \right)}{3d}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]

[Out] (a*Sec[c + d*x]^(3/2)*(-6*(A + B)*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2] + 2*(3*A + B)*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + 2*(B + 3*(A + B)*Cos[c + d*x])*Sin[c + d*x]))/(3*d)

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Ba \sec(dx + c)^2 + (A + B)a \sec(dx + c) + Aa\right)\sqrt{\sec(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x, algorithm="fricas")

[Out] integral((B*a*sec(d*x + c)^2 + (A + B)*a*sec(d*x + c) + A*a)*sqrt(sec(d*x + c)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(a \sec(dx + c) + a)\sqrt{\sec(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)*sqrt(sec(d*x + c)), x)

maple [B] time = 10.07, size = 427, normalized size = 3.16

$$a \sqrt{-2 \left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right) \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right) \right)} \left(\frac{2A \sqrt{\frac{1 - \cos(dx+c)}{2}} \sqrt{-2 \left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)} \text{EllipticF}\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\sqrt{-2 \left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right) + \sin^2\left(\frac{dx}{2} + \frac{c}{2}\right) \right)}} + 2B \right) -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(d*x+c))*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x)

[Out] $-a*(-(-2*\cos(1/2*d*x+1/2*c)^2+1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*A*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)})+2*B*(-1/6*\cos(1/2*d*x+1/2*c)*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/(-1/2+\cos(1/2*d*x+1/2*c)^2)^2+1/3*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)}))+4*(1/2*A+1/2*B)*(-(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)})+2*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^2)/\sin(1/2*d*x+1/2*c)^2/(2*\sin(1/2*d*x+1/2*c)^2-1))/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(a \sec(dx + c) + a)\sqrt{\sec(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)*sqrt(sec(d*x + c)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A + \frac{B}{\cos(c + dx)} \right) \left(a + \frac{a}{\cos(c + dx)} \right) \sqrt{\frac{1}{\cos(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))*(1/cos(c + d*x))^(1/2),x)

[Out] int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))*(1/cos(c + d*x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a \left(\int A \sqrt{\sec(c + dx)} dx + \int A \sec^{\frac{3}{2}}(c + dx) dx + \int B \sec^{\frac{3}{2}}(c + dx) dx + \int B \sec^{\frac{5}{2}}(c + dx) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))*sec(d*x+c)**(1/2),x)

[Out] a*(Integral(A*sqrt(sec(c + d*x)), x) + Integral(A*sec(c + d*x)**(3/2), x) + Integral(B*sec(c + d*x)**(3/2), x) + Integral(B*sec(c + d*x)**(5/2), x))

$$3.182 \quad \int \frac{(a+a \sec(c+dx))(A+B \sec(c+dx))}{\sqrt{\sec(c+dx)}} dx$$

Optimal. Leaf size=106

$$\frac{2a(A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

[Out] $2*a*B*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/d+2*a*(A-B)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c),2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d+2*a*(A+B)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c),2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d$

Rubi [A] time = 0.13, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {3997, 3787, 3771, 2639, 2641}

$$\frac{2a(A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

[In] Int[((a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]))/Sqrt[Sec[c + d*x]], x]

[Out] $(2*a*(A - B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/d + (2*a*(A + B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticF}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/d + (2*a*B*\text{Sqrt}[\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/d$

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 3771

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_.)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_.))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] :> -Simp[(b*B*Cot[
e + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && !LeQ[n,
-1]
```

Rubi steps

$$\begin{aligned} \int \frac{(a + a \sec(c + dx))(A + B \sec(c + dx))}{\sqrt{\sec(c + dx)}} dx &= \frac{2aB\sqrt{\sec(c + dx)} \sin(c + dx)}{d} + 2 \int \frac{\frac{1}{2}a(A - B) + \frac{1}{2}a(A + B) \sec(c + dx)}{\sqrt{\sec(c + dx)}} dx \\ &= \frac{2aB\sqrt{\sec(c + dx)} \sin(c + dx)}{d} + (a(A - B)) \int \frac{1}{\sqrt{\sec(c + dx)}} dx + \\ &= \frac{2aB\sqrt{\sec(c + dx)} \sin(c + dx)}{d} + (a(A - B)\sqrt{\cos(c + dx)}\sqrt{\sec(c + dx)} \\ &= \frac{2a(A - B)\sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{\sec(c + dx)}}{d} + \frac{2a(A + B)\sqrt{\sec(c + dx)}}{d} \end{aligned}$$

Mathematica [A] time = 0.31, size = 77, normalized size = 0.73

$$\frac{2a\sqrt{\sec(c + dx)} \left((A + B)\sqrt{\cos(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right) + (A - B)\sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) + B \sin(c + dx) \right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]))/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*a*Sqrt[Sec[c + d*x]]*((A - B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2,
2] + (A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + B*Sin[c + d*x])
)/d
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{Ba \sec(dx + c)^2 + (A + B)a \sec(dx + c) + Aa}{\sqrt{\sec(dx + c)}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="fricas")

[Out] integral((B*a*sec(d*x + c)^2 + (A + B)*a*sec(d*x + c) + A*a)/sqrt(sec(d*x + c)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(a \sec(dx + c) + a)}{\sqrt{\sec(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)/sqrt(sec(d*x + c)), x)

maple [A] time = 4.86, size = 240, normalized size = 2.26

$$2a \left(A \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{2 \left(\sin^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) - 1} \operatorname{EllipticF} \left(\cos \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) - A \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{2 \left(\sin^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) - 1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x)

[Out] $-2*a*(A*(\sin(1/2*d*x+1/2*c))^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\operatorname{EllipticF}(\cos(1/2*d*x+1/2*c), 2^{(1/2)})-A*(\sin(1/2*d*x+1/2*c))^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\operatorname{EllipticE}(\cos(1/2*d*x+1/2*c), 2^{(1/2)})+B*(\sin(1/2*d*x+1/2*c))^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\operatorname{EllipticF}(\cos(1/2*d*x+1/2*c), 2^{(1/2)})+B*(\sin(1/2*d*x+1/2*c))^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\operatorname{EllipticE}(\cos(1/2*d*x+1/2*c), 2^{(1/2)})-2*B*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^2)/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(a \sec(dx + c) + a)}{\sqrt{\sec(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)/sqrt(sec(d*x + c)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(a + \frac{a}{\cos(c+dx)}\right)}{\sqrt{\frac{1}{\cos(c+dx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x)))/(1/cos(c + d*x))^(1/2),x)

[Out] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x)))/(1/cos(c + d*x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a \left(\int \frac{A}{\sqrt{\sec(c+dx)}} dx + \int A \sqrt{\sec(c+dx)} dx + \int B \sqrt{\sec(c+dx)} dx + \int B \sec^{\frac{3}{2}}(c+dx) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)**(1/2),x)

[Out] a*(Integral(A/sqrt(sec(c + d*x)), x) + Integral(A*sqrt(sec(c + d*x)), x) + Integral(B*sqrt(sec(c + d*x)), x) + Integral(B*sec(c + d*x)**(3/2), x))

$$3.183 \quad \int \frac{(a+a \sec(c+dx))(A+B \sec(c+dx))}{3 \sec^2(c+dx)} dx$$

Optimal. Leaf size=110

$$\frac{2a(A+3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2a(A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

[Out] $2/3*a*A*\sin(d*x+c)/d/\sec(d*x+c)^{(1/2)}+2*a*(A+B)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c),2^{(1/2)})*\cos(d*x+c)^{(1/2)}* \sec(d*x+c)^{(1/2)}/d+2/3*a*(A+3*B)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c),2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d$

Rubi [A] time = 0.13, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {3996, 3787, 3771, 2639, 2641}

$$\frac{2a(A+3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2a(A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\frac{(a + a*\text{Sec}[c + d*x])*(A + B*\text{Sec}[c + d*x])}{\text{Sec}[c + d*x]^{(3/2)}}, x]$

[Out] $(2*a*(A + B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/d + (2*a*(A + 3*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticF}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/(3*d) + (2*a*A*\text{Sin}[c + d*x])/(3*d*\text{Sqrt}[\text{Sec}[c + d*x]])$

Rule 2639

$\text{Int}[\text{Sqrt}[\sin[(c_.) + (d_.)*(x_)]]], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticE}[(1*(c - \text{Pi}/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 2641

$\text{Int}[1/\text{Sqrt}[\sin[(c_.) + (d_.)*(x_)]]], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticF}[(1*(c - \text{Pi}/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 3771

$\text{Int}[(\text{csc}[(c_.) + (d_.)*(x_)]*(b_.))^{(n_.)}, x_Symbol] \rightarrow \text{Dist}[(b*\text{Csc}[c + d*x])^{n_*}\text{Sin}[c + d*x]^n, \text{Int}[1/\text{Sin}[c + d*x]^n, x], x] /; \text{FreeQ}\{b, c, d\}, x] \&\&$

EqQ[n^2, 1/4]

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) +
(a_.)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) +
(a_.))*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := Simp[(A*a*Cot[e +
f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
+ 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{(a + a \sec(c + dx))(A + B \sec(c + dx))}{\sec^{\frac{3}{2}}(c + dx)} dx = \frac{2aA \sin(c + dx)}{3d\sqrt{\sec(c + dx)}} - \frac{2}{3} \int \frac{-\frac{3}{2}a(A + B) - \frac{1}{2}a(A + 3B) \sec(c + dx)}{\sqrt{\sec(c + dx)}} dx$$

$$= \frac{2aA \sin(c + dx)}{3d\sqrt{\sec(c + dx)}} + (a(A + B)) \int \frac{1}{\sqrt{\sec(c + dx)}} dx + \frac{1}{3}(a(A + 3B)) \int \frac{1}{\sqrt{\sec(c + dx)}} dx$$

$$= \frac{2aA \sin(c + dx)}{3d\sqrt{\sec(c + dx)}} + (a(A + B)\sqrt{\cos(c + dx)}\sqrt{\sec(c + dx)}) \int \sqrt{\cos(c + dx)} dx + \frac{1}{3}(a(A + 3B)\sqrt{\cos(c + dx)}\sqrt{\sec(c + dx)}) \int \sqrt{\cos(c + dx)} dx$$

$$= \frac{2a(A + B)\sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{\sec(c + dx)}}{d} + \frac{2a(A + 3B)\sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{\sec(c + dx)}}{3d}$$

Mathematica [A] time = 0.32, size = 83, normalized size = 0.75

$$\frac{a\sqrt{\sec(c + dx)} \left(2(A + 3B)\sqrt{\cos(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right) + 6(A + B)\sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) + A \sin(2(c + dx)) \right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(3/2),x]
```

```
[Out] (a*Sqrt[Sec[c + d*x]]*(6*(A + B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2,
2] + 2*(A + 3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + A*Sin[2*(c
+ d*x)]))/(3*d)
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{Ba \sec(dx + c)^2 + (A + B)a \sec(dx + c) + Aa}{\sec(dx + c)^{\frac{3}{2}}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="fricas")

[Out] integral((B*a*sec(d*x + c)^2 + (A + B)*a*sec(d*x + c) + A*a)/sec(d*x + c)^(3/2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(a \sec(dx + c) + a)}{\sec(dx + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)/sec(d*x + c)^(3/2), x)

maple [B] time = 4.98, size = 321, normalized size = 2.92

$$2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)\right)} a \left(4A \cos\left(\frac{dx}{2} + \frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + A\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{2\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x)

[Out]
$$-2/3*((2*\cos(1/2*d*x+1/2*c)^2-1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*a*(4*A*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^4+A*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)})-3*A*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)})-2*A*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^2+3*B*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)})-3*B*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)}))/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(a \sec(dx + c) + a)}{\sec(dx + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)/sec(d*x + c)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(a + \frac{a}{\cos(c+dx)}\right)}{\left(\frac{1}{\cos(c+dx)}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x)))/(1/cos(c + d*x))^(3/2),x)

[Out] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x)))/(1/cos(c + d*x))^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a \left(\int \frac{A}{\sec^{\frac{3}{2}}(c + dx)} dx + \int \frac{A}{\sqrt{\sec(c + dx)}} dx + \int \frac{B}{\sqrt{\sec(c + dx)}} dx + \int B \sqrt{\sec(c + dx)} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)**(3/2),x)

[Out] a*(Integral(A/sec(c + d*x)**(3/2), x) + Integral(A/sqrt(sec(c + d*x)), x) + Integral(B/sqrt(sec(c + d*x)), x) + Integral(B*sqrt(sec(c + d*x)), x))

$$3.184 \quad \int \frac{(a+a \sec(c+dx))(A+B \sec(c+dx))}{5 \sec^2(c+dx)} dx$$

Optimal. Leaf size=141

$$\frac{2a(A+B) \sin(c+dx)}{3d \sqrt{\sec(c+dx)}} + \frac{2a(A+B) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} F\left(\frac{1}{2}(c+dx) \middle| 2\right)}{3d} + \frac{2a(3A+5B) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{5d}$$

[Out] $2/5*a*A*\sin(d*x+c)/d/\sec(d*x+c)^{(3/2)}+2/3*a*(A+B)*\sin(d*x+c)/d/\sec(d*x+c)^{(1/2)}+2/5*a*(3*A+5*B)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d+2/3*a*(A+B)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d$

Rubi [A] time = 0.15, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {3996, 3787, 3769, 3771, 2641, 2639}

$$\frac{2a(A+B) \sin(c+dx)}{3d \sqrt{\sec(c+dx)}} + \frac{2a(A+B) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} F\left(\frac{1}{2}(c+dx) \middle| 2\right)}{3d} + \frac{2a(3A+5B) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{5d}$$

Antiderivative was successfully verified.

[In] Int[((a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(5/2), x]

[Out] $(2*a*(3*A + 5*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/(5*d) + (2*a*(A + B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticF}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/(3*d) + (2*a*A*\text{Sin}[c + d*x])/(5*d*\text{Sec}[c + d*x]^{(3/2)}) + (2*a*(A + B)*\text{Sin}[c + d*x])/(3*d*\text{Sqrt}[\text{Sec}[c + d*x]])$

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 3769

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +

$d*x])^{(n + 2), x], x] /; \text{FreeQ}[\{b, c, d\}, x] \ \&\& \ \text{LtQ}[n, -1] \ \&\& \ \text{IntegerQ}[2*n]$
]

Rule 3771

$\text{Int}[(\text{csc}[(c_.) + (d_.)*(x_)]*(b_.)^{(n_)}, x_Symbol] \ :> \ \text{Dist}[(b*\text{Csc}[c + d*x])^n * \text{Sin}[c + d*x]^n, \text{Int}[1/\text{Sin}[c + d*x]^n, x], x] /; \text{FreeQ}[\{b, c, d\}, x] \ \&\& \ \text{EqQ}[n^2, 1/4]$

Rule 3787

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_)]*(d_.)^{(n_.)} * (\text{csc}[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] \ :> \ \text{Dist}[a, \text{Int}[(d*\text{Csc}[e + f*x])^n, x], x] + \text{Dist}[b/d, \text{Int}[(d*\text{Csc}[e + f*x])^{(n + 1)}, x], x] /; \text{FreeQ}[\{a, b, d, e, f, n\}, x]$

Rule 3996

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_)]*(d_.)^{(n_)} * (\text{csc}[(e_.) + (f_.)*(x_)]*(b_.) + (a_)) * (\text{csc}[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] \ :> \ \text{Simp}[(A*a*\text{Cot}[e + f*x] * (d*\text{Csc}[e + f*x])^n) / (f*n), x] + \text{Dist}[1/(d*n), \text{Int}[(d*\text{Csc}[e + f*x])^{(n + 1)} * \text{Simp}[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*\text{Csc}[e + f*x], x], x], x] /; \text{FreeQ}[\{a, b, d, e, f, A, B\}, x] \ \&\& \ \text{NeQ}[A*b - a*B, 0] \ \&\& \ \text{LeQ}[n, -1]$

Rubi steps

$$\begin{aligned} \int \frac{(a + a \sec(c + dx))(A + B \sec(c + dx))}{\sec^{\frac{5}{2}}(c + dx)} dx &= \frac{2aA \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} - \frac{2}{5} \int \frac{-\frac{5}{2}a(A + B) - \frac{1}{2}a(3A + 5B) \sec(c + dx)}{\sec^{\frac{3}{2}}(c + dx)} dx \\ &= \frac{2aA \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} + (a(A + B)) \int \frac{1}{\sec^{\frac{3}{2}}(c + dx)} dx + \frac{1}{5}(a(3A + 5B)) \int \frac{1}{\sec^{\frac{3}{2}}(c + dx)} dx \\ &= \frac{2aA \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} + \frac{2a(A + B) \sin(c + dx)}{3d \sqrt{\sec(c + dx)}} + \frac{1}{3}(a(A + B)) \int \sqrt{\sec(c + dx)} dx \\ &= \frac{2a(3A + 5B) \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{\sec(c + dx)}}{5d} + \frac{2aA \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} \\ &= \frac{2a(3A + 5B) \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{\sec(c + dx)}}{5d} + \frac{2a(A + B) \sin(c + dx)}{3d \sqrt{\sec(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.59, size = 99, normalized size = 0.70

$$\frac{a\sqrt{\sec(c+dx)}\left(\sin(2(c+dx))(5(A+B)+3A\cos(c+dx))+10(A+B)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)+6(3A+B)\sin(c+dx)\right)}{15d}$$

Antiderivative was successfully verified.

[In] Integrate[((a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(5/2), x]

[Out] (a*Sqrt[Sec[c + d*x]]*(6*(3*A + 5*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 10*(A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (5*(A + B) + 3*A*Cos[c + d*x])*Sin[2*(c + d*x)])/(15*d)

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{Ba \sec(dx+c)^2 + (A+B)a \sec(dx+c) + Aa}{\sec(dx+c)^{\frac{5}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2), x, algorithm="fricas")

[Out] integral((B*a*sec(d*x + c)^2 + (A + B)*a*sec(d*x + c) + A*a)/sec(d*x + c)^(5/2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx+c) + A)(a \sec(dx+c) + a)}{\sec(dx+c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2), x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)/sec(d*x + c)^(5/2), x)

maple [B] time = 4.37, size = 355, normalized size = 2.52

$$2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} a \left(-24A \cos\left(\frac{dx}{2} + \frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + (44A + 20B)\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \dots\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x)`

[Out]
$$-2/15*((2*\cos(1/2*d*x+1/2*c)^2-1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*a*(-24*A*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^6+(44*A+20*B)*\sin(1/2*d*x+1/2*c)^4*\cos(1/2*d*x+1/2*c)+(-16*A-10*B)*\sin(1/2*d*x+1/2*c)^2*\cos(1/2*d*x+1/2*c)+5*A*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)})-9*A*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)})+5*B*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)})-15*B*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)})/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(a \sec(dx + c) + a)}{\sec(dx + c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="maxima")`

[Out] `integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)/sec(d*x + c)^(5/2), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(a + \frac{a}{\cos(c+dx)}\right)}{\left(\frac{1}{\cos(c+dx)}\right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x)))/(1/cos(c + d*x))^(5/2),x)`

[Out] `int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x)))/(1/cos(c + d*x))^(5/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a \left(\int \frac{A}{\sec^{\frac{5}{2}}(c + dx)} dx + \int \frac{A}{\sec^{\frac{3}{2}}(c + dx)} dx + \int \frac{B}{\sec^{\frac{3}{2}}(c + dx)} dx + \int \frac{B}{\sqrt{\sec(c + dx)}} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.


```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)**(5/2),x)
```

```
[Out] a*(Integral(A/sec(c + d*x)**(5/2), x) + Integral(A/sec(c + d*x)**(3/2), x)
+ Integral(B/sec(c + d*x)**(3/2), x) + Integral(B/sqrt(sec(c + d*x)), x))
```

$$3.185 \quad \int \frac{(a+a \sec(c+dx))(A+B \sec(c+dx))}{7 \sec^2(c+dx)} dx$$

Optimal. Leaf size=172

$$\frac{2a(A+B) \sin(c+dx)}{5d \sec^2(c+dx)} + \frac{2a(5A+7B) \sin(c+dx)}{21d \sqrt{\sec(c+dx)}} + \frac{2a(5A+7B) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} F\left(\frac{1}{2}(c+dx) \middle| 2\right)}{21d} + \frac{6a(A+B) \sin(c+dx)}{5d \sec^2(c+dx)}$$

[Out] $2/7*a*A*\sin(d*x+c)/d/\sec(d*x+c)^{(5/2)}+2/5*a*(A+B)*\sin(d*x+c)/d/\sec(d*x+c)^{(3/2)}+2/21*a*(5*A+7*B)*\sin(d*x+c)/d/\sec(d*x+c)^{(1/2)}+6/5*a*(A+B)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c),2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d+2/21*a*(5*A+7*B)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c),2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d$

Rubi [A] time = 0.16, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {3996, 3787, 3769, 3771, 2639, 2641}

$$\frac{2a(A+B) \sin(c+dx)}{5d \sec^2(c+dx)} + \frac{2a(5A+7B) \sin(c+dx)}{21d \sqrt{\sec(c+dx)}} + \frac{2a(5A+7B) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} F\left(\frac{1}{2}(c+dx) \middle| 2\right)}{21d} + \frac{6a(A+B) \sin(c+dx)}{5d \sec^2(c+dx)}$$

Antiderivative was successfully verified.

[In] Int[((a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(7/2), x]

[Out] $(6*a*(A+B)*\text{Sqrt}[\text{Cos}[c+d*x]]*\text{EllipticE}[(c+d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c+d*x]])/(5*d) + (2*a*(5*A+7*B)*\text{Sqrt}[\text{Cos}[c+d*x]]*\text{EllipticF}[(c+d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c+d*x]])/(21*d) + (2*a*A*\text{Sin}[c+d*x])/(7*d*\text{Sec}[c+d*x]^{(5/2)}) + (2*a*(A+B)*\text{Sin}[c+d*x])/(5*d*\text{Sec}[c+d*x]^{(3/2)}) + (2*a*(5*A+7*B)*\text{Sin}[c+d*x])/(21*d*\text{Sqrt}[\text{Sec}[c+d*x]])$

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]]], x_Symbol] := Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] := Simp[(A*a*Cot[e +
f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
+ 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + a \sec(c + dx))(A + B \sec(c + dx))}{\sec^{\frac{7}{2}}(c + dx)} dx &= \frac{2aA \sin(c + dx)}{7d \sec^{\frac{5}{2}}(c + dx)} - \frac{2}{7} \int \frac{-\frac{7}{2}a(A + B) - \frac{1}{2}a(5A + 7B) \sec(c + dx)}{\sec^{\frac{5}{2}}(c + dx)} \\
&= \frac{2aA \sin(c + dx)}{7d \sec^{\frac{5}{2}}(c + dx)} + (a(A + B)) \int \frac{1}{\sec^{\frac{5}{2}}(c + dx)} dx + \frac{1}{7}(a(5A + 7B)) \int \frac{\sec(c + dx)}{\sec^{\frac{5}{2}}(c + dx)} dx \\
&= \frac{2aA \sin(c + dx)}{7d \sec^{\frac{5}{2}}(c + dx)} + \frac{2a(A + B) \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} + \frac{2a(5A + 7B) \sin(c + dx)}{21d \sqrt{\sec(c + dx)}} \\
&= \frac{2aA \sin(c + dx)}{7d \sec^{\frac{5}{2}}(c + dx)} + \frac{2a(A + B) \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} + \frac{2a(5A + 7B) \sin(c + dx)}{21d \sqrt{\sec(c + dx)}} \\
&= \frac{6a(A + B) \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{\sec(c + dx)}}{5d} + \frac{2a(5A + 7B)}{21d}
\end{aligned}$$

Mathematica [A] time = 0.99, size = 113, normalized size = 0.66

$$a\sqrt{\sec(c+dx)} \left(\sin(2(c+dx))(42(A+B)\cos(c+dx) + 15A\cos(2(c+dx)) + 65A + 70B) + 20(5A+7B)\sqrt{\cos(c+dx)} \right)$$

$$210d$$

Antiderivative was successfully verified.

[In] Integrate[((a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(7/2), x]

[Out] (a*Sqrt[Sec[c + d*x]]*(252*(A + B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 20*(5*A + 7*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (65*A + 70*B + 42*(A + B)*Cos[c + d*x] + 15*A*Cos[2*(c + d*x)])*Sin[2*(c + d*x)])/(210*d)

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{Ba \sec(dx+c)^2 + (A+B)a \sec(dx+c) + Aa}{\sec(dx+c)^{\frac{7}{2}}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2), x, algorithm="fricas")

[Out] integral((B*a*sec(d*x + c)^2 + (A + B)*a*sec(d*x + c) + A*a)/sec(d*x + c)^(7/2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx+c) + A)(a \sec(dx+c) + a)}{\sec(dx+c)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2), x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)/sec(d*x + c)^(7/2), x)

maple [A] time = 4.20, size = 383, normalized size = 2.23

$$2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} a \left(240A \cos\left(\frac{dx}{2} + \frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + (-528A - 168B)\left(\sin^6\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \dots \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x)`

[Out]
$$-2/105*((2*\cos(1/2*d*x+1/2*c)^2-1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*a*(240*A*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^8+(-528*A-168*B)*\sin(1/2*d*x+1/2*c)^6*\cos(1/2*d*x+1/2*c)+(448*A+308*B)*\sin(1/2*d*x+1/2*c)^4*\cos(1/2*d*x+1/2*c)+(-122*A-112*B)*\sin(1/2*d*x+1/2*c)^2*\cos(1/2*d*x+1/2*c)+25*A*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)})-63*A*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)})+35*B*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)})-63*B*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)}))/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(a \sec(dx + c) + a)}{\sec(dx + c)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm="maxima")`

[Out] `integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)/sec(d*x + c)^(7/2), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(a + \frac{a}{\cos(c+dx)}\right)}{\left(\frac{1}{\cos(c+dx)}\right)^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x)))/(1/cos(c + d*x))^(7/2),x)`

[Out] `int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x)))/(1/cos(c + d*x))^(7/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a \left(\int \frac{A}{\sec^{\frac{7}{2}}(c + dx)} dx + \int \frac{A}{\sec^{\frac{5}{2}}(c + dx)} dx + \int \frac{B}{\sec^{\frac{5}{2}}(c + dx)} dx + \int \frac{B}{\sec^{\frac{3}{2}}(c + dx)} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)**(7/2),x)
```

```
[Out] a*(Integral(A/sec(c + d*x)**(7/2), x) + Integral(A/sec(c + d*x)**(5/2), x)
+ Integral(B/sec(c + d*x)**(5/2), x) + Integral(B/sec(c + d*x)**(3/2), x))
```

$$3.186 \quad \int \sec^2(c+dx)(a+a \sec(c+dx))^2(A+B \sec(c+dx)) dx$$

Optimal. Leaf size=234

$$\frac{2a^2(7A+9B) \sin(c+dx) \sec^{\frac{5}{2}}(c+dx)}{35d} + \frac{4a^2(7A+6B) \sin(c+dx) \sec^{\frac{3}{2}}(c+dx)}{21d} + \frac{4a^2(4A+3B) \sin(c+dx) \sqrt{\sec(c+dx)}}{5d}$$

[Out] $4/21*a^2*(7*A+6*B)*\sec(d*x+c)^{(3/2)}*\sin(d*x+c)/d+2/35*a^2*(7*A+9*B)*\sec(d*x+c)^{(5/2)}*\sin(d*x+c)/d+2/7*B*\sec(d*x+c)^{(5/2)}*(a^2+a^2*\sec(d*x+c))*\sin(d*x+c)/d+4/5*a^2*(4*A+3*B)*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/d-4/5*a^2*(4*A+3*B)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d+4/21*a^2*(7*A+6*B)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d$

Rubi [A] time = 0.34, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {4018, 3997, 3787, 3768, 3771, 2639, 2641}

$$\frac{2a^2(7A+9B) \sin(c+dx) \sec^{\frac{5}{2}}(c+dx)}{35d} + \frac{4a^2(7A+6B) \sin(c+dx) \sec^{\frac{3}{2}}(c+dx)}{21d} + \frac{4a^2(4A+3B) \sin(c+dx) \sqrt{\sec(c+dx)}}{5d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sec}[c+d*x]^{(3/2)}*(a+a*\text{Sec}[c+d*x])^2*(A+B*\text{Sec}[c+d*x]),x]$

[Out] $(-4*a^2*(4*A+3*B)*\text{Sqrt}[\text{Cos}[c+d*x]]*\text{EllipticE}[(c+d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c+d*x]])/(5*d) + (4*a^2*(7*A+6*B)*\text{Sqrt}[\text{Cos}[c+d*x]]*\text{EllipticF}[(c+d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c+d*x]])/(21*d) + (4*a^2*(4*A+3*B)*\text{Sqrt}[\text{Sec}[c+d*x]]*\text{Sin}[c+d*x])/(5*d) + (4*a^2*(7*A+6*B)*\text{Sec}[c+d*x]^{(3/2)}*\text{Sin}[c+d*x])/(21*d) + (2*a^2*(7*A+9*B)*\text{Sec}[c+d*x]^{(5/2)}*\text{Sin}[c+d*x])/(35*d) + (2*B*\text{Sec}[c+d*x]^{(5/2)}*(a^2+a^2*\text{Sec}[c+d*x])*\text{Sin}[c+d*x])/(7*d)$

Rule 2639

$\text{Int}[\text{Sqrt}[\sin[(c_.)+(d_.)*(x_)]], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticE}[(1*(c-Pi/2+d*x))/2, 2])/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 2641

$\text{Int}[1/\text{Sqrt}[\sin[(c_.)+(d_.)*(x_)]], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticF}[(1*(c-Pi/2+d*x))/2, 2])/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]
)*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] := -Simp[(b*B*Cot[
e + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[
e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && !LeQ[n,
-1]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] := -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && !LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2(A+B\sec(c+dx))dx &= \frac{2B\sec^{\frac{5}{2}}(c+dx)(a^2+a^2\sec(c+dx))\sin(c+dx)}{7d} + \\
&= \frac{2a^2(7A+9B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{35d} + \frac{2B\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d} \\
&= \frac{2a^2(7A+9B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{35d} + \frac{2B\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d} \\
&= \frac{4a^2(4A+3B)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d} + \frac{4a^2(7A+9B)\sqrt{\sec(c+dx)}\sin(c+dx)}{35d} \\
&= \frac{4a^2(4A+3B)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d} + \frac{4a^2(7A+9B)\sqrt{\sec(c+dx)}\sin(c+dx)}{35d} \\
&= -\frac{4a^2(4A+3B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}
\end{aligned}$$

Mathematica [C] time = 4.67, size = 463, normalized size = 1.98

$$a^2 \csc(c)e^{-idx} \cos^3(c+dx) \sec^4\left(\frac{1}{2}(c+dx)\right) (\sec(c+dx)+1)^2 (A+B\sec(c+dx)) \left(7\sqrt{2}(-1+e^{2ic})(4A+3B)e^{2ic}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
[Out] (a^2*Cos[c + d*x]^3*Csc[c]*Sec[(c + d*x)/2]^4*(7*Sqrt[2]*(4*A + 3*B)*E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]) - ((-1 + E^((2*I)*c))*(7*A*(-5 + 9*E^(I*(c + d*x))) - 5*E^((2*I)*(c + d*x)) + 36*E^((3*I)*(c + d*x)) + 5*E^((4*I)*(c + d*x)) + 39*E^((5*I)*(c + d*x)) + 5*E^((6*I)*(c + d*x)) + 12*E^((7*I)*(c + d*x))) + 3*B*(-10 + 7*E^(I*(c + d*x)) - 20*E^((2*I)*(c + d*x)) + 63*E^((3*I)*(c + d*x)) + 20*E^((4*I)*(c + d*x)) + 77*E^((5*I)*(c + d*x)) + 10*E^((6*I)*(c + d*x)) + 21*E^((7*I)*(c + d*x))) + (5*I)*(7*A + 6*B)*(1 + E^((2*I)*(c + d*x)))^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])*Sqrt[Sec[c + d*x]]/(E^(I*(c - d*x))*(1 + E^((2*I)*(c + d*x)))^3)*(1 + Sec[c + d*x])^2*(A + B*Sec[c + d*x]))/(210*d*E^(I*d*x)*(B + A*Cos[c + d*x]))
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Ba^2 \sec(dx+c)^4 + (A+2B)a^2 \sec(dx+c)^3 + (2A+B)a^2 \sec(dx+c)^2 + Aa^2 \sec(dx+c)\right)\sqrt{\sec(dx+c)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] integral((B*a^2*sec(d*x+c)^4 + (A+2*B)*a^2*sec(d*x+c)^3 + (2*A+B)*a^2*sec(d*x+c)^2 + A*a^2*sec(d*x+c))*sqrt(sec(d*x+c)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx+c) + A)(a \sec(dx+c) + a)^2 \sec(dx+c)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] integrate((B*sec(d*x+c) + A)*(a*sec(d*x+c) + a)^2*sec(d*x+c)^(3/2), x)

maple [B] time = 15.56, size = 852, normalized size = 3.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x)

[Out] $-a^2 * (-(-2 * \cos(1/2 * d * x + 1/2 * c) ^ 2 + 1) * \sin(1/2 * d * x + 1/2 * c) ^ 2) ^ (1/2) * (8 * (1/2 * A + 1/4 * B) * (-1/6 * \cos(1/2 * d * x + 1/2 * c) * (-2 * \sin(1/2 * d * x + 1/2 * c) ^ 4 + \sin(1/2 * d * x + 1/2 * c) ^ 2) ^ (1/2) / (-1/2 + \cos(1/2 * d * x + 1/2 * c) ^ 2) ^ 2 + 1/3 * (\sin(1/2 * d * x + 1/2 * c) ^ 2) ^ (1/2) * (-2 * \cos(1/2 * d * x + 1/2 * c) ^ 2 + 1) ^ (1/2) / (-2 * \sin(1/2 * d * x + 1/2 * c) ^ 4 + \sin(1/2 * d * x + 1/2 * c) ^ 2) ^ (1/2) * \text{EllipticF}(\cos(1/2 * d * x + 1/2 * c), 2 ^ (1/2))) + 2 * A * (-(-2 * \sin(1/2 * d * x + 1/2 * c) ^ 4 + \sin(1/2 * d * x + 1/2 * c) ^ 2) ^ (1/2) * (\sin(1/2 * d * x + 1/2 * c) ^ 2) ^ (1/2) * (2 * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 1) ^ (1/2) * \text{EllipticE}(\cos(1/2 * d * x + 1/2 * c), 2 ^ (1/2))) + 2 * (-2 * \sin(1/2 * d * x + 1/2 * c) ^ 4 + \sin(1/2 * d * x + 1/2 * c) ^ 2) ^ (1/2) * \cos(1/2 * d * x + 1/2 * c) * \sin(1/2 * d * x + 1/2 * c) ^ 2) / \sin(1/2 * d * x + 1/2 * c) ^ 2 / (2 * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 1) + 2 * B * (-1/56 * \cos(1/2 * d * x + 1/2 * c) * (-2 * \sin(1/2 * d * x + 1/2 * c) ^ 4 + \sin(1/2 * d * x + 1/2 * c) ^ 2) ^ (1/2) / (-1/2 + \cos(1/2 * d * x + 1/2 * c) ^ 2) ^ 4 - 5/42 * \cos(1/2 * d * x + 1/2 * c) * (-2 * \sin(1/2 * d * x + 1/2 * c) ^ 4 + \sin(1/2 * d * x + 1/2 * c) ^ 2) ^ (1/2) / (-1/2 + \cos(1/2 * d * x + 1/2 * c) ^ 2) ^ 2 + 5/21 * (\sin(1/2 * d * x + 1/2 * c) ^ 2) ^ (1/2) * (-2 * \cos(1/2 * d * x + 1/2 * c) ^ 2 + 1) ^ (1/2) / (-2 * \sin(1/2 * d * x + 1/2 * c) ^ 4 + \sin(1/2 * d * x + 1/2 * c) ^ 2) ^ (1/2) * \text{EllipticF}(\cos(1/2 * d * x + 1/2 * c), 2 ^ (1/2))) - 8/5 * (1/4 * A + 1/2 * B) / (8$

```
*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin
(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x
+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2))-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1
/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm
="maxima")
```

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(A + \frac{B}{\cos(c + dx)} \right) \left(a + \frac{a}{\cos(c + dx)} \right)^2 \left(\frac{1}{\cos(c + dx)} \right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^2*(1/cos(c + d*x))^(3/2),x)
```

```
[Out] int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^2*(1/cos(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(a+a*sec(d*x+c))**2*(A+B*sec(d*x+c)),x)
```

[Out] Timed out

$$3.187 \quad \int \sqrt{\sec(c + dx)} (a + a \sec(c + dx))^2 (A + B \sec(c + dx)) dx$$

Optimal. Leaf size=199

$$\frac{2a^2(5A + 7B) \sin(c + dx) \sec^{\frac{3}{2}}(c + dx)}{15d} + \frac{4a^2(5A + 4B) \sin(c + dx) \sqrt{\sec(c + dx)}}{5d} + \frac{4a^2(2A + B) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)}}{3d}$$

[Out] $2/15*a^2*(5*A+7*B)*\sec(d*x+c)^{(3/2)}*\sin(d*x+c)/d+2/5*B*\sec(d*x+c)^{(3/2)}*(a^2+a^2*\sec(d*x+c))*\sin(d*x+c)/d+4/5*a^2*(5*A+4*B)*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/d-4/5*a^2*(5*A+4*B)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c),2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d+4/3*a^2*(2*A+B)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c),2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d$

Rubi [A] time = 0.30, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {4018, 3997, 3787, 3771, 2641, 3768, 2639}

$$\frac{2a^2(5A + 7B) \sin(c + dx) \sec^{\frac{3}{2}}(c + dx)}{15d} + \frac{4a^2(5A + 4B) \sin(c + dx) \sqrt{\sec(c + dx)}}{5d} + \frac{4a^2(2A + B) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)}}{3d}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]

[Out] $(-4*a^2*(5*A + 4*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/(5*d) + (4*a^2*(2*A + B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticF}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/(3*d) + (4*a^2*(5*A + 4*B)*\text{Sqrt}[\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(5*d) + (2*a^2*(5*A + 7*B)*\text{Sec}[c + d*x]^{(3/2)}*\text{Sin}[c + d*x])/(15*d) + (2*B*\text{Sec}[c + d*x]^{(3/2)}*(a^2 + a^2*\text{Sec}[c + d*x])*\text{Sin}[c + d*x])/(5*d)$

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 3768

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x] * (b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3771

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x])^n * Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 3997

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] := -Simp[(b*B*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e + f*x])^n * Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && !LeQ[n, -1]

Rule 4018

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] := -Simp[(b*B*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n * Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && !LtQ[n, -1]

Rubi steps

$$\begin{aligned}
\int \sqrt{\sec(c+dx)} (a + a \sec(c+dx))^2 (A + B \sec(c+dx)) dx &= \frac{2B \sec^{\frac{3}{2}}(c+dx) (a^2 + a^2 \sec(c+dx)) \sin(c+dx)}{5d} + \\
&= \frac{2a^2(5A+7B) \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{15d} + \frac{2B \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{5d} \\
&= \frac{2a^2(5A+7B) \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{15d} + \frac{2B \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{5d} \\
&= \frac{4a^2(5A+4B) \sqrt{\sec(c+dx)} \sin(c+dx)}{5d} + \frac{2a^2(5A+7B) \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{15d} \\
&= \frac{4a^2(2A+B) \sqrt{\cos(c+dx)} F\left(\frac{1}{2}(c+dx) \middle| 2\right) \sqrt{\sec(c+dx)}}{3d} \\
&= -\frac{4a^2(5A+4B) \sqrt{\cos(c+dx)} E\left(\frac{1}{2}(c+dx) \middle| 2\right) \sqrt{\sec(c+dx)}}{5d}
\end{aligned}$$

Mathematica [C] time = 6.57, size = 321, normalized size = 1.61

$$\frac{a^2 e^{-ic} (-1 + e^{2ic}) \csc(c) \sec^4\left(\frac{1}{2}(c+dx)\right) (\sec(c+dx) + 1)^2 (A + B \sec(c+dx)) \left(2(5A+4B) e^{i(c+dx)} (1 + e^{2i(c+dx)})^{5/2}\right)}{5d}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]

[Out] (a^2*(-1 + E^((2*I)*c))*Csc[c]*(5*A + 10*B - 30*A*E^(I*(c + d*x)) - 18*B*E^(I*(c + d*x)) - 60*A*E^((3*I)*(c + d*x)) - 54*B*E^((3*I)*(c + d*x)) - 5*A*E^((4*I)*(c + d*x)) - 10*B*E^((4*I)*(c + d*x)) - 30*A*E^((5*I)*(c + d*x)) - 24*B*E^((5*I)*(c + d*x)) - (10*I)*(2*A + B)*(1 + E^((2*I)*(c + d*x))))^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 2*(5*A + 4*B)*E^(I*(c + d*x))*(1 + E^((2*I)*(c + d*x)))^(5/2)*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*Sec[(c + d*x)/2]^4*(1 + Sec[c + d*x])^2*(A + B*Sec[c + d*x]))/(60*d*E^(I*c)*(1 + E^((2*I)*(c + d*x)))^2*(B + A*Cos[c + d*x])*Sec[c + d*x]^(5/2))

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Ba^2 \sec(dx+c)^3 + (A+2B)a^2 \sec(dx+c)^2 + (2A+B)a^2 \sec(dx+c) + Aa^2\right)\sqrt{\sec(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*a^2*sec(d*x + c)^3 + (A + 2*B)*a^2*sec(d*x + c)^2 + (2*A + B)*a^2*sec(d*x + c) + A*a^2)*sqrt(sec(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(a \sec(dx + c) + a)^2 \sqrt{\sec(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^2*sqrt(sec(d*x + c)), x)
```

maple [B] time = 13.12, size = 743, normalized size = 3.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x)
```

```
[Out] -a^2*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+8*(1/4*A+1/2*B)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+8*(1/2*A+1/4*B)*(-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)-2/5*B/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2
```

$$\begin{aligned} & \left(\frac{1}{2} \right) - 8 \sin\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 \cos\left(\frac{1}{2}dx + \frac{1}{2}c\right) * \left(-2 \sin\left(\frac{1}{2}dx + \frac{1}{2}c\right)^4 \right. \\ & \left. + \sin\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 \right)^{\frac{1}{2}} / \sin\left(\frac{1}{2}dx + \frac{1}{2}c\right) / \left(2 \cos\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - 1 \right)^{\frac{1}{2}} / d \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(a \sec(dx + c) + a)^2 \sqrt{\sec(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^2*sqrt(sec(d*x + c)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A + \frac{B}{\cos(c + dx)} \right) \left(a + \frac{a}{\cos(c + dx)} \right)^2 \sqrt{\frac{1}{\cos(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^2*(1/cos(c + d*x))^(1/2),x)

[Out] int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^2*(1/cos(c + d*x))^(1/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))*2*(A+B*sec(d*x+c))*sec(d*x+c)**(1/2),x)

[Out] Timed out

$$3.188 \quad \int \frac{(a+a \sec(c+dx))^2(A+B \sec(c+dx))}{\sqrt{\sec(c+dx)}} dx$$

Optimal. Leaf size=160

$$\frac{2a^2(3A+5B) \sin(c+dx) \sqrt{\sec(c+dx)}}{3d} + \frac{4a^2(3A+2B) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} F\left(\frac{1}{2}(c+dx) \middle| 2\right)}{3d} + \frac{2B \sin(c+dx)}{3d}$$

[Out] $2/3*a^2*(3*A+5*B)*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/d+2/3*B*(a^2+a^2*\sec(d*x+c))*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/d-4*a^2*B*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c),2^{(1/2)})*\cos(d*x+c)^{(1/2)*\sec(d*x+c)^{(1/2)}/d+4/3*a^2*(3*A+2*B)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c),2^{(1/2)})*\cos(d*x+c)^{(1/2)*\sec(d*x+c)^{(1/2)}/d$

Rubi [A] time = 0.28, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4018, 3997, 3787, 3771, 2639, 2641}

$$\frac{2a^2(3A+5B) \sin(c+dx) \sqrt{\sec(c+dx)}}{3d} + \frac{4a^2(3A+2B) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} F\left(\frac{1}{2}(c+dx) \middle| 2\right)}{3d} + \frac{2B \sin(c+dx)}{3d}$$

Antiderivative was successfully verified.

[In] Int[((a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]))/Sqrt[Sec[c + d*x]], x]

[Out] $(-4*a^2*B*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/d + (4*a^2*(3*A + 2*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticF}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/(3*d) + (2*a^2*(3*A + 5*B)*\text{Sqrt}[\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(3*d) + (2*B*\text{Sqrt}[\text{Sec}[c + d*x]]*(a^2 + a^2*\text{Sec}[c + d*x])*\text{Sin}[c + d*x])/(3*d)$

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 3771

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^n, x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&

EqQ[n^2, 1/4]

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 3997

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> -Simp[(b*B*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && !LeQ[n, -1]

Rule 4018

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> -Simp[(b*B*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && !LtQ[n, -1]

Rubi steps

$$\begin{aligned}
\int \frac{(a + a \sec(c + dx))^2 (A + B \sec(c + dx))}{\sqrt{\sec(c + dx)}} dx &= \frac{2B\sqrt{\sec(c + dx)} (a^2 + a^2 \sec(c + dx)) \sin(c + dx)}{3d} + \frac{2}{3} \int \frac{(a + a \sec(c + dx))^2 (A + B \sec(c + dx))}{\sqrt{\sec(c + dx)}} dx \\
&= \frac{2a^2(3A + 5B)\sqrt{\sec(c + dx)} \sin(c + dx)}{3d} + \frac{2B\sqrt{\sec(c + dx)} (a^2 + a^2 \sec(c + dx)) \sin(c + dx)}{3d} \\
&= \frac{2a^2(3A + 5B)\sqrt{\sec(c + dx)} \sin(c + dx)}{3d} + \frac{2B\sqrt{\sec(c + dx)} (a^2 + a^2 \sec(c + dx)) \sin(c + dx)}{3d} \\
&= \frac{2a^2(3A + 5B)\sqrt{\sec(c + dx)} \sin(c + dx)}{3d} + \frac{2B\sqrt{\sec(c + dx)} (a^2 + a^2 \sec(c + dx)) \sin(c + dx)}{3d} \\
&= -\frac{4a^2B\sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{\sec(c + dx)}}{d} + \frac{4a^2(3A + 5B)\sqrt{\sec(c + dx)} \sin(c + dx)}{3d}
\end{aligned}$$

Mathematica [C] time = 3.21, size = 295, normalized size = 1.84

$$a^2 \sec^4\left(\frac{1}{2}(c + dx)\right) (\sec(c + dx) + 1)^2 (A + B \sec(c + dx)) \left(\frac{-3 \csc(c) \cos(dx) (A \cos(2c) - A - 4B) + 6A \cos(c) \sin(dx) + 2B \tan(c + dx)}{4d \sec^{\frac{5}{2}}(c + dx)} \right)$$

$$3(A \cos(c + dx) + B \sec(c + dx))$$

Antiderivative was successfully verified.

[In] Integrate[((a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]))/Sqrt[Sec[c + d*x]], x]

[Out] (a^2*Sec[(c + d*x)/2]^4*(1 + Sec[c + d*x])^2*(A + B*Sec[c + d*x])*(((-I)*Sqrt[2]*Cos[c + d*x]^3*(3*B*Sqrt[1 + E^((2*I)*(c + d*x))] + 3*B*(-1 + E^((2*I)*(c + d*x)))*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] + (3*A + 2*B)*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))])/(d*(-1 + E^((2*I)*c))*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))])*Sqrt[1 + E^((2*I)*(c + d*x))]) + (-3*(-A - 4*B + A*Cos[2*c])*Cos[d*x]*Csc[c] + 6*A*Cos[c]*Sin[d*x] + 2*B*Tan[c + d*x])/(4*d*Sec[c + d*x]^(5/2)))/(3*(B + A*Cos[c + d*x]))

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{Ba^2 \sec(dx + c)^3 + (A + 2B)a^2 \sec(dx + c)^2 + (2A + B)a^2 \sec(dx + c) + Aa^2}{\sqrt{\sec(dx + c)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="fricas")

[Out] integral((B*a^2*sec(d*x + c)^3 + (A + 2*B)*a^2*sec(d*x + c)^2 + (2*A + B)*a^2*sec(d*x + c) + A*a^2)/sqrt(sec(d*x + c)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(a \sec(dx + c) + a)^2}{\sqrt{\sec(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^2/sqrt(sec(d*x + c)), x)

maple [B] time = 5.14, size = 513, normalized size = 3.21

$$4a^2 \left(6\sqrt{-2 \left(\sin^4 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) + \sin^2 \left(\frac{dx}{2} + \frac{c}{2} \right)} (A + 2B) \cos \left(\frac{dx}{2} + \frac{c}{2} \right) \left(\sin^4 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) - \sqrt{-2 \left(\sin^4 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) + \sin^2 \left(\frac{dx}{2} + \frac{c}{2} \right)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x)

[Out]
$$\begin{aligned} & -4/3*a^2*(6*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(A+2*B)*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^4-(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(3*A+7*B)*\sin(1/2*d*x+1/2*c)^2*\cos(1/2*d*x+1/2*c)-2*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*(3*A*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})+2*B*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})+3*B*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)}))*\sin(1/2*d*x+1/2*c)^2+3*A*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}+2*B*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}+3*B*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)}))/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(3/2)}/\sin(1/2*d*x+1/2*c)/d \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(a \sec(dx + c) + a)^2}{\sqrt{\sec(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^2/sqrt(sec(d*x + c)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(a + \frac{a}{\cos(c+dx)}\right)^2}{\sqrt{\frac{1}{\cos(c+dx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^2)/(1/cos(c + d*x))^(1/2),x)

[Out] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^2)/(1/cos(c + d*x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2 \left(\int \frac{A}{\sqrt{\sec(c + dx)}} dx + \int 2A\sqrt{\sec(c + dx)} dx + \int A \sec^{\frac{3}{2}}(c + dx) dx + \int B\sqrt{\sec(c + dx)} dx + \int 2B \sec^{\frac{3}{2}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)**(1/2),x)

[Out] a**2*(Integral(A/sqrt(sec(c + d*x)), x) + Integral(2*A*sqrt(sec(c + d*x)), x) + Integral(A*sec(c + d*x)**(3/2), x) + Integral(B*sqrt(sec(c + d*x)), x) + Integral(2*B*sec(c + d*x)**(3/2), x) + Integral(B*sec(c + d*x)**(5/2), x))

$$3.189 \quad \int \frac{(a+a \sec(c+dx))^2(A+B \sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)} dx$$

Optimal. Leaf size=158

$$-\frac{2a^2(A-3B) \sin(c+dx) \sqrt{\sec(c+dx)}}{3d} + \frac{4a^2(2A+3B) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} F\left(\frac{1}{2}(c+dx) \middle| 2\right)}{3d} + \frac{2A \sin(c+dx)}{3d}$$

[Out] $2/3*A*(a^2+a^2*\sec(d*x+c))*\sin(d*x+c)/d/\sec(d*x+c)^{(1/2)}-2/3*a^2*(A-3*B)*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/d+4*a^2*A*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d+4/3*a^2*(2*A+3*B)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d$

Rubi [A] time = 0.26, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4017, 3997, 3787, 3771, 2639, 2641}

$$-\frac{2a^2(A-3B) \sin(c+dx) \sqrt{\sec(c+dx)}}{3d} + \frac{4a^2(2A+3B) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} F\left(\frac{1}{2}(c+dx) \middle| 2\right)}{3d} + \frac{2A \sin(c+dx)}{3d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + a*\text{Sec}[c + d*x])^2*(A + B*\text{Sec}[c + d*x])]/\text{Sec}[c + d*x]^{(3/2)}, x]$

[Out] $(4*a^2*A*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/d + (4*a^2*(2*A + 3*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticF}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/(3*d) - (2*a^2*(A - 3*B)*\text{Sqrt}[\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(3*d) + (2*A*(a^2 + a^2*\text{Sec}[c + d*x])*\text{Sin}[c + d*x])/(3*d*\text{Sqrt}[\text{Sec}[c + d*x]])$

Rule 2639

$\text{Int}[\text{Sqrt}[\sin[(c_.) + (d_.)*(x_.)]], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticE}[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 2641

$\text{Int}[1/\text{Sqrt}[\sin[(c_.) + (d_.)*(x_.)]], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticF}[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 3771

$\text{Int}[(\text{csc}[(c_.) + (d_.)*(x_.)]*(b_.))^{(n_.)}, x_Symbol] \rightarrow \text{Dist}[(b*\text{Csc}[c + d*x])^{(n)}*\text{Sin}[c + d*x]^n, \text{Int}[1/\text{Sin}[c + d*x]^n, x], x] /; \text{FreeQ}\{b, c, d\}, x] \&\&$

EqQ[n^2, 1/4]

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) +
(a_.)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) +
(a_.))*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> -Simp[(b*B*Cot[e
+ f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && !LeQ[n,
-1]
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) +
(a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + a \sec(c + dx))^2 (A + B \sec(c + dx))}{\sec^{\frac{3}{2}}(c + dx)} dx &= \frac{2A (a^2 + a^2 \sec(c + dx)) \sin(c + dx)}{3d \sqrt{\sec(c + dx)}} + \frac{2}{3} \int \frac{(a + a \sec(c + dx))}{\sec^{\frac{3}{2}}(c + dx)} dx \\
&= -\frac{2a^2 (A - 3B) \sqrt{\sec(c + dx)} \sin(c + dx)}{3d} + \frac{2A (a^2 + a^2 \sec(c + dx)) \sin(c + dx)}{3d \sqrt{\sec(c + dx)}} \\
&= -\frac{2a^2 (A - 3B) \sqrt{\sec(c + dx)} \sin(c + dx)}{3d} + \frac{2A (a^2 + a^2 \sec(c + dx)) \sin(c + dx)}{3d \sqrt{\sec(c + dx)}} \\
&= -\frac{2a^2 (A - 3B) \sqrt{\sec(c + dx)} \sin(c + dx)}{3d} + \frac{2A (a^2 + a^2 \sec(c + dx)) \sin(c + dx)}{3d \sqrt{\sec(c + dx)}} \\
&= \frac{4a^2 A \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{\sec(c + dx)}}{d} + \frac{4a^2 (2A + 3B) \sin(c + dx)}{3d}
\end{aligned}$$

Mathematica [C] time = 3.01, size = 299, normalized size = 1.89

$$a^2 \sec^4\left(\frac{1}{2}(c + dx)\right) (\sec(c + dx) + 1)^2 (A + B \sec(c + dx)) \left(\frac{-3(2A-B) \csc(c) \cos(dx) - 3(2A+B) \csc(c) \cos(2c+dx) + A \sin(2(c+dx))}{4d \sec^{\frac{5}{2}}(c+dx)} + \frac{3(A \cos(c + dx) + B)}{d} \right)$$

Antiderivative was successfully verified.

[In] Integrate[((a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(3/2), x]

[Out] (a^2*Sec[(c + d*x)/2]^4*(1 + Sec[c + d*x])^2*(A + B*Sec[c + d*x])*((I*Sqrt[2]*Cos[c + d*x]^3*(3*A*Sqrt[1 + E^((2*I)*(c + d*x))] + 3*A*(-1 + E^((2*I)*c))*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] - (2*A + 3*B)*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(d*(-1 + E^((2*I)*c))*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]) + (-3*(2*A - B)*Cos[d*x]*Csc[c] - 3*(2*A + B)*Cos[2*c + d*x]*Csc[c] + A*Sin[2*(c + d*x)])/(4*d*Sec[c + d*x]^(5/2)))/(3*(B + A*Cos[c + d*x]))

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{Ba^2 \sec(dx + c)^3 + (A + 2B)a^2 \sec(dx + c)^2 + (2A + B)a^2 \sec(dx + c) + Aa^2}{\sec(dx + c)^{\frac{3}{2}}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="fricas")

[Out] integral((B*a^2*sec(d*x + c)^3 + (A + 2*B)*a^2*sec(d*x + c)^2 + (2*A + B)*a^2*sec(d*x + c) + A*a^2)/sec(d*x + c)^(3/2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(a \sec(dx + c) + a)^2}{\sec(dx + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^2/sec(d*x + c)^(3/2), x)

maple [B] time = 4.64, size = 388, normalized size = 2.46

$$4a^2 \left(2A \sqrt{-2 \left(\sin^4 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) + \sin^2 \left(\frac{dx}{2} + \frac{c}{2} \right)} \cos \left(\frac{dx}{2} + \frac{c}{2} \right) \left(\sin^4 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) - \sqrt{-2 \left(\sin^4 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) + \sin^2 \left(\frac{dx}{2} \right)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x)

[Out] $-4/3*a^2*(2*A*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^4-(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(A+3*B)*\sin(1/2*d*x+1/2*c)^2*\cos(1/2*d*x+1/2*c)+2*A*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)})*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}-3*A*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)})+3*B*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)})*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/\sin(1/2*d*x+1/2*c)/((2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(a \sec(dx + c) + a)^2}{\sec(dx + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^2/sec(d*x + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(a + \frac{a}{\cos(c+dx)}\right)^2}{\left(\frac{1}{\cos(c+dx)}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^2)/(1/cos(c + d*x))^(3/2),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^2)/(1/cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2 \left(\int \frac{A}{\sec^{\frac{3}{2}}(c+dx)} dx + \int \frac{2A}{\sqrt{\sec(c+dx)}} dx + \int A\sqrt{\sec(c+dx)} dx + \int \frac{B}{\sqrt{\sec(c+dx)}} dx + \int 2B\sqrt{\sec(c+dx)} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*(A+B*sec(d*x+c))/sec(d*x+c)**(3/2),x)
```

```
[Out] a**2*(Integral(A/sec(c + d*x)**(3/2), x) + Integral(2*A/sqrt(sec(c + d*x)), x) + Integral(A*sqrt(sec(c + d*x)), x) + Integral(B/sqrt(sec(c + d*x)), x) + Integral(2*B*sqrt(sec(c + d*x)), x) + Integral(B*sec(c + d*x)**(3/2), x))
```

$$3.190 \quad \int \frac{(a+a \sec(c+dx))^2(A+B \sec(c+dx))}{\sec^{\frac{5}{2}}(c+dx)} dx$$

Optimal. Leaf size=166

$$\frac{2a^2(7A+5B)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}} + \frac{4a^2(A+2B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{4a^2(4A+5B)\sqrt{\cos(c+dx)}}{3d}$$

[Out] $2/5*A*(a^2+a^2*\sec(d*x+c))*\sin(d*x+c)/d/\sec(d*x+c)^{(3/2)}+2/15*a^2*(7*A+5*B)*\sin(d*x+c)/d/\sec(d*x+c)^{(1/2)}+4/5*a^2*(4*A+5*B)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c),2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d+4/3*a^2*(A+2*B)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c),2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d$

Rubi [A] time = 0.26, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4017, 3996, 3787, 3771, 2639, 2641}

$$\frac{2a^2(7A+5B)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}} + \frac{4a^2(A+2B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{4a^2(4A+5B)\sqrt{\cos(c+dx)}}{3d}$$

Antiderivative was successfully verified.

[In] Int[((a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(5/2), x]

[Out] $(4*a^2*(4*A+5*B)*\text{Sqrt}[\text{Cos}[c+d*x]]*\text{EllipticE}[(c+d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c+d*x]])/(5*d) + (4*a^2*(A+2*B)*\text{Sqrt}[\text{Cos}[c+d*x]]*\text{EllipticF}[(c+d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c+d*x]])/(3*d) + (2*a^2*(7*A+5*B)*\text{Sin}[c+d*x])/(15*d*\text{Sqrt}[\text{Sec}[c+d*x]]) + (2*A*(a^2+a^2*\text{Sec}[c+d*x])*\text{Sin}[c+d*x])/(5*d*\text{Sec}[c+d*x]^{(3/2)})$

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 3771

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^n], x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^n]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 3996

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^n]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] := Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]

Rule 4017

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^n]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^m*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] := Simp[(a*A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && LtQ[n, -1]

Rubi steps

$$\begin{aligned}
\int \frac{(a + a \sec(c + dx))^2 (A + B \sec(c + dx))}{\sec^{\frac{5}{2}}(c + dx)} dx &= \frac{2A (a^2 + a^2 \sec(c + dx)) \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} + \frac{2}{5} \int \frac{(a + a \sec(c + dx))}{\sec^{\frac{5}{2}}(c + dx)} dx \\
&= \frac{2a^2(7A + 5B) \sin(c + dx)}{15d \sqrt{\sec(c + dx)}} + \frac{2A (a^2 + a^2 \sec(c + dx)) \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} \\
&= \frac{2a^2(7A + 5B) \sin(c + dx)}{15d \sqrt{\sec(c + dx)}} + \frac{2A (a^2 + a^2 \sec(c + dx)) \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} \\
&= \frac{2a^2(7A + 5B) \sin(c + dx)}{15d \sqrt{\sec(c + dx)}} + \frac{2A (a^2 + a^2 \sec(c + dx)) \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} \\
&= \frac{4a^2(4A + 5B) \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{\sec(c + dx)}}{5d} + \dots
\end{aligned}$$

Mathematica [C] time = 1.80, size = 153, normalized size = 0.92

$$\frac{a^2 \sqrt{\sec(c + dx)} \left(-4i(4A + 5B) e^{i(c+dx)} \sqrt{1 + e^{2i(c+dx)}} {}_2F_1\left(\frac{1}{2}, \frac{3}{4}, \frac{7}{4}; -e^{2i(c+dx)}\right) + \cos(c + dx)(10(2A + B) \sin(c + dx)) \right)}{15d}$$

Antiderivative was successfully verified.

[In] Integrate[((a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(5/2), x]

[Out] (a^2*Sqrt[Sec[c + d*x]]*(20*(A + 2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] - (4*I)*(4*A + 5*B)*E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))] + Cos[c + d*x]*((48*I)*A + (60*I)*B + 10*(2*A + B)*Sin[c + d*x] + 3*A*Ssin[2*(c + d*x)])))/(15*d)

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{Ba^2 \sec(dx + c)^3 + (A + 2B)a^2 \sec(dx + c)^2 + (2A + B)a^2 \sec(dx + c) + Aa^2}{\sec(dx + c)^{\frac{5}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="fricas")

[Out] integral((B*a^2*sec(d*x + c)^3 + (A + 2*B)*a^2*sec(d*x + c)^2 + (2*A + B)*a^2*sec(d*x + c) + A*a^2)/sec(d*x + c)^(5/2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(a \sec(dx + c) + a)^2}{\sec(dx + c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^2/sec(d*x + c)^(5/2), x)

maple [A] time = 4.53, size = 357, normalized size = 2.15

$$4\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} a^2 \left(-12A \cos\left(\frac{dx}{2} + \frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + (32A + 10B)\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x)

[Out] -4/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(-12*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(32*A+10*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-13*A-5*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-12*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+10*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-15*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(a \sec(dx + c) + a)^2}{\sec(dx + c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^2/sec(d*x + c)^(5/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(a + \frac{a}{\cos(c+dx)}\right)^2}{\left(\frac{1}{\cos(c+dx)}\right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^2)/(1/cos(c + d*x))^(5/2),x)

[Out] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^2)/(1/cos(c + d*x))^(5/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2 \left(\int \frac{A}{\sec^{\frac{5}{2}}(c+dx)} dx + \int \frac{2A}{\sec^{\frac{3}{2}}(c+dx)} dx + \int \frac{A}{\sqrt{\sec(c+dx)}} dx + \int \frac{B}{\sec^{\frac{3}{2}}(c+dx)} dx + \int \frac{2B}{\sqrt{\sec(c+dx)}} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)**(5/2),x)

[Out] a**2*(Integral(A/sec(c + d*x)**(5/2), x) + Integral(2*A/sec(c + d*x)**(3/2), x) + Integral(A/sqrt(sec(c + d*x)), x) + Integral(B/sec(c + d*x)**(3/2), x) + Integral(2*B/sqrt(sec(c + d*x)), x) + Integral(B*sqrt(sec(c + d*x)), x))

$$3.191 \quad \int \frac{(a+a \sec(c+dx))^2(A+B \sec(c+dx))}{7 \sec^2(c+dx)} dx$$

Optimal. Leaf size=201

$$\frac{2a^2(9A+7B) \sin(c+dx)}{35d \sec^{\frac{3}{2}}(c+dx)} + \frac{4a^2(6A+7B) \sin(c+dx)}{21d \sqrt{\sec(c+dx)}} + \frac{4a^2(6A+7B) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} F\left(\frac{1}{2}(c+dx) \middle| 2\right)}{21d}$$

[Out] 2/35*a^2*(9*A+7*B)*sin(d*x+c)/d/sec(d*x+c)^(3/2)+2/7*A*(a^2+a^2*sec(d*x+c))*sin(d*x+c)/d/sec(d*x+c)^(5/2)+4/21*a^2*(6*A+7*B)*sin(d*x+c)/d/sec(d*x+c)^(1/2)+4/5*a^2*(3*A+4*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+4/21*a^2*(6*A+7*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d

Rubi [A] time = 0.29, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {4017, 3996, 3787, 3769, 3771, 2641, 2639}

$$\frac{2a^2(9A+7B) \sin(c+dx)}{35d \sec^{\frac{3}{2}}(c+dx)} + \frac{4a^2(6A+7B) \sin(c+dx)}{21d \sqrt{\sec(c+dx)}} + \frac{4a^2(6A+7B) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} F\left(\frac{1}{2}(c+dx) \middle| 2\right)}{21d}$$

Antiderivative was successfully verified.

[In] Int[((a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(7/2),x]

[Out] (4*a^2*(3*A + 4*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (4*a^2*(6*A + 7*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*a^2*(9*A + 7*B)*Sin[c + d*x])/(35*d*Sec[c + d*x]^(3/2)) + (4*a^2*(6*A + 7*B)*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x]]) + (2*A*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2))

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 3769


```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] := Simp[(A*a*Cot[e +
f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
+ 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] := Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + a \sec(c + dx))^2 (A + B \sec(c + dx))}{\sec^{\frac{7}{2}}(c + dx)} dx &= \frac{2A (a^2 + a^2 \sec(c + dx)) \sin(c + dx)}{7d \sec^{\frac{5}{2}}(c + dx)} + \frac{2}{7} \int \frac{(a + a \sec(c + dx))}{\sec^{\frac{5}{2}}(c + dx)} dx \\
&= \frac{2a^2(9A + 7B) \sin(c + dx)}{35d \sec^{\frac{3}{2}}(c + dx)} + \frac{2A (a^2 + a^2 \sec(c + dx)) \sin(c + dx)}{7d \sec^{\frac{5}{2}}(c + dx)} \\
&= \frac{2a^2(9A + 7B) \sin(c + dx)}{35d \sec^{\frac{3}{2}}(c + dx)} + \frac{2A (a^2 + a^2 \sec(c + dx)) \sin(c + dx)}{7d \sec^{\frac{5}{2}}(c + dx)} \\
&= \frac{2a^2(9A + 7B) \sin(c + dx)}{35d \sec^{\frac{3}{2}}(c + dx)} + \frac{4a^2(6A + 7B) \sin(c + dx)}{21d \sqrt{\sec(c + dx)}} + \frac{2A (a^2 + a^2 \sec(c + dx)) \sin(c + dx)}{7d \sec^{\frac{5}{2}}(c + dx)} \\
&= \frac{4a^2(3A + 4B) \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{\sec(c + dx)}}{5d} + \frac{2A (a^2 + a^2 \sec(c + dx)) \sin(c + dx)}{7d \sec^{\frac{5}{2}}(c + dx)} \\
&= \frac{4a^2(3A + 4B) \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{\sec(c + dx)}}{5d} + \frac{4a^2(3A + 4B) \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{\sec(c + dx)}}{5d} + \frac{2A (a^2 + a^2 \sec(c + dx)) \sin(c + dx)}{7d \sec^{\frac{5}{2}}(c + dx)}
\end{aligned}$$

Mathematica [C] time = 2.38, size = 193, normalized size = 0.96

$$a^2 e^{-idx} \sqrt{\sec(c + dx)} (\cos(dx) + i \sin(dx)) \left(-56i(3A + 4B) e^{i(c+dx)} \sqrt{1 + e^{2i(c+dx)}} {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{7}{4}; -e^{2i(c+dx)}\right) + \cos(c + dx) \right)$$

Antiderivative was successfully verified.

[In] Integrate[((a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(7/2), x]

[Out] (a^2*Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])*(40*(6*A + 7*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] - (56*I)*(3*A + 4*B)*E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]) + Cos[c + d*x]*((504*I)*A + (672*I)*B + 5*(51*A + 56*B)*Sin[c + d*x] + 42*(2*A + B)*Sin[2*(c + d*x)] + 15*A*Sin[3*(c + d*x)])))/(210*d*E^(I*d*x))

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{Ba^2 \sec(dx + c)^3 + (A + 2B)a^2 \sec(dx + c)^2 + (2A + B)a^2 \sec(dx + c) + Aa^2}{\sec(dx + c)^{\frac{7}{2}}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm="fricas")

[Out] integral((B*a^2*sec(d*x + c)^3 + (A + 2*B)*a^2*sec(d*x + c)^2 + (2*A + B)*a^2*sec(d*x + c) + A*a^2)/sec(d*x + c)^(7/2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(a \sec(dx + c) + a)^2}{\sec(dx + c)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^2/sec(d*x + c)^(7/2), x)

maple [A] time = 4.43, size = 385, normalized size = 1.92

$$4\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} a^2 \left(120A \cos\left(\frac{dx}{2} + \frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + (-348A - 84B)\left(\sin^6\left(\frac{dx}{2}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x)

[Out]
$$\begin{aligned} & -4/105*((2*\cos(1/2*d*x+1/2*c)^2-1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*a^2*(120*A*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^8+(-348*A-84*B)*\sin(1/2*d*x+1/2*c)^6*\cos(1/2*d*x+1/2*c)+(378*A+224*B)*\sin(1/2*d*x+1/2*c)^4*\cos(1/2*d*x+1/2*c)+(-17*A-91*B)*\sin(1/2*d*x+1/2*c)^2*\cos(1/2*d*x+1/2*c)+30*A*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)})-63*A*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)})+35*B*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)})-84*B*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)}))/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(a \sec(dx + c) + a)^2}{\sec(dx + c)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^2/sec(d*x + c)^(7/2), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(a + \frac{a}{\cos(c+dx)}\right)^2}{\left(\frac{1}{\cos(c+dx)}\right)^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^2)/(1/cos(c + d*x))^(7/2),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^2)/(1/cos(c + d*x))^(7/2), x
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2 \left(\int \frac{A}{\sec^{\frac{7}{2}}(c+dx)} dx + \int \frac{2A}{\sec^{\frac{5}{2}}(c+dx)} dx + \int \frac{A}{\sec^{\frac{3}{2}}(c+dx)} dx + \int \frac{B}{\sec^{\frac{5}{2}}(c+dx)} dx + \int \frac{2B}{\sec^{\frac{3}{2}}(c+dx)} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*(A+B*sec(d*x+c))/sec(d*x+c)**(7/2),x)
```

```
[Out] a**2*(Integral(A/sec(c + d*x)**(7/2), x) + Integral(2*A/sec(c + d*x)**(5/2)
, x) + Integral(A/sec(c + d*x)**(3/2), x) + Integral(B/sec(c + d*x)**(5/2),
x) + Integral(2*B/sec(c + d*x)**(3/2), x) + Integral(B/sqrt(sec(c + d*x)),
x))
```

$$3.192 \quad \int \frac{(a+a \sec(c+dx))^2(A+B \sec(c+dx))}{9 \sec^2(c+dx)} dx$$

Optimal. Leaf size=234

$$\frac{4a^2(8A+9B) \sin(c+dx)}{45d \sec^3(c+dx)} + \frac{2a^2(11A+9B) \sin(c+dx)}{63d \sec^5(c+dx)} + \frac{4a^2(5A+6B) \sin(c+dx)}{21d \sqrt{\sec(c+dx)}} + \frac{4a^2(5A+6B) \sqrt{\cos(c+dx)}}{2}$$

[Out] $2/63*a^2*(11*A+9*B)*\sin(d*x+c)/d/\sec(d*x+c)^(5/2)+4/45*a^2*(8*A+9*B)*\sin(d*x+c)/d/\sec(d*x+c)^(3/2)+2/9*A*(a^2+a^2*\sec(d*x+c))*\sin(d*x+c)/d/\sec(d*x+c)^(7/2)+4/21*a^2*(5*A+6*B)*\sin(d*x+c)/d/\sec(d*x+c)^(1/2)+4/15*a^2*(8*A+9*B)*(\cos(1/2*d*x+1/2*c)^2)^(1/2)/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^(1/2))*\cos(d*x+c)^(1/2)*\sec(d*x+c)^(1/2)/d+4/21*a^2*(5*A+6*B)*(\cos(1/2*d*x+1/2*c)^2)^(1/2)/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^(1/2))*\cos(d*x+c)^(1/2)*\sec(d*x+c)^(1/2)/d$

Rubi [A] time = 0.32, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {4017, 3996, 3787, 3769, 3771, 2639, 2641}

$$\frac{4a^2(8A+9B) \sin(c+dx)}{45d \sec^3(c+dx)} + \frac{2a^2(11A+9B) \sin(c+dx)}{63d \sec^5(c+dx)} + \frac{4a^2(5A+6B) \sin(c+dx)}{21d \sqrt{\sec(c+dx)}} + \frac{4a^2(5A+6B) \sqrt{\cos(c+dx)}}{2}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + a*\text{Sec}[c + d*x])^2*(A + B*\text{Sec}[c + d*x])]/\text{Sec}[c + d*x]^(9/2), x]$

[Out] $(4*a^2*(8*A + 9*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/(15*d) + (4*a^2*(5*A + 6*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticF}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/(21*d) + (2*a^2*(11*A + 9*B)*\text{Sin}[c + d*x])/(63*d*\text{Sec}[c + d*x]^(5/2)) + (4*a^2*(8*A + 9*B)*\text{Sin}[c + d*x])/(45*d*\text{Sec}[c + d*x]^(3/2)) + (4*a^2*(5*A + 6*B)*\text{Sin}[c + d*x])/(21*d*\text{Sqrt}[\text{Sec}[c + d*x]]) + (2*A*(a^2 + a^2*\text{Sec}[c + d*x])*\text{Sin}[c + d*x])/(9*d*\text{Sec}[c + d*x]^(7/2))$

Rule 2639

$\text{Int}[\text{Sqrt}[\sin[(c_.) + (d_.)*(x_.)]], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticE}[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 2641

$\text{Int}[1/\text{Sqrt}[\sin[(c_.) + (d_.)*(x_.)]], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticF}[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.))^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x]
)^(n)*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) +
(a_.)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) +
(a_.))*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := Simp[(A*a*Cot[e +
f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
+ 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) +
(a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + a \sec(c + dx))^2 (A + B \sec(c + dx))}{\sec^{\frac{9}{2}}(c + dx)} dx &= \frac{2A (a^2 + a^2 \sec(c + dx)) \sin(c + dx)}{9d \sec^{\frac{7}{2}}(c + dx)} + \frac{2}{9} \int \frac{(a + a \sec(c + dx))}{\sec^{\frac{9}{2}}(c + dx)} dx \\
&= \frac{2a^2(11A + 9B) \sin(c + dx)}{63d \sec^{\frac{5}{2}}(c + dx)} + \frac{2A (a^2 + a^2 \sec(c + dx)) \sin(c + dx)}{9d \sec^{\frac{7}{2}}(c + dx)} \\
&= \frac{2a^2(11A + 9B) \sin(c + dx)}{63d \sec^{\frac{5}{2}}(c + dx)} + \frac{2A (a^2 + a^2 \sec(c + dx)) \sin(c + dx)}{9d \sec^{\frac{7}{2}}(c + dx)} \\
&= \frac{2a^2(11A + 9B) \sin(c + dx)}{63d \sec^{\frac{5}{2}}(c + dx)} + \frac{4a^2(8A + 9B) \sin(c + dx)}{45d \sec^{\frac{3}{2}}(c + dx)} + \frac{4a^2}{15d} \\
&= \frac{2a^2(11A + 9B) \sin(c + dx)}{63d \sec^{\frac{5}{2}}(c + dx)} + \frac{4a^2(8A + 9B) \sin(c + dx)}{45d \sec^{\frac{3}{2}}(c + dx)} + \frac{4a^2}{15d} \\
&= \frac{4a^2(8A + 9B) \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{\sec(c + dx)}}{15d} + \dots
\end{aligned}$$

Mathematica [C] time = 3.00, size = 217, normalized size = 0.93

$$a^2 e^{-idx} \sqrt{\sec(c + dx)} (\cos(dx) + i \sin(dx)) \left(-112i(8A + 9B) e^{i(c+dx)} \sqrt{1 + e^{2i(c+dx)}} {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{7}{4}; -e^{2i(c+dx)}\right) + \cos(c + dx) \right)$$

Antiderivative was successfully verified.

[In] Integrate[((a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(9/2), x]

[Out] (a^2*Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])*(240*(5*A + 6*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] - (112*I)*(8*A + 9*B)*E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]) + Cos[c + d*x]*((2688*I)*A + (3024*I)*B + 30*(46*A + 51*B)*Sin[c + d*x] + 14*(37*A + 36*B)*Sin[2*(c + d*x)] + 180*A*Sin[3*(c + d*x)] + 90*B*Sin[3*(c + d*x)] + 35*A*Sin[4*(c + d*x)])))/(1260*d*I*E^(I*d*x))

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{Ba^2 \sec(dx + c)^3 + (A + 2B)a^2 \sec(dx + c)^2 + (2A + B)a^2 \sec(dx + c) + Aa^2}{\sec(dx + c)^{\frac{9}{2}}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x, algorithm="fricas")

[Out] integral((B*a^2*sec(d*x + c)^3 + (A + 2*B)*a^2*sec(d*x + c)^2 + (2*A + B)*a^2*sec(d*x + c) + A*a^2)/sec(d*x + c)^(9/2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(a \sec(dx + c) + a)^2}{\sec(dx + c)^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^2/sec(d*x + c)^(9/2), x)

maple [A] time = 4.72, size = 413, normalized size = 1.76

$$4\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} a^2 \left(-560A \cos\left(\frac{dx}{2} + \frac{c}{2}\right)\left(\sin^{10}\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + (1840A + 360B)\left(\sin^8\left(\frac{dx}{2} + \frac{c}{2}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x)

[Out] -4/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(-560*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(1840*A+360*B)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-2368*A-1044*B)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(1568*A+1134*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-387*A-351*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+75*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-168*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+90*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-189*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(a \sec(dx + c) + a)^2}{\sec(dx + c)^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^2/sec(d*x + c)^(9/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(a + \frac{a}{\cos(c+dx)}\right)^2}{\left(\frac{1}{\cos(c+dx)}\right)^{9/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^2)/(1/cos(c + d*x))^(9/2),x)

[Out] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^2)/(1/cos(c + d*x))^(9/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))**2*(A+B*sec(d*x+c))/sec(d*x+c)**(9/2),x)

[Out] Timed out

$$3.193 \quad \int \sec^2(c+dx)(a+a \sec(c+dx))^3(A+B \sec(c+dx)) dx$$

Optimal. Leaf size=277

$$\frac{4a^3(24A+23B) \sin(c+dx) \sec^5(c+dx)}{105d} + \frac{4a^3(13A+11B) \sin(c+dx) \sec^3(c+dx)}{21d} + \frac{2(9A+13B) \sin(c+dx) \sec(c+dx)}{d}$$

[Out] 4/21*a^3*(13*A+11*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d+4/105*a^3*(24*A+23*B)*sec(d*x+c)^(5/2)*sin(d*x+c)/d+2/9*a*B*sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^2*sin(d*x+c)/d+2/63*(9*A+13*B)*sec(d*x+c)^(5/2)*(a^3+a^3*sec(d*x+c))*sin(d*x+c)/d+4/15*a^3*(21*A+17*B)*sin(d*x+c)*sec(d*x+c)^(1/2)/d-4/15*a^3*(21*A+17*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+4/21*a^3*(13*A+11*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d

Rubi [A] time = 0.54, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {4018, 3997, 3787, 3768, 3771, 2639, 2641}

$$\frac{4a^3(24A+23B) \sin(c+dx) \sec^5(c+dx)}{105d} + \frac{4a^3(13A+11B) \sin(c+dx) \sec^3(c+dx)}{21d} + \frac{2(9A+13B) \sin(c+dx) \sec(c+dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]

[Out] (-4*a^3*(21*A + 17*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(15*d) + (4*a^3*(13*A + 11*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (4*a^3*(21*A + 17*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d) + (4*a^3*(13*A + 11*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(21*d) + (4*a^3*(24*A + 23*B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(105*d) + (2*a*B*Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^2*Ssin[c + d*x])/(9*d) + (2*(9*A + 13*B)*Sec[c + d*x]^(5/2)*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x])/(63*d)

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 3768

Int[(csc[(c_.) + (d_.)*(x_)])*(b_.))^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x] * (b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3771

Int[(csc[(c_.) + (d_.)*(x_)])*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x])^n * Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_)])*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)])*(b_.) + (a_), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 3997

Int[(csc[(e_.) + (f_.)*(x_)])*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)])*(b_.) + (a_.)*(csc[(e_.) + (f_.)*(x_)])*(B_.) + (A_.)), x_Symbol] := -Simp[(b*B*Cot[e + f*x] * (d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e + f*x])^n * Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && !LeQ[n, -1]

Rule 4018

Int[(csc[(e_.) + (f_.)*(x_)])*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)])*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)])*(B_.) + (A_.)), x_Symbol] := -Simp[(b*B*Cot[e + f*x] * (a + b*Csc[e + f*x])^(m - 1) * (d*Csc[e + f*x])^n)/(f*(m + n)), x] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1) * (d*Csc[e + f*x])^n * Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && !LtQ[n, -1]

Rubi steps

$$\begin{aligned}
\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3(A+B\sec(c+dx))dx &= \frac{2aB\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2\sin(c+dx)}{9d} + \frac{2a^3\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{9d} \\
&= \frac{2aB\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2\sin(c+dx)}{9d} + \frac{2a^3\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{9d} \\
&= \frac{4a^3(24A+23B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{105d} + \frac{2aB\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{105d} \\
&= \frac{4a^3(24A+23B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{105d} + \frac{2aB\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{105d} \\
&= \frac{4a^3(21A+17B)\sqrt{\sec(c+dx)}\sin(c+dx)}{15d} + \frac{4a^3(13A+9B)\sqrt{\sec(c+dx)}\sin(c+dx)}{15d} \\
&= \frac{4a^3(21A+17B)\sqrt{\sec(c+dx)}\sin(c+dx)}{15d} + \frac{4a^3(13A+9B)\sqrt{\sec(c+dx)}\sin(c+dx)}{15d} \\
&= -\frac{4a^3(21A+17B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d}
\end{aligned}$$

Mathematica [C] time = 6.96, size = 793, normalized size = 2.86

$$\frac{7A\csc(c)e^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\left((-1+e^{2ic})e^{2idx}{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)-3\sqrt{1+e^{2i(c+dx)}}\right)\cos^4(c+dx)\sec^{\frac{3}{2}}(c+dx)}{30\sqrt{2}d(A\cos(c+dx)+B)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]

[Out] (7*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c + d*x]^4*Csc[c]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]))/(30*Sqrt[2]*d*E^(I*d*x)*(B + A*Cos[c + d*x])) + (17*B*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c + d*x]^4*Csc[c]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]))/(90*Sqrt[2]*d*E^(I*d*x)*(B + A*Cos[c + d*x])) + (13*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sec[c/2 +

$$\begin{aligned} & \left(\frac{d*x}{2}\right)^6*(a + a*\text{Sec}[c + d*x])^3*(A + B*\text{Sec}[c + d*x])/((42*d*(B + A*\text{Cos}[c + d*x])*\text{Sec}[c + d*x]^{(7/2)} + (11*B*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticF}[(c + d*x)/2, 2]*\text{Sec}[c/2 + (d*x)/2]^6*(a + a*\text{Sec}[c + d*x])^3*(A + B*\text{Sec}[c + d*x]))/(42*d*(B + A*\text{Cos}[c + d*x])*\text{Sec}[c + d*x]^{(7/2)} + (\text{Sec}[c/2 + (d*x)/2]^6*(a + a*\text{Sec}[c + d*x])^3*(A + B*\text{Sec}[c + d*x])*((21*A + 17*B)*\text{Cos}[d*x]*\text{Csc}[c])/(30*d) + (B*\text{Sec}[c]*\text{Sec}[c + d*x]^4*\text{Sin}[d*x])/(36*d) + (\text{Sec}[c]*\text{Sec}[c + d*x]^3*(7*B*\text{Sin}[c] + 9*A*\text{Sin}[d*x] + 27*B*\text{Sin}[d*x]))/(252*d) + (\text{Sec}[c]*\text{Sec}[c + d*x]^2*(45*A*\text{Sin}[c] + 135*B*\text{Sin}[c] + 189*A*\text{Sin}[d*x] + 238*B*\text{Sin}[d*x]))/(1260*d) + (\text{Sec}[c]*\text{Sec}[c + d*x]*(189*A*\text{Sin}[c] + 238*B*\text{Sin}[c] + 390*A*\text{Sin}[d*x] + 330*B*\text{Sin}[d*x]))/(1260*d) + ((13*A + 11*B)*\text{Tan}[c])/(42*d)))/(B + A*\text{Cos}[c + d*x])* \text{Sec}[c + d*x]^{(7/2)} \end{aligned}$$

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Ba^3 \sec(dx + c)^5 + (A + 3B)a^3 \sec(dx + c)^4 + 3(A + B)a^3 \sec(dx + c)^3 + (3A + B)a^3 \sec(dx + c)^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] integral((B*a^3*sec(d*x + c)^5 + (A + 3*B)*a^3*sec(d*x + c)^4 + 3*(A + B)*a^3*sec(d*x + c)^3 + (3*A + B)*a^3*sec(d*x + c)^2 + A*a^3*sec(d*x + c))*sqrt(sec(d*x + c)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(a \sec(dx + c) + a)^3 \sec(dx + c)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^3*sec(d*x + c)^(3/2), x)

maple [B] time = 18.52, size = 1180, normalized size = 4.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x)

[Out] -a^3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(16*(3/8*A+1/8*B)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^

$$2)^{(1/2)} / (-1/2 + \cos(1/2 * d * x + 1/2 * c))^2 + 1/3 * (\sin(1/2 * d * x + 1/2 * c))^2)^{(1/2)} * (-2 * \cos(1/2 * d * x + 1/2 * c)^2 + 1)^{(1/2)} / (-2 * \sin(1/2 * d * x + 1/2 * c)^4 + \sin(1/2 * d * x + 1/2 * c)^2)^{(1/2)} * \text{EllipticF}(\cos(1/2 * d * x + 1/2 * c), 2^{(1/2)}) + 2 * A * (-2 * \sin(1/2 * d * x + 1/2 * c)^4 + \sin(1/2 * d * x + 1/2 * c)^2)^{(1/2)} * (\sin(1/2 * d * x + 1/2 * c))^2)^{(1/2)} * (2 * \sin(1/2 * d * x + 1/2 * c)^2 - 1)^{(1/2)} * \text{EllipticE}(\cos(1/2 * d * x + 1/2 * c), 2^{(1/2)}) + 2 * (-2 * \sin(1/2 * d * x + 1/2 * c)^4 + \sin(1/2 * d * x + 1/2 * c)^2)^{(1/2)} * \cos(1/2 * d * x + 1/2 * c) * \sin(1/2 * d * x + 1/2 * c)^2 / \sin(1/2 * d * x + 1/2 * c)^2 / (2 * \sin(1/2 * d * x + 1/2 * c)^2 - 1) - 16/5 * (3/8 * A + 3/8 * B) / (8 * \sin(1/2 * d * x + 1/2 * c)^6 - 12 * \sin(1/2 * d * x + 1/2 * c)^4 + 6 * \sin(1/2 * d * x + 1/2 * c)^2 - 1) / \sin(1/2 * d * x + 1/2 * c)^2 * (12 * \text{EllipticE}(\cos(1/2 * d * x + 1/2 * c), 2^{(1/2)}) * (2 * \sin(1/2 * d * x + 1/2 * c)^2 - 1)^{(1/2)} * (\sin(1/2 * d * x + 1/2 * c))^2)^{(1/2)} * \sin(1/2 * d * x + 1/2 * c)^4 - 24 * \cos(1/2 * d * x + 1/2 * c) * \sin(1/2 * d * x + 1/2 * c)^6 - 12 * \text{EllipticE}(\cos(1/2 * d * x + 1/2 * c), 2^{(1/2)}) * (2 * \sin(1/2 * d * x + 1/2 * c)^2 - 1)^{(1/2)} * (\sin(1/2 * d * x + 1/2 * c))^2)^{(1/2)} * \sin(1/2 * d * x + 1/2 * c)^2 + 24 * \sin(1/2 * d * x + 1/2 * c)^4 * \cos(1/2 * d * x + 1/2 * c) + 3 * (\sin(1/2 * d * x + 1/2 * c))^2)^{(1/2)} * (2 * \sin(1/2 * d * x + 1/2 * c)^2 - 1)^{(1/2)} * \text{EllipticE}(\cos(1/2 * d * x + 1/2 * c), 2^{(1/2)}) - 8 * \sin(1/2 * d * x + 1/2 * c)^2 * \cos(1/2 * d * x + 1/2 * c) * (-2 * \sin(1/2 * d * x + 1/2 * c)^4 + \sin(1/2 * d * x + 1/2 * c)^2)^{(1/2)} + 16 * (1/8 * A + 3/8 * B) * (-1/56 * \cos(1/2 * d * x + 1/2 * c) * (-2 * \sin(1/2 * d * x + 1/2 * c)^4 + \sin(1/2 * d * x + 1/2 * c)^2)^{(1/2)} + 16 * (1/8 * A + 3/8 * B) * (-1/56 * \cos(1/2 * d * x + 1/2 * c) * (-2 * \sin(1/2 * d * x + 1/2 * c)^4 + \sin(1/2 * d * x + 1/2 * c)^2)^{(1/2)} / (-1/2 + \cos(1/2 * d * x + 1/2 * c))^2)^4 - 5/42 * \cos(1/2 * d * x + 1/2 * c) * (-2 * \sin(1/2 * d * x + 1/2 * c)^4 + \sin(1/2 * d * x + 1/2 * c)^2)^{(1/2)} / (-1/2 + \cos(1/2 * d * x + 1/2 * c))^2)^2 + 5/21 * (\sin(1/2 * d * x + 1/2 * c))^2)^{(1/2)} * (-2 * \cos(1/2 * d * x + 1/2 * c)^2 + 1)^{(1/2)} / (-2 * \sin(1/2 * d * x + 1/2 * c)^4 + \sin(1/2 * d * x + 1/2 * c)^2)^{(1/2)} * \text{EllipticF}(\cos(1/2 * d * x + 1/2 * c), 2^{(1/2)}) + 2 * B * (-1/144 * \cos(1/2 * d * x + 1/2 * c) * (-2 * \sin(1/2 * d * x + 1/2 * c)^4 + \sin(1/2 * d * x + 1/2 * c)^2)^{(1/2)} / (-1/2 + \cos(1/2 * d * x + 1/2 * c))^2)^5 - 7/180 * \cos(1/2 * d * x + 1/2 * c) * (-2 * \sin(1/2 * d * x + 1/2 * c)^4 + \sin(1/2 * d * x + 1/2 * c)^2)^{(1/2)} / (-1/2 + \cos(1/2 * d * x + 1/2 * c))^2)^3 - 14/15 * \sin(1/2 * d * x + 1/2 * c)^2 * \cos(1/2 * d * x + 1/2 * c) / (-2 * \cos(1/2 * d * x + 1/2 * c)^2 + 1) * \sin(1/2 * d * x + 1/2 * c)^2)^{(1/2)} + 7/15 * (\sin(1/2 * d * x + 1/2 * c))^2)^{(1/2)} * (-2 * \cos(1/2 * d * x + 1/2 * c)^2 + 1)^{(1/2)} / (-2 * \sin(1/2 * d * x + 1/2 * c)^4 + \sin(1/2 * d * x + 1/2 * c)^2)^{(1/2)} * \text{EllipticF}(\cos(1/2 * d * x + 1/2 * c), 2^{(1/2)}) - 7/15 * (\sin(1/2 * d * x + 1/2 * c))^2)^{(1/2)} * (-2 * \cos(1/2 * d * x + 1/2 * c)^2 + 1)^{(1/2)} / (-2 * \sin(1/2 * d * x + 1/2 * c)^4 + \sin(1/2 * d * x + 1/2 * c)^2)^{(1/2)} * (\text{EllipticF}(\cos(1/2 * d * x + 1/2 * c), 2^{(1/2)}) - \text{EllipticE}(\cos(1/2 * d * x + 1/2 * c), 2^{(1/2)}))) / \sin(1/2 * d * x + 1/2 * c) / (2 * \cos(1/2 * d * x + 1/2 * c)^2 - 1)^{(1/2)} / d$$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(A + \frac{B}{\cos(c + dx)} \right) \left(a + \frac{a}{\cos(c + dx)} \right)^3 \left(\frac{1}{\cos(c + dx)} \right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^3*(1/cos(c + d*x))^(3/2),x)
```

```
[Out] int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^3*(1/cos(c + d*x))^(3/2), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(a+a*sec(d*x+c))**3*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

$$3.194 \quad \int \sqrt{\sec(c + dx)} (a + a \sec(c + dx))^3 (A + B \sec(c + dx)) dx$$

Optimal. Leaf size=244

$$\frac{4a^3(42A + 41B) \sin(c + dx) \sec^{\frac{3}{2}}(c + dx)}{105d} + \frac{2(7A + 11B) \sin(c + dx) \sec^{\frac{3}{2}}(c + dx) (a^3 \sec(c + dx) + a^3)}{35d} + \frac{4a^3(9A + 7B) \sin(c + dx) \sec^{\frac{3}{2}}(c + dx)}{35d}$$

[Out] $4/105*a^3*(42*A+41*B)*\sec(d*x+c)^{(3/2)}*\sin(d*x+c)/d+2/7*a*B*\sec(d*x+c)^{(3/2)}*(a+a*\sec(d*x+c))^2*\sin(d*x+c)/d+2/35*(7*A+11*B)*\sec(d*x+c)^{(3/2)}*(a^3+a^3*\sec(d*x+c))*\sin(d*x+c)/d+4/5*a^3*(9*A+7*B)*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/d-4/5*a^3*(9*A+7*B)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d+4/21*a^3*(2*1*A+13*B)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d$

Rubi [A] time = 0.44, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {4018, 3997, 3787, 3771, 2641, 3768, 2639}

$$\frac{4a^3(42A + 41B) \sin(c + dx) \sec^{\frac{3}{2}}(c + dx)}{105d} + \frac{2(7A + 11B) \sin(c + dx) \sec^{\frac{3}{2}}(c + dx) (a^3 \sec(c + dx) + a^3)}{35d} + \frac{4a^3(9A + 7B) \sin(c + dx) \sec^{\frac{3}{2}}(c + dx)}{35d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sqrt}[\text{Sec}[c + d*x]]*(a + a*\text{Sec}[c + d*x])^3*(A + B*\text{Sec}[c + d*x]), x]$

[Out] $(-4*a^3*(9*A + 7*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/(5*d) + (4*a^3*(21*A + 13*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticF}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/(21*d) + (4*a^3*(9*A + 7*B)*\text{Sqrt}[\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(5*d) + (4*a^3*(42*A + 41*B)*\text{Sec}[c + d*x]^{(3/2)}*\text{Sin}[c + d*x])/(105*d) + (2*a*B*\text{Sec}[c + d*x]^{(3/2)}*(a + a*\text{Sec}[c + d*x])^2*\text{Sin}[c + d*x])/(7*d) + (2*(7*A + 11*B)*\text{Sec}[c + d*x]^{(3/2)}*(a^3 + a^3*\text{Sec}[c + d*x])*\text{Sin}[c + d*x])/(35*d)$

Rule 2639

$\text{Int}[\text{Sqrt}[\sin[(c_.) + (d_.)*(x_.)]], x_Symbol] := \text{Simp}[(2*\text{EllipticE}[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 2641

$\text{Int}[1/\text{Sqrt}[\sin[(c_.) + (d_.)*(x_.)]], x_Symbol] := \text{Simp}[(2*\text{EllipticF}[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 3768

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x] * (b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3771

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 3997

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] :> -Simp[(b*B*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && !LeQ[n, -1]

Rule 4018

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] :> -Simp[(b*B*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && !LtQ[n, -1]

Rubi steps

$$\begin{aligned}
\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3(A+B\sec(c+dx))dx &= \frac{2aB\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2\sin(c+dx)}{7d} + \\
&= \frac{2aB\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2\sin(c+dx)}{7d} + \\
&= \frac{4a^3(42A+41B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{105d} + \frac{2aB\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{7d} + \\
&= \frac{4a^3(42A+41B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{105d} + \frac{2aB\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{7d} + \\
&= \frac{4a^3(9A+7B)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d} + \frac{4a^3(42A+41B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{105d} + \\
&= \frac{4a^3(21A+13B)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{21d} + \\
&= -\frac{4a^3(9A+7B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}
\end{aligned}$$

Mathematica [C] time = 5.23, size = 465, normalized size = 1.91

$$a^3 \csc(c)e^{-idx} \cos^4(c+dx) \sec^6\left(\frac{1}{2}(c+dx)\right) (\sec(c+dx)+1)^3 (A+B\sec(c+dx)) \left(7\sqrt{2}(-1+e^{2ic})(9A+7B)e^{2idx}\right)$$

Antiderivative was successfully verified.

```

[In] Integrate[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
[Out] (a^3*Cos[c + d*x]^4*Csc[c]*Sec[(c + d*x)/2]^6*(7*Sqrt[2]*(9*A + 7*B)*E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))] - ((-1 + E^((2*I)*c))*(21*A*(-5 + 16*E^(I*(c + d*x))) - 5*E^((2*I)*(c + d*x))) + 54*E^((3*I)*(c + d*x)) + 5*E^((4*I)*(c + d*x)) + 56*E^((5*I)*(c + d*x)) + 5*E^((6*I)*(c + d*x)) + 18*E^((7*I)*(c + d*x))) + 2*B*(-65 + 8*4*E^(I*(c + d*x)) - 95*E^((2*I)*(c + d*x)) + 441*E^((3*I)*(c + d*x)) + 95*E^((4*I)*(c + d*x)) + 504*E^((5*I)*(c + d*x)) + 65*E^((6*I)*(c + d*x)) + 147*E^((7*I)*(c + d*x))) + (10*I)*(21*A + 13*B)*(1 + E^((2*I)*(c + d*x)))^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])*Sqrt[Sec[c + d*x]])/(2*E^(I*(c

```

$-d*x)) * (1 + E^{((2*I)*(c + d*x))})^3) * (1 + \text{Sec}[c + d*x])^3 * (A + B*\text{Sec}[c + d*x]) / (420*d*E^{(I*d*x)}*(B + A*\text{Cos}[c + d*x]))$

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$\text{integral}((Ba^3 \sec(dx + c)^4 + (A + 3B)a^3 \sec(dx + c)^3 + 3(A + B)a^3 \sec(dx + c)^2 + (3A + B)a^3 \sec(dx + c) +$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x, algorithm="fricas")`

[Out] `integral((B*a^3*sec(d*x + c)^4 + (A + 3*B)*a^3*sec(d*x + c)^3 + 3*(A + B)*a^3*sec(d*x + c)^2 + (3*A + B)*a^3*sec(d*x + c) + A*a^3)*sqrt(sec(d*x + c)), x)`

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(a \sec(dx + c) + a)^3 \sqrt{\sec(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x, algorithm="giac")`

[Out] `integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^3*sqrt(sec(d*x + c)), x)`

maple [B] time = 15.40, size = 931, normalized size = 3.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x)`

[Out] `-a^3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+16*(3/8*A+3/8*B)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+16*(3/8*A+1/8*B)*(-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)`

) $\sin(1/2*d*x+1/2*c)^2/\sin(1/2*d*x+1/2*c)^2/(2*\sin(1/2*d*x+1/2*c)^2-1)-16/5*(1/8*A+3/8*B)/(8*\sin(1/2*d*x+1/2*c)^6-12*\sin(1/2*d*x+1/2*c)^4+6*\sin(1/2*d*x+1/2*c)^2-1)/\sin(1/2*d*x+1/2*c)^2*(12*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)}))*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*\sin(1/2*d*x+1/2*c)^4-24*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^6-12*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)})*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*\sin(1/2*d*x+1/2*c)^2+24*\sin(1/2*d*x+1/2*c)^4*\cos(1/2*d*x+1/2*c)+3*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)})-8*\sin(1/2*d*x+1/2*c)^2*\cos(1/2*d*x+1/2*c))*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}+2*B*(-1/56*\cos(1/2*d*x+1/2*c)*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/(-1/2+\cos(1/2*d*x+1/2*c)^2)^4-5/42*\cos(1/2*d*x+1/2*c)*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/(-1/2+\cos(1/2*d*x+1/2*c)^2)^2+5/21*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})))/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(a \sec(dx + c) + a)^3 \sqrt{\sec(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^3*sqrt(sec(d*x + c)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(A + \frac{B}{\cos(c + dx)} \right) \left(a + \frac{a}{\cos(c + dx)} \right)^3 \sqrt{\frac{1}{\cos(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^3*(1/cos(c + d*x))^(1/2),x)

[Out] int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^3*(1/cos(c + d*x))^(1/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*(A+B*sec(d*x+c))*sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

$$3.195 \quad \int \frac{(a+a \sec(c+dx))^3(A+B \sec(c+dx))}{\sqrt{\sec(c+dx)}} dx$$

Optimal. Leaf size=211

$$\frac{4a^3(20A + 21B) \sin(c + dx)\sqrt{\sec(c + dx)}}{15d} + \frac{2(5A + 9B) \sin(c + dx)\sqrt{\sec(c + dx)} (a^3 \sec(c + dx) + a^3)}{15d} + \frac{4a^3(5A + 9B) \sin(c + dx)\sqrt{\sec(c + dx)}}{15d}$$

[Out] 4/15*a^3*(20*A+21*B)*sin(d*x+c)*sec(d*x+c)^(1/2)/d+2/5*a*B*(a+a*sec(d*x+c))^2*sin(d*x+c)*sec(d*x+c)^(1/2)/d+2/15*(5*A+9*B)*(a^3+a^3*sec(d*x+c))*sin(d*x+c)*sec(d*x+c)^(1/2)/d-4/5*a^3*(5*A+9*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+4/3*a^3*(5*A+3*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d

Rubi [A] time = 0.42, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4018, 3997, 3787, 3771, 2639, 2641}

$$\frac{4a^3(20A + 21B) \sin(c + dx)\sqrt{\sec(c + dx)}}{15d} + \frac{2(5A + 9B) \sin(c + dx)\sqrt{\sec(c + dx)} (a^3 \sec(c + dx) + a^3)}{15d} + \frac{4a^3(5A + 9B) \sin(c + dx)\sqrt{\sec(c + dx)}}{15d}$$

Antiderivative was successfully verified.

[In] Int[((a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]))/Sqrt[Sec[c + d*x]],x]

[Out] (-4*a^3*(5*A + 9*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (4*a^3*(5*A + 3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (4*a^3*(20*A + 21*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d) + (2*a*B*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^2*Sin[c + d*x])/(5*d) + (2*(5*A + 9*B)*Sqrt[Sec[c + d*x]]*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x])/(15*d)

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] := -Simp[(b*B*Cot[e
+ f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && !LeQ[n,
-1]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] := -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && !LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + a \sec(c + dx))^3 (A + B \sec(c + dx))}{\sqrt{\sec(c + dx)}} dx &= \frac{2aB\sqrt{\sec(c + dx)} (a + a \sec(c + dx))^2 \sin(c + dx)}{5d} + \frac{2}{5} \int \frac{(a + a \sec(c + dx))^3 (A + B \sec(c + dx))}{\sqrt{\sec(c + dx)}} dx \\
&= \frac{2aB\sqrt{\sec(c + dx)} (a + a \sec(c + dx))^2 \sin(c + dx)}{5d} + \frac{2(5A + 9B)}{5} \int \frac{(a + a \sec(c + dx))^3}{\sqrt{\sec(c + dx)}} dx \\
&= \frac{4a^3(20A + 21B)\sqrt{\sec(c + dx)} \sin(c + dx)}{15d} + \frac{2aB\sqrt{\sec(c + dx)} (a + a \sec(c + dx))^2 \sin(c + dx)}{5d} \\
&= \frac{4a^3(20A + 21B)\sqrt{\sec(c + dx)} \sin(c + dx)}{15d} + \frac{2aB\sqrt{\sec(c + dx)} (a + a \sec(c + dx))^2 \sin(c + dx)}{5d} \\
&= \frac{4a^3(20A + 21B)\sqrt{\sec(c + dx)} \sin(c + dx)}{15d} + \frac{2aB\sqrt{\sec(c + dx)} (a + a \sec(c + dx))^2 \sin(c + dx)}{5d} \\
&= -\frac{4a^3(5A + 9B)\sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{\sec(c + dx)}}{5d} + \dots
\end{aligned}$$

Mathematica [C] time = 2.41, size = 244, normalized size = 1.16

$$\frac{a^3 e^{-idx} \sec^{\frac{5}{2}}(c + dx) (\cos(dx) + i \sin(dx)) \left(2i(5A + 9B) e^{-i(c+dx)} \left(1 + e^{2i(c+dx)} \right)^{5/2} {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{7}{4}; -e^{2i(c+dx)}\right) + 40(5A + \dots \right)}{\dots}$$

Antiderivative was successfully verified.

[In] Integrate[((a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]))/Sqrt[Sec[c + d*x]], x]

[Out] (a^3*Sec[c + d*x]^(5/2)*(Cos[d*x] + I*Sin[d*x])*((-90*I)*A*Cos[c + d*x] - (162*I)*B*Cos[c + d*x] - (30*I)*A*Cos[3*(c + d*x)] - (54*I)*B*Cos[3*(c + d*x)]) + 40*(5*A + 3*B)*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] + ((2*I)*(5*A + 9*B)*(1 + E^((2*I)*(c + d*x)))^(5/2)*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]/E^(I*(c + d*x)) + 45*A*Sin[c + d*x] + 66*B*Sin[c + d*x] + 10*A*Sin[2*(c + d*x)] + 30*B*Sin[2*(c + d*x)] + 45*A*Sin[3*(c + d*x)] + 54*B*Sin[3*(c + d*x)]))/(30*d*E^(I*d*x))

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{Ba^3 \sec(dx + c)^4 + (A + 3B)a^3 \sec(dx + c)^3 + 3(A + B)a^3 \sec(dx + c)^2 + (3A + B)a^3 \sec(dx + c) + Aa^3}{\sqrt{\sec(dx + c)}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="fricas")

[Out] integral((B*a^3*sec(d*x + c)^4 + (A + 3*B)*a^3*sec(d*x + c)^3 + 3*(A + B)*a^3*sec(d*x + c)^2 + (3*A + B)*a^3*sec(d*x + c) + A*a^3)/sqrt(sec(d*x + c)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(a \sec(dx + c) + a)^3}{\sqrt{\sec(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^3/sqrt(sec(d*x + c)), x)

maple [B] time = 12.88, size = 916, normalized size = 4.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x)

[Out] 4/15*a^3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^3*(100*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4+60*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-180*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+60*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4+108*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-216*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-100*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2-60*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+190*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-60*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2-108*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+246*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+25*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)

```
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-50*A*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+15*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+27*B*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))-72*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(
1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(a \sec(dx + c) + a)^3}{\sqrt{\sec(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^3/sqrt(sec(d*x + c)), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(a + \frac{a}{\cos(c+dx)}\right)^3}{\sqrt{\frac{1}{\cos(c+dx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^3)/(1/cos(c + d*x))^(1/2),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^3)/(1/cos(c + d*x))^(1/2), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*(A+B*sec(d*x+c))/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

$$3.196 \quad \int \frac{(a+a \sec(c+dx))^3 (A+B \sec(c+dx))}{\sec^2(c+dx)} dx$$

Optimal. Leaf size=199

$$\frac{4a^3(A+4B) \sin(c+dx) \sqrt{\sec(c+dx)}}{3d} - \frac{2(A-B) \sin(c+dx) \sqrt{\sec(c+dx)} (a^3 \sec(c+dx) + a^3)}{3d} + \frac{20a^3(A+B) \sqrt{\sec(c+dx)}}{3d}$$

[Out] $2/3*a*A*(a+a*\sec(d*x+c))^2*\sin(d*x+c)/d/\sec(d*x+c)^{(1/2)}+4/3*a^3*(A+4*B)*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/d-2/3*(A-B)*(a^3+a^3*\sec(d*x+c))*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/d+4*a^3*(A-B)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c),2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d+20/3*a^3*(A+B)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c),2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d$

Rubi [A] time = 0.41, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {4017, 4018, 3997, 3787, 3771, 2639, 2641}

$$\frac{4a^3(A+4B) \sin(c+dx) \sqrt{\sec(c+dx)}}{3d} - \frac{2(A-B) \sin(c+dx) \sqrt{\sec(c+dx)} (a^3 \sec(c+dx) + a^3)}{3d} + \frac{20a^3(A+B) \sqrt{\sec(c+dx)}}{3d}$$

Antiderivative was successfully verified.

[In] Int[((a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(3/2),x]

[Out] $(4*a^3*(A-B)*\text{Sqrt}[\text{Cos}[c+d*x]]*\text{EllipticE}[(c+d*x)/2,2]*\text{Sqrt}[\text{Sec}[c+d*x]])/d+(20*a^3*(A+B)*\text{Sqrt}[\text{Cos}[c+d*x]]*\text{EllipticF}[(c+d*x)/2,2]*\text{Sqrt}[\text{Sec}[c+d*x]])/(3*d)+(4*a^3*(A+4*B)*\text{Sqrt}[\text{Sec}[c+d*x]]*\text{Sin}[c+d*x])/(3*d)+(2*a*A*(a+a*\text{Sec}[c+d*x])^2*\text{Sin}[c+d*x])/(3*d*\text{Sqrt}[\text{Sec}[c+d*x]])-(2*(A-B)*\text{Sqrt}[\text{Sec}[c+d*x]]*(a^3+a^3*\text{Sec}[c+d*x])*\text{Sin}[c+d*x])/(3*d)$

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.))^(n_), x_Symbol] :=> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) +
(a_.)), x_Symbol] :=> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) +
(a_.))*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :=> -Simp[(b*B*Cot[
e + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && !LeQ[n,
-1]
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (
a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :=> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (
a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :=> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && !LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + a \sec(c + dx))^3 (A + B \sec(c + dx))}{\sec^{\frac{3}{2}}(c + dx)} dx &= \frac{2aA(a + a \sec(c + dx))^2 \sin(c + dx)}{3d\sqrt{\sec(c + dx)}} + \frac{2}{3} \int \frac{(a + a \sec(c + dx))^2}{\sec^{\frac{3}{2}}(c + dx)} dx \\
&= \frac{2aA(a + a \sec(c + dx))^2 \sin(c + dx)}{3d\sqrt{\sec(c + dx)}} - \frac{2(A - B)\sqrt{\sec(c + dx)}}{3d} \int \frac{1}{\sec^{\frac{3}{2}}(c + dx)} dx \\
&= \frac{4a^3(A + 4B)\sqrt{\sec(c + dx)} \sin(c + dx)}{3d} + \frac{2aA(a + a \sec(c + dx))}{3d\sqrt{\sec(c + dx)}} \\
&= \frac{4a^3(A + 4B)\sqrt{\sec(c + dx)} \sin(c + dx)}{3d} + \frac{2aA(a + a \sec(c + dx))}{3d\sqrt{\sec(c + dx)}} \\
&= \frac{4a^3(A + 4B)\sqrt{\sec(c + dx)} \sin(c + dx)}{3d} + \frac{2aA(a + a \sec(c + dx))}{3d\sqrt{\sec(c + dx)}} \\
&= \frac{4a^3(A - B)\sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{\sec(c + dx)}}{d} + \frac{20a^3}{d}
\end{aligned}$$

Mathematica [C] time = 2.05, size = 202, normalized size = 1.02

$$\frac{a^3 e^{-idx} \sec^{\frac{3}{2}}(c + dx) (\cos(dx) + i \sin(dx)) \left(-4i(A - B) \left(1 + e^{2i(c+dx)} \right)^{3/2} {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{7}{4}; -e^{2i(c+dx)}\right) + 40(A + B) \cos^{\frac{3}{2}}(c + dx) \right)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[((a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(3/2), x]

[Out] (a^3*Sec[c + d*x]^(3/2)*(Cos[d*x] + I*Sin[d*x])*((12*I)*A - (12*I)*B + (12*I)*A*Cos[2*(c + d*x)] - (12*I)*B*Cos[2*(c + d*x)] + 40*(A + B)*Cos[c + d*x])^(3/2)*EllipticF[(c + d*x)/2, 2] - (4*I)*(A - B)*(1 + E^((2*I)*(c + d*x)))^(3/2)*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))] + A*Sin[c + d*x] + 4*B*Sin[c + d*x] + 6*A*Sin[2*(c + d*x)] + 18*B*Sin[2*(c + d*x)] + A*Sin[3*(c + d*x)])/(6*d*E^(I*d*x))

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{Ba^3 \sec(dx + c)^4 + (A + 3B)a^3 \sec(dx + c)^3 + 3(A + B)a^3 \sec(dx + c)^2 + (3A + B)a^3 \sec(dx + c) + 2a^3}{\sec(dx + c)^{\frac{3}{2}}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="fricas")

[Out] integral((B*a^3*sec(d*x + c)^4 + (A + 3*B)*a^3*sec(d*x + c)^3 + 3*(A + B)*a^3*sec(d*x + c)^2 + (3*A + B)*a^3*sec(d*x + c) + A*a^3)/sec(d*x + c)^(3/2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(a \sec(dx + c) + a)^3}{\sec(dx + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^3/sec(d*x + c)^(3/2), x)

maple [B] time = 5.72, size = 654, normalized size = 3.29

$$4a^3 \left(-4A \sqrt{-2 \left(\sin^4 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) + \sin^2 \left(\frac{dx}{2} + \frac{c}{2} \right)} \cos \left(\frac{dx}{2} + \frac{c}{2} \right) \left(\sin^6 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) + 2 \sqrt{-2 \left(\sin^4 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) + \sin^2 \left(\frac{dx}{2} + \frac{c}{2} \right)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x)

[Out] $-4/3*a^3*(-4*A*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^6+2*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(5*A+9*B)*\sin(1/2*d*x+1/2*c)^4*\cos(1/2*d*x+1/2*c)-2*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*A+5*B)*\sin(1/2*d*x+1/2*c)^2*\cos(1/2*d*x+1/2*c)-2*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(5*A*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})-3*A*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)})+5*B*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})+3*B*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)})))*\sin(1/2*d*x+1/2*c)^2+5*A*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}-3*A*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)})+5*B*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}$

$n(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}+3*B*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)})/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(3/2)}/\sin(1/2*d*x+1/2*c)/d$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(a \sec(dx + c) + a)^3}{\sec(dx + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^3/sec(d*x + c)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(a + \frac{a}{\cos(c+dx)}\right)^3}{\left(\frac{1}{\cos(c+dx)}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^3)/(1/cos(c + d*x))^(3/2),x)

[Out] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^3)/(1/cos(c + d*x))^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3 \left(\int \frac{A}{\sec^{\frac{3}{2}}(c + dx)} dx + \int \frac{3A}{\sqrt{\sec(c + dx)}} dx + \int 3A\sqrt{\sec(c + dx)} dx + \int A \sec^{\frac{3}{2}}(c + dx) dx + \int \frac{B}{\sqrt{\sec(c + dx)}} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))*3*(A+B*sec(d*x+c))/sec(d*x+c)**(3/2),x)

[Out] a**3*(Integral(A/sec(c + d*x)**(3/2), x) + Integral(3*A/sqrt(sec(c + d*x)), x) + Integral(3*A*sqrt(sec(c + d*x)), x) + Integral(A*sec(c + d*x)**(3/2), x) + Integral(B/sqrt(sec(c + d*x)), x) + Integral(3*B*sqrt(sec(c + d*x)), x) + Integral(3*B*sec(c + d*x)**(3/2), x) + Integral(B*sec(c + d*x)**(5/2), x))

$$3.197 \quad \int \frac{(a+a \sec(c+dx))^3 (A+B \sec(c+dx))}{\sec^2(c+dx)} dx$$

Optimal. Leaf size=211

$$\frac{4a^3(6A-5B) \sin(c+dx) \sqrt{\sec(c+dx)}}{15d} + \frac{2(9A+5B) \sin(c+dx) (a^3 \sec(c+dx) + a^3)}{15d \sqrt{\sec(c+dx)}} + \frac{4a^3(3A+5B) \sqrt{\cos(c+dx)}}{15d}$$

[Out] $\frac{2}{5} a A (a + a \sec(dx+c))^2 \sin(dx+c) / d \sec(dx+c)^{(3/2)} + \frac{2}{15} (9A+5B) (a^3 + a^3 \sec(dx+c)) \sin(dx+c) / d \sec(dx+c)^{(1/2)} - \frac{4}{15} a^3 (6A-5B) \sin(dx+c) \sec(dx+c)^{(1/2)} / d + \frac{4}{5} a^3 (9A+5B) (\cos(1/2 dx + 1/2 c))^2)^{(1/2)} / \cos(1/2 dx + 1/2 c) * \text{EllipticE}(\sin(1/2 dx + 1/2 c), 2^{(1/2)}) * \cos(dx+c)^{(1/2)} \sec(dx+c)^{(1/2)} / d + \frac{4}{3} a^3 (3A+5B) (\cos(1/2 dx + 1/2 c))^2)^{(1/2)} / \cos(1/2 dx + 1/2 c) * \text{EllipticF}(\sin(1/2 dx + 1/2 c), 2^{(1/2)}) * \cos(dx+c)^{(1/2)} \sec(dx+c)^{(1/2)} / d$

Rubi [A] time = 0.41, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4017, 3997, 3787, 3771, 2639, 2641}

$$\frac{4a^3(6A-5B) \sin(c+dx) \sqrt{\sec(c+dx)}}{15d} + \frac{2(9A+5B) \sin(c+dx) (a^3 \sec(c+dx) + a^3)}{15d \sqrt{\sec(c+dx)}} + \frac{4a^3(3A+5B) \sqrt{\cos(c+dx)}}{15d}$$

Antiderivative was successfully verified.

[In] Int[((a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(5/2), x]

[Out] $(4a^3(9A+5B) \sqrt{\cos(c+dx)} * \text{EllipticE}[(c+dx)/2, 2] * \sqrt{\sec(c+dx)}) / (5d) + (4a^3(3A+5B) \sqrt{\cos(c+dx)} * \text{EllipticF}[(c+dx)/2, 2] * \sqrt{\sec(c+dx)}) / (3d) - (4a^3(6A-5B) \sqrt{\sec(c+dx)} * \sin(c+dx)) / (15d) + (2aA(a+a \sec(c+dx))^2 \sin(c+dx)) / (5d \sec(c+dx)^{(3/2)}) + (2(9A+5B)(a^3 + a^3 \sec(c+dx)) \sin(c+dx)) / (15d * \sqrt{\sec(c+dx)})$

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]]], x_Symbol] := Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 3771

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :=> Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :=> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 3997

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :=> -Simp[(b*B*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && !LeQ[n, -1]

Rule 4017

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :=> Simp[(a*A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && LtQ[n, -1]

Rubi steps

$$\begin{aligned}
\int \frac{(a + a \sec(c + dx))^3 (A + B \sec(c + dx))}{\sec^{\frac{5}{2}}(c + dx)} dx &= \frac{2aA(a + a \sec(c + dx))^2 \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} + \frac{2}{5} \int \frac{(a + a \sec(c + dx))^2}{\sec^{\frac{5}{2}}(c + dx)} dx \\
&= \frac{2aA(a + a \sec(c + dx))^2 \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} + \frac{2(9A + 5B)(a^3 + a^3 \sec(c + dx))}{15d \sqrt{\sec(c + dx)}} \\
&= -\frac{4a^3(6A - 5B)\sqrt{\sec(c + dx)} \sin(c + dx)}{15d} + \frac{2aA(a + a \sec(c + dx))^2 \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} \\
&= -\frac{4a^3(6A - 5B)\sqrt{\sec(c + dx)} \sin(c + dx)}{15d} + \frac{2aA(a + a \sec(c + dx))^2 \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} \\
&= -\frac{4a^3(6A - 5B)\sqrt{\sec(c + dx)} \sin(c + dx)}{15d} + \frac{2aA(a + a \sec(c + dx))^2 \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} \\
&= \frac{4a^3(9A + 5B)\sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{\sec(c + dx)}}{5d} + \frac{4a^3(9A + 5B)\sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{\sec(c + dx)}}{5d} + \frac{4a^3(9A + 5B)\sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{\sec(c + dx)}}{5d} + \frac{4a^3(9A + 5B)\sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{\sec(c + dx)}}{5d}
\end{aligned}$$

Mathematica [C] time = 1.77, size = 207, normalized size = 0.98

$$a^3 e^{-idx} \sqrt{\sec(c + dx)} (\cos(dx) + i \sin(dx)) \left(-8i(9A + 5B) e^{i(c+dx)} \sqrt{1 + e^{2i(c+dx)}} {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{7}{4}; -e^{2i(c+dx)}\right) + 40(3A + 5B) \right)$$

Antiderivative was successfully verified.

[In] Integrate[((a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(5/2), x]

[Out] (a^3*Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])*((216*I)*A*Cos[c + d*x] + (120*I)*B*Cos[c + d*x] + 40*(3*A + 5*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] - (8*I)*(9*A + 5*B)*E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))])*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))] + 3*A*Sin[c + d*x] + 60*B*Sin[c + d*x] + 30*A*Sin[2*(c + d*x)] + 10*B*Sin[2*(c + d*x)] + 3*A*Sin[3*(c + d*x)])/(30*d*E^(I*d*x))

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{Ba^3 \sec(dx + c)^4 + (A + 3B)a^3 \sec(dx + c)^3 + 3(A + B)a^3 \sec(dx + c)^2 + (3A + B)a^3 \sec(dx + c) + Aa^3}{\sec(dx + c)^{\frac{5}{2}}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="fricas")

[Out] integral((B*a^3*sec(d*x + c)^4 + (A + 3*B)*a^3*sec(d*x + c)^3 + 3*(A + B)*a^3*sec(d*x + c)^2 + (3*A + B)*a^3*sec(d*x + c) + A*a^3)/sec(d*x + c)^(5/2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(a \sec(dx + c) + a)^3}{\sec(dx + c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^3/sec(d*x + c)^(5/2), x)

maple [B] time = 4.78, size = 519, normalized size = 2.46

$$4a^3 \left(-12A \sqrt{-2 \left(\sin^4 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) + \sin^2 \left(\frac{dx}{2} + \frac{c}{2} \right)} \cos \left(\frac{dx}{2} + \frac{c}{2} \right) \left(\sin^6 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) + 2 \sqrt{-2 \left(\sin^4 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) + \sin^2 \left(\frac{dx}{2} + \frac{c}{2} \right)} \right) + \sin^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x)

[Out]
$$-4/15*a^3*(-12*A*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^6+2*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(21*A+5*B)*\sin(1/2*d*x+1/2*c)^4*\cos(1/2*d*x+1/2*c)-2*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(9*A+10*B)*\sin(1/2*d*x+1/2*c)^2*\cos(1/2*d*x+1/2*c)+15*A*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}-27*A*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)})+25*B*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}-15*B*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)}))/(-2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}$$

$c)^4 + \sin(1/2*d*x + 1/2*c)^2)^{1/2} / \sin(1/2*d*x + 1/2*c) / (2*\cos(1/2*d*x + 1/2*c)^2 - 1)^{1/2} / d$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(a \sec(dx + c) + a)^3}{\sec(dx + c)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^3/sec(d*x + c)^(5/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(a + \frac{a}{\cos(c+dx)}\right)^3}{\left(\frac{1}{\cos(c+dx)}\right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^3)/(1/cos(c + d*x))^(5/2),x)

[Out] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^3)/(1/cos(c + d*x))^(5/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3 \left(\int \frac{A}{\sec^{5/2}(c+dx)} dx + \int \frac{3A}{\sec^{3/2}(c+dx)} dx + \int \frac{3A}{\sqrt{\sec(c+dx)}} dx + \int A \sqrt{\sec(c+dx)} dx + \int \frac{B}{\sec^{3/2}(c+dx)} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))**3*(A+B*sec(d*x+c))/sec(d*x+c)**(5/2),x)

[Out] a**3*(Integral(A/sec(c + d*x)**(5/2), x) + Integral(3*A/sec(c + d*x)**(3/2), x) + Integral(3*A/sqrt(sec(c + d*x)), x) + Integral(A*sqrt(sec(c + d*x)), x) + Integral(B/sec(c + d*x)**(3/2), x) + Integral(3*B/sqrt(sec(c + d*x)), x) + Integral(3*B*sqrt(sec(c + d*x)), x) + Integral(B*sec(c + d*x)**(3/2), x))

$$3.198 \quad \int \frac{(a+a \sec(c+dx))^3 (A+B \sec(c+dx))}{7 \sec^2(c+dx)} dx$$

Optimal. Leaf size=211

$$\frac{2(11A + 7B) \sin(c + dx) (a^3 \sec(c + dx) + a^3)}{35d \sec^2(c + dx)} + \frac{4a^3(41A + 42B) \sin(c + dx)}{105d \sqrt{\sec(c + dx)}} + \frac{4a^3(13A + 21B) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)}}{21d}$$

[Out] $\frac{2}{7} a A (a + a \sec(dx+c))^2 \sin(dx+c) / d \sec(dx+c)^{(5/2)} + \frac{2}{35} (11A + 7B) (a^3 + a^3 \sec(dx+c)) \sin(dx+c) / d \sec(dx+c)^{(3/2)} + \frac{4}{105} a^3 (41A + 42B) \sin(dx+c) / d \sec(dx+c)^{(1/2)} + \frac{4}{5} a^3 (7A + 9B) (\cos(1/2 dx + 1/2 c))^2 \cos(1/2 dx + 1/2 c) \operatorname{EllipticE}(\sin(1/2 dx + 1/2 c), 2^{(1/2)}) \cos(dx+c)^{(1/2)} \sec(dx+c)^{(1/2)} / d + \frac{4}{21} a^3 (13A + 21B) (\cos(1/2 dx + 1/2 c))^2 \cos(1/2 dx + 1/2 c) \operatorname{EllipticF}(\sin(1/2 dx + 1/2 c), 2^{(1/2)}) \cos(dx+c)^{(1/2)} \sec(dx+c)^{(1/2)} / d$

Rubi [A] time = 0.44, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4017, 3996, 3787, 3771, 2639, 2641}

$$\frac{2(11A + 7B) \sin(c + dx) (a^3 \sec(c + dx) + a^3)}{35d \sec^2(c + dx)} + \frac{4a^3(41A + 42B) \sin(c + dx)}{105d \sqrt{\sec(c + dx)}} + \frac{4a^3(13A + 21B) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)}}{21d}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(a + a \operatorname{Sec}[c + d*x])^3 (A + B \operatorname{Sec}[c + d*x]) / \operatorname{Sec}[c + d*x]^{(7/2)}, x]$

[Out] $(4a^3(7A + 9B) \operatorname{Sqrt}[\operatorname{Cos}[c + d*x]] \operatorname{EllipticE}[(c + d*x)/2, 2] \operatorname{Sqrt}[\operatorname{Sec}[c + d*x]]) / (5d) + (4a^3(13A + 21B) \operatorname{Sqrt}[\operatorname{Cos}[c + d*x]] \operatorname{EllipticF}[(c + d*x)/2, 2] \operatorname{Sqrt}[\operatorname{Sec}[c + d*x]]) / (21d) + (4a^3(41A + 42B) \operatorname{Sin}[c + d*x]) / (105d \operatorname{Sqrt}[\operatorname{Sec}[c + d*x]]) + (2aA(a + a \operatorname{Sec}[c + d*x])^2 \operatorname{Sin}[c + d*x]) / (7d \operatorname{Sec}[c + d*x]^{(5/2)}) + (2(11A + 7B)(a^3 + a^3 \operatorname{Sec}[c + d*x]) \operatorname{Sin}[c + d*x]) / (35d \operatorname{Sec}[c + d*x]^{(3/2)})$

Rule 2639

$\operatorname{Int}[\operatorname{Sqrt}[\sin[(c_.) + (d_.)(x_.)]], x_Symbol] \rightarrow \operatorname{Simp}[(2 \operatorname{EllipticE}[(1*(c - P i/2 + d*x))/2, 2]) / d, x] /; \operatorname{FreeQ}\{c, d\}, x]$

Rule 2641

$\operatorname{Int}[1/\operatorname{Sqrt}[\sin[(c_.) + (d_.)(x_.)]], x_Symbol] \rightarrow \operatorname{Simp}[(2 \operatorname{EllipticF}[(1*(c - P i/2 + d*x))/2, 2]) / d, x] /; \operatorname{FreeQ}\{c, d\}, x]$

Rule 3771

Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 3996

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]

Rule 4017

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := Simp[(a*A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && LtQ[n, -1]

Rubi steps

$$\begin{aligned}
\int \frac{(a + a \sec(c + dx))^3 (A + B \sec(c + dx))}{\sec^{\frac{7}{2}}(c + dx)} dx &= \frac{2aA(a + a \sec(c + dx))^2 \sin(c + dx)}{7d \sec^{\frac{5}{2}}(c + dx)} + \frac{2}{7} \int \frac{(a + a \sec(c + dx))^3}{\sec^{\frac{7}{2}}(c + dx)} dx \\
&= \frac{2aA(a + a \sec(c + dx))^2 \sin(c + dx)}{7d \sec^{\frac{5}{2}}(c + dx)} + \frac{2(11A + 7B)(a^3 + a^3 \sec^3(c + dx))}{35d \sec^{\frac{3}{2}}(c + dx)} \\
&= \frac{4a^3(41A + 42B) \sin(c + dx)}{105d \sqrt{\sec(c + dx)}} + \frac{2aA(a + a \sec(c + dx))^2 \sin(c + dx)}{7d \sec^{\frac{5}{2}}(c + dx)} \\
&= \frac{4a^3(41A + 42B) \sin(c + dx)}{105d \sqrt{\sec(c + dx)}} + \frac{2aA(a + a \sec(c + dx))^2 \sin(c + dx)}{7d \sec^{\frac{5}{2}}(c + dx)} \\
&= \frac{4a^3(41A + 42B) \sin(c + dx)}{105d \sqrt{\sec(c + dx)}} + \frac{2aA(a + a \sec(c + dx))^2 \sin(c + dx)}{7d \sec^{\frac{5}{2}}(c + dx)} \\
&= \frac{4a^3(7A + 9B) \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{\sec(c + dx)}}{5d} + \dots
\end{aligned}$$

Mathematica [C] time = 2.70, size = 194, normalized size = 0.92

$$a^3 e^{-idx} \sqrt{\sec(c + dx)} (\cos(dx) + i \sin(dx)) \left(-56i(7A + 9B) e^{i(c+dx)} \sqrt{1 + e^{2i(c+dx)}} {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{7}{4}; -e^{2i(c+dx)}\right) + \cos(c + dx) \right)$$

Antiderivative was successfully verified.

[In] Integrate[((a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(7/2), x]

[Out] (a^3*Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x]))*(40*(13*A + 21*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] - (56*I)*(7*A + 9*B)*E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]) + Cos[c + d*x]*((168*I)*(7*A + 9*B) + 5*(107*A + 84*B)*Sin[c + d*x] + 42*(3*A + B)*Sin[2*(c + d*x)] + 15*A*Sin[3*(c + d*x)])))/(210*d*E^(I*d*x))

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{Ba^3 \sec(dx + c)^4 + (A + 3B)a^3 \sec(dx + c)^3 + 3(A + B)a^3 \sec(dx + c)^2 + (3A + B)a^3 \sec(dx + c) + \dots}{\sec(dx + c)^{\frac{7}{2}}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm="fricas")

[Out] integral((B*a^3*sec(d*x + c)^4 + (A + 3*B)*a^3*sec(d*x + c)^3 + 3*(A + B)*a^3*sec(d*x + c)^2 + (3*A + B)*a^3*sec(d*x + c) + A*a^3)/sec(d*x + c)^(7/2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(a \sec(dx + c) + a)^3}{\sec(dx + c)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^3/sec(d*x + c)^(7/2), x)

maple [A] time = 5.02, size = 385, normalized size = 1.82

$$4\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} a^3 \left(120A \cos\left(\frac{dx}{2} + \frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + (-432A - 84B)\left(\sin^6\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \dots\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x)

[Out] -4/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(120*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-432*A-84*B)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(602*A+294*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-208*A-126*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+65*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+105*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-189*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(a \sec(dx + c) + a)^3}{\sec(dx + c)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^3/sec(d*x + c)^(7/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(a + \frac{a}{\cos(c+dx)}\right)^3}{\left(\frac{1}{\cos(c+dx)}\right)^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^3)/(1/cos(c + d*x))^(7/2),x)

[Out] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^3)/(1/cos(c + d*x))^(7/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3 \left(\int \frac{A}{\sec^{\frac{7}{2}}(c + dx)} dx + \int \frac{3A}{\sec^{\frac{5}{2}}(c + dx)} dx + \int \frac{3A}{\sec^{\frac{3}{2}}(c + dx)} dx + \int \frac{A}{\sqrt{\sec(c + dx)}} dx + \int \frac{B}{\sec^{\frac{5}{2}}(c + dx)} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))**3*(A+B*sec(d*x+c))/sec(d*x+c)**(7/2),x)

[Out] a**3*(Integral(A/sec(c + d*x)**(7/2), x) + Integral(3*A/sec(c + d*x)**(5/2), x) + Integral(3*A/sec(c + d*x)**(3/2), x) + Integral(A/sqrt(sec(c + d*x)), x) + Integral(B/sec(c + d*x)**(5/2), x) + Integral(3*B/sec(c + d*x)**(3/2), x) + Integral(3*B/sqrt(sec(c + d*x)), x) + Integral(B*sqrt(sec(c + d*x)), x))

$$3.199 \quad \int \frac{(a+a \sec(c+dx))^3 (A+B \sec(c+dx))}{9 \sec^2(c+dx)} dx$$

Optimal. Leaf size=244

$$\frac{4a^3(23A + 24B) \sin(c + dx)}{105d \sec^3(c + dx)} + \frac{2(13A + 9B) \sin(c + dx) (a^3 \sec(c + dx) + a^3)}{63d \sec^5(c + dx)} + \frac{4a^3(11A + 13B) \sin(c + dx)}{21d \sqrt{\sec(c + dx)}} + \frac{4a^3(11A + 13B) \sin(c + dx)}{21d \sqrt{\sec(c + dx)}} + \dots$$

[Out] $4/105*a^3*(23*A+24*B)*\sin(d*x+c)/d/\sec(d*x+c)^{(3/2)}+2/9*a*A*(a+a*\sec(d*x+c))^2*\sin(d*x+c)/d/\sec(d*x+c)^{(7/2)}+2/63*(13*A+9*B)*(a^3+a^3*\sec(d*x+c))*\sin(d*x+c)/d/\sec(d*x+c)^{(5/2)}+4/21*a^3*(11*A+13*B)*\sin(d*x+c)/d/\sec(d*x+c)^{(1/2)}+4/15*a^3*(17*A+21*B)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c),2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d+4/21*a^3*(11*A+13*B)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c),2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d$

Rubi [A] time = 0.48, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {4017, 3996, 3787, 3769, 3771, 2641, 2639}

$$\frac{4a^3(23A + 24B) \sin(c + dx)}{105d \sec^3(c + dx)} + \frac{2(13A + 9B) \sin(c + dx) (a^3 \sec(c + dx) + a^3)}{63d \sec^5(c + dx)} + \frac{4a^3(11A + 13B) \sin(c + dx)}{21d \sqrt{\sec(c + dx)}} + \frac{4a^3(11A + 13B) \sin(c + dx)}{21d \sqrt{\sec(c + dx)}} + \dots$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + a*\text{Sec}[c + d*x])^3*(A + B*\text{Sec}[c + d*x])]/\text{Sec}[c + d*x]^{(9/2)}, x]$

[Out] $(4*a^3*(17*A + 21*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/(15*d) + (4*a^3*(11*A + 13*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticF}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/(21*d) + (4*a^3*(23*A + 24*B)*\text{Sin}[c + d*x])/(105*d*\text{Sec}[c + d*x]^{(3/2)}) + (4*a^3*(11*A + 13*B)*\text{Sin}[c + d*x])/(21*d*\text{Sqrt}[\text{Sec}[c + d*x]]) + (2*a*A*(a + a*\text{Sec}[c + d*x])^2*\text{Sin}[c + d*x])/(9*d*\text{Sec}[c + d*x]^{(7/2)}) + (2*(13*A + 9*B)*(a^3 + a^3*\text{Sec}[c + d*x])*\text{Sin}[c + d*x])/(63*d*\text{Sec}[c + d*x]^{(5/2)})$

Rule 2639

$\text{Int}[\text{Sqrt}[\sin[(c_.) + (d_.)*(x_.)]], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticE}[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 2641

Int[1/Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 3769

Int[(csc[(c_) + (d_)*(x_)])*(b_)^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 3771

Int[(csc[(c_) + (d_)*(x_)])*(b_)^(n_), x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 3787

Int[(csc[(e_) + (f_)*(x_)])*(d_)^(n_)*(csc[(e_) + (f_)*(x_)])*(b_) + (a_), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 3996

Int[(csc[(e_) + (f_)*(x_)])*(d_)^(n_)*(csc[(e_) + (f_)*(x_)])*(b_) + (a_)*(csc[(e_) + (f_)*(x_)])*(B_) + (A_)), x_Symbol] := Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]

Rule 4017

Int[(csc[(e_) + (f_)*(x_)])*(d_)^(n_)*(csc[(e_) + (f_)*(x_)])*(b_) + (a_)^(m_)*(csc[(e_) + (f_)*(x_)])*(B_) + (A_)), x_Symbol] := Simp[(a*A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && LtQ[n, -1]

Rubi steps

$$\begin{aligned}
\int \frac{(a + a \sec(c + dx))^3 (A + B \sec(c + dx))}{\sec^{\frac{9}{2}}(c + dx)} dx &= \frac{2aA(a + a \sec(c + dx))^2 \sin(c + dx)}{9d \sec^{\frac{7}{2}}(c + dx)} + \frac{2}{9} \int \frac{(a + a \sec(c + dx))^2}{\sec^{\frac{9}{2}}(c + dx)} dx \\
&= \frac{2aA(a + a \sec(c + dx))^2 \sin(c + dx)}{9d \sec^{\frac{7}{2}}(c + dx)} + \frac{2(13A + 9B)(a^3 + a^3 \sec^2(c + dx))}{63d \sec^{\frac{5}{2}}(c + dx)} \\
&= \frac{4a^3(23A + 24B) \sin(c + dx)}{105d \sec^{\frac{3}{2}}(c + dx)} + \frac{2aA(a + a \sec(c + dx))^2 \sin(c + dx)}{9d \sec^{\frac{7}{2}}(c + dx)} \\
&= \frac{4a^3(23A + 24B) \sin(c + dx)}{105d \sec^{\frac{3}{2}}(c + dx)} + \frac{2aA(a + a \sec(c + dx))^2 \sin(c + dx)}{9d \sec^{\frac{7}{2}}(c + dx)} \\
&= \frac{4a^3(23A + 24B) \sin(c + dx)}{105d \sec^{\frac{3}{2}}(c + dx)} + \frac{4a^3(11A + 13B) \sin(c + dx)}{21d \sqrt{\sec(c + dx)}} + \frac{2aA(a + a \sec(c + dx))^2 \sin(c + dx)}{9d \sec^{\frac{7}{2}}(c + dx)} \\
&= \frac{4a^3(17A + 21B) \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{\sec(c + dx)}}{15d} + \frac{4a^3(17A + 21B) \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{\sec(c + dx)}}{15d} + \frac{2aA(a + a \sec(c + dx))^2 \sin(c + dx)}{9d \sec^{\frac{7}{2}}(c + dx)}
\end{aligned}$$

Mathematica [C] time = 3.17, size = 196, normalized size = 0.80

$$a^3 \sqrt{\sec(c + dx)} \left(-112i(17A + 21B) e^{i(c+dx)} \sqrt{1 + e^{2i(c+dx)}} {}_2F_1\left(\frac{1}{2}, \frac{3}{4}, \frac{7}{4}; -e^{2i(c+dx)}\right) + \cos(c + dx)(30(97A + 107B) \sin(c + dx) + 14(73A + 54B) \sin[2(c + dx)] + 270A \sin[3(c + dx)] + 90B \sin[3(c + dx)] + 35A \sin[4(c + dx)]) \right) / (1260d)$$

Antiderivative was successfully verified.

[In] Integrate[((a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(9/2), x]

[Out] (a^3*Sqrt[Sec[c + d*x]]*(240*(11*A + 13*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] - (112*I)*(17*A + 21*B)*E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))] + Cos[c + d*x]*((5712*I)*A + (7056*I)*B + 30*(97*A + 107*B)*Sin[c + d*x] + 14*(73*A + 54*B)*Sin[2*(c + d*x)] + 270*A*Ssin[3*(c + d*x)] + 90*B*Ssin[3*(c + d*x)] + 35*A*Ssin[4*(c + d*x)])))/(1260*d)

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{Ba^3 \sec(dx+c)^4 + (A+3B)a^3 \sec(dx+c)^3 + 3(A+B)a^3 \sec(dx+c)^2 + (3A+B)a^3 \sec(dx+c) + Aa^3}{\sec(dx+c)^{\frac{9}{2}}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x, algorithm="fricas")

[Out] integral((B*a^3*sec(d*x + c)^4 + (A + 3*B)*a^3*sec(d*x + c)^3 + 3*(A + B)*a^3*sec(d*x + c)^2 + (3*A + B)*a^3*sec(d*x + c) + A*a^3)/sec(d*x + c)^(9/2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx+c) + A)(a \sec(dx+c) + a)^3}{\sec(dx+c)^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^3/sec(d*x + c)^(9/2), x)

maple [A] time = 4.46, size = 413, normalized size = 1.69

$$4\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} a^3 \left(-560A \cos\left(\frac{dx}{2} + \frac{c}{2}\right)\left(\sin^{10}\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + (2200A + 360B)\left(\sin^8\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x)

[Out] -4/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(-560*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(2200*A+360*B)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-3412*A-1296*B)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(2702*A+1806*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-738*A-624*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+165*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-357*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(co

$s(1/2*d*x+1/2*c), 2^{(1/2)})+195*B*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c), 2^{(1/2)})-441*B*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c), 2^{(1/2)})/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(a + \frac{a}{\cos(c+dx)}\right)^3}{\left(\frac{1}{\cos(c+dx)}\right)^{9/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^3)/(1/cos(c + d*x))^(9/2),x)

[Out] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^3)/(1/cos(c + d*x))^(9/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))**3*(A+B*sec(d*x+c))/sec(d*x+c)**(9/2),x)

[Out] Timed out

$$3.200 \quad \int \frac{(a+a \sec(c+dx))^3 (A+B \sec(c+dx))}{\sec^{\frac{11}{2}}(c+dx)} dx$$

Optimal. Leaf size=277

$$\frac{4a^3(15A + 17B) \sin(c + dx)}{45d \sec^{\frac{3}{2}}(c + dx)} + \frac{20a^3(21A + 22B) \sin(c + dx)}{693d \sec^{\frac{5}{2}}(c + dx)} + \frac{2(15A + 11B) \sin(c + dx) (a^3 \sec(c + dx) + a^3)}{99d \sec^{\frac{7}{2}}(c + dx)} + \dots$$

[Out] 20/693*a^3*(21*A+22*B)*sin(d*x+c)/d/sec(d*x+c)^(5/2)+4/45*a^3*(15*A+17*B)*sin(d*x+c)/d/sec(d*x+c)^(3/2)+2/11*a*A*(a+a*sec(d*x+c))^2*sin(d*x+c)/d/sec(d*x+c)^(9/2)+2/99*(15*A+11*B)*(a^3+a^3*sec(d*x+c))*sin(d*x+c)/d/sec(d*x+c)^(7/2)+4/231*a^3*(105*A+121*B)*sin(d*x+c)/d/sec(d*x+c)^(1/2)+4/15*a^3*(15*A+17*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+4/231*a^3*(105*A+121*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d

Rubi [A] time = 0.51, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {4017, 3996, 3787, 3769, 3771, 2639, 2641}

$$\frac{4a^3(15A + 17B) \sin(c + dx)}{45d \sec^{\frac{3}{2}}(c + dx)} + \frac{20a^3(21A + 22B) \sin(c + dx)}{693d \sec^{\frac{5}{2}}(c + dx)} + \frac{2(15A + 11B) \sin(c + dx) (a^3 \sec(c + dx) + a^3)}{99d \sec^{\frac{7}{2}}(c + dx)} + \dots$$

Antiderivative was successfully verified.

[In] Int[((a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(11/2),x]

[Out] (4*a^3*(15*A + 17*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(15*d) + (4*a^3*(105*A + 121*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(231*d) + (20*a^3*(21*A + 22*B)*Sin[c + d*x])/(693*d*Sec[c + d*x]^(5/2)) + (4*a^3*(15*A + 17*B)*Sin[c + d*x])/(45*d*Sec[c + d*x]^(3/2)) + (4*a^3*(105*A + 121*B)*Sin[c + d*x])/(231*d*Sqrt[Sec[c + d*x]]) + (2*a*A*(a + a*Sec[c + d*x])^2*Ssin[c + d*x])/(11*d*Sec[c + d*x]^(9/2)) + (2*(15*A + 11*B)*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x])/(99*d*Sec[c + d*x]^(7/2))

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_.)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 3769

Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.))^n, x_Symbol] := Simp[(Cos[c + d*x]*(b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 3771

Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.))^n, x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 3996

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]

Rule 4017

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := Simp[(a*A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && LtQ[n, -1]

Rubi steps

$$\begin{aligned}
\int \frac{(a + a \sec(c + dx))^3 (A + B \sec(c + dx))}{\sec^{\frac{11}{2}}(c + dx)} dx &= \frac{2aA(a + a \sec(c + dx))^2 \sin(c + dx)}{11d \sec^{\frac{9}{2}}(c + dx)} + \frac{2}{11} \int \frac{(a + a \sec(c + dx))}{\sec^{\frac{7}{2}}(c + dx)} dx \\
&= \frac{2aA(a + a \sec(c + dx))^2 \sin(c + dx)}{11d \sec^{\frac{9}{2}}(c + dx)} + \frac{2(15A + 11B)(a^3 + a^3 \sec^2(c + dx))}{99d \sec^{\frac{7}{2}}(c + dx)} \\
&= \frac{20a^3(21A + 22B) \sin(c + dx)}{693d \sec^{\frac{5}{2}}(c + dx)} + \frac{2aA(a + a \sec(c + dx))^2 \sin(c + dx)}{11d \sec^{\frac{9}{2}}(c + dx)} \\
&= \frac{20a^3(21A + 22B) \sin(c + dx)}{693d \sec^{\frac{5}{2}}(c + dx)} + \frac{2aA(a + a \sec(c + dx))^2 \sin(c + dx)}{11d \sec^{\frac{9}{2}}(c + dx)} \\
&= \frac{20a^3(21A + 22B) \sin(c + dx)}{693d \sec^{\frac{5}{2}}(c + dx)} + \frac{4a^3(15A + 17B) \sin(c + dx)}{45d \sec^{\frac{3}{2}}(c + dx)} \\
&= \frac{20a^3(21A + 22B) \sin(c + dx)}{693d \sec^{\frac{5}{2}}(c + dx)} + \frac{4a^3(15A + 17B) \sin(c + dx)}{45d \sec^{\frac{3}{2}}(c + dx)} \\
&= \frac{4a^3(15A + 17B) \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{\sec(c + dx)}}{15d} + \dots
\end{aligned}$$

Mathematica [C] time = 3.74, size = 239, normalized size = 0.86

$$\frac{a^3 e^{-idx} \sqrt{\sec(c + dx)} (\cos(dx) + i \sin(dx)) \left(-2464i(15A + 17B) e^{i(c+dx)} \sqrt{1 + e^{2i(c+dx)}} {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{7}{4}; -e^{2i(c+dx)}\right) + \cos(dx) \right)}{15d}$$

Antiderivative was successfully verified.

[In] Integrate[((a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(11/2), x]

[Out] (a^3*Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])*(480*(105*A + 121*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] - (2464*I)*(15*A + 17*B)*E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))] + Cos[c + d*x]*((110880*I)*A + (125664*I)*B + 30*(1953*A + 2134*B)*Sin[c + d*x] + 308*(75*A + 73*B)*Sin[2*(c + d*x)] + 8505*A*Sin[3*(c + d*x)] + 5940*B*Sin[3*(c + d*x)] + 2310*A*Sin[4*(c + d*x)] + 770*B*Sin[4*(c + d*x)] + 315*A*Sin[5*(c + d*x)])))/(27720*d*E^(I*d*x))

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{Ba^3 \sec(dx+c)^4 + (A+3B)a^3 \sec(dx+c)^3 + 3(A+B)a^3 \sec(dx+c)^2 + (3A+B)a^3 \sec(dx+c) + Aa^3}{\sec(dx+c)^{\frac{11}{2}}} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(11/2),x, algorithm="fricas")

[Out] integral((B*a^3*sec(d*x + c)^4 + (A + 3*B)*a^3*sec(d*x + c)^3 + 3*(A + B)*a^3*sec(d*x + c)^2 + (3*A + B)*a^3*sec(d*x + c) + A*a^3)/sec(d*x + c)^(11/2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx+c) + A)(a \sec(dx+c) + a)^3}{\sec(dx+c)^{\frac{11}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(11/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^3/sec(d*x + c)^(11/2), x)

maple [A] time = 4.48, size = 441, normalized size = 1.59

$$4\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} a^3 \left(10080A \cos\left(\frac{dx}{2} + \frac{c}{2}\right)\left(\sin^{12}\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + (-43680A - 6160B)\left(\sin^{10}\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + (77280A + 24200B)\sin^8\left(\frac{dx}{2} + \frac{c}{2}\right) + (-72240A - 37532B)\sin^6\left(\frac{dx}{2} + \frac{c}{2}\right) + (39270A + 29722B)\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right) + (-8820A - 8118B)\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right) + 1575A\right) \sqrt{\sin\left(\frac{dx}{2} + \frac{c}{2}\right)} - 3465A \left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^{-1/2} \text{EllipticF}\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), 2^{1/2}\right) - 3465A \left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^{-1/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(11/2),x)

[Out] -4/3465*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(10080*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12+(-43680*A-6160*B)*sin(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+(77280*A+24200*B)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-72240*A-37532*B)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(39270*A+29722*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-8820*A-8118*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+1575*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3465*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)

$\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)})+1815*B*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)})-3927*B*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)})/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(11/2),x, algorithm m="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(a + \frac{a}{\cos(c+dx)}\right)^3}{\left(\frac{1}{\cos(c+dx)}\right)^{11/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^3)/(1/cos(c + d*x))^(11/2),x)

[Out] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^3)/(1/cos(c + d*x))^(11/2),x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))**3*(A+B*sec(d*x+c))/sec(d*x+c)**(11/2),x)

[Out] Timed out

$$3.201 \quad \int \frac{\sec^{\frac{7}{2}}(c+dx)(A+B \sec(c+dx))}{a+a \sec(c+dx)} dx$$

Optimal. Leaf size=229

$$\frac{(A-B) \sin(c+dx) \sec^{\frac{7}{2}}(c+dx)}{d(a \sec(c+dx) + a)} - \frac{(5A-7B) \sin(c+dx) \sec^{\frac{5}{2}}(c+dx)}{5ad} + \frac{5(A-B) \sin(c+dx) \sec^{\frac{3}{2}}(c+dx)}{3ad} - \frac{3(5A-7B) \sin(c+dx) \sec^{\frac{1}{2}}(c+dx)}{3ad}$$

[Out] 5/3*(A-B)*sec(d*x+c)^(3/2)*sin(d*x+c)/a/d-1/5*(5*A-7*B)*sec(d*x+c)^(5/2)*sin(d*x+c)/a/d+(A-B)*sec(d*x+c)^(7/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))-3/5*(5*A-7*B)*sin(d*x+c)*sec(d*x+c)^(1/2)/a/d+3/5*(5*A-7*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d+5/3*(A-B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d

Rubi [A] time = 0.25, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4019, 3787, 3768, 3771, 2641, 2639}

$$\frac{(A-B) \sin(c+dx) \sec^{\frac{7}{2}}(c+dx)}{d(a \sec(c+dx) + a)} - \frac{(5A-7B) \sin(c+dx) \sec^{\frac{5}{2}}(c+dx)}{5ad} + \frac{5(A-B) \sin(c+dx) \sec^{\frac{3}{2}}(c+dx)}{3ad} - \frac{3(5A-7B) \sin(c+dx) \sec^{\frac{1}{2}}(c+dx)}{3ad}$$

Antiderivative was successfully verified.

[In] Int[(Sec[c + d*x]^(7/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x]),x]

[Out] (3*(5*A - 7*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*a*d) + (5*(A - B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a*d) - (3*(5*A - 7*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*a*d) + (5*(A - B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a*d) - ((5*A - 7*B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*a*d) + ((A - B)*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]))

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x]
)*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^(n)*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^{\frac{7}{2}}(c+dx)(A+B\sec(c+dx))}{a+a\sec(c+dx)} dx &= \frac{(A-B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} + \frac{\int \sec^{\frac{5}{2}}(c+dx)\left(\frac{5}{2}a(A-B) - \frac{1}{2}a(5A+B)\right) dx}{a^2} \\
&= \frac{(A-B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{(5A-7B)\int \sec^{\frac{7}{2}}(c+dx) dx}{2a} + \frac{(5A+B)\int \sec^{\frac{5}{2}}(c+dx) dx}{2a} \\
&= \frac{5(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3ad} - \frac{(5A-7B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5ad} \\
&= -\frac{3(5A-7B)\sqrt{\sec(c+dx)}\sin(c+dx)}{5ad} + \frac{5(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3ad} \\
&= \frac{5(A-B)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3ad} - \frac{3(5A-7B)\sqrt{\sec(c+dx)}}{5ad} \\
&= \frac{3(5A-7B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5ad} + \frac{5(A-B)\sqrt{\cos(c+dx)}}{5ad}
\end{aligned}$$

Mathematica [C] time = 7.70, size = 814, normalized size = 3.55

$$\frac{Ae^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\csc\left(\frac{c}{2}\right)\left(e^{2idx}\left(-1+e^{2ic}\right){}_2F_1\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)-3\sqrt{1+e^{2i(c+dx)}}\right)\sec\left(\frac{c}{2}\right)(A+B\sec(c+dx))}{\sqrt{2}d(B+A\cos(c+dx))(\sec(c+dx)a+a)}$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[c + d*x]^(7/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x]),x]

[Out] -((A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*Sec[c/2]*(A + B*Sec[c + d*x]))/(Sqrt[2]*d*E^(I*d*x)*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])) + (7*B*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*Sec[c/2]*(A + B*Sec[c + d*x]))/(5*Sqrt[2]*d*E^(I*d*x)*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])) + (5*A*Cos[c/2 + (d*x)/2]^2*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c/2]*Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x])*Sin[c]/(3*d*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])) - (5*B*Cos[c/2 + (d*x)/2]^2*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c/2]*Sqrt[Sec[c + d*x]]*(A + B

$$\text{Sec}[c + d*x] * \text{Sin}[c] / (3*d*(B + A*\text{Cos}[c + d*x])*(a + a*\text{Sec}[c + d*x])) + (\text{Cos}[c/2 + (d*x)/2]^2 * \text{Sqrt}[\text{Sec}[c + d*x]] * (A + B*\text{Sec}[c + d*x]) * ((3*(-5*A + 7*B) * \text{Cos}[d*x] * \text{Csc}[c/2] * \text{Sec}[c/2]) / (5*d) - ((-A + B) * \text{Sec}[c/2] * \text{Sec}[c] * (-\text{Sin}[c/2] + 5*\text{Sin}[(3*c)/2])) / (3*d) - (2*\text{Sec}[c/2] * \text{Sec}[c/2 + (d*x)/2] * (-A*\text{Sin}[(d*x)/2]) + B*\text{Sin}[(d*x)/2])) / d + (4*B*\text{Sec}[c] * \text{Sec}[c + d*x]^2 * \text{Sin}[d*x]) / (5*d) + (4*\text{Sec}[c] * \text{Sec}[c + d*x] * (3*B*\text{Sin}[c] + 5*A*\text{Sin}[d*x] - 5*B*\text{Sin}[d*x])) / (15*d)) / ((B + A*\text{Cos}[c + d*x])*(a + a*\text{Sec}[c + d*x]))$$

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{(B \sec(dx + c)^4 + A \sec(dx + c)^3) \sqrt{\sec(dx + c)}}{a \sec(dx + c) + a}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="fricas")

[Out] integral((B*sec(d*x + c)^4 + A*sec(d*x + c)^3)*sqrt(sec(d*x + c))/(a*sec(d*x + c) + a), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sec(dx + c)^{\frac{7}{2}}}{a \sec(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(7/2)/(a*sec(d*x + c) + a), x)

maple [B] time = 14.32, size = 806, normalized size = 3.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x)

[Out]
$$-(-(-2*\cos(1/2*d*x+1/2*c)^2+1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/a*((2*A-2*B)*(-1/6*\cos(1/2*d*x+1/2*c)*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/(-1/2+\cos(1/2*d*x+1/2*c)^2)^2+1/3*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}* \text{EllipticF}(\cos(1/2*d*x+1/2*c), 2^{(1/2)})) + (-2*A+2*B)*(-(-2*\sin(1/2*d*x+1/2*c)^2)^{(1/2)})$$

$$4 + \sin(1/2 dx + 1/2 c)^2)^{1/2} * (\sin(1/2 dx + 1/2 c)^2)^{1/2} * (2 \sin(1/2 dx + 1/2 c)^2 - 1)^{1/2} * \text{EllipticE}(\cos(1/2 dx + 1/2 c), 2^{1/2}) + 2 * (-2 \sin(1/2 dx + 1/2 c)^4 + \sin(1/2 dx + 1/2 c)^2)^{1/2} * \cos(1/2 dx + 1/2 c) * \sin(1/2 dx + 1/2 c)^2 / \sin(1/2 dx + 1/2 c)^2 / (2 \sin(1/2 dx + 1/2 c)^2 - 1) - 2/5 * B / (8 \sin(1/2 dx + 1/2 c)^6 - 12 \sin(1/2 dx + 1/2 c)^4 + 6 \sin(1/2 dx + 1/2 c)^2 - 1) / \sin(1/2 dx + 1/2 c)^2 * (12 * \text{EllipticE}(\cos(1/2 dx + 1/2 c), 2^{1/2})) * (2 \sin(1/2 dx + 1/2 c)^2 - 1)^{1/2} * (\sin(1/2 dx + 1/2 c)^2)^{1/2} * \sin(1/2 dx + 1/2 c)^4 - 24 * \cos(1/2 dx + 1/2 c) * \sin(1/2 dx + 1/2 c)^6 - 12 * \text{EllipticE}(\cos(1/2 dx + 1/2 c), 2^{1/2}) * (2 \sin(1/2 dx + 1/2 c)^2 - 1)^{1/2} * (\sin(1/2 dx + 1/2 c)^2)^{1/2} * \sin(1/2 dx + 1/2 c)^2 + 24 * \sin(1/2 dx + 1/2 c)^4 * \cos(1/2 dx + 1/2 c) + 3 * (\sin(1/2 dx + 1/2 c)^2)^{1/2} * (2 \sin(1/2 dx + 1/2 c)^2 - 1)^{1/2} * \text{EllipticE}(\cos(1/2 dx + 1/2 c), 2^{1/2}) - 8 * \sin(1/2 dx + 1/2 c)^2 * \cos(1/2 dx + 1/2 c) * (-2 \sin(1/2 dx + 1/2 c)^4 + \sin(1/2 dx + 1/2 c)^2)^{1/2} + (A - B) * (\cos(1/2 dx + 1/2 c) * (\sin(1/2 dx + 1/2 c)^2)^{1/2} * (2 \sin(1/2 dx + 1/2 c)^2 - 1)^{1/2} * (\text{EllipticF}(\cos(1/2 dx + 1/2 c), 2^{1/2}) - \text{EllipticE}(\cos(1/2 dx + 1/2 c), 2^{1/2}))) - 2 * \sin(1/2 dx + 1/2 c)^4 + \sin(1/2 dx + 1/2 c)^2) / \cos(1/2 dx + 1/2 c) / (-2 \sin(1/2 dx + 1/2 c)^4 + \sin(1/2 dx + 1/2 c)^2)^{1/2} / \sin(1/2 dx + 1/2 c) / (2 * \cos(1/2 dx + 1/2 c)^2 - 1)^{1/2} / d$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sec(dx + c)^{7/2}}{a \sec(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(dx+c)^(7/2)*(A+B*sec(dx+c))/(a+a*sec(dx+c)),x, algorithm="maxima")

[Out] integrate((B*sec(dx + c) + A)*sec(dx + c)^(7/2)/(a*sec(dx + c) + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(\frac{1}{\cos(c+dx)}\right)^{7/2}}{a + \frac{a}{\cos(c+dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + dx))*(1/cos(c + dx))^(7/2))/(a + a/cos(c + dx)),x)

[Out] int(((A + B/cos(c + dx))*(1/cos(c + dx))^(7/2))/(a + a/cos(c + dx)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.


```
[In] integrate(sec(d*x+c)**(7/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

$$3.202 \quad \int \frac{\sec^2(c+dx)(A+B \sec(c+dx))}{a+a \sec(c+dx)} dx$$

Optimal. Leaf size=192

$$\frac{(A-B) \sin(c+dx) \sec^2(c+dx)}{d(a \sec(c+dx) + a)} - \frac{(3A-5B) \sin(c+dx) \sec^2(c+dx)}{3ad} + \frac{3(A-B) \sin(c+dx) \sqrt{\sec(c+dx)}}{ad} - \frac{(3A-5B) \sin(c+dx) \sqrt{\sec(c+dx)}}{3ad} \quad (3A-5B)$$

[Out] $-1/3*(3*A-5*B)*\sec(d*x+c)^{(3/2)}*\sin(d*x+c)/a/d+(A-B)*\sec(d*x+c)^{(5/2)}*\sin(d*x+c)/d/(a+a*\sec(d*x+c))+3*(A-B)*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/a/d-3*(A-B)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/a/d-1/3*(3*A-5*B)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/a/d$

Rubi [A] time = 0.23, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4019, 3787, 3768, 3771, 2639, 2641}

$$\frac{(A-B) \sin(c+dx) \sec^2(c+dx)}{d(a \sec(c+dx) + a)} - \frac{(3A-5B) \sin(c+dx) \sec^2(c+dx)}{3ad} + \frac{3(A-B) \sin(c+dx) \sqrt{\sec(c+dx)}}{ad} - \frac{(3A-5B) \sin(c+dx) \sqrt{\sec(c+dx)}}{3ad} \quad (3A-5B)$$

Antiderivative was successfully verified.

[In] Int[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x]),x]

[Out] $(-3*(A-B)*\text{Sqrt}[\text{Cos}[c+d*x]]*\text{EllipticE}[(c+d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c+d*x]])/(a*d) - ((3*A-5*B)*\text{Sqrt}[\text{Cos}[c+d*x]]*\text{EllipticF}[(c+d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c+d*x]])/(3*a*d) + (3*(A-B)*\text{Sqrt}[\text{Sec}[c+d*x]]*\text{Sin}[c+d*x])/(a*d) - ((3*A-5*B)*\text{Sec}[c+d*x]^{(3/2)}*\text{Sin}[c+d*x])/(3*a*d) + ((A-B)*\text{Sec}[c+d*x]^{(5/2)}*\text{Sin}[c+d*x])/(d*(a+a*\text{Sec}[c+d*x]))$

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]
)*(b*Csc[c + d*x])^(n - 1)/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x]
)^(n)*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] := Simp[(d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{a+a\sec(c+dx)} dx &= \frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} + \frac{\int \sec^{\frac{3}{2}}(c+dx)\left(\frac{3}{2}a(A-B) - \frac{1}{2}a(3A-B)\right) dx}{a^2} \\
&= \frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{(3A-5B)\int \sec^{\frac{5}{2}}(c+dx) dx}{2a} + \frac{(3A-B)\int \sec^{\frac{3}{2}}(c+dx) dx}{a^2} \\
&= \frac{3(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{ad} - \frac{(3A-5B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3ad} \\
&= \frac{3(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{ad} - \frac{(3A-5B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3ad} \\
&= -\frac{3(A-B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} - \frac{(3A-5B)\sqrt{\cos(c+dx)}}{ad}
\end{aligned}$$

Mathematica [C] time = 3.53, size = 372, normalized size = 1.94

$$e^{-\frac{1}{2}i(c+dx)} \cos\left(\frac{1}{2}(c+dx)\right) \sqrt{\sec(c+dx)} (A+B\sec(c+dx)) \left(i \left(3(A-B)e^{i(c+dx)} \sqrt{1+e^{2i(c+dx)}} (e^{i(c+dx)} + e^{2i(c+dx)} + e^{3i(c+dx)}) \right) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x]),x]
[Out] (Cos[(c + d*x)/2]*(-(3*A - 5*B)*(1 + E^(I*(c + d*x)) + E^((2*I)*(c + d*x)) + E^((3*I)*(c + d*x))))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]) + I*(-3*A + 5*B - 6*A*E^(I*(c + d*x)) + 8*B*E^(I*(c + d*x)) - 12*A*E^((2*I)*(c + d*x)) + 10*B*E^((2*I)*(c + d*x)) - 6*A*E^((3*I)*(c + d*x)) + 4*B*E^((3*I)*(c + d*x)) - 9*A*E^((4*I)*(c + d*x)) + 9*B*E^((4*I)*(c + d*x)) + 3*(A - B)*E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*(1 + E^(I*(c + d*x)) + E^((2*I)*(c + d*x)) + E^((3*I)*(c + d*x)))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x]))/(3*a*d*E^((I/2)*(c + d*x))*(1 + E^((2*I)*(c + d*x)))*(B + A*Cos[c + d*x])*(1 + Sec[c + d*x])))
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B\sec(dx+c)^3 + A\sec(dx+c)^2)\sqrt{\sec(dx+c)}}{a\sec(dx+c) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="fricas")

[Out] integral((B*sec(d*x + c)^3 + A*sec(d*x + c)^2)*sqrt(sec(d*x + c))/(a*sec(d*x + c) + a), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sec(dx + c)^{\frac{5}{2}}}{a \sec(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(5/2)/(a*sec(d*x + c) + a), x)

maple [B] time = 11.62, size = 493, normalized size = 2.57

$$\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \left(2B \left(-\frac{\cos\left(\frac{dx}{2} + \frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)}}{6\left(-\frac{1}{2} + \cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^2} + \frac{\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2} + \frac{c}{2}\right)}}{3\sqrt{-2\cos\left(\frac{dx}{2} + \frac{c}{2}\right)}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x)

[Out]
$$\begin{aligned} & -(-(-2*\cos(1/2*d*x+1/2*c)^2+1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/a*(2*B*(-1/6*\cos \\ & (1/2*d*x+1/2*c)*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/(-1/2+ \\ & \cos(1/2*d*x+1/2*c)^2)^2+1/3*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)*(-2*\cos(1/2*d*x+1/ \\ & 2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)*\text{Elliptic} \\ & \text{icF}(\cos(1/2*d*x+1/2*c), 2^{(1/2)})})+(2*A-2*B)*(-(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1 \\ & /2*d*x+1/2*c)^2)^{(1/2)*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)*(2*\sin(1/2*d*x+1/2*c)^2 \\ & -1)^{(1/2)*\text{EllipticE}(\cos(1/2*d*x+1/2*c), 2^{(1/2)})}+2*(-2*\sin(1/2*d*x+1/2*c)^4+ \\ & \sin(1/2*d*x+1/2*c)^2)^{(1/2)*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^2}/\sin(1/ \\ & 2*d*x+1/2*c)^2/(2*\sin(1/2*d*x+1/2*c)^2-1)+(-A+B)*(\cos(1/2*d*x+1/2*c)*(\sin(1 \\ & /2*d*x+1/2*c)^2)^{(1/2)*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)*(\text{EllipticF}(\cos(1/2* \\ & d*x+1/2*c), 2^{(1/2)})-\text{EllipticE}(\cos(1/2*d*x+1/2*c), 2^{(1/2)})})-2*\sin(1/2*d*x+1/ \\ & 2*c)^4+\sin(1/2*d*x+1/2*c)^2)/\cos(1/2*d*x+1/2*c)/(-2*\sin(1/2*d*x+1/2*c)^4+\sin \\ & (1/2*d*x+1/2*c)^2)^{(1/2)}/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1 \\ & /2)}/d \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sec(dx + c)^{\frac{5}{2}}}{a \sec(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(5/2)/(a*sec(d*x + c) + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(\frac{1}{\cos(c+dx)}\right)^{5/2}}{a + \frac{a}{\cos(c+dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + a/cos(c + d*x)),x)

[Out] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + a/cos(c + d*x)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x)

[Out] Timed out

$$3.203 \quad \int \frac{\sec^3(c+dx)(A+B \sec(c+dx))}{a+a \sec(c+dx)} dx$$

Optimal. Leaf size=153

$$\frac{(A-B) \sin(c+dx) \sec^3(c+dx)}{d(a \sec(c+dx) + a)} - \frac{(A-3B) \sin(c+dx) \sqrt{\sec(c+dx)}}{ad} + \frac{(A-B) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} F\left(\frac{1}{2}\right)}{ad}$$

[Out] (A-B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))-(A-3*B)*sin(d*x+c)*sec(d*x+c)^(1/2)/a/d+(A-3*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d+(A-B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d

Rubi [A] time = 0.19, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4019, 3787, 3771, 2641, 3768, 2639}

$$\frac{(A-B) \sin(c+dx) \sec^3(c+dx)}{d(a \sec(c+dx) + a)} - \frac{(A-3B) \sin(c+dx) \sqrt{\sec(c+dx)}}{ad} + \frac{(A-B) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} F\left(\frac{1}{2}\right)}{ad}$$

Antiderivative was successfully verified.

[In] Int[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x]),x]

[Out] ((A - 3*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d) + ((A - B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d) - ((A - 3*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*d) + ((A - B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]))

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 3768

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x])* (b*Csc[c + d*x])^(n - 1)]/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I

$\text{Int}[(b*\text{Csc}[c + d*x])^{(n - 2)}, x], x] /; \text{FreeQ}\{b, c, d\}, x] \ \&\& \ \text{GtQ}[n, 1] \ \&\& \ \text{IntegerQ}[2*n]$

Rule 3771

$\text{Int}[(\text{csc}[c] + d*x)*(b)]^{(n)}, x_Symbol] \ :> \ \text{Dist}[(b*\text{Csc}[c + d*x])^{(n)}*\text{Sin}[c + d*x]^n, \text{Int}[1/\text{Sin}[c + d*x]^n, x], x] /; \text{FreeQ}\{b, c, d\}, x] \ \&\& \ \text{EqQ}[n^2, 1/4]$

Rule 3787

$\text{Int}[(\text{csc}[e] + (f*x)*d)^{(n)}*(\text{csc}[e] + (f*x)*b) + (a)], x_Symbol] \ :> \ \text{Dist}[a, \text{Int}[(d*\text{Csc}[e + f*x])^n, x], x] + \text{Dist}[b/d, \text{Int}[(d*\text{Csc}[e + f*x])^{(n + 1)}, x], x] /; \text{FreeQ}\{a, b, d, e, f, n\}, x]$

Rule 4019

$\text{Int}[(\text{csc}[e] + (f*x)*d)^{(n)}*(\text{csc}[e] + (f*x)*b) + (a)]^{(m)}*(\text{csc}[e] + (f*x)*B) + (A)], x_Symbol] \ :> \ \text{Simp}[(d*(A*b - a*B)*\text{Cot}[e + f*x]*(a + b*\text{Csc}[e + f*x])^m*(d*\text{Csc}[e + f*x])^{(n - 1)})/(a*f*(2*m + 1)), x] - \text{Dist}[1/(a*b*(2*m + 1)), \text{Int}[(a + b*\text{Csc}[e + f*x])^{(m + 1)}*(d*\text{Csc}[e + f*x])^{(n - 1)}*\text{Simp}[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m - n + 1) + A*b*(m + n))*\text{Csc}[e + f*x], x], x], x] /; \text{FreeQ}\{a, b, d, e, f, A, B\}, x] \ \&\& \ \text{NeQ}[A*b - a*B, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{LtQ}[m, -2^{(-1)}] \ \&\& \ \text{GtQ}[n, 0]$

Rubi steps

$$\begin{aligned}
\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{a+a\sec(c+dx)} dx &= \frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} + \frac{\int \sqrt{\sec(c+dx)} \left(\frac{1}{2}a(A-B) - \frac{1}{2}a\right)}{a^2} \\
&= \frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{(A-3B)\int \sec^{\frac{3}{2}}(c+dx) dx}{2a} + \frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{ad} \\
&= -\frac{(A-3B)\sqrt{\sec(c+dx)}\sin(c+dx)}{ad} + \frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} \\
&= \frac{(A-B)\sqrt{\cos(c+dx)} F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} - \frac{(A-3B)\sqrt{\sec(c+dx)}}{ad} \\
&= \frac{(A-3B)\sqrt{\cos(c+dx)} E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} + \frac{(A-B)\sqrt{\cos(c+dx)}}{ad}
\end{aligned}$$

Mathematica [C] time = 4.83, size = 420, normalized size = 2.75

$$\cos^2\left(\frac{1}{2}(c+dx)\right)(A+B\sec(c+dx))\left(-6\sqrt{\sec(c+dx)}\left(2(B-A)\tan\left(\frac{1}{2}(c+dx)\right)+2(A-3B)\csc(c)\cos(dx)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x]),x]
[Out] (Cos[(c + d*x)/2]^2*(A + B*Sec[c + d*x])*((-2*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))]/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Csc[c]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))]] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])/E^(I*d*x) + (6*Sqrt[2]*B*Sqrt[E^(I*(c + d*x))]/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Csc[c]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))]] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])/E^(I*d*x) + 12*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]] - 12*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]] - 6*Sqrt[Sec[c + d*x]]*(2*(A - 3*B)*Cos[d*x]*Csc[c] + 2*(-A + B)*Tan[(c + d*x)/2]))/(6*a*d*(B + A*Cos[c + d*x])*(1 + Sec[c + d*x]))
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B\sec(dx+c)^2 + A\sec(dx+c))\sqrt{\sec(dx+c)}}{a\sec(dx+c) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(dx+c)^(3/2)*(A+B*sec(dx+c))/(a+a*sec(dx+c)),x, algorithm="fricas")

[Out] integral((B*sec(dx + c)^2 + A*sec(dx + c))*sqrt(sec(dx + c))/(a*sec(dx + c) + a), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sec(dx + c)^{\frac{3}{2}}}{a \sec(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(dx+c)^(3/2)*(A+B*sec(dx+c))/(a+a*sec(dx+c)),x, algorithm="giac")

[Out] integrate((B*sec(dx + c) + A)*sec(dx + c)^(3/2)/(a*sec(dx + c) + a), x)

maple [A] time = 8.62, size = 318, normalized size = 2.08

$$\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \left(-\cos\left(\frac{dx}{2} + \frac{c}{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} - 1\sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}\right) + s$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(dx+c)^(3/2)*(A+B*sec(dx+c))/(a+a*sec(dx+c)),x)

[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a*(-cos(1/2*d*x+1/2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(A-3*B)*sin(1/2*d*x+1/2*c)^4-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(A-5*B)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^3/(2*sin(1/2*d*x+1/2*c)^2-1)/cos(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sec(dx + c)^{\frac{3}{2}}}{a \sec(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(3/2)/(a*sec(d*x + c) + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(\frac{1}{\cos(c+dx)}\right)^{3/2}}{a + \frac{a}{\cos(c+dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + a/cos(c + d*x)),x)

[Out] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + a/cos(c + d*x)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A \sec^2(c+dx)}{\sec(c+dx)+1} dx + \int \frac{B \sec^2(c+dx)}{\sec(c+dx)+1} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x)

[Out] (Integral(A*sec(c + d*x)**(3/2)/(sec(c + d*x) + 1), x) + Integral(B*sec(c + d*x)**(5/2)/(sec(c + d*x) + 1), x))/a

$$3.204 \quad \int \frac{\sqrt{\sec(c+dx)} (A+B \sec(c+dx))}{a+a \sec(c+dx)} dx$$

Optimal. Leaf size=123

$$\frac{(A-B) \sin(c+dx) \sqrt{\sec(c+dx)}}{d(a \sec(c+dx) + a)} + \frac{(A+B) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} F\left(\frac{1}{2}(c+dx) \middle| 2\right)}{ad} - \frac{(A-B) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{ad}$$

[Out] (A-B)*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))- (A-B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d+(A+B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d

Rubi [A] time = 0.17, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {4019, 3787, 3771, 2639, 2641}

$$\frac{(A-B) \sin(c+dx) \sqrt{\sec(c+dx)}}{d(a \sec(c+dx) + a)} + \frac{(A+B) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} F\left(\frac{1}{2}(c+dx) \middle| 2\right)}{ad} - \frac{(A-B) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{ad}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x]),x]

[Out] -(((A - B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d) + ((A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d) + ((A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]))

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 3771

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^n, x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&

EqQ[n^2, 1/4]

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) +
(a_.)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (
a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> Simp[(d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{\sec(c+dx)}(A+B\sec(c+dx))}{a+a\sec(c+dx)} dx &= \frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{d(a+a\sec(c+dx))} + \frac{\int \frac{-\frac{1}{2}a(A-B)+\frac{1}{2}a(A+B)\sec(c+dx)}{\sqrt{\sec(c+dx)}} dx}{a^2} \\ &= \frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{(A-B)\int \frac{1}{\sqrt{\sec(c+dx)}} dx}{2a} + \frac{(A+B)\sqrt{\sec(c+dx)}}{2a} \\ &= \frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{((A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)})}{2a} \\ &= -\frac{(A-B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} + \frac{(A+B)\sqrt{\sec(c+dx)}}{2a} \end{aligned}$$

Mathematica [C] time = 1.18, size = 200, normalized size = 1.63

$$\frac{(-1 + e^{2ic})e^{-\frac{1}{2}i(4c+dx)}\left(\csc\left(\frac{c}{2}\right) + i\sec\left(\frac{c}{2}\right)\right)\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left((A-B)\left(e^{i(c+dx)}\left(1 + e^{i(c+dx)}\right)\sqrt{1 + e^{2i(c+dx)}}\right)\right)}{24ad}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x]),x]
 [Out] $((-1 + E^{(2I)c}) * ((-3I)(A + B)(1 + E^{I(c + dx)})) * \text{Sqrt}[\text{Cos}[c + dx]] * \text{EllipticF}[(c + dx)/2, 2] + (A - B) * (-3(1 + E^{(2I)(c + dx)}) + E^{I(c + dx)}) * (1 + E^{I(c + dx)}) * \text{Sqrt}[1 + E^{(2I)(c + dx)}]) * \text{Hypergeometric2F1}[1/2, 3/4, 7/4, -E^{(2I)(c + dx)}]) * (\text{Csc}[c/2] + I * \text{Sec}[c/2]) * \text{Sec}[(c + dx)/2] * \text{Sqrt}[\text{Sec}[c + dx]]) / (24 * a * d * E^{(I/2)(4c + dx)})$
fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{(B \sec(dx + c) + A) \sqrt{\sec(dx + c)}}{a \sec(dx + c) + a}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
 [Out] integral((B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(a*sec(d*x + c) + a), x)
giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sqrt{\sec(dx + c)}}{a \sec(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
 [Out] integrate((B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(a*sec(d*x + c) + a), x)
maple [A] time = 4.81, size = 243, normalized size = 1.98

$$\frac{\sqrt{\left(2 \left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right) \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) \left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right) \sqrt{2 \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1} \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}}\right) \left(A \text{EllipticF}\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), 2\right) + A \text{EllipticE}\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), 2\right) + B \text{EllipticF}\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), 2\right) - B \text{EllipticE}\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), 2\right)\right)}{a \cos\left(\frac{dx}{2} + \frac{c}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x)
 [Out] $-((2 * \cos(1/2 * d * x + 1/2 * c))^2 - 1) * \sin(1/2 * d * x + 1/2 * c)^2)^{(1/2)} * (\cos(1/2 * d * x + 1/2 * c) * (2 * \sin(1/2 * d * x + 1/2 * c)^2 - 1)^{(1/2)} * (\sin(1/2 * d * x + 1/2 * c)^2)^{(1/2)} * (A * \text{EllipticF}(\cos(1/2 * d * x + 1/2 * c), 2^{(1/2)}) + A * \text{EllipticE}(\cos(1/2 * d * x + 1/2 * c), 2^{(1/2)}) + B * \text{EllipticF}(\cos(1/2 * d * x + 1/2 * c), 2^{(1/2)}) - B * \text{EllipticE}(\cos(1/2 * d * x + 1/2 * c), 2^{(1/2)})))$

$$+(2*A-2*B)*\sin(1/2*d*x+1/2*c)^4+(-A+B)*\sin(1/2*d*x+1/2*c)^2)/a/\cos(1/2*d*x+1/2*c)/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)\sqrt{\sec(dx + c)}}{a \sec(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(a*sec(d*x + c) + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \sqrt{\frac{1}{\cos(c+dx)}}}{a + \frac{a}{\cos(c+dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + a/cos(c + d*x)),x)

[Out] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + a/cos(c + d*x)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A\sqrt{\sec(c+dx)}}{\sec(c+dx)+1} dx + \int \frac{B \sec^{\frac{3}{2}}(c+dx)}{\sec(c+dx)+1} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)**(1/2)/(a+a*sec(d*x+c)),x)

[Out] (Integral(A*sqrt(sec(c + d*x))/(sec(c + d*x) + 1), x) + Integral(B*sec(c + d*x)**(3/2)/(sec(c + d*x) + 1), x))/a

$$3.205 \quad \int \frac{A+B \sec(c+dx)}{\sqrt{\sec(c+dx)} (a+a \sec(c+dx))} dx$$

Optimal. Leaf size=128

$$\frac{(A-B) \sin(c+dx) \sqrt{\sec(c+dx)}}{d(a \sec(c+dx)+a)} - \frac{(A-B) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} F\left(\frac{1}{2}(c+dx) \middle| 2\right)}{ad} + \frac{(3A-B) \sqrt{\cos(c+dx)}}{ad}$$

[Out] $-(A-B) \sin(d*x+c) \sec(d*x+c)^{(1/2)} / d / (a+a \sec(d*x+c)) + (3*A-B) * (\cos(1/2*d*x+1/2*c)^2)^{(1/2)} / \cos(1/2*d*x+1/2*c) * \text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)}) * \cos(d*x+c)^{(1/2)} * \sec(d*x+c)^{(1/2)} / a/d - (A-B) * (\cos(1/2*d*x+1/2*c)^2)^{(1/2)} / \cos(1/2*d*x+1/2*c) * \text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)}) * \cos(d*x+c)^{(1/2)} * \sec(d*x+c)^{(1/2)} / a/d$

Rubi [A] time = 0.18, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {4020, 3787, 3771, 2639, 2641}

$$\frac{(A-B) \sin(c+dx) \sqrt{\sec(c+dx)}}{d(a \sec(c+dx)+a)} - \frac{(A-B) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} F\left(\frac{1}{2}(c+dx) \middle| 2\right)}{ad} + \frac{(3A-B) \sqrt{\cos(c+dx)}}{ad}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(A + B*\text{Sec}[c + d*x]) / (\text{Sqrt}[\text{Sec}[c + d*x]] * (a + a*\text{Sec}[c + d*x])), x]$

[Out] $((3*A - B) * \text{Sqrt}[\text{Cos}[c + d*x]] * \text{EllipticE}[(c + d*x)/2, 2] * \text{Sqrt}[\text{Sec}[c + d*x]]) / (a*d) - ((A - B) * \text{Sqrt}[\text{Cos}[c + d*x]] * \text{EllipticF}[(c + d*x)/2, 2] * \text{Sqrt}[\text{Sec}[c + d*x]]) / (a*d) - ((A - B) * \text{Sqrt}[\text{Sec}[c + d*x]] * \text{Sin}[c + d*x]) / (d * (a + a*\text{Sec}[c + d*x]))$

Rule 2639

$\text{Int}[\text{Sqrt}[\sin[(c_.) + (d_.)*(x_.)]], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticE}[(1*(c - \text{Pi}/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 2641

$\text{Int}[1/\text{Sqrt}[\sin[(c_.) + (d_.)*(x_.)]], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticF}[(1*(c - \text{Pi}/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 3771

$\text{Int}[(\text{csc}[(c_.) + (d_.)*(x_.)] * (b_.))^{(n_.)}, x_Symbol] \rightarrow \text{Dist}[(b * \text{Csc}[c + d*x])^n * \text{Sin}[c + d*x]^n, \text{Int}[1/\text{Sin}[c + d*x]^n, x], x] /; \text{FreeQ}\{b, c, d\}, x] \&\&$

EqQ[n^2, 1/4]

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) +
(a_.), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (
a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
+ f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{A + B \sec(c + dx)}{\sqrt{\sec(c + dx)} (a + a \sec(c + dx))} dx &= -\frac{(A - B)\sqrt{\sec(c + dx)} \sin(c + dx)}{d(a + a \sec(c + dx))} + \frac{\int \frac{\frac{1}{2}a(3A-B) - \frac{1}{2}a(A-B) \sec(c+dx)}{\sqrt{\sec(c+dx)}} dx}{a^2} \\ &= -\frac{(A - B)\sqrt{\sec(c + dx)} \sin(c + dx)}{d(a + a \sec(c + dx))} - \frac{(A - B) \int \sqrt{\sec(c + dx)} dx}{2a} + \frac{(3A - B) \int \sqrt{\sec(c + dx)} dx}{2a} \\ &= -\frac{(A - B)\sqrt{\sec(c + dx)} \sin(c + dx)}{d(a + a \sec(c + dx))} - \frac{((A - B)\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} + (3A - B)\sqrt{\sec(c + dx)})}{2a} \\ &= \frac{(3A - B)\sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{\sec(c + dx)}}{ad} - \frac{(A - B)\sqrt{\cos(c + dx)}}{2a} \end{aligned}$$

Mathematica [C] time = 2.78, size = 445, normalized size = 3.48

$$\cos^2\left(\frac{1}{2}(c + dx)\right) (A + B \sec(c + dx)) \left(-\frac{6 \csc\left(\frac{c}{2}\right) \sec\left(\frac{c}{2}\right) \sec\left(\frac{1}{2}(c + dx)\right) \left((2A - B) \cos\left(\frac{1}{2}(c - dx)\right) + A \cos\left(\frac{1}{2}(3c + dx)\right) \right)}{\sqrt{\sec(c + dx)}} - 6\sqrt{2} A \csc(c) e \right)$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sec[c + d*x])/(Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])),x]

[Out] (Cos[(c + d*x)/2]^2*((-6*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Csc[c]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))]) + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])/E^(I*d*x) + (2*Sqrt[2]*B*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Csc[c]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))]) + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])/E^(I*d*x) - (6*((2*A - B)*Cos[(c - d*x)/2] + A*Cos[(3*c + d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/2])/Sqrt[Sec[c + d*x]] - 12*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]] + 12*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x]))/(6*a*d*(B + A*Cos[c + d*x])*(1 + Sec[c + d*x]))

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B \sec(dx + c) + A)\sqrt{\sec(dx + c)}}{a \sec(dx + c)^2 + a \sec(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="fricas")

[Out] integral((B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(a*sec(d*x + c)^2 + a*sec(d*x + c)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(a \sec(dx + c) + a)\sqrt{\sec(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)*sqrt(sec(d*x + c))), x)

maple [A] time = 4.41, size = 244, normalized size = 1.91

$$\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1}\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}}\left(A \text{EllipticF}\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right)\right), \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}}\right) + B \text{EllipticF}\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right)\right), \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}}\right)}{a \cos\left(\frac{dx}{2} + \frac{c}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*sec(d*x+c))/(a+a*sec(d*x+c))/sec(d*x+c)^(1/2),x)`

[Out] $((2*\cos(1/2*d*x+1/2*c)^2-1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(\cos(1/2*d*x+1/2*c)* (2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(A*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})+3*A*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)})-B*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})-B*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)}))+(2*A-2*B)*\sin(1/2*d*x+1/2*c)^4+(-A+B)*\sin(1/2*d*x+1/2*c)^2)/a/\cos(1/2*d*x+1/2*c)/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(a \sec(dx + c) + a) \sqrt{\sec(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="maxima")`

[Out] `integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)*sqrt(sec(d*x + c))), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\left(a + \frac{a}{\cos(c+dx)}\right) \sqrt{\frac{1}{\cos(c+dx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B/cos(c + d*x))/((a + a/cos(c + d*x))*(1/cos(c + d*x))^(1/2)),x)`

[Out] `int((A + B/cos(c + d*x))/((a + a/cos(c + d*x))*(1/cos(c + d*x))^(1/2)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A}{\sec^{\frac{3}{2}}(c+dx) + \sqrt{\sec(c+dx)}} dx + \int \frac{B \sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx) + \sqrt{\sec(c+dx)}} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))/sec(d*x+c)**(1/2),x)`

[Out] `(Integral(A/(sec(c + d*x)**(3/2) + sqrt(sec(c + d*x))), x) + Integral(B*sec(c + d*x)/(sec(c + d*x)**(3/2) + sqrt(sec(c + d*x))), x))/a`

$$3.206 \quad \int \frac{A+B \sec(c+dx)}{\sec^2(c+dx)(a+a \sec(c+dx))} dx$$

Optimal. Leaf size=164

$$\frac{(5A-3B)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} - \frac{(A-B)\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)} + \frac{(5A-3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}$$

[Out] 1/3*(5*A-3*B)*sin(d*x+c)/a/d/sec(d*x+c)^(1/2)-(A-B)*sin(d*x+c)/d/(a+a*sec(d*x+c))/sec(d*x+c)^(1/2)-3*(A-B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d+1/3*(5*A-3*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d

Rubi [A] time = 0.19, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4020, 3787, 3769, 3771, 2641, 2639}

$$\frac{(5A-3B)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} - \frac{(A-B)\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)} + \frac{(5A-3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])),x]

[Out] (-3*(A - B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d) + ((5*A - 3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a*d) + ((5*A - 3*B)*Sin[c + d*x])/(3*a*d*Sqrt[Sec[c + d*x]]) - ((A - B)*Sin[c + d*x])/(d*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x]))

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 3769

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^n, x_Symbol] := Simp[(Cos[c + d*x]*(b*Csc[c + d*x])^(n + 1))/(b*d^n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +

$d*x])^{(n + 2)}, x], x] /; \text{FreeQ}[\{b, c, d\}, x] \ \&\& \ \text{LtQ}[n, -1] \ \&\& \ \text{IntegerQ}[2*n]$
]

Rule 3771

$\text{Int}[(\text{csc}[c_.] + (d_.)*(x_)]*(b_.)^{(n_)}, x_Symbol] \text{:>} \text{Dist}[(b*\text{Csc}[c + d*x])^{n*}*\text{Sin}[c + d*x]^{n}, \text{Int}[1/\text{Sin}[c + d*x]^{n}, x], x] /; \text{FreeQ}[\{b, c, d\}, x] \ \&\& \ \text{EqQ}[n^2, 1/4]$

Rule 3787

$\text{Int}[(\text{csc}[e_.] + (f_.)*(x_)]*(d_.)^{(n_.)}*(\text{csc}[e_.] + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] \text{:>} \text{Dist}[a, \text{Int}[(d*\text{Csc}[e + f*x])^{n}, x], x] + \text{Dist}[b/d, \text{Int}[(d*\text{Csc}[e + f*x])^{(n + 1)}, x], x] /; \text{FreeQ}[\{a, b, d, e, f, n\}, x]$

Rule 4020

$\text{Int}[(\text{csc}[e_.] + (f_.)*(x_)]*(d_.)^{(n_.)}*(\text{csc}[e_.] + (f_.)*(x_)]*(b_.) + (a_))^{(m_)}*(\text{csc}[e_.] + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] \text{:>} -\text{Simp}[(A*b - a*B)*\text{Cot}[e + f*x]*(a + b*\text{Csc}[e + f*x])^{m*}*(d*\text{Csc}[e + f*x])^{n}]/(b*f*(2*m + 1)), x] - \text{Dist}[1/(a^2*(2*m + 1)), \text{Int}[(a + b*\text{Csc}[e + f*x])^{(m + 1)}*(d*\text{Csc}[e + f*x])^{n*}*\text{Simp}[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*\text{Csc}[e + f*x], x], x], x] /; \text{FreeQ}[\{a, b, d, e, f, A, B, n\}, x] \ \&\& \ \text{NeQ}[A*b - a*B, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{LtQ}[m, -2^{(-1)}] \ \&\& \ !\text{GtQ}[n, 0]$

Rubi steps

$$\begin{aligned}
\int \frac{A + B \sec(c + dx)}{\sec^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))} dx &= -\frac{(A - B) \sin(c + dx)}{d\sqrt{\sec(c + dx)}(a + a \sec(c + dx))} + \frac{\int \frac{\frac{1}{2}a(5A-3B) - \frac{3}{2}a(A-B) \sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)} dx}{a^2} \\
&= -\frac{(A - B) \sin(c + dx)}{d\sqrt{\sec(c + dx)}(a + a \sec(c + dx))} + \frac{(5A - 3B) \int \frac{1}{\sec^{\frac{3}{2}}(c+dx)} dx}{2a} - \frac{(3(A - B) \sin(c + dx))}{2a} \\
&= \frac{(5A - 3B) \sin(c + dx)}{3ad\sqrt{\sec(c + dx)}} - \frac{(A - B) \sin(c + dx)}{d\sqrt{\sec(c + dx)}(a + a \sec(c + dx))} + \frac{(5A - 3B)}{2a} \\
&= -\frac{3(A - B)\sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{\sec(c + dx)}}{ad} + \frac{(5A - 3B) \sin(c + dx)}{3ad\sqrt{\sec(c + dx)}} \\
&= -\frac{3(A - B)\sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{\sec(c + dx)}}{ad} + \frac{(5A - 3B)\sqrt{\cos(c + dx)}}{3ad}
\end{aligned}$$

Mathematica [C] time = 2.42, size = 232, normalized size = 1.41

$$\frac{e^{-idx} \cos\left(\frac{1}{2}(c + dx)\right) \sec^{\frac{3}{2}}(c + dx) (\cos(dx) + i \sin(dx)) \left(3i(A - B)e^{\frac{1}{2}i(c+dx)} (1 + e^{i(c+dx)}) \sqrt{1 + e^{2i(c+dx)}} {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{7}{4}; \dots\right)\right)}{\dots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])),x]
[Out] (Cos[(c + d*x)/2]*Sec[c + d*x]^(3/2)*(Cos[d*x] + I*Sin[d*x])*(2*(5*A - 3*B)
*Cos[(c + d*x)/2]*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (3*I)*(A -
B)*E^((I/2)*(c + d*x))*(1 + E^(I*(c + d*x)))*Sqrt[1 + E^((2*I)*(c + d*x))])
*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]) + 2*Cos[c + d*x]*((
-9*I)*(A - B)*Cos[(c + d*x)/2] + (5*A - 3*B + 2*A*Cos[c + d*x])*Sin[(c + d*
x)/2]))/(3*a*d*E^(I*d*x)*(1 + Sec[c + d*x]))
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B \sec(dx + c) + A)\sqrt{\sec(dx + c)}}{a \sec(dx + c)^3 + a \sec(dx + c)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm="
fricas")
```

[Out] integral((B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(a*sec(d*x + c)^3 + a*sec(d*x + c)^2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(a \sec(dx + c) + a) \sec(dx + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)*sec(d*x + c)^(3/2)), x)

maple [A] time = 5.00, size = 262, normalized size = 1.60

$$\sqrt{\left(2 \left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right) \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right) \sqrt{2 \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1} \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \left(5A \text{EllipticF}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x)

[Out] -1/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1/2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-8*A*sin(1/2*d*x+1/2*c)^6+(18*A-6*B)*sin(1/2*d*x+1/2*c)^4+(-7*A+3*B)*sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(a \sec(dx + c) + a) \sec(dx + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)*sec(d*x + c)^(3/2)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\left(a + \frac{a}{\cos(c+dx)}\right) \left(\frac{1}{\cos(c+dx)}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/((a + a/cos(c + d*x))*(1/cos(c + d*x))^(3/2)), x)

[Out] int((A + B/cos(c + d*x))/((a + a/cos(c + d*x))*(1/cos(c + d*x))^(3/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A}{\sec^{\frac{5}{2}}(c+dx) + \sec^{\frac{3}{2}}(c+dx)} dx + \int \frac{B \sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx) + \sec^{\frac{3}{2}}(c+dx)} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)**(3/2)/(a+a*sec(d*x+c)), x)

[Out] (Integral(A/(sec(c + d*x)**(5/2) + sec(c + d*x)**(3/2)), x) + Integral(B*sec(c + d*x)/(sec(c + d*x)**(5/2) + sec(c + d*x)**(3/2)), x))/a

$$3.207 \quad \int \frac{A+B \sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+a \sec(c+dx))} dx$$

Optimal. Leaf size=197

$$-\frac{(A-B) \sin(c+dx)}{d \sec^{\frac{3}{2}}(c+dx)(a \sec(c+dx)+a)} + \frac{(7A-5B) \sin(c+dx)}{5ad \sec^{\frac{3}{2}}(c+dx)} - \frac{5(A-B) \sin(c+dx)}{3ad \sqrt{\sec(c+dx)}} - \frac{5(A-B) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{3ad}$$

[Out] $1/5*(7*A-5*B)*\sin(d*x+c)/a/d/\sec(d*x+c)^{(3/2)}-(A-B)*\sin(d*x+c)/d/\sec(d*x+c)^{(3/2)}/(a+a*\sec(d*x+c))-5/3*(A-B)*\sin(d*x+c)/a/d/\sec(d*x+c)^{(1/2)}+3/5*(7*A-5*B)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c),2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/a/d-5/3*(A-B)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c),2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/a/d$

Rubi [A] time = 0.21, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4020, 3787, 3769, 3771, 2639, 2641}

$$-\frac{(A-B) \sin(c+dx)}{d \sec^{\frac{3}{2}}(c+dx)(a \sec(c+dx)+a)} + \frac{(7A-5B) \sin(c+dx)}{5ad \sec^{\frac{3}{2}}(c+dx)} - \frac{5(A-B) \sin(c+dx)}{3ad \sqrt{\sec(c+dx)}} - \frac{5(A-B) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{3ad}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])),x]

[Out] $(3*(7*A-5*B)*\text{Sqrt}[\text{Cos}[c+d*x]]*\text{EllipticE}[(c+d*x)/2,2]*\text{Sqrt}[\text{Sec}[c+d*x]])/(5*a*d)-(5*(A-B)*\text{Sqrt}[\text{Cos}[c+d*x]]*\text{EllipticF}[(c+d*x)/2,2]*\text{Sqrt}[\text{Sec}[c+d*x]])/(3*a*d)+((7*A-5*B)*\text{Sin}[c+d*x])/(5*a*d*\text{Sec}[c+d*x]^{(3/2)})-(5*(A-B)*\text{Sin}[c+d*x])/(3*a*d*\text{Sqrt}[\text{Sec}[c+d*x]])-((A-B)*\text{Sin}[c+d*x])/(d*\text{Sec}[c+d*x]^{(3/2)}*(a+a*\text{Sec}[c+d*x]))$

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.))^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x]
)^(n)*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) +
(a_.)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (
a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
+ f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{A + B \sec(c + dx)}{\sec^{\frac{5}{2}}(c + dx)(a + a \sec(c + dx))} dx &= -\frac{(A - B) \sin(c + dx)}{d \sec^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))} + \int \frac{\frac{1}{2}a(7A-5B) - \frac{5}{2}a(A-B) \sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)} dx \\
&= -\frac{(A - B) \sin(c + dx)}{d \sec^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))} + \frac{(7A - 5B) \int \frac{1}{\sec^{\frac{5}{2}}(c+dx)} dx}{2a} \quad (5A) \\
&= \frac{(7A - 5B) \sin(c + dx)}{5ad \sec^{\frac{3}{2}}(c + dx)} - \frac{5(A - B) \sin(c + dx)}{3ad \sqrt{\sec(c + dx)}} - \frac{(A - B) \sin(c + dx)}{d \sec^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))} \\
&= \frac{(7A - 5B) \sin(c + dx)}{5ad \sec^{\frac{3}{2}}(c + dx)} - \frac{5(A - B) \sin(c + dx)}{3ad \sqrt{\sec(c + dx)}} - \frac{(A - B) \sin(c + dx)}{d \sec^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))} \\
&= \frac{3(7A - 5B) \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{\sec(c + dx)}}{5ad} - \frac{5(A - B) \sqrt{\cos(c + dx)}}{5ad}
\end{aligned}$$

Mathematica [C] time = 3.98, size = 540, normalized size = 2.74

$$\cos^2\left(\frac{1}{2}(c + dx)\right) (A + B \sec(c + dx)) \left(\sqrt{\sec(c + dx)} \left(-40(A - B) \sin(2c) \cos(2dx) + 12(33A - 20B) \cos(c) \sin(2dx) \right) \right)$$

Antiderivative was successfully verified.

```

[In] Integrate[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])),x]
[Out] (Cos[(c + d*x)/2]^2*(A + B*Sec[c + d*x])*((-84*sqrt[2]*A*sqrt[E^(I*(c + d*x))]/(1 + E^((2*I)*(c + d*x)))]*sqrt[1 + E^((2*I)*(c + d*x))]*Csc[c]*(-3*sqrt[1 + E^((2*I)*(c + d*x))]] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])/E^(I*d*x) + (60*sqrt[2]*B*sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*sqrt[1 + E^((2*I)*(c + d*x))]*Csc[c]*(-3*sqrt[1 + E^((2*I)*(c + d*x))]] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])/E^(I*d*x) - 200*A*sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*sqrt[Sec[c + d*x]] + 200*B*sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*sqrt[Sec[c + d*x]] + sqrt[Sec[c + d*x]]*(-3*(51*A - 40*B + (33*A - 20*B)*Cos[2*c])*Cos[d*x]*Csc[c/2]*Sec[c/2] - 40*(A - B)*Cos[2*d*x]*Sin[2*c] + 12*A*Cos[3*d*x]*Sin[3*c] + 120*(A - B)*Sec[c/2]*Sec[(c + d*x)/2]*Sin[(d*x)/2] + 12*(33*A - 20*B)*Cos[c]*Sin[d*x] -

```

$40*(A - B)*\cos[2*c]*\sin[2*d*x] + 12*A*\cos[3*c]*\sin[3*d*x] + 120*(A - B)*\tan[c/2]))/(60*a*d*(B + A*\cos[c + d*x])*(1 + \sec[c + d*x]))$

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B \sec(dx + c) + A)\sqrt{\sec(dx + c)}}{a \sec(dx + c)^4 + a \sec(dx + c)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")

[Out] integral((B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(a*sec(d*x + c)^4 + a*sec(d*x + c)^3), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(a \sec(dx + c) + a) \sec(dx + c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)*sec(d*x + c)^(5/2)), x)

maple [A] time = 4.90, size = 282, normalized size = 1.43

$$\sqrt{2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \left(-\cos\left(\frac{dx}{2} + \frac{c}{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1}\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}}\right) \left(25A \text{ EllipticF}\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), 2\right) + 63A \text{ EllipticE}\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), 2\right) - 25B \text{ EllipticF}\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), 2\right) - 45B \text{ EllipticE}\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), 2\right)\right) + 48A \sin\left(\frac{dx}{2} + \frac{c}{2}\right)^8 + (-56A - 40B) \sin\left(\frac{dx}{2} + \frac{c}{2}\right)^6 + (-30A + 90B) \sin\left(\frac{dx}{2} + \frac{c}{2}\right)^4 + (23A - 35B) \sin\left(\frac{dx}{2} + \frac{c}{2}\right)^2 / a/c$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x)

[Out] -1/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-cos(1/2*d*x+1/2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(25*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+63*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-25*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-45*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+48*A*sin(1/2*d*x+1/2*c)^8+(-56*A-40*B)*sin(1/2*d*x+1/2*c)^6+(-30*A+90*B)*sin(1/2*d*x+1/2*c)^4+(23*A-35*B)*sin(1/2*d*x+1/2*c)^2)/a/c

$\cos(1/2*d*x+1/2*c)/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(a \sec(dx + c) + a) \sec(dx + c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)*sec(d*x + c)^(5/2)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\left(a + \frac{a}{\cos(c+dx)}\right) \left(\frac{1}{\cos(c+dx)}\right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/((a + a/cos(c + d*x))*(1/cos(c + d*x))^(5/2)),x)

[Out] int((A + B/cos(c + d*x))/((a + a/cos(c + d*x))*(1/cos(c + d*x))^(5/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A}{\sec^{\frac{7}{2}}(c+dx)+\sec^{\frac{5}{2}}(c+dx)} dx + \int \frac{B \sec(c+dx)}{\sec^{\frac{7}{2}}(c+dx)+\sec^{\frac{5}{2}}(c+dx)} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)**(5/2)/(a+a*sec(d*x+c)),x)

[Out] (Integral(A/(sec(c + d*x)**(7/2) + sec(c + d*x)**(5/2)), x) + Integral(B*sec(c + d*x)/(sec(c + d*x)**(7/2) + sec(c + d*x)**(5/2)), x))/a

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x]
)^(n)*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_.)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] := -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
+ f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{A + B \sec(c + dx)}{\sec^{\frac{7}{2}}(c + dx)(a + a \sec(c + dx))} dx &= -\frac{(A - B) \sin(c + dx)}{d \sec^{\frac{5}{2}}(c + dx)(a + a \sec(c + dx))} + \frac{\int \frac{\frac{1}{2}a(9A-7B) - \frac{7}{2}a(A-B) \sec(c+dx)}{\sec^{\frac{7}{2}}(c+dx)} dx}{a^2} \\
&= -\frac{(A - B) \sin(c + dx)}{d \sec^{\frac{5}{2}}(c + dx)(a + a \sec(c + dx))} + \frac{(9A - 7B) \int \frac{1}{\sec^{\frac{7}{2}}(c+dx)} dx}{2a} - \frac{7(A - B) \sin(c + dx)}{5ad \sec^{\frac{3}{2}}(c + dx)} \\
&= \frac{(9A - 7B) \sin(c + dx)}{7ad \sec^{\frac{5}{2}}(c + dx)} - \frac{7(A - B) \sin(c + dx)}{5ad \sec^{\frac{3}{2}}(c + dx)} - \frac{(A - B) \sin(c + dx)}{d \sec^{\frac{5}{2}}(c + dx)(a + a \sec(c + dx))} \\
&= \frac{(9A - 7B) \sin(c + dx)}{7ad \sec^{\frac{5}{2}}(c + dx)} - \frac{7(A - B) \sin(c + dx)}{5ad \sec^{\frac{3}{2}}(c + dx)} + \frac{5(9A - 7B) \sin(c + dx)}{21ad \sqrt{\sec(c + dx)}} \\
&= -\frac{21(A - B) \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{\sec(c + dx)}}{5ad} + \frac{(9A - 7B) \sin(c + dx)}{7ad \sec^{\frac{5}{2}}(c + dx)} \\
&= -\frac{21(A - B) \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{\sec(c + dx)}}{5ad} + \frac{5(9A - 7B) \sqrt{\sec(c + dx)}}{7ad \sec^{\frac{5}{2}}(c + dx)}
\end{aligned}$$

Mathematica [C] time = 4.16, size = 568, normalized size = 2.47

$$\cos^2\left(\frac{1}{2}(c + dx)\right) (A + B \sec(c + dx)) \left(\sqrt{\sec(c + dx)} \left(20(27A - 14B) \sin(2c) \cos(2dx) - 84(A - B) \sin(3c) \cos(3dx) \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(7/2)*(a + a*Sec[c + d*x])),x]

[Out] (Cos[(c + d*x)/2]^2*(A + B*Sec[c + d*x])*((588*sqrt[2]*A*sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*sqrt[1 + E^((2*I)*(c + d*x))]*Csc[c]*(-3*sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])/E^(I*d*x) - (588*sqrt[2]*B*sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*sqrt[1 + E^((2*I)*(c + d*x))]*Csc[c]*(-3*sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])/E^(I*d*x) + 1800*A*sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*sqrt[Sec[c + d*x]] - 1400*B*sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*sqrt[Sec[c + d*x]] + sqrt[Sec[c

+ d*x]]*(63*(A - B)*(17 + 11*Cos[2*c])*Cos[d*x]*Csc[c/2]*Sec[c/2] + 20*(27*A - 14*B)*Cos[2*d*x]*Sin[2*c] - 84*(A - B)*Cos[3*d*x]*Sin[3*c] + 30*A*Cos[4*d*x]*Sin[4*c] - 840*(A - B)*Sec[c/2]*Sec[(c + d*x)/2]*Sin[(d*x)/2] - 2772*(A - B)*Cos[c]*Sin[d*x] + 20*(27*A - 14*B)*Cos[2*c]*Sin[2*d*x] - 84*(A - B)*Cos[3*c]*Sin[3*d*x] + 30*A*Cos[4*c]*Sin[4*d*x] - 840*(A - B)*Tan[c/2]))/(420*a*d*(B + A*Cos[c + d*x])*(1 + Sec[c + d*x]))

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B \sec(dx + c) + A)\sqrt{\sec(dx + c)}}{a \sec(dx + c)^5 + a \sec(dx + c)^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(7/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")

[Out] integral((B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(a*sec(d*x + c)^5 + a*sec(d*x + c)^4), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(a \sec(dx + c) + a) \sec(dx + c)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(7/2)/(a+a*sec(d*x+c)),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)*sec(d*x + c)^(7/2)), x)

maple [A] time = 5.00, size = 300, normalized size = 1.30

$$\sqrt{\left(2 \left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right) \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right) \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{2 \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1}\right) \left(225A \text{ Elliptic}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))/sec(d*x+c)^(7/2)/(a+a*sec(d*x+c)),x)

[Out] -1/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1/2*c)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(225*

$A*\text{EllipticF}(\cos(1/2*d*x+1/2*c), 2^{(1/2)})+441*A*\text{EllipticE}(\cos(1/2*d*x+1/2*c), 2^{(1/2)})-175*B*\text{EllipticF}(\cos(1/2*d*x+1/2*c), 2^{(1/2)})-441*B*\text{EllipticE}(\cos(1/2*d*x+1/2*c), 2^{(1/2)})-480*A*\sin(1/2*d*x+1/2*c)^{10}+(864*A+336*B)*\sin(1/2*d*x+1/2*c)^8+(-888*A-392*B)*\sin(1/2*d*x+1/2*c)^6+(930*A-210*B)*\sin(1/2*d*x+1/2*c)^4+(-321*A+161*B)*\sin(1/2*d*x+1/2*c)^2)/a/\cos(1/2*d*x+1/2*c)/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(a \sec(dx + c) + a) \sec(dx + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(7/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)*sec(d*x + c)^(7/2)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\left(a + \frac{a}{\cos(c+dx)}\right) \left(\frac{1}{\cos(c+dx)}\right)^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/((a + a/cos(c + d*x))*(1/cos(c + d*x))^(7/2)),x)

[Out] int((A + B/cos(c + d*x))/((a + a/cos(c + d*x))*(1/cos(c + d*x))^(7/2)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)**(7/2)/(a+a*sec(d*x+c)),x)

[Out] Timed out

$$3.209 \quad \int \frac{\sec^2(c+dx)(A+B \sec(c+dx))}{(a+a \sec(c+dx))^2} dx$$

Optimal. Leaf size=237

$$\frac{(4A-7B) \sin(c+dx) \sec^{\frac{5}{2}}(c+dx)}{3a^2d(\sec(c+dx)+1)} - \frac{5(A-2B) \sin(c+dx) \sec^{\frac{3}{2}}(c+dx)}{3a^2d} + \frac{(4A-7B) \sin(c+dx) \sqrt{\sec(c+dx)}}{a^2d}$$

[Out] $-5/3*(A-2*B)*\sec(d*x+c)^{(3/2)}*\sin(d*x+c)/a^2/d+1/3*(4*A-7*B)*\sec(d*x+c)^{(5/2)}*\sin(d*x+c)/a^2/d/(1+\sec(d*x+c))+1/3*(A-B)*\sec(d*x+c)^{(7/2)}*\sin(d*x+c)/d/(a+a*\sec(d*x+c))^2+(4*A-7*B)*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/a^2/d-(4*A-7*B)*(c \cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/a^2/d-5/3*(A-2*B)*(c \cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/a^2/d$

Rubi [A] time = 0.37, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4019, 3787, 3768, 3771, 2639, 2641}

$$\frac{(4A-7B) \sin(c+dx) \sec^{\frac{5}{2}}(c+dx)}{3a^2d(\sec(c+dx)+1)} - \frac{5(A-2B) \sin(c+dx) \sec^{\frac{3}{2}}(c+dx)}{3a^2d} + \frac{(4A-7B) \sin(c+dx) \sqrt{\sec(c+dx)}}{a^2d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(\text{Sec}[c+d*x]^{(7/2)}*(A+B*\text{Sec}[c+d*x]))/(a+a*\text{Sec}[c+d*x])^2,x]$

[Out] $-(((4*A-7*B)*\text{Sqrt}[\text{Cos}[c+d*x]]*\text{EllipticE}[(c+d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c+d*x]])/(a^2*d) - (5*(A-2*B)*\text{Sqrt}[\text{Cos}[c+d*x]]*\text{EllipticF}[(c+d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c+d*x]])/(3*a^2*d) + ((4*A-7*B)*\text{Sqrt}[\text{Sec}[c+d*x]]*\text{Sin}[c+d*x])/(a^2*d) - (5*(A-2*B)*\text{Sec}[c+d*x]^{(3/2)}*\text{Sin}[c+d*x])/(3*a^2*d) + ((4*A-7*B)*\text{Sec}[c+d*x]^{(5/2)}*\text{Sin}[c+d*x])/(3*a^2*d*(1+\text{Sec}[c+d*x])) + ((A-B)*\text{Sec}[c+d*x]^{(7/2)}*\text{Sin}[c+d*x])/(3*d*(a+a*\text{Sec}[c+d*x])^2)$

Rule 2639

$\text{Int}[\text{Sqrt}[\sin[(c_.)+(d_.)*(x_)]], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticE}[(1*(c-Pi/2+d*x))/2, 2])/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 2641

$\text{Int}[1/\text{Sqrt}[\sin[(c_.)+(d_.)*(x_)]], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticF}[(1*(c-Pi/2+d*x))/2, 2])/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := -Simp[(b*Csc[c + d*x]
)*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] := Simp[(d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^{\frac{7}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^2} dx &= \frac{(A-B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} + \int \frac{\sec^{\frac{5}{2}}(c+dx)\left(\frac{5}{2}a(A-B)-\frac{3}{2}a(A-3B)\sec(c+dx)\right)}{a+a\sec(c+dx)} \frac{1}{3a^2} \\
&= \frac{(4A-7B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} + \frac{(A-B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} \\
&= \frac{(4A-7B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} + \frac{(A-B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} \\
&= \frac{(4A-7B)\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2d} - \frac{5(A-2B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d} \\
&= \frac{(4A-7B)\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2d} - \frac{5(A-2B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d} \\
&= -\frac{(4A-7B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} - \frac{5(A-2B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d}
\end{aligned}$$

Mathematica [C] time = 8.03, size = 865, normalized size = 3.65

$$\frac{4\sqrt{2}Ae^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\csc\left(\frac{c}{2}\right)\left(e^{2idx}\left(-1+e^{2ic}\right){}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)-3\sqrt{1+e^{2i(c+dx)}}\right)\sec\left(\frac{c}{2}\right)}{3d(B+A\cos(c+dx))(\sec(c+dx)a+a)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[c + d*x]^(7/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^2, x]

[Out] (4*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x]))/(3*d*E^(I*d*x)*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^2) - (7*Sqrt[2]*B*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x]))/(3*d*E^(I*d*x)*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^2) - (10*A*Cos[c/2 + (d*x)/2]^4*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c/2]*Sec[c + d*x]^(3/2)*(A + B*Sec[c

+ d*x))*Sin[c]]/(3*d*(B + A*Cos[c + d*x]))*(a + a*Sec[c + d*x])^2) + (20*B*Cos[c/2 + (d*x)/2]^4*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c/2]*Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x])*Sin[c]]/(3*d*(B + A*Cos[c + d*x]))*(a + a*Sec[c + d*x])^2) + (Cos[c/2 + (d*x)/2]^4*Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x])*((-2*(-4*A + 7*B)*Cos[d*x]*Csc[c/2]*Sec[c/2])/d + (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(-(A*Sin[(d*x)/2]) + B*Sin[(d*x)/2])))/(3*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(-5*A*Sin[(d*x)/2] + 8*B*Sin[(d*x)/2]))/(3*d) + (8*B*Sec[c]*Sec[c + d*x]*Sin[d*x])/(3*d) + (4*(2*B - 5*A*Cos[c] + 10*B*Cos[c])*Sec[c]*Tan[c/2])/(3*d) + (2*(-A + B)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(3*d)))/((B + A*Cos[c + d*x]))*(a + a*Sec[c + d*x])^2)

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B \sec(dx+c)^4 + A \sec(dx+c)^3)\sqrt{\sec(dx+c)}}{a^2 \sec(dx+c)^2 + 2a^2 \sec(dx+c) + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="fricas")

[Out] integral((B*sec(d*x + c)^4 + A*sec(d*x + c)^3)*sqrt(sec(d*x + c))/(a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx+c) + A) \sec(dx+c)^{\frac{7}{2}}}{(a \sec(dx+c) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(7/2)/(a*sec(d*x + c) + a)^2, x)

maple [B] time = 15.80, size = 750, normalized size = 3.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x)

[Out] -1/2*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a^2*(4*B*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/

```
(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*
d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/3*(-A+B)*(2*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))-3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*c)*sin(1/2*d
*x+1/2*c)^2-2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2)))*cos(1/2*d*x+1/2*c)-12*sin(1/2*d*x+1/2*c)^6+20*sin(1/2*d*x+1/2*c)^4-
7*sin(1/2*d*x+1/2*c)^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
)/cos(1/2*d*x+1/2*c)/(-1+sin(1/2*d*x+1/2*c)^2)+(4*A-8*B)*(-(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/
2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)+(-2*A+4*B)*(cos(1/2
*d*x+1/2*c)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
)-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)/cos(1/2*d*x+1/2*c)/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*
d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm
="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(\frac{1}{\cos(c+dx)}\right)^{7/2}}{\left(a + \frac{a}{\cos(c+dx)}\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(7/2))/(a + a/cos(c + d*x))^2,x)

[Out] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(7/2))/(a + a/cos(c + d*x))^2, x
)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```


$$3.210 \quad \int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B \sec(c+dx))}{(a+a \sec(c+dx))^2} dx$$

Optimal. Leaf size=204

$$\frac{(2A-5B) \sin(c+dx) \sec^{\frac{3}{2}}(c+dx)}{3a^2d(\sec(c+dx)+1)} - \frac{(A-4B) \sin(c+dx) \sqrt{\sec(c+dx)}}{a^2d} + \frac{(2A-5B) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{3a^2d}$$

[Out] 1/3*(2*A-5*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/a^2/d/(1+sec(d*x+c))+1/3*(A-B)*sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^2-(A-4*B)*sin(d*x+c)*sec(d*x+c)^(1/2)/a^2/d+(A-4*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/d+1/3*(2*A-5*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/d

Rubi [A] time = 0.35, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4019, 3787, 3771, 2641, 3768, 2639}

$$\frac{(2A-5B) \sin(c+dx) \sec^{\frac{3}{2}}(c+dx)}{3a^2d(\sec(c+dx)+1)} - \frac{(A-4B) \sin(c+dx) \sqrt{\sec(c+dx)}}{a^2d} + \frac{(2A-5B) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{3a^2d}$$

Antiderivative was successfully verified.

[In] Int[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^2,x]

[Out] ((A - 4*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*d) + ((2*A - 5*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a^2*d) - ((A - 4*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a^2*d) + ((2*A - 5*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a^2*d*(1 + Sec[c + d*x])) + ((A - B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2)

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]
)*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] := Simp[(d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^2} dx &= \frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} + \int \frac{\sec^{\frac{3}{2}}(c+dx)\left(\frac{3}{2}a(A-B)-\frac{1}{2}a(A-7B)\sec(c+dx)\right)}{a+a\sec(c+dx)} dx \\
&= \frac{(2A-5B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} + \frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} \\
&= \frac{(2A-5B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} + \frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} \\
&= -\frac{(A-4B)\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2d} + \frac{(2A-5B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} \\
&= \frac{(2A-5B)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3a^2d} - \frac{(A-4B)\sqrt{\sec(c+dx)}}{a^2d} \\
&= \frac{(A-4B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} + \frac{(2A-5B)\sqrt{\cos(c+dx)}}{a^2d}
\end{aligned}$$

Mathematica [C] time = 7.13, size = 455, normalized size = 2.23

$$\cos^4\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)(A+B\sec(c+dx))\left(-2\sqrt{\sec(c+dx)}\left(6(A-4B)\csc(c)\cos(dx)-\tan\left(\frac{1}{2}(c+dx)\right)\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^2, x]

[Out] (Cos[(c + d*x)/2]^4*Sec[c + d*x]*(A + B*Sec[c + d*x])*((-2*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Csc[c]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))]] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])/E^(I*d*x) + (8*Sqrt[2]*B*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Csc[c]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))]] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])/E^(I*d*x) + 8*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]] - 20*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]] - 2*Sqrt[Sec[c + d*x]]*(6*(A - 4*B)*Cos[d*x]*Csc[c] - (3*(A - 2*B) + (2*A - 5*B)*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*a^2*d*(B + A*Cos[c + d*x])*(1 + Sec[c + d*x])^2)

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{(B \sec(dx+c)^3 + A \sec(dx+c)^2) \sqrt{\sec(dx+c)}}{a^2 \sec(dx+c)^2 + 2a^2 \sec(dx+c) + a^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="fricas")

[Out] integral((B*sec(d*x + c)^3 + A*sec(d*x + c)^2)*sqrt(sec(d*x + c))/(a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx+c) + A) \sec(dx+c)^{\frac{5}{2}}}{(a \sec(dx+c) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(5/2)/(a*sec(d*x + c) + a)^2, x)

maple [B] time = 5.88, size = 492, normalized size = 2.41

$$2\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)} \sqrt{2\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1} \left(2A \text{EllipticF}\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x)

[Out] 1/6*(2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(2*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-5*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+12*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(2*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-5*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+12*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))

$d*x+1/2*c), 2^{(1/2)}) * \cos(1/2*d*x+1/2*c) - 12 * (-2*\sin(1/2*d*x+1/2*c)^4 + \sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (A-4*B) * \sin(1/2*d*x+1/2*c)^6 + 2 * (-2*\sin(1/2*d*x+1/2*c)^4 + \sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (10*A-43*B) * \sin(1/2*d*x+1/2*c)^4 - (-2*\sin(1/2*d*x+1/2*c)^4 + \sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (7*A-37*B) * \sin(1/2*d*x+1/2*c)^2) / a^2 / \cos(1/2*d*x+1/2*c)^3 / (-2*\sin(1/2*d*x+1/2*c)^4 + \sin(1/2*d*x+1/2*c)^2)^{(1/2)} / \sin(1/2*d*x+1/2*c) / (2*\cos(1/2*d*x+1/2*c)^2 - 1)^{(1/2)} / d$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(\frac{1}{\cos(c+dx)}\right)^{5/2}}{\left(a + \frac{a}{\cos(c+dx)}\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + a/cos(c + d*x))^2,x)

[Out] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + a/cos(c + d*x))^2, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**2,x)

[Out] Timed out

$$3.211 \quad \int \frac{\sec^3(c+dx)(A+B \sec(c+dx))}{(a+a \sec(c+dx))^2} dx$$

Optimal. Leaf size=161

$$\frac{(A+2B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} - \frac{B \sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d(\sec(c+dx)+1)} + \frac{B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{a^2d}$$

[Out] 1/3*(A-B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^2-B*sin(d*x+c)*sec(d*x+c)^(1/2)/a^2/d/(1+sec(d*x+c))+B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/d+1/3*(A+2*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/d

Rubi [A] time = 0.30, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {4019, 3787, 3771, 2639, 2641}

$$\frac{(A+2B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} - \frac{B \sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d(\sec(c+dx)+1)} + \frac{B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{a^2d}$$

Antiderivative was successfully verified.

[In] Int[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^2,x]

[Out] (B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*d) + ((A + 2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a^2*d) - (B*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a^2*d*(1 + Sec[c + d*x])) + ((A - B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2)

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 3771

Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.))^(n_.), x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 4019

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := Simp[(d*(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{\sec^{\frac{3}{2}}(c + dx)(A + B \sec(c + dx))}{(a + a \sec(c + dx))^2} dx &= \frac{(A - B) \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{3d(a + a \sec(c + dx))^2} + \int \frac{\sqrt{\sec(c + dx)} \left(\frac{1}{2}a(A - B) + \frac{1}{2}a(A + 5B) \sec(c + dx) \right)}{a + a \sec(c + dx)} dx \\
 &= -\frac{B\sqrt{\sec(c + dx)} \sin(c + dx)}{a^2d(1 + \sec(c + dx))} + \frac{(A - B) \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{3d(a + a \sec(c + dx))^2} + \int \frac{B \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{a + a \sec(c + dx)} dx \\
 &= -\frac{B\sqrt{\sec(c + dx)} \sin(c + dx)}{a^2d(1 + \sec(c + dx))} + \frac{(A - B) \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{3d(a + a \sec(c + dx))^2} + \frac{B \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{a + a \sec(c + dx)} \\
 &= -\frac{B\sqrt{\sec(c + dx)} \sin(c + dx)}{a^2d(1 + \sec(c + dx))} + \frac{(A - B) \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{3d(a + a \sec(c + dx))^2} + \frac{B \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{a + a \sec(c + dx)} \\
 &= \frac{B\sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{\sec(c + dx)}}{a^2d} + \frac{(A + 2B)\sqrt{\cos(c + dx)}}{3d(a + a \sec(c + dx))^2}
 \end{aligned}$$

Mathematica [C] time = 2.95, size = 256, normalized size = 1.59

$$e^{-idx} \cos\left(\frac{1}{2}(c + dx)\right) \sec^5(c + dx) \left(\cos\left(\frac{1}{2}(c + 3dx)\right) + i \sin\left(\frac{1}{2}(c + 3dx)\right) \right) \left(2i \cos(c + dx) (-i(A - B) \sin(c + dx) + \dots \right)$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^2, x]

[Out] (Cos[(c + d*x)/2]*Sec[c + d*x]^(5/2)*(((-I)*B*(1 + E^(I*(c + d*x)))^3*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])/E^(I*(c + d*x)) + 8*(A + 2*B)*Cos[(c + d*x)/2]^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*(Cos[(c + d*x)/2] - I*Sin[(c + d*x)/2]) + (2*I)*Cos[c + d*x]*(-A + 7*B + (A + 5*B)*Cos[c + d*x] - I*(A - B)*Sin[c + d*x]))*(Cos[(c + 3*d*x)/2] + I*Sin[(c + 3*d*x)/2]))/(6*a^2*d*E^(I*d*x)*(1 + Sec[c + d*x])^2)

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{(B \sec(dx + c)^2 + A \sec(dx + c)) \sqrt{\sec(dx + c)}}{a^2 \sec(dx + c)^2 + 2 a^2 \sec(dx + c) + a^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="fricas")

[Out] integral((B*sec(d*x + c)^2 + A*sec(d*x + c))*sqrt(sec(d*x + c))/(a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sec(dx + c)^{\frac{3}{2}}}{(a \sec(dx + c) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(3/2)/(a*sec(d*x + c) + a)^2, x)

maple [A] time = 5.03, size = 350, normalized size = 2.17

$$\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \left(2A\left(\cos^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 1}\right) \text{Ellip}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x)`

[Out] `-1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-12*B*cos(1/2*d*x+1/2*c)^6+4*B*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-6*B*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*A*cos(1/2*d*x+1/2*c)^4+16*B*cos(1/2*d*x+1/2*c)^4-3*A*cos(1/2*d*x+1/2*c)^2-3*B*cos(1/2*d*x+1/2*c)^2+A-B)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d`

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="maxima")`

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(\frac{1}{\cos(c+dx)}\right)^{3/2}}{\left(a + \frac{a}{\cos(c+dx)}\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + a/cos(c + d*x))^2,x)`

[Out] `int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + a/cos(c + d*x))^2, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A \sec^{\frac{3}{2}}(c+dx)}{\sec^2(c+dx)+2 \sec(c+dx)+1} dx + \int \frac{B \sec^{\frac{5}{2}}(c+dx)}{\sec^2(c+dx)+2 \sec(c+dx)+1} dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**2,x)

[Out] (Integral(A*sec(c + d*x)**(3/2)/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)
+ Integral(B*sec(c + d*x)**(5/2)/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)
)/a**2

$$3.212 \quad \int \frac{\sqrt{\sec(c+dx)} (A+B \sec(c+dx))}{(a+a \sec(c+dx))^2} dx$$

Optimal. Leaf size=168

$$\frac{(2A+B) \sin(c+dx) \sqrt{\sec(c+dx)}}{3a^2 d (\sec(c+dx)+1)} + \frac{(2A+B) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} F\left(\frac{1}{2}(c+dx) \middle| 2\right)}{3a^2 d} - \frac{A \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{3a^2 d}$$

[Out] 1/3*(2*A+B)*sin(d*x+c)*sec(d*x+c)^(1/2)/a^2/d/(1+sec(d*x+c))+1/3*(A-B)*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^2-A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/d+1/3*(2*A+B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/d

Rubi [A] time = 0.31, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4019, 4020, 3787, 3771, 2639, 2641}

$$\frac{(2A+B) \sin(c+dx) \sqrt{\sec(c+dx)}}{3a^2 d (\sec(c+dx)+1)} + \frac{(2A+B) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} F\left(\frac{1}{2}(c+dx) \middle| 2\right)}{3a^2 d} - \frac{A \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{3a^2 d}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^2,x]

[Out] -((A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*d)) + ((2*A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a^2*d) + ((2*A + B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*a^2*d*(1 + Sec[c + d*x])) + ((A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2)

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 3771

Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 4019

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := Simp[(d*(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0]

Rule 4020

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := -Simp[((A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m + 1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0]

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\sec(c+dx)}(A+B\sec(c+dx))}{(a+a\sec(c+dx))^2} dx &= \frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\int \frac{-\frac{1}{2}a(A-B)+\frac{3}{2}a(A+B)\sec(c+dx)}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))} dx}{3a^2} \\
&= \frac{(2A+B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3a^2d(1+\sec(c+dx))} + \frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d(a+a\sec(c+dx))^2} \\
&= \frac{(2A+B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3a^2d(1+\sec(c+dx))} + \frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d(a+a\sec(c+dx))^2} \\
&= \frac{(2A+B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3a^2d(1+\sec(c+dx))} + \frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d(a+a\sec(c+dx))^2} \\
&= -\frac{A\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} + \frac{(2A+B)\sqrt{\cos(c+dx)}}{3d(a+a\sec(c+dx))^2}
\end{aligned}$$

Mathematica [C] time = 3.65, size = 256, normalized size = 1.52

$$e^{-idx} \cos\left(\frac{1}{2}(c+dx)\right) \sec^{\frac{5}{2}}(c+dx) \left(\cos\left(\frac{1}{2}(c+3dx)\right) + i \sin\left(\frac{1}{2}(c+3dx)\right)\right) \left(i \left(Ae^{-i(c+dx)} (1 + e^{i(c+dx)})^3 \sqrt{1 + e^{2i(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^2, x]

[Out] (Cos[(c + d*x)/2]*Sec[c + d*x]^(5/2)*(8*(2*A + B)*Cos[(c + d*x)/2]^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*(Cos[(c + d*x)/2] - I*Sin[(c + d*x)/2]) + I*((A*(1 + E^(I*(c + d*x))))^3*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])/E^(I*(c + d*x)) - 2*Cos[c + d*x]*(5*A + B + (7*A - B)*Cos[c + d*x] - I*(A - B)*Sin[c + d*x]))*(Cos[(c + 3*d*x)/2] + I*Sin[(c + 3*d*x)/2]))/(6*a^2*d*E^(I*d*x)*(1 + Sec[c + d*x])^2)

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B \sec(dx+c) + A)\sqrt{\sec(dx+c)}}{a^2 \sec(dx+c)^2 + 2a^2 \sec(dx+c) + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")

[Out] integral((B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)\sqrt{\sec(dx + c)}}{(a \sec(dx + c) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(a*sec(d*x + c) + a)^2, x)

maple [A] time = 5.66, size = 350, normalized size = 2.08

$$\sqrt{\left(2 \left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right) \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \left(12A \left(\cos^6\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 4A \left(\cos^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)\right) \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{-2 \left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x)

[Out] -1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*A*cos(1/2*d*x+1/2*c)^6+4*A*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+6*A*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*B*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-20*A*cos(1/2*d*x+1/2*c)^4+2*B*cos(1/2*d*x+1/2*c)^4+9*A*cos(1/2*d*x+1/2*c)^2-3*B*cos(1/2*d*x+1/2*c)^2-A+B)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \sqrt{\frac{1}{\cos(c+dx)}}}{\left(a + \frac{a}{\cos(c+dx)}\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + a/cos(c + d*x))^2,x)

[Out] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + a/cos(c + d*x))^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A\sqrt{\sec(c+dx)}}{\sec^2(c+dx)+2\sec(c+dx)+1} dx + \int \frac{B\sec^{\frac{3}{2}}(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1} dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)**(1/2)/(a+a*sec(d*x+c))**2,x)

[Out] (Integral(A*sqrt(sec(c + d*x))/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x) + Integral(B*sec(c + d*x)**(3/2)/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x))/a**2

$$3.213 \quad \int \frac{A+B \sec(c+dx)}{\sqrt{\sec(c+dx)} (a+a \sec(c+dx))^2} dx$$

Optimal. Leaf size=177

$$\frac{(5A-2B) \sin(c+dx) \sqrt{\sec(c+dx)}}{3a^2 d (\sec(c+dx)+1)} - \frac{(5A-2B) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} F\left(\frac{1}{2}(c+dx) \middle| 2\right)}{3a^2 d} + \frac{(4A-B) \sqrt{\cos(c+dx)}}{3a^2 d}$$

[Out] $-1/3*(5*A-2*B)*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/a^2/d/(1+\sec(d*x+c))-1/3*(A-B)*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/d/(a+a*\sec(d*x+c))^2+(4*A-B)*(\cos(1/2*d*x+1/2*c))^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)*}\sec(d*x+c)^{(1/2)}/a^2/d-1/3*(5*A-2*B)*(\cos(1/2*d*x+1/2*c))^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)*}\sec(d*x+c)^{(1/2)}/a^2/d$

Rubi [A] time = 0.33, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {4020, 3787, 3771, 2639, 2641}

$$\frac{(5A-2B) \sin(c+dx) \sqrt{\sec(c+dx)}}{3a^2 d (\sec(c+dx)+1)} - \frac{(5A-2B) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} F\left(\frac{1}{2}(c+dx) \middle| 2\right)}{3a^2 d} + \frac{(4A-B) \sqrt{\cos(c+dx)}}{3a^2 d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(A + B*\text{Sec}[c + d*x])]/(\text{Sqrt}[\text{Sec}[c + d*x]]*(a + a*\text{Sec}[c + d*x])^2), x]$

[Out] $((4*A - B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/(a^2*d) - ((5*A - 2*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticF}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/(3*a^2*d) - ((5*A - 2*B)*\text{Sqrt}[\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(3*a^2*d*(1 + \text{Sec}[c + d*x])) - ((A - B)*\text{Sqrt}[\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(3*d*(a + a*\text{Sec}[c + d*x])^2)$

Rule 2639

$\text{Int}[\text{Sqrt}[\sin[(c_.) + (d_.)*(x_.)]], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticE}[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 2641

$\text{Int}[1/\text{Sqrt}[\sin[(c_.) + (d_.)*(x_.)]], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticF}[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 3771

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 4020

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] := -Simp[((A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m + 1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{A + B \sec(c + dx)}{\sqrt{\sec(c + dx)} (a + a \sec(c + dx))^2} dx &= -\frac{(A - B)\sqrt{\sec(c + dx)} \sin(c + dx)}{3d(a + a \sec(c + dx))^2} + \frac{\int \frac{\frac{1}{2}a(7A-B) - \frac{3}{2}a(A-B)\sec(c+dx)}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))} dx}{3a^2} \\
 &= -\frac{(5A - 2B)\sqrt{\sec(c + dx)} \sin(c + dx)}{3a^2d(1 + \sec(c + dx))} - \frac{(A - B)\sqrt{\sec(c + dx)} \sin(c + dx)}{3d(a + a \sec(c + dx))^2} \\
 &= -\frac{(5A - 2B)\sqrt{\sec(c + dx)} \sin(c + dx)}{3a^2d(1 + \sec(c + dx))} - \frac{(A - B)\sqrt{\sec(c + dx)} \sin(c + dx)}{3d(a + a \sec(c + dx))^2} \\
 &= -\frac{(5A - 2B)\sqrt{\sec(c + dx)} \sin(c + dx)}{3a^2d(1 + \sec(c + dx))} - \frac{(A - B)\sqrt{\sec(c + dx)} \sin(c + dx)}{3d(a + a \sec(c + dx))^2} \\
 &= \frac{(4A - B)\sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{\sec(c + dx)}}{a^2d} - \frac{(5A - 2B)\sqrt{\sec(c + dx)} \sin(c + dx)}{3d(a + a \sec(c + dx))^2}
 \end{aligned}$$

Mathematica [C] time = 6.88, size = 854, normalized size = 4.82

$$\frac{4\sqrt{2} A e^{-idx} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \sqrt{1+e^{2i(c+dx)}} \csc\left(\frac{c}{2}\right) \left(e^{2idx} (-1+e^{2ic}) {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{7}{4}; -e^{2i(c+dx)}\right) - 3\sqrt{1+e^{2i(c+dx)}} \right) \sec\left(\frac{c}{2}\right)}{3d(B+A\cos(c+dx))(\sec(c+dx)a+a)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sec[c + d*x])/(Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^2), x]

[Out] (-4*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x])]/(3*d*E^(I*d*x)*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^2) + (Sqrt[2]*B*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x])]/(3*d*E^(I*d*x)*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^2) - (10*A*Cos[c/2 + (d*x)/2]^4*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c/2]*Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x])*Sin[c]/(3*d*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^2) + (4*B*Cos[c/2 + (d*x)/2]^4*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c/2]*Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x])*Sin[c]/(3*d*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^2) + (Cos[c/2 + (d*x)/2]^4*Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x])*((-2*(3*A - B + A*Cos[2*c])*Cos[d*x]*Csc[c/2]*Sec[c/2])/d + (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(-(A*Sin[(d*x)/2]) + B*Sin[(d*x)/2]))/(3*d) - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(-7*A*Sin[(d*x)/2] + 4*B*Sin[(d*x)/2]))/(3*d) + (8*A*Cos[c]*Sin[d*x])/d - (4*(-7*A + 4*B)*Tan[c/2])/ (3*d) + (2*(-A + B)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/ (3*d)))/((B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^2)

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B \sec(dx + c) + A)\sqrt{\sec(dx + c)}}{a^2 \sec(dx + c)^3 + 2a^2 \sec(dx + c)^2 + a^2 \sec(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="fricas")

[Out] integral((B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(a^2*sec(d*x + c)^3 + 2*a^2*sec(d*x + c)^2 + a^2*sec(d*x + c)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(a \sec(dx + c) + a)^2 \sqrt{\sec(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^2*sqrt(sec(d*x + c))), x)

maple [A] time = 5.82, size = 421, normalized size = 2.38

$$\sqrt{\left(2 \left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right) \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) \left(24A \left(\cos^6\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 10A \left(\cos^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)\right) \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{-2 \left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2/sec(d*x+c)^(1/2),x)

[Out] 1/6/a^2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(24*A*cos(1/2*d*x+1/2*c)^6+10*A*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+24*A*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-12*B*cos(1/2*d*x+1/2*c)^6-4*B*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-6*B*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-38*A*cos(1/2*d*x+1/2*c)^4+20*B*cos(1/2*d*x+1/2*c)^4+15*A*cos(1/2*d*x+1/2*c)^2-9*B*cos(1/2*d*x+1/2*c)^2-A+B)/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(a \sec(dx + c) + a)^2 \sqrt{\sec(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^2*sqrt(sec(d*x + c))), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\left(a + \frac{a}{\cos(c+dx)}\right)^2 \sqrt{\frac{1}{\cos(c+dx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/((a + a/cos(c + d*x))^2*(1/cos(c + d*x))^(1/2)), x)

[Out] int((A + B/cos(c + d*x))/((a + a/cos(c + d*x))^2*(1/cos(c + d*x))^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A}{\sec^{\frac{5}{2}}(c+dx) + 2\sec^{\frac{3}{2}}(c+dx) + \sqrt{\sec(c+dx)}} dx + \int \frac{B \sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx) + 2\sec^{\frac{3}{2}}(c+dx) + \sqrt{\sec(c+dx)}} dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2/sec(d*x+c)**(1/2), x)

[Out] (Integral(A/(sec(c + d*x)**(5/2) + 2*sec(c + d*x)**(3/2) + sqrt(sec(c + d*x))), x) + Integral(B*sec(c + d*x)/(sec(c + d*x)**(5/2) + 2*sec(c + d*x)**(3/2) + sqrt(sec(c + d*x))), x))/a**2

$$3.214 \quad \int \frac{A+B \sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a \sec(c+dx))^2} dx$$

Optimal. Leaf size=211

$$\frac{5(2A - B) \sin(c + dx)}{3a^2 d \sqrt{\sec(c + dx)}} - \frac{(7A - 4B) \sin(c + dx)}{3a^2 d \sqrt{\sec(c + dx)} (\sec(c + dx) + 1)} + \frac{5(2A - B) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} F\left(\frac{1}{2}(c + dx)\right)}{3a^2 d}$$

[Out] $5/3*(2*A-B)*\sin(d*x+c)/a^2/d/\sec(d*x+c)^{(1/2)}-1/3*(7*A-4*B)*\sin(d*x+c)/a^2/d/(1+\sec(d*x+c))/\sec(d*x+c)^{(1/2)}-1/3*(A-B)*\sin(d*x+c)/d/(a+a*\sec(d*x+c))^2/\sec(d*x+c)^{(1/2)}-(7*A-4*B)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c),2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/a^2/d+5/3*(2*A-B)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c),2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/a^2/d$

Rubi [A] time = 0.36, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4020, 3787, 3769, 3771, 2641, 2639}

$$\frac{5(2A - B) \sin(c + dx)}{3a^2 d \sqrt{\sec(c + dx)}} - \frac{(7A - 4B) \sin(c + dx)}{3a^2 d \sqrt{\sec(c + dx)} (\sec(c + dx) + 1)} + \frac{5(2A - B) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} F\left(\frac{1}{2}(c + dx)\right)}{3a^2 d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(A + B*\text{Sec}[c + d*x])/(\text{Sec}[c + d*x]^{(3/2)}*(a + a*\text{Sec}[c + d*x])^2), x]$

[Out] $-(((7*A - 4*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/(a^2*d) + (5*(2*A - B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticF}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/(3*a^2*d) + (5*(2*A - B)*\text{Sin}[c + d*x])/(3*a^2*d*\text{Sqrt}[\text{Sec}[c + d*x]]) - ((7*A - 4*B)*\text{Sin}[c + d*x])/(3*a^2*d*\text{Sqrt}[\text{Sec}[c + d*x]])*(1 + \text{Sec}[c + d*x]) - ((A - B)*\text{Sin}[c + d*x])/(3*d*\text{Sqrt}[\text{Sec}[c + d*x]])*(a + a*\text{Sec}[c + d*x])^2)$

Rule 2639

$\text{Int}[\text{Sqrt}[\sin[(c_.) + (d_.)*(x_)]]], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticE}[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}[\{c, d\}, x]$

Rule 2641

$\text{Int}[1/\text{Sqrt}[\sin[(c_.) + (d_.)*(x_)]]], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticF}[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}[\{c, d\}, x]$

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x]
)^(n)*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_.)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] := -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
+ f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{A + B \sec(c + dx)}{\sec^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))^2} dx &= -\frac{(A - B) \sin(c + dx)}{3d\sqrt{\sec(c + dx)}(a + a \sec(c + dx))^2} + \int \frac{\frac{3}{2}a(3A-B) - \frac{5}{2}a(A-B) \sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a \sec(c+dx))} dx \\
&= -\frac{(7A - 4B) \sin(c + dx)}{3a^2d\sqrt{\sec(c + dx)}(1 + \sec(c + dx))} - \frac{(A - B) \sin(c + dx)}{3d\sqrt{\sec(c + dx)}(a + a \sec(c + dx))} \\
&= -\frac{(7A - 4B) \sin(c + dx)}{3a^2d\sqrt{\sec(c + dx)}(1 + \sec(c + dx))} - \frac{(A - B) \sin(c + dx)}{3d\sqrt{\sec(c + dx)}(a + a \sec(c + dx))} \\
&= \frac{5(2A - B) \sin(c + dx)}{3a^2d\sqrt{\sec(c + dx)}} - \frac{(7A - 4B) \sin(c + dx)}{3a^2d\sqrt{\sec(c + dx)}(1 + \sec(c + dx))} - \frac{(A - B) \sin(c + dx)}{3d\sqrt{\sec(c + dx)}(a + a \sec(c + dx))} \\
&= -\frac{(7A - 4B)\sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{\sec(c + dx)}}{a^2d} + \frac{5(2A - B) \sin(c + dx)}{3a^2d\sqrt{\sec(c + dx)}} \\
&= -\frac{(7A - 4B)\sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{\sec(c + dx)}}{a^2d} + \frac{5(2A - B) \sin(c + dx)}{3d\sqrt{\sec(c + dx)}(a + a \sec(c + dx))}
\end{aligned}$$

Mathematica [C] time = 6.91, size = 899, normalized size = 4.26

$$\frac{7\sqrt{2} A e^{-idx} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \sqrt{1 + e^{2i(c+dx)}} \csc\left(\frac{c}{2}\right) \left(e^{2idx} (-1 + e^{2ic}) {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{7}{4}; -e^{2i(c+dx)}\right) - 3\sqrt{1 + e^{2i(c+dx)}} \right) \sec\left(\frac{c}{2}\right) \sec(c + dx)}{3d(B + A \cos(c + dx))(\sec(c + dx)a + a)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2), x]

[Out] (7*sqrt[2]*A*sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*(-3*sqrt[1 + E^((2*I)*(c + d*x))]] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x]))/(3*d*E^(I*d*x)*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^2) - (4*sqrt[2]*B*sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*(-3*sqrt[1 + E^((2*I)*(c + d*x))]] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x]))/(3*d*E^(I*d*x)*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^2)

$$\begin{aligned} &]*(a + a*\text{Sec}[c + d*x])^2) + (20*A*\text{Cos}[c/2 + (d*x)/2]^4*\text{Sqrt}[\text{Cos}[c + d*x]]* \\ & \text{Csc}[c/2]*\text{EllipticF}[(c + d*x)/2, 2]*\text{Sec}[c/2]*\text{Sec}[c + d*x]^{(3/2)}*(A + B*\text{Sec}[c \\ & + d*x])*\text{Sin}[c])/(3*d*(B + A*\text{Cos}[c + d*x])*(a + a*\text{Sec}[c + d*x])^2) - (10*B* \\ & \text{Cos}[c/2 + (d*x)/2]^4*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Csc}[c/2]*\text{EllipticF}[(c + d*x)/2, 2]* \\ & \text{Sec}[c/2]*\text{Sec}[c + d*x]^{(3/2)}*(A + B*\text{Sec}[c + d*x])*\text{Sin}[c])/(3*d*(B + A*\text{Cos}[c \\ & + d*x])*(a + a*\text{Sec}[c + d*x])^2) + (\text{Cos}[c/2 + (d*x)/2]^4*\text{Sec}[c + d*x]^{(3/2)}* \\ & (A + B*\text{Sec}[c + d*x])*((-2*(-5*A + 3*B - 2*A*\text{Cos}[2*c] + B*\text{Cos}[2*c])*\text{Cos}[d*x] \\ & *\text{Csc}[c/2]*\text{Sec}[c/2])/d + (4*A*\text{Cos}[2*d*x]*\text{Sin}[2*c])/(3*d) - (2*\text{Sec}[c/2]*\text{Sec}[c \\ & /2 + (d*x)/2]^3*(-(A*\text{Sin}[(d*x)/2]) + B*\text{Sin}[(d*x)/2]))/(3*d) + (4*\text{Sec}[c/2]*\text{S} \\ & \text{ec}[c/2 + (d*x)/2]*(-10*A*\text{Sin}[(d*x)/2] + 7*B*\text{Sin}[(d*x)/2]))/(3*d) + (8*(-2*A \\ & + B)*\text{Cos}[c]*\text{Sin}[d*x])/d + (4*A*\text{Cos}[2*c]*\text{Sin}[2*d*x])/(3*d) + (4*(-10*A + 7* \\ & B)*\text{Tan}[c/2])/(3*d) - (2*(-A + B)*\text{Sec}[c/2 + (d*x)/2]^2*\text{Tan}[c/2])/(3*d)))/((B \\ & + A*\text{Cos}[c + d*x])*(a + a*\text{Sec}[c + d*x])^2) \end{aligned}$$

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B \sec(dx + c) + A)\sqrt{\sec(dx + c)}}{a^2 \sec(dx + c)^4 + 2a^2 \sec(dx + c)^3 + a^2 \sec(dx + c)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")

[Out] integral((B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(a^2*sec(d*x + c)^4 + 2*a^2*sec(d*x + c)^3 + a^2*sec(d*x + c)^2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(a \sec(dx + c) + a)^2 \sec(dx + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^2*sec(d*x + c)^(3/2)), x)

maple [A] time = 6.28, size = 435, normalized size = 2.06

$$\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \left(16A\left(\cos^8\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 12A\left(\cos^6\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 20A\left(\cos^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x)`

[Out]
$$-1/6/a^2*((2*\cos(1/2*d*x+1/2*c)^2-1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(16*A*\cos(1/2*d*x+1/2*c)^8+12*A*\cos(1/2*d*x+1/2*c)^6+20*A*\cos(1/2*d*x+1/2*c)^3*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})+42*A*\cos(1/2*d*x+1/2*c)^3*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)})-24*B*\cos(1/2*d*x+1/2*c)^6-10*B*\cos(1/2*d*x+1/2*c)^3*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})-24*B*\cos(1/2*d*x+1/2*c)^3*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)})-48*A*\cos(1/2*d*x+1/2*c)^4+38*B*\cos(1/2*d*x+1/2*c)^4+21*A*\cos(1/2*d*x+1/2*c)^2-15*B*\cos(1/2*d*x+1/2*c)^2-A+B)/\cos(1/2*d*x+1/2*c)^3/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(a \sec(dx + c) + a)^2 \sec(dx + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")`

[Out] `integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^2*sec(d*x + c)^(3/2)), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\left(a + \frac{a}{\cos(c+dx)}\right)^2 \left(\frac{1}{\cos(c+dx)}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B/cos(c + d*x))/((a + a/cos(c + d*x))^2*(1/cos(c + d*x))^(3/2)),x)`

[Out] `int((A + B/cos(c + d*x))/((a + a/cos(c + d*x))^2*(1/cos(c + d*x))^(3/2)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A}{\sec^{\frac{7}{2}}(c+dx)+2\sec^{\frac{5}{2}}(c+dx)+\sec^{\frac{3}{2}}(c+dx)} dx + \int \frac{B \sec(c+dx)}{\sec^{\frac{7}{2}}(c+dx)+2\sec^{\frac{5}{2}}(c+dx)+\sec^{\frac{3}{2}}(c+dx)} dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)**(3/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] (Integral(A/(sec(c + d*x)**(7/2) + 2*sec(c + d*x)**(5/2) + sec(c + d*x)**(3/2)), x) + Integral(B*sec(c + d*x)/(sec(c + d*x)**(7/2) + 2*sec(c + d*x)**(5/2) + sec(c + d*x)**(3/2)), x))/a**2
```

$$3.215 \quad \int \frac{A+B \sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+a \sec(c+dx))^2} dx$$

Optimal. Leaf size=244

$$-\frac{(3A-2B)\sin(c+dx)}{a^2 d \sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)+1)} + \frac{7(8A-5B)\sin(c+dx)}{15a^2 d \sec^{\frac{3}{2}}(c+dx)} - \frac{5(3A-2B)\sin(c+dx)}{3a^2 d \sqrt{\sec(c+dx)}} - \frac{5(3A-2B)\sqrt{\cos(c+dx)}}{a^2 d \sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)+1)}$$

[Out] $7/15*(8*A-5*B)*\sin(d*x+c)/a^2/d/\sec(d*x+c)^{(3/2)}-(3*A-2*B)*\sin(d*x+c)/a^2/d/\sec(d*x+c)^{(3/2)}/(1+\sec(d*x+c))-1/3*(A-B)*\sin(d*x+c)/d/\sec(d*x+c)^{(3/2)}/(a+a*\sec(d*x+c))^2-5/3*(3*A-2*B)*\sin(d*x+c)/a^2/d/\sec(d*x+c)^{(1/2)}+7/5*(8*A-5*B)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c),2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/a^2/d-5/3*(3*A-2*B)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c),2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/a^2/d$

Rubi [A] time = 0.38, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4020, 3787, 3769, 3771, 2639, 2641}

$$-\frac{(3A-2B)\sin(c+dx)}{a^2 d \sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)+1)} + \frac{7(8A-5B)\sin(c+dx)}{15a^2 d \sec^{\frac{3}{2}}(c+dx)} - \frac{5(3A-2B)\sin(c+dx)}{3a^2 d \sqrt{\sec(c+dx)}} - \frac{5(3A-2B)\sqrt{\cos(c+dx)}}{a^2 d \sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)+1)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(A + B*\text{Sec}[c + d*x])]/(\text{Sec}[c + d*x]^{(5/2)}*(a + a*\text{Sec}[c + d*x])^2), x]$

[Out] $(7*(8*A - 5*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/(5*a^2*d) - (5*(3*A - 2*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticF}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/(3*a^2*d) + (7*(8*A - 5*B)*\text{Sin}[c + d*x])/(15*a^2*d*\text{Sec}[c + d*x]^{(3/2)}) - (5*(3*A - 2*B)*\text{Sin}[c + d*x])/(3*a^2*d*\text{Sqrt}[\text{Sec}[c + d*x]]) - ((3*A - 2*B)*\text{Sin}[c + d*x])/(a^2*d*\text{Sec}[c + d*x]^{(3/2)}*(1 + \text{Sec}[c + d*x])) - ((A - B)*\text{Sin}[c + d*x])/(3*d*\text{Sec}[c + d*x]^{(3/2)}*(a + a*\text{Sec}[c + d*x])^2)$

Rule 2639

$\text{Int}[\text{Sqrt}[\sin[(c_.) + (d_.)*(x_.)]], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticE}[(1*(c - \text{Pi}/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 2641

$\text{Int}[1/\text{Sqrt}[\sin[(c_.) + (d_.)*(x_.)]], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticF}[(1*(c - \text{Pi}/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x]
)^(n)*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_.)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] := -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
+ f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{A + B \sec(c + dx)}{\sec^{\frac{5}{2}}(c + dx)(a + a \sec(c + dx))^2} dx &= -\frac{(A - B) \sin(c + dx)}{3d \sec^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))^2} + \frac{\int \frac{\frac{1}{2}a(11A-5B) - \frac{7}{2}a(A-B) \sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+a \sec(c+dx))} dx}{3a^2} \\
&= -\frac{(3A - 2B) \sin(c + dx)}{a^2 d \sec^{\frac{3}{2}}(c + dx)(1 + \sec(c + dx))} - \frac{(A - B) \sin(c + dx)}{3d \sec^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))} \\
&= -\frac{(3A - 2B) \sin(c + dx)}{a^2 d \sec^{\frac{3}{2}}(c + dx)(1 + \sec(c + dx))} - \frac{(A - B) \sin(c + dx)}{3d \sec^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))} \\
&= \frac{7(8A - 5B) \sin(c + dx)}{15a^2 d \sec^{\frac{3}{2}}(c + dx)} - \frac{5(3A - 2B) \sin(c + dx)}{3a^2 d \sqrt{\sec(c + dx)}} - \frac{(3A - 2B) \sin(c + dx)}{a^2 d \sec^{\frac{3}{2}}(c + dx)} \\
&= \frac{7(8A - 5B) \sin(c + dx)}{15a^2 d \sec^{\frac{3}{2}}(c + dx)} - \frac{5(3A - 2B) \sin(c + dx)}{3a^2 d \sqrt{\sec(c + dx)}} - \frac{(3A - 2B) \sin(c + dx)}{a^2 d \sec^{\frac{3}{2}}(c + dx)} \\
&= \frac{7(8A - 5B) \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{\sec(c + dx)}}{5a^2 d} - \frac{5(3A - 2B) \sin(c + dx)}{3a^2 d}
\end{aligned}$$

Mathematica [C] time = 7.15, size = 946, normalized size = 3.88

$$\frac{56\sqrt{2} A e^{-idx} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \sqrt{1+e^{2i(c+dx)}} \csc\left(\frac{c}{2}\right) \left(e^{2idx} (-1+e^{2ic}) {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{7}{4}; -e^{2i(c+dx)}\right) - 3\sqrt{1+e^{2i(c+dx)}} \right) \sec\left(\frac{c}{2}\right)}{15d(B+A\cos(c+dx))(\sec(c+dx)a+a)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^2), x]

[Out] (-56*sqrt(2)*A*sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*(-3*sqrt[1 + E^((2*I)*(c + d*x))]] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x])]/(15*d*E^(I*d*x)*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^2) + (7*sqrt(2)*B*sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*(-3*sqrt[1 + E^((2*I)*(c + d*x))]] + E^((2*I)*d*x))

```

*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]
*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x))/(3*d*E^(I*d*x)*(B + A*Cos[c +
d*x]))*(a + a*Sec[c + d*x])^2) - (10*A*Cos[c/2 + (d*x)/2]^4*Sqrt[Cos[c + d*x
]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c/2]*Sec[c + d*x]^(3/2)*(A + B*Se
c[c + d*x])*Sin[c])/(d*(B + A*Cos[c + d*x]))*(a + a*Sec[c + d*x])^2) + (20*B
*Cos[c/2 + (d*x)/2]^4*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]
*Sec[c/2]*Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x])*Sin[c])/(3*d*(B + A*Cos[c
+ d*x]))*(a + a*Sec[c + d*x])^2) + (Cos[c/2 + (d*x)/2]^4*Sec[c + d*x]^(3/2)
*(A + B*Sec[c + d*x])*((( -151*A + 100*B - 73*A*Cos[2*c] + 40*B*Cos[2*c])*Co
s[d*x]*Csc[c/2]*Sec[c/2]))/(10*d) + (4*(-2*A + B)*Cos[2*d*x]*Sin[2*c])/(3*d)
+ (2*A*Cos[3*d*x]*Sin[3*c])/(5*d) + (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(-(A
Sin[(d*x)/2]) + B*Sin[(d*x)/2]))/(3*d) - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(-1
3*A*Sin[(d*x)/2] + 10*B*Sin[(d*x)/2]))/(3*d) - (2*(-73*A + 40*B)*Cos[c]*Sin
[d*x])/(5*d) + (4*(-2*A + B)*Cos[2*c]*Sin[2*d*x])/(3*d) + (2*A*Cos[3*c]*Sin
[3*d*x])/(5*d) - (4*(-13*A + 10*B)*Tan[c/2])/(3*d) + (2*(-A + B)*Sec[c/2 +
(d*x)/2]^2*Tan[c/2])/(3*d)))/(B + A*Cos[c + d*x))*(a + a*Sec[c + d*x])^2)

```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{(B \sec(dx + c) + A) \sqrt{\sec(dx + c)}}{a^2 \sec(dx + c)^5 + 2a^2 \sec(dx + c)^4 + a^2 \sec(dx + c)^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```

[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm
="fricas")

```

```

[Out] integral((B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(a^2*sec(d*x + c)^5 + 2*a^
2*sec(d*x + c)^4 + a^2*sec(d*x + c)^3), x)

```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(a \sec(dx + c) + a)^2 \sec(dx + c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```

[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm
="giac")

```

```

[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^2*sec(d*x + c)^(5/2)),
x)

```

maple [A] time = 5.62, size = 465, normalized size = 1.91

$$\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)\left(96A\left(\cos^{10}\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 352A\left(\cos^8\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 80B\left(\cos^8\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \dots\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x)

[Out]
$$-1/30/a^2*((2*\cos(1/2*d*x+1/2*c)^2-1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(96*A*\cos(1/2*d*x+1/2*c)^{10}-352*A*\cos(1/2*d*x+1/2*c)^8+80*B*\cos(1/2*d*x+1/2*c)^8+120*A*\cos(1/2*d*x+1/2*c)^6-150*A*\cos(1/2*d*x+1/2*c)^3*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)})-336*A*\cos(1/2*d*x+1/2*c)^3*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)})+60*B*\cos(1/2*d*x+1/2*c)^6+100*B*\cos(1/2*d*x+1/2*c)^3*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)})+210*B*\cos(1/2*d*x+1/2*c)^3*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)})+266*A*\cos(1/2*d*x+1/2*c)^4-240*B*\cos(1/2*d*x+1/2*c)^4-135*A*\cos(1/2*d*x+1/2*c)^2+105*B*\cos(1/2*d*x+1/2*c)^2+5*A-5*B)/\cos(1/2*d*x+1/2*c)^3/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(a \sec(dx + c) + a)^2 \sec(dx + c)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^2*sec(d*x + c)^(5/2)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\left(a + \frac{a}{\cos(c+dx)}\right)^2 \left(\frac{1}{\cos(c+dx)}\right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/((a + a/cos(c + d*x))^2*(1/cos(c + d*x))^(5/2)),x)
[Out] int((A + B/cos(c + d*x))/((a + a/cos(c + d*x))^2*(1/cos(c + d*x))^(5/2)), x
)
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)**(5/2)/(a+a*sec(d*x+c))**2,x)
[Out] Timed out
```


$$3.216 \quad \int \frac{\sec^2(c+dx)(A+B \sec(c+dx))^9}{(a+a \sec(c+dx))^3} dx$$

Optimal. Leaf size=292

$$\frac{7(7A - 17B) \sin(c + dx) \sec^{\frac{5}{2}}(c + dx)}{30d (a^3 \sec(c + dx) + a^3)} - \frac{(13A - 33B) \sin(c + dx) \sec^{\frac{3}{2}}(c + dx)}{6a^3d} + \frac{7(7A - 17B) \sin(c + dx) \sqrt{\sec(c + dx)}}{10a^3d}$$

[Out] $-1/6*(13*A-33*B)*\sec(d*x+c)^{(3/2)}*\sin(d*x+c)/a^3/d+1/5*(A-B)*\sec(d*x+c)^{(9/2)}*\sin(d*x+c)/d/(a+a*\sec(d*x+c))^3+1/3*(A-2*B)*\sec(d*x+c)^{(7/2)}*\sin(d*x+c)/a/d/(a+a*\sec(d*x+c))^2+7/30*(7*A-17*B)*\sec(d*x+c)^{(5/2)}*\sin(d*x+c)/d/(a^3+a^3*\sec(d*x+c))+7/10*(7*A-17*B)*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/a^3/d-7/10*(7*A-17*B)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/a^3/d-1/6*(13*A-33*B)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/a^3/d$

Rubi [A] time = 0.56, antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4019, 3787, 3768, 3771, 2639, 2641}

$$\frac{7(7A - 17B) \sin(c + dx) \sec^{\frac{5}{2}}(c + dx)}{30d (a^3 \sec(c + dx) + a^3)} - \frac{(13A - 33B) \sin(c + dx) \sec^{\frac{3}{2}}(c + dx)}{6a^3d} + \frac{7(7A - 17B) \sin(c + dx) \sqrt{\sec(c + dx)}}{10a^3d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(\text{Sec}[c + d*x]^{(9/2)}*(A + B*\text{Sec}[c + d*x]))/(a + a*\text{Sec}[c + d*x])^3, x]$

[Out] $(-7*(7*A - 17*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/(10*a^3*d) - ((13*A - 33*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticF}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/(6*a^3*d) + (7*(7*A - 17*B)*\text{Sqrt}[\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(10*a^3*d) - ((13*A - 33*B)*\text{Sec}[c + d*x]^{(3/2)}*\text{Sin}[c + d*x])/(6*a^3*d) + ((A - B)*\text{Sec}[c + d*x]^{(9/2)}*\text{Sin}[c + d*x])/(5*d*(a + a*\text{Sec}[c + d*x])^3) + ((A - 2*B)*\text{Sec}[c + d*x]^{(7/2)}*\text{Sin}[c + d*x])/(3*a*d*(a + a*\text{Sec}[c + d*x])^2) + (7*(7*A - 17*B)*\text{Sec}[c + d*x]^{(5/2)}*\text{Sin}[c + d*x])/(30*d*(a^3 + a^3*\text{Sec}[c + d*x]))$

Rule 2639

$\text{Int}[\text{Sqrt}[\sin[(c_.) + (d_.)*(x_.)]], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticE}[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_.)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 3768

Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.))^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3771

Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 4019

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := Simp[(d*(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0]

Rubi steps

$$\begin{aligned}
\int \frac{\sec^{\frac{9}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^3} dx &= \frac{(A-B)\sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \int \frac{\sec^{\frac{7}{2}}(c+dx)\left(\frac{7}{2}a(A-B)-\frac{1}{2}a(3A-13B)\sec(c+dx)\right)}{(a+a\sec(c+dx))^2} \frac{1}{5a^2} \\
&= \frac{(A-B)\sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(A-2B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3ad(a+a\sec(c+dx))^2} \\
&= \frac{(A-B)\sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(A-2B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3ad(a+a\sec(c+dx))^2} \\
&= \frac{(A-B)\sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(A-2B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3ad(a+a\sec(c+dx))^2} \\
&= \frac{7(7A-17B)\sqrt{\sec(c+dx)}\sin(c+dx)}{10a^3d} - \frac{(13A-33B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6a^3d} \\
&= \frac{7(7A-17B)\sqrt{\sec(c+dx)}\sin(c+dx)}{10a^3d} - \frac{(13A-33B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6a^3d} \\
&= -\frac{7(7A-17B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} - \frac{(13A-33B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6a^3d}
\end{aligned}$$

Mathematica [C] time = 8.50, size = 953, normalized size = 3.26

$$\frac{49\sqrt{2}Ae^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\csc\left(\frac{c}{2}\right)\left(e^{2idx}(-1+e^{2ic}){}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)-3\sqrt{1+e^{2i(c+dx)}}\right)\sec\left(\frac{c}{2}\right)}{15d(B+A\cos(c+dx))(\sec(c+dx)a+a)^3}$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[c + d*x]^(9/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^3, x]

[Out] (49*sqrt[2]*A*sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]*Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(15*d*E^(I*d*x)*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^3 - (119*sqrt[2]*B*sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*sqrt[1 + E^((2*I)*(c + d*x))])*C

$\cos[c/2 + (d*x)/2]^6 * \text{Csc}[c/2] * (-3 * \text{Sqrt}[1 + E^{((2*I)*(c + d*x))}] + E^{((2*I)*d*x)} * (-1 + E^{((2*I)*c)}) * \text{Hypergeometric2F1}[1/2, 3/4, 7/4, -E^{((2*I)*(c + d*x))}] * \text{Sec}[c/2] * \text{Sec}[c + d*x]^2 * (A + B * \text{Sec}[c + d*x])) / (15 * d * E^{(I*d*x)} * (B + A * \text{Cos}[c + d*x]) * (a + a * \text{Sec}[c + d*x])^3) - (26 * A * \text{Cos}[c/2 + (d*x)/2]^6 * \text{Sqrt}[\text{Cos}[c + d*x]] * \text{Csc}[c/2] * \text{EllipticF}[(c + d*x)/2, 2] * \text{Sec}[c/2] * \text{Sec}[c + d*x]^{(5/2)} * (A + B * \text{Sec}[c + d*x]) * \text{Sin}[c]) / (3 * d * (B + A * \text{Cos}[c + d*x]) * (a + a * \text{Sec}[c + d*x])^3) + (22 * B * \text{Cos}[c/2 + (d*x)/2]^6 * \text{Sqrt}[\text{Cos}[c + d*x]] * \text{Csc}[c/2] * \text{EllipticF}[(c + d*x)/2, 2] * \text{Sec}[c/2] * \text{Sec}[c + d*x]^{(5/2)} * (A + B * \text{Sec}[c + d*x]) * \text{Sin}[c]) / (d * (B + A * \text{Cos}[c + d*x]) * (a + a * \text{Sec}[c + d*x])^3) + (\text{Cos}[c/2 + (d*x)/2]^6 * \text{Sec}[c + d*x]^{(5/2)} * (A + B * \text{Sec}[c + d*x]) * ((-14 * (-7 * A + 17 * B) * \text{Cos}[d*x] * \text{Csc}[c/2] * \text{Sec}[c/2]) / (5 * d) + (2 * \text{Sec}[c/2] * \text{Sec}[c/2 + (d*x)/2]^5 * (-A * \text{Sin}[(d*x)/2]) + B * \text{Sin}[(d*x)/2])) / (5 * d) + (4 * \text{Sec}[c/2] * \text{Sec}[c/2 + (d*x)/2]^3 * (-8 * A * \text{Sin}[(d*x)/2] + 13 * B * \text{Sin}[(d*x)/2])) / (15 * d) + (4 * \text{Sec}[c/2] * \text{Sec}[c/2 + (d*x)/2] * (-13 * A * \text{Sin}[(d*x)/2] + 2 * 9 * B * \text{Sin}[(d*x)/2])) / (3 * d) + (16 * B * \text{Sec}[c] * \text{Sec}[c + d*x] * \text{Sin}[d*x]) / (3 * d) + (4 * (4 * B - 13 * A * \text{Cos}[c] + 33 * B * \text{Cos}[c]) * \text{Sec}[c] * \text{Tan}[c/2]) / (3 * d) + (4 * (-8 * A + 13 * B) * \text{Sec}[c/2 + (d*x)/2]^2 * \text{Tan}[c/2]) / (15 * d) + (2 * (-A + B) * \text{Sec}[c/2 + (d*x)/2]^4 * \text{Tan}[c/2]) / (5 * d)) / ((B + A * \text{Cos}[c + d*x]) * (a + a * \text{Sec}[c + d*x])^3)$

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{(B \sec(dx + c)^5 + A \sec(dx + c)^4) \sqrt{\sec(dx + c)}}{a^3 \sec(dx + c)^3 + 3a^3 \sec(dx + c)^2 + 3a^3 \sec(dx + c) + a^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(9/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="fricas")

[Out] integral((B*sec(d*x + c)^5 + A*sec(d*x + c)^4)*sqrt(sec(d*x + c))/(a^3*sec(d*x + c)^3 + 3*a^3*sec(d*x + c)^2 + 3*a^3*sec(d*x + c) + a^3), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sec(dx + c)^{\frac{9}{2}}}{(a \sec(dx + c) + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(9/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(9/2)/(a*sec(d*x + c) + a)^3, x)

maple [B] time = 6.97, size = 876, normalized size = 3.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(\sec(dx+c)^{(9/2)}*(A+B*\sec(dx+c))/(a+a*\sec(dx+c))^3,x)$

[Out]
$$\begin{aligned} & -1/60*(-4*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d \\ & *x+1/2*c)^2-1)^{(1/2)}*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(147*A*\text{EllipticE}(\cos(1/2* \\ & d*x+1/2*c),2^{(1/2)})-65*A*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})-357*B*\text{Elliptic} \\ & \text{icE}(\cos(1/2*d*x+1/2*c),2^{(1/2)})+165*B*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})) \\ &)*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^6+10*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1 \\ & /2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*(\sin(1/2*d*x+1/2*c) \\ & ^2)^{(1/2)}*(147*A*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)})-65*A*\text{EllipticF}(\cos(1 \\ & /2*d*x+1/2*c),2^{(1/2)})-357*B*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)})+165*B*\text{El} \\ & \text{lipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)}))*\sin(1/2*d*x+1/2*c)^4*\cos(1/2*d*x+1/2*c \\ &)-8*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2 \\ & *c)^2-1)^{(1/2)}*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(147*A*\text{EllipticE}(\cos(1/2*d*x+1/ \\ & 2*c),2^{(1/2)})-65*A*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})-357*B*\text{EllipticE}(\co \\ & s(1/2*d*x+1/2*c),2^{(1/2)})+165*B*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)}))*\sin(\\ & 1/2*d*x+1/2*c)^2*\cos(1/2*d*x+1/2*c)+2*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+ \\ & 1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*(\sin(1/2*d*x+1/2*c)^2)^{(1/ \\ & 2)}*(147*A*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)})-65*A*\text{EllipticF}(\cos(1/2*d*x+ \\ & 1/2*c),2^{(1/2)})-357*B*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)})+165*B*\text{EllipticF} \\ & (\cos(1/2*d*x+1/2*c),2^{(1/2)}))*\cos(1/2*d*x+1/2*c)-168*(-2*\sin(1/2*d*x+1/2*c) \\ & ^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(7*A-17*B)*\sin(1/2*d*x+1/2*c)^{10}+8*(-2*\sin(1 \\ & /2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(482*A-1167*B)*\sin(1/2*d*x+1/2* \\ & c)^8-10*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(461*A-1111*B) \\ & *\sin(1/2*d*x+1/2*c)^6+14*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/ \\ & 2)}*(169*A-404*B)*\sin(1/2*d*x+1/2*c)^4-(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+ \\ & 1/2*c)^2)^{(1/2)}*(439*A-1029*B)*\sin(1/2*d*x+1/2*c)^2)/(2*\cos(1/2*d*x+1/2*c) \\ & ^2-1)^{(3/2)}/a^3/\cos(1/2*d*x+1/2*c)^5/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/ \\ & 2*c)^2)^{(1/2)}/\sin(1/2*d*x+1/2*c)/d \end{aligned}$$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(\sec(dx+c)^{(9/2)}*(A+B*\sec(dx+c))/(a+a*\sec(dx+c))^3,x, \text{algorithm} = \text{"maxima"})$

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(\frac{1}{\cos(c+dx)}\right)^{9/2}}{\left(a + \frac{a}{\cos(c+dx)}\right)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(9/2))/(a + a/cos(c + d*x))^3,x)
```

```
[Out] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(9/2))/(a + a/cos(c + d*x))^3, x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(9/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

$$3.217 \quad \int \frac{\sec^{\frac{7}{2}}(c+dx)(A+B \sec(c+dx))}{(a+a \sec(c+dx))^3} dx$$

Optimal. Leaf size=261

$$\frac{(3A-13B) \sin(c+dx) \sec^{\frac{3}{2}}(c+dx)}{6d(a^3 \sec(c+dx) + a^3)} - \frac{(9A-49B) \sin(c+dx) \sqrt{\sec(c+dx)}}{10a^3d} + \frac{(3A-13B) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{6a^3d}$$

[Out] 1/5*(A-B)*sec(d*x+c)^(7/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^3+1/15*(3*A-8*B)*sec(d*x+c)^(5/2)*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^2+1/6*(3*A-13*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a^3+a^3*sec(d*x+c))-1/10*(9*A-49*B)*sin(d*x+c)*sec(d*x+c)^(1/2)/a^3/d+1/10*(9*A-49*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d+1/6*(3*A-13*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d

Rubi [A] time = 0.54, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4019, 3787, 3771, 2641, 3768, 2639}

$$\frac{(3A-13B) \sin(c+dx) \sec^{\frac{3}{2}}(c+dx)}{6d(a^3 \sec(c+dx) + a^3)} - \frac{(9A-49B) \sin(c+dx) \sqrt{\sec(c+dx)}}{10a^3d} + \frac{(3A-13B) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{6a^3d}$$

Antiderivative was successfully verified.

[In] Int[(Sec[c + d*x]^(7/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^3,x]

[Out] ((9*A - 49*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(10*a^3*d) + ((3*A - 13*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(6*a^3*d) - ((9*A - 49*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(10*a^3*d) + ((A - B)*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) + ((3*A - 8*B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) + ((3*A - 13*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(6*d*(a^3 + a^3*Sec[c + d*x]))

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_.)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 3768

Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.))^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3771

Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 4019

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := Simp[(d*(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0]

Rubi steps

$$\begin{aligned}
\int \frac{\sec^{\frac{7}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^3} dx &= \frac{(A-B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \int \frac{\sec^{\frac{5}{2}}(c+dx)\left(\frac{5}{2}a(A-B)-\frac{1}{2}a(A-11B)\sec(c+dx)\right)}{(a+a\sec(c+dx))^2} dx \\
&= \frac{(A-B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(3A-8B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} \\
&= \frac{(A-B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(3A-8B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} \\
&= \frac{(A-B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(3A-8B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} \\
&= -\frac{(9A-49B)\sqrt{\sec(c+dx)}\sin(c+dx)}{10a^3d} + \frac{(A-B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} \\
&= \frac{(3A-13B)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{6a^3d} - \frac{(9A-49B)\sqrt{\sec(c+dx)}\sin(c+dx)}{10a^3d} \\
&= \frac{(9A-49B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} + \frac{(3A-13B)\sqrt{\sec(c+dx)}\sin(c+dx)}{10a^3d}
\end{aligned}$$

Mathematica [C] time = 7.52, size = 924, normalized size = 3.54

$$\frac{3\sqrt{2}Ae^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\csc\left(\frac{c}{2}\right)\left(e^{2idx}\left(-1+e^{2ic}\right){}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)-3\sqrt{1+e^{2i(c+dx)}}\right)\sec\left(\frac{c}{2}\right)}{5d(B+A\cos(c+dx))(\sec(c+dx)a+a)^3}$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[c + d*x]^(7/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^3, x]

[Out] (-3*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]*Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(5*d*E^(I*d*x)*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^3) + (49*Sqrt[2]*B*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos

$$\begin{aligned} & [c/2 + (d*x)/2]^6 * \text{Csc}[c/2] * (-3 * \text{Sqrt}[1 + E^{((2*I)*(c + d*x))}] + E^{((2*I)*d*x)} \\ &) * (-1 + E^{((2*I)*c)}) * \text{Hypergeometric2F1}[1/2, 3/4, 7/4, -E^{((2*I)*(c + d*x))}] \\ &) * \text{Sec}[c/2] * \text{Sec}[c + d*x]^2 * (A + B * \text{Sec}[c + d*x]) / (15 * d * E^{(I*d*x)} * (B + A * \text{Cos}[\\ & c + d*x]) * (a + a * \text{Sec}[c + d*x])^3) + (2 * A * \text{Cos}[c/2 + (d*x)/2]^6 * \text{Sqrt}[\text{Cos}[c + \\ & d*x]] * \text{Csc}[c/2] * \text{EllipticF}[(c + d*x)/2, 2] * \text{Sec}[c/2] * \text{Sec}[c + d*x]^{(5/2)} * (A + B \\ & * \text{Sec}[c + d*x]) * \text{Sin}[c]) / (d * (B + A * \text{Cos}[c + d*x]) * (a + a * \text{Sec}[c + d*x])^3) - (2 \\ & 6 * B * \text{Cos}[c/2 + (d*x)/2]^6 * \text{Sqrt}[\text{Cos}[c + d*x]] * \text{Csc}[c/2] * \text{EllipticF}[(c + d*x)/2, \\ & 2] * \text{Sec}[c/2] * \text{Sec}[c + d*x]^{(5/2)} * (A + B * \text{Sec}[c + d*x]) * \text{Sin}[c]) / (3 * d * (B + A * \text{Co} \\ & s[c + d*x]) * (a + a * \text{Sec}[c + d*x])^3) + (\text{Cos}[c/2 + (d*x)/2]^6 * \text{Sec}[c + d*x]^{(5 \\ & /2)} * (A + B * \text{Sec}[c + d*x]) * ((2 * (-9 * A + 49 * B) * \text{Cos}[d*x] * \text{Csc}[c/2] * \text{Sec}[c/2]) / (5 * d \\ &) - (2 * \text{Sec}[c/2] * \text{Sec}[c/2 + (d*x)/2]^5 * (-A * \text{Sin}[(d*x)/2]) + B * \text{Sin}[(d*x)/2])) / \\ & (5 * d) - (4 * \text{Sec}[c/2] * \text{Sec}[c/2 + (d*x)/2]^3 * (-3 * A * \text{Sin}[(d*x)/2] + 8 * B * \text{Sin}[(d*x) \\ & /2])) / (15 * d) - (4 * \text{Sec}[c/2] * \text{Sec}[c/2 + (d*x)/2] * (-3 * A * \text{Sin}[(d*x)/2] + 13 * B * \text{Sin} \\ & [(d*x)/2])) / (3 * d) - (4 * (-3 * A + 13 * B) * \text{Tan}[c/2]) / (3 * d) - (4 * (-3 * A + 8 * B) * \text{Sec} \\ & [c/2 + (d*x)/2]^2 * \text{Tan}[c/2]) / (15 * d) - (2 * (-A + B) * \text{Sec}[c/2 + (d*x)/2]^4 * \text{Tan}[c \\ & /2]) / (5 * d)) / ((B + A * \text{Cos}[c + d*x]) * (a + a * \text{Sec}[c + d*x])^3) \end{aligned}$$

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{(B \sec(dx + c)^4 + A \sec(dx + c)^3) \sqrt{\sec(dx + c)}}{a^3 \sec(dx + c)^3 + 3a^3 \sec(dx + c)^2 + 3a^3 \sec(dx + c) + a^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="fricas")

[Out] integral((B*sec(d*x + c)^4 + A*sec(d*x + c)^3)*sqrt(sec(d*x + c))/(a^3*sec(d*x + c)^3 + 3*a^3*sec(d*x + c)^2 + 3*a^3*sec(d*x + c) + a^3), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sec(dx + c)^{\frac{7}{2}}}{(a \sec(dx + c) + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(7/2)/(a*sec(d*x + c) + a)^3, x)

maple [B] time = 6.02, size = 685, normalized size = 2.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(\sec(dx+c)^{(7/2)}*(A+B*\sec(dx+c))/(a+a*\sec(dx+c))^3,x)$

[Out] $\frac{1}{60}*(-2*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(15*A*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})-27*A*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)})-65*B*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})+147*B*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)}))*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^4+4*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(15*A*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})-27*A*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)})-65*B*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})+147*B*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)}))*\sin(1/2*d*x+1/2*c)^2*\cos(1/2*d*x+1/2*c)-2*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(15*A*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})-27*A*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)})-65*B*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})+147*B*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)}))*\cos(1/2*d*x+1/2*c)+12*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(9*A-49*B)*\sin(1/2*d*x+1/2*c)^8-2*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(147*A-817*B)*\sin(1/2*d*x+1/2*c)^6+6*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(43*A-248*B)*\sin(1/2*d*x+1/2*c)^4-(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(69*A-439*B)*\sin(1/2*d*x+1/2*c)^2)/a^3/\cos(1/2*d*x+1/2*c)^5/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(\sec(dx+c)^{(7/2)}*(A+B*\sec(dx+c))/(a+a*\sec(dx+c))^3,x, \text{algorithm}="maxima")$

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(\frac{1}{\cos(c+dx)}\right)^{7/2}}{\left(a + \frac{a}{\cos(c+dx)}\right)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(((A + B/\cos(c + d*x))*(1/\cos(c + d*x))^{(7/2)}))/(a + a/\cos(c + d*x))^3,x)$

```
[Out] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(7/2))/(a + a/cos(c + d*x))^3, x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

$$3.218 \quad \int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B \sec(c+dx))}{(a+a \sec(c+dx))^3} dx$$

Optimal. Leaf size=220

$$\frac{(A+9B) \sin(c+dx) \sqrt{\sec(c+dx)}}{10d(a^3 \sec(c+dx) + a^3)} + \frac{(A+3B) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} F\left(\frac{1}{2}(c+dx) \middle| 2\right)}{6a^3 d} + \frac{(A+9B) \sqrt{\cos(c+dx)}}{6a^3 d}$$

```
[Out] 1/5*(A-B)*sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^3+1/15*(A-6*B)*sec
(d*x+c)^(3/2)*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^2-1/10*(A+9*B)*sin(d*x+c)*sec
(d*x+c)^(1/2)/d/(a^3+a^3*sec(d*x+c))+1/10*(A+9*B)*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/
2)*sec(d*x+c)^(1/2)/a^3/d+1/6*(A+3*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*
d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c
)^(1/2)/a^3/d
```

Rubi [A] time = 0.49, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {4019, 3787, 3771, 2639, 2641}

$$\frac{(A+9B) \sin(c+dx) \sqrt{\sec(c+dx)}}{10d(a^3 \sec(c+dx) + a^3)} + \frac{(A+3B) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} F\left(\frac{1}{2}(c+dx) \middle| 2\right)}{6a^3 d} + \frac{(A+9B) \sqrt{\cos(c+dx)}}{6a^3 d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] ((A + 9*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(10*a^3*d) + ((A + 3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[
Sec[c + d*x]])/(6*a^3*d) + ((A - B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d*(
a + a*Sec[c + d*x])^3) + ((A - 6*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*a*
d*(a + a*Sec[c + d*x])^2) - ((A + 9*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(10
*d*(a^3 + a^3*Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c -
Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.))^(n_), x_Symbol] :=> Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] :=> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 4019

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :=> Simp[(d*(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0]

Rubi steps

$$\begin{aligned}
\int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^3} dx &= \frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \int \frac{\sec^{\frac{3}{2}}(c+dx)\left(\frac{3}{2}a(A-B)+\frac{1}{2}a(A+9B)\sec(c+dx)\right)}{(a+a\sec(c+dx))^2} dx \\
&= \frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(A-6B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} \\
&= \frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(A-6B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} \\
&= \frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(A-6B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} \\
&= \frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(A-6B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} \\
&= \frac{(A+9B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}+(A+3B)\sqrt{\cos(c+dx)}}{10a^3d}
\end{aligned}$$

Mathematica [C] time = 7.11, size = 919, normalized size = 4.18

$$\frac{\sqrt{2} A e^{-i d x} \sqrt{\frac{e^{i(c+d x)}}{1+e^{2 i(c+d x)}}} \sqrt{1+e^{2 i(c+d x)}} \csc\left(\frac{c}{2}\right)\left(e^{2 i d x}\left(-1+e^{2 i c}\right) {}_2 F_1\left(\frac{1}{2}, \frac{3}{4}, \frac{7}{4}; -e^{2 i(c+d x)}\right)-3 \sqrt{1+e^{2 i(c+d x)}}\right) \sec\left(\frac{c}{2}\right)}{15 d(B+A \cos(c+d x))(\sec(c+d x) a+a)^3}$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^3, x]

[Out] -1/15*(Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(d*E^(I*d*x)*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^3) - (3*Sqrt[2]*B*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(5*d*E^(I*d*x)*(B + A*Cos[c

+ d*x))*(a + a*Sec[c + d*x])^3) + (2*A*Cos[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c/2]*Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x])*Sin[c])/(3*d*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^3) + (2*B*Cos[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c/2]*Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x])*Sin[c])/(d*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^3) + (Cos[c/2 + (d*x)/2]^6*Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x])*((-2*(A + 9*B)*Cos[d*x]*Csc[c/2]*Sec[c/2])/(5*d) + (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^5*(-(A*Sin[(d*x)/2]) + B*Sin[(d*x)/2]))/(5*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(A*Sin[(d*x)/2] + 3*B*Sin[(d*x)/2]))/(3*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(2*A*Sin[(d*x)/2] + 3*B*Sin[(d*x)/2]))/(15*d) + (4*(A + 3*B)*Tan[c/2])/(3*d) + (4*(2*A + 3*B)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(15*d) + (2*(-A + B)*Sec[c/2 + (d*x)/2]^4*Tan[c/2])/(5*d)))/((B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^3)

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{(B \sec(dx+c)^3 + A \sec(dx+c)^2) \sqrt{\sec(dx+c)}}{a^3 \sec(dx+c)^3 + 3a^3 \sec(dx+c)^2 + 3a^3 \sec(dx+c) + a^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="fricas")

[Out] integral((B*sec(d*x + c)^3 + A*sec(d*x + c)^2)*sqrt(sec(d*x + c))/(a^3*sec(d*x + c)^3 + 3*a^3*sec(d*x + c)^2 + 3*a^3*sec(d*x + c) + a^3), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx+c) + A) \sec(dx+c)^{\frac{5}{2}}}{(a \sec(dx+c) + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(5/2)/(a*sec(d*x + c) + a)^3, x)

maple [A] time = 5.15, size = 451, normalized size = 2.05

$$\sqrt{\left(2 \left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right) \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \left(12A \left(\cos^8\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 10A \left(\cos^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right)\right) \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{-2 \left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x)`

[Out] $\frac{1}{60} \left((2 \cos(1/2 dx + 1/2 c) - 1) \sin(1/2 dx + 1/2 c)^2 \right)^{1/2} (12 A \cos(1/2 dx + 1/2 c)^8 - 10 A \cos(1/2 dx + 1/2 c)^5 (\sin(1/2 dx + 1/2 c)^2)^{1/2} (-2 \cos(1/2 dx + 1/2 c)^{2+1})^{1/2} \text{EllipticF}(\cos(1/2 dx + 1/2 c), 2^{1/2}) + 6 A \cos(1/2 dx + 1/2 c)^5 (\sin(1/2 dx + 1/2 c)^2)^{1/2} (-2 \cos(1/2 dx + 1/2 c)^{2+1})^{1/2} \text{EllipticE}(\cos(1/2 dx + 1/2 c), 2^{1/2}) + 108 B \cos(1/2 dx + 1/2 c)^8 - 30 B \cos(1/2 dx + 1/2 c)^5 (\sin(1/2 dx + 1/2 c)^2)^{1/2} (-2 \cos(1/2 dx + 1/2 c)^{2+1})^{1/2} \text{EllipticF}(\cos(1/2 dx + 1/2 c), 2^{1/2}) + 54 B \cos(1/2 dx + 1/2 c)^5 (\sin(1/2 dx + 1/2 c)^2)^{1/2} (-2 \cos(1/2 dx + 1/2 c)^{2+1})^{1/2} \text{EllipticE}(\cos(1/2 dx + 1/2 c), 2^{1/2}) - 22 A \cos(1/2 dx + 1/2 c)^6 - 138 B \cos(1/2 dx + 1/2 c)^6 + 6 A \cos(1/2 dx + 1/2 c)^4 + 24 B \cos(1/2 dx + 1/2 c)^4 + 7 A \cos(1/2 dx + 1/2 c)^2 + 3 B \cos(1/2 dx + 1/2 c)^2 - 3 A + 3 B) / a^3 / \cos(1/2 dx + 1/2 c)^5 / (-2 \sin(1/2 dx + 1/2 c)^4 + \sin(1/2 dx + 1/2 c)^2)^{1/2} / \sin(1/2 dx + 1/2 c) / (2 \cos(1/2 dx + 1/2 c)^2 - 1)^{1/2} / d$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="maxima")`

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)} \right) \left(\frac{1}{\cos(c+dx)} \right)^{5/2}}{\left(a + \frac{a}{\cos(c+dx)} \right)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + a/cos(c + d*x))^3,x)`

[Out] `int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + a/cos(c + d*x))^3, x)`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

$$3.219 \quad \int \frac{\sec^2(c+dx)(A+B \sec(c+dx))}{(a+a \sec(c+dx))^3} dx$$

Optimal. Leaf size=216

$$\frac{(A+B) \sin(c+dx) \sqrt{\sec(c+dx)}}{6d(a^3 \sec(c+dx) + a^3)} + \frac{(A+B) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} F\left(\frac{1}{2}(c+dx) \middle| 2\right)}{6a^3 d} - \frac{(A-B) \sqrt{\cos(c+dx)}}{6a^3 d}$$

[Out] 1/5*(A-B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^3-1/15*(A+4*B)*sin(d*x+c)*sec(d*x+c)^(1/2)/a/d/(a+a*sec(d*x+c))^2+1/6*(A+B)*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a^3+a^3*sec(d*x+c))-1/10*(A-B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d+1/6*(A+B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d

Rubi [A] time = 0.48, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4019, 4020, 3787, 3771, 2639, 2641}

$$\frac{(A+B) \sin(c+dx) \sqrt{\sec(c+dx)}}{6d(a^3 \sec(c+dx) + a^3)} + \frac{(A+B) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} F\left(\frac{1}{2}(c+dx) \middle| 2\right)}{6a^3 d} - \frac{(A-B) \sqrt{\cos(c+dx)}}{6a^3 d}$$

Antiderivative was successfully verified.

[In] Int[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^3,x]

[Out] -((A - B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(10*a^3*d) + ((A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(6*a^3*d) + ((A - B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) - ((A + 4*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) + ((A + B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(6*d*(a^3 + a^3*Sec[c + d*x]))

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 3771

Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 4019

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := Simp[(d*(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0]

Rule 4020

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := -Simp[((A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m + 1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0]

Rubi steps

$$\begin{aligned}
\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^3} dx &= \frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{\int \frac{\sqrt{\sec(c+dx)}\left(\frac{1}{2}a(A-B)+\frac{1}{2}a(3A+7B)\sec(c+dx)\right)}{(a+a\sec(c+dx))^2} dx}{5a^2} \\
&= \frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(A+4B)\sqrt{\sec(c+dx)}\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} \\
&= \frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(A+4B)\sqrt{\sec(c+dx)}\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} \\
&= \frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(A+4B)\sqrt{\sec(c+dx)}\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} \\
&= \frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(A+4B)\sqrt{\sec(c+dx)}\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} \\
&= -\frac{(A-B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} + \frac{(A+B)\sqrt{\cos(c+dx)}}{10a^3d}
\end{aligned}$$

Mathematica [C] time = 6.94, size = 918, normalized size = 4.25

$$\frac{\sqrt{2} A e^{-i d x} \sqrt{\frac{e^{i(c+d x)}}{1+e^{2 i(c+d x)}}} \sqrt{1+e^{2 i(c+d x)}} \operatorname{csc}\left(\frac{c}{2}\right)\left(e^{2 i d x}\left(-1+e^{2 i c}\right) {}_2 F_1\left(\frac{1}{2}, \frac{3}{4}, \frac{7}{4}; -e^{2 i(c+d x)}\right)-3 \sqrt{1+e^{2 i(c+d x)}}\right) \sec\left(\frac{c}{2}\right) \sec(c+dx)}{15 d(B+A \cos(c+dx))(\sec(c+dx) a+a)^3}$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^3, x]

[Out] (Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))]) + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]*Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(15*d*E^(I*d*x)*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^3) - (Sqrt[2]*B*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))]) + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]*Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(15*d*E^(I*d*x)*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^3)

x))(a + a*Sec[c + d*x])^3) + (2*A*Cos[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c/2]*Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x])*Sin[c])/(3*d*(B + A*Cos[c + d*x]))*(a + a*Sec[c + d*x])^3) + (2*B*Cos[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c/2]*Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x])*Sin[c])/(3*d*(B + A*Cos[c + d*x]))*(a + a*Sec[c + d*x])^3) + (Cos[c/2 + (d*x)/2]^6*Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x])*((-2*(-A + B)*Cos[d*x]*Csc[c/2]*Sec[c/2])/(5*d) - (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^5*(-(A*Sin[(d*x)/2]) + B*Sin[(d*x)/2]))/(5*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(A*Sin[(d*x)/2] + B*Sin[(d*x)/2]))/(3*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(-7*A*Sin[(d*x)/2] + 2*B*Sin[(d*x)/2]))/(15*d) + (4*(A + B)*Tan[c/2])/(3*d) + (4*(-7*A + 2*B)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(15*d) - (2*(-A + B)*Sec[c/2 + (d*x)/2]^4*Tan[c/2])/(5*d)))/((B + A*Cos[c + d*x]))*(a + a*Sec[c + d*x])^3)

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{(B \sec(dx + c)^2 + A \sec(dx + c)) \sqrt{\sec(dx + c)}}{a^3 \sec(dx + c)^3 + 3a^3 \sec(dx + c)^2 + 3a^3 \sec(dx + c) + a^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="fricas")

[Out] integral((B*sec(d*x + c)^2 + A*sec(d*x + c))*sqrt(sec(d*x + c))/(a^3*sec(d*x + c)^3 + 3*a^3*sec(d*x + c)^2 + 3*a^3*sec(d*x + c) + a^3), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sec(dx + c)^{\frac{3}{2}}}{(a \sec(dx + c) + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(3/2)/(a*sec(d*x + c) + a)^3, x)

maple [A] time = 4.83, size = 451, normalized size = 2.09

$$\sqrt{\left(2 \left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right) \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \left(12A \left(\cos^8\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 10A \left(\cos^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right)\right) \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{-2 \left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x)`

[Out]
$$-1/60*((2*\cos(1/2*d*x+1/2*c)^2-1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(12*A*\cos(1/2*d*x+1/2*c)^8+10*A*\cos(1/2*d*x+1/2*c)^5*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})+6*A*\cos(1/2*d*x+1/2*c)^5*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)})-12*B*\cos(1/2*d*x+1/2*c)^8+10*B*\cos(1/2*d*x+1/2*c)^5*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})-6*B*\cos(1/2*d*x+1/2*c)^5*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)})-2*A*\cos(1/2*d*x+1/2*c)^6+22*B*\cos(1/2*d*x+1/2*c)^6-24*A*\cos(1/2*d*x+1/2*c)^4-6*B*\cos(1/2*d*x+1/2*c)^4+17*A*\cos(1/2*d*x+1/2*c)^2-7*B*\cos(1/2*d*x+1/2*c)^2-3*A+3*B)/a^3/\cos(1/2*d*x+1/2*c)^5/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d$$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="maxima")`

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(\frac{1}{\cos(c+dx)}\right)^{3/2}}{\left(a + \frac{a}{\cos(c+dx)}\right)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + a/cos(c + d*x))^3,x)`

[Out] `int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + a/cos(c + d*x))^3, x)`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```


$$3.220 \quad \int \frac{\sqrt{\sec(c+dx)} (A+B \sec(c+dx))}{(a+a \sec(c+dx))^3} dx$$

Optimal. Leaf size=222

$$\frac{(3A+B) \sin(c+dx) \sqrt{\sec(c+dx)}}{6d (a^3 \sec(c+dx) + a^3)} + \frac{(3A+B) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} F\left(\frac{1}{2}(c+dx) \middle| 2\right)}{6a^3 d} - \frac{(9A+B) \sqrt{\cos(c+dx)}}{6a^3 d}$$

[Out] 1/5*(A-B)*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^3+1/15*(3*A+2*B)*sin(d*x+c)*sec(d*x+c)^(1/2)/a/d/(a+a*sec(d*x+c))^2+1/6*(3*A+B)*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a^3+a^3*sec(d*x+c))-1/10*(9*A+B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d+1/6*(3*A+B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d

Rubi [A] time = 0.49, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4019, 4020, 3787, 3771, 2639, 2641}

$$\frac{(3A+B) \sin(c+dx) \sqrt{\sec(c+dx)}}{6d (a^3 \sec(c+dx) + a^3)} + \frac{(3A+B) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} F\left(\frac{1}{2}(c+dx) \middle| 2\right)}{6a^3 d} - \frac{(9A+B) \sqrt{\cos(c+dx)}}{6a^3 d}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^3,x]

[Out] -((9*A + B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(10*a^3*d) + ((3*A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(6*a^3*d) + ((A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) + ((3*A + 2*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) + ((3*A + B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(6*d*(a^3 + a^3*Sec[c + d*x]))

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 3771

Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.))^(n_), x_Symbol] :=> Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] :=> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 4019

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :=> Simp[(d*(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0]

Rule 4020

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :=> -Simp[((A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m + 1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0]

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\sec(c+dx)}(A+B\sec(c+dx))}{(a+a\sec(c+dx))^3} dx &= \frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{\int \frac{-\frac{1}{2}a(A-B)+\frac{5}{2}a(A+B)\sec(c+dx)}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2} dx}{5a^2} \\
&= \frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(3A+2B)\sqrt{\sec(c+dx)}\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} \\
&= \frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(3A+2B)\sqrt{\sec(c+dx)}\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} \\
&= \frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(3A+2B)\sqrt{\sec(c+dx)}\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} \\
&= \frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(3A+2B)\sqrt{\sec(c+dx)}\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} \\
&= -\frac{(9A+B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} + \frac{(3A+B)\sqrt{\sec(c+dx)}}{15ad}
\end{aligned}$$

Mathematica [C] time = 7.10, size = 919, normalized size = 4.14

$$\frac{3\sqrt{2}Ae^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\csc\left(\frac{c}{2}\right)\left(e^{2idx}\left(-1+e^{2ic}\right){}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)-3\sqrt{1+e^{2i(c+dx)}}\right)\sec\left(\frac{c}{2}\right)}{5d(B+A\cos(c+dx))(\sec(c+dx)a+a)^3}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^3, x]

[Out] (3*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]*Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(5*d*E^(I*d*x)*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^3) + (Sqrt[2]*B*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(15*d*E^(I*d*x)*(B + A*Cos[c +

$d*x])*(a + a*\text{Sec}[c + d*x])^3 + (2*A*\text{Cos}[c/2 + (d*x)/2]^6*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Csc}[c/2]*\text{EllipticF}[(c + d*x)/2, 2]*\text{Sec}[c/2]*\text{Sec}[c + d*x]^{(5/2)}*(A + B*\text{Sec}[c + d*x])* \text{Sin}[c])/(d*(B + A*\text{Cos}[c + d*x])*(a + a*\text{Sec}[c + d*x])^3 + (2*B*\text{Cos}[c/2 + (d*x)/2]^6*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Csc}[c/2]*\text{EllipticF}[(c + d*x)/2, 2]*\text{Sec}[c/2]*\text{Sec}[c + d*x]^{(5/2)}*(A + B*\text{Sec}[c + d*x])* \text{Sin}[c])/(3*d*(B + A*\text{Cos}[c + d*x])*(a + a*\text{Sec}[c + d*x])^3 + (\text{Cos}[c/2 + (d*x)/2]^6*\text{Sec}[c + d*x]^{(5/2)}*(A + B*\text{Sec}[c + d*x])*((2*(9*A + B)*\text{Cos}[d*x]*\text{Csc}[c/2]*\text{Sec}[c/2])/(5*d) + (4*\text{Sec}[c/2]*\text{Sec}[c/2 + (d*x)/2]*(-9*A*\text{Sin}[(d*x)/2] + B*\text{Sin}[(d*x)/2]))/(3*d) + (2*\text{Sec}[c/2]*\text{Sec}[c/2 + (d*x)/2]^5*(-(A*\text{Sin}[(d*x)/2]) + B*\text{Sin}[(d*x)/2]))/(5*d) - (4*\text{Sec}[c/2]*\text{Sec}[c/2 + (d*x)/2]^3*(-12*A*\text{Sin}[(d*x)/2] + 7*B*\text{Sin}[(d*x)/2]))/(15*d) + (4*(-9*A + B)*\text{Tan}[c/2])/(3*d) - (4*(-12*A + 7*B)*\text{Sec}[c/2 + (d*x)/2]^2*\text{Tan}[c/2])/(15*d) + (2*(-A + B)*\text{Sec}[c/2 + (d*x)/2]^4*\text{Tan}[c/2])/(5*d)))/(B + A*\text{Cos}[c + d*x])*(a + a*\text{Sec}[c + d*x])^3$

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B \sec(dx + c) + A)\sqrt{\sec(dx + c)}}{a^3 \sec(dx + c)^3 + 3a^3 \sec(dx + c)^2 + 3a^3 \sec(dx + c) + a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")

[Out] integral((B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(a^3*sec(d*x + c)^3 + 3*a^3*sec(d*x + c)^2 + 3*a^3*sec(d*x + c) + a^3), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)\sqrt{\sec(dx + c)}}{(a \sec(dx + c) + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(a*sec(d*x + c) + a)^3, x)

maple [A] time = 6.00, size = 451, normalized size = 2.03

$$\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \left(108A\left(\cos^8\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 30A\left(\cos^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right)\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}}\sqrt{-2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x)`

[Out]
$$-1/60/a^3*((2*\cos(1/2*d*x+1/2*c)^2-1)*\sin(1/2*d*x+1/2*c)^2)^(1/2)*(108*A*\cos(1/2*d*x+1/2*c)^8+30*A*\cos(1/2*d*x+1/2*c)^5*(\sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*\cos(1/2*d*x+1/2*c)^2+1)^(1/2)*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^(1/2))+54*A*\cos(1/2*d*x+1/2*c)^5*(\sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*\cos(1/2*d*x+1/2*c)^2+1)^(1/2)*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^(1/2))+12*B*\cos(1/2*d*x+1/2*c)^8+10*B*\cos(1/2*d*x+1/2*c)^5*(\sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*\cos(1/2*d*x+1/2*c)^2+1)^(1/2)*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^(1/2))+6*B*\cos(1/2*d*x+1/2*c)^5*(\sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*\cos(1/2*d*x+1/2*c)^2+1)^(1/2)*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^(1/2))-198*A*\cos(1/2*d*x+1/2*c)^6-2*B*\cos(1/2*d*x+1/2*c)^6+114*A*\cos(1/2*d*x+1/2*c)^4-24*B*\cos(1/2*d*x+1/2*c)^4-27*A*\cos(1/2*d*x+1/2*c)^2+17*B*\cos(1/2*d*x+1/2*c)^2+3*A-3*B)/\cos(1/2*d*x+1/2*c)^5/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^(1/2)/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d$$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")`

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \sqrt{\frac{1}{\cos(c+dx)}}}{\left(a + \frac{a}{\cos(c+dx)}\right)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + a/cos(c + d*x))^3,x)`

[Out] `int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + a/cos(c + d*x))^3, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A\sqrt{\sec(c+dx)}}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1} dx + \int \frac{B\sec^3(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1} dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)**(1/2)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] (Integral(A*sqrt(sec(c + d*x))/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d*x) + 1), x) + Integral(B*sec(c + d*x)**(3/2)/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d*x) + 1), x))/a**3
```

$$3.221 \quad \int \frac{A+B \sec(c+dx)}{\sqrt{\sec(c+dx)} (a+a \sec(c+dx))^3} dx$$

Optimal. Leaf size=228

$$\frac{(13A - 3B) \sin(c + dx) \sqrt{\sec(c + dx)}}{6d (a^3 \sec(c + dx) + a^3)} - \frac{(13A - 3B) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{6a^3 d} + \frac{(49A - 9B) \sqrt{\cos(c + dx)}}{6a^3 d}$$

[Out] $-1/5*(A-B)*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/d/(a+a*\sec(d*x+c))^{-3}-1/15*(8*A-3*B)*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/a/d/(a+a*\sec(d*x+c))^{-2}-1/6*(13*A-3*B)*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/d/(a^3+a^3*\sec(d*x+c))+1/10*(49*A-9*B)*(\cos(1/2*d*x+1/2*c))^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/a^3/d-1/6*(13*A-3*B)*(\cos(1/2*d*x+1/2*c))^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/a^3/d$

Rubi [A] time = 0.50, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {4020, 3787, 3771, 2639, 2641}

$$\frac{(13A - 3B) \sin(c + dx) \sqrt{\sec(c + dx)}}{6d (a^3 \sec(c + dx) + a^3)} - \frac{(13A - 3B) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{6a^3 d} + \frac{(49A - 9B) \sqrt{\cos(c + dx)}}{6a^3 d}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sec[c + d*x])/(Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^3), x]

[Out] $((49*A - 9*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/(10*a^3*d) - ((13*A - 3*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticF}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/(6*a^3*d) - ((A - B)*\text{Sqrt}[\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(5*d*(a + a*\text{Sec}[c + d*x])^3) - ((8*A - 3*B)*\text{Sqrt}[\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(15*a*d*(a + a*\text{Sec}[c + d*x])^2) - ((13*A - 3*B)*\text{Sqrt}[\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(6*d*(a^3 + a^3*\text{Sec}[c + d*x]))$

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) +
(a_.)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (
a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
+ f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{A + B \sec(c + dx)}{\sqrt{\sec(c + dx)} (a + a \sec(c + dx))^3} dx &= -\frac{(A - B)\sqrt{\sec(c + dx)} \sin(c + dx)}{5d(a + a \sec(c + dx))^3} + \frac{\int \frac{\frac{1}{2}a(11A-B) - \frac{5}{2}a(A-B)\sec(c+dx)}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2} dx}{5a^2} \\
&= -\frac{(A - B)\sqrt{\sec(c + dx)} \sin(c + dx)}{5d(a + a \sec(c + dx))^3} - \frac{(8A - 3B)\sqrt{\sec(c + dx)} \sin(c + dx)}{15ad(a + a \sec(c + dx))^2} \\
&= -\frac{(A - B)\sqrt{\sec(c + dx)} \sin(c + dx)}{5d(a + a \sec(c + dx))^3} - \frac{(8A - 3B)\sqrt{\sec(c + dx)} \sin(c + dx)}{15ad(a + a \sec(c + dx))^2} \\
&= -\frac{(A - B)\sqrt{\sec(c + dx)} \sin(c + dx)}{5d(a + a \sec(c + dx))^3} - \frac{(8A - 3B)\sqrt{\sec(c + dx)} \sin(c + dx)}{15ad(a + a \sec(c + dx))^2} \\
&= -\frac{(A - B)\sqrt{\sec(c + dx)} \sin(c + dx)}{5d(a + a \sec(c + dx))^3} - \frac{(8A - 3B)\sqrt{\sec(c + dx)} \sin(c + dx)}{15ad(a + a \sec(c + dx))^2} \\
&= \frac{(49A - 9B)\sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{\sec(c + dx)}}{10a^3d} - \frac{(13A - 3B)\sqrt{\sec(c + dx)} \sin(c + dx)}{15ad(a + a \sec(c + dx))^2}
\end{aligned}$$

Mathematica [C] time = 7.16, size = 943, normalized size = 4.14

$$\frac{49\sqrt{2} A e^{-idx} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \sqrt{1+e^{2i(c+dx)}} \csc\left(\frac{c}{2}\right) \left(e^{2idx} (-1+e^{2ic}) {}_2F_1\left(\frac{1}{2}, \frac{3}{4}, \frac{7}{4}; -e^{2i(c+dx)}\right) - 3\sqrt{1+e^{2i(c+dx)}} \right) \sec\left(\frac{c}{2} + \frac{(c+dx)}{2}\right)}{15d(B + A \cos(c + dx))(\sec(c + dx)a + a)^3}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sec[c + d*x])/(Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^3), x]

[Out] (-49*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]*Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(15*d*E^(I*d*x)*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^3) + (3*Sqrt[2]*B*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]*Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(5*d*E^(I*d*x)*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^3)

$c + dx)) * (a + a * \sec[c + dx])^3 - (26 * A * \cos[c/2 + (dx)/2]^6 * \sqrt{\cos[c + dx]} * \csc[c/2] * \text{EllipticF}[(c + dx)/2, 2] * \sec[c/2] * \sec[c + dx]^{(5/2)} * (A + B * \sec[c + dx]) * \sin[c]) / (3 * d * (B + A * \cos[c + dx]) * (a + a * \sec[c + dx])^3) + (2 * B * \cos[c/2 + (dx)/2]^6 * \sqrt{\cos[c + dx]} * \csc[c/2] * \text{EllipticF}[(c + dx)/2, 2] * \sec[c/2] * \sec[c + dx]^{(5/2)} * (A + B * \sec[c + dx]) * \sin[c]) / (d * (B + A * \cos[c + dx]) * (a + a * \sec[c + dx])^3) + (\cos[c/2 + (dx)/2]^6 * \sec[c + dx]^{(5/2)} * (A + B * \sec[c + dx]) * ((-2 * (39 * A - 9 * B + 10 * A * \cos[2 * c]) * \cos[dx] * \csc[c/2] * \sec[c/2]) / (5 * d) - (2 * \sec[c/2] * \sec[c/2 + (dx)/2]^5 * (-A * \sin[(dx)/2]) + B * \sin[(dx)/2])) / (5 * d) - (4 * \sec[c/2] * \sec[c/2 + (dx)/2] * (-23 * A * \sin[(dx)/2] + 9 * B * \sin[(dx)/2])) / (3 * d) + (4 * \sec[c/2] * \sec[c/2 + (dx)/2]^3 * (-17 * A * \sin[(dx)/2] + 12 * B * \sin[(dx)/2])) / (15 * d) + (16 * A * \cos[c] * \sin[dx]) / d - (4 * (-23 * A + 9 * B) * \tan[c/2]) / (3 * d) + (4 * (-17 * A + 12 * B) * \sec[c/2 + (dx)/2]^2 * \tan[c/2]) / (15 * d) - (2 * (-A + B) * \sec[c/2 + (dx)/2]^4 * \tan[c/2]) / (5 * d)) / ((B + A * \cos[c + dx]) * (a + a * \sec[c + dx])^3)$

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B \sec(dx + c) + A) \sqrt{\sec(dx + c)}}{a^3 \sec(dx + c)^4 + 3 a^3 \sec(dx + c)^3 + 3 a^3 \sec(dx + c)^2 + a^3 \sec(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="fricas")

[Out] integral((B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(a^3*sec(d*x + c)^4 + 3*a^3*sec(d*x + c)^3 + 3*a^3*sec(d*x + c)^2 + a^3*sec(d*x + c)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(a \sec(dx + c) + a)^3 \sqrt{\sec(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^3*sqrt(sec(d*x + c))), x)

maple [A] time = 5.19, size = 451, normalized size = 1.98

$$\sqrt{\left(2 \left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right) \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \left(348 A \left(\cos^8\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 130 A \left(\cos^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right) \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{-2 \left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3/sec(d*x+c)^(1/2),x)`

[Out] $\frac{1}{60} \frac{1}{a^3} \left((2 \cos(\frac{1}{2} d x + \frac{1}{2} c))^2 - 1 \right) \sin(\frac{1}{2} d x + \frac{1}{2} c)^2 \sqrt{\frac{1}{2} d x + \frac{1}{2} c} \left(348 A \cos(\frac{1}{2} d x + \frac{1}{2} c)^8 + 130 A \cos(\frac{1}{2} d x + \frac{1}{2} c)^5 \left(\sin(\frac{1}{2} d x + \frac{1}{2} c)^2 \right)^{\frac{1}{2}} \left(-2 \cos(\frac{1}{2} d x + \frac{1}{2} c)^2 + 1 \right)^{\frac{1}{2}} \operatorname{EllipticF}\left(\cos(\frac{1}{2} d x + \frac{1}{2} c), 2^{\frac{1}{2}}\right) + 294 A \cos(\frac{1}{2} d x + \frac{1}{2} c)^5 \left(\sin(\frac{1}{2} d x + \frac{1}{2} c)^2 \right)^{\frac{1}{2}} \left(-2 \cos(\frac{1}{2} d x + \frac{1}{2} c)^2 + 1 \right)^{\frac{1}{2}} \operatorname{EllipticE}\left(\cos(\frac{1}{2} d x + \frac{1}{2} c), 2^{\frac{1}{2}}\right) - 108 B \cos(\frac{1}{2} d x + \frac{1}{2} c)^8 - 30 B \cos(\frac{1}{2} d x + \frac{1}{2} c)^5 \left(\sin(\frac{1}{2} d x + \frac{1}{2} c)^2 \right)^{\frac{1}{2}} \left(-2 \cos(\frac{1}{2} d x + \frac{1}{2} c)^2 + 1 \right)^{\frac{1}{2}} \operatorname{EllipticF}\left(\cos(\frac{1}{2} d x + \frac{1}{2} c), 2^{\frac{1}{2}}\right) - 54 B \cos(\frac{1}{2} d x + \frac{1}{2} c)^5 \left(\sin(\frac{1}{2} d x + \frac{1}{2} c)^2 \right)^{\frac{1}{2}} \left(-2 \cos(\frac{1}{2} d x + \frac{1}{2} c)^2 + 1 \right)^{\frac{1}{2}} \operatorname{EllipticE}\left(\cos(\frac{1}{2} d x + \frac{1}{2} c), 2^{\frac{1}{2}}\right) - 578 A \cos(\frac{1}{2} d x + \frac{1}{2} c)^6 + 198 B \cos(\frac{1}{2} d x + \frac{1}{2} c)^6 + 264 A \cos(\frac{1}{2} d x + \frac{1}{2} c)^4 - 114 B \cos(\frac{1}{2} d x + \frac{1}{2} c)^4 - 37 A \cos(\frac{1}{2} d x + \frac{1}{2} c)^2 + 27 B \cos(\frac{1}{2} d x + \frac{1}{2} c)^2 + 3 A - 3 B \right) \frac{1}{\cos(\frac{1}{2} d x + \frac{1}{2} c)^5} \frac{1}{(-2 \sin(\frac{1}{2} d x + \frac{1}{2} c)^4 + \sin(\frac{1}{2} d x + \frac{1}{2} c)^2)^{\frac{1}{2}}} \frac{1}{\sin(\frac{1}{2} d x + \frac{1}{2} c)} \frac{1}{(2 \cos(\frac{1}{2} d x + \frac{1}{2} c)^2 - 1)^{\frac{1}{2}}} \frac{1}{d}$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="maxima")`

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\left(a + \frac{a}{\cos(c+dx)}\right)^3 \sqrt{\frac{1}{\cos(c+dx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B/cos(c + d*x))/((a + a/cos(c + d*x))^3*(1/cos(c + d*x))^(1/2)),x)`

[Out] `int((A + B/cos(c + d*x))/((a + a/cos(c + d*x))^3*(1/cos(c + d*x))^(1/2)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A}{\sec^{\frac{7}{2}}(c+dx) + 3 \sec^{\frac{5}{2}}(c+dx) + 3 \sec^{\frac{3}{2}}(c+dx) + \sqrt{\sec(c+dx)}} dx + \int \frac{B \sec(c+dx)}{\sec^{\frac{7}{2}}(c+dx) + 3 \sec^{\frac{5}{2}}(c+dx) + 3 \sec^{\frac{3}{2}}(c+dx) + \sqrt{\sec(c+dx)}} dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))**3/sec(d*x+c)**(1/2),x)
```

```
[Out] (Integral(A/(sec(c + d*x)**(7/2) + 3*sec(c + d*x)**(5/2) + 3*sec(c + d*x)**(3/2) + sqrt(sec(c + d*x))), x) + Integral(B*sec(c + d*x)/(sec(c + d*x)**(7/2) + 3*sec(c + d*x)**(5/2) + 3*sec(c + d*x)**(3/2) + sqrt(sec(c + d*x))), x))/a**3
```

$$3.222 \quad \int \frac{A+B \sec(c+dx)}{\sec^2(c+dx)(a+a \sec(c+dx))^3} dx$$

Optimal. Leaf size=261

$$\frac{(33A - 13B) \sin(c + dx)}{6a^3 d \sqrt{\sec(c + dx)}} - \frac{7(17A - 7B) \sin(c + dx)}{30d \sqrt{\sec(c + dx)} (a^3 \sec(c + dx) + a^3)} + \frac{(33A - 13B) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} F\left(\frac{1}{2}\right)}{6a^3 d}$$

[Out] $1/6*(33*A-13*B)*\sin(d*x+c)/a^3/d/\sec(d*x+c)^{(1/2)}-1/5*(A-B)*\sin(d*x+c)/d/(a+a*\sec(d*x+c))^3/\sec(d*x+c)^{(1/2)}-1/3*(2*A-B)*\sin(d*x+c)/a/d/(a+a*\sec(d*x+c))^2/\sec(d*x+c)^{(1/2)}-7/30*(17*A-7*B)*\sin(d*x+c)/d/(a^3+a^3*\sec(d*x+c))/\sec(d*x+c)^{(1/2)}-7/10*(17*A-7*B)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/a^3/d+1/6*(33*A-13*B)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/a^3/d$

Rubi [A] time = 0.55, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4020, 3787, 3769, 3771, 2641, 2639}

$$\frac{(33A - 13B) \sin(c + dx)}{6a^3 d \sqrt{\sec(c + dx)}} - \frac{7(17A - 7B) \sin(c + dx)}{30d \sqrt{\sec(c + dx)} (a^3 \sec(c + dx) + a^3)} + \frac{(33A - 13B) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} F\left(\frac{1}{2}\right)}{6a^3 d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(A + B*\text{Sec}[c + d*x])]/(\text{Sec}[c + d*x]^{(3/2)}*(a + a*\text{Sec}[c + d*x])^3), x]$

[Out] $(-7*(17*A - 7*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/(10*a^3*d) + ((33*A - 13*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticF}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/(6*a^3*d) + ((33*A - 13*B)*\text{Sin}[c + d*x])/(6*a^3*d*\text{Sqrt}[\text{Sec}[c + d*x]]) - ((A - B)*\text{Sin}[c + d*x])/(5*d*\text{Sqrt}[\text{Sec}[c + d*x]])*(a + a*\text{Sec}[c + d*x])^3 - ((2*A - B)*\text{Sin}[c + d*x])/(3*a*d*\text{Sqrt}[\text{Sec}[c + d*x]])*(a + a*\text{Sec}[c + d*x])^2 - (7*(17*A - 7*B)*\text{Sin}[c + d*x])/(30*d*\text{Sqrt}[\text{Sec}[c + d*x]])*(a^3 + a^3*\text{Sec}[c + d*x])$

Rule 2639

$\text{Int}[\text{Sqrt}[\sin[(c_.) + (d_.)*(x_.)]], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticE}[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_.)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 3769

Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.))^n, x_Symbol] := Simp[(Cos[c + d*x]*(b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 3771

Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.))^n, x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 4020

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := -Simp[((A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m + 1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0]

Rubi steps

$$\begin{aligned}
\int \frac{A + B \sec(c + dx)}{\sec^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))^3} dx &= -\frac{(A - B) \sin(c + dx)}{5d\sqrt{\sec(c + dx)}(a + a \sec(c + dx))^3} + \int \frac{\frac{1}{2}a(13A-3B) - \frac{7}{2}a(A-B)\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a \sec(c+dx))^2} dx \\
&= -\frac{(A - B) \sin(c + dx)}{5d\sqrt{\sec(c + dx)}(a + a \sec(c + dx))^3} - \frac{(2A - B) \sin(c + dx)}{3ad\sqrt{\sec(c + dx)}(a + a \sec(c + dx))^2} \\
&= -\frac{(A - B) \sin(c + dx)}{5d\sqrt{\sec(c + dx)}(a + a \sec(c + dx))^3} - \frac{(2A - B) \sin(c + dx)}{3ad\sqrt{\sec(c + dx)}(a + a \sec(c + dx))^2} \\
&= -\frac{(A - B) \sin(c + dx)}{5d\sqrt{\sec(c + dx)}(a + a \sec(c + dx))^3} - \frac{(2A - B) \sin(c + dx)}{3ad\sqrt{\sec(c + dx)}(a + a \sec(c + dx))^2} \\
&= \frac{(33A - 13B) \sin(c + dx)}{6a^3d\sqrt{\sec(c + dx)}} - \frac{(A - B) \sin(c + dx)}{5d\sqrt{\sec(c + dx)}(a + a \sec(c + dx))^3} - \frac{(2A - B) \sin(c + dx)}{3ad\sqrt{\sec(c + dx)}(a + a \sec(c + dx))^2} \\
&= -\frac{7(17A - 7B)\sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{\sec(c + dx)}}{10a^3d} + \frac{(33A - 13B) \sin(c + dx)}{6a^3d} \\
&= -\frac{7(17A - 7B)\sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{\sec(c + dx)}}{10a^3d} + \frac{(33A - 13B) \sin(c + dx)}{6a^3d}
\end{aligned}$$

Mathematica [C] time = 7.28, size = 988, normalized size = 3.79

$$\frac{119\sqrt{2} A e^{-idx} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \sqrt{1+e^{2i(c+dx)}} \csc\left(\frac{c}{2}\right) \left(e^{2idx} (-1+e^{2ic}) {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{7}{4}; -e^{2i(c+dx)}\right) - 3\sqrt{1+e^{2i(c+dx)}} \right) \sec\left(\frac{c}{2}\right)}{15d(B+A \cos(c+dx))(\sec(c+dx)a+a)^3}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^3), x]

[Out] (119*sqrt(2)*A*sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*sqrt[1 + E^((2*I)*(c + d*x))]] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]*Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(15*d*(B + A*cos(c + d*x))*(sec(c + d*x)*a + a)^3)

$$E^{(I*d*x)}*(B + A*\cos[c + d*x])*(a + a*\sec[c + d*x])^3 - (49*\sqrt{2}*B*\sqrt{E^{(I*(c + d*x))}/(1 + E^{((2*I)*(c + d*x))})}*\sqrt{1 + E^{((2*I)*(c + d*x))}}*\cos[c/2 + (d*x)/2]^6*\csc[c/2]*(-3*\sqrt{1 + E^{((2*I)*(c + d*x))}} + E^{((2*I)*d*x)}*(-1 + E^{((2*I)*c)})*\text{Hypergeometric2F1}[1/2, 3/4, 7/4, -E^{((2*I)*(c + d*x))}])*\sec[c/2]*\sec[c + d*x]^2*(A + B*\sec[c + d*x]))/(15*d*E^{(I*d*x)}*(B + A*\cos[c + d*x])*(a + a*\sec[c + d*x])^3) + (22*A*\cos[c/2 + (d*x)/2]^6*\sqrt{\cos[c + d*x]}*\csc[c/2]*\text{EllipticF}[(c + d*x)/2, 2]*\sec[c/2]*\sec[c + d*x]^{(5/2)}*(A + B*\sec[c + d*x])*sin[c])/((d*(B + A*\cos[c + d*x])*(a + a*\sec[c + d*x])^3) - (26*B*\cos[c/2 + (d*x)/2]^6*\sqrt{\cos[c + d*x]}*\csc[c/2]*\text{EllipticF}[(c + d*x)/2, 2]*\sec[c/2]*\sec[c + d*x]^{(5/2)}*(A + B*\sec[c + d*x])*sin[c])/(3*d*(B + A*\cos[c + d*x])*(a + a*\sec[c + d*x])^3) + (\cos[c/2 + (d*x)/2]^6*\sec[c + d*x]^{(5/2)}*(A + B*\sec[c + d*x])*((-2*(-89*A + 39*B - 30*A*\cos[2*c] + 10*B*\cos[2*c]))*\cos[d*x]*\csc[c/2]*\sec[c/2])/(5*d) + (8*A*\cos[2*d*x]*sin[2*c])/(3*d) + (2*\sec[c/2]*\sec[c/2 + (d*x)/2]^5*(-(A*\sin[(d*x)/2]) + B*\sin[(d*x)/2]))/(5*d) - (4*\sec[c/2]*\sec[c/2 + (d*x)/2]^3*(-22*A*\sin[(d*x)/2] + 17*B*\sin[(d*x)/2]))/(15*d) + (4*\sec[c/2]*\sec[c/2 + (d*x)/2]*(-43*A*\sin[(d*x)/2] + 23*B*\sin[(d*x)/2]))/(3*d) + (16*(-3*A + B)*\cos[c]*sin[d*x])/d + (8*A*\cos[2*c]*sin[2*d*x])/(3*d) + (4*(-43*A + 23*B)*tan[c/2])/(3*d) - (4*(-22*A + 17*B)*\sec[c/2 + (d*x)/2]^2*tan[c/2])/(15*d) + (2*(-A + B)*\sec[c/2 + (d*x)/2]^4*tan[c/2])/(5*d)))/((B + A*\cos[c + d*x])*(a + a*\sec[c + d*x])^3)$$

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B \sec(dx + c) + A)\sqrt{\sec(dx + c)}}{a^3 \sec(dx + c)^5 + 3a^3 \sec(dx + c)^4 + 3a^3 \sec(dx + c)^3 + a^3 \sec(dx + c)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")

[Out] integral((B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(a^3*sec(d*x + c)^5 + 3*a^3*sec(d*x + c)^4 + 3*a^3*sec(d*x + c)^3 + a^3*sec(d*x + c)^2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(a \sec(dx + c) + a)^3 \sec(dx + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^3*sec(d*x + c)^(3/2)), x)

maple [A] time = 5.30, size = 465, normalized size = 1.78

$$\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)\left(160A\left(\cos^{10}\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 468A\left(\cos^8\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 330A\left(\cos^5\left(\frac{dx}{2}\right)\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x)`

[Out]
$$-1/60/a^3*((2*\cos(1/2*d*x+1/2*c)^2-1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(160*A*\cos(1/2*d*x+1/2*c)^{10}+468*A*\cos(1/2*d*x+1/2*c)^8+330*A*\cos(1/2*d*x+1/2*c)^5*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})+714*A*\cos(1/2*d*x+1/2*c)^5*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)})-348*B*\cos(1/2*d*x+1/2*c)^8-130*B*\cos(1/2*d*x+1/2*c)^5*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})-294*B*\cos(1/2*d*x+1/2*c)^5*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)})-1058*A*\cos(1/2*d*x+1/2*c)^6+578*B*\cos(1/2*d*x+1/2*c)^6+474*A*\cos(1/2*d*x+1/2*c)^4-264*B*\cos(1/2*d*x+1/2*c)^4-47*A*\cos(1/2*d*x+1/2*c)^2+37*B*\cos(1/2*d*x+1/2*c)^2+3*A-3*B)/\cos(1/2*d*x+1/2*c)^5/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is undefined.

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\left(a + \frac{a}{\cos(c+dx)}\right)^3 \left(\frac{1}{\cos(c+dx)}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B/cos(c + d*x))/((a + a/cos(c + d*x))^3*(1/cos(c + d*x))^(3/2)),x)`

```
[Out] int((A + B/cos(c + d*x))/((a + a/cos(c + d*x))^3*(1/cos(c + d*x))^(3/2)), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)**(3/2)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

$$3.223 \quad \int \frac{A+B \sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+a \sec(c+dx))^3} dx$$

Optimal. Leaf size=294

$$\frac{3(21A - 11B) \sin(c + dx)}{10d \sec^{\frac{3}{2}}(c + dx) (a^3 \sec(c + dx) + a^3)} + \frac{7(33A - 17B) \sin(c + dx)}{30a^3 d \sec^{\frac{3}{2}}(c + dx)} - \frac{(21A - 11B) \sin(c + dx)}{2a^3 d \sqrt{\sec(c + dx)}} - \frac{(21A - 11B) \sqrt{\cos(c + dx)}}{2a^3 d}$$

[Out] 7/30*(33*A-17*B)*sin(d*x+c)/a^3/d/sec(d*x+c)^(3/2)-1/5*(A-B)*sin(d*x+c)/d/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3-1/15*(12*A-7*B)*sin(d*x+c)/a/d/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2-3/10*(21*A-11*B)*sin(d*x+c)/d/sec(d*x+c)^(3/2)/(a^3+a^3*sec(d*x+c))-1/2*(21*A-11*B)*sin(d*x+c)/a^3/d/sec(d*x+c)^(1/2)+7/10*(33*A-17*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d-1/2*(21*A-11*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d

Rubi [A] time = 0.57, antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4020, 3787, 3769, 3771, 2639, 2641}

$$\frac{3(21A - 11B) \sin(c + dx)}{10d \sec^{\frac{3}{2}}(c + dx) (a^3 \sec(c + dx) + a^3)} + \frac{7(33A - 17B) \sin(c + dx)}{30a^3 d \sec^{\frac{3}{2}}(c + dx)} - \frac{(21A - 11B) \sin(c + dx)}{2a^3 d \sqrt{\sec(c + dx)}} - \frac{(21A - 11B) \sqrt{\cos(c + dx)}}{2a^3 d}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^3),x]

[Out] (7*(33*A - 17*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(10*a^3*d) - ((21*A - 11*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(2*a^3*d) + (7*(33*A - 17*B)*Sin[c + d*x])/(30*a^3*d*Sec[c + d*x]^(3/2)) - ((21*A - 11*B)*Sin[c + d*x])/(2*a^3*d*Sqrt[Sec[c + d*x]]) - ((A - B)*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^3) - ((12*A - 7*B)*Sin[c + d*x])/(15*a*d*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2) - (3*(21*A - 11*B)*Sin[c + d*x])/(10*d*Sec[c + d*x]^(3/2)*(a^3 + a^3*Sec[c + d*x]))

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_.)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 3769

Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.))^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 3771

Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 4020

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := -Simp[((A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m + 1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0]

Rubi steps

$$\begin{aligned}
\int \frac{A + B \sec(c + dx)}{\sec^{\frac{5}{2}}(c + dx)(a + a \sec(c + dx))^3} dx &= -\frac{(A - B) \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))^3} + \frac{\int \frac{\frac{5}{2}a(3A-B) - \frac{9}{2}a(A-B) \sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+a \sec(c+dx))^2} dx}{5a^2} \\
&= -\frac{(A - B) \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))^3} - \frac{(12A - 7B) \sin(c + dx)}{15ad \sec^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))^3} \\
&= -\frac{(A - B) \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))^3} - \frac{(12A - 7B) \sin(c + dx)}{15ad \sec^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))^3} \\
&= -\frac{(A - B) \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))^3} - \frac{(12A - 7B) \sin(c + dx)}{15ad \sec^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))^3} \\
&= \frac{7(33A - 17B) \sin(c + dx)}{30a^3d \sec^{\frac{3}{2}}(c + dx)} - \frac{(21A - 11B) \sin(c + dx)}{2a^3d \sqrt{\sec(c + dx)}} - \frac{(A - B) \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} \\
&= \frac{7(33A - 17B) \sin(c + dx)}{30a^3d \sec^{\frac{3}{2}}(c + dx)} - \frac{(21A - 11B) \sin(c + dx)}{2a^3d \sqrt{\sec(c + dx)}} - \frac{(A - B) \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} \\
&= \frac{7(33A - 17B) \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{\sec(c + dx)} - (21A - 11B) \sin(c + dx)}{10a^3d}
\end{aligned}$$

Mathematica [C] time = 7.59, size = 1032, normalized size = 3.51

$$\frac{77\sqrt{2} A e^{-idx} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \sqrt{1 + e^{2i(c+dx)}} \csc\left(\frac{c}{2}\right) \left(e^{2idx} (-1 + e^{2ic}) {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{7}{4}; -e^{2i(c+dx)}\right) - 3\sqrt{1 + e^{2i(c+dx)}} \right) \sec\left(\frac{c}{2} + \frac{(c+dx)}{2}\right)}{5d(B + A \cos(c + dx))(\sec(c + dx)a + a)^3}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^3), x]

[Out] (-77*sqrt[2]*A*sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*sqrt[1 + E^((2*I)*(c + d*x))])/(5*d*(B + A*cos(c + d*x))*(sec(c + d*x)*a + a)^3)

$d*x)) + E^{((2*I)*d*x)*(-1 + E^{((2*I)*c)}) * \text{Hypergeometric2F1}[1/2, 3/4, 7/4, -E^{((2*I)*(c + d*x))}] * \text{Sec}[c/2] * \text{Sec}[c + d*x]^2 * (A + B * \text{Sec}[c + d*x]) / (5*d * E^{(I*d*x)} * (B + A * \text{Cos}[c + d*x]) * (a + a * \text{Sec}[c + d*x])^3) + (119 * \text{Sqrt}[2] * B * \text{Sqrt}[E^{(I*(c + d*x))} / (1 + E^{((2*I)*(c + d*x))})] * \text{Sqrt}[1 + E^{((2*I)*(c + d*x))}] * \text{Cos}[c/2 + (d*x)/2]^6 * \text{Csc}[c/2] * (-3 * \text{Sqrt}[1 + E^{((2*I)*(c + d*x))}] + E^{((2*I)*d*x)} * (-1 + E^{((2*I)*c)}) * \text{Hypergeometric2F1}[1/2, 3/4, 7/4, -E^{((2*I)*(c + d*x))}] * \text{Sec}[c/2] * \text{Sec}[c + d*x]^2 * (A + B * \text{Sec}[c + d*x]) / (15*d * E^{(I*d*x)} * (B + A * \text{Cos}[c + d*x]) * (a + a * \text{Sec}[c + d*x])^3) - (42 * A * \text{Cos}[c/2 + (d*x)/2]^6 * \text{Sqrt}[\text{Cos}[c + d*x]] * \text{Csc}[c/2] * \text{EllipticF}[(c + d*x)/2, 2] * \text{Sec}[c/2] * \text{Sec}[c + d*x]^{(5/2)} * (A + B * \text{Sec}[c + d*x]) * \text{Sin}[c]) / (d * (B + A * \text{Cos}[c + d*x]) * (a + a * \text{Sec}[c + d*x])^3) + (22 * B * \text{Cos}[c/2 + (d*x)/2]^6 * \text{Sqrt}[\text{Cos}[c + d*x]] * \text{Csc}[c/2] * \text{EllipticF}[(c + d*x)/2, 2] * \text{Sec}[c/2] * \text{Sec}[c + d*x]^{(5/2)} * (A + B * \text{Sec}[c + d*x]) * \text{Sin}[c]) / (d * (B + A * \text{Cos}[c + d*x]) * (a + a * \text{Sec}[c + d*x])^3) + (\text{Cos}[c/2 + (d*x)/2]^6 * \text{Sec}[c + d*x]^{(5/2)} * (A + B * \text{Sec}[c + d*x]) * ((-329 * A + 178 * B - 133 * A * \text{Cos}[2*c] + 60 * B * \text{Cos}[2*c]) * \text{Cos}[d*x] * \text{Csc}[c/2] * \text{Sec}[c/2]) / (5*d) + (8 * (-3 * A + B) * \text{Cos}[2*d*x] * \text{Sin}[2*c]) / (3*d) + (4 * A * \text{Cos}[3*d*x] * \text{Sin}[3*c]) / (5*d) - (2 * \text{Sec}[c/2] * \text{Sec}[c/2 + (d*x)/2]^5 * (-A * \text{Sin}[(d*x)/2] + B * \text{Sin}[(d*x)/2])) / (5*d) + (4 * \text{Sec}[c/2] * \text{Sec}[c/2 + (d*x)/2]^3 * (-27 * A * \text{Sin}[(d*x)/2] + 22 * B * \text{Sin}[(d*x)/2])) / (15*d) - (4 * \text{Sec}[c/2] * \text{Sec}[c/2 + (d*x)/2] * (-69 * A * \text{Sin}[(d*x)/2] + 43 * B * \text{Sin}[(d*x)/2])) / (3*d) - (4 * (-133 * A + 60 * B) * \text{Cos}[c] * \text{Sin}[d*x]) / (5*d) + (8 * (-3 * A + B) * \text{Cos}[2*c] * \text{Sin}[2*d*x]) / (3*d) + (4 * A * \text{Cos}[3*c] * \text{Sin}[3*d*x]) / (5*d) - (4 * (-69 * A + 43 * B) * \text{Tan}[c/2]) / (3*d) + (4 * (-27 * A + 22 * B) * \text{Sec}[c/2 + (d*x)/2]^2 * \text{Tan}[c/2]) / (15*d) - (2 * (-A + B) * \text{Sec}[c/2 + (d*x)/2]^4 * \text{Tan}[c/2]) / (5*d)) / ((B + A * \text{Cos}[c + d*x]) * (a + a * \text{Sec}[c + d*x])^3)$

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{(B \sec(dx + c) + A) \sqrt{\sec(dx + c)}}{a^3 \sec(dx + c)^6 + 3a^3 \sec(dx + c)^5 + 3a^3 \sec(dx + c)^4 + a^3 \sec(dx + c)^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")

[Out] integral((B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(a^3*sec(d*x + c)^6 + 3*a^3*sec(d*x + c)^5 + 3*a^3*sec(d*x + c)^4 + a^3*sec(d*x + c)^3), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(a \sec(dx + c) + a)^3 \sec(dx + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^3*sec(d*x + c)^(5/2)), x)

maple [A] time = 5.90, size = 493, normalized size = 1.68

$$\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)\left(192A\left(\cos^{12}\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 864A\left(\cos^{10}\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 160B\left(\cos^{10}\left(\frac{dx}{2} + \frac{c}{2}\right)\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x)

[Out] -1/60/a^3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(192*A*cos(1/2*d*x+1/2*c)^12-864*A*cos(1/2*d*x+1/2*c)^10+160*B*cos(1/2*d*x+1/2*c)^10-228*A*cos(1/2*d*x+1/2*c)^8-630*A*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1386*A*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+468*B*cos(1/2*d*x+1/2*c)^8+330*B*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+714*B*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+1590*A*cos(1/2*d*x+1/2*c)^6-1058*B*cos(1/2*d*x+1/2*c)^6-744*A*cos(1/2*d*x+1/2*c)^4+474*B*cos(1/2*d*x+1/2*c)^4+57*A*cos(1/2*d*x+1/2*c)^2-47*B*cos(1/2*d*x+1/2*c)^2-3*A+3*B)/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\left(a + \frac{a}{\cos(c+dx)}\right)^3 \left(\frac{1}{\cos(c+dx)}\right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/((a + a/cos(c + d*x))^3*(1/cos(c + d*x))^(5/2)),x)
```

```
[Out] int((A + B/cos(c + d*x))/((a + a/cos(c + d*x))^3*(1/cos(c + d*x))^(5/2)), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)**(5/2)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```


$$3.224 \quad \int \sec^2(c+dx) \sqrt{a + a \sec(c+dx)} (A+B \sec(c+dx)) dx$$

Optimal. Leaf size=176

$$\frac{a(6A+5B) \sin(c+dx) \sec^{\frac{5}{2}}(c+dx)}{12d\sqrt{a \sec(c+dx)+a}} + \frac{a(6A+5B) \sin(c+dx) \sec^{\frac{3}{2}}(c+dx)}{8d\sqrt{a \sec(c+dx)+a}} + \frac{\sqrt{a}(6A+5B) \sinh^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}}\right)}{8d}$$

[Out] 1/8*(6*A+5*B)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*a^(1/2)/d+1/8*a*(6*A+5*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/12*a*(6*A+5*B)*sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/3*a*B*sec(d*x+c)^(7/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)

Rubi [A] time = 0.29, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {4016, 3803, 3801, 215}

$$\frac{a(6A+5B) \sin(c+dx) \sec^{\frac{5}{2}}(c+dx)}{12d\sqrt{a \sec(c+dx)+a}} + \frac{a(6A+5B) \sin(c+dx) \sec^{\frac{3}{2}}(c+dx)}{8d\sqrt{a \sec(c+dx)+a}} + \frac{\sqrt{a}(6A+5B) \sinh^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}}\right)}{8d}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]

[Out] (Sqrt[a]*(6*A + 5*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]])/(8*d) + (a*(6*A + 5*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(8*d*Sqrt[a + a*Sec[c + d*x]]) + (a*(6*A + 5*B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(12*d*Sqrt[a + a*Sec[c + d*x]]) + (a*B*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]])

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 3801

Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 + x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]

Rule 3803

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.)
+ (a_)], x_Symbol] :> Simp[(-2*b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/
(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(2*a*d*(n - 1))/(b*(2*n -
1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.)
+ (a_)]*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
+ Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x]
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && !
LtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \sec^{\frac{5}{2}}(c + dx) \sqrt{a + a \sec(c + dx)} (A + B \sec(c + dx)) dx &= \frac{aB \sec^{\frac{7}{2}}(c + dx) \sin(c + dx)}{3d\sqrt{a + a \sec(c + dx)}} + \frac{1}{6}(6A + 5B) \int \sec^{\frac{5}{2}}(c + dx) \sqrt{a + a \sec(c + dx)} dx \\
&= \frac{a(6A + 5B) \sec^{\frac{5}{2}}(c + dx) \sin(c + dx)}{12d\sqrt{a + a \sec(c + dx)}} + \frac{aB \sec^{\frac{7}{2}}(c + dx) \sin(c + dx)}{3d\sqrt{a + a \sec(c + dx)}} \\
&= \frac{a(6A + 5B) \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{8d\sqrt{a + a \sec(c + dx)}} + \frac{a(6A + 5B) \sec^{\frac{5}{2}}(c + dx) \sin(c + dx)}{12d\sqrt{a + a \sec(c + dx)}} \\
&= \frac{a(6A + 5B) \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{8d\sqrt{a + a \sec(c + dx)}} + \frac{a(6A + 5B) \sec^{\frac{5}{2}}(c + dx) \sin(c + dx)}{12d\sqrt{a + a \sec(c + dx)}} \\
&= \frac{\sqrt{a} (6A + 5B) \sinh^{-1}\left(\frac{\sqrt{a} \tan(c + dx)}{\sqrt{a + a \sec(c + dx)}}\right)}{8d} + \frac{a(6A + 5B) \sec^{\frac{5}{2}}(c + dx) \sin(c + dx)}{8d\sqrt{a + a \sec(c + dx)}}
\end{aligned}$$

Mathematica [A] time = 1.55, size = 131, normalized size = 0.74

$$\frac{\sqrt{a(\sec(c + dx) + 1)} \left(\tan\left(\frac{1}{2}(c + dx)\right) \sec^3(c + dx)(4(6A + 5B) \cos(c + dx) + 3(6A + 5B) \cos(2(c + dx))) + 18A + 15B \right)}{48d\sqrt{\sec(c + dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),
x]
```

```
[Out] (Sqrt[a*(1 + Sec[c + d*x])]*(3*Sqrt[2]*(6*A + 5*B)*ArcTanh[Sqrt[2]*Sin[(c +
d*x)/2]]*Sec[(c + d*x)/2] + (18*A + 31*B + 4*(6*A + 5*B)*Cos[c + d*x] + 3*
(6*A + 5*B)*Cos[2*(c + d*x)])*Sec[c + d*x]^3*Tan[(c + d*x)/2]))/(48*d*Sqrt[
Sec[c + d*x]])
```

fricas [A] time = 0.56, size = 448, normalized size = 2.55

$$\frac{3 \left((6A + 5B) \cos(dx + c)^3 + (6A + 5B) \cos(dx + c)^2 \right) \sqrt{a} \log \left(\frac{a \cos(dx + c)^3 - 7a \cos(dx + c)^2 - \frac{4(\cos(dx + c)^2 - 2 \cos(dx + c)) \sqrt{a} \sqrt{a}}{\sqrt{\cos(dx + c)}}}{\cos(dx + c)^3 + \cos(dx + c)^2} \right)}{96 \left(d \cos(dx + c)^3 + d \cos(dx + c)^2 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/96*(3*((6*A + 5*B)*cos(d*x + c)^3 + (6*A + 5*B)*cos(d*x + c)^2)*sqrt(a)*
log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x
+ c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos
(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(3*(6*A + 5*B)*cos
(d*x + c)^2 + 2*(6*A + 5*B)*cos(d*x + c) + 8*B)*sqrt((a*cos(d*x + c) + a)/c
os(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^3 + d*cos(d*x
+ c)^2), 1/48*(3*((6*A + 5*B)*cos(d*x + c)^3 + (6*A + 5*B)*cos(d*x + c)^2)
*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(co
s(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) + 2*(3*
(6*A + 5*B)*cos(d*x + c)^2 + 2*(6*A + 5*B)*cos(d*x + c) + 8*B)*sqrt((a*cos(
d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c
)^3 + d*cos(d*x + c)^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A) \sqrt{a \sec(dx + c) + a} \sec(dx + c)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*sqrt(a*sec(d*x + c) + a)*sec(d*x + c)^(5/2), x)

maple [B] time = 2.43, size = 408, normalized size = 2.32

$$\left(18A \left(\cos^3(dx + c) \right) \sqrt{2} \arctan \left(\frac{\sqrt{\frac{2}{1+\cos(dx+c)}} (\cos(dx+c)+1+\sin(dx+c)) \sqrt{2}}{4} \right) \right) - 18A \left(\cos^3(dx + c) \right) \sqrt{2} \arctan \left(\frac{\sqrt{\frac{2}{1+\cos(dx+c)}} (\cos(dx+c)+1+\sin(dx+c)) \sqrt{2}}{4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2),x)

[Out] 1/96/d*(18*A*cos(d*x+c)^3*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c))))^(1/2)*(cos(d*x+c)+1+sin(d*x+c))*2^(1/2))-18*A*cos(d*x+c)^3*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c))))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))*2^(1/2))+15*B*cos(d*x+c)^3*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c))))^(1/2)*(cos(d*x+c)+1+sin(d*x+c))*2^(1/2))-15*B*cos(d*x+c)^3*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c))))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))*2^(1/2))+36*A*sin(d*x+c)*cos(d*x+c)^2*(-2/(1+cos(d*x+c)))^(1/2)+30*B*sin(d*x+c)*cos(d*x+c)^2*(-2/(1+cos(d*x+c)))^(1/2)+24*A*sin(d*x+c)*cos(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)+20*B*sin(d*x+c)*cos(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)+16*B*(-2/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*(1/cos(d*x+c))^(5/2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-2/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^2*(cos(d*x+c)^2-1)

maxima [B] time = 2.23, size = 3342, normalized size = 18.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")

[Out] -1/96*(6*(12*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(7/2*arctan2(sin(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(d*x + c), cos(d*x + c)))) - 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(d*x + c), cos(d*x + c))) - 12*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 3*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)

$$\begin{aligned}
& c)^2 + 4\cos(2dx + 2c) + 1) \log(2\cos(1/2\arctan2(\sin(dx + c), \cos(dx + c)))^2 + 2\sin(1/2\arctan2(\sin(dx + c), \cos(dx + c)))^2 + 2\sqrt{2}\cos(1/2\arctan2(\sin(dx + c), \cos(dx + c))) + 2\sqrt{2}\sin(1/2\arctan2(\sin(dx + c), \cos(dx + c))) + 2) + 3*(2*(2\cos(2dx + 2c) + 1)\cos(4dx + 4c) + \cos(4dx + 4c)^2 + 4\cos(2dx + 2c)^2 + \sin(4dx + 4c)^2 + 4\sin(4dx + 4c)\sin(2dx + 2c) + 4\sin(2dx + 2c)^2 + 4\cos(2dx + 2c) + 1) \log(2\cos(1/2\arctan2(\sin(dx + c), \cos(dx + c)))^2 + 2\sin(1/2\arctan2(\sin(dx + c), \cos(dx + c)))^2 + 2\sqrt{2}\cos(1/2\arctan2(\sin(dx + c), \cos(dx + c))) - 2\sqrt{2}\sin(1/2\arctan2(\sin(dx + c), \cos(dx + c))) + 2) - 3*(2*(2\cos(2dx + 2c) + 1)\cos(4dx + 4c) + \cos(4dx + 4c)^2 + 4\cos(2dx + 2c)^2 + \sin(4dx + 4c)^2 + 4\sin(4dx + 4c)\sin(2dx + 2c) + 4\sin(2dx + 2c)^2 + 4\cos(2dx + 2c) + 1) \log(2\cos(1/2\arctan2(\sin(dx + c), \cos(dx + c)))^2 + 2\sin(1/2\arctan2(\sin(dx + c), \cos(dx + c)))^2 - 2\sqrt{2}\cos(1/2\arctan2(\sin(dx + c), \cos(dx + c))) + 2\sqrt{2}\sin(1/2\arctan2(\sin(dx + c), \cos(dx + c))) + 2) + 3*(2*(2\cos(2dx + 2c) + 1)\cos(4dx + 4c) + \cos(4dx + 4c)^2 + 4\cos(2dx + 2c)^2 + \sin(4dx + 4c)^2 + 4\sin(4dx + 4c)\sin(2dx + 2c) + 4\sin(2dx + 2c)^2 + 4\cos(2dx + 2c) + 1) \log(2\cos(1/2\arctan2(\sin(dx + c), \cos(dx + c)))^2 + 2\sin(1/2\arctan2(\sin(dx + c), \cos(dx + c)))^2 - 2\sqrt{2}\cos(1/2\arctan2(\sin(dx + c), \cos(dx + c))) - 2\sqrt{2}\sin(1/2\arctan2(\sin(dx + c), \cos(dx + c))) + 2) - 12*(\sqrt{2}\cos(4dx + 4c) + 2\sqrt{2}\cos(2dx + 2c) + \sqrt{2})\sin(7/2\arctan2(\sin(dx + c), \cos(dx + c))) - 4*(\sqrt{2}\cos(4dx + 4c) + 2\sqrt{2}\cos(2dx + 2c) + \sqrt{2})\sin(5/2\arctan2(\sin(dx + c), \cos(dx + c))) + 4*(\sqrt{2}\cos(4dx + 4c) + 2\sqrt{2}\cos(2dx + 2c) + \sqrt{2})\sin(3/2\arctan2(\sin(dx + c), \cos(dx + c))) + 12*(\sqrt{2}\cos(4dx + 4c) + 2\sqrt{2}\cos(2dx + 2c) + \sqrt{2})\sin(1/2\arctan2(\sin(dx + c), \cos(dx + c))))*A\sqrt{a}/(2*(2\cos(2dx + 2c) + 1)\cos(4dx + 4c) + \cos(4dx + 4c)^2 + 4\cos(2dx + 2c)^2 + \sin(4dx + 4c)^2 + 4\sin(4dx + 4c)\sin(2dx + 2c) + 4\sin(2dx + 2c)^2 + 4\cos(2dx + 2c) + 1) + (60*(\sqrt{2}\sin(6dx + 6c) + 3\sqrt{2}\sin(4dx + 4c) + 3\sqrt{2}\sin(2dx + 2c))*\cos(11/2\arctan2(\sin(dx + c), \cos(dx + c))) + 20*(\sqrt{2}\sin(6dx + 6c) + 3\sqrt{2}\sin(4dx + 4c) + 3\sqrt{2}\sin(2dx + 2c))*\cos(9/2\arctan2(\sin(dx + c), \cos(dx + c))) + 168*(\sqrt{2}\sin(6dx + 6c) + 3\sqrt{2}\sin(4dx + 4c) + 3\sqrt{2}\sin(2dx + 2c))*\cos(7/2\arctan2(\sin(dx + c), \cos(dx + c))) - 168*(\sqrt{2}\sin(6dx + 6c) + 3\sqrt{2}\sin(4dx + 4c) + 3\sqrt{2}\sin(2dx + 2c))*\cos(5/2\arctan2(\sin(dx + c), \cos(dx + c))) - 20*(\sqrt{2}\sin(6dx + 6c) + 3\sqrt{2}\sin(4dx + 4c) + 3\sqrt{2}\sin(2dx + 2c))*\cos(3/2\arctan2(\sin(dx + c), \cos(dx + c))) - 60*(\sqrt{2}\sin(6dx + 6c) + 3\sqrt{2}\sin(4dx + 4c) + 3\sqrt{2}\sin(2dx + 2c))*\cos(1/2\arctan2(\sin(dx + c), \cos(dx + c))) - 15*(2*(3\cos(4dx + 4c) + 3\cos(2dx + 2c) + 1)\cos(6dx + 6c) + \cos(6dx + 6c)^2 + 6*(3\cos(2dx + 2c) + 1)\cos(4dx + 4c) + 9\cos(4dx + 4c)^2 + 9\cos(2dx + 2c)^2 + 6*(\sin(4dx + 4c) + \sin(2dx + 2c))*\sin(6dx + 6c) + \sin(6dx + 6c)^2 + 9\sin(4dx + 4c)^2 + 18\sin(4dx + 4c)\sin(2dx + 2c) + 9\sin(2dx + 2c)^2 + 6\cos(2dx +
\end{aligned}$$

$$\begin{aligned}
& 2*c) + 1)*\log(2*\cos(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c)))^2 + 2*\sin(1/2* \\
& \arctan2(\sin(d*x + c), \cos(d*x + c)))^2 + 2*\sqrt{2}*\cos(1/2*\arctan2(\sin(d*x \\
& + c), \cos(d*x + c))) + 2*\sqrt{2}*\sin(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c) \\
&)) + 2) + 15*(2*(3*\cos(4*d*x + 4*c) + 3*\cos(2*d*x + 2*c) + 1)*\cos(6*d*x + 6 \\
& *c) + \cos(6*d*x + 6*c)^2 + 6*(3*\cos(2*d*x + 2*c) + 1)*\cos(4*d*x + 4*c) + 9* \\
& \cos(4*d*x + 4*c)^2 + 9*\cos(2*d*x + 2*c)^2 + 6*(\sin(4*d*x + 4*c) + \sin(2*d*x \\
& + 2*c))*\sin(6*d*x + 6*c) + \sin(6*d*x + 6*c)^2 + 9*\sin(4*d*x + 4*c)^2 + 18* \\
& \sin(4*d*x + 4*c)*\sin(2*d*x + 2*c) + 9*\sin(2*d*x + 2*c)^2 + 6*\cos(2*d*x + 2* \\
& c) + 1)*\log(2*\cos(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c)))^2 + 2*\sin(1/2*ar \\
& ctan2(\sin(d*x + c), \cos(d*x + c)))^2 + 2*\sqrt{2}*\cos(1/2*\arctan2(\sin(d*x + \\
& c), \cos(d*x + c))) - 2*\sqrt{2}*\sin(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c))) \\
& + 2) - 15*(2*(3*\cos(4*d*x + 4*c) + 3*\cos(2*d*x + 2*c) + 1)*\cos(6*d*x + 6*c \\
&) + \cos(6*d*x + 6*c)^2 + 6*(3*\cos(2*d*x + 2*c) + 1)*\cos(4*d*x + 4*c) + 9*co \\
& s(4*d*x + 4*c)^2 + 9*\cos(2*d*x + 2*c)^2 + 6*(\sin(4*d*x + 4*c) + \sin(2*d*x + \\
& 2*c))*\sin(6*d*x + 6*c) + \sin(6*d*x + 6*c)^2 + 9*\sin(4*d*x + 4*c)^2 + 18*si \\
& n(4*d*x + 4*c)*\sin(2*d*x + 2*c) + 9*\sin(2*d*x + 2*c)^2 + 6*\cos(2*d*x + 2*c) \\
& + 1)*\log(2*\cos(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c)))^2 + 2*\sin(1/2*arct \\
& an2(\sin(d*x + c), \cos(d*x + c)))^2 - 2*\sqrt{2}*\cos(1/2*\arctan2(\sin(d*x + c) \\
& , \cos(d*x + c))) + 2*\sqrt{2}*\sin(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c))) + \\
& 2) + 15*(2*(3*\cos(4*d*x + 4*c) + 3*\cos(2*d*x + 2*c) + 1)*\cos(6*d*x + 6*c) \\
& + \cos(6*d*x + 6*c)^2 + 6*(3*\cos(2*d*x + 2*c) + 1)*\cos(4*d*x + 4*c) + 9*\cos(\\
& 4*d*x + 4*c)^2 + 9*\cos(2*d*x + 2*c)^2 + 6*(\sin(4*d*x + 4*c) + \sin(2*d*x + 2 \\
& *c))*\sin(6*d*x + 6*c) + \sin(6*d*x + 6*c)^2 + 9*\sin(4*d*x + 4*c)^2 + 18*\sin(\\
& 4*d*x + 4*c)*\sin(2*d*x + 2*c) + 9*\sin(2*d*x + 2*c)^2 + 6*\cos(2*d*x + 2*c) + \\
& 1)*\log(2*\cos(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c)))^2 + 2*\sin(1/2*arctan \\
& 2(\sin(d*x + c), \cos(d*x + c)))^2 - 2*\sqrt{2}*\cos(1/2*\arctan2(\sin(d*x + c) \\
& , \cos(d*x + c))) - 2*\sqrt{2}*\sin(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c))) + 2 \\
&) - 60*(\sqrt{2}*\cos(6*d*x + 6*c) + 3*\sqrt{2}*\cos(4*d*x + 4*c) + 3*\sqrt{2}*\cos \\
& (2*d*x + 2*c) + \sqrt{2})*\sin(11/2*\arctan2(\sin(d*x + c), \cos(d*x + c))) - \\
& 20*(\sqrt{2}*\cos(6*d*x + 6*c) + 3*\sqrt{2}*\cos(4*d*x + 4*c) + 3*\sqrt{2}*\cos(2 \\
& *d*x + 2*c) + \sqrt{2})*\sin(9/2*\arctan2(\sin(d*x + c), \cos(d*x + c))) - 168*(\\
& \sqrt{2}*\cos(6*d*x + 6*c) + 3*\sqrt{2}*\cos(4*d*x + 4*c) + 3*\sqrt{2}*\cos(2*d*x \\
& + 2*c) + \sqrt{2})*\sin(7/2*\arctan2(\sin(d*x + c), \cos(d*x + c))) + 168*(\sqrt{2} \\
&)*\cos(6*d*x + 6*c) + 3*\sqrt{2}*\cos(4*d*x + 4*c) + 3*\sqrt{2}*\cos(2*d*x + 2 \\
& *c) + \sqrt{2})*\sin(5/2*\arctan2(\sin(d*x + c), \cos(d*x + c))) + 20*(\sqrt{2})*c \\
& os(6*d*x + 6*c) + 3*\sqrt{2}*\cos(4*d*x + 4*c) + 3*\sqrt{2}*\cos(2*d*x + 2*c) + \\
& \sqrt{2})*\sin(3/2*\arctan2(\sin(d*x + c), \cos(d*x + c))) + 60*(\sqrt{2})*\cos(6* \\
& d*x + 6*c) + 3*\sqrt{2}*\cos(4*d*x + 4*c) + 3*\sqrt{2}*\cos(2*d*x + 2*c) + \sqrt{2} \\
&)*\sin(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c))))*B*\sqrt{a}/(2*(3*\cos(4*d* \\
& x + 4*c) + 3*\cos(2*d*x + 2*c) + 1)*\cos(6*d*x + 6*c) + \cos(6*d*x + 6*c)^2 + \\
& 6*(3*\cos(2*d*x + 2*c) + 1)*\cos(4*d*x + 4*c) + 9*\cos(4*d*x + 4*c)^2 + 9*\cos(\\
& 2*d*x + 2*c)^2 + 6*(\sin(4*d*x + 4*c) + \sin(2*d*x + 2*c))*\sin(6*d*x + 6*c) + \\
& \sin(6*d*x + 6*c)^2 + 9*\sin(4*d*x + 4*c)^2 + 18*\sin(4*d*x + 4*c)*\sin(2*d*x \\
& + 2*c) + 9*\sin(2*d*x + 2*c)^2 + 6*\cos(2*d*x + 2*c) + 1))/d
\end{aligned}$$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A + \frac{B}{\cos(c + dx)} \right) \sqrt{a + \frac{a}{\cos(c + dx)}} \left(\frac{1}{\cos(c + dx)} \right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(5/2), x)

[Out] int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(5/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**(5/2)*(A+B*sec(d*x+c))*(a+a*sec(d*x+c))**(1/2), x)

[Out] Timed out

$$3.225 \quad \int \sec^{\frac{3}{2}}(c+dx) \sqrt{a + a \sec(c + dx)} (A+B \sec(c+dx)) dx$$

Optimal. Leaf size=131

$$\frac{a(4A + 3B) \sin(c + dx) \sec^{\frac{3}{2}}(c + dx)}{4d\sqrt{a \sec(c + dx) + a}} + \frac{\sqrt{a} (4A + 3B) \sinh^{-1} \left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}} \right)}{4d} + \frac{aB \sin(c + dx) \sec^{\frac{5}{2}}(c + dx)}{2d\sqrt{a \sec(c + dx) + a}}$$

[Out] 1/4*(4*A+3*B)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*a^(1/2)/d+1/4*a*(4*A+3*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/2*a*B*sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)

Rubi [A] time = 0.24, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {4016, 3803, 3801, 215}

$$\frac{a(4A + 3B) \sin(c + dx) \sec^{\frac{3}{2}}(c + dx)}{4d\sqrt{a \sec(c + dx) + a}} + \frac{\sqrt{a} (4A + 3B) \sinh^{-1} \left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}} \right)}{4d} + \frac{aB \sin(c + dx) \sec^{\frac{5}{2}}(c + dx)}{2d\sqrt{a \sec(c + dx) + a}}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]

[Out] (Sqrt[a]*(4*A + 3*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]])/(4*d) + (a*(4*A + 3*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(4*d*Sqrt[a + a*Sec[c + d*x]]) + (a*B*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(2*d*Sqrt[a + a*Sec[c + d*x]])

Rule 215

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 3801

Int[Sqrt[csc[(e_) + (f_)*(x_)]*(d_)]*Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)], x_Symbol] := Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 + x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]

Rule 3803


```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.)
+ (a_.)], x_Symbol] := Simp[(-2*b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/
(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(2*a*d*(n - 1))/(b*(2*n -
1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.)
+ (a_.)]*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
+ Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && !
LtQ[n, 0]
```

Rubi steps

$$\begin{aligned} \int \sec^{\frac{3}{2}}(c + dx) \sqrt{a + a \sec(c + dx)} (A + B \sec(c + dx)) dx &= \frac{aB \sec^{\frac{5}{2}}(c + dx) \sin(c + dx)}{2d\sqrt{a + a \sec(c + dx)}} + \frac{1}{4}(4A + 3B) \int \sec^{\frac{3}{2}} \\ &= \frac{a(4A + 3B) \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{4d\sqrt{a + a \sec(c + dx)}} + \frac{aB \sec^{\frac{5}{2}}(c + dx)}{2d\sqrt{a + a \sec(c + dx)}} \\ &= \frac{a(4A + 3B) \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{4d\sqrt{a + a \sec(c + dx)}} + \frac{aB \sec^{\frac{5}{2}}(c + dx)}{2d\sqrt{a + a \sec(c + dx)}} \\ &= \frac{\sqrt{a} (4A + 3B) \sinh^{-1}\left(\frac{\sqrt{a} \tan(c + dx)}{\sqrt{a + a \sec(c + dx)}}\right)}{4d} + \frac{a(4A + 3B)}{4d\sqrt{a + a \sec(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.52, size = 106, normalized size = 0.81

$$\frac{\sec\left(\frac{1}{2}(c + dx)\right) \sqrt{a(\sec(c + dx) + 1)} \left(\sqrt{2} (4A + 3B) \tanh^{-1}\left(\sqrt{2} \sin\left(\frac{1}{2}(c + dx)\right)\right) + 2 \sin\left(\frac{1}{2}(c + dx)\right)\right) \sec(c + dx)}{8d\sqrt{\sec(c + dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),
x]
```

[Out] (Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(Sqrt[2]*(4*A + 3*B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sec[c + d*x]*(4*A + 3*B + 2*B*Sec[c + d*x])*Sin[(c + d*x)/2]))/(8*d*Sqrt[Sec[c + d*x]])

fricas [A] time = 0.56, size = 402, normalized size = 3.07

$$\frac{\left((4A + 3B) \cos(dx + c)^2 + (4A + 3B) \cos(dx + c) \right) \sqrt{a} \log \left(\frac{a \cos(dx+c)^3 - 7a \cos(dx+c)^2 - \frac{4(\cos(dx+c)^2 - 2 \cos(dx+c)) \sqrt{a} \sqrt{\frac{a \cos(dx+c)}{\cos(dx+c)}}}{\sqrt{\cos(dx+c)}}}{\cos(dx+c)^3 + \cos(dx+c)^2} \right)}{16 \left(d \cos(dx + c)^2 + d \cos(dx + c) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")

[Out] [1/16*(((4*A + 3*B)*cos(d*x + c)^2 + (4*A + 3*B)*cos(d*x + c))*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(((4*A + 3*B)*cos(d*x + c) + 2*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^2 + d*cos(d*x + c)), 1/8*(((4*A + 3*B)*cos(d*x + c)^2 + (4*A + 3*B)*cos(d*x + c))*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) + 2*(((4*A + 3*B)*cos(d*x + c) + 2*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^2 + d*cos(d*x + c))]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A) \sqrt{a \sec(dx + c) + a} \sec(dx + c)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*sqrt(a*sec(d*x + c) + a)*sec(d*x + c)^(3/2), x)

maple [B] time = 2.48, size = 344, normalized size = 2.63

$$(-1 + \cos(dx + c)) \left(4A (\cos^2(dx + c)) \arctan \left(\frac{\sqrt{\frac{2}{1 + \cos(dx + c)}} (\cos(dx + c) + 1 + \sin(dx + c)) \sqrt{2}}{4} \right) \sqrt{2} - 4A (\cos^2(dx + c)) a \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2),x)

[Out]
$$-1/8/d*(-1+\cos(d*x+c))*(4*A*\cos(d*x+c)^2*\arctan(1/4*(-2/(1+\cos(d*x+c))))^(1/2)*(\cos(d*x+c)+1+\sin(d*x+c))*2^(1/2))*2^(1/2)-4*A*\cos(d*x+c)^2*\arctan(1/4*(-2/(1+\cos(d*x+c))))^(1/2)*(\cos(d*x+c)+1-\sin(d*x+c))*2^(1/2))*2^(1/2)+3*B*\cos(d*x+c)^2*\arctan(1/4*(-2/(1+\cos(d*x+c))))^(1/2)*(\cos(d*x+c)+1+\sin(d*x+c))*2^(1/2))*2^(1/2)-3*B*\cos(d*x+c)^2*\arctan(1/4*(-2/(1+\cos(d*x+c))))^(1/2)*(\cos(d*x+c)+1-\sin(d*x+c))*2^(1/2))*2^(1/2)+8*A*\sin(d*x+c)*\cos(d*x+c)*(-2/(1+\cos(d*x+c)))^(1/2)+6*B*\sin(d*x+c)*\cos(d*x+c)*(-2/(1+\cos(d*x+c)))^(1/2)+4*B*(-2/(1+\cos(d*x+c)))^(1/2)*\sin(d*x+c))*(1/\cos(d*x+c))^(3/2)*(a*(1+\cos(d*x+c))/\cos(d*x+c))^(1/2)/(-2/(1+\cos(d*x+c)))^(1/2)/\sin(d*x+c)^2$$

maxima [B] time = 1.71, size = 1927, normalized size = 14.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")

[Out]
$$-1/16*(4*(4*\sqrt{2}*\cos(3/2*\arctan2(\sin(d*x + c), \cos(d*x + c))))*\sin(2*d*x + 2*c) - 4*\sqrt{2}*\cos(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c))))*\sin(2*d*x + 2*c) - (\cos(2*d*x + 2*c)^2 + \sin(2*d*x + 2*c)^2 + 2*\cos(2*d*x + 2*c) + 1)*\log(2*\cos(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c))))^2 + 2*\sin(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c))))^2 + 2*\sqrt{2}*\cos(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c))) + 2*\sqrt{2}*\sin(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c))) + 2) + (\cos(2*d*x + 2*c)^2 + \sin(2*d*x + 2*c)^2 + 2*\cos(2*d*x + 2*c) + 1)*\log(2*\cos(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c))))^2 + 2*\sin(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c))))^2 + 2*\sqrt{2}*\cos(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c))) - 2*\sqrt{2}*\sin(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c))) + 2) - (\cos(2*d*x + 2*c)^2 + \sin(2*d*x + 2*c)^2 + 2*\cos(2*d*x + 2*c) + 1)*\log(2*\cos(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c))))^2 + 2*\sin(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c))))^2 - 2*\sqrt{2}*\cos(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c))) + 2*\sqrt{2}*\sin(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c))) + 2) + (\cos(2*d*x + 2*c)^2 + \sin(2*d*x + 2*c)^2 + 2*\cos(2*d*x + 2*c) + 1)*\log(2*\cos(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c))))^2 + 2*\sin(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c))))^2 + 2*\sqrt{2}*\cos(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c))) + 2*\sqrt{2}*\sin(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c))) + 2)$$

$$\begin{aligned}
& n(dx + c), \cos(dx + c)) \wedge 2 + 2 \sin(1/2 \arctan 2(\sin(dx + c), \cos(dx + c))) \wedge 2 - 2 \sqrt{2} \cos(1/2 \arctan 2(\sin(dx + c), \cos(dx + c))) - 2 \sqrt{2} \sin(1/2 \arctan 2(\sin(dx + c), \cos(dx + c))) + 2) - 4 (\sqrt{2} \cos(2dx + 2c) + \sqrt{2}) \sin(3/2 \arctan 2(\sin(dx + c), \cos(dx + c))) + 4 (\sqrt{2} \cos(2dx + 2c) + \sqrt{2}) \sin(1/2 \arctan 2(\sin(dx + c), \cos(dx + c))) \cdot A \sqrt{a} / (\cos(2dx + 2c) \wedge 2 + \sin(2dx + 2c) \wedge 2 + 2 \cos(2dx + 2c) + 1) + \\
& (12 \sqrt{2} \sin(4dx + 4c) + 2 \sqrt{2} \sin(2dx + 2c)) \cos(7/2 \arctan 2(\sin(dx + c), \cos(dx + c))) + 4 (\sqrt{2} \sin(4dx + 4c) + 2 \sqrt{2} \sin(2dx + 2c)) \cos(5/2 \arctan 2(\sin(dx + c), \cos(dx + c))) - 4 (\sqrt{2} \sin(4dx + 4c) + 2 \sqrt{2} \sin(2dx + 2c)) \cos(3/2 \arctan 2(\sin(dx + c), \cos(dx + c))) - 12 (\sqrt{2} \sin(4dx + 4c) + 2 \sqrt{2} \sin(2dx + 2c)) \cos(1/2 \arctan 2(\sin(dx + c), \cos(dx + c))) - 3 (2 (2 \cos(2dx + 2c) + 1) \cos(4dx + 4c) + \cos(4dx + 4c) \wedge 2 + 4 \cos(2dx + 2c) \wedge 2 + \sin(4dx + 4c) \wedge 2 + 4 \sin(4dx + 4c) \sin(2dx + 2c) + 4 \sin(2dx + 2c) \wedge 2 + 4 \cos(2dx + 2c) + 1) \log(2 \cos(1/2 \arctan 2(\sin(dx + c), \cos(dx + c))) \wedge 2 + 2 \sin(1/2 \arctan 2(\sin(dx + c), \cos(dx + c))) \wedge 2 + 2 \sqrt{2} \cos(1/2 \arctan 2(\sin(dx + c), \cos(dx + c))) + 2 \sqrt{2} \sin(1/2 \arctan 2(\sin(dx + c), \cos(dx + c))) + 2) + 3 (2 (2 \cos(2dx + 2c) + 1) \cos(4dx + 4c) + \cos(4dx + 4c) \wedge 2 + 4 \cos(2dx + 2c) \wedge 2 + \sin(4dx + 4c) \wedge 2 + 4 \sin(4dx + 4c) \sin(2dx + 2c) + 4 \sin(2dx + 2c) \wedge 2 + 4 \cos(2dx + 2c) + 1) \log(2 \cos(1/2 \arctan 2(\sin(dx + c), \cos(dx + c))) \wedge 2 + 2 \sin(1/2 \arctan 2(\sin(dx + c), \cos(dx + c))) \wedge 2 + 2 \sqrt{2} \cos(1/2 \arctan 2(\sin(dx + c), \cos(dx + c))) - 2 \sqrt{2} \sin(1/2 \arctan 2(\sin(dx + c), \cos(dx + c))) + 2) - 3 (2 (2 \cos(2dx + 2c) + 1) \cos(4dx + 4c) + \cos(4dx + 4c) \wedge 2 + 4 \cos(2dx + 2c) \wedge 2 + \sin(4dx + 4c) \wedge 2 + 4 \sin(4dx + 4c) \sin(2dx + 2c) + 4 \sin(2dx + 2c) \wedge 2 + 4 \cos(2dx + 2c) + 1) \log(2 \cos(1/2 \arctan 2(\sin(dx + c), \cos(dx + c))) \wedge 2 + 2 \sin(1/2 \arctan 2(\sin(dx + c), \cos(dx + c))) \wedge 2 - 2 \sqrt{2} \cos(1/2 \arctan 2(\sin(dx + c), \cos(dx + c))) + 2 \sqrt{2} \sin(1/2 \arctan 2(\sin(dx + c), \cos(dx + c))) + 2) + 3 (2 (2 \cos(2dx + 2c) + 1) \cos(4dx + 4c) + \cos(4dx + 4c) \wedge 2 + 4 \cos(2dx + 2c) \wedge 2 + \sin(4dx + 4c) \wedge 2 + 4 \sin(4dx + 4c) \sin(2dx + 2c) + 4 \sin(2dx + 2c) \wedge 2 + 4 \cos(2dx + 2c) + 1) \log(2 \cos(1/2 \arctan 2(\sin(dx + c), \cos(dx + c))) \wedge 2 + 2 \sin(1/2 \arctan 2(\sin(dx + c), \cos(dx + c))) \wedge 2 - 2 \sqrt{2} \cos(1/2 \arctan 2(\sin(dx + c), \cos(dx + c))) - 2 \sqrt{2} \sin(1/2 \arctan 2(\sin(dx + c), \cos(dx + c))) + 2) - 12 (\sqrt{2} \cos(4dx + 4c) + 2 \sqrt{2} \cos(2dx + 2c) + \sqrt{2}) \sin(7/2 \arctan 2(\sin(dx + c), \cos(dx + c))) - 4 (\sqrt{2} \cos(4dx + 4c) + 2 \sqrt{2} \cos(2dx + 2c) + \sqrt{2}) \sin(5/2 \arctan 2(\sin(dx + c), \cos(dx + c))) + 4 (\sqrt{2} \cos(4dx + 4c) + 2 \sqrt{2} \cos(2dx + 2c) + \sqrt{2}) \sin(3/2 \arctan 2(\sin(dx + c), \cos(dx + c))) + 12 (\sqrt{2} \cos(4dx + 4c) + 2 \sqrt{2} \cos(2dx + 2c) + \sqrt{2}) \sin(1/2 \arctan 2(\sin(dx + c), \cos(dx + c))) \cdot B \sqrt{a} / (2 (2 \cos(2dx + 2c) + 1) \cos(4dx + 4c) + \cos(4dx + 4c) \wedge 2 + 4 \cos(2dx + 2c) \wedge 2 + \sin(4dx + 4c) \wedge 2 + 4 \sin(4dx + 4c) \sin(2dx + 2c) + 4 \sin(2dx + 2c) \wedge 2 + 4 \cos(2dx + 2c) + 1) / d
\end{aligned}$$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A + \frac{B}{\cos(c + dx)} \right) \sqrt{a + \frac{a}{\cos(c + dx)}} \left(\frac{1}{\cos(c + dx)} \right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(3/2), x)

[Out] int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(3/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**(3/2)*(A+B*sec(d*x+c))*(a+a*sec(d*x+c))**(1/2), x)

[Out] Timed out

$$3.226 \quad \int \sqrt{\sec(c + dx)} \sqrt{a + a \sec(c + dx)} (A + B \sec(c + dx)) dx$$

Optimal. Leaf size=78

$$\frac{\sqrt{a}(2A + B) \sinh^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}}\right)}{d} + \frac{aB \sin(c + dx) \sec^{\frac{3}{2}}(c + dx)}{d\sqrt{a \sec(c + dx) + a}}$$

[Out] (2*A+B)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*a^(1/2)/d+a*B*sec(c(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)

Rubi [A] time = 0.16, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {4016, 3801, 215}

$$\frac{\sqrt{a}(2A + B) \sinh^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}}\right)}{d} + \frac{aB \sin(c + dx) \sec^{\frac{3}{2}}(c + dx)}{d\sqrt{a \sec(c + dx) + a}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]

[Out] (Sqrt[a]*(2*A + B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]])/d + (a*B*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])

Rule 215

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 3801

Int[Sqrt[csc[(e_) + (f_)*(x_)]*(d_)]*Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)], x_Symbol] := Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 + x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]

Rule 4016

Int[(csc[(e_) + (f_)*(x_)]*(d_))^(n_)*Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)]*(csc[(e_) + (f_)*(x_)]*(B_) + (A_)), x_Symbol] := Simp[(-2*b*B*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x]

]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && ! LtQ[n, 0]

Rubi steps

$$\int \sqrt{\sec(c + dx)} \sqrt{a + a \sec(c + dx)} (A + B \sec(c + dx)) dx = \frac{aB \sec^3(c + dx) \sin(c + dx)}{d\sqrt{a + a \sec(c + dx)}} + \frac{1}{2}(2A + B) \int \sqrt{\sec(c + dx)} dx$$

$$= \frac{aB \sec^3(c + dx) \sin(c + dx)}{d\sqrt{a + a \sec(c + dx)}} - \frac{(2A + B) \operatorname{Subst} \left(\int \sqrt{\sec(c + dx)} dx \right)}{d} + \frac{\sqrt{a} (2A + B) \sinh^{-1} \left(\frac{\sqrt{a} \tan(c + dx)}{\sqrt{a + a \sec(c + dx)}} \right)}{d} + \frac{aB \sec^3(c + dx) \sin(c + dx)}{d\sqrt{a + a \sec(c + dx)}}$$

Mathematica [A] time = 0.29, size = 89, normalized size = 1.14

$$\frac{\sec\left(\frac{1}{2}(c + dx)\right) \sqrt{\sec(c + dx)} \sqrt{a(\sec(c + dx) + 1)} \left(\sqrt{2} (2A + B) \cos(c + dx) \tanh^{-1} \left(\sqrt{2} \sin\left(\frac{1}{2}(c + dx)\right) \right) \right) + 2B \sqrt{a} \sec^3(c + dx) \sin(c + dx)}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]), x]

[Out] (Sec[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])]*(Sqrt[2]*(2*A + B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x] + 2*B*Sin[(c + d*x)/2]))/(2*d)

fricas [B] time = 0.75, size = 322, normalized size = 4.13

$$\left[\frac{((2A + B) \cos(dx + c) + 2A + B) \sqrt{a} \log \left(\frac{a \cos(dx + c)^3 - 7a \cos(dx + c)^2 - \frac{4(\cos(dx + c)^2 - 2 \cos(dx + c)) \sqrt{a} \sqrt{\frac{a \cos(dx + c) + a}{\cos(dx + c)}} \sin(dx + c)}{\sqrt{\cos(dx + c)}} + 8a}{\cos(dx + c)^3 + \cos(dx + c)^2} \right)}{4(d \cos(dx + c) + d)} \right] +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")

[Out] [1/4*(((2*A + B)*cos(d*x + c) + 2*A + B)*sqrt(a)*log((a*cos(d*x + c))^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*B*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d), 1/2*(((2*A + B)*cos(d*x + c) + 2*A + B)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) + 2*B*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A) \sqrt{a \sec(dx + c) + a} \sqrt{\sec(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*sqrt(a*sec(d*x + c) + a)*sqrt(sec(d*x + c)), x)

maple [B] time = 2.19, size = 277, normalized size = 3.55

$$\sqrt{\frac{1}{\cos(dx+c)}} \sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}} (-1 + \cos(dx + c)) \left(2A \arctan \left(\frac{\sqrt{\frac{-2}{1+\cos(dx+c)}} (\cos(dx+c)+1-\sin(dx+c)) \sqrt{2}}{4}} \right) \right) \sqrt{2} \cos(dx + c)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(1/2),x)

[Out] 1/2/d*(1/cos(d*x+c))^(1/2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(2*A*arctan(1/4*(-2/(1+cos(d*x+c))))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))*2^(1/2))*2^(1/2)*cos(d*x+c)-2*A*arctan(1/4*(-2/(1+cos(d*x+c))))^(1/2)*(cos(d*x+c)+1+sin(d*x+c))*2^(1/2))*2^(1/2)*cos(d*x+c)+B*arctan(1/4*(-2/(1+cos(d*x+c))))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))*2^(1/2))*2^(1/2)*cos(d*x+c)-B*arctan(1/4*(-2/(1+cos(d*x+c))))^(1/2)*(cos(d*x+c)+1+sin(d*x+c))*2^(1/2))*2^(1/2)*cos(d

$*x+c)-2*B*(-2/(1+\cos(d*x+c)))^{(1/2)*\sin(d*x+c))/\sin(d*x+c)^2/(-2/(1+\cos(d*x+c)))^{(1/2)}$

maxima [B] time = 1.59, size = 905, normalized size = 11.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(1/2),x, algorith="maxima")

[Out] $\frac{1}{4} * (2 * A * \sqrt{a} * (\log(2 * \cos(1/2 * d * x + 1/2 * c))^2 + 2 * \sin(1/2 * d * x + 1/2 * c)^2 + 2 * \sqrt{2} * \cos(1/2 * d * x + 1/2 * c) + 2 * \sqrt{2} * \sin(1/2 * d * x + 1/2 * c) + 2) - \log(2 * \cos(1/2 * d * x + 1/2 * c)^2 + 2 * \sin(1/2 * d * x + 1/2 * c)^2 + 2 * \sqrt{2} * \cos(1/2 * d * x + 1/2 * c) - 2 * \sqrt{2} * \sin(1/2 * d * x + 1/2 * c) + 2) + \log(2 * \cos(1/2 * d * x + 1/2 * c)^2 + 2 * \sin(1/2 * d * x + 1/2 * c)^2 - 2 * \sqrt{2} * \cos(1/2 * d * x + 1/2 * c) + 2 * \sqrt{2} * \sin(1/2 * d * x + 1/2 * c) + 2) - \log(2 * \cos(1/2 * d * x + 1/2 * c)^2 + 2 * \sin(1/2 * d * x + 1/2 * c)^2 - 2 * \sqrt{2} * \cos(1/2 * d * x + 1/2 * c) - 2 * \sqrt{2} * \sin(1/2 * d * x + 1/2 * c) + 2)) - (4 * \sqrt{2} * \cos(3/2 * \arctan2(\sin(d * x + c), \cos(d * x + c))) * \sin(2 * d * x + 2 * c) - 4 * \sqrt{2} * \cos(1/2 * \arctan2(\sin(d * x + c), \cos(d * x + c))) * \sin(2 * d * x + 2 * c) - (\cos(2 * d * x + 2 * c)^2 + \sin(2 * d * x + 2 * c)^2 + 2 * \cos(2 * d * x + 2 * c) + 1) * \log(2 * \cos(1/2 * \arctan2(\sin(d * x + c), \cos(d * x + c)))^2 + 2 * \sin(1/2 * \arctan2(\sin(d * x + c), \cos(d * x + c)))^2 + 2 * \sqrt{2} * \cos(1/2 * \arctan2(\sin(d * x + c), \cos(d * x + c))) + 2 * \sqrt{2} * \sin(1/2 * \arctan2(\sin(d * x + c), \cos(d * x + c))) + 2) + (\cos(2 * d * x + 2 * c)^2 + \sin(2 * d * x + 2 * c)^2 + 2 * \cos(2 * d * x + 2 * c) + 1) * \log(2 * \cos(1/2 * \arctan2(\sin(d * x + c), \cos(d * x + c)))^2 + 2 * \sin(1/2 * \arctan2(\sin(d * x + c), \cos(d * x + c)))^2 + 2 * \sqrt{2} * \cos(1/2 * \arctan2(\sin(d * x + c), \cos(d * x + c))) + 2 * \sqrt{2} * \sin(1/2 * \arctan2(\sin(d * x + c), \cos(d * x + c))) + 2) - (\cos(2 * d * x + 2 * c)^2 + \sin(2 * d * x + 2 * c)^2 + 2 * \cos(2 * d * x + 2 * c) + 1) * \log(2 * \cos(1/2 * \arctan2(\sin(d * x + c), \cos(d * x + c)))^2 + 2 * \sin(1/2 * \arctan2(\sin(d * x + c), \cos(d * x + c)))^2 - 2 * \sqrt{2} * \cos(1/2 * \arctan2(\sin(d * x + c), \cos(d * x + c))) + 2 * \sqrt{2} * \sin(1/2 * \arctan2(\sin(d * x + c), \cos(d * x + c))) + 2) + (\cos(2 * d * x + 2 * c)^2 + \sin(2 * d * x + 2 * c)^2 + 2 * \cos(2 * d * x + 2 * c) + 1) * \log(2 * \cos(1/2 * \arctan2(\sin(d * x + c), \cos(d * x + c)))^2 + 2 * \sin(1/2 * \arctan2(\sin(d * x + c), \cos(d * x + c)))^2 - 2 * \sqrt{2} * \cos(1/2 * \arctan2(\sin(d * x + c), \cos(d * x + c))) - 2 * \sqrt{2} * \sin(1/2 * \arctan2(\sin(d * x + c), \cos(d * x + c))) + 2) - 4 * (\sqrt{2} * \cos(2 * d * x + 2 * c) + \sqrt{2}) * \sin(3/2 * \arctan2(\sin(d * x + c), \cos(d * x + c))) + 4 * (\sqrt{2} * \cos(2 * d * x + 2 * c) + \sqrt{2}) * \sin(1/2 * \arctan2(\sin(d * x + c), \cos(d * x + c)))) * B * \sqrt{a} / (\cos(2 * d * x + 2 * c)^2 + \sin(2 * d * x + 2 * c)^2 + 2 * \cos(2 * d * x + 2 * c) + 1)) / d$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A + \frac{B}{\cos(c + dx)} \right) \sqrt{a + \frac{a}{\cos(c + dx)}} \sqrt{\frac{1}{\cos(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/2),
x)
```

```
[Out] int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/2),
x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a(\sec(c + dx) + 1)} (A + B \sec(c + dx)) \sqrt{\sec(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)**(1/2)*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*(A + B*sec(c + d*x))*sqrt(sec(c + d*x))
, x)
```

$$3.227 \quad \int \frac{\sqrt{a+a \sec(c+dx)} (A+B \sec(c+dx))}{\sqrt{\sec(c+dx)}} dx$$

Optimal. Leaf size=76

$$\frac{2aA \sin(c+dx) \sqrt{\sec(c+dx)}}{d \sqrt{a \sec(c+dx) + a}} + \frac{2\sqrt{a} B \sinh^{-1} \left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx) + a}} \right)}{d}$$

[Out] $2*B*\operatorname{arcsinh}(a^{(1/2)}*\tan(d*x+c)/(a+a*\sec(d*x+c))^{(1/2)})*a^{(1/2)}/d+2*a*A*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/d/(a+a*\sec(d*x+c))^{(1/2)}$

Rubi [A] time = 0.16, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {4015, 3801, 215}

$$\frac{2aA \sin(c+dx) \sqrt{\sec(c+dx)}}{d \sqrt{a \sec(c+dx) + a}} + \frac{2\sqrt{a} B \sinh^{-1} \left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx) + a}} \right)}{d}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(\operatorname{Sqrt}[a + a*\operatorname{Sec}[c + d*x]]*(A + B*\operatorname{Sec}[c + d*x]))/\operatorname{Sqrt}[\operatorname{Sec}[c + d*x]], x]$

[Out] $(2*\operatorname{Sqrt}[a]*B*\operatorname{ArcSinh}[(\operatorname{Sqrt}[a]*\operatorname{Tan}[c + d*x])/\operatorname{Sqrt}[a + a*\operatorname{Sec}[c + d*x]]])/d + (2*a*A*\operatorname{Sqrt}[\operatorname{Sec}[c + d*x]]*\operatorname{Sin}[c + d*x])/(d*\operatorname{Sqrt}[a + a*\operatorname{Sec}[c + d*x]])$

Rule 215

$\operatorname{Int}[1/\operatorname{Sqrt}[(a_) + (b_)*(x_)^2], x_Symbol] \rightarrow \operatorname{Simp}[\operatorname{ArcSinh}[(\operatorname{Rt}[b, 2]*x)/\operatorname{Sqrt}[a]]/\operatorname{Rt}[b, 2], x] /; \operatorname{FreeQ}\{a, b\}, x] \ \&\& \operatorname{GtQ}[a, 0] \ \&\& \operatorname{PosQ}[b]$

Rule 3801

$\operatorname{Int}[\operatorname{Sqrt}[\operatorname{csc}[(e_) + (f_)*(x_)]*(d_)]*\operatorname{Sqrt}[\operatorname{csc}[(e_) + (f_)*(x_)]*(b_) + (a_)], x_Symbol] \rightarrow \operatorname{Dist}[(-2*a*\operatorname{Sqrt}[(a*d)/b])/(b*f), \operatorname{Subst}[\operatorname{Int}[1/\operatorname{Sqrt}[1 + x^2/a], x], x, (b*\operatorname{Cot}[e + f*x])/\operatorname{Sqrt}[a + b*\operatorname{Csc}[e + f*x]]], x] /; \operatorname{FreeQ}\{a, b, d, e, f\}, x] \ \&\& \operatorname{EqQ}[a^2 - b^2, 0] \ \&\& \operatorname{GtQ}[(a*d)/b, 0]$

Rule 4015

$\operatorname{Int}[(\operatorname{csc}[(e_) + (f_)*(x_)]*(d_))^{(n_)}*\operatorname{Sqrt}[\operatorname{csc}[(e_) + (f_)*(x_)]*(b_) + (a_)]*(\operatorname{csc}[(e_) + (f_)*(x_)]*(B_) + (A_)), x_Symbol] \rightarrow \operatorname{Simp}[(A*b^2*\operatorname{Cot}[e + f*x]*(d*\operatorname{Csc}[e + f*x])^n)/(a*f*n*\operatorname{Sqrt}[a + b*\operatorname{Csc}[e + f*x]]), x] + \operatorname{Dist}[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), \operatorname{Int}[\operatorname{Sqrt}[a + b*\operatorname{Csc}[e + f*x]]*(d*\operatorname{Csc}[e + f*x])^{(n + 1)}, x], x] /; \operatorname{FreeQ}\{a, b, d, e, f, A, B\}, x] \ \&\& \operatorname{NeQ}[A*b - a*$

B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{a + a \sec(c + dx)} (A + B \sec(c + dx))}{\sqrt{\sec(c + dx)}} dx &= \frac{2aA\sqrt{\sec(c + dx)} \sin(c + dx)}{d\sqrt{a + a \sec(c + dx)}} + B \int \sqrt{\sec(c + dx)} \sqrt{a + a \sec(c + dx)} dx \\ &= \frac{2aA\sqrt{\sec(c + dx)} \sin(c + dx)}{d\sqrt{a + a \sec(c + dx)}} - \frac{(2B) \text{Subst} \left(\int \frac{1}{\sqrt{1 + \frac{x^2}{a}}} dx, x, -\frac{a}{\sqrt{a}} \right)}{d} \\ &= \frac{2\sqrt{a} B \sinh^{-1} \left(\frac{\sqrt{a} \tan(c + dx)}{\sqrt{a + a \sec(c + dx)}} \right)}{d} + \frac{2aA\sqrt{\sec(c + dx)} \sin(c + dx)}{d\sqrt{a + a \sec(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.47, size = 83, normalized size = 1.09

$$\frac{2a \left(A \sin(c + dx) \sqrt{-((\sec(c + dx) - 1) \sec(c + dx))} - B \tan(c + dx) \sin^{-1} \left(\sqrt{\sec(c + dx)} \right) \right)}{d \sqrt{1 - \sec(c + dx)} \sqrt{a(\sec(c + dx) + 1)}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]))/Sqrt[Sec[c + d*x]], x]

[Out] (2*a*(A*Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])]*Sin[c + d*x] - B*ArcSin[Sqrt[Sec[c + d*x]]*Tan[c + d*x]])/(d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])

fricas [B] time = 0.47, size = 307, normalized size = 4.04

$$\left[\frac{4 A \sqrt{\frac{a \cos(dx+c)+a}{\cos(dx+c)}} \sqrt{\cos(dx+c)} \sin(dx+c) + (B \cos(dx+c) + B) \sqrt{a} \log \left(\frac{a \cos(dx+c)^3 - 7 a \cos(dx+c)^2 - \frac{4(\cos(dx+c)^2 - 2 \cos(dx+c) + 1)}{\cos(dx+c)^3 + \cos(dx+c)}}{2(d \cos(dx+c) + d)} \right)}{2(d \cos(dx+c) + d)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")

[Out] [1/2*(4*A*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + (B*cos(d*x + c) + B)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(d*cos(d*x + c) + d), (2*A*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + (B*cos(d*x + c) + B)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*x + c) + d)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sqrt{a \sec(dx + c) + a}}{\sqrt{\sec(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*sqrt(a*sec(d*x + c) + a)/sqrt(sec(d*x + c)), x)

maple [B] time = 2.52, size = 178, normalized size = 2.34

$$\frac{\left(B\sqrt{2} \arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}} (\cos(dx+c)+1+\sin(dx+c))\sqrt{2}}{4} \right) \sqrt{-\frac{2}{1+\cos(dx+c)}} \sin(dx+c) - B\sqrt{2} \arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}} (\cos(dx+c)+1+\sin(dx+c))\sqrt{2}}{4} \right) \right)}{2d \sin(dx+c) \sqrt{\frac{1}{\cos(dx+c)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x)

[Out] -1/2/d*(B*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d*x+c))*2^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-B*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))*2^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+4*A*cos(d*x+c)-4*A)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)/(1/cos(d*x+c))^(1/2)

maxima [B] time = 1.20, size = 262, normalized size = 3.45

$$4\sqrt{2}A\sqrt{a}\sin\left(\frac{1}{2}dx + \frac{1}{2}c\right) + B\sqrt{a}\left(\log\left(2\cos\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + 2\sin\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2\right) + 2\sqrt{2}\cos\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 2\sqrt{2}\sin\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")

[Out] 1/2*(4*sqrt(2)*A*sqrt(a)*sin(1/2*d*x + 1/2*c) + B*sqrt(a)*(log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))))/d

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \sqrt{a + \frac{a}{\cos(c+dx)}}}{\sqrt{\frac{1}{\cos(c+dx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(1/2))/(1/cos(c + d*x))^(1/2),x)

[Out] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(1/2))/(1/cos(c + d*x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a(\sec(c+dx)+1)}(A+B\sec(c+dx))}{\sqrt{\sec(c+dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*(a+a*sec(d*x+c))**(1/2)/sec(d*x+c)**(1/2),x)

[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*(A + B*sec(c + d*x))/sqrt(sec(c + d*x)), x)

$$3.228 \quad \int \frac{\sqrt{a+a \sec(c+dx)} (A+B \sec(c+dx))}{3 \sec^2(c+dx)} dx$$

Optimal. Leaf size=82

$$\frac{2a(A+3B) \sin(c+dx) \sqrt{\sec(c+dx)}}{3d\sqrt{a \sec(c+dx)+a}} + \frac{2A \sin(c+dx) \sqrt{a \sec(c+dx)+a}}{3d\sqrt{\sec(c+dx)}}$$

[Out] $2/3*a*(A+3*B)*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/d/(a+a*\sec(d*x+c))^{(1/2)}+2/3*A*\sin(d*x+c)*(a+a*\sec(d*x+c))^{(1/2)}/d/\sec(d*x+c)^{(1/2)}$

Rubi [A] time = 0.16, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {4013, 3804}

$$\frac{2a(A+3B) \sin(c+dx) \sqrt{\sec(c+dx)}}{3d\sqrt{a \sec(c+dx)+a}} + \frac{2A \sin(c+dx) \sqrt{a \sec(c+dx)+a}}{3d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(3/2),x]

[Out] $(2*a*(A+3*B)*\text{Sqrt}[\text{Sec}[c+d*x]]*\text{Sin}[c+d*x])/(3*d*\text{Sqrt}[a+a*\text{Sec}[c+d*x]]) + (2*A*\text{Sqrt}[a+a*\text{Sec}[c+d*x]]*\text{Sin}[c+d*x])/(3*d*\text{Sqrt}[\text{Sec}[c+d*x]])$

Rule 3804

Int[Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 4013

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && !LeQ[m, -1]

Rubi steps

$$\int \frac{\sqrt{a + a \sec(c + dx)} (A + B \sec(c + dx))}{\sec^{\frac{3}{2}}(c + dx)} dx = \frac{2A\sqrt{a + a \sec(c + dx)} \sin(c + dx)}{3d\sqrt{\sec(c + dx)}} + \frac{1}{3}(A + 3B) \int \frac{\sqrt{a + a \sec(c + dx)}}{\sqrt{\sec(c + dx)}} dx$$

$$= \frac{2a(A + 3B)\sqrt{\sec(c + dx)} \sin(c + dx)}{3d\sqrt{a + a \sec(c + dx)}} + \frac{2A\sqrt{a + a \sec(c + dx)} \sin(c + dx)}{3d\sqrt{\sec(c + dx)}}$$

Mathematica [A] time = 0.24, size = 56, normalized size = 0.68

$$\frac{2 \tan\left(\frac{1}{2}(c + dx)\right) \sqrt{a(\sec(c + dx) + 1)} (A \cos(c + dx) + 2A + 3B)}{3d\sqrt{\sec(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(3/2), x]

[Out] (2*(2*A + 3*B + A*Cos[c + d*x])*Sqrt[a*(1 + Sec[c + d*x]))*Tan[(c + d*x)/2])/(3*d*Sqrt[Sec[c + d*x]])

fricas [A] time = 0.42, size = 74, normalized size = 0.90

$$\frac{2 \left(A \cos(dx + c)^2 + (2A + 3B) \cos(dx + c) \right) \sqrt{\frac{a \cos(dx + c) + a}{\cos(dx + c)}} \sin(dx + c)}{3(d \cos(dx + c) + d) \sqrt{\cos(dx + c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2), x, algorithm="fricas")

[Out] 2/3*(A*cos(d*x + c)^2 + (2*A + 3*B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/((d*cos(d*x + c) + d)*sqrt(cos(d*x + c)))

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sqrt{a \sec(dx + c) + a}}{\sec^{\frac{3}{2}}(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2), x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*sqrt(a*sec(d*x + c) + a)/sec(d*x + c)^(3/2), x)

maple [A] time = 2.60, size = 75, normalized size = 0.91

$$\frac{2(-1 + \cos(dx + c))(A \cos(dx + c) + 2A + 3B) \sqrt{\frac{a(1 + \cos(dx + c))}{\cos(dx + c)}} (\cos^2(dx + c)) \left(\frac{1}{\cos(dx + c)}\right)^{\frac{3}{2}}}{3d \sin(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2), x)

[Out] $-2/3/d*(-1+\cos(d*x+c))*(A*\cos(d*x+c)+2*A+3*B)*(a*(1+\cos(d*x+c))/\cos(d*x+c))^{\wedge}(1/2)*\cos(d*x+c)^{\wedge}2*(1/\cos(d*x+c))^{\wedge}(3/2)/\sin(d*x+c)$

maxima [A] time = 1.27, size = 134, normalized size = 1.63

$$\frac{\sqrt{2} \left(3 \cos\left(\frac{2}{3} \arctan\left(\sin\left(\frac{3}{2} dx + \frac{3}{2} c\right), \cos\left(\frac{3}{2} dx + \frac{3}{2} c\right)\right)\right) \sin\left(\frac{3}{2} dx + \frac{3}{2} c\right) - 3 \cos\left(\frac{3}{2} dx + \frac{3}{2} c\right) \sin\left(\frac{2}{3} \arctan\left(\sin\left(\frac{3}{2} dx + \frac{3}{2} c\right), \cos\left(\frac{3}{2} dx + \frac{3}{2} c\right)\right)\right) \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2), x, algorithm="maxima")

[Out] $1/6*(\sqrt{2}*(3*\cos(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) * \sin(3/2*d*x + 3/2*c) - 3*\cos(3/2*d*x + 3/2*c)*\sin(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))) + 2*\sin(3/2*d*x + 3/2*c) + 3*\sin(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))))*A*\sqrt{a} + 12*\sqrt{2}*B*\sqrt{a}*\sin(1/2*d*x + 1/2*c))/d$

mupad [B] time = 2.79, size = 81, normalized size = 0.99

$$\frac{\cos(c + dx) \sqrt{\frac{1}{\cos(c + dx)}} \sqrt{\frac{a(\cos(c + dx) + 1)}{\cos(c + dx)}} (4A \sin(c + dx) + 6B \sin(c + dx) + A \sin(2c + 2dx))}{3d (\cos(c + dx) + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(1/2))/(1/cos(c + d*x))^(3/2), x)

[Out] $(\cos(c + d*x)*(1/\cos(c + d*x))^{\wedge}(1/2)*((a*(\cos(c + d*x) + 1))/\cos(c + d*x))^{\wedge}(1/2)*(4*A*\sin(c + d*x) + 6*B*\sin(c + d*x) + A*\sin(2*c + 2*d*x)))/(3*d*(\cos(c + d*x) + 1))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a(\sec(c+dx)+1)}(A+B\sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*(a+a*sec(d*x+c))**(1/2)/sec(d*x+c)**(3/2),x)

[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*(A + B*sec(c + d*x))/sec(c + d*x)**(3/2), x)

$$3.229 \quad \int \frac{\sqrt{a+a \sec(c+dx)} (A+B \sec(c+dx))}{5 \sec^2(c+dx)} dx$$

Optimal. Leaf size=130

$$\frac{4a(4A + 5B) \sin(c + dx) \sqrt{\sec(c + dx)}}{15d \sqrt{a \sec(c + dx) + a}} + \frac{2a(4A + 5B) \sin(c + dx)}{15d \sqrt{\sec(c + dx)} \sqrt{a \sec(c + dx) + a}} + \frac{2aA \sin(c + dx)}{5d \sec^2(c + dx) \sqrt{a \sec(c + dx) + a}}$$

[Out] $2/5*a*A*\sin(d*x+c)/d/\sec(d*x+c)^{(3/2)}/(a+a*\sec(d*x+c))^{(1/2)}+2/15*a*(4*A+5*B)*\sin(d*x+c)/d/\sec(d*x+c)^{(1/2)}/(a+a*\sec(d*x+c))^{(1/2)}+4/15*a*(4*A+5*B)*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/d/(a+a*\sec(d*x+c))^{(1/2)}$

Rubi [A] time = 0.22, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {4015, 3805, 3804}

$$\frac{4a(4A + 5B) \sin(c + dx) \sqrt{\sec(c + dx)}}{15d \sqrt{a \sec(c + dx) + a}} + \frac{2a(4A + 5B) \sin(c + dx)}{15d \sqrt{\sec(c + dx)} \sqrt{a \sec(c + dx) + a}} + \frac{2aA \sin(c + dx)}{5d \sec^2(c + dx) \sqrt{a \sec(c + dx) + a}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(\text{Sqrt}[a + a*\text{Sec}[c + d*x]])*(A + B*\text{Sec}[c + d*x])]/\text{Sec}[c + d*x]^{(5/2)}, x]$

[Out] $(2*a*A*\text{Sin}[c + d*x])/((5*d*\text{Sec}[c + d*x]^{(3/2)}*\text{Sqrt}[a + a*\text{Sec}[c + d*x]]) + (2*a*(4*A + 5*B)*\text{Sin}[c + d*x])/((15*d*\text{Sqrt}[\text{Sec}[c + d*x]]*\text{Sqrt}[a + a*\text{Sec}[c + d*x]]) + (4*a*(4*A + 5*B)*\text{Sqrt}[\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/((15*d*\text{Sqrt}[a + a*\text{Sec}[c + d*x]])$

Rule 3804

$\text{Int}[\text{Sqrt}[\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]/\text{Sqrt}[\text{csc}[(e_.) + (f_.)*(x_.)]*(d_.)], x_Symbol] \rightarrow \text{Simp}[(-2*a*\text{Cot}[e + f*x])/(f*\text{Sqrt}[a + b*\text{Csc}[e + f*x]]*\text{Sqrt}[d*\text{Csc}[e + f*x]]), x] /; \text{FreeQ}\{a, b, d, e, f\}, x] \ \&\& \ \text{EqQ}[a^2 - b^2, 0]$

Rule 3805

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(d_.))^{(n_.)}*\text{Sqrt}[\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] \rightarrow \text{Simp}[(a*\text{Cot}[e + f*x]*(d*\text{Csc}[e + f*x])^n)/(f*n*\text{Sqrt}[a + b*\text{Csc}[e + f*x]]), x] + \text{Dist}[(a*(2*n + 1))/(2*b*d*n), \text{Int}[\text{Sqrt}[a + b*\text{Csc}[e + f*x]]*(d*\text{Csc}[e + f*x])^{(n + 1)}, x], x] /; \text{FreeQ}\{a, b, d, e, f\}, x] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{LtQ}[n, -2^{(-1)}] \ \&\& \ \text{IntegerQ}[2*n]$

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.)
+ (a_.)]*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> Simp[(A*b^2*Co
t[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
+ f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{a + a \sec(c + dx)} (A + B \sec(c + dx))}{\sec^{\frac{5}{2}}(c + dx)} dx &= \frac{2aA \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx) \sqrt{a + a \sec(c + dx)}} + \frac{1}{5}(4A + 5B) \int \frac{\sqrt{a + a \sec(c + dx)}}{\sec^{\frac{3}{2}}(c + dx)} dx \\ &= \frac{2aA \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx) \sqrt{a + a \sec(c + dx)}} + \frac{2a(4A + 5B) \sin(c + dx)}{15d \sqrt{\sec(c + dx)} \sqrt{a + a \sec(c + dx)}} \\ &= \frac{2aA \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx) \sqrt{a + a \sec(c + dx)}} + \frac{2a(4A + 5B) \sin(c + dx)}{15d \sqrt{\sec(c + dx)} \sqrt{a + a \sec(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.32, size = 71, normalized size = 0.55

$$\frac{a \sin(c + dx) \sqrt{\sec(c + dx)} (2(4A + 5B) \cos(c + dx) + 3A \cos(2(c + dx))) + 19A + 20B}{15d \sqrt{a(\sec(c + dx) + 1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(5/2), x]
```

```
[Out] (a*(19*A + 20*B + 2*(4*A + 5*B)*Cos[c + d*x] + 3*A*Cos[2*(c + d*x)])*Sqrt[Sec[c + d*x]]*Sin[c + d*x]/(15*d*Sqrt[a*(1 + Sec[c + d*x])])
```

fricas [A] time = 0.41, size = 92, normalized size = 0.71

$$\frac{2 \left(3 A \cos(dx + c)^3 + (4 A + 5 B) \cos(dx + c)^2 + 2 (4 A + 5 B) \cos(dx + c) \right) \sqrt{\frac{a \cos(dx + c) + a}{\cos(dx + c)}} \sin(dx + c)}{15 (d \cos(dx + c) + d) \sqrt{\cos(dx + c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2), x, algorithm="fricas")
```

[Out] $2/15*(3*A*\cos(dx + c)^3 + (4*A + 5*B)*\cos(dx + c)^2 + 2*(4*A + 5*B)*\cos(dx + c))*\sqrt{(a*\cos(dx + c) + a)/\cos(dx + c)}*\sin(dx + c)/((d*\cos(dx + c) + d)*\sqrt{\cos(dx + c)})$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)\sqrt{a \sec(dx + c) + a}}{\sec(dx + c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2),x, algorithm="giac")`

[Out] `integrate((B*sec(d*x + c) + A)*sqrt(a*sec(d*x + c) + a)/sec(d*x + c)^(5/2), x)`

maple [A] time = 2.48, size = 96, normalized size = 0.74

$$\frac{2(-1 + \cos(dx + c))\left(3A\left(\cos^2(dx + c)\right) + 4A\cos(dx + c) + 5B\cos(dx + c) + 8A + 10B\right)\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}}{15d\sin(dx + c)} \left(\cos(dx + c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2),x)`

[Out] $-2/15/d*(-1+\cos(dx+c))*(3*A*\cos(dx+c)^2+4*A*\cos(dx+c)+5*B*\cos(dx+c)+8*A+10*B)*(a*(1+\cos(dx+c))/\cos(dx+c))^(1/2)*\cos(dx+c)^3*(1/\cos(dx+c))^(5/2)/\sin(dx+c)$

maxima [B] time = 1.26, size = 317, normalized size = 2.44

$$\sqrt{2}\left(30\cos\left(\frac{4}{5}\arctan\left(\sin\left(\frac{5}{2}dx + \frac{5}{2}c\right), \cos\left(\frac{5}{2}dx + \frac{5}{2}c\right)\right)\right)\sin\left(\frac{5}{2}dx + \frac{5}{2}c\right) + 5\cos\left(\frac{2}{5}\arctan\left(\sin\left(\frac{5}{2}dx + \frac{5}{2}c\right), \cos\left(\frac{5}{2}dx + \frac{5}{2}c\right)\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2),x, algorithm="maxima")`

[Out] $1/60*(\sqrt{2}*(30*\cos(4/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) * \sin(5/2*d*x + 5/2*c) + 5*\cos(2/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) * \sin(5/2*d*x + 5/2*c) - 30*\cos(5/2*d*x + 5/2*c)*\sin(4/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) - 5*\cos(5/2*d*x + 5/2*c)*\sin(5/2*d*x + 5/2*c))$

$n(2/5 \arctan2(\sin(5/2 dx + 5/2 c), \cos(5/2 dx + 5/2 c))) + 6 \sin(5/2 dx + 5/2 c) + 5 \sin(3/5 \arctan2(\sin(5/2 dx + 5/2 c), \cos(5/2 dx + 5/2 c))) + 30 \sin(1/5 \arctan2(\sin(5/2 dx + 5/2 c), \cos(5/2 dx + 5/2 c))) * A \sqrt{a} + 10 \sqrt{2} * (3 \cos(2/3 \arctan2(\sin(3/2 dx + 3/2 c), \cos(3/2 dx + 3/2 c))) * \sin(3/2 dx + 3/2 c) - 3 \cos(3/2 dx + 3/2 c) * \sin(2/3 \arctan2(\sin(3/2 dx + 3/2 c), \cos(3/2 dx + 3/2 c)))) + 2 \sin(3/2 dx + 3/2 c) + 3 \sin(1/3 \arctan2(\sin(3/2 dx + 3/2 c), \cos(3/2 dx + 3/2 c))) * B \sqrt{a}) / d$

mupad [B] time = 3.38, size = 106, normalized size = 0.82

$$\frac{\cos(c + dx) \sqrt{\frac{1}{\cos(c+dx)}} \sqrt{\frac{a(\cos(c+dx)+1)}{\cos(c+dx)}} (35 A \sin(c + dx) + 40 B \sin(c + dx) + 8 A \sin(2c + 2dx) + 3 A \sin(3c + 3dx) + 10 B \sin(2c + 2dx))}{30 d (\cos(c + dx) + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(1/2))/(1/cos(c + d*x))^(5/2), x)

[Out] (cos(c + d*x)*(1/cos(c + d*x))^(1/2)*((a*(cos(c + d*x) + 1))/cos(c + d*x))^(1/2)*(35*A*sin(c + d*x) + 40*B*sin(c + d*x) + 8*A*sin(2*c + 2*d*x) + 3*A*sin(3*c + 3*d*x) + 10*B*sin(2*c + 2*d*x)))/(30*d*(cos(c + d*x) + 1))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a(\sec(c + dx) + 1)} (A + B \sec(c + dx))}{\sec^{\frac{5}{2}}(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*(a+a*sec(d*x+c))**(1/2)/sec(d*x+c)**(5/2), x)

[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*(A + B*sec(c + d*x))/sec(c + d*x)**(5/2), x)

$$3.230 \quad \int \frac{\sqrt{a+a \sec(c+dx)} (A+B \sec(c+dx))}{7 \sec^2(c+dx)} dx$$

Optimal. Leaf size=175

$$\frac{2a(6A+7B) \sin(c+dx)}{35d \sec^2(c+dx) \sqrt{a \sec(c+dx)+a}} + \frac{16a(6A+7B) \sin(c+dx) \sqrt{\sec(c+dx)}}{105d \sqrt{a \sec(c+dx)+a}} + \frac{8a(6A+7B) \sin(c+dx)}{105d \sqrt{\sec(c+dx)} \sqrt{a \sec(c+dx)+a}}$$

[Out] $2/7*a*A*\sin(d*x+c)/d/\sec(d*x+c)^{(5/2)}/(a+a*\sec(d*x+c))^{(1/2)}+2/35*a*(6*A+7*B)*\sin(d*x+c)/d/\sec(d*x+c)^{(3/2)}/(a+a*\sec(d*x+c))^{(1/2)}+8/105*a*(6*A+7*B)*\sin(d*x+c)/d/\sec(d*x+c)^{(1/2)}/(a+a*\sec(d*x+c))^{(1/2)}+16/105*a*(6*A+7*B)*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/d/(a+a*\sec(d*x+c))^{(1/2)}$

Rubi [A] time = 0.29, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {4015, 3805, 3804}

$$\frac{2a(6A+7B) \sin(c+dx)}{35d \sec^2(c+dx) \sqrt{a \sec(c+dx)+a}} + \frac{16a(6A+7B) \sin(c+dx) \sqrt{\sec(c+dx)}}{105d \sqrt{a \sec(c+dx)+a}} + \frac{8a(6A+7B) \sin(c+dx)}{105d \sqrt{\sec(c+dx)} \sqrt{a \sec(c+dx)+a}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(7/2), x]

[Out] $(2*a*A*\sin[c + d*x])/(7*d*\sec[c + d*x]^{(5/2)}*\sqrt{a + a*\sec[c + d*x]}) + (2*a*(6*A + 7*B)*\sin[c + d*x])/(35*d*\sec[c + d*x]^{(3/2)}*\sqrt{a + a*\sec[c + d*x]}) + (8*a*(6*A + 7*B)*\sin[c + d*x])/(105*d*\sqrt{\sec[c + d*x]}*\sqrt{a + a*\sec[c + d*x]}) + (16*a*(6*A + 7*B)*\sqrt{\sec[c + d*x]}*\sin[c + d*x])/(105*d*\sqrt{a + a*\sec[c + d*x]})$

Rule 3804

Int[Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 3805

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]

Rule 4015

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> Simp[(A*b^2*Cosot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{a + a \sec(c + dx)} (A + B \sec(c + dx))}{\sec^{\frac{7}{2}}(c + dx)} dx &= \frac{2aA \sin(c + dx)}{7d \sec^{\frac{5}{2}}(c + dx) \sqrt{a + a \sec(c + dx)}} + \frac{1}{7}(6A + 7B) \int \frac{\sqrt{a + a \sec(c + dx)}}{\sec^{\frac{5}{2}}(c + dx)} dx \\ &= \frac{2aA \sin(c + dx)}{7d \sec^{\frac{5}{2}}(c + dx) \sqrt{a + a \sec(c + dx)}} + \frac{2a(6A + 7B) \sin(c + dx)}{35d \sec^{\frac{3}{2}}(c + dx) \sqrt{a + a \sec(c + dx)}} \\ &= \frac{2aA \sin(c + dx)}{7d \sec^{\frac{5}{2}}(c + dx) \sqrt{a + a \sec(c + dx)}} + \frac{2a(6A + 7B) \sin(c + dx)}{35d \sec^{\frac{3}{2}}(c + dx) \sqrt{a + a \sec(c + dx)}} \\ &= \frac{2aA \sin(c + dx)}{7d \sec^{\frac{5}{2}}(c + dx) \sqrt{a + a \sec(c + dx)}} + \frac{2a(6A + 7B) \sin(c + dx)}{35d \sec^{\frac{3}{2}}(c + dx) \sqrt{a + a \sec(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.35, size = 91, normalized size = 0.52

$$\frac{2a \sin(c + dx) (8(6A + 7B) \sec^3(c + dx) + 4(6A + 7B) \sec^2(c + dx) + 3(6A + 7B) \sec(c + dx) + 15A)}{105d \sec^{\frac{5}{2}}(c + dx) \sqrt{a(\sec(c + dx) + 1)}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(7/2), x]

[Out] (2*a*(15*A + 3*(6*A + 7*B)*Sec[c + d*x] + 4*(6*A + 7*B)*Sec[c + d*x]^2 + 8*(6*A + 7*B)*Sec[c + d*x]^3)*Sin[c + d*x]/(105*d*Sec[c + d*x]^(5/2)*Sqrt[a*(1 + Sec[c + d*x])])

fricas [A] time = 0.41, size = 110, normalized size = 0.63

$$\frac{2(15A \cos(dx + c)^4 + 3(6A + 7B) \cos(dx + c)^3 + 4(6A + 7B) \cos(dx + c)^2 + 8(6A + 7B) \cos(dx + c)) \sqrt{\frac{a \cos(dx + c)}{c}}}{105(d \cos(dx + c) + d) \sqrt{\cos(dx + c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2),x, algorithm="fricas")

[Out] 2/105*(15*A*cos(d*x + c)^4 + 3*(6*A + 7*B)*cos(d*x + c)^3 + 4*(6*A + 7*B)*cos(d*x + c)^2 + 8*(6*A + 7*B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/((d*cos(d*x + c) + d)*sqrt(cos(d*x + c)))

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sqrt{a \sec(dx + c) + a}}{\sec(dx + c)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*sqrt(a*sec(d*x + c) + a)/sec(d*x + c)^(7/2), x)

maple [A] time = 2.95, size = 118, normalized size = 0.67

$$\frac{2(-1 + \cos(dx + c)) \left(15A \left(\cos^3(dx + c) \right) + 18A \left(\cos^2(dx + c) \right) + 21B \left(\cos^2(dx + c) \right) + 24A \cos(dx + c) + 2 \right)}{105d \sin(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2),x)

[Out] -2/105/d*(-1+cos(d*x+c))*(15*A*cos(d*x+c)^3+18*A*cos(d*x+c)^2+21*B*cos(d*x+c)^2+24*A*cos(d*x+c)+28*B*cos(d*x+c)+48*A+56*B)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(7/2)*cos(d*x+c)^4/sin(d*x+c)

maxima [B] time = 1.32, size = 498, normalized size = 2.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2),x, algorithm="maxima")

[Out] 1/840*(3*sqrt(2)*(105*cos(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) + 35*cos(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos

```
(7/2*d*x + 7/2*c))) * sin(7/2*d*x + 7/2*c) + 7*cos(2/7*arctan2(sin(7/2*d*x +
7/2*c), cos(7/2*d*x + 7/2*c))) * sin(7/2*d*x + 7/2*c) - 105*cos(7/2*d*x + 7/2
*c) * sin(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) - 35*cos(7
/2*d*x + 7/2*c) * sin(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))
) - 7*cos(7/2*d*x + 7/2*c) * sin(2/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*
x + 7/2*c))) + 10*sin(7/2*d*x + 7/2*c) + 7*sin(5/7*arctan2(sin(7/2*d*x + 7/
2*c), cos(7/2*d*x + 7/2*c))) + 35*sin(3/7*arctan2(sin(7/2*d*x + 7/2*c), cos
(7/2*d*x + 7/2*c))) + 105*sin(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x
+ 7/2*c))) * A*sqrt(a) + 14*sqrt(2)*(30*cos(4/5*arctan2(sin(5/2*d*x + 5/2*c
), cos(5/2*d*x + 5/2*c))) * sin(5/2*d*x + 5/2*c) + 5*cos(2/5*arctan2(sin(5/2*
d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) * sin(5/2*d*x + 5/2*c) - 30*cos(5/2*d*x
+ 5/2*c) * sin(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) - 5*c
os(5/2*d*x + 5/2*c) * sin(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2
*c))) + 6*sin(5/2*d*x + 5/2*c) + 5*sin(3/5*arctan2(sin(5/2*d*x + 5/2*c), co
s(5/2*d*x + 5/2*c))) + 30*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x
+ 5/2*c)))) * B*sqrt(a))/d
```

mupad [B] time = 4.19, size = 130, normalized size = 0.74

$$\frac{\cos(c + dx) \sqrt{\frac{1}{\cos(c+dx)}} \sqrt{\frac{a(\cos(c+dx)+1)}{\cos(c+dx)}} (420 A \sin(c + dx) + 490 B \sin(c + dx) + 126 A \sin(2c + 2dx) + 36 A \sin(3c + 3dx) + 15 A \sin(4c + 4dx) + 112 B \sin(2c + 2dx) + 42 B \sin(3c + 3dx))}{420 d (\cos(c + dx) + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(1/2))/(1/cos(c + d*x))^(7/2),x)
```

```
[Out] (cos(c + d*x)*(1/cos(c + d*x))^(1/2)*((a*(cos(c + d*x) + 1))/cos(c + d*x))^(1/2)*(420*A*sin(c + d*x) + 490*B*sin(c + d*x) + 126*A*sin(2*c + 2*d*x) + 36*A*sin(3*c + 3*d*x) + 15*A*sin(4*c + 4*d*x) + 112*B*sin(2*c + 2*d*x) + 42*B*sin(3*c + 3*d*x)))/(420*d*(cos(c + d*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+a*sec(d*x+c))**(1/2)/sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

$$3.231 \quad \int \sec^2(c+dx)(a+a \sec(c+dx))^{3/2}(A+B \sec(c+dx)) dx$$

Optimal. Leaf size=227

$$\frac{a^{3/2}(88A + 75B) \sinh^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}}\right)}{64d} + \frac{a^2(8A + 9B) \sin(c+dx) \sec^2(c+dx)}{24d\sqrt{a \sec(c+dx)+a}} + \frac{a^2(88A + 75B) \sin(c+dx) \sec^2(c+dx)}{96d\sqrt{a \sec(c+dx)+a}}$$

[Out] 1/64*a^(3/2)*(88*A+75*B)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+1/64*a^2*(88*A+75*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/96*a^2*(88*A+75*B)*sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/24*a^2*(8*A+9*B)*sec(d*x+c)^(7/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/4*a*B*sec(d*x+c)^(7/2)*sin(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d

Rubi [A] time = 0.55, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4018, 4016, 3803, 3801, 215}

$$\frac{a^2(8A + 9B) \sin(c+dx) \sec^2(c+dx)}{24d\sqrt{a \sec(c+dx)+a}} + \frac{a^2(88A + 75B) \sin(c+dx) \sec^2(c+dx)}{96d\sqrt{a \sec(c+dx)+a}} + \frac{a^2(88A + 75B) \sin(c+dx) \sec^2(c+dx)}{64d\sqrt{a \sec(c+dx)+a}}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]

[Out] (a^(3/2)*(88*A + 75*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]])/(64*d) + (a^2*(88*A + 75*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(64*d*Sqrt[a + a*Sec[c + d*x]]) + (a^2*(88*A + 75*B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(96*d*Sqrt[a + a*Sec[c + d*x]]) + (a^2*(8*A + 9*B)*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(24*d*Sqrt[a + a*Sec[c + d*x]]) + (a*B*Sec[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(4*d)

Rule 215

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Simp[ArcSinh[Rt[b, 2]*x]/Sqrt[a]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 3801

Int[Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] := Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 + x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,

b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]

Rule 3803

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^ (n_.)*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] :> Simp[(-2*b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(2*a*d*(n - 1))/(b*(2*n - 1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]

Rule 4016

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^ (n_.)*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> Simp[(-2*b*B*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && !LtQ[n, 0]

Rule 4018

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^ (n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^ (m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> -Simp[(b*B*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n *Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && !LtQ[n, -1]

Rubi steps

$$\begin{aligned}
\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))dx &= \frac{aB\sec^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{4d} + \\
&= \frac{a^2(8A+9B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{24d\sqrt{a+a\sec(c+dx)}} + \frac{aB\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4d} \\
&= \frac{a^2(88A+75B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{96d\sqrt{a+a\sec(c+dx)}} + \frac{a^2(8A+9B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{24d\sqrt{a+a\sec(c+dx)}} \\
&= \frac{a^2(88A+75B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{64d\sqrt{a+a\sec(c+dx)}} + \frac{a^2(88A+75B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{96d\sqrt{a+a\sec(c+dx)}} \\
&= \frac{a^2(88A+75B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{64d\sqrt{a+a\sec(c+dx)}} + \frac{a^2(88A+75B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{96d\sqrt{a+a\sec(c+dx)}} \\
&= \frac{a^2(88A+75B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{64d} + \frac{a^2(88A+75B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{64d\sqrt{a+a\sec(c+dx)}}
\end{aligned}$$

Mathematica [A] time = 1.50, size = 153, normalized size = 0.67

$$a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(6\sqrt{2}(88A+75B)\tanh^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\sec^4(c+dx)\right)$$

Antiderivative was successfully verified.

[In] Integrate[Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]), x]

[Out] (a*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(6*Sqrt[2]*(88*A + 75*B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]] + (352*A + 492*B + (1048*A + 1155*B)*Cos[c + d*x] + 4*(88*A + 75*B)*Cos[2*(c + d*x)] + 264*A*Cos[3*(c + d*x)] + 225*B*Cos[3*(c + d*x)])*Sec[c + d*x]^4*Sin[(c + d*x)/2]))/(768*d*Sqrt[Sec[c + d*x]])

fricas [A] time = 0.68, size = 494, normalized size = 2.18

$$\left[\frac{3 \left((88A + 75B)a \cos(dx + c)^4 + (88A + 75B)a \cos(dx + c)^3 \right) \sqrt{a} \log \left(\frac{a \cos(dx + c)^3 - 7a \cos(dx + c)^2 - \frac{4(\cos(dx + c)^2 - 2 \cos(dx + c))}{\sqrt{c}}}{\cos(dx + c)^3 + \cos(dx + c)} \right)}{768 \left(d \cos(dx + c) \right)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] [1/768*(3*((88*A + 75*B)*a*cos(d*x + c)^4 + (88*A + 75*B)*a*cos(d*x + c)^3)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(3*(88*A + 75*B)*a*cos(d*x + c)^3 + 2*(88*A + 75*B)*a*cos(d*x + c)^2 + 8*(8*A + 15*B)*a*cos(d*x + c) + 48*B*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3), 1/384*(3*((88*A + 75*B)*a*cos(d*x + c)^4 + (88*A + 75*B)*a*cos(d*x + c)^3)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) + 2*(3*(88*A + 75*B)*a*cos(d*x + c)^3 + 2*(88*A + 75*B)*a*cos(d*x + c)^2 + 8*(8*A + 15*B)*a*cos(d*x + c) + 48*B*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(a \sec(dx + c) + a)^{\frac{3}{2}} \sec(dx + c)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(5/2), x)

maple [B] time = 2.42, size = 479, normalized size = 2.11

$$\left(264A \left(\cos^4(dx+c) \right) \sqrt{2} \arctan \left(\frac{\sqrt{\frac{2}{1+\cos(dx+c)}} (\cos(dx+c)+1+\sin(dx+c)) \sqrt{2}}{4} \right) - 264A \left(\cos^4(dx+c) \right) \sqrt{2} \arctan \left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}} (\cos(dx+c)+1-\sin(dx+c)) \sqrt{2}}{4} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x)`

[Out] `1/768/d*(264*A*cos(d*x+c)^4*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c))))^(1/2)*(cos(d*x+c)+1+sin(d*x+c))*2^(1/2))-264*A*cos(d*x+c)^4*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c))))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))*2^(1/2))+225*B*cos(d*x+c)^4*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c))))^(1/2)*(cos(d*x+c)+1+sin(d*x+c))*2^(1/2))-225*B*cos(d*x+c)^4*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c))))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))*2^(1/2))+528*A*cos(d*x+c)^3*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)+450*B*cos(d*x+c)^3*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)+352*A*sin(d*x+c)*cos(d*x+c)^2*(-2/(1+cos(d*x+c)))^(1/2)+300*B*sin(d*x+c)*cos(d*x+c)^2*(-2/(1+cos(d*x+c)))^(1/2)+128*A*sin(d*x+c)*cos(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)+240*B*sin(d*x+c)*cos(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)+96*B*(-2/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(5/2)*(-2/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^2/cos(d*x+c)*(cos(d*x+c)^2-1)*a`

maxima [B] time = 2.62, size = 5879, normalized size = 25.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="maxima")`

[Out] `-1/768*(8*(132*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) + 3*sqrt(2)*a*sin(2*d*x + 2*c))*cos(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 44*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) + 3*sqrt(2)*a*sin(2*d*x + 2*c))*cos(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 216*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) + 3*sqrt(2)*a*sin(2*d*x + 2*c))*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 216*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) + 3*sqrt(2)*a*sin(2*d*x + 2*c))*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 44*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) + 3*sqrt(2)*a*sin(2*d*x + 2*c))*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 132*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) + 3*sqrt(2)*a*sin(2*d*x + 2*c))*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))`

$$\begin{aligned}
& 2*c))) - 33*(a*\cos(6*d*x + 6*c)^2 + 9*a*\cos(4*d*x + 4*c)^2 + 9*a*\cos(2*d*x \\
& + 2*c)^2 + a*\sin(6*d*x + 6*c)^2 + 9*a*\sin(4*d*x + 4*c)^2 + 18*a*\sin(4*d*x \\
& + 4*c)*\sin(2*d*x + 2*c) + 9*a*\sin(2*d*x + 2*c)^2 + 2*(3*a*\cos(4*d*x + 4*c) \\
& + 3*a*\cos(2*d*x + 2*c) + a)*\cos(6*d*x + 6*c) + 6*(3*a*\cos(2*d*x + 2*c) + a) \\
& *\cos(4*d*x + 4*c) + 6*a*\cos(2*d*x + 2*c) + 6*(a*\sin(4*d*x + 4*c) + a*\sin(2*d* \\
& x + 2*c))*\sin(6*d*x + 6*c) + a)*\log(2*\cos(1/4*\arctan2(\sin(2*d*x + 2*c), c \\
& os(2*d*x + 2*c)))^2 + 2*\sin(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)) \\
&)^2 + 2*\sqrt{2}*\cos(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 2*\sqrt{2} \\
& *\sin(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 2) + 33*(a*\cos \\
& (6*d*x + 6*c)^2 + 9*a*\cos(4*d*x + 4*c)^2 + 9*a*\cos(2*d*x + 2*c)^2 + a*\sin(6 \\
& *d*x + 6*c)^2 + 9*a*\sin(4*d*x + 4*c)^2 + 18*a*\sin(4*d*x + 4*c)*\sin(2*d*x + \\
& 2*c) + 9*a*\sin(2*d*x + 2*c)^2 + 2*(3*a*\cos(4*d*x + 4*c) + 3*a*\cos(2*d*x + 2 \\
& *c) + a)*\cos(6*d*x + 6*c) + 6*(3*a*\cos(2*d*x + 2*c) + a)*\cos(4*d*x + 4*c) + \\
& 6*a*\cos(2*d*x + 2*c) + 6*(a*\sin(4*d*x + 4*c) + a*\sin(2*d*x + 2*c))*\sin(6*d \\
& *x + 6*c) + a)*\log(2*\cos(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 \\
& + 2*\sin(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + 2*\sqrt{2}*\cos \\
& (1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) - 2*\sqrt{2}*\sin(1/4*\arcta \\
& n2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 2) - 33*(a*\cos(6*d*x + 6*c)^2 + 9 \\
& *a*\cos(4*d*x + 4*c)^2 + 9*a*\cos(2*d*x + 2*c)^2 + a*\sin(6*d*x + 6*c)^2 + 9*a \\
& *\sin(4*d*x + 4*c)^2 + 18*a*\sin(4*d*x + 4*c)*\sin(2*d*x + 2*c) + 9*a*\sin(2*d* \\
& x + 2*c)^2 + 2*(3*a*\cos(4*d*x + 4*c) + 3*a*\cos(2*d*x + 2*c) + a)*\cos(6*d*x \\
& + 6*c) + 6*(3*a*\cos(2*d*x + 2*c) + a)*\cos(4*d*x + 4*c) + 6*a*\cos(2*d*x + 2* \\
& c) + 6*(a*\sin(4*d*x + 4*c) + a*\sin(2*d*x + 2*c))*\sin(6*d*x + 6*c) + a)*\log(\\
& 2*\cos(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + 2*\sin(1/4*\arctan \\
& 2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 - 2*\sqrt{2}*\cos(1/4*\arctan2(\sin(2* \\
& d*x + 2*c), \cos(2*d*x + 2*c))) + 2*\sqrt{2}*\sin(1/4*\arctan2(\sin(2*d*x + 2*c) \\
& , \cos(2*d*x + 2*c))) + 2) + 33*(a*\cos(6*d*x + 6*c)^2 + 9*a*\cos(4*d*x + 4*c) \\
& ^2 + 9*a*\cos(2*d*x + 2*c)^2 + a*\sin(6*d*x + 6*c)^2 + 9*a*\sin(4*d*x + 4*c)^2 \\
& + 18*a*\sin(4*d*x + 4*c)*\sin(2*d*x + 2*c) + 9*a*\sin(2*d*x + 2*c)^2 + 2*(3*a \\
& *\cos(4*d*x + 4*c) + 3*a*\cos(2*d*x + 2*c) + a)*\cos(6*d*x + 6*c) + 6*(3*a*\cos \\
& (2*d*x + 2*c) + a)*\cos(4*d*x + 4*c) + 6*a*\cos(2*d*x + 2*c) + 6*(a*\sin(4*d*x \\
& + 4*c) + a*\sin(2*d*x + 2*c))*\sin(6*d*x + 6*c) + a)*\log(2*\cos(1/4*\arctan2(s \\
& in(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + 2*\sin(1/4*\arctan2(\sin(2*d*x + 2*c), \\
& \cos(2*d*x + 2*c)))^2 - 2*\sqrt{2}*\cos(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d \\
& *x + 2*c))) - 2*\sqrt{2}*\sin(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)) \\
&) + 2) - 132*(\sqrt{2})*a*\cos(6*d*x + 6*c) + 3*\sqrt{2})*a*\cos(4*d*x + 4*c) + 3 \\
& *\sqrt{2})*a*\cos(2*d*x + 2*c) + \sqrt{2})*a*\sin(11/4*\arctan2(\sin(2*d*x + 2*c), \\
& \cos(2*d*x + 2*c))) - 44*(\sqrt{2})*a*\cos(6*d*x + 6*c) + 3*\sqrt{2})*a*\cos(4*d* \\
& x + 4*c) + 3*\sqrt{2})*a*\cos(2*d*x + 2*c) + \sqrt{2})*a*\sin(9/4*\arctan2(\sin(2* \\
& d*x + 2*c), \cos(2*d*x + 2*c))) - 216*(\sqrt{2})*a*\cos(6*d*x + 6*c) + 3*\sqrt{2} \\
&)*a*\cos(4*d*x + 4*c) + 3*\sqrt{2})*a*\cos(2*d*x + 2*c) + \sqrt{2})*a*\sin(7/4*ar \\
& ctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 216*(\sqrt{2})*a*\cos(6*d*x + 6*c \\
&) + 3*\sqrt{2})*a*\cos(4*d*x + 4*c) + 3*\sqrt{2})*a*\cos(2*d*x + 2*c) + \sqrt{2})*a \\
&)*\sin(5/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 44*(\sqrt{2})*a*\cos(\\
& 6*d*x + 6*c) + 3*\sqrt{2})*a*\cos(4*d*x + 4*c) + 3*\sqrt{2})*a*\cos(2*d*x + 2*c)
\end{aligned}$$

$$\begin{aligned}
& + \sqrt{2} * a) * \sin(3/4 * \arctan2(\sin(2 * d * x + 2 * c), \cos(2 * d * x + 2 * c))) + 132 * (\sqrt{2} * a * \cos(6 * d * x + 6 * c) + 3 * \sqrt{2} * a * \cos(4 * d * x + 4 * c) + 3 * \sqrt{2} * a * \cos(2 * d * x + 2 * c) + \sqrt{2} * a) * \sin(1/4 * \arctan2(\sin(2 * d * x + 2 * c), \cos(2 * d * x + 2 * c))) * A * \sqrt{a} / (2 * (3 * \cos(4 * d * x + 4 * c) + 3 * \cos(2 * d * x + 2 * c) + 1) * \cos(6 * d * x + 6 * c) + \cos(6 * d * x + 6 * c)^2 + 6 * (3 * \cos(2 * d * x + 2 * c) + 1) * \cos(4 * d * x + 4 * c) + 9 * \cos(4 * d * x + 4 * c)^2 + 9 * \cos(2 * d * x + 2 * c)^2 + 6 * (\sin(4 * d * x + 4 * c) + \sin(2 * d * x + 2 * c)) * \sin(6 * d * x + 6 * c) + \sin(6 * d * x + 6 * c)^2 + 9 * \sin(4 * d * x + 4 * c)^2 + 18 * \sin(4 * d * x + 4 * c) * \sin(2 * d * x + 2 * c) + 9 * \sin(2 * d * x + 2 * c)^2 + 6 * \cos(2 * d * x + 2 * c) + 1) + 3 * (300 * (\sqrt{2} * a * \sin(8 * d * x + 8 * c) + 4 * \sqrt{2} * a * \sin(6 * d * x + 6 * c) + 6 * \sqrt{2} * a * \sin(4 * d * x + 4 * c) + 4 * \sqrt{2} * a * \sin(2 * d * x + 2 * c)) * \cos(15/4 * \arctan2(\sin(2 * d * x + 2 * c), \cos(2 * d * x + 2 * c))) + 100 * (\sqrt{2} * a * \sin(8 * d * x + 8 * c) + 4 * \sqrt{2} * a * \sin(6 * d * x + 6 * c) + 6 * \sqrt{2} * a * \sin(4 * d * x + 4 * c) + 4 * \sqrt{2} * a * \sin(2 * d * x + 2 * c)) * \cos(13/4 * \arctan2(\sin(2 * d * x + 2 * c), \cos(2 * d * x + 2 * c))) + 1140 * (\sqrt{2} * a * \sin(8 * d * x + 8 * c) + 4 * \sqrt{2} * a * \sin(6 * d * x + 6 * c) + 6 * \sqrt{2} * a * \sin(4 * d * x + 4 * c) + 4 * \sqrt{2} * a * \sin(2 * d * x + 2 * c)) * \cos(11/4 * \arctan2(\sin(2 * d * x + 2 * c), \cos(2 * d * x + 2 * c))) - 228 * (\sqrt{2} * a * \sin(8 * d * x + 8 * c) + 4 * \sqrt{2} * a * \sin(6 * d * x + 6 * c) + 6 * \sqrt{2} * a * \sin(4 * d * x + 4 * c) + 4 * \sqrt{2} * a * \sin(2 * d * x + 2 * c)) * \cos(9/4 * \arctan2(\sin(2 * d * x + 2 * c), \cos(2 * d * x + 2 * c))) + 228 * (\sqrt{2} * a * \sin(8 * d * x + 8 * c) + 4 * \sqrt{2} * a * \sin(6 * d * x + 6 * c) + 6 * \sqrt{2} * a * \sin(4 * d * x + 4 * c) + 4 * \sqrt{2} * a * \sin(2 * d * x + 2 * c)) * \cos(7/4 * \arctan2(\sin(2 * d * x + 2 * c), \cos(2 * d * x + 2 * c))) - 1140 * (\sqrt{2} * a * \sin(8 * d * x + 8 * c) + 4 * \sqrt{2} * a * \sin(6 * d * x + 6 * c) + 6 * \sqrt{2} * a * \sin(4 * d * x + 4 * c) + 4 * \sqrt{2} * a * \sin(2 * d * x + 2 * c)) * \cos(5/4 * \arctan2(\sin(2 * d * x + 2 * c), \cos(2 * d * x + 2 * c))) - 100 * (\sqrt{2} * a * \sin(8 * d * x + 8 * c) + 4 * \sqrt{2} * a * \sin(6 * d * x + 6 * c) + 6 * \sqrt{2} * a * \sin(4 * d * x + 4 * c) + 4 * \sqrt{2} * a * \sin(2 * d * x + 2 * c)) * \cos(3/4 * \arctan2(\sin(2 * d * x + 2 * c), \cos(2 * d * x + 2 * c))) - 300 * (\sqrt{2} * a * \sin(8 * d * x + 8 * c) + 4 * \sqrt{2} * a * \sin(6 * d * x + 6 * c) + 6 * \sqrt{2} * a * \sin(4 * d * x + 4 * c) + 4 * \sqrt{2} * a * \sin(2 * d * x + 2 * c)) * \cos(1/4 * \arctan2(\sin(2 * d * x + 2 * c), \cos(2 * d * x + 2 * c))) - 75 * (a * \cos(8 * d * x + 8 * c)^2 + 16 * a * \cos(6 * d * x + 6 * c)^2 + 36 * a * \cos(4 * d * x + 4 * c)^2 + 16 * a * \cos(2 * d * x + 2 * c)^2 + a * \sin(8 * d * x + 8 * c)^2 + 16 * a * \sin(6 * d * x + 6 * c)^2 + 36 * a * \sin(4 * d * x + 4 * c)^2 + 48 * a * \sin(4 * d * x + 4 * c) * \sin(2 * d * x + 2 * c) + 16 * a * \sin(2 * d * x + 2 * c)^2 + 2 * (4 * a * \cos(6 * d * x + 6 * c) + 6 * a * \cos(4 * d * x + 4 * c) + 4 * a * \cos(2 * d * x + 2 * c) + a) * \cos(8 * d * x + 8 * c) + 8 * (6 * a * \cos(4 * d * x + 4 * c) + 4 * a * \cos(2 * d * x + 2 * c) + a) * \cos(6 * d * x + 6 * c) + 12 * (4 * a * \cos(2 * d * x + 2 * c) + a) * \cos(4 * d * x + 4 * c) + 8 * a * \cos(2 * d * x + 2 * c) + 4 * (2 * a * \sin(6 * d * x + 6 * c) + 3 * a * \sin(4 * d * x + 4 * c) + 2 * a * \sin(2 * d * x + 2 * c)) * \sin(8 * d * x + 8 * c) + 16 * (3 * a * \sin(4 * d * x + 4 * c) + 2 * a * \sin(2 * d * x + 2 * c)) * \sin(6 * d * x + 6 * c) + a) * \log(2 * \cos(1/4 * \arctan2(\sin(2 * d * x + 2 * c), \cos(2 * d * x + 2 * c)))^2 + 2 * \sin(1/4 * \arctan2(\sin(2 * d * x + 2 * c), \cos(2 * d * x + 2 * c)))^2 + 2 * \sqrt{2} * \cos(1/4 * \arctan2(\sin(2 * d * x + 2 * c), \cos(2 * d * x + 2 * c))) + 2 * \sqrt{2} * \sin(1/4 * \arctan2(\sin(2 * d * x + 2 * c), \cos(2 * d * x + 2 * c))) + 2) + 75 * (a * \cos(8 * d * x + 8 * c)^2 + 16 * a * \cos(6 * d * x + 6 * c)^2 + 36 * a * \cos(4 * d * x + 4 * c)^2 + 16 * a * \cos(2 * d * x + 2 * c)^2 + a * \sin(8 * d * x + 8 * c)^2 + 16 * a * \sin(6 * d * x + 6 * c)^2 + 36 * a * \sin(4 * d * x + 4 * c)^2 + 48 * a * \sin(4 * d * x + 4 * c) * \sin(2 * d * x + 2 * c) + 16 * a * \sin(2 * d * x + 2 * c)^2 + 2 * (4 * a * \cos(6 * d * x + 6 * c) + 6 * a * \cos(4 * d * x + 4 * c) + 4 * a * \cos(2 * d * x + 2 * c) + a) * \cos(8 * d * x + 8 * c) + 8 * (6 * a * \cos(4 * d * x + 4 * c) + 4 * a * \cos(2 * d * x + 2 * c) + a) * \cos(6 * d * x + 6 *
\end{aligned}$$

$$\begin{aligned}
& *c) + 12*(4*a*cos(2*d*x + 2*c) + a)*cos(4*d*x + 4*c) + 8*a*cos(2*d*x + 2*c) \\
& + 4*(2*a*sin(6*d*x + 6*c) + 3*a*sin(4*d*x + 4*c) + 2*a*sin(2*d*x + 2*c))*s \\
& in(8*d*x + 8*c) + 16*(3*a*sin(4*d*x + 4*c) + 2*a*sin(2*d*x + 2*c))*sin(6*d* \\
& x + 6*c) + a)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 \\
& + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(\\
& 1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan \\
& 2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 75*(a*cos(8*d*x + 8*c)^2 + 16 \\
& *a*cos(6*d*x + 6*c)^2 + 36*a*cos(4*d*x + 4*c)^2 + 16*a*cos(2*d*x + 2*c)^2 + \\
& a*sin(8*d*x + 8*c)^2 + 16*a*sin(6*d*x + 6*c)^2 + 36*a*sin(4*d*x + 4*c)^2 + \\
& 48*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*a*sin(2*d*x + 2*c)^2 + 2*(4*a* \\
& cos(6*d*x + 6*c) + 6*a*cos(4*d*x + 4*c) + 4*a*cos(2*d*x + 2*c) + a)*cos(8*d \\
& *x + 8*c) + 8*(6*a*cos(4*d*x + 4*c) + 4*a*cos(2*d*x + 2*c) + a)*cos(6*d*x + \\
& 6*c) + 12*(4*a*cos(2*d*x + 2*c) + a)*cos(4*d*x + 4*c) + 8*a*cos(2*d*x + 2* \\
& c) + 4*(2*a*sin(6*d*x + 6*c) + 3*a*sin(4*d*x + 4*c) + 2*a*sin(2*d*x + 2*c)) \\
& *sin(8*d*x + 8*c) + 16*(3*a*sin(4*d*x + 4*c) + 2*a*sin(2*d*x + 2*c))*sin(6* \\
& d*x + 6*c) + a)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^ \\
& 2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*co \\
& s(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arct \\
& an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 75*(a*cos(8*d*x + 8*c)^2 + \\
& 16*a*cos(6*d*x + 6*c)^2 + 36*a*cos(4*d*x + 4*c)^2 + 16*a*cos(2*d*x + 2*c)^2 \\
& + a*sin(8*d*x + 8*c)^2 + 16*a*sin(6*d*x + 6*c)^2 + 36*a*sin(4*d*x + 4*c)^2 \\
& + 48*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*a*sin(2*d*x + 2*c)^2 + 2*(4* \\
& a*cos(6*d*x + 6*c) + 6*a*cos(4*d*x + 4*c) + 4*a*cos(2*d*x + 2*c) + a)*cos(8 \\
& *d*x + 8*c) + 8*(6*a*cos(4*d*x + 4*c) + 4*a*cos(2*d*x + 2*c) + a)*cos(6*d*x \\
& + 6*c) + 12*(4*a*cos(2*d*x + 2*c) + a)*cos(4*d*x + 4*c) + 8*a*cos(2*d*x + \\
& 2*c) + 4*(2*a*sin(6*d*x + 6*c) + 3*a*sin(4*d*x + 4*c) + 2*a*sin(2*d*x + 2*c \\
&))*sin(8*d*x + 8*c) + 16*(3*a*sin(4*d*x + 4*c) + 2*a*sin(2*d*x + 2*c))*sin(\\
& 6*d*x + 6*c) + a)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)) \\
&)^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)* \\
& cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*ar \\
& ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 300*(sqrt(2)*a*cos(8*d*x \\
& + 8*c) + 4*sqrt(2)*a*cos(6*d*x + 6*c) + 6*sqrt(2)*a*cos(4*d*x + 4*c) + 4*sq \\
& rt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(15/4*arctan2(sin(2*d*x + 2*c), co \\
& s(2*d*x + 2*c))) - 100*(sqrt(2)*a*cos(8*d*x + 8*c) + 4*sqrt(2)*a*cos(6*d*x \\
& + 6*c) + 6*sqrt(2)*a*cos(4*d*x + 4*c) + 4*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt \\
& (2)*a)*sin(13/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1140*(sqrt(2 \\
&)*a*cos(8*d*x + 8*c) + 4*sqrt(2)*a*cos(6*d*x + 6*c) + 6*sqrt(2)*a*cos(4*d*x \\
& + 4*c) + 4*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(11/4*arctan2(sin(2* \\
& d*x + 2*c), cos(2*d*x + 2*c))) + 228*(sqrt(2)*a*cos(8*d*x + 8*c) + 4*sqrt(2 \\
&)*a*cos(6*d*x + 6*c) + 6*sqrt(2)*a*cos(4*d*x + 4*c) + 4*sqrt(2)*a*cos(2*d*x \\
& + 2*c) + sqrt(2)*a)*sin(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - \\
& 228*(sqrt(2)*a*cos(8*d*x + 8*c) + 4*sqrt(2)*a*cos(6*d*x + 6*c) + 6*sqrt(2 \\
&)*a*cos(4*d*x + 4*c) + 4*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(7/4*arc \\
& tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1140*(sqrt(2)*a*cos(8*d*x + 8*c \\
&) + 4*sqrt(2)*a*cos(6*d*x + 6*c) + 6*sqrt(2)*a*cos(4*d*x + 4*c) + 4*sqrt(2)
\end{aligned}$$

```
*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + 100*(sqrt(2)*a*cos(8*d*x + 8*c) + 4*sqrt(2)*a*cos(6*d*x + 6*c)
+ 6*sqrt(2)*a*cos(4*d*x + 4*c) + 4*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a
*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 300*(sqrt(2)*a*cos(
8*d*x + 8*c) + 4*sqrt(2)*a*cos(6*d*x + 6*c) + 6*sqrt(2)*a*cos(4*d*x + 4*c)
+ 4*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(1/4*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))) *B*sqrt(a)/(2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c
) + 4*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*co
s(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x +
6*c)^2 + 12*(4*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)
^2 + 16*cos(2*d*x + 2*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2
*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x
+ 4*c) + 2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*
sin(4*d*x + 4*c)^2 + 48*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x +
2*c)^2 + 8*cos(2*d*x + 2*c) + 1))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(A + \frac{B}{\cos(c + dx)} \right) \left(a + \frac{a}{\cos(c + dx)} \right)^{3/2} \left(\frac{1}{\cos(c + dx)} \right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(5/2),
x)
```

```
[Out] int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(5/2),
x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

$$3.232 \quad \int \sec^{\frac{3}{2}}(c+dx)(a+a \sec(c+dx))^{\frac{3}{2}}(A+B \sec(c+dx)) dx$$

Optimal. Leaf size=180

$$\frac{a^{3/2}(14A + 11B) \sinh^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}}\right)}{8d} + \frac{a^2(6A + 7B) \sin(c + dx) \sec^{\frac{5}{2}}(c + dx)}{12d\sqrt{a \sec(c + dx) + a}} + \frac{a^2(14A + 11B) \sin(c + dx) \sec^{\frac{3}{2}}(c + dx)}{8d\sqrt{a \sec(c + dx) + a}}$$

[Out] $1/8*a^{(3/2)}*(14*A+11*B)*\operatorname{arcsinh}(a^{(1/2)}*\tan(d*x+c)/(a+a*\sec(d*x+c))^{(1/2)})/d+1/8*a^2*(14*A+11*B)*\sec(d*x+c)^{(3/2)}*\sin(d*x+c)/d/(a+a*\sec(d*x+c))^{(1/2)}+1/12*a^2*(6*A+7*B)*\sec(d*x+c)^{(5/2)}*\sin(d*x+c)/d/(a+a*\sec(d*x+c))^{(1/2)}+1/3*a*B*\sec(d*x+c)^{(5/2)}*\sin(d*x+c)*(a+a*\sec(d*x+c))^{(1/2)}/d$

Rubi [A] time = 0.42, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4018, 4016, 3803, 3801, 215}

$$\frac{a^2(6A + 7B) \sin(c + dx) \sec^{\frac{5}{2}}(c + dx)}{12d\sqrt{a \sec(c + dx) + a}} + \frac{a^2(14A + 11B) \sin(c + dx) \sec^{\frac{3}{2}}(c + dx)}{8d\sqrt{a \sec(c + dx) + a}} + \frac{a^{3/2}(14A + 11B) \sinh^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}}\right)}{8d}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Sec}[c + d*x]^{(3/2)}*(a + a*\operatorname{Sec}[c + d*x])^{(3/2)}*(A + B*\operatorname{Sec}[c + d*x]), x]$

[Out] $(a^{(3/2)}*(14*A + 11*B)*\operatorname{ArcSinh}[(\operatorname{Sqrt}[a]*\operatorname{Tan}[c + d*x])/(\operatorname{Sqrt}[a + a*\operatorname{Sec}[c + d*x]])]/(8*d) + (a^2*(14*A + 11*B)*\operatorname{Sec}[c + d*x]^{(3/2)}*\operatorname{Sin}[c + d*x])/(8*d*\operatorname{Sqrt}[a + a*\operatorname{Sec}[c + d*x]]) + (a^2*(6*A + 7*B)*\operatorname{Sec}[c + d*x]^{(5/2)}*\operatorname{Sin}[c + d*x])/(12*d*\operatorname{Sqrt}[a + a*\operatorname{Sec}[c + d*x]]) + (a*B*\operatorname{Sec}[c + d*x]^{(5/2)}*\operatorname{Sqrt}[a + a*\operatorname{Sec}[c + d*x]]*\operatorname{Sin}[c + d*x])/(3*d)$

Rule 215

$\operatorname{Int}[1/\operatorname{Sqrt}[(a_) + (b_)*(x_)^2], x_Symbol] \rightarrow \operatorname{Simp}[\operatorname{ArcSinh}[(\operatorname{Rt}[b, 2]*x)/\operatorname{Sqrt}[a]]/\operatorname{Rt}[b, 2], x] /; \operatorname{FreeQ}\{a, b, x\} \&\& \operatorname{GtQ}[a, 0] \&\& \operatorname{PosQ}[b]$

Rule 3801

$\operatorname{Int}[\operatorname{Sqrt}[\operatorname{csc}[(e_) + (f_)*(x_)]*(d_)]*\operatorname{Sqrt}[\operatorname{csc}[(e_) + (f_)*(x_)]*(b_) + (a_)], x_Symbol] \rightarrow \operatorname{Dist}[(-2*a*\operatorname{Sqrt}[(a*d)/b])/(b*f), \operatorname{Subst}[\operatorname{Int}[1/\operatorname{Sqrt}[1 + x^2/a], x], x, (b*\operatorname{Cot}[e + f*x])/(\operatorname{Sqrt}[a + b*\operatorname{Csc}[e + f*x]])], x] /; \operatorname{FreeQ}\{a, b, d, e, f\}, x\} \&\& \operatorname{EqQ}[a^2 - b^2, 0] \&\& \operatorname{GtQ}[(a*d)/b, 0]$

Rule 3803

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.)
+ (a_.)], x_Symbol] := Simp[(-2*b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/
(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(2*a*d*(n - 1))/(b*(2*n -
1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.)
+ (a_.)]*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
+ Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && !
LtQ[n, 0]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (
a_.))^(m_)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && !LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))dx &= \frac{aB\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{3d} + \frac{1}{3} \\
&= \frac{a^2(6A+7B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}} + \frac{aB\sec^{\frac{5}{2}}(c+dx)}{3d} \\
&= \frac{a^2(14A+11B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}} + \frac{a^2(6A+7B)}{12d} \\
&= \frac{a^2(14A+11B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}} + \frac{a^2(6A+7B)}{12d} \\
&= \frac{a^{3/2}(14A+11B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{8d} + \frac{a^2(14A+11B)}{8d}
\end{aligned}$$

Mathematica [A] time = 1.55, size = 134, normalized size = 0.74

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(3\sqrt{2}(14A+11B)\tanh^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)}{48d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]), x]

[Out] (a*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(3*Sqrt[2]*(14*A + 11*B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]] + (7*(6*A + 7*B) + 4*(6*A + 11*B)*Cos[c + d*x] + (42*A + 33*B)*Cos[2*(c + d*x)])*Sec[c + d*x]^3*Ssin[(c + d*x)/2]))/(48*d*Sqrt[Sec[c + d*x]])

fricas [A] time = 0.59, size = 458, normalized size = 2.54

$$\left[\frac{3\left((14A+11B)a\cos(dx+c)^3+(14A+11B)a\cos(dx+c)^2\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-\frac{4(\cos(dx+c)^2-2\cos(dx+c))}{\sqrt{a}}}{\cos(dx+c)^3+\cos(dx+c)^2}\right)}{96\left(d\cos(dx+c)^3+d\cos(dx+c)\right)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] [1/96*(3*((14*A + 11*B)*a*cos(d*x + c)^3 + (14*A + 11*B)*a*cos(d*x + c)^2)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(3*(14*A + 11*B)*a*cos(d*x + c)^2 + 2*(6*A + 11*B)*a*cos(d*x + c) + 8*B*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2), 1/48*(3*((14*A + 11*B)*a*cos(d*x + c)^3 + (14*A + 11*B)*a*cos(d*x + c)^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) + 2*(3*(14*A + 11*B)*a*cos(d*x + c)^2 + 2*(6*A + 11*B)*a*cos(d*x + c) + 8*B*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(a \sec(dx + c) + a)^{\frac{3}{2}} \sec(dx + c)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/2), x)

maple [B] time = 2.42, size = 415, normalized size = 2.31

$$(-1 + \cos(dx + c)) \left(42A (\cos^3(dx + c)) \sqrt{2} \arctan \left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}} (\cos(dx+c)+1+\sin(dx+c))\sqrt{2}}{4} \right) \right) - 42A (\cos^3(dx + c))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x)

[Out] -1/48/d*(-1+cos(d*x+c))*(42*A*cos(d*x+c)^3*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c))))^(1/2)*(cos(d*x+c)+1+sin(d*x+c))*2^(1/2))-42*A*cos(d*x+c)^3*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c))))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))*2^(1/2))+33*B*cos(d*x+c)^3*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c))))^(1/2)*(cos(d*x+c)+1+si

$$\begin{aligned} & n(d*x+c))*2^{(1/2)}-33*B*\cos(d*x+c)^3*2^{(1/2)}*\arctan(1/4*(-2/(1+\cos(d*x+c))) \\ & ^{(1/2)}*(\cos(d*x+c)+1-\sin(d*x+c))*2^{(1/2)}+84*A*\sin(d*x+c)*\cos(d*x+c)^2*(-2/ \\ & (1+\cos(d*x+c)))^{(1/2)}+66*B*\sin(d*x+c)*\cos(d*x+c)^2*(-2/(1+\cos(d*x+c)))^{(1/2)} \\ &)+24*A*\sin(d*x+c)*\cos(d*x+c)*(-2/(1+\cos(d*x+c)))^{(1/2)}+44*B*\sin(d*x+c)*\cos \\ & (d*x+c)*(-2/(1+\cos(d*x+c)))^{(1/2)}+16*B*(-2/(1+\cos(d*x+c)))^{(1/2)}*\sin(d*x+c)) \\ & *(a*(1+\cos(d*x+c))/\cos(d*x+c))^{(1/2)}*(1/\cos(d*x+c))^{(3/2)}/(-2/(1+\cos(d*x+c) \\ &))^{(1/2)}/\sin(d*x+c)^2/\cos(d*x+c)*a \end{aligned}$$

maxima [B] time = 1.95, size = 4606, normalized size = 25.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out]
$$\begin{aligned} & -1/96*(6*(56*\sqrt{2}*a*\cos(7/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + \\ & 3/2*c))))*\sin(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) - 24* \\ & \sqrt{2}*a*\cos(5/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))*\sin(\\ & 4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) - 12*\sqrt{2}*a*\sin \\ & (3/2*d*x + 3/2*c) + 28*\sqrt{2}*a*\sin(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(\\ & 3/2*d*x + 3/2*c))) - 4*(3*\sqrt{2}*a*\sin(3/2*d*x + 3/2*c) + 7*\sqrt{2}*a*\sin(\\ & 7/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) - 3*\sqrt{2}*a*\sin(\\ & 5/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) - 7*\sqrt{2}*a*\sin(\\ & 1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))))*\cos(8/3*\arctan2(s \\ & in(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) - 8*(3*\sqrt{2}*a*\sin(3/2*d*x + \\ & 3/2*c) - 7*\sqrt{2}*a*\sin(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/ \\ & 2*c))))*\cos(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) - 7*(a \\ & * \cos(8/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + 4*a*\cos(4 \\ & /3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + a*\sin(8/3*\arcta \\ & n2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + 4*a*\sin(8/3*\arctan2(\sin \\ & (3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))*\sin(4/3*\arctan2(\sin(3/2*d*x + 3/2 \\ & *c), \cos(3/2*d*x + 3/2*c))) + 4*a*\sin(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos \\ & (3/2*d*x + 3/2*c)))^2 + 2*(2*a*\cos(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/ \\ & 2*d*x + 3/2*c))) + a)*\cos(8/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3 \\ & /2*c))) + 4*a*\cos(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) \\ & + a)*\log(2*\cos(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + \\ & 2*\sin(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + 2*\sqrt{2} \\ & * \cos(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 2*\sqrt{2} \\ & * \sin(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 2) + 7*(a* \\ & \cos(8/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + 4*a*\cos(4/3 \\ & * \arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + a*\sin(8/3*\arctan2 \\ & (\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + 4*a*\sin(8/3*\arctan2(\sin(3 \\ & /2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))*\sin(4/3*\arctan2(\sin(3/2*d*x + 3/2*c \\ &), \cos(3/2*d*x + 3/2*c))) + 4*a*\sin(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3 \end{aligned}$$

$$\begin{aligned}
& /2*d*x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2* \\
& d*x + 3/2*c))) + a*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2 \\
& *c))) + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + \\
& a*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2 \\
& *sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2) \\
& *cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*s \\
& in(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 7*(a*cos \\
& (8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*cos(4/3*a \\
& rctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*sin(8/3*arctan2(s \\
& in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3*arctan2(sin(3/2 \\
& *d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), \\
& cos(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2 \\
& *d*x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d* \\
& x + 3/2*c))) + a*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c \\
&))) + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + a) \\
& *log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*s \\
& in(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*c \\
& os(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin \\
& (1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 7*(a*cos(8 \\
& /3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*cos(4/3*arc \\
& tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*sin(8/3*arctan2(sin \\
& (3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3*arctan2(sin(3/2*d \\
& *x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), c \\
& os(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d \\
& *x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x \\
& + 3/2*c))) + a*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)) \\
&) + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + a)*l \\
& og(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin \\
& (1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos \\
& (1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1 \\
& /3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 4*(3*sqrt(2) \\
& *a*cos(3/2*d*x + 3/2*c) + 7*sqrt(2)*a*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c), \\
& cos(3/2*d*x + 3/2*c))) - 3*sqrt(2)*a*cos(5/3*arctan2(sin(3/2*d*x + 3/2*c), \\
& cos(3/2*d*x + 3/2*c))) - 7*sqrt(2)*a*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), \\
& cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x \\
& + 3/2*c))) - 28*(2*sqrt(2)*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2* \\
& d*x + 3/2*c))) + sqrt(2)*a)*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d \\
& *x + 3/2*c))) + 12*(2*sqrt(2)*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3 \\
& /2*d*x + 3/2*c))) + sqrt(2)*a)*sin(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/ \\
& 2*d*x + 3/2*c))) + 8*(3*sqrt(2)*a*cos(3/2*d*x + 3/2*c) - 7*sqrt(2)*a*cos(1/ \\
& 3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(4/3*arctan2(sin \\
& (3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) *A*sqrt(a)/(2*(2*cos(4/3*arctan2(\\
& sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*cos(8/3*arctan2(sin(3/2*d \\
& *x + 3/2*c), cos(3/2*d*x + 3/2*c))) + cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), \\
& cos(3/2*d*x + 3/2*c)))^2 + 4*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
\end{aligned}$$

$$\begin{aligned}
& *d*x + 3/2*c))^{2} + \sin(8/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^{2} + 4*\sin(8/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) * \sin(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 4*\sin(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^{2} + 4*\cos(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 1) + (132*(\sqrt{2})*a*\sin(6*d*x + 6*c) + 3*\sqrt{2})*a*\sin(4*d*x + 4*c) + 3*\sqrt{2})*a*\sin(2*d*x + 2*c))*\cos(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 44*(\sqrt{2})*a*\sin(6*d*x + 6*c) + 3*\sqrt{2})*a*\sin(4*d*x + 4*c) + 3*\sqrt{2})*a*\sin(2*d*x + 2*c))*\cos(9/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 216*(\sqrt{2})*a*\sin(6*d*x + 6*c) + 3*\sqrt{2})*a*\sin(4*d*x + 4*c) + 3*\sqrt{2})*a*\sin(2*d*x + 2*c))*\cos(7/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) - 216*(\sqrt{2})*a*\sin(6*d*x + 6*c) + 3*\sqrt{2})*a*\sin(4*d*x + 4*c) + 3*\sqrt{2})*a*\sin(2*d*x + 2*c))*\cos(5/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) - 44*(\sqrt{2})*a*\sin(6*d*x + 6*c) + 3*\sqrt{2})*a*\sin(4*d*x + 4*c) + 3*\sqrt{2})*a*\sin(2*d*x + 2*c))*\cos(3/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) - 132*(\sqrt{2})*a*\sin(6*d*x + 6*c) + 3*\sqrt{2})*a*\sin(4*d*x + 4*c) + 3*\sqrt{2})*a*\sin(2*d*x + 2*c))*\cos(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) - 33*(a*\cos(6*d*x + 6*c)^{2} + 9*a*\cos(4*d*x + 4*c)^{2} + 9*a*\cos(2*d*x + 2*c)^{2} + a*\sin(6*d*x + 6*c)^{2} + 9*a*\sin(4*d*x + 4*c)^{2} + 18*a*\sin(4*d*x + 4*c)*\sin(2*d*x + 2*c) + 9*a*\sin(2*d*x + 2*c)^{2} + 2*(3*a*\cos(4*d*x + 4*c) + 3*a*\cos(2*d*x + 2*c) + a)*\cos(6*d*x + 6*c) + 6*(3*a*\cos(2*d*x + 2*c) + a)*\cos(4*d*x + 4*c) + 6*a*\cos(2*d*x + 2*c) + 6*(a*\sin(4*d*x + 4*c) + a*\sin(2*d*x + 2*c))*\sin(6*d*x + 6*c) + a)*\log(2*\cos(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^{2} + 2*\sin(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^{2} + 2*\sqrt{2})*\cos(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 2*\sqrt{2})*\sin(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 2) + 33*(a*\cos(6*d*x + 6*c)^{2} + 9*a*\cos(4*d*x + 4*c)^{2} + 9*a*\cos(2*d*x + 2*c)^{2} + a*\sin(6*d*x + 6*c)^{2} + 9*a*\sin(4*d*x + 4*c)^{2} + 18*a*\sin(4*d*x + 4*c)*\sin(2*d*x + 2*c) + 9*a*\sin(2*d*x + 2*c)^{2} + 2*(3*a*\cos(4*d*x + 4*c) + 3*a*\cos(2*d*x + 2*c) + a)*\cos(6*d*x + 6*c) + 6*(3*a*\cos(2*d*x + 2*c) + a)*\cos(4*d*x + 4*c) + 6*a*\cos(2*d*x + 2*c) + 6*(a*\sin(4*d*x + 4*c) + a*\sin(2*d*x + 2*c))*\sin(6*d*x + 6*c) + a)*\log(2*\cos(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^{2} + 2*\sin(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^{2} + 2*\sqrt{2})*\cos(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) - 2*\sqrt{2})*\sin(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 2) - 33*(a*\cos(6*d*x + 6*c)^{2} + 9*a*\cos(4*d*x + 4*c)^{2} + 9*a*\cos(2*d*x + 2*c)^{2} + a*\sin(6*d*x + 6*c)^{2} + 9*a*\sin(4*d*x + 4*c)^{2} + 18*a*\sin(4*d*x + 4*c)*\sin(2*d*x + 2*c) + 9*a*\sin(2*d*x + 2*c)^{2} + 2*(3*a*\cos(4*d*x + 4*c) + 3*a*\cos(2*d*x + 2*c) + a)*\cos(6*d*x + 6*c) + 6*(3*a*\cos(2*d*x + 2*c) + a)*\cos(4*d*x + 4*c) + 6*a*\cos(2*d*x + 2*c) + 6*(a*\sin(4*d*x + 4*c) + a*\sin(2*d*x + 2*c))*\sin(6*d*x + 6*c) + a)*\log(2*\cos(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^{2} + 2*\sin(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^{2} - 2*\sqrt{2})*\cos(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 2*\sqrt{2})*\sin(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 2) + 33*(a*\cos(6*d*x + 6*c)^{2} + 9*a*\cos(4*d*x + 4*c)^{2} + 9*a*\cos(2*d*x + 2*c)^{2} + a*\sin(6*d*x + 6*c)^{2} + 9*a*\sin(4*d*x + 4*c)^{2} + 18*a*\sin(4*d*x + 4*c)*\sin(2*d*x +
\end{aligned}$$

$2*c) + 9*a*\sin(2*d*x + 2*c)^2 + 2*(3*a*\cos(4*d*x + 4*c) + 3*a*\cos(2*d*x + 2*c) + a)*\cos(6*d*x + 6*c) + 6*(3*a*\cos(2*d*x + 2*c) + a)*\cos(4*d*x + 4*c) + 6*a*\cos(2*d*x + 2*c) + 6*(a*\sin(4*d*x + 4*c) + a*\sin(2*d*x + 2*c))*\sin(6*d*x + 6*c) + a*\log(2*\cos(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + 2*\sin(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))))^2 - 2*\sqrt{2}*\cos(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) - 2*\sqrt{2}*\sin(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 2) - 132*(\sqrt{2}*a*\cos(6*d*x + 6*c) + 3*\sqrt{2}*a*\cos(4*d*x + 4*c) + 3*\sqrt{2}*a*\cos(2*d*x + 2*c) + \sqrt{2}*a)*\sin(11/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) - 44*(\sqrt{2}*a*\cos(6*d*x + 6*c) + 3*\sqrt{2}*a*\cos(4*d*x + 4*c) + 3*\sqrt{2}*a*\cos(2*d*x + 2*c) + \sqrt{2}*a)*\sin(9/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) - 216*(\sqrt{2}*a*\cos(6*d*x + 6*c) + 3*\sqrt{2}*a*\cos(4*d*x + 4*c) + 3*\sqrt{2}*a*\cos(2*d*x + 2*c) + \sqrt{2}*a)*\sin(7/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 216*(\sqrt{2}*a*\cos(6*d*x + 6*c) + 3*\sqrt{2}*a*\cos(4*d*x + 4*c) + 3*\sqrt{2}*a*\cos(2*d*x + 2*c) + \sqrt{2}*a)*\sin(5/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 44*(\sqrt{2}*a*\cos(6*d*x + 6*c) + 3*\sqrt{2}*a*\cos(4*d*x + 4*c) + 3*\sqrt{2}*a*\cos(2*d*x + 2*c) + \sqrt{2}*a)*\sin(3/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 132*(\sqrt{2}*a*\cos(6*d*x + 6*c) + 3*\sqrt{2}*a*\cos(4*d*x + 4*c) + 3*\sqrt{2}*a*\cos(2*d*x + 2*c) + \sqrt{2}*a)*\sin(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))))*B*\sqrt{a}/(2*(3*\cos(4*d*x + 4*c) + 3*\cos(2*d*x + 2*c) + 1)*\cos(6*d*x + 6*c) + \cos(6*d*x + 6*c)^2 + 6*(3*\cos(2*d*x + 2*c) + 1)*\cos(4*d*x + 4*c) + 9*\cos(4*d*x + 4*c)^2 + 9*\cos(2*d*x + 2*c)^2 + 6*(\sin(4*d*x + 4*c) + \sin(2*d*x + 2*c))*\sin(6*d*x + 6*c) + \sin(6*d*x + 6*c)^2 + 9*\sin(4*d*x + 4*c)^2 + 18*\sin(4*d*x + 4*c)*\sin(2*d*x + 2*c) + 9*\sin(2*d*x + 2*c)^2 + 6*\cos(2*d*x + 2*c) + 1))/d$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A + \frac{B}{\cos(c + dx)} \right) \left(a + \frac{a}{\cos(c + dx)} \right)^{3/2} \left(\frac{1}{\cos(c + dx)} \right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(3/2), x)

[Out] int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(3/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**(3/2)*(a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)), x)

[Out] Timed out

$$3.233 \quad \int \sqrt{\sec(c+dx)} (a+a \sec(c+dx))^{3/2} (A+B \sec(c+dx)) dx$$

Optimal. Leaf size=133

$$\frac{a^{3/2}(12A+7B) \sinh^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}}\right)}{4d} + \frac{a^2(4A+5B) \sin(c+dx) \sec^2(c+dx)}{4d\sqrt{a \sec(c+dx)+a}} + \frac{aB \sin(c+dx) \sec^2(c+dx)\sqrt{a \sec(c+dx)+a}}{2d}$$

[Out] $1/4*a^{(3/2)}*(12*A+7*B)*\operatorname{arcsinh}(a^{(1/2)}*\tan(d*x+c)/(a+a*\sec(d*x+c))^{(1/2)})/d$
 $+1/4*a^2*(4*A+5*B)*\sec(d*x+c)^{(3/2)}*\sin(d*x+c)/d/(a+a*\sec(d*x+c))^{(1/2)+1/2}$
 $*a*B*\sec(d*x+c)^{(3/2)}*\sin(d*x+c)*(a+a*\sec(d*x+c))^{(1/2)}/d$

Rubi [A] time = 0.34, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {4018, 4016, 3801, 215}

$$\frac{a^2(4A+5B) \sin(c+dx) \sec^2(c+dx)}{4d\sqrt{a \sec(c+dx)+a}} + \frac{a^{3/2}(12A+7B) \sinh^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}}\right)}{4d} + \frac{aB \sin(c+dx) \sec^2(c+dx)\sqrt{a \sec(c+dx)+a}}{2d}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Sqrt}[\operatorname{Sec}[c+d*x]]*(a+a*\operatorname{Sec}[c+d*x])^{(3/2)}*(A+B*\operatorname{Sec}[c+d*x]),x]$

[Out] $(a^{(3/2)}*(12*A+7*B)*\operatorname{ArcSinh}[(\operatorname{Sqrt}[a]*\operatorname{Tan}[c+d*x])/(\operatorname{Sqrt}[a+a*\operatorname{Sec}[c+d*x]])]/(4*d) + (a^2*(4*A+5*B)*\operatorname{Sec}[c+d*x]^{(3/2)}*\operatorname{Sin}[c+d*x])/(4*d*\operatorname{Sqrt}[a+a*\operatorname{Sec}[c+d*x]]) + (a*B*\operatorname{Sec}[c+d*x]^{(3/2)}*\operatorname{Sqrt}[a+a*\operatorname{Sec}[c+d*x]]*\operatorname{Sin}[c+d*x])/(2*d)$

Rule 215

$\operatorname{Int}[1/\operatorname{Sqrt}[(a_) + (b_)*(x_)^2], x_Symbol] \rightarrow \operatorname{Simp}[\operatorname{ArcSinh}[(\operatorname{Rt}[b, 2]*x)/\operatorname{Sqrt}[a]]/\operatorname{Rt}[b, 2], x] /; \operatorname{FreeQ}\{a, b\}, x \ \&\& \operatorname{GtQ}[a, 0] \ \&\& \operatorname{PosQ}[b]$

Rule 3801

$\operatorname{Int}[\operatorname{Sqrt}[\operatorname{csc}[(e_.) + (f_)*(x_)]*(d_.)]*\operatorname{Sqrt}[\operatorname{csc}[(e_.) + (f_)*(x_)]*(b_.) + (a_.)], x_Symbol] \rightarrow \operatorname{Dist}[(-2*a*\operatorname{Sqrt}[(a*d)/b])/(b*f), \operatorname{Subst}[\operatorname{Int}[1/\operatorname{Sqrt}[1+x^2/a], x], x, (b*\operatorname{Cot}[e+f*x])/(\operatorname{Sqrt}[a+b*\operatorname{Csc}[e+f*x]])], x] /; \operatorname{FreeQ}\{a, b, d, e, f\}, x \ \&\& \operatorname{EqQ}[a^2 - b^2, 0] \ \&\& \operatorname{GtQ}[(a*d)/b, 0]$

Rule 4016

$\operatorname{Int}[(\operatorname{csc}[(e_.) + (f_)*(x_)]*(d_.))^{(n_)}*\operatorname{Sqrt}[\operatorname{csc}[(e_.) + (f_)*(x_)]*(b_.) + (a_.)]*(\operatorname{csc}[(e_.) + (f_)*(x_)]*(B_.) + (A_.)), x_Symbol] \rightarrow \operatorname{Simp}[(-2*b*B*$

```

Cot[e + f*x]*(d*Csc[e + f*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
+ Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x]]
]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && !
LtQ[n, 0]

```

Rule 4018

```

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (
a_))^m*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && !LtQ[n, -1]

```

Rubi steps

$$\begin{aligned}
\int \sqrt{\sec(c + dx)} (a + a \sec(c + dx))^{3/2} (A + B \sec(c + dx)) dx &= \frac{aB \sec^2(c + dx) \sqrt{a + a \sec(c + dx)} \sin(c + dx)}{2d} + \\
&= \frac{a^2(4A + 5B) \sec^2(c + dx) \sin(c + dx)}{4d \sqrt{a + a \sec(c + dx)}} + \frac{aB \sec^2(c + dx)}{4d} \\
&= \frac{a^2(4A + 5B) \sec^2(c + dx) \sin(c + dx)}{4d \sqrt{a + a \sec(c + dx)}} + \frac{aB \sec^2(c + dx)}{4d} \\
&= \frac{a^{3/2}(12A + 7B) \sinh^{-1}\left(\frac{\sqrt{a} \tan(c + dx)}{\sqrt{a + a \sec(c + dx)}}\right)}{4d} + \frac{a^2(4A + 5B) \sec^2(c + dx) \sin(c + dx)}{4d \sqrt{a + a \sec(c + dx)}} + \frac{aB \sec^2(c + dx)}{4d}
\end{aligned}$$

Mathematica [A] time = 0.73, size = 107, normalized size = 0.80

$$\frac{a \sec\left(\frac{1}{2}(c + dx)\right) \sqrt{a(\sec(c + dx) + 1)} \left(\sqrt{2}(12A + 7B) \tanh^{-1}\left(\sqrt{2} \sin\left(\frac{1}{2}(c + dx)\right)\right)\right) + 2 \sin\left(\frac{1}{2}(c + dx)\right) \sec(c + dx)}{8d \sqrt{\sec(c + dx)}}$$

Antiderivative was successfully verified.

```

[In] Integrate[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]), x]

```

```
[Out] (a*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(Sqrt[2]*(12*A + 7*B)*ArcTan
h[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sec[c + d*x]*(4*A + 7*B + 2*B*Sec[c + d*x])
*Sin[(c + d*x)/2]))/(8*d*Sqrt[Sec[c + d*x]])
```

fricas [A] time = 0.61, size = 410, normalized size = 3.08

$$\frac{\left((12A + 7B)a \cos(dx + c)^2 + (12A + 7B)a \cos(dx + c) \right) \sqrt{a} \log \left(\frac{a \cos(dx+c)^3 - 7a \cos(dx+c)^2 - \frac{4(\cos(dx+c)^2 - 2\cos(dx+c))\sqrt{a}}{\sqrt{\cos(dx+c)}}}{\cos(dx+c)^3 + \cos(dx+c)^2} \right)}{16 \left(d \cos(dx + c)^2 + d \cos(dx + c) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x, algo
rithm="fricas")
```

```
[Out] [1/16*(((12*A + 7*B)*a*cos(d*x + c)^2 + (12*A + 7*B)*a*cos(d*x + c))*sqrt(a)
)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*
x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(c
os(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*((4*A + 7*B)*a*c
os(d*x + c) + 2*B*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/s
qrt(cos(d*x + c)))/(d*cos(d*x + c)^2 + d*cos(d*x + c)), 1/8*(((12*A + 7*B)*
a*cos(d*x + c)^2 + (12*A + 7*B)*a*cos(d*x + c))*sqrt(-a)*arctan(2*sqrt(-a)*
sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*
cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) + 2*((4*A + 7*B)*a*cos(d*x + c) + 2
*B*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c
)))/(d*cos(d*x + c)^2 + d*cos(d*x + c))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(a \sec(dx + c) + a)^{\frac{3}{2}} \sqrt{\sec(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x, algo
rithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(3/2)*sqrt(sec(d*x + c)
), x)
```

maple [B] time = 2.11, size = 353, normalized size = 2.65

$$(-1 + \cos(dx + c)) \left(12A (\cos^2(dx + c)) \arctan \left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}} (\cos(dx+c)+1-\sin(dx+c))\sqrt{2}}{4} \right) \sqrt{2} - 12A (\cos^2(dx + c)) \right) a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x)

[Out] 1/8/d*(-1+cos(d*x+c))*(12*A*cos(d*x+c)^2*arctan(1/4*(-2/(1+cos(d*x+c))))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))*2^(1/2))*2^(1/2)-12*A*cos(d*x+c)^2*arctan(1/4*(-2/(1+cos(d*x+c))))^(1/2)*(cos(d*x+c)+1+sin(d*x+c))*2^(1/2))*2^(1/2)+7*B*cos(d*x+c)^2*arctan(1/4*(-2/(1+cos(d*x+c))))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))*2^(1/2))*2^(1/2)-7*B*cos(d*x+c)^2*arctan(1/4*(-2/(1+cos(d*x+c))))^(1/2)*(cos(d*x+c)+1+sin(d*x+c))*2^(1/2))*2^(1/2)-8*A*sin(d*x+c)*cos(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)-14*B*sin(d*x+c)*cos(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)-4*B*(-2/(1+cos(d*x+c)))^(1/2)*sin(d*x+c))*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(1/2)/cos(d*x+c)/sin(d*x+c)^2/(-2/(1+cos(d*x+c)))^(1/2)*a

maxima [B] time = 1.52, size = 3389, normalized size = 25.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x, algorith="maxima")

[Out] 1/16*(4*(3*(a*log(2*cos(1/2*d*x + 1/2*c))^2 + 2*sin(1/2*d*x + 1/2*c))^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a*log(2*cos(1/2*d*x + 1/2*c))^2 + 2*sin(1/2*d*x + 1/2*c))^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a*log(2*cos(1/2*d*x + 1/2*c))^2 + 2*sin(1/2*d*x + 1/2*c))^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a*log(2*cos(1/2*d*x + 1/2*c))^2 + 2*sin(1/2*d*x + 1/2*c))^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c)^2 + 3*(a*log(2*cos(1/2*d*x + 1/2*c))^2 + 2*sin(1/2*d*x + 1/2*c))^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a*log(2*cos(1/2*d*x + 1/2*c))^2 + 2*sin(1/2*d*x + 1/2*c))^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a*log(2*cos(1/2*d*x + 1/2*c))^2 + 2*sin(1/2*d*x + 1/2*c))^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a*log(2*cos(1/2*d*x + 1/2*c))^2 + 2*sin(1/2*d*x + 1/2*c))^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*a*sin(3/2*d*x + 3/2*c)

$$\begin{aligned}
& x + 3/2*c) - 4*\sqrt{2}*a*\sin(1/2*d*x + 1/2*c) + 2*(2*\sqrt{2}*a*\sin(3/2*d*x \\
& + 3/2*c) - 2*\sqrt{2}*a*\sin(1/2*d*x + 1/2*c) + 3*a*\log(2*\cos(1/2*d*x + 1/2*c) \\
&)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2} \\
& *\sin(1/2*d*x + 1/2*c) + 2) - 3*a*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d \\
& *x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/ \\
& 2*c) + 2) + 3*a*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2 \\
& *\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 3*a*\log \\
& (2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2* \\
& d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2))*\cos(2*d*x + 2*c) + 3*a* \\
& \log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2 \\
& *d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 3*a*\log(2*\cos(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2 \\
& *\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + 3*a*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*s \\
& in(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2* \\
& d*x + 1/2*c) + 2) - 3*a*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2* \\
& c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) \\
& - 4*(\sqrt{2}*a*\cos(3/2*d*x + 3/2*c) - \sqrt{2}*a*\cos(1/2*d*x + 1/2*c))*\sin(\\
& 2*d*x + 2*c))*A*\sqrt{a}/(\cos(2*d*x + 2*c)^2 + \sin(2*d*x + 2*c)^2 + 2*\cos(2* \\
& d*x + 2*c) + 1) - (56*\sqrt{2}*a*\cos(7/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3 \\
& /2*d*x + 3/2*c)))*\sin(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c) \\
&))) - 24*\sqrt{2}*a*\cos(5/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2* \\
& c)))*\sin(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) - 12*\sqrt{2} \\
& (2)*a*\sin(3/2*d*x + 3/2*c) + 28*\sqrt{2}*a*\sin(1/3*\arctan2(\sin(3/2*d*x + 3/2 \\
& *c), \cos(3/2*d*x + 3/2*c))) - 4*(3*\sqrt{2}*a*\sin(3/2*d*x + 3/2*c) + 7*\sqrt{2} \\
& (2)*a*\sin(7/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) - 3*\sqrt{2} \\
& (2)*a*\sin(5/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) - 7*\sqrt{2} \\
& (2)*a*\sin(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))))*\cos(8/3* \\
& arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) - 8*(3*\sqrt{2}*a*\sin(3 \\
& /2*d*x + 3/2*c) - 7*\sqrt{2}*a*\sin(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2 \\
& *d*x + 3/2*c))))*\cos(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c) \\
&)) - 7*(a*\cos(8/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + \\
& 4*a*\cos(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + a*\sin(\\
& 8/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + 4*a*\sin(8/3*ar \\
& ctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))*\sin(4/3*\arctan2(\sin(3/2* \\
& d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 4*a*\sin(4/3*\arctan2(\sin(3/2*d*x + 3/ \\
& 2*c), \cos(3/2*d*x + 3/2*c)))^2 + 2*(2*a*\cos(4/3*\arctan2(\sin(3/2*d*x + 3/2*c) \\
&), \cos(3/2*d*x + 3/2*c))) + a*\cos(8/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/ \\
& 2*d*x + 3/2*c))) + 4*a*\cos(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + \\
& 3/2*c))) + a*\log(2*\cos(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2 \\
& *c))))^2 + 2*\sin(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 \\
& + 2*\sqrt{2}*\cos(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + \\
& 2*\sqrt{2}*\sin(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 2) \\
& + 7*(a*\cos(8/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + 4* \\
& a*\cos(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + a*\sin(8/ \\
& 3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + 4*a*\sin(8/3*arct
\end{aligned}$$

$$\begin{aligned}
& \text{an2}(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)) * \sin(4/3*\arctan2(\sin(3/2*d*x \\
& + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 4*a*\sin(4/3*\arctan2(\sin(3/2*d*x + 3/2* \\
& c), \cos(3/2*d*x + 3/2*c)))^2 + 2*(2*a*\cos(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \\
& \cos(3/2*d*x + 3/2*c))) + a)*\cos(8/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2* \\
& d*x + 3/2*c))) + 4*a*\cos(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/ \\
& 2*c))) + a)*\log(2*\cos(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c) \\
&)))^2 + 2*\sin(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + \\
& 2*\sqrt{2}*\cos(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) - 2* \\
& \sqrt{2}*\sin(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 2) - \\
& 7*(a*\cos(8/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + 4*a* \\
& \cos(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + a*\sin(8/3* \\
& \arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + 4*a*\sin(8/3*\arctan \\
& 2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) * \sin(4/3*\arctan2(\sin(3/2*d*x \\
& + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 4*a*\sin(4/3*\arctan2(\sin(3/2*d*x + 3/2*c) \\
& , \cos(3/2*d*x + 3/2*c)))^2 + 2*(2*a*\cos(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), c \\
& os(3/2*d*x + 3/2*c))) + a)*\cos(8/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d* \\
& x + 3/2*c))) + 4*a*\cos(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2* \\
& c))) + a)*\log(2*\cos(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c) \\
&)))^2 + 2*\sin(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 - 2* \\
& \sqrt{2}*\cos(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 2*\sqrt{ \\
& rt(2)*\sin(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 2) + 7 \\
& *(a*\cos(8/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + 4*a*co \\
& s(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + a*\sin(8/3*ar \\
& ctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + 4*a*\sin(8/3*\arctan2(\\
& \sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) * \sin(4/3*\arctan2(\sin(3/2*d*x + \\
& 3/2*c), \cos(3/2*d*x + 3/2*c))) + 4*a*\sin(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \\
& \cos(3/2*d*x + 3/2*c)))^2 + 2*(2*a*\cos(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos \\
& (3/2*d*x + 3/2*c))) + a)*\cos(8/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x \\
& + 3/2*c))) + 4*a*\cos(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c) \\
&)) + a)*\log(2*\cos(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^ \\
& 2 + 2*\sin(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 - 2*\sqrt{ \\
& rt(2)*\cos(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) - 2*\sqrt{ \\
& (2)*\sin(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 2) + 4*(\\
& 3*\sqrt{2}*a*\cos(3/2*d*x + 3/2*c) + 7*\sqrt{2}*a*\cos(7/3*\arctan2(\sin(3/2*d*x \\
& + 3/2*c), \cos(3/2*d*x + 3/2*c))) - 3*\sqrt{2}*a*\cos(5/3*\arctan2(\sin(3/2*d*x \\
& + 3/2*c), \cos(3/2*d*x + 3/2*c))) - 7*\sqrt{2}*a*\cos(1/3*\arctan2(\sin(3/2*d*x \\
& + 3/2*c), \cos(3/2*d*x + 3/2*c)))) * \sin(8/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos \\
& (3/2*d*x + 3/2*c))) - 28*(2*\sqrt{2}*a*\cos(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \\
& \cos(3/2*d*x + 3/2*c))) + \sqrt{2}*a)*\sin(7/3*\arctan2(\sin(3/2*d*x + 3/2*c), \\
& \cos(3/2*d*x + 3/2*c))) + 12*(2*\sqrt{2}*a*\cos(4/3*\arctan2(\sin(3/2*d*x + 3/2* \\
& c), \cos(3/2*d*x + 3/2*c))) + \sqrt{2}*a)*\sin(5/3*\arctan2(\sin(3/2*d*x + 3/2*c) \\
&), \cos(3/2*d*x + 3/2*c))) + 8*(3*\sqrt{2}*a*\cos(3/2*d*x + 3/2*c) - 7*\sqrt{2} \\
& *a*\cos(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))) * \sin(4/3*ar \\
& ctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) * B*\sqrt{a}/(2*(2*\cos(4/3 \\
& *\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 1)*\cos(8/3*\arctan2(
\end{aligned}$$

$\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)) + \cos(8/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + 4*\cos(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + \sin(8/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + 4*\sin(8/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))*\sin(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 4*\sin(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + 4*\cos(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 1)/d$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A + \frac{B}{\cos(c + dx)} \right) \left(a + \frac{a}{\cos(c + dx)} \right)^{3/2} \sqrt{\frac{1}{\cos(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(1/2), x)

[Out] int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(1/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c))*sec(d*x+c)**(1/2), x)

[Out] Timed out

$$3.234 \quad \int \frac{(a+a \sec(c+dx))^{3/2}(A+B \sec(c+dx))}{\sqrt{\sec(c+dx)}} dx$$

Optimal. Leaf size=124

$$\frac{a^{3/2}(2A+3B) \sinh^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}}\right)}{d} + \frac{a^2(2A-B) \sin(c+dx) \sqrt{\sec(c+dx)}}{d \sqrt{a \sec(c+dx)+a}} + \frac{aB \sin(c+dx) \sqrt{\sec(c+dx)} \sqrt{a \sec(c+dx)}}{d}$$

[Out] $a^{(3/2)}*(2*A+3*B)*\operatorname{arcsinh}(a^{(1/2)}*\tan(d*x+c)/(a+a*\sec(d*x+c))^{(1/2)})/d+a^2*(2*A-B)*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/d/(a+a*\sec(d*x+c))^{(1/2)}+a*B*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}*(a+a*\sec(d*x+c))^{(1/2)}/d$

Rubi [A] time = 0.31, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {4018, 4015, 3801, 215}

$$\frac{a^2(2A-B) \sin(c+dx) \sqrt{\sec(c+dx)}}{d \sqrt{a \sec(c+dx)+a}} + \frac{a^{3/2}(2A+3B) \sinh^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}}\right)}{d} + \frac{aB \sin(c+dx) \sqrt{\sec(c+dx)} \sqrt{a \sec(c+dx)}}{d}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(a+a*\operatorname{Sec}[c+d*x])^{(3/2)}*(A+B*\operatorname{Sec}[c+d*x])]/\operatorname{Sqrt}[\operatorname{Sec}[c+d*x]],x]$
 [Out] $(a^{(3/2)}*(2*A+3*B)*\operatorname{ArcSinh}[(\operatorname{Sqrt}[a]*\operatorname{Tan}[c+d*x])/\operatorname{Sqrt}[a+a*\operatorname{Sec}[c+d*x]])]/d+(a^2*(2*A-B)*\operatorname{Sqrt}[\operatorname{Sec}[c+d*x]]*\operatorname{Sin}[c+d*x])/(d*\operatorname{Sqrt}[a+a*\operatorname{Sec}[c+d*x]])+(a*B*\operatorname{Sqrt}[\operatorname{Sec}[c+d*x]]*\operatorname{Sqrt}[a+a*\operatorname{Sec}[c+d*x]]*\operatorname{Sin}[c+d*x])/d$

Rule 215

$\operatorname{Int}[1/\operatorname{Sqrt}[(a_)+(b_)*(x_)^2], x_Symbol] \rightarrow \operatorname{Simp}[\operatorname{ArcSinh}[(\operatorname{Rt}[b, 2]*x)/\operatorname{Sqrt}[a]]/\operatorname{Rt}[b, 2], x] /; \operatorname{FreeQ}\{a, b, x\} \&\& \operatorname{GtQ}[a, 0] \&\& \operatorname{PosQ}[b]$

Rule 3801

$\operatorname{Int}[\operatorname{Sqrt}[\operatorname{csc}[(e_)+(f_)*(x_)]*(d_)]*\operatorname{Sqrt}[\operatorname{csc}[(e_)+(f_)*(x_)]*(b_)+(a_)], x_Symbol] \rightarrow \operatorname{Dist}[(-2*a*\operatorname{Sqrt}[(a*d)/b])/(b*f), \operatorname{Subst}[\operatorname{Int}[1/\operatorname{Sqrt}[1+x^2/a], x], x, (b*\operatorname{Cot}[e+f*x])/\operatorname{Sqrt}[a+b*\operatorname{Csc}[e+f*x]]], x] /; \operatorname{FreeQ}\{a, b, d, e, f\}, x\} \&\& \operatorname{EqQ}[a^2-b^2, 0] \&\& \operatorname{GtQ}[(a*d)/b, 0]$

Rule 4015

$\operatorname{Int}[(\operatorname{csc}[(e_)+(f_)*(x_)]*(d_))^{(n_)}*\operatorname{Sqrt}[\operatorname{csc}[(e_)+(f_)*(x_)]*(b_)+(a_)], x_Symbol] \rightarrow \operatorname{Simp}[(A*b^2*C$

```
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
+ f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (
a_.))^m*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && !LtQ[n, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{(a + a \sec(c + dx))^{3/2} (A + B \sec(c + dx))}{\sqrt{\sec(c + dx)}} dx &= \frac{aB \sqrt{\sec(c + dx)} \sqrt{a + a \sec(c + dx)} \sin(c + dx)}{d} + \int \frac{\sqrt{a + a \sec(c + dx)}}{\sqrt{\sec(c + dx)}} dx \\ &= \frac{a^2 (2A - B) \sqrt{\sec(c + dx)} \sin(c + dx)}{d \sqrt{a + a \sec(c + dx)}} + \frac{aB \sqrt{\sec(c + dx)} \sqrt{a + a \sec(c + dx)}}{d \sqrt{a + a \sec(c + dx)}} \\ &= \frac{a^2 (2A - B) \sqrt{\sec(c + dx)} \sin(c + dx)}{d \sqrt{a + a \sec(c + dx)}} + \frac{aB \sqrt{\sec(c + dx)} \sqrt{a + a \sec(c + dx)}}{d \sqrt{a + a \sec(c + dx)}} \\ &= \frac{a^{3/2} (2A + 3B) \sinh^{-1} \left(\frac{\sqrt{a} \tan(c + dx)}{\sqrt{a + a \sec(c + dx)}} \right)}{d} + \frac{a^2 (2A - B) \sqrt{\sec(c + dx)}}{d \sqrt{a + a \sec(c + dx)}} \end{aligned}$$

Mathematica [A] time = 1.74, size = 107, normalized size = 0.86

$$\frac{a^2 \tan(c + dx) \left(\sqrt{(\cos(c + dx) - 1) \sec^2(c + dx)} (2A \cos(c + dx) + B) + 2A \sin^{-1} \left(\sqrt{1 - \sec(c + dx)} \right) - 3B \sin^{-1} \left(\sqrt{1 - \sec(c + dx)} \right) \right)}{d \sqrt{1 - \sec(c + dx)} \sqrt{a(\sec(c + dx) + 1)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]))/Sqrt[Sec[c + d*
x]], x]
```

[Out] $(a^2(2A \operatorname{ArcSin}[\sqrt{1 - \operatorname{Sec}[c + dx]]] - 3B \operatorname{ArcSin}[\sqrt{\operatorname{Sec}[c + dx]}]) + (B + 2A \cos[c + dx]) \sqrt{(-1 + \cos[c + dx]) \operatorname{Sec}[c + dx]^2}) \operatorname{Tan}[c + dx]) / (d \sqrt{1 - \operatorname{Sec}[c + dx]} \sqrt{a(1 + \operatorname{Sec}[c + dx])})$

fricas [A] time = 0.54, size = 364, normalized size = 2.94

$$\frac{((2A + 3B)a \cos(dx + c) + (2A + 3B)a) \sqrt{a} \log \left(\frac{a \cos(dx+c)^3 - 7a \cos(dx+c)^2 - \frac{4(\cos(dx+c)^2 - 2\cos(dx+c)) \sqrt{a} \sqrt{\frac{a \cos(dx+c)+a}{\cos(dx+c)}} \sin(dx+c)}{\cos(dx+c)^3 + \cos(dx+c)^2}}{\sqrt{\cos(dx+c)}} \right)}{4(d \cos(dx + c) + d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="fricas")`

[Out] $[1/4 * (((2A + 3B) * a * \cos(dx + c) + (2A + 3B) * a) * \sqrt{a} * \log((a * \cos(dx + c)^3 - 7 * a * \cos(dx + c)^2 - 4 * (\cos(dx + c)^2 - 2 * \cos(dx + c)) * \sqrt{a} * \sqrt{\frac{a * \cos(dx + c) + a}{\cos(dx + c)}} * \sin(dx + c) / \sqrt{\cos(dx + c)} + 8 * a) / (\cos(dx + c)^3 + \cos(dx + c)^2)) + 4 * (2 * A * a * \cos(dx + c) + B * a) * \sqrt{(a * \cos(dx + c) + a) / \cos(dx + c)} * \sin(dx + c) / \sqrt{\cos(dx + c)}) / (d * \cos(dx + c) + d), 1/2 * (((2A + 3B) * a * \cos(dx + c) + (2A + 3B) * a) * \sqrt{-a} * \arctan(2 * \sqrt{-a} * \sqrt{(a * \cos(dx + c) + a) / \cos(dx + c)} * \sqrt{\cos(dx + c)}) * \sin(dx + c) / (a * \cos(dx + c)^2 - a * \cos(dx + c) - 2 * a)) + 2 * (2 * A * a * \cos(dx + c) + B * a) * \sqrt{(a * \cos(dx + c) + a) / \cos(dx + c)} * \sin(dx + c) / \sqrt{\cos(dx + c)}) / (d * \cos(dx + c) + d)]$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(a \sec(dx + c) + a)^{\frac{3}{2}}}{\sqrt{\sec(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="giac")`

[Out] `integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(3/2)/sqrt(sec(d*x + c)), x)`

maple [B] time = 2.54, size = 346, normalized size = 2.79

$$\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}} \left(2A \cos(dx+c) \sin(dx+c) \sqrt{\frac{2}{1+\cos(dx+c)}} \arctan \left(\frac{\sqrt{\frac{2}{1+\cos(dx+c)}} (\cos(dx+c)+1+\sin(dx+c)) \sqrt{2}}{4}} \right) \sqrt{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x)

[Out] -1/4/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(2*A*cos(d*x+c)*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d*x+c))*2^(1/2))*2^(1/2)-2*A*cos(d*x+c)*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))*2^(1/2))*2^(1/2)+3*B*cos(d*x+c)*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d*x+c))*2^(1/2))*2^(1/2)-3*B*cos(d*x+c)*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))*2^(1/2))*2^(1/2)+8*A*cos(d*x+c)^2-8*A*cos(d*x+c)+4*B*cos(d*x+c)-4*B)*(1/cos(d*x+c))^(1/2)/sin(d*x+c)*a

maxima [B] time = 1.30, size = 1417, normalized size = 11.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorith="maxima")

[Out] 1/4*(sqrt(2)*(sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 8*a*sin(1/2*d*x + 1/2*c))*A*sqrt(a) + (3*(a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c)^2 + 3*(a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))

$2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*a*sin(3/2*d*x + 3/2*c) - 4*sqrt(2)*a*sin(1/2*d*x + 1/2*c) + 2*(2*sqrt(2)*a*sin(3/2*d*x + 3/2*c) - 2*sqrt(2)*a*sin(1/2*d*x + 1/2*c) + 3*a*log(2*cos(1/2*d*x + 1/2*c))^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 4*(sqrt(2)*a*cos(3/2*d*x + 3/2*c) - sqrt(2)*a*cos(1/2*d*x + 1/2*c))*sin(2*d*x + 2*c))*B*sqrt(a)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1))/d$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(a + \frac{a}{\cos(c+dx)}\right)^{3/2}}{\sqrt{\frac{1}{\cos(c+dx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(3/2))/(1/cos(c + d*x))^(1/2), x)

[Out] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(3/2))/(1/cos(c + d*x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a(\sec(c + dx) + 1))^{\frac{3}{2}} (A + B \sec(c + dx))}{\sqrt{\sec(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**(3/2)*(A + B*sec(c + d*x))/sqrt(sec(c + d*x)), x)
```

$$3.235 \quad \int \frac{(a+a \sec(c+dx))^{3/2}(A+B \sec(c+dx))}{\sec^2(c+dx)} dx$$

Optimal. Leaf size=125

$$\frac{2a^{3/2}B \sinh^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}}\right)}{d} + \frac{2a^2(4A+3B) \sin(c+dx) \sqrt{\sec(c+dx)}}{3d\sqrt{a \sec(c+dx)+a}} + \frac{2aA \sin(c+dx) \sqrt{a \sec(c+dx)+a}}{3d\sqrt{\sec(c+dx)}}$$

[Out] $2a^{3/2}B \operatorname{arcsinh}(a^{1/2} \tan(dx+c) / (a+a \sec(dx+c))^{1/2}) / d + 2/3 a^2 (4A+3B) \sin(dx+c) \sec(dx+c)^{1/2} / d (a+a \sec(dx+c))^{1/2} + 2/3 a A \sin(dx+c) (a+a \sec(dx+c))^{1/2} / d \sec(dx+c)^{1/2}$

Rubi [A] time = 0.34, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {4017, 4015, 3801, 215}

$$\frac{2a^2(4A+3B) \sin(c+dx) \sqrt{\sec(c+dx)}}{3d\sqrt{a \sec(c+dx)+a}} + \frac{2a^{3/2}B \sinh^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}}\right)}{d} + \frac{2aA \sin(c+dx) \sqrt{a \sec(c+dx)+a}}{3d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(a + a \operatorname{Sec}[c + d*x])^{3/2} (A + B \operatorname{Sec}[c + d*x]) / \operatorname{Sec}[c + d*x]^{3/2}, x]$

[Out] $(2*a^{3/2}*B*\operatorname{ArcSinh}[(\operatorname{Sqrt}[a]*\operatorname{Tan}[c + d*x])/\operatorname{Sqrt}[a + a*\operatorname{Sec}[c + d*x]])/d + (2*a^2*(4*A + 3*B)*\operatorname{Sqrt}[\operatorname{Sec}[c + d*x]]*\operatorname{Sin}[c + d*x])/(3*d*\operatorname{Sqrt}[a + a*\operatorname{Sec}[c + d*x]]) + (2*a*A*\operatorname{Sqrt}[a + a*\operatorname{Sec}[c + d*x]]*\operatorname{Sin}[c + d*x])/(3*d*\operatorname{Sqrt}[\operatorname{Sec}[c + d*x]])$

Rule 215

$\operatorname{Int}[1/\operatorname{Sqrt}[(a_) + (b_)*(x_)^2], x_Symbol] \rightarrow \operatorname{Simp}[\operatorname{ArcSinh}[\operatorname{Rt}[b, 2]*x]/\operatorname{Sqrt}[a]/\operatorname{Rt}[b, 2], x] /; \operatorname{FreeQ}\{a, b\}, x \ \&\& \operatorname{GtQ}[a, 0] \ \&\& \operatorname{PosQ}[b]$

Rule 3801

$\operatorname{Int}[\operatorname{Sqrt}[\operatorname{csc}[(e_) + (f_)*(x_)]*(d_)]*\operatorname{Sqrt}[\operatorname{csc}[(e_) + (f_)*(x_)]*(b_) + (a_)], x_Symbol] \rightarrow \operatorname{Dist}[(-2*a*\operatorname{Sqrt}[(a*d)/b])/(b*f), \operatorname{Subst}[\operatorname{Int}[1/\operatorname{Sqrt}[1 + x^2/a], x], x, (b*\operatorname{Cot}[e + f*x])/\operatorname{Sqrt}[a + b*\operatorname{Csc}[e + f*x]]], x] /; \operatorname{FreeQ}\{a, b, d, e, f\}, x \ \&\& \operatorname{EqQ}[a^2 - b^2, 0] \ \&\& \operatorname{GtQ}[(a*d)/b, 0]$

Rule 4015

$\operatorname{Int}[(\operatorname{csc}[(e_) + (f_)*(x_)]*(d_))^{(n_)}*\operatorname{Sqrt}[\operatorname{csc}[(e_) + (f_)*(x_)]*(b_) + (a_)]*(\operatorname{csc}[(e_) + (f_)*(x_)]*(B_) + (A_)), x_Symbol] \rightarrow \operatorname{Simp}[(A*b^2*c$

ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
 [(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
 B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]

Rule 4017

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> Simp[(a*A*Co
 t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
 t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
 [a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
 ; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]

Rubi steps

$$\int \frac{(a + a \sec(c + dx))^{3/2} (A + B \sec(c + dx))}{\sec^2(c + dx)} dx = \frac{2aA\sqrt{a + a \sec(c + dx)} \sin(c + dx)}{3d\sqrt{\sec(c + dx)}} + \frac{2}{3} \int \frac{\sqrt{a + a \sec(c + dx)}}{\sec(c + dx)} dx$$

$$= \frac{2a^2(4A + 3B)\sqrt{\sec(c + dx)} \sin(c + dx)}{3d\sqrt{a + a \sec(c + dx)}} + \frac{2aA\sqrt{a + a \sec(c + dx)}}{3d\sqrt{\sec(c + dx)}}$$

$$= \frac{2a^2(4A + 3B)\sqrt{\sec(c + dx)} \sin(c + dx)}{3d\sqrt{a + a \sec(c + dx)}} + \frac{2aA\sqrt{a + a \sec(c + dx)}}{3d\sqrt{\sec(c + dx)}}$$

$$= \frac{2a^{3/2}B \sinh^{-1}\left(\frac{\sqrt{a} \tan(c + dx)}{\sqrt{a + a \sec(c + dx)}}\right)}{d} + \frac{2a^2(4A + 3B)\sqrt{\sec(c + dx)} \sin(c + dx)}{3d\sqrt{a + a \sec(c + dx)}}$$

Mathematica [A] time = 0.66, size = 109, normalized size = 0.87

$$\frac{2a^2 \tan(c + dx) \left(\sqrt{1 - \sec(c + dx)} (A \cos(c + dx) + 5A + 3B) + 3B \sqrt{\sec(c + dx)} \sin^{-1} \left(\sqrt{1 - \sec(c + dx)} \right) \right)}{3d \sqrt{-((\sec(c + dx) - 1) \sec(c + dx))} \sqrt{a(\sec(c + dx) + 1)}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[((a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(3/2), x]

[Out] $(2*a^2*((5*A + 3*B + A*\cos[c + d*x])*Sqrt[1 - \text{Sec}[c + d*x]] + 3*B*\text{ArcSin}[Sqrt[1 - \text{Sec}[c + d*x]]]*Sqrt[\text{Sec}[c + d*x]]*\text{Tan}[c + d*x])/(3*d*Sqrt[-((-1 + \text{Sec}[c + d*x])*\text{Sec}[c + d*x]])*Sqrt[a*(1 + \text{Sec}[c + d*x])])$

fricas [A] time = 0.47, size = 368, normalized size = 2.94

$$\frac{3(Ba \cos(dx + c) + Ba)\sqrt{a} \log\left(\frac{a \cos(dx+c)^3 - 7a \cos(dx+c)^2 - \frac{4(\cos(dx+c)^2 - 2 \cos(dx+c))\sqrt{a} \sqrt{\frac{a \cos(dx+c)+a}{\cos(dx+c)}} \sin(dx+c)}{\sqrt{\cos(dx+c)}} + 8a}{\cos(dx+c)^3 + \cos(dx+c)^2}\right) + \frac{4(Aa \cos(dx+c) + A)}{6(d \cos(dx+c) + d)}}{6(d \cos(dx+c) + d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="fricas")`

[Out] $[1/6*(3*(B*a*\cos(d*x + c) + B*a)*\text{sqrt}(a)*\log((a*\cos(d*x + c))^3 - 7*a*\cos(d*x + c)^2 - 4*(\cos(d*x + c)^2 - 2*\cos(d*x + c))*\text{sqrt}(a)*\text{sqrt}((a*\cos(d*x + c) + a)/\cos(d*x + c))*\sin(d*x + c)/\text{sqrt}(\cos(d*x + c)) + 8*a)/(\cos(d*x + c)^3 + \cos(d*x + c)^2)) + 4*(A*a*\cos(d*x + c)^2 + (5*A + 3*B)*a*\cos(d*x + c))*\text{sqrt}((a*\cos(d*x + c) + a)/\cos(d*x + c))*\sin(d*x + c)/\text{sqrt}(\cos(d*x + c)))/(d*\cos(d*x + c) + d), 1/3*(3*(B*a*\cos(d*x + c) + B*a)*\text{sqrt}(-a)*\arctan(2*\text{sqrt}(-a))*\text{sqrt}((a*\cos(d*x + c) + a)/\cos(d*x + c))*\text{sqrt}(\cos(d*x + c))*\sin(d*x + c)/(a*\cos(d*x + c)^2 - a*\cos(d*x + c) - 2*a)) + 2*(A*a*\cos(d*x + c)^2 + (5*A + 3*B)*a*\cos(d*x + c))*\text{sqrt}((a*\cos(d*x + c) + a)/\cos(d*x + c))*\sin(d*x + c)/\text{sqrt}(\cos(d*x + c)))/(d*\cos(d*x + c) + d)]$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(a \sec(dx + c) + a)^{\frac{3}{2}}}{\sec(dx + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="giac")`

[Out] `integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(3/2), x)`

maple [A] time = 2.61, size = 211, normalized size = 1.69

$$\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}} \left(3B\sqrt{2} \arctan \left(\frac{\sqrt{\frac{2}{1+\cos(dx+c)}} (\cos(dx+c)+1+\sin(dx+c))\sqrt{2}}{4}} \right) \sqrt{\frac{2}{1+\cos(dx+c)}} \sin(dx+c) - 3B\sqrt{2} \arctan \left(\frac{\sqrt{\frac{2}{1+\cos(dx+c)}} (\cos(dx+c)-1-\sin(dx+c))\sqrt{2}}{4}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x)

[Out] -1/6/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(3*B*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c))))^(1/2)*(cos(d*x+c)+1+sin(d*x+c))*2^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-3*B*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c))))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))*2^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+4*A*cos(d*x+c)^2+16*A*cos(d*x+c)+12*B*cos(d*x+c)-20*A-12*B)*cos(d*x+c)^2*(1/cos(d*x+c))^(3/2)/sin(d*x+c)*a

maxima [B] time = 1.53, size = 314, normalized size = 2.51

$$3\sqrt{2} \left(\sqrt{2} a \log \left(2 \cos \left(\frac{1}{2} dx + \frac{1}{2} c \right)^2 + 2 \sin \left(\frac{1}{2} dx + \frac{1}{2} c \right)^2 + 2 \sqrt{2} \cos \left(\frac{1}{2} dx + \frac{1}{2} c \right) + 2 \sqrt{2} \sin \left(\frac{1}{2} dx + \frac{1}{2} c \right) + 2 \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="maxima")

[Out] 1/12*(3*sqrt(2)*(sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 8*a*sin(1/2*d*x + 1/2*c))*B*sqrt(a) + 4*(sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 9*sqrt(2)*a*sin(1/2*d*x + 1/2*c))*A*sqrt(a))/d

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)} \right) \left(a + \frac{a}{\cos(c+dx)} \right)^{3/2}}{\left(\frac{1}{\cos(c+dx)} \right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(3/2))/(1/cos(c + d*x))^(3/2), x)`

[Out] `int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(3/2))/(1/cos(c + d*x))^(3/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a(\sec(c + dx) + 1))^{\frac{3}{2}}(A + B \sec(c + dx))}{\sec^{\frac{3}{2}}(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)**(3/2), x)`

[Out] `Integral((a*(sec(c + d*x) + 1))**(3/2)*(A + B*sec(c + d*x))/sec(c + d*x)**(3/2), x)`

$$3.236 \quad \int \frac{(a+a \sec(c+dx))^{3/2}(A+B \sec(c+dx))}{5 \sec^2(c+dx)} dx$$

Optimal. Leaf size=131

$$\frac{8a^2(3A+5B) \sin(c+dx) \sqrt{\sec(c+dx)}}{15d \sqrt{a \sec(c+dx)+a}} + \frac{2a(3A+5B) \sin(c+dx) \sqrt{a \sec(c+dx)+a}}{15d \sqrt{\sec(c+dx)}} + \frac{2A \sin(c+dx)(a \sec(c+dx))^{3/2}}{5d \sec^2(c+dx)}$$

[Out] 2/5*A*(a+a*sec(d*x+c))^(3/2)*sin(d*x+c)/d/sec(d*x+c)^(3/2)+8/15*a^2*(3*A+5*B)*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^(1/2)+2/15*a*(3*A+5*B)*sin(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d/sec(d*x+c)^(1/2)

Rubi [A] time = 0.26, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {4013, 3809, 3804}

$$\frac{8a^2(3A+5B) \sin(c+dx) \sqrt{\sec(c+dx)}}{15d \sqrt{a \sec(c+dx)+a}} + \frac{2a(3A+5B) \sin(c+dx) \sqrt{a \sec(c+dx)+a}}{15d \sqrt{\sec(c+dx)}} + \frac{2A \sin(c+dx)(a \sec(c+dx))^{3/2}}{5d \sec^2(c+dx)}$$

Antiderivative was successfully verified.

[In] Int[((a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(5/2), x]

[Out] (8*a^2*(3*A + 5*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*(3*A + 5*B)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[Sec[c + d*x]]) + (2*A*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2))

Rule 3804

Int[Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 3809

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.), x_Symbol] :> -Simp[(a*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*m), x] + Dist[(b*(2*m - 1))/(d*m), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && GtQ[m, 1/2] && IntegerQ[2*m]

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{(a + a \sec(c + dx))^{3/2} (A + B \sec(c + dx))}{\sec^2(c + dx)} dx &= \frac{2A(a + a \sec(c + dx))^{3/2} \sin(c + dx)}{5d \sec^2(c + dx)} + \frac{1}{5}(3A + 5B) \int \frac{(a + a \sec(c + dx))^{3/2}}{\sec^2(c + dx)} dx \\ &= \frac{2a(3A + 5B)\sqrt{a + a \sec(c + dx)} \sin(c + dx)}{15d\sqrt{\sec(c + dx)}} + \frac{2A(a + a \sec(c + dx))^{3/2}}{5d \sec^2(c + dx)} \\ &= \frac{8a^2(3A + 5B)\sqrt{\sec(c + dx)} \sin(c + dx)}{15d\sqrt{a + a \sec(c + dx)}} + \frac{2a(3A + 5B)\sqrt{a + a \sec(c + dx)}}{15d\sqrt{\sec(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.51, size = 73, normalized size = 0.56

$$\frac{a^2 \sin(c + dx) \sqrt{\sec(c + dx)} (2(9A + 5B) \cos(c + dx) + 3A \cos(2(c + dx)) + 39A + 50B)}{15d\sqrt{a(\sec(c + dx) + 1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(5/2), x]
```

```
[Out] (a^2*(39*A + 50*B + 2*(9*A + 5*B)*Cos[c + d*x] + 3*A*Cos[2*(c + d*x)])*Sqrt[Sec[c + d*x]]*Sin[c + d*x]/(15*d*Sqrt[a*(1 + Sec[c + d*x])])
```

fricas [A] time = 0.42, size = 94, normalized size = 0.72

$$\frac{2 \left(3 A a \cos(dx + c)^3 + (9 A + 5 B) a \cos(dx + c)^2 + (18 A + 25 B) a \cos(dx + c) \right) \sqrt{\frac{a \cos(dx + c) + a}{\cos(dx + c)}} \sin(dx + c)}{15 (d \cos(dx + c) + d) \sqrt{\cos(dx + c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2), x, algorith="fricas")
```


[Out] $2/15*(3*A*a*\cos(d*x + c)^3 + (9*A + 5*B)*a*\cos(d*x + c)^2 + (18*A + 25*B)*a*\cos(d*x + c))*\sqrt{((a*\cos(d*x + c) + a)/\cos(d*x + c))*\sin(d*x + c)/((d*\cos(d*x + c) + d)*\sqrt{\cos(d*x + c)})}$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(a \sec(dx + c) + a)^{\frac{3}{2}}}{\sec(dx + c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="giac")`

[Out] `integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(5/2), x)`

maple [A] time = 2.62, size = 97, normalized size = 0.74

$$\frac{2(-1 + \cos(dx + c)) \left(3A \left(\cos^2(dx + c) \right) + 9A \cos(dx + c) + 5B \cos(dx + c) + 18A + 25B \right) \sqrt{\frac{a(1 + \cos(dx + c))}{\cos(dx + c)}}}{15d \sin(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x)`

[Out] $-2/15/d*(-1+\cos(d*x+c))*(3*A*\cos(d*x+c)^2+9*A*\cos(d*x+c)+5*B*\cos(d*x+c)+18*A+25*B)*(a*(1+\cos(d*x+c))/\cos(d*x+c))^{(1/2)}*\cos(d*x+c)^3*(1/\cos(d*x+c))^{(5/2)}/\sin(d*x+c)*a$

maxima [B] time = 1.50, size = 250, normalized size = 1.91

$$3\sqrt{2} \left(20a \cos\left(\frac{4}{5} \arctan\left(\sin\left(\frac{5}{2}dx + \frac{5}{2}c\right), \cos\left(\frac{5}{2}dx + \frac{5}{2}c\right)\right)\right) \sin\left(\frac{5}{2}dx + \frac{5}{2}c\right) + 5a \cos\left(\frac{2}{5} \arctan\left(\sin\left(\frac{5}{2}dx + \frac{5}{2}c\right), \cos\left(\frac{5}{2}dx + \frac{5}{2}c\right)\right)\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="maxima")`

[Out] $1/60*(3*\sqrt{2}*(20*a*\cos(4/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c)))*\sin(5/2*d*x + 5/2*c) + 5*a*\cos(2/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c)))*\sin(5/2*d*x + 5/2*c) - 20*a*\cos(5/2*d*x + 5/2*c)*\sin(4/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) - 5*a*\cos(5/2*d*x + 5/2*c))$

$$\frac{5}{2}c) \sin\left(\frac{2}{5} \arctan\left(\frac{\sin\left(\frac{5}{2}dx + \frac{5}{2}c\right)}{\cos\left(\frac{5}{2}dx + \frac{5}{2}c\right)}\right)\right) + 2a \sin\left(\frac{5}{2}dx + \frac{5}{2}c\right) + 5a \sin\left(\frac{3}{5} \arctan\left(\frac{\sin\left(\frac{5}{2}dx + \frac{5}{2}c\right)}{\cos\left(\frac{5}{2}dx + \frac{5}{2}c\right)}\right)\right) + 20a \sin\left(\frac{1}{5} \arctan\left(\frac{\sin\left(\frac{5}{2}dx + \frac{5}{2}c\right)}{\cos\left(\frac{5}{2}dx + \frac{5}{2}c\right)}\right)\right) \sqrt{a} + 20\left(\sqrt{2}a \sin\left(\frac{3}{2}dx + \frac{3}{2}c\right) + 9\sqrt{2}a \sin\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right) B \sqrt{a} \Big/ d$$

mupad [B] time = 3.40, size = 107, normalized size = 0.82

$$\frac{a \cos(c + dx) \sqrt{\frac{1}{\cos(c+dx)}} \sqrt{\frac{a(\cos(c+dx)+1)}{\cos(c+dx)}} (75 A \sin(c + dx) + 100 B \sin(c + dx) + 18 A \sin(2c + 2dx) + 3 A \sin(3c + 3dx) + 10 B \sin(2c + 2dx))}{30 d (\cos(c + dx) + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(3/2))/(1/cos(c + d*x))^(5/2), x)

[Out] (a*cos(c + d*x)*(1/cos(c + d*x))^(1/2)*((a*(cos(c + d*x) + 1))/cos(c + d*x))^(1/2)*(75*A*sin(c + d*x) + 100*B*sin(c + d*x) + 18*A*sin(2*c + 2*d*x) + 3*A*sin(3*c + 3*d*x) + 10*B*sin(2*c + 2*d*x)))/(30*d*(cos(c + d*x) + 1))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)**(5/2), x)

[Out] Timed out

$$3.237 \quad \int \frac{(a+a \sec(c+dx))^{3/2}(A+B \sec(c+dx))}{7 \sec^2(c+dx)} dx$$

Optimal. Leaf size=181

$$\frac{2a^2(8A+7B) \sin(c+dx)}{35d \sec^2(c+dx) \sqrt{a \sec(c+dx)+a}} + \frac{4a^2(52A+63B) \sin(c+dx) \sqrt{\sec(c+dx)}}{105d \sqrt{a \sec(c+dx)+a}} + \frac{2a^2(52A+63B) \sin(c+dx)}{105d \sqrt{\sec(c+dx)} \sqrt{a \sec(c+dx)+a}}$$

[Out] $2/35*a^2*(8*A+7*B)*\sin(d*x+c)/d/\sec(d*x+c)^{(3/2)}/(a+a*\sec(d*x+c))^{(1/2)+2/1}$
 $05*a^2*(52*A+63*B)*\sin(d*x+c)/d/\sec(d*x+c)^{(1/2)}/(a+a*\sec(d*x+c))^{(1/2)+4/1}$
 $05*a^2*(52*A+63*B)*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/d/(a+a*\sec(d*x+c))^{(1/2)+2/7}$
 $*a*A*\sin(d*x+c)*(a+a*\sec(d*x+c))^{(1/2)}/d/\sec(d*x+c)^{(5/2)}$

Rubi [A] time = 0.44, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {4017, 4015, 3805, 3804}

$$\frac{2a^2(8A+7B) \sin(c+dx)}{35d \sec^2(c+dx) \sqrt{a \sec(c+dx)+a}} + \frac{4a^2(52A+63B) \sin(c+dx) \sqrt{\sec(c+dx)}}{105d \sqrt{a \sec(c+dx)+a}} + \frac{2a^2(52A+63B) \sin(c+dx)}{105d \sqrt{\sec(c+dx)} \sqrt{a \sec(c+dx)+a}}$$

Antiderivative was successfully verified.

[In] Int[((a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(7/2), x]

[Out] $(2*a^2*(8*A+7*B)*\sin[c+d*x])/(35*d*\sec[c+d*x]^{(3/2)}*\sqrt{a+a*\sec[c+d*x]}) + (2*a^2*(52*A+63*B)*\sin[c+d*x])/(105*d*\sqrt{\sec[c+d*x]}*\sqrt{a+a*\sec[c+d*x]}) + (4*a^2*(52*A+63*B)*\sqrt{\sec[c+d*x]}*\sin[c+d*x])/(105*d*\sqrt{a+a*\sec[c+d*x]}) + (2*a*A*\sqrt{a+a*\sec[c+d*x]}*\sin[c+d*x])/(7*d*\sec[c+d*x]^{(5/2)})$

Rule 3804

Int[Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 3805

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.)
+ (a_)]*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_)), x_Symbol] := Simp[(A*b^2*Co
t[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
+ f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_)), x_Symbol] := Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a + a \sec(c + dx))^{3/2} (A + B \sec(c + dx))}{\sec^2(c + dx)} dx = \frac{2aA \sqrt{a + a \sec(c + dx)} \sin(c + dx)}{7d \sec^5(c + dx)} + \frac{2}{7} \int \frac{\sqrt{a + a \sec(c + dx)}}{\sec^2(c + dx)} dx$$

$$= \frac{2a^2(8A + 7B) \sin(c + dx)}{35d \sec^3(c + dx) \sqrt{a + a \sec(c + dx)}} + \frac{2aA \sqrt{a + a \sec(c + dx)}}{7d \sec^2(c + dx)}$$

$$= \frac{2a^2(8A + 7B) \sin(c + dx)}{35d \sec^3(c + dx) \sqrt{a + a \sec(c + dx)}} + \frac{2a^2(52A + 63B) \sin(c + dx)}{105d \sqrt{\sec(c + dx)} \sqrt{a + a \sec(c + dx)}}$$

$$= \frac{2a^2(8A + 7B) \sin(c + dx)}{35d \sec^3(c + dx) \sqrt{a + a \sec(c + dx)}} + \frac{2a^2(52A + 63B) \sin(c + dx)}{105d \sqrt{\sec(c + dx)} \sqrt{a + a \sec(c + dx)}}$$

Mathematica [A] time = 0.49, size = 92, normalized size = 0.51

$$\frac{2a^2 \sin(c + dx) \left((2(52A + 63B) \sec^3(c + dx) + (52A + 63B) \sec^2(c + dx) + 3(13A + 7B) \sec(c + dx) + 15A) \right)}{105d \sec^2(c + dx) \sqrt{a(\sec(c + dx) + 1)}}$$

Antiderivative was successfully verified.

[In] Integrate[((a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(7/2), x]

[Out] (2*a^2*(15*A + 3*(13*A + 7*B)*Sec[c + d*x] + (52*A + 63*B)*Sec[c + d*x]^2 + 2*(52*A + 63*B)*Sec[c + d*x]^3)*Sin[c + d*x]/(105*d*Sec[c + d*x]^(5/2)*Sqrt[a*(1 + Sec[c + d*x])])

fricas [A] time = 0.42, size = 113, normalized size = 0.62

$$\frac{2(15Aa \cos(dx+c)^4 + 3(13A+7B)a \cos(dx+c)^3 + (52A+63B)a \cos(dx+c)^2 + 2(52A+63B)a \cos(dx+c))}{105(d \cos(dx+c) + d)\sqrt{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2), x, algorithm="fricas")

[Out] 2/105*(15*A*a*cos(d*x + c)^4 + 3*(13*A + 7*B)*a*cos(d*x + c)^3 + (52*A + 63*B)*a*cos(d*x + c)^2 + 2*(52*A + 63*B)*a*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/((d*cos(d*x + c) + d)*sqrt(cos(d*x + c)))

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx+c) + A)(a \sec(dx+c) + a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2), x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(7/2), x)

maple [A] time = 2.60, size = 119, normalized size = 0.66

$$\frac{2(-1 + \cos(dx+c))\left(15A\left(\cos^3(dx+c)\right) + 39A\left(\cos^2(dx+c)\right) + 21B\left(\cos^2(dx+c)\right) + 52A\cos(dx+c) + 63B\right)}{105d \sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2), x)

[Out] $-2/105/d*(-1+\cos(dx+c))*(15A*\cos(dx+c)^3+39A*\cos(dx+c)^2+21B*\cos(dx+c)^2+52A*\cos(dx+c)+63B*\cos(dx+c)+104A+126B)*(a*(1+\cos(dx+c))/\cos(dx+c))^{1/2}*(1/\cos(dx+c))^{7/2}*\cos(dx+c)^4/\sin(dx+c)*a$

maxima [B] time = 1.27, size = 514, normalized size = 2.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sec(dx+c))^(3/2)*(A+B*sec(dx+c))/sec(dx+c)^(7/2),x, algorithm="maxima")`

[Out] $1/840*(\sqrt{2}*(735*a*\cos(6/7*\arctan2(\sin(7/2*d*x + 7/2*c), \cos(7/2*d*x + 7/2*c))) * \sin(7/2*d*x + 7/2*c) + 175*a*\cos(4/7*\arctan2(\sin(7/2*d*x + 7/2*c), \cos(7/2*d*x + 7/2*c))) * \sin(7/2*d*x + 7/2*c) + 63*a*\cos(2/7*\arctan2(\sin(7/2*d*x + 7/2*c), \cos(7/2*d*x + 7/2*c))) * \sin(7/2*d*x + 7/2*c) - 735*a*\cos(7/2*d*x + 7/2*c)*\sin(6/7*\arctan2(\sin(7/2*d*x + 7/2*c), \cos(7/2*d*x + 7/2*c))) - 175*a*\cos(7/2*d*x + 7/2*c)*\sin(4/7*\arctan2(\sin(7/2*d*x + 7/2*c), \cos(7/2*d*x + 7/2*c))) - 63*a*\cos(7/2*d*x + 7/2*c)*\sin(2/7*\arctan2(\sin(7/2*d*x + 7/2*c), \cos(7/2*d*x + 7/2*c))) + 30*a*\sin(7/2*d*x + 7/2*c) + 63*a*\sin(5/7*\arctan2(\sin(7/2*d*x + 7/2*c), \cos(7/2*d*x + 7/2*c))) + 175*a*\sin(3/7*\arctan2(\sin(7/2*d*x + 7/2*c), \cos(7/2*d*x + 7/2*c))) + 735*a*\sin(1/7*\arctan2(\sin(7/2*d*x + 7/2*c), \cos(7/2*d*x + 7/2*c))))*A*\sqrt{a} + 42*\sqrt{2}*(20*a*\cos(4/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) * \sin(5/2*d*x + 5/2*c) + 5*a*\cos(2/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) * \sin(5/2*d*x + 5/2*c) - 20*a*\cos(5/2*d*x + 5/2*c)*\sin(4/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) - 5*a*\cos(5/2*d*x + 5/2*c)*\sin(2/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) + 2*a*\sin(5/2*d*x + 5/2*c) + 5*a*\sin(3/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) + 20*a*\sin(1/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))))*B*\sqrt{a})/d$

mupad [B] time = 4.34, size = 131, normalized size = 0.72

$$\frac{a \cos(c + dx) \sqrt{\frac{1}{\cos(c+dx)}} \sqrt{\frac{a(\cos(c+dx)+1)}{\cos(c+dx)}} (910 A \sin(c + dx) + 1050 B \sin(c + dx) + 238 A \sin(2c + 2dx) + 420 d (\cos(c + dx) + 1))}{420 d (\cos(c + dx) + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(3/2))/(1/cos(c + d*x))^(7/2),x)`

[Out] $(a*\cos(c + d*x)*(1/\cos(c + d*x))^{1/2}*((a*(\cos(c + d*x) + 1))/\cos(c + d*x))^{1/2}*(910A*\sin(c + d*x) + 1050B*\sin(c + d*x) + 238A*\sin(2*c + 2*d*x) + 78A*\sin(3*c + 3*d*x) + 15A*\sin(4*c + 4*d*x) + 252*B*\sin(2*c + 2*d*x) + 42*B*\sin(3*c + 3*d*x)))/(420*d*(\cos(c + d*x) + 1))$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)**(7/2),x)

[Out] Timed out

$$3.238 \quad \int \frac{(a+a \sec(c+dx))^{3/2}(A+B \sec(c+dx))}{\sec^2(c+dx)} dx$$

Optimal. Leaf size=228

$$\frac{2a^2(34A+39B)\sin(c+dx)}{105d \sec^2(c+dx)\sqrt{a \sec(c+dx)+a}} + \frac{2a^2(10A+9B)\sin(c+dx)}{63d \sec^2(c+dx)\sqrt{a \sec(c+dx)+a}} + \frac{16a^2(34A+39B)\sin(c+dx)\sqrt{\sec(c+dx)}}{315d\sqrt{a \sec(c+dx)+a}}$$

[Out] 2/63*a^2*(10*A+9*B)*sin(d*x+c)/d/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2)+2/105*a^2*(34*A+39*B)*sin(d*x+c)/d/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2)+8/315*a^2*(34*A+39*B)*sin(d*x+c)/d/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+16/315*a^2*(34*A+39*B)*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^(1/2)+2/9*a*A*sin(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d/sec(d*x+c)^(7/2)

Rubi [A] time = 0.51, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {4017, 4015, 3805, 3804}

$$\frac{2a^2(34A+39B)\sin(c+dx)}{105d \sec^2(c+dx)\sqrt{a \sec(c+dx)+a}} + \frac{2a^2(10A+9B)\sin(c+dx)}{63d \sec^2(c+dx)\sqrt{a \sec(c+dx)+a}} + \frac{16a^2(34A+39B)\sin(c+dx)\sqrt{\sec(c+dx)}}{315d\sqrt{a \sec(c+dx)+a}}$$

Antiderivative was successfully verified.

[In] Int[((a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(9/2), x]

[Out] (2*a^2*(10*A + 9*B)*Sin[c + d*x])/(63*d*Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2*(34*A + 39*B)*Sin[c + d*x])/(105*d*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (8*a^2*(34*A + 39*B)*Sin[c + d*x])/(315*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (16*a^2*(34*A + 39*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(315*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*A*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2))

Rule 3804

Int[Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 3805

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&

EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]

Rule 4015

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := Simp[(A*b^2*Co
 t[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
 [(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
 B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]

Rule 4017

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := Simp[(a*A*Co
 t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
 t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
 [a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
 ; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]

Rubi steps

$$\int \frac{(a + a \sec(c + dx))^{3/2} (A + B \sec(c + dx))}{\sec^2(c + dx)} dx = \frac{2aA\sqrt{a + a \sec(c + dx)} \sin(c + dx)}{9d \sec^2(c + dx)} + \frac{2}{9} \int \frac{\sqrt{a + a \sec(c + dx)}}{\sec^2(c + dx)} dx$$

$$= \frac{2a^2(10A + 9B) \sin(c + dx)}{63d \sec^2(c + dx) \sqrt{a + a \sec(c + dx)}} + \frac{2aA\sqrt{a + a \sec(c + dx)}}{9d \sec^2(c + dx)}$$

$$= \frac{2a^2(10A + 9B) \sin(c + dx)}{63d \sec^2(c + dx) \sqrt{a + a \sec(c + dx)}} + \frac{2a^2(34A + 39B) \sin(c + dx)}{105d \sec^2(c + dx) \sqrt{a + a \sec(c + dx)}}$$

$$= \frac{2a^2(10A + 9B) \sin(c + dx)}{63d \sec^2(c + dx) \sqrt{a + a \sec(c + dx)}} + \frac{2a^2(34A + 39B) \sin(c + dx)}{105d \sec^2(c + dx) \sqrt{a + a \sec(c + dx)}}$$

$$= \frac{2a^2(10A + 9B) \sin(c + dx)}{63d \sec^2(c + dx) \sqrt{a + a \sec(c + dx)}} + \frac{2a^2(34A + 39B) \sin(c + dx)}{105d \sec^2(c + dx) \sqrt{a + a \sec(c + dx)}}$$

Mathematica [A] time = 0.67, size = 110, normalized size = 0.48

$$\frac{2a^2 \sin(c + dx) \left(8(34A + 39B) \sec^4(c + dx) + 4(34A + 39B) \sec^3(c + dx) + 3(34A + 39B) \sec^2(c + dx) + 5(17A + 9B) \sec(c + dx) \right)}{315d \sec^{\frac{7}{2}}(c + dx) \sqrt{a(\sec(c + dx) + 1)}}$$

Antiderivative was successfully verified.

[In] Integrate[((a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(9/2), x]

[Out] (2*a^2*(35*A + 5*(17*A + 9*B)*Sec[c + d*x] + 3*(34*A + 39*B)*Sec[c + d*x]^2 + 4*(34*A + 39*B)*Sec[c + d*x]^3 + 8*(34*A + 39*B)*Sec[c + d*x]^4)*Sin[c + d*x]/(315*d*Sec[c + d*x]^(7/2)*Sqrt[a*(1 + Sec[c + d*x])])

fricas [A] time = 0.43, size = 132, normalized size = 0.58

$$\frac{2 \left(35 A a \cos(dx + c)^5 + 5(17 A + 9 B) a \cos(dx + c)^4 + 3(34 A + 39 B) a \cos(dx + c)^3 + 4(34 A + 39 B) a \cos(dx + c)^2 + 5(17 A + 9 B) a \cos(dx + c) \right)}{315 (d \cos(dx + c) + d) \sqrt{\cos(dx + c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2), x, algorithm="fricas")

[Out] 2/315*(35*A*a*cos(d*x + c)^5 + 5*(17*A + 9*B)*a*cos(d*x + c)^4 + 3*(34*A + 39*B)*a*cos(d*x + c)^3 + 4*(34*A + 39*B)*a*cos(d*x + c)^2 + 8*(34*A + 39*B)*a*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/((d*cos(d*x + c) + d)*sqrt(cos(d*x + c)))

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(a \sec(dx + c) + a)^{\frac{3}{2}}}{\sec(dx + c)^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2), x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(9/2), x)

maple [A] time = 3.12, size = 141, normalized size = 0.62

$$2(-1 + \cos(dx + c)) \left(35A \left(\cos^4(dx + c) \right) + 85A \left(\cos^3(dx + c) \right) + 45B \left(\cos^3(dx + c) \right) + 102A \left(\cos^2(dx + c) \right) \right) -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x)
```

```
[Out] -2/315/d*(-1+cos(d*x+c))*(35*A*cos(d*x+c)^4+85*A*cos(d*x+c)^3+45*B*cos(d*x+c)^3+102*A*cos(d*x+c)^2+117*B*cos(d*x+c)^2+136*A*cos(d*x+c)+156*B*cos(d*x+c)+272*A+312*B)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^5*(1/cos(d*x+c))^(9/2)/sin(d*x+c)*a
```

maxima [B] time = 1.35, size = 700, normalized size = 3.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] 1/5040*(sqrt(2)*(3780*a*cos(8/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) * sin(9/2*d*x + 9/2*c) + 1050*a*cos(2/3*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) * sin(9/2*d*x + 9/2*c) + 378*a*cos(4/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) * sin(9/2*d*x + 9/2*c) + 135*a*cos(2/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) * sin(9/2*d*x + 9/2*c) - 3780*a*cos(9/2*d*x + 9/2*c) * sin(8/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) - 1050*a*cos(9/2*d*x + 9/2*c) * sin(2/3*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) - 378*a*cos(9/2*d*x + 9/2*c) * sin(4/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) - 135*a*cos(9/2*d*x + 9/2*c) * sin(2/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) + 70*a*sin(9/2*d*x + 9/2*c) + 135*a*sin(7/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) + 378*a*sin(5/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) + 1050*a*sin(1/3*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) + 3780*a*sin(1/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))) * A * sqrt(a) + 6*sqrt(2)*(735*a*cos(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) * sin(7/2*d*x + 7/2*c) + 175*a*cos(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) * sin(7/2*d*x + 7/2*c) + 63*a*cos(2/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) * sin(7/2*d*x + 7/2*c) - 735*a*cos(7/2*d*x + 7/2*c) * sin(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) - 175*a*cos(7/2*d*x + 7/2*c) * sin(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) - 63*a*cos(7/2*d*x + 7/2*c) * sin(2/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 30*a*sin(7/2*d*x + 7/2*c) + 63*a*sin(5/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 175*a*sin(3/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 735*a*sin(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))) * B * sqrt(a))/d
```

mupad [B] time = 5.30, size = 155, normalized size = 0.68

$$a \cos(c + dx) \sqrt{\frac{1}{\cos(c+dx)}} \sqrt{\frac{a(\cos(c+dx)+1)}{\cos(c+dx)}} (4830 A \sin(c + dx) + 5460 B \sin(c + dx) + 1428 A \sin(2c + 2dx))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(3/2))/(1/cos(c + d*x))^(9/2),x)

[Out] (a*cos(c + d*x)*(1/cos(c + d*x))^(1/2)*((a*(cos(c + d*x) + 1))/cos(c + d*x))^(1/2)*(4830*A*sin(c + d*x) + 5460*B*sin(c + d*x) + 1428*A*sin(2*c + 2*d*x) + 513*A*sin(3*c + 3*d*x) + 170*A*sin(4*c + 4*d*x) + 35*A*sin(5*c + 5*d*x) + 1428*B*sin(2*c + 2*d*x) + 468*B*sin(3*c + 3*d*x) + 90*B*sin(4*c + 4*d*x)))/(2520*d*(cos(c + d*x) + 1))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)**(9/2),x)

[Out] Timed out

$$3.239 \quad \int \sec^2(c+dx)(a+a \sec(c+dx))^{5/2}(A+B \sec(c+dx)) dx$$

Optimal. Leaf size=274

$$\frac{a^{5/2}(326A + 283B) \sinh^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}}\right)}{128d} + \frac{a^3(170A + 157B) \sin(c+dx) \sec^2(c+dx)}{240d\sqrt{a \sec(c+dx)+a}} + \frac{a^3(326A + 283B) \sin(c+dx)}{192d\sqrt{a \sec(c+dx)+a}}$$

[Out] 1/128*a^(5/2)*(326*A+283*B)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+1/5*a*B*sec(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(3/2)*sin(d*x+c)/d+1/128*a^3*(326*A+283*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/192*a^3*(326*A+283*B)*sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/240*a^3*(170*A+157*B)*sec(d*x+c)^(7/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/40*a^2*(10*A+13*B)*sec(d*x+c)^(7/2)*sin(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d

Rubi [A] time = 0.69, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4018, 4016, 3803, 3801, 215}

$$\frac{a^2(10A + 13B) \sin(c+dx) \sec^2(c+dx) \sqrt{a \sec(c+dx)+a}}{40d} + \frac{a^3(170A + 157B) \sin(c+dx) \sec^2(c+dx)}{240d\sqrt{a \sec(c+dx)+a}} + \frac{a^3(326A + 283B) \sin(c+dx)}{192d\sqrt{a \sec(c+dx)+a}}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]

[Out] (a^(5/2)*(326*A + 283*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]])/(128*d) + (a^3*(326*A + 283*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(128*d*Sqrt[a + a*Sec[c + d*x]]) + (a^3*(326*A + 283*B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(192*d*Sqrt[a + a*Sec[c + d*x]]) + (a^3*(170*A + 157*B)*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(240*d*Sqrt[a + a*Sec[c + d*x]]) + (a^2*(10*A + 13*B)*Sec[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(40*d) + (a*B*Sec[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 3801

Int[Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] := Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +

x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]

Rule 3803

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^n]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] :> Simp[(-2*b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(2*a*d*(n - 1))/(b*(2*n - 1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]

Rule 4016

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^n]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] :> Simp[(-2*b*B*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && !LtQ[n, 0]

Rule 4018

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^n]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^m*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] :> -Simp[(b*B*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && !LtQ[n, -1]

Rubi steps

$$\begin{aligned}
\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))dx &= \frac{aB\sec^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{5d} \\
&= \frac{a^2(10A+13B)\sec^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{40d} \\
&= \frac{a^3(170A+157B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{240d\sqrt{a+a\sec(c+dx)}} + \frac{a^2(10A+13B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{240d} \\
&= \frac{a^3(326A+283B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{192d\sqrt{a+a\sec(c+dx)}} + \frac{a^2(10A+13B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{192d} \\
&= \frac{a^3(326A+283B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{128d\sqrt{a+a\sec(c+dx)}} + \frac{a^2(10A+13B)\sec^{\frac{1}{2}}(c+dx)\sin(c+dx)}{128d} \\
&= \frac{a^3(326A+283B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{128d\sqrt{a+a\sec(c+dx)}} + \frac{a^3(326A+283B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{128d} + \frac{a^2(10A+13B)\sec^{\frac{1}{2}}(c+dx)\sin(c+dx)}{128d}
\end{aligned}$$

Mathematica [A] time = 2.28, size = 178, normalized size = 0.65

$$\frac{a^2 \sec\left(\frac{1}{2}(c+dx)\right) \sqrt{a(\sec(c+dx)+1)} \left(60\sqrt{2}(326A+283B) \tanh^{-1}\left(\sqrt{2} \sin\left(\frac{1}{2}(c+dx)\right)\right) + \sin\left(\frac{1}{2}(c+dx)\right) \sec\left(\frac{1}{2}(c+dx)\right)\right)}{15360d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]), x]

[Out] (a^2*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(60*Sqrt[2]*(326*A + 283*B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]] + (22030*A + 24863*B + 36*(650*A + 781*B)*Cos[c + d*x] + 4*(6730*A + 6509*B)*Cos[2*(c + d*x)] + 6520*A*Cos[3*(c + d*x)] + 5660*B*Cos[3*(c + d*x)] + 4890*A*Cos[4*(c + d*x)] + 4245*B*Cos[4*(c + d*x)])*Sec[c + d*x]^5*Sin[(c + d*x)/2]))/(15360*d*Sqrt[Sec[c + d*x]])

fricas [A] time = 0.68, size = 558, normalized size = 2.04

$$\left[\frac{15 \left((326A + 283B)a^2 \cos(dx + c)^5 + (326A + 283B)a^2 \cos(dx + c)^4 \right) \sqrt{a} \log \left(\frac{a \cos(dx+c)^3 - 7a \cos(dx+c)^2 - 4(\cos(dx+c))^2}{\cos(dx+c)^3} \right)}{\dots} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] [1/7680*(15*((326*A + 283*B)*a^2*cos(d*x + c)^5 + (326*A + 283*B)*a^2*cos(d*x + c)^4)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(15*(326*A + 283*B)*a^2*cos(d*x + c)^4 + 10*(326*A + 283*B)*a^2*cos(d*x + c)^3 + 8*(230*A + 283*B)*a^2*cos(d*x + c)^2 + 48*(10*A + 29*B)*a^2*cos(d*x + c) + 384*B*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^5 + d*cos(d*x + c)^4), 1/3840*(15*((326*A + 283*B)*a^2*cos(d*x + c)^5 + (326*A + 283*B)*a^2*cos(d*x + c)^4)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) + 2*(15*(326*A + 283*B)*a^2*cos(d*x + c)^4 + 10*(326*A + 283*B)*a^2*cos(d*x + c)^3 + 8*(230*A + 283*B)*a^2*cos(d*x + c)^2 + 48*(10*A + 29*B)*a^2*cos(d*x + c) + 384*B*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^5 + d*cos(d*x + c)^4)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(a \sec(dx + c) + a)^{\frac{5}{2}} \sec(dx + c)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(5/2), x)

maple [B] time = 2.46, size = 543, normalized size = 1.98

$$\left(-4890A \arctan \left(\frac{\sqrt{\frac{2}{1+\cos(dx+c)}} (\cos(dx+c)+1-\sin(dx+c))\sqrt{2}}{4} \right) (\cos^5(dx+c))\sqrt{2} + 4890A \arctan \left(\frac{\sqrt{\frac{2}{1+\cos(dx+c)}} (\cos(dx+c)+1+\sin(dx+c))\sqrt{2}}{4} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x)`

[Out] $\frac{1}{7680d}(-4890A \arctan(1/4*(-2/(1+\cos(d*x+c))))^{1/2}(\cos(d*x+c)+1-\sin(d*x+c))^2^{1/2} \cos(d*x+c)^5 2^{1/2} + 4890A \arctan(1/4*(-2/(1+\cos(d*x+c))))^{1/2}(\cos(d*x+c)+1+\sin(d*x+c))^2^{1/2} \cos(d*x+c)^5 2^{1/2} - 4245B \arctan(1/4*(-2/(1+\cos(d*x+c))))^{1/2}(\cos(d*x+c)+1-\sin(d*x+c))^2^{1/2} \cos(d*x+c)^5 2^{1/2} + 4245B \arctan(1/4*(-2/(1+\cos(d*x+c))))^{1/2}(\cos(d*x+c)+1+\sin(d*x+c))^2^{1/2} \cos(d*x+c)^5 2^{1/2} + 9780A(-2/(1+\cos(d*x+c)))^{1/2} \cos(d*x+c)^4 \sin(d*x+c) + 8490B(-2/(1+\cos(d*x+c)))^{1/2} \cos(d*x+c)^4 \sin(d*x+c) + 6520A \cos(d*x+c)^3 \sin(d*x+c) (-2/(1+\cos(d*x+c)))^{1/2} + 5660B \cos(d*x+c)^3 \sin(d*x+c) (-2/(1+\cos(d*x+c)))^{1/2} + 3680A \sin(d*x+c) \cos(d*x+c)^2 (-2/(1+\cos(d*x+c)))^{1/2} + 4528B \sin(d*x+c) \cos(d*x+c)^2 (-2/(1+\cos(d*x+c)))^{1/2} + 960A \sin(d*x+c) \cos(d*x+c) (-2/(1+\cos(d*x+c)))^{1/2} + 2784B \sin(d*x+c) \cos(d*x+c) (-2/(1+\cos(d*x+c)))^{1/2} + 768B(-2/(1+\cos(d*x+c)))^{1/2} \sin(d*x+c)) (a(1+\cos(d*x+c))/\cos(d*x+c))^{1/2} (1/\cos(d*x+c))^{5/2} (-2/(1+\cos(d*x+c)))^{1/2} / \cos(d*x+c)^2 / \sin(d*x+c)^2 (\cos(d*x+c)^2 - 1) a^2$

maxima [B] time = 5.20, size = 9242, normalized size = 33.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="maxima")`

[Out] $-1/7680*(10*(1956*(\sqrt{2})a^2 \sin(8d*x + 8c) + 4\sqrt{2})a^2 \sin(6d*x + 6c) + 6\sqrt{2})a^2 \sin(4d*x + 4c) + 4\sqrt{2})a^2 \sin(2d*x + 2c)) \cos(15/4 \arctan^2(\sin(2d*x + 2c), \cos(2d*x + 2c))) + 652*(\sqrt{2})a^2 \sin(8d*x + 8c) + 4\sqrt{2})a^2 \sin(6d*x + 6c) + 6\sqrt{2})a^2 \sin(4d*x + 4c) + 4\sqrt{2})a^2 \sin(2d*x + 2c)) \cos(13/4 \arctan^2(\sin(2d*x + 2c), \cos(2d*x + 2c))) + 6204*(\sqrt{2})a^2 \sin(8d*x + 8c) + 4\sqrt{2})a^2 \sin(6d*x + 6c) + 6\sqrt{2})a^2 \sin(4d*x + 4c) + 4\sqrt{2})a^2 \sin(2d*x + 2c)) \cos(11/4 \arctan^2(\sin(2d*x + 2c), \cos(2d*x + 2c))) - 2060*(\sqrt{2})a^2 \sin(8d*x + 8c) + 4\sqrt{2})a^2 \sin(6d*x + 6c) + 6\sqrt{2})a^2 \sin(4d*x + 4c) + 4\sqrt{2})a^2 \sin(2d*x + 2c)) \cos(9/4 \arctan^2(\sin(2d*x + 2c), \cos(2d*x + 2c))) + 2060*(\sqrt{2})a^2 \sin(8d*x + 8c) + 4\sqrt{2})a^2$

$$\begin{aligned}
& * \sin(6*d*x + 6*c) + 6*\sqrt{2}*a^2*\sin(4*d*x + 4*c) + 4*\sqrt{2}*a^2*\sin(2*d*x + 2*c)) * \cos(7/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) - 6204*(\sqrt{2}*a^2*\sin(8*d*x + 8*c) + 4*\sqrt{2}*a^2*\sin(6*d*x + 6*c) + 6*\sqrt{2}*a^2*\sin(4*d*x + 4*c) + 4*\sqrt{2}*a^2*\sin(2*d*x + 2*c)) * \cos(5/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) - 652*(\sqrt{2}*a^2*\sin(8*d*x + 8*c) + 4*\sqrt{2}*a^2*\sin(6*d*x + 6*c) + 6*\sqrt{2}*a^2*\sin(4*d*x + 4*c) + 4*\sqrt{2}*a^2*\sin(2*d*x + 2*c)) * \cos(3/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) - 1956*(\sqrt{2}*a^2*\sin(8*d*x + 8*c) + 4*\sqrt{2}*a^2*\sin(6*d*x + 6*c) + 6*\sqrt{2}*a^2*\sin(4*d*x + 4*c) + 4*\sqrt{2}*a^2*\sin(2*d*x + 2*c)) * \cos(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) - 489*(a^2*\cos(8*d*x + 8*c)^2 + 16*a^2*\cos(6*d*x + 6*c)^2 + 36*a^2*\cos(4*d*x + 4*c)^2 + 16*a^2*\cos(2*d*x + 2*c)^2 + a^2*\sin(8*d*x + 8*c)^2 + 16*a^2*\sin(6*d*x + 6*c)^2 + 36*a^2*\sin(4*d*x + 4*c)^2 + 48*a^2*\sin(4*d*x + 4*c)*\sin(2*d*x + 2*c) + 16*a^2*\sin(2*d*x + 2*c)^2 + 8*a^2*\cos(2*d*x + 2*c) + a^2 + 2*(4*a^2*\cos(6*d*x + 6*c) + 6*a^2*\cos(4*d*x + 4*c) + 4*a^2*\cos(2*d*x + 2*c) + a^2)*\cos(8*d*x + 8*c) + 8*(6*a^2*\cos(4*d*x + 4*c) + 4*a^2*\cos(2*d*x + 2*c) + a^2)*\cos(6*d*x + 6*c) + 12*(4*a^2*\cos(2*d*x + 2*c) + a^2)*\cos(4*d*x + 4*c) + 4*(2*a^2*\sin(6*d*x + 6*c) + 3*a^2*\sin(4*d*x + 4*c) + 2*a^2*\sin(2*d*x + 2*c))*\sin(8*d*x + 8*c) + 16*(3*a^2*\sin(4*d*x + 4*c) + 2*a^2*\sin(2*d*x + 2*c))*\sin(6*d*x + 6*c))*\log(2*\cos(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + 2*\sin(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + 2*\sqrt{2}*\cos(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 2*\sqrt{2}*\sin(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))) + 2) + 489*(a^2*\cos(8*d*x + 8*c)^2 + 16*a^2*\cos(6*d*x + 6*c)^2 + 36*a^2*\cos(4*d*x + 4*c)^2 + 16*a^2*\cos(2*d*x + 2*c)^2 + a^2*\sin(8*d*x + 8*c)^2 + 16*a^2*\sin(6*d*x + 6*c)^2 + 36*a^2*\sin(4*d*x + 4*c)^2 + 48*a^2*\sin(4*d*x + 4*c)*\sin(2*d*x + 2*c) + 16*a^2*\sin(2*d*x + 2*c)^2 + 8*a^2*\cos(2*d*x + 2*c) + a^2 + 2*(4*a^2*\cos(6*d*x + 6*c) + 6*a^2*\cos(4*d*x + 4*c) + 4*a^2*\cos(2*d*x + 2*c) + a^2)*\cos(8*d*x + 8*c) + 8*(6*a^2*\cos(4*d*x + 4*c) + 4*a^2*\cos(2*d*x + 2*c) + a^2)*\cos(6*d*x + 6*c) + 12*(4*a^2*\cos(2*d*x + 2*c) + a^2)*\cos(4*d*x + 4*c) + 4*(2*a^2*\sin(6*d*x + 6*c) + 3*a^2*\sin(4*d*x + 4*c) + 2*a^2*\sin(2*d*x + 2*c))*\sin(8*d*x + 8*c) + 16*(3*a^2*\sin(4*d*x + 4*c) + 2*a^2*\sin(2*d*x + 2*c))*\sin(6*d*x + 6*c))*\log(2*\cos(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + 2*\sin(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + 2*\sqrt{2}*\cos(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) - 2*\sqrt{2}*\sin(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 2) - 489*(a^2*\cos(8*d*x + 8*c)^2 + 16*a^2*\cos(6*d*x + 6*c)^2 + 36*a^2*\cos(4*d*x + 4*c)^2 + 16*a^2*\cos(2*d*x + 2*c)^2 + a^2*\sin(8*d*x + 8*c)^2 + 16*a^2*\sin(6*d*x + 6*c)^2 + 36*a^2*\sin(4*d*x + 4*c)^2 + 48*a^2*\sin(4*d*x + 4*c)*\sin(2*d*x + 2*c) + 16*a^2*\sin(2*d*x + 2*c)^2 + 8*a^2*\cos(2*d*x + 2*c) + a^2 + 2*(4*a^2*\cos(6*d*x + 6*c) + 6*a^2*\cos(4*d*x + 4*c) + 4*a^2*\cos(2*d*x + 2*c) + a^2)*\cos(8*d*x + 8*c) + 8*(6*a^2*\cos(4*d*x + 4*c) + 4*a^2*\cos(2*d*x + 2*c) + a^2)*\cos(6*d*x + 6*c) + 12*(4*a^2*\cos(2*d*x + 2*c) + a^2)*\cos(4*d*x + 4*c) + 4*(2*a^2*\sin(6*d*x + 6*c) + 3*a^2*\sin(4*d*x + 4*c) + 2*a^2*\sin(2*d*x + 2*c))*\sin(8*d*x + 8*c) + 16*(3*a^2*\sin(4*d*x + 4*c) + 2*a^2*\sin(2*d*x + 2*c))*\sin(6*d*x + 6*c))*\log(2*\cos(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + 2
\end{aligned}$$

$$\begin{aligned}
& * \sin(1/4 * \arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 - 2 * \sqrt{2} * \cos(1/4 \\
& * \arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 2 * \sqrt{2} * \sin(1/4 * \arctan2(\sin(2*d*x + 2*c), \\
& \cos(2*d*x + 2*c))) + 2) + 489 * (a^2 * \cos(8*d*x + 8*c)^2 + 16 \\
& * a^2 * \cos(6*d*x + 6*c)^2 + 36 * a^2 * \cos(4*d*x + 4*c)^2 + 16 * a^2 * \cos(2*d*x + 2*c)^2 \\
& + a^2 * \sin(8*d*x + 8*c)^2 + 16 * a^2 * \sin(6*d*x + 6*c)^2 + 36 * a^2 * \sin(4*d*x + 4*c)^2 \\
& + 48 * a^2 * \sin(4*d*x + 4*c) * \sin(2*d*x + 2*c) + 16 * a^2 * \sin(2*d*x + 2*c)^2 \\
& + 8 * a^2 * \cos(2*d*x + 2*c) + a^2 + 2 * (4 * a^2 * \cos(6*d*x + 6*c) + 6 * a^2 * \cos(4*d*x + 4*c) \\
& + 4 * a^2 * \cos(2*d*x + 2*c) + a^2) * \cos(8*d*x + 8*c) + 8 * (6 * a^2 * \cos(4*d*x + 4*c) \\
& + 4 * a^2 * \cos(2*d*x + 2*c) + a^2) * \cos(6*d*x + 6*c) + 12 * (4 * a^2 * \cos(2*d*x + 2*c) \\
& + a^2) * \cos(4*d*x + 4*c) + 4 * (2 * a^2 * \sin(6*d*x + 6*c) + 3 * a^2 * \sin(4*d*x + 4*c) \\
& + 2 * a^2 * \sin(2*d*x + 2*c)) * \sin(8*d*x + 8*c) + 16 * (3 * a^2 * \sin(4*d*x + 4*c) \\
& + 2 * a^2 * \sin(2*d*x + 2*c)) * \sin(6*d*x + 6*c)) * \log(2 * \cos(1/4 * \arctan2(\sin(2*d*x + 2*c), \\
& \cos(2*d*x + 2*c)))^2 + 2 * \sin(1/4 * \arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 - 2 * \sqrt{2} * \cos(1/4 * \arctan2(\sin(2*d*x + 2*c), \\
& \cos(2*d*x + 2*c))) - 2 * \sqrt{2} * \sin(1/4 * \arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 2) - 1956 * (\sqrt{2} * a^2 * \cos(8*d*x + 8*c) \\
& + 4 * \sqrt{2} * a^2 * \cos(6*d*x + 6*c) + 6 * \sqrt{2} * a^2 * \cos(4*d*x + 4*c) + 4 * \sqrt{2} * a^2 * \cos(2*d*x + 2*c) \\
& + \sqrt{2} * a^2) * \sin(15/4 * \arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) - 652 * (\sqrt{2} * a^2 * \cos(8*d*x + 8*c) \\
& + 4 * \sqrt{2} * a^2 * \cos(6*d*x + 6*c) + 6 * \sqrt{2} * a^2 * \cos(4*d*x + 4*c) + 4 * \sqrt{2} * a^2 * \cos(2*d*x + 2*c) \\
& + \sqrt{2} * a^2) * \sin(13/4 * \arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) - 6204 * (\sqrt{2} * a^2 * \cos(8*d*x + 8*c) \\
& + 4 * \sqrt{2} * a^2 * \cos(6*d*x + 6*c) + 6 * \sqrt{2} * a^2 * \cos(4*d*x + 4*c) + 4 * \sqrt{2} * a^2 * \cos(2*d*x + 2*c) \\
& + \sqrt{2} * a^2) * \sin(11/4 * \arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 2060 * (\sqrt{2} * a^2 * \cos(8*d*x + 8*c) \\
& + 4 * \sqrt{2} * a^2 * \cos(6*d*x + 6*c) + 6 * \sqrt{2} * a^2 * \cos(4*d*x + 4*c) + 4 * \sqrt{2} * a^2 * \cos(2*d*x + 2*c) \\
& + \sqrt{2} * a^2) * \sin(9/4 * \arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) - 2060 * (\sqrt{2} * a^2 * \cos(8*d*x + 8*c) \\
& + 4 * \sqrt{2} * a^2 * \cos(6*d*x + 6*c) + 6 * \sqrt{2} * a^2 * \cos(4*d*x + 4*c) + 4 * \sqrt{2} * a^2 * \cos(2*d*x + 2*c) \\
& + \sqrt{2} * a^2) * \sin(7/4 * \arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 6204 * (\sqrt{2} * a^2 * \cos(8*d*x + 8*c) \\
& + 4 * \sqrt{2} * a^2 * \cos(6*d*x + 6*c) + 6 * \sqrt{2} * a^2 * \cos(4*d*x + 4*c) + 4 * \sqrt{2} * a^2 * \cos(2*d*x + 2*c) \\
& + \sqrt{2} * a^2) * \sin(5/4 * \arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 652 * (\sqrt{2} * a^2 * \cos(8*d*x + 8*c) \\
& + 4 * \sqrt{2} * a^2 * \cos(6*d*x + 6*c) + 6 * \sqrt{2} * a^2 * \cos(4*d*x + 4*c) + 4 * \sqrt{2} * a^2 * \cos(2*d*x + 2*c) \\
& + \sqrt{2} * a^2) * \sin(3/4 * \arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 1956 * (\sqrt{2} * a^2 * \cos(8*d*x + 8*c) \\
& + 4 * \sqrt{2} * a^2 * \cos(6*d*x + 6*c) + 6 * \sqrt{2} * a^2 * \cos(4*d*x + 4*c) + 4 * \sqrt{2} * a^2 * \cos(2*d*x + 2*c) \\
& + \sqrt{2} * a^2) * \sin(1/4 * \arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) * A * \sqrt{a} / (2 * (4 * \cos(6*d*x + 6*c) \\
& + 6 * \cos(4*d*x + 4*c) + 4 * \cos(2*d*x + 2*c) + 1) * \cos(8*d*x + 8*c) + \cos(8*d*x + 8*c)^2 + 8 * (6 * \cos(4*d*x + 4*c) \\
& + 4 * \cos(2*d*x + 2*c) + 1) * \cos(6*d*x + 6*c) + 16 * \cos(6*d*x + 6*c)^2 + 12 * (4 * \cos(2*d*x + 2*c) + 1) * \cos(4*d*x + 4*c) \\
& + 36 * \cos(4*d*x + 4*c)^2 + 16 * \cos(2*d*x + 2*c)^2 + 4 * (2 * \sin(6*d*x + 6*c) + 3 * \sin(4*d*x + 4*c) \\
& + 2 * \sin(2*d*x + 2*c)) * \sin(8*d*x + 8*c) + \sin(8*d*x + 8*c)^2 + 16 * (3 * \sin(4*d*x + 4*c) + 2 * \sin(2*d*x + 2*c)) * \sin(6*d*x + 6*c) \\
& + 16 * \sin(6*d*x + 6*c)^2 + 36 * \sin(4*d*x + 4*c)^2 + 48 * \sin(4*d*x + 4*c) * \sin(2*d*x + 2*c) + 16 * \sin(2*d*x + 2*c)^2 + 8 * \cos(2*d*x
\end{aligned}$$

$$\begin{aligned}
& + 2*c) + 1) + (16980*(\sqrt{2})*a^2*\sin(10*d*x + 10*c) + 5*\sqrt{2})*a^2*\sin(8 \\
& *d*x + 8*c) + 10*\sqrt{2})*a^2*\sin(6*d*x + 6*c) + 10*\sqrt{2})*a^2*\sin(4*d*x + \\
& 4*c) + 5*\sqrt{2})*a^2*\sin(2*d*x + 2*c))*\cos(19/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 5660*(\sqrt{2})*a^2*\sin(10*d*x + 10*c) + 5*\sqrt{2})*a^2*\sin(8*d*x + 8*c) + 10*\sqrt{2})*a^2*\sin(6*d*x + 6*c) + 10*\sqrt{2})*a^2*\sin(4*d*x + 4*c) + 5*\sqrt{2})*a^2*\sin(2*d*x + 2*c))*\cos(17/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 81504*(\sqrt{2})*a^2*\sin(10*d*x + 10*c) + 5*\sqrt{2})*a^2*\sin(8*d*x + 8*c) + 10*\sqrt{2})*a^2*\sin(6*d*x + 6*c) + 10*\sqrt{2})*a^2*\sin(4*d*x + 4*c) + 5*\sqrt{2})*a^2*\sin(2*d*x + 2*c))*\cos(15/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 8320*(\sqrt{2})*a^2*\sin(10*d*x + 10*c) + 5*\sqrt{2})*a^2*\sin(8*d*x + 8*c) + 10*\sqrt{2})*a^2*\sin(6*d*x + 6*c) + 10*\sqrt{2})*a^2*\sin(4*d*x + 4*c) + 5*\sqrt{2})*a^2*\sin(2*d*x + 2*c))*\cos(13/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 86440*(\sqrt{2})*a^2*\sin(10*d*x + 10*c) + 5*\sqrt{2})*a^2*\sin(8*d*x + 8*c) + 10*\sqrt{2})*a^2*\sin(6*d*x + 6*c) + 10*\sqrt{2})*a^2*\sin(4*d*x + 4*c) + 5*\sqrt{2})*a^2*\sin(2*d*x + 2*c))*\cos(11/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) - 86440*(\sqrt{2})*a^2*\sin(10*d*x + 10*c) + 5*\sqrt{2})*a^2*\sin(8*d*x + 8*c) + 10*\sqrt{2})*a^2*\sin(6*d*x + 6*c) + 10*\sqrt{2})*a^2*\sin(4*d*x + 4*c) + 5*\sqrt{2})*a^2*\sin(2*d*x + 2*c))*\cos(9/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) - 8320*(\sqrt{2})*a^2*\sin(10*d*x + 10*c) + 5*\sqrt{2})*a^2*\sin(8*d*x + 8*c) + 10*\sqrt{2})*a^2*\sin(6*d*x + 6*c) + 10*\sqrt{2})*a^2*\sin(4*d*x + 4*c) + 5*\sqrt{2})*a^2*\sin(2*d*x + 2*c))*\cos(7/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) - 81504*(\sqrt{2})*a^2*\sin(10*d*x + 10*c) + 5*\sqrt{2})*a^2*\sin(8*d*x + 8*c) + 10*\sqrt{2})*a^2*\sin(6*d*x + 6*c) + 10*\sqrt{2})*a^2*\sin(4*d*x + 4*c) + 5*\sqrt{2})*a^2*\sin(2*d*x + 2*c))*\cos(5/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) - 5660*(\sqrt{2})*a^2*\sin(10*d*x + 10*c) + 5*\sqrt{2})*a^2*\sin(8*d*x + 8*c) + 10*\sqrt{2})*a^2*\sin(6*d*x + 6*c) + 10*\sqrt{2})*a^2*\sin(4*d*x + 4*c) + 5*\sqrt{2})*a^2*\sin(2*d*x + 2*c))*\cos(3/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) - 16980*(\sqrt{2})*a^2*\sin(10*d*x + 10*c) + 5*\sqrt{2})*a^2*\sin(8*d*x + 8*c) + 10*\sqrt{2})*a^2*\sin(6*d*x + 6*c) + 10*\sqrt{2})*a^2*\sin(4*d*x + 4*c) + 5*\sqrt{2})*a^2*\sin(2*d*x + 2*c))*\cos(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) - 4245*(a^2*\cos(10*d*x + 10*c))^2 + 25*a^2*\cos(8*d*x + 8*c)^2 + 100*a^2*\cos(6*d*x + 6*c)^2 + 100*a^2*\cos(4*d*x + 4*c)^2 + 25*a^2*\cos(2*d*x + 2*c)^2 + a^2*\sin(10*d*x + 10*c)^2 + 25*a^2*\sin(8*d*x + 8*c)^2 + 100*a^2*\sin(6*d*x + 6*c)^2 + 100*a^2*\sin(4*d*x + 4*c)^2 + 100*a^2*\sin(4*d*x + 4*c)*\sin(2*d*x + 2*c) + 25*a^2*\sin(2*d*x + 2*c)^2 + 10*a^2*\cos(2*d*x + 2*c) + a^2 + 2*(5*a^2*\cos(8*d*x + 8*c) + 10*a^2*\cos(6*d*x + 6*c) + 10*a^2*\cos(4*d*x + 4*c) + 5*a^2*\cos(2*d*x + 2*c) + a^2)*\cos(10*d*x + 10*c) + 10*(10*a^2*\cos(6*d*x + 6*c) + 10*a^2*\cos(4*d*x + 4*c) + 5*a^2*\cos(2*d*x + 2*c) + a^2)*\cos(8*d*x + 8*c) + 20*(10*a^2*\cos(4*d*x + 4*c) + 5*a^2*\cos(2*d*x + 2*c) + a^2)*\cos(6*d*x + 6*c) + 20*(5*a^2*\cos(2*d*x + 2*c) + a^2)*\cos(4*d*x + 4*c) + 10*(a^2*\sin(8*d*x + 8*c) + 2*a^2*\sin(6*d*x + 6*c) + 2*a^2*\sin(4*d*x + 4*c) + a^2*\sin(2*d*x + 2*c))*\sin(10*d*x + 10*c) + 50*(2*a^2*\sin(6*d*x + 6*c) + 2*a^2*\sin(4*d*x + 4*c) + a^2*\sin(2*d*x + 2*c))*\sin(8*d*x + 8*c) + 100*(2*a^2*\sin(4*d*x + 4*c) + a^2*\sin(2*d*x + 2*c))*\sin(6*d*x + 6*c))*\log(2*\cos(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))))^2
\end{aligned}$$

$$\begin{aligned}
& + 2*\sin(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + 2*\sqrt{2}*\cos(\\
& 1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 2*\sqrt{2}*\sin(1/4*\arctan \\
& 2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 2) + 4245*(a^2*\cos(10*d*x + 10*c)^ \\
& 2 + 25*a^2*\cos(8*d*x + 8*c)^2 + 100*a^2*\cos(6*d*x + 6*c)^2 + 100*a^2*\cos(4* \\
& d*x + 4*c)^2 + 25*a^2*\cos(2*d*x + 2*c)^2 + a^2*\sin(10*d*x + 10*c)^2 + 25*a^ \\
& 2*\sin(8*d*x + 8*c)^2 + 100*a^2*\sin(6*d*x + 6*c)^2 + 100*a^2*\sin(4*d*x + 4*c \\
&)^2 + 100*a^2*\sin(4*d*x + 4*c)*\sin(2*d*x + 2*c) + 25*a^2*\sin(2*d*x + 2*c)^2 \\
& + 10*a^2*\cos(2*d*x + 2*c) + a^2 + 2*(5*a^2*\cos(8*d*x + 8*c) + 10*a^2*\cos(6 \\
& *d*x + 6*c) + 10*a^2*\cos(4*d*x + 4*c) + 5*a^2*\cos(2*d*x + 2*c) + a^2)*\cos(1 \\
& 0*d*x + 10*c) + 10*(10*a^2*\cos(6*d*x + 6*c) + 10*a^2*\cos(4*d*x + 4*c) + 5*a \\
& ^2*\cos(2*d*x + 2*c) + a^2)*\cos(8*d*x + 8*c) + 20*(10*a^2*\cos(4*d*x + 4*c) + \\
& 5*a^2*\cos(2*d*x + 2*c) + a^2)*\cos(6*d*x + 6*c) + 20*(5*a^2*\cos(2*d*x + 2*c \\
&) + a^2)*\cos(4*d*x + 4*c) + 10*(a^2*\sin(8*d*x + 8*c) + 2*a^2*\sin(6*d*x + 6* \\
& c) + 2*a^2*\sin(4*d*x + 4*c) + a^2*\sin(2*d*x + 2*c))*\sin(10*d*x + 10*c) + 50 \\
& *(2*a^2*\sin(6*d*x + 6*c) + 2*a^2*\sin(4*d*x + 4*c) + a^2*\sin(2*d*x + 2*c))*\sin \\
& (8*d*x + 8*c) + 100*(2*a^2*\sin(4*d*x + 4*c) + a^2*\sin(2*d*x + 2*c))*\sin(6 \\
& *d*x + 6*c))*\log(2*\cos(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + \\
& 2*\sin(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + 2*\sqrt{2}*\cos(1 \\
& /4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) - 2*\sqrt{2}*\sin(1/4*\arctan2 \\
& (\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 2) - 4245*(a^2*\cos(10*d*x + 10*c)^2 \\
& + 25*a^2*\cos(8*d*x + 8*c)^2 + 100*a^2*\cos(6*d*x + 6*c)^2 + 100*a^2*\cos(4*d \\
& *x + 4*c)^2 + 25*a^2*\cos(2*d*x + 2*c)^2 + a^2*\sin(10*d*x + 10*c)^2 + 25*a^2 \\
& *\sin(8*d*x + 8*c)^2 + 100*a^2*\sin(6*d*x + 6*c)^2 + 100*a^2*\sin(4*d*x + 4*c) \\
& ^2 + 100*a^2*\sin(4*d*x + 4*c)*\sin(2*d*x + 2*c) + 25*a^2*\sin(2*d*x + 2*c)^2 \\
& + 10*a^2*\cos(2*d*x + 2*c) + a^2 + 2*(5*a^2*\cos(8*d*x + 8*c) + 10*a^2*\cos(6* \\
& d*x + 6*c) + 10*a^2*\cos(4*d*x + 4*c) + 5*a^2*\cos(2*d*x + 2*c) + a^2)*\cos(10 \\
& *d*x + 10*c) + 10*(10*a^2*\cos(6*d*x + 6*c) + 10*a^2*\cos(4*d*x + 4*c) + 5*a^ \\
& 2*\cos(2*d*x + 2*c) + a^2)*\cos(8*d*x + 8*c) + 20*(10*a^2*\cos(4*d*x + 4*c) + \\
& 5*a^2*\cos(2*d*x + 2*c) + a^2)*\cos(6*d*x + 6*c) + 20*(5*a^2*\cos(2*d*x + 2*c) \\
& + a^2)*\cos(4*d*x + 4*c) + 10*(a^2*\sin(8*d*x + 8*c) + 2*a^2*\sin(6*d*x + 6*c \\
&) + 2*a^2*\sin(4*d*x + 4*c) + a^2*\sin(2*d*x + 2*c))*\sin(10*d*x + 10*c) + 50* \\
& (2*a^2*\sin(6*d*x + 6*c) + 2*a^2*\sin(4*d*x + 4*c) + a^2*\sin(2*d*x + 2*c))*\sin \\
& (8*d*x + 8*c) + 100*(2*a^2*\sin(4*d*x + 4*c) + a^2*\sin(2*d*x + 2*c))*\sin(6* \\
& d*x + 6*c))*\log(2*\cos(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + \\
& 2*\sin(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 - 2*\sqrt{2}*\cos(1/ \\
& 4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 2*\sqrt{2}*\sin(1/4*\arctan2(\\
& \sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 2) + 4245*(a^2*\cos(10*d*x + 10*c)^2 \\
& + 25*a^2*\cos(8*d*x + 8*c)^2 + 100*a^2*\cos(6*d*x + 6*c)^2 + 100*a^2*\cos(4*d* \\
& x + 4*c)^2 + 25*a^2*\cos(2*d*x + 2*c)^2 + a^2*\sin(10*d*x + 10*c)^2 + 25*a^2* \\
& \sin(8*d*x + 8*c)^2 + 100*a^2*\sin(6*d*x + 6*c)^2 + 100*a^2*\sin(4*d*x + 4*c)^ \\
& 2 + 100*a^2*\sin(4*d*x + 4*c)*\sin(2*d*x + 2*c) + 25*a^2*\sin(2*d*x + 2*c)^2 + \\
& 10*a^2*\cos(2*d*x + 2*c) + a^2 + 2*(5*a^2*\cos(8*d*x + 8*c) + 10*a^2*\cos(6*d \\
& *x + 6*c) + 10*a^2*\cos(4*d*x + 4*c) + 5*a^2*\cos(2*d*x + 2*c) + a^2)*\cos(10* \\
& d*x + 10*c) + 10*(10*a^2*\cos(6*d*x + 6*c) + 10*a^2*\cos(4*d*x + 4*c) + 5*a^2 \\
& *\cos(2*d*x + 2*c) + a^2)*\cos(8*d*x + 8*c) + 20*(10*a^2*\cos(4*d*x + 4*c) + 5
\end{aligned}$$

$$\begin{aligned}
& a^2 \cos(2dx + 2c) + a^2 \cos(6dx + 6c) + 20(5a^2 \cos(2dx + 2c) \\
& + a^2 \cos(4dx + 4c) + 10(a^2 \sin(8dx + 8c) + 2a^2 \sin(6dx + 6c) \\
& + 2a^2 \sin(4dx + 4c) + a^2 \sin(2dx + 2c)) \sin(10dx + 10c) + 50(\\
& 2a^2 \sin(6dx + 6c) + 2a^2 \sin(4dx + 4c) + a^2 \sin(2dx + 2c)) \sin \\
& (8dx + 8c) + 100(2a^2 \sin(4dx + 4c) + a^2 \sin(2dx + 2c)) \sin(6dx \\
& + 6c)) \log(2 \cos(1/4 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c)))^2 + 2 \\
& \sin(1/4 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c)))^2 - 2\sqrt{2} \cos(1/4 \\
& \arctan2(\sin(2dx + 2c), \cos(2dx + 2c))) - 2\sqrt{2} \sin(1/4 \arctan2(s \\
& \sin(2dx + 2c), \cos(2dx + 2c))) + 2) - 16980(\sqrt{2} a^2 \cos(10dx + \\
& 10c) + 5\sqrt{2} a^2 \cos(8dx + 8c) + 10\sqrt{2} a^2 \cos(6dx + 6c) + \\
& 10\sqrt{2} a^2 \cos(4dx + 4c) + 5\sqrt{2} a^2 \cos(2dx + 2c) + \sqrt{2} a^2 \\
& \sin(19/4 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c))) - 5660(\sqrt{2} a^2 \cos(10dx + 10c) \\
& + 5\sqrt{2} a^2 \cos(8dx + 8c) + 10\sqrt{2} a^2 \cos(6dx + 6c) + 10\sqrt{2} a^2 \cos(4dx + 4c) \\
& + 5\sqrt{2} a^2 \cos(2dx + 2c) + \sqrt{2} a^2 \sin(17/4 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c))) \\
& - 81504(\sqrt{2} a^2 \cos(10dx + 10c) + 5\sqrt{2} a^2 \cos(8dx + 8c) + \\
& 10\sqrt{2} a^2 \cos(6dx + 6c) + 10\sqrt{2} a^2 \cos(4dx + 4c) + 5\sqrt{2} \\
& (2) a^2 \cos(2dx + 2c) + \sqrt{2} a^2 \sin(15/4 \arctan2(\sin(2dx + 2c), \\
& \cos(2dx + 2c))) - 8320(\sqrt{2} a^2 \cos(10dx + 10c) + 5\sqrt{2} a^2 \cos(8dx + 8c) \\
& + 10\sqrt{2} a^2 \cos(6dx + 6c) + 10\sqrt{2} a^2 \cos(4dx + 4c) + 5\sqrt{2} a^2 \cos(2dx + 2c) \\
& + \sqrt{2} a^2 \sin(13/4 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c))) - 86440(\sqrt{2} a^2 \cos(10dx + 10c) \\
& + 5\sqrt{2} a^2 \cos(8dx + 8c) + 10\sqrt{2} a^2 \cos(6dx + 6c) + 10\sqrt{2} a^2 \cos(4dx + 4c) \\
& + 5\sqrt{2} a^2 \cos(2dx + 2c) + \sqrt{2} a^2 \sin(11/4 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c))) \\
& + 86440(\sqrt{2} a^2 \cos(10dx + 10c) + 5\sqrt{2} a^2 \cos(8dx + 8c) + 10\sqrt{2} a^2 \cos(6dx + 6c) \\
& + 10\sqrt{2} a^2 \cos(4dx + 4c) + 5\sqrt{2} a^2 \cos(2dx + 2c) + \sqrt{2} a^2 \sin(9/4 \arctan2(\sin(2dx + 2c), \\
& \cos(2dx + 2c))) + 8320(\sqrt{2} a^2 \cos(10dx + 10c) + 5\sqrt{2} a^2 \cos(8dx + 8c) + 10\sqrt{2} a^2 \cos(6dx + 6c) \\
& + 10\sqrt{2} a^2 \cos(4dx + 4c) + 5\sqrt{2} a^2 \cos(2dx + 2c) + \sqrt{2} a^2 \sin(7/4 \arctan2(\sin(2dx + 2c), \\
& \cos(2dx + 2c))) + 81504(\sqrt{2} a^2 \cos(10dx + 10c) + 5\sqrt{2} a^2 \cos(8dx + 8c) \\
& + 10\sqrt{2} a^2 \cos(6dx + 6c) + 10\sqrt{2} a^2 \cos(4dx + 4c) + 5\sqrt{2} a^2 \cos(2dx + 2c) \\
& + \sqrt{2} a^2 \sin(5/4 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c))) + 5660(\sqrt{2} a^2 \cos(10dx + 10c) \\
& + 5\sqrt{2} a^2 \cos(8dx + 8c) + 10\sqrt{2} a^2 \cos(6dx + 6c) + 10\sqrt{2} a^2 \cos(4dx + 4c) \\
& + 5\sqrt{2} a^2 \cos(2dx + 2c) + \sqrt{2} a^2 \sin(3/4 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c))) \\
& + 16980(\sqrt{2} a^2 \cos(10dx + 10c) + 5\sqrt{2} a^2 \cos(8dx + 8c) + 10\sqrt{2} a^2 \cos(6dx + 6c) \\
& + 10\sqrt{2} a^2 \cos(4dx + 4c) + 5\sqrt{2} a^2 \cos(2dx + 2c) + \sqrt{2} a^2 \sin(1/4 \arctan2(\sin(2dx + 2c), \\
& \cos(2dx + 2c)))) B \sqrt{a} / (2 \\
& (5 \cos(8dx + 8c) + 10 \cos(6dx + 6c) + 10 \cos(4dx + 4c) + 5 \cos(2dx + 2c) + 1) \cos(10dx + 10c) \\
& + \cos(10dx + 10c)^2 + 10(10 \cos(6dx + 6c) + 10 \cos(4dx + 4c) + 5 \cos(2dx + 2c) + 1) \cos(8dx + 8c) \\
& + 25 \cos(8dx + 8c)^2 + 20(10 \cos(4dx + 4c) + 5 \cos(2dx + 2c) + 1) *
\end{aligned}$$

$\cos(6dx + 6c) + 100\cos(6dx + 6c)^2 + 20(5\cos(2dx + 2c) + 1)\cos(4dx + 4c) + 100\cos(4dx + 4c)^2 + 25\cos(2dx + 2c)^2 + 10(\sin(8dx + 8c) + 2\sin(6dx + 6c) + 2\sin(4dx + 4c) + \sin(2dx + 2c))\sin(10dx + 10c) + \sin(10dx + 10c)^2 + 50(2\sin(6dx + 6c) + 2\sin(4dx + 4c) + \sin(2dx + 2c))\sin(8dx + 8c) + 25\sin(8dx + 8c)^2 + 100(2\sin(4dx + 4c) + \sin(2dx + 2c))\sin(6dx + 6c) + 100\sin(6dx + 6c)^2 + 100\sin(4dx + 4c)^2 + 100\sin(4dx + 4c)\sin(2dx + 2c) + 25\sin(2dx + 2c)^2 + 10\cos(2dx + 2c) + 1)/d$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(A + \frac{B}{\cos(c + dx)} \right) \left(a + \frac{a}{\cos(c + dx)} \right)^{5/2} \left(\frac{1}{\cos(c + dx)} \right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(5/2), x)

[Out] int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(5/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**(5/2)*(a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)), x)

[Out] Timed out

$$3.240 \quad \int \sec^{\frac{3}{2}}(c+dx)(a+a \sec(c+dx))^{5/2}(A+B \sec(c+dx)) dx$$

Optimal. Leaf size=227

$$\frac{a^{5/2}(200A + 163B) \sinh^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}}\right)}{64d} + \frac{a^3(104A + 95B) \sin(c + dx) \sec^{\frac{5}{2}}(c + dx)}{96d\sqrt{a \sec(c + dx) + a}} + \frac{a^3(200A + 163B) \sin(c + dx)}{64d\sqrt{a \sec(c + dx) + a}}$$

[Out] $1/64*a^{(5/2)}*(200*A+163*B)*\operatorname{arcsinh}(a^{(1/2)}*\tan(d*x+c)/(a+a*\sec(d*x+c))^{(1/2)})/d+1/4*a*B*\sec(d*x+c)^{(5/2)}*(a+a*\sec(d*x+c))^{(3/2)}*\sin(d*x+c)/d+1/64*a^3*(200*A+163*B)*\sec(d*x+c)^{(3/2)}*\sin(d*x+c)/d/(a+a*\sec(d*x+c))^{(1/2)}+1/96*a^3*(104*A+95*B)*\sec(d*x+c)^{(5/2)}*\sin(d*x+c)/d/(a+a*\sec(d*x+c))^{(1/2)}+1/24*a^2*(8*A+11*B)*\sec(d*x+c)^{(5/2)}*\sin(d*x+c)*(a+a*\sec(d*x+c))^{(1/2)}/d$

Rubi [A] time = 0.59, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4018, 4016, 3803, 3801, 215}

$$\frac{a^3(104A + 95B) \sin(c + dx) \sec^{\frac{5}{2}}(c + dx)}{96d\sqrt{a \sec(c + dx) + a}} + \frac{a^3(200A + 163B) \sin(c + dx) \sec^{\frac{3}{2}}(c + dx)}{64d\sqrt{a \sec(c + dx) + a}} + \frac{a^2(8A + 11B) \sin(c + dx)}{64d\sqrt{a \sec(c + dx) + a}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Sec}[c + d*x]^{(3/2)}*(a + a*\operatorname{Sec}[c + d*x])^{(5/2)}*(A + B*\operatorname{Sec}[c + d*x]), x]$

[Out] $(a^{(5/2)}*(200*A + 163*B)*\operatorname{ArcSinh}[(\operatorname{Sqrt}[a]*\operatorname{Tan}[c + d*x])/(\operatorname{Sqrt}[a + a*\operatorname{Sec}[c + d*x]])]/(64*d) + (a^3*(200*A + 163*B)*\operatorname{Sec}[c + d*x]^{(3/2)}*\operatorname{Sin}[c + d*x])/((64*d*\operatorname{Sqrt}[a + a*\operatorname{Sec}[c + d*x]]) + (a^3*(104*A + 95*B)*\operatorname{Sec}[c + d*x]^{(5/2)}*\operatorname{Sin}[c + d*x])/((96*d*\operatorname{Sqrt}[a + a*\operatorname{Sec}[c + d*x]]) + (a^2*(8*A + 11*B)*\operatorname{Sec}[c + d*x]^{(5/2)}*\operatorname{Sqrt}[a + a*\operatorname{Sec}[c + d*x]]*\operatorname{Sin}[c + d*x])/((24*d) + (a*B*\operatorname{Sec}[c + d*x]^{(5/2)}*(a + a*\operatorname{Sec}[c + d*x])^{(3/2)}*\operatorname{Sin}[c + d*x]))/(4*d)$

Rule 215

$\operatorname{Int}[1/\operatorname{Sqrt}[(a_) + (b_)*(x_)^2], x_Symbol] := \operatorname{Simp}[\operatorname{ArcSinh}[(\operatorname{Rt}[b, 2]*x)/\operatorname{Sqrt}[a]]/\operatorname{Rt}[b, 2], x] /;$ $\operatorname{FreeQ}\{a, b\}, x \&\& \operatorname{GtQ}[a, 0] \&\& \operatorname{PosQ}[b]$

Rule 3801

$\operatorname{Int}[\operatorname{Sqrt}[\operatorname{csc}[(e_) + (f_)*(x_)]*(d_)]*\operatorname{Sqrt}[\operatorname{csc}[(e_) + (f_)*(x_)]*(b_) + (a_)], x_Symbol] := \operatorname{Dist}[(-2*a*\operatorname{Sqrt}[(a*d)/b])/((b*f) * \operatorname{Subst}[\operatorname{Int}[1/\operatorname{Sqrt}[1 + x^2/a], x], x, (b*\operatorname{Cot}[e + f*x])/(\operatorname{Sqrt}[a + b*\operatorname{Csc}[e + f*x]])], x] /;$ $\operatorname{FreeQ}\{a,$

b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]

Rule 3803

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] := Simp[(-2*b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(2*a*d*(n - 1))/(b*(2*n - 1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]

Rule 4016

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := Simp[(-2*b*B*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && !LtQ[n, 0]

Rule 4018

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := -Simp[(b*B*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && !LtQ[n, -1]

Rubi steps

$$\begin{aligned}
\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))dx &= \frac{aB\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{4d} + \\
&= \frac{a^2(8A+11B)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{24d} \\
&= \frac{a^3(104A+95B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{96d\sqrt{a+a\sec(c+dx)}} + \frac{a^2(8A}{ \\
&= \frac{a^3(200A+163B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{64d\sqrt{a+a\sec(c+dx)}} + \frac{a^3(10}{ \\
&= \frac{a^3(200A+163B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{64d\sqrt{a+a\sec(c+dx)}} + \frac{a^3(10}{ \\
&= \frac{a^{5/2}(200A+163B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{64d} + \frac{a^3(20}
\end{aligned}$$

Mathematica [A] time = 1.59, size = 154, normalized size = 0.68

$$\frac{a^2 \sec\left(\frac{1}{2}(c+dx)\right) \sqrt{a(\sec(c+dx)+1)} \left(6\sqrt{2}(200A+163B) \tanh^{-1}\left(\sqrt{2} \sin\left(\frac{1}{2}(c+dx)\right)\right) + \sin\left(\frac{1}{2}(c+dx)\right) \sec^4\right)}{64d}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]), x]

[Out] (a^2*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(6*Sqrt[2]*(200*A + 163*B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]] + (544*A + 844*B + (2056*A + 2203*B)*Cos[c + d*x] + (544*A + 652*B)*Cos[2*(c + d*x)] + 600*A*Cos[3*(c + d*x)] + 489*B*Cos[3*(c + d*x)])*Sec[c + d*x]^4*Sin[(c + d*x)/2))/(768*d*Sqrt[Sec[c + d*x]])

fricas [A] time = 0.70, size = 518, normalized size = 2.28

$$\frac{3 \left((200A + 163B)a^2 \cos(dx + c)^4 + (200A + 163B)a^2 \cos(dx + c)^3 \right) \sqrt{a} \log \left(\frac{a \cos(dx + c)^3 - 7a \cos(dx + c)^2 - \frac{4(\cos(dx + c)^2)}{\cos(dx + c)^3}}{\cos(dx + c)^3} \right)}{768 (d \cos(dx + c)^4 + d \cos(dx + c)^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(dx+c)^(3/2)*(a+a*sec(dx+c))^(5/2)*(A+B*sec(dx+c)),x, algorithm="fricas")

[Out] [1/768*(3*((200*A + 163*B)*a^2*cos(dx + c)^4 + (200*A + 163*B)*a^2*cos(dx + c)^3)*sqrt(a)*log((a*cos(dx + c)^3 - 7*a*cos(dx + c)^2 - 4*(cos(dx + c)^2 - 2*cos(dx + c))*sqrt(a)*sqrt((a*cos(dx + c) + a)/cos(dx + c))*sin(dx + c)/sqrt(cos(dx + c)) + 8*a)/(cos(dx + c)^3 + cos(dx + c)^2)) + 4*(3*(200*A + 163*B)*a^2*cos(dx + c)^3 + 2*(136*A + 163*B)*a^2*cos(dx + c)^2 + 8*(8*A + 23*B)*a^2*cos(dx + c) + 48*B*a^2)*sqrt((a*cos(dx + c) + a)/cos(dx + c))*sin(dx + c)/sqrt(cos(dx + c)))/(d*cos(dx + c)^4 + d*cos(dx + c)^3), 1/384*(3*((200*A + 163*B)*a^2*cos(dx + c)^4 + (200*A + 163*B)*a^2*cos(dx + c)^3)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(dx + c) + a)/cos(dx + c))*sqrt(cos(dx + c))*sin(dx + c)/(a*cos(dx + c)^2 - a*cos(dx + c) - 2*a)) + 2*(3*(200*A + 163*B)*a^2*cos(dx + c)^3 + 2*(136*A + 163*B)*a^2*cos(dx + c)^2 + 8*(8*A + 23*B)*a^2*cos(dx + c) + 48*B*a^2)*sqrt((a*cos(dx + c) + a)/cos(dx + c))*sin(dx + c)/sqrt(cos(dx + c)))/(d*cos(dx + c)^4 + d*cos(dx + c)^3)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(a \sec(dx + c) + a)^{\frac{5}{2}} \sec(dx + c)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(dx+c)^(3/2)*(a+a*sec(dx+c))^(5/2)*(A+B*sec(dx+c)),x, algorithm="giac")

[Out] integrate((B*sec(dx + c) + A)*(a*sec(dx + c) + a)^(5/2)*sec(dx + c)^(3/2), x)

maple [B] time = 2.46, size = 479, normalized size = 2.11

$$(-1 + \cos(dx + c)) \left(-600A (\cos^4(dx + c)) \sqrt{2} \arctan \left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}} (\cos(dx+c)+1-\sin(dx+c)) \sqrt{2}}{4} \right) + 600A (\cos^4(dx + c)) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x)

[Out]
$$\begin{aligned} & -1/384/d*(-1+\cos(d*x+c))*(-600*A*\cos(d*x+c)^4*2^{(1/2)}*\arctan(1/4*(-2/(1+\cos(d*x+c))))^{(1/2)}*(\cos(d*x+c)+1-\sin(d*x+c))*2^{(1/2)}+600*A*\cos(d*x+c)^4*2^{(1/2)} \\ & * \arctan(1/4*(-2/(1+\cos(d*x+c))))^{(1/2)}*(\cos(d*x+c)+1+\sin(d*x+c))*2^{(1/2)}-489*B*\cos(d*x+c)^4*2^{(1/2)} \\ & * \arctan(1/4*(-2/(1+\cos(d*x+c))))^{(1/2)}*(\cos(d*x+c)+1-\sin(d*x+c))*2^{(1/2)}+489*B*\cos(d*x+c)^4*2^{(1/2)} \\ & * \arctan(1/4*(-2/(1+\cos(d*x+c))))^{(1/2)}*(\cos(d*x+c)+1+\sin(d*x+c))*2^{(1/2)}+1200*A*\cos(d*x+c)^3*\sin(d*x+c) \\ & * (-2/(1+\cos(d*x+c)))^{(1/2)}+978*B*\cos(d*x+c)^3*\sin(d*x+c)*(-2/(1+\cos(d*x+c)))^{(1/2)} \\ & +544*A*\sin(d*x+c)*\cos(d*x+c)^2*(-2/(1+\cos(d*x+c)))^{(1/2)}+652*B*\sin(d*x+c)*\cos(d*x+c)^2 \\ & * (-2/(1+\cos(d*x+c)))^{(1/2)}+128*A*\sin(d*x+c)*\cos(d*x+c)*(-2/(1+\cos(d*x+c)))^{(1/2)} \\ & +368*B*\sin(d*x+c)*\cos(d*x+c)*(-2/(1+\cos(d*x+c)))^{(1/2)}+96*B*(-2/(1+\cos(d*x+c)))^{(1/2)} \\ & *\sin(d*x+c)*(a*(1+\cos(d*x+c))/\cos(d*x+c))^{(1/2)}*(1/\cos(d*x+c))^{(3/2)}/\sin(d*x+c)^2/(-2/(1+\cos(d*x+c)))^{(1/2)}/\cos(d*x+c)^2*a^2 \end{aligned}$$

maxima [B] time = 3.09, size = 7331, normalized size = 32.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out]
$$\begin{aligned} & 1/768*(8*(300*\sqrt{2})*a^2*\cos(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) \\ & * \sin(6*d*x + 6*c) - 28*\sqrt{2})*a^2*\sin(9/2*d*x + 9/2*c) + 28*\sqrt{2})*a^2*\sin(3/2*d*x + 3/2*c) \\ & - 28*(\sqrt{2})*a^2*\sin(9/2*d*x + 9/2*c) - \sqrt{2})*a^2*\sin(3/2*d*x + 3/2*c))*\cos(6*d*x + 6*c) - 300*(\sqrt{2})*a^2*\sin(6*d*x + 6*c) \\ & + 3*\sqrt{2})*a^2*\sin(8/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 3*\sqrt{2})*a^2*\sin(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) \\ & * \cos(11/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) - 12*(7*\sqrt{2})*a^2*\sin(9/2*d*x + 9/2*c) \\ & - 7*\sqrt{2})*a^2*\sin(3/2*d*x + 3/2*c) - 114*\sqrt{2})*a^2*\sin(7/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) \\ & + 114*\sqrt{2})*a^2*\sin(5/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 75*\sqrt{2})*a^2*\sin(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) \end{aligned}$$

$$\begin{aligned}
& *x + 3/2*c))))*\cos(8/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) \\
& - 456*(\sqrt{2}*a^2*\sin(6*d*x + 6*c) + 3*\sqrt{2}*a^2*\sin(4/3*\arctan2(\sin(3/ \\
& 2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))))*\cos(7/3*\arctan2(\sin(3/2*d*x + 3/2*c \\
&), \cos(3/2*d*x + 3/2*c))) + 456*(\sqrt{2}*a^2*\sin(6*d*x + 6*c) + 3*\sqrt{2}*a \\
& ^2*\sin(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))))*\cos(5/3*\ar \\
& ctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) - 12*(7*\sqrt{2}*a^2*\sin(\\
& 9/2*d*x + 9/2*c) - 7*\sqrt{2}*a^2*\sin(3/2*d*x + 3/2*c) + 75*\sqrt{2}*a^2*\sin(\\
& 1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))))*\cos(4/3*\arctan2(s \\
& in(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 75*(a^2*\cos(6*d*x + 6*c)^2 + \\
& 9*a^2*\cos(8/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + 9*a^ \\
& 2*\cos(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + a^2*\sin(\\
& 6*d*x + 6*c)^2 + 9*a^2*\sin(8/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + \\
& 3/2*c)))^2 + 6*a^2*\sin(6*d*x + 6*c)*\sin(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), c \\
& os(3/2*d*x + 3/2*c))) + 9*a^2*\sin(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2 \\
& *d*x + 3/2*c)))^2 + 2*a^2*\cos(6*d*x + 6*c) + a^2 + 6*(a^2*\cos(6*d*x + 6*c) \\
& + 3*a^2*\cos(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + a^2) \\
& *\cos(8/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 6*(a^2*\cos(\\
& 6*d*x + 6*c) + a^2)*\cos(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2 \\
& *c))) + 6*(a^2*\sin(6*d*x + 6*c) + 3*a^2*\sin(4/3*\arctan2(\sin(3/2*d*x + 3/2*c \\
&), \cos(3/2*d*x + 3/2*c))))*\sin(8/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d* \\
& x + 3/2*c))))*\log(2*\cos(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2 \\
& *c)))^2 + 2*\sin(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 \\
& + 2*\sqrt{2}*\cos(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + \\
& 2*\sqrt{2}*\sin(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 2) \\
& - 75*(a^2*\cos(6*d*x + 6*c)^2 + 9*a^2*\cos(8/3*\arctan2(\sin(3/2*d*x + 3/2*c), \\
& \cos(3/2*d*x + 3/2*c)))^2 + 9*a^2*\cos(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos \\
& (3/2*d*x + 3/2*c)))^2 + a^2*\sin(6*d*x + 6*c)^2 + 9*a^2*\sin(8/3*\arctan2(\sin(\\
& 3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + 6*a^2*\sin(6*d*x + 6*c)*\sin(4/3 \\
& *\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 9*a^2*\sin(4/3*\arcta \\
& n2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + 2*a^2*\cos(6*d*x + 6*c) \\
& + a^2 + 6*(a^2*\cos(6*d*x + 6*c) + 3*a^2*\cos(4/3*\arctan2(\sin(3/2*d*x + 3/2*c \\
&), \cos(3/2*d*x + 3/2*c))) + a^2)*\cos(8/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(\\
& 3/2*d*x + 3/2*c))) + 6*(a^2*\cos(6*d*x + 6*c) + a^2)*\cos(4/3*\arctan2(\sin(3/2 \\
& *d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 6*(a^2*\sin(6*d*x + 6*c) + 3*a^2*\sin \\
& (4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))))*\sin(8/3*\arctan2(\\
& sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))))*\log(2*\cos(1/3*\arctan2(\sin(3/2 \\
& *d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + 2*\sin(1/3*\arctan2(\sin(3/2*d*x + 3 \\
& /2*c), \cos(3/2*d*x + 3/2*c)))^2 + 2*\sqrt{2}*\cos(1/3*\arctan2(\sin(3/2*d*x + 3 \\
& /2*c), \cos(3/2*d*x + 3/2*c))) - 2*\sqrt{2}*\sin(1/3*\arctan2(\sin(3/2*d*x + 3/2 \\
& *c), \cos(3/2*d*x + 3/2*c))) + 2) + 75*(a^2*\cos(6*d*x + 6*c)^2 + 9*a^2*\cos(8 \\
& /3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + 9*a^2*\cos(4/3*a \\
& rctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + a^2*\sin(6*d*x + 6*c \\
&)^2 + 9*a^2*\sin(8/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 \\
& + 6*a^2*\sin(6*d*x + 6*c)*\sin(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x \\
& + 3/2*c))) + 9*a^2*\sin(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*
\end{aligned}$$

$$\begin{aligned}
& c)))^2 + 2a^2 \cos(6dx + 6c) + a^2 + 6(a^2 \cos(6dx + 6c) + 3a^2 \cos(4/3 \arctan 2(\sin(3/2 dx + 3/2 c), \cos(3/2 dx + 3/2 c)))) + a^2 \cos(8/3 \arctan 2(\sin(3/2 dx + 3/2 c), \cos(3/2 dx + 3/2 c))) + 6(a^2 \cos(6dx + 6c) + a^2 \cos(4/3 \arctan 2(\sin(3/2 dx + 3/2 c), \cos(3/2 dx + 3/2 c)))) + 6(a^2 \sin(6dx + 6c) + 3a^2 \sin(4/3 \arctan 2(\sin(3/2 dx + 3/2 c), \cos(3/2 dx + 3/2 c)))) \sin(8/3 \arctan 2(\sin(3/2 dx + 3/2 c), \cos(3/2 dx + 3/2 c))) \log(2 \cos(1/3 \arctan 2(\sin(3/2 dx + 3/2 c), \cos(3/2 dx + 3/2 c))))^2 + 2 \sin(1/3 \arctan 2(\sin(3/2 dx + 3/2 c), \cos(3/2 dx + 3/2 c)))^2 - 2 \sqrt{2} \cos(1/3 \arctan 2(\sin(3/2 dx + 3/2 c), \cos(3/2 dx + 3/2 c))) + 2 \sqrt{2} \sin(1/3 \arctan 2(\sin(3/2 dx + 3/2 c), \cos(3/2 dx + 3/2 c))) + 2) - 75(a^2 \cos(6dx + 6c))^2 + 9a^2 \cos(8/3 \arctan 2(\sin(3/2 dx + 3/2 c), \cos(3/2 dx + 3/2 c)))^2 + 9a^2 \cos(4/3 \arctan 2(\sin(3/2 dx + 3/2 c), \cos(3/2 dx + 3/2 c)))^2 + a^2 \sin(6dx + 6c)^2 + 9a^2 \sin(8/3 \arctan 2(\sin(3/2 dx + 3/2 c), \cos(3/2 dx + 3/2 c)))^2 + 6a^2 \sin(6dx + 6c) \sin(4/3 \arctan 2(\sin(3/2 dx + 3/2 c), \cos(3/2 dx + 3/2 c))) + 9a^2 \sin(4/3 \arctan 2(\sin(3/2 dx + 3/2 c), \cos(3/2 dx + 3/2 c)))^2 + 2a^2 \cos(6dx + 6c) + a^2 + 6(a^2 \cos(6dx + 6c) + 3a^2 \cos(4/3 \arctan 2(\sin(3/2 dx + 3/2 c), \cos(3/2 dx + 3/2 c)))) + a^2 \cos(8/3 \arctan 2(\sin(3/2 dx + 3/2 c), \cos(3/2 dx + 3/2 c))) + 6(a^2 \cos(6dx + 6c) + a^2 \cos(4/3 \arctan 2(\sin(3/2 dx + 3/2 c), \cos(3/2 dx + 3/2 c)))) + 6(a^2 \sin(6dx + 6c) + 3a^2 \sin(4/3 \arctan 2(\sin(3/2 dx + 3/2 c), \cos(3/2 dx + 3/2 c)))) \sin(8/3 \arctan 2(\sin(3/2 dx + 3/2 c), \cos(3/2 dx + 3/2 c))) \log(2 \cos(1/3 \arctan 2(\sin(3/2 dx + 3/2 c), \cos(3/2 dx + 3/2 c))))^2 + 2 \sin(1/3 \arctan 2(\sin(3/2 dx + 3/2 c), \cos(3/2 dx + 3/2 c)))^2 - 2 \sqrt{2} \cos(1/3 \arctan 2(\sin(3/2 dx + 3/2 c), \cos(3/2 dx + 3/2 c))) - 2 \sqrt{2} \sin(1/3 \arctan 2(\sin(3/2 dx + 3/2 c), \cos(3/2 dx + 3/2 c))) + 2) + 28(\sqrt{2} a^2 \cos(9/2 dx + 9/2 c) - \sqrt{2} a^2 \cos(3/2 dx + 3/2 c)) \sin(6dx + 6c) + 300(\sqrt{2} a^2 \cos(6dx + 6c) + 3\sqrt{2} a^2 \cos(8/3 \arctan 2(\sin(3/2 dx + 3/2 c), \cos(3/2 dx + 3/2 c))) + 3\sqrt{2} a^2 \cos(4/3 \arctan 2(\sin(3/2 dx + 3/2 c), \cos(3/2 dx + 3/2 c)))) + \sqrt{2} a^2 \sin(11/3 \arctan 2(\sin(3/2 dx + 3/2 c), \cos(3/2 dx + 3/2 c))) + 12(7\sqrt{2} a^2 \cos(9/2 dx + 9/2 c) - 7\sqrt{2} a^2 \cos(3/2 dx + 3/2 c) - 114\sqrt{2} a^2 \cos(7/3 \arctan 2(\sin(3/2 dx + 3/2 c), \cos(3/2 dx + 3/2 c)))) + 114\sqrt{2} a^2 \cos(5/3 \arctan 2(\sin(3/2 dx + 3/2 c), \cos(3/2 dx + 3/2 c))) + 75\sqrt{2} a^2 \cos(1/3 \arctan 2(\sin(3/2 dx + 3/2 c), \cos(3/2 dx + 3/2 c))) \sin(8/3 \arctan 2(\sin(3/2 dx + 3/2 c), \cos(3/2 dx + 3/2 c))) + 456(\sqrt{2} a^2 \cos(6dx + 6c) + 3\sqrt{2} a^2 \cos(4/3 \arctan 2(\sin(3/2 dx + 3/2 c), \cos(3/2 dx + 3/2 c)))) + \sqrt{2} a^2 \sin(7/3 \arctan 2(\sin(3/2 dx + 3/2 c), \cos(3/2 dx + 3/2 c))) - 456(\sqrt{2} a^2 \cos(6dx + 6c) + 3\sqrt{2} a^2 \cos(4/3 \arctan 2(\sin(3/2 dx + 3/2 c), \cos(3/2 dx + 3/2 c)))) + \sqrt{2} a^2 \sin(5/3 \arctan 2(\sin(3/2 dx + 3/2 c), \cos(3/2 dx + 3/2 c))) + 12(7\sqrt{2} a^2 \cos(9/2 dx + 9/2 c) - 7\sqrt{2} a^2 \cos(3/2 dx + 3/2 c) + 75\sqrt{2} a^2 \cos(1/3 \arctan 2(\sin(3/2 dx + 3/2 c), \cos(3/2 dx + 3/2 c)))) \sin(4/3 \arctan 2(\sin(3/2 dx + 3/2 c), \cos(3/2 dx + 3/2 c))) - 300(\sqrt{2} a^2 \cos(6dx + 6c) + \sqrt{2} a^2 \sin(1/3 \arctan 2(\sin(3/2 dx + 3/2 c), \cos(3/2 dx + 3/2 c)))) A \sqrt{a} / (\cos(6dx + 6c))^2 + 6
\end{aligned}$$

$$\begin{aligned}
& (\cos(6dx + 6c) + 3\cos(4/3\arctan2(\sin(3/2dx + 3/2c), \cos(3/2dx + 3/2c))) + 1)\cos(8/3\arctan2(\sin(3/2dx + 3/2c), \cos(3/2dx + 3/2c))) + \\
& 9\cos(8/3\arctan2(\sin(3/2dx + 3/2c), \cos(3/2dx + 3/2c)))^2 + 6(\cos(6dx + 6c) + 1)\cos(4/3\arctan2(\sin(3/2dx + 3/2c), \cos(3/2dx + 3/2c))) + \\
& 9\cos(4/3\arctan2(\sin(3/2dx + 3/2c), \cos(3/2dx + 3/2c)))^2 + \sin(6dx + 6c)^2 + 6(\sin(6dx + 6c) + 3\sin(4/3\arctan2(\sin(3/2dx + 3/2c), \cos(3/2dx + 3/2c))))\sin(8/3\arctan2(\sin(3/2dx + 3/2c), \cos(3/2dx + 3/2c))) + \\
& 9\sin(8/3\arctan2(\sin(3/2dx + 3/2c), \cos(3/2dx + 3/2c)))^2 + 6\sin(6dx + 6c)\sin(4/3\arctan2(\sin(3/2dx + 3/2c), \cos(3/2dx + 3/2c))) + \\
& 9\sin(4/3\arctan2(\sin(3/2dx + 3/2c), \cos(3/2dx + 3/2c)))^2 + 2\cos(6dx + 6c) + 1 - (1956(\sqrt{2})a^2\sin(8dx + 8c) + 4\sqrt{2})a^2\sin(6dx + 6c) + 6\sqrt{2})a^2\sin(4dx + 4c) + 4\sqrt{2})a^2\sin(2dx + 2c))\cos(15/4\arctan2(\sin(2dx + 2c), \cos(2dx + 2c))) + \\
& 652(\sqrt{2})a^2\sin(8dx + 8c) + 4\sqrt{2})a^2\sin(6dx + 6c) + 6\sqrt{2})a^2\sin(4dx + 4c) + 4\sqrt{2})a^2\sin(2dx + 2c))\cos(13/4\arctan2(\sin(2dx + 2c), \cos(2dx + 2c))) + \\
& 6204(\sqrt{2})a^2\sin(8dx + 8c) + 4\sqrt{2})a^2\sin(6dx + 6c) + 6\sqrt{2})a^2\sin(4dx + 4c) + 4\sqrt{2})a^2\sin(2dx + 2c))\cos(11/4\arctan2(\sin(2dx + 2c), \cos(2dx + 2c))) - \\
& 2060(\sqrt{2})a^2\sin(8dx + 8c) + 4\sqrt{2})a^2\sin(6dx + 6c) + 6\sqrt{2})a^2\sin(4dx + 4c) + 4\sqrt{2})a^2\sin(2dx + 2c))\cos(9/4\arctan2(\sin(2dx + 2c), \cos(2dx + 2c))) + \\
& 2060(\sqrt{2})a^2\sin(8dx + 8c) + 4\sqrt{2})a^2\sin(6dx + 6c) + 6\sqrt{2})a^2\sin(4dx + 4c) + 4\sqrt{2})a^2\sin(2dx + 2c))\cos(7/4\arctan2(\sin(2dx + 2c), \cos(2dx + 2c))) - \\
& 6204(\sqrt{2})a^2\sin(8dx + 8c) + 4\sqrt{2})a^2\sin(6dx + 6c) + 6\sqrt{2})a^2\sin(4dx + 4c) + 4\sqrt{2})a^2\sin(2dx + 2c))\cos(5/4\arctan2(\sin(2dx + 2c), \cos(2dx + 2c))) - \\
& 652(\sqrt{2})a^2\sin(8dx + 8c) + 4\sqrt{2})a^2\sin(6dx + 6c) + 6\sqrt{2})a^2\sin(4dx + 4c) + 4\sqrt{2})a^2\sin(2dx + 2c))\cos(3/4\arctan2(\sin(2dx + 2c), \cos(2dx + 2c))) - \\
& 1956(\sqrt{2})a^2\sin(8dx + 8c) + 4\sqrt{2})a^2\sin(6dx + 6c) + 6\sqrt{2})a^2\sin(4dx + 4c) + 4\sqrt{2})a^2\sin(2dx + 2c))\cos(1/4\arctan2(\sin(2dx + 2c), \cos(2dx + 2c))) - \\
& 489(a^2\cos(8dx + 8c))^2 + 16a^2\cos(6dx + 6c)^2 + 36a^2\cos(4dx + 4c)^2 + 16a^2\cos(2dx + 2c)^2 + a^2\sin(8dx + 8c)^2 + 16a^2\sin(6dx + 6c)^2 + 36a^2\sin(4dx + 4c)^2 + \\
& 48a^2\sin(4dx + 4c)\sin(2dx + 2c) + 16a^2\sin(2dx + 2c)^2 + 8a^2\cos(2dx + 2c) + a^2 + 2(4a^2\cos(6dx + 6c) + 6a^2\cos(4dx + 4c) + 4a^2\cos(2dx + 2c) + a^2)\cos(8dx + 8c) + \\
& 8(6a^2\cos(4dx + 4c) + 4a^2\cos(2dx + 2c) + a^2)\cos(6dx + 6c) + 12(4a^2\cos(2dx + 2c) + a^2)\cos(4dx + 4c) + 4(2a^2\sin(6dx + 6c) + 3a^2\sin(4dx + 4c) + 2a^2\sin(2dx + 2c))\sin(8dx + 8c) + \\
& 16(3a^2\sin(4dx + 4c) + 2a^2\sin(2dx + 2c))\sin(6dx + 6c))\log(2\cos(1/4\arctan2(\sin(2dx + 2c), \cos(2dx + 2c)))^2 + 2\sin(1/4\arctan2(\sin(2dx + 2c), \cos(2dx + 2c))))^2 + 2\sqrt{2})\cos(1/4\arctan2(\sin(2dx + 2c), \cos(2dx + 2c))) + 2\sqrt{2})\sin(1/4\arctan2(\sin(2dx + 2c), \cos(2dx + 2c))) + 2) + 489(a^2\cos(8dx + 8c))^2 + 16a^2\cos(6dx + 6c)^2 + 36a^2\cos(4dx + 4c)^2 + 16a^2\cos(2dx + 2c)^2 + a
\end{aligned}$$

$$\begin{aligned}
&^2\sin(8*d*x + 8*c)^2 + 16*a^2\sin(6*d*x + 6*c)^2 + 36*a^2\sin(4*d*x + 4*c) \\
&^2 + 48*a^2\sin(4*d*x + 4*c)*\sin(2*d*x + 2*c) + 16*a^2\sin(2*d*x + 2*c)^2 + \\
&8*a^2\cos(2*d*x + 2*c) + a^2 + 2*(4*a^2\cos(6*d*x + 6*c) + 6*a^2\cos(4*d*x \\
&+ 4*c) + 4*a^2\cos(2*d*x + 2*c) + a^2)*\cos(8*d*x + 8*c) + 8*(6*a^2\cos(4*d \\
&*x + 4*c) + 4*a^2\cos(2*d*x + 2*c) + a^2)*\cos(6*d*x + 6*c) + 12*(4*a^2\cos(\\
&2*d*x + 2*c) + a^2)*\cos(4*d*x + 4*c) + 4*(2*a^2\sin(6*d*x + 6*c) + 3*a^2\sin \\
&n(4*d*x + 4*c) + 2*a^2\sin(2*d*x + 2*c))*\sin(8*d*x + 8*c) + 16*(3*a^2\sin(4 \\
&*d*x + 4*c) + 2*a^2\sin(2*d*x + 2*c))*\sin(6*d*x + 6*c))*\log(2*\cos(1/4*\arcta \\
&n2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + 2*\sin(1/4*\arctan2(\sin(2*d*x + 2 \\
&*c), \cos(2*d*x + 2*c)))^2 + 2*\sqrt{2}*\cos(1/4*\arctan2(\sin(2*d*x + 2*c), \cos \\
&(2*d*x + 2*c))) - 2*\sqrt{2}*\sin(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2 \\
&*c))) + 2) - 489*(a^2\cos(8*d*x + 8*c)^2 + 16*a^2\cos(6*d*x + 6*c)^2 + 36*a \\
&^2\cos(4*d*x + 4*c)^2 + 16*a^2\cos(2*d*x + 2*c)^2 + a^2\sin(8*d*x + 8*c)^2 \\
&+ 16*a^2\sin(6*d*x + 6*c)^2 + 36*a^2\sin(4*d*x + 4*c)^2 + 48*a^2\sin(4*d*x \\
&+ 4*c)*\sin(2*d*x + 2*c) + 16*a^2\sin(2*d*x + 2*c)^2 + 8*a^2\cos(2*d*x + 2*c \\
&) + a^2 + 2*(4*a^2\cos(6*d*x + 6*c) + 6*a^2\cos(4*d*x + 4*c) + 4*a^2\cos(2* \\
&d*x + 2*c) + a^2)*\cos(8*d*x + 8*c) + 8*(6*a^2\cos(4*d*x + 4*c) + 4*a^2\cos(\\
&2*d*x + 2*c) + a^2)*\cos(6*d*x + 6*c) + 12*(4*a^2\cos(2*d*x + 2*c) + a^2)*\co \\
&s(4*d*x + 4*c) + 4*(2*a^2\sin(6*d*x + 6*c) + 3*a^2\sin(4*d*x + 4*c) + 2*a^2 \\
&*sin(2*d*x + 2*c))*\sin(8*d*x + 8*c) + 16*(3*a^2\sin(4*d*x + 4*c) + 2*a^2\sin \\
&>n(2*d*x + 2*c))*\sin(6*d*x + 6*c))*\log(2*\cos(1/4*\arctan2(\sin(2*d*x + 2*c), c \\
&>os(2*d*x + 2*c)))^2 + 2*\sin(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)) \\
&)^2 - 2*\sqrt{2}*\cos(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 2*\sq \\
&rt(2)*\sin(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 2) + 489*(a^2* \\
&>cos(8*d*x + 8*c)^2 + 16*a^2\cos(6*d*x + 6*c)^2 + 36*a^2\cos(4*d*x + 4*c)^2 \\
&+ 16*a^2\cos(2*d*x + 2*c)^2 + a^2\sin(8*d*x + 8*c)^2 + 16*a^2\sin(6*d*x + 6 \\
&>*c)^2 + 36*a^2\sin(4*d*x + 4*c)^2 + 48*a^2\sin(4*d*x + 4*c)*\sin(2*d*x + 2*c \\
&) + 16*a^2\sin(2*d*x + 2*c)^2 + 8*a^2\cos(2*d*x + 2*c) + a^2 + 2*(4*a^2\cos \\
&(6*d*x + 6*c) + 6*a^2\cos(4*d*x + 4*c) + 4*a^2\cos(2*d*x + 2*c) + a^2)*\cos(\\
&8*d*x + 8*c) + 8*(6*a^2\cos(4*d*x + 4*c) + 4*a^2\cos(2*d*x + 2*c) + a^2)*\co \\
&>s(6*d*x + 6*c) + 12*(4*a^2\cos(2*d*x + 2*c) + a^2)*\cos(4*d*x + 4*c) + 4*(2* \\
&>a^2\sin(6*d*x + 6*c) + 3*a^2\sin(4*d*x + 4*c) + 2*a^2\sin(2*d*x + 2*c))*\sin \\
&(8*d*x + 8*c) + 16*(3*a^2\sin(4*d*x + 4*c) + 2*a^2\sin(2*d*x + 2*c))*\sin(6* \\
&>d*x + 6*c))*\log(2*\cos(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + \\
&2*\sin(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 - 2*\sqrt{2}*\cos(1/ \\
&4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) - 2*\sqrt{2}*\sin(1/4*\arctan2(\\
&>\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 2) - 1956*(\sqrt{2}*a^2\cos(8*d*x + 8 \\
&>*c) + 4*\sqrt{2}*a^2\cos(6*d*x + 6*c) + 6*\sqrt{2}*a^2\cos(4*d*x + 4*c) + 4*s \\
&>qrt(2)*a^2\cos(2*d*x + 2*c) + \sqrt{2}*a^2*\sin(15/4*\arctan2(\sin(2*d*x + 2*c \\
&), \cos(2*d*x + 2*c))) - 652*(\sqrt{2}*a^2\cos(8*d*x + 8*c) + 4*\sqrt{2}*a^2*c \\
&>os(6*d*x + 6*c) + 6*\sqrt{2}*a^2\cos(4*d*x + 4*c) + 4*\sqrt{2}*a^2\cos(2*d*x \\
&+ 2*c) + \sqrt{2}*a^2*\sin(13/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) \\
&- 6204*(\sqrt{2}*a^2\cos(8*d*x + 8*c) + 4*\sqrt{2}*a^2\cos(6*d*x + 6*c) + 6* \\
&>\sqrt{2}*a^2\cos(4*d*x + 4*c) + 4*\sqrt{2}*a^2\cos(2*d*x + 2*c) + \sqrt{2}*a^2 \\
&)*\sin(11/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 2060*(\sqrt{2}*a^2
\end{aligned}$$


```

*cos(8*d*x + 8*c) + 4*sqrt(2)*a^2*cos(6*d*x + 6*c) + 6*sqrt(2)*a^2*cos(4*d*
x + 4*c) + 4*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2*sin(9/4*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2060*(sqrt(2)*a^2*cos(8*d*x + 8*c) + 4
*sqrt(2)*a^2*cos(6*d*x + 6*c) + 6*sqrt(2)*a^2*cos(4*d*x + 4*c) + 4*sqrt(2)*
a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2*sin(7/4*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + 6204*(sqrt(2)*a^2*cos(8*d*x + 8*c) + 4*sqrt(2)*a^2*cos(6*d*
x + 6*c) + 6*sqrt(2)*a^2*cos(4*d*x + 4*c) + 4*sqrt(2)*a^2*cos(2*d*x + 2*c)
+ sqrt(2)*a^2*sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 652*(
sqrt(2)*a^2*cos(8*d*x + 8*c) + 4*sqrt(2)*a^2*cos(6*d*x + 6*c) + 6*sqrt(2)*a
^2*cos(4*d*x + 4*c) + 4*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2*sin(3/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1956*(sqrt(2)*a^2*cos(8*d*x
+ 8*c) + 4*sqrt(2)*a^2*cos(6*d*x + 6*c) + 6*sqrt(2)*a^2*cos(4*d*x + 4*c) +
4*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2*sin(1/4*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c))))*B*sqrt(a)/(2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x +
4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6
*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x
+ 6*c)^2 + 12*(4*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4
*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c)
+ 2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d
*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 +
36*sin(4*d*x + 4*c)^2 + 48*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x
+ 2*c)^2 + 8*cos(2*d*x + 2*c) + 1))/d

```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(A + \frac{B}{\cos(c + dx)} \right) \left(a + \frac{a}{\cos(c + dx)} \right)^{5/2} \left(\frac{1}{\cos(c + dx)} \right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(3/2), x)

[Out] int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(3/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**(3/2)*(a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)), x)

[Out] Timed out

$$3.241 \quad \int \sqrt{\sec(c + dx)} (a + a \sec(c + dx))^{5/2} (A + B \sec(c + dx)) dx$$

Optimal. Leaf size=180

$$\frac{a^{5/2}(38A + 25B) \sinh^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}}\right)}{8d} + \frac{a^3(54A + 49B) \sin(c + dx) \sec^2(c + dx)}{24d\sqrt{a \sec(c + dx) + a}} + \frac{a^2(2A + 3B) \sin(c + dx) \sec^2(c + dx)}{4d}$$

[Out] 1/8*a^(5/2)*(38*A+25*B)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+1/3*a*B*sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2)*sin(d*x+c)/d+1/24*a^3*(54*A+49*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/4*a^2*(2*A+3*B)*sec(d*x+c)^(3/2)*sin(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d

Rubi [A] time = 0.51, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {4018, 4016, 3801, 215}

$$\frac{a^3(54A + 49B) \sin(c + dx) \sec^2(c + dx)}{24d\sqrt{a \sec(c + dx) + a}} + \frac{a^2(2A + 3B) \sin(c + dx) \sec^2(c + dx)\sqrt{a \sec(c + dx) + a}}{4d} + \frac{a^{5/2}(38A + 25B) \sinh^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}}\right)}{8d}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]

[Out] (a^(5/2)*(38*A + 25*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]])/(8*d) + (a^3*(54*A + 49*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(24*d*Sqrt[a + a*Sec[c + d*x]]) + (a^2*(2*A + 3*B)*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(4*d) + (a*B*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3*d)

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[Rt[b, 2]*x]/Sqrt[a]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 3801

Int[Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] := Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 + x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.)
+ (a_.)]*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
+ Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x]
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && !
LtQ[n, 0]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (
a_.))^(m_)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && !LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{\sec(c+dx)} (a+a \sec(c+dx))^{5/2} (A+B \sec(c+dx)) dx &= \frac{aB \sec^{\frac{3}{2}}(c+dx) (a+a \sec(c+dx))^{3/2} \sin(c+dx)}{3d} \\
&= \frac{a^2(2A+3B) \sec^{\frac{3}{2}}(c+dx) \sqrt{a+a \sec(c+dx)} \sin(c+dx)}{4d} \\
&= \frac{a^3(54A+49B) \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{24d \sqrt{a+a \sec(c+dx)}} + \frac{a^2(2A+3B) \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{4d} \\
&= \frac{a^3(54A+49B) \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{24d \sqrt{a+a \sec(c+dx)}} + \frac{a^2(2A+3B) \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{4d} \\
&= \frac{a^{5/2}(38A+25B) \sinh^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a+a \sec(c+dx)}}\right)}{8d} + \frac{a^3(54A+49B) \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{24d \sqrt{a+a \sec(c+dx)}}
\end{aligned}$$

Mathematica [A] time = 1.45, size = 133, normalized size = 0.74

$$\frac{a^2 \sec\left(\frac{1}{2}(c+dx)\right) \sqrt{a(\sec(c+dx)+1)} \left(3\sqrt{2}(38A+25B) \tanh^{-1}\left(\sqrt{2} \sin\left(\frac{1}{2}(c+dx)\right)\right) + \sin\left(\frac{1}{2}(c+dx)\right) \sec^3(c+dx)\right)}{48d \sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]

[Out] (a^2*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(3*Sqrt[2]*(38*A + 25*B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]] + (66*A + 91*B + 4*(6*A + 17*B)*Cos[c + d*x] + (66*A + 75*B)*Cos[2*(c + d*x)])*Sec[c + d*x]^3*Sin[(c + d*x)/2]))/(48*d*Sqrt[Sec[c + d*x]])

fricas [A] time = 0.59, size = 478, normalized size = 2.66

$$\frac{3 \left((38A + 25B)a^2 \cos(dx + c)^3 + (38A + 25B)a^2 \cos(dx + c)^2 \right) \sqrt{a} \log \left(\frac{a \cos(dx+c)^3 - 7a \cos(dx+c)^2 - \frac{4(\cos(dx+c)^2 - 2 \cos(dx+c) + 1)}{\cos(dx+c)^3 + \cos(dx+c)^2}}{96(d \cos(dx+c)^3 + d \cos(dx+c)^2)} \right)}{96(d \cos(dx+c)^3 + d \cos(dx+c)^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x, algorithm="fricas")

[Out] [1/96*(3*((38*A + 25*B)*a^2*cos(d*x + c)^3 + (38*A + 25*B)*a^2*cos(d*x + c)^2)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(3*(22*A + 25*B)*a^2*cos(d*x + c)^2 + 2*(6*A + 17*B)*a^2*cos(d*x + c) + 8*B*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2), 1/48*(3*((38*A + 25*B)*a^2*cos(d*x + c)^3 + (38*A + 25*B)*a^2*cos(d*x + c)^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) + 2*(3*(22*A + 25*B)*a^2*cos(d*x + c)^2 + 2*(6*A + 17*B)*a^2*cos(d*x + c) + 8*B*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(a \sec(dx + c) + a)^{\frac{5}{2}} \sqrt{\sec(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(5/2)*sqrt(sec(d*x + c)), x)

maple [B] time = 2.16, size = 419, normalized size = 2.33

$$\left(114A \left(\cos^3(dx + c) \right) \sqrt{2} \arctan \left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}} (\cos(dx+c)+1+\sin(dx+c))\sqrt{2}}{4} \right) - 114A \left(\cos^3(dx + c) \right) \sqrt{2} \arctan \left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}} (\cos(dx+c)+1-\sin(dx+c))\sqrt{2}}{4} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x)

[Out] 1/96/d*(114*A*cos(d*x+c)^3*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c))))^(1/2)*(cos(d*x+c)+1+sin(d*x+c))*2^(1/2))-114*A*cos(d*x+c)^3*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c))))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))*2^(1/2))+75*B*cos(d*x+c)^3*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c))))^(1/2)*(cos(d*x+c)+1+sin(d*x+c))*2^(1/2))-75*B*cos(d*x+c)^3*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c))))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))*2^(1/2))+132*A*sin(d*x+c)*cos(d*x+c)^2*(-2/(1+cos(d*x+c)))^(1/2)+150*B*sin(d*x+c)*cos(d*x+c)^2*(-2/(1+cos(d*x+c)))^(1/2)+24*A*sin(d*x+c)*cos(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)+68*B*sin(d*x+c)*cos(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)+16*B*(-2/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(1/2)*(-2/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)^2/sin(d*x+c)^2*(cos(d*x+c)^2-1)*a^2

maxima [B] time = 8.72, size = 6297, normalized size = 34.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x, algorithm="maxima")

[Out] -1/96*(6*(88*sqrt(2)*a^2*cos(7/2*d*x + 7/2*c)*sin(2*d*x + 2*c) - 56*sqrt(2)*a^2*cos(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c) - 28*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) + 44*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) - 19*(a^2*log(2*cos(1/2*d*x + 1/2*c))^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c))^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a^2*log(2*cos(1/2*d*x + 1/2*c))^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a

$$\begin{aligned}
& /2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x \\
& + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 \\
& - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*co \\
& s(2*d*x + 2*c))*cos(4*d*x + 4*c) - 4*(14*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) - \\
& 22*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 \\
& + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin \\
& (1/2*d*x + 1/2*c) + 2) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d* \\
& x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2 \\
& *c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - \\
& 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 19* \\
& a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos \\
& (1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c) + \\
& 4*(11*sqrt(2)*a^2*cos(7/2*d*x + 7/2*c) - 7*sqrt(2)*a^2*cos(5/2*d*x + 5/2*c \\
&) + 7*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c) - 11*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c \\
&) - 19*(a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt \\
& (2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2 \\
& *cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x \\
& + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a^2*log(2*cos(1/2*d*x + 1/ \\
& 2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt \\
& (2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/ \\
& 2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + \\
& 1/2*c) + 2))*sin(2*d*x + 2*c))*sin(4*d*x + 4*c) - 44*(2*sqrt(2)*a^2*cos(2* \\
& d*x + 2*c) + sqrt(2)*a^2)*sin(7/2*d*x + 7/2*c) + 28*(2*sqrt(2)*a^2*cos(2*d* \\
& x + 2*c) + sqrt(2)*a^2)*sin(5/2*d*x + 5/2*c) + 8*(7*sqrt(2)*a^2*cos(3/2*d*x \\
& + 3/2*c) - 11*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c))*sin(2*d*x + 2*c))*A*sqrt(a \\
&)/(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos \\
& (2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) \\
& + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1) - (300*sqrt(2)*a^2*cos(1/3 \\
& *arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(6*d*x + 6*c) - 28 \\
& *sqrt(2)*a^2*sin(9/2*d*x + 9/2*c) + 28*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) - 2 \\
& 8*(sqrt(2)*a^2*sin(9/2*d*x + 9/2*c) - sqrt(2)*a^2*sin(3/2*d*x + 3/2*c))*cos \\
& (6*d*x + 6*c) - 300*(sqrt(2)*a^2*sin(6*d*x + 6*c) + 3*sqrt(2)*a^2*sin(8/3*a \\
& rctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 3*sqrt(2)*a^2*sin(4/3 \\
& *arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(11/3*arctan2(sin \\
& (3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 12*(7*sqrt(2)*a^2*sin(9/2*d*x + \\
& 9/2*c) - 7*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) - 114*sqrt(2)*a^2*sin(7/3*arct \\
& an2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 114*sqrt(2)*a^2*sin(5/3* \\
& arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 75*sqrt(2)*a^2*sin(1 \\
& /3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(8/3*arctan2(si \\
& n(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 456*(sqrt(2)*a^2*sin(6*d*x + 6 \\
& *c) + 3*sqrt(2)*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2 \\
& *c))))*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 456*(\\
& sqrt(2)*a^2*sin(6*d*x + 6*c) + 3*sqrt(2)*a^2*sin(4/3*arctan2(sin(3/2*d*x + \\
& 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3 \\
& /2*d*x + 3/2*c))) - 12*(7*sqrt(2)*a^2*sin(9/2*d*x + 9/2*c) - 7*sqrt(2)*a^2*
\end{aligned}$$

$$\begin{aligned}
& \sin(3/2*d*x + 3/2*c) + 75*\sqrt{2}*a^2*\sin(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \\
& \cos(3/2*d*x + 3/2*c))) * \cos(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x \\
& + 3/2*c))) + 75*(a^2*\cos(6*d*x + 6*c)^2 + 9*a^2*\cos(8/3*\arctan2(\sin(3/2*d*x \\
& + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + 9*a^2*\cos(4/3*\arctan2(\sin(3/2*d*x + 3 \\
& /2*c), \cos(3/2*d*x + 3/2*c)))^2 + a^2*\sin(6*d*x + 6*c)^2 + 9*a^2*\sin(8/3*\ar \\
& ctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + 6*a^2*\sin(6*d*x + 6* \\
& c)*\sin(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 9*a^2*\sin \\
& (4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + 2*a^2*\cos(6*d \\
& *x + 6*c) + a^2 + 6*(a^2*\cos(6*d*x + 6*c) + 3*a^2*\cos(4/3*\arctan2(\sin(3/2*d \\
& *x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + a^2)*\cos(8/3*\arctan2(\sin(3/2*d*x + 3/ \\
& 2*c), \cos(3/2*d*x + 3/2*c))) + 6*(a^2*\cos(6*d*x + 6*c) + a^2)*\cos(4/3*\arcta \\
& n2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 6*(a^2*\sin(6*d*x + 6*c) + \\
& 3*a^2*\sin(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))))*\sin(8/ \\
& 3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) * \log(2*\cos(1/3*\arcta \\
& n2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + 2*\sin(1/3*\arctan2(\sin(3 \\
& /2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + 2*\sqrt{2}*\cos(1/3*\arctan2(\sin(3 \\
& /2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 2*\sqrt{2}*\sin(1/3*\arctan2(\sin(3/2 \\
& *d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 2) - 75*(a^2*\cos(6*d*x + 6*c)^2 + 9 \\
& *a^2*\cos(8/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + 9*a^2 \\
& *\cos(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + a^2*\sin(6 \\
& *d*x + 6*c)^2 + 9*a^2*\sin(8/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3 \\
& /2*c)))^2 + 6*a^2*\sin(6*d*x + 6*c)*\sin(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), co \\
& s(3/2*d*x + 3/2*c))) + 9*a^2*\sin(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2* \\
& d*x + 3/2*c)))^2 + 2*a^2*\cos(6*d*x + 6*c) + a^2 + 6*(a^2*\cos(6*d*x + 6*c) + \\
& 3*a^2*\cos(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + a^2)* \\
& \cos(8/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 6*(a^2*\cos(6 \\
& *d*x + 6*c) + a^2)*\cos(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2* \\
& c))) + 6*(a^2*\sin(6*d*x + 6*c) + 3*a^2*\sin(4/3*\arctan2(\sin(3/2*d*x + 3/2*c) \\
& , \cos(3/2*d*x + 3/2*c))))*\sin(8/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x \\
& + 3/2*c))) * \log(2*\cos(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2* \\
& c)))^2 + 2*\sin(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + \\
& 2*\sqrt{2}*\cos(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) - 2 \\
& *\sqrt{2}*\sin(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 2) \\
& + 75*(a^2*\cos(6*d*x + 6*c)^2 + 9*a^2*\cos(8/3*\arctan2(\sin(3/2*d*x + 3/2*c), \\
& \cos(3/2*d*x + 3/2*c)))^2 + 9*a^2*\cos(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(\\
& 3/2*d*x + 3/2*c)))^2 + a^2*\sin(6*d*x + 6*c)^2 + 9*a^2*\sin(8/3*\arctan2(\sin(3 \\
& /2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + 6*a^2*\sin(6*d*x + 6*c)*\sin(4/3* \\
& arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 9*a^2*\sin(4/3*\arctan \\
& 2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + 2*a^2*\cos(6*d*x + 6*c) + \\
& a^2 + 6*(a^2*\cos(6*d*x + 6*c) + 3*a^2*\cos(4/3*\arctan2(\sin(3/2*d*x + 3/2*c) \\
& , \cos(3/2*d*x + 3/2*c))) + a^2)*\cos(8/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3 \\
& /2*d*x + 3/2*c))) + 6*(a^2*\cos(6*d*x + 6*c) + a^2)*\cos(4/3*\arctan2(\sin(3/2* \\
& d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 6*(a^2*\sin(6*d*x + 6*c) + 3*a^2*\sin(\\
& 4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))))*\sin(8/3*\arctan2(s \\
& in(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) * \log(2*\cos(1/3*\arctan2(\sin(3/2*
\end{aligned}$$

$$\begin{aligned}
& d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + 2*\sin(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 - 2*\sqrt{2}*\cos(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 2*\sqrt{2}*\sin(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 2) - 75*(a^2*\cos(6*d*x + 6*c))^2 + 9*a^2*\cos(8/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + 9*a^2*\cos(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + a^2*\sin(6*d*x + 6*c))^2 + 9*a^2*\sin(8/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + 6*a^2*\sin(6*d*x + 6*c)*\sin(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 9*a^2*\sin(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))))^2 + 2*a^2*\cos(6*d*x + 6*c) + a^2 + 6*(a^2*\cos(6*d*x + 6*c) + 3*a^2*\cos(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))) + a^2*\cos(8/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 6*(a^2*\cos(6*d*x + 6*c) + a^2*\cos(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))) + 6*(a^2*\sin(6*d*x + 6*c) + 3*a^2*\sin(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))))*\sin(8/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))))*\log(2*\cos(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + 2*\sin(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 - 2*\sqrt{2}*\cos(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) - 2*\sqrt{2}*\sin(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 2) + 28*(\sqrt{2}*a^2*\cos(9/2*d*x + 9/2*c) - \sqrt{2}*a^2*\cos(3/2*d*x + 3/2*c))*\sin(6*d*x + 6*c) + 300*(\sqrt{2}*a^2*\cos(6*d*x + 6*c) + 3*\sqrt{2}*a^2*\cos(8/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))) + 3*\sqrt{2}*a^2*\cos(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))) + \sqrt{2}*a^2*\sin(11/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))) + 12*(7*\sqrt{2}*a^2*\cos(9/2*d*x + 9/2*c) - 7*\sqrt{2}*a^2*\cos(3/2*d*x + 3/2*c) - 114*\sqrt{2}*a^2*\cos(7/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))) + 114*\sqrt{2}*a^2*\cos(5/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))) + 75*\sqrt{2}*a^2*\cos(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))))*\sin(8/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))) + 456*(\sqrt{2}*a^2*\cos(6*d*x + 6*c) + 3*\sqrt{2}*a^2*\cos(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))) + \sqrt{2}*a^2*\sin(7/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))) - 456*(\sqrt{2}*a^2*\cos(6*d*x + 6*c) + 3*\sqrt{2}*a^2*\cos(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))) + \sqrt{2}*a^2*\sin(5/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))) + 12*(7*\sqrt{2}*a^2*\cos(9/2*d*x + 9/2*c) - 7*\sqrt{2}*a^2*\cos(3/2*d*x + 3/2*c) + 75*\sqrt{2}*a^2*\cos(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))))*\sin(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))) - 300*(\sqrt{2}*a^2*\cos(6*d*x + 6*c) + \sqrt{2}*a^2*\sin(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))))*B*\sqrt{a}/(\cos(6*d*x + 6*c))^2 + 6*(\cos(6*d*x + 6*c) + 3*\cos(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))) + 1)*\cos(8/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))))^2 + 6*(\cos(6*d*x + 6*c) + 1)*\cos(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))) + 9*\cos(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))))^2 + \sin(6*d*x + 6*c))^2 + 6*(\sin(6*d*x + 6*c) + 3*\sin(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))))*\sin
\end{aligned}$$

```
(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 9*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 6*sin(6*d*x + 6*c)*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 9*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*cos(6*d*x + 6*c) + 1)) /d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A + \frac{B}{\cos(c + dx)} \right) \left(a + \frac{a}{\cos(c + dx)} \right)^{5/2} \sqrt{\frac{1}{\cos(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(1/2), x)
```

```
[Out] int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c))*sec(d*x+c)**(1/2), x)
```

```
[Out] Timed out
```

$$3.242 \quad \int \frac{(a+a \sec(c+dx))^{5/2}(A+B \sec(c+dx))}{\sqrt{\sec(c+dx)}} dx$$

Optimal. Leaf size=180

$$\frac{a^{5/2}(20A + 19B) \sinh^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}}\right)}{4d} + \frac{a^3(4A - 9B) \sin(c + dx) \sqrt{\sec(c + dx)}}{4d \sqrt{a \sec(c + dx) + a}} + \frac{a^2(4A + 7B) \sin(c + dx) \sqrt{\sec(c + dx)}}{4d}$$

[Out] 1/4*a^(5/2)*(20*A+19*B)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+1/2*a*B*(a+a*sec(d*x+c))^(3/2)*sin(d*x+c)*sec(d*x+c)^(1/2)/d+1/4*a^3*(4*A-9*B)*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^(1/2)+1/4*a^2*(4*A+7*B)*sin(d*x+c)*sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(1/2)/d

Rubi [A] time = 0.50, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {4018, 4015, 3801, 215}

$$\frac{a^3(4A - 9B) \sin(c + dx) \sqrt{\sec(c + dx)}}{4d \sqrt{a \sec(c + dx) + a}} + \frac{a^2(4A + 7B) \sin(c + dx) \sqrt{\sec(c + dx)} \sqrt{a \sec(c + dx) + a}}{4d} + \frac{a^{5/2}(20A + 19B) \sinh^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}}\right)}{4d}$$

Antiderivative was successfully verified.

[In] Int[((a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Sqrt[Sec[c + d*x]], x]

[Out] (a^(5/2)*(20*A + 19*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]])/(4*d) + (a^3*(4*A - 9*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[a + a*Sec[c + d*x]]) + (a^2*(4*A + 7*B)*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(4*d) + (a*B*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(2*d)

Rule 215

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 3801

Int[Sqrt[csc[(e_) + (f_)*(x_)]*(d_)]*Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 + x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n)*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.)
+ (a_.)]*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> Simp[(A*b^2*Co
t[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
+ f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (
a_.))^m*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> -Simp[(b*B*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && !LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + a \sec(c + dx))^{5/2} (A + B \sec(c + dx))}{\sqrt{\sec(c + dx)}} dx &= \frac{aB \sqrt{\sec(c + dx)} (a + a \sec(c + dx))^{3/2} \sin(c + dx)}{2d} + \frac{1}{2} \int \frac{(a + a \sec(c + dx))^{5/2} (A + B \sec(c + dx))}{\sqrt{\sec(c + dx)}} dx \\
&= \frac{a^2(4A + 7B) \sqrt{\sec(c + dx)} \sqrt{a + a \sec(c + dx)} \sin(c + dx)}{4d} + \frac{a^3(4A - 9B) \sqrt{\sec(c + dx)} \sin(c + dx)}{4d \sqrt{a + a \sec(c + dx)}} + \frac{a^2(4A + 7B) \sqrt{\sec(c + dx)}}{4d \sqrt{a + a \sec(c + dx)}} \\
&= \frac{a^3(4A - 9B) \sqrt{\sec(c + dx)} \sin(c + dx)}{4d \sqrt{a + a \sec(c + dx)}} + \frac{a^2(4A + 7B) \sqrt{\sec(c + dx)}}{4d \sqrt{a + a \sec(c + dx)}} \\
&= \frac{a^{5/2} (20A + 19B) \sinh^{-1} \left(\frac{\sqrt{a} \tan(c + dx)}{\sqrt{a + a \sec(c + dx)}} \right)}{4d} + \frac{a^3(4A - 9B) \sqrt{\sec(c + dx)}}{4d \sqrt{a + a \sec(c + dx)}}
\end{aligned}$$

Mathematica [A] time = 2.27, size = 137, normalized size = 0.76

$$\frac{a^3 \left(\sqrt{-((\sec(c + dx) - 1) \sec(c + dx))} (\tan(c + dx)(4A + 2B \sec(c + dx) + 11B) + 8A \sin(c + dx)) + 20A \tan(c + dx) \right)}{4d \sqrt{1 - \sec(c + dx)} \sqrt{a(\sec(c + dx) + 1)}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[((a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Sqrt[Sec[c + d*x]], x]

[Out] (a^3*(20*A*ArcSin[Sqrt[1 - Sec[c + d*x]]]*Tan[c + d*x] - 19*B*ArcSin[Sqrt[Sec[c + d*x]]]*Tan[c + d*x] + Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])]*(8*A*Sin[c + d*x] + (4*A + 11*B + 2*B*Sec[c + d*x])*Tan[c + d*x])))/(4*d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])

fricas [A] time = 0.56, size = 454, normalized size = 2.52

$$\frac{\left((20A + 19B)a^2 \cos(dx + c)^2 + (20A + 19B)a^2 \cos(dx + c) \right) \sqrt{a} \log \left(\frac{a \cos(dx+c)^3 - 7a \cos(dx+c)^2 - \frac{4(\cos(dx+c)^2 - 2 \cos(dx+c) + 1)}{\cos(dx+c)^3 + \cos(dx+c)}}{\sqrt{\cos(dx+c)^3 + \cos(dx+c)}} \right)}{16 \left(d \cos(dx + c)^2 + d \cos(dx + c) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2), x, algorithm="fricas")

[Out] [1/16*(((20*A + 19*B)*a^2*cos(d*x + c)^2 + (20*A + 19*B)*a^2*cos(d*x + c))*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(8*A*a^2*cos(d*x + c)^2 + (4*A + 11*B)*a^2*cos(d*x + c) + 2*B*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^2 + d*cos(d*x + c)), 1/8*(((20*A + 19*B)*a^2*cos(d*x + c)^2 + (20*A + 19*B)*a^2*cos(d*x + c))*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) + 2*(8*A*a^2*cos(d*x + c)^2 + (4*A + 11*B)*a^2*cos(d*x + c) + 2*B*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^2 + d*cos(d*x + c))]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(a \sec(dx + c) + a)^{\frac{5}{2}}}{\sqrt{\sec(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(5/2)/sqrt(sec(d*x + c)), x)

maple [B] time = 2.99, size = 386, normalized size = 2.14

$$\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}} \left(20A \sin(dx+c) \sqrt{\frac{2}{1+\cos(dx+c)}} \arctan\left(\frac{\sqrt{\frac{2}{1+\cos(dx+c)}} (\cos(dx+c)+1+\sin(dx+c))\sqrt{2}}{4}}\right) \right) (\cos^2(dx+c)) \sqrt{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x)

[Out] -1/16/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(20*A*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d*x+c))*2^(1/2))*cos(d*x+c)^2*2^(1/2)-20*A*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))*2^(1/2))*cos(d*x+c)^2*2^(1/2)+19*B*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d*x+c))*2^(1/2))*cos(d*x+c)^2*2^(1/2)-19*B*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))*2^(1/2))*cos(d*x+c)^2*2^(1/2)+32*A*cos(d*x+c)^3-16*A*cos(d*x+c)^2+44*B*cos(d*x+c)^2-16*A*cos(d*x+c)-36*B*cos(d*x+c)-8*B*(1/cos(d*x+c))^(1/2)/sin(d*x+c)/cos(d*x+c)*a^2

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(a + \frac{a}{\cos(c+dx)}\right)^{5/2}}{\sqrt{\frac{1}{\cos(c+dx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2))/(1/cos(c + d*x))^(1/2),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2))/(1/cos(c + d*x))^(1/2), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

$$3.243 \quad \int \frac{(a+a \sec(c+dx))^{5/2}(A+B \sec(c+dx))}{\sec^2(c+dx)} dx$$

Optimal. Leaf size=177

$$\frac{a^{5/2}(2A+5B) \sinh^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}}\right)}{d} + \frac{a^3(14A+3B) \sin(c+dx) \sqrt{\sec(c+dx)}}{3d \sqrt{a \sec(c+dx)+a}} - \frac{a^2(2A-3B) \sin(c+dx) \sqrt{\sec(c+dx)}}{3d}$$

[Out] $a^{5/2}*(2*A+5*B)*\operatorname{arcsinh}(a^{1/2}*\tan(d*x+c)/(a+a*\sec(d*x+c))^{1/2})/d+2/3*a*A*(a+a*\sec(d*x+c))^{3/2}*\sin(d*x+c)/d/\sec(d*x+c)^{1/2}+1/3*a^3*(14*A+3*B)*\sin(d*x+c)*\sec(d*x+c)^{1/2}/d/(a+a*\sec(d*x+c))^{1/2}-1/3*a^2*(2*A-3*B)*\sin(d*x+c)*\sec(d*x+c)^{1/2}*(a+a*\sec(d*x+c))^{1/2}/d$

Rubi [A] time = 0.51, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4017, 4018, 4015, 3801, 215}

$$\frac{a^3(14A+3B) \sin(c+dx) \sqrt{\sec(c+dx)}}{3d \sqrt{a \sec(c+dx)+a}} - \frac{a^2(2A-3B) \sin(c+dx) \sqrt{\sec(c+dx)}}{3d} + \frac{a^{5/2}(2A+5B)}{3d}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(a+a*\operatorname{Sec}[c+d*x])^{5/2}*(A+B*\operatorname{Sec}[c+d*x])]/\operatorname{Sec}[c+d*x]^{3/2},x]$

[Out] $(a^{5/2}*(2*A+5*B)*\operatorname{ArcSinh}[(\operatorname{Sqrt}[a]*\operatorname{Tan}[c+d*x])/\operatorname{Sqrt}[a+a*\operatorname{Sec}[c+d*x]])]/d+(a^3*(14*A+3*B)*\operatorname{Sqrt}[\operatorname{Sec}[c+d*x]]*\operatorname{Sin}[c+d*x])/(3*d*\operatorname{Sqrt}[a+a*\operatorname{Sec}[c+d*x]])-(a^2*(2*A-3*B)*\operatorname{Sqrt}[\operatorname{Sec}[c+d*x]]*\operatorname{Sqrt}[a+a*\operatorname{Sec}[c+d*x]]*\operatorname{Sin}[c+d*x])/(3*d)+(2*a*A*(a+a*\operatorname{Sec}[c+d*x])^{3/2}*\operatorname{Sin}[c+d*x])/(3*d*\operatorname{Sqrt}[\operatorname{Sec}[c+d*x]])$

Rule 215

$\operatorname{Int}[1/\operatorname{Sqrt}[(a_.)+(b_.)*(x_)^2],x_Symbol] \rightarrow \operatorname{Simp}[\operatorname{ArcSinh}[\operatorname{Rt}[b,2]*x]/\operatorname{Sqrt}[a]/\operatorname{Rt}[b,2],x] /; \operatorname{FreeQ}\{a,b\},x \ \&\& \operatorname{GtQ}[a,0] \ \&\& \operatorname{PosQ}[b]$

Rule 3801

$\operatorname{Int}[\operatorname{Sqrt}[\operatorname{csc}[e_.]+(f_.)*(x_.)]*(d_.)]*\operatorname{Sqrt}[\operatorname{csc}[e_.]+(f_.)*(x_.)]*(b_.)+(a_.),x_Symbol] \rightarrow \operatorname{Dist}[(-2*a*\operatorname{Sqrt}[(a*d)/b])/(b*f),\operatorname{Subst}[\operatorname{Int}[1/\operatorname{Sqrt}[1+x^2/a],x],x,(b*\operatorname{Cot}[e+f*x])/\operatorname{Sqrt}[a+b*\operatorname{Csc}[e+f*x]]],x] /; \operatorname{FreeQ}\{a,b,d,e,f\},x \ \&\& \operatorname{EqQ}[a^2-b^2,0] \ \&\& \operatorname{GtQ}[(a*d)/b,0]$

Rule 4015


```

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*Co
t[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
+ f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]

```

Rule 4017

```

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1]

```

Rule 4018

```

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && !LtQ[n, -1]

```

Rubi steps

$$\begin{aligned}
\int \frac{(a + a \sec(c + dx))^{5/2} (A + B \sec(c + dx))}{\sec^3(c + dx)} dx &= \frac{2aA(a + a \sec(c + dx))^{3/2} \sin(c + dx)}{3d\sqrt{\sec(c + dx)}} + \frac{2}{3} \int \frac{(a + a \sec(c + dx))^{5/2} (A + B \sec(c + dx))}{\sec^3(c + dx)} dx \\
&= -\frac{a^2(2A - 3B)\sqrt{\sec(c + dx)} \sqrt{a + a \sec(c + dx)} \sin(c + dx)}{3d} + \frac{2}{3} \int \frac{(a + a \sec(c + dx))^{5/2} (A + B \sec(c + dx))}{\sec^3(c + dx)} dx \\
&= \frac{a^3(14A + 3B)\sqrt{\sec(c + dx)} \sin(c + dx)}{3d\sqrt{a + a \sec(c + dx)}} - \frac{a^2(2A - 3B)\sqrt{\sec(c + dx)} \sqrt{a + a \sec(c + dx)} \sin(c + dx)}{3d} \\
&= \frac{a^3(14A + 3B)\sqrt{\sec(c + dx)} \sin(c + dx)}{3d\sqrt{a + a \sec(c + dx)}} - \frac{a^2(2A - 3B)\sqrt{\sec(c + dx)} \sqrt{a + a \sec(c + dx)} \sin(c + dx)}{3d} \\
&= \frac{a^{5/2}(2A + 5B) \sinh^{-1}\left(\frac{\sqrt{a} \tan(c + dx)}{\sqrt{a + a \sec(c + dx)}}\right)}{d} + \frac{a^3(14A + 3B)\sqrt{\sec(c + dx)} \sin(c + dx)}{3d\sqrt{a + a \sec(c + dx)}}
\end{aligned}$$

Mathematica [A] time = 1.05, size = 133, normalized size = 0.75

$$\frac{a^3 \left(3(2A + 5B) \sin(c + dx) \sec^{\frac{3}{2}}(c + dx) \sin^{-1}\left(\sqrt{1 - \sec(c + dx)}\right) + \sqrt{1 - \sec(c + dx)} (\tan(c + dx)(16A + 3B \sec(c + dx)) + \sqrt{a(\sec(c + dx) + 1)}) \right)}{3d\sqrt{-((\sec(c + dx) - 1) \sec(c + dx))} \sqrt{a(\sec(c + dx) + 1)}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[((a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(3/2),x]

[Out] (a^3*(3*(2*A + 5*B)*ArcSin[Sqrt[1 - Sec[c + d*x]]]*Sec[c + d*x]^(3/2)*Sin[c + d*x] + Sqrt[1 - Sec[c + d*x]]*(2*A*SIN[c + d*x] + (16*A + 6*B + 3*B*Sec[c + d*x])*Tan[c + d*x]))/(3*d*Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])]*Sqrt[a*(1 + Sec[c + d*x])])

fricas [A] time = 0.60, size = 424, normalized size = 2.40

$$\left[\frac{3 \left((2A + 5B)a^2 \cos(dx + c) + (2A + 5B)a^2 \right) \sqrt{a} \log \left(\frac{a \cos(dx+c)^3 - 7a \cos(dx+c)^2 - \frac{4(\cos(dx+c)^2 - 2\cos(dx+c)) \sqrt{a} \sqrt{\frac{a \cos(dx+c)+a}{\cos(dx+c)}} \sin(dx+c)}{\sqrt{\cos(dx+c)}}}{\cos(dx+c)^3 + \cos(dx+c)^2} \right)}{12(d \cos(dx + c) + d)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="fricas")

[Out] [1/12*(3*((2*A + 5*B)*a^2*cos(d*x + c) + (2*A + 5*B)*a^2)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(2*A*a^2*cos(d*x + c)^2 + 2*(8*A + 3*B)*a^2*cos(d*x + c) + 3*B*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d), 1/6*(3*((2*A + 5*B)*a^2*cos(d*x + c) + (2*A + 5*B)*a^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) + 2*(2*A*a^2*cos(d*x + c)^2 + 2*(8*A + 3*B)*a^2*cos(d*x + c) + 3*B*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(a \sec(dx + c) + a)^{\frac{5}{2}}}{\sec(dx + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(3/2), x)

maple [B] time = 3.29, size = 376, normalized size = 2.12

$$\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}} \left(6A \cos(dx+c) \sin(dx+c) \sqrt{\frac{2}{1+\cos(dx+c)}} \arctan\left(\frac{\sqrt{\frac{2}{1+\cos(dx+c)}} (\cos(dx+c)+1+\sin(dx+c))\sqrt{2}}{4}}\right) \sqrt{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x)

[Out] -1/12/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(6*A*cos(d*x+c)*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d*x+c))*2^(1/2))*2^(1/2)-6*A*cos(d*x+c)*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))*2^(1/2))

```
*2^(1/2)+15*B*cos(d*x+c)*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d*x+c))*2^(1/2))*2^(1/2)-15*B*cos(d*x+c)*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))*2^(1/2))*2^(1/2)+8*A*cos(d*x+c)^3+56*A*cos(d*x+c)^2+24*B*cos(d*x+c)^2-64*A*cos(d*x+c)-12*B*cos(d*x+c)-12*B)*cos(d*x+c)*(1/cos(d*x+c))^(3/2)/sin(d*x+c)*a^2
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(a + \frac{a}{\cos(c+dx)}\right)^{5/2}}{\left(\frac{1}{\cos(c+dx)}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2))/(1/cos(c + d*x))^(3/2),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2))/(1/cos(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)**(3/2),x)
```

[Out] Timed out

$$3.244 \quad \int \frac{(a+a \sec(c+dx))^{5/2}(A+B \sec(c+dx))}{5 \sec^2(c+dx)} dx$$

Optimal. Leaf size=172

$$\frac{2a^{5/2}B \sinh^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}}\right)}{d} + \frac{2a^3(32A+35B) \sin(c+dx) \sqrt{\sec(c+dx)}}{15d \sqrt{a \sec(c+dx)+a}} + \frac{2a^2(8A+5B) \sin(c+dx) \sqrt{a \sec(c+dx)+a}}{15d \sqrt{\sec(c+dx)}}$$

[Out] $2*a^{(5/2)}*B*\operatorname{arcsinh}(a^{(1/2)}*\tan(d*x+c)/(a+a*\sec(d*x+c))^{(1/2)})/d+2/5*a*A*(a+a*\sec(d*x+c))^{(3/2)}*\sin(d*x+c)/d/\sec(d*x+c)^{(3/2)}+2/15*a^3*(32*A+35*B)*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/d/(a+a*\sec(d*x+c))^{(1/2)}+2/15*a^2*(8*A+5*B)*\sin(d*x+c)*(a+a*\sec(d*x+c))^{(1/2)}/d/\sec(d*x+c)^{(1/2)}$

Rubi [A] time = 0.49, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {4017, 4015, 3801, 215}

$$\frac{2a^3(32A+35B) \sin(c+dx) \sqrt{\sec(c+dx)}}{15d \sqrt{a \sec(c+dx)+a}} + \frac{2a^2(8A+5B) \sin(c+dx) \sqrt{a \sec(c+dx)+a}}{15d \sqrt{\sec(c+dx)}} + \frac{2a^{5/2}B \sinh^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(a+a*\operatorname{Sec}[c+d*x])^{(5/2)}*(A+B*\operatorname{Sec}[c+d*x])]/\operatorname{Sec}[c+d*x]^{(5/2)},x]$

[Out] $(2*a^{(5/2)}*B*\operatorname{ArcSinh}[(\operatorname{Sqrt}[a]*\operatorname{Tan}[c+d*x])/\operatorname{Sqrt}[a+a*\operatorname{Sec}[c+d*x]])]/d+(2*a^3*(32*A+35*B)*\operatorname{Sqrt}[\operatorname{Sec}[c+d*x]]*\operatorname{Sin}[c+d*x])/(15*d*\operatorname{Sqrt}[a+a*\operatorname{Sec}[c+d*x]])+(2*a^2*(8*A+5*B)*\operatorname{Sqrt}[a+a*\operatorname{Sec}[c+d*x]]*\operatorname{Sin}[c+d*x])/(15*d*\operatorname{Sqrt}[\operatorname{Sec}[c+d*x]])+(2*a*A*(a+a*\operatorname{Sec}[c+d*x])^{(3/2)}*\operatorname{Sin}[c+d*x])/(5*d*\operatorname{Sec}[c+d*x]^{(3/2)})$

Rule 215

$\operatorname{Int}[1/\operatorname{Sqrt}[(a_+)+(b_+)*(x_+)^2], x_Symbol] \rightarrow \operatorname{Simp}[\operatorname{ArcSinh}[(\operatorname{Rt}[b, 2]*x)/\operatorname{Sqrt}[a]]/\operatorname{Rt}[b, 2], x] /; \operatorname{FreeQ}\{a, b\}, x \ \&\& \operatorname{GtQ}[a, 0] \ \&\& \operatorname{PosQ}[b]$

Rule 3801

$\operatorname{Int}[\operatorname{Sqrt}[\operatorname{csc}[e_+]+(f_+)*(x_+)]*(d_+)]*\operatorname{Sqrt}[\operatorname{csc}[e_+]+(f_+)*(x_+)]*(b_+)+(a_+), x_Symbol] \rightarrow \operatorname{Dist}[(-2*a*\operatorname{Sqrt}[(a*d)/b])/(b*f), \operatorname{Subst}[\operatorname{Int}[1/\operatorname{Sqrt}[1+x^2/a], x], x, (b*\operatorname{Cot}[e+f*x])/\operatorname{Sqrt}[a+b*\operatorname{Csc}[e+f*x]]], x] /; \operatorname{FreeQ}\{a, b, d, e, f\}, x \ \&\& \operatorname{EqQ}[a^2-b^2, 0] \ \&\& \operatorname{GtQ}[(a*d)/b, 0]$

Rule 4015

```

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.)
+ (a_)]*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*Co
t[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
+ f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]

```

Rule 4017

```

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1]

```

Rubi steps

$$\begin{aligned}
\int \frac{(a + a \sec(c + dx))^{5/2} (A + B \sec(c + dx))}{\sec^2(c + dx)} dx &= \frac{2aA(a + a \sec(c + dx))^{3/2} \sin(c + dx)}{5d \sec^3(c + dx)} + \frac{2}{5} \int \frac{(a + a \sec(c + dx))}{\sec^2(c + dx)} dx \\
&= \frac{2a^2(8A + 5B)\sqrt{a + a \sec(c + dx)} \sin(c + dx)}{15d\sqrt{\sec(c + dx)}} + \frac{2aA(a + a \sec(c + dx))^{3/2} \sin(c + dx)}{5d \sec^3(c + dx)} \\
&= \frac{2a^3(32A + 35B)\sqrt{\sec(c + dx)} \sin(c + dx)}{15d\sqrt{a + a \sec(c + dx)}} + \frac{2a^2(8A + 5B)\sqrt{a + a \sec(c + dx)} \sin(c + dx)}{15d \sec^3(c + dx)} \\
&= \frac{2a^3(32A + 35B)\sqrt{\sec(c + dx)} \sin(c + dx)}{15d\sqrt{a + a \sec(c + dx)}} + \frac{2a^2(8A + 5B)\sqrt{a + a \sec(c + dx)} \sin(c + dx)}{15d \sec^3(c + dx)} \\
&= \frac{2a^{5/2}B \sinh^{-1}\left(\frac{\sqrt{a} \tan(c + dx)}{\sqrt{a + a \sec(c + dx)}}\right)}{d} + \frac{2a^3(32A + 35B)\sqrt{\sec(c + dx)} \sin(c + dx)}{15d\sqrt{a + a \sec(c + dx)}}
\end{aligned}$$

Mathematica [A] time = 1.96, size = 127, normalized size = 0.74

$$\frac{a^3 \tan(c + dx) \left(\sqrt{1 - \sec(c + dx)} (2(14A + 5B) \cos(c + dx) + 3A \cos(2(c + dx))) + 89A + 80B \right) + 30B \sqrt{\sec(c + dx)}}{15d \sqrt{-((\sec(c + dx) - 1) \sec(c + dx))} \sqrt{a(\sec(c + dx) + 1)}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[((a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(5/2),x]

[Out] (a^3*((89*A + 80*B + 2*(14*A + 5*B)*Cos[c + d*x] + 3*A*Cos[2*(c + d*x)])*Sqrt[1 - Sec[c + d*x]] + 30*B*ArcSin[Sqrt[1 - Sec[c + d*x]]]*Sqrt[Sec[c + d*x]])*Tan[c + d*x]/(15*d*Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])]*Sqrt[a*(1 + Sec[c + d*x])])

fricas [A] time = 0.48, size = 424, normalized size = 2.47

$$\left[\frac{15 \left(B a^2 \cos(dx + c) + B a^2 \right) \sqrt{a} \log \left(\frac{a \cos(dx+c)^3 - 7 a \cos(dx+c)^2 - \frac{4(\cos(dx+c)^2 - 2 \cos(dx+c)) \sqrt{a} \sqrt{\frac{a \cos(dx+c)+a}{\cos(dx+c)} \sin(dx+c)} + 8 a}{\sqrt{\cos(dx+c)}}}{\cos(dx+c)^3 + \cos(dx+c)^2} \right)}{30(d \cos(dx + c) + d)} + \frac{4(3 A a^2 \cos(dx + c) + B a^2) \sqrt{a} \arctan\left(\frac{2 \sqrt{-a} \sqrt{\frac{a \cos(dx+c)+a}{\cos(dx+c)} \sin(dx+c)}}{\cos(dx+c)}\right)}{30(d \cos(dx + c) + d)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="fricas")

[Out] [1/30*(15*(B*a^2*cos(d*x + c) + B*a^2)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(3*A*a^2*cos(d*x + c)^3 + (14*A + 5*B)*a^2*cos(d*x + c)^2 + (43*A + 40*B)*a^2*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d), 1/15*(15*(B*a^2*cos(d*x + c) + B*a^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) + 2*(3*A*a^2*cos(d*x + c)^3 + (14*A + 5*B)*a^2*cos(d*x + c)^2 + (43*A + 40*B)*a^2*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(a \sec(dx + c) + a)^{\frac{5}{2}}}{\sec(dx + c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(5/2), x)

maple [A] time = 2.72, size = 235, normalized size = 1.37

$$\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}} \left(15B\sqrt{2} \arctan \left(\frac{\sqrt{\frac{2}{1+\cos(dx+c)}} (\cos(dx+c)+1+\sin(dx+c))\sqrt{2}}{4} \right) \right) \sqrt{\frac{2}{1+\cos(dx+c)}} \sin(dx+c) - 15B\sqrt{2} \arctan \left(\frac{\sqrt{\frac{2}{1+\cos(dx+c)}} (\cos(dx+c)+1+\sin(dx+c))\sqrt{2}}{4} \right)}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x)

[Out] $-1/30/d*(a*(1+\cos(d*x+c))/\cos(d*x+c))^{(1/2)}*(15*B*2^{(1/2)}*\arctan(1/4*(-2/(1+\cos(d*x+c))))^{(1/2)}*(\cos(d*x+c)+1+\sin(d*x+c))*2^{(1/2)}*(-2/(1+\cos(d*x+c)))^{(1/2)}*\sin(d*x+c)-15*B*2^{(1/2)}*\arctan(1/4*(-2/(1+\cos(d*x+c))))^{(1/2)}*(\cos(d*x+c)+1-\sin(d*x+c))*2^{(1/2)}*(-2/(1+\cos(d*x+c)))^{(1/2)}*\sin(d*x+c)+12*A*\cos(d*x+c)^3+44*A*\cos(d*x+c)^2+20*B*\cos(d*x+c)^2+116*A*\cos(d*x+c)+140*B*\cos(d*x+c)-172*A-160*B)*\cos(d*x+c)^3*(1/\cos(d*x+c))^{(5/2)}/\sin(d*x+c)*a^2$

maxima [B] time = 1.66, size = 655, normalized size = 3.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="maxima")

[Out] $1/60*(5*\sqrt{2}*(30*a^2*\cos(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))*\sin(3/2*d*x + 3/2*c) - 30*a^2*\cos(3/2*d*x + 3/2*c)*\sin(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 3*\sqrt{2}*a^2*\log(2*\cos(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))))^2 + 2*\sin(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + 2*\sqrt{2}*\cos(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 2*\sqrt{2}*\sin(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 2) - 3*\sqrt{2}*a^2*\log(2*\cos(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))))^2 + 2*\sin(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + 2*\sqrt{2}*\cos(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) - 2*\sqrt{2}*\sin(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 2) + 3*\sqrt{2}*a^2*\log(2*\cos(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))))^2 + 2*\sin(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 - 2*\sqrt{2}*\cos(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 2*\sqrt{2}*\sin(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))))$


```
ctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 3*sqrt(2)*a^2*log
(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 4*a^2*sin(3/2*
d*x + 3/2*c) + 30*a^2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c))))*B*sqrt(a) + 2*(3*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) + 25*sqrt(2)*a^2
*sin(3/2*d*x + 3/2*c) + 150*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*A*sqrt(a))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(a + \frac{a}{\cos(c+dx)}\right)^{5/2}}{\left(\frac{1}{\cos(c+dx)}\right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2))/(1/cos(c + d*x))^(5/2
),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2))/(1/cos(c + d*x))^(5/2
), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

$$3.245 \quad \int \frac{(a+a \sec(c+dx))^{5/2}(A+B \sec(c+dx))}{\sec^2(c+dx)} dx$$

Optimal. Leaf size=178

$$\frac{64a^3(5A+7B) \sin(c+dx) \sqrt{\sec(c+dx)}}{105d \sqrt{a \sec(c+dx)+a}} + \frac{16a^2(5A+7B) \sin(c+dx) \sqrt{a \sec(c+dx)+a}}{105d \sqrt{\sec(c+dx)}} + \frac{2a(5A+7B) \sin(c+dx)}{35d \sec^3(c+dx)}$$

[Out] $2/35*a*(5*A+7*B)*(a+a*\sec(d*x+c))^{(3/2)}*\sin(d*x+c)/d/\sec(d*x+c)^{(3/2)}+2/7*A*(a+a*\sec(d*x+c))^{(5/2)}*\sin(d*x+c)/d/\sec(d*x+c)^{(5/2)}+64/105*a^3*(5*A+7*B)*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/d/(a+a*\sec(d*x+c))^{(1/2)}+16/105*a^2*(5*A+7*B)*\sin(d*x+c)*(a+a*\sec(d*x+c))^{(1/2)}/d/\sec(d*x+c)^{(1/2)}$

Rubi [A] time = 0.32, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {4013, 3809, 3804}

$$\frac{64a^3(5A+7B) \sin(c+dx) \sqrt{\sec(c+dx)}}{105d \sqrt{a \sec(c+dx)+a}} + \frac{16a^2(5A+7B) \sin(c+dx) \sqrt{a \sec(c+dx)+a}}{105d \sqrt{\sec(c+dx)}} + \frac{2a(5A+7B) \sin(c+dx)}{35d \sec^3(c+dx)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + a*\text{Sec}[c + d*x])^{(5/2)}*(A + B*\text{Sec}[c + d*x])]/\text{Sec}[c + d*x]^{(7/2)}, x]$

[Out] $(64*a^3*(5*A + 7*B)*\text{Sqrt}[\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(105*d*\text{Sqrt}[a + a*\text{Sec}[c + d*x]]) + (16*a^2*(5*A + 7*B)*\text{Sqrt}[a + a*\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(105*d*\text{Sqrt}[\text{Sec}[c + d*x]]) + (2*a*(5*A + 7*B)*(a + a*\text{Sec}[c + d*x])^{(3/2)}*\text{Sin}[c + d*x])/(35*d*\text{Sec}[c + d*x]^{(3/2)}) + (2*A*(a + a*\text{Sec}[c + d*x])^{(5/2)}*\text{Sin}[c + d*x])/(7*d*\text{Sec}[c + d*x]^{(5/2)})$

Rule 3804

$\text{Int}[\text{Sqrt}[\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]/\text{Sqrt}[\text{csc}[(e_.) + (f_.)*(x_.)]*(d_.)], x_Symbol] \rightarrow \text{Simp}[(-2*a*\text{Cot}[e + f*x])/(f*\text{Sqrt}[a + b*\text{Csc}[e + f*x]]*\text{Sqrt}[d*\text{Csc}[e + f*x]]), x] /; \text{FreeQ}[\{a, b, d, e, f\}, x] \&\& \text{EqQ}[a^2 - b^2, 0]$

Rule 3809

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(d_.))^{(n)}*(\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^{(m)}, x_Symbol] \rightarrow -\text{Simp}[(a*\text{Cot}[e + f*x]*(a + b*\text{Csc}[e + f*x])^{(m-1)}*(d*\text{Csc}[e + f*x])^n)/(f*m), x] + \text{Dist}[(b*(2*m - 1))/(d*m), \text{Int}[(a + b*\text{Csc}[e + f*x])^{(m-1)}*(d*\text{Csc}[e + f*x])^{(n+1)}, x], x] /; \text{FreeQ}[\{a, b, d, e, f, m, n\}, x] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{EqQ}[m + n, 0] \&\& \text{GtQ}[m, 1/2] \&\& \text{IntegerQ}[2*m]$

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (
a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
- b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] && !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{(a + a \sec(c + dx))^{5/2} (A + B \sec(c + dx))}{\sec^2(c + dx)} dx &= \frac{2A(a + a \sec(c + dx))^{5/2} \sin(c + dx)}{7d \sec^2(c + dx)} + \frac{1}{7}(5A + 7B) \int \frac{(a + a \sec(c + dx))^{5/2} \sin(c + dx)}{\sec^2(c + dx)} dx \\ &= \frac{2a(5A + 7B)(a + a \sec(c + dx))^{3/2} \sin(c + dx)}{35d \sec^2(c + dx)} + \frac{2A(a + a \sec(c + dx))^{5/2} \sin(c + dx)}{7d \sec^2(c + dx)} \\ &= \frac{16a^2(5A + 7B)\sqrt{a + a \sec(c + dx)} \sin(c + dx)}{105d \sqrt{\sec(c + dx)}} + \frac{2a(5A + 7B)(a + a \sec(c + dx))^{3/2} \sin(c + dx)}{105d \sqrt{\sec(c + dx)}} \\ &= \frac{64a^3(5A + 7B)\sqrt{\sec(c + dx)} \sin(c + dx)}{105d \sqrt{a + a \sec(c + dx)}} + \frac{16a^2(5A + 7B)\sqrt{a + a \sec(c + dx)} \sin(c + dx)}{105d \sqrt{\sec(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.57, size = 91, normalized size = 0.51

$$\frac{2a^3 \sin(c + dx) \left((230A + 301B) \sec^3(c + dx) + (115A + 98B) \sec^2(c + dx) + 3(20A + 7B) \sec(c + dx) + 15A \right)}{105d \sec^2(c + dx) \sqrt{a(\sec(c + dx) + 1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(7/2), x]
```

```
[Out] (2*a^3*(15*A + 3*(20*A + 7*B)*Sec[c + d*x] + (115*A + 98*B)*Sec[c + d*x]^2 + (230*A + 301*B)*Sec[c + d*x]^3)*Sin[c + d*x]/(105*d*Sec[c + d*x]^(5/2)*Sqrt[a*(1 + Sec[c + d*x])])
```

fricas [A] time = 0.44, size = 120, normalized size = 0.67

$$\frac{2(15Aa^2 \cos(dx + c)^4 + 3(20A + 7B)a^2 \cos(dx + c)^3 + (115A + 98B)a^2 \cos(dx + c)^2 + (230A + 301B)a^2 \cos(dx + c) + 15A)}{105(d \cos(dx + c) + d)\sqrt{\cos(dx + c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm="fricas")

[Out] 2/105*(15*A*a^2*cos(d*x + c)^4 + 3*(20*A + 7*B)*a^2*cos(d*x + c)^3 + (115*A + 98*B)*a^2*cos(d*x + c)^2 + (230*A + 301*B)*a^2*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/((d*cos(d*x + c) + d)*sqrt(cos(d*x + c)))

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(a \sec(dx + c) + a)^{\frac{5}{2}}}{\sec(dx + c)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(7/2), x)

maple [A] time = 2.61, size = 121, normalized size = 0.68

$$\frac{2(-1 + \cos(dx + c))(15A(\cos^3(dx + c)) + 60A(\cos^2(dx + c)) + 21B(\cos^2(dx + c)) + 115A \cos(dx + c) + 98B)}{105d \sin(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x)

[Out] -2/105/d*(-1+cos(d*x+c))*(15*A*cos(d*x+c)^3+60*A*cos(d*x+c)^2+21*B*cos(d*x+c)^2+115*A*cos(d*x+c)+98*B*cos(d*x+c)+230*A+301*B)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^4*(1/cos(d*x+c))^(7/2)/sin(d*x+c)*a^2

maxima [B] time = 1.39, size = 385, normalized size = 2.16

$$5\sqrt{2} \left(315 a^2 \cos \left(\frac{6}{7} \arctan \left(\sin \left(\frac{7}{2} dx + \frac{7}{2} c \right), \cos \left(\frac{7}{2} dx + \frac{7}{2} c \right) \right) \right) \sin \left(\frac{7}{2} dx + \frac{7}{2} c \right) + 77 a^2 \cos \left(\frac{4}{7} \arctan \left(\sin \left(\frac{7}{2} dx + \frac{7}{2} c \right), \cos \left(\frac{7}{2} dx + \frac{7}{2} c \right) \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm="maxima")

```
[Out] 1/840*(5*sqrt(2)*(315*a^2*cos(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) + 77*a^2*cos(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) + 21*a^2*cos(2/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) - 315*a^2*cos(7/2*d*x + 7/2*c)*sin(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) - 77*a^2*cos(7/2*d*x + 7/2*c)*sin(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) - 21*a^2*cos(7/2*d*x + 7/2*c)*sin(2/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 6*a^2*sin(7/2*d*x + 7/2*c) + 21*a^2*sin(5/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 77*a^2*sin(3/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 315*a^2*sin(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))))*A*sqrt(a) + 28*(3*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) + 25*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) + 150*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*B*sqrt(a))/d
```

mupad [B] time = 4.35, size = 133, normalized size = 0.75

$$\frac{a^2 \cos(c + dx) \sqrt{\frac{1}{\cos(c+dx)}} \sqrt{\frac{a(\cos(c+dx)+1)}{\cos(c+dx)}} (1960 A \sin(c + dx) + 2450 B \sin(c + dx) + 490 A \sin(2c + 2dx) + 120 A \sin(3c + 3dx) + 15 A \sin(4c + 4dx) + 392 B \sin(2c + 2dx) + 42 B \sin(3c + 3dx))}{420 d (\cos(c + dx) + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2))/(1/cos(c + d*x))^(7/2), x)
```

```
[Out] (a^2*cos(c + d*x)*(1/cos(c + d*x))^(1/2)*((a*(cos(c + d*x) + 1))/cos(c + d*x))^(1/2)*(1960*A*sin(c + d*x) + 2450*B*sin(c + d*x) + 490*A*sin(2*c + 2*d*x) + 120*A*sin(3*c + 3*d*x) + 15*A*sin(4*c + 4*d*x) + 392*B*sin(2*c + 2*d*x) + 42*B*sin(3*c + 3*d*x)))/(420*d*(cos(c + d*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)**(7/2), x)
```

```
[Out] Timed out
```

$$3.246 \quad \int \frac{(a+a \sec(c+dx))^{5/2}(A+B \sec(c+dx))}{9 \sec^2(c+dx)} dx$$

Optimal. Leaf size=228

$$\frac{2a^3(124A + 135B) \sin(c + dx)}{315d \sec^2(c + dx) \sqrt{a \sec(c + dx) + a}} + \frac{4a^3(292A + 345B) \sin(c + dx) \sqrt{\sec(c + dx)}}{315d \sqrt{a \sec(c + dx) + a}} + \frac{2a^3(292A + 345B) \sin(c + dx)}{315d \sqrt{\sec(c + dx)} \sqrt{a \sec(c + dx) + a}}$$

[Out] 2/9*a*A*(a+a*sec(d*x+c))^(3/2)*sin(d*x+c)/d/sec(d*x+c)^(7/2)+2/315*a^3*(124*A+135*B)*sin(d*x+c)/d/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2)+2/315*a^3*(2*92*A+345*B)*sin(d*x+c)/d/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+4/315*a^3*(292*A+345*B)*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^(1/2)+2/21*a^2*(4*A+3*B)*sin(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d/sec(d*x+c)^(5/2)

Rubi [A] time = 0.63, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {4017, 4015, 3805, 3804}

$$\frac{2a^3(124A + 135B) \sin(c + dx)}{315d \sec^2(c + dx) \sqrt{a \sec(c + dx) + a}} + \frac{2a^2(4A + 3B) \sin(c + dx) \sqrt{a \sec(c + dx) + a}}{21d \sec^2(c + dx)} + \frac{4a^3(292A + 345B) \sin(c + dx)}{315d \sqrt{a \sec(c + dx) + a}}$$

Antiderivative was successfully verified.

[In] Int[((a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(9/2), x]

[Out] (2*a^3*(124*A + 135*B)*Sin[c + d*x])/(315*d*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (2*a^3*(292*A + 345*B)*Sin[c + d*x])/(315*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (4*a^3*(292*A + 345*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(315*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2*(4*A + 3*B)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(21*d*Sec[c + d*x]^(5/2)) + (2*a*A*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2))

Rule 3804

Int[Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 3805

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&

EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]

Rule 4015

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^ (n_.)*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> Simp[(A*b^2*Co t[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist [(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]

Rule 4017

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^ (n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^ (m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> Simp[(a*A*Co t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp [a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && LtQ[n, -1]

Rubi steps

$$\begin{aligned} \int \frac{(a + a \sec(c + dx))^{5/2} (A + B \sec(c + dx))}{\sec^2(c + dx)} dx &= \frac{2aA(a + a \sec(c + dx))^{3/2} \sin(c + dx)}{9d \sec^2(c + dx)} + \frac{2}{9} \int \frac{(a + a \sec(c + dx))^{5/2} (A + B \sec(c + dx))}{\sec^2(c + dx)} dx \\ &= \frac{2a^2(4A + 3B)\sqrt{a + a \sec(c + dx)} \sin(c + dx)}{21d \sec^2(c + dx)} + \frac{2aA(a + a \sec(c + dx))^{3/2} \sin(c + dx)}{9d \sec^2(c + dx)} \\ &= \frac{2a^3(124A + 135B) \sin(c + dx)}{315d \sec^2(c + dx)\sqrt{a + a \sec(c + dx)}} + \frac{2a^2(4A + 3B)\sqrt{a + a \sec(c + dx)} \sin(c + dx)}{21d \sec^2(c + dx)} \\ &= \frac{2a^3(124A + 135B) \sin(c + dx)}{315d \sec^2(c + dx)\sqrt{a + a \sec(c + dx)}} + \frac{2a^3(292A + 345B) \sin(c + dx)}{315d \sqrt{\sec(c + dx)} \sqrt{a + a \sec(c + dx)}} \\ &= \frac{2a^3(124A + 135B) \sin(c + dx)}{315d \sec^2(c + dx)\sqrt{a + a \sec(c + dx)}} + \frac{2a^3(292A + 345B) \sin(c + dx)}{315d \sqrt{\sec(c + dx)} \sqrt{a + a \sec(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.71, size = 108, normalized size = 0.47

$$\frac{2a^3 \sin(c + dx) \left((584A + 690B) \sec^4(c + dx) + (292A + 345B) \sec^3(c + dx) + 3(73A + 60B) \sec^2(c + dx) + 5(26A + 9B) \sec(c + dx) + 2 \right)}{315d \sec^{\frac{7}{2}}(c + dx) \sqrt{a(\sec(c + dx) + 1)}}$$

Antiderivative was successfully verified.

[In] Integrate[((a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(9/2), x]

[Out] (2*a^3*(35*A + 5*(26*A + 9*B)*Sec[c + d*x] + 3*(73*A + 60*B)*Sec[c + d*x]^2 + (292*A + 345*B)*Sec[c + d*x]^3 + (584*A + 690*B)*Sec[c + d*x]^4)*Sin[c + d*x]/(315*d*Sec[c + d*x]^(7/2)*Sqrt[a*(1 + Sec[c + d*x])])

fricas [A] time = 0.45, size = 141, normalized size = 0.62

$$\frac{2 \left(35 A a^2 \cos(dx + c)^5 + 5(26 A + 9 B) a^2 \cos(dx + c)^4 + 3(73 A + 60 B) a^2 \cos(dx + c)^3 + (292 A + 345 B) a^2 \cos(dx + c)^2 + 2(292 A + 345 B) a^2 \cos(dx + c) + 2 \right)}{315 (d \cos(dx + c) + d) \sqrt{\cos(dx + c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2), x, algorithm="fricas")

[Out] 2/315*(35*A*a^2*cos(d*x + c)^5 + 5*(26*A + 9*B)*a^2*cos(d*x + c)^4 + 3*(73*A + 60*B)*a^2*cos(d*x + c)^3 + (292*A + 345*B)*a^2*cos(d*x + c)^2 + 2*(292*A + 345*B)*a^2*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/((d*cos(d*x + c) + d)*sqrt(cos(d*x + c)))

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(a \sec(dx + c) + a)^{\frac{5}{2}}}{\sec(dx + c)^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2), x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(9/2), x)

maple [A] time = 2.68, size = 143, normalized size = 0.63

$$2(-1 + \cos(dx + c)) \left(35A \left(\cos^4(dx + c) \right) + 130A \left(\cos^3(dx + c) \right) + 45B \left(\cos^3(dx + c) \right) + 219A \left(\cos^2(dx + c) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((a+a*\sec(d*x+c))^{5/2}*(A+B*\sec(d*x+c))/\sec(d*x+c)^{9/2},x)$

[Out] $-2/315/d*(-1+\cos(d*x+c))*(35*A*\cos(d*x+c)^4+130*A*\cos(d*x+c)^3+45*B*\cos(d*x+c)^3+219*A*\cos(d*x+c)^2+180*B*\cos(d*x+c)^2+292*A*\cos(d*x+c)+345*B*\cos(d*x+c)+584*A+690*B)*(a*(1+\cos(d*x+c))/\cos(d*x+c))^{1/2}*\cos(d*x+c)^5*(1/\cos(d*x+c))^{9/2}/\sin(d*x+c)*a^2$

maxima [B] time = 1.49, size = 746, normalized size = 3.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((a+a*\sec(d*x+c))^{5/2}*(A+B*\sec(d*x+c))/\sec(d*x+c)^{9/2},x, \text{algorithm}="maxima")$

[Out] $1/5040*(\sqrt{2}*(8190*a^2*\cos(8/9*\arctan2(\sin(9/2*d*x + 9/2*c), \cos(9/2*d*x + 9/2*c))) * \sin(9/2*d*x + 9/2*c) + 2100*a^2*\cos(2/3*\arctan2(\sin(9/2*d*x + 9/2*c), \cos(9/2*d*x + 9/2*c))) * \sin(9/2*d*x + 9/2*c) + 756*a^2*\cos(4/9*\arctan2(\sin(9/2*d*x + 9/2*c), \cos(9/2*d*x + 9/2*c))) * \sin(9/2*d*x + 9/2*c) + 225*a^2*\cos(2/9*\arctan2(\sin(9/2*d*x + 9/2*c), \cos(9/2*d*x + 9/2*c))) * \sin(9/2*d*x + 9/2*c) - 8190*a^2*\cos(9/2*d*x + 9/2*c) * \sin(8/9*\arctan2(\sin(9/2*d*x + 9/2*c), \cos(9/2*d*x + 9/2*c))) - 2100*a^2*\cos(9/2*d*x + 9/2*c) * \sin(2/3*\arctan2(\sin(9/2*d*x + 9/2*c), \cos(9/2*d*x + 9/2*c))) - 756*a^2*\cos(9/2*d*x + 9/2*c) * \sin(4/9*\arctan2(\sin(9/2*d*x + 9/2*c), \cos(9/2*d*x + 9/2*c))) - 225*a^2*\cos(9/2*d*x + 9/2*c) * \sin(2/9*\arctan2(\sin(9/2*d*x + 9/2*c), \cos(9/2*d*x + 9/2*c)))) + 70*a^2*\sin(9/2*d*x + 9/2*c) + 225*a^2*\sin(7/9*\arctan2(\sin(9/2*d*x + 9/2*c), \cos(9/2*d*x + 9/2*c))) + 756*a^2*\sin(5/9*\arctan2(\sin(9/2*d*x + 9/2*c), \cos(9/2*d*x + 9/2*c))) + 2100*a^2*\sin(1/3*\arctan2(\sin(9/2*d*x + 9/2*c), \cos(9/2*d*x + 9/2*c))) + 8190*a^2*\sin(1/9*\arctan2(\sin(9/2*d*x + 9/2*c), \cos(9/2*d*x + 9/2*c)))) * A * \sqrt{a} + 30*\sqrt{2}*(315*a^2*\cos(6/7*\arctan2(\sin(7/2*d*x + 7/2*c), \cos(7/2*d*x + 7/2*c))) * \sin(7/2*d*x + 7/2*c) + 77*a^2*\cos(4/7*\arctan2(\sin(7/2*d*x + 7/2*c), \cos(7/2*d*x + 7/2*c))) * \sin(7/2*d*x + 7/2*c) + 21*a^2*\cos(2/7*\arctan2(\sin(7/2*d*x + 7/2*c), \cos(7/2*d*x + 7/2*c))) * \sin(7/2*d*x + 7/2*c) - 315*a^2*\cos(7/2*d*x + 7/2*c) * \sin(6/7*\arctan2(\sin(7/2*d*x + 7/2*c), \cos(7/2*d*x + 7/2*c))) - 77*a^2*\cos(7/2*d*x + 7/2*c) * \sin(4/7*\arctan2(\sin(7/2*d*x + 7/2*c), \cos(7/2*d*x + 7/2*c))) - 21*a^2*\cos(7/2*d*x + 7/2*c) * \sin(2/7*\arctan2(\sin(7/2*d*x + 7/2*c), \cos(7/2*d*x + 7/2*c))) + 6*a^2*\sin(7/2*d*x + 7/2*c) + 21*a^2*\sin(5/7*\arctan2(\sin(7/2*d*x + 7/2*c), \cos(7/2*d*x + 7/2*c))) + 77*a^2*\sin(3/7*\arctan2(\sin(7/2*d*x + 7/2*c), \cos(7/2*d*x + 7/2*c))) + 315*a^2*\sin(1/7*\arctan2(\sin(7/2*d*x + 7/2*c), \cos(7/2*d*x + 7/2*c)))) * B * \sqrt{a})/d$

mupad [B] time = 5.55, size = 157, normalized size = 0.69

$$a^2 \cos(c + dx) \sqrt{\frac{1}{\cos(c+dx)}} \sqrt{\frac{a(\cos(c+dx)+1)}{\cos(c+dx)}} (10290 A \sin(c + dx) + 11760 B \sin(c + dx) + 2856 A \sin(2c + 2dx))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2))/(1/cos(c + d*x))^(9/2),x)

[Out] (a^2*cos(c + d*x)*(1/cos(c + d*x))^(1/2)*((a*(cos(c + d*x) + 1))/cos(c + d*x))^(1/2)*(10290*A*sin(c + d*x) + 11760*B*sin(c + d*x) + 2856*A*sin(2*c + 2*d*x) + 981*A*sin(3*c + 3*d*x) + 260*A*sin(4*c + 4*d*x) + 35*A*sin(5*c + 5*d*x) + 2940*B*sin(2*c + 2*d*x) + 720*B*sin(3*c + 3*d*x) + 90*B*sin(4*c + 4*d*x)))/(2520*d*(cos(c + d*x) + 1))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)**(9/2),x)

[Out] Timed out

$$3.247 \quad \int \frac{(a+a \sec(c+dx))^{5/2}(A+B \sec(c+dx))}{\sec^2(c+dx)} dx$$

Optimal. Leaf size=275

$$\frac{2a^3(710A + 803B) \sin(c + dx)}{1155d \sec^2(c + dx) \sqrt{a \sec(c + dx) + a}} + \frac{2a^3(194A + 209B) \sin(c + dx)}{693d \sec^2(c + dx) \sqrt{a \sec(c + dx) + a}} + \frac{16a^3(710A + 803B) \sin(c + dx)}{3465d \sqrt{a \sec(c + dx)}}$$

[Out] 2/11*a*A*(a+a*sec(d*x+c))^(3/2)*sin(d*x+c)/d/sec(d*x+c)^(9/2)+2/693*a^3*(194*A+209*B)*sin(d*x+c)/d/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2)+2/1155*a^3*(710*A+803*B)*sin(d*x+c)/d/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2)+8/3465*a^3*(710*A+803*B)*sin(d*x+c)/d/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+16/3465*a^3*(710*A+803*B)*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^(1/2)+2/99*a^2*(14*A+11*B)*sin(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d/sec(d*x+c)^(7/2)

Rubi [A] time = 0.70, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {4017, 4015, 3805, 3804}

$$\frac{2a^3(710A + 803B) \sin(c + dx)}{1155d \sec^2(c + dx) \sqrt{a \sec(c + dx) + a}} + \frac{2a^3(194A + 209B) \sin(c + dx)}{693d \sec^2(c + dx) \sqrt{a \sec(c + dx) + a}} + \frac{2a^2(14A + 11B) \sin(c + dx) \sqrt{a}}{99d \sec^2(c + dx)}$$

Antiderivative was successfully verified.

[In] Int[((a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(11/2), x]

[Out] (2*a^3*(194*A + 209*B)*Sin[c + d*x])/(693*d*Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) + (2*a^3*(710*A + 803*B)*Sin[c + d*x])/(1155*d*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (8*a^3*(710*A + 803*B)*Sin[c + d*x])/(3465*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (16*a^3*(710*A + 803*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3465*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2*(14*A + 11*B)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(99*d*Sec[c + d*x]^(7/2)) + (2*a*A*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(11*d*Sec[c + d*x]^(9/2))

Rule 3804

Int[Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]])*Sqrt[d*Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.)
+ (a_.)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
+ b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.)
+ (a_.)]*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
+ f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (
a_.))^(m_)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + a \sec(c + dx))^{5/2} (A + B \sec(c + dx))}{\sec^{\frac{11}{2}}(c + dx)} dx &= \frac{2aA(a + a \sec(c + dx))^{3/2} \sin(c + dx)}{11d \sec^{\frac{9}{2}}(c + dx)} + \frac{2}{11} \int \frac{(a + a \sec(c + dx))^{5/2} (A + B \sec(c + dx))}{\sec^{\frac{11}{2}}(c + dx)} dx \\
&= \frac{2a^2(14A + 11B)\sqrt{a + a \sec(c + dx)} \sin(c + dx)}{99d \sec^{\frac{7}{2}}(c + dx)} + \frac{2aA(a + a \sec(c + dx))^{3/2} \sin(c + dx)}{11d \sec^{\frac{9}{2}}(c + dx)} \\
&= \frac{2a^3(194A + 209B) \sin(c + dx)}{693d \sec^{\frac{5}{2}}(c + dx)\sqrt{a + a \sec(c + dx)}} + \frac{2a^2(14A + 11B)\sqrt{a + a \sec(c + dx)} \sin(c + dx)}{99d \sec^{\frac{7}{2}}(c + dx)} \\
&= \frac{2a^3(194A + 209B) \sin(c + dx)}{693d \sec^{\frac{5}{2}}(c + dx)\sqrt{a + a \sec(c + dx)}} + \frac{2a^3(710A + 803B) \sin(c + dx)}{1155d \sec^{\frac{3}{2}}(c + dx)\sqrt{a + a \sec(c + dx)}} \\
&= \frac{2a^3(194A + 209B) \sin(c + dx)}{693d \sec^{\frac{5}{2}}(c + dx)\sqrt{a + a \sec(c + dx)}} + \frac{2a^3(710A + 803B) \sin(c + dx)}{1155d \sec^{\frac{3}{2}}(c + dx)\sqrt{a + a \sec(c + dx)}} \\
&= \frac{2a^3(194A + 209B) \sin(c + dx)}{693d \sec^{\frac{5}{2}}(c + dx)\sqrt{a + a \sec(c + dx)}} + \frac{2a^3(710A + 803B) \sin(c + dx)}{1155d \sec^{\frac{3}{2}}(c + dx)\sqrt{a + a \sec(c + dx)}}
\end{aligned}$$

Mathematica [A] time = 4.25, size = 127, normalized size = 0.46

$$\frac{2a^3 \sin(c + dx) (8(710A + 803B) \sec^5(c + dx) + 4(710A + 803B) \sec^4(c + dx) + 3(710A + 803B) \sec^3(c + dx) + 2(710A + 803B) \sec^2(c + dx) + (710A + 803B) \sec(c + dx) + 1)}{3465d \sec^{\frac{9}{2}}(c + dx) \sqrt{a(\sec(c + dx) + 1)}}$$

Antiderivative was successfully verified.

[In] Integrate[(((a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(1/2)), x]

[Out] (2*a^3*(315*A + 35*(32*A + 11*B))*Sec[c + d*x] + 5*(355*A + 286*B)*Sec[c + d*x]^2 + 3*(710*A + 803*B)*Sec[c + d*x]^3 + 4*(710*A + 803*B)*Sec[c + d*x]^4 + 8*(710*A + 803*B)*Sec[c + d*x]^5*Sin[c + d*x])/(3465*d*Sec[c + d*x]^(9/2)*Sqrt[a*(1 + Sec[c + d*x])])

fricas [A] time = 0.48, size = 162, normalized size = 0.59

$$\frac{2(315 A a^2 \cos(dx + c)^6 + 35(32 A + 11 B) a^2 \cos(dx + c)^5 + 5(355 A + 286 B) a^2 \cos(dx + c)^4 + 3(710 A + 803 B) a^2 \cos(dx + c)^3 + 2(710 A + 803 B) a^2 \cos(dx + c)^2 + (710 A + 803 B) a^2 \cos(dx + c) + 1)}{3465(d \cos(dx + c) + 1) \sqrt{a(d \cos(dx + c) + 1)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(11/2),x, algorithm="fricas")

[Out] $\frac{2}{3465} \cdot (315 \cdot A \cdot a^2 \cdot \cos(d \cdot x + c)^6 + 35 \cdot (32 \cdot A + 11 \cdot B) \cdot a^2 \cdot \cos(d \cdot x + c)^5 + 5 \cdot (355 \cdot A + 286 \cdot B) \cdot a^2 \cdot \cos(d \cdot x + c)^4 + 3 \cdot (710 \cdot A + 803 \cdot B) \cdot a^2 \cdot \cos(d \cdot x + c)^3 + 4 \cdot (710 \cdot A + 803 \cdot B) \cdot a^2 \cdot \cos(d \cdot x + c)^2 + 8 \cdot (710 \cdot A + 803 \cdot B) \cdot a^2 \cdot \cos(d \cdot x + c)) \cdot \sqrt{\frac{a \cdot \cos(d \cdot x + c) + a}{\cos(d \cdot x + c)}} \cdot \sin(d \cdot x + c) / ((d \cdot \cos(d \cdot x + c) + d) \cdot \sqrt{\cos(d \cdot x + c)})$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(a \sec(dx + c) + a)^{\frac{5}{2}}}{\sec(dx + c)^{\frac{11}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(11/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(11/2), x)

maple [A] time = 2.79, size = 165, normalized size = 0.60

$$2(-1 + \cos(dx + c)) \left(315A \left(\cos^5(dx + c) \right) + 1120A \left(\cos^4(dx + c) \right) + 385B \left(\cos^4(dx + c) \right) + 1775A \left(\cos^3(dx + c) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(11/2),x)

[Out] $-2/3465/d \cdot (-1 + \cos(d \cdot x + c)) \cdot (315 \cdot A \cdot \cos(d \cdot x + c)^5 + 1120 \cdot A \cdot \cos(d \cdot x + c)^4 + 385 \cdot B \cdot \cos(d \cdot x + c)^4 + 1775 \cdot A \cdot \cos(d \cdot x + c)^3 + 1430 \cdot B \cdot \cos(d \cdot x + c)^3 + 2130 \cdot A \cdot \cos(d \cdot x + c)^2 + 2409 \cdot B \cdot \cos(d \cdot x + c)^2 + 2840 \cdot A \cdot \cos(d \cdot x + c) + 3212 \cdot B \cdot \cos(d \cdot x + c) + 5680 \cdot A + 6424 \cdot B) \cdot (a \cdot (1 + \cos(d \cdot x + c)) / \cos(d \cdot x + c))^{\frac{1}{2}} \cdot \cos(d \cdot x + c)^6 \cdot (1 / \cos(d \cdot x + c))^{\frac{11}{2}} / \sin(d \cdot x + c) \cdot a^2$

maxima [B] time = 1.51, size = 945, normalized size = 3.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(11/2),x, algorith="maxima")

[Out] 1/110880*(5*sqrt(2)*(31878*a^2*cos(10/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) * sin(11/2*d*x + 11/2*c) + 8778*a^2*cos(8/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) * sin(11/2*d*x + 11/2*c) + 3465*a^2*cos(6/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) * sin(11/2*d*x + 11/2*c) + 1287*a^2*cos(4/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) * sin(11/2*d*x + 11/2*c) + 385*a^2*cos(2/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) * sin(11/2*d*x + 11/2*c) - 31878*a^2*cos(11/2*d*x + 11/2*c)*sin(10/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) - 8778*a^2*cos(11/2*d*x + 11/2*c)*sin(8/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) - 3465*a^2*cos(11/2*d*x + 11/2*c)*sin(6/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) - 1287*a^2*cos(11/2*d*x + 11/2*c)*sin(4/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) - 385*a^2*cos(11/2*d*x + 11/2*c)*sin(2/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) + 126*a^2*sin(11/2*d*x + 11/2*c) + 385*a^2*sin(9/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) + 1287*a^2*sin(7/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) + 3465*a^2*sin(5/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) + 8778*a^2*sin(3/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) + 31878*a^2*sin(1/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))))*A*sqrt(a) + 22*sqrt(2)*(8190*a^2*cos(8/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) * sin(9/2*d*x + 9/2*c) + 2100*a^2*cos(2/3*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) * sin(9/2*d*x + 9/2*c) + 756*a^2*cos(4/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) * sin(9/2*d*x + 9/2*c) + 225*a^2*cos(2/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) * sin(9/2*d*x + 9/2*c) - 8190*a^2*cos(9/2*d*x + 9/2*c)*sin(8/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) - 2100*a^2*cos(9/2*d*x + 9/2*c)*sin(2/3*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) - 756*a^2*cos(9/2*d*x + 9/2*c)*sin(4/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) - 225*a^2*cos(9/2*d*x + 9/2*c)*sin(2/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) + 70*a^2*sin(9/2*d*x + 9/2*c) + 225*a^2*sin(7/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) + 756*a^2*sin(5/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) + 2100*a^2*sin(1/3*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) + 8190*a^2*sin(1/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))))*B*sqrt(a))/d

mupad [B] time = 8.77, size = 392, normalized size = 1.43

$$\sqrt{a - \frac{a}{2 \sin\left(\frac{c}{2} + \frac{dx}{2}\right)^2 - 1}} \left(2 \sin\left(\frac{11c}{4} + \frac{11dx}{4}\right)^2 + \sin\left(\frac{11c}{2} + \frac{11dx}{2}\right) \operatorname{li} - 1 \right) \left(\frac{A a^2 \sin\left(\frac{11c}{2} + \frac{11dx}{2}\right) \left(-2 \sin\left(\frac{11c}{4} + \frac{11dx}{4}\right)^2 + \sin\left(\frac{11c}{2} + \frac{11dx}{2}\right) \right)}{88d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2))/(1/cos(c + d*x))^(11/2),x)
```

```
[Out] ((a - a/(2*sin(c/2 + (d*x)/2)^2 - 1))^(1/2)*(sin((11*c)/2 + (11*d*x)/2)*1i + 2*sin((11*c)/4 + (11*d*x)/4)^2 - 1)*((A*a^2*sin((11*c)/2 + (11*d*x)/2)*(sin((11*c)/2 + (11*d*x)/2)*1i - 2*sin((11*c)/4 + (11*d*x)/4)^2 + 1))/(88*d) + (a^2*sin((9*c)/2 + (9*d*x)/2)*(5*A + 2*B)*(sin((11*c)/2 + (11*d*x)/2)*1i - 2*sin((11*c)/4 + (11*d*x)/4)^2 + 1))/(72*d) + (a^2*sin((7*c)/2 + (7*d*x)/2)*(13*A + 10*B)*(sin((11*c)/2 + (11*d*x)/2)*1i - 2*sin((11*c)/4 + (11*d*x)/4)^2 + 1))/(56*d) + (a^2*sin((3*c)/2 + (3*d*x)/2)*(19*A + 20*B)*(sin((11*c)/2 + (11*d*x)/2)*1i - 2*sin((11*c)/4 + (11*d*x)/4)^2 + 1))/(12*d) + (a^2*sin(c/2 + (d*x)/2)*(23*A + 26*B)*(sin((11*c)/2 + (11*d*x)/2)*1i - 2*sin((11*c)/4 + (11*d*x)/4)^2 + 1))/(4*d) + (a^2*sin((5*c)/2 + (5*d*x)/2)*(25*A + 24*B)*(sin((11*c)/2 + (11*d*x)/2)*1i - 2*sin((11*c)/4 + (11*d*x)/4)^2 + 1))/(40*d)))/(2*(-1/(2*sin(c/2 + (d*x)/2)^2 - 1))^(1/2)*(2*sin(c/4 + (d*x)/4)^2 - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```


$$3.248 \quad \int \frac{\sec^2(c+dx)(A+B \sec(c+dx))}{\sqrt{a+a \sec(c+dx)}} dx$$

Optimal. Leaf size=190

$$\frac{(4A - B) \sin(c + dx) \sec^2(c + dx)}{4d\sqrt{a \sec(c + dx) + a}} + \frac{\sqrt{2} (A - B) \tanh^{-1} \left(\frac{\sqrt{a} \sin(c+dx) \sqrt{\sec(c+dx)}}{\sqrt{2} \sqrt{a \sec(c+dx) + a}} \right)}{\sqrt{a} d} - \frac{(4A - 7B) \sinh^{-1} \left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx) + a}} \right)}{4\sqrt{a} d}$$

[Out] $-1/4*(4*A-7*B)*\operatorname{arcsinh}(a^{(1/2)}*\tan(d*x+c)/(a+a*\sec(d*x+c))^{(1/2)})/d/a^{(1/2)}$
 $+ (A-B)*\operatorname{arctanh}(1/2*\sin(d*x+c)*a^{(1/2)}*\sec(d*x+c)^{(1/2)}*2^{(1/2)}/(a+a*\sec(d*x+c))^{(1/2)})*2^{(1/2)}/d/a^{(1/2)}+1/4*(4*A-B)*\sec(d*x+c)^{(3/2)}*\sin(d*x+c)/d/(a+a*\sec(d*x+c))^{(1/2)}+1/2*B*\sec(d*x+c)^{(5/2)}*\sin(d*x+c)/d/(a+a*\sec(d*x+c))^{(1/2)}$

Rubi [A] time = 0.57, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {4021, 4023, 3808, 206, 3801, 215}

$$\frac{(4A - B) \sin(c + dx) \sec^2(c + dx)}{4d\sqrt{a \sec(c + dx) + a}} + \frac{\sqrt{2} (A - B) \tanh^{-1} \left(\frac{\sqrt{a} \sin(c+dx) \sqrt{\sec(c+dx)}}{\sqrt{2} \sqrt{a \sec(c+dx) + a}} \right)}{\sqrt{a} d} - \frac{(4A - 7B) \sinh^{-1} \left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx) + a}} \right)}{4\sqrt{a} d}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(\operatorname{Sec}[c + d*x]^{(5/2)}*(A + B*\operatorname{Sec}[c + d*x]))/\operatorname{Sqrt}[a + a*\operatorname{Sec}[c + d*x]], x]$

[Out] $-((4*A - 7*B)*\operatorname{ArcSinh}[(\operatorname{Sqrt}[a]*\operatorname{Tan}[c + d*x])/\operatorname{Sqrt}[a + a*\operatorname{Sec}[c + d*x]])/(4*\operatorname{Sqrt}[a]*d) + (\operatorname{Sqrt}[2]*(A - B)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Sqrt}[\operatorname{Sec}[c + d*x]]*\operatorname{Sin}[c + d*x])/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a + a*\operatorname{Sec}[c + d*x]])]/(\operatorname{Sqrt}[a]*d) + ((4*A - B)*\operatorname{Sec}[c + d*x]^{(3/2)}*\operatorname{Sin}[c + d*x])/(4*d*\operatorname{Sqrt}[a + a*\operatorname{Sec}[c + d*x]]) + (B*\operatorname{Sec}[c + d*x]^{(5/2)}*\operatorname{Sin}[c + d*x])/(2*d*\operatorname{Sqrt}[a + a*\operatorname{Sec}[c + d*x]])$

Rule 206

$\operatorname{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1*\operatorname{ArcTanh}[(\operatorname{Rt}[-b, 2]*x)/\operatorname{Rt}[a, 2]])/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /; \operatorname{FreeQ}\{a, b\}, x] \ \&\& \operatorname{NegQ}[a/b] \ \&\& (\operatorname{GtQ}[a, 0] \ || \ \operatorname{LtQ}[b, 0])$

Rule 215

$\operatorname{Int}[1/\operatorname{Sqrt}[(a_ + (b_)*(x_)^2)], x_Symbol] \rightarrow \operatorname{Simp}[\operatorname{ArcSinh}[(\operatorname{Rt}[b, 2]*x)/\operatorname{Sqrt}[a]]/\operatorname{Rt}[b, 2], x] /; \operatorname{FreeQ}\{a, b\}, x] \ \&\& \operatorname{GtQ}[a, 0] \ \&\& \operatorname{PosQ}[b]$

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.)
+ (a_.)], x_Symbol] :=> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.)
+ (a_.)], x_Symbol] :=> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4021

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (
a_))^m*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_)), x_Symbol] :=> -Simp[(B*d*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x
] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)
*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &&
GtQ[n, 1]
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (
a_))^m*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_)), x_Symbol] :=> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{\sqrt{a+a\sec(c+dx)}} dx &= \frac{B\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}} + \frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(\frac{3aB}{2} + \frac{1}{2}a(4A-B)\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}} dx}{2a} \\
&= \frac{(4A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{B\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}} + \int \frac{\sec^{\frac{3}{2}}(c+dx)\left(\frac{3aB}{2} + \frac{1}{2}a(4A-B)\sec(c+dx)\right)}{2a\sqrt{a+a\sec(c+dx)}} dx \\
&= \frac{(4A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{B\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}} - \frac{(4A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} \\
&= \frac{(4A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{B\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}} + \frac{(4A-7B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4\sqrt{a}d} + \frac{\sqrt{2}(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d}
\end{aligned}$$

Mathematica [A] time = 0.89, size = 125, normalized size = 0.66

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(8(A-B)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) - \sqrt{2}(4A-7B)\tanh^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)}{4d\sqrt{a}(\sec(c+dx)+1)}$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/Sqrt[a + a*Sec[c + d*x]], x]

[Out] (Cos[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(8*(A - B)*ArcTanh[Sin[(c + d*x)/2]] - Sqrt[2]*(4*A - 7*B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sec[c + d*x]*(4*A - B + 2*B*Sec[c + d*x])*Sin[(c + d*x)/2]))/(4*d*Sqrt[a*(1 + Sec[c + d*x])])

fricas [A] time = 0.65, size = 617, normalized size = 3.25

$$\left((4A - 7B) \cos(dx + c)^2 + (4A - 7B) \cos(dx + c) \right) \sqrt{a} \log \left(\frac{a \cos(dx+c)^3 - 7a \cos(dx+c)^2 - \frac{4(\cos(dx+c)^2 - 2\cos(dx+c)) \sqrt{a} \sqrt{\frac{a \cos(dx+c)}{\cos(dx+c)}}}{\sqrt{\cos(dx+c)}}}{\cos(dx+c)^3 + \cos(dx+c)^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")

[Out] [-1/16*(((4*A - 7*B)*cos(d*x + c)^2 + (4*A - 7*B)*cos(d*x + c))*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 8*sqrt(2)*((A - B)*a*cos(d*x + c)^2 + (A - B)*a*cos(d*x + c))*log(-(cos(d*x + c)^2 + 2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/sqrt(a) - 4*((4*A - B)*cos(d*x + c) + 2*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c)^2 + a*d*cos(d*x + c)), -1/8*(8*sqrt(2)*((A - B)*a*cos(d*x + c)^2 + (A - B)*a*cos(d*x + c))*sqrt(-1/a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*sqrt(cos(d*x + c))/sin(d*x + c)) + ((4*A - 7*B)*cos(d*x + c)^2 + (4*A - 7*B)*cos(d*x + c))*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) - 2*((4*A - B)*cos(d*x + c) + 2*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c)^2 + a*d*cos(d*x + c))]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sec(dx + c)^{\frac{5}{2}}}{\sqrt{a \sec(dx + c) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(5/2)/sqrt(a*sec(d*x + c) + a), x)

maple [B] time = 2.64, size = 423, normalized size = 2.23

$$\left(-4A \left(\cos^2(dx + c) \right) \arctan \left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}} (\cos(dx+c)+1+\sin(dx+c))\sqrt{2}}{4} \right) \right) \sqrt{2} + 4A \left(\cos^2(dx + c) \right) \arctan \left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}}{1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x)

[Out] 1/16/d*(-4*A*cos(d*x+c)^2*arctan(1/4*(-2/(1+cos(d*x+c))))^(1/2)*(cos(d*x+c)+1+sin(d*x+c))*2^(1/2))*2^(1/2)+4*A*cos(d*x+c)^2*arctan(1/4*(-2/(1+cos(d*x+c))))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))*2^(1/2))*2^(1/2)+7*B*cos(d*x+c)^2*arctan(1/4*(-2/(1+cos(d*x+c))))^(1/2)*(cos(d*x+c)+1+sin(d*x+c))*2^(1/2))*2^(1/2)-7*B*cos(d*x+c)^2*arctan(1/4*(-2/(1+cos(d*x+c))))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))*2^(1/2))*2^(1/2)+8*A*sin(d*x+c)*cos(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)+16*A*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c))))^(1/2)*cos(d*x+c)^2-2*B*sin(d*x+c)*cos(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)-16*B*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c))))^(1/2)*cos(d*x+c)^2+4*B*(-2/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)*(1/cos(d*x+c))^(5/2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-2/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^2*(cos(d*x+c)^2-1)/a

maxima [B] time = 1.64, size = 2524, normalized size = 13.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")

[Out] -1/16*(4*(4*sqrt(2)*cos(3/2*arctan2(sin(d*x + c), cos(d*x + c))))*sin(2*d*x + 2*c) - 4*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))))*sin(2*d*x + 2*c) + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2)

$c), \cos(dx + c))$ ² + 2*sqrt(2)*cos(1/2*arctan2(sin(dx + c), cos(dx + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(dx + c), cos(dx + c))) + 2) + (cos(2*d*x + 2*c)² + sin(2*d*x + 2*c)² + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(dx + c), cos(dx + c)))² + 2*sin(1/2*arctan2(sin(dx + c), cos(dx + c)))² - 2*sqrt(2)*cos(1/2*arctan2(sin(dx + c), cos(dx + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(dx + c), cos(dx + c))) + 2) - (cos(2*d*x + 2*c)² + sin(2*d*x + 2*c)² + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(dx + c), cos(dx + c)))² + 2*sin(1/2*arctan2(sin(dx + c), cos(dx + c)))² - 2*sqrt(2)*cos(1/2*arctan2(sin(dx + c), cos(dx + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(dx + c), cos(dx + c))) + 2) - 2*(sqrt(2)*cos(2*d*x + 2*c)² + sqrt(2)*sin(2*d*x + 2*c)² + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(cos(1/2*arctan2(sin(dx + c), cos(dx + c)))² + sin(1/2*arctan2(sin(dx + c), cos(dx + c)))² + 1) + 2*(sqrt(2)*cos(2*d*x + 2*c)² + sqrt(2)*sin(2*d*x + 2*c)² + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(cos(1/2*arctan2(sin(dx + c), cos(dx + c)))² + sin(1/2*arctan2(sin(dx + c), cos(dx + c)))² - 2*sin(1/2*arctan2(sin(dx + c), cos(dx + c))) + 1) - 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(3/2*arctan2(sin(dx + c), cos(dx + c))) + 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(1/2*arctan2(sin(dx + c), cos(dx + c))))*A/((cos(2*d*x + 2*c)² + sin(2*d*x + 2*c)² + 2*cos(2*d*x + 2*c) + 1)*sqrt(a)) - (4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(7/2*arctan2(sin(dx + c), cos(dx + c))) - 20*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(dx + c), cos(dx + c))) + 20*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(dx + c), cos(dx + c))) - 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(dx + c), cos(dx + c))) + 7*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)² + 4*cos(2*d*x + 2*c)² + sin(4*d*x + 4*c)² + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)² + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(dx + c), cos(dx + c)))² + 2*sin(1/2*arctan2(sin(dx + c), cos(dx + c)))² + 2*sqrt(2)*cos(1/2*arctan2(sin(dx + c), cos(dx + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(dx + c), cos(dx + c))) + 2) - 7*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)² + 4*cos(2*d*x + 2*c)² + sin(4*d*x + 4*c)² + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)² + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(dx + c), cos(dx + c)))² + 2*sin(1/2*arctan2(sin(dx + c), cos(dx + c)))² + 2*sqrt(2)*cos(1/2*arctan2(sin(dx + c), cos(dx + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(dx + c), cos(dx + c))) + 2) + 7*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)² + 4*cos(2*d*x + 2*c)² + sin(4*d*x + 4*c)² + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)² + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(dx + c), cos(dx + c)))² + 2*sin(1/2*arctan2(sin(dx + c), cos(dx + c)))² - 2*sqrt(2)*cos(1/2*arctan2(sin(dx + c), cos(dx + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(dx + c), cos(dx + c))) + 2) - 7*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)² + 4*cos(2*d*x + 2*c)² + sin(4*d*x + 4*c)² + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)² + 4*cos(2*d*x + 2*c)

) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 8*(sqrt(2)*cos(4*d*x + 4*c)^2 + 4*sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(4*d*x + 4*c)^2 + 4*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sqrt(2)*sin(2*d*x + 2*c)^2 + 2*(2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 4*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 1) + 8*(sqrt(2)*cos(4*d*x + 4*c)^2 + 4*sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(4*d*x + 4*c)^2 + 4*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sqrt(2)*sin(2*d*x + 2*c)^2 + 2*(2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 4*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 1) - 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(7/2*arctan2(sin(d*x + c), cos(d*x + c))) + 20*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(5/2*arctan2(sin(d*x + c), cos(d*x + c))) - 20*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(3/2*arctan2(sin(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))))*B/((2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*sqrt(a))/d

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(\frac{1}{\cos(c+dx)}\right)^{5/2}}{\sqrt{a + \frac{a}{\cos(c+dx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + a/cos(c + d*x))^(1/2), x)

[Out] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + a/cos(c + d*x))^(1/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(1/2), x)

[Out] Timed out

$$3.249 \quad \int \frac{\sec^2(c+dx)(A+B \sec(c+dx))}{\sqrt{a+a \sec(c+dx)}} dx$$

Optimal. Leaf size=141

$$\frac{\sqrt{2}(A-B) \tanh^{-1}\left(\frac{\sqrt{a} \sin(c+dx) \sqrt{\sec(c+dx)}}{\sqrt{2} \sqrt{a \sec(c+dx)+a}}\right)}{\sqrt{a} d} + \frac{(2A-B) \sinh^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}}\right)}{\sqrt{a} d} + \frac{B \sin(c+dx) \sec^2(c+dx)}{d \sqrt{a \sec(c+dx)+a}}$$

[Out] (2*A-B)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d/a^(1/2)-(A-B)*arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2)/d/a^(1/2)+B*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)

Rubi [A] time = 0.39, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {4021, 4023, 3808, 206, 3801, 215}

$$\frac{\sqrt{2}(A-B) \tanh^{-1}\left(\frac{\sqrt{a} \sin(c+dx) \sqrt{\sec(c+dx)}}{\sqrt{2} \sqrt{a \sec(c+dx)+a}}\right)}{\sqrt{a} d} + \frac{(2A-B) \sinh^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}}\right)}{\sqrt{a} d} + \frac{B \sin(c+dx) \sec^2(c+dx)}{d \sqrt{a \sec(c+dx)+a}}$$

Antiderivative was successfully verified.

[In] Int[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/Sqrt[a + a*Sec[c + d*x]], x]

[Out] ((2*A - B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]])/(Sqrt[a]*d) - (Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(Sqrt[a]*d) + (B*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 215

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 3801

Int[Sqrt[csc[(e_) + (f_)*(x_)]*(d_)]*Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)], x_Symbol] := Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +

x^2/a , x , x , $(b \cdot \cot[e + f \cdot x]) / \sqrt{a + b \cdot \csc[e + f \cdot x]}$, x /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]

Rule 3808

Int[Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x])*Sqrt[d*Csc[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 4021

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> -Simp[(B*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && GtQ[n, 1]

Rule 4023

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> Dist[(A*b - a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned}
\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{\sqrt{a+a\sec(c+dx)}} dx &= \frac{B\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{\int \frac{\sqrt{\sec(c+dx)}\left(\frac{aB}{2} + \frac{1}{2}a(2A-B)\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}} dx}{a} \\
&= \frac{B\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{(2A-B)\int \sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)} dx}{2a} \\
&= \frac{B\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{(2A-B)\text{Subst}\left(\int \frac{1}{2a-x^2} dx, x, -\frac{a\sqrt{\sec(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\right)}{d} \\
&= \frac{(2A-B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d} - \frac{\sqrt{2}(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d}
\end{aligned}$$

Mathematica [A] time = 0.42, size = 106, normalized size = 0.75

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(-2(A-B)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right) + \sqrt{2}(2A-B)\tanh^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/Sqrt[a + a*Sec[c + d*x]], x]

[Out] (Cos[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(-2*(A - B)*ArcTanh[Sin[(c + d*x)/2]] + Sqrt[2]*(2*A - B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]] + 2*B*Sec[c + d*x]*Sin[(c + d*x)/2])/(d*Sqrt[a*(1 + Sec[c + d*x])])

fricas [A] time = 0.60, size = 531, normalized size = 3.77

$$\frac{((2A - B)\cos(dx + c) + 2A - B)\sqrt{a} \log\left(\frac{a\cos(dx+c)^3 - 7a\cos(dx+c)^2 + \frac{4(\cos(dx+c)^2 - 2\cos(dx+c))\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}} + 8a}{\cos(dx+c)^3 + \cos(dx+c)^2}\right)}{4(ad\cos(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")

[Out] [-1/4*(((2*A - B)*cos(d*x + c) + 2*A - B)*sqrt(a)*log((a*cos(d*x + c))^3 - 7*a*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 2*sqrt(2)*((A - B)*a*cos(d*x + c) + (A - B)*a)*log(-(cos(d*x + c)^2 - 2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/sqrt(a) - 4*B*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c) + a*d), 1/2*(2*sqrt(2)*((A - B)*a*cos(d*x + c) + (A - B)*a)*sqrt(-1/a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*sqrt(cos(d*x + c))/sin(d*x + c)) + ((2*A - B)*cos(d*x + c) + 2*A - B)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) + 2*B*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c) + a*d)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sec(dx + c)^{\frac{3}{2}}}{\sqrt{a \sec(dx + c) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(3/2)/sqrt(a*sec(d*x + c) + a), x)

maple [B] time = 2.59, size = 352, normalized size = 2.50

$$\left(2A \arctan \left(\frac{\sqrt{\frac{2}{1+\cos(dx+c)}} (\cos(dx+c)+1+\sin(dx+c)) \sqrt{2}}{4} \right) \sqrt{2} \cos(dx + c) - 2A \arctan \left(\frac{\sqrt{\frac{2}{1+\cos(dx+c)}} (\cos(dx+c)+1-\sin(dx+c))}{4} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x)

[Out] 1/4/d*(2*A*arctan(1/4*(-2/(1+cos(d*x+c))))^(1/2)*(cos(d*x+c)+1+sin(d*x+c))*2^(1/2))*2^(1/2)*cos(d*x+c)-2*A*arctan(1/4*(-2/(1+cos(d*x+c))))^(1/2)*(cos(d*

$$x+c)+1-\sin(d*x+c))*2^{(1/2)}*2^{(1/2)}*\cos(d*x+c)-B*\arctan(1/4*(-2/(1+\cos(d*x+c)))^{(1/2)}*(\cos(d*x+c)+1+\sin(d*x+c))*2^{(1/2)}*2^{(1/2)}*\cos(d*x+c)+B*\arctan(1/4*(-2/(1+\cos(d*x+c)))^{(1/2)}*(\cos(d*x+c)+1-\sin(d*x+c))*2^{(1/2)}*2^{(1/2)}*\cos(d*x+c)-4*A*\cos(d*x+c)*\arctan(1/2*\sin(d*x+c)*(-2/(1+\cos(d*x+c)))^{(1/2)}))+4*B*\cos(d*x+c)*\arctan(1/2*\sin(d*x+c)*(-2/(1+\cos(d*x+c)))^{(1/2)}))+2*B*(-2/(1+\cos(d*x+c)))^{(1/2)}*\sin(d*x+c))*\cos(d*x+c)*(1/\cos(d*x+c))^{(3/2)}*(a*(1+\cos(d*x+c))/\cos(d*x+c))^{(1/2)}*(-2/(1+\cos(d*x+c)))^{(1/2)}/\sin(d*x+c)^2*(\cos(d*x+c)^2-1)/a$$

maxima [B] time = 1.56, size = 1353, normalized size = 9.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")

[Out]
$$-1/4*(2*(\sqrt{2}*\log(\cos(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c))))^2 + \sin(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c))))^2 + 2*\sin(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c))) + 1) - \sqrt{2}*\log(\cos(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c))))^2 + \sin(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c))))^2 - 2*\sin(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c))) + 1) - \log(2*\cos(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c))))^2 + 2*\sin(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c))))^2 + 2*\sqrt{2}*(2*\cos(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c)))) + 2*\sqrt{2}*\sin(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c)))) + 2) + \log(2*\cos(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c))))^2 + 2*\sin(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c))))^2 + 2*\sqrt{2}*(2*\cos(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c)))) - 2*\sqrt{2}*\sin(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c)))) + 2) - \log(2*\cos(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c))))^2 + 2*\sin(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c))))^2 - 2*\sqrt{2}*(2*\cos(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c)))) + 2*\sqrt{2}*\sin(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c)))) + 2) + \log(2*\cos(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c))))^2 + 2*\sin(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c))))^2 - 2*\sqrt{2}*(2*\cos(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c)))) - 2*\sqrt{2}*\sin(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c)))) + 2))*A/\sqrt{a} + (4*\sqrt{2}*\cos(3/2*\arctan2(\sin(d*x + c), \cos(d*x + c))))*\sin(2*d*x + 2*c) - 4*\sqrt{2}*\cos(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c))))*\sin(2*d*x + 2*c) + (\cos(2*d*x + 2*c))^2 + \sin(2*d*x + 2*c))^2 + 2*\cos(2*d*x + 2*c) + 1)*\log(2*\cos(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c))))^2 + 2*\sin(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c))))^2 + 2*\sqrt{2}*(2*\cos(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c)))) + 2*\sqrt{2}*\sin(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c)))) + 2) - (\cos(2*d*x + 2*c))^2 + \sin(2*d*x + 2*c))^2 + 2*\cos(2*d*x + 2*c) + 1)*\log(2*\cos(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c))))^2 + 2*\sin(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c))))^2 + 2*\sqrt{2}*(2*\cos(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c)))) - 2*\sqrt{2}*\sin(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c)))) + 2) + (\cos(2*d*x + 2*c))^2 + \sin(2*d*x + 2*c))^2 + 2*\cos(2*d*x + 2*c) + 1)*\log(2*\cos(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c))))^2$$

+ 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 2*(sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 1) + 2*(sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 1) - 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(3/2*arctan2(sin(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))))*B/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sqrt(a))/d

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(\frac{1}{\cos(c+dx)}\right)^{3/2}}{\sqrt{a + \frac{a}{\cos(c+dx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + a/cos(c + d*x))^(1/2), x)

[Out] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + a/cos(c + d*x))^(1/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(1/2), x)

[Out] Timed out

$$3.250 \quad \int \frac{\sqrt{\sec(c+dx)} (A+B \sec(c+dx))}{\sqrt{a+a \sec(c+dx)}} dx$$

Optimal. Leaf size=100

$$\frac{\sqrt{2} (A - B) \tanh^{-1} \left(\frac{\sqrt{a} \sin(c+dx) \sqrt{\sec(c+dx)}}{\sqrt{2} \sqrt{a \sec(c+dx)+a}} \right)}{\sqrt{a} d} + \frac{2B \sinh^{-1} \left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}} \right)}{\sqrt{a} d}$$

[Out] $2*B*\operatorname{arcsinh}(a^{(1/2)}*\tan(d*x+c)/(a+a*\sec(d*x+c))^{(1/2)})/d/a^{(1/2)}+(A-B)*\operatorname{arctanh}(1/2*\sin(d*x+c)*a^{(1/2)}*\sec(d*x+c)^{(1/2)}*2^{(1/2)}/(a+a*\sec(d*x+c))^{(1/2)})*2^{(1/2)}/d/a^{(1/2)}$

Rubi [A] time = 0.23, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4023, 3808, 206, 3801, 215}

$$\frac{\sqrt{2} (A - B) \tanh^{-1} \left(\frac{\sqrt{a} \sin(c+dx) \sqrt{\sec(c+dx)}}{\sqrt{2} \sqrt{a \sec(c+dx)+a}} \right)}{\sqrt{a} d} + \frac{2B \sinh^{-1} \left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}} \right)}{\sqrt{a} d}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(\operatorname{Sqrt}[\operatorname{Sec}[c + d*x]]*(A + B*\operatorname{Sec}[c + d*x]))/\operatorname{Sqrt}[a + a*\operatorname{Sec}[c + d*x]],x]$

[Out] $(2*B*\operatorname{ArcSinh}[(\operatorname{Sqrt}[a]*\operatorname{Tan}[c + d*x])/(\operatorname{Sqrt}[a + a*\operatorname{Sec}[c + d*x]])]/(\operatorname{Sqrt}[a]*d) + (\operatorname{Sqrt}[2]*(A - B)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Sqrt}[\operatorname{Sec}[c + d*x]]*\operatorname{Sin}[c + d*x])/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a + a*\operatorname{Sec}[c + d*x]])])]/(\operatorname{Sqrt}[a]*d)$

Rule 206

$\operatorname{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1*\operatorname{ArcTanh}[(\operatorname{Rt}[-b, 2]*x)/\operatorname{Rt}[a, 2]])/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /; \operatorname{FreeQ}\{a, b\}, x] \ \&\& \operatorname{NegQ}[a/b] \ \&\& (\operatorname{GtQ}[a, 0] \ || \ \operatorname{LtQ}[b, 0])$

Rule 215

$\operatorname{Int}[1/\operatorname{Sqrt}[(a_ + (b_)*(x_)^2), x_Symbol] \rightarrow \operatorname{Simp}[\operatorname{ArcSinh}[(\operatorname{Rt}[b, 2]*x)/\operatorname{Sqrt}[a]]/\operatorname{Rt}[b, 2], x] /; \operatorname{FreeQ}\{a, b\}, x] \ \&\& \operatorname{GtQ}[a, 0] \ \&\& \operatorname{PosQ}[b]$

Rule 3801

$\operatorname{Int}[\operatorname{Sqrt}[\operatorname{csc}[(e_ + (f_)*(x_)]*(d_)]*\operatorname{Sqrt}[\operatorname{csc}[(e_ + (f_)*(x_)]*(b_ + a_)]], x_Symbol] \rightarrow \operatorname{Dist}[(-2*a*\operatorname{Sqrt}[(a*d)/b])/(b*f), \operatorname{Subst}[\operatorname{Int}[1/\operatorname{Sqrt}[1 + x^2/a], x], x, (b*\operatorname{Cot}[e + f*x])/(\operatorname{Sqrt}[a + b*\operatorname{Csc}[e + f*x]])], x] /; \operatorname{FreeQ}\{a,$

b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]

Rule 3808

Int[Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] := Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 4023

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := Dist[(A*b - a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}(A+B\sec(c+dx))}{\sqrt{a+a\sec(c+dx)}} dx = (A-B) \int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a\sec(c+dx)}} dx + \frac{B \int \sqrt{\sec(c+dx)} \sqrt{a+a\sec(c+dx)}}{a}$$

$$= \frac{(2(A-B)) \operatorname{Subst}\left(\int \frac{1}{2a-x^2} dx, x, -\frac{a\sqrt{\sec(c+dx)} \sin(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} - \frac{(2B) \operatorname{Subst}\left(\int \frac{1}{2a-x^2} dx, x, -\frac{a\sqrt{\sec(c+dx)} \sin(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= \frac{2B \sinh^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a} d} + \frac{\sqrt{2}(A-B) \tanh^{-1}\left(\frac{\sqrt{a} \sqrt{\sec(c+dx)} \sin(c+dx)}{\sqrt{2} \sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a} d}$$

Mathematica [A] time = 0.20, size = 95, normalized size = 0.95

$$\frac{\tan(c+dx) \left(\sqrt{2}(B-A) \tan^{-1}\left(\frac{\sqrt{2} \sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right) - 2B \sin^{-1}\left(\sqrt{\sec(c+dx)}\right) \right)}{d \sqrt{1-\sec(c+dx)} \sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x]))/Sqrt[a + a*Sec[c + d*x]], x]

[Out] $((-2*B*ArcSin[Sqrt[Sec[c + d*x]]) + Sqrt[2]*(-A + B)*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]])*Tan[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]])*Sqrt[a*(1 + Sec[c + d*x]))$

fricas [A] time = 0.52, size = 366, normalized size = 3.66

$$\frac{\sqrt{2}(A - B)\sqrt{a} \log\left(\frac{\cos(dx+c)^2 + \frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{\sqrt{a}} - 2\cos(dx+c) - 3}{\cos(dx+c)^2 + 2\cos(dx+c) + 1}\right) - B\sqrt{a} \log\left(\frac{a\cos(dx+c)^3 - 7a\cos(dx+c)^2}{\dots}\right)}{2ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")`

[Out] $[-1/2*(\sqrt{2}*(A - B)*\sqrt{a}*\log(-(\cos(d*x + c))^2 + 2*\sqrt{2}*\sqrt{(a*\cos(d*x + c) + a)/\cos(d*x + c)}*\sqrt{\cos(d*x + c)}*\sin(d*x + c)/\sqrt{a} - 2*\cos(d*x + c) - 3)/(\cos(d*x + c)^2 + 2*\cos(d*x + c) + 1)) - B*\sqrt{a}*\log((a*\cos(d*x + c)^3 - 7*a*\cos(d*x + c)^2 - 4*(\cos(d*x + c)^2 - 2*\cos(d*x + c))*\sqrt{a}*\sqrt{(a*\cos(d*x + c) + a)/\cos(d*x + c)}*\sin(d*x + c)/\sqrt{\cos(d*x + c)}) + 8*a)/(\cos(d*x + c)^3 + \cos(d*x + c)^2)))/(a*d), -(\sqrt{2}*(A - B)*\sqrt{a}*\sqrt{-1/a}*\arctan(\sqrt{2}*\sqrt{(a*\cos(d*x + c) + a)/\cos(d*x + c)}*\sqrt{-1/a}*\sqrt{\cos(d*x + c)})/\sin(d*x + c)) - B*\sqrt{-a}*\arctan(2*\sqrt{-a}*\sqrt{(a*\cos(d*x + c) + a)/\cos(d*x + c)}*\sqrt{\cos(d*x + c)}*\sin(d*x + c)/(a*\cos(d*x + c)^2 - a*\cos(d*x + c) - 2*a)))/(a*d)]$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)\sqrt{\sec(dx + c)}}{\sqrt{a \sec(dx + c) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")`

[Out] `integrate((B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/sqrt(a*sec(d*x + c) + a), x)`

maple [B] time = 2.35, size = 211, normalized size = 2.11

$$\sqrt{\frac{1}{\cos(dx+c)}} \sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}} \cos(dx+c) \left(B\sqrt{2} \arctan\left(\frac{\sqrt{\frac{2}{1+\cos(dx+c)}} (\cos(dx+c)+1+\sin(dx+c))\sqrt{2}}{4}}\right) - B\sqrt{2} \arctan\left(\frac{\sqrt{\frac{2}{1+\cos(dx+c)}} (\cos(dx+c)+1-\sin(dx+c))\sqrt{2}}{4}}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x)

[Out] 1/2/d*(1/cos(d*x+c))^(1/2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)*(B*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d*x+c))*2^(1/2))-B*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))*2^(1/2))+2*A*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))-2*B*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)))*(-2/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^2*(cos(d*x+c)^2-1)/a

maxima [B] time = 1.58, size = 567, normalized size = 5.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")

[Out] 1/2*((sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*A/sqrt(a) - (sqrt(2)*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 1) - sqrt(2)*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 1) - log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2)

$\cos(dx + c))) - 2\sqrt{2}\sin(1/2\arctan2(\sin(dx + c), \cos(dx + c))) + 2))\cdot B/\sqrt{a})/d$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \sqrt{\frac{1}{\cos(c+dx)}}}{\sqrt{a + \frac{a}{\cos(c+dx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + a/cos(c + d*x))^(1/2), x)

[Out] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + a/cos(c + d*x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \sqrt{\sec(c + dx)}}{\sqrt{a (\sec(c + dx) + 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)**(1/2)/(a+a*sec(d*x+c))**(1/2), x)

[Out] Integral((A + B*sec(c + d*x))*sqrt(sec(c + d*x))/sqrt(a*(sec(c + d*x) + 1)), x)

$$3.251 \quad \int \frac{A+B \sec(c+dx)}{\sqrt{\sec(c+dx)} \sqrt{a+a \sec(c+dx)}} dx$$

Optimal. Leaf size=99

$$\frac{2A \sin(c+dx) \sqrt{\sec(c+dx)}}{d \sqrt{a \sec(c+dx) + a}} - \frac{\sqrt{2} (A-B) \tanh^{-1} \left(\frac{\sqrt{a} \sin(c+dx) \sqrt{\sec(c+dx)}}{\sqrt{2} \sqrt{a \sec(c+dx) + a}} \right)}{\sqrt{a} d}$$

[Out] $-(A-B) \operatorname{arctanh}(1/2 \sin(dx+c) a^{1/2} \sec(dx+c)^{1/2} 2^{1/2} / (a+a \sec(dx+c))^{1/2}) 2^{1/2} / d a^{1/2} + 2A \sin(dx+c) \sec(dx+c)^{1/2} / d (a+a \sec(dx+c))^{1/2}$

Rubi [A] time = 0.19, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {4013, 3808, 206}

$$\frac{2A \sin(c+dx) \sqrt{\sec(c+dx)}}{d \sqrt{a \sec(c+dx) + a}} - \frac{\sqrt{2} (A-B) \tanh^{-1} \left(\frac{\sqrt{a} \sin(c+dx) \sqrt{\sec(c+dx)}}{\sqrt{2} \sqrt{a \sec(c+dx) + a}} \right)}{\sqrt{a} d}$$

Antiderivative was successfully verified.

[In] `Int[(A + B*Sec[c + d*x])/(Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]),x]`

[Out] $-\left(\frac{\sqrt{2} (A-B) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{\sec(c+dx)} \sin(c+dx)}{\sqrt{2} \sqrt{a \sec(c+dx) + a}}\right]}{\sqrt{2} \sqrt{a \sec(c+dx) + a}}\right) / (\sqrt{a} d) + (2A \sqrt{\sec(c+dx)} \sin(c+dx)) / (d \sqrt{a \sec(c+dx) + a})$

Rule 206

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])`

Rule 3808

`Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] := Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]`

Rule 4013

`Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] := Simp[(A*Cot[`

$e + f*x]*(a + b*\text{Csc}[e + f*x])^m*(d*\text{Csc}[e + f*x])^n)/(f*n), x] - \text{Dist}[(a*A*m - b*B*n)/(b*d*n), \text{Int}[(a + b*\text{Csc}[e + f*x])^m*(d*\text{Csc}[e + f*x])^{n+1}, x], x] /; \text{FreeQ}[\{a, b, d, e, f, A, B, m, n\}, x] \&\& \text{NeQ}[A*b - a*B, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{EqQ}[m + n + 1, 0] \&\& !\text{LeQ}[m, -1]$

Rubi steps

$$\begin{aligned} \int \frac{A + B \sec(c + dx)}{\sqrt{\sec(c + dx)} \sqrt{a + a \sec(c + dx)}} dx &= \frac{2A\sqrt{\sec(c + dx)} \sin(c + dx)}{d\sqrt{a + a \sec(c + dx)}} + (-A + B) \int \frac{\sqrt{\sec(c + dx)}}{\sqrt{a + a \sec(c + dx)}} dx \\ &= \frac{2A\sqrt{\sec(c + dx)} \sin(c + dx)}{d\sqrt{a + a \sec(c + dx)}} + \frac{(2(A - B)) \text{Subst}\left(\int \frac{1}{2a-x^2} dx, x, -\frac{a\sqrt{\sec(c + dx)}}{\sqrt{a + a \sec(c + dx)}}\right)}{d} \\ &= -\frac{\sqrt{2}(A - B) \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c + dx)} \sin(c + dx)}{\sqrt{2}\sqrt{a + a \sec(c + dx)}}\right)}{\sqrt{a}d} + \frac{2A\sqrt{\sec(c + dx)} \sin(c + dx)}{d\sqrt{a + a \sec(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.27, size = 114, normalized size = 1.15

$$\frac{\tan(c + dx) \left(\sqrt{2}(A - B)\sqrt{\sec(c + dx)} \tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sec(c + dx)}}{\sqrt{1 - \sec(c + dx)}}\right) + 2A\sqrt{1 - \sec(c + dx)} \right)}{d\sqrt{-((\sec(c + dx) - 1)\sec(c + dx))} \sqrt{a(\sec(c + dx) + 1)}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(A + B*Sec[c + d*x])/(Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]), x]

[Out] ((2*A*Sqrt[1 - Sec[c + d*x]] + Sqrt[2]*(A - B)*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]])*Sqrt[Sec[c + d*x]]*Tan[c + d*x])/(d*Sqrt[(-(-1 + Sec[c + d*x])*Sec[c + d*x])*Sqrt[a*(1 + Sec[c + d*x])]])

fricas [A] time = 0.47, size = 306, normalized size = 3.09

$$\left[\frac{4A\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}} \sqrt{\cos(dx+c)} \sin(dx+c) - \frac{\sqrt{2}((A-B)a\cos(dx+c)+(A-B)a) \log\left(\frac{\cos(dx+c)^2 - \frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}} \sqrt{\cos(dx+c)} \sin(dx+c)}{\sqrt{a}}}{\cos(dx+c)^2 + 2\cos(dx+c) + 1}\right)}{\sqrt{a}}}{2(ad\cos(dx+c) + ad)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")

[Out] [1/2*(4*A*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - sqrt(2)*((A - B)*a*cos(d*x + c) + (A - B)*a)*log(-(cos(d*x + c))^2 - 2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/sqrt(a)/(a*d*cos(d*x + c) + a*d), (sqrt(2)*((A - B)*a*cos(d*x + c) + (A - B)*a)*sqrt(-1/a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*sqrt(cos(d*x + c))/sin(d*x + c)) + 2*A*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{\sqrt{a \sec(dx + c) + a} \sqrt{\sec(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)/(sqrt(a*sec(d*x + c) + a)*sqrt(sec(d*x + c))), x)

maple [A] time = 2.37, size = 150, normalized size = 1.52

$$\frac{\left(\arctan \left(\frac{\sin(dx+c) \sqrt{-\frac{2}{1+\cos(dx+c)}}}{2} \right) \sqrt{-\frac{2}{1+\cos(dx+c)}} A \sin(dx+c) - \arctan \left(\frac{\sin(dx+c) \sqrt{-\frac{2}{1+\cos(dx+c)}}}{2} \right) \sqrt{-\frac{2}{1+\cos(dx+c)}} B \sin(dx+c) \right)}{d \sqrt{\frac{1}{\cos(dx+c)}} \sin(dx+c) a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x)

[Out] 1/d*(arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c))))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*A*sin(d*x+c)-arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c))))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*B*sin(d*x+c)-2*A*cos(d*x+c)+2*A)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/(1/cos(d*x+c))^(1/2)/sin(d*x+c)/a

maxima [B] time = 1.36, size = 195, normalized size = 1.97

$$\frac{\left(\sqrt{2} \log \left(\cos \left(\frac{1}{2} dx + \frac{1}{2} c \right)^2 + \sin \left(\frac{1}{2} dx + \frac{1}{2} c \right)^2 + 2 \sin \left(\frac{1}{2} dx + \frac{1}{2} c \right) + 1 \right) - \sqrt{2} \log \left(\cos \left(\frac{1}{2} dx + \frac{1}{2} c \right)^2 + \sin \left(\frac{1}{2} dx + \frac{1}{2} c \right)^2 - 2 \sin \left(\frac{1}{2} dx + \frac{1}{2} c \right) + 1 \right) - 4 \sqrt{2} \sin \left(\frac{1}{2} dx + \frac{1}{2} c \right) \right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")

[Out]
$$-1/2*((\sqrt{2})\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 4*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*A/\sqrt{a} - (\sqrt{2})\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*B/\sqrt{a})/d$$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\sqrt{a + \frac{a}{\cos(c+dx)}} \sqrt{\frac{1}{\cos(c+dx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/((a + a/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/2)),x)

[Out] int((A + B/cos(c + d*x))/((a + a/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \sec(c + dx)}{\sqrt{a (\sec(c + dx) + 1)} \sqrt{\sec(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)**(1/2)/(a+a*sec(d*x+c))**(1/2),x)

[Out] Integral((A + B*sec(c + d*x))/(sqrt(a*(sec(c + d*x) + 1))*sqrt(sec(c + d*x))), x)

$$3.252 \quad \int \frac{A+B \sec(c+dx)}{\sec^2(c+dx) \sqrt{a+a \sec(c+dx)}} dx$$

Optimal. Leaf size=142

$$-\frac{2(A-3B) \sin(c+dx) \sqrt{\sec(c+dx)}}{3d \sqrt{a \sec(c+dx)+a}} + \frac{\sqrt{2}(A-B) \tanh^{-1}\left(\frac{\sqrt{a} \sin(c+dx) \sqrt{\sec(c+dx)}}{\sqrt{2} \sqrt{a \sec(c+dx)+a}}\right)}{\sqrt{a} d} + \frac{2A \sin(c+dx)}{3d \sqrt{\sec(c+dx)} \sqrt{a \sec(c+dx)}}$$

[Out] (A-B)*arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2)/d/a^(1/2)+2/3*A*sin(d*x+c)/d/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)-2/3*(A-3*B)*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^(1/2)

Rubi [A] time = 0.33, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {4022, 4013, 3808, 206}

$$-\frac{2(A-3B) \sin(c+dx) \sqrt{\sec(c+dx)}}{3d \sqrt{a \sec(c+dx)+a}} + \frac{\sqrt{2}(A-B) \tanh^{-1}\left(\frac{\sqrt{a} \sin(c+dx) \sqrt{\sec(c+dx)}}{\sqrt{2} \sqrt{a \sec(c+dx)+a}}\right)}{\sqrt{a} d} + \frac{2A \sin(c+dx)}{3d \sqrt{\sec(c+dx)} \sqrt{a \sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]),x]

[Out] (Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(Sqrt[a]*d) + (2*A*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) - (2*(A - 3*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 3808

Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] := Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 4013


```
Int[(csc[e_] + (f_)*(x_))*(d_)^(n_)*(csc[e_] + (f_)*(x_))*(b_) + (
a_)^(m_)*(csc[e_] + (f_)*(x_))*(B_) + (A_)), x_Symbol] := Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
- b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] && !LeQ[m, -1]
```

Rule 4022

```
Int[(csc[e_] + (f_)*(x_))*(d_)^(n_)*(csc[e_] + (f_)*(x_))*(b_) + (
a_)^(m_)*(csc[e_] + (f_)*(x_))*(B_) + (A_)), x_Symbol] := Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
- A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{A + B \sec(c + dx)}{\sec^2(c + dx) \sqrt{a + a \sec(c + dx)}} dx &= \frac{2A \sin(c + dx)}{3d \sqrt{\sec(c + dx)} \sqrt{a + a \sec(c + dx)}} + \frac{2 \int \frac{-\frac{1}{2}a(A-3B) + aA \sec(c+dx)}{\sqrt{\sec(c+dx)} \sqrt{a+a \sec(c+dx)}} dx}{3a} \\ &= \frac{2A \sin(c + dx)}{3d \sqrt{\sec(c + dx)} \sqrt{a + a \sec(c + dx)}} - \frac{2(A - 3B) \sqrt{\sec(c + dx)} \sin(c + dx)}{3d \sqrt{a + a \sec(c + dx)}} \\ &= \frac{2A \sin(c + dx)}{3d \sqrt{\sec(c + dx)} \sqrt{a + a \sec(c + dx)}} - \frac{2(A - 3B) \sqrt{\sec(c + dx)} \sin(c + dx)}{3d \sqrt{a + a \sec(c + dx)}} \\ &= \frac{\sqrt{2}(A - B) \tanh^{-1}\left(\frac{\sqrt{a} \sqrt{\sec(c+dx)} \sin(c+dx)}{\sqrt{2} \sqrt{a+a \sec(c+dx)}}\right)}{\sqrt{a} d} + \frac{2A \sin(c + dx)}{3d \sqrt{\sec(c + dx)} \sqrt{a + a \sec(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.39, size = 132, normalized size = 0.93

$$\frac{\tan(c + dx) \left(2\sqrt{1 - \sec(c + dx)} (A \cos(c + dx) - A + 3B) - 3\sqrt{2} (A - B) \sqrt{\sec(c + dx)} \tan^{-1} \left(\frac{\sqrt{2} \sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}} \right) \right)}{3d \sqrt{-((\sec(c + dx) - 1) \sec(c + dx))} \sqrt{a(\sec(c + dx) + 1)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]
), x]
```

[Out] $((2*(-A + 3*B + A*\cos[c + d*x])*Sqrt[1 - \text{Sec}[c + d*x]] - 3*Sqrt[2]*(A - B)*ArcTan[(Sqrt[2]*Sqrt[\text{Sec}[c + d*x]])/Sqrt[1 - \text{Sec}[c + d*x]]]*Sqrt[\text{Sec}[c + d*x]])*Tan[c + d*x])/(3*d*Sqrt[-((-1 + \text{Sec}[c + d*x])*Sec[c + d*x])]*Sqrt[a*(1 + \text{Sec}[c + d*x])])$

fricas [A] time = 0.46, size = 354, normalized size = 2.49

$$\frac{3\sqrt{2}((A-B)a\cos(dx+c)+(A-B)a)\log\left(\frac{\cos(dx+c)^2 + \frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{\sqrt{a}} - 2\cos(dx+c) - 3}{\cos(dx+c)^2 + 2\cos(dx+c) + 1}\right)}{\sqrt{a}} - \frac{4(A\cos(dx+c)^2 - (A-3B)\cos(dx+c))}{\sqrt{\cos(dx+c)}}$$

$$6(ad\cos(dx+c) + ad)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")`

[Out] $[-1/6*(3*\sqrt{2})*((A - B)*a*\cos(d*x + c) + (A - B)*a)*\log(-(\cos(d*x + c))^2 + 2*\sqrt{2}*\sqrt{(a*\cos(d*x + c) + a)/\cos(d*x + c)}*\sqrt{\cos(d*x + c)}*\sin(d*x + c)/\sqrt{a} - 2*\cos(d*x + c) - 3)/(\cos(d*x + c)^2 + 2*\cos(d*x + c) + 1))/\sqrt{a} - 4*(A*\cos(d*x + c)^2 - (A - 3*B)*\cos(d*x + c))*\sqrt{(a*\cos(d*x + c) + a)/\cos(d*x + c)}*\sin(d*x + c)/\sqrt{\cos(d*x + c)}}/(a*d*\cos(d*x + c) + a*d), -1/3*(3*\sqrt{2})*((A - B)*a*\cos(d*x + c) + (A - B)*a)*\sqrt{-1/a}*\arctan(\sqrt{2}*\sqrt{(a*\cos(d*x + c) + a)/\cos(d*x + c)}*\sqrt{-1/a}*\sqrt{\cos(d*x + c)})/\sin(d*x + c) - 2*(A*\cos(d*x + c)^2 - (A - 3*B)*\cos(d*x + c))*\sqrt{(a*\cos(d*x + c) + a)/\cos(d*x + c)}*\sin(d*x + c)/\sqrt{\cos(d*x + c)}}/(a*d*\cos(d*x + c) + a*d)]$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{\sqrt{a \sec(dx + c) + a} \sec(dx + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")`

[Out] `integrate((B*sec(d*x + c) + A)/(sqrt(a*sec(d*x + c) + a)*sec(d*x + c)^(3/2)), x)`

maple [A] time = 2.52, size = 183, normalized size = 1.29

$$\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}} \left(3 \arctan \left(\frac{\sin(dx+c) \sqrt{-\frac{2}{1+\cos(dx+c)}}}{2} \right) \sqrt{-\frac{2}{1+\cos(dx+c)}} A \sin(dx+c) - 3 \arctan \left(\frac{\sin(dx+c) \sqrt{-\frac{2}{1+\cos(dx+c)}}}{2} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x)

[Out] $-1/3/d*(a*(1+\cos(d*x+c))/\cos(d*x+c))^{(1/2)}*(3*\arctan(1/2*\sin(d*x+c))*(-2/(1+\cos(d*x+c)))^{(1/2)})*(-2/(1+\cos(d*x+c)))^{(1/2)}*A*\sin(d*x+c)-3*\arctan(1/2*\sin(d*x+c))*(-2/(1+\cos(d*x+c)))^{(1/2)})*(-2/(1+\cos(d*x+c)))^{(1/2)}*B*\sin(d*x+c)+2*A*\cos(d*x+c)^2-4*A*\cos(d*x+c)+6*B*\cos(d*x+c)+2*A-6*B)*\cos(d*x+c)^2*(1/\cos(d*x+c))^{(3/2)}/\sin(d*x+c)/a$

maxima [B] time = 1.23, size = 387, normalized size = 2.73

$$\left(3 \sqrt{2} \cos\left(\frac{2}{3} \arctan\left(\sin\left(\frac{3}{2} dx + \frac{3}{2} c\right), \cos\left(\frac{3}{2} dx + \frac{3}{2} c\right)\right)\right) \sin\left(\frac{3}{2} dx + \frac{3}{2} c\right) - 3 \sqrt{2} \cos\left(\frac{3}{2} dx + \frac{3}{2} c\right) \sin\left(\frac{2}{3} \arctan\left(\sin\left(\frac{3}{2} dx + \frac{3}{2} c\right), \cos\left(\frac{3}{2} dx + \frac{3}{2} c\right)\right)\right) - 3 \sqrt{2} \log\left(\cos\left(\frac{1}{3} \arctan\left(\sin\left(\frac{3}{2} dx + \frac{3}{2} c\right), \cos\left(\frac{3}{2} dx + \frac{3}{2} c\right)\right)\right) + 1\right) \right) / d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")

[Out] $-1/6*((3*\sqrt{2})*\cos(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) * \sin(3/2*d*x + 3/2*c) - 3*\sqrt{2}*\cos(3/2*d*x + 3/2*c)*\sin(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) - 3*\sqrt{2}*\log(\cos(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + \sin(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + 2*\sin(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 1) + 3*\sqrt{2}*\log(\cos(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + \sin(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 - 2*\sin(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 1) - 2*\sqrt{2}*\sin(3/2*d*x + 3/2*c) + 3*\sqrt{2}*\sin(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))))*A/\sqrt{a} + 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 4*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*B/\sqrt{a))/d$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\sqrt{a + \frac{a}{\cos(c+dx)} \left(\frac{1}{\cos(c+dx)} \right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/((a + a/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(3/2)), x)
```

```
[Out] int((A + B/cos(c + d*x))/((a + a/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \sec(c + dx)}{\sqrt{a(\sec(c + dx) + 1)} \sec^{\frac{3}{2}}(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)**(3/2)/(a+a*sec(d*x+c))**(1/2), x)
```

```
[Out] Integral((A + B*sec(c + d*x))/(sqrt(a*(sec(c + d*x) + 1))*sec(c + d*x)**(3/2)), x)
```

$$3.253 \quad \int \frac{A+B \sec(c+dx)}{\sec^2(c+dx) \sqrt{a+a \sec(c+dx)}} dx$$

Optimal. Leaf size=187

$$\frac{2(13A - 5B) \sin(c + dx) \sqrt{\sec(c + dx)}}{15d \sqrt{a \sec(c + dx) + a}} - \frac{2(A - 5B) \sin(c + dx)}{15d \sqrt{\sec(c + dx)} \sqrt{a \sec(c + dx) + a}} - \frac{\sqrt{2} (A - B) \tanh^{-1} \left(\frac{\sqrt{a} \sin(c+dx) \sqrt{\sec(c+dx)}}{\sqrt{2} \sqrt{a \sec(c+dx) + a}} \right)}{\sqrt{a} d}$$

[Out] $-(A-B) \operatorname{arctanh}(1/2 \sin(dx+c)) a^{1/2} \sec(dx+c)^{1/2} 2^{1/2} / (a+a \sec(dx+c))^{1/2} 2^{1/2} / d a^{1/2} + 2/5 A \sin(dx+c) / d \sec(dx+c)^{3/2} / (a+a \sec(dx+c))^{1/2} - 2/15 (A-5B) \sin(dx+c) / d \sec(dx+c)^{1/2} / (a+a \sec(dx+c))^{1/2} + 2/15 (13A-5B) \sin(dx+c) \sec(dx+c)^{1/2} / d (a+a \sec(dx+c))^{1/2}$

Rubi [A] time = 0.51, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {4022, 4013, 3808, 206}

$$\frac{2(13A - 5B) \sin(c + dx) \sqrt{\sec(c + dx)}}{15d \sqrt{a \sec(c + dx) + a}} - \frac{2(A - 5B) \sin(c + dx)}{15d \sqrt{\sec(c + dx)} \sqrt{a \sec(c + dx) + a}} - \frac{\sqrt{2} (A - B) \tanh^{-1} \left(\frac{\sqrt{a} \sin(c+dx) \sqrt{\sec(c+dx)}}{\sqrt{2} \sqrt{a \sec(c+dx) + a}} \right)}{\sqrt{a} d}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]),x]

[Out] $-((\operatorname{Sqrt}[2] * (A - B) * \operatorname{ArcTanh}[(\operatorname{Sqrt}[a] * \operatorname{Sqrt}[\operatorname{Sec}[c + d*x]] * \operatorname{Sin}[c + d*x]) / (\operatorname{Sqrt}[2] * \operatorname{Sqrt}[a + a * \operatorname{Sec}[c + d*x]])]) / (\operatorname{Sqrt}[a] * d)) + (2 * A * \operatorname{Sin}[c + d*x]) / (5 * d * \operatorname{Sec}[c + d*x]^{3/2} * \operatorname{Sqrt}[a + a * \operatorname{Sec}[c + d*x]]) - (2 * (A - 5 * B) * \operatorname{Sin}[c + d*x]) / (15 * d * \operatorname{Sqrt}[\operatorname{Sec}[c + d*x]] * \operatorname{Sqrt}[a + a * \operatorname{Sec}[c + d*x]]) + (2 * (13 * A - 5 * B) * \operatorname{Sqrt}[\operatorname{Sec}[c + d*x]] * \operatorname{Sin}[c + d*x]) / (15 * d * \operatorname{Sqrt}[a + a * \operatorname{Sec}[c + d*x]])$

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 3808

Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_)), x_Symbol] := Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && !LeQ[m, -1]
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_)), x_Symbol] := Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{A + B \sec(c + dx)}{\sec^{\frac{5}{2}}(c + dx) \sqrt{a + a \sec(c + dx)}} dx = \frac{2A \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx) \sqrt{a + a \sec(c + dx)}} + \frac{2 \int \frac{-\frac{1}{2}a(A-5B)+2aA \sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx) \sqrt{a+a \sec(c+dx)}} dx}{5a}$$

$$= \frac{2A \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx) \sqrt{a + a \sec(c + dx)}} - \frac{2(A - 5B) \sin(c + dx)}{15d \sqrt{\sec(c + dx)} \sqrt{a + a \sec(c + dx)}}$$

$$= \frac{2A \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx) \sqrt{a + a \sec(c + dx)}} - \frac{2(A - 5B) \sin(c + dx)}{15d \sqrt{\sec(c + dx)} \sqrt{a + a \sec(c + dx)}}$$

$$= \frac{2A \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx) \sqrt{a + a \sec(c + dx)}} - \frac{2(A - 5B) \sin(c + dx)}{15d \sqrt{\sec(c + dx)} \sqrt{a + a \sec(c + dx)}}$$

$$= -\frac{\sqrt{2}(A - B) \tanh^{-1}\left(\frac{\sqrt{a} \sqrt{\sec(c+dx)} \sin(c+dx)}{\sqrt{2} \sqrt{a+a \sec(c+dx)}}\right)}{\sqrt{a} d} + \frac{2A \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx) \sqrt{a + a \sec(c + dx)}}$$

Mathematica [A] time = 1.19, size = 133, normalized size = 0.71

$$\frac{15\sqrt{2}(A-B)\tan(c+dx)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)}{\sqrt{1-\sec(c+dx)}} + \frac{\sin(c+dx)\sqrt{\sec(c+dx)}(-2(A-5B)\cos(c+dx) + 3A\cos(2(c+dx)) + 2)}{15d\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]), x]

[Out] ((29*A - 10*B - 2*(A - 5*B)*Cos[c + d*x] + 3*A*Cos[2*(c + d*x)])*Sqrt[Sec[c + d*x]]*Sin[c + d*x] + (15*Sqrt[2]*(A - B)*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Tan[c + d*x])/Sqrt[1 - Sec[c + d*x]])/(15*d*Sqrt[a*(1 + Sec[c + d*x])])

fricas [A] time = 0.46, size = 388, normalized size = 2.07

$$\frac{15\sqrt{2}((A-B)a\cos(dx+c)+(A-B)a)\log\left(\frac{\cos(dx+c)^2 - \frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{\sqrt{a}} - 2\cos(dx+c) - 3}{\cos(dx+c)^2 + 2\cos(dx+c) + 1}\right)}{\sqrt{a}} - \frac{4(3A\cos(dx+c)^3 - (A-5B)\cos(dx+c))}{30(ad\cos(dx+c) + ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2), x, algorithm="fricas")

[Out] [-1/30*(15*sqrt(2)*((A - B)*a*cos(d*x + c) + (A - B)*a)*log(-(cos(d*x + c))^2 - 2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/sqrt(a) - 4*(3*A*cos(d*x + c)^3 - (A - 5*B)*cos(d*x + c)^2 + (13*A - 5*B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c) + a*d), 1/15*(15*sqrt(2)*((A - B)*a*cos(d*x + c) + (A - B)*a)*sqrt(-1/a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*sqrt(cos(d*x + c))/sin(d*x + c)) + 2*(3*A*cos(d*x + c)^3 - (A - 5*B)*cos(d*x + c)^2 + (13*A - 5*B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c) + a*d)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{\sqrt{a \sec(dx + c) + a} \sec(dx + c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)/(sqrt(a*sec(d*x + c) + a)*sec(d*x + c)^(5/2)), x)

maple [A] time = 2.67, size = 205, normalized size = 1.10

$$\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}} \left(6A \left(\cos^3(dx+c) \right) - 15 \arctan \left(\frac{\sin(dx+c) \sqrt{\frac{2}{1+\cos(dx+c)}}}{2} \right) \right) \sqrt{\frac{2}{1+\cos(dx+c)}} A \sin(dx+c) + 15 \arctan$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x)

[Out] -1/15/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(6*A*cos(d*x+c)^3-15*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*A*sin(d*x+c)+15*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*B*sin(d*x+c)-8*A*cos(d*x+c)^2+10*B*cos(d*x+c)^2+28*A*cos(d*x+c)-20*B*cos(d*x+c)-26*A+10*B)*cos(d*x+c)^3*(1/cos(d*x+c))^(5/2)/sin(d*x+c)/a

maxima [B] time = 1.39, size = 640, normalized size = 3.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")

[Out] 1/60*(sqrt(2)*(60*cos(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) * sin(5/2*d*x + 5/2*c) - 5*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) * sin(5/2*d*x + 5/2*c) - 60*cos(5/2*d*x + 5/2*c) * sin(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 5*cos(5/2*d*x + 5/2*c) * sin(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) - 30*log(cos(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + 2*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + 2*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2


```
*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 1) + 30*log(cos(1/5*arctan2(sin(5/2*d
*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + sin(1/5*arctan2(sin(5/2*d*x + 5/2*c
), cos(5/2*d*x + 5/2*c)))^2 - 2*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5
/2*d*x + 5/2*c))) + 1) + 6*sin(5/2*d*x + 5/2*c) - 5*sin(3/5*arctan2(sin(5/2
*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 60*sin(1/5*arctan2(sin(5/2*d*x + 5/
2*c), cos(5/2*d*x + 5/2*c))))*A/sqrt(a) - 10*(3*sqrt(2)*cos(2/3*arctan2(sin
(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(3/2*d*x + 3/2*c) - 3*sqrt(2)*
cos(3/2*d*x + 3/2*c)*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/
2*c))) - 3*sqrt(2)*log(cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/
2*c)))^2 + sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2
+ 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) + 3*
sqrt(2)*log(cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2
+ sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sin(1/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) - 2*sqrt(2)*sin
(3/2*d*x + 3/2*c) + 3*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c))))*B/sqrt(a))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\sqrt{a + \frac{a}{\cos(c+dx)}} \left(\frac{1}{\cos(c+dx)}\right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/((a + a/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(5/2)
),x)
```

```
[Out] int((A + B/cos(c + d*x))/((a + a/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(5/2)
), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)**(5/2)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

$$3.254 \quad \int \frac{A+B \sec(c+dx)}{\sec^2(c+dx) \sqrt{a+a \sec(c+dx)}} dx$$

Optimal. Leaf size=230

$$\frac{2(A-7B) \sin(c+dx)}{35d \sec^2(c+dx) \sqrt{a \sec(c+dx)} + a} - \frac{2(43A-91B) \sin(c+dx) \sqrt{\sec(c+dx)}}{105d \sqrt{a \sec(c+dx)} + a} + \frac{2(31A-7B) \sin(c+dx)}{105d \sqrt{\sec(c+dx)} \sqrt{a \sec(c+dx)}}$$

[Out] (A-B)*arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2)/d/a^(1/2)+2/7*A*sin(d*x+c)/d/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2)-2/35*(A-7*B)*sin(d*x+c)/d/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2)+2/105*(31*A-7*B)*sin(d*x+c)/d/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)-2/105*(43*A-91*B)*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^(1/2)

Rubi [A] time = 0.69, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {4022, 4013, 3808, 206}

$$\frac{2(A-7B) \sin(c+dx)}{35d \sec^2(c+dx) \sqrt{a \sec(c+dx)} + a} - \frac{2(43A-91B) \sin(c+dx) \sqrt{\sec(c+dx)}}{105d \sqrt{a \sec(c+dx)} + a} + \frac{2(31A-7B) \sin(c+dx)}{105d \sqrt{\sec(c+dx)} \sqrt{a \sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]),x]

[Out] (Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(Sqrt[a]*d) + (2*A*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) - (2*(A - 7*B)*Sin[c + d*x])/(35*d*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (2*(31*A - 7*B)*Sin[c + d*x])/(105*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) - (2*(43*A - 91*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(105*d*Sqrt[a + a*Sec[c + d*x]])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 3808

Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] := Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;

FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 4013

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && !LeQ[m, -1]

Rule 4022

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]

Rubi steps

$$\begin{aligned}
\int \frac{A + B \sec(c + dx)}{\sec^{\frac{7}{2}}(c + dx) \sqrt{a + a \sec(c + dx)}} dx &= \frac{2A \sin(c + dx)}{7d \sec^{\frac{5}{2}}(c + dx) \sqrt{a + a \sec(c + dx)}} + \frac{2 \int \frac{-\frac{1}{2}a(A-7B)+3aA \sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx) \sqrt{a+a \sec(c+dx)}} dx}{7a} \\
&= \frac{2A \sin(c + dx)}{7d \sec^{\frac{5}{2}}(c + dx) \sqrt{a + a \sec(c + dx)}} - \frac{2(A - 7B) \sin(c + dx)}{35d \sec^{\frac{3}{2}}(c + dx) \sqrt{a + a \sec(c + dx)}} \\
&= \frac{2A \sin(c + dx)}{7d \sec^{\frac{5}{2}}(c + dx) \sqrt{a + a \sec(c + dx)}} - \frac{2(A - 7B) \sin(c + dx)}{35d \sec^{\frac{3}{2}}(c + dx) \sqrt{a + a \sec(c + dx)}} \\
&= \frac{2A \sin(c + dx)}{7d \sec^{\frac{5}{2}}(c + dx) \sqrt{a + a \sec(c + dx)}} - \frac{2(A - 7B) \sin(c + dx)}{35d \sec^{\frac{3}{2}}(c + dx) \sqrt{a + a \sec(c + dx)}} \\
&= \frac{2A \sin(c + dx)}{7d \sec^{\frac{5}{2}}(c + dx) \sqrt{a + a \sec(c + dx)}} - \frac{2(A - 7B) \sin(c + dx)}{35d \sec^{\frac{3}{2}}(c + dx) \sqrt{a + a \sec(c + dx)}} \\
&= \frac{\sqrt{2} (A - B) \tanh^{-1} \left(\frac{\sqrt{a} \sqrt{\sec(c+dx)} \sin(c+dx)}{\sqrt{2} \sqrt{a+a \sec(c+dx)}} \right)}{\sqrt{a} d} + \frac{2A \sin(c + dx)}{7d \sec^{\frac{5}{2}}(c + dx) \sqrt{a + a \sec(c + dx)}}
\end{aligned}$$

Mathematica [A] time = 1.64, size = 152, normalized size = 0.66

$$\frac{2 \sin(c+dx) \left((43A-91B) \sec^3(c+dx) + (7B-31A) \sec^2(c+dx) + 3(A-7B) \sec(c+dx) - 15A \right)}{\sec^{\frac{5}{2}}(c+dx)} - \frac{105 \sqrt{2} (A-B) \tan(c+dx) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}} \right)}{\sqrt{1-\sec(c+dx)}}}{105d \sqrt{a(\sec(c+dx) + 1)}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]), x]

[Out] ((-2*(-15*A + 3*(A - 7*B)*Sec[c + d*x] + (-31*A + 7*B)*Sec[c + d*x]^2 + (43*A - 91*B)*Sec[c + d*x]^3)*Sin[c + d*x])/Sec[c + d*x]^(5/2) - (105*Sqrt[2]*(A - B)*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]])*Tan[c + d*x])/Sqrt[1 - Sec[c + d*x]]/(105*d*Sqrt[a*(1 + Sec[c + d*x])])

fricas [A] time = 0.48, size = 422, normalized size = 1.83

$$\frac{105 \sqrt{2} ((A-B)a \cos(dx+c) + (A-B)a) \log \left(\frac{\cos(dx+c)^2 + \frac{2\sqrt{2} \sqrt{\frac{a \cos(dx+c)+a}{\cos(dx+c)}} \sqrt{\cos(dx+c)} \sin(dx+c)}{\sqrt{a}} - 2 \cos(dx+c) - 3}{\cos(dx+c)^2 + 2 \cos(dx+c) + 1} \right)}{\sqrt{a}} - \frac{4(15A \cos(dx+c)^4 - 3(A-7B) \cos(dx+c)^3 + (31A - 7B) \cos(dx+c)^2 - (43A - 91B) \cos(dx+c)) \sqrt{\frac{a \cos(dx+c)+a}{\cos(dx+c)}} \sin(dx+c) / \sqrt{\cos(dx+c)}}{210(ad \cos(dx+c) + ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")

[Out] [-1/210*(105*sqrt(2)*((A - B)*a*cos(d*x + c) + (A - B)*a)*log(-(cos(d*x + c))^2 + 2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/sqrt(a) - 4*(15*A*cos(d*x + c)^4 - 3*(A - 7*B)*cos(d*x + c)^3 + (31*A - 7*B)*cos(d*x + c)^2 - (43*A - 91*B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c) + a*d), -1/105*(105*sqrt(2)*((A - B)*a*cos(d*x + c) + (A - B)*a)*sqrt(-1/a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*sqrt(cos(d*x + c))/sin(d*x + c)) - 2*(15*A*cos(d*x + c)^4 - 3*(A - 7*B)*cos(d*x + c)^3 + (31*A - 7*B)*cos(d*x + c)^2 - (43*A - 91*B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c) + a*d)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{\sqrt{a \sec(dx + c) + a} \sec(dx + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)/(sqrt(a*sec(d*x + c) + a)*sec(d*x + c)^(7/2)), x)

maple [A] time = 2.84, size = 227, normalized size = 0.99

$$\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}} \left(30A \left(\cos^4(dx+c) \right) + 105 \arctan \left(\frac{\sin(dx+c) \sqrt{\frac{2}{1+\cos(dx+c)}}}{2} \right) \right) \sqrt{\frac{2}{1+\cos(dx+c)}} A \sin(dx+c) - 36A(c)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))/sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(1/2),x)

[Out] -1/105/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(30*A*cos(d*x+c)^4+105*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c))))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*A*sin(d*x+c)-36*A*cos(d*x+c)^3-105*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c))))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*B*sin(d*x+c)+42*B*cos(d*x+c)^3+68*A*cos(d*x+c)^2-56*B*cos(d*x+c)^2-148*A*cos(d*x+c)+196*B*cos(d*x+c)+86*A-182*B)*cos(d*x+c)^4*(1/cos(d*x+c))^(7/2)/sin(d*x+c)/a

maxima [B] time = 2.13, size = 805, normalized size = 3.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")

[Out] -1/840*(sqrt(2)*(525*cos(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))))*sin(7/2*d*x + 7/2*c) - 175*cos(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) + 21*cos(2/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) - 525*cos(7/2*d*x + 7/2*c)*sin(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))) + 175*cos(7/2*d*x + 7/2*c)*sin(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))) - 21*cos(7/2*d*x + 7/2*c)*sin(2/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))) - 420*log(cos(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))))^2 + sin(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))))^2 + 2*sin(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))) + 1) + 420*log(cos(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))))^2 + sin(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))))^2 - 2*sin(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))) + 1) - 30*sin(7/2*d*x + 7/2*c) + 21*sin(5/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))) - 175*sin(3/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))) + 525*sin(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))))*A/sqrt(a) - 14*sqrt(2)*(60*cos(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))))*sin(5/2*d*x + 5/2*c) - 5*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))))*sin(5/2*d*x + 5/2*c) - 60*cos(5/2*d*x + 5/2*c)*sin(4/5*arctan2(sin(

$5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) + 5*\cos(5/2*d*x + 5/2*c)*\sin(2/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) - 30*\log(\cos(1/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))))^2 + \sin(1/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c)))^2 + 2*\sin(1/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) + 1) + 30*\log(\cos(1/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))))^2 + \sin(1/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))))^2 - 2*\sin(1/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) + 1) + 6*\sin(5/2*d*x + 5/2*c) - 5*\sin(3/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) + 60*\sin(1/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))))*B/\sqrt{a})/d$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\sqrt{a + \frac{a}{\cos(c+dx)} \left(\frac{1}{\cos(c+dx)}\right)^{7/2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/((a + a/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(7/2)), x)

[Out] int((A + B/cos(c + d*x))/((a + a/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(7/2)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)**(7/2)/(a+a*sec(d*x+c))**(1/2), x)

[Out] Timed out

$$3.255 \quad \int \frac{\sec^2(c+dx)(A+B \sec(c+dx))}{(a+a \sec(c+dx))^{3/2}} dx$$

Optimal. Leaf size=247

$$\frac{(9A - 13B) \tanh^{-1} \left(\frac{\sqrt{a} \sin(c+dx) \sqrt{\sec(c+dx)}}{\sqrt{2} \sqrt{a \sec(c+dx)+a}} \right)}{2\sqrt{2} a^{3/2} d} - \frac{(12A - 19B) \sinh^{-1} \left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}} \right)}{4a^{3/2} d} + \frac{(A - B) \sin(c + dx) \sec^2(c + dx)^{7/2}}{2d(a \sec(c + dx) + a)^{3/2}}$$

[Out] $-1/4*(12*A-19*B)*\operatorname{arcsinh}(a^{1/2}*\tan(d*x+c)/(a+a*\sec(d*x+c))^{1/2})/a^{3/2}/d+1/2*(A-B)*\sec(d*x+c)^{7/2}*\sin(d*x+c)/d/(a+a*\sec(d*x+c))^{3/2}+1/4*(9*A-13*B)*\operatorname{arctanh}(1/2*\sin(d*x+c)*a^{1/2}*\sec(d*x+c)^{1/2}*2^{1/2}/(a+a*\sec(d*x+c))^{1/2})/a^{3/2}/d*2^{1/2}+1/4*(6*A-7*B)*\sec(d*x+c)^{3/2}*\sin(d*x+c)/a/d/(a+a*\sec(d*x+c))^{1/2}-1/2*(A-2*B)*\sec(d*x+c)^{5/2}*\sin(d*x+c)/a/d/(a+a*\sec(d*x+c))^{1/2}$

Rubi [A] time = 0.78, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4019, 4021, 4023, 3808, 206, 3801, 215}

$$\frac{(9A - 13B) \tanh^{-1} \left(\frac{\sqrt{a} \sin(c+dx) \sqrt{\sec(c+dx)}}{\sqrt{2} \sqrt{a \sec(c+dx)+a}} \right)}{2\sqrt{2} a^{3/2} d} - \frac{(12A - 19B) \sinh^{-1} \left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}} \right)}{4a^{3/2} d} + \frac{(A - B) \sin(c + dx) \sec^2(c + dx)^{7/2}}{2d(a \sec(c + dx) + a)^{3/2}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(\operatorname{Sec}[c + d*x]^{7/2}*(A + B*\operatorname{Sec}[c + d*x]))/(a + a*\operatorname{Sec}[c + d*x]^{3/2}), x]$

[Out] $-((12*A - 19*B)*\operatorname{ArcSinh}[(\operatorname{Sqrt}[a]*\operatorname{Tan}[c + d*x])/(\operatorname{Sqrt}[a + a*\operatorname{Sec}[c + d*x]])])/(4*a^{3/2}*d) + ((9*A - 13*B)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Sqrt}[\operatorname{Sec}[c + d*x]]*\operatorname{Sin}[c + d*x])/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a + a*\operatorname{Sec}[c + d*x]])])/(2*\operatorname{Sqrt}[2]*a^{3/2}*d) + ((A - B)*\operatorname{Sec}[c + d*x]^{7/2}*\operatorname{Sin}[c + d*x])/(2*d*(a + a*\operatorname{Sec}[c + d*x]^{3/2})) + ((6*A - 7*B)*\operatorname{Sec}[c + d*x]^{3/2}*\operatorname{Sin}[c + d*x])/(4*a*d*\operatorname{Sqrt}[a + a*\operatorname{Sec}[c + d*x]]) - ((A - 2*B)*\operatorname{Sec}[c + d*x]^{5/2}*\operatorname{Sin}[c + d*x])/(2*a*d*\operatorname{Sqrt}[a + a*\operatorname{Sec}[c + d*x]])$

Rule 206

$\operatorname{Int}[(a_) + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1*\operatorname{ArcTanh}[(\operatorname{Rt}[-b, 2]*x)/\operatorname{Rt}[a, 2]])/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /; \operatorname{FreeQ}\{a, b\}, x \ \&\& \operatorname{NegQ}[a/b] \ \&\& (\operatorname{GtQ}[a, 0] \ || \ \operatorname{LtQ}[b, 0])$

Rule 215

$\operatorname{Int}[1/\operatorname{Sqrt}[(a_) + (b_)*(x_)^2], x_Symbol] \rightarrow \operatorname{Simp}[\operatorname{ArcSinh}[(\operatorname{Rt}[b, 2]*x)/\operatorname{Sqrt}[a]]/\operatorname{Rt}[b, 2], x] /; \operatorname{FreeQ}\{a, b\}, x \ \&\& \operatorname{GtQ}[a, 0] \ \&\& \operatorname{PosQ}[b]$

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4021

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x
] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)
*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &&
GtQ[n, 1]
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^{\frac{7}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{\frac{3}{2}}} dx &= \frac{(A-B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{\frac{3}{2}}} + \int \frac{\sec^{\frac{5}{2}}(c+dx)\left(\frac{5}{2}a(A-B)-2a(A-2B)\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}} \frac{1}{2a^2} \\
&= \frac{(A-B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{\frac{3}{2}}} - \frac{(A-2B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2ad\sqrt{a+a\sec(c+dx)}} + \\
&= \frac{(A-B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{\frac{3}{2}}} + \frac{(6A-7B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4ad\sqrt{a+a\sec(c+dx)}} \\
&= \frac{(A-B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{\frac{3}{2}}} + \frac{(6A-7B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4ad\sqrt{a+a\sec(c+dx)}} \\
&= \frac{(A-B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{\frac{3}{2}}} + \frac{(6A-7B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4ad\sqrt{a+a\sec(c+dx)}} \\
&= \frac{(A-B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{\frac{3}{2}}} + \frac{(6A-7B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4ad\sqrt{a+a\sec(c+dx)}} \\
&= -\frac{(12A-19B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4a^{\frac{3}{2}}d} + \frac{(9A-13B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{\frac{3}{2}}d}
\end{aligned}$$

Mathematica [B] time = 4.64, size = 497, normalized size = 2.01

$$4(6A-7B)\sin\left(\frac{1}{2}(c+dx)\right)\cos^3\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\sin^{-1}\left(\sqrt{1-\sec(c+dx)}\right)+8(9A-13B)\sin\left(\frac{1}{2}(c+dx)\right)$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[c + d*x]^(7/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(3/2), x]

[Out] (4*(6*A - 7*B)*ArcSin[Sqrt[1 - Sec[c + d*x]]]*Cos[(c + d*x)/2]^3*Sec[c + d*x]^2*Sin[(c + d*x)/2] + 8*(9*A - 13*B)*ArcSin[Sqrt[Sec[c + d*x]]]*Cos[(c + d*x)/2]^3*Sec[c + d*x]^2*Sin[(c + d*x)/2] + 6*A*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x] - 7*B*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x] + 4*A*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x] - 3*B*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x] + 2*B*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(7/2)*Sin[c + d*x] - 9*Sqrt[2]*A*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Tan[c + d*x] + 13*Sqrt[2]*B*A

```
rcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]*Tan[c + d*x] - 9
*Sqrt[2]*A*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sec[
c + d*x]*Tan[c + d*x] + 13*Sqrt[2]*B*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sq
rt[1 - Sec[c + d*x]]]*Sec[c + d*x]*Tan[c + d*x])/(4*d*Sqrt[1 - Sec[c + d*x]
]*(a*(1 + Sec[c + d*x]))^(3/2))
```

fricas [A] time = 0.67, size = 761, normalized size = 3.08

$$\frac{2\sqrt{2}\left((9A - 13B)\cos(dx + c)^3 + 2(9A - 13B)\cos(dx + c)^2 + (9A - 13B)\cos(dx + c)\right)\sqrt{a}\log\left(-\frac{a\cos(dx + c)}{\dots}\right)}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] [-1/16*(2*sqrt(2)*((9*A - 13*B)*cos(d*x + c)^3 + 2*(9*A - 13*B)*cos(d*x + c)
)^2 + (9*A - 13*B)*cos(d*x + c))*sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)
)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x
+ c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + ((
12*A - 19*B)*cos(d*x + c)^3 + 2*(12*A - 19*B)*cos(d*x + c)^2 + (12*A - 19*B)
*cos(d*x + c))*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos
(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c
))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)
) - 4*((6*A - 7*B)*cos(d*x + c)^2 + (4*A - 3*B)*cos(d*x + c) + 2*B)*sqrt((a
*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*co
s(d*x + c)^3 + 2*a^2*d*cos(d*x + c)^2 + a^2*d*cos(d*x + c)), -1/8*(2*sqrt(2)
)*((9*A - 13*B)*cos(d*x + c)^3 + 2*(9*A - 13*B)*cos(d*x + c)^2 + (9*A - 13*
B)*cos(d*x + c))*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)
/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) + ((12*A - 19*B)*cos(d*
x + c)^3 + 2*(12*A - 19*B)*cos(d*x + c)^2 + (12*A - 19*B)*cos(d*x + c))*sqr
t(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*
x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) - 2*((6*A -
7*B)*cos(d*x + c)^2 + (4*A - 3*B)*cos(d*x + c) + 2*B)*sqrt((a*cos(d*x + c)
+ a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^3
+ 2*a^2*d*cos(d*x + c)^2 + a^2*d*cos(d*x + c))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sec(dx + c)^{\frac{7}{2}}}{(a \sec(dx + c) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algorith="giac")

[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(7/2)/(a*sec(d*x + c) + a)^(3/2), x)

maple [B] time = 2.36, size = 541, normalized size = 2.19

$$(-1 + \cos(dx + c)) \left(12A\sqrt{2} \arctan \left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}} (\cos(dx+c)+1-\sin(dx+c))\sqrt{2}}{4} \right) \right) \sin(dx + c) (\cos^2(dx + c)) - 12A\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x)

[Out] -1/8/d*(-1+cos(d*x+c))*(12*A*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c))))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))*2^(1/2))*sin(d*x+c)*cos(d*x+c)^2-12*A*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c))))^(1/2)*(cos(d*x+c)+1+sin(d*x+c))*2^(1/2))*sin(d*x+c)*cos(d*x+c)^2-19*B*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c))))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))*2^(1/2))*sin(d*x+c)*cos(d*x+c)^2+19*B*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c))))^(1/2)*(cos(d*x+c)+1+sin(d*x+c))*2^(1/2))*sin(d*x+c)*cos(d*x+c)^2+36*A*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c))))^(1/2))*sin(d*x+c)*cos(d*x+c)^2-12*A*(-2/(1+cos(d*x+c))))^(1/2))*cos(d*x+c)^3-52*B*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c))))^(1/2))*sin(d*x+c)*cos(d*x+c)^2+14*B*(-2/(1+cos(d*x+c))))^(1/2))*cos(d*x+c)^3+4*A*(-2/(1+cos(d*x+c))))^(1/2))*cos(d*x+c)^2-8*B*(-2/(1+cos(d*x+c))))^(1/2))*cos(d*x+c)^2+8*A*(-2/(1+cos(d*x+c))))^(1/2))*cos(d*x+c)-10*B*(-2/(1+cos(d*x+c))))^(1/2))*cos(d*x+c)+4*B*(-2/(1+cos(d*x+c))))^(1/2))*cos(d*x+c)^2*(1/cos(d*x+c))^(7/2)/(-2/(1+cos(d*x+c))))^(1/2)/sin(d*x+c)^3/a^2

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(\frac{1}{\cos(c+dx)}\right)^{7/2}}{\left(a + \frac{a}{\cos(c+dx)}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(7/2))/(a + a/cos(c + d*x))^(3/2),x)

[Out] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(7/2))/(a + a/cos(c + d*x))^(3/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**(7/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(3/2),x)

[Out] Timed out

$$3.256 \quad \int \frac{\sec^2(c+dx)(A+B \sec(c+dx))}{(a+a \sec(c+dx))^{3/2}} dx$$

Optimal. Leaf size=197

$$\frac{(5A-9B) \tanh^{-1}\left(\frac{\sqrt{a} \sin(c+dx) \sqrt{\sec(c+dx)}}{\sqrt{2} \sqrt{a \sec(c+dx)+a}}\right)}{2\sqrt{2} a^{3/2} d} + \frac{(2A-3B) \sinh^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}}\right)}{a^{3/2} d} + \frac{(A-B) \sin(c+dx) \sec^2(c+dx)}{2d(a \sec(c+dx)+a)^{3/2}}$$

[Out] (2*A-3*B)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)/d+1/2*(A-B)*sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(3/2)-1/4*(5*A-9*B)*arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2)-1/2*(A-3*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^(1/2)

Rubi [A] time = 0.60, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4019, 4021, 4023, 3808, 206, 3801, 215}

$$\frac{(5A-9B) \tanh^{-1}\left(\frac{\sqrt{a} \sin(c+dx) \sqrt{\sec(c+dx)}}{\sqrt{2} \sqrt{a \sec(c+dx)+a}}\right)}{2\sqrt{2} a^{3/2} d} + \frac{(2A-3B) \sinh^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}}\right)}{a^{3/2} d} + \frac{(A-B) \sin(c+dx) \sec^2(c+dx)}{2d(a \sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(3/2), x]

[Out] ((2*A - 3*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]])/(a^(3/2)*d) - ((5*A - 9*B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]/(2*Sqrt[2]*a^(3/2)*d) + ((A - B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(2*d*(a + a*Sec[c + d*x])^(3/2)) - ((A - 3*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(2*a*d*Sqrt[a + a*Sec[c + d*x]])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_.)], x_Symbol] :=> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_.)], x_Symbol] :=> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] :=> Simp[(d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*(
2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4021

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] :=> -Simp[(B*d*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x
] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)
*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &&
GtQ[n, 1]
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] :=> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{\frac{3}{2}}} dx &= \frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{\frac{3}{2}}} + \frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(\frac{3}{2}a(A-B)-a(A-3B)\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}}{2a^2} \\
&= \frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{\frac{3}{2}}} - \frac{(A-3B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2ad\sqrt{a+a\sec(c+dx)}} + \\
&= \frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{\frac{3}{2}}} - \frac{(A-3B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2ad\sqrt{a+a\sec(c+dx)}} \\
&= \frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{\frac{3}{2}}} - \frac{(A-3B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2ad\sqrt{a+a\sec(c+dx)}} + \\
&= \frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{\frac{3}{2}}} - \frac{(A-3B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2ad\sqrt{a+a\sec(c+dx)}} \\
&= \frac{(2A-3B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{\frac{3}{2}}d} - \frac{(5A-9B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{\frac{3}{2}}d}
\end{aligned}$$

Mathematica [A] time = 1.94, size = 132, normalized size = 0.67

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left((9B-5A)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sqrt{2}(2A-3B)\tanh^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)}{2ad\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(3/2), x]

[Out] (Cos[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*((-5*A + 9*B)*ArcTanh[Sin[(c + d*x)/2]] + 2*Sqrt[2]*(2*A - 3*B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]] + Sec[(c + d*x)/2]*(-A + 3*B + 2*B*Sec[c + d*x])*Tan[(c + d*x)/2))/(2*a*d*Sqrt[a*(1 + Sec[c + d*x])])

fricas [A] time = 0.66, size = 669, normalized size = 3.40

$$\sqrt{2} \left((5A - 9B) \cos(dx + c)^2 + 2(5A - 9B) \cos(dx + c) + 5A - 9B \right) \sqrt{a} \log \left(-\frac{a \cos(dx+c)^2 - 2\sqrt{2}\sqrt{a} \sqrt{\frac{a \cos(dx+c)+a}{\cos(dx+c)}}}{\cos(dx+c)^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")

[Out] [-1/8*(sqrt(2)*((5*A - 9*B)*cos(d*x + c)^2 + 2*(5*A - 9*B)*cos(d*x + c) + 5*A - 9*B)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 2*((2*A - 3*B)*cos(d*x + c)^2 + 2*(2*A - 3*B)*cos(d*x + c) + 2*A - 3*B)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*((A - 3*B)*cos(d*x + c) - 2*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), 1/4*(sqrt(2)*((5*A - 9*B)*cos(d*x + c)^2 + 2*(5*A - 9*B)*cos(d*x + c) + 5*A - 9*B)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c)))/(a*sin(d*x + c))) + 2*((2*A - 3*B)*cos(d*x + c)^2 + 2*(2*A - 3*B)*cos(d*x + c) + 2*A - 3*B)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) - 2*((A - 3*B)*cos(d*x + c) - 2*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sec(dx + c)^{\frac{5}{2}}}{(a \sec(dx + c) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(5/2)/(a*sec(d*x + c) + a)^(3/2), x)

maple [B] time = 2.38, size = 479, normalized size = 2.43

$$\left(\frac{1}{\cos(dx+c)}\right)^{\frac{5}{2}} \left(\cos^2(dx+c)\right) \sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}} \left(-2A \cos(dx+c) \sin(dx+c) \sqrt{2} \arctan\left(\frac{\sqrt{\frac{2}{1+\cos(dx+c)}} (\cos(dx+c)+1-\sin(dx+c))}{4}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2), x)

[Out] 1/4/d*(1/cos(d*x+c))^(5/2)*cos(d*x+c)^2*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-2*A*cos(d*x+c)*sin(d*x+c)*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c))))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))*2^(1/2))+2*A*cos(d*x+c)*sin(d*x+c)*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c))))^(1/2)*(cos(d*x+c)+1+sin(d*x+c))*2^(1/2))+3*B*cos(d*x+c)*sin(d*x+c)*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c))))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))*2^(1/2))-3*B*cos(d*x+c)*sin(d*x+c)*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c))))^(1/2)*(cos(d*x+c)+1+sin(d*x+c))*2^(1/2))+A*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2-5*A*cos(d*x+c)*sin(d*x+c)*arctan(1/2*sin(d*x+c))*(-2/(1+cos(d*x+c)))^(1/2))-3*B*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2+9*B*cos(d*x+c)*sin(d*x+c)*arctan(1/2*sin(d*x+c))*(-2/(1+cos(d*x+c)))^(1/2))-A*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)+B*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)+2*B*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^3*(cos(d*x+c)^2-1)/a^2

maxima [B] time = 4.03, size = 7057, normalized size = 35.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2), x, algorithm="maxima")

[Out] 1/4*((4*(sin(2*d*x + 2*c) + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 2*(sqrt(2)*cos(2*d*x + 2*c)^2 + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))^2 + 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))^2 + 2*sqrt(2)*cos(2*d*x + 2*c)

$$\begin{aligned}
& 2(\sin(2dx + 2c), \cos(2dx + 2c))^2 + 2(2\sqrt{2}\cos(2dx + 2c) + \\
& \sqrt{2})\cos(4dx + 4c) + 4(\sqrt{2}\cos(4dx + 4c) + 2\sqrt{2}\cos(2dx \\
& + 2c) + 2\sqrt{2}\cos(1/2\arctan2(\sin(2dx + 2c), \cos(2dx + 2c))) \\
& + \sqrt{2})\cos(3/2\arctan2(\sin(2dx + 2c), \cos(2dx + 2c))) + 4(\sqrt{2} \\
&)\cos(4dx + 4c) + 2\sqrt{2}\cos(2dx + 2c) + \sqrt{2})\cos(1/2\arctan2(\\
& \sin(2dx + 2c), \cos(2dx + 2c))) + 4(\sqrt{2}\sin(4dx + 4c) + 2\sqrt{2} \\
& (2)\sin(2dx + 2c) + 2\sqrt{2}\sin(1/2\arctan2(\sin(2dx + 2c), \cos(2dx \\
& + 2c))))\sin(3/2\arctan2(\sin(2dx + 2c), \cos(2dx + 2c))) + 4(\sqrt{2} \\
& (2)\sin(4dx + 4c) + 2\sqrt{2}\sin(2dx + 2c))\sin(1/2\arctan2(\sin(2dx \\
& + 2c), \cos(2dx + 2c))) + 4\sqrt{2}\cos(2dx + 2c) + \sqrt{2})\log(2c \\
& \cos(1/4\arctan2(\sin(2dx + 2c), \cos(2dx + 2c)))^2 + 2\sin(1/4\arctan2(s \\
& \sin(2dx + 2c), \cos(2dx + 2c)))^2 + 2\sqrt{2}\cos(1/4\arctan2(\sin(2dx \\
& + 2c), \cos(2dx + 2c))) - 2\sqrt{2}\sin(1/4\arctan2(\sin(2dx + 2c), c \\
& \cos(2dx + 2c))) + 2) + 3(\sqrt{2}\cos(4dx + 4c))^2 + 4\sqrt{2}\cos(2dx \\
& + 2c)^2 + 4\sqrt{2}\cos(3/2\arctan2(\sin(2dx + 2c), \cos(2dx + 2c))) \\
& ^2 + 4\sqrt{2}\cos(1/2\arctan2(\sin(2dx + 2c), \cos(2dx + 2c)))^2 + sqr \\
& t(2)\sin(4dx + 4c)^2 + 4\sqrt{2}\sin(4dx + 4c)\sin(2dx + 2c) + 4s \\
& qrt(2)\sin(2dx + 2c)^2 + 4\sqrt{2}\sin(3/2\arctan2(\sin(2dx + 2c), \cos \\
& (2dx + 2c)))^2 + 4\sqrt{2}\sin(1/2\arctan2(\sin(2dx + 2c), \cos(2dx + \\
& 2c)))^2 + 2(2\sqrt{2}\cos(2dx + 2c) + \sqrt{2})\cos(4dx + 4c) + 4(\\
& \sqrt{2}\cos(4dx + 4c) + 2\sqrt{2}\cos(2dx + 2c) + 2\sqrt{2}\cos(1/2a \\
& rctan2(\sin(2dx + 2c), \cos(2dx + 2c))) + \sqrt{2})\cos(3/2\arctan2(\sin(\\
& 2dx + 2c), \cos(2dx + 2c))) + 4(\sqrt{2}\cos(4dx + 4c) + 2\sqrt{2}) \\
& \cos(2dx + 2c) + \sqrt{2})\cos(1/2\arctan2(\sin(2dx + 2c), \cos(2dx + 2 \\
& *c))) + 4(\sqrt{2}\sin(4dx + 4c) + 2\sqrt{2}\sin(2dx + 2c) + 2\sqrt{2} \\
&)\sin(1/2\arctan2(\sin(2dx + 2c), \cos(2dx + 2c))))\sin(3/2\arctan2(\sin \\
& (2dx + 2c), \cos(2dx + 2c))) + 4(\sqrt{2}\sin(4dx + 4c) + 2\sqrt{2} \\
&)\sin(2dx + 2c))\sin(1/2\arctan2(\sin(2dx + 2c), \cos(2dx + 2c))) + 4 \\
& *sqrt(2)\cos(2dx + 2c) + sqrt(2))\log(2\cos(1/4\arctan2(\sin(2dx + 2c) \\
& , \cos(2dx + 2c)))^2 + 2\sin(1/4\arctan2(\sin(2dx + 2c), \cos(2dx + 2 \\
& c)))^2 - 2\sqrt{2}\cos(1/4\arctan2(\sin(2dx + 2c), \cos(2dx + 2c))) + 2 \\
& *sqrt(2)\sin(1/4\arctan2(\sin(2dx + 2c), \cos(2dx + 2c))) + 2) - 3(sqr \\
& t(2)\cos(4dx + 4c))^2 + 4\sqrt{2}\cos(2dx + 2c)^2 + 4\sqrt{2}\cos(3/2 \\
& arctan2(\sin(2dx + 2c), \cos(2dx + 2c)))^2 + 4\sqrt{2}\cos(1/2\arctan2(\\
& \sin(2dx + 2c), \cos(2dx + 2c)))^2 + sqrt(2)\sin(4dx + 4c)^2 + 4\sqr \\
& t(2)\sin(4dx + 4c)\sin(2dx + 2c) + 4\sqrt{2}\sin(2dx + 2c)^2 + 4s \\
& qrt(2)\sin(3/2\arctan2(\sin(2dx + 2c), \cos(2dx + 2c)))^2 + 4\sqrt{2}\s \\
& in(1/2\arctan2(\sin(2dx + 2c), \cos(2dx + 2c)))^2 + 2(2\sqrt{2}\cos(2 \\
& dx + 2c) + sqrt(2))\cos(4dx + 4c) + 4(\sqrt{2}\cos(4dx + 4c) + 2sq \\
& rt(2)\cos(2dx + 2c) + 2\sqrt{2}\cos(1/2\arctan2(\sin(2dx + 2c), \cos(2 \\
& dx + 2c))) + sqrt(2))\cos(3/2\arctan2(\sin(2dx + 2c), \cos(2dx + 2c)) \\
&) + 4(\sqrt{2}\cos(4dx + 4c) + 2\sqrt{2}\cos(2dx + 2c) + sqrt(2))\cos \\
& (1/2\arctan2(\sin(2dx + 2c), \cos(2dx + 2c))) + 4(\sqrt{2}\sin(4dx + \\
& 4c) + 2\sqrt{2}\sin(2dx + 2c) + 2\sqrt{2}\sin(1/2\arctan2(\sin(2dx + 2 \\
& *c), \cos(2dx + 2c))))\sin(3/2\arctan2(\sin(2dx + 2c), \cos(2dx + 2c)
\end{aligned}$$

$$\begin{aligned}
&)) + 4*(\sqrt{2}*\sin(4*d*x + 4*c) + 2*\sqrt{2}*\sin(2*d*x + 2*c))*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 4*\sqrt{2}*\cos(2*d*x + 2*c) + \sqrt{2}*\log(2*\cos(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))))^2 + 2*\sin(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 - 2*\sqrt{2}*\cos(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) - 2*\sqrt{2}*\sin(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 2) - 9*(2*(2*\cos(2*d*x + 2*c) + 1)*\cos(4*d*x + 4*c) + \cos(4*d*x + 4*c)^2 + 4*\cos(2*d*x + 2*c)^2 + 4*(\cos(4*d*x + 4*c) + 2*\cos(2*d*x + 2*c) + 2*\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))) + 1)*\cos(3/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 4*\cos(3/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + 4*(\cos(4*d*x + 4*c) + 2*\cos(2*d*x + 2*c) + 1)*\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 4*\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + \sin(4*d*x + 4*c)^2 + 4*\sin(4*d*x + 4*c)*\sin(2*d*x + 2*c) + 4*\sin(2*d*x + 2*c)^2 + 4*(\sin(4*d*x + 4*c) + 2*\sin(2*d*x + 2*c) + 2*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))))*\sin(3/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 4*\sin(3/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + 4*(\sin(4*d*x + 4*c) + 2*\sin(2*d*x + 2*c))*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 4*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + 4*\cos(2*d*x + 2*c) + 1)*\log(\cos(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + \sin(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + 2*\sin(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 1) + 9*(2*(2*\cos(2*d*x + 2*c) + 1)*\cos(4*d*x + 4*c) + \cos(4*d*x + 4*c)^2 + 4*\cos(2*d*x + 2*c)^2 + 4*(\cos(4*d*x + 4*c) + 2*\cos(2*d*x + 2*c) + 2*\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))) + 1)*\cos(3/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 4*\cos(3/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + 4*(\cos(4*d*x + 4*c) + 2*\cos(2*d*x + 2*c) + 1)*\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 4*\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + \sin(4*d*x + 4*c)^2 + 4*\sin(4*d*x + 4*c)*\sin(2*d*x + 2*c) + 4*\sin(2*d*x + 2*c)^2 + 4*(\sin(4*d*x + 4*c) + 2*\sin(2*d*x + 2*c) + 2*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))))*\sin(3/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 4*\sin(3/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + 4*(\sin(4*d*x + 4*c) + 2*\sin(2*d*x + 2*c))*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 4*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + 4*\cos(2*d*x + 2*c) + 1)*\log(\cos(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + \sin(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 - 2*\sin(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 1) - 12*(\cos(4*d*x + 4*c) + 2*\cos(2*d*x + 2*c) + 2*\cos(3/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))) + 2*\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 1)*\sin(7/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 8*(\cos(5/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) - \cos(3/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))) - 3*\cos(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))))*\sin(3/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) - 4*(\cos(4*d*x + 4*c) + 2*\cos(2*d*x + 2*c) + 2*\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))) + 1)*\sin(5/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 4*(\cos(4*d*x + 4*c) + 2*\cos(2*d*x + 2*c) + 2*\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))) + 1)*\sin(3/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))
\end{aligned}$$

```
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 24*cos(1/4*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) + 12*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(1/4*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) + 24*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*B/((sqrt
(2)*a*cos(4*d*x + 4*c)^2 + 4*sqrt(2)*a*cos(2*d*x + 2*c)^2 + 4*sqrt(2)*a*cos
(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*sqrt(2)*a*cos(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*a*sin(4*d*x + 4*c)^
2 + 4*sqrt(2)*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sqrt(2)*a*sin(2*d*x +
2*c)^2 + 4*sqrt(2)*a*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^
2 + 4*sqrt(2)*a*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*
sqrt(2)*a*cos(2*d*x + 2*c) + 2*(2*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*c
os(4*d*x + 4*c) + 4*(sqrt(2)*a*cos(4*d*x + 4*c) + 2*sqrt(2)*a*cos(2*d*x + 2
*c) + 2*sqrt(2)*a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sq
rt(2)*a)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(sqrt(2)*
a*cos(4*d*x + 4*c) + 2*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*cos(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(sqrt(2)*a*sin(4*d*x + 4*c) +
2*sqrt(2)*a*sin(2*d*x + 2*c) + 2*sqrt(2)*a*sin(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
+ 4*(sqrt(2)*a*sin(4*d*x + 4*c) + 2*sqrt(2)*a*sin(2*d*x + 2*c))*sin(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2)*a)*sqrt(a))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(\frac{1}{\cos(c+dx)}\right)^{5/2}}{\left(a + \frac{a}{\cos(c+dx)}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + a/cos(c + d*x))^(3/2),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + a/cos(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

$$3.257 \quad \int \frac{\sec^2(c+dx)(A+B \sec(c+dx))}{(a+a \sec(c+dx))^{3/2}} dx$$

Optimal. Leaf size=145

$$\frac{(A-5B) \tanh^{-1}\left(\frac{\sqrt{a} \sin(c+dx) \sqrt{\sec(c+dx)}}{\sqrt{2} \sqrt{a \sec(c+dx)+a}}\right)}{2\sqrt{2} a^{3/2} d} + \frac{2B \sinh^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}}\right)}{a^{3/2} d} + \frac{(A-B) \sin(c+dx) \sec^2(c+dx)}{2d(a \sec(c+dx)+a)^{3/2}}$$

[Out] $2*B*\operatorname{arcsinh}(a^{1/2}*\tan(d*x+c)/(a+a*\sec(d*x+c))^{1/2})/a^{3/2}/d+1/2*(A-B)*\sec(d*x+c)^{3/2}*\sin(d*x+c)/d/(a+a*\sec(d*x+c))^{3/2}+1/4*(A-5*B)*\operatorname{arctanh}(1/2*\sin(d*x+c)*a^{1/2}*\sec(d*x+c)^{1/2}*2^{1/2}/(a+a*\sec(d*x+c))^{1/2})/a^{3/2}/d*2^{1/2}$

Rubi [A] time = 0.39, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {4019, 4023, 3808, 206, 3801, 215}

$$\frac{(A-5B) \tanh^{-1}\left(\frac{\sqrt{a} \sin(c+dx) \sqrt{\sec(c+dx)}}{\sqrt{2} \sqrt{a \sec(c+dx)+a}}\right)}{2\sqrt{2} a^{3/2} d} + \frac{2B \sinh^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}}\right)}{a^{3/2} d} + \frac{(A-B) \sin(c+dx) \sec^2(c+dx)}{2d(a \sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(\operatorname{Sec}[c+d*x]^{3/2}*(A+B*\operatorname{Sec}[c+d*x]))/(a+a*\operatorname{Sec}[c+d*x]^{3/2}),x]$

[Out] $(2*B*\operatorname{ArcSinh}[(\operatorname{Sqrt}[a]*\operatorname{Tan}[c+d*x])/(\operatorname{Sqrt}[a+a*\operatorname{Sec}[c+d*x]])]/(a^{3/2}*d) + ((A-5*B)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Sqrt}[\operatorname{Sec}[c+d*x]]*\operatorname{Sin}[c+d*x])/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a+a*\operatorname{Sec}[c+d*x]])])/(2*\operatorname{Sqrt}[2]*a^{3/2}*d) + ((A-B)*\operatorname{Sec}[c+d*x]^{3/2}*\operatorname{Sin}[c+d*x])/(2*d*(a+a*\operatorname{Sec}[c+d*x]^{3/2}))$

Rule 206

$\operatorname{Int}[(a_+ + (b_+)*(x_+)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1*\operatorname{ArcTanh}[(\operatorname{Rt}[-b, 2]*x)/\operatorname{Rt}[a, 2]])/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /; \operatorname{FreeQ}\{a, b\}, x \ \&\& \operatorname{NegQ}[a/b] \ \&\& (\operatorname{GtQ}[a, 0] \ || \ \operatorname{LtQ}[b, 0])$

Rule 215

$\operatorname{Int}[1/\operatorname{Sqrt}[(a_+ + (b_+)*(x_+)^2)], x_Symbol] \rightarrow \operatorname{Simp}[\operatorname{ArcSinh}[(\operatorname{Rt}[b, 2]*x)/\operatorname{Sqrt}[a]]/\operatorname{Rt}[b, 2], x] /; \operatorname{FreeQ}\{a, b\}, x \ \&\& \operatorname{GtQ}[a, 0] \ \&\& \operatorname{PosQ}[b]$

Rule 3801

$\operatorname{Int}[\operatorname{Sqrt}[\operatorname{csc}[(e_+ + (f_+)*(x_+)]*(d_+)]*\operatorname{Sqrt}[\operatorname{csc}[(e_+ + (f_+)*(x_+)]*(b_+ + (a_+))], x_Symbol] \rightarrow \operatorname{Dist}[(-2*a*\operatorname{Sqrt}[(a*d)/b])/(\operatorname{b*f}), \operatorname{Subst}[\operatorname{Int}[1/\operatorname{Sqrt}[1 +$

x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]

Rule 3808

Int[Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x])*Sqrt[d*Csc[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 4019

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> Simp[(d*(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0]

Rule 4023

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> Dist[(A*b - a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned}
\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{\frac{3}{2}}} dx &= \frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{\frac{3}{2}}} + \frac{\int \frac{\sqrt{\sec(c+dx)}\left(\frac{1}{2}a(A-B)+2aB\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}} dx}{2a^2} \\
&= \frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{\frac{3}{2}}} + \frac{(A-5B)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a\sec(c+dx)}} dx}{4a} + \frac{B\int \frac{1}{\sqrt{a+a\sec(c+dx)}} dx}{2a} \\
&= \frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{\frac{3}{2}}} - \frac{(A-5B)\text{Subst}\left(\int \frac{1}{2a-x^2} dx, x, -\frac{a\sqrt{\sec(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\right)}{2ad} \\
&= \frac{2B\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{\frac{3}{2}}d} + \frac{(A-5B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{\frac{3}{2}}d} + \frac{B\int \frac{1}{\sqrt{a+a\sec(c+dx)}} dx}{2a}
\end{aligned}$$

Mathematica [A] time = 0.86, size = 113, normalized size = 0.78

$$\frac{\sqrt{\sec(c+dx)}\left((A-B)\tan\left(\frac{1}{2}(c+dx)\right) + (A-5B)\cos\left(\frac{1}{2}(c+dx)\right)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right) + 4\sqrt{2}B\cos\left(\frac{1}{2}(c+dx)\right)}{2ad\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(3/2), x]

[Out] (Sqrt[Sec[c + d*x]]*((A - 5*B)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2] + 4*Sqrt[2]*B*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[(c + d*x)/2] + (A - B)*Tan[(c + d*x)/2]))/(2*a*d*Sqrt[a*(1 + Sec[c + d*x])])

fricas [B] time = 0.54, size = 601, normalized size = 4.14

$$\left[\frac{\sqrt{2}\left((A-5B)\cos(dx+c)^2 + 2(A-5B)\cos(dx+c) + A-5B\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^2 + 2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}}{\cos(dx+c)^2 + 2\cos(dx+c)}\right)}{\dots} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")

[Out] [-1/8*(sqrt(2)*((A - 5*B)*cos(d*x + c)^2 + 2*(A - 5*B)*cos(d*x + c) + A - 5*B)*sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*(A - B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 4*(B*cos(d*x + c)^2 + 2*B*cos(d*x + c) + B)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), -1/4*(sqrt(2)*((A - 5*B)*cos(d*x + c)^2 + 2*(A - 5*B)*cos(d*x + c) + A - 5*B)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) - 2*(A - B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 4*(B*cos(d*x + c)^2 + 2*B*cos(d*x + c) + B)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sec(dx + c)^{\frac{3}{2}}}{(a \sec(dx + c) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(3/2)/(a*sec(d*x + c) + a)^(3/2), x)

maple [B] time = 2.56, size = 313, normalized size = 2.16

$$\left(2B\sqrt{2} \arctan \left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}} (\cos(dx+c)+1+\sin(dx+c))\sqrt{2}}{4} \right) \sin(dx + c) - 2B\sqrt{2} \arctan \left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}} (\cos(dx+c)+1-\sin(dx+c))\sqrt{2}}{4} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x)

```
[Out] -1/2/d*(2*B*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d*x+c))*2^(1/2))*sin(d*x+c)-2*B*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))*2^(1/2))*sin(d*x+c)+A*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*sin(d*x+c)-A*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)-5*B*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*sin(d*x+c)+B*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)+A*(-2/(1+cos(d*x+c)))^(1/2)-B*(-2/(1+cos(d*x+c)))^(1/2))*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*cos(d*x+c)^2*(1/cos(d*x+c))^(3/2)/sin(d*x+c)^3/(-2/(1+cos(d*x+c)))^(1/2)/a^2
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(\frac{1}{\cos(c+dx)}\right)^{3/2}}{\left(a + \frac{a}{\cos(c+dx)}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + a/cos(c + d*x))^(3/2),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + a/cos(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(3/2),x)
```

[Out] Timed out

$$3.258 \quad \int \frac{\sqrt{\sec(c+dx)} (A+B \sec(c+dx))}{(a+a \sec(c+dx))^{3/2}} dx$$

Optimal. Leaf size=107

$$\frac{(3A+B) \tanh^{-1}\left(\frac{\sqrt{a} \sin(c+dx) \sqrt{\sec(c+dx)}}{\sqrt{2} \sqrt{a \sec(c+dx)+a}}\right)}{2\sqrt{2} a^{3/2} d} - \frac{(A-B) \sin(c+dx) \sec^{\frac{3}{2}}(c+dx)}{2d(a \sec(c+dx)+a)^{3/2}}$$

[Out] $-1/2*(A-B)*\sec(d*x+c)^{(3/2)}*\sin(d*x+c)/d/(a+a*\sec(d*x+c))^{(3/2)}+1/4*(3*A+B)*\operatorname{arctanh}(1/2*\sin(d*x+c)*a^{(1/2)}*\sec(d*x+c)^{(1/2)}*2^{(1/2)/(a+a*\sec(d*x+c))^{(1/2)})/a^{(3/2)}/d*2^{(1/2)}$

Rubi [A] time = 0.19, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {4012, 3808, 206}

$$\frac{(3A+B) \tanh^{-1}\left(\frac{\sqrt{a} \sin(c+dx) \sqrt{\sec(c+dx)}}{\sqrt{2} \sqrt{a \sec(c+dx)+a}}\right)}{2\sqrt{2} a^{3/2} d} - \frac{(A-B) \sin(c+dx) \sec^{\frac{3}{2}}(c+dx)}{2d(a \sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(\operatorname{Sqrt}[\operatorname{Sec}[c+d*x]]*(A+B*\operatorname{Sec}[c+d*x]))/(a+a*\operatorname{Sec}[c+d*x])^{(3/2)},x]$

[Out] $((3*A+B)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Sqrt}[\operatorname{Sec}[c+d*x]]*\operatorname{Sin}[c+d*x])/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a+a*\operatorname{Sec}[c+d*x]])])/(2*\operatorname{Sqrt}[2]*a^{(3/2)}*d) - ((A-B)*\operatorname{Sec}[c+d*x]^{(3/2)}*\operatorname{Sin}[c+d*x])/(2*d*(a+a*\operatorname{Sec}[c+d*x])^{(3/2)})$

Rule 206

$\operatorname{Int}[(a_+ + (b_+)*(x_+)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1*\operatorname{ArcTanh}[(\operatorname{Rt}[-b, 2]*x)/\operatorname{Rt}[a, 2]])/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /; \operatorname{FreeQ}\{a, b\}, x] \&\& \operatorname{NegQ}[a/b] \&\& (\operatorname{GtQ}[a, 0] \parallel \operatorname{LtQ}[b, 0])$

Rule 3808

$\operatorname{Int}[\operatorname{Sqrt}[\operatorname{csc}[e_+ + (f_+)*(x_+)]*(d_+)]/\operatorname{Sqrt}[\operatorname{csc}[e_+ + (f_+)*(x_+)]*(b_+ + a_+)], x_Symbol] \rightarrow \operatorname{Dist}[(-2*b*d)/(a*f), \operatorname{Subst}[\operatorname{Int}[1/(2*b - d*x^2), x], x, (b*\operatorname{Cot}[e + f*x])]/(\operatorname{Sqrt}[a + b*\operatorname{Csc}[e + f*x]]*\operatorname{Sqrt}[d*\operatorname{Csc}[e + f*x]])], x] /; \operatorname{FreeQ}\{a, b, d, e, f\}, x] \&\& \operatorname{EqQ}[a^2 - b^2, 0]$

Rule 4012

$\operatorname{Int}[(\operatorname{csc}[e_+ + (f_+)*(x_+)]*(d_+))^{(n_+)}*(\operatorname{csc}[e_+ + (f_+)*(x_+)]*(b_+ + a_+))^{(m_+)}*(\operatorname{csc}[e_+ + (f_+)*(x_+)]*(B_+ + A_+)], x_Symbol] \rightarrow -\operatorname{Simp}[(A*b$

- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m + 1)), x] + Dist[(a*A*m + b*B*(m + 1))/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && LeQ[m, -1]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{\sec(c+dx)}(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{3/2}} dx &= -\frac{(A-B)\sec^3(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(3A+B)\int\frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a\sec(c+dx)}}dx}{4a} \\ &= -\frac{(A-B)\sec^3(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(3A+B)\text{Subst}\left(\int\frac{1}{2a-x^2}dx, x, -\frac{a}{\sqrt{a+a\sec(c+dx)}}\right)}{2ad} \\ &= \frac{(3A+B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{(A-B)\sec^3(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} \end{aligned}$$

Mathematica [A] time = 0.29, size = 84, normalized size = 0.79

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left((B-A)\sin\left(\frac{1}{2}(c+dx)\right) + (3A+B)\cos^2\left(\frac{1}{2}(c+dx)\right)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)}{d(a(\sec(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(3/2), x]

[Out] (Cos[(c + d*x)/2]*Sec[c + d*x]^(3/2)*((3*A + B)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^2 + (-A + B)*Sin[(c + d*x)/2]))/(d*(a*(1 + Sec[c + d*x]))^(3/2))

fricas [A] time = 0.46, size = 376, normalized size = 3.51

$$\left[\frac{\sqrt{2}\left((3A+B)\cos(dx+c)^2 + 2(3A+B)\cos(dx+c) + 3A+B\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2 - 2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}}{\cos(dx+c)^2 + 2\cos(dx+c)}\right)}{8(a^2d\cos(dx+c)^2 + 2a^2d\cos(dx+c))} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")

[Out] [1/8*(sqrt(2)*((3*A + B)*cos(d*x + c)^2 + 2*(3*A + B)*cos(d*x + c) + 3*A + B)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*(A - B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), -1/4*(sqrt(2)*((3*A + B)*cos(d*x + c)^2 + 2*(3*A + B)*cos(d*x + c) + 3*A + B)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) + 2*(A - B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sqrt{\sec(dx + c)}}{(a \sec(dx + c) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(a*sec(d*x + c) + a)^(3/2), x)

maple [B] time = 2.18, size = 219, normalized size = 2.05

$$\sqrt{\frac{1}{\cos(dx+c)}} \sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}} \cos(dx+c) \left(A \sqrt{\frac{2}{1+\cos(dx+c)}} \cos(dx+c) + 3A \arctan\left(\frac{\sin(dx+c) \sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right) \sin(dx+c) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(3/2),x)

[Out] 1/4/d*(1/cos(d*x+c))^(1/2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)*(A*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)+3*A*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*sin(d*x+c)-B*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)+B*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*sin(d*x+c)-A*(-2/(1+cos(d*x+c)))^(1/2)+B*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^3*(cos(d*x+c)^2-1)/a^2

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \sqrt{\frac{1}{\cos(c+dx)}}}{\left(a + \frac{a}{\cos(c+dx)}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + a/cos(c + d*x))^(3/2),x)

[Out] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + a/cos(c + d*x))^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \sqrt{\sec(c + dx)}}{(a(\sec(c + dx) + 1))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)**(1/2)/(a+a*sec(d*x+c))**(3/2),x)

[Out] Integral((A + B*sec(c + d*x))*sqrt(sec(c + d*x))/(a*(sec(c + d*x) + 1))**(3/2), x)

$$3.259 \quad \int \frac{A+B \sec(c+dx)}{\sqrt{\sec(c+dx)} (a+a \sec(c+dx))^{3/2}} dx$$

Optimal. Leaf size=156

$$\frac{(7A-3B) \tanh^{-1}\left(\frac{\sqrt{a} \sin(c+dx) \sqrt{\sec(c+dx)}}{\sqrt{2} \sqrt{a \sec(c+dx)+a}}\right)}{2\sqrt{2} a^{3/2} d} + \frac{(5A-B) \sin(c+dx) \sqrt{\sec(c+dx)}}{2ad \sqrt{a \sec(c+dx)+a}} - \frac{(A-B) \sin(c+dx) \sqrt{\sec(c+dx)}}{2d(a \sec(c+dx)+a)^{3/2}}$$

[Out] $-1/4*(7*A-3*B)*\operatorname{arctanh}(1/2*\sin(d*x+c)*a^{(1/2)}*\sec(d*x+c)^{(1/2)}*2^{(1/2)}/(a+a*\sec(d*x+c))^{(1/2)})/a^{(3/2)}/d*2^{(1/2)}-1/2*(A-B)*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/d/(a+a*\sec(d*x+c))^{(3/2)}+1/2*(5*A-B)*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/a/d/(a+a*\sec(d*x+c))^{(1/2)}$

Rubi [A] time = 0.36, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {4020, 4013, 3808, 206}

$$\frac{(7A-3B) \tanh^{-1}\left(\frac{\sqrt{a} \sin(c+dx) \sqrt{\sec(c+dx)}}{\sqrt{2} \sqrt{a \sec(c+dx)+a}}\right)}{2\sqrt{2} a^{3/2} d} + \frac{(5A-B) \sin(c+dx) \sqrt{\sec(c+dx)}}{2ad \sqrt{a \sec(c+dx)+a}} - \frac{(A-B) \sin(c+dx) \sqrt{\sec(c+dx)}}{2d(a \sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(A + B*\operatorname{Sec}[c + d*x])]/(\operatorname{Sqrt}[\operatorname{Sec}[c + d*x]]*(a + a*\operatorname{Sec}[c + d*x])^{(3/2)}), x]$

[Out] $-((7*A - 3*B)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Sqrt}[\operatorname{Sec}[c + d*x]]*\operatorname{Sin}[c + d*x])]/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a + a*\operatorname{Sec}[c + d*x]])])/(2*\operatorname{Sqrt}[2]*a^{(3/2)}*d) - ((A - B)*\operatorname{Sqrt}[\operatorname{Sec}[c + d*x]]*\operatorname{Sin}[c + d*x])/(2*d*(a + a*\operatorname{Sec}[c + d*x])^{(3/2)}) + ((5*A - B)*\operatorname{Sqrt}[\operatorname{Sec}[c + d*x]]*\operatorname{Sin}[c + d*x])/(2*a*d*\operatorname{Sqrt}[a + a*\operatorname{Sec}[c + d*x]])$

Rule 206

$\operatorname{Int}[(a_) + (b_)*(x_)^2]^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1*\operatorname{ArcTanh}[(\operatorname{Rt}[-b, 2]*x)/\operatorname{Rt}[a, 2]])/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /;$ FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 3808

$\operatorname{Int}[\operatorname{Sqrt}[\operatorname{csc}[e_] + (f_)*(x_)]*(d_)]/\operatorname{Sqrt}[\operatorname{csc}[e_] + (f_)*(x_)]*(b_) + (a_)], x_Symbol] \rightarrow \operatorname{Dist}[(-2*b*d)/(a*f), \operatorname{Subst}[\operatorname{Int}[1/(2*b - d*x^2), x], x, (b*\operatorname{Cot}[e + f*x])]/(\operatorname{Sqrt}[a + b*\operatorname{Csc}[e + f*x]]*\operatorname{Sqrt}[d*\operatorname{Csc}[e + f*x]])], x] /;$ FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && !LeQ[m, -1]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m + 1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{A + B \sec(c + dx)}{\sqrt{\sec(c + dx)} (a + a \sec(c + dx))^{3/2}} dx &= -\frac{(A - B)\sqrt{\sec(c + dx)} \sin(c + dx)}{2d(a + a \sec(c + dx))^{3/2}} + \frac{\int \frac{\frac{1}{2}a(5A - B) - a(A - B)\sec(c + dx)}{\sqrt{\sec(c + dx)} \sqrt{a + a \sec(c + dx)}} dx}{2a^2} \\ &= -\frac{(A - B)\sqrt{\sec(c + dx)} \sin(c + dx)}{2d(a + a \sec(c + dx))^{3/2}} + \frac{(5A - B)\sqrt{\sec(c + dx)} \sin(c + dx)}{2ad\sqrt{a + a \sec(c + dx)}} \\ &= -\frac{(A - B)\sqrt{\sec(c + dx)} \sin(c + dx)}{2d(a + a \sec(c + dx))^{3/2}} + \frac{(5A - B)\sqrt{\sec(c + dx)} \sin(c + dx)}{2ad\sqrt{a + a \sec(c + dx)}} \\ &= -\frac{(7A - 3B) \tanh^{-1}\left(\frac{\sqrt{a} \sqrt{\sec(c + dx)} \sin(c + dx)}{\sqrt{2} \sqrt{a + a \sec(c + dx)}}\right)}{2\sqrt{2} a^{3/2} d} - \frac{(A - B)\sqrt{\sec(c + dx)} \sin(c + dx)}{2d(a + a \sec(c + dx))^{3/2}} \end{aligned}$$

Mathematica [A] time = 1.52, size = 174, normalized size = 1.12

$$\frac{\sin(c + dx) \left((5A - B)\sqrt{1 - \sec(c + dx)} \sec^{\frac{3}{2}}(c + dx) + 4A\sqrt{-((\sec(c + dx) - 1)\sec(c + dx))} \right) + 2\sqrt{2}(7A - 3B)\sin(c + dx)}{2d\sqrt{1 - \sec(c + dx)}(a(\sec(c + dx) + 1))^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(3/2)), x]
```

```
[Out] (2*sqrt(2)*(7*A - 3*B)*ArcTan[(sqrt(2)*sqrt(sec(c + d*x)))/sqrt(1 - sec(c + d*x))]*cos((c + d*x)/2)^3*sec(c + d*x)^2*sin((c + d*x)/2) + ((5*A - B)*sqrt(1 - sec(c + d*x))*sec(c + d*x)^(3/2) + 4*A*sqrt(-((-1 + sec(c + d*x))*sec(c + d*x))))*sin(c + d*x))/(2*d*sqrt(1 - sec(c + d*x))*(a*(1 + sec(c + d*x)))^(3/2))
```

fricas [A] time = 0.46, size = 430, normalized size = 2.76

$$\frac{\sqrt{2} \left((7A - 3B) \cos(dx + c)^2 + 2(7A - 3B) \cos(dx + c) + 7A - 3B \right) \sqrt{a} \log \left(-\frac{a \cos(dx+c)^2 - 2\sqrt{2}\sqrt{a} \sqrt{\frac{a \cos(dx+c)+a}{\cos(dx+c)}}}{\cos(dx+c)^2} \right)}{8 \left(a^2 d \cos(dx + c)^2 + 2 a^2 d \cos(dx + c) + a^2 d \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(sqrt(2))*((7*A - 3*B)*cos(d*x + c)^2 + 2*(7*A - 3*B)*cos(d*x + c) + 7*A - 3*B)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*(4*A*cos(d*x + c)^2 + (5*A - B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), 1/4*(sqrt(2))*((7*A - 3*B)*cos(d*x + c)^2 + 2*(7*A - 3*B)*cos(d*x + c) + 7*A - 3*B)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) + 2*(4*A*cos(d*x + c)^2 + (5*A - B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(a \sec(dx + c) + a)^{\frac{3}{2}} \sqrt{\sec(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^(3/2)*sqrt(sec(d*x + c))), x)
```

maple [B] time = 2.46, size = 287, normalized size = 1.84

$$(-1 + \cos(dx + c)) \left(7A \arctan \left(\frac{\sin(dx+c) \sqrt{-\frac{2}{1+\cos(dx+c)}}}{2} \right) \sqrt{-\frac{2}{1+\cos(dx+c)}} \cos(dx + c) \sin(dx + c) - 3B \arctan \left(\frac{\sin(dx+c)}{\dots} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x)`

[Out]
$$\begin{aligned} & -1/4/d*(-1+\cos(d*x+c))*(7*A*\arctan(1/2*\sin(d*x+c)*(-2/(1+\cos(d*x+c))))^{(1/2)} \\ &)*(-2/(1+\cos(d*x+c)))^{(1/2)}*\cos(d*x+c)*\sin(d*x+c)-3*B*\arctan(1/2*\sin(d*x+c) \\ &)*(-2/(1+\cos(d*x+c)))^{(1/2)}*(-2/(1+\cos(d*x+c)))^{(1/2)}*\cos(d*x+c)*\sin(d*x+c) \\ & +7*\arctan(1/2*\sin(d*x+c)*(-2/(1+\cos(d*x+c)))^{(1/2)}*(-2/(1+\cos(d*x+c)))^{(1/2)} \\ &)*A*\sin(d*x+c)-3*\arctan(1/2*\sin(d*x+c)*(-2/(1+\cos(d*x+c)))^{(1/2)}*(-2/(1+c \\ & os(d*x+c)))^{(1/2)}*B*\sin(d*x+c)-8*A*\cos(d*x+c)^2-2*A*\cos(d*x+c)+2*B*\cos(d*x+ \\ & c)+10*A-2*B)*(a*(1+\cos(d*x+c))/\cos(d*x+c))^{(1/2)}/\sin(d*x+c)^3/(1/\cos(d*x+c) \\ &)^{(1/2)}/a^2 \end{aligned}$$

maxima [B] time = 1.26, size = 8208, normalized size = 52.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")`

[Out]
$$\begin{aligned} & -1/4*((4*(7*\log(\cos(1/2*d*x + 1/2*c))^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2 \\ & *d*x + 1/2*c) + 1) - 7*\log(\cos(1/2*d*x + 1/2*c))^2 + \sin(1/2*d*x + 1/2*c)^2 \\ & - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sin(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c \\ &)^4 + 63*(\log(\cos(1/2*d*x + 1/2*c))^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d \\ & *x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c))^2 + \sin(1/2*d*x + 1/2*c)^2 - 2* \\ & \sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^4 + 4*(7*\log(\cos(1/2*d*x + \\ & 1/2*c))^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 7*\log(\cos \\ & (1/2*d*x + 1/2*c))^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) \\ & - 8*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c)^4 + 70*(\log(\cos(1/2*d*x + 1/ \\ & 2*c))^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2 \\ & *d*x + 1/2*c))^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos \\ & (1/2*d*x + 1/2*c)^2*\sin(1/2*d*x + 1/2*c)^2 + 7*(\log(\cos(1/2*d*x + 1/2*c))^2 \\ & + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + \\ & 1/2*c))^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d* \\ & x + 1/2*c)^4 - 8*\sin(1/2*d*x + 1/2*c)^5 + 28*(7*(\log(\cos(1/2*d*x + 1/2*c))^2 \\ & + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + \end{aligned}$$

$$\begin{aligned}
& \frac{1}{2}c)^2 + \sin(\frac{1}{2}d*x + \frac{1}{2}c)^2 - 2*\sin(\frac{1}{2}d*x + \frac{1}{2}c) + 1))*\cos(\frac{1}{2}d \\
& *x + \frac{1}{2}c) - 8*\cos(\frac{1}{2}d*x + \frac{1}{2}c)*\sin(\frac{1}{2}d*x + \frac{1}{2}c))*\cos(\frac{3}{2}d*x + \frac{3}{ \\
& 2*c)^3 + 4*(21*(\log(\cos(\frac{1}{2}d*x + \frac{1}{2}c)^2 + \sin(\frac{1}{2}d*x + \frac{1}{2}c)^2 + 2*\sin \\
& (\frac{1}{2}d*x + \frac{1}{2}c) + 1) - \log(\cos(\frac{1}{2}d*x + \frac{1}{2}c)^2 + \sin(\frac{1}{2}d*x + \frac{1}{2}c)^ \\
& 2 - 2*\sin(\frac{1}{2}d*x + \frac{1}{2}c) + 1))*\sin(\frac{1}{2}d*x + \frac{1}{2}c) - 24*\sin(\frac{1}{2}d*x + \frac{1}{ \\
& 2*c)^2 - 20)*\sin(\frac{3}{2}d*x + \frac{3}{2}c)^3 - 8*(10*\cos(\frac{1}{2}d*x + \frac{1}{2}c)^2 + 3)*\sin \\
& (\frac{1}{2}d*x + \frac{1}{2}c)^3 + ((7*\log(\cos(\frac{1}{2}d*x + \frac{1}{2}c)^2 + \sin(\frac{1}{2}d*x + \frac{1}{2}c) \\
& ^2 + 2*\sin(\frac{1}{2}d*x + \frac{1}{2}c) + 1) - 7*\log(\cos(\frac{1}{2}d*x + \frac{1}{2}c)^2 + \sin(\frac{1}{2}d \\
& *x + \frac{1}{2}c)^2 - 2*\sin(\frac{1}{2}d*x + \frac{1}{2}c) + 1) - 8*\sin(\frac{1}{2}d*x + \frac{1}{2}c))*\cos(3 \\
& /2*d*x + 3/2*c)^2 + 63*(\log(\cos(\frac{1}{2}d*x + \frac{1}{2}c)^2 + \sin(\frac{1}{2}d*x + \frac{1}{2}c)^2 \\
& + 2*\sin(\frac{1}{2}d*x + \frac{1}{2}c) + 1) - \log(\cos(\frac{1}{2}d*x + \frac{1}{2}c)^2 + \sin(\frac{1}{2}d*x + \\
& \frac{1}{2}c)^2 - 2*\sin(\frac{1}{2}d*x + \frac{1}{2}c) + 1))*\cos(\frac{1}{2}d*x + \frac{1}{2}c)^2 + (7*\log(co \\
& s(\frac{1}{2}d*x + \frac{1}{2}c)^2 + \sin(\frac{1}{2}d*x + \frac{1}{2}c)^2 + 2*\sin(\frac{1}{2}d*x + \frac{1}{2}c) + 1) \\
& - 7*\log(\cos(\frac{1}{2}d*x + \frac{1}{2}c)^2 + \sin(\frac{1}{2}d*x + \frac{1}{2}c)^2 - 2*\sin(\frac{1}{2}d*x + \\
& \frac{1}{2}c) + 1) - 8*\sin(\frac{1}{2}d*x + \frac{1}{2}c))*\sin(\frac{3}{2}d*x + \frac{3}{2}c)^2 + 7*(\log(\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c \\
&) + 1))*\sin(1/2*d*x + 1/2*c)^2 - 8*\sin(1/2*d*x + 1/2*c)^3 + 6*(7*(\log(\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c \\
&) + 1))*\cos(1/2*d*x + 1/2*c) - 8*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c)) \\
& *\cos(3/2*d*x + 3/2*c) + 2*(7*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/ \\
& 2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2 \\
& *d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) - 8*\sin \\
& (1/2*d*x + 1/2*c)^2 - 8)*\sin(3/2*d*x + 3/2*c) - 8*(9*\cos(1/2*d*x + 1/2*c)^2 \\
& + 2)*\sin(1/2*d*x + 1/2*c))*\cos(5/2*d*x + 5/2*c)^2 + (427*(\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) \\
& *\cos(1/2*d*x + 1/2*c)^2 + 35*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/ \\
& 2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2 \\
& *d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 - 40* \\
& \sin(1/2*d*x + 1/2*c)^3 - 8*(61*\cos(1/2*d*x + 1/2*c)^2 + 9)*\sin(1/2*d*x + 1/ \\
& 2*c))*\cos(3/2*d*x + 3/2*c)^2 + ((7*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 7*\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sin(1/2*d*x + 1/2* \\
& c))*\cos(3/2*d*x + 3/2*c)^2 + 63*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& \frac{1}{2}c)^2 + 2*\sin(\frac{1}{2}d*x + \frac{1}{2}c) + 1) - \log(\cos(\frac{1}{2}d*x + \frac{1}{2}c)^2 + \sin(\\
& \frac{1}{2}d*x + \frac{1}{2}c)^2 - 2*\sin(\frac{1}{2}d*x + \frac{1}{2}c) + 1))*\cos(\frac{1}{2}d*x + \frac{1}{2}c)^2 + \\
& (7*\log(\cos(\frac{1}{2}d*x + \frac{1}{2}c)^2 + \sin(\frac{1}{2}d*x + \frac{1}{2}c)^2 + 2*\sin(\frac{1}{2}d*x + 1/ \\
& 2*c) + 1) - 7*\log(\cos(\frac{1}{2}d*x + \frac{1}{2}c)^2 + \sin(\frac{1}{2}d*x + \frac{1}{2}c)^2 - 2*\sin(1 \\
& /2*d*x + 1/2*c) + 1) - 8*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c)^2 + 7*(\\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c \\
&) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d* \\
& x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 - 8*\sin(1/2*d*x + 1/2*c)^3 + 6*(7*(\\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c
\end{aligned}$$

$$\begin{aligned}
&) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c) - 8*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x \\
& + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(7*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2 \\
& *d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c \\
&) - 8*\sin(1/2*d*x + 1/2*c)^2 - 8)*\sin(3/2*d*x + 3/2*c) - 8*(9*\cos(1/2*d*x + \\
& 1/2*c)^2 + 2)*\sin(1/2*d*x + 1/2*c))*\sin(5/2*d*x + 5/2*c)^2 + (8*(7*\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) \\
& - 7*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1 \\
& /2*c) + 1) - 8*\sin(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c)^2 + 259*(\log(\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2* \\
& c) + 1))*\cos(1/2*d*x + 1/2*c)^2 + 91*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2* \\
& d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) \\
& ^2 - 104*\sin(1/2*d*x + 1/2*c)^3 + 28*(7*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1 \\
& /2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^ \\
& 2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2 \\
& *c) - 8*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) - 8 \\
& *(37*\cos(1/2*d*x + 1/2*c)^2 + 21)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c \\
&)^2 + 2*(2*(7*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1 \\
& /2*d*x + 1/2*c) + 1) - 7*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^ \\
& 2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sin(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2 \\
& *c)^3 + 63*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2 \\
& *d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - \\
& 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^3 + 7*(\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*c \\
& os(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c)^2 - 8*\cos(1/2*d*x + 1/2*c)*\sin(1/2 \\
& *d*x + 1/2*c)^3 + 13*(7*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^ \\
& 2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c) - 8*\cos(1/2* \\
& d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c)^2 + (2*(7*\log(\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& 7*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1) - 8*\sin(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 7*(\log(\cos(1/2*d* \\
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(c \\
& os(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1 \\
&))*\cos(1/2*d*x + 1/2*c) - 8*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c))*\sin(\\
& 3/2*d*x + 3/2*c)^2 + 2*(84*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2* \\
& c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d \\
& *x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 + 7*(lo \\
& g(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 - 8*\sin(1/2*d*x + 1/2*c)^3 - 16*(6*co
\end{aligned}$$

$$\begin{aligned}
& s(1/2*d*x + 1/2*c)^2 + 1)*\sin(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(7 \\
& *(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2* \\
& d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 8*\cos(1/2*d* \\
& x + 1/2*c)*\sin(1/2*d*x + 1/2*c)^2 + 2*(7*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(\\
& 1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/ \\
& 2*c) - 8*\sin(1/2*d*x + 1/2*c)^2 - 8)*\cos(3/2*d*x + 3/2*c) - 8*\cos(1/2*d*x + \\
& 1/2*c))*\sin(3/2*d*x + 3/2*c) - 8*(9*\cos(1/2*d*x + 1/2*c)^3 + 2*\cos(1/2*d*x \\
& + 1/2*c))*\sin(1/2*d*x + 1/2*c))*\cos(5/2*d*x + 5/2*c) + 2*(147*(\log(\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - lo \\
& g(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) \\
& + 1))*\cos(1/2*d*x + 1/2*c)^3 + 35*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \si \\
& n(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)*\si \\
& n(1/2*d*x + 1/2*c)^2 - 40*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c)^3 - 56* \\
& (3*\cos(1/2*d*x + 1/2*c)^3 + \cos(1/2*d*x + 1/2*c))*\sin(1/2*d*x + 1/2*c))*\cos \\
& (3/2*d*x + 3/2*c) + 2*(2*(7*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2* \\
& c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 7*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2 \\
& *d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sin(1/2*d*x + 1/2*c))*\sin \\
& (3/2*d*x + 3/2*c)^3 + 63*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
& ^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2*\sin(1/2*d \\
& *x + 1/2*c) + 7*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\si \\
& n(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
& ^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^3 - 8*\sin(1/2*d*x + \\
& 1/2*c)^4 + (7*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(\\
& 1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) - 8*\sin(1/2*d*x + 1/2* \\
& c)^2 - 4)*\cos(3/2*d*x + 3/2*c)^2 + (35*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2* \\
& c) - 40*\sin(1/2*d*x + 1/2*c)^2 - 36)*\sin(3/2*d*x + 3/2*c)^2 - 4*(18*\cos(1/2 \\
& *d*x + 1/2*c)^2 + 5)*\sin(1/2*d*x + 1/2*c)^2 + 6*(7*(\log(\cos(1/2*d*x + 1/2*c) \\
&)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d* \\
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/ \\
& 2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 8*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + \\
& 1/2*c)^2 - 4*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) - 36*\cos(1/2*d*x + \\
& 1/2*c)^2 + 2*((7*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\si \\
& n(1/2*d*x + 1/2*c) + 1) - 7*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2* \\
& c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sin(1/2*d*x + 1/2*c))*\cos(3/2*d*x + \\
& 3/2*c)^2 + 63*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(\\
& 1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 + 14*(\log(\cos(1/2*d* \\
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(c
\end{aligned}$$

$$\begin{aligned}
& \cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1 \\
&))*\sin(1/2*d*x + 1/2*c)^2 - 16*\sin(1/2*d*x + 1/2*c)^3 + 6*(7*(\log(\cos(1/2*d \\
& *x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\\
& \cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + \\
& 1))*\cos(1/2*d*x + 1/2*c) - 8*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c))*\cos \\
& (3/2*d*x + 3/2*c) - 4*(18*\cos(1/2*d*x + 1/2*c)^2 + 7)*\sin(1/2*d*x + 1/2*c)) \\
& *\sin(3/2*d*x + 3/2*c))*\sin(5/2*d*x + 5/2*c) + 2*(133*(\log(\cos(1/2*d*x + 1/2 \\
& *c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2* \\
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(\\
& 1/2*d*x + 1/2*c)^2*\sin(1/2*d*x + 1/2*c) + 21*(\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x \\
& + 1/2*c)^3 - 24*\sin(1/2*d*x + 1/2*c)^4 + 2*(21*(\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d* \\
& x + 1/2*c) - 24*\sin(1/2*d*x + 1/2*c)^2 - 20)*\cos(3/2*d*x + 3/2*c)^2 - 8*(19 \\
& *\cos(1/2*d*x + 1/2*c)^2 + 7)*\sin(1/2*d*x + 1/2*c)^2 + 16*(7*(\log(\cos(1/2*d* \\
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(c \\
& os(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1 \\
&))*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 8*\cos(1/2*d*x + 1/2*c)*\sin(1 \\
& /2*d*x + 1/2*c)^2 - 5*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) - 80*\cos(1 \\
& /2*d*x + 1/2*c)^2)*\sin(3/2*d*x + 3/2*c) - 8*(9*\cos(1/2*d*x + 1/2*c)^4 + 11* \\
& \cos(1/2*d*x + 1/2*c)^2)*\sin(1/2*d*x + 1/2*c))*A*\sqrt{a}/(4*\sqrt{2})*a^2*\cos(\\
& 3/2*d*x + 3/2*c)^4 + 28*\sqrt{2})*a^2*\cos(3/2*d*x + 3/2*c)^3*\cos(1/2*d*x + 1/ \\
& 2*c) + 9*\sqrt{2})*a^2*\cos(1/2*d*x + 1/2*c)^4 + 4*\sqrt{2})*a^2*\sin(3/2*d*x + 3 \\
& /2*c)^4 + 12*\sqrt{2})*a^2*\sin(3/2*d*x + 3/2*c)^3*\sin(1/2*d*x + 1/2*c) + 10*s \\
& \sqrt{2})*a^2*\cos(1/2*d*x + 1/2*c)^2*\sin(1/2*d*x + 1/2*c)^2 + \sqrt{2})*a^2*\sin(\\
& 1/2*d*x + 1/2*c)^4 + (\sqrt{2})*a^2*\cos(3/2*d*x + 3/2*c)^2 + 6*\sqrt{2})*a^2*co \\
& s(3/2*d*x + 3/2*c)*\cos(1/2*d*x + 1/2*c) + 9*\sqrt{2})*a^2*\cos(1/2*d*x + 1/2*c \\
&)^2 + \sqrt{2})*a^2*\sin(3/2*d*x + 3/2*c)^2 + 2*\sqrt{2})*a^2*\sin(3/2*d*x + 3/2* \\
& c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2})*a^2*\sin(1/2*d*x + 1/2*c)^2)*\cos(5/2*d*x + \\
& 5/2*c)^2 + (61*\sqrt{2})*a^2*\cos(1/2*d*x + 1/2*c)^2 + 5*\sqrt{2})*a^2*\sin(1/2* \\
& d*x + 1/2*c)^2)*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2})*a^2*\cos(3/2*d*x + 3/2*c)^ \\
& 2 + 6*\sqrt{2})*a^2*\cos(3/2*d*x + 3/2*c)*\cos(1/2*d*x + 1/2*c) + 9*\sqrt{2})*a^2 \\
& *\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*a^2*\sin(3/2*d*x + 3/2*c)^2 + 2*\sqrt{2})*a^ \\
& 2*\sin(3/2*d*x + 3/2*c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2})*a^2*\sin(1/2*d*x + 1/2 \\
& *c)^2)*\sin(5/2*d*x + 5/2*c)^2 + (8*\sqrt{2})*a^2*\cos(3/2*d*x + 3/2*c)^2 + 28* \\
& \sqrt{2})*a^2*\cos(3/2*d*x + 3/2*c)*\cos(1/2*d*x + 1/2*c) + 37*\sqrt{2})*a^2*\cos(\\
& 1/2*d*x + 1/2*c)^2 + 13*\sqrt{2})*a^2*\sin(1/2*d*x + 1/2*c)^2)*\sin(3/2*d*x + 3 \\
& /2*c)^2 + 2*(2*\sqrt{2})*a^2*\cos(3/2*d*x + 3/2*c)^3 + 13*\sqrt{2})*a^2*\cos(3/2* \\
& d*x + 3/2*c)^2*\cos(1/2*d*x + 1/2*c) + 9*\sqrt{2})*a^2*\cos(1/2*d*x + 1/2*c)^3 \\
& + \sqrt{2})*a^2*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c)^2 + (2*\sqrt{2})*a^2* \\
& \cos(3/2*d*x + 3/2*c) + \sqrt{2})*a^2*\cos(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2* \\
& c)^2 + 2*(12*\sqrt{2})*a^2*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*a^2*\sin(1/2*d*x + \\
& 1/2*c)^2)*\cos(3/2*d*x + 3/2*c) + 2*(2*\sqrt{2})*a^2*\cos(3/2*d*x + 3/2*c)*\sin
\end{aligned}$$

$$\begin{aligned}
& (1/2*d*x + 1/2*c) + \text{sqrt}(2)*a^2*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c))* \\
& \sin(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c) + 2*(21*\text{sqrt}(2)*a^2*\cos(1/2*d*x \\
& + 1/2*c)^3 + 5*\text{sqrt}(2)*a^2*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c)^2)*\cos \\
& (3/2*d*x + 3/2*c) + 2*(2*\text{sqrt}(2)*a^2*\sin(3/2*d*x + 3/2*c)^3 + \text{sqrt}(2)*a^2*c \\
& \cos(3/2*d*x + 3/2*c)^2*\sin(1/2*d*x + 1/2*c) + 6*\text{sqrt}(2)*a^2*\cos(3/2*d*x + 3/ \\
& 2*c)*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) + 9*\text{sqrt}(2)*a^2*\cos(1/2*d*x \\
& + 1/2*c)^2*\sin(1/2*d*x + 1/2*c) + 5*\text{sqrt}(2)*a^2*\sin(3/2*d*x + 3/2*c)^2*\sin(\\
& 1/2*d*x + 1/2*c) + \text{sqrt}(2)*a^2*\sin(1/2*d*x + 1/2*c)^3 + 2*(\text{sqrt}(2)*a^2*\cos(\\
& 3/2*d*x + 3/2*c)^2 + 6*\text{sqrt}(2)*a^2*\cos(3/2*d*x + 3/2*c)*\cos(1/2*d*x + 1/2*c \\
&) + 9*\text{sqrt}(2)*a^2*\cos(1/2*d*x + 1/2*c)^2 + 2*\text{sqrt}(2)*a^2*\sin(1/2*d*x + 1/2* \\
& c)^2)*\sin(3/2*d*x + 3/2*c))*\sin(5/2*d*x + 5/2*c) + 2*(6*\text{sqrt}(2)*a^2*\cos(3/2 \\
& *d*x + 3/2*c)^2*\sin(1/2*d*x + 1/2*c) + 16*\text{sqrt}(2)*a^2*\cos(3/2*d*x + 3/2*c)* \\
& \cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) + 19*\text{sqrt}(2)*a^2*\cos(1/2*d*x + 1/ \\
& 2*c)^2*\sin(1/2*d*x + 1/2*c) + 3*\text{sqrt}(2)*a^2*\sin(1/2*d*x + 1/2*c)^3)*\sin(3/2 \\
& *d*x + 3/2*c)) - (3*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/ \\
& 2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(2*d*x + 2*c)^2 + 12*(\log(\cos(1/2* \\
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + \\
& 1))*\cos(d*x + c)^2 + 3*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^ \\
& 2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(2*d*x + 2*c)^2 + 12*(\log(\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2* \\
& c) + 1))*\sin(d*x + c)^2 + 2*(6*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1 \\
& /2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 3*\log(\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1) - 2*\sin(3/2*d*x + 3/2*c) + 2*\sin(1/2*d*x + 1/2*c))*\cos(2*d*x + 2*c \\
&) + 4*(3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d* \\
& x + 1/2*c) + 1) - 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 \\
& *\sin(1/2*d*x + 1/2*c) + 1) + 2*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) + 4*(3*(1 \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& + 1/2*c) + 1))*\sin(d*x + c) + \cos(3/2*d*x + 3/2*c) - \cos(1/2*d*x + 1/2*c)) \\
& *\sin(2*d*x + 2*c) - 4*(2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c) + 8*\cos(3/2 \\
& *d*x + 3/2*c)*\sin(d*x + c) - 8*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + 3*\log(co \\
& s(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) \\
& - 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) + 4*\sin(1/2*d*x + 1/2*c))*B/((\text{sqrt}(2)*a*\cos(2*d*x + 2*c)^2 + 4* \\
& \text{sqrt}(2)*a*\cos(d*x + c)^2 + \text{sqrt}(2)*a*\sin(2*d*x + 2*c)^2 + 4*\text{sqrt}(2)*a*\sin(2 \\
& *d*x + 2*c)*\sin(d*x + c) + 4*\text{sqrt}(2)*a*\sin(d*x + c)^2 + 4*\text{sqrt}(2)*a*\cos(d*x \\
& + c) + 2*(2*\text{sqrt}(2)*a*\cos(d*x + c) + \text{sqrt}(2)*a)*\cos(2*d*x + 2*c) + \text{sqrt}(2) \\
& *a)*\text{sqrt}(a))/d
\end{aligned}$$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\left(a + \frac{a}{\cos(c+dx)}\right)^{3/2} \sqrt{\frac{1}{\cos(c+dx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/((a + a/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(1/2)),x)

[Out] int((A + B/cos(c + d*x))/((a + a/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \sec(c + dx)}{(a(\sec(c + dx) + 1))^{\frac{3}{2}} \sqrt{\sec(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(3/2)/sec(d*x+c)**(1/2),x)

[Out] Integral((A + B*sec(c + d*x))/((a*(sec(c + d*x) + 1))**(3/2)*sqrt(sec(c + d*x))), x)

$$3.260 \quad \int \frac{A+B \sec(c+dx)}{\sec^2(c+dx)(a+a \sec(c+dx))^{3/2}} dx$$

Optimal. Leaf size=203

$$\frac{(11A - 7B) \tanh^{-1} \left(\frac{\sqrt{a} \sin(c+dx) \sqrt{\sec(c+dx)}}{\sqrt{2} \sqrt{a \sec(c+dx)+a}} \right)}{2\sqrt{2} a^{3/2} d} - \frac{(19A - 15B) \sin(c+dx) \sqrt{\sec(c+dx)}}{6ad \sqrt{a \sec(c+dx)+a}} + \frac{(7A - 3B) \sin(c+dx)}{6ad \sqrt{\sec(c+dx)} \sqrt{a \sec(c+dx)}}$$

[Out] $1/4*(11*A-7*B)*\operatorname{arctanh}(1/2*\sin(d*x+c)*a^{1/2}*\sec(d*x+c)^{(1/2)}*2^{(1/2)}/(a+a*\sec(d*x+c))^{(1/2)})/a^{(3/2)}/d*2^{(1/2)}-1/2*(A-B)*\sin(d*x+c)/d/(a+a*\sec(d*x+c))^{(3/2)}/\sec(d*x+c)^{(1/2)}+1/6*(7*A-3*B)*\sin(d*x+c)/a/d/\sec(d*x+c)^{(1/2)}/(a+a*\sec(d*x+c))^{(1/2)}-1/6*(19*A-15*B)*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/a/d/(a+a*\sec(d*x+c))^{(1/2)}$

Rubi [A] time = 0.55, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4020, 4022, 4013, 3808, 206}

$$\frac{(11A - 7B) \tanh^{-1} \left(\frac{\sqrt{a} \sin(c+dx) \sqrt{\sec(c+dx)}}{\sqrt{2} \sqrt{a \sec(c+dx)+a}} \right)}{2\sqrt{2} a^{3/2} d} - \frac{(19A - 15B) \sin(c+dx) \sqrt{\sec(c+dx)}}{6ad \sqrt{a \sec(c+dx)+a}} + \frac{(7A - 3B) \sin(c+dx)}{6ad \sqrt{\sec(c+dx)} \sqrt{a \sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)),x]

[Out] $((11*A - 7*B)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Sqrt}[\operatorname{Sec}[c + d*x]]*\operatorname{Sin}[c + d*x])/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a + a*\operatorname{Sec}[c + d*x]])])/(2*\operatorname{Sqrt}[2]*a^{(3/2)}*d) - ((A - B)*\operatorname{Sin}[c + d*x])/(2*d*\operatorname{Sqrt}[\operatorname{Sec}[c + d*x]]*(a + a*\operatorname{Sec}[c + d*x])^{(3/2)}) + ((7*A - 3*B)*\operatorname{Sin}[c + d*x])/(6*a*d*\operatorname{Sqrt}[\operatorname{Sec}[c + d*x]]*\operatorname{Sqrt}[a + a*\operatorname{Sec}[c + d*x]]) - ((19*A - 15*B)*\operatorname{Sqrt}[\operatorname{Sec}[c + d*x]]*\operatorname{Sin}[c + d*x])/(6*a*d*\operatorname{Sqrt}[a + a*\operatorname{Sec}[c + d*x]])$

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]]/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 3808

Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] := Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (
a_))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_)), x_Symbol] := Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
- b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] && !LeQ[m, -1]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (
a_))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_)), x_Symbol] := -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
+ f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0]
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (
a_))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_)), x_Symbol] := Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
- A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{A + B \sec(c + dx)}{\sec^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))^{3/2}} dx &= -\frac{(A - B) \sin(c + dx)}{2d\sqrt{\sec(c + dx)}(a + a \sec(c + dx))^{3/2}} + \frac{\int \frac{\frac{1}{2}a(7A-3B)-2a(A-B)\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} dx}{2a^2} \\
&= -\frac{(A - B) \sin(c + dx)}{2d\sqrt{\sec(c + dx)}(a + a \sec(c + dx))^{3/2}} + \frac{(7A - 3B) \sin(c + dx)}{6ad\sqrt{\sec(c + dx)}\sqrt{a + a \sec(c + dx)}} \\
&= -\frac{(A - B) \sin(c + dx)}{2d\sqrt{\sec(c + dx)}(a + a \sec(c + dx))^{3/2}} + \frac{(7A - 3B) \sin(c + dx)}{6ad\sqrt{\sec(c + dx)}\sqrt{a + a \sec(c + dx)}} \\
&= -\frac{(A - B) \sin(c + dx)}{2d\sqrt{\sec(c + dx)}(a + a \sec(c + dx))^{3/2}} + \frac{(7A - 3B) \sin(c + dx)}{6ad\sqrt{\sec(c + dx)}\sqrt{a + a \sec(c + dx)}} \\
&= \frac{(11A - 7B) \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{(A - B) \sin(c + dx)}{2d\sqrt{\sec(c + dx)}(a + a \sec(c + dx))^{3/2}}
\end{aligned}$$

Mathematica [A] time = 1.80, size = 173, normalized size = 0.85

$$\frac{\tan(c + dx)\sqrt{1 - \sec(c + dx)}(\sec(c + dx)(2A \cos(2(c + dx)) - 17A + 15B) + 12(B - A)) - 6\sqrt{2}(11A - 7B) \sin(c + dx)}{6d\sqrt{-((\sec(c + dx) - 1)\sec(c + dx))}(a(\sec(c + dx) + 1))^{3/2}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)), x]

[Out] (-6*Sqrt[2]*(11*A - 7*B)*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]])*Cos[(c + d*x)/2]^3*Sec[c + d*x]^(5/2)*Sin[(c + d*x)/2] + Sqrt[1 - Sec[c + d*x]]*(12*(-A + B) + (-17*A + 15*B + 2*A*Cos[2*(c + d*x)])*Sec[c + d*x])*Tan[c + d*x])/(6*d*Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])]*(a*(1 + Sec[c + d*x]))^(3/2))

fricas [A] time = 0.46, size = 464, normalized size = 2.29

$$\left[\frac{3\sqrt{2}\left((11A - 7B)\cos(dx + c)^2 + 2(11A - 7B)\cos(dx + c) + 11A - 7B\right)\sqrt{a} \log\left(\frac{a\cos(dx+c)^2 + 2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(a)}{\cos(dx+c)}}}{\cos(dx+c)}\right)}{24\left(a^2d\cos(dx + c)^2 + 2a\cos(dx + c) + 1\right)^{3/2}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")

[Out] [-1/24*(3*sqrt(2)*((11*A - 7*B)*cos(d*x + c)^2 + 2*(11*A - 7*B)*cos(d*x + c) + 11*A - 7*B)*sqrt(a)*log(-(a*cos(d*x + c))^2 + 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*(4*A*cos(d*x + c)^3 - 12*(A - B)*cos(d*x + c)^2 - (19*A - 15*B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), -1/12*(3*sqrt(2)*((11*A - 7*B)*cos(d*x + c)^2 + 2*(11*A - 7*B)*cos(d*x + c) + 11*A - 7*B)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c)))/(a*sin(d*x + c))) - 2*(4*A*cos(d*x + c)^3 - 12*(A - B)*cos(d*x + c)^2 - (19*A - 15*B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(a \sec(dx + c) + a)^{\frac{3}{2}} \sec(dx + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/2)), x)

maple [A] time = 2.50, size = 317, normalized size = 1.56

$$\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}} (-1 + \cos(dx + c)) \left(33A \arctan \left(\frac{\sin(dx+c) \sqrt{-\frac{2}{1+\cos(dx+c)}}}{2} \right) \sqrt{-\frac{2}{1+\cos(dx+c)}} \cos(dx + c) \sin(dx + c) - \dots \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x)

[Out] 1/12/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(33*A*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)-21*B*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)+8*A*cos(d*x+c)^3+33*arctan(1/2*sin(d

$x+c)*(-2/(1+\cos(d*x+c)))^{(1/2)}*(-2/(1+\cos(d*x+c)))^{(1/2)}*A*\sin(d*x+c)-21*\arctan(1/2*\sin(d*x+c)*(-2/(1+\cos(d*x+c)))^{(1/2)}*(-2/(1+\cos(d*x+c)))^{(1/2)}*B*\sin(d*x+c)-32*A*\cos(d*x+c)^2+24*B*\cos(d*x+c)^2-14*A*\cos(d*x+c)+6*B*\cos(d*x+c)+38*A-30*B)*\cos(d*x+c)^2*(1/\cos(d*x+c))^{(3/2)}/\sin(d*x+c)^3/a^2$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\left(a + \frac{a}{\cos(c+dx)}\right)^{3/2} \left(\frac{1}{\cos(c+dx)}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/((a + a/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(3/2)), x)

[Out] int((A + B/cos(c + d*x))/((a + a/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(3/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \sec(c + dx)}{(a(\sec(c + dx) + 1))^{\frac{3}{2}} \sec^{\frac{3}{2}}(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)**(3/2)/(a+a*sec(d*x+c))**(3/2),x)

[Out] Integral((A + B*sec(c + d*x))/((a*(sec(c + d*x) + 1))**(3/2)*sec(c + d*x)**(3/2)), x)

$$3.261 \quad \int \frac{A+B \sec(c+dx)}{\sec^2(c+dx)(a+a \sec(c+dx))^{3/2}} dx$$

Optimal. Leaf size=250

$$-\frac{(15A-11B) \tanh^{-1}\left(\frac{\sqrt{a} \sin(c+dx) \sqrt{\sec(c+dx)}}{\sqrt{2} \sqrt{a \sec(c+dx)+a}}\right)}{2\sqrt{2} a^{3/2} d} + \frac{(9A-5B) \sin(c+dx)}{10ad \sec^2(c+dx) \sqrt{a \sec(c+dx)+a}} - \frac{(A-B) \sin(c+dx)}{2d \sec^2(c+dx) (a \sec(c+dx)+a)^{3/2}}$$

[Out] $-1/2*(A-B)*\sin(d*x+c)/d/\sec(d*x+c)^{(3/2)}/(a+a*\sec(d*x+c))^{(3/2)}-1/4*(15*A-11*B)*\operatorname{arctanh}(1/2*\sin(d*x+c)*a^{(1/2)}*\sec(d*x+c)^{(1/2)}*2^{(1/2)}/(a+a*\sec(d*x+c))^{(1/2)})/a^{(3/2)}/d*2^{(1/2)}+1/10*(9*A-5*B)*\sin(d*x+c)/a/d/\sec(d*x+c)^{(3/2)}/(a+a*\sec(d*x+c))^{(1/2)}-1/30*(39*A-35*B)*\sin(d*x+c)/a/d/\sec(d*x+c)^{(1/2)}/(a+a*\sec(d*x+c))^{(1/2)}+1/30*(147*A-95*B)*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/a/d/(a+a*\sec(d*x+c))^{(1/2)}$

Rubi [A] time = 0.73, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4020, 4022, 4013, 3808, 206}

$$-\frac{(15A-11B) \tanh^{-1}\left(\frac{\sqrt{a} \sin(c+dx) \sqrt{\sec(c+dx)}}{\sqrt{2} \sqrt{a \sec(c+dx)+a}}\right)}{2\sqrt{2} a^{3/2} d} + \frac{(9A-5B) \sin(c+dx)}{10ad \sec^2(c+dx) \sqrt{a \sec(c+dx)+a}} - \frac{(A-B) \sin(c+dx)}{2d \sec^2(c+dx) (a \sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(A+B*\operatorname{Sec}[c+d*x])/(\operatorname{Sec}[c+d*x]^{(5/2)}*(a+a*\operatorname{Sec}[c+d*x])^{(3/2)}),x]$

[Out] $-((15*A-11*B)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Sqrt}[\operatorname{Sec}[c+d*x]]*\operatorname{Sin}[c+d*x])/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a+a*\operatorname{Sec}[c+d*x]])])/(2*\operatorname{Sqrt}[2]*a^{(3/2)}*d)-((A-B)*\operatorname{Sin}[c+d*x])/((2*d*\operatorname{Sec}[c+d*x]^{(3/2)}*(a+a*\operatorname{Sec}[c+d*x])^{(3/2)})+((9*A-5*B)*\operatorname{Sin}[c+d*x]))/(10*a*d*\operatorname{Sec}[c+d*x]^{(3/2)}*\operatorname{Sqrt}[a+a*\operatorname{Sec}[c+d*x]])-((39*A-35*B)*\operatorname{Sin}[c+d*x])/((30*a*d*\operatorname{Sqrt}[\operatorname{Sec}[c+d*x]]*\operatorname{Sqrt}[a+a*\operatorname{Sec}[c+d*x]])+((147*A-95*B)*\operatorname{Sqrt}[\operatorname{Sec}[c+d*x]]*\operatorname{Sin}[c+d*x]))/(30*a*d*\operatorname{Sqrt}[a+a*\operatorname{Sec}[c+d*x]])$

Rule 206

$\operatorname{Int}[(a_+ + (b_+)*(x_+)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1*\operatorname{ArcTanh}[(\operatorname{Rt}[-b, 2]*x)/\operatorname{Rt}[a, 2]])/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /; \operatorname{FreeQ}\{a, b\}, x] \ \&\& \operatorname{NegQ}[a/b] \ \&\& (\operatorname{Gt} Q[a, 0] \ || \ \operatorname{Lt} Q[b, 0])$

Rule 3808

$\operatorname{Int}[\operatorname{Sqrt}[\operatorname{csc}[e_+ + (f_+)*(x_+)]*(d_+)]/\operatorname{Sqrt}[\operatorname{csc}[e_+ + (f_+)*(x_+)]*(b_+ + (a_+))], x_Symbol] \rightarrow \operatorname{Dist}[(-2*b*d)/(a*f), \operatorname{Subst}[\operatorname{Int}[1/(2*b - d*x^2), x], x]$

, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]), x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 4013

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && !LeQ[m, -1]

Rule 4020

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m + 1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0]

Rule 4022

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]

Rubi steps

$$\begin{aligned}
\int \frac{A + B \sec(c + dx)}{\sec^{\frac{5}{2}}(c + dx)(a + a \sec(c + dx))^{\frac{3}{2}}} dx &= -\frac{(A - B) \sin(c + dx)}{2d \sec^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))^{\frac{3}{2}}} + \int \frac{\frac{1}{2}a(9A-5B)-3a(A-B) \sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a \sec(c+dx)}} dx \\
&= -\frac{(A - B) \sin(c + dx)}{2d \sec^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))^{\frac{3}{2}}} + \frac{(9A - 5B) \sin(c + dx)}{10ad \sec^{\frac{3}{2}}(c + dx)\sqrt{a + a \sec(c + dx)}} \\
&= -\frac{(A - B) \sin(c + dx)}{2d \sec^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))^{\frac{3}{2}}} + \frac{(9A - 5B) \sin(c + dx)}{10ad \sec^{\frac{3}{2}}(c + dx)\sqrt{a + a \sec(c + dx)}} \\
&= -\frac{(A - B) \sin(c + dx)}{2d \sec^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))^{\frac{3}{2}}} + \frac{(9A - 5B) \sin(c + dx)}{10ad \sec^{\frac{3}{2}}(c + dx)\sqrt{a + a \sec(c + dx)}} \\
&= -\frac{(A - B) \sin(c + dx)}{2d \sec^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))^{\frac{3}{2}}} + \frac{(9A - 5B) \sin(c + dx)}{10ad \sec^{\frac{3}{2}}(c + dx)\sqrt{a + a \sec(c + dx)}} \\
&= -\frac{(15A - 11B) \tanh^{-1}\left(\frac{\sqrt{a} \sqrt{\sec(c+dx)} \sin(c+dx)}{\sqrt{2} \sqrt{a+a \sec(c+dx)}}\right)}{2\sqrt{2} a^{\frac{3}{2}} d} - \frac{(A - B) \sin(c + dx)}{2d \sec^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))^{\frac{3}{2}}}
\end{aligned}$$

Mathematica [A] time = 1.50, size = 171, normalized size = 0.68

$$\frac{\sec(c + dx) \left(\frac{15\sqrt{2}(15A-11B) \cos^2\left(\frac{1}{2}(c+dx)\right) \tan(c+dx) \tan^{-1}\left(\frac{\sqrt{2} \sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)}{\sqrt{1-\sec(c+dx)}} + \sin(c + dx) \sqrt{\sec(c + dx)} (3(39A - 20B) \cos(c + dx) + 30d(a(\sec(c + dx) + 1))^{\frac{3}{2}}) \right)}{30d(a(\sec(c + dx) + 1))^{\frac{3}{2}}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/2)), x]

[Out] (Sec[c + d*x]*((141*A - 85*B + 3*(39*A - 20*B)*Cos[c + d*x] + (-6*A + 10*B)*Cos[2*(c + d*x)] + 3*A*Cos[3*(c + d*x)])*Sqrt[Sec[c + d*x]]*Sin[c + d*x] + (15*Sqrt[2]*(15*A - 11*B)*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]])*Cos[(c + d*x)/2]^2*Tan[c + d*x])/Sqrt[1 - Sec[c + d*x]])/(30*d*(a*(1 + Sec[c + d*x]))^(3/2))

fricas [A] time = 0.49, size = 500, normalized size = 2.00

$$\frac{15\sqrt{2}\left((15A-11B)\cos(dx+c)^2+2(15A-11B)\cos(dx+c)+15A-11B\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a}\sqrt{\cos(dx+c)+a}}{120(a^2d\cos(dx+c)+a^2d)}\right)}{120(a^2d\cos(dx+c)+a^2d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")

[Out] [-1/120*(15*sqrt(2)*((15*A - 11*B)*cos(d*x + c)^2 + 2*(15*A - 11*B)*cos(d*x + c) + 15*A - 11*B)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*(12*A*cos(d*x + c)^4 - 4*(3*A - 5*B)*cos(d*x + c)^3 + 12*(9*A - 5*B)*cos(d*x + c)^2 + (147*A - 95*B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), 1/60*(15*sqrt(2)*((15*A - 11*B)*cos(d*x + c)^2 + 2*(15*A - 11*B)*cos(d*x + c) + 15*A - 11*B)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) + 2*(12*A*cos(d*x + c)^4 - 4*(3*A - 5*B)*cos(d*x + c)^3 + 12*(9*A - 5*B)*cos(d*x + c)^2 + (147*A - 95*B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx+c) + A}{(a \sec(dx+c) + a)^{\frac{3}{2}} \sec(dx+c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(5/2)), x)

maple [A] time = 2.80, size = 339, normalized size = 1.36

$$\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}} (-1 + \cos(dx+c)) \left(225A \arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right) \sqrt{-\frac{2}{1+\cos(dx+c)}} \cos(dx+c) \sin(dx+c) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x)

[Out] -1/60/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(225*A*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)-24*A*cos(d*x+c)^4-165*B*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c))))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)+225*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*A*sin(d*x+c)+48*A*cos(d*x+c)^3-165*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*B*sin(d*x+c)-40*B*cos(d*x+c)^3-240*A*cos(d*x+c)^2+160*B*cos(d*x+c)^2-78*A*cos(d*x+c)+70*B*cos(d*x+c)+294*A-190*B)*cos(d*x+c)^3*(1/cos(d*x+c))^(5/2)/sin(d*x+c)^3/a^2

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is undefined.

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\left(a + \frac{a}{\cos(c+dx)}\right)^{3/2} \left(\frac{1}{\cos(c+dx)}\right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/((a + a/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(5/2)),x)

[Out] int((A + B/cos(c + d*x))/((a + a/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(5/2)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)**(5/2)/(a+a*sec(d*x+c))**(3/2),x)

[Out] Timed out

$$3.262 \quad \int \frac{\sec^2(c+dx)(A+B \sec(c+dx))}{(a+a \sec(c+dx))^{5/2}} dx$$

Optimal. Leaf size=246

$$\frac{(43A - 115B) \tanh^{-1}\left(\frac{\sqrt{a} \sin(c+dx) \sqrt{\sec(c+dx)}}{\sqrt{2} \sqrt{a \sec(c+dx)+a}}\right)}{16\sqrt{2} a^{5/2} d} + \frac{(2A - 5B) \sinh^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}}\right)}{a^{5/2} d} - \frac{(11A - 35B) \sin(c+dx) \sec^2(c+dx)}{16a^2 d \sqrt{a \sec(c+dx)+a}}$$

[Out] (2*A-5*B)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(5/2)/d+1/4*(A-B)*sec(d*x+c)^(7/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(5/2)+1/16*(7*A-15*B)*sec(d*x+c)^(5/2)*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^(3/2)-1/32*(43*A-115*B)*arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2)-1/16*(11*A-35*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/a^2/d/(a+a*sec(d*x+c))^(1/2)

Rubi [A] time = 0.82, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4019, 4021, 4023, 3808, 206, 3801, 215}

$$\frac{(11A - 35B) \sin(c+dx) \sec^2(c+dx)}{16a^2 d \sqrt{a \sec(c+dx)+a}} - \frac{(43A - 115B) \tanh^{-1}\left(\frac{\sqrt{a} \sin(c+dx) \sqrt{\sec(c+dx)}}{\sqrt{2} \sqrt{a \sec(c+dx)+a}}\right)}{16\sqrt{2} a^{5/2} d} + \frac{(2A - 5B) \sinh^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}}\right)}{a^{5/2} d}$$

Antiderivative was successfully verified.

[In] Int[(Sec[c + d*x]^(7/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(5/2), x]

[Out] ((2*A - 5*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]])/(a^(5/2)*d) - ((43*A - 115*B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) + ((A - B)*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(4*d*(a + a*Sec[c + d*x])^(5/2)) + ((7*A - 15*B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/2)) - ((11*A - 35*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(16*a^2*d*Sqrt[a + a*Sec[c + d*x]])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4021

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x
] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)
*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &&
GtQ[n, 1]
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^{\frac{7}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{5/2}} dx &= \frac{(A-B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \int \frac{\sec^{\frac{5}{2}}(c+dx)\left(\frac{5}{2}a(A-B)-a(A-5B)\sec(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}} dx \\
&= \frac{(A-B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(7A-15B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} \\
&= \frac{(A-B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(7A-15B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} \\
&= \frac{(A-B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(7A-15B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} \\
&= \frac{(A-B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(7A-15B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} \\
&= \frac{(2A-5B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d} - \frac{(43A-115B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d}
\end{aligned}$$

Mathematica [B] time = 6.17, size = 941, normalized size = 3.83

$$\frac{7B(\sec(c+dx)+1)\sin(c+dx)\sec^{\frac{11}{2}}(c+dx)}{16d(a(\sec(c+dx)+1))^{5/2}} - \frac{B\sin(c+dx)\sec^{\frac{11}{2}}(c+dx)}{4d(a(\sec(c+dx)+1))^{5/2}} - \frac{7B(\sec(c+dx)+1)^2\sin(c+dx)\sec^{\frac{9}{2}}(c+dx)}{16d(a(\sec(c+dx)+1))^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[c + d*x]^(7/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(5/2), x]

[Out] -1/4*(A*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(d*(a*(1 + Sec[c + d*x]))^(5/2)) - (B*Sec[c + d*x]^(11/2)*Sin[c + d*x])/(4*d*(a*(1 + Sec[c + d*x]))^(5/2)) + (3*A*Sec[c + d*x]^(9/2)*(1 + Sec[c + d*x])*Sin[c + d*x])/(16*d*(a*(1 + Sec[c + d*x]))^(5/2)) + (7*B*Sec[c + d*x]^(11/2)*(1 + Sec[c + d*x])*Sin[c + d*x])/(16*d*(a*(1 + Sec[c + d*x]))^(5/2)) - (11*A*Sec[c + d*x]^(3/2)*(1 + Sec[c + d*x])^2*Sin[c + d*x])/(16*d*(a*(1 + Sec[c + d*x]))^(5/2)) + (35*B*Sec[c + d*x]^(3/2)*(1 + Sec[c + d*x])^2*Sin[c + d*x])/(16*d*(a*(1 + Sec[c + d*x]))^(5/2)) + (7*A*Sec[c + d*x]^(5/2)*(1 + Sec[c + d*x])^2*Sin[c + d*x])/(16*

$$\begin{aligned}
& d*(a*(1 + \text{Sec}[c + d*x]))^{(5/2)} - (15*B*\text{Sec}[c + d*x]^{(5/2)}*(1 + \text{Sec}[c + d*x] \\
&)^2*\text{Sin}[c + d*x])/(16*d*(a*(1 + \text{Sec}[c + d*x]))^{(5/2)} - (3*A*\text{Sec}[c + d*x]^{(7/2)}*(1 + \text{Sec}[c + d*x] \\
&)^2*\text{Sin}[c + d*x])/(16*d*(a*(1 + \text{Sec}[c + d*x]))^{(5/2)} + (11*B*\text{Sec}[c + d*x]^{(7/2)}*(1 + \text{Sec}[c + d*x] \\
&)^2*\text{Sin}[c + d*x])/(16*d*(a*(1 + \text{Sec}[c + d*x]))^{(5/2)} - (7*B*\text{Sec}[c + d*x]^{(9/2)}*(1 + \text{Sec}[c + d*x] \\
&)^2*\text{Sin}[c + d*x])/(16*d*(a*(1 + \text{Sec}[c + d*x]))^{(5/2)} - (11*A*\text{ArcSin}[\text{Sqrt}[1 - \text{Sec}[c + d*x]]) \\
&]*(1 + \text{Sec}[c + d*x])^2*\text{Tan}[c + d*x])/(16*d*\text{Sqrt}[1 - \text{Sec}[c + d*x]]*(a*(1 + \text{Sec}[c + d*x] \\
&)^{(5/2)} + (35*B*\text{ArcSin}[\text{Sqrt}[1 - \text{Sec}[c + d*x]])*(1 + \text{Sec}[c + d*x])^2*\text{Tan}[c + d*x])/(16*d*\text{Sqrt}[1 - \text{Sec}[c + d*x] \\
&]*(a*(1 + \text{Sec}[c + d*x]))^{(5/2)} - (43*A*\text{ArcSin}[\text{Sqrt}[\text{Sec}[c + d*x]])*(1 + \text{Sec}[c + d*x])^2*\text{Tan}[c + d*x] \\
&)/(16*d*\text{Sqrt}[1 - \text{Sec}[c + d*x]]*(a*(1 + \text{Sec}[c + d*x]))^{(5/2)} + (115*B*\text{ArcSin}[\text{Sqrt}[\text{Sec}[c + d*x]]) \\
&]*(1 + \text{Sec}[c + d*x])^2*\text{Tan}[c + d*x])/(16*d*\text{Sqrt}[1 - \text{Sec}[c + d*x]]*(a*(1 + \text{Sec}[c + d*x]))^{(5/2)} + (43*A*\text{ArcTan}[(\text{Sqrt}[2]*\text{Sqrt} \\
& [\text{Sec}[c + d*x]])/\text{Sqrt}[1 - \text{Sec}[c + d*x]])*(1 + \text{Sec}[c + d*x])^2*\text{Tan}[c + d*x])/(16*\text{Sqrt}[2]*d*\text{Sqrt}[1 - \text{Sec}[c + d*x] \\
&]*(a*(1 + \text{Sec}[c + d*x]))^{(5/2)} - (115*B*\text{ArcTan}[(\text{Sqrt}[2]*\text{Sqrt}[\text{Sec}[c + d*x]])/\text{Sqrt}[1 - \text{Sec}[c + d*x]]) \\
&]*(1 + \text{Sec}[c + d*x])^2*\text{Tan}[c + d*x])/(16*\text{Sqrt}[2]*d*\text{Sqrt}[1 - \text{Sec}[c + d*x]]*(a*(1 + \text{Sec}[c + d*x] \\
&)^{(5/2)}))^{(5/2)}
\end{aligned}$$

fricas [A] time = 0.72, size = 803, normalized size = 3.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")

[Out]
$$\begin{aligned}
& [-1/64*(\text{sqrt}(2)*((43*A - 115*B)*\cos(d*x + c)^3 + 3*(43*A - 115*B)*\cos(d*x + c)^2 + 3*(43*A - 115*B)*\cos(d*x + c) + 43*A - 115*B)*\text{sqrt}(a)*\log(-(a*\cos(d*x + c))^2 - 2*\text{sqrt}(2)*\text{sqrt}(a)*\text{sqrt}((a*\cos(d*x + c) + a)/\cos(d*x + c))*\text{sqrt}(\cos(d*x + c))*\sin(d*x + c) - 2*a*\cos(d*x + c) - 3*a)/(\cos(d*x + c)^2 + 2*\cos(d*x + c) + 1)) + 16*((2*A - 5*B)*\cos(d*x + c)^3 + 3*(2*A - 5*B)*\cos(d*x + c)^2 + 3*(2*A - 5*B)*\cos(d*x + c) + 2*A - 5*B)*\text{sqrt}(a)*\log((a*\cos(d*x + c))^3 - 7*a*\cos(d*x + c)^2 + 4*(\cos(d*x + c)^2 - 2*\cos(d*x + c))*\text{sqrt}(a)*\text{sqrt}((a*\cos(d*x + c) + a)/\cos(d*x + c))*\sin(d*x + c)/\text{sqrt}(\cos(d*x + c)) + 8*a)/(\cos(d*x + c)^3 + \cos(d*x + c)^2)) + 4*((11*A - 35*B)*\cos(d*x + c)^2 + 5*(3*A - 11*B)*\cos(d*x + c) - 16*B)*\text{sqrt}((a*\cos(d*x + c) + a)/\cos(d*x + c))*\sin(d*x + c)/\text{sqrt}(\cos(d*x + c)))/(a^3*d*\cos(d*x + c)^3 + 3*a^3*d*\cos(d*x + c)^2 + 3*a^3*d*\cos(d*x + c) + a^3*d), 1/32*(\text{sqrt}(2)*((43*A - 115*B)*\cos(d*x + c)^3 + 3*(43*A - 115*B)*\cos(d*x + c)^2 + 3*(43*A - 115*B)*\cos(d*x + c) + 43*A - 115*B)*\text{sqrt}(-a)*\arctan(\text{sqrt}(2)*\text{sqrt}(-a)*\text{sqrt}((a*\cos(d*x + c) + a)/\cos(d*x + c))*\text{sqrt}(\cos(d*x + c))/(a*\sin(d*x + c))) + 16*((2*A - 5*B)*\cos(d*x + c)^3 + 3*(2*A - 5*B)*\cos(d*x + c)^2 + 3*(2*A - 5*B)*\cos(d*x + c) + 2*A - 5*B)*\text{sqrt}(-a)*\arctan(2*\text{sqrt}(-a)*\text{sqrt}((a*\cos(d*x + c) + a)/\cos(d*x + c))*\text{sqrt}(\cos(d*x + c))*\sin(d*x + c)/(a*\cos(d*x + c)^2 - a*\cos(d*x + c) - 2*a)) - 2*((
\end{aligned}$$

$11*A - 35*B)*\cos(d*x + c)^2 + 5*(3*A - 11*B)*\cos(d*x + c) - 16*B)*\sqrt{(a*\cos(d*x + c) + a)/\cos(d*x + c)}*\sin(d*x + c)/\sqrt{\cos(d*x + c)})/(a^3*d*\cos(d*x + c)^3 + 3*a^3*d*\cos(d*x + c)^2 + 3*a^3*d*\cos(d*x + c) + a^3*d)]$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sec(dx + c)^{\frac{7}{2}}}{(a \sec(dx + c) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algorith="giac")

[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(7/2)/(a*sec(d*x + c) + a)^(5/2), x)

maple [B] time = 2.64, size = 831, normalized size = 3.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x)

[Out] $-1/16/d*(1/\cos(d*x+c))^{7/2}*\cos(d*x+c)^3*(a*(1+\cos(d*x+c))/\cos(d*x+c))^{1/2}*(-1+\cos(d*x+c))^{2*(1/2)*\arctan(1/4*(-2/(1+\cos(d*x+c))))^{1/2}*(\cos(d*x+c)+1-\sin(d*x+c))*2^{1/2})*\sin(d*x+c)*\cos(d*x+c)^2-16*A*2^{1/2)*\arctan(1/4*(-2/(1+\cos(d*x+c))))^{1/2}*(\cos(d*x+c)+1+\sin(d*x+c))*2^{1/2})*\sin(d*x+c)*\cos(d*x+c)^2-40*B*2^{1/2)*\arctan(1/4*(-2/(1+\cos(d*x+c))))^{1/2}*(\cos(d*x+c)+1-\sin(d*x+c))*2^{1/2})*\sin(d*x+c)*\cos(d*x+c)^2+40*B*2^{1/2)*\arctan(1/4*(-2/(1+\cos(d*x+c))))^{1/2}*(\cos(d*x+c)+1+\sin(d*x+c))*2^{1/2})*\sin(d*x+c)*\cos(d*x+c)^2+16*A*\cos(d*x+c)*\sin(d*x+c)*2^{1/2)*\arctan(1/4*(-2/(1+\cos(d*x+c))))^{1/2}*(\cos(d*x+c)+1-\sin(d*x+c))*2^{1/2}))-16*A*\cos(d*x+c)*\sin(d*x+c)*2^{1/2)*\arctan(1/4*(-2/(1+\cos(d*x+c))))^{1/2}*(\cos(d*x+c)+1+\sin(d*x+c))*2^{1/2}))+43*A*\arctan(1/2*\sin(d*x+c)*(-2/(1+\cos(d*x+c))))^{1/2})*\sin(d*x+c)*\cos(d*x+c)^2-11*A*(-2/(1+\cos(d*x+c))))^{1/2})*\cos(d*x+c)^3-40*B*\cos(d*x+c)*\sin(d*x+c)*2^{1/2)*\arctan(1/4*(-2/(1+\cos(d*x+c))))^{1/2}*(\cos(d*x+c)+1-\sin(d*x+c))*2^{1/2}))+40*B*\cos(d*x+c)*\sin(d*x+c)*2^{1/2)*\arctan(1/4*(-2/(1+\cos(d*x+c))))^{1/2}*(\cos(d*x+c)+1+\sin(d*x+c))*2^{1/2}))-115*B*\arctan(1/2*\sin(d*x+c)*(-2/(1+\cos(d*x+c))))^{1/2})*\sin(d*x+c)*\cos(d*x+c)^2+35*B*(-2/(1+\cos(d*x+c))))^{1/2})*\cos(d*x+c)^3+43*A*\cos(d*x+c)*\sin(d*x+c)*\arctan(1/2*\sin(d*x+c)*(-2/(1+\cos(d*x+c))))^{1/2}))-4*A*(-2/(1+\cos(d*x+c))))^{1/2})*\cos(d*x+c)^2-115*B*\cos(d*x+c)*\sin(d*x+c)*\arctan(1/2*\sin(d*x+c)*(-2/(1+\cos(d*x+c))))^{1/2}))+20*B*(-2/(1+\cos(d*x+c))))^{1/2})*\cos(d*x+c)^2+15*A*(-2/(1+\cos(d*x+c))))^{1/2})*\cos(d*x+c)-39*B*(-2/(1+$

$\cos(d*x+c))^{(1/2)*\cos(d*x+c)-16*B*(-2/(1+\cos(d*x+c)))^{(1/2)}}/(-2/(1+\cos(d*x+c)))^{(1/2)}/\sin(d*x+c)^5/a^3$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(\frac{1}{\cos(c+dx)}\right)^{7/2}}{\left(a + \frac{a}{\cos(c+dx)}\right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(7/2))/(a + a/cos(c + d*x))^(5/2),x)

[Out] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(7/2))/(a + a/cos(c + d*x))^(5/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**(7/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(5/2),x)

[Out] Timed out

$$3.263 \quad \int \frac{\sec^2(c+dx)(A+B \sec(c+dx))}{(a+a \sec(c+dx))^{5/2}} dx$$

Optimal. Leaf size=194

$$\frac{(3A - 43B) \tanh^{-1} \left(\frac{\sqrt{a} \sin(c+dx) \sqrt{\sec(c+dx)}}{\sqrt{2} \sqrt{a \sec(c+dx)+a}} \right)}{16\sqrt{2} a^{5/2} d} + \frac{2B \sinh^{-1} \left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}} \right)}{a^{5/2} d} + \frac{(A - B) \sin(c + dx) \sec^2(c + dx)}{4d(a \sec(c + dx) + a)^{5/2}} + \frac{(3A - 11B) \sec^2(c + dx)}{16d(a \sec(c + dx) + a)^{5/2}}$$

[Out] $2*B*\operatorname{arcsinh}(a^{1/2}*\tan(d*x+c)/(a+a*\sec(d*x+c))^{1/2})/a^{5/2}/d+1/4*(A-B)*\sec(d*x+c)^{5/2}*\sin(d*x+c)/d/(a+a*\sec(d*x+c))^{5/2}+1/16*(3*A-11*B)*\sec(d*x+c)^{3/2}*\sin(d*x+c)/a/d/(a+a*\sec(d*x+c))^{3/2}+1/32*(3*A-43*B)*\operatorname{arctanh}(1/2*\sin(d*x+c)*a^{1/2}*\sec(d*x+c)^{1/2}*2^{1/2}/(a+a*\sec(d*x+c))^{1/2})/a^{5/2}/d*2^{1/2}$

Rubi [A] time = 0.59, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {4019, 4023, 3808, 206, 3801, 215}

$$\frac{(3A - 43B) \tanh^{-1} \left(\frac{\sqrt{a} \sin(c+dx) \sqrt{\sec(c+dx)}}{\sqrt{2} \sqrt{a \sec(c+dx)+a}} \right)}{16\sqrt{2} a^{5/2} d} + \frac{2B \sinh^{-1} \left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}} \right)}{a^{5/2} d} + \frac{(A - B) \sin(c + dx) \sec^2(c + dx)}{4d(a \sec(c + dx) + a)^{5/2}} + \frac{(3A - 11B) \sec^2(c + dx)}{16d(a \sec(c + dx) + a)^{5/2}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(\operatorname{Sec}[c + d*x]^{5/2}*(A + B*\operatorname{Sec}[c + d*x]))/(a + a*\operatorname{Sec}[c + d*x]^{5/2}), x]$

[Out] $(2*B*\operatorname{ArcSinh}[(\operatorname{Sqrt}[a]*\operatorname{Tan}[c + d*x])/(\operatorname{Sqrt}[a + a*\operatorname{Sec}[c + d*x]])]/(a^{5/2}*d) + ((3*A - 43*B)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Sqrt}[\operatorname{Sec}[c + d*x]]*\operatorname{Sin}[c + d*x])/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a + a*\operatorname{Sec}[c + d*x]])])/(16*\operatorname{Sqrt}[2]*a^{5/2}*d) + ((A - B)*\operatorname{Sec}[c + d*x]^{5/2}*\operatorname{Sin}[c + d*x])/(4*d*(a + a*\operatorname{Sec}[c + d*x]^{5/2})) + ((3*A - 11*B)*\operatorname{Sec}[c + d*x]^{3/2}*\operatorname{Sin}[c + d*x])/(16*a*d*(a + a*\operatorname{Sec}[c + d*x]^{3/2}))$

Rule 206

$\operatorname{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1*\operatorname{ArcTanh}[(\operatorname{Rt}[-b, 2]*x)/\operatorname{Rt}[a, 2]])/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /; \operatorname{FreeQ}\{a, b\}, x] \ \&\& \operatorname{NegQ}[a/b] \ \&\& (\operatorname{GtQ}[a, 0] \ || \ \operatorname{LtQ}[b, 0])$

Rule 215

$\operatorname{Int}[1/\operatorname{Sqrt}[(a_ + (b_)*(x_)^2)], x_Symbol] \rightarrow \operatorname{Simp}[\operatorname{ArcSinh}[(\operatorname{Rt}[b, 2]*x)/\operatorname{Sqrt}[a]]/\operatorname{Rt}[b, 2], x] /; \operatorname{FreeQ}\{a, b\}, x] \ \&\& \operatorname{GtQ}[a, 0] \ \&\& \operatorname{PosQ}[b]$

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_.)], x_Symbol] :=> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_.)], x_Symbol] :=> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] :=> Simp[(d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] :=> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{\frac{5}{2}}} dx &= \frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{\frac{5}{2}}} + \frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(\frac{3}{2}a(A-B)+4aB\sec(c+dx)\right)}{(a+a\sec(c+dx))^{\frac{3}{2}}} dx}{4a^2} \\
&= \frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{\frac{5}{2}}} + \frac{(3A-11B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{\frac{3}{2}}} \\
&= \frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{\frac{5}{2}}} + \frac{(3A-11B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{\frac{3}{2}}} \\
&= \frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{\frac{5}{2}}} + \frac{(3A-11B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{\frac{3}{2}}} \\
&= \frac{2B\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{\frac{5}{2}}d} + \frac{(3A-43B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{\frac{5}{2}}d}
\end{aligned}$$

Mathematica [B] time = 6.16, size = 845, normalized size = 4.36

$$\frac{3B(\sec(c+dx)+1)\sin(c+dx)\sec^{\frac{9}{2}}(c+dx)}{16d(a(\sec(c+dx)+1))^{\frac{5}{2}}} - \frac{B\sin(c+dx)\sec^{\frac{9}{2}}(c+dx)}{4d(a(\sec(c+dx)+1))^{\frac{5}{2}}} - \frac{3B(\sec(c+dx)+1)^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{16d(a(\sec(c+dx)+1))^{\frac{5}{2}}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(5/2), x]

[Out] -1/4*(A*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(d*(a*(1 + Sec[c + d*x]))^(5/2)) - (B*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(4*d*(a*(1 + Sec[c + d*x]))^(5/2)) - (A*Sec[c + d*x]^(7/2)*(1 + Sec[c + d*x])*Sin[c + d*x])/(16*d*(a*(1 + Sec[c + d*x]))^(5/2)) + (3*B*Sec[c + d*x]^(9/2)*(1 + Sec[c + d*x])*Sin[c + d*x])/(16*d*(a*(1 + Sec[c + d*x]))^(5/2)) + (3*A*Sec[c + d*x]^(3/2)*(1 + Sec[c + d*x])^2*Sin[c + d*x])/(16*d*(a*(1 + Sec[c + d*x]))^(5/2)) - (11*B*Sec[c + d*x]^(3/2)*(1 + Sec[c + d*x])^2*Sin[c + d*x])/(16*d*(a*(1 + Sec[c + d*x]))^(5/2)) + (A*Sec[c + d*x]^(5/2)*(1 + Sec[c + d*x])^2*Sin[c + d*x])/(16*d*(a*(1 + Sec[c + d*x]))^(5/2)) + (7*B*Sec[c + d*x]^(5/2)*(1 + Sec[c + d*x])^2*Sin[c + d*x])/(16*d*(a*(1 + Sec[c + d*x]))^(5/2)) - (3*B*Sec[c + d*x]^(7/2)*(1 + Sec[c + d*x])^2*Sin[c + d*x])/(16*d*(a*(1 + Sec[c + d*x]))^(5/2)) + (3*A*ArcSin[Sqrt[1 - Sec[c + d*x]]]*(1 + Sec[c + d*x])^2*Tan[c + d*x])/(16*d*Sq

$$\begin{aligned} & \text{rt}[1 - \text{Sec}[c + d*x]]*(a*(1 + \text{Sec}[c + d*x]))^{(5/2)} - (11*B*\text{ArcSin}[\text{Sqrt}[1 - \\ & \text{Sec}[c + d*x]]]*(1 + \text{Sec}[c + d*x])^2*\text{Tan}[c + d*x])/(16*d*\text{Sqrt}[1 - \text{Sec}[c + d* \\ & x]]*(a*(1 + \text{Sec}[c + d*x]))^{(5/2)} + (3*A*\text{ArcSin}[\text{Sqrt}[\text{Sec}[c + d*x]]]*(1 + \text{Se} \\ & c[c + d*x])^2*\text{Tan}[c + d*x])/(16*d*\text{Sqrt}[1 - \text{Sec}[c + d*x]]*(a*(1 + \text{Sec}[c + d* \\ & x]))^{(5/2)} - (43*B*\text{ArcSin}[\text{Sqrt}[\text{Sec}[c + d*x]]]*(1 + \text{Sec}[c + d*x])^2*\text{Tan}[c + \\ & d*x])/(16*d*\text{Sqrt}[1 - \text{Sec}[c + d*x]]*(a*(1 + \text{Sec}[c + d*x]))^{(5/2)} - (3*A*Ar \\ & cTan[(\text{Sqrt}[2]*\text{Sqrt}[\text{Sec}[c + d*x]])/\text{Sqrt}[1 - \text{Sec}[c + d*x]]]*(1 + \text{Sec}[c + d*x] \\ &)^2*\text{Tan}[c + d*x])/(16*\text{Sqrt}[2]*d*\text{Sqrt}[1 - \text{Sec}[c + d*x]]*(a*(1 + \text{Sec}[c + d*x] \\ &))^{(5/2)} + (43*B*\text{ArcTan}[(\text{Sqrt}[2]*\text{Sqrt}[\text{Sec}[c + d*x]])/\text{Sqrt}[1 - \text{Sec}[c + d*x] \\ &]]*(1 + \text{Sec}[c + d*x])^2*\text{Tan}[c + d*x])/(16*\text{Sqrt}[2]*d*\text{Sqrt}[1 - \text{Sec}[c + d*x]]* \\ & (a*(1 + \text{Sec}[c + d*x]))^{(5/2)} \end{aligned}$$

fricas [B] time = 0.57, size = 749, normalized size = 3.86

$$\sqrt{2} \left((3A - 43B) \cos(dx + c)^3 + 3(3A - 43B) \cos(dx + c)^2 + 3(3A - 43B) \cos(dx + c) + 3A - 43B \right) \sqrt{a} \ln$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algorith="fricas")

[Out]
$$\begin{aligned} & [-1/64*(\text{sqrt}(2)*((3*A - 43*B)*\cos(d*x + c)^3 + 3*(3*A - 43*B)*\cos(d*x + c)^2 + 3*(3*A - 43*B)*\cos(d*x + c) + 3*A - 43*B)*\text{sqrt}(a)*\log(-(a*\cos(d*x + c))^2 + 2*\text{sqrt}(2)*\text{sqrt}(a)*\text{sqrt}((a*\cos(d*x + c) + a)/\cos(d*x + c))*\text{sqrt}(\cos(d*x + c)))*\sin(d*x + c) - 2*a*\cos(d*x + c) - 3*a)/(\cos(d*x + c)^2 + 2*\cos(d*x + c) + 1)) - 32*(B*\cos(d*x + c)^3 + 3*B*\cos(d*x + c)^2 + 3*B*\cos(d*x + c) + B)*\text{sqrt}(a)*\log((a*\cos(d*x + c)^3 - 7*a*\cos(d*x + c)^2 - 4*(\cos(d*x + c)^2 - 2*\cos(d*x + c))*\text{sqrt}(a)*\text{sqrt}((a*\cos(d*x + c) + a)/\cos(d*x + c))*\sin(d*x + c))/\text{sqrt}(\cos(d*x + c)) + 8*a)/(\cos(d*x + c)^3 + \cos(d*x + c)^2) - 4*((3*A - 11*B)*\cos(d*x + c)^2 + (7*A - 15*B)*\cos(d*x + c))*\text{sqrt}((a*\cos(d*x + c) + a)/\cos(d*x + c))*\sin(d*x + c)/\text{sqrt}(\cos(d*x + c)))/(a^3*d*\cos(d*x + c)^3 + 3*a^3*d*\cos(d*x + c)^2 + 3*a^3*d*\cos(d*x + c) + a^3*d), -1/32*(\text{sqrt}(2)*((3*A - 43*B)*\cos(d*x + c)^3 + 3*(3*A - 43*B)*\cos(d*x + c)^2 + 3*(3*A - 43*B)*\cos(d*x + c) + 3*A - 43*B)*\text{sqrt}(-a)*\arctan(\text{sqrt}(2)*\text{sqrt}(-a)*\text{sqrt}((a*\cos(d*x + c) + a)/\cos(d*x + c))*\text{sqrt}(\cos(d*x + c)))/(a*\sin(d*x + c))) - 32*(B*\cos(d*x + c)^3 + 3*B*\cos(d*x + c)^2 + 3*B*\cos(d*x + c) + B)*\text{sqrt}(-a)*\arctan(2*\text{sqrt}(-a)*\text{sqrt}((a*\cos(d*x + c) + a)/\cos(d*x + c))*\text{sqrt}(\cos(d*x + c))*\sin(d*x + c)/(a*\cos(d*x + c)^2 - a*\cos(d*x + c) - 2*a)) - 2*((3*A - 11*B)*\cos(d*x + c)^2 \end{aligned}$$

+ (7*A - 15*B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sec(dx + c)^{\frac{5}{2}}}{(a \sec(dx + c) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algorith="giac")

[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(5/2)/(a*sec(d*x + c) + a)^(5/2), x)

maple [B] time = 2.82, size = 550, normalized size = 2.84

$$(-1 + \cos(dx + c))^2 \left(16B \cos(dx + c) \sin(dx + c) \sqrt{2} \arctan \left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}} (\cos(dx+c)+1+\sin(dx+c)) \sqrt{2}}{4} \right) - 16B \cos(dx + c) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x)

[Out] 1/16/d*(-1+cos(d*x+c))^2*(16*B*cos(d*x+c)*sin(d*x+c)*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d*x+c))*2^(1/2))-16*B*cos(d*x+c)*sin(d*x+c)*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))*2^(1/2))+3*A*cos(d*x+c)*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))-3*A*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2+16*B*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d*x+c))*2^(1/2))*sin(d*x+c)-16*B*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))*2^(1/2))*sin(d*x+c)-43*B*cos(d*x+c)*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))+11*B*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2+3*A*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*sin(d*x+c)-4*A*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)-43*B*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*sin(d*x+c)+4*B*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)+7*A*(-2/(1+cos(d*x+c)))^(1/2)-15*B*(-2/(1+cos(d*x+c)))^(1/2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^3*(1/cos(d*x+c))^(5/2)/sin(d*x+c)^5/(-2/(1+cos(d*x+c)))^(1/2)/a^3

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(\frac{1}{\cos(c+dx)}\right)^{5/2}}{\left(a + \frac{a}{\cos(c+dx)}\right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + a/cos(c + d*x))^(5/2),x)

[Out] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + a/cos(c + d*x))^(5/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(5/2),x)

[Out] Timed out

$$3.264 \quad \int \frac{\sec^3(c+dx)(A+B \sec(c+dx))}{(a+a \sec(c+dx))^{5/2}} dx$$

Optimal. Leaf size=156

$$\frac{(5A + 3B) \tanh^{-1} \left(\frac{\sqrt{a} \sin(c+dx) \sqrt{\sec(c+dx)}}{\sqrt{2} \sqrt{a \sec(c+dx)+a}} \right)}{16\sqrt{2} a^{5/2} d} - \frac{(A - B) \sin(c + dx) \sec^5(c + dx)}{4d(a \sec(c + dx) + a)^{5/2}} + \frac{(5A + 3B) \sin(c + dx) \sec^3(c + dx)}{16ad(a \sec(c + dx) + a)^{3/2}}$$

[Out] $-1/4*(A-B)*\sec(d*x+c)^{(5/2)}*\sin(d*x+c)/d/(a+a*\sec(d*x+c))^{(5/2)}+1/16*(5*A+3*B)*\sec(d*x+c)^{(3/2)}*\sin(d*x+c)/a/d/(a+a*\sec(d*x+c))^{(3/2)}+1/32*(5*A+3*B)*\operatorname{rctanh}(1/2*\sin(d*x+c)*a^{(1/2)}*\sec(d*x+c)^{(1/2)}*2^{(1/2)}/(a+a*\sec(d*x+c))^{(1/2)})/a^{(5/2)}/d*2^{(1/2)}$

Rubi [A] time = 0.27, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {4012, 3810, 3808, 206}

$$\frac{(5A + 3B) \tanh^{-1} \left(\frac{\sqrt{a} \sin(c+dx) \sqrt{\sec(c+dx)}}{\sqrt{2} \sqrt{a \sec(c+dx)+a}} \right)}{16\sqrt{2} a^{5/2} d} - \frac{(A - B) \sin(c + dx) \sec^5(c + dx)}{4d(a \sec(c + dx) + a)^{5/2}} + \frac{(5A + 3B) \sin(c + dx) \sec^3(c + dx)}{16ad(a \sec(c + dx) + a)^{3/2}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(\operatorname{Sec}[c + d*x]^{(3/2)}*(A + B*\operatorname{Sec}[c + d*x]))/(a + a*\operatorname{Sec}[c + d*x])^{(5/2)}, x]$

[Out] $((5*A + 3*B)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Sqrt}[\operatorname{Sec}[c + d*x]]*\operatorname{Sin}[c + d*x])]/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a + a*\operatorname{Sec}[c + d*x]])))/(16*\operatorname{Sqrt}[2]*a^{(5/2)}*d) - ((A - B)*\operatorname{Sec}[c + d*x]^{(5/2)}*\operatorname{Sin}[c + d*x])/(4*d*(a + a*\operatorname{Sec}[c + d*x])^{(5/2)}) + ((5*A + 3*B)*\operatorname{Sec}[c + d*x]^{(3/2)}*\operatorname{Sin}[c + d*x])/(16*a*d*(a + a*\operatorname{Sec}[c + d*x])^{(3/2)})$

Rule 206

$\operatorname{Int}[(a_.) + (b_.)*(x_)^2)^{-1}, x_Symbol] :> \operatorname{Simp}[(1*\operatorname{ArcTanh}[(\operatorname{Rt}[-b, 2]*x)/\operatorname{Rt}[a, 2]])/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /; \operatorname{FreeQ}\{a, b\}, x \ \&\& \operatorname{NegQ}[a/b] \ \&\& (\operatorname{GtQ}[a, 0] \ || \operatorname{LtQ}[b, 0])$

Rule 3808

$\operatorname{Int}[\operatorname{Sqrt}[\operatorname{csc}[(e_.) + (f_.)*(x_)]*(d_.)]/\operatorname{Sqrt}[\operatorname{csc}[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> \operatorname{Dist}[(-2*b*d)/(a*f), \operatorname{Subst}[\operatorname{Int}[1/(2*b - d*x^2), x], x, (b*\operatorname{Cot}[e + f*x])]/(\operatorname{Sqrt}[a + b*\operatorname{Csc}[e + f*x]]*\operatorname{Sqrt}[d*\operatorname{Csc}[e + f*x]])], x] /; \operatorname{FreeQ}\{a, b, d, e, f\}, x \ \&\& \operatorname{EqQ}[a^2 - b^2, 0]$

Rule 3810

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (
a_.))^(m_.), x_Symbol] :> Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc
c[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[(d*(m + 1))/(b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[
{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && LtQ[m, -
2^(-1)] && IntegerQ[2*m]
```

Rule 4012

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (
a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] + Dist[(a*A*m + b*B*(m + 1))/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*
x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x
] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && LeQ[m,
-1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^3(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{5/2}} dx &= -\frac{(A-B)\sec^5(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(5A+3B)\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^{3/2}} dx}{8a} \\
&= -\frac{(A-B)\sec^5(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(5A+3B)\sec^3(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} \\
&= -\frac{(A-B)\sec^5(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(5A+3B)\sec^3(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} \\
&= \frac{(5A+3B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{(A-B)\sec^5(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.81, size = 106, normalized size = 0.68

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\left(\frac{1}{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\left((5A+3B)\cos(c+dx)+A+7B\right)+(5A+3B)\cos^4\left(\frac{1}{2}(c+dx)\right)}{4d(a(\sec(c+dx)+1))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(5
/2), x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c + d*x]^(5/2)*((5*A + 3*B)*ArcTanh[Sin[(c + d*x)/2]]
*Cos[(c + d*x)/2]^4 + ((A + 7*B + (5*A + 3*B)*Cos[c + d*x])*Sin[(c + d*x)/2
])/2))/(4*d*(a*(1 + Sec[c + d*x]))^(5/2))
```

fricas [A] time = 0.47, size = 498, normalized size = 3.19

$$\frac{\sqrt{2} \left((5A + 3B) \cos(dx + c)^3 + 3(5A + 3B) \cos(dx + c)^2 + 3(5A + 3B) \cos(dx + c) + 5A + 3B \right) \sqrt{a} \log \left(-\frac{a \cos(dx + c)}{\dots} \right)}{64 \left(a^3 d \cos(dx + c)^3 + 3 a^3 d \cos(dx + c)^2 + \dots \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] [1/64*(sqrt(2)*((5*A + 3*B)*cos(d*x + c)^3 + 3*(5*A + 3*B)*cos(d*x + c)^2 +
3*(5*A + 3*B)*cos(d*x + c) + 5*A + 3*B)*sqrt(a)*log(-(a*cos(d*x + c))^2 - 2
*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))
*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) +
1)) + 4*((5*A + 3*B)*cos(d*x + c)^2 + (A + 7*B)*cos(d*x + c))*sqrt((a*cos(d
*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x
+ c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1/32*(sqr
t(2)*((5*A + 3*B)*cos(d*x + c)^3 + 3*(5*A + 3*B)*cos(d*x + c)^2 + 3*(5*A +
3*B)*cos(d*x + c) + 5*A + 3*B)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) - 2*((5*A
+ 3*B)*cos(d*x + c)^2 + (A + 7*B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/
cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^
3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sec(dx + c)^{\frac{3}{2}}}{(a \sec(dx + c) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(3/2)/(a*sec(d*x + c) + a)^(5/2
), x)
```

maple [B] time = 2.63, size = 350, normalized size = 2.24

$$(-1 + \cos(dx + c))^2 \left(5A \sqrt{\frac{2}{1 + \cos(dx + c)}} (\cos^2(dx + c)) - 5A \cos(dx + c) \sin(dx + c) \arctan \left(\frac{\sin(dx + c) \sqrt{\frac{2}{1 + \cos(dx + c)}}}{2} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x)`

[Out] `-1/16/d*(-1+cos(d*x+c))^2*(5*A*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2-5*A*cos(d*x+c)*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))+3*B*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2-3*B*cos(d*x+c)*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))-4*A*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)-5*A*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*sin(d*x+c)+4*B*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)-3*B*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*sin(d*x+c)-A*(-2/(1+cos(d*x+c)))^(1/2)-7*B*(-2/(1+cos(d*x+c)))^(1/2))*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^2*(1/cos(d*x+c))^(3/2)/(-2/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^5/a^3`

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")`

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(\frac{1}{\cos(c+dx)}\right)^{3/2}}{\left(a + \frac{a}{\cos(c+dx)}\right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + a/cos(c + d*x))^(5/2),x)`

[Out] `int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + a/cos(c + d*x))^(5/2), x)`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(5/2),x)

[Out] Timed out

$$3.265 \quad \int \frac{\sqrt{\sec(c+dx)} (A+B \sec(c+dx))}{(a+a \sec(c+dx))^{5/2}} dx$$

Optimal. Leaf size=156

$$\frac{(19A + 5B) \tanh^{-1} \left(\frac{\sqrt{a} \sin(c+dx) \sqrt{\sec(c+dx)}}{\sqrt{2} \sqrt{a \sec(c+dx)+a}} \right)}{16\sqrt{2} a^{5/2} d} - \frac{(9A - B) \sin(c + dx) \sec^3(c + dx)}{16ad(a \sec(c + dx) + a)^{3/2}} - \frac{(A - B) \sin(c + dx) \sec^3(c + dx)}{4d(a \sec(c + dx) + a)^{5/2}}$$

[Out] $-1/4*(A-B)*\sec(d*x+c)^{(3/2)}*\sin(d*x+c)/d/(a+a*\sec(d*x+c))^{(5/2)}-1/16*(9*A-B)*\sec(d*x+c)^{(3/2)}*\sin(d*x+c)/a/d/(a+a*\sec(d*x+c))^{(3/2)}+1/32*(19*A+5*B)*\operatorname{arctanh}(1/2*\sin(d*x+c)*a^{(1/2)}*\sec(d*x+c)^{(1/2)}*2^{(1/2)}/(a+a*\sec(d*x+c))^{(1/2)})/a^{(5/2)}/d*2^{(1/2)}$

Rubi [A] time = 0.57, antiderivative size = 203, normalized size of antiderivative = 1.30, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4019, 4020, 4013, 3808, 206}

$$-\frac{(9A - B) \sin(c + dx) \sqrt{\sec(c + dx)}}{16a^2 d \sqrt{a \sec(c + dx) + a}} + \frac{(19A + 5B) \tanh^{-1} \left(\frac{\sqrt{a} \sin(c+dx) \sqrt{\sec(c+dx)}}{\sqrt{2} \sqrt{a \sec(c+dx)+a}} \right)}{16\sqrt{2} a^{5/2} d} + \frac{(5A + 3B) \sin(c + dx) \sqrt{\sec(c + dx)}}{16ad(a \sec(c + dx) + a)^{3/2}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(\operatorname{Sqrt}[\operatorname{Sec}[c + d*x]])*(A + B*\operatorname{Sec}[c + d*x])]/(a + a*\operatorname{Sec}[c + d*x])^{(5/2)}, x]$

[Out] $((19*A + 5*B)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Sqrt}[\operatorname{Sec}[c + d*x]]*\operatorname{Sin}[c + d*x])]/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a + a*\operatorname{Sec}[c + d*x]])))/(16*\operatorname{Sqrt}[2]*a^{(5/2)}*d) + ((A - B)*\operatorname{Sqrt}[\operatorname{Sec}[c + d*x]]*\operatorname{Sin}[c + d*x])/(4*d*(a + a*\operatorname{Sec}[c + d*x])^{(5/2)}) + ((5*A + 3*B)*\operatorname{Sqrt}[\operatorname{Sec}[c + d*x]]*\operatorname{Sin}[c + d*x])/(16*a*d*(a + a*\operatorname{Sec}[c + d*x])^{(3/2)}) - ((9*A - B)*\operatorname{Sqrt}[\operatorname{Sec}[c + d*x]]*\operatorname{Sin}[c + d*x])/(16*a^2*d*\operatorname{Sqrt}[a + a*\operatorname{Sec}[c + d*x]])$

Rule 206

$\operatorname{Int}[(a_.) + (b_.)*(x_)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1*\operatorname{ArcTanh}[(\operatorname{Rt}[-b, 2]*x)/\operatorname{Rt}[a, 2]])/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /;$ FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 3808

$\operatorname{Int}[\operatorname{Sqrt}[\operatorname{csc}[e_.) + (f_.)*(x_)]*(d_.)]/\operatorname{Sqrt}[\operatorname{csc}[e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] \rightarrow \operatorname{Dist}[(-2*b*d)/(a*f), \operatorname{Subst}[\operatorname{Int}[1/(2*b - d*x^2), x], x, (b*\operatorname{Cot}[e + f*x])]/(\operatorname{Sqrt}[a + b*\operatorname{Csc}[e + f*x]]*\operatorname{Sqrt}[d*\operatorname{Csc}[e + f*x]])], x] /;$ FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && !LeQ[m, -1]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := Simp[(d*(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := -Simp[((A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m + 1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\sec(c+dx)}(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{5/2}} dx &= \frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{\int \frac{-\frac{1}{2}a(A-B)+2a(A+B)\sec(c+dx)}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}} dx}{4a^2} \\
&= \frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(5A+3B)\sqrt{\sec(c+dx)}\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} \\
&= \frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(5A+3B)\sqrt{\sec(c+dx)}\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} \\
&= \frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(5A+3B)\sqrt{\sec(c+dx)}\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} \\
&= \frac{(19A+5B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} + \frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}}
\end{aligned}$$

Mathematica [A] time = 1.16, size = 103, normalized size = 0.66

$$\frac{\sqrt{\sec(c+dx)}\left(\tan\left(\frac{1}{2}(c+dx)\right)\left((B-9A)\sec(c+dx)-13A+5B\right)+2(19A+5B)\cos^3\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)}{16ad(a(\sec(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(5/2), x]

[Out] (Sqrt[Sec[c + d*x]]*(2*(19*A + 5*B)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^3*Sec[c + d*x] + (-13*A + 5*B + (-9*A + B)*Sec[c + d*x])*Tan[(c + d*x)/2]))/(16*a*d*(a*(1 + Sec[c + d*x]))^(3/2))

fricas [A] time = 0.71, size = 502, normalized size = 3.22

$$\left[\frac{\sqrt{2}\left((19A+5B)\cos(dx+c)^3+3(19A+5B)\cos(dx+c)^2+3(19A+5B)\cos(dx+c)+19A+5B\right)\sqrt{a}\log\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{64\left(a^3d\cos(dx+c)^3+3a^3d\right)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")

[Out] [1/64*(sqrt(2)*((19*A + 5*B)*cos(d*x + c)^3 + 3*(19*A + 5*B)*cos(d*x + c)^2 + 3*(19*A + 5*B)*cos(d*x + c) + 19*A + 5*B)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*((13*A - 5*B)*cos(d*x + c)^2 + (9*A - B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1/32*(sqrt(2)*((19*A + 5*B)*cos(d*x + c)^3 + 3*(19*A + 5*B)*cos(d*x + c)^2 + 3*(19*A + 5*B)*cos(d*x + c) + 19*A + 5*B)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) + 2*((13*A - 5*B)*cos(d*x + c)^2 + (9*A - B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sqrt{\sec(dx + c)}}{(a \sec(dx + c) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(a*sec(d*x + c) + a)^(5/2), x)

maple [B] time = 2.38, size = 347, normalized size = 2.22

$$\sqrt{\frac{1}{\cos(dx+c)}} \sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}} \cos(dx+c) (-1 + \cos(dx+c))^2 \left(13A \sqrt{-\frac{2}{1+\cos(dx+c)}} (\cos^2(dx+c)) + 19A \cos(dx+c) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(5/2),x)

[Out] 1/16/d*(1/cos(d*x+c))^(1/2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)*(-1+cos(d*x+c))^2*(13*A*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2+19*A*cos(d*x+c)*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))-5*B*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2+5*B*cos(d*x+c)*sin(d*x+c)*arctan(1/2*sin(d*x+c)))

$$x+c)*(-2/(1+\cos(dx+c)))^{(1/2)}-4*A*(-2/(1+\cos(dx+c)))^{(1/2)}*\cos(dx+c)+19$$

$$*A*\arctan(1/2*\sin(dx+c)*(-2/(1+\cos(dx+c)))^{(1/2)})*\sin(dx+c)+4*B*(-2/(1+\cos(dx+c)))^{(1/2)}*\cos(dx+c)+5*B*\arctan(1/2*\sin(dx+c)*(-2/(1+\cos(dx+c)))^{(1/2)})*\sin(dx+c)-9*A*(-2/(1+\cos(dx+c)))^{(1/2)}+B*(-2/(1+\cos(dx+c)))^{(1/2)})/\sin(dx+c)^5/(-2/(1+\cos(dx+c)))^{(1/2)}/a^3$$

maxima [B] time = 2.73, size = 5924, normalized size = 37.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(dx+c))*sec(dx+c)^(1/2)/(a+a*sec(dx+c))^(5/2),x, algorithm="maxima")

[Out] 1/32*((19*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(4*d*x + 4*c)^2 + 304*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(3*d*x + 3*c)^2 + 684*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(2*d*x + 2*c)^2 + 304*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(dx + c)^2 + 19*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(4*d*x + 4*c)^2 + 304*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(3*d*x + 3*c)^2 + 684*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(2*d*x + 2*c)^2 + 304*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(dx + c)^2 + 2*(76*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(3*d*x + 3*c) + 114*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(2*d*x + 2*c) + 76*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(dx + c) + 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 26*sin(7/2*d*x + 7/2*c) - 10*sin(5/2*d*x

$$\begin{aligned}
& + 5/2*c) + 10*\sin(3/2*d*x + 3/2*c) + 26*\sin(1/2*d*x + 1/2*c))*\cos(4*d*x + \\
& 4*c) + 104*(2*\sin(3*d*x + 3*c) + 3*\sin(2*d*x + 2*c) + 2*\sin(d*x + c))*\cos(7 \\
& /2*d*x + 7/2*c) + 8*(114*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
& ^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(2*d*x + 2*c) + 76*(\log(\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c \\
&) + 1))*\cos(d*x + c) + 19*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
& ^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 19*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2* \\
& d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 10*\sin(5/2*d*x + 5/2*c) + 10 \\
& *sin(3/2*d*x + 3/2*c) + 26*\sin(1/2*d*x + 1/2*c))*\cos(3*d*x + 3*c) + 40*(3*s \\
& in(2*d*x + 2*c) + 2*\sin(d*x + c))*\cos(5/2*d*x + 5/2*c) + 12*(76*(\log(\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - l \\
& og(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) \\
& + 1))*\cos(d*x + c) + 19*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^ \\
& 2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 19*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d \\
& *x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) + 10*\sin(3/2*d*x + 3/2*c) + 26* \\
& sin(1/2*d*x + 1/2*c))*\cos(2*d*x + 2*c) + 8*(19*\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 19*\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) + 26*\sin(\\
& 1/2*d*x + 1/2*c))*\cos(d*x + c) + 4*(38*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(3*d*x + 3*c) + \\
& 57*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1 \\
& /2*d*x + 1/2*c) + 1))*\sin(2*d*x + 2*c) + 38*(\log(\cos(1/2*d*x + 1/2*c)^2 + s \\
& in(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2 \\
& *c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c) \\
& + 13*\cos(7/2*d*x + 7/2*c) + 5*\cos(5/2*d*x + 5/2*c) - 5*\cos(3/2*d*x + 3/2*c) \\
& - 13*\cos(1/2*d*x + 1/2*c))*\sin(4*d*x + 4*c) - 52*(4*\cos(3*d*x + 3*c) + 6*c \\
& os(2*d*x + 2*c) + 4*\cos(d*x + c) + 1)*\sin(7/2*d*x + 7/2*c) + 16*(57*(\log(co \\
& s(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) \\
& - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/ \\
& 2*c) + 1))*\sin(2*d*x + 2*c) + 38*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c) + 5*\cos(5/2 \\
& *d*x + 5/2*c) - 5*\cos(3/2*d*x + 3/2*c) - 13*\cos(1/2*d*x + 1/2*c))*\sin(3*d*x \\
& + 3*c) - 20*(6*\cos(2*d*x + 2*c) + 4*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c) \\
& + 24*(38*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2* \\
& d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 \\
& *sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c) - 5*\cos(3/2*d*x + 3/2*c) - 13*\cos(\\
& 1/2*d*x + 1/2*c))*\sin(2*d*x + 2*c) + 20*(4*\cos(d*x + c) + 1)*\sin(3/2*d*x + \\
& 3/2*c) - 80*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) - 208*\cos(1/2*d*x + 1/2*c)*si \\
& n(d*x + c) + 19*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin \\
& (1/2*d*x + 1/2*c) + 1) - 19*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*
\end{aligned}$$

$$\begin{aligned}
& c)^2 - 2\sin(1/2*d*x + 1/2*c) + 1) + 52\sin(1/2*d*x + 1/2*c))*A/((\sqrt{2})*a \\
& ^2*\cos(4*d*x + 4*c)^2 + 16*\sqrt{2})*a^2*\cos(3*d*x + 3*c)^2 + 36*\sqrt{2})*a^2* \\
& \cos(2*d*x + 2*c)^2 + 16*\sqrt{2})*a^2*\cos(d*x + c)^2 + \sqrt{2})*a^2*\sin(4*d*x \\
& + 4*c)^2 + 16*\sqrt{2})*a^2*\sin(3*d*x + 3*c)^2 + 36*\sqrt{2})*a^2*\sin(2*d*x + 2 \\
& *c)^2 + 48*\sqrt{2})*a^2*\sin(2*d*x + 2*c)*\sin(d*x + c) + 16*\sqrt{2})*a^2*\sin(d \\
& *x + c)^2 + 8*\sqrt{2})*a^2*\cos(d*x + c) + \sqrt{2})*a^2 + 2*(4*\sqrt{2})*a^2*\cos \\
& (3*d*x + 3*c) + 6*\sqrt{2})*a^2*\cos(2*d*x + 2*c) + 4*\sqrt{2})*a^2*\cos(d*x + c) \\
& + \sqrt{2})*a^2*\cos(4*d*x + 4*c) + 8*(6*\sqrt{2})*a^2*\cos(2*d*x + 2*c) + 4*sq \\
& rt(2)*a^2*\cos(d*x + c) + \sqrt{2})*a^2*\cos(3*d*x + 3*c) + 12*(4*\sqrt{2})*a^2* \\
& \cos(d*x + c) + \sqrt{2})*a^2*\cos(2*d*x + 2*c) + 4*(2*\sqrt{2})*a^2*\sin(3*d*x + \\
& 3*c) + 3*\sqrt{2})*a^2*\sin(2*d*x + 2*c) + 2*\sqrt{2})*a^2*\sin(d*x + c))*\sin(4* \\
& d*x + 4*c) + 16*(3*\sqrt{2})*a^2*\sin(2*d*x + 2*c) + 2*\sqrt{2})*a^2*\sin(d*x + c \\
&))*\sin(3*d*x + 3*c))*\sqrt{a}) + (4*(3*\sin(3/2*d*x + 3/2*c) + 5*\sin(7/3*\arct \\
& an2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) - 3*\sin(5/3*\arctan2(\sin(3/ \\
& 2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) - 5*\sin(1/3*\arctan2(\sin(3/2*d*x + 3/ \\
& 2*c), \cos(3/2*d*x + 3/2*c))))*\cos(8/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2 \\
& *d*x + 3/2*c))) - 40*(2*\sin(3*d*x + 3*c) + 3*\sin(4/3*\arctan2(\sin(3/2*d*x + \\
& 3/2*c), \cos(3/2*d*x + 3/2*c))) + 2*\sin(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), co \\
& s(3/2*d*x + 3/2*c))))*\cos(7/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3 \\
& /2*c))) + 24*(2*\sin(3*d*x + 3*c) + 3*\sin(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \\
& \cos(3/2*d*x + 3/2*c))) + 2*\sin(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d* \\
& x + 3/2*c))))*\cos(5/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) \\
& + 24*(3*\sin(3/2*d*x + 3/2*c) - 5*\sin(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(\\
& 3/2*d*x + 3/2*c))))*\cos(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2 \\
& *c))) + 16*(3*\sin(3/2*d*x + 3/2*c) - 5*\sin(1/3*\arctan2(\sin(3/2*d*x + 3/2*c) \\
& , \cos(3/2*d*x + 3/2*c))))*\cos(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x \\
& + 3/2*c))) + 5*(16*\cos(3*d*x + 3*c)^2 + 2*(4*\cos(3*d*x + 3*c) + 6*\cos(4/3* \\
& arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 4*\cos(2/3*\arctan2(si \\
& n(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 1)*\cos(8/3*\arctan2(\sin(3/2*d*x \\
& + 3/2*c), \cos(3/2*d*x + 3/2*c))) + \cos(8/3*\arctan2(\sin(3/2*d*x + 3/2*c), c \\
& os(3/2*d*x + 3/2*c)))^2 + 12*(4*\cos(3*d*x + 3*c) + 4*\cos(2/3*\arctan2(\sin(3/ \\
& 2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 1)*\cos(4/3*\arctan2(\sin(3/2*d*x + 3 \\
& /2*c), \cos(3/2*d*x + 3/2*c))) + 36*\cos(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), co \\
& s(3/2*d*x + 3/2*c)))^2 + 8*(4*\cos(3*d*x + 3*c) + 1)*\cos(2/3*\arctan2(\sin(3/2 \\
& *d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 16*\cos(2/3*\arctan2(\sin(3/2*d*x + 3/ \\
& 2*c), \cos(3/2*d*x + 3/2*c)))^2 + 16*\sin(3*d*x + 3*c)^2 + 4*(2*\sin(3*d*x + 3 \\
& *c) + 3*\sin(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 2*si \\
& n(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))))*\sin(8/3*\arctan2 \\
& (\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + \sin(8/3*\arctan2(\sin(3/2*d*x \\
& + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + 48*(\sin(3*d*x + 3*c) + \sin(2/3*\arctan \\
& 2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))))*\sin(4/3*\arctan2(\sin(3/2*d*x \\
& + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 36*\sin(4/3*\arctan2(\sin(3/2*d*x + 3/2*c) \\
& , \cos(3/2*d*x + 3/2*c)))^2 + 32*\sin(3*d*x + 3*c)*\sin(2/3*\arctan2(\sin(3/2*d* \\
& x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 16*\sin(2/3*\arctan2(\sin(3/2*d*x + 3/2*c \\
&), \cos(3/2*d*x + 3/2*c)))^2 + 8*\cos(3*d*x + 3*c) + 1)*\log(\cos(1/3*\arctan2(s
\end{aligned}$$

$$\begin{aligned}
& \sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))\wedge 2 + \sin(1/3*\arctan2(\sin(3/2*d*x \\
& + 3/2*c), \cos(3/2*d*x + 3/2*c))\wedge 2 + 2*\sin(1/3*\arctan2(\sin(3/2*d*x + 3/2*c \\
&), \cos(3/2*d*x + 3/2*c))) + 1) - 5*(16*\cos(3*d*x + 3*c)\wedge 2 + 2*(4*\cos(3*d*x \\
& + 3*c) + 6*\cos(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 4 \\
& *\cos(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 1)*\cos(8/3* \\
& \arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + \cos(8/3*\arctan2(\sin(\\
& 3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))\wedge 2 + 12*(4*\cos(3*d*x + 3*c) + 4*\cos \\
& (2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 1)*\cos(4/3*\arct \\
& an2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 36*\cos(4/3*\arctan2(\sin(3 \\
& /2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))\wedge 2 + 8*(4*\cos(3*d*x + 3*c) + 1)*\cos(\\
& 2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 16*\cos(2/3*\arcta \\
& n2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))\wedge 2 + 16*\sin(3*d*x + 3*c)\wedge 2 + \\
& 4*(2*\sin(3*d*x + 3*c) + 3*\sin(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d* \\
& x + 3/2*c))) + 2*\sin(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c) \\
&)))*\sin(8/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + \sin(8/3* \\
& \arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))\wedge 2 + 48*(\sin(3*d*x + 3* \\
& c) + \sin(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))))*\sin(4/3* \\
& \arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 36*\sin(4/3*\arctan2(s \\
& in(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))\wedge 2 + 32*\sin(3*d*x + 3*c)*\sin(2/3 \\
& *\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 16*\sin(2/3*\arctan2(\\
& \sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))\wedge 2 + 8*\cos(3*d*x + 3*c) + 1)*lo \\
& g(\cos(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))\wedge 2 + \sin(1/3* \\
& \arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))\wedge 2 - 2*\sin(1/3*\arctan2(\\
& \sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 1) - 48*\cos(3/2*d*x + 3/2*c) \\
& *\sin(3*d*x + 3*c) + 80*\cos(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + \\
& 3/2*c)))*\sin(3*d*x + 3*c) + 48*\cos(3*d*x + 3*c)*\sin(3/2*d*x + 3/2*c) - 4*(3 \\
& *\cos(3/2*d*x + 3/2*c) + 5*\cos(7/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x \\
& + 3/2*c))) - 3*\cos(5/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)) \\
&) - 5*\cos(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))))*\sin(8/3 \\
& *\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 20*(4*\cos(3*d*x + 3 \\
& *c) + 6*\cos(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 4*co \\
& s(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 1)*\sin(7/3*arc \\
& tan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) - 12*(4*\cos(3*d*x + 3*c) \\
& + 6*\cos(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 4*\cos(2/ \\
& 3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 1)*\sin(5/3*\arctan2 \\
& (\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) - 24*(3*\cos(3/2*d*x + 3/2*c) \\
& - 5*\cos(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))))*\sin(4/3*a \\
& rctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) - 16*(3*\cos(3/2*d*x + 3 \\
& /2*c) - 5*\cos(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))))*\sin \\
& (2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) - 20*(4*\cos(3*d*x \\
& + 3*c) + 1)*\sin(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + \\
& 12*\sin(3/2*d*x + 3/2*c))*B/((16*\sqrt{2})*a\wedge 2*\cos(3*d*x + 3*c)\wedge 2 + \sqrt{2})*a \\
& \wedge 2*\cos(8/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))\wedge 2 + 36*\sqrt{ \\
& 2})*a\wedge 2*\cos(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))\wedge 2 + 16 \\
& *\sqrt{2})*a\wedge 2*\cos(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))\wedge 2
\end{aligned}$$

+ 16*sqrt(2)*a^2*sin(3*d*x + 3*c)^2 + sqrt(2)*a^2*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 36*sqrt(2)*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 32*sqrt(2)*a^2*sin(3*d*x + 3*c)*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 16*sqrt(2)*a^2*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 8*sqrt(2)*a^2*cos(3*d*x + 3*c) + sqrt(2)*a^2 + 2*(4*sqrt(2)*a^2*cos(3*d*x + 3*c) + 6*sqrt(2)*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))) + 4*sqrt(2)*a^2*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sqrt(2)*a^2*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 12*(4*sqrt(2)*a^2*cos(3*d*x + 3*c) + 4*sqrt(2)*a^2*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))) + sqrt(2)*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 8*(4*sqrt(2)*a^2*cos(3*d*x + 3*c) + sqrt(2)*a^2*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))) + 4*(2*sqrt(2)*a^2*sin(3*d*x + 3*c) + 3*sqrt(2)*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*a^2*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 48*(sqrt(2)*a^2*sin(3*d*x + 3*c) + sqrt(2)*a^2*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sqrt(a))/d

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \sqrt{\frac{1}{\cos(c+dx)}}}{\left(a + \frac{a}{\cos(c+dx)}\right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + a/cos(c + d*x))^(5/2), x)

[Out] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + a/cos(c + d*x))^(5/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \sqrt{\sec(c + dx)}}{(a (\sec(c + dx) + 1))^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)**(1/2)/(a+a*sec(d*x+c))**(5/2), x)

[Out] Integral((A + B*sec(c + d*x))*sqrt(sec(c + d*x))/(a*(sec(c + d*x) + 1))**(5/2), x)

$$3.266 \quad \int \frac{A+B \sec(c+dx)}{\sqrt{\sec(c+dx)} (a+a \sec(c+dx))^{5/2}} dx$$

Optimal. Leaf size=203

$$\frac{(75A - 19B) \tanh^{-1} \left(\frac{\sqrt{a} \sin(c+dx) \sqrt{\sec(c+dx)}}{\sqrt{2} \sqrt{a \sec(c+dx)+a}} \right)}{16\sqrt{2} a^{5/2} d} + \frac{(49A - 9B) \sin(c + dx) \sqrt{\sec(c + dx)}}{16a^2 d \sqrt{a \sec(c + dx) + a}} - \frac{(13A - 5B) \sin(c + dx) \sqrt{\sec(c + dx)}}{16ad(a \sec(c + dx) + a)^{3/2}}$$

[Out] $-1/32*(75*A-19*B)*\operatorname{arctanh}(1/2*\sin(d*x+c)*a^{(1/2)}*\sec(d*x+c)^{(1/2)}*2^{(1/2)})/(a+a*\sec(d*x+c))^{(1/2)}/a^{(5/2)}/d*2^{(1/2)}-1/4*(A-B)*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/d/(a+a*\sec(d*x+c))^{(5/2)}-1/16*(13*A-5*B)*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/a/d/(a+a*\sec(d*x+c))^{(3/2)}+1/16*(49*A-9*B)*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/a^2/d/(a+a*\sec(d*x+c))^{(1/2)}$

Rubi [A] time = 0.57, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {4020, 4013, 3808, 206}

$$\frac{(49A - 9B) \sin(c + dx) \sqrt{\sec(c + dx)}}{16a^2 d \sqrt{a \sec(c + dx) + a}} - \frac{(75A - 19B) \tanh^{-1} \left(\frac{\sqrt{a} \sin(c+dx) \sqrt{\sec(c+dx)}}{\sqrt{2} \sqrt{a \sec(c+dx)+a}} \right)}{16\sqrt{2} a^{5/2} d} - \frac{(13A - 5B) \sin(c + dx) \sqrt{\sec(c + dx)}}{16ad(a \sec(c + dx) + a)^{3/2}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(A + B*\operatorname{Sec}[c + d*x])/(\operatorname{Sqrt}[\operatorname{Sec}[c + d*x]]*(a + a*\operatorname{Sec}[c + d*x])^{(5/2)})], x]$

[Out] $-((75*A - 19*B)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Sqrt}[\operatorname{Sec}[c + d*x]]*\operatorname{Sin}[c + d*x])/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a + a*\operatorname{Sec}[c + d*x]])])/(16*\operatorname{Sqrt}[2]*a^{(5/2)}*d) - ((A - B)*\operatorname{Sqrt}[\operatorname{Sec}[c + d*x]]*\operatorname{Sin}[c + d*x])/(4*d*(a + a*\operatorname{Sec}[c + d*x])^{(5/2)}) - ((13*A - 5*B)*\operatorname{Sqrt}[\operatorname{Sec}[c + d*x]]*\operatorname{Sin}[c + d*x])/(16*a*d*(a + a*\operatorname{Sec}[c + d*x])^{(3/2)}) + ((49*A - 9*B)*\operatorname{Sqrt}[\operatorname{Sec}[c + d*x]]*\operatorname{Sin}[c + d*x])/(16*a^2*d*\operatorname{Sqrt}[a + a*\operatorname{Sec}[c + d*x]])$

Rule 206

$\operatorname{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1*\operatorname{ArcTanh}[(\operatorname{Rt}[-b, 2]*x)/\operatorname{Rt}[a, 2]])/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /; \operatorname{FreeQ}\{a, b\}, x] \ \&\& \operatorname{NegQ}[a/b] \ \&\& (\operatorname{GtQ}[a, 0] \ || \ \operatorname{LtQ}[b, 0])$

Rule 3808

$\operatorname{Int}[\operatorname{Sqrt}[\operatorname{csc}[e_ + (f_)*(x_)]*(d_)]/\operatorname{Sqrt}[\operatorname{csc}[e_ + (f_)*(x_)]*(b_ + (a_))], x_Symbol] \rightarrow \operatorname{Dist}[(-2*b*d)/(a*f), \operatorname{Subst}[\operatorname{Int}[1/(2*b - d*x^2), x], x, (b*\operatorname{Cot}[e + f*x])/(\operatorname{Sqrt}[a + b*\operatorname{Csc}[e + f*x]]*\operatorname{Sqrt}[d*\operatorname{Csc}[e + f*x]])], x] /; \operatorname{FreeQ}\{a, b, d, e, f\}, x] \ \&\& \operatorname{EqQ}[a^2 - b^2, 0]$

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (
a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
- b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] && !LeQ[m, -1]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (
a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
+ f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
f*x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0]
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{A + B \sec(c + dx)}{\sqrt{\sec(c + dx)} (a + a \sec(c + dx))^{5/2}} dx &= -\frac{(A - B)\sqrt{\sec(c + dx)} \sin(c + dx)}{4d(a + a \sec(c + dx))^{5/2}} + \frac{\int \frac{\frac{1}{2}a(9A - B) - 2a(A - B)\sec(c + dx)}{\sqrt{\sec(c + dx)} (a + a \sec(c + dx))^{3/2}} dx}{4a^2} \\ &= -\frac{(A - B)\sqrt{\sec(c + dx)} \sin(c + dx)}{4d(a + a \sec(c + dx))^{5/2}} - \frac{(13A - 5B)\sqrt{\sec(c + dx)} \sin(c + dx)}{16ad(a + a \sec(c + dx))^3} \\ &= -\frac{(A - B)\sqrt{\sec(c + dx)} \sin(c + dx)}{4d(a + a \sec(c + dx))^{5/2}} - \frac{(13A - 5B)\sqrt{\sec(c + dx)} \sin(c + dx)}{16ad(a + a \sec(c + dx))^3} \\ &= -\frac{(A - B)\sqrt{\sec(c + dx)} \sin(c + dx)}{4d(a + a \sec(c + dx))^{5/2}} - \frac{(13A - 5B)\sqrt{\sec(c + dx)} \sin(c + dx)}{16ad(a + a \sec(c + dx))^3} \\ &= -\frac{(75A - 19B) \tanh^{-1}\left(\frac{\sqrt{a} \sqrt{\sec(c + dx)} \sin(c + dx)}{\sqrt{2} \sqrt{a + a \sec(c + dx)}}\right)}{16\sqrt{2} a^{5/2} d} - \frac{(A - B)\sqrt{\sec(c + dx)} \sin(c + dx)}{4d(a + a \sec(c + dx))^{5/2}} \end{aligned}$$

Mathematica [A] time = 2.61, size = 206, normalized size = 1.01

$$\frac{\sin(c + dx) \left((49A - 9B)\sqrt{1 - \sec(c + dx)} \sec^5(c + dx) + (85A - 13B)\sqrt{1 - \sec(c + dx)} \sec^3(c + dx) + 32A\sqrt{1 - \sec(c + dx)} \right)}{16d\sqrt{1 - \sec(c + dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(5/2)),x]
```

```
[Out] (4*Sqrt[2]*(75*A - 19*B)*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]])*Cos[(c + d*x)/2]^5*Sec[c + d*x]^3*Sin[(c + d*x)/2] + ((85*A - 13*B)*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(3/2) + (49*A - 9*B)*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(5/2) + 32*A*Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])]*Sin[c + d*x])/(16*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(5/2))
```

fricas [A] time = 0.48, size = 524, normalized size = 2.58

$$\frac{\sqrt{2} \left((75A - 19B) \cos(dx + c)^3 + 3(75A - 19B) \cos(dx + c)^2 + 3(75A - 19B) \cos(dx + c) + 75A - 19B \right) \sqrt{\sec(dx + c)}}{64 \left(a^3 d \cos(dx + c) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/64*(sqrt(2)*((75*A - 19*B)*cos(d*x + c)^3 + 3*(75*A - 19*B)*cos(d*x + c)^2 + 3*(75*A - 19*B)*cos(d*x + c) + 75*A - 19*B)*sqrt(a)*log(-(a*cos(d*x + c))^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*(32*A*cos(d*x + c)^3 + (85*A - 13*B)*cos(d*x + c)^2 + (49*A - 9*B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), 1/32*(sqrt(2)*((75*A - 19*B)*cos(d*x + c)^3 + 3*(75*A - 19*B)*cos(d*x + c)^2 + 3*(75*A - 19*B)*cos(d*x + c) + 75*A - 19*B)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) + 2*(32*A*cos(d*x + c)^3 + (85*A - 13*B)*cos(d*x + c)^2 + (49*A - 9*B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(a \sec(dx + c) + a)^{\frac{5}{2}} \sqrt{\sec(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^(5/2)*sqrt(sec(d*x + c))), x)

maple [B] time = 2.74, size = 419, normalized size = 2.06

$$(-1 + \cos(dx + c))^2 \left(-75A (\cos^2(dx + c)) \arctan \left(\frac{\sin(dx+c) \sqrt{\frac{2}{1+\cos(dx+c)}}}{2} \right) \sqrt{\frac{2}{1+\cos(dx+c)}} \sin(dx + c) + 19B (\cos(dx + c)) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x)

[Out]
$$-1/32/d*(-1+\cos(d*x+c))^2*(-75*A*\cos(d*x+c)^2*\arctan(1/2*\sin(d*x+c)*(-2/(1+\cos(d*x+c)))^(1/2))*(-2/(1+\cos(d*x+c)))^(1/2)*\sin(d*x+c)+19*B*\cos(d*x+c)^2*\arctan(1/2*\sin(d*x+c)*(-2/(1+\cos(d*x+c)))^(1/2))*(-2/(1+\cos(d*x+c)))^(1/2)*\sin(d*x+c)-150*A*\arctan(1/2*\sin(d*x+c)*(-2/(1+\cos(d*x+c)))^(1/2))*(-2/(1+\cos(d*x+c)))^(1/2)*\cos(d*x+c)*\sin(d*x+c)+38*B*\arctan(1/2*\sin(d*x+c)*(-2/(1+\cos(d*x+c)))^(1/2))*(-2/(1+\cos(d*x+c)))^(1/2)*\cos(d*x+c)*\sin(d*x+c)+64*A*\cos(d*x+c)^3-75*\arctan(1/2*\sin(d*x+c)*(-2/(1+\cos(d*x+c)))^(1/2))*(-2/(1+\cos(d*x+c)))^(1/2)*A*\sin(d*x+c)+19*\arctan(1/2*\sin(d*x+c)*(-2/(1+\cos(d*x+c)))^(1/2))*(-2/(1+\cos(d*x+c)))^(1/2)*B*\sin(d*x+c)+106*A*\cos(d*x+c)^2-26*B*\cos(d*x+c)^2-72*A*\cos(d*x+c)+8*B*\cos(d*x+c)-98*A+18*B)*(a*(1+\cos(d*x+c))/\cos(d*x+c))^(1/2)/\sin(d*x+c)^5/(1/\cos(d*x+c))^(1/2)/a^3$$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\left(a + \frac{a}{\cos(c+dx)}\right)^{5/2} \sqrt{\frac{1}{\cos(c+dx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/((a + a/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(1/2)),x)
```

```
[Out] int((A + B/cos(c + d*x))/((a + a/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(1/2)), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(5/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

$$3.267 \quad \int \frac{A+B \sec(c+dx)}{\sec^2(c+dx)(a+a \sec(c+dx))^{5/2}} dx$$

Optimal. Leaf size=250

$$\frac{(163A - 75B) \tanh^{-1} \left(\frac{\sqrt{a} \sin(c+dx) \sqrt{\sec(c+dx)}}{\sqrt{2} \sqrt{a \sec(c+dx)+a}} \right)}{16\sqrt{2} a^{5/2} d} - \frac{(299A - 147B) \sin(c+dx) \sqrt{\sec(c+dx)}}{48a^2 d \sqrt{a \sec(c+dx)+a}} + \frac{(95A - 39B) \sin(c+dx)}{48a^2 d \sqrt{\sec(c+dx)} \sqrt{a}}$$

[Out] $1/32*(163*A-75*B)*\operatorname{arctanh}(1/2*\sin(d*x+c)*a^{1/2}*\sec(d*x+c)^{(1/2)}*2^{(1/2)}/(a+a*\sec(d*x+c))^{(1/2)})/a^{(5/2)}/d*2^{(1/2)}-1/4*(A-B)*\sin(d*x+c)/d/(a+a*\sec(d*x+c))^{(5/2)}/\sec(d*x+c)^{(1/2)}-1/16*(17*A-9*B)*\sin(d*x+c)/a/d/(a+a*\sec(d*x+c))^{(3/2)}/\sec(d*x+c)^{(1/2)}+1/48*(95*A-39*B)*\sin(d*x+c)/a^2/d/\sec(d*x+c)^{(1/2)}/(a+a*\sec(d*x+c))^{(1/2)}-1/48*(299*A-147*B)*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/a^2/d/(a+a*\sec(d*x+c))^{(1/2)}$

Rubi [A] time = 0.76, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4020, 4022, 4013, 3808, 206}

$$-\frac{(299A - 147B) \sin(c+dx) \sqrt{\sec(c+dx)}}{48a^2 d \sqrt{a \sec(c+dx)+a}} + \frac{(95A - 39B) \sin(c+dx)}{48a^2 d \sqrt{\sec(c+dx)} \sqrt{a \sec(c+dx)+a}} + \frac{(163A - 75B) \tanh^{-1} \left(\frac{\sqrt{a} \sin(c+dx) \sqrt{\sec(c+dx)}}{\sqrt{2} \sqrt{a \sec(c+dx)+a}} \right)}{16\sqrt{2} a^{5/2} d}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(A + B*\operatorname{Sec}[c + d*x])/(\operatorname{Sec}[c + d*x]^{(3/2)}*(a + a*\operatorname{Sec}[c + d*x])^{(5/2)}), x]$

[Out] $((163*A - 75*B)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Sqrt}[\operatorname{Sec}[c + d*x]]*\operatorname{Sin}[c + d*x])/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a + a*\operatorname{Sec}[c + d*x]])])/((16*\operatorname{Sqrt}[2]*a^{(5/2)}*d) - ((A - B)*\operatorname{Sin}[c + d*x]))/(4*d*\operatorname{Sqrt}[\operatorname{Sec}[c + d*x]]*(a + a*\operatorname{Sec}[c + d*x])^{(5/2)}) - ((17*A - 9*B)*\operatorname{Sin}[c + d*x])/((16*a*d*\operatorname{Sqrt}[\operatorname{Sec}[c + d*x]]*(a + a*\operatorname{Sec}[c + d*x])^{(3/2)}) + ((95*A - 39*B)*\operatorname{Sin}[c + d*x])/((48*a^2*d*\operatorname{Sqrt}[\operatorname{Sec}[c + d*x]]*\operatorname{Sqrt}[a + a*\operatorname{Sec}[c + d*x]]) - ((299*A - 147*B)*\operatorname{Sqrt}[\operatorname{Sec}[c + d*x]]*\operatorname{Sin}[c + d*x])/((48*a^2*d*\operatorname{Sqrt}[a + a*\operatorname{Sec}[c + d*x]]))$

Rule 206

$\operatorname{Int}(((a_) + (b_)*(x_)^2)^{-1}, x_Symbol] := \operatorname{Simp}[(1*\operatorname{ArcTanh}[(\operatorname{Rt}[-b, 2]*x)/\operatorname{Rt}[a, 2]])/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /; \operatorname{FreeQ}\{a, b\}, x] \ \&\& \operatorname{NegQ}[a/b] \ \&\& (\operatorname{GtQ}[a, 0] \ || \operatorname{LtQ}[b, 0])$

Rule 3808

$\operatorname{Int}[\operatorname{Sqrt}[\operatorname{csc}[(e_) + (f_)*(x_)]*(d_)]/\operatorname{Sqrt}[\operatorname{csc}[(e_) + (f_)*(x_)]*(b_) + (a_)], x_Symbol] := \operatorname{Dist}[(-2*b*d)/(a*f), \operatorname{Subst}[\operatorname{Int}[1/(2*b - d*x^2), x], x]$

, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]), x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 4013

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && !LeQ[m, -1]

Rule 4020

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m + 1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0]

Rule 4022

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]

Rubi steps

$$\begin{aligned}
\int \frac{A + B \sec(c + dx)}{\sec^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))^{5/2}} dx &= -\frac{(A - B) \sin(c + dx)}{4d\sqrt{\sec(c + dx)}(a + a \sec(c + dx))^{5/2}} + \int \frac{\frac{1}{2}a(11A-3B)-3a(A-B)\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} dx \\
&= -\frac{(A - B) \sin(c + dx)}{4d\sqrt{\sec(c + dx)}(a + a \sec(c + dx))^{5/2}} - \frac{(17A - 9B) \sin(c + dx)}{16ad\sqrt{\sec(c + dx)}(a + a \sec(c + dx))^{5/2}} \\
&= -\frac{(A - B) \sin(c + dx)}{4d\sqrt{\sec(c + dx)}(a + a \sec(c + dx))^{5/2}} - \frac{(17A - 9B) \sin(c + dx)}{16ad\sqrt{\sec(c + dx)}(a + a \sec(c + dx))^{5/2}} \\
&= -\frac{(A - B) \sin(c + dx)}{4d\sqrt{\sec(c + dx)}(a + a \sec(c + dx))^{5/2}} - \frac{(17A - 9B) \sin(c + dx)}{16ad\sqrt{\sec(c + dx)}(a + a \sec(c + dx))^{5/2}} \\
&= -\frac{(A - B) \sin(c + dx)}{4d\sqrt{\sec(c + dx)}(a + a \sec(c + dx))^{5/2}} - \frac{(17A - 9B) \sin(c + dx)}{16ad\sqrt{\sec(c + dx)}(a + a \sec(c + dx))^{5/2}} \\
&= \frac{(163A - 75B) \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{(A - B) \sin(c + dx)}{4d\sqrt{\sec(c + dx)}(a + a \sec(c + dx))^{5/2}}
\end{aligned}$$

Mathematica [A] time = 1.89, size = 193, normalized size = 0.77

$$\frac{2 \tan(c + dx) \sqrt{1 - \sec(c + dx)} \sec^2(c + dx) ((255B - 479A) \cos(c + dx) + (48B - 80A) \cos(2(c + dx)) + 8A \cos(3(c + dx)))}{96d \sqrt{-((\sec(c + dx) - 1) \sec(c + dx))}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(5/2)), x]

[Out] (-12*sqrt(2)*(163*A - 75*B)*ArcTan[(sqrt(2)*sqrt[Sec[c + d*x]])/sqrt[1 - Sec[c + d*x]]]*Cos[(c + d*x)/2]^4*Sec[c + d*x]^(7/2)*Sin[c + d*x] + 2*(-379*A + 195*B + (-479*A + 255*B)*Cos[c + d*x] + (-80*A + 48*B)*Cos[2*(c + d*x)] + 8*A*Cos[3*(c + d*x)])*sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^2*Tan[c + d*x])/(96*d*sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])]*(a*(1 + Sec[c + d*x]))^(5/2))

fricas [A] time = 0.48, size = 564, normalized size = 2.26

$$3\sqrt{2}\left((163A - 75B)\cos(dx + c)^3 + 3(163A - 75B)\cos(dx + c)^2 + 3(163A - 75B)\cos(dx + c) + 163A - 75B\right)$$

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Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")

[Out] [-1/192*(3*sqrt(2)*((163*A - 75*B)*cos(d*x + c)^3 + 3*(163*A - 75*B)*cos(d*x + c)^2 + 3*(163*A - 75*B)*cos(d*x + c) + 163*A - 75*B)*sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*(32*A*cos(d*x + c)^4 - 32*(5*A - 3*B)*cos(d*x + c)^3 - (503*A - 255*B)*cos(d*x + c)^2 - (299*A - 147*B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1/96*(3*sqrt(2)*((163*A - 75*B)*cos(d*x + c)^3 + 3*(163*A - 75*B)*cos(d*x + c)^2 + 3*(163*A - 75*B)*cos(d*x + c) + 163*A - 75*B)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c)))/(a*sin(d*x + c))) - 2*(32*A*cos(d*x + c)^4 - 32*(5*A - 3*B)*cos(d*x + c)^3 - (503*A - 255*B)*cos(d*x + c)^2 - (299*A - 147*B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(a \sec(dx + c) + a)^{\frac{5}{2}} \sec(dx + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(3/2)), x)

maple [B] time = 2.88, size = 449, normalized size = 1.80

$$\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}} (-1 + \cos(dx+c))^2 \left(489A (\cos^2(dx+c)) \arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right) \sqrt{-\frac{2}{1+\cos(dx+c)}} \sin(dx+c) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x)

[Out]
$$-1/96/d*(a*(1+\cos(d*x+c))/\cos(d*x+c))^{1/2}*(-1+\cos(d*x+c))^{2*(489*A*\cos(d*x+c)^2*\arctan(1/2*\sin(d*x+c)*(-2/(1+\cos(d*x+c)))^{1/2})*(-2/(1+\cos(d*x+c)))^{1/2}*\sin(d*x+c)-225*B*\cos(d*x+c)^2*\arctan(1/2*\sin(d*x+c)*(-2/(1+\cos(d*x+c)))^{1/2})*(-2/(1+\cos(d*x+c)))^{1/2}*\sin(d*x+c)+978*A*\arctan(1/2*\sin(d*x+c)*(-2/(1+\cos(d*x+c)))^{1/2})*(-2/(1+\cos(d*x+c)))^{1/2}*\cos(d*x+c)*\sin(d*x+c)+64*A*\cos(d*x+c)^4-450*B*\arctan(1/2*\sin(d*x+c)*(-2/(1+\cos(d*x+c)))^{1/2})*(-2/(1+\cos(d*x+c)))^{1/2}*\cos(d*x+c)*\sin(d*x+c)+489*\arctan(1/2*\sin(d*x+c)*(-2/(1+\cos(d*x+c)))^{1/2})*(-2/(1+\cos(d*x+c)))^{1/2}*A*\sin(d*x+c)-384*A*\cos(d*x+c)^3-225*\arctan(1/2*\sin(d*x+c)*(-2/(1+\cos(d*x+c)))^{1/2})*(-2/(1+\cos(d*x+c)))^{1/2}*B*\sin(d*x+c)+192*B*\cos(d*x+c)^3-686*A*\cos(d*x+c)^2+318*B*\cos(d*x+c)^2+408*A*\cos(d*x+c)-216*B*\cos(d*x+c)+598*A-294*B)*\cos(d*x+c)^2*(1/\cos(d*x+c))^{3/2}/\sin(d*x+c)^5/a^3$$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x, algorith="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\left(a + \frac{a}{\cos(c+dx)}\right)^{5/2} \left(\frac{1}{\cos(c+dx)}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/((a + a/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(3/2)),x)

```
[Out] int((A + B/cos(c + d*x))/((a + a/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(3/2)), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)**(3/2)/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

$$3.268 \quad \int \frac{A+B \sec(c+dx)}{\sec^2(c+dx)(a+a \sec(c+dx))^{5/2}} dx$$

Optimal. Leaf size=297

$$\frac{(283A - 163B) \tanh^{-1}\left(\frac{\sqrt{a} \sin(c+dx) \sqrt{\sec(c+dx)}}{\sqrt{2} \sqrt{a \sec(c+dx)+a}}\right)}{16\sqrt{2} a^{5/2} d} + \frac{(157A - 85B) \sin(c+dx)}{80a^2 d \sec^2(c+dx) \sqrt{a \sec(c+dx)+a}} + \frac{(2671A - 1495B) \sin(c+dx)}{240a^2 d \sqrt{a \sec(c+dx)+a}}$$

[Out] $-1/4*(A-B)*\sin(d*x+c)/d/\sec(d*x+c)^{(3/2)}/(a+a*\sec(d*x+c))^{(5/2)}-1/16*(21*A-13*B)*\sin(d*x+c)/a/d/\sec(d*x+c)^{(3/2)}/(a+a*\sec(d*x+c))^{(3/2)}-1/32*(283*A-163*B)*\operatorname{arctanh}(1/2*\sin(d*x+c)*a^{(1/2)}*\sec(d*x+c)^{(1/2)}*2^{(1/2)}/(a+a*\sec(d*x+c))^{(1/2)})/a^{(5/2)}/d*2^{(1/2)}+1/80*(157*A-85*B)*\sin(d*x+c)/a^2/d/\sec(d*x+c)^{(3/2)}/(a+a*\sec(d*x+c))^{(1/2)}-1/240*(787*A-475*B)*\sin(d*x+c)/a^2/d/\sec(d*x+c)^{(1/2)}/(a+a*\sec(d*x+c))^{(1/2)}+1/240*(2671*A-1495*B)*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/a^2/d/(a+a*\sec(d*x+c))^{(1/2)}$

Rubi [A] time = 0.96, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4020, 4022, 4013, 3808, 206}

$$\frac{(157A - 85B) \sin(c+dx)}{80a^2 d \sec^2(c+dx) \sqrt{a \sec(c+dx)+a}} + \frac{(2671A - 1495B) \sin(c+dx) \sqrt{\sec(c+dx)}}{240a^2 d \sqrt{a \sec(c+dx)+a}} - \frac{(787A - 475B) \sin(c+dx)}{240a^2 d \sqrt{\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/2)),x]

[Out] $-((283*A - 163*B)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Sqrt}[\operatorname{Sec}[c + d*x]]*\operatorname{Sin}[c + d*x])]/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a + a*\operatorname{Sec}[c + d*x]])))/(16*\operatorname{Sqrt}[2]*a^{(5/2)}*d) - ((A - B)*\operatorname{Sin}[c + d*x])/((4*d*\operatorname{Sec}[c + d*x]^{(3/2)}*(a + a*\operatorname{Sec}[c + d*x])^{(5/2)}) - ((21*A - 13*B)*\operatorname{Sin}[c + d*x])/(16*a*d*\operatorname{Sec}[c + d*x]^{(3/2)}*(a + a*\operatorname{Sec}[c + d*x])^{(3/2)}) + ((157*A - 85*B)*\operatorname{Sin}[c + d*x])/(80*a^2*d*\operatorname{Sec}[c + d*x]^{(3/2)}*\operatorname{Sqrt}[a + a*\operatorname{Sec}[c + d*x]]) - ((787*A - 475*B)*\operatorname{Sin}[c + d*x])/(240*a^2*d*\operatorname{Sqrt}[\operatorname{Sec}[c + d*x]]*\operatorname{Sqrt}[a + a*\operatorname{Sec}[c + d*x]]) + ((2671*A - 1495*B)*\operatorname{Sqrt}[\operatorname{Sec}[c + d*x]]*\operatorname{Sin}[c + d*x])/(240*a^2*d*\operatorname{Sqrt}[a + a*\operatorname{Sec}[c + d*x]])$

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] := Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && !LeQ[m, -1]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := -Simp[((A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m + 1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0]
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{A + B \sec(c + dx)}{\sec^{\frac{5}{2}}(c + dx)(a + a \sec(c + dx))^{5/2}} dx &= -\frac{(A - B) \sin(c + dx)}{4d \sec^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))^{5/2}} + \int \frac{\frac{1}{2}a(13A-5B)-4a(A-B) \sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+a \sec(c+dx))^{3/2}} dx \\
&= -\frac{(A - B) \sin(c + dx)}{4d \sec^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))^{5/2}} - \frac{(21A - 13B) \sin(c + dx)}{16ad \sec^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))^{5/2}} \\
&= -\frac{(A - B) \sin(c + dx)}{4d \sec^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))^{5/2}} - \frac{(21A - 13B) \sin(c + dx)}{16ad \sec^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))^{5/2}} \\
&= -\frac{(A - B) \sin(c + dx)}{4d \sec^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))^{5/2}} - \frac{(21A - 13B) \sin(c + dx)}{16ad \sec^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))^{5/2}} \\
&= -\frac{(A - B) \sin(c + dx)}{4d \sec^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))^{5/2}} - \frac{(21A - 13B) \sin(c + dx)}{16ad \sec^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))^{5/2}} \\
&= -\frac{(A - B) \sin(c + dx)}{4d \sec^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))^{5/2}} - \frac{(21A - 13B) \sin(c + dx)}{16ad \sec^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))^{5/2}} \\
&= -\frac{(A - B) \sin(c + dx)}{4d \sec^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))^{5/2}} - \frac{(21A - 13B) \sin(c + dx)}{16ad \sec^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))^{5/2}} \\
&= -\frac{(283A - 163B) \tanh^{-1}\left(\frac{\sqrt{a} \sqrt{\sec(c+dx)} \sin(c+dx)}{\sqrt{2} \sqrt{a+a \sec(c+dx)}}\right)}{16\sqrt{2} a^{5/2} d} - \frac{(A - B) \sin(c + dx)}{4d \sec^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))^{5/2}}
\end{aligned}$$

Mathematica [A] time = 2.40, size = 196, normalized size = 0.66

$$\sec^2(c + dx) \left(\frac{30\sqrt{2}(283A-163B) \cos^4\left(\frac{1}{2}(c+dx)\right) \tan(c+dx) \tan^{-1}\left(\frac{\sqrt{2} \sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)}{\sqrt{1-\sec(c+dx)}} + \sin(c + dx) \sqrt{\sec(c + dx)} (5(887A - 479B) \cos^2(c + dx) - 240a) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/2)), x]

[Out] (Sec[c + d*x]^2*((3491*A - 1895*B + 5*(887*A - 479*B)*Cos[c + d*x] + 16*(52*A - 25*B)*Cos[2*(c + d*x)] - 40*A*Cos[3*(c + d*x)] + 40*B*Cos[3*(c + d*x)]

+ 12*A*Cos[4*(c + d*x)]*Sqrt[Sec[c + d*x]]*Sin[c + d*x] + (30*Sqrt[2]*(283*A - 163*B)*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Cos[(c + d*x)/2]^4*Tan[c + d*x])/Sqrt[1 - Sec[c + d*x]]))^(5/2))

fricas [A] time = 0.47, size = 592, normalized size = 1.99

$$\frac{15\sqrt{2}\left((283A - 163B)\cos(dx + c)^3 + 3(283A - 163B)\cos(dx + c)^2 + 3(283A - 163B)\cos(dx + c) + 283A - 163B\right)\sqrt{a}\log\left(\frac{(a\cos(dx + c)^2 - 2\sqrt{2}\sqrt{a}\sqrt{(a\cos(dx + c) + a)/\cos(dx + c)})\sqrt{\cos(dx + c)}\sin(dx + c) - 2a\cos(dx + c) - 3a)}{(a^3d\cos(dx + c)^3 + 3a^3d\cos(dx + c)^2 + 3a^3d\cos(dx + c) + a^3d)}\right) + 2(96A\cos(dx + c)^5 - 160(A - B)\cos(dx + c)^4 + 32(49A - 25B)\cos(dx + c)^3 + 5(911A - 503B)\cos(dx + c)^2 + (2671A - 1495B)\cos(dx + c))\sqrt{(a\cos(dx + c) + a)/\cos(dx + c)}\sin(dx + c)/\sqrt{\cos(dx + c)}}}{(a^3d\cos(dx + c)^3 + 3a^3d\cos(dx + c)^2 + 3a^3d\cos(dx + c) + a^3d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(5/2),x, algorith="fricas")

[Out] [-1/960*(15*sqrt(2)*((283*A - 163*B)*cos(d*x + c)^3 + 3*(283*A - 163*B)*cos(d*x + c)^2 + 3*(283*A - 163*B)*cos(d*x + c) + 283*A - 163*B)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*(96*A*cos(d*x + c)^5 - 160*(A - B)*cos(d*x + c)^4 + 32*(49*A - 25*B)*cos(d*x + c)^3 + 5*(911*A - 503*B)*cos(d*x + c)^2 + (2671*A - 1495*B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), 1/480*(15*sqrt(2)*((283*A - 163*B)*cos(d*x + c)^3 + 3*(283*A - 163*B)*cos(d*x + c)^2 + 3*(283*A - 163*B)*cos(d*x + c) + 283*A - 163*B)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) + 2*(96*A*cos(d*x + c)^5 - 160*(A - B)*cos(d*x + c)^4 + 32*(49*A - 25*B)*cos(d*x + c)^3 + 5*(911*A - 503*B)*cos(d*x + c)^2 + (2671*A - 1495*B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(a \sec(dx + c) + a)^{\frac{5}{2}} \sec(dx + c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(5/2),x, algorith="giac")

[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(5/2)), x)

maple [A] time = 3.02, size = 471, normalized size = 1.59

$$\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}} (-1 + \cos(dx + c))^2 \left(4245A (\cos^2(dx + c)) \arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right) \sqrt{-\frac{2}{1+\cos(dx+c)}} \sin(dx + c) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(5/2),x)

[Out] 1/480/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))^2*(4245*A*cos(d*x+c)^2*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c))))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-192*A*cos(d*x+c)^5-2445*B*cos(d*x+c)^2*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c))))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+8490*A*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c))))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)+512*A*cos(d*x+c)^4-4890*B*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c))))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)-320*B*cos(d*x+c)^4+4245*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c))))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*A*sin(d*x+c)-3456*A*cos(d*x+c)^3-2445*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c))))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*B*sin(d*x+c)+1920*B*cos(d*x+c)^3-5974*A*cos(d*x+c)^2+3430*B*cos(d*x+c)^2+3768*A*cos(d*x+c)-2040*B*cos(d*x+c)+5342*A-2990*B)*cos(d*x+c)^3*(1/cos(d*x+c))^(5/2)/sin(d*x+c)^5/a^3

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\left(a + \frac{a}{\cos(c+dx)}\right)^{5/2} \left(\frac{1}{\cos(c+dx)}\right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/((a + a/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(5/2)),x)
```

```
[Out] int((A + B/cos(c + d*x))/((a + a/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(5/2)), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)**(5/2)/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```


$$3.269 \quad \int (a + a \sec(c + dx))^{2/3} (A + B \sec(c + dx)) dx$$

Optimal. Leaf size=406

$$\frac{3\sqrt{2} A \tan(c + dx)(a \sec(c + dx) + a)^{2/3} F_1\left(\frac{7}{6}; \frac{1}{2}, 1; \frac{13}{6}; \frac{1}{2}(\sec(c + dx) + 1), \sec(c + dx) + 1\right)}{7d\sqrt{1 - \sec(c + dx)}} + \frac{3B \tan(c + dx)(a \sec(c + dx) + a)^{2/3}}{2d(\sec(c + dx) + 1)}$$

```
[Out] 3/2*B*(a+a*sec(d*x+c))^(2/3)*tan(d*x+c)/d/(1+sec(d*x+c))+3/7*A*AppellF1(7/6,
,1,1/2,13/6,1+sec(d*x+c),1/2+1/2*sec(d*x+c))*(a+a*sec(d*x+c))^(2/3)*2^(1/2)
*tan(d*x+c)/d/(1-sec(d*x+c))^(1/2)-1/4*3^(3/4)*B*((2^(1/3)-(1+sec(d*x+c))^(1/3)
*(1-3^(1/2)))^2/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)/(2^(1/3)
-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))*(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)
(1/2)))*EllipticF((1-(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))^2/(2^(1/3)
-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(a+a*se
c(d*x+c))^(2/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3))*((2^(2/3)+2^(1/3)*(1+sec(d*x
+c))^(1/3)+(1+sec(d*x+c))^(2/3))/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)
)^2)^(1/2)*tan(d*x+c)*2^(2/3)/d/(1-sec(d*x+c))/(1+sec(d*x+c))/(-(1+sec(d*x+c)
)^(1/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3)))/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(
1/2)))^2)^(1/2)
```

Rubi [A] time = 0.63, antiderivative size = 406, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3924, 3779, 3778, 136, 3828, 3827, 50, 63, 225}

$$\frac{3\sqrt{2} A \tan(c + dx)(a \sec(c + dx) + a)^{2/3} F_1\left(\frac{7}{6}; \frac{1}{2}, 1; \frac{13}{6}; \frac{1}{2}(\sec(c + dx) + 1), \sec(c + dx) + 1\right)}{7d\sqrt{1 - \sec(c + dx)}} + \frac{3B \tan(c + dx)(a \sec(c + dx) + a)^{2/3}}{2d(\sec(c + dx) + 1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(2/3)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (3*sqrt[2]*A*AppellF1[7/6, 1/2, 1, 13/6, (1 + Sec[c + d*x])/2, 1 + Sec[c +
d*x])*(a + a*Sec[c + d*x])^(2/3)*Tan[c + d*x]/(7*d*sqrt[1 - Sec[c + d*x]])
+ (3*B*(a + a*Sec[c + d*x])^(2/3)*Tan[c + d*x]/(2*d*(1 + Sec[c + d*x])) -
(3^(3/4)*B*EllipticF[ArcCos[(2^(1/3) - (1 - sqrt[3])*(1 + Sec[c + d*x])^(1/3)
)/2^(1/3) - (1 + sqrt[3])*(1 + Sec[c + d*x])^(1/3)], (2 + sqrt[3])/4]*
(a + a*Sec[c + d*x])^(2/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*sqrt[(2^(2/3)
+ 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 +
Sec[c + d*x])^(1/3)*(1 + 3^(1/2)))^2]^(1/2)]/d/(1 - Sec[c + d*x])/(1 + Sec[c +
d*x])/(1 + Sec[c + d*x])^(1/3)
```

$$3) + 2^{(1/3)} * (1 + \text{Sec}[c + d*x])^{(1/3)} + (1 + \text{Sec}[c + d*x])^{(2/3)} / (2^{(1/3)} - (1 + \text{Sqrt}[3]) * (1 + \text{Sec}[c + d*x])^{(1/3)})^2 * \text{Tan}[c + d*x] / (2 * 2^{(1/3)} * d * (1 - \text{Sec}[c + d*x]) * (1 + \text{Sec}[c + d*x]) * \text{Sqrt}[-((1 + \text{Sec}[c + d*x])^{(1/3)} * (2^{(1/3)} - (1 + \text{Sec}[c + d*x])^{(1/3)})) / (2^{(1/3)} - (1 + \text{Sqrt}[3]) * (1 + \text{Sec}[c + d*x])^{(1/3)})^2])]$$

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] := Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))]/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n},
x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
0] && !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] := With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 3778

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] := Dist[(a^n*Cot
[c + d*x])/(d*Sqrt[1 + Csc[c + d*x]]*Sqrt[1 - Csc[c + d*x]]), Subst[Int[(1
+ (b*x)/a)^(n - 1/2)/(x*Sqrt[1 - (b*x)/a]), x], x, Csc[c + d*x]], x] /; Fre
```

$eQ[\{a, b, c, d, n\}, x] \&\& EqQ[a^2 - b^2, 0] \&\& !IntegerQ[2*n] \&\& GtQ[a, 0]$

Rule 3779

$Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.) + (a_.))^(n_.), x_Symbol] :> Dist[(a^{IntPart[n]}*(a + b*Csc[c + d*x])^{FracPart[n]})/(1 + (b*Csc[c + d*x])/a)^{FracPart[n]}, Int[(1 + (b*Csc[c + d*x])/a)^n, x], x] /; FreeQ[\{a, b, c, d, n\}, x] \&\& EqQ[a^2 - b^2, 0] \&\& !IntegerQ[2*n] \&\& !GtQ[a, 0]$

Rule 3827

$Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*sqrt[a + b*Csc[e + f*x]])*sqrt[a - b*Csc[e + f*x]], Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2))/sqrt[a - b*x], x], x, Csc[e + f*x], x] /; FreeQ[\{a, b, d, e, f, m, n\}, x] \&\& EqQ[a^2 - b^2, 0] \&\& !IntegerQ[m] \&\& GtQ[a, 0]$

Rule 3828

$Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.), x_Symbol] :> Dist[(a^{IntPart[m]}*(a + b*Csc[e + f*x])^{FracPart[m]})/(1 + (b*Csc[e + f*x])/a)^{FracPart[m]}, Int[(1 + (b*Csc[e + f*x])/a)^m*(d*Csc[e + f*x])^n, x], x] /; FreeQ[\{a, b, d, e, f, m, n\}, x] \&\& EqQ[a^2 - b^2, 0] \&\& !IntegerQ[m] \&\& !GtQ[a, 0]$

Rule 3924

$Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.)), x_Symbol] :> Dist[c, Int[(a + b*Csc[e + f*x])^m, x], x] + Dist[d, Int[(a + b*Csc[e + f*x])^m*Csc[e + f*x], x], x] /; FreeQ[\{a, b, c, d, e, f, m\}, x] \&\& NeQ[b*c - a*d, 0] \&\& !IntegerQ[2*m]$

Rubi steps

$$\begin{aligned}
\int (a + a \sec(c + dx))^{2/3} (A + B \sec(c + dx)) dx &= A \int (a + a \sec(c + dx))^{2/3} dx + B \int \sec(c + dx) (a + a \sec(c + dx))^{2/3} dx \\
&= \frac{(A(a + a \sec(c + dx))^{2/3}) \int (1 + \sec(c + dx))^{2/3} dx}{(1 + \sec(c + dx))^{2/3}} + \frac{(B(a + a \sec(c + dx))^{2/3}) \int \sec(c + dx) dx}{(1 + \sec(c + dx))^{2/3}} \\
&= \frac{(A(a + a \sec(c + dx))^{2/3} \tan(c + dx)) \operatorname{Subst}\left(\int \frac{\sqrt[6]{1+x}}{\sqrt{1-x}} dx, x, \sec(c + dx)\right)}{d\sqrt{1 - \sec(c + dx)} (1 + \sec(c + dx))^{7/6}} \\
&= \frac{3\sqrt{2} AF_1\left(\frac{7}{6}; \frac{1}{2}, 1; \frac{13}{6}; \frac{1}{2}(1 + \sec(c + dx)), 1 + \sec(c + dx)\right) (a + a \sec(c + dx))^{2/3}}{7d\sqrt{1 - \sec(c + dx)}} \\
&= \frac{3\sqrt{2} AF_1\left(\frac{7}{6}; \frac{1}{2}, 1; \frac{13}{6}; \frac{1}{2}(1 + \sec(c + dx)), 1 + \sec(c + dx)\right) (a + a \sec(c + dx))^{2/3}}{7d\sqrt{1 - \sec(c + dx)}} \\
&= \frac{3\sqrt{2} AF_1\left(\frac{7}{6}; \frac{1}{2}, 1; \frac{13}{6}; \frac{1}{2}(1 + \sec(c + dx)), 1 + \sec(c + dx)\right) (a + a \sec(c + dx))^{2/3}}{7d\sqrt{1 - \sec(c + dx)}}
\end{aligned}$$

Mathematica [B] time = 20.72, size = 4445, normalized size = 10.95

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(a + a*Sec[c + d*x])^(2/3)*(A + B*Sec[c + d*x]),x]

[Out] (3*B*Cos[c + d*x]*((1 + Cos[c + d*x])*Sec[c + d*x])^(2/3)*(a*(1 + Sec[c + d*x]))^(2/3)*(A + B*Sec[c + d*x])*Tan[(c + d*x)/2])/(2*d*(B + A*Cos[c + d*x])*(1 + Sec[c + d*x])^(2/3)) + (Cos[c + d*x]*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(2/3)*(a*(1 + Sec[c + d*x]))^(2/3)*(A + B*Sec[c + d*x])*((A*Cos[c + d*x]*Sec[(c + d*x)/2]^2*(1 + Sec[c + d*x])^(2/3))/2 + Sec[(c + d*x)/2]^2*((A*(1 + Sec[c + d*x])^(2/3))/2 + (B*(1 + Sec[c + d*x])^(2/3))/4))*Tan[(c + d*x)/2]*(2*B*AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(-3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2))*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3)*Tan[(c + d*x)/2]^4 + 9*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2)*(3*(4*A + B)*Cos[(c + d*x)/2]^2 + B*AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2)*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3)*Tan[(c + d*x)/2]^2))/(3*2^(1

$$\begin{aligned}
& /3) * d * (B + A * \cos[c + d * x]) * (1 + \sec[c + d * x])^{2/3} * (9 * \text{AppellF1}[1/2, 2/3, 1, \\
& 3/2, \tan[(c + d * x)/2]^2, -\tan[(c + d * x)/2]^2] + 2 * (-3 * \text{AppellF1}[3/2, 2/3, \\
& 2, 5/2, \tan[(c + d * x)/2]^2, -\tan[(c + d * x)/2]^2] + 2 * \text{AppellF1}[3/2, 5/3, 1, \\
& 5/2, \tan[(c + d * x)/2]^2, -\tan[(c + d * x)/2]^2]) * \tan[(c + d * x)/2]^2 * ((\sec[(c \\
& + d * x)/2]^2 * (\cos[(c + d * x)/2]^2 * \sec[c + d * x])^{2/3} * (2 * B * \text{AppellF1}[3/2, 2/3 \\
& , 1, 5/2, \tan[(c + d * x)/2]^2, -\tan[(c + d * x)/2]^2] * (-3 * \text{AppellF1}[3/2, 2/3, 2 \\
& , 5/2, \tan[(c + d * x)/2]^2, -\tan[(c + d * x)/2]^2] + 2 * \text{AppellF1}[3/2, 5/3, 1, 5 \\
& /2, \tan[(c + d * x)/2]^2, -\tan[(c + d * x)/2]^2]) * (\cos[c + d * x] * \sec[(c + d * x)/2 \\
&]^2)^{2/3} * \tan[(c + d * x)/2]^4 + 9 * \text{AppellF1}[1/2, 2/3, 1, 3/2, \tan[(c + d * x)/ \\
& 2]^2, -\tan[(c + d * x)/2]^2] * (3 * (4 * A + B) * \cos[(c + d * x)/2]^2 + B * \text{AppellF1}[3/2 \\
& , 2/3, 1, 5/2, \tan[(c + d * x)/2]^2, -\tan[(c + d * x)/2]^2] * (\cos[c + d * x] * \sec[(c \\
& + d * x)/2]^2)^{2/3} * \tan[(c + d * x)/2]^2)) / (6 * 2^{1/3} * (9 * \text{AppellF1}[1/2, 2/3, \\
& 1, 3/2, \tan[(c + d * x)/2]^2, -\tan[(c + d * x)/2]^2] + 2 * (-3 * \text{AppellF1}[3/2, 2/3 \\
& , 2, 5/2, \tan[(c + d * x)/2]^2, -\tan[(c + d * x)/2]^2] + 2 * \text{AppellF1}[3/2, 5/3, 1 \\
& , 5/2, \tan[(c + d * x)/2]^2, -\tan[(c + d * x)/2]^2]) * \tan[(c + d * x)/2]^2) - ((\cos \\
& [(c + d * x)/2]^2 * \sec[c + d * x])^{2/3} * \tan[(c + d * x)/2] * (2 * B * \text{AppellF1}[3/2, 2 \\
& /3, 1, 5/2, \tan[(c + d * x)/2]^2, -\tan[(c + d * x)/2]^2] * (-3 * \text{AppellF1}[3/2, 2/3, \\
& 2, 5/2, \tan[(c + d * x)/2]^2, -\tan[(c + d * x)/2]^2] + 2 * \text{AppellF1}[3/2, 5/3, 1, \\
& 5/2, \tan[(c + d * x)/2]^2, -\tan[(c + d * x)/2]^2]) * (\cos[c + d * x] * \sec[(c + d * x) \\
& /2]^2)^{2/3} * \tan[(c + d * x)/2]^4 + 9 * \text{AppellF1}[1/2, 2/3, 1, 3/2, \tan[(c + d * x) \\
&)/2]^2, -\tan[(c + d * x)/2]^2] * (3 * (4 * A + B) * \cos[(c + d * x)/2]^2 + B * \text{AppellF1}[3 \\
& /2, 2/3, 1, 5/2, \tan[(c + d * x)/2]^2, -\tan[(c + d * x)/2]^2] * (\cos[c + d * x] * \sec \\
& [(c + d * x)/2]^2)^{2/3} * \tan[(c + d * x)/2]^2) * (2 * (-3 * \text{AppellF1}[3/2, 2/3, 2, 5/ \\
& 2, \tan[(c + d * x)/2]^2, -\tan[(c + d * x)/2]^2] + 2 * \text{AppellF1}[3/2, 5/3, 1, 5/2, \\
& \tan[(c + d * x)/2]^2, -\tan[(c + d * x)/2]^2]) * \sec[(c + d * x)/2]^2 * \tan[(c + d * x)/ \\
& 2] + 9 * (-1/3 * (\text{AppellF1}[3/2, 2/3, 2, 5/2, \tan[(c + d * x)/2]^2, -\tan[(c + d * x) \\
& /2]^2] * \sec[(c + d * x)/2]^2 * \tan[(c + d * x)/2]) + (2 * \text{AppellF1}[3/2, 5/3, 1, 5/2, \\
& \tan[(c + d * x)/2]^2, -\tan[(c + d * x)/2]^2] * \sec[(c + d * x)/2]^2 * \tan[(c + d * x)/ \\
& 2])) / 9) + 2 * \tan[(c + d * x)/2]^2 * (-3 * ((-6 * \text{AppellF1}[5/2, 2/3, 3, 7/2, \tan[(c + \\
& d * x)/2]^2, -\tan[(c + d * x)/2]^2] * \sec[(c + d * x)/2]^2 * \tan[(c + d * x)/2]) / 5 + (2 \\
& * \text{AppellF1}[5/2, 5/3, 2, 7/2, \tan[(c + d * x)/2]^2, -\tan[(c + d * x)/2]^2] * \sec[(c \\
& + d * x)/2]^2 * \tan[(c + d * x)/2]) / 5) + 2 * ((-3 * \text{AppellF1}[5/2, 5/3, 2, 7/2, \tan[(c \\
& + d * x)/2]^2, -\tan[(c + d * x)/2]^2] * \sec[(c + d * x)/2]^2 * \tan[(c + d * x)/2]) / 5 \\
& + \text{AppellF1}[5/2, 8/3, 1, 7/2, \tan[(c + d * x)/2]^2, -\tan[(c + d * x)/2]^2] * \sec[(c \\
& + d * x)/2]^2 * \tan[(c + d * x)/2])))) / (3 * 2^{1/3} * (9 * \text{AppellF1}[1/2, 2/3, 1, 3/2, \\
& \tan[(c + d * x)/2]^2, -\tan[(c + d * x)/2]^2] + 2 * (-3 * \text{AppellF1}[3/2, 2/3, 2, 5/2 \\
& , \tan[(c + d * x)/2]^2, -\tan[(c + d * x)/2]^2] + 2 * \text{AppellF1}[3/2, 5/3, 1, 5/2, \tan \\
& [(c + d * x)/2]^2, -\tan[(c + d * x)/2]^2]) * \tan[(c + d * x)/2]^2) + ((\cos[(c \\
& + d * x)/2]^2 * \sec[c + d * x])^{2/3} * \tan[(c + d * x)/2] * (4 * B * \text{AppellF1}[3/2, 2/3, 1, \\
& 5/2, \tan[(c + d * x)/2]^2, -\tan[(c + d * x)/2]^2] * (-3 * \text{AppellF1}[3/2, 2/3, 2, 5/ \\
& 2, \tan[(c + d * x)/2]^2, -\tan[(c + d * x)/2]^2] + 2 * \text{AppellF1}[3/2, 5/3, 1, 5/2, \\
& \tan[(c + d * x)/2]^2, -\tan[(c + d * x)/2]^2]) * \sec[(c + d * x)/2]^2 * (\cos[c + d * x] * \\
& \sec[(c + d * x)/2]^2)^{2/3} * \tan[(c + d * x)/2]^3 + 2 * B * (-3 * \text{AppellF1}[3/2, 2/3, 2 \\
& , 5/2, \tan[(c + d * x)/2]^2, -\tan[(c + d * x)/2]^2] + 2 * \text{AppellF1}[3/2, 5/3, 1, 5 \\
& /2, \tan[(c + d * x)/2]^2, -\tan[(c + d * x)/2]^2]) * (\cos[c + d * x] * \sec[(c + d * x)/2
\end{aligned}$$

$$\begin{aligned}
&]^2)^{(2/3)} * \tan[(c + dx)/2]^4 * ((-3 * \text{AppellF1}[5/2, 2/3, 2, 7/2, \tan[(c + dx)/2]^2, -\tan[(c + dx)/2]^2] * \sec[(c + dx)/2]^2 * \tan[(c + dx)/2]) / 5 + (2 * \text{AppellF1}[5/2, 5/3, 1, 7/2, \tan[(c + dx)/2]^2, -\tan[(c + dx)/2]^2] * \sec[(c + dx)/2]^2 * \tan[(c + dx)/2]) / 5 + (4 * B * \text{AppellF1}[3/2, 2/3, 1, 5/2, \tan[(c + dx)/2]^2, -\tan[(c + dx)/2]^2] * (-3 * \text{AppellF1}[3/2, 2/3, 2, 5/2, \tan[(c + dx)/2]^2, -\tan[(c + dx)/2]^2] + 2 * \text{AppellF1}[3/2, 5/3, 1, 5/2, \tan[(c + dx)/2]^2, -\tan[(c + dx)/2]^2]) * \tan[(c + dx)/2]^4 * (-(\sec[(c + dx)/2]^2 * \sin[c + dx]) + \cos[c + dx] * \sec[(c + dx)/2]^2 * \tan[(c + dx)/2])) / (3 * (\cos[c + dx] * \sec[(c + dx)/2]^2)^{(1/3)}) + 9 * (-1/3 * (\text{AppellF1}[3/2, 2/3, 2, 5/2, \tan[(c + dx)/2]^2, -\tan[(c + dx)/2]^2] * \sec[(c + dx)/2]^2 * \tan[(c + dx)/2]) + (2 * \text{AppellF1}[3/2, 5/3, 1, 5/2, \tan[(c + dx)/2]^2, -\tan[(c + dx)/2]^2] * \sec[(c + dx)/2]^2 * \tan[(c + dx)/2]) / 9) * (3 * (4 * A + B) * \cos[(c + dx)/2]^2 + B * \text{AppellF1}[3/2, 2/3, 1, 5/2, \tan[(c + dx)/2]^2, -\tan[(c + dx)/2]^2] * (\cos[c + dx] * \sec[(c + dx)/2]^2)^{(2/3)} * \tan[(c + dx)/2]^2) + 2 * B * \text{AppellF1}[3/2, 2/3, 1, 5/2, \tan[(c + dx)/2]^2, -\tan[(c + dx)/2]^2] * (\cos[c + dx] * \sec[(c + dx)/2]^2)^{(2/3)} * \tan[(c + dx)/2]^4 * (-3 * ((-6 * \text{AppellF1}[5/2, 2/3, 3, 7/2, \tan[(c + dx)/2]^2, -\tan[(c + dx)/2]^2] * \sec[(c + dx)/2]^2 * \tan[(c + dx)/2]) / 5 + (2 * \text{AppellF1}[5/2, 5/3, 2, 7/2, \tan[(c + dx)/2]^2, -\tan[(c + dx)/2]^2] * \sec[(c + dx)/2]^2 * \tan[(c + dx)/2]) / 5 + 2 * ((-3 * \text{AppellF1}[5/2, 5/3, 2, 7/2, \tan[(c + dx)/2]^2, -\tan[(c + dx)/2]^2] * \sec[(c + dx)/2]^2 * \tan[(c + dx)/2]) / 5 + \text{AppellF1}[5/2, 8/3, 1, 7/2, \tan[(c + dx)/2]^2, -\tan[(c + dx)/2]^2] * \sec[(c + dx)/2]^2 * \tan[(c + dx)/2])) + 9 * \text{AppellF1}[1/2, 2/3, 1, 3/2, \tan[(c + dx)/2]^2, -\tan[(c + dx)/2]^2] * (-3 * (4 * A + B) * \cos[(c + dx)/2] * \sin[(c + dx)/2] + B * \text{AppellF1}[3/2, 2/3, 1, 5/2, \tan[(c + dx)/2]^2, -\tan[(c + dx)/2]^2] * \sec[(c + dx)/2]^2 * (\cos[c + dx] * \sec[(c + dx)/2]^2)^{(2/3)} * \tan[(c + dx)/2] + B * (\cos[c + dx] * \sec[(c + dx)/2]^2)^{(2/3)} * \tan[(c + dx)/2]^2 * ((-3 * \text{AppellF1}[5/2, 2/3, 2, 7/2, \tan[(c + dx)/2]^2, -\tan[(c + dx)/2]^2] * \sec[(c + dx)/2]^2 * \tan[(c + dx)/2]) / 5 + (2 * \text{AppellF1}[5/2, 5/3, 1, 7/2, \tan[(c + dx)/2]^2, -\tan[(c + dx)/2]^2] * \sec[(c + dx)/2]^2 * \tan[(c + dx)/2]) / 5) + (2 * B * \text{AppellF1}[3/2, 2/3, 1, 5/2, \tan[(c + dx)/2]^2, -\tan[(c + dx)/2]^2] * \tan[(c + dx)/2]^2 * (-(\sec[(c + dx)/2]^2 * \sin[c + dx]) + \cos[c + dx] * \sec[(c + dx)/2]^2 * \tan[(c + dx)/2])) / (3 * (\cos[c + dx] * \sec[(c + dx)/2]^2)^{(1/3)})) / (3 * 2^{(1/3)} * (9 * \text{AppellF1}[1/2, 2/3, 1, 3/2, \tan[(c + dx)/2]^2, -\tan[(c + dx)/2]^2] + 2 * (-3 * \text{AppellF1}[3/2, 2/3, 2, 5/2, \tan[(c + dx)/2]^2, -\tan[(c + dx)/2]^2] + 2 * \text{AppellF1}[3/2, 5/3, 1, 5/2, \tan[(c + dx)/2]^2, -\tan[(c + dx)/2]^2]) * \tan[(c + dx)/2]^2) + (2^{(2/3)} * \tan[(c + dx)/2] * (2 * B * \text{AppellF1}[3/2, 2/3, 1, 5/2, \tan[(c + dx)/2]^2, -\tan[(c + dx)/2]^2] * (-3 * \text{AppellF1}[3/2, 2/3, 2, 5/2, \tan[(c + dx)/2]^2, -\tan[(c + dx)/2]^2] + 2 * \text{AppellF1}[3/2, 5/3, 1, 5/2, \tan[(c + dx)/2]^2, -\tan[(c + dx)/2]^2]) * (\cos[c + dx] * \sec[(c + dx)/2]^2)^{(2/3)} * \tan[(c + dx)/2]^4 + 9 * \text{AppellF1}[1/2, 2/3, 1, 3/2, \tan[(c + dx)/2]^2, -\tan[(c + dx)/2]^2] * (3 * (4 * A + B) * \cos[(c + dx)/2]^2 + B * \text{AppellF1}[3/2, 2/3, 1, 5/2, \tan[(c + dx)/2]^2, -\tan[(c + dx)/2]^2] * (\cos[c + dx] * \sec[(c + dx)/2]^2)^{(2/3)} * \tan[(c + dx)/2]^2) * (-(\cos[(c + dx)/2] * \sec[c + dx] * \sin[(c + dx)/2]) + \cos[(c + dx)/2]^2 * \sec[c + dx] * \tan[c + dx])) / (9 * (\cos[(c + dx)/2]^2 * \sec[c + dx])^{(1/3)} * (9 * \text{AppellF1}[1/2, 2/3, 1, 3/2, \tan[(c + dx)/2]
\end{aligned}$$

$^2, -\tan[(c + dx)/2]^2 + 2*(-3*\text{AppellF1}[3/2, 2/3, 2, 5/2, \tan[(c + dx)/2]^2, -\tan[(c + dx)/2]^2 + 2*\text{AppellF1}[3/2, 5/3, 1, 5/2, \tan[(c + dx)/2]^2, -\tan[(c + dx)/2]^2])*\tan[(c + dx)/2]^2))$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(dx+c))^(2/3)*(A+B*sec(dx+c)),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(a \sec(dx + c) + a)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(dx+c))^(2/3)*(A+B*sec(dx+c)),x, algorithm="giac")

[Out] integrate((B*sec(dx + c) + A)*(a*sec(dx + c) + a)^(2/3), x)

maple [F] time = 1.25, size = 0, normalized size = 0.00

$$\int (a + a \sec(dx + c))^{\frac{2}{3}} (A + B \sec(dx + c)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(dx+c))^(2/3)*(A+B*sec(dx+c)),x)

[Out] int((a+a*sec(dx+c))^(2/3)*(A+B*sec(dx+c)),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(a \sec(dx + c) + a)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(dx+c))^(2/3)*(A+B*sec(dx+c)),x, algorithm="maxima")

[Out] integrate((B*sec(dx + c) + A)*(a*sec(dx + c) + a)^(2/3), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(A + \frac{B}{\cos(c + dx)} \right) \left(a + \frac{a}{\cos(c + dx)} \right)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(2/3), x)`

[Out] `int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(2/3), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sec(c + dx) + 1))^{\frac{2}{3}} (A + B \sec(c + dx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sec(d*x+c))**(2/3)*(A+B*sec(d*x+c)), x)`

[Out] `Integral((a*(sec(c + d*x) + 1))**(2/3)*(A + B*sec(c + d*x)), x)`

$$3.270 \quad \int \frac{A+B \sec(c+dx)}{\sqrt[3]{a+a \sec(c+dx)}} dx$$

Optimal. Leaf size=354

$$\frac{3\sqrt{2} A \tan(c+dx) F_1\left(\frac{1}{6}; \frac{1}{2}, 1; \frac{7}{6}; \frac{1}{2}(\sec(c+dx)+1), \sec(c+dx)+1\right)}{d\sqrt{1-\sec(c+dx)} \sqrt[3]{a \sec(c+dx)+a}} \frac{3^{3/4} B \tan(c+dx) \left(\sqrt[3]{2} - \sqrt[3]{\sec(c+dx)+1}\right)}{\sqrt[3]{2} d(1-\sec(c+dx))}$$

[Out] 3*A*AppellF1(1/6, 1, 1/2, 7/6, 1+sec(d*x+c), 1/2+1/2*sec(d*x+c))*2^(1/2)*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/3)/(1-sec(d*x+c))^(1/2)-1/2*3^(3/4)*B*((2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))^2/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2))))^2^(1/2)/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))*(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))*EllipticF((1-(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2))))^2/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2))))^2^(1/2), 1/4*6^(1/2)+1/4*2^(1/2))*(2^(1/3)-(1+sec(d*x+c))^(1/3))*((2^(2/3)+2^(1/3)*(1+sec(d*x+c))^(1/3)+(1+sec(d*x+c))^(2/3))/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2))))^2^(1/2)*tan(d*x+c)*2^(2/3)/d/(1-sec(d*x+c))/(a+a*sec(d*x+c))^(1/3)/(-(1+sec(d*x+c))^(1/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3)))/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2))))^2^(1/2)

Rubi [A] time = 0.37, antiderivative size = 354, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3924, 3779, 3778, 136, 3828, 3827, 63, 225}

$$\frac{3\sqrt{2} A \tan(c+dx) F_1\left(\frac{1}{6}; \frac{1}{2}, 1; \frac{7}{6}; \frac{1}{2}(\sec(c+dx)+1), \sec(c+dx)+1\right)}{d\sqrt{1-\sec(c+dx)} \sqrt[3]{a \sec(c+dx)+a}} \frac{3^{3/4} B \tan(c+dx) \left(\sqrt[3]{2} - \sqrt[3]{\sec(c+dx)+1}\right)}{\sqrt[3]{2} d(1-\sec(c+dx))}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sec[c + d*x])/(a + a*Sec[c + d*x])^(1/3), x]

[Out] (3*sqrt[2]*A*AppellF1[1/6, 1/2, 1, 7/6, (1 + Sec[c + d*x])/2, 1 + Sec[c + d*x])*Tan[c + d*x])/(d*sqrt[1 - Sec[c + d*x]]*(a + a*Sec[c + d*x])^(1/3)) - (3^(3/4)*B*EllipticF[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))]/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))]

$$d*x))^{(1/3)}^2*\text{Tan}[c + d*x]/(2^{(1/3)}*d*(1 - \text{Sec}[c + d*x])*(a + a*\text{Sec}[c + d*x])^{(1/3)}*\text{Sqrt}[-(((1 + \text{Sec}[c + d*x])^{(1/3)}*(2^{(1/3)} - (1 + \text{Sec}[c + d*x])^{(1/3)})))/(2^{(1/3)} - (1 + \text{Sqrt}[3])*(1 + \text{Sec}[c + d*x])^{(1/3)})^2])]$$
Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] := Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))]/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
0] && !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] := With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x] /; FreeQ[{a, b}, x
]
```

Rule 3778

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] := Dist[(a^n*Cot
[c + d*x])/(d*Sqrt[1 + Csc[c + d*x]]*Sqrt[1 - Csc[c + d*x]]), Subst[Int[(1
+ (b*x)/a)^(n - 1/2)/(x*Sqrt[1 - (b*x)/a]), x], x, Csc[c + d*x]], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 3779

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] := Dist[(a^IntPa
rt[n]*(a + b*Csc[c + d*x])^FracPart[n])/(1 + (b*Csc[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Csc[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] && !IntegerQ[2*n] && !GtQ[a, 0]
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_.))^(m_), x_Symbol] := Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] && !IntegerQ[m] && GtQ[a, 0]
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_.))^(m_), x_Symbol] := Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/((1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a]^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] && !IntegerQ[m] && !GtQ[a, 0]
```

Rule 3924

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_.)), x_Symbol] := Dist[c, Int[(a + b*Csc[e + f*x])^m, x], x] + Dist
[d, Int[(a + b*Csc[e + f*x])^m*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d, e
, f, m}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
\int \frac{A + B \sec(c + dx)}{\sqrt[3]{a + a \sec(c + dx)}} dx &= A \int \frac{1}{\sqrt[3]{a + a \sec(c + dx)}} dx + B \int \frac{\sec(c + dx)}{\sqrt[3]{a + a \sec(c + dx)}} dx \\
&= \frac{(A \sqrt[3]{1 + \sec(c + dx)}) \int \frac{1}{\sqrt[3]{1 + \sec(c + dx)}} dx}{\sqrt[3]{a + a \sec(c + dx)}} + \frac{(B \sqrt[3]{1 + \sec(c + dx)}) \int \frac{\sec(c + dx)}{\sqrt[3]{1 + \sec(c + dx)}} dx}{\sqrt[3]{a + a \sec(c + dx)}} \\
&= \frac{(A \tan(c + dx)) \operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x} x(1+x)^{5/6}} dx, x, \sec(c + dx)\right)}{d \sqrt{1 - \sec(c + dx)} \sqrt[6]{1 + \sec(c + dx)} \sqrt[3]{a + a \sec(c + dx)}} - \frac{(B \tan(c + dx)) \operatorname{Subst}\left(\int \frac{\sec(c + dx)}{\sqrt{1-x} x(1+x)^{5/6}} dx, x, \sec(c + dx)\right)}{d \sqrt{1 - \sec(c + dx)} \sqrt[6]{1 + \sec(c + dx)} \sqrt[3]{a + a \sec(c + dx)}} \\
&= \frac{3\sqrt{2} AF_1\left(\frac{1}{6}; \frac{1}{2}, 1; \frac{7}{6}; \frac{1}{2}(1 + \sec(c + dx)), 1 + \sec(c + dx)\right) \tan(c + dx)}{d \sqrt{1 - \sec(c + dx)} \sqrt[3]{a + a \sec(c + dx)}} - \frac{(6B \tan(c + dx)) \operatorname{Subst}\left(\int \frac{\sec(c + dx)}{\sqrt{1-x} x(1+x)^{5/6}} dx, x, \sec(c + dx)\right)}{d \sqrt{1 - \sec(c + dx)} \sqrt[6]{1 + \sec(c + dx)} \sqrt[3]{a + a \sec(c + dx)}} \\
&= \frac{3\sqrt{2} AF_1\left(\frac{1}{6}; \frac{1}{2}, 1; \frac{7}{6}; \frac{1}{2}(1 + \sec(c + dx)), 1 + \sec(c + dx)\right) \tan(c + dx)}{d \sqrt{1 - \sec(c + dx)} \sqrt[3]{a + a \sec(c + dx)}} - \frac{3^{3/4} BF\left(\cos\left(\frac{c + dx}{2}\right)\right)}{d \sqrt{1 - \sec(c + dx)} \sqrt[3]{a + a \sec(c + dx)}}
\end{aligned}$$

Mathematica [B] time = 19.16, size = 2709, normalized size = 7.65

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(A + B*Sec[c + d*x])/(a + a*Sec[c + d*x])^(1/3), x]

[Out] $(2^{2/3} \cos[c + dx] (\cos[(c + dx)/2]^{2/3} \sec[c + dx]^{2/3} (1 + \sec[c + dx])^{1/3} (A + B \sec[c + dx]) ((B \sec[(c + dx)/2]^{2/3} (1 + \sec[c + dx])^{2/3})/2 + (A \cos[c + dx] \sec[(c + dx)/2]^{2/3} (1 + \sec[c + dx])^{2/3})/2) \tan[(c + dx)/2] ((-A + B) \text{AppellF1}[3/2, 2/3, 1, 5/2, \tan[(c + dx)/2]^2, -\tan[(c + dx)/2]^2] (\cos[c + dx] \sec[(c + dx)/2]^{2/3} \tan[(c + dx)/2]^2 + (27(A + B) \text{AppellF1}[1/2, 2/3, 1, 3/2, \tan[(c + dx)/2]^2, -\tan[(c + dx)/2]^2] \cos[(c + dx)/2]^2) / (9 \text{AppellF1}[1/2, 2/3, 1, 3/2, \tan[(c + dx)/2]^2, -\tan[(c + dx)/2]^2] + 2(-3 \text{AppellF1}[3/2, 2/3, 2, 5/2, \tan[(c + dx)/2]^2, -\tan[(c + dx)/2]^2] + 2 \text{AppellF1}[3/2, 5/3, 1, 5/2, \tan[(c + dx)/2]^2, -\tan[(c + dx)/2]^2]) \tan[(c + dx)/2]^2)) / (3d(B + A \cos[c + dx]) (a(1 + \sec[c + dx]))^{1/3} (\sec[(c + dx)/2]^{2/3} \cos[(c + dx)/2]^{2/3} \sec[c + dx]^{2/3} ((-A + B) \text{AppellF1}[3/2, 2/3, 1, 5/2, \tan[(c + dx)/2]^2, -\tan[(c + dx)/2]^2] (\cos[c + dx] \sec[(c + dx)/2]^{2/3} \tan[(c + dx)/2]^2 + (27(A + B) \text{AppellF1}[1/2, 2/3, 1, 3/2, \tan[(c + dx)/2]^2, -\tan[(c + dx)/2]^2] \cos[(c + dx)/2]^2) / (9 \text{AppellF1}[1/2, 2/3, 1, 3/2, \tan[(c + dx)/2]^2, -\tan[(c + dx)/2]^2] + 2(-3 \text{AppellF1}[3/2, 2/3, 2, 5/2, \tan[(c + dx)/2]^2, -\tan[(c + dx)/2]^2] + 2 \text{AppellF1}[3/2, 5/3, 1, 5/2, \tan[(c + dx)/2]^2, -\tan[(c + dx)/2]^2]) \tan[(c + dx)/2]^2)) / (3 \cdot 2^{1/3}) + (2^{2/3} (\cos[(c + dx)/2]^{2/3} \sec[c + dx]^{2/3} \tan[(c + dx)/2] ((-A + B) \text{AppellF1}[3/2, 2/3, 1, 5/2, \tan[(c + dx)/2]^2, -\tan[(c + dx)/2]^2] \sec[(c + dx)/2]^{2/3} (\cos[c + dx] \sec[(c + dx)/2]^{2/3} \tan[(c + dx)/2] + (-A + B) (\cos[c + dx] \sec[(c + dx)/2]^{2/3} \tan[(c + dx)/2]^2 ((-3 \text{AppellF1}[5/2, 2/3, 2, 7/2, \tan[(c + dx)/2]^2, -\tan[(c + dx)/2]^2] \sec[(c + dx)/2]^{2/3} \tan[(c + dx)/2]) / 5 + (2 \text{AppellF1}[5/2, 5/3, 1, 7/2, \tan[(c + dx)/2]^2, -\tan[(c + dx)/2]^2] \sec[(c + dx)/2]^{2/3} \tan[(c + dx)/2]) / 5) + (2(-A + B) \text{AppellF1}[3/2, 2/3, 1, 5/2, \tan[(c + dx)/2]^2, -\tan[(c + dx)/2]^2] \tan[(c + dx)/2]^2 ((-\sec[(c + dx)/2]^{2/3} \sin[c + dx]) + \cos[c + dx] \sec[(c + dx)/2]^{2/3} \tan[(c + dx)/2])) / (3 (\cos[c + dx] \sec[(c + dx)/2]^{2/3})^{1/3} - (27(A + B) \text{AppellF1}[1/2, 2/3, 1, 3/2, \tan[(c + dx)/2]^2, -\tan[(c + dx)/2]^2] \cos[(c + dx)/2] \sin[(c + dx)/2]) / (9 \text{AppellF1}[1/2, 2/3, 1, 3/2, \tan[(c + dx)/2]^2, -\tan[(c + dx)/2]^2] + 2(-3 \text{AppellF1}[3/2, 2/3, 2, 5/2, \tan[(c + dx)/2]^2, -\tan[(c + dx)/2]^2] + 2 \text{AppellF1}[3/2, 5/3, 1, 5/2, \tan[(c + dx)/2]^2, -\tan[(c + dx)/2]^2]) \tan[(c + dx)/2]^2 + (27(A + B) \cos[(c + dx)/2]^{2/3} (-1/3 \text{AppellF1}[3/2, 2/3, 2, 5/2, \tan[(c + dx)/2]^2, -\tan[(c + dx)/2]^2] \sec[(c + dx)/2]^{2/3} \tan[(c + dx)/2] + (2 \text{AppellF1}[3/2, 5/3, 1, 5/2, \tan[(c + dx)/2]^2, -\tan[(c + dx)/2]^2] \sec[(c + dx)/2]^{2/3} \tan[(c + dx)/2]) / 9)) / (9 \text{AppellF1}[1/2, 2/3, 1, 3/2, \tan[(c + dx)/2]^2, -\tan[(c + dx)/2]^2] + 2*$

$$\begin{aligned}
& -3 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{2}{3}, 2, \frac{5}{2}, \tan\left(\frac{c+dx}{2}\right)^2, -\tan\left(\frac{c+dx}{2}\right)^2\right] + 2 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{5}{3}, 1, \frac{5}{2}, \tan\left(\frac{c+dx}{2}\right)^2, -\tan\left(\frac{c+dx}{2}\right)^2\right] \cdot \tan\left(\frac{c+dx}{2}\right)^2 \\
& - (27(A+B) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{2}{3}, 1, \frac{3}{2}, \tan\left(\frac{c+dx}{2}\right)^2, -\tan\left(\frac{c+dx}{2}\right)^2\right] \cdot \cos\left(\frac{c+dx}{2}\right)^2 \cdot (2 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{2}{3}, 2, \frac{5}{2}, \tan\left(\frac{c+dx}{2}\right)^2, -\tan\left(\frac{c+dx}{2}\right)^2\right] \right. \\
& \left. + 2 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{5}{3}, 1, \frac{5}{2}, \tan\left(\frac{c+dx}{2}\right)^2, -\tan\left(\frac{c+dx}{2}\right)^2\right] \cdot \sec\left(\frac{c+dx}{2}\right)^2 \cdot \tan\left(\frac{c+dx}{2}\right) + 9 \cdot (-1/3 \cdot (\operatorname{AppellF1}\left[\frac{3}{2}, \frac{2}{3}, 2, \frac{5}{2}, \tan\left(\frac{c+dx}{2}\right)^2, -\tan\left(\frac{c+dx}{2}\right)^2\right] \right. \right. \\
& \left. \left. \cdot \sec\left(\frac{c+dx}{2}\right)^2 \cdot \tan\left(\frac{c+dx}{2}\right)\right) + (2 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{5}{3}, 1, \frac{5}{2}, \tan\left(\frac{c+dx}{2}\right)^2, -\tan\left(\frac{c+dx}{2}\right)^2\right] \cdot \sec\left(\frac{c+dx}{2}\right)^2 \cdot \tan\left(\frac{c+dx}{2}\right)) / 9 \right. \\
& \left. + 2 \tan\left(\frac{c+dx}{2}\right)^2 \cdot (-3 \cdot ((-6 \operatorname{AppellF1}\left[\frac{5}{2}, \frac{2}{3}, 3, \frac{7}{2}, \tan\left(\frac{c+dx}{2}\right)^2, -\tan\left(\frac{c+dx}{2}\right)^2\right] \cdot \sec\left(\frac{c+dx}{2}\right)^2 \cdot \tan\left(\frac{c+dx}{2}\right)) / 5 + (2 \operatorname{AppellF1}\left[\frac{5}{2}, \frac{5}{3}, 2, \frac{7}{2}, \tan\left(\frac{c+dx}{2}\right)^2, -\tan\left(\frac{c+dx}{2}\right)^2\right] \cdot \sec\left(\frac{c+dx}{2}\right)^2 \cdot \tan\left(\frac{c+dx}{2}\right)) / 5) \right. \\
& \left. + 2 \cdot ((-3 \operatorname{AppellF1}\left[\frac{5}{2}, \frac{5}{3}, 2, \frac{7}{2}, \tan\left(\frac{c+dx}{2}\right)^2, -\tan\left(\frac{c+dx}{2}\right)^2\right] \cdot \sec\left(\frac{c+dx}{2}\right)^2 \cdot \tan\left(\frac{c+dx}{2}\right)) / 5 + \operatorname{AppellF1}\left[\frac{5}{2}, \frac{8}{3}, 1, \frac{7}{2}, \tan\left(\frac{c+dx}{2}\right)^2, -\tan\left(\frac{c+dx}{2}\right)^2\right] \cdot \sec\left(\frac{c+dx}{2}\right)^2 \cdot \tan\left(\frac{c+dx}{2}\right))\right) \right. \\
& \left. \right) / (9 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{2}{3}, 1, \frac{3}{2}, \tan\left(\frac{c+dx}{2}\right)^2, -\tan\left(\frac{c+dx}{2}\right)^2\right] + 2 \cdot (-3 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{2}{3}, 2, \frac{5}{2}, \tan\left(\frac{c+dx}{2}\right)^2, -\tan\left(\frac{c+dx}{2}\right)^2\right] \cdot \tan\left(\frac{c+dx}{2}\right)^2) \right. \\
& \left. + (2 \cdot 2^{(2/3)} \cdot \tan\left(\frac{c+dx}{2}\right) \cdot ((-A+B) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{2}{3}, 1, \frac{5}{2}, \tan\left(\frac{c+dx}{2}\right)^2, -\tan\left(\frac{c+dx}{2}\right)^2\right] \cdot \cos\left[\frac{c+dx}{2}\right] \cdot \sec\left(\frac{c+dx}{2}\right)^2)^{(2/3)} \cdot \tan\left(\frac{c+dx}{2}\right)^2 + (27(A+B) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{2}{3}, 1, \frac{3}{2}, \tan\left(\frac{c+dx}{2}\right)^2, -\tan\left(\frac{c+dx}{2}\right)^2\right] \cdot \cos\left[\frac{c+dx}{2}\right]) / (9 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{2}{3}, 1, \frac{3}{2}, \tan\left(\frac{c+dx}{2}\right)^2, -\tan\left(\frac{c+dx}{2}\right)^2\right] + 2 \cdot (-3 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{2}{3}, 2, \frac{5}{2}, \tan\left(\frac{c+dx}{2}\right)^2, -\tan\left(\frac{c+dx}{2}\right)^2\right] \cdot \tan\left(\frac{c+dx}{2}\right)^2) \right. \right. \\
& \left. \left. + 2 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{5}{3}, 1, \frac{5}{2}, \tan\left(\frac{c+dx}{2}\right)^2, -\tan\left(\frac{c+dx}{2}\right)^2\right] \cdot \tan\left(\frac{c+dx}{2}\right)^2\right) \cdot (-\cos\left[\frac{c+dx}{2}\right] \cdot \sec\left[\frac{c+dx}{2}\right] \cdot \sin\left(\frac{c+dx}{2}\right) + \cos\left[\frac{c+dx}{2}\right]^2 \cdot \sec\left[\frac{c+dx}{2}\right] \cdot \tan\left[\frac{c+dx}{2}\right])\right) / (9 \cdot (\cos\left[\frac{c+dx}{2}\right]^2 \cdot \sec\left[\frac{c+dx}{2}\right])^{(1/3)}))
\end{aligned}$$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(dx+c))/(a+a*sec(dx+c))^(1/3),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx+c) + A}{(a \sec(dx+c) + a)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/3),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)/(a*sec(d*x + c) + a)^(1/3), x)

maple [F] time = 1.32, size = 0, normalized size = 0.00

$$\int \frac{A + B \sec(dx + c)}{(a + a \sec(dx + c))^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/3),x)

[Out] int((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/3),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(a \sec(dx + c) + a)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/3),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)/(a*sec(d*x + c) + a)^(1/3), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\left(a + \frac{a}{\cos(c+dx)}\right)^{1/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/(a + a/cos(c + d*x))^(1/3),x)

[Out] int((A + B/cos(c + d*x))/(a + a/cos(c + d*x))^(1/3), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \sec(c + dx)}{\sqrt[3]{a (\sec(c + dx) + 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(1/3),x)

[Out] Integral((A + B*sec(c + d*x))/(a*(sec(c + d*x) + 1))**(1/3), x)

$$3.271 \quad \int \frac{A+B \sec(c+dx)}{(a+a \sec(c+dx))^{4/3}} dx$$

Optimal. Leaf size=415

$$\frac{3\sqrt{2} A \tan(c+dx) F_1\left(-\frac{5}{6}; \frac{1}{2}, 1; \frac{1}{6}; \frac{1}{2}(\sec(c+dx)+1), \sec(c+dx)+1\right)}{5ad\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)\sqrt[3]{a \sec(c+dx)+a}} + \frac{3B \tan(c+dx)}{5ad(\sec(c+dx)+1)\sqrt[3]{a \sec(c+dx)+a}}$$

[Out] $3/5*B*\tan(d*x+c)/a/d/(1+\sec(d*x+c))/(a+a*\sec(d*x+c))^{(1/3)}-3/5*A*AppellF1(-5/6,1,1/2,1/6,1+\sec(d*x+c),1/2+1/2*\sec(d*x+c))*2^{(1/2)}*\tan(d*x+c)/a/d/(1+\sec(d*x+c))/(a+a*\sec(d*x+c))^{(1/3)}/(1-\sec(d*x+c))^{(1/2)}-1/10*3^{(3/4)}*B*((2^{(1/3)}-(1+\sec(d*x+c))^{(1/3)}*(1-3^{(1/2))))^2/(2^{(1/3)}-(1+\sec(d*x+c))^{(1/3)}*(1+3^{(1/2))))^2)^{(1/2)}/(2^{(1/3)}-(1+\sec(d*x+c))^{(1/3)}*(1-3^{(1/2))))*(2^{(1/3)}-(1+\sec(d*x+c))^{(1/3)}*(1+3^{(1/2))))*EllipticF((1-(2^{(1/3)}-(1+\sec(d*x+c))^{(1/3)}*(1-3^{(1/2))))^2/(2^{(1/3)}-(1+\sec(d*x+c))^{(1/3)}*(1+3^{(1/2))))^2)^{(1/2)},1/4*6^{(1/2)}+1/4*2^{(1/2)}*(2^{(1/3)}-(1+\sec(d*x+c))^{(1/3)})*((2^{(2/3)}+2^{(1/3)}*(1+\sec(d*x+c))^{(1/3)}+(1+\sec(d*x+c))^{(2/3)})/(2^{(1/3)}-(1+\sec(d*x+c))^{(1/3)}*(1+3^{(1/2))))^2)^{(1/2)}*\tan(d*x+c)*2^{(2/3)}/a/d/(1-\sec(d*x+c))/(a+a*\sec(d*x+c))^{(1/3)}/(-1+\sec(d*x+c))^{(1/3)}*(2^{(1/3)}-(1+\sec(d*x+c))^{(1/3)})/(2^{(1/3)}-(1+\sec(d*x+c))^{(1/3)}*(1+3^{(1/2))))^2)^{(1/2)}$

Rubi [A] time = 0.43, antiderivative size = 415, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3924, 3779, 3778, 136, 3828, 3827, 51, 63, 225}

$$\frac{3\sqrt{2} A \tan(c+dx) F_1\left(-\frac{5}{6}; \frac{1}{2}, 1; \frac{1}{6}; \frac{1}{2}(\sec(c+dx)+1), \sec(c+dx)+1\right)}{5ad\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)\sqrt[3]{a \sec(c+dx)+a}} + \frac{3B \tan(c+dx)}{5ad(\sec(c+dx)+1)\sqrt[3]{a \sec(c+dx)+a}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sec[c + d*x])/(a + a*Sec[c + d*x])^(4/3), x]

[Out] $(3*B*\tan[c + d*x])/(5*a*d*(1 + \sec[c + d*x])*(a + a*\sec[c + d*x])^{(1/3)}) - (3*\sqrt{2}*A*AppellF1[-5/6, 1/2, 1, 1/6, (1 + \sec[c + d*x])/2, 1 + \sec[c + d*x]]*\tan[c + d*x])/(5*a*d*\sqrt{1 - \sec[c + d*x]}*(1 + \sec[c + d*x])*(a + a*\sec[c + d*x])^{(1/3)}) - (3^{(3/4)}*B*EllipticF[ArcCos[(2^{(1/3)} - (1 - \sqrt{3})*(1 + \sec[c + d*x])^{(1/3)})/(2^{(1/3)} - (1 + \sqrt{3})*(1 + \sec[c + d*x])^{(1/3)})])*(1 + \sec[c + d*x])^{(1/3)})/(2^{(1/3)} - (1 + \sqrt{3})*(1 + \sec[c + d*x])^{(1/3)})$

3))] , (2 + Sqrt[3])/4*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3]))*(1 + Sec[c + d*x])^(1/3))^2]*Tan[c + d*x]/(5*2^(1/3)*a*d*(1 - Sec[c + d*x])*(a + a*Sec[c + d*x])^(1/3)*Sqrt[-(((1 + Sec[c + d*x])^(1/3))*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3]))*(1 + Sec[c + d*x])^(1/3))^2]))

Rule 51

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 136

Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))]/(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d), 0] && !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])

Rule 225

Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x] /; FreeQ[{a, b}, x]

Rule 3778

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] := Dist[(a^n*Cot[c + d*x])/(d*Sqrt[1 + Csc[c + d*x]]*Sqrt[1 - Csc[c + d*x]]), Subst[Int[(1

+ (b*x)/a)^(n - 1/2)/(x*Sqrt[1 - (b*x)/a]), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[2*n] && GtQ[a, 0]

Rule 3779

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_.))^(n_), x_Symbol] := Dist[(a^IntPart[n]*(a + b*Csc[c + d*x])^FracPart[n])/(1 + (b*Csc[c + d*x])/a)^FracPart[n], Int[(1 + (b*Csc[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[2*n] && !GtQ[a, 0]

Rule 3827

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_), x_Symbol] := Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2))/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m] && GtQ[a, 0]

Rule 3828

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_), x_Symbol] := Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m] && !GtQ[a, 0]

Rule 3924

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.)), x_Symbol] := Dist[c, Int[(a + b*Csc[e + f*x])^m, x], x] + Dist[d, Int[(a + b*Csc[e + f*x])^m*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[2*m]

Rubi steps

$$\begin{aligned}
\int \frac{A + B \sec(c + dx)}{(a + a \sec(c + dx))^{4/3}} dx &= A \int \frac{1}{(a + a \sec(c + dx))^{4/3}} dx + B \int \frac{\sec(c + dx)}{(a + a \sec(c + dx))^{4/3}} dx \\
&= \frac{(A \sqrt[3]{1 + \sec(c + dx)}) \int \frac{1}{(1 + \sec(c + dx))^{4/3}} dx}{a \sqrt[3]{a + a \sec(c + dx)}} + \frac{(B \sqrt[3]{1 + \sec(c + dx)}) \int \frac{\sec(c + dx)}{(1 + \sec(c + dx))^{4/3}} dx}{a \sqrt[3]{a + a \sec(c + dx)}} \\
&= \frac{(A \tan(c + dx)) \operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x} x(1+x)^{11/6}} dx, x, \sec(c + dx)\right)}{ad \sqrt{1 - \sec(c + dx)} \sqrt[6]{1 + \sec(c + dx)} \sqrt[3]{a + a \sec(c + dx)}} - \frac{(B \tan(c + dx)) \operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x} x(1+x)^{11/6}} dx, x, \sec(c + dx)\right)}{ad \sqrt{1 - \sec(c + dx)} \sqrt[6]{1 + \sec(c + dx)} \sqrt[3]{a + a \sec(c + dx)}} \\
&= \frac{3B \tan(c + dx)}{5ad(1 + \sec(c + dx)) \sqrt[3]{a + a \sec(c + dx)}} - \frac{3\sqrt{2} AF_1\left(-\frac{5}{6}; \frac{1}{2}, 1; \frac{1}{6}; \frac{1}{2}(1 + \sec(c + dx))\right)}{5ad \sqrt{1 - \sec(c + dx)} (1 + \sec(c + dx))} \\
&= \frac{3B \tan(c + dx)}{5ad(1 + \sec(c + dx)) \sqrt[3]{a + a \sec(c + dx)}} - \frac{3\sqrt{2} AF_1\left(-\frac{5}{6}; \frac{1}{2}, 1; \frac{1}{6}; \frac{1}{2}(1 + \sec(c + dx))\right)}{5ad \sqrt{1 - \sec(c + dx)} (1 + \sec(c + dx))} \\
&= \frac{3B \tan(c + dx)}{5ad(1 + \sec(c + dx)) \sqrt[3]{a + a \sec(c + dx)}} - \frac{3\sqrt{2} AF_1\left(-\frac{5}{6}; \frac{1}{2}, 1; \frac{1}{6}; \frac{1}{2}(1 + \sec(c + dx))\right)}{5ad \sqrt{1 - \sec(c + dx)} (1 + \sec(c + dx))}
\end{aligned}$$

Mathematica [B] time = 19.56, size = 2901, normalized size = 6.99

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(A + B*Sec[c + d*x])/(a + a*Sec[c + d*x])^(4/3), x]

[Out] (Cos[c + d*x]*((1 + Cos[c + d*x])*Sec[c + d*x])^(2/3)*(1 + Sec[c + d*x])^(4/3)*(A + B*Sec[c + d*x])*((3*Sec[(c + d*x)/2]*(-A*Sin[(c + d*x)/2]) + B*Sin[(c + d*x)/2]))/5 - (3*Sec[(c + d*x)/2]^3*(-A*Sin[(c + d*x)/2]) + B*Sin[(c + d*x)/2])/10)/(d*(B + A*Cos[c + d*x])*(a*(1 + Sec[c + d*x]))^(4/3)) + (2^(2/3)*Cos[c + d*x]*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(2/3)*(1 + Sec[c + d*x])^(4/3)*(A + B*Sec[c + d*x])*((A*Cos[c + d*x]*Sec[(c + d*x)/2]^2*(1 + Sec[c + d*x])^(2/3))/2 + Sec[(c + d*x)/2]^2*(-1/10*(A*(1 + Sec[c + d*x])^(2/3)) + (B*(1 + Sec[c + d*x])^(2/3))/10))*Tan[(c + d*x)/2]*((-6*A + B)*AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3)*Tan[(c + d*x)/2]^2 + (27*(4*A + B)*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Cos[(c + d*x)/2]^2)/(9*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*

$$\frac{((c + dx)/2)))/((9 \operatorname{AppellF1}[1/2, 2/3, 1, 3/2, \tan[(c + dx)/2]^2, -\tan[(c + dx)/2]^2] + 2(-3 \operatorname{AppellF1}[3/2, 2/3, 2, 5/2, \tan[(c + dx)/2]^2, -\tan[(c + dx)/2]^2] + 2 \operatorname{AppellF1}[3/2, 5/3, 1, 5/2, \tan[(c + dx)/2]^2, -\tan[(c + dx)/2]^2]) \tan[(c + dx)/2]^2) / 15 + (2 \cdot 2^{2/3} \tan[(c + dx)/2] \cdot ((-6A + B) \operatorname{AppellF1}[3/2, 2/3, 1, 5/2, \tan[(c + dx)/2]^2, -\tan[(c + dx)/2]^2] \cdot (\cos[c + dx] \operatorname{Sec}[(c + dx)/2]^2)^{2/3} \tan[(c + dx)/2]^2 + (27(4A + B) \operatorname{AppellF1}[1/2, 2/3, 1, 3/2, \tan[(c + dx)/2]^2, -\tan[(c + dx)/2]^2] \cdot \cos[(c + dx)/2]^2) / (9 \operatorname{AppellF1}[1/2, 2/3, 1, 3/2, \tan[(c + dx)/2]^2, -\tan[(c + dx)/2]^2] + 2(-3 \operatorname{AppellF1}[3/2, 2/3, 2, 5/2, \tan[(c + dx)/2]^2, -\tan[(c + dx)/2]^2] + 2 \operatorname{AppellF1}[3/2, 5/3, 1, 5/2, \tan[(c + dx)/2]^2, -\tan[(c + dx)/2]^2]) \tan[(c + dx)/2]^2) \cdot (-\cos[(c + dx)/2] \operatorname{Sec}[c + dx] \operatorname{Sin}[(c + dx)/2]) + \cos[(c + dx)/2]^2 \operatorname{Sec}[c + dx] \tan[c + dx])) / (45 \cdot (\cos[(c + dx)/2]^2 \operatorname{Sec}[c + dx])^{1/3}))$$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(dx+c))/(a+a*sec(dx+c))^(4/3),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(a \sec(dx + c) + a)^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(dx+c))/(a+a*sec(dx+c))^(4/3),x, algorithm="giac")

[Out] integrate((B*sec(dx + c) + A)/(a*sec(dx + c) + a)^(4/3), x)

maple [F] time = 1.52, size = 0, normalized size = 0.00

$$\int \frac{A + B \sec(dx + c)}{(a + a \sec(dx + c))^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(dx+c))/(a+a*sec(dx+c))^(4/3),x)

[Out] int((A+B*sec(dx+c))/(a+a*sec(dx+c))^(4/3),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(a \sec(dx + c) + a)^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(4/3),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)/(a*sec(d*x + c) + a)^(4/3), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\left(a + \frac{a}{\cos(c+dx)}\right)^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/(a + a/cos(c + d*x))^(4/3),x)

[Out] int((A + B/cos(c + d*x))/(a + a/cos(c + d*x))^(4/3), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \sec(c + dx)}{(a (\sec(c + dx) + 1))^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(4/3),x)

[Out] Integral((A + B*sec(c + d*x))/(a*(sec(c + d*x) + 1))**(4/3), x)

$$3.272 \quad \int (a + a \sec(c + dx))^{4/3} (A + B \sec(c + dx)) dx$$

Optimal. Leaf size=787

$$\frac{3\sqrt{2} a A \tan(c + dx) (\sec(c + dx) + 1) \sqrt[3]{a \sec(c + dx) + a} F_1\left(\frac{11}{6}; \frac{1}{2}, 1; \frac{17}{6}; \frac{1}{2} (\sec(c + dx) + 1), \sec(c + dx) + 1\right) + 3aB}{11d\sqrt{1 - \sec(c + dx)}}$$

[Out]
$$\begin{aligned} & 3/4 * a * B * (a + a * \sec(d * x + c))^{1/3} * \tan(d * x + c) / d - 15/4 * a * B * (a + a * \sec(d * x + c))^{1/3} \\ & * (1 + 3^{1/2}) * \tan(d * x + c) / d / (1 + \sec(d * x + c))^{2/3} / (2^{1/3} - (1 + \sec(d * x + c))^{1/3} \\ &) * (1 + 3^{1/2})) + 3/11 * a * A * \text{AppellF1}(11/6, 1, 1/2, 17/6, 1 + \sec(d * x + c), 1/2 + 1/2 * \sec(d \\ & * x + c)) * (1 + \sec(d * x + c)) * (a + a * \sec(d * x + c))^{1/3} * 2^{1/2} * \tan(d * x + c) / d / (1 - \sec(d * \\ & x + c))^{1/2} + 15/4 * 3^{1/4} * a * B * ((2^{1/3} - (1 + \sec(d * x + c))^{1/3}) * (1 - 3^{1/2}))^2 / \\ & (2^{1/3} - (1 + \sec(d * x + c))^{1/3}) * (1 + 3^{1/2}))^2)^{1/2} / (2^{1/3} - (1 + \sec(d * x + c)) \\ & ^{1/3}) * (1 - 3^{1/2})) * (2^{1/3} - (1 + \sec(d * x + c))^{1/3}) * (1 + 3^{1/2})) * \text{EllipticE}((1 \\ & - (2^{1/3} - (1 + \sec(d * x + c))^{1/3}) * (1 - 3^{1/2}))^2 / (2^{1/3} - (1 + \sec(d * x + c))^{1/3} \\ & * (1 + 3^{1/2}))^2)^{1/2}, 1/4 * 6^{1/2} + 1/4 * 2^{1/2}) * (a + a * \sec(d * x + c))^{1/3} * (2^{1/3} - (1 + \sec(d * x + c))^{1/3}) * ((2^{2/3} + 2^{1/3}) * (1 + \sec(d * x + c))^{1/3} + (1 + \sec(d * x + c))^{2/3}) / (2^{1/3} - (1 + \sec(d * x + c))^{1/3}) * (1 + 3^{1/2}))^2)^{1/2} * \tan(d * x + c) * 2^{1/3} / d / (1 - \sec(d * x + c)) / (1 + \sec(d * x + c))^{2/3} / ((- (1 + \sec(d * x + c))^{1/3}) * (2^{1/3} - (1 + \sec(d * x + c))^{1/3}) / (2^{1/3} - (1 + \sec(d * x + c))^{1/3}) * (1 + 3^{1/2}))^2)^{1/2} + 5/8 * 3^{3/4} * a * B * ((2^{1/3} - (1 + \sec(d * x + c))^{1/3}) * (1 - 3^{1/2}))^2 / (2^{1/3} - (1 + \sec(d * x + c))^{1/3}) * (1 + 3^{1/2}))^2)^{1/2} / (2^{1/3} - (1 + \sec(d * x + c))^{1/3}) * (1 - 3^{1/2})) * (2^{1/3} - (1 + \sec(d * x + c))^{1/3}) * (1 + 3^{1/2})) * \text{EllipticF}((1 - (2^{1/3} - (1 + \sec(d * x + c))^{1/3}) * (1 - 3^{1/2}))^2 / (2^{1/3} - (1 + \sec(d * x + c))^{1/3}) * (1 + 3^{1/2}))^2)^{1/2}, 1/4 * 6^{1/2} + 1/4 * 2^{1/2}) * (a + a * \sec(d * x + c))^{1/3} * (2^{1/3} - (1 + \sec(d * x + c))^{1/3}) * (1 - 3^{1/2}) * ((2^{2/3} + 2^{1/3}) * (1 + \sec(d * x + c))^{1/3} + (1 + \sec(d * x + c))^{2/3}) / (2^{1/3} - (1 + \sec(d * x + c))^{1/3}) * (1 + 3^{1/2}))^2)^{1/2} * \tan(d * x + c) * 2^{1/3} / d / (1 - \sec(d * x + c)) / (1 + \sec(d * x + c))^{2/3} / ((- (1 + \sec(d * x + c))^{1/3}) * (2^{1/3} - (1 + \sec(d * x + c))^{1/3}) / (2^{1/3} - (1 + \sec(d * x + c))^{1/3}) * (1 + 3^{1/2}))^2)^{1/2} \end{aligned}$$

Rubi [A] time = 0.84, antiderivative size = 787, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3924, 3779, 3778, 136, 3828, 3827, 50, 63, 308, 225,

1881}

$$\frac{3\sqrt{2}aA \tan(c+dx)(\sec(c+dx)+1)\sqrt[3]{a \sec(c+dx)+a} F_1\left(\frac{11}{6}; \frac{1}{2}, 1; \frac{17}{6}; \frac{1}{2}(\sec(c+dx)+1), \sec(c+dx)+1\right)}{11d\sqrt{1-\sec(c+dx)}} + 3a$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sec[c + d*x])^(4/3)*(A + B*Sec[c + d*x]),x]

[Out] (3*a*B*(a + a*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(4*d) + (3*Sqrt[2]*a*A*AppellF1[11/6, 1/2, 1, 17/6, (1 + Sec[c + d*x])/2, 1 + Sec[c + d*x]]*(1 + Sec[c + d*x])*(a + a*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(11*d*Sqrt[1 - Sec[c + d*x]]) - (15*(1 + Sqrt[3])*a*B*(a + a*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(4*d*(1 + Sec[c + d*x])^(2/3)*(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))) + (15*3^(1/4)*a*B*EllipticE[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))]/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(a + a*Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(2*2^(2/3)*d*(1 - Sec[c + d*x])*(1 + Sec[c + d*x])^(2/3)*Sqrt[-((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2])) + (5*3^(3/4)*(1 - Sqrt[3])*a*B*EllipticF[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))]/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(a + a*Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(4*2^(2/3)*d*(1 - Sec[c + d*x])*(1 + Sec[c + d*x])^(2/3)*Sqrt[-((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]))

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +

$(d*x^p)/b)^n, x], x, (a + b*x)^{(1/p)}, x]] /; \text{FreeQ}[\{a, b, c, d\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{LtQ}[-1, m, 0] \&\& \text{LeQ}[-1, n, 0] \&\& \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \&\& \text{IntLinearQ}[a, b, c, d, m, n, x]$

Rule 136

$\text{Int}[(a_ + (b_)*(x_))^{(m_)*((c_ + (d_)*(x_))^{(n_)*((e_ + (f_)*(x_))^{(p_)}, x_Symbol]} :> \text{Simp}[(b*e - a*f)^p*(a + b*x)^{(m + 1)*\text{AppellF1}[m + 1, -n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -(f*(a + b*x)/(b*e - a*f))]/(b^{(p + 1)*(m + 1)*(b/(b*c - a*d))^n}, x] /; \text{FreeQ}[\{a, b, c, d, e, f, m, n\}, x] \&\& \text{IntegerQ}[m] \&\& \text{IntegerQ}[n] \&\& \text{IntegerQ}[p] \&\& \text{GtQ}[b/(b*c - a*d), 0] \&\& \text{!(GtQ}[d/(d*a - c*b), 0] \&\& \text{SimplerQ}[c + d*x, a + b*x])$

Rule 225

$\text{Int}[1/\text{Sqrt}[(a_ + (b_)*(x_)^6], x_Symbol]} :> \text{With}[\{r = \text{Numer}[\text{Rt}[b/a, 3]], s = \text{Denom}[\text{Rt}[b/a, 3]]\}, \text{Simp}[(x*(s + r*x^2)*\text{Sqrt}[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + \text{Sqrt}[3])*r*x^2)^2]*\text{EllipticF}[\text{ArcCos}[(s + (1 - \text{Sqrt}[3])*r*x^2)/(s + (1 + \text{Sqrt}[3])*r*x^2)], (2 + \text{Sqrt}[3])/4])/(2*3^{(1/4)*s*\text{Sqrt}[a + b*x^6]*\text{Sqrt}[(r*x^2*(s + r*x^2))/(s + (1 + \text{Sqrt}[3])*r*x^2)^2]), x]] /; \text{FreeQ}[\{a, b\}, x]$

Rule 308

$\text{Int}[(x_)^4/\text{Sqrt}[(a_ + (b_)*(x_)^6], x_Symbol]} :> \text{With}[\{r = \text{Numer}[\text{Rt}[b/a, 3]], s = \text{Denom}[\text{Rt}[b/a, 3]]\}, \text{Dist}[(\text{Sqrt}[3] - 1)*s^2/(2*r^2), \text{Int}[1/\text{Sqrt}[a + b*x^6], x], x] - \text{Dist}[1/(2*r^2), \text{Int}[(\text{Sqrt}[3] - 1)*s^2 - 2*r^2*x^4]/\text{Sqrt}[a + b*x^6], x], x]] /; \text{FreeQ}[\{a, b\}, x]$

Rule 1881

$\text{Int}[(c_ + (d_)*(x_)^4)/\text{Sqrt}[(a_ + (b_)*(x_)^6], x_Symbol]} :> \text{With}[\{r = \text{Numer}[\text{Rt}[b/a, 3]], s = \text{Denom}[\text{Rt}[b/a, 3]]\}, \text{Simp}[(\text{Sqrt}[3] + 1)*d*s^3*\text{Sqrt}[a + b*x^6]/(2*a*r^2*(s + (1 + \text{Sqrt}[3])*r*x^2)), x] - \text{Simp}[(3^{(1/4)*d*s*x*(s + r*x^2)*\text{Sqrt}[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + \text{Sqrt}[3])*r*x^2)^2]*\text{EllipticE}[\text{ArcCos}[(s + (1 - \text{Sqrt}[3])*r*x^2)/(s + (1 + \text{Sqrt}[3])*r*x^2)], (2 + \text{Sqrt}[3])/4])/(2*r^2*\text{Sqrt}[(r*x^2*(s + r*x^2))/(s + (1 + \text{Sqrt}[3])*r*x^2)^2]*\text{Sqrt}[a + b*x^6]), x]] /; \text{FreeQ}[\{a, b, c, d\}, x] \&\& \text{EqQ}[2*\text{Rt}[b/a, 3]^2*c - (1 - \text{Sqrt}[3])*d, 0]$

Rule 3778

$\text{Int}[(\text{csc}[(c_ + (d_)*(x_)]*(b_ + (a_))^{(n_)}, x_Symbol]} :> \text{Dist}[(a^n*\text{Cot}[c + d*x])/(\text{d*Sqrt}[1 + \text{Csc}[c + d*x]]*\text{Sqrt}[1 - \text{Csc}[c + d*x]]), \text{Subst}[\text{Int}[(1 + (b*x)/a)^{(n - 1/2)}/(x*\text{Sqrt}[1 - (b*x)/a]), x], x, \text{Csc}[c + d*x]], x] /; \text{Fre$

$eQ[\{a, b, c, d, n\}, x] \&\& EqQ[a^2 - b^2, 0] \&\& !IntegerQ[2*n] \&\& GtQ[a, 0]$

Rule 3779

$Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.) + (a_.))^(n_.), x_Symbol] :> Dist[(a^{IntPart[n]}*(a + b*Csc[c + d*x])^{FracPart[n]})/(1 + (b*Csc[c + d*x])/a)^{FracPart[n]}, Int[(1 + (b*Csc[c + d*x])/a)^n, x], x] /; FreeQ[\{a, b, c, d, n\}, x] \&\& EqQ[a^2 - b^2, 0] \&\& !IntegerQ[2*n] \&\& !GtQ[a, 0]$

Rule 3827

$Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]])*Sqrt[a - b*Csc[e + f*x]], Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2))/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[\{a, b, d, e, f, m, n\}, x] \&\& EqQ[a^2 - b^2, 0] \&\& !IntegerQ[m] \&\& GtQ[a, 0]$

Rule 3828

$Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.), x_Symbol] :> Dist[(a^{IntPart[m]}*(a + b*Csc[e + f*x])^{FracPart[m]})/(1 + (b*Csc[e + f*x])/a)^{FracPart[m]}, Int[(1 + (b*Csc[e + f*x])/a)^m*(d*Csc[e + f*x])^n, x], x] /; FreeQ[\{a, b, d, e, f, m, n\}, x] \&\& EqQ[a^2 - b^2, 0] \&\& !IntegerQ[m] \&\& !GtQ[a, 0]$

Rule 3924

$Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.)), x_Symbol] :> Dist[c, Int[(a + b*Csc[e + f*x])^m, x], x] + Dist[d, Int[(a + b*Csc[e + f*x])^m*Csc[e + f*x], x], x] /; FreeQ[\{a, b, c, d, e, f, m\}, x] \&\& NeQ[b*c - a*d, 0] \&\& !IntegerQ[2*m]$

Rubi steps

$$\begin{aligned}
\int (a + a \sec(c + dx))^{4/3} (A + B \sec(c + dx)) dx &= A \int (a + a \sec(c + dx))^{4/3} dx + B \int \sec(c + dx) (a + a \sec(c + dx))^{4/3} dx \\
&= \frac{(aA \sqrt[3]{a + a \sec(c + dx)}) \int (1 + \sec(c + dx))^{4/3} dx}{\sqrt[3]{1 + \sec(c + dx)}} + \frac{(aB \sqrt[3]{a + a \sec(c + dx)}) \int \sec(c + dx) (1 + \sec(c + dx))^{4/3} dx}{\sqrt[3]{1 + \sec(c + dx)}} \\
&= -\frac{(aA \sqrt[3]{a + a \sec(c + dx)} \tan(c + dx)) \operatorname{Subst}\left(\int \frac{(1+x)^{5/6}}{\sqrt{1-x}} dx, x, \sec(c + dx)\right)}{d \sqrt{1 - \sec(c + dx)} (1 + \sec(c + dx))^{5/6}} \\
&= \frac{3aB \sqrt[3]{a + a \sec(c + dx)} \tan(c + dx)}{4d} + \frac{3\sqrt{2} a AF_1\left(\frac{11}{6}; \frac{1}{2}, 1; \frac{17}{6}; \frac{1}{2}\right)}{4d} \\
&= \frac{3aB \sqrt[3]{a + a \sec(c + dx)} \tan(c + dx)}{4d} + \frac{3\sqrt{2} a AF_1\left(\frac{11}{6}; \frac{1}{2}, 1; \frac{17}{6}; \frac{1}{2}\right)}{4d} \\
&= \frac{3aB \sqrt[3]{a + a \sec(c + dx)} \tan(c + dx)}{4d} + \frac{3\sqrt{2} a AF_1\left(\frac{11}{6}; \frac{1}{2}, 1; \frac{17}{6}; \frac{1}{2}\right)}{4d} \\
&= \frac{3aB \sqrt[3]{a + a \sec(c + dx)} \tan(c + dx)}{4d} + \frac{3\sqrt{2} a AF_1\left(\frac{11}{6}; \frac{1}{2}, 1; \frac{17}{6}; \frac{1}{2}\right)}{4d}
\end{aligned}$$

Mathematica [B] time = 19.61, size = 4110, normalized size = 5.22

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(a + a*Sec[c + d*x])^(4/3)*(A + B*Sec[c + d*x]),x]

[Out] (Cos[c + d*x]*((1 + Cos[c + d*x])*Sec[c + d*x])^(1/3)*(a*(1 + Sec[c + d*x])^(4/3)*(A + B*Sec[c + d*x])*((3*(4*A + 5*B)*Sin[c + d*x])/4 + (3*B*Tan[c + d*x])/4))/(d*(B + A*Cos[c + d*x])*(1 + Sec[c + d*x])^(4/3)) + (Cos[c + d*x])*(a*(1 + Sec[c + d*x])^(4/3)*(A + B*Sec[c + d*x])*(2*A*(1 + Sec[c + d*x])^(1/3) + (5*B*(1 + Sec[c + d*x])^(1/3))/4 + Cos[c + d*x]*(-3*A*(1 + Sec[c + d*x])^(1/3) - (15*B*(1 + Sec[c + d*x])^(1/3))/4))*Tan[(c + d*x)/2]*(-((4*A + 5*B)*AppellF1[3/2, 1/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2)/(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3)) - (9*(3*AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(-4*A + 5*B

$$\begin{aligned}
& + 5*(4*A + 7*B)*\text{Cos}[c + d*x]) - 4*(4*A + 5*B)*(3*\text{AppellF1}[3/2, 1/3, 2, 5/2, \\
& \text{Tan}[(c + d*x)/2]^2, -\text{Tan}[(c + d*x)/2]^2] - \text{AppellF1}[3/2, 4/3, 1, 5/2, \text{Tan} \\
& [(c + d*x)/2]^2, -\text{Tan}[(c + d*x)/2]^2])*\text{Cos}[c + d*x]*\text{Tan}[(c + d*x)/2]^2)/(2 \\
& *(-1 + \text{Tan}[(c + d*x)/2]^2)*(-9*\text{AppellF1}[1/2, 1/3, 1, 3/2, \text{Tan}[(c + d*x)/2]^2, \\
& -\text{Tan}[(c + d*x)/2]^2] + 2*(3*\text{AppellF1}[3/2, 1/3, 2, 5/2, \text{Tan}[(c + d*x)/2]^2, \\
& -\text{Tan}[(c + d*x)/2]^2] - \text{AppellF1}[3/2, 4/3, 1, 5/2, \text{Tan}[(c + d*x)/2]^2, -\text{T} \\
& \text{an}[(c + d*x)/2]^2])*\text{Tan}[(c + d*x)/2]^2)))/(6*2^{(2/3)}*d*(B + A*\text{Cos}[c + d*x] \\
&)*(\text{Cos}[(c + d*x)/2]^2*\text{Sec}[c + d*x])^{(2/3)}*(1 + \text{Sec}[c + d*x])^{(4/3)}*((\text{Sec}[(c \\
& + d*x)/2]^2*(-((4*A + 5*B)*\text{AppellF1}[3/2, 1/3, 1, 5/2, \text{Tan}[(c + d*x)/2]^2, \\
& -\text{Tan}[(c + d*x)/2]^2]*\text{Tan}[(c + d*x)/2]^2)/(\text{Cos}[c + d*x]*\text{Sec}[(c + d*x)/2]^2) \\
& ^{(2/3)}) - (9*(3*\text{AppellF1}[1/2, 1/3, 1, 3/2, \text{Tan}[(c + d*x)/2]^2, -\text{Tan}[(c + d* \\
& x)/2]^2)*(-4*A + 5*B + 5*(4*A + 7*B)*\text{Cos}[c + d*x]) - 4*(4*A + 5*B)*(3*\text{Appel \\
& lF1}[3/2, 1/3, 2, 5/2, \text{Tan}[(c + d*x)/2]^2, -\text{Tan}[(c + d*x)/2]^2] - \text{AppellF1}[3 \\
& /2, 4/3, 1, 5/2, \text{Tan}[(c + d*x)/2]^2, -\text{Tan}[(c + d*x)/2]^2])*\text{Cos}[c + d*x]*\text{Tan} \\
& [(c + d*x)/2]^2))/(2*(-1 + \text{Tan}[(c + d*x)/2]^2)*(-9*\text{AppellF1}[1/2, 1/3, 1, 3/ \\
& 2, \text{Tan}[(c + d*x)/2]^2, -\text{Tan}[(c + d*x)/2]^2] + 2*(3*\text{AppellF1}[3/2, 1/3, 2, 5/ \\
& 2, \text{Tan}[(c + d*x)/2]^2, -\text{Tan}[(c + d*x)/2]^2] - \text{AppellF1}[3/2, 4/3, 1, 5/2, \text{T}a \\
& n[(c + d*x)/2]^2, -\text{Tan}[(c + d*x)/2]^2])*\text{Tan}[(c + d*x)/2]^2)))/(12*2^{(2/3)}* \\
& (\text{Cos}[(c + d*x)/2]^2*\text{Sec}[c + d*x])^{(2/3)}) + (\text{Tan}[(c + d*x)/2]*(-((4*A + 5*B) \\
&)*\text{AppellF1}[3/2, 1/3, 1, 5/2, \text{Tan}[(c + d*x)/2]^2, -\text{Tan}[(c + d*x)/2]^2]*\text{Sec}[(c \\
& + d*x)/2]^2*\text{Tan}[(c + d*x)/2]))/(\text{Cos}[c + d*x]*\text{Sec}[(c + d*x)/2]^2)^{(2/3)}) - \\
& ((4*A + 5*B)*\text{Tan}[(c + d*x)/2]^2*(-3*\text{AppellF1}[5/2, 1/3, 2, 7/2, \text{Tan}[(c + d* \\
& x)/2]^2, -\text{Tan}[(c + d*x)/2]^2]*\text{Sec}[(c + d*x)/2]^2*\text{Tan}[(c + d*x)/2])/5 + (\text{App} \\
& ellF1[5/2, 4/3, 1, 7/2, \text{Tan}[(c + d*x)/2]^2, -\text{Tan}[(c + d*x)/2]^2]*\text{Sec}[(c + d \\
& *x)/2]^2*\text{Tan}[(c + d*x)/2])/5))/(\text{Cos}[c + d*x]*\text{Sec}[(c + d*x)/2]^2)^{(2/3)} + (2 \\
& *(4*A + 5*B)*\text{AppellF1}[3/2, 1/3, 1, 5/2, \text{Tan}[(c + d*x)/2]^2, -\text{Tan}[(c + d*x)/ \\
& 2]^2]*\text{Tan}[(c + d*x)/2]^2*(-(\text{Sec}[(c + d*x)/2]^2*\text{Sin}[c + d*x]) + \text{Cos}[c + d*x] \\
&)*\text{Sec}[(c + d*x)/2]^2*\text{Tan}[(c + d*x)/2]))/(3*(\text{Cos}[c + d*x]*\text{Sec}[(c + d*x)/2]^2) \\
& ^{(5/3)}) + (9*\text{Sec}[(c + d*x)/2]^2*\text{Tan}[(c + d*x)/2]*(3*\text{AppellF1}[1/2, 1/3, 1, 3 \\
& /2, \text{Tan}[(c + d*x)/2]^2, -\text{Tan}[(c + d*x)/2]^2)*(-4*A + 5*B + 5*(4*A + 7*B)*\text{Co} \\
& s[c + d*x]) - 4*(4*A + 5*B)*(3*\text{AppellF1}[3/2, 1/3, 2, 5/2, \text{Tan}[(c + d*x)/2]^2, \\
& -\text{Tan}[(c + d*x)/2]^2] - \text{AppellF1}[3/2, 4/3, 1, 5/2, \text{Tan}[(c + d*x)/2]^2, -\text{T} \\
& \text{an}[(c + d*x)/2]^2])*\text{Cos}[c + d*x]*\text{Tan}[(c + d*x)/2]^2))/(2*(-1 + \text{Tan}[(c + d*x] \\
&)/2]^2)^2*(-9*\text{AppellF1}[1/2, 1/3, 1, 3/2, \text{Tan}[(c + d*x)/2]^2, -\text{Tan}[(c + d*x) \\
& /2]^2] + 2*(3*\text{AppellF1}[3/2, 1/3, 2, 5/2, \text{Tan}[(c + d*x)/2]^2, -\text{Tan}[(c + d*x) \\
& /2]^2] - \text{AppellF1}[3/2, 4/3, 1, 5/2, \text{Tan}[(c + d*x)/2]^2, -\text{Tan}[(c + d*x)/2]^2 \\
&])*\text{Tan}[(c + d*x)/2]^2)) + (9*(3*\text{AppellF1}[1/2, 1/3, 1, 3/2, \text{Tan}[(c + d*x)/2] \\
& ^2, -\text{Tan}[(c + d*x)/2]^2)*(-4*A + 5*B + 5*(4*A + 7*B)*\text{Cos}[c + d*x]) - 4*(4*A \\
& + 5*B)*(3*\text{AppellF1}[3/2, 1/3, 2, 5/2, \text{Tan}[(c + d*x)/2]^2, -\text{Tan}[(c + d*x)/2] \\
& ^2] - \text{AppellF1}[3/2, 4/3, 1, 5/2, \text{Tan}[(c + d*x)/2]^2, -\text{Tan}[(c + d*x)/2]^2])* \\
& \text{Cos}[c + d*x]*\text{Tan}[(c + d*x)/2]^2*(2*(3*\text{AppellF1}[3/2, 1/3, 2, 5/2, \text{Tan}[(c + \\
& d*x)/2]^2, -\text{Tan}[(c + d*x)/2]^2] - \text{AppellF1}[3/2, 4/3, 1, 5/2, \text{Tan}[(c + d*x)/ \\
& 2]^2, -\text{Tan}[(c + d*x)/2]^2])*\text{Sec}[(c + d*x)/2]^2*\text{Tan}[(c + d*x)/2] - 9*(-1/3*(\\
& \text{AppellF1}[3/2, 1/3, 2, 5/2, \text{Tan}[(c + d*x)/2]^2, -\text{Tan}[(c + d*x)/2]^2]*\text{Sec}[(c \\
& + d*x)/2]^2*\text{Tan}[(c + d*x)/2]) + (\text{AppellF1}[3/2, 4/3, 1, 5/2, \text{Tan}[(c + d*x)/2]
\end{aligned}$$

$$\begin{aligned}
&]^2, -\tan[(c + dx)/2]^2 * \sec[(c + dx)/2]^2 * \tan[(c + dx)/2]) / 9) + 2 * \tan[(c + dx)/2]^2 * ((3 * \text{AppellF1}[5/2, 4/3, 2, 7/2, \tan[(c + dx)/2]^2, -\tan[(c + dx)/2]^2] * \sec[(c + dx)/2]^2 * \tan[(c + dx)/2]) / 5 - (4 * \text{AppellF1}[5/2, 7/3, 1, 7/2, \tan[(c + dx)/2]^2, -\tan[(c + dx)/2]^2] * \sec[(c + dx)/2]^2 * \tan[(c + dx)/2]) / 5 + 3 * ((-6 * \text{AppellF1}[5/2, 1/3, 3, 7/2, \tan[(c + dx)/2]^2, -\tan[(c + dx)/2]^2] * \sec[(c + dx)/2]^2 * \tan[(c + dx)/2]) / 5 + (\text{AppellF1}[5/2, 4/3, 2, 7/2, \tan[(c + dx)/2]^2, -\tan[(c + dx)/2]^2] * \sec[(c + dx)/2]^2 * \tan[(c + dx)/2]) / 5))) / (2 * (-1 + \tan[(c + dx)/2]^2) * (-9 * \text{AppellF1}[1/2, 1/3, 1, 3/2, \tan[(c + dx)/2]^2, -\tan[(c + dx)/2]^2] + 2 * (3 * \text{AppellF1}[3/2, 1/3, 2, 5/2, \tan[(c + dx)/2]^2, -\tan[(c + dx)/2]^2] - \text{AppellF1}[3/2, 4/3, 1, 5/2, \tan[(c + dx)/2]^2, -\tan[(c + dx)/2]^2]) * \tan[(c + dx)/2]^2) - (9 * (-15 * (4 * A + 7 * B) * \text{AppellF1}[1/2, 1/3, 1, 3/2, \tan[(c + dx)/2]^2, -\tan[(c + dx)/2]^2] * \sin[c + dx] - 4 * (4 * A + 5 * B) * (3 * \text{AppellF1}[3/2, 1/3, 2, 5/2, \tan[(c + dx)/2]^2, -\tan[(c + dx)/2]^2] - \text{AppellF1}[3/2, 4/3, 1, 5/2, \tan[(c + dx)/2]^2, -\tan[(c + dx)/2]^2]) * \cos[c + dx] * \sec[(c + dx)/2]^2 * \tan[(c + dx)/2] + 4 * (4 * A + 5 * B) * (3 * \text{AppellF1}[3/2, 1/3, 2, 5/2, \tan[(c + dx)/2]^2, -\tan[(c + dx)/2]^2] - \text{AppellF1}[3/2, 4/3, 1, 5/2, \tan[(c + dx)/2]^2, -\tan[(c + dx)/2]^2]) * \sin[c + dx] * \tan[(c + dx)/2]^2 + 3 * (-4 * A + 5 * B + 5 * (4 * A + 7 * B) * \cos[c + dx]) * (-1/3 * (\text{AppellF1}[3/2, 1/3, 2, 5/2, \tan[(c + dx)/2]^2, -\tan[(c + dx)/2]^2] * \sec[(c + dx)/2]^2 * \tan[(c + dx)/2]) + (\text{AppellF1}[3/2, 4/3, 1, 5/2, \tan[(c + dx)/2]^2, -\tan[(c + dx)/2]^2] * \sec[(c + dx)/2]^2 * \tan[(c + dx)/2]) / 9) - 4 * (4 * A + 5 * B) * \cos[c + dx] * \tan[(c + dx)/2]^2 * ((3 * \text{AppellF1}[5/2, 4/3, 2, 7/2, \tan[(c + dx)/2]^2, -\tan[(c + dx)/2]^2] * \sec[(c + dx)/2]^2 * \tan[(c + dx)/2]) / 5 - (4 * \text{AppellF1}[5/2, 7/3, 1, 7/2, \tan[(c + dx)/2]^2, -\tan[(c + dx)/2]^2] * \sec[(c + dx)/2]^2 * \tan[(c + dx)/2]) / 5 + 3 * ((-6 * \text{AppellF1}[5/2, 1/3, 3, 7/2, \tan[(c + dx)/2]^2, -\tan[(c + dx)/2]^2] * \sec[(c + dx)/2]^2 * \tan[(c + dx)/2]) / 5 + (\text{AppellF1}[5/2, 4/3, 2, 7/2, \tan[(c + dx)/2]^2, -\tan[(c + dx)/2]^2] * \sec[(c + dx)/2]^2 * \tan[(c + dx)/2]) / 5))) / (2 * (-1 + \tan[(c + dx)/2]^2) * (-9 * \text{AppellF1}[1/2, 1/3, 1, 3/2, \tan[(c + dx)/2]^2, -\tan[(c + dx)/2]^2] + 2 * (3 * \text{AppellF1}[3/2, 1/3, 2, 5/2, \tan[(c + dx)/2]^2, -\tan[(c + dx)/2]^2] - \text{AppellF1}[3/2, 4/3, 1, 5/2, \tan[(c + dx)/2]^2, -\tan[(c + dx)/2]^2]) * \tan[(c + dx)/2]^2))) / (6 * 2^(2/3) * (\cos[(c + dx)/2]^2 * \sec[c + dx])^(2/3)) - (\tan[(c + dx)/2] * (-((4 * A + 5 * B) * \text{AppellF1}[3/2, 1/3, 1, 5/2, \tan[(c + dx)/2]^2, -\tan[(c + dx)/2]^2] * \tan[(c + dx)/2]^2) / (\cos[c + dx] * \sec[(c + dx)/2]^2)^(2/3)) - (9 * (3 * \text{AppellF1}[1/2, 1/3, 1, 3/2, \tan[(c + dx)/2]^2, -\tan[(c + dx)/2]^2] * (-4 * A + 5 * B + 5 * (4 * A + 7 * B) * \cos[c + dx]) - 4 * (4 * A + 5 * B) * (3 * \text{AppellF1}[3/2, 1/3, 2, 5/2, \tan[(c + dx)/2]^2, -\tan[(c + dx)/2]^2] - \text{AppellF1}[3/2, 4/3, 1, 5/2, \tan[(c + dx)/2]^2, -\tan[(c + dx)/2]^2]) * \cos[c + dx] * \tan[(c + dx)/2]^2)) / (2 * (-1 + \tan[(c + dx)/2]^2) * (-9 * \text{AppellF1}[1/2, 1/3, 1, 3/2, \tan[(c + dx)/2]^2, -\tan[(c + dx)/2]^2] + 2 * (3 * \text{AppellF1}[3/2, 1/3, 2, 5/2, \tan[(c + dx)/2]^2, -\tan[(c + dx)/2]^2] - \text{AppellF1}[3/2, 4/3, 1, 5/2, \tan[(c + dx)/2]^2, -\tan[(c + dx)/2]^2]) * \tan[(c + dx)/2]^2))) * (-\cos[(c + dx)/2] * \sec[c + dx] * \sin[(c + dx)/2]) + \cos[(c + dx)/2]^2 * \sec[c + dx] * \tan[c + dx]) / (9 * 2^(2/3) * (\cos[(c + dx)/2]^2 * \sec[c + dx])^(5/3))
\end{aligned}$$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(a \sec(dx + c) + a)^{\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(4/3), x)

maple [F] time = 1.73, size = 0, normalized size = 0.00

$$\int (a + a \sec(dx + c))^{\frac{4}{3}} (A + B \sec(dx + c)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x)

[Out] int((a+a*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(a \sec(dx + c) + a)^{\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(4/3), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(A + \frac{B}{\cos(c + dx)} \right) \left(a + \frac{a}{\cos(c + dx)} \right)^{\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(4/3), x)`

[Out] `int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(4/3), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sec(c + dx) + 1))^{\frac{4}{3}} (A + B \sec(c + dx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sec(d*x+c))**(4/3)*(A+B*sec(d*x+c)), x)`

[Out] `Integral((a*(sec(c + d*x) + 1))**(4/3)*(A + B*sec(c + d*x)), x)`

$$3.273 \quad \int \sqrt[3]{a + a \sec(c + dx)} (A + B \sec(c + dx)) dx$$

Optimal. Leaf size=739

$$\frac{3\sqrt{2} A \tan(c + dx) \sqrt[3]{a \sec(c + dx) + a} F_1\left(\frac{5}{6}; \frac{1}{2}, 1; \frac{11}{6}; \frac{1}{2}(\sec(c + dx) + 1), \sec(c + dx) + 1\right)}{5d\sqrt{1 - \sec(c + dx)}} - \frac{3(1 + \sqrt{3}) B \tan(c + dx)}{d(\sec(c + dx) + 1)^{2/3}}$$

[Out] $-3*B*(a+a*\sec(d*x+c))^{(1/3)}*(1+3^{(1/2)})*\tan(d*x+c)/d/(1+\sec(d*x+c))^{(2/3)}/(2^{(1/3)}-(1+\sec(d*x+c))^{(1/3)}*(1+3^{(1/2)}))+3/5*A*\text{AppellF1}(5/6, 1, 1/2, 11/6, 1+\sec(d*x+c), 1/2+1/2*\sec(d*x+c))*(a+a*\sec(d*x+c))^{(1/3)}*2^{(1/2)}*\tan(d*x+c)/d/(1-\sec(d*x+c))^{(1/2)}+3*2^{(1/3)}*3^{(1/4)}*B*((2^{(1/3)}-(1+\sec(d*x+c))^{(1/3)}*(1-3^{(1/2)}))^{(1/2)}+(1+\sec(d*x+c))^{(1/3)}*(1-3^{(1/2)}))^{(1/2)}/(2^{(1/3)}-(1+\sec(d*x+c))^{(1/3)}*(1-3^{(1/2)}))^{(1/2)}+(1+\sec(d*x+c))^{(1/3)}*(1-3^{(1/2)}))^{(1/2)})*2^{(1/3)}-(1+\sec(d*x+c))^{(1/3)}*(1+3^{(1/2)}))^{(1/2)}*E\text{llipticE}((1-(2^{(1/3)}-(1+\sec(d*x+c))^{(1/3)}*(1-3^{(1/2)}))^{(1/2)}+(1+\sec(d*x+c))^{(1/3)}*(1-3^{(1/2)}))^{(1/2)}), 1/4*6^{(1/2)}+1/4*2^{(1/2)})*(a+a*\sec(d*x+c))^{(1/3)}*(2^{(1/3)}-(1+\sec(d*x+c))^{(1/3)})*((2^{(2/3)}+2^{(1/3)}*(1+\sec(d*x+c))^{(1/3)}+(1+\sec(d*x+c))^{(2/3)})/(2^{(1/3)}-(1+\sec(d*x+c))^{(1/3)}*(1+3^{(1/2)}))^{(1/2)})*\tan(d*x+c)/d/(1-\sec(d*x+c))/(1+\sec(d*x+c))^{(2/3)}/(-(1+\sec(d*x+c))^{(1/3)}*(2^{(1/3)}-(1+\sec(d*x+c))^{(1/3)}))/(2^{(1/3)}-(1+\sec(d*x+c))^{(1/3)}*(1+3^{(1/2)}))^{(1/2)}+(1+\sec(d*x+c))^{(1/3)}*(1-3^{(1/2)}))^{(1/2)}/(2^{(1/3)}-(1+\sec(d*x+c))^{(1/3)}*(1-3^{(1/2)}))^{(1/2)})*2^{(1/3)}-(1+\sec(d*x+c))^{(1/3)}*(1+3^{(1/2)}))^{(1/2)}*E\text{llipticF}((1-(2^{(1/3)}-(1+\sec(d*x+c))^{(1/3)}*(1-3^{(1/2)}))^{(1/2)}+(1+\sec(d*x+c))^{(1/3)}*(1-3^{(1/2)}))^{(1/2)}), 1/4*6^{(1/2)}+1/4*2^{(1/2)})*(a+a*\sec(d*x+c))^{(1/3)}*(2^{(1/3)}-(1+\sec(d*x+c))^{(1/3)})*((2^{(2/3)}+2^{(1/3)}*(1+\sec(d*x+c))^{(1/3)}+(1+\sec(d*x+c))^{(2/3)})/(2^{(1/3)}-(1+\sec(d*x+c))^{(1/3)}*(1+3^{(1/2)}))^{(1/2)})*\tan(d*x+c)*2^{(1/3)}/d/(1-\sec(d*x+c))/(1+\sec(d*x+c))^{(2/3)}/(-(1+\sec(d*x+c))^{(1/3)}*(2^{(1/3)}-(1+\sec(d*x+c))^{(1/3)}))/(2^{(1/3)}-(1+\sec(d*x+c))^{(1/3)}*(1+3^{(1/2)}))^{(1/2)}+(1+\sec(d*x+c))^{(1/3)}*(1-3^{(1/2)}))^{(1/2)}$

Rubi [A] time = 0.70, antiderivative size = 739, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3924, 3779, 3778, 136, 3828, 3827, 63, 308, 225, 1881}

$$\frac{3\sqrt{2} A \tan(c + dx) \sqrt[3]{a \sec(c + dx) + a} F_1\left(\frac{5}{6}; \frac{1}{2}, 1; \frac{11}{6}; \frac{1}{2}(\sec(c + dx) + 1), \sec(c + dx) + 1\right)}{5d\sqrt{1 - \sec(c + dx)}} - \frac{3(1 + \sqrt{3}) B \tan(c + dx)}{d(\sec(c + dx) + 1)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(1/3)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (3*Sqrt[2]*A*AppellF1[5/6, 1/2, 1, 11/6, (1 + Sec[c + d*x])/2, 1 + Sec[c + d*x])*(a + a*Sec[c + d*x])^(1/3)*Tan[c + d*x]/(5*d*Sqrt[1 - Sec[c + d*x]]) - (3*(1 + Sqrt[3])*B*(a + a*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(d*(1 + Sec[c + d*x])^(2/3)*(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))) + (3*2^(1/3)*3^(1/4)*B*EllipticE[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))]/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(a + a*Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(d*(1 - Sec[c + d*x])*(1 + Sec[c + d*x])^(2/3)*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]]) + (3^(3/4)*(1 - Sqrt[3])*B*EllipticF[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))]/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(a + a*Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(2^(2/3)*d*(1 - Sec[c + d*x])*(1 + Sec[c + d*x])^(2/3)*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))]/(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d), 0] && !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
```


$\text{Int}[(r*x^2*(s + r*x^2))/(s + (1 + \text{Sqrt}[3])*r*x^2)^2], x] /; \text{FreeQ}\{a, b\}, x]$

Rule 308

$\text{Int}[(x_)^4/\text{Sqrt}[(a_) + (b_)*(x_)^6], x_Symbol] := \text{With}\{r = \text{Numer}[\text{Rt}[b/a, 3]], s = \text{Denom}[\text{Rt}[b/a, 3]]\}, \text{Dist}[\text{((Sqrt}[3] - 1)*s^2)/(2*r^2), \text{Int}[1/\text{Sqrt}[a + b*x^6], x], x] - \text{Dist}[1/(2*r^2), \text{Int}[\text{((Sqrt}[3] - 1)*s^2 - 2*r^2*x^4)/\text{Sqrt}[a + b*x^6], x], x]] /; \text{FreeQ}\{a, b\}, x]$

Rule 1881

$\text{Int}[\text{((c_) + (d_)*(x_)^4)/\text{Sqrt}[(a_) + (b_)*(x_)^6], x_Symbol] := \text{With}\{r = \text{Numer}[\text{Rt}[b/a, 3]], s = \text{Denom}[\text{Rt}[b/a, 3]]\}, \text{Simp}[\text{((1 + Sqrt}[3])*d*s^3*x*\text{Sqrt}[a + b*x^6])/(2*a*r^2*(s + (1 + \text{Sqrt}[3])*r*x^2)), x] - \text{Simp}[\text{(3^(1/4)*d*s*x*(s + r*x^2)*\text{Sqrt}[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + \text{Sqrt}[3])*r*x^2)^2]*\text{EllipticE}[\text{ArcCos}[(s + (1 - \text{Sqrt}[3])*r*x^2)/(s + (1 + \text{Sqrt}[3])*r*x^2)], (2 + \text{Sqrt}[3])/4])/(2*r^2*\text{Sqrt}[(r*x^2*(s + r*x^2))/(s + (1 + \text{Sqrt}[3])*r*x^2)^2]*\text{Sqrt}[a + b*x^6]), x]] /; \text{FreeQ}\{a, b, c, d\}, x] \&\& \text{EqQ}[2*\text{Rt}[b/a, 3]^2*c - (1 - \text{Sqrt}[3])*d, 0]$

Rule 3778

$\text{Int}[(\text{csc}[(c_) + (d_)*(x_)]*(b_) + (a_))^{(n_)}, x_Symbol] := \text{Dist}[(a^n*\text{Cot}[c + d*x])/(d*\text{Sqrt}[1 + \text{Csc}[c + d*x]]*\text{Sqrt}[1 - \text{Csc}[c + d*x]]), \text{Subst}[\text{Int}[(1 + (b*x)/a)^{(n - 1/2)}/(x*\text{Sqrt}[1 - (b*x)/a]), x], x, \text{Csc}[c + d*x]], x] /; \text{FreeQ}\{a, b, c, d, n\}, x] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& !\text{IntegerQ}[2*n] \&\& \text{GtQ}[a, 0]$

Rule 3779

$\text{Int}[(\text{csc}[(c_) + (d_)*(x_)]*(b_) + (a_))^{(n_)}, x_Symbol] := \text{Dist}[(a^{\text{IntPart}[n]}*(a + b*\text{Csc}[c + d*x])^{\text{FracPart}[n]})/(1 + (b*\text{Csc}[c + d*x])/a)^{\text{FracPart}[n]}, \text{Int}[(1 + (b*\text{Csc}[c + d*x])/a)^n, x], x] /; \text{FreeQ}\{a, b, c, d, n\}, x] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& !\text{IntegerQ}[2*n] \&\& !\text{GtQ}[a, 0]$

Rule 3827

$\text{Int}[(\text{csc}[(e_) + (f_)*(x_)]*(d_))^{(n_)}*(\text{csc}[(e_) + (f_)*(x_)]*(b_) + (a_))^{(m_)}, x_Symbol] := \text{Dist}[(a^2*d*\text{Cot}[e + f*x])/(f*\text{Sqrt}[a + b*\text{Csc}[e + f*x]]*\text{Sqrt}[a - b*\text{Csc}[e + f*x]]), \text{Subst}[\text{Int}[\text{((d*x)^(n - 1)*(a + b*x)^(m - 1/2))/\text{Sqrt}[a - b*x], x], x, \text{Csc}[e + f*x]], x] /; \text{FreeQ}\{a, b, d, e, f, m, n\}, x] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& !\text{IntegerQ}[m] \&\& \text{GtQ}[a, 0]$

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) +
(a_.)^(m_.), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/ (1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] && !IntegerQ[m] && !GtQ[a, 0]
```

Rule 3924

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(d
_.) + (c_.)), x_Symbol] :> Dist[c, Int[(a + b*Csc[e + f*x])^m, x], x] + Dist
[d, Int[(a + b*Csc[e + f*x])^m*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d, e
, f, m}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
\int \sqrt[3]{a + a \sec(c + dx)} (A + B \sec(c + dx)) dx &= A \int \sqrt[3]{a + a \sec(c + dx)} dx + B \int \sec(c + dx) \sqrt[3]{a + a \sec(c + dx)} dx \\
&= \frac{(A \sqrt[3]{a + a \sec(c + dx)}) \int \sqrt[3]{1 + \sec(c + dx)} dx}{\sqrt[3]{1 + \sec(c + dx)}} + \frac{(B \sqrt[3]{a + a \sec(c + dx)}) \int \sec(c + dx) \sqrt[3]{1 + \sec(c + dx)} dx}{\sqrt[3]{1 + \sec(c + dx)}} \\
&= -\frac{(A \sqrt[3]{a + a \sec(c + dx)} \tan(c + dx)) \operatorname{Subst}\left(\int \frac{1}{\sqrt{1-xx} \sqrt[6]{1+xx}} dx, x, \sec(c + dx)\right)}{d \sqrt{1 - \sec(c + dx)} (1 + \sec(c + dx))^{5/6}} \\
&= \frac{3\sqrt{2} AF_1\left(\frac{5}{6}; \frac{1}{2}, 1; \frac{11}{6}; \frac{1}{2}(1 + \sec(c + dx)), 1 + \sec(c + dx)\right) \sqrt[3]{a + a \sec(c + dx)}}{5d \sqrt{1 - \sec(c + dx)}} \\
&= \frac{3\sqrt{2} AF_1\left(\frac{5}{6}; \frac{1}{2}, 1; \frac{11}{6}; \frac{1}{2}(1 + \sec(c + dx)), 1 + \sec(c + dx)\right) \sqrt[3]{a + a \sec(c + dx)}}{5d \sqrt{1 - \sec(c + dx)}} \\
&= \frac{3\sqrt{2} AF_1\left(\frac{5}{6}; \frac{1}{2}, 1; \frac{11}{6}; \frac{1}{2}(1 + \sec(c + dx)), 1 + \sec(c + dx)\right) \sqrt[3]{a + a \sec(c + dx)}}{5d \sqrt{1 - \sec(c + dx)}}
\end{aligned}$$

Mathematica [B] time = 21.34, size = 5094, normalized size = 6.89

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(a + a*Sec[c + d*x])^(1/3)*(A + B*Sec[c + d*x]),x]

[Out] Result too large to show

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^(1/3)*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(a \sec(dx + c) + a)^{\frac{1}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^(1/3)*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(1/3), x)

maple [F] time = 1.46, size = 0, normalized size = 0.00

$$\int (a + a \sec(dx + c))^{\frac{1}{3}} (A + B \sec(dx + c)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(d*x+c))^(1/3)*(A+B*sec(d*x+c)),x)

[Out] int((a+a*sec(d*x+c))^(1/3)*(A+B*sec(d*x+c)),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(a \sec(dx + c) + a)^{\frac{1}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^(1/3)*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(1/3), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(A + \frac{B}{\cos(c + dx)} \right) \left(a + \frac{a}{\cos(c + dx)} \right)^{\frac{1}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(1/3), x)`

[Out] `int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(1/3), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{a(\sec(c + dx) + 1)} (A + B \sec(c + dx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sec(d*x+c))**(1/3)*(A+B*sec(d*x+c)), x)`

[Out] `Integral((a*(sec(c + d*x) + 1))**(1/3)*(A + B*sec(c + d*x)), x)`

$$3.274 \quad \int \frac{A+B \sec(c+dx)}{(a+a \sec(c+dx))^{2/3}} dx$$

Optimal. Leaf size=764

$$\frac{3\sqrt{2} A \tan(c+dx) F_1\left(-\frac{1}{6}; \frac{1}{2}, 1; \frac{5}{6}; \frac{1}{2}(\sec(c+dx)+1), \sec(c+dx)+1\right)}{d\sqrt{1-\sec(c+dx)}(a \sec(c+dx)+a)^{2/3}} + \frac{3B \tan(c+dx)}{d(a \sec(c+dx)+a)^{2/3}} + \frac{3(1+\sqrt{2})}{d(\sqrt{2}-1)}$$

[Out] $3*B*\tan(d*x+c)/d/(a+a*\sec(d*x+c))^{(2/3)}+3*B*(1+\sec(d*x+c))^{(1/3)}*(1+3^{(1/2)})*\tan(d*x+c)/d/(a+a*\sec(d*x+c))^{(2/3)}/(2^{(1/3)}-(1+\sec(d*x+c))^{(1/3)}*(1+3^{(1/2)}))-3*A*AppellF1(-1/6, 1, 1/2, 5/6, 1+\sec(d*x+c), 1/2+1/2*\sec(d*x+c))*2^{(1/2)}*\tan(d*x+c)/d/(a+a*\sec(d*x+c))^{(2/3)}/(1-\sec(d*x+c))^{(1/2)}-3*2^{(1/3)}*3^{(1/4)}*B*((2^{(1/3)}-(1+\sec(d*x+c))^{(1/3)}*(1-3^{(1/2)}))^{(1/2)})^2/(2^{(1/3)}-(1+\sec(d*x+c))^{(1/3)}*(1+3^{(1/2)}))^{(1/2)})^2/(2^{(1/3)}-(1+\sec(d*x+c))^{(1/3)}*(1-3^{(1/2)}))*(2^{(1/3)}-(1+\sec(d*x+c))^{(1/3)}*(1+3^{(1/2)}))^{(1/2)})*EllipticE((1-(2^{(1/3)}-(1+\sec(d*x+c))^{(1/3)}*(1-3^{(1/2)}))^{(1/2)})^2/(2^{(1/3)}-(1+\sec(d*x+c))^{(1/3)}*(1+3^{(1/2)}))^{(1/2)}), 1/4*6^{(1/2)}+1/4*2^{(1/2)}*(1+\sec(d*x+c))^{(1/3)}*(2^{(1/3)}-(1+\sec(d*x+c))^{(1/3)}))*((2^{(2/3)}+2^{(1/3)}*(1+\sec(d*x+c))^{(1/3)}+(1+\sec(d*x+c))^{(2/3)})/(2^{(1/3)}-(1+\sec(d*x+c))^{(1/3)}*(1+3^{(1/2)}))^{(1/2)})^2*\tan(d*x+c)/d/(1-\sec(d*x+c))/(a+a*\sec(d*x+c))^{(2/3)}/(-(1+\sec(d*x+c))^{(1/3)}*(2^{(1/3)}-(1+\sec(d*x+c))^{(1/3)}))/(2^{(1/3)}-(1+\sec(d*x+c))^{(1/3)}*(1+3^{(1/2)}))^{(1/2)}-1/2*3^{(3/4)}*B*((2^{(1/3)}-(1+\sec(d*x+c))^{(1/3)}*(1-3^{(1/2)}))^{(1/2)})^2/(2^{(1/3)}-(1+\sec(d*x+c))^{(1/3)}*(1+3^{(1/2)}))^{(1/2)})^2/(2^{(1/3)}-(1+\sec(d*x+c))^{(1/3)}*(1-3^{(1/2)}))*(2^{(1/3)}-(1+\sec(d*x+c))^{(1/3)}*(1+3^{(1/2)}))^{(1/2)})*EllipticF((1-(2^{(1/3)}-(1+\sec(d*x+c))^{(1/3)}*(1-3^{(1/2)}))^{(1/2)})^2/(2^{(1/3)}-(1+\sec(d*x+c))^{(1/3)}*(1+3^{(1/2)}))^{(1/2)}), 1/4*6^{(1/2)}+1/4*2^{(1/2)}*(1+\sec(d*x+c))^{(1/3)}*(2^{(1/3)}-(1+\sec(d*x+c))^{(1/3)}))*(1-3^{(1/2)})*((2^{(2/3)}+2^{(1/3)}*(1+\sec(d*x+c))^{(1/3)}+(1+\sec(d*x+c))^{(2/3)})/(2^{(1/3)}-(1+\sec(d*x+c))^{(1/3)}*(1+3^{(1/2)}))^{(1/2)})^2*\tan(d*x+c)*2^{(1/3)}/d/(1-\sec(d*x+c))/(a+a*\sec(d*x+c))^{(2/3)}/(-(1+\sec(d*x+c))^{(1/3)}*(2^{(1/3)}-(1+\sec(d*x+c))^{(1/3)}))/(2^{(1/3)}-(1+\sec(d*x+c))^{(1/3)}*(1+3^{(1/2)}))^{(1/2)}$

Rubi [A] time = 0.73, antiderivative size = 764, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3924, 3779, 3778, 136, 3828, 3827, 51, 63, 308, 225, 1881}

$$\frac{3\sqrt{2} A \tan(c+dx) F_1\left(-\frac{1}{6}; \frac{1}{2}, 1; \frac{5}{6}; \frac{1}{2}(\sec(c+dx)+1), \sec(c+dx)+1\right)}{d\sqrt{1-\sec(c+dx)}(a \sec(c+dx)+a)^{2/3}} + \frac{3B \tan(c+dx)}{d(a \sec(c+dx)+a)^{2/3}} + \frac{3(1+\sqrt{2})}{d(\sqrt{2}-1)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(a + a*Sec[c + d*x])^(2/3),x]
```

```
[Out] (3*B*Tan[c + d*x])/(d*(a + a*Sec[c + d*x])^(2/3)) - (3*Sqrt[2]*A*AppellF1[-
1/6, 1/2, 1, 5/6, (1 + Sec[c + d*x])/2, 1 + Sec[c + d*x]]*Tan[c + d*x])/(d*
Sqrt[1 - Sec[c + d*x]]*(a + a*Sec[c + d*x])^(2/3)) + (3*(1 + Sqrt[3])*B*(1
+ Sec[c + d*x])^(1/3)*Tan[c + d*x])/(d*(a + a*Sec[c + d*x])^(2/3)*(2^(1/3)
- (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))) - (3*2^(1/3)*3^(1/4)*B*EllipticE
[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 +
Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(1 + Sec[c + d*x])^(1
/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c
+ d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Se
c[c + d*x])^(1/3))^2]*Tan[c + d*x])/(d*(1 - Sec[c + d*x])*(a + a*Sec[c + d*
x])^(2/3)*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1
/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]]) - (3^(3/4)*(1
- Sqrt[3])*B*EllipticF[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^
(1/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4
]*(1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2
/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3)
- (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(2^(2/3)*d*(1 -
Sec[c + d*x])*(a + a*Sec[c + d*x])^(2/3)*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*
(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c +
d*x])^(1/3))^2]])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] := Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))]/
```

$(b^{(p+1)}(m+1)(b/(b*c - a*d))^n), x] /;$ FreeQ[{a, b, c, d, e, f, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d), 0] && !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])

Rule 225

Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x]

Rule 308

Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqrt[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]

Rule 1881

Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqrt[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1 - Sqrt[3])*d, 0]

Rule 3778

Int[(csc[(c_) + (d_.)*(x_)]*(b_) + (a_))^(n_), x_Symbol] := Dist[(a^n*Cot[c + d*x])/(d*Sqrt[1 + Csc[c + d*x]]*Sqrt[1 - Csc[c + d*x]]), Subst[Int[(1 + (b*x)/a)^(n - 1/2)/(x*Sqrt[1 - (b*x)/a]), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[2*n] && GtQ[a, 0]

Rule 3779

Int[(csc[(c_) + (d_.)*(x_)]*(b_) + (a_))^(n_), x_Symbol] := Dist[(a^IntPart[n]*(a + b*Csc[c + d*x])^FracPart[n])/(1 + (b*Csc[c + d*x])/a)^FracPart[n], Int[(1 + (b*Csc[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[2*n] && !GtQ[a, 0]

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_.)^(m_.), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] && !IntegerQ[m] && GtQ[a, 0]
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_.)^(m_.), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] && !IntegerQ[m] && !GtQ[a, 0]
```

Rule 3924

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_.)), x_Symbol] :> Dist[c, Int[(a + b*Csc[e + f*x])^m, x], x] + Dist
[d, Int[(a + b*Csc[e + f*x])^m*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d, e
, f, m}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
\int \frac{A + B \sec(c + dx)}{(a + a \sec(c + dx))^{2/3}} dx &= A \int \frac{1}{(a + a \sec(c + dx))^{2/3}} dx + B \int \frac{\sec(c + dx)}{(a + a \sec(c + dx))^{2/3}} dx \\
&= \frac{(A(1 + \sec(c + dx))^{2/3}) \int \frac{1}{(1 + \sec(c + dx))^{2/3}} dx}{(a + a \sec(c + dx))^{2/3}} + \frac{(B(1 + \sec(c + dx))^{2/3}) \int \frac{\sec(c + dx)}{(1 + \sec(c + dx))^{2/3}} dx}{(a + a \sec(c + dx))^{2/3}} \\
&= \frac{(A\sqrt[6]{1 + \sec(c + dx)} \tan(c + dx)) \operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x}x(1+x)^{7/6}} dx, x, \sec(c + dx)\right)}{d\sqrt{1 - \sec(c + dx)}(a + a \sec(c + dx))^{2/3}} - \frac{(B\sqrt[6]{1 + \sec(c + dx)} \tan(c + dx)) \operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x}x(1+x)^{7/6}} dx, x, \sec(c + dx)\right)}{d\sqrt{1 - \sec(c + dx)}(a + a \sec(c + dx))^{2/3}} \\
&= \frac{3B \tan(c + dx)}{d(a + a \sec(c + dx))^{2/3}} - \frac{3\sqrt{2} AF_1\left(-\frac{1}{6}; \frac{1}{2}, 1; \frac{5}{6}; \frac{1}{2}(1 + \sec(c + dx)), 1 + \sec(c + dx)\right)}{d\sqrt{1 - \sec(c + dx)}(a + a \sec(c + dx))^{2/3}} \\
&= \frac{3B \tan(c + dx)}{d(a + a \sec(c + dx))^{2/3}} - \frac{3\sqrt{2} AF_1\left(-\frac{1}{6}; \frac{1}{2}, 1; \frac{5}{6}; \frac{1}{2}(1 + \sec(c + dx)), 1 + \sec(c + dx)\right)}{d\sqrt{1 - \sec(c + dx)}(a + a \sec(c + dx))^{2/3}} \\
&= \frac{3B \tan(c + dx)}{d(a + a \sec(c + dx))^{2/3}} - \frac{3\sqrt{2} AF_1\left(-\frac{1}{6}; \frac{1}{2}, 1; \frac{5}{6}; \frac{1}{2}(1 + \sec(c + dx)), 1 + \sec(c + dx)\right)}{d\sqrt{1 - \sec(c + dx)}(a + a \sec(c + dx))^{2/3}} \\
&= \frac{3B \tan(c + dx)}{d(a + a \sec(c + dx))^{2/3}} - \frac{3\sqrt{2} AF_1\left(-\frac{1}{6}; \frac{1}{2}, 1; \frac{5}{6}; \frac{1}{2}(1 + \sec(c + dx)), 1 + \sec(c + dx)\right)}{d\sqrt{1 - \sec(c + dx)}(a + a \sec(c + dx))^{2/3}}
\end{aligned}$$

Mathematica [B] time = 19.36, size = 4066, normalized size = 5.32

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(A + B*Sec[c + d*x])/(a + a*Sec[c + d*x])^(2/3), x]

[Out] (Cos[c + d*x]*((1 + Cos[c + d*x])*Sec[c + d*x])^(1/3)*(1 + Sec[c + d*x])^(2/3)*(A + B*Sec[c + d*x])*(3*Sec[(c + d*x)/2]*(-A*Sin[(c + d*x)/2]) + B*Sin[(c + d*x)/2]) - 3*(-A + B)*Sin[c + d*x]))/(d*(B + A*Cos[c + d*x])*(a*(1 + Sec[c + d*x]))^(2/3)) - (2^(1/3)*Cos[c + d*x]*(1 + Sec[c + d*x])^(2/3)*(A + B*Sec[c + d*x])*(2*A*(1 + Sec[c + d*x])^(1/3) - B*(1 + Sec[c + d*x])^(1/3) + Cos[c + d*x]*(-3*A*(1 + Sec[c + d*x])^(1/3) + 3*B*(1 + Sec[c + d*x])^(1/3))))*Tan[(c + d*x)/2]*((A - B)*AppellF1[3/2, 1/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2)/(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3) - (9*(3*AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c +

$$\begin{aligned}
& d*x)/2]^2)*(A + B + (-5*A + 7*B)*Cos[c + d*x]) + 4*(A - B)*(3*AppellF1[3/2, \\
& 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - AppellF1[3/2, 4/3, \\
& 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Cos[c + d*x]*Tan[(c + d* \\
& x)/2]^2))/(2*(-1 + Tan[(c + d*x)/2]^2)*(-9*AppellF1[1/2, 1/3, 1, 3/2, Tan[(c \\
& + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c \\
& + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d \\
& *x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)))/(3*d*(B + A*Cos[c + \\
& d*x])*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(2/3)*(a*(1 + Sec[c + d*x]))^(2/3)* \\
& (-1/3*(Sec[(c + d*x)/2]^2*((A - B)*AppellF1[3/2, 1/3, 1, 5/2, Tan[(c + d*x) \\
&)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2)/(Cos[c + d*x]*Sec[(c + d*x) \\
&)/2]^2)^(2/3) - (9*(3*AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c \\
& + d*x)/2]^2)*(A + B + (-5*A + 7*B)*Cos[c + d*x]) + 4*(A - B)*(3*AppellF1[\\
& 3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - AppellF1[3/2, \\
& 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Cos[c + d*x]*Tan[(c \\
& + d*x)/2]^2))/(2*(-1 + Tan[(c + d*x)/2]^2)*(-9*AppellF1[1/2, 1/3, 1, 3/2, T \\
& an[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(3*AppellF1[3/2, 1/3, 2, 5/2, T \\
& an[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - AppellF1[3/2, 4/3, 1, 5/2, Tan[(c \\
& + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)))/(2^(2/3)*(Cos[(c \\
& + d*x)/2]^2*Sec[c + d*x])^(2/3)) - (2^(1/3)*Tan[(c + d*x)/2]*((A - B)*App \\
& ellF1[3/2, 1/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d \\
& *x)/2]^2*Tan[(c + d*x)/2])/(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3) + ((A - \\
& B)*Tan[(c + d*x)/2]^2*(-3*AppellF1[5/2, 1/3, 2, 7/2, Tan[(c + d*x)/2]^2, - \\
& Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 + (AppellF1[5/2, \\
& 4/3, 1, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*T \\
& an[(c + d*x)/2])/5)/(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3) - (2*(A - B)*A \\
& ppellF1[3/2, 1/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(c + \\
& d*x)/2]^2*(-(Sec[(c + d*x)/2]^2*Sin[c + d*x]) + Cos[c + d*x]*Sec[(c + d*x) \\
& /2]^2*Tan[(c + d*x)/2]))/(3*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(5/3)) + (9*S \\
& ec[(c + d*x)/2]^2*Tan[(c + d*x)/2]*(3*AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d \\
& *x)/2]^2, -Tan[(c + d*x)/2]^2)*(A + B + (-5*A + 7*B)*Cos[c + d*x]) + 4*(A - \\
& B)*(3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] \\
& - AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Cos[\\
& c + d*x]*Tan[(c + d*x)/2]^2))/(2*(-1 + Tan[(c + d*x)/2]^2)^2*(-9*AppellF1[1 \\
& /2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(3*AppellF1[3 \\
& /2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - AppellF1[3/2, 4 \\
& /3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)) \\
& + (9*(3*AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] \\
& *(A + B + (-5*A + 7*B)*Cos[c + d*x]) + 4*(A - B)*(3*AppellF1[3/2, 1/3, 2, 5 \\
& /2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - AppellF1[3/2, 4/3, 1, 5/2, T \\
& an[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Cos[c + d*x]*Tan[(c + d*x)/2]^2)*(\\
& 2*(3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - \\
& AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Sec[(c \\
& + d*x)/2]^2*Tan[(c + d*x)/2] - 9*(-1/3*(AppellF1[3/2, 1/3, 2, 5/2, Tan[(c \\
& + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]) + (A \\
& ppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c +
\end{aligned}$$

$$\begin{aligned}
& d*x)/2]^2*\text{Tan}[(c + d*x)/2])/9) + 2*\text{Tan}[(c + d*x)/2]^2*((3*\text{AppellF1}[5/2, 4/ \\
& 3, 2, 7/2, \text{Tan}[(c + d*x)/2]^2, -\text{Tan}[(c + d*x)/2]^2]*\text{Sec}[(c + d*x)/2]^2*\text{Tan} \\
& (c + d*x)/2])/5 - (4*\text{AppellF1}[5/2, 7/3, 1, 7/2, \text{Tan}[(c + d*x)/2]^2, -\text{Tan}[(c \\
& + d*x)/2]^2]*\text{Sec}[(c + d*x)/2]^2*\text{Tan}[(c + d*x)/2])/5 + 3*((-6*\text{AppellF1}[5/2, \\
& 1/3, 3, 7/2, \text{Tan}[(c + d*x)/2]^2, -\text{Tan}[(c + d*x)/2]^2]*\text{Sec}[(c + d*x)/2]^2*\text{T} \\
& \text{an}[(c + d*x)/2])/5 + (\text{AppellF1}[5/2, 4/3, 2, 7/2, \text{Tan}[(c + d*x)/2]^2, -\text{Tan}[(c \\
& + d*x)/2]^2]*\text{Sec}[(c + d*x)/2]^2*\text{Tan}[(c + d*x)/2])/5))))/(2*(-1 + \text{Tan}[(c + \\
& d*x)/2]^2)*(-9*\text{AppellF1}[1/2, 1/3, 1, 3/2, \text{Tan}[(c + d*x)/2]^2, -\text{Tan}[(c + d* \\
& x)/2]^2] + 2*(3*\text{AppellF1}[3/2, 1/3, 2, 5/2, \text{Tan}[(c + d*x)/2]^2, -\text{Tan}[(c + d* \\
& x)/2]^2] - \text{AppellF1}[3/2, 4/3, 1, 5/2, \text{Tan}[(c + d*x)/2]^2, -\text{Tan}[(c + d*x)/2] \\
& ^2])* \text{Tan}[(c + d*x)/2]^2) - (9*(-3*(-5*A + 7*B)*\text{AppellF1}[1/2, 1/3, 1, 3/2 \\
& , \text{Tan}[(c + d*x)/2]^2, -\text{Tan}[(c + d*x)/2]^2]*\text{Sin}[c + d*x] + 4*(A - B)*(3*\text{Appel} \\
& \text{llF1}[3/2, 1/3, 2, 5/2, \text{Tan}[(c + d*x)/2]^2, -\text{Tan}[(c + d*x)/2]^2] - \text{AppellF1}[\\
& 3/2, 4/3, 1, 5/2, \text{Tan}[(c + d*x)/2]^2, -\text{Tan}[(c + d*x)/2]^2])* \text{Cos}[c + d*x]*\text{Se} \\
& \text{c}[(c + d*x)/2]^2*\text{Tan}[(c + d*x)/2] - 4*(A - B)*(3*\text{AppellF1}[3/2, 1/3, 2, 5/2, \\
& \text{Tan}[(c + d*x)/2]^2, -\text{Tan}[(c + d*x)/2]^2] - \text{AppellF1}[3/2, 4/3, 1, 5/2, \text{Tan} \\
& (c + d*x)/2]^2, -\text{Tan}[(c + d*x)/2]^2])* \text{Sin}[c + d*x]*\text{Tan}[(c + d*x)/2]^2 + 3*(\\
& A + B + (-5*A + 7*B)*\text{Cos}[c + d*x])*(-1/3*(\text{AppellF1}[3/2, 1/3, 2, 5/2, \text{Tan}[(c \\
& + d*x)/2]^2, -\text{Tan}[(c + d*x)/2]^2]*\text{Sec}[(c + d*x)/2]^2*\text{Tan}[(c + d*x)/2]) + (\\
& \text{AppellF1}[3/2, 4/3, 1, 5/2, \text{Tan}[(c + d*x)/2]^2, -\text{Tan}[(c + d*x)/2]^2]*\text{Sec}[(c \\
& + d*x)/2]^2*\text{Tan}[(c + d*x)/2])/9) + 4*(A - B)*\text{Cos}[c + d*x]*\text{Tan}[(c + d*x)/2]^ \\
& 2*((3*\text{AppellF1}[5/2, 4/3, 2, 7/2, \text{Tan}[(c + d*x)/2]^2, -\text{Tan}[(c + d*x)/2]^2]*\text{S} \\
& \text{ec}[(c + d*x)/2]^2*\text{Tan}[(c + d*x)/2])/5 - (4*\text{AppellF1}[5/2, 7/3, 1, 7/2, \text{Tan}[(c \\
& + d*x)/2]^2, -\text{Tan}[(c + d*x)/2]^2]*\text{Sec}[(c + d*x)/2]^2*\text{Tan}[(c + d*x)/2])/5 \\
& + 3*((-6*\text{AppellF1}[5/2, 1/3, 3, 7/2, \text{Tan}[(c + d*x)/2]^2, -\text{Tan}[(c + d*x)/2]^2] \\
& *\text{Sec}[(c + d*x)/2]^2*\text{Tan}[(c + d*x)/2])/5 + (\text{AppellF1}[5/2, 4/3, 2, 7/2, \text{Tan} \\
& (c + d*x)/2]^2, -\text{Tan}[(c + d*x)/2]^2]*\text{Sec}[(c + d*x)/2]^2*\text{Tan}[(c + d*x)/2])/5 \\
&)))))/(2*(-1 + \text{Tan}[(c + d*x)/2]^2)*(-9*\text{AppellF1}[1/2, 1/3, 1, 3/2, \text{Tan}[(c + d \\
& *x)/2]^2, -\text{Tan}[(c + d*x)/2]^2] + 2*(3*\text{AppellF1}[3/2, 1/3, 2, 5/2, \text{Tan}[(c + d \\
& *x)/2]^2, -\text{Tan}[(c + d*x)/2]^2] - \text{AppellF1}[3/2, 4/3, 1, 5/2, \text{Tan}[(c + d*x)/2] \\
& ^2, -\text{Tan}[(c + d*x)/2]^2])* \text{Tan}[(c + d*x)/2]^2)))/(3*(\text{Cos}[(c + d*x)/2]^2*\text{Se} \\
& \text{c}[c + d*x])^(2/3)) + (2*2^(1/3)*\text{Tan}[(c + d*x)/2]*((A - B)*\text{AppellF1}[3/2, 1/ \\
& 3, 1, 5/2, \text{Tan}[(c + d*x)/2]^2, -\text{Tan}[(c + d*x)/2]^2]*\text{Tan}[(c + d*x)/2]^2)/(\text{Co} \\
& \text{s}[c + d*x]*\text{Sec}[(c + d*x)/2]^2)^(2/3) - (9*(3*\text{AppellF1}[1/2, 1/3, 1, 3/2, \text{Tan} \\
& [(c + d*x)/2]^2, -\text{Tan}[(c + d*x)/2]^2]*(A + B + (-5*A + 7*B)*\text{Cos}[c + d*x]) + \\
& 4*(A - B)*(3*\text{AppellF1}[3/2, 1/3, 2, 5/2, \text{Tan}[(c + d*x)/2]^2, -\text{Tan}[(c + d*x) \\
& /2]^2] - \text{AppellF1}[3/2, 4/3, 1, 5/2, \text{Tan}[(c + d*x)/2]^2, -\text{Tan}[(c + d*x)/2]^2] \\
&])* \text{Cos}[c + d*x]*\text{Tan}[(c + d*x)/2]^2))/ (2*(-1 + \text{Tan}[(c + d*x)/2]^2)*(-9*\text{Appel} \\
& \text{llF1}[1/2, 1/3, 1, 3/2, \text{Tan}[(c + d*x)/2]^2, -\text{Tan}[(c + d*x)/2]^2] + 2*(3*\text{Appel} \\
& \text{llF1}[3/2, 1/3, 2, 5/2, \text{Tan}[(c + d*x)/2]^2, -\text{Tan}[(c + d*x)/2]^2] - \text{AppellF1}[3 \\
& /2, 4/3, 1, 5/2, \text{Tan}[(c + d*x)/2]^2, -\text{Tan}[(c + d*x)/2]^2])* \text{Tan}[(c + d*x)/2] \\
& ^2)))*(-(\text{Cos}[(c + d*x)/2]*\text{Sec}[c + d*x]*\text{Sin}[(c + d*x)/2]) + \text{Cos}[(c + d*x)/2] \\
& ^2*\text{Sec}[c + d*x]*\text{Tan}[c + d*x]))/(9*(\text{Cos}[(c + d*x)/2]^2*\text{Sec}[c + d*x])^(5/3))) \\
&)
\end{aligned}$$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(2/3),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(a \sec(dx + c) + a)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(2/3),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)/(a*sec(d*x + c) + a)^(2/3), x)

maple [F] time = 1.32, size = 0, normalized size = 0.00

$$\int \frac{A + B \sec(dx + c)}{(a + a \sec(dx + c))^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(2/3),x)

[Out] int((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(2/3),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(a \sec(dx + c) + a)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(2/3),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)/(a*sec(d*x + c) + a)^(2/3), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\left(a + \frac{a}{\cos(c+dx)}\right)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B/cos(c + d*x))/(a + a/cos(c + d*x))^(2/3), x)`

[Out] `int((A + B/cos(c + d*x))/(a + a/cos(c + d*x))^(2/3), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \sec(c + dx)}{(a(\sec(c + dx) + 1))^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(2/3), x)`

[Out] `Integral((A + B*sec(c + d*x))/(a*(sec(c + d*x) + 1))**(2/3), x)`

$$3.275 \quad \int (c \sec(e+fx))^n (a+a \sec(e+fx))^m (A+B \sec(e+fx)) dx$$

Optimal. Leaf size=197

$$\frac{(A-B) \tan(e+fx) (\sec(e+fx)+1)^{-m-\frac{1}{2}} (a \sec(e+fx)+a)^m (c \sec(e+fx))^n F_1\left(n; \frac{1}{2}, \frac{1}{2}-m; n+1; \sec(e+fx)\right)}{fn \sqrt{1-\sec(e+fx)}}$$

[Out] -B*AppellF1(n, -1/2-m, 1/2, 1+n, -sec(f*x+e), sec(f*x+e))*(c*sec(f*x+e))^n*(1+sec(f*x+e))^(1/2-m)*(a+a*sec(f*x+e))^m*tan(f*x+e)/f/n/(1-sec(f*x+e))^(1/2)-(A-B)*AppellF1(n, 1/2-m, 1/2, 1+n, -sec(f*x+e), sec(f*x+e))*(c*sec(f*x+e))^n*(1+sec(f*x+e))^(1/2-m)*(a+a*sec(f*x+e))^m*tan(f*x+e)/f/n/(1-sec(f*x+e))^(1/2)

Rubi [A] time = 0.36, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {4023, 3828, 3827, 133}

$$\frac{(A-B) \tan(e+fx) (\sec(e+fx)+1)^{-m-\frac{1}{2}} (a \sec(e+fx)+a)^m (c \sec(e+fx))^n F_1\left(n; \frac{1}{2}, \frac{1}{2}-m; n+1; \sec(e+fx)\right)}{fn \sqrt{1-\sec(e+fx)}}$$

Antiderivative was successfully verified.

[In] Int[(c*Sec[e + f*x])^n*(a + a*Sec[e + f*x])^m*(A + B*Sec[e + f*x]),x]

[Out] -((B*AppellF1[n, 1/2, -1/2 - m, 1 + n, Sec[e + f*x], -Sec[e + f*x]]*(c*Sec[e + f*x])^n*(1 + Sec[e + f*x])^(1/2 - m)*(a + a*Sec[e + f*x])^m*Tan[e + f*x])/(f*n*Sqrt[1 - Sec[e + f*x]]) - ((A - B)*AppellF1[n, 1/2, 1/2 - m, 1 + n, Sec[e + f*x], -Sec[e + f*x]]*(c*Sec[e + f*x])^n*(1 + Sec[e + f*x])^(1/2 - m)*(a + a*Sec[e + f*x])^m*Tan[e + f*x])/(f*n*Sqrt[1 - Sec[e + f*x]])

Rule 133

Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_Symbol] := Simp[(c^n*e^p*(b*x)^(m+1)*AppellF1[m+1, -n, -p, m+2, -((d*x)/c), -((f*x)/e)]/(b*(m+1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])

Rule 3827

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_), x_Symbol] := Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n-1)*(a + b*x)^(m-1/2))/Sqrt[a - b*x], x], x, Csc[e + f*x], x] /; FreeQ[{a, b, d, e, f, m, n}, x]

] && EqQ[a^2 - b^2, 0] && !IntegerQ[m] && GtQ[a, 0]

Rule 3828

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m])/ (1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m] && !GtQ[a, 0]

Rule 4023

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> Dist[(A*b - a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned} \int (c \sec(e + fx))^n (a + a \sec(e + fx))^m (A + B \sec(e + fx)) dx &= (A - B) \int (c \sec(e + fx))^n (a + a \sec(e + fx))^m dx \\ &= ((A - B)(1 + \sec(e + fx))^{-m} (a + a \sec(e + fx))^m) \int (c \sec(e + fx))^n dx \\ &= \frac{\left((A - B)c(1 + \sec(e + fx))^{-\frac{1}{2} - m} (a + a \sec(e + fx))^m \right) \operatorname{arctan}\left(\frac{c \sec(e + fx)}{1 + \sec(e + fx)}\right)}{c} \\ &= \frac{BF_1\left(n; \frac{1}{2}, -\frac{1}{2} - m; 1 + n; \sec(e + fx), -\sec(e + fx)\right)}{c} \end{aligned}$$

Mathematica [B] time = 23.07, size = 4897, normalized size = 24.86

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(c*Sec[e + f*x])^n*(a + a*Sec[e + f*x])^m*(A + B*Sec[e + f*x]),x]
 [Out] (2^(1 + m)*(Sec[(e + f*x)/2]^2)^n*Sec[e + f*x]^(-1 - n)*(c*Sec[e + f*x])^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(m + n)*(a*(1 + Sec[e + f*x]))^m*(A + B*S

$$\begin{aligned}
& \text{ec}[e + f*x])*(A*\text{Sec}[e + f*x]^n*(1 + \text{Sec}[e + f*x])^m + B*\text{Sec}[e + f*x]^{(1 + n)} \\
& *(1 + \text{Sec}[e + f*x])^m)*\text{Tan}[(e + f*x)/2]*((-3*A*\text{AppellF1}[1/2, m + n, 1 - n, \\
& 3/2, \text{Tan}[(e + f*x)/2]^2, -\text{Tan}[(e + f*x)/2]^2)*\text{Cos}[e + f*x])/(3*\text{AppellF1}[1/ \\
& 2, m + n, 1 - n, 3/2, \text{Tan}[(e + f*x)/2]^2, -\text{Tan}[(e + f*x)/2]^2] + 2*((-1 + n \\
&)*\text{AppellF1}[3/2, m + n, 2 - n, 5/2, \text{Tan}[(e + f*x)/2]^2, -\text{Tan}[(e + f*x)/2]^2] \\
& + (m + n)*\text{AppellF1}[3/2, 1 + m + n, 1 - n, 5/2, \text{Tan}[(e + f*x)/2]^2, -\text{Tan}[(e \\
& + f*x)/2]^2])*\text{Tan}[(e + f*x)/2]^2) - (B*\text{AppellF1}[1/2, 1 + m + n, -n, 3/2, \text{T} \\
& \text{an}[(e + f*x)/2]^2, -\text{Tan}[(e + f*x)/2]^2])/(AppellF1[1/2, 1 + m + n, -n, 3/2, \\
& \text{Tan}[(e + f*x)/2]^2, -\text{Tan}[(e + f*x)/2]^2] + (2*(n*\text{AppellF1}[3/2, 1 + m + n, \\
& 1 - n, 5/2, \text{Tan}[(e + f*x)/2]^2, -\text{Tan}[(e + f*x)/2]^2] + (1 + m + n)*\text{AppellF1} \\
& [3/2, 2 + m + n, -n, 5/2, \text{Tan}[(e + f*x)/2]^2, -\text{Tan}[(e + f*x)/2]^2])*\text{Tan}[(e \\
& + f*x)/2]^2/3)))/(f*(B + A*\text{Cos}[e + f*x])*(1 + \text{Sec}[e + f*x])^m*(-1 + \text{Tan}[(e \\
& + f*x)/2]^2)*(-((2^(1 + m)*(\text{Sec}[(e + f*x)/2]^2)^(1 + n)*(\text{Cos}[(e + f*x)/2]^ \\
& 2*\text{Sec}[e + f*x])^(m + n)*\text{Tan}[(e + f*x)/2]^2*((-3*A*\text{AppellF1}[1/2, m + n, 1 - \\
& n, 3/2, \text{Tan}[(e + f*x)/2]^2, -\text{Tan}[(e + f*x)/2]^2)*\text{Cos}[e + f*x])/(3*\text{AppellF1}[\\
& 1/2, m + n, 1 - n, 3/2, \text{Tan}[(e + f*x)/2]^2, -\text{Tan}[(e + f*x)/2]^2] + 2*((-1 + n \\
&)*\text{AppellF1}[3/2, m + n, 2 - n, 5/2, \text{Tan}[(e + f*x)/2]^2, -\text{Tan}[(e + f*x)/2]^ \\
& 2] + (m + n)*\text{AppellF1}[3/2, 1 + m + n, 1 - n, 5/2, \text{Tan}[(e + f*x)/2]^2, -\text{Tan} \\
& (e + f*x)/2]^2])*\text{Tan}[(e + f*x)/2]^2) - (B*\text{AppellF1}[1/2, 1 + m + n, -n, 3/2, \\
& \text{Tan}[(e + f*x)/2]^2, -\text{Tan}[(e + f*x)/2]^2])/(AppellF1[1/2, 1 + m + n, -n, 3/ \\
& 2, \text{Tan}[(e + f*x)/2]^2, -\text{Tan}[(e + f*x)/2]^2] + (2*(n*\text{AppellF1}[3/2, 1 + m + n \\
& , 1 - n, 5/2, \text{Tan}[(e + f*x)/2]^2, -\text{Tan}[(e + f*x)/2]^2] + (1 + m + n)*\text{Appell} \\
& \text{F1}[3/2, 2 + m + n, -n, 5/2, \text{Tan}[(e + f*x)/2]^2, -\text{Tan}[(e + f*x)/2]^2])*\text{Tan}[(\\
& e + f*x)/2]^2/3)))/(-1 + \text{Tan}[(e + f*x)/2]^2)^2 + (2^m*(\text{Sec}[(e + f*x)/2]^2) \\
&)^(1 + n)*(\text{Cos}[(e + f*x)/2]^2*\text{Sec}[e + f*x])^(m + n)*((-3*A*\text{AppellF1}[1/2, m \\
& + n, 1 - n, 3/2, \text{Tan}[(e + f*x)/2]^2, -\text{Tan}[(e + f*x)/2]^2)*\text{Cos}[e + f*x])/(3* \\
& \text{AppellF1}[1/2, m + n, 1 - n, 3/2, \text{Tan}[(e + f*x)/2]^2, -\text{Tan}[(e + f*x)/2]^2] + \\
& 2*((-1 + n)*\text{AppellF1}[3/2, m + n, 2 - n, 5/2, \text{Tan}[(e + f*x)/2]^2, -\text{Tan}[(e + \\
& f*x)/2]^2] + (m + n)*\text{AppellF1}[3/2, 1 + m + n, 1 - n, 5/2, \text{Tan}[(e + f*x)/2] \\
& ^2, -\text{Tan}[(e + f*x)/2]^2])*\text{Tan}[(e + f*x)/2]^2) - (B*\text{AppellF1}[1/2, 1 + m + n, \\
& -n, 3/2, \text{Tan}[(e + f*x)/2]^2, -\text{Tan}[(e + f*x)/2]^2])/(AppellF1[1/2, 1 + m + \\
& n, -n, 3/2, \text{Tan}[(e + f*x)/2]^2, -\text{Tan}[(e + f*x)/2]^2] + (2*(n*\text{AppellF1}[3/2, \\
& 1 + m + n, 1 - n, 5/2, \text{Tan}[(e + f*x)/2]^2, -\text{Tan}[(e + f*x)/2]^2] + (1 + m + \\
& n)*\text{AppellF1}[3/2, 2 + m + n, -n, 5/2, \text{Tan}[(e + f*x)/2]^2, -\text{Tan}[(e + f*x)/2]^ \\
& 2])*\text{Tan}[(e + f*x)/2]^2/3)))/(-1 + \text{Tan}[(e + f*x)/2]^2) + (2^(1 + m)*n*(\text{Sec} \\
& (e + f*x)/2]^2)^n*(\text{Cos}[(e + f*x)/2]^2*\text{Sec}[e + f*x])^(m + n)*\text{Tan}[(e + f*x)/2 \\
&]^2*((-3*A*\text{AppellF1}[1/2, m + n, 1 - n, 3/2, \text{Tan}[(e + f*x)/2]^2, -\text{Tan}[(e + f \\
& *x)/2]^2)*\text{Cos}[e + f*x])/(3*\text{AppellF1}[1/2, m + n, 1 - n, 3/2, \text{Tan}[(e + f*x)/2 \\
&]^2, -\text{Tan}[(e + f*x)/2]^2] + 2*((-1 + n)*\text{AppellF1}[3/2, m + n, 2 - n, 5/2, \text{T} \\
& \text{an}[(e + f*x)/2]^2, -\text{Tan}[(e + f*x)/2]^2] + (m + n)*\text{AppellF1}[3/2, 1 + m + n, 1 \\
& - n, 5/2, \text{Tan}[(e + f*x)/2]^2, -\text{Tan}[(e + f*x)/2]^2])*\text{Tan}[(e + f*x)/2]^2) - \\
& (B*\text{AppellF1}[1/2, 1 + m + n, -n, 3/2, \text{Tan}[(e + f*x)/2]^2, -\text{Tan}[(e + f*x)/2]^ \\
& 2])/(AppellF1[1/2, 1 + m + n, -n, 3/2, \text{Tan}[(e + f*x)/2]^2, -\text{Tan}[(e + f*x)/2 \\
&]^2] + (2*(n*\text{AppellF1}[3/2, 1 + m + n, 1 - n, 5/2, \text{Tan}[(e + f*x)/2]^2, -\text{Tan} \\
& (e + f*x)/2]^2] + (1 + m + n)*\text{AppellF1}[3/2, 2 + m + n, -n, 5/2, \text{Tan}[(e + f*
\end{aligned}$$

$$\begin{aligned}
& x)/2]^2, -\text{Tan}[(e + f*x)/2]^2])*\text{Tan}[(e + f*x)/2]^2/3)))/(-1 + \text{Tan}[(e + f*x)/2]^2) + (2^{(1 + m)}*(\text{Sec}[(e + f*x)/2]^2)^n*(\text{Cos}[(e + f*x)/2]^2*\text{Sec}[e + f*x])^{(m + n)}*\text{Tan}[(e + f*x)/2]^2*((3*A*\text{AppellF1}[1/2, m + n, 1 - n, 3/2, \text{Tan}[(e + f*x)/2]^2, -\text{Tan}[(e + f*x)/2]^2]*\text{Sin}[e + f*x])/(3*\text{AppellF1}[1/2, m + n, 1 - n, 3/2, \text{Tan}[(e + f*x)/2]^2, -\text{Tan}[(e + f*x)/2]^2] + 2*((-1 + n)*\text{AppellF1}[3/2, m + n, 2 - n, 5/2, \text{Tan}[(e + f*x)/2]^2, -\text{Tan}[(e + f*x)/2]^2] + (m + n)*\text{AppellF1}[3/2, 1 + m + n, 1 - n, 5/2, \text{Tan}[(e + f*x)/2]^2, -\text{Tan}[(e + f*x)/2]^2])* \text{Tan}[(e + f*x)/2]^2) - (3*A*\text{Cos}[e + f*x]*(-1/3*((1 - n)*\text{AppellF1}[3/2, m + n, 2 - n, 5/2, \text{Tan}[(e + f*x)/2]^2, -\text{Tan}[(e + f*x)/2]^2]*\text{Sec}[(e + f*x)/2]^2*\text{Tan}[(e + f*x)/2]) + ((m + n)*\text{AppellF1}[3/2, 1 + m + n, 1 - n, 5/2, \text{Tan}[(e + f*x)/2]^2, -\text{Tan}[(e + f*x)/2]^2]*\text{Sec}[(e + f*x)/2]^2*\text{Tan}[(e + f*x)/2])/3)))/(3*A*\text{AppellF1}[1/2, m + n, 1 - n, 3/2, \text{Tan}[(e + f*x)/2]^2, -\text{Tan}[(e + f*x)/2]^2] + 2*((-1 + n)*\text{AppellF1}[3/2, m + n, 2 - n, 5/2, \text{Tan}[(e + f*x)/2]^2, -\text{Tan}[(e + f*x)/2]^2] + (m + n)*\text{AppellF1}[3/2, 1 + m + n, 1 - n, 5/2, \text{Tan}[(e + f*x)/2]^2, -\text{Tan}[(e + f*x)/2]^2])* \text{Tan}[(e + f*x)/2]^2) - (B*((n)*\text{AppellF1}[3/2, 1 + m + n, 1 - n, 5/2, \text{Tan}[(e + f*x)/2]^2, -\text{Tan}[(e + f*x)/2]^2]*\text{Sec}[(e + f*x)/2]^2*\text{Tan}[(e + f*x)/2])/3 + ((1 + m + n)*\text{AppellF1}[3/2, 2 + m + n, -n, 5/2, \text{Tan}[(e + f*x)/2]^2, -\text{Tan}[(e + f*x)/2]^2]*\text{Sec}[(e + f*x)/2]^2*\text{Tan}[(e + f*x)/2])/3)))/(\text{AppellF1}[1/2, 1 + m + n, -n, 3/2, \text{Tan}[(e + f*x)/2]^2, -\text{Tan}[(e + f*x)/2]^2] + (2*(n)*\text{AppellF1}[3/2, 1 + m + n, 1 - n, 5/2, \text{Tan}[(e + f*x)/2]^2, -\text{Tan}[(e + f*x)/2]^2] + (1 + m + n)*\text{AppellF1}[3/2, 2 + m + n, -n, 5/2, \text{Tan}[(e + f*x)/2]^2, -\text{Tan}[(e + f*x)/2]^2])* \text{Tan}[(e + f*x)/2]^2/3) + (3*A*\text{AppellF1}[1/2, m + n, 1 - n, 3/2, \text{Tan}[(e + f*x)/2]^2, -\text{Tan}[(e + f*x)/2]^2]*\text{Cos}[e + f*x]*(2*((-1 + n)*\text{AppellF1}[3/2, m + n, 2 - n, 5/2, \text{Tan}[(e + f*x)/2]^2, -\text{Tan}[(e + f*x)/2]^2] + (m + n)*\text{AppellF1}[3/2, 1 + m + n, 1 - n, 5/2, \text{Tan}[(e + f*x)/2]^2, -\text{Tan}[(e + f*x)/2]^2])* \text{Sec}[(e + f*x)/2]^2*\text{Tan}[(e + f*x)/2] + 3*(-1/3*((1 - n)*\text{AppellF1}[3/2, m + n, 2 - n, 5/2, \text{Tan}[(e + f*x)/2]^2, -\text{Tan}[(e + f*x)/2]^2]*\text{Sec}[(e + f*x)/2]^2*\text{Tan}[(e + f*x)/2]) + ((m + n)*\text{AppellF1}[3/2, 1 + m + n, 1 - n, 5/2, \text{Tan}[(e + f*x)/2]^2, -\text{Tan}[(e + f*x)/2]^2]*\text{Sec}[(e + f*x)/2]^2*\text{Tan}[(e + f*x)/2])/3) + 2*\text{Tan}[(e + f*x)/2]^2*((-1 + n)*((-3*(2 - n)*\text{AppellF1}[5/2, m + n, 3 - n, 7/2, \text{Tan}[(e + f*x)/2]^2, -\text{Tan}[(e + f*x)/2]^2]*\text{Sec}[(e + f*x)/2]^2*\text{Tan}[(e + f*x)/2])/5 + (3*(m + n)*\text{AppellF1}[5/2, 1 + m + n, 2 - n, 7/2, \text{Tan}[(e + f*x)/2]^2, -\text{Tan}[(e + f*x)/2]^2]*\text{Sec}[(e + f*x)/2]^2*\text{Tan}[(e + f*x)/2])/5) + (m + n)*((-3*(1 - n)*\text{AppellF1}[5/2, 1 + m + n, 2 - n, 7/2, \text{Tan}[(e + f*x)/2]^2, -\text{Tan}[(e + f*x)/2]^2]*\text{Sec}[(e + f*x)/2]^2*\text{Tan}[(e + f*x)/2])/5 + (3*(1 + m + n)*\text{AppellF1}[5/2, 2 + m + n, 1 - n, 7/2, \text{Tan}[(e + f*x)/2]^2, -\text{Tan}[(e + f*x)/2]^2]*\text{Sec}[(e + f*x)/2]^2*\text{Tan}[(e + f*x)/2])/5)))/((3*\text{AppellF1}[1/2, m + n, 1 - n, 3/2, \text{Tan}[(e + f*x)/2]^2, -\text{Tan}[(e + f*x)/2]^2] + 2*((-1 + n)*\text{AppellF1}[3/2, m + n, 2 - n, 5/2, \text{Tan}[(e + f*x)/2]^2, -\text{Tan}[(e + f*x)/2]^2] + (m + n)*\text{AppellF1}[3/2, 1 + m + n, 1 - n, 5/2, \text{Tan}[(e + f*x)/2]^2, -\text{Tan}[(e + f*x)/2]^2])* \text{Tan}[(e + f*x)/2]^2)^2 + (B*\text{AppellF1}[1/2, 1 + m + n, -n, 3/2, \text{Tan}[(e + f*x)/2]^2, -\text{Tan}[(e + f*x)/2]^2]*((n)*\text{AppellF1}[3/2, 1 + m + n, 1 - n, 5/2, \text{Tan}[(e + f*x)/2]^2, -\text{Tan}[(e + f*x)/2]^2]*\text{Sec}[(e + f*x)/2]^2*\text{Tan}[(e + f*x)/2])/3 + ((1 + m + n)*\text{AppellF1}[3/2, 2 + m + n, -n, 5/2, \text{Tan}[(e + f*x)/2]^2, -\text{Tan}[(e + f*x)/2]^2]*\text{Sec}[(e + f*x)/2]^2*\text{Tan}[(e + f*x)/2])/3 + (2*(n*A
\end{aligned}$$

```

ppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2
] + (1 + m + n)*AppellF1[3/2, 2 + m + n, -n, 5/2, Tan[(e + f*x)/2]^2, -Tan[
(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3 + (2*Tan[(e + f*x)/
2]^2*(n*((-3*(1 - n)*AppellF1[5/2, 1 + m + n, 2 - n, 7/2, Tan[(e + f*x)/2]^
2, -Tan[(e + f*x)/2]^2)*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(1 + m
+ n)*AppellF1[5/2, 2 + m + n, 1 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x
)/2]^2)*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5) + (1 + m + n)*((3*n*AppellF
1[5/2, 2 + m + n, 1 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2)*Sec[
(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(2 + m + n)*AppellF1[5/2, 3 + m + n
, -n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2)*Sec[(e + f*x)/2]^2*Tan[
(e + f*x)/2])/5))/3)/(AppellF1[1/2, 1 + m + n, -n, 3/2, Tan[(e + f*x)/2]^
2, -Tan[(e + f*x)/2]^2] + (2*(n*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e
+ f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (1 + m + n)*AppellF1[3/2, 2 + m + n, -
n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)/3)^2)
)/(-1 + Tan[(e + f*x)/2]^2) + (2^(1 + m)*(m + n)*(Sec[(e + f*x)/2]^2)^n*(Co
s[(e + f*x)/2]^2*Sec[e + f*x])^(-1 + m + n)*Tan[(e + f*x)/2]*((-3*A*AppellF
1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2)*Cos[e +
f*x]))/(3*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x
)/2]^2] + 2*((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2,
-Tan[(e + f*x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e
+ f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (B*AppellF1[1/2, 1
+ m + n, -n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2))/(AppellF1[1/2,
1 + m + n, -n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(n*Appel
lF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] +
(1 + m + n)*AppellF1[3/2, 2 + m + n, -n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e +
f*x)/2]^2])*Tan[(e + f*x)/2]^2)/3)*(-(Cos[(e + f*x)/2]*Sec[e + f*x]*Sin[(
e + f*x)/2]) + Cos[(e + f*x)/2]^2*Sec[e + f*x]*Tan[e + f*x]))/(-1 + Tan[(e
+ f*x)/2]^2)))

```

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(B \sec(fx + e) + A\right)\left(a \sec(fx + e) + a\right)^m \left(c \sec(fx + e)\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```

[In] integrate((c*sec(f*x+e))^n*(a+a*sec(f*x+e))^m*(A+B*sec(f*x+e)),x, algorithm
="fricas")

```

```

[Out] integral((B*sec(f*x + e) + A)*(a*sec(f*x + e) + a)^m*(c*sec(f*x + e))^n, x)

```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(fx + e) + A)(a \sec(fx + e) + a)^m (c \sec(fx + e))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*sec(f*x+e))^n*(a+a*sec(f*x+e))^m*(A+B*sec(f*x+e)),x, algorithm="giac")

[Out] integrate((B*sec(f*x + e) + A)*(a*sec(f*x + e) + a)^m*(c*sec(f*x + e))^n, x)

maple [F] time = 5.06, size = 0, normalized size = 0.00

$$\int (c \sec(fx + e))^n (a + a \sec(fx + e))^m (A + B \sec(fx + e)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*sec(f*x+e))^n*(a+a*sec(f*x+e))^m*(A+B*sec(f*x+e)),x)

[Out] int((c*sec(f*x+e))^n*(a+a*sec(f*x+e))^m*(A+B*sec(f*x+e)),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(fx + e) + A)(a \sec(fx + e) + a)^m (c \sec(fx + e))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*sec(f*x+e))^n*(a+a*sec(f*x+e))^m*(A+B*sec(f*x+e)),x, algorithm="maxima")

[Out] integrate((B*sec(f*x + e) + A)*(a*sec(f*x + e) + a)^m*(c*sec(f*x + e))^n, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A + \frac{B}{\cos(e + fx)} \right) \left(a + \frac{a}{\cos(e + fx)} \right)^m \left(\frac{c}{\cos(e + fx)} \right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(e + f*x))*(a + a/cos(e + f*x))^m*(c/cos(e + f*x))^n,x)

[Out] int((A + B/cos(e + f*x))*(a + a/cos(e + f*x))^m*(c/cos(e + f*x))^n, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sec(e + fx) + 1))^m (c \sec(e + fx))^n (A + B \sec(e + fx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*sec(f*x+e))^n*(a+a*sec(f*x+e))^m*(A+B*sec(f*x+e)),x)

[Out] Integral((a*(sec(e + f*x) + 1))^m*(c*sec(e + f*x))^n*(A + B*sec(e + f*x)), x)

$$3.276 \quad \int \sec^{-1-n}(c+dx)(a+a \sec(c+dx))^n(A+B \sec(c+dx)) dx$$

Optimal. Leaf size=164

$$\frac{(An + Bn + B) \sin(c + dx) \sec^{1-n}(c + dx) \left(\frac{\sec(c+dx)+1}{1-\sec(c+dx)}\right)^{\frac{1}{2}-n} (a \sec(c + dx) + a)^n {}_2F_1\left(\frac{1}{2} - n, -n; 1 - n; -\frac{2 \sec(c+dx)}{1-\sec(c+dx)}\right)}{dn(n + 1)(\sec(c + dx) + 1)}$$

[Out] A*(a+a*sec(d*x+c))^n*sin(d*x+c)/d/(1+n)/(sec(d*x+c)^n)+(A*n+B*n+B)*hypergeom([-n, 1/2-n], [1-n], -2*sec(d*x+c)/(1-sec(d*x+c)))*sec(d*x+c)^(1-n)*((1+sec(d*x+c))/(1-sec(d*x+c)))^(1/2-n)*(a+a*sec(d*x+c))^n*sin(d*x+c)/d/n/(1+n)/(1+sec(d*x+c))

Rubi [A] time = 0.26, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {4013, 3828, 3825, 132}

$$\frac{(An + Bn + B) \sin(c + dx) \sec^{1-n}(c + dx) \left(\frac{\sec(c+dx)+1}{1-\sec(c+dx)}\right)^{\frac{1}{2}-n} (a \sec(c + dx) + a)^n {}_2F_1\left(\frac{1}{2} - n, -n; 1 - n; -\frac{2 \sec(c+dx)}{1-\sec(c+dx)}\right)}{dn(n + 1)(\sec(c + dx) + 1)}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d*x]^(-1 - n)*(a + a*Sec[c + d*x])^n*(A + B*Sec[c + d*x]), x]

[Out] (A*(a + a*Sec[c + d*x])^n*Sin[c + d*x])/(d*(1 + n)*Sec[c + d*x]^n) + ((B + A*n + B*n)*Hypergeometric2F1[1/2 - n, -n, 1 - n, (-2*Sec[c + d*x])/(1 - Sec[c + d*x])]*Sec[c + d*x]^(1 - n)*((1 + Sec[c + d*x])/(1 - Sec[c + d*x]))^(1/2 - n)*(a + a*Sec[c + d*x])^n*Sin[c + d*x])/(d*n*(1 + n)*(1 + Sec[c + d*x]))

Rule 132

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x))))]/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n + p + 2, 0] && !IntegerQ[n]

Rule 3825

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_), x_Symbol] :> -Dist[(((a*d)/b)^n*Cot[e + f*x])/(a^(n - 2)*f*Sqrt[

$a + b \operatorname{Csc}[e + f x] \operatorname{Sqrt}[a - b \operatorname{Csc}[e + f x]]$, $\operatorname{Subst}[\operatorname{Int}[(a - x)^{(n-1)} (2a - x)^{(m-1/2)}] / \operatorname{Sqrt}[x], x], x, a - b \operatorname{Csc}[e + f x], x] /;$ $\operatorname{FreeQ}[\{a, b, d, e, f, m, n\}, x] \ \&\& \ \operatorname{EqQ}[a^2 - b^2, 0] \ \&\& \ !\operatorname{IntegerQ}[m] \ \&\& \ \operatorname{GtQ}[a, 0] \ \&\& \ !\operatorname{IntegerQ}[n] \ \&\& \ \operatorname{GtQ}[(a*d)/b, 0]$

Rule 3828

$\operatorname{Int}[(\operatorname{csc}[e] + (f)(x))(d)^{(n)}(\operatorname{csc}[e] + (f)(x))(b) + (a))^{(m)}, x_{\text{Symbol}}] \rightarrow \operatorname{Dist}[(a^{\operatorname{IntPart}[m]}(a + b \operatorname{Csc}[e + f x])^{\operatorname{FracPart}[m]}) / (1 + (b \operatorname{Csc}[e + f x])/a)^{\operatorname{FracPart}[m]}, \operatorname{Int}[(1 + (b \operatorname{Csc}[e + f x])/a)^m (d \operatorname{Csc}[e + f x])^n, x], x] /;$ $\operatorname{FreeQ}[\{a, b, d, e, f, m, n\}, x] \ \&\& \ \operatorname{EqQ}[a^2 - b^2, 0] \ \&\& \ !\operatorname{IntegerQ}[m] \ \&\& \ !\operatorname{GtQ}[a, 0]$

Rule 4013

$\operatorname{Int}[(\operatorname{csc}[e] + (f)(x))(d)^{(n)}(\operatorname{csc}[e] + (f)(x))(b) + (a))^{(m)}(\operatorname{csc}[e] + (f)(x))(B) + (A)), x_{\text{Symbol}}] \rightarrow \operatorname{Simp}[(A \operatorname{Cot}[e + f x](a + b \operatorname{Csc}[e + f x])^m (d \operatorname{Csc}[e + f x])^n) / (f n), x] - \operatorname{Dist}[(a A m - b B n) / (b d n), \operatorname{Int}[(a + b \operatorname{Csc}[e + f x])^m (d \operatorname{Csc}[e + f x])^{(n+1)}, x], x] /;$ $\operatorname{FreeQ}[\{a, b, d, e, f, A, B, m, n\}, x] \ \&\& \ \operatorname{NeQ}[A b - a B, 0] \ \&\& \ \operatorname{EqQ}[a^2 - b^2, 0] \ \&\& \ \operatorname{EqQ}[m + n + 1, 0] \ \&\& \ !\operatorname{LeQ}[m, -1]$

Rubi steps

$$\begin{aligned} \int \sec^{-1-n}(c + dx)(a + a \sec(c + dx))^n (A + B \sec(c + dx)) dx &= \frac{A \sec^{-n}(c + dx)(a + a \sec(c + dx))^n \sin(c + dx)}{d(1 + n)} \\ &= \frac{A \sec^{-n}(c + dx)(a + a \sec(c + dx))^n \sin(c + dx)}{d(1 + n)} \\ &= \frac{A \sec^{-n}(c + dx)(a + a \sec(c + dx))^n \sin(c + dx)}{d(1 + n)} \\ &= \frac{A \sec^{-n}(c + dx)(a + a \sec(c + dx))^n \sin(c + dx)}{d(1 + n)} \end{aligned}$$

Mathematica [A] time = 1.13, size = 111, normalized size = 0.68

$$\frac{\sin(c + dx) \sec^{-n}(c + dx)(a(\sec(c + dx) + 1))^n \left(\frac{(An+Bn+B) \left(-\cot^2\left(\frac{1}{2}(c+dx)\right) \right)^{\frac{1}{2}-n} {}_2F_1\left(\frac{1}{2}-n, -n; 1-n; \csc^2\left(\frac{1}{2}(c+dx)\right)\right)}{n(\cos(c+dx)+1)} + A \right)}{d(n+1)}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[c + d*x]^(-1 - n)*(a + a*Sec[c + d*x])^n*(A + B*Sec[c + d*x]), x]

[Out] ((A + ((B + A*n + B*n)*(-Cot[(c + d*x)/2]^2)^(1/2 - n)*Hypergeometric2F1[1/2 - n, -n, 1 - n, Csc[(c + d*x)/2]^2])/(n*(1 + Cos[c + d*x])))*(a*(1 + Sec[c + d*x]))^n*Sin[c + d*x]/(d*(1 + n)*Sec[c + d*x]^n)

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}((B \sec(dx + c) + A)(a \sec(dx + c) + a)^n \sec(dx + c)^{-n-1}, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(-1-n)*(a+a*sec(d*x+c))^n*(A+B*sec(d*x+c)), x, algorithm="fricas")

[Out] integral((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^n*sec(d*x + c)^(-n - 1), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(a \sec(dx + c) + a)^n \sec(dx + c)^{-n-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(-1-n)*(a+a*sec(d*x+c))^n*(A+B*sec(d*x+c)), x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^n*sec(d*x + c)^(-n - 1), x)

maple [F] time = 4.56, size = 0, normalized size = 0.00

$$\int (\sec^{-1-n}(dx + c))(a + a \sec(dx + c))^n (A + B \sec(dx + c)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sec(d*x+c)^(-1-n)*(a+a*sec(d*x+c))^n*(A+B*sec(d*x+c)),x)`

[Out] `int(sec(d*x+c)^(-1-n)*(a+a*sec(d*x+c))^n*(A+B*sec(d*x+c)),x)`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(a \sec(dx + c) + a)^n \sec(dx + c)^{-n-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^(-1-n)*(a+a*sec(d*x+c))^n*(A+B*sec(d*x+c)),x, algorithm m="maxima")`

[Out] `integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^n*sec(d*x + c)^(-n - 1), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(a + \frac{a}{\cos(c+dx)}\right)^n}{\left(\frac{1}{\cos(c+dx)}\right)^{n+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^n)/(1/cos(c + d*x))^(n + 1), x)`

[Out] `int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^n)/(1/cos(c + d*x))^(n + 1), x)`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)**(-1-n)*(a+a*sec(d*x+c))**n*(A+B*sec(d*x+c)),x)`

[Out] Timed out

$$3.277 \quad \int \sec^3(c + dx)(a + b \sec(c + dx))(A + B \sec(c + dx)) dx$$

Optimal. Leaf size=114

$$\frac{(aB + Ab) \tan^3(c + dx)}{3d} + \frac{(aB + Ab) \tan(c + dx)}{d} + \frac{(4aA + 3bB) \tanh^{-1}(\sin(c + dx))}{8d} + \frac{(4aA + 3bB) \tan(c + dx) \sec(c + dx)}{8d}$$

[Out] 1/8*(4*A*a+3*B*b)*arctanh(sin(d*x+c))/d+(A*b+B*a)*tan(d*x+c)/d+1/8*(4*A*a+3*B*b)*sec(d*x+c)*tan(d*x+c)/d+1/4*b*B*sec(d*x+c)^3*tan(d*x+c)/d+1/3*(A*b+B*a)*tan(d*x+c)^3/d

Rubi [A] time = 0.15, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {3997, 3787, 3768, 3770, 3767}

$$\frac{(aB + Ab) \tan^3(c + dx)}{3d} + \frac{(aB + Ab) \tan(c + dx)}{d} + \frac{(4aA + 3bB) \tanh^{-1}(\sin(c + dx))}{8d} + \frac{(4aA + 3bB) \tan(c + dx) \sec(c + dx)}{8d}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d*x]^3*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]), x]

[Out] ((4*a*A + 3*b*B)*ArcTanh[Sin[c + d*x]]/(8*d) + ((A*b + a*B)*Tan[c + d*x])/d + ((4*a*A + 3*b*B)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (b*B*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + ((A*b + a*B)*Tan[c + d*x]^3)/(3*d)

Rule 3767

Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 3768

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x])*(b*Csc[c + d*x])^(n - 1)/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 3997

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> -Simp[(b*B*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && !LeQ[n, -1]

Rubi steps

$$\begin{aligned} \int \sec^3(c + dx)(a + b \sec(c + dx))(A + B \sec(c + dx)) dx &= \frac{bB \sec^3(c + dx) \tan(c + dx)}{4d} + \frac{1}{4} \int \sec^3(c + dx)(4aA + 3bB) dx \\ &= \frac{bB \sec^3(c + dx) \tan(c + dx)}{4d} + (Ab + aB) \int \sec^4(c + dx) dx \\ &= \frac{(4aA + 3bB) \sec(c + dx) \tan(c + dx)}{8d} + \frac{bB \sec^3(c + dx)}{4d} \\ &= \frac{(4aA + 3bB) \tanh^{-1}(\sin(c + dx))}{8d} + \frac{(Ab + aB) \tan(c + dx)}{d} \end{aligned}$$

Mathematica [A] time = 0.63, size = 85, normalized size = 0.75

$$\frac{3(4aA + 3bB) \tanh^{-1}(\sin(c + dx)) + \tan(c + dx) \sec(c + dx) (8(aB + Ab)(\cos(2(c + dx)) + 2) \sec(c + dx) + 12aA)}{24d}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[c + d*x]^3*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]

[Out] (3*(4*a*A + 3*b*B)*ArcTanh[Sin[c + d*x]] + Sec[c + d*x]*(12*a*A + 9*b*B + 8*(A*b + a*B)*(2 + Cos[2*(c + d*x)]))*Sec[c + d*x] + 6*b*B*Sec[c + d*x]^2)*Tan[c + d*x]/(24*d)

fricas [A] time = 0.97, size = 136, normalized size = 1.19

$$\frac{3(4Aa + 3Bb) \cos(dx + c)^4 \log(\sin(dx + c) + 1) - 3(4Aa + 3Bb) \cos(dx + c)^4 \log(-\sin(dx + c) + 1) + 2(12aA + 9bB + 8(Ab + aB)(2 + \cos(2(dx + c))) \sec(dx + c) + 6bB \sec(dx + c)^2) \tan(dx + c)}{48d \cos(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] $\frac{1}{48}*(3*(4*A*a + 3*B*b)*\cos(d*x + c)^4*\log(\sin(d*x + c) + 1) - 3*(4*A*a + 3*B*b)*\cos(d*x + c)^4*\log(-\sin(d*x + c) + 1) + 2*(16*(B*a + A*b)*\cos(d*x + c)^3 + 3*(4*A*a + 3*B*b)*\cos(d*x + c)^2 + 6*B*b + 8*(B*a + A*b)*\cos(d*x + c))*\sin(d*x + c))/(d*\cos(d*x + c)^4)$

giac [B] time = 0.30, size = 304, normalized size = 2.67

$$3(4Aa + 3Bb) \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right|\right) - 3(4Aa + 3Bb) \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1\right|\right) + \frac{2\left(12Aa \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right)^7 - 24A^2B \tan^7\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 24A^2B^2 \tan^7\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 15B^3 \tan^7\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 12A^3 \tan^5\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 40A^2B \tan^5\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 40A^2B^2 \tan^5\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 9B^3 \tan^5\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 12A^3 \tan^3\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 40A^2B \tan^3\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 40A^2B^2 \tan^3\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 9B^3 \tan^3\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 12A^3 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 24A^2B \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 24A^2B^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 15B^3 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right)^2 - 1}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] $\frac{1}{24}*(3*(4*A*a + 3*B*b)*\log(\text{abs}(\tan(1/2*d*x + 1/2*c) + 1)) - 3*(4*A*a + 3*B*b)*\log(\text{abs}(\tan(1/2*d*x + 1/2*c) - 1)) + 2*(12*A*a*\tan(1/2*d*x + 1/2*c)^7 - 24*B*a*\tan(1/2*d*x + 1/2*c)^7 - 24*A*b*\tan(1/2*d*x + 1/2*c)^7 + 15*B*b*\tan(1/2*d*x + 1/2*c)^7 - 12*A*a*\tan(1/2*d*x + 1/2*c)^5 + 40*B*a*\tan(1/2*d*x + 1/2*c)^5 + 40*A*b*\tan(1/2*d*x + 1/2*c)^5 + 9*B*b*\tan(1/2*d*x + 1/2*c)^5 - 12*A*a*\tan(1/2*d*x + 1/2*c)^3 - 40*B*a*\tan(1/2*d*x + 1/2*c)^3 - 40*A*b*\tan(1/2*d*x + 1/2*c)^3 + 9*B*b*\tan(1/2*d*x + 1/2*c)^3 + 12*A*a*\tan(1/2*d*x + 1/2*c) + 24*B*a*\tan(1/2*d*x + 1/2*c) + 24*A*b*\tan(1/2*d*x + 1/2*c) + 15*B*b*\tan(1/2*d*x + 1/2*c))/(\tan(1/2*d*x + 1/2*c)^2 - 1)^4/d$

maple [A] time = 1.25, size = 171, normalized size = 1.50

$$\frac{aA \sec(dx + c) \tan(dx + c)}{2d} + \frac{aA \ln(\sec(dx + c) + \tan(dx + c))}{2d} + \frac{2aB \tan(dx + c)}{3d} + \frac{aB \tan(dx + c) (\sec^2(dx + c) + \tan^2(dx + c))}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^3*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x)

[Out] $\frac{1}{2}*a*A*\sec(d*x+c)*\tan(d*x+c)/d + \frac{1}{2}*d*a*A*\ln(\sec(d*x+c) + \tan(d*x+c)) + \frac{2}{3}*d*a*B*\tan(d*x+c) + \frac{1}{3}*d*a*B*\tan(d*x+c)*\sec(d*x+c)^2 + \frac{2}{3}*A*b*\tan(d*x+c)/d + \frac{1}{3}*A*b*\sec(d*x+c)^2*\tan(d*x+c)/d + \frac{1}{4}*b*B*\sec(d*x+c)^3*\tan(d*x+c)/d + \frac{3}{8}*b*B*\sec(d*x+c)*\tan(d*x+c)/d + \frac{3}{8}*d*B*b*\ln(\sec(d*x+c) + \tan(d*x+c))$

maxima [A] time = 0.88, size = 163, normalized size = 1.43

$$16 \left(\tan(dx+c)^3 + 3 \tan(dx+c) \right) Ba + 16 \left(\tan(dx+c)^3 + 3 \tan(dx+c) \right) Ab - 3 Bb \left(\frac{2(3 \sin(dx+c)^3 - 5 \sin(dx+c))}{\sin(dx+c)^4 - 2 \sin(dx+c)^2 + 1} - 3 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] 1/48*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a + 16*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*b - 3*B*b*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 12*A*a*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)))/d

mupad [B] time = 5.73, size = 194, normalized size = 1.70

$$\frac{\left(Aa - 2Ab - 2Ba + \frac{5Bb}{4} \right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^7 + \left(\frac{10Ab}{3} - Aa + \frac{10Ba}{3} + \frac{3Bb}{4} \right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5 + \left(\frac{3Bb}{4} - \frac{10Ab}{3} - \frac{10Ba}{3} \right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 + \left(\frac{10Ab}{3} - Aa + \frac{10Ba}{3} + \frac{3Bb}{4} \right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right) + \left(Aa - 2Ab - 2Ba + \frac{5Bb}{4} \right)}{d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^8 - 4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^6 + 6 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 - 4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + 1 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x)))/cos(c + d*x)^3,x)

[Out] (tan(c/2 + (d*x)/2)*(A*a + 2*A*b + 2*B*a + (5*B*b)/4) + tan(c/2 + (d*x)/2)^7*(A*a - 2*A*b - 2*B*a + (5*B*b)/4) - tan(c/2 + (d*x)/2)^3*(A*a + (10*A*b)/3 + (10*B*a)/3 - (3*B*b)/4) + tan(c/2 + (d*x)/2)^5*((10*A*b)/3 - A*a + (10*B*a)/3 + (3*B*b)/4))/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1)) + (atanh(tan(c/2 + (d*x)/2))*(A*a + (3*B*b)/4))/d

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \sec(c + dx)) (a + b \sec(c + dx)) \sec^3(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**3*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x)

[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))*sec(c + d*x)**3, x)

$$3.278 \quad \int \sec^2(c + dx)(a + b \sec(c + dx))(A + B \sec(c + dx)) dx$$

Optimal. Leaf size=93

$$\frac{(3aA + 2bB) \tan(c + dx)}{3d} + \frac{(aB + Ab) \tanh^{-1}(\sin(c + dx))}{2d} + \frac{(aB + Ab) \tan(c + dx) \sec(c + dx)}{2d} + \frac{bB \tan(c + dx) \sec(c + dx)}{3d}$$

[Out] $1/2*(A*b+B*a)*\operatorname{arctanh}(\sin(d*x+c))/d+1/3*(3*A*a+2*B*b)*\tan(d*x+c)/d+1/2*(A*b+B*a)*\sec(d*x+c)*\tan(d*x+c)/d+1/3*b*B*\sec(d*x+c)^2*\tan(d*x+c)/d$

Rubi [A] time = 0.13, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {3997, 3787, 3767, 8, 3768, 3770}

$$\frac{(3aA + 2bB) \tan(c + dx)}{3d} + \frac{(aB + Ab) \tanh^{-1}(\sin(c + dx))}{2d} + \frac{(aB + Ab) \tan(c + dx) \sec(c + dx)}{2d} + \frac{bB \tan(c + dx) \sec(c + dx)}{3d}$$

Antiderivative was successfully verified.

[In] `Int[Sec[c + d*x]^2*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]`

[Out] $((A*b + a*B)*\operatorname{ArcTanh}[\operatorname{Sin}[c + d*x]])/(2*d) + ((3*a*A + 2*b*B)*\operatorname{Tan}[c + d*x])/(3*d) + ((A*b + a*B)*\operatorname{Sec}[c + d*x]*\operatorname{Tan}[c + d*x])/(2*d) + (b*B*\operatorname{Sec}[c + d*x]^2*\operatorname{Tan}[c + d*x])/(3*d)$

Rule 8

`Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

Rule 3767

`Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]`

Rule 3768

`Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := -Simp[(b*Csc[c + d*x]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]`

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x]
  /; FreeQ[{c, d}, x]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)])*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)])*(b_.) +
(a_), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)])*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)])*(b_.) +
(a_)*(csc[(e_.) + (f_.)*(x_)])*(B_.) + (A_), x_Symbol] := -Simp[(b*B*Cot[e
+ f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && !LeQ[n,
-1]
```

Rubi steps

$$\begin{aligned} \int \sec^2(c + dx)(a + b \sec(c + dx))(A + B \sec(c + dx)) dx &= \frac{bB \sec^2(c + dx) \tan(c + dx)}{3d} + \frac{1}{3} \int \sec^2(c + dx)(3a + b \sec(c + dx)) dx \\ &= \frac{bB \sec^2(c + dx) \tan(c + dx)}{3d} + (Ab + aB) \int \sec^3(c + dx) dx \\ &= \frac{(Ab + aB) \sec(c + dx) \tan(c + dx)}{2d} + \frac{bB \sec^2(c + dx) \tan(c + dx)}{3d} \\ &= \frac{(Ab + aB) \tanh^{-1}(\sin(c + dx))}{2d} + \frac{(3aA + 2bB) \tan(c + dx)}{3d} \end{aligned}$$

Mathematica [A] time = 0.29, size = 67, normalized size = 0.72

$$\frac{3(aB + Ab) \tanh^{-1}(\sin(c + dx)) + \tan(c + dx) (3(aB + Ab) \sec(c + dx) + 6aA + 2bB \tan^2(c + dx) + 6bB)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]), x]
```

```
[Out] (3*(A*b + a*B)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(6*a*A + 6*b*B + 3*(A*b
+ a*B)*Sec[c + d*x] + 2*b*B*Tan[c + d*x]^2))/(6*d)
```

fricas [A] time = 0.45, size = 115, normalized size = 1.24

$$\frac{3(Ba + Ab) \cos(dx + c)^3 \log(\sin(dx + c) + 1) - 3(Ba + Ab) \cos(dx + c)^3 \log(-\sin(dx + c) + 1) + 2(2(3Aa + 2Bb) \cos(dx + c)^2 + 2B^2b + 3(Ba + Ab) \cos(dx + c) \sin(dx + c)) / (d \cos(dx + c)^3)}{12d \cos(dx + c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] 1/12*(3*(B*a + A*b)*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*(B*a + A*b)*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + 2*(2*(3*A*a + 2*B*b)*cos(d*x + c)^2 + 2*B*b + 3*(B*a + A*b)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)

giac [B] time = 0.27, size = 210, normalized size = 2.26

$$3(Ba + Ab) \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right|\right) - 3(Ba + Ab) \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1\right|\right) - \frac{2\left(6Aa \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5 - 3Ba \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] 1/6*(3*(B*a + A*b)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(B*a + A*b)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(6*A*a*tan(1/2*d*x + 1/2*c)^5 - 3*B*a*tan(1/2*d*x + 1/2*c)^5 - 3*A*b*tan(1/2*d*x + 1/2*c)^5 + 6*B*b*tan(1/2*d*x + 1/2*c)^5 - 12*A*a*tan(1/2*d*x + 1/2*c)^3 - 4*B*b*tan(1/2*d*x + 1/2*c)^3 + 6*A*a*tan(1/2*d*x + 1/2*c) + 3*B*a*tan(1/2*d*x + 1/2*c) + 3*A*b*tan(1/2*d*x + 1/2*c) + 6*B*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3/d

maple [A] time = 1.22, size = 128, normalized size = 1.38

$$\frac{aA \tan(dx + c)}{d} + \frac{aB \sec(dx + c) \tan(dx + c)}{2d} + \frac{aB \ln(\sec(dx + c) + \tan(dx + c))}{2d} + \frac{Ab \sec(dx + c) \tan(dx + c)}{2d} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^2*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x)

[Out] a*A*tan(d*x+c)/d+1/2/d*a*B*sec(d*x+c)*tan(d*x+c)+1/2/d*a*B*ln(sec(d*x+c)+tan(d*x+c))+1/2*A*b*sec(d*x+c)*tan(d*x+c)/d+1/2/d*A*b*ln(sec(d*x+c)+tan(d*x+c))+2/3*b*B*tan(d*x+c)/d+1/3*b*B*sec(d*x+c)^2*tan(d*x+c)/d

maxima [A] time = 0.66, size = 127, normalized size = 1.37

$$\frac{4 \left(\tan(dx+c)^3 + 3 \tan(dx+c) \right) Bb - 3 Ba \left(\frac{2 \sin(dx+c)}{\sin(dx+c)^2-1} - \log(\sin(dx+c)+1) + \log(\sin(dx+c)-1) \right) - 3 Ab}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*b - 3*B*a*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 3*A*b*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 12*A*a*tan(d*x + c))/d

mupad [B] time = 4.39, size = 145, normalized size = 1.56

$$\frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right) (Ab + Ba) (2Aa - Ab - Ba + 2Bb) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5 + \left(-4Aa - \frac{4Bb}{3}\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3}{d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^6 - 3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 + 3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 - 1 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x)))/cos(c + d*x)^2,x)

[Out] (atanh(tan(c/2 + (d*x)/2))*(A*b + B*a))/d - (tan(c/2 + (d*x)/2)*(2*A*a + A*b + B*a + 2*B*b) - tan(c/2 + (d*x)/2)^3*(4*A*a + (4*B*b)/3) + tan(c/2 + (d*x)/2)^5*(2*A*a - A*b - B*a + 2*B*b))/(d*(3*tan(c/2 + (d*x)/2)^2 - 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \sec(c + dx))(a + b \sec(c + dx)) \sec^2(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**2*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x)

[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))*sec(c + d*x)**2, x)

$$3.279 \quad \int \sec(c + dx)(a + b \sec(c + dx))(A + B \sec(c + dx)) dx$$

Optimal. Leaf size=61

$$\frac{(aB + Ab) \tan(c + dx)}{d} + \frac{(2aA + bB) \tanh^{-1}(\sin(c + dx))}{2d} + \frac{bB \tan(c + dx) \sec(c + dx)}{2d}$$

[Out] $1/2*(2*A*a+B*b)*\arctanh(\sin(d*x+c))/d+(A*b+B*a)*\tan(d*x+c)/d+1/2*b*B*\sec(d*x+c)*\tan(d*x+c)/d$

Rubi [A] time = 0.08, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {3997, 3787, 3770, 3767, 8}

$$\frac{(aB + Ab) \tan(c + dx)}{d} + \frac{(2aA + bB) \tanh^{-1}(\sin(c + dx))}{2d} + \frac{bB \tan(c + dx) \sec(c + dx)}{2d}$$

Antiderivative was successfully verified.

[In] `Int[Sec[c + d*x]*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]`

[Out] $((2*a*A + b*B)*\text{ArcTanh}[\text{Sin}[c + d*x]])/(2*d) + ((A*b + a*B)*\text{Tan}[c + d*x])/d + (b*B*\text{Sec}[c + d*x]*\text{Tan}[c + d*x])/(2*d)$

Rule 8

`Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

Rule 3767

`Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]`

Rule 3770

`Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]`

Rule 3787

`Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[`

$(d \cdot \text{Csc}[e + f \cdot x])^{(n + 1)}, x], x] /; \text{FreeQ}[\{a, b, d, e, f, n\}, x]$

Rule 3997

$\text{Int}[(\text{csc}[(e_.) + (f_.) \cdot (x_)] \cdot (d_.)^{(n_.)} \cdot (\text{csc}[(e_.) + (f_.) \cdot (x_)] \cdot (b_.) + (a_.) \cdot (\text{csc}[(e_.) + (f_.) \cdot (x_)] \cdot (B_.) + (A_.)], x_Symbol] :> -\text{Simp}[(b \cdot B \cdot \text{Cot}[e + f \cdot x] \cdot (d \cdot \text{Csc}[e + f \cdot x])^n) / (f \cdot (n + 1)), x] + \text{Dist}[1 / (n + 1), \text{Int}[(d \cdot \text{Csc}[e + f \cdot x])^n \cdot \text{Simp}[A \cdot a \cdot (n + 1) + B \cdot b \cdot n + (A \cdot b + B \cdot a) \cdot (n + 1) \cdot \text{Csc}[e + f \cdot x], x], x] /; \text{FreeQ}[\{a, b, d, e, f, A, B\}, x] \&\& \text{NeQ}[A \cdot b - a \cdot B, 0] \&\& !\text{LeQ}[n, -1]$

Rubi steps

$$\begin{aligned} \int \sec(c + dx)(a + b \sec(c + dx))(A + B \sec(c + dx)) dx &= \frac{bB \sec(c + dx) \tan(c + dx)}{2d} + \frac{1}{2} \int \sec(c + dx)(2aA + \\ &= \frac{bB \sec(c + dx) \tan(c + dx)}{2d} + (Ab + aB) \int \sec^2(c + dx) dx \\ &= \frac{(2aA + bB) \tanh^{-1}(\sin(c + dx))}{2d} + \frac{bB \sec(c + dx) \tan(c + dx)}{2d} \\ &= \frac{(2aA + bB) \tanh^{-1}(\sin(c + dx))}{2d} + \frac{(Ab + aB) \tan(c + dx)}{d} \end{aligned}$$

Mathematica [A] time = 0.03, size = 75, normalized size = 1.23

$$\frac{aA \tanh^{-1}(\sin(c + dx))}{d} + \frac{aB \tan(c + dx)}{d} + \frac{Ab \tan(c + dx)}{d} + \frac{bB \tanh^{-1}(\sin(c + dx))}{2d} + \frac{bB \tan(c + dx) \sec(c + dx)}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[c + d*x]*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]

[Out] (a*A*ArcTanh[Sin[c + d*x]])/d + (b*B*ArcTanh[Sin[c + d*x]])/(2*d) + (A*b*Tan[c + d*x])/d + (a*B*Tan[c + d*x])/d + (b*B*Sec[c + d*x]*Tan[c + d*x])/(2*d)

fricas [A] time = 0.44, size = 96, normalized size = 1.57

$$\frac{(2Aa + Bb) \cos(dx + c)^2 \log(\sin(dx + c) + 1) - (2Aa + Bb) \cos(dx + c)^2 \log(-\sin(dx + c) + 1) + 2(Bb + 2Aa) \sin(dx + c)}{4d \cos(dx + c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] $\frac{1}{4} * ((2 * A * a + B * b) * \cos(d * x + c) ^ 2 * \log(\sin(d * x + c) + 1) - (2 * A * a + B * b) * \cos(d * x + c) ^ 2 * \log(-\sin(d * x + c) + 1) + 2 * (B * b + 2 * (B * a + A * b) * \cos(d * x + c)) * \sin(d * x + c)) / (d * \cos(d * x + c) ^ 2)$

giac [B] time = 0.24, size = 153, normalized size = 2.51

$$(2 A a + B b) \log \left(\left| \tan \left(\frac{1}{2} d x + \frac{1}{2} c \right) + 1 \right| \right) - (2 A a + B b) \log \left(\left| \tan \left(\frac{1}{2} d x + \frac{1}{2} c \right) - 1 \right| \right) - \frac{2 \left(2 B a \tan \left(\frac{1}{2} d x + \frac{1}{2} c \right)^3 + 2 A b \tan \left(\frac{1}{2} d x + \frac{1}{2} c \right) \right)}{2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] $\frac{1}{2} * ((2 * A * a + B * b) * \log(\operatorname{abs}(\tan(1/2 * d * x + 1/2 * c) + 1)) - (2 * A * a + B * b) * \log(\operatorname{abs}(\tan(1/2 * d * x + 1/2 * c) - 1)) - 2 * (2 * B * a * \tan(1/2 * d * x + 1/2 * c) ^ 3 + 2 * A * b * \tan(1/2 * d * x + 1/2 * c) ^ 3 - B * b * \tan(1/2 * d * x + 1/2 * c) ^ 3 - 2 * B * a * \tan(1/2 * d * x + 1/2 * c) - 2 * A * b * \tan(1/2 * d * x + 1/2 * c) - B * b * \tan(1/2 * d * x + 1/2 * c)) / (\tan(1/2 * d * x + 1/2 * c) ^ 2 - 1) ^ 2) / d$

maple [A] time = 0.96, size = 86, normalized size = 1.41

$$\frac{a A \ln(\sec(dx+c) + \tan(dx+c))}{d} + \frac{a B \tan(dx+c)}{d} + \frac{A b \tan(dx+c)}{d} + \frac{b B \sec(dx+c) \tan(dx+c)}{2 d} + \frac{B b \ln(\sec(dx+c) + \tan(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x)

[Out] $\frac{1}{d} * a * A * \ln(\sec(d * x + c) + \tan(d * x + c)) + \frac{1}{d} * a * B * \tan(d * x + c) + A * b * \tan(d * x + c) / d + \frac{1}{2} * b * B * \sec(d * x + c) * \tan(d * x + c) / d + \frac{1}{2} * d * B * b * \ln(\sec(d * x + c) + \tan(d * x + c))$

maxima [A] time = 0.61, size = 88, normalized size = 1.44

$$\frac{B b \left(\frac{2 \sin(dx+c)}{\sin(dx+c)^2-1} - \log(\sin(dx+c) + 1) + \log(\sin(dx+c) - 1) \right) - 4 A a \log(\sec(dx+c) + \tan(dx+c)) - 4 B a \tan(dx+c)}{4 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] $-1/4*(B*b*(2*\sin(d*x + c)/(\sin(d*x + c)^2 - 1) - \log(\sin(d*x + c) + 1) + \log(\sin(d*x + c) - 1)) - 4*A*a*\log(\sec(d*x + c) + \tan(d*x + c)) - 4*B*a*\tan(d*x + c) - 4*A*b*\tan(d*x + c))/d$

mupad [B] time = 3.14, size = 104, normalized size = 1.70

$$\frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right) (2Ab + 2Ba + Bb) - \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 (2Ab + 2Ba - Bb) \operatorname{atanh}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right) (2Aa + Bb)}{d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 - 2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + 1 \right)} + \frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right) (2Aa + Bb)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x)))/cos(c + d*x), x)`

[Out] $(\tan(c/2 + (d*x)/2)*(2*A*b + 2*B*a + B*b) - \tan(c/2 + (d*x)/2)^3*(2*A*b + 2*B*a - B*b))/(d*(\tan(c/2 + (d*x)/2)^4 - 2*\tan(c/2 + (d*x)/2)^2 + 1)) + (\operatorname{atanh}(\tan(c/2 + (d*x)/2))*(2*A*a + B*b))/d$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \sec(c + dx))(a + b \sec(c + dx)) \sec(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)), x)`

[Out] `Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))*sec(c + d*x), x)`

3.280 $\int (a + b \sec(c + dx))(A + B \sec(c + dx)) dx$

Optimal. Leaf size=35

$$\frac{(aB + Ab) \tanh^{-1}(\sin(c + dx))}{d} + aAx + \frac{bB \tan(c + dx)}{d}$$

[Out] a*A*x+(A*b+B*a)*arctanh(sin(d*x+c))/d+b*B*tan(d*x+c)/d

Rubi [A] time = 0.03, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3914, 3767, 8, 3770}

$$\frac{(aB + Ab) \tanh^{-1}(\sin(c + dx))}{d} + aAx + \frac{bB \tan(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]

[Out] a*A*x + ((A*b + a*B)*ArcTanh[Sin[c + d*x]])/d + (b*B*Tan[c + d*x])/d

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 3767

Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3914

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.)), x_Symbol] := Simp[a*c*x, x] + (Dist[b*d, Int[Csc[e + f*x]^2, x], x] + Dist[b*c + a*d, Int[Csc[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]

Rubi steps

$$\begin{aligned}
\int (a + b \sec(c + dx))(A + B \sec(c + dx)) dx &= aAx + (bB) \int \sec^2(c + dx) dx + (Ab + aB) \int \sec(c + dx) dx \\
&= aAx + \frac{(Ab + aB) \tanh^{-1}(\sin(c + dx))}{d} - \frac{(bB) \operatorname{Subst}(\int 1 dx, x, -t)}{d} \\
&= aAx + \frac{(Ab + aB) \tanh^{-1}(\sin(c + dx))}{d} + \frac{bB \tan(c + dx)}{d}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 43, normalized size = 1.23

$$aAx + \frac{aB \tanh^{-1}(\sin(c + dx))}{d} + \frac{Ab \tanh^{-1}(\sin(c + dx))}{d} + \frac{bB \tan(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]

[Out] a*A*x + (A*b*ArcTanh[Sin[c + d*x]])/d + (a*B*ArcTanh[Sin[c + d*x]])/d + (b*B*Tan[c + d*x])/d

fricas [B] time = 0.48, size = 85, normalized size = 2.43

$$\frac{2 A a d x \cos(dx + c) + (B a + A b) \cos(dx + c) \log(\sin(dx + c) + 1) - (B a + A b) \cos(dx + c) \log(-\sin(dx + c) + 1)}{2 d \cos(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] 1/2*(2*A*a*d*x*cos(d*x + c) + (B*a + A*b)*cos(d*x + c)*log(sin(d*x + c) + 1) - (B*a + A*b)*cos(d*x + c)*log(-sin(d*x + c) + 1) + 2*B*b*sin(d*x + c))/(d*cos(d*x + c))

giac [B] time = 0.26, size = 84, normalized size = 2.40

$$\frac{(dx + c)Aa + (Ba + Ab) \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 1\right|\right) - (Ba + Ab) \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 1\right|\right) - \frac{2 B b \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] $((d*x + c)*A*a + (B*a + A*b)*\log(\text{abs}(\tan(1/2*d*x + 1/2*c) + 1)) - (B*a + A*b)*\log(\text{abs}(\tan(1/2*d*x + 1/2*c) - 1)) - 2*B*b*\tan(1/2*d*x + 1/2*c)/(\tan(1/2*d*x + 1/2*c)^2 - 1))/d$

maple [A] time = 0.73, size = 65, normalized size = 1.86

$$aAx + \frac{Ab \ln(\sec(dx + c) + \tan(dx + c))}{d} + \frac{Aac}{d} + \frac{aB \ln(\sec(dx + c) + \tan(dx + c))}{d} + \frac{bB \tan(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x)`

[Out] $a*A*x + 1/d*A*b*\ln(\sec(d*x+c)+\tan(d*x+c)) + 1/d*A*a*c + 1/d*a*B*\ln(\sec(d*x+c)+\tan(d*x+c)) + b*B*\tan(d*x+c)/d$

maxima [A] time = 0.84, size = 56, normalized size = 1.60

$$\frac{(dx + c)Aa + Ba \log(\sec(dx + c) + \tan(dx + c)) + Ab \log(\sec(dx + c) + \tan(dx + c)) + Bb \tan(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="maxima")`

[Out] $((d*x + c)*A*a + B*a*\log(\sec(d*x + c) + \tan(d*x + c)) + A*b*\log(\sec(d*x + c) + \tan(d*x + c)) + B*b*\tan(d*x + c))/d$

mupad [B] time = 2.24, size = 114, normalized size = 3.26

$$\frac{2Aa \operatorname{atan}\left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)}\right)}{d} + \frac{Bb \sin(c + dx)}{d \cos(c + dx)} - \frac{Ab \operatorname{atan}\left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right) 1i}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)}\right) 2i}{d} - \frac{Ba \operatorname{atan}\left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right) 1i}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)}\right) 2i}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B/cos(c + d*x))*(a + b/cos(c + d*x)),x)`

[Out] $(2*A*a*\operatorname{atan}(\sin(c/2 + (d*x)/2)/\cos(c/2 + (d*x)/2)))/d - (A*b*\operatorname{atan}((\sin(c/2 + (d*x)/2)*1i)/\cos(c/2 + (d*x)/2))*2i)/d - (B*a*\operatorname{atan}((\sin(c/2 + (d*x)/2)*1i)/\cos(c/2 + (d*x)/2))*2i)/d + (B*b*\sin(c + d*x))/(d*\cos(c + d*x))$

sympy [A] time = 7.96, size = 71, normalized size = 2.03

$$\begin{cases} \frac{Aa(c+dx) + Ab \log(\tan(c+dx) + \sec(c+dx)) + Ba \log(\tan(c+dx) + \sec(c+dx)) + Bb \tan(c+dx)}{d} & \text{for } d \neq 0 \\ x(A + B \sec(c))(a + b \sec(c)) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] Piecewise(((A*a*(c + d*x) + A*b*log(tan(c + d*x) + sec(c + d*x)) + B*a*log(
tan(c + d*x) + sec(c + d*x)) + B*b*tan(c + d*x))/d, Ne(d, 0)), (x*(A + B*se
c(c))*(a + b*sec(c)), True))
```

$$3.281 \quad \int \cos(c + dx)(a + b \sec(c + dx))(A + B \sec(c + dx)) dx$$

Optimal. Leaf size=35

$$x(aB + Ab) + \frac{aA \sin(c + dx)}{d} + \frac{bB \tanh^{-1}(\sin(c + dx))}{d}$$

[Out] (A*b+B*a)*x+b*B*arctanh(sin(d*x+c))/d+a*A*sin(d*x+c)/d

Rubi [A] time = 0.05, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {3996, 3770}

$$x(aB + Ab) + \frac{aA \sin(c + dx)}{d} + \frac{bB \tanh^{-1}(\sin(c + dx))}{d}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]

[Out] (A*b + a*B)*x + (b*B*ArcTanh[Sin[c + d*x]])/d + (a*A*Sin[c + d*x])/d

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3996

Int[(csc[(e_.) + (f_.)*(x_)])*(d_.)^(n_.)*(csc[(e_.) + (f_.)*(x_)])*(b_.) + (a_.)*(csc[(e_.) + (f_.)*(x_)])*(B_.) + (A_.)), x_Symbol] := Simp[(A*a*Cot[e + f*x])*(d*Csc[e + f*x])^n/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]

Rubi steps

$$\begin{aligned} \int \cos(c + dx)(a + b \sec(c + dx))(A + B \sec(c + dx)) dx &= \frac{aA \sin(c + dx)}{d} - \int (-Ab - aB - bB \sec(c + dx)) dx \\ &= (Ab + aB)x + \frac{aA \sin(c + dx)}{d} + (bB) \int \sec(c + dx) dx \\ &= (Ab + aB)x + \frac{bB \tanh^{-1}(\sin(c + dx))}{d} + \frac{aA \sin(c + dx)}{d} \end{aligned}$$

Mathematica [A] time = 0.03, size = 46, normalized size = 1.31

$$\frac{aA \sin(c) \cos(dx)}{d} + \frac{aA \cos(c) \sin(dx)}{d} + aBx + Abx + \frac{bB \tanh^{-1}(\sin(c + dx))}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]

[Out] A*b*x + a*B*x + (b*B*ArcTanh[Sin[c + d*x]])/d + (a*A*Cos[d*x]*Sin[c])/d + (a*A*Cos[c]*Sin[d*x])/d

fricas [A] time = 0.46, size = 54, normalized size = 1.54

$$\frac{2(Ba + Ab)dx + Bb \log(\sin(dx + c) + 1) - Bb \log(-\sin(dx + c) + 1) + 2Aa \sin(dx + c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] 1/2*(2*(B*a + A*b)*d*x + B*b*log(sin(d*x + c) + 1) - B*b*log(-sin(d*x + c) + 1) + 2*A*a*sin(d*x + c))/d

giac [B] time = 0.47, size = 79, normalized size = 2.26

$$\frac{Bb \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right|\right) - Bb \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1\right|\right) + (Ba + Ab)(dx + c) + \frac{2Aa \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)}{\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + 1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] (B*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - B*b*log(abs(tan(1/2*d*x + 1/2*c) - 1))) + (B*a + A*b)*(d*x + c) + 2*A*a*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 + 1))/d

maple [A] time = 0.80, size = 56, normalized size = 1.60

$$Abx + Bxa + \frac{aA \sin(dx + c)}{d} + \frac{Abc}{d} + \frac{Bb \ln(\sec(dx + c) + \tan(dx + c))}{d} + \frac{Bac}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x)

[Out] $A*b*x+B*x*a+a*A*\sin(d*x+c)/d+1/d*A*b*c+1/d*B*b*\ln(\sec(d*x+c)+\tan(d*x+c))+1/d*B*a*c$

maxima [A] time = 0.44, size = 58, normalized size = 1.66

$$\frac{2(dx+c)Ba + 2(dx+c)Ab + Bb(\log(\sin(dx+c)+1) - \log(\sin(dx+c)-1)) + 2Aa\sin(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="maxima")`

[Out] $1/2*(2*(d*x+c)*B*a + 2*(d*x+c)*A*b + B*b*(\log(\sin(d*x+c)+1) - \log(\sin(d*x+c)-1)) + 2*A*a*\sin(d*x+c))/d$

mupad [B] time = 2.19, size = 100, normalized size = 2.86

$$\frac{Aa\sin(c+dx)}{d} + \frac{2Ab\operatorname{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{d} + \frac{2Ba\operatorname{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{d} + \frac{2Bb\operatorname{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(c+d*x)*(A+B/cos(c+d*x))*(a+b/cos(c+d*x)),x)`

[Out] $(A*a*\sin(c+dx))/d + (2*A*b*\operatorname{atan}(\sin(c/2+(dx)/2)/\cos(c/2+(dx)/2)))/d + (2*B*a*\operatorname{atan}(\sin(c/2+(dx)/2)/\cos(c/2+(dx)/2)))/d + (2*B*b*\operatorname{atanh}(\sin(c/2+(dx)/2)/\cos(c/2+(dx)/2)))/d$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \sec(c + dx))(a + b \sec(c + dx)) \cos(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x)`

[Out] `Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))*cos(c + d*x), x)`

$$3.282 \quad \int \cos^2(c + dx)(a + b \sec(c + dx))(A + B \sec(c + dx)) dx$$

Optimal. Leaf size=52

$$\frac{(aB + Ab) \sin(c + dx)}{d} + \frac{1}{2}x(aA + 2bB) + \frac{aA \sin(c + dx) \cos(c + dx)}{2d}$$

[Out] 1/2*(A*a+2*B*b)*x+(A*b+B*a)*sin(d*x+c)/d+1/2*a*A*cos(d*x+c)*sin(d*x+c)/d

Rubi [A] time = 0.10, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {3996, 3787, 2637, 8}

$$\frac{(aB + Ab) \sin(c + dx)}{d} + \frac{1}{2}x(aA + 2bB) + \frac{aA \sin(c + dx) \cos(c + dx)}{2d}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^2*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]

[Out] ((a*A + 2*b*B)*x)/2 + ((A*b + a*B)*Sin[c + d*x])/d + (a*A*Cos[c + d*x]*Sin[c + d*x])/(2*d)

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_)])*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)])*(b_.) + (a_.), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 3996

Int[(csc[(e_.) + (f_.)*(x_)])*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)])*(b_.) + (a_.)*(csc[(e_.) + (f_.)*(x_)])*(B_.) + (A_.), x_Symbol] := Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x] /

; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]

Rubi steps

$$\begin{aligned} \int \cos^2(c + dx)(a + b \sec(c + dx))(A + B \sec(c + dx)) dx &= \frac{aA \cos(c + dx) \sin(c + dx)}{2d} - \frac{1}{2} \int \cos(c + dx)(-2(Ab + aB) \sec(c + dx) + A) dx \\ &= \frac{aA \cos(c + dx) \sin(c + dx)}{2d} - (-Ab - aB) \int \cos(c + dx) dx + \frac{A}{2} \int \cos(c + dx) dx \\ &= \frac{1}{2}(aA + 2bB)x + \frac{(Ab + aB) \sin(c + dx)}{d} + \frac{aA \cos(c + dx)}{2} \end{aligned}$$

Mathematica [A] time = 0.09, size = 51, normalized size = 0.98

$$\frac{4(aB + Ab) \sin(c + dx) + aA \sin(2(c + dx)) + 2aAc + 2aAdx + 4bBdx}{4d}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]^2*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]

[Out] (2*a*A*c + 2*a*A*d*x + 4*b*B*d*x + 4*(A*b + a*B)*Sin[c + d*x] + a*A*Ssin[2*(c + d*x)])/(4*d)

fricas [A] time = 0.46, size = 42, normalized size = 0.81

$$\frac{(Aa + 2Bb)dx + (Aa \cos(dx + c) + 2Ba + 2Ab) \sin(dx + c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] 1/2*((A*a + 2*B*b)*d*x + (A*a*cos(d*x + c) + 2*B*a + 2*A*b)*sin(d*x + c))/d

giac [B] time = 0.22, size = 121, normalized size = 2.33

$$\frac{(Aa + 2Bb)(dx + c) - \frac{2 \left(Aa \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 - 2Ba \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 - 2Ab \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 - Aa \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 2Ba \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 2Ab \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) \right)}{\left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + 1 \right)^2}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] $\frac{1}{2} * ((A*a + 2*B*b) * (d*x + c) - 2 * (A*a * \tan(1/2*d*x + 1/2*c)^3 - 2*B*a * \tan(1/2*d*x + 1/2*c)^3 - A*a * \tan(1/2*d*x + 1/2*c) - 2*B*a * \tan(1/2*d*x + 1/2*c) - 2*A*b * \tan(1/2*d*x + 1/2*c)) / (\tan(1/2*d*x + 1/2*c)^2 + 1)^2 / d$

maple [A] time = 0.78, size = 57, normalized size = 1.10

$$\frac{aA \left(\frac{\cos(dx+c) \sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2} \right) + Ab \sin(dx+c) + aB \sin(dx+c) + B(dx+c)b}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^2*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x)

[Out] $\frac{1}{d} * (a*A*(1/2*\cos(d*x+c)*\sin(d*x+c)+1/2*d*x+1/2*c)+A*b*\sin(d*x+c)+a*B*\sin(d*x+c)+B*(d*x+c)*b)$

maxima [A] time = 0.78, size = 55, normalized size = 1.06

$$\frac{(2dx + 2c + \sin(2dx + 2c))Aa + 4(dx + c)Bb + 4Ba \sin(dx + c) + 4Ab \sin(dx + c)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] $\frac{1}{4} * ((2*d*x + 2*c + \sin(2*d*x + 2*c)) * A*a + 4 * (d*x + c) * B*b + 4 * B*a * \sin(d*x + c) + 4 * A*b * \sin(d*x + c)) / d$

mupad [B] time = 2.03, size = 50, normalized size = 0.96

$$\frac{Aax}{2} + Bbx + \frac{Ab \sin(c + dx)}{d} + \frac{Ba \sin(c + dx)}{d} + \frac{Aa \sin(2c + 2dx)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^2*(A + B/cos(c + d*x))*(a + b/cos(c + d*x)),x)

[Out] $(A*a*x)/2 + B*b*x + (A*b*\sin(c + d*x))/d + (B*a*\sin(c + d*x))/d + (A*a*\sin(2*c + 2*d*x))/(4*d)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \sec(c + dx)) (a + b \sec(c + dx)) \cos^2(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))*cos(c + d*x)**2, x)
```

$$3.283 \quad \int \cos^3(c + dx)(a + b \sec(c + dx))(A + B \sec(c + dx)) dx$$

Optimal. Leaf size=84

$$\frac{(2aA + 3bB) \sin(c + dx)}{3d} + \frac{(aB + Ab) \sin(c + dx) \cos(c + dx)}{2d} + \frac{1}{2}x(aB + Ab) + \frac{aA \sin(c + dx) \cos^2(c + dx)}{3d}$$

[Out] $1/2*(A*b+B*a)*x+1/3*(2*A*a+3*B*b)*\sin(d*x+c)/d+1/2*(A*b+B*a)*\cos(d*x+c)*\sin(d*x+c)/d+1/3*a*A*\cos(d*x+c)^2*\sin(d*x+c)/d$

Rubi [A] time = 0.13, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {3996, 3787, 2635, 8, 2637}

$$\frac{(2aA + 3bB) \sin(c + dx)}{3d} + \frac{(aB + Ab) \sin(c + dx) \cos(c + dx)}{2d} + \frac{1}{2}x(aB + Ab) + \frac{aA \sin(c + dx) \cos^2(c + dx)}{3d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Cos}[c + d*x]^3*(a + b*\text{Sec}[c + d*x])*(A + B*\text{Sec}[c + d*x]), x]$

[Out] $((A*b + a*B)*x)/2 + ((2*a*A + 3*b*B)*\text{Sin}[c + d*x])/(3*d) + ((A*b + a*B)*\text{Cos}[c + d*x]*\text{Sin}[c + d*x])/(2*d) + (a*A*\text{Cos}[c + d*x]^2*\text{Sin}[c + d*x])/(3*d)$

Rule 8

$\text{Int}[a_, x_Symbol] \text{ :> } \text{Simp}[a*x, x] \text{ /; } \text{FreeQ}[a, x]$

Rule 2635

$\text{Int}[(b_*\sin[(c_*) + (d_*)(x_)])^{(n_*)}, x_Symbol] \text{ :> } -\text{Simp}[(b*\text{Cos}[c + d*x])*(b*\text{Sin}[c + d*x])^{(n-1)}]/(d*n), x] + \text{Dist}[(b^2*(n-1))/n, \text{Int}[(b*\text{Sin}[c + d*x])^{(n-2)}, x], x] \text{ /; } \text{FreeQ}\{b, c, d\}, x] \ \&\& \ \text{GtQ}[n, 1] \ \&\& \ \text{IntegerQ}[2*n]$

Rule 2637

$\text{Int}[\sin[\text{Pi}/2 + (c_*) + (d_*)(x_)], x_Symbol] \text{ :> } \text{Simp}[\text{Sin}[c + d*x]/d, x] \text{ /; } \text{FreeQ}\{c, d\}, x]$

Rule 3787

$\text{Int}[(\text{csc}[(e_*) + (f_*)(x_)]*(d_*)^{(n_*)}*(\text{csc}[(e_*) + (f_*)(x_)]*(b_*) + (a_*)), x_Symbol] \text{ :> } \text{Dist}[a, \text{Int}[(d*\text{Csc}[e + f*x])^n, x], x] + \text{Dist}[b/d, \text{Int}[\text{Csc}[e + f*x], x], x]$

$(d \cdot \text{Csc}[e + f \cdot x])^{(n + 1)}, x], x] /; \text{FreeQ}[\{a, b, d, e, f, n\}, x]$

Rule 3996

$\text{Int}[(\text{csc}[(e_.) + (f_.)(x_.)](d_.))^{(n_.)}(\text{csc}[(e_.) + (f_.)(x_.)](b_.) + (a_.))(\text{csc}[(e_.) + (f_.)(x_.)](B_.) + (A_.)), x_Symbol] :> \text{Simp}[(A*a*\text{Cot}[e + f*x]*(d*\text{Csc}[e + f*x])^n)/(f*n), x] + \text{Dist}[1/(d*n), \text{Int}[(d*\text{Csc}[e + f*x])^{(n + 1)}*\text{Simp}[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*\text{Csc}[e + f*x], x], x], x] /; \text{FreeQ}[\{a, b, d, e, f, A, B\}, x] \&\& \text{NeQ}[A*b - a*B, 0] \&\& \text{LeQ}[n, -1]$

Rubi steps

$$\begin{aligned} \int \cos^3(c + dx)(a + b \sec(c + dx))(A + B \sec(c + dx)) dx &= \frac{aA \cos^2(c + dx) \sin(c + dx)}{3d} - \frac{1}{3} \int \cos^2(c + dx)(-3(a + b \sec(c + dx))) dx \\ &= \frac{aA \cos^2(c + dx) \sin(c + dx)}{3d} - (-Ab - aB) \int \cos^2(c + dx) dx \\ &= \frac{(2aA + 3bB) \sin(c + dx)}{3d} + \frac{(Ab + aB) \cos(c + dx) \sin(c + dx)}{2d} \\ &= \frac{1}{2}(Ab + aB)x + \frac{(2aA + 3bB) \sin(c + dx)}{3d} + \frac{(Ab + aB) \cos(c + dx) \sin(c + dx)}{2d} \end{aligned}$$

Mathematica [A] time = 0.16, size = 75, normalized size = 0.89

$$\frac{3(3aA + 4bB) \sin(c + dx) + 3(aB + Ab) \sin(2(c + dx)) + aA \sin(3(c + dx)) + 6aBc + 6aBdx + 6Abc + 6Abdx}{12d}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]^3*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]

[Out] (6*A*b*c + 6*a*B*c + 6*A*b*d*x + 6*a*B*d*x + 3*(3*a*A + 4*b*B)*Sin[c + d*x] + 3*(A*b + a*B)*Sin[2*(c + d*x)] + a*A*Ssin[3*(c + d*x)])/(12*d)

fricas [A] time = 0.43, size = 60, normalized size = 0.71

$$\frac{3(Ba + Ab)dx + (2Aa \cos(dx + c)^2 + 4Aa + 6Bb + 3(Ba + Ab) \cos(dx + c)) \sin(dx + c)}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] $1/6*(3*(B*a + A*b)*d*x + (2*A*a*cos(d*x + c)^2 + 4*A*a + 6*B*b + 3*(B*a + A*b)*cos(d*x + c))*sin(d*x + c))/d$

giac [B] time = 0.79, size = 180, normalized size = 2.14

$$3(Ba + Ab)(dx + c) + \frac{2\left(6Aa \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5 - 3Ba \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5 - 3Ab \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5 + 6Bb \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5 + 4Aa \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 + 12Ba \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 + 12Ab \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 + 6Bb \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3\right)}{\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right)^2 + 1} \cdot \frac{1}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="giac")`

[Out] $1/6*(3*(B*a + A*b)*(d*x + c) + 2*(6*A*a*tan(1/2*d*x + 1/2*c)^5 - 3*B*a*tan(1/2*d*x + 1/2*c)^5 - 3*A*b*tan(1/2*d*x + 1/2*c)^5 + 6*B*b*tan(1/2*d*x + 1/2*c)^5 + 4*A*a*tan(1/2*d*x + 1/2*c)^3 + 12*B*b*tan(1/2*d*x + 1/2*c)^3 + 6*A*a*tan(1/2*d*x + 1/2*c) + 3*B*a*tan(1/2*d*x + 1/2*c) + 3*A*b*tan(1/2*d*x + 1/2*c) + 6*B*b*tan(1/2*d*x + 1/2*c)))/(tan(1/2*d*x + 1/2*c)^2 + 1)^3/d$

maple [A] time = 1.31, size = 85, normalized size = 1.01

$$\frac{aA(2+\cos^2(dx+c))\sin(dx+c)}{3} + Ab\left(\frac{\cos(dx+c)\sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2}\right) + aB\left(\frac{\cos(dx+c)\sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2}\right) + B\sin(dx+c)b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(d*x+c)^3*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x)`

[Out] $1/d*(1/3*a*A*(2+\cos(d*x+c)^2)*\sin(d*x+c)+A*b*(1/2*\cos(d*x+c)*\sin(d*x+c)+1/2*d*x+1/2*c)+a*B*(1/2*\cos(d*x+c)*\sin(d*x+c)+1/2*d*x+1/2*c)+B*\sin(d*x+c)*b)$

maxima [A] time = 0.91, size = 79, normalized size = 0.94

$$\frac{4(\sin(dx+c)^3 - 3\sin(dx+c))Aa - 3(2dx + 2c + \sin(2dx + 2c))Ba - 3(2dx + 2c + \sin(2dx + 2c))Ab - 3(2dx + 2c + \sin(2dx + 2c))Aa - 3(2dx + 2c + \sin(2dx + 2c))Ba - 3(2dx + 2c + \sin(2dx + 2c))Ab - 3(2dx + 2c + \sin(2dx + 2c))Aa - 3(2dx + 2c + \sin(2dx + 2c))Ba - 3(2dx + 2c + \sin(2dx + 2c))Ab}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="maxima")`

[Out] $-1/12*(4*(\sin(d*x + c)^3 - 3*\sin(d*x + c))*A*a - 3*(2*d*x + 2*c + \sin(2*d*x + 2*c))*B*a - 3*(2*d*x + 2*c + \sin(2*d*x + 2*c))*A*b - 12*B*b*\sin(d*x + c))/d$

mupad [B] time = 2.07, size = 84, normalized size = 1.00

$$\frac{A b x}{2} + \frac{B a x}{2} + \frac{3 A a \sin(c + d x)}{4 d} + \frac{B b \sin(c + d x)}{d} + \frac{A a \sin(3 c + 3 d x)}{12 d} + \frac{A b \sin(2 c + 2 d x)}{4 d} + \frac{B a \sin(2 c + 2 d x)}{4 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(c + d*x)^3*(A + B/cos(c + d*x))*(a + b/cos(c + d*x)),x)`

[Out] `(A*b*x)/2 + (B*a*x)/2 + (3*A*a*sin(c + d*x))/(4*d) + (B*b*sin(c + d*x))/d + (A*a*sin(3*c + 3*d*x))/(12*d) + (A*b*sin(2*c + 2*d*x))/(4*d) + (B*a*sin(2*c + 2*d*x))/(4*d)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \sec(c + dx)) (a + b \sec(c + dx)) \cos^3(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)**3*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x)`

[Out] `Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))*cos(c + d*x)**3, x)`

$$3.284 \quad \int \cos^4(c + dx)(a + b \sec(c + dx))(A + B \sec(c + dx)) dx$$

Optimal. Leaf size=105

$$-\frac{(aB + Ab) \sin^3(c + dx)}{3d} + \frac{(aB + Ab) \sin(c + dx)}{d} + \frac{(3aA + 4bB) \sin(c + dx) \cos(c + dx)}{8d} + \frac{1}{8}x(3aA + 4bB) + \frac{aA \sin(c + dx)}{d}$$

[Out] $1/8*(3*A*a+4*B*b)*x+(A*b+B*a)*\sin(d*x+c)/d+1/8*(3*A*a+4*B*b)*\cos(d*x+c)*\sin(d*x+c)/d+1/4*a*A*\cos(d*x+c)^3*\sin(d*x+c)/d-1/3*(A*b+B*a)*\sin(d*x+c)^3/d$

Rubi [A] time = 0.14, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {3996, 3787, 2633, 2635, 8}

$$-\frac{(aB + Ab) \sin^3(c + dx)}{3d} + \frac{(aB + Ab) \sin(c + dx)}{d} + \frac{(3aA + 4bB) \sin(c + dx) \cos(c + dx)}{8d} + \frac{1}{8}x(3aA + 4bB) + \frac{aA \sin(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] `Int[Cos[c + d*x]^4*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]`

[Out] $((3*a*A + 4*b*B)*x)/8 + ((A*b + a*B)*\sin[c + d*x])/d + ((3*a*A + 4*b*B)*\cos[c + d*x]*\sin[c + d*x])/(8*d) + (a*A*\cos[c + d*x]^3*\sin[c + d*x])/(4*d) - ((A*b + a*B)*\sin[c + d*x]^3)/(3*d)$

Rule 8

`Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

Rule 2633

`Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]`

Rule 2635

`Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x])*(b*Sine[c + d*x])^(n - 1)/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sine[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]`

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_.)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_.))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] := Simp[(A*a*Cot[e +
f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
+ 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rubi steps

$$\begin{aligned} \int \cos^4(c + dx)(a + b \sec(c + dx))(A + B \sec(c + dx)) dx &= \frac{aA \cos^3(c + dx) \sin(c + dx)}{4d} - \frac{1}{4} \int \cos^3(c + dx)(-4A + B \sec(c + dx)) dx \\ &= \frac{aA \cos^3(c + dx) \sin(c + dx)}{4d} - (-Ab - aB) \int \cos^3(c + dx) dx \\ &= \frac{(3aA + 4bB) \cos(c + dx) \sin(c + dx)}{8d} + \frac{aA \cos^3(c + dx)}{4d} \\ &= \frac{1}{8}(3aA + 4bB)x + \frac{(Ab + aB) \sin(c + dx)}{d} + \frac{(3aA + 4bB) \cos(c + dx) \sin(c + dx)}{8d} \end{aligned}$$

Mathematica [A] time = 0.24, size = 91, normalized size = 0.87

$$\frac{-32(aB + Ab) \sin^3(c + dx) + 96(aB + Ab) \sin(c + dx) + 24(aA + bB) \sin(2(c + dx)) + 3aA \sin(4(c + dx)) + 36aA \cos^3(c + dx)}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (36*a*A*c + 48*b*B*c + 36*a*A*d*x + 48*b*B*d*x + 96*(A*b + a*B)*Sin[c + d*x]
] - 32*(A*b + a*B)*Sin[c + d*x]^3 + 24*(a*A + b*B)*Sin[2*(c + d*x)] + 3*a*A
*Sin[4*(c + d*x)]/(96*d)
```

fricas [A] time = 0.44, size = 81, normalized size = 0.77

$$\frac{3(3Aa + 4Bb)dx + (6Aa \cos(dx + c))^3 + 8(Ba + Ab) \cos(dx + c)^2 + 16Ba + 16Ab + 3(3Aa + 4Bb) \cos(dx + c)}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] 1/24*(3*(3*A*a + 4*B*b)*d*x + (6*A*a*cos(d*x + c)^3 + 8*(B*a + A*b)*cos(d*x + c)^2 + 16*B*a + 16*A*b + 3*(3*A*a + 4*B*b)*cos(d*x + c))*sin(d*x + c))/d

giac [B] time = 0.24, size = 272, normalized size = 2.59

$$3(3Aa + 4Bb)(dx + c) - \frac{2\left(15Aa \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^7 - 24Ba \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^7 - 24Ab \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^7 + 12Bb \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^7 - 9Aa \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5 - 24Aa \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 - 40Aa \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 24Ba \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5 - 40Ba \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 - 40Ab \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5 - 12Ab \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 - 15Ab \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 12Bb \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5 - 12Bb \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 - 12Bb \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right)}{(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + 1)^4} / d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] 1/24*(3*(3*A*a + 4*B*b)*(d*x + c) - 2*(15*A*a*tan(1/2*d*x + 1/2*c)^7 - 24*B*a*tan(1/2*d*x + 1/2*c)^7 - 24*A*b*tan(1/2*d*x + 1/2*c)^7 + 12*B*b*tan(1/2*d*x + 1/2*c)^7 - 9*A*a*tan(1/2*d*x + 1/2*c)^5 - 40*B*a*tan(1/2*d*x + 1/2*c)^5 - 40*A*b*tan(1/2*d*x + 1/2*c)^5 + 12*B*b*tan(1/2*d*x + 1/2*c)^5 + 9*A*a*tan(1/2*d*x + 1/2*c)^3 - 40*B*a*tan(1/2*d*x + 1/2*c)^3 - 40*A*b*tan(1/2*d*x + 1/2*c)^3 - 12*B*b*tan(1/2*d*x + 1/2*c)^3 - 15*A*a*tan(1/2*d*x + 1/2*c) - 24*B*a*tan(1/2*d*x + 1/2*c) - 24*A*b*tan(1/2*d*x + 1/2*c) - 12*B*b*tan(1/2*d*x + 1/2*c))/tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d

maple [A] time = 1.52, size = 107, normalized size = 1.02

$$aA \left(\frac{\left(\cos^3(dx+c) + \frac{3\cos(dx+c)}{2} \right) \sin(dx+c)}{4} + \frac{3dx}{8} + \frac{3c}{8} \right) + \frac{Ab(2+\cos^2(dx+c))\sin(dx+c)}{3} + \frac{aB(2+\cos^2(dx+c))\sin(dx+c)}{3} + Bb \left(\frac{\cos(dx+c)\sin(dx+c)}{2} \right) / d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^4*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x)

[Out] 1/d*(a*A*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3*A*b*(2+cos(d*x+c)^2)*sin(d*x+c)+1/3*a*B*(2+cos(d*x+c)^2)*sin(d*x+c)+B*b*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))

maxima [A] time = 2.05, size = 101, normalized size = 0.96

$$3(12dx + 12c + \sin(4dx + 4c) + 8 \sin(2dx + 2c))Aa - 32(\sin(dx + c)^3 - 3 \sin(dx + c))Ba - 32(\sin(dx + c) - \cos(dx + c)) / 96d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] $\frac{1}{96}*(3*(12*d*x + 12*c + \sin(4*d*x + 4*c)) + 8*\sin(2*d*x + 2*c))*A*a - 32*(\sin(d*x + c)^3 - 3*\sin(d*x + c))*B*a - 32*(\sin(d*x + c)^3 - 3*\sin(d*x + c))*A*b + 24*(2*d*x + 2*c + \sin(2*d*x + 2*c))*B*b)/d$

mupad [B] time = 2.13, size = 117, normalized size = 1.11

$$\frac{3Aax}{8} + \frac{Bbx}{2} + \frac{3Ab \sin(c+dx)}{4d} + \frac{3Ba \sin(c+dx)}{4d} + \frac{Aa \sin(2c+2dx)}{4d} + \frac{Aa \sin(4c+4dx)}{32d} + \frac{Ab \sin(3c+3dx)}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^4*(A + B/cos(c + d*x))*(a + b/cos(c + d*x)),x)

[Out] $(3*A*a*x)/8 + (B*b*x)/2 + (3*A*b*\sin(c + d*x))/(4*d) + (3*B*a*\sin(c + d*x))/(4*d) + (A*a*\sin(2*c + 2*d*x))/(4*d) + (A*a*\sin(4*c + 4*d*x))/(32*d) + (A*b*\sin(3*c + 3*d*x))/(12*d) + (B*a*\sin(3*c + 3*d*x))/(12*d) + (B*b*\sin(2*c + 2*d*x))/(4*d)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \sec(c + dx))(a + b \sec(c + dx)) \cos^4(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**4*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x)

[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))*cos(c + d*x)**4, x)

$$3.285 \quad \int \sec^3(c+dx)(a+b \sec(c+dx))^2(A+B \sec(c+dx)) dx$$

Optimal. Leaf size=198

$$\frac{(4a^2A + 6abB + 3Ab^2) \tanh^{-1}(\sin(c+dx))}{8d} + \frac{(4a^2A + 6abB + 3Ab^2) \tan(c+dx) \sec(c+dx)}{8d} + \frac{(5a(aB + 2Ab) + \dots)}{8d}$$

[Out] 1/8*(4*A*a^2+3*A*b^2+6*B*a*b)*arctanh(sin(d*x+c))/d+1/5*(4*b^2*B+5*a*(2*A*b+B*a))*tan(d*x+c)/d+1/8*(4*A*a^2+3*A*b^2+6*B*a*b)*sec(d*x+c)*tan(d*x+c)/d+1/20*b*(5*A*b+6*B*a)*sec(d*x+c)^3*tan(d*x+c)/d+1/5*b*B*sec(d*x+c)^3*(a+b*sec(d*x+c))*tan(d*x+c)/d+1/15*(4*b^2*B+5*a*(2*A*b+B*a))*tan(d*x+c)^3/d

Rubi [A] time = 0.29, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4026, 4047, 3767, 4046, 3768, 3770}

$$\frac{(4a^2A + 6abB + 3Ab^2) \tanh^{-1}(\sin(c+dx))}{8d} + \frac{(4a^2A + 6abB + 3Ab^2) \tan(c+dx) \sec(c+dx)}{8d} + \frac{(5a(aB + 2Ab) + \dots)}{8d}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]

[Out] ((4*a^2*A + 3*A*b^2 + 6*a*b*B)*ArcTanh[Sin[c + d*x]])/(8*d) + ((4*b^2*B + 5*a*(2*A*b + a*B))*Tan[c + d*x])/(5*d) + ((4*a^2*A + 3*A*b^2 + 6*a*b*B)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (b*(5*A*b + 6*a*B)*Sec[c + d*x]^3*Tan[c + d*x])/(20*d) + (b*B*Sec[c + d*x]^3*(a + b*Sec[c + d*x])*Tan[c + d*x])/(5*d) + ((4*b^2*B + 5*a*(2*A*b + a*B))*Tan[c + d*x]^3)/(15*d)

Rule 3767

Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 3768

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := -Simp[(b*Csc[c + d*x]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]

Rule 4026

Int[(csc[(e_.) + (f_.)*(x_)])*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)])*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_)])*(B_.) + (A_.), x_Symbol] := -Simp[(b*B*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x] + Dist[1/(m + n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*Simp[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B)*(m + n) + b^2*B*(m + n - 1))*Csc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && !IGtQ[n, 1] && !IntegerQ[m]

Rule 4046

Int[(csc[(e_.) + (f_.)*(x_)])*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)])^2*(C_.) + (A_.), x_Symbol] := -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]

Rule 4047

Int[(csc[(e_.) + (f_.)*(x_)])*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)])*(B_.) + csc[(e_.) + (f_.)*(x_)])^2*(C_.), x_Symbol] := Dist[B/b, Int[(b*Csc[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2), x] /; FreeQ[{b, e, f, A, B, C, m}, x]

Rubi steps

$$\begin{aligned}
 \int \sec^3(c + dx)(a + b \sec(c + dx))^2(A + B \sec(c + dx)) dx &= \frac{bB \sec^3(c + dx)(a + b \sec(c + dx)) \tan(c + dx)}{5d} + \frac{1}{5} \int \\
 &= \frac{bB \sec^3(c + dx)(a + b \sec(c + dx)) \tan(c + dx)}{5d} + \frac{1}{5} \int \\
 &= \frac{b(5Ab + 6aB) \sec^3(c + dx) \tan(c + dx)}{20d} + \frac{bB \sec^3(c + dx)}{5d} \\
 &= \frac{(4b^2B + 5a(2Ab + aB)) \tan(c + dx)}{5d} + \frac{(4a^2A + 3Ab^2)}{5d} \\
 &= \frac{(4a^2A + 3Ab^2 + 6abB) \tanh^{-1}(\sin(c + dx))}{8d} + \frac{(4b^2B + 5a(2Ab + aB)) \tan(c + dx)}{5d}
 \end{aligned}$$

Mathematica [A] time = 1.56, size = 150, normalized size = 0.76

$$\frac{15(4a^2A + 6abB + 3Ab^2) \tanh^{-1}(\sin(c + dx)) + \tan(c + dx) \left(8 \left(5(a^2B + 2aAb + 2b^2B) \tan^2(c + dx) + 15(a^2B + 2aAb + 2b^2B) \right) \right)}{120}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]

[Out] (15*(4*a^2*A + 3*A*b^2 + 6*a*b*B)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(15*(4*a^2*A + 3*A*b^2 + 6*a*b*B)*Sec[c + d*x] + 30*b*(A*b + 2*a*B)*Sec[c + d*x]^3 + 8*(15*(2*a*A*b + a^2*B + b^2*B) + 5*(2*a*A*b + a^2*B + 2*b^2*B)*Tan[c + d*x]^2 + 3*b^2*B*Tan[c + d*x]^4)))/(120*d)

fricas [A] time = 0.54, size = 208, normalized size = 1.05

$$\frac{15(4Aa^2 + 6Bab + 3Ab^2) \cos(dx + c)^5 \log(\sin(dx + c) + 1) - 15(4Aa^2 + 6Bab + 3Ab^2) \cos(dx + c)^5 \log(-\sin(dx + c) + 1)}{120}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] 1/240*(15*(4*A*a^2 + 6*B*a*b + 3*A*b^2)*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 15*(4*A*a^2 + 6*B*a*b + 3*A*b^2)*cos(d*x + c)^5*log(-sin(d*x + c) + 1) + 2*(16*(5*B*a^2 + 10*A*a*b + 4*B*b^2)*cos(d*x + c)^4 + 15*(4*A*a^2 + 6*B*a*b + 3*A*b^2)*cos(d*x + c)^3 + 24*B*b^2 + 8*(5*B*a^2 + 10*A*a*b + 4*B*b^2)*cos(d*x + c)^2 + 30*(2*B*a*b + A*b^2)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^5)

giac [B] time = 0.33, size = 528, normalized size = 2.67

$$15(4Aa^2 + 6Bab + 3Ab^2) \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right|\right) - 15(4Aa^2 + 6Bab + 3Ab^2) \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] 1/120*(15*(4*A*a^2 + 6*B*a*b + 3*A*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(4*A*a^2 + 6*B*a*b + 3*A*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(60*A*a^2*tan(1/2*d*x + 1/2*c)^9 - 120*B*a^2*tan(1/2*d*x + 1/2*c)^9 - 240*A*b

$$\begin{aligned}
& a*b*\tan(1/2*d*x + 1/2*c)^9 + 150*B*a*b*\tan(1/2*d*x + 1/2*c)^9 + 75*A*b^2*\tan(1/2*d*x + 1/2*c)^9 - 120*B*b^2*\tan(1/2*d*x + 1/2*c)^9 - 120*A*a^2*\tan(1/2*d*x + 1/2*c)^7 + 320*B*a^2*\tan(1/2*d*x + 1/2*c)^7 + 640*A*a*b*\tan(1/2*d*x + 1/2*c)^7 - 60*B*a*b*\tan(1/2*d*x + 1/2*c)^7 - 30*A*b^2*\tan(1/2*d*x + 1/2*c)^7 + 160*B*b^2*\tan(1/2*d*x + 1/2*c)^7 - 400*B*a^2*\tan(1/2*d*x + 1/2*c)^5 - 800*A*a*b*\tan(1/2*d*x + 1/2*c)^5 - 464*B*b^2*\tan(1/2*d*x + 1/2*c)^5 + 120*A*a^2*\tan(1/2*d*x + 1/2*c)^3 + 320*B*a^2*\tan(1/2*d*x + 1/2*c)^3 + 640*A*a*b*\tan(1/2*d*x + 1/2*c)^3 + 60*B*a*b*\tan(1/2*d*x + 1/2*c)^3 + 30*A*b^2*\tan(1/2*d*x + 1/2*c)^3 + 160*B*b^2*\tan(1/2*d*x + 1/2*c)^3 - 60*A*a^2*\tan(1/2*d*x + 1/2*c) - 120*B*a^2*\tan(1/2*d*x + 1/2*c) - 240*A*a*b*\tan(1/2*d*x + 1/2*c) - 150*B*a*b*\tan(1/2*d*x + 1/2*c) - 75*A*b^2*\tan(1/2*d*x + 1/2*c) - 120*B*b^2*\tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
\end{aligned}$$

maple [A] time = 1.52, size = 312, normalized size = 1.58

$$\frac{a^2 A \sec(dx+c) \tan(dx+c)}{2d} + \frac{a^2 A \ln(\sec(dx+c) + \tan(dx+c))}{2d} + \frac{2a^2 B \tan(dx+c)}{3d} + \frac{a^2 B (\sec^2(dx+c)) \tan(dx+c)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^3*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x)

[Out] 1/2/d*a^2*A*sec(d*x+c)*tan(d*x+c)+1/2/d*a^2*A*ln(sec(d*x+c)+tan(d*x+c))+2/3*a^2*B*tan(d*x+c)/d+1/3*a^2*B*sec(d*x+c)^2*tan(d*x+c)/d+4/3*a*A*b*tan(d*x+c)/d+2/3*a*A*b*sec(d*x+c)^2*tan(d*x+c)/d+1/2/d*B*a*b*tan(d*x+c)*sec(d*x+c)^3+3/4/d*B*a*b*sec(d*x+c)*tan(d*x+c)+3/4/d*B*a*b*ln(sec(d*x+c)+tan(d*x+c))+1/4/d*A*b^2*tan(d*x+c)*sec(d*x+c)^3+3/8/d*A*b^2*sec(d*x+c)*tan(d*x+c)+3/8/d*A*b^2*ln(sec(d*x+c)+tan(d*x+c))+8/15*b^2*B*tan(d*x+c)/d+1/5/d*b^2*B*tan(d*x+c)*sec(d*x+c)^4+4/15/d*b^2*B*tan(d*x+c)*sec(d*x+c)^2

maxima [A] time = 0.67, size = 276, normalized size = 1.39

$$80(\tan(dx+c)^3 + 3 \tan(dx+c))Ba^2 + 160(\tan(dx+c)^3 + 3 \tan(dx+c))Aab + 16(3 \tan(dx+c)^5 + 10 \tan(dx+c)^3 + 15 \tan(dx+c) \tan^3(dx+c))Bb^2 - 30Bab(2(3 \sin(dx+c)^3 - 5 \sin(dx+c)) / (\sin(dx+c)^4 - 2 \sin(dx+c)^2 + 1) - 3 \log(\sin(dx+c) + 1) + 3 \log(\sin(dx+c) - 1)) - 15Ab^2(2(3 \sin(dx+c)^3 - 5 \sin(dx+c)) / (\sin(dx+c)^4 -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] 1/240*(80*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a^2 + 160*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a*b + 16*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c)*tan^3(d*x + c))*B*b^2 - 30*B*a*b*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 15*A*b^2*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 -

$2*\sin(d*x + c)^2 + 1) - 3*\log(\sin(d*x + c) + 1) + 3*\log(\sin(d*x + c) - 1))$
 $- 60*A*a^2*(2*\sin(d*x + c)/(\sin(d*x + c)^2 - 1) - \log(\sin(d*x + c) + 1) + \log(\sin(d*x + c) - 1)))/d$

mupad [B] time = 5.71, size = 359, normalized size = 1.81

$$\frac{\operatorname{atanh}\left(\frac{4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{Aa^2}{2}+\frac{3Bab}{4}+\frac{3Ab^2}{8}\right)}{2Aa^2+3Bab+\frac{3Ab^2}{2}}\right)\left(Aa^2+\frac{3Bab}{2}+\frac{3Ab^2}{4}\right)\left(2Ba^2-\frac{5Ab^2}{4}-Aa^2+2Bb^2+4Aab-\frac{5Bab}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^2)/cos(c + d*x)^3,x)`

[Out] $(\operatorname{atanh}((4*\tan(c/2 + (d*x)/2))*((A*a^2)/2 + (3*A*b^2)/8 + (3*B*a*b)/4)))/(2*A*a^2 + (3*A*b^2)/2 + 3*B*a*b))*((A*a^2 + (3*A*b^2)/4 + (3*B*a*b)/2))/d - (\tan(c/2 + (d*x)/2)^5*((20*B*a^2)/3 + (116*B*b^2)/15 + (40*A*a*b)/3) - \tan(c/2 + (d*x)/2)^9*(A*a^2 + (5*A*b^2)/4 - 2*B*a^2 - 2*B*b^2 - 4*A*a*b + (5*B*a*b)/2) - \tan(c/2 + (d*x)/2)^3*(2*A*a^2 + (A*b^2)/2 + (16*B*a^2)/3 + (8*B*b^2)/3 + (32*A*a*b)/3 + B*a*b) + \tan(c/2 + (d*x)/2)^7*(2*A*a^2 + (A*b^2)/2 - (16*B*a^2)/3 - (8*B*b^2)/3 - (32*A*a*b)/3 + B*a*b) + \tan(c/2 + (d*x)/2)*(A*a^2 + (5*A*b^2)/4 + 2*B*a^2 + 2*B*b^2 + 4*A*a*b + (5*B*a*b)/2))/(d*(5*\tan(c/2 + (d*x)/2)^2 - 10*\tan(c/2 + (d*x)/2)^4 + 10*\tan(c/2 + (d*x)/2)^6 - 5*\tan(c/2 + (d*x)/2)^8 + \tan(c/2 + (d*x)/2)^{10} - 1))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \sec(c + dx))(a + b \sec(c + dx))^2 \sec^3(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)**3*(a+b*sec(d*x+c))**2*(A+B*sec(d*x+c)),x)`

[Out] `Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**2*sec(c + d*x)**3, x)`

$$3.286 \quad \int \sec^2(c+dx)(a+b \sec(c+dx))^2(A+B \sec(c+dx)) dx$$

Optimal. Leaf size=179

$$\frac{(4a^2B + 8aAb + 3b^2B) \tanh^{-1}(\sin(c+dx))}{8d} + \frac{(-2a^2B + 8aAb + 9b^2B) \tan(c+dx) \sec(c+dx)}{24d} + \frac{(a^3(-B) + 4a^2Ab + 4aAb^2 + 3b^3) \tan(c+dx)}{6bd}$$

[Out] 1/8*(8*A*a*b+4*B*a^2+3*B*b^2)*arctanh(sin(d*x+c))/d+1/6*(4*A*a^2*b+4*A*b^3-B*a^3+8*B*a*b^2)*tan(d*x+c)/b/d+1/24*(8*A*a*b-2*B*a^2+9*B*b^2)*sec(d*x+c)*tan(d*x+c)/d+1/12*(4*A*b-B*a)*(a+b*sec(d*x+c))^2*tan(d*x+c)/b/d+1/4*B*(a+b*sec(d*x+c))^3*tan(d*x+c)/b/d

Rubi [A] time = 0.32, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {4010, 4002, 3997, 3787, 3770, 3767, 8}

$$\frac{(4a^2Ab + a^3(-B) + 8ab^2B + 4Ab^3) \tan(c+dx)}{6bd} + \frac{(4a^2B + 8aAb + 3b^2B) \tanh^{-1}(\sin(c+dx))}{8d} + \frac{(-2a^2B + 8aAb + 9b^2B) \tan(c+dx) \sec(c+dx)}{24d}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]

[Out] ((8*a*A*b + 4*a^2*B + 3*b^2*B)*ArcTanh[Sin[c + d*x]]/(8*d) + ((4*a^2*A*b + 4*A*b^3 - a^3*B + 8*a*b^2*B)*Tan[c + d*x])/(6*b*d) + ((8*a*A*b - 2*a^2*B + 9*b^2*B)*Sec[c + d*x]*Tan[c + d*x])/(24*d) + ((4*A*b - a*B)*(a + b*Sec[c + d*x])^2*Tan[c + d*x])/(12*b*d) + (B*(a + b*Sec[c + d*x])^3*Tan[c + d*x])/(4*b*d)

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 3767

Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) +
(a_.)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) +
(a_.))*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := -Simp[(b*B*Cot[e
+ f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && !LeQ[n,
-1]
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_.)]*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(cs
c[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_.)]^2*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(
csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Cs
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
0] && !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \sec^2(c+dx)(a+b\sec(c+dx))^2(A+B\sec(c+dx))dx &= \frac{B(a+b\sec(c+dx))^3 \tan(c+dx)}{4bd} + \frac{\int \sec(c+dx)(a+b\sec(c+dx))^2(A+B\sec(c+dx))dx}{4bd} \\
&= \frac{(4Ab-aB)(a+b\sec(c+dx))^2 \tan(c+dx)}{12bd} + \frac{B(a+b\sec(c+dx))^3 \tan(c+dx)}{4bd} \\
&= \frac{(8aAb-2a^2B+9b^2B)\sec(c+dx)\tan(c+dx)}{24d} + \frac{(4a^2A+2aAb+3b^2B)\sec^3(c+dx)\tan(c+dx)}{24d} \\
&= \frac{(8aAb-2a^2B+9b^2B)\sec(c+dx)\tan(c+dx)}{24d} + \frac{(4a^2A+2aAb+3b^2B)\sec^3(c+dx)\tan(c+dx)}{24d} \\
&= \frac{(8aAb+4a^2B+3b^2B)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{(8aAb+4a^2B+3b^2B)\sec^3(c+dx)\tan(c+dx)}{24d} \\
&= \frac{(8aAb+4a^2B+3b^2B)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{(4a^2A+2aAb+3b^2B)\sec^3(c+dx)\tan(c+dx)}{24d}
\end{aligned}$$

Mathematica [A] time = 0.75, size = 120, normalized size = 0.67

$$\frac{3(4a^2B+8aAb+3b^2B)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)(3(4a^2B+8aAb+3b^2B)\sec(c+dx)+24(a^2A+2aAb+3b^2B)\sec^3(c+dx)\tan(c+dx))}{24d}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]

[Out] (3*(8*a*A*b + 4*a^2*B + 3*b^2*B)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(24*(a^2*A + A*b^2 + 2*a*b*B) + 3*(8*a*A*b + 4*a^2*B + 3*b^2*B)*Sec[c + d*x] + 6*b^2*B*Sec[c + d*x]^3 + 8*b*(A*b + 2*a*B)*Tan[c + d*x]^2))/(24*d)

fricas [A] time = 0.45, size = 180, normalized size = 1.01

$$\frac{3(4Ba^2+8Aab+3Bb^2)\cos(dx+c)^4\log(\sin(dx+c)+1)-3(4Ba^2+8Aab+3Bb^2)\cos(dx+c)^4\log(-\sin(dx+c)+1)+2*(8*(3Aa^2+4Bab+2Ab^2)\cos(dx+c)^3+6Bb^2+3*(4Ba^2+8Aab+3Bb^2)\cos(dx+c)^2+8*(2Bab+Ab^2)\cos(dx+c))*\sin(dx+c)}{(d*\cos(dx+c))^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] 1/48*(3*(4*B*a^2 + 8*A*a*b + 3*B*b^2)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(4*B*a^2 + 8*A*a*b + 3*B*b^2)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(8*(3*A*a^2 + 4*B*a*b + 2*A*b^2)*cos(d*x + c)^3 + 6*B*b^2 + 3*(4*B*a^2 + 8*A*a*b + 3*B*b^2)*cos(d*x + c)^2 + 8*(2*B*a*b + A*b^2)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^4)

giac [B] time = 0.29, size = 478, normalized size = 2.67

$$3(4Ba^2 + 8Aab + 3Bb^2) \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right|\right) - 3(4Ba^2 + 8Aab + 3Bb^2) \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1\right|\right) -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] $\frac{1}{24} \cdot (3 \cdot (4 \cdot B \cdot a^2 + 8 \cdot A \cdot a \cdot b + 3 \cdot B \cdot b^2) \cdot \log(\text{abs}(\tan(1/2 \cdot d \cdot x + 1/2 \cdot c) + 1)) - 3 \cdot (4 \cdot B \cdot a^2 + 8 \cdot A \cdot a \cdot b + 3 \cdot B \cdot b^2) \cdot \log(\text{abs}(\tan(1/2 \cdot d \cdot x + 1/2 \cdot c) - 1)) - 2 \cdot (24 \cdot A \cdot a^2 \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^7 - 12 \cdot B \cdot a^2 \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^7 - 24 \cdot A \cdot a \cdot b \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^7 + 48 \cdot B \cdot a \cdot b \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^7 + 24 \cdot A \cdot b^2 \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^7 - 15 \cdot B \cdot b^2 \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^7 - 72 \cdot A \cdot a^2 \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^5 + 12 \cdot B \cdot a^2 \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^5 + 24 \cdot A \cdot a \cdot b \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^5 - 80 \cdot B \cdot a \cdot b \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^5 - 40 \cdot A \cdot b^2 \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^5 - 9 \cdot B \cdot b^2 \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^5 + 72 \cdot A \cdot a^2 \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^3 + 12 \cdot B \cdot a^2 \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^3 + 24 \cdot A \cdot a \cdot b \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^3 + 80 \cdot B \cdot a \cdot b \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^3 + 40 \cdot A \cdot b^2 \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^3 - 9 \cdot B \cdot b^2 \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^3 - 24 \cdot A \cdot a^2 \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c) - 12 \cdot B \cdot a^2 \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c) - 24 \cdot A \cdot a \cdot b \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c) - 48 \cdot B \cdot a \cdot b \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c) - 24 \cdot A \cdot b^2 \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c) - 15 \cdot B \cdot b^2 \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)) / (\tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^2 - 1)^4 / d$

maple [A] time = 1.43, size = 241, normalized size = 1.35

$$\frac{a^2 A \tan(dx+c)}{d} + \frac{a^2 B \sec(dx+c) \tan(dx+c)}{2d} + \frac{B a^2 \ln(\sec(dx+c) + \tan(dx+c))}{2d} + \frac{a A b \sec(dx+c) \tan(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^2*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x)

[Out] $a^2 \cdot A \cdot \tan(d \cdot x + c) / d + 1/2 \cdot a^2 \cdot B \cdot \sec(d \cdot x + c) \cdot \tan(d \cdot x + c) / d + 1/2 \cdot d \cdot B \cdot a^2 \cdot \ln(\sec(d \cdot x + c) + \tan(d \cdot x + c)) + a \cdot A \cdot b \cdot \sec(d \cdot x + c) \cdot \tan(d \cdot x + c) / d + 1/d \cdot A \cdot a \cdot b \cdot \ln(\sec(d \cdot x + c) + \tan(d \cdot x + c)) + 4/3 \cdot d \cdot B \cdot a \cdot b \cdot \tan(d \cdot x + c) + 2/3 \cdot d \cdot B \cdot a \cdot b \cdot \tan(d \cdot x + c) \cdot \sec(d \cdot x + c)^2 + 2/3 \cdot d \cdot A \cdot b^2 \cdot \tan(d \cdot x + c) + 1/3 \cdot d \cdot A \cdot b^2 \cdot \tan(d \cdot x + c) \cdot \sec(d \cdot x + c)^2 + 1/4 \cdot d \cdot b^2 \cdot B \cdot \tan(d \cdot x + c) \cdot \sec(d \cdot x + c)^3 + 3/8 \cdot d \cdot b^2 \cdot B \cdot \sec(d \cdot x + c) \cdot \tan(d \cdot x + c) + 3/8 \cdot d \cdot b^2 \cdot B \cdot \ln(\sec(d \cdot x + c) + \tan(d \cdot x + c))$

maxima [A] time = 0.96, size = 228, normalized size = 1.27

$$32 \left(\tan(dx+c)^3 + 3 \tan(dx+c) \right) Bab + 16 \left(\tan(dx+c)^3 + 3 \tan(dx+c) \right) Ab^2 - 3 Bb^2 \left(\frac{2(3 \sin(dx+c)^3 - 5 \sin(dx+c))}{\sin(dx+c)^4 - 2 \sin(dx+c)^2 + 1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] $\frac{1}{48}*(32*(\tan(d*x + c))^3 + 3*\tan(d*x + c))*B*a*b + 16*(\tan(d*x + c))^3 + 3*\tan(d*x + c))*A*b^2 - 3*B*b^2*(2*(3*\sin(d*x + c))^3 - 5*\sin(d*x + c))/(\sin(d*x + c)^4 - 2*\sin(d*x + c)^2 + 1) - 3*\log(\sin(d*x + c) + 1) + 3*\log(\sin(d*x + c) - 1) - 12*B*a^2*(2*\sin(d*x + c))/(\sin(d*x + c)^2 - 1) - \log(\sin(d*x + c) + 1) + \log(\sin(d*x + c) - 1) - 24*A*a*b*(2*\sin(d*x + c))/(\sin(d*x + c)^2 - 1) - \log(\sin(d*x + c) + 1) + \log(\sin(d*x + c) - 1) + 48*A*a^2*\tan(d*x + c))/d$

mupad [B] time = 5.69, size = 317, normalized size = 1.77

$$\frac{\operatorname{atanh}\left(\frac{4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{Ba^2}{2}+Aab+\frac{3Bb^2}{8}\right)}{2Ba^2+4Aab+\frac{3Bb^2}{2}}\right)\left(Ba^2+2Aab+\frac{3Bb^2}{4}\right)\left(2Aa^2+2Ab^2-Ba^2-\frac{5Bb^2}{4}-2Aab+4Bab\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^2)/cos(c + d*x)^2,x)

[Out] $(\operatorname{atanh}((4*\tan(c/2 + (d*x)/2))*((B*a^2)/2 + (3*B*b^2)/8 + A*a*b)))/(2*B*a^2 + (3*B*b^2)/2 + 4*A*a*b))*(B*a^2 + (3*B*b^2)/4 + 2*A*a*b))/d - (\tan(c/2 + (d*x)/2))^7*(2*A*a^2 + 2*A*b^2 - B*a^2 - (5*B*b^2)/4 - 2*A*a*b + 4*B*a*b) + \tan(c/2 + (d*x)/2)^3*(6*A*a^2 + (10*A*b^2)/3 + B*a^2 - (3*B*b^2)/4 + 2*A*a*b + (20*B*a*b)/3) - \tan(c/2 + (d*x)/2)^5*(6*A*a^2 + (10*A*b^2)/3 - B*a^2 + (3*B*b^2)/4 - 2*A*a*b + (20*B*a*b)/3) - \tan(c/2 + (d*x)/2)*(2*A*a^2 + 2*A*b^2 + B*a^2 + (5*B*b^2)/4 + 2*A*a*b + 4*B*a*b))/(d*(6*\tan(c/2 + (d*x)/2)^4 - 4*\tan(c/2 + (d*x)/2)^2 - 4*\tan(c/2 + (d*x)/2)^6 + \tan(c/2 + (d*x)/2)^8 + 1))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \sec(c + dx))(a + b \sec(c + dx))^2 \sec^2(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**2*(a+b*sec(d*x+c))**2*(A+B*sec(d*x+c)),x)

[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**2*sec(c + d*x)**2, x)

$$3.287 \quad \int \sec(c + dx)(a + b \sec(c + dx))^2(A + B \sec(c + dx)) dx$$

Optimal. Leaf size=116

$$\frac{2(a^2B + 3aAb + b^2B) \tan(c + dx)}{3d} + \frac{(2a^2A + 2abB + Ab^2) \tanh^{-1}(\sin(c + dx))}{2d} + \frac{b(2aB + 3Ab) \tan(c + dx) \sec(c + dx)}{6d}$$

[Out] 1/2*(2*A*a^2+A*b^2+2*B*a*b)*arctanh(sin(d*x+c))/d+2/3*(3*A*a*b+B*a^2+B*b^2)*tan(d*x+c)/d+1/6*b*(3*A*b+2*B*a)*sec(d*x+c)*tan(d*x+c)/d+1/3*B*(a+b*sec(d*x+c))^2*tan(d*x+c)/d

Rubi [A] time = 0.18, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {4002, 3997, 3787, 3770, 3767, 8}

$$\frac{2(a^2B + 3aAb + b^2B) \tan(c + dx)}{3d} + \frac{(2a^2A + 2abB + Ab^2) \tanh^{-1}(\sin(c + dx))}{2d} + \frac{b(2aB + 3Ab) \tan(c + dx) \sec(c + dx)}{6d}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d*x]*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]

[Out] ((2*a^2*A + A*b^2 + 2*a*b*B)*ArcTanh[Sin[c + d*x]])/(2*d) + (2*(3*a*A*b + a^2*B + b^2*B)*Tan[c + d*x])/(3*d) + (b*(3*A*b + 2*a*B)*Sec[c + d*x]*Tan[c + d*x])/(6*d) + (B*(a + b*Sec[c + d*x])^2*Tan[c + d*x])/(3*d)

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 3767

Int[csc[(c_) + (d_)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 3770

Int[csc[(c_) + (d_)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) +
(a_.)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) +
(a_.))*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> -Simp[(b*B*Cot[e
+ f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && !LeQ[n,
-1]
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_.)]*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(cs
c[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
 \int \sec(c + dx)(a + b \sec(c + dx))^2(A + B \sec(c + dx)) dx &= \frac{B(a + b \sec(c + dx))^2 \tan(c + dx)}{3d} + \frac{1}{3} \int \sec(c + dx)(a + b \sec(c + dx))^2(A + B \sec(c + dx)) dx \\
 &= \frac{b(3Ab + 2aB) \sec(c + dx) \tan(c + dx)}{6d} + \frac{B(a + b \sec(c + dx))^2 \tan(c + dx)}{3d} \\
 &= \frac{b(3Ab + 2aB) \sec(c + dx) \tan(c + dx)}{6d} + \frac{B(a + b \sec(c + dx))^2 \tan(c + dx)}{3d} \\
 &= \frac{(2a^2A + Ab^2 + 2abB) \tanh^{-1}(\sin(c + dx))}{2d} + \frac{b(3Ab + 2aB) \sec(c + dx) \tan(c + dx)}{6d} \\
 &= \frac{(2a^2A + Ab^2 + 2abB) \tanh^{-1}(\sin(c + dx))}{2d} + \frac{2(3aAb + 2a^2B) \sec(c + dx) \tan(c + dx)}{6d}
 \end{aligned}$$

Mathematica [A] time = 0.49, size = 92, normalized size = 0.79

$$\frac{3(2a^2A + 2abB + Ab^2) \tanh^{-1}(\sin(c + dx)) + \tan(c + dx) (2(3a^2B + 6aAb + b^2B \tan^2(c + dx) + 3b^2B) + 3b(2aAb + 2a^2B) \sec(c + dx) \tan(c + dx))}{6d}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[c + d*x]*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]

[Out] (3*(2*a^2*A + A*b^2 + 2*a*b*B)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(3*b*(A*b + 2*a*B)*Sec[c + d*x] + 2*(6*a*A*b + 3*a^2*B + 3*b^2*B + b^2*B*Tan[c + d*x]^2)))/(6*d)

fricas [A] time = 0.45, size = 150, normalized size = 1.29

$$\frac{3(2Aa^2 + 2Bab + Ab^2) \cos(dx + c)^3 \log(\sin(dx + c) + 1) - 3(2Aa^2 + 2Bab + Ab^2) \cos(dx + c)^3 \log(-\sin(dx + c) + 1)}{12d \cos(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] 1/12*(3*(2*A*a^2 + 2*B*a*b + A*b^2)*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*(2*A*a^2 + 2*B*a*b + A*b^2)*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + 2*(2*B*b^2 + 2*(3*B*a^2 + 6*A*a*b + 2*B*b^2)*cos(d*x + c)^2 + 3*(2*B*a*b + A*b^2)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)

giac [B] time = 0.43, size = 294, normalized size = 2.53

$$3(2Aa^2 + 2Bab + Ab^2) \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right|\right) - 3(2Aa^2 + 2Bab + Ab^2) \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1\right|\right) - \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] 1/6*(3*(2*A*a^2 + 2*B*a*b + A*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(2*A*a^2 + 2*B*a*b + A*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(6*B*a^2*tan(1/2*d*x + 1/2*c)^5 + 12*A*a*b*tan(1/2*d*x + 1/2*c)^5 - 6*B*a*b*tan(1/2*d*x + 1/2*c)^5 - 3*A*b^2*tan(1/2*d*x + 1/2*c)^5 + 6*B*b^2*tan(1/2*d*x + 1/2*c)^5 - 12*B*a^2*tan(1/2*d*x + 1/2*c)^3 - 24*A*a*b*tan(1/2*d*x + 1/2*c)^3 - 4*B*b^2*tan(1/2*d*x + 1/2*c)^3 + 6*B*a^2*tan(1/2*d*x + 1/2*c) + 12*A*a*b*tan(1/2*d*x + 1/2*c) + 6*B*a*b*tan(1/2*d*x + 1/2*c) + 3*A*b^2*tan(1/2*d*x + 1/2*c) + 6*B*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3/d

maple [A] time = 1.18, size = 174, normalized size = 1.50

$$\frac{a^2 A \ln(\sec(dx + c) + \tan(dx + c))}{d} + \frac{a^2 B \tan(dx + c)}{d} + \frac{2aAb \tan(dx + c)}{d} + \frac{Bab \sec(dx + c) \tan(dx + c)}{d} + \frac{Bab^2 \sec(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sec(d*x+c)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x)`

[Out] $1/d*a^2*A*\ln(\sec(d*x+c)+\tan(d*x+c))+a^2*B*\tan(d*x+c)/d+2*a*A*b*\tan(d*x+c)/d+1/d*B*a*b*\sec(d*x+c)*\tan(d*x+c)+1/d*B*a*b*\ln(\sec(d*x+c)+\tan(d*x+c))+1/2/d*A*b^2*\sec(d*x+c)*\tan(d*x+c)+1/2/d*A*b^2*\ln(\sec(d*x+c)+\tan(d*x+c))+2/3*b^2*B*\tan(d*x+c)/d+1/3/d*b^2*B*\tan(d*x+c)*\sec(d*x+c)^2$

maxima [A] time = 1.66, size = 165, normalized size = 1.42

$$4 \left(\tan(dx+c)^3 + 3 \tan(dx+c) \right) B b^2 - 6 B a b \left(\frac{2 \sin(dx+c)}{\sin(dx+c)^2-1} - \log(\sin(dx+c)+1) + \log(\sin(dx+c)-1) \right) - 3 A b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="maxima")`

[Out] $1/12*(4*(\tan(dx+c)^3+3*\tan(dx+c))*B*b^2-6*B*a*b*(2*\sin(dx+c)/(\sin(dx+c)^2-1)-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1))-3*A*b^2*(2*\sin(dx+c)/(\sin(dx+c)^2-1)-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1))+12*A*a^2*\log(\sec(dx+c)+\tan(dx+c))+12*B*a^2*\tan(dx+c)+24*A*a*b*\tan(dx+c))/d$

mupad [B] time = 5.44, size = 227, normalized size = 1.96

$$\frac{\operatorname{atanh}\left(\frac{4 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(A a^2+B a b+\frac{A b^2}{2}\right)}{4 A a^2+4 B a b+2 A b^2}\right)\left(2 A a^2+2 B a b+A b^2\right)}{d} - \frac{\left(2 B a^2-A b^2+2 B b^2+4 A a b-2 B a b\right) \tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A+B/cos(c+d*x))*(a+b/cos(c+d*x))^2)/cos(c+d*x),x)`

[Out] $(\operatorname{atanh}((4*\tan(c/2+(d*x)/2)*(A*a^2+(A*b^2)/2+B*a*b))/(4*A*a^2+2*A*b^2+4*B*a*b))*(2*A*a^2+A*b^2+2*B*a*b))/d - (\tan(c/2+(d*x)/2)*(A*b^2+2*B*a^2+2*B*b^2+4*A*a*b+2*B*a*b) - \tan(c/2+(d*x)/2)^3*(4*B*a^2+(4*B*b^2)/3+8*A*a*b) + \tan(c/2+(d*x)/2)^5*(2*B*a^2-A*b^2+2*B*b^2+4*A*a*b-2*B*a*b))/(d*(3*\tan(c/2+(d*x)/2)^2-3*\tan(c/2+(d*x)/2)^4+\tan(c/2+(d*x)/2)^6-1))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \sec(c + dx)) (a + b \sec(c + dx))^2 \sec(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))**2*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**2*sec(c + d*x), x)
```

3.288 $\int (a + b \sec(c + dx))^2 (A + B \sec(c + dx)) dx$

Optimal. Leaf size=86

$$\frac{(2a^2B + 4aAb + b^2B) \tanh^{-1}(\sin(c + dx))}{2d} + a^2Ax + \frac{b(3aB + 2Ab) \tan(c + dx)}{2d} + \frac{bB \tan(c + dx)(a + b \sec(c + dx))}{2d}$$

[Out] $a^2Ax + \frac{1}{2} \frac{(4Aab + 2Bb^2 + 2Ba^2) \operatorname{arctanh}(\sin(dx+c))}{d} + \frac{1}{2} \frac{b(2Ab + 3Ba) \tan(dx+c)}{d} + \frac{1}{2} \frac{bB \tan(dx+c) \sec(dx+c)}{d}$

Rubi [A] time = 0.08, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3918, 3770, 3767, 8}

$$\frac{(2a^2B + 4aAb + b^2B) \tanh^{-1}(\sin(c + dx))}{2d} + a^2Ax + \frac{b(3aB + 2Ab) \tan(c + dx)}{2d} + \frac{bB \tan(c + dx)(a + b \sec(c + dx))}{2d}$$

Antiderivative was successfully verified.

[In] `Int[(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]`

[Out] $a^2Ax + \frac{(4aAb + 2a^2B + b^2B) \operatorname{ArcTanh}[\sin[c + d*x]]}{2d} + \frac{b(2Ab + 3aB) \tan[c + d*x]}{2d} + \frac{bB \sec[c + d*x] \tan[c + d*x]}{2d}$

Rule 8

`Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

Rule 3767

`Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]`

Rule 3770

`Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]`

Rule 3918

`Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.)), x_Symbol] := -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1))/(f*m), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m - 2)*Simp[a^2*c*m + (b^2*d*(m - 1) + 2*a*b*c*m + a^2*d*m)*Csc[e + f*x] + b*(b*c*m + a*d*(2*m -`

1)) * Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && GtQ[m, 1] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m]

Rubi steps

$$\begin{aligned} \int (a + b \sec(c + dx))^2 (A + B \sec(c + dx)) dx &= \frac{bB(a + b \sec(c + dx)) \tan(c + dx)}{2d} + \frac{1}{2} \int (2a^2 A + (4aAb + 2a^2 B) \sec(c + dx)) dx \\ &= a^2 Ax + \frac{bB(a + b \sec(c + dx)) \tan(c + dx)}{2d} + \frac{1}{2} (b(2Ab + 3aB)) \int \sec(c + dx) dx \\ &= a^2 Ax + \frac{(4aAb + 2a^2 B + b^2 B) \tanh^{-1}(\sin(c + dx))}{2d} + \frac{bB(a + b \sec(c + dx)) \tan(c + dx)}{2d} \\ &= a^2 Ax + \frac{(4aAb + 2a^2 B + b^2 B) \tanh^{-1}(\sin(c + dx))}{2d} + \frac{b(2Ab + 3aB) \sec(c + dx)}{2d} \end{aligned}$$

Mathematica [A] time = 0.28, size = 67, normalized size = 0.78

$$\frac{(2a^2 B + 4aAb + b^2 B) \tanh^{-1}(\sin(c + dx)) + 2a^2 A dx + b \tan(c + dx)(4aB + 2Ab + bB \sec(c + dx))}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]

[Out] (2*a^2*A*d*x + (4*a*A*b + 2*a^2*B + b^2*B)*ArcTanh[Sin[c + d*x]] + b*(2*A*b + 4*a*B + b*B*Sec[c + d*x])*Tan[c + d*x])/(2*d)

fricas [A] time = 0.46, size = 136, normalized size = 1.58

$$\frac{4 A a^2 dx \cos(dx + c)^2 + (2 B a^2 + 4 A a b + B b^2) \cos(dx + c)^2 \log(\sin(dx + c) + 1) - (2 B a^2 + 4 A a b + B b^2) \cos(dx + c)}{4 d \cos(dx + c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] 1/4*(4*A*a^2*d*x*cos(d*x + c)^2 + (2*B*a^2 + 4*A*a*b + B*b^2)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (2*B*a^2 + 4*A*a*b + B*b^2)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(B*b^2 + 2*(2*B*a*b + A*b^2)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2)

giac [B] time = 0.29, size = 192, normalized size = 2.23

$$2(dx+c)Aa^2 + (2Ba^2 + 4Aab + Bb^2) \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right|\right) - (2Ba^2 + 4Aab + Bb^2) \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] $\frac{1}{2}*(2*(d*x + c)*A*a^2 + (2*B*a^2 + 4*A*a*b + B*b^2)*\log(\text{abs}(\tan(1/2*d*x + 1/2*c) + 1)) - (2*B*a^2 + 4*A*a*b + B*b^2)*\log(\text{abs}(\tan(1/2*d*x + 1/2*c) - 1))) - 2*(4*B*a*b*\tan(1/2*d*x + 1/2*c)^3 + 2*A*b^2*\tan(1/2*d*x + 1/2*c)^3 - B*b^2*\tan(1/2*d*x + 1/2*c)^3 - 4*B*a*b*\tan(1/2*d*x + 1/2*c) - 2*A*b^2*\tan(1/2*d*x + 1/2*c) - B*b^2*\tan(1/2*d*x + 1/2*c))/(\tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d$

maple [A] time = 0.94, size = 133, normalized size = 1.55

$$a^2 Ax + \frac{Aa^2c}{d} + \frac{Ba^2 \ln(\sec(dx+c) + \tan(dx+c))}{d} + \frac{2Aab \ln(\sec(dx+c) + \tan(dx+c))}{d} + \frac{2Bab \tan(dx+c)}{d} + \frac{Aa^2}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x)

[Out] $a^2Ax + 1/dAa^2c + 1/dBa^2 \ln(\sec(d*x+c) + \tan(d*x+c)) + 2/dAa*b \ln(\sec(d*x+c) + \tan(d*x+c)) + 2/dBab \tan(d*x+c) + 1/dAa^2 \tan(d*x+c) + 1/2/dB^2 \ln(\sec(d*x+c) + \tan(d*x+c)) + 1/2/dB^2 \tan(d*x+c) + 1/2/dB^2 \ln(\sec(d*x+c) + \tan(d*x+c))$

maxima [A] time = 1.39, size = 126, normalized size = 1.47

$$\frac{4(dx+c)Aa^2 - Bb^2 \left(\frac{2 \sin(dx+c)}{\sin(dx+c)^2 - 1} - \log(\sin(dx+c) + 1) + \log(\sin(dx+c) - 1) \right) + 4Ba^2 \log(\sec(dx+c) + \tan(dx+c))}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] $\frac{1}{4}*(4*(d*x + c)*A*a^2 - B*b^2*(2*\sin(d*x + c)/(\sin(d*x + c)^2 - 1) - \log(\sin(d*x + c) + 1) + \log(\sin(d*x + c) - 1)) + 4*B*a^2*\log(\sec(d*x + c) + \tan(d*x + c)) + 8*A*a*b*\log(\sec(d*x + c) + \tan(d*x + c)) + 8*B*a*b*\tan(d*x + c) + 4*A*b^2*\tan(d*x + c))/d$

mupad [B] time = 2.74, size = 176, normalized size = 2.05

$$2 \left(A a^2 \operatorname{atan} \left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)} \right) + B a^2 \operatorname{atanh} \left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)} \right) + \frac{B b^2 \operatorname{atanh} \left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)} \right)}{2} + 2 A a b \operatorname{atanh} \left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)} \right) \right) \frac{1}{d} + \frac{A b^2 \sin(2c + 2dx)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^2, x)`

[Out] `(2*(A*a^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) + B*a^2*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) + (B*b^2*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/2 + 2*A*a*b*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + ((A*b^2*sin(2*c + 2*d*x))/2 + (B*b^2*sin(c + d*x))/2 + B*a*b*sin(2*c + 2*d*x))/(d*(cos(2*c + 2*d*x)/2 + 1/2))`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \sec(c + dx))(a + b \sec(c + dx))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*sec(d*x+c))*2*(A+B*sec(d*x+c)), x)`

[Out] `Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))*2, x)`

$$3.289 \quad \int \cos(c + dx)(a + b \sec(c + dx))^2(A + B \sec(c + dx)) dx$$

Optimal. Leaf size=60

$$\frac{a^2 A \sin(c + dx)}{d} + \frac{b(2aB + Ab) \tanh^{-1}(\sin(c + dx))}{d} + ax(aB + 2Ab) + \frac{b^2 B \tan(c + dx)}{d}$$

[Out] $a*(2*A*b+B*a)*x+b*(A*b+2*B*a)*\arctanh(\sin(d*x+c))/d+a^2*A*\sin(d*x+c)/d+b^2*B*\tan(d*x+c)/d$

Rubi [A] time = 0.10, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {4024, 3770, 3767, 8}

$$\frac{a^2 A \sin(c + dx)}{d} + \frac{b(2aB + Ab) \tanh^{-1}(\sin(c + dx))}{d} + ax(aB + 2Ab) + \frac{b^2 B \tan(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] `Int[Cos[c + d*x]*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]`

[Out] $a*(2*A*b + a*B)*x + (b*(A*b + 2*a*B)*\text{ArcTanh}[\text{Sin}[c + d*x]])/d + (a^2*A*\text{Sin}[c + d*x])/d + (b^2*B*\text{Tan}[c + d*x])/d$

Rule 8

`Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

Rule 3767

`Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]`

Rule 3770

`Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]`

Rule 4024

`Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^2*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] := Simp[(a^2*A*Cos[e + f*x]*(d*Csc[e + f*x])^(n + 1))/(d*f*n), x] + Dist[1/(d*n), Int[(d*Csc[`

$e + f*x])^{(n + 1)*(a*(2*A*b + a*B)*n + (2*a*b*B*n + A*(b^2*n + a^2*(n + 1)))}$
 $*Csc[e + f*x] + b^2*B*n*Csc[e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, A,$
 $, B}, x] \&\& NeQ[A*b - a*B, 0] \&\& NeQ[a^2 - b^2, 0] \&\& LeQ[n, -1]$

Rubi steps

$$\begin{aligned} \int \cos(c + dx)(a + b \sec(c + dx))^2(A + B \sec(c + dx)) dx &= \frac{a^2 A \sin(c + dx)}{d} - \int (-a(2Ab + aB) + (-Ab^2 - 2ab \\ &= a(2Ab + aB)x + \frac{a^2 A \sin(c + dx)}{d} + (b^2 B) \int \sec^2(c + dx) dx \\ &= a(2Ab + aB)x + \frac{b(Ab + 2aB) \tanh^{-1}(\sin(c + dx))}{d} + \\ &= a(2Ab + aB)x + \frac{b(Ab + 2aB) \tanh^{-1}(\sin(c + dx))}{d} + \end{aligned}$$

Mathematica [A] time = 0.49, size = 109, normalized size = 1.82

$$\frac{a^2 A \sin(c + dx) + a(c + dx)(aB + 2Ab) - b(2aB + Ab) \log\left(\cos\left(\frac{1}{2}(c + dx)\right) - \sin\left(\frac{1}{2}(c + dx)\right)\right) + b(2aB + Ab) \log\left(\cos\left(\frac{1}{2}(c + dx)\right) + \sin\left(\frac{1}{2}(c + dx)\right)\right)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]

[Out] (a*(2*A*b + a*B)*(c + d*x) - b*(A*b + 2*a*B)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + b*(A*b + 2*a*B)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + a^2 *A*Sin[c + d*x] + b^2*B*Tan[c + d*x])/d

fricas [A] time = 0.48, size = 117, normalized size = 1.95

$$\frac{2(Ba^2 + 2Aab)dx \cos(dx + c) + (2Bab + Ab^2) \cos(dx + c) \log(\sin(dx + c) + 1) - (2Bab + Ab^2) \cos(dx + c) \log(\sin(dx + c) - 1)}{2d \cos(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] 1/2*(2*(B*a^2 + 2*A*a*b)*d*x*cos(d*x + c) + (2*B*a*b + A*b^2)*cos(d*x + c)*log(sin(d*x + c) + 1) - (2*B*a*b + A*b^2)*cos(d*x + c)*log(-sin(d*x + c) + 1) + 2*(A*a^2*cos(d*x + c) + B*b^2)*sin(d*x + c))/(d*cos(d*x + c))

giac [B] time = 0.28, size = 154, normalized size = 2.57

$$\frac{(Ba^2 + 2Aab)(dx + c) + (2Bab + Ab^2) \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right|\right) - (2Bab + Ab^2) \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1\right|\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] ((B*a^2 + 2*A*a*b)*(d*x + c) + (2*B*a*b + A*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - (2*B*a*b + A*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(A*a^2*tan(1/2*d*x + 1/2*c)^3 - B*b^2*tan(1/2*d*x + 1/2*c)^3 - A*a^2*tan(1/2*d*x + 1/2*c) - B*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^4 - 1)/d

maple [A] time = 0.87, size = 104, normalized size = 1.73

$$2Axab+a^2Bx+\frac{a^2A \sin(dx+c)}{d}+\frac{Ab^2 \ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{2Aabc}{d}+\frac{b^2B \tan(dx+c)}{d}+\frac{2Bab \ln(\sec(dx+c)+\tan(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x)

[Out] 2*A*x*a*b+a^2*B*x+1/d*a^2*A*sin(d*x+c)+1/d*A*b^2*ln(sec(d*x+c)+tan(d*x+c))+2/d*A*a*b*c+b^2*B*tan(d*x+c)/d+2/d*B*a*b*ln(sec(d*x+c)+tan(d*x+c))+1/d*B*a^2*c

maxima [A] time = 0.91, size = 103, normalized size = 1.72

$$\frac{2(dx+c)Ba^2+4(dx+c)Aab+2Bab(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))+Ab^2(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] 1/2*(2*(d*x + c)*B*a^2 + 4*(d*x + c)*A*a*b + 2*B*a*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + A*b^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 2*A*a^2*sin(d*x + c) + 2*B*b^2*tan(d*x + c))/d

mapad [B] time = 2.55, size = 163, normalized size = 2.72

$$\frac{Bb^2 \tan(c+dx)}{d} + \frac{2Ba^2 \operatorname{atan}\left(\frac{\sin\left(\frac{c+dx}{2}\right)}{\cos\left(\frac{c+dx}{2}\right)}\right)}{d} + \frac{2Ab^2 \operatorname{atanh}\left(\frac{\sin\left(\frac{c+dx}{2}\right)}{\cos\left(\frac{c+dx}{2}\right)}\right)}{d} + \frac{Aa^2 \sin(2c+2dx)}{2d \cos(c+dx)} + \frac{4Aab \operatorname{atan}\left(\frac{\sin\left(\frac{c+dx}{2}\right)}{\cos\left(\frac{c+dx}{2}\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^2,x)
```

```
[Out] (B*b^2*tan(c + d*x))/d + (2*B*a^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (2*A*b^2*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (A*a^2*sin(2*c + 2*d*x))/(2*d*cos(c + d*x)) + (4*A*a*b*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (4*B*a*b*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \sec(c + dx))(a + b \sec(c + dx))^2 \cos(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))**2*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**2*cos(c + d*x), x)
```

$$3.290 \quad \int \cos^2(c+dx)(a+b \sec(c+dx))^2(A+B \sec(c+dx)) dx$$

Optimal. Leaf size=80

$$\frac{1}{2}x(a^2A + 4abB + 2Ab^2) + \frac{a^2A \sin(c+dx) \cos(c+dx)}{2d} + \frac{a(aB + 2Ab) \sin(c+dx)}{d} + \frac{b^2B \tanh^{-1}(\sin(c+dx))}{d}$$

[Out] $1/2*(A*a^2+2*A*b^2+4*B*a*b)*x+b^2*B*\arctanh(\sin(d*x+c))/d+a*(2*A*b+B*a)*\sin(d*x+c)/d+1/2*a^2*A*\cos(d*x+c)*\sin(d*x+c)/d$

Rubi [A] time = 0.17, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {4024, 4047, 8, 4045, 3770}

$$\frac{1}{2}x(a^2A + 4abB + 2Ab^2) + \frac{a^2A \sin(c+dx) \cos(c+dx)}{2d} + \frac{a(aB + 2Ab) \sin(c+dx)}{d} + \frac{b^2B \tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

[In] `Int[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]`

[Out] $((a^2*A + 2*A*b^2 + 4*a*b*B)*x)/2 + (b^2*B*\text{ArcTanh}[\text{Sin}[c + d*x]])/d + (a*(2*A*b + a*B)*\text{Sin}[c + d*x])/d + (a^2*A*\text{Cos}[c + d*x]*\text{Sin}[c + d*x])/(2*d)$

Rule 8

`Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

Rule 3770

`Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]`

Rule 4024

`Int[(csc[(e_.) + (f_.)*(x_)])*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)])*(b_.) + (a_.))^(2*(csc[(e_.) + (f_.)*(x_)])*(B_.) + (A_.)), x_Symbol] := Simp[(a^2*A*Cos[e + f*x]*(d*Csc[e + f*x])^(n + 1))/(d*f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*(a*(2*A*b + a*B)*n + (2*a*b*B*n + A*(b^2*n + a^2*(n + 1)))*Csc[e + f*x] + b^2*B*n*Csc[e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]`

Rule 4045

```
Int[(csc[(e_) + (f_)*(x_)]*(b_))^(m_)*(csc[(e_) + (f_)*(x_)]^2*(C_)
+ (A_)), x_Symbol] := Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 4047

```
Int[(csc[(e_) + (f_)*(x_)]*(b_))^(m_)*((A_) + csc[(e_) + (f_)*(x_)]*
(B_) + csc[(e_) + (f_)*(x_)]^2*(C_)), x_Symbol] := Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\begin{aligned} \int \cos^2(c + dx)(a + b \sec(c + dx))^2(A + B \sec(c + dx)) dx &= \frac{a^2 A \cos(c + dx) \sin(c + dx)}{2d} - \frac{1}{2} \int \cos(c + dx) (-2) \\ &= \frac{a^2 A \cos(c + dx) \sin(c + dx)}{2d} - \frac{1}{2} \int \cos(c + dx) (-2) \\ &= \frac{1}{2} (a^2 A + 2Ab^2 + 4abB) x + \frac{a(2Ab + aB) \sin(c + dx)}{d} \\ &= \frac{1}{2} (a^2 A + 2Ab^2 + 4abB) x + \frac{b^2 B \tanh^{-1}(\sin(c + dx))}{d} \end{aligned}$$

Mathematica [A] time = 0.23, size = 120, normalized size = 1.50

$$\frac{2(c + dx)(a^2 A + 4abB + 2Ab^2) + a^2 A \sin(2(c + dx)) + 4a(aB + 2Ab) \sin(c + dx) - 4b^2 B \log\left(\cos\left(\frac{1}{2}(c + dx)\right)\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*(a^2*A + 2*A*b^2 + 4*a*b*B)*(c + d*x) - 4*b^2*B*Log[Cos[(c + d*x)/2]] - S
in[(c + d*x)/2]] + 4*b^2*B*Log[Cos[(c + d*x)/2]] + Sin[(c + d*x)/2]] + 4*a*(
2*A*b + a*B)*Sin[c + d*x] + a^2*A*Sin[2*(c + d*x)]/(4*d)
```

fricas [A] time = 0.47, size = 87, normalized size = 1.09

$$\frac{Bb^2 \log(\sin(dx + c) + 1) - Bb^2 \log(-\sin(dx + c) + 1) + (Aa^2 + 4Bab + 2Ab^2)dx + (Aa^2 \cos(dx + c) + 2Ba^2 \sin(dx + c))}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] 1/2*(B*b^2*log(sin(d*x + c) + 1) - B*b^2*log(-sin(d*x + c) + 1) + (A*a^2 + 4*B*a*b + 2*A*b^2)*d*x + (A*a^2*cos(d*x + c) + 2*B*a^2 + 4*A*a*b)*sin(d*x + c))/d

giac [B] time = 0.45, size = 178, normalized size = 2.22

$$\frac{2 B b^2 \log \left(\left| \tan \left(\frac{1}{2} d x + \frac{1}{2} c \right) + 1 \right| \right) - 2 B b^2 \log \left(\left| \tan \left(\frac{1}{2} d x + \frac{1}{2} c \right) - 1 \right| \right) + (A a^2 + 4 B a b + 2 A b^2)(d x + c) - \frac{2 \left(A a^2 \tan \left(\frac{1}{2} d x + \frac{1}{2} c \right) + 1 \right)}{2 d}}{2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] 1/2*(2*B*b^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 2*B*b^2*log(abs(tan(1/2*d*x + 1/2*c) - 1))) + (A*a^2 + 4*B*a*b + 2*A*b^2)*(d*x + c) - 2*(A*a^2*tan(1/2*d*x + 1/2*c)^3 - 2*B*a^2*tan(1/2*d*x + 1/2*c)^3 - 4*A*a*b*tan(1/2*d*x + 1/2*c)^3 - A*a^2*tan(1/2*d*x + 1/2*c) - 2*B*a^2*tan(1/2*d*x + 1/2*c) - 4*A*a*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2/d

maple [A] time = 0.71, size = 120, normalized size = 1.50

$$\frac{a^2 A \cos(dx + c) \sin(dx + c)}{2d} + \frac{a^2 A x}{2} + \frac{A a^2 c}{2d} + \frac{B a^2 \sin(dx + c)}{d} + \frac{2 A a b \sin(dx + c)}{d} + 2 B x a b + \frac{2 B a b c}{d} + A x b^2 + \frac{A b^2 c}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^2*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x)

[Out] 1/2*a^2*A*cos(d*x+c)*sin(d*x+c)/d+1/2*a^2*A*x+1/2/d*A*a^2*c+1/d*B*a^2*sin(d*x+c)+2/d*A*a*b*sin(d*x+c)+2*B*x*a*b+2/d*B*a*b*c+A*x*b^2+1/d*A*b^2*c+1/d*b^2*B*ln(sec(d*x+c)+tan(d*x+c))

maxima [A] time = 0.88, size = 99, normalized size = 1.24

$$\frac{(2 d x + 2 c + \sin(2 d x + 2 c)) A a^2 + 8 (d x + c) B a b + 4 (d x + c) A b^2 + 2 B b^2 (\log(\sin(dx + c) + 1) - \log(\sin(dx + c) - 1))}{4 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] $1/4*((2*d*x + 2*c + \sin(2*d*x + 2*c))*A*a^2 + 8*(d*x + c)*B*a*b + 4*(d*x + c)*A*b^2 + 2*B*b^2*(\log(\sin(d*x + c) + 1) - \log(\sin(d*x + c) - 1)) + 4*B*a^2*\sin(d*x + c) + 8*A*a*b*\sin(d*x + c))/d$

mupad [B] time = 2.38, size = 169, normalized size = 2.11

$$\frac{B a^2 \sin(c + d x)}{d} + \frac{A a^2 \operatorname{atan}\left(\frac{\sin\left(\frac{c}{2} + \frac{d x}{2}\right)}{\cos\left(\frac{c}{2} + \frac{d x}{2}\right)}\right)}{d} + \frac{2 A b^2 \operatorname{atan}\left(\frac{\sin\left(\frac{c}{2} + \frac{d x}{2}\right)}{\cos\left(\frac{c}{2} + \frac{d x}{2}\right)}\right)}{d} + \frac{2 B b^2 \operatorname{atanh}\left(\frac{\sin\left(\frac{c}{2} + \frac{d x}{2}\right)}{\cos\left(\frac{c}{2} + \frac{d x}{2}\right)}\right)}{d} + \frac{A a^2 \sin(2 c + 2 d x)}{4 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(c + d*x)^2*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^2,x)`

[Out] $(B*a^2*\sin(c + d*x))/d + (A*a^2*\operatorname{atan}(\sin(c/2 + (d*x)/2)/\cos(c/2 + (d*x)/2)))/d + (2*A*b^2*\operatorname{atan}(\sin(c/2 + (d*x)/2)/\cos(c/2 + (d*x)/2)))/d + (2*B*b^2*\operatorname{atanh}(\sin(c/2 + (d*x)/2)/\cos(c/2 + (d*x)/2)))/d + (A*a^2*\sin(2*c + 2*d*x))/(4*d) + (2*A*a*b*\sin(c + d*x))/d + (4*B*a*b*\operatorname{atan}(\sin(c/2 + (d*x)/2)/\cos(c/2 + (d*x)/2)))/d$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \sec(c + dx))(a + b \sec(c + dx))^2 \cos^2(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)**2*(a+b*sec(d*x+c))**2*(A+B*sec(d*x+c)),x)`

[Out] `Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**2*cos(c + d*x)**2, x)`

$$3.291 \quad \int \cos^3(c+dx)(a+b \sec(c+dx))^2(A+B \sec(c+dx)) dx$$

Optimal. Leaf size=107

$$\frac{(2a^2A + 6abB + 3Ab^2) \sin(c+dx)}{3d} + \frac{1}{2}x(a^2B + 2aAb + 2b^2B) + \frac{a^2A \sin(c+dx) \cos^2(c+dx)}{3d} + \frac{a(aB + 2Ab) \sin(c+dx)}{2d}$$

[Out] $1/2*(2*A*a*b+B*a^2+2*B*b^2)*x+1/3*(2*A*a^2+3*A*b^2+6*B*a*b)*\sin(dx+c)/d+1/2*a*(2*A*b+B*a)*\cos(dx+c)*\sin(dx+c)/d+1/3*a^2*A*\cos(dx+c)^2*\sin(dx+c)/d$

Rubi [A] time = 0.22, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {4024, 4047, 2637, 4045, 8}

$$\frac{(2a^2A + 6abB + 3Ab^2) \sin(c+dx)}{3d} + \frac{1}{2}x(a^2B + 2aAb + 2b^2B) + \frac{a^2A \sin(c+dx) \cos^2(c+dx)}{3d} + \frac{a(aB + 2Ab) \sin(c+dx)}{2d}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]), x]

[Out] $((2*a*A*b + a^2*B + 2*b^2*B)*x)/2 + ((2*a^2*A + 3*A*b^2 + 6*a*b*B)*\sin[c + d*x])/(3*d) + (a*(2*A*b + a*B)*\cos[c + d*x]*\sin[c + d*x])/(2*d) + (a^2*A*\cos[c + d*x]^2*\sin[c + d*x])/(3*d)$

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_.)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 4024

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^2*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := Simp[(a^2*A*Cos[e + f*x]*(d*Csc[e + f*x])^(n + 1))/(d*f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*(a*(2*A*b + a*B)*n + (2*a*b*B*n + A*(b^2*n + a^2*(n + 1)))*Csc[e + f*x] + b^2*B*n*Csc[e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]

Rule 4045

Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]^2*(C_.) + (A_.)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] + Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]

Rule 4047

Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2), x] /; FreeQ[{b, e, f, A, B, C, m}, x]

Rubi steps

$$\begin{aligned} \int \cos^3(c + dx)(a + b \sec(c + dx))^2(A + B \sec(c + dx)) dx &= \frac{a^2 A \cos^2(c + dx) \sin(c + dx)}{3d} - \frac{1}{3} \int \cos^2(c + dx) (a + b \sec(c + dx))^2(A + B \sec(c + dx)) dx \\ &= \frac{a^2 A \cos^2(c + dx) \sin(c + dx)}{3d} - \frac{1}{3} \int \cos^2(c + dx) (a + b \sec(c + dx))^2(A + B \sec(c + dx)) dx \\ &= \frac{(2a^2 A + 3Ab^2 + 6abB) \sin(c + dx)}{3d} + \frac{a(2Ab + aB)}{3d} \int \cos^2(c + dx) (a + b \sec(c + dx))^2(A + B \sec(c + dx)) dx \\ &= \frac{1}{2} (2aAb + a^2B + 2b^2B) x + \frac{(2a^2 A + 3Ab^2 + 6abB) \sin(c + dx) + a(2Ab + aB) \int \cos^2(c + dx) (a + b \sec(c + dx))^2(A + B \sec(c + dx)) dx}{3d} \end{aligned}$$

Mathematica [A] time = 0.24, size = 90, normalized size = 0.84

$$\frac{6(c + dx)(a^2B + 2aAb + 2b^2B) + 3(3a^2A + 8abB + 4Ab^2) \sin(c + dx) + a^2A \sin(3(c + dx)) + 3a(aB + 2Ab) \int \cos^2(c + dx) (a + b \sec(c + dx))^2(A + B \sec(c + dx)) dx}{12d}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]

[Out] (6*(2*a*A*b + a^2*B + 2*b^2*B)*(c + d*x) + 3*(3*a^2*A + 4*A*b^2 + 8*a*b*B)*Sin[c + d*x] + 3*a*(2*A*b + a*B)*Sin[2*(c + d*x)] + a^2*A*Ssin[3*(c + d*x)])/(12*d)

fricas [A] time = 0.47, size = 85, normalized size = 0.79

$$\frac{3(Ba^2 + 2Aab + 2Bb^2)dx + (2Aa^2 \cos(dx + c)^2 + 4Aa^2 + 12Bab + 6Ab^2 + 3(Ba^2 + 2Aab) \cos(dx + c)) \int \cos^2(c + dx) (a + b \sec(c + dx))^2(A + B \sec(c + dx)) dx}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] 1/6*(3*(B*a^2 + 2*A*a*b + 2*B*b^2)*d*x + (2*A*a^2*cos(d*x + c)^2 + 4*A*a^2 + 12*B*a*b + 6*A*b^2 + 3*(B*a^2 + 2*A*a*b)*cos(d*x + c))*sin(d*x + c))/d

giac [B] time = 0.49, size = 254, normalized size = 2.37

$$3(Ba^2 + 2Aab + 2Bb^2)(dx + c) + \frac{2\left(6Aa^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5 - 3Ba^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5 - 6Aab \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5 + 12Bab \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5 + 6Ab^2\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] 1/6*(3*(B*a^2 + 2*A*a*b + 2*B*b^2)*(d*x + c) + 2*(6*A*a^2*tan(1/2*d*x + 1/2*c)^5 - 3*B*a^2*tan(1/2*d*x + 1/2*c)^5 - 6*A*a*b*tan(1/2*d*x + 1/2*c)^5 + 12*B*a*b*tan(1/2*d*x + 1/2*c)^5 + 6*A*b^2*tan(1/2*d*x + 1/2*c)^5 + 4*A*a^2*tan(1/2*d*x + 1/2*c)^3 + 24*B*a*b*tan(1/2*d*x + 1/2*c)^3 + 12*A*b^2*tan(1/2*d*x + 1/2*c)^3 + 6*A*a^2*tan(1/2*d*x + 1/2*c) + 3*B*a^2*tan(1/2*d*x + 1/2*c) + 6*A*a*b*tan(1/2*d*x + 1/2*c) + 12*B*a*b*tan(1/2*d*x + 1/2*c) + 6*A*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^3/d

maple [A] time = 1.06, size = 114, normalized size = 1.07

$$\frac{a^2 A (2 + \cos^2(dx+c)) \sin(dx+c)}{3} + 2Aab \left(\frac{\cos(dx+c) \sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2} \right) + B a^2 \left(\frac{\cos(dx+c) \sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2} \right) + A b^2 \sin(dx+c)$$

$$d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^3*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x)

[Out] 1/d*(1/3*a^2*A*(2+cos(d*x+c)^2)*sin(d*x+c)+2*A*a*b*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+B*a^2*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+A*b^2*sin(d*x+c)+2*B*a*b*sin(d*x+c)+B*(d*x+c)*b^2)

maxima [A] time = 1.17, size = 108, normalized size = 1.01

$$\frac{4(\sin(dx+c)^3 - 3\sin(dx+c))Aa^2 - 3(2dx+2c+\sin(2dx+2c))Ba^2 - 6(2dx+2c+\sin(2dx+2c))Aab}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] -1/12*(4*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^2 - 3*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^2 - 6*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a*b - 12*(d*x + c)*B*b^2 - 24*B*a*b*sin(d*x + c) - 12*A*b^2*sin(d*x + c))/d

mupad [B] time = 2.10, size = 115, normalized size = 1.07

$$\frac{B a^2 x}{2} + B b^2 x + \frac{3 A a^2 \sin(c + d x)}{4 d} + \frac{A b^2 \sin(c + d x)}{d} + A a b x + \frac{A a^2 \sin(3 c + 3 d x)}{12 d} + \frac{B a^2 \sin(2 c + 2 d x)}{4 d} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^3*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^2,x)

[Out] (B*a^2*x)/2 + B*b^2*x + (3*A*a^2*sin(c + d*x))/(4*d) + (A*b^2*sin(c + d*x))/d + A*a*b*x + (A*a^2*sin(3*c + 3*d*x))/(12*d) + (B*a^2*sin(2*c + 2*d*x))/(4*d) + (2*B*a*b*sin(c + d*x))/d + (A*a*b*sin(2*c + 2*d*x))/(2*d)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \sec(c + dx)) (a + b \sec(c + dx))^2 \cos^3(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**3*(a+b*sec(d*x+c))**2*(A+B*sec(d*x+c)),x)

[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**2*cos(c + d*x)**3, x)

$$3.292 \quad \int \cos^4(c+dx)(a+b \sec(c+dx))^2(A+B \sec(c+dx)) dx$$

Optimal. Leaf size=136

$$\frac{(a^2B + 2aAb + b^2B) \sin(c+dx)}{d} + \frac{(3a^2A + 8abB + 4Ab^2) \sin(c+dx) \cos(c+dx)}{8d} + \frac{1}{8}x(3a^2A + 8abB + 4Ab^2) + \dots$$

[Out] 1/8*(3*A*a^2+4*A*b^2+8*B*a*b)*x+(2*A*a*b+B*a^2+B*b^2)*sin(d*x+c)/d+1/8*(3*A*a^2+4*A*b^2+8*B*a*b)*cos(d*x+c)*sin(d*x+c)/d+1/4*a^2*A*cos(d*x+c)^3*sin(d*x+c)/d-1/3*a*(2*A*b+B*a)*sin(d*x+c)^3/d

Rubi [A] time = 0.26, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4024, 4047, 2635, 8, 4044, 3013}

$$\frac{(a^2B + 2aAb + b^2B) \sin(c+dx)}{d} + \frac{(3a^2A + 8abB + 4Ab^2) \sin(c+dx) \cos(c+dx)}{8d} + \frac{1}{8}x(3a^2A + 8abB + 4Ab^2) + \dots$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]), x]

[Out] ((3*a^2*A + 4*A*b^2 + 8*a*b*B)*x)/8 + ((2*a*A*b + a^2*B + b^2*B)*Sin[c + d*x])/d + ((3*a^2*A + 4*A*b^2 + 8*a*b*B)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a^2*A*cos[c + d*x]^3*sin[c + d*x])/(4*d) - (a*(2*A*b + a*B)*sin[c + d*x]^3)/(3*d)

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[(b*cos[c + d*x])*(b*sin[c + d*x])^(n-1)/(d*n), x] + Dist[(b^2*(n-1))/n, Int[(b*sin[c + d*x])^(n-2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3013

Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] := -Dist[f^(-1), Subst[Int[(1 - x^2)^((m-1)/2)*(A + C - C*x^2)], x], x, Cos[e + f*x], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m+1)/2, 0]

Rule 4024

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (
a_))^(2*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := Simp[(a^2*A*Cos
[e + f*x]*(d*Csc[e + f*x])^(n + 1))/(d*f*n), x] + Dist[1/(d*n), Int[(d*Csc[
e + f*x])^(n + 1)*(a*(2*A*b + a*B)*n + (2*a*b*B*n + A*(b^2*n + a^2*(n + 1))
)*Csc[e + f*x] + b^2*B*n*Csc[e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4044

```
Int[csc[(e_.) + (f_.)*(x_.)]^(m_.)*(csc[(e_.) + (f_.)*(x_.)]^2*(C_.) + (A_.)),
x_Symbol] := Int[(C + A*Sin[e + f*x]^2)/Sin[e + f*x]^(m + 2), x] /; FreeQ[
{e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && ILtQ[(m + 1)/2, 0]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_.)]*
(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.)), x_Symbol] := Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\begin{aligned}
\int \cos^4(c + dx)(a + b \sec(c + dx))^2(A + B \sec(c + dx)) dx &= \frac{a^2 A \cos^3(c + dx) \sin(c + dx)}{4d} - \frac{1}{4} \int \cos^3(c + dx) (a + b \sec(c + dx))^2(A + B \sec(c + dx)) dx \\
&= \frac{a^2 A \cos^3(c + dx) \sin(c + dx)}{4d} - \frac{1}{4} \int \cos^3(c + dx) (a + b \sec(c + dx))^2(A + B \sec(c + dx)) dx \\
&= \frac{(3a^2 A + 4Ab^2 + 8abB) \cos(c + dx) \sin(c + dx)}{8d} + \frac{1}{4} \int \cos^3(c + dx) (a + b \sec(c + dx))^2(A + B \sec(c + dx)) dx \\
&= \frac{1}{8} (3a^2 A + 4Ab^2 + 8abB) x + \frac{(3a^2 A + 4Ab^2 + 8abB) \cos(c + dx) \sin(c + dx)}{8d} \\
&= \frac{1}{8} (3a^2 A + 4Ab^2 + 8abB) x + \frac{(2aAb + a^2 B + b^2 B) \cos(c + dx) \sin(c + dx)}{d}
\end{aligned}$$

Mathematica [A] time = 0.46, size = 118, normalized size = 0.87

$$\frac{12(c + dx)(3a^2 A + 8abB + 4Ab^2) + 24(3a^2 B + 6aAb + 4b^2 B) \sin(c + dx) + 24(a^2 A + 2abB + Ab^2) \sin(2(c + dx))}{96d}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]

[Out] (12*(3*a^2*A + 4*A*b^2 + 8*a*b*B)*(c + d*x) + 24*(6*a*A*b + 3*a^2*B + 4*b^2*B)*Sin[c + d*x] + 24*(a^2*A + A*b^2 + 2*a*b*B)*Sin[2*(c + d*x)] + 8*a*(2*A*b + a*B)*Sin[3*(c + d*x)] + 3*a^2*A*Ssin[4*(c + d*x)])/(96*d)

fricas [A] time = 0.45, size = 114, normalized size = 0.84

$$\frac{3(3Aa^2 + 8Bab + 4Ab^2)dx + (6Aa^2 \cos(dx + c)^3 + 16Ba^2 + 32Aab + 24Bb^2 + 8(Ba^2 + 2Aab) \cos(dx + c)^2)}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] 1/24*(3*(3*A*a^2 + 8*B*a*b + 4*A*b^2)*d*x + (6*A*a^2*cos(d*x + c)^3 + 16*B*a^2 + 32*A*a*b + 24*B*b^2 + 8*(B*a^2 + 2*A*a*b)*cos(d*x + c)^2 + 3*(3*A*a^2 + 8*B*a*b + 4*A*b^2)*cos(d*x + c))*sin(d*x + c))/d

giac [B] time = 0.26, size = 437, normalized size = 3.21

$$3(3Aa^2 + 8Bab + 4Ab^2)(dx + c) - \frac{2\left(15Aa^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^7 - 24Ba^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^7 - 48Aab \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^7 + 24Bab \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^7\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] 1/24*(3*(3*A*a^2 + 8*B*a*b + 4*A*b^2)*(d*x + c) - 2*(15*A*a^2*tan(1/2*d*x + 1/2*c)^7 - 24*B*a^2*tan(1/2*d*x + 1/2*c)^7 - 48*A*a*b*tan(1/2*d*x + 1/2*c)^7 + 24*B*a*b*tan(1/2*d*x + 1/2*c)^7 + 12*A*b^2*tan(1/2*d*x + 1/2*c)^7 - 24*B*b^2*tan(1/2*d*x + 1/2*c)^7 - 9*A*a^2*tan(1/2*d*x + 1/2*c)^5 - 40*B*a^2*tan(1/2*d*x + 1/2*c)^5 - 80*A*a*b*tan(1/2*d*x + 1/2*c)^5 + 24*B*a*b*tan(1/2*d*x + 1/2*c)^5 + 12*A*b^2*tan(1/2*d*x + 1/2*c)^5 - 72*B*b^2*tan(1/2*d*x + 1/2*c)^5 + 9*A*a^2*tan(1/2*d*x + 1/2*c)^3 - 40*B*a^2*tan(1/2*d*x + 1/2*c)^3 - 80*A*a*b*tan(1/2*d*x + 1/2*c)^3 - 24*B*a*b*tan(1/2*d*x + 1/2*c)^3 - 12*A*b^2*tan(1/2*d*x + 1/2*c)^3 - 72*B*b^2*tan(1/2*d*x + 1/2*c)^3 - 15*A*a^2*tan(1/2*d*x + 1/2*c) - 24*B*a^2*tan(1/2*d*x + 1/2*c) - 48*A*a*b*tan(1/2*d*x + 1/2*c) - 24*B*a*b*tan(1/2*d*x + 1/2*c) - 12*A*b^2*tan(1/2*d*x + 1/2*c) - 24*B*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d

maple [A] time = 1.41, size = 152, normalized size = 1.12

$$\frac{a^2 A \left(\frac{\left(\cos^3(dx+c) + \frac{3 \cos(dx+c)}{2} \right) \sin(dx+c)}{4} + \frac{3dx}{8} + \frac{3c}{8} \right) + \frac{B a^2 (2 + \cos^2(dx+c)) \sin(dx+c)}{3} + \frac{2Aab(2 + \cos^2(dx+c)) \sin(dx+c)}{3} + 2Bab \left(\frac{\cos(dx+c)}{2} \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^4*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x)

[Out] 1/d*(a^2*A*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3*B*a^2*(2+cos(d*x+c)^2)*sin(d*x+c)+2/3*A*a*b*(2+cos(d*x+c)^2)*sin(d*x+c)+2*B*a*b*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+A*b^2*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+b^2*B*sin(d*x+c))

maxima [A] time = 1.67, size = 142, normalized size = 1.04

$$\frac{3(12dx + 12c + \sin(4dx + 4c) + 8 \sin(2dx + 2c))Aa^2 - 32(\sin(dx + c)^3 - 3 \sin(dx + c))Ba^2 - 64(\sin(dx + c)^3 - 3 \sin(dx + c))Aab + 48(2dx + 2c + \sin(2dx + 2c))Bab + 24(2dx + 2c + \sin(2dx + 2c))Aab^2 + 96Bb^2 \sin(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] 1/96*(3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^2 - 32*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^2 - 64*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a*b + 48*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a*b + 24*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*b^2 + 96*B*b^2*sin(d*x + c))/d

mupad [B] time = 2.18, size = 169, normalized size = 1.24

$$\frac{3Aa^2x}{8} + \frac{Ab^2x}{2} + \frac{3Ba^2 \sin(c + dx)}{4d} + \frac{Bb^2 \sin(c + dx)}{d} + Babx + \frac{Aa^2 \sin(2c + 2dx)}{4d} + \frac{Aa^2 \sin(4c + 4dx)}{32d} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^4*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^2,x)

[Out] (3*A*a^2*x)/8 + (A*b^2*x)/2 + (3*B*a^2*sin(c + d*x))/(4*d) + (B*b^2*sin(c + d*x))/d + B*a*b*x + (A*a^2*sin(2*c + 2*d*x))/(4*d) + (A*a^2*sin(4*c + 4*d*x))/(32*d) + (A*b^2*sin(2*c + 2*d*x))/(4*d) + (B*a^2*sin(3*c + 3*d*x))/(12*d) + (3*A*a*b*sin(c + d*x))/(2*d) + (A*a*b*sin(3*c + 3*d*x))/(6*d) + (B*a*b*sin(2*c + 2*d*x))/(2*d)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+b*sec(d*x+c))**2*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

$$3.293 \quad \int \cos^5(c+dx)(a+b \sec(c+dx))^2(A+B \sec(c+dx)) dx$$

Optimal. Leaf size=180

$$\frac{(4a^2A + 10abB + 5Ab^2) \sin^3(c+dx)}{15d} + \frac{(4a^2A + 10abB + 5Ab^2) \sin(c+dx)}{5d} + \frac{(3a^2B + 6aAb + 4b^2B) \sin(c+dx)}{8d}$$

[Out] 1/8*(6*A*a*b+3*B*a^2+4*B*b^2)*x+1/5*(4*A*a^2+5*A*b^2+10*B*a*b)*sin(d*x+c)/d+1/8*(6*A*a*b+3*B*a^2+4*B*b^2)*cos(d*x+c)*sin(d*x+c)/d+1/4*a*(2*A*b+B*a)*cos(d*x+c)^3*sin(d*x+c)/d+1/5*a^2*A*cos(d*x+c)^4*sin(d*x+c)/d-1/15*(4*A*a^2+5*A*b^2+10*B*a*b)*sin(d*x+c)^3/d

Rubi [A] time = 0.27, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4024, 4047, 2633, 4045, 2635, 8}

$$\frac{(4a^2A + 10abB + 5Ab^2) \sin^3(c+dx)}{15d} + \frac{(4a^2A + 10abB + 5Ab^2) \sin(c+dx)}{5d} + \frac{(3a^2B + 6aAb + 4b^2B) \sin(c+dx)}{8d}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^5*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]

[Out] ((6*a*A*b + 3*a^2*B + 4*b^2*B)*x)/8 + ((4*a^2*A + 5*A*b^2 + 10*a*b*B)*Sin[c + d*x])/(5*d) + ((6*a*A*b + 3*a^2*B + 4*b^2*B)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a*(2*A*b + a*B)*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) + (a^2*A*Cos[c + d*x]^4*Sin[c + d*x])/(5*d) - ((4*a^2*A + 5*A*b^2 + 10*a*b*B)*Sin[c + d*x]^3)/(15*d)

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2633

Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[Expand[(1 - x^2)^(n-1)/2], x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n-1)/2, 0]

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x])*(b*Ssin[c + d*x])^(n-1)/(d*n), x] + Dist[(b^2*(n-1))/n, Int[(b*Ssin[c + d*x])^(n-2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

]

Rule 4024

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^2*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> Simp[(a^2*A*Cos[e + f*x]*(d*Csc[e + f*x])^(n + 1))/(d*f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*(a*(2*A*b + a*B)*n + (2*a*b*B*n + A*(b^2*n + a^2*(n + 1)))*Csc[e + f*x] + b^2*B*n*Csc[e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]

Rule 4045

Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]^2*(C_.) + (A_.)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] + Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]

Rule 4047

Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2), x] /; FreeQ[{b, e, f, A, B, C, m}, x]

Rubi steps

$$\begin{aligned}
 \int \cos^5(c + dx)(a + b \sec(c + dx))^2(A + B \sec(c + dx)) dx &= \frac{a^2 A \cos^4(c + dx) \sin(c + dx)}{5d} - \frac{1}{5} \int \cos^4(c + dx) (-) \\
 &= \frac{a^2 A \cos^4(c + dx) \sin(c + dx)}{5d} - \frac{1}{5} \int \cos^4(c + dx) (-) \\
 &= \frac{a(2Ab + aB) \cos^3(c + dx) \sin(c + dx)}{4d} + \frac{a^2 A \cos^4(c + dx)}{5d} \\
 &= \frac{(4a^2 A + 5Ab^2 + 10abB) \sin(c + dx)}{5d} + \frac{(6aAb + 3a^2 B)}{5d} \\
 &= \frac{1}{8} (6aAb + 3a^2 B + 4b^2 B) x + \frac{(4a^2 A + 5Ab^2 + 10abB) \sin(c + dx)}{5d}
 \end{aligned}$$

Mathematica [A] time = 0.55, size = 146, normalized size = 0.81

$$\frac{60(c + dx) (3a^2B + 6aAb + 4b^2B) + 60 (5a^2A + 12abB + 6Ab^2) \sin(c + dx) + 120 (a^2B + 2aAb + b^2B) \sin(2(c + dx))}{480d}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]^5*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]

[Out] (60*(6*a*A*b + 3*a^2*B + 4*b^2*B)*(c + d*x) + 60*(5*a^2*A + 6*A*b^2 + 12*a*b*B)*Sin[c + d*x] + 120*(2*a*A*b + a^2*B + b^2*B)*Sin[2*(c + d*x)] + 10*(5*a^2*A + 4*A*b^2 + 8*a*b*B)*Sin[3*(c + d*x)] + 15*a*(2*A*b + a*B)*Sin[4*(c + d*x)] + 6*a^2*A*Ssin[5*(c + d*x)])/(480*d)

fricas [A] time = 0.48, size = 142, normalized size = 0.79

$$\frac{15 (3Ba^2 + 6Aab + 4Bb^2)dx + (24Aa^2 \cos(dx + c)^4 + 30 (Ba^2 + 2Aab) \cos(dx + c)^3 + 64Aa^2 + 160Bab + 80Ab^2) \sin(dx + c)}{120d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] 1/120*(15*(3*B*a^2 + 6*A*a*b + 4*B*b^2)*d*x + (24*A*a^2*cos(d*x + c)^4 + 30*(B*a^2 + 2*A*a*b)*cos(d*x + c)^3 + 64*A*a^2 + 160*B*a*b + 80*A*b^2 + 8*(4*A*a^2 + 10*B*a*b + 5*A*b^2)*cos(d*x + c)^2 + 15*(3*B*a^2 + 6*A*a*b + 4*B*b^2)*cos(d*x + c))*sin(d*x + c))/d

giac [B] time = 0.64, size = 487, normalized size = 2.71

$$15 (3Ba^2 + 6Aab + 4Bb^2)(dx + c) + \frac{2 \left(120Aa^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^9 - 75Ba^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^9 - 150Aab \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^9 + 240Bab \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^9 + 120Aa^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^7 - 60Aa^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5 + 160Aa^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 - 60Aa^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 120Aa^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) \right)}{120d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] 1/120*(15*(3*B*a^2 + 6*A*a*b + 4*B*b^2)*(d*x + c) + 2*(120*A*a^2*tan(1/2*d*x + 1/2*c)^9 - 75*B*a^2*tan(1/2*d*x + 1/2*c)^9 - 150*A*a*b*tan(1/2*d*x + 1/2*c)^9 + 240*B*a*b*tan(1/2*d*x + 1/2*c)^9 + 120*A*b^2*tan(1/2*d*x + 1/2*c)^9 - 60*B*b^2*tan(1/2*d*x + 1/2*c)^9 + 160*A*a^2*tan(1/2*d*x + 1/2*c)^7 - 30*B*a^2*tan(1/2*d*x + 1/2*c)^7 - 60*A*a*b*tan(1/2*d*x + 1/2*c)^7 + 640*B*a*b

$$\begin{aligned} & * \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^7 + 320A^2b^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^7 - 120B^2b^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^7 \\ & + 464A^2a^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5 + 800B^2a^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5 + 400A^2b^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5 \\ & + 160A^2a^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 + 30B^2a^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 + 60A^2a^2b \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 \\ & + 640B^2a^2b \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 + 320A^2b^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 + 120B^2b^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 \\ & + 120A^2a^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 75B^2a^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 150A^2a^2b \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) \\ & + 240B^2a^2b \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 120A^2b^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 60B^2b^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) \\ & \left. \right) / (\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + 1)^5 / d \end{aligned}$$

maple [A] time = 1.58, size = 184, normalized size = 1.02

$$\frac{a^2 A \left(\frac{8}{3} + \cos^4(dx+c) + \frac{4(\cos^2(dx+c))}{3} \right) \sin(dx+c)}{5} + B a^2 \left(\frac{(\cos^3(dx+c) + \frac{3\cos(dx+c)}{2}) \sin(dx+c)}{4} + \frac{3dx}{8} + \frac{3c}{8} \right) + 2Aab \left(\frac{(\cos^3(dx+c) + \frac{3\cos(dx+c)}{2}) \sin(dx+c)}{4} \right) \sin(dx+c)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(d*x+c)^5*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x)`

[Out] $\frac{1}{d} \left(\frac{1}{5} a^2 A (8/3 + \cos^4(dx+c) + 4/3 \cos^2(dx+c)) \sin(dx+c) + B a^2 (1/4 (\cos^3(dx+c) + 3/2 \cos(dx+c)) \sin(dx+c) + 3/8 dx + 3/8 c) + 2 A a^2 b (1/4 (\cos^3(dx+c) + 3/2 \cos(dx+c)) \sin(dx+c) + 3/8 dx + 3/8 c) + 2/3 B a^2 b (2 + \cos^2(dx+c)) \sin(dx+c) + 1/3 A b^2 (2 + \cos^2(dx+c)) \sin(dx+c) + b^2 B (1/2 \cos(dx+c) \sin(dx+c) + 1/2 dx + 1/2 c) \right)$

maxima [A] time = 0.96, size = 176, normalized size = 0.98

$$32 \left(3 \sin(dx+c)^5 - 10 \sin(dx+c)^3 + 15 \sin(dx+c) \right) A a^2 + 15 (12 dx + 12 c + \sin(4 dx + 4 c) + 8 \sin(2 dx + 2 c)) B a^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="maxima")`

[Out] $\frac{1}{480} \left(32 (3 \sin(dx+c)^5 - 10 \sin(dx+c)^3 + 15 \sin(dx+c)) A a^2 + 15 (12 dx + 12 c + \sin(4 dx + 4 c) + 8 \sin(2 dx + 2 c)) B a^2 + 30 (12 dx + 12 c + \sin(4 dx + 4 c) + 8 \sin(2 dx + 2 c)) A a^2 b - 320 (\sin(dx+c)^3 - 3 \sin(dx+c)) B a^2 b - 160 (\sin(dx+c)^3 - 3 \sin(dx+c)) A b^2 + 120 (2 dx + 2 c + \sin(2 dx + 2 c)) B b^2 \right) / d$

mupad [B] time = 5.81, size = 307, normalized size = 1.71

$$\frac{x \left(\frac{3B a^2}{4} + \frac{3A a b}{2} + B b^2 \right)}{2} + \frac{\left(2A a^2 + 2A b^2 - \frac{5B a^2}{4} - B b^2 - \frac{5A a b}{2} + 4B a b \right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^9 + \left(\frac{8A a^2}{3} + \frac{16A b^2}{3} - \frac{B a^2}{2} \right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(c + d*x)^5*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^2,x)`

[Out] $(x*((3*B*a^2)/4 + B*b^2 + (3*A*a*b)/2))/2 + (\tan(c/2 + (d*x)/2)^5*((116*A*a^2)/15 + (20*A*b^2)/3 + (40*B*a*b)/3) + \tan(c/2 + (d*x)/2)^9*(2*A*a^2 + 2*A*b^2 - (5*B*a^2)/4 - B*b^2 - (5*A*a*b)/2 + 4*B*a*b) + \tan(c/2 + (d*x)/2)^3*((8*A*a^2)/3 + (16*A*b^2)/3 + (B*a^2)/2 + 2*B*b^2 + A*a*b + (32*B*a*b)/3) + \tan(c/2 + (d*x)/2)^7*((8*A*a^2)/3 + (16*A*b^2)/3 - (B*a^2)/2 - 2*B*b^2 - A*a*b + (32*B*a*b)/3) + \tan(c/2 + (d*x)/2)*(2*A*a^2 + 2*A*b^2 + (5*B*a^2)/4 + B*b^2 + (5*A*a*b)/2 + 4*B*a*b)/(d*(5*\tan(c/2 + (d*x)/2)^2 + 10*\tan(c/2 + (d*x)/2)^4 + 10*\tan(c/2 + (d*x)/2)^6 + 5*\tan(c/2 + (d*x)/2)^8 + \tan(c/2 + (d*x)/2)^{10} + 1))$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)**5*(a+b*sec(d*x+c))**2*(A+B*sec(d*x+c)),x)`

[Out] Timed out

$$3.294 \quad \int \sec^2(c+dx)(a+b \sec(c+dx))^3(A+B \sec(c+dx)) dx$$

Optimal. Leaf size=252

$$\frac{(-3a^2B + 15aAb + 16b^2B) \tan(c+dx)(a+b \sec(c+dx))^2}{60bd} + \frac{(4a^3B + 12a^2Ab + 9ab^2B + 3Ab^3) \tanh^{-1}(\sin(c+dx))}{8d}$$

[Out] 1/8*(12*A*a^2*b+3*A*b^3+4*B*a^3+9*B*a*b^2)*arctanh(sin(d*x+c))/d+1/30*(15*A*a^3*b+60*A*a*b^3-3*B*a^4+52*B*a^2*b^2+16*B*b^4)*tan(d*x+c)/b/d+1/120*(30*A*a^2*b+45*A*b^3-6*B*a^3+71*B*a*b^2)*sec(d*x+c)*tan(d*x+c)/d+1/60*(15*A*a*b-3*B*a^2+16*B*b^2)*(a+b*sec(d*x+c))^2*tan(d*x+c)/b/d+1/20*(5*A*b-B*a)*(a+b*sec(d*x+c))^3*tan(d*x+c)/b/d+1/5*B*(a+b*sec(d*x+c))^4*tan(d*x+c)/b/d

Rubi [A] time = 0.48, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {4010, 4002, 3997, 3787, 3770, 3767, 8}

$$\frac{(15a^3Ab + 52a^2b^2B - 3a^4B + 60aAb^3 + 16b^4B) \tan(c+dx)}{30bd} + \frac{(12a^2Ab + 4a^3B + 9ab^2B + 3Ab^3) \tanh^{-1}(\sin(c+dx))}{8d}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]), x]

[Out] ((12*a^2*A*b + 3*A*b^3 + 4*a^3*B + 9*a*b^2*B)*ArcTanh[Sin[c + d*x]])/(8*d) + ((15*a^3*A*b + 60*a*A*b^3 - 3*a^4*B + 52*a^2*b^2*B + 16*b^4*B)*Tan[c + d*x])/(30*b*d) + ((30*a^2*A*b + 45*A*b^3 - 6*a^3*B + 71*a*b^2*B)*Sec[c + d*x]*Tan[c + d*x])/(120*d) + ((15*a*A*b - 3*a^2*B + 16*b^2*B)*(a + b*Sec[c + d*x])^2*Tan[c + d*x])/(60*b*d) + ((5*A*b - a*B)*(a + b*Sec[c + d*x])^3*Tan[c + d*x])/(20*b*d) + (B*(a + b*Sec[c + d*x])^4*Tan[c + d*x])/(5*b*d)

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 3767

Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 3770


```
Int[csc[(c_.) + (d_.)*(x_.)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x]
  /; FreeQ[{c, d}, x]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) +
(a_.)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) +
(a_.))*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := -Simp[(b*B*Cot[e
+ f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && !LeQ[n,
-1]
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_.)]*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(cs
c[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_.)]^2*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(
csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Csc
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
0] && !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \sec^2(c + dx)(a + b \sec(c + dx))^3(A + B \sec(c + dx)) dx &= \frac{B(a + b \sec(c + dx))^4 \tan(c + dx)}{5bd} + \frac{\int \sec(c + dx)(a + b \sec(c + dx))^3(A + B \sec(c + dx)) dx}{5bd} \\
&= \frac{(5Ab - aB)(a + b \sec(c + dx))^3 \tan(c + dx)}{20bd} + \frac{B(a + b \sec(c + dx))^4 \tan(c + dx)}{5bd} \\
&= \frac{(15aAb - 3a^2B + 16b^2B)(a + b \sec(c + dx))^2 \tan(c + dx)}{60bd} + \frac{B(a + b \sec(c + dx))^4 \tan(c + dx)}{5bd} \\
&= \frac{(30a^2Ab + 45Ab^3 - 6a^3B + 71ab^2B) \sec(c + dx) \tan(c + dx)}{120d} + \frac{B(a + b \sec(c + dx))^4 \tan(c + dx)}{5bd} \\
&= \frac{(30a^2Ab + 45Ab^3 - 6a^3B + 71ab^2B) \sec(c + dx) \tan(c + dx)}{120d} + \frac{B(a + b \sec(c + dx))^4 \tan(c + dx)}{5bd} \\
&= \frac{(12a^2Ab + 3Ab^3 + 4a^3B + 9ab^2B) \tanh^{-1}(\sin(c + dx))}{8d} + \frac{B(a + b \sec(c + dx))^4 \tan(c + dx)}{5bd} \\
&= \frac{(12a^2Ab + 3Ab^3 + 4a^3B + 9ab^2B) \tanh^{-1}(\sin(c + dx))}{8d} + \frac{B(a + b \sec(c + dx))^4 \tan(c + dx)}{5bd}
\end{aligned}$$

Mathematica [A] time = 3.50, size = 181, normalized size = 0.72

$$\frac{15(4a^3B + 12a^2Ab + 9ab^2B + 3Ab^3) \tanh^{-1}(\sin(c + dx)) + \tan(c + dx) \left(8(5b(3a^2B + 3aAb + 2b^2B)) \tan^2(c + dx) + \dots \right)}{120d}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]

[Out] (15*(12*a^2*A*b + 3*A*b^3 + 4*a^3*B + 9*a*b^2*B)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(15*(12*a^2*A*b + 3*A*b^3 + 4*a^3*B + 9*a*b^2*B)*Sec[c + d*x] + 30*b^2*(A*b + 3*a*B)*Sec[c + d*x]^3 + 8*(15*(a^3*A + 3*a*A*b^2 + 3*a^2*b*B + b^3*B) + 5*b*(3*a*A*b + 3*a^2*B + 2*b^2*B)*Tan[c + d*x]^2 + 3*b^3*B*Tan[c + d*x]^4)))/(120*d)

fricas [A] time = 0.54, size = 249, normalized size = 0.99

$$\frac{15(4Ba^3 + 12Aa^2b + 9Bab^2 + 3Ab^3) \cos(dx + c)^5 \log(\sin(dx + c) + 1) - 15(4Ba^3 + 12Aa^2b + 9Bab^2 + 3Ab^3) \cos(dx + c)^5 \log(\sin(dx + c) - 1)}{120d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="fricas")

```
[Out] 1/240*(15*(4*B*a^3 + 12*A*a^2*b + 9*B*a*b^2 + 3*A*b^3)*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 15*(4*B*a^3 + 12*A*a^2*b + 9*B*a*b^2 + 3*A*b^3)*cos(d*x + c)^5*log(-sin(d*x + c) + 1) + 2*(8*(15*A*a^3 + 30*B*a^2*b + 30*A*a*b^2 + 8*B*b^3)*cos(d*x + c)^4 + 24*B*b^3 + 15*(4*B*a^3 + 12*A*a^2*b + 9*B*a*b^2 + 3*A*b^3)*cos(d*x + c)^3 + 8*(15*B*a^2*b + 15*A*a*b^2 + 4*B*b^3)*cos(d*x + c)^2 + 30*(3*B*a*b^2 + A*b^3)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^5)
```

giac [B] time = 0.82, size = 722, normalized size = 2.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/120*(15*(4*B*a^3 + 12*A*a^2*b + 9*B*a*b^2 + 3*A*b^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(4*B*a^3 + 12*A*a^2*b + 9*B*a*b^2 + 3*A*b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(120*A*a^3*tan(1/2*d*x + 1/2*c)^9 - 60*B*a^3*tan(1/2*d*x + 1/2*c)^9 - 180*A*a^2*b*tan(1/2*d*x + 1/2*c)^9 + 360*B*a^2*b*tan(1/2*d*x + 1/2*c)^9 + 360*A*a*b^2*tan(1/2*d*x + 1/2*c)^9 - 225*B*a*b^2*tan(1/2*d*x + 1/2*c)^9 - 75*A*b^3*tan(1/2*d*x + 1/2*c)^9 + 120*B*b^3*tan(1/2*d*x + 1/2*c)^9 - 480*A*a^3*tan(1/2*d*x + 1/2*c)^7 + 120*B*a^3*tan(1/2*d*x + 1/2*c)^7 + 360*A*a^2*b*tan(1/2*d*x + 1/2*c)^7 - 960*B*a^2*b*tan(1/2*d*x + 1/2*c)^7 - 960*A*a*b^2*tan(1/2*d*x + 1/2*c)^7 + 90*B*a*b^2*tan(1/2*d*x + 1/2*c)^7 + 30*A*b^3*tan(1/2*d*x + 1/2*c)^7 - 160*B*b^3*tan(1/2*d*x + 1/2*c)^7 + 720*A*a^3*tan(1/2*d*x + 1/2*c)^5 + 1200*B*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 1200*A*a*b^2*tan(1/2*d*x + 1/2*c)^5 + 464*B*b^3*tan(1/2*d*x + 1/2*c)^5 - 480*A*a^3*tan(1/2*d*x + 1/2*c)^3 - 120*B*a^3*tan(1/2*d*x + 1/2*c)^3 - 360*A*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 960*B*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 960*A*a*b^2*tan(1/2*d*x + 1/2*c)^3 - 90*B*a*b^2*tan(1/2*d*x + 1/2*c)^3 - 30*A*b^3*tan(1/2*d*x + 1/2*c)^3 - 160*B*b^3*tan(1/2*d*x + 1/2*c)^3 + 120*A*a^3*tan(1/2*d*x + 1/2*c) + 60*B*a^3*tan(1/2*d*x + 1/2*c) + 180*A*a^2*b*tan(1/2*d*x + 1/2*c) + 360*B*a^2*b*tan(1/2*d*x + 1/2*c) + 360*A*a*b^2*tan(1/2*d*x + 1/2*c) + 225*B*a*b^2*tan(1/2*d*x + 1/2*c) + 75*A*b^3*tan(1/2*d*x + 1/2*c) + 120*B*b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

maple [A] time = 1.71, size = 382, normalized size = 1.52

$$\frac{A a^3 \tan(dx + c)}{d} + \frac{a^3 B \sec(dx + c) \tan(dx + c)}{2d} + \frac{a^3 B \ln(\sec(dx + c) + \tan(dx + c))}{2d} + \frac{3A a^2 b \sec(dx + c) \tan(dx + c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x)
```

```
[Out] 1/d*A*a^3*tan(d*x+c)+1/2/d*a^3*B*sec(d*x+c)*tan(d*x+c)+1/2/d*a^3*B*ln(sec(d
*x+c)+tan(d*x+c))+3/2/d*A*a^2*b*sec(d*x+c)*tan(d*x+c)+3/2/d*A*a^2*b*ln(sec(
d*x+c)+tan(d*x+c))+2/d*a^2*b*B*tan(d*x+c)+1/d*a^2*b*B*tan(d*x+c)*sec(d*x+c)
^2+2/d*A*a*b^2*tan(d*x+c)+1/d*A*a*b^2*tan(d*x+c)*sec(d*x+c)^2+3/4/d*B*a*b^2
*tan(d*x+c)*sec(d*x+c)^3+9/8/d*B*a*b^2*sec(d*x+c)*tan(d*x+c)+9/8/d*B*a*b^2*
ln(sec(d*x+c)+tan(d*x+c))+1/4/d*A*b^3*tan(d*x+c)*sec(d*x+c)^3+3/8/d*A*b^3*s
ec(d*x+c)*tan(d*x+c)+3/8/d*A*b^3*ln(sec(d*x+c)+tan(d*x+c))+8/15/d*b^3*B*tan
(d*x+c)+1/5/d*b^3*B*tan(d*x+c)*sec(d*x+c)^4+4/15/d*b^3*B*tan(d*x+c)*sec(d*x
+c)^2
```

maxima [A] time = 0.83, size = 341, normalized size = 1.35

$$240 \left(\tan(dx+c)^3 + 3 \tan(dx+c) \right) B a^2 b + 240 \left(\tan(dx+c)^3 + 3 \tan(dx+c) \right) A a b^2 + 16 \left(3 \tan(dx+c)^5 + 10 \tan(dx+c)^3 + 15 \tan(dx+c) \right) B b^3 - 45 B a b^2 \left(2 \left(3 \sin(dx+c)^3 - 5 \sin(dx+c) \right) / \left(\sin(dx+c)^4 - 2 \sin(dx+c)^2 + 1 \right) - 3 \log(\sin(dx+c) + 1) + 3 \log(\sin(dx+c) - 1) \right) - 15 A b^3 \left(2 \left(3 \sin(dx+c)^3 - 5 \sin(dx+c) \right) / \left(\sin(dx+c)^4 - 2 \sin(dx+c)^2 + 1 \right) - 3 \log(\sin(dx+c) + 1) + 3 \log(\sin(dx+c) - 1) \right) - 60 B a^3 \left(2 \sin(dx+c) / \left(\sin(dx+c)^2 - 1 \right) - \log(\sin(dx+c) + 1) + \log(\sin(dx+c) - 1) \right) - 180 A a^2 b \left(2 \sin(dx+c) / \left(\sin(dx+c)^2 - 1 \right) - \log(\sin(dx+c) + 1) + \log(\sin(dx+c) - 1) \right) + 240 A a^3 \tan(dx+c) / d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/240*(240*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a^2*b + 240*(tan(d*x + c)^3
+ 3*tan(d*x + c))*A*a*b^2 + 16*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*t
an(d*x + c))*B*b^3 - 45*B*a*b^2*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin
(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d
*x + c) - 1)) - 15*A*b^3*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x +
c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c)
- 1)) - 60*B*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) +
1) + log(sin(d*x + c) - 1)) - 180*A*a^2*b*(2*sin(d*x + c)/(sin(d*x + c)^2
- 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 240*A*a^3*tan(d*x +
c))/d
```

mupad [B] time = 5.79, size = 470, normalized size = 1.87

$$\frac{\operatorname{atanh}\left(\frac{4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right) \left(\frac{B a^3}{2} + \frac{3 A a^2 b}{2} + \frac{9 B a b^2}{8} + \frac{3 A b^3}{8}\right)}{2 B a^3 + 6 A a^2 b + \frac{9 B a b^2}{2} + \frac{3 A b^3}{2}}\right) \left(B a^3 + 3 A a^2 b + \frac{9 B a b^2}{4} + \frac{3 A b^3}{4}\right) \left(2 A a^3 - \frac{5 A b^3}{4} - B a^3 + 2 B b^3 + \dots\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^3)/cos(c + d*x)^2,x)
```

```
[Out] (atanh((4*tan(c/2 + (d*x)/2)*((3*A*b^3)/8 + (B*a^3)/2 + (3*A*a^2*b)/2 + (9*
B*a*b^2)/8)))/((3*A*b^3)/2 + 2*B*a^3 + 6*A*a^2*b + (9*B*a*b^2)/2))*((3*A*b^3
```

```

)/4 + B*a^3 + 3*A*a^2*b + (9*B*a*b^2)/4))/d - (tan(c/2 + (d*x)/2)*(2*A*a^3
+ (5*A*b^3)/4 + B*a^3 + 2*B*b^3 + 6*A*a*b^2 + 3*A*a^2*b + (15*B*a*b^2)/4 +
6*B*a^2*b) + tan(c/2 + (d*x)/2)^5*(12*A*a^3 + (116*B*b^3)/15 + 20*A*a*b^2 +
20*B*a^2*b) + tan(c/2 + (d*x)/2)^9*(2*A*a^3 - (5*A*b^3)/4 - B*a^3 + 2*B*b^
3 + 6*A*a*b^2 - 3*A*a^2*b - (15*B*a*b^2)/4 + 6*B*a^2*b) - tan(c/2 + (d*x)/2
)^3*(8*A*a^3 + (A*b^3)/2 + 2*B*a^3 + (8*B*b^3)/3 + 16*A*a*b^2 + 6*A*a^2*b +
(3*B*a*b^2)/2 + 16*B*a^2*b) - tan(c/2 + (d*x)/2)^7*(8*A*a^3 - (A*b^3)/2 -
2*B*a^3 + (8*B*b^3)/3 + 16*A*a*b^2 - 6*A*a^2*b - (3*B*a*b^2)/2 + 16*B*a^2*b
))/(d*(5*tan(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x
)/2)^6 - 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1))

```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \sec(c + dx))(a + b \sec(c + dx))^3 \sec^2(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**2*(a+b*sec(d*x+c))**3*(A+B*sec(d*x+c)), x)

[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**3*sec(c + d*x)**2, x)

$$3.295 \quad \int \sec(c + dx)(a + b \sec(c + dx))^3(A + B \sec(c + dx)) dx$$

Optimal. Leaf size=180

$$\frac{b(6a^2B + 20aAb + 9b^2B) \tan(c + dx) \sec(c + dx)}{24d} + \frac{(3a^3B + 16a^2Ab + 12ab^2B + 4Ab^3) \tan(c + dx)}{6d} + \frac{(8a^3A + 12a^2bB + 12aAb^2 + 3b^3B) \tanh^{-1}(\sin(c + dx))}{8d} + \frac{b(6a^2A + 20aAb + 9b^2A) \tan(c + dx) \sec(c + dx)}{24d}$$

[Out] 1/8*(8*A*a^3+12*A*a*b^2+12*B*a^2*b+3*B*b^3)*arctanh(sin(d*x+c))/d+1/6*(16*A*a^2*b+4*A*b^3+3*B*a^3+12*B*a*b^2)*tan(d*x+c)/d+1/24*b*(20*A*a*b+6*B*a^2+9*B*b^2)*sec(d*x+c)*tan(d*x+c)/d+1/12*(4*A*b+3*B*a)*(a+b*sec(d*x+c))^2*tan(d*x+c)/d+1/4*B*(a+b*sec(d*x+c))^3*tan(d*x+c)/d

Rubi [A] time = 0.33, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {4002, 3997, 3787, 3770, 3767, 8}

$$\frac{(16a^2Ab + 3a^3B + 12ab^2B + 4Ab^3) \tan(c + dx)}{6d} + \frac{(8a^3A + 12a^2bB + 12aAb^2 + 3b^3B) \tanh^{-1}(\sin(c + dx))}{8d} + \frac{b(6a^2A + 20aAb + 9b^2A) \tan(c + dx) \sec(c + dx)}{24d}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d*x]*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]

[Out] ((8*a^3*A + 12*a*A*b^2 + 12*a^2*b*B + 3*b^3*B)*ArcTanh[Sin[c + d*x]])/(8*d) + ((16*a^2*A*b + 4*A*b^3 + 3*a^3*B + 12*a*b^2*B)*Tan[c + d*x])/(6*d) + (b*(20*a*A*b + 6*a^2*B + 9*b^2*B)*Sec[c + d*x]*Tan[c + d*x])/(24*d) + ((4*A*b + 3*a*B)*(a + b*Sec[c + d*x])^2*Tan[c + d*x])/(12*d) + (B*(a + b*Sec[c + d*x])^3*Tan[c + d*x])/(4*d)

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 3767

Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 3997

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := -Simp[(b*B*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && !LeQ[n, -1]

Rule 4002

Int[csc[(e_.) + (f_.)*(x_.)]*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := -Simp[(B*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]

Rubi steps

$$\begin{aligned}
 \int \sec(c + dx)(a + b \sec(c + dx))^3(A + B \sec(c + dx)) dx &= \frac{B(a + b \sec(c + dx))^3 \tan(c + dx)}{4d} + \frac{1}{4} \int \sec(c + dx) \\
 &= \frac{(4Ab + 3aB)(a + b \sec(c + dx))^2 \tan(c + dx)}{12d} + \frac{B(a + b \sec(c + dx)) \tan(c + dx)}{4d} \\
 &= \frac{b(20aAb + 6a^2B + 9b^2B) \sec(c + dx) \tan(c + dx)}{24d} + \frac{B(a + b \sec(c + dx)) \tan(c + dx)}{4d} \\
 &= \frac{b(20aAb + 6a^2B + 9b^2B) \sec(c + dx) \tan(c + dx)}{24d} + \frac{(8a^3A + 12aAb^2 + 12a^2bB + 3b^3B) \tanh^{-1}(\sin(c + dx))}{8d} \\
 &= \frac{(8a^3A + 12aAb^2 + 12a^2bB + 3b^3B) \tanh^{-1}(\sin(c + dx))}{8d}
 \end{aligned}$$

Mathematica [A] time = 0.95, size = 140, normalized size = 0.78

$$\frac{3(8a^3A + 12a^2bB + 12aAb^2 + 3b^3B) \tanh^{-1}(\sin(c + dx)) + \tan(c + dx) (9b(4a^2B + 4aAb + b^2B) \sec(c + dx) + 24d)}{24d}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[c + d*x]*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]

[Out] (3*(8*a^3*A + 12*a*A*b^2 + 12*a^2*b*B + 3*b^3*B)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(24*(3*a^2*A*b + A*b^3 + a^3*B + 3*a*b^2*B) + 9*b*(4*a*A*b + 4*a^2*B + b^2*B)*Sec[c + d*x] + 6*b^3*B*Sec[c + d*x]^3 + 8*b^2*(A*b + 3*a*B)*Tan[c + d*x]^2))/(24*d)

fricas [A] time = 0.48, size = 211, normalized size = 1.17

$$\frac{3(8Aa^3 + 12Ba^2b + 12Aab^2 + 3Bb^3) \cos(dx + c)^4 \log(\sin(dx + c) + 1) - 3(8Aa^3 + 12Ba^2b + 12Aab^2 + 3Bb^3) \log(\sin(dx + c) - 1)}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] 1/48*(3*(8*A*a^3 + 12*B*a^2*b + 12*A*a*b^2 + 3*B*b^3)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(8*A*a^3 + 12*B*a^2*b + 12*A*a*b^2 + 3*B*b^3)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(6*B*b^3 + 8*(3*B*a^3 + 9*A*a^2*b + 6*B*a*b^2 + 2*A*b^3)*cos(d*x + c)^3 + 9*(4*B*a^2*b + 4*A*a*b^2 + B*b^3)*cos(d*x + c)^2 + 8*(3*B*a*b^2 + A*b^3)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^4)

giac [B] time = 0.70, size = 586, normalized size = 3.26

$$3(8Aa^3 + 12Ba^2b + 12Aab^2 + 3Bb^3) \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right|\right) - 3(8Aa^3 + 12Ba^2b + 12Aab^2 + 3Bb^3) \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] 1/24*(3*(8*A*a^3 + 12*B*a^2*b + 12*A*a*b^2 + 3*B*b^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(8*A*a^3 + 12*B*a^2*b + 12*A*a*b^2 + 3*B*b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(24*B*a^3*tan(1/2*d*x + 1/2*c)^7 + 72*A*a^2*b*tan(1/2*d*x + 1/2*c)^6 + 144*A*a*b^2*tan(1/2*d*x + 1/2*c)^5 + 144*B*b^3*tan(1/2*d*x + 1/2*c)^4 + 72*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 + 144*A*a*b^3*tan(1/2*d*x + 1/2*c)^2 + 72*B*b^4*tan(1/2*d*x + 1/2*c) + 72*A*a^2*b^3))

$$\begin{aligned} & \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^7 - 36B^2a^2b \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^7 - 36A^2ab^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^7 + 72B^2ab^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^7 + 24A^2ab^3 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^7 - 15B^2b^3 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^7 - 72B^2a^3 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5 - 216A^2a^2b \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5 + 36B^2a^2b \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5 + 36A^2ab^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5 - 120B^2ab^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5 - 40A^2ab^3 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5 - 9B^2b^3 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5 + 72B^2a^3 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 + 216A^2a^2b \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 + 36B^2a^2b \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 + 36A^2ab^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 + 120B^2ab^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 + 40A^2ab^3 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 - 9B^2b^3 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 - 24B^2a^3 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 72A^2a^2b \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 36B^2a^2b \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 36A^2ab^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 72B^2ab^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 24A^2ab^3 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 15B^2b^3 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) \Big/ (\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - 1)^4 \Big/ d \end{aligned}$$

maple [A] time = 1.38, size = 290, normalized size = 1.61

$$\frac{Aa^3 \ln(\sec(dx+c) + \tan(dx+c))}{d} + \frac{a^3B \tan(dx+c)}{d} + \frac{3Aa^2b \tan(dx+c)}{d} + \frac{3a^2bB \sec(dx+c) \tan(dx+c)}{2d} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x)

[Out] $\frac{1}{d}A^2a^3 \ln(\sec(dx+c) + \tan(dx+c)) + \frac{1}{d}a^3B \tan(dx+c) + \frac{3}{d}A^2a^2b \tan(dx+c) + \frac{3}{2} \frac{A^2a^2bB \sec(dx+c) \tan(dx+c)}{d} + \frac{3}{2} \frac{A^2a^2bB \ln(\sec(dx+c) + \tan(dx+c))}{d} + \frac{3}{2} \frac{A^2a^2b^2 \sec(dx+c) \tan(dx+c)}{d} + \frac{3}{2} \frac{A^2a^2b^2 \ln(\sec(dx+c) + \tan(dx+c))}{d} + \frac{2}{d}B^2a^2b \tan(dx+c) + \frac{1}{d}B^2a^2b^2 \tan(dx+c) \sec(dx+c)^2 + \frac{2}{3} \frac{A^2ab^3 \tan(dx+c)}{d} + \frac{1}{3} \frac{A^2ab^3 \tan(dx+c) \sec(dx+c)^2}{d} + \frac{1}{4} \frac{d^2b^3B \tan(dx+c) \sec(dx+c)^3}{d} + \frac{3}{8} \frac{d^2b^3B \sec(dx+c) \tan(dx+c)}{d} + \frac{3}{8} \frac{d^2b^3B \ln(\sec(dx+c) + \tan(dx+c))}{d}$

maxima [A] time = 0.74, size = 266, normalized size = 1.48

$$\frac{48 \left(\tan(dx+c)^3 + 3 \tan(dx+c) \right) Bab^2 + 16 \left(\tan(dx+c)^3 + 3 \tan(dx+c) \right) Ab^3 - 3Bb^3 \left(\frac{2(3 \sin(dx+c)^3 - 5 \sin(dx+c))}{\sin(dx+c)^4 - 2 \sin(dx+c)^2 + 1} \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] $\frac{1}{48} \left(48 \left(\tan(dx+c)^3 + 3 \tan(dx+c) \right) B^2ab^2 + 16 \left(\tan(dx+c)^3 + 3 \tan(dx+c) \right) Ab^3 - 3B^2b^3 \left(\frac{2(3 \sin(dx+c)^3 - 5 \sin(dx+c))}{\sin(dx+c)^4 - 2 \sin(dx+c)^2 + 1} \right) - 3 \log(\sin(dx+c) + 1) + 3 \log(\sin(dx+c) - 1) \right) - 36B^2a^2b \left(\frac{2 \sin(dx+c)}{\sin(dx+c)^2 - 1} - \log(\sin(dx+c)) \right)$

$x + c) + 1) + \log(\sin(dx + c) - 1) - 36Aa^2b^2(2\sin(dx + c)/(\sin(dx + c)^2 - 1) - \log(\sin(dx + c) + 1) + \log(\sin(dx + c) - 1)) + 48Aa^3 \log(\sec(dx + c) + \tan(dx + c)) + 48Ba^3 \tan(dx + c) + 144Aa^2b \tan(dx + c))/d$

mupad [B] time = 6.01, size = 395, normalized size = 2.19

$$\frac{\operatorname{atanh}\left(\frac{4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right) \left(Aa^3 + \frac{3Ba^2b}{2} + \frac{3Aab^2}{2} + \frac{3Bb^3}{8}\right)}{4Aa^3 + 6Ba^2b + 6Aab^2 + \frac{3Bb^3}{2}}\right) \left(2Aa^3 + 3Ba^2b + 3Aab^2 + \frac{3Bb^3}{4}\right) \left(2Ab^3 + 2Ba^3 - \frac{5Bb^3}{4} - 3Aa^3\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B/cos(c + dx))*(a + b/cos(c + dx))^3)/cos(c + dx), x)`

[Out] $(\operatorname{atanh}((4 \tan(c/2 + (dx)/2) * (Aa^3 + (3Bb^3)/8 + (3Aa^2b)/2 + (3Bb^2)/2)) / (4Aa^3 + (3Bb^3)/2 + 6Aa^2b + 6Ba^2b)) * (2Aa^3 + (3Bb^3)/4 + 3Aa^2b + 3Ba^2b)) / d - (\tan(c/2 + (dx)/2)^7 * (2Aa^3 + 2Ba^3 - (5Bb^3)/4 - 3Aa^2b + 6Aa^2b + 6Ba^2b - 3Ba^2b) + \tan(c/2 + (dx)/2)^3 * ((10Aa^3)/3 + 6Ba^3 - (3Bb^3)/4 + 3Aa^2b + 18Aa^2b + 10Ba^2b + 3Ba^2b) - \tan(c/2 + (dx)/2)^5 * ((10Aa^3)/3 + 6Ba^3 + (3Bb^3)/4 - 3Aa^2b + 18Aa^2b + 10Ba^2b - 3Ba^2b) - \tan(c/2 + (dx)/2) * (2Aa^3 + 2Ba^3 + (5Bb^3)/4 + 3Aa^2b + 6Aa^2b + 6Ba^2b + 3Ba^2b)) / (d * (6 \tan(c/2 + (dx)/2)^4 - 4 \tan(c/2 + (dx)/2)^2 - 4 \tan(c/2 + (dx)/2)^6 + \tan(c/2 + (dx)/2)^8 + 1))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \sec(c + dx))(a + b \sec(c + dx))^3 \sec(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(dx+c)*(a+b*sec(dx+c))**3*(A+B*sec(dx+c)), x)`

[Out] `Integral((A + B*sec(c + dx))*(a + b*sec(c + dx))**3*sec(c + dx), x)`

3.296 $\int (a + b \sec(c + dx))^3 (A + B \sec(c + dx)) dx$

Optimal. Leaf size=137

$$a^3 Ax + \frac{b(8a^2B + 9aAb + 2b^2B) \tan(c + dx)}{3d} + \frac{(2a^3B + 6a^2Ab + 3ab^2B + Ab^3) \tanh^{-1}(\sin(c + dx))}{2d} + \frac{b^2(5aB + 3B^2)}{3d}$$

[Out] $a^3Ax + \frac{1}{2} * (6Aa^2b + Ab^3 + 2Ba^3 + 3Bab^2) * \arctanh(\sin(dx+c)) / d + \frac{1}{3} * b * (9Aab + 8Ba^2 + 2Bb^2) * \tan(dx+c) / d + \frac{1}{6} * b^2 * (3Ab + 5Ba) * \sec(dx+c) * \tan(dx+c) / d + \frac{1}{3} * b * B * (a + b \sec(dx+c))^2 * \tan(dx+c) / d$

Rubi [A] time = 0.19, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3918, 4048, 3770, 3767, 8}

$$\frac{b(8a^2B + 9aAb + 2b^2B) \tan(c + dx)}{3d} + \frac{(6a^2Ab + 2a^3B + 3ab^2B + Ab^3) \tanh^{-1}(\sin(c + dx))}{2d} + a^3 Ax + \frac{b^2(5aB + 3B^2)}{3d}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]

[Out] $a^3Ax + ((6a^2Ab + Ab^3 + 2a^3B + 3ab^2B) * \text{ArcTanh}[\text{Sin}[c + d*x]]) / (2*d) + (b * (9aAb + 8a^2B + 2b^2B) * \text{Tan}[c + d*x]) / (3*d) + (b^2 * (3Ab + 5Ba) * \text{Sec}[c + d*x] * \text{Tan}[c + d*x]) / (6*d) + (b * B * (a + b * \text{Sec}[c + d*x])^2 * \text{Tan}[c + d*x]) / (3*d)$

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 3767

Int[csc[(c_) + (d_)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 3770

Int[csc[(c_) + (d_)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3918

Int[(csc[(e_) + (f_)*(x_)]*(b_) + (a_))^(m_)*(csc[(e_) + (f_)*(x_)]*(d_) + (c_)), x_Symbol] := -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1) + c*(a + b*Csc[e + f*x])^(m - 1))/d, x]

1))/ (f*m), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m - 2)*Simp[a^2*c*m + (b^2*d*(m - 1) + 2*a*b*c*m + a^2*d*m)*Csc[e + f*x] + b*(b*c*m + a*d*(2*m - 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && GtQ[m, 1] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m]

Rule 4048

Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] := -Simp[(b*C*Csc[e + f*x]*Cot[e + f*x])/(2*f), x] + Dist[1/2, Int[Simp[2*A*a + (2*B*a + b*(2*A + C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x]

Rubi steps

$$\begin{aligned}
 \int (a + b \sec(c + dx))^3 (A + B \sec(c + dx)) dx &= \frac{bB(a + b \sec(c + dx))^2 \tan(c + dx)}{3d} + \frac{1}{3} \int (a + b \sec(c + dx)) (3a^2 \\
 &= \frac{b^2(3Ab + 5aB) \sec(c + dx) \tan(c + dx)}{6d} + \frac{bB(a + b \sec(c + dx))^2 \tan(c + dx)}{3d} \\
 &= a^3 Ax + \frac{b^2(3Ab + 5aB) \sec(c + dx) \tan(c + dx)}{6d} + \frac{bB(a + b \sec(c + dx))^2 \tan(c + dx)}{3d} \\
 &= a^3 Ax + \frac{(6a^2 Ab + Ab^3 + 2a^3 B + 3ab^2 B) \tanh^{-1}(\sin(c + dx))}{2d} + \frac{bB(a + b \sec(c + dx))^2 \tan(c + dx)}{3d} \\
 &= a^3 Ax + \frac{(6a^2 Ab + Ab^3 + 2a^3 B + 3ab^2 B) \tanh^{-1}(\sin(c + dx))}{2d} + \frac{bB(a + b \sec(c + dx))^2 \tan(c + dx)}{3d}
 \end{aligned}$$

Mathematica [A] time = 0.59, size = 108, normalized size = 0.79

$$\frac{6a^3 Adx + 3b \tan(c + dx) (6a^2 B + b(3aB + Ab) \sec(c + dx) + 6aAb + 2b^2 B) + 3 (2a^3 B + 6a^2 Ab + 3ab^2 B + Ab^3) \tan(c + dx)}{6d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]), x]

[Out] (6*a^3*A*d*x + 3*(6*a^2*A*b + A*b^3 + 2*a^3*B + 3*a*b^2*B)*ArcTanh[Sin[c + d*x]] + 3*b*(6*a*A*b + 6*a^2*B + 2*b^2*B + b*(A*b + 3*a*B)*Sec[c + d*x])*Tan[c + d*x] + 2*b^3*B*Tan[c + d*x]^3)/(6*d)

fricas [A] time = 0.47, size = 189, normalized size = 1.38

$$\frac{12 Aa^3 dx \cos(dx + c)^3 + 3 (2Ba^3 + 6Aa^2b + 3Bab^2 + Ab^3) \cos(dx + c)^3 \log(\sin(dx + c) + 1) - 3 (2Ba^3 + 6Aa^2b + 3Bab^2 + Ab^3) \cos(dx + c)^3 \log(\sin(dx + c) - 1)}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] $\frac{1}{12}*(12*A*a^3*d*x*\cos(d*x + c)^3 + 3*(2*B*a^3 + 6*A*a^2*b + 3*B*a*b^2 + A*b^3)*\cos(d*x + c)^3*\log(\sin(d*x + c) + 1) - 3*(2*B*a^3 + 6*A*a^2*b + 3*B*a*b^2 + A*b^3)*\cos(d*x + c)^3*\log(-\sin(d*x + c) + 1) + 2*(2*B*b^3 + 2*(9*B*a^2*b + 9*A*a*b^2 + 2*B*b^3)*\cos(d*x + c)^2 + 3*(3*B*a*b^2 + A*b^3)*\cos(d*x + c))*\sin(d*x + c))/(d*\cos(d*x + c)^3)$

giac [B] time = 0.65, size = 336, normalized size = 2.45

$$6(dx+c)Aa^3 + 3(2Ba^3 + 6Aa^2b + 3Bab^2 + Ab^3) \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right|\right) - 3(2Ba^3 + 6Aa^2b + 3Bab^2 +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] $\frac{1}{6}*(6*(d*x + c)*A*a^3 + 3*(2*B*a^3 + 6*A*a^2*b + 3*B*a*b^2 + A*b^3)*\log(\operatorname{abs}(\tan(1/2*d*x + 1/2*c) + 1)) - 3*(2*B*a^3 + 6*A*a^2*b + 3*B*a*b^2 + A*b^3)*\log(\operatorname{abs}(\tan(1/2*d*x + 1/2*c) - 1)) - 2*(18*B*a^2*b*\tan(1/2*d*x + 1/2*c)^5 + 18*A*a*b^2*\tan(1/2*d*x + 1/2*c)^5 - 9*B*a*b^2*\tan(1/2*d*x + 1/2*c)^5 - 3*A*b^3*\tan(1/2*d*x + 1/2*c)^5 + 6*B*b^3*\tan(1/2*d*x + 1/2*c)^5 - 36*B*a^2*b*\tan(1/2*d*x + 1/2*c)^3 - 36*A*a*b^2*\tan(1/2*d*x + 1/2*c)^3 - 4*B*b^3*\tan(1/2*d*x + 1/2*c)^3 + 18*B*a^2*b*\tan(1/2*d*x + 1/2*c) + 18*A*a*b^2*\tan(1/2*d*x + 1/2*c) + 9*B*a*b^2*\tan(1/2*d*x + 1/2*c) + 3*A*b^3*\tan(1/2*d*x + 1/2*c) + 6*B*b^3*\tan(1/2*d*x + 1/2*c))/(\tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d$

maple [A] time = 1.18, size = 223, normalized size = 1.63

$$a^3Ax + \frac{Aa^3c}{d} + \frac{a^3B \ln(\sec(dx+c) + \tan(dx+c))}{d} + \frac{3Aa^2b \ln(\sec(dx+c) + \tan(dx+c))}{d} + \frac{3a^2bB \tan(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x)

[Out] $a^3Ax + 1/d*A*a^3c + 1/d*a^3B*\ln(\sec(d*x+c) + \tan(d*x+c)) + 3/d*A*a^2*b*\ln(\sec(d*x+c) + \tan(d*x+c)) + 3/d*a^2*b*B*\tan(d*x+c) + 3/d*A*a*b^2*\tan(d*x+c) + 3/2/d*B*a*b^2*\sec(d*x+c)*\tan(d*x+c) + 3/2/d*B*a*b^2*\ln(\sec(d*x+c) + \tan(d*x+c)) + 1/2/d*A*b^3*\sec(d*x+c)*\tan(d*x+c) + 1/2/d*A*b^3*\ln(\sec(d*x+c) + \tan(d*x+c)) + 2/3/d*b^3*B*\tan(d*x+c) + 1/3/d*b^3*B*\tan(d*x+c)*\sec(d*x+c)^2$

maxima [A] time = 0.32, size = 202, normalized size = 1.47

$$12(dx+c)Aa^3 + 4(\tan(dx+c)^3 + 3\tan(dx+c))Bb^3 - 9Bab^2\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1} - \log(\sin(dx+c)+1) + \log(\sin(dx+c)-1)\right) - 3A^2b^3\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1} - \log(\sin(dx+c)+1) + \log(\sin(dx+c)-1)\right) + 12B^2a^3\log(\sec(dx+c)+\tan(dx+c)) + 36A^2a^2b\log(\sec(dx+c)+\tan(dx+c)) + 36B^2a^2b\tan(dx+c) + 36A^2a^2b^2\tan(dx+c)/d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] 1/12*(12*(d*x + c)*A*a^3 + 4*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*b^3 - 9*B*a*b^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 3*A*b^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 12*B*a^3*log(sec(d*x + c) + tan(d*x + c)) + 36*A*a^2*b*log(sec(d*x + c) + tan(d*x + c)) + 36*B*a^2*b*tan(d*x + c) + 36*A*a*b^2*tan(d*x + c))/d

mupad [B] time = 4.03, size = 526, normalized size = 3.84

$$\frac{A b^3 \sin(2c+2dx)}{4} + \frac{B b^3 \sin(3c+3dx)}{6} + \frac{B b^3 \sin(c+dx)}{2} + \frac{3 A a b^2 \sin(c+dx)}{4} + \frac{3 B a^2 b \sin(c+dx)}{4} + \frac{3 A a^3 \cos(c+dx) \operatorname{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^3,x)

[Out] ((A*b^3*sin(2*c + 2*d*x))/4 + (B*b^3*sin(3*c + 3*d*x))/6 + (B*b^3*sin(c + d*x))/2 + (3*A*a*b^2*sin(c + d*x))/4 + (3*B*a^2*b*sin(c + d*x))/4 + (3*A*a^3*cos(c + d*x)*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/2 - (A*b^3*cos(c + d*x)*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*3i)/4 - (B*a^3*cos(c + d*x)*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*3i)/2 + (3*A*a*b^2*sin(3*c + 3*d*x))/4 + (3*B*a*b^2*sin(2*c + 2*d*x))/4 + (3*B*a^2*b*sin(3*c + 3*d*x))/4 + (A*a^3*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(3*c + 3*d*x))/2 - (A*b^3*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*cos(3*c + 3*d*x)*1i)/4 - (B*a^3*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*cos(3*c + 3*d*x)*1i)/2 - (A*a^2*b*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*cos(3*c + 3*d*x)*3i)/2 - (B*a*b^2*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*cos(3*c + 3*d*x)*3i)/4 - (A*a^2*b*cos(c + d*x)*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*9i)/2 - (B*a*b^2*cos(c + d*x)*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*9i)/4)/(d*((3*cos(c + d*x))/4 + cos(3*c + 3*d*x)/4))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \sec(c + dx))(a + b \sec(c + dx))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**3*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**3, x)
```

$$3.297 \quad \int \cos(c + dx)(a + b \sec(c + dx))^3(A + B \sec(c + dx)) dx$$

Optimal. Leaf size=119

$$\frac{b(6a^2B + 6aAb + b^2B) \tanh^{-1}(\sin(c + dx))}{2d} + \frac{a^2(2aA - bB) \sin(c + dx)}{2d} + a^2x(aB + 3Ab) + \frac{b^2(2aB + Ab) \tan(c + dx)}{d}$$

[Out] $a^2(3A*b+B*a)*x+1/2*b*(6*A*a*b+6*B*a^2+B*b^2)*\operatorname{arctanh}(\sin(d*x+c))/d+1/2*a^2*(2*A*a-B*b)*\sin(d*x+c)/d+1/2*b*B*(a+b*\sec(d*x+c))^2*\sin(d*x+c)/d+b^2*(A*b+2*B*a)*\tan(d*x+c)/d$

Rubi [A] time = 0.22, antiderivative size = 131, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {4025, 4048, 3770, 3767, 8}

$$\frac{b(2a^2A - 3abB - Ab^2) \tan(c + dx)}{d} + \frac{b(6a^2B + 6aAb + b^2B) \tanh^{-1}(\sin(c + dx))}{2d} + a^2x(aB + 3Ab) - \frac{b^2(2aA - bB)}{d}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Cos}[c + d*x]*(a + b*\operatorname{Sec}[c + d*x])^3*(A + B*\operatorname{Sec}[c + d*x]), x]$

[Out] $a^2(3A*b + a*B)*x + (b*(6*a*A*b + 6*a^2*B + b^2*B)*\operatorname{ArcTanh}[\operatorname{Sin}[c + d*x]])/(2*d) + (a*A*(a + b*\operatorname{Sec}[c + d*x])^2*\operatorname{Sin}[c + d*x])/d - (b*(2*a^2*A - A*b^2 - 3*a*b*B)*\operatorname{Tan}[c + d*x])/d - (b^2*(2*a*A - b*B)*\operatorname{Sec}[c + d*x]*\operatorname{Tan}[c + d*x])/(2*d)$

Rule 8

$\operatorname{Int}[a_, x_Symbol] \rightarrow \operatorname{Simp}[a*x, x] /; \operatorname{FreeQ}[a, x]$

Rule 3767

$\operatorname{Int}[\operatorname{csc}[(c_.) + (d_.)*(x_)]^{(n_)}, x_Symbol] \rightarrow -\operatorname{Dist}[d^{(-1)}, \operatorname{Subst}[\operatorname{Int}[\operatorname{ExpandIntegrand}[(1 + x^2)^{(n/2 - 1)}, x], x], x, \operatorname{Cot}[c + d*x]], x] /; \operatorname{FreeQ}\{c, d\}, x \&\& \operatorname{IGtQ}[n/2, 0]$

Rule 3770

$\operatorname{Int}[\operatorname{csc}[(c_.) + (d_.)*(x_)], x_Symbol] \rightarrow -\operatorname{Simp}[\operatorname{ArcTanh}[\operatorname{Cos}[c + d*x]]/d, x] /; \operatorname{FreeQ}\{c, d\}, x]$

Rule 4025


```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> Simp[(a*A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e + f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LeQ[n, -1]
```

Rule 4048

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] :> -Simp[(b*C*Csc[e + f*x]*Cot[e + f*x])/(2*f), x] + Dist[1/2, Int[Simp[2*A*a + (2*B*a + b*(2*A + C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x]
```

Rubi steps

$$\begin{aligned} \int \cos(c + dx)(a + b \sec(c + dx))^3(A + B \sec(c + dx)) dx &= \frac{aA(a + b \sec(c + dx))^2 \sin(c + dx)}{d} - \int (a + b \sec(c + dx)) \cos(c + dx) dx \\ &= \frac{aA(a + b \sec(c + dx))^2 \sin(c + dx)}{d} - \frac{b^2(2aA - bB) \sin(c + dx)}{d} \\ &= a^2(3Ab + aB)x + \frac{aA(a + b \sec(c + dx))^2 \sin(c + dx)}{d} \\ &= a^2(3Ab + aB)x + \frac{b(6aAb + 6a^2B + b^2B) \tanh^{-1}(\sin(c + dx))}{2d} \\ &= a^2(3Ab + aB)x + \frac{b(6aAb + 6a^2B + b^2B) \tanh^{-1}(\sin(c + dx))}{2d} \end{aligned}$$

Mathematica [B] time = 0.98, size = 399, normalized size = 3.35

$$\frac{\sec^2(c + dx) \left((a^3 A + 2b^3 B) \sin(c + dx) + a^3 A \sin(3(c + dx)) + 2a^3 B c + 2a^3 B dx + \cos(2(c + dx)) \right) \left(-b(6a^2 B + 6a^2 B) \right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```

```
[Out] (Sec[c + d*x]^2*(6*a^2*A*b*c + 2*a^3*B*c + 6*a^2*A*b*d*x + 2*a^3*B*d*x - 6*a*A*b^2*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 6*a^2*b*B*Log[Cos[(c + d*x)/2]]
```

$x)/2] - \sin[(c + dx)/2] - b^3 B \log[\cos[(c + dx)/2] - \sin[(c + dx)/2]] + 6a^2 b^2 \log[\cos[(c + dx)/2] + \sin[(c + dx)/2]] + 6a^2 b^2 B \log[\cos[(c + dx)/2] + \sin[(c + dx)/2]] + b^3 B \log[\cos[(c + dx)/2] + \sin[(c + dx)/2]] + \cos[2(c + dx)] * (2a^2(3Ab + a^2 B)(c + dx) - b(6a^2 B + b^2 B)) \log[\cos[(c + dx)/2] - \sin[(c + dx)/2]] + b(6a^2 B + b^2 B) \log[\cos[(c + dx)/2] + \sin[(c + dx)/2]] + (a^3 A + 2b^3 B) \sin[c + dx] + 2Ab^3 \sin[2(c + dx)] + 6a^2 b^2 B \sin[2(c + dx)] + a^3 A \sin[3(c + dx)])) / (4d)$

fricas [A] time = 0.47, size = 167, normalized size = 1.40

$$\frac{4(Ba^3 + 3Aa^2b)dx \cos(dx + c)^2 + (6Ba^2b + 6Aab^2 + Bb^3) \cos(dx + c)^2 \log(\sin(dx + c) + 1) - (6Ba^2b + 6Aa^2b + Bb^3) \cos(dx + c)^2 \log(-\sin(dx + c) + 1) + 2(2Aa^3 \cos(dx + c)^2 + Bb^3 + 2(3Ba^2b + Ab^3) \cos(dx + c)) \sin(dx + c)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(dx+c)*(a+b*sec(dx+c))^3*(A+B*sec(dx+c)),x, algorithm="fricas")

[Out] $\frac{1}{4} * (4 * (B * a^3 + 3 * A * a^2 * b) * dx * \cos(dx + c)^2 + (6 * B * a^2 * b + 6 * A * a * b^2 + B * b^3) * \cos(dx + c)^2 * \log(\sin(dx + c) + 1) - (6 * B * a^2 * b + 6 * A * a * b^2 + B * b^3) * \cos(dx + c)^2 * \log(-\sin(dx + c) + 1) + 2 * (2 * A * a^3 * \cos(dx + c)^2 + B * b^3 + 2 * (3 * B * a^2 * b + A * b^3) * \cos(dx + c)) * \sin(dx + c)) / (d * \cos(dx + c)^2)$

giac [B] time = 0.38, size = 241, normalized size = 2.03

$$\frac{4Aa^3 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + 1} + 2(Ba^3 + 3Aa^2b)(dx + c) + (6Ba^2b + 6Aab^2 + Bb^3) \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 1\right|\right) - (6Ba^2b + 6Aa^2b + Bb^3) \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(dx+c)*(a+b*sec(dx+c))^3*(A+B*sec(dx+c)),x, algorithm="giac")

[Out] $\frac{1}{2} * (4 * A * a^3 * \tan(1/2 * dx + 1/2 * c) / (\tan(1/2 * dx + 1/2 * c)^2 + 1) + 2 * (B * a^3 + 3 * A * a^2 * b) * (dx + c) + (6 * B * a^2 * b + 6 * A * a * b^2 + B * b^3) * \log(\text{abs}(\tan(1/2 * dx + 1/2 * c) + 1)) - (6 * B * a^2 * b + 6 * A * a * b^2 + B * b^3) * \log(\text{abs}(\tan(1/2 * dx + 1/2 * c) - 1)) - 2 * (6 * B * a^2 * b^2 * \tan(1/2 * dx + 1/2 * c)^3 + 2 * A * b^3 * \tan(1/2 * dx + 1/2 * c)^3 - B * b^3 * \tan(1/2 * dx + 1/2 * c)^3 - 6 * B * a * b^2 * \tan(1/2 * dx + 1/2 * c) - 2 * A * b^3 * \tan(1/2 * dx + 1/2 * c) - B * b^3 * \tan(1/2 * dx + 1/2 * c)) / (\tan(1/2 * dx + 1/2 * c)^2 - 1)^2) / d$

maple [A] time = 1.17, size = 172, normalized size = 1.45

$$\frac{a^3 A \sin(dx + c)}{d} + a^3 B x + \frac{a^3 B c}{d} + 3A x a^2 b + \frac{3A a^2 b c}{d} + \frac{3a^2 b B \ln(\sec(dx + c) + \tan(dx + c))}{d} + \frac{3A a b^2 \ln(\sec(dx + c) + \tan(dx + c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(d*x+c)*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x)`

[Out] $a^3 A \sin(d*x+c)/d + a^3 B*x + 1/d*a^3*B*c + 3*A*x*a^2*b + 3/d*A*a^2*b*c + 3/d*a^2*b*B*\ln(\sec(d*x+c)+\tan(d*x+c)) + 3/d*A*a*b^2*\ln(\sec(d*x+c)+\tan(d*x+c)) + 3/d*B*a*b^2*\tan(d*x+c) + 1/d*A*b^3*\tan(d*x+c) + 1/2/d*b^3*B*sec(d*x+c)*\tan(d*x+c) + 1/2/d*b^3*B*\ln(\sec(d*x+c)+\tan(d*x+c))$

maxima [A] time = 0.77, size = 169, normalized size = 1.42

$$4(dx+c)Ba^3 + 12(dx+c)Aa^2b - Bb^3 \left(\frac{2 \sin(dx+c)}{\sin(dx+c)^2-1} - \log(\sin(dx+c)+1) + \log(\sin(dx+c)-1) \right) + 6Ba^2b(\log(\sec(dx+c)+\tan(dx+c)))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="maxima")`

[Out] $1/4*(4*(d*x+c)*B*a^3 + 12*(d*x+c)*A*a^2*b - B*b^3*(2*\sin(d*x+c)/(\sin(d*x+c)^2-1) - \log(\sin(d*x+c)+1) + \log(\sin(d*x+c)-1)) + 6*B*a^2*b*(\log(\sin(d*x+c)+1) - \log(\sin(d*x+c)-1)) + 6*A*a*b^2*(\log(\sin(d*x+c)+1) - \log(\sin(d*x+c)-1)) + 4*A*a^3*\sin(d*x+c) + 12*B*a*b^2*\tan(d*x+c) + 4*A*b^3*\tan(d*x+c))/d$

mupad [B] time = 3.60, size = 249, normalized size = 2.09

$$\frac{\frac{Aa^3 \sin(3c+3dx)}{4} + \frac{Ab^3 \sin(2c+2dx)}{2} + \frac{Aa^3 \sin(c+dx)}{4} + \frac{Bb^3 \sin(c+dx)}{2} + \frac{3Bab^2 \sin(2c+2dx)}{2}}{d \left(\frac{\cos(2c+2dx)}{2} + \frac{1}{2} \right)} - 2 \left(-Ba^3 \operatorname{atan} \left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)} \right) \right) +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(c+d*x)*(A+B/cos(c+d*x))*(a+b/cos(c+d*x))^3,x)`

[Out] $((A*a^3*\sin(3*c+3*d*x))/4 + (A*b^3*\sin(2*c+2*d*x))/2 + (A*a^3*\sin(c+d*x))/4 + (B*b^3*\sin(c+d*x))/2 + (3*B*a*b^2*\sin(2*c+2*d*x))/2)/(d*(\cos(2*c+2*d*x)/2+1/2)) - (2*((B*b^3*\operatorname{atan}((\sin(c/2+(d*x)/2)*1i)/\cos(c/2+(d*x)/2))*1i)/2 - B*a^3*\operatorname{atan}(\sin(c/2+(d*x)/2)/\cos(c/2+(d*x)/2)) - 3*A*a^2*b*\operatorname{atan}(\sin(c/2+(d*x)/2)/\cos(c/2+(d*x)/2)) + A*a*b^2*\operatorname{atan}((\sin(c/2+(d*x)/2)*1i)/\cos(c/2+(d*x)/2))*3i + B*a^2*b*\operatorname{atan}((\sin(c/2+(d*x)/2)*1i)/\cos(c/2+(d*x)/2))*3i))/d$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \sec(c + dx)) (a + b \sec(c + dx))^3 \cos(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)*(a+b*sec(d*x+c))**3*(A+B*sec(d*x+c)),x)`

[Out] `Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**3*cos(c + d*x), x)`

$$3.298 \quad \int \cos^2(c+dx)(a+b \sec(c+dx))^3(A+B \sec(c+dx)) dx$$

Optimal. Leaf size=124

$$\frac{1}{2}ax(a^2A + 6abB + 6Ab^2) + \frac{a^2(aB + 2Ab) \sin(c+dx)}{d} - \frac{b^2(aA - 2bB) \tan(c+dx)}{2d} + \frac{b^2(3aB + Ab) \tanh^{-1}(\sin(c+dx))}{d}$$

[Out] 1/2*a*(A*a^2+6*A*b^2+6*B*a*b)*x+b^2*(A*b+3*B*a)*arctanh(sin(d*x+c))/d+a^2*(2*A*b+B*a)*sin(d*x+c)/d+1/2*a*A*cos(d*x+c)*(a+b*sec(d*x+c))^2*sin(d*x+c)/d-1/2*b^2*(A*a-2*B*b)*tan(d*x+c)/d

Rubi [A] time = 0.33, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4025, 4076, 4047, 8, 4045, 3770}

$$\frac{1}{2}ax(a^2A + 6abB + 6Ab^2) + \frac{a^2(aB + 2Ab) \sin(c+dx)}{d} - \frac{b^2(aA - 2bB) \tan(c+dx)}{2d} + \frac{b^2(3aB + Ab) \tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]

[Out] (a*(a^2*A + 6*A*b^2 + 6*a*b*B)*x)/2 + (b^2*(A*b + 3*a*B)*ArcTanh[Sin[c + d*x]])/d + (a^2*(2*A*b + a*B)*Sin[c + d*x])/d + (a*A*Cos[c + d*x]*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(2*d) - (b^2*(a*A - 2*b*B)*Tan[c + d*x])/(2*d)

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 3770

Int[csc[(c_) + (d_)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rule 4025

Int[(csc[(e_) + (f_)*(x_)]*(d_.))^n*(csc[(e_) + (f_)*(x_)]*(b_.) + (a_.))^m*(csc[(e_) + (f_)*(x_)]*(B_.) + (A_.)), x_Symbol] := Simp[(a*A*Co t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dis t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a *(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e + f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&

LeQ[n, -1]

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]^2*(C_.)
+ (A_.)), x_Symbol] := Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_.)]*
(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.)), x_Symbol] := Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 4076

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.)
)*(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (
a_.)), x_Symbol] := -Simp[(b*C*Csc[e + f*x]*Cot[e + f*x]*(d*Csc[e + f*x])^n)
/(f*(n + 2)), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2)
+ (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n +
2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &&
!LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \cos^2(c + dx)(a + b \sec(c + dx))^3(A + B \sec(c + dx)) dx &= \frac{aA \cos(c + dx)(a + b \sec(c + dx))^2 \sin(c + dx)}{2d} - \frac{1}{2} \int \cos^2(c + dx)(a + b \sec(c + dx))^2(A + B \sec(c + dx)) dx \\
&= \frac{aA \cos(c + dx)(a + b \sec(c + dx))^2 \sin(c + dx)}{2d} - \frac{b^2}{2d} \int \cos^2(c + dx)(a + b \sec(c + dx)) dx \\
&= \frac{aA \cos(c + dx)(a + b \sec(c + dx))^2 \sin(c + dx)}{2d} - \frac{b^2}{2d} \int \cos^2(c + dx) dx \\
&= \frac{1}{2} a (a^2 A + 6Ab^2 + 6abB) x + \frac{a^2(2Ab + aB) \sin(c + dx)}{d} \\
&= \frac{1}{2} a (a^2 A + 6Ab^2 + 6abB) x + \frac{b^2(Ab + 3aB) \tanh^{-1}(\sec(c + dx))}{d}
\end{aligned}$$

Mathematica [A] time = 0.71, size = 217, normalized size = 1.75

$$a^3 A \sin(2(c + dx)) + 2a(c + dx) (a^2 A + 6abB + 6Ab^2) + 4a^2(aB + 3Ab) \sin(c + dx) - 4b^2(3aB + Ab) \log(\cos(\dots))$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]

[Out] (2*a*(a^2*A + 6*A*b^2 + 6*a*b*B)*(c + d*x) - 4*b^2*(A*b + 3*a*B)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 4*b^2*(A*b + 3*a*B)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (4*b^3*B*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) + (4*b^3*B*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) + 4*a^2*(3*A*b + a*B)*Sin[c + d*x] + a^3*A*Sin[2*(c + d*x)]/(4*d)

fricas [A] time = 0.47, size = 152, normalized size = 1.23

$$\frac{(Aa^3 + 6Ba^2b + 6Aab^2)dx \cos(dx + c) + (3Bab^2 + Ab^3) \cos(dx + c) \log(\sin(dx + c) + 1) - (3Bab^2 + Ab^3) \cos(dx + c) \log(\sin(dx + c) - 1)}{2d \cos(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] 1/2*((A*a^3 + 6*B*a^2*b + 6*A*a*b^2)*d*x*cos(d*x + c) + (3*B*a*b^2 + A*b^3)*cos(d*x + c)*log(sin(d*x + c) + 1) - (3*B*a*b^2 + A*b^3)*cos(d*x + c)*log(-sin(d*x + c) + 1) + (A*a^3*cos(d*x + c)^2 + 2*B*b^3 + 2*(B*a^3 + 3*A*a^2*b)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))

giac [A] time = 0.35, size = 234, normalized size = 1.89

$$\frac{4Bb^3 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)}{\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - 1} - (Aa^3 + 6Ba^2b + 6Aab^2)(dx + c) - 2(3Bab^2 + Ab^3) \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right|\right) + 2(3Bab^2 + Ab^3) \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] -1/2*(4*B*b^3*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1) - (A*a^3 + 6*B*a^2*b + 6*A*a*b^2)*(d*x + c) - 2*(3*B*a*b^2 + A*b^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) + 2*(3*B*a*b^2 + A*b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)))/(d*cos(d*x + c))

$$\frac{x + 1/2*c) + 1)) + 2*(3*B*a*b^2 + A*b^3)*\log(\text{abs}(\tan(1/2*d*x + 1/2*c) - 1)) + 2*(A*a^3*\tan(1/2*d*x + 1/2*c)^3 - 2*B*a^3*\tan(1/2*d*x + 1/2*c)^3 - 6*A*a^2*b*\tan(1/2*d*x + 1/2*c)^3 - A*a^3*\tan(1/2*d*x + 1/2*c) - 2*B*a^3*\tan(1/2*d*x + 1/2*c) - 6*A*a^2*b*\tan(1/2*d*x + 1/2*c))/(\tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d$$

maple [A] time = 0.95, size = 168, normalized size = 1.35

$$\frac{A a^3 \cos(dx + c) \sin(dx + c)}{2d} + \frac{a^3 A x}{2} + \frac{A a^3 c}{2d} + \frac{a^3 B \sin(dx + c)}{d} + \frac{3A a^2 b \sin(dx + c)}{d} + 3B x a^2 b + \frac{3B a^2 b c}{d} + 3A x a b^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(d*x+c)^2*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)), x)`

[Out] `1/2/d*A*a^3*cos(d*x+c)*sin(d*x+c)+1/2*a^3*A*x+1/2/d*A*a^3*c+a^3*B*sin(d*x+c)/d+3/d*A*a^2*b*sin(d*x+c)+3*B*x*a^2*b+3/d*B*a^2*b*c+3*A*x*a*b^2+3/d*A*a*b^2*c+3/d*B*a*b^2*ln(sec(d*x+c)+tan(d*x+c))+1/d*A*b^3*ln(sec(d*x+c)+tan(d*x+c))+1/d*b^3*B*tan(d*x+c)`

maxima [A] time = 0.81, size = 144, normalized size = 1.16

$$\frac{(2dx + 2c + \sin(2dx + 2c))Aa^3 + 12(dx + c)Ba^2b + 12(dx + c)Aab^2 + 6Bab^2(\log(\sin(dx + c) + 1) - \log(\sin(dx + c) - 1))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)), x, algorithm="maxima")`

[Out] `1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*A*a^3 + 12*(d*x + c)*B*a^2*b + 12*(d*x + c)*A*a*b^2 + 6*B*a*b^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 2*A*b^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 4*B*a^3*sin(d*x + c) + 12*A*a^2*b*sin(d*x + c) + 4*B*b^3*tan(d*x + c))/d`

mupad [B] time = 3.33, size = 236, normalized size = 1.90

$$\frac{A a^3 \operatorname{atan}\left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)}\right) - A b^3 \operatorname{atan}\left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right) 1i}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)}\right) 2i + 6 A a b^2 \operatorname{atan}\left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)}\right) + 6 B a^2 b \operatorname{atan}\left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)}\right) - B a b^2}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(c + d*x)^2*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^3, x)`

[Out] `(A*a^3*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) - A*b^3*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*2i + 6*A*a*b^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))`


```
(c/2 + (d*x)/2)) + 6*B*a^2*b*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) -
B*a*b^2*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*6i)/d + ((A*a^3*si
n(3*c + 3*d*x))/8 + (B*a^3*sin(2*c + 2*d*x))/2 + (A*a^3*sin(c + d*x))/8 + B
*b^3*sin(c + d*x) + (3*A*a^2*b*sin(2*c + 2*d*x))/2)/(d*cos(c + d*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \sec(c + dx)) (a + b \sec(c + dx))^3 \cos^2(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**2*(a+b*sec(d*x+c))**3*(A+B*sec(d*x+c)),x)

[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**3*cos(c + d*x)**2, x)

$$3.299 \quad \int \cos^3(c+dx)(a+b \sec(c+dx))^3(A+B \sec(c+dx)) dx$$

Optimal. Leaf size=145

$$\frac{a(2a^2A + 9abB + 8Ab^2) \sin(c+dx)}{3d} + \frac{a^2(3aB + 5Ab) \sin(c+dx) \cos(c+dx)}{6d} + \frac{1}{2}x(a^3B + 3a^2Ab + 6ab^2B + 2Ab^3)$$

[Out] $1/2*(3*A*a^2*b+2*A*b^3+B*a^3+6*B*a*b^2)*x+b^3*B*\operatorname{arctanh}(\sin(d*x+c))/d+1/3*a*(2*A*a^2+8*A*b^2+9*B*a*b)*\sin(d*x+c)/d+1/6*a^2*(5*A*b+3*B*a)*\cos(d*x+c)*\sin(d*x+c)/d+1/3*a*A*\cos(d*x+c)^2*(a+b*\sec(d*x+c))^2*\sin(d*x+c)/d$

Rubi [A] time = 0.35, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4025, 4074, 4047, 8, 4045, 3770}

$$\frac{a(2a^2A + 9abB + 8Ab^2) \sin(c+dx)}{3d} + \frac{1}{2}x(3a^2Ab + a^3B + 6ab^2B + 2Ab^3) + \frac{a^2(3aB + 5Ab) \sin(c+dx) \cos(c+dx)}{6d}$$

Antiderivative was successfully verified.

[In] `Int[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]`

[Out] $((3*a^2*A*b + 2*A*b^3 + a^3*B + 6*a*b^2*B)*x)/2 + (b^3*B*\operatorname{ArcTanh}[\sin[c + d*x]])/d + (a*(2*a^2*A + 8*A*b^2 + 9*a*b*B)*\sin[c + d*x])/(3*d) + (a^2*(5*A*b + 3*a*B)*\cos[c + d*x]*\sin[c + d*x])/(6*d) + (a*A*\cos[c + d*x]^2*(a + b*\sec[c + d*x])^2*\sin[c + d*x])/(3*d)$

Rule 8

`Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

Rule 3770

`Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]`

Rule 4025

`Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^m*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] := Simp[(a*A*Co t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dis t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e + f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x] /; FreeQ[{a, b, d`

, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LeQ[n, -1]

Rule 4045

Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]^2*(C_.) + (A_.)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] + Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]

Rule 4047

Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2), x] /; FreeQ[{b, e, f, A, B, C, m}, x]

Rule 4074

Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] :> Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]

Rubi steps

$$\begin{aligned}
 \int \cos^3(c + dx)(a + b \sec(c + dx))^3(A + B \sec(c + dx)) dx &= \frac{aA \cos^2(c + dx)(a + b \sec(c + dx))^2 \sin(c + dx)}{3d} - \frac{1}{3} \\
 &= \frac{a^2(5Ab + 3aB) \cos(c + dx) \sin(c + dx)}{6d} + \frac{aA \cos^2(c + dx)}{3} \\
 &= \frac{a^2(5Ab + 3aB) \cos(c + dx) \sin(c + dx)}{6d} + \frac{aA \cos^2(c + dx)}{3} \\
 &= \frac{1}{2} (3a^2Ab + 2Ab^3 + a^3B + 6ab^2B) x + \frac{a(2a^2A + 8a^2B)}{6d} \\
 &= \frac{1}{2} (3a^2Ab + 2Ab^3 + a^3B + 6ab^2B) x + \frac{b^3B \tanh^{-1}\left(\frac{a + b \sec(c + dx)}{a}\right)}{a}
 \end{aligned}$$

Mathematica [A] time = 0.38, size = 159, normalized size = 1.10

$$a^3 A \sin(3(c + dx)) + 9a (a^2 A + 4abB + 4Ab^2) \sin(c + dx) + 3a^2 (aB + 3Ab) \sin(2(c + dx)) + 6(c + dx) (a^3 B + 3a^2 A)$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]

[Out] (6*(3*a^2*A*b + 2*A*b^3 + a^3*B + 6*a*b^2*B)*(c + d*x) - 12*b^3*B*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 12*b^3*B*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 9*a*(a^2*A + 4*A*b^2 + 4*a*b*B)*Sin[c + d*x] + 3*a^2*(3*A*b + a*B)*Sin[2*(c + d*x)] + a^3*A*Sin[3*(c + d*x)])/(12*d)

fricas [A] time = 0.51, size = 131, normalized size = 0.90

$$3 B b^3 \log(\sin(dx + c) + 1) - 3 B b^3 \log(-\sin(dx + c) + 1) + 3 (B a^3 + 3 A a^2 b + 6 B a b^2 + 2 A b^3) dx + (2 A a^3 \cos(dx + c) + 3 A a^2 b \sin(dx + c) + 3 A a b^2 \cos(dx + c) + 3 A b^3 \sin(dx + c)) / d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] 1/6*(3*B*b^3*log(sin(d*x + c) + 1) - 3*B*b^3*log(-sin(d*x + c) + 1) + 3*(B*a^3 + 3*A*a^2*b + 6*B*a*b^2 + 2*A*b^3)*d*x + (2*A*a^3*cos(d*x + c)^2 + 4*A*a^3 + 18*B*a^2*b + 18*A*a*b^2 + 3*(B*a^3 + 3*A*a^2*b)*cos(d*x + c))*sin(d*x + c))/d

giac [B] time = 0.67, size = 314, normalized size = 2.17

$$6 B b^3 \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 1\right|\right) - 6 B b^3 \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 1\right|\right) + 3 (B a^3 + 3 A a^2 b + 6 B a b^2 + 2 A b^3) (dx + c) + (2 A a^3 \cos(dx + c) + 3 A a^2 b \sin(dx + c) + 3 A a b^2 \cos(dx + c) + 3 A b^3 \sin(dx + c)) / d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] 1/6*(6*B*b^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 6*B*b^3*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 3*(B*a^3 + 3*A*a^2*b + 6*B*a*b^2 + 2*A*b^3)*(d*x + c) + 2*(6*A*a^3*tan(1/2*d*x + 1/2*c)^5 - 3*B*a^3*tan(1/2*d*x + 1/2*c)^5 - 9*A*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 18*B*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 18*A*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 9*B*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 3*A*b^3*tan(1/2*d*x + 1/2*c)^5 + 3*B*b^3*tan(1/2*d*x + 1/2*c)^5)/(12*d)

$$\frac{2*\tan(1/2*d*x + 1/2*c)^5 + 4*A*a^3*\tan(1/2*d*x + 1/2*c)^3 + 36*B*a^2*b*\tan(1/2*d*x + 1/2*c)^3 + 36*A*a*b^2*\tan(1/2*d*x + 1/2*c)^3 + 6*A*a^3*\tan(1/2*d*x + 1/2*c) + 3*B*a^3*\tan(1/2*d*x + 1/2*c) + 9*A*a^2*b*\tan(1/2*d*x + 1/2*c) + 18*B*a^2*b*\tan(1/2*d*x + 1/2*c) + 18*A*a*b^2*\tan(1/2*d*x + 1/2*c))/(\tan(1/2*d*x + 1/2*c)^2 + 1)^3/d$$

maple [A] time = 2.00, size = 207, normalized size = 1.43

$$\frac{A(\cos^2(dx+c))\sin(dx+c)a^3}{3d} + \frac{2a^3A\sin(dx+c)}{3d} + \frac{a^3B\cos(dx+c)\sin(dx+c)}{2d} + \frac{a^3Bx}{2} + \frac{a^3Bc}{2d} + \frac{3Aa^2b\cos(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^3*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x)

[Out] 1/3/d*A*cos(d*x+c)^2*sin(d*x+c)*a^3+2/3*a^3*A*sin(d*x+c)/d+1/2/d*a^3*B*cos(d*x+c)*sin(d*x+c)+1/2*a^3*B*x+1/2/d*a^3*B*c+3/2/d*A*a^2*b*cos(d*x+c)*sin(d*x+c)+3/2*A*x*a^2*b+3/2/d*A*a^2*b*c+3/d*a^2*b*B*sin(d*x+c)+3/d*A*a*b^2*sin(d*x+c)+3*B*x*a*b^2+3/d*B*a*b^2*c+A*x*b^3+1/d*A*b^3*c+1/d*b^3*B*ln(sec(d*x+c)+tan(d*x+c))

maxima [A] time = 0.70, size = 152, normalized size = 1.05

$$\frac{4(\sin(dx+c)^3 - 3\sin(dx+c))Aa^3 - 3(2dx+2c+\sin(2dx+2c))Ba^3 - 9(2dx+2c+\sin(2dx+2c))Aa^2b}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] -1/12*(4*(sin(d*x+c)^3 - 3*sin(d*x+c))*A*a^3 - 3*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^3 - 9*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a^2*b - 36*(d*x + c)*B*a*b^2 - 12*(d*x + c)*A*b^3 - 6*B*b^3*(log(sin(d*x+c)+1) - log(sin(d*x+c)-1)) - 36*B*a^2*b*sin(d*x+c) - 36*A*a*b^2*sin(d*x+c))/d

mupad [B] time = 3.90, size = 1924, normalized size = 13.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c+d*x)^3*(A+B/cos(c+d*x))*(a+b/cos(c+d*x))^3,x)

[Out] (tan(c/2+(d*x)/2)*(2*A*a^3+B*a^3+6*A*a*b^2+3*A*a^2*b+6*B*a^2*b)+tan(c/2+(d*x)/2)^3*((4*A*a^3)/3+12*A*a*b^2+12*B*a^2*b)+tan(c/2+(d*x)/2)^5*(2*A*a^3-B*a^3+6*A*a*b^2-3*A*a^2*b+6*B*a^2*b))/(d*(3*tan(c/2+(d*x)/2)+tan(c/2+(d*x)/2)^3))

$$\begin{aligned}
& c/2 + (d*x)/2)^2 + 3*\tan(c/2 + (d*x)/2)^4 + \tan(c/2 + (d*x)/2)^6 + 1)) + (a \\
& \tan((((A*b^3*1i + (B*a^3*1i)/2 + (A*a^2*b*3i)/2 + B*a*b^2*3i)*(32*A*b^3 + 1 \\
& 6*B*a^3 + 32*B*b^3 + 48*A*a^2*b + 96*B*a*b^2) + \tan(c/2 + (d*x)/2)*(32*A^2* \\
& b^6 + 8*B^2*a^6 + 32*B^2*b^6 + 96*A^2*a^2*b^4 + 72*A^2*a^4*b^2 + 288*B^2*a^ \\
& 2*b^4 + 96*B^2*a^4*b^2 + 192*A*B*a*b^5 + 48*A*B*a^5*b + 320*A*B*a^3*b^3))*(\\
& A*b^3*1i + (B*a^3*1i)/2 + (A*a^2*b*3i)/2 + B*a*b^2*3i)*1i - ((A*b^3*1i + (B \\
& *a^3*1i)/2 + (A*a^2*b*3i)/2 + B*a*b^2*3i)*(32*A*b^3 + 16*B*a^3 + 32*B*b^3 + \\
& 48*A*a^2*b + 96*B*a*b^2) - \tan(c/2 + (d*x)/2)*(32*A^2*b^6 + 8*B^2*a^6 + 32 \\
& *B^2*b^6 + 96*A^2*a^2*b^4 + 72*A^2*a^4*b^2 + 288*B^2*a^2*b^4 + 96*B^2*a^4*b \\
& ^2 + 192*A*B*a*b^5 + 48*A*B*a^5*b + 320*A*B*a^3*b^3))*(A*b^3*1i + (B*a^3*1i \\
&)/2 + (A*a^2*b*3i)/2 + B*a*b^2*3i)*1i)/(((A*b^3*1i + (B*a^3*1i)/2 + (A*a^2* \\
& b*3i)/2 + B*a*b^2*3i)*(32*A*b^3 + 16*B*a^3 + 32*B*b^3 + 48*A*a^2*b + 96*B*a \\
& *b^2) + \tan(c/2 + (d*x)/2)*(32*A^2*b^6 + 8*B^2*a^6 + 32*B^2*b^6 + 96*A^2*a^ \\
& 2*b^4 + 72*A^2*a^4*b^2 + 288*B^2*a^2*b^4 + 96*B^2*a^4*b^2 + 192*A*B*a*b^5 + \\
& 48*A*B*a^5*b + 320*A*B*a^3*b^3))*(A*b^3*1i + (B*a^3*1i)/2 + (A*a^2*b*3i)/2 \\
& + B*a*b^2*3i) + ((A*b^3*1i + (B*a^3*1i)/2 + (A*a^2*b*3i)/2 + B*a*b^2*3i)*(\\
& 32*A*b^3 + 16*B*a^3 + 32*B*b^3 + 48*A*a^2*b + 96*B*a*b^2) - \tan(c/2 + (d*x) \\
& /2)*(32*A^2*b^6 + 8*B^2*a^6 + 32*B^2*b^6 + 96*A^2*a^2*b^4 + 72*A^2*a^4*b^2 \\
& + 288*B^2*a^2*b^4 + 96*B^2*a^4*b^2 + 192*A*B*a*b^5 + 48*A*B*a^5*b + 320*A*B \\
& *a^3*b^3))*(A*b^3*1i + (B*a^3*1i)/2 + (A*a^2*b*3i)/2 + B*a*b^2*3i) - 64*A*B \\
& ^2*b^9 + 64*A^2*B*b^9 - 192*B^3*a*b^8 + 576*B^3*a^2*b^7 - 32*B^3*a^3*b^6 + \\
& 192*B^3*a^4*b^5 + 16*B^3*a^6*b^3 + 384*A*B^2*a*b^8 - 96*A*B^2*a^2*b^7 + 640 \\
& *A*B^2*a^3*b^6 + 96*A*B^2*a^5*b^4 + 192*A^2*B*a^2*b^7 + 144*A^2*B*a^4*b^5)) \\
& *(2*A*b^3 + B*a^3 + 3*A*a^2*b + 6*B*a*b^2))/d - (B*b^3*atan((B*b^3*(\tan(c/2 \\
& + (d*x)/2)*(32*A^2*b^6 + 8*B^2*a^6 + 32*B^2*b^6 + 96*A^2*a^2*b^4 + 72*A^2* \\
& a^4*b^2 + 288*B^2*a^2*b^4 + 96*B^2*a^4*b^2 + 192*A*B*a*b^5 + 48*A*B*a^5*b + \\
& 320*A*B*a^3*b^3) + B*b^3*(32*A*b^3 + 16*B*a^3 + 32*B*b^3 + 48*A*a^2*b + 96 \\
& *B*a*b^2))*1i + B*b^3*(\tan(c/2 + (d*x)/2)*(32*A^2*b^6 + 8*B^2*a^6 + 32*B^2* \\
& b^6 + 96*A^2*a^2*b^4 + 72*A^2*a^4*b^2 + 288*B^2*a^2*b^4 + 96*B^2*a^4*b^2 + \\
& 192*A*B*a*b^5 + 48*A*B*a^5*b + 320*A*B*a^3*b^3) - B*b^3*(32*A*b^3 + 16*B*a^ \\
& 3 + 32*B*b^3 + 48*A*a^2*b + 96*B*a*b^2))*1i)/(64*A^2*B*b^9 - 64*A*B^2*b^9 - \\
& 192*B^3*a*b^8 + B*b^3*(\tan(c/2 + (d*x)/2)*(32*A^2*b^6 + 8*B^2*a^6 + 32*B^2 \\
& *b^6 + 96*A^2*a^2*b^4 + 72*A^2*a^4*b^2 + 288*B^2*a^2*b^4 + 96*B^2*a^4*b^2 + \\
& 192*A*B*a*b^5 + 48*A*B*a^5*b + 320*A*B*a^3*b^3) + B*b^3*(32*A*b^3 + 16*B*a \\
& ^3 + 32*B*b^3 + 48*A*a^2*b + 96*B*a*b^2)) - B*b^3*(\tan(c/2 + (d*x)/2)*(32*A \\
& ^2*b^6 + 8*B^2*a^6 + 32*B^2*b^6 + 96*A^2*a^2*b^4 + 72*A^2*a^4*b^2 + 288*B^2 \\
& *a^2*b^4 + 96*B^2*a^4*b^2 + 192*A*B*a*b^5 + 48*A*B*a^5*b + 320*A*B*a^3*b^3) \\
& - B*b^3*(32*A*b^3 + 16*B*a^3 + 32*B*b^3 + 48*A*a^2*b + 96*B*a*b^2)) + 576* \\
& B^3*a^2*b^7 - 32*B^3*a^3*b^6 + 192*B^3*a^4*b^5 + 16*B^3*a^6*b^3 + 384*A*B^2 \\
& *a*b^8 - 96*A*B^2*a^2*b^7 + 640*A*B^2*a^3*b^6 + 96*A*B^2*a^5*b^4 + 192*A^2* \\
& B*a^2*b^7 + 144*A^2*B*a^4*b^5))*2i)/d
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*sec(d*x+c))**3*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

$$3.300 \quad \int \cos^4(c+dx)(a+b \sec(c+dx))^3(A+B \sec(c+dx)) dx$$

Optimal. Leaf size=179

$$\frac{a(3a^2A + 12abB + 10Ab^2) \sin(c+dx) \cos(c+dx)}{8d} + \frac{a^2(2aB + 3Ab) \sin(c+dx) \cos^2(c+dx)}{6d} + \frac{(2a^3B + 6a^2Ab + 3a^3A) \sin^3(c+dx)}{8d}$$

[Out] $1/8*(3*A*a^3+12*A*a*b^2+12*B*a^2*b+8*B*b^3)*x+1/3*(6*A*a^2*b+3*A*b^3+2*B*a^3+9*B*a*b^2)*\sin(d*x+c)/d+1/8*a*(3*A*a^2+10*A*b^2+12*B*a*b)*\cos(d*x+c)*\sin(d*x+c)/d+1/6*a^2*(3*A*b+2*B*a)*\cos(d*x+c)^2*\sin(d*x+c)/d+1/4*a*A*\cos(d*x+c)^3*(a+b*\sec(d*x+c))^2*\sin(d*x+c)/d$

Rubi [A] time = 0.42, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4025, 4074, 4047, 2637, 4045, 8}

$$\frac{(6a^2Ab + 2a^3B + 9ab^2B + 3Ab^3) \sin(c+dx)}{3d} + \frac{a(3a^2A + 12abB + 10Ab^2) \sin(c+dx) \cos(c+dx)}{8d} + \frac{1}{8}x(3a^3A + 9a^2Ab + 3a^3B)$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]), x]

[Out] $((3*a^3*A + 12*a*A*b^2 + 12*a^2*b*B + 8*b^3*B)*x)/8 + ((6*a^2*A*b + 3*A*b^3 + 2*a^3*B + 9*a*b^2*B)*\sin[c + d*x])/(3*d) + (a*(3*a^2*A + 10*A*b^2 + 12*a*b*B)*\cos[c + d*x]*\sin[c + d*x])/(8*d) + (a^2*(3*A*b + 2*a*B)*\cos[c + d*x]^2*\sin[c + d*x])/(6*d) + (a*A*\cos[c + d*x]^3*(a + b*\sec[c + d*x])^2*\sin[c + d*x])/(4*d)$

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_.)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 4025

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := Simp[(a*A*Cos[t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n]/(f*n), x] + Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a

$*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e + f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x] /;$ FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LeQ[n, -1]

Rule 4045

Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]^2*(C_.) + (A_.)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] + Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]

Rule 4047

Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2), x] /; FreeQ[{b, e, f, A, B, C, m}, x]

Rule 4074

Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] :> Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]

Rubi steps

$$\begin{aligned} \int \cos^4(c + dx)(a + b \sec(c + dx))^3(A + B \sec(c + dx)) dx &= \frac{aA \cos^3(c + dx)(a + b \sec(c + dx))^2 \sin(c + dx)}{4d} - \frac{a^2(3Ab + 2aB) \cos^2(c + dx) \sin(c + dx)}{6d} + \frac{aA \cos^3(c + dx)}{6d} \\ &= \frac{a^2(3Ab + 2aB) \cos^2(c + dx) \sin(c + dx)}{6d} + \frac{aA \cos^3(c + dx)}{6d} \\ &= \frac{(6a^2Ab + 3Ab^3 + 2a^3B + 9ab^2B) \sin(c + dx)}{3d} + \frac{aA \cos^3(c + dx)}{6d} \\ &= \frac{1}{8} (3a^3A + 12aAb^2 + 12a^2bB + 8b^3B) x + \frac{(6a^2Ab + 3Ab^3 + 2a^3B + 9ab^2B) \sin(c + dx)}{3d} + \frac{aA \cos^3(c + dx)}{6d} \end{aligned}$$

Mathematica [A] time = 0.43, size = 140, normalized size = 0.78

$$\frac{3a^3 A \sin(4(c + dx)) + 24a(a^2 A + 3abB + 3Ab^2) \sin(2(c + dx)) + 8a^2(aB + 3Ab) \sin(3(c + dx)) + 12(c + dx)(3a^3 A^2 + 4a^2 A^2 b + 3a^2 A b^2 + 3a^2 B^2) \cos(4(c + dx))}{96d}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]

[Out] (12*(3*a^3*A + 12*a*A*b^2 + 12*a^2*b*B + 8*b^3*B)*(c + d*x) + 24*(9*a^2*A*b + 4*A*b^3 + 3*a^3*B + 12*a*b^2*B)*Sin[c + d*x] + 24*a*(a^2*A + 3*A*b^2 + 3*a*b*B)*Sin[2*(c + d*x)] + 8*a^2*(3*A*b + a*B)*Sin[3*(c + d*x)] + 3*a^3*A*Sin[4*(c + d*x)])/(96*d)

fricas [A] time = 0.46, size = 136, normalized size = 0.76

$$\frac{3(3Aa^3 + 12Ba^2b + 12Aab^2 + 8Bb^3)dx + (6Aa^3 \cos(dx + c)^3 + 16Ba^3 + 48Aa^2b + 72Bab^2 + 24Ab^3 + 8(Ba^3 + 3Aa^2b + 3Aab^2 + Bb^3) \sin(dx + c)) \sin(dx + c)}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] 1/24*(3*(3*A*a^3 + 12*B*a^2*b + 12*A*a*b^2 + 8*B*b^3)*d*x + (6*A*a^3*cos(d*x + c)^3 + 16*B*a^3 + 48*A*a^2*b + 72*B*a*b^2 + 24*A*b^3 + 8*(B*a^3 + 3*A*a^2*b)*cos(d*x + c)^2 + 9*(A*a^3 + 4*B*a^2*b + 4*A*a*b^2)*cos(d*x + c))*sin(d*x + c))/d

giac [B] time = 0.32, size = 536, normalized size = 2.99

$$3(3Aa^3 + 12Ba^2b + 12Aab^2 + 8Bb^3)(dx + c) - \frac{2\left(15Aa^3 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^7 - 24Ba^3 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^7 - 72Aa^2b \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^7 + 36Ba^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^7 + 36Aa^2b \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^7 - 72Bab^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^7 - 24Ab^3 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^7 - 9Aa^3 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5 - 40Ba^3 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5 + 40Aa^2b \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5 + 40Bab^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5 - 40Ab^3 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5\right)}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] 1/24*(3*(3*A*a^3 + 12*B*a^2*b + 12*A*a*b^2 + 8*B*b^3)*(d*x + c) - 2*(15*A*a^3*tan(1/2*d*x + 1/2*c)^7 - 24*B*a^3*tan(1/2*d*x + 1/2*c)^7 - 72*A*a^2*b*tan(1/2*d*x + 1/2*c)^7 + 36*B*a^2*b*tan(1/2*d*x + 1/2*c)^7 + 36*A*a*b^2*tan(1/2*d*x + 1/2*c)^7 - 72*B*a*b^2*tan(1/2*d*x + 1/2*c)^7 - 24*A*b^3*tan(1/2*d*x + 1/2*c)^7 - 9*A*a^3*tan(1/2*d*x + 1/2*c)^5 - 40*B*a^3*tan(1/2*d*x + 1/2*c)^5 + 40*A*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 40*B*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 40*A*b^3*tan(1/2*d*x + 1/2*c)^5))/d

$$\begin{aligned} & c)^5 - 120Aa^2b\tan(1/2dx + 1/2c)^5 + 36B^2a^2b\tan(1/2dx + 1/2c) \\ & ^5 + 36Aa^2b^2\tan(1/2dx + 1/2c)^5 - 216B^2a^2b^2\tan(1/2dx + 1/2c)^5 \\ & - 72A^2b^3\tan(1/2dx + 1/2c)^5 + 9A^2a^3\tan(1/2dx + 1/2c)^3 - 40B^2 \\ & a^3\tan(1/2dx + 1/2c)^3 - 120A^2a^2b\tan(1/2dx + 1/2c)^3 - 36B^2a^2 \\ & b\tan(1/2dx + 1/2c)^3 - 36A^2a^2b^2\tan(1/2dx + 1/2c)^3 - 216B^2a^2b^2 \\ & \tan(1/2dx + 1/2c)^3 - 72A^2b^3\tan(1/2dx + 1/2c)^3 - 15A^2a^3\tan(1/2 \\ & dx + 1/2c) - 24B^2a^3\tan(1/2dx + 1/2c) - 72A^2a^2b\tan(1/2dx + 1/ \\ & 2c) - 36B^2a^2b\tan(1/2dx + 1/2c) - 36A^2a^2b^2\tan(1/2dx + 1/2c) - \\ & 72B^2a^2b^2\tan(1/2dx + 1/2c) - 24A^2b^3\tan(1/2dx + 1/2c))/(\tan(1/2d \\ & *x + 1/2c)^2 + 1)^4)/d \end{aligned}$$

maple [A] time = 1.62, size = 180, normalized size = 1.01

$$Aa^3 \left(\frac{\left(\cos^3(dx+c) + \frac{3\cos(dx+c)}{2} \right) \sin(dx+c)}{4} + \frac{3dx}{8} + \frac{3c}{8} \right) + Aa^2b \left(2 + \cos^2(dx+c) \right) \sin(dx+c) + \frac{a^3B(2+\cos^2(dx+c))\sin(dx+c)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(dx+c)^4*(a+b*sec(dx+c))^3*(A+B*sec(dx+c)),x)

[Out] 1/d*(Aa^3*(1/4*(cos(dx+c)^3+3/2*cos(dx+c))*sin(dx+c)+3/8*dx+3/8*c)+Aa^2*b*(2+cos(dx+c)^2)*sin(dx+c)+1/3*a^3*B*(2+cos(dx+c)^2)*sin(dx+c)+3Aa^2*b^2*(1/2*cos(dx+c)*sin(dx+c)+1/2*d*x+1/2*c)+3a^2*b*B*(1/2*cos(dx+c)*sin(dx+c)+1/2*d*x+1/2*c)+A*b^3*sin(dx+c)+3B*a^2*b^2*sin(dx+c)+B*(d*x+c)*b^3)

maxima [A] time = 0.67, size = 171, normalized size = 0.96

$$3(12dx + 12c + \sin(4dx + 4c) + 8\sin(2dx + 2c))Aa^3 - 32(\sin(dx + c)^3 - 3\sin(dx + c))Ba^3 - 96(\sin(dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(dx+c)^4*(a+b*sec(dx+c))^3*(A+B*sec(dx+c)),x, algorithm="maxima")

[Out] 1/96*(3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*Aa^3 - 32*(sin(d*x + c)^3 - 3*sin(d*x + c))*Ba^3 - 96*(sin(d*x + c)^3 - 3*sin(d*x + c))*Aa^2*b + 72*(2*d*x + 2*c + sin(2*d*x + 2*c))*B^2a^2*b + 72*(2*d*x + 2*c + sin(2*d*x + 2*c))*Aa^2*b^2 + 96*(d*x + c)*B^2b^3 + 288*B^2a^2*b^2*sin(d*x + c) + 96*A^2b^3*sin(d*x + c))/d

mupad [B] time = 2.49, size = 202, normalized size = 1.13

$$\frac{3Aa^3x}{8} + Bb^3x + \frac{3Aab^2x}{2} + \frac{3Ba^2bx}{2} + \frac{Ab^3\sin(c+dx)}{d} + \frac{3Ba^3\sin(c+dx)}{4d} + \frac{Aa^3\sin(2c+2dx)}{4d} + \frac{Aa^3\sin(2c+2dx)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^3,x)
```

```
[Out] (3*A*a^3*x)/8 + B*b^3*x + (3*A*a*b^2*x)/2 + (3*B*a^2*b*x)/2 + (A*b^3*sin(c
+ d*x))/d + (3*B*a^3*sin(c + d*x))/(4*d) + (A*a^3*sin(2*c + 2*d*x))/(4*d) +
(A*a^3*sin(4*c + 4*d*x))/(32*d) + (B*a^3*sin(3*c + 3*d*x))/(12*d) + (3*A*a
*b^2*sin(2*c + 2*d*x))/(4*d) + (A*a^2*b*sin(3*c + 3*d*x))/(4*d) + (3*B*a^2*
b*sin(2*c + 2*d*x))/(4*d) + (9*A*a^2*b*sin(c + d*x))/(4*d) + (3*B*a*b^2*sin
(c + d*x))/d
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+b*sec(d*x+c))**3*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

3.301 $\int \cos^5(c+dx)(a+b \sec(c+dx))^3(A+B \sec(c+dx)) dx$

Optimal. Leaf size=221

$$\frac{a(4a^2A + 15abB + 12Ab^2) \sin^3(c+dx)}{15d} + \frac{a^2(5aB + 7Ab) \sin(c+dx) \cos^3(c+dx)}{20d} + \frac{(4a^3A + 15a^2bB + 14aAb^2) \sin(c+dx)}{5d}$$

[Out] 1/8*(9*A*a^2*b+4*A*b^3+3*B*a^3+12*B*a*b^2)*x+1/5*(4*A*a^3+14*A*a*b^2+15*B*a^2*b+5*B*b^3)*sin(d*x+c)/d+1/8*(9*A*a^2*b+4*A*b^3+3*B*a^3+12*B*a*b^2)*cos(d*x+c)*sin(d*x+c)/d+1/20*a^2*(7*A*b+5*B*a)*cos(d*x+c)^3*sin(d*x+c)/d+1/5*a*A*cos(d*x+c)^4*(a+b*sec(d*x+c))^2*sin(d*x+c)/d-1/15*a*(4*A*a^2+12*A*b^2+15*B*a*b)*sin(d*x+c)^3/d

Rubi [A] time = 0.49, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {4025, 4074, 4047, 2635, 8, 4044, 3013}

$$\frac{a(4a^2A + 15abB + 12Ab^2) \sin^3(c+dx)}{15d} + \frac{(4a^3A + 15a^2bB + 14aAb^2 + 5b^3B) \sin(c+dx)}{5d} + \frac{(9a^2Ab + 3a^3B + 14aAb^2) \sin(c+dx)}{5d}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^5*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]), x]

[Out] ((9*a^2*A*b + 4*A*b^3 + 3*a^3*B + 12*a*b^2*B)*x)/8 + ((4*a^3*A + 14*a*A*b^2 + 15*a^2*b*B + 5*b^3*B)*Sin[c + d*x])/(5*d) + ((9*a^2*A*b + 4*A*b^3 + 3*a^3*B + 12*a*b^2*B)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a^2*(7*A*b + 5*a*B)*Cos[c + d*x]^3*Ssin[c + d*x])/(20*d) + (a*A*Cos[c + d*x]^4*(a + b*Sec[c + d*x])^2*Ssin[c + d*x])/(5*d) - (a*(4*a^2*A + 12*A*b^2 + 15*a*b*B)*Sin[c + d*x]^3)/(15*d)

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x])*(b*Ssin[c + d*x])^(n - 1)]/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Ssin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3013

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
  x_Symbol] :> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^m*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
LeQ[n, -1]
```

Rule 4044

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_)),
  x_Symbol] :> Int[(C + A*Sin[e + f*x]^2)/Sin[e + f*x]^(m + 2), x] /; FreeQ[
{e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && ILtQ[(m + 1)/2, 0]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^m*(A_. + csc[(e_.) + (f_.)*(x_)]*(
B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
  x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 4074

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*csc[(e_.) + (f_.)*(x_)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \cos^5(c + dx)(a + b \sec(c + dx))^3(A + B \sec(c + dx)) dx &= \frac{aA \cos^4(c + dx)(a + b \sec(c + dx))^2 \sin(c + dx)}{5d} - \frac{a^2(7Ab + 5aB) \cos^3(c + dx) \sin(c + dx)}{20d} + \frac{aA \cos^4(c + dx)}{20d} \\
&= \frac{a^2(7Ab + 5aB) \cos^3(c + dx) \sin(c + dx)}{20d} + \frac{aA \cos^4(c + dx)}{20d} \\
&= \frac{(9a^2Ab + 4Ab^3 + 3a^3B + 12ab^2B) \cos(c + dx) \sin(c + dx)}{8d} \\
&= \frac{1}{8} (9a^2Ab + 4Ab^3 + 3a^3B + 12ab^2B) x + \frac{(9a^2Ab + 4Ab^3 + 3a^3B + 12ab^2B) \cos(c + dx) \sin(c + dx)}{8d} \\
&= \frac{1}{8} (9a^2Ab + 4Ab^3 + 3a^3B + 12ab^2B) x + \frac{(4a^3A + 12a^2bB) \cos(c + dx) \sin(c + dx)}{8d}
\end{aligned}$$

Mathematica [A] time = 0.73, size = 176, normalized size = 0.80

$$\frac{6a^3A \sin(5(c + dx)) + 10a(5a^2A + 12abB + 12Ab^2) \sin(3(c + dx)) + 15a^2(aB + 3Ab) \sin(4(c + dx)) + 60(c + dx) \cos(5(c + dx))}{480d}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]^5*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]

[Out] (60*(9*a^2*A*b + 4*A*b^3 + 3*a^3*B + 12*a*b^2*B)*(c + d*x) + 60*(5*a^3*A + 18*a*A*b^2 + 18*a^2*b*B + 8*b^3*B)*Sin[c + d*x] + 120*(3*a^2*A*b + A*b^3 + a^3*B + 3*a*b^2*B)*Sin[2*(c + d*x)] + 10*a*(5*a^2*A + 12*A*b^2 + 12*a*b*B)*Sin[3*(c + d*x)] + 15*a^2*(3*A*b + a*B)*Sin[4*(c + d*x)] + 6*a^3*A*Ssin[5*(c + d*x)])/(480*d)

fricas [A] time = 0.45, size = 174, normalized size = 0.79

$$\frac{15(3Ba^3 + 9Aa^2b + 12Bab^2 + 4Ab^3)dx + (24Aa^3 \cos(dx + c)^4 + 64Aa^3 + 240Ba^2b + 240Aab^2 + 120Bb^3 + 30Aa^3 \cos(dx + c)^3 + 8(4Aa^3 + 15Bb^2) \cos(dx + c))}{480d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] 1/120*(15*(3*B*a^3 + 9*A*a^2*b + 12*B*a*b^2 + 4*A*b^3)*d*x + (24*A*a^3*cos(d*x + c)^4 + 64*A*a^3 + 240*B*a^2*b + 240*A*a*b^2 + 120*B*b^3 + 30*(B*a^3 + 3*A*a^2*b)*cos(d*x + c)^3 + 8*(4*A*a^3 + 15*B*b^2)*cos(d*x + c))

+ c)^2 + 15*(3*B*a^3 + 9*A*a^2*b + 12*B*a*b^2 + 4*A*b^3)*cos(d*x + c))*sin(d*x + c))/d

giac [B] time = 0.34, size = 672, normalized size = 3.04

$$15 \left(3 B a^3 + 9 A a^2 b + 12 B a b^2 + 4 A b^3 \right) (d x + c) + \frac{2 \left(120 A a^3 \tan \left(\frac{1}{2} d x + \frac{1}{2} c \right)^9 - 75 B a^3 \tan \left(\frac{1}{2} d x + \frac{1}{2} c \right)^9 - 225 A a^2 b \tan \left(\frac{1}{2} d x + \frac{1}{2} c \right)^9 + 360 B a^2 b \tan \left(\frac{1}{2} d x + \frac{1}{2} c \right)^9 + 360 A a b^2 \tan \left(\frac{1}{2} d x + \frac{1}{2} c \right)^9 - 180 B a b^2 \tan \left(\frac{1}{2} d x + \frac{1}{2} c \right)^9 - 60 A b^3 \tan \left(\frac{1}{2} d x + \frac{1}{2} c \right)^9 + 120 B b^3 \tan \left(\frac{1}{2} d x + \frac{1}{2} c \right)^9 + 160 A a^3 \tan \left(\frac{1}{2} d x + \frac{1}{2} c \right)^7 - 30 B a^3 \tan \left(\frac{1}{2} d x + \frac{1}{2} c \right)^7 - 90 A a^2 b \tan \left(\frac{1}{2} d x + \frac{1}{2} c \right)^7 + 960 B a^2 b \tan \left(\frac{1}{2} d x + \frac{1}{2} c \right)^7 + 960 A a b^2 \tan \left(\frac{1}{2} d x + \frac{1}{2} c \right)^7 - 360 B a b^2 \tan \left(\frac{1}{2} d x + \frac{1}{2} c \right)^7 - 120 A b^3 \tan \left(\frac{1}{2} d x + \frac{1}{2} c \right)^7 + 480 B b^3 \tan \left(\frac{1}{2} d x + \frac{1}{2} c \right)^7 + 464 A a^3 \tan \left(\frac{1}{2} d x + \frac{1}{2} c \right)^5 + 1200 B a^2 b \tan \left(\frac{1}{2} d x + \frac{1}{2} c \right)^5 + 1200 A a b^2 \tan \left(\frac{1}{2} d x + \frac{1}{2} c \right)^5 + 720 B b^3 \tan \left(\frac{1}{2} d x + \frac{1}{2} c \right)^5 + 160 A a^3 \tan \left(\frac{1}{2} d x + \frac{1}{2} c \right)^3 + 30 B a^3 \tan \left(\frac{1}{2} d x + \frac{1}{2} c \right)^3 + 90 A a^2 b \tan \left(\frac{1}{2} d x + \frac{1}{2} c \right)^3 + 960 B a^2 b \tan \left(\frac{1}{2} d x + \frac{1}{2} c \right)^3 + 960 A a b^2 \tan \left(\frac{1}{2} d x + \frac{1}{2} c \right)^3 + 360 B a b^2 \tan \left(\frac{1}{2} d x + \frac{1}{2} c \right)^3 + 120 A b^3 \tan \left(\frac{1}{2} d x + \frac{1}{2} c \right)^3 + 480 B b^3 \tan \left(\frac{1}{2} d x + \frac{1}{2} c \right)^3 + 120 A a^3 \tan \left(\frac{1}{2} d x + \frac{1}{2} c \right) + 75 B a^3 \tan \left(\frac{1}{2} d x + \frac{1}{2} c \right) + 225 A a^2 b \tan \left(\frac{1}{2} d x + \frac{1}{2} c \right) + 360 B a^2 b \tan \left(\frac{1}{2} d x + \frac{1}{2} c \right) + 360 A a b^2 \tan \left(\frac{1}{2} d x + \frac{1}{2} c \right) + 180 B a b^2 \tan \left(\frac{1}{2} d x + \frac{1}{2} c \right) + 60 A b^3 \tan \left(\frac{1}{2} d x + \frac{1}{2} c \right) + 120 B b^3 \tan \left(\frac{1}{2} d x + \frac{1}{2} c \right)}{\left(\tan \left(\frac{1}{2} d x + \frac{1}{2} c \right)^2 + 1 \right)^5} / d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] 1/120*(15*(3*B*a^3 + 9*A*a^2*b + 12*B*a*b^2 + 4*A*b^3)*(d*x + c) + 2*(120*A*a^3*tan(1/2*d*x + 1/2*c)^9 - 75*B*a^3*tan(1/2*d*x + 1/2*c)^9 - 225*A*a^2*b*tan(1/2*d*x + 1/2*c)^9 + 360*B*a^2*b*tan(1/2*d*x + 1/2*c)^9 + 360*A*a*b^2*tan(1/2*d*x + 1/2*c)^9 - 180*B*a*b^2*tan(1/2*d*x + 1/2*c)^9 - 60*A*b^3*tan(1/2*d*x + 1/2*c)^9 + 120*B*b^3*tan(1/2*d*x + 1/2*c)^9 + 160*A*a^3*tan(1/2*d*x + 1/2*c)^7 - 30*B*a^3*tan(1/2*d*x + 1/2*c)^7 - 90*A*a^2*b*tan(1/2*d*x + 1/2*c)^7 + 960*B*a^2*b*tan(1/2*d*x + 1/2*c)^7 + 960*A*a*b^2*tan(1/2*d*x + 1/2*c)^7 - 360*B*a*b^2*tan(1/2*d*x + 1/2*c)^7 - 120*A*b^3*tan(1/2*d*x + 1/2*c)^7 + 480*B*b^3*tan(1/2*d*x + 1/2*c)^7 + 464*A*a^3*tan(1/2*d*x + 1/2*c)^5 + 1200*B*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 1200*A*a*b^2*tan(1/2*d*x + 1/2*c)^5 + 720*B*b^3*tan(1/2*d*x + 1/2*c)^5 + 160*A*a^3*tan(1/2*d*x + 1/2*c)^3 + 30*B*a^3*tan(1/2*d*x + 1/2*c)^3 + 90*A*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 960*B*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 960*A*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 360*B*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 120*A*b^3*tan(1/2*d*x + 1/2*c)^3 + 480*B*b^3*tan(1/2*d*x + 1/2*c)^3 + 120*A*a^3*tan(1/2*d*x + 1/2*c) + 75*B*a^3*tan(1/2*d*x + 1/2*c) + 225*A*a^2*b*tan(1/2*d*x + 1/2*c) + 360*B*a^2*b*tan(1/2*d*x + 1/2*c) + 360*A*a*b^2*tan(1/2*d*x + 1/2*c) + 180*B*a*b^2*tan(1/2*d*x + 1/2*c) + 60*A*b^3*tan(1/2*d*x + 1/2*c) + 120*B*b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^5)/d

maple [A] time = 2.11, size = 227, normalized size = 1.03

$$\frac{A a^3 \left(\frac{8}{3} + \cos^4(dx+c) + \frac{4(\cos^2(dx+c))}{3} \right) \sin(dx+c)}{5} + a^3 B \left(\frac{\left(\cos^3(dx+c) + \frac{3 \cos(dx+c)}{2} \right) \sin(dx+c)}{4} + \frac{3dx}{8} + \frac{3c}{8} \right) + 3A a^2 b \left(\frac{\left(\cos^3(dx+c) + \frac{3 \cos(dx+c)}{2} \right)}{4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^5*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x)


```
[Out] 1/d*(1/5*A*a^3*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+a^3*B*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+3*A*a^2*b*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+a^2*b*B*(2+cos(d*x+c)^2)*sin(d*x+c)+A*a*b^2*(2+cos(d*x+c)^2)*sin(d*x+c)+3*B*a*b^2*(1/2*cos(d*x+c))*sin(d*x+c)+1/2*d*x+1/2*c)+A*b^3*(1/2*cos(d*x+c))*sin(d*x+c)+1/2*d*x+1/2*c)+b^3*B*sin(d*x+c))
```

maxima [A] time = 0.68, size = 217, normalized size = 0.98

$$32 \left(3 \sin(dx + c)^5 - 10 \sin(dx + c)^3 + 15 \sin(dx + c) \right) Aa^3 + 15 (12 dx + 12 c + \sin(4 dx + 4 c) + 8 \sin(2 dx + 2 c)) B^3 a^3 + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/480*(32*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*A*a^3 + 15*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^3 + 45*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^2*b - 480*(sin(d*x + c))^3 - 3*sin(d*x + c))*B*a^2*b - 480*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a*b^2 + 360*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a*b^2 + 120*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*b^3 + 480*B*b^3*sin(d*x + c))/d
```

mupad [B] time = 2.73, size = 277, normalized size = 1.25

$$\frac{A b^3 x}{2} + \frac{3 B a^3 x}{8} + \frac{9 A a^2 b x}{8} + \frac{3 B a b^2 x}{2} + \frac{5 A a^3 \sin(c + dx)}{8d} + \frac{B b^3 \sin(c + dx)}{d} + \frac{5 A a^3 \sin(3c + 3dx)}{48d} + \frac{A a^3 \sin(5c + 5dx)}{80d} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^3,x)
```

```
[Out] (A*b^3*x)/2 + (3*B*a^3*x)/8 + (9*A*a^2*b*x)/8 + (3*B*a*b^2*x)/2 + (5*A*a^3*sin(c + d*x))/(8*d) + (B*b^3*sin(c + d*x))/d + (5*A*a^3*sin(3*c + 3*d*x))/(48*d) + (A*a^3*sin(5*c + 5*d*x))/(80*d) + (A*b^3*sin(2*c + 2*d*x))/(4*d) + (B*a^3*sin(2*c + 2*d*x))/(4*d) + (B*a^3*sin(4*c + 4*d*x))/(32*d) + (3*A*a^2*b*sin(2*c + 2*d*x))/(4*d) + (A*a*b^2*sin(3*c + 3*d*x))/(4*d) + (3*A*a^2*b*sin(4*c + 4*d*x))/(32*d) + (3*B*a*b^2*sin(2*c + 2*d*x))/(4*d) + (B*a^2*b*sin(3*c + 3*d*x))/(4*d) + (9*A*a*b^2*sin(c + d*x))/(4*d) + (9*B*a^2*b*sin(c + d*x))/(4*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+b*sec(d*x+c))**3*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

3.302 $\int \sec^2(c+dx)(a+b \sec(c+dx))^4(A+B \sec(c+dx)) dx$

Optimal. Leaf size=334

$$\frac{(-4a^2B + 24aAb + 25b^2B) \tan(c+dx)(a+b \sec(c+dx))^3}{120bd} + \frac{(-4a^3B + 24a^2Ab + 53ab^2B + 32Ab^3) \tan(c+dx)}{120bd}$$

[Out] $1/16*(32*A*a^3*b+24*A*a*b^3+8*B*a^4+36*B*a^2*b^2+5*B*b^4)*\operatorname{arctanh}(\sin(d*x+c))/d+1/60*(24*A*a^4*b+224*A*a^2*b^3+32*A*b^5-4*B*a^5+121*B*a^3*b^2+128*B*a*b^4)*\tan(d*x+c)/b/d+1/240*(48*A*a^3*b+232*A*a*b^3-8*B*a^4+178*B*a^2*b^2+75*B*b^4)*\sec(d*x+c)*\tan(d*x+c)/d+1/120*(24*A*a^2*b+32*A*b^3-4*B*a^3+53*B*a*b^2)*(a+b*\sec(d*x+c))^2*\tan(d*x+c)/b/d+1/120*(24*A*a*b-4*B*a^2+25*B*b^2)*(a+b*\sec(d*x+c))^3*\tan(d*x+c)/b/d+1/30*(6*A*b-B*a)*(a+b*\sec(d*x+c))^4*\tan(d*x+c)/b/d+1/6*B*(a+b*\sec(d*x+c))^5*\tan(d*x+c)/b/d$

Rubi [A] time = 0.71, antiderivative size = 334, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {4010, 4002, 3997, 3787, 3770, 3767, 8}

$$\frac{(224a^2Ab^3 + 24a^4Ab + 121a^3b^2B - 4a^5B + 128ab^4B + 32Ab^5) \tan(c+dx)}{60bd} + \frac{(32a^3Ab + 36a^2b^2B + 8a^4B + 24a^5B) \tan(c+dx)}{16bd}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Sec}[c+dx]^2(a+b*\operatorname{Sec}[c+dx])^4(A+B*\operatorname{Sec}[c+dx]),x]$

[Out] $((32*a^3*A*b + 24*a*A*b^3 + 8*a^4*B + 36*a^2*b^2*B + 5*b^4*B)*\operatorname{ArcTanh}[\operatorname{Sin}[c+dx]])/(16*d) + ((24*a^4*A*b + 224*a^2*A*b^3 + 32*A*b^5 - 4*a^5*B + 121*a^3*b^2*B + 128*a*b^4*B)*\operatorname{Tan}[c+dx])/(60*b*d) + ((48*a^3*A*b + 232*a*A*b^3 - 8*a^4*B + 178*a^2*b^2*B + 75*b^4*B)*\operatorname{Sec}[c+dx]*\operatorname{Tan}[c+dx])/(240*d) + ((24*a^2*A*b + 32*A*b^3 - 4*a^3*B + 53*a*b^2*B)*(a+b*\operatorname{Sec}[c+dx])^2*\operatorname{Tan}[c+dx])/(120*b*d) + ((24*a*A*b - 4*a^2*B + 25*b^2*B)*(a+b*\operatorname{Sec}[c+dx])^3*\operatorname{Tan}[c+dx])/(120*b*d) + ((6*A*b - a*B)*(a+b*\operatorname{Sec}[c+dx])^4*\operatorname{Tan}[c+dx])/(30*b*d) + (B*(a+b*\operatorname{Sec}[c+dx])^5*\operatorname{Tan}[c+dx])/(6*b*d)$

Rule 8

$\operatorname{Int}[a_, x_Symbol] := \operatorname{Simp}[a*x, x] /; \operatorname{FreeQ}[a, x]$

Rule 3767

$\operatorname{Int}[\operatorname{csc}[(c_.) + (d_.)*(x_)]^{(n_)}, x_Symbol] := -\operatorname{Dist}[d^{(-1)}, \operatorname{Subst}[\operatorname{Int}[\operatorname{ExpandIntegrand}[(1+x^2)^{(n/2-1)}, x], x], x, \operatorname{Cot}[c+dx]], x] /; \operatorname{FreeQ}\{c,$

d}, x] && IGtQ[n/2, 0]

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x]
/; FreeQ[{c, d}, x]

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_)])*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)])*(b_.) +
(a_), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 3997

Int[(csc[(e_.) + (f_.)*(x_)])*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)])*(b_.) +
(a_)*(csc[(e_.) + (f_.)*(x_)])*(B_.) + (A_)), x_Symbol] := -Simp[(b*B*Cot[e
+ f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && !LeQ[n,
-1]

Rule 4002

Int[csc[(e_.) + (f_.)*(x_)])*(csc[(e_.) + (f_.)*(x_)])*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)])*(B_.) + (A_)), x_Symbol] := -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]

Rule 4010

Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)])*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)])*(B_.) + (A_)), x_Symbol] := -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Cs
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
0] && !LtQ[m, -1]

Rubi steps

$$\begin{aligned}
\int \sec^2(c + dx)(a + b \sec(c + dx))^4(A + B \sec(c + dx)) dx &= \frac{B(a + b \sec(c + dx))^5 \tan(c + dx)}{6bd} + \frac{\int \sec(c + dx)(a + b \sec(c + dx))^4 dx}{6bd} \\
&= \frac{(6Ab - aB)(a + b \sec(c + dx))^4 \tan(c + dx)}{30bd} + \frac{B(a + b \sec(c + dx))^5}{6bd} \\
&= \frac{(24aAb - 4a^2B + 25b^2B)(a + b \sec(c + dx))^3 \tan(c + dx)}{120bd} + \frac{B(a + b \sec(c + dx))^5}{6bd} \\
&= \frac{(24a^2Ab + 32Ab^3 - 4a^3B + 53ab^2B)(a + b \sec(c + dx))^2 \tan(c + dx)}{120bd} + \frac{B(a + b \sec(c + dx))^5}{6bd} \\
&= \frac{(48a^3Ab + 232aAb^3 - 8a^4B + 178a^2b^2B + 75b^4B) \tan(c + dx)}{240d} + \frac{B(a + b \sec(c + dx))^5}{6bd} \\
&= \frac{(48a^3Ab + 232aAb^3 - 8a^4B + 178a^2b^2B + 75b^4B) \tan(c + dx)}{240d} + \frac{B(a + b \sec(c + dx))^5}{6bd} \\
&= \frac{(32a^3Ab + 24aAb^3 + 8a^4B + 36a^2b^2B + 5b^4B) \tan(c + dx)}{16d} + \frac{B(a + b \sec(c + dx))^5}{6bd} \\
&= \frac{(32a^3Ab + 24aAb^3 + 8a^4B + 36a^2b^2B + 5b^4B) \tan(c + dx)}{16d} + \frac{B(a + b \sec(c + dx))^5}{6bd}
\end{aligned}$$

Mathematica [A] time = 2.85, size = 244, normalized size = 0.73

$$\frac{15(8a^4B + 32a^3Ab + 36a^2b^2B + 24aAb^3 + 5b^4B) \tanh^{-1}(\sin(c + dx)) + \tan(c + dx) (10b^2(36a^2B + 24aAb + 5b^4B))}{16d}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]

[Out] (15*(32*a^3*A*b + 24*a*A*b^3 + 8*a^4*B + 36*a^2*b^2*B + 5*b^4*B)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(240*(a^4*A + 6*a^2*A*b^2 + A*b^4 + 4*a^3*b*B + 4*a*b^3*B) + 15*(32*a^3*A*b + 24*a*A*b^3 + 8*a^4*B + 36*a^2*b^2*B + 5*b^4*B)*Sec[c + d*x] + 10*b^2*(24*a*A*b + 36*a^2*B + 5*b^2*B)*Sec[c + d*x]^3 + 40*b^4*B*Sec[c + d*x]^5 + 160*b*(3*a^2*A*b + A*b^3 + 2*a^3*B + 4*a*b^2*B)*Tan[c + d*x]^2 + 48*b^3*(A*b + 4*a*B)*Tan[c + d*x]^4)/(240*d)

fricas [A] time = 0.49, size = 327, normalized size = 0.98

$$\frac{15(8Ba^4 + 32Aa^3b + 36Ba^2b^2 + 24Aab^3 + 5Bb^4) \cos(dx + c)^6 \log(\sin(dx + c) + 1) - 15(8Ba^4 + 32Aa^3b + 36Ba^2b^2 + 24Aab^3 + 5Bb^4) \tan(dx + c)}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] $\frac{1}{480} \cdot (15 \cdot (8 \cdot B \cdot a^4 + 32 \cdot A \cdot a^3 \cdot b + 36 \cdot B \cdot a^2 \cdot b^2 + 24 \cdot A \cdot a \cdot b^3 + 5 \cdot B \cdot b^4) \cdot \cos(d \cdot x + c)^6 \cdot \log(\sin(d \cdot x + c) + 1) - 15 \cdot (8 \cdot B \cdot a^4 + 32 \cdot A \cdot a^3 \cdot b + 36 \cdot B \cdot a^2 \cdot b^2 + 24 \cdot A \cdot a \cdot b^3 + 5 \cdot B \cdot b^4) \cdot \cos(d \cdot x + c)^6 \cdot \log(-\sin(d \cdot x + c) + 1) + 2 \cdot (16 \cdot (15 \cdot A \cdot a^4 + 40 \cdot B \cdot a^3 \cdot b + 60 \cdot A \cdot a^2 \cdot b^2 + 32 \cdot B \cdot a \cdot b^3 + 8 \cdot A \cdot b^4) \cdot \cos(d \cdot x + c)^5 + 40 \cdot B \cdot b^4 + 15 \cdot (8 \cdot B \cdot a^4 + 32 \cdot A \cdot a^3 \cdot b + 36 \cdot B \cdot a^2 \cdot b^2 + 24 \cdot A \cdot a \cdot b^3 + 5 \cdot B \cdot b^4) \cdot \cos(d \cdot x + c)^4 + 32 \cdot (10 \cdot B \cdot a^3 \cdot b + 15 \cdot A \cdot a^2 \cdot b^2 + 8 \cdot B \cdot a \cdot b^3 + 2 \cdot A \cdot b^4) \cdot \cos(d \cdot x + c)^3 + 10 \cdot (36 \cdot B \cdot a^2 \cdot b^2 + 24 \cdot A \cdot a \cdot b^3 + 5 \cdot B \cdot b^4) \cdot \cos(d \cdot x + c)^2 + 48 \cdot (4 \cdot B \cdot a \cdot b^3 + A \cdot b^4) \cdot \cos(d \cdot x + c)) \cdot \sin(d \cdot x + c)) / (d \cdot \cos(d \cdot x + c)^6)$

giac [B] time = 2.28, size = 1186, normalized size = 3.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] $\frac{1}{240} \cdot (15 \cdot (8 \cdot B \cdot a^4 + 32 \cdot A \cdot a^3 \cdot b + 36 \cdot B \cdot a^2 \cdot b^2 + 24 \cdot A \cdot a \cdot b^3 + 5 \cdot B \cdot b^4) \cdot \log(\abs{\tan(1/2 \cdot d \cdot x + 1/2 \cdot c) + 1}) - 15 \cdot (8 \cdot B \cdot a^4 + 32 \cdot A \cdot a^3 \cdot b + 36 \cdot B \cdot a^2 \cdot b^2 + 24 \cdot A \cdot a \cdot b^3 + 5 \cdot B \cdot b^4) \cdot \log(\abs{\tan(1/2 \cdot d \cdot x + 1/2 \cdot c) - 1}) - 2 \cdot (240 \cdot A \cdot a^4 \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^{11} - 120 \cdot B \cdot a^4 \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^{11} - 480 \cdot A \cdot a^3 \cdot b \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^{11} + 960 \cdot B \cdot a^3 \cdot b \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^{11} + 1440 \cdot A \cdot a^2 \cdot b^2 \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^{11} - 900 \cdot B \cdot a^2 \cdot b^2 \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^{11} - 600 \cdot A \cdot a \cdot b^3 \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^{11} + 960 \cdot B \cdot a \cdot b^3 \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^{11} + 240 \cdot A \cdot b^4 \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^{11} - 165 \cdot B \cdot b^4 \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^{11} - 1200 \cdot A \cdot a^4 \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^9 + 360 \cdot B \cdot a^4 \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^9 + 1440 \cdot A \cdot a^3 \cdot b \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^9 - 3520 \cdot B \cdot a^3 \cdot b \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^9 - 5280 \cdot A \cdot a^2 \cdot b^2 \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^9 + 1260 \cdot B \cdot a^2 \cdot b^2 \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^9 + 840 \cdot A \cdot a \cdot b^3 \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^9 - 2240 \cdot B \cdot a \cdot b^3 \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^9 - 560 \cdot A \cdot b^4 \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^9 - 25 \cdot B \cdot b^4 \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^9 + 2400 \cdot A \cdot a^4 \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^7 - 240 \cdot B \cdot a^4 \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^7 - 960 \cdot A \cdot a^3 \cdot b \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^7 + 5760 \cdot B \cdot a^3 \cdot b \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^7 + 8640 \cdot A \cdot a^2 \cdot b^2 \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^7 - 360 \cdot B \cdot a^2 \cdot b^2 \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^7 - 240 \cdot A \cdot a \cdot b^3 \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^7 + 4992 \cdot B \cdot a \cdot b^3 \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^7 + 1248 \cdot A \cdot b^4 \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^7 - 450 \cdot B \cdot b^4 \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^7 - 2400 \cdot A \cdot a^4 \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^5 - 240 \cdot B \cdot a^4 \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^5 - 960 \cdot A \cdot a^3 \cdot b \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^5 - 5760 \cdot B \cdot a^3 \cdot b \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^5 - 8640 \cdot A \cdot a^2 \cdot b^2 \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^5 - 360 \cdot B \cdot a^2 \cdot b^2 \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^5 - 240 \cdot A \cdot a \cdot b^3 \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^5 - 4992 \cdot B \cdot a \cdot b^3 \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^5 - 1248 \cdot A \cdot b^4 \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^5 - 450 \cdot B \cdot b^4 \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^5 + 1200 \cdot A \cdot a^4 \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^3 + 360 \cdot B \cdot a^4 \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^3 + 1440 \cdot A \cdot a^3 \cdot b \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^3 + 3520 \cdot B \cdot a^3 \cdot b \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^3 + 5280 \cdot A \cdot a^2 \cdot b^2 \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^3)$

$$\begin{aligned}
& + 1/2*c)^3 + 1260*B*a^2*b^2*\tan(1/2*d*x + 1/2*c)^3 + 840*A*a*b^3*\tan(1/2*d \\
& *x + 1/2*c)^3 + 2240*B*a*b^3*\tan(1/2*d*x + 1/2*c)^3 + 560*A*b^4*\tan(1/2*d*x \\
& + 1/2*c)^3 - 25*B*b^4*\tan(1/2*d*x + 1/2*c)^3 - 240*A*a^4*\tan(1/2*d*x + 1/2 \\
& *c) - 120*B*a^4*\tan(1/2*d*x + 1/2*c) - 480*A*a^3*b*\tan(1/2*d*x + 1/2*c) - 9 \\
& 60*B*a^3*b*\tan(1/2*d*x + 1/2*c) - 1440*A*a^2*b^2*\tan(1/2*d*x + 1/2*c) - 900 \\
& *B*a^2*b^2*\tan(1/2*d*x + 1/2*c) - 600*A*a*b^3*\tan(1/2*d*x + 1/2*c) - 960*B* \\
& a*b^3*\tan(1/2*d*x + 1/2*c) - 240*A*b^4*\tan(1/2*d*x + 1/2*c) - 165*B*b^4*\tan \\
& (1/2*d*x + 1/2*c))/(\tan(1/2*d*x + 1/2*c)^2 - 1)^6/d
\end{aligned}$$

maple [A] time = 1.85, size = 550, normalized size = 1.65

$$\frac{A b^4 \tan(dx+c) \left(\sec^4(dx+c)\right)}{5d} + \frac{4A b^4 \tan(dx+c) \left(\sec^2(dx+c)\right)}{15d} + \frac{A a^4 \tan(dx+c)}{d} + \frac{a^4 B \ln(\sec(dx+c))}{2d} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^2*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)), x)

[Out] 1/d*A*a^4*tan(d*x+c)+1/2/d*a^4*B*ln(sec(d*x+c)+tan(d*x+c))+9/4/d*a^2*b^2*B*ln(sec(d*x+c)+tan(d*x+c))+3/2/d*a*A*b^3*ln(sec(d*x+c)+tan(d*x+c))+1/6/d*B*b^4*tan(d*x+c)*sec(d*x+c)^5+5/24/d*B*b^4*tan(d*x+c)*sec(d*x+c)^3+5/16/d*B*b^4*sec(d*x+c)*tan(d*x+c)+32/15/d*B*a*b^3*tan(d*x+c)+8/3/d*B*a^3*b*tan(d*x+c)+4/d*A*a^2*b^2*tan(d*x+c)+5/16/d*B*b^4*ln(sec(d*x+c)+tan(d*x+c))+8/15/d*A*b^4*tan(d*x+c)+3/2/d*a*A*b^3*sec(d*x+c)*tan(d*x+c)+4/5/d*B*a*b^3*tan(d*x+c)*sec(d*x+c)^4+16/15/d*B*a*b^3*tan(d*x+c)*sec(d*x+c)^2+4/3/d*B*a^3*b*tan(d*x+c)*sec(d*x+c)^2+2/d*A*a^3*b*sec(d*x+c)*tan(d*x+c)+3/2/d*a^2*b^2*B*tan(d*x+c)*sec(d*x+c)^3+9/4/d*a^2*b^2*B*sec(d*x+c)*tan(d*x+c)+1/d*a*A*b^3*tan(d*x+c)*sec(d*x+c)^3+2/d*A*a^2*b^2*tan(d*x+c)*sec(d*x+c)^2+1/2/d*a^4*B*sec(d*x+c)*tan(d*x+c)+2/d*A*a^3*b*ln(sec(d*x+c)+tan(d*x+c))+1/5/d*A*b^4*tan(d*x+c)*sec(d*x+c)^4+4/15/d*A*b^4*tan(d*x+c)*sec(d*x+c)^2

maxima [A] time = 0.76, size = 474, normalized size = 1.42

$$640 \left(\tan(dx+c)^3 + 3 \tan(dx+c) \right) B a^3 b + 960 \left(\tan(dx+c)^3 + 3 \tan(dx+c) \right) A a^2 b^2 + 128 \left(3 \tan(dx+c)^5 + \dots \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)), x, algorithm="maxima")

[Out] 1/480*(640*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a^3*b + 960*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^2*b^2 + 128*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*B*a*b^3 + 32*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*A*b^4 - 5*B*b^4*(2*(15*sin(d*x + c)^5 - 40*sin(d*x + c)^3 + 33*s

$$\frac{\ln(d*x + c)/(\sin(d*x + c)^6 - 3*\sin(d*x + c)^4 + 3*\sin(d*x + c)^2 - 1) - 15*\log(\sin(d*x + c) + 1) + 15*\log(\sin(d*x + c) - 1) - 180*B*a^2*b^2*(2*(3*\sin(d*x + c)^3 - 5*\sin(d*x + c)))/(\sin(d*x + c)^4 - 2*\sin(d*x + c)^2 + 1) - 3*\log(\sin(d*x + c) + 1) + 3*\log(\sin(d*x + c) - 1) - 120*A*a*b^3*(2*(3*\sin(d*x + c)^3 - 5*\sin(d*x + c)))/(\sin(d*x + c)^4 - 2*\sin(d*x + c)^2 + 1) - 3*\log(\sin(d*x + c) + 1) + 3*\log(\sin(d*x + c) - 1) - 120*B*a^4*(2*\sin(d*x + c)/(\sin(d*x + c)^2 - 1) - \log(\sin(d*x + c) + 1) + \log(\sin(d*x + c) - 1)) - 480*A*a^3*b*(2*\sin(d*x + c)/(\sin(d*x + c)^2 - 1) - \log(\sin(d*x + c) + 1) + \log(\sin(d*x + c) - 1)) + 480*A*a^4*\tan(d*x + c))/d$$

mupad [B] time = 5.74, size = 709, normalized size = 2.12

$$\frac{\operatorname{atanh}\left(\frac{4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{Ba^4}{2}+2Aa^3b+\frac{9Ba^2b^2}{4}+\frac{3Aab^3}{2}+\frac{5Bb^4}{16}\right)}{2Ba^4+8Aa^3b+9Ba^2b^2+6Aab^3+\frac{5Bb^4}{4}}\right)\left(Ba^4+4Aa^3b+\frac{9Ba^2b^2}{2}+3Aab^3+\frac{5Bb^4}{8}\right)}{d} + \frac{\left(Ba^4-2Aab^4\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^4)/cos(c + d*x)^2,x)`

[Out]
$$\frac{\operatorname{atanh}\left(\frac{4*\tan(c/2 + (d*x)/2)*((B*a^4)/2 + (5*B*b^4)/16 + (9*B*a^2*b^2)/4 + (3*A*a*b^3)/2 + 2*A*a^3*b)}{(2*B*a^4 + (5*B*b^4)/4 + 9*B*a^2*b^2 + 6*A*a*b^3 + 8*A*a^3*b)}\right)*(B*a^4 + (5*B*b^4)/8 + (9*B*a^2*b^2)/2 + 3*A*a*b^3 + 4*A*a^3*b)}{d} + \frac{\tan(c/2 + (d*x)/2)*((2*A*a^4 + 2*A*b^4 + B*a^4 + (11*B*b^4)/8 + 12*A*a^2*b^2 + (15*B*a^2*b^2)/2 + 5*A*a*b^3 + 4*A*a^3*b + 8*B*a*b^3 + 8*B*a^3*b) - \tan(c/2 + (d*x)/2)^{11}*(2*A*a^4 + 2*A*b^4 - B*a^4 - (11*B*b^4)/8 + 12*A*a^2*b^2 - (15*B*a^2*b^2)/2 - 5*A*a*b^3 - 4*A*a^3*b + 8*B*a*b^3 + 8*B*a^3*b) - \tan(c/2 + (d*x)/2)^3*(10*A*a^4 + (14*A*b^4)/3 + 3*B*a^4 - (5*B*b^4)/24 + 44*A*a^2*b^2 + (21*B*a^2*b^2)/2 + 7*A*a*b^3 + 12*A*a^3*b + (56*B*a*b^3)/3 + (88*B*a^3*b)/3) + \tan(c/2 + (d*x)/2)^9*(10*A*a^4 + (14*A*b^4)/3 - 3*B*a^4 + (5*B*b^4)/24 + 44*A*a^2*b^2 - (21*B*a^2*b^2)/2 - 7*A*a*b^3 - 12*A*a^3*b + (56*B*a*b^3)/3 + (88*B*a^3*b)/3) + \tan(c/2 + (d*x)/2)^5*(20*A*a^4 + (52*A*b^4)/5 + 2*B*a^4 + (15*B*b^4)/4 + 72*A*a^2*b^2 + 3*B*a^2*b^2 + 2*A*a*b^3 + 8*A*a^3*b + (208*B*a*b^3)/5 + 48*B*a^3*b) - \tan(c/2 + (d*x)/2)^7*(20*A*a^4 + (52*A*b^4)/5 - 2*B*a^4 - (15*B*b^4)/4 + 72*A*a^2*b^2 - 3*B*a^2*b^2 - 2*A*a*b^3 - 8*A*a^3*b + (208*B*a*b^3)/5 + 48*B*a^3*b)}}{(d*(15*\tan(c/2 + (d*x)/2)^4 - 6*\tan(c/2 + (d*x)/2)^2 - 20*\tan(c/2 + (d*x)/2)^6 + 15*\tan(c/2 + (d*x)/2)^8 - 6*\tan(c/2 + (d*x)/2)^{10} + \tan(c/2 + (d*x)/2)^{12} + 1)}$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \sec(c + dx))(a + b \sec(c + dx))^4 \sec^2(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*sec(d*x+c))**4*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**4*sec(c + d*x)**2, x)
```

3.303 $\int \sec(c + dx)(a + b \sec(c + dx))^4(A + B \sec(c + dx)) dx$

Optimal. Leaf size=250

$$\frac{(12a^2B + 35aAb + 16b^2B) \tan(c + dx)(a + b \sec(c + dx))^2}{60d} + \frac{b(24a^3B + 130a^2Ab + 116ab^2B + 45Ab^3) \tan(c + dx)}{120d}$$

[Out] $\frac{1}{8} * (8 * A * a^4 + 24 * A * a^2 * b^2 + 3 * A * b^4 + 16 * B * a^3 * b + 12 * B * a * b^3) * \operatorname{arctanh}(\sin(dx + c)) / d + \frac{1}{30} * (95 * A * a^3 * b + 80 * A * a * b^3 + 12 * B * a^4 + 112 * B * a^2 * b^2 + 16 * B * b^4) * \tan(dx + c) / d + \frac{1}{120} * b * (130 * A * a^2 * b + 45 * A * b^3 + 24 * B * a^3 + 116 * B * a * b^2) * \sec(dx + c) * \tan(dx + c) / d + \frac{1}{60} * (35 * A * a * b + 12 * B * a^2 + 16 * B * b^2) * (a + b * \sec(dx + c))^2 * \tan(dx + c) / d + \frac{1}{20} * (5 * A * b + 4 * B * a) * (a + b * \sec(dx + c))^3 * \tan(dx + c) / d + \frac{1}{5} * B * (a + b * \sec(dx + c))^4 * \tan(dx + c) / d$

Rubi [A] time = 0.52, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {4002, 3997, 3787, 3770, 3767, 8}

$$\frac{(95a^3Ab + 112a^2b^2B + 12a^4B + 80aAb^3 + 16b^4B) \tan(c + dx)}{30d} + \frac{(24a^2Ab^2 + 8a^4A + 16a^3bB + 12ab^3B + 3Ab^4) \tan(c + dx)}{8d}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Sec}[c + dx] * (a + b * \operatorname{Sec}[c + dx])^4 * (A + B * \operatorname{Sec}[c + dx]), x]$

[Out] $((8 * a^4 * A + 24 * a^2 * A * b^2 + 3 * A * b^4 + 16 * a^3 * b * B + 12 * a * b^3 * B) * \operatorname{ArcTanh}[\operatorname{Sin}[c + dx]]) / (8 * d) + ((95 * a^3 * A * b + 80 * a * A * b^3 + 12 * a^4 * B + 112 * a^2 * b^2 * B + 16 * b^4 * B) * \operatorname{Tan}[c + dx]) / (30 * d) + (b * (130 * a^2 * A * b + 45 * A * b^3 + 24 * a^3 * B + 116 * a * b^2 * B) * \operatorname{Sec}[c + dx] * \operatorname{Tan}[c + dx]) / (120 * d) + ((35 * a * A * b + 12 * a^2 * B + 16 * b^2 * B) * (a + b * \operatorname{Sec}[c + dx])^2 * \operatorname{Tan}[c + dx]) / (60 * d) + ((5 * A * b + 4 * a * B) * (a + b * \operatorname{Sec}[c + dx])^3 * \operatorname{Tan}[c + dx]) / (20 * d) + (B * (a + b * \operatorname{Sec}[c + dx])^4 * \operatorname{Tan}[c + dx]) / (5 * d)$

Rule 8

$\operatorname{Int}[a_, x_Symbol] \rightarrow \operatorname{Simp}[a * x, x] / ; \operatorname{FreeQ}[a, x]$

Rule 3767

$\operatorname{Int}[\operatorname{csc}[(c_.) + (d_.) * (x_.)]^{(n_.)}, x_Symbol] \rightarrow -\operatorname{Dist}[d^{(-1)}, \operatorname{Subst}[\operatorname{Int}[\operatorname{ExpandIntegrand}[(1 + x^2)^{(n/2 - 1)}, x], x], x, \operatorname{Cot}[c + dx]], x] / ; \operatorname{FreeQ}[\{c, d\}, x] \ \&\& \ \operatorname{IGtQ}[n/2, 0]$

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_.)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x]
  /; FreeQ[{c, d}, x]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) +
(a_.)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) +
(a_.))*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := -Simp[(b*B*Cot[e
+ f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && !LeQ[n,
-1]
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_.)]*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(cs
c[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \sec(c + dx)(a + b \sec(c + dx))^4(A + B \sec(c + dx)) dx &= \frac{B(a + b \sec(c + dx))^4 \tan(c + dx)}{5d} + \frac{1}{5} \int \sec(c + dx)(a + b \sec(c + dx))^4(A + B \sec(c + dx)) dx \\
&= \frac{(5Ab + 4aB)(a + b \sec(c + dx))^3 \tan(c + dx)}{20d} + \frac{B(a + b \sec(c + dx))^4 \tan(c + dx)}{5d} \\
&= \frac{(35aAb + 12a^2B + 16b^2B)(a + b \sec(c + dx))^2 \tan(c + dx)}{60d} \\
&= \frac{b(130a^2Ab + 45Ab^3 + 24a^3B + 116ab^2B) \sec(c + dx) \tan(c + dx)}{120d} \\
&= \frac{b(130a^2Ab + 45Ab^3 + 24a^3B + 116ab^2B) \sec(c + dx) \tan(c + dx)}{120d} \\
&= \frac{(8a^4A + 24a^2Ab^2 + 3Ab^4 + 16a^3bB + 12ab^3B) \tanh^{-1}(\sin(c + dx)) + \tan(c + dx) (80b^2(3a^2B + 2aAb + b^2B))}{8d} \\
&= \frac{(8a^4A + 24a^2Ab^2 + 3Ab^4 + 16a^3bB + 12ab^3B) \tanh^{-1}(\sin(c + dx)) + \tan(c + dx) (80b^2(3a^2B + 2aAb + b^2B))}{8d}
\end{aligned}$$

Mathematica [A] time = 3.94, size = 198, normalized size = 0.79

$$\frac{15(8a^4A + 16a^3bB + 24a^2Ab^2 + 12ab^3B + 3Ab^4) \tanh^{-1}(\sin(c + dx)) + \tan(c + dx) (80b^2(3a^2B + 2aAb + b^2B))}{8d}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[c + d*x]*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]

[Out] (15*(8*a^4*A + 24*a^2*A*b^2 + 3*A*b^4 + 16*a^3*b*B + 12*a*b^3*B)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(120*(4*a^3*A*b + 4*a*A*b^3 + a^4*B + 6*a^2*b^2*B + b^4*B) + 15*b*(24*a^2*A*b + 3*A*b^3 + 16*a^3*B + 12*a*b^2*B)*Sec[c + d*x] + 30*b^3*(A*b + 4*a*B)*Sec[c + d*x]^3 + 80*b^2*(2*a*A*b + 3*a^2*B + b^2*B)*Tan[c + d*x]^2 + 24*b^4*B*Tan[c + d*x]^4))/(120*d)

fricas [A] time = 0.48, size = 281, normalized size = 1.12

$$\frac{15(8Aa^4 + 16Ba^3b + 24Aa^2b^2 + 12Bab^3 + 3Ab^4) \cos(dx + c)^5 \log(\sin(dx + c) + 1) - 15(8Aa^4 + 16Ba^3b + 24Aa^2b^2 + 12Bab^3 + 3Ab^4) \sin(dx + c)^5}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="fricas")

```
[Out] 1/240*(15*(8*A*a^4 + 16*B*a^3*b + 24*A*a^2*b^2 + 12*B*a*b^3 + 3*A*b^4)*cos(
d*x + c)^5*log(sin(d*x + c) + 1) - 15*(8*A*a^4 + 16*B*a^3*b + 24*A*a^2*b^2
+ 12*B*a*b^3 + 3*A*b^4)*cos(d*x + c)^5*log(-sin(d*x + c) + 1) + 2*(24*B*b^4
+ 8*(15*B*a^4 + 60*A*a^3*b + 60*B*a^2*b^2 + 40*A*a*b^3 + 8*B*b^4)*cos(d*x
+ c)^4 + 15*(16*B*a^3*b + 24*A*a^2*b^2 + 12*B*a*b^3 + 3*A*b^4)*cos(d*x + c)
^3 + 16*(15*B*a^2*b^2 + 10*A*a*b^3 + 2*B*b^4)*cos(d*x + c)^2 + 30*(4*B*a*b^
3 + A*b^4)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^5)
```

giac [B] time = 1.08, size = 850, normalized size = 3.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/120*(15*(8*A*a^4 + 16*B*a^3*b + 24*A*a^2*b^2 + 12*B*a*b^3 + 3*A*b^4)*log(
abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(8*A*a^4 + 16*B*a^3*b + 24*A*a^2*b^2 +
12*B*a*b^3 + 3*A*b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(120*B*a^4*tan
(1/2*d*x + 1/2*c)^9 + 480*A*a^3*b*tan(1/2*d*x + 1/2*c)^9 - 240*B*a^3*b*tan(
1/2*d*x + 1/2*c)^9 - 360*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^9 + 720*B*a^2*b^2*t
an(1/2*d*x + 1/2*c)^9 + 480*A*a*b^3*tan(1/2*d*x + 1/2*c)^9 - 300*B*a*b^3*ta
n(1/2*d*x + 1/2*c)^9 - 75*A*b^4*tan(1/2*d*x + 1/2*c)^9 + 120*B*b^4*tan(1/2*
d*x + 1/2*c)^9 - 480*B*a^4*tan(1/2*d*x + 1/2*c)^7 - 1920*A*a^3*b*tan(1/2*d*
x + 1/2*c)^7 + 480*B*a^3*b*tan(1/2*d*x + 1/2*c)^7 + 720*A*a^2*b^2*tan(1/2*d
*x + 1/2*c)^7 - 1920*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 - 1280*A*a*b^3*tan(1/
2*d*x + 1/2*c)^7 + 120*B*a*b^3*tan(1/2*d*x + 1/2*c)^7 + 30*A*b^4*tan(1/2*d*
x + 1/2*c)^7 - 160*B*b^4*tan(1/2*d*x + 1/2*c)^7 + 720*B*a^4*tan(1/2*d*x + 1
/2*c)^5 + 2880*A*a^3*b*tan(1/2*d*x + 1/2*c)^5 + 2400*B*a^2*b^2*tan(1/2*d*x
+ 1/2*c)^5 + 1600*A*a*b^3*tan(1/2*d*x + 1/2*c)^5 + 464*B*b^4*tan(1/2*d*x +
1/2*c)^5 - 480*B*a^4*tan(1/2*d*x + 1/2*c)^3 - 1920*A*a^3*b*tan(1/2*d*x + 1/
2*c)^3 - 480*B*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 720*A*a^2*b^2*tan(1/2*d*x + 1
/2*c)^3 - 1920*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 1280*A*a*b^3*tan(1/2*d*x
+ 1/2*c)^3 - 120*B*a*b^3*tan(1/2*d*x + 1/2*c)^3 - 30*A*b^4*tan(1/2*d*x + 1/
2*c)^3 - 160*B*b^4*tan(1/2*d*x + 1/2*c)^3 + 120*B*a^4*tan(1/2*d*x + 1/2*c)
+ 480*A*a^3*b*tan(1/2*d*x + 1/2*c) + 240*B*a^3*b*tan(1/2*d*x + 1/2*c) + 360
*A*a^2*b^2*tan(1/2*d*x + 1/2*c) + 720*B*a^2*b^2*tan(1/2*d*x + 1/2*c) + 480*
A*a*b^3*tan(1/2*d*x + 1/2*c) + 300*B*a*b^3*tan(1/2*d*x + 1/2*c) + 75*A*b^4*
tan(1/2*d*x + 1/2*c) + 120*B*b^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)
^2 - 1)^5)/d
```

maple [A] time = 1.67, size = 431, normalized size = 1.72

$$\frac{A a^4 \ln(\sec(dx + c) + \tan(dx + c))}{d} + \frac{a^4 B \tan(dx + c)}{d} + \frac{4A a^3 b \tan(dx + c)}{d} + \frac{2B a^3 b \sec(dx + c) \tan(dx + c)}{d} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sec(d*x+c)*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x)`

[Out] $\frac{1}{d}Aa^4\ln(\sec(dx+c)+\tan(dx+c))+\frac{1}{d}a^4B\tan(dx+c)+\frac{4}{d}Aa^3b\tan(dx+c)+\frac{2}{d}Ba^3b^2\sec(dx+c)\tan(dx+c)+\frac{2}{d}Ba^3b^2\ln(\sec(dx+c)+\tan(dx+c))+\frac{3}{d}Aa^2b^2\sec(dx+c)\tan(dx+c)+\frac{3}{d}Aa^2b^2\ln(\sec(dx+c)+\tan(dx+c))+\frac{4}{d}a^2b^2B\tan(dx+c)+\frac{2}{d}a^2b^2B\tan(dx+c)\sec(dx+c)^2+\frac{8}{3}dAa^3b^3\tan(dx+c)+\frac{4}{3}dAa^3b^3\tan(dx+c)\sec(dx+c)^2+\frac{1}{d}B^3a^3b^3\tan(dx+c)\sec(dx+c)^3+\frac{3}{2}dBa^3b^3\sec(dx+c)\tan(dx+c)+\frac{3}{2}dBa^3b^3\ln(\sec(dx+c)+\tan(dx+c))+\frac{1}{4}dA^4b^4\tan(dx+c)\sec(dx+c)^3+\frac{3}{8}dA^4b^4\sec(dx+c)\tan(dx+c)+\frac{3}{8}dA^4b^4\ln(\sec(dx+c)+\tan(dx+c))+\frac{8}{15}dB^4b^4\tan(dx+c)+\frac{1}{5}dB^4b^4\tan(dx+c)\sec(dx+c)^4+\frac{4}{15}dB^4b^4\tan(dx+c)\sec(dx+c)^2$

maxima [A] time = 1.02, size = 379, normalized size = 1.52

$$\frac{480(\tan(dx+c)^3+3\tan(dx+c))Ba^2b^2+320(\tan(dx+c)^3+3\tan(dx+c))Aab^3+16(3\tan(dx+c)^5+10\tan(dx+c)^3+15\tan(dx+c))B^3b^4}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="maxima")`

[Out] $\frac{1}{240}*(480*(\tan(dx+c)^3+3*\tan(dx+c))*Ba^2b^2+320*(\tan(dx+c)^3+3*\tan(dx+c))*Aa^3b^3+16*(3*\tan(dx+c)^5+10*\tan(dx+c)^3+15*\tan(dx+c))*B^3b^4-60*Ba^3b^3*(2*(3*\sin(dx+c)^3-5*\sin(dx+c))/(\sin(dx+c)^4-2*\sin(dx+c)^2+1)-3*\log(\sin(dx+c)+1)+3*\log(\sin(dx+c)-1))-15*A^4b^4*(2*(3*\sin(dx+c)^3-5*\sin(dx+c))/(\sin(dx+c)^4-2*\sin(dx+c)^2+1)-3*\log(\sin(dx+c)+1)+3*\log(\sin(dx+c)-1))-240*B^3a^3b^3*(2*\sin(dx+c)/(\sin(dx+c)^2-1)-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1))-360*Aa^2b^2*(2*\sin(dx+c)/(\sin(dx+c)^2-1)-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1))+240*Aa^4*\log(\sec(dx+c)+\tan(dx+c))+240*B^4a^4*\tan(dx+c)+960*Aa^3b^3*\tan(dx+c))/d$

mupad [B] time = 6.01, size = 555, normalized size = 2.22

$$\frac{\operatorname{atanh}\left(\frac{4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(Aa^4+2Ba^3b+3Aa^2b^2+\frac{3Bab^3}{2}+\frac{3Ab^4}{8}\right)}{4Aa^4+8Ba^3b+12Aa^2b^2+6Bab^3+\frac{3Ab^4}{2}}\right)}{d}\left(2Aa^4+4Ba^3b+6Aa^2b^2+3Bab^3+\frac{3Ab^4}{4}\right)\left(2Ba^4-\frac{5}{2}B^2a^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^4)/cos(c + d*x),x)`

[Out]
$$\frac{\operatorname{atanh}\left(\frac{4\tan\left(\frac{c}{2} + \frac{d*x}{2}\right)\left(A*a^4 + \frac{3A*b^4}{8} + 3A*a^2*b^2 + \frac{3B*a*b^3}{2} + 2B*a^3*b\right)}{4A*a^4 + \frac{3A*b^4}{2} + 12A*a^2*b^2 + 6B*a*b^3 + 8B*a^3*b}\right)\left(2A*a^4 + \frac{3A*b^4}{4} + 6A*a^2*b^2 + 3B*a*b^3 + 4B*a^3*b\right)}{d} - \left(\tan\left(\frac{c}{2} + \frac{d*x}{2}\right)\left(\frac{5A*b^4}{4} + 2B*a^4 + 2B*b^4 + 6A*a^2*b^2 + 12B*a^2*b^2 + 8A*a*b^3 + 8A*a^3*b + 5B*a*b^3 + 4B*a^3*b\right) + \tan\left(\frac{c}{2} + \frac{d*x}{2}\right)^5\left(\frac{12B*a^4}{15} + \frac{116B*b^4}{15} + 40B*a^2*b^2 + \frac{80A*a*b^3}{3} + 48A*a^3*b\right) + \tan\left(\frac{c}{2} + \frac{d*x}{2}\right)^9\left(2B*a^4 - \frac{5A*b^4}{4} + 2B*b^4 - 6A*a^2*b^2 + 12B*a^2*b^2 + 8A*a*b^3 + 8A*a^3*b - 5B*a*b^3 - 4B*a^3*b\right) - \tan\left(\frac{c}{2} + \frac{d*x}{2}\right)^3\left(\frac{A*b^4}{2} + 8B*a^4 + \frac{8B*b^4}{3} + 12A*a^2*b^2 + 32B*a^2*b^2 + \frac{64A*a*b^3}{3} + 32A*a^3*b + 2B*a*b^3 + 8B*a^3*b\right) - \tan\left(\frac{c}{2} + \frac{d*x}{2}\right)^7\left(8B*a^4 - \frac{A*b^4}{2} + \frac{8B*b^4}{3} - 12A*a^2*b^2 + 32B*a^2*b^2 + \frac{64A*a*b^3}{3} + 32A*a^3*b - 2B*a*b^3 - 8B*a^3*b\right)}{d\left(5\tan\left(\frac{c}{2} + \frac{d*x}{2}\right)^2 - 10\tan\left(\frac{c}{2} + \frac{d*x}{2}\right)^4 + 10\tan\left(\frac{c}{2} + \frac{d*x}{2}\right)^6 - 5\tan\left(\frac{c}{2} + \frac{d*x}{2}\right)^8 + \tan\left(\frac{c}{2} + \frac{d*x}{2}\right)^{10} - 1\right)}$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \sec(c + dx))(a + b \sec(c + dx))^4 \sec(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)*(a+b*sec(d*x+c))**4*(A+B*sec(d*x+c)),x)`

[Out] `Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**4*sec(c + d*x), x)`

3.304 $\int (a + b \sec(c + dx))^4 (A + B \sec(c + dx)) dx$

Optimal. Leaf size=200

$$a^4 Ax + \frac{b^2 (26a^2 B + 32aAb + 9b^2 B) \tan(c + dx) \sec(c + dx)}{24d} + \frac{b (19a^3 B + 34a^2 Ab + 16ab^2 B + 4Ab^3) \tan(c + dx)}{6d} +$$

[Out] $a^4 A x + 1/8 * (32 * A * a^3 * b + 16 * A * a * b^3 + 8 * B * a^4 + 24 * B * a^2 * b^2 + 3 * B * b^4) * \operatorname{arctanh}(\sin(d * x + c)) / d + 1/6 * b * (34 * A * a^2 * b + 4 * A * b^3 + 19 * B * a^3 + 16 * B * a * b^2) * \tan(d * x + c) / d + 1/2 * 4 * b^2 * (32 * A * a * b + 26 * B * a^2 + 9 * B * b^2) * \sec(d * x + c) * \tan(d * x + c) / d + 1/12 * b * (4 * A * b + 7 * B * a) * (a + b * \sec(d * x + c))^2 * \tan(d * x + c) / d + 1/4 * b * B * (a + b * \sec(d * x + c))^3 * \tan(d * x + c) / d$

Rubi [A] time = 0.33, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3918, 4056, 4048, 3770, 3767, 8}

$$\frac{b (34a^2 Ab + 19a^3 B + 16ab^2 B + 4Ab^3) \tan(c + dx)}{6d} + \frac{(32a^3 Ab + 24a^2 b^2 B + 8a^4 B + 16aAb^3 + 3b^4 B) \tanh^{-1}(\sin(c + dx))}{8d}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(a + b \operatorname{Sec}[c + d * x])^4 (A + B \operatorname{Sec}[c + d * x]), x]$

[Out] $a^4 A x + ((32 * a^3 * A * b + 16 * a * A * b^3 + 8 * a^4 * B + 24 * a^2 * b^2 * B + 3 * b^4 * B) * \operatorname{ArcTanh}[\operatorname{Sin}[c + d * x]]) / (8 * d) + (b * (34 * a^2 * A * b + 4 * A * b^3 + 19 * a^3 * B + 16 * a * b^2 * B) * \operatorname{Tan}[c + d * x]) / (6 * d) + (b^2 * (32 * a * A * b + 26 * a^2 * B + 9 * b^2 * B) * \operatorname{Sec}[c + d * x] * \operatorname{Tan}[c + d * x]) / (24 * d) + (b * (4 * A * b + 7 * a * B) * (a + b * \operatorname{Sec}[c + d * x])^2 * \operatorname{Tan}[c + d * x]) / (12 * d) + (b * B * (a + b * \operatorname{Sec}[c + d * x])^3 * \operatorname{Tan}[c + d * x]) / (4 * d)$

Rule 8

$\operatorname{Int}[a_, x_Symbol] := \operatorname{Simp}[a * x, x] /; \operatorname{FreeQ}[a, x]$

Rule 3767

$\operatorname{Int}[\operatorname{csc}[(c_.) + (d_.) * (x_.)]^{(n_.)}, x_Symbol] := -\operatorname{Dist}[d^{(-1)}, \operatorname{Subst}[\operatorname{Int}[\operatorname{ExpandIntegrand}[(1 + x^2)^{(n/2 - 1)}, x], x], x, \operatorname{Cot}[c + d * x]], x] /; \operatorname{FreeQ}[\{c, d\}, x] \ \&\amp; \operatorname{IGtQ}[n/2, 0]$

Rule 3770

$\operatorname{Int}[\operatorname{csc}[(c_.) + (d_.) * (x_.)], x_Symbol] := -\operatorname{Simp}[\operatorname{ArcTanh}[\operatorname{Cos}[c + d * x]] / d, x] /; \operatorname{FreeQ}[\{c, d\}, x]$

Rule 3918


```
Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.)), x_Symbol] := -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1))/(f*m), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m - 2)*Simp[a^2*c*m + (b^2*d*(m - 1) + 2*a*b*c*m + a^2*d*m)*Csc[e + f*x] + b*(b*c*m + a*d*(2*m - 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && GtQ[m, 1] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 4048

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] := -Simp[(b*C*Csc[e + f*x]*Cot[e + f*x])/(2*f), x] + Dist[1/2, Int[Simp[2*A*a + (2*B*a + b*(2*A + C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x]
```

Rule 4056

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.), x_Symbol] := -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Csc[e + f*x])^(m - 1)*Simp[a*A*(m + 1) + ((A*b + a*B)*(m + 1) + b*C*m)*Csc[e + f*x] + (b*B*(m + 1) + a*C*m)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && IGtQ[2*m, 0]
```

Rubi steps

$$\begin{aligned}
 \int (a + b \sec(c + dx))^4 (A + B \sec(c + dx)) dx &= \frac{bB(a + b \sec(c + dx))^3 \tan(c + dx)}{4d} + \frac{1}{4} \int (a + b \sec(c + dx))^2 (4A + b \sec(c + dx)) dx \\
 &= \frac{b(4Ab + 7aB)(a + b \sec(c + dx))^2 \tan(c + dx)}{12d} + \frac{bB(a + b \sec(c + dx)) \tan(c + dx)}{4d} \\
 &= \frac{b^2 (32aAb + 26a^2B + 9b^2B) \sec(c + dx) \tan(c + dx)}{24d} + \frac{b(4Ab + 7aB) \tan(c + dx)}{4d} \\
 &= a^4 Ax + \frac{b^2 (32aAb + 26a^2B + 9b^2B) \sec(c + dx) \tan(c + dx)}{24d} + \frac{b(4Ab + 7aB) \tan(c + dx)}{4d} \\
 &= a^4 Ax + \frac{(32a^3Ab + 16aAb^3 + 8a^4B + 24a^2b^2B + 3b^4B) \tanh^{-1}(\sec(c + dx))}{8d} \\
 &= a^4 Ax + \frac{(32a^3Ab + 16aAb^3 + 8a^4B + 24a^2b^2B + 3b^4B) \tanh^{-1}(\sec(c + dx))}{8d}
 \end{aligned}$$

Mathematica [A] time = 1.03, size = 160, normalized size = 0.80

$$\frac{24a^4 Adx + 3b \tan(c + dx) \left(b \left(24a^2 B + 16aAb + 3b^2 B \right) \sec(c + dx) + 8 \left(4a^3 B + 6a^2 Ab + 4ab^2 B + Ab^3 \right) + 2b^3 B \sec(c + dx) \right)}{2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]

[Out] (24*a^4*A*d*x + 3*(32*a^3*A*b + 16*a*A*b^3 + 8*a^4*B + 24*a^2*b^2*B + 3*b^4*B)*ArcTanh[Sin[c + d*x]] + 3*b*(8*(6*a^2*A*b + A*b^3 + 4*a^3*B + 4*a*b^2*B) + b*(16*a*A*b + 24*a^2*B + 3*b^2*B)*Sec[c + d*x] + 2*b^3*B*Sec[c + d*x]^3)*Tan[c + d*x] + 8*b^3*(A*b + 4*a*B)*Tan[c + d*x]^3)/(24*d)

fricas [A] time = 0.49, size = 250, normalized size = 1.25

$$\frac{48 Aa^4 dx \cos(dx + c)^4 + 3 \left(8Ba^4 + 32Aa^3b + 24Ba^2b^2 + 16Aab^3 + 3Bb^4 \right) \cos(dx + c)^4 \log(\sin(dx + c) + 1) - 3 \left(8Ba^4 + 32Aa^3b + 24Ba^2b^2 + 16Aab^3 + 3Bb^4 \right) \cos(dx + c)^4 \log(\sin(dx + c) - 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] 1/48*(48*A*a^4*d*x*cos(d*x + c)^4 + 3*(8*B*a^4 + 32*A*a^3*b + 24*B*a^2*b^2 + 16*A*a*b^3 + 3*B*b^4)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(8*B*a^4 + 32*A*a^3*b + 24*B*a^2*b^2 + 16*A*a*b^3 + 3*B*b^4)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(6*B*b^4 + 16*(6*B*a^3*b + 9*A*a^2*b^2 + 4*B*a*b^3 + A*b^4)*cos(d*x + c)^3 + 3*(24*B*a^2*b^2 + 16*A*a*b^3 + 3*B*b^4)*cos(d*x + c)^2 + 8*(4*B*a*b^3 + A*b^4)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^4)

giac [B] time = 2.39, size = 635, normalized size = 3.18

$$24(dx + c)Aa^4 + 3 \left(8Ba^4 + 32Aa^3b + 24Ba^2b^2 + 16Aab^3 + 3Bb^4 \right) \log \left(\left| \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) + 1 \right| \right) - 3 \left(8Ba^4 + 32Aa^3b + 24Ba^2b^2 + 16Aab^3 + 3Bb^4 \right) \log \left(\left| \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) - 1 \right| \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] 1/24*(24*(d*x + c)*A*a^4 + 3*(8*B*a^4 + 32*A*a^3*b + 24*B*a^2*b^2 + 16*A*a*b^3 + 3*B*b^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(8*B*a^4 + 32*A*a^3*b + 24*B*a^2*b^2 + 16*A*a*b^3 + 3*B*b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(96*B*a^3*b*tan(1/2*d*x + 1/2*c)^7 + 144*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 - 72*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 - 48*A*a*b^3*tan(1/2*d*x + 1/2*c)^7 - 3*B*b^4*tan(1/2*d*x + 1/2*c)^7))/(d*cos(d*x + c)^4)

$$7 + 96*B*a*b^3*\tan(1/2*d*x + 1/2*c)^7 + 24*A*b^4*\tan(1/2*d*x + 1/2*c)^7 - 15*B*b^4*\tan(1/2*d*x + 1/2*c)^7 - 288*B*a^3*b*\tan(1/2*d*x + 1/2*c)^5 - 432*A*a^2*b^2*\tan(1/2*d*x + 1/2*c)^5 + 72*B*a^2*b^2*\tan(1/2*d*x + 1/2*c)^5 + 48*A*a*b^3*\tan(1/2*d*x + 1/2*c)^5 - 160*B*a*b^3*\tan(1/2*d*x + 1/2*c)^5 - 40*A*a*b^4*\tan(1/2*d*x + 1/2*c)^5 - 9*B*b^4*\tan(1/2*d*x + 1/2*c)^5 + 288*B*a^3*b*\tan(1/2*d*x + 1/2*c)^3 + 432*A*a^2*b^2*\tan(1/2*d*x + 1/2*c)^3 + 72*B*a^2*b^2*\tan(1/2*d*x + 1/2*c)^3 + 48*A*a*b^3*\tan(1/2*d*x + 1/2*c)^3 + 160*B*a*b^3*\tan(1/2*d*x + 1/2*c)^3 + 40*A*b^4*\tan(1/2*d*x + 1/2*c)^3 - 9*B*b^4*\tan(1/2*d*x + 1/2*c)^3 - 96*B*a^3*b*\tan(1/2*d*x + 1/2*c) - 144*A*a^2*b^2*\tan(1/2*d*x + 1/2*c) - 72*B*a^2*b^2*\tan(1/2*d*x + 1/2*c) - 48*A*a*b^3*\tan(1/2*d*x + 1/2*c) - 96*B*a*b^3*\tan(1/2*d*x + 1/2*c) - 24*A*b^4*\tan(1/2*d*x + 1/2*c) - 15*B*b^4*\tan(1/2*d*x + 1/2*c))/(\tan(1/2*d*x + 1/2*c)^2 - 1)^4/d$$

maple [A] time = 1.38, size = 338, normalized size = 1.69

$$Aa^4x + \frac{Aa^4c}{d} + \frac{a^4B \ln(\sec(dx+c) + \tan(dx+c))}{d} + \frac{4Aa^3b \ln(\sec(dx+c) + \tan(dx+c))}{d} + \frac{4Ba^3b \tan(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x)

[Out] $Aa^4x + 1/dAa^4c + 1/dA^4B \ln(\sec(d*x+c) + \tan(d*x+c)) + 4/dAa^3b \ln(\sec(d*x+c) + \tan(d*x+c)) + 4/dB^3a^3b \tan(d*x+c) + 6/dAa^2b^2 \tan(d*x+c) + 3/dA^2b^2B \sec(d*x+c) \tan(d*x+c) + 3/dA^2b^2B \ln(\sec(d*x+c) + \tan(d*x+c)) + 2/dAa^2b^3 \sec(d*x+c) \tan(d*x+c) + 2/dAa^2b^3 \ln(\sec(d*x+c) + \tan(d*x+c)) + 8/3/dB^3a^3b^3 \tan(d*x+c) + 4/3/dB^3a^3b^3 \tan(d*x+c) \sec(d*x+c)^2 + 2/3/dAa^2b^4 \tan(d*x+c) + 1/3/dAa^2b^4 \tan(d*x+c) \sec(d*x+c)^2 + 1/4/dB^2b^4 \tan(d*x+c) \sec(d*x+c)^3 + 3/8/dB^2b^4 \sec(d*x+c) \tan(d*x+c) + 3/8/dB^2b^4 \ln(\sec(d*x+c) + \tan(d*x+c))$

maxima [A] time = 0.84, size = 303, normalized size = 1.52

$$48(dx+c)Aa^4 + 64(\tan(dx+c)^3 + 3 \tan(dx+c))Bab^3 + 16(\tan(dx+c)^3 + 3 \tan(dx+c))Ab^4 - 3Bb^4 \left(\frac{2(3}{\sin} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] $1/48*(48*(d*x + c)*Aa^4 + 64*(\tan(d*x + c)^3 + 3*\tan(d*x + c))*B^3a^3b^3 + 16*(\tan(d*x + c)^3 + 3*\tan(d*x + c))*A^2b^4 - 3*B^2b^4*(2*(3*\sin(d*x + c)^3 - 5*\sin(d*x + c))/(\sin(d*x + c)^4 - 2*\sin(d*x + c)^2 + 1) - 3*\log(\sin(d*x + c) + 1) + 3*\log(\sin(d*x + c) - 1)) - 72*B^2a^2b^2*(2*\sin(d*x + c))/(\sin(d*x + c)^2 - 1) - \log(\sin(d*x + c) + 1) + \log(\sin(d*x + c) - 1)) - 48*Aa^2b^3*(2*\sin(d*x + c))/(\sin(d*x + c)^2 - 1) - \log(\sin(d*x + c) + 1) + \log(\sin(d*x +$

c) - 1)) + 48*B*a^4*log(sec(d*x + c) + tan(d*x + c)) + 192*A*a^3*b*log(sec(d*x + c) + tan(d*x + c)) + 192*B*a^3*b*tan(d*x + c) + 288*A*a^2*b^2*tan(d*x + c))/d

mupad [B] time = 5.01, size = 1969, normalized size = 9.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^4,x)

[Out] (9*B*a^4*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) + (27*B*b^4*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/8 + 4*A*b^4*sin(2*c + 2*d*x) + A*b^4*sin(4*c + 4*d*x) + (9*B*b^4*sin(3*c + 3*d*x))/8 + 9*A*a^4*atan((64*A^2*a^8*sin(c/2 + (d*x)/2) + 64*B^2*a^8*sin(c/2 + (d*x)/2) + 9*B^2*b^8*sin(c/2 + (d*x)/2) + 256*A^2*a^2*b^6*sin(c/2 + (d*x)/2) + 1024*A^2*a^4*b^4*sin(c/2 + (d*x)/2) + 1024*A^2*a^6*b^2*sin(c/2 + (d*x)/2) + 144*B^2*a^2*b^6*sin(c/2 + (d*x)/2) + 624*B^2*a^4*b^4*sin(c/2 + (d*x)/2) + 384*B^2*a^6*b^2*sin(c/2 + (d*x)/2) + 960*A*B*a^3*b^5*sin(c/2 + (d*x)/2) + 1792*A*B*a^5*b^3*sin(c/2 + (d*x)/2) + 96*A*B*a*b^7*sin(c/2 + (d*x)/2) + 512*A*B*a^7*b*sin(c/2 + (d*x)/2))/((cos(c/2 + (d*x)/2)*(64*A^2*a^8 + 64*B^2*a^8 + 9*B^2*b^8 + 256*A^2*a^2*b^6 + 1024*A^2*a^4*b^4 + 1024*A^2*a^6*b^2 + 144*B^2*a^2*b^6 + 624*B^2*a^4*b^4 + 384*B^2*a^6*b^2 + 96*A*B*a*b^7 + 512*A*B*a^7*b + 960*A*B*a^3*b^5 + 1792*A*B*a^5*b^3))) + (33*B*b^4*sin(c + d*x))/8 + 12*A*a^4*cos(2*c + 2*d*x)*atan((64*A^2*a^8*sin(c/2 + (d*x)/2) + 64*B^2*a^8*sin(c/2 + (d*x)/2) + 9*B^2*b^8*sin(c/2 + (d*x)/2) + 256*A^2*a^2*b^6*sin(c/2 + (d*x)/2) + 1024*A^2*a^4*b^4*sin(c/2 + (d*x)/2) + 1024*A^2*a^6*b^2*sin(c/2 + (d*x)/2) + 144*B^2*a^2*b^6*sin(c/2 + (d*x)/2) + 624*B^2*a^4*b^4*sin(c/2 + (d*x)/2) + 384*B^2*a^6*b^2*sin(c/2 + (d*x)/2) + 960*A*B*a^3*b^5*sin(c/2 + (d*x)/2) + 1792*A*B*a^5*b^3*sin(c/2 + (d*x)/2) + 96*A*B*a*b^7*sin(c/2 + (d*x)/2) + 512*A*B*a^7*b*sin(c/2 + (d*x)/2))/((cos(c/2 + (d*x)/2)*(64*A^2*a^8 + 64*B^2*a^8 + 9*B^2*b^8 + 256*A^2*a^2*b^6 + 1024*A^2*a^4*b^4 + 1024*A^2*a^6*b^2 + 144*B^2*a^2*b^6 + 624*B^2*a^4*b^4 + 384*B^2*a^6*b^2 + 96*A*B*a*b^7 + 512*A*B*a^7*b + 960*A*B*a^3*b^5 + 1792*A*B*a^5*b^3))) + 3*A*a^4*cos(4*c + 4*d*x)*atan((64*A^2*a^8*sin(c/2 + (d*x)/2) + 64*B^2*a^8*sin(c/2 + (d*x)/2) + 9*B^2*b^8*sin(c/2 + (d*x)/2) + 256*A^2*a^2*b^6*sin(c/2 + (d*x)/2) + 1024*A^2*a^4*b^4*sin(c/2 + (d*x)/2) + 1024*A^2*a^6*b^2*sin(c/2 + (d*x)/2) + 144*B^2*a^2*b^6*sin(c/2 + (d*x)/2) + 624*B^2*a^4*b^4*sin(c/2 + (d*x)/2) + 384*B^2*a^6*b^2*sin(c/2 + (d*x)/2) + 960*A*B*a^3*b^5*sin(c/2 + (d*x)/2) + 1792*A*B*a^5*b^3*sin(c/2 + (d*x)/2) + 96*A*B*a*b^7*sin(c/2 + (d*x)/2) + 512*A*B*a^7*b*sin(c/2 + (d*x)/2))/((cos(c/2 + (d*x)/2)*(64*A^2*a^8 + 64*B^2*a^8 + 9*B^2*b^8 + 256*A^2*a^2*b^6 + 1024*A^2*a^4*b^4 + 1024*A^2*a^6*b^2 + 144*B^2*a^2*b^6 + 624*B^2*a^4*b^4 + 384*B^2*a^6*b^2 + 96*A*B*a*b^7 + 512*A*B*a^7*b + 960*A*B*a^3*b^5 + 1792*A*B*a^5*b^3))) + 6*A*a*b^3*sin(c + d*x) + 18*A*a*b^3*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) + 36*A*a^3*b*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) +

```

6*A*a*b^3*sin(3*c + 3*d*x) + 16*B*a*b^3*sin(2*c + 2*d*x) + 12*B*a^3*b*sin(
2*c + 2*d*x) + 4*B*a*b^3*sin(4*c + 4*d*x) + 6*B*a^3*b*sin(4*c + 4*d*x) + 9*
B*a^2*b^2*sin(c + d*x) + 12*B*a^4*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/
2))*cos(2*c + 2*d*x) + 3*B*a^4*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))
*cos(4*c + 4*d*x) + 27*B*a^2*b^2*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2
)) + (9*B*b^4*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(2*c + 2*d*x)
)/2 + (9*B*b^4*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(4*c + 4*d*x
))/8 + 18*A*a^2*b^2*sin(2*c + 2*d*x) + 9*A*a^2*b^2*sin(4*c + 4*d*x) + 9*B*a
^2*b^2*sin(3*c + 3*d*x) + 24*A*a*b^3*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*
x)/2))*cos(2*c + 2*d*x) + 48*A*a^3*b*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*
x)/2))*cos(2*c + 2*d*x) + 6*A*a*b^3*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x
)/2))*cos(4*c + 4*d*x) + 12*A*a^3*b*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x
)/2))*cos(4*c + 4*d*x) + 36*B*a^2*b^2*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d
*x)/2))*cos(2*c + 2*d*x) + 9*B*a^2*b^2*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (
d*x)/2))*cos(4*c + 4*d*x))/(12*d*(cos(2*c + 2*d*x)/2 + cos(4*c + 4*d*x)/8 +
3/8))

```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \sec(c + dx))(a + b \sec(c + dx))^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))*4*(A+B*sec(d*x+c)),x)

[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))*4, x)

$$3.305 \quad \int \cos(c + dx)(a + b \sec(c + dx))^4 (A + B \sec(c + dx)) dx$$

Optimal. Leaf size=195

$$a^3 x(aB + 4Ab) - \frac{b^2 (6a^2 A - 8abB - 3Ab^2) \tan(c + dx) \sec(c + dx)}{6d} - \frac{b (6a^3 A - 17a^2 bB - 12aAb^2 - 2b^3 B) \tan(c + dx)}{3d}$$

[Out] $a^3 x(4A*b + B*a) * x + 1/2 * b * (12 * A * a^2 * b + A * b^3 + 8 * B * a^3 + 4 * B * a * b^2) * \operatorname{arctanh}(\sin(d * x + c)) / d + a * A * (a + b * \sec(d * x + c))^3 * \sin(d * x + c) / d - 1/3 * b * (6 * A * a^3 - 12 * A * a * b^2 - 17 * B * a^2 * b - 2 * B * b^3) * \tan(d * x + c) / d - 1/6 * b^2 * (6 * A * a^2 - 3 * A * b^2 - 8 * B * a * b) * \sec(d * x + c) * \tan(d * x + c) / d - 1/3 * b * (3 * A * a - B * b) * (a + b * \sec(d * x + c))^2 * \tan(d * x + c) / d$

Rubi [A] time = 0.37, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {4025, 4056, 4048, 3770, 3767, 8}

$$\frac{b (6a^3 A - 17a^2 bB - 12aAb^2 - 2b^3 B) \tan(c + dx)}{3d} + \frac{b (12a^2 Ab + 8a^3 B + 4ab^2 B + Ab^3) \tanh^{-1}(\sin(c + dx))}{2d} - \frac{b^2 (6a^3 A - 17a^2 bB - 12aAb^2 - 2b^3 B) \tan(c + dx)}{3d}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]

[Out] $a^3 x(4A*b + a*B) * x + (b * (12 * a^2 * A * b + A * b^3 + 8 * a^3 * B + 4 * a * b^2 * B) * \operatorname{ArcTanh}[\sin[c + d * x]]) / (2 * d) + (a * A * (a + b * \sec[c + d * x])^3 * \sin[c + d * x]) / d - (b * (6 * a^3 * A - 12 * a * A * b^2 - 17 * a^2 * b * B - 2 * b^3 * B) * \tan[c + d * x]) / (3 * d) - (b^2 * (6 * a^2 * A - 3 * A * b^2 - 8 * a * b * B) * \sec[c + d * x] * \tan[c + d * x]) / (6 * d) - (b * (3 * a * A - b * B) * (a + b * \sec[c + d * x])^2 * \tan[c + d * x]) / (3 * d)$

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 3767

Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (
a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> Simp[(a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
LeQ[n, -1]
```

Rule 4048

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] :> -Simp[(b*C*Csc[e +
f*x]*Cot[e + f*x])/(2*f), x] + Dist[1/2, Int[Simp[2*A*a + (2*B*a + b*(2*A +
C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, e, f, A, B, C}, x]
```

Rule 4056

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.), x_Symbol] :> -Simp[(C*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[(a +
b*Csc[e + f*x])^(m - 1)*Simp[a*A*(m + 1) + ((A*b + a*B)*(m + 1) + b*C*m)*C
sc[e + f*x] + (b*B*(m + 1) + a*C*m)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && IGtQ[2*m, 0]
```

Rubi steps

$$\begin{aligned}
\int \cos(c + dx)(a + b \sec(c + dx))^4(A + B \sec(c + dx)) dx &= \frac{aA(a + b \sec(c + dx))^3 \sin(c + dx)}{d} - \int (a + b \sec(c + dx))^4(A + B \sec(c + dx)) dx \\
&= \frac{aA(a + b \sec(c + dx))^3 \sin(c + dx)}{d} - \frac{b(3aA - bB)(a + b \sec(c + dx))^4}{d} \\
&= \frac{aA(a + b \sec(c + dx))^3 \sin(c + dx)}{d} - \frac{b^2(6a^2A - 3Ab^2)}{d} \\
&= a^3(4Ab + aB)x + \frac{aA(a + b \sec(c + dx))^3 \sin(c + dx)}{d} \\
&= a^3(4Ab + aB)x + \frac{b(12a^2Ab + Ab^3 + 8a^3B + 4ab^2B)}{2d} \\
&= a^3(4Ab + aB)x + \frac{b(12a^2Ab + Ab^3 + 8a^3B + 4ab^2B)}{2d}
\end{aligned}$$

Mathematica [B] time = 6.29, size = 1051, normalized size = 5.39

$$\frac{(-Ab^4 - 4aBb^3 - 12a^2Ab^2 - 8a^3Bb) \log\left(\cos\left(\frac{1}{2}(c + dx)\right) - \sin\left(\frac{1}{2}(c + dx)\right)\right) (a + b \sec(c + dx))^4 (A + B \sec(c + dx))}{2d(b + a \cos(c + dx))^4 (B + A \cos(c + dx))}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]

[Out] (a^3*(4*A*b + a*B)*(c + d*x)*Cos[c + d*x]^5*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x]))/(d*(b + a*Cos[c + d*x])^4*(B + A*Cos[c + d*x])) + ((-12*a^2*A*b^2 - A*b^4 - 8*a^3*b*B - 4*a*b^3*B)*Cos[c + d*x]^5*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x]))/(2*d*(b + a*Cos[c + d*x])^4*(B + A*Cos[c + d*x])) + ((12*a^2*A*b^2 + A*b^4 + 8*a^3*b*B + 4*a*b^3*B)*Cos[c + d*x]^5*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x]))/(2*d*(b + a*Cos[c + d*x])^4*(B + A*Cos[c + d*x])) + ((3*A*b^4 + 12*a*b^3*B + b^4*B)*Cos[c + d*x]^5*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x]))/(12*d*(b + a*Cos[c + d*x])^4*(B + A*Cos[c + d*x]))*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + (b^4*B*Cos[c + d*x]^5*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x]))*Sin[(c + d*x)/2]/(6*d*(b + a*Cos[c + d*x])^4*(B + A*Cos[c + d*x]))*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3 + (b^4*B*Cos[c + d*x]^5*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x]))*Sin[(c + d*x)/2]/(6*d*(b + a*Cos[c + d*x])^4*(B + A*Cos[c + d*x]))*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3 + ((-3*A*b^4 - 12*a*b^3*B - b^4*B)*Cos[c + d*x]^5*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x]))/(12*d*(b + a*Cos[c + d*x])^4

$$\begin{aligned} &*(B + A*\cos[c + d*x])*(\cos[(c + d*x)/2] + \sin[(c + d*x)/2])^2) + (2*\cos[c + \\ &d*x]^5*(a + b*\sec[c + d*x])^4*(A + B*\sec[c + d*x])*(6*a*A*b^3*\sin[(c + d*x) \\ &)/2] + 9*a^2*b^2*B*\sin[(c + d*x)/2] + b^4*B*\sin[(c + d*x)/2]))/(3*d*(b + a* \\ &\cos[c + d*x])^4*(B + A*\cos[c + d*x])*(\cos[(c + d*x)/2] - \sin[(c + d*x)/2])) \\ &+ (2*\cos[c + d*x]^5*(a + b*\sec[c + d*x])^4*(A + B*\sec[c + d*x])*(6*a*A*b^3 \\ &*\sin[(c + d*x)/2] + 9*a^2*b^2*B*\sin[(c + d*x)/2] + b^4*B*\sin[(c + d*x)/2])) \\ &/(3*d*(b + a*\cos[c + d*x])^4*(B + A*\cos[c + d*x])*(\cos[(c + d*x)/2] + \sin[(c \\ &+ d*x)/2])) + (a^4*A*\cos[c + d*x]^5*(a + b*\sec[c + d*x])^4*(A + B*\sec[c + \\ &d*x])* \sin[c + d*x])/(d*(b + a*\cos[c + d*x])^4*(B + A*\cos[c + d*x])) \end{aligned}$$

fricas [A] time = 0.48, size = 219, normalized size = 1.12

$$\frac{12(Ba^4 + 4Aa^3b)dx \cos(dx + c)^3 + 3(8Ba^3b + 12Aa^2b^2 + 4Bab^3 + Ab^4) \cos(dx + c)^3 \log(\sin(dx + c) + 1) - \dots}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] 1/12*(12*(B*a^4 + 4*A*a^3*b)*d*x*cos(d*x + c)^3 + 3*(8*B*a^3*b + 12*A*a^2*b^2 + 4*B*a*b^3 + A*b^4)*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*(8*B*a^3*b + 12*A*a^2*b^2 + 4*B*a*b^3 + A*b^4)*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + 2*(6*A*a^4*cos(d*x + c)^3 + 2*B*b^4 + 4*(9*B*a^2*b^2 + 6*A*a*b^3 + B*b^4)*cos(d*x + c)^2 + 3*(4*B*a*b^3 + A*b^4)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)

giac [B] time = 0.37, size = 387, normalized size = 1.98

$$\frac{12Aa^4 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)}{\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + 1} + 6(Ba^4 + 4Aa^3b)(dx + c) + 3(8Ba^3b + 12Aa^2b^2 + 4Bab^3 + Ab^4) \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right| + 1\right) + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] 1/6*(12*A*a^4*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 + 1) + 6*(B*a^4 + 4*A*a^3*b)*(d*x + c) + 3*(8*B*a^3*b + 12*A*a^2*b^2 + 4*B*a*b^3 + A*b^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(8*B*a^3*b + 12*A*a^2*b^2 + 4*B*a*b^3 + A*b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(36*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 + 24*A*a*b^3*tan(1/2*d*x + 1/2*c)^5 - 12*B*a*b^3*tan(1/2*d*x + 1/2*c)^5 - 3*A*b^4*tan(1/2*d*x + 1/2*c)^5 + 6*B*b^4*tan(1/2*d*x + 1/2*c)^5 - \dots)

$$\frac{72Ba^2b^2\tan(1/2dx + 1/2c)^3 - 48Aab^3\tan(1/2dx + 1/2c)^3 - 4Bb^4\tan(1/2dx + 1/2c)^3 + 36Ba^2b^2\tan(1/2dx + 1/2c) + 24Aab^3\tan(1/2dx + 1/2c) + 12Bba^3\tan(1/2dx + 1/2c) + 3Ab^4\tan(1/2dx + 1/2c) + 6Bb^4\tan(1/2dx + 1/2c)}{(\tan(1/2dx + 1/2c)^2 - 1)^3/d}$$

maple [A] time = 1.48, size = 262, normalized size = 1.34

$$\frac{Aa^4 \sin(dx+c)}{d} + a^4 Bx + \frac{a^4 Bc}{d} + 4Aa^3bx + \frac{4Aa^3bc}{d} + \frac{4Ba^3b \ln(\sec(dx+c) + \tan(dx+c))}{d} + \frac{6Aa^2b^2 \ln(\sec(dx+c) + \tan(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(dx+c)*(a+b*sec(dx+c))^4*(A+B*sec(dx+c)),x)

[Out] $\frac{1}{d}Aa^4\sin(dx+c) + a^4Bx + \frac{1}{d}a^4Bc + 4Aa^3bx + \frac{4}{d}Aa^3bc + \frac{4}{d}Ba^3b \ln(\sec(dx+c) + \tan(dx+c)) + \frac{6}{d}Aa^2b^2 \ln(\sec(dx+c) + \tan(dx+c)) + \frac{6}{d}Aa^2b^2B \tan(dx+c) + \frac{4}{d}a^4A \tan(dx+c) + \frac{2}{d}Bba^3 \sec(dx+c) \tan(dx+c) + \frac{1}{2}dAab^4 \sec(dx+c) \tan(dx+c) + \frac{1}{2}dAab^4 \ln(\sec(dx+c) + \tan(dx+c)) + \frac{2}{3}dBb^4 \tan(dx+c) + \frac{1}{3}dBb^4 \tan(dx+c) \sec(dx+c)^2$

maxima [A] time = 0.63, size = 245, normalized size = 1.26

$$\frac{12(dx+c)Ba^4 + 48(dx+c)Aa^3b + 4(\tan(dx+c)^3 + 3 \tan(dx+c))Bb^4 - 12Bab^3 \left(\frac{2 \sin(dx+c)}{\sin(dx+c)^2 - 1} - \log(\sin(dx+c) + 1) + \log(\sin(dx+c) - 1) \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(dx+c)*(a+b*sec(dx+c))^4*(A+B*sec(dx+c)),x, algorithm="maxima")

[Out] $\frac{1}{12} \left(12(dx+c)Ba^4 + 48(dx+c)Aa^3b + 4(\tan(dx+c)^3 + 3 \tan(dx+c))Bb^4 - 12Bab^3 \left(\frac{2 \sin(dx+c)}{\sin(dx+c)^2 - 1} - \log(\sin(dx+c) + 1) + \log(\sin(dx+c) - 1) \right) - 3Aab^4 \left(\frac{2 \sin(dx+c)}{\sin(dx+c)^2 - 1} - \log(\sin(dx+c) + 1) + \log(\sin(dx+c) - 1) \right) + 24Bba^3b \left(\log(\sin(dx+c) + 1) - \log(\sin(dx+c) - 1) \right) + 36Aa^2b^2 \left(\log(\sin(dx+c) + 1) - \log(\sin(dx+c) - 1) \right) + 12Aa^4 \sin(dx+c) + 72Ba^2b^2 \tan(dx+c) + 48Aa^3b \tan(dx+c) \right) / d$

mupad [B] time = 4.90, size = 636, normalized size = 3.26

$$\frac{Aa^4 \sin(2c+2dx)}{4} + \frac{Aa^4 \sin(4c+4dx)}{8} + \frac{Ab^4 \sin(2c+2dx)}{4} + \frac{Bb^4 \sin(3c+3dx)}{6} + \frac{Bb^4 \sin(c+dx)}{2} + Aab^3 \sin(c+dx) + \frac{3Ba^4 \cos(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(c + d*x)*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^4,x)`

[Out]
$$\begin{aligned} & ((Aa^4\sin(2c + 2dx))/4 + (Aa^4\sin(4c + 4dx))/8 + (Ab^4\sin(2c + 2dx))/4 + (Bb^4\sin(3c + 3dx))/6 + (Bb^4\sin(c + dx))/2 + Aab^3\sin(c + dx) + (3Ba^4\cos(c + dx)*\operatorname{atan}(\sin(c/2 + (dx)/2)/\cos(c/2 + (dx)/2)))/2 - (Ab^4\cos(c + dx)*\operatorname{atan}((\sin(c/2 + (dx)/2)*i)/\cos(c/2 + (dx)/2))*3i)/4 + Aab^3\sin(3c + 3dx) + Baa^3\sin(2c + 2dx) + (3Ba^2b^2\sin(c + dx))/2 + (Ba^4*\operatorname{atan}(\sin(c/2 + (dx)/2)/\cos(c/2 + (dx)/2))*\cos(3c + 3dx))/2 - (Ab^4*\operatorname{atan}((\sin(c/2 + (dx)/2)*i)/\cos(c/2 + (dx)/2))*\cos(3c + 3dx)*i)/4 + (3Ba^2b^2\sin(3c + 3dx))/2 + 2Aa^3b*\operatorname{atan}(\sin(c/2 + (dx)/2)/\cos(c/2 + (dx)/2))*\cos(3c + 3dx) - Aa^2b^2*\cos(c + dx)*\operatorname{atan}((\sin(c/2 + (dx)/2)*i)/\cos(c/2 + (dx)/2))*9i - Baa^3*\operatorname{atan}((\sin(c/2 + (dx)/2)*i)/\cos(c/2 + (dx)/2))*\cos(3c + 3dx)*i - Baa^3b*\operatorname{atan}((\sin(c/2 + (dx)/2)*i)/\cos(c/2 + (dx)/2))*\cos(3c + 3dx)*2i - Aa^2b^2*\operatorname{atan}((\sin(c/2 + (dx)/2)*i)/\cos(c/2 + (dx)/2))*\cos(3c + 3dx)*3i + 6Aa^3b*\cos(c + dx)*\operatorname{atan}(\sin(c/2 + (dx)/2)/\cos(c/2 + (dx)/2)) - Baa^3b^3*\cos(c + dx)*\operatorname{atan}((\sin(c/2 + (dx)/2)*i)/\cos(c/2 + (dx)/2))*3i - Baa^3b*\cos(c + dx)*\operatorname{atan}((\sin(c/2 + (dx)/2)*i)/\cos(c/2 + (dx)/2))*6i)/(d*((3*\cos(c + dx))/4 + \cos(3c + 3dx)/4)) \end{aligned}$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \sec(c + dx))(a + b \sec(c + dx))^4 \cos(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)*(a+b*sec(d*x+c))**4*(A+B*sec(d*x+c)),x)`

[Out] `Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**4*cos(c + d*x), x)`

$$3.306 \quad \int \cos^2(c+dx)(a+b \sec(c+dx))^4(A+B \sec(c+dx)) dx$$

Optimal. Leaf size=209

$$\frac{b^2(12a^2B + 8aAb + b^2B) \tanh^{-1}(\sin(c+dx))}{2d} - \frac{b^2(2a^2B + 6aAb - b^2B) \tan(c+dx) \sec(c+dx)}{2d} + \frac{1}{2}a^2x(a^2A + 8aAb + 8B^2)$$

[Out] $\frac{1}{2}a^2x(Aa^2 + 12Aab + 8B^2) + \frac{1}{2}b^2(8Aab + 12B^2) \operatorname{arctanh}(\sin(dx+c))/d + \frac{1}{2}a(5Ab + 2B^2)(a+b \sec(dx+c))^2 \sin(dx+c)/d + \frac{1}{2}aA \cos(dx+c)(a+b \sec(dx+c))^3 \sin(dx+c)/d - \frac{1}{2}b(13Aa^2b - 2Ab^3 + 4B^2a^3 - 8B^2ab^2) \tan(dx+c)/d - \frac{1}{2}b^2(6Aab + 2B^2 - B^2) \sec(dx+c) \tan(dx+c)/d$

Rubi [A] time = 0.46, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4025, 4094, 4048, 3770, 3767, 8}

$$-\frac{b(13a^2Ab + 4a^3B - 8ab^2B - 2Ab^3) \tan(c+dx)}{2d} + \frac{b^2(12a^2B + 8aAb + b^2B) \tanh^{-1}(\sin(c+dx))}{2d} - \frac{b^2(2a^2B + 6aAb - b^2B) \tan(c+dx) \sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Cos}[c + dx]^2(a + b \operatorname{Sec}[c + dx])^4(A + B \operatorname{Sec}[c + dx]), x]$

[Out] $(a^2(a^2A + 12Aab + 8a^2B) + b^2(8Aab + 12a^2B + b^2B)) \operatorname{ArcTanh}[\operatorname{Sin}[c + dx]]/(2d) + (a(5Ab + 2B^2)(a + b \operatorname{Sec}[c + dx])^2 \operatorname{Sin}[c + dx])/(2d) + (aA \operatorname{Cos}[c + dx](a + b \operatorname{Sec}[c + dx])^3 \operatorname{Sin}[c + dx])/(2d) - (b(13a^2Ab - 2Ab^3 + 4a^3B - 8a^2Bb) \operatorname{Tan}[c + dx])/(2d) - (b^2(6Aab + 2a^2B - b^2B) \operatorname{Sec}[c + dx] \operatorname{Tan}[c + dx])/(2d)$

Rule 8

$\operatorname{Int}[a_, x_Symbol] \rightarrow \operatorname{Simp}[a*x, x] /; \operatorname{FreeQ}[a, x]$

Rule 3767

$\operatorname{Int}[\operatorname{csc}[(c_.) + (d_.)*(x_)]^{(n_)}, x_Symbol] \rightarrow -\operatorname{Dist}[d^{(-1)}, \operatorname{Subst}[\operatorname{Int}[\operatorname{ExpandIntegrand}[(1 + x^2)^{(n/2 - 1)}, x], x], x, \operatorname{Cot}[c + dx]], x] /; \operatorname{FreeQ}[\{c, d\}, x] \ \&\amp; \operatorname{IGtQ}[n/2, 0]$

Rule 3770

$\operatorname{Int}[\operatorname{csc}[(c_.) + (d_.)*(x_)], x_Symbol] \rightarrow -\operatorname{Simp}[\operatorname{ArcTanh}[\operatorname{Cos}[c + dx]]/d, x] /; \operatorname{FreeQ}[\{c, d\}, x]$

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (
a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
LeQ[n, -1]
```

Rule 4048

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] := -Simp[(b*C*Csc[e +
f*x]*Cot[e + f*x])/(2*f), x] + Dist[1/2, Int[Simp[2*A*a + (2*B*a + b*(2*A +
C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, e, f, A, B, C}, x]
```

Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a
_))^(m_.), x_Symbol] := Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
+ f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \cos^2(c + dx)(a + b \sec(c + dx))^4(A + B \sec(c + dx)) dx &= \frac{aA \cos(c + dx)(a + b \sec(c + dx))^3 \sin(c + dx)}{2d} - \frac{1}{2} \int \frac{aA \cos^2(c + dx)(a + b \sec(c + dx))^4(A + B \sec(c + dx)) dx}{\cos(c + dx)} \\
&= \frac{a(5Ab + 2aB)(a + b \sec(c + dx))^2 \sin(c + dx)}{2d} + \frac{aA \cos(c + dx)(a + b \sec(c + dx))^3 \sin(c + dx)}{2d} \\
&= \frac{a(5Ab + 2aB)(a + b \sec(c + dx))^2 \sin(c + dx)}{2d} + \frac{aA \cos(c + dx)(a + b \sec(c + dx))^3 \sin(c + dx)}{2d} \\
&= \frac{1}{2} a^2 (a^2 A + 12Ab^2 + 8abB) x + \frac{a(5Ab + 2aB)(a + b \sec(c + dx))^2 \sin(c + dx)}{2d} \\
&= \frac{1}{2} a^2 (a^2 A + 12Ab^2 + 8abB) x + \frac{b^2 (8aAb + 12a^2 B + 8abB) \sin(c + dx)}{2d} \\
&= \frac{1}{2} a^2 (a^2 A + 12Ab^2 + 8abB) x + \frac{b^2 (8aAb + 12a^2 B + 8abB) \sin(c + dx)}{2d}
\end{aligned}$$

Mathematica [A] time = 2.01, size = 310, normalized size = 1.48

$$a^4 A \sin(2(c + dx)) + 4a^3(aB + 4Ab) \sin(c + dx) + 2a^2(c + dx)(a^2 A + 8abB + 12Ab^2) - 2b^2(12a^2 B + 8aAb + b^2)$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]

[Out] (2*a^2*(a^2*A + 12*A*b^2 + 8*a*b*B)*(c + d*x) - 2*b^2*(8*a*A*b + 12*a^2*B + b^2*B)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 2*b^2*(8*a*A*b + 12*a^2*B + B + b^2*B)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (b^4*B)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + (4*b^3*(A*b + 4*a*B)*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) - (b^4*B)/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + (4*b^3*(A*b + 4*a*B)*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) + 4*a^3*(4*A*b + a*B)*Sin[c + d*x] + a^4*A*Sin[2*(c + d*x)]/(4*d)

fricas [A] time = 0.49, size = 202, normalized size = 0.97

$$2 \left(Aa^4 + 8Ba^3b + 12Aa^2b^2 \right) dx \cos(dx + c)^2 + \left(12Ba^2b^2 + 8Aab^3 + Bb^4 \right) \cos(dx + c)^2 \log(\sin(dx + c) + 1) - \left(\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] $\frac{1}{4} \cdot (2 \cdot (A \cdot a^4 + 8 \cdot B \cdot a^3 \cdot b + 12 \cdot A \cdot a^2 \cdot b^2) \cdot d \cdot x \cdot \cos(d \cdot x + c)^2 + (12 \cdot B \cdot a^2 \cdot b^2 + 8 \cdot A \cdot a \cdot b^3 + B \cdot b^4) \cdot \cos(d \cdot x + c)^2 \cdot \log(\sin(d \cdot x + c) + 1) - (12 \cdot B \cdot a^2 \cdot b^2 + 8 \cdot A \cdot a \cdot b^3 + B \cdot b^4) \cdot \cos(d \cdot x + c)^2 \cdot \log(-\sin(d \cdot x + c) + 1) + 2 \cdot (A \cdot a^4 \cdot \cos(d \cdot x + c)^3 + B \cdot b^4 + 2 \cdot (B \cdot a^4 + 4 \cdot A \cdot a^3 \cdot b) \cdot \cos(d \cdot x + c)^2 + 2 \cdot (4 \cdot B \cdot a \cdot b^3 + A \cdot b^4) \cdot \cos(d \cdot x + c)) \cdot \sin(d \cdot x + c)) / (d \cdot \cos(d \cdot x + c)^2)$

giac [B] time = 2.11, size = 528, normalized size = 2.53

$$(Aa^4 + 8Ba^3b + 12Aa^2b^2)(dx + c) + (12Ba^2b^2 + 8Aab^3 + Bb^4) \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right|\right) - (12Ba^2b^2 + 8A$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] $\frac{1}{2} \cdot ((A \cdot a^4 + 8 \cdot B \cdot a^3 \cdot b + 12 \cdot A \cdot a^2 \cdot b^2) \cdot (d \cdot x + c) + (12 \cdot B \cdot a^2 \cdot b^2 + 8 \cdot A \cdot a \cdot b^3 + B \cdot b^4) \cdot \log(\text{abs}(\tan(1/2 \cdot d \cdot x + 1/2 \cdot c) + 1)) - (12 \cdot B \cdot a^2 \cdot b^2 + 8 \cdot A \cdot a \cdot b^3 + B \cdot b^4) \cdot \log(\text{abs}(\tan(1/2 \cdot d \cdot x + 1/2 \cdot c) - 1)) - 2 \cdot (A \cdot a^4 \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^7 - 2 \cdot B \cdot a^4 \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^7 - 8 \cdot A \cdot a^3 \cdot b \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^7 + 8 \cdot B \cdot a \cdot b^3 \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^7 + 2 \cdot A \cdot b^4 \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^7 - B \cdot b^4 \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^7 - 3 \cdot A \cdot a^4 \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^5 + 2 \cdot B \cdot a^4 \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^5 + 8 \cdot A \cdot a^3 \cdot b \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^5 + 8 \cdot B \cdot a \cdot b^3 \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^5 + 2 \cdot A \cdot b^4 \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^5 - 3 \cdot B \cdot b^4 \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^5 + 3 \cdot A \cdot a^4 \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^3 + 2 \cdot B \cdot a^4 \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^3 + 8 \cdot A \cdot a^3 \cdot b \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^3 - 8 \cdot B \cdot a \cdot b^3 \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^3 - 2 \cdot A \cdot b^4 \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^3 - 3 \cdot B \cdot b^4 \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^3 - A \cdot a^4 \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c) - 2 \cdot B \cdot a^4 \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c) - 8 \cdot A \cdot a^3 \cdot b \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c) - 8 \cdot B \cdot a \cdot b^3 \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c) - 2 \cdot A \cdot b^4 \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c) - B \cdot b^4 \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)) / (\tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^4 - 1)^2 / d$

maple [A] time = 1.07, size = 236, normalized size = 1.13

$$\frac{Aa^4 \cos(dx + c) \sin(dx + c)}{2d} + \frac{Aa^4 x}{2} + \frac{Aa^4 c}{2d} + \frac{a^4 B \sin(dx + c)}{d} + \frac{4Aa^3 b \sin(dx + c)}{d} + 4Bx a^3 b + \frac{4B a^3 b c}{d} + 6Ax a^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^2*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x)

[Out] $\frac{1}{2} \cdot d \cdot A \cdot a^4 \cdot \cos(d \cdot x + c) \cdot \sin(d \cdot x + c) + \frac{1}{2} \cdot A \cdot a^4 \cdot x + \frac{1}{2} \cdot d \cdot A \cdot a^4 \cdot c + \frac{1}{d} \cdot a^4 \cdot B \cdot \sin(d \cdot x + c) + \frac{4}{d} \cdot A \cdot a^3 \cdot b \cdot \sin(d \cdot x + c) + 4 \cdot B \cdot x \cdot a^3 \cdot b + \frac{4}{d} \cdot B \cdot a^3 \cdot b \cdot c + 6 \cdot A \cdot x \cdot a^2 \cdot b^2 + \frac{6}{d} \cdot A \cdot a^2 \cdot b^2 \cdot c + \frac{6}{d} \cdot a^2 \cdot b^2 \cdot B \cdot \ln(\sec(d \cdot x + c) + \tan(d \cdot x + c)) + \frac{4}{d} \cdot a \cdot A \cdot b^3 \cdot \ln(\sec(d \cdot x + c))$

$+\tan(dx+c))+4/d*B*a*b^3*\tan(dx+c)+1/d*A*b^4*\tan(dx+c)+1/2/d*B*b^4*\sec(dx+c)*\tan(dx+c)+1/2/d*B*b^4*\ln(\sec(dx+c)+\tan(dx+c))$

maxima [A] time = 0.72, size = 209, normalized size = 1.00

$$(2dx + 2c + \sin(2dx + 2c))Aa^4 + 16(dx + c)Ba^3b + 24(dx + c)Aa^2b^2 - Bb^4 \left(\frac{2 \sin(dx+c)}{\sin(dx+c)^2 - 1} - \log(\sin(dx + c) + 1) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(dx+c)^2*(a+b*sec(dx+c))^4*(A+B*sec(dx+c)),x, algorithm="maxima")

[Out] $\frac{1}{4} * ((2dx + 2c + \sin(2dx + 2c)) * Aa^4 + 16(dx + c) * Ba^3b + 24(dx + c) * Aa^2b^2 - Bb^4 * (2 * \sin(dx + c) / (\sin(dx + c)^2 - 1) - \log(\sin(dx + c) + 1) + \log(\sin(dx + c) - 1)) + 12 * Ba^2b^2 * (\log(\sin(dx + c) + 1) - \log(\sin(dx + c) - 1)) + 8 * Aa * b^3 * (\log(\sin(dx + c) + 1) - \log(\sin(dx + c) - 1)) + 4 * Ba^4 * \sin(dx + c) + 16 * Aa^3 * b * \sin(dx + c) + 16 * Ba * b^3 * \tan(dx + c) + 4 * A * b^4 * \tan(dx + c)) / d$

mupad [B] time = 4.39, size = 330, normalized size = 1.58

$$2 \left(\frac{Aa^4 \operatorname{atan}\left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)}\right)}{2} + \frac{Bb^4 \operatorname{atanh}\left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)}\right)}{2} + 4Aa^3b \operatorname{atanh}\left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)}\right) + 4Ba^3b \operatorname{atan}\left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)}\right) + 6Aa^2b^2 \operatorname{atan}\left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)}\right) \right) / d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + dx)^2*(A + B/cos(c + dx))*(a + b/cos(c + dx))^4,x)

[Out] $(2 * ((Aa^4 * \operatorname{atan}(\sin(c/2 + (dx)/2) / \cos(c/2 + (dx)/2))) / 2 + (Bb^4 * \operatorname{atanh}(\sin(c/2 + (dx)/2) / \cos(c/2 + (dx)/2))) / 2 + 4 * Aa^3 * b * \operatorname{atanh}(\sin(c/2 + (dx)/2) / \cos(c/2 + (dx)/2)) + 4 * Ba^3 * b * \operatorname{atan}(\sin(c/2 + (dx)/2) / \cos(c/2 + (dx)/2)) + 6 * Aa^2 * b^2 * \operatorname{atan}(\sin(c/2 + (dx)/2) / \cos(c/2 + (dx)/2)) + 6 * Ba^2 * b^2 * \operatorname{atanh}(\sin(c/2 + (dx)/2) / \cos(c/2 + (dx)/2)))) / d + ((Aa^4 * \sin(2c + 2dx)) / 8 + (Aa^4 * \sin(4c + 4dx)) / 16 + (Ab^4 * \sin(2c + 2dx)) / 2 + (Ba^4 * \sin(3c + 3dx)) / 4 + (Ba^4 * \sin(c + dx)) / 4 + (Bb^4 * \sin(c + dx)) / 2 + Aa^3 * b * \sin(c + dx) + Aa^3 * b * \sin(3c + 3dx) + 2 * Ba * b^3 * \sin(2c + 2dx)) / (d * (\cos(2c + 2dx) / 2 + 1/2))$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*sec(d*x+c))**4*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

$$3.307 \quad \int \cos^3(c+dx)(a+b \sec(c+dx))^4(A+B \sec(c+dx)) dx$$

Optimal. Leaf size=198

$$\frac{a^2(2a^2A + 9abB + 9Ab^2) \sin(c+dx)}{3d} - \frac{b^2(3a^2B + 8aAb - 6b^2B) \tan(c+dx)}{6d} + \frac{1}{2}ax(a^3B + 4a^2Ab + 12ab^2B + 8A$$

[Out] $\frac{1}{2}a*(4*A*a^2*b+8*A*b^3+B*a^3+12*B*a*b^2)*x+b^3*(A*b+4*B*a)*\operatorname{arctanh}(\sin(d*x+c))/d+1/3*a^2*(2*A*a^2+9*A*b^2+9*B*a*b)*\sin(d*x+c)/d+1/2*a*(2*A*b+B*a)*\cos(d*x+c)*(a+b*\sec(d*x+c))^2*\sin(d*x+c)/d+1/3*a*A*\cos(d*x+c)^2*(a+b*\sec(d*x+c))^3*\sin(d*x+c)/d-1/6*b^2*(8*A*a*b+3*B*a^2-6*B*b^2)*\tan(d*x+c)/d$

Rubi [A] time = 0.59, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {4025, 4094, 4076, 4047, 8, 4045, 3770}

$$\frac{a^2(2a^2A + 9abB + 9Ab^2) \sin(c+dx)}{3d} - \frac{b^2(3a^2B + 8aAb - 6b^2B) \tan(c+dx)}{6d} + \frac{1}{2}ax(4a^2Ab + a^3B + 12ab^2B + 8A$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Cos}[c+d*x]^3*(a+b*\operatorname{Sec}[c+d*x])^4*(A+B*\operatorname{Sec}[c+d*x]),x]$

[Out] $(a*(4*a^2*A*b + 8*A*b^3 + a^3*B + 12*a*b^2*B)*x)/2 + (b^3*(A*b + 4*a*B)*\operatorname{ArcTanh}[\operatorname{Sin}[c+d*x]])/d + (a^2*(2*a^2*A + 9*A*b^2 + 9*a*b*B)*\operatorname{Sin}[c+d*x])/(3*d) + (a*(2*A*b + a*B)*\operatorname{Cos}[c+d*x]*(a+b*\operatorname{Sec}[c+d*x])^2*\operatorname{Sin}[c+d*x])/(2*d) + (a*A*\operatorname{Cos}[c+d*x]^2*(a+b*\operatorname{Sec}[c+d*x])^3*\operatorname{Sin}[c+d*x])/(3*d) - (b^2*(8*a*A*b + 3*a^2*B - 6*b^2*B)*\operatorname{Tan}[c+d*x])/(6*d)$

Rule 8

$\operatorname{Int}[a_, x_Symbol] \rightarrow \operatorname{Simp}[a*x, x] /; \operatorname{FreeQ}[a, x]$

Rule 3770

$\operatorname{Int}[\operatorname{csc}[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow -\operatorname{Simp}[\operatorname{ArcTanh}[\operatorname{Cos}[c+d*x]]/d, x] /; \operatorname{FreeQ}\{c, d\}, x]$

Rule 4025

$\operatorname{Int}[(\operatorname{csc}[(e_.) + (f_.)*(x_.)]*(d_.))^n*(\operatorname{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m*(\operatorname{csc}[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] \rightarrow \operatorname{Simp}[(a*A*\operatorname{Cot}[e+f*x]*(a+b*\operatorname{Csc}[e+f*x])^{m-1}*(d*\operatorname{Csc}[e+f*x])^n)/(f*n), x] + \operatorname{Dist}[1/(d*n), \operatorname{Int}[(a+b*\operatorname{Csc}[e+f*x])^{m-2}*(d*\operatorname{Csc}[e+f*x])^{n+1}]*\operatorname{Simp}[a$

```

*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
LeQ[n, -1]

```

Rule 4045

```

Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]^2*(C_.)
+ (A_.)), x_Symbol] := Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]

```

Rule 4047

```

Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_.)]*
(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.)), x_Symbol] := Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
x] /; FreeQ[{b, e, f, A, B, C, m}, x]

```

Rule 4076

```

Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.)
)*csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (
a_.)), x_Symbol] := -Simp[(b*C*Csc[e + f*x]*Cot[e + f*x]*(d*Csc[e + f*x])^n)
/(f*(n + 2)), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2)
+ (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n +
2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &&
!LtQ[n, -1]

```

Rule 4094

```

Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.)
)*csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a
_.))^(m_.), x_Symbol] := Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
+ f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]

```

Rubi steps

$$\begin{aligned}
\int \cos^3(c + dx)(a + b \sec(c + dx))^4(A + B \sec(c + dx)) dx &= \frac{aA \cos^2(c + dx)(a + b \sec(c + dx))^3 \sin(c + dx)}{3d} - \frac{1}{3} \\
&= \frac{a(2Ab + aB) \cos(c + dx)(a + b \sec(c + dx))^2 \sin(c + dx)}{2d} \\
&= \frac{a(2Ab + aB) \cos(c + dx)(a + b \sec(c + dx))^2 \sin(c + dx)}{2d} \\
&= \frac{a(2Ab + aB) \cos(c + dx)(a + b \sec(c + dx))^2 \sin(c + dx)}{2d} \\
&= \frac{1}{2}a(4a^2Ab + 8Ab^3 + a^3B + 12ab^2B)x + \frac{a^2(2a^2A + 3a^2B)}{2d} \\
&= \frac{1}{2}a(4a^2Ab + 8Ab^3 + a^3B + 12ab^2B)x + \frac{b^3(Ab + 4a^2B)}{2d}
\end{aligned}$$

Mathematica [A] time = 1.12, size = 257, normalized size = 1.30

$$a^4A \sin(3(c + dx)) + 3a^3(aB + 4Ab) \sin(2(c + dx)) + 3a^2(3a^2A + 16abB + 24Ab^2) \sin(c + dx) + 6a(c + dx)(a^3B$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]

[Out] (6*a*(4*a^2*A*b + 8*A*b^3 + a^3*B + 12*a*b^2*B)*(c + d*x) - 12*b^3*(A*b + 4*a*B)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 12*b^3*(A*b + 4*a*B)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (12*b^4*B*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) + (12*b^4*B*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) + 3*a^2*(3*a^2*A + 24*A*b^2 + 16*a*b*B)*Sin[c + d*x] + 3*a^3*(4*A*b + a*B)*Sin[2*(c + d*x)] + a^4*A*Sin[3*(c + d*x)]/(12*d)

fricas [A] time = 0.49, size = 196, normalized size = 0.99

$$3(Ba^4 + 4Aa^3b + 12Ba^2b^2 + 8Aab^3)dx \cos(dx + c) + 3(4Bab^3 + Ab^4) \cos(dx + c) \log(\sin(dx + c) + 1) - 3(4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] $\frac{1}{6} \cdot (3 \cdot (B \cdot a^4 + 4 \cdot A \cdot a^3 \cdot b + 12 \cdot B \cdot a^2 \cdot b^2 + 8 \cdot A \cdot a \cdot b^3) \cdot d \cdot x \cdot \cos(d \cdot x + c) + 3 \cdot (4 \cdot B \cdot a \cdot b^3 + A \cdot b^4) \cdot \cos(d \cdot x + c) \cdot \log(\sin(d \cdot x + c) + 1) - 3 \cdot (4 \cdot B \cdot a \cdot b^3 + A \cdot b^4) \cdot \cos(d \cdot x + c) \cdot \log(-\sin(d \cdot x + c) + 1) + (2 \cdot A \cdot a^4 \cdot \cos(d \cdot x + c)^3 + 6 \cdot B \cdot b^4 + 3 \cdot (B \cdot a^4 + 4 \cdot A \cdot a^3 \cdot b) \cdot \cos(d \cdot x + c)^2 + 4 \cdot (A \cdot a^4 + 6 \cdot B \cdot a^3 \cdot b + 9 \cdot A \cdot a^2 \cdot b^2) \cdot \cos(d \cdot x + c)) \cdot \sin(d \cdot x + c)) / (d \cdot \cos(d \cdot x + c))$

giac [A] time = 1.86, size = 371, normalized size = 1.87

$$\frac{12 B b^4 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 1} - 3 \left(B a^4 + 4 A a^3 b + 12 B a^2 b^2 + 8 A a b^3 \right) (dx + c) - 6 \left(4 B a b^3 + A b^4 \right) \log\left(\left| \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) \right| + 1\right) + 6 \left(4 B a b^3 + A b^4 \right) \log\left(\left| \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) \right| - 1\right) - 2 \cdot (6 \cdot A \cdot a^4 \cdot \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 - 3 \cdot B \cdot a^4 \cdot \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 - 12 \cdot A \cdot a^3 \cdot b \cdot \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 + 24 \cdot B \cdot a^3 \cdot b \cdot \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 + 36 \cdot A \cdot a^2 \cdot b^2 \cdot \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 + 4 \cdot A \cdot a^4 \cdot \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 + 48 \cdot B \cdot a^3 \cdot b \cdot \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 + 72 \cdot A \cdot a^2 \cdot b^2 \cdot \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 + 6 \cdot A \cdot a^4 \cdot \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 3 \cdot B \cdot a^4 \cdot \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 12 \cdot A \cdot a^3 \cdot b \cdot \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 24 \cdot B \cdot a^3 \cdot b \cdot \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 36 \cdot A \cdot a^2 \cdot b^2 \cdot \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)) / (\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + 1)^3 / d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="giac")`

[Out]
$$-1/6 \cdot (12 \cdot B \cdot b^4 \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c) / (\tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^2 - 1) - 3 \cdot (B \cdot a^4 + 4 \cdot A \cdot a^3 \cdot b + 12 \cdot B \cdot a^2 \cdot b^2 + 8 \cdot A \cdot a \cdot b^3) \cdot (d \cdot x + c) - 6 \cdot (4 \cdot B \cdot a \cdot b^3 + A \cdot b^4) \cdot \log(\text{abs}(\tan(1/2 \cdot d \cdot x + 1/2 \cdot c) + 1)) + 6 \cdot (4 \cdot B \cdot a \cdot b^3 + A \cdot b^4) \cdot \log(\text{abs}(\tan(1/2 \cdot d \cdot x + 1/2 \cdot c) - 1)) - 2 \cdot (6 \cdot A \cdot a^4 \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^5 - 3 \cdot B \cdot a^4 \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^5 - 12 \cdot A \cdot a^3 \cdot b \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^5 + 24 \cdot B \cdot a^3 \cdot b \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^5 + 36 \cdot A \cdot a^2 \cdot b^2 \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^5 + 4 \cdot A \cdot a^4 \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^3 + 48 \cdot B \cdot a^3 \cdot b \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^3 + 72 \cdot A \cdot a^2 \cdot b^2 \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^3 + 6 \cdot A \cdot a^4 \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c) + 3 \cdot B \cdot a^4 \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c) + 12 \cdot A \cdot a^3 \cdot b \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c) + 24 \cdot B \cdot a^3 \cdot b \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c) + 36 \cdot A \cdot a^2 \cdot b^2 \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)) / (\tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^2 + 1)^3 / d$$

maple [A] time = 1.16, size = 255, normalized size = 1.29

$$\frac{A \sin(dx + c) (\cos^2(dx + c)) a^4}{3d} + \frac{2A a^4 \sin(dx + c)}{3d} + \frac{a^4 B \cos(dx + c) \sin(dx + c)}{2d} + \frac{a^4 B x}{2} + \frac{a^4 B c}{2d} + \frac{2A a^3 b \sin(dx + c)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(d*x+c)^3*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x)`

[Out]
$$1/3 \cdot d \cdot A \cdot \sin(d \cdot x + c) \cdot \cos(d \cdot x + c)^2 \cdot a^4 + 2/3 \cdot d \cdot A \cdot a^4 \cdot \sin(d \cdot x + c) + 1/2 \cdot d \cdot a^4 \cdot B \cdot \cos(d \cdot x + c) \cdot \sin(d \cdot x + c) + 1/2 \cdot a^4 \cdot B \cdot x + 1/2 \cdot d \cdot a^4 \cdot B \cdot c + 2 \cdot d \cdot A \cdot a^3 \cdot b \cdot \sin(d \cdot x + c) \cdot \cos(d \cdot x + c) + 2 \cdot A \cdot a^3 \cdot b \cdot x + 2 \cdot d \cdot A \cdot a^3 \cdot b \cdot c + 4 \cdot d \cdot B \cdot a^3 \cdot b \cdot \sin(d \cdot x + c) + 6 \cdot d \cdot A \cdot a^2 \cdot b^2 \cdot \sin(d \cdot x + c) + 6 \cdot B \cdot a^2 \cdot b^2 \cdot x + 6 \cdot d \cdot B \cdot a^2 \cdot b^2 \cdot c + 4 \cdot A \cdot a \cdot b^3 \cdot x + 4 \cdot d \cdot A \cdot a \cdot b^3 \cdot c + 4 \cdot d \cdot B \cdot a \cdot b^3 \cdot \ln(\sec(d \cdot x + c) + \tan(d \cdot x + c)) + 1 \cdot d \cdot A \cdot b^4 \cdot \ln(\sec(d \cdot x + c) + \tan(d \cdot x + c)) + 1 \cdot d \cdot B \cdot b^4 \cdot \tan(d \cdot x + c)$$

maxima [A] time = 0.72, size = 197, normalized size = 0.99

$$\frac{4(\sin(dx+c)^3 - 3\sin(dx+c))Aa^4 - 3(2dx+2c+\sin(2dx+2c))Ba^4 - 12(2dx+2c+\sin(2dx+2c))Aa}{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out]
$$-1/12*(4*(\sin(d*x+c)^3 - 3*\sin(d*x+c))*A*a^4 - 3*(2*d*x+2*c+\sin(2*d*x+2*c))*B*a^4 - 12*(2*d*x+2*c+\sin(2*d*x+2*c))*A*a^3*b - 72*(d*x+c)*B*a^2*b^2 - 48*(d*x+c)*A*a*b^3 - 24*B*a*b^3*(\log(\sin(d*x+c)+1) - \log(\sin(d*x+c)-1)) - 6*A*b^4*(\log(\sin(d*x+c)+1) - \log(\sin(d*x+c)-1)) - 48*B*a^3*b*\sin(d*x+c) - 72*A*a^2*b^2*\sin(d*x+c) - 12*B*b^4*\tan(d*x+c))/d$$

mupad [B] time = 4.31, size = 2523, normalized size = 12.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c+d*x)^3*(A+B/cos(c+d*x))*(a+b/cos(c+d*x))^4,x)

[Out]
$$\begin{aligned} & -(\tan(c/2+(d*x)/2)^7*(2*A*a^4 - B*a^4 - 2*B*b^4 + 12*A*a^2*b^2 - 4*A*a^3*b + 8*B*a^3*b) - \tan(c/2+(d*x)/2)*(2*A*a^4 + B*a^4 + 2*B*b^4 + 12*A*a^2*b^2 + 4*A*a^3*b + 8*B*a^3*b) + \tan(c/2+(d*x)/2)^3*((2*A*a^4)/3 + B*a^4 - 6*B*b^4 - 12*A*a^2*b^2 + 4*A*a^3*b - 8*B*a^3*b) + \tan(c/2+(d*x)/2)^5*(B*a^4 - (2*A*a^4)/3 - 6*B*b^4 + 12*A*a^2*b^2 + 4*A*a^3*b + 8*B*a^3*b))/((d*(2*\tan(c/2+(d*x)/2)^2 - 2*\tan(c/2+(d*x)/2)^6 - \tan(c/2+(d*x)/2)^8 + 1)) - \\ & (\operatorname{atan}(((A*b^4 + 4*B*a*b^3)*(A*b^4 + 4*B*a*b^3)*(32*A*b^4 + 16*B*a^4 + 192*B*a^2*b^2 + 128*A*a*b^3 + 64*A*a^3*b + 128*B*a*b^3) + \tan(c/2+(d*x)/2)*(32*A^2*b^8 + 8*B^2*a^8 + 512*A^2*a^2*b^6 + 512*A^2*a^4*b^4 + 128*A^2*a^6*b^2 + 512*B^2*a^2*b^6 + 1152*B^2*a^4*b^4 + 192*B^2*a^6*b^2 + 256*A*B*a*b^7 + 64*A*B*a^7*b + 1536*A*B*a^3*b^5 + 896*A*B*a^5*b^3))*1i - (A*b^4 + 4*B*a*b^3)*(A*b^4 + 4*B*a*b^3)*(32*A*b^4 + 16*B*a^4 + 192*B*a^2*b^2 + 128*A*a*b^3 + 64*A*a^3*b + 128*B*a*b^3) - \tan(c/2+(d*x)/2)*(32*A^2*b^8 + 8*B^2*a^8 + 512*A^2*a^2*b^6 + 512*A^2*a^4*b^4 + 128*A^2*a^6*b^2 + 512*B^2*a^2*b^6 + 1152*B^2*a^4*b^4 + 192*B^2*a^6*b^2 + 256*A*B*a*b^7 + 64*A*B*a^7*b + 1536*A*B*a^3*b^5 + 896*A*B*a^5*b^3))*1i)/((A*b^4 + 4*B*a*b^3)*(A*b^4 + 4*B*a*b^3)*(32*A*b^4 + 16*B*a^4 + 192*B*a^2*b^2 + 128*A*a*b^3 + 64*A*a^3*b + 128*B*a*b^3) + \tan(c/2+(d*x)/2)*(32*A^2*b^8 + 8*B^2*a^8 + 512*A^2*a^2*b^6 + 512*A^2*a^4*b^4 + 128*A^2*a^6*b^2 + 512*B^2*a^2*b^6 + 1152*B^2*a^4*b^4 + 192*B^2*a^6*b^2 + 256*A*B*a*b^7 + 64*A*B*a^7*b + 1536*A*B*a^3*b^5 + 896*A*B*a^5*b^3)) + (A*b^4 + 4*B*a*b^3)*(A*b^4 + 4*B*a*b^3)*(32*A*b^4 + 16*B*a^4 + 192*B*a^2*b^2 + 128*A*a*b^3 + 64*A*a^3*b + 128*B*a*b^3) \end{aligned}$$

$$\begin{aligned}
& *b^2 + 128*A*a*b^3 + 64*A*a^3*b + 128*B*a*b^3) - \tan(c/2 + (d*x)/2)*(32*A^2 \\
& *b^8 + 8*B^2*a^8 + 512*A^2*a^2*b^6 + 512*A^2*a^4*b^4 + 128*A^2*a^6*b^2 + 51 \\
& 2*B^2*a^2*b^6 + 1152*B^2*a^4*b^4 + 192*B^2*a^6*b^2 + 256*A*B*a*b^7 + 64*A*B \\
& *a^7*b + 1536*A*B*a^3*b^5 + 896*A*B*a^5*b^3)) - 256*A^3*a*b^11 + 1024*A^3*a \\
& ^2*b^10 - 128*A^3*a^3*b^9 + 1024*A^3*a^4*b^8 + 256*A^3*a^6*b^6 - 6144*B^3*a \\
& ^4*b^8 + 9216*B^3*a^5*b^7 - 512*B^3*a^6*b^6 + 1536*B^3*a^7*b^5 + 64*B^3*a^9 \\
& *b^3 - 7168*A*B^2*a^3*b^9 + 14592*A*B^2*a^4*b^8 - 2304*A*B^2*a^5*b^7 + 7552 \\
& *A*B^2*a^6*b^6 + 528*A*B^2*a^8*b^4 - 2432*A^2*B*a^2*b^10 + 7168*A^2*B*a^3*b \\
& ^9 - 1056*A^2*B*a^4*b^8 + 5888*A^2*B*a^5*b^7 + 1152*A^2*B*a^7*b^5))*(A*b^4* \\
& 2i + B*a*b^3*8i))/d - (a*atan(((a*(\tan(c/2 + (d*x)/2)*(32*A^2*b^8 + 8*B^2*a \\
& ^8 + 512*A^2*a^2*b^6 + 512*A^2*a^4*b^4 + 128*A^2*a^6*b^2 + 512*B^2*a^2*b^6 \\
& + 1152*B^2*a^4*b^4 + 192*B^2*a^6*b^2 + 256*A*B*a*b^7 + 64*A*B*a^7*b + 1536* \\
& A*B*a^3*b^5 + 896*A*B*a^5*b^3) - (a*(8*A*b^3 + B*a^3 + 4*A*a^2*b + 12*B*a*b \\
& ^2)*(32*A*b^4 + 16*B*a^4 + 192*B*a^2*b^2 + 128*A*a*b^3 + 64*A*a^3*b + 128*B \\
& *a*b^3)*1i)/2)*(8*A*b^3 + B*a^3 + 4*A*a^2*b + 12*B*a*b^2))/2 + (a*(\tan(c/2 \\
& + (d*x)/2)*(32*A^2*b^8 + 8*B^2*a^8 + 512*A^2*a^2*b^6 + 512*A^2*a^4*b^4 + 12 \\
& 8*A^2*a^6*b^2 + 512*B^2*a^2*b^6 + 1152*B^2*a^4*b^4 + 192*B^2*a^6*b^2 + 256* \\
& A*B*a*b^7 + 64*A*B*a^7*b + 1536*A*B*a^3*b^5 + 896*A*B*a^5*b^3) + (a*(8*A*b^ \\
& 3 + B*a^3 + 4*A*a^2*b + 12*B*a*b^2)*(32*A*b^4 + 16*B*a^4 + 192*B*a^2*b^2 + \\
& 128*A*a*b^3 + 64*A*a^3*b + 128*B*a*b^3)*1i)/2)*(8*A*b^3 + B*a^3 + 4*A*a^2*b \\
& + 12*B*a*b^2))/2)/(1024*A^3*a^2*b^10 - 256*A^3*a*b^11 - 128*A^3*a^3*b^9 + \\
& 1024*A^3*a^4*b^8 + 256*A^3*a^6*b^6 - 6144*B^3*a^4*b^8 + 9216*B^3*a^5*b^7 - \\
& 512*B^3*a^6*b^6 + 1536*B^3*a^7*b^5 + 64*B^3*a^9*b^3 - (a*(\tan(c/2 + (d*x)/2 \\
&)*(32*A^2*b^8 + 8*B^2*a^8 + 512*A^2*a^2*b^6 + 512*A^2*a^4*b^4 + 128*A^2*a^6 \\
& *b^2 + 512*B^2*a^2*b^6 + 1152*B^2*a^4*b^4 + 192*B^2*a^6*b^2 + 256*A*B*a*b^7 \\
& + 64*A*B*a^7*b + 1536*A*B*a^3*b^5 + 896*A*B*a^5*b^3) - (a*(8*A*b^3 + B*a^3 \\
& + 4*A*a^2*b + 12*B*a*b^2)*(32*A*b^4 + 16*B*a^4 + 192*B*a^2*b^2 + 128*A*a*b \\
& ^3 + 64*A*a^3*b + 128*B*a*b^3)*1i)/2)*(8*A*b^3 + B*a^3 + 4*A*a^2*b + 12*B*a \\
& *b^2)*1i)/2 + (a*(\tan(c/2 + (d*x)/2)*(32*A^2*b^8 + 8*B^2*a^8 + 512*A^2*a^2* \\
& b^6 + 512*A^2*a^4*b^4 + 128*A^2*a^6*b^2 + 512*B^2*a^2*b^6 + 1152*B^2*a^4*b^ \\
& 4 + 192*B^2*a^6*b^2 + 256*A*B*a*b^7 + 64*A*B*a^7*b + 1536*A*B*a^3*b^5 + 896 \\
& *A*B*a^5*b^3) + (a*(8*A*b^3 + B*a^3 + 4*A*a^2*b + 12*B*a*b^2)*(32*A*b^4 + 1 \\
& 6*B*a^4 + 192*B*a^2*b^2 + 128*A*a*b^3 + 64*A*a^3*b + 128*B*a*b^3)*1i)/2)*(8 \\
& *A*b^3 + B*a^3 + 4*A*a^2*b + 12*B*a*b^2)*1i)/2 - 7168*A*B^2*a^3*b^9 + 14592 \\
& *A*B^2*a^4*b^8 - 2304*A*B^2*a^5*b^7 + 7552*A*B^2*a^6*b^6 + 528*A*B^2*a^8*b^ \\
& 4 - 2432*A^2*B*a^2*b^10 + 7168*A^2*B*a^3*b^9 - 1056*A^2*B*a^4*b^8 + 5888*A^ \\
& 2*B*a^5*b^7 + 1152*A^2*B*a^7*b^5))*(8*A*b^3 + B*a^3 + 4*A*a^2*b + 12*B*a*b^ \\
& 2))/d
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**3*(a+b*sec(d*x+c))**4*(A+B*sec(d*x+c)),x)

[Out] Timed out

$$3.308 \quad \int \cos^4(c+dx)(a+b \sec(c+dx))^4(A+B \sec(c+dx)) dx$$

Optimal. Leaf size=216

$$\frac{a^2(9a^2A + 32abB + 26Ab^2) \sin(c+dx) \cos(c+dx)}{24d} + \frac{a(4a^3B + 16a^2Ab + 34ab^2B + 19Ab^3) \sin(c+dx)}{6d} + \frac{1}{8}x$$

[Out] $\frac{1}{8}*(3*A*a^4+24*A*a^2*b^2+8*A*b^4+16*B*a^3*b+32*B*a*b^3)*x+b^4*B*\operatorname{arctanh}(\sin(d*x+c))/d+1/6*a*(16*A*a^2*b+19*A*b^3+4*B*a^3+34*B*a*b^2)*\sin(d*x+c)/d+1/24*a^2*(9*A*a^2+26*A*b^2+32*B*a*b)*\cos(d*x+c)*\sin(d*x+c)/d+1/12*a*(7*A*b+4*B*a)*\cos(d*x+c)^2*(a+b*\sec(d*x+c))^2*\sin(d*x+c)/d+1/4*a*A*\cos(d*x+c)^3*(a+b*\sec(d*x+c))^3*\sin(d*x+c)/d$

Rubi [A] time = 0.61, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {4025, 4094, 4074, 4047, 8, 4045, 3770}

$$\frac{a(16a^2Ab + 4a^3B + 34ab^2B + 19Ab^3) \sin(c+dx)}{6d} + \frac{a^2(9a^2A + 32abB + 26Ab^2) \sin(c+dx) \cos(c+dx)}{24d} + \frac{1}{8}x$$

Antiderivative was successfully verified.

[In] `Int[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]`

[Out] $((3*a^4*A + 24*a^2*A*b^2 + 8*A*b^4 + 16*a^3*b*B + 32*a*b^3*B)*x)/8 + (b^4*B*\operatorname{ArcTanh}[\operatorname{Sin}[c + d*x]])/d + (a*(16*a^2*A*b + 19*A*b^3 + 4*a^3*B + 34*a*b^2*B)*\operatorname{Sin}[c + d*x])/(6*d) + (a^2*(9*a^2*A + 26*A*b^2 + 32*a*b*B)*\operatorname{Cos}[c + d*x]*\operatorname{Sin}[c + d*x])/(24*d) + (a*(7*A*b + 4*a*B)*\operatorname{Cos}[c + d*x]^2*(a + b*\operatorname{Sec}[c + d*x])^2*\operatorname{Sin}[c + d*x])/(12*d) + (a*A*\operatorname{Cos}[c + d*x]^3*(a + b*\operatorname{Sec}[c + d*x])^3*\operatorname{Sin}[c + d*x])/(4*d)$

Rule 8

`Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

Rule 3770

`Int[csc[(c_.) + (d_.)*(x_.)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]`

Rule 4025

`Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := Simp[(a*A*Co`

$$\int [e + f*x]*(a + b*\text{Csc}[e + f*x])^{(m-1)}*(d*\text{Csc}[e + f*x])^n/(f*n), x] + \text{Dist}[1/(d*n), \text{Int}[(a + b*\text{Csc}[e + f*x])^{(m-2)}*(d*\text{Csc}[e + f*x])^{(n+1)}*\text{Simp}[a*(a*B*n - A*b*(m-n-1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1+n)))*\text{Csc}[e + f*x] + b*(b*B*n + a*A*(m+n))*\text{Csc}[e + f*x]^2, x], x], x] /;$$

$$\text{FreeQ}\{a, b, d, e, f, A, B\}, x\} \ \&\& \ \text{NeQ}[A*b - a*B, 0] \ \&\& \ \text{NeQ}[a^2 - b^2, 0] \ \&\& \ \text{GtQ}[m, 1] \ \&\& \ \text{LeQ}[n, -1]$$

Rule 4045

$$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.))^{(m_.)}*(\text{csc}[(e_.) + (f_.)*(x_.)]^2*(C_.) + (A_.)), x_Symbol] \ :> \ \text{Simp}[(A*\text{Cot}[e + f*x]*(b*\text{Csc}[e + f*x])^m)/(f*m), x] + \text{Dist}[(C*m + A*(m+1))/(b^2*m), \text{Int}[(b*\text{Csc}[e + f*x])^{(m+2)}, x], x] /;$$

$$\text{FreeQ}\{b, e, f, A, C\}, x\} \ \&\& \ \text{NeQ}[C*m + A*(m+1), 0] \ \&\& \ \text{LeQ}[m, -1]$$

Rule 4047

$$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.))^{(m_.)}*((A_.) + \text{csc}[(e_.) + (f_.)*(x_.)]*(B_.) + \text{csc}[(e_.) + (f_.)*(x_.)]^2*(C_.)), x_Symbol] \ :> \ \text{Dist}[B/b, \text{Int}[(b*\text{Csc}[e + f*x])^{(m+1)}, x], x] + \text{Int}[(b*\text{Csc}[e + f*x])^m*(A + C*\text{Csc}[e + f*x]^2), x] /;$$

$$\text{FreeQ}\{b, e, f, A, B, C, m\}, x\}$$

Rule 4074

$$\text{Int}[(A_.) + \text{csc}[(e_.) + (f_.)*(x_.)]*(B_.) + \text{csc}[(e_.) + (f_.)*(x_.)]^2*(C_.) + \text{csc}[(e_.) + (f_.)*(x_.)]*(d_.)]^{(n_.)}*(\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] \ :> \ \text{Simp}[(A*a*\text{Cot}[e + f*x]*(d*\text{Csc}[e + f*x])^n)/(f*n), x] + \text{Dist}[1/(d*n), \text{Int}[(d*\text{Csc}[e + f*x])^{(n+1)}*\text{Simp}[n*(B*a + A*b) + (n*(a*C + B*b) + A*a*(n+1))*\text{Csc}[e + f*x] + b*C*n*\text{Csc}[e + f*x]^2, x], x], x] /;$$

$$\text{FreeQ}\{a, b, d, e, f, A, B, C\}, x\} \ \&\& \ \text{LtQ}[n, -1]$$

Rule 4094

$$\text{Int}[(A_.) + \text{csc}[(e_.) + (f_.)*(x_.)]*(B_.) + \text{csc}[(e_.) + (f_.)*(x_.)]^2*(C_.) + \text{csc}[(e_.) + (f_.)*(x_.)]*(d_.)]^{(n_.)}*(\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^{(m_.)}, x_Symbol] \ :> \ \text{Simp}[(A*\text{Cot}[e + f*x]*(a + b*\text{Csc}[e + f*x])^m*(d*\text{Csc}[e + f*x])^n)/(f*n), x] - \text{Dist}[1/(d*n), \text{Int}[(a + b*\text{Csc}[e + f*x])^{(m-1)}*(d*\text{Csc}[e + f*x])^{(n+1)}*\text{Simp}[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n+1)))*\text{Csc}[e + f*x] - b*(C*n + A*(m+n+1))*\text{Csc}[e + f*x]^2, x], x], x] /;$$

$$\text{FreeQ}\{a, b, d, e, f, A, B, C\}, x\} \ \&\& \ \text{NeQ}[a^2 - b^2, 0] \ \&\& \ \text{GtQ}[m, 0] \ \&\& \ \text{LeQ}[n, -1]$$

Rubi steps

$$\begin{aligned}
\int \cos^4(c + dx)(a + b \sec(c + dx))^4(A + B \sec(c + dx)) dx &= \frac{aA \cos^3(c + dx)(a + b \sec(c + dx))^3 \sin(c + dx)}{4d} - \frac{1}{4} \\
&= \frac{a(7Ab + 4aB) \cos^2(c + dx)(a + b \sec(c + dx))^2 \sin(c + dx)}{12d} \\
&= \frac{a^2 (9a^2 A + 26Ab^2 + 32abB) \cos(c + dx) \sin(c + dx)}{24d} \\
&= \frac{a^2 (9a^2 A + 26Ab^2 + 32abB) \cos(c + dx) \sin(c + dx)}{24d} \\
&= \frac{1}{8} (3a^4 A + 24a^2 Ab^2 + 8Ab^4 + 16a^3 bB + 32ab^3 B) x \\
&= \frac{1}{8} (3a^4 A + 24a^2 Ab^2 + 8Ab^4 + 16a^3 bB + 32ab^3 B) x
\end{aligned}$$

Mathematica [A] time = 0.62, size = 210, normalized size = 0.97

$$3a^4 A \sin(4(c + dx)) + 8a^3(aB + 4Ab) \sin(3(c + dx)) + 24a^2(a^2 A + 4abB + 6Ab^2) \sin(2(c + dx)) + 24a(3a^3 B +$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]

[Out] (12*(3*a^4*A + 24*a^2*A*b^2 + 8*A*b^4 + 16*a^3*b*B + 32*a*b^3*B)*(c + d*x) - 96*b^4*B*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 96*b^4*B*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 24*a*(12*a^2*A*b + 16*A*b^3 + 3*a^3*B + 24*a*b^2*B)*Sin[c + d*x] + 24*a^2*(a^2*A + 6*A*b^2 + 4*a*b*B)*Sin[2*(c + d*x)] + 8*a^3*(4*A*b + a*B)*Sin[3*(c + d*x)] + 3*a^4*A*Sin[4*(c + d*x)])/(96*d)

fricas [A] time = 0.48, size = 183, normalized size = 0.85

$$12 Bb^4 \log(\sin(dx + c) + 1) - 12 Bb^4 \log(-\sin(dx + c) + 1) + 3(3 Aa^4 + 16 Ba^3b + 24 Aa^2b^2 + 32 Bab^3 + 8 A$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] 1/24*(12*B*b^4*log(sin(d*x + c) + 1) - 12*B*b^4*log(-sin(d*x + c) + 1) + 3*(3*A*a^4 + 16*B*a^3*b + 24*A*a^2*b^2 + 32*B*a*b^3 + 8*A*b^4)*d*x + (6*A*a^4

$$*\cos(dx + c)^3 + 16*B*a^4 + 64*A*a^3*b + 144*B*a^2*b^2 + 96*A*a*b^3 + 8*(B*a^4 + 4*A*a^3*b)*\cos(dx + c)^2 + 3*(3*A*a^4 + 16*B*a^3*b + 24*A*a^2*b^2)*\cos(dx + c)*\sin(dx + c))/d$$

giac [B] time = 0.42, size = 603, normalized size = 2.79

$$24 B b^4 \log \left(\left| \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) + 1 \right| \right) - 24 B b^4 \log \left(\left| \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) - 1 \right| \right) + 3 \left(3 A a^4 + 16 B a^3 b + 24 A a^2 b^2 + 32 B a b^3 + 8 (B a^4 + 4 A a^3 b) \cos(dx + c)^2 + 3 (3 A a^4 + 16 B a^3 b + 24 A a^2 b^2) \cos(dx + c) \sin(dx + c) \right) / d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(dx+c)^4*(a+b*sec(dx+c))^4*(A+B*sec(dx+c)),x, algorithm="giac")

[Out] 1/24*(24*B*b^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 24*B*b^4*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 3*(3*A*a^4 + 16*B*a^3*b + 24*A*a^2*b^2 + 32*B*a*b^3 + 8*A*b^4)*(d*x + c) - 2*(15*A*a^4*tan(1/2*d*x + 1/2*c)^7 - 24*B*a^4*tan(1/2*d*x + 1/2*c)^7 - 96*A*a^3*b*tan(1/2*d*x + 1/2*c)^7 + 48*B*a^3*b*tan(1/2*d*x + 1/2*c)^7 + 72*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 - 144*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 - 96*A*a*b^3*tan(1/2*d*x + 1/2*c)^7 - 9*A*a^4*tan(1/2*d*x + 1/2*c)^5 - 40*B*a^4*tan(1/2*d*x + 1/2*c)^5 - 160*A*a^3*b*tan(1/2*d*x + 1/2*c)^5 + 48*B*a^3*b*tan(1/2*d*x + 1/2*c)^5 + 72*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 - 432*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 - 288*A*a*b^3*tan(1/2*d*x + 1/2*c)^5 + 9*A*a^4*tan(1/2*d*x + 1/2*c)^3 - 40*B*a^4*tan(1/2*d*x + 1/2*c)^3 - 160*A*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 48*B*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 72*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 432*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 288*A*a*b^3*tan(1/2*d*x + 1/2*c)^3 - 15*A*a^4*tan(1/2*d*x + 1/2*c) - 24*B*a^4*tan(1/2*d*x + 1/2*c) - 96*A*a^3*b*tan(1/2*d*x + 1/2*c) - 48*B*a^3*b*tan(1/2*d*x + 1/2*c) - 72*A*a^2*b^2*tan(1/2*d*x + 1/2*c) - 144*B*a^2*b^2*tan(1/2*d*x + 1/2*c) - 96*A*a*b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d

maple [A] time = 1.18, size = 319, normalized size = 1.48

$$\frac{A a^4 \sin(dx + c) (\cos^3(dx + c))}{4d} + \frac{3A a^4 \cos(dx + c) \sin(dx + c)}{8d} + \frac{3A a^4 x}{8} + \frac{3A a^4 c}{8d} + \frac{B \sin(dx + c) (\cos^2(dx + c))}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(dx+c)^4*(a+b*sec(dx+c))^4*(A+B*sec(dx+c)),x)

[Out] 1/4/d*A*a^4*sin(dx+c)*cos(dx+c)^3+3/8/d*A*a^4*cos(dx+c)*sin(dx+c)+3/8*A*a^4*x+3/8/d*A*a^4*c+1/3/d*B*sin(dx+c)*cos(dx+c)^2*a^4+2/3/d*a^4*B*sin(dx+c)+4/3/d*A*sin(dx+c)*cos(dx+c)^2*a^3*b+8/3/d*A*a^3*b*sin(dx+c)+2/d*B*a

$^3*b*\sin(dx+c)*\cos(dx+c)+2*B*x*a^3*b+2/d*B*a^3*b*c+3/d*A*a^2*b^2*\sin(dx+c)*\cos(dx+c)+3*A*x*a^2*b^2+3/d*A*a^2*b^2*c+6/d*a^2*b^2*B*\sin(dx+c)+4/d*a^2*b^2*B*\sin(dx+c)+4*B*x*a*b^3+4/d*B*a*b^3*c+A*x*b^4+1/d*A*b^4*c+1/d*B*b^4*\ln(\sec(dx+c)+\tan(dx+c))$

maxima [A] time = 0.70, size = 215, normalized size = 1.00

$$\frac{3(12dx + 12c + \sin(4dx + 4c) + 8\sin(2dx + 2c))Aa^4 - 32(\sin(dx + c)^3 - 3\sin(dx + c))Ba^4 - 128(\sin(dx + c)^3 - 3\sin(dx + c))Ba^4 - 128(\sin(dx + c)^3 - 3\sin(dx + c))Ba^4}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(dx+c)^4*(a+b*sec(dx+c))^4*(A+B*sec(dx+c)),x, algorithm="maxima")

[Out] $\frac{1}{96}*(3*(12*dx + 12*c + \sin(4*dx + 4*c)) + 8*\sin(2*dx + 2*c))*A*a^4 - 32*(\sin(dx + c)^3 - 3*\sin(dx + c))*B*a^4 - 128*(\sin(dx + c)^3 - 3*\sin(dx + c))*A*a^3*b + 96*(2*d*x + 2*c + \sin(2*d*x + 2*c))*B*a^3*b + 144*(2*d*x + 2*c + \sin(2*d*x + 2*c))*A*a^2*b^2 + 384*(d*x + c)*B*a*b^3 + 96*(d*x + c)*A*b^4 + 48*B*b^4*(\log(\sin(dx + c) + 1) - \log(\sin(dx + c) - 1)) + 576*B*a^2*b^2*\sin(dx + c) + 384*A*a*b^3*\sin(dx + c))/d$

mupad [B] time = 3.38, size = 369, normalized size = 1.71

$$\frac{3Ba^4 \sin(c + dx)}{4d} + \frac{3Aa^4 \operatorname{atan}\left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)}\right)}{4d} + \frac{2Ab^4 \operatorname{atan}\left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)}\right)}{d} + \frac{2Bb^4 \operatorname{atanh}\left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)}\right)}{d} + \frac{Aa^4 \sin(2c)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + dx)^4*(A + B/cos(c + dx))*(a + b/cos(c + dx))^4,x)

[Out] $(3Ba^4*\sin(c + dx))/(4*d) + (3Aa^4*\operatorname{atan}(\sin(c/2 + (dx)/2)/\cos(c/2 + (dx)/2)))/(4*d) + (2Aa^4*b^4*\operatorname{atan}(\sin(c/2 + (dx)/2)/\cos(c/2 + (dx)/2)))/d + (2Bb^4*\operatorname{atanh}(\sin(c/2 + (dx)/2)/\cos(c/2 + (dx)/2)))/d + (Aa^4*\sin(2*c + 2*d*x))/(4*d) + (Aa^4*\sin(4*c + 4*d*x))/(32*d) + (Ba^4*\sin(3*c + 3*d*x))/(12*d) + (8Ba^4*b^3*\operatorname{atan}(\sin(c/2 + (dx)/2)/\cos(c/2 + (dx)/2)))/d + (4Ba^4*b^3*\operatorname{atan}(\sin(c/2 + (dx)/2)/\cos(c/2 + (dx)/2)))/d + (Aa^4*b^3*\sin(3*c + 3*d*x))/(3*d) + (Ba^4*b^3*\sin(2*c + 2*d*x))/d + (6Ba^4*b^2*\sin(c + d*x))/d + (6Aa^4*b^2*\operatorname{atan}(\sin(c/2 + (dx)/2)/\cos(c/2 + (dx)/2)))/d + (3Aa^4*b^2*\sin(2*c + 2*d*x))/(2*d) + (4Aa^4*b^3*\sin(c + d*x))/d + (3Aa^4*b^3*\sin(c + d*x))/d$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+b*sec(d*x+c))**4*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

$$3.309 \quad \int \cos^5(c+dx)(a+b \sec(c+dx))^4(A+B \sec(c+dx)) dx$$

Optimal. Leaf size=258

$$\frac{a^2(8a^2A + 25abB + 18Ab^2) \sin(c+dx) \cos^2(c+dx)}{30d} + \frac{a(15a^3B + 60a^2Ab + 110ab^2B + 56Ab^3) \sin(c+dx) \cos^2(c+dx)}{40d}$$

[Out] 1/8*(12*A*a^3*b+16*A*a*b^3+3*B*a^4+24*B*a^2*b^2+8*B*b^4)*x+1/15*(8*A*a^4+60*A*a^2*b^2+15*A*b^4+40*B*a^3*b+60*B*a*b^3)*sin(d*x+c)/d+1/40*a*(60*A*a^2*b+56*A*b^3+15*B*a^3+110*B*a*b^2)*cos(d*x+c)*sin(d*x+c)/d+1/30*a^2*(8*A*a^2+18*A*b^2+25*B*a*b)*cos(d*x+c)^2*sin(d*x+c)/d+1/20*a*(8*A*b+5*B*a)*cos(d*x+c)^3*(a+b*sec(d*x+c))^2*sin(d*x+c)/d+1/5*a*A*cos(d*x+c)^4*(a+b*sec(d*x+c))^3*sin(d*x+c)/d

Rubi [A] time = 0.69, antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {4025, 4094, 4074, 4047, 2637, 4045, 8}

$$\frac{(60a^2Ab^2 + 8a^4A + 40a^3bB + 60ab^3B + 15Ab^4) \sin(c+dx)}{15d} + \frac{a^2(8a^2A + 25abB + 18Ab^2) \sin(c+dx) \cos^2(c+dx)}{30d}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^5*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]

[Out] ((12*a^3*A*b + 16*a*A*b^3 + 3*a^4*B + 24*a^2*b^2*B + 8*b^4*B)*x)/8 + ((8*a^4*A + 60*a^2*A*b^2 + 15*A*b^4 + 40*a^3*b*B + 60*a*b^3*B)*Sin[c + d*x])/(15*d) + (a*(60*a^2*A*b + 56*A*b^3 + 15*a^3*B + 110*a*b^2*B)*Cos[c + d*x]*Sin[c + d*x])/(40*d) + (a^2*(8*a^2*A + 18*A*b^2 + 25*a*b*B)*Cos[c + d*x]^2*SIn[c + d*x])/(30*d) + (a*(8*A*b + 5*a*B)*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^2*SIn[c + d*x])/(20*d) + (a*A*cos[c + d*x]^4*(a + b*Sec[c + d*x])^3*SIn[c + d*x])/(5*d)

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] :> Simp[(a*A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e + f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LeQ[n, -1]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_.)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] + Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2), x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 4074

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] :> Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \cos^5(c + dx)(a + b \sec(c + dx))^4(A + B \sec(c + dx)) dx &= \frac{aA \cos^4(c + dx)(a + b \sec(c + dx))^3 \sin(c + dx)}{5d} - \frac{1}{5} \\
&= \frac{a(8Ab + 5aB) \cos^3(c + dx)(a + b \sec(c + dx))^2 \sin(c + dx)}{20d} \\
&= \frac{a^2 (8a^2 A + 18Ab^2 + 25abB) \cos^2(c + dx) \sin(c + dx)}{30d} \\
&= \frac{a^2 (8a^2 A + 18Ab^2 + 25abB) \cos^2(c + dx) \sin(c + dx)}{30d} \\
&= \frac{(8a^4 A + 60a^2 Ab^2 + 15Ab^4 + 40a^3 bB + 60ab^3 B) \sin(c + dx)}{15d} \\
&= \frac{1}{8} (12a^3 Ab + 16aAb^3 + 3a^4 B + 24a^2 b^2 B + 8b^4 B) x
\end{aligned}$$

Mathematica [A] time = 0.65, size = 263, normalized size = 1.02

$$50a^4 A \sin(3(c + dx)) + 6a^4 A \sin(5(c + dx)) + 15a^4 B \sin(4(c + dx)) + 180a^4 Bc + 180a^4 Bdx + 60a^3 Ab \sin(4(c + dx))$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]^5*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]

[Out] (720*a^3*A*b*c + 960*a*A*b^3*c + 180*a^4*B*c + 1440*a^2*b^2*B*c + 480*b^4*B*c + 720*a^3*A*b*d*x + 960*a*A*b^3*d*x + 180*a^4*B*d*x + 1440*a^2*b^2*B*d*x + 480*b^4*B*d*x + 60*(5*a^4*A + 36*a^2*A*b^2 + 8*A*b^4 + 24*a^3*b*B + 32*a*b^3*B)*Sin[c + d*x] + 120*a*(4*a^2*A*b + 4*A*b^3 + a^3*B + 6*a*b^2*B)*Sin[2*(c + d*x)] + 50*a^4*A*Ssin[3*(c + d*x)] + 240*a^2*A*b^2*Ssin[3*(c + d*x)] + 160*a^3*b*B*Ssin[3*(c + d*x)] + 60*a^3*A*b*Ssin[4*(c + d*x)] + 15*a^4*B*Ssin[4*(c + d*x)] + 6*a^4*A*Ssin[5*(c + d*x)])/(480*d)

fricas [A] time = 0.46, size = 197, normalized size = 0.76

$$15 (3 Ba^4 + 12 Aa^3 b + 24 Ba^2 b^2 + 16 Aab^3 + 8 Bb^4) dx + (24 Aa^4 \cos(dx + c)^4 + 64 Aa^4 + 320 Ba^3 b + 480 Aa^2 b^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] 1/120*(15*(3*B*a^4 + 12*A*a^3*b + 24*B*a^2*b^2 + 16*A*a*b^3 + 8*B*b^4)*d*x + (24*A*a^4*cos(d*x + c)^4 + 64*A*a^4 + 320*B*a^3*b + 480*A*a^2*b^2 + 480*B

$$*a*b^3 + 120*A*b^4 + 30*(B*a^4 + 4*A*a^3*b)*\cos(d*x + c)^3 + 16*(2*A*a^4 + 10*B*a^3*b + 15*A*a^2*b^2)*\cos(d*x + c)^2 + 15*(3*B*a^4 + 12*A*a^3*b + 24*B*a^2*b^2 + 16*A*a*b^3)*\cos(d*x + c))*\sin(d*x + c))/d$$

giac [B] time = 0.32, size = 791, normalized size = 3.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out]
$$\frac{1}{120}*(15*(3*B*a^4 + 12*A*a^3*b + 24*B*a^2*b^2 + 16*A*a*b^3 + 8*B*b^4)*(d*x + c) + 2*(120*A*a^4*\tan(1/2*d*x + 1/2*c)^9 - 75*B*a^4*\tan(1/2*d*x + 1/2*c)^9 - 300*A*a^3*b*\tan(1/2*d*x + 1/2*c)^9 + 480*B*a^3*b*\tan(1/2*d*x + 1/2*c)^9 + 720*A*a^2*b^2*\tan(1/2*d*x + 1/2*c)^9 - 360*B*a^2*b^2*\tan(1/2*d*x + 1/2*c)^9 - 240*A*a*b^3*\tan(1/2*d*x + 1/2*c)^9 + 480*B*a*b^3*\tan(1/2*d*x + 1/2*c)^9 + 120*A*b^4*\tan(1/2*d*x + 1/2*c)^9 + 160*A*a^4*\tan(1/2*d*x + 1/2*c)^7 - 30*B*a^4*\tan(1/2*d*x + 1/2*c)^7 - 120*A*a^3*b*\tan(1/2*d*x + 1/2*c)^7 + 1280*B*a^3*b*\tan(1/2*d*x + 1/2*c)^7 + 1920*A*a^2*b^2*\tan(1/2*d*x + 1/2*c)^7 - 720*B*a^2*b^2*\tan(1/2*d*x + 1/2*c)^7 - 480*A*a*b^3*\tan(1/2*d*x + 1/2*c)^7 + 1920*B*a*b^3*\tan(1/2*d*x + 1/2*c)^7 + 480*A*b^4*\tan(1/2*d*x + 1/2*c)^7 + 464*A*a^4*\tan(1/2*d*x + 1/2*c)^5 + 1600*B*a^3*b*\tan(1/2*d*x + 1/2*c)^5 + 2400*A*a^2*b^2*\tan(1/2*d*x + 1/2*c)^5 + 2880*B*a*b^3*\tan(1/2*d*x + 1/2*c)^5 + 720*A*b^4*\tan(1/2*d*x + 1/2*c)^5 + 160*A*a^4*\tan(1/2*d*x + 1/2*c)^3 + 30*B*a^4*\tan(1/2*d*x + 1/2*c)^3 + 120*A*a^3*b*\tan(1/2*d*x + 1/2*c)^3 + 1280*B*a^3*b*\tan(1/2*d*x + 1/2*c)^3 + 1920*A*a^2*b^2*\tan(1/2*d*x + 1/2*c)^3 + 720*B*a^2*b^2*\tan(1/2*d*x + 1/2*c)^3 + 480*A*a*b^3*\tan(1/2*d*x + 1/2*c)^3 + 1920*B*a*b^3*\tan(1/2*d*x + 1/2*c)^3 + 480*A*b^4*\tan(1/2*d*x + 1/2*c)^3 + 120*A*a^4*\tan(1/2*d*x + 1/2*c) + 75*B*a^4*\tan(1/2*d*x + 1/2*c) + 300*A*a^3*b*\tan(1/2*d*x + 1/2*c) + 480*B*a^3*b*\tan(1/2*d*x + 1/2*c) + 720*A*a^2*b^2*\tan(1/2*d*x + 1/2*c) + 360*B*a^2*b^2*\tan(1/2*d*x + 1/2*c) + 240*A*a*b^3*\tan(1/2*d*x + 1/2*c) + 480*B*a*b^3*\tan(1/2*d*x + 1/2*c) + 120*A*b^4*\tan(1/2*d*x + 1/2*c))/(\tan(1/2*d*x + 1/2*c)^2 + 1)^5)/d$$

maple [A] time = 1.56, size = 258, normalized size = 1.00

$$\frac{Aa^4\left(\frac{8}{3} + \cos^4(dx+c) + \frac{4(\cos^2(dx+c))}{3}\right)\sin(dx+c)}{5} + 4Aa^3b\left(\frac{\left(\cos^3(dx+c) + \frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4} + \frac{3dx}{8} + \frac{3c}{8}\right) + a^4B\left(\frac{\left(\cos^3(dx+c) + \frac{3\cos(dx+c)}{2}\right)}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^5*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x)

[Out] $1/d*(1/5*A*a^4*(8/3+\cos(d*x+c)^4+4/3*\cos(d*x+c)^2)*\sin(d*x+c)+4*A*a^3*b*(1/4*(\cos(d*x+c)^3+3/2*\cos(d*x+c))*\sin(d*x+c)+3/8*d*x+3/8*c)+a^4*B*(1/4*(\cos(d*x+c)^3+3/2*\cos(d*x+c))*\sin(d*x+c)+3/8*d*x+3/8*c)+2*A*a^2*b^2*(2+\cos(d*x+c)^2)*\sin(d*x+c)+4/3*B*a^3*b*(2+\cos(d*x+c)^2)*\sin(d*x+c)+4*a*A*b^3*(1/2*\cos(d*x+c)*\sin(d*x+c)+1/2*d*x+1/2*c)+6*a^2*b^2*B*(1/2*\cos(d*x+c)*\sin(d*x+c)+1/2*d*x+1/2*c)+A*b^4*\sin(d*x+c)+4*B*a*b^3*\sin(d*x+c)+B*b^4*(d*x+c))$

maxima [A] time = 0.75, size = 246, normalized size = 0.95

$$32 \left(3 \sin(dx + c)^5 - 10 \sin(dx + c)^3 + 15 \sin(dx + c) \right) Aa^4 + 15 (12 dx + 12 c + \sin(4 dx + 4 c) + 8 \sin(2 dx + 2 c)) B a^4 + 60 (12 dx + 12 c + \sin(4 dx + 4 c) + 8 \sin(2 dx + 2 c)) A a^3 b - 640 (\sin(dx + c))^3 - 3 \sin(dx + c) B a^3 b - 960 (\sin(dx + c))^3 - 3 \sin(dx + c) A a^2 b^2 + 720 (2 dx + 2 c + \sin(2 dx + 2 c)) B a^2 b^2 + 480 (2 dx + 2 c + \sin(2 dx + 2 c)) A a b^3 + 480 (d x + c) B b^4 + 1920 B a b^3 \sin(dx + c) + 480 A b^4 \sin(dx + c) / d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="maxima")`

[Out] $1/480*(32*(3*\sin(d*x + c)^5 - 10*\sin(d*x + c)^3 + 15*\sin(d*x + c))*A*a^4 + 15*(12*d*x + 12*c + \sin(4*d*x + 4*c) + 8*\sin(2*d*x + 2*c))*B*a^4 + 60*(12*d*x + 12*c + \sin(4*d*x + 4*c) + 8*\sin(2*d*x + 2*c))*A*a^3*b - 640*(\sin(d*x + c))^3 - 3*\sin(d*x + c)*B*a^3*b - 960*(\sin(d*x + c))^3 - 3*\sin(d*x + c))*A*a^2*b^2 + 720*(2*d*x + 2*c + \sin(2*d*x + 2*c))*B*a^2*b^2 + 480*(2*d*x + 2*c + \sin(2*d*x + 2*c))*A*a*b^3 + 480*(d*x + c)*B*b^4 + 1920*B*a*b^3*\sin(d*x + c) + 480*A*b^4*\sin(d*x + c))/d$

mupad [B] time = 2.71, size = 307, normalized size = 1.19

$$\frac{3 B a^4 x}{8} + B b^4 x + 2 A a b^3 x + \frac{3 A a^3 b x}{2} + \frac{5 A a^4 \sin(c + d x)}{8 d} + \frac{A b^4 \sin(c + d x)}{d} + 3 B a^2 b^2 x + \frac{5 A a^4 \sin(3 c + 3 d x)}{48 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(c + d*x)^5*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^4,x)`

[Out] $(3*B*a^4*x)/8 + B*b^4*x + 2*A*a*b^3*x + (3*A*a^3*b*x)/2 + (5*A*a^4*\sin(c + d*x))/(8*d) + (A*b^4*\sin(c + d*x))/d + 3*B*a^2*b^2*x + (5*A*a^4*\sin(3*c + 3*d*x))/(48*d) + (A*a^4*\sin(5*c + 5*d*x))/(80*d) + (B*a^4*\sin(2*c + 2*d*x))/(4*d) + (B*a^4*\sin(4*c + 4*d*x))/(32*d) + (A*a*b^3*\sin(2*c + 2*d*x))/d + (A*a^3*b*\sin(2*c + 2*d*x))/d + (A*a^3*b*\sin(4*c + 4*d*x))/(8*d) + (9*A*a^2*b^2*\sin(c + d*x))/(2*d) + (B*a^3*b*\sin(3*c + 3*d*x))/(3*d) + (A*a^2*b^2*\sin(3*c + 3*d*x))/(2*d) + (3*B*a^2*b^2*\sin(2*c + 2*d*x))/(2*d) + (4*B*a*b^3*\sin(c + d*x))/d + (3*B*a^3*b*\sin(c + d*x))/d$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+b*sec(d*x+c))**4*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

$$3.310 \quad \int \cos^6(c+dx)(a+b \sec(c+dx))^4(A+B \sec(c+dx)) dx$$

Optimal. Leaf size=309

$$\frac{a^2(25a^2A + 72abB + 48Ab^2) \sin(c+dx) \cos^3(c+dx)}{120d} - \frac{a(4a^3B + 16a^2Ab + 27ab^2B + 13Ab^3) \sin^3(c+dx)}{15d} + \dots$$

[Out] $\frac{1}{16}*(5*A*a^4+36*A*a^2*b^2+8*A*b^4+24*B*a^3*b+32*B*a*b^3)*x + \frac{1}{15}*(48*A*a^3*b+53*A*a*b^3+12*B*a^4+87*B*a^2*b^2+15*B*b^4)*\sin(d*x+c)/d + \frac{1}{16}*(5*A*a^4+36*A*a^2*b^2+8*A*b^4+24*B*a^3*b+32*B*a*b^3)*\cos(d*x+c)*\sin(d*x+c)/d + \frac{1}{120}*a^2*(25*A*a^2+48*A*b^2+72*B*a*b)*\cos(d*x+c)^3*\sin(d*x+c)/d + \frac{1}{10}*a*(3*A*b+2*B*a)*\cos(d*x+c)^4*(a+b*\sec(d*x+c))^2*\sin(d*x+c)/d + \frac{1}{6}*a*A*\cos(d*x+c)^5*(a+b*\sec(d*x+c))^3*\sin(d*x+c)/d - \frac{1}{15}*a*(16*A*a^2*b+13*A*b^3+4*B*a^3+27*B*a*b^2)*\sin(d*x+c)^3/d$

Rubi [A] time = 0.82, antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {4025, 4094, 4074, 4047, 2635, 8, 4044, 3013}

$$\frac{a(16a^2Ab + 4a^3B + 27ab^2B + 13Ab^3) \sin^3(c+dx)}{15d} + \frac{(48a^3Ab + 87a^2b^2B + 12a^4B + 53aAb^3 + 15b^4B) \sin(c+dx)}{15d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Cos}[c + d*x]^6*(a + b*\text{Sec}[c + d*x])^4*(A + B*\text{Sec}[c + d*x]), x]$

[Out] $((5*a^4*A + 36*a^2*A*b^2 + 8*A*b^4 + 24*a^3*b*B + 32*a*b^3*B)*x)/16 + ((48*a^3*A*b + 53*a*A*b^3 + 12*a^4*B + 87*a^2*b^2*B + 15*b^4*B)*\text{Sin}[c + d*x])/(15*d) + ((5*a^4*A + 36*a^2*A*b^2 + 8*A*b^4 + 24*a^3*b*B + 32*a*b^3*B)*\text{Cos}[c + d*x]*\text{Sin}[c + d*x])/(16*d) + (a^2*(25*a^2*A + 48*A*b^2 + 72*a*b*B)*\text{Cos}[c + d*x]^3*\text{Sin}[c + d*x])/(120*d) + (a*(3*A*b + 2*a*B)*\text{Cos}[c + d*x]^4*(a + b*\text{Sec}[c + d*x])^2*\text{Sin}[c + d*x])/(10*d) + (a*A*\text{Cos}[c + d*x]^5*(a + b*\text{Sec}[c + d*x])^3*\text{Sin}[c + d*x])/(6*d) - (a*(16*a^2*A*b + 13*A*b^3 + 4*a^3*B + 27*a*b^2*B)*\text{Sin}[c + d*x]^3)/(15*d)$

Rule 8

$\text{Int}[a_, x_Symbol] \rightarrow \text{Simp}[a*x, x] /; \text{FreeQ}[a, x]$

Rule 2635

$\text{Int}(((b_.)*\sin[(c_.) + (d_.)*(x_.)])^{(n_.)}, x_Symbol) \rightarrow -\text{Simp}[(b*\text{Cos}[c + d*x])* (b*\text{Sin}[c + d*x])^{(n-1)}]/(d*n), x] + \text{Dist}[(b^2*(n-1))/n, \text{Int}[(b*\text{Sin}[c + d*x])^{(n-2)}, x], x] /; \text{FreeQ}\{b, c, d\}, x] \&\& \text{GtQ}[n, 1] \&\& \text{IntegerQ}[2*n]$

]

Rule 3013

Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2),
 x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]

Rule 4025

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^m*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] :> Simp[(a*A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e + f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LeQ[n, -1]

Rule 4044

Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_.)), x_Symbol] :> Int[(C + A*Sin[e + f*x]^2)/Sin[e + f*x]^(m + 2), x] /; FreeQ[{e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && ILtQ[(m + 1)/2, 0]

Rule 4047

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^m*(A_. + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2), x] /; FreeQ[{b, e, f, A, B, C, m}, x]

Rule 4074

Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] :> Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]

Rule 4094

Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] :> Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]

```

_)^(m_), x_Symbol] := Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
+ f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]

```

Rubi steps

$$\begin{aligned}
 \int \cos^6(c + dx)(a + b \sec(c + dx))^4(A + B \sec(c + dx)) dx &= \frac{aA \cos^5(c + dx)(a + b \sec(c + dx))^3 \sin(c + dx)}{6d} \\
 &= \frac{a(3Ab + 2aB) \cos^4(c + dx)(a + b \sec(c + dx))^2 \sin(c + dx)}{10d} \\
 &= \frac{a^2 (25a^2 A + 48Ab^2 + 72abB) \cos^3(c + dx) \sin(c + dx)}{120d} \\
 &= \frac{a^2 (25a^2 A + 48Ab^2 + 72abB) \cos^3(c + dx) \sin(c + dx)}{120d} \\
 &= \frac{(5a^4 A + 36a^2 Ab^2 + 8Ab^4 + 24a^3 bB + 32ab^3 B) \cos^3(c + dx) \sin(c + dx)}{16d} \\
 &= \frac{1}{16} (5a^4 A + 36a^2 Ab^2 + 8Ab^4 + 24a^3 bB + 32ab^3 B) \cos^3(c + dx) \sin(c + dx) \\
 &= \frac{1}{16} (5a^4 A + 36a^2 Ab^2 + 8Ab^4 + 24a^3 bB + 32ab^3 B) \cos^3(c + dx) \sin(c + dx)
 \end{aligned}$$

Mathematica [A] time = 1.24, size = 333, normalized size = 1.08

$$\frac{45a^4 A \sin(4(c + dx)) + 5a^4 A \sin(6(c + dx)) + 300a^4 A c + 300a^4 A dx + 100a^4 B \sin(3(c + dx)) + 12a^4 B \sin(5(c + dx))}{16}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]^6*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]

[Out] (300*a^4*A*c + 2160*a^2*A*b^2*c + 480*A*b^4*c + 1440*a^3*b*B*c + 1920*a*b^3*B*c + 300*a^4*A*d*x + 2160*a^2*A*b^2*d*x + 480*A*b^4*d*x + 1440*a^3*b*B*d*x + 1920*a*b^3*B*d*x + 120*(20*a^3*A*b + 24*a*A*b^3 + 5*a^4*B + 36*a^2*b^2*B + 8*b^4*B)*Sin[c + d*x] + 15*(15*a^4*A + 96*a^2*A*b^2 + 16*A*b^4 + 64*a^3*b*B + 64*a*b^3*B)*Sin[2*(c + d*x)] + 400*a^3*A*b*Ssin[3*(c + d*x)] + 320*a*A*b^3*Ssin[3*(c + d*x)] + 100*a^4*B*Ssin[3*(c + d*x)] + 480*a^2*b^2*B*Ssin[3*(c + d*x)] + 45*a^4*A*Ssin[4*(c + d*x)] + 180*a^2*A*b^2*Ssin[4*(c + d*x)] + 12

$$0*a^3*b*B*\sin[4*(c + d*x)] + 48*a^3*A*b*\sin[5*(c + d*x)] + 12*a^4*B*\sin[5*(c + d*x)] + 5*a^4*A*\sin[6*(c + d*x)]/(960*d)$$

fricas [A] time = 0.47, size = 243, normalized size = 0.79

$$15 \left(5 Aa^4 + 24 Ba^3b + 36 Aa^2b^2 + 32 Bab^3 + 8 Ab^4 \right) dx + \left(40 Aa^4 \cos(dx + c)^5 + 128 Ba^4 + 512 Aa^3b + 960 Ba^2b^2 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^6*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] 1/240*(15*(5*A*a^4 + 24*B*a^3*b + 36*A*a^2*b^2 + 32*B*a*b^3 + 8*A*b^4)*d*x + (40*A*a^4*cos(d*x + c)^5 + 128*B*a^4 + 512*A*a^3*b + 960*B*a^2*b^2 + 640*A*a*b^3 + 240*B*b^4 + 48*(B*a^4 + 4*A*a^3*b)*cos(d*x + c)^4 + 10*(5*A*a^4 + 24*B*a^3*b + 36*A*a^2*b^2)*cos(d*x + c)^3 + 32*(2*B*a^4 + 8*A*a^3*b + 15*B*a^2*b^2 + 10*A*a*b^3)*cos(d*x + c)^2 + 15*(5*A*a^4 + 24*B*a^3*b + 36*A*a^2*b^2 + 32*B*a*b^3 + 8*A*b^4)*cos(d*x + c))*sin(d*x + c))/d

giac [B] time = 3.05, size = 1127, normalized size = 3.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^6*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] 1/240*(15*(5*A*a^4 + 24*B*a^3*b + 36*A*a^2*b^2 + 32*B*a*b^3 + 8*A*b^4)*(d*x + c) - 2*(165*A*a^4*tan(1/2*d*x + 1/2*c)^11 - 240*B*a^4*tan(1/2*d*x + 1/2*c)^11 - 960*A*a^3*b*tan(1/2*d*x + 1/2*c)^11 + 600*B*a^3*b*tan(1/2*d*x + 1/2*c)^11 + 900*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^11 - 1440*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^11 - 960*A*a*b^3*tan(1/2*d*x + 1/2*c)^11 + 480*B*a*b^3*tan(1/2*d*x + 1/2*c)^11 + 120*A*b^4*tan(1/2*d*x + 1/2*c)^11 - 240*B*b^4*tan(1/2*d*x + 1/2*c)^11 - 25*A*a^4*tan(1/2*d*x + 1/2*c)^9 - 560*B*a^4*tan(1/2*d*x + 1/2*c)^9 - 2240*A*a^3*b*tan(1/2*d*x + 1/2*c)^9 + 840*B*a^3*b*tan(1/2*d*x + 1/2*c)^9 + 1260*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^9 - 5280*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^9 - 3520*A*a*b^3*tan(1/2*d*x + 1/2*c)^9 + 1440*B*a*b^3*tan(1/2*d*x + 1/2*c)^9 + 360*A*b^4*tan(1/2*d*x + 1/2*c)^9 - 1200*B*b^4*tan(1/2*d*x + 1/2*c)^9 + 450*A*a^4*tan(1/2*d*x + 1/2*c)^7 - 1248*B*a^4*tan(1/2*d*x + 1/2*c)^7 - 4992*A*a^3*b*tan(1/2*d*x + 1/2*c)^7 + 240*B*a^3*b*tan(1/2*d*x + 1/2*c)^7 + 360*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 - 8640*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 - 5760*A*a*b^3*tan(1/2*d*x + 1/2*c)^7 + 960*B*a*b^3*tan(1/2*d*x + 1/2*c)^7 + 240*A*b^4*tan(1/2*d*x + 1/2*c)^7 - 2400*B*b^4*tan(1/2*d*x + 1/2*c)^7 - 450*A*a^4*tan(1/2*d*x + 1/2*c)^5 - 1248*B*a^4*tan(1/2*d*x + 1/2*c)^5 -

$$4992A^3b^3\tan(1/2dx + 1/2c)^5 - 240B^3a^3b^3\tan(1/2dx + 1/2c)^5 - 360A^2b^2\tan(1/2dx + 1/2c)^5 - 8640B^2a^2b^2\tan(1/2dx + 1/2c)^5 - 5760A^2b^3\tan(1/2dx + 1/2c)^5 - 960B^2a^2b^3\tan(1/2dx + 1/2c)^5 - 240Ab^4\tan(1/2dx + 1/2c)^5 - 2400B^2b^4\tan(1/2dx + 1/2c)^5 + 25A^4\tan(1/2dx + 1/2c)^3 - 560B^4\tan(1/2dx + 1/2c)^3 - 2240A^3b\tan(1/2dx + 1/2c)^3 - 840B^3a^3b\tan(1/2dx + 1/2c)^3 - 1260A^2b^2\tan(1/2dx + 1/2c)^3 - 5280B^2a^2b^2\tan(1/2dx + 1/2c)^3 - 3520A^2b^3\tan(1/2dx + 1/2c)^3 - 1440B^2a^2b^3\tan(1/2dx + 1/2c)^3 - 360Ab^4\tan(1/2dx + 1/2c)^3 - 1200B^2b^4\tan(1/2dx + 1/2c)^3 - 165A^4\tan(1/2dx + 1/2c) - 240B^4\tan(1/2dx + 1/2c) - 960A^3b\tan(1/2dx + 1/2c) - 600B^3a^3b\tan(1/2dx + 1/2c) - 900A^2b^2\tan(1/2dx + 1/2c) - 1440B^2a^2b^2\tan(1/2dx + 1/2c) - 960A^2b^3\tan(1/2dx + 1/2c) - 480B^2a^2b^3\tan(1/2dx + 1/2c) - 120Ab^4\tan(1/2dx + 1/2c) - 240B^2b^4\tan(1/2dx + 1/2c))/(\tan(1/2dx + 1/2c)^2 + 1)^6/d$$

maple [A] time = 1.85, size = 316, normalized size = 1.02

$$Aa^4 \left(\frac{\left(\cos^5(dx+c) + \frac{5(\cos^3(dx+c))}{4} + \frac{15\cos(dx+c)}{8} \right) \sin(dx+c)}{6} + \frac{5dx}{16} + \frac{5c}{16} \right) + \frac{a^4 B \left(\frac{8}{3} + \cos^4(dx+c) + \frac{4(\cos^2(dx+c))}{3} \right) \sin(dx+c)}{5} + \frac{4Aa^3b \left(\frac{8}{3} + \cos^4(dx+c) \right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^6*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)), x)

[Out] 1/d*(A*a^4*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/16*d*x+5/16*c)+1/5*a^4*B*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+4/5*A*a^3*b*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+4*B*a^3*b*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+6*A*a^2*b^2*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+2*a^2*b^2*B*(2+cos(d*x+c)^2)*sin(d*x+c)+4/3*a*A*b^3*(2+cos(d*x+c)^2)*sin(d*x+c)+4*B*a^2*b^3*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+A*b^4*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+B*b^4*sin(d*x+c))

maxima [A] time = 0.87, size = 307, normalized size = 0.99

$$\frac{5(4 \sin(2dx + 2c)^3 - 60dx - 60c - 9 \sin(4dx + 4c) - 48 \sin(2dx + 2c))Aa^4 - 64(3 \sin(dx + c)^5 - 10 \sin(dx + c))B^2a^2b^2}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^6*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)), x, algorithm="maxima")

```
[Out] -1/960*(5*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*
sin(2*d*x + 2*c))*A*a^4 - 64*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(
d*x + c))*B*a^4 - 256*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x +
c))*A*a^3*b - 120*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B
*a^3*b - 180*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^2*
b^2 + 1920*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^2*b^2 + 1280*(sin(d*x + c)
^3 - 3*sin(d*x + c))*A*a*b^3 - 960*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a*b^3
- 240*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*b^4 - 960*B*b^4*sin(d*x + c))/d
```

mupad [B] time = 3.18, size = 403, normalized size = 1.30

$$\frac{5 A a^4 x}{16} + \frac{A b^4 x}{2} + 2 B a b^3 x + \frac{3 B a^3 b x}{2} + \frac{5 B a^4 \sin(c + d x)}{8 d} + \frac{B b^4 \sin(c + d x)}{d} + \frac{9 A a^2 b^2 x}{4} + \frac{15 A a^4 \sin(2 c + 2 d x)}{64 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^4, x)
```

```
[Out] (5*A*a^4*x)/16 + (A*b^4*x)/2 + 2*B*a*b^3*x + (3*B*a^3*b*x)/2 + (5*B*a^4*sin
(c + d*x))/(8*d) + (B*b^4*sin(c + d*x))/d + (9*A*a^2*b^2*x)/4 + (15*A*a^4*sin
in(2*c + 2*d*x))/(64*d) + (3*A*a^4*sin(4*c + 4*d*x))/(64*d) + (A*a^4*sin(6*
c + 6*d*x))/(192*d) + (A*b^4*sin(2*c + 2*d*x))/(4*d) + (5*B*a^4*sin(3*c + 3
*d*x))/(48*d) + (B*a^4*sin(5*c + 5*d*x))/(80*d) + (A*a*b^3*sin(3*c + 3*d*x)
)/(3*d) + (5*A*a^3*b*sin(3*c + 3*d*x))/(12*d) + (A*a^3*b*sin(5*c + 5*d*x))/
(20*d) + (B*a*b^3*sin(2*c + 2*d*x))/d + (B*a^3*b*sin(2*c + 2*d*x))/d + (B*a
^3*b*sin(4*c + 4*d*x))/(8*d) + (9*B*a^2*b^2*sin(c + d*x))/(2*d) + (3*A*a^2*
b^2*sin(2*c + 2*d*x))/(2*d) + (3*A*a^2*b^2*sin(4*c + 4*d*x))/(16*d) + (B*a
^2*b^2*sin(3*c + 3*d*x))/(2*d) + (3*A*a*b^3*sin(c + d*x))/d + (5*A*a^3*b*sin
(c + d*x))/(2*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+b*sec(d*x+c))**4*(A+B*sec(d*x+c)), x)
```

```
[Out] Timed out
```

$$3.311 \quad \int \frac{\sec^4(c+dx)(A+B \sec(c+dx))}{a+b \sec(c+dx)} dx$$

Optimal. Leaf size=187

$$\frac{2a^3(Ab - aB) \tanh^{-1}\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^4 d \sqrt{a-b} \sqrt{a+b}} + \frac{(2a^2 + b^2)(Ab - aB) \tanh^{-1}(\sin(c + dx))}{2b^4 d} - \frac{(-3a^2B + 3aAb - 2b^2B)}{3b^3 d}$$

[Out] 1/2*(2*a^2+b^2)*(A*b-B*a)*arctanh(sin(d*x+c))/b^4/d-2*a^3*(A*b-B*a)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/b^4/d/(a-b)^(1/2)/(a+b)^(1/2)-1/3*(3*A*a*b-3*B*a^2-2*B*b^2)*tan(d*x+c)/b^3/d+1/2*(A*b-B*a)*sec(d*x+c)*tan(d*x+c)/b^2/d+1/3*B*sec(d*x+c)^2*tan(d*x+c)/b/d

Rubi [A] time = 0.68, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {4033, 4092, 4082, 3998, 3770, 3831, 2659, 208}

$$\frac{(-3a^2B + 3aAb - 2b^2B) \tan(c + dx)}{3b^3 d} + \frac{(2a^2 + b^2)(Ab - aB) \tanh^{-1}(\sin(c + dx))}{2b^4 d} - \frac{2a^3(Ab - aB) \tanh^{-1}\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^4 d \sqrt{a-b} \sqrt{a+b}}$$

Antiderivative was successfully verified.

[In] Int[(Sec[c + d*x]^4*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x]),x]

[Out] ((2*a^2 + b^2)*(A*b - a*B)*ArcTanh[Sin[c + d*x]]/(2*b^4*d) - (2*a^3*(A*b - a*B)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b^4*Sqrt[a + b]*d) - ((3*a*A*b - 3*a^2*B - 2*b^2*B)*Tan[c + d*x])/(3*b^3*d) + ((A*b - a*B)*Sec[c + d*x]*Tan[c + d*x])/(2*b^2*d) + (B*Sec[c + d*x]^2*Tan[c + d*x])/(3*b*d)

Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 2659

Int[((a_) + (b_)*sin[Pi/2 + (c_) + (d_)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 3770

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x]
/; FreeQ[{c, d}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)))/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := Dist[B/b, Int[Csc[e + f*x], x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 4033

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] := -Simp[(B*d^2 * Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2))/(b*f*(m + n)), x] + Dist[d^2/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 2)*Simp[a*B*(n - 2) + B*b*(m + n - 1)*Csc[e + f*x] + (A*b*(m + n) - a*B*(n - 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && NeQ[m + n, 0] && !IGtQ[m, 1]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_), x_Symbol] := -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]
```

Rule 4092

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_), x_Symbol] := -Simp[(C*Csc[e + f*x]*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[a*C + b*(C*(m + 2) + A*(m + 3))*Csc[e + f*x] - (2*a*C - b*B*(m
```

+ 3)) * Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && N
eQ[a^2 - b^2, 0] && !LtQ[m, -1]

Rubi steps

$$\begin{aligned}
 \int \frac{\sec^4(c+dx)(A+B\sec(c+dx))}{a+b\sec(c+dx)} dx &= \frac{B\sec^2(c+dx)\tan(c+dx)}{3bd} + \frac{\int \frac{\sec^2(c+dx)(2aB+2bB\sec(c+dx)+3(Ab-aB)\sec^2(c+dx))}{a+b\sec(c+dx)} dx}{3b} \\
 &= \frac{(Ab-aB)\sec(c+dx)\tan(c+dx)}{2b^2d} + \frac{B\sec^2(c+dx)\tan(c+dx)}{3bd} + \int \frac{\sec^2(c+dx)(2aB+2bB\sec(c+dx)+3(Ab-aB)\sec^2(c+dx))}{a+b\sec(c+dx)} dx \\
 &= -\frac{(3aAb-3a^2B-2b^2B)\tan(c+dx)}{3b^3d} + \frac{(Ab-aB)\sec(c+dx)\tan(c+dx)}{2b^2d} \\
 &= -\frac{(3aAb-3a^2B-2b^2B)\tan(c+dx)}{3b^3d} + \frac{(Ab-aB)\sec(c+dx)\tan(c+dx)}{2b^2d} \\
 &= \frac{(2a^2+b^2)(Ab-aB)\tanh^{-1}(\sin(c+dx))}{2b^4d} - \frac{(3aAb-3a^2B-2b^2B)\tan(c+dx)}{3b^3d} \\
 &= \frac{(2a^2+b^2)(Ab-aB)\tanh^{-1}(\sin(c+dx))}{2b^4d} - \frac{(3aAb-3a^2B-2b^2B)\tan(c+dx)}{3b^3d} \\
 &= \frac{(2a^2+b^2)(Ab-aB)\tanh^{-1}(\sin(c+dx))}{2b^4d} - \frac{2a^3(Ab-aB)\tanh^{-1}\left(\frac{\sqrt{a-b}\sqrt{a+b}}{\sqrt{a-b}b^4\sqrt{a+b}}\right)}{\sqrt{a-b}b^4\sqrt{a+b}}
 \end{aligned}$$

Mathematica [B] time = 3.01, size = 422, normalized size = 2.26

$$\frac{4b(3a^2B-3aAb+2b^2B)\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)} + \frac{4b(3a^2B-3aAb+2b^2B)\sin\left(\frac{1}{2}(c+dx)\right)}{\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)} + 6(2a^2+b^2)(aB-Ab)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) - \sin\left(\frac{1}{2}(c+dx)\right)$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[c + d*x]^4*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x]), x]

[Out] ((24*a^3*(A*b - a*B)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] + 6*(2*a^2 + b^2)*(-(A*b) + a*B)*Log[Cos[(c + d*x)/2] - Sin

$$\begin{aligned} & \left[\frac{(c + dx)/2}{2} \right] - 6 \cdot (2a^2 + b^2) \cdot (-A \cdot b + a \cdot B) \cdot \text{Log} \left[\frac{\cos[(c + dx)/2] + \sin[(c + dx)/2]}{\cos[(c + dx)/2] - \sin[(c + dx)/2]} \right] \\ & + \frac{(b^2 \cdot (3A \cdot b + (-3a + b) \cdot B))}{(\cos[(c + dx)/2] - \sin[(c + dx)/2])^2} + \frac{(2 \cdot b^3 \cdot B \cdot \sin[(c + dx)/2])}{(\cos[(c + dx)/2] - \sin[(c + dx)/2])^3} \\ & + \frac{(4 \cdot b \cdot (-3a \cdot A \cdot b + 3a^2 \cdot B + 2 \cdot b^2 \cdot B) \cdot \sin[(c + dx)/2])}{(\cos[(c + dx)/2] - \sin[(c + dx)/2])} \\ & + \frac{(2 \cdot b^3 \cdot B \cdot \sin[(c + dx)/2])}{(\cos[(c + dx)/2] + \sin[(c + dx)/2])} - \frac{(b^2 \cdot (3A \cdot b + (-3a + b) \cdot B))}{(\cos[(c + dx)/2] + \sin[(c + dx)/2])^2} \\ & + \frac{(4 \cdot b \cdot (-3a \cdot A \cdot b + 3a^2 \cdot B + 2 \cdot b^2 \cdot B) \cdot \sin[(c + dx)/2])}{(\cos[(c + dx)/2] + \sin[(c + dx)/2])} \end{aligned}$$

fricas [B] time = 1.18, size = 743, normalized size = 3.97

$$\left[\frac{6 \left(B a^4 - A a^3 b \right) \sqrt{a^2 - b^2} \cos(dx + c)^3 \log \left(\frac{2 a b \cos(dx + c) - (a^2 - 2 b^2) \cos(dx + c)^2 - 2 \sqrt{a^2 - b^2} (b \cos(dx + c) + a) \sin(dx + c) + 2 a^2 - b^2}{a^2 \cos(dx + c)^2 + 2 a b \cos(dx + c) + b^2} \right) + \dots \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(dx+c)^4*(A+B*sec(dx+c))/(a+b*sec(dx+c)),x, algorithm="fricas")

[Out]
$$\begin{aligned} & \left[-\frac{1}{12} \cdot (6 \cdot (B \cdot a^4 - A \cdot a^3 \cdot b) \cdot \sqrt{a^2 - b^2} \cdot \cos(dx + c)^3 \cdot \log \left(\frac{2 \cdot a \cdot b \cdot \cos(dx + c) - (a^2 - 2 \cdot b^2) \cdot \cos(dx + c)^2 - 2 \cdot \sqrt{a^2 - b^2} \cdot (b \cdot \cos(dx + c) + a) \cdot \sin(dx + c) + 2 \cdot a^2 - b^2}{a^2 \cdot \cos(dx + c)^2 + 2 \cdot a \cdot b \cdot \cos(dx + c) + b^2} \right) \right. \\ & + 3 \cdot (2 \cdot B \cdot a^5 - 2 \cdot A \cdot a^4 \cdot b - B \cdot a^3 \cdot b^2 + A \cdot a^2 \cdot b^3 - B \cdot a \cdot b^4 + A \cdot b^5) \cdot \cos(dx + c)^3 \cdot \log(\sin(dx + c) + 1) \\ & - 3 \cdot (2 \cdot B \cdot a^5 - 2 \cdot A \cdot a^4 \cdot b - B \cdot a^3 \cdot b^2 + A \cdot a^2 \cdot b^3 - B \cdot a \cdot b^4 + A \cdot b^5) \cdot \cos(dx + c)^3 \cdot \log(-\sin(dx + c) + 1) \\ & - 2 \cdot (2 \cdot B \cdot a^2 \cdot b^3 - 2 \cdot B \cdot b^5 + 2 \cdot (3 \cdot B \cdot a^4 \cdot b - 3 \cdot A \cdot a^3 \cdot b^2 - B \cdot a^2 \cdot b^3 + 3 \cdot A \cdot a \cdot b^4 - 2 \cdot B \cdot b^5) \cdot \cos(dx + c)^2 \\ & - 3 \cdot (B \cdot a^3 \cdot b^2 - A \cdot a^2 \cdot b^3 - B \cdot a \cdot b^4 + A \cdot b^5) \cdot \cos(dx + c)) \cdot \sin(dx + c) \left. \right] / ((a^2 \cdot b^4 - b^6) \cdot d \cdot \cos(dx + c)^3) \\ & + \frac{1}{12} \cdot (12 \cdot (B \cdot a^4 - A \cdot a^3 \cdot b) \cdot \sqrt{-a^2 + b^2} \cdot \arctan(-\sqrt{-a^2 + b^2} \cdot (b \cdot \cos(dx + c) + a) / ((a^2 - b^2) \cdot \sin(dx + c))) \cdot \cos(dx + c)^3 \\ & - 3 \cdot (2 \cdot B \cdot a^5 - 2 \cdot A \cdot a^4 \cdot b - B \cdot a^3 \cdot b^2 + A \cdot a^2 \cdot b^3 - B \cdot a \cdot b^4 + A \cdot b^5) \cdot \cos(dx + c)^3 \cdot \log(\sin(dx + c) + 1) \\ & + 3 \cdot (2 \cdot B \cdot a^5 - 2 \cdot A \cdot a^4 \cdot b - B \cdot a^3 \cdot b^2 + A \cdot a^2 \cdot b^3 - B \cdot a \cdot b^4 + A \cdot b^5) \cdot \cos(dx + c)^3 \cdot \log(-\sin(dx + c) + 1) \\ & + 2 \cdot (2 \cdot B \cdot a^2 \cdot b^3 - 2 \cdot B \cdot b^5 + 2 \cdot (3 \cdot B \cdot a^4 \cdot b - 3 \cdot A \cdot a^3 \cdot b^2 - B \cdot a^2 \cdot b^3 + 3 \cdot A \cdot a \cdot b^4 - 2 \cdot B \cdot b^5) \cdot \cos(dx + c)^2 \\ & - 3 \cdot (B \cdot a^3 \cdot b^2 - A \cdot a^2 \cdot b^3 - B \cdot a \cdot b^4 + A \cdot b^5) \cdot \cos(dx + c)) \cdot \sin(dx + c) \left. \right] / ((a^2 \cdot b^4 - b^6) \cdot d \cdot \cos(dx + c)^3) \end{aligned}$$

giac [B] time = 0.28, size = 412, normalized size = 2.20

$$\frac{3 \left(2 B a^3 - 2 A a^2 b + B a b^2 - A b^3 \right) \log \left(\left| \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) + 1 \right| \right)}{b^4} - \frac{3 \left(2 B a^3 - 2 A a^2 b + B a b^2 - A b^3 \right) \log \left(\left| \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) - 1 \right| \right)}{b^4} - \frac{12 \left(B a^4 - A a^3 b \right) \left(\pi \left[\frac{dx + c}{2\pi} + \frac{1}{2} \right] \right) \operatorname{sgn} \left(\frac{dx + c}{2\pi} + \frac{1}{2} \right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="giac")

[Out]
$$\begin{aligned} & -1/6*(3*(2*B*a^3 - 2*A*a^2*b + B*a*b^2 - A*b^3)*\log(\text{abs}(\tan(1/2*d*x + 1/2*c) + 1))/b^4 - 3*(2*B*a^3 - 2*A*a^2*b + B*a*b^2 - A*b^3)*\log(\text{abs}(\tan(1/2*d*x + 1/2*c) - 1))/b^4 - 12*(B*a^4 - A*a^3*b)*(pi*\text{floor}(1/2*(d*x + c)/pi + 1/2)*\text{sgn}(-2*a + 2*b) + \arctan(-(a*\tan(1/2*d*x + 1/2*c) - b*\tan(1/2*d*x + 1/2*c))/\sqrt{-a^2 + b^2}))/(\sqrt{-a^2 + b^2}*b^4) + 2*(6*B*a^2*\tan(1/2*d*x + 1/2*c)^5 - 6*A*a*b*\tan(1/2*d*x + 1/2*c)^5 + 3*B*a*b*\tan(1/2*d*x + 1/2*c)^5 - 3*A*b^2*\tan(1/2*d*x + 1/2*c)^5 + 6*B*b^2*\tan(1/2*d*x + 1/2*c)^5 - 12*B*a^2*\tan(1/2*d*x + 1/2*c)^3 + 12*A*a*b*\tan(1/2*d*x + 1/2*c)^3 - 4*B*b^2*\tan(1/2*d*x + 1/2*c)^3 + 6*B*a^2*\tan(1/2*d*x + 1/2*c) - 6*A*a*b*\tan(1/2*d*x + 1/2*c) - 3*B*a*b*\tan(1/2*d*x + 1/2*c) + 3*A*b^2*\tan(1/2*d*x + 1/2*c) + 6*B*b^2*\tan(1/2*d*x + 1/2*c))/((\tan(1/2*d*x + 1/2*c)^2 - 1)^3*b^3))/d \end{aligned}$$

maple [B] time = 0.58, size = 688, normalized size = 3.68

$$\frac{2a^4 \operatorname{arctanh}\left(\frac{\tan\left(\frac{dx}{2} + \frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right) B}{db^4 \sqrt{(a-b)(a+b)}} - \frac{2a^3 \operatorname{arctanh}\left(\frac{\tan\left(\frac{dx}{2} + \frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right) A}{db^3 \sqrt{(a-b)(a+b)}} - \frac{\ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right) a^3 B}{db^4} - \frac{aB}{2db^2 \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x)

[Out]
$$\begin{aligned} & 2/d*a^4/b^4/((a-b)*(a+b))^{(1/2)}*\operatorname{arctanh}(\tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^{(1/2)})*B - 2/d*a^3/b^3/((a-b)*(a+b))^{(1/2)}*\operatorname{arctanh}(\tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^{(1/2)})*A - 1/d/b^4*\ln(\tan(1/2*d*x+1/2*c)+1)*a^3*B - 1/2/d/b^2/(\tan(1/2*d*x+1/2*c)-1)^2*a*B + 1/2/d/b/(\tan(1/2*d*x+1/2*c)+1)^2*B + 1/2/d/b*\ln(\tan(1/2*d*x+1/2*c)+1)*A + 1/2/d/b/(\tan(1/2*d*x+1/2*c)+1)*A - 1/d/b/(\tan(1/2*d*x+1/2*c)+1)*B + 1/2/d/b/(\tan(1/2*d*x+1/2*c)-1)*A - 1/d/b/(\tan(1/2*d*x+1/2*c)-1)*B - 1/3/d*B/b/(\tan(1/2*d*x+1/2*c)+1)^3 - 1/2/d/b*\ln(\tan(1/2*d*x+1/2*c)-1)*A + 1/d/b^2/(\tan(1/2*d*x+1/2*c)-1)*A*a - 1/3/d*B/b/(\tan(1/2*d*x+1/2*c)-1)^3 + 1/2/d/b/(\tan(1/2*d*x+1/2*c)-1)^2*A - 1/2/d/b/(\tan(1/2*d*x+1/2*c)-1)^2*B - 1/2/d/b/(\tan(1/2*d*x+1/2*c)+1)^2*A + 1/d/b^3*\ln(\tan(1/2*d*x+1/2*c)+1)*A*a^2 + 1/2/d/b^2/(\tan(1/2*d*x+1/2*c)+1)^2*a*B - 1/d/b^3/(\tan(1/2*d*x+1/2*c)+1)*a^2*B - 1/2/d/b^2/(\tan(1/2*d*x+1/2*c)-1)*B*a - 1/d/b^3*\ln(\tan(1/2*d*x+1/2*c)-1)*A*a^2 - 1/d/b^3/(\tan(1/2*d*x+1/2*c)-1)*a^2*B + 1/2/d/b^2*\ln(\tan(1/2*d*x+1/2*c)-1)*B*a - 1/2/d/b^2*\ln(\tan(1/2*d*x+1/2*c)+1)*B*a + 1/d/b^2/(\tan(1/2*d*x+1/2*c)+1)*A*a - 1/2/d/b^2/(\tan(1/2*d*x+1/2*c)+1)*B*a + 1/d/b^4*\ln(\tan(1/2*d*x+1/2*c)-1)*a^3*B \end{aligned}$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 6.93, size = 4667, normalized size = 24.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^4*(a + b/cos(c + d*x))),x)
```

```
[Out] - ((tan(c/2 + (d*x)/2)*(A*b^2 + 2*B*a^2 + 2*B*b^2 - 2*A*a*b - B*a*b))/b^3 + (tan(c/2 + (d*x)/2)^5*(2*B*a^2 - A*b^2 + 2*B*b^2 - 2*A*a*b + B*a*b))/b^3 - (4*tan(c/2 + (d*x)/2)^3*(3*B*a^2 + B*b^2 - 3*A*a*b))/(3*b^3))/(d*(3*tan(c/2 + (d*x)/2)^2 - 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1)) - (atan((((8*tan(c/2 + (d*x)/2)*(A^2*b^9 - 8*B^2*a^9 - 3*A^2*a*b^8 + 16*B^2*a^8*b + 7*A^2*a^2*b^7 - 13*A^2*a^3*b^6 + 16*A^2*a^4*b^5 - 16*A^2*a^5*b^4 + 16*A^2*a^6*b^3 - 8*A^2*a^7*b^2 + B^2*a^2*b^7 - 3*B^2*a^3*b^6 + 7*B^2*a^4*b^5 - 13*B^2*a^5*b^4 + 16*B^2*a^6*b^3 - 16*B^2*a^7*b^2 - 2*A*B*a*b^8 + 16*A*B*a^8*b + 6*A*B*a^2*b^7 - 14*A*B*a^3*b^6 + 26*A*B*a^4*b^5 - 32*A*B*a^5*b^4 + 32*A*B*a^6*b^3 - 32*A*B*a^7*b^2))/b^6 - (((8*(2*A*b^13 + 2*A*a^2*b^11 - 6*A*a^3*b^10 + 4*A*a^4*b^9 + 2*B*a^2*b^11 - 2*B*a^3*b^10 + 6*B*a^4*b^9 - 4*B*a^5*b^8 - 2*A*a*b^12 - 2*B*a*b^12))/b^9 - (4*tan(c/2 + (d*x)/2)*(8*a*b^10 - 16*a^2*b^9 + 8*a^3*b^8)*(A*b^3 - 2*B*a^3 + 2*A*a^2*b - B*a*b^2))/b^10)*(A*b^3 - 2*B*a^3 + 2*A*a^2*b - B*a*b^2))/(2*b^4))*(A*b^3 - 2*B*a^3 + 2*A*a^2*b - B*a*b^2)*1i)/(2*b^4) + (((8*tan(c/2 + (d*x)/2)*(A^2*b^9 - 8*B^2*a^9 - 3*A^2*a*b^8 + 16*B^2*a^8*b + 7*A^2*a^2*b^7 - 13*A^2*a^3*b^6 + 16*A^2*a^4*b^5 - 16*A^2*a^5*b^4 + 16*A^2*a^6*b^3 - 8*A^2*a^7*b^2 + B^2*a^2*b^7 - 3*B^2*a^3*b^6 + 7*B^2*a^4*b^5 - 13*B^2*a^5*b^4 + 16*B^2*a^6*b^3 - 16*B^2*a^7*b^2 - 2*A*B*a*b^8 + 16*A*B*a^8*b + 6*A*B*a^2*b^7 - 14*A*B*a^3*b^6 + 26*A*B*a^4*b^5 - 32*A*B*a^5*b^4 + 32*A*B*a^6*b^3 - 32*A*B*a^7*b^2))/b^6 + (((8*(2*A*b^13 + 2*A*a^2*b^11 - 6*A*a^3*b^10 + 4*A*a^4*b^9 + 2*B*a^2*b^11 - 2*B*a^3*b^10 + 6*B*a^4*b^9 - 4*B*a^5*b^8 - 2*A*a*b^12 - 2*B*a*b^12))/b^9 + (4*tan(c/2 + (d*x)/2)*(8*a*b^10 - 16*a^2*b^9 + 8*a^3*b^8)*(A*b^3 - 2*B*a^3 + 2*A*a^2*b - B*a*b^2))/b^10)*(A*b^3 - 2*B*a^3 + 2*A*a^2*b - B*a*b^2))/(2*b^4))*(A*b^3 - 2*B*a^3 + 2*A*a^2*b - B*a*b^2)*1i)/(2*b^4))/((16*(4*B^3*a^11 - 6*B^3*a^10*b + A^3*a^3*b^8 - 2*A^3*a^4*b^7 + 5*A^3*a^5*b^6 - 6*A^3*a^6*b^5 + 6*A^3*a^7*b^4 - 4*A^3*a^8*b^3 - B^3*a^6*b^5 + 2*B^3*a^7*b^4 - 5*B^3*a^8*b^3 + 6*B^3*a^9*b^2 - 12*A*B^2*a^10*b + 3*A*B^2*a^5*b^6 - 6*A*B^2*a^6*b^5 + 15*A*B^2*a^7*b^4
```


$$\begin{aligned}
& - 18AB^2a^8b^3 + 18AB^2a^9b^2 - 3A^2B^4a^4b^7 + 6A^2B^5a^5b^6 \\
& - 15A^2B^6a^6b^5 + 18A^2B^7a^7b^4 - 18A^2B^8a^8b^3 + 12A^2B^9a^9b^2 \\
&))/b^9 - (((8\tan(c/2 + (d*x)/2)*(A^2b^9 - 8B^2a^9 - 3A^2a^8b + 16B^2a^8b + 7A^2a^2b^7 - 13A^2a^3b^6 + 16A^2a^4b^5 - 16A^2a^5b^4 \\
& + 16A^2a^6b^3 - 8A^2a^7b^2 + B^2a^2b^7 - 3B^2a^3b^6 + 7B^2a^4b^5 - 13B^2a^5b^4 + 16B^2a^6b^3 - 16B^2a^7b^2 - 2AB^4a^8b + 16A \\
& *B^5a^8b + 6AB^6a^2b^7 - 14AB^7a^3b^6 + 26AB^8a^4b^5 - 32AB^9a^5b^4 \\
& + 32AB^6a^6b^3 - 32AB^7a^7b^2))/b^6 - (((8*(2Ab^13 + 2Aa^2b^11 - \\
& 6Aa^3b^10 + 4Aa^4b^9 + 2Ba^2b^11 - 2Ba^3b^10 + 6Ba^4b^9 - 4B \\
& *a^5b^8 - 2Aa^6b^12 - 2Ba^6b^12))/b^9 - (4\tan(c/2 + (d*x)/2)*(8a^6b^10 \\
& - 16a^2b^9 + 8a^3b^8)*(Ab^3 - 2Ba^3 + 2Aa^2b - Ba^2b^2))/b^10)*(\\
& Ab^3 - 2Ba^3 + 2Aa^2b - Ba^2b^2))/(2b^4))*(Ab^3 - 2Ba^3 + 2Aa^2 \\
& *b - Ba^2b^2))/(2b^4) + (((8\tan(c/2 + (d*x)/2)*(A^2b^9 - 8B^2a^9 - 3A \\
& ^2a^8b + 16B^2a^8b + 7A^2a^2b^7 - 13A^2a^3b^6 + 16A^2a^4b^5 - \\
& 16A^2a^5b^4 + 16A^2a^6b^3 - 8A^2a^7b^2 + B^2a^2b^7 - 3B^2a^3b^6 \\
& + 7B^2a^4b^5 - 13B^2a^5b^4 + 16B^2a^6b^3 - 16B^2a^7b^2 - 2 \\
& AB^4a^8b + 16AB^5a^8b + 6AB^6a^2b^7 - 14AB^7a^3b^6 + 26AB^8a^4b^5 \\
& - 32AB^9a^5b^4 + 32AB^6a^6b^3 - 32AB^7a^7b^2))/b^6 + (((8*(2Ab^13 + \\
& 2Aa^2b^11 - 6Aa^3b^10 + 4Aa^4b^9 + 2Ba^2b^11 - 2Ba^3b^10 + \\
& 6Ba^4b^9 - 4Ba^5b^8 - 2Aa^6b^12 - 2Ba^6b^12))/b^9 + (4\tan(c/2 + (d \\
& *x)/2)*(8a^6b^10 - 16a^2b^9 + 8a^3b^8)*(Ab^3 - 2Ba^3 + 2Aa^2b - B \\
& *a^2b^2))/b^10)*(Ab^3 - 2Ba^3 + 2Aa^2b - Ba^2b^2))/(2b^4))*(Ab^3 - 2 \\
& *Ba^3 + 2Aa^2b - Ba^2b^2))/(2b^4))*((Ab^3 - 2Ba^3 + 2Aa^2b - Ba \\
& *b^2)*i)/(b^4*d) - (a^3*atan(((a^3*((a + b)*(a - b))^(1/2)*(Ab - Ba))*((8 \\
& *tan(c/2 + (d*x)/2)*(A^2b^9 - 8B^2a^9 - 3A^2a^8b + 16B^2a^8b + 7A \\
& ^2a^2b^7 - 13A^2a^3b^6 + 16A^2a^4b^5 - 16A^2a^5b^4 + 16A^2a^6b^3 \\
& - 8A^2a^7b^2 + B^2a^2b^7 - 3B^2a^3b^6 + 7B^2a^4b^5 - 13B^2a^5b^4 \\
& + 16B^2a^6b^3 - 16B^2a^7b^2 - 2AB^4a^8b + 16AB^5a^8b + 6 \\
& AB^6a^2b^7 - 14AB^7a^3b^6 + 26AB^8a^4b^5 - 32AB^9a^5b^4 + 32AB^6a^6 \\
& *b^3 - 32AB^7a^7b^2))/b^6 + (a^3*((a + b)*(a - b))^(1/2))*((8*(2Ab^13 + \\
& 2Aa^2b^11 - 6Aa^3b^10 + 4Aa^4b^9 + 2Ba^2b^11 - 2Ba^3b^10 + 6 \\
& *Ba^4b^9 - 4Ba^5b^8 - 2Aa^6b^12 - 2Ba^6b^12))/b^9 + (8a^3*tan(c/2 + \\
& (d*x)/2)*((a + b)*(a - b))^(1/2)*(Ab - Ba)*(8a^6b^10 - 16a^2b^9 + 8a^ \\
& 3b^8))/(b^6*(b^6 - a^2b^4)))*(Ab - Ba))/(b^6 - a^2b^4)*i)/(b^6 - a^2 \\
& *b^4) + (a^3*((a + b)*(a - b))^(1/2)*(Ab - Ba))*((8\tan(c/2 + (d*x)/2)*(A^ \\
& 2b^9 - 8B^2a^9 - 3A^2a^8b + 16B^2a^8b + 7A^2a^2b^7 - 13A^2a^3 \\
& *b^6 + 16A^2a^4b^5 - 16A^2a^5b^4 + 16A^2a^6b^3 - 8A^2a^7b^2 + B \\
& ^2a^2b^7 - 3B^2a^3b^6 + 7B^2a^4b^5 - 13B^2a^5b^4 + 16B^2a^6b^ \\
& 3 - 16B^2a^7b^2 - 2AB^4a^8b + 16AB^5a^8b + 6AB^6a^2b^7 - 14AB^7a^ \\
& 3b^6 + 26AB^8a^4b^5 - 32AB^9a^5b^4 + 32AB^6a^6b^3 - 32AB^7a^7b^2)) \\
& /b^6 - (a^3*((a + b)*(a - b))^(1/2))*((8*(2Ab^13 + 2Aa^2b^11 - 6Aa^3b^ \\
& 10 + 4Aa^4b^9 + 2Ba^2b^11 - 2Ba^3b^10 + 6Ba^4b^9 - 4Ba^5b^8 - 2Aa^6b^ \\
& 12 - 2Ba^6b^12))/b^9 - (8a^3*tan(c/2 + (d*x)/2)*((a + b)*(a - \\
& b))^(1/2)*(Ab - Ba)*(8a^6b^10 - 16a^2b^9 + 8a^3b^8))/(b^6*(b^6 - a^2 \\
& *b^4)))*(Ab - Ba))/(b^6 - a^2b^4))*i)/(b^6 - a^2b^4))/((16*(4B^3a^11
\end{aligned}$$

$$\begin{aligned}
& - 6*B^3*a^{10}*b + A^3*a^3*b^8 - 2*A^3*a^4*b^7 + 5*A^3*a^5*b^6 - 6*A^3*a^6*b^5 + 6*A^3*a^7*b^4 - 4*A^3*a^8*b^3 - B^3*a^6*b^5 + 2*B^3*a^7*b^4 - 5*B^3*a^8*b^3 + 6*B^3*a^9*b^2 - 12*A*B^2*a^{10}*b + 3*A*B^2*a^5*b^6 - 6*A*B^2*a^6*b^5 + 15*A*B^2*a^7*b^4 - 18*A*B^2*a^8*b^3 + 18*A*B^2*a^9*b^2 - 3*A^2*B*a^4*b^7 + 6*A^2*B*a^5*b^6 - 15*A^2*B*a^6*b^5 + 18*A^2*B*a^7*b^4 - 18*A^2*B*a^8*b^3 + 12*A^2*B*a^9*b^2)/b^9 + (a^3*((a + b)*(a - b))^(1/2)*(A*b - B*a))*((8*tan(c/2 + (d*x)/2)*(A^2*b^9 - 8*B^2*a^9 - 3*A^2*a*b^8 + 16*B^2*a^8*b + 7*A^2*a^2*b^7 - 13*A^2*a^3*b^6 + 16*A^2*a^4*b^5 - 16*A^2*a^5*b^4 + 16*A^2*a^6*b^3 - 8*A^2*a^7*b^2 + B^2*a^2*b^7 - 3*B^2*a^3*b^6 + 7*B^2*a^4*b^5 - 13*B^2*a^5*b^4 + 16*B^2*a^6*b^3 - 16*B^2*a^7*b^2 - 2*A*B*a*b^8 + 16*A*B*a^8*b + 6*A*B*a^2*b^7 - 14*A*B*a^3*b^6 + 26*A*B*a^4*b^5 - 32*A*B*a^5*b^4 + 32*A*B*a^6*b^3 - 32*A*B*a^7*b^2))/b^6 + (a^3*((a + b)*(a - b))^(1/2))*((8*(2*A*b^13 + 2*A*a^2*b^11 - 6*A*a^3*b^10 + 4*A*a^4*b^9 + 2*B*a^2*b^11 - 2*B*a^3*b^10 + 6*B*a^4*b^9 - 4*B*a^5*b^8 - 2*A*a*b^12 - 2*B*a*b^12))/b^9 + (8*a^3*tan(c/2 + (d*x)/2))*((a + b)*(a - b))^(1/2)*(A*b - B*a)*(8*a*b^10 - 16*a^2*b^9 + 8*a^3*b^8)))/(b^6*(b^6 - a^2*b^4)))*(A*b - B*a))/(b^6 - a^2*b^4)) - (a^3*((a + b)*(a - b))^(1/2)*(A*b - B*a))*((8*tan(c/2 + (d*x)/2)*(A^2*b^9 - 8*B^2*a^9 - 3*A^2*a*b^8 + 16*B^2*a^8*b + 7*A^2*a^2*b^7 - 13*A^2*a^3*b^6 + 16*A^2*a^4*b^5 - 16*A^2*a^5*b^4 + 16*A^2*a^6*b^3 - 8*A^2*a^7*b^2 + B^2*a^2*b^7 - 3*B^2*a^3*b^6 + 7*B^2*a^4*b^5 - 13*B^2*a^5*b^4 + 16*B^2*a^6*b^3 - 16*B^2*a^7*b^2 - 2*A*B*a*b^8 + 16*A*B*a^8*b + 6*A*B*a^2*b^7 - 14*A*B*a^3*b^6 + 26*A*B*a^4*b^5 - 32*A*B*a^5*b^4 + 32*A*B*a^6*b^3 - 32*A*B*a^7*b^2))/b^6 - (a^3*((a + b)*(a - b))^(1/2))*((8*(2*A*b^13 + 2*A*a^2*b^11 - 6*A*a^3*b^10 + 4*A*a^4*b^9 + 2*B*a^2*b^11 - 2*B*a^3*b^10 + 6*B*a^4*b^9 - 4*B*a^5*b^8 - 2*A*a*b^12 - 2*B*a*b^12))/b^9 - (8*a^3*tan(c/2 + (d*x)/2))*((a + b)*(a - b))^(1/2)*(A*b - B*a)*(8*a*b^10 - 16*a^2*b^9 + 8*a^3*b^8)))/(b^6*(b^6 - a^2*b^4)))*(A*b - B*a))/(b^6 - a^2*b^4)))*((a + b)*(a - b))^(1/2)*(A*b - B*a)*2i)/(d*(b^6 - a^2*b^4))
\end{aligned}$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \sec^4(c + dx)}{a + b \sec(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**4*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x)

[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**4/(a + b*sec(c + d*x)), x)

$$3.312 \quad \int \frac{\sec^3(c+dx)(A+B \sec(c+dx))}{a+b \sec(c+dx)} dx$$

Optimal. Leaf size=143

$$\frac{2a^2(Ab - aB) \tanh^{-1}\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3 d \sqrt{a-b} \sqrt{a+b}} - \frac{(-2a^2B + 2aAb - b^2B) \tanh^{-1}(\sin(c+dx))}{2b^3 d} + \frac{(Ab - aB) \tan(c+dx)}{b^2 d} + \dots$$

[Out] $-1/2*(2*A*a*b-2*B*a^2-B*b^2)*\operatorname{arctanh}(\sin(d*x+c))/b^3/d+2*a^2*(A*b-B*a)*\operatorname{arctanh}((a-b)^{(1/2)}*\tan(1/2*d*x+1/2*c)/(a+b)^{(1/2)})/b^3/d/(a-b)^{(1/2)}/(a+b)^{(1/2)}+(A*b-B*a)*\tan(d*x+c)/b^2/d+1/2*B*\sec(d*x+c)*\tan(d*x+c)/b/d$

Rubi [A] time = 0.40, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {4033, 4082, 3998, 3770, 3831, 2659, 208}

$$\frac{(-2a^2B + 2aAb - b^2B) \tanh^{-1}(\sin(c+dx))}{2b^3 d} + \frac{2a^2(Ab - aB) \tanh^{-1}\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3 d \sqrt{a-b} \sqrt{a+b}} + \frac{(Ab - aB) \tan(c+dx)}{b^2 d} + \dots$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(\operatorname{Sec}[c + d*x])^3*(A + B*\operatorname{Sec}[c + d*x])]/(a + b*\operatorname{Sec}[c + d*x]), x]$

[Out] $-((2*a*A*b - 2*a^2*B - b^2*B)*\operatorname{ArcTanh}[\operatorname{Sin}[c + d*x]])/(2*b^3*d) + (2*a^2*(A*b - a*B)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a - b]*\operatorname{Tan}[(c + d*x)/2])/(\operatorname{Sqrt}[a + b])]/(\operatorname{Sqrt}[a - b]*b^3*\operatorname{Sqrt}[a + b]*d) + ((A*b - a*B)*\operatorname{Tan}[c + d*x])/(b^2*d) + (B*\operatorname{Sec}[c + d*x]*\operatorname{Tan}[c + d*x])/(2*b*d)$

Rule 208

$\operatorname{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[-(a/b), 2]*\operatorname{ArcTanh}[x/\operatorname{Rt}[-(a/b), 2]])/a, x] /; \operatorname{FreeQ}\{a, b\}, x] \ \&\& \operatorname{NegQ}[a/b]$

Rule 2659

$\operatorname{Int}[(a_ + (b_)*\sin[\operatorname{Pi}/2 + (c_.) + (d_)*(x_)])^{-1}, x_Symbol] \rightarrow \operatorname{With}\{e = \operatorname{FreeFactors}[\operatorname{Tan}[(c + d*x)/2], x]\}, \operatorname{Dist}[(2*e)/d, \operatorname{Subst}[\operatorname{Int}[1/(a + b + (a - b)*e^2*x^2), x], x, \operatorname{Tan}[(c + d*x)/2]/e], x] /; \operatorname{FreeQ}\{a, b, c, d\}, x] \ \&\& \operatorname{NeQ}[a^2 - b^2, 0]$

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x]
  /; FreeQ[{c, d}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] := Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] := Dist[B/b, Int[Csc[e + f*x], x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 4033

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] := -Simp[(B*d^2 * Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2))/(b*f*(m + n)), x] + Dist[d^2/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 2)*Simp[a*B*(n - 2) + B*b*(m + n - 1)*Csc[e + f*x] + (A*b*(m + n) - a*B*(n - 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && NeQ[m + n, 0] && !IGtQ[m, 1]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] := -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^3(c+dx)(A+B\sec(c+dx))}{a+b\sec(c+dx)} dx &= \frac{B\sec(c+dx)\tan(c+dx)}{2bd} + \frac{\int \frac{\sec(c+dx)(aB+bB\sec(c+dx)+2(Ab-aB)\sec^2(c+dx))}{a+b\sec(c+dx)}}{2b} \\
&= \frac{(Ab-aB)\tan(c+dx)}{b^2d} + \frac{B\sec(c+dx)\tan(c+dx)}{2bd} + \frac{\int \frac{\sec(c+dx)(abB-(2a^2B-2aAb+b^2B)\sec^2(c+dx))}{a+b\sec(c+dx)}}{2b} \\
&= \frac{(Ab-aB)\tan(c+dx)}{b^2d} + \frac{B\sec(c+dx)\tan(c+dx)}{2bd} + \frac{(a^2(Ab-aB)) \int \frac{\sec(c+dx)}{a+b\sec(c+dx)}}{b^3} \\
&= -\frac{(2aAb-2a^2B-b^2B)\tanh^{-1}(\sin(c+dx))}{2b^3d} + \frac{(Ab-aB)\tan(c+dx)}{b^2d} \\
&= -\frac{(2aAb-2a^2B-b^2B)\tanh^{-1}(\sin(c+dx))}{2b^3d} + \frac{(Ab-aB)\tan(c+dx)}{b^2d} \\
&= -\frac{(2aAb-2a^2B-b^2B)\tanh^{-1}(\sin(c+dx))}{2b^3d} + \frac{2a^2(Ab-aB)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a-b}b^3\sqrt{a}}
\end{aligned}$$

Mathematica [B] time = 1.92, size = 300, normalized size = 2.10

$$\frac{8a^2(aB-Ab)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} - 2(2a^2B-2aAb+b^2B)\log\left(\cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+dx)\right)\right) + 2(2a^2B-2aAb+b^2B)\log\left(\cos\left(\frac{1}{2}(c+dx)\right) + \sin\left(\frac{1}{2}(c+dx)\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x]),x]

[Out] ((8*a^2*(-(A*b) + a*B)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] - 2*(-2*a*A*b + 2*a^2*B + b^2*B)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 2*(-2*a*A*b + 2*a^2*B + b^2*B)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (b^2*B)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + (4*b*(A*b - a*B)*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) - (b^2*B)/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + (4*b*(A*b - a*B)*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])/(4*b^3*d)

fricas [B] time = 4.83, size = 609, normalized size = 4.26

$$\left[\frac{2(Ba^3 - Aa^2b)\sqrt{a^2 - b^2} \cos(dx + c)^2 \log\left(\frac{2ab \cos(dx+c) - (a^2 - 2b^2)\cos(dx+c)^2 + 2\sqrt{a^2 - b^2}(b \cos(dx+c) + a) \sin(dx+c) + 2a^2 - b^2}{a^2 \cos(dx+c)^2 + 2ab \cos(dx+c) + b^2}\right)}{\dots} \right] -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="fricas")

[Out] [-1/4*(2*(B*a^3 - A*a^2*b)*sqrt(a^2 - b^2)*cos(d*x + c)^2*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - (2*B*a^4 - 2*A*a^3*b - B*a^2*b^2 + 2*A*a*b^3 - B*b^4)*cos(d*x + c)^2*log(sin(d*x + c) + 1) + (2*B*a^4 - 2*A*a^3*b - B*a^2*b^2 + 2*A*a*b^3 - B*b^4)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) - 2*(B*a^2*b^2 - B*b^4 - 2*(B*a^3*b - A*a^2*b^2 - B*a*b^3 + A*b^4)*cos(d*x + c))*sin(d*x + c))/((a^2*b^3 - b^5)*d*cos(d*x + c)^2), -1/4*(4*(B*a^3 - A*a^2*b)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c)))*cos(d*x + c)^2 - (2*B*a^4 - 2*A*a^3*b - B*a^2*b^2 + 2*A*a*b^3 - B*b^4)*cos(d*x + c)^2*log(sin(d*x + c) + 1) + (2*B*a^4 - 2*A*a^3*b - B*a^2*b^2 + 2*A*a*b^3 - B*b^4)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) - 2*(B*a^2*b^2 - B*b^4 - 2*(B*a^3*b - A*a^2*b^2 - B*a*b^3 + A*b^4)*cos(d*x + c))*sin(d*x + c))/((a^2*b^3 - b^5)*d*cos(d*x + c)^2)]

giac [B] time = 0.62, size = 269, normalized size = 1.88

$$\frac{(2Ba^2 - 2Aab + Bb^2) \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right|\right)}{b^3} - \frac{(2Ba^2 - 2Aab + Bb^2) \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1\right|\right)}{b^3} - \frac{4(Ba^3 - Aa^2b) \left(\pi \left[\frac{dx+c}{2\pi} + \frac{1}{2} \right] \operatorname{sgn}(-2a+2b) + \arctan\left(-\frac{a}{\sqrt{-a^2+b^2}} \right) \right)}{\sqrt{-a^2+b^2} b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="giac")

[Out] 1/2*((2*B*a^2 - 2*A*a*b + B*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^3 - (2*B*a^2 - 2*A*a*b + B*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^3 - 4*(B*a^3 - A*a^2*b)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/(sqrt(-a^2 + b^2)*b^3) + 2*(2*B*a*tan(1/2*d*x + 1/2*c)^3 - 2*A*b*tan(1/2*d*x + 1/2*c)^3 + B*b*tan(1/2*d*x + 1/2*c)^3 - 2*B*a*tan(1/2*d*x + 1/2*c) + 2*A*b*tan(

$\frac{1}{2}dx + \frac{1}{2}c) + B \cdot b \cdot \tan(\frac{1}{2}dx + \frac{1}{2}c) / ((\tan(\frac{1}{2}dx + \frac{1}{2}c))^2 - 1)^{-2} \cdot b^2) / d$

maple [B] time = 0.65, size = 410, normalized size = 2.87

$$\frac{2a^2 \operatorname{arctanh}\left(\frac{\tan\left(\frac{dx}{2} + \frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right) A}{db^2 \sqrt{(a-b)(a+b)}} - \frac{2a^3 \operatorname{arctanh}\left(\frac{\tan\left(\frac{dx}{2} + \frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right) B}{db^3 \sqrt{(a-b)(a+b)}} + \frac{B}{2db \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)^2} - \frac{A}{db \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x)`

[Out] $2/d \cdot a^2/b^2 / ((a-b) \cdot (a+b))^{1/2} \cdot \operatorname{arctanh}(\tan(\frac{1}{2}dx + \frac{1}{2}c) \cdot (a-b) / ((a-b) \cdot (a+b))^{1/2}) \cdot A - 2/d \cdot a^3/b^3 / ((a-b) \cdot (a+b))^{1/2} \cdot \operatorname{arctanh}(\tan(\frac{1}{2}dx + \frac{1}{2}c) \cdot (a-b) / ((a-b) \cdot (a+b))^{1/2}) \cdot B + 1/2 \cdot d/b / (\tan(\frac{1}{2}dx + \frac{1}{2}c) - 1)^{2B-1} \cdot d/b / (\tan(\frac{1}{2}dx + \frac{1}{2}c) - 1) \cdot A + 1/d \cdot b^2 / (\tan(\frac{1}{2}dx + \frac{1}{2}c) - 1) \cdot B \cdot a + 1/2 \cdot d/b / (\tan(\frac{1}{2}dx + \frac{1}{2}c) - 1) \cdot B + 1/d \cdot b^2 \cdot \ln(\tan(\frac{1}{2}dx + \frac{1}{2}c) - 1) \cdot A \cdot a - 1/d \cdot b^3 \cdot \ln(\tan(\frac{1}{2}dx + \frac{1}{2}c) - 1) \cdot a^2 \cdot B - 1/2 \cdot d/b \cdot \ln(\tan(\frac{1}{2}dx + \frac{1}{2}c) - 1) \cdot B - 1/2 \cdot d/b / (\tan(\frac{1}{2}dx + \frac{1}{2}c) + 1)^{2B-1} \cdot d/b / (\tan(\frac{1}{2}dx + \frac{1}{2}c) + 1) \cdot A + 1/d \cdot b^2 / (\tan(\frac{1}{2}dx + \frac{1}{2}c) + 1) \cdot B \cdot a + 1/2 \cdot d/b / (\tan(\frac{1}{2}dx + \frac{1}{2}c) + 1) \cdot B - 1/d \cdot b^2 \cdot \ln(\tan(\frac{1}{2}dx + \frac{1}{2}c) + 1) \cdot A \cdot a + 1/d \cdot b^3 \cdot \ln(\tan(\frac{1}{2}dx + \frac{1}{2}c) + 1) \cdot a^2 \cdot B + 1/2 \cdot d/b \cdot \ln(\tan(\frac{1}{2}dx + \frac{1}{2}c) + 1) \cdot B$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for more details) Is 4*a^2-4*b^2 positive or negative?

mupad [B] time = 6.08, size = 4047, normalized size = 28.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B/cos(c + d*x))/(cos(c + d*x)^3*(a + b/cos(c + d*x))),x)`

```
[Out] (B*a*sin(2*c + 2*d*x))/(2*d*(a^2 - b^2)*(cos(2*c + 2*d*x)/2 + 1/2)) - (A*b*
sin(2*c + 2*d*x))/(2*d*(a^2 - b^2)*(cos(2*c + 2*d*x)/2 + 1/2)) - (B*b*sin(c
+ d*x))/(2*d*(a^2 - b^2)*(cos(2*c + 2*d*x)/2 + 1/2)) - (A*a*atan((sin(c/2
+ (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*1i)/(d*(a^2 - b^2)*(cos(2*c + 2*d*x)/2 +
1/2)) + (B*b*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*1i)/(2*d*(a^
2 - b^2)*(cos(2*c + 2*d*x)/2 + 1/2)) + (A*a^3*atan((sin(c/2 + (d*x)/2)*1i)/
cos(c/2 + (d*x)/2))*1i)/(b^2*d*(a^2 - b^2)*(cos(2*c + 2*d*x)/2 + 1/2)) + (B
*a^2*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*1i)/(2*b*d*(a^2 - b^2
)*(cos(2*c + 2*d*x)/2 + 1/2)) - (B*a^4*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2
+ (d*x)/2))*1i)/(b^3*d*(a^2 - b^2)*(cos(2*c + 2*d*x)/2 + 1/2)) + (A*a^2*si
n(2*c + 2*d*x))/(2*b*d*(a^2 - b^2)*(cos(2*c + 2*d*x)/2 + 1/2)) - (B*a^3*sin
(2*c + 2*d*x))/(2*b^2*d*(a^2 - b^2)*(cos(2*c + 2*d*x)/2 + 1/2)) - (A*a*atan
((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*cos(2*c + 2*d*x)*1i)/(d*(a^2 -
b^2)*(cos(2*c + 2*d*x)/2 + 1/2)) + (B*b*atan((sin(c/2 + (d*x)/2)*1i)/cos(c
/2 + (d*x)/2))*cos(2*c + 2*d*x)*1i)/(2*d*(a^2 - b^2)*(cos(2*c + 2*d*x)/2 +
1/2)) + (B*a^2*sin(c + d*x))/(2*b*d*(a^2 - b^2)*(cos(2*c + 2*d*x)/2 + 1/2))
+ (A*a^3*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*cos(2*c + 2*d*x)
*1i)/(b^2*d*(a^2 - b^2)*(cos(2*c + 2*d*x)/2 + 1/2)) + (B*a^2*atan((sin(c/2
+ (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*cos(2*c + 2*d*x)*1i)/(2*b*d*(a^2 - b^2)*
(cos(2*c + 2*d*x)/2 + 1/2)) - (B*a^4*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 +
(d*x)/2))*cos(2*c + 2*d*x)*1i)/(b^3*d*(a^2 - b^2)*(cos(2*c + 2*d*x)/2 + 1/
2)) + (A*a^2*atan(((8*B^2*a^7*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(3/2) - 8*B^2*
a^9*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) + B^2*b^9*sin(c/2 + (d*x)/2)*(a^2
- b^2)^(1/2) - B^2*a*b^8*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) + 4*A^2*a^2*b
^7*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) - 4*A^2*a^3*b^6*sin(c/2 + (d*x)/2)*
(a^2 - b^2)^(1/2) - 4*A^2*a^4*b^5*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) + 8*
A^2*a^5*b^4*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(3/2) + 12*A^2*a^5*b^4*sin(c/2 +
(d*x)/2)*(a^2 - b^2)^(1/2) - 8*A^2*a^7*b^2*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(
1/2) + 2*B^2*a^2*b^7*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) - 2*B^2*a^3*b^6*
sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) - 3*B^2*a^4*b^5*sin(c/2 + (d*x)/2)*(a^
2 - b^2)^(1/2) + 3*B^2*a^5*b^4*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) + 8*B^2
*a^7*b^2*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) - 4*A*B*a*b^8*sin(c/2 + (d*x)
/2)*(a^2 - b^2)^(1/2) - 16*A*B*a^6*b*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(3/2) +
16*A*B*a^8*b*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) + 4*A*B*a^2*b^7*sin(c/2
+ (d*x)/2)*(a^2 - b^2)^(1/2) + 4*A*B*a^5*b^4*sin(c/2 + (d*x)/2)*(a^2 - b^2)
^(1/2) - 20*A*B*a^6*b^3*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2))*1i)/(cos(c/2
+ (d*x)/2)*(a^2*b - b^3)*(B^2*b^7 + 4*A^2*a^2*b^5 - 4*A^2*a^4*b^3 + 2*B^2*a
^2*b^5 - 3*B^2*a^4*b^3 - 4*A*B*a*b^6 + 4*A*B*a^5*b^2)))*((a + b)*(a - b))^(
1/2)*1i)/(b^2*d*(a^2 - b^2)*(cos(2*c + 2*d*x)/2 + 1/2)) - (B*a^3*atan(((8*B
^2*a^7*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(3/2) - 8*B^2*a^9*sin(c/2 + (d*x)/2)*
(a^2 - b^2)^(1/2) + B^2*b^9*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) - B^2*a*b^
8*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) + 4*A^2*a^2*b^7*sin(c/2 + (d*x)/2)*
(a^2 - b^2)^(1/2) - 4*A^2*a^3*b^6*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) - 4*A
^2*a^4*b^5*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) + 8*A^2*a^5*b^4*sin(c/2 + (
d*x)/2)*(a^2 - b^2)^(3/2) + 12*A^2*a^5*b^4*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(
```


$$\begin{aligned}
& 1/2) - 8A^2a^7b^2\sin(c/2 + (d*x)/2)*(a^2 - b^2)^{(1/2)} + 2B^2a^2b^7\sin(c/2 + (d*x)/2)*(a^2 - b^2)^{(1/2)} - 2B^2a^3b^6\sin(c/2 + (d*x)/2)*(a^2 - b^2)^{(1/2)} - 3B^2a^4b^5\sin(c/2 + (d*x)/2)*(a^2 - b^2)^{(1/2)} + 3B^2a^5b^4\sin(c/2 + (d*x)/2)*(a^2 - b^2)^{(1/2)} + 8B^2a^7b^2\sin(c/2 + (d*x)/2)*(a^2 - b^2)^{(1/2)} - 4A*B*a*b^8\sin(c/2 + (d*x)/2)*(a^2 - b^2)^{(1/2)} - 16A*B*a^6*b*\sin(c/2 + (d*x)/2)*(a^2 - b^2)^{(3/2)} + 16A*B*a^8*b*\sin(c/2 + (d*x)/2)*(a^2 - b^2)^{(1/2)} + 4A*B*a^2*b^7\sin(c/2 + (d*x)/2)*(a^2 - b^2)^{(1/2)} + 4A*B*a^5*b^4\sin(c/2 + (d*x)/2)*(a^2 - b^2)^{(1/2)} - 20A*B*a^6*b^3*\sin(c/2 + (d*x)/2)*(a^2 - b^2)^{(1/2))*1i)/(\cos(c/2 + (d*x)/2)*(a^2*b - b^3))*(B^2*b^7 + 4A^2*a^2*b^5 - 4A^2*a^4*b^3 + 2B^2*a^2*b^5 - 3B^2*a^4*b^3 - 4A*B*a*b^6 + 4A*B*a^5*b^2))*((a + b)*(a - b))^{(1/2)*1i)/(b^3*d*(a^2 - b^2)*(cos(2*c + 2*d*x)/2 + 1/2)) + (A*a^2*atan(((8*B^2*a^7*\sin(c/2 + (d*x)/2)*(a^2 - b^2)^{(3/2)} - 8*B^2*a^9*\sin(c/2 + (d*x)/2)*(a^2 - b^2)^{(1/2)} + B^2*b^9*\sin(c/2 + (d*x)/2)*(a^2 - b^2)^{(1/2)} - B^2*a*b^8*\sin(c/2 + (d*x)/2)*(a^2 - b^2)^{(1/2)} + 4A^2*a^2*b^7*\sin(c/2 + (d*x)/2)*(a^2 - b^2)^{(1/2)} - 4A^2*a^3*b^6*\sin(c/2 + (d*x)/2)*(a^2 - b^2)^{(1/2)} - 4A^2*a^4*b^5*\sin(c/2 + (d*x)/2)*(a^2 - b^2)^{(1/2)} + 8A^2*a^5*b^2*\sin(c/2 + (d*x)/2)*(a^2 - b^2)^{(3/2)} + 12A^2*a^5*b^4*\sin(c/2 + (d*x)/2)*(a^2 - b^2)^{(1/2)} - 8A^2*a^7*b^2*\sin(c/2 + (d*x)/2)*(a^2 - b^2)^{(1/2)} + 2B^2*a^2*b^7*\sin(c/2 + (d*x)/2)*(a^2 - b^2)^{(1/2)} - 2B^2*a^3*b^6*\sin(c/2 + (d*x)/2)*(a^2 - b^2)^{(1/2)} - 3B^2*a^4*b^5*\sin(c/2 + (d*x)/2)*(a^2 - b^2)^{(1/2)} + 3B^2*a^5*b^4*\sin(c/2 + (d*x)/2)*(a^2 - b^2)^{(1/2)} + 8B^2*a^7*b^2*\sin(c/2 + (d*x)/2)*(a^2 - b^2)^{(1/2)} - 4A*B*a*b^8*\sin(c/2 + (d*x)/2)*(a^2 - b^2)^{(1/2)} - 16A*B*a^6*b*\sin(c/2 + (d*x)/2)*(a^2 - b^2)^{(3/2)} + 16A*B*a^8*b*\sin(c/2 + (d*x)/2)*(a^2 - b^2)^{(1/2)} + 4A*B*a^2*b^7*\sin(c/2 + (d*x)/2)*(a^2 - b^2)^{(1/2)} + 4A*B*a^5*b^4*\sin(c/2 + (d*x)/2)*(a^2 - b^2)^{(1/2)} - 20A*B*a^6*b^3*\sin(c/2 + (d*x)/2)*(a^2 - b^2)^{(1/2))*1i)/(\cos(c/2 + (d*x)/2)*(a^2*b - b^3)*(B^2*b^7 + 4A^2*a^2*b^5 - 4A^2*a^4*b^3 + 2B^2*a^2*b^5 - 3B^2*a^4*b^3 - 4A*B*a*b^6 + 4A*B*a^5*b^2))*cos(2*c + 2*d*x))*((a + b)*(a - b))^{(1/2)*1i)/(b^2*d*(a^2 - b^2)*(cos(2*c + 2*d*x)/2 + 1/2)) - (B*a^3*atan(((8*B^2*a^7*\sin(c/2 + (d*x)/2)*(a^2 - b^2)^{(3/2)} - 8*B^2*a^9*\sin(c/2 + (d*x)/2)*(a^2 - b^2)^{(1/2)} + B^2*b^9*\sin(c/2 + (d*x)/2)*(a^2 - b^2)^{(1/2)} - B^2*a*b^8*\sin(c/2 + (d*x)/2)*(a^2 - b^2)^{(1/2)} + 4A^2*a^2*b^7*\sin(c/2 + (d*x)/2)*(a^2 - b^2)^{(1/2)} - 4A^2*a^3*b^6*\sin(c/2 + (d*x)/2)*(a^2 - b^2)^{(1/2)} - 4A^2*a^4*b^5*\sin(c/2 + (d*x)/2)*(a^2 - b^2)^{(1/2)} + 8A^2*a^5*b^2*\sin(c/2 + (d*x)/2)*(a^2 - b^2)^{(3/2)} + 12A^2*a^5*b^4*\sin(c/2 + (d*x)/2)*(a^2 - b^2)^{(1/2)} - 8A^2*a^7*b^2*\sin(c/2 + (d*x)/2)*(a^2 - b^2)^{(1/2)} + 2B^2*a^2*b^7*\sin(c/2 + (d*x)/2)*(a^2 - b^2)^{(1/2)} - 2B^2*a^3*b^6*\sin(c/2 + (d*x)/2)*(a^2 - b^2)^{(1/2)} - 3B^2*a^4*b^5*\sin(c/2 + (d*x)/2)*(a^2 - b^2)^{(1/2)} + 3B^2*a^5*b^4*\sin(c/2 + (d*x)/2)*(a^2 - b^2)^{(1/2)} + 8B^2*a^7*b^2*\sin(c/2 + (d*x)/2)*(a^2 - b^2)^{(1/2)} - 4A*B*a*b^8*\sin(c/2 + (d*x)/2)*(a^2 - b^2)^{(1/2)} - 16A*B*a^6*b*\sin(c/2 + (d*x)/2)*(a^2 - b^2)^{(3/2)} + 16A*B*a^8*b*\sin(c/2 + (d*x)/2)*(a^2 - b^2)^{(1/2)} + 4A*B*a^2*b^7*\sin(c/2 + (d*x)/2)*(a^2 - b^2)^{(1/2)} + 4A*B*a^5*b^4*\sin(c/2 + (d*x)/2)*(a^2 - b^2)^{(1/2)} - 20A*B*a^6*b^3*\sin(c/2 + (d*x)/2)*(a^2 - b^2)^{(1/2))*1i)/(\cos(c/2 + (d*x)/2)*(a^2*b - b^3)*(B^2*b^7 + 4A^2*a^2*b^5 -
\end{aligned}$$

```
4*A^2*a^4*b^3 + 2*B^2*a^2*b^5 - 3*B^2*a^4*b^3 - 4*A*B*a*b^6 + 4*A*B*a^5*b^2
)))*cos(2*c + 2*d*x)*((a + b)*(a - b))^(1/2)*1i)/(b^3*d*(a^2 - b^2)*(cos(2*
c + 2*d*x)/2 + 1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \sec^3(c + dx)}{a + b \sec(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**3/(a + b*sec(c + d*x)), x)
```

$$3.313 \quad \int \frac{\sec^2(c+dx)(A+B \sec(c+dx))}{a+b \sec(c+dx)} dx$$

Optimal. Leaf size=98

$$\frac{(Ab - aB) \tanh^{-1}(\sin(c + dx))}{b^2 d} - \frac{2a(Ab - aB) \tanh^{-1}\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2 d \sqrt{a-b} \sqrt{a+b}} + \frac{B \tan(c + dx)}{bd}$$

[Out] (A*b-B*a)*arctanh(sin(d*x+c))/b^2/d-2*a*(A*b-B*a)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/b^2/d/(a-b)^(1/2)/(a+b)^(1/2)+B*tan(d*x+c)/b/d

Rubi [A] time = 0.23, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {4010, 12, 3789, 3770, 3831, 2659, 208}

$$\frac{(Ab - aB) \tanh^{-1}(\sin(c + dx))}{b^2 d} - \frac{2a(Ab - aB) \tanh^{-1}\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2 d \sqrt{a-b} \sqrt{a+b}} + \frac{B \tan(c + dx)}{bd}$$

Antiderivative was successfully verified.

[In] Int[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x]),x]

[Out] ((A*b - a*B)*ArcTanh[Sin[c + d*x]]/(b^2*d) - (2*a*(A*b - a*B)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]]/(Sqrt[a - b]*b^2*Sqrt[a + b]*d) + (B*Tan[c + d*x])/(b*d)

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 2659

Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x]
  /; FreeQ[{c, d}, x]
```

Rule 3789

```
Int[csc[(e_.) + (f_.)*(x_)]^2/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] := Dist[1/b, Int[Csc[e + f*x], x], x] - Dist[a/b, Int[Csc[e + f*x]/(a
+ b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] := Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] := -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Cs
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
0] && !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^2(c+dx)(A+B\sec(c+dx))}{a+b\sec(c+dx)} dx &= \frac{B \tan(c+dx)}{bd} + \frac{\int \frac{(Ab-aB)\sec^2(c+dx)}{a+b\sec(c+dx)} dx}{b} \\
&= \frac{B \tan(c+dx)}{bd} + \frac{(Ab-aB) \int \frac{\sec^2(c+dx)}{a+b\sec(c+dx)} dx}{b} \\
&= \frac{B \tan(c+dx)}{bd} + \frac{(Ab-aB) \int \sec(c+dx) dx}{b^2} - \frac{(a(Ab-aB)) \int \frac{\sec(c+dx)}{a+b\sec(c+dx)} dx}{b^2} \\
&= \frac{(Ab-aB) \tanh^{-1}(\sin(c+dx))}{b^2d} + \frac{B \tan(c+dx)}{bd} - \frac{(a(Ab-aB)) \int \frac{\sec(c+dx)}{1+\frac{a}{b}\sec(c+dx)} dx}{b^3} \\
&= \frac{(Ab-aB) \tanh^{-1}(\sin(c+dx))}{b^2d} + \frac{B \tan(c+dx)}{bd} - \frac{(2a(Ab-aB)) \operatorname{Subst}\left(\int \frac{\sec(u)}{1+\frac{a}{b}\sec(u)} du\right)}{b^3} \\
&= \frac{(Ab-aB) \tanh^{-1}(\sin(c+dx))}{b^2d} - \frac{2a(Ab-aB) \tanh^{-1}\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b} b^2 \sqrt{a+b} d}
\end{aligned}$$

Mathematica [A] time = 0.71, size = 130, normalized size = 1.33

$$\frac{2a(aB-Ab) \tanh^{-1}\left(\frac{(b-a) \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} - \frac{(Ab-aB) \left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) - \sin\left(\frac{1}{2}(c+dx)\right)\right) - \log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) + \cos\left(\frac{1}{2}(c+dx)\right)}{b^2d}$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x]),x]

[Out] ((-2*a*(-(A*b) + a*B)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] - (A*b - a*B)*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + b*B*Tan[c + d*x]/(b^2*d)

fricas [B] time = 0.62, size = 472, normalized size = 4.82

$$\left[\frac{(Ba^2 - Aab)\sqrt{a^2 - b^2} \cos(dx + c) \log\left(\frac{2ab \cos(dx+c) - (a^2 - 2b^2) \cos(dx+c)^2 - 2\sqrt{a^2 - b^2} (b \cos(dx+c) + a) \sin(dx+c) + 2a^2 - b^2}{a^2 \cos(dx+c)^2 + 2ab \cos(dx+c) + b^2}\right) + (B(a^2 - b^2) \cos(dx+c) - aB \sin(dx+c))}{b^2d} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="fricas")

[Out] [-1/2*((B*a^2 - A*a*b)*sqrt(a^2 - b^2)*cos(d*x + c)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + (B*a^3 - A*a^2*b - B*a*b^2 + A*b^3)*cos(d*x + c)*log(sin(d*x + c) + 1) - (B*a^3 - A*a^2*b - B*a*b^2 + A*b^3)*cos(d*x + c)*log(-sin(d*x + c) + 1) - 2*(B*a^2*b - B*b^3)*sin(d*x + c)/((a^2*b^2 - b^4)*d*cos(d*x + c)), 1/2*(2*(B*a^2 - A*a*b)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c)))*cos(d*x + c) - (B*a^3 - A*a^2*b - B*a*b^2 + A*b^3)*cos(d*x + c)*log(sin(d*x + c) + 1) + (B*a^3 - A*a^2*b - B*a*b^2 + A*b^3)*cos(d*x + c)*log(-sin(d*x + c) + 1) + 2*(B*a^2*b - B*b^3)*sin(d*x + c)/((a^2*b^2 - b^4)*d*cos(d*x + c))]

giac [A] time = 0.63, size = 176, normalized size = 1.80

$$\frac{(Ba-Ab) \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 1\right|\right)}{b^2} - \frac{(Ba-Ab) \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 1\right|\right)}{b^2} + \frac{2B \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{\left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 1\right)b} - \frac{2(Ba^2 - Ab^2) \left(\pi \left[\frac{dx+c}{2\pi} + \frac{1}{2}\right] \operatorname{sgn}(-2a+2b) + \arctan\left(\frac{b \cos\left(\frac{1}{2} dx + \frac{1}{2} c\right) + a}{\sqrt{-a^2 + b^2}}\right)\right)}{\sqrt{-a^2 + b^2} b^2}$$

d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="giac")

[Out] -((B*a - A*b)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^2 - (B*a - A*b)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^2 + 2*B*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 - 1)*b) - 2*(B*a^2 - A*a*b)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/sqrt(-a^2 + b^2))/d

maple [B] time = 0.53, size = 228, normalized size = 2.33

$$\frac{2a \operatorname{arctanh}\left(\frac{\tan\left(\frac{dx}{2} + \frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right) A}{db \sqrt{(a-b)(a+b)}} + \frac{2a^2 \operatorname{arctanh}\left(\frac{\tan\left(\frac{dx}{2} + \frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right) B}{d b^2 \sqrt{(a-b)(a+b)}} - \frac{B}{db \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)} - \frac{\ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right) A}{db}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x)

[Out] -2/d*a/b/((a-b)*(a+b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b)))^(1/2))*A+2/d*a^2/b^2/((a-b)*(a+b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/

$((a-b)*(a+b))^{(1/2)} * B - 1/d/b / (\tan(1/2*d*x+1/2*c) - 1) * B - 1/d/b * \ln(\tan(1/2*d*x+1/2*c) - 1) * A + 1/d/b^2 * \ln(\tan(1/2*d*x+1/2*c) - 1) * B * a - 1/d/b / (\tan(1/2*d*x+1/2*c) + 1) * B + 1/d/b * \ln(\tan(1/2*d*x+1/2*c) + 1) * A - 1/d/b^2 * \ln(\tan(1/2*d*x+1/2*c) + 1) * B * a$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for more details) Is 4*a^2-4*b^2 positive or negative?

mupad [B] time = 2.97, size = 719, normalized size = 7.34

$$\frac{2 B a \operatorname{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d x}{2}\right)}\right)}{d\left(a^2-b^2\right)} - \frac{2 A b \operatorname{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d x}{2}\right)}\right)}{d\left(a^2-b^2\right)} - \frac{B b \tan(c+d x)}{d\left(a^2-b^2\right)} - \frac{B a^2 \ln\left(\frac{a \sin\left(\frac{c}{2}+\frac{d x}{2}\right)-b \sin\left(\frac{c}{2}+\frac{d x}{2}\right)+\cos\left(\frac{c}{2}+\frac{d x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d x}{2}\right)}\right) \sqrt{a^2-b^2}}{d\left(a^2-b^2\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^2*(a + b/cos(c + d*x))),x)

[Out] $(2*B*a*\operatorname{atanh}(\sin(c/2 + (d*x)/2)/\cos(c/2 + (d*x)/2)))/(d*(a^2 - b^2)) - (2*A*b*\operatorname{atanh}(\sin(c/2 + (d*x)/2)/\cos(c/2 + (d*x)/2)))/(d*(a^2 - b^2)) - (B*b*\tan(c + d*x))/(d*(a^2 - b^2)) - (B*a^2*\log((a*\sin(c/2 + (d*x)/2) - b*\sin(c/2 + (d*x)/2) + \cos(c/2 + (d*x)/2)*(a^2 - b^2)^{(1/2)})/\cos(c/2 + (d*x)/2)))/(d*(a^2 - b^2)^{(3/2)}) + (2*A*a^2*\operatorname{atanh}(\sin(c/2 + (d*x)/2)/\cos(c/2 + (d*x)/2)))/(b*d*(a^2 - b^2)) - (2*B*a^3*\operatorname{atanh}(\sin(c/2 + (d*x)/2)/\cos(c/2 + (d*x)/2)))/(b^2*d*(a^2 - b^2)) - (A*a^3*\log((a*\sin(c/2 + (d*x)/2) - b*\sin(c/2 + (d*x)/2) + \cos(c/2 + (d*x)/2)*(a^2 - b^2)^{(1/2)})/\cos(c/2 + (d*x)/2)))/(b*d*(a^2 - b^2)^{(3/2)}) + (B*a^4*\log((a*\sin(c/2 + (d*x)/2) - b*\sin(c/2 + (d*x)/2) + \cos(c/2 + (d*x)/2)*(a^2 - b^2)^{(1/2)})/\cos(c/2 + (d*x)/2)))/(b^2*d*(a^2 - b^2)^{(3/2)}) + (A*a*b*\log((a*\sin(c/2 + (d*x)/2) - b*\sin(c/2 + (d*x)/2) + \cos(c/2 + (d*x)/2)*(a^2 - b^2)^{(1/2)})/\cos(c/2 + (d*x)/2)))/(d*(a^2 - b^2)^{(3/2)}) + (B*a^2*\tan(c + d*x))/(b*d*(a^2 - b^2)) - (B*a^2*\log((b*\sin(c/2 + (d*x)/2) - a*\sin(c/2 + (d*x)/2) + \cos(c/2 + (d*x)/2)*(a^2 - b^2)^{(1/2)})/\cos(c/2 + (d*x)/2))*((a + b)*(a - b))^{(1/2)})/(b^2*d*(a^2 - b^2)) + (A*a*\log((b*\sin(c/2 + (d*x)/2) - a*\sin(c/2 + (d*x)/2) + \cos(c/2 + (d*x)/2)*(a^2 - b^2)^{(1/2)})/\cos(c/2 + (d*x)/2))*((a + b)*(a - b))^{(1/2)})/(b*d*(a^2 - b^2))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \sec^2(c + dx)}{a + b \sec(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)), x)

[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**2/(a + b*sec(c + d*x)), x)

$$3.314 \quad \int \frac{\sec(c+dx)(A+B \sec(c+dx))}{a+b \sec(c+dx)} dx$$

Optimal. Leaf size=76

$$\frac{2(Ab - aB) \tanh^{-1}\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{bd\sqrt{a-b}\sqrt{a+b}} + \frac{B \tanh^{-1}(\sin(c+dx))}{bd}$$

[Out] B*arctanh(sin(d*x+c))/b/d+2*(A*b-B*a)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/b/d/(a-b)^(1/2)/(a+b)^(1/2)

Rubi [A] time = 0.13, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {3998, 3770, 3831, 2659, 208}

$$\frac{2(Ab - aB) \tanh^{-1}\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{bd\sqrt{a-b}\sqrt{a+b}} + \frac{B \tanh^{-1}(\sin(c+dx))}{bd}$$

Antiderivative was successfully verified.

[In] Int[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x]),x]

[Out] (B*ArcTanh[Sin[c + d*x]]/(b*d) + (2*(A*b - a*B)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b*Sqrt[a + b]*d)

Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 2659

Int[((a_) + (b_)*sin[Pi/2 + (c_) + (d_)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 3770

Int[csc[(c_) + (d_)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3831

Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 3998

Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)))/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := Dist[B/b, Int[Csc[e + f*x], x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]

Rubi steps

$$\begin{aligned} \int \frac{\sec(c + dx)(A + B \sec(c + dx))}{a + b \sec(c + dx)} dx &= \frac{B \int \sec(c + dx) dx}{b} + \frac{(Ab - aB) \int \frac{\sec(c + dx)}{a + b \sec(c + dx)} dx}{b} \\ &= \frac{B \tanh^{-1}(\sin(c + dx))}{bd} + \frac{(Ab - aB) \int \frac{1}{1 + \frac{a \cos(c + dx)}{b}} dx}{b^2} \\ &= \frac{B \tanh^{-1}(\sin(c + dx))}{bd} + \frac{(2(Ab - aB)) \text{Subst}\left(\int \frac{1}{1 + \frac{a}{b} + (1 - \frac{a}{b})x^2} dx, x, \tan\left(\frac{1}{2}\right)\right)}{b^2 d} \\ &= \frac{B \tanh^{-1}(\sin(c + dx))}{bd} + \frac{2(Ab - aB) \tanh^{-1}\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c + dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b} b \sqrt{a+b} d} \end{aligned}$$

Mathematica [A] time = 0.19, size = 112, normalized size = 1.47

$$\frac{2(aB - Ab) \tanh^{-1}\left(\frac{(b-a) \tan\left(\frac{1}{2}(c + dx)\right)}{\sqrt{a^2 - b^2}}\right)}{\sqrt{a^2 - b^2}} + B \left(\log\left(\sin\left(\frac{1}{2}(c + dx)\right) + \cos\left(\frac{1}{2}(c + dx)\right)\right) - \log\left(\cos\left(\frac{1}{2}(c + dx)\right) - \sin\left(\frac{1}{2}(c + dx)\right)\right) \right) / bd$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x]),x]

[Out] ((2*(-(A*b) + a*B)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] + B*(-Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]))/(b*d)

fricas [A] time = 1.04, size = 316, normalized size = 4.16

$$\left[\frac{(Ba - Ab)\sqrt{a^2 - b^2} \log\left(\frac{2ab \cos(dx+c) - (a^2 - 2b^2) \cos(dx+c)^2 + 2\sqrt{a^2 - b^2} (b \cos(dx+c) + a) \sin(dx+c) + 2a^2 - b^2}{a^2 \cos(dx+c)^2 + 2ab \cos(dx+c) + b^2}\right) - (Ba^2 - Bb^2) \log\left(\frac{\sin(dx+c) + 1}{\sin(dx+c) - 1}\right)}{2(a^2b - b^3)d} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="fricas")

[Out] [-1/2*((B*a - A*b)*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - (B*a^2 - B*b^2)*log(sin(d*x + c) + 1) + (B*a^2 - B*b^2)*log(-sin(d*x + c) + 1))/((a^2*b - b^3)*d), -1/2*(2*(B*a - A*b)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c)))) - (B*a^2 - B*b^2)*log(sin(d*x + c) + 1) + (B*a^2 - B*b^2)*log(-sin(d*x + c) + 1))/((a^2*b - b^3)*d)]

giac [A] time = 1.33, size = 127, normalized size = 1.67

$$\frac{B \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 1\right|\right)}{b} - \frac{B \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 1\right|\right)}{b} + \frac{2 \left(\pi \left\lfloor \frac{dx+c}{2\pi} + \frac{1}{2} \right\rfloor \operatorname{sgn}(2a-2b) + \arctan\left(\frac{a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - b \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{\sqrt{-a^2 + b^2}}\right) \right) (Ba - Ab)}{\sqrt{-a^2 + b^2} d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="giac")

[Out] (B*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b - B*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b + 2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*(B*a - A*b)/(sqrt(-a^2 + b^2)*b))/d

maple [A] time = 0.75, size = 135, normalized size = 1.78

$$\frac{2 \operatorname{arctanh}\left(\frac{\tan\left(\frac{dx}{2} + \frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right) A}{d \sqrt{(a-b)(a+b)}} - \frac{2 \operatorname{arctanh}\left(\frac{\tan\left(\frac{dx}{2} + \frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right) a B}{db \sqrt{(a-b)(a+b)}} - \frac{\ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right) B}{db} + \frac{\ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right) B}{db}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x)

[Out] $2/d/((a-b)*(a+b))^{(1/2)}*\operatorname{arctanh}(\tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^{(1/2)})*A-2/d/b/((a-b)*(a+b))^{(1/2)}*\operatorname{arctanh}(\tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^{(1/2)})*a*B-1/d/b*\ln(\tan(1/2*d*x+1/2*c)-1)*B+1/d/b*\ln(\tan(1/2*d*x+1/2*c)+1)*B$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see 'assume?' for more details)Is 4*a^2-4*b^2 positive or negative?

mupad [B] time = 3.09, size = 573, normalized size = 7.54

$$\frac{A a^2 \ln\left(\frac{a \sin\left(\frac{c}{2} + \frac{dx}{2}\right) - b \sin\left(\frac{c}{2} + \frac{dx}{2}\right) + \cos\left(\frac{c}{2} + \frac{dx}{2}\right) \sqrt{a^2 - b^2}}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)}\right) + A \ln\left(\frac{a \cos\left(\frac{c}{2} + \frac{dx}{2}\right) + b \cos\left(\frac{c}{2} + \frac{dx}{2}\right) - \sin\left(\frac{c}{2} + \frac{dx}{2}\right) \sqrt{a^2 - b^2}}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)}\right) \sqrt{(a+b)(a-b)}}{d(a^2 - b^2)^{3/2} + d(a^2 - b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B/cos(c + d*x))/(cos(c + d*x)*(a + b/cos(c + d*x))),x)`

[Out] $(A*a^2*\log((a*\sin(c/2 + (d*x)/2) - b*\sin(c/2 + (d*x)/2) + \cos(c/2 + (d*x)/2)*(a^2 - b^2)^{(1/2)})/\cos(c/2 + (d*x)/2)))/(d*(a^2 - b^2)^{(3/2)}) - (A*\log((a*\cos(c/2 + (d*x)/2) + b*\cos(c/2 + (d*x)/2) - \sin(c/2 + (d*x)/2)*(a^2 - b^2)^{(1/2)})/\cos(c/2 + (d*x)/2))*((a + b)*(a - b))^{(1/2)})/(d*(a^2 - b^2)) - (2*B*b*\operatorname{atanh}(\sin(c/2 + (d*x)/2)/\cos(c/2 + (d*x)/2)))/(d*(a^2 - b^2)) - (A*b^2*\log((a*\sin(c/2 + (d*x)/2) - b*\sin(c/2 + (d*x)/2) + \cos(c/2 + (d*x)/2)*(a^2 - b^2)^{(1/2)})/\cos(c/2 + (d*x)/2)))/(d*(a^2 - b^2)^{(3/2)}) + (2*B*a^2*\operatorname{atanh}(\sin(c/2 + (d*x)/2)/\cos(c/2 + (d*x)/2)))/(b*d*(a^2 - b^2)) - (B*a^3*\log((a*\sin(c/2 + (d*x)/2) - b*\sin(c/2 + (d*x)/2) + \cos(c/2 + (d*x)/2)*(a^2 - b^2)^{(1/2)})/\cos(c/2 + (d*x)/2)))/(b*d*(a^2 - b^2)^{(3/2)}) + (B*a*b*\log((a*\sin(c/2 + (d*x)/2) - b*\sin(c/2 + (d*x)/2) + \cos(c/2 + (d*x)/2)*(a^2 - b^2)^{(1/2)})/\cos(c/2 + (d*x)/2)))/(d*(a^2 - b^2)^{(3/2)}) + (B*a*\log((a*\cos(c/2 + (d*x)/2) + b*\cos(c/2 + (d*x)/2) - \sin(c/2 + (d*x)/2)*(a^2 - b^2)^{(1/2)})/\cos(c/2 + (d*x)/2))*((a + b)*(a - b))^{(1/2)})/(b*d*(a^2 - b^2))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \sec(c + dx)}{a + b \sec(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)/(a + b*sec(c + d*x)), x)
```

$$3.315 \quad \int \frac{A+B \sec(c+dx)}{a+b \sec(c+dx)} dx$$

Optimal. Leaf size=67

$$\frac{Ax}{a} - \frac{2(Ab - aB) \tanh^{-1} \left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}} \right)}{ad\sqrt{a-b}\sqrt{a+b}}$$

[Out] $A*x/a - 2*(A*b - B*a)*\operatorname{arctanh}((a-b)^{(1/2)}*\tan(1/2*d*x + 1/2*c)/(a+b)^{(1/2)})/a/d/((a-b)^{(1/2)})/(a+b)^{(1/2)}$

Rubi [A] time = 0.10, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3919, 3831, 2659, 208}

$$\frac{Ax}{a} - \frac{2(Ab - aB) \tanh^{-1} \left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}} \right)}{ad\sqrt{a-b}\sqrt{a+b}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(A + B*\operatorname{Sec}[c + d*x])/(a + b*\operatorname{Sec}[c + d*x]), x]$

[Out] $(A*x)/a - (2*(A*b - a*B)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a - b]*\operatorname{Tan}[(c + d*x)/2])/\operatorname{Sqrt}[a + b]])/(a*\operatorname{Sqrt}[a - b]*\operatorname{Sqrt}[a + b]*d)$

Rule 208

$\operatorname{Int}(((a_) + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[-(a/b), 2]*\operatorname{ArcTanh}[x/\operatorname{Rt}[-(a/b), 2]])/a, x] /; \operatorname{FreeQ}\{a, b\}, x] \ \&\& \operatorname{NegQ}[a/b]$

Rule 2659

$\operatorname{Int}(((a_) + (b_)*\sin[\operatorname{Pi}/2 + (c_) + (d_)*(x_)])^{-1}, x_Symbol] \rightarrow \operatorname{With}\{e = \operatorname{FreeFactors}[\operatorname{Tan}[(c + d*x)/2], x]\}, \operatorname{Dist}[(2*e)/d, \operatorname{Subst}[\operatorname{Int}[1/(a + b + (a - b)*e^2*x^2), x], x, \operatorname{Tan}[(c + d*x)/2]/e], x] /; \operatorname{FreeQ}\{a, b, c, d\}, x] \ \&\& \operatorname{NeQ}[a^2 - b^2, 0]$

Rule 3831

$\operatorname{Int}[\operatorname{csc}[(e_) + (f_)*(x_)]/(\operatorname{csc}[(e_) + (f_)*(x_)]*(b_) + (a_)), x_Symbol] \rightarrow \operatorname{Dist}[1/b, \operatorname{Int}[1/(1 + (a*\operatorname{Sin}[e + f*x])/b), x], x] /; \operatorname{FreeQ}\{a, b, e, f\}, x] \ \&\& \operatorname{NeQ}[a^2 - b^2, 0]$

Rule 3919

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))/(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rubi steps

$$\begin{aligned} \int \frac{A + B \sec(c + dx)}{a + b \sec(c + dx)} dx &= \frac{Ax}{a} - \frac{(Ab - aB) \int \frac{\sec(c+dx)}{a+b \sec(c+dx)} dx}{a} \\ &= \frac{Ax}{a} - \frac{(Ab - aB) \int \frac{1}{1 + \frac{a \cos(c+dx)}{b}} dx}{ab} \\ &= \frac{Ax}{a} - \frac{(2(Ab - aB)) \text{Subst} \left(\int \frac{1}{1 + \frac{a}{b} + (1 - \frac{a}{b})x^2} dx, x, \tan \left(\frac{1}{2}(c + dx) \right) \right)}{abd} \\ &= \frac{Ax}{a} - \frac{2(Ab - aB) \tanh^{-1} \left(\frac{\sqrt{a-b} \tan \left(\frac{1}{2}(c+dx) \right)}{\sqrt{a+b}} \right)}{a\sqrt{a-b}\sqrt{a+b}d} \end{aligned}$$

Mathematica [A] time = 0.13, size = 68, normalized size = 1.01

$$\frac{2(Ab - aB) \tanh^{-1} \left(\frac{(b-a) \tan \left(\frac{1}{2}(c+dx) \right)}{\sqrt{a^2 - b^2}} \right)}{\sqrt{a^2 - b^2}} + A(c + dx)$$

ad

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sec[c + d*x])/(a + b*Sec[c + d*x]), x]

[Out] (A*(c + d*x) + (2*(A*b - a*B)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2]/(a*d)

fricas [A] time = 0.47, size = 250, normalized size = 3.73

$$\left[\frac{2(Aa^2 - Ab^2)dx - (Ba - Ab)\sqrt{a^2 - b^2} \log \left(\frac{2ab \cos(dx+c) - (a^2 - 2b^2) \cos(dx+c)^2 - 2\sqrt{a^2 - b^2} (b \cos(dx+c) + a) \sin(dx+c) + 2a^2 - b^2}{a^2 \cos(dx+c)^2 + 2ab \cos(dx+c) + b^2} \right)}{2(a^3 - ab^2)d} \right],$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="fricas")

[Out] [1/2*(2*(A*a^2 - A*b^2)*d*x - (B*a - A*b)*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)))/((a^3 - a*b^2)*d), ((A*a^2 - A*b^2)*d*x + (B*a - A*b)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c)))/((a^3 - a*b^2)*d)]

giac [B] time = 0.43, size = 274, normalized size = 4.09

$$\frac{\left(\sqrt{-a^2+b^2} A(a-2b)|-a+b|+\sqrt{-a^2+b^2} B a|-a+b|-\sqrt{-a^2+b^2} A|a||-a+b|+\sqrt{-a^2+b^2} B|a||-a+b|\right)\left(\pi\left[\frac{dx+c}{2\pi}+\frac{1}{2}\right]+\arctan\left(\frac{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{\frac{b+\sqrt{(a+b)(a-b)+b^2}}{a-b}}}\right)\right)}{(a^2-2ab+b^2)a^2+(a^2b-2ab^2+b^3)|a|} + \frac{(Aa+Ba)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="giac")

[Out] ((sqrt(-a^2 + b^2)*A*(a - 2*b)*abs(-a + b) + sqrt(-a^2 + b^2)*B*a*abs(-a + b) - sqrt(-a^2 + b^2)*A*abs(a)*abs(-a + b) + sqrt(-a^2 + b^2)*B*abs(a)*abs(-a + b))*(pi*floor(1/2*(d*x + c)/pi + 1/2) + arctan(tan(1/2*d*x + 1/2*c)/sqrt(-(b + sqrt((a + b)*(a - b) + b^2))/(a - b))))/((a^2 - 2*a*b + b^2)*a^2 + (a^2*b - 2*a*b^2 + b^3)*abs(a)) + (A*a + B*a - 2*A*b + A*abs(a) - B*abs(a))*(pi*floor(1/2*(d*x + c)/pi + 1/2) + arctan(tan(1/2*d*x + 1/2*c)/sqrt(-(b - sqrt((a + b)*(a - b) + b^2))/(a - b))))/(a^2 - b*abs(a))/d

maple [A] time = 0.80, size = 113, normalized size = 1.69

$$-\frac{2 \operatorname{arctanh}\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right) A b}{d a \sqrt{(a-b)(a+b)}} + \frac{2 \operatorname{arctanh}\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right) B}{d \sqrt{(a-b)(a+b)}} + \frac{2 A \operatorname{arctan}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x)

[Out] -2/d/a/((a-b)*(a+b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*A*b+2/d/((a-b)*(a+b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*B+2/d*A/a*arctan(tan(1/2*d*x+1/2*c))

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for more details)Is 4*a^2-4*b^2 positive or negative?

mupad [B] time = 3.20, size = 573, normalized size = 8.55

$$\frac{2 A a \operatorname{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d x}{2}\right)}\right) B \ln\left(\frac{a \cos\left(\frac{c}{2}+\frac{d x}{2}\right)+b \cos\left(\frac{c}{2}+\frac{d x}{2}\right)-\sin\left(\frac{c}{2}+\frac{d x}{2}\right) \sqrt{a^2-b^2}}{\cos\left(\frac{c}{2}+\frac{d x}{2}\right)}\right) \sqrt{(a+b)(a-b)} B a^2 \ln\left(\frac{a \sin\left(\frac{c}{2}+\frac{d x}{2}\right)-b}{\cos\left(\frac{c}{2}+\frac{d x}{2}\right)}\right)}{d\left(a^2-b^2\right) d\left(a^2-b^2\right) d\left(a^2-b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B/cos(c + d*x))/(a + b/cos(c + d*x)),x)`

[Out] $(2*A*a*\operatorname{atan}(\sin(c/2 + (d*x)/2)/\cos(c/2 + (d*x)/2)))/(d*(a^2 - b^2)) - (B*\log((a*\cos(c/2 + (d*x)/2) + b*\cos(c/2 + (d*x)/2) - \sin(c/2 + (d*x)/2)*(a^2 - b^2)^{(1/2)})/\cos(c/2 + (d*x)/2))*((a + b)*(a - b))^{(1/2)})/(d*(a^2 - b^2)) + (B*a^2*\log((a*\sin(c/2 + (d*x)/2) - b*\sin(c/2 + (d*x)/2) + \cos(c/2 + (d*x)/2)*(a^2 - b^2)^{(1/2)})/\cos(c/2 + (d*x)/2)))/(d*(a^2 - b^2)^{(3/2)}) - (B*b^2*\log((a*\sin(c/2 + (d*x)/2) - b*\sin(c/2 + (d*x)/2) + \cos(c/2 + (d*x)/2)*(a^2 - b^2)^{(1/2)})/\cos(c/2 + (d*x)/2)))/(d*(a^2 - b^2)^{(3/2)}) - (2*A*b^2*\operatorname{atan}(\sin(c/2 + (d*x)/2)/\cos(c/2 + (d*x)/2)))/(a*d*(a^2 - b^2)) + (A*b^3*\log((a*\sin(c/2 + (d*x)/2) - b*\sin(c/2 + (d*x)/2) + \cos(c/2 + (d*x)/2)*(a^2 - b^2)^{(1/2)})/\cos(c/2 + (d*x)/2)))/(a*d*(a^2 - b^2)^{(3/2)}) - (A*a*b*\log((a*\sin(c/2 + (d*x)/2) - b*\sin(c/2 + (d*x)/2) + \cos(c/2 + (d*x)/2)*(a^2 - b^2)^{(1/2)})/\cos(c/2 + (d*x)/2)))/(d*(a^2 - b^2)^{(3/2)}) + (A*b*\log((a*\cos(c/2 + (d*x)/2) + b*\cos(c/2 + (d*x)/2) - \sin(c/2 + (d*x)/2)*(a^2 - b^2)^{(1/2)})/\cos(c/2 + (d*x)/2))*((a + b)*(a - b))^{(1/2)})/(a*d*(a^2 - b^2))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \sec(c + dx)}{a + b \sec(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x)`

[Out] `Integral((A + B*sec(c + d*x))/(a + b*sec(c + d*x)), x)`

$$3.316 \quad \int \frac{\cos(c+dx)(A+B \sec(c+dx))}{a+b \sec(c+dx)} dx$$

Optimal. Leaf size=90

$$\frac{2b(Ab - aB) \tanh^{-1} \left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}} \right)}{a^2 d \sqrt{a-b} \sqrt{a+b}} - \frac{x(Ab - aB)}{a^2} + \frac{A \sin(c + dx)}{ad}$$

[Out] $-(A*b-B*a)*x/a^2+A*\sin(d*x+c)/a/d+2*b*(A*b-B*a)*\operatorname{arctanh}((a-b)^{(1/2)}*\tan(1/2*d*x+1/2*c)/(a+b)^{(1/2}))/a^2/d/(a-b)^{(1/2)/(a+b)^{(1/2)}$

Rubi [A] time = 0.15, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {4034, 12, 3783, 2659, 208}

$$\frac{2b(Ab - aB) \tanh^{-1} \left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}} \right)}{a^2 d \sqrt{a-b} \sqrt{a+b}} - \frac{x(Ab - aB)}{a^2} + \frac{A \sin(c + dx)}{ad}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(\operatorname{Cos}[c + d*x]*(A + B*\operatorname{Sec}[c + d*x]))/(a + b*\operatorname{Sec}[c + d*x]), x]$

[Out] $-\left(\frac{(A*b - a*B)*x}{a^2}\right) + \left(\frac{2*b*(A*b - a*B)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a - b]*\operatorname{Tan}[(c + d*x)/2])/(\operatorname{Sqrt}[a + b])]}{a^2*\operatorname{Sqrt}[a - b]*\operatorname{Sqrt}[a + b]*d}\right) + \left(\frac{A*\operatorname{Sin}[c + d*x]}{a*d}\right)$

Rule 12

$\operatorname{Int}[(a_)*(u_), x_Symbol] := \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /;$ FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 208

$\operatorname{Int}[(a_)+(b_.)*(x_)^2)^{-1}, x_Symbol] := \operatorname{Simp}[(\operatorname{Rt}[-(a/b), 2]*\operatorname{ArcTanh}[x/\operatorname{Rt}[-(a/b), 2]])/a, x] /;$ FreeQ[{a, b}, x] && NegQ[a/b]

Rule 2659

$\operatorname{Int}[(a_)+(b_.)*\sin[\operatorname{Pi}/2 + (c_.) + (d_.)*(x_)])^{-1}, x_Symbol] := \operatorname{With}[{e = \operatorname{FreeFactors}[\operatorname{Tan}[(c + d*x)/2], x]}, \operatorname{Dist}[(2*e)/d, \operatorname{Subst}[\operatorname{Int}[1/(a + b + (a - b)*e^2*x^2), x], x, \operatorname{Tan}[(c + d*x)/2]/e], x]] /;$ FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 3783

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_.))⁽⁻¹⁾, x_Symbol] := Simp[x/a, x] - Dist[1/a, Int[1/(1 + (a*Sin[c + d*x])/b), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a² - b², 0]

Rule 4034

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] := Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])ⁿ)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + A*a*(n + 1)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]², x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && NeQ[a² - b², 0] && LeQ[n, -1]

Rubi steps

$$\begin{aligned}
 \int \frac{\cos(c + dx)(A + B \sec(c + dx))}{a + b \sec(c + dx)} dx &= \frac{A \sin(c + dx)}{ad} - \frac{\int \frac{Ab - aB}{a + b \sec(c + dx)} dx}{a} \\
 &= \frac{A \sin(c + dx)}{ad} - \frac{(Ab - aB) \int \frac{1}{a + b \sec(c + dx)} dx}{a} \\
 &= -\frac{(Ab - aB)x}{a^2} + \frac{A \sin(c + dx)}{ad} + \frac{(Ab - aB) \int \frac{1}{1 + \frac{a \cos(c + dx)}{b}} dx}{a^2} \\
 &= -\frac{(Ab - aB)x}{a^2} + \frac{A \sin(c + dx)}{ad} + \frac{(2(Ab - aB)) \text{Subst} \left(\int \frac{1}{1 + \frac{a}{b} + \left(\frac{1-a}{b}\right)x^2} dx, \right)}{a^2 d} \\
 &= -\frac{(Ab - aB)x}{a^2} + \frac{2b(Ab - aB) \tanh^{-1} \left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c + dx)\right)}{\sqrt{a+b}} \right)}{a^2 \sqrt{a-b} \sqrt{a+b} d} + \frac{A \sin(c + dx)}{ad}
 \end{aligned}$$

Mathematica [A] time = 0.23, size = 85, normalized size = 0.94

$$\frac{2b(Ab - aB) \tanh^{-1} \left(\frac{(b-a) \tan\left(\frac{1}{2}(c + dx)\right)}{\sqrt{a^2 - b^2}} \right)}{\sqrt{a^2 - b^2}} + \frac{(c + dx)(aB - Ab) + aA \sin(c + dx)}{a^2 d}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x]),x]

[Out] ((- (A*b) + a*B)*(c + d*x) - (2*b*(A*b - a*B)*ArcTanh[((-a + b)*Tan[(c + d*x)])/2])/Sqrt[a^2 - b^2])/Sqrt[a^2 - b^2] + a*A*Sin[c + d*x]/(a^2*d)

fricas [A] time = 0.48, size = 328, normalized size = 3.64

$$\frac{2(Ba^3 - Aa^2b - Bab^2 + Ab^3)dx - (Bab - Ab^2)\sqrt{a^2 - b^2} \log\left(\frac{2ab \cos(dx+c) - (a^2 - 2b^2)\cos(dx+c)^2 + 2\sqrt{a^2 - b^2}(b \cos(dx+c) + a)}{a^2 \cos(dx+c)^2 + 2ab \cos(dx+c) + b^2}\right)}{2(a^4 - a^2b^2)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="fricas")

[Out] [1/2*(2*(B*a^3 - A*a^2*b - B*a*b^2 + A*b^3)*d*x - (B*a*b - A*b^2)*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + 2*(A*a^3 - A*a*b^2)*sin(d*x + c))/((a^4 - a^2*b^2)*d), ((B*a^3 - A*a^2*b - B*a*b^2 + A*b^3)*d*x - (B*a*b - A*b^2)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) + (A*a^3 - A*a*b^2)*sin(d*x + c))/((a^4 - a^2*b^2)*d)]

giac [A] time = 0.92, size = 141, normalized size = 1.57

$$\frac{\frac{(Ba - Ab)(dx + c)}{a^2} + \frac{2A \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right)^2 + 1}a - \frac{2(Bab - Ab^2)\left(\pi\left[\frac{dx+c}{2\pi} + \frac{1}{2}\right] \operatorname{sgn}(-2a+2b) + \arctan\left(-\frac{a \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - b \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)}{\sqrt{-a^2 + b^2}}\right)\right)}{\sqrt{-a^2 + b^2} a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="giac")

[Out] ((B*a - A*b)*(d*x + c)/a^2 + 2*A*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 + 1)*a) - 2*(B*a*b - A*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((sqrt(-a^2 + b^2)*a^2))/d

maple [B] time = 1.25, size = 172, normalized size = 1.91

$$\frac{2b^2 \operatorname{arctanh}\left(\frac{\tan\left(\frac{dx}{2} + \frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)A}{da^2\sqrt{(a-b)(a+b)}} - \frac{2b \operatorname{arctanh}\left(\frac{\tan\left(\frac{dx}{2} + \frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)B}{da\sqrt{(a-b)(a+b)}} + \frac{2A \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{da\left(1 + \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} - \frac{2A \operatorname{arctan}\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{da^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x)`

[Out] $2/d*b^2/a^2/((a-b)*(a+b))^{(1/2)}*\operatorname{arctanh}(\tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^{(1/2)})*A-2/d*b/a/((a-b)*(a+b))^{(1/2)}*\operatorname{arctanh}(\tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^{(1/2)})*B+2/d/a*A*\tan(1/2*d*x+1/2*c)/(1+\tan(1/2*d*x+1/2*c)^2)-2/d/a^2*A*\operatorname{arctan}(\tan(1/2*d*x+1/2*c))*b+2/a/d*\operatorname{arctan}(\tan(1/2*d*x+1/2*c))*B$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for more details)Is 4*a^2-4*b^2 positive or negative?

mupad [B] time = 3.36, size = 740, normalized size = 8.22

$$\frac{2 A b^3 \operatorname{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d x}{2}\right)}\right)}{d\left(a^4-a^2 b^2\right)}+\frac{2 B a^3 \operatorname{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d x}{2}\right)}\right)}{d\left(a^4-a^2 b^2\right)}+\frac{A a^3 \sin(c+d x)}{d\left(a^4-a^2 b^2\right)}-\frac{A a b^2 \sin(c+d x)}{d\left(a^4-a^2 b^2\right)}+\frac{A b^2 \operatorname{atan}\left(\frac{-a^5 \sin\left(\frac{c}{2}+\frac{d x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d x}{2}\right)}\right)}{d\left(a^4-a^2 b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((cos(c + d*x)*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x)),x)`

[Out] $(2*A*b^3*\operatorname{atan}(\sin(c/2 + (d*x)/2)/\cos(c/2 + (d*x)/2)))/(d*(a^4 - a^2*b^2)) + (2*B*a^3*\operatorname{atan}(\sin(c/2 + (d*x)/2)/\cos(c/2 + (d*x)/2)))/(d*(a^4 - a^2*b^2)) + (A*a^3*\sin(c + d*x))/(d*(a^4 - a^2*b^2)) - (A*a*b^2*\sin(c + d*x))/(d*(a^4 - a^2*b^2)) + (A*b^2*\operatorname{atan}((b^3*\sin(c/2 + (d*x)/2)*(a^2 - b^2)^{(3/2)*2i} - a^5*\sin(c/2 + (d*x)/2)*(a^2 - b^2)^{(1/2)*1i} + b^5*\sin(c/2 + (d*x)/2)*(a^2 - b^2)^{(1/2)*2i} - a^2*b^3*\sin(c/2 + (d*x)/2)*(a^2 - b^2)^{(1/2)*3i} + a^3*b^2*\sin(c/2 + (d*x)/2)*(a^2 - b^2)^{(1/2)*1i} + a^4*b*\sin(c/2 + (d*x)/2)*(a^2 - b^2)^{(1/2)*1i})/(a^6*\cos(c/2 + (d*x)/2) + a^2*b^4*\cos(c/2 + (d*x)/2) - 2*a^4*b^2*\cos(c/2 + (d*x)/2)))/(d*(a^4 - a^2*b^2)) - (2*A*a^2*b*\operatorname{atan}(\sin(c/2 + (d*x)/2)/\cos(c/2 + (d*x)/2)))/(d*(a^4 - a^2*b^2)) - (2*B*a*b^2*\operatorname{atan}(\sin(c/2 + (d*x)/2)/\cos(c/2 + (d*x)/2)))/(d*(a^4 - a^2*b^2)) - (B*a*b*\operatorname{atan}((b^3*\sin(c/2 + (d*x)/2)*(a^2 - b^2)^{(3/2)*2i} - a^5*\sin(c/2 + (d*x)/2)*(a^2 - b^2)^{(1/2)*1i} + b^5*\sin(c/2 + (d*x)/2)*(a^2 - b^2)^{(1/2)*2i} -$

```
a^2*b^3*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2)*3i + a^3*b^2*sin(c/2 + (d*x)/2)
)*(a^2 - b^2)^(1/2)*1i + a^4*b*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2)*1i)/(a^
6*cos(c/2 + (d*x)/2) + a^2*b^4*cos(c/2 + (d*x)/2) - 2*a^4*b^2*cos(c/2 + (d*
x)/2)))*(a^2 - b^2)^(1/2)*2i)/(d*(a^4 - a^2*b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \cos(c + dx)}{a + b \sec(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*cos(c + d*x)/(a + b*sec(c + d*x)), x)
```

$$3.317 \quad \int \frac{\cos^2(c+dx)(A+B \sec(c+dx))}{a+b \sec(c+dx)} dx$$

Optimal. Leaf size=134

$$\frac{2b^2(Ab - aB) \tanh^{-1}\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3 d \sqrt{a-b} \sqrt{a+b}} - \frac{(Ab - aB) \sin(c+dx)}{a^2 d} + \frac{x(a^2 A - 2abB + 2Ab^2)}{2a^3} + \frac{A \sin(c+dx) \cos(c+dx)}{2ad}$$

[Out] $1/2*(A*a^2+2*A*b^2-2*B*a*b)*x/a^3-(A*b-B*a)*\sin(d*x+c)/a^2/d+1/2*A*\cos(d*x+c)*\sin(d*x+c)/a/d-2*b^2*(A*b-B*a)*\operatorname{arctanh}((a-b)^{(1/2)}*\tan(1/2*d*x+1/2*c)/(a+b)^{(1/2)})/a^3/d/(a-b)^{(1/2)}/(a+b)^{(1/2)}$

Rubi [A] time = 0.40, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4034, 4104, 3919, 3831, 2659, 208}

$$\frac{2b^2(Ab - aB) \tanh^{-1}\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3 d \sqrt{a-b} \sqrt{a+b}} + \frac{x(a^2 A - 2abB + 2Ab^2)}{2a^3} - \frac{(Ab - aB) \sin(c+dx)}{a^2 d} + \frac{A \sin(c+dx) \cos(c+dx)}{2ad}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(\operatorname{Cos}[c + d*x]^2*(A + B*\operatorname{Sec}[c + d*x]))/(a + b*\operatorname{Sec}[c + d*x]), x]$

[Out] $((a^2*A + 2*A*b^2 - 2*a*b*B)*x)/(2*a^3) - (2*b^2*(A*b - a*B)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a - b]*\operatorname{Tan}[(c + d*x)/2])/(\operatorname{Sqrt}[a + b])])/(a^3*\operatorname{Sqrt}[a - b]*\operatorname{Sqrt}[a + b]*d) - ((A*b - a*B)*\operatorname{Sin}[c + d*x])/(a^2*d) + (A*\operatorname{Cos}[c + d*x]*\operatorname{Sin}[c + d*x])/(2*a*d)$

Rule 208

$\operatorname{Int}[(a + b*x^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[-(a/b), 2]*\operatorname{ArcTanh}[x/\operatorname{Rt}[-(a/b), 2]])/a, x] /;$ $\operatorname{FreeQ}\{a, b, x\} \ \&\& \ \operatorname{NegQ}[a/b]$

Rule 2659

$\operatorname{Int}[(a + b*\sin[\operatorname{Pi}/2 + (c + d*x)])^{-1}, x_Symbol] \rightarrow \operatorname{With}\{e = \operatorname{FreeFactors}[\operatorname{Tan}[(c + d*x)/2], x]\}, \operatorname{Dist}[(2*e)/d, \operatorname{Subst}[\operatorname{Int}[1/(a + b + (a - b)*e^2*x^2), x], x, \operatorname{Tan}[(c + d*x)/2]/e], x] /;$ $\operatorname{FreeQ}\{a, b, c, d, x\} \ \&\& \ \operatorname{NeQ}[a^2 - b^2, 0]$

Rule 3831

$\operatorname{Int}[\operatorname{csc}[e + f*x]/(\operatorname{csc}[e + f*x]*(b + a)), x_Symbol] \rightarrow \operatorname{Dist}[1/b, \operatorname{Int}[1/(1 + (a*\operatorname{Sin}[e + f*x])/b), x], x] /;$ $\operatorname{FreeQ}\{a, b, e, f\}$

}, x] && NeQ[a^2 - b^2, 0]

Rule 3919

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))/(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 4034

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + A*a*(n + 1)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]

Rule 4104

Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m, x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(c+dx)(A+B\sec(c+dx))}{a+b\sec(c+dx)} dx &= \frac{A\cos(c+dx)\sin(c+dx)}{2ad} - \frac{\int \frac{\cos(c+dx)(2(Ab-aB)-aA\sec(c+dx)-Ab\sec^2(c+dx))}{a+b\sec(c+dx)}}{2a} \\
&= -\frac{(Ab-aB)\sin(c+dx)}{a^2d} + \frac{A\cos(c+dx)\sin(c+dx)}{2ad} + \frac{\int \frac{a^2A+2Ab^2-2abB}{a+b\sec(c+dx)}}{2} \\
&= \frac{(a^2A+2Ab^2-2abB)x}{2a^3} - \frac{(Ab-aB)\sin(c+dx)}{a^2d} + \frac{A\cos(c+dx)\sin(c+dx)}{2ad} \\
&= \frac{(a^2A+2Ab^2-2abB)x}{2a^3} - \frac{(Ab-aB)\sin(c+dx)}{a^2d} + \frac{A\cos(c+dx)\sin(c+dx)}{2ad} \\
&= \frac{(a^2A+2Ab^2-2abB)x}{2a^3} - \frac{(Ab-aB)\sin(c+dx)}{a^2d} + \frac{A\cos(c+dx)\sin(c+dx)}{2ad} \\
&= \frac{(a^2A+2Ab^2-2abB)x}{2a^3} - \frac{2b^2(Ab-aB)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3\sqrt{a-b}\sqrt{a+b}d} - \dots
\end{aligned}$$

Mathematica [A] time = 0.36, size = 121, normalized size = 0.90

$$\frac{2(c+dx)(a^2A-2abB+2Ab^2) + \frac{8b^2(Ab-aB)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + a^2A\sin(2(c+dx)) + 4a(aB-Ab)\sin(c+dx)}{4a^3d}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x]),x]

[Out] (2*(a^2*A + 2*A*b^2 - 2*a*b*B)*(c + d*x) + (8*b^2*(A*b - a*B)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] + 4*a*(-(A*b) + a*B)*Sin[c + d*x] + a^2*A*Sin[2*(c + d*x)]/(4*a^3*d)

fricas [A] time = 0.50, size = 427, normalized size = 3.19

$$\left[\frac{(Aa^4 - 2Ba^3b + Aa^2b^2 + 2Bab^3 - 2Ab^4)dx - (Bab^2 - Ab^3)\sqrt{a^2 - b^2} \log\left(\frac{2ab\cos(dx+c) - (a^2 - 2b^2)\cos(dx+c)^2 - 2\sqrt{a^2 - b^2}\sin(dx+c)}{a^2\cos(dx+c)^2 + 2ab\sin(dx+c)}\right)}{2(a^5 - \dots)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="fricas")

[Out] [1/2*((A*a^4 - 2*B*a^3*b + A*a^2*b^2 + 2*B*a*b^3 - 2*A*b^4)*d*x - (B*a*b^2 - A*b^3)*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c))^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + (2*B*a^4 - 2*A*a^3*b - 2*B*a^2*b^2 + 2*A*a*b^3 + (A*a^4 - A*a^2*b^2)*cos(d*x + c))*sin(d*x + c))/((a^5 - a^3*b^2)*d), 1/2*((A*a^4 - 2*B*a^3*b + A*a^2*b^2 + 2*B*a*b^3 - 2*A*b^4)*d*x + 2*(B*a*b^2 - A*b^3)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) + (2*B*a^4 - 2*A*a^3*b - 2*B*a^2*b^2 + 2*A*a*b^3 + (A*a^4 - A*a^2*b^2)*cos(d*x + c))*sin(d*x + c))/((a^5 - a^3*b^2)*d)]

giac [A] time = 0.27, size = 227, normalized size = 1.69

$$\frac{(Aa^2 - 2Bab + 2Ab^2)(dx+c)}{a^3} + \frac{4(Bab^2 - Ab^3) \left(\pi \left[\frac{dx+c}{2\pi} + \frac{1}{2} \right] \operatorname{sgn}(-2a+2b) + \arctan \left(-\frac{a \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - b \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)}{\sqrt{-a^2+b^2}} \right) \right)}{\sqrt{-a^2+b^2} a^3} - \frac{2 \left(Aa \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) \right)^3 - 2Ba \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="giac")

[Out] 1/2*((A*a^2 - 2*B*a*b + 2*A*b^2)*(d*x + c)/a^3 + 4*(B*a*b^2 - A*b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/(sqrt(-a^2 + b^2)*a^3) - 2*(A*a*tan(1/2*d*x + 1/2*c)^3 - 2*B*a*tan(1/2*d*x + 1/2*c)^3 + 2*A*b*tan(1/2*d*x + 1/2*c)^3 - A*a*tan(1/2*d*x + 1/2*c) - 2*B*a*tan(1/2*d*x + 1/2*c) + 2*A*b*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^2)/d

maple [B] time = 1.14, size = 367, normalized size = 2.74

$$\frac{2b^3 \operatorname{arctanh} \left(\frac{\tan\left(\frac{dx}{2} + \frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}} \right) A}{d a^3 \sqrt{(a-b)(a+b)}} + \frac{2b^2 \operatorname{arctanh} \left(\frac{\tan\left(\frac{dx}{2} + \frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}} \right) B}{d a^2 \sqrt{(a-b)(a+b)}} - \frac{\left(\tan^3 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) A}{da \left(1 + \tan^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)^2} - \frac{2 \left(\tan^3 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) A}{d a^2 \left(1 + \tan^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x)

[Out] -2/d*b^3/a^3/((a-b)*(a+b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*A+2/d*b^2/a^2/((a-b)*(a+b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(a

$$-b)/((a-b)*(a+b))^{(1/2)})*B-1/d/a/(1+\tan(1/2*d*x+1/2*c)^2)^2*\tan(1/2*d*x+1/2*c)^3*A-2/d/a^2/(1+\tan(1/2*d*x+1/2*c)^2)^2*\tan(1/2*d*x+1/2*c)^3*A*b+2/d/a/(1+\tan(1/2*d*x+1/2*c)^2)^2*\tan(1/2*d*x+1/2*c)^3*B+1/d/a/(1+\tan(1/2*d*x+1/2*c)^2)^2*\tan(1/2*d*x+1/2*c)*A-2/d/a^2/(1+\tan(1/2*d*x+1/2*c)^2)^2*\tan(1/2*d*x+1/2*c)*A*b+2/d/a/(1+\tan(1/2*d*x+1/2*c)^2)^2*\tan(1/2*d*x+1/2*c)*B+1/d*A/a*\arctan(\tan(1/2*d*x+1/2*c))+2/d/a^3*\arctan(\tan(1/2*d*x+1/2*c))*A*b^2-2/d/a^2*a*\arctan(\tan(1/2*d*x+1/2*c))*B*b$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for more details)Is 4*a^2-4*b^2 positive or negative?

mupad [B] time = 6.02, size = 3740, normalized size = 27.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)^2*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x)),x)

[Out] ((tan(c/2 + (d*x)/2)*(A*a - 2*A*b + 2*B*a))/a^2 - (tan(c/2 + (d*x)/2)^3*(A*a + 2*A*b - 2*B*a))/a^2)/(d*(2*tan(c/2 + (d*x)/2)^2 + tan(c/2 + (d*x)/2)^4 + 1)) - (atan(((((((8*(2*A*a^10 + 4*A*a^6*b^4 - 6*A*a^7*b^3 + 2*A*a^8*b^2 - 4*B*a^7*b^3 + 8*B*a^8*b^2 - 2*A*a^9*b - 4*B*a^9*b))/a^6 - (4*tan(c/2 + (d*x)/2)*(A*a^2*1i + A*b^2*2i - B*a*b*2i))*(8*a^8*b + 8*a^6*b^3 - 16*a^7*b^2))/a^7)*(A*a^2*1i + A*b^2*2i - B*a*b*2i))/(2*a^3) + (8*tan(c/2 + (d*x)/2)*(A^2*a^7 - 8*A^2*b^7 + 16*A^2*a*b^6 - 3*A^2*a^6*b - 16*A^2*a^2*b^5 + 16*A^2*a^3*b^4 - 13*A^2*a^4*b^3 + 7*A^2*a^5*b^2 - 8*B^2*a^2*b^5 + 16*B^2*a^3*b^4 - 12*B^2*a^4*b^3 + 4*B^2*a^5*b^2 + 16*A*B*a*b^6 - 4*A*B*a^6*b - 32*A*B*a^2*b^5 + 28*A*B*a^3*b^4 - 20*A*B*a^4*b^3 + 12*A*B*a^5*b^2)))/a^4)*(A*a^2*1i + A*b^2*2i - B*a*b*2i)*1i)/(2*a^3) - ((((((8*(2*A*a^10 + 4*A*a^6*b^4 - 6*A*a^7*b^3 + 2*A*a^8*b^2 - 4*B*a^7*b^3 + 8*B*a^8*b^2 - 2*A*a^9*b - 4*B*a^9*b))/a^6 + (4*tan(c/2 + (d*x)/2)*(A*a^2*1i + A*b^2*2i - B*a*b*2i))*(8*a^8*b + 8*a^6*b^3 - 16*a^7*b^2))/a^7)*(A*a^2*1i + A*b^2*2i - B*a*b*2i))/(2*a^3) - (8*tan(c/2 + (d*x)/2)*(A^2*a^7 - 8*A^2*b^7 + 16*A^2*a*b^6 - 3*A^2*a^6*b - 16*A^2*a^2*b^5 + 16*A^2*a^3*b^4 - 13*A^2*a^4*b^3 + 7*A^2*a^5*b^2 - 8*B^2*a^2*b^5 + 16*B^2*a^3*b^4 - 12*B^2*a^4*b^3 + 4*B^2*a^5*b^2 + 16*A*B*a*b^6 - 4*A*B*a^6*b -

$$\begin{aligned}
& (32*A*B*a^2*b^5 + 28*A*B*a^3*b^4 - 20*A*B*a^4*b^3 + 12*A*B*a^5*b^2))/a^4)*(A \\
& *a^2*1i + A*b^2*2i - B*a*b*2i)*1i)/(2*a^3))/((16*(4*A^3*b^8 - 6*A^3*a*b^7 + \\
& 6*A^3*a^2*b^6 - 5*A^3*a^3*b^5 + 2*A^3*a^4*b^4 - A^3*a^5*b^3 - 4*B^3*a^3*b^ \\
& 5 + 4*B^3*a^4*b^4 - 12*A^2*B*a*b^7 + 12*A*B^2*a^2*b^6 - 14*A*B^2*a^3*b^5 + \\
& 6*A*B^2*a^4*b^4 - 4*A*B^2*a^5*b^3 + 16*A^2*B*a^2*b^6 - 12*A^2*B*a^3*b^5 + 9 \\
& *A^2*B*a^4*b^4 - 2*A^2*B*a^5*b^3 + A^2*B*a^6*b^2))/a^6 + (((((8*(2*A*a^10 + \\
& 4*A*a^6*b^4 - 6*A*a^7*b^3 + 2*A*a^8*b^2 - 4*B*a^7*b^3 + 8*B*a^8*b^2 - 2*A* \\
& a^9*b - 4*B*a^9*b))/a^6 - (4*tan(c/2 + (d*x)/2)*(A*a^2*1i + A*b^2*2i - B*a* \\
& b*2i))*(8*a^8*b + 8*a^6*b^3 - 16*a^7*b^2))/a^7)*(A*a^2*1i + A*b^2*2i - B*a*b \\
& *2i))/(2*a^3) + (8*tan(c/2 + (d*x)/2)*(A^2*a^7 - 8*A^2*b^7 + 16*A^2*a*b^6 - \\
& 3*A^2*a^6*b - 16*A^2*a^2*b^5 + 16*A^2*a^3*b^4 - 13*A^2*a^4*b^3 + 7*A^2*a^5 \\
& *b^2 - 8*B^2*a^2*b^5 + 16*B^2*a^3*b^4 - 12*B^2*a^4*b^3 + 4*B^2*a^5*b^2 + 16 \\
& *A*B*a*b^6 - 4*A*B*a^6*b - 32*A*B*a^2*b^5 + 28*A*B*a^3*b^4 - 20*A*B*a^4*b^3 \\
& + 12*A*B*a^5*b^2))/a^4)*(A*a^2*1i + A*b^2*2i - B*a*b*2i))/(2*a^3) + (((((8 \\
& *(2*A*a^10 + 4*A*a^6*b^4 - 6*A*a^7*b^3 + 2*A*a^8*b^2 - 4*B*a^7*b^3 + 8*B*a^ \\
& 8*b^2 - 2*A*a^9*b - 4*B*a^9*b))/a^6 + (4*tan(c/2 + (d*x)/2)*(A*a^2*1i + A*b \\
& ^2*2i - B*a*b*2i))*(8*a^8*b + 8*a^6*b^3 - 16*a^7*b^2))/a^7)*(A*a^2*1i + A*b^ \\
& ^2*2i - B*a*b*2i))/(2*a^3) - (8*tan(c/2 + (d*x)/2)*(A^2*a^7 - 8*A^2*b^7 + 16 \\
& *A^2*a*b^6 - 3*A^2*a^6*b - 16*A^2*a^2*b^5 + 16*A^2*a^3*b^4 - 13*A^2*a^4*b^3 \\
& + 7*A^2*a^5*b^2 - 8*B^2*a^2*b^5 + 16*B^2*a^3*b^4 - 12*B^2*a^4*b^3 + 4*B^2* \\
& a^5*b^2 + 16*A*B*a*b^6 - 4*A*B*a^6*b - 32*A*B*a^2*b^5 + 28*A*B*a^3*b^4 - 20 \\
& *A*B*a^4*b^3 + 12*A*B*a^5*b^2))/a^4)*(A*a^2*1i + A*b^2*2i - B*a*b*2i))/(2*a \\
& ^3)))*(A*a^2*1i + A*b^2*2i - B*a*b*2i)*1i)/(a^3*d) - (b^2*atan(((b^2*((a + \\
& b)*(a - b))^(1/2))*((8*tan(c/2 + (d*x)/2)*(A^2*a^7 - 8*A^2*b^7 + 16*A^2*a*b^ \\
& 6 - 3*A^2*a^6*b - 16*A^2*a^2*b^5 + 16*A^2*a^3*b^4 - 13*A^2*a^4*b^3 + 7*A^2* \\
& a^5*b^2 - 8*B^2*a^2*b^5 + 16*B^2*a^3*b^4 - 12*B^2*a^4*b^3 + 4*B^2*a^5*b^2 + \\
& 16*A*B*a*b^6 - 4*A*B*a^6*b - 32*A*B*a^2*b^5 + 28*A*B*a^3*b^4 - 20*A*B*a^4* \\
& b^3 + 12*A*B*a^5*b^2))/a^4 + (b^2*((a + b)*(a - b))^(1/2)*(A*b - B*a))*((8*(\\
& 2*A*a^10 + 4*A*a^6*b^4 - 6*A*a^7*b^3 + 2*A*a^8*b^2 - 4*B*a^7*b^3 + 8*B*a^8* \\
& b^2 - 2*A*a^9*b - 4*B*a^9*b))/a^6 - (8*b^2*tan(c/2 + (d*x)/2))*((a + b)*(a - \\
& b))^(1/2)*(A*b - B*a)*(8*a^8*b + 8*a^6*b^3 - 16*a^7*b^2))/(a^4*(a^5 - a^3* \\
& b^2))))/(a^5 - a^3*b^2))*(A*b - B*a)*1i)/(a^5 - a^3*b^2) + (b^2*((a + b)*(a \\
& - b))^(1/2))*((8*tan(c/2 + (d*x)/2)*(A^2*a^7 - 8*A^2*b^7 + 16*A^2*a*b^6 - 3 \\
& *A^2*a^6*b - 16*A^2*a^2*b^5 + 16*A^2*a^3*b^4 - 13*A^2*a^4*b^3 + 7*A^2*a^5*b \\
& ^2 - 8*B^2*a^2*b^5 + 16*B^2*a^3*b^4 - 12*B^2*a^4*b^3 + 4*B^2*a^5*b^2 + 16*A \\
& *B*a*b^6 - 4*A*B*a^6*b - 32*A*B*a^2*b^5 + 28*A*B*a^3*b^4 - 20*A*B*a^4*b^3 + \\
& 12*A*B*a^5*b^2))/a^4 - (b^2*((a + b)*(a - b))^(1/2)*(A*b - B*a))*((8*(2*A*a \\
& ^10 + 4*A*a^6*b^4 - 6*A*a^7*b^3 + 2*A*a^8*b^2 - 4*B*a^7*b^3 + 8*B*a^8*b^2 - \\
& 2*A*a^9*b - 4*B*a^9*b))/a^6 + (8*b^2*tan(c/2 + (d*x)/2))*((a + b)*(a - b)) \\
& ^{1/2}*(A*b - B*a)*(8*a^8*b + 8*a^6*b^3 - 16*a^7*b^2))/(a^4*(a^5 - a^3*b^2)) \\
&))/(a^5 - a^3*b^2))*(A*b - B*a)*1i)/(a^5 - a^3*b^2))/((16*(4*A^3*b^8 - 6*A^ \\
& 3*a*b^7 + 6*A^3*a^2*b^6 - 5*A^3*a^3*b^5 + 2*A^3*a^4*b^4 - A^3*a^5*b^3 - 4*B \\
& ^3*a^3*b^5 + 4*B^3*a^4*b^4 - 12*A^2*B*a*b^7 + 12*A*B^2*a^2*b^6 - 14*A*B^2*a \\
& ^3*b^5 + 6*A*B^2*a^4*b^4 - 4*A*B^2*a^5*b^3 + 16*A^2*B*a^2*b^6 - 12*A^2*B*a^ \\
& 3*b^5 + 9*A^2*B*a^4*b^4 - 2*A^2*B*a^5*b^3 + A^2*B*a^6*b^2))/a^6 + (b^2*((a
\end{aligned}$$

$$\begin{aligned}
& + b)(a - b))^{(1/2)} * ((8 * \tan(c/2 + (d*x)/2) * (A^2 * a^7 - 8 * A^2 * b^7 + 16 * A^2 * a * \\
& b^6 - 3 * A^2 * a^6 * b - 16 * A^2 * a^2 * b^5 + 16 * A^2 * a^3 * b^4 - 13 * A^2 * a^4 * b^3 + 7 * A^ \\
& 2 * a^5 * b^2 - 8 * B^2 * a^2 * b^5 + 16 * B^2 * a^3 * b^4 - 12 * B^2 * a^4 * b^3 + 4 * B^2 * a^5 * b^2 \\
& + 16 * A * B * a * b^6 - 4 * A * B * a^6 * b - 32 * A * B * a^2 * b^5 + 28 * A * B * a^3 * b^4 - 20 * A * B * a^ \\
& 4 * b^3 + 12 * A * B * a^5 * b^2)) / a^4 + (b^2 * ((a + b) * (a - b))^{(1/2)} * (A * b - B * a) * ((8 \\
& * (2 * A * a^{10} + 4 * A * a^6 * b^4 - 6 * A * a^7 * b^3 + 2 * A * a^8 * b^2 - 4 * B * a^7 * b^3 + 8 * B * a^ \\
& 8 * b^2 - 2 * A * a^9 * b - 4 * B * a^9 * b)) / a^6 - (8 * b^2 * \tan(c/2 + (d*x)/2) * ((a + b) * (a \\
& - b))^{(1/2)} * (A * b - B * a) * (8 * a^8 * b + 8 * a^6 * b^3 - 16 * a^7 * b^2)) / (a^4 * (a^5 - a^ \\
& 3 * b^2))) / (a^5 - a^3 * b^2)) * (A * b - B * a) / (a^5 - a^3 * b^2) - (b^2 * ((a + b) * (a \\
& - b))^{(1/2)} * ((8 * \tan(c/2 + (d*x)/2) * (A^2 * a^7 - 8 * A^2 * b^7 + 16 * A^2 * a * b^6 - 3 * \\
& A^2 * a^6 * b - 16 * A^2 * a^2 * b^5 + 16 * A^2 * a^3 * b^4 - 13 * A^2 * a^4 * b^3 + 7 * A^2 * a^5 * b^ \\
& 2 - 8 * B^2 * a^2 * b^5 + 16 * B^2 * a^3 * b^4 - 12 * B^2 * a^4 * b^3 + 4 * B^2 * a^5 * b^2 + 16 * A * \\
& B * a * b^6 - 4 * A * B * a^6 * b - 32 * A * B * a^2 * b^5 + 28 * A * B * a^3 * b^4 - 20 * A * B * a^4 * b^3 + \\
& 12 * A * B * a^5 * b^2)) / a^4 - (b^2 * ((a + b) * (a - b))^{(1/2)} * (A * b - B * a) * ((8 * (2 * A * a^ \\
& 10 + 4 * A * a^6 * b^4 - 6 * A * a^7 * b^3 + 2 * A * a^8 * b^2 - 4 * B * a^7 * b^3 + 8 * B * a^8 * b^2 - \\
& 2 * A * a^9 * b - 4 * B * a^9 * b)) / a^6 + (8 * b^2 * \tan(c/2 + (d*x)/2) * ((a + b) * (a - b))^{(\\
& 1/2)} * (A * b - B * a) * (8 * a^8 * b + 8 * a^6 * b^3 - 16 * a^7 * b^2)) / (a^4 * (a^5 - a^3 * b^2))) \\
&) / (a^5 - a^3 * b^2)) * (A * b - B * a) / (a^5 - a^3 * b^2)) * ((a + b) * (a - b))^{(1/2)} * (\\
& A * b - B * a) * 2i) / (d * (a^5 - a^3 * b^2))
\end{aligned}$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \cos^2(c + dx)}{a + b \sec(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x)

[Out] Integral((A + B*sec(c + d*x))*cos(c + d*x)**2/(a + b*sec(c + d*x)), x)

$$3.318 \quad \int \frac{\cos^3(c+dx)(A+B \sec(c+dx))}{a+b \sec(c+dx)} dx$$

Optimal. Leaf size=178

$$\frac{2b^3(Ab - aB) \tanh^{-1}\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4 d \sqrt{a-b} \sqrt{a+b}} - \frac{(Ab - aB) \sin(c+dx) \cos(c+dx)}{2a^2 d} - \frac{x(a^2 + 2b^2)(Ab - aB)}{2a^4} + \frac{(2a^2 A - 3abB + 3Ab^2) \sin(c+dx)}{3a^3 d}$$

[Out] $-1/2*(a^2+2*b^2)*(A*b-B*a)*x/a^4+1/3*(2*A*a^2+3*A*b^2-3*B*a*b)*\sin(d*x+c)/a^3/d-1/2*(A*b-B*a)*\cos(d*x+c)*\sin(d*x+c)/a^2/d+1/3*A*\cos(d*x+c)^2*\sin(d*x+c)/a/d+2*b^3*(A*b-B*a)*\operatorname{arctanh}((a-b)^{(1/2)}*\tan(1/2*d*x+1/2*c)/(a+b)^{(1/2)})/a^4/d/(a-b)^{(1/2)}/(a+b)^{(1/2)}$

Rubi [A] time = 0.64, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4034, 4104, 3919, 3831, 2659, 208}

$$\frac{(2a^2 A - 3abB + 3Ab^2) \sin(c+dx)}{3a^3 d} + \frac{2b^3(Ab - aB) \tanh^{-1}\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4 d \sqrt{a-b} \sqrt{a+b}} - \frac{x(a^2 + 2b^2)(Ab - aB)}{2a^4} - \frac{(Ab - aB)}{2a^4}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x]),x]

[Out] $-((a^2 + 2*b^2)*(A*b - a*B)*x)/(2*a^4) + (2*b^3*(A*b - a*B)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a - b]*\operatorname{Tan}[(c + d*x)/2])/(\operatorname{Sqrt}[a + b])])/(a^4*\operatorname{Sqrt}[a - b]*\operatorname{Sqrt}[a + b]*d) + ((2*a^2*A + 3*A*b^2 - 3*a*b*B)*\operatorname{Sin}[c + d*x])/(3*a^3*d) - ((A*b - a*B)*\operatorname{Cos}[c + d*x]*\operatorname{Sin}[c + d*x])/(2*a^2*d) + (A*\operatorname{Cos}[c + d*x]^2*\operatorname{Sin}[c + d*x])/(3*a*d)$

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 2659

Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol]
:> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol]
:> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b*c - a*d, 0]
```

Rule 4034

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^m*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol]
:> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + A*a*(n + 1)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x]
&& NeQ[A*b - a*B, 0]
&& NeQ[a^2 - b^2, 0]
&& LeQ[n, -1]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^m, x_Symbol]
:> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x]
&& NeQ[a^2 - b^2, 0]
&& LeQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^3(c+dx)(A+B\sec(c+dx))}{a+b\sec(c+dx)} dx &= \frac{A\cos^2(c+dx)\sin(c+dx)}{3ad} - \frac{\int \frac{\cos^2(c+dx)(3(Ab-aB)-2aA\sec(c+dx)-2Ab\sec^2(c+dx))}{a+b\sec(c+dx)} dx}{3a} \\
&= -\frac{(Ab-aB)\cos(c+dx)\sin(c+dx)}{2a^2d} + \frac{A\cos^2(c+dx)\sin(c+dx)}{3ad} + \int \frac{\cos^2(c+dx)(3(Ab-aB)-2aA\sec(c+dx)-2Ab\sec^2(c+dx))}{3a(a+b\sec(c+dx))} dx \\
&= \frac{(2a^2A+3Ab^2-3abB)\sin(c+dx)}{3a^3d} - \frac{(Ab-aB)\cos(c+dx)\sin(c+dx)}{2a^2d} \\
&= -\frac{(a^2+2b^2)(Ab-aB)x}{2a^4} + \frac{(2a^2A+3Ab^2-3abB)\sin(c+dx)}{3a^3d} - \frac{(Ab-aB)\cos(c+dx)\sin(c+dx)}{2a^2d} \\
&= -\frac{(a^2+2b^2)(Ab-aB)x}{2a^4} + \frac{(2a^2A+3Ab^2-3abB)\sin(c+dx)}{3a^3d} - \frac{(Ab-aB)\cos(c+dx)\sin(c+dx)}{2a^2d} \\
&= -\frac{(a^2+2b^2)(Ab-aB)x}{2a^4} + \frac{(2a^2A+3Ab^2-3abB)\sin(c+dx)}{3a^3d} - \frac{(Ab-aB)\cos(c+dx)\sin(c+dx)}{2a^2d} \\
&= -\frac{(a^2+2b^2)(Ab-aB)x}{2a^4} + \frac{2b^3(Ab-aB)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4\sqrt{a-b}\sqrt{a+b}d} + \frac{(2a^2A+3Ab^2-3abB)\sin(c+dx)}{3a^3d} - \frac{(Ab-aB)\cos(c+dx)\sin(c+dx)}{2a^2d}
\end{aligned}$$

Mathematica [A] time = 0.54, size = 152, normalized size = 0.85

$$\frac{a^3A\sin(3(c+dx)) + 6(a^2+2b^2)(c+dx)(aB-Ab) + 3a(3a^2A-4abB+4Ab^2)\sin(c+dx) - \frac{24b^3(Ab-aB)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a^2-b^2}}}{12a^4d}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x]),x]

[Out] (6*(a^2 + 2*b^2)*(-(A*b) + a*B)*(c + d*x) - (24*b^3*(A*b - a*B)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] + 3*a*(3*a^2*A + 4*A*b^2 - 4*a*b*B)*Sin[c + d*x] + 3*a^2*(-(A*b) + a*B)*Sin[2*(c + d*x)] + a^3*A*Ssin[3*(c + d*x)]/(12*a^4*d)

fricas [A] time = 0.56, size = 547, normalized size = 3.07

$$\left[\frac{3(Ba^5 - Aa^4b + Ba^3b^2 - Aa^2b^3 - 2Bab^4 + 2Ab^5)dx - 3(Bab^3 - Ab^4)\sqrt{a^2 - b^2} \log\left(\frac{2ab \cos(dx+c) - (a^2 - 2b^2)\cos(dx+c)}{a^2 \cos(dx+c)}\right)}{\dots} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="fricas")

[Out] [1/6*(3*(B*a^5 - A*a^4*b + B*a^3*b^2 - A*a^2*b^3 - 2*B*a*b^4 + 2*A*b^5)*d*x - 3*(B*a*b^3 - A*b^4)*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + (4*A*a^5 - 6*B*a^4*b + 2*A*a^3*b^2 + 6*B*a^2*b^3 - 6*A*a*b^4 + 2*(A*a^5 - A*a^3*b^2)*cos(d*x + c)^2 + 3*(B*a^5 - A*a^4*b - B*a^3*b^2 + A*a^2*b^3)*cos(d*x + c))*sin(d*x + c))/((a^6 - a^4*b^2)*d), 1/6*(3*(B*a^5 - A*a^4*b + B*a^3*b^2 - A*a^2*b^3 - 2*B*a*b^4 + 2*A*b^5)*d*x - 6*(B*a*b^3 - A*b^4)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) + (4*A*a^5 - 6*B*a^4*b + 2*A*a^3*b^2 + 6*B*a^2*b^3 - 6*A*a*b^4 + 2*(A*a^5 - A*a^3*b^2)*cos(d*x + c)^2 + 3*(B*a^5 - A*a^4*b - B*a^3*b^2 + A*a^2*b^3)*cos(d*x + c))*sin(d*x + c))/((a^6 - a^4*b^2)*d)]

giac [B] time = 0.31, size = 360, normalized size = 2.02

$$\frac{3(Ba^3 - Aa^2b + 2Bab^2 - 2Ab^3)(dx+c)}{a^4} - \frac{12(Bab^3 - Ab^4)\left(\pi\left[\frac{dx+c}{2\pi} + \frac{1}{2}\right] \operatorname{sgn}(-2a+2b) + \arctan\left(-\frac{a \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - b \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\sqrt{-a^2+b^2}a^4} + \frac{2\left(6Aa^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="giac")

[Out] 1/6*(3*(B*a^3 - A*a^2*b + 2*B*a*b^2 - 2*A*b^3)*(d*x + c)/a^4 - 12*(B*a*b^3 - A*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/sqrt(-a^2 + b^2)*a^4 + 2*(6*A*a^2*tan(1/2*d*x + 1/2*c)^5 - 3*B*a^2*tan(1/2*d*x + 1/2*c)^5 + 3*A*a*b*tan(1/2*d*x + 1/2*c)^5 - 6*B*a*b*tan(1/2*d*x + 1/2*c)^5 + 6*A*b^2*tan(1/2*d*x + 1/2*c)^5 + 4*A*a^2*tan(1/2*d*x + 1/2*c)^3 - 12*B*a*b*tan(1/2*d*x + 1/2*c)^3 + 12*A*b^2*tan(1/2*d*x + 1/2*c)^3 + 6*A*a^2*tan(1/2*d*x + 1/2*c) + 3*B*a^2*tan(1/2*d*x + 1/2*c) - 3*A*a*b*tan(1/2*d*x + 1/2*c) -

$6*B*a*b*\tan(1/2*d*x + 1/2*c) + 6*A*b^2*\tan(1/2*d*x + 1/2*c))/((\tan(1/2*d*x + 1/2*c)^2 + 1)^3*a^3))/d$

maple [B] time = 1.22, size = 641, normalized size = 3.60

$$\frac{2b^4 \operatorname{arctanh}\left(\frac{\tan\left(\frac{dx}{2} + \frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right) A}{d a^4 \sqrt{(a-b)(a+b)}} - \frac{2b^3 \operatorname{arctanh}\left(\frac{\tan\left(\frac{dx}{2} + \frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right) B}{d a^3 \sqrt{(a-b)(a+b)}} + \frac{2\left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right) A}{d a \left(1 + \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^3} + \frac{\left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right) A b}{d a^2 \left(1 + \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x)`

[Out] $2/d*b^4/a^4/((a-b)*(a+b))^{(1/2)*\operatorname{arctanh}(\tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^{(1/2)})} * A - 2/d*b^3/a^3/((a-b)*(a+b))^{(1/2)*\operatorname{arctanh}(\tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^{(1/2)})} * B + 2/d/a/(1+\tan(1/2*d*x+1/2*c)^2)^3 * \tan(1/2*d*x+1/2*c)^5 * A + 1/d/a^2/(1+\tan(1/2*d*x+1/2*c)^2)^3 * \tan(1/2*d*x+1/2*c)^5 * A * b + 2/d/a^3/(1+\tan(1/2*d*x+1/2*c)^2)^3 * \tan(1/2*d*x+1/2*c)^5 * A * b^2 - 1/d/a/(1+\tan(1/2*d*x+1/2*c)^2)^3 * \tan(1/2*d*x+1/2*c)^5 * B - 2/d/a^2/(1+\tan(1/2*d*x+1/2*c)^2)^3 * \tan(1/2*d*x+1/2*c)^5 * b * B + 4/3/d/a/(1+\tan(1/2*d*x+1/2*c)^2)^3 * \tan(1/2*d*x+1/2*c)^3 * A + 4/d/a^3/(1+\tan(1/2*d*x+1/2*c)^2)^3 * \tan(1/2*d*x+1/2*c)^3 * A * b^2 - 4/d/a^2/(1+\tan(1/2*d*x+1/2*c)^2)^3 * \tan(1/2*d*x+1/2*c)^3 * b * B + 2/d/a/(1+\tan(1/2*d*x+1/2*c)^2)^3 * \tan(1/2*d*x+1/2*c)^2 * A + 2/d/a^3/(1+\tan(1/2*d*x+1/2*c)^2)^3 * \tan(1/2*d*x+1/2*c)^2 * A * b^2 - 2/d/a^2/(1+\tan(1/2*d*x+1/2*c)^2)^3 * \tan(1/2*d*x+1/2*c)^2 * b * B - 1/d/a^2/(1+\tan(1/2*d*x+1/2*c)^2)^3 * \tan(1/2*d*x+1/2*c)^2 * A * b + 1/d/a/(1+\tan(1/2*d*x+1/2*c)^2)^3 * \tan(1/2*d*x+1/2*c)^2 * B - 1/d/a^2 * A * \operatorname{arctan}(\tan(1/2*d*x+1/2*c)) * b - 2/d/a^4 * \operatorname{arctan}(\tan(1/2*d*x+1/2*c)) * A * b^3 + 1/a/d * \operatorname{arctan}(\tan(1/2*d*x+1/2*c)) * B + 2/d/a^3 * \operatorname{arctan}(\tan(1/2*d*x+1/2*c)) * B * b^2$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for more details) Is 4*a^2-4*b^2 positive or negative?

mupad [B] time = 6.90, size = 4572, normalized size = 25.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((\cos(c + d*x))^3*(A + B/\cos(c + d*x)))/(a + b/\cos(c + d*x)),x)$

[Out] $((\tan(c/2 + (d*x)/2)*(2*A*a^2 + 2*A*b^2 + B*a^2 - A*a*b - 2*B*a*b))/a^3 + (\tan(c/2 + (d*x)/2)^5*(2*A*a^2 + 2*A*b^2 - B*a^2 + A*a*b - 2*B*a*b))/a^3 + (4*\tan(c/2 + (d*x)/2)^3*(A*a^2 + 3*A*b^2 - 3*B*a*b))/(3*a^3))/(d*(3*\tan(c/2 + (d*x)/2)^2 + 3*\tan(c/2 + (d*x)/2)^4 + \tan(c/2 + (d*x)/2)^6 + 1)) - (\text{atan}(((a^2 + 2*b^2)*(A*b - B*a)*((8*\tan(c/2 + (d*x)/2)*(8*A^2*b^9 - B^2*a^9 - 16*A^2*a*b^8 + 3*B^2*a^8*b + 16*A^2*a^2*b^7 - 16*A^2*a^3*b^6 + 13*A^2*a^4*b^5 - 7*A^2*a^5*b^4 + 3*A^2*a^6*b^3 - A^2*a^7*b^2 + 8*B^2*a^2*b^7 - 16*B^2*a^3*b^6 + 16*B^2*a^4*b^5 - 16*B^2*a^5*b^4 + 13*B^2*a^6*b^3 - 7*B^2*a^7*b^2 - 16*A*B*a*b^8 + 2*A*B*a^8*b + 32*A*B*a^2*b^7 - 32*A*B*a^3*b^6 + 32*A*B*a^4*b^5 - 26*A*B*a^5*b^4 + 14*A*B*a^6*b^3 - 6*A*B*a^7*b^2)))/a^6 - ((a^2 + 2*b^2)*(A*b - B*a)*((8*(2*B*a^13 - 4*A*a^8*b^5 + 6*A*a^9*b^4 - 2*A*a^10*b^3 + 2*A*a^11*b^2 + 4*B*a^9*b^4 - 6*B*a^10*b^3 + 2*B*a^11*b^2 - 2*A*a^12*b - 2*B*a^12*b))/a^9 - (\tan(c/2 + (d*x)/2)*(a^2 + 2*b^2)*(A*b - B*a)*(8*a^10*b + 8*a^8*b^3 - 16*a^9*b^2)*4i)/a^10)*1i)/(2*a^4)))/(2*a^4) + ((a^2 + 2*b^2)*(A*b - B*a)*((8*\tan(c/2 + (d*x)/2)*(8*A^2*b^9 - B^2*a^9 - 16*A^2*a*b^8 + 3*B^2*a^8*b + 16*A^2*a^2*b^7 - 16*A^2*a^3*b^6 + 13*A^2*a^4*b^5 - 7*A^2*a^5*b^4 + 3*A^2*a^6*b^3 - A^2*a^7*b^2 + 8*B^2*a^2*b^7 - 16*B^2*a^3*b^6 + 16*B^2*a^4*b^5 - 16*B^2*a^5*b^4 + 13*B^2*a^6*b^3 - 7*B^2*a^7*b^2 - 16*A*B*a*b^8 + 2*A*B*a^8*b + 32*A*B*a^2*b^7 - 32*A*B*a^3*b^6 + 32*A*B*a^4*b^5 - 26*A*B*a^5*b^4 + 14*A*B*a^6*b^3 - 6*A*B*a^7*b^2))/a^6 + ((a^2 + 2*b^2)*(A*b - B*a)*((8*(2*B*a^13 - 4*A*a^8*b^5 + 6*A*a^9*b^4 - 2*A*a^10*b^3 + 2*A*a^11*b^2 + 4*B*a^9*b^4 - 6*B*a^10*b^3 + 2*B*a^11*b^2 - 2*A*a^12*b - 2*B*a^12*b))/a^9 + (\tan(c/2 + (d*x)/2)*(a^2 + 2*b^2)*(A*b - B*a)*(8*a^10*b + 8*a^8*b^3 - 16*a^9*b^2)*4i)/a^10)*1i)/(2*a^4)))/(2*a^4))/((16*(4*A^3*b^11 - 6*A^3*a*b^10 + 6*A^3*a^2*b^9 - 5*A^3*a^3*b^8 + 2*A^3*a^4*b^7 - A^3*a^5*b^6 - 4*B^3*a^3*b^8 + 6*B^3*a^4*b^7 - 6*B^3*a^5*b^6 + 5*B^3*a^6*b^5 - 2*B^3*a^7*b^4 + B^3*a^8*b^3 - 12*A^2*B*a*b^10 + 12*A*B^2*a^2*b^9 - 18*A*B^2*a^3*b^8 + 18*A*B^2*a^4*b^7 - 15*A*B^2*a^5*b^6 + 6*A*B^2*a^6*b^5 - 3*A*B^2*a^7*b^4 + 18*A^2*B*a^2*b^9 - 18*A^2*B*a^3*b^8 + 15*A^2*B*a^4*b^7 - 6*A^2*B*a^5*b^6 + 3*A^2*B*a^6*b^5))/a^9 + ((a^2 + 2*b^2)*(A*b - B*a)*((8*\tan(c/2 + (d*x)/2)*(8*A^2*b^9 - B^2*a^9 - 16*A^2*a*b^8 + 3*B^2*a^8*b + 16*A^2*a^2*b^7 - 16*A^2*a^3*b^6 + 13*A^2*a^4*b^5 - 7*A^2*a^5*b^4 + 3*A^2*a^6*b^3 - A^2*a^7*b^2 + 8*B^2*a^2*b^7 - 16*B^2*a^3*b^6 + 16*B^2*a^4*b^5 - 16*B^2*a^5*b^4 + 13*B^2*a^6*b^3 - 7*B^2*a^7*b^2 - 16*A*B*a*b^8 + 2*A*B*a^8*b + 32*A*B*a^2*b^7 - 32*A*B*a^3*b^6 + 32*A*B*a^4*b^5 - 26*A*B*a^5*b^4 + 14*A*B*a^6*b^3 - 6*A*B*a^7*b^2))/a^6 - ((a^2 + 2*b^2)*(A*b - B*a)*((8*(2*B*a^13 - 4*A*a^8*b^5 + 6*A*a^9*b^4 - 2*A*a^10*b^3 + 2*A*a^11*b^2 + 4*B*a^9*b^4 - 6*B*a^10*b^3 + 2*B*a^11*b^2 - 2*A*a^12*b - 2*B*a^12*b))/a^9 - (\tan(c/2 + (d*x)/2)*(a^2 + 2*b^2)*(A*b - B*a)*(8*a^10*b + 8*a^8*b^3 - 16*a^9*b^2)*4i)/a^10)*1i)/(2*a^4)))/(2*a^4) - ((a^2 + 2*b^2)*(A*b - B*a)*((8*\tan(c/2 + (d*x)/2)*(8*A^2*b^9 - B^2*a^9 - 16*A^2*a*b^8 + 3*B^2*a^8*b + 16*A^2*a^2*b^7 - 16*A^2*a^3*b^6 + 13*A^2*a^4*b^5 - 7*A^2*a^5*b^4 + 3*A^2*a^6*b^3 - A^2*a^7*b^2 + 8*B^2*a^2*b^7 - 16*B^2*a^3*b^6 + 13*A^2*a^4*b^5 - 7*A^2*a^5*b^4 +$

$$\begin{aligned}
& 3A^2a^6b^3 - A^2a^7b^2 + 8B^2a^2b^7 - 16B^2a^3b^6 + 16B^2a^4b^5 - 16B^2a^5b^4 + 13B^2a^6b^3 - 7B^2a^7b^2 - 16ABa^8b + 2ABa^8b + 32ABa^2b^7 - 32ABa^3b^6 + 32ABa^4b^5 - 26ABa^5b^4 + 14ABa^6b^3 - 6ABa^7b^2)/a^6 + ((a^2 + 2b^2)(Ab - Ba)((8(2B^2a^{13} - 4A^2a^8b^5 + 6A^2a^9b^4 - 2A^2a^{10}b^3 + 2A^2a^{11}b^2 + 4B^2a^9b^4 - 6B^2a^{10}b^3 + 2B^2a^{11}b^2 - 2A^2a^{12}b - 2B^2a^{12}b))/a^9 + (\tan(c/2 + (d*x)/2)(a^2 + 2b^2)(Ab - Ba)(8a^{10}b + 8a^8b^3 - 16a^9b^2)*4i)/a^{10}*i)/(2a^4))*i)/(2a^4)))(a^2 + 2b^2)(Ab - Ba))/(a^4*d) - (b^3*atan(((b^3*((a + b)(a - b))^{1/2})(Ab - Ba)((8\tan(c/2 + (d*x)/2)*(8A^2b^9 - B^2a^9 - 16A^2a^8b + 3B^2a^8b + 16A^2a^2b^7 - 16A^2a^3b^6 + 13A^2a^4b^5 - 7A^2a^5b^4 + 3A^2a^6b^3 - A^2a^7b^2 + 8B^2a^2b^7 - 16B^2a^3b^6 + 16B^2a^4b^5 - 16B^2a^5b^4 + 13B^2a^6b^3 - 7B^2a^7b^2 - 16ABa^8b + 2ABa^8b + 32ABa^2b^7 - 32ABa^3b^6 + 32ABa^4b^5 - 26ABa^5b^4 + 14ABa^6b^3 - 6ABa^7b^2)))/a^6 + (b^3*((a + b)(a - b))^{1/2})((8(2B^2a^{13} - 4A^2a^8b^5 + 6A^2a^9b^4 - 2A^2a^{10}b^3 + 2A^2a^{11}b^2 + 4B^2a^9b^4 - 6B^2a^{10}b^3 + 2B^2a^{11}b^2 - 2A^2a^{12}b - 2B^2a^{12}b))/a^9 + (8b^3\tan(c/2 + (d*x)/2)((a + b)(a - b))^{1/2})(Ab - Ba)(8a^{10}b + 8a^8b^3 - 16a^9b^2))/(a^6(a^6 - a^4b^2)))(Ab - Ba))/(a^6 - a^4b^2))*i)/(a^6 - a^4b^2) + (b^3*((a + b)(a - b))^{1/2})(Ab - Ba)((8\tan(c/2 + (d*x)/2)*(8A^2b^9 - B^2a^9 - 16A^2a^8b + 3B^2a^8b + 16A^2a^2b^7 - 16A^2a^3b^6 + 13A^2a^4b^5 - 7A^2a^5b^4 + 3A^2a^6b^3 - A^2a^7b^2 + 8B^2a^2b^7 - 16B^2a^3b^6 + 16B^2a^4b^5 - 16B^2a^5b^4 + 13B^2a^6b^3 - 7B^2a^7b^2 - 16ABa^8b + 2ABa^8b + 32ABa^2b^7 - 32ABa^3b^6 + 32ABa^4b^5 - 26ABa^5b^4 + 14ABa^6b^3 - 6ABa^7b^2))/a^6 - (b^3*((a + b)(a - b))^{1/2})((8(2B^2a^{13} - 4A^2a^8b^5 + 6A^2a^9b^4 - 2A^2a^{10}b^3 + 2A^2a^{11}b^2 + 4B^2a^9b^4 - 6B^2a^{10}b^3 + 2B^2a^{11}b^2 - 2A^2a^{12}b - 2B^2a^{12}b))/a^9 - (8b^3\tan(c/2 + (d*x)/2)((a + b)(a - b))^{1/2})(Ab - Ba)(8a^{10}b + 8a^8b^3 - 16a^9b^2))/(a^6(a^6 - a^4b^2)))(Ab - Ba))/(a^6 - a^4b^2))*i)/(a^6 - a^4b^2))/((16(4A^3b^{11} - 6A^3a^2b^{10} + 6A^3a^2b^9 - 5A^3a^3b^8 + 2A^3a^4b^7 - A^3a^5b^6 - 4B^3a^3b^8 + 6B^3a^4b^7 - 6B^3a^5b^6 + 5B^3a^6b^5 - 2B^3a^7b^4 + B^3a^8b^3 - 12A^2B^2a^2b^9 - 18AB^2a^3b^8 + 18AB^2a^4b^7 - 15AB^2a^5b^6 + 6AB^2a^6b^5 - 3AB^2a^7b^4 + 18A^2B^2a^2b^9 - 18A^2B^2a^3b^8 + 15A^2B^2a^4b^7 - 6A^2B^2a^5b^6 + 3A^2B^2a^6b^5)))/a^9 - (b^3*((a + b)(a - b))^{1/2})(Ab - Ba)((8\tan(c/2 + (d*x)/2)*(8A^2b^9 - B^2a^9 - 16A^2a^8b + 3B^2a^8b + 16A^2a^2b^7 - 16A^2a^3b^6 + 13A^2a^4b^5 - 7A^2a^5b^4 + 3A^2a^6b^3 - A^2a^7b^2 + 8B^2a^2b^7 - 16B^2a^3b^6 + 16B^2a^4b^5 - 16B^2a^5b^4 + 13B^2a^6b^3 - 7B^2a^7b^2 - 16ABa^8b + 2ABa^8b + 32ABa^2b^7 - 32ABa^3b^6 + 32ABa^4b^5 - 26ABa^5b^4 + 14ABa^6b^3 - 6ABa^7b^2))/a^6 + (b^3*((a + b)(a - b))^{1/2})((8(2B^2a^{13} - 4A^2a^8b^5 + 6A^2a^9b^4 - 2A^2a^{10}b^3 + 2A^2a^{11}b^2 + 4B^2a^9b^4 - 6B^2a^{10}b^3 + 2B^2a^{11}b^2 - 2A^2a^{12}b - 2B^2a^{12}b))/a^9 + (8b^3\tan(c/2 + (d*x)/2)((a + b)(a - b))^{1/2})(Ab - Ba)(8a^{10}b + 8a^8b^3 - 16a^9b^2))/(a^6(a^6 - a^4b^2)
\end{aligned}$$

$$\begin{aligned} & b^2)) * (A*b - B*a)) / (a^6 - a^4*b^2)) / (a^6 - a^4*b^2) + (b^3 * ((a + b) * (a - \\ & b))^{(1/2)} * (A*b - B*a) * ((8*\tan(c/2 + (d*x)/2) * (8*A^2*b^9 - B^2*a^9 - 16*A^2* \\ & a*b^8 + 3*B^2*a^8*b + 16*A^2*a^2*b^7 - 16*A^2*a^3*b^6 + 13*A^2*a^4*b^5 - 7* \\ & A^2*a^5*b^4 + 3*A^2*a^6*b^3 - A^2*a^7*b^2 + 8*B^2*a^2*b^7 - 16*B^2*a^3*b^6 \\ & + 16*B^2*a^4*b^5 - 16*B^2*a^5*b^4 + 13*B^2*a^6*b^3 - 7*B^2*a^7*b^2 - 16*A*B \\ & *a*b^8 + 2*A*B*a^8*b + 32*A*B*a^2*b^7 - 32*A*B*a^3*b^6 + 32*A*B*a^4*b^5 - 2 \\ & 6*A*B*a^5*b^4 + 14*A*B*a^6*b^3 - 6*A*B*a^7*b^2)) / a^6 - (b^3 * ((a + b) * (a - b \\ &))^{(1/2)} * ((8*(2*B*a^13 - 4*A*a^8*b^5 + 6*A*a^9*b^4 - 2*A*a^10*b^3 + 2*A*a^1 \\ & 1*b^2 + 4*B*a^9*b^4 - 6*B*a^10*b^3 + 2*B*a^11*b^2 - 2*A*a^12*b - 2*B*a^12*b \\ &)) / a^9 - (8*b^3*\tan(c/2 + (d*x)/2) * ((a + b) * (a - b))^{(1/2)} * (A*b - B*a) * (8*a \\ & ^10*b + 8*a^8*b^3 - 16*a^9*b^2)) / (a^6 * (a^6 - a^4*b^2))) * (A*b - B*a)) / (a^6 - \\ & a^4*b^2)) / (a^6 - a^4*b^2)) * ((a + b) * (a - b))^{(1/2)} * (A*b - B*a) * 2i) / (d * (a \\ & ^6 - a^4*b^2)) \end{aligned}$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \cos^3(c + dx)}{a + b \sec(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)), x)

[Out] Integral((A + B*sec(c + d*x))*cos(c + d*x)**3/(a + b*sec(c + d*x)), x)

$$3.319 \quad \int \frac{\cos^4(c+dx)(A+B \sec(c+dx))}{a+b \sec(c+dx)} dx$$

Optimal. Leaf size=240

$$\frac{2b^4(Ab - aB) \tanh^{-1}\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5 d \sqrt{a-b} \sqrt{a+b}} - \frac{(Ab - aB) \sin(c+dx) \cos^2(c+dx)}{3a^2 d} - \frac{(2a^2 + 3b^2)(Ab - aB) \sin(c+dx)}{3a^4 d}$$

[Out] $1/8*(3*A*a^4+4*A*a^2*b^2+8*A*b^4-4*B*a^3*b-8*B*a*b^3)*x/a^5-1/3*(2*a^2+3*b^2)*(A*b-B*a)*\sin(d*x+c)/a^4/d+1/8*(3*A*a^2+4*A*b^2-4*B*a*b)*\cos(d*x+c)*\sin(d*x+c)/a^3/d-1/3*(A*b-B*a)*\cos(d*x+c)^2*\sin(d*x+c)/a^2/d+1/4*A*\cos(d*x+c)^3*\sin(d*x+c)/a/d-2*b^4*(A*b-B*a)*\operatorname{arctanh}((a-b)^{(1/2)}*\tan(1/2*d*x+1/2*c)/(a+b)^{(1/2}))/a^5/d/(a-b)^{(1/2)/(a+b)^{(1/2)}$

Rubi [A] time = 0.98, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4034, 4104, 3919, 3831, 2659, 208}

$$\frac{(2a^2 + 3b^2)(Ab - aB) \sin(c+dx)}{3a^4 d} + \frac{(3a^2 A - 4abB + 4Ab^2) \sin(c+dx) \cos(c+dx)}{8a^3 d} - \frac{2b^4(Ab - aB) \tanh^{-1}\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5 d \sqrt{a-b} \sqrt{a+b}}$$

Antiderivative was successfully verified.

[In] Int[((Cos[c + d*x]^4*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x]),x]

[Out] $((3*a^4*A + 4*a^2*A*b^2 + 8*A*b^4 - 4*a^3*b*B - 8*a*b^3*B)*x)/(8*a^5) - (2*b^4*(A*b - a*B)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a - b]*\operatorname{Tan}[(c + d*x)/2]]/\operatorname{Sqrt}[a + b])/(a^5*\operatorname{Sqrt}[a - b]*\operatorname{Sqrt}[a + b]*d) - ((2*a^2 + 3*b^2)*(A*b - a*B)*\operatorname{Sin}[c + d*x])/(3*a^4*d) + ((3*a^2*A + 4*A*b^2 - 4*a*b*B)*\operatorname{Cos}[c + d*x]*\operatorname{Sin}[c + d*x])/(8*a^3*d) - ((A*b - a*B)*\operatorname{Cos}[c + d*x]^2*\operatorname{Sin}[c + d*x])/(3*a^2*d) + (A*\operatorname{Cos}[c + d*x]^3*\operatorname{Sin}[c + d*x])/(4*a*d)$

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 2659

Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]

&& NeQ[a^2 - b^2, 0]

Rule 3831

Int[csc[(e_.) + (f_.)*(x_.)]/(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] := Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 3919

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))/(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] := Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 4034

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + A*a*(n + 1)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]

Rule 4104

Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m, x_Symbol] := Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]

Rubi steps

$$\begin{aligned}
\int \frac{\cos^4(c+dx)(A+B\sec(c+dx))}{a+b\sec(c+dx)} dx &= \frac{A\cos^3(c+dx)\sin(c+dx)}{4ad} - \int \frac{\cos^3(c+dx)(4(Ab-aB)-3aA\sec(c+dx)-3Ab\sec^2(c+dx))}{a+b\sec(c+dx)} dx \\
&= -\frac{(Ab-aB)\cos^2(c+dx)\sin(c+dx)}{3a^2d} + \frac{A\cos^3(c+dx)\sin(c+dx)}{4ad} + \int \frac{\cos^3(c+dx)(4(Ab-aB)-3aA\sec(c+dx)-3Ab\sec^2(c+dx))}{a+b\sec(c+dx)} dx \\
&= \frac{(3a^2A+4Ab^2-4abB)\cos(c+dx)\sin(c+dx)}{8a^3d} - \frac{(Ab-aB)\cos^2(c+dx)\sin(c+dx)}{3a^2d} \\
&= -\frac{(2a^2+3b^2)(Ab-aB)\sin(c+dx)}{3a^4d} + \frac{(3a^2A+4Ab^2-4abB)\cos(c+dx)\sin(c+dx)}{8a^3d} \\
&= \frac{(3a^4A+4a^2Ab^2+8Ab^4-4a^3bB-8ab^3B)x}{8a^5} - \frac{(2a^2+3b^2)(Ab-aB)\sin(c+dx)}{3a^4d} \\
&= \frac{(3a^4A+4a^2Ab^2+8Ab^4-4a^3bB-8ab^3B)x}{8a^5} - \frac{(2a^2+3b^2)(Ab-aB)\sin(c+dx)}{3a^4d} \\
&= \frac{(3a^4A+4a^2Ab^2+8Ab^4-4a^3bB-8ab^3B)x}{8a^5} - \frac{(2a^2+3b^2)(Ab-aB)\sin(c+dx)}{3a^4d} \\
&= \frac{(3a^4A+4a^2Ab^2+8Ab^4-4a^3bB-8ab^3B)x}{8a^5} - \frac{2b^4(Ab-aB)\tanh^{-1}\left(\frac{a+b\sec(c+dx)}{a-b}\right)}{a^5\sqrt{a-b}\sqrt{a+b}}
\end{aligned}$$

Mathematica [A] time = 0.67, size = 202, normalized size = 0.84

$$3a^4A\sin(4(c+dx)) + 8a^3(ab - Ab)\sin(3(c+dx)) + 24a^2(a^2A - abB + Ab^2)\sin(2(c+dx)) + 24a(3a^2 + 4b^2)(A - b)\sin(c+dx) - \frac{2b^4(Ab - aB)\operatorname{arctanh}\left(\frac{a+b\sec(c+dx)}{a-b}\right)}{a^5\sqrt{a-b}\sqrt{a+b}}$$

96a

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]^4*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x]),x]

[Out] (12*(3*a^4*A + 4*a^2*A*b^2 + 8*A*b^4 - 4*a^3*b*B - 8*a*b^3*B)*(c + d*x) + (192*b^4*(A*b - a*B)*ArcTanh[(-a + b)*Tan[(c + d*x)/2]]/Sqrt[a^2 - b^2])/Sqrt[a^2 - b^2] + 24*a*(3*a^2 + 4*b^2)*(-(A*b) + a*B)*Sin[c + d*x] + 24*a^2*(a^2*A + A*b^2 - a*b*B)*Sin[2*(c + d*x)] + 8*a^3*(-(A*b) + a*B)*Sin[3*(c + d*x)] + 3*a^4*A*Ssin[4*(c + d*x)]/(96*a^5*d)

fricas [A] time = 0.55, size = 685, normalized size = 2.85

$$\frac{3 \left(3 A a^6 - 4 B a^5 b + A a^4 b^2 - 4 B a^3 b^3 + 4 A a^2 b^4 + 8 B a b^5 - 8 A b^6 \right) dx - 12 \left(B a b^4 - A b^5 \right) \sqrt{a^2 - b^2} \log \left(\frac{2 a b \cos(d x + c) - (a^2 - 2 b^2) \cos(d x + c) - 2 \sqrt{a^2 - b^2} (b \cos(d x + c) + a) \sin(d x + c) + 2 a^2 - b^2}{(a^2 \cos(d x + c)^2 + 2 a b \cos(d x + c) + b^2)} \right)}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="fricas")

[Out] [1/24*(3*(3*A*a^6 - 4*B*a^5*b + A*a^4*b^2 - 4*B*a^3*b^3 + 4*A*a^2*b^4 + 8*B*a*b^5 - 8*A*b^6)*d*x - 12*(B*a*b^4 - A*b^5)*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + (16*B*a^6 - 16*A*a^5*b + 8*B*a^4*b^2 - 8*A*a^3*b^3 - 24*B*a^2*b^4 + 24*A*a*b^5 + 6*(A*a^6 - A*a^4*b^2)*cos(d*x + c)^3 + 8*(B*a^6 - A*a^5*b - B*a^4*b^2 + A*a^3*b^3)*cos(d*x + c)^2 + 3*(3*A*a^6 - 4*B*a^5*b + A*a^4*b^2 + 4*B*a^3*b^3 - 4*A*a^2*b^4)*cos(d*x + c))*sin(d*x + c))/((a^7 - a^5*b^2)*d), 1/24*(3*(3*A*a^6 - 4*B*a^5*b + A*a^4*b^2 - 4*B*a^3*b^3 + 4*A*a^2*b^4 + 8*B*a*b^5 - 8*A*b^6)*d*x + 24*(B*a*b^4 - A*b^5)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) + (16*B*a^6 - 16*A*a^5*b + 8*B*a^4*b^2 - 8*A*a^3*b^3 - 24*B*a^2*b^4 + 24*A*a*b^5 + 6*(A*a^6 - A*a^4*b^2)*cos(d*x + c)^3 + 8*(B*a^6 - A*a^5*b - B*a^4*b^2 + A*a^3*b^3)*cos(d*x + c)^2 + 3*(3*A*a^6 - 4*B*a^5*b + A*a^4*b^2 + 4*B*a^3*b^3 - 4*A*a^2*b^4)*cos(d*x + c))*sin(d*x + c))/((a^7 - a^5*b^2)*d)]

giac [B] time = 0.30, size = 642, normalized size = 2.68

$$\frac{3 \left(3 A a^4 - 4 B a^3 b + 4 A a^2 b^2 - 8 B a b^3 + 8 A b^4 \right) (d x + c)}{a^5} + \frac{48 \left(B a b^4 - A b^5 \right) \left(\pi \left\lfloor \frac{d x + c}{2 \pi} + \frac{1}{2} \right\rfloor \operatorname{sgn}(-2 a + 2 b) + \arctan \left(-\frac{a \tan \left(\frac{1}{2} d x + \frac{1}{2} c \right) - b \tan \left(\frac{1}{2} d x + \frac{1}{2} c \right)}{\sqrt{-a^2 + b^2}} \right) \right)}{\sqrt{-a^2 + b^2} a^5} - \frac{2 \left(15 A a^3 \tan \left(\frac{1}{2} d x + \frac{1}{2} c \right)^7 - 24 B a^3 \tan \left(\frac{1}{2} d x + \frac{1}{2} c \right)^7 + 24 A a^2 b \tan \left(\frac{1}{2} d x + \frac{1}{2} c \right)^7 - 12 B a^2 b \tan \left(\frac{1}{2} d x + \frac{1}{2} c \right)^7 \right)}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="giac")

[Out] 1/24*(3*(3*A*a^4 - 4*B*a^3*b + 4*A*a^2*b^2 - 8*B*a*b^3 + 8*A*b^4)*(d*x + c)/a^5 + 48*(B*a*b^4 - A*b^5)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/sqrt(-a^2 + b^2)*a^5) - 2*(15*A*a^3*tan(1/2*d*x + 1/2*c)^7 - 24*B*a^3*tan(1/2*d*x + 1/2*c)^7 + 24*A*a^2*b*tan(1/2*d*x + 1/2*c)^7 - 12*B*a^2*b*tan(1/2*d*x + 1/2*c)^7) - 12*(15*A*a^3*tan(1/2*d*x + 1/2*c)^7 - 24*B*a^3*tan(1/2*d*x + 1/2*c)^7 + 24*A*a^2*b*tan(1/2*d*x + 1/2*c)^7 - 12*B*a^2*b*tan(1/2*d*x + 1/2*c)^7)

$$\begin{aligned} & \tan(1/2*d*x + 1/2*c)^7 + 12*A*a*b^2*\tan(1/2*d*x + 1/2*c)^7 - 24*B*a*b^2*\tan \\ & (1/2*d*x + 1/2*c)^7 + 24*A*b^3*\tan(1/2*d*x + 1/2*c)^7 - 9*A*a^3*\tan(1/2*d*x \\ & + 1/2*c)^5 - 40*B*a^3*\tan(1/2*d*x + 1/2*c)^5 + 40*A*a^2*b*\tan(1/2*d*x + 1/ \\ & 2*c)^5 - 12*B*a^2*b*\tan(1/2*d*x + 1/2*c)^5 + 12*A*a*b^2*\tan(1/2*d*x + 1/2*c \\ &)^5 - 72*B*a*b^2*\tan(1/2*d*x + 1/2*c)^5 + 72*A*b^3*\tan(1/2*d*x + 1/2*c)^5 + \\ & 9*A*a^3*\tan(1/2*d*x + 1/2*c)^3 - 40*B*a^3*\tan(1/2*d*x + 1/2*c)^3 + 40*A*a^2 \\ & *b*\tan(1/2*d*x + 1/2*c)^3 + 12*B*a^2*b*\tan(1/2*d*x + 1/2*c)^3 - 12*A*a*b^2 \\ & *\tan(1/2*d*x + 1/2*c)^3 - 72*B*a*b^2*\tan(1/2*d*x + 1/2*c)^3 + 72*A*b^3*\tan(\\ & 1/2*d*x + 1/2*c)^3 - 15*A*a^3*\tan(1/2*d*x + 1/2*c) - 24*B*a^3*\tan(1/2*d*x + \\ & 1/2*c) + 24*A*a^2*b*\tan(1/2*d*x + 1/2*c) + 12*B*a^2*b*\tan(1/2*d*x + 1/2*c) \\ & - 12*A*a*b^2*\tan(1/2*d*x + 1/2*c) - 24*B*a*b^2*\tan(1/2*d*x + 1/2*c) + 24*A \\ & *b^3*\tan(1/2*d*x + 1/2*c))/((\tan(1/2*d*x + 1/2*c)^2 + 1)^4*a^4))/d \end{aligned}$$

maple [B] time = 1.18, size = 1212, normalized size = 5.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(\cos(d*x+c)^4*(A+B*\sec(d*x+c))/(a+b*\sec(d*x+c)), x)$

[Out]
$$\begin{aligned} & 2/d/a^3/(1+\tan(1/2*d*x+1/2*c)^2)^4*\tan(1/2*d*x+1/2*c)*b^2*B+10/3/d/a/(1+\tan \\ & (1/2*d*x+1/2*c)^2)^4*\tan(1/2*d*x+1/2*c)^3*B+5/4/d/a/(1+\tan(1/2*d*x+1/2*c)^2 \\ &)^4*\tan(1/2*d*x+1/2*c)*A-2/d/a^4*\arctan(\tan(1/2*d*x+1/2*c))*B*b^3+2/d/a/(1+ \\ & \tan(1/2*d*x+1/2*c)^2)^4*\tan(1/2*d*x+1/2*c)^7*B+3/4/d/a/(1+\tan(1/2*d*x+1/2*c \\ &)^2)^4*\tan(1/2*d*x+1/2*c)^5*A+2/d/a/(1+\tan(1/2*d*x+1/2*c)^2)^4*\tan(1/2*d*x+ \\ & 1/2*c)*B+1/d/a^2/(1+\tan(1/2*d*x+1/2*c)^2)^4*\tan(1/2*d*x+1/2*c)^5*B*b+2/d*b^ \\ & 4/a^4/((a-b)*(a+b))^{(1/2)}*\operatorname{arctanh}(\tan(1/2*d*x+1/2*c))*(a-b)/((a-b)*(a+b))^{(1/ \\ & 2)})*B+3/4/d*A/a*\arctan(\tan(1/2*d*x+1/2*c))-2/d*b^5/a^5/((a-b)*(a+b))^{(1/2)} \\ & *\operatorname{arctanh}(\tan(1/2*d*x+1/2*c))*(a-b)/((a-b)*(a+b))^{(1/2)}*A+10/3/d/a/(1+\tan(1/ \\ & 2*d*x+1/2*c)^2)^4*\tan(1/2*d*x+1/2*c)^5*B-3/4/d/a/(1+\tan(1/2*d*x+1/2*c)^2)^4 \\ & *\tan(1/2*d*x+1/2*c)^3*A+2/d/a^5*\arctan(\tan(1/2*d*x+1/2*c))*A*b^4+1/d/a^3/(1 \\ & +\tan(1/2*d*x+1/2*c)^2)^4*\tan(1/2*d*x+1/2*c)*A*b^2-2/d/a^2/(1+\tan(1/2*d*x+1/ \\ & 2*c)^2)^4*\tan(1/2*d*x+1/2*c)*A*b-10/3/d/a^2/(1+\tan(1/2*d*x+1/2*c)^2)^4*\tan(\\ & 1/2*d*x+1/2*c)^5*A*b-6/d/a^4/(1+\tan(1/2*d*x+1/2*c)^2)^4*\tan(1/2*d*x+1/2*c)^ \\ & 5*A*b^3+2/d/a^3/(1+\tan(1/2*d*x+1/2*c)^2)^4*\tan(1/2*d*x+1/2*c)^7*b^2*B-1/d/a \\ & ^3/(1+\tan(1/2*d*x+1/2*c)^2)^4*\tan(1/2*d*x+1/2*c)^7*A*b^2-2/d/a^4/(1+\tan(1/2 \\ & *d*x+1/2*c)^2)^4*\tan(1/2*d*x+1/2*c)^7*A*b^3+1/d/a^3/(1+\tan(1/2*d*x+1/2*c)^2 \\ &)^4*\tan(1/2*d*x+1/2*c)^3*A*b^2-1/d/a^2/(1+\tan(1/2*d*x+1/2*c)^2)^4*\tan(1/2*d \\ & *x+1/2*c)*B*b-2/d/a^4/(1+\tan(1/2*d*x+1/2*c)^2)^4*\tan(1/2*d*x+1/2*c)*A*b^3+6 \\ & /d/a^3/(1+\tan(1/2*d*x+1/2*c)^2)^4*\tan(1/2*d*x+1/2*c)^3*b^2*B-10/3/d/a^2/(1+ \\ & \tan(1/2*d*x+1/2*c)^2)^4*\tan(1/2*d*x+1/2*c)^3*A*b-1/d/a^3/(1+\tan(1/2*d*x+1/2 \\ & *c)^2)^4*\tan(1/2*d*x+1/2*c)^5*A*b^2-6/d/a^4/(1+\tan(1/2*d*x+1/2*c)^2)^4*\tan(\\ & 1/2*d*x+1/2*c)^3*A*b^3+1/d/a^3*\arctan(\tan(1/2*d*x+1/2*c))*A*b^2-1/d/a^2*\operatorname{arc} \\ & \tan(\tan(1/2*d*x+1/2*c))*B*b+6/d/a^3/(1+\tan(1/2*d*x+1/2*c)^2)^4*\tan(1/2*d*x+ \\ & 1/2*c)^5*b^2*B-1/d/a^2/(1+\tan(1/2*d*x+1/2*c)^2)^4*\tan(1/2*d*x+1/2*c)^3*B*b \end{aligned}$$

$$\frac{1}{d/a^2/(1+\tan(1/2*d*x+1/2*c))^2} \tan(1/2*d*x+1/2*c)^7 * B*b^{-2} / \frac{1}{d/a^2/(1+\tan(1/2*d*x+1/2*c))^2} \tan(1/2*d*x+1/2*c)^4 * \tan(1/2*d*x+1/2*c)^7 * A*b^{-5} / \frac{4}{d/a/(1+\tan(1/2*d*x+1/2*c))^2} \tan(1/2*d*x+1/2*c)^7 * A$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for more details) Is 4*a^2-4*b^2 positive or negative?

mupad [B] time = 8.64, size = 5903, normalized size = 24.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)^4*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x)),x)

[Out] ((tan(c/2 + (d*x)/2)*(5*A*a^3 - 8*A*b^3 + 8*B*a^3 + 4*A*a*b^2 - 8*A*a^2*b + 8*B*a*b^2 - 4*B*a^2*b))/(4*a^4) - (tan(c/2 + (d*x)/2)^7*(5*A*a^3 + 8*A*b^3 - 8*B*a^3 + 4*A*a*b^2 + 8*A*a^2*b - 8*B*a*b^2 - 4*B*a^2*b))/(4*a^4) - (tan(c/2 + (d*x)/2)^3*(9*A*a^3 + 72*A*b^3 - 40*B*a^3 - 12*A*a*b^2 + 40*A*a^2*b - 72*B*a*b^2 + 12*B*a^2*b))/(12*a^4) + (tan(c/2 + (d*x)/2)^5*(9*A*a^3 - 72*A*b^3 + 40*B*a^3 - 12*A*a*b^2 - 40*A*a^2*b + 72*B*a*b^2 + 12*B*a^2*b))/(12*a^4))/(d*(4*tan(c/2 + (d*x)/2)^2 + 6*tan(c/2 + (d*x)/2)^4 + 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1)) - (atan((((((((12*A*a^16 + 32*A*a^10*b^6 - 48*A*a^11*b^5 + 16*A*a^12*b^4 - 4*A*a^13*b^3 + 4*A*a^14*b^2 - 32*B*a^11*b^5 + 48*B*a^12*b^4 - 16*B*a^13*b^3 + 16*B*a^14*b^2 - 12*A*a^15*b - 16*B*a^15*b)/a^12 - (tan(c/2 + (d*x)/2)*(128*a^12*b + 128*a^10*b^3 - 256*a^11*b^2)*(A*a^4*3i + A*b^4*8i + A*a^2*b^2*4i - B*a*b^3*8i - B*a^3*b*4i))/(16*a^13))*(A*a^4*3i + A*b^4*8i + A*a^2*b^2*4i - B*a*b^3*8i - B*a^3*b*4i))/(8*a^5) + (tan(c/2 + (d*x)/2)*(9*A^2*a^11 - 128*A^2*b^11 + 256*A^2*a*b^10 - 27*A^2*a^10*b - 256*A^2*a^2*b^9 + 256*A^2*a^3*b^8 - 256*A^2*a^4*b^7 + 256*A^2*a^5*b^6 - 216*A^2*a^6*b^5 + 136*A^2*a^7*b^4 - 81*A^2*a^8*b^3 + 51*A^2*a^9*b^2 - 128*B^2*a^2*b^9 + 256*B^2*a^3*b^8 - 256*B^2*a^4*b^7 + 256*B^2*a^5*b^6 - 208*B^2*a^6*b^5 + 112*B^2*a^7*b^4 - 48*B^2*a^8*b^3 + 16*B^2*a^9*b^2 + 256*A*B*a*b^10 - 24*A*B*a^10*b - 512*A*B*a^2*b^9 + 512*A*B*a^3*b^8 - 512*A*B*a^4*b^7 + 464*A*B*a^5*b^6 - 368*A*B*a^6*b^5 + 264*A*B*a^7*b^4 - 152*A*B*a^8*b^3 + 72*A*B*a^9*b^2))/(2*a^8))*(A*a^4*3i + A*b^4*8i + A*a^2*b^2*4i - B*a*b^3*8i

$$\begin{aligned}
& - B*a^3*b^4i)*i)/(8*a^5) - (((((12*A*a^16 + 32*A*a^10*b^6 - 48*A*a^11*b^5 \\
& + 16*A*a^12*b^4 - 4*A*a^13*b^3 + 4*A*a^14*b^2 - 32*B*a^11*b^5 + 48*B*a^12* \\
& b^4 - 16*B*a^13*b^3 + 16*B*a^14*b^2 - 12*A*a^15*b - 16*B*a^15*b)/a^12 + (ta \\
& n(c/2 + (d*x)/2)*(128*a^12*b + 128*a^10*b^3 - 256*a^11*b^2)*(A*a^4*3i + A*b \\
& ^4*8i + A*a^2*b^2*4i - B*a*b^3*8i - B*a^3*b^4i))/(16*a^13))*(A*a^4*3i + A*b \\
& ^4*8i + A*a^2*b^2*4i - B*a*b^3*8i - B*a^3*b^4i))/(8*a^5) - (tan(c/2 + (d*x) \\
& /2)*(9*A^2*a^11 - 128*A^2*b^11 + 256*A^2*a*b^10 - 27*A^2*a^10*b - 256*A^2*a \\
& ^2*b^9 + 256*A^2*a^3*b^8 - 256*A^2*a^4*b^7 + 256*A^2*a^5*b^6 - 216*A^2*a^6* \\
& b^5 + 136*A^2*a^7*b^4 - 81*A^2*a^8*b^3 + 51*A^2*a^9*b^2 - 128*B^2*a^2*b^9 + \\
& 256*B^2*a^3*b^8 - 256*B^2*a^4*b^7 + 256*B^2*a^5*b^6 - 208*B^2*a^6*b^5 + 11 \\
& 2*B^2*a^7*b^4 - 48*B^2*a^8*b^3 + 16*B^2*a^9*b^2 + 256*A*B*a*b^10 - 24*A*B*a \\
& ^10*b - 512*A*B*a^2*b^9 + 512*A*B*a^3*b^8 - 512*A*B*a^4*b^7 + 464*A*B*a^5*b \\
& ^6 - 368*A*B*a^6*b^5 + 264*A*B*a^7*b^4 - 152*A*B*a^8*b^3 + 72*A*B*a^9*b^2)) \\
& /((2*a^8))*(A*a^4*3i + A*b^4*8i + A*a^2*b^2*4i - B*a*b^3*8i - B*a^3*b^4i)*i \\
&))/(8*a^5))/((64*A^3*b^14 - 96*A^3*a*b^13 + 96*A^3*a^2*b^12 - 104*A^3*a^3*b^ \\
& 11 + 104*A^3*a^4*b^10 - 88*A^3*a^5*b^9 + 48*A^3*a^6*b^8 - 33*A^3*a^7*b^7 + \\
& 18*A^3*a^8*b^6 - 9*A^3*a^9*b^5 - 64*B^3*a^3*b^11 + 96*B^3*a^4*b^10 - 96*B^3 \\
& *a^5*b^9 + 80*B^3*a^6*b^8 - 32*B^3*a^7*b^7 + 16*B^3*a^8*b^6 - 192*A^2*B*a*b \\
& ^13 + 192*A*B^2*a^2*b^12 - 288*A*B^2*a^3*b^11 + 288*A*B^2*a^4*b^10 - 264*A* \\
& B^2*a^5*b^9 + 168*A*B^2*a^6*b^8 - 120*A*B^2*a^7*b^7 + 48*A*B^2*a^8*b^6 - 24 \\
& *A*B^2*a^9*b^5 + 288*A^2*B*a^2*b^12 - 288*A^2*B*a^3*b^11 + 288*A^2*B*a^4*b^ \\
& 10 - 240*A^2*B*a^5*b^9 + 192*A^2*B*a^6*b^8 - 96*A^2*B*a^7*b^7 + 57*A^2*B*a^ \\
& 8*b^6 - 18*A^2*B*a^9*b^5 + 9*A^2*B*a^10*b^4)/a^12 + (((((12*A*a^16 + 32*A*a \\
& ^10*b^6 - 48*A*a^11*b^5 + 16*A*a^12*b^4 - 4*A*a^13*b^3 + 4*A*a^14*b^2 - 32* \\
& B*a^11*b^5 + 48*B*a^12*b^4 - 16*B*a^13*b^3 + 16*B*a^14*b^2 - 12*A*a^15*b - \\
& 16*B*a^15*b)/a^12 - (tan(c/2 + (d*x)/2)*(128*a^12*b + 128*a^10*b^3 - 256*a^ \\
& 11*b^2)*(A*a^4*3i + A*b^4*8i + A*a^2*b^2*4i - B*a*b^3*8i - B*a^3*b^4i))/(16 \\
& *a^13))*(A*a^4*3i + A*b^4*8i + A*a^2*b^2*4i - B*a*b^3*8i - B*a^3*b^4i))/(8* \\
& a^5) + (tan(c/2 + (d*x)/2)*(9*A^2*a^11 - 128*A^2*b^11 + 256*A^2*a*b^10 - 27 \\
& *A^2*a^10*b - 256*A^2*a^2*b^9 + 256*A^2*a^3*b^8 - 256*A^2*a^4*b^7 + 256*A^2 \\
& *a^5*b^6 - 216*A^2*a^6*b^5 + 136*A^2*a^7*b^4 - 81*A^2*a^8*b^3 + 51*A^2*a^9* \\
& b^2 - 128*B^2*a^2*b^9 + 256*B^2*a^3*b^8 - 256*B^2*a^4*b^7 + 256*B^2*a^5*b^6 \\
& - 208*B^2*a^6*b^5 + 112*B^2*a^7*b^4 - 48*B^2*a^8*b^3 + 16*B^2*a^9*b^2 + 25 \\
& 6*A*B*a*b^10 - 24*A*B*a^10*b - 512*A*B*a^2*b^9 + 512*A*B*a^3*b^8 - 512*A*B* \\
& a^4*b^7 + 464*A*B*a^5*b^6 - 368*A*B*a^6*b^5 + 264*A*B*a^7*b^4 - 152*A*B*a^8 \\
& *b^3 + 72*A*B*a^9*b^2))/((2*a^8))*(A*a^4*3i + A*b^4*8i + A*a^2*b^2*4i - B*a* \\
& b^3*8i - B*a^3*b^4i))/(8*a^5) + (((((12*A*a^16 + 32*A*a^10*b^6 - 48*A*a^11* \\
& b^5 + 16*A*a^12*b^4 - 4*A*a^13*b^3 + 4*A*a^14*b^2 - 32*B*a^11*b^5 + 48*B*a^ \\
& 12*b^4 - 16*B*a^13*b^3 + 16*B*a^14*b^2 - 12*A*a^15*b - 16*B*a^15*b)/a^12 + \\
& (tan(c/2 + (d*x)/2)*(128*a^12*b + 128*a^10*b^3 - 256*a^11*b^2)*(A*a^4*3i + \\
& A*b^4*8i + A*a^2*b^2*4i - B*a*b^3*8i - B*a^3*b^4i))/(16*a^13))*(A*a^4*3i + \\
& A*b^4*8i + A*a^2*b^2*4i - B*a*b^3*8i - B*a^3*b^4i))/(8*a^5) - (tan(c/2 + (d \\
& *x)/2)*(9*A^2*a^11 - 128*A^2*b^11 + 256*A^2*a*b^10 - 27*A^2*a^10*b - 256*A^ \\
& 2*a^2*b^9 + 256*A^2*a^3*b^8 - 256*A^2*a^4*b^7 + 256*A^2*a^5*b^6 - 216*A^2*a \\
& ^6*b^5 + 136*A^2*a^7*b^4 - 81*A^2*a^8*b^3 + 51*A^2*a^9*b^2 - 128*B^2*a^2*b^
\end{aligned}$$

$$\begin{aligned}
& 9 + 256B^2a^3b^8 - 256B^2a^4b^7 + 256B^2a^5b^6 - 208B^2a^6b^5 + \\
& 112B^2a^7b^4 - 48B^2a^8b^3 + 16B^2a^9b^2 + 256AB^2a^10b - 24A^2B^2a^10b - 512A^2B^2a^2b^9 + 512A^2B^2a^3b^8 - 512A^2B^2a^4b^7 + 464A^2B^2a^5b^6 - 368A^2B^2a^6b^5 + 264A^2B^2a^7b^4 - 152A^2B^2a^8b^3 + 72A^2B^2a^9b^2) \\
&)/(2a^8)) * (A^2a^4b^3i + A^2b^4b^8i + A^2a^2b^2b^4i - B^2a^3b^3b^8i - B^2a^3b^4b^4i) \\
&)/(8a^5)) * (A^2a^4b^3i + A^2b^4b^8i + A^2a^2b^2b^4i - B^2a^3b^3b^8i - B^2a^3b^4b^4i) * \\
& 1i)/(4a^5d) - (b^4 * \operatorname{atan}((b^4 * ((a + b) * (a - b))^{1/2}) * (Ab - B^2a) * ((\tan(c/2 + (d*x)/2) * (9A^2a^{11} - 128A^2b^{11} + 256A^2a^2b^{10} - 27A^2a^{10}b - 256A^2a^2b^9 + 256A^2a^3b^8 - 256A^2a^4b^7 + 256A^2a^5b^6 - 216A^2a^6b^5 + 136A^2a^7b^4 - 81A^2a^8b^3 + 51A^2a^9b^2 - 128B^2a^2b^9 + 256B^2a^3b^8 - 256B^2a^4b^7 + 256B^2a^5b^6 - 208B^2a^6b^5 + 112B^2a^7b^4 - 48B^2a^8b^3 + 16B^2a^9b^2 + 256AB^2a^10b - 24A^2B^2a^10b - 512A^2B^2a^2b^9 + 512A^2B^2a^3b^8 - 512A^2B^2a^4b^7 + 464A^2B^2a^5b^6 - 368A^2B^2a^6b^5 + 264A^2B^2a^7b^4 - 152A^2B^2a^8b^3 + 72A^2B^2a^9b^2)))/(2a^8) + (b^4 * ((a + b) * (a - b))^{1/2}) * (Ab - B^2a) * ((12A^2a^{16} + 32A^2a^{10}b^6 - 48A^2a^{11}b^5 + 16A^2a^{12}b^4 - 4A^2a^{13}b^3 + 4A^2a^{14}b^2 - 32B^2a^{11}b^5 + 48B^2a^{12}b^4 - 16B^2a^{13}b^3 + 16B^2a^{14}b^2 - 12A^2a^{15}b - 16B^2a^{15}b)/a^{12} - (b^4 * \tan(c/2 + (d*x)/2) * ((a + b) * (a - b))^{1/2}) * (Ab - B^2a) * (128a^{12}b + 128a^{10}b^3 - 256a^{11}b^2)))/(2a^8 * (a^7 - a^5b^2))) / (a^7 - a^5b^2)) * 1i) / (a^7 - a^5b^2) + (b^4 * ((a + b) * (a - b))^{1/2}) * (Ab - B^2a) * ((\tan(c/2 + (d*x)/2) * (9A^2a^{11} - 128A^2b^{11} + 256A^2a^2b^{10} - 27A^2a^{10}b - 256A^2a^2b^9 + 256A^2a^3b^8 - 256A^2a^4b^7 + 256A^2a^5b^6 - 216A^2a^6b^5 + 136A^2a^7b^4 - 81A^2a^8b^3 + 51A^2a^9b^2 - 128B^2a^2b^9 + 256B^2a^3b^8 - 256B^2a^4b^7 + 256B^2a^5b^6 - 208B^2a^6b^5 + 112B^2a^7b^4 - 48B^2a^8b^3 + 16B^2a^9b^2 + 256AB^2a^10b - 24A^2B^2a^10b - 512A^2B^2a^2b^9 + 512A^2B^2a^3b^8 - 512A^2B^2a^4b^7 + 464A^2B^2a^5b^6 - 368A^2B^2a^6b^5 + 264A^2B^2a^7b^4 - 152A^2B^2a^8b^3 + 72A^2B^2a^9b^2)))/(2a^8) - (b^4 * ((a + b) * (a - b))^{1/2}) * (Ab - B^2a) * ((12A^2a^{16} + 32A^2a^{10}b^6 - 48A^2a^{11}b^5 + 16A^2a^{12}b^4 - 4A^2a^{13}b^3 + 4A^2a^{14}b^2 - 32B^2a^{11}b^5 + 48B^2a^{12}b^4 - 16B^2a^{13}b^3 + 16B^2a^{14}b^2 - 12A^2a^{15}b - 16B^2a^{15}b)/a^{12} + (b^4 * \tan(c/2 + (d*x)/2) * ((a + b) * (a - b))^{1/2}) * (Ab - B^2a) * (128a^{12}b + 128a^{10}b^3 - 256a^{11}b^2)) / (2a^8 * (a^7 - a^5b^2))) / (a^7 - a^5b^2)) * 1i) / (a^7 - a^5b^2)) / ((64A^3b^{14} - 96A^3a^3b^{13} + 96A^3a^2b^{12} - 104A^3a^3b^{11} + 104A^3a^4b^{10} - 88A^3a^5b^9 + 48A^3a^6b^8 - 33A^3a^7b^7 + 18A^3a^8b^6 - 9A^3a^9b^5 - 64B^3a^3b^{11} + 96B^3a^4b^{10} - 96B^3a^5b^9 + 80B^3a^6b^8 - 32B^3a^7b^7 + 16B^3a^8b^6 - 192A^2B^2a^2b^{13} + 192A^2B^2a^2b^{12} - 288A^2B^2a^3b^{11} + 288A^2B^2a^4b^{10} - 264A^2B^2a^5b^9 + 168A^2B^2a^6b^8 - 120A^2B^2a^7b^7 + 48A^2B^2a^8b^6 - 24A^2B^2a^9b^5 + 288A^2B^2a^2b^{12} - 288A^2B^2a^3b^{11} + 288A^2B^2a^4b^{10} - 240A^2B^2a^5b^9 + 192A^2B^2a^6b^8 - 96A^2B^2a^7b^7 + 57A^2B^2a^8b^6 - 18A^2B^2a^9b^5 + 9A^2B^2a^{10}b^4)/a^{12} + (b^4 * ((a + b) * (a - b))^{1/2}) * (Ab - B^2a) * ((\tan(c/2 + (d*x)/2) * (9A^2a^{11} - 128A^2b^{11} + 256A^2a^2b^{10} - 27A^2a^{10}b - 256A^2a^2b^9 + 256A^2a^3b^8 - 256A^2a^4b^7 + 256A^2a^5b^6 - 216A^2a^6b^5 + 136A^2a^7b^4 - 81A^2a^8b^3 + 51A^2a^9b^2 - 128*
\end{aligned}$$

$$\begin{aligned}
& B^2 a^2 b^9 + 256 B^2 a^3 b^8 - 256 B^2 a^4 b^7 + 256 B^2 a^5 b^6 - 208 B^2 \\
& a^6 b^5 + 112 B^2 a^7 b^4 - 48 B^2 a^8 b^3 + 16 B^2 a^9 b^2 + 256 A B a b^9 \\
& - 24 A B a^{10} b - 512 A B a^2 b^9 + 512 A B a^3 b^8 - 512 A B a^4 b^7 + \\
& 464 A B a^5 b^6 - 368 A B a^6 b^5 + 264 A B a^7 b^4 - 152 A B a^8 b^3 + 72 A \\
& A B a^9 b^2) / (2 a^8) + (b^4 ((a + b)(a - b))^{1/2} (A b - B a) * ((12 A a^{16} \\
& + 32 A a^{10} b^6 - 48 A a^{11} b^5 + 16 A a^{12} b^4 - 4 A a^{13} b^3 + 4 A a^{14} \\
& b^2 - 32 B a^{11} b^5 + 48 B a^{12} b^4 - 16 B a^{13} b^3 + 16 B a^{14} b^2 - 12 A \\
& a^{15} b - 16 B a^{15} b) / a^{12} - (b^4 \tan(c/2 + (d*x)/2) * ((a + b)(a - b))^{1/2} \\
& (A b - B a) * (128 a^{12} b + 128 a^{10} b^3 - 256 a^{11} b^2)) / (2 a^8 * (a^7 - a^5 b^2))) \\
& / (a^7 - a^5 b^2)) / (a^7 - a^5 b^2) - (b^4 ((a + b)(a - b))^{1/2} (A b - \\
& B a) * ((\tan(c/2 + (d*x)/2) * (9 A^2 a^{11} - 128 A^2 b^{11} + 256 A^2 a b^{10} \\
& - 27 A^2 a^{10} b - 256 A^2 a^2 b^9 + 256 A^2 a^3 b^8 - 256 A^2 a^4 b^7 + 2 \\
& 56 A^2 a^5 b^6 - 216 A^2 a^6 b^5 + 136 A^2 a^7 b^4 - 81 A^2 a^8 b^3 + 51 A^2 \\
& a^9 b^2 - 128 B^2 a^2 b^9 + 256 B^2 a^3 b^8 - 256 B^2 a^4 b^7 + 256 B^2 a^5 b^6 \\
& - 208 B^2 a^6 b^5 + 112 B^2 a^7 b^4 - 48 B^2 a^8 b^3 + 16 B^2 a^9 b^2 \\
& + 256 A B a b^{10} - 24 A B a^{10} b - 512 A B a^2 b^9 + 512 A B a^3 b^8 - 51 \\
& 2 A B a^4 b^7 + 464 A B a^5 b^6 - 368 A B a^6 b^5 + 264 A B a^7 b^4 - 152 A \\
& B a^8 b^3 + 72 A B a^9 b^2)) / (2 a^8) - (b^4 ((a + b)(a - b))^{1/2} (A b - \\
& B a) * ((12 A a^{16} + 32 A a^{10} b^6 - 48 A a^{11} b^5 + 16 A a^{12} b^4 - 4 A a^{13} \\
& b^3 + 4 A a^{14} b^2 - 32 B a^{11} b^5 + 48 B a^{12} b^4 - 16 B a^{13} b^3 + 16 B \\
& a^{14} b^2 - 12 A a^{15} b - 16 B a^{15} b) / a^{12} + (b^4 \tan(c/2 + (d*x)/2) * ((a + \\
& b)(a - b))^{1/2} (A b - B a) * (128 a^{12} b + 128 a^{10} b^3 - 256 a^{11} b^2)) / \\
& (2 a^8 * (a^7 - a^5 b^2))) / (a^7 - a^5 b^2)) / (a^7 - a^5 b^2)) * ((a + b)(a - \\
& b))^{1/2} (A b - B a) * 2i) / (d * (a^7 - a^5 b^2))
\end{aligned}$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \cos^4(c + dx)}{a + b \sec(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**4*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x)

[Out] Integral((A + B*sec(c + d*x))*cos(c + d*x)**4/(a + b*sec(c + d*x)), x)

$$3.320 \quad \int \frac{\sec^4(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^2} dx$$

Optimal. Leaf size=272

$$\frac{a(Ab - aB) \tan(c + dx) \sec^2(c + dx)}{bd(a^2 - b^2)(a + b \sec(c + dx))} - \frac{(-3a^2B + 2aAb + b^2B) \tan(c + dx) \sec(c + dx)}{2b^2d(a^2 - b^2)} - \frac{(-6a^2B + 4aAb - b^2B) \tan(c + dx) \sec^3(c + dx)}{2b^4d}$$

[Out] $-1/2*(4*A*a*b-6*B*a^2-B*b^2)*\operatorname{arctanh}(\sin(d*x+c))/b^4/d+2*a^2*(2*A*a^2*b-3*A*b^3-3*B*a^3+4*B*a*b^2)*\operatorname{arctanh}((a-b)^{(1/2)}*\tan(1/2*d*x+1/2*c)/(a+b)^{(1/2)})/(a-b)^{(3/2)}/b^4/(a+b)^{(3/2)}/d+(2*A*a^2*b-A*b^3-3*B*a^3+2*B*a*b^2)*\tan(d*x+c)/b^3/(a^2-b^2)/d-1/2*(2*A*a*b-3*B*a^2+B*b^2)*\sec(d*x+c)*\tan(d*x+c)/b^2/(a^2-b^2)/d+a*(A*b-B*a)*\sec(d*x+c)^2*\tan(d*x+c)/b/(a^2-b^2)/d/(a+b*\sec(d*x+c))$

Rubi [A] time = 0.87, antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {4029, 4092, 4082, 3998, 3770, 3831, 2659, 208}

$$\frac{(2a^2Ab - 3a^3B + 2ab^2B - Ab^3) \tan(c + dx)}{b^3d(a^2 - b^2)} - \frac{(-6a^2B + 4aAb - b^2B) \tanh^{-1}(\sin(c + dx))}{2b^4d} + \frac{2a^2(2a^2Ab - 3a^3B - 2ab^2B + Ab^3) \tan(c + dx) \sec^2(c + dx)}{2b^4d}$$

Antiderivative was successfully verified.

[In] Int[(Sec[c + d*x]^4*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^2,x]

[Out] $-((4*a*A*b - 6*a^2*B - b^2*B)*\operatorname{ArcTanh}[\sin[c + d*x]])/(2*b^4*d) + (2*a^2*(2*a^2*A*b - 3*A*b^3 - 3*a^3*B + 4*a*b^2*B)*\operatorname{ArcTanh}[(\sqrt{a - b})*\tan[(c + d*x)/2]]/\sqrt{a + b})/((a - b)^{(3/2)}*b^4*(a + b)^{(3/2)*d} + ((2*a^2*A*b - A*b^3 - 3*a^3*B + 2*a*b^2*B)*\tan[c + d*x])/(b^3*(a^2 - b^2)*d) - ((2*a*A*b - 3*a^2*B + b^2*B)*\sec[c + d*x]*\tan[c + d*x])/(2*b^2*(a^2 - b^2)*d) + (a*(A*b - a*B)*\sec[c + d*x]^2*\tan[c + d*x])/(b*(a^2 - b^2)*d*(a + b*\sec[c + d*x]))$

Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 2659

Int[((a_) + (b_)*sin[Pi/2 + (c_) + (d_)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (

$a - b)e^{2x^2}$, x , $\tan[(c + dx)/2]/e$, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 3770

Int[csc[(e_.) + (f_.)(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + dx]]/d, x] /; FreeQ[{c, d}, x]

Rule 3831

Int[csc[(e_.) + (f_.)(x_)]/(csc[(e_.) + (f_.)(x_)]*(b_.) + (a_.)), x_Symbol] := Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 3998

Int[(csc[(e_.) + (f_.)(x_)]*(csc[(e_.) + (f_.)(x_)]*(B_.) + (A_.)))/(csc[(e_.) + (f_.)(x_)]*(b_.) + (a_.)), x_Symbol] := Dist[B/b, Int[Csc[e + f*x], x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]

Rule 4029

Int[(csc[(e_.) + (f_.)(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)(x_)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)(x_)]*(B_.) + (A_.)), x_Symbol] := Simp[(a*d^2*(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[d/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(n - 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2*(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 1]

Rule 4082

Int[csc[(e_.) + (f_.)(x_)]*((A_.) + csc[(e_.) + (f_.)(x_)]*(B_.) + csc[(e_.) + (f_.)(x_)]^2*(C_.))*(csc[(e_.) + (f_.)(x_)]*(b_.) + (a_.))^(m_), x_Symbol] := -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rule 4092


```

Int[csc[(e_.) + (f_.)*(x_.)]^2*((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[
(e_.) + (f_.)*(x_.)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)^(m_)), x
_Symbol] :> -Simp[(C*Csc[e + f*x]*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
)/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[Csc[e + f*x]*(a + b*Csc[e + f
*x])^m*Simp[a*C + b*(C*(m + 2) + A*(m + 3))*Csc[e + f*x] - (2*a*C - b*B*(m
+ 3))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && N
eQ[a^2 - b^2, 0] && !LtQ[m, -1]

```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^4(c + dx)(A + B \sec(c + dx))}{(a + b \sec(c + dx))^2} dx &= \frac{a(Ab - aB) \sec^2(c + dx) \tan(c + dx)}{b(a^2 - b^2)d(a + b \sec(c + dx))} + \int \frac{\sec^2(c + dx)(2a(Ab - aB) - b(Ab - aB) \sec(c + dx))}{a + b \sec(c + dx)} dx \\
&= -\frac{(2aAb - 3a^2B + b^2B) \sec(c + dx) \tan(c + dx)}{2b^2(a^2 - b^2)d} + \frac{a(Ab - aB) \sec^2(c + dx)}{b(a^2 - b^2)d(a + b \sec(c + dx))} \\
&= \frac{(2a^2Ab - Ab^3 - 3a^3B + 2ab^2B) \tan(c + dx)}{b^3(a^2 - b^2)d} - \frac{(2aAb - 3a^2B + b^2B) \sec(c + dx)}{2b^2(a^2 - b^2)d(a + b \sec(c + dx))} \\
&= \frac{(2a^2Ab - Ab^3 - 3a^3B + 2ab^2B) \tan(c + dx)}{b^3(a^2 - b^2)d} - \frac{(2aAb - 3a^2B + b^2B) \sec(c + dx)}{2b^2(a^2 - b^2)d(a + b \sec(c + dx))} \\
&= -\frac{(4aAb - 6a^2B - b^2B) \tanh^{-1}(\sin(c + dx))}{2b^4d} + \frac{(2a^2Ab - Ab^3 - 3a^3B + 2ab^2B) \tan(c + dx)}{b^3(a^2 - b^2)d} \\
&= -\frac{(4aAb - 6a^2B - b^2B) \tanh^{-1}(\sin(c + dx))}{2b^4d} + \frac{(2a^2Ab - Ab^3 - 3a^3B + 2ab^2B) \tan(c + dx)}{b^3(a^2 - b^2)d} \\
&= -\frac{(4aAb - 6a^2B - b^2B) \tanh^{-1}(\sin(c + dx))}{2b^4d} + \frac{2a^2(2a^2Ab - 3Ab^3 - 3a^3B + 2ab^2B) \tan(c + dx)}{2b^4d}
\end{aligned}$$

Mathematica [A] time = 6.30, size = 438, normalized size = 1.61

$$\frac{(-6a^2B + 4aAb - b^2B) \log\left(\cos\left(\frac{1}{2}(c + dx)\right) - \sin\left(\frac{1}{2}(c + dx)\right)\right)}{2b^4d} + \frac{(6a^2B - 4aAb + b^2B) \log\left(\sin\left(\frac{1}{2}(c + dx)\right) + \cos\left(\frac{1}{2}(c + dx)\right)\right)}{2b^4d}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(Sec[c + d*x]^4*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^2,x]

[Out] $(-2*a^2*(-2*a^2*A*b + 3*A*b^3 + 3*a^3*B - 4*a*b^2*B)*\text{ArcTanh}[((-a + b)*\text{Tan}[(c + d*x)/2])/\text{Sqrt}[a^2 - b^2]])/(b^4*\text{Sqrt}[a^2 - b^2]*(-a^2 + b^2)*d) + ((4*a*A*b - 6*a^2*B - b^2*B)*\text{Log}[\text{Cos}[(c + d*x)/2] - \text{Sin}[(c + d*x)/2]])/(2*b^4*d) + ((-4*a*A*b + 6*a^2*B + b^2*B)*\text{Log}[\text{Cos}[(c + d*x)/2] + \text{Sin}[(c + d*x)/2]])/(2*b^4*d) + B/(4*b^2*d*(\text{Cos}[(c + d*x)/2] - \text{Sin}[(c + d*x)/2])^2) - B/(4*b^2*d*(\text{Cos}[(c + d*x)/2] + \text{Sin}[(c + d*x)/2])^2) + (A*b*\text{Sin}[(c + d*x)/2] - 2*a*B*\text{Sin}[(c + d*x)/2])/(b^3*d*(\text{Cos}[(c + d*x)/2] - \text{Sin}[(c + d*x)/2])) + (A*b*\text{Sin}[(c + d*x)/2] - 2*a*B*\text{Sin}[(c + d*x)/2])/(b^3*d*(\text{Cos}[(c + d*x)/2] + \text{Sin}[(c + d*x)/2])) + (-a^3*A*b*\text{Sin}[c + d*x]) + a^4*B*\text{Sin}[c + d*x])/(b^3*(-a + b)*(a + b)*d*(b + a*\text{Cos}[c + d*x]))$

fricas [B] time = 20.61, size = 1343, normalized size = 4.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm="fricas")

[Out] $[1/4*(2*((3*B*a^6 - 2*A*a^5*b - 4*B*a^4*b^2 + 3*A*a^3*b^3)*\cos(d*x + c)^3 + (3*B*a^5*b - 2*A*a^4*b^2 - 4*B*a^3*b^3 + 3*A*a^2*b^4)*\cos(d*x + c)^2)*\text{sqrt}(a^2 - b^2)*\log((2*a*b*\cos(d*x + c) - (a^2 - 2*b^2)*\cos(d*x + c)^2 - 2*\text{sqrt}(a^2 - b^2)*(b*\cos(d*x + c) + a)*\sin(d*x + c) + 2*a^2 - b^2)/(a^2*\cos(d*x + c)^2 + 2*a*b*\cos(d*x + c) + b^2)) + ((6*B*a^7 - 4*A*a^6*b - 11*B*a^5*b^2 + 8*A*a^4*b^3 + 4*B*a^3*b^4 - 4*A*a^2*b^5 + B*a*b^6)*\cos(d*x + c)^3 + (6*B*a^6*b - 4*A*a^5*b^2 - 11*B*a^4*b^3 + 8*A*a^3*b^4 + 4*B*a^2*b^5 - 4*A*a*b^6 + B*b^7)*\cos(d*x + c)^2)*\log(\sin(d*x + c) + 1) - ((6*B*a^7 - 4*A*a^6*b - 11*B*a^5*b^2 + 8*A*a^4*b^3 + 4*B*a^3*b^4 - 4*A*a^2*b^5 + B*a*b^6)*\cos(d*x + c)^3 + (6*B*a^6*b - 4*A*a^5*b^2 - 11*B*a^4*b^3 + 8*A*a^3*b^4 + 4*B*a^2*b^5 - 4*A*a*b^6 + B*b^7)*\cos(d*x + c)^2)*\log(-\sin(d*x + c) + 1) + 2*(B*a^4*b^3 - 2*B*a^2*b^5 + B*b^7 - 2*(3*B*a^6*b - 2*A*a^5*b^2 - 5*B*a^4*b^3 + 3*A*a^3*b^4 + 2*B*a^2*b^5 - A*a*b^6)*\cos(d*x + c)^2 - (3*B*a^5*b^2 - 2*A*a^4*b^3 - 6*B*a^3*b^4 + 4*A*a^2*b^5 + 3*B*a*b^6 - 2*A*b^7)*\cos(d*x + c))*\sin(d*x + c)]/((a^5*b^4 - 2*a^3*b^6 + a*b^8)*d*\cos(d*x + c)^3 + (a^4*b^5 - 2*a^2*b^7 + b^9)*d*\cos(d*x + c)^2), -1/4*(4*((3*B*a^6 - 2*A*a^5*b - 4*B*a^4*b^2 + 3*A*a^3*b^3)*\cos(d*x + c)^3 + (3*B*a^5*b - 2*A*a^4*b^2 - 4*B*a^3*b^3 + 3*A*a^2*b^4)*\cos(d*x + c)^2)*\text{sqrt}(-a^2 + b^2)*\arctan(-\text{sqrt}(-a^2 + b^2)*(b*\cos(d*x + c) + a)/((a^2 - b^2)*\sin(d*x + c))) - ((6*B*a^7 - 4*A*a^6*b - 11*B*a^5*b^2 + 8*A*a^4*b^3 + 4*B*a^3*b^4 - 4*A*a^2*b^5 + B*a*b^6)*\cos(d*x + c)^3 + (6*B*a^6*b - 4*A*a^5*b^2 - 11*B*a^4*b^3 + 8*A*a^3*b^4 + 4*B*a^2*b^5 - 4*A*a*b^6 + B*b^7)*\cos(d*x + c)^2)*\log(\sin(d*x + c) + 1) + ((6*B*a^7 - 4*A*a^6*b - 11*B$

$a^5b^2 + 8Aa^4b^3 + 4B^2a^3b^4 - 4A^2a^2b^5 + B^2ab^6) \cos(dx + c)^3 + (6B^2a^6b - 4A^2a^5b^2 - 11B^2a^4b^3 + 8A^2a^3b^4 + 4B^2a^2b^5 - 4A^2ab^6 + B^2b^7) \cos(dx + c)^2 \log(-\sin(dx + c) + 1) - 2(B^2a^4b^3 - 2B^2a^2b^5 + B^2b^7 - 2(3B^2a^6b - 2A^2a^5b^2 - 5B^2a^4b^3 + 3A^2a^3b^4 + 2B^2a^2b^5 - A^2ab^6) \cos(dx + c)^2 - (3B^2a^5b^2 - 2A^2a^4b^3 - 6B^2a^3b^4 + 4A^2a^2b^5 + 3B^2ab^6 - 2A^2b^7) \cos(dx + c)) \sin(dx + c) / ((a^5b^4 - 2a^3b^6 + a^2b^8) d \cos(dx + c)^3 + (a^4b^5 - 2a^2b^7 + b^9) d \cos(dx + c)^2]$

giac [A] time = 0.34, size = 384, normalized size = 1.41

$$\frac{4(3Ba^5 - 2Aa^4b - 4Ba^3b^2 + 3Aa^2b^3) \left(\pi \left[\frac{dx+c}{2\pi} + \frac{1}{2} \right] \operatorname{sgn}(-2a+2b) + \arctan \left(-\frac{a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - b \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{\sqrt{-a^2+b^2}} \right) \right)}{(a^2b^4 - b^6) \sqrt{-a^2+b^2}} - \frac{4(Ba^4 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - Aa^3b \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right))}{(a^2b^3 - b^5) \left(a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - b \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) \right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(dx+c)^4*(A+B*sec(dx+c))/(a+b*sec(dx+c))^2,x, algorithm="giac")

[Out] $-1/2*(4*(3B^2a^5 - 2A^2a^4b - 4B^2a^3b^2 + 3A^2a^2b^3)*(pi*floor(1/2*(dx + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^2*b^4 - b^6)*sqrt(-a^2 + b^2)) - 4*(B^2a^4*tan(1/2*d*x + 1/2*c) - A^2a^3*b*tan(1/2*d*x + 1/2*c))/((a^2*b^3 - b^5)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)) - (6B^2a^2 - 4A^2a*b + B^2b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^4 + (6B^2a^2 - 4A^2a*b + B^2b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^4 - 2*(4B^2a*tan(1/2*d*x + 1/2*c)^3 - 2A^2b*tan(1/2*d*x + 1/2*c)^3 + B^2b*tan(1/2*d*x + 1/2*c)^3 - 4B^2a*tan(1/2*d*x + 1/2*c) + 2A^2b*tan(1/2*d*x + 1/2*c) + B^2b*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*b^3))/d$

maple [B] time = 0.57, size = 698, normalized size = 2.57

$$\frac{2a^3 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) A}{d b^2 (a^2 - b^2) \left(a \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right) \right) - \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right) \right) b - a - b \right)} + \frac{2a^4 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) B}{d b^3 (a^2 - b^2) \left(a \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right) \right) - \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right) \right) b - a - b \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(dx+c)^4*(A+B*sec(dx+c))/(a+b*sec(dx+c))^2,x)

[Out] $-2/d*a^3/b^2/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)*A+2/d*a^4/b^3/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)*B$

$$x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)*B+4/d*a^4/b^3/(a-b)/(a+b)/((a-b)*(a+b))^{(1/2)*\operatorname{arctanh}(\tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^{(1/2)})*A-6/d*a^2/b/(a-b)/(a+b)/((a-b)*(a+b))^{(1/2)*\operatorname{arctanh}(\tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^{(1/2)})*A-6/d*a^5/b^4/(a-b)/(a+b)/((a-b)*(a+b))^{(1/2)*\operatorname{arctanh}(\tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^{(1/2)})*B+8/d*a^3/b^2/(a-b)/(a+b)/((a-b)*(a+b))^{(1/2)*\operatorname{arctanh}(\tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^{(1/2)})*B+1/2*d*B/b^2/(\tan(1/2*d*x+1/2*c)-1)^2-1/d/b^2/(\tan(1/2*d*x+1/2*c)-1)*A+2/d/b^3/(\tan(1/2*d*x+1/2*c)-1)*a*B+1/2/d/b^2/(\tan(1/2*d*x+1/2*c)-1)*B+2/d/b^3*\ln(\tan(1/2*d*x+1/2*c)-1)*A*a-3/d/b^4*\ln(\tan(1/2*d*x+1/2*c)-1)*a^2*B-1/2/d/b^2*\ln(\tan(1/2*d*x+1/2*c)-1)*B-1/2/d*B/b^2/(\tan(1/2*d*x+1/2*c)+1)^2-1/d/b^2/(\tan(1/2*d*x+1/2*c)+1)*A+2/d/b^3/(\tan(1/2*d*x+1/2*c)+1)*a*B+1/2/d/b^2/(\tan(1/2*d*x+1/2*c)+1)*B-2/d/b^3*\ln(\tan(1/2*d*x+1/2*c)+1)*A*a+3/d/b^4*\ln(\tan(1/2*d*x+1/2*c)+1)*a^2*B+1/2/d/b^2*\ln(\tan(1/2*d*x+1/2*c)+1)*B$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for more details)Is 4*a^2-4*b^2 positive or negative?

mupad [B] time = 11.17, size = 6678, normalized size = 24.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^4*(a + b/cos(c + d*x))^2),x)

[Out] (atan(-((((8*tan(c/2 + (d*x)/2)*(72*B^2*a^10 + B^2*b^10 - 2*B^2*a*b^9 - 72*B^2*a^9*b + 16*A^2*a^2*b^8 - 32*A^2*a^3*b^7 + 20*A^2*a^4*b^6 + 64*A^2*a^5*b^5 - 64*A^2*a^6*b^4 - 32*A^2*a^7*b^3 + 32*A^2*a^8*b^2 + 11*B^2*a^2*b^8 - 20*B^2*a^3*b^7 + 23*B^2*a^4*b^6 - 26*B^2*a^5*b^5 + 17*B^2*a^6*b^4 + 120*B^2*a^7*b^3 - 120*B^2*a^8*b^2 - 8*A*B*a*b^9 - 96*A*B*a^9*b + 16*A*B*a^2*b^8 - 40*A*B*a^3*b^7 + 64*A*B*a^4*b^6 - 40*A*B*a^5*b^5 - 176*A*B*a^6*b^4 + 176*A*B*a^7*b^3 + 96*A*B*a^8*b^2)))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) - (((8*(2*B*b^15 + 12*A*a^2*b^13 + 12*A*a^3*b^12 - 20*A*a^4*b^11 - 4*A*a^5*b^10 + 8*A*a^6*b^9 + 6*B*a^2*b^13 - 16*B*a^3*b^12 - 14*B*a^4*b^11 + 28*B*a^5*b^10 + 6*B*a^6*b^9 - 12*B*a^7*b^8 - 8*A*a*b^14)))/(a*b^11 + b^12 - a^2*b^10 - a^3*b^9) - (4*tan(c/2 + (d*x)/2)*(6*B*a^2 + B*b^2 - 4*A*a*b)*(8*a*b^13 - 8*a^2*b^12 -

$$\begin{aligned}
& \left(4*b^{11} + 28*B*a^5*b^{10} + 6*B*a^6*b^9 - 12*B*a^7*b^8 - 8*A*a*b^{14} \right) / \left(a*b^{11} \right. \\
& \left. + b^{12} - a^2*b^{10} - a^3*b^9 \right) + \left(4*\tan(c/2 + (d*x)/2) * \left(6*B*a^2 + B*b^2 - 4* \right. \right. \\
& \left. \left. A*a*b \right) * \left(8*a*b^{13} - 8*a^2*b^{12} - 16*a^3*b^{11} + 16*a^4*b^{10} + 8*a^5*b^9 - 8*a^6*b^8 \right) \right) / \left(b^4 * \left(a*b^8 + b^9 - a^2*b^7 - a^3*b^6 \right) \right) * \left(6*B*a^2 + B*b^2 - 4*A*a*b \right) / \left(2*b^4 \right) * \left(6*B*a^2 + B*b^2 - 4*A*a*b \right) / \left(2*b^4 \right) * \left(6*B*a^2 + B*b^2 - 4*A*a*b \right) * i / \left(b^4 * d \right) - \left(\left(\tan(c/2 + (d*x)/2) \right)^5 * \left(6*B*a^4 - 2*A*b^4 + B*b^4 + 2*A*a^2*b^2 - 5*B*a^2*b^2 + 2*A*a*b^3 - 4*A*a^3*b + 3*B*a*b^3 - 3*B*a^3*b \right) \right) / \left(\left(a*b^3 - b^4 \right) * \left(a + b \right) \right) + \left(2*\tan(c/2 + (d*x)/2) \right)^3 * \left(B*b^4 - 6*B*a^4 + 3*B*a^2*b^2 - 2*A*a*b^3 + 4*A*a^3*b \right) / \left(b * \left(a*b^2 - b^3 \right) * \left(a + b \right) \right) + \left(\tan(c/2 + (d*x)/2) \right) * \left(2*A*b^4 + 6*B*a^4 + B*b^4 - 2*A*a^2*b^2 - 5*B*a^2*b^2 + 2*A*a*b^3 - 4*A*a^3*b - 3*B*a*b^3 + 3*B*a^3*b \right) / \left(b^3 * \left(a + b \right) * \left(a - b \right) \right) / \left(d * \left(a + b - \tan(c/2 + (d*x)/2) \right)^2 * \left(3*a + b \right) - \tan(c/2 + (d*x)/2) \right)^6 * \left(a - b \right) + \tan(c/2 + (d*x)/2) \right)^4 * \left(3*a - b \right) \right) - \left(a^2 * \operatorname{atan} \left(\left(a^2 * \left(a + b \right)^3 * \left(a - b \right)^3 \right)^{1/2} * \left(8*\tan(c/2 + (d*x)/2) * \left(72*B^2*a^{10} + B^2*b^{10} - 2*B^2*a*b^9 - 72*B^2*a^9*b + 16*A^2*a^2*b^8 - 32*A^2*a^3*b^7 + 20*A^2*a^4*b^6 + 64*A^2*a^5*b^5 - 64*A^2*a^6*b^4 - 32*A^2*a^7*b^3 + 32*A^2*a^8*b^2 + 11*B^2*a^2*b^8 - 20*B^2*a^3*b^7 + 23*B^2*a^4*b^6 - 26*B^2*a^5*b^5 + 17*B^2*a^6*b^4 + 120*B^2*a^7*b^3 - 120*B^2*a^8*b^2 - 8*A*B*a*b^9 - 96*A*B*a^9*b + 16*A*B*a^2*b^8 - 40*A*B*a^3*b^7 + 64*A*B*a^4*b^6 - 40*A*B*a^5*b^5 - 176*A*B*a^6*b^4 + 176*A*B*a^7*b^3 + 96*A*B*a^8*b^2 \right) \right) / \left(a*b^8 + b^9 - a^2*b^7 - a^3*b^6 \right) + \left(a^2 * \left(\left(8 * \left(2*B*b^{15} + 12*A*a^2*b^{13} + 12*A*a^3*b^{12} - 20*A*a^4*b^{11} - 4*A*a^5*b^{10} + 8*A*a^6*b^9 + 6*B*a^2*b^{13} - 16*B*a^3*b^{12} - 14*B*a^4*b^{11} + 28*B*a^5*b^{10} + 6*B*a^6*b^9 - 12*B*a^7*b^8 - 8*A*a*b^{14} \right) \right) / \left(a*b^{11} + b^{12} - a^2*b^{10} - a^3*b^9 \right) + \left(8*a^2 * \tan(c/2 + (d*x)/2) * \left(a + b \right)^3 * \left(a - b \right)^3 \right)^{1/2} * \left(3*A*b^3 + 3*B*a^3 - 2*A*a^2*b - 4*B*a*b^2 \right) * \left(8*a*b^{13} - 8*a^2*b^{12} - 16*a^3*b^{11} + 16*a^4*b^{10} + 8*a^5*b^9 - 8*a^6*b^8 \right) \right) / \left(\left(a*b^8 + b^9 - a^2*b^7 - a^3*b^6 \right) * \left(b^{10} - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4 \right) \right) * \left(a + b \right)^3 * \left(a - b \right)^3 \right)^{1/2} * \left(3*A*b^3 + 3*B*a^3 - 2*A*a^2*b - 4*B*a*b^2 \right) / \left(b^{10} - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4 \right) * \left(3*A*b^3 + 3*B*a^3 - 2*A*a^2*b - 4*B*a*b^2 \right) * i / \left(b^{10} - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4 \right) + \left(a^2 * \left(a + b \right)^3 * \left(a - b \right)^3 \right)^{1/2} * \left(8*\tan(c/2 + (d*x)/2) * \left(72*B^2*a^{10} + B^2*b^{10} - 2*B^2*a*b^9 - 72*B^2*a^9*b + 16*A^2*a^2*b^8 - 32*A^2*a^3*b^7 + 20*A^2*a^4*b^6 + 64*A^2*a^5*b^5 - 64*A^2*a^6*b^4 - 32*A^2*a^7*b^3 + 32*A^2*a^8*b^2 + 11*B^2*a^2*b^8 - 20*B^2*a^3*b^7 + 23*B^2*a^4*b^6 - 26*B^2*a^5*b^5 + 17*B^2*a^6*b^4 + 120*B^2*a^7*b^3 - 120*B^2*a^8*b^2 - 8*A*B*a*b^9 - 96*A*B*a^9*b + 16*A*B*a^2*b^8 - 40*A*B*a^3*b^7 + 64*A*B*a^4*b^6 - 40*A*B*a^5*b^5 - 176*A*B*a^6*b^4 + 176*A*B*a^7*b^3 + 96*A*B*a^8*b^2 \right) \right) / \left(a*b^8 + b^9 - a^2*b^7 - a^3*b^6 \right) - \left(a^2 * \left(\left(8 * \left(2*B*b^{15} + 12*A*a^2*b^{13} + 12*A*a^3*b^{12} - 20*A*a^4*b^{11} - 4*A*a^5*b^{10} + 8*A*a^6*b^9 + 6*B*a^2*b^{13} - 16*B*a^3*b^{12} - 14*B*a^4*b^{11} + 28*B*a^5*b^{10} + 6*B*a^6*b^9 - 12*B*a^7*b^8 - 8*A*a*b^{14} \right) \right) / \left(a*b^{11} + b^{12} - a^2*b^{10} - a^3*b^9 \right) - \left(8*a^2 * \tan(c/2 + (d*x)/2) * \left(a + b \right)^3 * \left(a - b \right)^3 \right)^{1/2} * \left(3*A*b^3 + 3*B*a^3 - 2*A*a^2*b - 4*B*a*b^2 \right) * \left(8*a*b^{13} - 8*a^2*b^{12} - 16*a^3*b^{11} + 16*a^4*b^{10} + 8*a^5*b^9 - 8*a^6*b^8 \right) \right) / \left(\left(a*b^8 + b^9 - a^2*b^7 - a^3*b^6 \right) * \left(b^{10} - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4 \right) \right) * \left(a + b \right)^3 * \left(a - b \right)^3 \right)^{1/2} * \left(3*A*b^3 + 3*B*a^3 - 2*A*a^2*b - 4*B*a*b^2 \right) / \left(b^{10} - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4 \right) * \left(3*A*b^3 + 3*B*a^3 - 2*A*a^2*b - 4*B*a*b^2 \right) * i / \left(b^{10} - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4 \right) \right)
\end{aligned}$$

$$\begin{aligned}
&^8 + 3a^4b^6 - a^6b^4)) / ((16(108B^3a^{11} - 54B^3a^{10}b - 48A^3a^4b^7 - 24A^3a^5b^6 + 80A^3a^6b^5 + 16A^3a^7b^4 - 32A^3a^8b^3 + 4 \\
&*B^3a^3b^8 - 4B^3a^4b^7 + 41B^3a^5b^6 - 9B^3a^6b^5 + 63B^3a^7b^4 + 81B^3a^8b^3 - 216B^3a^9b^2 - 216AB^2a^{10}b - 3AB^2a^2b^9 \\
&+ 3AB^2a^3b^8 - 63AB^2a^4b^7 + 15AB^2a^5b^6 - 186AB^2a^6b^5 - 162AB^2a^7b^4 + 468AB^2a^8b^3 + 108AB^2a^9b^2 + 24A^2Ba^3b^8 - 6A^2Ba^4b^7 + 168A^2Ba^5b^6 + 108A^2Ba^6b^5 - 336A^2Ba \\
&a^7b^4 - 72A^2Ba^8b^3 + 144A^2Ba^9b^2)) / (ab^{11} + b^{12} - a^2b^{10} \\
&- a^3b^9) + (a^2((a + b)^3(a - b)^3)^{(1/2)} * ((8 \tan(c/2 + (dx)/2)) * (72B \\
&^2a^{10} + B^2b^{10} - 2B^2a^2b^9 - 72B^2a^9b + 16A^2a^2b^8 - 32A^2a^3b^7 + 20A^2a^4b^6 + 64A^2a^5b^5 - 64A^2a^6b^4 - 32A^2a^7b^3 \\
&+ 32A^2a^8b^2 + 11B^2a^2b^8 - 20B^2a^3b^7 + 23B^2a^4b^6 - 26B^2a^5b^5 + 17B^2a^6b^4 + 120B^2a^7b^3 - 120B^2a^8b^2 - 8ABa^2b^9 - 96ABa^9b + 16ABa^2b^8 - 40ABa^3b^7 + 64ABa^4b^6 - 40AA \\
&Ba^5b^5 - 176ABa^6b^4 + 176ABa^7b^3 + 96ABa^8b^2)) / (ab^8 + b \\
&^9 - a^2b^7 - a^3b^6) + (a^2((8 * (2Bb^{15} + 12Aa^2b^{13} + 12Aa^3b^{12} \\
&- 20Aa^4b^{11} - 4Aa^5b^{10} + 8Aa^6b^9 + 6Ba^2b^{13} - 16Ba^3b^{12} \\
&- 14Ba^4b^{11} + 28Ba^5b^{10} + 6Ba^6b^9 - 12Ba^7b^8 - 8Aa^2b^{14}))) / (ab^{11} + b^{12} - a^2b^{10} - a^3b^9) + (8a^2 \tan(c/2 + (dx)/2) * ((a + \\
&b)^3(a - b)^3)^{(1/2)} * (3A^2b^3 + 3B^2a^3 - 2A^2a^2b - 4B^2a^2b^2) * (8a^2b^{13} \\
&- 8a^2b^{12} - 16a^3b^{11} + 16a^4b^{10} + 8a^5b^9 - 8a^6b^8)) / ((ab^8 \\
&+ b^9 - a^2b^7 - a^3b^6) * (b^{10} - 3a^2b^8 + 3a^4b^6 - a^6b^4)) * ((a \\
&+ b)^3(a - b)^3)^{(1/2)} * (3A^2b^3 + 3B^2a^3 - 2A^2a^2b - 4B^2a^2b^2)) / (b^{10} \\
&- 3a^2b^8 + 3a^4b^6 - a^6b^4) * (3A^2b^3 + 3B^2a^3 - 2A^2a^2b - 4B^2a^2b^2)) / (b^{10} - 3a^2b^8 + 3a^4b^6 - a^6b^4) - (a^2((a + b)^3(a - b)^3) \\
&^{(1/2)} * ((8 \tan(c/2 + (dx)/2)) * (72B^2a^{10} + B^2b^{10} - 2B^2a^2b^9 - 72B^2a^9b + 16A^2a^2b^8 - 32A^2a^3b^7 + 20A^2a^4b^6 + 64A^2a^5b^5 \\
&- 64A^2a^6b^4 - 32A^2a^7b^3 + 32A^2a^8b^2 + 11B^2a^2b^8 - 20B^2a^3b^7 + 23B^2a^4b^6 - 26B^2a^5b^5 + 17B^2a^6b^4 + 120B^2a^7b^3 - 120B^2a^8b^2 - 8ABa^2b^9 - 96ABa^9b + 16ABa^2b^8 - 40AA \\
&Ba^3b^7 + 64ABa^4b^6 - 40ABa^5b^5 - 176ABa^6b^4 + 176ABa^7b^3 + 96ABa^8b^2)) / (ab^8 + b^9 - a^2b^7 - a^3b^6) - (a^2((8 * (2Bb \\
&b^{15} + 12Aa^2b^{13} + 12Aa^3b^{12} - 20Aa^4b^{11} - 4Aa^5b^{10} + 8Aa^6b^9 + 6Ba^2b^{13} - 16Ba^3b^{12} - 14Ba^4b^{11} + 28Ba^5b^{10} + 6Ba^6b^9 - 12Ba^7b^8 - 8Aa^2b^{14}))) / (ab^{11} + b^{12} - a^2b^{10} - a^3b^9) \\
&- (8a^2 \tan(c/2 + (dx)/2) * ((a + b)^3(a - b)^3)^{(1/2)} * (3A^2b^3 + 3B^2a^3 \\
&- 2A^2a^2b - 4B^2a^2b^2) * (8a^2b^{13} - 8a^2b^{12} - 16a^3b^{11} + 16a^4b^{10} \\
&+ 8a^5b^9 - 8a^6b^8)) / ((ab^8 + b^9 - a^2b^7 - a^3b^6) * (b^{10} - 3a^2b^8 + 3a^4b^6 - a^6b^4)) * ((a + b)^3(a - b)^3)^{(1/2)} * (3A^2b^3 + 3B^2a^3 \\
&- 2A^2a^2b - 4B^2a^2b^2)) / (b^{10} - 3a^2b^8 + 3a^4b^6 - a^6b^4) * (3A^2b^3 + 3B^2a^3 - 2A^2a^2b - 4B^2a^2b^2)) / (b^{10} - 3a^2b^8 + 3a^4b^6 - a^6b^4)) * ((a + b)^3(a - b)^3)^{(1/2)} * (3A^2b^3 + 3B^2a^3 - 2A^2a^2b - 4B^2a^2b^2) * 2i) / (d * (b^{10} - 3a^2b^8 + 3a^4b^6 - a^6b^4))
\end{aligned}$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \sec^4(c + dx)}{(a + b \sec(c + dx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**4*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**2,x)

[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**4/(a + b*sec(c + d*x))**2, x)

$$3.321 \quad \int \frac{\sec^3(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^2} dx$$

Optimal. Leaf size=164

$$\frac{a^2(Ab - aB) \tan(c + dx)}{b^2d(a^2 - b^2)(a + b \sec(c + dx))} - \frac{2a(-2a^3B + a^2Ab + 3ab^2B - 2Ab^3) \tanh^{-1}\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d(a-b)^{3/2}(a+b)^{3/2}} + \frac{(Ab - 2aB) \tan(c + dx)}{b^2d(a^2 - b^2)(a + b \sec(c + dx))}$$

[Out] (A*b-2*B*a)*arctanh(sin(d*x+c))/b^3/d-2*a*(A*a^2*b-2*A*b^3-2*B*a^3+3*B*a*b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(3/2)/b^3/(a+b)^(3/2)/d+B*tan(d*x+c)/b^2/d-a^2*(A*b-B*a)*tan(d*x+c)/b^2/(a^2-b^2)/d/(a+b*sec(d*x+c))

Rubi [A] time = 0.58, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {4028, 4082, 3998, 3770, 3831, 2659, 208}

$$\frac{2a(a^2Ab - 2a^3B + 3ab^2B - 2Ab^3) \tanh^{-1}\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d(a-b)^{3/2}(a+b)^{3/2}} - \frac{a^2(Ab - aB) \tan(c + dx)}{b^2d(a^2 - b^2)(a + b \sec(c + dx))} + \frac{(Ab - 2aB) \tan(c + dx)}{b^2d(a^2 - b^2)(a + b \sec(c + dx))}$$

Antiderivative was successfully verified.

[In] Int[(Sec[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^2,x]

[Out] ((A*b - 2*a*B)*ArcTanh[Sin[c + d*x]]/(b^3*d) - (2*a*(a^2*A*b - 2*A*b^3 - 2*a^3*B + 3*a*b^2*B)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(3/2)*b^3*(a + b)^(3/2)*d) + (B*Tan[c + d*x]/(b^2*d) - (a^2*(A*b - a*B)*Tan[c + d*x])/(b^2*(a^2 - b^2)*d*(a + b*Sec[c + d*x])))

Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 2659

Int[((a_) + (b_)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x]
  /; FreeQ[{c, d}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] := Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] := Dist[B/b, Int[Csc[e + f*x], x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 4028

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] := -Simp[(a^2*(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[a*b*(A*b - a*B)*(m + 1) - (A*b - a*B)*(a^2 + b^2*(m + 1))*Csc[e + f*x] + b*B*(m + 1)*(a^2 - b^2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] := -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^3(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^2} dx &= -\frac{a^2(Ab-aB)\tan(c+dx)}{b^2(a^2-b^2)d(a+b\sec(c+dx))} - \frac{\int \frac{\sec(c+dx)(-ab(Ab-aB)-(a^2-b^2)(Ab-aB))}{a+b\sec(c+dx)}}{b^2(a^2-b^2)} \\
&= \frac{B\tan(c+dx)}{b^2d} - \frac{a^2(Ab-aB)\tan(c+dx)}{b^2(a^2-b^2)d(a+b\sec(c+dx))} - \frac{\int \frac{\sec(c+dx)(-ab^2(Ab-aB))}{a+b\sec(c+dx)}}{b^2(a^2-b^2)} \\
&= \frac{B\tan(c+dx)}{b^2d} - \frac{a^2(Ab-aB)\tan(c+dx)}{b^2(a^2-b^2)d(a+b\sec(c+dx))} + \frac{(Ab-2aB)\int \sec(c+dx)}{b^3} \\
&= \frac{(Ab-2aB)\tanh^{-1}(\sin(c+dx))}{b^3d} + \frac{B\tan(c+dx)}{b^2d} - \frac{a^2(Ab-aB)\tan(c+dx)}{b^2(a^2-b^2)d(a+b\sec(c+dx))} \\
&= \frac{(Ab-2aB)\tanh^{-1}(\sin(c+dx))}{b^3d} + \frac{B\tan(c+dx)}{b^2d} - \frac{a^2(Ab-aB)\tan(c+dx)}{b^2(a^2-b^2)d(a+b\sec(c+dx))} \\
&= \frac{(Ab-2aB)\tanh^{-1}(\sin(c+dx))}{b^3d} - \frac{2a(a^2Ab-2Ab^3-2a^3B+3ab^2B)}{(a-b)^{3/2}b^3(a+b)}
\end{aligned}$$

Mathematica [A] time = 2.23, size = 240, normalized size = 1.46

$$\frac{a^2b(aB-Ab)\sin(c+dx)}{(a-b)(a+b)(a\cos(c+dx)+b)} - \frac{2a(2a^3B-a^2Ab-3ab^2B+2Ab^3)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}} + 2aB\log\left(\cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+dx)\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^2,x]

[Out] ((-2*a*(-(a^2*A*b) + 2*A*b^3 + 2*a^3*B - 3*a*b^2*B)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2) - A*b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 2*a*B*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + A*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 2*a*B*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (a^2*b*(-(A*b) + a*B)*Sin[c + d*x])/((a - b)*(a + b)*(b + a*Cos[c + d*x])) + b*B*Tan[c + d*x]/(b^3*d)

fricas [B] time = 13.16, size = 1114, normalized size = 6.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm="fricas")

[Out] [1/2*(((2*B*a^5 - A*a^4*b - 3*B*a^3*b^2 + 2*A*a^2*b^3)*cos(d*x + c)^2 + (2*B*a^4*b - A*a^3*b^2 - 3*B*a^2*b^3 + 2*A*a*b^4)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2))*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - ((2*B*a^6 - A*a^5*b - 4*B*a^4*b^2 + 2*A*a^3*b^3 + 2*B*a^2*b^4 - A*a*b^5)*cos(d*x + c)^2 + (2*B*a^5*b - A*a^4*b^2 - 4*B*a^3*b^3 + 2*A*a^2*b^4 + 2*B*a*b^5 - A*b^6)*cos(d*x + c))*log(sin(d*x + c) + 1) + ((2*B*a^6 - A*a^5*b - 4*B*a^4*b^2 + 2*A*a^3*b^3 + 2*B*a^2*b^4 - A*a*b^5)*cos(d*x + c)^2 + (2*B*a^5*b - A*a^4*b^2 - 4*B*a^3*b^3 + 2*A*a^2*b^4 + 2*B*a*b^5 - A*b^6)*cos(d*x + c))*log(-sin(d*x + c) + 1) + 2*(B*a^4*b^2 - 2*B*a^2*b^4 + B*b^6 + (2*B*a^5*b - A*a^4*b^2 - 3*B*a^3*b^3 + A*a^2*b^4 + B*a*b^5)*cos(d*x + c))*sin(d*x + c)/((a^5*b^3 - 2*a^3*b^5 + a*b^7)*d*cos(d*x + c)^2 + (a^4*b^4 - 2*a^2*b^6 + b^8)*d*cos(d*x + c)), 1/2*(2*((2*B*a^5 - A*a^4*b - 3*B*a^3*b^2 + 2*A*a^2*b^3)*cos(d*x + c)^2 + (2*B*a^4*b - A*a^3*b^2 - 3*B*a^2*b^3 + 2*A*a*b^4)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - ((2*B*a^6 - A*a^5*b - 4*B*a^4*b^2 + 2*A*a^3*b^3 + 2*B*a^2*b^4 - A*a*b^5)*cos(d*x + c)^2 + (2*B*a^5*b - A*a^4*b^2 - 4*B*a^3*b^3 + 2*A*a^2*b^4 + 2*B*a*b^5 - A*b^6)*cos(d*x + c))*log(sin(d*x + c) + 1) + ((2*B*a^6 - A*a^5*b - 4*B*a^4*b^2 + 2*A*a^3*b^3 + 2*B*a^2*b^4 - A*a*b^5)*cos(d*x + c)^2 + (2*B*a^5*b - A*a^4*b^2 - 4*B*a^3*b^3 + 2*A*a^2*b^4 + 2*B*a*b^5 - A*b^6)*cos(d*x + c))*log(-sin(d*x + c) + 1) + 2*(B*a^4*b^2 - 2*B*a^2*b^4 + B*b^6 + (2*B*a^5*b - A*a^4*b^2 - 3*B*a^3*b^3 + A*a^2*b^4 + B*a*b^5)*cos(d*x + c))*sin(d*x + c)/((a^5*b^3 - 2*a^3*b^5 + a*b^7)*d*cos(d*x + c)^2 + (a^4*b^4 - 2*a^2*b^6 + b^8)*d*cos(d*x + c))]

giac [B] time = 0.36, size = 404, normalized size = 2.46

$$\frac{2(2Ba^4 - Aa^3b - 3Ba^2b^2 + 2Aab^3) \left(\pi \left[\frac{dx+c}{2\pi} + \frac{1}{2} \right] \operatorname{sgn}(-2a+2b) + \arctan \left(-\frac{a \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - b \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)}{\sqrt{-a^2+b^2}} \right) \right)}{(a^2b^3 - b^5) \sqrt{-a^2+b^2}} - \frac{2 \left(2Ba^3 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 - Aa^2b \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) \right)}{\sqrt{-a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm="giac")

[Out] (2*(2*B*a^4 - A*a^3*b - 3*B*a^2*b^2 + 2*A*a*b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^2*b^3 - b^5)*sqrt(-a^2 + b^2)) - 2*(2*B*a^3

$$\begin{aligned} & * \tan\left(\frac{1}{2}d*x + \frac{1}{2}c\right)^3 - A*a^2*b*\tan\left(\frac{1}{2}d*x + \frac{1}{2}c\right)^3 - B*a^2*b*\tan\left(\frac{1}{2}d*x + \frac{1}{2}c\right)^3 - B*a*b^2*\tan\left(\frac{1}{2}d*x + \frac{1}{2}c\right)^3 + B*b^3*\tan\left(\frac{1}{2}d*x + \frac{1}{2}c\right)^3 - 2*B*a^3*\tan\left(\frac{1}{2}d*x + \frac{1}{2}c\right) + A*a^2*b*\tan\left(\frac{1}{2}d*x + \frac{1}{2}c\right) - B*a^2*b*\tan\left(\frac{1}{2}d*x + \frac{1}{2}c\right) + B*a*b^2*\tan\left(\frac{1}{2}d*x + \frac{1}{2}c\right) + B*b^3*\tan\left(\frac{1}{2}d*x + \frac{1}{2}c\right) \\ &) / ((a*\tan\left(\frac{1}{2}d*x + \frac{1}{2}c\right)^4 - b*\tan\left(\frac{1}{2}d*x + \frac{1}{2}c\right)^4 - 2*a*\tan\left(\frac{1}{2}d*x + \frac{1}{2}c\right)^2 + a + b)*(a^2*b^2 - b^4)) - (2*B*a - A*b)*\log(\text{abs}(\tan\left(\frac{1}{2}d*x + \frac{1}{2}c\right) + 1)) / b^3 + (2*B*a - A*b)*\log(\text{abs}(\tan\left(\frac{1}{2}d*x + \frac{1}{2}c\right) - 1)) / b^3) / d \end{aligned}$$

maple [B] time = 0.66, size = 510, normalized size = 3.11

$$\frac{2a^2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) A \quad 2a^3 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) B}{db(a^2 - b^2) \left(a \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right) \right) - \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right) \right) b - a - b \right) - db^2(a^2 - b^2) \left(a \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right) \right) - \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right) \right) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x)

[Out] $2/d*a^2/b/(a^2-b^2)*\tan(1/2*d*x+1/2*c)/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)*A-2/d*a^3/b^2/(a^2-b^2)*\tan(1/2*d*x+1/2*c)/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)*B-2/d*a^3/b^2/(a-b)/(a+b)/((a-b)*(a+b))^{1/2}*\text{arctanh}(\tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^{1/2})*A+4/d*a/(a-b)/(a+b)/((a-b)*(a+b))^{1/2}*\text{arctanh}(\tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^{1/2})*A+4/d*a^4/b^3/(a-b)/(a+b)/((a-b)*(a+b))^{1/2}*\text{arctanh}(\tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^{1/2})*B-6/d*a^2/b/(a-b)/(a+b)/((a-b)*(a+b))^{1/2}*\text{arctanh}(\tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^{1/2})*B-1/d/b^2/(\tan(1/2*d*x+1/2*c)-1)*B-1/d/b^2*\ln(\tan(1/2*d*x+1/2*c)-1)*A+2/d/b^3*\ln(\tan(1/2*d*x+1/2*c)-1)*a*B-1/d/b^2/(\tan(1/2*d*x+1/2*c)+1)*B+1/d/b^2*\ln(\tan(1/2*d*x+1/2*c)+1)*A-2/d/b^3*\ln(\tan(1/2*d*x+1/2*c)+1)*a*B$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for more details) Is 4*a^2-4*b^2 positive or negative?

mupad [B] time = 10.26, size = 5436, normalized size = 33.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((A + B/\cos(c + dx))/(\cos(c + dx)^3*(a + b/\cos(c + dx))^2), x)$

[Out]
$$\frac{((2*\tan(c/2 + (dx)/2)^3*(A*a^2*b - B*b^3 - 2*B*a^3 + B*a*b^2 + B*a^2*b))/((b^2*(a + b)*(a - b)) - (2*\tan(c/2 + (dx)/2)*(B*b^3 - 2*B*a^3 + A*a^2*b + B*a*b^2 - B*a^2*b)))/(b^2*(a + b)*(a - b)))/(d*(a + b + \tan(c/2 + (dx)/2)^4*(a - b) - 2*a*\tan(c/2 + (dx)/2)^2)) + (\text{atan}(\frac{((32*\tan(c/2 + (dx)/2)*(A^2*b^8 + 8*B^2*a^8 - 2*A^2*a*b^7 - 8*B^2*a^7*b + 3*A^2*a^2*b^6 + 4*A^2*a^3*b^5 - 5*A^2*a^4*b^4 - 2*A^2*a^5*b^3 + 2*A^2*a^6*b^2 + 4*B^2*a^2*b^6 - 8*B^2*a^3*b^5 + 5*B^2*a^4*b^4 + 16*B^2*a^5*b^3 - 16*B^2*a^6*b^2 - 4*A*B*a*b^7 - 8*A*B*a^7*b + 8*A*B*a^2*b^6 - 8*A*B*a^3*b^5 - 16*A*B*a^4*b^4 + 18*A*B*a^5*b^3 + 8*A*B*a^6*b^2))}{(a*b^6 + b^7 - a^2*b^5 - a^3*b^4)} + \frac{((32*(A*a^2*b^{10} - A*b^{12} - 3*A*a^3*b^9 + A*a^5*b^7 - 3*B*a^2*b^{10} - 3*B*a^3*b^9 + 5*B*a^4*b^8 + B*a^5*b^7 - 2*B*a^6*b^6 + 2*A*a*b^{11} + 2*B*a*b^{11}))}{(a*b^8 + b^9 - a^2*b^7 - a^3*b^6)} + (32*\tan(c/2 + (dx)/2)*(A*b - 2*B*a)*(2*a*b^{11} - 2*a^2*b^{10} - 4*a^3*b^9 + 4*a^4*b^8 + 2*a^5*b^7 - 2*a^6*b^6)))/(b^3*(a*b^6 + b^7 - a^2*b^5 - a^3*b^4)))*(A*b - 2*B*a))/b^3)*(A*b - 2*B*a)*1i)/b^3 + \frac{((32*\tan(c/2 + (dx)/2)*(A^2*b^8 + 8*B^2*a^8 - 2*A^2*a*b^7 - 8*B^2*a^7*b + 3*A^2*a^2*b^6 + 4*A^2*a^3*b^5 - 5*A^2*a^4*b^4 - 2*A^2*a^5*b^3 + 2*A^2*a^6*b^2 + 4*B^2*a^2*b^6 - 8*B^2*a^3*b^5 + 5*B^2*a^4*b^4 + 16*B^2*a^5*b^3 - 16*B^2*a^6*b^2 - 4*A*B*a*b^7 - 8*A*B*a^7*b + 8*A*B*a^2*b^6 - 8*A*B*a^3*b^5 - 16*A*B*a^4*b^4 + 18*A*B*a^5*b^3 + 8*A*B*a^6*b^2))}{(a*b^6 + b^7 - a^2*b^5 - a^3*b^4)} - \frac{((32*(A*a^2*b^{10} - A*b^{12} - 3*A*a^3*b^9 + A*a^5*b^7 - 3*B*a^2*b^{10} - 3*B*a^3*b^9 + 5*B*a^4*b^8 + B*a^5*b^7 - 2*B*a^6*b^6 + 2*A*a*b^{11} + 2*B*a*b^{11}))}{(a*b^8 + b^9 - a^2*b^7 - a^3*b^6)} - (32*\tan(c/2 + (dx)/2)*(A*b - 2*B*a)*(2*a*b^{11} - 2*a^2*b^{10} - 4*a^3*b^9 + 4*a^4*b^8 + 2*a^5*b^7 - 2*a^6*b^6)))/(b^3*(a*b^6 + b^7 - a^2*b^5 - a^3*b^4)))*(A*b - 2*B*a))/b^3)*(A*b - 2*B*a)*1i)/b^3)/((64*(8*B^3*a^8 - 2*A^3*a*b^7 - 4*B^3*a^7*b - 2*A^3*a^2*b^6 + 3*A^3*a^3*b^5 + A^3*a^4*b^4 - A^3*a^5*b^3 + 12*B^3*a^4*b^4 + 6*B^3*a^5*b^3 - 20*B^3*a^6*b^2 - 12*A*B^2*a^7*b - 20*A*B^2*a^3*b^5 - 13*A*B^2*a^4*b^4 + 32*A*B^2*a^5*b^3 + 8*A*B^2*a^6*b^2 + 11*A^2*B*a^2*b^6 + 9*A^2*B*a^3*b^5 - 17*A^2*B*a^4*b^4 - 5*A^2*B*a^5*b^3 + 6*A^2*B*a^6*b^2)))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6)} + \frac{((32*\tan(c/2 + (dx)/2)*(A^2*b^8 + 8*B^2*a^8 - 2*A^2*a*b^7 - 8*B^2*a^7*b + 3*A^2*a^2*b^6 + 4*A^2*a^3*b^5 - 5*A^2*a^4*b^4 - 2*A^2*a^5*b^3 + 2*A^2*a^6*b^2 + 4*B^2*a^2*b^6 - 8*B^2*a^3*b^5 + 5*B^2*a^4*b^4 + 16*B^2*a^5*b^3 - 16*B^2*a^6*b^2 - 4*A*B*a*b^7 - 8*A*B*a^7*b + 8*A*B*a^2*b^6 - 8*A*B*a^3*b^5 - 16*A*B*a^4*b^4 + 18*A*B*a^5*b^3 + 8*A*B*a^6*b^2))}{(a*b^6 + b^7 - a^2*b^5 - a^3*b^4)} + \frac{((32*(A*a^2*b^{10} - A*b^{12} - 3*A*a^3*b^9 + A*a^5*b^7 - 3*B*a^2*b^{10} - 3*B*a^3*b^9 + 5*B*a^4*b^8 + B*a^5*b^7 - 2*B*a^6*b^6 + 2*A*a*b^{11} + 2*B*a*b^{11}))}{(a*b^8 + b^9 - a^2*b^7 - a^3*b^6)} + (32*\tan(c/2 + (dx)/2)*(A*b -$$

$$\begin{aligned}
& 2*B*a)*(2*a*b^{11} - 2*a^2*b^{10} - 4*a^3*b^9 + 4*a^4*b^8 + 2*a^5*b^7 - 2*a^6*b^6))/b^3*(a*b^6 + b^7 - a^2*b^5 - a^3*b^4))*((32*\tan(c/2 + (d*x)/2)*(A^2*b^8 + 8*B^2*a^8 - 2*A^2*a*b^7 - 8*B^2*a^7*b + 3*A^2*a^2*b^6 + 4*A^2*a^3*b^5 - 5*A^2*a^4*b^4 - 2*A^2*a^5*b^3 + 2*A^2*a^6*b^2 + 4*B^2*a^2*b^6 - 8*B^2*a^3*b^5 + 5*B^2*a^4*b^4 + 16*B^2*a^5*b^3 - 16*B^2*a^6*b^2 - 4*A*B*a*b^7 - 8*A*B*a^7*b + 8*A*B*a^2*b^6 - 8*A*B*a^3*b^5 - 16*A*B*a^4*b^4 + 18*A*B*a^5*b^3 + 8*A*B*a^6*b^2))/(a*b^6 + b^7 - a^2*b^5 - a^3*b^4) - (((32*(A*a^2*b^10 - A*b^12 - 3*A*a^3*b^9 + A*a^5*b^7 - 3*B*a^2*b^10 - 3*B*a^3*b^9 + 5*B*a^4*b^8 + B*a^5*b^7 - 2*B*a^6*b^6 + 2*A*a*b^11 + 2*B*a*b^11))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) - (32*\tan(c/2 + (d*x)/2)*(A*b - 2*B*a)*(2*a*b^{11} - 2*a^2*b^{10} - 4*a^3*b^9 + 4*a^4*b^8 + 2*a^5*b^7 - 2*a^6*b^6))/b^3*(a*b - 2*B*a))/b^3))*(A*b - 2*B*a)*2i)/(b^3*d) + (a*atan(((a*((32*\tan(c/2 + (d*x)/2)*(A^2*b^8 + 8*B^2*a^8 - 2*A^2*a*b^7 - 8*B^2*a^7*b + 3*A^2*a^2*b^6 + 4*A^2*a^3*b^5 - 5*A^2*a^4*b^4 - 2*A^2*a^5*b^3 + 2*A^2*a^6*b^2 + 4*B^2*a^2*b^6 - 8*B^2*a^3*b^5 + 5*B^2*a^4*b^4 + 16*B^2*a^5*b^3 - 16*B^2*a^6*b^2 - 4*A*B*a*b^7 - 8*A*B*a^7*b + 8*A*B*a^2*b^6 - 8*A*B*a^3*b^5 - 16*A*B*a^4*b^4 + 18*A*B*a^5*b^3 + 8*A*B*a^6*b^2))/(a*b^6 + b^7 - a^2*b^5 - a^3*b^4) + (a*((32*(A*a^2*b^10 - A*b^12 - 3*A*a^3*b^9 + A*a^5*b^7 - 3*B*a^2*b^10 - 3*B*a^3*b^9 + 5*B*a^4*b^8 + B*a^5*b^7 - 2*B*a^6*b^6 + 2*A*a*b^11 + 2*B*a*b^11)))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) + (32*a*tan(c/2 + (d*x)/2)*((a + b)^3*(a - b)^3)^(1/2)*(2*A*b^3 + 2*B*a^3 - A*a^2*b - 3*B*a*b^2)*(2*a*b^{11} - 2*a^2*b^{10} - 4*a^3*b^9 + 4*a^4*b^8 + 2*a^5*b^7 - 2*a^6*b^6))/((a*b^6 + b^7 - a^2*b^5 - a^3*b^4)*(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3)))*((a + b)^3*(a - b)^3)^(1/2)*(2*A*b^3 + 2*B*a^3 - A*a^2*b - 3*B*a*b^2))/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3))*((a + b)^3*(a - b)^3)^(1/2)*(2*A*b^3 + 2*B*a^3 - A*a^2*b - 3*B*a*b^2)*1i)/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3) + (a*((32*\tan(c/2 + (d*x)/2)*(A^2*b^8 + 8*B^2*a^8 - 2*A^2*a*b^7 - 8*B^2*a^7*b + 3*A^2*a^2*b^6 + 4*A^2*a^3*b^5 - 5*A^2*a^4*b^4 - 2*A^2*a^5*b^3 + 2*A^2*a^6*b^2 + 4*B^2*a^2*b^6 - 8*B^2*a^3*b^5 + 5*B^2*a^4*b^4 + 16*B^2*a^5*b^3 - 16*B^2*a^6*b^2 - 4*A*B*a*b^7 - 8*A*B*a^7*b + 8*A*B*a^2*b^6 - 8*A*B*a^3*b^5 - 16*A*B*a^4*b^4 + 18*A*B*a^5*b^3 + 8*A*B*a^6*b^2))/(a*b^6 + b^7 - a^2*b^5 - a^3*b^4) - (a*((32*(A*a^2*b^10 - A*b^12 - 3*A*a^3*b^9 + A*a^5*b^7 - 3*B*a^2*b^10 - 3*B*a^3*b^9 + 5*B*a^4*b^8 + B*a^5*b^7 - 2*B*a^6*b^6 + 2*A*a*b^11 + 2*B*a*b^11)))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) - (32*a*tan(c/2 + (d*x)/2)*((a + b)^3*(a - b)^3)^(1/2)*(2*A*b^3 + 2*B*a^3 - A*a^2*b - 3*B*a*b^2)*(2*a*b^{11} - 2*a^2*b^{10} - 4*a^3*b^9 + 4*a^4*b^8 + 2*a^5*b^7 - 2*a^6*b^6))/((a*b^6 + b^7 - a^2*b^5 - a^3*b^4)*(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3)))*((a + b)^3*(a - b)^3)^(1/2)*(2*A*b^3 + 2*B*a^3 - A*a^2*b - 3*B*a*b^2))/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3))*((a + b)^3*(a - b)^3)^(1/2)*(2*A*b^3 + 2*B*a^3 - A*a^2*b - 3*B*a*b^2)*1i)/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3))/((64*(8*B^3*a^8 - 2*A^3*a*b^7 - 4*B^3*a^7*b - 2*A^3*a^2*b^6 + 3*A^3*a^3*b^5 + A^3*a^4*b^4 - A^3*a^5*b^3 + 12*B^3*a^4*b^4 + 6*B^3*a^5*b^3 - 20*B^3*a^6*b^2 - 12*A*B^2*a^7*b - 20*A*B^2*a^3*b^5 - 13*A*B^2*a^4*b^4 + 32*A*B^2*a^5*b^3 + 8*A*B^2*a^6*b^2 + 11*A^2*B*a^2*b^6 + 9*A^2*B*a^3*b^5 - 17*A^2*B*a^4*b^4 - 5*A^2*B*a^5*b^3
\end{aligned}$$

$$\begin{aligned}
& + 6*A^2*B*a^6*b^2)/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) + (a*((32*\tan(c/2 + (d*x)/2)*(A^2*b^8 + 8*B^2*a^8 - 2*A^2*a*b^7 - 8*B^2*a^7*b + 3*A^2*a^2*b^6 + 4*A^2*a^3*b^5 - 5*A^2*a^4*b^4 - 2*A^2*a^5*b^3 + 2*A^2*a^6*b^2 + 4*B^2*a^2*b^6 - 8*B^2*a^3*b^5 + 5*B^2*a^4*b^4 + 16*B^2*a^5*b^3 - 16*B^2*a^6*b^2 - 4*A*B*a*b^7 - 8*A*B*a^7*b + 8*A*B*a^2*b^6 - 8*A*B*a^3*b^5 - 16*A*B*a^4*b^4 + 18*A*B*a^5*b^3 + 8*A*B*a^6*b^2)))/(a*b^6 + b^7 - a^2*b^5 - a^3*b^4) + (a*((32*(A*a^2*b^10 - A*b^12 - 3*A*a^3*b^9 + A*a^5*b^7 - 3*B*a^2*b^10 - 3*B*a^3*b^9 + 5*B*a^4*b^8 + B*a^5*b^7 - 2*B*a^6*b^6 + 2*A*a*b^11 + 2*B*a*b^11)))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) + (32*a*\tan(c/2 + (d*x)/2)*((a + b)^3*(a - b)^3)^(1/2)*(2*A*b^3 + 2*B*a^3 - A*a^2*b - 3*B*a*b^2)*(2*a*b^11 - 2*a^2*b^10 - 4*a^3*b^9 + 4*a^4*b^8 + 2*a^5*b^7 - 2*a^6*b^6)))/((a*b^6 + b^7 - a^2*b^5 - a^3*b^4)*(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3)))*((a + b)^3*(a - b)^3)^(1/2)*(2*A*b^3 + 2*B*a^3 - A*a^2*b - 3*B*a*b^2))/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3))*((a + b)^3*(a - b)^3)^(1/2)*(2*A*b^3 + 2*B*a^3 - A*a^2*b - 3*B*a*b^2))/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3) - (a*((32*\tan(c/2 + (d*x)/2)*(A^2*b^8 + 8*B^2*a^8 - 2*A^2*a*b^7 - 8*B^2*a^7*b + 3*A^2*a^2*b^6 + 4*A^2*a^3*b^5 - 5*A^2*a^4*b^4 - 2*A^2*a^5*b^3 + 2*A^2*a^6*b^2 + 4*B^2*a^2*b^6 - 8*B^2*a^3*b^5 + 5*B^2*a^4*b^4 + 16*B^2*a^5*b^3 - 16*B^2*a^6*b^2 - 4*A*B*a*b^7 - 8*A*B*a^7*b + 8*A*B*a^2*b^6 - 8*A*B*a^3*b^5 - 16*A*B*a^4*b^4 + 18*A*B*a^5*b^3 + 8*A*B*a^6*b^2)))/(a*b^6 + b^7 - a^2*b^5 - a^3*b^4) - (a*((32*(A*a^2*b^10 - A*b^12 - 3*A*a^3*b^9 + A*a^5*b^7 - 3*B*a^2*b^10 - 3*B*a^3*b^9 + 5*B*a^4*b^8 + B*a^5*b^7 - 2*B*a^6*b^6 + 2*A*a*b^11 + 2*B*a*b^11)))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) - (32*a*\tan(c/2 + (d*x)/2)*((a + b)^3*(a - b)^3)^(1/2)*(2*A*b^3 + 2*B*a^3 - A*a^2*b - 3*B*a*b^2)*(2*a*b^11 - 2*a^2*b^10 - 4*a^3*b^9 + 4*a^4*b^8 + 2*a^5*b^7 - 2*a^6*b^6)))/((a*b^6 + b^7 - a^2*b^5 - a^3*b^4)*(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3)))*((a + b)^3*(a - b)^3)^(1/2)*(2*A*b^3 + 2*B*a^3 - A*a^2*b - 3*B*a*b^2))/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3))*((a + b)^3*(a - b)^3)^(1/2)*(2*A*b^3 + 2*B*a^3 - A*a^2*b - 3*B*a*b^2))/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3))*((a + b)^3*(a - b)^3)^(1/2)*(2*A*b^3 + 2*B*a^3 - A*a^2*b - 3*B*a*b^2)*2i)/(d*(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3))
\end{aligned}$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \sec^3(c + dx)}{(a + b \sec(c + dx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**2,x)

[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**3/(a + b*sec(c + d*x))**2, x)

$$3.322 \quad \int \frac{\sec^2(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^2} dx$$

Optimal. Leaf size=131

$$\frac{2(a^3B - 2ab^2B + Ab^3) \tanh^{-1}\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d(a-b)^{3/2}(a+b)^{3/2}} + \frac{a(Ab - aB) \tan(c+dx)}{bd(a^2 - b^2)(a+b \sec(c+dx))} + \frac{B \tanh^{-1}(\sin(c+dx))}{b^2d}$$

[Out] B*arctanh(sin(d*x+c))/b^2/d-2*(A*b^3+B*a^3-2*B*a*b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(3/2)/b^2/(a+b)^(3/2)/d+a*(A*b-B*a)*tan(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))

Rubi [A] time = 0.30, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4009, 3998, 3770, 3831, 2659, 208}

$$\frac{2(a^3B - 2ab^2B + Ab^3) \tanh^{-1}\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d(a-b)^{3/2}(a+b)^{3/2}} + \frac{a(Ab - aB) \tan(c+dx)}{bd(a^2 - b^2)(a+b \sec(c+dx))} + \frac{B \tanh^{-1}(\sin(c+dx))}{b^2d}$$

Antiderivative was successfully verified.

[In] Int[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^2,x]

[Out] (B*ArcTanh[Sin[c + d*x]])/(b^2*d) - (2*(A*b^3 + a^3*B - 2*a*b^2*B)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(3/2)*b^2*(a + b)^(3/2)*d) + (a*(A*b - a*B)*Tan[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))

Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 2659

Int[((a_) + (b_)*sin[Pi/2 + (c_) + (d_)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x]
  /; FreeQ[{c, d}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] := Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] := Dist[B/b, Int[Csc[e + f*x], x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 4009

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] := Simp[(a*(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(A*b - a*B)*(m + 1) - (a*A*b*(m + 2) - B*(a^2 + b^2*(m + 1)))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^2(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^2} dx &= \frac{a(Ab-aB)\tan(c+dx)}{b(a^2-b^2)d(a+b\sec(c+dx))} + \frac{\int \frac{\sec(c+dx)(-b(Ab-aB)+(a^2-b^2)B\sec(c+dx))}{a+b\sec(c+dx)} dx}{b(a^2-b^2)} \\
&= \frac{a(Ab-aB)\tan(c+dx)}{b(a^2-b^2)d(a+b\sec(c+dx))} + \frac{B \int \sec(c+dx) dx}{b^2} - \frac{(Ab^3+a(a^2-b^2))}{b^2} \\
&= \frac{B \tanh^{-1}(\sin(c+dx))}{b^2 d} + \frac{a(Ab-aB)\tan(c+dx)}{b(a^2-b^2)d(a+b\sec(c+dx))} - \frac{(Ab^3+a(a^2-b^2))}{b^2} \\
&= \frac{B \tanh^{-1}(\sin(c+dx))}{b^2 d} + \frac{a(Ab-aB)\tan(c+dx)}{b(a^2-b^2)d(a+b\sec(c+dx))} - \frac{2(Ab^3+a^3)}{b^2} \\
&= \frac{B \tanh^{-1}(\sin(c+dx))}{b^2 d} - \frac{2(Ab^3+a^3B-2ab^2B)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}b^2(a+b)^{3/2}d}
\end{aligned}$$

Mathematica [A] time = 0.72, size = 191, normalized size = 1.46

$$\cos(c+dx)(A+B\sec(c+dx)) \left(\frac{2(aB(a^2-2b^2)+Ab^3)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}} + \frac{ab(aB-Ab)\sin(c+dx)}{(b-a)(a+b)(a\cos(c+dx)+b)} - B \log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) \right)$$

$$b^2d(A\cos(c+dx)+B)$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^2, x]

[Out] (Cos[c + d*x]*(A + B*Sec[c + d*x])*((2*(A*b^3 + a*(a^2 - 2*b^2)*B)*ArcTanh[(-a + b)*Tan[(c + d*x)/2]]/Sqrt[a^2 - b^2]))/(a^2 - b^2)^(3/2) - B*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + B*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (a*b*(-(A*b) + a*B)*Sin[c + d*x])/((-a + b)*(a + b)*(b + a*Cos[c + d*x]))/(b^2*d*(B + A*Cos[c + d*x]))

fricas [B] time = 4.06, size = 694, normalized size = 5.30

$$\left[\frac{(Ba^3b - 2Bab^3 + Ab^4 + (Ba^4 - 2Ba^2b^2 + Aab^3)\cos(dx+c))\sqrt{a^2-b^2} \log\left(\frac{2ab\cos(dx+c)-(a^2-2b^2)\cos(dx+c)^2-2\sqrt{a^2-b^2}\sin(dx+c)}{a^2\cos(dx+c)^2+2ab\sin(dx+c)}\right)}{\dots} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm="fricas")

[Out] [1/2*((B*a^3*b - 2*B*a*b^3 + A*b^4 + (B*a^4 - 2*B*a^2*b^2 + A*a*b^3)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c))^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + (B*a^4*b - 2*B*a^2*b^3 + B*b^5 + (B*a^5 - 2*B*a^3*b^2 + B*a*b^4)*cos(d*x + c))*log(sin(d*x + c) + 1) - (B*a^4*b - 2*B*a^2*b^3 + B*b^5 + (B*a^5 - 2*B*a^3*b^2 + B*a*b^4)*cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(B*a^4*b - A*a^3*b^2 - B*a^2*b^3 + A*a*b^4)*sin(d*x + c)/((a^5*b^2 - 2*a^3*b^4 + a*b^6)*d*cos(d*x + c) + (a^4*b^3 - 2*a^2*b^5 + b^7)*d), -1/2*(2*(B*a^3*b - 2*B*a*b^3 + A*b^4 + (B*a^4 - 2*B*a^2*b^2 + A*a*b^3)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - (B*a^4*b - 2*B*a^2*b^3 + B*b^5 + (B*a^5 - 2*B*a^3*b^2 + B*a*b^4)*cos(d*x + c))*log(sin(d*x + c) + 1) + (B*a^4*b - 2*B*a^2*b^3 + B*b^5 + (B*a^5 - 2*B*a^3*b^2 + B*a*b^4)*cos(d*x + c))*log(-sin(d*x + c) + 1) + 2*(B*a^4*b - A*a^3*b^2 - B*a^2*b^3 + A*a*b^4)*sin(d*x + c)/((a^5*b^2 - 2*a^3*b^4 + a*b^6)*d*cos(d*x + c) + (a^4*b^3 - 2*a^2*b^5 + b^7)*d)]

giac [A] time = 0.31, size = 231, normalized size = 1.76

$$\frac{2(Ba^3 - 2Bab^2 + Ab^3) \left(\pi \left[\frac{dx+c}{2\pi} + \frac{1}{2} \right] \operatorname{sgn}(-2a+2b) + \arctan \left(-\frac{a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - b \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{\sqrt{-a^2+b^2}} \right) \right)}{(a^2b^2 - b^4) \sqrt{-a^2+b^2}} - \frac{B \log \left(\left| \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 1 \right| \right)}{b^2} + \frac{B \log \left(\left| \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 1 \right| \right)}{b^2}$$

d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm="giac")

[Out] -(2*(B*a^3 - 2*B*a*b^2 + A*b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^2*b^2 - b^4)*sqrt(-a^2 + b^2)) - B*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^2 + B*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^2 - 2*(B*a^2*tan(1/2*d*x + 1/2*c) - A*a*b*tan(1/2*d*x + 1/2*c))/((a^2*b - b^3)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b))/d

maple [B] time = 0.75, size = 350, normalized size = 2.67

$$\frac{2a \tan\left(\frac{dx}{2} + \frac{c}{2}\right) A}{d(a^2 - b^2) \left(a \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right) \right) - \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right) \right) b - a - b \right)} + \frac{2a^2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) B}{db(a^2 - b^2) \left(a \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right) \right) - \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right) \right) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x)

[Out] $-2/d*a/(a^2-b^2)*\tan(1/2*d*x+1/2*c)/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)*A+2/d/b*a^2/(a^2-b^2)*\tan(1/2*d*x+1/2*c)/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)*B-2/d*b/(a-b)/(a+b)/((a-b)*(a+b))^{1/2}*\operatorname{arctanh}(\tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^{1/2})*A-2/d*a^3/b^2/(a-b)/(a+b)/((a-b)*(a+b))^{1/2}*\operatorname{arctanh}(\tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^{1/2})*B+4/d/(a-b)/(a+b)/((a-b)*(a+b))^{1/2}*\operatorname{arctanh}(\tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^{1/2})*B*a-1/d/b^2*\ln(\tan(1/2*d*x+1/2*c)-1)*B+1/d/b^2*\ln(\tan(1/2*d*x+1/2*c)+1)*B$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see 'assume?' for more details) Is 4*a^2-4*b^2 positive or negative?

mupad [B] time = 9.65, size = 3751, normalized size = 28.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^2*(a + b/cos(c + d*x))^2),x)

[Out] $-(B*\operatorname{atan}\left(\frac{B*((B*((32*(A*a^2*b^7 - B*b^9 - A*b^9 - A*a^3*b^6 + B*a^2*b^7 - 3*B*a^3*b^6 + B*a^5*b^4 + A*a*b^8 + 2*B*a*b^8)))/(a*b^5 + b^6 - a^2*b^4 - a^3*b^3) - (32*B*\tan(c/2 + (d*x)/2)*(2*a*b^9 - 2*a^2*b^8 - 4*a^3*b^7 + 4*a^4*b^6 + 2*a^5*b^5 - 2*a^6*b^4)))/(b^2*(a*b^4 + b^5 - a^2*b^3 - a^3*b^2))\right)}{b^2}$

$$\begin{aligned}
& 2 - (32*\tan(c/2 + (d*x)/2)*(A^2*b^6 + 2*B^2*a^6 + B^2*b^6 - 2*B^2*a*b^5 - 2 \\
& *B^2*a^5*b + 3*B^2*a^2*b^4 + 4*B^2*a^3*b^3 - 5*B^2*a^4*b^2 - 4*A*B*a*b^5 + \\
& 2*A*B*a^3*b^3))/(a*b^4 + b^5 - a^2*b^3 - a^3*b^2))*1i)/b^2 - (B*((B*((32*(A \\
& *a^2*b^7 - B*b^9 - A*b^9 - A*a^3*b^6 + B*a^2*b^7 - 3*B*a^3*b^6 + B*a^5*b^4 \\
& + A*a*b^8 + 2*B*a*b^8)))/(a*b^5 + b^6 - a^2*b^4 - a^3*b^3) + (32*B*tan(c/2 + \\
& (d*x)/2)*(2*a*b^9 - 2*a^2*b^8 - 4*a^3*b^7 + 4*a^4*b^6 + 2*a^5*b^5 - 2*a^6* \\
& b^4))/(b^2*(a*b^4 + b^5 - a^2*b^3 - a^3*b^2))))/b^2 + (32*tan(c/2 + (d*x)/2 \\
&)*(A^2*b^6 + 2*B^2*a^6 + B^2*b^6 - 2*B^2*a*b^5 - 2*B^2*a^5*b + 3*B^2*a^2*b^ \\
& 4 + 4*B^2*a^3*b^3 - 5*B^2*a^4*b^2 - 4*A*B*a*b^5 + 2*A*B*a^3*b^3))/(a*b^4 + \\
& b^5 - a^2*b^3 - a^3*b^2))*1i)/b^2)/((B*((B*((32*(A*a^2*b^7 - B*b^9 - A*b^9 \\
& - A*a^3*b^6 + B*a^2*b^7 - 3*B*a^3*b^6 + B*a^5*b^4 + A*a*b^8 + 2*B*a*b^8)))/(\\
& a*b^5 + b^6 - a^2*b^4 - a^3*b^3) - (32*B*tan(c/2 + (d*x)/2)*(2*a*b^9 - 2*a^ \\
& 2*b^8 - 4*a^3*b^7 + 4*a^4*b^6 + 2*a^5*b^5 - 2*a^6*b^4))/(b^2*(a*b^4 + b^5 - \\
& a^2*b^3 - a^3*b^2))))/b^2 - (32*tan(c/2 + (d*x)/2)*(A^2*b^6 + 2*B^2*a^6 + \\
& B^2*b^6 - 2*B^2*a*b^5 - 2*B^2*a^5*b + 3*B^2*a^2*b^4 + 4*B^2*a^3*b^3 - 5*B^2 \\
& *a^4*b^2 - 4*A*B*a*b^5 + 2*A*B*a^3*b^3))/(a*b^4 + b^5 - a^2*b^3 - a^3*b^2)) \\
&)/b^2 - (64*(B^3*a^5 - A*B^2*b^5 + A^2*B*b^5 + 2*B^3*a*b^4 - B^3*a^4*b + 2* \\
& B^3*a^2*b^3 - 3*B^3*a^3*b^2 - 3*A*B^2*a*b^4 + A*B^2*a^2*b^3 + A*B^2*a^3*b^2 \\
&))/(a*b^5 + b^6 - a^2*b^4 - a^3*b^3) + (B*((B*((32*(A*a^2*b^7 - B*b^9 - A*b \\
& ^9 - A*a^3*b^6 + B*a^2*b^7 - 3*B*a^3*b^6 + B*a^5*b^4 + A*a*b^8 + 2*B*a*b^8) \\
&))/(a*b^5 + b^6 - a^2*b^4 - a^3*b^3) + (32*B*tan(c/2 + (d*x)/2)*(2*a*b^9 - 2 \\
& *a^2*b^8 - 4*a^3*b^7 + 4*a^4*b^6 + 2*a^5*b^5 - 2*a^6*b^4))/(b^2*(a*b^4 + b^ \\
& 5 - a^2*b^3 - a^3*b^2))))/b^2 + (32*tan(c/2 + (d*x)/2)*(A^2*b^6 + 2*B^2*a^6 \\
& + B^2*b^6 - 2*B^2*a*b^5 - 2*B^2*a^5*b + 3*B^2*a^2*b^4 + 4*B^2*a^3*b^3 - 5* \\
& B^2*a^4*b^2 - 4*A*B*a*b^5 + 2*A*B*a^3*b^3))/(a*b^4 + b^5 - a^2*b^3 - a^3*b^ \\
& 2)))/b^2))*2i)/(b^2*d) - (atan((((32*tan(c/2 + (d*x)/2)*(A^2*b^6 + 2*B^2*a \\
& ^6 + B^2*b^6 - 2*B^2*a*b^5 - 2*B^2*a^5*b + 3*B^2*a^2*b^4 + 4*B^2*a^3*b^3 - \\
& 5*B^2*a^4*b^2 - 4*A*B*a*b^5 + 2*A*B*a^3*b^3))/(a*b^4 + b^5 - a^2*b^3 - a^3* \\
& b^2) + (((32*(A*a^2*b^7 - B*b^9 - A*b^9 - A*a^3*b^6 + B*a^2*b^7 - 3*B*a^3*b^ \\
& ^6 + B*a^5*b^4 + A*a*b^8 + 2*B*a*b^8)))/(a*b^5 + b^6 - a^2*b^4 - a^3*b^3) + \\
& (32*tan(c/2 + (d*x)/2)*((a + b)^3*(a - b)^3)^(1/2)*(A*b^3 + B*a^3 - 2*B*a*b \\
& ^2)*(2*a*b^9 - 2*a^2*b^8 - 4*a^3*b^7 + 4*a^4*b^6 + 2*a^5*b^5 - 2*a^6*b^4))/ \\
& ((a*b^4 + b^5 - a^2*b^3 - a^3*b^2))*(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2)) \\
&))*((a + b)^3*(a - b)^3)^(1/2)*(A*b^3 + B*a^3 - 2*B*a*b^2))/(b^8 - 3*a^2*b^6 \\
& + 3*a^4*b^4 - a^6*b^2))*((a + b)^3*(a - b)^3)^(1/2)*(A*b^3 + B*a^3 - 2*B*a \\
& *b^2)*1i)/(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2) + (((32*tan(c/2 + (d*x)/2 \\
&)*(A^2*b^6 + 2*B^2*a^6 + B^2*b^6 - 2*B^2*a*b^5 - 2*B^2*a^5*b + 3*B^2*a^2*b^ \\
& 4 + 4*B^2*a^3*b^3 - 5*B^2*a^4*b^2 - 4*A*B*a*b^5 + 2*A*B*a^3*b^3))/(a*b^4 + \\
& b^5 - a^2*b^3 - a^3*b^2) - (((32*(A*a^2*b^7 - B*b^9 - A*b^9 - A*a^3*b^6 + B \\
& *a^2*b^7 - 3*B*a^3*b^6 + B*a^5*b^4 + A*a*b^8 + 2*B*a*b^8)))/(a*b^5 + b^6 - a \\
& ^2*b^4 - a^3*b^3) - (32*tan(c/2 + (d*x)/2)*((a + b)^3*(a - b)^3)^(1/2)*(A*b \\
& ^3 + B*a^3 - 2*B*a*b^2)*(2*a*b^9 - 2*a^2*b^8 - 4*a^3*b^7 + 4*a^4*b^6 + 2*a^ \\
& 5*b^5 - 2*a^6*b^4))/((a*b^4 + b^5 - a^2*b^3 - a^3*b^2))*(b^8 - 3*a^2*b^6 + 3 \\
& *a^4*b^4 - a^6*b^2)))*((a + b)^3*(a - b)^3)^(1/2)*(A*b^3 + B*a^3 - 2*B*a*b \\
& ^2))/(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2))*((a + b)^3*(a - b)^3)^(1/2)*(A
\end{aligned}$$

```

*b^3 + B*a^3 - 2*B*a*b^2)*1i)/(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2))/((64
*(B^3*a^5 - A*B^2*b^5 + A^2*B*b^5 + 2*B^3*a*b^4 - B^3*a^4*b + 2*B^3*a^2*b^3
- 3*B^3*a^3*b^2 - 3*A*B^2*a*b^4 + A*B^2*a^2*b^3 + A*B^2*a^3*b^2))/(a*b^5 +
b^6 - a^2*b^4 - a^3*b^3) - (((32*tan(c/2 + (d*x)/2)*(A^2*b^6 + 2*B^2*a^6 +
B^2*b^6 - 2*B^2*a*b^5 - 2*B^2*a^5*b + 3*B^2*a^2*b^4 + 4*B^2*a^3*b^3 - 5*B^
2*a^4*b^2 - 4*A*B*a*b^5 + 2*A*B*a^3*b^3))/(a*b^4 + b^5 - a^2*b^3 - a^3*b^2)
+ (((32*(A*a^2*b^7 - B*b^9 - A*b^9 - A*a^3*b^6 + B*a^2*b^7 - 3*B*a^3*b^6 +
B*a^5*b^4 + A*a*b^8 + 2*B*a*b^8))/(a*b^5 + b^6 - a^2*b^4 - a^3*b^3) + (32*
tan(c/2 + (d*x)/2)*((a + b)^3*(a - b)^3)^(1/2)*(A*b^3 + B*a^3 - 2*B*a*b^2)*
(2*a*b^9 - 2*a^2*b^8 - 4*a^3*b^7 + 4*a^4*b^6 + 2*a^5*b^5 - 2*a^6*b^4))/((a*
b^4 + b^5 - a^2*b^3 - a^3*b^2)*(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2)))*((
a + b)^3*(a - b)^3)^(1/2)*(A*b^3 + B*a^3 - 2*B*a*b^2))/(b^8 - 3*a^2*b^6 + 3
*a^4*b^4 - a^6*b^2))*((a + b)^3*(a - b)^3)^(1/2)*(A*b^3 + B*a^3 - 2*B*a*b^2
))/(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2) + (((32*tan(c/2 + (d*x)/2)*(A^2*
b^6 + 2*B^2*a^6 + B^2*b^6 - 2*B^2*a*b^5 - 2*B^2*a^5*b + 3*B^2*a^2*b^4 + 4*B
^2*a^3*b^3 - 5*B^2*a^4*b^2 - 4*A*B*a*b^5 + 2*A*B*a^3*b^3))/(a*b^4 + b^5 - a
^2*b^3 - a^3*b^2) - (((32*(A*a^2*b^7 - B*b^9 - A*b^9 - A*a^3*b^6 + B*a^2*b^
7 - 3*B*a^3*b^6 + B*a^5*b^4 + A*a*b^8 + 2*B*a*b^8))/(a*b^5 + b^6 - a^2*b^4
- a^3*b^3) - (32*tan(c/2 + (d*x)/2)*((a + b)^3*(a - b)^3)^(1/2)*(A*b^3 + B*
a^3 - 2*B*a*b^2)*(2*a*b^9 - 2*a^2*b^8 - 4*a^3*b^7 + 4*a^4*b^6 + 2*a^5*b^5 -
2*a^6*b^4))/((a*b^4 + b^5 - a^2*b^3 - a^3*b^2)*(b^8 - 3*a^2*b^6 + 3*a^4*b^
4 - a^6*b^2)))*((a + b)^3*(a - b)^3)^(1/2)*(A*b^3 + B*a^3 - 2*B*a*b^2))/(b^
8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2))*((a + b)^3*(a - b)^3)^(1/2)*(A*b^3 +
B*a^3 - 2*B*a*b^2))/(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2))*((a + b)^3*(a
- b)^3)^(1/2)*(A*b^3 + B*a^3 - 2*B*a*b^2)*2i)/(d*(b^8 - 3*a^2*b^6 + 3*a^4*
b^4 - a^6*b^2) - (2*tan(c/2 + (d*x)/2)*(B*a^2 - A*a*b))/(d*(a + b)*(a*b -
b^2)*(a + b - tan(c/2 + (d*x)/2)^2*(a - b)))

```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \sec^2(c + dx)}{(a + b \sec(c + dx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**2,x)

[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**2/(a + b*sec(c + d*x))**2, x)

$$3.323 \quad \int \frac{\sec(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^2} dx$$

Optimal. Leaf size=100

$$\frac{2(aA - bB) \tanh^{-1} \left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}} \right)}{d(a-b)^{3/2}(a+b)^{3/2}} - \frac{(Ab - aB) \tan(c+dx)}{d(a^2 - b^2)(a+b \sec(c+dx))}$$

[Out] $2*(A*a-B*b)*\operatorname{arctanh}((a-b)^{(1/2)}*\tan(1/2*d*x+1/2*c)/(a+b)^{(1/2)})/(a-b)^{(3/2)}/(a+b)^{(3/2)}/d-(A*b-B*a)*\tan(d*x+c)/(a^2-b^2)/d/(a+b*\sec(d*x+c))$

Rubi [A] time = 0.13, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {4003, 12, 3831, 2659, 208}

$$\frac{2(aA - bB) \tanh^{-1} \left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}} \right)}{d(a-b)^{3/2}(a+b)^{3/2}} - \frac{(Ab - aB) \tan(c+dx)}{d(a^2 - b^2)(a+b \sec(c+dx))}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(\operatorname{Sec}[c + d*x]*(A + B*\operatorname{Sec}[c + d*x]))/(a + b*\operatorname{Sec}[c + d*x])^2, x]$

[Out] $(2*(a*A - b*B)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a - b]*\operatorname{Tan}[(c + d*x)/2])/(\operatorname{Sqrt}[a + b])]/((a - b)^{(3/2)}*(a + b)^{(3/2)}*d) - ((A*b - a*B)*\operatorname{Tan}[c + d*x])/((a^2 - b^2)*d*(a + b*\operatorname{Sec}[c + d*x]))$

Rule 12

$\operatorname{Int}[(a_)*(u_), x_Symbol] := \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /; \operatorname{FreeQ}[a, x] \ \&\& \ !\operatorname{Match} Q[u, (b_)*(v_)] /; \operatorname{FreeQ}[b, x]$

Rule 208

$\operatorname{Int}[(a_)+(b_.)*(x_)^2)^{-1}, x_Symbol] := \operatorname{Simp}[(\operatorname{Rt}[-(a/b), 2]*\operatorname{ArcTanh}[x/\operatorname{Rt}[-(a/b), 2]])/a, x] /; \operatorname{FreeQ}\{a, b\}, x \ \&\& \ \operatorname{NegQ}[a/b]$

Rule 2659

$\operatorname{Int}[(a_)+(b_.)*\sin[\operatorname{Pi}/2 + (c_.) + (d_.)*(x_)])^{-1}, x_Symbol] := \operatorname{With}\{e = \operatorname{FreeFactors}[\operatorname{Tan}[(c + d*x)/2], x]\}, \operatorname{Dist}[(2*e)/d, \operatorname{Subst}[\operatorname{Int}[1/(a + b + (a - b)*e^2*x^2), x], x, \operatorname{Tan}[(c + d*x)/2]/e], x] /; \operatorname{FreeQ}\{a, b, c, d\}, x \ \&\& \ \operatorname{NeQ}[a^2 - b^2, 0]$

Rule 3831

Int[csc[(e_.) + (f_.)*(x_.)]/(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] := Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 4003

Int[csc[(e_.) + (f_.)*(x_.)]*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := -Simp[((A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(m + 1)*(a^2 - b^2), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[(a*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]

Rubi steps

$$\begin{aligned}
 \int \frac{\sec(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^2} dx &= -\frac{(Ab-aB)\tan(c+dx)}{(a^2-b^2)d(a+b\sec(c+dx))} + \frac{\int \frac{(-aA+bB)\sec(c+dx)}{a+b\sec(c+dx)} dx}{-a^2+b^2} \\
 &= -\frac{(Ab-aB)\tan(c+dx)}{(a^2-b^2)d(a+b\sec(c+dx))} + \frac{(aA-bB)\int \frac{\sec(c+dx)}{a+b\sec(c+dx)} dx}{a^2-b^2} \\
 &= -\frac{(Ab-aB)\tan(c+dx)}{(a^2-b^2)d(a+b\sec(c+dx))} + \frac{(aA-bB)\int \frac{1}{1+\frac{a\cos(c+dx)}{b}} dx}{b(a^2-b^2)} \\
 &= -\frac{(Ab-aB)\tan(c+dx)}{(a^2-b^2)d(a+b\sec(c+dx))} + \frac{(2(aA-bB))\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}+(1-\frac{a}{b})x^2} dx\right)}{b(a^2-b^2)d} \\
 &= \frac{2(aA-bB)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}d} - \frac{(Ab-aB)\tan(c+dx)}{(a^2-b^2)d(a+b\sec(c+dx))}
 \end{aligned}$$

Mathematica [A] time = 0.37, size = 97, normalized size = 0.97

$$\frac{(aB-Ab)\sin(c+dx)}{(a-b)(a+b)(a\cos(c+dx)+b)} - \frac{2(aA-bB)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}$$

d

Antiderivative was successfully verified.

[In] Integrate[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^2,x]

[Out] $\frac{((-2*(a*A - b*B)*\text{ArcTanh}[\frac{(-a + b)*\text{Tan}[(c + d*x)/2]}{\sqrt{a^2 - b^2}}])/\sqrt{a^2 - b^2})/(a^2 - b^2)^{3/2} + ((-(A*b) + a*B)*\text{Sin}[c + d*x])/((a - b)*(a + b)*(b + a*\text{Cos}[c + d*x]))}{d}$

fricas [A] time = 0.48, size = 389, normalized size = 3.89

$$\frac{\left((Aab - Bb^2 + (Aa^2 - Bab) \cos(dx + c)) \sqrt{a^2 - b^2} \log \left(\frac{2ab \cos(dx+c) - (a^2 - 2b^2) \cos(dx+c)^2 + 2\sqrt{a^2 - b^2} (b \cos(dx+c) + a) \sin(dx+c)}{a^2 \cos(dx+c)^2 + 2ab \cos(dx+c) + b^2} \right) \right)}{2 \left((a^5 - 2a^3b^2 + ab^4) d \cos(dx + c) + (a^4b - 2a^2b^3 + b^5) d \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm="fricas")

[Out] $\frac{1}{2} * \left((A*a*b - B*b^2 + (A*a^2 - B*a*b)*\cos(d*x + c)) * \sqrt{a^2 - b^2} * \log \left(\frac{a*b*\cos(d*x + c) - (a^2 - 2*b^2)*\cos(d*x + c)^2 + 2*\sqrt{a^2 - b^2}*(b*\cos(d*x + c) + a)*\sin(d*x + c) + 2*a^2 - b^2}{(a^2*\cos(d*x + c)^2 + 2*a*b*\cos(d*x + c) + b^2)} \right) + 2*(B*a^3 - A*a^2*b - B*a*b^2 + A*b^3)*\sin(d*x + c) \right) / \left((a^5 - 2*a^3*b^2 + a*b^4)*d*\cos(d*x + c) + (a^4*b - 2*a^2*b^3 + b^5)*d \right), \left((A*a*b - B*b^2 + (A*a^2 - B*a*b)*\cos(d*x + c))*\sqrt{-a^2 + b^2} * \arctan \left(\frac{-\sqrt{-a^2 + b^2}*(b*\cos(d*x + c) + a)}{(a^2 - b^2)*\sin(d*x + c)} \right) + (B*a^3 - A*a^2*b - B*a*b^2 + A*b^3)*\sin(d*x + c) \right) / \left((a^5 - 2*a^3*b^2 + a*b^4)*d*\cos(d*x + c) + (a^4*b - 2*a^2*b^3 + b^5)*d \right)$

giac [A] time = 0.33, size = 172, normalized size = 1.72

$$\frac{2 \left(\frac{\left(\pi \left[\frac{dx+c}{2\pi} + \frac{1}{2} \right] \text{sgn}(2a-2b) + \arctan \left(\frac{a \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) - b \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)}{\sqrt{-a^2 + b^2}} \right) \right) (Aa - Bb)}{(a^2 - b^2) \sqrt{-a^2 + b^2}} + \frac{Ba \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) - Ab \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)}{\left(a \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^2 - b \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^2 - a - b \right) (a^2 - b^2)} \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm="giac")

[Out] $-2 * \left((\pi * \text{floor}(1/2 * (d*x + c) / \pi + 1/2)) * \text{sgn}(2*a - 2*b) + \arctan \left(\frac{a * \tan(1/2 * d*x + 1/2 * c) - b * \tan(1/2 * d*x + 1/2 * c)}{\sqrt{-a^2 + b^2}} \right) \right) * (A*a - B*b) / \left((a^2 - b^2) * \sqrt{-a^2 + b^2} \right) + (B*a * \tan(1/2 * d*x + 1/2 * c) - A*b * \tan(1/2 * d*x + 1/2 * c)) / \left((a^5 - 2*a^3*b^2 + a*b^4) * d * \cos(d*x + c) + (a^4*b - 2*a^2*b^3 + b^5) * d \right)$

$*c)/((a*\tan(1/2*d*x + 1/2*c)^2 - b*\tan(1/2*d*x + 1/2*c)^2 - a - b)*(a^2 - b^2))/d$

maple [A] time = 0.72, size = 132, normalized size = 1.32

$$\frac{2(Ab-aB)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{(a^2-b^2)\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b-a-b\right)} + \frac{2(aA-Bb)\operatorname{arctanh}\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)(a+b)\sqrt{(a-b)(a+b)}}$$

d

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x)`

[Out] $1/d*(2*(A*b-B*a)/(a^2-b^2)*\tan(1/2*d*x+1/2*c)/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)+2*(A*a-B*b)/(a-b)/(a+b)/((a-b)*(a+b))^{1/2}*\operatorname{arctanh}(\tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^{1/2}))$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for more details)Is 4*a^2-4*b^2 positive or negative?

mupad [B] time = 2.42, size = 106, normalized size = 1.06

$$\frac{2 \operatorname{atanh}\left(\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a-b}}{\sqrt{a+b}}\right)(Aa-Bb)}{d(a+b)^{3/2}(a-b)^{3/2}} - \frac{2 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)(Ab-Ba)}{d(a+b)(a-b)\left((b-a)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+a+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B/cos(c + d*x))/(cos(c + d*x)*(a + b/cos(c + d*x))^2),x)`

[Out] $(2*\operatorname{atanh}((\tan(c/2 + (d*x)/2)*(a - b)^{1/2})/(a + b)^{1/2})*(A*a - B*b))/(d*(a + b)^{3/2}*(a - b)^{3/2}) - (2*\tan(c/2 + (d*x)/2)*(A*b - B*a))/(d*(a + b)*(a - b)*(a + b - \tan(c/2 + (d*x)/2)^2*(a - b)))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \sec(c + dx)}{(a + b \sec(c + dx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)/(a + b*sec(c + d*x))**2, x)
```

$$3.324 \quad \int \frac{A+B \sec(c+dx)}{(a+b \sec(c+dx))^2} dx$$

Optimal. Leaf size=124

$$\frac{b(Ab - aB) \tan(c + dx)}{ad(a^2 - b^2)(a + b \sec(c + dx))} + \frac{Ax}{a^2} - \frac{2(a^3(-B) + 2a^2Ab - Ab^3) \tanh^{-1}\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d(a-b)^{3/2}(a+b)^{3/2}}$$

[Out] $A*x/a^2 - 2*(2*A*a^2*b - A*b^3 - B*a^3)*\operatorname{arctanh}((a-b)^{(1/2)}*\tan(1/2*d*x + 1/2*c)/(a+b)^{(1/2)})/a^2/(a-b)^{(3/2)}/(a+b)^{(3/2)}/d + b*(A*b - B*a)*\tan(d*x + c)/a/(a^2 - b^2)/d/(a+b*\sec(d*x + c))$

Rubi [A] time = 0.21, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3923, 3919, 3831, 2659, 208}

$$-\frac{2(2a^2Ab + a^3(-B) - Ab^3) \tanh^{-1}\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d(a-b)^{3/2}(a+b)^{3/2}} + \frac{b(Ab - aB) \tan(c + dx)}{ad(a^2 - b^2)(a + b \sec(c + dx))} + \frac{Ax}{a^2}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(A + B*\operatorname{Sec}[c + d*x])/(a + b*\operatorname{Sec}[c + d*x])^2, x]$

[Out] $(A*x)/a^2 - (2*(2*a^2*A*b - A*b^3 - a^3*B)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a - b]*\operatorname{Tan}[(c + d*x)/2])/\operatorname{Sqrt}[a + b]])/(a^2*(a - b)^{(3/2)}*(a + b)^{(3/2)*d} + (b*(A*b - a*B)*\operatorname{Tan}[c + d*x])/(a*(a^2 - b^2)*d*(a + b*\operatorname{Sec}[c + d*x]))$

Rule 208

$\operatorname{Int}[(a_.) + (b_.)*(x_.)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[-(a/b), 2]*\operatorname{ArcTanh}[x/\operatorname{Rt}[-(a/b), 2]])/a, x] /;$ $\operatorname{FreeQ}\{a, b, x\} \ \&\& \ \operatorname{NegQ}[a/b]$

Rule 2659

$\operatorname{Int}[(a_.) + (b_.)*\sin[\operatorname{Pi}/2 + (c_.) + (d_.)*(x_.)])^{-1}, x_Symbol] \rightarrow \operatorname{With}\{e = \operatorname{FreeFactors}[\operatorname{Tan}[(c + d*x)/2], x]\}, \operatorname{Dist}[(2*e)/d, \operatorname{Subst}[\operatorname{Int}[1/(a + b + (a - b)*e^2*x^2), x], x, \operatorname{Tan}[(c + d*x)/2]/e], x] /;$ $\operatorname{FreeQ}\{a, b, c, d, x\} \ \&\& \ \operatorname{NeQ}[a^2 - b^2, 0]$

Rule 3831

$\operatorname{Int}[\operatorname{csc}[(e_.) + (f_.)*(x_.)]/(\operatorname{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_)), x_Symbol] \rightarrow \operatorname{Dist}[1/b, \operatorname{Int}[1/(1 + (a*\sin[e + f*x])/b), x], x] /;$ $\operatorname{FreeQ}\{a, b, e, f\}$

}, x] && NeQ[a^2 - b^2, 0]

Rule 3919

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))/(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 3923

Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.)), x_Symbol] :> Simp[(b*(b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[c*(a^2 - b^2)*(m + 1) - (a*(b*c - a*d)*(m + 1))*Csc[e + f*x] + b*(b*c - a*d)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m]

Rubi steps

$$\begin{aligned}
 \int \frac{A + B \sec(c + dx)}{(a + b \sec(c + dx))^2} dx &= \frac{b(Ab - aB) \tan(c + dx)}{a(a^2 - b^2) d(a + b \sec(c + dx))} - \frac{\int \frac{-A(a^2 - b^2) + a(Ab - aB) \sec(c + dx)}{a + b \sec(c + dx)} dx}{a(a^2 - b^2)} \\
 &= \frac{Ax}{a^2} + \frac{b(Ab - aB) \tan(c + dx)}{a(a^2 - b^2) d(a + b \sec(c + dx))} - \frac{(2a^2Ab - Ab^3 - a^3B) \int \frac{\sec(c + dx)}{a + b \sec(c + dx)} dx}{a^2(a^2 - b^2)} \\
 &= \frac{Ax}{a^2} + \frac{b(Ab - aB) \tan(c + dx)}{a(a^2 - b^2) d(a + b \sec(c + dx))} - \frac{(2a^2Ab - Ab^3 - a^3B) \int \frac{1}{1 + \frac{a \cos(c + dx)}{b}} dx}{a^2b(a^2 - b^2)} \\
 &= \frac{Ax}{a^2} + \frac{b(Ab - aB) \tan(c + dx)}{a(a^2 - b^2) d(a + b \sec(c + dx))} - \frac{(2(2a^2Ab - Ab^3 - a^3B)) \text{Subst}\left(\int \frac{1}{1 + \frac{a}{b} + (1 - \frac{a}{b})t^2} dt\right)}{a^2b(a^2 - b^2) d} \\
 &= \frac{Ax}{a^2} - \frac{2(2a^2Ab - Ab^3 - a^3B) \tanh^{-1}\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c + dx)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{3/2}(a+b)^{3/2}d} + \frac{b(Ab - aB) \tan(c + dx)}{a(a^2 - b^2) d(a + b \sec(c + dx))}
 \end{aligned}$$

Mathematica [A] time = 0.67, size = 155, normalized size = 1.25

$$\frac{Ab(a^2-b^2)(c+dx)+aA(a^2-b^2)(c+dx)\cos(c+dx)-ab(aB-Ab)\sin(c+dx)}{a\cos(c+dx)+b} - \frac{2(a^3B-2a^2Ab+Ab^3)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

$$a^2d(a-b)(a+b)$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sec[c + d*x])/(a + b*Sec[c + d*x])^2,x]

[Out] ((-2*(-2*a^2*A*b + A*b^3 + a^3*B)*ArcTanh[((-a + b)*Tan[(c + d*x)/2]])/Sqrt[a^2 - b^2])/Sqrt[a^2 - b^2] + (A*b*(a^2 - b^2)*(c + d*x) + a*A*(a^2 - b^2)*(c + d*x)*Cos[c + d*x] - a*b*(-(A*b) + a*B)*Sin[c + d*x])/(b + a*Cos[c + d*x]))/(a^2*(a - b)*(a + b)*d)

fricas [B] time = 0.54, size = 561, normalized size = 4.52

$$\left[\frac{2(Aa^5 - 2Aa^3b^2 + Aab^4)dx \cos(dx + c) + 2(Aa^4b - 2Aa^2b^3 + Ab^5)dx - (Ba^3b - 2Aa^2b^2 + Ab^4 + (Ba^4 - 2Aa^5 - 2Aa^3b^2 + Aab^4)d) \cos(dx + c) + (Aa^4b - 2Aa^2b^3 + Ab^5)d}{2((a^7 - 2a^5b^2 + a^3b^4)d \cos(dx + c) + (a^6b - 2a^4b^3 + a^2b^5)d)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm="fricas")

[Out] [1/2*(2*(A*a^5 - 2*A*a^3*b^2 + A*a*b^4)*d*x*cos(d*x + c) + 2*(A*a^4*b - 2*A*a^2*b^3 + A*b^5)*d*x - (B*a^3*b - 2*A*a^2*b^2 + A*b^4 + (B*a^4 - 2*A*a^3*b + A*a*b^3)*cos(d*x + c))*sqrt(a^2 - b^2)*log(((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 2*(B*a^4*b - A*a^3*b^2 - B*a^2*b^3 + A*a*b^4)*sin(d*x + c))/((a^7 - 2*a^5*b^2 + a^3*b^4)*d*cos(d*x + c) + (a^6*b - 2*a^4*b^3 + a^2*b^5)*d), ((A*a^5 - 2*A*a^3*b^2 + A*a*b^4)*d*x*cos(d*x + c) + (A*a^4*b - 2*A*a^2*b^3 + A*b^5)*d*x + (B*a^3*b - 2*A*a^2*b^2 + A*b^4 + (B*a^4 - 2*A*a^3*b + A*a*b^3)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - (B*a^4*b - A*a^3*b^2 - B*a^2*b^3 + A*a*b^4)*sin(d*x + c))/((a^7 - 2*a^5*b^2 + a^3*b^4)*d*cos(d*x + c) + (a^6*b - 2*a^4*b^3 + a^2*b^5)*d)]

giac [A] time = 0.30, size = 201, normalized size = 1.62

$$\frac{2(Ba^3 - 2Aa^2b + Ab^3)\left(\pi\left[\frac{dx+c}{2\pi} + \frac{1}{2}\right]\operatorname{sgn}(-2a+2b) + \arctan\left(-\frac{a\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - b\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{(a^4 - a^2b^2)\sqrt{-a^2+b^2}} + \frac{(dx+c)A}{a^2} + \frac{2(Bab\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - Ab^2\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right))}{(a^3 - ab^2)\left(a\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - b\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right)^2}$$

$$d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm="giac")

[Out] $(2*(B*a^3 - 2*A*a^2*b + A*b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + \arctan(-(a*\tan(1/2*d*x + 1/2*c) - b*\tan(1/2*d*x + 1/2*c))/\sqrt{-a^2 + b^2}))/((a^4 - a^2*b^2)*\sqrt{-a^2 + b^2}) + (d*x + c)*A/a^2 + 2*(B*a*b*\tan(1/2*d*x + 1/2*c) - A*b^2*\tan(1/2*d*x + 1/2*c))/((a^3 - a*b^2)*(a*\tan(1/2*d*x + 1/2*c)^2 - b*\tan(1/2*d*x + 1/2*c)^2 - a - b))/d$

maple [B] time = 0.79, size = 328, normalized size = 2.65

$$\frac{2b^2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) A}{da(a^2 - b^2) \left(a \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right) \right) - \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right) \right) b - a - b \right)} + \frac{2b \tan\left(\frac{dx}{2} + \frac{c}{2}\right) B}{d(a^2 - b^2) \left(a \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right) \right) - \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right) \right) b \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x)

[Out] $-2/d/a*b^2/(a^2-b^2)*\tan(1/2*d*x+1/2*c)/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)*A+2/d*b/(a^2-b^2)*\tan(1/2*d*x+1/2*c)/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)*B-4/d*b/(a-b)/(a+b)/((a-b)*(a+b))^{1/2}*\arctanh(\tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^{1/2})*A+2/d/a^2/(a-b)/(a+b)/((a-b)*(a+b))^{1/2}*\operatorname{arctanh}(\tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^{1/2})*A*b^3+2/d/(a-b)/(a+b)/((a-b)*(a+b))^{1/2}*\operatorname{arctanh}(\tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^{1/2})*B*a+2/d/a^2*\operatorname{arctan}(\tan(1/2*d*x+1/2*c))*A$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for more details)Is 4*a^2-4*b^2 positive or negative?

mupad [B] time = 9.66, size = 3763, normalized size = 30.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

$$\begin{aligned}
& A*a^4*b^5 - B*a^9 - A*a^9 - 3*A*a^6*b^3 + A*a^7*b^2 - B*a^6*b^3 + B*a^7*b^2 \\
& + 2*A*a^8*b + B*a^8*b)/(a^5*b + a^6 - a^3*b^3 - a^4*b^2) + (32*\tan(c/2 + \\
& (d*x)/2)*((a + b)^3*(a - b)^3)^{(1/2)}*(A*b^3 + B*a^3 - 2*A*a^2*b)*(2*a^9*b - \\
& 2*a^4*b^6 + 2*a^5*b^5 + 4*a^6*b^4 - 4*a^7*b^3 - 2*a^8*b^2))/((a^4*b + a^5 \\
& - a^2*b^3 - a^3*b^2)*(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2)))*((a + b)^3*(\\
& a - b)^3)^{(1/2)}*(A*b^3 + B*a^3 - 2*A*a^2*b))/(a^8 - a^2*b^6 + 3*a^4*b^4 - 3 \\
& *a^6*b^2))*(A*b^3 + B*a^3 - 2*A*a^2*b)*1i)/(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a \\
& ^6*b^2))/((64*(A^3*b^5 + A*B^2*a^5 - A^2*B*a^5 - A^3*a*b^4 + 2*A^3*a^4*b - \\
& 3*A^3*a^2*b^3 + 2*A^3*a^3*b^2 - 3*A^2*B*a^4*b + A^2*B*a^2*b^3 + A^2*B*a^3*b \\
& ^2)))/(a^5*b + a^6 - a^3*b^3 - a^4*b^2) + (((a + b)^3*(a - b)^3)^{(1/2)}*((32* \\
& \tan(c/2 + (d*x)/2)*(A^2*a^6 + 2*A^2*b^6 + B^2*a^6 - 2*A^2*a*b^5 - 2*A^2*a^5 \\
& *b - 5*A^2*a^2*b^4 + 4*A^2*a^3*b^3 + 3*A^2*a^4*b^2 - 4*A*B*a^5*b + 2*A*B*a^ \\
& 3*b^3)))/(a^4*b + a^5 - a^2*b^3 - a^3*b^2) + (((32*(A*a^4*b^5 - B*a^9 - A*a^ \\
& 9 - 3*A*a^6*b^3 + A*a^7*b^2 - B*a^6*b^3 + B*a^7*b^2 + 2*A*a^8*b + B*a^8*b)) \\
& / (a^5*b + a^6 - a^3*b^3 - a^4*b^2) - (32*\tan(c/2 + (d*x)/2)*((a + b)^3*(a - \\
& b)^3)^{(1/2)}*(A*b^3 + B*a^3 - 2*A*a^2*b)*(2*a^9*b - 2*a^4*b^6 + 2*a^5*b^5 + \\
& 4*a^6*b^4 - 4*a^7*b^3 - 2*a^8*b^2))/((a^4*b + a^5 - a^2*b^3 - a^3*b^2)*(a^ \\
& 8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2)))*((a + b)^3*(a - b)^3)^{(1/2)}*(A*b^3 + \\
& B*a^3 - 2*A*a^2*b))/(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2))*(A*b^3 + B*a^ \\
& 3 - 2*A*a^2*b))/(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2) - (((a + b)^3*(a - \\
& b)^3)^{(1/2)}*((32*\tan(c/2 + (d*x)/2)*(A^2*a^6 + 2*A^2*b^6 + B^2*a^6 - 2*A^2* \\
& a*b^5 - 2*A^2*a^5*b - 5*A^2*a^2*b^4 + 4*A^2*a^3*b^3 + 3*A^2*a^4*b^2 - 4*A*B \\
& *a^5*b + 2*A*B*a^3*b^3)))/(a^4*b + a^5 - a^2*b^3 - a^3*b^2) - (((32*(A*a^4*b \\
& ^5 - B*a^9 - A*a^9 - 3*A*a^6*b^3 + A*a^7*b^2 - B*a^6*b^3 + B*a^7*b^2 + 2*A* \\
& a^8*b + B*a^8*b)))/(a^5*b + a^6 - a^3*b^3 - a^4*b^2) + (32*\tan(c/2 + (d*x)/2 \\
&)*((a + b)^3*(a - b)^3)^{(1/2)}*(A*b^3 + B*a^3 - 2*A*a^2*b)*(2*a^9*b - 2*a^4* \\
& b^6 + 2*a^5*b^5 + 4*a^6*b^4 - 4*a^7*b^3 - 2*a^8*b^2))/((a^4*b + a^5 - a^2*b \\
& ^3 - a^3*b^2)*(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2)))*((a + b)^3*(a - b)^ \\
& 3)^{(1/2)}*(A*b^3 + B*a^3 - 2*A*a^2*b))/(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^ \\
& 2))*(A*b^3 + B*a^3 - 2*A*a^2*b))/(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2))* \\
& ((a + b)^3*(a - b)^3)^{(1/2)}*(A*b^3 + B*a^3 - 2*A*a^2*b)*2i)/(d*(a^8 - a^2*b \\
& ^6 + 3*a^4*b^4 - 3*a^6*b^2) - (2*\tan(c/2 + (d*x)/2)*(A*b^2 - B*a*b))/(d*(a \\
& + b)*(a*b - a^2)*(a + b - \tan(c/2 + (d*x)/2)^2*(a - b)))
\end{aligned}$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \sec(c + dx)}{(a + b \sec(c + dx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))**2,x)

[Out] Integral((A + B*sec(c + d*x))/(a + b*sec(c + d*x))**2, x)

$$3.325 \quad \int \frac{\cos(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^2} dx$$

Optimal. Leaf size=180

$$\frac{x(2Ab - aB)}{a^3} + \frac{(a^2A + abB - 2Ab^2) \sin(c + dx)}{a^2d(a^2 - b^2)} + \frac{b(Ab - aB) \sin(c + dx)}{ad(a^2 - b^2)(a + b \sec(c + dx))} + \frac{2b(-2a^3B + 3a^2Ab + ab^2B)}{a^3d(a - b)}$$

[Out] $-(2A*b-B*a)*x/a^3+2*b*(3*A*a^2*b-2*A*b^3-2*B*a^3+B*a*b^2)*\operatorname{arctanh}((a-b)^{(1/2)}*\tan(1/2*d*x+1/2*c)/(a+b)^{(1/2)})/a^3/(a-b)^{(3/2)/(a+b)^{(3/2)/d+(A*a^2-2*A*b^2+B*a*b)*\sin(d*x+c)/a^2/(a^2-b^2)/d+b*(A*b-B*a)*\sin(d*x+c)/a/(a^2-b^2)/d/(a+b*\sec(d*x+c))}$

Rubi [A] time = 0.57, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {4030, 4104, 3919, 3831, 2659, 208}

$$\frac{(a^2A + abB - 2Ab^2) \sin(c + dx)}{a^2d(a^2 - b^2)} + \frac{2b(3a^2Ab - 2a^3B + ab^2B - 2Ab^3) \tanh^{-1}\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d(a-b)^{3/2}(a+b)^{3/2}} + \frac{b(Ab - aB)}{ad(a^2 - b^2)(a - b)}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^2,x]

[Out] $-(((2A*b - a*B)*x)/a^3) + (2*b*(3*a^2*A*b - 2*A*b^3 - 2*a^3*B + a*b^2*B)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a - b]*\operatorname{Tan}[(c + d*x)/2])/(\operatorname{Sqrt}[a + b])]/(a^3*(a - b)^{(3/2)}*(a + b)^{(3/2)*d) + ((a^2*A - 2*A*b^2 + a*b*B)*\operatorname{Sin}[c + d*x])/(a^2*(a^2 - b^2)*d) + (b*(A*b - a*B)*\operatorname{Sin}[c + d*x])/(a*(a^2 - b^2)*d*(a + b*\operatorname{Sec}[c + d*x]))$

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 2659

Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol]
:= Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol]
:= Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b*c - a*d, 0]
```

Rule 4030

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^m*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol]
:= Simp[(b*(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[A*(a^2*(m + 1) - b^2*(m + n + 1)) + a*b*B*n - a*(A*b - a*B)*(m + 1)*Csc[e + f*x] + b*(A*b - a*B)*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x]
&& NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^m, x_Symbol]
:= Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x]
&& NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^2} dx &= \frac{b(Ab-aB)\sin(c+dx)}{a(a^2-b^2)d(a+b\sec(c+dx))} - \int \frac{\cos(c+dx)(-a^2A+2Ab^2-abB+a(Ab-aB)\sec(c+dx))}{a+b\sec(c+dx)} dx \\
&= \frac{(a^2A-2Ab^2+abB)\sin(c+dx)}{a^2(a^2-b^2)d} + \frac{b(Ab-aB)\sin(c+dx)}{a(a^2-b^2)d(a+b\sec(c+dx))} + \int \frac{\cos(c+dx)(-a^2A+2Ab^2-abB+a(Ab-aB)\sec(c+dx))}{a+b\sec(c+dx)} dx \\
&= -\frac{(2Ab-aB)x}{a^3} + \frac{(a^2A-2Ab^2+abB)\sin(c+dx)}{a^2(a^2-b^2)d} + \frac{b(Ab-aB)\sin(c+dx)}{a(a^2-b^2)d(a+b\sec(c+dx))} \\
&= -\frac{(2Ab-aB)x}{a^3} + \frac{(a^2A-2Ab^2+abB)\sin(c+dx)}{a^2(a^2-b^2)d} + \frac{b(Ab-aB)\sin(c+dx)}{a(a^2-b^2)d(a+b\sec(c+dx))} \\
&= -\frac{(2Ab-aB)x}{a^3} + \frac{(a^2A-2Ab^2+abB)\sin(c+dx)}{a^2(a^2-b^2)d} + \frac{b(Ab-aB)\sin(c+dx)}{a(a^2-b^2)d(a+b\sec(c+dx))} \\
&= -\frac{(2Ab-aB)x}{a^3} + \frac{2b(3a^2Ab-2Ab^3-2a^3B+ab^2B)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3(a-b)^{3/2}(a+b)^{3/2}d}
\end{aligned}$$

Mathematica [A] time = 1.13, size = 221, normalized size = 1.23

$$\frac{(a\cos(c+dx)+b)(A+B\sec(c+dx))\left(\frac{2b(2a^3B-3a^2Ab-ab^2B+2Ab^3)\sec(c+dx)(a\cos(c+dx)+b)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}\right)+\frac{ab^2(aB-a^2)}{(a^2-b^2)^{3/2}}}{a^3d(a+b\sec(c+dx))^2(A+B\sec(c+dx))}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^2, x]

[Out] ((b + a*Cos[c + d*x])*(A + B*Sec[c + d*x]))*((-2*A*b + a*B)*(c + d*x)*(b + a*Cos[c + d*x])*Sec[c + d*x] + (2*b*(-3*a^2*A*b + 2*A*b^3 + 2*a^3*B - a*b^2*B)*ArcTanh[(-a + b)*Tan[(c + d*x)/2]]/Sqrt[a^2 - b^2])*(b + a*Cos[c + d*x])*Sec[c + d*x])/(a^2 - b^2)^(3/2) + (a*b^2*(-(A*b) + a*B)*Tan[c + d*x])/(a - b)*(a + b) + a*A*(b + a*Cos[c + d*x])*Tan[c + d*x])/(a^3*d*(B + A*Cos[c + d*x])*(a + b*Sec[c + d*x])^2)

fricas [B] time = 0.56, size = 788, normalized size = 4.38

$$\int 2 \left(Ba^6 - 2Aa^5b - 2Ba^4b^2 + 4Aa^3b^3 + Ba^2b^4 - 2Aab^5 \right) dx \cos(dx + c) + 2 \left(Ba^5b - 2Aa^4b^2 - 2Ba^3b^3 + 4Aa^2b^4 - 2Aab^5 \right) dx \cos(dx + c)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm="fricas")

[Out] [1/2*(2*(B*a^6 - 2*A*a^5*b - 2*B*a^4*b^2 + 4*A*a^3*b^3 + B*a^2*b^4 - 2*A*a*b^5)*d*x*cos(d*x + c) + 2*(B*a^5*b - 2*A*a^4*b^2 - 2*B*a^3*b^3 + 4*A*a^2*b^4 + B*a*b^5 - 2*A*b^6)*d*x + (2*B*a^3*b^2 - 3*A*a^2*b^3 - B*a*b^4 + 2*A*b^5 + (2*B*a^4*b - 3*A*a^3*b^2 - B*a^2*b^3 + 2*A*a*b^4)*cos(d*x + c))*sqrt(a^2 - b^2)*log(((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c))^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + 2*(A*a^5*b + B*a^4*b^2 - 3*A*a^3*b^3 - B*a^2*b^4 + 2*A*a*b^5 + (A*a^6 - 2*A*a^4*b^2 + A*a^2*b^4)*cos(d*x + c))*sin(d*x + c))/((a^8 - 2*a^6*b^2 + a^4*b^4)*d*cos(d*x + c) + (a^7*b - 2*a^5*b^3 + a^3*b^5)*d), ((B*a^6 - 2*A*a^5*b - 2*B*a^4*b^2 + 4*A*a^3*b^3 + B*a^2*b^4 - 2*A*a*b^5)*d*x*cos(d*x + c) + (B*a^5*b - 2*A*a^4*b^2 - 2*B*a^3*b^3 + 4*A*a^2*b^4 + B*a*b^5 - 2*A*b^6)*d*x - (2*B*a^3*b^2 - 3*A*a^2*b^3 - B*a*b^4 + 2*A*b^5 + (2*B*a^4*b - 3*A*a^3*b^2 - B*a^2*b^3 + 2*A*a*b^4)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) + (A*a^5*b + B*a^4*b^2 - 3*A*a^3*b^3 - B*a^2*b^4 + 2*A*a*b^5 + (A*a^6 - 2*A*a^4*b^2 + A*a^2*b^4)*cos(d*x + c))*sin(d*x + c))/((a^8 - 2*a^6*b^2 + a^4*b^4)*d*cos(d*x + c) + (a^7*b - 2*a^5*b^3 + a^3*b^5)*d)]

giac [B] time = 0.49, size = 1107, normalized size = 6.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm="giac")

[Out] ((B*a^8 - 2*A*a^7*b - 3*B*a^7*b + 5*A*a^6*b^2 - 2*B*a^6*b^2 + 4*A*a^5*b^3 + 5*B*a^5*b^3 - 9*A*a^4*b^4 + B*a^4*b^4 - 2*A*a^3*b^5 - 2*B*a^3*b^5 + 4*A*a^2*b^6 - B*a^3*abs(-a^5 + a^3*b^2) + 2*A*a^2*b*abs(-a^5 + a^3*b^2) - B*a^2*b*abs(-a^5 + a^3*b^2) + A*a*b^2*abs(-a^5 + a^3*b^2) + B*a*b^2*abs(-a^5 + a^3*b^2) - 2*A*b^3*abs(-a^5 + a^3*b^2))*(pi*floor(1/2*(d*x + c)/pi + 1/2) + arctan(tan(1/2*d*x + 1/2*c)/sqrt(-(a^4*b - a^2*b^3 + sqrt((a^5 + a^4*b - a^3*b^2 - a^2*b^3)*(a^5 - a^4*b - a^3*b^2 + a^2*b^3) + (a^4*b - a^2*b^3)^2)))/(a

$$\begin{aligned} & \sqrt[5]{-a^4b - a^3b^2 + a^2b^3}) / (a^4b \operatorname{abs}(-a^5 + a^3b^2) - a^2b^3 \operatorname{abs} \\ & (-a^5 + a^3b^2) + (a^5 - a^3b^2)^2) - ((2a^2b + ab^2 - 2b^3) \sqrt{-a^2 + b^2} * A \operatorname{abs}(-a^5 + a^3b^2) \operatorname{abs}(-a + b) - (a^3 + a^2b - ab^2) \sqrt{-a^2 + b^2} * B \operatorname{abs}(-a^5 + a^3b^2) \operatorname{abs}(-a + b) + (2a^7b - 5a^6b^2 - 4a^5b^3 + 9a^4b^4 + 2a^3b^5 - 4a^2b^6) \sqrt{-a^2 + b^2} * A \operatorname{abs}(-a + b) - (a^8 - 3a^7b - 2a^6b^2 + 5a^5b^3 + a^4b^4 - 2a^3b^5) \sqrt{-a^2 + b^2} * B \operatorname{abs}(-a + b)) * (\pi * \operatorname{floor}(1/2 * (dx + c) / \pi + 1/2) + \arctan(\tan(1/2 * dx + 1/2 * c) / \sqrt{-(a^4b - a^2b^3 - \sqrt{(a^5 + a^4b - a^3b^2 - a^2b^3)} * (a^5 - a^4b - a^3b^2 + a^2b^3))}) / ((a^5 - a^3b^2)^2 * (a^2 - 2ab + b^2) - (a^6b - 2a^5b^2 + 2a^3b^4 - a^2b^5) \operatorname{abs}(-a^5 + a^3b^2))) + 2 * (A * a^3 * \tan(1/2 * dx + 1/2 * c)^3 - A * a^2 * b * \tan(1/2 * dx + 1/2 * c)^3 - A * a * b^2 * \tan(1/2 * dx + 1/2 * c)^3 - B * a * b^2 * \tan(1/2 * dx + 1/2 * c)^3 + 2 * A * b^3 * \tan(1/2 * dx + 1/2 * c)^3 - A * a^3 * \tan(1/2 * dx + 1/2 * c) - A * a^2 * b * \tan(1/2 * dx + 1/2 * c) + A * a * b^2 * \tan(1/2 * dx + 1/2 * c) - B * a * b^2 * \tan(1/2 * dx + 1/2 * c) + 2 * A * b^3 * \tan(1/2 * dx + 1/2 * c)) / ((a * \tan(1/2 * dx + 1/2 * c))^4 - b * \tan(1/2 * dx + 1/2 * c)^4 - 2 * b * \tan(1/2 * dx + 1/2 * c)^2 - a - b) * (a^4 - a^2 * b^2)) / d \end{aligned}$$

maple [B] time = 1.15, size = 453, normalized size = 2.52

$$\frac{2b^3 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) A}{da^2 (a^2 - b^2) \left(a \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) b - a - b\right)} - \frac{2b^2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) B}{da (a^2 - b^2) \left(a \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(dx+c)*(A+B*sec(dx+c))/(a+b*sec(dx+c))^2,x)`

[Out] $2/d/a^2b^3/(a^2-b^2)*\tan(1/2*dx+1/2*c)/(a*\tan(1/2*dx+1/2*c)^2-\tan(1/2*dx+1/2*c)^2*b-a-b)*A-2/d/a*b^2/(a^2-b^2)*\tan(1/2*dx+1/2*c)/(a*\tan(1/2*dx+1/2*c)^2-\tan(1/2*dx+1/2*c)^2*b-a-b)*B+6/d/a*b^2/(a-b)/(a+b)/((a-b)*(a+b))^{(1/2)*\operatorname{arctanh}(\tan(1/2*dx+1/2*c)*(a-b)/((a-b)*(a+b))^{(1/2)})}*A-4/d/a^3b^4/(a-b)/(a+b)/((a-b)*(a+b))^{(1/2)*\operatorname{arctanh}(\tan(1/2*dx+1/2*c)*(a-b)/((a-b)*(a+b))^{(1/2)})}*A-4/d*b/(a-b)/(a+b)/((a-b)*(a+b))^{(1/2)*\operatorname{arctanh}(\tan(1/2*dx+1/2*c)*(a-b)/((a-b)*(a+b))^{(1/2)})}*B+2/d/a^2b^3/(a-b)/(a+b)/((a-b)*(a+b))^{(1/2)*\operatorname{arctanh}(\tan(1/2*dx+1/2*c)*(a-b)/((a-b)*(a+b))^{(1/2)})}*B+2/d/a^2*A*\tan(1/2*dx+1/2*c)/((1+\tan(1/2*dx+1/2*c)^2)-4/d/a^3*A*\operatorname{arctan}(\tan(1/2*dx+1/2*c))*b+2/d/a^2*\operatorname{arctan}(\tan(1/2*dx+1/2*c))*B$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation may help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see 'assume?' for more details)Is 4*a^2-4*b^2 positive or negative?

mupad [B] time = 7.07, size = 3264, normalized size = 18.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x))^2,x)

[Out]
$$\frac{\left((2 \tan\left(\frac{c}{2} + \frac{d x}{2}\right))^3 (A a b^2 - 2 A^2 b^3 - A a^3 + A a^2 b + B a b^2) \right) / \left(a^2 (a + b) (a - b) \right) + \left(2 \tan\left(\frac{c}{2} + \frac{d x}{2}\right) (A a^3 - 2 A^2 b^3 - A a b^2 + A a^2 b + B a b^2) \right) / \left(a^2 (a + b) (a - b) \right) / \left(d (a + b - \tan\left(\frac{c}{2} + \frac{d x}{2}\right))^4 (a - b) + 2 b \tan\left(\frac{c}{2} + \frac{d x}{2}\right)^2 \right) + \left(\log\left(\tan\left(\frac{c}{2} + \frac{d x}{2}\right) - 1\right) (2 A b - B a) + \log\left(\tan\left(\frac{c}{2} + \frac{d x}{2}\right) + 1\right) (A b^2 - B a) \right) / \left(a^3 d \right) - \left(\log\left(\tan\left(\frac{c}{2} + \frac{d x}{2}\right) + 1\right) (A b^2 - B a) \right) / \left(a^3 d \right) - \left(b \operatorname{atan}\left(\frac{(32 \tan\left(\frac{c}{2} + \frac{d x}{2}\right) (8 A^2 b^8 + B^2 a^8 - 8 A^2 a b^7 - 2 B^2 a^7 b - 16 A^2 a^2 b^6 + 16 A^2 a^3 b^5 + 5 A^2 a^4 b^4 - 8 A^2 a^5 b^3 + 4 A^2 a^6 b^2 + 2 B^2 a^2 b^6 - 2 B^2 a^3 b^5 - 5 B^2 a^4 b^4 + 4 B^2 a^5 b^3 + 3 B^2 a^6 b^2 - 8 A B a b^7 - 4 A B a^7 b + 8 A B a^2 b^6 + 18 A B a^3 b^5 - 16 A B a^4 b^4 - 8 A B a^5 b^3 + 8 A B a^6 b^2)}{(a^6 b + a^7 - a^4 b^3 - a^5 b^2) + (b((32(A a^7 b^5 - 2 A a^6 b^6 - B a^{12} + 5 A a^8 b^4 - 3 A a^9 b^3 - 3 A a^{10} b^2 + B a^7 b^5 - 3 B a^9 b^3 + B a^{10} b^2 + 2 A a^{11} b + 2 B a^{11} b))}{(a^8 b + a^9 - a^6 b^3 - a^7 b^2) - (32 b \tan\left(\frac{c}{2} + \frac{d x}{2}\right) ((a + b)^3 (a - b)^3)^{(1/2)} (2 A b^3 + 2 B a^3 - 3 A a^2 b - B a b^2)}{(2 a^{11} b - 2 a^6 b^6 + 2 a^7 b^5 + 4 a^8 b^4 - 4 a^9 b^3 - 2 a^{10} b^2)} \right) / \left((a^6 b + a^7 - a^4 b^3 - a^5 b^2) (a^9 - a^3 b^6 + 3 a^5 b^4 - 3 a^7 b^2) \right) \right) * \left((a + b)^3 (a - b)^3 \right)^{(1/2)} (2 A b^3 + 2 B a^3 - 3 A a^2 b - B a b^2) \right) / \left(a^9 - a^3 b^6 + 3 a^5 b^4 - 3 a^7 b^2 \right) + \left(b \left((32 \tan\left(\frac{c}{2} + \frac{d x}{2}\right) (8 A^2 b^8 + B^2 a^8 - 8 A^2 a b^7 - 2 B^2 a^7 b - 16 A^2 a^2 b^6 + 16 A^2 a^3 b^5 + 5 A^2 a^4 b^4 - 8 A^2 a^5 b^3 + 4 A^2 a^6 b^2 + 2 B^2 a^2 b^6 - 2 B^2 a^3 b^5 - 5 B^2 a^4 b^4 + 4 B^2 a^5 b^3 + 3 B^2 a^6 b^2 - 8 A B a b^7 - 4 A B a^7 b + 8 A B a^2 b^6 + 18 A B a^3 b^5 - 16 A B a^4 b^4 - 8 A B a^5 b^3 + 8 A B a^6 b^2) \right) / \left(a^6 b + a^7 - a^4 b^3 - a^5 b^2 \right) - \left(b \left((32 (A a^7 b^5 - 2 A a^6 b^6 - B a^{12} + 5 A a^8 b^4 - 3 A a^9 b^3 - 3 A a^{10} b^2 + B a^7 b^5 - 3 B a^9 b^3 + B a^{10} b^2 + 2 A a^{11} b + 2 B a^{11} b)) \right) / \left(a^8 b + a^9 - a^6 b^3 - a^7 b^2 \right) + \left(32 b \tan\left(\frac{c}{2} + \frac{d x}{2}\right) ((a + b)^3 (a - b)^3)^{(1/2)} (2 A b^3 + 2 B a^3 - 3 A a^2 b - B a b^2) \right) / \left(a^9 - a^3 b^6 + 3 a^5 b^4 - 3 a^7 b^2 \right) \right) * \left((a + b)^3 (a - b)^3 \right)^{(1/2)} (2 A b^3 + 2 B a^3 - 3 A a^2 b - B a b^2) \right) / \left(a^9 - a^3 b^6 + 3 a^5 b^4 - 3 a^7 b^2 \right) + \left(b \left((32 \tan\left(\frac{c}{2} + \frac{d x}{2}\right) (8 A^2 b^8 + B^2 a^8 - 8 A^2 a b^7 - 2 B^2 a^7 b - 16 A^2 a^2 b^6 + 16 A^2 a^3 b^5 + 5 A^2 a^4 b^4 - 8 A^2 a^5 b^3 + 4 A^2 a^6 b^2 + 2 B^2 a^2 b^6 - 2 B^2 a^3 b^5 - 5 B^2 a^4 b^4 + 4 B^2 a^5 b^3 + 3 B^2 a^6 b^2 - 8 A B a b^7 - 4 A B a^7 b + 8 A B a^2 b^6 + 18 A B a^3 b^5 - 16 A B a^4 b^4 - 8 A B a^5 b^3 + 8 A B a^6 b^2) \right) / \left(a^6 b + a^7 - a^4 b^3 - a^5 b^2 \right) - \left(b \left((32 (A a^7 b^5 - 2 A a^6 b^6 - B a^{12} + 5 A a^8 b^4 - 3 A a^9 b^3 - 3 A a^{10} b^2 + B a^7 b^5 - 3 B a^9 b^3 + B a^{10} b^2 + 2 A a^{11} b + 2 B a^{11} b)) \right) / \left(a^8 b + a^9 - a^6 b^3 - a^7 b^2 \right) + \left(32 b \tan\left(\frac{c}{2} + \frac{d x}{2}\right) ((a + b)^3 (a - b)^3)^{(1/2)} (2 A b^3 + 2 B a^3 - 3 A a^2 b - B a b^2) \right) / \left(a^9 - a^3 b^6 + 3 a^5 b^4 - 3 a^7 b^2 \right) \right) * \left((a + b)^3 (a - b)^3 \right)^{(1/2)} (2 A b^3 + 2 B a^3 - 3 A a^2 b - B a b^2) \right) / \left(a^9 - a^3 b^6 + 3 a^5 b^4 - 3 a^7 b^2 \right) \right)$$


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^2))))*((a + b)^3*(a - b)^3)^(1/2)*(2*A*b^3 + 2*B*a^3 - 3*A*a^2*b - B*a*b^2)
)/(a^9 - a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2))*((a + b)^3*(a - b)^3)^(1/2)*(2*A
*b^3 + 2*B*a^3 - 3*A*a^2*b - B*a*b^2)*1i)/(a^9 - a^3*b^6 + 3*a^5*b^4 - 3*a^
7*b^2))/((64*(8*A^3*b^8 - 4*A^3*a*b^7 - 2*B^3*a^7*b - 20*A^3*a^2*b^6 + 6*A^
3*a^3*b^5 + 12*A^3*a^4*b^4 - B^3*a^3*b^5 + B^3*a^4*b^4 + 3*B^3*a^5*b^3 - 2*
B^3*a^6*b^2 - 12*A^2*B*a*b^7 + 6*A*B^2*a^2*b^6 - 5*A*B^2*a^3*b^5 - 17*A*B^2
*a^4*b^4 + 9*A*B^2*a^5*b^3 + 11*A*B^2*a^6*b^2 + 8*A^2*B*a^2*b^6 + 32*A^2*B*
a^3*b^5 - 13*A^2*B*a^4*b^4 - 20*A^2*B*a^5*b^3))/(a^8*b + a^9 - a^6*b^3 - a^
7*b^2) - (b*((32*tan(c/2 + (d*x)/2)*(8*A^2*b^8 + B^2*a^8 - 8*A^2*a*b^7 - 2*
B^2*a^7*b - 16*A^2*a^2*b^6 + 16*A^2*a^3*b^5 + 5*A^2*a^4*b^4 - 8*A^2*a^5*b^3
+ 4*A^2*a^6*b^2 + 2*B^2*a^2*b^6 - 2*B^2*a^3*b^5 - 5*B^2*a^4*b^4 + 4*B^2*a^
5*b^3 + 3*B^2*a^6*b^2 - 8*A*B*a*b^7 - 4*A*B*a^7*b + 8*A*B*a^2*b^6 + 18*A*B*
a^3*b^5 - 16*A*B*a^4*b^4 - 8*A*B*a^5*b^3 + 8*A*B*a^6*b^2)))/(a^6*b + a^7 - a
^4*b^3 - a^5*b^2) + (b*((32*(A*a^7*b^5 - 2*A*a^6*b^6 - B*a^12 + 5*A*a^8*b^4
- 3*A*a^9*b^3 - 3*A*a^10*b^2 + B*a^7*b^5 - 3*B*a^9*b^3 + B*a^10*b^2 + 2*A*
a^11*b + 2*B*a^11*b)))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2) - (32*b*tan(c/2 + (
d*x)/2))*((a + b)^3*(a - b)^3)^(1/2)*(2*A*b^3 + 2*B*a^3 - 3*A*a^2*b - B*a*b^
2)*(2*a^11*b - 2*a^6*b^6 + 2*a^7*b^5 + 4*a^8*b^4 - 4*a^9*b^3 - 2*a^10*b^2))
)/((a^6*b + a^7 - a^4*b^3 - a^5*b^2)*(a^9 - a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2)
))*((a + b)^3*(a - b)^3)^(1/2)*(2*A*b^3 + 2*B*a^3 - 3*A*a^2*b - B*a*b^2))/
(a^9 - a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2))*((a + b)^3*(a - b)^3)^(1/2)*(2*A*b^
3 + 2*B*a^3 - 3*A*a^2*b - B*a*b^2))/(a^9 - a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2)
+ (b*((32*tan(c/2 + (d*x)/2)*(8*A^2*b^8 + B^2*a^8 - 8*A^2*a*b^7 - 2*B^2*a^
7*b - 16*A^2*a^2*b^6 + 16*A^2*a^3*b^5 + 5*A^2*a^4*b^4 - 8*A^2*a^5*b^3 + 4*A
^2*a^6*b^2 + 2*B^2*a^2*b^6 - 2*B^2*a^3*b^5 - 5*B^2*a^4*b^4 + 4*B^2*a^5*b^3
+ 3*B^2*a^6*b^2 - 8*A*B*a*b^7 - 4*A*B*a^7*b + 8*A*B*a^2*b^6 + 18*A*B*a^3*b^
5 - 16*A*B*a^4*b^4 - 8*A*B*a^5*b^3 + 8*A*B*a^6*b^2))/(a^6*b + a^7 - a^4*b^3
- a^5*b^2) - (b*((32*(A*a^7*b^5 - 2*A*a^6*b^6 - B*a^12 + 5*A*a^8*b^4 - 3*A
*a^9*b^3 - 3*A*a^10*b^2 + B*a^7*b^5 - 3*B*a^9*b^3 + B*a^10*b^2 + 2*A*a^11*b
+ 2*B*a^11*b)))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2) + (32*b*tan(c/2 + (d*x)/2
))*((a + b)^3*(a - b)^3)^(1/2)*(2*A*b^3 + 2*B*a^3 - 3*A*a^2*b - B*a*b^2)*(2*
a^11*b - 2*a^6*b^6 + 2*a^7*b^5 + 4*a^8*b^4 - 4*a^9*b^3 - 2*a^10*b^2))/((a^6
*b + a^7 - a^4*b^3 - a^5*b^2)*(a^9 - a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2))))*((a
+ b)^3*(a - b)^3)^(1/2)*(2*A*b^3 + 2*B*a^3 - 3*A*a^2*b - B*a*b^2))/(a^9 -
a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2))*((a + b)^3*(a - b)^3)^(1/2)*(2*A*b^3 + 2*
B*a^3 - 3*A*a^2*b - B*a*b^2))/(a^9 - a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2))))*((a
+ b)^3*(a - b)^3)^(1/2)*(2*A*b^3 + 2*B*a^3 - 3*A*a^2*b - B*a*b^2)*2i)/(d*(
a^9 - a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2))

```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \cos(c + dx)}{(a + b \sec(c + dx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((A + B*sec(c + d*x))*cos(c + d*x)/(a + b*sec(c + d*x))**2, x)
```

$$3.326 \quad \int \frac{\cos^2(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^2} dx$$

Optimal. Leaf size=261

$$\frac{(a^2A + 2abB - 3Ab^2) \sin(c+dx) \cos(c+dx)}{2a^2d(a^2 - b^2)} + \frac{b(Ab - aB) \sin(c+dx) \cos(c+dx)}{ad(a^2 - b^2)(a + b \sec(c+dx))} + \frac{x(a^2A - 4abB + 6Ab^2)}{2a^4} - \frac{2b^2(4a^2Ab - 3a^3B + 2a^2b^2B - 3Ab^3) \sin(c+dx)}{a^3d(a^2 - b^2)} + \frac{(a^2A + 2abB - 3Ab^2) \sin(c+dx) \cos(c+dx)}{2a^2d(a^2 - b^2)}$$

[Out] $\frac{1}{2} * (A * a^2 + 6 * A * b^2 - 4 * B * a * b) * x / a^4 - 2 * b^2 * (4 * A * a^2 * b - 3 * A * b^3 - 3 * B * a^3 + 2 * B * a * b^2) * \operatorname{arctanh}((a-b)^{1/2} * \tan(1/2 * d * x + 1/2 * c) / (a+b)^{1/2}) / a^4 / (a-b)^{3/2} / (a+b)^{3/2} / d - (2 * A * a^2 * b - 3 * A * b^3 - B * a^3 + 2 * B * a * b^2) * \sin(d * x + c) / a^3 / (a^2 - b^2) / d + 1/2 * (A * a^2 - 3 * A * b^2 + 2 * B * a * b) * \cos(d * x + c) * \sin(d * x + c) / a^2 / (a^2 - b^2) / d + b * (A * b - B * a) * \cos(d * x + c) * \sin(d * x + c) / a / (a^2 - b^2) / d / (a + b * \sec(d * x + c))$

Rubi [A] time = 0.89, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4030, 4104, 3919, 3831, 2659, 208}

$$\frac{(2a^2Ab + a^3(-B) + 2ab^2B - 3Ab^3) \sin(c+dx)}{a^3d(a^2 - b^2)} + \frac{(a^2A + 2abB - 3Ab^2) \sin(c+dx) \cos(c+dx)}{2a^2d(a^2 - b^2)} - \frac{2b^2(4a^2Ab - 3a^3B + 2a^2b^2B - 3Ab^3) \sin(c+dx)}{a^3d(a^2 - b^2)}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(\operatorname{Cos}[c + d * x])^2 * (A + B * \operatorname{Sec}[c + d * x])] / (a + b * \operatorname{Sec}[c + d * x])^2, x]$

[Out] $((a^2 * A + 6 * A * b^2 - 4 * a * b * B) * x) / (2 * a^4) - (2 * b^2 * (4 * a^2 * A * b - 3 * A * b^3 - 3 * a^3 * B + 2 * a * b^2 * B) * \operatorname{ArcTanh}[\operatorname{Sqrt}[a - b] * \operatorname{Tan}[(c + d * x) / 2] / \operatorname{Sqrt}[a + b]]) / (a^4 * (a - b)^{3/2} * (a + b)^{3/2} * d) - ((2 * a^2 * A * b - 3 * A * b^3 - a^3 * B + 2 * a * b^2 * B) * \operatorname{Sin}[c + d * x]) / (a^3 * (a^2 - b^2) * d) + ((a^2 * A - 3 * A * b^2 + 2 * a * b * B) * \operatorname{Cos}[c + d * x] * \operatorname{Sin}[c + d * x]) / (2 * a^2 * (a^2 - b^2) * d) + (b * (A * b - a * B) * \operatorname{Cos}[c + d * x] * \operatorname{Sin}[c + d * x]) / (a * (a^2 - b^2) * d * (a + b * \operatorname{Sec}[c + d * x]))$

Rule 208

$\operatorname{Int}[(a + b * x^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[-(a/b), 2] * \operatorname{ArcTanh}[x / \operatorname{Rt}[-(a/b), 2]]) / a, x] /;$ $\operatorname{FreeQ}\{a, b\}, x$ && $\operatorname{NegQ}[a/b]$

Rule 2659

$\operatorname{Int}[(a + b * \sin[\operatorname{Pi}/2 + (c + d * x)])^{-1}, x_Symbol] \rightarrow \operatorname{With}\{e = \operatorname{FreeFactors}[\operatorname{Tan}[(c + d * x) / 2], x]\}, \operatorname{Dist}[(2 * e) / d, \operatorname{Subst}[\operatorname{Int}[1 / (a + b + (a - b) * e^2 * x^2), x], x, \operatorname{Tan}[(c + d * x) / 2] / e], x] /;$ $\operatorname{FreeQ}\{a, b, c, d\}, x$

&& NeQ[a^2 - b^2, 0]

Rule 3831

Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 3919

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 4030

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] :> Simp[(b*(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[A*(a^2*(m + 1) - b^2*(m + n + 1)) + a*b*B*n - a*(A*b - a*B)*(m + 1)*Csc[e + f*x] + b*(A*b - a*B)*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])

Rule 4104

Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^2} dx &= \frac{b(Ab-aB)\cos(c+dx)\sin(c+dx)}{a(a^2-b^2)d(a+b\sec(c+dx))} - \int \frac{\cos^2(c+dx)(-a^2A+3Ab^2-2abB+a(Ab-))}{a+b\sec(c+dx)} dx \\
&= \frac{(a^2A-3Ab^2+2abB)\cos(c+dx)\sin(c+dx)}{2a^2(a^2-b^2)d} + \frac{b(Ab-aB)\cos(c+dx)\sin(c+dx)}{a(a^2-b^2)d(a+b\sec(c+dx))} \\
&= -\frac{(2a^2Ab-3Ab^3-a^3B+2ab^2B)\sin(c+dx)}{a^3(a^2-b^2)d} + \frac{(a^2A-3Ab^2+2abB)\cos(c+dx)\sin(c+dx)}{2a^2(a^2-b^2)d} \\
&= \frac{(a^2A+6Ab^2-4abB)x}{2a^4} - \frac{(2a^2Ab-3Ab^3-a^3B+2ab^2B)\sin(c+dx)}{a^3(a^2-b^2)d} \\
&= \frac{(a^2A+6Ab^2-4abB)x}{2a^4} - \frac{(2a^2Ab-3Ab^3-a^3B+2ab^2B)\sin(c+dx)}{a^3(a^2-b^2)d} \\
&= \frac{(a^2A+6Ab^2-4abB)x}{2a^4} - \frac{(2a^2Ab-3Ab^3-a^3B+2ab^2B)\sin(c+dx)}{a^3(a^2-b^2)d} \\
&= \frac{(a^2A+6Ab^2-4abB)x}{2a^4} - \frac{2b^2(4a^2Ab-3Ab^3-3a^3B+2ab^2B)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^4(a-b)^{3/2}(a+b)^{3/2}d}
\end{aligned}$$

Mathematica [A] time = 1.14, size = 184, normalized size = 0.70

$$\frac{2(c+dx)(a^2A-4abB+6Ab^2)+a^2A\sin(2(c+dx))-\frac{8b^2(3a^3B-4a^2Ab-2ab^2B+3Ab^3)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}-\frac{4ab^3(aB-)}{(a-b)(a+b)}}{4a^4d}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^2,x]

[Out] (2*(a^2*A + 6*A*b^2 - 4*a*b*B)*(c + d*x) - (8*b^2*(-4*a^2*A*b + 3*A*b^3 + 3*a^3*B - 2*a*b^2*B)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2) + 4*a*(-2*A*b + a*B)*Sin[c + d*x] - (4*a*b^3*(-(A*b) + a*B)*Sin[c + d*x])/((a - b)*(a + b)*(b + a*Cos[c + d*x])) + a^2*A*Ssin[2*(c + d*x)]/(4*a^4*d)

fricas [A] time = 0.60, size = 970, normalized size = 3.72

$$\left[\frac{\left(Aa^7 - 4Ba^6b + 4Aa^5b^2 + 8Ba^4b^3 - 11Aa^3b^4 - 4Ba^2b^5 + 6Aab^6 \right) dx \cos(dx + c) + \left(Aa^6b - 4Ba^5b^2 + 4Aa^4b^3 \right)}{\dots} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm="fricas")

[Out] [1/2*((A*a^7 - 4*B*a^6*b + 4*A*a^5*b^2 + 8*B*a^4*b^3 - 11*A*a^3*b^4 - 4*B*a^2*b^5 + 6*A*a*b^6)*d*x*cos(d*x + c) + (A*a^6*b - 4*B*a^5*b^2 + 4*A*a^4*b^3 + 8*B*a^3*b^4 - 11*A*a^2*b^5 - 4*B*a*b^6 + 6*A*b^7)*d*x + (3*B*a^3*b^3 - 4*A*a^2*b^4 - 2*B*a*b^5 + 3*A*b^6 + (3*B*a^4*b^2 - 4*A*a^3*b^3 - 2*B*a^2*b^4 + 3*A*a*b^5)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + (2*B*a^6*b - 4*A*a^5*b^2 - 6*B*a^4*b^3 + 10*A*a^3*b^4 + 4*B*a^2*b^5 - 6*A*a*b^6 + (A*a^7 - 2*A*a^5*b^2 + A*a^3*b^4)*cos(d*x + c)^2 + (2*B*a^7 - 3*A*a^6*b - 4*B*a^5*b^2 + 6*A*a^4*b^3 + 2*B*a^3*b^4 - 3*A*a^2*b^5)*cos(d*x + c))*sin(d*x + c))/((a^9 - 2*a^7*b^2 + a^5*b^4)*d*cos(d*x + c) + (a^8*b - 2*a^6*b^3 + a^4*b^5)*d), 1/2*((A*a^7 - 4*B*a^6*b + 4*A*a^5*b^2 + 8*B*a^4*b^3 - 11*A*a^3*b^4 - 4*B*a^2*b^5 + 6*A*a*b^6)*d*x*cos(d*x + c) + (A*a^6*b - 4*B*a^5*b^2 + 4*A*a^4*b^3 + 8*B*a^3*b^4 - 11*A*a^2*b^5 - 4*B*a*b^6 + 6*A*b^7)*d*x + 2*(3*B*a^3*b^3 - 4*A*a^2*b^4 - 2*B*a*b^5 + 3*A*b^6 + (3*B*a^4*b^2 - 4*A*a^3*b^3 - 2*B*a^2*b^4 + 3*A*a*b^5)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) + (2*B*a^6*b - 4*A*a^5*b^2 - 6*B*a^4*b^3 + 10*A*a^3*b^4 + 4*B*a^2*b^5 - 6*A*a*b^6 + (A*a^7 - 2*A*a^5*b^2 + A*a^3*b^4)*cos(d*x + c)^2 + (2*B*a^7 - 3*A*a^6*b - 4*B*a^5*b^2 + 6*A*a^4*b^3 + 2*B*a^3*b^4 - 3*A*a^2*b^5)*cos(d*x + c))*sin(d*x + c))/((a^9 - 2*a^7*b^2 + a^5*b^4)*d*cos(d*x + c) + (a^8*b - 2*a^6*b^3 + a^4*b^5)*d)]

giac [A] time = 1.16, size = 340, normalized size = 1.30

$$\frac{4(3Ba^3b^2 - 4Aa^2b^3 - 2Bab^4 + 3Ab^5) \left(\pi \left[\frac{dx+c}{2\pi} + \frac{1}{2} \right] \operatorname{sgn}(-2a+2b) + \arctan \left(-\frac{a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - b \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{\sqrt{-a^2+b^2}} \right) \right)}{(a^6 - a^4b^2) \sqrt{-a^2+b^2}} + \frac{4 \left(Bab^3 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - Ab^4 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) \right)}{(a^5 - a^3b^2) \left(a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - b \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) \right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm="giac")

[Out] $\frac{1}{2}*(4*(3*B*a^3*b^2 - 4*A*a^2*b^3 - 2*B*a*b^4 + 3*A*b^5)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*\tan(1/2*d*x + 1/2*c) - b*\tan(1/2*d*x + 1/2*c))/\sqrt{-a^2 + b^2}))/((a^6 - a^4*b^2)*\sqrt{-a^2 + b^2}) + 4*(B*a*b^3*\tan(1/2*d*x + 1/2*c) - A*b^4*\tan(1/2*d*x + 1/2*c))/((a^5 - a^3*b^2)*(a*\tan(1/2*d*x + 1/2*c)^2 - b*\tan(1/2*d*x + 1/2*c)^2 - a - b)) + (A*a^2 - 4*B*a*b + 6*A*b^2)*(d*x + c)/a^4 - 2*(A*a*\tan(1/2*d*x + 1/2*c)^3 - 2*B*a*\tan(1/2*d*x + 1/2*c)^3 + 4*A*b*\tan(1/2*d*x + 1/2*c)^3 - A*a*\tan(1/2*d*x + 1/2*c) - 2*B*a*\tan(1/2*d*x + 1/2*c) + 4*A*b*\tan(1/2*d*x + 1/2*c))/((\tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^3))/d$

maple [B] time = 1.10, size = 651, normalized size = 2.49

$$\frac{2b^4 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) A}{d a^3 (a^2 - b^2) \left(a \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right) \right) - \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right) \right) b - a - b \right)} + \frac{2b^3 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) B}{d a^2 (a^2 - b^2) \left(a \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right) \right) - \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right) \right) - \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right) \right) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x)

[Out] $-2/d*b^4/a^3/(a^2-b^2)*\tan(1/2*d*x+1/2*c)/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)*A+2/d*b^3/a^2/(a^2-b^2)*\tan(1/2*d*x+1/2*c)/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)*B-8/d/a^2/(a-b)/(a+b)/((a-b)*(a+b))^{1/2}*\operatorname{arctanh}(\tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^{1/2})*A*b^3+6/d*b^5/a^4/(a-b)/(a+b)/((a-b)*(a+b))^{1/2}*\operatorname{arctanh}(\tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^{1/2})*A+6/d*b^2/a/(a-b)/(a+b)/((a-b)*(a+b))^{1/2}*\operatorname{arctanh}(\tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^{1/2})*B-4/d*b^4/a^3/(a-b)/(a+b)/((a-b)*(a+b))^{1/2}*\operatorname{arctanh}(\tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^{1/2})*B-1/d/a^2/(1+\tan(1/2*d*x+1/2*c)^2)^2*\tan(1/2*d*x+1/2*c)^3*A-4/d/a^3/(1+\tan(1/2*d*x+1/2*c)^2)^2*\tan(1/2*d*x+1/2*c)^3*A*b+2/d/a^2/(1+\tan(1/2*d*x+1/2*c)^2)^2*B*\tan(1/2*d*x+1/2*c)^3+1/d/a^2/(1+\tan(1/2*d*x+1/2*c)^2)^2*A*\tan(1/2*d*x+1/2*c)-4/d/a^3/(1+\tan(1/2*d*x+1/2*c)^2)^2*\tan(1/2*d*x+1/2*c)*A*b+2/d/a^2/(1+\tan(1/2*d*x+1/2*c)^2)^2*B*\tan(1/2*d*x+1/2*c)+1/d/a^2*\operatorname{arctan}(\tan(1/2*d*x+1/2*c))*A+6/d/a^4*\operatorname{arctan}(\tan(1/2*d*x+1/2*c))*A*b^2-4/d/a^3*\operatorname{arctan}(\tan(1/2*d*x+1/2*c))*B*b$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation may help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see 'assume?' for more details) Is 4*a^2-4*b^2 positive or negative?

mupad [B] time = 11.11, size = 6731, normalized size = 25.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)^2*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x))^2,x)

[Out] (b^2*atan(((b^2*((a + b)^3*(a - b)^3)^(1/2))*((8*tan(c/2 + (d*x)/2)*(A^2*a^10 + 72*A^2*b^10 - 72*A^2*a*b^9 - 2*A^2*a^9*b - 120*A^2*a^2*b^8 + 120*A^2*a^3*b^7 + 17*A^2*a^4*b^6 - 26*A^2*a^5*b^5 + 23*A^2*a^6*b^4 - 20*A^2*a^7*b^3 + 11*A^2*a^8*b^2 + 32*B^2*a^2*b^8 - 32*B^2*a^3*b^7 - 64*B^2*a^4*b^6 + 64*B^2*a^5*b^5 + 20*B^2*a^6*b^4 - 32*B^2*a^7*b^3 + 16*B^2*a^8*b^2 - 96*A*B*a*b^9 - 8*A*B*a^9*b + 96*A*B*a^2*b^8 + 176*A*B*a^3*b^7 - 176*A*B*a^4*b^6 - 40*A*B*a^5*b^5 + 64*A*B*a^6*b^4 - 40*A*B*a^7*b^3 + 16*A*B*a^8*b^2)))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2) + (b^2*((8*(2*A*a^15 - 12*A*a^8*b^7 + 6*A*a^9*b^6 + 28*A*a^10*b^5 - 14*A*a^11*b^4 - 16*A*a^12*b^3 + 6*A*a^13*b^2 + 8*B*a^9*b^6 - 4*B*a^10*b^5 - 20*B*a^11*b^4 + 12*B*a^12*b^3 + 12*B*a^13*b^2 - 8*B*a^14*b)))/(a^11*b + a^12 - a^9*b^3 - a^10*b^2) - (8*b^2*tan(c/2 + (d*x)/2))*((a + b)^3*(a - b)^3)^(1/2)*(3*A*b^3 + 3*B*a^3 - 4*A*a^2*b - 2*B*a*b^2)*(8*a^13*b - 8*a^8*b^6 + 8*a^9*b^5 + 16*a^10*b^4 - 16*a^11*b^3 - 8*a^12*b^2)))/((a^8*b + a^9 - a^6*b^3 - a^7*b^2)*(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2)))*((a + b)^3*(a - b)^3)^(1/2)*(3*A*b^3 + 3*B*a^3 - 4*A*a^2*b - 2*B*a*b^2))/(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2))*((a + b)^3*(a - b)^3)^(1/2))*((8*tan(c/2 + (d*x)/2)*(A^2*a^10 + 72*A^2*b^10 - 72*A^2*a*b^9 - 2*A^2*a^9*b - 120*A^2*a^2*b^8 + 120*A^2*a^3*b^7 + 17*A^2*a^4*b^6 - 26*A^2*a^5*b^5 + 23*A^2*a^6*b^4 - 20*A^2*a^7*b^3 + 11*A^2*a^8*b^2 + 32*B^2*a^2*b^8 - 32*B^2*a^3*b^7 - 64*B^2*a^4*b^6 + 64*B^2*a^5*b^5 + 20*B^2*a^6*b^4 - 32*B^2*a^7*b^3 + 16*B^2*a^8*b^2 - 96*A*B*a*b^9 - 8*A*B*a^9*b + 96*A*B*a^2*b^8 + 176*A*B*a^3*b^7 - 176*A*B*a^4*b^6 - 40*A*B*a^5*b^5 + 64*A*B*a^6*b^4 - 40*A*B*a^7*b^3 + 16*A*B*a^8*b^2)))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2) - (b^2*((8*(2*A*a^15 - 12*A*a^8*b^7 + 6*A*a^9*b^6 + 28*A*a^10*b^5 - 14*A*a^11*b^4 - 16*A*a^12*b^3 + 6*A*a^13*b^2 + 8*B*a^9*b^6 - 4*B*a^10*b^5 - 20*B*a^11*b^4 + 12*B*a^12*b^3 + 12*B*a^13*b^2 - 8*B*a^14*b)))/(a^11*b + a^12 - a^9*b^3 - a^10*b^2) + (8*b^2*tan(c/2 + (d*x)/2))*((a + b)^3*(a - b)^3)^(1/2)*(3*A*b^3 + 3*B*a^3 - 4*A*a^2*b - 2*B*a*b^2)*(8*a^13*b - 8*a^8*b^6 + 8*a^9*b^5 + 16*a^10*b^4 - 16*a^11*b^3 - 8*a^12*b^2)))/((a^8*b + a^9 - a^6*b^3 - a^7*b^2)*(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2)))

$$\begin{aligned}
& b^6 + 3a^6b^4 - 3a^8b^2)) * ((a + b)^3 * (a - b)^3)^{(1/2)} * (3A*b^3 + 3B*a^3 - 4A*a^2*b - 2B*a*b^2)) / (a^{10} - a^4*b^6 + 3a^6*b^4 - 3a^8*b^2)) * (3A*b^3 + 3B*a^3 - 4A*a^2*b - 2B*a*b^2) * i) / (a^{10} - a^4*b^6 + 3a^6*b^4 - 3a^8*b^2)) / ((16*(108*A^3*b^{11} - 54*A^3*a*b^{10} - 216*A^3*a^2*b^9 + 81*A^3*a^3*b^8 + 63*A^3*a^4*b^7 - 9*A^3*a^5*b^6 + 41*A^3*a^6*b^5 - 4*A^3*a^7*b^4 + 4*A^3*a^8*b^3 - 32*B^3*a^3*b^8 + 16*B^3*a^4*b^7 + 80*B^3*a^5*b^6 - 24*B^3*a^6*b^5 - 48*B^3*a^7*b^4 - 216*A^2*B*a*b^{10} + 144*A*B^2*a^2*b^9 - 72*A*B^2*a^3*b^8 - 336*A*B^2*a^4*b^7 + 108*A*B^2*a^5*b^6 + 168*A*B^2*a^6*b^5 - 6*A*B^2*a^7*b^4 + 24*A*B^2*a^8*b^3 + 108*A^2*B*a^2*b^9 + 468*A^2*B*a^3*b^8 - 162*A^2*B*a^4*b^7 - 186*A^2*B*a^5*b^6 + 15*A^2*B*a^6*b^5 - 63*A^2*B*a^7*b^4 + 3*A^2*B*a^8*b^3 - 3*A^2*B*a^9*b^2)) / (a^{11}*b + a^{12} - a^9*b^3 - a^{10}*b^2) - (b^2*((a + b)^3*(a - b)^3)^{(1/2)}*((8*tan(c/2 + (d*x)/2)*(A^2*a^{10} + 72*A^2*b^{10} - 72*A^2*a*b^9 - 2*A^2*a^9*b - 120*A^2*a^2*b^8 + 120*A^2*a^3*b^7 + 17*A^2*a^4*b^6 - 26*A^2*a^5*b^5 + 23*A^2*a^6*b^4 - 20*A^2*a^7*b^3 + 11*A^2*a^8*b^2 + 32*B^2*a^2*b^8 - 32*B^2*a^3*b^7 - 64*B^2*a^4*b^6 + 64*B^2*a^5*b^5 + 20*B^2*a^6*b^4 - 32*B^2*a^7*b^3 + 16*B^2*a^8*b^2 - 96*A*B*a*b^9 - 8*A*B*a^9*b + 96*A*B*a^2*b^8 + 176*A*B*a^3*b^7 - 176*A*B*a^4*b^6 - 40*A*B*a^5*b^5 + 64*A*B*a^6*b^4 - 40*A*B*a^7*b^3 + 16*A*B*a^8*b^2)) / (a^8*b + a^9 - a^6*b^3 - a^7*b^2) + (b^2*((8*(2*A*a^{15} - 12*A*a^8*b^7 + 6*A*a^9*b^6 + 28*A*a^{10}*b^5 - 14*A*a^{11}*b^4 - 16*A*a^{12}*b^3 + 6*A*a^{13}*b^2 + 8*B*a^9*b^6 - 4*B*a^{10}*b^5 - 20*B*a^{11}*b^4 + 12*B*a^{12}*b^3 + 12*B*a^{13}*b^2 - 8*B*a^{14}*b)) / (a^{11}*b + a^{12} - a^9*b^3 - a^{10}*b^2) - (8*b^2*tan(c/2 + (d*x)/2)*((a + b)^3*(a - b)^3)^{(1/2)}*(3A*b^3 + 3B*a^3 - 4A*a^2*b - 2B*a*b^2)*(8*a^{13}*b - 8*a^8*b^6 + 8*a^9*b^5 + 16*a^{10}*b^4 - 16*a^{11}*b^3 - 8*a^{12}*b^2)) / ((a^8*b + a^9 - a^6*b^3 - a^7*b^2)*(a^{10} - a^4*b^6 + 3a^6*b^4 - 3a^8*b^2))) * ((a + b)^3*(a - b)^3)^{(1/2)}*(3A*b^3 + 3B*a^3 - 4A*a^2*b - 2B*a*b^2)) / (a^{10} - a^4*b^6 + 3a^6*b^4 - 3a^8*b^2)) * (3A*b^3 + 3B*a^3 - 4A*a^2*b - 2B*a*b^2)) / (a^{10} - a^4*b^6 + 3a^6*b^4 - 3a^8*b^2) + (b^2*((a + b)^3*(a - b)^3)^{(1/2)}*((8*tan(c/2 + (d*x)/2)*(A^2*a^{10} + 72*A^2*b^{10} - 72*A^2*a*b^9 - 2*A^2*a^9*b - 120*A^2*a^2*b^8 + 120*A^2*a^3*b^7 + 17*A^2*a^4*b^6 - 26*A^2*a^5*b^5 + 23*A^2*a^6*b^4 - 20*A^2*a^7*b^3 + 11*A^2*a^8*b^2 + 32*B^2*a^2*b^8 - 32*B^2*a^3*b^7 - 64*B^2*a^4*b^6 + 64*B^2*a^5*b^5 + 20*B^2*a^6*b^4 - 32*B^2*a^7*b^3 + 16*B^2*a^8*b^2 - 96*A*B*a*b^9 - 8*A*B*a^9*b + 96*A*B*a^2*b^8 + 176*A*B*a^3*b^7 - 176*A*B*a^4*b^6 - 40*A*B*a^5*b^5 + 64*A*B*a^6*b^4 - 40*A*B*a^7*b^3 + 16*A*B*a^8*b^2)) / (a^8*b + a^9 - a^6*b^3 - a^7*b^2) - (b^2*((8*(2*A*a^{15} - 12*A*a^8*b^7 + 6*A*a^9*b^6 + 28*A*a^{10}*b^5 - 14*A*a^{11}*b^4 - 16*A*a^{12}*b^3 + 6*A*a^{13}*b^2 + 8*B*a^9*b^6 - 4*B*a^{10}*b^5 - 20*B*a^{11}*b^4 + 12*B*a^{12}*b^3 + 12*B*a^{13}*b^2 - 8*B*a^{14}*b)) / (a^{11}*b + a^{12} - a^9*b^3 - a^{10}*b^2) + (8*b^2*tan(c/2 + (d*x)/2)*((a + b)^3*(a - b)^3)^{(1/2)}*(3A*b^3 + 3B*a^3 - 4A*a^2*b - 2*B*a*b^2)*(8*a^{13}*b - 8*a^8*b^6 + 8*a^9*b^5 + 16*a^{10}*b^4 - 16*a^{11}*b^3 - 8*a^{12}*b^2)) / ((a^8*b + a^9 - a^6*b^3 - a^7*b^2)*(a^{10} - a^4*b^6 + 3a^6*b^4 - 3a^8*b^2))) * ((a + b)^3*(a - b)^3)^{(1/2)}*(3A*b^3 + 3B*a^3 - 4A*a^2*b - 2*B*a*b^2)) / (a^{10} - a^4*b^6 + 3a^6*b^4 - 3a^8*b^2)) * (3A*b^3 + 3B*a^3 - 4A*a^2*b - 2*B*a*b^2)) / (a^{10} - a^4*b^6 + 3a^6*b^4 - 3a^8*b^2)) * ((a + b)^3*(a - b)^3)^{(1/2)}*(3A*b^3 + 3B*a^3 - 4A*a^2*b - 2*B*a*b^2)) / (d*(a^
\end{aligned}$$

$$\begin{aligned}
& 10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2)) - (\operatorname{atan}(-(((8*\tan(c/2 + (d*x)/2)*(A \\
& ^2*a^{10} + 72*A^2*b^{10} - 72*A^2*a*b^9 - 2*A^2*a^9*b - 120*A^2*a^2*b^8 + 120* \\
& A^2*a^3*b^7 + 17*A^2*a^4*b^6 - 26*A^2*a^5*b^5 + 23*A^2*a^6*b^4 - 20*A^2*a^7 \\
& *b^3 + 11*A^2*a^8*b^2 + 32*B^2*a^2*b^8 - 32*B^2*a^3*b^7 - 64*B^2*a^4*b^6 + \\
& 64*B^2*a^5*b^5 + 20*B^2*a^6*b^4 - 32*B^2*a^7*b^3 + 16*B^2*a^8*b^2 - 96*A*B* \\
& a*b^9 - 8*A*B*a^9*b + 96*A*B*a^2*b^8 + 176*A*B*a^3*b^7 - 176*A*B*a^4*b^6 - \\
& 40*A*B*a^5*b^5 + 64*A*B*a^6*b^4 - 40*A*B*a^7*b^3 + 16*A*B*a^8*b^2)))/(a^8*b \\
& + a^9 - a^6*b^3 - a^7*b^2) + (((8*(2*A*a^{15} - 12*A*a^8*b^7 + 6*A*a^9*b^6 + \\
& 28*A*a^{10}*b^5 - 14*A*a^{11}*b^4 - 16*A*a^{12}*b^3 + 6*A*a^{13}*b^2 + 8*B*a^9*b^6 \\
& - 4*B*a^{10}*b^5 - 20*B*a^{11}*b^4 + 12*B*a^{12}*b^3 + 12*B*a^{13}*b^2 - 8*B*a^{14}*b \\
&)))/(a^{11}*b + a^{12} - a^9*b^3 - a^{10}*b^2) - (4*\tan(c/2 + (d*x)/2)*(A*a^2*1i + \\
& A*b^2*6i - B*a*b*4i)*(8*a^{13}*b - 8*a^8*b^6 + 8*a^9*b^5 + 16*a^{10}*b^4 - 16* \\
& a^{11}*b^3 - 8*a^{12}*b^2))/(a^4*(a^8*b + a^9 - a^6*b^3 - a^7*b^2)))*(A*a^2*1i \\
& + A*b^2*6i - B*a*b*4i))/(2*a^4))*(A*a^2*1i + A*b^2*6i - B*a*b*4i)*1i)/(2*a^ \\
& 4) + (((8*\tan(c/2 + (d*x)/2)*(A^2*a^{10} + 72*A^2*b^{10} - 72*A^2*a*b^9 - 2*A^2 \\
& *a^9*b - 120*A^2*a^2*b^8 + 120*A^2*a^3*b^7 + 17*A^2*a^4*b^6 - 26*A^2*a^5*b^ \\
& 5 + 23*A^2*a^6*b^4 - 20*A^2*a^7*b^3 + 11*A^2*a^8*b^2 + 32*B^2*a^2*b^8 - 32* \\
& B^2*a^3*b^7 - 64*B^2*a^4*b^6 + 64*B^2*a^5*b^5 + 20*B^2*a^6*b^4 - 32*B^2*a^7 \\
& *b^3 + 16*B^2*a^8*b^2 - 96*A*B*a*b^9 - 8*A*B*a^9*b + 96*A*B*a^2*b^8 + 176*A \\
& *B*a^3*b^7 - 176*A*B*a^4*b^6 - 40*A*B*a^5*b^5 + 64*A*B*a^6*b^4 - 40*A*B*a^7 \\
& *b^3 + 16*A*B*a^8*b^2))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2) - (((8*(2*A*a^{15} \\
& - 12*A*a^8*b^7 + 6*A*a^9*b^6 + 28*A*a^{10}*b^5 - 14*A*a^{11}*b^4 - 16*A*a^{12}*b^ \\
& 3 + 6*A*a^{13}*b^2 + 8*B*a^9*b^6 - 4*B*a^{10}*b^5 - 20*B*a^{11}*b^4 + 12*B*a^{12}*b \\
& ^3 + 12*B*a^{13}*b^2 - 8*B*a^{14}*b)))/(a^{11}*b + a^{12} - a^9*b^3 - a^{10}*b^2) + (4 \\
& *tan(c/2 + (d*x)/2)*(A*a^2*1i + A*b^2*6i - B*a*b*4i)*(8*a^{13}*b - 8*a^8*b^6 \\
& + 8*a^9*b^5 + 16*a^{10}*b^4 - 16*a^{11}*b^3 - 8*a^{12}*b^2))/(a^4*(a^8*b + a^9 - \\
& a^6*b^3 - a^7*b^2)))*(A*a^2*1i + A*b^2*6i - B*a*b*4i))/(2*a^4))*(A*a^2*1i + \\
& A*b^2*6i - B*a*b*4i)*1i)/(2*a^4))/((16*(108*A^3*b^{11} - 54*A^3*a*b^{10} - 216 \\
& *A^3*a^2*b^9 + 81*A^3*a^3*b^8 + 63*A^3*a^4*b^7 - 9*A^3*a^5*b^6 + 41*A^3*a^6 \\
& *b^5 - 4*A^3*a^7*b^4 + 4*A^3*a^8*b^3 - 32*B^3*a^3*b^8 + 16*B^3*a^4*b^7 + 80 \\
& *B^3*a^5*b^6 - 24*B^3*a^6*b^5 - 48*B^3*a^7*b^4 - 216*A^2*B*a*b^{10} + 144*A*B \\
& ^2*a^2*b^9 - 72*A*B^2*a^3*b^8 - 336*A*B^2*a^4*b^7 + 108*A*B^2*a^5*b^6 + 168 \\
& *A*B^2*a^6*b^5 - 6*A*B^2*a^7*b^4 + 24*A*B^2*a^8*b^3 + 108*A^2*B*a^2*b^9 + 4 \\
& 68*A^2*B*a^3*b^8 - 162*A^2*B*a^4*b^7 - 186*A^2*B*a^5*b^6 + 15*A^2*B*a^6*b^5 \\
& - 63*A^2*B*a^7*b^4 + 3*A^2*B*a^8*b^3 - 3*A^2*B*a^9*b^2))/(a^{11}*b + a^{12} - \\
& a^9*b^3 - a^{10}*b^2) - (((8*\tan(c/2 + (d*x)/2)*(A^2*a^{10} + 72*A^2*b^{10} - 72* \\
& A^2*a*b^9 - 2*A^2*a^9*b - 120*A^2*a^2*b^8 + 120*A^2*a^3*b^7 + 17*A^2*a^4*b^ \\
& 6 - 26*A^2*a^5*b^5 + 23*A^2*a^6*b^4 - 20*A^2*a^7*b^3 + 11*A^2*a^8*b^2 + 32* \\
& B^2*a^2*b^8 - 32*B^2*a^3*b^7 - 64*B^2*a^4*b^6 + 64*B^2*a^5*b^5 + 20*B^2*a^6 \\
& *b^4 - 32*B^2*a^7*b^3 + 16*B^2*a^8*b^2 - 96*A*B*a*b^9 - 8*A*B*a^9*b + 96*A* \\
& B*a^2*b^8 + 176*A*B*a^3*b^7 - 176*A*B*a^4*b^6 - 40*A*B*a^5*b^5 + 64*A*B*a^6 \\
& *b^4 - 40*A*B*a^7*b^3 + 16*A*B*a^8*b^2))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2) \\
& + (((8*(2*A*a^{15} - 12*A*a^8*b^7 + 6*A*a^9*b^6 + 28*A*a^{10}*b^5 - 14*A*a^{11}*b \\
& ^4 - 16*A*a^{12}*b^3 + 6*A*a^{13}*b^2 + 8*B*a^9*b^6 - 4*B*a^{10}*b^5 - 20*B*a^{11}* \\
& b^4 + 12*B*a^{12}*b^3 + 12*B*a^{13}*b^2 - 8*B*a^{14}*b)))/(a^{11}*b + a^{12} - a^9*b^3
\end{aligned}$$

```

- a^10*b^2) - (4*tan(c/2 + (d*x)/2)*(A*a^2*1i + A*b^2*6i - B*a*b*4i)*(8*a^
13*b - 8*a^8*b^6 + 8*a^9*b^5 + 16*a^10*b^4 - 16*a^11*b^3 - 8*a^12*b^2))/(a^
4*(a^8*b + a^9 - a^6*b^3 - a^7*b^2)))*(A*a^2*1i + A*b^2*6i - B*a*b*4i))/(2*
a^4))*(A*a^2*1i + A*b^2*6i - B*a*b*4i))/(2*a^4) + (((8*tan(c/2 + (d*x)/2)*(
A^2*a^10 + 72*A^2*b^10 - 72*A^2*a*b^9 - 2*A^2*a^9*b - 120*A^2*a^2*b^8 + 120
*A^2*a^3*b^7 + 17*A^2*a^4*b^6 - 26*A^2*a^5*b^5 + 23*A^2*a^6*b^4 - 20*A^2*a^
7*b^3 + 11*A^2*a^8*b^2 + 32*B^2*a^2*b^8 - 32*B^2*a^3*b^7 - 64*B^2*a^4*b^6 +
64*B^2*a^5*b^5 + 20*B^2*a^6*b^4 - 32*B^2*a^7*b^3 + 16*B^2*a^8*b^2 - 96*A*B
*a*b^9 - 8*A*B*a^9*b + 96*A*B*a^2*b^8 + 176*A*B*a^3*b^7 - 176*A*B*a^4*b^6 -
40*A*B*a^5*b^5 + 64*A*B*a^6*b^4 - 40*A*B*a^7*b^3 + 16*A*B*a^8*b^2))/(a^8*b
+ a^9 - a^6*b^3 - a^7*b^2) - (((8*(2*A*a^15 - 12*A*a^8*b^7 + 6*A*a^9*b^6 +
28*A*a^10*b^5 - 14*A*a^11*b^4 - 16*A*a^12*b^3 + 6*A*a^13*b^2 + 8*B*a^9*b^6
- 4*B*a^10*b^5 - 20*B*a^11*b^4 + 12*B*a^12*b^3 + 12*B*a^13*b^2 - 8*B*a^14*
b)))/(a^11*b + a^12 - a^9*b^3 - a^10*b^2) + (4*tan(c/2 + (d*x)/2)*(A*a^2*1i
+ A*b^2*6i - B*a*b*4i)*(8*a^13*b - 8*a^8*b^6 + 8*a^9*b^5 + 16*a^10*b^4 - 16
*a^11*b^3 - 8*a^12*b^2))/(a^4*(a^8*b + a^9 - a^6*b^3 - a^7*b^2)))*(A*a^2*1i
+ A*b^2*6i - B*a*b*4i))/(2*a^4))*(A*a^2*1i + A*b^2*6i - B*a*b*4i))/(2*a^4)
))*(A*a^2*1i + A*b^2*6i - B*a*b*4i)*1i)/(a^4*d) - ((tan(c/2 + (d*x)/2)^5*(A
*a^4 + 6*A*b^4 - 2*B*a^4 - 5*A*a^2*b^2 + 2*B*a^2*b^2 - 3*A*a*b^3 + 3*A*a^3*
b - 4*B*a*b^3 + 2*B*a^3*b))/((a^3*b - a^4)*(a + b)) + (tan(c/2 + (d*x)/2)*(
A*a^4 + 6*A*b^4 + 2*B*a^4 - 5*A*a^2*b^2 - 2*B*a^2*b^2 + 3*A*a*b^3 - 3*A*a^3
*b - 4*B*a*b^3 + 2*B*a^3*b))/((a^3*b - a^4)*(a + b)) - (2*tan(c/2 + (d*x)/2
)^3*(A*a^4 - 6*A*b^4 + 3*A*a^2*b^2 + 4*B*a*b^3 - 2*B*a^3*b))/(a*(a^2*b - a^
3)*(a + b)))/(d*(a + b + tan(c/2 + (d*x)/2)^2*(a + 3*b) - tan(c/2 + (d*x)/2
)^4*(a - 3*b) - tan(c/2 + (d*x)/2)^6*(a - b)))

```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \cos^2(c + dx)}{(a + b \sec(c + dx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**2, x)

[Out] Integral((A + B*sec(c + d*x))*cos(c + d*x)**2/(a + b*sec(c + d*x))**2, x)

$$3.327 \quad \int \frac{\cos^3(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^2} dx$$

Optimal. Leaf size=346

$$\frac{(a^2 A + 3abB - 4Ab^2) \sin(c + dx) \cos^2(c + dx)}{3a^2 d (a^2 - b^2)} + \frac{b(Ab - aB) \sin(c + dx) \cos^2(c + dx)}{ad (a^2 - b^2) (a + b \sec(c + dx))} - \frac{(a^3(-B) + 2a^2 Ab + 3ab^2 B)}{2a^3 c}$$

[Out] $-1/2*(2*A*a^2*b+8*A*b^3-B*a^3-6*B*a*b^2)*x/a^5+2*b^3*(5*A*a^2*b-4*A*b^3-4*B*a^3+3*B*a*b^2)*\operatorname{arctanh}((a-b)^{(1/2)}*\tan(1/2*d*x+1/2*c)/(a+b)^{(1/2)})/a^5/(a-b)^{(3/2)}/(a+b)^{(3/2)}/d+1/3*(2*A*a^4+7*A*a^2*b^2-12*A*b^4-6*B*a^3*b+9*B*a*b^3)*\sin(d*x+c)/a^4/(a^2-b^2)/d-1/2*(2*A*a^2*b-4*A*b^3-B*a^3+3*B*a*b^2)*\cos(d*x+c)*\sin(d*x+c)/a^3/(a^2-b^2)/d+1/3*(A*a^2-4*A*b^2+3*B*a*b)*\cos(d*x+c)^2*\sin(d*x+c)/a^2/(a^2-b^2)/d+b*(A*b-B*a)*\cos(d*x+c)^2*\sin(d*x+c)/a/(a^2-b^2)/d/(a+b*\sec(d*x+c))$

Rubi [A] time = 1.27, antiderivative size = 346, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4030, 4104, 3919, 3831, 2659, 208}

$$\frac{(7a^2 Ab^2 + 2a^4 A - 6a^3 bB + 9ab^3 B - 12Ab^4) \sin(c + dx)}{3a^4 d (a^2 - b^2)} + \frac{(a^2 A + 3abB - 4Ab^2) \sin(c + dx) \cos^2(c + dx)}{3a^2 d (a^2 - b^2)} - \frac{(2a^3 B)}{2a^3 c}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(\operatorname{Cos}[c + d*x]^3*(A + B*\operatorname{Sec}[c + d*x]))/(a + b*\operatorname{Sec}[c + d*x])^2, x]$

[Out] $-((2*a^2*A*b + 8*A*b^3 - a^3*B - 6*a*b^2*B)*x)/(2*a^5) + (2*b^3*(5*a^2*A*b - 4*A*b^3 - 4*a^3*B + 3*a*b^2*B)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a - b]*\operatorname{Tan}[(c + d*x)/2])/(\operatorname{Sqrt}[a + b])])/(a^5*(a - b)^{(3/2)}*(a + b)^{(3/2)}*d) + ((2*a^4*A + 7*a^2*A*b^2 - 12*A*b^4 - 6*a^3*b*B + 9*a*b^3*B)*\operatorname{Sin}[c + d*x])/(3*a^4*(a^2 - b^2)*d) - ((2*a^2*A*b - 4*A*b^3 - a^3*B + 3*a*b^2*B)*\operatorname{Cos}[c + d*x]*\operatorname{Sin}[c + d*x])/(2*a^3*(a^2 - b^2)*d) + ((a^2*A - 4*A*b^2 + 3*a*b*B)*\operatorname{Cos}[c + d*x]^2*\operatorname{Sin}[c + d*x])/(3*a^2*(a^2 - b^2)*d) + (b*(A*b - a*B)*\operatorname{Cos}[c + d*x]^2*\operatorname{Sin}[c + d*x])/(a*(a^2 - b^2)*d*(a + b*\operatorname{Sec}[c + d*x]))$

Rule 208

$\operatorname{Int}[(a + b*x^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[-(a/b), 2]*\operatorname{ArcTanh}[x/\operatorname{Rt}[-(a/b), 2]])/a, x] /; \operatorname{FreeQ}\{a, b\}, x] \ \&\& \ \operatorname{NegQ}[a/b]$

Rule 2659

```
Int[((a_) + (b_)*sin[Pi/2 + (c_) + (d_)*(x_)])^(-1), x_Symbol] := With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3831

```
Int[csc[(e_) + (f_)*(x_)]/(csc[(e_) + (f_)*(x_)]*(b_) + (a_)), x_Symbol] := Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3919

```
Int[(csc[(e_) + (f_)*(x_)]*(d_) + (c_))/(csc[(e_) + (f_)*(x_)]*(b_) + (a_)), x_Symbol] := Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 4030

```
Int[(csc[(e_) + (f_)*(x_)]*(d_)^(n_)*(csc[(e_) + (f_)*(x_)]*(b_) + (a_))^(m_)*(csc[(e_) + (f_)*(x_)]*(B_) + (A_)), x_Symbol] := Simp[(b*(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[A*(a^2*(m + 1) - b^2*(m + n + 1)) + a*b*B*n - a*(A*b - a*B)*(m + 1)*Csc[e + f*x] + b*(A*b - a*B)*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4104

```
Int[((A_) + csc[(e_) + (f_)*(x_)]*(B_) + csc[(e_) + (f_)*(x_)]^2*(C_))*(csc[(e_) + (f_)*(x_)]*(d_)^(n_)*(csc[(e_) + (f_)*(x_)]*(b_) + (a_))^(m_)), x_Symbol] := Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^3(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^2} dx &= \frac{b(Ab-aB)\cos^2(c+dx)\sin(c+dx)}{a(a^2-b^2)d(a+b\sec(c+dx))} - \int \frac{\cos^3(c+dx)(-a^2A+4Ab^2-3abB+a(Ab-aB))}{a+b\sec(c+dx)} dx \\
&= \frac{(a^2A-4Ab^2+3abB)\cos^2(c+dx)\sin(c+dx)}{3a^2(a^2-b^2)d} + \frac{b(Ab-aB)\cos^2(c+dx)}{a(a^2-b^2)d(a+b\sec(c+dx))} \\
&= -\frac{(2a^2Ab-4Ab^3-a^3B+3ab^2B)\cos(c+dx)\sin(c+dx)}{2a^3(a^2-b^2)d} + \frac{(a^2A-4Ab^2+3abB)\cos^2(c+dx)\sin(c+dx)}{3a^2(a^2-b^2)d} \\
&= \frac{(2a^4A+7a^2Ab^2-12Ab^4-6a^3bB+9ab^3B)\sin(c+dx)}{3a^4(a^2-b^2)d} - \frac{(2a^2Ab-4Ab^3-a^3B+3ab^2B)\cos(c+dx)\sin(c+dx)}{3a^2(a^2-b^2)d} \\
&= -\frac{(2a^2Ab+8Ab^3-a^3B-6ab^2B)x}{2a^5} + \frac{(2a^4A+7a^2Ab^2-12Ab^4-6a^3bB+9ab^3B)\sin(c+dx)}{3a^4(a^2-b^2)d} \\
&= -\frac{(2a^2Ab+8Ab^3-a^3B-6ab^2B)x}{2a^5} + \frac{(2a^4A+7a^2Ab^2-12Ab^4-6a^3bB+9ab^3B)\sin(c+dx)}{3a^4(a^2-b^2)d} \\
&= -\frac{(2a^2Ab+8Ab^3-a^3B-6ab^2B)x}{2a^5} + \frac{(2a^4A+7a^2Ab^2-12Ab^4-6a^3bB+9ab^3B)\sin(c+dx)}{3a^4(a^2-b^2)d} \\
&= -\frac{(2a^2Ab+8Ab^3-a^3B-6ab^2B)x}{2a^5} + \frac{2b^3(5a^2Ab-4Ab^3-4a^3B+3ab^2B)\sin(c+dx)}{a^5(a-b)^{3/2}(a+b\sec(c+dx))}
\end{aligned}$$

Mathematica [A] time = 1.36, size = 224, normalized size = 0.65

$$\frac{a^3A\sin(3(c+dx)) + 3a(3a^2A - 8abB + 12Ab^2)\sin(c+dx) + 3a^2(aB - 2Ab)\sin(2(c+dx)) + 6(c+dx)(a^3B - 2Ab^3)}{12a^5d}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^2,x]

[Out] (6*(-2*a^2*A*b - 8*A*b^3 + a^3*B + 6*a*b^2*B)*(c + d*x) + (24*b^3*(-5*a^2*A*b + 4*A*b^3 + 4*a^3*B - 3*a*b^2*B)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqr

$$\frac{t[a^2 - b^2]}{(a^2 - b^2)^{3/2}} + \frac{3*a*(3*a^2*A + 12*A*b^2 - 8*a*b*B)*\sin[c + d*x] + (12*a*b^4*(-(A*b) + a*B)*\sin[c + d*x])}{((a - b)*(a + b)*(b + a*\cos[c + d*x]))} + \frac{3*a^2*(-2*A*b + a*B)*\sin[2*(c + d*x)] + a^3*A*\sin[3*(c + d*x)]}{(12*a^5*d)}$$

fricas [A] time = 0.62, size = 1167, normalized size = 3.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm="fricas")

[Out]
$$\frac{1}{6}*(3*(B*a^8 - 2*A*a^7*b + 4*B*a^6*b^2 - 4*A*a^5*b^3 - 11*B*a^4*b^4 + 14*A*a^3*b^5 + 6*B*a^2*b^6 - 8*A*a*b^7)*d*x*\cos(d*x + c) + 3*(B*a^7*b - 2*A*a^6*b^2 + 4*B*a^5*b^3 - 4*A*a^4*b^4 - 11*B*a^3*b^5 + 14*A*a^2*b^6 + 6*B*a*b^7 - 8*A*b^8)*d*x + 3*(4*B*a^3*b^4 - 5*A*a^2*b^5 - 3*B*a*b^6 + 4*A*b^7 + (4*B*a^4*b^3 - 5*A*a^3*b^4 - 3*B*a^2*b^5 + 4*A*a*b^6)*\cos(d*x + c))*\sqrt{a^2 - b^2}*\log((2*a*b*\cos(d*x + c) - (a^2 - 2*b^2)*\cos(d*x + c)^2 - 2*\sqrt{a^2 - b^2}*(b*\cos(d*x + c) + a)*\sin(d*x + c) + 2*a^2 - b^2)/(a^2*\cos(d*x + c)^2 + 2*a*b*\cos(d*x + c) + b^2)) + (4*A*a^7*b - 12*B*a^6*b^2 + 10*A*a^5*b^3 + 30*B*a^4*b^4 - 38*A*a^3*b^5 - 18*B*a^2*b^6 + 24*A*a*b^7 + 2*(A*a^8 - 2*A*a^6*b^2 + A*a^4*b^4)*\cos(d*x + c)^3 + (3*B*a^8 - 4*A*a^7*b - 6*B*a^6*b^2 + 8*A*a^5*b^3 + 3*B*a^4*b^4 - 4*A*a^3*b^5)*\cos(d*x + c)^2 + (4*A*a^8 - 9*B*a^7*b + 4*A*a^6*b^2 + 18*B*a^5*b^3 - 20*A*a^4*b^4 - 9*B*a^3*b^5 + 12*A*a^2*b^6)*\cos(d*x + c)*\sin(d*x + c))/(a^{10} - 2*a^8*b^2 + a^6*b^4)*d*\cos(d*x + c) + (a^9*b - 2*a^7*b^3 + a^5*b^5)*d), \frac{1}{6}*(3*(B*a^8 - 2*A*a^7*b + 4*B*a^6*b^2 - 4*A*a^5*b^3 - 11*B*a^4*b^4 + 14*A*a^3*b^5 + 6*B*a^2*b^6 - 8*A*a*b^7)*d*x*\cos(d*x + c) + 3*(B*a^7*b - 2*A*a^6*b^2 + 4*B*a^5*b^3 - 4*A*a^4*b^4 - 11*B*a^3*b^5 + 14*A*a^2*b^6 + 6*B*a*b^7 - 8*A*b^8)*d*x - 6*(4*B*a^3*b^4 - 5*A*a^2*b^5 - 3*B*a*b^6 + 4*A*b^7 + (4*B*a^4*b^3 - 5*A*a^3*b^4 - 3*B*a^2*b^5 + 4*A*a*b^6)*\cos(d*x + c))*\sqrt{-a^2 + b^2}*\arctan(-\sqrt{-a^2 + b^2}*(b*\cos(d*x + c) + a)/((a^2 - b^2)*\sin(d*x + c))) + (4*A*a^7*b - 12*B*a^6*b^2 + 10*A*a^5*b^3 + 30*B*a^4*b^4 - 38*A*a^3*b^5 - 18*B*a^2*b^6 + 24*A*a*b^7 + 2*(A*a^8 - 2*A*a^6*b^2 + A*a^4*b^4)*\cos(d*x + c)^3 + (3*B*a^8 - 4*A*a^7*b - 6*B*a^6*b^2 + 8*A*a^5*b^3 + 3*B*a^4*b^4 - 4*A*a^3*b^5)*\cos(d*x + c)^2 + (4*A*a^8 - 9*B*a^7*b + 4*A*a^6*b^2 + 18*B*a^5*b^3 - 20*A*a^4*b^4 - 9*B*a^3*b^5 + 12*A*a^2*b^6)*\cos(d*x + c)*\sin(d*x + c))/(a^{10} - 2*a^8*b^2 + a^6*b^4)*d*\cos(d*x + c) + (a^9*b - 2*a^7*b^3 + a^5*b^5)*d)]$$

giac [A] time = 3.26, size = 473, normalized size = 1.37

$$\frac{12(4Ba^3b^3 - 5Aa^2b^4 - 3Bab^5 + 4Ab^6) \left(\pi \left[\frac{dx+c}{2\pi} + \frac{1}{2} \right] \operatorname{sgn}(-2a+2b) + \arctan \left(-\frac{a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - b \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{\sqrt{-a^2+b^2}} \right) \right)}{(a^7 - a^5b^2) \sqrt{-a^2+b^2}} + \frac{12(Bab^4 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - Ab^5 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right))}{(a^6 - a^4b^2) \left(a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - b \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) \right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm="giac")

[Out]
$$-1/6*(12*(4*B*a^3*b^3 - 5*A*a^2*b^4 - 3*B*a*b^5 + 4*A*b^6)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^7 - a^5*b^2)*sqrt(-a^2 + b^2)) + 12*(B*a*b^4*tan(1/2*d*x + 1/2*c) - A*b^5*tan(1/2*d*x + 1/2*c))/((a^6 - a^4*b^2)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)) - 3*(B*a^3 - 2*A*a^2*b + 6*B*a*b^2 - 8*A*b^3)*(d*x + c)/a^5 - 2*(6*A*a^2*tan(1/2*d*x + 1/2*c)^5 - 3*B*a^2*tan(1/2*d*x + 1/2*c)^5 + 6*A*a*b*tan(1/2*d*x + 1/2*c)^5 - 12*B*a*b*tan(1/2*d*x + 1/2*c)^5 + 18*A*b^2*tan(1/2*d*x + 1/2*c)^5 + 4*A*a^2*tan(1/2*d*x + 1/2*c)^3 - 24*B*a*b*tan(1/2*d*x + 1/2*c)^3 + 36*A*b^2*tan(1/2*d*x + 1/2*c)^3 + 6*A*a^2*tan(1/2*d*x + 1/2*c) + 3*B*a^2*tan(1/2*d*x + 1/2*c) - 6*A*a*b*tan(1/2*d*x + 1/2*c) - 12*B*a*b*tan(1/2*d*x + 1/2*c) + 18*A*b^2*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^3*a^4)/d$$

maple [B] time = 1.40, size = 926, normalized size = 2.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x)

[Out]
$$2/d*b^5/a^4/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)*A-2/d*b^4/a^3/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)*B+10/d/a^3*b^4/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*A-8/d*b^6/a^5/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*A-8/d/a^2*b^3/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*B+6/d*b^5/a^4/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*B+2/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^5*A+2/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^5*A*b+6/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^5*A*b^2-1/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^5*B-4/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^5*b*B+4/3/d/a^2/($$

$$1 + \tan\left(\frac{1}{2}d*x + \frac{1}{2}c\right)^2 \wedge 3 * \tan\left(\frac{1}{2}d*x + \frac{1}{2}c\right)^3 * A + \frac{12}{d/a^4} / \left(1 + \tan\left(\frac{1}{2}d*x + \frac{1}{2}c\right)^2\right) \wedge 3 * \tan\left(\frac{1}{2}d*x + \frac{1}{2}c\right)^3 * A * b^2 - \frac{8}{d/a^3} / \left(1 + \tan\left(\frac{1}{2}d*x + \frac{1}{2}c\right)^2\right) \wedge 3 * \tan\left(\frac{1}{2}d*x + \frac{1}{2}c\right)^3 * b * B + \frac{2}{d/a^2} / \left(1 + \tan\left(\frac{1}{2}d*x + \frac{1}{2}c\right)^2\right) \wedge 3 * \tan\left(\frac{1}{2}d*x + \frac{1}{2}c\right) * A + \frac{6}{d/a^4} / \left(1 + \tan\left(\frac{1}{2}d*x + \frac{1}{2}c\right)^2\right) \wedge 3 * \tan\left(\frac{1}{2}d*x + \frac{1}{2}c\right) * A * b^2 - \frac{4}{d/a^3} / \left(1 + \tan\left(\frac{1}{2}d*x + \frac{1}{2}c\right)^2\right) \wedge 3 * \tan\left(\frac{1}{2}d*x + \frac{1}{2}c\right) * b * B - \frac{2}{d/a^3} / \left(1 + \tan\left(\frac{1}{2}d*x + \frac{1}{2}c\right)^2\right) \wedge 3 * \tan\left(\frac{1}{2}d*x + \frac{1}{2}c\right) * A * b + \frac{1}{d/a^2} / \left(1 + \tan\left(\frac{1}{2}d*x + \frac{1}{2}c\right)^2\right) \wedge 3 * \tan\left(\frac{1}{2}d*x + \frac{1}{2}c\right) * B - \frac{2}{d/a^3} * A * \arctan\left(\tan\left(\frac{1}{2}d*x + \frac{1}{2}c\right)\right) * b - \frac{8}{d/a^5} * \arctan\left(\tan\left(\frac{1}{2}d*x + \frac{1}{2}c\right)\right) * A * b^3 + \frac{1}{d/a^2} * \arctan\left(\tan\left(\frac{1}{2}d*x + \frac{1}{2}c\right)\right) * B + \frac{6}{d/a^4} * \arctan\left(\tan\left(\frac{1}{2}d*x + \frac{1}{2}c\right)\right) * B * b^2$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for more details) Is 4*a^2-4*b^2 positive or negative?

mupad [B] time = 11.66, size = 7763, normalized size = 22.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)^3*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x))^2,x)

[Out]
$$-\left(\left(\tan\left(\frac{c}{2} + \frac{d*x}{2}\right)\right)^7 * \left(2*A*a^5 + 8*A*b^5 - B*a^5 - 6*A*a^2*b^3 + 2*A*a^3*b^2 + 3*B*a^2*b^3 + 5*B*a^3*b^2 - 4*A*a*b^4 - 6*B*a*b^4 - 3*B*a^4*b\right)\right) / \left(a^4 * (a + b) * (a - b)\right) - \left(\tan\left(\frac{c}{2} + \frac{d*x}{2}\right)\right) * \left(2*A*a^5 - 8*A*b^5 + B*a^5 + 6*A*a^2*b^3 + 2*A*a^3*b^2 + 3*B*a^2*b^3 - 5*B*a^3*b^2 - 4*A*a*b^4 + 6*B*a*b^4 - 3*B*a^4*b\right) / \left(a^4 * (a + b) * (a - b)\right) + \left(\tan\left(\frac{c}{2} + \frac{d*x}{2}\right)\right)^3 * \left(2*A*a^5 + 72*A*b^5 + 3*B*a^5 - 38*A*a^2*b^3 - 14*A*a^3*b^2 - 9*B*a^2*b^3 + 33*B*a^3*b^2 + 12*A*a*b^4 - 16*A*a^4*b - 54*B*a*b^4 + 9*B*a^4*b\right) / \left(3*a^4 * (a + b) * (a - b)\right) - \left(\tan\left(\frac{c}{2} + \frac{d*x}{2}\right)\right)^5 * \left(2*A*a^5 - 72*A*b^5 - 3*B*a^5 + 38*A*a^2*b^3 - 14*A*a^3*b^2 - 9*B*a^2*b^3 - 33*B*a^3*b^2 + 12*A*a*b^4 + 16*A*a^4*b + 54*B*a*b^4 + 9*B*a^4*b\right) / \left(3*a^4 * (a + b) * (a - b)\right) / \left(d * (a + b - \tan\left(\frac{c}{2} + \frac{d*x}{2}\right))^8 * (a - b) + \tan\left(\frac{c}{2} + \frac{d*x}{2}\right)^2 * (2*a + 4*b) - \tan\left(\frac{c}{2} + \frac{d*x}{2}\right)^6 * (2*a - 4*b) + 6*b * \tan\left(\frac{c}{2} + \frac{d*x}{2}\right)^4\right) - \left(\operatorname{atan}\left(\left(\left(\left(\left(\left(8 * (2*B*a^18 + 16*A*a^10*b^8 - 8*A*a^11*b^7 - 36*A*a^12*b^6 + 16*A*a^13*b^5 + 20*A*a^14*b^4 - 4*A*a^15*b^3 - 12*B*a^11*b^7 + 6*B*a^12*b^6 + 28*B*a^13*b^5 - 14*B*a^14*b^4 - 16*B*a^15*b^3 + 6*B*a^16*b^2 - 4*A*a^17*b)\right)\right)\right)\right)\right)\right) / \left(a^{14} * b + a^{15} - a^{12} * b^3 - a^{13} * b^2\right) - (8$$

$$\begin{aligned}
& * \tan(c/2 + (d*x)/2) * (A*b^3*4i - (B*a^3*1i))/2 + A*a^2*b*1i - B*a*b^2*3i) * (8*a^15*b - 8*a^10*b^6 + 8*a^11*b^5 + 16*a^12*b^4 - 16*a^13*b^3 - 8*a^14*b^2) \\
& / (a^5*(a^10*b + a^11 - a^8*b^3 - a^9*b^2)) * (A*b^3*4i - (B*a^3*1i))/2 + A*a^2*b*1i - B*a*b^2*3i) / a^5 + (8*\tan(c/2 + (d*x)/2) * (128*A^2*b^12 + B^2*a^12 - 128*A^2*a*b^11 - 2*B^2*a^11*b - 192*A^2*a^2*b^10 + 192*A^2*a^3*b^9 + 8*A^2*a^4*b^8 - 8*A^2*a^5*b^7 + 28*A^2*a^6*b^6 - 48*A^2*a^7*b^5 + 28*A^2*a^8*b^4 - 8*A^2*a^9*b^3 + 4*A^2*a^10*b^2 + 72*B^2*a^2*b^10 - 72*B^2*a^3*b^9 - 120*B^2*a^4*b^8 + 120*B^2*a^5*b^7 + 17*B^2*a^6*b^6 - 26*B^2*a^7*b^5 + 23*B^2*a^8*b^4 - 20*B^2*a^9*b^3 + 11*B^2*a^10*b^2 - 192*A*B*a*b^11 - 4*A*B*a^11*b + 192*A*B*a^2*b^10 + 304*A*B*a^3*b^9 - 304*A*B*a^4*b^8 - 28*A*B*a^5*b^7 + 40*A*B*a^6*b^6 - 52*A*B*a^7*b^5 + 64*A*B*a^8*b^4 - 36*A*B*a^9*b^3 + 8*A*B*a^10*b^2) / (a^10*b + a^11 - a^8*b^3 - a^9*b^2)) * (A*b^3*4i - (B*a^3*1i))/2 + A*a^2*b*1i - B*a*b^2*3i) * 1i) / a^5 - (((((8*(2*B*a^18 + 16*A*a^10*b^8 - 8*A*a^11*b^7 - 36*A*a^12*b^6 + 16*A*a^13*b^5 + 20*A*a^14*b^4 - 4*A*a^15*b^3 - 12*B*a^11*b^7 + 6*B*a^12*b^6 + 28*B*a^13*b^5 - 14*B*a^14*b^4 - 16*B*a^15*b^3 + 6*B*a^16*b^2 - 4*A*a^17*b) / (a^14*b + a^15 - a^12*b^3 - a^13*b^2) + (8*\tan(c/2 + (d*x)/2) * (A*b^3*4i - (B*a^3*1i))/2 + A*a^2*b*1i - B*a*b^2*3i) * (8*a^15*b - 8*a^10*b^6 + 8*a^11*b^5 + 16*a^12*b^4 - 16*a^13*b^3 - 8*a^14*b^2) / (a^5*(a^10*b + a^11 - a^8*b^3 - a^9*b^2)) * (A*b^3*4i - (B*a^3*1i))/2 + A*a^2*b*1i - B*a*b^2*3i) / a^5 - (8*\tan(c/2 + (d*x)/2) * (128*A^2*b^12 + B^2*a^12 - 128*A^2*a*b^11 - 2*B^2*a^11*b - 192*A^2*a^2*b^10 + 192*A^2*a^3*b^9 + 8*A^2*a^4*b^8 - 8*A^2*a^5*b^7 + 28*A^2*a^6*b^6 - 48*A^2*a^7*b^5 + 28*A^2*a^8*b^4 - 8*A^2*a^9*b^3 + 4*A^2*a^10*b^2 + 72*B^2*a^2*b^10 - 72*B^2*a^3*b^9 - 120*B^2*a^4*b^8 + 120*B^2*a^5*b^7 + 17*B^2*a^6*b^6 - 26*B^2*a^7*b^5 + 23*B^2*a^8*b^4 - 20*B^2*a^9*b^3 + 11*B^2*a^10*b^2 - 192*A*B*a*b^11 - 4*A*B*a^11*b + 192*A*B*a^2*b^10 + 304*A*B*a^3*b^9 - 304*A*B*a^4*b^8 - 28*A*B*a^5*b^7 + 40*A*B*a^6*b^6 - 52*A*B*a^7*b^5 + 64*A*B*a^8*b^4 - 36*A*B*a^9*b^3 + 8*A*B*a^10*b^2) / (a^10*b + a^11 - a^8*b^3 - a^9*b^2)) * (A*b^3*4i - (B*a^3*1i))/2 + A*a^2*b*1i - B*a*b^2*3i) * 1i) / a^5) / ((16*(256*A^3*b^14 - 128*A^3*a*b^13 - 448*A^3*a^2*b^12 + 192*A^3*a^3*b^11 + 48*A^3*a^4*b^10 - 24*A^3*a^5*b^9 + 124*A^3*a^6*b^8 - 20*A^3*a^7*b^7 + 20*A^3*a^8*b^6 - 108*B^3*a^3*b^11 + 54*B^3*a^4*b^10 + 216*B^3*a^5*b^9 - 81*B^3*a^6*b^8 - 63*B^3*a^7*b^7 + 9*B^3*a^8*b^6 - 41*B^3*a^9*b^5 + 4*B^3*a^10*b^4 - 4*B^3*a^11*b^3 - 576*A^2*B*a*b^13 + 432*A*B^2*a^2*b^12 - 216*A*B^2*a^3*b^11 - 828*A*B^2*a^4*b^10 + 324*A*B^2*a^5*b^9 + 192*A*B^2*a^6*b^8 - 39*A*B^2*a^7*b^7 + 183*A*B^2*a^8*b^6 - 21*A*B^2*a^9*b^5 + 21*A*B^2*a^10*b^4 + 288*A^2*B*a^2*b^12 + 1056*A^2*B*a^3*b^11 - 432*A^2*B*a^4*b^10 - 180*A^2*B*a^5*b^9 + 54*A^2*B*a^6*b^8 - 264*A^2*B*a^7*b^7 + 36*A^2*B*a^8*b^6 - 36*A^2*B*a^9*b^5) / (a^14*b + a^15 - a^12*b^3 - a^13*b^2) + (((((8*(2*B*a^18 + 16*A*a^10*b^8 - 8*A*a^11*b^7 - 36*A*a^12*b^6 + 16*A*a^13*b^5 + 20*A*a^14*b^4 - 4*A*a^15*b^3 - 12*B*a^11*b^7 + 6*B*a^12*b^6 + 28*B*a^13*b^5 - 14*B*a^14*b^4 - 16*B*a^15*b^3 + 6*B*a^16*b^2 - 4*A*a^17*b) / (a^14*b + a^15 - a^12*b^3 - a^13*b^2) - (8*\tan(c/2 + (d*x)/2) * (A*b^3*4i - (B*a^3*1i))/2 + A*a^2*b*1i - B*a*b^2*3i) * (8*a^15*b - 8*a^10*b^6 + 8*a^11*b^5 + 16*a^12*b^4 - 16*a^13*b^3 - 8*a^14*b^2) / (a^5*(a^10*b + a^11 - a^8*b^3 - a^9*b^2))) * (A*b^3*4i - (B*a^3*1i))/2 + A*a^2*b*1i - B*a*b^2*3i) / a^5 + (8*\tan(c/2 + (d
\end{aligned}$$

$$\begin{aligned}
& *x)/2)*(128*A^2*b^12 + B^2*a^12 - 128*A^2*a*b^11 - 2*B^2*a^11*b - 192*A^2*a \\
& ^2*b^10 + 192*A^2*a^3*b^9 + 8*A^2*a^4*b^8 - 8*A^2*a^5*b^7 + 28*A^2*a^6*b^6 \\
& - 48*A^2*a^7*b^5 + 28*A^2*a^8*b^4 - 8*A^2*a^9*b^3 + 4*A^2*a^10*b^2 + 72*B^2 \\
& *a^2*b^10 - 72*B^2*a^3*b^9 - 120*B^2*a^4*b^8 + 120*B^2*a^5*b^7 + 17*B^2*a^6 \\
& *b^6 - 26*B^2*a^7*b^5 + 23*B^2*a^8*b^4 - 20*B^2*a^9*b^3 + 11*B^2*a^10*b^2 - \\
& 192*A*B*a*b^11 - 4*A*B*a^11*b + 192*A*B*a^2*b^10 + 304*A*B*a^3*b^9 - 304*A \\
& *B*a^4*b^8 - 28*A*B*a^5*b^7 + 40*A*B*a^6*b^6 - 52*A*B*a^7*b^5 + 64*A*B*a^8* \\
& b^4 - 36*A*B*a^9*b^3 + 8*A*B*a^10*b^2))/ (a^10*b + a^11 - a^8*b^3 - a^9*b^2) \\
&)*(A*b^3*4i - (B*a^3*1i)/2 + A*a^2*b*1i - B*a*b^2*3i))/a^5 + (((((8*(2*B*a^ \\
& 18 + 16*A*a^10*b^8 - 8*A*a^11*b^7 - 36*A*a^12*b^6 + 16*A*a^13*b^5 + 20*A*a^ \\
& 14*b^4 - 4*A*a^15*b^3 - 12*B*a^11*b^7 + 6*B*a^12*b^6 + 28*B*a^13*b^5 - 14*B \\
& *a^14*b^4 - 16*B*a^15*b^3 + 6*B*a^16*b^2 - 4*A*a^17*b))/ (a^14*b + a^15 - a^ \\
& 12*b^3 - a^13*b^2) + (8*tan(c/2 + (d*x)/2)*(A*b^3*4i - (B*a^3*1i)/2 + A*a^2 \\
& *b*1i - B*a*b^2*3i)*(8*a^15*b - 8*a^10*b^6 + 8*a^11*b^5 + 16*a^12*b^4 - 16* \\
& a^13*b^3 - 8*a^14*b^2))/ (a^5*(a^10*b + a^11 - a^8*b^3 - a^9*b^2)))*(A*b^3*4 \\
& i - (B*a^3*1i)/2 + A*a^2*b*1i - B*a*b^2*3i))/a^5 - (8*tan(c/2 + (d*x)/2)*(1 \\
& 28*A^2*b^12 + B^2*a^12 - 128*A^2*a*b^11 - 2*B^2*a^11*b - 192*A^2*a^2*b^10 + \\
& 192*A^2*a^3*b^9 + 8*A^2*a^4*b^8 - 8*A^2*a^5*b^7 + 28*A^2*a^6*b^6 - 48*A^2* \\
& a^7*b^5 + 28*A^2*a^8*b^4 - 8*A^2*a^9*b^3 + 4*A^2*a^10*b^2 + 72*B^2*a^2*b^10 \\
& - 72*B^2*a^3*b^9 - 120*B^2*a^4*b^8 + 120*B^2*a^5*b^7 + 17*B^2*a^6*b^6 - 26 \\
& *B^2*a^7*b^5 + 23*B^2*a^8*b^4 - 20*B^2*a^9*b^3 + 11*B^2*a^10*b^2 - 192*A*B* \\
& a*b^11 - 4*A*B*a^11*b + 192*A*B*a^2*b^10 + 304*A*B*a^3*b^9 - 304*A*B*a^4*b^ \\
& 8 - 28*A*B*a^5*b^7 + 40*A*B*a^6*b^6 - 52*A*B*a^7*b^5 + 64*A*B*a^8*b^4 - 36* \\
& A*B*a^9*b^3 + 8*A*B*a^10*b^2))/ (a^10*b + a^11 - a^8*b^3 - a^9*b^2))*(A*b^3* \\
& 4i - (B*a^3*1i)/2 + A*a^2*b*1i - B*a*b^2*3i))/a^5))*(A*b^3*4i - (B*a^3*1i)/ \\
& 2 + A*a^2*b*1i - B*a*b^2*3i)*2i)/(a^5*d) - (b^3*atan(((b^3*((a + b)^3*(a - \\
& b)^3)^(1/2))*((8*tan(c/2 + (d*x)/2)*(128*A^2*b^12 + B^2*a^12 - 128*A^2*a*b^1 \\
& 1 - 2*B^2*a^11*b - 192*A^2*a^2*b^10 + 192*A^2*a^3*b^9 + 8*A^2*a^4*b^8 - 8*A \\
& ^2*a^5*b^7 + 28*A^2*a^6*b^6 - 48*A^2*a^7*b^5 + 28*A^2*a^8*b^4 - 8*A^2*a^9*b \\
& ^3 + 4*A^2*a^10*b^2 + 72*B^2*a^2*b^10 - 72*B^2*a^3*b^9 - 120*B^2*a^4*b^8 + \\
& 120*B^2*a^5*b^7 + 17*B^2*a^6*b^6 - 26*B^2*a^7*b^5 + 23*B^2*a^8*b^4 - 20*B^2 \\
& *a^9*b^3 + 11*B^2*a^10*b^2 - 192*A*B*a*b^11 - 4*A*B*a^11*b + 192*A*B*a^2*b^ \\
& 10 + 304*A*B*a^3*b^9 - 304*A*B*a^4*b^8 - 28*A*B*a^5*b^7 + 40*A*B*a^6*b^6 - \\
& 52*A*B*a^7*b^5 + 64*A*B*a^8*b^4 - 36*A*B*a^9*b^3 + 8*A*B*a^10*b^2))/ (a^10*b \\
& + a^11 - a^8*b^3 - a^9*b^2) + (b^3*((8*(2*B*a^18 + 16*A*a^10*b^8 - 8*A*a^1 \\
& 1*b^7 - 36*A*a^12*b^6 + 16*A*a^13*b^5 + 20*A*a^14*b^4 - 4*A*a^15*b^3 - 12*B \\
& *a^11*b^7 + 6*B*a^12*b^6 + 28*B*a^13*b^5 - 14*B*a^14*b^4 - 16*B*a^15*b^3 + \\
& 6*B*a^16*b^2 - 4*A*a^17*b))/ (a^14*b + a^15 - a^12*b^3 - a^13*b^2) - (8*b^3* \\
& tan(c/2 + (d*x)/2)*((a + b)^3*(a - b)^3)^(1/2)*(4*A*b^3 + 4*B*a^3 - 5*A*a^2 \\
& *b - 3*B*a*b^2)*(8*a^15*b - 8*a^10*b^6 + 8*a^11*b^5 + 16*a^12*b^4 - 16*a^13 \\
& *b^3 - 8*a^14*b^2))/ ((a^10*b + a^11 - a^8*b^3 - a^9*b^2)*(a^11 - a^5*b^6 + \\
& 3*a^7*b^4 - 3*a^9*b^2)))*((a + b)^3*(a - b)^3)^(1/2)*(4*A*b^3 + 4*B*a^3 - 5 \\
& *A*a^2*b - 3*B*a*b^2))/ (a^11 - a^5*b^6 + 3*a^7*b^4 - 3*a^9*b^2))*(4*A*b^3 + \\
& 4*B*a^3 - 5*A*a^2*b - 3*B*a*b^2)*1i)/(a^11 - a^5*b^6 + 3*a^7*b^4 - 3*a^9*b \\
& ^2) + (b^3*((a + b)^3*(a - b)^3)^(1/2))*((8*tan(c/2 + (d*x)/2)*(128*A^2*b^12
\end{aligned}$$

$$\begin{aligned}
& + B^2 a^{12} - 128 A^2 a^4 b^{11} - 2 B^2 a^{11} b - 192 A^2 a^2 b^{10} + 192 A^2 a^3 b^9 + 8 A^2 a^4 b^8 - 8 A^2 a^5 b^7 + 28 A^2 a^6 b^6 - 48 A^2 a^7 b^5 + 28 A^2 a^8 b^4 - 8 A^2 a^9 b^3 + 4 A^2 a^{10} b^2 + 72 B^2 a^2 b^{10} - 72 B^2 a^3 b^9 - 120 B^2 a^4 b^8 + 120 B^2 a^5 b^7 + 17 B^2 a^6 b^6 - 26 B^2 a^7 b^5 + 23 B^2 a^8 b^4 - 20 B^2 a^9 b^3 + 11 B^2 a^{10} b^2 - 192 A B a^4 b^{11} - 4 A B a^5 b^{10} + 192 A B a^2 b^{10} + 304 A B a^3 b^9 - 304 A B a^4 b^8 - 28 A B a^5 b^7 + 40 A B a^6 b^6 - 52 A B a^7 b^5 + 64 A B a^8 b^4 - 36 A B a^9 b^3 + 8 A B a^{10} b^2) / (a^{10} b + a^{11} - a^8 b^3 - a^9 b^2) - (b^3 ((8 (2 B a^{18} + 16 A a^{10} b^8 - 8 A a^{11} b^7 - 36 A a^{12} b^6 + 16 A a^{13} b^5 + 20 A a^{14} b^4 - 4 A a^{15} b^3 - 12 B a^{11} b^7 + 6 B a^{12} b^6 + 28 B a^{13} b^5 - 14 B a^{14} b^4 - 16 B a^{15} b^3 + 6 B a^{16} b^2 - 4 A a^{17} b) / (a^{14} b + a^{15} - a^{12} b^3 - a^{13} b^2) + (8 b^3 \tan(c/2 + (d*x)/2) ((a+b)^3 (a-b)^3)^{1/2} (4 A a^3 b^3 + 4 B a^3 - 5 A a^2 b - 3 B a b^2) (8 a^{15} b - 8 a^{10} b^6 + 8 a^{11} b^5 + 16 a^{12} b^4 - 16 a^{13} b^3 - 8 a^{14} b^2)) / ((a^{10} b + a^{11} - a^8 b^3 - a^9 b^2) (a^{11} - a^5 b^6 + 3 a^7 b^4 - 3 a^9 b^2))) * ((a+b)^3 (a-b)^3)^{1/2} (4 A a^3 b^3 + 4 B a^3 - 5 A a^2 b - 3 B a b^2)) / (a^{11} - a^5 b^6 + 3 a^7 b^4 - 3 a^9 b^2)) * (4 A a^3 b^3 + 4 B a^3 - 5 A a^2 b - 3 B a b^2) * i) / (a^{11} - a^5 b^6 + 3 a^7 b^4 - 3 a^9 b^2)) / ((16 (256 A^3 b^{14} - 128 A^3 a^3 b^{13} - 448 A^3 a^2 b^{12} + 192 A^3 a^3 b^{11} + 48 A^3 a^4 b^{10} - 24 A^3 a^5 b^9 + 124 A^3 a^6 b^8 - 20 A^3 a^7 b^7 + 20 A^3 a^8 b^6 - 108 B^3 a^3 b^{11} + 54 B^3 a^4 b^{10} + 216 B^3 a^5 b^9 - 81 B^3 a^6 b^8 - 63 B^3 a^7 b^7 + 9 B^3 a^8 b^6 - 41 B^3 a^9 b^5 + 4 B^3 a^{10} b^4 - 4 B^3 a^{11} b^3 - 576 A^2 B a^4 b^{13} + 432 A^2 B a^5 b^{12} - 216 A^2 B a^6 b^{11} - 828 A^2 B a^7 b^{10} + 324 A^2 B a^8 b^9 + 192 A^2 B a^9 b^8 - 39 A^2 B a^{10} b^7 + 183 A^2 B a^{11} b^6 - 21 A^2 B a^{12} b^5 + 21 A^2 B a^{13} b^4 + 288 A^2 B a^{14} b^3 + 1056 A^2 B a^{15} b^2 - 432 A^2 B a^{16} b - 180 A^2 B a^{17} b) / (a^{14} b + a^{15} - a^{12} b^3 - a^{13} b^2) + (b^3 ((a+b)^3 (a-b)^3)^{1/2} ((8 \tan(c/2 + (d*x)/2) (128 A^2 b^{12} + B^2 a^{12} - 128 A^2 a^4 b^{11} - 2 B^2 a^{11} b - 192 A^2 a^2 b^{10} + 192 A^2 a^3 b^9 + 8 A^2 a^4 b^8 - 8 A^2 a^5 b^7 + 28 A^2 a^6 b^6 - 48 A^2 a^7 b^5 + 28 A^2 a^8 b^4 - 8 A^2 a^9 b^3 + 4 A^2 a^{10} b^2 + 72 B^2 a^2 b^{10} - 72 B^2 a^3 b^9 - 120 B^2 a^4 b^8 + 120 B^2 a^5 b^7 + 17 B^2 a^6 b^6 - 26 B^2 a^7 b^5 + 23 B^2 a^8 b^4 - 20 B^2 a^9 b^3 + 11 B^2 a^{10} b^2 - 192 A B a^4 b^{11} - 4 A B a^5 b^{10} + 192 A B a^2 b^{10} + 304 A B a^3 b^9 - 304 A B a^4 b^8 - 28 A B a^5 b^7 + 40 A B a^6 b^6 - 52 A B a^7 b^5 + 64 A B a^8 b^4 - 36 A B a^9 b^3 + 8 A B a^{10} b^2)) / (a^{10} b + a^{11} - a^8 b^3 - a^9 b^2) + (b^3 ((8 (2 B a^{18} + 16 A a^{10} b^8 - 8 A a^{11} b^7 - 36 A a^{12} b^6 + 16 A a^{13} b^5 + 20 A a^{14} b^4 - 4 A a^{15} b^3 - 12 B a^{11} b^7 + 6 B a^{12} b^6 + 28 B a^{13} b^5 - 14 B a^{14} b^4 - 16 B a^{15} b^3 + 6 B a^{16} b^2 - 4 A a^{17} b) / (a^{14} b + a^{15} - a^{12} b^3 - a^{13} b^2) - (8 b^3 \tan(c/2 + (d*x)/2) ((a+b)^3 (a-b)^3)^{1/2} (4 A a^3 b^3 + 4 B a^3 - 5 A a^2 b - 3 B a b^2) (8 a^{15} b - 8 a^{10} b^6 + 8 a^{11} b^5 + 16 a^{12} b^4 - 16 a^{13} b^3 - 8 a^{14} b^2)) / ((a^{10} b + a^{11} - a^8 b^3 - a^9 b^2) (a^{11} - a^5 b^6 + 3 a^7 b^4 - 3 a^9 b^2))) * ((a+b)^3 (a-b)^3)^{1/2} (4 A a^3 b^3 + 4 B a^3 - 5 A a^2 b - 3 B a b^2)) / (a^{11} - a^5 b^6 + 3 a^7 b^4 - 3 a^9 b^2)) * (4 A a^3 b^3 + 4 B a^3 - 5 A a^2 b - 3 B a b^2)) / (a^{11} - a^5 b^6 +
\end{aligned}$$

$$\begin{aligned}
& 3a^7b^4 - 3a^9b^2) - (b^3((a+b)^3(a-b)^3)^{1/2}((8\tan(c/2 + (d \\
& *x)/2)*(128A^2b^{12} + B^2a^{12} - 128A^2a*b^{11} - 2B^2a^{11}*b - 192A^2a \\
& ^2*b^{10} + 192A^2a^3*b^9 + 8A^2a^4*b^8 - 8A^2a^5*b^7 + 28A^2a^6*b^6 \\
& - 48A^2a^7*b^5 + 28A^2a^8*b^4 - 8A^2a^9*b^3 + 4A^2a^{10}*b^2 + 72B^2 \\
& *a^2*b^{10} - 72B^2a^3*b^9 - 120B^2a^4*b^8 + 120B^2a^5*b^7 + 17B^2a^6 \\
& *b^6 - 26B^2a^7*b^5 + 23B^2a^8*b^4 - 20B^2a^9*b^3 + 11B^2a^{10}*b^2 - \\
& 192A*B*a*b^{11} - 4A*B*a^{11}*b + 192A*B*a^2*b^{10} + 304A*B*a^3*b^9 - 304A \\
& *B*a^4*b^8 - 28A*B*a^5*b^7 + 40A*B*a^6*b^6 - 52A*B*a^7*b^5 + 64A*B*a^8 \\
& b^4 - 36A*B*a^9*b^3 + 8A*B*a^{10}*b^2)))/(a^{10}*b + a^{11} - a^8*b^3 - a^9*b^2) \\
& - (b^3((8*(2B*a^{18} + 16A*a^{10}*b^8 - 8A*a^{11}*b^7 - 36A*a^{12}*b^6 + 16A \\
& *a^{13}*b^5 + 20A*a^{14}*b^4 - 4A*a^{15}*b^3 - 12B*a^{11}*b^7 + 6B*a^{12}*b^6 + 2 \\
& 8B*a^{13}*b^5 - 14B*a^{14}*b^4 - 16B*a^{15}*b^3 + 6B*a^{16}*b^2 - 4A*a^{17}*b)))/ \\
& (a^{14}*b + a^{15} - a^{12}*b^3 - a^{13}*b^2) + (8*b^3*\tan(c/2 + (d*x)/2)*((a+b)^ \\
& 3*(a-b)^3)^{1/2}*(4A*b^3 + 4B*a^3 - 5A*a^2*b - 3B*a*b^2)*(8*a^{15}*b - \\
& 8*a^{10}*b^6 + 8*a^{11}*b^5 + 16*a^{12}*b^4 - 16*a^{13}*b^3 - 8*a^{14}*b^2))/((a^{10}*b \\
& + a^{11} - a^8*b^3 - a^9*b^2)*(a^{11} - a^5*b^6 + 3*a^7*b^4 - 3*a^9*b^2))*((a \\
& + b)^3*(a-b)^3)^{1/2}*(4A*b^3 + 4B*a^3 - 5A*a^2*b - 3B*a*b^2))/(a^{11} \\
& - a^5*b^6 + 3*a^7*b^4 - 3*a^9*b^2))*((4A*b^3 + 4B*a^3 - 5A*a^2*b - 3B*a \\
& *b^2))/(a^{11} - a^5*b^6 + 3*a^7*b^4 - 3*a^9*b^2))*((a+b)^3*(a-b)^3)^{1/2} \\
& *(4A*b^3 + 4B*a^3 - 5A*a^2*b - 3B*a*b^2)*2i)/(d*(a^{11} - a^5*b^6 + 3*a \\
& ^7*b^4 - 3*a^9*b^2))
\end{aligned}$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \cos^3(c + dx)}{(a + b \sec(c + dx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**2,x)

[Out] Integral((A + B*sec(c + d*x))*cos(c + d*x)**3/(a + b*sec(c + d*x))**2, x)

$$3.328 \quad \int \frac{\sec^5(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^3} dx$$

Optimal. Leaf size=407

$$\frac{a(Ab - aB) \tan(c + dx) \sec^3(c + dx)}{2bd(a^2 - b^2)(a + b \sec(c + dx))^2} - \frac{(-12a^2B + 6aAb - b^2B) \tanh^{-1}(\sin(c + dx))}{2b^5d} + \frac{a(-4a^3B + 2a^2Ab + 7ab^2B)}{2b^2d(a^2 - b^2)^2}$$

[Out] $-1/2*(6*A*a*b-12*B*a^2-B*b^2)*\operatorname{arctanh}(\sin(d*x+c))/b^5/d+a^2*(6*A*a^4*b-15*A*a^2*b^3+12*A*b^5-12*B*a^5+29*B*a^3*b^2-20*B*a*b^4)*\operatorname{arctanh}((a-b)^{(1/2)}*\tan(1/2*d*x+1/2*c)/(a+b)^{(1/2)))/(a-b)^{(5/2)}/b^5/(a+b)^{(5/2)}/d+1/2*(6*A*a^4*b-11*A*a^2*b^3+2*A*b^5-12*B*a^5+21*B*a^3*b^2-6*B*a*b^4)*\tan(d*x+c)/b^4/(a^2-b^2)^2/d-1/2*(3*A*a^3*b-6*A*a*b^3-6*B*a^4+10*B*a^2*b^2-B*b^4)*\sec(d*x+c)*\tan(d*x+c)/b^3/(a^2-b^2)^2/d+1/2*a*(A*b-B*a)*\sec(d*x+c)^3*\tan(d*x+c)/b/(a^2-b^2)^2/d/(a+b*\sec(d*x+c))^2+1/2*a*(2*A*a^2*b-5*A*b^3-4*B*a^3+7*B*a*b^2)*\sec(d*x+c)^2*\tan(d*x+c)/b^2/(a^2-b^2)^2/d/(a+b*\sec(d*x+c))$

Rubi [A] time = 1.96, antiderivative size = 407, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.290$, Rules used = {4029, 4098, 4092, 4082, 3998, 3770, 3831, 2659, 208}

$$\frac{(-11a^2Ab^3 + 6a^4Ab + 21a^3b^2B - 12a^5B - 6ab^4B + 2Ab^5) \tan(c + dx)}{2b^4d(a^2 - b^2)^2} - \frac{(-12a^2B + 6aAb - b^2B) \tanh^{-1}(\sin(c + dx))}{2b^5d}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(\operatorname{Sec}[c + d*x]^5*(A + B*\operatorname{Sec}[c + d*x]))/(a + b*\operatorname{Sec}[c + d*x])^3, x]$

[Out] $-((6*a*A*b - 12*a^2*B - b^2*B)*\operatorname{ArcTanh}[\operatorname{Sin}[c + d*x]])/(2*b^5*d) + (a^2*(6*a^4*A*b - 15*a^2*A*b^3 + 12*A*b^5 - 12*a^5*B + 29*a^3*b^2*B - 20*a*b^4*B)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a - b]*\operatorname{Tan}[(c + d*x)/2])/(\operatorname{Sqrt}[a + b])]/((a - b)^{(5/2)}*b^5*(a + b)^{(5/2)*d} + ((6*a^4*A*b - 11*a^2*A*b^3 + 2*A*b^5 - 12*a^5*B + 21*a^3*b^2*B - 6*a*b^4*B)*\operatorname{Tan}[c + d*x])/(2*b^4*(a^2 - b^2)^2*d) - ((3*a^3*A*b - 6*a*A*b^3 - 6*a^4*B + 10*a^2*b^2*B - b^4*B)*\operatorname{Sec}[c + d*x]*\operatorname{Tan}[c + d*x])/(2*b^3*(a^2 - b^2)^2*d) + (a*(A*b - a*B)*\operatorname{Sec}[c + d*x]^3*\operatorname{Tan}[c + d*x])/(2*b*(a^2 - b^2)^2*d*(a + b*\operatorname{Sec}[c + d*x])^2) + (a*(2*a^2*A*b - 5*A*b^3 - 4*a^3*B + 7*a*b^2*B)*\operatorname{Sec}[c + d*x]^2*\operatorname{Tan}[c + d*x])/(2*b^2*(a^2 - b^2)^2*d*(a + b*\operatorname{Sec}[c + d*x]))$

Rule 208

$\operatorname{Int}[(a + b*x^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[-(a/b), 2]*\operatorname{ArcTanh}[x/\operatorname{Rt}[-(a/b), 2]])/a, x] /; \operatorname{FreeQ}\{a, b\}, x] \&\& \operatorname{NegQ}[a/b]$

Rule 2659

```
Int[((a_) + (b_)*sin[Pi/2 + (c_) + (d_)*(x_)])^(-1), x_Symbol] := With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3770

```
Int[csc[(c_) + (d_)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x]
/; FreeQ[{c, d}, x]
```

Rule 3831

```
Int[csc[(e_) + (f_)*(x_)]/(csc[(e_) + (f_)*(x_)]*(b_) + (a_)), x_Symbol] := Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3998

```
Int[(csc[(e_) + (f_)*(x_)]*(csc[(e_) + (f_)*(x_)]*(B_) + (A_)))/(csc[(e_) + (f_)*(x_)]*(b_) + (a_)), x_Symbol] := Dist[B/b, Int[Csc[e + f*x], x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 4029

```
Int[(csc[(e_) + (f_)*(x_)]*(d_)^(n_)*(csc[(e_) + (f_)*(x_)]*(b_) + (a_))^(m_)*(csc[(e_) + (f_)*(x_)]*(B_) + (A_)), x_Symbol] := Simp[(a*d^2*(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[d/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(n - 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2*(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 1]
```

Rule 4082

```
Int[csc[(e_) + (f_)*(x_)]*((A_) + csc[(e_) + (f_)*(x_)]*(B_) + csc[(e_) + (f_)*(x_)]^2*(C_))*(csc[(e_) + (f_)*(x_)]*(b_) + (a_))^(m_), x_Symbol] := -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
```

eeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rule 4092

Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_), x_Symbol] :> -Simp[(C*Csc[e + f*x]*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[a*C + b*(C*(m + 2) + A*(m + 3))*Csc[e + f*x] - (2*a*C - b*B*(m + 3))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && !LtQ[m, -1]

Rule 4098

Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_), x_Symbol] :> -Simp[(d*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1))/(b*f*(a^2 - b^2)*(m + 1)), x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A - b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]

Rubi steps

$$\begin{aligned}
\int \frac{\sec^5(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^3} dx &= \frac{a(Ab-aB)\sec^3(c+dx)\tan(c+dx)}{2b(a^2-b^2)d(a+b\sec(c+dx))^2} + \frac{\int \frac{\sec^3(c+dx)(3a(Ab-aB)-2b(Ab-aB)\sec(c+dx))}{(a+b\sec(c+dx))^2} dx}{2b(a^2-b^2)d} \\
&= \frac{a(Ab-aB)\sec^3(c+dx)\tan(c+dx)}{2b(a^2-b^2)d(a+b\sec(c+dx))^2} + \frac{a(2a^2Ab-5Ab^3-4a^3B+7ab^2)}{2b^2(a^2-b^2)^2d} \\
&= -\frac{(3a^3Ab-6aAb^3-6a^4B+10a^2b^2B-b^4B)\sec(c+dx)\tan(c+dx)}{2b^3(a^2-b^2)^2d} + \frac{a^2(6a^4Ab-15a^2Ab^3-6a^4B+10a^2b^2B-b^4B)\tan(c+dx)}{2b^3(a^2-b^2)^2d} \\
&= \frac{(6a^4Ab-11a^2Ab^3+2Ab^5-12a^5B+21a^3b^2B-6ab^4B)\tan(c+dx)}{2b^4(a^2-b^2)^2d} - \frac{a^2(6a^4Ab-15a^2Ab^3-6a^4B+10a^2b^2B-b^4B)\tan(c+dx)}{2b^3(a^2-b^2)^2d} \\
&= \frac{(6a^4Ab-11a^2Ab^3+2Ab^5-12a^5B+21a^3b^2B-6ab^4B)\tan(c+dx)}{2b^4(a^2-b^2)^2d} \\
&= -\frac{(6aAb-12a^2B-b^2B)\tanh^{-1}(\sin(c+dx))}{2b^5d} + \frac{(6a^4Ab-11a^2Ab^3+2Ab^5-12a^5B+21a^3b^2B-6ab^4B)\tan(c+dx)}{2b^4(a^2-b^2)^2d} \\
&= -\frac{(6aAb-12a^2B-b^2B)\tanh^{-1}(\sin(c+dx))}{2b^5d} + \frac{(6a^4Ab-11a^2Ab^3+2Ab^5-12a^5B+21a^3b^2B-6ab^4B)\tan(c+dx)}{2b^4(a^2-b^2)^2d} \\
&= -\frac{(6aAb-12a^2B-b^2B)\tanh^{-1}(\sin(c+dx))}{2b^5d} + \frac{a^2(6a^4Ab-15a^2Ab^3-6a^4B+10a^2b^2B-b^4B)\tan(c+dx)}{2b^3(a^2-b^2)^2d}
\end{aligned}$$

Mathematica [A] time = 3.08, size = 507, normalized size = 1.25

$$-8(12a^2B-6aAb+b^2B)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+8(12a^2B-6aAb+b^2B)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[c + d*x]^5*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^3,x]

```
[Out] ((16*a^2*(-6*a^4*A*b + 15*a^2*A*b^3 - 12*A*b^5 + 12*a^5*B - 29*a^3*b^2*B +
20*a*b^4*B)*ArcTanh[(-a + b)*Tan[(c + d*x)/2]]/Sqrt[a^2 - b^2])/(a^2 - b^
2)^(5/2) - 8*(-6*a*A*b + 12*a^2*B + b^2*B)*Log[Cos[(c + d*x)/2] - Sin[(c +
d*x)/2]] + 8*(-6*a*A*b + 12*a^2*B + b^2*B)*Log[Cos[(c + d*x)/2] + Sin[(c +
d*x)/2]] + (2*b*(18*a^5*A*b^2 - 32*a^3*A*b^4 + 8*a*A*b^6 - 36*a^6*b*B + 68*
a^4*b^3*B - 30*a^2*b^5*B + 4*b^7*B + (18*a^6*A*b - 25*a^4*A*b^3 - 10*a^2*A*
b^5 + 8*A*b^7 - 36*a^7*B + 47*a^5*b^2*B + 14*a^3*b^4*B - 16*a*b^6*B)*Cos[c
+ d*x] - 2*a*b*(-9*a^4*A*b + 16*a^2*A*b^3 - 4*A*b^5 + 18*a^5*B - 32*a^3*b^2
*B + 11*a*b^4*B)*Cos[2*(c + d*x)] + 6*a^6*A*b*Cos[3*(c + d*x)] - 11*a^4*A*b
^3*Cos[3*(c + d*x)] + 2*a^2*A*b^5*Cos[3*(c + d*x)] - 12*a^7*B*Cos[3*(c + d
x)] + 21*a^5*b^2*B*Cos[3*(c + d*x)] - 6*a^3*b^4*B*Cos[3*(c + d*x)])*Sec[c +
d*x]*Tan[c + d*x])/((a^2 - b^2)^2*(b + a*Cos[c + d*x])^2)/(16*b^5*d)
```

fricas [B] time = 72.77, size = 2444, normalized size = 6.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] [-1/4*(((12*B*a^9 - 6*A*a^8*b - 29*B*a^7*b^2 + 15*A*a^6*b^3 + 20*B*a^5*b^4
- 12*A*a^4*b^5)*cos(d*x + c)^4 + 2*(12*B*a^8*b - 6*A*a^7*b^2 - 29*B*a^6*b^3
+ 15*A*a^5*b^4 + 20*B*a^4*b^5 - 12*A*a^3*b^6)*cos(d*x + c)^3 + (12*B*a^7*b
^2 - 6*A*a^6*b^3 - 29*B*a^5*b^4 + 15*A*a^4*b^5 + 20*B*a^3*b^6 - 12*A*a^2*b^
7)*cos(d*x + c)^2)*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*
cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^
2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - ((12*B*a^10 - 6
*A*a^9*b - 35*B*a^8*b^2 + 18*A*a^7*b^3 + 33*B*a^6*b^4 - 18*A*a^5*b^5 - 9*B*
a^4*b^6 + 6*A*a^3*b^7 - B*a^2*b^8)*cos(d*x + c)^4 + 2*(12*B*a^9*b - 6*A*a^8
*b^2 - 35*B*a^7*b^3 + 18*A*a^6*b^4 + 33*B*a^5*b^5 - 18*A*a^4*b^6 - 9*B*a^3*
b^7 + 6*A*a^2*b^8 - B*a*b^9)*cos(d*x + c)^3 + (12*B*a^8*b^2 - 6*A*a^7*b^3 -
35*B*a^6*b^4 + 18*A*a^5*b^5 + 33*B*a^4*b^6 - 18*A*a^3*b^7 - 9*B*a^2*b^8 +
6*A*a*b^9 - B*b^10)*cos(d*x + c)^2)*log(sin(d*x + c) + 1) + ((12*B*a^10 - 6
*A*a^9*b - 35*B*a^8*b^2 + 18*A*a^7*b^3 + 33*B*a^6*b^4 - 18*A*a^5*b^5 - 9*B*
a^4*b^6 + 6*A*a^3*b^7 - B*a^2*b^8)*cos(d*x + c)^4 + 2*(12*B*a^9*b - 6*A*a^8
*b^2 - 35*B*a^7*b^3 + 18*A*a^6*b^4 + 33*B*a^5*b^5 - 18*A*a^4*b^6 - 9*B*a^3*
b^7 + 6*A*a^2*b^8 - B*a*b^9)*cos(d*x + c)^3 + (12*B*a^8*b^2 - 6*A*a^7*b^3 -
35*B*a^6*b^4 + 18*A*a^5*b^5 + 33*B*a^4*b^6 - 18*A*a^3*b^7 - 9*B*a^2*b^8 +
6*A*a*b^9 - B*b^10)*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) - 2*(B*a^6*b^4 -
3*B*a^4*b^6 + 3*B*a^2*b^8 - B*b^10 - (12*B*a^9*b - 6*A*a^8*b^2 - 33*B*a^7*
b^3 + 17*A*a^6*b^4 + 27*B*a^5*b^5 - 13*A*a^4*b^6 - 6*B*a^3*b^7 + 2*A*a^2*b^
8)*cos(d*x + c)^3 - (18*B*a^8*b^2 - 9*A*a^7*b^3 - 50*B*a^6*b^4 + 25*A*a^5*b
^5 + 43*B*a^4*b^6 - 20*A*a^3*b^7 - 11*B*a^2*b^8 + 4*A*a*b^9)*cos(d*x + c)^2
- 2*(2*B*a^7*b^3 - A*a^6*b^4 - 6*B*a^5*b^5 + 3*A*a^4*b^6 + 6*B*a^3*b^7 - 3
```

$$\begin{aligned} & *A*a^2*b^8 - 2*B*a*b^9 + A*b^{10})*\cos(d*x + c))*\sin(d*x + c))/((a^8*b^5 - 3* \\ & a^6*b^7 + 3*a^4*b^9 - a^2*b^{11})*d*\cos(d*x + c)^4 + 2*(a^7*b^6 - 3*a^5*b^8 + \\ & 3*a^3*b^{10} - a*b^{12})*d*\cos(d*x + c)^3 + (a^6*b^7 - 3*a^4*b^9 + 3*a^2*b^{11} \\ & - b^{13})*d*\cos(d*x + c)^2), -1/4*(2*((12*B*a^9 - 6*A*a^8*b - 29*B*a^7*b^2 + \\ & 15*A*a^6*b^3 + 20*B*a^5*b^4 - 12*A*a^4*b^5)*\cos(d*x + c)^4 + 2*(12*B*a^8*b \\ & - 6*A*a^7*b^2 - 29*B*a^6*b^3 + 15*A*a^5*b^4 + 20*B*a^4*b^5 - 12*A*a^3*b^6)* \\ & \cos(d*x + c)^3 + (12*B*a^7*b^2 - 6*A*a^6*b^3 - 29*B*a^5*b^4 + 15*A*a^4*b^5 \\ & + 20*B*a^3*b^6 - 12*A*a^2*b^7)*\cos(d*x + c)^2)*\sqrt{-a^2 + b^2}*\arctan(-\sqrt{ \\ & t(-a^2 + b^2)*(b*\cos(d*x + c) + a)/((a^2 - b^2)*\sin(d*x + c))} - ((12*B*a^1 \\ & 0 - 6*A*a^9*b - 35*B*a^8*b^2 + 18*A*a^7*b^3 + 33*B*a^6*b^4 - 18*A*a^5*b^5 - \\ & 9*B*a^4*b^6 + 6*A*a^3*b^7 - B*a^2*b^8)*\cos(d*x + c)^4 + 2*(12*B*a^9*b - 6* \\ & A*a^8*b^2 - 35*B*a^7*b^3 + 18*A*a^6*b^4 + 33*B*a^5*b^5 - 18*A*a^4*b^6 - 9*B \\ & *a^3*b^7 + 6*A*a^2*b^8 - B*a*b^9)*\cos(d*x + c)^3 + (12*B*a^8*b^2 - 6*A*a^7* \\ & b^3 - 35*B*a^6*b^4 + 18*A*a^5*b^5 + 33*B*a^4*b^6 - 18*A*a^3*b^7 - 9*B*a^2*b \\ & ^8 + 6*A*a*b^9 - B*b^{10})*\cos(d*x + c)^2)*\log(\sin(d*x + c) + 1) + ((12*B*a^1 \\ & 0 - 6*A*a^9*b - 35*B*a^8*b^2 + 18*A*a^7*b^3 + 33*B*a^6*b^4 - 18*A*a^5*b^5 - \\ & 9*B*a^4*b^6 + 6*A*a^3*b^7 - B*a^2*b^8)*\cos(d*x + c)^4 + 2*(12*B*a^9*b - 6* \\ & A*a^8*b^2 - 35*B*a^7*b^3 + 18*A*a^6*b^4 + 33*B*a^5*b^5 - 18*A*a^4*b^6 - 9*B \\ & *a^3*b^7 + 6*A*a^2*b^8 - B*a*b^9)*\cos(d*x + c)^3 + (12*B*a^8*b^2 - 6*A*a^7* \\ & b^3 - 35*B*a^6*b^4 + 18*A*a^5*b^5 + 33*B*a^4*b^6 - 18*A*a^3*b^7 - 9*B*a^2*b \\ & ^8 + 6*A*a*b^9 - B*b^{10})*\cos(d*x + c)^2)*\log(-\sin(d*x + c) + 1) - 2*(B*a^6* \\ & b^4 - 3*B*a^4*b^6 + 3*B*a^2*b^8 - B*b^{10} - (12*B*a^9*b - 6*A*a^8*b^2 - 33*B \\ & *a^7*b^3 + 17*A*a^6*b^4 + 27*B*a^5*b^5 - 13*A*a^4*b^6 - 6*B*a^3*b^7 + 2*A*a \\ & ^2*b^8)*\cos(d*x + c)^3 - (18*B*a^8*b^2 - 9*A*a^7*b^3 - 50*B*a^6*b^4 + 25*A* \\ & a^5*b^5 + 43*B*a^4*b^6 - 20*A*a^3*b^7 - 11*B*a^2*b^8 + 4*A*a*b^9)*\cos(d*x + \\ & c)^2 - 2*(2*B*a^7*b^3 - A*a^6*b^4 - 6*B*a^5*b^5 + 3*A*a^4*b^6 + 6*B*a^3*b^ \\ & 7 - 3*A*a^2*b^8 - 2*B*a*b^9 + A*b^{10})*\cos(d*x + c))*\sin(d*x + c))/((a^8*b^5 \\ & - 3*a^6*b^7 + 3*a^4*b^9 - a^2*b^{11})*d*\cos(d*x + c)^4 + 2*(a^7*b^6 - 3*a^5* \\ & b^8 + 3*a^3*b^{10} - a*b^{12})*d*\cos(d*x + c)^3 + (a^6*b^7 - 3*a^4*b^9 + 3*a^2* \\ & b^{11} - b^{13})*d*\cos(d*x + c)^2)] \end{aligned}$$

giac [B] time = 0.50, size = 1391, normalized size = 3.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^5*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm="giac")

[Out]
$$\begin{aligned} & -1/2*(2*(12*B*a^7 - 6*A*a^6*b - 29*B*a^5*b^2 + 15*A*a^4*b^3 + 20*B*a^3*b^4 \\ & - 12*A*a^2*b^5)*(pi*\text{floor}(1/2*(d*x + c)/pi + 1/2)*\text{sgn}(-2*a + 2*b) + \arctan(\\ & -(a*\tan(1/2*d*x + 1/2*c) - b*\tan(1/2*d*x + 1/2*c))/\sqrt{-a^2 + b^2}))/((a^4 \\ & *b^5 - 2*a^2*b^7 + b^9)*\sqrt{-a^2 + b^2}) - 2*(12*B*a^7*\tan(1/2*d*x + 1/2*c) \\ &)^7 - 6*A*a^6*b*\tan(1/2*d*x + 1/2*c)^7 - 18*B*a^6*b*\tan(1/2*d*x + 1/2*c)^7 \\ & + 9*A*a^5*b^2*\tan(1/2*d*x + 1/2*c)^7 - 17*B*a^5*b^2*\tan(1/2*d*x + 1/2*c)^7 \end{aligned}$$

$$\begin{aligned}
& + 9*A*a^4*b^3*\tan(1/2*d*x + 1/2*c)^7 + 33*B*a^4*b^3*\tan(1/2*d*x + 1/2*c)^7 \\
& - 16*A*a^3*b^4*\tan(1/2*d*x + 1/2*c)^7 - 2*B*a^3*b^4*\tan(1/2*d*x + 1/2*c)^7 \\
& + 2*A*a^2*b^5*\tan(1/2*d*x + 1/2*c)^7 - 13*B*a^2*b^5*\tan(1/2*d*x + 1/2*c)^7 \\
& + 4*A*a*b^6*\tan(1/2*d*x + 1/2*c)^7 + 4*B*a*b^6*\tan(1/2*d*x + 1/2*c)^7 - 2*A \\
& *b^7*\tan(1/2*d*x + 1/2*c)^7 + B*b^7*\tan(1/2*d*x + 1/2*c)^7 - 36*B*a^7*\tan(1 \\
& /2*d*x + 1/2*c)^5 + 18*A*a^6*b*\tan(1/2*d*x + 1/2*c)^5 + 18*B*a^6*b*\tan(1/2* \\
& d*x + 1/2*c)^5 - 9*A*a^5*b^2*\tan(1/2*d*x + 1/2*c)^5 + 67*B*a^5*b^2*\tan(1/2* \\
& d*x + 1/2*c)^5 - 35*A*a^4*b^3*\tan(1/2*d*x + 1/2*c)^5 - 29*B*a^4*b^3*\tan(1/2 \\
& *d*x + 1/2*c)^5 + 16*A*a^3*b^4*\tan(1/2*d*x + 1/2*c)^5 - 26*B*a^3*b^4*\tan(1/ \\
& 2*d*x + 1/2*c)^5 + 10*A*a^2*b^5*\tan(1/2*d*x + 1/2*c)^5 + 5*B*a^2*b^5*\tan(1/ \\
& 2*d*x + 1/2*c)^5 - 4*A*a*b^6*\tan(1/2*d*x + 1/2*c)^5 + 4*B*a*b^6*\tan(1/2*d*x \\
& + 1/2*c)^5 - 2*A*b^7*\tan(1/2*d*x + 1/2*c)^5 + 3*B*b^7*\tan(1/2*d*x + 1/2*c) \\
& ^5 + 36*B*a^7*\tan(1/2*d*x + 1/2*c)^3 - 18*A*a^6*b*\tan(1/2*d*x + 1/2*c)^3 + \\
& 18*B*a^6*b*\tan(1/2*d*x + 1/2*c)^3 - 9*A*a^5*b^2*\tan(1/2*d*x + 1/2*c)^3 - 67 \\
& *B*a^5*b^2*\tan(1/2*d*x + 1/2*c)^3 + 35*A*a^4*b^3*\tan(1/2*d*x + 1/2*c)^3 - 2 \\
& 9*B*a^4*b^3*\tan(1/2*d*x + 1/2*c)^3 + 16*A*a^3*b^4*\tan(1/2*d*x + 1/2*c)^3 + \\
& 26*B*a^3*b^4*\tan(1/2*d*x + 1/2*c)^3 - 10*A*a^2*b^5*\tan(1/2*d*x + 1/2*c)^3 + \\
& 5*B*a^2*b^5*\tan(1/2*d*x + 1/2*c)^3 - 4*A*a*b^6*\tan(1/2*d*x + 1/2*c)^3 - 4* \\
& B*a*b^6*\tan(1/2*d*x + 1/2*c)^3 + 2*A*b^7*\tan(1/2*d*x + 1/2*c)^3 + 3*B*b^7*t \\
& an(1/2*d*x + 1/2*c)^3 - 12*B*a^7*\tan(1/2*d*x + 1/2*c) + 6*A*a^6*b*\tan(1/2*d \\
& *x + 1/2*c) - 18*B*a^6*b*\tan(1/2*d*x + 1/2*c) + 9*A*a^5*b^2*\tan(1/2*d*x + 1 \\
& /2*c) + 17*B*a^5*b^2*\tan(1/2*d*x + 1/2*c) - 9*A*a^4*b^3*\tan(1/2*d*x + 1/2*c) \\
&) + 33*B*a^4*b^3*\tan(1/2*d*x + 1/2*c) - 16*A*a^3*b^4*\tan(1/2*d*x + 1/2*c) + \\
& 2*B*a^3*b^4*\tan(1/2*d*x + 1/2*c) - 2*A*a^2*b^5*\tan(1/2*d*x + 1/2*c) - 13*B \\
& *a^2*b^5*\tan(1/2*d*x + 1/2*c) + 4*A*a*b^6*\tan(1/2*d*x + 1/2*c) - 4*B*a*b^6* \\
& \tan(1/2*d*x + 1/2*c) + 2*A*b^7*\tan(1/2*d*x + 1/2*c) + B*b^7*\tan(1/2*d*x + 1 \\
& /2*c))/((a^4*b^4 - 2*a^2*b^6 + b^8)*(a*\tan(1/2*d*x + 1/2*c)^4 - b*\tan(1/2*d \\
& *x + 1/2*c)^4 - 2*a*\tan(1/2*d*x + 1/2*c)^2 + a + b)^2) - (12*B*a^2 - 6*A*a* \\
& b + B*b^2)*\log(\text{abs}(\tan(1/2*d*x + 1/2*c) + 1))/b^5 + (12*B*a^2 - 6*A*a*b + B \\
& *b^2)*\log(\text{abs}(\tan(1/2*d*x + 1/2*c) - 1))/b^5)/d
\end{aligned}$$

maple [B] time = 0.88, size = 1599, normalized size = 3.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\int (\sec(dx+c)^5(A+B\sec(dx+c)))/(a+b\sec(dx+c))^3, x$

[Out]
$$\begin{aligned}
& -3/d/b^4*\ln(\tan(1/2*d*x+1/2*c)+1)*A+a^6/d/b^5*\ln(\tan(1/2*d*x+1/2*c)+1)*a^2* \\
& B-10/d*a^4/b^2/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/ \\
& (a^2+2*a*b+b^2)*\tan(1/2*d*x+1/2*c)^3*B+1/d*a^4/b^2/(a*\tan(1/2*d*x+1/2*c)^2- \\
& \tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)*\tan(1/2*d*x+1/2*c)^3*A+ \\
& 4/d*a^5/b^3/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a+b)/(a- \\
& b)^2*\tan(1/2*d*x+1/2*c)*A-4/d*a^5/b^3/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1 \\
& /2*c)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)*\tan(1/2*d*x+1/2*c)^3*A+8/d*a^3/b/(a
\end{aligned}$$

$$\begin{aligned} & \tan(1/2*d*x+1/2*c)^2 - \tan(1/2*d*x+1/2*c)^2 * b - a - b)^2 / (a-b) / (a^2 + 2*a*b + b^2) * \tan(1/2*d*x+1/2*c)^3 * A + 1/d*a^4/b^2 / (a*\tan(1/2*d*x+1/2*c)^2 - \tan(1/2*d*x+1/2*c)^2 * b - a - b)^2 / (a+b) / (a-b)^2 * \tan(1/2*d*x+1/2*c) * A + 6/d*a^6/b^4 / (a*\tan(1/2*d*x+1/2*c)^2 - \tan(1/2*d*x+1/2*c)^2 * b - a - b)^2 / (a-b) / (a^2 + 2*a*b + b^2) * \tan(1/2*d*x+1/2*c)^3 * B - 1/d*a^5/b^3 / (a*\tan(1/2*d*x+1/2*c)^2 - \tan(1/2*d*x+1/2*c)^2 * b - a - b)^2 / (a-b) / (a^2 + 2*a*b + b^2) * \tan(1/2*d*x+1/2*c)^3 * B - 6/d*a^6/b^4 / (a*\tan(1/2*d*x+1/2*c)^2 - \tan(1/2*d*x+1/2*c)^2 * b - a - b)^2 / (a+b) / (a-b)^2 * \tan(1/2*d*x+1/2*c) * B - 1/d*a^5/b^3 / (a*\tan(1/2*d*x+1/2*c)^2 - \tan(1/2*d*x+1/2*c)^2 * b - a - b)^2 / (a+b) / (a-b)^2 * \tan(1/2*d*x+1/2*c) * B + 10/d*a^4/b^2 / (a*\tan(1/2*d*x+1/2*c)^2 - \tan(1/2*d*x+1/2*c)^2 * b - a - b)^2 / (a+b) / (a-b)^2 * \tan(1/2*d*x+1/2*c) * B - 8/d*a^3/b / (a*\tan(1/2*d*x+1/2*c)^2 - \tan(1/2*d*x+1/2*c)^2 * b - a - b)^2 / (a+b) / (a-b)^2 * \tan(1/2*d*x+1/2*c) * A - 20/d*a^3/b / (a^4 - 2*a^2*b^2 + b^4) / ((a-b)*(a+b))^(1/2) * \operatorname{arctanh}(\tan(1/2*d*x+1/2*c) * (a-b) / ((a-b)*(a+b))^(1/2)) * B - 15/d*a^4/b^2 / (a^4 - 2*a^2*b^2 + b^4) / ((a-b)*(a+b))^(1/2) * \operatorname{arctanh}(\tan(1/2*d*x+1/2*c) * (a-b) / ((a-b)*(a+b))^(1/2)) * A + 1/2*d*B/b^3 / (\tan(1/2*d*x+1/2*c) - 1)^2 - 1/d/b^3 / (\tan(1/2*d*x+1/2*c) - 1) * A + 1/2/d/b^3 / (\tan(1/2*d*x+1/2*c) - 1) * B - 1/2/d/b^3 * \ln(\tan(1/2*d*x+1/2*c) - 1) * B - 1/2/d*B/b^3 / (\tan(1/2*d*x+1/2*c) + 1)^2 - 1/d/b^3 / (\tan(1/2*d*x+1/2*c) + 1) * A + 1/2/d/b^3 / (\tan(1/2*d*x+1/2*c) + 1) * B + 1/2/d/b^3 * \ln(\tan(1/2*d*x+1/2*c) + 1) * B - 12/d*a^7/b^5 / (a^4 - 2*a^2*b^2 + b^4) / ((a-b)*(a+b))^(1/2) * \operatorname{arctanh}(\tan(1/2*d*x+1/2*c) * (a-b) / ((a-b)*(a+b))^(1/2)) * B + 29/d*a^5/b^3 / (a^4 - 2*a^2*b^2 + b^4) / ((a-b)*(a+b))^(1/2) * \operatorname{arctanh}(\tan(1/2*d*x+1/2*c) * (a-b) / ((a-b)*(a+b))^(1/2)) * B + 6/d*a^6/b^4 / (a^4 - 2*a^2*b^2 + b^4) / ((a-b)*(a+b))^(1/2) * \operatorname{arctanh}(\tan(1/2*d*x+1/2*c) * (a-b) / ((a-b)*(a+b))^(1/2)) * A + 3/d/b^4 * \ln(\tan(1/2*d*x+1/2*c) - 1) * A * a + 3/d/b^4 / (\tan(1/2*d*x+1/2*c) - 1) * a * B + 3/d/b^4 / (\tan(1/2*d*x+1/2*c) + 1) * a * B + 12/d*a^2 / (a^4 - 2*a^2*b^2 + b^4) / ((a-b)*(a+b))^(1/2) * \operatorname{arctanh}(\tan(1/2*d*x+1/2*c) * (a-b) / ((a-b)*(a+b))^(1/2)) * A - 6/d/b^5 * \ln(\tan(1/2*d*x+1/2*c) - 1) * a^2 * B \end{aligned}$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^5*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for more details)Is 4*a^2-4*b^2 positive or negative?

mupad [B] time = 14.23, size = 10533, normalized size = 25.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((A + B/\cos(c + d*x))/(\cos(c + d*x)^5*(a + b/\cos(c + d*x))^3), x)$

[Out] $((\tan(c/2 + (d*x)/2)^3*(2*A*b^7 + 36*B*a^7 + 3*B*b^7 - 10*A*a^2*b^5 + 16*A*a^3*b^4 + 35*A*a^4*b^3 - 9*A*a^5*b^2 + 5*B*a^2*b^5 + 26*B*a^3*b^4 - 29*B*a^4*b^3 - 67*B*a^5*b^2 - 4*A*a*b^6 - 18*A*a^6*b - 4*B*a*b^6 + 18*B*a^6*b))/((a + b)^2*(b^6 - 2*a*b^5 + a^2*b^4)) + (\tan(c/2 + (d*x)/2)^5*(3*B*b^7 - 36*B*a^7 - 2*A*b^7 + 10*A*a^2*b^5 + 16*A*a^3*b^4 - 35*A*a^4*b^3 - 9*A*a^5*b^2 + 5*B*a^2*b^5 - 26*B*a^3*b^4 - 29*B*a^4*b^3 + 67*B*a^5*b^2 - 4*A*a*b^6 + 18*A*a^6*b + 4*B*a*b^6 + 18*B*a^6*b))/((a + b)^2*(b^6 - 2*a*b^5 + a^2*b^4)) - (\tan(c/2 + (d*x)/2)^7*(B*b^6 - 12*B*a^6 - 2*A*b^6 + 4*A*a^2*b^4 - 12*A*a^3*b^3 - 3*A*a^4*b^2 - 8*B*a^2*b^4 - 10*B*a^3*b^3 + 23*B*a^4*b^2 + 2*A*a*b^5 + 6*A*a^5*b + 5*B*a*b^5 + 6*B*a^5*b))/((a*b^4 - b^5)*(a + b)^2) + (\tan(c/2 + (d*x)/2)*(2*A*b^6 - 12*B*a^6 + B*b^6 - 4*A*a^2*b^4 - 12*A*a^3*b^3 + 3*A*a^4*b^2 - 8*B*a^2*b^4 + 10*B*a^3*b^3 + 23*B*a^4*b^2 + 2*A*a*b^5 + 6*A*a^5*b - 5*B*a*b^5 - 6*B*a^5*b))/((a + b)*(b^6 - 2*a*b^5 + a^2*b^4)))/(d*(2*a*b + \tan(c/2 + (d*x)/2)^4*(6*a^2 - 2*b^2) - \tan(c/2 + (d*x)/2)^2*(4*a*b + 4*a^2) + \tan(c/2 + (d*x)/2)^6*(4*a*b - 4*a^2) + \tan(c/2 + (d*x)/2)^8*(a^2 - 2*a*b + b^2) + a^2 + b^2)) - (\text{atan}((((8*\tan(c/2 + (d*x)/2)*(288*B^2*a^14 + B^2*b^14 - 2*B^2*a*b^13 - 288*B^2*a^13*b + 36*A^2*a^2*b^12 - 72*A^2*a^3*b^11 + 36*A^2*a^4*b^10 + 288*A^2*a^5*b^9 - 288*A^2*a^6*b^8 - 432*A^2*a^7*b^7 + 441*A^2*a^8*b^6 + 288*A^2*a^9*b^5 - 288*A^2*a^10*b^4 - 72*A^2*a^11*b^3 + 72*A^2*a^12*b^2 + 21*B^2*a^2*b^12 - 40*B^2*a^3*b^11 + 74*B^2*a^4*b^10 - 108*B^2*a^5*b^9 + 18*B^2*a^6*b^8 + 872*B^2*a^7*b^7 - 827*B^2*a^8*b^6 - 1538*B^2*a^9*b^5 + 1538*B^2*a^10*b^4 + 1104*B^2*a^11*b^3 - 1104*B^2*a^12*b^2 - 12*A*B*a*b^13 - 288*A*B*a^13*b + 24*A*B*a^2*b^12 - 108*A*B*a^3*b^11 + 192*A*B*a^4*b^10 - 72*A*B*a^5*b^9 - 1008*A*B*a^6*b^8 + 984*A*B*a^7*b^7 + 1632*A*B*a^8*b^6 - 1650*A*B*a^9*b^5 - 1128*A*B*a^10*b^4 + 1128*A*B*a^11*b^3 + 288*A*B*a^12*b^2)))/(a*b^14 + b^15 - 3*a^2*b^13 - 3*a^3*b^12 + 3*a^4*b^11 + 3*a^5*b^10 - a^6*b^9 - a^7*b^8) - (((4*(4*B*b^21 + 48*A*a^2*b^19 + 72*A*a^3*b^18 - 156*A*a^4*b^17 - 84*A*a^5*b^16 + 192*A*a^6*b^15 + 48*A*a^7*b^14 - 108*A*a^8*b^13 - 12*A*a^9*b^12 + 24*A*a^10*b^11 + 28*B*a^2*b^19 - 80*B*a^3*b^18 - 120*B*a^4*b^17 + 276*B*a^5*b^16 + 164*B*a^6*b^15 - 360*B*a^7*b^14 - 100*B*a^8*b^13 + 212*B*a^9*b^12 + 24*B*a^10*b^11 - 48*B*a^11*b^10 - 24*A*a*b^20)))/(a*b^18 + b^19 - 3*a^2*b^17 - 3*a^3*b^16 + 3*a^4*b^15 + 3*a^5*b^14 - a^6*b^13 - a^7*b^12) - (4*\tan(c/2 + (d*x)/2)*(12*B*a^2 + B*b^2 - 6*A*a*b)*(8*a*b^19 - 8*a^2*b^18 - 32*a^3*b^17 + 32*a^4*b^16 + 48*a^5*b^15 - 48*a^6*b^14 - 32*a^7*b^13 + 32*a^8*b^12 + 8*a^9*b^11 - 8*a^10*b^10))/(b^5*(a*b^14 + b^15 - 3*a^2*b^13 - 3*a^3*b^12 + 3*a^4*b^11 + 3*a^5*b^10 - a^6*b^9 - a^7*b^8)))*(12*B*a^2 + B*b^2 - 6*A*a*b))/(2*b^5))*(12*B*a^2 + B*b^2 - 6*A*a*b)*1i)/(2*b^5) + (((8*\tan(c/2 + (d*x)/2)*(288*B^2*a^14 + B^2*b^14 - 2*B^2*a*b^13 - 288*B^2*a^13*b + 36*A^2*a^2*b^12 - 72*A^2*a^3*b^11 + 36*A^2*a^4*b^10 + 288*A^2*a^5*b^9 - 288*A^2*a^6*b^8 - 432*A^2*a^7*b^7 + 441*A^2*a^8*b^6 + 288*A^2*a^9*b^5 - 288*A^2*a^10*b^4 - 72*A^2*a^11*b^3 + 72*A^2*a^12*b^2 + 21*B^2*a^2*b^12 - 40*B^2*a^3*b^11 + 74*B^2*a^4*b^10 - 108*B^2*a^5*b^9 + 18*B^2*a^6*b^8 + 872*B^2*a^7*b^7 - 827*B^2*a^8*b^6 - 1538*B^2*a^9*b^5 + 1538*B^2*a^10*b^4 + 1104*$

$$\begin{aligned}
& B^2 a^{11} b^3 - 1104 B^2 a^{12} b^2 - 12 A B a b^{13} - 288 A B a^{13} b + 24 A B a^2 b^{12} - 108 A B a^3 b^{11} + 192 A B a^4 b^{10} - 72 A B a^5 b^9 - 1008 A B a^6 b^8 + 984 A B a^7 b^7 + 1632 A B a^8 b^6 - 1650 A B a^9 b^5 - 1128 A B a^{10} b^4 + 1128 A B a^{11} b^3 + 288 A B a^{12} b^2) / (a b^{14} + b^{15} - 3 a^2 b^{13} - 3 a^3 b^{12} + 3 a^4 b^{11} + 3 a^5 b^{10} - a^6 b^9 - a^7 b^8) + (((4(4 B b^{21} + 48 A a^2 b^{19} + 72 A a^3 b^{18} - 156 A a^4 b^{17} - 84 A a^5 b^{16} + 192 A a^6 b^{15} + 48 A a^7 b^{14} - 108 A a^8 b^{13} - 12 A a^9 b^{12} + 24 A a^{10} b^{11} + 28 B a^2 b^{19} - 80 B a^3 b^{18} - 120 B a^4 b^{17} + 276 B a^5 b^{16} + 164 B a^6 b^{15} - 360 B a^7 b^{14} - 100 B a^8 b^{13} + 212 B a^9 b^{12} + 24 B a^{10} b^{11} - 48 B a^{11} b^{10} - 24 A a b^{20})) / (a b^{18} + b^{19} - 3 a^2 b^{17} - 3 a^3 b^{16} + 3 a^4 b^{15} + 3 a^5 b^{14} - a^6 b^{13} - a^7 b^{12}) + (4 \tan(c/2 + (d*x)/2) * (12 B a^2 + B b^2 - 6 A a b) * (8 a b^{19} - 8 a^2 b^{18} - 32 a^3 b^{17} + 32 a^4 b^{16} + 48 a^5 b^{15} - 48 a^6 b^{14} - 32 a^7 b^{13} + 32 a^8 b^{12} + 8 a^9 b^{11} - 8 a^{10} b^{10})) / (b^5 (a b^{14} + b^{15} - 3 a^2 b^{13} - 3 a^3 b^{12} + 3 a^4 b^{11} + 3 a^5 b^{10} - a^6 b^9 - a^7 b^8)) * (12 B a^2 + B b^2 - 6 A a b) / (2 b^5)) * (12 B a^2 + B b^2 - 6 A a b) * 1i) / (2 b^5)) / ((8(1728 B^3 a^{15} - 864 B^3 a^{14} b - 432 A^3 a^4 b^{11} - 432 A^3 a^5 b^{10} + 1404 A^3 a^6 b^9 + 756 A^3 a^7 b^8 - 1728 A^3 a^8 b^7 - 486 A^3 a^9 b^6 + 972 A^3 a^{10} b^5 + 108 A^3 a^{11} b^4 - 216 A^3 a^{12} b^3 + 20 B^3 a^3 b^{12} - 20 B^3 a^4 b^{11} + 411 B^3 a^5 b^{10} - 11 B^3 a^6 b^9 + 1314 B^3 a^7 b^8 + 2326 B^3 a^8 b^7 - 7829 B^3 a^9 b^6 - 4770 B^3 a^{10} b^5 + 11700 B^3 a^{11} b^4 + 3456 B^3 a^{12} b^3 - 7344 B^3 a^{13} b^2 - 2592 A B^2 a^{14} b - 12 A B^2 a^2 b^{13} + 12 A B^2 a^3 b^{12} - 489 A B^2 a^4 b^{11} + 9 A B^2 a^5 b^{10} - 2892 A B^2 a^6 b^9 - 3972 A B^2 a^7 b^8 + 13347 A B^2 a^8 b^7 + 7767 A B^2 a^9 b^6 - 18594 A B^2 a^{10} b^5 - 5400 A B^2 a^{11} b^4 + 11232 A B^2 a^{12} b^3 + 1296 A B^2 a^{13} b^2 + 144 A^2 B a^3 b^{12} + 1980 A^2 B a^5 b^{10} + 2268 A^2 B a^6 b^9 - 7524 A^2 B a^7 b^8 - 4203 A^2 B a^8 b^7 + 9828 A^2 B a^9 b^6 + 2808 A^2 B a^{10} b^5 - 5724 A^2 B a^{11} b^4 - 648 A^2 B a^{12} b^3 + 1296 A^2 B a^{13} b^2)) / (a b^{18} + b^{19} - 3 a^2 b^{17} - 3 a^3 b^{16} + 3 a^4 b^{15} + 3 a^5 b^{14} - a^6 b^{13} - a^7 b^{12}) - (((8 \tan(c/2 + (d*x)/2) * (288 B^2 a^{14} + B^2 b^{14} - 2 B^2 a b^{13} - 288 B^2 a^{13} b + 36 A^2 a^2 b^{12} - 72 A^2 a^3 b^{11} + 36 A^2 a^4 b^{10} + 288 A^2 a^5 b^9 - 288 A^2 a^6 b^8 - 432 A^2 a^7 b^7 + 441 A^2 a^8 b^6 + 288 A^2 a^9 b^5 - 288 A^2 a^{10} b^4 - 72 A^2 a^{11} b^3 + 72 A^2 a^{12} b^2 + 21 B^2 a^2 b^{12} - 40 B^2 a^3 b^{11} + 74 B^2 a^4 b^{10} - 108 B^2 a^5 b^9 + 18 B^2 a^6 b^8 + 872 B^2 a^7 b^7 - 827 B^2 a^8 b^6 - 1538 B^2 a^9 b^5 + 1538 B^2 a^{10} b^4 + 1104 B^2 a^{11} b^3 - 1104 B^2 a^{12} b^2 - 12 A B a b^{13} - 288 A B a^{13} b + 24 A B a^2 b^{12} - 108 A B a^3 b^{11} + 192 A B a^4 b^{10} - 72 A B a^5 b^9 - 1008 A B a^6 b^8 + 984 A B a^7 b^7 + 1632 A B a^8 b^6 - 1650 A B a^9 b^5 - 1128 A B a^{10} b^4 + 1128 A B a^{11} b^3 + 288 A B a^{12} b^2)) / (a b^{14} + b^{15} - 3 a^2 b^{13} - 3 a^3 b^{12} + 3 a^4 b^{11} + 3 a^5 b^{10} - a^6 b^9 - a^7 b^8) - (((4(4 B b^{21} + 48 A a^2 b^{19} + 72 A a^3 b^{18} - 156 A a^4 b^{17} - 84 A a^5 b^{16} + 192 A a^6 b^{15} + 48 A a^7 b^{14} - 108 A a^8 b^{13} - 12 A a^9 b^{12} + 24 A a^{10} b^{11} + 28 B a^2 b^{19} - 80 B a^3 b^{18} - 120 B a^4 b^{17} + 276 B a^5 b^{16} + 164 B a^6 b^{15} - 360 B a^7 b^{14} - 100 B a^8 b^{13} + 212 B a^9 b^{12} + 24 B a^{10} b^{11} - 48 B a^{11} b^{10} - 24 A a b^{20})) / (a b^{18} + b^{19} - 3 a^2 b^{17} - 3 a^3 b^{16} + 3 a^4
\end{aligned}$$

$$\begin{aligned}
& 4*b^{15} + 3*a^5*b^{14} - a^6*b^{13} - a^7*b^{12}) - (4*\tan(c/2 + (d*x)/2)*(12*B*a^2 + B*b^2 - 6*A*a*b)*(8*a*b^{19} - 8*a^2*b^{18} - 32*a^3*b^{17} + 32*a^4*b^{16} + 4 \\
& 8*a^5*b^{15} - 48*a^6*b^{14} - 32*a^7*b^{13} + 32*a^8*b^{12} + 8*a^9*b^{11} - 8*a^{10}*b^{10}))/ (b^5*(a*b^{14} + b^{15} - 3*a^2*b^{13} - 3*a^3*b^{12} + 3*a^4*b^{11} + 3*a^5*b \\
& ^{10} - a^6*b^9 - a^7*b^8)))*(12*B*a^2 + B*b^2 - 6*A*a*b))/(2*b^5))*(12*B*a^2 + B*b^2 - 6*A*a*b))/(2*b^5) + (((8*\tan(c/2 + (d*x)/2)*(288*B^2*a^{14} + B^2* \\
& b^{14} - 2*B^2*a*b^{13} - 288*B^2*a^{13}*b + 36*A^2*a^2*b^{12} - 72*A^2*a^3*b^{11} + 36*A^2*a^4*b^{10} + 288*A^2*a^5*b^9 - 288*A^2*a^6*b^8 - 432*A^2*a^7*b^7 + 441 \\
& *A^2*a^8*b^6 + 288*A^2*a^9*b^5 - 288*A^2*a^{10}*b^4 - 72*A^2*a^{11}*b^3 + 72*A^2*a^{12}*b^2 + 21*B^2*a^2*b^{12} - 40*B^2*a^3*b^{11} + 74*B^2*a^4*b^{10} - 108*B^2* \\
& a^5*b^9 + 18*B^2*a^6*b^8 + 872*B^2*a^7*b^7 - 827*B^2*a^8*b^6 - 1538*B^2*a^9*b^5 + 1538*B^2*a^{10}*b^4 + 1104*B^2*a^{11}*b^3 - 1104*B^2*a^{12}*b^2 - 12*A*B*a \\
& *b^{13} - 288*A*B*a^{13}*b + 24*A*B*a^2*b^{12} - 108*A*B*a^3*b^{11} + 192*A*B*a^4*b^{10} - 72*A*B*a^5*b^9 - 1008*A*B*a^6*b^8 + 984*A*B*a^7*b^7 + 1632*A*B*a^8*b^6 - 1650*A*B*a^9*b^5 - 1128*A*B*a^{10}*b^4 + 1128*A*B*a^{11}*b^3 + 288*A*B*a^{12} \\
& *b^2))/ (a*b^{14} + b^{15} - 3*a^2*b^{13} - 3*a^3*b^{12} + 3*a^4*b^{11} + 3*a^5*b^{10} - a^6*b^9 - a^7*b^8) + (((4*(4*B*b^{21} + 48*A*a^2*b^{19} + 72*A*a^3*b^{18} - 156* \\
& A*a^4*b^{17} - 84*A*a^5*b^{16} + 192*A*a^6*b^{15} + 48*A*a^7*b^{14} - 108*A*a^8*b^{13} - 12*A*a^9*b^{12} + 24*A*a^{10}*b^{11} + 28*B*a^2*b^{19} - 80*B*a^3*b^{18} - 120*B* \\
& a^4*b^{17} + 276*B*a^5*b^{16} + 164*B*a^6*b^{15} - 360*B*a^7*b^{14} - 100*B*a^8*b^{13} + 212*B*a^9*b^{12} + 24*B*a^{10}*b^{11} - 48*B*a^{11}*b^{10} - 24*A*a*b^{20}))/ (a*b^{18} + b^{19} - 3*a^2*b^{17} - 3*a^3*b^{16} + 3*a^4*b^{15} + 3*a^5*b^{14} - a^6*b^{13} - a \\
& ^7*b^{12}) + (4*\tan(c/2 + (d*x)/2)*(12*B*a^2 + B*b^2 - 6*A*a*b)*(8*a*b^{19} - 8 \\
& *a^2*b^{18} - 32*a^3*b^{17} + 32*a^4*b^{16} + 48*a^5*b^{15} - 48*a^6*b^{14} - 32*a^7* \\
& b^{13} + 32*a^8*b^{12} + 8*a^9*b^{11} - 8*a^{10}*b^{10}))/ (b^5*(a*b^{14} + b^{15} - 3*a^2 \\
& *b^{13} - 3*a^3*b^{12} + 3*a^4*b^{11} + 3*a^5*b^{10} - a^6*b^9 - a^7*b^8)))*(12*B*a \\
& ^2 + B*b^2 - 6*A*a*b))/(2*b^5))*(12*B*a^2 + B*b^2 - 6*A*a*b))/(2*b^5))*(12 \\
& *B*a^2 + B*b^2 - 6*A*a*b)*i)/(b^5*d) - (a^2*atan(((a^2*((a + b)^5*(a - b) \\
& ^5)^(1/2))*((8*\tan(c/2 + (d*x)/2)*(288*B^2*a^{14} + B^2*b^{14} - 2*B^2*a*b^{13} - 2 \\
& 88*B^2*a^{13}*b + 36*A^2*a^2*b^{12} - 72*A^2*a^3*b^{11} + 36*A^2*a^4*b^{10} + 288*A \\
& ^2*a^5*b^9 - 288*A^2*a^6*b^8 - 432*A^2*a^7*b^7 + 441*A^2*a^8*b^6 + 288*A^2* \\
& a^9*b^5 - 288*A^2*a^{10}*b^4 - 72*A^2*a^{11}*b^3 + 72*A^2*a^{12}*b^2 + 21*B^2*a^2 \\
& *b^{12} - 40*B^2*a^3*b^{11} + 74*B^2*a^4*b^{10} - 108*B^2*a^5*b^9 + 18*B^2*a^6*b^8 \\
& + 872*B^2*a^7*b^7 - 827*B^2*a^8*b^6 - 1538*B^2*a^9*b^5 + 1538*B^2*a^{10}*b^4 \\
& + 1104*B^2*a^{11}*b^3 - 1104*B^2*a^{12}*b^2 - 12*A*B*a*b^{13} - 288*A*B*a^{13}*b \\
& + 24*A*B*a^2*b^{12} - 108*A*B*a^3*b^{11} + 192*A*B*a^4*b^{10} - 72*A*B*a^5*b^9 - \\
& 1008*A*B*a^6*b^8 + 984*A*B*a^7*b^7 + 1632*A*B*a^8*b^6 - 1650*A*B*a^9*b^5 - \\
& 1128*A*B*a^{10}*b^4 + 1128*A*B*a^{11}*b^3 + 288*A*B*a^{12}*b^2))/ (a*b^{14} + b^{15} - \\
& 3*a^2*b^{13} - 3*a^3*b^{12} + 3*a^4*b^{11} + 3*a^5*b^{10} - a^6*b^9 - a^7*b^8) - (\\
& a^2*((4*(4*B*b^{21} + 48*A*a^2*b^{19} + 72*A*a^3*b^{18} - 156*A*a^4*b^{17} - 84*A*a \\
& ^5*b^{16} + 192*A*a^6*b^{15} + 48*A*a^7*b^{14} - 108*A*a^8*b^{13} - 12*A*a^9*b^{12} + \\
& 24*A*a^{10}*b^{11} + 28*B*a^2*b^{19} - 80*B*a^3*b^{18} - 120*B*a^4*b^{17} + 276*B*a^ \\
& 5*b^{16} + 164*B*a^6*b^{15} - 360*B*a^7*b^{14} - 100*B*a^8*b^{13} + 212*B*a^9*b^{12} \\
& + 24*B*a^{10}*b^{11} - 48*B*a^{11}*b^{10} - 24*A*a*b^{20}))/ (a*b^{18} + b^{19} - 3*a^2*b^ \\
& ^{17} - 3*a^3*b^{16} + 3*a^4*b^{15} + 3*a^5*b^{14} - a^6*b^{13} - a^7*b^{12}) - (4*a^2*t
\end{aligned}$$

$$\begin{aligned}
& \text{an}(c/2 + (d*x)/2)*((a + b)^5*(a - b)^5)^{(1/2)}*(12*A*b^5 - 12*B*a^5 - 15*A*a^2*b^3 + 29*B*a^3*b^2 + 6*A*a^4*b - 20*B*a*b^4)*(8*a*b^19 - 8*a^2*b^18 - 32*a^3*b^17 + 32*a^4*b^16 + 48*a^5*b^15 - 48*a^6*b^14 - 32*a^7*b^13 + 32*a^8*b^12 + 8*a^9*b^11 - 8*a^10*b^10))/((b^15 - 5*a^2*b^13 + 10*a^4*b^11 - 10*a^6*b^9 + 5*a^8*b^7 - a^10*b^5)*(a*b^14 + b^15 - 3*a^2*b^13 - 3*a^3*b^12 + 3*a^4*b^11 + 3*a^5*b^10 - a^6*b^9 - a^7*b^8)))*((a + b)^5*(a - b)^5)^{(1/2)}*(12*A*b^5 - 12*B*a^5 - 15*A*a^2*b^3 + 29*B*a^3*b^2 + 6*A*a^4*b - 20*B*a*b^4))/(2*(b^15 - 5*a^2*b^13 + 10*a^4*b^11 - 10*a^6*b^9 + 5*a^8*b^7 - a^10*b^5)))*((8*tan(c/2 + (d*x)/2)*(288*B^2*a^14 + B^2*b^14 - 2*B^2*a*b^13 - 288*B^2*a^13*b + 36*A^2*a^2*b^12 - 72*A^2*a^3*b^11 + 36*A^2*a^4*b^10 + 288*A^2*a^5*b^9 - 288*A^2*a^6*b^8 - 432*A^2*a^7*b^7 + 441*A^2*a^8*b^6 + 288*A^2*a^9*b^5 - 288*A^2*a^10*b^4 - 72*A^2*a^11*b^3 + 72*A^2*a^12*b^2 + 21*B^2*a^2*b^12 - 40*B^2*a^3*b^11 + 74*B^2*a^4*b^10 - 108*B^2*a^5*b^9 + 18*B^2*a^6*b^8 + 872*B^2*a^7*b^7 - 827*B^2*a^8*b^6 - 1538*B^2*a^9*b^5 + 1538*B^2*a^10*b^4 + 1104*B^2*a^11*b^3 - 1104*B^2*a^12*b^2 - 12*A*B*a*b^13 - 288*A*B*a^13*b + 24*A*B*a^2*b^12 - 108*A*B*a^3*b^11 + 192*A*B*a^4*b^10 - 72*A*B*a^5*b^9 - 1008*A*B*a^6*b^8 + 984*A*B*a^7*b^7 + 1632*A*B*a^8*b^6 - 1650*A*B*a^9*b^5 - 1128*A*B*a^10*b^4 + 1128*A*B*a^11*b^3 + 288*A*B*a^12*b^2))/(a*b^14 + b^15 - 3*a^2*b^13 - 3*a^3*b^12 + 3*a^4*b^11 + 3*a^5*b^10 - a^6*b^9 - a^7*b^8) + (a^2*((4*(4*B*b^21 + 48*A*a^2*b^19 + 72*A*a^3*b^18 - 156*A*a^4*b^17 - 84*A*a^5*b^16 + 192*A*a^6*b^15 + 48*A*a^7*b^14 - 108*A*a^8*b^13 - 12*A*a^9*b^12 + 24*A*a^10*b^11 + 28*B*a^2*b^19 - 80*B*a^3*b^18 - 120*B*a^4*b^17 + 276*B*a^5*b^16 + 164*B*a^6*b^15 - 360*B*a^7*b^14 - 100*B*a^8*b^13 + 212*B*a^9*b^12 + 24*B*a^10*b^11 - 48*B*a^11*b^10 - 24*A*a*b^20)))/(a*b^18 + b^19 - 3*a^2*b^17 - 3*a^3*b^16 + 3*a^4*b^15 + 3*a^5*b^14 - a^6*b^13 - a^7*b^12) + (4*a^2*tan(c/2 + (d*x)/2)*((a + b)^5*(a - b)^5)^{(1/2)}*(12*A*b^5 - 12*B*a^5 - 15*A*a^2*b^3 + 29*B*a^3*b^2 + 6*A*a^4*b - 20*B*a*b^4)*(8*a*b^19 - 8*a^2*b^18 - 32*a^3*b^17 + 32*a^4*b^16 + 48*a^5*b^15 - 48*a^6*b^14 - 32*a^7*b^13 + 32*a^8*b^12 + 8*a^9*b^11 - 8*a^10*b^10))/((b^15 - 5*a^2*b^13 + 10*a^4*b^11 - 10*a^6*b^9 + 5*a^8*b^7 - a^10*b^5)*(a*b^14 + b^15 - 3*a^2*b^13 - 3*a^3*b^12 + 3*a^4*b^11 + 3*a^5*b^10 - a^6*b^9 - a^7*b^8)))*((a + b)^5*(a - b)^5)^{(1/2)}*(12*A*b^5 - 12*B*a^5 - 15*A*a^2*b^3 + 29*B*a^3*b^2 + 6*A*a^4*b - 20*B*a*b^4))/(2*(b^15 - 5*a^2*b^13 + 10*a^4*b^11 - 10*a^6*b^9 + 5*a^8*b^7 - a^10*b^5)))*((8*(1728*B^3*a^15 - 864*B^3*a^14*b - 432*A^3*a^4*b^11 - 432*A^3*a^5*b^10 + 1404*A^3*a^6*b^9 + 756*A^3*a^7*b^8 - 1728*A^3*a^8*b^7 - 486*A^3*a^9*b^6 + 972*A^3*a^10*b^5 + 108*A^3*a^11*b^4 - 216*A^3*a^12*b^3 + 20*B^3*a^3*b^12 - 20*B^3*a^4*b^11 + 411*B^3*a^5*b^10 - 11*B^3*a^6*b^9 + 1314*B^3*a^7*b^8 + 2326*B^3*a^8*b^7 - 7829*B^3*a^9*b^6 - 4770*B^3*a^10*b^5 + 11700*B^3*a^11*b^4 + 3456*B^3*a^12*b^3 - 7344*B^3*a^13*b^2 - 2592*A*B^2*a^14*b - 12*A*B^2*a^2*b^13 + 12*A*B^2*a^3*b^12 - 489*A*B^2*a^4*b^11 + 9*A*B^2*a^5*b^10 - 2892*A*B^2*a^6*b^9 - 3972*A*B^2*a^7*b^8 + 13
\end{aligned}$$

$$\begin{aligned}
& 347*A*B^2*a^8*b^7 + 7767*A*B^2*a^9*b^6 - 18594*A*B^2*a^{10}*b^5 - 5400*A*B^2* \\
& a^{11}*b^4 + 11232*A*B^2*a^{12}*b^3 + 1296*A*B^2*a^{13}*b^2 + 144*A^2*B*a^3*b^{12} \\
& + 1980*A^2*B*a^5*b^{10} + 2268*A^2*B*a^6*b^9 - 7524*A^2*B*a^7*b^8 - 4203*A^2* \\
& B*a^8*b^7 + 9828*A^2*B*a^9*b^6 + 2808*A^2*B*a^{10}*b^5 - 5724*A^2*B*a^{11}*b^4 \\
& - 648*A^2*B*a^{12}*b^3 + 1296*A^2*B*a^{13}*b^2)/(a*b^{18} + b^{19} - 3*a^2*b^{17} - \\
& 3*a^3*b^{16} + 3*a^4*b^{15} + 3*a^5*b^{14} - a^6*b^{13} - a^7*b^{12}) - (a^2*((a + b) \\
& ^5*(a - b)^5)^{(1/2)}*((8*\tan(c/2 + (d*x)/2))*(288*B^2*a^{14} + B^2*b^{14} - 2*B^2 \\
& *a*b^{13} - 288*B^2*a^{13}*b + 36*A^2*a^2*b^{12} - 72*A^2*a^3*b^{11} + 36*A^2*a^4*b \\
& ^{10} + 288*A^2*a^5*b^9 - 288*A^2*a^6*b^8 - 432*A^2*a^7*b^7 + 441*A^2*a^8*b^6 \\
& + 288*A^2*a^9*b^5 - 288*A^2*a^{10}*b^4 - 72*A^2*a^{11}*b^3 + 72*A^2*a^{12}*b^2 + \\
& 21*B^2*a^2*b^{12} - 40*B^2*a^3*b^{11} + 74*B^2*a^4*b^{10} - 108*B^2*a^5*b^9 + 18 \\
& *B^2*a^6*b^8 + 872*B^2*a^7*b^7 - 827*B^2*a^8*b^6 - 1538*B^2*a^9*b^5 + 1538* \\
& B^2*a^{10}*b^4 + 1104*B^2*a^{11}*b^3 - 1104*B^2*a^{12}*b^2 - 12*A*B*a*b^{13} - 288* \\
& A*B*a^{13}*b + 24*A*B*a^2*b^{12} - 108*A*B*a^3*b^{11} + 192*A*B*a^4*b^{10} - 72*A*B \\
& *a^5*b^9 - 1008*A*B*a^6*b^8 + 984*A*B*a^7*b^7 + 1632*A*B*a^8*b^6 - 1650*A*B \\
& *a^9*b^5 - 1128*A*B*a^{10}*b^4 + 1128*A*B*a^{11}*b^3 + 288*A*B*a^{12}*b^2))/(a*b^ \\
& 14 + b^{15} - 3*a^2*b^{13} - 3*a^3*b^{12} + 3*a^4*b^{11} + 3*a^5*b^{10} - a^6*b^9 - a \\
& ^7*b^8) - (a^2*((4*(4*B*b^{21} + 48*A*a^2*b^{19} + 72*A*a^3*b^{18} - 156*A*a^4*b^ \\
& 17 - 84*A*a^5*b^{16} + 192*A*a^6*b^{15} + 48*A*a^7*b^{14} - 108*A*a^8*b^{13} - 12*A \\
& *a^9*b^{12} + 24*A*a^{10}*b^{11} + 28*B*a^2*b^{19} - 80*B*a^3*b^{18} - 120*B*a^4*b^{17} \\
& + 276*B*a^5*b^{16} + 164*B*a^6*b^{15} - 360*B*a^7*b^{14} - 100*B*a^8*b^{13} + 212* \\
& B*a^9*b^{12} + 24*B*a^{10}*b^{11} - 48*B*a^{11}*b^{10} - 24*A*a*b^{20}))/((a*b^{18} + b^{19} \\
& - 3*a^2*b^{17} - 3*a^3*b^{16} + 3*a^4*b^{15} + 3*a^5*b^{14} - a^6*b^{13} - a^7*b^{12}) \\
& - (4*a^2*\tan(c/2 + (d*x)/2))*((a + b)^5*(a - b)^5)^{(1/2)}*(12*A*b^5 - 12*B*a \\
& ^5 - 15*A*a^2*b^3 + 29*B*a^3*b^2 + 6*A*a^4*b - 20*B*a*b^4)*(8*a*b^{19} - 8*a^ \\
& 2*b^{18} - 32*a^3*b^{17} + 32*a^4*b^{16} + 48*a^5*b^{15} - 48*a^6*b^{14} - 32*a^7*b^{1 \\
& 3 + 32*a^8*b^{12} + 8*a^9*b^{11} - 8*a^{10}*b^{10}))/((b^{15} - 5*a^2*b^{13} + 10*a^4*b \\
& ^{11} - 10*a^6*b^9 + 5*a^8*b^7 - a^{10}*b^5)*(a*b^{14} + b^{15} - 3*a^2*b^{13} - 3*a^ \\
& 3*b^{12} + 3*a^4*b^{11} + 3*a^5*b^{10} - a^6*b^9 - a^7*b^8)))*((a + b)^5*(a - b) \\
& ^5)^{(1/2)}*(12*A*b^5 - 12*B*a^5 - 15*A*a^2*b^3 + 29*B*a^3*b^2 + 6*A*a^4*b - 2 \\
& 0*B*a*b^4))/(2*(b^{15} - 5*a^2*b^{13} + 10*a^4*b^{11} - 10*a^6*b^9 + 5*a^8*b^7 - \\
& a^{10}*b^5))*((12*A*b^5 - 12*B*a^5 - 15*A*a^2*b^3 + 29*B*a^3*b^2 + 6*A*a^4*b \\
& - 20*B*a*b^4))/(2*(b^{15} - 5*a^2*b^{13} + 10*a^4*b^{11} - 10*a^6*b^9 + 5*a^8*b^7 \\
& - a^{10}*b^5)) + (a^2*((a + b)^5*(a - b)^5)^{(1/2)}*((8*\tan(c/2 + (d*x)/2))*(28 \\
& 8*B^2*a^{14} + B^2*b^{14} - 2*B^2*a*b^{13} - 288*B^2*a^{13}*b + 36*A^2*a^2*b^{12} - 7 \\
& 2*A^2*a^3*b^{11} + 36*A^2*a^4*b^{10} + 288*A^2*a^5*b^9 - 288*A^2*a^6*b^8 - 432* \\
& A^2*a^7*b^7 + 441*A^2*a^8*b^6 + 288*A^2*a^9*b^5 - 288*A^2*a^{10}*b^4 - 72*A^2 \\
& *a^{11}*b^3 + 72*A^2*a^{12}*b^2 + 21*B^2*a^2*b^{12} - 40*B^2*a^3*b^{11} + 74*B^2*a^ \\
& 4*b^{10} - 108*B^2*a^5*b^9 + 18*B^2*a^6*b^8 + 872*B^2*a^7*b^7 - 827*B^2*a^8*b \\
& ^6 - 1538*B^2*a^9*b^5 + 1538*B^2*a^{10}*b^4 + 1104*B^2*a^{11}*b^3 - 1104*B^2*a^ \\
& 12*b^2 - 12*A*B*a*b^{13} - 288*A*B*a^{13}*b + 24*A*B*a^2*b^{12} - 108*A*B*a^3*b^1 \\
& 1 + 192*A*B*a^4*b^{10} - 72*A*B*a^5*b^9 - 1008*A*B*a^6*b^8 + 984*A*B*a^7*b^7 \\
& + 1632*A*B*a^8*b^6 - 1650*A*B*a^9*b^5 - 1128*A*B*a^{10}*b^4 + 1128*A*B*a^{11}*b \\
& ^3 + 288*A*B*a^{12}*b^2))/(a*b^{14} + b^{15} - 3*a^2*b^{13} - 3*a^3*b^{12} + 3*a^4*b^ \\
& 11 + 3*a^5*b^{10} - a^6*b^9 - a^7*b^8) + (a^2*((4*(4*B*b^{21} + 48*A*a^2*b^{19} +
\end{aligned}$$

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72*A*a^3*b^18 - 156*A*a^4*b^17 - 84*A*a^5*b^16 + 192*A*a^6*b^15 + 48*A*a^7
*b^14 - 108*A*a^8*b^13 - 12*A*a^9*b^12 + 24*A*a^10*b^11 + 28*B*a^2*b^19 - 8
0*B*a^3*b^18 - 120*B*a^4*b^17 + 276*B*a^5*b^16 + 164*B*a^6*b^15 - 360*B*a^7
*b^14 - 100*B*a^8*b^13 + 212*B*a^9*b^12 + 24*B*a^10*b^11 - 48*B*a^11*b^10 -
24*A*a*b^20))/(a*b^18 + b^19 - 3*a^2*b^17 - 3*a^3*b^16 + 3*a^4*b^15 + 3*a^
5*b^14 - a^6*b^13 - a^7*b^12) + (4*a^2*tan(c/2 + (d*x)/2)*((a + b)^5*(a - b
)^5)^(1/2)*(12*A*b^5 - 12*B*a^5 - 15*A*a^2*b^3 + 29*B*a^3*b^2 + 6*A*a^4*b -
20*B*a*b^4)*(8*a*b^19 - 8*a^2*b^18 - 32*a^3*b^17 + 32*a^4*b^16 + 48*a^5*b^
15 - 48*a^6*b^14 - 32*a^7*b^13 + 32*a^8*b^12 + 8*a^9*b^11 - 8*a^10*b^10))/(
(b^15 - 5*a^2*b^13 + 10*a^4*b^11 - 10*a^6*b^9 + 5*a^8*b^7 - a^10*b^5)*(a*b^
14 + b^15 - 3*a^2*b^13 - 3*a^3*b^12 + 3*a^4*b^11 + 3*a^5*b^10 - a^6*b^9 - a
^7*b^8)))*((a + b)^5*(a - b)^5)^(1/2)*(12*A*b^5 - 12*B*a^5 - 15*A*a^2*b^3 +
29*B*a^3*b^2 + 6*A*a^4*b - 20*B*a*b^4))/(2*(b^15 - 5*a^2*b^13 + 10*a^4*b^1
1 - 10*a^6*b^9 + 5*a^8*b^7 - a^10*b^5)))*(12*A*b^5 - 12*B*a^5 - 15*A*a^2*b^
3 + 29*B*a^3*b^2 + 6*A*a^4*b - 20*B*a*b^4))/(2*(b^15 - 5*a^2*b^13 + 10*a^4*
b^11 - 10*a^6*b^9 + 5*a^8*b^7 - a^10*b^5))))*((a + b)^5*(a - b)^5)^(1/2)*(1
2*A*b^5 - 12*B*a^5 - 15*A*a^2*b^3 + 29*B*a^3*b^2 + 6*A*a^4*b - 20*B*a*b^4)*
1i)/(d*(b^15 - 5*a^2*b^13 + 10*a^4*b^11 - 10*a^6*b^9 + 5*a^8*b^7 - a^10*b^5
))

```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \sec^5(c + dx)}{(a + b \sec(c + dx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**5*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**3,x)

[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**5/(a + b*sec(c + d*x))**3, x)

$$3.329 \quad \int \frac{\sec^4(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^3} dx$$

Optimal. Leaf size=289

$$\frac{a(Ab - aB) \tan(c + dx) \sec^2(c + dx)}{2bd(a^2 - b^2)(a + b \sec(c + dx))^2} - \frac{(-3a^2B + aAb + 2b^2B) \tan(c + dx)}{2b^3d(a^2 - b^2)} - \frac{a^2(-3a^3B + a^2Ab + 6ab^2B - 4Ab^3) \tan(c + dx)}{2b^3d(a^2 - b^2)^2(a + b \sec(c + dx))}$$

[Out] (A*b-3*B*a)*arctanh(sin(d*x+c))/b^4/d-a*(2*A*a^4*b-5*A*a^2*b^3+6*A*b^5-6*B*a^5+15*B*a^3*b^2-12*B*a*b^4)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(5/2)/b^4/(a+b)^(5/2)/d-1/2*(A*a*b-3*B*a^2+2*B*b^2)*tan(d*x+c)/b^3/(a^2-b^2)/d+1/2*a*(A*b-B*a)*sec(d*x+c)^2*tan(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^2-1/2*a^2*(A*a^2*b-4*A*b^3-3*B*a^3+6*B*a*b^2)*tan(d*x+c)/b^3/(a^2-b^2)^2/d/(a+b*sec(d*x+c))

Rubi [A] time = 1.42, antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {4029, 4090, 4082, 3998, 3770, 3831, 2659, 208}

$$\frac{(-3a^2B + aAb + 2b^2B) \tan(c + dx)}{2b^3d(a^2 - b^2)} - \frac{a(-5a^2Ab^3 + 2a^4Ab + 15a^3b^2B - 6a^5B - 12ab^4B + 6Ab^5) \tanh^{-1}\left(\frac{\sqrt{a-b} \tan(c+dx)}{\sqrt{a+b}}\right)}{b^4d(a-b)^{5/2}(a+b)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(Sec[c + d*x]^4*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^3,x]

[Out] ((A*b - 3*a*B)*ArcTanh[Sin[c + d*x]]/(b^4*d) - (a*(2*a^4*A*b - 5*a^2*A*b^3 + 6*A*b^5 - 6*a^5*B + 15*a^3*b^2*B - 12*a*b^4*B)*ArcTanh[(Sqrt[a - b]*Tan[c + d*x]/2)]/Sqrt[a + b]))/((a - b)^(5/2)*b^4*(a + b)^(5/2)*d) - ((a*A*b - 3*a^2*B + 2*b^2*B)*Tan[c + d*x])/(2*b^3*(a^2 - b^2)*d) + (a*(A*b - a*B)*Sec[c + d*x]^2*Tan[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) - (a^2*(a^2*A*b - 4*A*b^3 - 3*a^3*B + 6*a*b^2*B)*Tan[c + d*x])/(2*b^3*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 2659

```
Int[((a_) + (b_)*sin[Pi/2 + (c_) + (d_)*(x_)])^(-1), x_Symbol] := With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3770

```
Int[csc[(c_) + (d_)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x]
/; FreeQ[{c, d}, x]
```

Rule 3831

```
Int[csc[(e_) + (f_)*(x_)]/(csc[(e_) + (f_)*(x_)]*(b_) + (a_)), x_Symbol]
:= Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3998

```
Int[(csc[(e_) + (f_)*(x_)]*(csc[(e_) + (f_)*(x_)]*(B_) + (A_)))/(csc[(e_) + (f_)*(x_)]*(b_) + (a_)), x_Symbol] := Dist[B/b, Int[Csc[e + f*x], x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 4029

```
Int[(csc[(e_) + (f_)*(x_)]*(d_)^(n_)*(csc[(e_) + (f_)*(x_)]*(b_) + (a_))^(m_)*(csc[(e_) + (f_)*(x_)]*(B_) + (A_)), x_Symbol] := Simp[(a*d^2*(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[d/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(n - 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2*(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 1]
```

Rule 4082

```
Int[csc[(e_) + (f_)*(x_)]*((A_) + csc[(e_) + (f_)*(x_)]*(B_) + csc[(e_) + (f_)*(x_)]^2*(C_))*(csc[(e_) + (f_)*(x_)]*(b_) + (a_))^(m_), x_Symbol] := -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]
```

Rule 4090

```

Int[csc[(e_.) + (f_.)*(x_.)]^2*((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[
(e_.) + (f_.)*(x_.)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_), x
_Symbol] := Simp[(a*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dist[1/(b^2*(m + 1)*(a^2 - b^
2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(m + 1)*(-(a*(b*B
- a*C)) + A*b^2) + (b*B*(a^2 + b^2*(m + 1)) - a*(A*b^2*(m + 2) + C*(a^2 +
b^2*(m + 1)))]*Csc[e + f*x] - b*C*(m + 1)*(a^2 - b^2)*Csc[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]

```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^4(c + dx)(A + B \sec(c + dx))}{(a + b \sec(c + dx))^3} dx &= \frac{a(Ab - aB) \sec^2(c + dx) \tan(c + dx)}{2b(a^2 - b^2)d(a + b \sec(c + dx))^2} + \frac{\int \frac{\sec^2(c + dx)(2a(Ab - aB) - 2b(Ab - aB) \sec(c + dx))}{(a + b \sec(c + dx))^2} dx}{2b(a^2 - b^2)} \\
&= \frac{a(Ab - aB) \sec^2(c + dx) \tan(c + dx)}{2b(a^2 - b^2)d(a + b \sec(c + dx))^2} - \frac{a^2(a^2 Ab - 4Ab^3 - 3a^3 B + 6ab^2 B)}{2b^3(a^2 - b^2)^2 d(a + b \sec(c + dx))} \\
&= -\frac{(aAb - 3a^2 B + 2b^2 B) \tan(c + dx)}{2b^3(a^2 - b^2)d} + \frac{a(Ab - aB) \sec^2(c + dx) \tan(c + dx)}{2b(a^2 - b^2)d(a + b \sec(c + dx))} \\
&= -\frac{(aAb - 3a^2 B + 2b^2 B) \tan(c + dx)}{2b^3(a^2 - b^2)d} + \frac{a(Ab - aB) \sec^2(c + dx) \tan(c + dx)}{2b(a^2 - b^2)d(a + b \sec(c + dx))} \\
&= \frac{(Ab - 3aB) \tanh^{-1}(\sin(c + dx))}{b^4 d} - \frac{(aAb - 3a^2 B + 2b^2 B) \tan(c + dx)}{2b^3(a^2 - b^2)d} + \\
&= \frac{(Ab - 3aB) \tanh^{-1}(\sin(c + dx))}{b^4 d} - \frac{(aAb - 3a^2 B + 2b^2 B) \tan(c + dx)}{2b^3(a^2 - b^2)d} + \\
&= \frac{(Ab - 3aB) \tanh^{-1}(\sin(c + dx))}{b^4 d} - \frac{a(2a^4 Ab - 5a^2 Ab^3 + 6Ab^5 - 6a^5 B + 6ab^4 B)}{(a - b)^2 d}
\end{aligned}$$

Mathematica [A] time = 6.52, size = 418, normalized size = 1.45

$$\frac{a^2 Ab \sin(c + dx) - a^3 B \sin(c + dx)}{2b^2 d(b - a)(a + b)(a \cos(c + dx) + b)^2} + \frac{4a^5 B \sin(c + dx) - 2a^4 Ab \sin(c + dx) - 7a^3 b^2 B \sin(c + dx) + 5a^2 Ab^3 \sin(c + dx)}{2b^3 d(b - a)^2(a + b)^2(a \cos(c + dx) + b)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(Sec[c + d*x]^4*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^3,x]

[Out] (a*(2*a^4*A*b - 5*a^2*A*b^3 + 6*A*b^5 - 6*a^5*B + 15*a^3*b^2*B - 12*a*b^4*B)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]/(b^4*Sqrt[a^2 - b^2]*(-a^2 + b^2)^2*d) + ((-(A*b) + 3*a*B)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/(b^4*d) + ((A*b - 3*a*B)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/(b^4*d) + (B*Sin[(c + d*x)/2])/(b^3*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) + (B*Sin[(c + d*x)/2])/(b^3*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])) + (a^2*A*b*Sin[c + d*x] - a^3*B*Sin[c + d*x])/(2*b^2*(-a + b)*(a + b)*d*(b + a*Cos[c + d*x])^2) + (-2*a^4*A*b*Sin[c + d*x] + 5*a^2*A*b^3*Sin[c + d*x] + 4*a^5*B*Sin[c + d*x] - 7*a^3*b^2*B*Sin[c + d*x])/(2*b^3*(-a + b)^2*(a + b)^2*d*(b + a*Cos[c + d*x]))

fricas [B] time = 48.00, size = 2111, normalized size = 7.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm="fricas")

[Out] [-1/4*(((6*B*a^8 - 2*A*a^7*b - 15*B*a^6*b^2 + 5*A*a^5*b^3 + 12*B*a^4*b^4 - 6*A*a^3*b^5)*cos(d*x + c)^3 + 2*(6*B*a^7*b - 2*A*a^6*b^2 - 15*B*a^5*b^3 + 5*A*a^4*b^4 + 12*B*a^3*b^5 - 6*A*a^2*b^6)*cos(d*x + c)^2 + (6*B*a^6*b^2 - 2*A*a^5*b^3 - 15*B*a^4*b^4 + 5*A*a^3*b^5 + 12*B*a^2*b^6 - 6*A*a*b^7)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c))^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + 2*((3*B*a^9 - A*a^8*b - 9*B*a^7*b^2 + 3*A*a^6*b^3 + 9*B*a^5*b^4 - 3*A*a^4*b^5 - 3*B*a^3*b^6 + A*a^2*b^7)*cos(d*x + c)^3 + 2*(3*B*a^8*b - A*a^7*b^2 - 9*B*a^6*b^3 + 3*A*a^5*b^4 + 9*B*a^4*b^5 - 3*A*a^3*b^6 - 3*B*a^2*b^7 + A*a*b^8)*cos(d*x + c)^2 + (3*B*a^7*b^2 - A*a^6*b^3 - 9*B*a^5*b^4 + 3*A*a^4*b^5 + 9*B*a^3*b^6 - 3*A*a^2*b^7 - 3*B*a*b^8 + A*b^9)*cos(d*x + c))*log(sin(d*x + c) + 1) - 2*((3*B*a^9 - A*a^8*b - 9*B*a^7*b^2 + 3*A*a^6*b^3 + 9*B*a^5*b^4 - 3*A*a^4*b^5 - 3*B*a^3*b^6 + A*a^2*b^7)*cos(d*x + c)^3 + 2*(3*B*a^8*b - A*a^7*b^2 - 9*B*a^6*b^3 + 3*A*a^5*b^4 + 9*B*a^4*b^5 - 3*A*a^3*b^6 - 3*B*a^2*b^7 + A*a*b^8)*cos(d*x + c)^2

+ (3*B*a^7*b^2 - A*a^6*b^3 - 9*B*a^5*b^4 + 3*A*a^4*b^5 + 9*B*a^3*b^6 - 3*A*a^2*b^7 - 3*B*a*b^8 + A*b^9)*cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(2*B*a^6*b^3 - 6*B*a^4*b^5 + 6*B*a^2*b^7 - 2*B*b^9 + (6*B*a^8*b - 2*A*a^7*b^2 - 17*B*a^6*b^3 + 7*A*a^5*b^4 + 13*B*a^4*b^5 - 5*A*a^3*b^6 - 2*B*a^2*b^7)*cos(d*x + c)^2 + (9*B*a^7*b^2 - 3*A*a^6*b^3 - 25*B*a^5*b^4 + 9*A*a^4*b^5 + 20*B*a^3*b^6 - 6*A*a^2*b^7 - 4*B*a*b^8)*cos(d*x + c))*sin(d*x + c))/((a^8*b^4 - 3*a^6*b^6 + 3*a^4*b^8 - a^2*b^10)*d*cos(d*x + c)^3 + 2*(a^7*b^5 - 3*a^5*b^7 + 3*a^3*b^9 - a*b^11)*d*cos(d*x + c)^2 + (a^6*b^6 - 3*a^4*b^8 + 3*a^2*b^10 - b^12)*d*cos(d*x + c)), 1/2*((6*B*a^8 - 2*A*a^7*b - 15*B*a^6*b^2 + 5*A*a^5*b^3 + 12*B*a^4*b^4 - 6*A*a^3*b^5)*cos(d*x + c)^3 + 2*(6*B*a^7*b - 2*A*a^6*b^2 - 15*B*a^5*b^3 + 5*A*a^4*b^4 + 12*B*a^3*b^5 - 6*A*a^2*b^6)*cos(d*x + c)^2 + (6*B*a^6*b^2 - 2*A*a^5*b^3 - 15*B*a^4*b^4 + 5*A*a^3*b^5 + 12*B*a^2*b^6 - 6*A*a*b^7)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - ((3*B*a^9 - A*a^8*b - 9*B*a^7*b^2 + 3*A*a^6*b^3 + 9*B*a^5*b^4 - 3*A*a^4*b^5 - 3*B*a^3*b^6 + A*a^2*b^7)*cos(d*x + c)^3 + 2*(3*B*a^8*b - A*a^7*b^2 - 9*B*a^6*b^3 + 3*A*a^5*b^4 + 9*B*a^4*b^5 - 3*A*a^3*b^6 - 3*B*a^2*b^7 + A*a*b^8)*cos(d*x + c)^2 + (3*B*a^7*b^2 - A*a^6*b^3 - 9*B*a^5*b^4 + 3*A*a^4*b^5 + 9*B*a^3*b^6 - 3*A*a^2*b^7 - 3*B*a*b^8 + A*b^9)*cos(d*x + c))*log(sin(d*x + c) + 1) + ((3*B*a^9 - A*a^8*b - 9*B*a^7*b^2 + 3*A*a^6*b^3 + 9*B*a^5*b^4 - 3*A*a^4*b^5 - 3*B*a^3*b^6 + A*a^2*b^7)*cos(d*x + c)^3 + 2*(3*B*a^8*b - A*a^7*b^2 - 9*B*a^6*b^3 + 3*A*a^5*b^4 + 9*B*a^4*b^5 - 3*A*a^3*b^6 - 3*B*a^2*b^7 + A*a*b^8)*cos(d*x + c)^2 + (3*B*a^7*b^2 - A*a^6*b^3 - 9*B*a^5*b^4 + 3*A*a^4*b^5 + 9*B*a^3*b^6 - 3*A*a^2*b^7 - 3*B*a*b^8 + A*b^9)*cos(d*x + c))*log(-sin(d*x + c) + 1) + (2*B*a^6*b^3 - 6*B*a^4*b^5 + 6*B*a^2*b^7 - 2*B*b^9 + (6*B*a^8*b - 2*A*a^7*b^2 - 17*B*a^6*b^3 + 7*A*a^5*b^4 + 13*B*a^4*b^5 - 5*A*a^3*b^6 - 2*B*a^2*b^7)*cos(d*x + c)^2 + (9*B*a^7*b^2 - 3*A*a^6*b^3 - 25*B*a^5*b^4 + 9*A*a^4*b^5 + 20*B*a^3*b^6 - 6*A*a^2*b^7 - 4*B*a*b^8)*cos(d*x + c))*sin(d*x + c))/((a^8*b^4 - 3*a^6*b^6 + 3*a^4*b^8 - a^2*b^10)*d*cos(d*x + c)^3 + 2*(a^7*b^5 - 3*a^5*b^7 + 3*a^3*b^9 - a*b^11)*d*cos(d*x + c)^2 + (a^6*b^6 - 3*a^4*b^8 + 3*a^2*b^10 - b^12)*d*cos(d*x + c))]

giac [B] time = 0.46, size = 581, normalized size = 2.01

$$\frac{(6Ba^6 - 2Aa^5b - 15Ba^4b^2 + 5Aa^3b^3 + 12Ba^2b^4 - 6Aab^5) \left(\pi \left[\frac{dx+c}{2\pi} + \frac{1}{2} \right] \operatorname{sgn}(-2a+2b) + \arctan \left(-\frac{a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - b \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{\sqrt{-a^2+b^2}} \right) \right)}{(a^4b^4 - 2a^2b^6 + b^8) \sqrt{-a^2+b^2}} - \frac{4Ba^6 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3}{\sqrt{-a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm="giac")

[Out] ((6*B*a^6 - 2*A*a^5*b - 15*B*a^4*b^2 + 5*A*a^3*b^3 + 12*B*a^2*b^4 - 6*A*a*b

$$\begin{aligned} &^5) * (\pi * \text{floor}(1/2 * (d * x + c) / \pi + 1/2) * \text{sgn}(-2 * a + 2 * b) + \arctan(-(a * \tan(1/2 * \\ &d * x + 1/2 * c) - b * \tan(1/2 * d * x + 1/2 * c)) / \sqrt{-a^2 + b^2})) / ((a^4 * b^4 - 2 * a^2 \\ &* b^6 + b^8) * \sqrt{-a^2 + b^2}) - (4 * B * a^6 * \tan(1/2 * d * x + 1/2 * c)^3 - 2 * A * a^5 * b \\ &* \tan(1/2 * d * x + 1/2 * c)^3 - 5 * B * a^5 * b * \tan(1/2 * d * x + 1/2 * c)^3 + 3 * A * a^4 * b^2 * \tan \\ &(1/2 * d * x + 1/2 * c)^3 - 7 * B * a^4 * b^2 * \tan(1/2 * d * x + 1/2 * c)^3 + 5 * A * a^3 * b^3 * \tan \\ &(1/2 * d * x + 1/2 * c)^3 + 8 * B * a^3 * b^3 * \tan(1/2 * d * x + 1/2 * c)^3 - 6 * A * a^2 * b^4 * \tan(\\ &1/2 * d * x + 1/2 * c)^3 - 4 * B * a^6 * \tan(1/2 * d * x + 1/2 * c) + 2 * A * a^5 * b * \tan(1/2 * d * x + \\ &1/2 * c) - 5 * B * a^5 * b * \tan(1/2 * d * x + 1/2 * c) + 3 * A * a^4 * b^2 * \tan(1/2 * d * x + 1/2 * c) \\ &+ 7 * B * a^4 * b^2 * \tan(1/2 * d * x + 1/2 * c) - 5 * A * a^3 * b^3 * \tan(1/2 * d * x + 1/2 * c) + 8 * \\ &B * a^3 * b^3 * \tan(1/2 * d * x + 1/2 * c) - 6 * A * a^2 * b^4 * \tan(1/2 * d * x + 1/2 * c)) / ((a^4 * b^ \\ &3 - 2 * a^2 * b^5 + b^7) * (a * \tan(1/2 * d * x + 1/2 * c)^2 - b * \tan(1/2 * d * x + 1/2 * c)^2 - \\ &a - b)^2) - (3 * B * a - A * b) * \log(\text{abs}(\tan(1/2 * d * x + 1/2 * c) + 1)) / b^4 + (3 * B * a \\ &- A * b) * \log(\text{abs}(\tan(1/2 * d * x + 1/2 * c) - 1)) / b^4 - 2 * B * \tan(1/2 * d * x + 1/2 * c) / ((\\ &\tan(1/2 * d * x + 1/2 * c)^2 - 1) * b^3) / d \end{aligned}$$

maple [B] time = 0.69, size = 1406, normalized size = 4.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(\sec(d*x+c)^4 * (A+B*\sec(d*x+c)) / (a+b*\sec(d*x+c))^3, x)$

[Out]
$$\begin{aligned} &2/d * a^4 / b^2 / (a * \tan(1/2 * d * x + 1/2 * c)^2 - \tan(1/2 * d * x + 1/2 * c)^2 * b - a - b)^2 / (a - b) / (a^ \\ &2 + 2 * a * b + b^2) * \tan(1/2 * d * x + 1/2 * c)^3 * A - 1/d * a^3 / b / (a * \tan(1/2 * d * x + 1/2 * c)^2 - \tan(1 \\ &/2 * d * x + 1/2 * c)^2 * b - a - b)^2 / (a - b) / (a^2 + 2 * a * b + b^2) * \tan(1/2 * d * x + 1/2 * c)^3 * A - 6/d * a \\ &^2 / (a * \tan(1/2 * d * x + 1/2 * c)^2 - \tan(1/2 * d * x + 1/2 * c)^2 * b - a - b)^2 / (a - b) / (a^2 + 2 * a * b + b \\ &^2) * \tan(1/2 * d * x + 1/2 * c)^3 * A - 4/d * a^5 / b^3 / (a * \tan(1/2 * d * x + 1/2 * c)^2 - \tan(1/2 * d * x + \\ &1/2 * c)^2 * b - a - b)^2 / (a - b) / (a^2 + 2 * a * b + b^2) * \tan(1/2 * d * x + 1/2 * c)^3 * B + 1/d * a^4 / b^2 / \\ &(a * \tan(1/2 * d * x + 1/2 * c)^2 - \tan(1/2 * d * x + 1/2 * c)^2 * b - a - b)^2 / (a - b) / (a^2 + 2 * a * b + b^2) \\ &* \tan(1/2 * d * x + 1/2 * c)^3 * B + 8/d * a^3 / b / (a * \tan(1/2 * d * x + 1/2 * c)^2 - \tan(1/2 * d * x + 1/2 * c \\ &)^2 * b - a - b)^2 / (a - b) / (a^2 + 2 * a * b + b^2) * \tan(1/2 * d * x + 1/2 * c)^3 * B - 2/d * a^4 / b^2 / (a * \tan \\ &(1/2 * d * x + 1/2 * c)^2 - \tan(1/2 * d * x + 1/2 * c)^2 * b - a - b)^2 / (a + b) / (a - b)^2 * \tan(1/2 * d * x + \\ &1/2 * c) * A - 1/d * a^3 / b / (a * \tan(1/2 * d * x + 1/2 * c)^2 - \tan(1/2 * d * x + 1/2 * c)^2 * b - a - b)^2 / (a \\ &+ b) / (a - b)^2 * \tan(1/2 * d * x + 1/2 * c) * A + 6/d * a^2 / (a * \tan(1/2 * d * x + 1/2 * c)^2 - \tan(1/2 * d * \\ &x + 1/2 * c)^2 * b - a - b)^2 / (a + b) / (a - b)^2 * \tan(1/2 * d * x + 1/2 * c) * A + 4/d * a^5 / b^3 / (a * \tan(1 \\ &/2 * d * x + 1/2 * c)^2 - \tan(1/2 * d * x + 1/2 * c)^2 * b - a - b)^2 / (a + b) / (a - b)^2 * \tan(1/2 * d * x + 1/2 \\ &* c) * B + 1/d * a^4 / b^2 / (a * \tan(1/2 * d * x + 1/2 * c)^2 - \tan(1/2 * d * x + 1/2 * c)^2 * b - a - b)^2 / (a + \\ &b) / (a - b)^2 * \tan(1/2 * d * x + 1/2 * c) * B - 8/d * a^3 / b / (a * \tan(1/2 * d * x + 1/2 * c)^2 - \tan(1/2 * d \\ &* x + 1/2 * c)^2 * b - a - b)^2 / (a + b) / (a - b)^2 * \tan(1/2 * d * x + 1/2 * c) * B - 2/d * a^5 / b^3 / (a^4 - 2 * \\ &a^2 * b^2 + b^4) / ((a - b) * (a + b))^(1/2) * \text{arctanh}(\tan(1/2 * d * x + 1/2 * c) * (a - b) / ((a - b) * (a \\ &+ b))^(1/2)) * A + 5/d * a^3 / b / (a^4 - 2 * a^2 * b^2 + b^4) / ((a - b) * (a + b))^(1/2) * \text{arctanh}(\tan \\ &(1/2 * d * x + 1/2 * c) * (a - b) / ((a - b) * (a + b))^(1/2)) * A - 6/d * a * b / (a^4 - 2 * a^2 * b^2 + b^4) / ((\\ &a - b) * (a + b))^(1/2) * \text{arctanh}(\tan(1/2 * d * x + 1/2 * c) * (a - b) / ((a - b) * (a + b))^(1/2)) * A + 6 \\ &/d * a^6 / b^4 / (a^4 - 2 * a^2 * b^2 + b^4) / ((a - b) * (a + b))^(1/2) * \text{arctanh}(\tan(1/2 * d * x + 1/2 * \\ &c) * (a - b) / ((a - b) * (a + b))^(1/2)) * B - 15/d * a^4 / b^2 / (a^4 - 2 * a^2 * b^2 + b^4) / ((a - b) * (a + \end{aligned}$$

$$b)^{(1/2)} \cdot \operatorname{arctanh}(\tan(1/2 \cdot d \cdot x + 1/2 \cdot c) \cdot (a-b) / ((a-b) \cdot (a+b))^{(1/2)}) \cdot B + 12/d \cdot a^2 / (a^4 - 2 \cdot a^2 \cdot b^2 + b^4) / ((a-b) \cdot (a+b))^{(1/2)} \cdot \operatorname{arctanh}(\tan(1/2 \cdot d \cdot x + 1/2 \cdot c) \cdot (a-b) / ((a-b) \cdot (a+b))^{(1/2)}) \cdot B - 1/d/b^3 / (\tan(1/2 \cdot d \cdot x + 1/2 \cdot c) - 1) \cdot B - 1/d/b^3 \cdot \ln(\tan(1/2 \cdot d \cdot x + 1/2 \cdot c) - 1) \cdot A + 3/d/b^4 \cdot \ln(\tan(1/2 \cdot d \cdot x + 1/2 \cdot c) - 1) \cdot a \cdot B - 1/d/b^3 / (\tan(1/2 \cdot d \cdot x + 1/2 \cdot c) + 1) \cdot B + 1/d/b^3 \cdot \ln(\tan(1/2 \cdot d \cdot x + 1/2 \cdot c) + 1) \cdot A - 3/d/b^4 \cdot \ln(\tan(1/2 \cdot d \cdot x + 1/2 \cdot c) + 1) \cdot a \cdot B$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for more details)Is 4*a^2-4*b^2 positive or negative?

mupad [B] time = 14.54, size = 9286, normalized size = 32.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^4*(a + b/cos(c + d*x))^3),x)

[Out]
$$\begin{aligned} & ((\tan(c/2 + (d \cdot x)/2)^5 \cdot (6 \cdot B \cdot a^5 - 2 \cdot B \cdot b^5 + 6 \cdot A \cdot a^2 \cdot b^3 + A \cdot a^3 \cdot b^2 + 4 \cdot B \cdot a^2 \cdot b^3 - 12 \cdot B \cdot a^3 \cdot b^2 - 2 \cdot A \cdot a^4 \cdot b + 2 \cdot B \cdot a \cdot b^4 - 3 \cdot B \cdot a^4 \cdot b)) / ((a \cdot b^3 - b^4) \cdot (a + b)^2) \\ & + (\tan(c/2 + (d \cdot x)/2) \cdot (6 \cdot B \cdot a^5 + 2 \cdot B \cdot b^5 + 6 \cdot A \cdot a^2 \cdot b^3 - A \cdot a^3 \cdot b^2 - 4 \cdot B \cdot a^2 \cdot b^3 - 12 \cdot B \cdot a^3 \cdot b^2 - 2 \cdot A \cdot a^4 \cdot b + 2 \cdot B \cdot a \cdot b^4 + 3 \cdot B \cdot a^4 \cdot b)) / ((a + b) \cdot (b^5 - 2 \cdot a \cdot b^4 + a^2 \cdot b^3)) - (2 \cdot \tan(c/2 + (d \cdot x)/2)^3 \cdot (6 \cdot B \cdot a^6 - 2 \cdot B \cdot b^6 + 5 \cdot A \cdot a^3 \cdot b^3 + 6 \cdot B \cdot a^2 \cdot b^4 - 13 \cdot B \cdot a^4 \cdot b^2 - 2 \cdot A \cdot a^5 \cdot b)) / (b \cdot (a \cdot b^2 - b^3) \cdot (a + b)^2 \cdot (a - b)) \\ & / (d \cdot (2 \cdot a \cdot b - \tan(c/2 + (d \cdot x)/2)^2 \cdot (2 \cdot a \cdot b + 3 \cdot a^2 - b^2) - \tan(c/2 + (d \cdot x)/2)^6 \cdot (a^2 - 2 \cdot a \cdot b + b^2) + a^2 + b^2 - \tan(c/2 + (d \cdot x)/2)^4 \cdot (2 \cdot a \cdot b - 3 \cdot a^2 + b^2)) \\ & + (\operatorname{atan}(\frac{(8 \cdot \tan(c/2 + (d \cdot x)/2) \cdot (4 \cdot A^2 \cdot b^{12} + 7 \cdot 2 \cdot B^2 \cdot a^{12} - 8 \cdot A^2 \cdot a \cdot b^{11} - 72 \cdot B^2 \cdot a^{11} \cdot b + 24 \cdot A^2 \cdot a^2 \cdot b^{10} + 32 \cdot A^2 \cdot a^3 \cdot b^9 - 52 \cdot A^2 \cdot a^4 \cdot b^8 - 48 \cdot A^2 \cdot a^5 \cdot b^7 + 57 \cdot A^2 \cdot a^6 \cdot b^6 + 32 \cdot A^2 \cdot a^7 \cdot b^5 - 32 \cdot A^2 \cdot a^8 \cdot b^4 - 8 \cdot A^2 \cdot a^9 \cdot b^3 + 8 \cdot A^2 \cdot a^{10} \cdot b^2 + 36 \cdot B^2 \cdot a^2 \cdot b^{10} - 72 \cdot B^2 \cdot a^3 \cdot b^9 + 36 \cdot B^2 \cdot a^4 \cdot b^8 + 288 \cdot B^2 \cdot a^5 \cdot b^7 - 288 \cdot B^2 \cdot a^6 \cdot b^6 - 432 \cdot B^2 \cdot a^7 \cdot b^5 + 441 \cdot B^2 \cdot a^8 \cdot b^4 + 288 \cdot B^2 \cdot a^9 \cdot b^3 - 288 \cdot B^2 \cdot a^{10} \cdot b^2 - 24 \cdot A \cdot B \cdot a \cdot b^{11} - 4 \cdot 8 \cdot A \cdot B \cdot a^{11} \cdot b + 48 \cdot A \cdot B \cdot a^2 \cdot b^{10} - 72 \cdot A \cdot B \cdot a^3 \cdot b^9 - 192 \cdot A \cdot B \cdot a^4 \cdot b^8 + 252 \cdot A \cdot B \cdot a^5 \cdot b^7 + 288 \cdot A \cdot B \cdot a^6 \cdot b^6 - 318 \cdot A \cdot B \cdot a^7 \cdot b^5 - 192 \cdot A \cdot B \cdot a^8 \cdot b^4 + 192 \cdot A \cdot B \cdot a^9 \cdot b^3 + 48 \cdot A \cdot B \cdot a^{10} \cdot b^2)} / (a \cdot b^{12} + b^{13} - 3 \cdot a^2 \cdot b^{11} - 3 \cdot a^3 \cdot b^{10} + 3 \cdot a^4 \cdot b^9 + 3 \cdot a^5 \cdot b^8 - a^6 \cdot b^7 - a^7 \cdot b^6) + (((8 \cdot (4 \cdot A \cdot b^{18} - 8 \cdot A \cdot a^2 \cdot b^{16} + 34 \cdot A \end{aligned}$$

$$\begin{aligned}
& a^3b^{15} + 6Aa^4b^{14} - 36A^2a^5b^{13} - 4A^3a^6b^{12} + 18A^4a^7b^{11} + 2 \\
& A^5a^8b^{10} - 4A^6a^9b^9 + 24B^2a^{12}b^{16} + 36B^3a^{13}b^{15} - 78B^4a^{14}b^{14} - \\
& 42B^5a^{15}b^{13} + 96B^6a^{16}b^{12} + 24B^7a^{17}b^{11} - 54B^8a^{18}b^{10} - 6B^9a^{19}b^9 \\
& + 12B^{10}a^{20}b^8 - 12A^2a^{17}b^{17} - 12B^2a^{17}b^{17}) / (a^2b^{15} + b^{16} - 3a^2b^{14} \\
& - 3a^3b^{13} + 3a^4b^{12} + 3a^5b^{11} - a^6b^{10} - a^7b^9) + (8\tan(c/2 \\
& + (dx)/2) * (Ab - 3Ba) * (8a^{17}b^{17} - 8a^{16}b^{16} - 32a^{15}b^{15} + 32a^{14}b^{14} \\
& + 48a^{13}b^{13} - 48a^{12}b^{12} - 32a^{11}b^{11} + 32a^{10}b^{10} + 8a^9b^9 - 8a^{10}b^8) \\
&) / (b^4 * (a^2b^{12} + b^{13} - 3a^2b^{11} - 3a^3b^{10} + 3a^4b^9 + 3a^5b^8 - a^6b^7 - a^7b^6)) * (Ab - 3Ba) / b^4 + (((8 \\
& * \tan(c/2 + (dx)/2) * (4A^2b^{12} + 72B^2a^{12} - 8A^2a^{11}b^{11} - 72B^2a^{11}b^{11} * \\
& b + 24A^2a^{10}b^{10} + 32A^2a^{10}b^9 - 52A^2a^{10}b^8 - 48A^2a^{10}b^7 + 57 \\
& A^2a^{10}b^6 + 32A^2a^{10}b^5 - 32A^2a^{10}b^4 - 8A^2a^{10}b^3 + 8A^2a^{10}b^2 + 36B^2a^{10}b^{10} \\
& - 72B^2a^{10}b^9 + 36B^2a^{10}b^8 + 288B^2a^{10}b^7 - 288B^2a^{10}b^6 - 432B^2a^{10}b^5 + 441B^2a^{10}b^4 \\
& + 288B^2a^{10}b^3 - 288B^2a^{10}b^2 - 24A^2a^{10}b^{11} - 48A^2a^{10}b^{11} * b + 48A^2a^{10}b^{10} - 72A^2a^{10}b^9 \\
& - 192A^2a^{10}b^8 + 252A^2a^{10}b^7 + 288A^2a^{10}b^6 - 318A^2a^{10}b^5 - 192A^2a^{10}b^4 + 192A^2a^{10}b^3 \\
& + 48A^2a^{10}b^2)) / (a^2b^{12} + b^{13} - 3a^2b^{11} - 3a^3b^{10} + 3a^4b^9 + 3a^5b^8 - a^6b^7 - a^7b^6) - (\\
& ((8 * (4A^2b^{18} - 8A^2a^{12}b^{16} + 34A^2a^{13}b^{15} + 6A^2a^{14}b^{14} - 36A^2a^{15}b^{13} \\
& - 4A^2a^{16}b^{12} + 18A^2a^{17}b^{11} + 2A^2a^{18}b^{10} - 4A^2a^{19}b^9 + 24B^2a^{12}b^{16} \\
& + 36B^2a^{13}b^{15} - 78B^2a^{14}b^{14} - 42B^2a^{15}b^{13} + 96B^2a^{16}b^{12} + 24B^2a^{17}b^{11} \\
& - 54B^2a^{18}b^{10} - 6B^2a^{19}b^9 + 12B^2a^{20}b^8 - 12A^2a^{17}b^{17} - 12B^2a^{17}b^{17} \\
& * b^{17})) / (a^2b^{15} + b^{16} - 3a^2b^{14} - 3a^3b^{13} + 3a^4b^{12} + 3a^5b^{11} \\
& - a^6b^{10} - a^7b^9) - (8 * \tan(c/2 + (dx)/2) * (Ab - 3Ba) * (8a^{17}b^{17} - 8a^{16}b^{16} \\
& - 32a^{15}b^{15} + 32a^{14}b^{14} + 48a^{13}b^{13} - 48a^{12}b^{12} - 32a^{11}b^{11} + 32a^{10}b^{10} + 8a^9b^9 \\
& - 8a^{10}b^8) / (b^4 * (a^2b^{12} + b^{13} - 3a^2b^{11} - 3a^3b^{10} + 3a^4b^9 + 3a^5b^8 - a^6b^7 - a^7b^6)) * (Ab - 3Ba) \\
&) / b^4 + ((16 * (108B^3a^{12} - 12A^3a^{11}b^{11} - 54B^3a^{11}b^{11} * b - 24A^3a^{10}b^{10} \\
& + 34A^3a^{10}b^9 + 26A^3a^{10}b^8 - 36A^3a^{10}b^7 - 13A^3a^{10}b^6 + 18A^3a^{10}b^5 + 2A^3a^{10}b^4 \\
& - 4A^3a^{10}b^3 + 216B^3a^{10}b^8 + 216B^3a^{10}b^7 - 702B^3a^{10}b^6 - 378B^3a^{10}b^5 + 864B^3a^{10}b^4 \\
& + 243B^3a^{10}b^3 - 486B^3a^{10}b^2 - 108A^2a^{11}b^{11} - 252A^2a^{11}b^{11} * b^9 - 324A^2a^{11}b^8 \\
& + 774A^2a^{11}b^7 + 486A^2a^{11}b^6 - 900A^2a^{11}b^5 - 279A^2a^{11}b^4 + 486A^2a^{11}b^3 + 54A^2a^{11}b^2 \\
& + 96A^2a^{11}b^{10} + 156A^2a^{11}b^9 - 282A^2a^{11}b^8 - 198A^2a^{11}b^7 + 312A^2a^{11}b^6 + 105A^2a^{11}b^5 \\
& - 162A^2a^{11}b^4 - 18A^2a^{11}b^3 + 36A^2a^{11}b^2)) / (a^2b^{15} + b^{16} - 3a^2b^{14} - 3a^3b^{13} + 3a^4b^{12} \\
& + 3a^5b^{11} - a^6b^{10} - a^7b^9) - (((8 * \tan(c/2 + (dx)/2) * (4A^2b^{12} + 72B^2a^{12} - 8A^2a^{11}b^{11} \\
& - 72B^2a^{11}b^{11} * b + 24A^2a^{10}b^{10} + 32A^2a^{10}b^9 - 52A^2a^{10}b^8 - 48A^2a^{10}b^7 + 57A^2a^{10}b^6 \\
& + 32A^2a^{10}b^5 - 32A^2a^{10}b^4 - 8A^2a^{10}b^3 + 8A^2a^{10}b^2 + 36B^2a^{10}b^{10} - 72B^2a^{10}b^9 \\
& + 36B^2a^{10}b^8 + 288B^2a^{10}b^7 - 288B^2a^{10}b^6 - 432B^2a^{10}b^5 + 441B^2a^{10}b^4 + 288B^2a^{10}b^3 \\
& - 288B^2a^{10}b^2 - 24A^2a^{10}b^{11} - 48A^2a^{10}b^{11} * b + 48A^2a^{10}b^{10} - 72A^2a^{10}b^9 - 192A^2a^{10}b^8 \\
& + 252A^2a^{10}b^7 + 288A^2a^{10}b^6 - 318A^2a^{10}b^5 - 192A^2a^{10}b^4 + 19
\end{aligned}$$

$$\begin{aligned}
& 2*A*B*a^9*b^3 + 48*A*B*a^{10}*b^2)) / (a*b^{12} + b^{13} - 3*a^2*b^{11} - 3*a^3*b^{10} \\
& + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6) + (((8*(4*A*b^{18} - 8*A*a^2*b^{16} \\
& + 34*A*a^3*b^{15} + 6*A*a^4*b^{14} - 36*A*a^5*b^{13} - 4*A*a^6*b^{12} + 18*A*a^7*b^{11} \\
& + 2*A*a^8*b^{10} - 4*A*a^9*b^9 + 24*B*a^2*b^{16} + 36*B*a^3*b^{15} - 78*B*a^4*b^{14} \\
& - 42*B*a^5*b^{13} + 96*B*a^6*b^{12} + 24*B*a^7*b^{11} - 54*B*a^8*b^{10} - 6*B*a^9*b^9 \\
& + 12*B*a^{10}*b^8 - 12*A*a*b^{17} - 12*B*a*b^{17}))) / (a*b^{15} + b^{16} - 3*a^2*b^{14} \\
& - 3*a^3*b^{13} + 3*a^4*b^{12} + 3*a^5*b^{11} - a^6*b^{10} - a^7*b^9) + (8*\tan(c/2 + (d*x)/2) * (A*b - 3*B*a) * (8*a*b^{17} - 8*a^2*b^{16} - 32*a^3*b^{15} + 32*a^4*b^{14} \\
& + 48*a^5*b^{13} - 48*a^6*b^{12} - 32*a^7*b^{11} + 32*a^8*b^{10} + 8*a^9*b^9 - 8*a^{10}*b^8)) / (b^4*(a*b^{12} + b^{13} - 3*a^2*b^{11} - 3*a^3*b^{10} + 3*a^4*b^9 \\
& + 3*a^5*b^8 - a^6*b^7 - a^7*b^6))) * (A*b - 3*B*a)) / b^4 + (((8*\tan(c/2 + (d*x)/2) * (4*A^2*b^{12} + 72*B^2*a^{12} - 8*A^2*a*b^{11} - 72*B^2*a^{11}*b \\
& + 24*A^2*a^2*b^{10} + 32*A^2*a^3*b^9 - 52*A^2*a^4*b^8 - 48*A^2*a^5*b^7 + 57*A^2*a^6*b^6 + 32*A^2*a^7*b^5 - 32*A^2*a^8*b^4 - 8*A^2*a^9*b^3 + 8*A^2*a^{10}*b^2 \\
& + 36*B^2*a^2*b^{10} - 72*B^2*a^3*b^9 + 36*B^2*a^4*b^8 + 288*B^2*a^5*b^7 - 288*B^2*a^6*b^6 - 432*B^2*a^7*b^5 + 441*B^2*a^8*b^4 + 288*B^2*a^9*b^3 - 288*B^2*a^{10}*b^2 \\
& - 24*A*B*a*b^{11} - 48*A*B*a^{11}*b + 48*A*B*a^2*b^{10} - 72*A*B*a^3*b^9 - 192*A*B*a^4*b^8 + 252*A*B*a^5*b^7 + 288*A*B*a^6*b^6 - 318*A*B*a^7*b^5 - 192*A*B*a^8*b^4 \\
& + 192*A*B*a^9*b^3 + 48*A*B*a^{10}*b^2)) / (a*b^{12} + b^{13} - 3*a^2*b^{11} - 3*a^3*b^{10} + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6) - (((8*(4*A*b^{18} - 8*A*a^2*b^{16} \\
& + 34*A*a^3*b^{15} + 6*A*a^4*b^{14} - 36*A*a^5*b^{13} - 4*A*a^6*b^{12} + 18*A*a^7*b^{11} + 2*A*a^8*b^{10} - 4*A*a^9*b^9 + 24*B*a^2*b^{16} + 36*B*a^3*b^{15} \\
& - 78*B*a^4*b^{14} - 42*B*a^5*b^{13} + 96*B*a^6*b^{12} + 24*B*a^7*b^{11} - 54*B*a^8*b^{10} - 6*B*a^9*b^9 + 12*B*a^{10}*b^8 - 12*A*a*b^{17} - 12*B*a*b^{17}))) / (a*b^{15} + b^{16} - 3*a^2*b^{14} \\
& - 3*a^3*b^{13} + 3*a^4*b^{12} + 3*a^5*b^{11} - a^6*b^{10} - a^7*b^9) - (8*\tan(c/2 + (d*x)/2) * (A*b - 3*B*a) * (8*a*b^{17} - 8*a^2*b^{16} - 32*a^3*b^{15} + 32*a^4*b^{14} \\
& + 48*a^5*b^{13} - 48*a^6*b^{12} - 32*a^7*b^{11} + 32*a^8*b^{10} + 8*a^9*b^9 - 8*a^{10}*b^8)) / (b^4*(a*b^{12} + b^{13} - 3*a^2*b^{11} - 3*a^3*b^{10} + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6))) * (A*b - 3*B*a)) / b^4 * (A*b - 3*B*a)) / b^4 * (A*b - 3*B*a) * 2i) / (b^4*d) + (a*\atan(((a*((8*\tan(c/2 + (d*x)/2) * (4*A^2*b^{12} + 72*B^2*a^{12} - 8*A^2*a*b^{11} - 72*B^2*a^{11}*b \\
& + 24*A^2*a^2*b^{10} + 32*A^2*a^3*b^9 - 52*A^2*a^4*b^8 - 48*A^2*a^5*b^7 + 57*A^2*a^6*b^6 + 32*A^2*a^7*b^5 - 32*A^2*a^8*b^4 - 8*A^2*a^9*b^3 + 8*A^2*a^{10}*b^2 + 36*B^2*a^2*b^{10} - 72*B^2*a^3*b^9 \\
& + 36*B^2*a^4*b^8 + 288*B^2*a^5*b^7 - 288*B^2*a^6*b^6 - 432*B^2*a^7*b^5 + 441*B^2*a^8*b^4 + 288*B^2*a^9*b^3 - 288*B^2*a^{10}*b^2 - 24*A*B*a*b^{11} - 48*A*B*a^{11}*b + 48*A*B*a^2*b^{10} - 72*A*B*a^3*b^9 \\
& - 192*A*B*a^4*b^8 + 252*A*B*a^5*b^7 + 288*A*B*a^6*b^6 - 318*A*B*a^7*b^5 - 192*A*B*a^8*b^4 + 192*A*B*a^9*b^3 + 48*A*B*a^{10}*b^2)) / (a*b^{12} + b^{13} - 3*a^2*b^{11} - 3*a^3*b^{10} \\
& + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6) - (a*((8*(4*A*b^{18} - 8*A*a^2*b^{16} + 34*A*a^3*b^{15} + 6*A*a^4*b^{14} - 36*A*a^5*b^{13} - 4*A*a^6*b^{12} + 18*A*a^7*b^{11} \\
& + 2*A*a^8*b^{10} - 4*A*a^9*b^9 + 24*B*a^2*b^{16} + 36*B*a^3*b^{15} - 78*B*a^4*b^{14} - 42*B*a^5*b^{13} + 96*B*a^6*b^{12} + 24*B*a^7*b^{11} - 54*B*a^8*b^{10} - 6*B*a^9*b^9 \\
& + 12*B*a^{10}*b^8 - 12*A*a*b^{17} - 12*B*a*b^{17}))) / (a*b^{15} + b^{16} - 3*a^2*b^{14} - 3*a^3*b^{13} + 3*a^4*b^{12} + 3*a^5*b^{11} - a^6*b^{10} - a^7*b^9) - (4*a*\tan(c/2 + (d*x)/2) * ((a + b)^5 * (a - b)^5)^(1
\end{aligned}$$

$$\begin{aligned}
& /2) * (6 * A * b^5 - 6 * B * a^5 - 5 * A * a^2 * b^3 + 15 * B * a^3 * b^2 + 2 * A * a^4 * b - 12 * B * a * b^4) * (8 * a * b^17 - 8 * a^2 * b^16 - 32 * a^3 * b^15 + 32 * a^4 * b^14 + 48 * a^5 * b^13 - 48 * a^6 * b^12 - 32 * a^7 * b^11 + 32 * a^8 * b^10 + 8 * a^9 * b^9 - 8 * a^{10} * b^8) / ((b^{14} - 5 * a^2 * b^{12} + 10 * a^4 * b^{10} - 10 * a^6 * b^8 + 5 * a^8 * b^6 - a^{10} * b^4) * (a * b^{12} + b^{13} - 3 * a^2 * b^{11} - 3 * a^3 * b^{10} + 3 * a^4 * b^9 + 3 * a^5 * b^8 - a^6 * b^7 - a^7 * b^6)) * ((a + b)^5 * (a - b)^5)^{(1/2)} * (6 * A * b^5 - 6 * B * a^5 - 5 * A * a^2 * b^3 + 15 * B * a^3 * b^2 + 2 * A * a^4 * b - 12 * B * a * b^4) / (2 * (b^{14} - 5 * a^2 * b^{12} + 10 * a^4 * b^{10} - 10 * a^6 * b^8 + 5 * a^8 * b^6 - a^{10} * b^4)) * ((a + b)^5 * (a - b)^5)^{(1/2)} * (6 * A * b^5 - 6 * B * a^5 - 5 * A * a^2 * b^3 + 15 * B * a^3 * b^2 + 2 * A * a^4 * b - 12 * B * a * b^4) * i) / (2 * (b^{14} - 5 * a^2 * b^{12} + 10 * a^4 * b^{10} - 10 * a^6 * b^8 + 5 * a^8 * b^6 - a^{10} * b^4)) + (a * ((8 * \tan(c/2 + (d * x)/2) * (4 * A^2 * b^{12} + 72 * B^2 * a^{12} - 8 * A^2 * a * b^{11} - 72 * B^2 * a^{11} * b + 24 * A^2 * a^2 * b^{10} + 32 * A^2 * a^3 * b^9 - 52 * A^2 * a^4 * b^8 - 48 * A^2 * a^5 * b^7 + 57 * A^2 * a^6 * b^6 + 32 * A^2 * a^7 * b^5 - 32 * A^2 * a^8 * b^4 - 8 * A^2 * a^9 * b^3 + 8 * A^2 * a^{10} * b^2 + 36 * B^2 * a^2 * b^{10} - 72 * B^2 * a^3 * b^9 + 36 * B^2 * a^4 * b^8 + 288 * B^2 * a^5 * b^7 - 288 * B^2 * a^6 * b^6 - 432 * B^2 * a^7 * b^5 + 441 * B^2 * a^8 * b^4 + 288 * B^2 * a^9 * b^3 - 288 * B^2 * a^{10} * b^2 - 24 * A * B * a * b^{11} - 48 * A * B * a^{11} * b + 48 * A * B * a^2 * b^{10} - 72 * A * B * a^3 * b^9 - 192 * A * B * a^4 * b^8 + 252 * A * B * a^5 * b^7 + 288 * A * B * a^6 * b^6 - 318 * A * B * a^7 * b^5 - 192 * A * B * a^8 * b^4 + 192 * A * B * a^9 * b^3 + 48 * A * B * a^{10} * b^2)) / (a * b^{12} + b^{13} - 3 * a^2 * b^{11} - 3 * a^3 * b^{10} + 3 * a^4 * b^9 + 3 * a^5 * b^8 - a^6 * b^7 - a^7 * b^6) + (a * ((8 * (4 * A * b^{18} - 8 * A * a^2 * b^{16} + 34 * A * a^3 * b^{15} + 6 * A * a^4 * b^{14} - 36 * A * a^5 * b^{13} - 4 * A * a^6 * b^{12} + 18 * A * a^7 * b^{11} + 2 * A * a^8 * b^{10} - 4 * A * a^9 * b^9 + 24 * B * a^2 * b^{16} + 36 * B * a^3 * b^{15} - 78 * B * a^4 * b^{14} - 42 * B * a^5 * b^{13} + 96 * B * a^6 * b^{12} + 24 * B * a^7 * b^{11} - 54 * B * a^8 * b^{10} - 6 * B * a^9 * b^9 + 12 * B * a^{10} * b^8 - 12 * A * a * b^{17} - 12 * B * a * b^{17}))) / (a * b^{15} + b^{16} - 3 * a^2 * b^{14} - 3 * a^3 * b^{13} + 3 * a^4 * b^{12} + 3 * a^5 * b^{11} - a^6 * b^{10} - a^7 * b^9) + (4 * a * \tan(c/2 + (d * x)/2) * ((a + b)^5 * (a - b)^5)^{(1/2)} * (6 * A * b^5 - 6 * B * a^5 - 5 * A * a^2 * b^3 + 15 * B * a^3 * b^2 + 2 * A * a^4 * b - 12 * B * a * b^4) * (8 * a * b^{17} - 8 * a^2 * b^{16} - 32 * a^3 * b^{15} + 32 * a^4 * b^{14} + 48 * a^5 * b^{13} - 48 * a^6 * b^{12} - 32 * a^7 * b^{11} + 32 * a^8 * b^{10} + 8 * a^9 * b^9 - 8 * a^{10} * b^8) / ((b^{14} - 5 * a^2 * b^{12} + 10 * a^4 * b^{10} - 10 * a^6 * b^8 + 5 * a^8 * b^6 - a^{10} * b^4) * (a * b^{12} + b^{13} - 3 * a^2 * b^{11} - 3 * a^3 * b^{10} + 3 * a^4 * b^9 + 3 * a^5 * b^8 - a^6 * b^7 - a^7 * b^6)) * ((a + b)^5 * (a - b)^5)^{(1/2)} * (6 * A * b^5 - 6 * B * a^5 - 5 * A * a^2 * b^3 + 15 * B * a^3 * b^2 + 2 * A * a^4 * b - 12 * B * a * b^4) / (2 * (b^{14} - 5 * a^2 * b^{12} + 10 * a^4 * b^{10} - 10 * a^6 * b^8 + 5 * a^8 * b^6 - a^{10} * b^4)) * ((a + b)^5 * (a - b)^5)^{(1/2)} * (6 * A * b^5 - 6 * B * a^5 - 5 * A * a^2 * b^3 + 15 * B * a^3 * b^2 + 2 * A * a^4 * b - 12 * B * a * b^4) * i) / (2 * (b^{14} - 5 * a^2 * b^{12} + 10 * a^4 * b^{10} - 10 * a^6 * b^8 + 5 * a^8 * b^6 - a^{10} * b^4)) / ((16 * (108 * B^3 * a^{12} - 12 * A^3 * a * b^{11} - 54 * B^3 * a^{11} * b - 24 * A^3 * a^2 * b^{10} + 34 * A^3 * a^3 * b^9 + 26 * A^3 * a^4 * b^8 - 36 * A^3 * a^5 * b^7 - 13 * A^3 * a^6 * b^6 + 18 * A^3 * a^7 * b^5 + 2 * A^3 * a^8 * b^4 - 4 * A^3 * a^9 * b^3 + 216 * B^3 * a^4 * b^8 + 216 * B^3 * a^5 * b^7 - 702 * B^3 * a^6 * b^6 - 378 * B^3 * a^7 * b^5 + 864 * B^3 * a^8 * b^4 + 243 * B^3 * a^9 * b^3 - 486 * B^3 * a^{10} * b^2 - 108 * A * B^2 * a^{11} * b - 252 * A * B^2 * a^3 * b^9 - 324 * A * B^2 * a^4 * b^8 + 774 * A * B^2 * a^5 * b^7 + 486 * A * B^2 * a^6 * b^6 - 900 * A * B^2 * a^7 * b^5 - 279 * A * B^2 * a^8 * b^4 + 486 * A * B^2 * a^9 * b^3 + 54 * A * B^2 * a^{10} * b^2 + 96 * A^2 * B * a^2 * b^{10} + 156 * A^2 * B * a^3 * b^9 - 282 * A^2 * B * a^4 * b^8 - 198 * A^2 * B * a^5 * b^7 + 312 * A^2 * B * a^6 * b^6 + 105 * A^2 * B * a^7 * b^5 - 162 * A^2 * B * a^8 * b^4 - 18 * A^2 * B * a^9 * b^3 + 36 * A^2 * B * a^{10} * b^2)) / (a * b^{15} + b^{16} - 3 * a^2 * b^{14} - 3 * a^3 * b^{13} + 3 * a^4 * b^{12} + 3 * a^5 * b^{11} - a^6 * b^{10} - a^7 * b^9) + (a * ((8 * \tan(c/2 + (d *
\end{aligned}$$

$$\begin{aligned}
& x)/2)*(4*A^2*b^{12} + 72*B^2*a^{12} - 8*A^2*a*b^{11} - 72*B^2*a^{11}*b + 24*A^2*a^2* \\
& *b^{10} + 32*A^2*a^3*b^9 - 52*A^2*a^4*b^8 - 48*A^2*a^5*b^7 + 57*A^2*a^6*b^6 + \\
& 32*A^2*a^7*b^5 - 32*A^2*a^8*b^4 - 8*A^2*a^9*b^3 + 8*A^2*a^{10}*b^2 + 36*B^2* \\
& a^2*b^{10} - 72*B^2*a^3*b^9 + 36*B^2*a^4*b^8 + 288*B^2*a^5*b^7 - 288*B^2*a^6* \\
& b^6 - 432*B^2*a^7*b^5 + 441*B^2*a^8*b^4 + 288*B^2*a^9*b^3 - 288*B^2*a^{10}*b^2 \\
& - 24*A*B*a*b^{11} - 48*A*B*a^{11}*b + 48*A*B*a^2*b^{10} - 72*A*B*a^3*b^9 - 192* \\
& A*B*a^4*b^8 + 252*A*B*a^5*b^7 + 288*A*B*a^6*b^6 - 318*A*B*a^7*b^5 - 192*A*B \\
& *a^8*b^4 + 192*A*B*a^9*b^3 + 48*A*B*a^{10}*b^2))/(a*b^{12} + b^{13} - 3*a^2*b^{11} \\
& - 3*a^3*b^{10} + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6) - (a*((8*(4*A*b^{11} \\
& 8 - 8*A*a^2*b^{16} + 34*A*a^3*b^{15} + 6*A*a^4*b^{14} - 36*A*a^5*b^{13} - 4*A*a^6*b \\
& ^{12} + 18*A*a^7*b^{11} + 2*A*a^8*b^{10} - 4*A*a^9*b^9 + 24*B*a^2*b^{16} + 36*B*a^3 \\
& *b^{15} - 78*B*a^4*b^{14} - 42*B*a^5*b^{13} + 96*B*a^6*b^{12} + 24*B*a^7*b^{11} - 54* \\
& B*a^8*b^{10} - 6*B*a^9*b^9 + 12*B*a^{10}*b^8 - 12*A*a*b^{17} - 12*B*a*b^{17}))/((a*b^{15} + b^{16} - 3*a^2*b^{14} - 3*a^3*b^{13} + 3*a^4*b^{12} + 3*a^5*b^{11} - a^6*b^{10} - \\
& a^7*b^9) - (4*a*tan(c/2 + (d*x)/2))*((a + b)^5*(a - b)^5)^{(1/2)}*(6*A*b^5 - \\
& 6*B*a^5 - 5*A*a^2*b^3 + 15*B*a^3*b^2 + 2*A*a^4*b - 12*B*a*b^4)*(8*a*b^{17} - \\
& 8*a^2*b^{16} - 32*a^3*b^{15} + 32*a^4*b^{14} + 48*a^5*b^{13} - 48*a^6*b^{12} - 32*a^7 \\
& *b^{11} + 32*a^8*b^{10} + 8*a^9*b^9 - 8*a^{10}*b^8))/((b^{14} - 5*a^2*b^{12} + 10*a^4 \\
& *b^{10} - 10*a^6*b^8 + 5*a^8*b^6 - a^{10}*b^4)*(a*b^{12} + b^{13} - 3*a^2*b^{11} - 3* \\
& a^3*b^{10} + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6)))*((a + b)^5*(a - b)^ \\
& 5)^{(1/2)}*(6*A*b^5 - 6*B*a^5 - 5*A*a^2*b^3 + 15*B*a^3*b^2 + 2*A*a^4*b - 12*B \\
& *a*b^4))/(2*(b^{14} - 5*a^2*b^{12} + 10*a^4*b^{10} - 10*a^6*b^8 + 5*a^8*b^6 - a^{10} \\
& *b^4)))*((a + b)^5*(a - b)^5)^{(1/2)}*(6*A*b^5 - 6*B*a^5 - 5*A*a^2*b^3 + 15* \\
& B*a^3*b^2 + 2*A*a^4*b - 12*B*a*b^4))/(2*(b^{14} - 5*a^2*b^{12} + 10*a^4*b^{10} - \\
& 10*a^6*b^8 + 5*a^8*b^6 - a^{10}*b^4)) - (a*((8*tan(c/2 + (d*x)/2))*(4*A^2*b^{12} \\
& + 72*B^2*a^{12} - 8*A^2*a*b^{11} - 72*B^2*a^{11}*b + 24*A^2*a^2*b^{10} + 32*A^2*a^3 \\
& *b^9 - 52*A^2*a^4*b^8 - 48*A^2*a^5*b^7 + 57*A^2*a^6*b^6 + 32*A^2*a^7*b^5 - \\
& 32*A^2*a^8*b^4 - 8*A^2*a^9*b^3 + 8*A^2*a^{10}*b^2 + 36*B^2*a^2*b^{10} - 72*B^2 \\
& *a^3*b^9 + 36*B^2*a^4*b^8 + 288*B^2*a^5*b^7 - 288*B^2*a^6*b^6 - 432*B^2*a^7 \\
& *b^5 + 441*B^2*a^8*b^4 + 288*B^2*a^9*b^3 - 288*B^2*a^{10}*b^2 - 24*A*B*a*b^{11} \\
& - 48*A*B*a^{11}*b + 48*A*B*a^2*b^{10} - 72*A*B*a^3*b^9 - 192*A*B*a^4*b^8 + 252 \\
& *A*B*a^5*b^7 + 288*A*B*a^6*b^6 - 318*A*B*a^7*b^5 - 192*A*B*a^8*b^4 + 192*A* \\
& B*a^9*b^3 + 48*A*B*a^{10}*b^2))/(a*b^{12} + b^{13} - 3*a^2*b^{11} - 3*a^3*b^{10} + 3* \\
& a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6) + (a*((8*(4*A*b^{18} - 8*A*a^2*b^{16} \\
& + 34*A*a^3*b^{15} + 6*A*a^4*b^{14} - 36*A*a^5*b^{13} - 4*A*a^6*b^{12} + 18*A*a^7*b^ \\
& ^{11} + 2*A*a^8*b^{10} - 4*A*a^9*b^9 + 24*B*a^2*b^{16} + 36*B*a^3*b^{15} - 78*B*a^4* \\
& b^{14} - 42*B*a^5*b^{13} + 96*B*a^6*b^{12} + 24*B*a^7*b^{11} - 54*B*a^8*b^{10} - 6*B* \\
& a^9*b^9 + 12*B*a^{10}*b^8 - 12*A*a*b^{17} - 12*B*a*b^{17}))/((a*b^{15} + b^{16} - 3*a^2 \\
& *b^{14} - 3*a^3*b^{13} + 3*a^4*b^{12} + 3*a^5*b^{11} - a^6*b^{10} - a^7*b^9) + (4*a* \\
& tan(c/2 + (d*x)/2))*((a + b)^5*(a - b)^5)^{(1/2)}*(6*A*b^5 - 6*B*a^5 - 5*A*a^2 \\
& *b^3 + 15*B*a^3*b^2 + 2*A*a^4*b - 12*B*a*b^4)*(8*a*b^{17} - 8*a^2*b^{16} - 32*a \\
& ^3*b^{15} + 32*a^4*b^{14} + 48*a^5*b^{13} - 48*a^6*b^{12} - 32*a^7*b^{11} + 32*a^8*b^ \\
& ^{10} + 8*a^9*b^9 - 8*a^{10}*b^8))/((b^{14} - 5*a^2*b^{12} + 10*a^4*b^{10} - 10*a^6*b^ \\
& ^8 + 5*a^8*b^6 - a^{10}*b^4)*(a*b^{12} + b^{13} - 3*a^2*b^{11} - 3*a^3*b^{10} + 3*a^4* \\
& b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6)))*((a + b)^5*(a - b)^5)^{(1/2)}*(6*A*b^5
\end{aligned}$$

$$\frac{-6Ba^5 - 5Aa^2b^3 + 15Ba^3b^2 + 2Aa^4b - 12Bab^4}{(2(b^{14} - 5a^2b^{12} + 10a^4b^{10} - 10a^6b^8 + 5a^8b^6 - a^{10}b^4))} \cdot ((a+b)^5(a-b)^5)^{1/2} \cdot \frac{-6Ab^5 - 6Ba^5 - 5Aa^2b^3 + 15Ba^3b^2 + 2Aa^4b - 12Bab^4}{(2(b^{14} - 5a^2b^{12} + 10a^4b^{10} - 10a^6b^8 + 5a^8b^6 - a^{10}b^4))} \cdot ((a+b)^5(a-b)^5)^{1/2} \cdot \frac{-6Ab^5 - 6Ba^5 - 5Aa^2b^3 + 15Ba^3b^2 + 2Aa^4b - 12Bab^4}{(2(b^{14} - 5a^2b^{12} + 10a^4b^{10} - 10a^6b^8 + 5a^8b^6 - a^{10}b^4))} \cdot 1i) / (d(b^{14} - 5a^2b^{12} + 10a^4b^{10} - 10a^6b^8 + 5a^8b^6 - a^{10}b^4))$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \sec^4(c + dx)}{(a + b \sec(c + dx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**4*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**3,x)

[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**4/(a + b*sec(c + d*x))**3, x)

$$3.330 \quad \int \frac{\sec^3(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^3} dx$$

Optimal. Leaf size=220

$$\frac{a^2(Ab - aB) \tan(c + dx)}{2b^2d(a^2 - b^2)(a + b \sec(c + dx))^2} + \frac{a(-3a^3B + a^2Ab + 6ab^2B - 4Ab^3) \tan(c + dx)}{2b^2d(a^2 - b^2)^2(a + b \sec(c + dx))} + \frac{(-2a^5B + 5a^3b^2B + a^2Ab^3)}{b^3}$$

[Out] B*arctanh(sin(d*x+c))/b^3/d+(A*a^2*b^3+2*A*b^5-2*B*a^5+5*B*a^3*b^2-6*B*a*b^4)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(5/2)/b^3/(a+b)^(5/2)/d-1/2*a^2*(A*b-B*a)*tan(d*x+c)/b^2/(a^2-b^2)/d/(a+b*sec(d*x+c))^2+1/2*a*(A*a^2*b-4*A*b^3-3*B*a^3+6*B*a*b^2)*tan(d*x+c)/b^2/(a^2-b^2)^2/d/(a+b*sec(d*x+c))

Rubi [A] time = 0.69, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {4028, 4080, 3998, 3770, 3831, 2659, 208}

$$\frac{(a^2Ab^3 + 5a^3b^2B - 2a^5B - 6ab^4B + 2Ab^5) \tanh^{-1}\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d(a-b)^{5/2}(a+b)^{5/2}} - \frac{a^2(Ab - aB) \tan(c + dx)}{2b^2d(a^2 - b^2)(a + b \sec(c + dx))^2} + \frac{a(a^2Ab^3 + 5a^3b^2B - 2a^5B - 6ab^4B + 2Ab^5)}{b^3}$$

Antiderivative was successfully verified.

[In] Int[(Sec[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^3,x]

[Out] (B*ArcTanh[Sin[c + d*x]])/(b^3*d) + ((a^2*A*b^3 + 2*A*b^5 - 2*a^5*B + 5*a^3*b^2*B - 6*a*b^4*B)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(5/2)*b^3*(a + b)^(5/2)*d) - (a^2*(A*b - a*B)*Tan[c + d*x])/(2*b^2*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) + (a*(a^2*A*b - 4*A*b^3 - 3*a^3*B + 6*a*b^2*B)*Tan[c + d*x])/(2*b^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 2659

Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x]

&& NeQ[a^2 - b^2, 0]

Rule 3770

Int[csc[(c_.) + (d_.)*(x_.)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3831

Int[csc[(e_.) + (f_.)*(x_.)]/(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] := Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 3998

Int[(csc[(e_.) + (f_.)*(x_.)]*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)))/(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] := Dist[B/b, Int[Csc[e + f*x], x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]

Rule 4028

Int[csc[(e_.) + (f_.)*(x_.)]^3*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := -Simp[(a^2*(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[a*b*(A*b - a*B)*(m + 1) - (A*b - a*B)*(a^2 + b^2*(m + 1))*Csc[e + f*x] + b*B*(m + 1)*(a^2 - b^2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]

Rule 4080

Int[csc[(e_.) + (f_.)*(x_.)]*((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_), x_Symbol] := -Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C))*(m + 1))*Csc[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned}
\int \frac{\sec^3(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^3} dx &= -\frac{a^2(Ab-aB)\tan(c+dx)}{2b^2(a^2-b^2)d(a+b\sec(c+dx))^2} - \frac{\int \frac{\sec(c+dx)(-2ab(Ab-aB)-(a^2-2b^2)(Ab-aB))}{(a+b\sec(c+dx))^2} dx}{2b^2(a^2-b^2)} \\
&= -\frac{a^2(Ab-aB)\tan(c+dx)}{2b^2(a^2-b^2)d(a+b\sec(c+dx))^2} + \frac{a(a^2Ab-4Ab^3-3a^3B+6ab^2B)}{2b^2(a^2-b^2)^2d(a+b\sec(c+dx))} \\
&= -\frac{a^2(Ab-aB)\tan(c+dx)}{2b^2(a^2-b^2)d(a+b\sec(c+dx))^2} + \frac{a(a^2Ab-4Ab^3-3a^3B+6ab^2B)}{2b^2(a^2-b^2)^2d(a+b\sec(c+dx))} \\
&= \frac{B \tanh^{-1}(\sin(c+dx))}{b^3d} - \frac{a^2(Ab-aB)\tan(c+dx)}{2b^2(a^2-b^2)d(a+b\sec(c+dx))^2} + \frac{a(a^2Ab-4Ab^3-3a^3B+6ab^2B)}{2b^2(a^2-b^2)^2d(a+b\sec(c+dx))} \\
&= \frac{B \tanh^{-1}(\sin(c+dx))}{b^3d} - \frac{a^2(Ab-aB)\tan(c+dx)}{2b^2(a^2-b^2)d(a+b\sec(c+dx))^2} + \frac{a(a^2Ab-4Ab^3-3a^3B+6ab^2B)}{2b^2(a^2-b^2)^2d(a+b\sec(c+dx))} \\
&= \frac{B \tanh^{-1}(\sin(c+dx))}{b^3d} + \frac{(a^2Ab^3+2Ab^5-2a^5B+5a^3b^2B-6ab^4B)\tan(c+dx)}{(a-b)^{5/2}b^3(a+b)^{5/2}d}
\end{aligned}$$

Mathematica [A] time = 2.05, size = 270, normalized size = 1.23

$$\cos(c+dx)(A+B\sec(c+dx)) \left(\frac{ab(-2a^3B+5ab^2B-3Ab^3)\sin(c+dx)}{(a-b)^2(a+b)^2(a\cos(c+dx)+b)} + \frac{2(2a^5B-5a^3b^2B-a^2Ab^3+6ab^4B-2Ab^5)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}} \right)$$

$2b^3d(A\cos(c+dx)+B)$

Antiderivative was successfully verified.

[In] Integrate[(Sec[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^3, x]

[Out] (Cos[c + d*x]*(A + B*Sec[c + d*x])*((2*(-(a^2*A*b^3) - 2*A*b^5 + 2*a^5*B - 5*a^3*b^2*B + 6*a*b^4*B)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2) - 2*B*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 2*B*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (a*b^2*(-(A*b) + a*B)*Sin[c + d*x])/((-a + b)*(a + b)*(b + a*Cos[c + d*x])^2) + (a*b*(-3*A*b^3 - 2*a^3*B +

$5*a*b^2*B*\sin[c + d*x])/((a - b)^2*(a + b)^2*(b + a*\cos[c + d*x])))/(2*b^3*d*(B + A*\cos[c + d*x]))$

fricas [B] time = 18.23, size = 1419, normalized size = 6.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm="fricas")

[Out]
$$[-1/4*((2*B*a^5*b^2 - 5*B*a^3*b^4 - A*a^2*b^5 + 6*B*a*b^6 - 2*A*b^7 + (2*B*a^7 - 5*B*a^5*b^2 - A*a^4*b^3 + 6*B*a^3*b^4 - 2*A*a^2*b^5)*\cos(d*x + c)^2 + 2*(2*B*a^6*b - 5*B*a^4*b^3 - A*a^3*b^4 + 6*B*a^2*b^5 - 2*A*a*b^6)*\cos(d*x + c))*\sqrt{a^2 - b^2}*\log((2*a*b*\cos(d*x + c) - (a^2 - 2*b^2)*\cos(d*x + c))^2 + 2*\sqrt{a^2 - b^2}*(b*\cos(d*x + c) + a)*\sin(d*x + c) + 2*a^2 - b^2)/(a^2*\cos(d*x + c)^2 + 2*a*b*\cos(d*x + c) + b^2)) - 2*(B*a^6*b^2 - 3*B*a^4*b^4 + 3*B*a^2*b^6 - B*b^8 + (B*a^8 - 3*B*a^6*b^2 + 3*B*a^4*b^4 - B*a^2*b^6)*\cos(d*x + c)^2 + 2*(B*a^7*b - 3*B*a^5*b^3 + 3*B*a^3*b^5 - B*a*b^7)*\cos(d*x + c))*\log(\sin(d*x + c) + 1) + 2*(B*a^6*b^2 - 3*B*a^4*b^4 + 3*B*a^2*b^6 - B*b^8 + (B*a^8 - 3*B*a^6*b^2 + 3*B*a^4*b^4 - B*a^2*b^6)*\cos(d*x + c)^2 + 2*(B*a^7*b - 3*B*a^5*b^3 + 3*B*a^3*b^5 - B*a*b^7)*\cos(d*x + c))*\log(-\sin(d*x + c) + 1) + 2*(3*B*a^6*b^2 - A*a^5*b^3 - 9*B*a^4*b^4 + 5*A*a^3*b^5 + 6*B*a^2*b^6 - 4*A*a*b^7 + (2*B*a^7*b - 7*B*a^5*b^3 + 3*A*a^4*b^4 + 5*B*a^3*b^5 - 3*A*a^2*b^6)*\cos(d*x + c))*\sin(d*x + c))/((a^8*b^3 - 3*a^6*b^5 + 3*a^4*b^7 - a^2*b^9)*d*\cos(d*x + c)^2 + 2*(a^7*b^4 - 3*a^5*b^6 + 3*a^3*b^8 - a*b^10)*d*\cos(d*x + c) + (a^6*b^5 - 3*a^4*b^7 + 3*a^2*b^9 - b^11)*d), -1/2*((2*B*a^5*b^2 - 5*B*a^3*b^4 - A*a^2*b^5 + 6*B*a*b^6 - 2*A*b^7 + (2*B*a^7 - 5*B*a^5*b^2 - A*a^4*b^3 + 6*B*a^3*b^4 - 2*A*a^2*b^5)*\cos(d*x + c)^2 + 2*(2*B*a^6*b - 5*B*a^4*b^3 - A*a^3*b^4 + 6*B*a^2*b^5 - 2*A*a*b^6)*\cos(d*x + c))*\sqrt{-a^2 + b^2})*\arctan(-\sqrt{-a^2 + b^2}*(b*\cos(d*x + c) + a)/((a^2 - b^2)*\sin(d*x + c))) - (B*a^6*b^2 - 3*B*a^4*b^4 + 3*B*a^2*b^6 - B*b^8 + (B*a^8 - 3*B*a^6*b^2 + 3*B*a^4*b^4 - B*a^2*b^6)*\cos(d*x + c)^2 + 2*(B*a^7*b - 3*B*a^5*b^3 + 3*B*a^3*b^5 - B*a*b^7)*\cos(d*x + c))*\log(\sin(d*x + c) + 1) + (B*a^6*b^2 - 3*B*a^4*b^4 + 3*B*a^2*b^6 - B*b^8 + (B*a^8 - 3*B*a^6*b^2 + 3*B*a^4*b^4 - B*a^2*b^6)*\cos(d*x + c)^2 + 2*(B*a^7*b - 3*B*a^5*b^3 + 3*B*a^3*b^5 - B*a*b^7)*\cos(d*x + c))*\log(-\sin(d*x + c) + 1) + (3*B*a^6*b^2 - A*a^5*b^3 - 9*B*a^4*b^4 + 5*A*a^3*b^5 + 6*B*a^2*b^6 - 4*A*a*b^7 + (2*B*a^7*b - 7*B*a^5*b^3 + 3*A*a^4*b^4 + 5*B*a^3*b^5 - 3*A*a^2*b^6)*\cos(d*x + c))*\sin(d*x + c))/((a^8*b^3 - 3*a^6*b^5 + 3*a^4*b^7 - a^2*b^9)*d*\cos(d*x + c)^2 + 2*(a^7*b^4 - 3*a^5*b^6 + 3*a^3*b^8 - a*b^10)*d*\cos(d*x + c) + (a^6*b^5 - 3*a^4*b^7 + 3*a^2*b^9 - b^11)*d)]$$

giac [B] time = 0.48, size = 486, normalized size = 2.21

$$\frac{(2Ba^5 - 5Ba^3b^2 - Aa^2b^3 + 6Bab^4 - 2Ab^5) \left(\pi \left[\frac{dx+c}{2\pi} + \frac{1}{2} \right] \operatorname{sgn}(-2a+2b) + \arctan \left(-\frac{a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - b \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{\sqrt{-a^2+b^2}} \right) \right)}{(a^4b^3 - 2a^2b^5 + b^7) \sqrt{-a^2+b^2}} - \frac{B \log \left(\left| \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 1 \right| \right)}{b^3} + \frac{B \log \left(\left| \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 1 \right| \right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm="giac")

[Out] $-\left((2Ba^5 - 5Ba^3b^2 - Aa^2b^3 + 6Bab^4 - 2Ab^5) \cdot (\pi \cdot \operatorname{floor}\left(\frac{1}{2}(dx+c)\right) / \pi + \frac{1}{2}) \cdot \operatorname{sgn}(-2a+2b) + \arctan\left(-\frac{a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - b \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{\sqrt{-a^2+b^2}}\right)\right) / \left((a^4b^3 - 2a^2b^5 + b^7) \cdot \sqrt{-a^2+b^2}\right) - B \cdot \log\left(\frac{\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 1\right|}{b^3}\right) + B \cdot \log\left(\frac{\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 1\right|}{b^3}\right) - (2Ba^5 \tan^3\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 3Ba^4b \tan^2\left(\frac{1}{2} dx + \frac{1}{2} c\right) + Aa^3b^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 5Ba^3b^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 3Aa^2b^3 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 6Ba^2b^3 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 1/2*c)^3 - 4Aa^2b^4 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 2Ba^5 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 3Ba^4b \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + Aa^3b^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 5Ba^3b^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 3Aa^2b^3 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 6Ba^2b^3 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 4Aa^2b^4 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)) / \left((a^4b^2 - 2a^2b^4 + b^6) \cdot (a \tan^2\left(\frac{1}{2} dx + \frac{1}{2} c\right) - b \tan^2\left(\frac{1}{2} dx + \frac{1}{2} c\right) - a - b)^2\right) / d$

maple [B] time = 0.80, size = 1085, normalized size = 4.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x)

[Out] $\frac{1/d \cdot a^2 \cdot (a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - b \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - a - b)^2 \cdot (a-b)}{(a^2 + 2ab + b^2) \cdot \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 \cdot A + 4/d \cdot b \cdot (a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - b \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - a - b)^2 \cdot a \cdot (a-b)}{(a^2 + 2ab + b^2) \cdot \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 \cdot A + 2/d \cdot a^4 \cdot b^2} \cdot \frac{1}{(a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - b \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - a - b)^2 \cdot (a-b)} \cdot \frac{1}{(a^2 + 2ab + b^2) \cdot \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 \cdot B - 1/d \cdot a^3 \cdot b} \cdot \frac{1}{(a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - b \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - a - b)^2 \cdot (a-b)} \cdot \frac{1}{(a^2 + 2ab + b^2) \cdot \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 \cdot B - 6/d} \cdot \frac{1}{(a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - b \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - a - b)^2 \cdot a^2 \cdot (a-b)} \cdot \frac{1}{(a^2 + 2ab + b^2) \cdot \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 \cdot B + 1/d \cdot a^2} \cdot \frac{1}{(a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - b \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - a - b)^2 \cdot (a+b)} \cdot \frac{1}{(a-b)^2 \cdot \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) \cdot A - 4/d \cdot b} \cdot \frac{1}{(a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - b \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - a - b)^2 \cdot a \cdot (a+b)} \cdot \frac{1}{(a-b)^2 \cdot \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) \cdot A - 2/d \cdot a^4 \cdot b^2} \cdot \frac{1}{(a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - b \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - a - b)^2 \cdot (a+b)} \cdot \frac{1}{(a-b)^2 \cdot \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}$

$$\begin{aligned} & x+1/2*c)*B-1/d*a^3/b/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)^2/ \\ & (a+b)/(a-b)^2*\tan(1/2*d*x+1/2*c)*B+6/d/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+ \\ & 1/2*c)^2*b-a-b)^2*a^2/(a+b)/(a-b)^2*\tan(1/2*d*x+1/2*c)*B+1/d*a^2/(a^4-2*a^2 \\ & *b^2+b^4)/((a-b)*(a+b))^{(1/2)*\operatorname{arctanh}(\tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b) \\ &)^{(1/2)})}*A+2/d*b^2/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^{(1/2)*\operatorname{arctanh}(\tan(1/2* \\ & d*x+1/2*c)*(a-b)/((a-b)*(a+b))^{(1/2)})}*A-2/d*a^5/b^3/(a^4-2*a^2*b^2+b^4)/((a \\ & -b)*(a+b))^{(1/2)*\operatorname{arctanh}(\tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^{(1/2)})}*B+5/ \\ & d*a^3/b/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^{(1/2)*\operatorname{arctanh}(\tan(1/2*d*x+1/2*c)* \\ & (a-b)/((a-b)*(a+b))^{(1/2)})}*B-6/d*b/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^{(1/2)* \\ & \operatorname{arctanh}(\tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^{(1/2)})}*a*B-1/d/b^3*\ln(\tan(1/ \\ & 2*d*x+1/2*c)-1)*B+1/d/b^3*\ln(\tan(1/2*d*x+1/2*c)+1)*B \end{aligned}$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for more details)Is 4*a^2-4*b^2 positive or negative?

mupad [B] time = 11.53, size = 6899, normalized size = 31.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^3*(a + b/cos(c + d*x))^3),x)

[Out]
$$\begin{aligned} & ((\tan(c/2 + (d*x)/2)^3*(2*B*a^4 + A*a^2*b^2 - 6*B*a^2*b^2 + 4*A*a*b^3 - B*a \\ & ^3*b))/((a*b^2 - b^3)*(a + b)^2) - (\tan(c/2 + (d*x)/2)*(2*B*a^4 - A*a^2*b^2 \\ & - 6*B*a^2*b^2 + 4*A*a*b^3 + B*a^3*b))/((a + b)*(b^4 - 2*a*b^3 + a^2*b^2))) \\ & /((d*(2*a*b - \tan(c/2 + (d*x)/2)^2*(2*a^2 - 2*b^2) + \tan(c/2 + (d*x)/2)^4*(a \\ & ^2 - 2*a*b + b^2) + a^2 + b^2) - (B*\operatorname{atan}(-((B*((B*((8*(4*A*b^15 + 4*B*b^15 \\ & - 6*A*a^2*b^13 + 6*A*a^3*b^12 + 2*A*a^6*b^9 - 2*A*a^7*b^8 - 8*B*a^2*b^13 + \\ & 34*B*a^3*b^12 + 6*B*a^4*b^11 - 36*B*a^5*b^10 - 4*B*a^6*b^9 + 18*B*a^7*b^8 \\ & + 2*B*a^8*b^7 - 4*B*a^9*b^6 - 4*A*a*b^14 - 12*B*a*b^14)))/(a*b^12 + b^13 - 3 \\ & *a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6) - (8*B* \\ & \tan(c/2 + (d*x)/2)*(8*a*b^15 - 8*a^2*b^14 - 32*a^3*b^13 + 32*a^4*b^12 + 48* \\ & a^5*b^11 - 48*a^6*b^10 - 32*a^7*b^9 + 32*a^8*b^8 + 8*a^9*b^7 - 8*a^10*b^6)) \\ & /((b^3*(a*b^10 + b^11 - 3*a^2*b^9 - 3*a^3*b^8 + 3*a^4*b^7 + 3*a^5*b^6 - a^6* \\ & b^5 - a^7*b^4))))/b^3 - (8*\tan(c/2 + (d*x)/2)*(4*A^2*b^10 + 8*B^2*a^10 + 4* \end{aligned}$$

$$\begin{aligned}
& B^2b^{10} - 8B^2a^9b^9 - 8B^2a^9b^9 + 4A^2a^2b^8 + A^2a^4b^6 + 24B^2 \\
& a^2b^8 + 32B^2a^3b^7 - 52B^2a^4b^6 - 48B^2a^5b^5 + 57B^2a^6b^4 \\
& + 32B^2a^7b^3 - 32B^2a^8b^2 - 24ABa^9b^9 + 8ABa^3b^7 + 2ABa^5b^5 \\
& - 4ABa^7b^3) / (a^2b^{10} + b^{11} - 3a^2b^9 - 3a^3b^8 + 3a^4b^7 \\
& + 3a^5b^6 - a^6b^5 - a^7b^4) * i) / b^3 - (B * (B * ((8 * (4A^2b^{15} + 4B^2b^{15} \\
& - 6A^2a^2b^{13} + 6A^2a^3b^{12} + 2A^2a^6b^9 - 2A^2a^7b^8 - 8B^2a^2b^{13} \\
& + 34B^2a^3b^{12} + 6B^2a^4b^{11} - 36B^2a^5b^{10} - 4B^2a^6b^9 + 18B^2a^7b^8 \\
& + 2B^2a^8b^7 - 4B^2a^9b^6 - 4A^2a^2b^{14} - 12B^2a^3b^{14})) / (a^2b^{12} + b^{13} - \\
& 3a^2b^{11} - 3a^3b^{10} + 3a^4b^9 + 3a^5b^8 - a^6b^7 - a^7b^6) + (8 * \\
& B * \tan(c/2 + (d*x)/2) * (8a^2b^{15} - 8a^2b^{14} - 32a^3b^{13} + 32a^4b^{12} + 4 \\
& 8a^5b^{11} - 48a^6b^{10} - 32a^7b^9 + 32a^8b^8 + 8a^9b^7 - 8a^{10}b^6 \\
&)) / (b^3 * (a^2b^{10} + b^{11} - 3a^2b^9 - 3a^3b^8 + 3a^4b^7 + 3a^5b^6 - a^6b^5 \\
& - a^7b^4))) / b^3 + (8 * \tan(c/2 + (d*x)/2) * (4A^2b^{10} + 8B^2a^{10} + \\
& 4B^2b^{10} - 8B^2a^9b^9 - 8B^2a^9b^9 + 4A^2a^2b^8 + A^2a^4b^6 + 24B^2 \\
& a^2b^8 + 32B^2a^3b^7 - 52B^2a^4b^6 - 48B^2a^5b^5 + 57B^2a^6b^4 \\
& + 32B^2a^7b^3 - 32B^2a^8b^2 - 24ABa^9b^9 + 8ABa^3b^7 + 2ABa^5b^5 \\
& - 4ABa^7b^3) / (a^2b^{10} + b^{11} - 3a^2b^9 - 3a^3b^8 + 3a^4b^7 \\
& + 3a^5b^6 - a^6b^5 - a^7b^4) * i) / b^3) / ((16 * (4B^3a^9 - 4AB^2b^9 \\
& + 4A^2B^2b^9 + 12B^3a^8b^8 - 2B^3a^8b^8 + 24B^3a^2b^7 - 34B^3a^3b^6 \\
& - 26B^3a^4b^5 + 36B^3a^5b^4 + 13B^3a^6b^3 - 18B^3a^7b^2 - 2 \\
& 0AB^2a^2b^8 + 6AB^2a^2b^7 + 2AB^2a^3b^6 + 2AB^2a^5b^4 - 2AB^2a^6b^3 \\
& - 2AB^2a^7b^2 + 4A^2B^2a^2b^7 + A^2B^2a^4b^5)) / (a^2b^{12} + \\
& b^{13} - 3a^2b^{11} - 3a^3b^{10} + 3a^4b^9 + 3a^5b^8 - a^6b^7 - a^7b^6) \\
& + (B * (B * ((8 * (4A^2b^{15} + 4B^2b^{15} - 6A^2a^2b^{13} + 6A^2a^3b^{12} + 2A^2a^6b^9 \\
& - 2A^2a^7b^8 - 8B^2a^2b^{13} + 34B^2a^3b^{12} + 6B^2a^4b^{11} - 36B^2a^5b^{10} \\
& - 4B^2a^6b^9 + 18B^2a^7b^8 + 2B^2a^8b^7 - 4B^2a^9b^6 - 4A^2a^2b^{14} \\
& - 12B^2a^3b^{14})) / (a^2b^{12} + b^{13} - 3a^2b^{11} - 3a^3b^{10} + 3a^4b^9 + 3a^5b^8 \\
& - a^6b^7 - a^7b^6) - (8 * B * \tan(c/2 + (d*x)/2) * (8a^2b^{15} - 8a^2b^{14} \\
& - 32a^3b^{13} + 32a^4b^{12} + 48a^5b^{11} - 48a^6b^{10} - 32a^7b^9 + 32a^8b^8 \\
& + 8a^9b^7 - 8a^{10}b^6)) / (b^3 * (a^2b^{10} + b^{11} - 3a^2b^9 - 3a^3b^8 \\
& + 3a^4b^7 + 3a^5b^6 - a^6b^5 - a^7b^4)))) / b^3 - (8 * \tan(c/2 + (d*x) \\
&) / 2) * (4A^2b^{10} + 8B^2a^{10} + 4B^2b^{10} - 8B^2a^9b^9 - 8B^2a^9b^9 + 4 \\
& A^2a^2b^8 + A^2a^4b^6 + 24B^2a^2b^8 + 32B^2a^3b^7 - 52B^2a^4b^6 \\
& - 48B^2a^5b^5 + 57B^2a^6b^4 + 32B^2a^7b^3 - 32B^2a^8b^2 - 24 \\
& ABa^9b^9 + 8ABa^3b^7 + 2ABa^5b^5 - 4ABa^7b^3) / (a^2b^{10} + b^{11} \\
& - 3a^2b^9 - 3a^3b^8 + 3a^4b^7 + 3a^5b^6 - a^6b^5 - a^7b^4)) / b^3 \\
& + (B * (B * ((8 * (4A^2b^{15} + 4B^2b^{15} - 6A^2a^2b^{13} + 6A^2a^3b^{12} + 2A^2a^6b^9 \\
& - 2A^2a^7b^8 - 8B^2a^2b^{13} + 34B^2a^3b^{12} + 6B^2a^4b^{11} - 36B^2a^5b^{10} \\
& - 4B^2a^6b^9 + 18B^2a^7b^8 + 2B^2a^8b^7 - 4B^2a^9b^6 - 4A^2a^2b^{14} - \\
& 12B^2a^3b^{14})) / (a^2b^{12} + b^{13} - 3a^2b^{11} - 3a^3b^{10} + 3a^4b^9 + 3a^5 \\
& b^8 - a^6b^7 - a^7b^6) + (8 * B * \tan(c/2 + (d*x)/2) * (8a^2b^{15} - 8a^2b^{14} \\
& - 32a^3b^{13} + 32a^4b^{12} + 48a^5b^{11} - 48a^6b^{10} - 32a^7b^9 + 32a^8b^8 \\
& + 8a^9b^7 - 8a^{10}b^6)) / (b^3 * (a^2b^{10} + b^{11} - 3a^2b^9 - 3a^3b^8 \\
& + 3a^4b^7 + 3a^5b^6 - a^6b^5 - a^7b^4)))) / b^3 + (8 * \tan(c/2 + (d*x) \\
&) / 2) * (4A^2b^{10} + 8B^2a^{10} + 4B^2b^{10} - 8B^2a^9b^9 - 8B^2a^9b^9 + 4A
\end{aligned}$$

$$\begin{aligned}
&^2*a^2*b^8 + A^2*a^4*b^6 + 24*B^2*a^2*b^8 + 32*B^2*a^3*b^7 - 52*B^2*a^4*b^6 \\
&- 48*B^2*a^5*b^5 + 57*B^2*a^6*b^4 + 32*B^2*a^7*b^3 - 32*B^2*a^8*b^2 - 24*A \\
&*B*a*b^9 + 8*A*B*a^3*b^7 + 2*A*B*a^5*b^5 - 4*A*B*a^7*b^3)) / (a*b^{10} + b^{11} - \\
&3*a^2*b^9 - 3*a^3*b^8 + 3*a^4*b^7 + 3*a^5*b^6 - a^6*b^5 - a^7*b^4)) / (b^3)) \\
&*2i) / (b^3*d) - (\operatorname{atan}(\frac{((a+b)^5*(a-b)^5)^{1/2} * ((8*\tan(c/2 + (d*x)/2) * (\\
&4*A^2*b^{10} + 8*B^2*a^{10} + 4*B^2*b^{10} - 8*B^2*a*b^9 - 8*B^2*a^9*b + 4*A^2*a^ \\
&2*b^8 + A^2*a^4*b^6 + 24*B^2*a^2*b^8 + 32*B^2*a^3*b^7 - 52*B^2*a^4*b^6 - 48 \\
&*B^2*a^5*b^5 + 57*B^2*a^6*b^4 + 32*B^2*a^7*b^3 - 32*B^2*a^8*b^2 - 24*A*B*a \\
&b^9 + 8*A*B*a^3*b^7 + 2*A*B*a^5*b^5 - 4*A*B*a^7*b^3)) / (a*b^{10} + b^{11} - 3*a^ \\
&2*b^9 - 3*a^3*b^8 + 3*a^4*b^7 + 3*a^5*b^6 - a^6*b^5 - a^7*b^4) - (((a+b)^ \\
&5*(a-b)^5)^{1/2} * ((8*(4*A*b^{15} + 4*B*b^{15} - 6*A*a^2*b^{13} + 6*A*a^3*b^{12} + \\
&2*A*a^6*b^9 - 2*A*a^7*b^8 - 8*B*a^2*b^{13} + 34*B*a^3*b^{12} + 6*B*a^4*b^{11} - \\
&36*B*a^5*b^{10} - 4*B*a^6*b^9 + 18*B*a^7*b^8 + 2*B*a^8*b^7 - 4*B*a^9*b^6 - 4* \\
&A*a*b^{14} - 12*B*a*b^{14})) / (a*b^{12} + b^{13} - 3*a^2*b^{11} - 3*a^3*b^{10} + 3*a^4*b \\
&^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6) - (4*\tan(c/2 + (d*x)/2) * ((a+b)^5*(a-b) \\
&^5)^{1/2} * (2*A*b^5 - 2*B*a^5 + A*a^2*b^3 + 5*B*a^3*b^2 - 6*B*a*b^4) * (8*a \\
&b^{15} - 8*a^2*b^{14} - 32*a^3*b^{13} + 32*a^4*b^{12} + 48*a^5*b^{11} - 48*a^6*b^{10} \\
&- 32*a^7*b^9 + 32*a^8*b^8 + 8*a^9*b^7 - 8*a^{10}*b^6)) / ((b^{13} - 5*a^2*b^{11} + \\
&10*a^4*b^9 - 10*a^6*b^7 + 5*a^8*b^5 - a^{10}*b^3) * (a*b^{10} + b^{11} - 3*a^2*b^9 \\
&- 3*a^3*b^8 + 3*a^4*b^7 + 3*a^5*b^6 - a^6*b^5 - a^7*b^4))) * (2*A*b^5 - 2*B*a \\
&^5 + A*a^2*b^3 + 5*B*a^3*b^2 - 6*B*a*b^4)) / (2*(b^{13} - 5*a^2*b^{11} + 10*a^4*b \\
&^9 - 10*a^6*b^7 + 5*a^8*b^5 - a^{10}*b^3))) * (2*A*b^5 - 2*B*a^5 + A*a^2*b^3 + \\
&5*B*a^3*b^2 - 6*B*a*b^4) * 1i) / (2*(b^{13} - 5*a^2*b^{11} + 10*a^4*b^9 - 10*a^6*b^ \\
&7 + 5*a^8*b^5 - a^{10}*b^3)) + (((a+b)^5*(a-b)^5)^{1/2} * ((8*\tan(c/2 + (d* \\
&x)/2) * (4*A^2*b^{10} + 8*B^2*a^{10} + 4*B^2*b^{10} - 8*B^2*a*b^9 - 8*B^2*a^9*b + 4 \\
&*A^2*a^2*b^8 + A^2*a^4*b^6 + 24*B^2*a^2*b^8 + 32*B^2*a^3*b^7 - 52*B^2*a^4*b \\
&^6 - 48*B^2*a^5*b^5 + 57*B^2*a^6*b^4 + 32*B^2*a^7*b^3 - 32*B^2*a^8*b^2 - 24 \\
&*A*B*a*b^9 + 8*A*B*a^3*b^7 + 2*A*B*a^5*b^5 - 4*A*B*a^7*b^3)) / (a*b^{10} + b^{11} \\
&- 3*a^2*b^9 - 3*a^3*b^8 + 3*a^4*b^7 + 3*a^5*b^6 - a^6*b^5 - a^7*b^4) + (((\\
&a+b)^5*(a-b)^5)^{1/2} * ((8*(4*A*b^{15} + 4*B*b^{15} - 6*A*a^2*b^{13} + 6*A*a^3 \\
&*b^{12} + 2*A*a^6*b^9 - 2*A*a^7*b^8 - 8*B*a^2*b^{13} + 34*B*a^3*b^{12} + 6*B*a^4* \\
&b^{11} - 36*B*a^5*b^{10} - 4*B*a^6*b^9 + 18*B*a^7*b^8 + 2*B*a^8*b^7 - 4*B*a^9*b \\
&^6 - 4*A*a*b^{14} - 12*B*a*b^{14})) / (a*b^{12} + b^{13} - 3*a^2*b^{11} - 3*a^3*b^{10} + \\
&3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6) + (4*\tan(c/2 + (d*x)/2) * ((a+b) \\
&^5*(a-b)^5)^{1/2} * (2*A*b^5 - 2*B*a^5 + A*a^2*b^3 + 5*B*a^3*b^2 - 6*B*a*b^ \\
&4) * (8*a*b^{15} - 8*a^2*b^{14} - 32*a^3*b^{13} + 32*a^4*b^{12} + 48*a^5*b^{11} - 48*a^ \\
&6*b^{10} - 32*a^7*b^9 + 32*a^8*b^8 + 8*a^9*b^7 - 8*a^{10}*b^6)) / ((b^{13} - 5*a^2* \\
&b^{11} + 10*a^4*b^9 - 10*a^6*b^7 + 5*a^8*b^5 - a^{10}*b^3) * (a*b^{10} + b^{11} - 3*a \\
&^2*b^9 - 3*a^3*b^8 + 3*a^4*b^7 + 3*a^5*b^6 - a^6*b^5 - a^7*b^4))) * (2*A*b^5 \\
&- 2*B*a^5 + A*a^2*b^3 + 5*B*a^3*b^2 - 6*B*a*b^4)) / (2*(b^{13} - 5*a^2*b^{11} + 1 \\
&0*a^4*b^9 - 10*a^6*b^7 + 5*a^8*b^5 - a^{10}*b^3))) * (2*A*b^5 - 2*B*a^5 + A*a^2 \\
&*b^3 + 5*B*a^3*b^2 - 6*B*a*b^4) * 1i) / (2*(b^{13} - 5*a^2*b^{11} + 10*a^4*b^9 - 10 \\
&*a^6*b^7 + 5*a^8*b^5 - a^{10}*b^3))) / ((16*(4*B^3*a^9 - 4*A*B^2*b^9 + 4*A^2*B* \\
&b^9 + 12*B^3*a*b^8 - 2*B^3*a^8*b + 24*B^3*a^2*b^7 - 34*B^3*a^3*b^6 - 26*B^3 \\
&*a^4*b^5 + 36*B^3*a^5*b^4 + 13*B^3*a^6*b^3 - 18*B^3*a^7*b^2 - 20*A*B^2*a*b^
\end{aligned}$$

$$\begin{aligned}
& 8 + 6*A*B^2*a^2*b^7 + 2*A*B^2*a^3*b^6 + 2*A*B^2*a^5*b^4 - 2*A*B^2*a^6*b^3 - \\
& 2*A*B^2*a^7*b^2 + 4*A^2*B*a^2*b^7 + A^2*B*a^4*b^5)/(a*b^12 + b^13 - 3*a^2 \\
& *b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6) - (((a + b) \\
& ^5*(a - b)^5)^{(1/2)}*((8*\tan(c/2 + (d*x)/2)*(4*A^2*b^10 + 8*B^2*a^10 + 4*B^2 \\
& *b^10 - 8*B^2*a*b^9 - 8*B^2*a^9*b + 4*A^2*a^2*b^8 + A^2*a^4*b^6 + 24*B^2*a^ \\
& 2*b^8 + 32*B^2*a^3*b^7 - 52*B^2*a^4*b^6 - 48*B^2*a^5*b^5 + 57*B^2*a^6*b^4 + \\
& 32*B^2*a^7*b^3 - 32*B^2*a^8*b^2 - 24*A*B*a*b^9 + 8*A*B*a^3*b^7 + 2*A*B*a^5 \\
& *b^5 - 4*A*B*a^7*b^3))/(a*b^10 + b^11 - 3*a^2*b^9 - 3*a^3*b^8 + 3*a^4*b^7 + \\
& 3*a^5*b^6 - a^6*b^5 - a^7*b^4) - (((a + b)^5*(a - b)^5)^{(1/2)}*((8*(4*A*b^1 \\
& 5 + 4*B*b^15 - 6*A*a^2*b^13 + 6*A*a^3*b^12 + 2*A*a^6*b^9 - 2*A*a^7*b^8 - 8* \\
& B*a^2*b^13 + 34*B*a^3*b^12 + 6*B*a^4*b^11 - 36*B*a^5*b^10 - 4*B*a^6*b^9 + 1 \\
& 8*B*a^7*b^8 + 2*B*a^8*b^7 - 4*B*a^9*b^6 - 4*A*a*b^14 - 12*B*a*b^14))/(a*b^1 \\
& 2 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7* \\
& b^6) - (4*\tan(c/2 + (d*x)/2)*((a + b)^5*(a - b)^5)^{(1/2)}*(2*A*b^5 - 2*B*a^5 \\
& + A*a^2*b^3 + 5*B*a^3*b^2 - 6*B*a*b^4)*(8*a*b^15 - 8*a^2*b^14 - 32*a^3*b^1 \\
& 3 + 32*a^4*b^12 + 48*a^5*b^11 - 48*a^6*b^10 - 32*a^7*b^9 + 32*a^8*b^8 + 8*a \\
& ^9*b^7 - 8*a^10*b^6))/(b^13 - 5*a^2*b^11 + 10*a^4*b^9 - 10*a^6*b^7 + 5*a^8 \\
& *b^5 - a^10*b^3)*(a*b^10 + b^11 - 3*a^2*b^9 - 3*a^3*b^8 + 3*a^4*b^7 + 3*a^5 \\
& *b^6 - a^6*b^5 - a^7*b^4)))*(2*A*b^5 - 2*B*a^5 + A*a^2*b^3 + 5*B*a^3*b^2 - \\
& 6*B*a*b^4))/(2*(b^13 - 5*a^2*b^11 + 10*a^4*b^9 - 10*a^6*b^7 + 5*a^8*b^5 - a \\
& ^10*b^3)))*(2*A*b^5 - 2*B*a^5 + A*a^2*b^3 + 5*B*a^3*b^2 - 6*B*a*b^4))/(2*(b \\
& ^13 - 5*a^2*b^11 + 10*a^4*b^9 - 10*a^6*b^7 + 5*a^8*b^5 - a^10*b^3)) + (((a \\
& + b)^5*(a - b)^5)^{(1/2)}*((8*\tan(c/2 + (d*x)/2)*(4*A^2*b^10 + 8*B^2*a^10 + 4 \\
& *B^2*b^10 - 8*B^2*a*b^9 - 8*B^2*a^9*b + 4*A^2*a^2*b^8 + A^2*a^4*b^6 + 24*B^ \\
& 2*a^2*b^8 + 32*B^2*a^3*b^7 - 52*B^2*a^4*b^6 - 48*B^2*a^5*b^5 + 57*B^2*a^6*b \\
& ^4 + 32*B^2*a^7*b^3 - 32*B^2*a^8*b^2 - 24*A*B*a*b^9 + 8*A*B*a^3*b^7 + 2*A*B \\
& *a^5*b^5 - 4*A*B*a^7*b^3))/(a*b^10 + b^11 - 3*a^2*b^9 - 3*a^3*b^8 + 3*a^4*b \\
& ^7 + 3*a^5*b^6 - a^6*b^5 - a^7*b^4) + (((a + b)^5*(a - b)^5)^{(1/2)}*((8*(4*A \\
& *b^15 + 4*B*b^15 - 6*A*a^2*b^13 + 6*A*a^3*b^12 + 2*A*a^6*b^9 - 2*A*a^7*b^8 \\
& - 8*B*a^2*b^13 + 34*B*a^3*b^12 + 6*B*a^4*b^11 - 36*B*a^5*b^10 - 4*B*a^6*b^9 \\
& + 18*B*a^7*b^8 + 2*B*a^8*b^7 - 4*B*a^9*b^6 - 4*A*a*b^14 - 12*B*a*b^14))/(a \\
& *b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - \\
& a^7*b^6) + (4*\tan(c/2 + (d*x)/2)*((a + b)^5*(a - b)^5)^{(1/2)}*(2*A*b^5 - 2*B \\
& *a^5 + A*a^2*b^3 + 5*B*a^3*b^2 - 6*B*a*b^4)*(8*a*b^15 - 8*a^2*b^14 - 32*a^3 \\
& *b^13 + 32*a^4*b^12 + 48*a^5*b^11 - 48*a^6*b^10 - 32*a^7*b^9 + 32*a^8*b^8 + \\
& 8*a^9*b^7 - 8*a^10*b^6))/(b^13 - 5*a^2*b^11 + 10*a^4*b^9 - 10*a^6*b^7 + 5 \\
& *a^8*b^5 - a^10*b^3)*(a*b^10 + b^11 - 3*a^2*b^9 - 3*a^3*b^8 + 3*a^4*b^7 + 3 \\
& *a^5*b^6 - a^6*b^5 - a^7*b^4)))*(2*A*b^5 - 2*B*a^5 + A*a^2*b^3 + 5*B*a^3*b^ \\
& 2 - 6*B*a*b^4))/(2*(b^13 - 5*a^2*b^11 + 10*a^4*b^9 - 10*a^6*b^7 + 5*a^8*b^5 \\
& - a^10*b^3)))*(2*A*b^5 - 2*B*a^5 + A*a^2*b^3 + 5*B*a^3*b^2 - 6*B*a*b^4))/(\\
& 2*(b^13 - 5*a^2*b^11 + 10*a^4*b^9 - 10*a^6*b^7 + 5*a^8*b^5 - a^10*b^3)))*(\\
& (a + b)^5*(a - b)^5)^{(1/2)}*(2*A*b^5 - 2*B*a^5 + A*a^2*b^3 + 5*B*a^3*b^2 - 6 \\
& *B*a*b^4)*ii)/(d*(b^13 - 5*a^2*b^11 + 10*a^4*b^9 - 10*a^6*b^7 + 5*a^8*b^5 - \\
& a^10*b^3))
\end{aligned}$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \sec^3(c + dx)}{(a + b \sec(c + dx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**3,x)

[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**3/(a + b*sec(c + d*x))**3, x)

$$3.331 \quad \int \frac{\sec^2(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^3} dx$$

Optimal. Leaf size=180

$$\frac{(a^2(-B) + 3aAb - 2b^2B) \tanh^{-1}\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}} + \frac{a(Ab - aB) \tan(c+dx)}{2bd(a^2 - b^2)(a+b \sec(c+dx))^2} + \frac{(a^3B + a^2Ab - 4ab^2B + a^3b^2)}{2bd(a^2 - b^2)^2(a+b \sec(c+dx))}$$

[Out] $-(3Aa^2b - B^2a^2 - 2B^2b^2) \operatorname{arctanh}\left(\frac{(a-b)^{1/2} \tan(1/2 dx + 1/2 c)}{(a+b)^{1/2}}\right) / (a-b)^{5/2} / (a+b)^{5/2} / d + 1/2 a (A^2 b - B^2 a) \tan(dx + c) / b / (a^2 - b^2) / d / (a + b \sec(dx + c))^2 + 1/2 (A^2 a^2 b + 2A^2 b^3 + B^2 a^3 - 4B^2 a^2 b) \tan(dx + c) / b / (a^2 - b^2)^2 / d / (a + b \sec(dx + c))$

Rubi [A] time = 0.34, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4009, 4003, 12, 3831, 2659, 208}

$$\frac{(a^2(-B) + 3aAb - 2b^2B) \tanh^{-1}\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}} + \frac{(a^2Ab + a^3B - 4ab^2B + 2Ab^3) \tan(c+dx)}{2bd(a^2 - b^2)^2(a+b \sec(c+dx))} + \frac{a(Ab - aB)}{2bd(a^2 - b^2)}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(\operatorname{Sec}[c + dx]^2(A + B \operatorname{Sec}[c + dx])) / (a + b \operatorname{Sec}[c + dx])^3, x]$

[Out] $-\left(\frac{(3a^2A^2b - a^2B^2 - 2b^2B^2) \operatorname{ArcTanh}\left[\frac{\sqrt{a-b} \tan\left(\frac{c+dx}{2}\right)}{\sqrt{a+b}}\right]}{(a-b)^{5/2}(a+b)^{5/2}d} + \frac{a(A^2b - a^2B) \tan[c+dx]}{2b(a^2 - b^2)d(a+b \operatorname{Sec}[c+dx])^2} + \frac{(a^2A^2b + 2A^2b^3 + a^3B - 4a^2b^2B) \tan[c+dx]}{2b(a^2 - b^2)^2d(a+b \operatorname{Sec}[c+dx])}\right)$

Rule 12

$\operatorname{Int}[(a_*)(u_), x_Symbol] \rightarrow \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /; \operatorname{FreeQ}[a, x] \ \&\& \ !\operatorname{Match}Q[u, (b_*)(v_)] /; \operatorname{FreeQ}[b, x]$

Rule 208

$\operatorname{Int}[(a_*) + (b_*)(x_)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[-(a/b), 2] \operatorname{ArcTanh}[x / \operatorname{Rt}[-(a/b), 2]]) / a, x] /; \operatorname{FreeQ}[\{a, b\}, x] \ \&\& \ \operatorname{NegQ}[a/b]$

Rule 2659

```
Int[((a_) + (b_)*sin[Pi/2 + (c_) + (d_)*(x_)])^(-1), x_Symbol] := With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3831

```
Int[csc[(e_) + (f_)*(x_)]/(csc[(e_) + (f_)*(x_)]*(b_) + (a_)), x_Symbol] := Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4003

```
Int[csc[(e_) + (f_)*(x_)]*(csc[(e_) + (f_)*(x_)]*(b_) + (a_))^(m_)*(csc[(e_) + (f_)*(x_)]*(B_) + (A_)), x_Symbol] := -Simp[((A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(m + 1)*(a^2 - b^2), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[(a*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4009

```
Int[csc[(e_) + (f_)*(x_)]^2*(csc[(e_) + (f_)*(x_)]*(b_) + (a_))^(m_)*(csc[(e_) + (f_)*(x_)]*(B_) + (A_)), x_Symbol] := Simp[(a*(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(A*b - a*B)*(m + 1) - (a*A*b*(m + 2) - B*(a^2 + b^2*(m + 1)))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^2(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^3} dx &= \frac{a(Ab-aB)\tan(c+dx)}{2b(a^2-b^2)d(a+b\sec(c+dx))^2} + \frac{\int \frac{\sec(c+dx)(-2b(Ab-aB)+(aAb+a^2B-2b^2B))}{(a+b\sec(c+dx))^2}}{2b(a^2-b^2)} \\
&= \frac{a(Ab-aB)\tan(c+dx)}{2b(a^2-b^2)d(a+b\sec(c+dx))^2} + \frac{(a^2Ab+2Ab^3+a^3B-4ab^2B)\tan(c+dx)}{2b(a^2-b^2)^2d(a+b\sec(c+dx))} \\
&= \frac{a(Ab-aB)\tan(c+dx)}{2b(a^2-b^2)d(a+b\sec(c+dx))^2} + \frac{(a^2Ab+2Ab^3+a^3B-4ab^2B)\tan(c+dx)}{2b(a^2-b^2)^2d(a+b\sec(c+dx))} \\
&= \frac{a(Ab-aB)\tan(c+dx)}{2b(a^2-b^2)d(a+b\sec(c+dx))^2} + \frac{(a^2Ab+2Ab^3+a^3B-4ab^2B)\tan(c+dx)}{2b(a^2-b^2)^2d(a+b\sec(c+dx))} \\
&= \frac{a(Ab-aB)\tan(c+dx)}{2b(a^2-b^2)d(a+b\sec(c+dx))^2} + \frac{(a^2Ab+2Ab^3+a^3B-4ab^2B)\tan(c+dx)}{2b(a^2-b^2)^2d(a+b\sec(c+dx))} \\
&= -\frac{(3aAb-a^2B-2b^2B)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} + \frac{a(Ab-aB)\tan(c+dx)}{2b(a^2-b^2)d(a+b\sec(c+dx))}
\end{aligned}$$

Mathematica [A] time = 0.73, size = 157, normalized size = 0.87

$$\frac{(2a^2A-3abB+Ab^2)\sin(c+dx)}{(a-b)^2(a+b)^2(a\cos(c+dx)+b)} - \frac{2(a^2B-3aAb+2b^2B)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}} + \frac{(aB-Ab)\sin(c+dx)}{(a-b)(a+b)(a\cos(c+dx)+b)^2}$$

$$2d$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^3,x]

[Out] ((-2*(-3*a*A*b + a^2*B + 2*b^2*B)*ArcTanh[(-a + b)*Tan[(c + d*x)/2]]/Sqrt[a^2 - b^2]))/(a^2 - b^2)^(5/2) + ((-(A*b) + a*B)*Sin[c + d*x])/((a - b)*(a + b)*(b + a*Cos[c + d*x])^2) + ((2*a^2*A + A*b^2 - 3*a*b*B)*Sin[c + d*x])/((a - b)^2*(a + b)^2*(b + a*Cos[c + d*x]))/(2*d)

fricas [B] time = 0.52, size = 750, normalized size = 4.17

$$\frac{\left(Ba^2b^2 - 3Aab^3 + 2Bb^4 + (Ba^4 - 3Aa^3b + 2Ba^2b^2) \cos(dx + c)^2 + 2(Ba^3b - 3Aa^2b^2 + 2Bab^3) \cos(dx + c) \right)}{4\left((a^8 - 3a^6b^2 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm="fricas")

[Out] [1/4*((B*a^2*b^2 - 3*A*a*b^3 + 2*B*b^4 + (B*a^4 - 3*A*a^3*b + 2*B*a^2*b^2)*cos(d*x + c)^2 + 2*(B*a^3*b - 3*A*a^2*b^2 + 2*B*a*b^3)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + 2*(B*a^5 + A*a^4*b - 5*B*a^3*b^2 + A*a^2*b^3 + 4*B*a*b^4 - 2*A*b^5 + (2*A*a^5 - 3*B*a^4*b - A*a^3*b^2 + 3*B*a^2*b^3 - A*a*b^4)*cos(d*x + c))*sin(d*x + c))/((a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*cos(d*x + c) + (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d), 1/2*((B*a^2*b^2 - 3*A*a*b^3 + 2*B*b^4 + (B*a^4 - 3*A*a^3*b + 2*B*a^2*b^2)*cos(d*x + c)^2 + 2*(B*a^3*b - 3*A*a^2*b^2 + 2*B*a*b^3)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) + (B*a^5 + A*a^4*b - 5*B*a^3*b^2 + A*a^2*b^3 + 4*B*a*b^4 - 2*A*b^5 + (2*A*a^5 - 3*B*a^4*b - A*a^3*b^2 + 3*B*a^2*b^3 - A*a*b^4)*cos(d*x + c))*sin(d*x + c))/((a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*cos(d*x + c) + (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d)]

giac [B] time = 3.44, size = 400, normalized size = 2.22

$$\frac{(Ba^2 - 3Aab + 2Bb^2) \left(\pi \left[\frac{dx+c}{2\pi} + \frac{1}{2} \right] \operatorname{sgn}(-2a+2b) + \arctan \left(-\frac{a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - b \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{\sqrt{-a^2+b^2}} \right) \right)}{(a^4 - 2a^2b^2 + b^4) \sqrt{-a^2+b^2}} - \frac{2Aa^3 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 - Ba^3 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 - Aa^2b \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3}{(a^4 - 2a^2b^2 + b^4) \sqrt{-a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm="giac")

[Out] ((B*a^2 - 3*A*a*b + 2*B*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^4 - 2*a^2*b^2 + b^4)*sqrt(-a^2 + b^2)) - (2*A*a^3*tan(1/2*d*x + 1/2*c)^3 - B*a^3*tan(1/2*d*x + 1/2*c)^3 - A*a^2*b*tan(1/2*d*x + 1/2*c)^3)/((a^4 - 2*a^2*b^2 + b^4)*sqrt(-a^2 + b^2)))

$$\frac{1}{2}c)^3 - B a^3 \tan\left(\frac{1}{2}d x + \frac{1}{2}c\right)^3 - A a^2 b \tan\left(\frac{1}{2}d x + \frac{1}{2}c\right)^3 - 3 B a^2 b \tan\left(\frac{1}{2}d x + \frac{1}{2}c\right)^3 + A a b^2 \tan\left(\frac{1}{2}d x + \frac{1}{2}c\right)^3 + 4 B a a b^2 \tan\left(\frac{1}{2}d x + \frac{1}{2}c\right)^3 - 2 A a b^3 \tan\left(\frac{1}{2}d x + \frac{1}{2}c\right)^3 - 2 A a^3 \tan\left(\frac{1}{2}d x + \frac{1}{2}c\right) - B a^3 \tan\left(\frac{1}{2}d x + \frac{1}{2}c\right) - A a^2 b \tan\left(\frac{1}{2}d x + \frac{1}{2}c\right) + 3 B a^2 b \tan\left(\frac{1}{2}d x + \frac{1}{2}c\right) - A a b^2 \tan\left(\frac{1}{2}d x + \frac{1}{2}c\right) + 4 B a a b^2 \tan\left(\frac{1}{2}d x + \frac{1}{2}c\right) - 2 A a b^3 \tan\left(\frac{1}{2}d x + \frac{1}{2}c\right) / ((a^4 - 2 a^2 b^2 + b^4) * (a \tan\left(\frac{1}{2}d x + \frac{1}{2}c\right)^2 - b \tan\left(\frac{1}{2}d x + \frac{1}{2}c\right)^2 - a - b)^2) / d$$

maple [A] time = 0.72, size = 238, normalized size = 1.32

$$\frac{\frac{(2a^2A + Aab + 2Ab^2 - a^2B - 4Bab)\left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + (2a^2A - Aab + 2Ab^2 + a^2B - 4Bab)\tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{(a-b)(a^2 + 2ab + b^2)} + \frac{(2a^2A - Aab + 2Ab^2 + a^2B - 4Bab)\tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{(a+b)(a^2 - 2ab + b^2)} - \frac{(3Aab - a^2B - 2b^2B)\operatorname{arctanh}\left(\frac{\tan\left(\frac{dx}{2} + \frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{(a^4 - 2a^2b^2 + b^4)\sqrt{(a-b)(a+b)}}}{\left(a\left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)b - a - b\right)^2} d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x)`

[Out] $\frac{1}{d} \left(\frac{2 \left(-\frac{1}{2} (2Aa^2 + Aab + 2Ab^2 - B a^2 - 4B a b) \right) / (a-b) / (a^2 + 2ab + b^2) \tan\left(\frac{1}{2}d x + \frac{1}{2}c\right)^3 + \frac{1}{2} (2Aa^2 - Aab + 2Ab^2 + B a^2 - 4B a b) / (a+b) / (a^2 - 2ab + b^2) \tan\left(\frac{1}{2}d x + \frac{1}{2}c\right)}{(a \tan\left(\frac{1}{2}d x + \frac{1}{2}c\right)^2 - \tan\left(\frac{1}{2}d x + \frac{1}{2}c\right)^2 b - a - b)^2} - \frac{(3Aab - B a^2 - 2B b^2) / (a^4 - 2a^2b^2 + b^4) / ((a-b) * (a+b))^{1/2} \operatorname{arctanh}\left(\tan\left(\frac{1}{2}d x + \frac{1}{2}c\right) * (a-b) / ((a-b) * (a+b))^{1/2}\right)}{(a-b) * (a+b)^{1/2}} \right)$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for more details) Is 4*a^2-4*b^2 positive or negative?

mupad [B] time = 5.42, size = 251, normalized size = 1.39

$$\frac{\operatorname{atanh}\left(\frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)(2a-2b)(a^2-2ab+b^2)}{2\sqrt{a+b}(a-b)^{5/2}}\right) (B a^2 - 3 A a b + 2 B b^2)}{d (a+b)^{5/2} (a-b)^{5/2}} - \frac{\frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 (2Aa^2 + 2Ab^2 - Ba^2 + Aab - 4Bab)}{(a+b)^2 (a-b)} - \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{\sqrt{a+b}}}{d \left(2ab - \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 (2a^2 - 2b^2) + \tan\left(\frac{c}{2} + \frac{dx}{2}\right) \sqrt{a+b} \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B/cos(c + d*x))/(cos(c + d*x)^2*(a + b/cos(c + d*x))^3),x)`

[Out] `(atanh((tan(c/2 + (d*x)/2)*(2*a - 2*b)*(a^2 - 2*a*b + b^2))/(2*(a + b)^(1/2)*(a - b)^(5/2)))*(B*a^2 + 2*B*b^2 - 3*A*a*b))/(d*(a + b)^(5/2)*(a - b)^(5/2)) - ((tan(c/2 + (d*x)/2)^3*(2*A*a^2 + 2*A*b^2 - B*a^2 + A*a*b - 4*B*a*b))/((a + b)^2*(a - b)) - (tan(c/2 + (d*x)/2)*(2*A*a^2 + 2*A*b^2 + B*a^2 - A*a*b - 4*B*a*b))/((a + b)*(a^2 - 2*a*b + b^2)))/(d*(2*a*b - tan(c/2 + (d*x)/2)^2*(2*a^2 - 2*b^2) + tan(c/2 + (d*x)/2)^4*(a^2 - 2*a*b + b^2) + a^2 + b^2))`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \sec^2(c + dx)}{(a + b \sec(c + dx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)**2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**3,x)`

[Out] `Integral((A + B*sec(c + d*x))*sec(c + d*x)**2/(a + b*sec(c + d*x))**3, x)`

$$3.332 \quad \int \frac{\sec(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^3} dx$$

Optimal. Leaf size=164

$$\frac{(2a^2A - 3abB + Ab^2) \tanh^{-1}\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}} - \frac{(a^2(-B) + 3aAb - 2b^2B) \tan(c+dx)}{2d(a^2 - b^2)^2(a+b \sec(c+dx))} - \frac{(Ab - aB) \tan(c+dx)}{2d(a^2 - b^2)(a+b \sec(c+dx))}$$

[Out] (2*A*a^2+A*b^2-3*B*a*b)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(5/2)/(a+b)^(5/2)/d-1/2*(A*b-B*a)*tan(d*x+c)/(a^2-b^2)/d/(a+b*sec(d*x+c))^2-1/2*(3*A*a*b-B*a^2-2*B*b^2)*tan(d*x+c)/(a^2-b^2)^2/d/(a+b*sec(d*x+c))

Rubi [A] time = 0.26, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {4003, 12, 3831, 2659, 208}

$$\frac{(2a^2A - 3abB + Ab^2) \tanh^{-1}\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}} - \frac{(a^2(-B) + 3aAb - 2b^2B) \tan(c+dx)}{2d(a^2 - b^2)^2(a+b \sec(c+dx))} - \frac{(Ab - aB) \tan(c+dx)}{2d(a^2 - b^2)(a+b \sec(c+dx))}$$

Antiderivative was successfully verified.

[In] Int[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^3,x]

[Out] ((2*a^2*A + A*b^2 - 3*a*b*B)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(5/2)*(a + b)^(5/2)*d) - ((A*b - a*B)*Tan[c + d*x])/(2*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) - ((3*a*A*b - a^2*B - 2*b^2*B)*Tan[c + d*x])/(2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 2659


```
Int[((a_) + (b_)*sin[Pi/2 + (c_) + (d_)*(x_)])^(-1), x_Symbol] := With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3831

```
Int[csc[(e_) + (f_)*(x_)]/(csc[(e_) + (f_)*(x_)]*(b_) + (a_)), x_Symbol] := Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4003

```
Int[csc[(e_) + (f_)*(x_)]*(csc[(e_) + (f_)*(x_)]*(b_) + (a_))^(m_)*(csc[(e_) + (f_)*(x_)]*(B_) + (A_)), x_Symbol] := -Simp[((A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(m + 1)*(a^2 - b^2), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[(a*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^3} dx &= -\frac{(Ab-aB)\tan(c+dx)}{2(a^2-b^2)d(a+b\sec(c+dx))^2} - \frac{\int \frac{\sec(c+dx)(-2(aA-bB)+(Ab-aB)\sec(c+dx))}{(a+b\sec(c+dx))^2}}{2(a^2-b^2)} \\
&= -\frac{(Ab-aB)\tan(c+dx)}{2(a^2-b^2)d(a+b\sec(c+dx))^2} - \frac{(3aAb-a^2B-2b^2B)\tan(c+dx)}{2(a^2-b^2)^2d(a+b\sec(c+dx))} + \\
&= -\frac{(Ab-aB)\tan(c+dx)}{2(a^2-b^2)d(a+b\sec(c+dx))^2} - \frac{(3aAb-a^2B-2b^2B)\tan(c+dx)}{2(a^2-b^2)^2d(a+b\sec(c+dx))} + \\
&= -\frac{(Ab-aB)\tan(c+dx)}{2(a^2-b^2)d(a+b\sec(c+dx))^2} - \frac{(3aAb-a^2B-2b^2B)\tan(c+dx)}{2(a^2-b^2)^2d(a+b\sec(c+dx))} + \\
&= -\frac{(Ab-aB)\tan(c+dx)}{2(a^2-b^2)d(a+b\sec(c+dx))^2} - \frac{(3aAb-a^2B-2b^2B)\tan(c+dx)}{2(a^2-b^2)^2d(a+b\sec(c+dx))} + \\
&= \frac{(2a^2A+Ab^2-3abB)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} - \frac{(Ab-aB)\tan(c+dx)}{2(a^2-b^2)d(a+b\sec(c+dx))}
\end{aligned}$$

Mathematica [A] time = 0.92, size = 172, normalized size = 1.05

$$\frac{2(2a^2A-3abB+Ab^2)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}} + \frac{(2a^3B-4a^2Ab+ab^2B+Ab^3)\sin(c+dx)}{a(a-b)^2(a+b)^2(a\cos(c+dx)+b)} + \frac{b(Ab-aB)\sin(c+dx)}{a(a-b)(a+b)(a\cos(c+dx)+b)^2}$$

2d

Antiderivative was successfully verified.

[In] Integrate[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^3,x]

[Out] ((-2*(2*a^2*A + A*b^2 - 3*a*b*B)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2) + (b*(A*b - a*B)*Sin[c + d*x])/(a*(a - b)*(a + b)*(b + a*Cos[c + d*x])^2) + ((-4*a^2*A*b + A*b^3 + 2*a^3*B + a*b^2*B)*Sin[c + d*x])/(a*(a - b)^2*(a + b)^2*(b + a*Cos[c + d*x]))/(2*d)

fricas [B] time = 0.51, size = 752, normalized size = 4.59

$$\left[\frac{(2 A a^2 b^2 - 3 B a b^3 + A b^4 + (2 A a^4 - 3 B a^3 b + A a^2 b^2) \cos(dx + c))^2 + 2(2 A a^3 b - 3 B a^2 b^2 + A a b^3) \cos(dx + c)}{4((a^8 - 3 a^6 b^2} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm="fricas")

[Out] [1/4*((2*A*a^2*b^2 - 3*B*a*b^3 + A*b^4 + (2*A*a^4 - 3*B*a^3*b + A*a^2*b^2)*cos(d*x + c)^2 + 2*(2*A*a^3*b - 3*B*a^2*b^2 + A*a*b^3)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + 2*(B*a^4*b - 3*A*a^3*b^2 + B*a^2*b^3 + 3*A*a*b^4 - 2*B*b^5 + (2*B*a^5 - 4*A*a^4*b - B*a^3*b^2 + 5*A*a^2*b^3 - B*a*b^4 - A*b^5)*cos(d*x + c))*sin(d*x + c))/((a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*cos(d*x + c) + (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d), 1/2*((2*A*a^2*b^2 - 3*B*a*b^3 + A*b^4 + (2*A*a^4 - 3*B*a^3*b + A*a^2*b^2)*cos(d*x + c)^2 + 2*(2*A*a^3*b - 3*B*a^2*b^2 + A*a*b^3)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) + (B*a^4*b - 3*A*a^3*b^2 + B*a^2*b^3 + 3*A*a*b^4 - 2*B*b^5 + (2*B*a^5 - 4*A*a^4*b - B*a^3*b^2 + 5*A*a^2*b^3 - B*a*b^4 - A*b^5)*cos(d*x + c))*sin(d*x + c))/((a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*cos(d*x + c) + (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d)]

giac [B] time = 0.38, size = 399, normalized size = 2.43

$$\frac{(2 A a^2 - 3 B a b + A b^2) \left(\pi \left[\frac{dx+c}{2\pi} + \frac{1}{2} \right] \operatorname{sgn}(-2 a+2 b)+\arctan\left(-\frac{a \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)-b \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\left(a^4-2 a^2 b^2+b^4\right) \sqrt{-a^2+b^2}} - \frac{2 B a^3 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^3-4 A a^2 b \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^3-B a^2 b \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)}{\left(a^4-2 a^2 b^2+b^4\right) \sqrt{-a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm="giac")

[Out] ((2*A*a^2 - 3*B*a*b + A*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^4 - 2*a^2*b^2 + b^4)*sqrt(-a^2 + b^2)) - (2*B*a^3*tan(1/2*d*x +

$$\frac{1}{2}c)^3 - 4Aa^2b \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 - Ba^2b \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 + 3Aab^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 + Babb^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 + Ab^3 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 - 2Bb^3 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 - 2Ba^3 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 4Aa^2b \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - Ba^2b \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 3Aab^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - Babb^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - Ab^3 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 2Bb^3 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) / ((a^4 - 2a^2b^2 + b^4) * (a \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - b \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - a - b)^2) / d$$

maple [A] time = 0.72, size = 236, normalized size = 1.44

$$\frac{2 \left(-\frac{(4Aab+Ab^2-2a^2B-Bab-2b^2B)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a-b)(a^2+2ab+b^2)} + \frac{(4Aab-Ab^2-2a^2B+Bab-2b^2B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)(a^2-2ab+b^2)} \right)}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b-a-b\right)^2} + \frac{(2a^2A+Ab^2-3Bab) \operatorname{arctanh}\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{(a^4-2a^2b^2+b^4)\sqrt{(a-b)(a+b)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x)`

[Out] $\frac{1}{d} \left(-2 \left(-\frac{1}{2} (4Aab+Ab^2-2a^2B-Bab-2b^2B) / (a-b) / (a^2+2ab+b^2) \right) \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 + \frac{1}{2} (4Aab-Ab^2-2a^2B+Bab-2b^2B) / (a+b) / (a^2-2ab+b^2) \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) \right) / (a \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 - \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 b - a - b)^2 + (2Aa^2+Ab^2-3Bab) / (a^4-2a^2b^2+b^4) / ((a-b)(a+b))^{1/2} \operatorname{arctanh}\left(\frac{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for more details) Is 4*a^2-4*b^2 positive or negative?

mupad [B] time = 5.35, size = 251, normalized size = 1.53

$$\frac{\operatorname{atanh}\left(\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(2a-2b)(a^2-2ab+b^2)}{2\sqrt{a+b}(a-b)^{5/2}}\right) (2Aa^2-3Bab+Ab^2)}{d(a+b)^{5/2}(a-b)^{5/2}} - \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3 (2Ba^2-Ab^2+2Bb^2-4Aab+Bab)}{(a+b)^2(a-b)} - \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(2ab - \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 (2a^2-2b^2) + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B/cos(c + d*x))/(cos(c + d*x)*(a + b/cos(c + d*x))^3),x)`

[Out] $(\operatorname{atanh}(\frac{\tan(c/2 + (d*x)/2)*(2*a - 2*b)*(a^2 - 2*a*b + b^2)}{2*(a + b)^{(1/2)}*(a - b)^{(5/2)}})) * (2*A*a^2 + A*b^2 - 3*B*a*b) / (d*(a + b)^{(5/2)}*(a - b)^{(5/2)}) - ((\tan(c/2 + (d*x)/2)^3*(2*B*a^2 - A*b^2 + 2*B*b^2 - 4*A*a*b + B*a*b)) / ((a + b)^2*(a - b)) - (\tan(c/2 + (d*x)/2)*(A*b^2 + 2*B*a^2 + 2*B*b^2 - 4*A*a*b - B*a*b)) / ((a + b)*(a^2 - 2*a*b + b^2))) / (d*(2*a*b - \tan(c/2 + (d*x)/2)^2*(2*a^2 - 2*b^2) + \tan(c/2 + (d*x)/2)^4*(a^2 - 2*a*b + b^2) + a^2 + b^2))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \sec(c + dx)}{(a + b \sec(c + dx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**3,x)`

[Out] `Integral((A + B*sec(c + d*x))*sec(c + d*x)/(a + b*sec(c + d*x))**3, x)`

$$3.333 \quad \int \frac{A+B \sec(c+dx)}{(a+b \sec(c+dx))^3} dx$$

Optimal. Leaf size=205

$$\frac{Ax}{a^3} + \frac{b(Ab - aB) \tan(c + dx)}{2ad(a^2 - b^2)(a + b \sec(c + dx))^2} + \frac{b(-3a^3B + 5a^2Ab - 2Ab^3) \tan(c + dx)}{2a^2d(a^2 - b^2)^2(a + b \sec(c + dx))} - \frac{(-2a^5B + 6a^4Ab - a^3b^2B - 5a^2b^2A)}{a^3d(a - b)^{5/2}(a + b)^{5/2}}$$

[Out] $A*x/a^3 - (6*A*a^4*b - 5*A*a^2*b^3 + 2*A*b^5 - 2*B*a^5 - B*a^3*b^2) * \operatorname{arctanh}((a-b)^{(1/2)} * \tan(1/2*d*x + 1/2*c) / (a+b)^{(1/2)}) / a^3 / (a-b)^{(5/2)} / (a+b)^{(5/2)} / d + 1/2*b*(A*b - B*a) * \tan(d*x+c) / a / (a^2 - b^2) / d / (a+b*\sec(d*x+c))^{2+1/2} * b*(5*A*a^2*b - 2*A*b^3 - 3*B*a^3) * \tan(d*x+c) / a^2 / (a^2 - b^2)^2 / d / (a+b*\sec(d*x+c))$

Rubi [A] time = 0.54, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3923, 4060, 3919, 3831, 2659, 208}

$$\frac{(-5a^2Ab^3 + 6a^4Ab - a^3b^2B - 2a^5B + 2Ab^5) \tanh^{-1}\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d(a-b)^{5/2}(a+b)^{5/2}} + \frac{b(5a^2Ab - 3a^3B - 2Ab^3) \tan(c + dx)}{2a^2d(a^2 - b^2)^2(a + b \sec(c + dx))}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(A + B*\operatorname{Sec}[c + d*x]) / (a + b*\operatorname{Sec}[c + d*x])^3, x]$

[Out] $(A*x)/a^3 - ((6*a^4*A*b - 5*a^2*A*b^3 + 2*A*b^5 - 2*a^5*B - a^3*b^2*B) * \operatorname{ArcTanh}[\operatorname{Sqrt}[a - b] * \operatorname{Tan}[(c + d*x)/2] / \operatorname{Sqrt}[a + b]]) / (a^3*(a - b)^{(5/2)}*(a + b)^{(5/2)}*d) + (b*(A*b - a*B) * \operatorname{Tan}[c + d*x]) / (2*a*(a^2 - b^2)*d*(a + b*\operatorname{Sec}[c + d*x])^2) + (b*(5*a^2*A*b - 2*A*b^3 - 3*a^3*B) * \operatorname{Tan}[c + d*x]) / (2*a^2*(a^2 - b^2)^2*d*(a + b*\operatorname{Sec}[c + d*x]))$

Rule 208

$\operatorname{Int}[(a_) + (b_.) * (x_)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[-(a/b), 2] * \operatorname{ArcTanh}[x / \operatorname{Rt}[-(a/b), 2]]) / a, x] /;$ $\operatorname{FreeQ}\{a, b\}, x \&\& \operatorname{NegQ}[a/b]$

Rule 2659

$\operatorname{Int}[(a_) + (b_.) * \sin[\operatorname{Pi}/2 + (c_.) + (d_.) * (x_)]])^{-1}, x_Symbol] \rightarrow \operatorname{With}\{e = \operatorname{FreeFactors}[\operatorname{Tan}[(c + d*x)/2], x]\}, \operatorname{Dist}[(2*e)/d, \operatorname{Subst}[\operatorname{Int}[1/(a + b + (a - b)*e^2*x^2), x], x, \operatorname{Tan}[(c + d*x)/2]/e], x]] /;$ $\operatorname{FreeQ}\{a, b, c, d\}, x \&\& \operatorname{NeQ}[a^2 - b^2, 0]$

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol]
:= Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_.)), x_Symbol] := Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x]
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
a*d, 0]
```

Rule 3923

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_.)), x_Symbol] := Simp[(b*(b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f
*x])^(m + 1))/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)
), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[c*(a^2 - b^2)*(m + 1) - (a*(b*c -
a*d)*(m + 1))*Csc[e + f*x] + b*(b*c - a*d)*(m + 2)*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && Ne
Q[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 4060

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_), x_Symbol] := Simp[((A*b^2 -
a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^
2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
+ 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
+ (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{A + B \sec(c + dx)}{(a + b \sec(c + dx))^3} dx &= \frac{b(Ab - aB) \tan(c + dx)}{2a(a^2 - b^2) d(a + b \sec(c + dx))^2} - \frac{\int \frac{-2A(a^2 - b^2) + 2a(Ab - aB) \sec(c + dx) - b(Ab - aB) \sec^2(c + dx)}{(a + b \sec(c + dx))^2} dx}{2a(a^2 - b^2)} \\
&= \frac{b(Ab - aB) \tan(c + dx)}{2a(a^2 - b^2) d(a + b \sec(c + dx))^2} + \frac{b(5a^2Ab - 2Ab^3 - 3a^3B) \tan(c + dx)}{2a^2(a^2 - b^2)^2 d(a + b \sec(c + dx))} + \frac{\int \frac{2A(a^2 - b^2)}{(a + b \sec(c + dx))^2} dx}{2a(a^2 - b^2)} \\
&= \frac{Ax}{a^3} + \frac{b(Ab - aB) \tan(c + dx)}{2a(a^2 - b^2) d(a + b \sec(c + dx))^2} + \frac{b(5a^2Ab - 2Ab^3 - 3a^3B) \tan(c + dx)}{2a^2(a^2 - b^2)^2 d(a + b \sec(c + dx))} - \frac{\int \frac{2A(a^2 - b^2)}{(a + b \sec(c + dx))^2} dx}{2a(a^2 - b^2)} \\
&= \frac{Ax}{a^3} + \frac{b(Ab - aB) \tan(c + dx)}{2a(a^2 - b^2) d(a + b \sec(c + dx))^2} + \frac{b(5a^2Ab - 2Ab^3 - 3a^3B) \tan(c + dx)}{2a^2(a^2 - b^2)^2 d(a + b \sec(c + dx))} - \frac{\int \frac{2A(a^2 - b^2)}{(a + b \sec(c + dx))^2} dx}{2a(a^2 - b^2)} \\
&= \frac{Ax}{a^3} + \frac{b(Ab - aB) \tan(c + dx)}{2a(a^2 - b^2) d(a + b \sec(c + dx))^2} + \frac{b(5a^2Ab - 2Ab^3 - 3a^3B) \tan(c + dx)}{2a^2(a^2 - b^2)^2 d(a + b \sec(c + dx))} - \frac{\int \frac{2A(a^2 - b^2)}{(a + b \sec(c + dx))^2} dx}{2a(a^2 - b^2)} \\
&= \frac{Ax}{a^3} - \frac{(6a^4Ab - 5a^2Ab^3 + 2Ab^5 - 2a^5B - a^3b^2B) \tanh^{-1}\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3(a-b)^{5/2}(a+b)^{5/2}d} + \frac{\int \frac{2A(a^2 - b^2)}{(a + b \sec(c + dx))^2} dx}{2a(a^2 - b^2)}
\end{aligned}$$

Mathematica [A] time = 1.47, size = 267, normalized size = 1.30

$$\frac{\sec^2(c + dx)(a \cos(c + dx) + b)(A + B \sec(c + dx)) \left(-\frac{ab(4a^3B - 6a^2Ab - ab^2B + 3Ab^3) \sin(c + dx)(a \cos(c + dx) + b)}{(a-b)^2(a+b)^2} - \frac{2(2a^5B - 6a^4Ab + a^3b^2B)}{2a^3d(a + b \sec(c + dx))^3} \right)}{2a^3d(a + b \sec(c + dx))^3}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sec[c + d*x])/(a + b*Sec[c + d*x])^3,x]

[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]^2*(A + B*Sec[c + d*x])*(2*A*(c + d*x)*(b + a*Cos[c + d*x])^2 - (2*(-6*a^4*A*b + 5*a^2*A*b^3 - 2*A*b^5 + 2*a^5*B + a^3*b^2*B)*ArcTanh[(-a + b)*Tan[(c + d*x)/2]]/Sqrt[a^2 - b^2])*(b + a*Cos[c + d*x])^2)/(a^2 - b^2)^(5/2) + (a*b^2*(-(A*b) + a*B)*Sin[c + d*x])/((a - b)*(a + b)) - (a*b*(-6*a^2*A*b + 3*A*b^3 + 4*a^3*B - a*b^2*B)*(b + a*Cos[c + d*x])^2)/(a^3*d*(a + b*Sec[c + d*x])^3)

$d*x])*\text{Sin}[c + d*x])/((a - b)^2*(a + b)^2)))/(2*a^3*d*(B + A*\text{Cos}[c + d*x])* (a + b*\text{Sec}[c + d*x])^3)$

fricas [B] time = 0.55, size = 1152, normalized size = 5.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm="fricas")

[Out] $[1/4*(4*(A*a^8 - 3*A*a^6*b^2 + 3*A*a^4*b^4 - A*a^2*b^6)*d*x*\cos(d*x + c)^2 + 8*(A*a^7*b - 3*A*a^5*b^3 + 3*A*a^3*b^5 - A*a*b^7)*d*x*\cos(d*x + c) + 4*(A*a^6*b^2 - 3*A*a^4*b^4 + 3*A*a^2*b^6 - A*b^8)*d*x - (2*B*a^5*b^2 - 6*A*a^4*b^3 + B*a^3*b^4 + 5*A*a^2*b^5 - 2*A*b^7 + (2*B*a^7 - 6*A*a^6*b + B*a^5*b^2 + 5*A*a^4*b^3 - 2*A*a^2*b^5)*\cos(d*x + c)^2 + 2*(2*B*a^6*b - 6*A*a^5*b^2 + B*a^4*b^3 + 5*A*a^3*b^4 - 2*A*a*b^6)*\cos(d*x + c))*\sqrt{a^2 - b^2}*\log((2*a*b*\cos(d*x + c) - (a^2 - 2*b^2)*\cos(d*x + c)^2 - 2*\sqrt{a^2 - b^2}*(b*\cos(d*x + c) + a)*\sin(d*x + c) + 2*a^2 - b^2)/(a^2*\cos(d*x + c)^2 + 2*a*b*\cos(d*x + c) + b^2)) - 2*(3*B*a^6*b^2 - 5*A*a^5*b^3 - 3*B*a^4*b^4 + 7*A*a^3*b^5 - 2*A*a*b^7 + (4*B*a^7*b - 6*A*a^6*b^2 - 5*B*a^5*b^3 + 9*A*a^4*b^4 + B*a^3*b^5 - 3*A*a^2*b^6)*\cos(d*x + c))*\sin(d*x + c))/((a^{11} - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*d*\cos(d*x + c)^2 + 2*(a^{10}*b - 3*a^8*b^3 + 3*a^6*b^5 - a^4*b^7)*d*\cos(d*x + c) + (a^9*b^2 - 3*a^7*b^4 + 3*a^5*b^6 - a^3*b^8)*d), 1/2*(2*(A*a^8 - 3*A*a^6*b^2 + 3*A*a^4*b^4 - A*a^2*b^6)*d*x*\cos(d*x + c)^2 + 4*(A*a^7*b - 3*A*a^5*b^3 + 3*A*a^3*b^5 - A*a*b^7)*d*x*\cos(d*x + c) + 2*(A*a^6*b^2 - 3*A*a^4*b^4 + 3*A*a^2*b^6 - A*b^8)*d*x + (2*B*a^5*b^2 - 6*A*a^4*b^3 + B*a^3*b^4 + 5*A*a^2*b^5 - 2*A*b^7 + (2*B*a^7 - 6*A*a^6*b + B*a^5*b^2 + 5*A*a^4*b^3 - 2*A*a^2*b^5)*\cos(d*x + c)^2 + 2*(2*B*a^6*b - 6*A*a^5*b^2 + B*a^4*b^3 + 5*A*a^3*b^4 - 2*A*a*b^6)*\cos(d*x + c))*\sqrt{-a^2 + b^2}*\arctan(-\sqrt{-a^2 + b^2}*(b*\cos(d*x + c) + a)/((a^2 - b^2)*\sin(d*x + c))) - (3*B*a^6*b^2 - 5*A*a^5*b^3 - 3*B*a^4*b^4 + 7*A*a^3*b^5 - 2*A*a*b^7 + (4*B*a^7*b - 6*A*a^6*b^2 - 5*B*a^5*b^3 + 9*A*a^4*b^4 + B*a^3*b^5 - 3*A*a^2*b^6)*\cos(d*x + c))*\sin(d*x + c))/((a^{11} - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*d*\cos(d*x + c)^2 + 2*(a^{10}*b - 3*a^8*b^3 + 3*a^6*b^5 - a^4*b^7)*d*\cos(d*x + c) + (a^9*b^2 - 3*a^7*b^4 + 3*a^5*b^6 - a^3*b^8)*d)]$

giac [B] time = 0.37, size = 457, normalized size = 2.23

$$\frac{(2Ba^5 - 6Aa^4b + Ba^3b^2 + 5Aa^2b^3 - 2Ab^5) \left(\pi \left[\frac{dx+c}{2\pi} + \frac{1}{2} \right] \text{sgn}(-2a+2b) + \arctan \left(-\frac{a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - b \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{\sqrt{-a^2+b^2}} \right) \right)}{(a^7 - 2a^5b^2 + a^3b^4) \sqrt{-a^2+b^2}} + \frac{(dx+c)A}{a^3} + \frac{4Ba^4b \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm="giac")

[Out]
$$\frac{\begin{aligned} & ((2*B*a^5 - 6*A*a^4*b + B*a^3*b^2 + 5*A*a^2*b^3 - 2*A*b^5)*(pi*\text{floor}(1/2*(d*x + c)/pi + 1/2)*\text{sgn}(-2*a + 2*b) + \arctan(-(a*\tan(1/2*d*x + 1/2*c) - b*\tan(1/2*d*x + 1/2*c))/\sqrt{-a^2 + b^2}))/((a^7 - 2*a^5*b^2 + a^3*b^4)*\sqrt{-a^2 + b^2}) + (d*x + c)*A/a^3 + (4*B*a^4*b*\tan(1/2*d*x + 1/2*c)^3 - 6*A*a^3*b^2*\tan(1/2*d*x + 1/2*c)^3 - 3*B*a^3*b^2*\tan(1/2*d*x + 1/2*c)^3 + 5*A*a^2*b^3*\tan(1/2*d*x + 1/2*c)^3 - B*a^2*b^3*\tan(1/2*d*x + 1/2*c)^3 + 3*A*a*b^4*\tan(1/2*d*x + 1/2*c)^3 - 2*A*b^5*\tan(1/2*d*x + 1/2*c)^3 - 4*B*a^4*b*\tan(1/2*d*x + 1/2*c) + 6*A*a^3*b^2*\tan(1/2*d*x + 1/2*c) - 3*B*a^3*b^2*\tan(1/2*d*x + 1/2*c) + 5*A*a^2*b^3*\tan(1/2*d*x + 1/2*c) + B*a^2*b^3*\tan(1/2*d*x + 1/2*c) - 3*A*a*b^4*\tan(1/2*d*x + 1/2*c) - 2*A*b^5*\tan(1/2*d*x + 1/2*c))/((a^6 - 2*a^4*b^2 + a^2*b^4)*(a*\tan(1/2*d*x + 1/2*c)^2 - b*\tan(1/2*d*x + 1/2*c)^2 - a - b)^2))/d \end{aligned}}$$

maple [B] time = 0.86, size = 1063, normalized size = 5.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x)

[Out]
$$\begin{aligned} & -6/d/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)^2*b^2/(a-b)/(a^2+2*a*b+b^2)*\tan(1/2*d*x+1/2*c)^3*A-1/d/a/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)^2*b^3/(a-b)/(a^2+2*a*b+b^2)*\tan(1/2*d*x+1/2*c)^3*A+2/d/a^2/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)^2*b^4/(a-b)/(a^2+2*a*b+b^2)*\tan(1/2*d*x+1/2*c)^3*A+4/d*a/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)^2*b/(a-b)/(a^2+2*a*b+b^2)*\tan(1/2*d*x+1/2*c)^3*B+1/d/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)^2*b^2/(a-b)/(a^2+2*a*b+b^2)*\tan(1/2*d*x+1/2*c)^3*B+6/d/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)^2*b^2/(a+b)/(a-b)^2*\tan(1/2*d*x+1/2*c)*A-1/d/a/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)^2*b^3/(a+b)/(a-b)^2*\tan(1/2*d*x+1/2*c)*A-2/d/a^2/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)^2*b^4/(a+b)/(a-b)^2*\tan(1/2*d*x+1/2*c)*A-4/d*a/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)^2*b/(a+b)/(a-b)^2*\tan(1/2*d*x+1/2*c)*B+1/d/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)^2*b^2/(a+b)/(a-b)^2*\tan(1/2*d*x+1/2*c)*B-6/d*a*b/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*\operatorname{arctanh}(\tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b)))^(1/2))*A+5/d/a/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*\operatorname{arctanh}(\tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b)))^(1/2))*A*b^3-2/d/a^3/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*\operatorname{arctanh}(\tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b)))^(1/2))*A*b^5+2/d*a^2/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*\operatorname{arctanh}(\tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b)))^(1/2))*B+1/d/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*\operatorname{arctanh}(\tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b)))^(1/2))*b^2*B+2/d/a^3*\operatorname{arctan}(\tan(1/2*d*x+1/2*c))*A \end{aligned}}$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for more details)Is 4*a^2-4*b^2 positive or negative?

mupad [B] time = 11.79, size = 6909, normalized size = 33.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/(a + b/cos(c + d*x))^3,x)

[Out]
$$\begin{aligned} & (2*A*atan(((A*((8*\tan(c/2 + (d*x)/2)*(4*A^2*a^{10} + 8*A^2*b^{10} + 4*B^2*a^{10} \\ & - 8*A^2*a*b^9 - 8*A^2*a^9*b - 32*A^2*a^2*b^8 + 32*A^2*a^3*b^7 + 57*A^2*a^4* \\ & b^6 - 48*A^2*a^5*b^5 - 52*A^2*a^6*b^4 + 32*A^2*a^7*b^3 + 24*A^2*a^8*b^2 + B \\ & ^2*a^6*b^4 + 4*B^2*a^8*b^2 - 24*A*B*a^9*b - 4*A*B*a^3*b^7 + 2*A*B*a^5*b^5 + \\ & 8*A*B*a^7*b^3)))/(a^{10}*b + a^{11} - a^4*b^7 - a^5*b^6 + 3*a^6*b^5 + 3*a^7*b^4 \\ & - 3*a^8*b^3 - 3*a^9*b^2) + (A*((8*(4*A*a^{15} + 4*B*a^{15} - 4*A*a^6*b^9 + 2*A \\ & *a^7*b^8 + 18*A*a^8*b^7 - 4*A*a^9*b^6 - 36*A*a^{10}*b^5 + 6*A*a^{11}*b^4 + 34*A \\ & *a^{12}*b^3 - 8*A*a^{13}*b^2 - 2*B*a^8*b^7 + 2*B*a^9*b^6 + 6*B*a^{12}*b^3 - 6*B*a \\ & ^{13}*b^2 - 12*A*a^{14}*b - 4*B*a^{14}*b)))/(a^{12}*b + a^{13} - a^6*b^7 - a^7*b^6 + 3 \\ & *a^8*b^5 + 3*a^9*b^4 - 3*a^{10}*b^3 - 3*a^{11}*b^2) - (A*\tan(c/2 + (d*x)/2)*(8* \\ & a^{15}*b - 8*a^6*b^{10} + 8*a^7*b^9 + 32*a^8*b^8 - 32*a^9*b^7 - 48*a^{10}*b^6 + 4 \\ & 8*a^{11}*b^5 + 32*a^{12}*b^4 - 32*a^{13}*b^3 - 8*a^{14}*b^2)*8i)/(a^3*(a^{10}*b + a^{11} \\ & - a^4*b^7 - a^5*b^6 + 3*a^6*b^5 + 3*a^7*b^4 - 3*a^8*b^3 - 3*a^9*b^2))) * 1i \\ &)/a^3) / a^3 + (A*((8*\tan(c/2 + (d*x)/2)*(4*A^2*a^{10} + 8*A^2*b^{10} + 4*B^2*a^{10} \\ & - 8*A^2*a*b^9 - 8*A^2*a^9*b - 32*A^2*a^2*b^8 + 32*A^2*a^3*b^7 + 57*A^2*a^4* \\ & b^6 - 48*A^2*a^5*b^5 - 52*A^2*a^6*b^4 + 32*A^2*a^7*b^3 + 24*A^2*a^8*b^2 \\ & + B^2*a^6*b^4 + 4*B^2*a^8*b^2 - 24*A*B*a^9*b - 4*A*B*a^3*b^7 + 2*A*B*a^5*b^5 + \\ & 8*A*B*a^7*b^3)))/(a^{10}*b + a^{11} - a^4*b^7 - a^5*b^6 + 3*a^6*b^5 + 3*a^7* \\ & b^4 - 3*a^8*b^3 - 3*a^9*b^2) - (A*((8*(4*A*a^{15} + 4*B*a^{15} - 4*A*a^6*b^9 + \\ & 2*A*a^7*b^8 + 18*A*a^8*b^7 - 4*A*a^9*b^6 - 36*A*a^{10}*b^5 + 6*A*a^{11}*b^4 + 3 \\ & 4*A*a^{12}*b^3 - 8*A*a^{13}*b^2 - 2*B*a^8*b^7 + 2*B*a^9*b^6 + 6*B*a^{12}*b^3 - 6* \\ & B*a^{13}*b^2 - 12*A*a^{14}*b - 4*B*a^{14}*b)))/(a^{12}*b + a^{13} - a^6*b^7 - a^7*b^6 \\ & + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^{10}*b^3 - 3*a^{11}*b^2) + (A*\tan(c/2 + (d*x)/2)* \\ & (8*a^{15}*b - 8*a^6*b^{10} + 8*a^7*b^9 + 32*a^8*b^8 - 32*a^9*b^7 - 48*a^{10}*b^6 \\ & + 48*a^{11}*b^5 + 32*a^{12}*b^4 - 32*a^{13}*b^3 - 8*a^{14}*b^2)*8i)/(a^3*(a^{10}*b + \end{aligned}$$

$$\begin{aligned}
& a^{11} - a^4 b^7 - a^5 b^6 + 3a^6 b^5 + 3a^7 b^4 - 3a^8 b^3 - 3a^9 b^2)) \\
& *1i)/a^3))/a^3)/((16*(4A^3 b^9 + 4AB^2 a^9 - 4A^2 B a^9 - 2A^3 a b^8 + \\
& 12A^3 a^8 b - 18A^3 a^2 b^7 + 13A^3 a^3 b^6 + 36A^3 a^4 b^5 - 26A^3 a^5 b^4 - \\
& 34A^3 a^6 b^3 + 24A^3 a^7 b^2 - 20A^2 B a^8 b + AB^2 a^5 b^4 + \\
& 4AB^2 a^7 b^2 - 2A^2 B a^2 b^7 - 2A^2 B a^3 b^6 + 2A^2 B a^4 b^5 + 2A^2 B a^6 b^3 + 6A^2 B a^7 b^2)) / (a^{12} b + a^{13} - a^6 b^7 - a^7 b^6 + 3a^8 b^5 + 3a^9 b^4 - 3a^{10} b^3 - 3a^{11} b^2) - (A((8 \tan(c/2 + (d*x)/2) * (4A^2 a^{10} + 8A^2 b^{10} + 4B^2 a^{10} - 8A^2 a b^9 - 8A^2 a^9 b - 32A^2 a^2 b^8 + 32A^2 a^3 b^7 + 57A^2 a^4 b^6 - 48A^2 a^5 b^5 - 52A^2 a^6 b^4 + 32A^2 a^7 b^3 + 24A^2 a^8 b^2 + B^2 a^6 b^4 + 4B^2 a^8 b^2 - 24AB a^9 b - 4AB a^3 b^7 + 2AB a^5 b^5 + 8AB a^7 b^3)) / (a^{10} b + a^{11} - a^4 b^7 - a^5 b^6 + 3a^6 b^5 + 3a^7 b^4 - 3a^8 b^3 - 3a^9 b^2) + (A((8(4A a^{15} + 4B a^{15} - 4A a^6 b^9 + 2A a^7 b^8 + 18A a^8 b^7 - 4A a^9 b^6 - 36A a^{10} b^5 + 6A a^{11} b^4 + 34A a^{12} b^3 - 8A a^{13} b^2 - 2B a^8 b^7 + 2B a^9 b^6 + 6B a^{12} b^3 - 6B a^{13} b^2 - 12A a^{14} b - 4B a^{14} b)) / (a^{12} b + a^{13} - a^6 b^7 - a^7 b^6 + 3a^8 b^5 + 3a^9 b^4 - 3a^{10} b^3 - 3a^{11} b^2) - (A \tan(c/2 + (d*x)/2) * (8a^{15} b - 8a^6 b^{10} + 8a^7 b^9 + 32a^8 b^8 - 32a^9 b^7 - 48a^{10} b^6 + 48a^{11} b^5 + 32a^{12} b^4 - 32a^{13} b^3 - 8a^{14} b^2) * 8i) / (a^3 (a^{10} b + a^{11} - a^4 b^7 - a^5 b^6 + 3a^6 b^5 + 3a^7 b^4 - 3a^8 b^3 - 3a^9 b^2))) * 1i) / a^3 + (A((8 \tan(c/2 + (d*x)/2) * (4A^2 a^{10} + 8A^2 b^{10} + 4B^2 a^{10} - 8A^2 a b^9 - 8A^2 a^9 b - 32A^2 a^2 b^8 + 32A^2 a^3 b^7 + 57A^2 a^4 b^6 - 48A^2 a^5 b^5 - 52A^2 a^6 b^4 + 32A^2 a^7 b^3 + 24A^2 a^8 b^2 + B^2 a^6 b^4 + 4B^2 a^8 b^2 - 24AB a^9 b - 4AB a^3 b^7 + 2AB a^5 b^5 + 8AB a^7 b^3)) / (a^{10} b + a^{11} - a^4 b^7 - a^5 b^6 + 3a^6 b^5 + 3a^7 b^4 - 3a^8 b^3 - 3a^9 b^2) - (A((8(4A a^{15} + 4B a^{15} - 4A a^6 b^9 + 2A a^7 b^8 + 18A a^8 b^7 - 4A a^9 b^6 - 36A a^{10} b^5 + 6A a^{11} b^4 + 34A a^{12} b^3 - 8A a^{13} b^2 - 2B a^8 b^7 + 2B a^9 b^6 + 6B a^{12} b^3 - 6B a^{13} b^2 - 12A a^{14} b - 4B a^{14} b)) / (a^{12} b + a^{13} - a^6 b^7 - a^7 b^6 + 3a^8 b^5 + 3a^9 b^4 - 3a^{10} b^3 - 3a^{11} b^2) + (A \tan(c/2 + (d*x)/2) * (8a^{15} b - 8a^6 b^{10} + 8a^7 b^9 + 32a^8 b^8 - 32a^9 b^7 - 48a^{10} b^6 + 48a^{11} b^5 + 32a^{12} b^4 - 32a^{13} b^3 - 8a^{14} b^2) * 8i) / (a^3 (a^{10} b + a^{11} - a^4 b^7 - a^5 b^6 + 3a^6 b^5 + 3a^7 b^4 - 3a^8 b^3 - 3a^9 b^2))) * 1i) / a^3))) / (a^3 d) - ((\tan(c/2 + (d*x)/2)^3 * (2A b^4 - 6A a^2 b^2 + B a^2 b^2 - A a b^3 + 4B a^3 b)) / ((a^2 b - a^3) * (a + b)^2) + (\tan(c/2 + (d*x)/2) * (2A b^4 - 6A a^2 b^2 - B a^2 b^2 + A a b^3 + 4B a^3 b)) / ((a + b) * (a^4 - 2a^3 b + a^2 b^2))) / (d * (2a b - \tan(c/2 + (d*x)/2)^2 * (2a^2 - 2b^2) + \tan(c/2 + (d*x)/2)^4 * (a^2 - 2a b + b^2) + a^2 + b^2)) + (\operatorname{atan}((((a + b)^5 * (a - b)^5)^{(1/2)} * ((8 \tan(c/2 + (d*x)/2) * (4A^2 a^{10} + 8A^2 b^{10} + 4B^2 a^{10} - 8A^2 a b^9 - 8A^2 a^9 b - 32A^2 a^2 b^8 + 32A^2 a^3 b^7 + 57A^2 a^4 b^6 - 48A^2 a^5 b^5 - 52A^2 a^6 b^4 + 32A^2 a^7 b^3 + 24A^2 a^8 b^2 + B^2 a^6 b^4 + 4B^2 a^8 b^2 - 24AB a^9 b - 4AB a^3 b^7 + 2AB a^5 b^5 + 8AB a^7 b^3)) / (a^{10} b + a^{11} - a^4 b^7 - a^5 b^6 + 3a^6 b^5 + 3a^7 b^4 - 3a^8 b^3 - 3a^9 b^2) + (((a + b)^5 * (a - b)^5)^{(1/2)} * ((8(4A a^{15} + 4B a^{15} - 4A a^6 b^9 + 2A a^7 b^8 + 18A a^8 b^7 - 4A a^9 b^6 - 36A a^{10} b^5 + 6A a^{11} b^4 + 34
\end{aligned}$$

$$\begin{aligned}
& *A*a^{12}*b^3 - 8*A*a^{13}*b^2 - 2*B*a^8*b^7 + 2*B*a^9*b^6 + 6*B*a^{12}*b^3 - 6*B \\
& *a^{13}*b^2 - 12*A*a^{14}*b - 4*B*a^{14}*b)) / (a^{12}*b + a^{13} - a^6*b^7 - a^7*b^6 + \\
& 3*a^8*b^5 + 3*a^9*b^4 - 3*a^{10}*b^3 - 3*a^{11}*b^2) - (4*\tan(c/2 + (d*x)/2)* \\
& (a + b)^5*(a - b)^5)^{(1/2)}*(2*B*a^5 - 2*A*b^5 + 5*A*a^2*b^3 + B*a^3*b^2 - 6 \\
& *A*a^4*b)*(8*a^{15}*b - 8*a^6*b^{10} + 8*a^7*b^9 + 32*a^8*b^8 - 32*a^9*b^7 - 48 \\
& *a^{10}*b^6 + 48*a^{11}*b^5 + 32*a^{12}*b^4 - 32*a^{13}*b^3 - 8*a^{14}*b^2)) / ((a^{13} - \\
& a^3*b^{10} + 5*a^5*b^8 - 10*a^7*b^6 + 10*a^9*b^4 - 5*a^{11}*b^2)*(a^{10}*b + a^1 \\
& 1 - a^4*b^7 - a^5*b^6 + 3*a^6*b^5 + 3*a^7*b^4 - 3*a^8*b^3 - 3*a^9*b^2))) * (2 \\
& *B*a^5 - 2*A*b^5 + 5*A*a^2*b^3 + B*a^3*b^2 - 6*A*a^4*b) / (2*(a^{13} - a^3*b^1 \\
& 0 + 5*a^5*b^8 - 10*a^7*b^6 + 10*a^9*b^4 - 5*a^{11}*b^2))) * (2*B*a^5 - 2*A*b^5 \\
& + 5*A*a^2*b^3 + B*a^3*b^2 - 6*A*a^4*b)*1i) / (2*(a^{13} - a^3*b^{10} + 5*a^5*b^8 \\
& - 10*a^7*b^6 + 10*a^9*b^4 - 5*a^{11}*b^2)) + (((a + b)^5*(a - b)^5)^{(1/2)}*((8 \\
& *tan(c/2 + (d*x)/2)*(4*A^2*a^{10} + 8*A^2*b^{10} + 4*B^2*a^{10} - 8*A^2*a*b^9 - 8 \\
& *A^2*a^9*b - 32*A^2*a^2*b^8 + 32*A^2*a^3*b^7 + 57*A^2*a^4*b^6 - 48*A^2*a^5* \\
& b^5 - 52*A^2*a^6*b^4 + 32*A^2*a^7*b^3 + 24*A^2*a^8*b^2 + B^2*a^6*b^4 + 4*B^ \\
& 2*a^8*b^2 - 24*A*B*a^9*b - 4*A*B*a^3*b^7 + 2*A*B*a^5*b^5 + 8*A*B*a^7*b^3)) / \\
& (a^{10}*b + a^{11} - a^4*b^7 - a^5*b^6 + 3*a^6*b^5 + 3*a^7*b^4 - 3*a^8*b^3 - 3* \\
& a^9*b^2) - (((a + b)^5*(a - b)^5)^{(1/2)}*((8*(4*A*a^{15} + 4*B*a^{15} - 4*A*a^6* \\
& b^9 + 2*A*a^7*b^8 + 18*A*a^8*b^7 - 4*A*a^9*b^6 - 36*A*a^{10}*b^5 + 6*A*a^{11}*b \\
& ^4 + 34*A*a^{12}*b^3 - 8*A*a^{13}*b^2 - 2*B*a^8*b^7 + 2*B*a^9*b^6 + 6*B*a^{12}*b^ \\
& 3 - 6*B*a^{13}*b^2 - 12*A*a^{14}*b - 4*B*a^{14}*b)) / (a^{12}*b + a^{13} - a^6*b^7 - a^ \\
& 7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^{10}*b^3 - 3*a^{11}*b^2) + (4*\tan(c/2 + (d* \\
& x)/2)*((a + b)^5*(a - b)^5)^{(1/2)}*(2*B*a^5 - 2*A*b^5 + 5*A*a^2*b^3 + B*a^3* \\
& b^2 - 6*A*a^4*b)*(8*a^{15}*b - 8*a^6*b^{10} + 8*a^7*b^9 + 32*a^8*b^8 - 32*a^9*b \\
& ^7 - 48*a^{10}*b^6 + 48*a^{11}*b^5 + 32*a^{12}*b^4 - 32*a^{13}*b^3 - 8*a^{14}*b^2)) / (\\
& (a^{13} - a^3*b^{10} + 5*a^5*b^8 - 10*a^7*b^6 + 10*a^9*b^4 - 5*a^{11}*b^2)*(a^{10}* \\
& b + a^{11} - a^4*b^7 - a^5*b^6 + 3*a^6*b^5 + 3*a^7*b^4 - 3*a^8*b^3 - 3*a^9*b^ \\
& 2))) * (2*B*a^5 - 2*A*b^5 + 5*A*a^2*b^3 + B*a^3*b^2 - 6*A*a^4*b) / (2*(a^{13} - \\
& a^3*b^{10} + 5*a^5*b^8 - 10*a^7*b^6 + 10*a^9*b^4 - 5*a^{11}*b^2))) * (2*B*a^5 - 2 \\
& *A*b^5 + 5*A*a^2*b^3 + B*a^3*b^2 - 6*A*a^4*b)*1i) / (2*(a^{13} - a^3*b^{10} + 5*a \\
& ^5*b^8 - 10*a^7*b^6 + 10*a^9*b^4 - 5*a^{11}*b^2))) / ((16*(4*A^3*b^9 + 4*A*B^2* \\
& a^9 - 4*A^2*B*a^9 - 2*A^3*a*b^8 + 12*A^3*a^8*b - 18*A^3*a^2*b^7 + 13*A^3*a^ \\
& 3*b^6 + 36*A^3*a^4*b^5 - 26*A^3*a^5*b^4 - 34*A^3*a^6*b^3 + 24*A^3*a^7*b^2 - \\
& 20*A^2*B*a^8*b + A*B^2*a^5*b^4 + 4*A*B^2*a^7*b^2 - 2*A^2*B*a^2*b^7 - 2*A^2 \\
& *B*a^3*b^6 + 2*A^2*B*a^4*b^5 + 2*A^2*B*a^6*b^3 + 6*A^2*B*a^7*b^2)) / (a^{12}*b \\
& + a^{13} - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^{10}*b^3 - 3*a^{11}*b^ \\
& 2) - (((a + b)^5*(a - b)^5)^{(1/2)}*((8*\tan(c/2 + (d*x)/2)*(4*A^2*a^{10} + 8*A^ \\
& 2*b^{10} + 4*B^2*a^{10} - 8*A^2*a*b^9 - 8*A^2*a^9*b - 32*A^2*a^2*b^8 + 32*A^2*a \\
& ^3*b^7 + 57*A^2*a^4*b^6 - 48*A^2*a^5*b^5 - 52*A^2*a^6*b^4 + 32*A^2*a^7*b^3 \\
& + 24*A^2*a^8*b^2 + B^2*a^6*b^4 + 4*B^2*a^8*b^2 - 24*A*B*a^9*b - 4*A*B*a^3*b \\
& ^7 + 2*A*B*a^5*b^5 + 8*A*B*a^7*b^3)) / (a^{10}*b + a^{11} - a^4*b^7 - a^5*b^6 + 3 \\
& *a^6*b^5 + 3*a^7*b^4 - 3*a^8*b^3 - 3*a^9*b^2) + (((a + b)^5*(a - b)^5)^{(1/2)} \\
&)*((8*(4*A*a^{15} + 4*B*a^{15} - 4*A*a^6*b^9 + 2*A*a^7*b^8 + 18*A*a^8*b^7 - 4*A \\
& *a^9*b^6 - 36*A*a^{10}*b^5 + 6*A*a^{11}*b^4 + 34*A*a^{12}*b^3 - 8*A*a^{13}*b^2 - 2* \\
& B*a^8*b^7 + 2*B*a^9*b^6 + 6*B*a^{12}*b^3 - 6*B*a^{13}*b^2 - 12*A*a^{14}*b - 4*B*a
\end{aligned}$$

$$\begin{aligned} & \left(\frac{a^{14}b}{(a^{12}b + a^{13} - a^6b^7 - a^7b^6 + 3a^8b^5 + 3a^9b^4 - 3a^{10}b^3 - 3a^{11}b^2) - (4\tan(c/2 + (d*x)/2)*((a+b)^5*(a-b)^5)^{(1/2)}*(2B*a^5 - 2A*b^5 + 5A*a^2*b^3 + B*a^3*b^2 - 6A*a^4*b)} \right) * (8a^{15}b - 8a^6b^{10} + 8a^7b^9 + 32a^8b^8 - 32a^9b^7 - 48a^{10}b^6 + 48a^{11}b^5 + 32a^{12}b^4 - 32a^{13}b^3 - 8a^{14}b^2) / ((a^{13} - a^3b^{10} + 5a^5b^8 - 10a^7b^6 + 10a^9b^4 - 5a^{11}b^2) * (a^{10}b + a^{11} - a^4b^7 - a^5b^6 + 3a^6b^5 + 3a^7b^4 - 3a^8b^3 - 3a^9b^2)) * (2B*a^5 - 2A*b^5 + 5A*a^2*b^3 + B*a^3*b^2 - 6A*a^4*b) / (2*(a^{13} - a^3b^{10} + 5a^5b^8 - 10a^7b^6 + 10a^9b^4 - 5a^{11}b^2)) * (2B*a^5 - 2A*b^5 + 5A*a^2*b^3 + B*a^3*b^2 - 6A*a^4*b) / (2*(a^{13} - a^3b^{10} + 5a^5b^8 - 10a^7b^6 + 10a^9b^4 - 5a^{11}b^2)) + (((a+b)^5*(a-b)^5)^{(1/2)} * ((8*\tan(c/2 + (d*x)/2) * (4*A^2*a^{10} + 8*A^2*b^{10} + 4*B^2*a^{10} - 8*A^2*a*b^9 - 8*A^2*a^9*b - 32*A^2*a^2*b^8 + 32*A^2*a^3*b^7 + 57*A^2*a^4*b^6 - 48*A^2*a^5*b^5 - 52*A^2*a^6*b^4 + 32*A^2*a^7*b^3 + 24*A^2*a^8*b^2 + B^2*a^6*b^4 + 4*B^2*a^8*b^2 - 24*A*B*a^9*b - 4*A*B*a^3*b^7 + 2*A*B*a^5*b^5 + 8*A*B*a^7*b^3)) / (a^{10}b + a^{11} - a^4b^7 - a^5b^6 + 3a^6b^5 + 3a^7b^4 - 3a^8b^3 - 3a^9b^2) - (((a+b)^5*(a-b)^5)^{(1/2)} * ((8*(4*A*a^{15} + 4*B*a^{15} - 4*A*a^6*b^9 + 2*A*a^7*b^8 + 18*A*a^8*b^7 - 4*A*a^9*b^6 - 36*A*a^{10}b^5 + 6*A*a^{11}b^4 + 34*A*a^{12}b^3 - 8*A*a^{13}b^2 - 2*B*a^8*b^7 + 2*B*a^9*b^6 + 6*B*a^{12}b^3 - 6*B*a^{13}b^2 - 12*A*a^{14}b - 4*B*a^{14}b)) / (a^{12}b + a^{13} - a^6b^7 - a^7b^6 + 3a^8b^5 + 3a^9b^4 - 3a^{10}b^3 - 3a^{11}b^2) + (4*\tan(c/2 + (d*x)/2) * ((a+b)^5*(a-b)^5)^{(1/2)} * (2B*a^5 - 2A*b^5 + 5A*a^2*b^3 + B*a^3*b^2 - 6A*a^4*b) * (8a^{15}b - 8a^6b^{10} + 8a^7b^9 + 32a^8b^8 - 32a^9b^7 - 48a^{10}b^6 + 48a^{11}b^5 + 32a^{12}b^4 - 32a^{13}b^3 - 8a^{14}b^2) / ((a^{13} - a^3b^{10} + 5a^5b^8 - 10a^7b^6 + 10a^9b^4 - 5a^{11}b^2) * (a^{10}b + a^{11} - a^4b^7 - a^5b^6 + 3a^6b^5 + 3a^7b^4 - 3a^8b^3 - 3a^9b^2)) * (2B*a^5 - 2A*b^5 + 5A*a^2*b^3 + B*a^3*b^2 - 6A*a^4*b) / (2*(a^{13} - a^3b^{10} + 5a^5b^8 - 10a^7b^6 + 10a^9b^4 - 5a^{11}b^2)) * (2B*a^5 - 2A*b^5 + 5A*a^2*b^3 + B*a^3*b^2 - 6A*a^4*b) / (2*(a^{13} - a^3b^{10} + 5a^5b^8 - 10a^7b^6 + 10a^9b^4 - 5a^{11}b^2)) * ((a+b)^5*(a-b)^5)^{(1/2)} * (2B*a^5 - 2A*b^5 + 5A*a^2*b^3 + B*a^3*b^2 - 6A*a^4*b) * i) / (d*(a^{13} - a^3b^{10} + 5a^5b^8 - 10a^7b^6 + 10a^9b^4 - 5a^{11}b^2)) \end{aligned}$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \sec(c + dx)}{(a + b \sec(c + dx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))**3,x)

[Out] Integral((A + B*sec(c + d*x))/(a + b*sec(c + d*x))**3, x)

$$3.334 \quad \int \frac{\cos(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^3} dx$$

Optimal. Leaf size=290

$$-\frac{x(3Ab - aB)}{a^4} + \frac{b(Ab - aB) \sin(c + dx)}{2ad(a^2 - b^2)(a + b \sec(c + dx))^2} + \frac{b(-4a^3B + 6a^2Ab + ab^2B - 3Ab^3) \sin(c + dx)}{2a^2d(a^2 - b^2)^2(a + b \sec(c + dx))} + \frac{(2a^4A + 5a^3bB - 2a^2b^2B - 2ab^3B + 6Ab^4) \sin(c + dx)}{a^4d(a - b)^{5/2}(a + b)^5}$$

[Out] $-(3A*b-B*a)*x/a^4+b*(12*A*a^4*b-15*A*a^2*b^3+6*A*b^5-6*B*a^5+5*B*a^3*b^2-2*B*a*b^4)*\operatorname{arctanh}((a-b)^{(1/2)}*\tan(1/2*d*x+1/2*c)/(a+b)^{(1/2)})/a^4/(a-b)^{(5/2)}/(a+b)^{(5/2)}/d+1/2*(2*A*a^4-11*A*a^2*b^2+6*A*b^4+5*B*a^3*b-2*B*a*b^3)*\sin(d*x+c)/a^3/(a^2-b^2)^2/d+1/2*b*(A*b-B*a)*\sin(d*x+c)/a/(a^2-b^2)/d/(a+b*\sec(d*x+c))^2+1/2*b*(6*A*a^2*b-3*A*b^3-4*B*a^3+B*a*b^2)*\sin(d*x+c)/a^2/(a^2-b^2)^2/d/(a+b*\sec(d*x+c))$

Rubi [A] time = 1.53, antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {4030, 4100, 4104, 3919, 3831, 2659, 208}

$$\frac{(-11a^2Ab^2 + 2a^4A + 5a^3bB - 2ab^3B + 6Ab^4) \sin(c + dx)}{2a^3d(a^2 - b^2)^2} + \frac{b(-15a^2Ab^3 + 12a^4Ab + 5a^3b^2B - 6a^5B - 2ab^4B + 6Ab^4) \sin(c + dx)}{a^4d(a - b)^{5/2}(a + b)^5}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(\operatorname{Cos}[c + d*x]*(A + B*\operatorname{Sec}[c + d*x]))/(a + b*\operatorname{Sec}[c + d*x])^3, x]$

[Out] $-(((3A*b - a*B)*x)/a^4) + (b*(12*a^4*A*b - 15*a^2*A*b^3 + 6*A*b^5 - 6*a^5*B + 5*a^3*b^2*B - 2*a*b^4*B)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a - b]*\operatorname{Tan}[(c + d*x)/2])/(\operatorname{Sqrt}[a + b])])/(a^4*(a - b)^{(5/2)}*(a + b)^{(5/2)}*d) + ((2*a^4*A - 11*a^2*A*b^2 + 6*A*b^4 + 5*a^3*b*B - 2*a*b^3*B)*\operatorname{Sin}[c + d*x])/(2*a^3*(a^2 - b^2)^2*d) + (b*(A*b - a*B)*\operatorname{Sin}[c + d*x])/(2*a*(a^2 - b^2)*d*(a + b*\operatorname{Sec}[c + d*x])^2) + (b*(6*a^2*A*b - 3*A*b^3 - 4*a^3*B + a*b^2*B)*\operatorname{Sin}[c + d*x])/(2*a^2*(a^2 - b^2)^2*d*(a + b*\operatorname{Sec}[c + d*x]))$

Rule 208

$\operatorname{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[-(a/b), 2]*\operatorname{ArcTanh}[x/\operatorname{Rt}[-(a/b), 2]])/a, x] /;$ FreeQ[{a, b}, x] && NegQ[a/b]

Rule 2659

```
Int[((a_) + (b_)*sin[Pi/2 + (c_) + (d_)*(x_)])^(-1), x_Symbol] := With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3831

```
Int[csc[(e_) + (f_)*(x_)]/(csc[(e_) + (f_)*(x_)]*(b_) + (a_)), x_Symbol] := Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3919

```
Int[(csc[(e_) + (f_)*(x_)]*(d_) + (c_))/(csc[(e_) + (f_)*(x_)]*(b_) + (a_)), x_Symbol] := Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 4030

```
Int[(csc[(e_) + (f_)*(x_)]*(d_))^(n_)*(csc[(e_) + (f_)*(x_)]*(b_) + (a_))^(m_)*(csc[(e_) + (f_)*(x_)]*(B_) + (A_)), x_Symbol] := Simp[(b*(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[A*(a^2*(m + 1) - b^2*(m + n + 1)) + a*b*B*n - a*(A*b - a*B)*(m + 1)*Csc[e + f*x] + b*(A*b - a*B)*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4100

```
Int[((A_) + csc[(e_) + (f_)*(x_)]*(B_) + csc[(e_) + (f_)*(x_)]^2*(C_))*(csc[(e_) + (f_)*(x_)]*(d_))^(n_)*(csc[(e_) + (f_)*(x_)]*(b_) + (a_))^(m_), x_Symbol] := Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4104

```
Int[((A_) + csc[(e_) + (f_)*(x_)]*(B_) + csc[(e_) + (f_)*(x_)]^2*(C_)
```


$$\text{Simp}[(A \cdot \text{Cot}[e + f \cdot x] \cdot (a + b \cdot \text{Csc}[e + f \cdot x])^{m+1} \cdot (d \cdot \text{Csc}[e + f \cdot x])^n) / (a \cdot f \cdot n), x] + \text{Dist}[1 / (a \cdot d \cdot n), \text{Int}[(a + b \cdot \text{Csc}[e + f \cdot x])^m \cdot (d \cdot \text{Csc}[e + f \cdot x])^{n+1} \cdot \text{Simp}[a \cdot B \cdot n - A \cdot b \cdot (m + n + 1) + a \cdot (A + A \cdot n + C \cdot n) \cdot \text{Csc}[e + f \cdot x] + A \cdot b \cdot (m + n + 2) \cdot \text{Csc}[e + f \cdot x]^2, x], x], x] /;$$

$$\text{FreeQ}[\{a, b, d, e, f, A, B, C, m\}, x] \ \&\& \ \text{NeQ}[a^2 - b^2, 0] \ \&\& \ \text{LeQ}[n, -1]$$

Rubi steps

$$\begin{aligned}
 \int \frac{\cos(c + dx)(A + B \sec(c + dx))}{(a + b \sec(c + dx))^3} dx &= \frac{b(Ab - aB) \sin(c + dx)}{2a(a^2 - b^2)d(a + b \sec(c + dx))^2} - \frac{\int \frac{\cos(c + dx)(-2a^2A + 3Ab^2 - abB + 2a(Ab - aB))}{(a + b \sec(c + dx))^2} dx}{2a(a^2 - b^2)} \\
 &= \frac{b(Ab - aB) \sin(c + dx)}{2a(a^2 - b^2)d(a + b \sec(c + dx))^2} + \frac{b(6a^2Ab - 3Ab^3 - 4a^3B + ab^2B) \sin(c + dx)}{2a^2(a^2 - b^2)^2d(a + b \sec(c + dx))} \\
 &= \frac{(2a^4A - 11a^2Ab^2 + 6Ab^4 + 5a^3bB - 2ab^3B) \sin(c + dx)}{2a^3(a^2 - b^2)^2d} + \frac{b(Ab - aB) \sin(c + dx)}{2a(a^2 - b^2)} \\
 &= -\frac{(3Ab - aB)x}{a^4} + \frac{(2a^4A - 11a^2Ab^2 + 6Ab^4 + 5a^3bB - 2ab^3B) \sin(c + dx)}{2a^3(a^2 - b^2)^2d} \\
 &= -\frac{(3Ab - aB)x}{a^4} + \frac{(2a^4A - 11a^2Ab^2 + 6Ab^4 + 5a^3bB - 2ab^3B) \sin(c + dx)}{2a^3(a^2 - b^2)^2d} \\
 &= -\frac{(3Ab - aB)x}{a^4} + \frac{(2a^4A - 11a^2Ab^2 + 6Ab^4 + 5a^3bB - 2ab^3B) \sin(c + dx)}{2a^3(a^2 - b^2)^2d} \\
 &= -\frac{(3Ab - aB)x}{a^4} + \frac{b(12a^4Ab - 15a^2Ab^3 + 6Ab^5 - 6a^5B + 5a^3b^2B - 2ab^4)}{a^4(a - b)^{5/2}(a + b)^{5/2}d}
 \end{aligned}$$

Mathematica [A] time = 2.07, size = 306, normalized size = 1.06

$$\sec^2(c + dx)(a \cos(c + dx) + b)(A + B \sec(c + dx)) \left(\frac{ab^2(6a^3B - 8a^2Ab - 3ab^2B + 5Ab^3) \sin(c + dx)(a \cos(c + dx) + b)}{(a-b)^2(a+b)^2} - \frac{2b(-6a^5B + 12a^4A}{(a-b)^2(a+b)^2} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^3,x]

[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]^2*(A + B*Sec[c + d*x])*(2*(-3*A*b + a*B)*(c + d*x)*(b + a*Cos[c + d*x])^2 - (2*b*(12*a^4*A*b - 15*a^2*A*b^3 + 6*A*b^5 - 6*a^5*B + 5*a^3*b^2*B - 2*a*b^4*B)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]*(b + a*Cos[c + d*x])^2)/(a^2 - b^2)^(5/2) + (a*b^3*(A*b - a*B)*Sin[c + d*x])/((a - b)*(a + b)) + (a*b^2*(-8*a^2*A*b + 5*A*b^3 + 6*a^3*B - 3*a*b^2*B)*(b + a*Cos[c + d*x])*Sin[c + d*x])/((a - b)^2*(a + b)^2) + 2*a*A*(b + a*Cos[c + d*x])^2*Ssin[c + d*x]))/(2*a^4*d*(B + A*Cos[c + d*x])*(a + b*Sec[c + d*x])^3)

fricas [B] time = 0.62, size = 1568, normalized size = 5.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm="fricas")

[Out] [1/4*(4*(B*a^9 - 3*A*a^8*b - 3*B*a^7*b^2 + 9*A*a^6*b^3 + 3*B*a^5*b^4 - 9*A*a^4*b^5 - B*a^3*b^6 + 3*A*a^2*b^7)*d*x*cos(d*x + c)^2 + 8*(B*a^8*b - 3*A*a^7*b^2 - 3*B*a^6*b^3 + 9*A*a^5*b^4 + 3*B*a^4*b^5 - 9*A*a^3*b^6 - B*a^2*b^7 + 3*A*a*b^8)*d*x*cos(d*x + c) + 4*(B*a^7*b^2 - 3*A*a^6*b^3 - 3*B*a^5*b^4 + 9*A*a^4*b^5 + 3*B*a^3*b^6 - 9*A*a^2*b^7 - B*a*b^8 + 3*A*b^9)*d*x - (6*B*a^5*b^3 - 12*A*a^4*b^4 - 5*B*a^3*b^5 + 15*A*a^2*b^6 + 2*B*a*b^7 - 6*A*b^8 + (6*B*a^7*b - 12*A*a^6*b^2 - 5*B*a^5*b^3 + 15*A*a^4*b^4 + 2*B*a^3*b^5 - 6*A*a^2*b^6)*cos(d*x + c)^2 + 2*(6*B*a^6*b^2 - 12*A*a^5*b^3 - 5*B*a^4*b^4 + 15*A*a^3*b^5 + 2*B*a^2*b^6 - 6*A*a*b^7)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + 2*(2*A*a^7*b^2 + 5*B*a^6*b^3 - 13*A*a^5*b^4 - 7*B*a^4*b^5 + 17*A*a^3*b^6 + 2*B*a^2*b^7 - 6*A*a*b^8 + 2*(A*a^9 - 3*A*a^7*b^2 + 3*A*a^5*b^4 - A*a^3*b^6)*cos(d*x + c)^2 + (4*A*a^8*b + 6*B*a^7*b^2 - 20*A*a^6*b^3 - 9*B*a^5*b^4 + 25*A*a^4*b^5 + 3*B*a^3*b^6 - 9*A*a^2*b^7)*cos(d*x + c))*sin(d*x + c))/((a^12 - 3*a^10*b^2 + 3*a^8*b^4 - a^6*b^6)*d*cos(d*x + c)^2 + 2*(a^

$11*b - 3*a^9*b^3 + 3*a^7*b^5 - a^5*b^7)*d*\cos(dx + c) + (a^{10}*b^2 - 3*a^8*b^4 + 3*a^6*b^6 - a^4*b^8)*d), 1/2*(2*(B*a^9 - 3*A*a^8*b - 3*B*a^7*b^2 + 9*A*a^6*b^3 + 3*B*a^5*b^4 - 9*A*a^4*b^5 - B*a^3*b^6 + 3*A*a^2*b^7)*d*x*\cos(dx + c)^2 + 4*(B*a^8*b - 3*A*a^7*b^2 - 3*B*a^6*b^3 + 9*A*a^5*b^4 + 3*B*a^4*b^5 - 9*A*a^3*b^6 - B*a^2*b^7 + 3*A*a*b^8)*d*x*\cos(dx + c) + 2*(B*a^7*b^2 - 3*A*a^6*b^3 - 3*B*a^5*b^4 + 9*A*a^4*b^5 + 3*B*a^3*b^6 - 9*A*a^2*b^7 - B*a*b^8 + 3*A*b^9)*d*x - (6*B*a^5*b^3 - 12*A*a^4*b^4 - 5*B*a^3*b^5 + 15*A*a^2*b^6 + 2*B*a*b^7 - 6*A*b^8 + (6*B*a^7*b - 12*A*a^6*b^2 - 5*B*a^5*b^3 + 15*A*a^4*b^4 + 2*B*a^3*b^5 - 6*A*a^2*b^6)*\cos(dx + c)^2 + 2*(6*B*a^6*b^2 - 12*A*a^5*b^3 - 5*B*a^4*b^4 + 15*A*a^3*b^5 + 2*B*a^2*b^6 - 6*A*a*b^7)*\cos(dx + c)))*\sqrt{-a^2 + b^2}*\arctan(-\sqrt{-a^2 + b^2}*(b*\cos(dx + c) + a)/((a^2 - b^2)*\sin(dx + c))) + (2*A*a^7*b^2 + 5*B*a^6*b^3 - 13*A*a^5*b^4 - 7*B*a^4*b^5 + 17*A*a^3*b^6 + 2*B*a^2*b^7 - 6*A*a*b^8 + 2*(A*a^9 - 3*A*a^7*b^2 + 3*A*a^5*b^4 - A*a^3*b^6)*\cos(dx + c)^2 + (4*A*a^8*b + 6*B*a^7*b^2 - 20*A*a^6*b^3 - 9*B*a^5*b^4 + 25*A*a^4*b^5 + 3*B*a^3*b^6 - 9*A*a^2*b^7)*\cos(dx + c))*\sin(dx + c))/((a^{12} - 3*a^{10}*b^2 + 3*a^8*b^4 - a^6*b^6)*d*\cos(dx + c)^2 + 2*(a^{11}*b - 3*a^9*b^3 + 3*a^7*b^5 - a^5*b^7)*d*\cos(dx + c) + (a^{10}*b^2 - 3*a^8*b^4 + 3*a^6*b^6 - a^4*b^8)*d)]$

giac [A] time = 0.40, size = 546, normalized size = 1.88

$$\frac{(6Ba^5b - 12Aa^4b^2 - 5Ba^3b^3 + 15Aa^2b^4 + 2Bab^5 - 6Ab^6) \left(\pi \left[\frac{dx+c}{2\pi} + \frac{1}{2} \right] \operatorname{sgn}(-2a+2b) + \arctan \left(-\frac{a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - b \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{\sqrt{-a^2+b^2}} \right) \right)}{(a^8 - 2a^6b^2 + a^4b^4) \sqrt{-a^2+b^2}} + \frac{6Ba^4b^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{\sqrt{-a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(dx+c)*(A+B*sec(dx+c))/(a+b*sec(dx+c))^3,x, algorithm="giac")

[Out] -((6*B*a^5*b - 12*A*a^4*b^2 - 5*B*a^3*b^3 + 15*A*a^2*b^4 + 2*B*a*b^5 - 6*A*b^6)*(pi*floor(1/2*(dx + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^8 - 2*a^6*b^2 + a^4*b^4)*sqrt(-a^2 + b^2)) + (6*B*a^4*b^2*tan(1/2*d*x + 1/2*c)^3 - 8*A*a^3*b^3*tan(1/2*d*x + 1/2*c)^3 - 5*B*a^3*b^3*tan(1/2*d*x + 1/2*c)^3 + 7*A*a^2*b^4*tan(1/2*d*x + 1/2*c)^3 - 3*B*a^2*b^4*tan(1/2*d*x + 1/2*c)^3 + 5*A*a*b^5*tan(1/2*d*x + 1/2*c)^3 + 2*B*a*b^5*tan(1/2*d*x + 1/2*c)^3 - 4*A*b^6*tan(1/2*d*x + 1/2*c)^3 - 6*B*a^4*b^2*tan(1/2*d*x + 1/2*c) + 8*A*a^3*b^3*tan(1/2*d*x + 1/2*c) - 5*B*a^3*b^3*tan(1/2*d*x + 1/2*c) + 7*A*a^2*b^4*tan(1/2*d*x + 1/2*c) + 3*B*a^2*b^4*tan(1/2*d*x + 1/2*c) - 5*A*a*b^5*tan(1/2*d*x + 1/2*c) + 2*B*a*b^5*tan(1/2*d*x + 1/2*c) - 4*A*b^6*tan(1/2*d*x + 1/2*c))/((a^7 - 2*a^5*b^2 + a^3*b^4)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^2) - (B*a - 3*A*b)*(dx + c)/a^4 - 2*A*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 + 1)*a^3))/d

maple [B] time = 1.18, size = 1349, normalized size = 4.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(\cos(dx+c) \cdot (A+B \cdot \sec(dx+c)) / (a+b \cdot \sec(dx+c))^3, x)$

[Out]
$$\begin{aligned} & 8/d/a / (a \cdot \tan(1/2 \cdot dx+1/2 \cdot c)^2 - \tan(1/2 \cdot dx+1/2 \cdot c)^2 \cdot b - a - b)^2 \cdot b^3 / (a-b) / (a^2 + 2 \cdot a \cdot b + b^2) \cdot \tan(1/2 \cdot dx+1/2 \cdot c)^3 \cdot A + 1/d/a^2 / (a \cdot \tan(1/2 \cdot dx+1/2 \cdot c)^2 - \tan(1/2 \cdot dx+1/2 \cdot c)^2 \cdot b - a - b)^2 \cdot b^4 / (a-b) / (a^2 + 2 \cdot a \cdot b + b^2) \cdot \tan(1/2 \cdot dx+1/2 \cdot c)^3 \cdot A - 4/d \cdot b^5/a^3 / (a \cdot \tan(1/2 \cdot dx+1/2 \cdot c)^2 - \tan(1/2 \cdot dx+1/2 \cdot c)^2 \cdot b - a - b)^2 / (a-b) / (a^2 + 2 \cdot a \cdot b + b^2) \cdot \tan(1/2 \cdot dx+1/2 \cdot c)^3 \cdot A - 6/d / (a \cdot \tan(1/2 \cdot dx+1/2 \cdot c)^2 - \tan(1/2 \cdot dx+1/2 \cdot c)^2 \cdot b - a - b)^2 \cdot b^2 / (a-b) / (a^2 + 2 \cdot a \cdot b + b^2) \cdot \tan(1/2 \cdot dx+1/2 \cdot c)^3 \cdot B - 1/d \cdot b^3/a / (a \cdot \tan(1/2 \cdot dx+1/2 \cdot c)^2 - \tan(1/2 \cdot dx+1/2 \cdot c)^2 \cdot b - a - b)^2 / (a-b) / (a^2 + 2 \cdot a \cdot b + b^2) \cdot \tan(1/2 \cdot dx+1/2 \cdot c)^3 \cdot B + 2/d \cdot b^4/a^2 / (a \cdot \tan(1/2 \cdot dx+1/2 \cdot c)^2 - \tan(1/2 \cdot dx+1/2 \cdot c)^2 \cdot b - a - b)^2 / (a-b) / (a^2 + 2 \cdot a \cdot b + b^2) \cdot \tan(1/2 \cdot dx+1/2 \cdot c)^3 \cdot B - 8/d/a / (a \cdot \tan(1/2 \cdot dx+1/2 \cdot c)^2 - \tan(1/2 \cdot dx+1/2 \cdot c)^2 \cdot b - a - b)^2 \cdot b^3 / (a+b) / (a-b)^2 \cdot \tan(1/2 \cdot dx+1/2 \cdot c) \cdot A + 1/d/a^2 / (a \cdot \tan(1/2 \cdot dx+1/2 \cdot c)^2 - \tan(1/2 \cdot dx+1/2 \cdot c)^2 \cdot b - a - b)^2 \cdot b^4 / (a+b) / (a-b)^2 \cdot \tan(1/2 \cdot dx+1/2 \cdot c) \cdot A + 4/d \cdot b^5/a^3 / (a \cdot \tan(1/2 \cdot dx+1/2 \cdot c)^2 - \tan(1/2 \cdot dx+1/2 \cdot c)^2 \cdot b - a - b)^2 / (a+b) / (a-b)^2 \cdot \tan(1/2 \cdot dx+1/2 \cdot c) \cdot A + 6/d / (a \cdot \tan(1/2 \cdot dx+1/2 \cdot c)^2 - \tan(1/2 \cdot dx+1/2 \cdot c)^2 \cdot b - a - b)^2 \cdot b^2 / (a+b) / (a-b)^2 \cdot \tan(1/2 \cdot dx+1/2 \cdot c) \cdot B - 1/d \cdot b^3/a / (a \cdot \tan(1/2 \cdot dx+1/2 \cdot c)^2 - \tan(1/2 \cdot dx+1/2 \cdot c)^2 \cdot b - a - b)^2 / (a+b) / (a-b)^2 \cdot \tan(1/2 \cdot dx+1/2 \cdot c) \cdot B - 2/d \cdot b^4/a^2 / (a \cdot \tan(1/2 \cdot dx+1/2 \cdot c)^2 - \tan(1/2 \cdot dx+1/2 \cdot c)^2 \cdot b - a - b)^2 / (a+b) / (a-b)^2 \cdot \tan(1/2 \cdot dx+1/2 \cdot c) \cdot B + 12/d \cdot b^2 / (a^4 - 2 \cdot a^2 \cdot b^2 + b^4) / ((a-b) \cdot (a+b))^{1/2} \cdot \text{arctanh}(\tan(1/2 \cdot dx+1/2 \cdot c) \cdot (a-b) / ((a-b) \cdot (a+b))^{1/2}) \cdot A - 15/d \cdot b^4/a^2 / (a^4 - 2 \cdot a^2 \cdot b^2 + b^4) / ((a-b) \cdot (a+b))^{1/2} \cdot \text{arctanh}(\tan(1/2 \cdot dx+1/2 \cdot c) \cdot (a-b) / ((a-b) \cdot (a+b))^{1/2}) \cdot A + 6/d \cdot b^6/a^4 / (a^4 - 2 \cdot a^2 \cdot b^2 + b^4) / ((a-b) \cdot (a+b))^{1/2} \cdot \text{arctanh}(\tan(1/2 \cdot dx+1/2 \cdot c) \cdot (a-b) / ((a-b) \cdot (a+b))^{1/2}) \cdot A - 6/d \cdot b / (a^4 - 2 \cdot a^2 \cdot b^2 + b^4) / ((a-b) \cdot (a+b))^{1/2} \cdot \text{arctanh}(\tan(1/2 \cdot dx+1/2 \cdot c) \cdot (a-b) / ((a-b) \cdot (a+b))^{1/2}) \cdot a \cdot B + 5/d \cdot b^3/a / (a^4 - 2 \cdot a^2 \cdot b^2 + b^4) / ((a-b) \cdot (a+b))^{1/2} \cdot \text{arctanh}(\tan(1/2 \cdot dx+1/2 \cdot c) \cdot (a-b) / ((a-b) \cdot (a+b))^{1/2}) \cdot B - 2/d \cdot b^5/a^3 / (a^4 - 2 \cdot a^2 \cdot b^2 + b^4) / ((a-b) \cdot (a+b))^{1/2} \cdot \text{arctanh}(\tan(1/2 \cdot dx+1/2 \cdot c) \cdot (a-b) / ((a-b) \cdot (a+b))^{1/2}) \cdot B + 2/d/a^3 \cdot A \cdot \tan(1/2 \cdot dx+1/2 \cdot c) / (1 + \tan(1/2 \cdot dx+1/2 \cdot c)^2) - 6/d/a^4 \cdot A \cdot \arctan(\tan(1/2 \cdot dx+1/2 \cdot c)) \cdot b + 2/d/a^3 \cdot \arctan(\tan(1/2 \cdot dx+1/2 \cdot c)) \cdot B \end{aligned}$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(\cos(dx+c) \cdot (A+B \cdot \sec(dx+c)) / (a+b \cdot \sec(dx+c))^3, x, \text{algorithm}="maxima")$

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* h

elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for more details) Is 4*a^2-4*b^2 positive or negative?

mupad [B] time = 9.73, size = 5530, normalized size = 19.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x))^3,x)

[Out] ((tan(c/2 + (d*x)/2)^5*(6*A*b^5 - 2*A*a^5 - 12*A*a^2*b^3 + 4*A*a^3*b^2 + B*a^2*b^3 + 6*B*a^3*b^2 - 3*A*a*b^4 + 2*A*a^4*b - 2*B*a*b^4))/((a^3*b - a^4)*(a + b)^2) + (tan(c/2 + (d*x)/2)*(2*A*a^5 + 6*A*b^5 - 12*A*a^2*b^3 - 4*A*a^3*b^2 - B*a^2*b^3 + 6*B*a^3*b^2 + 3*A*a*b^4 + 2*A*a^4*b - 2*B*a*b^4))/((a + b)*(a^5 - 2*a^4*b + a^3*b^2)) + (2*tan(c/2 + (d*x)/2)^3*(2*A*a^6 - 6*A*b^6 + 13*A*a^2*b^4 - 6*A*a^4*b^2 - 5*B*a^3*b^3 + 2*B*a*b^5))/((a + b)^2*(a - b)))/(d*(2*a*b + tan(c/2 + (d*x)/2)^2*(2*a*b - a^2 + 3*b^2) + tan(c/2 + (d*x)/2)^6*(a^2 - 2*a*b + b^2) + a^2 + b^2 - tan(c/2 + (d*x)/2)^4*(2*a*b + a^2 - 3*b^2)) + (log(tan(c/2 + (d*x)/2) - 1i)*(3*A*b - B*a)*1i)/(a^4*d) - (log(tan(c/2 + (d*x)/2) + 1i)*(A*b*3i - B*a*1i))/(a^4*d) - (b*a*tan(((b*((8*tan(c/2 + (d*x)/2)*(72*A^2*b^12 + 4*B^2*a^12 - 72*A^2*a*b^11 - 8*B^2*a^11*b - 288*A^2*a^2*b^10 + 288*A^2*a^3*b^9 + 441*A^2*a^4*b^8 - 432*A^2*a^5*b^7 - 288*A^2*a^6*b^6 + 288*A^2*a^7*b^5 + 36*A^2*a^8*b^4 - 72*A^2*a^9*b^3 + 36*A^2*a^10*b^2 + 8*B^2*a^2*b^10 - 8*B^2*a^3*b^9 - 32*B^2*a^4*b^8 + 32*B^2*a^5*b^7 + 57*B^2*a^6*b^6 - 48*B^2*a^7*b^5 - 52*B^2*a^8*b^4 + 32*B^2*a^9*b^3 + 24*B^2*a^10*b^2 - 48*A*B*a*b^11 - 24*A*B*a^11*b + 48*A*B*a^2*b^10 + 192*A*B*a^3*b^9 - 192*A*B*a^4*b^8 - 318*A*B*a^5*b^7 + 288*A*B*a^6*b^6 + 252*A*B*a^7*b^5 - 192*A*B*a^8*b^4 - 72*A*B*a^9*b^3 + 48*A*B*a^10*b^2)))/(a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2) + (b*((8*(4*B*a^18 + 12*A*a^8*b^10 - 6*A*a^9*b^9 - 54*A*a^10*b^8 + 24*A*a^11*b^7 + 96*A*a^12*b^6 - 42*A*a^13*b^5 - 78*A*a^14*b^4 + 36*A*a^15*b^3 + 24*A*a^16*b^2 - 4*B*a^9*b^9 + 2*B*a^10*b^8 + 18*B*a^11*b^7 - 4*B*a^12*b^6 - 36*B*a^13*b^5 + 6*B*a^14*b^4 + 34*B*a^15*b^3 - 8*B*a^16*b^2 - 12*A*a^17*b - 12*B*a^17*b)))/(a^15*b + a^16 - a^9*b^7 - a^10*b^6 + 3*a^11*b^5 + 3*a^12*b^4 - 3*a^13*b^3 - 3*a^14*b^2) - (4*b*tan(c/2 + (d*x)/2)*((a + b)^5*(a - b)^5)^(1/2)*(6*A*b^5 - 6*B*a^5 - 15*A*a^2*b^3 + 5*B*a^3*b^2 + 12*A*a^4*b - 2*B*a*b^4)*(8*a^17*b - 8*a^8*b^10 + 8*a^9*b^9 + 32*a^10*b^8 - 32*a^11*b^7 - 48*a^12*b^6 + 48*a^13*b^5 + 32*a^14*b^4 - 32*a^15*b^3 - 8*a^16*b^2)))/(a^14 - a^4*b^10 + 5*a^6*b^8 - 10*a^8*b^6 + 10*a^10*b^4 - 5*a^12*b^2)*(a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2)))*((a + b)^5*(a - b)^5)^(1/2)*(6*A*b^5 - 6*B*a^5 - 15*A*a^2*b^3 + 5*B*a^3*b^2 + 12*A*a^4*b - 2*B*a*b^4))/(2*(a^14 - a^4*b^10 + 5*a^6*b^8 - 10*a^8*b^6 + 10*a^10*b^4 - 5*a^12*b^2))*((a + b)^5*(a - b)^5)^(1/2)*(6*A*b^5 - 6*B*a^5 - 15*A*a^2*b^3 + 5*B*a^3*b^2 + 12*A*a^4*b - 2*B*a*b^4)*1i)/(2*(a^14 - a^4*b^10 + 5*a^6*b^8 - 10*a^8*b^6 + 10*a^10*b^4 - 5*a^12*b^2)) + (b*((8*ta

$$\begin{aligned}
& n(c/2 + (d*x)/2)*(72*A^2*b^12 + 4*B^2*a^12 - 72*A^2*a*b^11 - 8*B^2*a^11*b - \\
& 288*A^2*a^2*b^10 + 288*A^2*a^3*b^9 + 441*A^2*a^4*b^8 - 432*A^2*a^5*b^7 - 2 \\
& 88*A^2*a^6*b^6 + 288*A^2*a^7*b^5 + 36*A^2*a^8*b^4 - 72*A^2*a^9*b^3 + 36*A^2 \\
& *a^10*b^2 + 8*B^2*a^2*b^10 - 8*B^2*a^3*b^9 - 32*B^2*a^4*b^8 + 32*B^2*a^5*b^ \\
& 7 + 57*B^2*a^6*b^6 - 48*B^2*a^7*b^5 - 52*B^2*a^8*b^4 + 32*B^2*a^9*b^3 + 24* \\
& B^2*a^10*b^2 - 48*A*B*a*b^11 - 24*A*B*a^11*b + 48*A*B*a^2*b^10 + 192*A*B*a^ \\
& 3*b^9 - 192*A*B*a^4*b^8 - 318*A*B*a^5*b^7 + 288*A*B*a^6*b^6 + 252*A*B*a^7*b \\
& ^5 - 192*A*B*a^8*b^4 - 72*A*B*a^9*b^3 + 48*A*B*a^10*b^2))/(a^12*b + a^13 - \\
& a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2) - (b*(\\
& (8*(4*B*a^18 + 12*A*a^8*b^10 - 6*A*a^9*b^9 - 54*A*a^10*b^8 + 24*A*a^11*b^7 \\
& + 96*A*a^12*b^6 - 42*A*a^13*b^5 - 78*A*a^14*b^4 + 36*A*a^15*b^3 + 24*A*a^16 \\
& *b^2 - 4*B*a^9*b^9 + 2*B*a^10*b^8 + 18*B*a^11*b^7 - 4*B*a^12*b^6 - 36*B*a^1 \\
& 3*b^5 + 6*B*a^14*b^4 + 34*B*a^15*b^3 - 8*B*a^16*b^2 - 12*A*a^17*b - 12*B*a^ \\
& 17*b)))/(a^15*b + a^16 - a^9*b^7 - a^10*b^6 + 3*a^11*b^5 + 3*a^12*b^4 - 3*a^ \\
& 13*b^3 - 3*a^14*b^2) + (4*b*tan(c/2 + (d*x)/2)*((a + b)^5*(a - b)^5)^(1/2)* \\
& (6*A*b^5 - 6*B*a^5 - 15*A*a^2*b^3 + 5*B*a^3*b^2 + 12*A*a^4*b - 2*B*a*b^4)*(\\
& 8*a^17*b - 8*a^8*b^10 + 8*a^9*b^9 + 32*a^10*b^8 - 32*a^11*b^7 - 48*a^12*b^6 \\
& + 48*a^13*b^5 + 32*a^14*b^4 - 32*a^15*b^3 - 8*a^16*b^2))/((a^14 - a^4*b^10 \\
& + 5*a^6*b^8 - 10*a^8*b^6 + 10*a^10*b^4 - 5*a^12*b^2)*(a^12*b + a^13 - a^6* \\
& b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2)))*((a + b) \\
& ^5*(a - b)^5)^(1/2)*(6*A*b^5 - 6*B*a^5 - 15*A*a^2*b^3 + 5*B*a^3*b^2 + 12*A* \\
& a^4*b - 2*B*a*b^4))/(2*(a^14 - a^4*b^10 + 5*a^6*b^8 - 10*a^8*b^6 + 10*a^10* \\
& b^4 - 5*a^12*b^2)))*((a + b)^5*(a - b)^5)^(1/2)*(6*A*b^5 - 6*B*a^5 - 15*A*a \\
& ^2*b^3 + 5*B*a^3*b^2 + 12*A*a^4*b - 2*B*a*b^4)*i)/(2*(a^14 - a^4*b^10 + 5* \\
& a^6*b^8 - 10*a^8*b^6 + 10*a^10*b^4 - 5*a^12*b^2)))/((16*(108*A^3*b^12 - 54* \\
& A^3*a*b^11 - 12*B^3*a^11*b - 486*A^3*a^2*b^10 + 243*A^3*a^3*b^9 + 864*A^3*a \\
& ^4*b^8 - 378*A^3*a^5*b^7 - 702*A^3*a^6*b^6 + 216*A^3*a^7*b^5 + 216*A^3*a^8* \\
& b^4 - 4*B^3*a^3*b^9 + 2*B^3*a^4*b^8 + 18*B^3*a^5*b^7 - 13*B^3*a^6*b^6 - 36* \\
& B^3*a^7*b^5 + 26*B^3*a^8*b^4 + 34*B^3*a^9*b^3 - 24*B^3*a^10*b^2 - 108*A^2*B \\
& *a*b^11 + 36*A*B^2*a^2*b^10 - 18*A*B^2*a^3*b^9 - 162*A*B^2*a^4*b^8 + 105*A* \\
& B^2*a^5*b^7 + 312*A*B^2*a^6*b^6 - 198*A*B^2*a^7*b^5 - 282*A*B^2*a^8*b^4 + 1 \\
& 56*A*B^2*a^9*b^3 + 96*A*B^2*a^10*b^2 + 54*A^2*B*a^2*b^10 + 486*A^2*B*a^3*b^ \\
& 9 - 279*A^2*B*a^4*b^8 - 900*A^2*B*a^5*b^7 + 486*A^2*B*a^6*b^6 + 774*A^2*B*a \\
& ^7*b^5 - 324*A^2*B*a^8*b^4 - 252*A^2*B*a^9*b^3))/(a^15*b + a^16 - a^9*b^7 - \\
& a^10*b^6 + 3*a^11*b^5 + 3*a^12*b^4 - 3*a^13*b^3 - 3*a^14*b^2) + (b*((8*tan \\
& (c/2 + (d*x)/2)*(72*A^2*b^12 + 4*B^2*a^12 - 72*A^2*a*b^11 - 8*B^2*a^11*b - \\
& 288*A^2*a^2*b^10 + 288*A^2*a^3*b^9 + 441*A^2*a^4*b^8 - 432*A^2*a^5*b^7 - 28 \\
& 8*A^2*a^6*b^6 + 288*A^2*a^7*b^5 + 36*A^2*a^8*b^4 - 72*A^2*a^9*b^3 + 36*A^2* \\
& a^10*b^2 + 8*B^2*a^2*b^10 - 8*B^2*a^3*b^9 - 32*B^2*a^4*b^8 + 32*B^2*a^5*b^7 \\
& + 57*B^2*a^6*b^6 - 48*B^2*a^7*b^5 - 52*B^2*a^8*b^4 + 32*B^2*a^9*b^3 + 24*B \\
& ^2*a^10*b^2 - 48*A*B*a*b^11 - 24*A*B*a^11*b + 48*A*B*a^2*b^10 + 192*A*B*a^3 \\
& *b^9 - 192*A*B*a^4*b^8 - 318*A*B*a^5*b^7 + 288*A*B*a^6*b^6 + 252*A*B*a^7*b^ \\
& 5 - 192*A*B*a^8*b^4 - 72*A*B*a^9*b^3 + 48*A*B*a^10*b^2))/(a^12*b + a^13 - a \\
& ^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2) + (b*((\\
& 8*(4*B*a^18 + 12*A*a^8*b^10 - 6*A*a^9*b^9 - 54*A*a^10*b^8 + 24*A*a^11*b^7 +
\end{aligned}$$

$$\begin{aligned}
& 96Aa^{12}b^6 - 42Aa^{13}b^5 - 78Aa^{14}b^4 + 36Aa^{15}b^3 + 24Aa^{16}b^2 - 4Ba^9b^9 + 2Ba^{10}b^8 + 18Ba^{11}b^7 - 4Ba^{12}b^6 - 36Ba^{13}b^5 + 6Ba^{14}b^4 + 34Ba^{15}b^3 - 8Ba^{16}b^2 - 12Aa^{17}b - 12Ba^{17}b) \\
&) / (a^{15}b + a^{16} - a^9b^7 - a^{10}b^6 + 3a^{11}b^5 + 3a^{12}b^4 - 3a^{13}b^3 - 3a^{14}b^2) - (4b \tan(c/2 + (d*x)/2) * ((a + b)^5 * (a - b)^5)^{(1/2)} * (6Ab^5 - 6Ba^5 - 15Aa^2b^3 + 5Ba^3b^2 + 12Aa^4b - 2Bab^4) * (8a^{17}b - 8a^8b^{10} + 8a^9b^9 + 32a^{10}b^8 - 32a^{11}b^7 - 48a^{12}b^6 + 48a^{13}b^5 + 32a^{14}b^4 - 32a^{15}b^3 - 8a^{16}b^2)) / ((a^{14} - a^4b^{10} + 5a^6b^8 - 10a^8b^6 + 10a^{10}b^4 - 5a^{12}b^2) * (a^{12}b + a^{13} - a^6b^7 - a^7b^6 + 3a^8b^5 + 3a^9b^4 - 3a^{10}b^3 - 3a^{11}b^2))) * ((a + b)^5 * (a - b)^5)^{(1/2)} * (6Ab^5 - 6Ba^5 - 15Aa^2b^3 + 5Ba^3b^2 + 12Aa^4b - 2Bab^4) / (2 * (a^{14} - a^4b^{10} + 5a^6b^8 - 10a^8b^6 + 10a^{10}b^4 - 5a^{12}b^2))) * ((a + b)^5 * (a - b)^5)^{(1/2)} * (6Ab^5 - 6Ba^5 - 15Aa^2b^3 + 5Ba^3b^2 + 12Aa^4b - 2Bab^4) / (2 * (a^{14} - a^4b^{10} + 5a^6b^8 - 10a^8b^6 + 10a^{10}b^4 - 5a^{12}b^2)) - (b * ((8 * \tan(c/2 + (d*x)/2) * (72A^2b^{12} + 4B^2a^{12} - 72A^2a^2b^{11} - 8B^2a^{11}b - 288A^2a^2b^{10} + 288A^2a^3b^9 + 441A^2a^4b^8 - 432A^2a^5b^7 - 288A^2a^6b^6 + 288A^2a^7b^5 + 36A^2a^8b^4 - 72A^2a^9b^3 + 36A^2a^{10}b^2 + 8B^2a^2b^{10} - 8B^2a^3b^9 - 32B^2a^4b^8 + 32B^2a^5b^7 + 57B^2a^6b^6 - 48B^2a^7b^5 - 52B^2a^8b^4 + 32B^2a^9b^3 + 24B^2a^{10}b^2 - 48ABab^{11} - 24ABa^{11}b + 48ABa^2b^{10} + 192ABa^3b^9 - 192ABa^4b^8 - 318ABa^5b^7 + 288ABa^6b^6 + 252ABa^7b^5 - 192ABa^8b^4 - 72ABa^9b^3 + 48ABa^{10}b^2)) / (a^{12}b + a^{13} - a^6b^7 - a^7b^6 + 3a^8b^5 + 3a^9b^4 - 3a^{10}b^3 - 3a^{11}b^2) - (b * ((8 * (4Ba^{18} + 12Aa^8b^{10} - 6Aa^9b^9 - 54Aa^{10}b^8 + 24Aa^{11}b^7 + 96Aa^{12}b^6 - 42Aa^{13}b^5 - 78Aa^{14}b^4 + 36Aa^{15}b^3 + 24Aa^{16}b^2 - 4Ba^9b^9 + 2Ba^{10}b^8 + 18Ba^{11}b^7 - 4Ba^{12}b^6 - 36Ba^{13}b^5 + 6Ba^{14}b^4 + 34Ba^{15}b^3 - 8Ba^{16}b^2 - 12Aa^{17}b - 12Ba^{17}b)) / (a^{15}b + a^{16} - a^9b^7 - a^{10}b^6 + 3a^{11}b^5 + 3a^{12}b^4 - 3a^{13}b^3 - 3a^{14}b^2) + (4b \tan(c/2 + (d*x)/2) * ((a + b)^5 * (a - b)^5)^{(1/2)} * (6Ab^5 - 6Ba^5 - 15Aa^2b^3 + 5Ba^3b^2 + 12Aa^4b - 2Bab^4) * (8a^{17}b - 8a^8b^{10} + 8a^9b^9 + 32a^{10}b^8 - 32a^{11}b^7 - 48a^{12}b^6 + 48a^{13}b^5 + 32a^{14}b^4 - 32a^{15}b^3 - 8a^{16}b^2)) / ((a^{14} - a^4b^{10} + 5a^6b^8 - 10a^8b^6 + 10a^{10}b^4 - 5a^{12}b^2) * (a^{12}b + a^{13} - a^6b^7 - a^7b^6 + 3a^8b^5 + 3a^9b^4 - 3a^{10}b^3 - 3a^{11}b^2))) * ((a + b)^5 * (a - b)^5)^{(1/2)} * (6Ab^5 - 6Ba^5 - 15Aa^2b^3 + 5Ba^3b^2 + 12Aa^4b - 2Bab^4) / (2 * (a^{14} - a^4b^{10} + 5a^6b^8 - 10a^8b^6 + 10a^{10}b^4 - 5a^{12}b^2))) * ((a + b)^5 * (a - b)^5)^{(1/2)} * (6Ab^5 - 6Ba^5 - 15Aa^2b^3 + 5Ba^3b^2 + 12Aa^4b - 2Bab^4) / (2 * (a^{14} - a^4b^{10} + 5a^6b^8 - 10a^8b^6 + 10a^{10}b^4 - 5a^{12}b^2)) * i) / (d * (a^{14} - a^4b^{10} + 5a^6b^8 - 10a^8b^6 + 10a^{10}b^4 - 5a^{12}b^2))
\end{aligned}$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \cos(c + dx)}{(a + b \sec(c + dx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**3, x)
```

```
[Out] Integral((A + B*sec(c + d*x))*cos(c + d*x)/(a + b*sec(c + d*x))**3, x)
```


$$3.335 \quad \int \frac{\cos^2(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^3} dx$$

Optimal. Leaf size=393

$$\frac{b(Ab - aB) \sin(c + dx) \cos(c + dx)}{2ad(a^2 - b^2)(a + b \sec(c + dx))^2} + \frac{x(a^2A - 6abB + 12Ab^2)}{2a^5} + \frac{b(-5a^3B + 7a^2Ab + 2ab^2B - 4Ab^3) \sin(c + dx)}{2a^2d(a^2 - b^2)^2(a + b \sec(c + dx))}$$

[Out] $1/2*(A*a^2+12*A*b^2-6*B*a*b)*x/a^5-b^2*(20*A*a^4*b-29*A*a^2*b^3+12*A*b^5-12*B*a^5+15*B*a^3*b^2-6*B*a*b^4)*\operatorname{arctanh}((a-b)^{(1/2)}*\tan(1/2*d*x+1/2*c)/(a+b)^{(1/2}))/a^5/(a-b)^{(5/2)/(a+b)^{(5/2)/d-1/2*(6*A*a^4*b-21*A*a^2*b^3+12*A*b^5-2*B*a^5+11*B*a^3*b^2-6*B*a*b^4)*\sin(d*x+c)/a^4/(a^2-b^2)^2/d+1/2*(A*a^4-10*A*a^2*b^2+6*A*b^4+6*B*a^3*b-3*B*a*b^3)*\cos(d*x+c)*\sin(d*x+c)/a^3/(a^2-b^2)^2/d+1/2*b*(A*b-B*a)*\cos(d*x+c)*\sin(d*x+c)/a/(a^2-b^2)/d/(a+b*\sec(d*x+c))^2+1/2*b*(7*A*a^2*b-4*A*b^3-5*B*a^3+2*B*a*b^2)*\cos(d*x+c)*\sin(d*x+c)/a^2/(a^2-b^2)^2/d/(a+b*\sec(d*x+c))$

Rubi [A] time = 2.00, antiderivative size = 393, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {4030, 4100, 4104, 3919, 3831, 2659, 208}

$$\frac{(-21a^2Ab^3 + 6a^4Ab + 11a^3b^2B - 2a^5B - 6ab^4B + 12Ab^5) \sin(c + dx)}{2a^4d(a^2 - b^2)^2} + \frac{(-10a^2Ab^2 + a^4A + 6a^3bB - 3ab^3B + \dots)}{2a^3d(a^2 - b^2)}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(\operatorname{Cos}[c + d*x]^2*(A + B*\operatorname{Sec}[c + d*x]))/(a + b*\operatorname{Sec}[c + d*x])^3, x]$

[Out] $((a^2*A + 12*A*b^2 - 6*a*b*B)*x)/(2*a^5) - (b^2*(20*a^4*A*b - 29*a^2*A*b^3 + 12*A*b^5 - 12*a^5*B + 15*a^3*b^2*B - 6*a*b^4*B)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a - b]*\operatorname{Tan}[(c + d*x)/2])/(\operatorname{Sqrt}[a + b])])/(a^5*(a - b)^{(5/2)*(a + b)^{(5/2)*d}) - ((6*a^4*A*b - 21*a^2*A*b^3 + 12*A*b^5 - 2*a^5*B + 11*a^3*b^2*B - 6*a*b^4*B)*\operatorname{Sin}[c + d*x])/(2*a^4*(a^2 - b^2)^2*d) + ((a^4*A - 10*a^2*A*b^2 + 6*A*b^4 + 6*a^3*b*B - 3*a*b^3*B)*\operatorname{Cos}[c + d*x]*\operatorname{Sin}[c + d*x])/(2*a^3*(a^2 - b^2)^2*d) + (b*(A*b - a*B)*\operatorname{Cos}[c + d*x]*\operatorname{Sin}[c + d*x])/(2*a*(a^2 - b^2)*d*(a + b*\operatorname{Sec}[c + d*x])^2) + (b*(7*a^2*A*b - 4*A*b^3 - 5*a^3*B + 2*a*b^2*B)*\operatorname{Cos}[c + d*x]*\operatorname{Sin}[c + d*x])/(2*a^2*(a^2 - b^2)^2*d*(a + b*\operatorname{Sec}[c + d*x]))$

Rule 208

$\operatorname{Int}[(a + b*x^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[-(a/b), 2]*\operatorname{ArcTanh}[x/\operatorname{Rt}[-(a/b), 2]])/a, x] /; \operatorname{FreeQ}\{a, b\}, x \ \&\& \operatorname{NegQ}[a/b]$

Rule 2659

```
Int[((a_) + (b_)*sin[Pi/2 + (c_) + (d_)*(x_)])^(-1), x_Symbol] := With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3831

```
Int[csc[(e_) + (f_)*(x_)]/(csc[(e_) + (f_)*(x_)]*(b_) + (a_)), x_Symbol] := Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3919

```
Int[(csc[(e_) + (f_)*(x_)]*(d_) + (c_))/(csc[(e_) + (f_)*(x_)]*(b_) + (a_)), x_Symbol] := Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 4030

```
Int[(csc[(e_) + (f_)*(x_)]*(d_))^(n_)*(csc[(e_) + (f_)*(x_)]*(b_) + (a_))^(m_)*(csc[(e_) + (f_)*(x_)]*(B_) + (A_)), x_Symbol] := Simp[(b*(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[A*(a^2*(m + 1) - b^2*(m + n + 1)) + a*b*B*n - a*(A*b - a*B)*(m + 1)*Csc[e + f*x] + b*(A*b - a*B)*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4100

```
Int[((A_) + csc[(e_) + (f_)*(x_)]*(B_) + csc[(e_) + (f_)*(x_)]^2*(C_))*(csc[(e_) + (f_)*(x_)]*(d_))^(n_)*(csc[(e_) + (f_)*(x_)]*(b_) + (a_))^(m_), x_Symbol] := Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4104

```

Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]

```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(c + dx)(A + B \sec(c + dx))}{(a + b \sec(c + dx))^3} dx &= \frac{b(Ab - aB) \cos(c + dx) \sin(c + dx)}{2a(a^2 - b^2)d(a + b \sec(c + dx))^2} - \int \frac{\cos^2(c + dx)(-2(a^2A - 2Ab^2 + abB) + 2a(A + b \sec(c + dx)))}{2a(a + b \sec(c + dx))^3} dx \\
&= \frac{b(Ab - aB) \cos(c + dx) \sin(c + dx)}{2a(a^2 - b^2)d(a + b \sec(c + dx))^2} + \frac{b(7a^2Ab - 4Ab^3 - 5a^3B + 2ab^2B)}{2a^2(a^2 - b^2)^2d(a + b \sec(c + dx))} \\
&= \frac{(a^4A - 10a^2Ab^2 + 6Ab^4 + 6a^3bB - 3ab^3B) \cos(c + dx) \sin(c + dx)}{2a^3(a^2 - b^2)^2d} + \frac{b(7a^2Ab - 4Ab^3 - 5a^3B + 2ab^2B)}{2a^2(a^2 - b^2)^2d(a + b \sec(c + dx))} \\
&= -\frac{(6a^4Ab - 21a^2Ab^3 + 12Ab^5 - 2a^5B + 11a^3b^2B - 6ab^4B) \sin(c + dx)}{2a^4(a^2 - b^2)^2d} \\
&= \frac{(a^2A + 12Ab^2 - 6abB)x}{2a^5} - \frac{(6a^4Ab - 21a^2Ab^3 + 12Ab^5 - 2a^5B + 11a^3b^2B - 6ab^4B) \sin(c + dx)}{2a^4(a^2 - b^2)^2d} \\
&= \frac{(a^2A + 12Ab^2 - 6abB)x}{2a^5} - \frac{(6a^4Ab - 21a^2Ab^3 + 12Ab^5 - 2a^5B + 11a^3b^2B - 6ab^4B) \sin(c + dx)}{2a^4(a^2 - b^2)^2d} \\
&= \frac{(a^2A + 12Ab^2 - 6abB)x}{2a^5} - \frac{(6a^4Ab - 21a^2Ab^3 + 12Ab^5 - 2a^5B + 11a^3b^2B - 6ab^4B) \sin(c + dx)}{2a^4(a^2 - b^2)^2d} \\
&= \frac{(a^2A + 12Ab^2 - 6abB)x}{2a^5} - \frac{b^2(20a^4Ab - 29a^2Ab^3 + 12Ab^5 - 12a^5B + 11a^3b^2B - 6ab^4B) \sin(c + dx)}{a^5(a - b)^5}
\end{aligned}$$

Mathematica [A] time = 4.69, size = 734, normalized size = 1.87

$$\frac{16b^2(-12a^5B+20a^4Ab+15a^3b^2B-29a^2Ab^3-6ab^4B+12Ab^5) \tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}} + \frac{2a^8A \sin(2(c+dx))+a^8A \sin(4(c+dx))+4a^8Ac+4a^8Adx+4a^8A^2}{(a^2-b^2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^3,x]
[Out] ((16*b^2*(20*a^4*A*b - 29*a^2*A*b^3 + 12*A*b^5 - 12*a^5*B + 15*a^3*b^2*B - 6*a*b^4*B)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2) + (4*a^8*A*c + 48*a^6*A*b^2*c - 12*a^4*A*b^4*c - 136*a^2*A*b^6*c + 96*A*b^8*c - 24*a^7*b*B*c + 72*a^3*b^5*B*c - 48*a*b^7*B*c + 4*a^8*A*d*x + 4*8*a^6*A*b^2*d*x - 12*a^4*A*b^4*d*x - 136*a^2*A*b^6*d*x + 96*A*b^8*d*x - 24*a^7*b*B*d*x + 72*a^3*b^5*B*d*x - 48*a*b^7*B*d*x + 16*a*b*(a^2 - b^2)^2*(a^2*A + 12*A*b^2 - 6*a*b*B)*(c + d*x)*Cos[c + d*x] + 4*(a^3 - a*b^2)^2*(a^2*A + 12*A*b^2 - 6*a*b*B)*(c + d*x)*Cos[2*(c + d*x)] - 8*a^7*A*b*Sin[c + d*x] - 32*a^5*A*b^3*Sin[c + d*x] + 160*a^3*A*b^5*Sin[c + d*x] - 96*a*A*b^7*Sin[c + d*x] + 4*a^8*B*Sin[c + d*x] + 8*a^6*b^2*B*Sin[c + d*x] - 84*a^4*b^4*B*Sin[c + d*x] + 48*a^2*b^6*B*Sin[c + d*x] + 2*a^8*A*Sin[2*(c + d*x)] - 48*a^6*A*b^2*Sin[2*(c + d*x)] + 130*a^4*A*b^4*Sin[2*(c + d*x)] - 72*a^2*A*b^6*Sin[2*(c + d*x)] + 16*a^7*b*B*Sin[2*(c + d*x)] - 64*a^5*b^3*B*Sin[2*(c + d*x)] + 36*a^3*b^5*B*Sin[2*(c + d*x)] - 8*a^7*A*b*Sin[3*(c + d*x)] + 16*a^5*A*b^3*Sin[3*(c + d*x)] - 8*a^3*A*b^5*Sin[3*(c + d*x)] + 4*a^8*B*Sin[3*(c + d*x)] - 8*a^6*b^2*B*Sin[3*(c + d*x)] + 4*a^4*b^4*B*Sin[3*(c + d*x)] + a^8*A*Sin[4*(c + d*x)] - 2*a^6*A*b^2*Sin[4*(c + d*x)] + a^4*A*b^4*Sin[4*(c + d*x)]))/(a^2 - b^2)^2*(b + a*Cos[c + d*x])^2)/(16*a^5*d)
```

fricas [B] time = 0.70, size = 1811, normalized size = 4.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(A*a^10 - 6*B*a^9*b + 9*A*a^8*b^2 + 18*B*a^7*b^3 - 33*A*a^6*b^4 - 18*B*a^5*b^5 + 35*A*a^4*b^6 + 6*B*a^3*b^7 - 12*A*a^2*b^8)*d*x*cos(d*x + c)^2 + 4*(A*a^9*b - 6*B*a^8*b^2 + 9*A*a^7*b^3 + 18*B*a^6*b^4 - 33*A*a^5*b^5 - 18*B*a^4*b^6 + 35*A*a^3*b^7 + 6*B*a^2*b^8 - 12*A*a*b^9)*d*x*cos(d*x + c) + 2*(A*a^8*b^2 - 6*B*a^7*b^3 + 9*A*a^6*b^4 + 18*B*a^5*b^5 - 33*A*a^4*b^6 - 18*B*a^3*b^7 + 35*A*a^2*b^8 + 6*B*a*b^9 - 12*A*b^10)*d*x - (12*B*a^5*b^4 - 20*A*a^4*b^5 - 15*B*a^3*b^6 + 29*A*a^2*b^7 + 6*B*a*b^8 - 12*A*b^9 + (12*B*a^7*
```

$$\begin{aligned}
& b^2 - 20Aa^6b^3 - 15B^2a^5b^4 + 29A^2a^4b^5 + 6B^2a^3b^6 - 12A^2a^2b^7) \cos(dx + c)^2 + 2(12B^2a^6b^3 - 20A^2a^5b^4 - 15B^2a^4b^5 + 29A^2a^3b^6 + 6B^2a^2b^7 - 12A^2ab^8) \cos(dx + c) \sqrt{a^2 - b^2} \log((2ab \cos(dx + c) - (a^2 - 2b^2) \cos(dx + c)^2 - 2\sqrt{a^2 - b^2}(b \cos(dx + c) + a) \sin(dx + c) + 2a^2 - b^2) / (a^2 \cos(dx + c)^2 + 2ab \cos(dx + c) + b^2)) + 2(2B^2a^8b^2 - 6A^2a^7b^3 - 13B^2a^6b^4 + 27A^2a^5b^5 + 17B^2a^4b^6 - 33A^2a^3b^7 - 6B^2a^2b^8 + 12A^2ab^9 + (A^2a^{10} - 3A^2a^8b^2 + 3A^2a^6b^4 - A^2a^4b^6) \cos(dx + c)^3 + 2(B^2a^{10} - 2A^2a^9b - 3B^2a^8b^2 + 6A^2a^7b^3 + 3B^2a^6b^4 - 6A^2a^5b^5 - B^2a^4b^6 + 2A^2a^3b^7) \cos(dx + c)^2 + (4B^2a^9b - 11A^2a^8b^2 - 20B^2a^7b^3 + 43A^2a^6b^4 + 25B^2a^5b^5 - 50A^2a^4b^6 - 9B^2a^3b^7 + 18A^2a^2b^8) \cos(dx + c)) \sin(dx + c) / ((a^{13} - 3a^{11}b^2 + 3a^9b^4 - a^7b^6) d \cos(dx + c)^2 + 2(a^{12}b - 3a^{10}b^3 + 3a^8b^5 - a^6b^7) d \cos(dx + c) + (a^{11}b^2 - 3a^9b^4 + 3a^7b^6 - a^5b^8) d), 1/2((A^2a^{10} - 6B^2a^9b + 9A^2a^8b^2 + 18B^2a^7b^3 - 33A^2a^6b^4 - 18B^2a^5b^5 + 35A^2a^4b^6 + 6B^2a^3b^7 - 12A^2a^2b^8) d x \cos(dx + c)^2 + 2(A^2a^9b - 6B^2a^8b^2 + 9A^2a^7b^3 + 18B^2a^6b^4 - 33A^2a^5b^5 - 18B^2a^4b^6 + 35A^2a^3b^7 + 6B^2a^2b^8 - 12A^2ab^9) d x \cos(dx + c) + (A^2a^8b^2 - 6B^2a^7b^3 + 9A^2a^6b^4 + 18B^2a^5b^5 - 33A^2a^4b^6 - 18B^2a^3b^7 + 35A^2a^2b^8 + 6B^2ab^9 - 12A^2b^{10}) d x + (12B^2a^5b^4 - 20A^2a^4b^5 - 15B^2a^3b^6 + 29A^2a^2b^7 + 6B^2ab^8 - 12A^2b^9 + (12B^2a^7b^2 - 20A^2a^6b^3 - 15B^2a^5b^4 + 29A^2a^4b^5 + 6B^2a^3b^6 - 12A^2a^2b^7) \cos(dx + c)^2 + 2(12B^2a^6b^3 - 20A^2a^5b^4 - 15B^2a^4b^5 + 29A^2a^3b^6 + 6B^2a^2b^7 - 12A^2ab^8) \cos(dx + c)) \sqrt{-a^2 + b^2} \arctan(-\sqrt{-a^2 + b^2}(b \cos(dx + c) + a) / ((a^2 - b^2) \sin(dx + c))) + (2B^2a^8b^2 - 6A^2a^7b^3 - 13B^2a^6b^4 + 27A^2a^5b^5 + 17B^2a^4b^6 - 33A^2a^3b^7 - 6B^2a^2b^8 + 12A^2ab^9 + (A^2a^{10} - 3A^2a^8b^2 + 3A^2a^6b^4 - A^2a^4b^6) \cos(dx + c)^3 + 2(B^2a^{10} - 2A^2a^9b - 3B^2a^8b^2 + 6A^2a^7b^3 + 3B^2a^6b^4 - 6A^2a^5b^5 - B^2a^4b^6 + 2A^2a^3b^7) \cos(dx + c)^2 + (4B^2a^9b - 11A^2a^8b^2 - 20B^2a^7b^3 + 43A^2a^6b^4 + 25B^2a^5b^5 - 50A^2a^4b^6 - 9B^2a^3b^7 + 18A^2a^2b^8) \cos(dx + c)) \sin(dx + c) / ((a^{13} - 3a^{11}b^2 + 3a^9b^4 - a^7b^6) d \cos(dx + c)^2 + 2(a^{12}b - 3a^{10}b^3 + 3a^8b^5 - a^6b^7) d \cos(dx + c) + (a^{11}b^2 - 3a^9b^4 + 3a^7b^6 - a^5b^8) d)]
\end{aligned}$$

giac [B] time = 0.79, size = 2700, normalized size = 6.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(dx+c)^2*(A+B*sec(dx+c))/(a+b*sec(dx+c))^3,x, algorithm="giac")

[Out] -1/2*(((a^6 - a^5*b + 10*a^4*b^2 + 10*a^3*b^3 - 23*a^2*b^4 - 6*a*b^5 + 12*b^6)*sqrt(-a^2 + b^2)*A*abs(a^9 - 2*a^7*b^2 + a^5*b^4)*abs(-a + b) - 3*(2*a^5*b + 2*a^4*b^2 - 4*a^3*b^3 - a^2*b^4 + 2*a*b^5)*sqrt(-a^2 + b^2)*B*abs(a^9

$$\begin{aligned}
& - 2*a^7*b^2 + a^5*b^4)*abs(-a + b) - (a^{15} - a^{14}*b + 8*a^{13}*b^2 - 28*a^{12} \\
& *b^3 - 42*a^{11}*b^4 + 111*a^{10}*b^5 + 68*a^9*b^6 - 158*a^8*b^7 - 47*a^7*b^8 + \\
& 100*a^6*b^9 + 12*a^5*b^{10} - 24*a^4*b^{11})*sqrt(-a^2 + b^2)*A*abs(-a + b) + \\
& 3*(2*a^{14}*b - 6*a^{13}*b^2 - 8*a^{12}*b^3 + 21*a^{11}*b^4 + 12*a^{10}*b^5 - 28*a^9* \\
& b^6 - 8*a^8*b^7 + 17*a^7*b^8 + 2*a^6*b^9 - 4*a^5*b^{10})*sqrt(-a^2 + b^2)*B*a \\
& bs(-a + b))*(pi*floor(1/2*(d*x + c)/pi + 1/2) + arctan(tan(1/2*d*x + 1/2*c) \\
& /sqrt(-(a^8*b - 2*a^6*b^3 + a^4*b^5 + sqrt((a^9 + a^8*b - 2*a^7*b^2 - 2*a^6 \\
& *b^3 + a^5*b^4 + a^4*b^5)*(a^9 - a^8*b - 2*a^7*b^2 + 2*a^6*b^3 + a^5*b^4 - \\
& a^4*b^5) + (a^8*b - 2*a^6*b^3 + a^4*b^5)^2)))/(a^9 - a^8*b - 2*a^7*b^2 + 2*a \\
& ^6*b^3 + a^5*b^4 - a^4*b^5)))/((a^9 - 2*a^7*b^2 + a^5*b^4)^2*(a^2 - 2*a*b \\
& + b^2) + (a^{10}*b - 2*a^9*b^2 - a^8*b^3 + 4*a^7*b^4 - a^6*b^5 - 2*a^5*b^6 + \\
& a^4*b^7)*abs(a^9 - 2*a^7*b^2 + a^5*b^4)) + (A*a^{15} - A*a^{14}*b - 6*B*a^{14}*b \\
& + 8*A*a^{13}*b^2 + 18*B*a^{13}*b^2 - 28*A*a^{12}*b^3 + 24*B*a^{12}*b^3 - 42*A*a^{11}* \\
& b^4 - 63*B*a^{11}*b^4 + 111*A*a^{10}*b^5 - 36*B*a^{10}*b^5 + 68*A*a^9*b^6 + 84*B* \\
& a^9*b^6 - 158*A*a^8*b^7 + 24*B*a^8*b^7 - 47*A*a^7*b^8 - 51*B*a^7*b^8 + 100* \\
& A*a^6*b^9 - 6*B*a^6*b^9 + 12*A*a^5*b^{10} + 12*B*a^5*b^{10} - 24*A*a^4*b^{11} + A \\
& *a^6*abs(a^9 - 2*a^7*b^2 + a^5*b^4) - A*a^5*b*abs(a^9 - 2*a^7*b^2 + a^5*b^4 \\
&) - 6*B*a^5*b*abs(a^9 - 2*a^7*b^2 + a^5*b^4) + 10*A*a^4*b^2*abs(a^9 - 2*a^7 \\
& *b^2 + a^5*b^4) - 6*B*a^4*b^2*abs(a^9 - 2*a^7*b^2 + a^5*b^4) + 10*A*a^3*b^3 \\
& *abs(a^9 - 2*a^7*b^2 + a^5*b^4) + 12*B*a^3*b^3*abs(a^9 - 2*a^7*b^2 + a^5*b^ \\
& 4) - 23*A*a^2*b^4*abs(a^9 - 2*a^7*b^2 + a^5*b^4) + 3*B*a^2*b^4*abs(a^9 - 2* \\
& a^7*b^2 + a^5*b^4) - 6*A*a*b^5*abs(a^9 - 2*a^7*b^2 + a^5*b^4) - 6*B*a*b^5*a \\
& bs(a^9 - 2*a^7*b^2 + a^5*b^4) + 12*A*b^6*abs(a^9 - 2*a^7*b^2 + a^5*b^4))*(p \\
& i*floor(1/2*(d*x + c)/pi + 1/2) + arctan(tan(1/2*d*x + 1/2*c)/sqrt(-(a^8*b \\
& - 2*a^6*b^3 + a^4*b^5 - sqrt((a^9 + a^8*b - 2*a^7*b^2 - 2*a^6*b^3 + a^5*b^4 \\
& + a^4*b^5)*(a^9 - a^8*b - 2*a^7*b^2 + 2*a^6*b^3 + a^5*b^4 - a^4*b^5) + (a^ \\
& 8*b - 2*a^6*b^3 + a^4*b^5)^2)))/(a^9 - a^8*b - 2*a^7*b^2 + 2*a^6*b^3 + a^5*b \\
& ^4 - a^4*b^5)))/((a^8*b*abs(a^9 - 2*a^7*b^2 + a^5*b^4) - 2*a^6*b^3*abs(a^9 \\
& - 2*a^7*b^2 + a^5*b^4) + a^4*b^5*abs(a^9 - 2*a^7*b^2 + a^5*b^4) - (a^9 - 2* \\
& a^7*b^2 + a^5*b^4)^2) + 2*(A*a^7*tan(1/2*d*x + 1/2*c)^7 - 2*B*a^7*tan(1/2*d \\
& *x + 1/2*c)^7 + 4*A*a^6*b*tan(1/2*d*x + 1/2*c)^7 + 4*B*a^6*b*tan(1/2*d*x + \\
& 1/2*c)^7 - 13*A*a^5*b^2*tan(1/2*d*x + 1/2*c)^7 + 2*B*a^5*b^2*tan(1/2*d*x + \\
& 1/2*c)^7 - 2*A*a^4*b^3*tan(1/2*d*x + 1/2*c)^7 - 16*B*a^4*b^3*tan(1/2*d*x + \\
& 1/2*c)^7 + 33*A*a^3*b^4*tan(1/2*d*x + 1/2*c)^7 + 9*B*a^3*b^4*tan(1/2*d*x + \\
& 1/2*c)^7 - 17*A*a^2*b^5*tan(1/2*d*x + 1/2*c)^7 + 9*B*a^2*b^5*tan(1/2*d*x + \\
& 1/2*c)^7 - 18*A*a*b^6*tan(1/2*d*x + 1/2*c)^7 - 6*B*a*b^6*tan(1/2*d*x + 1/2* \\
& c)^7 + 12*A*b^7*tan(1/2*d*x + 1/2*c)^7 - 3*A*a^7*tan(1/2*d*x + 1/2*c)^5 + 2 \\
& *B*a^7*tan(1/2*d*x + 1/2*c)^5 - 4*A*a^6*b*tan(1/2*d*x + 1/2*c)^5 + 4*B*a^6* \\
& b*tan(1/2*d*x + 1/2*c)^5 - 5*A*a^5*b^2*tan(1/2*d*x + 1/2*c)^5 - 10*B*a^5*b^ \\
& 2*tan(1/2*d*x + 1/2*c)^5 + 26*A*a^4*b^3*tan(1/2*d*x + 1/2*c)^5 - 16*B*a^4*b \\
& ^3*tan(1/2*d*x + 1/2*c)^5 + 29*A*a^3*b^4*tan(1/2*d*x + 1/2*c)^5 + 35*B*a^3* \\
& b^4*tan(1/2*d*x + 1/2*c)^5 - 67*A*a^2*b^5*tan(1/2*d*x + 1/2*c)^5 + 9*B*a^2* \\
& b^5*tan(1/2*d*x + 1/2*c)^5 - 18*A*a*b^6*tan(1/2*d*x + 1/2*c)^5 - 18*B*a*b^6 \\
& *tan(1/2*d*x + 1/2*c)^5 + 36*A*b^7*tan(1/2*d*x + 1/2*c)^5 + 3*A*a^7*tan(1/2 \\
& *d*x + 1/2*c)^3 + 2*B*a^7*tan(1/2*d*x + 1/2*c)^3 - 4*A*a^6*b*tan(1/2*d*x +
\end{aligned}$$

$$\begin{aligned} & \frac{1}{2}c)^3 - 4B*a^6*b*\tan(1/2*d*x + 1/2*c)^3 + 5A*a^5*b^2*\tan(1/2*d*x + 1/2 \\ & *c)^3 - 10B*a^5*b^2*\tan(1/2*d*x + 1/2*c)^3 + 26A*a^4*b^3*\tan(1/2*d*x + 1/ \\ & 2*c)^3 + 16B*a^4*b^3*\tan(1/2*d*x + 1/2*c)^3 - 29A*a^3*b^4*\tan(1/2*d*x + 1 \\ & /2*c)^3 + 35B*a^3*b^4*\tan(1/2*d*x + 1/2*c)^3 - 67A*a^2*b^5*\tan(1/2*d*x + \\ & 1/2*c)^3 - 9B*a^2*b^5*\tan(1/2*d*x + 1/2*c)^3 + 18A*a*b^6*\tan(1/2*d*x + 1/ \\ & 2*c)^3 - 18B*a*b^6*\tan(1/2*d*x + 1/2*c)^3 + 36A*b^7*\tan(1/2*d*x + 1/2*c)^ \\ & 3 - A*a^7*\tan(1/2*d*x + 1/2*c) - 2B*a^7*\tan(1/2*d*x + 1/2*c) + 4A*a^6*b*t \\ & \tan(1/2*d*x + 1/2*c) - 4B*a^6*b*\tan(1/2*d*x + 1/2*c) + 13A*a^5*b^2*\tan(1/2 \\ & *d*x + 1/2*c) + 2B*a^5*b^2*\tan(1/2*d*x + 1/2*c) - 2A*a^4*b^3*\tan(1/2*d*x \\ & + 1/2*c) + 16B*a^4*b^3*\tan(1/2*d*x + 1/2*c) - 33A*a^3*b^4*\tan(1/2*d*x + 1 \\ & /2*c) + 9B*a^3*b^4*\tan(1/2*d*x + 1/2*c) - 17A*a^2*b^5*\tan(1/2*d*x + 1/2*c \\ &) - 9B*a^2*b^5*\tan(1/2*d*x + 1/2*c) + 18A*a*b^6*\tan(1/2*d*x + 1/2*c) - 6* \\ & B*a*b^6*\tan(1/2*d*x + 1/2*c) + 12A*b^7*\tan(1/2*d*x + 1/2*c))/((a^8 - 2*a^6 \\ & *b^2 + a^4*b^4)*(a*\tan(1/2*d*x + 1/2*c)^4 - b*\tan(1/2*d*x + 1/2*c)^4 - 2*b* \\ & \tan(1/2*d*x + 1/2*c)^2 - a - b)^2))/d \end{aligned}$$

maple [B] time = 1.32, size = 1552, normalized size = 3.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\int (\cos(dx+c)^2*(A+B*\sec(dx+c)))/(a+b*\sec(dx+c))^3, x$

[Out] $\frac{10/d/a^2/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)^2*b^4/(a+b)/(a-b)^2*\tan(1/2*d*x+1/2*c)*A-10/d/a^2/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)^2*b^4/(a-b)/(a^2+2*a*b+b^2)*\tan(1/2*d*x+1/2*c)^3*A+6/d*b^6/a^4/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)*\tan(1/2*d*x+1/2*c)^3*A-4/d*b^5/a^3/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)*\tan(1/2*d*x+1/2*c)^3*B-6/d*b^6/a^4/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a+b)/(a-b)^2*\tan(1/2*d*x+1/2*c)*A+4/d*b^5/a^3/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a+b)/(a-b)^2*\tan(1/2*d*x+1/2*c)*B-1/d/a^3/(1+\tan(1/2*d*x+1/2*c)^2)^2*\tan(1/2*d*x+1/2*c)^3*A+1/d/a^3*\arctan(\tan(1/2*d*x+1/2*c))*A-1/d*b^5/a^3/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)*\tan(1/2*d*x+1/2*c)^3*A+8/d*b^3/a/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)*\tan(1/2*d*x+1/2*c)^3*B+1/d*b^4/a^2/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)*\tan(1/2*d*x+1/2*c)^3*B-1/d*b^5/a^3/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a+b)/(a-b)^2*\tan(1/2*d*x+1/2*c)*A-8/d*b^3/a/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a+b)/(a-b)^2*\tan(1/2*d*x+1/2*c)*B+1/d*b^4/a^2/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a+b)/(a-b)^2*\tan(1/2*d*x+1/2*c)*B+12/d/a^5*\arctan(\tan(1/2*d*x+1/2*c))*A*b^2-6/d/a^4*\arctan(\tan(1/2*d*x+1/2*c))*B*b+2/d/a^3/(1+\tan(1/2*d*x+1/2*c)^2)^2*B*\tan(1/2*d*x+1/2*c)^3+1/d/a^3/(1+\tan(1/2*d*x+1/2*c)^2)^2*A*\tan(1/2*d*x+1/2*c)+2/d/a^3/(1+\tan(1/2*d*x+1/2*c)^2)^2*B*\tan(1/2*d*x+1/2*c)-12/d*b^7/a^5/(a^4-2*a^2*b^2+b^4)/($

$$(a-b)*(a+b))^{(1/2)}*\operatorname{arctanh}(\tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^{(1/2)})*A-15/d*b^4/a^2/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^{(1/2)}*\operatorname{arctanh}(\tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^{(1/2)})*B+6/d*b^6/a^4/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^{(1/2)}*\operatorname{arctanh}(\tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^{(1/2)})*B-20/d/a/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^{(1/2)}*\operatorname{arctanh}(\tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^{(1/2)})*A*b^3+29/d/a^3/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^{(1/2)}*\operatorname{arctanh}(\tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^{(1/2)})*A*b^5+12/d/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^{(1/2)}*\operatorname{arctanh}(\tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^{(1/2)})*b^2*B-6/d/a^4/(1+\tan(1/2*d*x+1/2*c)^2)^2*\tan(1/2*d*x+1/2*c)^3*A*b-6/d/a^4/(1+\tan(1/2*d*x+1/2*c)^2)^2*\tan(1/2*d*x+1/2*c)*A*b$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for more details)Is 4*a^2-4*b^2 positive or negative?

mupad [B] time = 14.17, size = 10586, normalized size = 26.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)^2*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x))^3,x)

[Out] ((tan(c/2 + (d*x)/2)^5*(3*A*a^7 - 36*A*b^7 - 2*B*a^7 + 67*A*a^2*b^5 - 29*A*a^3*b^4 - 26*A*a^4*b^3 + 5*A*a^5*b^2 - 9*B*a^2*b^5 - 35*B*a^3*b^4 + 16*B*a^4*b^3 + 10*B*a^5*b^2 + 18*A*a*b^6 + 4*A*a^6*b + 18*B*a*b^6 - 4*B*a^6*b))/(a + b)^2*(a^6 - 2*a^5*b + a^4*b^2) - (tan(c/2 + (d*x)/2)^3*(3*A*a^7 + 36*A*b^7 + 2*B*a^7 - 67*A*a^2*b^5 - 29*A*a^3*b^4 + 26*A*a^4*b^3 + 5*A*a^5*b^2 - 9*B*a^2*b^5 + 35*B*a^3*b^4 + 16*B*a^4*b^3 - 10*B*a^5*b^2 + 18*A*a*b^6 - 4*A*a^6*b - 18*B*a*b^6 - 4*B*a^6*b))/(a + b)^2*(a^6 - 2*a^5*b + a^4*b^2) + (tan(c/2 + (d*x)/2)^7*(A*a^6 - 12*A*b^6 - 2*B*a^6 + 23*A*a^2*b^4 - 10*A*a^3*b^3 - 8*A*a^4*b^2 - 3*B*a^2*b^4 - 12*B*a^3*b^3 + 4*B*a^4*b^2 + 6*A*a*b^5 + 5*A*a^5*b + 6*B*a*b^5 + 2*B*a^5*b))/(a^4*b - a^5)*(a + b)^2 + (tan(c/2 + (d*x)/2)*(A*a^6 - 12*A*b^6 + 2*B*a^6 + 23*A*a^2*b^4 + 10*A*a^3*b^3 - 8*A*a^4*b^2 + 3*B*a^2*b^4 - 12*B*a^3*b^3 - 4*B*a^4*b^2 - 6*A*a*b^5 - 5*A*a^5*b + 6*B*a*b^5 + 2*B*a^5*b))/(a + b)*(a^6 - 2*a^5*b + a^4*b^2))/(d*(2*a*b - tan(c/2 + (d*x)/2)^4*(2*a^2 - 6*b^2) + tan(c/2 + (d*x)/2)^2*(4*a*b + 4*b^2))

$$\begin{aligned}
& - \tan(c/2 + (d*x)/2)^6*(4*a*b - 4*b^2) + \tan(c/2 + (d*x)/2)^8*(a^2 - 2*a*b \\
& + b^2) + a^2 + b^2)) + (\operatorname{atan}(\frac{((8*\tan(c/2 + (d*x)/2)*(A^2*a^{14} + 288*A^2*b^{14} \\
& - 288*A^2*a*b^{13} - 2*A^2*a^{13}*b - 1104*A^2*a^2*b^{12} + 1104*A^2*a^3*b^{11} \\
& + 1538*A^2*a^4*b^{10} - 1538*A^2*a^5*b^9 - 827*A^2*a^6*b^8 + 872*A^2*a^7*b^7 \\
& + 18*A^2*a^8*b^6 - 108*A^2*a^9*b^5 + 74*A^2*a^{10}*b^4 - 40*A^2*a^{11}*b^3 + 2 \\
& 1*A^2*a^{12}*b^2 + 72*B^2*a^2*b^{12} - 72*B^2*a^3*b^{11} - 288*B^2*a^4*b^{10} + 288 \\
& *B^2*a^5*b^9 + 441*B^2*a^6*b^8 - 432*B^2*a^7*b^7 - 288*B^2*a^8*b^6 + 288*B^ \\
& 2*a^9*b^5 + 36*B^2*a^{10}*b^4 - 72*B^2*a^{11}*b^3 + 36*B^2*a^{12}*b^2 - 288*A*B*a \\
& *b^{13} - 12*A*B*a^{13}*b + 288*A*B*a^2*b^{12} + 1128*A*B*a^3*b^{11} - 1128*A*B*a^4 \\
& *b^{10} - 1650*A*B*a^5*b^9 + 1632*A*B*a^6*b^8 + 984*A*B*a^7*b^7 - 1008*A*B*a^ \\
& 8*b^6 - 72*A*B*a^9*b^5 + 192*A*B*a^{10}*b^4 - 108*A*B*a^{11}*b^3 + 24*A*B*a^{12} \\
& b^2)))/(a^{14}*b + a^{15} - a^8*b^7 - a^9*b^6 + 3*a^{10}*b^5 + 3*a^{11}*b^4 - 3*a^{12} \\
& *b^3 - 3*a^{13}*b^2) + (((4*(4*A*a^{21} - 48*A*a^{10}*b^{11} + 24*A*a^{11}*b^{10} + 212 \\
& *A*a^{12}*b^9 - 100*A*a^{13}*b^8 - 360*A*a^{14}*b^7 + 164*A*a^{15}*b^6 + 276*A*a^{16} \\
& *b^5 - 120*A*a^{17}*b^4 - 80*A*a^{18}*b^3 + 28*A*a^{19}*b^2 + 24*B*a^{11}*b^{10} - 12 \\
& *B*a^{12}*b^9 - 108*B*a^{13}*b^8 + 48*B*a^{14}*b^7 + 192*B*a^{15}*b^6 - 84*B*a^{16}*b \\
& ^5 - 156*B*a^{17}*b^4 + 72*B*a^{18}*b^3 + 48*B*a^{19}*b^2 - 24*B*a^{20}*b)))/(a^{18}*b \\
& + a^{19} - a^{12}*b^7 - a^{13}*b^6 + 3*a^{14}*b^5 + 3*a^{15}*b^4 - 3*a^{16}*b^3 - 3*a^{ \\
& 17}*b^2) - (4*\tan(c/2 + (d*x)/2)*(A*a^2*1i + A*b^2*12i - B*a*b*6i)*(8*a^{19}*b \\
& - 8*a^{10}*b^{10} + 8*a^{11}*b^9 + 32*a^{12}*b^8 - 32*a^{13}*b^7 - 48*a^{14}*b^6 + 48* \\
& a^{15}*b^5 + 32*a^{16}*b^4 - 32*a^{17}*b^3 - 8*a^{18}*b^2))/(a^5*(a^{14}*b + a^{15} - a \\
& ^8*b^7 - a^9*b^6 + 3*a^{10}*b^5 + 3*a^{11}*b^4 - 3*a^{12}*b^3 - 3*a^{13}*b^2)))*(A* \\
& a^2*1i + A*b^2*12i - B*a*b*6i))/(2*a^5))*(A*a^2*1i + A*b^2*12i - B*a*b*6i)* \\
& 1i)/(2*a^5) + (((8*\tan(c/2 + (d*x)/2)*(A^2*a^{14} + 288*A^2*b^{14} - 288*A^2*a* \\
& b^{13} - 2*A^2*a^{13}*b - 1104*A^2*a^2*b^{12} + 1104*A^2*a^3*b^{11} + 1538*A^2*a^4* \\
& b^{10} - 1538*A^2*a^5*b^9 - 827*A^2*a^6*b^8 + 872*A^2*a^7*b^7 + 18*A^2*a^8*b^ \\
& 6 - 108*A^2*a^9*b^5 + 74*A^2*a^{10}*b^4 - 40*A^2*a^{11}*b^3 + 21*A^2*a^{12}*b^2 + \\
& 72*B^2*a^2*b^{12} - 72*B^2*a^3*b^{11} - 288*B^2*a^4*b^{10} + 288*B^2*a^5*b^9 + 4 \\
& 41*B^2*a^6*b^8 - 432*B^2*a^7*b^7 - 288*B^2*a^8*b^6 + 288*B^2*a^9*b^5 + 36*B \\
& ^2*a^{10}*b^4 - 72*B^2*a^{11}*b^3 + 36*B^2*a^{12}*b^2 - 288*A*B*a*b^{13} - 12*A*B*a \\
& ^{13}*b + 288*A*B*a^2*b^{12} + 1128*A*B*a^3*b^{11} - 1128*A*B*a^4*b^{10} - 1650*A*B \\
& *a^5*b^9 + 1632*A*B*a^6*b^8 + 984*A*B*a^7*b^7 - 1008*A*B*a^8*b^6 - 72*A*B*a^ \\
& ^9*b^5 + 192*A*B*a^{10}*b^4 - 108*A*B*a^{11}*b^3 + 24*A*B*a^{12}*b^2)))/(a^{14}*b + \\
& a^{15} - a^8*b^7 - a^9*b^6 + 3*a^{10}*b^5 + 3*a^{11}*b^4 - 3*a^{12}*b^3 - 3*a^{13}*b^ \\
& ^2) - (((4*(4*A*a^{21} - 48*A*a^{10}*b^{11} + 24*A*a^{11}*b^{10} + 212*A*a^{12}*b^9 - 10 \\
& 0*A*a^{13}*b^8 - 360*A*a^{14}*b^7 + 164*A*a^{15}*b^6 + 276*A*a^{16}*b^5 - 120*A*a^{1 \\
& 7}*b^4 - 80*A*a^{18}*b^3 + 28*A*a^{19}*b^2 + 24*B*a^{11}*b^{10} - 12*B*a^{12}*b^9 - 10 \\
& 8*B*a^{13}*b^8 + 48*B*a^{14}*b^7 + 192*B*a^{15}*b^6 - 84*B*a^{16}*b^5 - 156*B*a^{17} \\
& *b^4 + 72*B*a^{18}*b^3 + 48*B*a^{19}*b^2 - 24*B*a^{20}*b)))/(a^{18}*b + a^{19} - a^{12}*b \\
& ^7 - a^{13}*b^6 + 3*a^{14}*b^5 + 3*a^{15}*b^4 - 3*a^{16}*b^3 - 3*a^{17}*b^2) + (4*\tan \\
& (c/2 + (d*x)/2)*(A*a^2*1i + A*b^2*12i - B*a*b*6i)*(8*a^{19}*b - 8*a^{10}*b^{10} + \\
& 8*a^{11}*b^9 + 32*a^{12}*b^8 - 32*a^{13}*b^7 - 48*a^{14}*b^6 + 48*a^{15}*b^5 + 32*a^ \\
& 16*b^4 - 32*a^{17}*b^3 - 8*a^{18}*b^2))/(a^5*(a^{14}*b + a^{15} - a^8*b^7 - a^9*b^6 \\
& + 3*a^{10}*b^5 + 3*a^{11}*b^4 - 3*a^{12}*b^3 - 3*a^{13}*b^2)))*(A*a^2*1i + A*b^2*1 \\
& 2i - B*a*b*6i))/(2*a^5))*(A*a^2*1i + A*b^2*12i - B*a*b*6i)*1i)/(2*a^5))/((8
\end{aligned}$$

$$\begin{aligned}
& * (1728*A^3*b^{15} - 864*A^3*a*b^{14} - 7344*A^3*a^2*b^{13} + 3456*A^3*a^3*b^{12} + \\
& 11700*A^3*a^4*b^{11} - 4770*A^3*a^5*b^{10} - 7829*A^3*a^6*b^9 + 2326*A^3*a^7*b^8 \\
& + 1314*A^3*a^8*b^7 - 11*A^3*a^9*b^6 + 411*A^3*a^{10}*b^5 - 20*A^3*a^{11}*b^4 \\
& + 20*A^3*a^{12}*b^3 - 216*B^3*a^3*b^{12} + 108*B^3*a^4*b^{11} + 972*B^3*a^5*b^{10} \\
& - 486*B^3*a^6*b^9 - 1728*B^3*a^7*b^8 + 756*B^3*a^8*b^7 + 1404*B^3*a^9*b^6 - \\
& 432*B^3*a^{10}*b^5 - 432*B^3*a^{11}*b^4 - 2592*A^2*B*a*b^{14} + 1296*A*B^2*a^2*b \\
& ^{13} - 648*A*B^2*a^3*b^{12} - 5724*A*B^2*a^4*b^{11} + 2808*A*B^2*a^5*b^{10} + 9828 \\
& *A*B^2*a^6*b^9 - 4203*A*B^2*a^7*b^8 - 7524*A*B^2*a^8*b^7 + 2268*A*B^2*a^9*b \\
& ^6 + 1980*A*B^2*a^{10}*b^5 + 144*A*B^2*a^{12}*b^3 + 1296*A^2*B*a^2*b^{13} + 11232 \\
& *A^2*B*a^3*b^{12} - 5400*A^2*B*a^4*b^{11} - 18594*A^2*B*a^5*b^{10} + 7767*A^2*B*a \\
& ^6*b^9 + 13347*A^2*B*a^7*b^8 - 3972*A^2*B*a^8*b^7 - 2892*A^2*B*a^9*b^6 + 9* \\
& A^2*B*a^{10}*b^5 - 489*A^2*B*a^{11}*b^4 + 12*A^2*B*a^{12}*b^3 - 12*A^2*B*a^{13}*b^2 \\
&)) / (a^{18}*b + a^{19} - a^{12}*b^7 - a^{13}*b^6 + 3*a^{14}*b^5 + 3*a^{15}*b^4 - 3*a^{16} \\
& b^3 - 3*a^{17}*b^2) - (((8*\tan(c/2 + (d*x)/2)*(A^2*a^{14} + 288*A^2*b^{14} - 288* \\
& A^2*a*b^{13} - 2*A^2*a^{13}*b - 1104*A^2*a^2*b^{12} + 1104*A^2*a^3*b^{11} + 1538*A^ \\
& 2*a^4*b^{10} - 1538*A^2*a^5*b^9 - 827*A^2*a^6*b^8 + 872*A^2*a^7*b^7 + 18*A^2* \\
& a^8*b^6 - 108*A^2*a^9*b^5 + 74*A^2*a^{10}*b^4 - 40*A^2*a^{11}*b^3 + 21*A^2*a^{12} \\
& *b^2 + 72*B^2*a^2*b^{12} - 72*B^2*a^3*b^{11} - 288*B^2*a^4*b^{10} + 288*B^2*a^5*b \\
& ^9 + 441*B^2*a^6*b^8 - 432*B^2*a^7*b^7 - 288*B^2*a^8*b^6 + 288*B^2*a^9*b^5 \\
& + 36*B^2*a^{10}*b^4 - 72*B^2*a^{11}*b^3 + 36*B^2*a^{12}*b^2 - 288*A*B*a*b^{13} - 12 \\
& *A*B*a^{13}*b + 288*A*B*a^2*b^{12} + 1128*A*B*a^3*b^{11} - 1128*A*B*a^4*b^{10} - 16 \\
& 50*A*B*a^5*b^9 + 1632*A*B*a^6*b^8 + 984*A*B*a^7*b^7 - 1008*A*B*a^8*b^6 - 72 \\
& *A*B*a^9*b^5 + 192*A*B*a^{10}*b^4 - 108*A*B*a^{11}*b^3 + 24*A*B*a^{12}*b^2)) / (a^{1 \\
& 4}*b + a^{15} - a^8*b^7 - a^9*b^6 + 3*a^{10}*b^5 + 3*a^{11}*b^4 - 3*a^{12}*b^3 - 3*a \\
& ^{13}*b^2) + (((4*(4*A*a^{21} - 48*A*a^{10}*b^{11} + 24*A*a^{11}*b^{10} + 212*A*a^{12}*b^ \\
& 9 - 100*A*a^{13}*b^8 - 360*A*a^{14}*b^7 + 164*A*a^{15}*b^6 + 276*A*a^{16}*b^5 - 120 \\
& *A*a^{17}*b^4 - 80*A*a^{18}*b^3 + 28*A*a^{19}*b^2 + 24*B*a^{11}*b^{10} - 12*B*a^{12}*b^ \\
& 9 - 108*B*a^{13}*b^8 + 48*B*a^{14}*b^7 + 192*B*a^{15}*b^6 - 84*B*a^{16}*b^5 - 156*B \\
& *a^{17}*b^4 + 72*B*a^{18}*b^3 + 48*B*a^{19}*b^2 - 24*B*a^{20}*b)) / (a^{18}*b + a^{19} - \\
& a^{12}*b^7 - a^{13}*b^6 + 3*a^{14}*b^5 + 3*a^{15}*b^4 - 3*a^{16}*b^3 - 3*a^{17}*b^2) - \\
& (4*\tan(c/2 + (d*x)/2)*(A*a^2*1i + A*b^2*12i - B*a*b*6i)*(8*a^{19}*b - 8*a^{10} \\
& b^{10} + 8*a^{11}*b^9 + 32*a^{12}*b^8 - 32*a^{13}*b^7 - 48*a^{14}*b^6 + 48*a^{15}*b^5 + \\
& 32*a^{16}*b^4 - 32*a^{17}*b^3 - 8*a^{18}*b^2)) / (a^5*(a^{14}*b + a^{15} - a^8*b^7 - a \\
& ^9*b^6 + 3*a^{10}*b^5 + 3*a^{11}*b^4 - 3*a^{12}*b^3 - 3*a^{13}*b^2))) * (A*a^2*1i + A \\
& *b^2*12i - B*a*b*6i)) / (2*a^5) * (A*a^2*1i + A*b^2*12i - B*a*b*6i)) / (2*a^5) + \\
& (((8*\tan(c/2 + (d*x)/2)*(A^2*a^{14} + 288*A^2*b^{14} - 288*A^2*a*b^{13} - 2*A^2* \\
& a^{13}*b - 1104*A^2*a^2*b^{12} + 1104*A^2*a^3*b^{11} + 1538*A^2*a^4*b^{10} - 1538*A^ \\
& 2*a^5*b^9 - 827*A^2*a^6*b^8 + 872*A^2*a^7*b^7 + 18*A^2*a^8*b^6 - 108*A^2*a^ \\
& 9*b^5 + 74*A^2*a^{10}*b^4 - 40*A^2*a^{11}*b^3 + 21*A^2*a^{12}*b^2 + 72*B^2*a^2*b \\
& ^{12} - 72*B^2*a^3*b^{11} - 288*B^2*a^4*b^{10} + 288*B^2*a^5*b^9 + 441*B^2*a^6*b^ \\
& 8 - 432*B^2*a^7*b^7 - 288*B^2*a^8*b^6 + 288*B^2*a^9*b^5 + 36*B^2*a^{10}*b^4 - \\
& 72*B^2*a^{11}*b^3 + 36*B^2*a^{12}*b^2 - 288*A*B*a*b^{13} - 12*A*B*a^{13}*b + 288*A \\
& *B*a^2*b^{12} + 1128*A*B*a^3*b^{11} - 1128*A*B*a^4*b^{10} - 1650*A*B*a^5*b^9 + 16 \\
& 32*A*B*a^6*b^8 + 984*A*B*a^7*b^7 - 1008*A*B*a^8*b^6 - 72*A*B*a^9*b^5 + 192* \\
& A*B*a^{10}*b^4 - 108*A*B*a^{11}*b^3 + 24*A*B*a^{12}*b^2)) / (a^{14}*b + a^{15} - a^8*b^
\end{aligned}$$

$$\begin{aligned}
& 7 - a^9 b^6 + 3a^{10} b^5 + 3a^{11} b^4 - 3a^{12} b^3 - 3a^{13} b^2) - (((4*(4* \\
& A*a^{21} - 48*A*a^{10} b^{11} + 24*A*a^{11} b^{10} + 212*A*a^{12} b^9 - 100*A*a^{13} b^8 \\
& - 360*A*a^{14} b^7 + 164*A*a^{15} b^6 + 276*A*a^{16} b^5 - 120*A*a^{17} b^4 - 80*A* \\
& a^{18} b^3 + 28*A*a^{19} b^2 + 24*B*a^{11} b^{10} - 12*B*a^{12} b^9 - 108*B*a^{13} b^8 \\
& + 48*B*a^{14} b^7 + 192*B*a^{15} b^6 - 84*B*a^{16} b^5 - 156*B*a^{17} b^4 + 72*B*a^{18} b^3 \\
& + 48*B*a^{19} b^2 - 24*B*a^{20} b)) / (a^{18} b + a^{19} - a^{12} b^7 - a^{13} b^6 \\
& + 3a^{14} b^5 + 3a^{15} b^4 - 3a^{16} b^3 - 3a^{17} b^2) + (4*\tan(c/2 + (d*x)/ \\
& 2)*(A*a^2*i + A*b^2*12i - B*a*b*6i)*(8*a^{19} b - 8*a^{10} b^{10} + 8*a^{11} b^9 + \\
& 32*a^{12} b^8 - 32*a^{13} b^7 - 48*a^{14} b^6 + 48*a^{15} b^5 + 32*a^{16} b^4 - 32*a^{17} b^3 \\
& - 8*a^{18} b^2)) / (a^5*(a^{14} b + a^{15} - a^8 b^7 - a^9 b^6 + 3a^{10} b^5 \\
& + 3a^{11} b^4 - 3a^{12} b^3 - 3a^{13} b^2)))*(A*a^2*i + A*b^2*12i - B*a*b*6i \\
&)) / (2*a^5)*(A*a^2*i + A*b^2*12i - B*a*b*6i)) / (2*a^5))*(A*a^2*i + A*b^2* \\
& 12i - B*a*b*6i)*i) / (a^5*d) + (b^2*atan(((b^2*((a + b)^5*(a - b)^5)^(1/2))* \\
& (8*\tan(c/2 + (d*x)/2)*(A^2*a^{14} + 288*A^2*b^{14} - 288*A^2*a*b^{13} - 2*A^2*a^1 \\
& 3*b - 1104*A^2*a^2*b^{12} + 1104*A^2*a^3*b^{11} + 1538*A^2*a^4*b^{10} - 1538*A^2* \\
& a^5*b^9 - 827*A^2*a^6*b^8 + 872*A^2*a^7*b^7 + 18*A^2*a^8*b^6 - 108*A^2*a^9* \\
& b^5 + 74*A^2*a^{10} b^4 - 40*A^2*a^{11} b^3 + 21*A^2*a^{12} b^2 + 72*B^2*a^2*b^{12} \\
& - 72*B^2*a^3*b^{11} - 288*B^2*a^4*b^{10} + 288*B^2*a^5*b^9 + 441*B^2*a^6*b^8 - \\
& 432*B^2*a^7*b^7 - 288*B^2*a^8*b^6 + 288*B^2*a^9*b^5 + 36*B^2*a^{10} b^4 - 72 \\
& *B^2*a^{11} b^3 + 36*B^2*a^{12} b^2 - 288*A*B*a*b^{13} - 12*A*B*a^{13} b + 288*A*B* \\
& a^2*b^{12} + 1128*A*B*a^3*b^{11} - 1128*A*B*a^4*b^{10} - 1650*A*B*a^5*b^9 + 1632* \\
& A*B*a^6*b^8 + 984*A*B*a^7*b^7 - 1008*A*B*a^8*b^6 - 72*A*B*a^9*b^5 + 192*A*B \\
& *a^{10} b^4 - 108*A*B*a^{11} b^3 + 24*A*B*a^{12} b^2)) / (a^{14} b + a^{15} - a^8 b^7 - \\
& a^9 b^6 + 3a^{10} b^5 + 3a^{11} b^4 - 3a^{12} b^3 - 3a^{13} b^2) + (b^2*((4*(4* \\
& *A*a^{21} - 48*A*a^{10} b^{11} + 24*A*a^{11} b^{10} + 212*A*a^{12} b^9 - 100*A*a^{13} b^8 \\
& - 360*A*a^{14} b^7 + 164*A*a^{15} b^6 + 276*A*a^{16} b^5 - 120*A*a^{17} b^4 - 80*A \\
& *a^{18} b^3 + 28*A*a^{19} b^2 + 24*B*a^{11} b^{10} - 12*B*a^{12} b^9 - 108*B*a^{13} b^8 \\
& + 48*B*a^{14} b^7 + 192*B*a^{15} b^6 - 84*B*a^{16} b^5 - 156*B*a^{17} b^4 + 72*B*a^{18} b^3 \\
& + 48*B*a^{19} b^2 - 24*B*a^{20} b)) / (a^{18} b + a^{19} - a^{12} b^7 - a^{13} b^6 \\
& + 3a^{14} b^5 + 3a^{15} b^4 - 3a^{16} b^3 - 3a^{17} b^2) - (4*b^2*\tan(c/2 + (\\
& d*x)/2)*((a + b)^5*(a - b)^5)^(1/2)*(12*A*b^5 - 12*B*a^5 - 29*A*a^2*b^3 + 1 \\
& 5*B*a^3*b^2 + 20*A*a^4*b - 6*B*a*b^4)*(8*a^{19} b - 8*a^{10} b^{10} + 8*a^{11} b^9 \\
& + 32*a^{12} b^8 - 32*a^{13} b^7 - 48*a^{14} b^6 + 48*a^{15} b^5 + 32*a^{16} b^4 - 32* \\
& a^{17} b^3 - 8*a^{18} b^2)) / ((a^{15} - a^5*b^{10} + 5*a^7*b^8 - 10*a^9*b^6 + 10*a^1 \\
& 1*b^4 - 5*a^{13} b^2)*(a^{14} b + a^{15} - a^8 b^7 - a^9 b^6 + 3a^{10} b^5 + 3a^1 \\
& 1*b^4 - 3a^{12} b^3 - 3a^{13} b^2)))*((a + b)^5*(a - b)^5)^(1/2)*(12*A*b^5 - \\
& 12*B*a^5 - 29*A*a^2*b^3 + 15*B*a^3*b^2 + 20*A*a^4*b - 6*B*a*b^4)) / (2*(a^{15} \\
& - a^5*b^{10} + 5*a^7*b^8 - 10*a^9*b^6 + 10*a^{11} b^4 - 5*a^{13} b^2)))*(12*A*b^5 \\
& - 12*B*a^5 - 29*A*a^2*b^3 + 15*B*a^3*b^2 + 20*A*a^4*b - 6*B*a*b^4)*1i) / (2* \\
& (a^{15} - a^5*b^{10} + 5*a^7*b^8 - 10*a^9*b^6 + 10*a^{11} b^4 - 5*a^{13} b^2)) + (b \\
& ^2*((a + b)^5*(a - b)^5)^(1/2)*((8*\tan(c/2 + (d*x)/2)*(A^2*a^{14} + 288*A^2*b \\
& ^{14} - 288*A^2*a*b^{13} - 2*A^2*a^{13} b - 1104*A^2*a^2*b^{12} + 1104*A^2*a^3*b^{11} \\
& + 1538*A^2*a^4*b^{10} - 1538*A^2*a^5*b^9 - 827*A^2*a^6*b^8 + 872*A^2*a^7*b^7 \\
& + 18*A^2*a^8*b^6 - 108*A^2*a^9*b^5 + 74*A^2*a^{10} b^4 - 40*A^2*a^{11} b^3 + 2 \\
& 1*A^2*a^{12} b^2 + 72*B^2*a^2*b^{12} - 72*B^2*a^3*b^{11} - 288*B^2*a^4*b^{10} + 288
\end{aligned}$$

$$\begin{aligned}
& *B^2*a^5*b^9 + 441*B^2*a^6*b^8 - 432*B^2*a^7*b^7 - 288*B^2*a^8*b^6 + 288*B^2*a^9*b^5 + 36*B^2*a^{10}*b^4 - 72*B^2*a^{11}*b^3 + 36*B^2*a^{12}*b^2 - 288*A*B*a*b^{13} - 12*A*B*a^{13}*b + 288*A*B*a^2*b^{12} + 1128*A*B*a^3*b^{11} - 1128*A*B*a^4*b^{10} - 1650*A*B*a^5*b^9 + 1632*A*B*a^6*b^8 + 984*A*B*a^7*b^7 - 1008*A*B*a^8*b^6 - 72*A*B*a^9*b^5 + 192*A*B*a^{10}*b^4 - 108*A*B*a^{11}*b^3 + 24*A*B*a^{12}*b^2) / (a^{14}*b + a^{15} - a^8*b^7 - a^9*b^6 + 3*a^{10}*b^5 + 3*a^{11}*b^4 - 3*a^{12}*b^3 - 3*a^{13}*b^2) - (b^2*((4*(4*A*a^{21} - 48*A*a^{10}*b^{11} + 24*A*a^{11}*b^{10} + 212*A*a^{12}*b^9 - 100*A*a^{13}*b^8 - 360*A*a^{14}*b^7 + 164*A*a^{15}*b^6 + 276*A*a^{16}*b^5 - 120*A*a^{17}*b^4 - 80*A*a^{18}*b^3 + 28*A*a^{19}*b^2 + 24*B*a^{11}*b^{10} - 12*B*a^{12}*b^9 - 108*B*a^{13}*b^8 + 48*B*a^{14}*b^7 + 192*B*a^{15}*b^6 - 84*B*a^{16}*b^5 - 156*B*a^{17}*b^4 + 72*B*a^{18}*b^3 + 48*B*a^{19}*b^2 - 24*B*a^{20}*b)) / (a^{18}*b + a^{19} - a^{12}*b^7 - a^{13}*b^6 + 3*a^{14}*b^5 + 3*a^{15}*b^4 - 3*a^{16}*b^3 - 3*a^{17}*b^2) + (4*b^2*tan(c/2 + (d*x)/2)*((a + b)^5*(a - b)^5)^{(1/2)}*(12*A*b^5 - 12*B*a^5 - 29*A*a^2*b^3 + 15*B*a^3*b^2 + 20*A*a^4*b - 6*B*a*b^4)*(8*a^{19}*b - 8*a^{10}*b^{10} + 8*a^{11}*b^9 + 32*a^{12}*b^8 - 32*a^{13}*b^7 - 48*a^{14}*b^6 + 48*a^{15}*b^5 + 32*a^{16}*b^4 - 32*a^{17}*b^3 - 8*a^{18}*b^2)) / ((a^{15} - a^5*b^{10} + 5*a^7*b^8 - 10*a^9*b^6 + 10*a^{11}*b^4 - 5*a^{13}*b^2)*(a^{14}*b + a^{15} - a^8*b^7 - a^9*b^6 + 3*a^{10}*b^5 + 3*a^{11}*b^4 - 3*a^{12}*b^3 - 3*a^{13}*b^2))) * ((a + b)^5*(a - b)^5)^{(1/2)}*(12*A*b^5 - 12*B*a^5 - 29*A*a^2*b^3 + 15*B*a^3*b^2 + 20*A*a^4*b - 6*B*a*b^4) / (2*(a^{15} - a^5*b^{10} + 5*a^7*b^8 - 10*a^9*b^6 + 10*a^{11}*b^4 - 5*a^{13}*b^2))) * (12*A*b^5 - 12*B*a^5 - 29*A*a^2*b^3 + 15*B*a^3*b^2 + 20*A*a^4*b - 6*B*a*b^4) * i) / (2*(a^{15} - a^5*b^{10} + 5*a^7*b^8 - 10*a^9*b^6 + 10*a^{11}*b^4 - 5*a^{13}*b^2))) / ((8*(1728*A^3*b^{15} - 864*A^3*a*b^{14} - 7344*A^3*a^2*b^{13} + 3456*A^3*a^3*b^{12} + 11700*A^3*a^4*b^{11} - 4770*A^3*a^5*b^{10} - 7829*A^3*a^6*b^9 + 2326*A^3*a^7*b^8 + 1314*A^3*a^8*b^7 - 11*A^3*a^9*b^6 + 411*A^3*a^{10}*b^5 - 20*A^3*a^{11}*b^4 + 20*A^3*a^{12}*b^3 - 216*B^3*a^3*b^{12} + 108*B^3*a^4*b^{11} + 972*B^3*a^5*b^{10} - 486*B^3*a^6*b^9 - 1728*B^3*a^7*b^8 + 756*B^3*a^8*b^7 + 1404*B^3*a^9*b^6 - 432*B^3*a^{10}*b^5 - 432*B^3*a^{11}*b^4 - 2592*A^2*B*a*b^{14} + 1296*A*B^2*a^2*b^{13} - 648*A*B^2*a^3*b^{12} - 5724*A*B^2*a^4*b^{11} + 2808*A*B^2*a^5*b^{10} + 9828*A*B^2*a^6*b^9 - 4203*A*B^2*a^7*b^8 - 7524*A*B^2*a^8*b^7 + 2268*A*B^2*a^9*b^6 + 1980*A*B^2*a^{10}*b^5 + 144*A*B^2*a^{12}*b^3 + 1296*A^2*B*a^2*b^{13} + 11232*A^2*B*a^3*b^{12} - 5400*A^2*B*a^4*b^{11} - 18594*A^2*B*a^5*b^{10} + 7767*A^2*B*a^6*b^9 + 13347*A^2*B*a^7*b^8 - 3972*A^2*B*a^8*b^7 - 2892*A^2*B*a^9*b^6 + 9*A^2*B*a^{10}*b^5 - 489*A^2*B*a^{11}*b^4 + 12*A^2*B*a^{12}*b^3 - 12*A^2*B*a^{13}*b^2)) / (a^{18}*b + a^{19} - a^{12}*b^7 - a^{13}*b^6 + 3*a^{14}*b^5 + 3*a^{15}*b^4 - 3*a^{16}*b^3 - 3*a^{17}*b^2) - (b^2*((a + b)^5*(a - b)^5)^{(1/2)}*((8*tan(c/2 + (d*x)/2)*(A^2*a^{14} + 288*A^2*b^{14} - 288*A^2*a*b^{13} - 2*A^2*a^{13}*b - 1104*A^2*a^2*b^{12} + 1104*A^2*a^3*b^{11} + 1538*A^2*a^4*b^{10} - 1538*A^2*a^5*b^9 - 827*A^2*a^6*b^8 + 872*A^2*a^7*b^7 + 18*A^2*a^8*b^6 - 108*A^2*a^9*b^5 + 74*A^2*a^{10}*b^4 - 40*A^2*a^{11}*b^3 + 21*A^2*a^{12}*b^2 + 72*B^2*a^2*b^{12} - 72*B^2*a^3*b^{11} - 288*B^2*a^4*b^{10} + 288*B^2*a^5*b^9 + 441*B^2*a^6*b^8 - 432*B^2*a^7*b^7 - 288*B^2*a^8*b^6 + 288*B^2*a^9*b^5 + 36*B^2*a^{10}*b^4 - 72*B^2*a^{11}*b^3 + 36*B^2*a^{12}*b^2 - 288*A*B*a*b^{13} - 12*A*B*a^{13}*b + 288*A*B*a^2*b^{12} + 1128*A*B*a^3*b^{11} - 1128*A*B*a^4*b^{10} - 1650*A*B*a^5*b^9 + 1632*A*B*a^6*b^8 + 984*A*B*a^7*b^7 - 1008*A*B*a^8*b^6 - 72*A*B*a^9*b^
\end{aligned}$$

$$\begin{aligned}
& 5 + 192* A * B * a^{10} * b^4 - 108 * A * B * a^{11} * b^3 + 24 * A * B * a^{12} * b^2) / (a^{14} * b + a^{15} \\
& - a^8 * b^7 - a^9 * b^6 + 3 * a^{10} * b^5 + 3 * a^{11} * b^4 - 3 * a^{12} * b^3 - 3 * a^{13} * b^2) + \\
& (b^2 * ((4 * (4 * A * a^{21} - 48 * A * a^{10} * b^{11} + 24 * A * a^{11} * b^{10} + 212 * A * a^{12} * b^9 - 100 \\
& * A * a^{13} * b^8 - 360 * A * a^{14} * b^7 + 164 * A * a^{15} * b^6 + 276 * A * a^{16} * b^5 - 120 * A * a^{17} \\
& * b^4 - 80 * A * a^{18} * b^3 + 28 * A * a^{19} * b^2 + 24 * B * a^{11} * b^{10} - 12 * B * a^{12} * b^9 - 108 \\
& * B * a^{13} * b^8 + 48 * B * a^{14} * b^7 + 192 * B * a^{15} * b^6 - 84 * B * a^{16} * b^5 - 156 * B * a^{17} * b \\
& ^4 + 72 * B * a^{18} * b^3 + 48 * B * a^{19} * b^2 - 24 * B * a^{20} * b)) / (a^{18} * b + a^{19} - a^{12} * b^7 \\
& - a^{13} * b^6 + 3 * a^{14} * b^5 + 3 * a^{15} * b^4 - 3 * a^{16} * b^3 - 3 * a^{17} * b^2) - (4 * b^2 * \\
& \tan(c/2 + (d*x)/2) * ((a + b)^5 * (a - b)^5)^{(1/2)} * (12 * A * b^5 - 12 * B * a^5 - 29 * A * \\
& a^2 * b^3 + 15 * B * a^3 * b^2 + 20 * A * a^4 * b - 6 * B * a * b^4) * (8 * a^{19} * b - 8 * a^{10} * b^{10} + \\
& 8 * a^{11} * b^9 + 32 * a^{12} * b^8 - 32 * a^{13} * b^7 - 48 * a^{14} * b^6 + 48 * a^{15} * b^5 + 32 * a^{16} * b^4 \\
& - 32 * a^{17} * b^3 - 8 * a^{18} * b^2)) / ((a^{15} - a^5 * b^{10} + 5 * a^7 * b^8 - 10 * a^9 * b^6 + 10 * a^{11} * b^4 - 5 * a^{13} * b^2) * \\
& (a^{14} * b + a^{15} - a^8 * b^7 - a^9 * b^6 + 3 * a^{10} * b^5 + 3 * a^{11} * b^4 - 3 * a^{12} * b^3 - 3 * a^{13} * b^2)) * ((a + b)^5 * (a - b)^5)^{(1/2)} * (\\
& 12 * A * b^5 - 12 * B * a^5 - 29 * A * a^2 * b^3 + 15 * B * a^3 * b^2 + 20 * A * a^4 * b - 6 * B * a * b^4) \\
&) / (2 * (a^{15} - a^5 * b^{10} + 5 * a^7 * b^8 - 10 * a^9 * b^6 + 10 * a^{11} * b^4 - 5 * a^{13} * b^2)) \\
&) * (12 * A * b^5 - 12 * B * a^5 - 29 * A * a^2 * b^3 + 15 * B * a^3 * b^2 + 20 * A * a^4 * b - 6 * B * a * b^4) \\
&) / (2 * (a^{15} - a^5 * b^{10} + 5 * a^7 * b^8 - 10 * a^9 * b^6 + 10 * a^{11} * b^4 - 5 * a^{13} * b^2)) + (b^2 * ((a + b)^5 * (a - b)^5)^{(1/2)} * ((8 * \tan(c/2 + (d*x)/2) * (A^2 * a^{14} + 2 \\
& 88 * A^2 * b^{14} - 288 * A^2 * a * b^{13} - 2 * A^2 * a^{13} * b - 1104 * A^2 * a^2 * b^{12} + 1104 * A^2 * \\
& a^3 * b^{11} + 1538 * A^2 * a^4 * b^{10} - 1538 * A^2 * a^5 * b^9 - 827 * A^2 * a^6 * b^8 + 872 * A^2 * \\
& a^7 * b^7 + 18 * A^2 * a^8 * b^6 - 108 * A^2 * a^9 * b^5 + 74 * A^2 * a^{10} * b^4 - 40 * A^2 * a^{11} \\
& * b^3 + 21 * A^2 * a^{12} * b^2 + 72 * B^2 * a^2 * b^{12} - 72 * B^2 * a^3 * b^{11} - 288 * B^2 * a^4 * b^{10} \\
& + 288 * B^2 * a^5 * b^9 + 441 * B^2 * a^6 * b^8 - 432 * B^2 * a^7 * b^7 - 288 * B^2 * a^8 * b^6 \\
& + 288 * B^2 * a^9 * b^5 + 36 * B^2 * a^{10} * b^4 - 72 * B^2 * a^{11} * b^3 + 36 * B^2 * a^{12} * b^2 - 2 \\
& 88 * A * B * a * b^{13} - 12 * A * B * a^{13} * b + 288 * A * B * a^2 * b^{12} + 1128 * A * B * a^3 * b^{11} - 1128 \\
& * A * B * a^4 * b^{10} - 1650 * A * B * a^5 * b^9 + 1632 * A * B * a^6 * b^8 + 984 * A * B * a^7 * b^7 - 100 \\
& 8 * A * B * a^8 * b^6 - 72 * A * B * a^9 * b^5 + 192 * A * B * a^{10} * b^4 - 108 * A * B * a^{11} * b^3 + 24 * A \\
& * B * a^{12} * b^2)) / (a^{14} * b + a^{15} - a^8 * b^7 - a^9 * b^6 + 3 * a^{10} * b^5 + 3 * a^{11} * b^4 \\
& - 3 * a^{12} * b^3 - 3 * a^{13} * b^2) - (b^2 * ((4 * (4 * A * a^{21} - 48 * A * a^{10} * b^{11} + 24 * A * a^{11} * b^{10} + 212 * A * a^{12} * b^9 - 100 * A * a^{13} * b^8 - 360 * A * a^{14} * b^7 + 164 * A * a^{15} * b^6 + 276 * A * a^{16} * b^5 - 120 * A * a^{17} * b^4 - 80 * A * a^{18} * b^3 + 28 * A * a^{19} * b^2 + 24 * B * a^{11} * b^{10} - 12 * B * a^{12} * b^9 - 108 * B * a^{13} * b^8 + 48 * B * a^{14} * b^7 + 192 * B * a^{15} * b^6 - 84 * B * a^{16} * b^5 - 156 * B * a^{17} * b^4 + 72 * B * a^{18} * b^3 + 48 * B * a^{19} * b^2 - 24 * B * a^{20} * b)) / (a^{18} * b + a^{19} - a^{12} * b^7 - a^{13} * b^6 + 3 * a^{14} * b^5 + 3 * a^{15} * b^4 - 3 * a^{16} * b^3 - 3 * a^{17} * b^2) + (4 * b^2 * \tan(c/2 + (d*x)/2) * ((a + b)^5 * (a - b)^5)^{(1/2)} * (12 * A * b^5 - 12 * B * a^5 - 29 * A * a^2 * b^3 + 15 * B * a^3 * b^2 + 20 * A * a^4 * b - 6 * B * a * b^4) * (8 * a^{19} * b - 8 * a^{10} * b^{10} + 8 * a^{11} * b^9 + 32 * a^{12} * b^8 - 32 * a^{13} * b^7 - 48 * a^{14} * b^6 + 48 * a^{15} * b^5 + 32 * a^{16} * b^4 - 32 * a^{17} * b^3 - 8 * a^{18} * b^2)) / ((a^{15} - a^5 * b^{10} + 5 * a^7 * b^8 - 10 * a^9 * b^6 + 10 * a^{11} * b^4 - 5 * a^{13} * b^2) * (a^{14} * b + a^{15} - a^8 * b^7 - a^9 * b^6 + 3 * a^{10} * b^5 + 3 * a^{11} * b^4 - 3 * a^{12} * b^3 - 3 * a^{13} * b^2)) * ((a + b)^5 * (a - b)^5)^{(1/2)} * (12 * A * b^5 - 12 * B * a^5 - 29 * A * a^2 * b^3 + 15 * B * a^3 * b^2 + 20 * A * a^4 * b - 6 * B * a * b^4) / (2 * (a^{15} - a^5 * b^{10} + 5 * a^7 * b^8 - 10 * a^9 * b^6 + 10 * a^{11} * b^4 - 5 * a^{13} * b^2)) * (12 * A * b^5 - 12 * B * a^5 - 29 * A * a^2 * b^3 + 15 * B * a^3 * b^2 + 20 * A * a^4 * b - 6 * B * a * b^4) / (2 * (a^{15} - a^5 * b^{10} + 5 * a^7 * b^8 - 10 * a^9 * b^6 + 10 * a^{11} * b^4 - 5 * a^{13} * b^2))
\end{aligned}$$

$b^6 + 10*a^{11}*b^4 - 5*a^{13}*b^2)))*((a + b)^5*(a - b)^5)^{(1/2)}*(12*A*b^5 - 12*B*a^5 - 29*A*a^2*b^3 + 15*B*a^3*b^2 + 20*A*a^4*b - 6*B*a*b^4)*1i)/(d*(a^{15} - a^5*b^{10} + 5*a^7*b^8 - 10*a^9*b^6 + 10*a^{11}*b^4 - 5*a^{13}*b^2))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \cos^2(c + dx)}{(a + b \sec(c + dx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**3,x)

[Out] Integral((A + B*sec(c + d*x))*cos(c + d*x)**2/(a + b*sec(c + d*x))**3, x)

$$3.336 \quad \int \frac{\sec^5(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^4} dx$$

Optimal. Leaf size=418

$$\frac{a(Ab - aB) \tan(c + dx) \sec^3(c + dx)}{3bd(a^2 - b^2)(a + b \sec(c + dx))^3} + \frac{a(-4a^3B + a^2Ab + 9ab^2B - 6Ab^3) \tan(c + dx) \sec^2(c + dx)}{6b^2d(a^2 - b^2)^2(a + b \sec(c + dx))^2} - \frac{(-12a^4B + \dots)}{b^5d(a - b)}$$

[Out] $(A*b-4*B*a)*\operatorname{arctanh}(\sin(d*x+c))/b^5/d-a*(2*A*a^6*b-7*A*a^4*b^3+8*A*a^2*b^5-8*A*b^7-8*B*a^7+28*B*a^5*b^2-35*B*a^3*b^4+20*B*a*b^6)*\operatorname{arctanh}((a-b)^{(1/2)}*\tan(1/2*d*x+1/2*c)/(a+b)^{(1/2)))/(a-b)^{(7/2)}/b^5/(a+b)^{(7/2)}/d-1/6*(3*A*a^3*b-8*A*a*b^3-12*B*a^4+23*B*a^2*b^2-6*B*b^4)*\tan(d*x+c)/b^4/(a^2-b^2)^2/d+1/3*a*(A*b-B*a)*\sec(d*x+c)^3*\tan(d*x+c)/b/(a^2-b^2)/d/(a+b*\sec(d*x+c))^3+1/6*a*(A*a^2*b-6*A*b^3-4*B*a^3+9*B*a*b^2)*\sec(d*x+c)^2*\tan(d*x+c)/b^2/(a^2-b^2)^2/d/(a+b*\sec(d*x+c))^2-1/2*a^2*(A*a^4*b-2*A*a^2*b^3+6*A*b^5-4*B*a^5+11*B*a^3*b^2-12*B*a*b^4)*\tan(d*x+c)/b^4/(a^2-b^2)^3/d/(a+b*\sec(d*x+c))$

Rubi [A] time = 5.27, antiderivative size = 418, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.290$, Rules used = {4029, 4098, 4090, 4082, 3998, 3770, 3831, 2659, 208}

$$\frac{(3a^3Ab + 23a^2b^2B - 12a^4B - 8aAb^3 - 6b^4B) \tan(c + dx)}{6b^4d(a^2 - b^2)^2} - \frac{a(-7a^4Ab^3 + 8a^2Ab^5 + 2a^6Ab + 28a^5b^2B - 35a^3b^4B)}{b^5d(a - b)}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(\operatorname{Sec}[c + d*x])^5*(A + B*\operatorname{Sec}[c + d*x])]/(a + b*\operatorname{Sec}[c + d*x])^4, x]$

[Out] $((A*b - 4*a*B)*\operatorname{ArcTanh}[\operatorname{Sin}[c + d*x]])/(b^5*d) - (a*(2*a^6*A*b - 7*a^4*A*b^3 + 8*a^2*A*b^5 - 8*A*b^7 - 8*a^7*B + 28*a^5*b^2*B - 35*a^3*b^4*B + 20*a*b^6*B)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a - b]*\operatorname{Tan}[(c + d*x)/2])/ \operatorname{Sqrt}[a + b]])/((a - b)^{(7/2)}*b^5*(a + b)^{(7/2)}*d) - ((3*a^3*A*b - 8*a*A*b^3 - 12*a^4*B + 23*a^2*b^2*B - 6*b^4*B)*\operatorname{Tan}[c + d*x])/(6*b^4*(a^2 - b^2)^2*d) + (a*(A*b - a*B)*\operatorname{Sec}[c + d*x]^3*\operatorname{Tan}[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*\operatorname{Sec}[c + d*x])^3) + (a*(a^2*A*b - 6*A*b^3 - 4*a^3*B + 9*a*b^2*B)*\operatorname{Sec}[c + d*x]^2*\operatorname{Tan}[c + d*x])/(6*b^2*(a^2 - b^2)^2*d*(a + b*\operatorname{Sec}[c + d*x])^2) - (a^2*(a^4*A*b - 2*a^2*A*b^3 + 6*A*b^5 - 4*a^5*B + 11*a^3*b^2*B - 12*a*b^4*B)*\operatorname{Tan}[c + d*x])/(2*b^4*(a^2 - b^2)^3*d*(a + b*\operatorname{Sec}[c + d*x]))$

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 2659

Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3831

Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] := Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 3998

Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] := Dist[B/b, Int[Csc[e + f*x], x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]

Rule 4029

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^m*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] := Simp[(a*d^2*(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[d/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(n - 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2*(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 1]

Rule 4082

Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^m, x_Symbol] := -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))

, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rule 4090

Int[csc[(e_.) + (f_.)*(x_.)]^2*((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_), x_Symbol] := Simp[(a*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(m + 1)*(-a*(b*B - a*C) + A*b^2) + (b*B*(a^2 + b^2*(m + 1)) - a*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1)))*Csc[e + f*x] - b*C*(m + 1)*(a^2 - b^2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]

Rule 4098

Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_), x_Symbol] := -Simp[(d*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1))/(b*f*(a^2 - b^2)*(m + 1)), x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A - b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]

Rubi steps

$$\begin{aligned}
\int \frac{\sec^5(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^4} dx &= \frac{a(Ab-aB)\sec^3(c+dx)\tan(c+dx)}{3b(a^2-b^2)d(a+b\sec(c+dx))^3} + \int \frac{\sec^3(c+dx)(3a(Ab-aB)-3b(Ab-aB)\sec(c+dx))}{(a+b\sec(c+dx))^4} dx \\
&= \frac{a(Ab-aB)\sec^3(c+dx)\tan(c+dx)}{3b(a^2-b^2)d(a+b\sec(c+dx))^3} + \frac{a(a^2Ab-6Ab^3-4a^3B+9ab^2B)}{6b^2(a^2-b^2)^2d(a+b\sec(c+dx))} \\
&= \frac{a(Ab-aB)\sec^3(c+dx)\tan(c+dx)}{3b(a^2-b^2)d(a+b\sec(c+dx))^3} + \frac{a(a^2Ab-6Ab^3-4a^3B+9ab^2B)}{6b^2(a^2-b^2)^2d(a+b\sec(c+dx))} \\
&= -\frac{(3a^3Ab-8aAb^3-12a^4B+23a^2b^2B-6b^4B)\tan(c+dx)}{6b^4(a^2-b^2)^2d} + \frac{a(Ab-aB)}{3b(a^2-b^2)} \\
&= -\frac{(3a^3Ab-8aAb^3-12a^4B+23a^2b^2B-6b^4B)\tan(c+dx)}{6b^4(a^2-b^2)^2d} + \frac{a(Ab-aB)}{3b(a^2-b^2)} \\
&= \frac{(Ab-4aB)\tanh^{-1}(\sin(c+dx))}{b^5d} - \frac{(3a^3Ab-8aAb^3-12a^4B+23a^2b^2B-6b^4B)\tan(c+dx)}{6b^4(a^2-b^2)^2d} \\
&= \frac{(Ab-4aB)\tanh^{-1}(\sin(c+dx))}{b^5d} - \frac{(3a^3Ab-8aAb^3-12a^4B+23a^2b^2B-6b^4B)\tan(c+dx)}{6b^4(a^2-b^2)^2d} \\
&= \frac{(Ab-4aB)\tanh^{-1}(\sin(c+dx))}{b^5d} - \frac{a(2a^6Ab-7a^4Ab^3+8a^2Ab^5-8Ab^7)}{6b^4(a^2-b^2)^2d}
\end{aligned}$$

Mathematica [A] time = 3.25, size = 548, normalized size = 1.31

$$\frac{48a(8a^7B-2a^6Ab-28a^5b^2B+7a^4Ab^3+35a^3b^4B-8a^2Ab^5-20ab^6B+8Ab^7)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2}} + \frac{2b\tan(c+dx)(-24a^9B\cos(3(c+dx))+6a^8Ab\cos(2(c+dx))+4a^7Ab^2\cos(c+dx))}{6b^4(a^2-b^2)^2d}$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[c + d*x]^5*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^4,x]

```
[Out] ((-48*a*(-2*a^6*A*b + 7*a^4*A*b^3 - 8*a^2*A*b^5 + 8*A*b^7 + 8*a^7*B - 28*a^5*b^2*B + 35*a^3*b^4*B - 20*a*b^6*B)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(7/2) - 48*(A*b - 4*a*B)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 48*(A*b - 4*a*B)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (2*b*(30*a^7*A*b^2 - 90*a^5*A*b^4 + 120*a^3*A*b^6 - 120*a^8*b*B + 318*a^6*b^3*B - 246*a^4*b^5*B - 36*a^2*b^7*B + 24*b^9*B + a*(18*a^7*A*b - 7*a^5*A*b^3 - 50*a^3*A*b^5 + 144*a*A*b^7 - 72*a^8*B + 28*a^6*b^2*B + 305*a^4*b^4*B - 438*a^2*b^6*B + 72*b^8*B)*Cos[c + d*x] - 6*a^2*b*(-5*a^5*A*b + 15*a^3*A*b^3 - 20*a*A*b^5 + 20*a^6*B - 57*a^4*b^2*B + 53*a^2*b^4*B - 6*b^6*B)*Cos[2*(c + d*x)] + 6*a^8*A*b*Cos[3*(c + d*x)] - 17*a^6*A*b^3*Cos[3*(c + d*x)] + 26*a^4*A*b^5*Cos[3*(c + d*x)] - 24*a^9*B*Cos[3*(c + d*x)] + 68*a^7*b^2*B*Cos[3*(c + d*x)] - 65*a^5*b^4*B*Cos[3*(c + d*x)] + 6*a^3*b^6*B*Cos[3*(c + d*x)])))*Tan[c + d*x])/((-a^2 + b^2)^3*(b + a*Cos[c + d*x])^3)/(48*b^5*d)
```

fricas [B] time = 137.71, size = 3434, normalized size = 8.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] [-1/12*(3*((8*B*a^11 - 2*A*a^10*b - 28*B*a^9*b^2 + 7*A*a^8*b^3 + 35*B*a^7*b^4 - 8*A*a^6*b^5 - 20*B*a^5*b^6 + 8*A*a^4*b^7)*cos(d*x + c)^4 + 3*(8*B*a^10*b - 2*A*a^9*b^2 - 28*B*a^8*b^3 + 7*A*a^7*b^4 + 35*B*a^6*b^5 - 8*A*a^5*b^6 - 20*B*a^4*b^7 + 8*A*a^3*b^8)*cos(d*x + c)^3 + 3*(8*B*a^9*b^2 - 2*A*a^8*b^3 - 28*B*a^7*b^4 + 7*A*a^6*b^5 + 35*B*a^5*b^6 - 8*A*a^4*b^7 - 20*B*a^3*b^8 + 8*A*a^2*b^9)*cos(d*x + c)^2 + (8*B*a^8*b^3 - 2*A*a^7*b^4 - 28*B*a^6*b^5 + 7*A*a^5*b^6 + 35*B*a^4*b^7 - 8*A*a^3*b^8 - 20*B*a^2*b^9 + 8*A*a*b^10)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c))^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + 6*((4*B*a^12 - A*a^11*b - 16*B*a^10*b^2 + 4*A*a^9*b^3 + 24*B*a^8*b^4 - 6*A*a^7*b^5 - 16*B*a^6*b^6 + 4*A*a^5*b^7 + 4*B*a^4*b^8 - A*a^3*b^9)*cos(d*x + c)^4 + 3*(4*B*a^11*b - A*a^10*b^2 - 16*B*a^9*b^3 + 4*A*a^8*b^4 + 24*B*a^7*b^5 - 6*A*a^6*b^6 - 16*B*a^5*b^7 + 4*A*a^4*b^8 + 4*B*a^3*b^9 - A*a^2*b^10)*cos(d*x + c)^3 + 3*(4*B*a^10*b^2 - A*a^9*b^3 - 16*B*a^8*b^4 + 4*A*a^7*b^5 + 24*B*a^6*b^6 - 6*A*a^5*b^7 - 16*B*a^4*b^8 + 4*A*a^3*b^9 + 4*B*a^2*b^10 - A*a*b^11)*cos(d*x + c)^2 + (4*B*a^9*b^3 - A*a^8*b^4 - 16*B*a^7*b^5 + 4*A*a^6*b^6 + 24*B*a^5*b^7 - 6*A*a^4*b^8 - 16*B*a^3*b^9 + 4*A*a^2*b^10 + 4*B*a*b^11 - A*b^12)*cos(d*x + c))*log(sin(d*x + c) + 1) - 6*((4*B*a^12 - A*a^11*b - 16*B*a^10*b^2 + 4*A*a^9*b^3 + 24*B*a^8*b^4 - 6*A*a^7*b^5 - 16*B*a^6*b^6 + 4*A*a^5*b^7 + 4*B*a^4*b^8 - A*a^3*b^9)*cos(d*x + c)^4 + 3*(4*B*a^11*b - A*a^10*b^2 - 16*B*a^9*b^3 + 4*A*a^8*b^4 + 24*B*a^7*b^5 - 6*A*a^6*b^6 - 16*B*a^5*b^7 + 4*A*a^4*b^8 + 4*B*a^3*b^9 - A*a^2*b^10)*cos(d*x + c)^3 + 3*(4*B*a^10*b^2 - A*a^9*b^3 - 16*B*a^8*b^4
```

$$\begin{aligned}
& 8*b^4 + 4*A*a^7*b^5 + 24*B*a^6*b^6 - 6*A*a^5*b^7 - 16*B*a^4*b^8 + 4*A*a^3*b^9 + 4*B*a^2*b^10 - A*a*b^11) * \cos(dx + c)^2 + (4*B*a^9*b^3 - A*a^8*b^4 - 16*B*a^7*b^5 + 4*A*a^6*b^6 + 24*B*a^5*b^7 - 6*A*a^4*b^8 - 16*B*a^3*b^9 + 4*A*a^2*b^10 + 4*B*a*b^11 - A*b^12) * \cos(dx + c) * \log(-\sin(dx + c) + 1) - 2*(6*B*a^8*b^4 - 24*B*a^6*b^6 + 36*B*a^4*b^8 - 24*B*a^2*b^10 + 6*B*b^12 + (24*B*a^11*b - 6*A*a^10*b^2 - 92*B*a^9*b^3 + 23*A*a^8*b^4 + 133*B*a^7*b^5 - 43*A*a^6*b^6 - 71*B*a^5*b^7 + 26*A*a^4*b^8 + 6*B*a^3*b^9) * \cos(dx + c)^3 + 3*(20*B*a^10*b^2 - 5*A*a^9*b^3 - 77*B*a^8*b^4 + 20*A*a^7*b^5 + 110*B*a^6*b^6 - 35*A*a^5*b^7 - 59*B*a^4*b^8 + 20*A*a^3*b^9 + 6*B*a^2*b^10) * \cos(dx + c)^2 + (44*B*a^9*b^3 - 11*A*a^8*b^4 - 169*B*a^7*b^5 + 43*A*a^6*b^6 + 239*B*a^5*b^7 - 68*A*a^4*b^8 - 132*B*a^3*b^9 + 36*A*a^2*b^10 + 18*B*a*b^11) * \cos(dx + c)) * \sin(dx + c) / ((a^11*b^5 - 4*a^9*b^7 + 6*a^7*b^9 - 4*a^5*b^11 + a^3*b^13) * d * \cos(dx + c)^4 + 3*(a^10*b^6 - 4*a^8*b^8 + 6*a^6*b^10 - 4*a^4*b^12 + a^2*b^14) * d * \cos(dx + c)^3 + 3*(a^9*b^7 - 4*a^7*b^9 + 6*a^5*b^11 - 4*a^3*b^13 + a*b^15) * d * \cos(dx + c)^2 + (a^8*b^8 - 4*a^6*b^10 + 6*a^4*b^12 - 4*a^2*b^14 + b^16) * d * \cos(dx + c)), 1/6*(3*((8*B*a^11 - 2*A*a^10*b - 28*B*a^9*b^2 + 7*A*a^8*b^3 + 35*B*a^7*b^4 - 8*A*a^6*b^5 - 20*B*a^5*b^6 + 8*A*a^4*b^7) * \cos(dx + c)^4 + 3*(8*B*a^10*b - 2*A*a^9*b^2 - 28*B*a^8*b^3 + 7*A*a^7*b^4 + 35*B*a^6*b^5 - 8*A*a^5*b^6 - 20*B*a^4*b^7 + 8*A*a^3*b^8) * \cos(dx + c)^3 + 3*(8*B*a^9*b^2 - 2*A*a^8*b^3 - 28*B*a^7*b^4 + 7*A*a^6*b^5 + 35*B*a^5*b^6 - 8*A*a^4*b^7 - 20*B*a^3*b^8 + 8*A*a^2*b^9) * \cos(dx + c)^2 + (8*B*a^8*b^3 - 2*A*a^7*b^4 - 28*B*a^6*b^5 + 7*A*a^5*b^6 + 35*B*a^4*b^7 - 8*A*a^3*b^8 - 20*B*a^2*b^9 + 8*A*a*b^10) * \cos(dx + c)) * \sqrt{-a^2 + b^2} * \arctan(-\sqrt{-a^2 + b^2}) * (b * \cos(dx + c) + a) / ((a^2 - b^2) * \sin(dx + c))) - 3*((4*B*a^12 - A*a^11*b - 16*B*a^10*b^2 + 4*A*a^9*b^3 + 24*B*a^8*b^4 - 6*A*a^7*b^5 - 16*B*a^6*b^6 + 4*A*a^5*b^7 + 4*B*a^4*b^8 - A*a^3*b^9) * \cos(dx + c)^4 + 3*(4*B*a^11*b - A*a^10*b^2 - 16*B*a^9*b^3 + 4*A*a^8*b^4 + 24*B*a^7*b^5 - 6*A*a^6*b^6 - 16*B*a^5*b^7 + 4*A*a^4*b^8 + 4*B*a^3*b^9 - A*a^2*b^10) * \cos(dx + c)^3 + 3*(4*B*a^10*b^2 - A*a^9*b^3 - 16*B*a^8*b^4 + 4*A*a^7*b^5 + 24*B*a^6*b^6 - 6*A*a^5*b^7 - 16*B*a^4*b^8 + 4*A*a^3*b^9 + 4*B*a^2*b^10 - A*a*b^11) * \cos(dx + c)^2 + (4*B*a^9*b^3 - A*a^8*b^4 - 16*B*a^7*b^5 + 4*A*a^6*b^6 + 24*B*a^5*b^7 - 6*A*a^4*b^8 - 16*B*a^3*b^9 + 4*A*a^2*b^10 + 4*B*a*b^11 - A*b^12) * \cos(dx + c) * \log(\sin(dx + c) + 1) + 3*((4*B*a^12 - A*a^11*b - 16*B*a^10*b^2 + 4*A*a^9*b^3 + 24*B*a^8*b^4 - 6*A*a^7*b^5 - 16*B*a^6*b^6 + 4*A*a^5*b^7 + 4*B*a^4*b^8 - A*a^3*b^9) * \cos(dx + c)^4 + 3*(4*B*a^11*b - A*a^10*b^2 - 16*B*a^9*b^3 + 4*A*a^8*b^4 + 24*B*a^7*b^5 - 6*A*a^6*b^6 - 16*B*a^5*b^7 + 4*A*a^4*b^8 + 4*B*a^3*b^9 - A*a^2*b^10) * \cos(dx + c)^3 + 3*(4*B*a^10*b^2 - A*a^9*b^3 - 16*B*a^8*b^4 + 4*A*a^7*b^5 + 24*B*a^6*b^6 - 6*A*a^5*b^7 - 16*B*a^4*b^8 + 4*A*a^3*b^9 + 4*B*a^2*b^10 - A*a*b^11) * \cos(dx + c)^2 + (4*B*a^9*b^3 - A*a^8*b^4 - 16*B*a^7*b^5 + 4*A*a^6*b^6 + 24*B*a^5*b^7 - 6*A*a^4*b^8 - 16*B*a^3*b^9 + 4*A*a^2*b^10 + 4*B*a*b^11 - A*b^12) * \cos(dx + c)) * \log(-\sin(dx + c) + 1) + (6*B*a^8*b^4 - 24*B*a^6*b^6 + 36*B*a^4*b^8 - 24*B*a^2*b^10 + 6*B*b^12 + (24*B*a^11*b - 6*A*a^10*b^2 - 92*B*a^9*b^3 + 23*A*a^8*b^4 + 133*B*a^7*b^5 - 43*A*a^6*b^6 - 71*B*a^5*b^7 + 26*A*a^4*b^8 + 6*B*a^3*b^9) * \cos(dx + c)^3 + 3*(20*B*a^10*b^2 - 5*A*a^9*b^3 - 77*B*a^8*b^4 + 20*A*a^7*b^5 + 110*B*a^6*b^6
\end{aligned}$$

$$- 35Aa^5b^7 - 59Ba^4b^8 + 20Aa^3b^9 + 6Ba^2b^{10})\cos(dx + c)^2 + (44Ba^9b^3 - 11Aa^8b^4 - 169Ba^7b^5 + 43Aa^6b^6 + 239Ba^5b^7 - 68Aa^4b^8 - 132Ba^3b^9 + 36Aa^2b^{10} + 18Ba^1b^{11})\cos(dx + c)\sin(dx + c)/((a^{11}b^5 - 4a^9b^7 + 6a^7b^9 - 4a^5b^{11} + a^3b^{13})d\cos(dx + c)^4 + 3(a^{10}b^6 - 4a^8b^8 + 6a^6b^{10} - 4a^4b^{12} + a^2b^{14})d\cos(dx + c)^3 + 3(a^9b^7 - 4a^7b^9 + 6a^5b^{11} - 4a^3b^{13} + ab^{15})d\cos(dx + c)^2 + (a^8b^8 - 4a^6b^{10} + 6a^4b^{12} - 4a^2b^{14} + b^{16})d\cos(dx + c))]$$

giac [B] time = 0.47, size = 1005, normalized size = 2.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(dx+c)^5*(A+B*sec(dx+c))/(a+b*sec(dx+c))^4,x, algorithm="giac")

[Out] $\frac{1}{3} \cdot (3 \cdot (8B^3a^8 - 2A^2a^7b - 28B^2a^6b^2 + 7A^2a^5b^3 + 35B^2a^4b^4 - 8A^2a^3b^5 - 20B^2a^2b^6 + 8A^2ab^7) \cdot (\pi \cdot \text{floor}(1/2 \cdot (dx + c)) / \pi + 1/2) \cdot \text{sgn}(-2a + 2b) + \arctan(-(a \cdot \tan(1/2 \cdot dx + 1/2 \cdot c) - b \cdot \tan(1/2 \cdot dx + 1/2 \cdot c)) / \sqrt{-a^2 + b^2})) / ((a^6b^5 - 3a^4b^7 + 3a^2b^9 - b^{11}) \cdot \sqrt{-a^2 + b^2}) - (18B^2a^9 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c)^5 - 6A^2a^8b \cdot \tan(1/2 \cdot dx + 1/2 \cdot c)^5 - 42B^2a^8b \cdot \tan(1/2 \cdot dx + 1/2 \cdot c)^5 + 15A^2a^7b^2 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c)^5 - 24B^2a^7b^2 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c)^5 + 6A^2a^6b^3 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c)^5 + 117B^2a^6b^3 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c)^5 - 45A^2a^5b^4 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c)^5 - 24B^2a^5b^4 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c)^5 + 6A^2a^4b^5 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c)^5 - 105B^2a^4b^5 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c)^5 + 60A^2a^3b^6 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c)^5 + 60B^2a^3b^6 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c)^5 - 36A^2a^2b^7 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c)^5 - 36B^2a^9 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c)^3 + 12A^2a^8b \cdot \tan(1/2 \cdot dx + 1/2 \cdot c)^3 + 152B^2a^7b^2 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c)^3 - 56A^2a^6b^3 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c)^3 - 236B^2a^5b^4 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c)^3 + 116A^2a^4b^5 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c)^3 + 120B^2a^3b^6 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c)^3 - 72A^2a^2b^7 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c)^3 + 18B^2a^9 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c) - 6A^2a^8b \cdot \tan(1/2 \cdot dx + 1/2 \cdot c) + 42B^2a^8b \cdot \tan(1/2 \cdot dx + 1/2 \cdot c) - 15A^2a^7b^2 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c) - 24B^2a^7b^2 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c) + 6A^2a^6b^3 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c) - 117B^2a^6b^3 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c) + 45A^2a^5b^4 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c) - 24B^2a^5b^4 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c) + 6A^2a^4b^5 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c) + 105B^2a^4b^5 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c) - 60A^2a^3b^6 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c) + 60B^2a^3b^6 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c) - 36A^2a^2b^7 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c)) / ((a^6b^4 - 3a^4b^6 + 3a^2b^8 - b^{10}) \cdot (a \cdot \tan(1/2 \cdot dx + 1/2 \cdot c)^2 - b \cdot \tan(1/2 \cdot dx + 1/2 \cdot c)^2 - a - b)^3) - 3 \cdot (4Ba - Ab) \cdot \log(\text{abs}(\tan(1/2 \cdot dx + 1/2 \cdot c) + 1)) / b^5 + 3 \cdot (4Ba - Ab) \cdot \log(\text{abs}(\tan(1/2 \cdot dx + 1/2 \cdot c) - 1)) / b^5 - 6B \cdot \tan(1/2 \cdot dx + 1/2 \cdot c) / ((\tan(1/2 \cdot dx + 1/2 \cdot c)^2 - 1) \cdot b^4)) / d$

$$\frac{1}{2c} \int \frac{(a+b)^{-3} (a^3 - 3a^2b + 3ab^2 - b^3) \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) A + 12/d a^2b / (a \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 b - a - b)^{-3} / (a-b) / (a^3 + 3a^2b + 3ab^2 + b^3) \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5 A - 116/3/d a^5/b^2 / (a \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 b - a - b)^{-3} / (a^2 - 2ab + b^2) / (a^2 + 2ab + b^2) \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 B + 12/d a^2b / (a \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 b - a - b)^{-3} / (a+b) / (a^3 - 3a^2b + 3ab^2 - b^3) \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) A + 2/d a^6/b^3 / (a \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 b - a - b)^{-3} / (a-b) / (a^3 + 3a^2b + 3ab^2 + b^3) \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5 B - 4/d a^6/b^3 / (a \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 b - a - b)^{-3} / (a^2 - 2ab + b^2) / (a^2 + 2ab + b^2) \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 A - 6/d a^7/b^4 / (a \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 b - a - b)^{-3} / (a-b) / (a^3 + 3a^2b + 3ab^2 + b^3) \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5 B - 6/d a^7/b^4 / (a \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 b - a - b)^{-3} / (a+b) / (a^3 - 3a^2b + 3ab^2 - b^3) \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) B - 24/d a^2b / (a \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 b - a - b)^{-3} / (a^2 - 2ab + b^2) / (a^2 + 2ab + b^2) \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 A - 5/d a^4/b / (a \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 b - a - b)^{-3} / (a-b) / (a^3 + 3a^2b + 3ab^2 + b^3) \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5 B - 8/d a^3 / (a^6 - 3a^4b^2 + 3a^2b^4 - b^6) / ((a-b) * (a+b))^{1/2} * \operatorname{arctanh}\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) * (a-b) / ((a-b) * (a+b))^{1/2}\right) A - 1/d B/b^4 / (\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1) - 1/d/b^4 * \ln\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1\right) A - 1/d B/b^4 / (\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1) + 1/d/b^4 * \ln\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right) A + 4/d/b^5 * \ln\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1\right) a * B - 4/d/b^5 * \ln\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right) a * B$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(dx+c)^5*(A+B*sec(dx+c))/(a+b*sec(dx+c))^4,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for more details)Is 4*a^2-4*b^2 positive or negative?

mupad [B] time = 19.96, size = 13092, normalized size = 31.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + dx))/(cos(c + dx)^5*(a + b/cos(c + dx))^4),x)

[Out] ((tan(c/2 + (dx)/2)^7*(12*A*a^2*b^5 - 2*B*b^7 - 8*B*a^7 + 4*A*a^3*b^4 - 6*A*a^4*b^3 - A*a^5*b^2 + 6*B*a^2*b^5 - 26*B*a^3*b^4 - 11*B*a^4*b^3 + 24*B*a^5*b^2 + 2*A*a^6*b + 2*B*a*b^6 + 4*B*a^6*b)) / (b^4*(a + b)^3*(a - b)) - (tan(c/2 + (dx)/2)^3*(72*B*a^8 + 18*B*b^8 + 36*A*a^2*b^6 - 96*A*a^3*b^5 - 14*A*

$$\begin{aligned}
 & a^4b^4 + 59Aa^5b^3 + 3Aa^6b^2 - 72B^2a^2b^6 - 60B^3a^3b^5 + 273B^4a^4b^4 + 47B^5a^5b^3 - 236B^6a^6b^2 - 18Aa^7b - 12B^7a^7b) / (3b^4(a + b)^2(a - b)^3) + (\tan(c/2 + (dx)/2)^5(72B^2a^8 + 18B^3b^8 - 36Aa^2b^6 - 96Aa^3b^5 + 14Aa^4b^4 + 59Aa^5b^3 - 3Aa^6b^2 - 72B^2a^2b^6 + 60B^3a^3b^5 + 273B^4a^4b^4 - 47B^5a^5b^3 - 236B^6a^6b^2 - 18Aa^7b + 12B^7a^7b)) / (3b^4(a + b)^3(a - b)^2) - (\tan(c/2 + (dx)/2) * (2B^7b^7 - 8B^8a^7 + 12Aa^2b^5 - 4Aa^3b^4 - 6Aa^4b^3 + Aa^5b^2 - 6B^2a^2b^5 - 26B^3a^3b^4 + 11B^4a^4b^3 + 24B^5a^5b^2 + 2Aa^6b + 2B^7a^7b^6 - 4B^8a^8b)) / (b^4(a + b)(a - b)^3)) / (d * (3a^2b^2 + 3a^2b - \tan(c/2 + (dx)/2)^4(6a^2b^2 - 6a^3) - \tan(c/2 + (dx)/2)^2(6a^2b + 4a^3 - 2b^3) - \tan(c/2 + (dx)/2)^6(4a^3 - 6a^2b + 2b^3) + a^3 + b^3 + \tan(c/2 + (dx)/2)^8(3a^2b^2 - 3a^2b + a^3 - b^3))) + (\operatorname{atan}((((((A^2b - 4B^2a) * ((8 * (4A^2b^24 - 12Aa^2b^22 + 64Aa^3b^21 + 20Aa^4b^20 - 110Aa^5b^19 - 30Aa^6b^18 + 110Aa^7b^17 + 30Aa^8b^16 - 70Aa^9b^15 - 14Aa^10b^14 + 26Aa^11b^13 + 2Aa^12b^12 - 4Aa^13b^11 + 40B^2a^2b^22 + 72B^3a^3b^21 - 190B^4a^4b^20 - 146B^5a^5b^19 + 386B^6a^6b^18 + 174B^7a^7b^17 - 434B^8a^8b^16 - 126B^9a^9b^15 + 286B^10a^10b^14 + 50B^11a^11b^13 - 104B^12a^12b^12 - 8B^13a^13b^11 + 16B^14a^14b^10 - 16Aa^2b^23 - 16B^7a^7b^23))) / (a^2b^22 + b^23 - 5a^2b^21 - 5a^3b^20 + 10a^4b^19 + 10a^5b^18 - 10a^6b^17 - 10a^7b^16 + 5a^8b^15 + 5a^9b^14 - a^10b^13 - a^11b^12) - (8 * \tan(c/2 + (dx)/2) * (A^2b - 4B^2a) * (8a^2b^23 - 8a^2b^22 - 48a^3b^21 + 48a^4b^20 + 120a^5b^19 - 120a^6b^18 - 160a^7b^17 + 160a^8b^16 + 120a^9b^15 - 120a^10b^14 - 48a^11b^13 + 48a^12b^12 + 8a^13b^11 - 8a^14b^10))) / (b^5(a^2b^18 + b^19 - 5a^2b^17 - 5a^3b^16 + 10a^4b^15 + 10a^5b^14 - 10a^6b^13 - 10a^7b^12 + 5a^8b^11 + 5a^9b^10 - a^10b^9 - a^11b^8)))) / b^5 - (8 * \tan(c/2 + (dx)/2) * (4A^2b^16 + 128B^2a^16 - 8A^2a^2b^15 - 128B^2a^15b + 44A^2a^2b^14 + 48A^2a^3b^13 - 92A^2a^4b^12 - 120A^2a^5b^11 + 156A^2a^6b^10 + 160A^2a^7b^9 - 164A^2a^8b^8 - 120A^2a^9b^7 + 117A^2a^10b^6 + 48A^2a^11b^5 - 48A^2a^12b^4 - 8A^2a^13b^3 + 8A^2a^14b^2 + 64B^2a^2b^14 - 128B^2a^3b^13 + 80B^2a^4b^12 + 768B^2a^5b^11 - 824B^2a^6b^10 - 1920B^2a^7b^9 + 2025B^2a^8b^8 + 2560B^2a^9b^7 - 2600B^2a^10b^6 - 1920B^2a^11b^5 + 1920B^2a^12b^4 + 768B^2a^13b^3 - 768B^2a^14b^2 - 32A^2a^2b^15 - 64A^2a^2b^15b + 64A^2a^2b^14 - 160A^2a^3b^13 - 384A^2a^4b^12 + 592A^2a^5b^11 + 960A^2a^6b^10 - 1128A^2a^7b^9 - 1280A^2a^8b^8 + 1306A^2a^9b^7 + 960A^2a^10b^6 - 948A^2a^11b^5 - 384A^2a^12b^4 + 384A^2a^13b^3 + 64A^2a^14b^2)) / (a^2b^18 + b^19 - 5a^2b^17 - 5a^3b^16 + 10a^4b^15 + 10a^5b^14 - 10a^6b^13 - 10a^7b^12 + 5a^8b^11 + 5a^9b^10 - a^10b^9 - a^11b^8)) * (A^2b - 4B^2a) * i) / b^5 - (((A^2b - 4B^2a) * ((8 * (4A^2b^24 - 12Aa^2b^22 + 64Aa^3b^21 + 20Aa^4b^20 - 110Aa^5b^19 - 30Aa^6b^18 + 110Aa^7b^17 + 30Aa^8b^16 - 70Aa^9b^15 - 14Aa^10b^14 + 26Aa^11b^13 + 2Aa^12b^12 - 4Aa^13b^11 + 40B^2a^2b^22 + 72B^3a^3b^21 - 190B^4a^4b^20 - 146B^5a^5b^19 + 386B^6a^6b^18 + 174B^7a^7b^17 - 434B^8a^8b^16 - 126B^9a^9b^15 + 286B^10a^10b^14 + 50B^11a^11b^13 - 104B^12a^12b^12 - 8B^13a^13b^11 + 16B^14a^14b^10 - 16A^2a^2b^23)
 \end{aligned}$$

$$\begin{aligned}
& \left(a^{23} - 16Bab^{23} \right) / \left(a^{22} + b^{23} - 5a^2b^{21} - 5a^3b^{20} + 10a^4b^{19} \right. \\
& + 10a^5b^{18} - 10a^6b^{17} - 10a^7b^{16} + 5a^8b^{15} + 5a^9b^{14} - a^{10}b^{13} \\
& - a^{11}b^{12} \left. \right) + \left(8 \tan(c/2 + (dx)/2) \cdot (Ab - 4B^2a) \cdot (8a^3b^{23} - 8a^2b^{22} \right. \\
& - 48a^3b^{21} + 48a^4b^{20} + 120a^5b^{19} - 120a^6b^{18} - 160a^7b^{17} \\
& + 160a^8b^{16} + 120a^9b^{15} - 120a^{10}b^{14} - 48a^{11}b^{13} + 48a^{12}b^{12} \\
& + 8a^{13}b^{11} - 8a^{14}b^{10} \left. \right) / \left(b^5 \cdot (a^{18} + b^{19} - 5a^2b^{17} - 5a^3b^{16} \right. \\
& + 10a^4b^{15} + 10a^5b^{14} - 10a^6b^{13} - 10a^7b^{12} + 5a^8b^{11} + 5a^9b^{10} \\
& - a^{10}b^9 - a^{11}b^8 \left. \right) \left. \right) / b^5 + \left(8 \tan(c/2 + (dx)/2) \cdot (4A^2b^{16} \right. \\
& + 128B^2a^{16} - 8A^2ab^{15} - 128B^2a^{15}b + 44A^2a^2b^{14} + 48A^2a^3b^{13} \\
& - 92A^2a^4b^{12} - 120A^2a^5b^{11} + 156A^2a^6b^{10} + 160A^2a^7b^9 \\
& - 164A^2a^8b^8 - 120A^2a^9b^7 + 117A^2a^{10}b^6 + 48A^2a^{11}b^5 \\
& - 48A^2a^{12}b^4 - 8A^2a^{13}b^3 + 8A^2a^{14}b^2 + 64B^2a^2b^{14} \\
& - 128B^2a^3b^{13} + 80B^2a^4b^{12} + 768B^2a^5b^{11} - 824B^2a^6b^{10} \\
& - 1920B^2a^7b^9 + 2025B^2a^8b^8 + 2560B^2a^9b^7 - 2600B^2a^{10}b^6 \\
& - 1920B^2a^{11}b^5 + 1920B^2a^{12}b^4 + 768B^2a^{13}b^3 - 768B^2a^{14}b^2 \\
& - 32ABab^{15} - 64ABa^{15}b + 64ABa^2b^{14} - 160ABa^3b^{13} \\
& - 384ABa^4b^{12} + 592ABa^5b^{11} + 960ABa^6b^{10} - 1128ABa^7b^9 \\
& - 1280ABa^8b^8 + 1306ABa^9b^7 + 960ABa^{10}b^6 - 948ABa^{11}b^5 \\
& - 384ABa^{12}b^4 + 384ABa^{13}b^3 + 64ABa^{14}b^2 \left. \right) / \left(a^{18} + b^{19} \right. \\
& - 5a^2b^{17} - 5a^3b^{16} + 10a^4b^{15} + 10a^5b^{14} - 10a^6b^{13} - 10a^7b^{12} \\
& + 5a^8b^{11} + 5a^9b^{10} - a^{10}b^9 - a^{11}b^8 \left. \right) \cdot (Ab - 4B^2a) \cdot i) \\
& / b^5 / \left(\left(\left(\left(Ab - 4B^2a \right) \cdot \left(8 \left(4A^4b^{24} - 12A^4a^2b^{22} + 64A^4a^3b^{21} + 20A^4a^4b^{20} \right. \right. \right. \right. \right. \right. \\
& - 110A^4a^5b^{19} - 30A^4a^6b^{18} + 110A^4a^7b^{17} + 30A^4a^8b^{16} - 70A^4a^9b^{15} \\
& - 14A^4a^{10}b^{14} + 26A^4a^{11}b^{13} + 2A^4a^{12}b^{12} - 4A^4a^{13}b^{11} + 40B^4a^2b^{22} \\
& + 72B^4a^3b^{21} - 190B^4a^4b^{20} - 146B^4a^5b^{19} + 386B^4a^6b^{18} + 174B^4a^7b^{17} \\
& - 434B^4a^8b^{16} - 126B^4a^9b^{15} + 286B^4a^{10}b^{14} + 50B^4a^{11}b^{13} - 104B^4a^{12}b^{12} \\
& - 8B^4a^{13}b^{11} + 16B^4a^{14}b^{10} - 16A^4ab^{23} - 16B^4ab^{23} \left. \right) \left. \right) / \left(a^{22} + b^{23} - 5a^2b^{21} - 5a^3b^{20} \right. \\
& + 10a^4b^{19} + 10a^5b^{18} - 10a^6b^{17} - 10a^7b^{16} + 5a^8b^{15} + 5a^9b^{14} \\
& - a^{10}b^{13} - a^{11}b^{12} \left. \right) - \left(8 \tan(c/2 + (dx)/2) \cdot (Ab - 4B^2a) \cdot (8a^3b^{23} \right. \\
& - 8a^2b^{22} - 48a^3b^{21} + 48a^4b^{20} + 120a^5b^{19} - 120a^6b^{18} - 160a^7b^{17} \\
& + 160a^8b^{16} + 120a^9b^{15} - 120a^{10}b^{14} - 48a^{11}b^{13} + 48a^{12}b^{12} \\
& + 8a^{13}b^{11} - 8a^{14}b^{10} \left. \right) / \left(b^5 \cdot (a^{18} + b^{19} - 5a^2b^{17} - 5a^3b^{16} \right. \\
& + 10a^4b^{15} + 10a^5b^{14} - 10a^6b^{13} - 10a^7b^{12} + 5a^8b^{11} + 5a^9b^{10} \\
& - a^{10}b^9 - a^{11}b^8 \left. \right) \left. \right) / b^5 - \left(8 \tan(c/2 + (dx)/2) \cdot (4A^2b^{16} \right. \\
& + 128B^2a^{16} - 8A^2ab^{15} - 128B^2a^{15}b + 44A^2a^2b^{14} + 48A^2a^3b^{13} \\
& - 92A^2a^4b^{12} - 120A^2a^5b^{11} + 156A^2a^6b^{10} + 160A^2a^7b^9 \\
& - 164A^2a^8b^8 - 120A^2a^9b^7 + 117A^2a^{10}b^6 + 48A^2a^{11}b^5 \\
& - 48A^2a^{12}b^4 - 8A^2a^{13}b^3 + 8A^2a^{14}b^2 + 64B^2a^2b^{14} \\
& - 128B^2a^3b^{13} + 80B^2a^4b^{12} + 768B^2a^5b^{11} - 824B^2a^6b^{10} \\
& - 1920B^2a^7b^9 + 2025B^2a^8b^8 + 2560B^2a^9b^7 - 2600B^2a^{10}b^6 \\
& - 1920B^2a^{11}b^5 + 1920B^2a^{12}b^4 + 768B^2a^{13}b^3 - 768B^2a^{14}b^2 \\
& - 32ABab^{15} - 64ABa^{15}b + 64ABa^2b^{14} - 160ABa^3b^{13} \\
& - 384ABa^4b^{12} + 592ABa^5b^{11} + 960ABa^6b^{10} - 1128ABa^7b^9 \\
& - 1280ABa^8b^8 + 1306ABa^9b^7 + 960ABa^{10}b^6 -
\end{aligned}$$

$$\begin{aligned}
& 948*A*B*a^{11}*b^5 - 384*A*B*a^{12}*b^4 + 384*A*B*a^{13}*b^3 + 64*A*B*a^{14}*b^2)) / \\
& (a*b^{18} + b^{19} - 5*a^2*b^{17} - 5*a^3*b^{16} + 10*a^4*b^{15} + 10*a^5*b^{14} - 10*a^6*b^{13} - 10*a^7*b^{12} + 5*a^8*b^{11} + 5*a^9*b^{10} - a^{10}*b^9 - a^{11}*b^8)) * (A*b - 4*B*a)) / b^5 - \\
& (16*(256*B^3*a^{16} - 16*A^3*a*b^{15} - 128*B^3*a^{15}*b - 48*A^3*a^2*b^{14} + 64*A^3*a^3*b^{13} + 64*A^3*a^4*b^{12} - 110*A^3*a^5*b^{11} - 66*A^3*a^6*b^{10} + 110*A^3*a^7*b^9 + 34*A^3*a^8*b^8 - 70*A^3*a^9*b^7 - 11*A^3*a^{10}*b^6 + 26*A^3*a^{11}*b^5 + 2*A^3*a^{12}*b^4 - 4*A^3*a^{13}*b^3 + 640*B^3*a^4*b^{12} + 960*B^3*a^5*b^{11} - 3040*B^3*a^6*b^{10} - 2560*B^3*a^7*b^9 + 6176*B^3*a^8*b^8 + 3204*B^3*a^9*b^7 - 6944*B^3*a^{10}*b^6 - 2176*B^3*a^{11}*b^5 + 4576*B^3*a^{12}*b^4 + 800*B^3*a^{13}*b^3 - 1664*B^3*a^{14}*b^2 - 192*A*B^2*a^{15}*b - 576*A*B^2*a^3*b^{13} - 1104*A*B^2*a^4*b^{12} + 2544*A*B^2*a^5*b^{11} + 2376*A*B^2*a^6*b^{10} - 4848*A*B^2*a^7*b^9 - 2649*A*B^2*a^8*b^8 + 5232*A*B^2*a^9*b^7 + 1632*A*B^2*a^{10}*b^6 - 3408*A*B^2*a^{11}*b^5 - 576*A*B^2*a^{12}*b^4 + 1248*A*B^2*a^{13}*b^3 + 96*A*B^2*a^{14}*b^2 + 168*A^2*B*a^2*b^{14} + 408*A^2*B*a^3*b^{13} - 702*A^2*B*a^4*b^{12} - 690*A^2*B*a^5*b^{11} + 1266*A^2*B*a^6*b^{10} + 726*A^2*B*a^7*b^9 - 1314*A^2*B*a^8*b^8 - 408*A^2*B*a^9*b^7 + 846*A^2*B*a^{10}*b^6 + 138*A^2*B*a^{11}*b^5 - 312*A^2*B*a^{12}*b^4 - 24*A^2*B*a^{13}*b^3 + 48*A^2*B*a^{14}*b^2)) / (a*b^2 + b^{23} - 5*a^2*b^{21} - 5*a^3*b^{20} + 10*a^4*b^{19} + 10*a^5*b^{18} - 10*a^6*b^{17} - 10*a^7*b^{16} + 5*a^8*b^{15} + 5*a^9*b^{14} - a^{10}*b^{13} - a^{11}*b^{12}) + (((A*b - 4*B*a)) * ((8*(4*A*b^{24} - 12*A*a^2*b^{22} + 64*A*a^3*b^{21} + 20*A*a^4*b^{20} - 110*A*a^5*b^{19} - 30*A*a^6*b^{18} + 110*A*a^7*b^{17} + 30*A*a^8*b^{16} - 70*A*a^9*b^{15} - 14*A*a^{10}*b^{14} + 26*A*a^{11}*b^{13} + 2*A*a^{12}*b^{12} - 4*A*a^{13}*b^{11} + 40*B*a^2*b^{22} + 72*B*a^3*b^{21} - 190*B*a^4*b^{20} - 146*B*a^5*b^{19} + 386*B*a^6*b^{18} + 174*B*a^7*b^{17} - 434*B*a^8*b^{16} - 126*B*a^9*b^{15} + 286*B*a^{10}*b^{14} + 50*B*a^{11}*b^{13} - 104*B*a^{12}*b^{12} - 8*B*a^{13}*b^{11} + 16*B*a^{14}*b^{10} - 16*A*a*b^{23} - 16*B*a*b^{23}))) / (a*b^{22} + b^{23} - 5*a^2*b^{21} - 5*a^3*b^{20} + 10*a^4*b^{19} + 10*a^5*b^{18} - 10*a^6*b^{17} - 10*a^7*b^{16} + 5*a^8*b^{15} + 5*a^9*b^{14} - a^{10}*b^{13} - a^{11}*b^{12}) + (8*tan(c/2 + (d*x)/2) * (A*b - 4*B*a)) * (8*a*b^{23} - 8*a^2*b^{22} - 48*a^3*b^{21} + 48*a^4*b^{20} + 120*a^5*b^{19} - 120*a^6*b^{18} - 160*a^7*b^{17} + 160*a^8*b^{16} + 120*a^9*b^{15} - 120*a^{10}*b^{14} - 48*a^{11}*b^{13} + 48*a^{12}*b^{12} + 8*a^{13}*b^{11} - 8*a^{14}*b^{10})) / (b^5 * (a*b^{18} + b^{19} - 5*a^2*b^{17} - 5*a^3*b^{16} + 10*a^4*b^{15} + 10*a^5*b^{14} - 10*a^6*b^{13} - 10*a^7*b^{12} + 5*a^8*b^{11} + 5*a^9*b^{10} - a^{10}*b^9 - a^{11}*b^8))) / b^5 + (8*tan(c/2 + (d*x)/2) * (4*A^2*b^{16} + 128*B^2*a^{16} - 8*A^2*a*b^{15} - 128*B^2*a^{15}*b + 44*A^2*a^2*b^{14} + 48*A^2*a^3*b^{13} - 92*A^2*a^4*b^{12} - 120*A^2*a^5*b^{11} + 156*A^2*a^6*b^{10} + 160*A^2*a^7*b^9 - 164*A^2*a^8*b^8 - 120*A^2*a^9*b^7 + 117*A^2*a^{10}*b^6 + 48*A^2*a^{11}*b^5 - 48*A^2*a^{12}*b^4 - 8*A^2*a^{13}*b^3 + 8*A^2*a^{14}*b^2 + 64*B^2*a^2*b^{14} - 128*B^2*a^3*b^{13} + 80*B^2*a^4*b^{12} + 768*B^2*a^5*b^{11} - 824*B^2*a^6*b^{10} - 1920*B^2*a^7*b^9 + 2025*B^2*a^8*b^8 + 2560*B^2*a^9*b^7 - 2600*B^2*a^{10}*b^6 - 1920*B^2*a^{11}*b^5 + 1920*B^2*a^{12}*b^4 + 768*B^2*a^{13}*b^3 - 768*B^2*a^{14}*b^2 - 32*A*B*a*b^{15} - 64*A*B*a^{15}*b + 64*A*B*a^2*b^{14} - 160*A*B*a^3*b^{13} - 384*A*B*a^4*b^{12} + 592*A*B*a^5*b^{11} + 960*A*B*a^6*b^{10} - 1128*A*B*a^7*b^9 - 1280*A*B*a^8*b^8 + 1306*A*B*a^9*b^7 + 960*A*B*a^{10}*b^6 - 948*A*B*a^{11}*b^5 - 384*A*B*a^{12}*b^4 + 384*A*B*a^{13}*b^3 + 64*A*B*a^{14}*b^2)) / (a*b^{18} + b^{19} - 5*a^2*b^{17} - 5*a^3*b^{16} + 10*a^4*b^{15} + 10*a^5*b^{14} - 10*a^6*b^{13} - 10*
\end{aligned}$$

$$\begin{aligned}
& a^7b^{12} + 5a^8b^{11} + 5a^9b^{10} - a^{10}b^9 - a^{11}b^8) \cdot (A \cdot b - 4B \cdot a) / b \\
& ^5) \cdot (A \cdot b - 4B \cdot a) \cdot 2i) / (b^5 \cdot d) + (a \cdot \operatorname{atan}(((a \cdot ((8 \cdot \tan(c/2 + (d \cdot x)/2) \cdot (4 \cdot A^2 \cdot \\
& b^{16} + 128 \cdot B^2 \cdot a^{16} - 8 \cdot A^2 \cdot a \cdot b^{15} - 128 \cdot B^2 \cdot a^{15} \cdot b + 44 \cdot A^2 \cdot a^2 \cdot b^{14} + 48 \cdot \\
& A^2 \cdot a^3 \cdot b^{13} - 92 \cdot A^2 \cdot a^4 \cdot b^{12} - 120 \cdot A^2 \cdot a^5 \cdot b^{11} + 156 \cdot A^2 \cdot a^6 \cdot b^{10} + 160 \cdot \\
& A^2 \cdot a^7 \cdot b^9 - 164 \cdot A^2 \cdot a^8 \cdot b^8 - 120 \cdot A^2 \cdot a^9 \cdot b^7 + 117 \cdot A^2 \cdot a^{10} \cdot b^6 + 48 \cdot A^2 \\
& \cdot a^{11} \cdot b^5 - 48 \cdot A^2 \cdot a^{12} \cdot b^4 - 8 \cdot A^2 \cdot a^{13} \cdot b^3 + 8 \cdot A^2 \cdot a^{14} \cdot b^2 + 64 \cdot B^2 \cdot a^2 \cdot \\
& b^{14} - 128 \cdot B^2 \cdot a^3 \cdot b^{13} + 80 \cdot B^2 \cdot a^4 \cdot b^{12} + 768 \cdot B^2 \cdot a^5 \cdot b^{11} - 824 \cdot B^2 \cdot a^6 \cdot \\
& b^{10} - 1920 \cdot B^2 \cdot a^7 \cdot b^9 + 2025 \cdot B^2 \cdot a^8 \cdot b^8 + 2560 \cdot B^2 \cdot a^9 \cdot b^7 - 2600 \cdot B^2 \cdot a^{10} \cdot b^6 - \\
& 1920 \cdot B^2 \cdot a^{11} \cdot b^5 + 1920 \cdot B^2 \cdot a^{12} \cdot b^4 + 768 \cdot B^2 \cdot a^{13} \cdot b^3 - 768 \cdot B^2 \\
& \cdot a^{14} \cdot b^2 - 32 \cdot A \cdot B \cdot a \cdot b^{15} - 64 \cdot A \cdot B \cdot a^{15} \cdot b + 64 \cdot A \cdot B \cdot a^2 \cdot b^{14} - 160 \cdot A \cdot B \cdot a^3 \cdot b^{13} \\
& ^{13} - 384 \cdot A \cdot B \cdot a^4 \cdot b^{12} + 592 \cdot A \cdot B \cdot a^5 \cdot b^{11} + 960 \cdot A \cdot B \cdot a^6 \cdot b^{10} - 1128 \cdot A \cdot B \cdot a^7 \\
& \cdot b^9 - 1280 \cdot A \cdot B \cdot a^8 \cdot b^8 + 1306 \cdot A \cdot B \cdot a^9 \cdot b^7 + 960 \cdot A \cdot B \cdot a^{10} \cdot b^6 - 948 \cdot A \cdot B \cdot a^{11} \\
& \cdot b^5 - 384 \cdot A \cdot B \cdot a^{12} \cdot b^4 + 384 \cdot A \cdot B \cdot a^{13} \cdot b^3 + 64 \cdot A \cdot B \cdot a^{14} \cdot b^2)) / (a \cdot b^{18} + b \\
& ^{19} - 5 \cdot a^2 \cdot b^{17} - 5 \cdot a^3 \cdot b^{16} + 10 \cdot a^4 \cdot b^{15} + 10 \cdot a^5 \cdot b^{14} - 10 \cdot a^6 \cdot b^{13} - 1 \\
& 0 \cdot a^7 \cdot b^{12} + 5 \cdot a^8 \cdot b^{11} + 5 \cdot a^9 \cdot b^{10} - a^{10} \cdot b^9 - a^{11} \cdot b^8) - (a \cdot ((a + b)^7 \\
& \cdot (a - b)^7)^{(1/2)} \cdot ((8 \cdot (4 \cdot A \cdot b^{24} - 12 \cdot A \cdot a^2 \cdot b^{22} + 64 \cdot A \cdot a^3 \cdot b^{21} + 20 \cdot A \cdot a^4 \cdot \\
& b^{20} - 110 \cdot A \cdot a^5 \cdot b^{19} - 30 \cdot A \cdot a^6 \cdot b^{18} + 110 \cdot A \cdot a^7 \cdot b^{17} + 30 \cdot A \cdot a^8 \cdot b^{16} - 70 \\
& \cdot A \cdot a^9 \cdot b^{15} - 14 \cdot A \cdot a^{10} \cdot b^{14} + 26 \cdot A \cdot a^{11} \cdot b^{13} + 2 \cdot A \cdot a^{12} \cdot b^{12} - 4 \cdot A \cdot a^{13} \cdot b^{11} \\
& + 40 \cdot B \cdot a^2 \cdot b^{22} + 72 \cdot B \cdot a^3 \cdot b^{21} - 190 \cdot B \cdot a^4 \cdot b^{20} - 146 \cdot B \cdot a^5 \cdot b^{19} + 386 \cdot \\
& B \cdot a^6 \cdot b^{18} + 174 \cdot B \cdot a^7 \cdot b^{17} - 434 \cdot B \cdot a^8 \cdot b^{16} - 126 \cdot B \cdot a^9 \cdot b^{15} + 286 \cdot B \cdot a^{10} \cdot \\
& b^{14} + 50 \cdot B \cdot a^{11} \cdot b^{13} - 104 \cdot B \cdot a^{12} \cdot b^{12} - 8 \cdot B \cdot a^{13} \cdot b^{11} + 16 \cdot B \cdot a^{14} \cdot b^{10} - \\
& 16 \cdot A \cdot a \cdot b^{23} - 16 \cdot B \cdot a \cdot b^{23})) / (a \cdot b^{22} + b^{23} - 5 \cdot a^2 \cdot b^{21} - 5 \cdot a^3 \cdot b^{20} + 10 \cdot a \\
& ^4 \cdot b^{19} + 10 \cdot a^5 \cdot b^{18} - 10 \cdot a^6 \cdot b^{17} - 10 \cdot a^7 \cdot b^{16} + 5 \cdot a^8 \cdot b^{15} + 5 \cdot a^9 \cdot b^{14} \\
& - a^{10} \cdot b^{13} - a^{11} \cdot b^{12}) - (4 \cdot a \cdot \tan(c/2 + (d \cdot x)/2) \cdot ((a + b)^7 \cdot (a - b)^7)^{(1/2)} \cdot (8 \cdot A \cdot b^7 + 8 \cdot B \cdot a^7 - 8 \cdot A \cdot a^2 \cdot b^5 + 7 \cdot A \cdot a^4 \cdot b^3 + 35 \cdot B \cdot a^3 \cdot b^4 - 28 \cdot B \cdot a^5 \cdot b^2 - 2 \cdot A \cdot a^6 \cdot b - 20 \cdot B \cdot a \cdot b^6) \cdot (8 \cdot a \cdot b^{23} - 8 \cdot a^2 \cdot b^{22} - 48 \cdot a^3 \cdot b^{21} + 48 \cdot a^4 \cdot b^{20} + 120 \cdot a^5 \cdot b^{19} - 120 \cdot a^6 \cdot b^{18} - 160 \cdot a^7 \cdot b^{17} + 160 \cdot a^8 \cdot b^{16} + 120 \cdot a^9 \cdot b^{15} - 120 \cdot a^{10} \cdot b^{14} - 48 \cdot a^{11} \cdot b^{13} + 48 \cdot a^{12} \cdot b^{12} + 8 \cdot a^{13} \cdot b^{11} - 8 \cdot a^{14} \cdot b^{10})) / ((b^{19} - 7 \cdot a^2 \cdot b^{17} + 21 \cdot a^4 \cdot b^{15} - 35 \cdot a^6 \cdot b^{13} + 35 \cdot a^8 \cdot b^{11} - 21 \cdot a^{10} \cdot b^9 + 7 \cdot a^{12} \cdot b^7 - a^{14} \cdot b^5) \cdot (a \cdot b^{18} + b^{19} - 5 \cdot a^2 \cdot b^{17} - 5 \cdot a^3 \cdot b^{16} + 10 \cdot a^4 \cdot b^{15} + 10 \cdot a^5 \cdot b^{14} - 10 \cdot a^6 \cdot b^{13} - 10 \cdot a^7 \cdot b^{12} + 5 \cdot a^8 \cdot b^{11} + 5 \cdot a^9 \cdot b^{10} - a^{10} \cdot b^9 - a^{11} \cdot b^8)) \cdot (8 \cdot A \cdot b^7 + 8 \cdot B \cdot a^7 - 8 \cdot A \cdot a^2 \cdot b^5 + 7 \cdot A \cdot a^4 \cdot b^3 + 35 \cdot B \cdot a^3 \cdot b^4 - 28 \cdot B \cdot a^5 \cdot b^2 - 2 \cdot A \cdot a^6 \cdot b - 20 \cdot B \cdot a \cdot b^6)) / (2 \cdot (b^{19} - 7 \cdot a^2 \cdot b^{17} + 21 \cdot a^4 \cdot b^{15} - 35 \cdot a^6 \cdot b^{13} + 35 \cdot a^8 \cdot b^{11} - 21 \cdot a^{10} \cdot b^9 + 7 \cdot a^{12} \cdot b^7 - a^{14} \cdot b^5))) \cdot ((a + b)^7 \cdot (a - b)^7)^{(1/2)} \cdot (8 \cdot A \cdot b^7 + 8 \cdot B \cdot a^7 - 8 \cdot A \cdot a^2 \cdot b^5 + 7 \cdot A \cdot a^4 \cdot b^3 + 35 \cdot B \cdot a^3 \cdot b^4 - 28 \cdot B \cdot a^5 \cdot b^2 - 2 \cdot A \cdot a^6 \cdot b - 20 \cdot B \cdot a \cdot b^6) \cdot i) / (2 \cdot (b^{19} - 7 \cdot a^2 \cdot b^{17} + 21 \cdot a^4 \cdot b^{15} - 35 \cdot a^6 \cdot b^{13} + 35 \cdot a^8 \cdot b^{11} - 21 \cdot a^{10} \cdot b^9 + 7 \cdot a^{12} \cdot b^7 - a^{14} \cdot b^5)) + (a \cdot ((8 \cdot \tan(c/2 + (d \cdot x)/2) \cdot (4 \cdot A^2 \cdot b^{16} + 128 \cdot B^2 \cdot a^{16} - 8 \cdot A^2 \cdot a \cdot b^{15} - 128 \cdot B^2 \cdot a^{15} \cdot b + 44 \cdot A^2 \cdot a^2 \cdot b^{14} + 48 \cdot A^2 \cdot a^3 \cdot b^{13} - 92 \cdot A^2 \cdot a^4 \cdot b^{12} - 120 \cdot A^2 \cdot a^5 \cdot b^{11} + 156 \cdot A^2 \cdot a^6 \cdot b^{10} + 160 \cdot A^2 \cdot a^7 \cdot b^9 - 164 \cdot A^2 \cdot a^8 \cdot b^8 - 120 \cdot A^2 \cdot a^9 \cdot b^7 + 117 \cdot A^2 \cdot a^{10} \cdot b^6 + 48 \cdot A^2 \cdot a^{11} \cdot b^5 - 48 \cdot A^2 \cdot a^{12} \cdot b^4 - 8 \cdot A^2 \cdot a^{13} \cdot b^3 + 8 \cdot A^2 \cdot a^{14} \cdot b^2 + 64 \cdot B^2 \cdot a^2 \cdot b^{14} - 128 \cdot B^2 \cdot a^3 \cdot b^{13} + 80 \cdot B^2 \cdot a^4 \cdot b^{12} + 768 \cdot B^2 \cdot a^5 \cdot b^{11} - 824 \cdot B^2 \cdot a^6 \cdot b^{10} - 1920 \cdot B^2 \cdot a^7 \cdot b^9 + 2025 \cdot B^2 \cdot a^8 \cdot b^8 + 2560 \cdot B^2 \cdot a^9 \cdot b^7 - 2600 \cdot B^2 \cdot a^{10} \cdot b^6 - 1920 \cdot B^2 \cdot a^{11} \cdot b^5 + 1920 \cdot B^2 \cdot a^{12} \cdot b^4 + 768 \cdot B^2 \cdot a^{13} \cdot b^3 - 768 \cdot B^2 \cdot a^{14} \cdot b^2
\end{aligned}$$

$$\begin{aligned}
& 2 - 32*A*B*a*b^{15} - 64*A*B*a^{15}*b + 64*A*B*a^2*b^{14} - 160*A*B*a^3*b^{13} - 38 \\
& 4*A*B*a^4*b^{12} + 592*A*B*a^5*b^{11} + 960*A*B*a^6*b^{10} - 1128*A*B*a^7*b^9 - 1 \\
& 280*A*B*a^8*b^8 + 1306*A*B*a^9*b^7 + 960*A*B*a^{10}*b^6 - 948*A*B*a^{11}*b^5 - \\
& 384*A*B*a^{12}*b^4 + 384*A*B*a^{13}*b^3 + 64*A*B*a^{14}*b^2)) / (a*b^{18} + b^{19} - 5* \\
& a^2*b^{17} - 5*a^3*b^{16} + 10*a^4*b^{15} + 10*a^5*b^{14} - 10*a^6*b^{13} - 10*a^7*b^{12} \\
& + 5*a^8*b^{11} + 5*a^9*b^{10} - a^{10}*b^9 - a^{11}*b^8) + (a*((a + b)^7*(a - b) \\
& ^7)^{(1/2)}*((8*(4*A*b^{24} - 12*A*a^2*b^{22} + 64*A*a^3*b^{21} + 20*A*a^4*b^{20} - 1 \\
& 10*A*a^5*b^{19} - 30*A*a^6*b^{18} + 110*A*a^7*b^{17} + 30*A*a^8*b^{16} - 70*A*a^9*b \\
& ^{15} - 14*A*a^{10}*b^{14} + 26*A*a^{11}*b^{13} + 2*A*a^{12}*b^{12} - 4*A*a^{13}*b^{11} + 40* \\
& B*a^2*b^{22} + 72*B*a^3*b^{21} - 190*B*a^4*b^{20} - 146*B*a^5*b^{19} + 386*B*a^6*b^{18} \\
& + 174*B*a^7*b^{17} - 434*B*a^8*b^{16} - 126*B*a^9*b^{15} + 286*B*a^{10}*b^{14} + 5 \\
& 0*B*a^{11}*b^{13} - 104*B*a^{12}*b^{12} - 8*B*a^{13}*b^{11} + 16*B*a^{14}*b^{10} - 16*A*a*b \\
& ^{23} - 16*B*a*b^{23})) / (a*b^{22} + b^{23} - 5*a^2*b^{21} - 5*a^3*b^{20} + 10*a^4*b^{19} \\
& + 10*a^5*b^{18} - 10*a^6*b^{17} - 10*a^7*b^{16} + 5*a^8*b^{15} + 5*a^9*b^{14} - a^{10}* \\
& b^{13} - a^{11}*b^{12}) + (4*a*tan(c/2 + (d*x)/2)*((a + b)^7*(a - b)^7)^{(1/2)}*(8* \\
& A*b^7 + 8*B*a^7 - 8*A*a^2*b^5 + 7*A*a^4*b^3 + 35*B*a^3*b^4 - 28*B*a^5*b^2 - \\
& 2*A*a^6*b - 20*B*a*b^6)*(8*a*b^{23} - 8*a^2*b^{22} - 48*a^3*b^{21} + 48*a^4*b^{20} \\
& + 120*a^5*b^{19} - 120*a^6*b^{18} - 160*a^7*b^{17} + 160*a^8*b^{16} + 120*a^9*b^{15} \\
& - 120*a^{10}*b^{14} - 48*a^{11}*b^{13} + 48*a^{12}*b^{12} + 8*a^{13}*b^{11} - 8*a^{14}*b^{10}) \\
&) / ((b^{19} - 7*a^2*b^{17} + 21*a^4*b^{15} - 35*a^6*b^{13} + 35*a^8*b^{11} - 21*a^{10}*b^9 \\
& + 7*a^{12}*b^7 - a^{14}*b^5)*(a*b^{18} + b^{19} - 5*a^2*b^{17} - 5*a^3*b^{16} + 10*a \\
& ^4*b^{15} + 10*a^5*b^{14} - 10*a^6*b^{13} - 10*a^7*b^{12} + 5*a^8*b^{11} + 5*a^9*b^{10} \\
& - a^{10}*b^9 - a^{11}*b^8)))*(8*A*b^7 + 8*B*a^7 - 8*A*a^2*b^5 + 7*A*a^4*b^3 + \\
& 35*B*a^3*b^4 - 28*B*a^5*b^2 - 2*A*a^6*b - 20*B*a*b^6)) / (2*(b^{19} - 7*a^2*b^{17} \\
& + 21*a^4*b^{15} - 35*a^6*b^{13} + 35*a^8*b^{11} - 21*a^{10}*b^9 + 7*a^{12}*b^7 - a^{14} \\
& *b^5)))*((a + b)^7*(a - b)^7)^{(1/2)}*(8*A*b^7 + 8*B*a^7 - 8*A*a^2*b^5 + 7* \\
& A*a^4*b^3 + 35*B*a^3*b^4 - 28*B*a^5*b^2 - 2*A*a^6*b - 20*B*a*b^6)*i) / (2*(b \\
& ^{19} - 7*a^2*b^{17} + 21*a^4*b^{15} - 35*a^6*b^{13} + 35*a^8*b^{11} - 21*a^{10}*b^9 + \\
& 7*a^{12}*b^7 - a^{14}*b^5)) / ((16*(256*B^3*a^{16} - 16*A^3*a*b^{15} - 128*B^3*a^{15}* \\
& b - 48*A^3*a^2*b^{14} + 64*A^3*a^3*b^{13} + 64*A^3*a^4*b^{12} - 110*A^3*a^5*b^{11} \\
& - 66*A^3*a^6*b^{10} + 110*A^3*a^7*b^9 + 34*A^3*a^8*b^8 - 70*A^3*a^9*b^7 - 11* \\
& A^3*a^{10}*b^6 + 26*A^3*a^{11}*b^5 + 2*A^3*a^{12}*b^4 - 4*A^3*a^{13}*b^3 + 640*B^3* \\
& a^4*b^{12} + 960*B^3*a^5*b^{11} - 3040*B^3*a^6*b^{10} - 2560*B^3*a^7*b^9 + 6176*B \\
& ^3*a^8*b^8 + 3204*B^3*a^9*b^7 - 6944*B^3*a^{10}*b^6 - 2176*B^3*a^{11}*b^5 + 457 \\
& 6*B^3*a^{12}*b^4 + 800*B^3*a^{13}*b^3 - 1664*B^3*a^{14}*b^2 - 192*A*B^2*a^{15}*b - \\
& 576*A*B^2*a^3*b^{13} - 1104*A*B^2*a^4*b^{12} + 2544*A*B^2*a^5*b^{11} + 2376*A*B^2 \\
& *a^6*b^{10} - 4848*A*B^2*a^7*b^9 - 2649*A*B^2*a^8*b^8 + 5232*A*B^2*a^9*b^7 + \\
& 1632*A*B^2*a^{10}*b^6 - 3408*A*B^2*a^{11}*b^5 - 576*A*B^2*a^{12}*b^4 + 1248*A*B^2 \\
& *a^{13}*b^3 + 96*A*B^2*a^{14}*b^2 + 168*A^2*B*a^2*b^{14} + 408*A^2*B*a^3*b^{13} - 7 \\
& 02*A^2*B*a^4*b^{12} - 690*A^2*B*a^5*b^{11} + 1266*A^2*B*a^6*b^{10} + 726*A^2*B*a^ \\
& 7*b^9 - 1314*A^2*B*a^8*b^8 - 408*A^2*B*a^9*b^7 + 846*A^2*B*a^{10}*b^6 + 138*A \\
& ^2*B*a^{11}*b^5 - 312*A^2*B*a^{12}*b^4 - 24*A^2*B*a^{13}*b^3 + 48*A^2*B*a^{14}*b^2) \\
&) / (a*b^{22} + b^{23} - 5*a^2*b^{21} - 5*a^3*b^{20} + 10*a^4*b^{19} + 10*a^5*b^{18} - 10 \\
& *a^6*b^{17} - 10*a^7*b^{16} + 5*a^8*b^{15} + 5*a^9*b^{14} - a^{10}*b^{13} - a^{11}*b^{12}) \\
& + (a*((8*tan(c/2 + (d*x)/2)*(4*A^2*b^{16} + 128*B^2*a^{16} - 8*A^2*a*b^{15} - 128
\end{aligned}$$

$$\begin{aligned}
& *B^2*a^{15}*b + 44*A^2*a^2*b^{14} + 48*A^2*a^3*b^{13} - 92*A^2*a^4*b^{12} - 120*A^2 \\
& *a^5*b^{11} + 156*A^2*a^6*b^{10} + 160*A^2*a^7*b^9 - 164*A^2*a^8*b^8 - 120*A^2* \\
& a^9*b^7 + 117*A^2*a^{10}*b^6 + 48*A^2*a^{11}*b^5 - 48*A^2*a^{12}*b^4 - 8*A^2*a^{13} \\
& *b^3 + 8*A^2*a^{14}*b^2 + 64*B^2*a^2*b^{14} - 128*B^2*a^3*b^{13} + 80*B^2*a^4*b^{12} \\
& + 768*B^2*a^5*b^{11} - 824*B^2*a^6*b^{10} - 1920*B^2*a^7*b^9 + 2025*B^2*a^8*b^8 \\
& + 2560*B^2*a^9*b^7 - 2600*B^2*a^{10}*b^6 - 1920*B^2*a^{11}*b^5 + 1920*B^2*a^{12}*b^4 \\
& + 768*B^2*a^{13}*b^3 - 768*B^2*a^{14}*b^2 - 32*A*B*a*b^{15} - 64*A*B*a^{15}* \\
& b + 64*A*B*a^2*b^{14} - 160*A*B*a^3*b^{13} - 384*A*B*a^4*b^{12} + 592*A*B*a^5*b^{11} \\
& + 960*A*B*a^6*b^{10} - 1128*A*B*a^7*b^9 - 1280*A*B*a^8*b^8 + 1306*A*B*a^9*b^7 \\
& + 960*A*B*a^{10}*b^6 - 948*A*B*a^{11}*b^5 - 384*A*B*a^{12}*b^4 + 384*A*B*a^{13}* \\
& b^3 + 64*A*B*a^{14}*b^2)/(a*b^{18} + b^{19} - 5*a^2*b^{17} - 5*a^3*b^{16} + 10*a^4*b \\
& ^{15} + 10*a^5*b^{14} - 10*a^6*b^{13} - 10*a^7*b^{12} + 5*a^8*b^{11} + 5*a^9*b^{10} - a \\
& ^{10}*b^9 - a^{11}*b^8) - (a*((a + b)^7*(a - b)^7)^{(1/2)}*((8*(4*A*b^{24} - 12*A*a \\
& ^2*b^{22} + 64*A*a^3*b^{21} + 20*A*a^4*b^{20} - 110*A*a^5*b^{19} - 30*A*a^6*b^{18} + \\
& 110*A*a^7*b^{17} + 30*A*a^8*b^{16} - 70*A*a^9*b^{15} - 14*A*a^{10}*b^{14} + 26*A*a^{11} \\
& *b^{13} + 2*A*a^{12}*b^{12} - 4*A*a^{13}*b^{11} + 40*B*a^2*b^{22} + 72*B*a^3*b^{21} - 190 \\
& *B*a^4*b^{20} - 146*B*a^5*b^{19} + 386*B*a^6*b^{18} + 174*B*a^7*b^{17} - 434*B*a^8* \\
& b^{16} - 126*B*a^9*b^{15} + 286*B*a^{10}*b^{14} + 50*B*a^{11}*b^{13} - 104*B*a^{12}*b^{12} \\
& - 8*B*a^{13}*b^{11} + 16*B*a^{14}*b^{10} - 16*A*a*b^{23} - 16*B*a*b^{23}))/ (a*b^{22} + b^{23} \\
& - 5*a^2*b^{21} - 5*a^3*b^{20} + 10*a^4*b^{19} + 10*a^5*b^{18} - 10*a^6*b^{17} - 10 \\
& *a^7*b^{16} + 5*a^8*b^{15} + 5*a^9*b^{14} - a^{10}*b^{13} - a^{11}*b^{12}) - (4*a*tan(c/2 \\
& + (d*x)/2)*((a + b)^7*(a - b)^7)^{(1/2)}*(8*A*b^7 + 8*B*a^7 - 8*A*a^2*b^5 + \\
& 7*A*a^4*b^3 + 35*B*a^3*b^4 - 28*B*a^5*b^2 - 2*A*a^6*b - 20*B*a*b^6))*(8*a*b^ \\
& 23 - 8*a^2*b^{22} - 48*a^3*b^{21} + 48*a^4*b^{20} + 120*a^5*b^{19} - 120*a^6*b^{18} - \\
& 160*a^7*b^{17} + 160*a^8*b^{16} + 120*a^9*b^{15} - 120*a^{10}*b^{14} - 48*a^{11}*b^{13} \\
& + 48*a^{12}*b^{12} + 8*a^{13}*b^{11} - 8*a^{14}*b^{10}))/((b^{19} - 7*a^2*b^{17} + 21*a^4*b \\
& ^{15} - 35*a^6*b^{13} + 35*a^8*b^{11} - 21*a^{10}*b^9 + 7*a^{12}*b^7 - a^{14}*b^5)*(a*b \\
& ^{18} + b^{19} - 5*a^2*b^{17} - 5*a^3*b^{16} + 10*a^4*b^{15} + 10*a^5*b^{14} - 10*a^6*b \\
& ^{13} - 10*a^7*b^{12} + 5*a^8*b^{11} + 5*a^9*b^{10} - a^{10}*b^9 - a^{11}*b^8))*(8*A*b \\
& ^7 + 8*B*a^7 - 8*A*a^2*b^5 + 7*A*a^4*b^3 + 35*B*a^3*b^4 - 28*B*a^5*b^2 - 2* \\
& A*a^6*b - 20*B*a*b^6))/(2*(b^{19} - 7*a^2*b^{17} + 21*a^4*b^{15} - 35*a^6*b^{13} + \\
& 35*a^8*b^{11} - 21*a^{10}*b^9 + 7*a^{12}*b^7 - a^{14}*b^5))*((a + b)^7*(a - b)^7)^{(1/2)} \\
& *(8*A*b^7 + 8*B*a^7 - 8*A*a^2*b^5 + 7*A*a^4*b^3 + 35*B*a^3*b^4 - 28*B* \\
& a^5*b^2 - 2*A*a^6*b - 20*B*a*b^6))/(2*(b^{19} - 7*a^2*b^{17} + 21*a^4*b^{15} - 35 \\
& *a^6*b^{13} + 35*a^8*b^{11} - 21*a^{10}*b^9 + 7*a^{12}*b^7 - a^{14}*b^5)) - (a*((8*ta \\
& n(c/2 + (d*x)/2)*(4*A^2*b^{16} + 128*B^2*a^{16} - 8*A^2*a*b^{15} - 128*B^2*a^{15}*b \\
& + 44*A^2*a^2*b^{14} + 48*A^2*a^3*b^{13} - 92*A^2*a^4*b^{12} - 120*A^2*a^5*b^{11} + \\
& 156*A^2*a^6*b^{10} + 160*A^2*a^7*b^9 - 164*A^2*a^8*b^8 - 120*A^2*a^9*b^7 + 1 \\
& 17*A^2*a^{10}*b^6 + 48*A^2*a^{11}*b^5 - 48*A^2*a^{12}*b^4 - 8*A^2*a^{13}*b^3 + 8*A^ \\
& 2*a^{14}*b^2 + 64*B^2*a^2*b^{14} - 128*B^2*a^3*b^{13} + 80*B^2*a^4*b^{12} + 768*B^2 \\
& *a^5*b^{11} - 824*B^2*a^6*b^{10} - 1920*B^2*a^7*b^9 + 2025*B^2*a^8*b^8 + 2560*B \\
& ^2*a^9*b^7 - 2600*B^2*a^{10}*b^6 - 1920*B^2*a^{11}*b^5 + 1920*B^2*a^{12}*b^4 + 76 \\
& 8*B^2*a^{13}*b^3 - 768*B^2*a^{14}*b^2 - 32*A*B*a*b^{15} - 64*A*B*a^{15}*b + 64*A*B* \\
& a^2*b^{14} - 160*A*B*a^3*b^{13} - 384*A*B*a^4*b^{12} + 592*A*B*a^5*b^{11} + 960*A*B \\
& *a^6*b^{10} - 1128*A*B*a^7*b^9 - 1280*A*B*a^8*b^8 + 1306*A*B*a^9*b^7 + 960*A*
\end{aligned}$$

```

B*a^10*b^6 - 948*A*B*a^11*b^5 - 384*A*B*a^12*b^4 + 384*A*B*a^13*b^3 + 64*A*
B*a^14*b^2))/(a*b^18 + b^19 - 5*a^2*b^17 - 5*a^3*b^16 + 10*a^4*b^15 + 10*a^
5*b^14 - 10*a^6*b^13 - 10*a^7*b^12 + 5*a^8*b^11 + 5*a^9*b^10 - a^10*b^9 - a
^11*b^8) + (a*((a + b)^7*(a - b)^7)^(1/2))*((8*(4*A*b^24 - 12*A*a^2*b^22 + 6
4*A*a^3*b^21 + 20*A*a^4*b^20 - 110*A*a^5*b^19 - 30*A*a^6*b^18 + 110*A*a^7*b
^17 + 30*A*a^8*b^16 - 70*A*a^9*b^15 - 14*A*a^10*b^14 + 26*A*a^11*b^13 + 2*A
*a^12*b^12 - 4*A*a^13*b^11 + 40*B*a^2*b^22 + 72*B*a^3*b^21 - 190*B*a^4*b^20
- 146*B*a^5*b^19 + 386*B*a^6*b^18 + 174*B*a^7*b^17 - 434*B*a^8*b^16 - 126*
B*a^9*b^15 + 286*B*a^10*b^14 + 50*B*a^11*b^13 - 104*B*a^12*b^12 - 8*B*a^13*
b^11 + 16*B*a^14*b^10 - 16*A*a*b^23 - 16*B*a*b^23))/(a*b^22 + b^23 - 5*a^2*
b^21 - 5*a^3*b^20 + 10*a^4*b^19 + 10*a^5*b^18 - 10*a^6*b^17 - 10*a^7*b^16 +
5*a^8*b^15 + 5*a^9*b^14 - a^10*b^13 - a^11*b^12) + (4*a*tan(c/2 + (d*x)/2)
*((a + b)^7*(a - b)^7)^(1/2))*(8*A*b^7 + 8*B*a^7 - 8*A*a^2*b^5 + 7*A*a^4*b^3
+ 35*B*a^3*b^4 - 28*B*a^5*b^2 - 2*A*a^6*b - 20*B*a*b^6)*(8*a*b^23 - 8*a^2*
b^22 - 48*a^3*b^21 + 48*a^4*b^20 + 120*a^5*b^19 - 120*a^6*b^18 - 160*a^7*b^
17 + 160*a^8*b^16 + 120*a^9*b^15 - 120*a^10*b^14 - 48*a^11*b^13 + 48*a^12*b
^12 + 8*a^13*b^11 - 8*a^14*b^10))/((b^19 - 7*a^2*b^17 + 21*a^4*b^15 - 35*a^
6*b^13 + 35*a^8*b^11 - 21*a^10*b^9 + 7*a^12*b^7 - a^14*b^5)*(a*b^18 + b^19
- 5*a^2*b^17 - 5*a^3*b^16 + 10*a^4*b^15 + 10*a^5*b^14 - 10*a^6*b^13 - 10*a^
7*b^12 + 5*a^8*b^11 + 5*a^9*b^10 - a^10*b^9 - a^11*b^8)))*(8*A*b^7 + 8*B*a^
7 - 8*A*a^2*b^5 + 7*A*a^4*b^3 + 35*B*a^3*b^4 - 28*B*a^5*b^2 - 2*A*a^6*b - 2
0*B*a*b^6))/(2*(b^19 - 7*a^2*b^17 + 21*a^4*b^15 - 35*a^6*b^13 + 35*a^8*b^11
- 21*a^10*b^9 + 7*a^12*b^7 - a^14*b^5))*((a + b)^7*(a - b)^7)^(1/2)*(8*A*
b^7 + 8*B*a^7 - 8*A*a^2*b^5 + 7*A*a^4*b^3 + 35*B*a^3*b^4 - 28*B*a^5*b^2 - 2
*A*a^6*b - 20*B*a*b^6))/(2*(b^19 - 7*a^2*b^17 + 21*a^4*b^15 - 35*a^6*b^13 +
35*a^8*b^11 - 21*a^10*b^9 + 7*a^12*b^7 - a^14*b^5))))*((a + b)^7*(a - b)^7
)^(1/2)*(8*A*b^7 + 8*B*a^7 - 8*A*a^2*b^5 + 7*A*a^4*b^3 + 35*B*a^3*b^4 - 28*
B*a^5*b^2 - 2*A*a^6*b - 20*B*a*b^6)*1i)/(d*(b^19 - 7*a^2*b^17 + 21*a^4*b^15
- 35*a^6*b^13 + 35*a^8*b^11 - 21*a^10*b^9 + 7*a^12*b^7 - a^14*b^5))

```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \sec^5(c + dx)}{(a + b \sec(c + dx))^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**5*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**4,x)

[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**5/(a + b*sec(c + d*x))**4, x)

$$3.337 \quad \int \frac{\sec^4(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^4} dx$$

Optimal. Leaf size=310

$$\frac{a(Ab - aB) \tan(c + dx) \sec^2(c + dx)}{3bd(a^2 - b^2)(a + b \sec(c + dx))^3} + \frac{a^2(3a^3B - 8ab^2B + 5Ab^3) \tan(c + dx)}{6b^3d(a^2 - b^2)^2(a + b \sec(c + dx))^2} - \frac{a(9a^5B - 28a^3b^2B + a^2Ab^3 + 34a^2b^4B - 8a^3b^4B + 2a^7B - 8ab^6B + 2Ab^7) \tan(c + dx)}{6b^3d(a^2 - b^2)^3(a + b \sec(c + dx))^4}$$

[Out] $B \operatorname{arctanh}(\sin(dx+c))/b^4/d - (3Aa^2b^5 + 2Ab^7 + 2Ba^7 - 7Ba^5b^2 + 8Ba^3b^4 - 3b^4 - 8Ba^3b^6) \operatorname{arctanh}((a-b)^{1/2} \tan(1/2 dx + 1/2 c)/(a+b)^{1/2})/(a-b)^{7/2}/b^4/(a+b)^{7/2}/d + 1/3 a (A^2 b - B^2 a) \sec(dx+c)^2 \tan(dx+c)/b/(a^2 - b^2)/d/(a+b \sec(dx+c))^3 + 1/6 a^2 (5A^2 b^3 + 3B^2 a^3 - 8A^2 b^2) \tan(dx+c)/b^3/(a^2 - b^2)^2/d/(a+b \sec(dx+c))^2 - 1/6 a (A^2 b^3 - 16A^2 b^5 + 9Ba^5 - 28Ba^3b^2 + 34Ba^2b^4) \tan(dx+c)/b^3/(a^2 - b^2)^3/d/(a+b \sec(dx+c))$

Rubi [A] time = 1.37, antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {4029, 4090, 4080, 3998, 3770, 3831, 2659, 208}

$$\frac{(3a^2Ab^5 - 7a^5b^2B + 8a^3b^4B + 2a^7B - 8ab^6B + 2Ab^7) \tanh^{-1}\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^4d(a-b)^{7/2}(a+b)^{7/2}} + \frac{a(Ab - aB) \tan(c + dx) \sec^2(c + dx)}{3bd(a^2 - b^2)(a + b \sec(c + dx))^4}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(\operatorname{Sec}[c + dx]^4(A + B \operatorname{Sec}[c + dx]))/(a + b \operatorname{Sec}[c + dx])^4, x]$

[Out] $(B \operatorname{ArcTanh}[\operatorname{Sin}[c + dx]])/(b^4 d) - ((3a^2Ab^5 + 2Ab^7 + 2a^7B - 7a^5b^2B + 8a^3b^4B - 8a^2b^6B) \operatorname{ArcTanh}[(\operatorname{Sqrt}[a - b] \operatorname{Tan}[(c + dx)/2])/ \operatorname{Sqrt}[a + b]])/(a - b)^{7/2} b^4 (a + b)^{7/2} d + (a(A^2 b - B^2 a) \operatorname{Sec}[c + dx]^2 \operatorname{Tan}[c + dx])/(3b(a^2 - b^2)d(a + b \operatorname{Sec}[c + dx])^3) + (a^2(5A^2 b^3 + 3a^3B - 8a^2b^2B) \operatorname{Tan}[c + dx])/(6b^3(a^2 - b^2)^2 d(a + b \operatorname{Sec}[c + dx])^2) - (a(a^2Ab^3 - 16A^2 b^5 + 9a^5B - 28a^3b^2B + 34a^2b^4B) \operatorname{Tan}[c + dx])/(6b^3(a^2 - b^2)^3 d(a + b \operatorname{Sec}[c + dx]))$

Rule 208

$\operatorname{Int}(((a_) + (b_)) \cdot (x_)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[-(a/b), 2] \operatorname{ArcTanh}[x/\operatorname{Rt}[-(a/b), 2]])/a, x] /; \operatorname{FreeQ}\{a, b\}, x] \ \&\& \ \operatorname{NegQ}[a/b]$

Rule 2659

```
Int[((a_) + (b_)*sin[Pi/2 + (c_) + (d_)*(x_)])^(-1), x_Symbol] := With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3770

```
Int[csc[(c_) + (d_)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x]
/; FreeQ[{c, d}, x]
```

Rule 3831

```
Int[csc[(e_) + (f_)*(x_)]/(csc[(e_) + (f_)*(x_)]*(b_) + (a_)), x_Symbol] := Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3998

```
Int[(csc[(e_) + (f_)*(x_)]*(csc[(e_) + (f_)*(x_)]*(B_) + (A_)))/(csc[(e_) + (f_)*(x_)]*(b_) + (a_)), x_Symbol] := Dist[B/b, Int[Csc[e + f*x], x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 4029

```
Int[(csc[(e_) + (f_)*(x_)]*(d_))^(n_)*(csc[(e_) + (f_)*(x_)]*(b_) + (a_))^(m_)*(csc[(e_) + (f_)*(x_)]*(B_) + (A_)), x_Symbol] := Simp[(a*d^2*(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[d/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(n - 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2*(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 1]
```

Rule 4080

```
Int[csc[(e_) + (f_)*(x_)]*((A_) + csc[(e_) + (f_)*(x_)]*(B_) + csc[(e_) + (f_)*(x_)]^2*(C_))*(csc[(e_) + (f_)*(x_)]*(b_) + (a_))^(m_), x_Symbol] := -Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C))*(m + 1))*Csc[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2,
```


0]

Rule 4090

```

Int[csc[(e_.) + (f_.)*(x_.)]^2*((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[
(e_.) + (f_.)*(x_.)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_), x
_Symbol] :> Simp[(a*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dist[1/(b^2*(m + 1)*(a^2 - b^
2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(m + 1)*(-(a*(b*B
- a*C)) + A*b^2) + (b*B*(a^2 + b^2*(m + 1)) - a*(A*b^2*(m + 2) + C*(a^2 +
b^2*(m + 1)))*Csc[e + f*x] - b*C*(m + 1)*(a^2 - b^2)*Csc[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]

```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^4(c + dx)(A + B \sec(c + dx))}{(a + b \sec(c + dx))^4} dx &= \frac{a(Ab - aB) \sec^2(c + dx) \tan(c + dx)}{3b(a^2 - b^2)d(a + b \sec(c + dx))^3} + \frac{\int \frac{\sec^2(c + dx)(2a(Ab - aB) - 3b(Ab - aB) \sec(c + dx))}{(a + b \sec(c + dx))^3} dx}{3b(a^2 - b^2)d(a + b \sec(c + dx))^3} \\
&= \frac{a(Ab - aB) \sec^2(c + dx) \tan(c + dx)}{3b(a^2 - b^2)d(a + b \sec(c + dx))^3} + \frac{a^2(5Ab^3 + 3a^3B - 8ab^2B) \tan(c + dx)}{6b^3(a^2 - b^2)^2 d(a + b \sec(c + dx))^3} \\
&= \frac{a(Ab - aB) \sec^2(c + dx) \tan(c + dx)}{3b(a^2 - b^2)d(a + b \sec(c + dx))^3} + \frac{a^2(5Ab^3 + 3a^3B - 8ab^2B) \tan(c + dx)}{6b^3(a^2 - b^2)^2 d(a + b \sec(c + dx))^3} \\
&= \frac{a(Ab - aB) \sec^2(c + dx) \tan(c + dx)}{3b(a^2 - b^2)d(a + b \sec(c + dx))^3} + \frac{a^2(5Ab^3 + 3a^3B - 8ab^2B) \tan(c + dx)}{6b^3(a^2 - b^2)^2 d(a + b \sec(c + dx))^3} \\
&= \frac{B \tanh^{-1}(\sin(c + dx))}{b^4 d} + \frac{a(Ab - aB) \sec^2(c + dx) \tan(c + dx)}{3b(a^2 - b^2)d(a + b \sec(c + dx))^3} + \frac{a^2(5Ab^3 + 3a^3B - 8ab^2B) \tan(c + dx)}{6b^3(a^2 - b^2)^2 d(a + b \sec(c + dx))^3} \\
&= \frac{B \tanh^{-1}(\sin(c + dx))}{b^4 d} + \frac{a(Ab - aB) \sec^2(c + dx) \tan(c + dx)}{3b(a^2 - b^2)d(a + b \sec(c + dx))^3} + \frac{a^2(5Ab^3 + 3a^3B - 8ab^2B) \tan(c + dx)}{6b^3(a^2 - b^2)^2 d(a + b \sec(c + dx))^3} \\
&= \frac{B \tanh^{-1}(\sin(c + dx))}{b^4 d} - \frac{(3a^2Ab^5 + 2Ab^7 + 2a^7B - 7a^5b^2B + 8a^3b^4B) \tan(c + dx)}{(a - b)^{7/2}b^4(a + b \sec(c + dx))^3}
\end{aligned}$$

Mathematica [A] time = 1.99, size = 369, normalized size = 1.19

$$\cos(c + dx)(A + B \sec(c + dx)) \left(\frac{24(2a^7B - 7a^5b^2B + 8a^3b^4B + 3a^2Ab^5 - 8ab^6B + 2Ab^7) \tanh^{-1}\left(\frac{(b-a) \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2}} - \frac{2ab \sin(c+dx)(-6a^7B - 5a^5b^2B + 8a^3b^4B + 3a^2Ab^5 - 8ab^6B + 2Ab^7)}{(a^2-b^2)^{7/2}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[c + d*x]^4*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^4,x]

[Out] (Cos[c + d*x]*(A + B*Sec[c + d*x])*((24*(3*a^2*A*b^5 + 2*A*b^7 + 2*a^7*B - 7*a^5*b^2*B + 8*a^3*b^4*B - 8*a*b^6*B)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(7/2) - 24*B*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 24*B*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - (2*a*b*(8*a^4*A*b^3 + a^2*A*b^5 + 36*A*b^7 - 6*a^7*B - 5*a^5*b^2*B + 38*a^3*b^4*B - 72*a*b^6*B - 6*a*b*(-a^2*A*b^3) - 9*A*b^5 + 5*a^5*B - 15*a^3*b^2*B + 20*a*b^4*B)*Cos[c + d*x] + a^2*(4*a^2*A*b^3 + 11*A*b^5 - 6*a^5*B + 17*a^3*b^2*B - 26*a*b^4*B)*Cos[2*(c + d*x)]*Sin[c + d*x])/((-a^2 + b^2)^3*(b + a*Cos[c + d*x])^3))/(24*b^4*d*(B + A*Cos[c + d*x]))

fricas [B] time = 48.76, size = 2278, normalized size = 7.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^4,x, algorithm="fricas")

[Out] [-1/12*(3*(2*B*a^7*b^3 - 7*B*a^5*b^5 + 8*B*a^3*b^7 + 3*A*a^2*b^8 - 8*B*a*b^9 + 2*A*b^10 + (2*B*a^10 - 7*B*a^8*b^2 + 8*B*a^6*b^4 + 3*A*a^5*b^5 - 8*B*a^4*b^6 + 2*A*a^3*b^7)*cos(d*x + c)^3 + 3*(2*B*a^9*b - 7*B*a^7*b^3 + 8*B*a^5*b^5 + 3*A*a^4*b^6 - 8*B*a^3*b^7 + 2*A*a^2*b^8)*cos(d*x + c)^2 + 3*(2*B*a^8*b^2 - 7*B*a^6*b^4 + 8*B*a^4*b^6 + 3*A*a^3*b^7 - 8*B*a^2*b^8 + 2*A*a*b^9)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 6*(B*a^8*b^3 - 4*B*a^6*b^5 + 6*B*a^4*b^7 - 4*B*a^2*b^9 + B*b^11 + (B*a^11 - 4*B*a^9*b^2 + 6*B*a^7*b^4 - 4*B*a^5*b^6 + B*a^3*b^8)*cos(d*x + c)^3 + 3*(B*a^10*b - 4*B*a^8*b^3 + 6*B*a^6*b^5 - 4*B*a^4*b^7 + B*a^2*b^9)*cos(d*x + c)^2 + 3*(B*a^9*b^2 - 4*B*a^7*b^4 + 6*B*a^5*b^6 - 4*B*a^3*b^8 + B*a*b^10)*cos(d*x + c))*log(sin(d*x + c) + 1) + 6*(B*a^8*b^3 - 4*B*a^6*b^5 + 6*B*a^4*b^7 - 4*B*a^2*b^9 + B*b^11 + (B*a^11 - 4*B*a^9*b^2 + 6*B*a^7*b^4 - 4*B*a^5*b^6 + B*a^3*b^8)*cos(d*x + c)^3 + 3*(B*a^10*b - 4*B*a^8*b^3 + 6*B*a^6*b^5 - 4*B*a^4*b^7 + B*a^2*b^9)

```

*cos(d*x + c)^2 + 3*(B*a^9*b^2 - 4*B*a^7*b^4 + 6*B*a^5*b^6 - 4*B*a^3*b^8 +
B*a*b^10)*cos(d*x + c)*log(-sin(d*x + c) + 1) + 2*(11*B*a^8*b^3 - 2*A*a^7*
b^4 - 43*B*a^6*b^5 + 7*A*a^5*b^6 + 68*B*a^4*b^7 - 23*A*a^3*b^8 - 36*B*a^2*b
^9 + 18*A*a*b^10 + (6*B*a^10*b - 23*B*a^8*b^3 - 4*A*a^7*b^4 + 43*B*a^6*b^5
- 7*A*a^5*b^6 - 26*B*a^4*b^7 + 11*A*a^3*b^8)*cos(d*x + c)^2 + 3*(5*B*a^9*b^
2 - 20*B*a^7*b^4 - A*a^6*b^5 + 35*B*a^5*b^6 - 8*A*a^4*b^7 - 20*B*a^3*b^8 +
9*A*a^2*b^9)*cos(d*x + c))*sin(d*x + c))/((a^11*b^4 - 4*a^9*b^6 + 6*a^7*b^8
- 4*a^5*b^10 + a^3*b^12)*d*cos(d*x + c)^3 + 3*(a^10*b^5 - 4*a^8*b^7 + 6*a^
6*b^9 - 4*a^4*b^11 + a^2*b^13)*d*cos(d*x + c)^2 + 3*(a^9*b^6 - 4*a^7*b^8 +
6*a^5*b^10 - 4*a^3*b^12 + a*b^14)*d*cos(d*x + c) + (a^8*b^7 - 4*a^6*b^9 + 6
*a^4*b^11 - 4*a^2*b^13 + b^15)*d), -1/6*(3*(2*B*a^7*b^3 - 7*B*a^5*b^5 + 8*B
*a^3*b^7 + 3*A*a^2*b^8 - 8*B*a*b^9 + 2*A*b^10 + (2*B*a^10 - 7*B*a^8*b^2 + 8
*B*a^6*b^4 + 3*A*a^5*b^5 - 8*B*a^4*b^6 + 2*A*a^3*b^7)*cos(d*x + c)^3 + 3*(2
*B*a^9*b - 7*B*a^7*b^3 + 8*B*a^5*b^5 + 3*A*a^4*b^6 - 8*B*a^3*b^7 + 2*A*a^2*
b^8)*cos(d*x + c)^2 + 3*(2*B*a^8*b^2 - 7*B*a^6*b^4 + 8*B*a^4*b^6 + 3*A*a^3*
b^7 - 8*B*a^2*b^8 + 2*A*a*b^9)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(
-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - 3*(B*a^8*b^3
- 4*B*a^6*b^5 + 6*B*a^4*b^7 - 4*B*a^2*b^9 + B*b^11 + (B*a^11 - 4*B*a^9*b^2
+ 6*B*a^7*b^4 - 4*B*a^5*b^6 + B*a^3*b^8)*cos(d*x + c)^3 + 3*(B*a^10*b - 4*
B*a^8*b^3 + 6*B*a^6*b^5 - 4*B*a^4*b^7 + B*a^2*b^9)*cos(d*x + c)^2 + 3*(B*a^
9*b^2 - 4*B*a^7*b^4 + 6*B*a^5*b^6 - 4*B*a^3*b^8 + B*a*b^10)*cos(d*x + c))*l
og(sin(d*x + c) + 1) + 3*(B*a^8*b^3 - 4*B*a^6*b^5 + 6*B*a^4*b^7 - 4*B*a^2*b
^9 + B*b^11 + (B*a^11 - 4*B*a^9*b^2 + 6*B*a^7*b^4 - 4*B*a^5*b^6 + B*a^3*b^8
)*cos(d*x + c)^3 + 3*(B*a^10*b - 4*B*a^8*b^3 + 6*B*a^6*b^5 - 4*B*a^4*b^7 +
B*a^2*b^9)*cos(d*x + c)^2 + 3*(B*a^9*b^2 - 4*B*a^7*b^4 + 6*B*a^5*b^6 - 4*B*
a^3*b^8 + B*a*b^10)*cos(d*x + c))*log(-sin(d*x + c) + 1) + (11*B*a^8*b^3 -
2*A*a^7*b^4 - 43*B*a^6*b^5 + 7*A*a^5*b^6 + 68*B*a^4*b^7 - 23*A*a^3*b^8 - 36
*B*a^2*b^9 + 18*A*a*b^10 + (6*B*a^10*b - 23*B*a^8*b^3 - 4*A*a^7*b^4 + 43*B*
a^6*b^5 - 7*A*a^5*b^6 - 26*B*a^4*b^7 + 11*A*a^3*b^8)*cos(d*x + c)^2 + 3*(5*
B*a^9*b^2 - 20*B*a^7*b^4 - A*a^6*b^5 + 35*B*a^5*b^6 - 8*A*a^4*b^7 - 20*B*a^
3*b^8 + 9*A*a^2*b^9)*cos(d*x + c))*sin(d*x + c))/((a^11*b^4 - 4*a^9*b^6 + 6
*a^7*b^8 - 4*a^5*b^10 + a^3*b^12)*d*cos(d*x + c)^3 + 3*(a^10*b^5 - 4*a^8*b^
7 + 6*a^6*b^9 - 4*a^4*b^11 + a^2*b^13)*d*cos(d*x + c)^2 + 3*(a^9*b^6 - 4*a^
7*b^8 + 6*a^5*b^10 - 4*a^3*b^12 + a*b^14)*d*cos(d*x + c) + (a^8*b^7 - 4*a^6
*b^9 + 6*a^4*b^11 - 4*a^2*b^13 + b^15)*d)]

```

giac [B] time = 0.46, size = 844, normalized size = 2.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```

[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^4,x, algorithm="gi
ac")

```

```

[Out] -1/3*(3*(2*B*a^7 - 7*B*a^5*b^2 + 8*B*a^3*b^4 + 3*A*a^2*b^5 - 8*B*a*b^6 + 2*

```

$$\begin{aligned}
& A*b^7*(\pi*\text{floor}(1/2*(d*x + c)/\pi + 1/2)*\text{sgn}(-2*a + 2*b) + \arctan(-(a*\tan(1/2*d*x + 1/2*c) - b*\tan(1/2*d*x + 1/2*c))/\sqrt{-a^2 + b^2}))/((a^6*b^4 - 3*a^4*b^6 + 3*a^2*b^8 - b^{10})*\sqrt{-a^2 + b^2}) - 3*B*\log(\text{abs}(\tan(1/2*d*x + 1/2*c) + 1))/b^4 + 3*B*\log(\text{abs}(\tan(1/2*d*x + 1/2*c) - 1))/b^4 - (6*B*a^8*\tan(1/2*d*x + 1/2*c)^5 - 15*B*a^7*b*\tan(1/2*d*x + 1/2*c)^5 - 6*B*a^6*b^2*\tan(1/2*d*x + 1/2*c)^5 - 6*A*a^5*b^3*\tan(1/2*d*x + 1/2*c)^5 + 45*B*a^5*b^3*\tan(1/2*d*x + 1/2*c)^5 + 3*A*a^4*b^4*\tan(1/2*d*x + 1/2*c)^5 - 6*B*a^4*b^4*\tan(1/2*d*x + 1/2*c)^5 - 6*A*a^3*b^5*\tan(1/2*d*x + 1/2*c)^5 - 60*B*a^3*b^5*\tan(1/2*d*x + 1/2*c)^5 + 27*A*a^2*b^6*\tan(1/2*d*x + 1/2*c)^5 + 36*B*a^2*b^6*\tan(1/2*d*x + 1/2*c)^5 - 18*A*a*b^7*\tan(1/2*d*x + 1/2*c)^5 - 12*B*a^8*\tan(1/2*d*x + 1/2*c)^3 + 56*B*a^6*b^2*\tan(1/2*d*x + 1/2*c)^3 + 4*A*a^5*b^3*\tan(1/2*d*x + 1/2*c)^3 - 116*B*a^4*b^4*\tan(1/2*d*x + 1/2*c)^3 + 32*A*a^3*b^5*\tan(1/2*d*x + 1/2*c)^3 + 72*B*a^2*b^6*\tan(1/2*d*x + 1/2*c)^3 - 36*A*a*b^7*\tan(1/2*d*x + 1/2*c)^3 + 6*B*a^8*\tan(1/2*d*x + 1/2*c) + 15*B*a^7*b*\tan(1/2*d*x + 1/2*c) - 6*B*a^6*b^2*\tan(1/2*d*x + 1/2*c) - 6*A*a^5*b^3*\tan(1/2*d*x + 1/2*c) - 45*B*a^5*b^3*\tan(1/2*d*x + 1/2*c) - 3*A*a^4*b^4*\tan(1/2*d*x + 1/2*c) - 6*B*a^4*b^4*\tan(1/2*d*x + 1/2*c) - 6*A*a^3*b^5*\tan(1/2*d*x + 1/2*c) + 60*B*a^3*b^5*\tan(1/2*d*x + 1/2*c) - 27*A*a^2*b^6*\tan(1/2*d*x + 1/2*c) + 36*B*a^2*b^6*\tan(1/2*d*x + 1/2*c) - 18*A*a*b^7*\tan(1/2*d*x + 1/2*c))/((a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*(a*\tan(1/2*d*x + 1/2*c)^2 - b*\tan(1/2*d*x + 1/2*c)^2 - a - b)^3))/d
\end{aligned}$$

maple [B] time = 0.74, size = 2264, normalized size = 7.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(\sec(d*x+c)^4*(A+B*\sec(d*x+c))/(a+b*\sec(d*x+c))^4, x)$

[Out] $\begin{aligned}
& 12/d*b^2/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)^3*a/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*\tan(1/2*d*x+1/2*c)^3*A+12/d*b/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)^3*a^2/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*\tan(1/2*d*x+1/2*c)*B-6/d*b^2/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)^3*a/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*\tan(1/2*d*x+1/2*c)^5*A+12/d*b/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)^3*a^2/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*\tan(1/2*d*x+1/2*c)^5*B-6/d*b^2/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)^3*a/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*\tan(1/2*d*x+1/2*c)*A-2/d*a^3/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*\tan(1/2*d*x+1/2*c)^5*A-4/d*a^3/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*\tan(1/2*d*x+1/2*c)*B+4/d*a^3/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*\tan(1/2*d*x+1/2*c)^5*B-2/d*a^3/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*\tan(1/2*d*x+1/2*c)*A+44/3/d/b/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)^3*a^4/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*\tan(1/2*d*x+1/2*c)^3*B-4/d/b^3/(a*\tan(1/2*d
\end{aligned}$

$$\begin{aligned}
& *x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)^3*a^6/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2) \\
& * \tan(1/2*d*x+1/2*c)^3*B-24/d*b/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2*c) \\
& ^2*b-a-b)^3*a^2/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*\tan(1/2*d*x+1/2*c)^3*B+1/d* \\
& a^5/b^2/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3* \\
& a^2*b+3*a*b^2-b^3)*\tan(1/2*d*x+1/2*c)*B-1/d*a^5/b^2/(a*\tan(1/2*d*x+1/2*c)^2 \\
& -\tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*\tan(1/2*d*x+ \\
& 1/2*c)^5*B-6/d*a^4/b/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)^3/ \\
& (a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*\tan(1/2*d*x+1/2*c)*B+2/d*a^6/b^3/(a*\tan(1/2 \\
& *d*x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3) \\
& *\tan(1/2*d*x+1/2*c)*B+4/3/d/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b- \\
& a-b)^3*a^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*\tan(1/2*d*x+1/2*c)^3*A-2/d/b^4/(\\
& a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*\operatorname{arctanh}(\tan(1/2*d*x+1/2*c) \\
& *(a-b)/((a-b)*(a+b))^(1/2))*a^7*B-3/d*b/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b) \\
& *(a+b))^(1/2)*\operatorname{arctanh}(\tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*a^2*A+ \\
& 7/d/b^2/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*\operatorname{arctanh}(\tan(1/2*d \\
& *x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*a^5*B-3/d*a^2*b/(a*\tan(1/2*d*x+1/2*c)^ \\
& 2-\tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*\tan(1/2*d*x \\
& +1/2*c)^5*A+3/d*a^2*b/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)^3 \\
& /(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*\tan(1/2*d*x+1/2*c)*A+2/d*a^6/b^3/(a*\tan(1/ \\
& 2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3) \\
&)*\tan(1/2*d*x+1/2*c)^5*B-6/d*a^4/b/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2* \\
& c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*\tan(1/2*d*x+1/2*c)^5*B+1/d*B/ \\
& b^4*\ln(\tan(1/2*d*x+1/2*c)+1)-1/d*B/b^4*\ln(\tan(1/2*d*x+1/2*c)-1)+8/d*b^2/(a^ \\
& 6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*\operatorname{arctanh}(\tan(1/2*d*x+1/2*c)*(\\
& a-b)/((a-b)*(a+b))^(1/2))*a*B-8/d/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b) \\
&))^(1/2)*\operatorname{arctanh}(\tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*a^3*B-2/d*b^ \\
& 3/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*\operatorname{arctanh}(\tan(1/2*d*x+1/2 \\
& *c)*(a-b)/((a-b)*(a+b))^(1/2))*A
\end{aligned}$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^4,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for more details)Is 4*a^2-4*b^2 positive or negative?

mupad [B] time = 14.14, size = 9713, normalized size = 31.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((A + B/\cos(c + d*x))/(\cos(c + d*x)^4*(a + b/\cos(c + d*x))^4), x)$

[Out]
$$- ((\tan(c/2 + (d*x)/2)*(2*B*a^6 + 3*A*a^2*b^4 - 2*A*a^3*b^3 + 12*B*a^2*b^4 - 4*B*a^3*b^3 - 6*B*a^4*b^2 - 6*A*a*b^5 + B*a^5*b))/((a + b)*(3*a*b^5 - b^6 - 3*a^2*b^4 + a^3*b^3)) - (\tan(c/2 + (d*x)/2)^5*(3*A*a^2*b^4 - 2*B*a^6 + 2*A*a^3*b^3 - 12*B*a^2*b^4 - 4*B*a^3*b^3 + 6*B*a^4*b^2 + 6*A*a*b^5 + B*a^5*b))/((a*b^3 - b^4)*(a + b)^3) + (4*\tan(c/2 + (d*x)/2)^3*(A*a^3*b^3 - 3*B*a^6 - 18*B*a^2*b^4 + 11*B*a^4*b^2 + 9*A*a*b^5))/(3*(a + b)^2*(b^5 - 2*a*b^4 + a^2*b^3)))/(d*(\tan(c/2 + (d*x)/2)^2*(3*a*b^2 - 3*a^2*b - 3*a^3 + 3*b^3) - \tan(c/2 + (d*x)/2)^4*(3*a*b^2 + 3*a^2*b - 3*a^3 - 3*b^3) + 3*a*b^2 + 3*a^2*b + a^3 + b^3 - \tan(c/2 + (d*x)/2)^6*(3*a*b^2 - 3*a^2*b + a^3 - b^3))) - (B*\text{atan}(((B*((8*\tan(c/2 + (d*x)/2)*(4*A^2*b^14 + 8*B^2*a^14 + 4*B^2*b^14 - 8*B^2*a*b^13 - 8*B^2*a^13*b + 12*A^2*a^2*b^12 + 9*A^2*a^4*b^10 + 44*B^2*a^2*b^12 + 48*B^2*a^3*b^11 - 92*B^2*a^4*b^10 - 120*B^2*a^5*b^9 + 156*B^2*a^6*b^8 + 160*B^2*a^7*b^7 - 164*B^2*a^8*b^6 - 120*B^2*a^9*b^5 + 117*B^2*a^10*b^4 + 48*B^2*a^11*b^3 - 48*B^2*a^12*b^2 - 32*A*B*a*b^13 - 16*A*B*a^3*b^11 + 20*A*B*a^5*b^9 - 34*A*B*a^7*b^7 + 12*A*B*a^9*b^5)))/(a*b^16 + b^17 - 5*a^2*b^15 - 5*a^3*b^14 + 10*a^4*b^13 + 10*a^5*b^12 - 10*a^6*b^11 - 10*a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - a^10*b^7 - a^11*b^6) + (B*((8*(4*A*b^21 + 4*B*b^21 - 6*A*a^2*b^19 + 6*A*a^3*b^18 - 6*A*a^4*b^17 + 6*A*a^5*b^16 + 14*A*a^6*b^15 - 14*A*a^7*b^14 - 6*A*a^8*b^13 + 6*A*a^9*b^12 - 12*B*a^2*b^19 + 64*B*a^3*b^18 + 20*B*a^4*b^17 - 110*B*a^5*b^16 - 30*B*a^6*b^15 + 110*B*a^7*b^14 + 30*B*a^8*b^13 - 70*B*a^9*b^12 - 14*B*a^10*b^11 + 26*B*a^11*b^10 + 2*B*a^12*b^9 - 4*B*a^13*b^8 - 4*A*a*b^20 - 16*B*a*b^20)))/(a*b^19 + b^20 - 5*a^2*b^18 - 5*a^3*b^17 + 10*a^4*b^16 + 10*a^5*b^15 - 10*a^6*b^14 - 10*a^7*b^13 + 5*a^8*b^12 + 5*a^9*b^11 - a^10*b^10 - a^11*b^9) + (8*B*\tan(c/2 + (d*x)/2)*(8*a*b^21 - 8*a^2*b^20 - 48*a^3*b^19 + 48*a^4*b^18 + 120*a^5*b^17 - 120*a^6*b^16 - 160*a^7*b^15 + 160*a^8*b^14 + 120*a^9*b^13 - 120*a^10*b^12 - 48*a^11*b^11 + 48*a^12*b^10 + 8*a^13*b^9 - 8*a^14*b^8)))/(b^4*(a*b^16 + b^17 - 5*a^2*b^15 - 5*a^3*b^14 + 10*a^4*b^13 + 10*a^5*b^12 - 10*a^6*b^11 - 10*a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - a^10*b^7 - a^11*b^6))))/b^4 + (B*((8*\tan(c/2 + (d*x)/2)*(4*A^2*b^14 + 8*B^2*a^14 + 4*B^2*b^14 - 8*B^2*a*b^13 - 8*B^2*a^13*b + 12*A^2*a^2*b^12 + 9*A^2*a^4*b^10 + 44*B^2*a^2*b^12 + 48*B^2*a^3*b^11 - 92*B^2*a^4*b^10 - 120*B^2*a^5*b^9 + 156*B^2*a^6*b^8 + 160*B^2*a^7*b^7 - 164*B^2*a^8*b^6 - 120*B^2*a^9*b^5 + 117*B^2*a^10*b^4 + 48*B^2*a^11*b^3 - 48*B^2*a^12*b^2 - 32*A*B*a*b^13 - 16*A*B*a^3*b^11 + 20*A*B*a^5*b^9 - 34*A*B*a^7*b^7 + 12*A*B*a^9*b^5)))/(a*b^16 + b^17 - 5*a^2*b^15 - 5*a^3*b^14 + 10*a^4*b^13 + 10*a^5*b^12 - 10*a^6*b^11 - 10*a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - a^10*b^7 - a^11*b^6) - (B*((8*(4*A*b^21 + 4*B*b^21 - 6*A*a^2*b^19 + 6*A*a^3*b^18 - 6*A*a^4*b^17 + 6*A*a^5*b^16 + 14*A*a^6*b^15 - 14*A*a^7*b^14 - 6*A*a^8*b^13 + 6*A*a^9*b^12 - 12*B*a^2*b^19 + 64*B*a^3*b^18 + 20*B*a^4*b^17 - 110*B*a^5*b^16 - 30*B*a^6*b^15 + 110*B*a^7*b^14 + 30*B*a^8*b^13 - 70*B*a^9*b^12 - 14*B*a^10*b^11 + 26*B*a^11*b^10 + 2*B*a^12*b^9 - 4*B*a^13*b^8 - 4*A*a*b^20 -$$

$$\begin{aligned}
& 16*B*a*b^{20}))/ (a*b^{19} + b^{20} - 5*a^2*b^{18} - 5*a^3*b^{17} + 10*a^4*b^{16} + 10*a^5*b^{15} - 10*a^6*b^{14} - 10*a^7*b^{13} + 5*a^8*b^{12} + 5*a^9*b^{11} - a^{10}*b^{10} - \\
& a^{11}*b^9) - (8*B*\tan(c/2 + (d*x)/2)*(8*a*b^{21} - 8*a^2*b^{20} - 48*a^3*b^{19} + 48*a^4*b^{18} + 120*a^5*b^{17} - 120*a^6*b^{16} - 160*a^7*b^{15} + 160*a^8*b^{14} + \\
& 120*a^9*b^{13} - 120*a^{10}*b^{12} - 48*a^{11}*b^{11} + 48*a^{12}*b^{10} + 8*a^{13}*b^9 - 8*a^{14}*b^8))/ (b^4*(a*b^{16} + b^{17} - 5*a^2*b^{15} - 5*a^3*b^{14} + 10*a^4*b^{13} + 1 \\
& 0*a^5*b^{12} - 10*a^6*b^{11} - 10*a^7*b^{10} + 5*a^8*b^9 + 5*a^9*b^8 - a^{10}*b^7 - a^{11}*b^6))))/b^4)*i)/b^4)/((16*(4*B^3*a^{13} - 4*A*B^2*b^{13} + 4*A^2*B*b^{13} \\
& + 16*B^3*a*b^{12} - 2*B^3*a^{12}*b + 48*B^3*a^2*b^{11} - 64*B^3*a^3*b^{10} - 64*B^3*a^4*b^9 + 110*B^3*a^5*b^8 + 66*B^3*a^6*b^7 - 110*B^3*a^7*b^6 - 34*B^3*a^8* \\
& b^5 + 70*B^3*a^9*b^4 + 11*B^3*a^{10}*b^3 - 26*B^3*a^{11}*b^2 - 28*A*B^2*a*b^{12} \\
& + 6*A*B^2*a^2*b^{11} - 22*A*B^2*a^3*b^{10} + 6*A*B^2*a^4*b^9 + 14*A*B^2*a^5*b^8 \\
& - 14*A*B^2*a^6*b^7 - 20*A*B^2*a^7*b^6 + 6*A*B^2*a^8*b^5 + 6*A*B^2*a^9*b^4 \\
& + 12*A^2*B*a^2*b^{11} + 9*A^2*B*a^4*b^9))/ (a*b^{19} + b^{20} - 5*a^2*b^{18} - 5*a^3*b^{17} + 10*a^4*b^{16} + 10*a^5*b^{15} - 10*a^6*b^{14} - 10*a^7*b^{13} + 5*a^8*b^{12} \\
& + 5*a^9*b^{11} - a^{10}*b^{10} - a^{11}*b^9) + (B*((8*\tan(c/2 + (d*x)/2)*(4*A^2*b^{14} + 8*B^2*a^{14} + 4*B^2*b^{14} - 8*B^2*a*b^{13} - 8*B^2*a^{13}*b + 12*A^2*a^2*b^{12} \\
& + 9*A^2*a^4*b^{10} + 44*B^2*a^2*b^{12} + 48*B^2*a^3*b^{11} - 92*B^2*a^4*b^{10} - 1 \\
& 20*B^2*a^5*b^9 + 156*B^2*a^6*b^8 + 160*B^2*a^7*b^7 - 164*B^2*a^8*b^6 - 120* \\
& B^2*a^9*b^5 + 117*B^2*a^{10}*b^4 + 48*B^2*a^{11}*b^3 - 48*B^2*a^{12}*b^2 - 32*A*B \\
& *a*b^{13} - 16*A*B*a^3*b^{11} + 20*A*B*a^5*b^9 - 34*A*B*a^7*b^7 + 12*A*B*a^9*b^5))/ (a*b^{16} + b^{17} - 5*a^2*b^{15} - 5*a^3*b^{14} + 10*a^4*b^{13} + 10*a^5*b^{12} - \\
& 10*a^6*b^{11} - 10*a^7*b^{10} + 5*a^8*b^9 + 5*a^9*b^8 - a^{10}*b^7 - a^{11}*b^6) + \\
& (B*((8*(4*A*b^{21} + 4*B*b^{21} - 6*A*a^2*b^{19} + 6*A*a^3*b^{18} - 6*A*a^4*b^{17} + \\
& 6*A*a^5*b^{16} + 14*A*a^6*b^{15} - 14*A*a^7*b^{14} - 6*A*a^8*b^{13} + 6*A*a^9*b^{12} \\
& - 12*B*a^2*b^{19} + 64*B*a^3*b^{18} + 20*B*a^4*b^{17} - 110*B*a^5*b^{16} - 30*B*a^6 \\
& *b^{15} + 110*B*a^7*b^{14} + 30*B*a^8*b^{13} - 70*B*a^9*b^{12} - 14*B*a^{10}*b^{11} + 2 \\
& 6*B*a^{11}*b^{10} + 2*B*a^{12}*b^9 - 4*B*a^{13}*b^8 - 4*A*a*b^{20} - 16*B*a*b^{20}))/ (a \\
& *b^{19} + b^{20} - 5*a^2*b^{18} - 5*a^3*b^{17} + 10*a^4*b^{16} + 10*a^5*b^{15} - 10*a^6 \\
& *b^{14} - 10*a^7*b^{13} + 5*a^8*b^{12} + 5*a^9*b^{11} - a^{10}*b^{10} - a^{11}*b^9) + (8* \\
& B*\tan(c/2 + (d*x)/2)*(8*a*b^{21} - 8*a^2*b^{20} - 48*a^3*b^{19} + 48*a^4*b^{18} + 1 \\
& 20*a^5*b^{17} - 120*a^6*b^{16} - 160*a^7*b^{15} + 160*a^8*b^{14} + 120*a^9*b^{13} - 1 \\
& 20*a^{10}*b^{12} - 48*a^{11}*b^{11} + 48*a^{12}*b^{10} + 8*a^{13}*b^9 - 8*a^{14}*b^8))/ (b^4 \\
& *(a*b^{16} + b^{17} - 5*a^2*b^{15} - 5*a^3*b^{14} + 10*a^4*b^{13} + 10*a^5*b^{12} - 10* \\
& a^6*b^{11} - 10*a^7*b^{10} + 5*a^8*b^9 + 5*a^9*b^8 - a^{10}*b^7 - a^{11}*b^6))))/b^4 \\
&))/b^4 - (B*((8*\tan(c/2 + (d*x)/2)*(4*A^2*b^{14} + 8*B^2*a^{14} + 4*B^2*b^{14} - \\
& 8*B^2*a*b^{13} - 8*B^2*a^{13}*b + 12*A^2*a^2*b^{12} + 9*A^2*a^4*b^{10} + 44*B^2*a^2 \\
& *b^{12} + 48*B^2*a^3*b^{11} - 92*B^2*a^4*b^{10} - 120*B^2*a^5*b^9 + 156*B^2*a^6* \\
& b^8 + 160*B^2*a^7*b^7 - 164*B^2*a^8*b^6 - 120*B^2*a^9*b^5 + 117*B^2*a^{10}*b^4 \\
& + 48*B^2*a^{11}*b^3 - 48*B^2*a^{12}*b^2 - 32*A*B*a*b^{13} - 16*A*B*a^3*b^{11} + 2 \\
& 0*A*B*a^5*b^9 - 34*A*B*a^7*b^7 + 12*A*B*a^9*b^5))/ (a*b^{16} + b^{17} - 5*a^2*b^{15} \\
& - 5*a^3*b^{14} + 10*a^4*b^{13} + 10*a^5*b^{12} - 10*a^6*b^{11} - 10*a^7*b^{10} + 5 \\
& *a^8*b^9 + 5*a^9*b^8 - a^{10}*b^7 - a^{11}*b^6) - (B*((8*(4*A*b^{21} + 4*B*b^{21} - \\
& 6*A*a^2*b^{19} + 6*A*a^3*b^{18} - 6*A*a^4*b^{17} + 6*A*a^5*b^{16} + 14*A*a^6*b^{15} \\
& - 14*A*a^7*b^{14} - 6*A*a^8*b^{13} + 6*A*a^9*b^{12} - 12*B*a^2*b^{19} + 64*B*a^3*b^
\end{aligned}$$

$$\begin{aligned}
& 18 + 20*B*a^4*b^17 - 110*B*a^5*b^16 - 30*B*a^6*b^15 + 110*B*a^7*b^14 + 30*B \\
& *a^8*b^13 - 70*B*a^9*b^12 - 14*B*a^10*b^11 + 26*B*a^11*b^10 + 2*B*a^12*b^9 \\
& - 4*B*a^13*b^8 - 4*A*a*b^20 - 16*B*a*b^20)/(a*b^19 + b^20 - 5*a^2*b^18 - 5 \\
& *a^3*b^17 + 10*a^4*b^16 + 10*a^5*b^15 - 10*a^6*b^14 - 10*a^7*b^13 + 5*a^8*b \\
& ^12 + 5*a^9*b^11 - a^10*b^10 - a^11*b^9) - (8*B*tan(c/2 + (d*x)/2)*(8*a*b^2 \\
& 1 - 8*a^2*b^20 - 48*a^3*b^19 + 48*a^4*b^18 + 120*a^5*b^17 - 120*a^6*b^16 - \\
& 160*a^7*b^15 + 160*a^8*b^14 + 120*a^9*b^13 - 120*a^10*b^12 - 48*a^11*b^11 + \\
& 48*a^12*b^10 + 8*a^13*b^9 - 8*a^14*b^8))/(b^4*(a*b^16 + b^17 - 5*a^2*b^15 \\
& - 5*a^3*b^14 + 10*a^4*b^13 + 10*a^5*b^12 - 10*a^6*b^11 - 10*a^7*b^10 + 5*a^ \\
& 8*b^9 + 5*a^9*b^8 - a^10*b^7 - a^11*b^6))))/b^4)/b^4))*2i)/(b^4*d) - (atan \\
& (((((8*tan(c/2 + (d*x)/2)*(4*A^2*b^14 + 8*B^2*a^14 + 4*B^2*b^14 - 8*B^2*a*b \\
& ^13 - 8*B^2*a^13*b + 12*A^2*a^2*b^12 + 9*A^2*a^4*b^10 + 44*B^2*a^2*b^12 + 4 \\
& 8*B^2*a^3*b^11 - 92*B^2*a^4*b^10 - 120*B^2*a^5*b^9 + 156*B^2*a^6*b^8 + 160* \\
& B^2*a^7*b^7 - 164*B^2*a^8*b^6 - 120*B^2*a^9*b^5 + 117*B^2*a^10*b^4 + 48*B^2 \\
& *a^11*b^3 - 48*B^2*a^12*b^2 - 32*A*B*a*b^13 - 16*A*B*a^3*b^11 + 20*A*B*a^5* \\
& b^9 - 34*A*B*a^7*b^7 + 12*A*B*a^9*b^5))/(a*b^16 + b^17 - 5*a^2*b^15 - 5*a^3 \\
& *b^14 + 10*a^4*b^13 + 10*a^5*b^12 - 10*a^6*b^11 - 10*a^7*b^10 + 5*a^8*b^9 + \\
& 5*a^9*b^8 - a^10*b^7 - a^11*b^6) - (((8*(4*A*b^21 + 4*B*b^21 - 6*A*a^2*b^1 \\
& 9 + 6*A*a^3*b^18 - 6*A*a^4*b^17 + 6*A*a^5*b^16 + 14*A*a^6*b^15 - 14*A*a^7*b \\
& ^14 - 6*A*a^8*b^13 + 6*A*a^9*b^12 - 12*B*a^2*b^19 + 64*B*a^3*b^18 + 20*B*a^ \\
& 4*b^17 - 110*B*a^5*b^16 - 30*B*a^6*b^15 + 110*B*a^7*b^14 + 30*B*a^8*b^13 - \\
& 70*B*a^9*b^12 - 14*B*a^10*b^11 + 26*B*a^11*b^10 + 2*B*a^12*b^9 - 4*B*a^13*b \\
& ^8 - 4*A*a*b^20 - 16*B*a*b^20))/(a*b^19 + b^20 - 5*a^2*b^18 - 5*a^3*b^17 + \\
& 10*a^4*b^16 + 10*a^5*b^15 - 10*a^6*b^14 - 10*a^7*b^13 + 5*a^8*b^12 + 5*a^9* \\
& b^11 - a^10*b^10 - a^11*b^9) - (4*tan(c/2 + (d*x)/2)*((a + b)^7*(a - b)^7) \\
& ^{(1/2)}*(2*A*b^7 + 2*B*a^7 + 3*A*a^2*b^5 + 8*B*a^3*b^4 - 7*B*a^5*b^2 - 8*B*a* \\
& b^6)*(8*a*b^21 - 8*a^2*b^20 - 48*a^3*b^19 + 48*a^4*b^18 + 120*a^5*b^17 - 12 \\
& 0*a^6*b^16 - 160*a^7*b^15 + 160*a^8*b^14 + 120*a^9*b^13 - 120*a^10*b^12 - 4 \\
& 8*a^11*b^11 + 48*a^12*b^10 + 8*a^13*b^9 - 8*a^14*b^8))/((b^18 - 7*a^2*b^16 \\
& + 21*a^4*b^14 - 35*a^6*b^12 + 35*a^8*b^10 - 21*a^10*b^8 + 7*a^12*b^6 - a^14 \\
& *b^4)*(a*b^16 + b^17 - 5*a^2*b^15 - 5*a^3*b^14 + 10*a^4*b^13 + 10*a^5*b^12 \\
& - 10*a^6*b^11 - 10*a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - a^10*b^7 - a^11*b^6)) \\
&)*((a + b)^7*(a - b)^7)^{(1/2)}*(2*A*b^7 + 2*B*a^7 + 3*A*a^2*b^5 + 8*B*a^3*b^ \\
& 4 - 7*B*a^5*b^2 - 8*B*a*b^6))/(2*(b^18 - 7*a^2*b^16 + 21*a^4*b^14 - 35*a^6* \\
& b^12 + 35*a^8*b^10 - 21*a^10*b^8 + 7*a^12*b^6 - a^14*b^4)))*((a + b)^7*(a - \\
& b)^7)^{(1/2)}*(2*A*b^7 + 2*B*a^7 + 3*A*a^2*b^5 + 8*B*a^3*b^4 - 7*B*a^5*b^2 - \\
& 8*B*a*b^6)*1i)/(2*(b^18 - 7*a^2*b^16 + 21*a^4*b^14 - 35*a^6*b^12 + 35*a^8* \\
& b^10 - 21*a^10*b^8 + 7*a^12*b^6 - a^14*b^4)) + (((8*tan(c/2 + (d*x)/2)*(4*A \\
& ^2*b^14 + 8*B^2*a^14 + 4*B^2*b^14 - 8*B^2*a*b^13 - 8*B^2*a^13*b + 12*A^2*a^ \\
& 2*b^12 + 9*A^2*a^4*b^10 + 44*B^2*a^2*b^12 + 48*B^2*a^3*b^11 - 92*B^2*a^4*b^ \\
& 10 - 120*B^2*a^5*b^9 + 156*B^2*a^6*b^8 + 160*B^2*a^7*b^7 - 164*B^2*a^8*b^6 \\
& - 120*B^2*a^9*b^5 + 117*B^2*a^10*b^4 + 48*B^2*a^11*b^3 - 48*B^2*a^12*b^2 - \\
& 32*A*B*a*b^13 - 16*A*B*a^3*b^11 + 20*A*B*a^5*b^9 - 34*A*B*a^7*b^7 + 12*A*B* \\
& a^9*b^5))/(a*b^16 + b^17 - 5*a^2*b^15 - 5*a^3*b^14 + 10*a^4*b^13 + 10*a^5*b \\
& ^12 - 10*a^6*b^11 - 10*a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - a^10*b^7 - a^11*b
\end{aligned}$$

$$\begin{aligned}
& ^6) + (((8*(4*A*b^{21} + 4*B*b^{21} - 6*A*a^2*b^{19} + 6*A*a^3*b^{18} - 6*A*a^4*b^{17} \\
& + 6*A*a^5*b^{16} + 14*A*a^6*b^{15} - 14*A*a^7*b^{14} - 6*A*a^8*b^{13} + 6*A*a^9*b \\
& ^{12} - 12*B*a^2*b^{19} + 64*B*a^3*b^{18} + 20*B*a^4*b^{17} - 110*B*a^5*b^{16} - 30*B \\
& *a^6*b^{15} + 110*B*a^7*b^{14} + 30*B*a^8*b^{13} - 70*B*a^9*b^{12} - 14*B*a^{10}*b^{11} \\
& + 26*B*a^{11}*b^{10} + 2*B*a^{12}*b^9 - 4*B*a^{13}*b^8 - 4*A*a*b^{20} - 16*B*a*b^{20}) \\
&)/(a*b^{19} + b^{20} - 5*a^2*b^{18} - 5*a^3*b^{17} + 10*a^4*b^{16} + 10*a^5*b^{15} - 10 \\
& *a^6*b^{14} - 10*a^7*b^{13} + 5*a^8*b^{12} + 5*a^9*b^{11} - a^{10}*b^{10} - a^{11}*b^9) + \\
& (4*\tan(c/2 + (d*x)/2)*((a + b)^7*(a - b)^7)^{(1/2)}*(2*A*b^7 + 2*B*a^7 + 3*A \\
& *a^2*b^5 + 8*B*a^3*b^4 - 7*B*a^5*b^2 - 8*B*a*b^6)*(8*a*b^{21} - 8*a^2*b^{20} - \\
& 48*a^3*b^{19} + 48*a^4*b^{18} + 120*a^5*b^{17} - 120*a^6*b^{16} - 160*a^7*b^{15} + 16 \\
& 0*a^8*b^{14} + 120*a^9*b^{13} - 120*a^{10}*b^{12} - 48*a^{11}*b^{11} + 48*a^{12}*b^{10} + 8 \\
& *a^{13}*b^9 - 8*a^{14}*b^8))/((b^{18} - 7*a^2*b^{16} + 21*a^4*b^{14} - 35*a^6*b^{12} + \\
& 35*a^8*b^{10} - 21*a^{10}*b^8 + 7*a^{12}*b^6 - a^{14}*b^4)*(a*b^{16} + b^{17} - 5*a^2*b \\
& ^{15} - 5*a^3*b^{14} + 10*a^4*b^{13} + 10*a^5*b^{12} - 10*a^6*b^{11} - 10*a^7*b^{10} + \\
& 5*a^8*b^9 + 5*a^9*b^8 - a^{10}*b^7 - a^{11}*b^6)))*((a + b)^7*(a - b)^7)^{(1/2)}* \\
& (2*A*b^7 + 2*B*a^7 + 3*A*a^2*b^5 + 8*B*a^3*b^4 - 7*B*a^5*b^2 - 8*B*a*b^6))/ \\
& (2*(b^{18} - 7*a^2*b^{16} + 21*a^4*b^{14} - 35*a^6*b^{12} + 35*a^8*b^{10} - 21*a^{10}*b \\
& ^8 + 7*a^{12}*b^6 - a^{14}*b^4))*((a + b)^7*(a - b)^7)^{(1/2)}*(2*A*b^7 + 2*B*a^7 \\
& + 3*A*a^2*b^5 + 8*B*a^3*b^4 - 7*B*a^5*b^2 - 8*B*a*b^6)*i)/(2*(b^{18} - 7*a \\
& ^2*b^{16} + 21*a^4*b^{14} - 35*a^6*b^{12} + 35*a^8*b^{10} - 21*a^{10}*b^8 + 7*a^{12}*b^ \\
& 6 - a^{14}*b^4))/((16*(4*B^3*a^{13} - 4*A*B^2*b^{13} + 4*A^2*B*b^{13} + 16*B^3*a*b \\
& ^{12} - 2*B^3*a^{12}*b + 48*B^3*a^2*b^{11} - 64*B^3*a^3*b^{10} - 64*B^3*a^4*b^9 + 1 \\
& 10*B^3*a^5*b^8 + 66*B^3*a^6*b^7 - 110*B^3*a^7*b^6 - 34*B^3*a^8*b^5 + 70*B^3 \\
& *a^9*b^4 + 11*B^3*a^{10}*b^3 - 26*B^3*a^{11}*b^2 - 28*A*B^2*a*b^{12} + 6*A*B^2*a^ \\
& 2*b^{11} - 22*A*B^2*a^3*b^{10} + 6*A*B^2*a^4*b^9 + 14*A*B^2*a^5*b^8 - 14*A*B^2* \\
& a^6*b^7 - 20*A*B^2*a^7*b^6 + 6*A*B^2*a^8*b^5 + 6*A*B^2*a^9*b^4 + 12*A^2*B*a \\
& ^2*b^{11} + 9*A^2*B*a^4*b^9))/(a*b^{19} + b^{20} - 5*a^2*b^{18} - 5*a^3*b^{17} + 10*a \\
& ^4*b^{16} + 10*a^5*b^{15} - 10*a^6*b^{14} - 10*a^7*b^{13} + 5*a^8*b^{12} + 5*a^9*b^{11} \\
& - a^{10}*b^{10} - a^{11}*b^9) - (((8*\tan(c/2 + (d*x)/2)*(4*A^2*b^{14} + 8*B^2*a^{14} \\
& + 4*B^2*b^{14} - 8*B^2*a*b^{13} - 8*B^2*a^{13}*b + 12*A^2*a^2*b^{12} + 9*A^2*a^4*b \\
& ^{10} + 44*B^2*a^2*b^{12} + 48*B^2*a^3*b^{11} - 92*B^2*a^4*b^{10} - 120*B^2*a^5*b^9 \\
& + 156*B^2*a^6*b^8 + 160*B^2*a^7*b^7 - 164*B^2*a^8*b^6 - 120*B^2*a^9*b^5 + \\
& 117*B^2*a^{10}*b^4 + 48*B^2*a^{11}*b^3 - 48*B^2*a^{12}*b^2 - 32*A*B*a*b^{13} - 16*A \\
& *B*a^3*b^{11} + 20*A*B*a^5*b^9 - 34*A*B*a^7*b^7 + 12*A*B*a^9*b^5))/(a*b^{16} + \\
& b^{17} - 5*a^2*b^{15} - 5*a^3*b^{14} + 10*a^4*b^{13} + 10*a^5*b^{12} - 10*a^6*b^{11} - \\
& 10*a^7*b^{10} + 5*a^8*b^9 + 5*a^9*b^8 - a^{10}*b^7 - a^{11}*b^6) - (((8*(4*A*b^{21} \\
& + 4*B*b^{21} - 6*A*a^2*b^{19} + 6*A*a^3*b^{18} - 6*A*a^4*b^{17} + 6*A*a^5*b^{16} + 1 \\
& 4*A*a^6*b^{15} - 14*A*a^7*b^{14} - 6*A*a^8*b^{13} + 6*A*a^9*b^{12} - 12*B*a^2*b^{19} \\
& + 64*B*a^3*b^{18} + 20*B*a^4*b^{17} - 110*B*a^5*b^{16} - 30*B*a^6*b^{15} + 110*B*a^ \\
& 7*b^{14} + 30*B*a^8*b^{13} - 70*B*a^9*b^{12} - 14*B*a^{10}*b^{11} + 26*B*a^{11}*b^{10} + \\
& 2*B*a^{12}*b^9 - 4*B*a^{13}*b^8 - 4*A*a*b^{20} - 16*B*a*b^{20))/(a*b^{19} + b^{20} - 5 \\
& *a^2*b^{18} - 5*a^3*b^{17} + 10*a^4*b^{16} + 10*a^5*b^{15} - 10*a^6*b^{14} - 10*a^7*b \\
& ^{13} + 5*a^8*b^{12} + 5*a^9*b^{11} - a^{10}*b^{10} - a^{11}*b^9) - (4*\tan(c/2 + (d*x)/ \\
& 2)*((a + b)^7*(a - b)^7)^{(1/2)}*(2*A*b^7 + 2*B*a^7 + 3*A*a^2*b^5 + 8*B*a^3*b \\
& ^4 - 7*B*a^5*b^2 - 8*B*a*b^6)*(8*a*b^{21} - 8*a^2*b^{20} - 48*a^3*b^{19} + 48*a^4
\end{aligned}$$

$$\begin{aligned}
& *b^{18} + 120*a^5*b^{17} - 120*a^6*b^{16} - 160*a^7*b^{15} + 160*a^8*b^{14} + 120*a^9 \\
& *b^{13} - 120*a^{10}*b^{12} - 48*a^{11}*b^{11} + 48*a^{12}*b^{10} + 8*a^{13}*b^9 - 8*a^{14}*b \\
& ^8)/((b^{18} - 7*a^2*b^{16} + 21*a^4*b^{14} - 35*a^6*b^{12} + 35*a^8*b^{10} - 21*a^{10} \\
& *b^8 + 7*a^{12}*b^6 - a^{14}*b^4)*(a*b^{16} + b^{17} - 5*a^2*b^{15} - 5*a^3*b^{14} + 1 \\
& 0*a^4*b^{13} + 10*a^5*b^{12} - 10*a^6*b^{11} - 10*a^7*b^{10} + 5*a^8*b^9 + 5*a^9*b^8 \\
& - a^{10}*b^7 - a^{11}*b^6)))*((a + b)^7*(a - b)^7)^{(1/2)}*(2*A*b^7 + 2*B*a^7 + \\
& 3*A*a^2*b^5 + 8*B*a^3*b^4 - 7*B*a^5*b^2 - 8*B*a*b^6))/(2*(b^{18} - 7*a^2*b^{16} \\
& + 21*a^4*b^{14} - 35*a^6*b^{12} + 35*a^8*b^{10} - 21*a^{10}*b^8 + 7*a^{12}*b^6 - a^{14} \\
& *b^4)))*((a + b)^7*(a - b)^7)^{(1/2)}*(2*A*b^7 + 2*B*a^7 + 3*A*a^2*b^5 + 8* \\
& B*a^3*b^4 - 7*B*a^5*b^2 - 8*B*a*b^6))/(2*(b^{18} - 7*a^2*b^{16} + 21*a^4*b^{14} - \\
& 35*a^6*b^{12} + 35*a^8*b^{10} - 21*a^{10}*b^8 + 7*a^{12}*b^6 - a^{14}*b^4)) + (((8*t \\
& an(c/2 + (d*x)/2)*(4*A^2*b^{14} + 8*B^2*a^{14} + 4*B^2*b^{14} - 8*B^2*a*b^{13} - 8* \\
& B^2*a^{13}*b + 12*A^2*a^2*b^{12} + 9*A^2*a^4*b^{10} + 44*B^2*a^2*b^{12} + 48*B^2*a^ \\
& 3*b^{11} - 92*B^2*a^4*b^{10} - 120*B^2*a^5*b^9 + 156*B^2*a^6*b^8 + 160*B^2*a^7* \\
& b^7 - 164*B^2*a^8*b^6 - 120*B^2*a^9*b^5 + 117*B^2*a^{10}*b^4 + 48*B^2*a^{11}*b^ \\
& 3 - 48*B^2*a^{12}*b^2 - 32*A*B*a*b^{13} - 16*A*B*a^3*b^{11} + 20*A*B*a^5*b^9 - 34 \\
& *A*B*a^7*b^7 + 12*A*B*a^9*b^5))/(a*b^{16} + b^{17} - 5*a^2*b^{15} - 5*a^3*b^{14} + \\
& 10*a^4*b^{13} + 10*a^5*b^{12} - 10*a^6*b^{11} - 10*a^7*b^{10} + 5*a^8*b^9 + 5*a^9*b^8 \\
& - a^{10}*b^7 - a^{11}*b^6) + (((8*(4*A*b^{21} + 4*B*b^{21} - 6*A*a^2*b^{19} + 6*A* \\
& a^3*b^{18} - 6*A*a^4*b^{17} + 6*A*a^5*b^{16} + 14*A*a^6*b^{15} - 14*A*a^7*b^{14} - 6* \\
& A*a^8*b^{13} + 6*A*a^9*b^{12} - 12*B*a^2*b^{19} + 64*B*a^3*b^{18} + 20*B*a^4*b^{17} - \\
& 110*B*a^5*b^{16} - 30*B*a^6*b^{15} + 110*B*a^7*b^{14} + 30*B*a^8*b^{13} - 70*B*a^9 \\
& *b^{12} - 14*B*a^{10}*b^{11} + 26*B*a^{11}*b^{10} + 2*B*a^{12}*b^9 - 4*B*a^{13}*b^8 - 4*A \\
& *a*b^{20} - 16*B*a*b^{20}))/((a*b^{19} + b^{20} - 5*a^2*b^{18} - 5*a^3*b^{17} + 10*a^4*b \\
& ^{16} + 10*a^5*b^{15} - 10*a^6*b^{14} - 10*a^7*b^{13} + 5*a^8*b^{12} + 5*a^9*b^{11} - a \\
& ^{10}*b^{10} - a^{11}*b^9) + (4*tan(c/2 + (d*x)/2)*((a + b)^7*(a - b)^7)^{(1/2)}*(2 \\
& *A*b^7 + 2*B*a^7 + 3*A*a^2*b^5 + 8*B*a^3*b^4 - 7*B*a^5*b^2 - 8*B*a*b^6)*(8* \\
& a*b^{21} - 8*a^2*b^{20} - 48*a^3*b^{19} + 48*a^4*b^{18} + 120*a^5*b^{17} - 120*a^6*b^{16} \\
& - 160*a^7*b^{15} + 160*a^8*b^{14} + 120*a^9*b^{13} - 120*a^{10}*b^{12} - 48*a^{11}*b \\
& ^{11} + 48*a^{12}*b^{10} + 8*a^{13}*b^9 - 8*a^{14}*b^8))/((b^{18} - 7*a^2*b^{16} + 21*a^4 \\
& *b^{14} - 35*a^6*b^{12} + 35*a^8*b^{10} - 21*a^{10}*b^8 + 7*a^{12}*b^6 - a^{14}*b^4)*(a \\
& *b^{16} + b^{17} - 5*a^2*b^{15} - 5*a^3*b^{14} + 10*a^4*b^{13} + 10*a^5*b^{12} - 10*a^6 \\
& *b^{11} - 10*a^7*b^{10} + 5*a^8*b^9 + 5*a^9*b^8 - a^{10}*b^7 - a^{11}*b^6)))*((a + \\
& b)^7*(a - b)^7)^{(1/2)}*(2*A*b^7 + 2*B*a^7 + 3*A*a^2*b^5 + 8*B*a^3*b^4 - 7*B* \\
& a^5*b^2 - 8*B*a*b^6))/(2*(b^{18} - 7*a^2*b^{16} + 21*a^4*b^{14} - 35*a^6*b^{12} + 3 \\
& 5*a^8*b^{10} - 21*a^{10}*b^8 + 7*a^{12}*b^6 - a^{14}*b^4)))*((a + b)^7*(a - b)^7)^{(\\
& 1/2)}*(2*A*b^7 + 2*B*a^7 + 3*A*a^2*b^5 + 8*B*a^3*b^4 - 7*B*a^5*b^2 - 8*B*a*b^ \\
& ^6))/(2*(b^{18} - 7*a^2*b^{16} + 21*a^4*b^{14} - 35*a^6*b^{12} + 35*a^8*b^{10} - 21*a \\
& ^{10}*b^8 + 7*a^{12}*b^6 - a^{14}*b^4)))*((a + b)^7*(a - b)^7)^{(1/2)}*(2*A*b^7 + \\
& 2*B*a^7 + 3*A*a^2*b^5 + 8*B*a^3*b^4 - 7*B*a^5*b^2 - 8*B*a*b^6)*1i)/(d*(b^{18} \\
& - 7*a^2*b^{16} + 21*a^4*b^{14} - 35*a^6*b^{12} + 35*a^8*b^{10} - 21*a^{10}*b^8 + 7*a \\
& ^{12}*b^6 - a^{14}*b^4))
\end{aligned}$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \sec^4(c + dx)}{(a + b \sec(c + dx))^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**4*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**4,x)

[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**4/(a + b*sec(c + d*x))**4, x)

$$3.338 \quad \int \frac{\sec^3(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^4} dx$$

Optimal. Leaf size=274

$$\frac{a^2(Ab - aB) \tan(c + dx)}{3b^2d(a^2 - b^2)(a + b \sec(c + dx))^3} + \frac{(a^3A - 3a^2bB + 4aAb^2 - 2b^3B) \tanh^{-1}\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}} + \frac{a(-4a^3B + a^2A)}{6b^2d(a^2 - b^2)}$$

[Out] (A*a^3+4*A*a*b^2-3*B*a^2*b-2*B*b^3)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(7/2)/(a+b)^(7/2)/d-1/3*a^2*(A*b-B*a)*tan(d*x+c)/b^2/(a^2-b^2)/d/(a+b*sec(d*x+c))^3+1/6*a*(A*a^2*b-6*A*b^3-4*B*a^3+9*B*a*b^2)*tan(d*x+c)/b^2/(a^2-b^2)^2/d/(a+b*sec(d*x+c))^2+1/6*(A*a^4*b-10*A*a^2*b^3-6*A*b^5+2*B*a^5-5*B*a^3*b^2+18*B*a*b^4)*tan(d*x+c)/b^2/(a^2-b^2)^3/d/(a+b*sec(d*x+c))

Rubi [A] time = 0.70, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {4028, 4080, 4003, 12, 3831, 2659, 208}

$$\frac{(a^3A - 3a^2bB + 4aAb^2 - 2b^3B) \tanh^{-1}\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}} - \frac{a^2(Ab - aB) \tan(c + dx)}{3b^2d(a^2 - b^2)(a + b \sec(c + dx))^3} + \frac{a(a^2Ab - 4a^3B + a^2A)}{6b^2d(a^2 - b^2)}$$

Antiderivative was successfully verified.

[In] Int[(Sec[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^4,x]

[Out] ((a^3*A + 4*a*A*b^2 - 3*a^2*b*B - 2*b^3*B)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(7/2)*(a + b)^(7/2)*d) - (a^2*(A*b - a*B)*Tan[c + d*x])/(3*b^2*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^3) + (a*(a^2*A*b - 6*A*b^3 - 4*a^3*B + 9*a*b^2*B)*Tan[c + d*x])/(6*b^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x])^2) + ((a^4*A*b - 10*a^2*A*b^3 - 6*A*b^5 + 2*a^5*B - 5*a^3*b^2*B + 18*a*b^4*B)*Tan[c + d*x])/(6*b^2*(a^2 - b^2)^3*d*(a + b*Sec[c + d*x]))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 2659

```
Int[((a_) + (b_)*sin[Pi/2 + (c_) + (d_)*(x_)])^(-1), x_Symbol] := With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3831

```
Int[csc[(e_) + (f_)*(x_)]/(csc[(e_) + (f_)*(x_)]*(b_) + (a_)), x_Symbol] := Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4003

```
Int[csc[(e_) + (f_)*(x_)]*(csc[(e_) + (f_)*(x_)]*(b_) + (a_))^(m_)*(csc[(e_) + (f_)*(x_)]*(B_) + (A_)), x_Symbol] := -Simp[((A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(m + 1)*(a^2 - b^2), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[(a*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4028

```
Int[csc[(e_) + (f_)*(x_)]^3*(csc[(e_) + (f_)*(x_)]*(b_) + (a_))^(m_)*(csc[(e_) + (f_)*(x_)]*(B_) + (A_)), x_Symbol] := -Simp[(a^2*(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[a*b*(A*b - a*B)*(m + 1) - (A*b - a*B)*(a^2 + b^2*(m + 1))*Csc[e + f*x] + b*B*(m + 1)*(a^2 - b^2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4080

```
Int[csc[(e_) + (f_)*(x_)]*((A_) + csc[(e_) + (f_)*(x_)]*(B_) + csc[(e_) + (f_)*(x_)]^2*(C_))*(csc[(e_) + (f_)*(x_)]*(b_) + (a_))^(m_), x_Symbol] := -Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C))*(m + 1))*Csc[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^3(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^4} dx &= -\frac{a^2(Ab-aB)\tan(c+dx)}{3b^2(a^2-b^2)d(a+b\sec(c+dx))^3} - \frac{\int \frac{\sec(c+dx)(-3ab(Ab-aB)-(a^2-3b^2)(Ab-aB))}{(a+b\sec(c+dx))^3} dx}{3b^2(a^2-b^2)} \\
&= -\frac{a^2(Ab-aB)\tan(c+dx)}{3b^2(a^2-b^2)d(a+b\sec(c+dx))^3} + \frac{a(a^2Ab-6Ab^3-4a^3B+9ab^2B)}{6b^2(a^2-b^2)^2d(a+b\sec(c+dx))^2} \\
&= -\frac{a^2(Ab-aB)\tan(c+dx)}{3b^2(a^2-b^2)d(a+b\sec(c+dx))^3} + \frac{a(a^2Ab-6Ab^3-4a^3B+9ab^2B)}{6b^2(a^2-b^2)^2d(a+b\sec(c+dx))^2} \\
&= -\frac{a^2(Ab-aB)\tan(c+dx)}{3b^2(a^2-b^2)d(a+b\sec(c+dx))^3} + \frac{a(a^2Ab-6Ab^3-4a^3B+9ab^2B)}{6b^2(a^2-b^2)^2d(a+b\sec(c+dx))^2} \\
&= -\frac{a^2(Ab-aB)\tan(c+dx)}{3b^2(a^2-b^2)d(a+b\sec(c+dx))^3} + \frac{a(a^2Ab-6Ab^3-4a^3B+9ab^2B)}{6b^2(a^2-b^2)^2d(a+b\sec(c+dx))^2} \\
&= -\frac{a^2(Ab-aB)\tan(c+dx)}{3b^2(a^2-b^2)d(a+b\sec(c+dx))^3} + \frac{a(a^2Ab-6Ab^3-4a^3B+9ab^2B)}{6b^2(a^2-b^2)^2d(a+b\sec(c+dx))^2} \\
&= \frac{(a^3A+4aAb^2-3a^2bB-2b^3B)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} - \frac{a^2(Ab-aB)}{3b^2(a^2-b^2)}
\end{aligned}$$

Mathematica [A] time = 2.83, size = 226, normalized size = 0.82

$$\frac{(3a^2A-5abB+2Ab^2)\sin(c+dx)}{(a-b)^2(a+b)^2(a\cos(c+dx)+b)^2} + \frac{(4a^3B-13a^2Ab+11ab^2B-2Ab^3)\sin(c+dx)}{(a-b)^3(a+b)^3(a\cos(c+dx)+b)} - \frac{6(a^3A-3a^2bB+4aAb^2-2b^3B)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2}} + \frac{2(aB)}{(a-b)(a+b)}$$

$6d$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^4,x]
[Out] ((-6*(a^3*A + 4*a*A*b^2 - 3*a^2*b*B - 2*b^3*B)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(7/2) + (2*(-(A*b) + a*B)*Sin[c + d*x])
```

$$\frac{1}{(a-b)(a+b)(b+a\cos[c+dx])^3} + \frac{(3a^2A + 2Ab^2 - 5a*b*B)\sin[c+dx]}{(a-b)^2(a+b)^2(b+a\cos[c+dx])^2} + \frac{(-13a^2A - b - 2Ab^3 + 4a^3B + 11a*b^2*B)\sin[c+dx]}{(a-b)^3(a+b)^3(b+a\cos[c+dx])} \Big/ (6*d)$$

fricas [B] time = 0.60, size = 1230, normalized size = 4.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(dx+c)^3*(A+B*sec(dx+c))/(a+b*sec(dx+c))^4,x, algorithm="fricas")

[Out] $\left[\frac{1}{12} \cdot (3(Aa^3b^3 - 3Ba^2b^4 + 4Aa*b^5 - 2Bb^6 + (Aa^6 - 3Ba^5b + 4Aa^4b^2 - 2Ba^3b^3) \cos(dx+c)^3 + 3(Aa^5b - 3Ba^4b^2 + 4Aa^3b^3 - 2Ba^2b^4) \cos(dx+c)^2 + 3(Aa^4b^2 - 3Ba^3b^3 + 4Aa^2b^4 - 2Ba*b^5) \cos(dx+c)) \sqrt{a^2 - b^2} \log((2a*b*\cos(dx+c) - (a^2 - 2b^2) \cos(dx+c)^2 + 2\sqrt{a^2 - b^2} (b*\cos(dx+c) + a) \sin(dx+c) + 2a^2 - b^2) / (a^2 \cos(dx+c)^2 + 2a*b*\cos(dx+c) + b^2)) + 2(2Ba^7 + Aa^6b - 7Ba^5b^2 - 11Aa^4b^3 + 23Ba^3b^4 + 4Aa^2b^5 - 18Ba*b^6 + 6A*b^7 + (4Ba^7 - 13Aa^6b + 7Ba^5b^2 + 11Aa^4b^3 - 11Ba^3b^4 + 2Aa^2b^5) \cos(dx+c)^2 + 3(Aa^7 + Ba^6b - 10Aa^5b^2 + 8Ba^4b^3 + 7Aa^3b^4 - 9Ba^2b^5 + 2Aa*b^6) \cos(dx+c)) \sin(dx+c) / ((a^{11} - 4a^9b^2 + 6a^7b^4 - 4a^5b^6 + a^3b^8) * d \cos(dx+c)^3 + 3(a^{10}b - 4a^8b^3 + 6a^6b^5 - 4a^4b^7 + a^2b^9) * d \cos(dx+c)^2 + 3(a^9b^2 - 4a^7b^4 + 6a^5b^6 - 4a^3b^8 + ab^{10}) * d \cos(dx+c) + (a^8b^3 - 4a^6b^5 + 6a^4b^7 - 4a^2b^9 + b^{11}) * d) \right. \\ \left. , \frac{1}{6} \cdot (3(Aa^3b^3 - 3Ba^2b^4 + 4Aa*b^5 - 2Bb^6 + (Aa^6 - 3Ba^5b + 4Aa^4b^2 - 2Ba^3b^3) \cos(dx+c)^3 + 3(Aa^5b - 3Ba^4b^2 + 4Aa^3b^3 - 2Ba^2b^4) \cos(dx+c)^2 + 3(Aa^4b^2 - 3Ba^3b^3 + 4Aa^2b^4 - 2Ba*b^5) \cos(dx+c)) \sqrt{-a^2 + b^2} \arctan(-\sqrt{-a^2 + b^2} (b*\cos(dx+c) + a) / ((a^2 - b^2) \sin(dx+c))) + (2Ba^7 + Aa^6b - 7Ba^5b^2 - 11Aa^4b^3 + 23Ba^3b^4 + 4Aa^2b^5 - 18Ba*b^6 + 6A*b^7 + (4Ba^7 - 13Aa^6b + 7Ba^5b^2 + 11Aa^4b^3 - 11Ba^3b^4 + 2Aa^2b^5) \cos(dx+c)^2 + 3(Aa^7 + Ba^6b - 10Aa^5b^2 + 8Ba^4b^3 + 7Aa^3b^4 - 9Ba^2b^5 + 2Aa*b^6) \cos(dx+c)) \sin(dx+c) / ((a^{11} - 4a^9b^2 + 6a^7b^4 - 4a^5b^6 + a^3b^8) * d \cos(dx+c)^3 + 3(a^{10}b - 4a^8b^3 + 6a^6b^5 - 4a^4b^7 + a^2b^9) * d \cos(dx+c)^2 + 3(a^9b^2 - 4a^7b^4 + 6a^5b^6 - 4a^3b^8 + ab^{10}) * d \cos(dx+c) + (a^8b^3 - 4a^6b^5 + 6a^4b^7 - 4a^2b^9 + b^{11}) * d) \right]$

giac [B] time = 0.42, size = 693, normalized size = 2.53

$$\frac{3 \left(Aa^3 - 3Ba^2b + 4Aab^2 - 2Bb^3 \right) \left(\pi \left[\frac{dx+c}{2\pi} + \frac{1}{2} \right] \operatorname{sgn}(-2a+2b) + \arctan \left(-\frac{a \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) - b \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)}{\sqrt{-a^2+b^2}} \right) \right)}{(a^6 - 3a^4b^2 + 3a^2b^4 - b^6) \sqrt{-a^2+b^2}} + \frac{3Aa^5 \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) - 6Ba^5 \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^5}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^4,x, algorithm="giac")

[Out] $\frac{1}{3} * (3 * (A * a^3 - 3 * B * a^2 * b + 4 * A * a * b^2 - 2 * B * b^3) * (\pi * \operatorname{floor}(1/2 * (d * x + c) / \pi + 1/2) * \operatorname{sgn}(-2 * a + 2 * b) + \arctan(- (a * \tan(1/2 * d * x + 1/2 * c) - b * \tan(1/2 * d * x + 1/2 * c)) / \sqrt{-a^2 + b^2}))) / ((a^6 - 3 * a^4 * b^2 + 3 * a^2 * b^4 - b^6) * \sqrt{-a^2 + b^2}) + (3 * A * a^5 * \tan(1/2 * d * x + 1/2 * c)^5 - 6 * B * a^5 * \tan(1/2 * d * x + 1/2 * c)^5 + 12 * A * a^4 * b * \tan(1/2 * d * x + 1/2 * c)^5 + 3 * B * a^4 * b * \tan(1/2 * d * x + 1/2 * c)^5 - 27 * A * a^3 * b^2 * \tan(1/2 * d * x + 1/2 * c)^5 - 6 * B * a^3 * b^2 * \tan(1/2 * d * x + 1/2 * c)^5 + 12 * A * a^2 * b^3 * \tan(1/2 * d * x + 1/2 * c)^5 + 27 * B * a^2 * b^3 * \tan(1/2 * d * x + 1/2 * c)^5 - 6 * A * a * b^4 * \tan(1/2 * d * x + 1/2 * c)^5 - 18 * B * a * b^4 * \tan(1/2 * d * x + 1/2 * c)^5 + 6 * A * b^5 * \tan(1/2 * d * x + 1/2 * c)^5 + 4 * B * a^5 * \tan(1/2 * d * x + 1/2 * c)^3 - 28 * A * a^4 * b * \tan(1/2 * d * x + 1/2 * c)^3 + 32 * B * a^3 * b^2 * \tan(1/2 * d * x + 1/2 * c)^3 + 16 * A * a^2 * b^3 * \tan(1/2 * d * x + 1/2 * c)^3 - 36 * B * a * b^4 * \tan(1/2 * d * x + 1/2 * c)^3 + 12 * A * b^5 * \tan(1/2 * d * x + 1/2 * c)^3 - 3 * A * a^5 * \tan(1/2 * d * x + 1/2 * c) - 6 * B * a^5 * \tan(1/2 * d * x + 1/2 * c) + 12 * A * a^4 * b * \tan(1/2 * d * x + 1/2 * c) - 3 * B * a^4 * b * \tan(1/2 * d * x + 1/2 * c) + 27 * A * a^3 * b^2 * \tan(1/2 * d * x + 1/2 * c) - 6 * B * a^3 * b^2 * \tan(1/2 * d * x + 1/2 * c) + 12 * A * a^2 * b^3 * \tan(1/2 * d * x + 1/2 * c) - 27 * B * a^2 * b^3 * \tan(1/2 * d * x + 1/2 * c) + 6 * A * a * b^4 * \tan(1/2 * d * x + 1/2 * c) - 18 * B * a * b^4 * \tan(1/2 * d * x + 1/2 * c) + 6 * A * b^5 * \tan(1/2 * d * x + 1/2 * c)) / ((a^6 - 3 * a^4 * b^2 + 3 * a^2 * b^4 - b^6) * (a * \tan(1/2 * d * x + 1/2 * c)^2 - b * \tan(1/2 * d * x + 1/2 * c)^2 - a - b)^3) / d$

maple [A] time = 0.81, size = 375, normalized size = 1.37

$$\frac{2 \left(\frac{(Aa^3 + 6Aa^2b + 2Aab^2 + 2Ab^3 - 2a^3B - 3a^2bB - 6Bab^2) \left(\tan^5 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{2(a-b)(a^3 + 3a^2b + 3b^2a + b^3)} + \frac{2(7Aa^2b + 3Ab^3 - a^3B - 9Bab^2) \left(\tan^3 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{3(a^2 + 2ab + b^2)(a^2 - 2ab + b^2)} + \frac{(Aa^3 - 6Aa^2b + 2Aab^2 - 2Ab^3 + 2a^3B - 3a^2bB + 6Bab^2) \left(\tan \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{2(a+b)(a^3 - 3a^2b + 3b^2a - b^3)} \right)}{\left(a \left(\tan^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) - \left(\tan^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) b - a - b \right)^3} d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^4,x)

[Out] $\frac{1}{d} * (-2 * (-1/2 * (A * a^3 + 6 * A * a^2 * b + 2 * A * a * b^2 + 2 * A * b^3 - 2 * B * a^3 - 3 * B * a^2 * b - 6 * B * a * b^2) / (a - b) / (a^3 + 3 * a^2 * b + 3 * a * b^2 + b^3) * \tan(1/2 * d * x + 1/2 * c)^5 + 2/3 * (7 * A * a^2 * b + 3 * A$

$$\frac{b^3 - B a^3 - 9 B a^2 b^2}{(a^2 + 2 a b + b^2)(a^2 - 2 a b + b^2)} \tan\left(\frac{1}{2} d x + \frac{1}{2} c\right)^3 + \frac{1}{2} \frac{(A a^3 - 6 A a^2 b + 2 A a b^2 - 2 A b^3 + 2 B a^3 - 3 B a^2 b + 6 B a b^2)}{(a+b)(a^3 - 3 a^2 b + 3 a b^2 - b^3)} \tan\left(\frac{1}{2} d x + \frac{1}{2} c\right) / \left(\frac{a \tan\left(\frac{1}{2} d x + \frac{1}{2} c\right)^2 - \tan\left(\frac{1}{2} d x + \frac{1}{2} c\right)^2 b - a b}{(a-b)^3 + (A a^3 + 4 A a^2 b^2 - 3 B a^2 b - 2 B b^3)} \right) / \left(\frac{a^6 - 3 a^4 b^2 + 3 a^2 b^4 - b^6}{((a-b)(a+b))^{1/2} \operatorname{arctanh}\left(\frac{\tan\left(\frac{1}{2} d x + \frac{1}{2} c\right)(a-b)}{(a-b)(a+b)}\right)} \right)$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(dx+c)^3*(A+B*sec(dx+c))/(a+b*sec(dx+c))^4,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for more details)Is 4*a^2-4*b^2 positive or negative?

mupad [B] time = 6.80, size = 439, normalized size = 1.60

$$\frac{4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 (-B a^3 + 7 A a^2 b - 9 B a b^2 + 3 A b^3)}{3(a+b)^2(a^2 - 2ab + b^2)} - \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5 (A a^3 + 2 A b^3 - 2 B a^3 + 2 A a b^2 + 6 A a^2 b - 6 B a b^2 - 3 B a^2 b)}{(a+b)^3(a-b)} + \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{3(a+b)^2(a^2 - 2ab + b^2)}$$

$$d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 (-3 a^3 - 3 a^2 b + 3 a b^2 + 3 b^3) - \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 (-3 a^3 + 3 a^2 b + 3 a b^2 - 3 b^3) + 3 a b^2 + 3 a^2 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + dx))/(cos(c + dx)^3*(a + b/cos(c + dx))^4),x)

[Out]
$$\frac{((4 \tan(c/2 + (dx)/2)^3 (3 A a^3 - B a^3 + 7 A a^2 b - 9 B a^2 b^2)) / (3 (a + b)^2 (a^2 - 2 a b + b^2)) - (\tan(c/2 + (dx)/2)^5 (A a^3 + 2 A b^3 - 2 B a^3 + 2 A a^2 b + 6 A a b^2 - 6 B a b^2 - 3 B a^2 b)) / ((a + b)^3 (a - b)) + (\tan(c/2 + (dx)/2) (A a^3 - 2 A b^3 + 2 B a^3 + 2 A a^2 b - 6 A a^2 b + 6 B a^2 b - 3 B a^2 b)) / ((a + b) (3 a^2 b^2 - 3 a^2 b + a^3 - b^3))}{(d (\tan(c/2 + (dx)/2)^2 (3 a^2 b^2 - 3 a^2 b - 3 a^3 + 3 b^3) - \tan(c/2 + (dx)/2)^4 (3 a^2 b^2 + 3 a^2 b - 3 a^3 - 3 b^3) + 3 a^2 b^2 + 3 a^2 b + a^3 + b^3 - \tan(c/2 + (dx)/2)^6 (3 a^2 b^2 - 3 a^2 b + a^3 - b^3)) + (\operatorname{atanh}(\frac{\tan(c/2 + (dx)/2)}{2} (2 a - 2 b) (3 a^2 b^2 - 3 a^2 b + a^3 - b^3)) / (2 (a + b)^{1/2} (a - b)^{7/2})) (A a^3 - 2 B b^3 + 4 A a^2 b - 3 B a^2 b)) / (d (a + b)^{7/2} (a - b)^{7/2}}$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \sec^3(c + dx)}{(a + b \sec(c + dx))^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**4,x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**3/(a + b*sec(c + d*x))**4, x)
```

$$3.339 \quad \int \frac{\sec^2(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^4} dx$$

Optimal. Leaf size=263

$$\frac{a(Ab - aB) \tan(c + dx)}{3bd(a^2 - b^2)(a + b \sec(c + dx))^3} - \frac{(a^3(-B) + 4a^2Ab - 4ab^2B + Ab^3) \tanh^{-1}\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}} + \frac{(a^3B + 2a^2Ab - 2aAb^2 - b^3B)}{6bd(a^2 - b^2)(a + b \sec(c + dx))}$$

[Out] $-(4Aa^2b + Ab^3 - B^3a^3 - 4A^2b^2) \operatorname{arctanh}\left(\frac{(a-b)^{1/2} \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)}{(a+b)^{1/2}}\right) / (a-b)^{7/2} / (a+b)^{7/2} / d + 1/3 a (Ab - Ba) \tan(dx + c) / b / (a^2 - b^2) / d / (a + b \sec(dx + c))^3 + 1/6 (2Aa^2b + 3A^2b^3 + B^3a^3 - 6A^2b^2) \tan(dx + c) / b / (a^2 - b^2)^2 / d / (a + b \sec(dx + c))^2 + 1/6 (2Aa^3b + 13A^2b^3 + B^4a^4 - 10A^2b^2 - 6B^4b^4) \tan(dx + c) / b / (a^2 - b^2)^3 / d / (a + b \sec(dx + c))$

Rubi [A] time = 0.62, antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4009, 4003, 12, 3831, 2659, 208}

$$\frac{(4a^2Ab + a^3(-B) - 4ab^2B + Ab^3) \tanh^{-1}\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}} + \frac{(2a^3Ab - 10a^2b^2B + a^4B + 13aAb^3 - 6b^4B) \tan(c + dx)}{6bd(a^2 - b^2)^3(a + b \sec(c + dx))}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(\operatorname{Sec}[c + dx])^2(A + B \operatorname{Sec}[c + dx])] / (a + b \operatorname{Sec}[c + dx])^4, x]$

[Out] $-\left(\frac{4a^2Ab + Ab^3 - a^3B - 4a^2b^2B}{(a-b)^{7/2}(a+b)^{7/2}d} \operatorname{ArcTanh}\left[\frac{\sqrt{a-b} \tan\left(\frac{c+dx}{2}\right)}{\sqrt{a+b}}\right] + \frac{a(Ab - aB) \tan(c + dx)}{(3b(a^2 - b^2)d(a + b \operatorname{Sec}[c + dx])^3} + \frac{(2a^2Ab + 3A^2b^3 + a^3B - 6a^2b^2B) \tan(c + dx)}{(6b(a^2 - b^2)^2d(a + b \operatorname{Sec}[c + dx])^2} + \frac{(2a^3Ab + 13a^2b^3 + a^4B - 10a^2b^2B - 6b^4B) \tan(c + dx)}{(6b(a^2 - b^2)^3d(a + b \operatorname{Sec}[c + dx])}\right)$

Rule 12

$\operatorname{Int}[(a_*) (u_*) , x_Symbol] \rightarrow \operatorname{Dist}[a, \operatorname{Int}[u, x], x] / ; \operatorname{FreeQ}[a, x] \ \&\& \ !\operatorname{Match} Q[u, (b_*) (v_*) / ; \operatorname{FreeQ}[b, x]]$

Rule 208

$\operatorname{Int}[(a_*) + (b_*) (x_*)^2]^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[-(a/b), 2] \operatorname{ArcTanh}[x / \operatorname{Rt}[-(a/b), 2]]) / a, x] / ; \operatorname{FreeQ}[\{a, b\}, x] \ \&\& \ \operatorname{NegQ}[a/b]$

Rule 2659

```
Int[((a_) + (b_)*sin[Pi/2 + (c_) + (d_)*(x_)])^(-1), x_Symbol] := With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3831

```
Int[csc[(e_) + (f_)*(x_)]/(csc[(e_) + (f_)*(x_)]*(b_) + (a_)), x_Symbol] := Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4003

```
Int[csc[(e_) + (f_)*(x_)]*(csc[(e_) + (f_)*(x_)]*(b_) + (a_))^(m_)*(csc[(e_) + (f_)*(x_)]*(B_) + (A_)), x_Symbol] := -Simp[((A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(m + 1)*(a^2 - b^2), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[(a*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4009

```
Int[csc[(e_) + (f_)*(x_)]^2*(csc[(e_) + (f_)*(x_)]*(b_) + (a_))^(m_)*(csc[(e_) + (f_)*(x_)]*(B_) + (A_)), x_Symbol] := Simp[(a*(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(A*b - a*B)*(m + 1) - (a*A*b*(m + 2) - B*(a^2 + b^2*(m + 1)))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^2(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^4} dx &= \frac{a(Ab-aB)\tan(c+dx)}{3b(a^2-b^2)d(a+b\sec(c+dx))^3} + \frac{\int \frac{\sec(c+dx)(-3b(Ab-aB)+(2aAb+a^2B-3b^2))}{(a+b\sec(c+dx))^3}}{3b(a^2-b^2)} \\
&= \frac{a(Ab-aB)\tan(c+dx)}{3b(a^2-b^2)d(a+b\sec(c+dx))^3} + \frac{(2a^2Ab+3Ab^3+a^3B-6ab^2B)\tan(c+dx)}{6b(a^2-b^2)^2d(a+b\sec(c+dx))^2} \\
&= \frac{a(Ab-aB)\tan(c+dx)}{3b(a^2-b^2)d(a+b\sec(c+dx))^3} + \frac{(2a^2Ab+3Ab^3+a^3B-6ab^2B)\tan(c+dx)}{6b(a^2-b^2)^2d(a+b\sec(c+dx))^2} \\
&= \frac{a(Ab-aB)\tan(c+dx)}{3b(a^2-b^2)d(a+b\sec(c+dx))^3} + \frac{(2a^2Ab+3Ab^3+a^3B-6ab^2B)\tan(c+dx)}{6b(a^2-b^2)^2d(a+b\sec(c+dx))^2} \\
&= \frac{a(Ab-aB)\tan(c+dx)}{3b(a^2-b^2)d(a+b\sec(c+dx))^3} + \frac{(2a^2Ab+3Ab^3+a^3B-6ab^2B)\tan(c+dx)}{6b(a^2-b^2)^2d(a+b\sec(c+dx))^2} \\
&= \frac{a(Ab-aB)\tan(c+dx)}{3b(a^2-b^2)d(a+b\sec(c+dx))^3} + \frac{(2a^2Ab+3Ab^3+a^3B-6ab^2B)\tan(c+dx)}{6b(a^2-b^2)^2d(a+b\sec(c+dx))^2} \\
&= \frac{a(Ab-aB)\tan(c+dx)}{3b(a^2-b^2)d(a+b\sec(c+dx))^3} + \frac{(2a^2Ab+3Ab^3+a^3B-6ab^2B)\tan(c+dx)}{6b(a^2-b^2)^2d(a+b\sec(c+dx))^2} \\
&= -\frac{(4a^2Ab+Ab^3-a^3B-4ab^2B)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} + \frac{a(Ab-aB)\tan(c+dx)}{3b(a^2-b^2)}
\end{aligned}$$

Mathematica [A] time = 1.27, size = 252, normalized size = 0.96

$$\frac{24(a^3B-4a^2Ab+4ab^2B-Ab^3)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{2\sin(c+dx)(-6a^5A+11a^4bB-14a^3Ab^2+22a^2b^3B+a(-6a^4A+13a^3bB-10a^2Ab^2+2ab^3B+3b^4))}{(a\cos(c+dx)(a+b\sec(c+dx))^3)}$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^4, x]

[Out] ((24*(-4*a^2*A*b - A*b^3 + a^3*B + 4*a*b^2*B)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] + (2*(-6*a^5*A - 14*a^3*A*b^2 - 25*a*A*b^4 + 11*a^4*b*B + 22*a^2*b^3*B + 12*b^5*B - 6*(2*a^4*A*b + 9*a^2*A*b^2 + 6*a*b^3)))/(a*cos(c + d*x)*(a + b*Sec[c + d*x])^3)

$$\frac{3 - A*b^5 + a^5*B - 9*a^3*b^2*B - 2*a*b^4*B)*\cos[c + d*x] + a*(-6*a^4*A - 10*a^2*A*b^2 + A*b^4 + 13*a^3*b*B + 2*a*b^3*B)*\cos[2*(c + d*x)]*\sin[c + d*x]}{(b + a*\cos[c + d*x])^3/(24*(-a^2 + b^2)^3*d)}$$

fricas [B] time = 0.58, size = 1242, normalized size = 4.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^4,x, algorithm="fricas")

[Out] [1/12*(3*(B*a^3*b^3 - 4*A*a^2*b^4 + 4*B*a*b^5 - A*b^6 + (B*a^6 - 4*A*a^5*b + 4*B*a^4*b^2 - A*a^3*b^3)*cos(d*x + c)^3 + 3*(B*a^5*b - 4*A*a^4*b^2 + 4*B*a^3*b^3 - A*a^2*b^4)*cos(d*x + c)^2 + 3*(B*a^4*b^2 - 4*A*a^3*b^3 + 4*B*a^2*b^4 - A*a*b^5)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + 2*(B*a^6*b + 2*A*a^5*b^2 - 11*B*a^4*b^3 + 11*A*a^3*b^4 + 4*B*a^2*b^5 - 13*A*a*b^6 + 6*B*b^7 + (6*A*a^7 - 13*B*a^6*b + 4*A*a^5*b^2 + 11*B*a^4*b^3 - 11*A*a^3*b^4 + 2*B*a^2*b^5 + A*a*b^6)*cos(d*x + c)^2 + 3*(B*a^7 + 2*A*a^6*b - 10*B*a^5*b^2 + 7*A*a^4*b^3 + 7*B*a^3*b^4 - 10*A*a^2*b^5 + 2*B*a*b^6 + A*b^7)*cos(d*x + c))*sin(d*x + c))/((a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d*cos(d*x + c)^3 + 3*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c)^2 + 3*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c) + (a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d), 1/6*(3*(B*a^3*b^3 - 4*A*a^2*b^4 + 4*B*a*b^5 - A*b^6 + (B*a^6 - 4*A*a^5*b + 4*B*a^4*b^2 - A*a^3*b^3)*cos(d*x + c)^3 + 3*(B*a^5*b - 4*A*a^4*b^2 + 4*B*a^3*b^3 - A*a^2*b^4)*cos(d*x + c)^2 + 3*(B*a^4*b^2 - 4*A*a^3*b^3 + 4*B*a^2*b^4 - A*a*b^5)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) + (B*a^6*b + 2*A*a^5*b^2 - 11*B*a^4*b^3 + 11*A*a^3*b^4 + 4*B*a^2*b^5 - 13*A*a*b^6 + 6*B*b^7 + (6*A*a^7 - 13*B*a^6*b + 4*A*a^5*b^2 + 11*B*a^4*b^3 - 11*A*a^3*b^4 + 2*B*a^2*b^5 + A*a*b^6)*cos(d*x + c)^2 + 3*(B*a^7 + 2*A*a^6*b - 10*B*a^5*b^2 + 7*A*a^4*b^3 + 7*B*a^3*b^4 - 10*A*a^2*b^5 + 2*B*a*b^6 + A*b^7)*cos(d*x + c))*sin(d*x + c))/((a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d*cos(d*x + c)^3 + 3*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c)^2 + 3*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c) + (a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d)]

giac [B] time = 1.99, size = 726, normalized size = 2.76

$$\frac{3 \left(B a^3 - 4 A a^2 b + 4 B a b^2 - A b^3 \right) \left(\pi \left[\frac{d x + c}{2 \pi} + \frac{1}{2} \right] \operatorname{sgn}(-2 a + 2 b) + \arctan \left(-\frac{a \tan \left(\frac{1}{2} d x + \frac{1}{2} c \right) - b \tan \left(\frac{1}{2} d x + \frac{1}{2} c \right)}{\sqrt{-a^2 + b^2}} \right) \right)}{\left(a^6 - 3 a^4 b^2 + 3 a^2 b^4 - b^6 \right) \sqrt{-a^2 + b^2}} - \frac{6 A a^5 \tan \left(\frac{1}{2} d x + \frac{1}{2} c \right)^5 - 3 B a^5 \tan \left(\frac{1}{2} d x + \frac{1}{2} c \right)^5}{\sqrt{-a^2 + b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^4,x, algorithm="giac")

[Out] 1/3*(3*(B*a^3 - 4*A*a^2*b + 4*B*a*b^2 - A*b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(-a^2 + b^2)) - (6*A*a^5*tan(1/2*d*x + 1/2*c)^5 - 3*B*a^5*tan(1/2*d*x + 1/2*c)^5 - 6*A*a^4*b*tan(1/2*d*x + 1/2*c)^5 - 12*B*a^4*b*tan(1/2*d*x + 1/2*c)^5 + 12*A*a^3*b^2*tan(1/2*d*x + 1/2*c)^5 + 27*B*a^3*b^2*tan(1/2*d*x + 1/2*c)^5 - 27*A*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 - 12*B*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 + 12*A*a*b^4*tan(1/2*d*x + 1/2*c)^5 + 6*B*a*b^4*tan(1/2*d*x + 1/2*c)^5 + 3*A*b^5*tan(1/2*d*x + 1/2*c)^5 - 6*B*b^5*tan(1/2*d*x + 1/2*c)^5 - 12*A*a^5*tan(1/2*d*x + 1/2*c)^3 + 28*B*a^4*b*tan(1/2*d*x + 1/2*c)^3 - 16*A*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 - 16*B*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 + 28*A*a*b^4*tan(1/2*d*x + 1/2*c)^3 - 12*B*b^5*tan(1/2*d*x + 1/2*c)^3 + 6*A*a^5*tan(1/2*d*x + 1/2*c) + 3*B*a^5*tan(1/2*d*x + 1/2*c) + 6*A*a^4*b*tan(1/2*d*x + 1/2*c) - 12*B*a^4*b*tan(1/2*d*x + 1/2*c) + 12*A*a^3*b^2*tan(1/2*d*x + 1/2*c) - 27*B*a^3*b^2*tan(1/2*d*x + 1/2*c) + 27*A*a^2*b^3*tan(1/2*d*x + 1/2*c) - 12*B*a^2*b^3*tan(1/2*d*x + 1/2*c) + 12*A*a*b^4*tan(1/2*d*x + 1/2*c) - 6*B*a*b^4*tan(1/2*d*x + 1/2*c) - 3*A*b^5*tan(1/2*d*x + 1/2*c) - 6*B*b^5*tan(1/2*d*x + 1/2*c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^3)/d

maple [A] time = 0.79, size = 388, normalized size = 1.48

$$\frac{\frac{(2 A a^3 + 2 A a^2 b + 6 A a b^2 + A b^3 - a^3 B - 6 a^2 b B - 2 B a b^2 - 2 b^3 B) \left(\tan^5 \left(\frac{d x}{2} + \frac{c}{2} \right) \right)}{(a-b)(a^3 + 3 a^2 b + 3 b^2 a + b^3)} + \frac{4(3 A a^3 + 7 A a b^2 - 7 a^2 b B - 3 b^3 B) \left(\tan^3 \left(\frac{d x}{2} + \frac{c}{2} \right) \right)}{3(a^2 + 2 a b + b^2)(a^2 - 2 a b + b^2)} - \frac{(2 A a^3 - 2 A a^2 b + 6 A a b^2 - A b^3 + a^3 B - 6 a^2 b B + 2 B a b^2 - 2 b^3 B) \left(\tan \left(\frac{d x}{2} + \frac{c}{2} \right) \right)}{(a+b)(a^3 - 3 a^2 b + 3 b^2 a - b^3)}}{d \left(a \left(\tan^2 \left(\frac{d x}{2} + \frac{c}{2} \right) \right) - \left(\tan^2 \left(\frac{d x}{2} + \frac{c}{2} \right) \right) b - a - b \right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^4,x)

[Out] 1/d*(2*(-1/2*(2*A*a^3+2*A*a^2*b+6*A*a*b^2+A*b^3-B*a^3-6*B*a^2*b-2*B*a*b^2-2*B*b^3)/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5+2/3*(3*A*a^3+7

$$\frac{(A^2 b^2 - 7 A B a^2 b + 3 B^2 b^3) / (a^2 + 2 A a b + b^2) / (a^2 - 2 A a b + b^2) \tan(1/2 d x + 1/2 c)^3 - 1/2 (2 A a^3 - 2 A a^2 b + 6 A a b^2 - A b^3 + B a^3 - 6 B a^2 b + 2 B a b^2 - 2 B b^3) / (a + b) / (a^3 - 3 a^2 b + 3 a b^2 - b^3) \tan(1/2 d x + 1/2 c)}{(a \tan(1/2 d x + 1/2 c))^2 - \tan(1/2 d x + 1/2 c)^2 b - a b)^3 - (4 A a^2 b + A b^3 - B a^3 - 4 B a b^2) / (a^6 - 3 a^4 b^2 + 3 a^2 b^4 - b^6) / ((a - b) (a + b))^{1/2} \operatorname{arctanh}(\tan(1/2 d x + 1/2 c) (a - b) / ((a - b) (a + b))^{1/2})}$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^4,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for more details)Is 4*a^2-4*b^2 positive or negative?

mupad [B] time = 6.65, size = 451, normalized size = 1.71

$$\frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right) (2 A a^3 - A b^3 + B a^3 - 2 B b^3 + 6 A a b^2 - 2 A a^2 b + 2 B a b^2 - 6 B a^2 b)}{(a+b)(a^3-3a^2b+3ab^2-b^3)} - \frac{4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 (3 A a^3 - 7 B a^2 b + 7 A a b^2 - 3 B b^3)}{3(a+b)^2(a^2-2ab+b^2)} + \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 (-3 a^3 - 3 a^2 b + 3 a b^2 + 3 b^3) - \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 (-3 a^3 + 3 a^2 b + 3 a b^2 - 3 b^3) + 3 a b^2 + 3 a^2 b \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^2*(a + b/cos(c + d*x))^4),x)

[Out] ((tan(c/2 + (d*x)/2)*(2*A*a^3 - A*b^3 + B*a^3 - 2*B*b^3 + 6*A*a*b^2 - 2*A*a^2*b + 2*B*a*b^2 - 6*B*a^2*b))/((a + b)*(3*a*b^2 - 3*a^2*b + a^3 - b^3)) - (4*tan(c/2 + (d*x)/2)^3*(3*A*a^3 - 3*B*b^3 + 7*A*a*b^2 - 7*B*a^2*b))/(3*(a + b)^2*(a^2 - 2*a*b + b^2)) + (tan(c/2 + (d*x)/2)^5*(2*A*a^3 + A*b^3 - B*a^3 - 2*B*b^3 + 6*A*a*b^2 + 2*A*a^2*b - 2*B*a*b^2 - 6*B*a^2*b))/((a + b)^3*(a - b)))/(d*(tan(c/2 + (d*x)/2)^2*(3*a*b^2 - 3*a^2*b - 3*a^3 + 3*b^3) - tan(c/2 + (d*x)/2)^4*(3*a*b^2 + 3*a^2*b - 3*a^3 - 3*b^3) + 3*a*b^2 + 3*a^2*b + a^3 + b^3 - tan(c/2 + (d*x)/2)^6*(3*a*b^2 - 3*a^2*b + a^3 - b^3))) - (atanh((tan(c/2 + (d*x)/2)*(2*a - 2*b)*(3*a*b^2 - 3*a^2*b + a^3 - b^3))/(2*(a + b))^(1/2)*(a - b)^(7/2)))*(A*b^3 - B*a^3 + 4*A*a^2*b - 4*B*a*b^2))/(d*(a + b)^(7/2)*(a - b)^(7/2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \sec^2(c + dx)}{(a + b \sec(c + dx))^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**4,x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**2/(a + b*sec(c + d*x))**4, x)
```

$$3.340 \quad \int \frac{\sec(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^4} dx$$

Optimal. Leaf size=237

$$\frac{(-2a^2B + 5aAb - 3b^2B) \tan(c + dx)}{6d(a^2 - b^2)^2 (a + b \sec(c + dx))^2} - \frac{(Ab - aB) \tan(c + dx)}{3d(a^2 - b^2) (a + b \sec(c + dx))^3} + \frac{(2a^3A - 4a^2bB + 3aAb^2 - b^3B) \tanh^{-1}}{d(a - b)^{7/2}(a + b)^{7/2}}$$

[Out] (2*A*a^3+3*A*a*b^2-4*B*a^2*b-B*b^3)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(7/2)/(a+b)^(7/2)/d-1/3*(A*b-B*a)*tan(d*x+c)/(a^2-b^2)/d/(a+b*sec(d*x+c))^3-1/6*(5*A*a*b-2*B*a^2-3*B*b^2)*tan(d*x+c)/(a^2-b^2)^2/d/(a+b*sec(d*x+c))^2-1/6*(11*A*a^2*b+4*A*b^3-2*B*a^3-13*B*a*b^2)*tan(d*x+c)/(a^2-b^2)^3/d/(a+b*sec(d*x+c))

Rubi [A] time = 0.51, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {4003, 12, 3831, 2659, 208}

$$\frac{(2a^3A - 4a^2bB + 3aAb^2 - b^3B) \tanh^{-1}\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}} - \frac{(11a^2Ab - 2a^3B - 13ab^2B + 4Ab^3) \tan(c + dx)}{6d(a^2 - b^2)^3 (a + b \sec(c + dx))} - \frac{(-2a^3A + 3a^2bB - 4aAb^2 + b^3B) \tan(c + dx)}{6d(a^2 - b^2)^3 (a + b \sec(c + dx))}$$

Antiderivative was successfully verified.

[In] Int[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^4,x]

[Out] ((2*a^3*A + 3*a*A*b^2 - 4*a^2*b*B - b^3*B)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a - b)^(7/2)*(a + b)^(7/2)*d - ((A*b - a*B)*Tan[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^3) - ((5*a*A*b - 2*a^2*B - 3*b^2*B)*Tan[c + d*x])/(6*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x])^2) - ((11*a^2*A*b + 4*A*b^3 - 2*a^3*B - 13*a*b^2*B)*Tan[c + d*x])/(6*(a^2 - b^2)^3*d*(a + b*Sec[c + d*x]))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 2659

```
Int[((a_) + (b_)*sin[Pi/2 + (c_) + (d_)*(x_)])^(-1), x_Symbol] := With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3831

```
Int[csc[(e_) + (f_)*(x_)]/(csc[(e_) + (f_)*(x_)]*(b_) + (a_)), x_Symbol] := Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4003

```
Int[csc[(e_) + (f_)*(x_)]*(csc[(e_) + (f_)*(x_)]*(b_) + (a_))^(m_)*(csc[(e_) + (f_)*(x_)]*(B_) + (A_)), x_Symbol] := -Simp[((A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(m + 1)*(a^2 - b^2), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[(a*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^4} dx &= -\frac{(Ab-aB)\tan(c+dx)}{3(a^2-b^2)d(a+b\sec(c+dx))^3} - \frac{\int \frac{\sec(c+dx)(-3(aA-bB)+2(Ab-aB)\sec(c+dx))}{(a+b\sec(c+dx))^3}}{3(a^2-b^2)} \\
&= -\frac{(Ab-aB)\tan(c+dx)}{3(a^2-b^2)d(a+b\sec(c+dx))^3} - \frac{(5aAb-2a^2B-3b^2B)\tan(c+dx)}{6(a^2-b^2)^2 d(a+b\sec(c+dx))^2} \\
&= -\frac{(Ab-aB)\tan(c+dx)}{3(a^2-b^2)d(a+b\sec(c+dx))^3} - \frac{(5aAb-2a^2B-3b^2B)\tan(c+dx)}{6(a^2-b^2)^2 d(a+b\sec(c+dx))^2} \\
&= -\frac{(Ab-aB)\tan(c+dx)}{3(a^2-b^2)d(a+b\sec(c+dx))^3} - \frac{(5aAb-2a^2B-3b^2B)\tan(c+dx)}{6(a^2-b^2)^2 d(a+b\sec(c+dx))^2} \\
&= -\frac{(Ab-aB)\tan(c+dx)}{3(a^2-b^2)d(a+b\sec(c+dx))^3} - \frac{(5aAb-2a^2B-3b^2B)\tan(c+dx)}{6(a^2-b^2)^2 d(a+b\sec(c+dx))^2} \\
&= -\frac{(Ab-aB)\tan(c+dx)}{3(a^2-b^2)d(a+b\sec(c+dx))^3} - \frac{(5aAb-2a^2B-3b^2B)\tan(c+dx)}{6(a^2-b^2)^2 d(a+b\sec(c+dx))^2} \\
&= \frac{(2a^3A+3aAb^2-4a^2bB-b^3B)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} - \frac{(Ab-aB)\tan(c+dx)}{3(a^2-b^2)d}
\end{aligned}$$

Mathematica [A] time = 1.11, size = 404, normalized size = 1.70

$$\sec^3(c+dx)(a\cos(c+dx)+b)(A+B\sec(c+dx)) \left(-6a^5B\sin(c+dx) - 6a^5B\sin(3(c+dx)) + 18a^4Ab\sin(c+dx) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^4, x]

[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]^3*(A + B*Sec[c + d*x]))*((24*(2*a^3*A + 3*a*A*b^2 - 4*a^2*b*B - b^3*B)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2

$$\begin{aligned} & - b^2] * (b + a * \cos[c + d * x])^3 / \sqrt{a^2 - b^2} + 18 * a^4 * A * b * \sin[c + d * x] + \\ & 39 * a^2 * A * b^3 * \sin[c + d * x] + 18 * A * b^5 * \sin[c + d * x] - 6 * a^5 * B * \sin[c + d * x] - \\ & 18 * a^3 * b^2 * B * \sin[c + d * x] - 51 * a * b^4 * B * \sin[c + d * x] + 54 * a^3 * A * b^2 * \sin[2 * (\\ & c + d * x)] + 6 * a * A * b^4 * \sin[2 * (c + d * x)] - 12 * a^4 * b * B * \sin[2 * (c + d * x)] - 54 * a \\ & ^2 * b^3 * B * \sin[2 * (c + d * x)] + 6 * b^5 * B * \sin[2 * (c + d * x)] + 18 * a^4 * A * b * \sin[3 * (c \\ & + d * x)] - 5 * a^2 * A * b^3 * \sin[3 * (c + d * x)] + 2 * A * b^5 * \sin[3 * (c + d * x)] - 6 * a^5 * B \\ & * \sin[3 * (c + d * x)] - 10 * a^3 * b^2 * B * \sin[3 * (c + d * x)] + a * b^4 * B * \sin[3 * (c + d * x) \\ &])) / (24 * (-a^2 + b^2)^3 * d * (B + A * \cos[c + d * x]) * (a + b * \sec[c + d * x])^4) \end{aligned}$$

fricas [B] time = 0.59, size = 1238, normalized size = 5.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^4,x, algorithm="fricas")

[Out] [1/12*(3*(2*A*a^3*b^3 - 4*B*a^2*b^4 + 3*A*a*b^5 - B*b^6 + (2*A*a^6 - 4*B*a^5*b + 3*A*a^4*b^2 - B*a^3*b^3)*cos(d*x + c)^3 + 3*(2*A*a^5*b - 4*B*a^4*b^2 + 3*A*a^3*b^3 - B*a^2*b^4)*cos(d*x + c)^2 + 3*(2*A*a^4*b^2 - 4*B*a^3*b^3 + 3*A*a^2*b^4 - B*a*b^5)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + 2*(2*B*a^5*b^2 - 11*A*a^4*b^3 + 11*B*a^3*b^4 + 7*A*a^2*b^5 - 13*B*a*b^6 + 4*A*b^7 + (6*B*a^7 - 18*A*a^6*b + 4*B*a^5*b^2 + 23*A*a^4*b^3 - 11*B*a^3*b^4 - 7*A*a^2*b^5 + B*a*b^6 + 2*A*b^7)*cos(d*x + c)^2 + 3*(2*B*a^6*b - 9*A*a^5*b^2 + 7*B*a^4*b^3 + 8*A*a^3*b^4 - 10*B*a^2*b^5 + A*a*b^6 + B*b^7)*cos(d*x + c))*sin(d*x + c))/((a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d*cos(d*x + c)^3 + 3*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c)^2 + 3*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c) + (a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d), 1/6*(3*(2*A*a^3*b^3 - 4*B*a^2*b^4 + 3*A*a*b^5 - B*b^6 + (2*A*a^6 - 4*B*a^5*b + 3*A*a^4*b^2 - B*a^3*b^3)*cos(d*x + c)^3 + 3*(2*A*a^5*b - 4*B*a^4*b^2 + 3*A*a^3*b^3 - B*a^2*b^4)*cos(d*x + c)^2 + 3*(2*A*a^4*b^2 - 4*B*a^3*b^3 + 3*A*a^2*b^4 - B*a*b^5)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) + (2*B*a^5*b^2 - 11*A*a^4*b^3 + 11*B*a^3*b^4 + 7*A*a^2*b^5 - 13*B*a*b^6 + 4*A*b^7 + (6*B*a^7 - 18*A*a^6*b + 4*B*a^5*b^2 + 23*A*a^4*b^3 - 11*B*a^3*b^4 - 7*A*a^2*b^5 + B*a*b^6 + 2*A*b^7)*cos(d*x + c)^2 + 3*(2*B*a^6*b - 9*A*a^5*b^2 + 7*B*a^4*b^3 + 8*A*a^3*b^4 - 10*B*a^2*b^5 + A*a*b^6 + B*b^7)*cos(d*x + c))*sin(d*x + c))/((a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d*cos(d*x + c)^3 + 3*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c)^2 + 3*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c) + (a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d)]

giac [B] time = 0.40, size = 693, normalized size = 2.92

$$\frac{3(2Aa^3 - 4Ba^2b + 3Aab^2 - Bb^3) \left(\pi \left[\frac{dx+c}{2\pi} + \frac{1}{2} \right] \operatorname{sgn}(2a-2b) + \arctan \left(\frac{a \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) - b \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)}{\sqrt{-a^2+b^2}} \right) \right)}{(a^6 - 3a^4b^2 + 3a^2b^4 - b^6) \sqrt{-a^2+b^2}} + \frac{6Ba^5 \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^5 - 18Aa^4b \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^4}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^4,x, algorithm="giac")

[Out]
$$\frac{-1/3*(3*(2*A*a^3 - 4*B*a^2*b + 3*A*a*b^2 - B*b^3)*(pi*\operatorname{floor}(1/2*(d*x + c)/\pi + 1/2)*\operatorname{sgn}(2*a - 2*b) + \arctan((a*\tan(1/2*d*x + 1/2*c) - b*\tan(1/2*d*x + 1/2*c))/\sqrt{-a^2 + b^2}))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*\sqrt{-a^2 + b^2}) + (6*B*a^5*\tan(1/2*d*x + 1/2*c)^5 - 18*A*a^4*b*\tan(1/2*d*x + 1/2*c)^5 - 6*B*a^4*b*\tan(1/2*d*x + 1/2*c)^5 + 27*A*a^3*b^2*\tan(1/2*d*x + 1/2*c)^5 + 12*B*a^3*b^2*\tan(1/2*d*x + 1/2*c)^5 - 6*A*a^2*b^3*\tan(1/2*d*x + 1/2*c)^5 - 27*B*a^2*b^3*\tan(1/2*d*x + 1/2*c)^5 + 3*A*a*b^4*\tan(1/2*d*x + 1/2*c)^5 + 12*B*a*b^4*\tan(1/2*d*x + 1/2*c)^5 - 6*A*b^5*\tan(1/2*d*x + 1/2*c)^5 + 3*B*b^5*\tan(1/2*d*x + 1/2*c)^5 - 12*B*a^5*\tan(1/2*d*x + 1/2*c)^3 + 36*A*a^4*b*\tan(1/2*d*x + 1/2*c)^3 - 16*B*a^3*b^2*\tan(1/2*d*x + 1/2*c)^3 - 32*A*a^2*b^3*\tan(1/2*d*x + 1/2*c)^3 + 28*B*a*b^4*\tan(1/2*d*x + 1/2*c)^3 - 4*A*b^5*\tan(1/2*d*x + 1/2*c)^3 + 6*B*a^5*\tan(1/2*d*x + 1/2*c) - 18*A*a^4*b*\tan(1/2*d*x + 1/2*c) + 6*B*a^4*b*\tan(1/2*d*x + 1/2*c) - 27*A*a^3*b^2*\tan(1/2*d*x + 1/2*c) + 12*B*a^3*b^2*\tan(1/2*d*x + 1/2*c) - 6*A*a^2*b^3*\tan(1/2*d*x + 1/2*c) + 27*B*a^2*b^3*\tan(1/2*d*x + 1/2*c) - 3*A*a*b^4*\tan(1/2*d*x + 1/2*c) + 12*B*a*b^4*\tan(1/2*d*x + 1/2*c) - 6*A*b^5*\tan(1/2*d*x + 1/2*c) - 3*B*b^5*\tan(1/2*d*x + 1/2*c)))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(a*\tan(1/2*d*x + 1/2*c)^2 - b*\tan(1/2*d*x + 1/2*c)^2 - a - b)^3)/d$$

maple [A] time = 0.73, size = 376, normalized size = 1.59

$$2 \left(\frac{(6Aa^2b + 3Aab^2 + 2Ab^3 - 2a^3B - 2a^2bB - 6Bab^2 - b^3B) \left(\tan^5 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{2(a-b)(a^3 + 3a^2b + 3b^2a + b^3)} + \frac{2(9Aa^2b + Ab^3 - 3a^3B - 7Bab^2) \left(\tan^3 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{3(a^2 + 2ab + b^2)(a^2 - 2ab + b^2)} - \frac{(6Aa^2b - 3Aab^2 + 2Ab^3 - 2a^3B + 2a^2bB - 6Bab^2 + b^3B)}{2(a+b)(a^3 - 3a^2b + 3b^2a - b^3)} \right) \frac{\left(a \left(\tan^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) - \left(\tan^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) b - a - b \right)^3}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^4,x)

[Out]
$$1/d*(-2*(-1/2*(6*A*a^2*b + 3*A*a*b^2 + 2*A*b^3 - 2*B*a^3 - 2*B*a^2*b - 6*B*a*b^2 - B*b^3)/(a-b)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\tan(1/2*d*x + 1/2*c)^5 + 2/3*(9*A*a^2*b + A*b^3 - 3*a^3*B - 7*B*a*b^2)/(a^2 + 2*a*b + b^2)*(a*\tan(1/2*d*x + 1/2*c)^2 - b*\tan(1/2*d*x + 1/2*c)^2 - a - b)^3)/d$$

$$\frac{3-3Ba^3-7Bab^2}{(a^2+2ab+b^2)(a^2-2ab+b^2)} \tan\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^3 - \frac{1}{2} \frac{(6Aa^2b-3Aab^2+2Ab^3-2Ba^3+2Ba^2b-6Bab^2+Bb^3)}{(a+b)} \frac{(a^3-3a^2b+3ab^2-b^3) \tan\left(\frac{1}{2}d*x+\frac{1}{2}c\right)}{(a \tan\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2 - \tan\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2 b - a - b)^3} + \frac{(2Aa^3+3Aab^2-4Ba^2b-Bb^3)}{(a^6-3a^4b^2+3a^2b^4-b^6)} \frac{1}{((a-b)(a+b))^{1/2} \operatorname{arctanh}\left(\frac{\tan\left(\frac{1}{2}d*x+\frac{1}{2}c\right)(a-b)}{(a-b)(a+b)}\right)^{1/2}}$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^4,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for more details) Is 4*a^2-4*b^2 positive or negative?

mupad [B] time = 6.67, size = 439, normalized size = 1.85

$$\frac{4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 (-3Ba^3 + 9Aa^2b - 7Bab^2 + Ab^3)}{3(a+b)^2(a^2 - 2ab + b^2)} + \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5 (2Ba^3 - 2Ab^3 + Bb^3 - 3Aa^2b - 6Aa^2b + 6Bab^2 + 2Ba^2b)}{(a+b)^3(a-b)} - \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 (-3a^3 - 3a^2b + 3ab^2 + 3b^3) - \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 (-3a^3 + 3a^2b + 3ab^2 - 3b^3) + 3ab^2 + 3a^2 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/(cos(c + d*x)*(a + b/cos(c + d*x))^4),x)

[Out]
$$\frac{((4 \tan\left(\frac{c}{2} + \frac{d*x}{2}\right)^3 (A*b^3 - 3B*a^3 + 9A*a^2*b - 7B*a*b^2)) / (3*(a + b)^2*(a^2 - 2*a*b + b^2)) + (\tan\left(\frac{c}{2} + \frac{d*x}{2}\right)^5 (2B*a^3 - 2A*b^3 + B*b^3 - 3A*a^2*b - 6A*a^2*b + 6B*a*b^2 + 2B*a^2*b)) / ((a + b)^3*(a - b)) - (\tan\left(\frac{c}{2} + \frac{d*x}{2}\right) * (2A*b^3 - 2B*a^3 + B*b^3 - 3A*a*b^2 + 6A*a^2*b - 6B*a*b^2 + 2B*a^2*b)) / ((a + b) * (3*a*b^2 - 3*a^2*b + a^3 - b^3))}{d * (\tan\left(\frac{c}{2} + \frac{d*x}{2}\right)^2 * (3*a*b^2 - 3*a^2*b - 3*a^3 + 3*b^3) - \tan\left(\frac{c}{2} + \frac{d*x}{2}\right)^4 * (3*a*b^2 + 3*a^2*b - 3*a^3 - 3*b^3) + 3*a*b^2 + 3*a^2*b + a^3 + b^3 - \tan\left(\frac{c}{2} + \frac{d*x}{2}\right)^6 * (3*a*b^2 - 3*a^2*b + a^3 - b^3))} + (\operatorname{atanh}\left(\frac{\tan\left(\frac{c}{2} + \frac{d*x}{2}\right) * (2*a - 2*b) * (3*a*b^2 - 3*a^2*b + a^3 - b^3)}{(2*(a + b))^{1/2} * (a - b)^{7/2}}\right)) * (2A*a^3 - B*b^3 + 3A*a*b^2 - 4B*a^2*b) / (d * (a + b)^{7/2} * (a - b)^{7/2})$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \sec(c + dx)}{(a + b \sec(c + dx))^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**4,x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)/(a + b*sec(c + d*x))**4, x)
```


$$3.341 \quad \int \frac{A+B \sec(c+dx)}{(a+b \sec(c+dx))^4} dx$$

Optimal. Leaf size=292

$$\frac{Ax}{a^4} + \frac{b(Ab - aB) \tan(c + dx)}{3ad(a^2 - b^2)(a + b \sec(c + dx))^3} + \frac{b(-5a^3B + 8a^2Ab - 3Ab^3) \tan(c + dx)}{6a^2d(a^2 - b^2)^2(a + b \sec(c + dx))^2} + \frac{b(-11a^5B + 26a^4Ab - 4a^3b^2B)}{6a^3d(a^2 - b^2)^3}$$

[Out] $A*x/a^4 - (8*A*a^6*b - 8*A*a^4*b^3 + 7*A*a^2*b^5 - 2*A*b^7 - 2*a^7*B - 3*a^5*b^2*B) * \text{arc tanh}((a-b)^{(1/2)} * \tan(1/2*d*x + 1/2*c) / (a+b)^{(1/2)}) / a^4 / (a-b)^{(7/2)} / (a+b)^{(7/2)} / d + 1/3*b*(A*b - B*a) * \tan(d*x + c) / a / (a^2 - b^2) / d / (a+b*\sec(d*x+c))^3 + 1/6*b*(8*A*a^2*b - 3*A*b^3 - 5*B*a^3) * \tan(d*x+c) / a^2 / (a^2 - b^2)^2 / d / (a+b*\sec(d*x+c))^2 + 1/6*b*(26*A*a^4*b - 17*A*a^2*b^3 + 6*A*b^5 - 11*B*a^5 - 4*B*a^3*b^2) * \tan(d*x+c) / a^3 / (a^2 - b^2)^3 / d / (a+b*\sec(d*x+c))$

Rubi [A] time = 1.07, antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3923, 4060, 3919, 3831, 2659, 208}

$$\frac{(-8a^4Ab^3 + 7a^2Ab^5 + 8a^6Ab - 3a^5b^2B - 2a^7B - 2Ab^7) \tanh^{-1}\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d(a-b)^{7/2}(a+b)^{7/2}} + \frac{b(-17a^2Ab^3 + 26a^4Ab - 4a^3b^2B)}{6a^3d(a^2 - b^2)^3}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sec[c + d*x])/(a + b*Sec[c + d*x])^4, x]

[Out] $(A*x)/a^4 - ((8*a^6*A*b - 8*a^4*A*b^3 + 7*a^2*A*b^5 - 2*A*b^7 - 2*a^7*B - 3*a^5*b^2*B) * \text{ArcTanh}[(\text{Sqrt}[a - b] * \text{Tan}[(c + d*x)/2]) / \text{Sqrt}[a + b]]) / (a^4 * (a - b)^{(7/2)} * (a + b)^{(7/2)} * d) + (b*(A*b - a*B) * \text{Tan}[c + d*x]) / (3*a*(a^2 - b^2) * d * (a + b*\text{Sec}[c + d*x])^3) + (b*(8*a^2*A*b - 3*A*b^3 - 5*a^3*B) * \text{Tan}[c + d*x]) / (6*a^2*(a^2 - b^2)^2 * d * (a + b*\text{Sec}[c + d*x])^2) + (b*(26*a^4*A*b - 17*a^2*A*b^3 + 6*A*b^5 - 11*a^5*B - 4*a^3*b^2*B) * \text{Tan}[c + d*x]) / (6*a^3*(a^2 - b^2)^3 * d * (a + b*\text{Sec}[c + d*x]))$

Rule 208

Int[((a_) + (b_) * (x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2] * ArcTanh[x / Rt[-(a/b), 2]]) / a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 2659

```
Int[((a_) + (b_)*sin[Pi/2 + (c_) + (d_)*(x_)])^(-1), x_Symbol] := With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3831

```
Int[csc[(e_) + (f_)*(x_)]/(csc[(e_) + (f_)*(x_)]*(b_) + (a_)), x_Symbol] := Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3919

```
Int[(csc[(e_) + (f_)*(x_)]*(d_) + (c_))/(csc[(e_) + (f_)*(x_)]*(b_) + (a_)), x_Symbol] := Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3923

```
Int[(csc[(e_) + (f_)*(x_)]*(b_) + (a_))^(m_)*(csc[(e_) + (f_)*(x_)]*(d_) + (c_)), x_Symbol] := Simp[(b*(b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[c*(a^2 - b^2)*(m + 1) - (a*(b*c - a*d)*(m + 1))*Csc[e + f*x] + b*(b*c - a*d)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 4060

```
Int[((A_) + csc[(e_) + (f_)*(x_)]*(B_) + csc[(e_) + (f_)*(x_)]^2*(C_))*(csc[(e_) + (f_)*(x_)]*(b_) + (a_))^(m_), x_Symbol] := Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{A + B \sec(c + dx)}{(a + b \sec(c + dx))^4} dx &= \frac{b(Ab - aB) \tan(c + dx)}{3a(a^2 - b^2) d(a + b \sec(c + dx))^3} - \frac{\int \frac{-3A(a^2 - b^2) + 3a(Ab - aB) \sec(c + dx) - 2b(Ab - aB) \sec^2(c + dx)}{(a + b \sec(c + dx))^3} dx}{3a(a^2 - b^2)} \\
&= \frac{b(Ab - aB) \tan(c + dx)}{3a(a^2 - b^2) d(a + b \sec(c + dx))^3} + \frac{b(8a^2 Ab - 3Ab^3 - 5a^3 B) \tan(c + dx)}{6a^2(a^2 - b^2)^2 d(a + b \sec(c + dx))^2} + \frac{\int \frac{6A}{(a + b \sec(c + dx))^3} dx}{6a^2(a^2 - b^2)^2} \\
&= \frac{b(Ab - aB) \tan(c + dx)}{3a(a^2 - b^2) d(a + b \sec(c + dx))^3} + \frac{b(8a^2 Ab - 3Ab^3 - 5a^3 B) \tan(c + dx)}{6a^2(a^2 - b^2)^2 d(a + b \sec(c + dx))^2} + \frac{b(26Ab - 15a^2 B)}{6a^2(a^2 - b^2)^2} \\
&= \frac{Ax}{a^4} + \frac{b(Ab - aB) \tan(c + dx)}{3a(a^2 - b^2) d(a + b \sec(c + dx))^3} + \frac{b(8a^2 Ab - 3Ab^3 - 5a^3 B) \tan(c + dx)}{6a^2(a^2 - b^2)^2 d(a + b \sec(c + dx))^2} + \frac{b(26Ab - 15a^2 B)}{6a^2(a^2 - b^2)^2} \\
&= \frac{Ax}{a^4} + \frac{b(Ab - aB) \tan(c + dx)}{3a(a^2 - b^2) d(a + b \sec(c + dx))^3} + \frac{b(8a^2 Ab - 3Ab^3 - 5a^3 B) \tan(c + dx)}{6a^2(a^2 - b^2)^2 d(a + b \sec(c + dx))^2} + \frac{b(26Ab - 15a^2 B)}{6a^2(a^2 - b^2)^2} \\
&= \frac{Ax}{a^4} + \frac{b(Ab - aB) \tan(c + dx)}{3a(a^2 - b^2) d(a + b \sec(c + dx))^3} + \frac{b(8a^2 Ab - 3Ab^3 - 5a^3 B) \tan(c + dx)}{6a^2(a^2 - b^2)^2 d(a + b \sec(c + dx))^2} + \frac{b(26Ab - 15a^2 B)}{6a^2(a^2 - b^2)^2} \\
&= \frac{Ax}{a^4} - \frac{(8a^6 Ab - 8a^4 Ab^3 + 7a^2 Ab^5 - 2Ab^7 - 2a^7 B - 3a^5 b^2 B) \tanh^{-1} \left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}} \right)}{a^4(a-b)^{7/2}(a+b)^{7/2}d}
\end{aligned}$$

Mathematica [B] time = 3.51, size = 769, normalized size = 2.63

$$\sec^3(c + dx)(a \cos(c + dx) + b)(A + B \sec(c + dx)) \left(\frac{6a^9 A c \cos(3(c+dx)) + 6a^9 A dx \cos(3(c+dx)) + 36a^8 A b c + 36a^8 A b dx - 18a^8 b B \sin(c + dx)}{a^4(a-b)^{7/2}(a+b)^{7/2}d} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sec[c + d*x])/(a + b*Sec[c + d*x])^4, x]

[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]^3*(A + B*Sec[c + d*x])*((-24*(-8*a^6*A*b + 8*a^4*A*b^3 - 7*a^2*A*b^5 + 2*A*b^7 + 2*a^7*B + 3*a^5*b^2*B)*ArcTanh[(-

$$\begin{aligned} & (a + b) \cdot \tan\left(\frac{c + d \cdot x}{2}\right) / \sqrt{a^2 - b^2} \cdot (b + a \cdot \cos[c + d \cdot x])^3 / (a^2 - b^2)^{7/2} + (36 \cdot a^8 \cdot A \cdot b \cdot c - 84 \cdot a^6 \cdot A \cdot b^3 \cdot c + 36 \cdot a^4 \cdot A \cdot b^5 \cdot c + 36 \cdot a^2 \cdot A \cdot b^7 \cdot c \\ & - 24 \cdot A \cdot b^9 \cdot c + 36 \cdot a^8 \cdot A \cdot b \cdot d \cdot x - 84 \cdot a^6 \cdot A \cdot b^3 \cdot d \cdot x + 36 \cdot a^4 \cdot A \cdot b^5 \cdot d \cdot x + 36 \cdot a^2 \cdot A \cdot b^7 \cdot d \cdot x - 24 \cdot A \cdot b^9 \cdot d \cdot x + 18 \cdot a \cdot A \cdot (a^2 - b^2)^3 \cdot (a^2 + 4 \cdot b^2) \cdot (c + d \cdot x) \cdot \\ & \cos[c + d \cdot x] + 36 \cdot a^2 \cdot A \cdot b \cdot (a^2 - b^2)^3 \cdot (c + d \cdot x) \cdot \cos[2 \cdot (c + d \cdot x)] + 6 \cdot a^9 \cdot A \cdot c \cdot \cos[3 \cdot (c + d \cdot x)] - 18 \cdot a^7 \cdot A \cdot b^2 \cdot c \cdot \cos[3 \cdot (c + d \cdot x)] + 18 \cdot a^5 \cdot A \cdot b^4 \cdot c \cdot \cos \\ & [3 \cdot (c + d \cdot x)] - 6 \cdot a^3 \cdot A \cdot b^6 \cdot c \cdot \cos[3 \cdot (c + d \cdot x)] + 6 \cdot a^9 \cdot A \cdot d \cdot x \cdot \cos[3 \cdot (c + d \cdot x)] - 18 \cdot a^7 \cdot A \cdot b^2 \cdot d \cdot x \cdot \cos[3 \cdot (c + d \cdot x)] + 18 \cdot a^5 \cdot A \cdot b^4 \cdot d \cdot x \cdot \cos[3 \cdot (c + d \cdot x)] \\ & - 6 \cdot a^3 \cdot A \cdot b^6 \cdot d \cdot x \cdot \cos[3 \cdot (c + d \cdot x)] + 36 \cdot a^7 \cdot A \cdot b^2 \cdot \sin[c + d \cdot x] + 72 \cdot a^5 \cdot A \cdot b^4 \cdot \sin[c + d \cdot x] - 57 \cdot a^3 \cdot A \cdot b^6 \cdot \sin[c + d \cdot x] + 24 \cdot a \cdot A \cdot b^8 \cdot \sin[c + d \cdot x] - 18 \cdot \\ & a^8 \cdot b \cdot B \cdot \sin[c + d \cdot x] - 39 \cdot a^6 \cdot b^3 \cdot B \cdot \sin[c + d \cdot x] - 18 \cdot a^4 \cdot b^5 \cdot B \cdot \sin[c + d \cdot x] + 120 \cdot a^6 \cdot A \cdot b^3 \cdot \sin[2 \cdot (c + d \cdot x)] - 90 \cdot a^4 \cdot A \cdot b^5 \cdot \sin[2 \cdot (c + d \cdot x)] + 30 \cdot a^2 \\ & \cdot A \cdot b^7 \cdot \sin[2 \cdot (c + d \cdot x)] - 54 \cdot a^7 \cdot b^2 \cdot B \cdot \sin[2 \cdot (c + d \cdot x)] - 6 \cdot a^5 \cdot b^4 \cdot B \cdot \sin[2 \cdot (c + d \cdot x)] + 36 \cdot a^7 \cdot A \cdot b^2 \cdot \sin[3 \cdot (c + d \cdot x)] - 32 \cdot a^5 \cdot A \cdot b^4 \cdot \sin[3 \cdot (c + d \cdot x)] \\ & + 11 \cdot a^3 \cdot A \cdot b^6 \cdot \sin[3 \cdot (c + d \cdot x)] - 18 \cdot a^8 \cdot b \cdot B \cdot \sin[3 \cdot (c + d \cdot x)] + 5 \cdot a^6 \cdot b^3 \cdot B \cdot \sin[3 \cdot (c + d \cdot x)] - 2 \cdot a^4 \cdot b^5 \cdot B \cdot \sin[3 \cdot (c + d \cdot x)] / (a^2 - b^2)^3) / (24 \cdot a^4 \cdot \\ & d \cdot (B + A \cdot \cos[c + d \cdot x]) \cdot (a + b \cdot \sec[c + d \cdot x])^4) \end{aligned}$$

fricas [B] time = 0.65, size = 1867, normalized size = 6.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^4,x, algorithm="fricas")

[Out] [1/12*(12*(A*a^11 - 4*A*a^9*b^2 + 6*A*a^7*b^4 - 4*A*a^5*b^6 + A*a^3*b^8)*d*x*cos(d*x + c)^3 + 36*(A*a^10*b - 4*A*a^8*b^3 + 6*A*a^6*b^5 - 4*A*a^4*b^7 + A*a^2*b^9)*d*x*cos(d*x + c)^2 + 36*(A*a^9*b^2 - 4*A*a^7*b^4 + 6*A*a^5*b^6 - 4*A*a^3*b^8 + A*a*b^10)*d*x*cos(d*x + c) + 12*(A*a^8*b^3 - 4*A*a^6*b^5 + 6*A*a^4*b^7 - 4*A*a^2*b^9 + A*b^11)*d*x - 3*(2*B*a^7*b^3 - 8*A*a^6*b^4 + 3*B*a^5*b^5 + 8*A*a^4*b^6 - 7*A*a^2*b^8 + 2*A*b^10 + (2*B*a^10 - 8*A*a^9*b + 3*B*a^8*b^2 + 8*A*a^7*b^3 - 7*A*a^5*b^5 + 2*A*a^3*b^7)*cos(d*x + c)^3 + 3*(2*B*a^9*b - 8*A*a^8*b^2 + 3*B*a^7*b^3 + 8*A*a^6*b^4 - 7*A*a^4*b^6 + 2*A*a^2*b^8)*cos(d*x + c)^2 + 3*(2*B*a^8*b^2 - 8*A*a^7*b^3 + 3*B*a^6*b^4 + 8*A*a^5*b^5 - 7*A*a^3*b^7 + 2*A*a*b^9)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 2*(11*B*a^8*b^3 - 26*A*a^7*b^4 - 7*B*a^6*b^5 + 43*A*a^5*b^6 - 4*B*a^4*b^7 - 23*A*a^3*b^8 + 6*A*a*b^10 + (18*B*a^10*b - 36*A*a^9*b^2 - 23*B*a^8*b^3 + 68*A*a^7*b^4 + 7*B*a^6*b^5 - 43*A*a^5*b^6 - 2*B*a^4*b^7 + 11*A*a^3*b^8)*cos(d*x + c)^2 + 3*(9*B*a^9*b^2 - 20*A*a^8*b^3 - 8*B*a^7*b^4 + 35*A*a^6*b^5 - B*a^5*b^6 - 20*A*a^4*b^7 + 5*A*a^2*b^9)*cos(d*x + c))*sin(d*x + c)]/(a^15 - 4*a^13*b^2 + 6*a^11*b^4 - 4*a^9*b^6 + a^7*b^8)*d*cos(d*x + c)^3 + 3*(a^14*b - 4*a^12*b^3 + 6*a^10*b^5 - 4*a^8*b^7 + a^6*b^9)*d*cos(d*x + c)^2 + 3*(a^13*b^2 - 4*a^11*b^4 + 6*a^9*b^6 - 4*a^7*b^8 + a^5*b^10)*d*cos(d

```

*x + c) + (a^12*b^3 - 4*a^10*b^5 + 6*a^8*b^7 - 4*a^6*b^9 + a^4*b^11)*d), 1/
6*(6*(A*a^11 - 4*A*a^9*b^2 + 6*A*a^7*b^4 - 4*A*a^5*b^6 + A*a^3*b^8)*d*x*cos
(d*x + c)^3 + 18*(A*a^10*b - 4*A*a^8*b^3 + 6*A*a^6*b^5 - 4*A*a^4*b^7 + A*a^
2*b^9)*d*x*cos(d*x + c)^2 + 18*(A*a^9*b^2 - 4*A*a^7*b^4 + 6*A*a^5*b^6 - 4*A
*a^3*b^8 + A*a*b^10)*d*x*cos(d*x + c) + 6*(A*a^8*b^3 - 4*A*a^6*b^5 + 6*A*a^
4*b^7 - 4*A*a^2*b^9 + A*b^11)*d*x + 3*(2*B*a^7*b^3 - 8*A*a^6*b^4 + 3*B*a^5*
b^5 + 8*A*a^4*b^6 - 7*A*a^2*b^8 + 2*A*b^10 + (2*B*a^10 - 8*A*a^9*b + 3*B*a^
8*b^2 + 8*A*a^7*b^3 - 7*A*a^5*b^5 + 2*A*a^3*b^7)*cos(d*x + c)^3 + 3*(2*B*a^
9*b - 8*A*a^8*b^2 + 3*B*a^7*b^3 + 8*A*a^6*b^4 - 7*A*a^4*b^6 + 2*A*a^2*b^8)*
cos(d*x + c)^2 + 3*(2*B*a^8*b^2 - 8*A*a^7*b^3 + 3*B*a^6*b^4 + 8*A*a^5*b^5 -
7*A*a^3*b^7 + 2*A*a*b^9)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2
+ b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - (11*B*a^8*b^3 - 2
6*A*a^7*b^4 - 7*B*a^6*b^5 + 43*A*a^5*b^6 - 4*B*a^4*b^7 - 23*A*a^3*b^8 + 6*A
*a*b^10 + (18*B*a^10*b - 36*A*a^9*b^2 - 23*B*a^8*b^3 + 68*A*a^7*b^4 + 7*B*a
^6*b^5 - 43*A*a^5*b^6 - 2*B*a^4*b^7 + 11*A*a^3*b^8)*cos(d*x + c)^2 + 3*(9*B
*a^9*b^2 - 20*A*a^8*b^3 - 8*B*a^7*b^4 + 35*A*a^6*b^5 - B*a^5*b^6 - 20*A*a^4
*b^7 + 5*A*a^2*b^9)*cos(d*x + c))*sin(d*x + c))/((a^15 - 4*a^13*b^2 + 6*a^1
1*b^4 - 4*a^9*b^6 + a^7*b^8)*d*cos(d*x + c)^3 + 3*(a^14*b - 4*a^12*b^3 + 6*
a^10*b^5 - 4*a^8*b^7 + a^6*b^9)*d*cos(d*x + c)^2 + 3*(a^13*b^2 - 4*a^11*b^4
+ 6*a^9*b^6 - 4*a^7*b^8 + a^5*b^10)*d*cos(d*x + c) + (a^12*b^3 - 4*a^10*b^
5 + 6*a^8*b^7 - 4*a^6*b^9 + a^4*b^11)*d)]

```

giac [B] time = 0.41, size = 814, normalized size = 2.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

```

[Out] 1/3*(3*(2*B*a^7 - 8*A*a^6*b + 3*B*a^5*b^2 + 8*A*a^4*b^3 - 7*A*a^2*b^5 + 2*A
*b^7)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/
2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^10 - 3*a^8*
b^2 + 3*a^6*b^4 - a^4*b^6)*sqrt(-a^2 + b^2)) + 3*(d*x + c)*A/a^4 + (18*B*a^
7*b*tan(1/2*d*x + 1/2*c)^5 - 36*A*a^6*b^2*tan(1/2*d*x + 1/2*c)^5 - 27*B*a^6
*b^2*tan(1/2*d*x + 1/2*c)^5 + 60*A*a^5*b^3*tan(1/2*d*x + 1/2*c)^5 + 6*B*a^5
*b^3*tan(1/2*d*x + 1/2*c)^5 + 6*A*a^4*b^4*tan(1/2*d*x + 1/2*c)^5 - 3*B*a^4*
b^4*tan(1/2*d*x + 1/2*c)^5 - 45*A*a^3*b^5*tan(1/2*d*x + 1/2*c)^5 + 6*B*a^3*
b^5*tan(1/2*d*x + 1/2*c)^5 + 6*A*a^2*b^6*tan(1/2*d*x + 1/2*c)^5 + 15*A*a*b^
7*tan(1/2*d*x + 1/2*c)^5 - 6*A*b^8*tan(1/2*d*x + 1/2*c)^5 - 36*B*a^7*b*tan(
1/2*d*x + 1/2*c)^3 + 72*A*a^6*b^2*tan(1/2*d*x + 1/2*c)^3 + 32*B*a^5*b^3*tan
(1/2*d*x + 1/2*c)^3 - 116*A*a^4*b^4*tan(1/2*d*x + 1/2*c)^3 + 4*B*a^3*b^5*ta
n(1/2*d*x + 1/2*c)^3 + 56*A*a^2*b^6*tan(1/2*d*x + 1/2*c)^3 - 12*A*b^8*tan(1
/2*d*x + 1/2*c)^3 + 18*B*a^7*b*tan(1/2*d*x + 1/2*c) - 36*A*a^6*b^2*tan(1/2*
d*x + 1/2*c) + 27*B*a^6*b^2*tan(1/2*d*x + 1/2*c) - 60*A*a^5*b^3*tan(1/2*d*x
+ 1/2*c) + 6*B*a^5*b^3*tan(1/2*d*x + 1/2*c) + 6*A*a^4*b^4*tan(1/2*d*x + 1/

```

$$2*c) + 3*B*a^4*b^4*\tan(1/2*d*x + 1/2*c) + 45*A*a^3*b^5*\tan(1/2*d*x + 1/2*c) + 6*B*a^3*b^5*\tan(1/2*d*x + 1/2*c) + 6*A*a^2*b^6*\tan(1/2*d*x + 1/2*c) - 15*A*a*b^7*\tan(1/2*d*x + 1/2*c) - 6*A*b^8*\tan(1/2*d*x + 1/2*c))/((a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*(a*\tan(1/2*d*x + 1/2*c)^2 - b*\tan(1/2*d*x + 1/2*c)^2 - a - b)^3))/d$$

maple [B] time = 0.89, size = 2242, normalized size = 7.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^4,x)

[Out]
$$\frac{6/d/a/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)^3*b^4/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*\tan(1/2*d*x+1/2*c)^5*A+24/d*b^2/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)^3*a/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*\tan(1/2*d*x+1/2*c)^3*A+6/d*b/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)^3*a^2/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*\tan(1/2*d*x+1/2*c)*B+4/d/a^3/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)^3*b^6/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*\tan(1/2*d*x+1/2*c)^3*A-3/d*a/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)^3*b^2/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*\tan(1/2*d*x+1/2*c)*B-12/d*b^2/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)^3*a/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*\tan(1/2*d*x+1/2*c)^5*A+6/d*b/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)^3*a^2/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*\tan(1/2*d*x+1/2*c)^5*B-12/d*b^2/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)^3*a/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*\tan(1/2*d*x+1/2*c)*A+2/d/a^4/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*\operatorname{arctanh}(\tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b)))^(1/2))*A*b^7+2/d/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)^3*b^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*\tan(1/2*d*x+1/2*c)^5*B+6/d/a/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)^3*b^4/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*\tan(1/2*d*x+1/2*c)*A-44/3/d/a/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)^3*b^5/(a+b)/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*\tan(1/2*d*x+1/2*c)^3*A-1/d/a^2/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)^3*b^6/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*\tan(1/2*d*x+1/2*c)*A-12/d*b/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)^3*a^2/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*\tan(1/2*d*x+1/2*c)^3*B+1/d/a^2/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)^3*b^5/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*\tan(1/2*d*x+1/2*c)^5*A-8/d*b/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*\operatorname{arctanh}(\tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b)))^(1/2))*a^2*A-2/d/a^3/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)^3*b^6/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*\tan(1/2*d*x+1/2*c)^5*A+3/d*a/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)^3*b^2/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*\tan(1/2*d*x+1/2*c)^5*B-4/d/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)^3*b^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*\tan(1/2*d*x+1/2*c)^5*A-7/d/a^2/(a^6-3*a^4*b^2+$$

$$3*a^2*b^4-b^6)/((a-b)*(a+b))^{(1/2)}*\operatorname{arctanh}(\tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^{(1/2)})*A*b^5+2/d*A/a^4*\operatorname{arctan}(\tan(1/2*d*x+1/2*c))+3/d*b^2/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^{(1/2)}*\operatorname{arctanh}(\tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^{(1/2)})*A*B+2/d/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)^3*b^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*\tan(1/2*d*x+1/2*c)*B+4/d/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)^3*b^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*\tan(1/2*d*x+1/2*c)*A-4/3/d/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)^3*b^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*\tan(1/2*d*x+1/2*c)^3*B+2/d/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^{(1/2)}*\operatorname{arctanh}(\tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^{(1/2)})*a^3*B+8/d*b^3/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^{(1/2)}*\operatorname{arctanh}(\tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^{(1/2)})*A$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^4,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for more details)Is 4*a^2-4*b^2 positive or negative?

mupad [B] time = 14.41, size = 9721, normalized size = 33.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/(a + b/cos(c + d*x))^4,x)

[Out] $((\tan(c/2 + (d*x)/2)^5*(6*A*a^2*b^4 - 2*A*b^6 - 4*A*a^3*b^3 - 12*A*a^4*b^2 + 2*B*a^3*b^3 + 3*B*a^4*b^2 + A*a*b^5 + 6*B*a^5*b))/((a^3*b - a^4)*(a + b)^3) - (\tan(c/2 + (d*x)/2)*(2*A*b^6 - 6*A*a^2*b^4 - 4*A*a^3*b^3 + 12*A*a^4*b^2 - 2*B*a^3*b^3 + 3*B*a^4*b^2 + A*a*b^5 - 6*B*a^5*b))/((a + b)*(3*a^5*b - a^6 + a^3*b^3 - 3*a^4*b^2)) + (4*\tan(c/2 + (d*x)/2)^3*(11*A*a^2*b^4 - 3*A*b^6 - 18*A*a^4*b^2 + B*a^3*b^3 + 9*B*a^5*b))/((3*(a + b)^2*(a^5 - 2*a^4*b + a^3*b^2)))/(d*(\tan(c/2 + (d*x)/2)^2*(3*a*b^2 - 3*a^2*b - 3*a^3 + 3*b^3) - \tan(c/2 + (d*x)/2)^4*(3*a*b^2 + 3*a^2*b - 3*a^3 - 3*b^3) + 3*a*b^2 + 3*a^2*b + a^3 + b^3 - \tan(c/2 + (d*x)/2)^6*(3*a*b^2 - 3*a^2*b + a^3 - b^3))) - (2*A*\operatorname{atan}(-((A*((8*\tan(c/2 + (d*x)/2)*(4*A^2*a^14 + 8*A^2*b^14 + 4*B^2*a^14 - 8*A^2*a*b^13 - 8*A^2*a^13*b - 48*A^2*a^2*b^12 + 48*A^2*a^3*b^11 + 117*A^2*a^4*b^10 - 120*A^2*a^5*b^9 - 164*A^2*a^6*b^8 + 160*A^2*a^7*b^7 + 156*A^2*a^8*b^6 - 120*A^2*a^9*b^5 - 92*A^2*a^10*b^4 + 48*A^2*a^11*b^3 + 44*A^2*a^12*b^2 + 9*B^2*a^10*b^4 + 12*B^2*a^12*b^2 - 32*A*B*a^13*b + 12*A*B*a^5*b^9 - 34*A$

$$\begin{aligned}
& B*a^7*b^7 + 20*A*B*a^9*b^5 - 16*A*B*a^{11}*b^3)) / (a^{16}*b + a^{17} - a^6*b^{11} - \\
& a^7*b^{10} + 5*a^8*b^9 + 5*a^9*b^8 - 10*a^{10}*b^7 - 10*a^{11}*b^6 + 10*a^{12}*b^5 \\
& + 10*a^{13}*b^4 - 5*a^{14}*b^3 - 5*a^{15}*b^2) + (A*((8*(4*A*a^{21} + 4*B*a^{21} - 4* \\
& A*a^8*b^{13} + 2*A*a^9*b^{12} + 26*A*a^{10}*b^{11} - 14*A*a^{11}*b^{10} - 70*A*a^{12}*b^9 \\
& + 30*A*a^{13}*b^8 + 110*A*a^{14}*b^7 - 30*A*a^{15}*b^6 - 110*A*a^{16}*b^5 + 20*A*a \\
& ^{17}*b^4 + 64*A*a^{18}*b^3 - 12*A*a^{19}*b^2 + 6*B*a^{12}*b^9 - 6*B*a^{13}*b^8 - 14* \\
& B*a^{14}*b^7 + 14*B*a^{15}*b^6 + 6*B*a^{16}*b^5 - 6*B*a^{17}*b^4 + 6*B*a^{18}*b^3 - 6 \\
& *B*a^{19}*b^2 - 16*A*a^{20}*b - 4*B*a^{20}*b)) / (a^{19}*b + a^{20} - a^9*b^{11} - a^{10}*b \\
& ^{10} + 5*a^{11}*b^9 + 5*a^{12}*b^8 - 10*a^{13}*b^7 - 10*a^{14}*b^6 + 10*a^{15}*b^5 + 1 \\
& 0*a^{16}*b^4 - 5*a^{17}*b^3 - 5*a^{18}*b^2) - (A*\tan(c/2 + (d*x)/2)*(8*a^{21}*b - 8 \\
& *a^8*b^{14} + 8*a^9*b^{13} + 48*a^{10}*b^{12} - 48*a^{11}*b^{11} - 120*a^{12}*b^{10} + 120* \\
& a^{13}*b^9 + 160*a^{14}*b^8 - 160*a^{15}*b^7 - 120*a^{16}*b^6 + 120*a^{17}*b^5 + 48*a \\
& ^{18}*b^4 - 48*a^{19}*b^3 - 8*a^{20}*b^2)*8i) / (a^4*(a^{16}*b + a^{17} - a^6*b^{11} - a^ \\
& 7*b^{10} + 5*a^8*b^9 + 5*a^9*b^8 - 10*a^{10}*b^7 - 10*a^{11}*b^6 + 10*a^{12}*b^5 + \\
& 10*a^{13}*b^4 - 5*a^{14}*b^3 - 5*a^{15}*b^2))) * 1i) / a^4) / a^4 + (A*((8*\tan(c/2 + (\\
& d*x)/2)*(4*A^2*a^{14} + 8*A^2*b^{14} + 4*B^2*a^{14} - 8*A^2*a*b^{13} - 8*A^2*a^{13}*b \\
& - 48*A^2*a^2*b^{12} + 48*A^2*a^3*b^{11} + 117*A^2*a^4*b^{10} - 120*A^2*a^5*b^9 - \\
& 164*A^2*a^6*b^8 + 160*A^2*a^7*b^7 + 156*A^2*a^8*b^6 - 120*A^2*a^9*b^5 - 92 \\
& *A^2*a^{10}*b^4 + 48*A^2*a^{11}*b^3 + 44*A^2*a^{12}*b^2 + 9*B^2*a^{10}*b^4 + 12*B^2 \\
& *a^{12}*b^2 - 32*A*B*a^{13}*b + 12*A*B*a^5*b^9 - 34*A*B*a^7*b^7 + 20*A*B*a^9*b^ \\
& 5 - 16*A*B*a^{11}*b^3)) / (a^{16}*b + a^{17} - a^6*b^{11} - a^7*b^{10} + 5*a^8*b^9 + 5* \\
& a^9*b^8 - 10*a^{10}*b^7 - 10*a^{11}*b^6 + 10*a^{12}*b^5 + 10*a^{13}*b^4 - 5*a^{14}*b^ \\
& 3 - 5*a^{15}*b^2) - (A*((8*(4*A*a^{21} + 4*B*a^{21} - 4*A*a^8*b^{13} + 2*A*a^9*b^{12} \\
& + 26*A*a^{10}*b^{11} - 14*A*a^{11}*b^{10} - 70*A*a^{12}*b^9 + 30*A*a^{13}*b^8 + 110*A \\
& a^{14}*b^7 - 30*A*a^{15}*b^6 - 110*A*a^{16}*b^5 + 20*A*a^{17}*b^4 + 64*A*a^{18}*b^3 - \\
& 12*A*a^{19}*b^2 + 6*B*a^{12}*b^9 - 6*B*a^{13}*b^8 - 14*B*a^{14}*b^7 + 14*B*a^{15}*b^ \\
& 6 + 6*B*a^{16}*b^5 - 6*B*a^{17}*b^4 + 6*B*a^{18}*b^3 - 6*B*a^{19}*b^2 - 16*A*a^{20}*b \\
& - 4*B*a^{20}*b)) / (a^{19}*b + a^{20} - a^9*b^{11} - a^{10}*b^{10} + 5*a^{11}*b^9 + 5*a^{12} \\
& *b^8 - 10*a^{13}*b^7 - 10*a^{14}*b^6 + 10*a^{15}*b^5 + 10*a^{16}*b^4 - 5*a^{17}*b^3 - \\
& 5*a^{18}*b^2) + (A*\tan(c/2 + (d*x)/2)*(8*a^{21}*b - 8*a^8*b^{14} + 8*a^9*b^{13} + \\
& 48*a^{10}*b^{12} - 48*a^{11}*b^{11} - 120*a^{12}*b^{10} + 120*a^{13}*b^9 + 160*a^{14}*b^8 - \\
& 160*a^{15}*b^7 - 120*a^{16}*b^6 + 120*a^{17}*b^5 + 48*a^{18}*b^4 - 48*a^{19}*b^3 - 8 \\
& *a^{20}*b^2)*8i) / (a^4*(a^{16}*b + a^{17} - a^6*b^{11} - a^7*b^{10} + 5*a^8*b^9 + 5*a^ \\
& 9*b^8 - 10*a^{10}*b^7 - 10*a^{11}*b^6 + 10*a^{12}*b^5 + 10*a^{13}*b^4 - 5*a^{14}*b^3 \\
& - 5*a^{15}*b^2))) * 1i) / a^4) / a^4) / ((16*(4*A^3*b^{13} + 4*A*B^2*a^{13} - 4*A^2*B*a^ \\
& 13 - 2*A^3*a*b^{12} + 16*A^3*a^{12}*b - 26*A^3*a^2*b^{11} + 11*A^3*a^3*b^{10} + 70* \\
& A^3*a^4*b^9 - 34*A^3*a^5*b^8 - 110*A^3*a^6*b^7 + 66*A^3*a^7*b^6 + 110*A^3*a^ \\
& 8*b^5 - 64*A^3*a^9*b^4 - 64*A^3*a^{10}*b^3 + 48*A^3*a^{11}*b^2 - 28*A^2*B*a^{12} \\
& *b + 9*A*B^2*a^9*b^4 + 12*A*B^2*a^{11}*b^2 + 6*A^2*B*a^4*b^9 + 6*A^2*B*a^5*b^ \\
& 8 - 20*A^2*B*a^6*b^7 - 14*A^2*B*a^7*b^6 + 14*A^2*B*a^8*b^5 + 6*A^2*B*a^9*b^ \\
& 4 - 22*A^2*B*a^{10}*b^3 + 6*A^2*B*a^{11}*b^2)) / (a^{19}*b + a^{20} - a^9*b^{11} - a^{10} \\
& *b^{10} + 5*a^{11}*b^9 + 5*a^{12}*b^8 - 10*a^{13}*b^7 - 10*a^{14}*b^6 + 10*a^{15}*b^5 + \\
& 10*a^{16}*b^4 - 5*a^{17}*b^3 - 5*a^{18}*b^2) - (A*((8*\tan(c/2 + (d*x)/2)*(4*A^2* \\
& a^{14} + 8*A^2*b^{14} + 4*B^2*a^{14} - 8*A^2*a*b^{13} - 8*A^2*a^{13}*b - 48*A^2*a^2*b \\
& ^{12} + 48*A^2*a^3*b^{11} + 117*A^2*a^4*b^{10} - 120*A^2*a^5*b^9 - 164*A^2*a^6*b^
\end{aligned}$$

$$\begin{aligned}
& 8 + 160A^2a^7b^7 + 156A^2a^8b^6 - 120A^2a^9b^5 - 92A^2a^{10}b^4 + 48A^2a^{11}b^3 + 44A^2a^{12}b^2 + 9B^2a^{10}b^4 + 12B^2a^{12}b^2 - 32A^2a^{13}b + 12ABa^5b^9 - 34ABa^7b^7 + 20ABa^9b^5 - 16ABa^{11}b^3) / (a^{16}b + a^{17} - a^6b^{11} - a^7b^{10} + 5a^8b^9 + 5a^9b^8 - 10a^{10}b^7 - 10a^{11}b^6 + 10a^{12}b^5 + 10a^{13}b^4 - 5a^{14}b^3 - 5a^{15}b^2) \\
& + (A((8(4Aa^{21} + 4Ba^{21} - 4Aa^8b^{13} + 2Aa^9b^{12} + 26Aa^{10}b^{11} - 14Aa^{11}b^{10} - 70Aa^{12}b^9 + 30Aa^{13}b^8 + 110Aa^{14}b^7 - 30Aa^{15}b^6 - 110Aa^{16}b^5 + 20Aa^{17}b^4 + 64Aa^{18}b^3 - 12Aa^{19}b^2 + 6Ba^{12}b^9 - 6Ba^{13}b^8 - 14Ba^{14}b^7 + 14Ba^{15}b^6 + 6Ba^{16}b^5 - 6Ba^{17}b^4 + 6Ba^{18}b^3 - 6Ba^{19}b^2 - 16Aa^{20}b - 4Ba^{20}b))) / (a^{19}b + a^{20} - a^9b^{11} - a^{10}b^{10} + 5a^{11}b^9 + 5a^{12}b^8 - 10a^{13}b^7 - 10a^{14}b^6 + 10a^{15}b^5 + 10a^{16}b^4 - 5a^{17}b^3 - 5a^{18}b^2) - (A \tan(c/2 + (dx)/2) * (8a^{21}b - 8a^8b^{14} + 8a^9b^{13} + 48a^{10}b^{12} - 48a^{11}b^{11} - 120a^{12}b^{10} + 120a^{13}b^9 + 160a^{14}b^8 - 160a^{15}b^7 - 120a^{16}b^6 + 120a^{17}b^5 + 48a^{18}b^4 - 48a^{19}b^3 - 8a^{20}b^2) * 8i) / (a^4 * (a^{16}b + a^{17} - a^6b^{11} - a^7b^{10} + 5a^8b^9 + 5a^9b^8 - 10a^{10}b^7 - 10a^{11}b^6 + 10a^{12}b^5 + 10a^{13}b^4 - 5a^{14}b^3 - 5a^{15}b^2))) * 1i) / a^4 + (A((8 \tan(c/2 + (dx)/2) * (4A^2a^{14} + 8A^2b^{14} + 4B^2a^{14} - 8A^2a^8b^{13} - 8A^2a^{13}b - 48A^2a^2b^{12} + 48A^2a^3b^{11} + 117A^2a^4b^{10} - 120A^2a^5b^9 - 164A^2a^6b^8 + 160A^2a^7b^7 + 156A^2a^8b^6 - 120A^2a^9b^5 - 92A^2a^{10}b^4 + 48A^2a^{11}b^3 + 44A^2a^{12}b^2 + 9B^2a^{10}b^4 + 12B^2a^{12}b^2 - 32ABa^{13}b + 12ABa^5b^9 - 34ABa^7b^7 + 20ABa^9b^5 - 16ABa^{11}b^3))) / (a^{16}b + a^{17} - a^6b^{11} - a^7b^{10} + 5a^8b^9 + 5a^9b^8 - 10a^{10}b^7 - 10a^{11}b^6 + 10a^{12}b^5 + 10a^{13}b^4 - 5a^{14}b^3 - 5a^{15}b^2) - (A((8(4Aa^{21} + 4Ba^{21} - 4Aa^8b^{13} + 2Aa^9b^{12} + 26Aa^{10}b^{11} - 14Aa^{11}b^{10} - 70Aa^{12}b^9 + 30Aa^{13}b^8 + 110Aa^{14}b^7 - 30Aa^{15}b^6 - 110Aa^{16}b^5 + 20Aa^{17}b^4 + 64Aa^{18}b^3 - 12Aa^{19}b^2 + 6Ba^{12}b^9 - 6Ba^{13}b^8 - 14Ba^{14}b^7 + 14Ba^{15}b^6 + 6Ba^{16}b^5 - 6Ba^{17}b^4 + 6Ba^{18}b^3 - 6Ba^{19}b^2 - 16Aa^{20}b - 4Ba^{20}b))) / (a^{19}b + a^{20} - a^9b^{11} - a^{10}b^{10} + 5a^{11}b^9 + 5a^{12}b^8 - 10a^{13}b^7 - 10a^{14}b^6 + 10a^{15}b^5 + 10a^{16}b^4 - 5a^{17}b^3 - 5a^{18}b^2) + (A \tan(c/2 + (dx)/2) * (8a^{21}b - 8a^8b^{14} + 8a^9b^{13} + 48a^{10}b^{12} - 48a^{11}b^{11} - 120a^{12}b^{10} + 120a^{13}b^9 + 160a^{14}b^8 - 160a^{15}b^7 - 120a^{16}b^6 + 120a^{17}b^5 + 48a^{18}b^4 - 48a^{19}b^3 - 8a^{20}b^2) * 8i) / (a^4 * (a^{16}b + a^{17} - a^6b^{11} - a^7b^{10} + 5a^8b^9 + 5a^9b^8 - 10a^{10}b^7 - 10a^{11}b^6 + 10a^{12}b^5 + 10a^{13}b^4 - 5a^{14}b^3 - 5a^{15}b^2))) * 1i) / a^4)) / (a^4 * d) + (\operatorname{atan}((((8 \tan(c/2 + (dx)/2) * (4A^2a^{14} + 8A^2b^{14} + 4B^2a^{14} - 8A^2a^8b^{13} - 8A^2a^{13}b - 48A^2a^2b^{12} + 48A^2a^3b^{11} + 117A^2a^4b^{10} - 120A^2a^5b^9 - 164A^2a^6b^8 + 160A^2a^7b^7 + 156A^2a^8b^6 - 120A^2a^9b^5 - 92A^2a^{10}b^4 + 48A^2a^{11}b^3 + 44A^2a^{12}b^2 + 9B^2a^{10}b^4 + 12B^2a^{12}b^2 - 32ABa^{13}b + 12ABa^5b^9 - 34ABa^7b^7 + 20ABa^9b^5 - 16ABa^{11}b^3))) / (a^{16}b + a^{17} - a^6b^{11} - a^7b^{10} + 5a^8b^9 + 5a^9b^8 - 10a^{10}b^7 - 10a^{11}b^6 + 10a^{12}b^5 + 10a^{13}b^4 - 5a^{14}b^3 - 5a^{15}b^2) + (((8(4Aa^{21} + 4Ba^{21}
\end{aligned}$$

$$\begin{aligned}
& - 4Aa^8b^{13} + 2Aa^9b^{12} + 26Aa^{10}b^{11} - 14Aa^{11}b^{10} - 70Aa^{12}b^9 + 30Aa^{13}b^8 + 110Aa^{14}b^7 - 30Aa^{15}b^6 - 110Aa^{16}b^5 + 20Aa^{17}b^4 + 64Aa^{18}b^3 - 12Aa^{19}b^2 + 6Ba^{12}b^9 - 6Ba^{13}b^8 \\
& - 14Ba^{14}b^7 + 14Ba^{15}b^6 + 6Ba^{16}b^5 - 6Ba^{17}b^4 + 6Ba^{18}b^3 - 6Ba^{19}b^2 - 16Aa^{20}b - 4Ba^{20}b) / (a^{19}b + a^{20} - a^9b^{11} - a^{10}b^{10} + 5a^{11}b^9 + 5a^{12}b^8 - 10a^{13}b^7 - 10a^{14}b^6 + 10a^{15}b^5 + 10a^{16}b^4 - 5a^{17}b^3 - 5a^{18}b^2) - (4\tan(c/2 + (d*x)/2) * ((a + b)^7 * (a - b)^7)^{(1/2)} * (2Ab^7 + 2Ba^7 - 7Aa^2b^5 + 8Aa^4b^3 + 3Ba^5b^2 - 8Aa^6b) * (8a^{21}b - 8a^8b^{14} + 8a^9b^{13} + 48a^{10}b^{12} - 48a^{11}b^{11} - 120a^{12}b^{10} + 120a^{13}b^9 + 160a^{14}b^8 - 160a^{15}b^7 - 120a^{16}b^6 + 120a^{17}b^5 + 48a^{18}b^4 - 48a^{19}b^3 - 8a^{20}b^2)) / ((a^{18} - a^4b^{14} + 7a^6b^{12} - 21a^8b^{10} + 35a^{10}b^8 - 35a^{12}b^6 + 21a^{14}b^4 - 7a^{16}b^2) * (a^{16}b + a^{17} - a^6b^{11} - a^7b^{10} + 5a^8b^9 + 5a^9b^8 - 10a^{10}b^7 - 10a^{11}b^6 + 10a^{12}b^5 + 10a^{13}b^4 - 5a^{14}b^3 - 5a^{15}b^2)) * ((a + b)^7 * (a - b)^7)^{(1/2)} * (2Ab^7 + 2Ba^7 - 7Aa^2b^5 + 8Aa^4b^3 + 3Ba^5b^2 - 8Aa^6b) / (2(a^{18} - a^4b^{14} + 7a^6b^{12} - 21a^8b^{10} + 35a^{10}b^8 - 35a^{12}b^6 + 21a^{14}b^4 - 7a^{16}b^2)) * ((a + b)^7 * (a - b)^7)^{(1/2)} * (2Ab^7 + 2Ba^7 - 7Aa^2b^5 + 8Aa^4b^3 + 3Ba^5b^2 - 8Aa^6b) * i) / (2(a^{18} - a^4b^{14} + 7a^6b^{12} - 21a^8b^{10} + 35a^{10}b^8 - 35a^{12}b^6 + 21a^{14}b^4 - 7a^{16}b^2)) + (((8\tan(c/2 + (d*x)/2) * (4A^2a^{14} + 8A^2b^{14} + 4B^2a^{14} - 8A^2a*b^{13} - 8A^2a^{13}*b - 48A^2a^2b^{12} + 48A^2a^3b^{11} + 117A^2a^4b^{10} - 120A^2a^5b^9 - 164A^2a^6b^8 + 160A^2a^7b^7 + 156A^2a^8b^6 - 120A^2a^9b^5 - 92A^2a^{10}b^4 + 48A^2a^{11}b^3 + 44A^2a^{12}b^2 + 9B^2a^{10}b^4 + 12B^2a^{12}b^2 - 32A*B*a^{13}b + 12A*B*a^5b^9 - 34A*B*a^7b^7 + 20A*B*a^9b^5 - 16A*B*a^{11}b^3)) / (a^{16}b + a^{17} - a^6b^{11} - a^7b^{10} + 5a^8b^9 + 5a^9b^8 - 10a^{10}b^7 - 10a^{11}b^6 + 10a^{12}b^5 + 10a^{13}b^4 - 5a^{14}b^3 - 5a^{15}b^2) - (((8(4Aa^{21} + 4Ba^{21} - 4Aa^8b^{13} + 2Aa^9b^{12} + 26Aa^{10}b^{11} - 14Aa^{11}b^{10} - 70Aa^{12}b^9 + 30Aa^{13}b^8 + 110Aa^{14}b^7 - 30Aa^{15}b^6 - 110Aa^{16}b^5 + 20Aa^{17}b^4 + 64Aa^{18}b^3 - 12Aa^{19}b^2 + 6Ba^{12}b^9 - 6Ba^{13}b^8 - 14Ba^{14}b^7 + 14Ba^{15}b^6 + 6Ba^{16}b^5 - 6Ba^{17}b^4 + 6Ba^{18}b^3 - 6Ba^{19}b^2 - 16Aa^{20}b - 4Ba^{20}b)) / (a^{19}b + a^{20} - a^9b^{11} - a^{10}b^{10} + 5a^{11}b^9 + 5a^{12}b^8 - 10a^{13}b^7 - 10a^{14}b^6 + 10a^{15}b^5 + 10a^{16}b^4 - 5a^{17}b^3 - 5a^{18}b^2) + (4\tan(c/2 + (d*x)/2) * ((a + b)^7 * (a - b)^7)^{(1/2)} * (2Ab^7 + 2Ba^7 - 7Aa^2b^5 + 8Aa^4b^3 + 3Ba^5b^2 - 8Aa^6b) * (8a^{21}b - 8a^8b^{14} + 8a^9b^{13} + 48a^{10}b^{12} - 48a^{11}b^{11} - 120a^{12}b^{10} + 120a^{13}b^9 + 160a^{14}b^8 - 160a^{15}b^7 - 120a^{16}b^6 + 120a^{17}b^5 + 48a^{18}b^4 - 48a^{19}b^3 - 8a^{20}b^2)) / ((a^{18} - a^4b^{14} + 7a^6b^{12} - 21a^8b^{10} + 35a^{10}b^8 - 35a^{12}b^6 + 21a^{14}b^4 - 7a^{16}b^2) * (a^{16}b + a^{17} - a^6b^{11} - a^7b^{10} + 5a^8b^9 + 5a^9b^8 - 10a^{10}b^7 - 10a^{11}b^6 + 10a^{12}b^5 + 10a^{13}b^4 - 5a^{14}b^3 - 5a^{15}b^2)) * ((a + b)^7 * (a - b)^7)^{(1/2)} * (2Ab^7 + 2Ba^7 - 7Aa^2b^5 + 8Aa^4b^3 + 3Ba^5b^2 - 8Aa^6b) / (2(a^{18} - a^4b^{14} + 7a^6b^{12} - 21a^8b^{10} + 35a^{10}b^8 - 35a^{12}b^6 + 21a^{14}b^4 - 7a^{16}b^2)) * ((a + b)^7 * (a - b)^7)^{(1/2)} * (2
\end{aligned}$$

$$\begin{aligned}
& A*b^7 + 2*B*a^7 - 7*A*a^2*b^5 + 8*A*a^4*b^3 + 3*B*a^5*b^2 - 8*A*a^6*b)*1i)/ \\
& (2*(a^{18} - a^4*b^{14} + 7*a^6*b^{12} - 21*a^8*b^{10} + 35*a^{10}*b^8 - 35*a^{12}*b^6 \\
& + 21*a^{14}*b^4 - 7*a^{16}*b^2)))/((16*(4*A^3*b^{13} + 4*A*B^2*a^{13} - 4*A^2*B*a^1 \\
& 3 - 2*A^3*a*b^{12} + 16*A^3*a^{12}*b - 26*A^3*a^2*b^{11} + 11*A^3*a^3*b^{10} + 70*A \\
& ^3*a^4*b^9 - 34*A^3*a^5*b^8 - 110*A^3*a^6*b^7 + 66*A^3*a^7*b^6 + 110*A^3*a^ \\
& 8*b^5 - 64*A^3*a^9*b^4 - 64*A^3*a^{10}*b^3 + 48*A^3*a^{11}*b^2 - 28*A^2*B*a^{12} \\
& b + 9*A*B^2*a^9*b^4 + 12*A*B^2*a^{11}*b^2 + 6*A^2*B*a^4*b^9 + 6*A^2*B*a^5*b^8 \\
& - 20*A^2*B*a^6*b^7 - 14*A^2*B*a^7*b^6 + 14*A^2*B*a^8*b^5 + 6*A^2*B*a^9*b^4 \\
& - 22*A^2*B*a^{10}*b^3 + 6*A^2*B*a^{11}*b^2))/(a^{19}*b + a^{20} - a^9*b^{11} - a^{10} \\
& b^{10} + 5*a^{11}*b^9 + 5*a^{12}*b^8 - 10*a^{13}*b^7 - 10*a^{14}*b^6 + 10*a^{15}*b^5 + \\
& 10*a^{16}*b^4 - 5*a^{17}*b^3 - 5*a^{18}*b^2) - (((8*\tan(c/2 + (d*x)/2)*(4*A^2*a^1 \\
& 4 + 8*A^2*b^{14} + 4*B^2*a^{14} - 8*A^2*a*b^{13} - 8*A^2*a^{13}*b - 48*A^2*a^2*b^{12} \\
& + 48*A^2*a^3*b^{11} + 117*A^2*a^4*b^{10} - 120*A^2*a^5*b^9 - 164*A^2*a^6*b^8 + \\
& 160*A^2*a^7*b^7 + 156*A^2*a^8*b^6 - 120*A^2*a^9*b^5 - 92*A^2*a^{10}*b^4 + 48 \\
& *A^2*a^{11}*b^3 + 44*A^2*a^{12}*b^2 + 9*B^2*a^{10}*b^4 + 12*B^2*a^{12}*b^2 - 32*A*B \\
& *a^{13}*b + 12*A*B*a^5*b^9 - 34*A*B*a^7*b^7 + 20*A*B*a^9*b^5 - 16*A*B*a^{11}*b^ \\
& 3)))/(a^{16}*b + a^{17} - a^6*b^{11} - a^7*b^{10} + 5*a^8*b^9 + 5*a^9*b^8 - 10*a^{10} \\
& b^7 - 10*a^{11}*b^6 + 10*a^{12}*b^5 + 10*a^{13}*b^4 - 5*a^{14}*b^3 - 5*a^{15}*b^2) + \\
& (((8*(4*A*a^{21} + 4*B*a^{21} - 4*A*a^8*b^{13} + 2*A*a^9*b^{12} + 26*A*a^{10}*b^{11} - \\
& 14*A*a^{11}*b^{10} - 70*A*a^{12}*b^9 + 30*A*a^{13}*b^8 + 110*A*a^{14}*b^7 - 30*A*a^{15} \\
& *b^6 - 110*A*a^{16}*b^5 + 20*A*a^{17}*b^4 + 64*A*a^{18}*b^3 - 12*A*a^{19}*b^2 + 6*B \\
& *a^{12}*b^9 - 6*B*a^{13}*b^8 - 14*B*a^{14}*b^7 + 14*B*a^{15}*b^6 + 6*B*a^{16}*b^5 - 6 \\
& *B*a^{17}*b^4 + 6*B*a^{18}*b^3 - 6*B*a^{19}*b^2 - 16*A*a^{20}*b - 4*B*a^{20}*b)))/(a^{1 \\
& 9}*b + a^{20} - a^9*b^{11} - a^{10}*b^{10} + 5*a^{11}*b^9 + 5*a^{12}*b^8 - 10*a^{13}*b^7 - \\
& 10*a^{14}*b^6 + 10*a^{15}*b^5 + 10*a^{16}*b^4 - 5*a^{17}*b^3 - 5*a^{18}*b^2) - (4*ta \\
& n(c/2 + (d*x)/2)*((a + b)^7*(a - b)^7)^{(1/2)}*(2*A*b^7 + 2*B*a^7 - 7*A*a^2*b \\
& ^5 + 8*A*a^4*b^3 + 3*B*a^5*b^2 - 8*A*a^6*b)*(8*a^{21}*b - 8*a^8*b^{14} + 8*a^9* \\
& b^{13} + 48*a^{10}*b^{12} - 48*a^{11}*b^{11} - 120*a^{12}*b^{10} + 120*a^{13}*b^9 + 160*a^{1 \\
& 4}*b^8 - 160*a^{15}*b^7 - 120*a^{16}*b^6 + 120*a^{17}*b^5 + 48*a^{18}*b^4 - 48*a^{19} \\
& b^3 - 8*a^{20}*b^2))/((a^{18} - a^4*b^{14} + 7*a^6*b^{12} - 21*a^8*b^{10} + 35*a^{10}*b \\
& ^8 - 35*a^{12}*b^6 + 21*a^{14}*b^4 - 7*a^{16}*b^2))*(a^{16}*b + a^{17} - a^6*b^{11} - a^ \\
& 7*b^{10} + 5*a^8*b^9 + 5*a^9*b^8 - 10*a^{10}*b^7 - 10*a^{11}*b^6 + 10*a^{12}*b^5 + \\
& 10*a^{13}*b^4 - 5*a^{14}*b^3 - 5*a^{15}*b^2)))*((a + b)^7*(a - b)^7)^{(1/2)}*(2*A*b \\
& ^7 + 2*B*a^7 - 7*A*a^2*b^5 + 8*A*a^4*b^3 + 3*B*a^5*b^2 - 8*A*a^6*b))/(2*(a^ \\
& 18 - a^4*b^{14} + 7*a^6*b^{12} - 21*a^8*b^{10} + 35*a^{10}*b^8 - 35*a^{12}*b^6 + 21*a \\
& ^14*b^4 - 7*a^{16}*b^2)))*((a + b)^7*(a - b)^7)^{(1/2)}*(2*A*b^7 + 2*B*a^7 - 7* \\
& A*a^2*b^5 + 8*A*a^4*b^3 + 3*B*a^5*b^2 - 8*A*a^6*b))/(2*(a^{18} - a^4*b^{14} + 7 \\
& *a^6*b^{12} - 21*a^8*b^{10} + 35*a^{10}*b^8 - 35*a^{12}*b^6 + 21*a^{14}*b^4 - 7*a^{16} \\
& b^2)) + (((8*\tan(c/2 + (d*x)/2)*(4*A^2*a^{14} + 8*A^2*b^{14} + 4*B^2*a^{14} - 8*A \\
& ^2*a*b^{13} - 8*A^2*a^{13}*b - 48*A^2*a^2*b^{12} + 48*A^2*a^3*b^{11} + 117*A^2*a^4* \\
& b^{10} - 120*A^2*a^5*b^9 - 164*A^2*a^6*b^8 + 160*A^2*a^7*b^7 + 156*A^2*a^8*b^ \\
& 6 - 120*A^2*a^9*b^5 - 92*A^2*a^{10}*b^4 + 48*A^2*a^{11}*b^3 + 44*A^2*a^{12}*b^2 + \\
& 9*B^2*a^{10}*b^4 + 12*B^2*a^{12}*b^2 - 32*A*B*a^{13}*b + 12*A*B*a^5*b^9 - 34*A*B \\
& *a^7*b^7 + 20*A*B*a^9*b^5 - 16*A*B*a^{11}*b^3))/(a^{16}*b + a^{17} - a^6*b^{11} - a \\
& ^7*b^{10} + 5*a^8*b^9 + 5*a^9*b^8 - 10*a^{10}*b^7 - 10*a^{11}*b^6 + 10*a^{12}*b^5 +
\end{aligned}$$

$$10*a^{13}*b^4 - 5*a^{14}*b^3 - 5*a^{15}*b^2) - (((8*(4*A*a^{21} + 4*B*a^{21} - 4*A*a^{18}*b^{13} + 2*A*a^{19}*b^{12} + 26*A*a^{10}*b^{11} - 14*A*a^{11}*b^{10} - 70*A*a^{12}*b^9 + 30*A*a^{13}*b^8 + 110*A*a^{14}*b^7 - 30*A*a^{15}*b^6 - 110*A*a^{16}*b^5 + 20*A*a^{17}*b^4 + 64*A*a^{18}*b^3 - 12*A*a^{19}*b^2 + 6*B*a^{12}*b^9 - 6*B*a^{13}*b^8 - 14*B*a^{14}*b^7 + 14*B*a^{15}*b^6 + 6*B*a^{16}*b^5 - 6*B*a^{17}*b^4 + 6*B*a^{18}*b^3 - 6*B*a^{19}*b^2 - 16*A*a^{20}*b - 4*B*a^{20}*b)))/(a^{19}*b + a^{20} - a^9*b^{11} - a^{10}*b^{10} + 5*a^{11}*b^9 + 5*a^{12}*b^8 - 10*a^{13}*b^7 - 10*a^{14}*b^6 + 10*a^{15}*b^5 + 10*a^{16}*b^4 - 5*a^{17}*b^3 - 5*a^{18}*b^2) + (4*\tan(c/2 + (d*x)/2)*((a + b)^7*(a - b)^7)^{(1/2)}*(2*A*b^7 + 2*B*a^7 - 7*A*a^2*b^5 + 8*A*a^4*b^3 + 3*B*a^5*b^2 - 8*A*a^6*b))*(8*a^{21}*b - 8*a^8*b^{14} + 8*a^9*b^{13} + 48*a^{10}*b^{12} - 48*a^{11}*b^{11} - 120*a^{12}*b^{10} + 120*a^{13}*b^9 + 160*a^{14}*b^8 - 160*a^{15}*b^7 - 120*a^{16}*b^6 + 120*a^{17}*b^5 + 48*a^{18}*b^4 - 48*a^{19}*b^3 - 8*a^{20}*b^2))/((a^{18} - a^4*b^{14} + 7*a^6*b^{12} - 21*a^8*b^{10} + 35*a^{10}*b^8 - 35*a^{12}*b^6 + 21*a^{14}*b^4 - 7*a^{16}*b^2)*(a^{16}*b + a^{17} - a^6*b^{11} - a^7*b^{10} + 5*a^8*b^9 + 5*a^9*b^8 - 10*a^{10}*b^7 - 10*a^{11}*b^6 + 10*a^{12}*b^5 + 10*a^{13}*b^4 - 5*a^{14}*b^3 - 5*a^{15}*b^2)))*((a + b)^7*(a - b)^7)^{(1/2)}*(2*A*b^7 + 2*B*a^7 - 7*A*a^2*b^5 + 8*A*a^4*b^3 + 3*B*a^5*b^2 - 8*A*a^6*b))/(2*(a^{18} - a^4*b^{14} + 7*a^6*b^{12} - 21*a^8*b^{10} + 35*a^{10}*b^8 - 35*a^{12}*b^6 + 21*a^{14}*b^4 - 7*a^{16}*b^2)))*((a + b)^7*(a - b)^7)^{(1/2)}*(2*A*b^7 + 2*B*a^7 - 7*A*a^2*b^5 + 8*A*a^4*b^3 + 3*B*a^5*b^2 - 8*A*a^6*b))/(2*(a^{18} - a^4*b^{14} + 7*a^6*b^{12} - 21*a^8*b^{10} + 35*a^{10}*b^8 - 35*a^{12}*b^6 + 21*a^{14}*b^4 - 7*a^{16}*b^2)))*((a + b)^7*(a - b)^7)^{(1/2)}*(2*A*b^7 + 2*B*a^7 - 7*A*a^2*b^5 + 8*A*a^4*b^3 + 3*B*a^5*b^2 - 8*A*a^6*b)*1i)/(d*(a^{18} - a^4*b^{14} + 7*a^6*b^{12} - 21*a^8*b^{10} + 35*a^{10}*b^8 - 35*a^{12}*b^6 + 21*a^{14}*b^4 - 7*a^{16}*b^2))$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \sec(c + dx)}{(a + b \sec(c + dx))^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))**4,x)

[Out] Integral((A + B*sec(c + d*x))/(a + b*sec(c + d*x))**4, x)

$$3.342 \quad \int \frac{\cos(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^4} dx$$

Optimal. Leaf size=411

$$\frac{x(4Ab - aB)}{a^5} + \frac{b(Ab - aB) \sin(c + dx)}{3ad(a^2 - b^2)(a + b \sec(c + dx))^3} + \frac{b(-6a^3B + 9a^2Ab + ab^2B - 4Ab^3) \sin(c + dx)}{6a^2d(a^2 - b^2)^2(a + b \sec(c + dx))^2} + \frac{b(-6a^5B + \dots)}{\dots}$$

[Out] $-(4A*b-B*a)*x/a^5+b*(20*A*a^6*b-35*A*a^4*b^3+28*A*a^2*b^5-8*A*b^7-8*B*a^7+8*B*a^5*b^2-7*B*a^3*b^4+2*B*a*b^6)*\operatorname{arctanh}((a-b)^{(1/2)}*\tan(1/2*d*x+1/2*c)/(a+b)^{(1/2}))/a^5/(a-b)^{(7/2)/(a+b)^{(7/2)/d+1/6*(6*A*a^6-65*A*a^4*b^2+68*A*a^2*b^4-24*A*b^6+26*B*a^5*b-17*B*a^3*b^3+6*B*a*b^5)*\sin(d*x+c)/a^4/(a^2-b^2)^3/d+1/3*b*(A*b-B*a)*\sin(d*x+c)/a/(a^2-b^2)/d/(a+b*\sec(d*x+c))^3+1/6*b*(9*A*a^2*b-4*A*b^3-6*B*a^3+B*a*b^2)*\sin(d*x+c)/a^2/(a^2-b^2)^2/d/(a+b*\sec(d*x+c))^2+1/2*b*(12*A*a^4*b-11*A*a^2*b^3+4*A*b^5-6*B*a^5+2*B*a^3*b^2-B*a*b^4)*\sin(d*x+c)/a^3/(a^2-b^2)^3/d/(a+b*\sec(d*x+c))$

Rubi [A] time = 5.60, antiderivative size = 411, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {4030, 4100, 4104, 3919, 3831, 2659, 208}

$$\frac{(-65a^4Ab^2 + 68a^2Ab^4 + 6a^6A - 17a^3b^3B + 26a^5bB + 6ab^5B - 24Ab^6) \sin(c + dx)}{6a^4d(a^2 - b^2)^3} + \frac{b(-35a^4Ab^3 + 28a^2Ab^5 + \dots)}{\dots}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^4,x]

[Out] $-(((4A*b - a*B)*x)/a^5) + (b*(20*a^6*A*b - 35*a^4*A*b^3 + 28*a^2*A*b^5 - 8*A*b^7 - 8*a^7*B + 8*a^5*b^2*B - 7*a^3*b^4*B + 2*a*b^6*B)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a - b]*\operatorname{Tan}[(c + d*x)/2])/(\operatorname{Sqrt}[a + b])]/(a^5*(a - b)^{(7/2)*(a + b)^{(7/2)*d}) + ((6*a^6*A - 65*a^4*A*b^2 + 68*a^2*A*b^4 - 24*A*b^6 + 26*a^5*b*B - 17*a^3*b^3*B + 6*a*b^5*B)*\operatorname{Sin}[c + d*x])/(6*a^4*(a^2 - b^2)^3*d) + (b*(A*b - a*B)*\operatorname{Sin}[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*\operatorname{Sec}[c + d*x])^3) + (b*(9*a^2*A*b - 4*A*b^3 - 6*a^3*B + a*b^2*B)*\operatorname{Sin}[c + d*x])/(6*a^2*(a^2 - b^2)^2*d*(a + b*\operatorname{Sec}[c + d*x])^2) + (b*(12*a^4*A*b - 11*a^2*A*b^3 + 4*A*b^5 - 6*a^5*B + 2*a^3*b^2*B - a*b^4*B)*\operatorname{Sin}[c + d*x])/(2*a^3*(a^2 - b^2)^3*d*(a + b*\operatorname{Sec}[c + d*x]))$

Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 2659

```
Int[((a_) + (b_)*sin[Pi/2 + (c_) + (d_)*(x_)])^(-1), x_Symbol] := With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3831

```
Int[csc[(e_) + (f_)*(x_)]/(csc[(e_) + (f_)*(x_)]*(b_) + (a_)), x_Symbol] := Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3919

```
Int[(csc[(e_) + (f_)*(x_)]*(d_) + (c_))/(csc[(e_) + (f_)*(x_)]*(b_) + (a_)), x_Symbol] := Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 4030

```
Int[(csc[(e_) + (f_)*(x_)]*(d_))^(n_)*(csc[(e_) + (f_)*(x_)]*(b_) + (a_))^(m_)*(csc[(e_) + (f_)*(x_)]*(B_) + (A_)), x_Symbol] := Simp[(b*(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[A*(a^2*(m + 1) - b^2*(m + n + 1)) + a*b*B*n - a*(A*b - a*B)*(m + 1)*Csc[e + f*x] + b*(A*b - a*B)*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4100

```
Int[((A_) + csc[(e_) + (f_)*(x_)]*(B_) + csc[(e_) + (f_)*(x_)]^2*(C_))*(csc[(e_) + (f_)*(x_)]*(d_))^(n_)*(csc[(e_) + (f_)*(x_)]*(b_) + (a_))^(m_), x_Symbol] := Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4104

```

Int[((A_.) + csc[(e_.) + (f_.)*(x_)])*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]

```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^4} dx &= \frac{b(Ab-aB)\sin(c+dx)}{3a(a^2-b^2)d(a+b\sec(c+dx))^3} - \int \frac{\cos(c+dx)(-3a^2A+4Ab^2-abB+3a(Ab-aB))}{(a+b\sec(c+dx))^3} dx \\
&= \frac{b(Ab-aB)\sin(c+dx)}{3a(a^2-b^2)d(a+b\sec(c+dx))^3} + \frac{b(9a^2Ab-4Ab^3-6a^3B+ab^2B)\sin(c+dx)}{6a^2(a^2-b^2)^2d(a+b\sec(c+dx))^2} \\
&= \frac{b(Ab-aB)\sin(c+dx)}{3a(a^2-b^2)d(a+b\sec(c+dx))^3} + \frac{b(9a^2Ab-4Ab^3-6a^3B+ab^2B)\sin(c+dx)}{6a^2(a^2-b^2)^2d(a+b\sec(c+dx))^2} \\
&= \frac{(6a^6A-65a^4Ab^2+68a^2Ab^4-24Ab^6+26a^5bB-17a^3b^3B+6ab^5B)\sin(c+dx)}{6a^4(a^2-b^2)^3d} \\
&= -\frac{(4Ab-aB)x}{a^5} + \frac{(6a^6A-65a^4Ab^2+68a^2Ab^4-24Ab^6+26a^5bB-17a^3b^3B+6ab^5B)\sin(c+dx)}{6a^4(a^2-b^2)^3d} \\
&= -\frac{(4Ab-aB)x}{a^5} + \frac{(6a^6A-65a^4Ab^2+68a^2Ab^4-24Ab^6+26a^5bB-17a^3b^3B+6ab^5B)\sin(c+dx)}{6a^4(a^2-b^2)^3d} \\
&= -\frac{(4Ab-aB)x}{a^5} + \frac{(6a^6A-65a^4Ab^2+68a^2Ab^4-24Ab^6+26a^5bB-17a^3b^3B+6ab^5B)\sin(c+dx)}{6a^4(a^2-b^2)^3d} \\
&= -\frac{(4Ab-aB)x}{a^5} + \frac{b(20a^6Ab-35a^4Ab^3+28a^2Ab^5-8Ab^7-8a^7B+8a^6b^2B)\sin(c+dx)}{a^5(a-b)^{7/2}(a+b)^{7/2}}
\end{aligned}$$

Mathematica [B] time = 6.36, size = 1205, normalized size = 2.93

$$(b + a \cos(c + dx)) \sec^3(c + dx)(A + B \sec(c + dx)) \left(\frac{24b(8Ba^7 - 20Aba^6 - 8b^2Ba^5 + 35Ab^3a^4 + 7b^4Ba^3 - 28Ab^5a^2 - 2b^6Ba + 8Ab^7) \tanh^{-1}\left(\frac{a+b \sec(c+dx)}{a^2-b^2}\right)}{(a^2-b^2)^{7/2}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^4,x]

[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]^3*(A + B*Sec[c + d*x])*((24*b*(-20*a^6*A*b + 35*a^4*A*b^3 - 28*a^2*A*b^5 + 8*A*b^7 + 8*a^7*B - 8*a^5*b^2*B + 7*a^3*b^4*B - 2*a*b^6*B)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2])*(b + a*Cos[c + d*x])^3)/(a^2 - b^2)^(7/2) + (-144*a^8*A*b^2*c + 336*a^6*A*b^4*c - 144*a^4*A*b^6*c - 144*a^2*A*b^8*c + 96*A*b^10*c + 36*a^9*b*B*c - 84*a^7*b^3*B*c + 36*a^5*b^5*B*c + 36*a^3*b^7*B*c - 24*a*b^9*B*c - 144*a^8*A*b^2*d*x + 336*a^6*A*b^4*d*x - 144*a^4*A*b^6*d*x - 144*a^2*A*b^8*d*x + 96*A*b^10*d*x + 36*a^9*b*B*d*x - 84*a^7*b^3*B*d*x + 36*a^5*b^5*B*d*x + 36*a^3*b^7*B*d*x - 24*a*b^9*B*d*x + 18*a*(a^2 - b^2)^3*(a^2 + 4*b^2)*(-4*A*b + a*B)*(c + d*x)*Cos[c + d*x] + 36*a^2*b*(a^2 - b^2)^3*(-4*A*b + a*B)*(c + d*x)*Cos[2*(c + d*x)] - 24*a^9*A*b*c*Cos[3*(c + d*x)] + 72*a^7*A*b^3*c*Cos[3*(c + d*x)] - 72*a^5*A*b^5*c*Cos[3*(c + d*x)] + 24*a^3*A*b^7*c*Cos[3*(c + d*x)] + 6*a^10*B*c*Cos[3*(c + d*x)] - 18*a^8*b^2*B*c*Cos[3*(c + d*x)] + 18*a^6*b^4*B*c*Cos[3*(c + d*x)] - 6*a^4*b^6*B*c*Cos[3*(c + d*x)] - 24*a^9*A*b*d*x*Cos[3*(c + d*x)] + 72*a^7*A*b^3*d*x*Cos[3*(c + d*x)] - 72*a^5*A*b^5*d*x*Cos[3*(c + d*x)] + 24*a^3*A*b^7*d*x*Cos[3*(c + d*x)] + 6*a^10*B*d*x*Cos[3*(c + d*x)] - 18*a^8*b^2*B*d*x*Cos[3*(c + d*x)] + 18*a^6*b^4*B*d*x*Cos[3*(c + d*x)] - 6*a^4*b^6*B*d*x*Cos[3*(c + d*x)] + 18*a^9*A*b*Sin[c + d*x] - 90*a^7*A*b^3*Sin[c + d*x] - 135*a^5*A*b^5*Sin[c + d*x] + 228*a^3*A*b^7*Sin[c + d*x] - 96*a^9*A*b^9*Sin[c + d*x] + 36*a^8*b^2*B*Sin[c + d*x] + 72*a^6*b^4*B*Sin[c + d*x] - 57*a^4*b^6*B*Sin[c + d*x] + 24*a^2*b^8*B*Sin[c + d*x] + 6*a^10*A*Sin[2*(c + d*x)] + 18*a^8*A*b^2*Sin[2*(c + d*x)] - 300*a^6*A*b^4*Sin[2*(c + d*x)] + 336*a^4*A*b^6*Sin[2*(c + d*x)] - 120*a^2*A*b^8*Sin[2*(c + d*x)] + 120*a^7*b^3*B*Sin[2*(c + d*x)] - 90*a^5*b^5*B*Sin[2*(c + d*x)] + 30*a^3*b^7*B*Sin[2*(c + d*x)] + 18*a^9*A*b*Sin[3*(c + d*x)] - 114*a^7*A*b^3*Sin[3*(c + d*x)] + 125*a^5*A*b^5*Sin[3*(c + d*x)] - 44*a^3*A*b^7*Sin[3*(c + d*x)] + 36*a^8*b^2*B*Sin[3*(c + d*x)] - 32*a^6*b^4*B*Sin[3*(c + d*x)] + 11*a^4*b^6*B*Sin[3*(c + d*x)] + 3*a^10*A*Sin[4*(c + d*x)] - 9*a^8*A*b^2*Sin[4*(c + d*x)] + 9*a^6*A*b^4*Sin[4*(c + d*x)] - 3*a^4*A*b^6*Sin[4*(c + d*x)])/(a^2 - b^2)^3)/(24*a^5*d*(B + A*Cos[c + d*x])*(a + b*Sec[c + d*x])^4)

fricas [B] time = 0.81, size = 2560, normalized size = 6.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^4,x, algorithm="fricas")

[Out]
$$\begin{aligned} & [1/12*(12*(B*a^{12} - 4*A*a^{11}*b - 4*B*a^{10}*b^2 + 16*A*a^9*b^3 + 6*B*a^8*b^4 \\ & - 24*A*a^7*b^5 - 4*B*a^6*b^6 + 16*A*a^5*b^7 + B*a^4*b^8 - 4*A*a^3*b^9)*d*x* \\ & \cos(d*x + c)^3 + 36*(B*a^{11}*b - 4*A*a^{10}*b^2 - 4*B*a^9*b^3 + 16*A*a^8*b^4 + \\ & 6*B*a^7*b^5 - 24*A*a^6*b^6 - 4*B*a^5*b^7 + 16*A*a^4*b^8 + B*a^3*b^9 - 4*A* \\ & a^2*b^{10})*d*x*\cos(d*x + c)^2 + 36*(B*a^{10}*b^2 - 4*A*a^9*b^3 - 4*B*a^8*b^4 + \\ & 16*A*a^7*b^5 + 6*B*a^6*b^6 - 24*A*a^5*b^7 - 4*B*a^4*b^8 + 16*A*a^3*b^9 + B \\ & *a^2*b^{10} - 4*A*a*b^{11})*d*x*\cos(d*x + c) + 12*(B*a^9*b^3 - 4*A*a^8*b^4 - 4* \\ & B*a^7*b^5 + 16*A*a^6*b^6 + 6*B*a^5*b^7 - 24*A*a^4*b^8 - 4*B*a^3*b^9 + 16*A* \\ & a^2*b^{10} + B*a*b^{11} - 4*A*b^{12})*d*x - 3*(8*B*a^7*b^4 - 20*A*a^6*b^5 - 8*B*a \\ & ^5*b^6 + 35*A*a^4*b^7 + 7*B*a^3*b^8 - 28*A*a^2*b^9 - 2*B*a*b^{10} + 8*A*b^{11} \\ & + (8*B*a^{10}*b - 20*A*a^9*b^2 - 8*B*a^8*b^3 + 35*A*a^7*b^4 + 7*B*a^6*b^5 - 2 \\ & 8*A*a^5*b^6 - 2*B*a^4*b^7 + 8*A*a^3*b^8)*\cos(d*x + c)^3 + 3*(8*B*a^9*b^2 - \\ & 20*A*a^8*b^3 - 8*B*a^7*b^4 + 35*A*a^6*b^5 + 7*B*a^5*b^6 - 28*A*a^4*b^7 - 2* \\ & B*a^3*b^8 + 8*A*a^2*b^9)*\cos(d*x + c)^2 + 3*(8*B*a^8*b^3 - 20*A*a^7*b^4 - 8 \\ & *B*a^6*b^5 + 35*A*a^5*b^6 + 7*B*a^4*b^7 - 28*A*a^3*b^8 - 2*B*a^2*b^9 + 8*A* \\ & a*b^{10})*\cos(d*x + c))*\sqrt{a^2 - b^2}*\log((2*a*b*\cos(d*x + c) - (a^2 - 2*b^ \\ & 2)*\cos(d*x + c)^2 + 2*\sqrt{a^2 - b^2}*(b*\cos(d*x + c) + a)*\sin(d*x + c) + 2 \\ & *a^2 - b^2)/(a^2*\cos(d*x + c)^2 + 2*a*b*\cos(d*x + c) + b^2)) + 2*(6*A*a^9*b \\ & ^3 + 26*B*a^8*b^4 - 71*A*a^7*b^5 - 43*B*a^6*b^6 + 133*A*a^5*b^7 + 23*B*a^4* \\ & b^8 - 92*A*a^3*b^9 - 6*B*a^2*b^{10} + 24*A*a*b^{11} + 6*(A*a^{12} - 4*A*a^{10}*b^2 \\ & + 6*A*a^8*b^4 - 4*A*a^6*b^6 + A*a^4*b^8)*\cos(d*x + c)^3 + (18*A*a^{11}*b + 36 \\ & *B*a^{10}*b^2 - 132*A*a^9*b^3 - 68*B*a^8*b^4 + 239*A*a^7*b^5 + 43*B*a^6*b^6 - \\ & 169*A*a^5*b^7 - 11*B*a^4*b^8 + 44*A*a^3*b^9)*\cos(d*x + c)^2 + 3*(6*A*a^{10}* \\ & b^2 + 20*B*a^9*b^3 - 59*A*a^8*b^4 - 35*B*a^7*b^5 + 110*A*a^6*b^6 + 20*B*a^5* \\ & b^7 - 77*A*a^4*b^8 - 5*B*a^3*b^9 + 20*A*a^2*b^{10})*\cos(d*x + c))*\sin(d*x + \\ & c))/((a^{16} - 4*a^{14}*b^2 + 6*a^{12}*b^4 - 4*a^{10}*b^6 + a^8*b^8)*d*\cos(d*x + c) \\ & ^3 + 3*(a^{15}*b - 4*a^{13}*b^3 + 6*a^{11}*b^5 - 4*a^9*b^7 + a^7*b^9)*d*\cos(d*x + \\ & c)^2 + 3*(a^{14}*b^2 - 4*a^{12}*b^4 + 6*a^{10}*b^6 - 4*a^8*b^8 + a^6*b^{10})*d*\cos \\ & (d*x + c) + (a^{13}*b^3 - 4*a^{11}*b^5 + 6*a^9*b^7 - 4*a^7*b^9 + a^5*b^{11})*d), \\ & 1/6*(6*(B*a^{12} - 4*A*a^{11}*b - 4*B*a^{10}*b^2 + 16*A*a^9*b^3 + 6*B*a^8*b^4 - 2 \\ & 4*A*a^7*b^5 - 4*B*a^6*b^6 + 16*A*a^5*b^7 + B*a^4*b^8 - 4*A*a^3*b^9)*d*x*\cos \\ & (d*x + c)^3 + 18*(B*a^{11}*b - 4*A*a^{10}*b^2 - 4*B*a^9*b^3 + 16*A*a^8*b^4 + 6* \\ & B*a^7*b^5 - 24*A*a^6*b^6 - 4*B*a^5*b^7 + 16*A*a^4*b^8 + B*a^3*b^9 - 4*A*a^2 \\ & *b^{10})*d*x*\cos(d*x + c)^2 + 18*(B*a^{10}*b^2 - 4*A*a^9*b^3 - 4*B*a^8*b^4 + 16 \\ & *A*a^7*b^5 + 6*B*a^6*b^6 - 24*A*a^5*b^7 - 4*B*a^4*b^8 + 16*A*a^3*b^9 + B*a^ \\ & 2*b^{10} - 4*A*a*b^{11})*d*x*\cos(d*x + c) + 6*(B*a^9*b^3 - 4*A*a^8*b^4 - 4*B*a^ \\ & 7*b^5 + 16*A*a^6*b^6 + 6*B*a^5*b^7 - 24*A*a^4*b^8 - 4*B*a^3*b^9 + 16*A*a^2* \\ & b^{10} + B*a*b^{11} - 4*A*b^{12})*d*x - 3*(8*B*a^7*b^4 - 20*A*a^6*b^5 - 8*B*a^5*b \\ & ^6 + 35*A*a^4*b^7 + 7*B*a^3*b^8 - 28*A*a^2*b^9 - 2*B*a*b^{10} + 8*A*b^{11} + (8 \\ & *B*a^{10}*b - 20*A*a^9*b^2 - 8*B*a^8*b^3 + 35*A*a^7*b^4 + 7*B*a^6*b^5 - 28*A* \end{aligned}$$

$$\begin{aligned}
& a^5 b^6 - 2 B a^4 b^7 + 8 A a^3 b^8) \cos(dx + c)^3 + 3(8 B a^9 b^2 - 20 A a^8 b^3 - 8 B a^7 b^4 + 35 A a^6 b^5 + 7 B a^5 b^6 - 28 A a^4 b^7 - 2 B a^3 b^8 + 8 A a^2 b^9) \cos(dx + c)^2 + 3(8 B a^8 b^3 - 20 A a^7 b^4 - 8 B a^6 b^5 + 35 A a^5 b^6 + 7 B a^4 b^7 - 28 A a^3 b^8 - 2 B a^2 b^9 + 8 A a b^{10}) \cos(dx + c) \sqrt{-a^2 + b^2} \arctan(-\sqrt{-a^2 + b^2} (b \cos(dx + c) + a) / ((a^2 - b^2) \sin(dx + c))) + (6 A a^9 b^3 + 26 B a^8 b^4 - 71 A a^7 b^5 - 43 B a^6 b^6 + 133 A a^5 b^7 + 23 B a^4 b^8 - 92 A a^3 b^9 - 6 B a^2 b^{10} + 24 A a b^{11} + 6(A a^{12} - 4 A a^{10} b^2 + 6 A a^8 b^4 - 4 A a^6 b^6 + A a^4 b^8) \cos(dx + c)^3 + (18 A a^{11} b + 36 B a^{10} b^2 - 132 A a^9 b^3 - 68 B a^8 b^4 + 239 A a^7 b^5 + 43 B a^6 b^6 - 169 A a^5 b^7 - 11 B a^4 b^8 + 44 A a^3 b^9) \cos(dx + c)^2 + 3(6 A a^{10} b^2 + 20 B a^9 b^3 - 59 A a^8 b^4 - 35 B a^7 b^5 + 110 A a^6 b^6 + 20 B a^5 b^7 - 77 A a^4 b^8 - 5 B a^3 b^9 + 20 A a^2 b^{10}) \cos(dx + c) \sin(dx + c)) / ((a^{16} - 4 a^{14} b^2 + 6 a^{12} b^4 - 4 a^{10} b^6 + a^8 b^8) d \cos(dx + c)^3 + 3(a^{15} b - 4 a^{13} b^3 + 6 a^{11} b^5 - 4 a^9 b^7 + a^7 b^9) d \cos(dx + c)^2 + 3(a^{14} b^2 - 4 a^{12} b^4 + 6 a^{10} b^6 - 4 a^8 b^8 + a^6 b^{10}) d \cos(dx + c) + (a^{13} b^3 - 4 a^{11} b^5 + 6 a^9 b^7 - 4 a^7 b^9 + a^5 b^{11}) d)
\end{aligned}$$

giac [B] time = 1.96, size = 966, normalized size = 2.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(dx+c)*(A+B*sec(dx+c))/(a+b*sec(dx+c))^4,x, algorithm="giac")

[Out]
$$\begin{aligned}
& -1/3(3(8 B a^7 b - 20 A a^6 b^2 - 8 B a^5 b^3 + 35 A a^4 b^4 + 7 B a^3 b^5 - 28 A a^2 b^6 - 2 B a b^7 + 8 A b^8) (\pi \operatorname{floor}(1/2(dx + c)) / \pi + 1/2) \operatorname{sgn}(-2a + 2b) + \arctan(-(a \tan(1/2 dx + 1/2 c) - b \tan(1/2 dx + 1/2 c)) / \sqrt{-a^2 + b^2})) / ((a^{11} - 3 a^9 b^2 + 3 a^7 b^4 - a^5 b^6) \sqrt{-a^2 + b^2}) + (36 B a^7 b^2 \tan(1/2 dx + 1/2 c)^5 - 60 A a^6 b^3 \tan(1/2 dx + 1/2 c)^5 - 60 B a^6 b^3 \tan(1/2 dx + 1/2 c)^5 + 105 A a^5 b^4 \tan(1/2 dx + 1/2 c)^5 - 6 B a^5 b^4 \tan(1/2 dx + 1/2 c)^5 + 24 A a^4 b^5 \tan(1/2 dx + 1/2 c)^5 + 45 B a^4 b^5 \tan(1/2 dx + 1/2 c)^5 - 117 A a^3 b^6 \tan(1/2 dx + 1/2 c)^5 - 6 B a^3 b^6 \tan(1/2 dx + 1/2 c)^5 + 24 A a^2 b^7 \tan(1/2 dx + 1/2 c)^5 - 15 B a^2 b^7 \tan(1/2 dx + 1/2 c)^5 + 42 A a b^8 \tan(1/2 dx + 1/2 c)^5 + 6 B a b^8 \tan(1/2 dx + 1/2 c)^5 - 18 A b^9 \tan(1/2 dx + 1/2 c)^5 - 72 B a^7 b^2 \tan(1/2 dx + 1/2 c)^3 + 120 A a^6 b^3 \tan(1/2 dx + 1/2 c)^3 + 116 B a^5 b^4 \tan(1/2 dx + 1/2 c)^3 - 236 A a^4 b^5 \tan(1/2 dx + 1/2 c)^3 - 56 B a^3 b^6 \tan(1/2 dx + 1/2 c)^3 + 152 A a^2 b^7 \tan(1/2 dx + 1/2 c)^3 + 12 B a b^8 \tan(1/2 dx + 1/2 c)^3 - 36 A b^9 \tan(1/2 dx + 1/2 c)^3 + 36 B a^7 b^2 \tan(1/2 dx + 1/2 c) - 60 A a^6 b^3 \tan(1/2 dx + 1/2 c) + 60 B a^6 b^3 \tan(1/2 dx + 1/2 c) - 105 A a^5 b^4 \tan(1/2 dx + 1/2 c) - 6 B a^5 b^4 \tan(1/2 dx + 1/2 c) + 24 A a^4 b^5 \tan(1/2 dx + 1/2 c) - 45 B a^4 b^5 \tan(1/2 dx + 1/2 c) + 117 A a^3 b^6 \tan(1/2 dx + 1/2 c) - 6 B
\end{aligned}$$

$$a^3b^6 \tan(1/2dx + 1/2c) + 24Aa^2b^7 \tan(1/2dx + 1/2c) + 15Bb^8 \tan(1/2dx + 1/2c) - 42Aab^8 \tan(1/2dx + 1/2c) + 6Bb^8 \tan(1/2dx + 1/2c) - 18Aab^9 \tan(1/2dx + 1/2c) / ((a^{10} - 3a^8b^2 + 3a^6b^4 - a^4b^6) * (a \tan(1/2dx + 1/2c)^2 - b \tan(1/2dx + 1/2c)^2 - a - b)^3) - 3(Ba - 4Ab)(dx + c) / a^5 - 6A \tan(1/2dx + 1/2c) / ((\tan(1/2dx + 1/2c)^2 + 1) * a^4) / d$$

maple [B] time = 1.41, size = 2891, normalized size = 7.03

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\int (\cos(dx+c) * (A+B*\sec(dx+c))) / (a+b*\sec(dx+c))^4, x$

[Out] $5/d/a / (a*\tan(1/2dx+1/2c)^2 - \tan(1/2dx+1/2c)^2 * b - a - b)^3 * b^4 / (a-b) / (a^3 + 3a^2b + 3ab^2 + b^3) * \tan(1/2dx+1/2c)^5 * A - 12/d*a / (a*\tan(1/2dx+1/2c)^2 - \tan(1/2dx+1/2c)^2 * b - a - b)^3 * b^2 / (a+b) / (a^3 - 3a^2b + 3ab^2 - b^3) * \tan(1/2dx+1/2c) * B + 2/d * b^7 / a^4 / (a^6 - 3a^4b^2 + 3a^2b^4 - b^6) / ((a-b) * (a+b))^{1/2} * \operatorname{arctanh}(\tan(1/2dx+1/2c) * (a-b) / ((a-b) * (a+b))^{1/2}) * B - 40/d * b^3 / (a*\tan(1/2dx+1/2c)^2 - \tan(1/2dx+1/2c)^2 * b - a - b)^3 / (a^2 - 2ab + b^2) / (a^2 + 2ab + b^2) * \tan(1/2dx+1/2c)^3 * A - 35/d * b^4 / a / (a^6 - 3a^4b^2 + 3a^2b^4 - b^6) / ((a-b) * (a+b))^{1/2} * \operatorname{arctanh}(\tan(1/2dx+1/2c) * (a-b) / ((a-b) * (a+b))^{1/2}) * A + 28/d * b^6 / a^3 / (a^6 - 3a^4b^2 + 3a^2b^4 - b^6) / ((a-b) * (a+b))^{1/2} * \operatorname{arctanh}(\tan(1/2dx+1/2c) * (a-b) / ((a-b) * (a+b))^{1/2}) * A - 8/d * b^8 / a^5 / (a^6 - 3a^4b^2 + 3a^2b^4 - b^6) / ((a-b) * (a+b))^{1/2} * \operatorname{arctanh}(\tan(1/2dx+1/2c) * (a-b) / ((a-b) * (a+b))^{1/2}) * A - 7/d * b^5 / a^2 / (a^6 - 3a^4b^2 + 3a^2b^4 - b^6) / ((a-b) * (a+b))^{1/2} * \operatorname{arctanh}(\tan(1/2dx+1/2c) * (a-b) / ((a-b) * (a+b))^{1/2}) * B - 8/d * a^2 * b / (a^6 - 3a^4b^2 + 3a^2b^4 - b^6) / ((a-b) * (a+b))^{1/2} * \operatorname{arctanh}(\tan(1/2dx+1/2c) * (a-b) / ((a-b) * (a+b))^{1/2}) * B + 20/d * a * b^2 / (a^6 - 3a^4b^2 + 3a^2b^4 - b^6) / ((a-b) * (a+b))^{1/2} * \operatorname{arctanh}(\tan(1/2dx+1/2c) * (a-b) / ((a-b) * (a+b))^{1/2}) * A - 4/d / (a*\tan(1/2dx+1/2c)^2 - \tan(1/2dx+1/2c)^2 * b - a - b)^3 * b^3 / (a-b) / (a^3 + 3a^2b + 3ab^2 + b^3) * \tan(1/2dx+1/2c)^5 * B - 5/d/a / (a*\tan(1/2dx+1/2c)^2 - \tan(1/2dx+1/2c)^2 * b - a - b)^3 * b^4 / (a+b) / (a^3 - 3a^2b + 3ab^2 - b^3) * \tan(1/2dx+1/2c) * A - 18/d/a^2 / (a*\tan(1/2dx+1/2c)^2 - \tan(1/2dx+1/2c)^2 * b - a - b)^3 * b^5 / (a+b) / (a^3 - 3a^2b + 3ab^2 - b^3) * \tan(1/2dx+1/2c) * A + 2/d/a^3 / (a*\tan(1/2dx+1/2c)^2 - \tan(1/2dx+1/2c)^2 * b - a - b)^3 * b^6 / (a+b) / (a^3 - 3a^2b + 3ab^2 - b^3) * \tan(1/2dx+1/2c) * A - 18/d/a^2 / (a*\tan(1/2dx+1/2c)^2 - \tan(1/2dx+1/2c)^2 * b - a - b)^3 * b^5 / (a-b) / (a^3 + 3a^2b + 3ab^2 + b^3) * \tan(1/2dx+1/2c)^5 * A + 2/d/a^4 * \operatorname{arctan}(\tan(1/2dx+1/2c)) * B - 2/d/a^3 / (a*\tan(1/2dx+1/2c)^2 - \tan(1/2dx+1/2c)^2 * b - a - b)^3 * b^6 / (a-b) / (a^3 + 3a^2b + 3ab^2 + b^3) * \tan(1/2dx+1/2c)^5 * A - 12/d*a / (a*\tan(1/2dx+1/2c)^2 - \tan(1/2dx+1/2c)^2 * b - a - b)^3 * b^2 / (a-b) / (a^3 + 3a^2b + 3ab^2 + b^3) * \tan(1/2dx+1/2c)^5 * B + 20/d / (a*\tan(1/2dx+1/2c)^2 - \tan(1/2dx+1/2c)^2 * b - a - b)^3 * b^3 / (a-b) / (a^3 + 3a^2b + 3ab^2 + b^3) * \tan(1/2dx+1/2c)^5 * A - 44/3/d * b^4 / a / (a*\tan(1/2dx+1/2c)^2 - \tan(1/2dx+1/2c)^2 * b - a - b)^3 / (a^2 - 2ab + b^2) * \tan(1/2dx+1/2c)^3 * B + 4/d * b^6 / a^3 / (a*\tan(1/2dx+1/2c)^2 - \tan(1/2dx+1/2c)^2 * b - a - b)^3$

$$\begin{aligned}
& 2 - \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 b - a - b)^3 / (a^2 - 2ab + b^2) / (a^2 + 2ab + b^2) * \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 B + 6/d * b^4/a / (a * \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 b - a - b)^3 \\
& / (a + b) / (a^3 - 3a^2b + 3ab^2 - b^3) * \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) * B + 116/3/d * b^5/a^2 / (a * \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 b - a - b)^3 / (a^2 - 2ab + b^2) / (a^2 + 2ab + b^2) * \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 A \\
& - 12/d * b^7/a^4 / (a * \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 b - a - b)^3 / (a^2 - 2ab + b^2) / (a^2 + 2ab + b^2) * \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 A \\
& + 24/d * b^2 * a / (a * \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 b - a - b)^3 / (a^2 - 2ab + b^2) / (a^2 + 2ab + b^2) * \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 B + 6/d * b^4/a / (a * \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 b - a - b)^3 / (a - b) / (a^3 + 3a^2b + 3ab^2 + b^3) * \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5 B - 1/d * b^5/a^2 / (a * \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 b - a - b)^3 / (a + b) / (a^3 - 3a^2b + 3ab^2 - b^3) * \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) * B + 1/d * b^5/a^2 / (a * \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 b - a - b)^3 / (a - b) / (a^3 + 3a^2b + 3ab^2 + b^3) * \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5 B - 2/d * b^6/a^3 / (a * \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 b - a - b)^3 / (a - b) / (a^3 + 3a^2b + 3ab^2 + b^3) * \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5 B + 6/d * b^7/a^4 / (a * \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 b - a - b)^3 / (a + b) / (a^3 - 3a^2b + 3ab^2 - b^3) * \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) * A + 6/d * b^7/a^4 / (a * \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 b - a - b)^3 / (a - b) / (a^3 + 3a^2b + 3ab^2 + b^3) * \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5 A - 2/d * b^6/a^3 / (a * \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 b - a - b)^3 / (a + b) / (a^3 - 3a^2b + 3ab^2 - b^3) * \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) * B + 4/d / (a * \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 b - a - b)^3 * b^3 / (a + b) / (a^3 - 3a^2b + 3ab^2 - b^3) * \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) * A - 8/d / a^5 * A * \arctan(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)) * b + 8/d * b^3 / (a^6 - 3a^4 * b^2 + 3a^2 * b^4 - b^6) / ((a - b) * (a + b))^(1/2) * \operatorname{arctanh}(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) * (a - b) / ((a - b) * (a + b))^(1/2)) * B + 2/d / a^4 * A * \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) / (1 + \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2)
\end{aligned}$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^4,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for more details) Is 4*a^2-4*b^2 positive or negative?

mupad [B] time = 14.20, size = 7863, normalized size = 19.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x))^4,x)

[Out] $(\log(\tan(c/2 + (d*x)/2) - 1i)*(4*A*b - B*a)*1i)/(a^5*d) - ((\tan(c/2 + (d*x)/2)*(2*A*a^7 - 8*A*b^7 + 24*A*a^2*b^5 + 11*A*a^3*b^4 - 26*A*a^4*b^3 - 6*A*a^5*b^2 + B*a^2*b^5 - 6*B*a^3*b^4 - 4*B*a^4*b^3 + 12*B*a^5*b^2 - 4*A*a*b^6 + 2*A*a^6*b + 2*B*a*b^6))/((a + b)*(3*a^6*b - a^7 + a^4*b^3 - 3*a^5*b^2)) - (\tan(c/2 + (d*x)/2)^3*(18*A*a^8 + 72*A*b^8 - 236*A*a^2*b^6 + 47*A*a^3*b^5 + 273*A*a^4*b^4 - 60*A*a^5*b^3 - 72*A*a^6*b^2 + 3*B*a^2*b^6 + 59*B*a^3*b^5 - 14*B*a^4*b^4 - 96*B*a^5*b^3 + 36*B*a^6*b^2 - 12*A*a*b^7 - 18*B*a*b^7))/(3*(a + b)^2*(3*a^6*b - a^7 + a^4*b^3 - 3*a^5*b^2)) + (\tan(c/2 + (d*x)/2)^7*(24*A*a^2*b^5 - 8*A*b^7 - 2*A*a^7 - 11*A*a^3*b^4 - 26*A*a^4*b^3 + 6*A*a^5*b^2 - B*a^2*b^5 - 6*B*a^3*b^4 + 4*B*a^4*b^3 + 12*B*a^5*b^2 + 4*A*a*b^6 + 2*A*a^6*b + 2*B*a*b^6))/((a^4*b - a^5)*(a + b)^3) + (\tan(c/2 + (d*x)/2)^5*(18*A*a^8 + 72*A*b^8 - 236*A*a^2*b^6 - 47*A*a^3*b^5 + 273*A*a^4*b^4 + 60*A*a^5*b^3 - 72*A*a^6*b^2 - 3*B*a^2*b^6 + 59*B*a^3*b^5 + 14*B*a^4*b^4 - 96*B*a^5*b^3 - 36*B*a^6*b^2 + 12*A*a*b^7 - 18*B*a*b^7))/(3*(a^4*b - a^5)*(a + b)^3*(a - b)))/(d*(3*a*b^2 + 3*a^2*b - \tan(c/2 + (d*x)/2)^4*(6*a^2*b - 6*b^3) + \tan(c/2 + (d*x)/2)^2*(6*a*b^2 - 2*a^3 + 4*b^3) + \tan(c/2 + (d*x)/2)^6*(2*a^3 - 6*a*b^2 + 4*b^3) + a^3 + b^3 - \tan(c/2 + (d*x)/2)^8*(3*a*b^2 - 3*a^2*b + a^3 - b^3))) - (\log(\tan(c/2 + (d*x)/2) + 1i)*(A*b*4i - B*a*1i))/(a^5*d) - (b*atan(((b*((8*\tan(c/2 + (d*x)/2)*(128*A^2*b^16 + 4*B^2*a^16 - 128*A^2*a*b^15 - 8*B^2*a^15*b - 768*A^2*a^2*b^14 + 768*A^2*a^3*b^13 + 1920*A^2*a^4*b^12 - 1920*A^2*a^5*b^11 - 2600*A^2*a^6*b^10 + 2560*A^2*a^7*b^9 + 2025*A^2*a^8*b^8 - 1920*A^2*a^9*b^7 - 824*A^2*a^10*b^6 + 768*A^2*a^11*b^5 + 80*A^2*a^12*b^4 - 128*A^2*a^13*b^3 + 64*A^2*a^14*b^2 + 8*B^2*a^2*b^14 - 8*B^2*a^3*b^13 - 48*B^2*a^4*b^12 + 48*B^2*a^5*b^11 + 117*B^2*a^6*b^10 - 120*B^2*a^7*b^9 - 164*B^2*a^8*b^8 + 160*B^2*a^9*b^7 + 156*B^2*a^10*b^6 - 120*B^2*a^11*b^5 - 92*B^2*a^12*b^4 + 48*B^2*a^13*b^3 + 44*B^2*a^14*b^2 - 64*A*B*a*b^15 - 32*A*B*a^15*b + 64*A*B*a^2*b^14 + 384*A*B*a^3*b^13 - 384*A*B*a^4*b^12 - 948*A*B*a^5*b^11 + 960*A*B*a^6*b^10 + 1306*A*B*a^7*b^9 - 1280*A*B*a^8*b^8 - 1128*A*B*a^9*b^7 + 960*A*B*a^10*b^6 + 592*A*B*a^11*b^5 - 384*A*B*a^12*b^4 - 160*A*B*a^13*b^3 + 64*A*B*a^14*b^2)))/(a^18*b + a^19 - a^8*b^11 - a^9*b^10 + 5*a^10*b^9 + 5*a^11*b^8 - 10*a^12*b^7 - 10*a^13*b^6 + 10*a^14*b^5 + 10*a^15*b^4 - 5*a^16*b^3 - 5*a^17*b^2) + (b*((a + b)^7*(a - b)^7)^(1/2))*((8*(4*B*a^24 + 16*A*a^10*b^14 - 8*A*a^11*b^13 - 104*A*a^12*b^12 + 50*A*a^13*b^11 + 286*A*a^14*b^10 - 126*A*a^15*b^9 - 434*A*a^16*b^8 + 174*A*a^17*b^7 + 386*A*a^18*b^6 - 146*A*a^19*b^5 - 190*A*a^20*b^4 + 72*A*a^21*b^3 + 40*A*a^22*b^2 - 4*B*a^11*b^13 + 2*B*a^12*b^12 + 26*B*a^13*b^11 - 14*B*a^14*b^10 - 70*B*a^15*b^9 + 30*B*a^16*b^8 + 110*B*a^17*b^7 - 30*B*a^18*b^6 - 110*B*a^19*b^5 + 20*B*a^20*b^4 + 64*B*a^21*b^3 - 12*B*a^22*b^2 - 16*A*a^23*b - 16*B*a^23*b)))/(a^22*b + a^23 - a^12*b^11 - a^13*b^10 + 5*a^14*b^9 + 5*a^15*b^8 - 10*a^16*b^7 - 10*a^17*b^6 + 10*a^18*b^5 + 10*a^19*b^4 - 5*a^20*b^3 - 5*a^21*b^2) - (4*b*\tan(c/2 + (d*x)/2)*((a + b)^7*(a - b)^7)^(1/2)*(8*A*b^7 + 8*B*a^7 - 28*A*a^2*b^5 + 35*A*a^4*b^3 + 7*B*a^3*b^4 - 8*B*a^5*b^2 - 20*A*a^6*b - 2*B*a*b^6)*(8*a^23*b - 8*a^10*b^14 + 8*a^11*b^13 + 48*a^12*b^12 - 48*a^13*b^11 - 120*a^14*b^10 + 120*a^15*b^9 + 160*a^16*b^8 - 160*a^17*b^7 - 120*a^18*b^6 + 120*a^19*b^5 + 48*a^20*b^4 - 48*a^21*b^3 - 8*a^22*b^2)))/((a^19 - a^5*b^14 + 7*a^7*$

$$\begin{aligned}
& b^{12} - 21a^9b^{10} + 35a^{11}b^8 - 35a^{13}b^6 + 21a^{15}b^4 - 7a^{17}b^2) * \\
& (a^{18}b + a^{19} - a^8b^{11} - a^9b^{10} + 5a^{10}b^9 + 5a^{11}b^8 - 10a^{12}b^7 - 10a^{13}b^6 + 10a^{14}b^5 + 10a^{15}b^4 - 5a^{16}b^3 - 5a^{17}b^2)) * (8 \\
& * A * b^7 + 8 * B * a^7 - 28 * A * a^2 * b^5 + 35 * A * a^4 * b^3 + 7 * B * a^3 * b^4 - 8 * B * a^5 * b^2 \\
& - 20 * A * a^6 * b - 2 * B * a * b^6)) / (2 * (a^{19} - a^5 * b^{14} + 7 * a^7 * b^{12} - 21 * a^9 * b^{10} + \\
& 35 * a^{11} * b^8 - 35 * a^{13} * b^6 + 21 * a^{15} * b^4 - 7 * a^{17} * b^2)) * ((a + b)^7 * (a - b) \\
& ^7)^{(1/2)} * (8 * A * b^7 + 8 * B * a^7 - 28 * A * a^2 * b^5 + 35 * A * a^4 * b^3 + 7 * B * a^3 * b^4 - \\
& 8 * B * a^5 * b^2 - 20 * A * a^6 * b - 2 * B * a * b^6) * i) / (2 * (a^{19} - a^5 * b^{14} + 7 * a^7 * b^{12} \\
& - 21 * a^9 * b^{10} + 35 * a^{11} * b^8 - 35 * a^{13} * b^6 + 21 * a^{15} * b^4 - 7 * a^{17} * b^2)) + (b \\
& * ((8 * \tan(c/2 + (d * x) / 2) * (128 * A^2 * b^{16} + 4 * B^2 * a^{16} - 128 * A^2 * a * b^{15} - 8 * B^2 \\
& * a^{15} * b - 768 * A^2 * a^2 * b^{14} + 768 * A^2 * a^3 * b^{13} + 1920 * A^2 * a^4 * b^{12} - 1920 * A^2 \\
& * a^5 * b^{11} - 2600 * A^2 * a^6 * b^{10} + 2560 * A^2 * a^7 * b^9 + 2025 * A^2 * a^8 * b^8 - 1920 \\
& * A^2 * a^9 * b^7 - 824 * A^2 * a^{10} * b^6 + 768 * A^2 * a^{11} * b^5 + 80 * A^2 * a^{12} * b^4 - 128 * \\
& A^2 * a^{13} * b^3 + 64 * A^2 * a^{14} * b^2 + 8 * B^2 * a^2 * b^{14} - 8 * B^2 * a^3 * b^{13} - 48 * B^2 * a^4 * b^{12} + 48 * B^2 * a^5 * b^{11} + 117 * B^2 * a^6 * b^{10} - 120 * B^2 * a^7 * b^9 - 164 * B^2 * a^8 * b^8 + 160 * B^2 * a^9 * b^7 + 156 * B^2 * a^{10} * b^6 - 120 * B^2 * a^{11} * b^5 - 92 * B^2 * a^{12} * b^4 + 48 * B^2 * a^{13} * b^3 + 44 * B^2 * a^{14} * b^2 - 64 * A * B * a * b^{15} - 32 * A * B * a^{15} * b + 64 * A * B * a^2 * b^{14} + 384 * A * B * a^3 * b^{13} - 384 * A * B * a^4 * b^{12} - 948 * A * B * a^5 * b^{11} + 960 * A * B * a^6 * b^{10} + 1306 * A * B * a^7 * b^9 - 1280 * A * B * a^8 * b^8 - 1128 * A * B * a^9 * b^7 + 960 * A * B * a^{10} * b^6 + 592 * A * B * a^{11} * b^5 - 384 * A * B * a^{12} * b^4 - 160 * A * B * a^{13} * b^3 + 64 * A * B * a^{14} * b^2)) / (a^{18} * b + a^{19} - a^8 * b^{11} - a^9 * b^{10} + 5 * a^{10} * b^9 + 5 * a^{11} * b^8 - 10 * a^{12} * b^7 - 10 * a^{13} * b^6 + 10 * a^{14} * b^5 + 10 * a^{15} * b^4 - 5 * a^{16} * b^3 - 5 * a^{17} * b^2) - (b * ((a + b)^7 * (a - b)^7)^{(1/2)} * ((8 * (4 * B * a^{24} + 16 * A * a^{10} * b^{14} - 8 * A * a^{11} * b^{13} - 104 * A * a^{12} * b^{12} + 50 * A * a^{13} * b^{11} + 286 * A * a^{14} * b^{10} - 126 * A * a^{15} * b^9 - 434 * A * a^{16} * b^8 + 174 * A * a^{17} * b^7 + 386 * A * a^{18} * b^6 - 146 * A * a^{19} * b^5 - 190 * A * a^{20} * b^4 + 72 * A * a^{21} * b^3 + 40 * A * a^{22} * b^2 - 4 * B * a^{11} * b^{13} + 2 * B * a^{12} * b^{12} + 26 * B * a^{13} * b^{11} - 14 * B * a^{14} * b^{10} - 70 * B * a^{15} * b^9 + 30 * B * a^{16} * b^8 + 110 * B * a^{17} * b^7 - 30 * B * a^{18} * b^6 - 110 * B * a^{19} * b^5 + 20 * B * a^{20} * b^4 + 64 * B * a^{21} * b^3 - 12 * B * a^{22} * b^2 - 16 * A * a^{23} * b - 16 * B * a^{23} * b)) / (a^{22} * b + a^{23} - a^{12} * b^{11} - a^{13} * b^{10} + 5 * a^{14} * b^9 + 5 * a^{15} * b^8 - 10 * a^{16} * b^7 - 10 * a^{17} * b^6 + 10 * a^{18} * b^5 + 10 * a^{19} * b^4 - 5 * a^{20} * b^3 - 5 * a^{21} * b^2) + (4 * b * \tan(c/2 + (d * x) / 2) * ((a + b)^7 * (a - b)^7)^{(1/2)} * (8 * A * b^7 + 8 * B * a^7 - 28 * A * a^2 * b^5 + 35 * A * a^4 * b^3 + 7 * B * a^3 * b^4 - 8 * B * a^5 * b^2 - 20 * A * a^6 * b - 2 * B * a * b^6) * (8 * a^{23} * b - 8 * a^{10} * b^{14} + 8 * a^{11} * b^{13} + 48 * a^{12} * b^{12} - 48 * a^{13} * b^{11} - 120 * a^{14} * b^{10} + 120 * a^{15} * b^9 + 160 * a^{16} * b^8 - 160 * a^{17} * b^7 - 120 * a^{18} * b^6 + 120 * a^{19} * b^5 + 48 * a^{20} * b^4 - 48 * a^{21} * b^3 - 8 * a^{22} * b^2)) / ((a^{19} - a^5 * b^{14} + 7 * a^7 * b^{12} - 21 * a^9 * b^{10} + 35 * a^{11} * b^8 - 35 * a^{13} * b^6 + 21 * a^{15} * b^4 - 7 * a^{17} * b^2)) * (a^{18} * b + a^{19} - a^8 * b^{11} - a^9 * b^{10} + 5 * a^{10} * b^9 + 5 * a^{11} * b^8 - 10 * a^{12} * b^7 - 10 * a^{13} * b^6 + 10 * a^{14} * b^5 + 10 * a^{15} * b^4 - 5 * a^{16} * b^3 - 5 * a^{17} * b^2)) * (8 * A * b^7 + 8 * B * a^7 - 28 * A * a^2 * b^5 + 35 * A * a^4 * b^3 + 7 * B * a^3 * b^4 - 8 * B * a^5 * b^2 - 20 * A * a^6 * b - 2 * B * a * b^6)) / (2 * (a^{19} - a^5 * b^{14} + 7 * a^7 * b^{12} - 21 * a^9 * b^{10} + 35 * a^{11} * b^8 - 35 * a^{13} * b^6 + 21 * a^{15} * b^4 - 7 * a^{17} * b^2)) * ((a + b)^7 * (a - b)^7)^{(1/2)} * (8 * A * b^7 + 8 * B * a^7 - 28 * A * a^2 * b^5 + 35 * A * a^4 * b^3 + 7 * B * a^3 * b^4 - 8 * B * a^5 * b^2 - 20 * A * a^6 * b - 2 * B * a * b^6) * i) / (2 * (a^{19} - a^5 * b^{14} + 7 * a^7 * b^{12} - 21 * a^9 * b^{10} + 35 * a^{11} * b^8 - 35 * a^{13} * b^6 + 21 * a^{15} * b^4 - 7 * a^{17} * b^2)) / ((16 * (256 * A
\end{aligned}$$

$$\begin{aligned}
& ^3b^{16} - 128A^3a^3b^{15} - 16B^3a^{15}b - 1664A^3a^2b^{14} + 800A^3a^3b^{13} + 4576A^3a^4b^{12} - 2176A^3a^5b^{11} - 6944A^3a^6b^{10} + 3204A^3a^7b^9 + 6176A^3a^8b^8 - 2560A^3a^9b^7 - 3040A^3a^{10}b^6 + 960A^3a^{11}b^5 + 640A^3a^{12}b^4 - 4B^3a^3b^{13} + 2B^3a^4b^{12} + 26B^3a^5b^{11} - 11B^3a^6b^{10} - 70B^3a^7b^9 + 34B^3a^8b^8 + 110B^3a^9b^7 - 66B^3a^{10}b^6 - 110B^3a^{11}b^5 + 64B^3a^{12}b^4 + 64B^3a^{13}b^3 - 48B^3a^{14}b^2 - 192A^2B^2a^2b^{14} - 24A^2B^2a^3b^{13} - 312A^2B^2a^4b^{12} + 138A^2B^2a^5b^{11} + 846A^2B^2a^6b^{10} - 408A^2B^2a^7b^9 - 1314A^2B^2a^8b^8 + 726A^2B^2a^9b^7 + 1266A^2B^2a^{10}b^6 - 690A^2B^2a^{11}b^5 - 702A^2B^2a^{12}b^4 + 408A^2B^2a^{13}b^3 + 168A^2B^2a^{14}b^2 + 96A^2B^2a^{15}b - 1248A^2B^2a^{16} \\
& + 1248A^2B^2a^3b^{13} - 576A^2B^2a^4b^{12} - 3408A^2B^2a^5b^{11} + 1632A^2B^2a^6b^{10} + 5232A^2B^2a^7b^9 - 2649A^2B^2a^8b^8 - 4848A^2B^2a^9b^7 + 2376A^2B^2a^{10}b^6 + 2544A^2B^2a^{11}b^5 - 1104A^2B^2a^{12}b^4 - 576A^2B^2a^{13}b^3) / (a^{22}b + a^{23} - a^{12}b^{11} - a^{13}b^{10} + 5a^{14}b^9 + 5a^{15}b^8 - 10a^{16}b^7 - 10a^{17}b^6 + 10a^{18}b^5 + 10a^{19}b^4 - 5a^{20}b^3 - 5a^{21}b^2) + (b((8\tan(c/2 + (d*x)/2)*(128A^2b^{16} + 4B^2a^{16} - 128A^2a^3b^{13} - 8B^2a^{15}b - 768A^2a^2b^{14} + 768A^2a^3b^{13} + 1920A^2a^4b^{12} - 1920A^2a^5b^{11} - 2600A^2a^6b^{10} + 2560A^2a^7b^9 + 2025A^2a^8b^8 - 1920A^2a^9b^7 - 824A^2a^{10}b^6 + 768A^2a^{11}b^5 + 80A^2a^{12}b^4 - 128A^2a^{13}b^3 + 64A^2a^{14}b^2 + 8B^2a^2b^{14} - 8B^2a^3b^{13} - 48B^2a^4b^{12} + 48B^2a^5b^{11} + 117B^2a^6b^{10} - 120B^2a^7b^9 - 164B^2a^8b^8 + 160B^2a^9b^7 + 156B^2a^{10}b^6 - 120B^2a^{11}b^5 - 92B^2a^{12}b^4 + 48B^2a^{13}b^3 + 44B^2a^{14}b^2 - 64A^2B^2a^3b^{13} - 32A^2B^2a^{15}b + 64A^2B^2a^2b^{14} + 384A^2B^2a^3b^{13} - 384A^2B^2a^4b^{12} - 948A^2B^2a^5b^{11} + 960A^2B^2a^6b^{10} + 1306A^2B^2a^7b^9 - 1280A^2B^2a^8b^8 - 1128A^2B^2a^9b^7 + 960A^2B^2a^{10}b^6 + 592A^2B^2a^{11}b^5 - 384A^2B^2a^{12}b^4 - 160A^2B^2a^{13}b^3 + 64A^2B^2a^{14}b^2) / (a^{18}b + a^{19} - a^8b^{11} - a^9b^{10} + 5a^{10}b^9 + 5a^{11}b^8 - 10a^{12}b^7 - 10a^{13}b^6 + 10a^{14}b^5 + 10a^{15}b^4 - 5a^{16}b^3 - 5a^{17}b^2) + (b((a + b)^7(a - b)^7)^{(1/2)}((8*(4B^2a^{24} + 16A^2a^{10}b^{14} - 8A^2a^{11}b^{13} - 104A^2a^{12}b^{12} + 50A^2a^{13}b^{11} + 286A^2a^{14}b^{10} - 126A^2a^{15}b^9 - 434A^2a^{16}b^8 + 174A^2a^{17}b^7 + 386A^2a^{18}b^6 - 146A^2a^{19}b^5 - 190A^2a^{20}b^4 + 72A^2a^{21}b^3 + 40A^2a^{22}b^2 - 4B^2a^{11}b^{13} + 2B^2a^{12}b^{12} + 26B^2a^{13}b^{11} - 14B^2a^{14}b^{10} - 70B^2a^{15}b^9 + 30B^2a^{16}b^8 + 110B^2a^{17}b^7 - 30B^2a^{18}b^6 - 110B^2a^{19}b^5 + 20B^2a^{20}b^4 + 64B^2a^{21}b^3 - 12B^2a^{22}b^2 - 16A^2a^{23}b - 16B^2a^{23}b) / (a^{22}b + a^{23} - a^{12}b^{11} - a^{13}b^{10} + 5a^{14}b^9 + 5a^{15}b^8 - 10a^{16}b^7 - 10a^{17}b^6 + 10a^{18}b^5 + 10a^{19}b^4 - 5a^{20}b^3 - 5a^{21}b^2) - (4b\tan(c/2 + (d*x)/2)*((a + b)^7(a - b)^7)^{(1/2)}((8A^2b^7 + 8B^2a^7 - 28A^2a^2b^5 + 35A^2a^4b^3 + 7B^2a^3b^4 - 8B^2a^5b^2 - 20A^2a^6b - 2B^2a^6b^6)*(8a^{23}b - 8a^{10}b^{14} + 8a^{11}b^{13} + 48a^{12}b^{12} - 48a^{13}b^{11} - 120a^{14}b^{10} + 120a^{15}b^9 + 160a^{16}b^8 - 160a^{17}b^7 - 120a^{18}b^6 + 120a^{19}b^5 + 48a^{20}b^4 - 48a^{21}b^3 - 8a^{22}b^2) / ((a^{19} - a^5b^{14} + 7a^7b^{12} - 21a^9b^{10} + 35a^{11}b^8 - 35a^{13}b^6 + 21a^{15}b^4 - 7a^{17}b^2)*(a^{18}b + a^{19} - a^8b^{11} - a^9b^{10} + 5a^{10}b^9 + 5a^{11}b^8 - 10a^{12}b^7 - 10a^{13}b^6 + 10a^{14}b^5 + 10a^{15}
\end{aligned}$$

$$\begin{aligned}
& b^4 - 5a^{16}b^3 - 5a^{17}b^2)) * (8A^7b + 8B^7a^7 - 28A^2b^5 + 35A^4b^3 + 7B^3a^3b^4 - 8B^5a^5b^2 - 20A^6b - 2B^6a^6b^6) / (2(a^{19} - a^5b^{14} + 7a^7b^{12} - 21a^9b^{10} + 35a^{11}b^8 - 35a^{13}b^6 + 21a^{15}b^4 - 7a^{17}b^2))) * ((a + b)^7 * (a - b)^7)^{(1/2)} * (8A^7b + 8B^7a^7 - 28A^2b^5 + 35A^4b^3 + 7B^3a^3b^4 - 8B^5a^5b^2 - 20A^6b - 2B^6a^6b^6) / (2(a^{19} - a^5b^{14} + 7a^7b^{12} - 21a^9b^{10} + 35a^{11}b^8 - 35a^{13}b^6 + 21a^{15}b^4 - 7a^{17}b^2)) - (b * ((8 * \tan(c/2 + (d*x)/2) * (128A^2b^{16} + 4B^2a^{16} - 128A^2a^2b^{15} - 8B^2a^{15}b - 768A^2a^2b^{14} + 768A^2a^3b^{13} + 1920A^2a^4b^{12} - 1920A^2a^5b^{11} - 2600A^2a^6b^{10} + 2560A^2a^7b^9 + 2025A^2a^8b^8 - 1920A^2a^9b^7 - 824A^2a^{10}b^6 + 768A^2a^{11}b^5 + 80A^2a^{12}b^4 - 128A^2a^{13}b^3 + 64A^2a^{14}b^2 + 8B^2a^2b^{14} - 8B^2a^3b^{13} - 48B^2a^4b^{12} + 48B^2a^5b^{11} + 117B^2a^6b^{10} - 120B^2a^7b^9 - 164B^2a^8b^8 + 160B^2a^9b^7 + 156B^2a^{10}b^6 - 120B^2a^{11}b^5 - 92B^2a^{12}b^4 + 48B^2a^{13}b^3 + 44B^2a^{14}b^2 - 64A^2B^2a^2b^{15} - 32A^2B^2a^3b^{14} + 64A^2B^2a^4b^{13} + 384A^2B^2a^5b^{12} - 384A^2B^2a^6b^{11} - 948A^2B^2a^7b^{10} + 960A^2B^2a^8b^9 + 1306A^2B^2a^9b^8 - 1280A^2B^2a^{10}b^7 - 1128A^2B^2a^{11}b^6 + 960A^2B^2a^{12}b^5 - 592A^2B^2a^{13}b^4 - 160A^2B^2a^{14}b^3 + 64A^2B^2a^{15}b^2)) / (a^{18}b + a^{19} - a^8b^{11} - a^9b^{10} + 5a^{10}b^9 + 5a^{11}b^8 - 10a^{12}b^7 - 10a^{13}b^6 + 10a^{14}b^5 + 10a^{15}b^4 - 5a^{16}b^3 - 5a^{17}b^2)) - (b * ((a + b)^7 * (a - b)^7)^{(1/2)} * ((8 * (4B^2a^{24} + 16A^2a^{10}b^{14} - 8A^2a^{11}b^{13} - 104A^2a^{12}b^{12} + 50A^2a^{13}b^{11} + 286A^2a^{14}b^{10} - 126A^2a^{15}b^9 - 434A^2a^{16}b^8 + 174A^2a^{17}b^7 + 386A^2a^{18}b^6 - 146A^2a^{19}b^5 - 190A^2a^{20}b^4 + 72A^2a^{21}b^3 + 40A^2a^{22}b^2 - 4B^2a^{11}b^{13} + 2B^2a^{12}b^{12} + 26B^2a^{13}b^{11} - 14B^2a^{14}b^{10} - 70B^2a^{15}b^9 + 30B^2a^{16}b^8 + 110B^2a^{17}b^7 - 30B^2a^{18}b^6 - 110B^2a^{19}b^5 + 20B^2a^{20}b^4 + 64B^2a^{21}b^3 - 12B^2a^{22}b^2 - 16A^2a^{23}b - 16B^2a^{23}b)) / (a^{22}b + a^{23} - a^{12}b^{11} - a^{13}b^{10} + 5a^{14}b^9 + 5a^{15}b^8 - 10a^{16}b^7 - 10a^{17}b^6 + 10a^{18}b^5 + 10a^{19}b^4 - 5a^{20}b^3 - 5a^{21}b^2) + (4 * b * \tan(c/2 + (d*x)/2) * ((a + b)^7 * (a - b)^7)^{(1/2)} * (8A^7b + 8B^7a^7 - 28A^2b^5 + 35A^4b^3 + 7B^3a^3b^4 - 8B^5a^5b^2 - 20A^6b - 2B^6a^6b^6) * (8a^{23}b - 8a^{10}b^{14} + 8a^{11}b^{13} + 48a^{12}b^{12} - 48a^{13}b^{11} - 120a^{14}b^{10} + 120a^{15}b^9 + 160a^{16}b^8 - 160a^{17}b^7 - 120a^{18}b^6 + 120a^{19}b^5 + 48a^{20}b^4 - 48a^{21}b^3 - 8a^{22}b^2)) / ((a^{19} - a^5b^{14} + 7a^7b^{12} - 21a^9b^{10} + 35a^{11}b^8 - 35a^{13}b^6 + 21a^{15}b^4 - 7a^{17}b^2)) * (a^{18}b + a^{19} - a^8b^{11} - a^9b^{10} + 5a^{10}b^9 + 5a^{11}b^8 - 10a^{12}b^7 - 10a^{13}b^6 + 10a^{14}b^5 + 10a^{15}b^4 - 5a^{16}b^3 - 5a^{17}b^2))) * (8A^7b + 8B^7a^7 - 28A^2b^5 + 35A^4b^3 + 7B^3a^3b^4 - 8B^5a^5b^2 - 20A^6b - 2B^6a^6b^6) / (2(a^{19} - a^5b^{14} + 7a^7b^{12} - 21a^9b^{10} + 35a^{11}b^8 - 35a^{13}b^6 + 21a^{15}b^4 - 7a^{17}b^2))) * ((a + b)^7 * (a - b)^7)^{(1/2)} * (8A^7b + 8B^7a^7 - 28A^2b^5 + 35A^4b^3 + 7B^3a^3b^4 - 8B^5a^5b^2 - 20A^6b - 2B^6a^6b^6) / (2(a^{19} - a^5b^{14} + 7a^7b^{12} - 21a^9b^{10} + 35a^{11}b^8 - 35a^{13}b^6 + 21a^{15}b^4 - 7a^{17}b^2))) * ((a + b)^7 * (a - b)^7)^{(1/2)} * (8A^7b + 8B^7a^7 - 28A^2b^5 + 35A^4b^3 + 7B^3a^3b^4 - 8B^5a^5b^2 - 20A^6b - 2B^6a^6b^6) * i) / (d * (a^{19} - a^5b^{14} + 7a^7b^{12} - 21a^9b^{10} + 35a^{11}b^8 - 35a^{13}b^6 + 21a^{15}b^4 - 7a^{17}b^2)))
\end{aligned}$$

$b^6 + 21*a^{15}*b^4 - 7*a^{17}*b^2))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \cos(c + dx)}{(a + b \sec(c + dx))^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**4,x)

[Out] Integral((A + B*sec(c + d*x))*cos(c + d*x)/(a + b*sec(c + d*x))**4, x)

$$3.343 \quad \int \frac{\cos^2(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^4} dx$$

Optimal. Leaf size=538

$$\frac{b(Ab - aB) \sin(c + dx) \cos(c + dx)}{3ad(a^2 - b^2)(a + b \sec(c + dx))^3} + \frac{x(a^2A - 8abB + 20Ab^2)}{2a^6} + \frac{b(-7a^3B + 10a^2Ab + 2ab^2B - 5Ab^3) \sin(c + dx)}{6a^2d(a^2 - b^2)^2(a + b \sec(c + dx))^2}$$

[Out] $\frac{1}{2}*(A*a^2+20*A*b^2-8*B*a*b)*x/a^6-b^2*(40*A*a^6*b-84*A*a^4*b^3+69*A*a^2*b^5-20*A*b^7-20*B*a^7+35*B*a^5*b^2-28*B*a^3*b^4+8*B*a*b^6)*\operatorname{arctanh}((a-b)^{(1/2)}*\tan(1/2*d*x+1/2*c)/(a+b)^{(1/2)})/a^6/(a-b)^{(7/2)}/(a+b)^{(7/2)}/d-1/6*(24*A*a^6*b-146*A*a^4*b^3+167*A*a^2*b^5-60*A*b^7-6*B*a^7+65*B*a^5*b^2-68*B*a^3*b^4+24*B*a*b^6)*\sin(d*x+c)/a^5/(a^2-b^2)^3/d+1/2*(A*a^6-23*A*a^4*b^2+27*A*a^2*b^4-10*A*b^6+12*B*a^5*b-11*B*a^3*b^3+4*B*a*b^5)*\cos(d*x+c)*\sin(d*x+c)/a^4/(a^2-b^2)^3/d+1/3*b*(A*b-B*a)*\cos(d*x+c)*\sin(d*x+c)/a/(a^2-b^2)/d/(a+b*\sec(d*x+c))^3+1/6*b*(10*A*a^2*b-5*A*b^3-7*B*a^3+2*B*a*b^2)*\cos(d*x+c)*\sin(d*x+c)/a^2/(a^2-b^2)^2/d/(a+b*\sec(d*x+c))^2+1/6*b*(48*A*a^4*b-53*A*a^2*b^3+20*A*b^5-27*B*a^5+20*B*a^3*b^2-8*B*a*b^4)*\cos(d*x+c)*\sin(d*x+c)/a^3/(a^2-b^2)^3/d/(a+b*\sec(d*x+c))$

Rubi [A] time = 6.84, antiderivative size = 538, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {4030, 4100, 4104, 3919, 3831, 2659, 208}

$$\frac{(-146a^4Ab^3 + 167a^2Ab^5 + 24a^6Ab + 65a^5b^2B - 68a^3b^4B - 6a^7B + 24ab^6B - 60Ab^7) \sin(c + dx)}{6a^5d(a^2 - b^2)^3} + \frac{(-23a^4Ab^2)}{6a^5d(a^2 - b^2)^3}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^4,x]

[Out] $((a^2*A + 20*A*b^2 - 8*a*b*B)*x)/(2*a^6) - (b^2*(40*a^6*A*b - 84*a^4*A*b^3 + 69*a^2*A*b^5 - 20*A*b^7 - 20*a^7*B + 35*a^5*b^2*B - 28*a^3*b^4*B + 8*a*b^6*B)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a - b]*\tan[(c + d*x)/2])/(\operatorname{Sqrt}[a + b])])/(a^6*(a - b)^{(7/2)}*(a + b)^{(7/2)*d} - ((24*a^6*A*b - 146*a^4*A*b^3 + 167*a^2*A*b^5 - 60*A*b^7 - 6*a^7*B + 65*a^5*b^2*B - 68*a^3*b^4*B + 24*a*b^6*B)*\sin[c + d*x])/(6*a^5*(a^2 - b^2)^3*d) + ((a^6*A - 23*a^4*A*b^2 + 27*a^2*A*b^4 - 10*A*b^6 + 12*a^5*b*B - 11*a^3*b^3*B + 4*a*b^5*B)*\cos[c + d*x]*\sin[c + d*x])/(2*a^4*(a^2 - b^2)^3*d) + (b*(A*b - a*B)*\cos[c + d*x]*\sin[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*\sec[c + d*x])^3) + (b*(10*a^2*A*b - 5*A*b^3 - 7*a^3*B + 2*a*b^2*B)*\cos[c + d*x]*\sin[c + d*x])/(6*a^2*(a^2 - b^2)^2*d*(a + b*\sec[c + d*x])^2) +$

$(b*(48*a^4*A*b - 53*a^2*A*b^3 + 20*A*b^5 - 27*a^5*B + 20*a^3*b^2*B - 8*a*b^4*B)*\text{Cos}[c + d*x]*\text{Sin}[c + d*x])/(6*a^3*(a^2 - b^2)^3*d*(a + b*\text{Sec}[c + d*x]))$

Rule 208

$\text{Int}[\frac{(a_1 + (b_1)x^2)^{-1}}{Rt[-(a/b), 2]}], x_Symbol] \rightarrow \text{Simp}[\frac{Rt[-(a/b), 2]*\text{ArcTanh}[x/Rt[-(a/b), 2]]}{a}, x] /; \text{FreeQ}\{a, b, x\} \ \&\& \ \text{NegQ}[a/b]$

Rule 2659

$\text{Int}[\frac{(a_1 + (b_1)\sin[\frac{\pi}{2} + (c_1) + (d_1)x])^{-1}}{x}], x_Symbol] \rightarrow \text{With}\{e = \text{FreeFactors}[\text{Tan}[(c + d*x)/2], x], \text{Dist}[(2*e)/d], \text{Subst}[\text{Int}[1/(a + b + (a - b)*e^2*x^2)], x], x, \text{Tan}[(c + d*x)/2]/e, x\} /; \text{FreeQ}\{a, b, c, d, x\} \ \&\& \ \text{NeQ}[a^2 - b^2, 0]$

Rule 3831

$\text{Int}[\frac{\text{csc}[e_1 + (f_1)x]}{\text{csc}[e_1 + (f_1)x]*(b_1) + (a_1)}], x_Symbol] \rightarrow \text{Dist}[1/b, \text{Int}[1/(1 + (a*\text{Sin}[e + f*x])/b)], x] /; \text{FreeQ}\{a, b, e, f, x\} \ \&\& \ \text{NeQ}[a^2 - b^2, 0]$

Rule 3919

$\text{Int}[\frac{\text{csc}[e_1 + (f_1)x]*(d_1) + (c_1)}{\text{csc}[e_1 + (f_1)x]*(b_1) + (a_1)}], x_Symbol] \rightarrow \text{Simp}[(c*x)/a, x] - \text{Dist}[(b*c - a*d)/a, \text{Int}[\text{Csc}[e + f*x]/(a + b*\text{Csc}[e + f*x]), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, x\} \ \&\& \ \text{NeQ}[b*c - a*d, 0]$

Rule 4030

$\text{Int}[\frac{\text{csc}[e_1 + (f_1)x]*(d_1)^{n_1}*(\text{csc}[e_1 + (f_1)x]*(b_1) + (a_1))^m}{\text{csc}[e_1 + (f_1)x]*(B_1) + (A_1)}], x_Symbol] \rightarrow \text{Simp}[(b*(A*b - a*B)*\text{Cot}[e + f*x]*(a + b*\text{Csc}[e + f*x])^{m+1}*(d*\text{Csc}[e + f*x])^n)/(a*f*(m+1)*(a^2 - b^2)), x] + \text{Dist}[1/(a*(m+1)*(a^2 - b^2)), \text{Int}[(a + b*\text{Csc}[e + f*x])^{m+1}*(d*\text{Csc}[e + f*x])^n*\text{Simp}[A*(a^2*(m+1) - b^2*(m+n+1)) + a*b*B*n - a*(A*b - a*B)*(m+1)*\text{Csc}[e + f*x] + b*(A*b - a*B)*(m+n+2)*\text{Csc}[e + f*x]^2, x], x], x] /; \text{FreeQ}\{a, b, d, e, f, A, B, n\}, x\} \ \&\& \ \text{NeQ}[A*b - a*B, 0] \ \&\& \ \text{NeQ}[a^2 - b^2, 0] \ \&\& \ \text{LtQ}[m, -1] \ \&\& \ !(\text{ILtQ}[m + 1/2, 0] \ \&\& \ \text{ILtQ}[n, 0])$

Rule 4100

$\text{Int}[\frac{(A_1 + \text{csc}[e_1 + (f_1)x]*(B_1) + \text{csc}[e_1 + (f_1)x]^2*(C_1))*(\text{csc}[e_1 + (f_1)x]*(d_1)^{n_1}*(\text{csc}[e_1 + (f_1)x]*(b_1) + (a_1))^m}{\text{csc}[e_1 + (f_1)x]*(B_1) + (A_1)}], x_Symbol] \rightarrow \text{Simp}[\frac{(A*b^2 - a*b*B + a^2*C)*\text{Cot}[e + f*x]*(a + b*\text{Cs}]{(A*b^2 - a*b*B + a^2*C)*\text{Cot}[e + f*x]*(a + b*\text{Cs}$

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c[e + f*x]^(m + 1)*(d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
&& NeQ[a^2 - b^2, 0] && LtQ[m, -1] && !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])

```

Rule 4104

```

Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]

```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^4} dx &= \frac{b(Ab-aB) \cos(c+dx) \sin(c+dx)}{3a(a^2-b^2)d(a+b \sec(c+dx))^3} - \int \frac{\cos^2(c+dx)(-3a^2A+5Ab^2-2abB+3a(AB))}{(a+b \sec(c+dx))^3} dx \\
&= \frac{b(Ab-aB) \cos(c+dx) \sin(c+dx)}{3a(a^2-b^2)d(a+b \sec(c+dx))^3} + \frac{b(10a^2Ab-5Ab^3-7a^3B+2ab^2B)}{6a^2(a^2-b^2)^2d(a+b \sec(c+dx))} \\
&= \frac{b(Ab-aB) \cos(c+dx) \sin(c+dx)}{3a(a^2-b^2)d(a+b \sec(c+dx))^3} + \frac{b(10a^2Ab-5Ab^3-7a^3B+2ab^2B)}{6a^2(a^2-b^2)^2d(a+b \sec(c+dx))} \\
&= \frac{(a^6A-23a^4Ab^2+27a^2Ab^4-10Ab^6+12a^5bB-11a^3b^3B+4ab^5B) \cos(c+dx) \sin(c+dx)}{2a^4(a^2-b^2)^3d} \\
&= -\frac{(24a^6Ab-146a^4Ab^3+167a^2Ab^5-60Ab^7-6a^7B+65a^5b^2B-68a^3b^4B)}{6a^5(a^2-b^2)^3d} \\
&= \frac{(a^2A+20Ab^2-8abB)x}{2a^6} - \frac{(24a^6Ab-146a^4Ab^3+167a^2Ab^5-60Ab^7-6a^7B+65a^5b^2B-68a^3b^4B)}{6a^5(a^2-b^2)^3d} \\
&= \frac{(a^2A+20Ab^2-8abB)x}{2a^6} - \frac{(24a^6Ab-146a^4Ab^3+167a^2Ab^5-60Ab^7-6a^7B+65a^5b^2B-68a^3b^4B)}{6a^5(a^2-b^2)^3d} \\
&= \frac{(a^2A+20Ab^2-8abB)x}{2a^6} - \frac{(24a^6Ab-146a^4Ab^3+167a^2Ab^5-60Ab^7-6a^7B+65a^5b^2B-68a^3b^4B)}{6a^5(a^2-b^2)^3d} \\
&= \frac{(a^2A+20Ab^2-8abB)x}{2a^6} - \frac{b^2(40a^6Ab-84a^4Ab^3+69a^2Ab^5-20Ab^7-6a^7B+65a^5b^2B-68a^3b^4B)}{2a^6(a^2-b^2)^3d}
\end{aligned}$$

Mathematica [B] time = 6.12, size = 1452, normalized size = 2.70

$$\frac{12Ac \cos(3(c+dx))a^{11}+12Adx \cos(3(c+dx))a^{11}+6A \sin(c+dx)a^{11}+24B \sin(2(c+dx))a^{11}+9A \sin(3(c+dx))a^{11}+12B \sin(4(c+dx))a^{11}+3A \sin(5(c+dx))a^{11}}{2a^6(a^2-b^2)^3d}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^4,x]

[Out] ((-96*b^2*(-40*a^6*A*b + 84*a^4*A*b^3 - 69*a^2*A*b^5 + 20*A*b^7 + 20*a^7*B - 35*a^5*b^2*B + 28*a^3*b^4*B - 8*a*b^6*B)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(7/2) + (72*a^10*A*b*c + 1272*a^8*A*b^3*c - 3288*a^6*A*b^5*c + 1512*a^4*A*b^7*c + 1392*a^2*A*b^9*c - 960*A*b^11*c - 576*a^9*b^2*B*c + 1344*a^7*b^4*B*c - 576*a^5*b^6*B*c - 576*a^3*b^8*B*c + 384*a*b^10*B*c + 72*a^10*A*b*d*x + 1272*a^8*A*b^3*d*x - 3288*a^6*A*b^5*d*x + 1512*a^4*A*b^7*d*x + 1392*a^2*A*b^9*d*x - 960*A*b^11*d*x - 576*a^9*b^2*B*d*x + 1344*a^7*b^4*B*d*x - 576*a^5*b^6*B*d*x - 576*a^3*b^8*B*d*x + 384*a*b^10*B*d*x + 36*a*(a^2 - b^2)^3*(a^2 + 4*b^2)*(a^2*A + 20*A*b^2 - 8*a*b*B)*(c + d*x)*Cos[c + d*x] + 72*a^2*b*(a^2 - b^2)^3*(a^2*A + 20*A*b^2 - 8*a*b*B)*(c + d*x)*Cos[2*(c + d*x)] + 12*a^11*A*c*Cos[3*(c + d*x)] + 204*a^9*A*b^2*c*Cos[3*(c + d*x)] - 684*a^7*A*b^4*c*Cos[3*(c + d*x)] + 708*a^5*A*b^6*c*Cos[3*(c + d*x)] - 240*a^3*A*b^8*c*Cos[3*(c + d*x)] - 96*a^10*b*B*c*Cos[3*(c + d*x)] + 288*a^8*b^3*B*c*Cos[3*(c + d*x)] - 288*a^6*b^5*B*c*Cos[3*(c + d*x)] + 96*a^4*b^7*B*c*Cos[3*(c + d*x)] + 12*a^11*A*d*x*Cos[3*(c + d*x)] + 204*a^9*A*b^2*d*x*Cos[3*(c + d*x)] - 684*a^7*A*b^4*d*x*Cos[3*(c + d*x)] + 708*a^5*A*b^6*d*x*Cos[3*(c + d*x)] - 240*a^3*A*b^8*d*x*Cos[3*(c + d*x)] - 96*a^10*b*B*d*x*Cos[3*(c + d*x)] + 288*a^8*b^3*B*d*x*Cos[3*(c + d*x)] - 288*a^6*b^5*B*d*x*Cos[3*(c + d*x)] + 96*a^4*b^7*B*d*x*Cos[3*(c + d*x)] + 6*a^11*A*Sin[c + d*x] - 270*a^9*A*b^2*Sin[c + d*x] + 750*a^7*A*b^4*Sin[c + d*x] + 1086*a^5*A*b^6*Sin[c + d*x] - 2232*a^3*A*b^8*Sin[c + d*x] + 960*a*A*b^10*Sin[c + d*x] + 72*a^10*b*B*Sin[c + d*x] - 360*a^8*b^3*B*Sin[c + d*x] - 540*a^6*b^5*B*Sin[c + d*x] + 912*a^4*b^7*B*Sin[c + d*x] - 384*a^2*b^9*B*Sin[c + d*x] - 60*a^10*A*b*Sin[2*(c + d*x)] - 372*a^8*A*b^3*Sin[2*(c + d*x)] + 2772*a^6*A*b^5*Sin[2*(c + d*x)] - 3300*a^4*A*b^7*Sin[2*(c + d*x)] + 1200*a^2*A*b^9*Sin[2*(c + d*x)] + 24*a^11*B*Sin[2*(c + d*x)] + 72*a^9*b^2*B*Sin[2*(c + d*x)] - 1200*a^7*b^4*B*Sin[2*(c + d*x)] + 1344*a^5*b^6*B*Sin[2*(c + d*x)] - 480*a^3*b^8*B*Sin[2*(c + d*x)] + 9*a^11*A*Sin[3*(c + d*x)] - 279*a^9*A*b^2*Sin[3*(c + d*x)] + 1143*a^7*A*b^4*Sin[3*(c + d*x)] - 1253*a^5*A*b^6*Sin[3*(c + d*x)] + 440*a^3*A*b^8*Sin[3*(c + d*x)] + 72*a^10*b*B*Sin[3*(c + d*x)] - 456*a^8*b^3*B*Sin[3*(c + d*x)] + 500*a^6*b^5*B*Sin[3*(c + d*x)] - 176*a^4*b^7*B*Sin[3*(c + d*x)] - 30*a^10*A*b*Sin[4*(c + d*x)] + 90*a^8*A*b^3*Sin[4*(c + d*x)] - 90*a^6*A*b^5*Sin[4*(c + d*x)] + 30*a^4*A*b^7*Sin[4*(c + d*x)] + 12*a^11*B*Sin[4*(c + d*x)] - 36*a^9*b^2*B*Sin[4*(c + d*x)] + 36*a^7*b^4*B*Sin[4*(c + d*x)] - 12*a^5*b^6*B*Sin[4*(c + d*x)] + 3*a^11*A*Sin[5*(c + d*x)] - 9*a^9*A*b^2*Sin[5*(c + d*x)] + 9*a^7*A*b^4*Sin[5*(c + d*x)] - 3*a^5*A*b^6*Sin[5*(c + d*x)]/((a^2 - b^2)^3*(b + a*Cos[c + d*x])^3))/(96*a^6*d)

fricas [B] time = 0.94, size = 2890, normalized size = 5.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^4,x, algorithm="fricas")

[Out]
$$\begin{aligned} & [1/12*(6*(A*a^{13} - 8*B*a^{12}*b + 16*A*a^{11}*b^2 + 32*B*a^{10}*b^3 - 74*A*a^9*b^4 - 48*B*a^8*b^5 + 116*A*a^7*b^6 + 32*B*a^6*b^7 - 79*A*a^5*b^8 - 8*B*a^4*b^9 + 20*A*a^3*b^{10})*d*x*\cos(d*x + c)^3 + 18*(A*a^{12}*b - 8*B*a^{11}*b^2 + 16*A*a^{10}*b^3 + 32*B*a^9*b^4 - 74*A*a^8*b^5 - 48*B*a^7*b^6 + 116*A*a^6*b^7 + 32*B*a^5*b^8 - 79*A*a^4*b^9 - 8*B*a^3*b^{10} + 20*A*a^2*b^{11})*d*x*\cos(d*x + c)^2 + 18*(A*a^{11}*b^2 - 8*B*a^{10}*b^3 + 16*A*a^9*b^4 + 32*B*a^8*b^5 - 74*A*a^7*b^6 - 48*B*a^6*b^7 + 116*A*a^5*b^8 + 32*B*a^4*b^9 - 79*A*a^3*b^{10} - 8*B*a^2*b^{11} + 20*A*a*b^{12})*d*x*\cos(d*x + c) + 6*(A*a^{10}*b^3 - 8*B*a^9*b^4 + 16*A*a^8*b^5 + 32*B*a^7*b^6 - 74*A*a^6*b^7 - 48*B*a^5*b^8 + 116*A*a^4*b^9 + 32*B*a^3*b^{10} - 79*A*a^2*b^{11} - 8*B*a*b^{12} + 20*A*b^{13})*d*x - 3*(20*B*a^7*b^5 - 40*A*a^6*b^6 - 35*B*a^5*b^7 + 84*A*a^4*b^8 + 28*B*a^3*b^9 - 69*A*a^2*b^{10} - 8*B*a*b^{11} + 20*A*b^{12} + (20*B*a^{10}*b^2 - 40*A*a^9*b^3 - 35*B*a^8*b^4 + 84*A*a^7*b^5 + 28*B*a^6*b^6 - 69*A*a^5*b^7 - 8*B*a^4*b^8 + 20*A*a^3*b^9)*\cos(d*x + c)^3 + 3*(20*B*a^9*b^3 - 40*A*a^8*b^4 - 35*B*a^7*b^5 + 84*A*a^6*b^6 + 28*B*a^5*b^7 - 69*A*a^4*b^8 - 8*B*a^3*b^9 + 20*A*a^2*b^{10})*\cos(d*x + c)^2 + 3*(20*B*a^8*b^4 - 40*A*a^7*b^5 - 35*B*a^6*b^6 + 84*A*a^5*b^7 + 28*B*a^4*b^8 - 69*A*a^3*b^9 - 8*B*a^2*b^{10} + 20*A*a*b^{11})*\cos(d*x + c))*\sqrt{a^2 - b^2}*\log((2*a*b*\cos(d*x + c) - (a^2 - 2*b^2)*\cos(d*x + c)^2 - 2*\sqrt{a^2 - b^2})*(b*\cos(d*x + c) + a)*\sin(d*x + c) + 2*a^2 - b^2)/(a^2*\cos(d*x + c)^2 + 2*a*b*\cos(d*x + c) + b^2)) + 2*(6*B*a^{10}*b^3 - 24*A*a^9*b^4 - 71*B*a^8*b^5 + 170*A*a^7*b^6 + 133*B*a^6*b^7 - 313*A*a^5*b^8 - 92*B*a^4*b^9 + 227*A*a^3*b^{10} + 24*B*a^2*b^{11} - 60*A*a*b^{12} + 3*(A*a^{13} - 4*A*a^{11}*b^2 + 6*A*a^9*b^4 - 4*A*a^7*b^6 + A*a^5*b^8)*\cos(d*x + c)^4 + 3*(2*B*a^{13} - 5*A*a^{12}*b - 8*B*a^{11}*b^2 + 20*A*a^{10}*b^3 + 12*B*a^9*b^4 - 30*A*a^8*b^5 - 8*B*a^7*b^6 + 20*A*a^6*b^7 + 2*B*a^5*b^8 - 5*A*a^4*b^9)*\cos(d*x + c)^3 + (18*B*a^{12}*b - 63*A*a^{11}*b^2 - 132*B*a^{10}*b^3 + 342*A*a^9*b^4 + 239*B*a^8*b^5 - 590*A*a^7*b^6 - 169*B*a^6*b^7 + 421*A*a^5*b^8 + 44*B*a^4*b^9 - 110*A*a^3*b^{10})*\cos(d*x + c)^2 + 3*(6*B*a^{11}*b^2 - 23*A*a^{10}*b^3 - 59*B*a^9*b^4 + 146*A*a^8*b^5 + 110*B*a^7*b^6 - 263*A*a^6*b^7 - 77*B*a^5*b^8 + 190*A*a^4*b^9 + 20*B*a^3*b^{10} - 50*A*a^2*b^{11})*\cos(d*x + c))*\sin(d*x + c))/((a^{17} - 4*a^{15}*b^2 + 6*a^{13}*b^4 - 4*a^{11}*b^6 + a^9*b^8)*d*\cos(d*x + c)^3 + 3*(a^{16}*b - 4*a^{14}*b^3 + 6*a^{12}*b^5 - 4*a^{10}*b^7 + a^8*b^9)*d*\cos(d*x + c)^2 + 3*(a^{15}*b^2 - 4*a^{13}*b^4 + 6*a^{11}*b^6 - 4*a^9*b^8 + a^7*b^{10})*d*\cos(d*x + c) + (a^{14}*b^3 - 4*a^{12}*b^5 + 6*a^{10}*b^7 - 4*a^8*b^9 + a^6*b^{11})*d), 1/6*(3*(A*a^{13} - 8*B*a^{12}*b + 16*A*a^{11}*b^2 + 32*B*a^{10}*b^3 - 74*A*a^9*b^4 - 48*B*a^8*b^5 + 116*A*a^7*b^6 + 32*B*a^6*b^7 - 79*A*a^5*b^8 - 8*B*a^4*b^9 + 20*A*a^3*b^{10})*d*x*\cos(d*x + c)^3 + 9*(A*a^{12}*b - 8*B*a^{11}*b^2 + 16*A*a^{10}*b^3 + 32*B*a^9*b^4 - 74*A*a^8*b^5 - 48*B*a^7*b^6 + 116*A*a^6*b^7 + 32*B*a^5*b^8 - 79*A*a^4*b^9 - 8*B*a^3*b^{10} + 20*A*a^2*b^{11})*d*x*\cos(d*x + c)^2 + 9*(A*a^{11}*b^2 - 8*B*a^{10}*b^3 + 16*A*a^9*b^4 + 32*B*a^8*b^5 - 74*A*a^7*b^6 - 48*B*a^6*b^7 + 116*A*a^5*b^8 + 32$$

```

*B*a^4*b^9 - 79*A*a^3*b^10 - 8*B*a^2*b^11 + 20*A*a*b^12)*d*x*cos(d*x + c) +
  3*(A*a^10*b^3 - 8*B*a^9*b^4 + 16*A*a^8*b^5 + 32*B*a^7*b^6 - 74*A*a^6*b^7 -
  48*B*a^5*b^8 + 116*A*a^4*b^9 + 32*B*a^3*b^10 - 79*A*a^2*b^11 - 8*B*a*b^12
+ 20*A*b^13)*d*x + 3*(20*B*a^7*b^5 - 40*A*a^6*b^6 - 35*B*a^5*b^7 + 84*A*a^4
*b^8 + 28*B*a^3*b^9 - 69*A*a^2*b^10 - 8*B*a*b^11 + 20*A*b^12 + (20*B*a^10*b
^2 - 40*A*a^9*b^3 - 35*B*a^8*b^4 + 84*A*a^7*b^5 + 28*B*a^6*b^6 - 69*A*a^5*b
^7 - 8*B*a^4*b^8 + 20*A*a^3*b^9)*cos(d*x + c)^3 + 3*(20*B*a^9*b^3 - 40*A*a^
8*b^4 - 35*B*a^7*b^5 + 84*A*a^6*b^6 + 28*B*a^5*b^7 - 69*A*a^4*b^8 - 8*B*a^3
*b^9 + 20*A*a^2*b^10)*cos(d*x + c)^2 + 3*(20*B*a^8*b^4 - 40*A*a^7*b^5 - 35*
B*a^6*b^6 + 84*A*a^5*b^7 + 28*B*a^4*b^8 - 69*A*a^3*b^9 - 8*B*a^2*b^10 + 20*
A*a*b^11)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*
x + c) + a)/((a^2 - b^2)*sin(d*x + c))) + (6*B*a^10*b^3 - 24*A*a^9*b^4 - 71
*B*a^8*b^5 + 170*A*a^7*b^6 + 133*B*a^6*b^7 - 313*A*a^5*b^8 - 92*B*a^4*b^9 +
  227*A*a^3*b^10 + 24*B*a^2*b^11 - 60*A*a*b^12 + 3*(A*a^13 - 4*A*a^11*b^2 +
  6*A*a^9*b^4 - 4*A*a^7*b^6 + A*a^5*b^8)*cos(d*x + c)^4 + 3*(2*B*a^13 - 5*A*a
^12*b - 8*B*a^11*b^2 + 20*A*a^10*b^3 + 12*B*a^9*b^4 - 30*A*a^8*b^5 - 8*B*a^
7*b^6 + 20*A*a^6*b^7 + 2*B*a^5*b^8 - 5*A*a^4*b^9)*cos(d*x + c)^3 + (18*B*a^
12*b - 63*A*a^11*b^2 - 132*B*a^10*b^3 + 342*A*a^9*b^4 + 239*B*a^8*b^5 - 590
*A*a^7*b^6 - 169*B*a^6*b^7 + 421*A*a^5*b^8 + 44*B*a^4*b^9 - 110*A*a^3*b^10)
*cos(d*x + c)^2 + 3*(6*B*a^11*b^2 - 23*A*a^10*b^3 - 59*B*a^9*b^4 + 146*A*a^
8*b^5 + 110*B*a^7*b^6 - 263*A*a^6*b^7 - 77*B*a^5*b^8 + 190*A*a^4*b^9 + 20*B
*a^3*b^10 - 50*A*a^2*b^11)*cos(d*x + c))*sin(d*x + c))/((a^17 - 4*a^15*b^2
+ 6*a^13*b^4 - 4*a^11*b^6 + a^9*b^8)*d*cos(d*x + c)^3 + 3*(a^16*b - 4*a^14*
b^3 + 6*a^12*b^5 - 4*a^10*b^7 + a^8*b^9)*d*cos(d*x + c)^2 + 3*(a^15*b^2 - 4
*a^13*b^4 + 6*a^11*b^6 - 4*a^9*b^8 + a^7*b^10)*d*cos(d*x + c) + (a^14*b^3 -
  4*a^12*b^5 + 6*a^10*b^7 - 4*a^8*b^9 + a^6*b^11)*d)]

```

giac [B] time = 0.41, size = 1052, normalized size = 1.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```

[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^4,x, algorithm="gi
ac")

```

```

[Out] 1/6*(6*(20*B*a^7*b^2 - 40*A*a^6*b^3 - 35*B*a^5*b^4 + 84*A*a^4*b^5 + 28*B*a^
3*b^6 - 69*A*a^2*b^7 - 8*B*a*b^8 + 20*A*b^9)*(pi*floor(1/2*(d*x + c)/pi + 1
/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2
*c))/sqrt(-a^2 + b^2)))/((a^12 - 3*a^10*b^2 + 3*a^8*b^4 - a^6*b^6)*sqrt(-a^
2 + b^2)) + 2*(60*B*a^7*b^3*tan(1/2*d*x + 1/2*c)^5 - 90*A*a^6*b^4*tan(1/2*d
*x + 1/2*c)^5 - 105*B*a^6*b^4*tan(1/2*d*x + 1/2*c)^5 + 162*A*a^5*b^5*tan(1/
2*d*x + 1/2*c)^5 - 24*B*a^5*b^5*tan(1/2*d*x + 1/2*c)^5 + 48*A*a^4*b^6*tan(1
/2*d*x + 1/2*c)^5 + 117*B*a^4*b^6*tan(1/2*d*x + 1/2*c)^5 - 213*A*a^3*b^7*ta
n(1/2*d*x + 1/2*c)^5 - 24*B*a^3*b^7*tan(1/2*d*x + 1/2*c)^5 + 48*A*a^2*b^8*ta
n(1/2*d*x + 1/2*c)^5 - 42*B*a^2*b^8*tan(1/2*d*x + 1/2*c)^5 + 81*A*a*b^9*ta

```


$$\begin{aligned} & n(1/2*d*x + 1/2*c)^5 + 18*B*a*b^9*\tan(1/2*d*x + 1/2*c)^5 - 36*A*b^{10}*\tan(1/2*d*x + 1/2*c)^5 - 120*B*a^7*b^3*\tan(1/2*d*x + 1/2*c)^3 + 180*A*a^6*b^4*\tan(1/2*d*x + 1/2*c)^3 + 236*B*a^5*b^5*\tan(1/2*d*x + 1/2*c)^3 - 392*A*a^4*b^6*\tan(1/2*d*x + 1/2*c)^3 - 152*B*a^3*b^7*\tan(1/2*d*x + 1/2*c)^3 + 284*A*a^2*b^8*\tan(1/2*d*x + 1/2*c)^3 + 36*B*a*b^9*\tan(1/2*d*x + 1/2*c)^3 - 72*A*b^{10}*\tan(1/2*d*x + 1/2*c)^3 + 60*B*a^7*b^3*\tan(1/2*d*x + 1/2*c) - 90*A*a^6*b^4*\tan(1/2*d*x + 1/2*c) + 105*B*a^6*b^4*\tan(1/2*d*x + 1/2*c) - 162*A*a^5*b^5*\tan(1/2*d*x + 1/2*c) - 24*B*a^5*b^5*\tan(1/2*d*x + 1/2*c) + 48*A*a^4*b^6*\tan(1/2*d*x + 1/2*c) - 117*B*a^4*b^6*\tan(1/2*d*x + 1/2*c) + 213*A*a^3*b^7*\tan(1/2*d*x + 1/2*c) - 24*B*a^3*b^7*\tan(1/2*d*x + 1/2*c) + 48*A*a^2*b^8*\tan(1/2*d*x + 1/2*c) + 42*B*a^2*b^8*\tan(1/2*d*x + 1/2*c) - 81*A*a*b^9*\tan(1/2*d*x + 1/2*c) + 18*B*a*b^9*\tan(1/2*d*x + 1/2*c) - 36*A*b^{10}*\tan(1/2*d*x + 1/2*c))/((a^{11} - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*(a*\tan(1/2*d*x + 1/2*c)^2 - b*\tan(1/2*d*x + 1/2*c)^2 - a - b)^3) + 3*(A*a^2 - 8*B*a*b + 20*A*b^2)*(d*x + c)/a^6 - 6*(A*a*\tan(1/2*d*x + 1/2*c)^3 - 2*B*a*\tan(1/2*d*x + 1/2*c)^3 + 8*A*b*\tan(1/2*d*x + 1/2*c)^3 - A*a*\tan(1/2*d*x + 1/2*c) - 2*B*a*\tan(1/2*d*x + 1/2*c) + 8*A*b*\tan(1/2*d*x + 1/2*c))/((\tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^5))/d \end{aligned}$$

maple [B] time = 1.15, size = 3099, normalized size = 5.76

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\int (\cos(dx+c)^2*(A+B*\sec(dx+c)))/(a+b*\sec(dx+c))^4, x$

[Out]
$$\begin{aligned} & -30/d/a/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)^3*b^4/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*\tan(1/2*d*x+1/2*c)^5*A-212/3/d/a^3/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)^3*b^6/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*\tan(1/2*d*x+1/2*c)^3*A-35/d*b^4/a/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^{(1/2)}*\operatorname{arctanh}(\tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^{(1/2)})*B+28/d*b^6/a^3/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^{(1/2)}*\operatorname{arctanh}(\tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^{(1/2)})*B-69/d/a^4/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^{(1/2)}*\operatorname{arctanh}(\tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^{(1/2)})*A*b^7+20/d/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)^3*b^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*\tan(1/2*d*x+1/2*c)^5*B-30/d/a/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)^3*b^4/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*\tan(1/2*d*x+1/2*c)*A+60/d/a/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)^3*b^4/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*\tan(1/2*d*x+1/2*c)^3*A+6/d/a^2/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)^3*b^5/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*\tan(1/2*d*x+1/2*c)*A+34/d/a^3/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)^3*b^6/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*\tan(1/2*d*x+1/2*c)*A-6/d/a^2/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)^3*b^5/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*\tan(1/2*d*x+1/2*c)^5*A-8/d*b^8/a^5/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^{(1/2)}*\operatorname{arctanh}(\tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^{(1/2)})*B+20/d*b^9/a^6/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^{(1/2)}*\operatorname{ar} \end{aligned}$$

$$\begin{aligned} & \operatorname{ctanh}(\tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^{(1/2)})*A+34/d/a^3/(a*\tan(1/2*d \\ & *x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)^3*b^6/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^ \\ & 3)*\tan(1/2*d*x+1/2*c)^5*A+84/d/a^2/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+ \\ & b))^{(1/2)}*\operatorname{arctanh}(\tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^{(1/2)})*A*b^5-12/d* \\ & b^7/a^4/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a^2-2*a*b+b^ \\ & 2)/(a^2+2*a*b+b^2)*\tan(1/2*d*x+1/2*c)^3*B+116/3/d*b^5/a^2/(a*\tan(1/2*d*x+1/ \\ & 2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*\tan(1/ \\ & 2*d*x+1/2*c)^3*B-12/d*b^8/a^5/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2* \\ & b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*\tan(1/2*d*x+1/2*c)*A-12/d*b^8/a^5/ \\ & (a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3* \\ & a*b^2+b^3)*\tan(1/2*d*x+1/2*c)^5*A+6/d*b^7/a^4/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1 \\ & /2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*\tan(1/2*d*x+1/2*c) \\ & *B-5/d*b^4/a/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a \\ & ^3-3*a^2*b+3*a*b^2-b^3)*\tan(1/2*d*x+1/2*c)*B+5/d*b^4/a/(a*\tan(1/2*d*x+1/2*c \\ &)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*\tan(1/2*d \\ & *x+1/2*c)^5*B-18/d*b^5/a^2/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a \\ & -b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*\tan(1/2*d*x+1/2*c)*B-18/d*b^5/a^2/(a* \\ & \tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b \\ & ^2+b^3)*\tan(1/2*d*x+1/2*c)^5*B-2/d*b^6/a^3/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2* \\ & d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*\tan(1/2*d*x+1/2*c)^5* \\ & B-3/d*b^7/a^4/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(\\ & a^3-3*a^2*b+3*a*b^2-b^3)*\tan(1/2*d*x+1/2*c)*A+3/d*b^7/a^4/(a*\tan(1/2*d*x+1/ \\ & 2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*\tan(1/ \\ & 2*d*x+1/2*c)^5*A+2/d*b^6/a^3/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b \\ & -a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*\tan(1/2*d*x+1/2*c)*B+1/d*A/a^4*\operatorname{arct} \\ & \operatorname{an}(\tan(1/2*d*x+1/2*c))+20/d*b^2/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b)) \\ & ^{(1/2)}*\operatorname{arctanh}(\tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^{(1/2)})*A*B+20/d/(a*\tan \\ & (1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)^3*b^3/(a+b)/(a^3-3*a^2*b+3*a \\ & *b^2-b^3)*\tan(1/2*d*x+1/2*c)*B-40/d/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2 \\ & *c)^2*b-a-b)^3*b^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*\tan(1/2*d*x+1/2*c)^3*B+6 \\ & /d*b^7/a^4/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3 \\ & +3*a^2*b+3*a*b^2+b^3)*\tan(1/2*d*x+1/2*c)^5*B-1/d/a^4/(1+\tan(1/2*d*x+1/2*c)^ \\ & 2)^2*\tan(1/2*d*x+1/2*c)^3*A-40/d*b^3/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(\\ & a+b))^{(1/2)}*\operatorname{arctanh}(\tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^{(1/2)})*A+24/d*b^ \\ & 8/a^5/(a*\tan(1/2*d*x+1/2*c)^2-\tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a^2-2*a*b+b^2) \\ & /(a^2+2*a*b+b^2)*\tan(1/2*d*x+1/2*c)^3*A+1/d/a^4/(1+\tan(1/2*d*x+1/2*c)^2)^2* \\ & \tan(1/2*d*x+1/2*c)*A+2/d/a^4/(1+\tan(1/2*d*x+1/2*c)^2)^2*\tan(1/2*d*x+1/2*c)* \\ & B+20/d/a^6*\operatorname{arctan}(\tan(1/2*d*x+1/2*c))*A*b^2-8/d/a^5*\operatorname{arctan}(\tan(1/2*d*x+1/2* \\ & c))*B*b-8/d/a^5/(1+\tan(1/2*d*x+1/2*c)^2)^2*\tan(1/2*d*x+1/2*c)*A*b-8/d/a^5/(\\ & 1+\tan(1/2*d*x+1/2*c)^2)^2*\tan(1/2*d*x+1/2*c)^3*A*b+2/d/a^4/(1+\tan(1/2*d*x+1 \\ & /2*c)^2)^2*\tan(1/2*d*x+1/2*c)^3*B \end{aligned}$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 15.82, size = 14438, normalized size = 26.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x))^4,x)
```

```
[Out] ((tan(c/2 + (d*x)/2)*(A*a^8 + 20*A*b^8 + 2*B*a^8 - 59*A*a^2*b^6 - 27*A*a^3*b^5 + 57*A*a^4*b^4 + 21*A*a^5*b^3 - 11*A*a^6*b^2 - 4*B*a^2*b^6 + 24*B*a^3*b^5 + 11*B*a^4*b^4 - 26*B*a^5*b^3 - 6*B*a^6*b^2 + 10*A*a*b^7 - 7*A*a^7*b - 8*B*a*b^7 + 2*B*a^7*b))/(a^5*(a + b)*(a - b)^3) + (2*tan(c/2 + (d*x)/2)^5*(9*A*a^10 + 180*A*b^10 - 611*A*a^2*b^8 + 740*A*a^4*b^6 - 324*A*a^6*b^4 + 36*A*a^8*b^2 + 248*B*a^3*b^7 - 320*B*a^5*b^5 + 132*B*a^7*b^3 - 72*B*a*b^9 - 18*B*a^9*b))/(3*a^5*(a + b)^3*(a - b)^3) + (tan(c/2 + (d*x)/2)^9*(A*a^8 + 20*A*b^8 - 2*B*a^8 - 59*A*a^2*b^6 + 27*A*a^3*b^5 + 57*A*a^4*b^4 - 21*A*a^5*b^3 - 11*A*a^6*b^2 + 4*B*a^2*b^6 + 24*B*a^3*b^5 - 11*B*a^4*b^4 - 26*B*a^5*b^3 + 6*B*a^6*b^2 - 10*A*a*b^7 + 7*A*a^7*b - 8*B*a*b^7 + 2*B*a^7*b))/(a^5*(a + b)^3*(a - b)) - (2*tan(c/2 + (d*x)/2)^3*(6*A*a^9 - 120*A*b^9 + 6*B*a^9 + 364*A*a^2*b^7 + 71*A*a^3*b^6 - 369*A*a^4*b^5 - 45*A*a^5*b^4 + 111*A*a^6*b^3 + 3*A*a^7*b^2 + 12*B*a^2*b^7 - 148*B*a^3*b^6 - 29*B*a^4*b^5 + 159*B*a^5*b^4 + 18*B*a^6*b^3 - 30*B*a^7*b^2 - 30*A*a*b^8 - 21*A*a^8*b + 48*B*a*b^8 - 6*B*a^8*b))/(3*a^5*(a + b)^2*(a - b)^3) - (2*tan(c/2 + (d*x)/2)^7*(6*A*a^9 + 120*A*b^9 - 6*B*a^9 - 364*A*a^2*b^7 + 71*A*a^3*b^6 + 369*A*a^4*b^5 - 45*A*a^5*b^4 - 111*A*a^6*b^3 + 3*A*a^7*b^2 + 12*B*a^2*b^7 + 148*B*a^3*b^6 - 29*B*a^4*b^5 - 159*B*a^5*b^4 + 18*B*a^6*b^3 + 30*B*a^7*b^2 - 30*A*a*b^8 + 21*A*a^8*b - 48*B*a*b^8 - 6*B*a^8*b))/(3*a^5*(a + b)^3*(a - b)^2))/(d*(tan(c/2 + (d*x)/2)^2*(9*a*b^2 + 3*a^2*b - a^3 + 5*b^3) + tan(c/2 + (d*x)/2)^4*(6*a*b^2 - 6*a^2*b - 2*a^3 + 10*b^3) - tan(c/2 + (d*x)/2)^6*(6*a*b^2 + 6*a^2*b - 2*a^3 - 10*b^3) + 3*a*b^2 + 3*a^2*b + a^3 + b^3 - tan(c/2 + (d*x)/2)^10*(3*a*b^2 - 3*a^2*b + a^3 - b^3) + tan(c/2 + (d*x)/2)^8*(3*a^2*b - 9*a*b^2 + a^3 + 5*b^3))) - (atan((((8*tan(c/2 + (d*x)/2)*(800*A^2*a*b^17 - 800*A^2*b^18 - A^2*a^18 + 2*A^2*a^17*b + 4720*A^2*a^2*b^16 - 4720*A^2*a^3*b^15 - 11522*A^2*a^4*b^14 + 11522*A^2*a^5*b^13 + 14837*A^2*a^6*b^12 - 14812*A^2*a^7*b^11 - 10385*A^2*a^8*b^10 + 10430*A^2*a^9*b^9 + 3325*A^2*a^10*b^8 - 3640*A^2*a^11*b^7 + 45*A^2*a^12*b^6 + 350*A^2*a^13*b^5 - 209*A^2*a^14*b^4 + 68*A^2*a^15*b
```

$$\begin{aligned}
& \sim^3 - 35A^2a^{16}b^2 - 128B^2a^2b^{16} + 128B^2a^3b^{15} + 768B^2a^4b^{14} - 768B^2a^5b^{13} - 1920B^2a^6b^{12} + 1920B^2a^7b^{11} + 2600B^2a^8b^{10} - 2560B^2a^9b^9 - 2025B^2a^{10}b^8 + 1920B^2a^{11}b^7 + 824B^2a^{12}b^6 - 768B^2a^{13}b^5 - 80B^2a^{14}b^4 + 128B^2a^{15}b^3 - 64B^2a^{16}b^2 + 640A^2B^2a^{17} + 16A^2B^2a^{17}b - 640A^2B^2a^{17}b^2 - 3808A^2B^2a^{17}b^3 + 3808A^2B^2a^{17}b^4 + 9408A^2B^2a^{17}b^5 - 9408A^2B^2a^{17}b^6 - 12430A^2B^2a^{17}b^7 + 12320A^2B^2a^{17}b^8 + 9200A^2B^2a^{17}b^9 - 8960A^2B^2a^{17}b^{10} - 3360A^2B^2a^{17}b^{11} + 3360A^2B^2a^{17}b^{12} + 144A^2B^2a^{17}b^{13} - 448A^2B^2a^{17}b^{14} + 240A^2B^2a^{17}b^{15} - 32A^2B^2a^{17}b^{16}) / (a^{20}b + a^{21} - a^{10}b^{11} - a^{11}b^{10} + 5a^{12}b^9 + 5a^{13}b^8 - 10a^{14}b^7 - 10a^{15}b^6 + 10a^{16}b^5 + 10a^{17}b^4 - 5a^{18}b^3 - 5a^{19}b^2) - (((4*(4A^2a^{27} - 80A^2a^{12}b^{15} + 40A^2a^{13}b^{14} + 516A^2a^{14}b^{13} - 248A^2a^{15}b^{12} - 1404A^2a^{16}b^{11} + 640A^2a^{17}b^{10} + 2076A^2a^{18}b^9 - 896A^2a^{19}b^8 - 1764A^2a^{20}b^7 + 724A^2a^{21}b^6 + 816A^2a^{22}b^5 - 316A^2a^{23}b^4 - 160A^2a^{24}b^3 + 52A^2a^{25}b^2 + 32B^2a^{13}b^{14} - 16B^2a^{14}b^{13} - 208B^2a^{15}b^{12} + 100B^2a^{16}b^{11} + 572B^2a^{17}b^{10} - 252B^2a^{18}b^9 - 868B^2a^{19}b^8 + 348B^2a^{20}b^7 + 772B^2a^{21}b^6 - 292B^2a^{22}b^5 - 380B^2a^{23}b^4 + 144B^2a^{24}b^3 + 80B^2a^{25}b^2 - 32B^2a^{26}b)) / (a^{25}b + a^{26} - a^{15}b^{11} - a^{16}b^{10} + 5a^{17}b^9 + 5a^{18}b^8 - 10a^{19}b^7 - 10a^{20}b^6 + 10a^{21}b^5 + 10a^{22}b^4 - 5a^{23}b^3 - 5a^{24}b^2) - (4*\tan(c/2 + (d*x)/2)*(A^2a^{21}i + A^2b^{20}i - B^2a^8i))*(8a^{25}b - 8a^{12}b^{14} + 8a^{13}b^{13} + 48a^{14}b^{12} - 48a^{15}b^{11} - 120a^{16}b^{10} + 120a^{17}b^9 + 160a^{18}b^8 - 160a^{19}b^7 - 120a^{20}b^6 + 120a^{21}b^5 + 48a^{22}b^4 - 48a^{23}b^3 - 8a^{24}b^2)) / (a^6*(a^{20}b + a^{21} - a^{10}b^{11} - a^{11}b^{10} + 5a^{12}b^9 + 5a^{13}b^8 - 10a^{14}b^7 - 10a^{15}b^6 + 10a^{16}b^5 + 10a^{17}b^4 - 5a^{18}b^3 - 5a^{19}b^2))) * (A^2a^{21}i + A^2b^{20}i - B^2a^8i)) / (2a^6) * (A^2a^{21}i + A^2b^{20}i - B^2a^8i) * 1) / (2a^6) + (((8*\tan(c/2 + (d*x)/2)*(800A^2a^2b^{17} - 800A^2a^2b^{18} - A^2a^{18} + 2A^2a^{17}b + 4720A^2a^2b^{16} - 4720A^2a^3b^{15} - 11522A^2a^4b^{14} + 11522A^2a^5b^{13} + 14837A^2a^6b^{12} - 14812A^2a^7b^{11} - 10385A^2a^8b^{10} + 10430A^2a^9b^9 + 3325A^2a^{10}b^8 - 3640A^2a^{11}b^7 + 45A^2a^{12}b^6 + 350A^2a^{13}b^5 - 209A^2a^{14}b^4 + 68A^2a^{15}b^3 - 35A^2a^{16}b^2 - 128B^2a^2b^{16} + 128B^2a^3b^{15} + 768B^2a^4b^{14} - 768B^2a^5b^{13} - 1920B^2a^6b^{12} + 1920B^2a^7b^{11} + 2600B^2a^8b^{10} - 2560B^2a^9b^9 - 2025B^2a^{10}b^8 + 1920B^2a^{11}b^7 + 824B^2a^{12}b^6 - 768B^2a^{13}b^5 - 80B^2a^{14}b^4 + 128B^2a^{15}b^3 - 64B^2a^{16}b^2 + 640A^2B^2a^{17} + 16A^2B^2a^{17}b - 640A^2B^2a^{17}b^2 - 3808A^2B^2a^{17}b^3 + 3808A^2B^2a^{17}b^4 + 9408A^2B^2a^{17}b^5 - 9408A^2B^2a^{17}b^6 - 12430A^2B^2a^{17}b^7 + 12320A^2B^2a^{17}b^8 + 9200A^2B^2a^{17}b^9 - 8960A^2B^2a^{17}b^{10} - 3360A^2B^2a^{17}b^{11} + 3360A^2B^2a^{17}b^{12} + 144A^2B^2a^{17}b^{13} - 448A^2B^2a^{17}b^{14} + 240A^2B^2a^{17}b^{15} - 32A^2B^2a^{17}b^{16})) / (a^{20}b + a^{21} - a^{10}b^{11} - a^{11}b^{10} + 5a^{12}b^9 + 5a^{13}b^8 - 10a^{14}b^7 - 10a^{15}b^6 + 10a^{16}b^5 + 10a^{17}b^4 - 5a^{18}b^3 - 5a^{19}b^2) + (((4*(4A^2a^{27} - 80A^2a^{12}b^{15} + 40A^2a^{13}b^{14} + 516A^2a^{14}b^{13} - 248A^2a^{15}b^{12} - 1404A^2a^{16}b^{11} + 640A^2a^{17}b^{10} + 2076A^2a^{18}b^9 - 896A^2a^{19}b^8 - 1764A^2a^{20}b^7 + 724A^2a^{21}b^6 + 816A^2a^{22}b^5 - 316A^2a^{23}b^4 - 160A^2a^{24}b^3 + 52A^2a^{25}b^2 + 32B^2a^{13}b^{14} - 16B^2a^{14}
\end{aligned}$$

$$\begin{aligned}
& *b^{13} - 208*B*a^{15}*b^{12} + 100*B*a^{16}*b^{11} + 572*B*a^{17}*b^{10} - 252*B*a^{18}*b^9 \\
& - 868*B*a^{19}*b^8 + 348*B*a^{20}*b^7 + 772*B*a^{21}*b^6 - 292*B*a^{22}*b^5 - 380 \\
& *B*a^{23}*b^4 + 144*B*a^{24}*b^3 + 80*B*a^{25}*b^2 - 32*B*a^{26}*b))/ (a^{25}*b + a^{26} \\
& - a^{15}*b^{11} - a^{16}*b^{10} + 5*a^{17}*b^9 + 5*a^{18}*b^8 - 10*a^{19}*b^7 - 10*a^{20}* \\
& b^6 + 10*a^{21}*b^5 + 10*a^{22}*b^4 - 5*a^{23}*b^3 - 5*a^{24}*b^2) + (4*\tan(c/2 + (\\
& d*x)/2)*(A*a^{2*1i} + A*b^{2*20i} - B*a*b*8i)*(8*a^{25}*b - 8*a^{12}*b^{14} + 8*a^{13}* \\
& b^{13} + 48*a^{14}*b^{12} - 48*a^{15}*b^{11} - 120*a^{16}*b^{10} + 120*a^{17}*b^9 + 160*a^{18}*b^8 \\
& - 160*a^{19}*b^7 - 120*a^{20}*b^6 + 120*a^{21}*b^5 + 48*a^{22}*b^4 - 48*a^{23}* \\
& b^3 - 8*a^{24}*b^2))/(a^6*(a^{20}*b + a^{21} - a^{10}*b^{11} - a^{11}*b^{10} + 5*a^{12}*b^9 \\
& + 5*a^{13}*b^8 - 10*a^{14}*b^7 - 10*a^{15}*b^6 + 10*a^{16}*b^5 + 10*a^{17}*b^4 - 5*a^{18}* \\
& b^3 - 5*a^{19}*b^2)))*(A*a^{2*1i} + A*b^{2*20i} - B*a*b*8i))/(2*a^6))*(A*a^{2* \\
& 1i} + A*b^{2*20i} - B*a*b*8i)*1i)/(2*a^6))/((8*(8000*A^3*b^{19} - 4000*A^3*a*b^{18} \\
& - 50800*A^3*a^2*b^{17} + 24400*A^3*a^3*b^{16} + 135260*A^3*a^4*b^{15} - 62030*A^3*a^5*b^{14} \\
& - 193689*A^3*a^6*b^{13} + 82337*A^3*a^7*b^{12} + 155991*A^3*a^8*b^{11} \\
& - 57345*A^3*a^9*b^{10} - 64479*A^3*a^{10}*b^9 + 16999*A^3*a^{11}*b^8 + 8281*A^3 \\
& *a^{12}*b^7 + 204*A^3*a^{13}*b^6 + 1396*A^3*a^{14}*b^5 - 40*A^3*a^{15}*b^4 + 40*A^3 \\
& *a^{16}*b^3 - 512*B^3*a^3*b^{16} + 256*B^3*a^4*b^{15} + 3328*B^3*a^5*b^{14} - 1600* \\
& B^3*a^6*b^{13} - 9152*B^3*a^7*b^{12} + 4352*B^3*a^8*b^{11} + 13888*B^3*a^9*b^{10} - \\
& 6408*B^3*a^{10}*b^9 - 12352*B^3*a^{11}*b^8 + 5120*B^3*a^{12}*b^7 + 6080*B^3*a^{13} \\
& *b^6 - 1920*B^3*a^{14}*b^5 - 1280*B^3*a^{15}*b^4 - 9600*A^2*B*a*b^{18} + 3840*A*B \\
& ^2*a^2*b^{17} - 1920*A*B^2*a^3*b^{16} - 24768*A*B^2*a^4*b^{15} + 11904*A*B^2*a^5* \\
& b^{14} + 67392*A*B^2*a^6*b^{13} - 31680*A*B^2*a^7*b^{12} - 100368*A*B^2*a^8*b^{11} \\
& + 45148*A*B^2*a^9*b^{10} + 86512*A*B^2*a^{10}*b^9 - 34567*A*B^2*a^{11}*b^8 - 4036 \\
& 8*A*B^2*a^{12}*b^7 + 11960*A*B^2*a^{13}*b^6 + 7440*A*B^2*a^{14}*b^5 + 80*A*B^2*a^{15}* \\
& b^4 + 320*A*B^2*a^{16}*b^3 + 4800*A^2*B*a^2*b^{17} + 61440*A^2*B*a^3*b^{16} - \\
& 29520*A^2*B*a^4*b^{15} - 165384*A^2*B*a^5*b^{14} + 76812*A^2*B*a^6*b^{13} + 24159 \\
& 6*A^2*B*a^7*b^{12} - 105755*A^2*B*a^8*b^{11} - 201479*A^2*B*a^9*b^{10} + 77359*A^2 \\
& *B*a^{10}*b^9 + 88721*A^2*B*a^{11}*b^8 - 24711*A^2*B*a^{12}*b^7 - 13929*A^2*B*a^{13} \\
& *b^6 - 255*A^2*B*a^{14}*b^5 - 1345*A^2*B*a^{15}*b^4 + 20*A^2*B*a^{16}*b^3 - 20* \\
& A^2*B*a^{17}*b^2))/(a^{25}*b + a^{26} - a^{15}*b^{11} - a^{16}*b^{10} + 5*a^{17}*b^9 + 5*a^{18} \\
& *b^8 - 10*a^{19}*b^7 - 10*a^{20}*b^6 + 10*a^{21}*b^5 + 10*a^{22}*b^4 - 5*a^{23}*b^3 \\
& - 5*a^{24}*b^2) + (((8*\tan(c/2 + (d*x)/2)*(800*A^2*a*b^{17} - 800*A^2*b^{18} - A^2 \\
& *a^{18} + 2*A^2*a^{17}*b + 4720*A^2*a^2*b^{16} - 4720*A^2*a^3*b^{15} - 11522*A^2* \\
& a^4*b^{14} + 11522*A^2*a^5*b^{13} + 14837*A^2*a^6*b^{12} - 14812*A^2*a^7*b^{11} - 1 \\
& 0385*A^2*a^8*b^{10} + 10430*A^2*a^9*b^9 + 3325*A^2*a^{10}*b^8 - 3640*A^2*a^{11}*b^7 \\
& + 45*A^2*a^{12}*b^6 + 350*A^2*a^{13}*b^5 - 209*A^2*a^{14}*b^4 + 68*A^2*a^{15}*b^3 \\
& - 35*A^2*a^{16}*b^2 - 128*B^2*a^2*b^{16} + 128*B^2*a^3*b^{15} + 768*B^2*a^4*b^{14} \\
& - 768*B^2*a^5*b^{13} - 1920*B^2*a^6*b^{12} + 1920*B^2*a^7*b^{11} + 2600*B^2*a^8 \\
& *b^{10} - 2560*B^2*a^9*b^9 - 2025*B^2*a^{10}*b^8 + 1920*B^2*a^{11}*b^7 + 824*B^2* \\
& a^{12}*b^6 - 768*B^2*a^{13}*b^5 - 80*B^2*a^{14}*b^4 + 128*B^2*a^{15}*b^3 - 64*B^2*a^{16} \\
& *b^2 + 640*A*B*a*b^{17} + 16*A*B*a^{17}*b - 640*A*B*a^2*b^{16} - 3808*A*B*a^3* \\
& b^{15} + 3808*A*B*a^4*b^{14} + 9408*A*B*a^5*b^{13} - 9408*A*B*a^6*b^{12} - 12430*A*B \\
& *a^7*b^{11} + 12320*A*B*a^8*b^{10} + 9200*A*B*a^9*b^9 - 8960*A*B*a^{10}*b^8 - 33 \\
& 60*A*B*a^{11}*b^7 + 3360*A*B*a^{12}*b^6 + 144*A*B*a^{13}*b^5 - 448*A*B*a^{14}*b^4 + \\
& 240*A*B*a^{15}*b^3 - 32*A*B*a^{16}*b^2))/(a^{20}*b + a^{21} - a^{10}*b^{11} - a^{11}*b^{11}
\end{aligned}$$

$$\begin{aligned}
& 0 + 5a^{12}b^9 + 5a^{13}b^8 - 10a^{14}b^7 - 10a^{15}b^6 + 10a^{16}b^5 + 10a^{17}b^4 - 5a^{18}b^3 - 5a^{19}b^2) - (((4*(4Aa^{27} - 80Aa^{12}b^{15} + 40Aa^{13}b^{14} + 516Aa^{14}b^{13} - 248Aa^{15}b^{12} - 1404Aa^{16}b^{11} + 640Aa^{17}b^{10} + 2076Aa^{18}b^9 - 896Aa^{19}b^8 - 1764Aa^{20}b^7 + 724Aa^{21}b^6 + 816Aa^{22}b^5 - 316Aa^{23}b^4 - 160Aa^{24}b^3 + 52Aa^{25}b^2 + 32Ba^{13}b^{14} - 16Ba^{14}b^{13} - 208Ba^{15}b^{12} + 100Ba^{16}b^{11} + 572Ba^{17}b^{10} - 252Ba^{18}b^9 - 868Ba^{19}b^8 + 348Ba^{20}b^7 + 772Ba^{21}b^6 - 292Ba^{22}b^5 - 380Ba^{23}b^4 + 144Ba^{24}b^3 + 80Ba^{25}b^2 - 32Ba^{26}b)) / (a^{25}b + a^{26} - a^{15}b^{11} - a^{16}b^{10} + 5a^{17}b^9 + 5a^{18}b^8 - 10a^{19}b^7 - 10a^{20}b^6 + 10a^{21}b^5 + 10a^{22}b^4 - 5a^{23}b^3 - 5a^{24}b^2) - (4*\tan(c/2 + (d*x)/2)*(Aa^2*1i + Ab^2*20i - B*a*b*8i)*(8a^{25}b - 8a^{12}b^{14} + 8a^{13}b^{13} + 48a^{14}b^{12} - 48a^{15}b^{11} - 120a^{16}b^{10} + 120a^{17}b^9 + 160a^{18}b^8 - 160a^{19}b^7 - 120a^{20}b^6 + 120a^{21}b^5 + 48a^{22}b^4 - 48a^{23}b^3 - 8a^{24}b^2)) / (a^6*(a^{20}b + a^{21} - a^{10}b^{11} - a^{11}b^{10} + 5a^{12}b^9 + 5a^{13}b^8 - 10a^{14}b^7 - 10a^{15}b^6 + 10a^{16}b^5 + 10a^{17}b^4 - 5a^{18}b^3 - 5a^{19}b^2))) * (Aa^2*1i + Ab^2*20i - B*a*b*8i)) / (2a^6) * (Aa^2*1i + Ab^2*20i - B*a*b*8i)) / (2a^6) - (((8*\tan(c/2 + (d*x)/2)*(800A^2*a*b^{17} - 800A^2*b^{18} - A^2*a^{18} + 2A^2*a^{17}b + 4720A^2*a^2*b^{16} - 4720A^2*a^3*b^{15} - 11522A^2*a^4*b^{14} + 11522A^2*a^5*b^{13} + 14837A^2*a^6*b^{12} - 14812A^2*a^7*b^{11} - 10385A^2*a^8*b^{10} + 10430A^2*a^9*b^9 + 3325A^2*a^{10}b^8 - 3640A^2*a^{11}b^7 + 45A^2*a^{12}b^6 + 350A^2*a^{13}b^5 - 209A^2*a^{14}b^4 + 68A^2*a^{15}b^3 - 35A^2*a^{16}b^2 - 128B^2*a^2*b^{16} + 128B^2*a^3*b^{15} + 768B^2*a^4*b^{14} - 768B^2*a^5*b^{13} - 1920B^2*a^6*b^{12} + 1920B^2*a^7*b^{11} + 2600B^2*a^8*b^{10} - 2560B^2*a^9*b^9 - 2025B^2*a^{10}b^8 + 1920B^2*a^{11}b^7 + 824B^2*a^{12}b^6 - 768B^2*a^{13}b^5 - 80B^2*a^{14}b^4 + 128B^2*a^{15}b^3 - 64B^2*a^{16}b^2 + 640A*B*a*b^{17} + 16A*B*a^{17}b - 640A*B*a^2*b^{16} - 3808A*B*a^3*b^{15} + 3808A*B*a^4*b^{14} + 9408A*B*a^5*b^{13} - 9408A*B*a^6*b^{12} - 12430A*B*a^7*b^{11} + 12320A*B*a^8*b^{10} + 9200A*B*a^9*b^9 - 8960A*B*a^{10}b^8 - 3360A*B*a^{11}b^7 + 3360A*B*a^{12}b^6 + 144A*B*a^{13}b^5 - 448A*B*a^{14}b^4 + 240A*B*a^{15}b^3 - 32A*B*a^{16}b^2)) / (a^{20}b + a^{21} - a^{10}b^{11} - a^{11}b^{10} + 5a^{12}b^9 + 5a^{13}b^8 - 10a^{14}b^7 - 10a^{15}b^6 + 10a^{16}b^5 + 10a^{17}b^4 - 5a^{18}b^3 - 5a^{19}b^2) + (((4*(4Aa^{27} - 80Aa^{12}b^{15} + 40Aa^{13}b^{14} + 516Aa^{14}b^{13} - 248Aa^{15}b^{12} - 1404Aa^{16}b^{11} + 640Aa^{17}b^{10} + 2076Aa^{18}b^9 - 896Aa^{19}b^8 - 1764Aa^{20}b^7 + 724Aa^{21}b^6 + 816Aa^{22}b^5 - 316Aa^{23}b^4 - 160Aa^{24}b^3 + 52Aa^{25}b^2 + 32Ba^{13}b^{14} - 16Ba^{14}b^{13} - 208Ba^{15}b^{12} + 100Ba^{16}b^{11} + 572Ba^{17}b^{10} - 252Ba^{18}b^9 - 868Ba^{19}b^8 + 348Ba^{20}b^7 + 772Ba^{21}b^6 - 292Ba^{22}b^5 - 380Ba^{23}b^4 + 144Ba^{24}b^3 + 80Ba^{25}b^2 - 32Ba^{26}b)) / (a^{25}b + a^{26} - a^{15}b^{11} - a^{16}b^{10} + 5a^{17}b^9 + 5a^{18}b^8 - 10a^{19}b^7 - 10a^{20}b^6 + 10a^{21}b^5 + 10a^{22}b^4 - 5a^{23}b^3 - 5a^{24}b^2) + (4*\tan(c/2 + (d*x)/2)*(Aa^2*1i + Ab^2*20i - B*a*b*8i)*(8a^{25}b - 8a^{12}b^{14} + 8a^{13}b^{13} + 48a^{14}b^{12} - 48a^{15}b^{11} - 120a^{16}b^{10} + 120a^{17}b^9 + 160a^{18}b^8 - 160a^{19}b^7 - 120a^{20}b^6 + 120a^{21}b^5 + 48a^{22}b^4 - 48a^{23}b^3 - 8a^{24}b^2)) / (a^6*(a^{20}b + a^{21} - a^{10}b^{11} - a^{11}b^{10} + 5a^{12}b^9 + 5
\end{aligned}$$

$$\begin{aligned}
& a^{13}b^8 - 10a^{14}b^7 - 10a^{15}b^6 + 10a^{16}b^5 + 10a^{17}b^4 - 5a^{18}b^3 - 5a^{19}b^2)) \cdot (Aa^2 \cdot 1i + Ab^2 \cdot 20i - B \cdot a \cdot b \cdot 8i) / (2a^6) \cdot (Aa^2 \cdot 1i + Ab^2 \cdot 20i - B \cdot a \cdot b \cdot 8i) / (2a^6)) \cdot (Aa^2 \cdot 1i + Ab^2 \cdot 20i - B \cdot a \cdot b \cdot 8i) \cdot 1i) / (a^6 \cdot d) - (b^2 \cdot \operatorname{atan}((b^2 \cdot ((8 \cdot \tan(c/2 + (d \cdot x)/2) \cdot (800A^2a^2b^{17} - 800A^2b^{18} - A^2a^{18} + 2A^2a^{17}b + 4720A^2a^2b^{16} - 4720A^2a^3b^{15} - 11522A^2a^4b^{14} + 11522A^2a^5b^{13} + 14837A^2a^6b^{12} - 14812A^2a^7b^{11} - 10385A^2a^8b^{10} + 10430A^2a^9b^9 + 3325A^2a^{10}b^8 - 3640A^2a^{11}b^7 + 45A^2a^{12}b^6 + 350A^2a^{13}b^5 - 209A^2a^{14}b^4 + 68A^2a^{15}b^3 - 35A^2a^{16}b^2 - 128B^2a^2b^{16} + 128B^2a^3b^{15} + 768B^2a^4b^{14} - 768B^2a^5b^{13} - 1920B^2a^6b^{12} + 1920B^2a^7b^{11} + 2600B^2a^8b^{10} - 2560B^2a^9b^9 - 2025B^2a^{10}b^8 + 1920B^2a^{11}b^7 + 824B^2a^{12}b^6 - 768B^2a^{13}b^5 - 80B^2a^{14}b^4 + 128B^2a^{15}b^3 - 64B^2a^{16}b^2 + 640A \cdot B \cdot a \cdot b^{17} + 16A \cdot B \cdot a^{17}b - 640A \cdot B \cdot a^2b^{16} - 3808A \cdot B \cdot a^3b^{15} + 3808A \cdot B \cdot a^4b^{14} + 9408A \cdot B \cdot a^5b^{13} - 9408A \cdot B \cdot a^6b^{12} - 12430A \cdot B \cdot a^7b^{11} + 12320A \cdot B \cdot a^8b^{10} + 9200A \cdot B \cdot a^9b^9 - 8960A \cdot B \cdot a^{10}b^8 - 3360A \cdot B \cdot a^{11}b^7 + 3360A \cdot B \cdot a^{12}b^6 + 144A \cdot B \cdot a^{13}b^5 - 448A \cdot B \cdot a^{14}b^4 + 240A \cdot B \cdot a^{15}b^3 - 32A \cdot B \cdot a^{16}b^2)) / (a^{20}b + a^{21} - a^{10}b^{11} - a^{11}b^{10} + 5a^{12}b^9 + 5a^{13}b^8 - 10a^{14}b^7 - 10a^{15}b^6 + 10a^{16}b^5 + 10a^{17}b^4 - 5a^{18}b^3 - 5a^{19}b^2) - (b^2 \cdot ((a + b)^7 \cdot (a - b)^7)^{(1/2)}) \cdot ((4 \cdot (4Aa^{27} - 80Aa^{12}b^{15} + 40Aa^{13}b^{14} + 516Aa^{14}b^{13} - 248Aa^{15}b^{12} - 1404Aa^{16}b^{11} + 640Aa^{17}b^{10} + 2076Aa^{18}b^9 - 896Aa^{19}b^8 - 1764Aa^{20}b^7 + 724Aa^{21}b^6 + 816Aa^{22}b^5 - 316Aa^{23}b^4 - 160Aa^{24}b^3 + 52Aa^{25}b^2 + 32B \cdot a^{13}b^{14} - 16B \cdot a^{14}b^{13} - 208B \cdot a^{15}b^{12} + 100B \cdot a^{16}b^{11} + 572B \cdot a^{17}b^{10} - 252B \cdot a^{18}b^9 - 868B \cdot a^{19}b^8 + 348B \cdot a^{20}b^7 + 772B \cdot a^{21}b^6 - 292B \cdot a^{22}b^5 - 380B \cdot a^{23}b^4 + 144B \cdot a^{24}b^3 + 80B \cdot a^{25}b^2 - 32B \cdot a^{26}b)) / (a^{25}b + a^{26} - a^{15}b^{11} - a^{16}b^{10} + 5a^{17}b^9 + 5a^{18}b^8 - 10a^{19}b^7 - 10a^{20}b^6 + 10a^{21}b^5 + 10a^{22}b^4 - 5a^{23}b^3 - 5a^{24}b^2) - (4b^2 \cdot \tan(c/2 + (d \cdot x)/2) \cdot ((a + b)^7 \cdot (a - b)^7)^{(1/2)} \cdot (20A \cdot b^7 + 20B \cdot a^7 - 69A \cdot a^2b^5 + 84A \cdot a^4b^3 + 28B \cdot a^3b^4 - 35B \cdot a^5b^2 - 40A \cdot a^6b - 8B \cdot a \cdot b^6) \cdot (8a^{25}b - 8a^{12}b^{14} + 8a^{13}b^{13} + 48a^{14}b^{12} - 48a^{15}b^{11} - 120a^{16}b^{10} + 120a^{17}b^9 + 160a^{18}b^8 - 160a^{19}b^7 - 120a^{20}b^6 + 120a^{21}b^5 + 48a^{22}b^4 - 48a^{23}b^3 - 8a^{24}b^2)) / ((a^{20} - a^6b^{14} + 7a^8b^{12} - 21a^{10}b^{10} + 35a^{12}b^8 - 35a^{14}b^6 + 21a^{16}b^4 - 7a^{18}b^2) \cdot (a^{20}b + a^{21} - a^{10}b^{11} - a^{11}b^{10} + 5a^{12}b^9 + 5a^{13}b^8 - 10a^{14}b^7 - 10a^{15}b^6 + 10a^{16}b^5 + 10a^{17}b^4 - 5a^{18}b^3 - 5a^{19}b^2)) \cdot (20A \cdot b^7 + 20B \cdot a^7 - 69A \cdot a^2b^5 + 84A \cdot a^4b^3 + 28B \cdot a^3b^4 - 35B \cdot a^5b^2 - 40A \cdot a^6b - 8B \cdot a \cdot b^6)) / (2 \cdot (a^{20} - a^6b^{14} + 7a^8b^{12} - 21a^{10}b^{10} + 35a^{12}b^8 - 35a^{14}b^6 + 21a^{16}b^4 - 7a^{18}b^2)) \cdot ((a + b)^7 \cdot (a - b)^7)^{(1/2)} \cdot (20A \cdot b^7 + 20B \cdot a^7 - 69A \cdot a^2b^5 + 84A \cdot a^4b^3 + 28B \cdot a^3b^4 - 35B \cdot a^5b^2 - 40A \cdot a^6b - 8B \cdot a \cdot b^6) \cdot 1i) / (2 \cdot (a^{20} - a^6b^{14} + 7a^8b^{12} - 21a^{10}b^{10} + 35a^{12}b^8 - 35a^{14}b^6 + 21a^{16}b^4 - 7a^{18}b^2)) + (b^2 \cdot ((8 \cdot \tan(c/2 + (d \cdot x)/2) \cdot (800A^2a^2b^{17} - 800A^2b^{18} - A^2a^{18} + 2A^2a^{17}b + 4720A^2a^2b^{16} - 4720A^2a^3b^{15} - 11522A^2a^4b^{14} + 11522A^2a^5b^{13} + 14837A^2a^6b^{12} - 14812A^2a^7b^{11} - 10385A^2a^8b^
\end{aligned}$$

$$\begin{aligned}
& 10 + 10430A^2a^9b^9 + 3325A^2a^{10}b^8 - 3640A^2a^{11}b^7 + 45A^2a^{12}b^6 + 350A^2a^{13}b^5 - 209A^2a^{14}b^4 + 68A^2a^{15}b^3 - 35A^2a^{16}b^2 - 128B^2a^2b^{16} + 128B^2a^3b^{15} + 768B^2a^4b^{14} - 768B^2a^5b^{13} - 1920B^2a^6b^{12} + 1920B^2a^7b^{11} + 2600B^2a^8b^{10} - 2560B^2a^9b^9 - 2025B^2a^{10}b^8 + 1920B^2a^{11}b^7 + 824B^2a^{12}b^6 - 768B^2a^{13}b^5 - 80B^2a^{14}b^4 + 128B^2a^{15}b^3 - 64B^2a^{16}b^2 + 640A^*B^*a^*b^{17} + 16A^*B^*a^{17}b - 640A^*B^*a^2b^{16} - 3808A^*B^*a^3b^{15} + 3808A^*B^*a^4b^{14} + 9408A^*B^*a^5b^{13} - 9408A^*B^*a^6b^{12} - 12430A^*B^*a^7b^{11} + 12320A^*B^*a^8b^{10} + 9200A^*B^*a^9b^9 - 8960A^*B^*a^{10}b^8 - 3360A^*B^*a^{11}b^7 + 3360A^*B^*a^{12}b^6 + 144A^*B^*a^{13}b^5 - 448A^*B^*a^{14}b^4 + 240A^*B^*a^{15}b^3 - 32A^*B^*a^{16}b^2)/(a^{20}b + a^{21} - a^{10}b^{11} - a^{11}b^{10} + 5a^{12}b^9 + 5a^{13}b^8 - 10a^{14}b^7 - 10a^{15}b^6 + 10a^{16}b^5 + 10a^{17}b^4 - 5a^{18}b^3 - 5a^{19}b^2) + (b^2((a + b)^7(a - b)^7)^{(1/2)}((4*(4A^*a^{27} - 80A^*a^{12}b^{15} + 40A^*a^{13}b^{14} + 516A^*a^{14}b^{13} - 248A^*a^{15}b^{12} - 1404A^*a^{16}b^{11} + 640A^*a^{17}b^{10} + 2076A^*a^{18}b^9 - 896A^*a^{19}b^8 - 1764A^*a^{20}b^7 + 724A^*a^{21}b^6 + 816A^*a^{22}b^5 - 316A^*a^{23}b^4 - 160A^*a^{24}b^3 + 52A^*a^{25}b^2 + 32B^*a^{13}b^{14} - 16B^*a^{14}b^{13} - 208B^*a^{15}b^{12} + 100B^*a^{16}b^{11} + 572B^*a^{17}b^{10} - 252B^*a^{18}b^9 - 868B^*a^{19}b^8 + 348B^*a^{20}b^7 + 772B^*a^{21}b^6 - 292B^*a^{22}b^5 - 380B^*a^{23}b^4 + 144B^*a^{24}b^3 + 80B^*a^{25}b^2 - 32B^*a^{26}b)))/(a^{25}b + a^{26} - a^{15}b^{11} - a^{16}b^{10} + 5a^{17}b^9 + 5a^{18}b^8 - 10a^{19}b^7 - 10a^{20}b^6 + 10a^{21}b^5 + 10a^{22}b^4 - 5a^{23}b^3 - 5a^{24}b^2) + (4b^2*tan(c/2 + (d*x)/2)*((a + b)^7(a - b)^7)^{(1/2)}*(20A^*b^7 + 20B^*a^7 - 69A^*a^2b^5 + 84A^*a^4b^3 + 28B^*a^3b^4 - 35B^*a^5b^2 - 40A^*a^6b - 8B^*a^*b^6)*(8a^{25}b - 8a^{12}b^{14} + 8a^{13}b^{13} + 48a^{14}b^{12} - 48a^{15}b^{11} - 120a^{16}b^{10} + 120a^{17}b^9 + 160a^{18}b^8 - 160a^{19}b^7 - 120a^{20}b^6 + 120a^{21}b^5 + 48a^{22}b^4 - 48a^{23}b^3 - 8a^{24}b^2))/((a^{20} - a^6b^{14} + 7a^8b^{12} - 21a^{10}b^{10} + 35a^{12}b^8 - 35a^{14}b^6 + 21a^{16}b^4 - 7a^{18}b^2))*(a^{20}b + a^{21} - a^{10}b^{11} - a^{11}b^{10} + 5a^{12}b^9 + 5a^{13}b^8 - 10a^{14}b^7 - 10a^{15}b^6 + 10a^{16}b^5 + 10a^{17}b^4 - 5a^{18}b^3 - 5a^{19}b^2))*((20A^*b^7 + 20B^*a^7 - 69A^*a^2b^5 + 84A^*a^4b^3 + 28B^*a^3b^4 - 35B^*a^5b^2 - 40A^*a^6b - 8B^*a^*b^6))/(2*(a^{20} - a^6b^{14} + 7a^8b^{12} - 21a^{10}b^{10} + 35a^{12}b^8 - 35a^{14}b^6 + 21a^{16}b^4 - 7a^{18}b^2))*((a + b)^7(a - b)^7)^{(1/2)}*(20A^*b^7 + 20B^*a^7 - 69A^*a^2b^5 + 84A^*a^4b^3 + 28B^*a^3b^4 - 35B^*a^5b^2 - 40A^*a^6b - 8B^*a^*b^6)*1i)/((2*(a^{20} - a^6b^{14} + 7a^8b^{12} - 21a^{10}b^{10} + 35a^{12}b^8 - 35a^{14}b^6 + 21a^{16}b^4 - 7a^{18}b^2)))/((8*(8000A^3a^3b^{19} - 4000A^3a^*b^{18} - 50800A^3a^2b^{17} + 24400A^3a^3b^{16} + 135260A^3a^4b^{15} - 62030A^3a^5b^{14} - 193689A^3a^6b^{13} + 82337A^3a^7b^{12} + 155991A^3a^8b^{11} - 57345A^3a^9b^{10} - 64479A^3a^{10}b^9 + 16999A^3a^{11}b^8 + 8281A^3a^{12}b^7 + 204A^3a^{13}b^6 + 1396A^3a^{14}b^5 - 40A^3a^{15}b^4 + 40A^3a^{16}b^3 - 512B^3a^3b^{16} + 256B^3a^4b^{15} + 3328B^3a^5b^{14} - 1600B^3a^6b^{13} - 9152B^3a^7b^{12} + 4352B^3a^8b^{11} + 13888B^3a^9b^{10} - 6408B^3a^{10}b^9 - 12352B^3a^{11}b^8 + 5120B^3a^{12}b^7 + 6080B^3a^{13}b^6 - 1920B^3a^{14}b^5 - 1280B^3a^{15}b^4 - 9600A^2B^*a^*b^{18} + 3840A^*B^2a^2b^{17} - 1920A^*B^2a^3b^{16} - 24768A^*B^2a^4b^{15} + 1190
\end{aligned}$$

$$\begin{aligned}
& 4*A*B^2*a^5*b^14 + 67392*A*B^2*a^6*b^13 - 31680*A*B^2*a^7*b^12 - 100368*A*B^2*a^8*b^11 + 45148*A*B^2*a^9*b^10 + 86512*A*B^2*a^10*b^9 - 34567*A*B^2*a^11*b^8 - 40368*A*B^2*a^12*b^7 + 11960*A*B^2*a^13*b^6 + 7440*A*B^2*a^14*b^5 + 80*A*B^2*a^15*b^4 + 320*A*B^2*a^16*b^3 + 4800*A^2*B*a^2*b^17 + 61440*A^2*B*a^3*b^16 - 29520*A^2*B*a^4*b^15 - 165384*A^2*B*a^5*b^14 + 76812*A^2*B*a^6*b^13 + 241596*A^2*B*a^7*b^12 - 105755*A^2*B*a^8*b^11 - 201479*A^2*B*a^9*b^10 + 77359*A^2*B*a^10*b^9 + 88721*A^2*B*a^11*b^8 - 24711*A^2*B*a^12*b^7 - 13929*A^2*B*a^13*b^6 - 255*A^2*B*a^14*b^5 - 1345*A^2*B*a^15*b^4 + 20*A^2*B*a^16*b^3 - 20*A^2*B*a^17*b^2)/(a^25*b + a^26 - a^15*b^11 - a^16*b^10 + 5*a^17*b^9 + 5*a^18*b^8 - 10*a^19*b^7 - 10*a^20*b^6 + 10*a^21*b^5 + 10*a^22*b^4 - 5*a^23*b^3 - 5*a^24*b^2) + (b^2*((8*tan(c/2 + (d*x)/2))*(800*A^2*a*b^17 - 800*A^2*b^18 - A^2*a^18 + 2*A^2*a^17*b + 4720*A^2*a^2*b^16 - 4720*A^2*a^3*b^15 - 11522*A^2*a^4*b^14 + 11522*A^2*a^5*b^13 + 14837*A^2*a^6*b^12 - 14812*A^2*a^7*b^11 - 10385*A^2*a^8*b^10 + 10430*A^2*a^9*b^9 + 3325*A^2*a^10*b^8 - 3640*A^2*a^11*b^7 + 45*A^2*a^12*b^6 + 350*A^2*a^13*b^5 - 209*A^2*a^14*b^4 + 68*A^2*a^15*b^3 - 35*A^2*a^16*b^2 - 128*B^2*a^2*b^16 + 128*B^2*a^3*b^15 + 768*B^2*a^4*b^14 - 768*B^2*a^5*b^13 - 1920*B^2*a^6*b^12 + 1920*B^2*a^7*b^11 + 2600*B^2*a^8*b^10 - 2560*B^2*a^9*b^9 - 2025*B^2*a^10*b^8 + 1920*B^2*a^11*b^7 + 824*B^2*a^12*b^6 - 768*B^2*a^13*b^5 - 80*B^2*a^14*b^4 + 128*B^2*a^15*b^3 - 64*B^2*a^16*b^2 + 640*A*B*a*b^17 + 16*A*B*a^17*b - 640*A*B*a^2*b^16 - 3808*A*B*a^3*b^15 + 3808*A*B*a^4*b^14 + 9408*A*B*a^5*b^13 - 9408*A*B*a^6*b^12 - 12430*A*B*a^7*b^11 + 12320*A*B*a^8*b^10 + 9200*A*B*a^9*b^9 - 8960*A*B*a^10*b^8 - 3360*A*B*a^11*b^7 + 3360*A*B*a^12*b^6 + 144*A*B*a^13*b^5 - 448*A*B*a^14*b^4 + 240*A*B*a^15*b^3 - 32*A*B*a^16*b^2))/(a^20*b + a^21 - a^10*b^11 - a^11*b^10 + 5*a^12*b^9 + 5*a^13*b^8 - 10*a^14*b^7 - 10*a^15*b^6 + 10*a^16*b^5 + 10*a^17*b^4 - 5*a^18*b^3 - 5*a^19*b^2) - (b^2*((a + b)^7*(a - b)^7)^(1/2))*((4*(4*A*a^27 - 80*A*a^12*b^15 + 40*A*a^13*b^14 + 516*A*a^14*b^13 - 248*A*a^15*b^12 - 1404*A*a^16*b^11 + 640*A*a^17*b^10 + 2076*A*a^18*b^9 - 896*A*a^19*b^8 - 1764*A*a^20*b^7 + 724*A*a^21*b^6 + 816*A*a^22*b^5 - 316*A*a^23*b^4 - 160*A*a^24*b^3 + 52*A*a^25*b^2 + 32*B*a^13*b^14 - 16*B*a^14*b^13 - 208*B*a^15*b^12 + 100*B*a^16*b^11 + 572*B*a^17*b^10 - 252*B*a^18*b^9 - 868*B*a^19*b^8 + 348*B*a^20*b^7 + 772*B*a^21*b^6 - 292*B*a^22*b^5 - 380*B*a^23*b^4 + 144*B*a^24*b^3 + 80*B*a^25*b^2 - 32*B*a^26*b))/(a^25*b + a^26 - a^15*b^11 - a^16*b^10 + 5*a^17*b^9 + 5*a^18*b^8 - 10*a^19*b^7 - 10*a^20*b^6 + 10*a^21*b^5 + 10*a^22*b^4 - 5*a^23*b^3 - 5*a^24*b^2) - (4*b^2*tan(c/2 + (d*x)/2))*((a + b)^7*(a - b)^7)^(1/2))*((20*A*b^7 + 20*B*a^7 - 69*A*a^2*b^5 + 84*A*a^4*b^3 + 28*B*a^3*b^4 - 35*B*a^5*b^2 - 40*A*a^6*b - 8*B*a*b^6)*(8*a^25*b - 8*a^12*b^14 + 8*a^13*b^13 + 48*a^14*b^12 - 48*a^15*b^11 - 120*a^16*b^10 + 120*a^17*b^9 + 160*a^18*b^8 - 160*a^19*b^7 - 120*a^20*b^6 + 120*a^21*b^5 + 48*a^22*b^4 - 48*a^23*b^3 - 8*a^24*b^2))/((a^20 - a^6*b^14 + 7*a^8*b^12 - 21*a^10*b^10 + 35*a^12*b^8 - 35*a^14*b^6 + 21*a^16*b^4 - 7*a^18*b^2))*(a^20*b + a^21 - a^10*b^11 - a^11*b^10 + 5*a^12*b^9 + 5*a^13*b^8 - 10*a^14*b^7 - 10*a^15*b^6 + 10*a^16*b^5 + 10*a^17*b^4 - 5*a^18*b^3 - 5*a^19*b^2))*((20*A*b^7 + 20*B*a^7 - 69*A*a^2*b^5 + 84*A*a^4*b^3 + 28*B*a^3*b^4 - 35*B*a^5*b^2 - 40*A*a^6*b - 8*B*a*b^6))/(2*(a^20 - a^6*b^14 + 7*a^8*b^12 - 21*a^10*
\end{aligned}$$

$$\begin{aligned}
& b^{10} + 35a^{12}b^8 - 35a^{14}b^6 + 21a^{16}b^4 - 7a^{18}b^2) \cdot ((a + b)^7 \cdot (a - b)^7)^{(1/2)} \cdot (20A^2b^7 + 20B^2a^7 - 69A^2a^2b^5 + 84A^2a^4b^3 + 28B^2a^3b^4 - 35B^2a^5b^2 - 40A^2a^6b - 8B^2a^2b^6) / (2(a^{20} - a^6b^{14} + 7a^8b^{12} - 21a^{10}b^{10} + 35a^{12}b^8 - 35a^{14}b^6 + 21a^{16}b^4 - 7a^{18}b^2)) - (b^2 \cdot ((8 \cdot \tan(c/2 + (d \cdot x)/2) \cdot (800A^2a^2b^{17} - 800A^2b^{18} - A^2a^{18} + 2A^2a^{17}b + 4720A^2a^2b^{16} - 4720A^2a^3b^{15} - 11522A^2a^4b^{14} + 11522A^2a^5b^{13} + 14837A^2a^6b^{12} - 14812A^2a^7b^{11} - 10385A^2a^8b^{10} + 10430A^2a^9b^9 + 3325A^2a^{10}b^8 - 3640A^2a^{11}b^7 + 45A^2a^{12}b^6 + 350A^2a^{13}b^5 - 209A^2a^{14}b^4 + 68A^2a^{15}b^3 - 35A^2a^{16}b^2 - 128B^2a^2b^{16} + 128B^2a^3b^{15} + 768B^2a^4b^{14} - 768B^2a^5b^{13} - 1920B^2a^6b^{12} + 1920B^2a^7b^{11} + 2600B^2a^8b^{10} - 2560B^2a^9b^9 - 2025B^2a^{10}b^8 + 1920B^2a^{11}b^7 + 824B^2a^{12}b^6 - 768B^2a^{13}b^5 - 80B^2a^{14}b^4 + 128B^2a^{15}b^3 - 64B^2a^{16}b^2 + 640A \cdot B \cdot a \cdot b^{17} + 16A \cdot B \cdot a^{17}b - 640A \cdot B \cdot a^2b^{16} - 3808A \cdot B \cdot a^3b^{15} + 3808A \cdot B \cdot a^4b^{14} + 9408A \cdot B \cdot a^5b^{13} - 9408A \cdot B \cdot a^6b^{12} - 12430A \cdot B \cdot a^7b^{11} + 12320A \cdot B \cdot a^8b^{10} + 9200A \cdot B \cdot a^9b^9 - 8960A \cdot B \cdot a^{10}b^8 - 3360A \cdot B \cdot a^{11}b^7 + 3360A \cdot B \cdot a^{12}b^6 + 144A \cdot B \cdot a^{13}b^5 - 448A \cdot B \cdot a^{14}b^4 + 240A \cdot B \cdot a^{15}b^3 - 32A \cdot B \cdot a^{16}b^2)) / (a^{20}b + a^{21} - a^{10}b^{11} - a^{11}b^{10} + 5a^{12}b^9 + 5a^{13}b^8 - 10a^{14}b^7 - 10a^{15}b^6 + 10a^{16}b^5 + 10a^{17}b^4 - 5a^{18}b^3 - 5a^{19}b^2) + (b^2 \cdot ((a + b)^7 \cdot (a - b)^7)^{(1/2)} \cdot ((4 \cdot (4A^2a^{27} - 80A^2a^{12}b^{15} + 40A^2a^{13}b^{14} + 516A^2a^{14}b^{13} - 248A^2a^{15}b^{12} - 1404A^2a^{16}b^{11} + 640A^2a^{17}b^{10} + 2076A^2a^{18}b^9 - 896A^2a^{19}b^8 - 1764A^2a^{20}b^7 + 724A^2a^{21}b^6 + 816A^2a^{22}b^5 - 316A^2a^{23}b^4 - 160A^2a^{24}b^3 + 52A^2a^{25}b^2 + 32B^2a^{13}b^{14} - 16B^2a^{14}b^{13} - 208B^2a^{15}b^{12} + 100B^2a^{16}b^{11} + 572B^2a^{17}b^{10} - 252B^2a^{18}b^9 - 868B^2a^{19}b^8 + 348B^2a^{20}b^7 + 772B^2a^{21}b^6 - 292B^2a^{22}b^5 - 380B^2a^{23}b^4 + 144B^2a^{24}b^3 + 80B^2a^{25}b^2 - 32B^2a^{26}b)) / (a^{25}b + a^{26} - a^{15}b^{11} - a^{16}b^{10} + 5a^{17}b^9 + 5a^{18}b^8 - 10a^{19}b^7 - 10a^{20}b^6 + 10a^{21}b^5 + 10a^{22}b^4 - 5a^{23}b^3 - 5a^{24}b^2) + (4b^2 \cdot \tan(c/2 + (d \cdot x)/2) \cdot ((a + b)^7 \cdot (a - b)^7)^{(1/2)} \cdot (20A^2b^7 + 20B^2a^7 - 69A^2a^2b^5 + 84A^2a^4b^3 + 28B^2a^3b^4 - 35B^2a^5b^2 - 40A^2a^6b - 8B^2a^2b^6) \cdot (8a^{25}b - 8a^{12}b^{14} + 8a^{13}b^{13} + 48a^{14}b^{12} - 48a^{15}b^{11} - 120a^{16}b^{10} + 120a^{17}b^9 + 160a^{18}b^8 - 160a^{19}b^7 - 120a^{20}b^6 + 120a^{21}b^5 + 48a^{22}b^4 - 48a^{23}b^3 - 8a^{24}b^2)) / ((a^{20} - a^6b^{14} + 7a^8b^{12} - 21a^{10}b^{10} + 35a^{12}b^8 - 35a^{14}b^6 + 21a^{16}b^4 - 7a^{18}b^2) \cdot (a^{20}b + a^{21} - a^{10}b^{11} - a^{11}b^{10} + 5a^{12}b^9 + 5a^{13}b^8 - 10a^{14}b^7 - 10a^{15}b^6 + 10a^{16}b^5 + 10a^{17}b^4 - 5a^{18}b^3 - 5a^{19}b^2))) \cdot (20A^2b^7 + 20B^2a^7 - 69A^2a^2b^5 + 84A^2a^4b^3 + 28B^2a^3b^4 - 35B^2a^5b^2 - 40A^2a^6b - 8B^2a^2b^6) / (2(a^{20} - a^6b^{14} + 7a^8b^{12} - 21a^{10}b^{10} + 35a^{12}b^8 - 35a^{14}b^6 + 21a^{16}b^4 - 7a^{18}b^2))) \cdot ((a + b)^7 \cdot (a - b)^7)^{(1/2)} \cdot (20A^2b^7 + 20B^2a^7 - 69A^2a^2b^5 + 84A^2a^4b^3 + 28B^2a^3b^4 - 35B^2a^5b^2 - 40A^2a^6b - 8B^2a^2b^6) / (2(a^{20} - a^6b^{14} + 7a^8b^{12} - 21a^{10}b^{10} + 35a^{12}b^8 - 35a^{14}b^6 + 21a^{16}b^4 - 7a^{18}b^2))) \cdot ((a + b)^7 \cdot (a - b)^7)^{(1/2)} \cdot (20A^2b^7 + 20B^2a^7 - 69A^2a^2b^5 + 84A^2a^4b^3 + 28B^2a^3b^4 - 35B^2a^5b^2 - 40A^2a^6b - 8B^2a^2b^6) \cdot i) / (d \cdot (a^{20} - a^6b^{14} + 7a^8b^{12} - 21a^{10}b^{10} + 35a^{12}b^8 - 35a^{14}b^6 + 21a^{16}b^4 - 7a^{18}b^2))
\end{aligned}$$

$b^{12} - 21*a^{10}*b^{10} + 35*a^{12}*b^8 - 35*a^{14}*b^6 + 21*a^{16}*b^4 - 7*a^{18}*b^2)$
 $)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \cos^2(c + dx)}{(a + b \sec(c + dx))^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**4,x)

[Out] Integral((A + B*sec(c + d*x))*cos(c + d*x)**2/(a + b*sec(c + d*x))**4, x)

$$3.344 \quad \int \frac{\frac{bB}{a} + B \sec(c+dx)}{a+b \sec(c+dx)} dx$$

Optimal. Leaf size=61

$$\frac{2B\sqrt{a-b}\sqrt{a+b} \tanh^{-1}\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d} + \frac{bBx}{a^2}$$

[Out] $b*B*x/a^2 + 2*B*\operatorname{arctanh}((a-b)^{(1/2)}*\tan(1/2*d*x + 1/2*c)/(a+b)^{(1/2}))* (a-b)^{(1/2)}*(a+b)^{(1/2)}/a^2/d$

Rubi [A] time = 0.11, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3919, 3831, 2659, 208}

$$\frac{2B\sqrt{a-b}\sqrt{a+b} \tanh^{-1}\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d} + \frac{bBx}{a^2}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(b*B)/a + B*\operatorname{Sec}[c + d*x])/(a + b*\operatorname{Sec}[c + d*x]), x]$

[Out] $(b*B*x)/a^2 + (2*\operatorname{Sqrt}[a - b]*\operatorname{Sqrt}[a + b]*B*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a - b]*\operatorname{Tan}[(c + d*x)/2])/ \operatorname{Sqrt}[a + b]])/(a^2*d)$

Rule 208

$\operatorname{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[-(a/b), 2]*\operatorname{ArcTanh}[x/\operatorname{Rt}[-(a/b), 2]])/a, x] /; \operatorname{FreeQ}\{a, b\}, x] \ \&\& \operatorname{NegQ}[a/b]$

Rule 2659

$\operatorname{Int}[(a_ + (b_)*\sin[\operatorname{Pi}/2 + (c_ + (d_)*(x_))])^{-1}, x_Symbol] \rightarrow \operatorname{With}\{e = \operatorname{FreeFactors}[\operatorname{Tan}[(c + d*x)/2], x]\}, \operatorname{Dist}[(2*e)/d, \operatorname{Subst}[\operatorname{Int}[1/(a + b + (a - b)*e^2*x^2), x], x, \operatorname{Tan}[(c + d*x)/2]/e], x]] /; \operatorname{FreeQ}\{a, b, c, d\}, x] \ \&\& \operatorname{NeQ}[a^2 - b^2, 0]$

Rule 3831

$\operatorname{Int}[\operatorname{csc}[(e_ + (f_)*(x_))]/(\operatorname{csc}[(e_ + (f_)*(x_))]*(b_ + (a_))], x_Symbol] \rightarrow \operatorname{Dist}[1/b, \operatorname{Int}[1/(1 + (a*\sin[e + f*x])/b), x], x] /; \operatorname{FreeQ}\{a, b, e, f\}, x] \ \&\& \operatorname{NeQ}[a^2 - b^2, 0]$

Rule 3919

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))/(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{\frac{bB}{a} + B \sec(c + dx)}{a + b \sec(c + dx)} dx &= \frac{bBx}{a^2} - \frac{\left(-aB + \frac{b^2B}{a}\right) \int \frac{\sec(c+dx)}{a+b \sec(c+dx)} dx}{a} \\
 &= \frac{bBx}{a^2} - \frac{\left(-aB + \frac{b^2B}{a}\right) \int \frac{1}{1+\frac{a \cos(c+dx)}{b}} dx}{ab} \\
 &= \frac{bBx}{a^2} - \frac{\left(2\left(-aB + \frac{b^2B}{a}\right)\right) \text{Subst}\left(\int \frac{1}{1+\frac{a}{b}+(1-\frac{a}{b})x^2} dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{abd} \\
 &= \frac{bBx}{a^2} + \frac{2\sqrt{a-b} \sqrt{a+b} B \tanh^{-1}\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d}
 \end{aligned}$$

Mathematica [A] time = 0.17, size = 61, normalized size = 1.00

$$\frac{B\left(b(c+dx) - 2\sqrt{a^2-b^2} \tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)\right)}{a^2d}$$

Antiderivative was successfully verified.

[In] Integrate[((b*B)/a + B*Sec[c + d*x])/(a + b*Sec[c + d*x]),x]

[Out] (B*(b*(c + d*x) - 2*Sqrt[a^2 - b^2]*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]))/(a^2*d)

fricas [A] time = 0.49, size = 197, normalized size = 3.23

$$\left[\frac{2Bbdx + \sqrt{a^2-b^2} B \log\left(\frac{2ab \cos(dx+c) - (a^2-2b^2) \cos(dx+c)^2 + 2\sqrt{a^2-b^2}(b \cos(dx+c)+a) \sin(dx+c) + 2a^2-b^2}{a^2 \cos(dx+c)^2 + 2ab \cos(dx+c) + b^2}\right)}{2a^2d}, \frac{Bbdx + \sqrt{-a^2 + b^2}}{a^2d} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*B/a+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="fricas")

[Out] [1/2*(2*B*b*d*x + sqrt(a^2 - b^2)*B*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)))/(a^2*d), (B*b*d*x + sqrt(-a^2 + b^2)*B*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))))/(a^2*d)]

giac [B] time = 0.71, size = 187, normalized size = 3.07

$$2 \left[\frac{\left(\sqrt{-a^2+b^2} \left(\pi \left[\frac{dx+c}{2\pi} + \frac{1}{2} \right] + \arctan \left(\frac{\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{\frac{ab + \sqrt{a^2 b^2 + (a^2 + ab)(a^2 - ab)}}{a^2 - ab}} \right) \right) B|-a+b| \right)}{a^3 - a^2 b} + \frac{\left(\pi \left[\frac{dx+c}{2\pi} + \frac{1}{2} \right] + \arctan \left(\frac{\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{\frac{ab - \sqrt{a^2 b^2 + (a^2 + ab)(a^2 - ab)}}{a^2 - ab}} \right) \right) Bb}{a^2} \right] \frac{1}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*B/a+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="giac")

[Out] 2*(sqrt(-a^2 + b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2) + arctan(tan(1/2*d*x + 1/2*c)/sqrt(-(a*b + sqrt(a^2*b^2 + (a^2 + a*b)*(a^2 - a*b)))/(a^2 - a*b))))*B*abs(-a + b)/(a^3 - a^2*b) + (pi*floor(1/2*(d*x + c)/pi + 1/2) + arctan(tan(1/2*d*x + 1/2*c)/sqrt(-(a*b - sqrt(a^2*b^2 + (a^2 + a*b)*(a^2 - a*b)))/(a^2 - a*b))))*B*b/a^2)/d

maple [B] time = 1.03, size = 116, normalized size = 1.90

$$\frac{2B \operatorname{arctanh}\left(\frac{\tan\left(\frac{dx}{2} + \frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{d\sqrt{(a-b)(a+b)}} - \frac{2b^2 \operatorname{arctanh}\left(\frac{\tan\left(\frac{dx}{2} + \frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)B}{d a^2 \sqrt{(a-b)(a+b)}} + \frac{2 \operatorname{arctan}\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right)\right) Bb}{d a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*B/a+B*sec(d*x+c))/(a+b*sec(d*x+c)),x)

[Out] 2/d*B/((a-b)*(a+b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))-2/d*b^2/a^2/((a-b)*(a+b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*B+2/d/a^2*arctan(tan(1/2*d*x+1/2*c))*B*b

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*B/a+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for more details)Is 4*a^2-4*b^2 positive or negative?

mupad [B] time = 2.56, size = 91, normalized size = 1.49

$$\frac{2 B b \operatorname{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d x}{2}\right)}\right)}{a^2 d} + \frac{2 B \operatorname{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d x}{2}\right) \sqrt{a^2-b^2}}{\cos\left(\frac{c}{2}+\frac{d x}{2}\right)(a+b)}\right) \sqrt{a^2-b^2}}{a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((B/cos(c + d*x) + (B*b)/a)/(a + b/cos(c + d*x)),x)`

[Out] $(2*B*b*\operatorname{atan}(\sin(c/2 + (d*x)/2)/\cos(c/2 + (d*x)/2)))/(a^2*d) + (2*B*\operatorname{atanh}(\sin(c/2 + (d*x)/2)*(a^2 - b^2)^{(1/2)}/(\cos(c/2 + (d*x)/2)*(a + b)))/(a^2 - b^2)^{(1/2)})/(a^2*d)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{B \left(\int \frac{b}{a+b \sec(c+dx)} dx + \int \frac{a \sec(c+dx)}{a+b \sec(c+dx)} dx \right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*B/a+B*sec(d*x+c))/(a+b*sec(d*x+c)),x)`

[Out] $B*(\operatorname{Integral}(b/(a + b*\sec(c + d*x)), x) + \operatorname{Integral}(a*\sec(c + d*x)/(a + b*\sec(c + d*x)), x))/a$

$$3.345 \quad \int \frac{\frac{aB}{b} + B \sec(c+dx)}{a+b \sec(c+dx)} dx$$

Optimal. Leaf size=6

$$\frac{Bx}{b}$$

[Out] B*x/b

Rubi [A] time = 0.00, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {21, 8}

$$\frac{Bx}{b}$$

Antiderivative was successfully verified.

[In] Int[((a*B)/b + B*Sec[c + d*x])/(a + b*Sec[c + d*x]),x]

[Out] (B*x)/b

Rule 8

Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]

Rule 21

Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :> Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplerQ[c + d*x, a + b*x])

Rubi steps

$$\int \frac{\frac{aB}{b} + B \sec(c + dx)}{a + b \sec(c + dx)} dx = \frac{B \int 1 dx}{b} = \frac{Bx}{b}$$

Mathematica [A] time = 0.00, size = 6, normalized size = 1.00

$$\frac{Bx}{b}$$

Antiderivative was successfully verified.

[In] Integrate[((a*B)/b + B*Sec[c + d*x])/(a + b*Sec[c + d*x]),x]

[Out] (B*x)/b

fricas [A] time = 0.40, size = 6, normalized size = 1.00

$$\frac{Bx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*B/b+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="fricas")

[Out] B*x/b

giac [B] time = 0.24, size = 13, normalized size = 2.17

$$\frac{(dx + c)B}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*B/b+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="giac")

[Out] (d*x + c)*B/(b*d)

maple [A] time = 0.04, size = 7, normalized size = 1.17

$$\frac{Bx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*B/b+B*sec(d*x+c))/(a+b*sec(d*x+c)),x)

[Out] B*x/b

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*B/b+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* h

elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see 'assume?' for more details) Is $4a^2-4b^2$ positive or negative?

mupad [B] time = 2.23, size = 6, normalized size = 1.00

$$\frac{Bx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((B/cos(c + d*x) + (B*a)/b)/(a + b/cos(c + d*x)),x)`

[Out] $(B*x)/b$

sympy [A] time = 5.35, size = 3, normalized size = 0.50

$$\frac{Bx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a*B/b+B*sec(d*x+c))/(a+b*sec(d*x+c)),x)`

[Out] $B*x/b$

$$3.346 \quad \int \frac{a+b \sec(c+dx)}{(b+a \sec(c+dx))^2} dx$$

Optimal. Leaf size=86

$$-\frac{2\sqrt{a-b}\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d} + \frac{ax}{b^2} - \frac{a \tan(c+dx)}{bd(a \sec(c+dx)+b)}$$

[Out] $a*x/b^2 - 2*\arctan((a-b)^{(1/2)}*\tan(1/2*d*x+1/2*c)/(a+b)^{(1/2}))* (a-b)^{(1/2)}*(a+b)^{(1/2)}/b^2/d - a*\tan(d*x+c)/b/d/(b+a*\sec(d*x+c))$

Rubi [A] time = 0.18, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3923, 3919, 3831, 2659, 205}

$$-\frac{2\sqrt{a-b}\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d} + \frac{ax}{b^2} - \frac{a \tan(c+dx)}{bd(a \sec(c+dx)+b)}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sec[c + d*x])/(b + a*Sec[c + d*x])^2, x]

[Out] $(a*x)/b^2 - (2*\text{Sqrt}[a - b]*\text{Sqrt}[a + b]*\text{ArcTan}[(\text{Sqrt}[a - b]*\text{Tan}[(c + d*x)/2])/\text{Sqrt}[a + b]])/(b^2*d) - (a*\text{Tan}[c + d*x])/(b*d*(b + a*\text{Sec}[c + d*x]))$

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 2659

Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 3831

Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 3919

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))/(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 3923

Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.)), x_Symbol] :> Simp[(b*(b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[c*(a^2 - b^2)*(m + 1) - (a*(b*c - a*d)*(m + 1))*Csc[e + f*x] + b*(b*c - a*d)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m]

Rubi steps

$$\begin{aligned}
 \int \frac{a + b \sec(c + dx)}{(b + a \sec(c + dx))^2} dx &= -\frac{a \tan(c + dx)}{bd(b + a \sec(c + dx))} + \frac{\int \frac{a(a^2 - b^2) + b(a^2 - b^2) \sec(c + dx)}{b + a \sec(c + dx)} dx}{b(a^2 - b^2)} \\
 &= \frac{ax}{b^2} - \frac{a \tan(c + dx)}{bd(b + a \sec(c + dx))} - \frac{(a^2 - b^2) \int \frac{\sec(c + dx)}{b + a \sec(c + dx)} dx}{b^2} \\
 &= \frac{ax}{b^2} - \frac{a \tan(c + dx)}{bd(b + a \sec(c + dx))} - \frac{(a^2 - b^2) \int \frac{1}{1 + \frac{b \cos(c + dx)}{a}} dx}{ab^2} \\
 &= \frac{ax}{b^2} - \frac{a \tan(c + dx)}{bd(b + a \sec(c + dx))} - \frac{(2(a^2 - b^2)) \text{Subst}\left(\int \frac{1}{1 + \frac{b}{a} + \left(1 - \frac{b}{a}\right)x^2} dx, x, \tan\left(\frac{1}{2}(c + dx)\right)\right)}{ab^2 d} \\
 &= \frac{ax}{b^2} - \frac{2\sqrt{a-b}\sqrt{a+b} \tan^{-1}\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c + dx)\right)}{\sqrt{a+b}}\right)}{b^2 d} - \frac{a \tan(c + dx)}{bd(b + a \sec(c + dx))}
 \end{aligned}$$

Mathematica [A] time = 0.38, size = 97, normalized size = 1.13

$$\frac{2\sqrt{b^2 - a^2} \tanh^{-1}\left(\frac{(b-a) \tan\left(\frac{1}{2}(c + dx)\right)}{\sqrt{b^2 - a^2}}\right) + \frac{a(ac + adx - b \sin(c + dx) + b(c + dx) \cos(c + dx))}{a + b \cos(c + dx)}}{b^2 d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sec[c + d*x])/(b + a*Sec[c + d*x])^2,x]

[Out] (2*sqrt[-a^2 + b^2]*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/sqrt[-a^2 + b^2]] + (a*(a*c + a*d*x + b*(c + d*x)*Cos[c + d*x] - b*Sin[c + d*x]))/(a + b*Cos[c + d*x]))/(b^2*d)

fricas [A] time = 0.48, size = 279, normalized size = 3.24

$$\frac{2 ab dx \cos(dx + c) + 2 a^2 dx - 2 ab \sin(dx + c) + \sqrt{-a^2 + b^2} (b \cos(dx + c) + a) \log\left(\frac{2 ab \cos(dx+c) + (2 a^2 - b^2) \cos(dx+c)}{b^2 \cos(dx+c)}\right)}{2 (b^3 d \cos(dx + c) + ab^2 d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))/(b+a*sec(d*x+c))^2,x, algorithm="fricas")

[Out] [1/2*(2*a*b*d*x*cos(d*x + c) + 2*a^2*d*x - 2*a*b*sin(d*x + c) + sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)))/(b^3*d*cos(d*x + c) + a*b^2*d), (a*b*d*x*cos(d*x + c) + a^2*d*x - a*b*sin(d*x + c) - sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c)))]/(b^3*d*cos(d*x + c) + a*b^2*d)]

giac [A] time = 0.77, size = 139, normalized size = 1.62

$$\frac{\frac{(dx+c)a}{b^2} - \frac{2a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{\left(a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - b \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + a + b\right)b}}{d} - \frac{2 \left(\pi \left\lfloor \frac{dx+c}{2\pi} + \frac{1}{2} \right\rfloor \operatorname{sgn}(2a-2b) + \arctan\left(\frac{a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - b \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{\sqrt{a^2 - b^2}}\right) \right) \sqrt{a^2 - b^2}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))/(b+a*sec(d*x+c))^2,x, algorithm="giac")

[Out] ((d*x + c)*a/b^2 - 2*a*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)*b) - 2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*sqrt(a^2 - b^2)/b^2)/d

maple [B] time = 0.80, size = 163, normalized size = 1.90

$$\frac{2a \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{db \left(a \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right) \right) - \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right) \right) b + a + b \right)} - \frac{2 \arctan\left(\frac{\tan\left(\frac{dx}{2} + \frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right) a^2}{d b^2 \sqrt{(a-b)(a+b)}} + \frac{2 \arctan\left(\frac{\tan\left(\frac{dx}{2} + \frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{d \sqrt{(a-b)(a+b)}} + \frac{2a}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(d*x+c))/(b+a*sec(d*x+c))^2,x)

[Out] -2/d/b*a*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)-2/d/b^2/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*a^2+2/d/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))+2/d*a/b^2*arctan(tan(1/2*d*x+1/2*c))

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))/(b+a*sec(d*x+c))^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for more details)Is 4*b^2-4*a^2 positive or negative?

mupad [B] time = 2.57, size = 444, normalized size = 5.16

$$\frac{2 \operatorname{atanh}\left(\frac{64 a^3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right) \sqrt{b^2 - a^2}}{64 a^4 - 128 a^3 b + 128 a b^3 - 64 b^4} - \frac{192 a^2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right) \sqrt{b^2 - a^2}}{128 a b^2 - 128 a^3 - 64 b^3 + \frac{64 a^4}{b}} + \frac{192 a \tan\left(\frac{c}{2} + \frac{dx}{2}\right) \sqrt{b^2 - a^2}}{128 a b - 64 b^2 - \frac{128 a^3}{b} + \frac{64 a^4}{b^2}} - \frac{64 b \tan\left(\frac{c}{2} + \frac{dx}{2}\right) \sqrt{b^2 - a^2}}{128 a b - 64 b^2 - \frac{128 a^3}{b} + \frac{64 a^4}{b^2}}\right) \sqrt{b^2 - a^2}}{b^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cos(c + d*x))/(b + a/cos(c + d*x))^2,x)

[Out] (2*atanh((64*a^3*tan(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))/(128*a*b^3 - 128*a^3*b + 64*a^4 - 64*b^4) - (192*a^2*tan(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))/(128*a*b^2 - 128*a^3 - 64*b^3 + (64*a^4)/b) + (192*a*tan(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))/(128*a*b - 64*b^2 - (128*a^3)/b + (64*a^4)/b^2) - (64*b*tan(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))/(128*a*b - 64*b^2 - (128*a^3)/b + (64*a^4)/b

$$\begin{aligned} &^2)) * (b^2 - a^2)^{(1/2)} / (b^2 * d) - (2 * a * \operatorname{atan}((64 * a^2 * \tan(c/2 + (d * x)/2))) / (64 \\ &* a * b - 64 * a^2 - (64 * a^3) / b + (64 * a^4) / b^2) + (64 * a^3 * \tan(c/2 + (d * x)/2)) / (6 \\ &4 * a * b^2 - 64 * a^2 * b - 64 * a^3 + (64 * a^4) / b) - (64 * a^4 * \tan(c/2 + (d * x)/2)) / (64 \\ &* a * b^3 - 64 * a^3 * b + 64 * a^4 - 64 * a^2 * b^2) - (64 * a * b * \tan(c/2 + (d * x)/2)) / (64 * \\ &a * b - 64 * a^2 - (64 * a^3) / b + (64 * a^4) / b^2)) / (b^2 * d) - (2 * a * \tan(c/2 + (d * x) / \\ &2)) / (b * d * (a + b + \tan(c/2 + (d * x) / 2)^2 * (a - b))) \end{aligned}$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \sec(c + dx)}{(a \sec(c + dx) + b)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))/(b+a*sec(d*x+c))**2,x)

[Out] Integral((a + b*sec(c + d*x))/(a*sec(c + d*x) + b)**2, x)

$$3.347 \quad \int \frac{3+\sec(c+dx)}{2-\sec(c+dx)} dx$$

Optimal. Leaf size=87

$$\frac{5 \log\left(\cos\left(\frac{1}{2}(c+dx)\right) - \sqrt{3} \sin\left(\frac{1}{2}(c+dx)\right)\right)}{2\sqrt{3}d} + \frac{5 \log\left(\sqrt{3} \sin\left(\frac{1}{2}(c+dx)\right) + \cos\left(\frac{1}{2}(c+dx)\right)\right)}{2\sqrt{3}d} + \frac{3x}{2}$$

[Out] $3/2*x - 5/6*\ln(\cos(1/2*d*x + 1/2*c) - \sin(1/2*d*x + 1/2*c)*3^{(1/2)})/d*3^{(1/2)} + 5/6*\ln(\cos(1/2*d*x + 1/2*c) + \sin(1/2*d*x + 1/2*c)*3^{(1/2)})/d*3^{(1/2)}$

Rubi [A] time = 0.07, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3919, 3831, 2659, 207}

$$\frac{5 \log\left(\cos\left(\frac{1}{2}(c+dx)\right) - \sqrt{3} \sin\left(\frac{1}{2}(c+dx)\right)\right)}{2\sqrt{3}d} + \frac{5 \log\left(\sqrt{3} \sin\left(\frac{1}{2}(c+dx)\right) + \cos\left(\frac{1}{2}(c+dx)\right)\right)}{2\sqrt{3}d} + \frac{3x}{2}$$

Antiderivative was successfully verified.

[In] Int[(3 + Sec[c + d*x])/(2 - Sec[c + d*x]),x]

[Out] $(3*x)/2 - (5*\text{Log}[\text{Cos}[(c + d*x)/2] - \text{Sqrt}[3]*\text{Sin}[(c + d*x)/2]])/(2*\text{Sqrt}[3]*d) + (5*\text{Log}[\text{Cos}[(c + d*x)/2] + \text{Sqrt}[3]*\text{Sin}[(c + d*x)/2]])/(2*\text{Sqrt}[3]*d)$

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 2659

Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 3831

Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 3919

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))/(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] := Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rubi steps

$$\begin{aligned} \int \frac{3 + \sec(c + dx)}{2 - \sec(c + dx)} dx &= \frac{3x}{2} + \frac{5}{2} \int \frac{\sec(c + dx)}{2 - \sec(c + dx)} dx \\ &= \frac{3x}{2} - \frac{5}{2} \int \frac{1}{1 - 2 \cos(c + dx)} dx \\ &= \frac{3x}{2} - \frac{5 \operatorname{Subst}\left(\int \frac{1}{-1+3x^2} dx, x, \tan\left(\frac{1}{2}(c + dx)\right)\right)}{d} \\ &= \frac{3x}{2} - \frac{5 \log\left(\cos\left(\frac{1}{2}(c + dx)\right) - \sqrt{3} \sin\left(\frac{1}{2}(c + dx)\right)\right)}{2\sqrt{3}d} + \frac{5 \log\left(\cos\left(\frac{1}{2}(c + dx)\right) + \sqrt{3} \sin\left(\frac{1}{2}(c + dx)\right)\right)}{2\sqrt{3}d} \end{aligned}$$

Mathematica [A] time = 0.10, size = 39, normalized size = 0.45

$$\frac{9(c + dx) + 10\sqrt{3} \tanh^{-1}\left(\sqrt{3} \tan\left(\frac{1}{2}(c + dx)\right)\right)}{6d}$$

Antiderivative was successfully verified.

[In] Integrate[(3 + Sec[c + d*x])/(2 - Sec[c + d*x]), x]

[Out] (9*(c + d*x) + 10*Sqrt[3]*ArcTanh[Sqrt[3]*Tan[(c + d*x)/2]])/(6*d)

fricas [A] time = 0.46, size = 84, normalized size = 0.97

$$\frac{18 dx + 5 \sqrt{3} \log\left(-\frac{2 \cos(dx+c)^2 + 2(\sqrt{3} \cos(dx+c) - 2\sqrt{3}) \sin(dx+c) + 4 \cos(dx+c) - 7}{4 \cos(dx+c)^2 - 4 \cos(dx+c) + 1}\right)}{12 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3+sec(d*x+c))/(2-sec(d*x+c)), x, algorithm="fricas")

[Out] 1/12*(18*d*x + 5*sqrt(3)*log(-(2*cos(d*x + c)^2 + 2*(sqrt(3)*cos(d*x + c) - 2*sqrt(3))*sin(d*x + c) + 4*cos(d*x + c) - 7)/(4*cos(d*x + c)^2 - 4*cos(d*x + c) + 1)))/d

giac [A] time = 0.32, size = 58, normalized size = 0.67

$$\frac{9 dx - 5 \sqrt{3} \log \left(\frac{\left| -2 \sqrt{3} + 6 \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) \right|}{\left| 2 \sqrt{3} + 6 \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) \right|} \right) + 9 c}{6 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3+sec(d*x+c))/(2-sec(d*x+c)),x, algorithm="giac")

[Out] 1/6*(9*d*x - 5*sqrt(3)*log(abs(-2*sqrt(3) + 6*tan(1/2*d*x + 1/2*c))/abs(2*sqrt(3) + 6*tan(1/2*d*x + 1/2*c))) + 9*c)/d

maple [A] time = 0.84, size = 39, normalized size = 0.45

$$\frac{5\sqrt{3} \operatorname{arctanh} \left(\tan \left(\frac{dx}{2} + \frac{c}{2} \right) \sqrt{3} \right)}{3d} + \frac{3 \operatorname{arctan} \left(\tan \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3+sec(d*x+c))/(2-sec(d*x+c)),x)

[Out] 5/3/d*3^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*3^(1/2))+3/d*arctan(tan(1/2*d*x+1/2*c))

maxima [A] time = 1.11, size = 80, normalized size = 0.92

$$\frac{5 \sqrt{3} \log \left(-\frac{\sqrt{3} - \frac{3 \sin(dx+c)}{\cos(dx+c)+1}}{\sqrt{3} + \frac{3 \sin(dx+c)}{\cos(dx+c)+1}} \right) - 18 \operatorname{arctan} \left(\frac{\sin(dx+c)}{\cos(dx+c)+1} \right)}{6 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3+sec(d*x+c))/(2-sec(d*x+c)),x, algorithm="maxima")

[Out] -1/6*(5*sqrt(3)*log(-(sqrt(3) - 3*sin(d*x + c)/(cos(d*x + c) + 1))/(sqrt(3) + 3*sin(d*x + c)/(cos(d*x + c) + 1))) - 18*arctan(sin(d*x + c)/(cos(d*x + c) + 1)))/d

mupad [B] time = 2.30, size = 26, normalized size = 0.30

$$\frac{3x}{2} + \frac{5 \sqrt{3} \operatorname{atanh} \left(\sqrt{3} \tan \left(\frac{c}{2} + \frac{dx}{2} \right) \right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-(1/cos(c + d*x) + 3)/(1/cos(c + d*x) - 2), x)`

[Out] $(3*x)/2 + (5*3^{(1/2)}*atanh(3^{(1/2)}*\tan(c/2 + (d*x)/2)))/(3*d)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sec(c + dx)}{\sec(c + dx) - 2} dx - \int \frac{3}{\sec(c + dx) - 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3+sec(d*x+c))/(2-sec(d*x+c)), x)`

[Out] $-\text{Integral}(\sec(c + d*x)/(\sec(c + d*x) - 2), x) - \text{Integral}(3/(\sec(c + d*x) - 2), x)$

$$3.348 \quad \int \sec^4(c+dx) \sqrt{a + b \sec(c + dx)} (A+B \sec(c+dx)) dx$$

Optimal. Leaf size=485

$$\frac{2(-6a^2B + 9aAb + 49b^2B) \tan(c + dx) \sec(c + dx) \sqrt{a + b \sec(c + dx)}}{315b^2d} - \frac{2(-8a^3B + 12a^2Ab - 13ab^2B - 75Ab^3) \tan(c + dx) \sec(c + dx) \sqrt{a + b \sec(c + dx)}}{315b^3d}$$

[Out] $-2/315*(a-b)*(24*A*a^3*b+57*A*a*b^3-16*B*a^4-24*B*a^2*b^2+147*B*b^4)*\cot(d*x+c)*\text{EllipticE}((a+b*\sec(d*x+c))^{1/2}/(a+b)^{1/2},((a+b)/(a-b))^{1/2})*(a+b)^{1/2}*(b*(1-\sec(d*x+c)))/(a+b)^{1/2}*(-b*(1+\sec(d*x+c)))/(a-b))^{1/2}/b^5/d-2/315*(a-b)*(3*b^3*(25*A-49*B)+18*a*b^2*(A-2*B)+12*a^2*b*(2*A-B)-16*a^3*B)*\cot(d*x+c)*\text{EllipticF}((a+b*\sec(d*x+c))^{1/2}/(a+b)^{1/2},((a+b)/(a-b))^{1/2})*(a+b)^{1/2}*(b*(1-\sec(d*x+c)))/(a+b)^{1/2}*(-b*(1+\sec(d*x+c)))/(a-b))^{1/2}/b^4/d-2/315*(12*A*a^2*b-75*A*b^3-8*B*a^3-13*B*a*b^2)*(a+b*\sec(d*x+c))^{1/2}*\tan(d*x+c)/b^3/d+2/315*(9*A*a*b-6*B*a^2+49*B*b^2)*\sec(d*x+c)*(a+b*\sec(d*x+c))^{1/2}*\tan(d*x+c)/b^2/d+2/63*(9*A*b+B*a)*\sec(d*x+c)^2*(a+b*\sec(d*x+c))^{1/2}*\tan(d*x+c)/b/d+2/9*B*\sec(d*x+c)^3*(a+b*\sec(d*x+c))^{1/2}*\tan(d*x+c)/d$

Rubi [A] time = 1.44, antiderivative size = 485, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {4031, 4102, 4092, 4082, 4005, 3832, 4004}

$$\frac{2(-6a^2B + 9aAb + 49b^2B) \tan(c + dx) \sec(c + dx) \sqrt{a + b \sec(c + dx)}}{315b^2d} - \frac{2(12a^2Ab - 8a^3B - 13ab^2B - 75Ab^3) \tan(c + dx) \sec(c + dx) \sqrt{a + b \sec(c + dx)}}{315b^3d}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d*x]^4*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]

[Out] $(-2*(a-b)*\text{Sqrt}[a+b]*(24*a^3*A*b+57*a*A*b^3-16*a^4*B-24*a^2*b^2*B+147*b^4*B)*\text{Cot}[c+d*x]*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[a+b*\text{Sec}[c+d*x]]]/\text{Sqrt}[a+b]],(a+b)/(a-b)*\text{Sqrt}[(b*(1-\text{Sec}[c+d*x]))/(a+b)]*\text{Sqrt}[-((b*(1+\text{Sec}[c+d*x]))/(a-b))]/(315*b^5*d)-(2*(a-b)*\text{Sqrt}[a+b]*(3*b^3*(25*A-49*B)+18*a*b^2*(A-2*B)+12*a^2*b*(2*A-B)-16*a^3*B)*\text{Cot}[c+d*x]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[a+b*\text{Sec}[c+d*x]]]/\text{Sqrt}[a+b]],(a+b)/(a-b)]*\text{Sqrt}[(b*(1-\text{Sec}[c+d*x]))/(a+b)]*\text{Sqrt}[-((b*(1+\text{Sec}[c+d*x]))/(a-b))]/(315*b^4*d)-(2*(12*a^2*A*b-75*A*b^3-8*a^3*B-13*a*b^2*B)*\text{Sqrt}[a+b*\text{Sec}[c+d*x]]*\text{Tan}[c+d*x]/(315*b^3*d)+(2*(9*a*A*b-6*a^2*B+49*b^2*B)*\text{Sec}[c+d*x]*\text{Sqrt}[a+b*\text{Sec}[c+d*x]]*\text{Tan}[c+d*x]/(315*b^2*d)+(2*(9*A*b+a*B)*\text{Sec}[c+d*x]^2*\text{Sqrt}[a+b*\text{Sec}[c+d*x]]*\text{Tan}[c+d*x]/(63*b*d)+(2*B*\text{Sec}[c+d*x]^3*\text{Sqrt}[a+b*\text{Sec}[c+d*x]]*\text{Tan}[c+d*x]))/(9*d)$

Rule 3832

Int[csc[(e_.) + (f_.)*(x_.)]/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)]/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 4004

Int[(csc[(e_.) + (f_.)*(x_.)]*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_)))/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A, 2]], (a*A + b*B)/(a*A - b*B)]/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]

Rule 4005

Int[(csc[(e_.) + (f_.)*(x_.)]*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_)))/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] :> Dist[A - B, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]

Rule 4031

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x] + Dist[d/(m + n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n - 1)*Simp[a*B*(n - 1) + (b*B*(m + n - 1) + a*A*(m + n))*Csc[e + f*x] + (a*B*m + A*b*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[0, m, 1] && GtQ[n, 0]

Rule 4082

Int[csc[(e_.) + (f_.)*(x_.)]*((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m, x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rule 4092

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_), x
_Symbol] :> -Simp[(C*Csc[e + f*x]*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
)/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[Csc[e + f*x]*(a + b*Csc[e + f
*x])^m*Simp[a*C + b*(C*(m + 2) + A*(m + 3))*Csc[e + f*x] - (2*a*C - b*B*(m
+ 3))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && N
eQ[a^2 - b^2, 0] && !LtQ[m, -1]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_.))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
 \int \sec^4(c + dx) \sqrt{a + b \sec(c + dx)} (A + B \sec(c + dx)) dx &= \frac{2B \sec^3(c + dx) \sqrt{a + b \sec(c + dx)} \tan(c + dx)}{9d} + \frac{2}{9} \int \sec^2(c + dx) \sqrt{a + b \sec(c + dx)} \tan(c + dx) dx \\
 &= \frac{2(9Ab + aB) \sec^2(c + dx) \sqrt{a + b \sec(c + dx)} \tan(c + dx)}{63bd} \\
 &= \frac{2(9aAb - 6a^2B + 49b^2B) \sec(c + dx) \sqrt{a + b \sec(c + dx)} \tan(c + dx)}{315b^2d} \\
 &= -\frac{2(12a^2Ab - 75Ab^3 - 8a^3B - 13ab^2B) \sqrt{a + b \sec(c + dx)} \tan(c + dx)}{315b^3d} \\
 &= -\frac{2(12a^2Ab - 75Ab^3 - 8a^3B - 13ab^2B) \sqrt{a + b \sec(c + dx)}}{315b^3d} \\
 &= -\frac{2(a - b) \sqrt{a + b} (24a^3Ab + 57aAb^3 - 16a^4B - 24a^2Bb^2)}{315b^3d}
 \end{aligned}$$

Mathematica [B] time = 26.14, size = 3734, normalized size = 7.70

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^4*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
[Out] (Sqrt[a + b*Sec[c + d*x]]*((2*(24*a^3*A*b + 57*a*A*b^3 - 16*a^4*B - 24*a^2*
b^2*B + 147*b^4*B)*Sin[c + d*x]))/(315*b^4) + (2*Sec[c + d*x]^3*(9*A*b*SIN[c
+ d*x] + a*B*SIN[c + d*x]))/(63*b) + (2*Sec[c + d*x]^2*(9*a*A*b*SIN[c + d*
x] - 6*a^2*B*SIN[c + d*x] + 49*b^2*B*SIN[c + d*x]))/(315*b^2) + (2*Sec[c +
d*x]*(-12*a^2*A*b*SIN[c + d*x] + 75*A*b^3*SIN[c + d*x] + 8*a^3*B*SIN[c + d*
x] + 13*a*b^2*B*SIN[c + d*x]))/(315*b^3) + (2*B*Sec[c + d*x]^3*Tan[c + d*x]
)/9)/d + (2*((-19*a*A)/(105*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) -
(8*a^3*A)/(105*b^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (16*a^4*
B)/(315*b^3*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (8*a^2*B)/(105*b
*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (7*b*B)/(15*Sqrt[b + a*Cos[
c + d*x]]*Sqrt[Sec[c + d*x]]) - (8*a^4*A*Sqrt[Sec[c + d*x]])/(105*b^3*Sqrt[
b + a*Cos[c + d*x]]) - (17*a^2*A*Sqrt[Sec[c + d*x]])/(105*b*Sqrt[b + a*Cos[
c + d*x]]) + (5*A*b*Sqrt[Sec[c + d*x]])/(21*Sqrt[b + a*Cos[c + d*x]]) - (4*
a*B*Sqrt[Sec[c + d*x]])/(35*Sqrt[b + a*Cos[c + d*x]]) + (16*a^5*B*Sqrt[Sec[
c + d*x]])/(315*b^4*Sqrt[b + a*Cos[c + d*x]]) + (4*a^3*B*Sqrt[Sec[c + d*x]
])/ (63*b^2*Sqrt[b + a*Cos[c + d*x]]) - (8*a^4*A*Cos[2*(c + d*x)]*Sqrt[Sec[c
+ d*x]])/(105*b^3*Sqrt[b + a*Cos[c + d*x]]) - (19*a^2*A*Cos[2*(c + d*x)]*Sq
rt[Sec[c + d*x]])/(105*b*Sqrt[b + a*Cos[c + d*x]]) - (7*a*B*Cos[2*(c + d*x)
]*Sqrt[Sec[c + d*x]])/(15*Sqrt[b + a*Cos[c + d*x]]) + (16*a^5*B*Cos[2*(c +
d*x)]*Sqrt[Sec[c + d*x]])/(315*b^4*Sqrt[b + a*Cos[c + d*x]]) + (8*a^3*B*Cos
[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(105*b^2*Sqrt[b + a*Cos[c + d*x]])*Sqrt[
Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*(2*(a + b)*(-24*a
^3*A*b - 57*a*A*b^3 + 16*a^4*B + 24*a^2*b^2*B - 147*b^4*B)*Sqrt[Cos[c + d*x]
]/(1 + Cos[c + d*x]))*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]
))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*b*(a + b)*(-16*
a^3*B + 12*a^2*b*(2*A + B) - 18*a*b^2*(A + 2*B) + 3*b^3*(25*A + 49*B))*Sqrt
[Cos[c + d*x]/(1 + Cos[c + d*x]))*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + C
os[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-24*
a^3*A*b - 57*a*A*b^3 + 16*a^4*B + 24*a^2*b^2*B - 147*b^4*B)*Cos[c + d*x]*(b
+ a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/ (315*b^4*d*(b + a*
Cos[c + d*x])*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Sec[c + d*x]]*((a*Sqrt[Cos[(c +
d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*x]*(2*(a + b)*(-24*a^3*A*b - 57*a*A*b^3
+ 16*a^4*B + 24*a^2*b^2*B - 147*b^4*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x]
)]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[
Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*b*(a + b)*(-16*a^3*B + 12*a^2*b*(2*
A + B) - 18*a*b^2*(A + 2*B) + 3*b^3*(25*A + 49*B))*Sqrt[Cos[c + d*x]/(1 + C
os[c + d*x]))*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Ellip
```

$$\begin{aligned}
& \text{ticF}[\text{ArcSin}[\text{Tan}[(c + d*x)/2]], (a - b)/(a + b)] + (-24*a^3*A*b - 57*a*A*b^3 \\
& + 16*a^4*B + 24*a^2*b^2*B - 147*b^4*B)*\text{Cos}[c + d*x]*(b + a*\text{Cos}[c + d*x])*S \\
& \text{ec}[(c + d*x)/2]^2*\text{Tan}[(c + d*x)/2])/((315*b^4*(b + a*\text{Cos}[c + d*x])^(3/2)*S \\
& \text{qrt}[\text{Sec}[(c + d*x)/2]^2]) - (\text{Sqrt}[\text{Cos}[(c + d*x)/2]^2*\text{Sec}[c + d*x]]*\text{Tan}[(c + d \\
& *x)/2]*(2*(a + b)*(-24*a^3*A*b - 57*a*A*b^3 + 16*a^4*B + 24*a^2*b^2*B - 147 \\
& *b^4*B)*\text{Sqrt}[\text{Cos}[c + d*x]/(1 + \text{Cos}[c + d*x])]*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/((a \\
& + b)*(1 + \text{Cos}[c + d*x]))]*\text{EllipticE}[\text{ArcSin}[\text{Tan}[(c + d*x)/2]], (a - b)/(a + \\
& b)] + 2*b*(a + b)*(-16*a^3*B + 12*a^2*b*(2*A + B) - 18*a*b^2*(A + 2*B) + 3 \\
& *b^3*(25*A + 49*B))*\text{Sqrt}[\text{Cos}[c + d*x]/(1 + \text{Cos}[c + d*x])]*\text{Sqrt}[(b + a*\text{Cos}[c \\
& + d*x])/((a + b)*(1 + \text{Cos}[c + d*x]))]*\text{EllipticF}[\text{ArcSin}[\text{Tan}[(c + d*x)/2]], \\
& (a - b)/(a + b)] + (-24*a^3*A*b - 57*a*A*b^3 + 16*a^4*B + 24*a^2*b^2*B - 14 \\
& 7*b^4*B)*\text{Cos}[c + d*x]*(b + a*\text{Cos}[c + d*x])*S\text{ec}[(c + d*x)/2]^2*\text{Tan}[(c + d*x) \\
& /2])/((315*b^4*\text{Sqrt}[b + a*\text{Cos}[c + d*x]]*\text{Sqrt}[\text{Sec}[(c + d*x)/2]^2]) + (2*\text{Sqrt} \\
& [\text{Cos}[(c + d*x)/2]^2*\text{Sec}[c + d*x]]*((-24*a^3*A*b - 57*a*A*b^3 + 16*a^4*B + \\
& 24*a^2*b^2*B - 147*b^4*B)*\text{Cos}[c + d*x]*(b + a*\text{Cos}[c + d*x])*S\text{ec}[(c + d*x)/2 \\
&]^4)/2 + ((a + b)*(-24*a^3*A*b - 57*a*A*b^3 + 16*a^4*B + 24*a^2*b^2*B - 147 \\
& *b^4*B)*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/((a + b)*(1 + \text{Cos}[c + d*x]))]*\text{EllipticE}[A \\
& \text{rcSin}[\text{Tan}[(c + d*x)/2]], (a - b)/(a + b)]*((\text{Cos}[c + d*x]*\text{Sin}[c + d*x])/((1 + \\
& \text{Cos}[c + d*x])^2 - \text{Sin}[c + d*x]/(1 + \text{Cos}[c + d*x])))/\text{Sqrt}[\text{Cos}[c + d*x]/(1 + \\
& \text{Cos}[c + d*x])] + (b*(a + b)*(-16*a^3*B + 12*a^2*b*(2*A + B) - 18*a*b^2*(A \\
& + 2*B) + 3*b^3*(25*A + 49*B))*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/((a + b)*(1 + \text{Cos}[c \\
& + d*x]))]*\text{EllipticF}[\text{ArcSin}[\text{Tan}[(c + d*x)/2]], (a - b)/(a + b)]*((\text{Cos}[c + d \\
& *x]*\text{Sin}[c + d*x])/((1 + \text{Cos}[c + d*x])^2 - \text{Sin}[c + d*x]/(1 + \text{Cos}[c + d*x])))/ \\
& \text{Sqrt}[\text{Cos}[c + d*x]/(1 + \text{Cos}[c + d*x])] + ((a + b)*(-24*a^3*A*b - 57*a*A*b^3 \\
& + 16*a^4*B + 24*a^2*b^2*B - 147*b^4*B)*\text{Sqrt}[\text{Cos}[c + d*x]/(1 + \text{Cos}[c + d*x]) \\
&]*\text{EllipticE}[\text{ArcSin}[\text{Tan}[(c + d*x)/2]], (a - b)/(a + b)]*(-((a*\text{Sin}[c + d*x])/ \\
& ((a + b)*(1 + \text{Cos}[c + d*x])))) + ((b + a*\text{Cos}[c + d*x])*S\text{in}[c + d*x])/((a + b \\
&)*(1 + \text{Cos}[c + d*x])^2))/\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/((a + b)*(1 + \text{Cos}[c + d \\
& *x]))] + (b*(a + b)*(-16*a^3*B + 12*a^2*b*(2*A + B) - 18*a*b^2*(A + 2*B) + \\
& 3*b^3*(25*A + 49*B))*\text{Sqrt}[\text{Cos}[c + d*x]/(1 + \text{Cos}[c + d*x])]*\text{EllipticF}[\text{ArcSin} \\
& [\text{Tan}[(c + d*x)/2]], (a - b)/(a + b)]*(-((a*\text{Sin}[c + d*x])/((a + b)*(1 + \text{Cos}[c \\
& + d*x])))) + ((b + a*\text{Cos}[c + d*x])*S\text{in}[c + d*x])/((a + b)*(1 + \text{Cos}[c + d*x] \\
&)^2))/\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/((a + b)*(1 + \text{Cos}[c + d*x]))] - a*(-24*a^ \\
& 3*A*b - 57*a*A*b^3 + 16*a^4*B + 24*a^2*b^2*B - 147*b^4*B)*\text{Cos}[c + d*x]*S\text{ec} \\
& [(c + d*x)/2]^2*\text{Sin}[c + d*x]*\text{Tan}[(c + d*x)/2] - (-24*a^3*A*b - 57*a*A*b^3 + \\
& 16*a^4*B + 24*a^2*b^2*B - 147*b^4*B)*(b + a*\text{Cos}[c + d*x])*S\text{ec}[(c + d*x)/2]^ \\
& 2*\text{Sin}[c + d*x]*\text{Tan}[(c + d*x)/2] + (-24*a^3*A*b - 57*a*A*b^3 + 16*a^4*B + 24 \\
& *a^2*b^2*B - 147*b^4*B)*\text{Cos}[c + d*x]*(b + a*\text{Cos}[c + d*x])*S\text{ec}[(c + d*x)/2]^ \\
& 2*\text{Tan}[(c + d*x)/2]^2 + (b*(a + b)*(-16*a^3*B + 12*a^2*b*(2*A + B) - 18*a*b^ \\
& 2*(A + 2*B) + 3*b^3*(25*A + 49*B))*\text{Sqrt}[\text{Cos}[c + d*x]/(1 + \text{Cos}[c + d*x])]*S \\
& \text{qrt}[(b + a*\text{Cos}[c + d*x])/((a + b)*(1 + \text{Cos}[c + d*x]))]*S\text{ec}[(c + d*x)/2]^2)/ \\
& (\text{Sqrt}[1 - \text{Tan}[(c + d*x)/2]^2]*\text{Sqrt}[1 - ((a - b)*\text{Tan}[(c + d*x)/2]^2)/(a + b) \\
&]) + ((a + b)*(-24*a^3*A*b - 57*a*A*b^3 + 16*a^4*B + 24*a^2*b^2*B - 147*b^4* \\
& B)*\text{Sqrt}[\text{Cos}[c + d*x]/(1 + \text{Cos}[c + d*x])]*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/((a + b) \\
& *(1 + \text{Cos}[c + d*x]))]*S\text{ec}[(c + d*x)/2]^2*\text{Sqrt}[1 - ((a - b)*\text{Tan}[(c + d*x)/2]
\end{aligned}$$

$\wedge 2)/(a + b)))/\text{Sqrt}[1 - \text{Tan}[(c + d*x)/2]^2)]/(315*b^4*\text{Sqrt}[b + a*\text{Cos}[c + d*x]]*\text{Sqrt}[\text{Sec}[(c + d*x)/2]^2]) + ((2*(a + b)*(-24*a^3*A*b - 57*a*A*b^3 + 16*a^4*B + 24*a^2*b^2*B - 147*b^4*B)*\text{Sqrt}[\text{Cos}[c + d*x]/(1 + \text{Cos}[c + d*x])] * \text{Sqrt}[(b + a*\text{Cos}[c + d*x])/((a + b)*(1 + \text{Cos}[c + d*x]))] * \text{EllipticE}[\text{ArcSin}[\text{Tan}[(c + d*x)/2]], (a - b)/(a + b)] + 2*b*(a + b)*(-16*a^3*B + 12*a^2*b*(2*A + B) - 18*a*b^2*(A + 2*B) + 3*b^3*(25*A + 49*B))*\text{Sqrt}[\text{Cos}[c + d*x]/(1 + \text{Cos}[c + d*x])] * \text{Sqrt}[(b + a*\text{Cos}[c + d*x])/((a + b)*(1 + \text{Cos}[c + d*x]))] * \text{EllipticF}[\text{ArcSin}[\text{Tan}[(c + d*x)/2]], (a - b)/(a + b)] + (-24*a^3*A*b - 57*a*A*b^3 + 16*a^4*B + 24*a^2*b^2*B - 147*b^4*B)*\text{Cos}[c + d*x]*(b + a*\text{Cos}[c + d*x])* \text{Sec}[(c + d*x)/2]^2*\text{Tan}[(c + d*x)/2]) * (-\text{Cos}[(c + d*x)/2]*\text{Sec}[c + d*x]*\text{Sin}[(c + d*x)/2]) + \text{Cos}[(c + d*x)/2]^2*\text{Sec}[c + d*x]*\text{Tan}[c + d*x]))/(315*b^4*\text{Sqrt}[b + a*\text{Cos}[c + d*x]]*\text{Sqrt}[\text{Sec}[(c + d*x)/2]^2]*\text{Sqrt}[\text{Cos}[(c + d*x)/2]^2*\text{Sec}[c + d*x]]))$

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}((B \sec(dx + c)^5 + A \sec(dx + c)^4) \sqrt{b \sec(dx + c) + a}, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")`

[Out] `integral((B*sec(d*x + c)^5 + A*sec(d*x + c)^4)*sqrt(b*sec(d*x + c) + a), x)`

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A) \sqrt{b \sec(dx + c) + a} \sec(dx + c)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")`

[Out] `integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^4, x)`

maple [B] time = 3.22, size = 4394, normalized size = 9.06

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sec(d*x+c)^4*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x)`

[Out] `-2/315/d*(1+cos(d*x+c))^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))^2*(-54*A*cos(d*x+c)^2*a*b^4+B*cos(d*x+c)^2*a^2*b^3-40*B*cos(d*x+c)*a*b^4`

$$\begin{aligned}
& -22*B*\cos(d*x+c)^3*a*b^4-30*A*\cos(d*x+c)^3*b^5-24*A*\cos(d*x+c)^5*(\cos(d*x+c) \\
&)/(1+\cos(d*x+c))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{(1/2)}*Ellip \\
& ticE((-1+\cos(d*x+c))/\sin(d*x+c),((a-b)/(a+b))^{(1/2)})*\sin(d*x+c)*a^4*b+8*B*c \\
& os(d*x+c)^6*a^4*b-24*B*\cos(d*x+c)^6*a^3*b^2+13*B*\cos(d*x+c)^6*a^2*b^3+147*B \\
& *cos(d*x+c)^6*a*b^4-16*B*\cos(d*x+c)^5*a^4*b+26*B*\cos(d*x+c)^5*a^3*b^2-24*B* \\
& cos(d*x+c)^5*a^2*b^3-85*B*\cos(d*x+c)^5*a*b^4+8*B*\cos(d*x+c)^4*a^4*b-35*B*b^ \\
& 5+24*A*\cos(d*x+c)^6*a^4*b-12*A*\cos(d*x+c)^6*a^3*b^2+57*A*\cos(d*x+c)^6*a^2*b \\
& ^3+75*A*\cos(d*x+c)^6*a*b^4-24*A*\cos(d*x+c)^5*a^4*b+24*A*\cos(d*x+c)^5*a^3*b^ \\
& 2-60*A*\cos(d*x+c)^5*a^2*b^3+57*A*\cos(d*x+c)^5*a*b^4-12*A*\cos(d*x+c)^4*a^3*b \\
& ^2-78*A*\cos(d*x+c)^4*a*b^4+3*A*\cos(d*x+c)^3*a^2*b^3+10*B*\cos(d*x+c)^4*a^2*b \\
& ^3-2*B*\cos(d*x+c)^3*a^3*b^2+75*A*\cos(d*x+c)^5*b^5+16*B*\cos(d*x+c)^5*a^5-16* \\
& B*\cos(d*x+c)^6*a^5-98*B*\cos(d*x+c)^4*b^5-45*A*\cos(d*x+c)*b^5+147*B*\cos(d*x+ \\
& c)^5*b^5-14*B*\cos(d*x+c)^2*b^5+24*B*\cos(d*x+c)^4*(\cos(d*x+c)/(1+\cos(d*x+c)) \\
&)^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{(1/2)}*EllipticE((-1+\cos(d*x \\
& +c))/\sin(d*x+c),((a-b)/(a+b))^{(1/2)})*\sin(d*x+c)*a^3*b^2+24*B*\cos(d*x+c)^4*(\\
& \cos(d*x+c)/(1+\cos(d*x+c))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{(1 \\
& /2)}*EllipticE((-1+\cos(d*x+c))/\sin(d*x+c),((a-b)/(a+b))^{(1/2)})*\sin(d*x+c)*a^ \\
& 2*b^3-147*B*\cos(d*x+c)^4*(\cos(d*x+c)/(1+\cos(d*x+c))^{(1/2)}*((b+a*\cos(d*x+c) \\
&)/(1+\cos(d*x+c))/(a+b))^{(1/2)}*EllipticE((-1+\cos(d*x+c))/\sin(d*x+c),((a-b)/(\\
& a+b))^{(1/2)})*\sin(d*x+c)*a*b^4-16*B*\cos(d*x+c)^4*(\cos(d*x+c)/(1+\cos(d*x+c)) \\
&)^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{(1/2)}*EllipticF((-1+\cos(d*x+ \\
& c))/\sin(d*x+c),((a-b)/(a+b))^{(1/2)})*\sin(d*x+c)*a^4*b-4*B*\cos(d*x+c)^4*(\cos(\\
& d*x+c)/(1+\cos(d*x+c))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{(1/2)}* \\
& EllipticF((-1+\cos(d*x+c))/\sin(d*x+c),((a-b)/(a+b))^{(1/2)})*\sin(d*x+c)*a^3*b^ \\
& 2-24*B*\cos(d*x+c)^4*(\cos(d*x+c)/(1+\cos(d*x+c))^{(1/2)}*((b+a*\cos(d*x+c))/(1+ \\
& \cos(d*x+c))/(a+b))^{(1/2)}*EllipticF((-1+\cos(d*x+c))/\sin(d*x+c),((a-b)/(a+b)) \\
& ^{(1/2)})*\sin(d*x+c)*a^2*b^3+111*B*\cos(d*x+c)^4*(\cos(d*x+c)/(1+\cos(d*x+c))^{(\\
& 1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{(1/2)}*EllipticF((-1+\cos(d*x+c) \\
&)/\sin(d*x+c),((a-b)/(a+b))^{(1/2)})*\sin(d*x+c)*a*b^4+75*A*\cos(d*x+c)^5*(\cos(d \\
& *x+c)/(1+\cos(d*x+c))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{(1/2)}*E \\
& llipticF((-1+\cos(d*x+c))/\sin(d*x+c),((a-b)/(a+b))^{(1/2)})*\sin(d*x+c)*b^5+16* \\
& B*\cos(d*x+c)^5*(\cos(d*x+c)/(1+\cos(d*x+c))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d \\
& *x+c))/(a+b))^{(1/2)}*EllipticE((-1+\cos(d*x+c))/\sin(d*x+c),((a-b)/(a+b))^{(1/2 \\
&))*\sin(d*x+c)*a^5-147*B*\cos(d*x+c)^5*(\cos(d*x+c)/(1+\cos(d*x+c))^{(1/2)}*((b+ \\
& a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{(1/2)}*EllipticE((-1+\cos(d*x+c))/\sin(d*x \\
& +c),((a-b)/(a+b))^{(1/2)})*\sin(d*x+c)*b^5+147*B*\cos(d*x+c)^5*(\cos(d*x+c)/(1+c \\
& os(d*x+c))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{(1/2)}*EllipticF((\\
& -1+\cos(d*x+c))/\sin(d*x+c),((a-b)/(a+b))^{(1/2)})*\sin(d*x+c)*b^5+75*A*\cos(d*x+ \\
& c)^4*(\cos(d*x+c)/(1+\cos(d*x+c))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+ \\
& b))^{(1/2)}*EllipticF((-1+\cos(d*x+c))/\sin(d*x+c),((a-b)/(a+b))^{(1/2)})*\sin(d*x \\
& +c)*b^5+16*B*\cos(d*x+c)^4*(\cos(d*x+c)/(1+\cos(d*x+c))^{(1/2)}*((b+a*\cos(d*x+c) \\
&)/(1+\cos(d*x+c))/(a+b))^{(1/2)}*EllipticE((-1+\cos(d*x+c))/\sin(d*x+c),((a-b)/ \\
& (a+b))^{(1/2)})*\sin(d*x+c)*a^5-147*B*\cos(d*x+c)^4*(\cos(d*x+c)/(1+\cos(d*x+c)) \\
&)^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{(1/2)}*EllipticE((-1+\cos(d*x+ \\
& c))/\sin(d*x+c),((a-b)/(a+b))^{(1/2)})*\sin(d*x+c)*b^5+147*B*\cos(d*x+c)^4*(\cos(
\end{aligned}$$

$$\begin{aligned}
& d*x+c)/(1+\cos(d*x+c))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}* \\
& \text{EllipticF}((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{1/2})*\sin(d*x+c)*b^{5-24} \\
& *A*\cos(d*x+c)^5*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(\\
& d*x+c))/(a+b))^{1/2}*\text{EllipticE}((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{1/2} \\
&)*\sin(d*x+c)*a^3*b^2-57*A*\cos(d*x+c)^5*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}* \\
& ((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*\text{EllipticE}((-1+\cos(d*x+c))/\sin \\
& (d*x+c), ((a-b)/(a+b))^{1/2})*\sin(d*x+c)*a^2*b^3-57*A*\cos(d*x+c)^5*(\cos(d*x+ \\
& c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*Elli \\
& pticE((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{1/2})*\sin(d*x+c)*a*b^4+24*A \\
& *cos(d*x+c)^5*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d* \\
& x+c))/(a+b))^{1/2}*\text{EllipticF}((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{1/2} \\
&)*\sin(d*x+c)*a^3*b^2+6*A*\cos(d*x+c)^5*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b \\
& +a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*\text{EllipticF}((-1+\cos(d*x+c))/\sin(d* \\
& x+c), ((a-b)/(a+b))^{1/2})*\sin(d*x+c)*a^2*b^3+57*A*\cos(d*x+c)^5*(\cos(d*x+c)/ \\
& (1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*\text{Ellipti} \\
& cF((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{1/2})*\sin(d*x+c)*a*b^4+16*B*co \\
& s(d*x+c)^5*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c) \\
&))/(a+b))^{1/2}*\text{EllipticE}((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{1/2})*s \\
& in(d*x+c)*a^4*b+24*B*\cos(d*x+c)^5*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*c \\
& os(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*\text{EllipticE}((-1+\cos(d*x+c))/\sin(d*x+c) \\
& , ((a-b)/(a+b))^{1/2})*\sin(d*x+c)*a^3*b^2+24*B*\cos(d*x+c)^5*(\cos(d*x+c)/(1+c \\
& os(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*\text{EllipticE}((\\
& -1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{1/2})*\sin(d*x+c)*a^2*b^3-147*B*\cos \\
& (d*x+c)^5*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c) \\
&))/(a+b))^{1/2}*\text{EllipticE}((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{1/2})*si \\
& n(d*x+c)*a*b^4-16*B*\cos(d*x+c)^5*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*co \\
& s(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*\text{EllipticF}((-1+\cos(d*x+c))/\sin(d*x+c), \\
& ((a-b)/(a+b))^{1/2})*\sin(d*x+c)*a^4*b-4*B*\cos(d*x+c)^5*(\cos(d*x+c)/(1+\cos(d \\
& *x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*\text{EllipticF}((-1+c \\
& os(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{1/2})*\sin(d*x+c)*a^3*b^2-24*B*\cos(d*x+ \\
& c)^5*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+ \\
& b))^{1/2}*\text{EllipticF}((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{1/2})*\sin(d*x \\
& +c)*a^2*b^3+111*B*\cos(d*x+c)^5*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(\\
& d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*\text{EllipticF}((-1+\cos(d*x+c))/\sin(d*x+c), ((\\
& a-b)/(a+b))^{1/2})*\sin(d*x+c)*a*b^4-24*A*\cos(d*x+c)^4*(\cos(d*x+c)/(1+\cos(d* \\
& x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*\text{EllipticE}((-1+co \\
& s(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{1/2})*\sin(d*x+c)*a^4*b-24*A*\cos(d*x+c)^ \\
& 4*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b)) \\
& ^{1/2}*\text{EllipticE}((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{1/2})*\sin(d*x+c) \\
& *a^3*b^2-57*A*\cos(d*x+c)^4*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+ \\
& c))/(1+\cos(d*x+c))/(a+b))^{1/2}*\text{EllipticE}((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b) \\
& / (a+b))^{1/2})*\sin(d*x+c)*a^2*b^3-57*A*\cos(d*x+c)^4*(\cos(d*x+c)/(1+\cos(d*x+ \\
& c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*\text{EllipticE}((-1+\cos(\\
& d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{1/2})*\sin(d*x+c)*a*b^4+24*A*\cos(d*x+c)^4* \\
& (\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}
\end{aligned}$$

$$\frac{1}{2} * \text{EllipticF}\left(\frac{-1 + \cos(dx+c)}{\sin(dx+c)}, \left(\frac{a-b}{a+b}\right)^{1/2}\right) * \sin(dx+c) * a^3 * b^2 + 6 * A * \cos(dx+c)^4 * \left(\frac{\cos(dx+c)}{1 + \cos(dx+c)}\right)^{1/2} * \left(\frac{b + a * \cos(dx+c)}{1 + \cos(dx+c)}\right)^{1/2} * \text{EllipticF}\left(\frac{-1 + \cos(dx+c)}{\sin(dx+c)}, \left(\frac{a-b}{a+b}\right)^{1/2}\right) * \sin(dx+c) * a^2 * b^3 + 57 * A * \cos(dx+c)^4 * \left(\frac{\cos(dx+c)}{1 + \cos(dx+c)}\right)^{1/2} * \left(\frac{b + a * \cos(dx+c)}{1 + \cos(dx+c)}\right)^{1/2} * \text{EllipticF}\left(\frac{-1 + \cos(dx+c)}{\sin(dx+c)}, \left(\frac{a-b}{a+b}\right)^{1/2}\right) * \sin(dx+c) * a * b^4 + 16 * B * \cos(dx+c)^4 * \left(\frac{\cos(dx+c)}{1 + \cos(dx+c)}\right)^{1/2} * \left(\frac{b + a * \cos(dx+c)}{1 + \cos(dx+c)}\right)^{1/2} * \text{EllipticE}\left(\frac{-1 + \cos(dx+c)}{\sin(dx+c)}, \left(\frac{a-b}{a+b}\right)^{1/2}\right) * \sin(dx+c) * a^4 * b / (b + a * \cos(dx+c)) / \cos(dx+c)^4 / \sin(dx+c)^5 / b^4$$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(dx+c)^4*(A+B*sec(dx+c))*(a+b*sec(dx+c))^(1/2),x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \sqrt{a + \frac{b}{\cos(c+dx)}}}{\cos(c+dx)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + dx))*(a + b/cos(c + dx))^(1/2))/cos(c + dx)^4,x)

[Out] int(((A + B/cos(c + dx))*(a + b/cos(c + dx))^(1/2))/cos(c + dx)^4, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \sec(c + dx)) \sqrt{a + b \sec(c + dx)} \sec^4(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(dx+c)**4*(A+B*sec(dx+c))*(a+b*sec(dx+c))**(1/2),x)

[Out] Integral((A + B*sec(c + dx))*sqrt(a + b*sec(c + dx))*sec(c + dx)**4, x)

$$3.349 \quad \int \sec^3(c+dx) \sqrt{a+b \sec(c+dx)} (A+B \sec(c+dx)) dx$$

Optimal. Leaf size=397

$$\frac{2(-4a^2B + 7aAb + 25b^2B) \tan(c+dx) \sqrt{a+b \sec(c+dx)}}{105b^2d} + \frac{2(a-b) \sqrt{a+b} (-8a^2B + 2ab(7A-3B) + b^2(63A$$

[Out] $2/105*(a-b)*(14*A*a^2*b-63*A*b^3-8*B*a^3-19*B*a*b^2)*\cot(d*x+c)*\text{EllipticE}((a+b*\sec(d*x+c))^{1/2}/(a+b)^{1/2},((a+b)/(a-b))^{1/2})*(a+b)^{1/2}*(b*(1-\sec(d*x+c))/(a+b))^{1/2}*(-b*(1+\sec(d*x+c))/(a-b))^{1/2}/b^4/d+2/105*(a-b)*(b^2*(63*A-25*B)+2*a*b*(7*A-3*B)-8*a^2*B)*\cot(d*x+c)*\text{EllipticF}((a+b*\sec(d*x+c))^{1/2}/(a+b)^{1/2},((a+b)/(a-b))^{1/2})*(a+b)^{1/2}*(b*(1-\sec(d*x+c))/(a+b))^{1/2}*(-b*(1+\sec(d*x+c))/(a-b))^{1/2}/b^3/d+2/105*(7*A*a*b-4*B*a^2+25*B*b^2)*(a+b*\sec(d*x+c))^{1/2}*\tan(d*x+c)/b^2/d+2/35*(7*A*b+B*a)*\sec(d*x+c)*(a+b*\sec(d*x+c))^{1/2}*\tan(d*x+c)/b/d+2/7*B*\sec(d*x+c)^2*(a+b*\sec(d*x+c))^{1/2}*\tan(d*x+c)/d$

Rubi [A] time = 0.93, antiderivative size = 397, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4031, 4092, 4082, 4005, 3832, 4004}

$$\frac{2(-4a^2B + 7aAb + 25b^2B) \tan(c+dx) \sqrt{a+b \sec(c+dx)}}{105b^2d} + \frac{2(a-b) \sqrt{a+b} (-8a^2B + 2ab(7A-3B) + b^2(63A$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d*x]^3*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]

[Out] $(2*(a-b)*\text{Sqrt}[a+b]*(14*a^2*A*b-63*A*b^3-8*a^3*B-19*a*b^2*B)*\text{Cot}[c+d*x]*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[a+b*\text{Sec}[c+d*x]]/\text{Sqrt}[a+b]],(a+b)/(a-b)]*\text{Sqrt}[(b*(1-\text{Sec}[c+d*x]))/(a+b)]*\text{Sqrt}[-((b*(1+\text{Sec}[c+d*x]))/(a-b))]/(105*b^4*d)+(2*(a-b)*\text{Sqrt}[a+b]*(b^2*(63*A-25*B)+2*a*b*(7*A-3*B)-8*a^2*B)*\text{Cot}[c+d*x]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[a+b*\text{Sec}[c+d*x]]/\text{Sqrt}[a+b]],(a+b)/(a-b)]*\text{Sqrt}[(b*(1-\text{Sec}[c+d*x]))/(a+b)]*\text{Sqrt}[-((b*(1+\text{Sec}[c+d*x]))/(a-b))]/(105*b^3*d)+(2*(7*a*A*b-4*a^2*B+25*b^2*B)*\text{Sqrt}[a+b*\text{Sec}[c+d*x]]*\text{Tan}[c+d*x]/(105*b^2*d)+(2*(7*A*b+a*B)*\text{Sec}[c+d*x]*\text{Sqrt}[a+b*\text{Sec}[c+d*x]]*\text{Tan}[c+d*x]/(35*b*d)+(2*B*\text{Sec}[c+d*x]^2*\text{Sqrt}[a+b*\text{Sec}[c+d*x]]*\text{Tan}[c+d*x])/(7*d)$

Rule 3832

Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-

```
((b*(1 + Csc[e + f*x]))/(a - b))*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)]/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)))/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A, 2]], (a*A + b*B)/(a*A - b*B))]/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)))/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] :> Dist[A - B, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 4031

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> -Simp[(B*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x] + Dist[d/(m + n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n - 1)*Simp[a*B*(n - 1) + (b*B*(m + n - 1) + a*A*(m + n))*Csc[e + f*x] + (a*B*m + A*b*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[0, m, 1] && GtQ[n, 0]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_.)]*((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m, x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]
```

Rule 4092

```
Int[csc[(e_.) + (f_.)*(x_.)]^2*((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m, x_Symbol] :> -Simp[(C*Csc[e + f*x]*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
```

)/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[a*C + b*(C*(m + 2) + A*(m + 3))*Csc[e + f*x] - (2*a*C - b*B*(m + 3))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && !LtQ[m, -1]

Rubi steps

$$\begin{aligned} \int \sec^3(c + dx)\sqrt{a + b \sec(c + dx)} (A + B \sec(c + dx)) dx &= \frac{2B \sec^2(c + dx)\sqrt{a + b \sec(c + dx)} \tan(c + dx)}{7d} + \frac{2}{7} \\ &= \frac{2(7Ab + aB) \sec(c + dx)\sqrt{a + b \sec(c + dx)} \tan(c + dx)}{35bd} \\ &= \frac{2(7aAb - 4a^2B + 25b^2B) \sqrt{a + b \sec(c + dx)} \tan(c + dx)}{105b^2d} \\ &= \frac{2(7aAb - 4a^2B + 25b^2B) \sqrt{a + b \sec(c + dx)} \tan(c + dx)}{105b^2d} \\ &= \frac{2(a - b)\sqrt{a + b} (14a^2Ab - 63Ab^3 - 8a^3B - 19ab^2B)}{105b^2d} \end{aligned}$$

Mathematica [B] time = 25.02, size = 3330, normalized size = 8.39

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^3*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
[Out] (Sqrt[a + b*Sec[c + d*x]]*((2*(-14*a^2*A*b + 63*A*b^3 + 8*a^3*B + 19*a*b^2*B)*Sin[c + d*x])/(105*b^3) + (2*Sec[c + d*x]^2*(7*A*b*Sin[c + d*x] + a*B*Sin[c + d*x]))/(35*b) + (2*Sec[c + d*x]*(7*a*A*b*Sin[c + d*x] - 4*a^2*B*Sin[c + d*x] + 25*b^2*B*Sin[c + d*x]))/(105*b^2) + (2*B*Sec[c + d*x]^2*Tan[c + d*x])/7))/d - (2*((2*a^2*A)/(15*b*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (3*A*b)/(5*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (19*a*B)/(105*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (8*a^3*B)/(105*b^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (2*a*A*Sqrt[Sec[c + d*x]])/(15*Sqrt[b + a*Cos[c + d*x]]) + (2*a^3*A*Sqrt[Sec[c + d*x]])/(15*b^2*Sqrt[b + a*Cos[c + d*x]]) - (8*a^4*B*Sqrt[Sec[c + d*x]])/(105*b^3*Sqrt[b + a*Cos[c + d*x]]) - (17*a^2*B*Sqrt[Sec[c + d*x]])/(105*b*Sqrt[b + a*Cos[c + d*x]]) + (5*b*
```

$$\begin{aligned}
& B\sqrt{\sec[c + dx]} / (21\sqrt{b + a\cos[c + dx]}) - (3a^2A\cos[2(c + dx)]\sqrt{\sec[c + dx]} / (5\sqrt{b + a\cos[c + dx]}) + (2a^3A\cos[2(c + dx)]\sqrt{\sec[c + dx]} / (15b^2\sqrt{b + a\cos[c + dx]}) - (8a^4B\cos[2(c + dx)]\sqrt{\sec[c + dx]} / (105b^3\sqrt{b + a\cos[c + dx]}) - (19a^2B\cos[2(c + dx)]\sqrt{\sec[c + dx]} / (105b\sqrt{b + a\cos[c + dx]}))\sqrt{\cos[(c + dx)/2]^2\sec[c + dx]}\sqrt{a + b\sec[c + dx]}(2(a + b)(-14a^2Ab + 63A^2b^3 + 8a^3B + 19a^2b^2B)\sqrt{\cos[c + dx]} / (1 + \cos[c + dx]))\sqrt{(b + a\cos[c + dx]) / ((a + b)(1 + \cos[c + dx]))}\text{EllipticE}[\text{ArcSin}[\text{Tan}[(c + dx)/2]], (a - b)/(a + b)] - 2b(a + b)(8a^2B - 2ab(7A + 3B) + b^2(63A + 25B))\sqrt{\cos[c + dx]} / (1 + \cos[c + dx])\sqrt{(b + a\cos[c + dx]) / ((a + b)(1 + \cos[c + dx]))}\text{EllipticF}[\text{ArcSin}[\text{Tan}[(c + dx)/2]], (a - b)/(a + b)] + (-14a^2Ab + 63A^2b^3 + 8a^3B + 19a^2b^2B)\cos[c + dx](b + a\cos[c + dx])\sec[(c + dx)/2]^2\tan[(c + dx)/2]) / (105b^3d(b + a\cos[c + dx])\sqrt{\sec[(c + dx)/2]^2}\sqrt{\sec[c + dx]}) * (-1/105(a\sqrt{\cos[(c + dx)/2]^2\sec[c + dx]}\sin[c + dx](2(a + b)(-14a^2Ab + 63A^2b^3 + 8a^3B + 19a^2b^2B)\sqrt{\cos[c + dx]} / (1 + \cos[c + dx]))\sqrt{(b + a\cos[c + dx]) / ((a + b)(1 + \cos[c + dx]))}\text{EllipticE}[\text{ArcSin}[\text{Tan}[(c + dx)/2]], (a - b)/(a + b)] - 2b(a + b)(8a^2B - 2ab(7A + 3B) + b^2(63A + 25B))\sqrt{\cos[c + dx]} / (1 + \cos[c + dx])\sqrt{(b + a\cos[c + dx]) / ((a + b)(1 + \cos[c + dx]))}\text{EllipticF}[\text{ArcSin}[\text{Tan}[(c + dx)/2]], (a - b)/(a + b)] + (-14a^2Ab + 63A^2b^3 + 8a^3B + 19a^2b^2B)\cos[c + dx](b + a\cos[c + dx])\sec[(c + dx)/2]^2\tan[(c + dx)/2]) / (b^3(b + a\cos[c + dx])^{3/2}\sqrt{\sec[(c + dx)/2]^2}) + (\sqrt{\cos[(c + dx)/2]^2\sec[c + dx]}\tan[(c + dx)/2](2(a + b)(-14a^2Ab + 63A^2b^3 + 8a^3B + 19a^2b^2B)\sqrt{\cos[c + dx]} / (1 + \cos[c + dx]))\sqrt{(b + a\cos[c + dx]) / ((a + b)(1 + \cos[c + dx]))}\text{EllipticE}[\text{ArcSin}[\text{Tan}[(c + dx)/2]], (a - b)/(a + b)] - 2b(a + b)(8a^2B - 2ab(7A + 3B) + b^2(63A + 25B))\sqrt{\cos[c + dx]} / (1 + \cos[c + dx])\sqrt{(b + a\cos[c + dx]) / ((a + b)(1 + \cos[c + dx]))}\text{EllipticF}[\text{ArcSin}[\text{Tan}[(c + dx)/2]], (a - b)/(a + b)] + (-14a^2Ab + 63A^2b^3 + 8a^3B + 19a^2b^2B)\cos[c + dx](b + a\cos[c + dx])\sec[(c + dx)/2]^2\tan[(c + dx)/2]) / (105b^3\sqrt{b + a\cos[c + dx]}\sqrt{\sec[(c + dx)/2]^2}) - (2\sqrt{\cos[(c + dx)/2]^2\sec[c + dx]} * (((-14a^2Ab + 63A^2b^3 + 8a^3B + 19a^2b^2B)\cos[c + dx](b + a\cos[c + dx])\sec[(c + dx)/2]^4) / 2 + ((a + b)(-14a^2Ab + 63A^2b^3 + 8a^3B + 19a^2b^2B)\sqrt{(b + a\cos[c + dx]) / ((a + b)(1 + \cos[c + dx]))}\text{EllipticE}[\text{ArcSin}[\text{Tan}[(c + dx)/2]], (a - b)/(a + b)] * ((\cos[c + dx]\sin[c + dx]) / (1 + \cos[c + dx])^2 - \sin[c + dx] / (1 + \cos[c + dx])) / \sqrt{\cos[c + dx]} / (1 + \cos[c + dx]) - (b(a + b)(8a^2B - 2ab(7A + 3B) + b^2(63A + 25B))\sqrt{(b + a\cos[c + dx]) / ((a + b)(1 + \cos[c + dx]))}\text{EllipticF}[\text{ArcSin}[\text{Tan}[(c + dx)/2]], (a - b)/(a + b)] * ((\cos[c + dx]\sin[c + dx]) / (1 + \cos[c + dx])^2 - \sin[c + dx] / (1 + \cos[c + dx])) / \sqrt{\cos[c + dx]} / (1 + \cos[c + dx]) + ((a + b)(-14a^2Ab + 63A^2b^3 + 8a^3B + 19a^2b^2B)\sqrt{\cos[c + dx]} / (1 + \cos[c + dx])\text{EllipticE}[\text{ArcSin}[\text{Tan}[(c + dx)/2]], (a - b)/(a + b)] * (-((a\sin[c + dx]) / ((a + b)(1 + \cos[c + dx]))) + ((b + a\cos[c + dx])\sin[c + dx]) / ((a + b)(1 + \cos[c + dx])^2))
\end{aligned}$$

)/Sqrt[(b + a*cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] - (b*(a + b)*(8*a^2*B - 2*a*b*(7*A + 3*B) + b^2*(63*A + 25*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*(-((a*sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])))) + ((b + a*cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2))/Sqrt[(b + a*cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] - a*(-14*a^2*A*b + 63*A*b^3 + 8*a^3*B + 19*a*b^2*B)*Cos[c + d*x]*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] - (-14*a^2*A*b + 63*A*b^3 + 8*a^3*B + 19*a*b^2*B)*(b + a*cos[c + d*x])*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] + (-14*a^2*A*b + 63*A*b^3 + 8*a^3*B + 19*a*b^2*B)*Cos[c + d*x]*(b + a*cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2 - (b*(a + b)*(8*a^2*B - 2*a*b*(7*A + 3*B) + b^2*(63*A + 25*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2)/(Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[1 - ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]) + ((a + b)*(-14*a^2*A*b + 63*A*b^3 + 8*a^3*B + 19*a*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2*Sqrt[1 - ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]/Sqrt[1 - Tan[(c + d*x)/2]^2])/(105*b^3*Sqrt[b + a*cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) - ((2*(a + b)*(-14*a^2*A*b + 63*A*b^3 + 8*a^3*B + 19*a*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(a + b)*(8*a^2*B - 2*a*b*(7*A + 3*B) + b^2*(63*A + 25*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-14*a^2*A*b + 63*A*b^3 + 8*a^3*B + 19*a*b^2*B)*Cos[c + d*x]*(b + a*cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(105*b^3*Sqrt[b + a*cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]))

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral} \left((B \sec(dx + c)^4 + A \sec(dx + c)^3) \sqrt{b \sec(dx + c) + a}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")

[Out] integral((B*sec(d*x + c)^4 + A*sec(d*x + c)^3)*sqrt(b*sec(d*x + c) + a), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A) \sqrt{b \sec(dx + c) + a} \sec(dx + c)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^3, x)

maple [B] time = 2.74, size = 3438, normalized size = 8.66

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^3*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x)

[Out]
$$-2/105/d*(1+\cos(d*x+c))^2*((b+a*\cos(d*x+c))/\cos(d*x+c))^{1/2}*(-1+\cos(d*x+c))^2*(-14*A*\cos(d*x+c)^5*a^3*b+7*A*\cos(d*x+c)^5*a^2*b^2+63*A*\cos(d*x+c)^5*a*b^3-4*B*\cos(d*x+c)^5*a^3*b+19*B*\cos(d*x+c)^5*a^2*b^2+14*A*\cos(d*x+c)^4*a^3*b-14*A*\cos(d*x+c)^4*a^2*b^2+8*B*\cos(d*x+c)^4*a^3*b+7*A*\cos(d*x+c)^3*a^2*b^2-4*B*\cos(d*x+c)^3*a^3*b-26*B*\cos(d*x+c)^3*a*b^3+B*\cos(d*x+c)^2*a^2*b^2-18*B*\cos(d*x+c)*a*b^3-28*A*\cos(d*x+c)^2*a*b^3-42*A*\cos(d*x+c)^3*b^4+25*B*\cos(d*x+c)^4*b^4-10*B*\cos(d*x+c)^2*b^4+19*B*\sin(d*x+c)*\cos(d*x+c)^4*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*EllipticF((-1+\cos(d*x+c))/\sin(d*x+c),((a-b)/(a+b))^{1/2})*a*b^3-35*A*\cos(d*x+c)^4*a*b^3+25*B*\cos(d*x+c)^5*a*b^3-20*B*\cos(d*x+c)^4*a^2*b^2-15*B*b^4-8*B*\sin(d*x+c)*\cos(d*x+c)^3*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*EllipticE((-1+\cos(d*x+c))/\sin(d*x+c),((a-b)/(a+b))^{1/2})*a^3*b-19*B*\sin(d*x+c)*\cos(d*x+c)^3*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*EllipticE((-1+\cos(d*x+c))/\sin(d*x+c),((a-b)/(a+b))^{1/2})*a^2*b^2-19*B*\sin(d*x+c)*\cos(d*x+c)^3*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*EllipticE((-1+\cos(d*x+c))/\sin(d*x+c),((a-b)/(a+b))^{1/2})*a*b^3+8*B*\sin(d*x+c)*\cos(d*x+c)^3*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*EllipticF((-1+\cos(d*x+c))/\sin(d*x+c),((a-b)/(a+b))^{1/2})*a^3*b+2*B*\sin(d*x+c)*\cos(d*x+c)^3*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*EllipticF((-1+\cos(d*x+c))/\sin(d*x+c),((a-b)/(a+b))^{1/2})*a^2*b^2+19*B*\sin(d*x+c)*\cos(d*x+c)^3*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*EllipticF((-1+\cos(d*x+c))/\sin(d*x+c),((a-b)/(a+b))^{1/2})*a*b^3+14*A*\sin(d*x+c)*\cos(d*x+c)^4*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*EllipticE((-1+\cos(d*x+c))/\sin(d*x+c),((a-b)/(a+b))^{1/2})*a^3*b+14*A*\sin(d*x+c)*\cos(d*x+c)^4*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*EllipticE((-1+\cos(d*x+c))/\sin(d*x+c),((a-b)/(a+b))^{1/2})*a^2*b^2-63*A*\sin(d*x+c)*\cos(d*x+c)^4*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*EllipticE((-1+\cos(d*x+c))/\sin(d*x+c),((a-b)/(a+b))^{1/2})*a*b^3-14*A*\sin(d*x+c)*\cos(d*x+c)^4*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*EllipticF((-1+\cos(d*x+c))/\sin(d*x+c),((a-b)/(a+b))^{1/2})*a^2*b^2+$$

$*a*b^3)/(b+a*\cos(d*x+c))/\cos(d*x+c)^3/\sin(d*x+c)^5/b^3$
maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \sqrt{a + \frac{b}{\cos(c+dx)}}}{\cos(c+dx)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(1/2))/cos(c + d*x)^3,x)

[Out] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(1/2))/cos(c + d*x)^3, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \sec(c + dx)) \sqrt{a + b \sec(c + dx)} \sec^3(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**3*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))**(1/2),x)

[Out] Integral((A + B*sec(c + d*x))*sqrt(a + b*sec(c + d*x))*sec(c + d*x)**3, x)

3.350 $\int \sec^2(c+dx) \sqrt{a+b \sec(c+dx)} (A+B \sec(c+dx)) dx$

Optimal. Leaf size=314

$$\frac{2(a-b)\sqrt{a+b}(-2a^2B+5aAb+9b^2B)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{\frac{-b(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b}\sec(c+dx)}{\sqrt{a+b}}\right)\right)}{15b^3d}$$

[Out] $-2/15*(a-b)*(5*A*a*b-2*B*a^2+9*B*b^2)*\cot(d*x+c)*\text{EllipticE}((a+b*\sec(d*x+c))^{1/2}/(a+b)^{1/2},((a+b)/(a-b))^{1/2})*(a+b)^{1/2}*(b*(1-\sec(d*x+c)))/(a+b)^{1/2}*(-b*(1+\sec(d*x+c)))/(a-b))^{1/2}/b^3/d-2/15*(a-b)*(5*A*b-2*B*a-9*B*b)*\cot(d*x+c)*\text{EllipticF}((a+b*\sec(d*x+c))^{1/2}/(a+b)^{1/2},((a+b)/(a-b))^{1/2})*(a+b)^{1/2}*(b*(1-\sec(d*x+c)))/(a+b)^{1/2}*(-b*(1+\sec(d*x+c)))/(a-b))^{1/2}/b^2/d+2/5*B*(a+b*\sec(d*x+c))^{3/2}*\tan(d*x+c)/b/d+2/15*(5*A*b-2*B*a)*(a+b*\sec(d*x+c))^{1/2}*\tan(d*x+c)/b/d$

Rubi [A] time = 0.60, antiderivative size = 314, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {4010, 4002, 4005, 3832, 4004}

$$\frac{2(a-b)\sqrt{a+b}(-2a^2B+5aAb+9b^2B)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{\frac{-b(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b}\sec(c+dx)}{\sqrt{a+b}}\right)\right)}{15b^3d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sec}[c+d*x]^2*\text{Sqrt}[a+b*\text{Sec}[c+d*x]]*(A+B*\text{Sec}[c+d*x]),x]$

[Out] $(-2*(a-b)*\text{Sqrt}[a+b]*(5*a*A*b-2*a^2*B+9*b^2*B)*\text{Cot}[c+d*x]*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[a+b*\text{Sec}[c+d*x]]/\text{Sqrt}[a+b]],(a+b)/(a-b)]*\text{Sqrt}[(b*(1-\text{Sec}[c+d*x]))/(a+b)]*\text{Sqrt}[-((b*(1+\text{Sec}[c+d*x]))/(a-b))]/(15*b^3*d)-(2*(a-b)*\text{Sqrt}[a+b]*(5*A*b-2*a*B-9*b*B)*\text{Cot}[c+d*x]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[a+b*\text{Sec}[c+d*x]]/\text{Sqrt}[a+b]],(a+b)/(a-b)]*\text{Sqrt}[(b*(1-\text{Sec}[c+d*x]))/(a+b)]*\text{Sqrt}[-((b*(1+\text{Sec}[c+d*x]))/(a-b))]/(15*b^2*d)+(2*(5*A*b-2*a*B)*\text{Sqrt}[a+b*\text{Sec}[c+d*x]]*\text{Tan}[c+d*x]/(15*b*d)+(2*B*(a+b*\text{Sec}[c+d*x])^{3/2}*\text{Tan}[c+d*x])/(5*b*d)$

Rule 3832

$\text{Int}[\text{csc}[(e_.)+(f_.)*(x_)]/\text{Sqrt}[\text{csc}[(e_.)+(f_.)*(x_)]*(b_.)+(a_)],x,\text{Symbol}]>\text{Simp}[(-2*\text{Rt}[a+b,2]*\text{Sqrt}[(b*(1-\text{Csc}[e+f*x]))/(a+b)]*\text{Sqrt}[-((b*(1+\text{Csc}[e+f*x]))/(a-b))]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[a+b*\text{Csc}[e+f*x]]/\text{Rt}[a+b,2]],(a+b)/(a-b))]/(b*f*\text{Cot}[e+f*x]),x]/;$ $\text{FreeQ}\{a,b,e,f,x\}$ && $\text{NeQ}[a^2-b^2,0]$

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] := -Simp[(B*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*Csc[e + f*x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] := Simp[(-2*(A*b - a*B)*Rt[a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A, 2]], (a*A + b*B)/(a*A - b*B)]/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] := Dist[A - B, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] := -Simp[(B*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Csc[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \sec^2(c + dx) \sqrt{a + b \sec(c + dx)} (A + B \sec(c + dx)) dx &= \frac{2B(a + b \sec(c + dx))^{3/2} \tan(c + dx)}{5bd} + \frac{2 \int \sec(c + dx) \sqrt{a + b \sec(c + dx)} dx}{5bd} \\
&= \frac{2(5Ab - 2aB) \sqrt{a + b \sec(c + dx)} \tan(c + dx)}{15bd} + \frac{2B \int \sec(c + dx) \sqrt{a + b \sec(c + dx)} dx}{15bd} \\
&= \frac{2(5Ab - 2aB) \sqrt{a + b \sec(c + dx)} \tan(c + dx)}{15bd} + \frac{2B \int \sec(c + dx) \sqrt{a + b \sec(c + dx)} dx}{15bd} \\
&= -\frac{2(a - b) \sqrt{a + b} (5aAb - 2a^2B + 9b^2B) \cot(c + dx)}{15bd}
\end{aligned}$$

Mathematica [B] time = 22.02, size = 2905, normalized size = 9.25

Result too large to show

Warning: Unable to verify antiderivative.

```

[In] Integrate[Sec[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
[Out] (Sqrt[a + b*Sec[c + d*x]]*((2*(5*a*A*b - 2*a^2*B + 9*b^2*B)*Sin[c + d*x])/
(15*b^2) + (2*Sec[c + d*x]*(5*A*b*Sin[c + d*x] + a*B*Sin[c + d*x]))/(15*b) +
(2*B*Sec[c + d*x]*Tan[c + d*x])/5))/d + (2*(-1/3*(a*A)/(Sqrt[b + a*Cos[c +
d*x]])*Sqrt[Sec[c + d*x]]) + (2*a^2*B)/(15*b*Sqrt[b + a*Cos[c + d*x]])*Sqrt[
Sec[c + d*x]]) - (3*b*B)/(5*Sqrt[b + a*Cos[c + d*x]])*Sqrt[Sec[c + d*x]]) -
(a^2*A*Sqrt[Sec[c + d*x]])/(3*b*Sqrt[b + a*Cos[c + d*x]]) + (A*b*Sqrt[Sec[c
+ d*x]])/(3*Sqrt[b + a*Cos[c + d*x]]) - (2*a*B*Sqrt[Sec[c + d*x]])/(15*Sqr
t[b + a*Cos[c + d*x]]) + (2*a^3*B*Sqrt[Sec[c + d*x]])/(15*b^2*Sqrt[b + a*Co
s[c + d*x]]) - (a^2*A*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*b*Sqrt[b + a*
Cos[c + d*x]]) - (3*a*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(5*Sqrt[b + a*
Cos[c + d*x]]) + (2*a^3*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(15*b^2*Sqrt
[b + a*Cos[c + d*x]])*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[a + b*Sec
[c + d*x]]*(2*(a + b)*(-5*a*A*b + 2*a^2*B - 9*b^2*B)*Sqrt[Cos[c + d*x]/(1 +
Cos[c + d*x]])*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Ell
ipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*b*(a + b)*(5*A*b - 2*
a*B + 9*b*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x]])*Sqrt[(b + a*Cos[c + d*x]
)/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)
/(a + b)] + (-5*a*A*b + 2*a^2*B - 9*b^2*B)*Cos[c + d*x]*(b + a*Cos[c + d*x]
)*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(15*b^2*d*(b + a*Cos[c + d*x])*Sqrt
[Sec[(c + d*x)/2]^2*Sqrt[Sec[c + d*x]]*((a*Sqrt[Cos[(c + d*x)/2]^2*Sec[c +
d*x]])*Sin[c + d*x]*(2*(a + b)*(-5*a*A*b + 2*a^2*B - 9*b^2*B)*Sqrt[Cos[c +

```

$$\begin{aligned}
& d*x)/(1 + \text{Cos}[c + d*x])]*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/((a + b)*(1 + \text{Cos}[c + d*x]))]*\text{EllipticE}[\text{ArcSin}[\text{Tan}[(c + d*x)/2]], (a - b)/(a + b)] + 2*b*(a + b)*(5 \\
& *A*b - 2*a*B + 9*b*B)*\text{Sqrt}[\text{Cos}[c + d*x]/(1 + \text{Cos}[c + d*x])]*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/((a + b)*(1 + \text{Cos}[c + d*x]))]*\text{EllipticF}[\text{ArcSin}[\text{Tan}[(c + d*x)/2]] \\
& , (a - b)/(a + b)] + (-5*a*A*b + 2*a^2*B - 9*b^2*B)*\text{Cos}[c + d*x]*(b + a*\text{Cos}[c + d*x])* \text{Sec}[(c + d*x)/2]^2*\text{Tan}[(c + d*x)/2])/((15*b^2*(b + a*\text{Cos}[c + d*x] \\
&)^(3/2)*\text{Sqrt}[\text{Sec}[(c + d*x)/2]^2]) - (\text{Sqrt}[\text{Cos}[(c + d*x)/2]^2*\text{Sec}[c + d*x]] \\
& *\text{Tan}[(c + d*x)/2]*(2*(a + b)*(-5*a*A*b + 2*a^2*B - 9*b^2*B)*\text{Sqrt}[\text{Cos}[c + d*x]/(1 + \text{Cos}[c + d*x])]*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/((a + b)*(1 + \text{Cos}[c + d*x] \\
&))]*\text{EllipticE}[\text{ArcSin}[\text{Tan}[(c + d*x)/2]], (a - b)/(a + b)] + 2*b*(a + b)*(5*A \\
& *b - 2*a*B + 9*b*B)*\text{Sqrt}[\text{Cos}[c + d*x]/(1 + \text{Cos}[c + d*x])]*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/((a + b)*(1 + \text{Cos}[c + d*x]))]*\text{EllipticF}[\text{ArcSin}[\text{Tan}[(c + d*x)/2]], \\
& (a - b)/(a + b)] + (-5*a*A*b + 2*a^2*B - 9*b^2*B)*\text{Cos}[c + d*x]*(b + a*\text{Cos}[c + d*x])* \text{Sec}[(c + d*x)/2]^2*\text{Tan}[(c + d*x)/2])/((15*b^2*\text{Sqrt}[b + a*\text{Cos}[c + d \\
& *x]]*\text{Sqrt}[\text{Sec}[(c + d*x)/2]^2]) + (2*\text{Sqrt}[\text{Cos}[(c + d*x)/2]^2*\text{Sec}[c + d*x]]*(\\
& ((-5*a*A*b + 2*a^2*B - 9*b^2*B)*\text{Cos}[c + d*x]*(b + a*\text{Cos}[c + d*x])* \text{Sec}[(c + \\
& d*x)/2]^4)/2 + ((a + b)*(-5*a*A*b + 2*a^2*B - 9*b^2*B)*\text{Sqrt}[(b + a*\text{Cos}[c + \\
& d*x])/((a + b)*(1 + \text{Cos}[c + d*x]))]*\text{EllipticE}[\text{ArcSin}[\text{Tan}[(c + d*x)/2]], (a \\
& - b)/(a + b)]*((\text{Cos}[c + d*x]*\text{Sin}[c + d*x])/(1 + \text{Cos}[c + d*x])^2 - \text{Sin}[c + d \\
& *x]/(1 + \text{Cos}[c + d*x])))/\text{Sqrt}[\text{Cos}[c + d*x]/(1 + \text{Cos}[c + d*x])] + (b*(a + b) \\
& *(5*A*b - 2*a*B + 9*b*B)*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/((a + b)*(1 + \text{Cos}[c + d*x] \\
&))]*\text{EllipticF}[\text{ArcSin}[\text{Tan}[(c + d*x)/2]], (a - b)/(a + b)]*((\text{Cos}[c + d*x]*\text{S \\
& in}[c + d*x])/(1 + \text{Cos}[c + d*x])^2 - \text{Sin}[c + d*x]/(1 + \text{Cos}[c + d*x])))/\text{Sqrt}[\\
& \text{Cos}[c + d*x]/(1 + \text{Cos}[c + d*x])] + ((a + b)*(-5*a*A*b + 2*a^2*B - 9*b^2*B)* \\
& \text{Sqrt}[\text{Cos}[c + d*x]/(1 + \text{Cos}[c + d*x])]*\text{EllipticE}[\text{ArcSin}[\text{Tan}[(c + d*x)/2]], (\\
& a - b)/(a + b)]*(-((a*\text{Sin}[c + d*x])/((a + b)*(1 + \text{Cos}[c + d*x])))) + ((b + a \\
& * \text{Cos}[c + d*x])* \text{Sin}[c + d*x])/((a + b)*(1 + \text{Cos}[c + d*x])^2))/\text{Sqrt}[(b + a*C \\
& os[c + d*x])/((a + b)*(1 + \text{Cos}[c + d*x]))] + (b*(a + b)*(5*A*b - 2*a*B + 9* \\
& b*B)*\text{Sqrt}[\text{Cos}[c + d*x]/(1 + \text{Cos}[c + d*x])]*\text{EllipticF}[\text{ArcSin}[\text{Tan}[(c + d*x)/2 \\
&]], (a - b)/(a + b)]*(-((a*\text{Sin}[c + d*x])/((a + b)*(1 + \text{Cos}[c + d*x])))) + ((\\
& b + a*\text{Cos}[c + d*x])* \text{Sin}[c + d*x])/((a + b)*(1 + \text{Cos}[c + d*x])^2))/\text{Sqrt}[(b \\
& + a*\text{Cos}[c + d*x])/((a + b)*(1 + \text{Cos}[c + d*x]))] - a*(-5*a*A*b + 2*a^2*B - 9 \\
& *b^2*B)*\text{Cos}[c + d*x]* \text{Sec}[(c + d*x)/2]^2*\text{Sin}[c + d*x]*\text{Tan}[(c + d*x)/2] - (-5 \\
& *a*A*b + 2*a^2*B - 9*b^2*B)*(b + a*\text{Cos}[c + d*x])* \text{Sec}[(c + d*x)/2]^2*\text{Sin}[c + \\
& d*x]*\text{Tan}[(c + d*x)/2] + (-5*a*A*b + 2*a^2*B - 9*b^2*B)*\text{Cos}[c + d*x]*(b + a \\
& * \text{Cos}[c + d*x])* \text{Sec}[(c + d*x)/2]^2*\text{Tan}[(c + d*x)/2]^2 + (b*(a + b)*(5*A*b - \\
& 2*a*B + 9*b*B)*\text{Sqrt}[\text{Cos}[c + d*x]/(1 + \text{Cos}[c + d*x])]*\text{Sqrt}[(b + a*\text{Cos}[c + d* \\
& x])/((a + b)*(1 + \text{Cos}[c + d*x]))]* \text{Sec}[(c + d*x)/2]^2)/(\text{Sqrt}[1 - \text{Tan}[(c + d* \\
& x)/2]^2]*\text{Sqrt}[1 - ((a - b)*\text{Tan}[(c + d*x)/2]^2)/(a + b)]) + ((a + b)*(-5*a*A \\
& *b + 2*a^2*B - 9*b^2*B)*\text{Sqrt}[\text{Cos}[c + d*x]/(1 + \text{Cos}[c + d*x])]*\text{Sqrt}[(b + a*C \\
& os[c + d*x])/((a + b)*(1 + \text{Cos}[c + d*x]))]* \text{Sec}[(c + d*x)/2]^2*\text{Sqrt}[1 - ((a \\
& - b)*\text{Tan}[(c + d*x)/2]^2)/(a + b)]/\text{Sqrt}[1 - \text{Tan}[(c + d*x)/2]^2])/((15*b^2*S \\
& qrt[b + a*\text{Cos}[c + d*x]]*\text{Sqrt}[\text{Sec}[(c + d*x)/2]^2]) + ((2*(a + b)*(-5*a*A*b + \\
& 2*a^2*B - 9*b^2*B)*\text{Sqrt}[\text{Cos}[c + d*x]/(1 + \text{Cos}[c + d*x])]*\text{Sqrt}[(b + a*\text{Cos}[c \\
& + d*x])/((a + b)*(1 + \text{Cos}[c + d*x]))]*\text{EllipticE}[\text{ArcSin}[\text{Tan}[(c + d*x)/2]],
\end{aligned}$$

$(a - b)/(a + b)] + 2*b*(a + b)*(5*A*b - 2*a*B + 9*b*B)*\text{Sqrt}[\text{Cos}[c + d*x]/(1 + \text{Cos}[c + d*x])]*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/((a + b)*(1 + \text{Cos}[c + d*x]))]*\text{EllipticF}[\text{ArcSin}[\text{Tan}[(c + d*x)/2]], (a - b)/(a + b)] + (-5*a*A*b + 2*a^2*B - 9*b^2*B)*\text{Cos}[c + d*x]*(b + a*\text{Cos}[c + d*x])* \text{Sec}[(c + d*x)/2]^2*\text{Tan}[(c + d*x)/2))*(-(\text{Cos}[(c + d*x)/2]*\text{Sec}[c + d*x]*\text{Sin}[(c + d*x)/2]) + \text{Cos}[(c + d*x)/2]^2*\text{Sec}[c + d*x]*\text{Tan}[c + d*x]))/(15*b^2*\text{Sqrt}[b + a*\text{Cos}[c + d*x]]*\text{Sqrt}[\text{Sec}[(c + d*x)/2]^2]*\text{Sqrt}[\text{Cos}[(c + d*x)/2]^2*\text{Sec}[c + d*x]]))$

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(B \sec(dx + c)^3 + A \sec(dx + c)^2\right)\sqrt{b \sec(dx + c) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")

[Out] integral((B*sec(d*x + c)^3 + A*sec(d*x + c)^2)*sqrt(b*sec(d*x + c) + a), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)\sqrt{b \sec(dx + c) + a} \sec(dx + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^2, x)

maple [B] time = 2.29, size = 2498, normalized size = 7.96

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^2*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x)

[Out] $-2/15/d*(1+\text{cos}(d*x+c))^2*((b+a*\text{cos}(d*x+c))/\text{cos}(d*x+c))^(1/2)*(-1+\text{cos}(d*x+c))^2*(-9*B*\text{sin}(d*x+c)*\text{cos}(d*x+c)^2*(\text{cos}(d*x+c)/(1+\text{cos}(d*x+c)))^(1/2)*((b+a*\text{cos}(d*x+c))/(1+\text{cos}(d*x+c)))/(a+b))^(1/2)*\text{EllipticE}((-1+\text{cos}(d*x+c))/\text{sin}(d*x+c), ((a-b)/(a+b))^(1/2))*a*b^2+5*A*\text{cos}(d*x+c)^3*b^3+2*B*\text{cos}(d*x+c)^3*a^3-3*b^3*B+9*B*\text{cos}(d*x+c)^3*b^3-6*B*\text{cos}(d*x+c)^2*b^3+5*A*\text{cos}(d*x+c)^3*a*b^2-10*A*\text{cos}(d*x+c)^2*a*b^2+9*B*\text{cos}(d*x+c)^4*a*b^2-2*B*\text{cos}(d*x+c)^3*a^2*b+B*\text{cos}(d*x+c)^2*a^2*b-4*B*\text{cos}(d*x+c)*a*b^2-5*A*\text{cos}(d*x+c)*b^3-5*B*\text{cos}(d*x+c)^3*a*b^2+5*A*\text{cos}(d*x+c)^4*a^2*b+5*A*\text{cos}(d*x+c)^4*a*b^2-5*A*\text{cos}(d*x+c)^3*a^2*b+B*\text{cos}(d*x$

$$\begin{aligned}
& +c)^4 a^2 b - 2B \cos(dx+c)^4 a^3 - 2B \sin(dx+c) \cos(dx+c)^3 (\cos(dx+c)/(1 \\
& +\cos(dx+c))^{1/2} * ((b+a \cos(dx+c))/(1+\cos(dx+c)) / (a+b))^{1/2} * \text{EllipticF} \\
& ((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{1/2}) * a^2 b + 7B \sin(dx+c) \cos(dx+c) \\
& ^3 (\cos(dx+c)/(1+\cos(dx+c))^{1/2} * ((b+a \cos(dx+c))/(1+\cos(dx+c)) / \\
& (a+b))^{1/2} * \text{EllipticF}((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{1/2}) * a b^2 \\
& + 2B \sin(dx+c) \cos(dx+c)^3 (\cos(dx+c)/(1+\cos(dx+c))^{1/2} * ((b+a \cos(dx+c) \\
& (dx+c))/(1+\cos(dx+c)) / (a+b))^{1/2} * \text{EllipticE}((-1+\cos(dx+c))/\sin(dx+c), ((a \\
& -b)/(a+b))^{1/2}) * a^2 b - 9B \sin(dx+c) \cos(dx+c)^3 (\cos(dx+c)/(1+\cos(dx+c) \\
& c))^{1/2} * ((b+a \cos(dx+c))/(1+\cos(dx+c)) / (a+b))^{1/2} * \text{EllipticE}((-1+\cos(dx+c) \\
& dx+c))/\sin(dx+c), ((a-b)/(a+b))^{1/2}) * a b^2 + 5A \sin(dx+c) \cos(dx+c)^2 (\\
& \cos(dx+c)/(1+\cos(dx+c))^{1/2} * ((b+a \cos(dx+c))/(1+\cos(dx+c)) / (a+b))^{1/2} \\
& * \text{EllipticF}((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{1/2}) * a b^2 - 5A \sin \\
& (dx+c) \cos(dx+c)^2 (\cos(dx+c)/(1+\cos(dx+c))^{1/2} * ((b+a \cos(dx+c))/(1 \\
& +\cos(dx+c)) / (a+b))^{1/2} * \text{EllipticE}((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b) \\
&)^{1/2}) * a^2 b - 5A \sin(dx+c) \cos(dx+c)^2 (\cos(dx+c)/(1+\cos(dx+c))^{1/2} \\
&) * ((b+a \cos(dx+c))/(1+\cos(dx+c)) / (a+b))^{1/2} * \text{EllipticE}((-1+\cos(dx+c))/s \\
& in(dx+c), ((a-b)/(a+b))^{1/2}) * a b^2 - 2B \sin(dx+c) \cos(dx+c)^2 (\cos(dx+c) \\
&) / (1+\cos(dx+c))^{1/2} * ((b+a \cos(dx+c))/(1+\cos(dx+c)) / (a+b))^{1/2} * \text{Ellip \\
& ticF}((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{1/2}) * a^2 b + 7B \sin(dx+c) * c \\
& \cos(dx+c)^2 (\cos(dx+c)/(1+\cos(dx+c))^{1/2} * ((b+a \cos(dx+c))/(1+\cos(dx+c) \\
& c)) / (a+b))^{1/2} * \text{EllipticF}((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{1/2}) * \\
& a b^2 + 2B \sin(dx+c) \cos(dx+c)^2 (\cos(dx+c)/(1+\cos(dx+c))^{1/2} * ((b+a * c \\
& \cos(dx+c))/(1+\cos(dx+c)) / (a+b))^{1/2} * \text{EllipticE}((-1+\cos(dx+c))/\sin(dx+c) \\
& , ((a-b)/(a+b))^{1/2}) * a^2 b + 5A \sin(dx+c) \cos(dx+c)^3 (\cos(dx+c)/(1+\cos(dx+c) \\
& dx+c))^{1/2} * ((b+a \cos(dx+c))/(1+\cos(dx+c)) / (a+b))^{1/2} * \text{EllipticF}((-1+ \\
& \cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{1/2}) * a b^2 - 5A \sin(dx+c) \cos(dx+c) \\
& ^3 (\cos(dx+c)/(1+\cos(dx+c))^{1/2} * ((b+a \cos(dx+c))/(1+\cos(dx+c)) / (a+b) \\
&)^{1/2} * \text{EllipticE}((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{1/2}) * a^2 b - 5A \\
& * \sin(dx+c) \cos(dx+c)^3 (\cos(dx+c)/(1+\cos(dx+c))^{1/2} * ((b+a \cos(dx+c) \\
&) / (1+\cos(dx+c)) / (a+b))^{1/2} * \text{EllipticE}((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(\\
& a+b))^{1/2}) * a b^2 + 2B \sin(dx+c) \cos(dx+c)^3 (\cos(dx+c)/(1+\cos(dx+c))^{1/2} \\
&) * ((b+a \cos(dx+c))/(1+\cos(dx+c)) / (a+b))^{1/2} * \text{EllipticE}((-1+\cos(dx+c) \\
&)) / \sin(dx+c), ((a-b)/(a+b))^{1/2}) * a^3 - 9B \sin(dx+c) \cos(dx+c)^3 (\cos(dx \\
& +c)/(1+\cos(dx+c))^{1/2} * ((b+a \cos(dx+c))/(1+\cos(dx+c)) / (a+b))^{1/2} * \text{Ell \\
& ipticE}((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{1/2}) * b^3 + 5A \sin(dx+c) * c \\
& \cos(dx+c)^2 (\cos(dx+c)/(1+\cos(dx+c))^{1/2} * ((b+a \cos(dx+c))/(1+\cos(dx+c) \\
& c)) / (a+b))^{1/2} * \text{EllipticF}((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{1/2}) * \\
& b^3 + 9B \sin(dx+c) \cos(dx+c)^2 (\cos(dx+c)/(1+\cos(dx+c))^{1/2} * ((b+a \cos \\
& (dx+c))/(1+\cos(dx+c)) / (a+b))^{1/2} * \text{EllipticF}((-1+\cos(dx+c))/\sin(dx+c), (\\
& (a-b)/(a+b))^{1/2}) * b^3 + 2B \sin(dx+c) \cos(dx+c)^2 (\cos(dx+c)/(1+\cos(dx+c) \\
& c))^{1/2} * ((b+a \cos(dx+c))/(1+\cos(dx+c)) / (a+b))^{1/2} * \text{EllipticE}((-1+\cos(dx+c) \\
& dx+c))/\sin(dx+c), ((a-b)/(a+b))^{1/2}) * a^3 - 9B \sin(dx+c) \cos(dx+c)^2 (\\
& \cos(dx+c)/(1+\cos(dx+c))^{1/2} * ((b+a \cos(dx+c))/(1+\cos(dx+c)) / (a+b))^{1/2} \\
&) * \text{EllipticE}((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{1/2}) * b^3 + 5A \sin(dx \\
& +c) \cos(dx+c)^3 (\cos(dx+c)/(1+\cos(dx+c))^{1/2} * ((b+a \cos(dx+c))/(1+\cos
\end{aligned}$$

$(d*x+c)/(a+b)^{(1/2)} * \text{EllipticF}((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{(1/2)}) * b^3 + 9*B*\sin(d*x+c)*\cos(d*x+c)^3 * (\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)} * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{(1/2)} * \text{EllipticF}((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{(1/2)}) * b^3 / (b+a*\cos(d*x+c))/\cos(d*x+c)^2 / \sin(d*x+c)^5 / b^2$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A) \sqrt{b \sec(dx + c) + a} \sec(dx + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^2, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \sqrt{a + \frac{b}{\cos(c+dx)}}}{\cos(c+dx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(1/2))/cos(c + d*x)^2,x)

[Out] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(1/2))/cos(c + d*x)^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \sec(c + dx)) \sqrt{a + b \sec(c + dx)} \sec^2(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**2*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))**(1/2),x)

[Out] Integral((A + B*sec(c + d*x))*sqrt(a + b*sec(c + d*x))*sec(c + d*x)**2, x)

$$3.351 \quad \int \sec(c+dx) \sqrt{a+b \sec(c+dx)} (A+B \sec(c+dx)) dx$$

Optimal. Leaf size=256

$$\frac{2(a-b)\sqrt{a+b}(aB+3Ab)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{\frac{-b(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b}\sec(c+dx)}{\sqrt{a+b}}\right)\middle|\frac{a+b}{a-b}\right)+2(a-b)\sqrt{a+b}}{3b^2d}$$

[Out] $-2/3*(a-b)*(3*A*b+B*a)*\cot(d*x+c)*\text{EllipticE}((a+b*\sec(d*x+c))^{1/2}/(a+b)^{1/2}),((a+b)/(a-b))^{1/2}*(a+b)^{1/2}*(b*(1-\sec(d*x+c))/(a+b))^{1/2}*(-b*(1+\sec(d*x+c))/(a-b))^{1/2}/b^2/d+2/3*(a-b)*(3*A-B)*\cot(d*x+c)*\text{EllipticF}((a+b*\sec(d*x+c))^{1/2}/(a+b)^{1/2}),((a+b)/(a-b))^{1/2}*(a+b)^{1/2}*(b*(1-\sec(d*x+c))/(a+b))^{1/2}*(-b*(1+\sec(d*x+c))/(a-b))^{1/2}/b/d+2/3*B*(a+b*\sec(d*x+c))^{1/2}*\tan(d*x+c)/d$

Rubi [A] time = 0.34, antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {4002, 4005, 3832, 4004}

$$\frac{2(a-b)\sqrt{a+b}(aB+3Ab)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{\frac{-b(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b}\sec(c+dx)}{\sqrt{a+b}}\right)\middle|\frac{a+b}{a-b}\right)+2(a-b)\sqrt{a+b}}{3b^2d}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]

[Out] $(-2*(a-b)*\text{Sqrt}[a+b]*(3*A*b+a*B)*\text{Cot}[c+d*x]*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[a+b*\text{Sec}[c+d*x]]/\text{Sqrt}[a+b]],(a+b)/(a-b)]*\text{Sqrt}[(b*(1-\text{Sec}[c+d*x]))/(a+b)]*\text{Sqrt}[-((b*(1+\text{Sec}[c+d*x]))/(a-b))]/(3*b^2*d)+(2*(a-b)*\text{Sqrt}[a+b]*(3*A-B)*\text{Cot}[c+d*x]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[a+b*\text{Sec}[c+d*x]]/\text{Sqrt}[a+b]],(a+b)/(a-b)]*\text{Sqrt}[(b*(1-\text{Sec}[c+d*x]))/(a+b)]*\text{Sqrt}[-((b*(1+\text{Sec}[c+d*x]))/(a-b))]/(3*b*d)+(2*B*\text{Sqrt}[a+b*\text{Sec}[c+d*x]]*\text{Tan}[c+d*x])/3*d)$

Rule 3832

Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)]/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_.)]*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := -Simp[(B*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)))/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] := Simp[(-2*(A*b - a*B)*Rt[a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]]/Rt[a + (b*B)/A, 2]], (a*A + b*B)/(a*A - b*B)]/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)))/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] := Dist[A - B, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rubi steps

$$\begin{aligned} \int \sec(c + dx) \sqrt{a + b \sec(c + dx)} (A + B \sec(c + dx)) dx &= \frac{2B \sqrt{a + b \sec(c + dx)} \tan(c + dx)}{3d} + \frac{2}{3} \int \frac{\sec(c + dx)}{\sqrt{a + b \sec(c + dx)}} dx \\ &= \frac{2B \sqrt{a + b \sec(c + dx)} \tan(c + dx)}{3d} + \frac{1}{3} ((a - b)(3A - B) \cot(c + dx) E\left(\sin^{-1}\left(\frac{\sqrt{a + b \sec(c + dx)}}{\sqrt{a + b}}\right)\right) \\ &= -\frac{2(a - b) \sqrt{a + b} (3Ab + aB) \cot(c + dx) E\left(\sin^{-1}\left(\frac{\sqrt{a + b \sec(c + dx)}}{\sqrt{a + b}}\right)\right)}{3b^2} \end{aligned}$$

Mathematica [A] time = 14.65, size = 408, normalized size = 1.59

$$\frac{\cos(c + dx) \sqrt{a + b \sec(c + dx)} (A + B \sec(c + dx)) \left(\frac{2(aB + 3Ab) \sin(c + dx)}{3b} + \frac{2}{3} B \tan(c + dx) \right)}{d(A \cos(c + dx) + B)} + \frac{2 \sqrt{\cos^2\left(\frac{1}{2}(c + dx)\right)} \sec\left(\frac{1}{2}(c + dx)\right)}{\sqrt{a + b \sec(c + dx)}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sec[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]

[Out] (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]))*(-2*(a + b)*(3*A*b + a*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] *Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*b*(a + b)*(3*A + B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] *Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - (3*A*b + a*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*b*d*(b + a*Cos[c + d*x])*(B + A*Cos[c + d*x])*Sqrt[Sec[(c + d*x)/2]^2*Sec[c + d*x]^(3/2)) + (Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]))*((2*(3*A*b + a*B)*Sin[c + d*x])/(3*b) + (2*B*Tan[c + d*x])/3))/(d*(B + A*Cos[c + d*x]))

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(B \sec(dx + c)^2 + A \sec(dx + c)\right)\sqrt{b \sec(dx + c) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")

[Out] integral((B*sec(d*x + c)^2 + A*sec(d*x + c))*sqrt(b*sec(d*x + c) + a), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)\sqrt{b \sec(dx + c) + a} \sec(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*sec(d*x + c), x)

maple [B] time = 2.01, size = 1752, normalized size = 6.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x)

[Out] -2/3/d*(-1+cos(d*x+c))^2*(-B*cos(d*x+c)^2*a^2+B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2

```

)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b-B*cos(d*x+c)
)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x
+c)))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))
)*a*b+B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*
x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-
b)/(a+b))^(1/2))*a*b-3*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b+3*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b-B*cos(d*x+c)^2
*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+
c)))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*
a*b-3*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d
*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a
-b)/(a+b))^(1/2))*a*b+3*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+
c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b+B*cos(d*x+c)^2*b^2-b^2*B+3*A*cos(d*
x+c)^2*b^2-3*A*cos(d*x+c)*b^2-3*A*cos(d*x+c)^2*a*b+B*cos(d*x+c)^3*a*b+B*cos
(d*x+c)^2*a*b-2*B*cos(d*x+c)*a*b+3*A*cos(d*x+c)^3*a*b+B*cos(d*x+c)^3*a^2-3*
A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c)
))/(1+cos(d*x+c)))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/
(a+b))^(1/2))*b^2+3*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2-B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*Ellipt
icE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2+B*cos(d*x+c)^2*sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(
a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2-3
*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c)
))/(1+cos(d*x+c)))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(
a+b))^(1/2))*b^2+3*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),((a-b)/(a+b))^(1/2))*b^2-B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2+B*cos(d*x+c)*sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(
1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2)*((b+a*c
os(d*x+c))/cos(d*x+c))^(1/2)*(1+cos(d*x+c))^2/(b+a*cos(d*x+c))/cos(d*x+c)/s
in(d*x+c)^5/b

```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A) \sqrt{b \sec(dx + c) + a} \sec(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*sec(d*x + c), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \sqrt{a + \frac{b}{\cos(c+dx)}}}{\cos(c+dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(1/2))/cos(c + d*x), x)

[Out] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(1/2))/cos(c + d*x), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \sec(c + dx)) \sqrt{a + b \sec(c + dx)} \sec(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))**(1/2), x)

[Out] Integral((A + B*sec(c + d*x))*sqrt(a + b*sec(c + d*x))*sec(c + d*x), x)

3.352 $\int \sqrt{a + b \sec(c + dx)} (A + B \sec(c + dx)) dx$

Optimal. Leaf size=320

$$\frac{2\sqrt{a+b}(B(a-b) + Ab) \cot(c + dx) \sqrt{\frac{b(1-\sec(c+dx))}{a+b}} \sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}} F\left(\sin^{-1}\left(\frac{\sqrt{a+b \sec(c+dx)}}{\sqrt{a+b}}\right) \middle| \frac{a+b}{a-b}\right) + 2A\sqrt{a+b} \cot(c + dx)}{bd}$$

[Out] $-2*(a-b)*B*\cot(d*x+c)*\text{EllipticE}((a+b*\sec(d*x+c))^{1/2}/(a+b)^{1/2}, ((a+b)/(a-b))^{1/2})*(a+b)^{1/2}*(b*(1-\sec(d*x+c)))/(a+b)^{1/2}*(-b*(1+\sec(d*x+c)))/(a-b)^{1/2}/b/d+2*(A*b+(a-b)*B)*\cot(d*x+c)*\text{EllipticF}((a+b*\sec(d*x+c))^{1/2}/(a+b)^{1/2}, ((a+b)/(a-b))^{1/2})*(a+b)^{1/2}*(b*(1-\sec(d*x+c)))/(a+b)^{1/2}*(-b*(1+\sec(d*x+c)))/(a-b)^{1/2}/b/d-2*A*\cot(d*x+c)*\text{EllipticPi}((a+b*\sec(d*x+c))^{1/2}/(a+b)^{1/2}, (a+b)/a, ((a+b)/(a-b))^{1/2})*(a+b)^{1/2}*(b*(1-\sec(d*x+c)))/(a+b)^{1/2}*(-b*(1+\sec(d*x+c)))/(a-b)^{1/2}/d$

Rubi [A] time = 0.29, antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3916, 3784, 4005, 3832, 4004}

$$\frac{2\sqrt{a+b}(B(a-b) + Ab) \cot(c + dx) \sqrt{\frac{b(1-\sec(c+dx))}{a+b}} \sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}} F\left(\sin^{-1}\left(\frac{\sqrt{a+b \sec(c+dx)}}{\sqrt{a+b}}\right) \middle| \frac{a+b}{a-b}\right) + 2A\sqrt{a+b} \cot(c + dx)}{bd}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sqrt}[a + b*\text{Sec}[c + d*x]]*(A + B*\text{Sec}[c + d*x]), x]$

[Out] $(-2*(a-b)*\text{Sqrt}[a+b]*B*\text{Cot}[c+d*x]*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[a+b*\text{Sec}[c+d*x]]/\text{Sqrt}[a+b]], (a+b)/(a-b)]*\text{Sqrt}[(b*(1-\text{Sec}[c+d*x]))/(a+b)]*\text{Sqrt}[-((b*(1+\text{Sec}[c+d*x]))/(a-b))]/(b*d) + (2*\text{Sqrt}[a+b]*(A*b+(a-b)*B)*\text{Cot}[c+d*x]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[a+b*\text{Sec}[c+d*x]]/\text{Sqrt}[a+b]], (a+b)/(a-b)]*\text{Sqrt}[(b*(1-\text{Sec}[c+d*x]))/(a+b)]*\text{Sqrt}[-((b*(1+\text{Sec}[c+d*x]))/(a-b))]/(b*d) - (2*A*\text{Sqrt}[a+b]*\text{Cot}[c+d*x]*\text{EllipticPi}[(a+b)/a, \text{ArcSin}[\text{Sqrt}[a+b*\text{Sec}[c+d*x]]/\text{Sqrt}[a+b]], (a+b)/(a-b)]*\text{Sqrt}[(b*(1-\text{Sec}[c+d*x]))/(a+b)]*\text{Sqrt}[-((b*(1+\text{Sec}[c+d*x]))/(a-b))]/d$

Rule 3784

$\text{Int}[1/\text{Sqrt}[\text{csc}[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] := \text{Simp}[(2*\text{Rt}[a + b, 2]*\text{Sqrt}[(b*(1 - \text{Csc}[c + d*x]))/(a + b)]*\text{Sqrt}[-((b*(1 + \text{Csc}[c + d*x]))/(a - b))]*\text{EllipticPi}[(a + b)/a, \text{ArcSin}[\text{Sqrt}[a + b*\text{Csc}[c + d*x]]/\text{Rt}[a + b, 2]], (a + b)/(a - b)]/(a*d*\text{Cot}[c + d*x]), x] /; \text{FreeQ}[\{a, b, c, d\}, x] \&\& \text{NeQ}[a^2 - b^2, 0]$

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol]
:> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)]/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3916

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol]
:> Dist[a*c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + Int[(Csc[e + f*x]*(b*c + a*d + b*d*Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol]
:> Simp[(-2*(A*b - a*B)*Rt[a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A, 2]], (a*A + b*B)/(a*A - b*B)]/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol]
:> Dist[A - B, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \sqrt{a + b \sec(c + dx)} (A + B \sec(c + dx)) dx = (aA) \int \frac{1}{\sqrt{a + b \sec(c + dx)}} dx + \int \frac{\sec(c + dx)(Ab + aB + bB \sec(c + dx))}{\sqrt{a + b \sec(c + dx)}} dx$$

$$= -\frac{2A\sqrt{a+b} \cot(c+dx) \Pi\left(\frac{a+b}{a}; \sin^{-1}\left(\frac{\sqrt{a+b \sec(c+dx)}}{\sqrt{a+b}}\right) \middle| \frac{a+b}{a-b}\right) \sqrt{b(1-\sec(c+dx))}}{d}$$

$$= -\frac{2(a-b)\sqrt{a+b} B \cot(c+dx) E\left(\sin^{-1}\left(\frac{\sqrt{a+b \sec(c+dx)}}{\sqrt{a+b}}\right) \middle| \frac{a+b}{a-b}\right) \sqrt{b(1-\sec(c+dx))}}{bd}$$

Mathematica [C] time = 17.96, size = 913, normalized size = 2.85

$$\frac{2B \cos(c + dx) \sqrt{a + b \sec(c + dx)} (A + B \sec(c + dx)) \sin(c + dx)}{d(B + A \cos(c + dx))} + \frac{2\sqrt{a + b \sec(c + dx)} (A + B \sec(c + dx))}{a \sqrt{a + b \sec(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]

[Out] (2*B*Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x])*Sin[c + d*x])/d*(B + A*Cos[c + d*x])) + (2*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]))*(a*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)/2] + b*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)/2] - 2*a*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)/2]^3 + a*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)/2]^5 - b*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)/2]^5 + (2*I)*a*A*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (2*I)*a*A*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - I*(a - b)*B*EllipticE[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - I*(a - b)*(A - B)*EllipticF[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)))/(Sqrt[(-a + b)/(a + b)]*d*Sqrt[b + a*Cos[c + d*x]]*(B + A*Cos[c + d*x])*Sec[c + d*x]^(3/2)*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(-1 + Tan[(c + d*x)/2]^2)*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)])

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\text{integral}\left((B \sec(dx + c) + A)\sqrt{b \sec(dx + c) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")

[Out] integral((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a), x)

$+c))^{1/2} * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)) / (a+b))^{1/2} * \sin(d*x+c) * \text{EllipticE}((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{1/2}) * a + B * (\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2} * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)) / (a+b))^{1/2} * \sin(d*x+c) * \text{EllipticE}((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{1/2}) * b - B * \cos(d*x+c)^2 * a + B * \cos(d*x+c) * a - b * B * \cos(d*x+c) + B * b) / \sin(d*x+c)^5 / (b+a*\cos(d*x+c))$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A) \sqrt{b \sec(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(A + \frac{B}{\cos(c + dx)} \right) \sqrt{a + \frac{b}{\cos(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(1/2),x)

[Out] int((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \sec(c + dx)) \sqrt{a + b \sec(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*(a+b*sec(d*x+c))**(1/2),x)

[Out] Integral((A + B*sec(c + d*x))*sqrt(a + b*sec(c + d*x)), x)

$$3.353 \quad \int \cos(c+dx) \sqrt{a+b \sec(c+dx)} (A+B \sec(c+dx)) dx$$

Optimal. Leaf size=344

$$\frac{\sqrt{a+b}(A+2B) \cot(c+dx) \sqrt{\frac{b(1-\sec(c+dx))}{a+b}} \sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}} F\left(\sin^{-1}\left(\frac{\sqrt{a+b \sec(c+dx)}}{\sqrt{a+b}}\right) \middle| \frac{a+b}{a-b}\right) + \sqrt{a+b}(2aB+Ab) \cot(c+dx)}{d}$$

[Out] A*(a-b)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2), ((a+b)/(a-b))^(1/2))* (a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b/d+(A+2*B)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2), ((a+b)/(a-b))^(1/2))* (a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/d-(A*b+2*B*a)*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2), (a+b)/a, ((a+b)/(a-b))^(1/2))* (a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a/d+A*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d

Rubi [A] time = 0.37, antiderivative size = 344, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4032, 4058, 3921, 3784, 3832, 4004}

$$\frac{\sqrt{a+b}(A+2B) \cot(c+dx) \sqrt{\frac{b(1-\sec(c+dx))}{a+b}} \sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}} F\left(\sin^{-1}\left(\frac{\sqrt{a+b \sec(c+dx)}}{\sqrt{a+b}}\right) \middle| \frac{a+b}{a-b}\right) + \sqrt{a+b}(2aB+Ab) \cot(c+dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]

[Out] (A*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))]/(b*d) + (Sqrt[a + b]*(A + 2*B)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))]/d - (Sqrt[a + b]*(A*b + 2*a*B)*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))]/(a*d) + (A*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/d

Rule 3784

Int[1/Sqrt[csc[(c_.) + (d_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] :> Simp[(2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b))]/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&

NeQ[a^2 - b^2, 0]

Rule 3832

Int[csc[(e_.) + (f_.)*(x_.)]/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)]/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 3921

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]

Rule 4004

Int[(csc[(e_.) + (f_.)*(x_.)]*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)))/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A, 2]], (a*A + b*B)/(a*A - b*B)]/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]

Rule 4032

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*A*(n + 1))*Csc[e + f*x] - A*b*(m + n + 1)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[0, m, 1] && LeQ[n, -1]

Rule 4058

Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.))/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] :> Int[(A + (B - C)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned}
\int \cos(c + dx) \sqrt{a + b \sec(c + dx)} (A + B \sec(c + dx)) dx &= \frac{A \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{d} + \int \frac{\frac{1}{2}(Ab + 2aB) + \dots}{\sqrt{a}} \\
&= \frac{A \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{d} - \frac{1}{2}(Ab) \int \frac{\sec(c + dx)}{\sqrt{a}} \\
&= \frac{A(a - b) \sqrt{a + b} \cot(c + dx) E\left(\sin^{-1}\left(\frac{\sqrt{a + b \sec(c + dx)}}{\sqrt{a + b}}\right) \middle| \frac{a}{a + b}\right)}{bd} \\
&= \frac{A(a - b) \sqrt{a + b} \cot(c + dx) E\left(\sin^{-1}\left(\frac{\sqrt{a + b \sec(c + dx)}}{\sqrt{a + b}}\right) \middle| \frac{a}{a + b}\right)}{bd}
\end{aligned}$$

Mathematica [C] time = 18.31, size = 1107, normalized size = 3.22

$$\frac{\sqrt{a + b \sec(c + dx)} \sqrt{\frac{-a \tan^2\left(\frac{1}{2}(c + dx)\right) + b \tan^2\left(\frac{1}{2}(c + dx)\right) + a + b}{\tan^2\left(\frac{1}{2}(c + dx)\right) + 1}}}{1} \left(aA \sqrt{\frac{b-a}{a+b}} \tan^5\left(\frac{1}{2}(c + dx)\right) - Ab \sqrt{\frac{b-a}{a+b}} \tan^5\left(\frac{1}{2}(c + dx)\right) - \dots \right)$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]

[Out] (Sqrt[a + b*Sec[c + d*x]]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]*(a*A*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2] + A*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2] - 2*a*A*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^3 + a*A*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 - A*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 - (2*I)*A*b*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (4*I)*a*B*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (2*I)*A*b*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (4*I)*a*B*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqr

$$\begin{aligned} & t[(-a + b)/(a + b)] * \text{Tan}[(c + d*x)/2], (a + b)/(a - b)] * \text{Tan}[(c + d*x)/2]^2 * \\ & \text{Sqrt}[1 - \text{Tan}[(c + d*x)/2]^2] * \text{Sqrt}[(a + b - a * \text{Tan}[(c + d*x)/2]^2 + b * \text{Tan}[(c \\ & + d*x)/2]^2)/(a + b)] - I * A * (a - b) * \text{EllipticE}[I * \text{ArcSinh}[\text{Sqrt}[(-a + b)/(a + \\ & b)] * \text{Tan}[(c + d*x)/2]], (a + b)/(a - b)] * \text{Sqrt}[1 - \text{Tan}[(c + d*x)/2]^2] * (1 + \text{T} \\ & \text{an}[(c + d*x)/2]^2) * \text{Sqrt}[(a + b - a * \text{Tan}[(c + d*x)/2]^2 + b * \text{Tan}[(c + d*x)/2]^ \\ & 2)/(a + b)] + (2 * I) * (a - b) * B * \text{EllipticF}[I * \text{ArcSinh}[\text{Sqrt}[(-a + b)/(a + b)] * \text{T} \\ & \text{an}[(c + d*x)/2]], (a + b)/(a - b)] * \text{Sqrt}[1 - \text{Tan}[(c + d*x)/2]^2] * (1 + \text{Tan}[(c \\ & + d*x)/2]^2) * \text{Sqrt}[(a + b - a * \text{Tan}[(c + d*x)/2]^2 + b * \text{Tan}[(c + d*x)/2]^2)/(a \\ & + b))] / (\text{Sqrt}[(-a + b)/(a + b)] * d * \text{Sqrt}[b + a * \text{Cos}[c + d*x]] * \text{Sqrt}[\text{Sec}[c + d*x \\ &] * \text{Sqrt}[(1 + \text{Tan}[(c + d*x)/2]^2)/(1 - \text{Tan}[(c + d*x)/2]^2)] * (b - b * \text{Tan}[(c + \\ & d*x)/2]^4 + a * (-1 + \text{Tan}[(c + d*x)/2]^2)^2)) \end{aligned}$$

fricas [F] time = 49.59, size = 0, normalized size = 0.00

$$\text{integral} \left((B \cos(dx + c) \sec(dx + c) + A \cos(dx + c)) \sqrt{b \sec(dx + c) + a}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")

[Out] integral((B*cos(d*x + c)*sec(d*x + c) + A*cos(d*x + c))*sqrt(b*sec(d*x + c) + a), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A) \sqrt{b \sec(dx + c) + a} \cos(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*cos(d*x + c), x)

maple [B] time = 1.98, size = 1389, normalized size = 4.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x)

[Out] 1/d*(-1+cos(d*x+c))^2*(2*A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b-A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/

```
(1+cos(d*x+c))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a-A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b-2*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b+2*B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a-2*B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b-4*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a+2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b-A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*sin(d*x+c)-A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b*sin(d*x+c)-2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b*sin(d*x+c)+2*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a-2*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b-4*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a*sin(d*x+c)-A*cos(d*x+c)^3+a*A*cos(d*x+c)^2*a-A*cos(d*x+c)^2*b+A*cos(d*x+c)*b*(1+cos(d*x+c))^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A) \sqrt{b \sec(dx + c) + a} \cos(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*cos(d*x + c), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c + dx) \left(A + \frac{B}{\cos(c + dx)} \right) \sqrt{a + \frac{b}{\cos(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(c + d*x)*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(1/2), x)`

[Out] `int(cos(c + d*x)*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(1/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \sec(c + dx)) \sqrt{a + b \sec(c + dx)} \cos(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))**(1/2), x)`

[Out] `Integral((A + B*sec(c + d*x))*sqrt(a + b*sec(c + d*x))*cos(c + d*x), x)`

$$3.354 \quad \int \cos^2(c+dx) \sqrt{a+b \sec(c+dx)} (A+B \sec(c+dx)) dx$$

Optimal. Leaf size=429

$$\frac{\sqrt{a+b} (4a^2A + 4abB - Ab^2) \cot(c+dx) \sqrt{\frac{b(1-\sec(c+dx))}{a+b}} \sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}} \Pi\left(\frac{a+b}{a}; \sin^{-1}\left(\frac{\sqrt{a+b \sec(c+dx)}}{\sqrt{a+b}}\right) \middle| \frac{a+b}{a-b}\right)}{4a^2d} + (4$$

[Out] 1/4*(a-b)*(A*b+4*B*a)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2), ((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c)))/(a-b)^(1/2)/a/b/d+1/4*(A*b+2*a*(A+2*B))*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2), ((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c)))/(a-b)^(1/2)/a/d-1/4*(4*A*a^2-A*b^2+4*B*a*b)*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2), (a+b)/a, ((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c)))/(a-b)^(1/2)/a^2/d+1/4*(A*b+4*B*a)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/a/d+1/2*A*cos(d*x+c)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d

Rubi [A] time = 0.73, antiderivative size = 429, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {4032, 4104, 4058, 3921, 3784, 3832, 4004}

$$\frac{\sqrt{a+b} (4a^2A + 4abB - Ab^2) \cot(c+dx) \sqrt{\frac{b(1-\sec(c+dx))}{a+b}} \sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}} \Pi\left(\frac{a+b}{a}; \sin^{-1}\left(\frac{\sqrt{a+b \sec(c+dx)}}{\sqrt{a+b}}\right) \middle| \frac{a+b}{a-b}\right)}{4a^2d} + (4$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]), x]

[Out] ((a - b)*Sqrt[a + b]*(A*b + 4*a*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))]/(4*a*b*d) + (Sqrt[a + b]*(A*b + 2*a*(A + 2*B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))]/(4*a*d) - (Sqrt[a + b]*(4*a^2*A - A*b^2 + 4*a*b*B)*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))]/(4*a^2*d) + ((A*b + 4*a*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(4*a*d) + (A*cos[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(2*d)

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_.)], x_Symbol] := Simp[(2*Rt[a
+ b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)]/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_S
ymbol] := Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)]/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_.)], x_Symbol] := Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] := Simp[(-2*(A*b - a*B)*Rt[a
+ (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
2]], (a*A + b*B)/(a*A - b*B)]/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4032

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_.))^m*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] := Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n
), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp[A*b*m - a
*B*n - (b*B*n + a*A*(n + 1))*Csc[e + f*x] - A*b*(m + n + 1)*Csc[e + f*x]^2,
x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[
a^2 - b^2, 0] && LtQ[0, m, 1] && LeQ[n, -1]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] := Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
```

+ Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0]

Rule 4104

Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m, x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]

Rubi steps

$$\begin{aligned}
 \int \cos^2(c + dx) \sqrt{a + b \sec(c + dx)} (A + B \sec(c + dx)) dx &= \frac{A \cos(c + dx) \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{2d} + \frac{1}{2} \int \\
 &= \frac{(Ab + 4aB) \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{4ad} + \frac{A \cos(c + dx)}{2d} \\
 &= \frac{(Ab + 4aB) \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{4ad} + \frac{A \cos(c + dx)}{2d} \\
 &= \frac{(a - b) \sqrt{a + b} (Ab + 4aB) \cot(c + dx) E \left(\sin^{-1} \left(\frac{\sqrt{a + b} \sin(c + dx)}{\sqrt{a + b \sec(c + dx)}} \right) \right)}{4abd} \\
 &= \frac{(a - b) \sqrt{a + b} (Ab + 4aB) \cot(c + dx) E \left(\sin^{-1} \left(\frac{\sqrt{a + b} \sin(c + dx)}{\sqrt{a + b \sec(c + dx)}} \right) \right)}{4abd}
 \end{aligned}$$

Mathematica [B] time = 18.94, size = 1149, normalized size = 2.68

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[Cos[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]

[Out] (A*Sqrt[a + b*Sec[c + d*x]]*Sin[2*(c + d*x)])/(4*d) + (Sqrt[a + b*Sec[c + d*x]]*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(a*A*b*Tan[(c + d*x)/2] + A*b^2*Tan[(c + d*x)/2] + 4*a^2*B*Tan[(c + d*x)/2] + 4*a*b*B*Tan[(c + d*x)/2] - 2*a*

$$\begin{aligned}
& A*b*\tan\left[\frac{c+dx}{2}\right]^3 - 8*a^2*B*\tan\left[\frac{c+dx}{2}\right]^3 + a*A*b*\tan\left[\frac{c+dx}{2}\right]^5 - A*b^2*\tan\left[\frac{c+dx}{2}\right]^5 + 4*a^2*B*\tan\left[\frac{c+dx}{2}\right]^5 - 4*a*b*B*\tan\left[\frac{c+dx}{2}\right]^5 + 8*a^2*A*\text{EllipticPi}\left[-1, \text{ArcSin}\left[\tan\left[\frac{c+dx}{2}\right]\right], \frac{a-b}{a+b}\right]*\sqrt{1 - \tan\left[\frac{c+dx}{2}\right]^2}*\sqrt{\frac{a+b - a*\tan\left[\frac{c+dx}{2}\right]^2 + b*\tan\left[\frac{c+dx}{2}\right]^2}{a+b}} - 2*A*b^2*\text{EllipticPi}\left[-1, \text{ArcSin}\left[\tan\left[\frac{c+dx}{2}\right]\right], \frac{a-b}{a+b}\right]*\sqrt{1 - \tan\left[\frac{c+dx}{2}\right]^2}*\sqrt{\frac{a+b - a*\tan\left[\frac{c+dx}{2}\right]^2 + b*\tan\left[\frac{c+dx}{2}\right]^2}{a+b}} + 8*a*b*B*\text{EllipticPi}\left[-1, \text{ArcSin}\left[\tan\left[\frac{c+dx}{2}\right]\right], \frac{a-b}{a+b}\right]*\sqrt{1 - \tan\left[\frac{c+dx}{2}\right]^2}*\sqrt{\frac{a+b - a*\tan\left[\frac{c+dx}{2}\right]^2 + b*\tan\left[\frac{c+dx}{2}\right]^2}{a+b}} + 8*a^2*A*\text{EllipticPi}\left[-1, \text{ArcSin}\left[\tan\left[\frac{c+dx}{2}\right]\right], \frac{a-b}{a+b}\right]*\tan\left[\frac{c+dx}{2}\right]^2*\sqrt{1 - \tan\left[\frac{c+dx}{2}\right]^2}*\sqrt{\frac{a+b - a*\tan\left[\frac{c+dx}{2}\right]^2 + b*\tan\left[\frac{c+dx}{2}\right]^2}{a+b}} - 2*A*b^2*\text{EllipticPi}\left[-1, \text{ArcSin}\left[\tan\left[\frac{c+dx}{2}\right]\right], \frac{a-b}{a+b}\right]*\tan\left[\frac{c+dx}{2}\right]^2*\sqrt{1 - \tan\left[\frac{c+dx}{2}\right]^2}*\sqrt{\frac{a+b - a*\tan\left[\frac{c+dx}{2}\right]^2 + b*\tan\left[\frac{c+dx}{2}\right]^2}{a+b}} + 8*a*b*B*\text{EllipticPi}\left[-1, \text{ArcSin}\left[\tan\left[\frac{c+dx}{2}\right]\right], \frac{a-b}{a+b}\right]*\tan\left[\frac{c+dx}{2}\right]^2*\sqrt{1 - \tan\left[\frac{c+dx}{2}\right]^2}*\sqrt{\frac{a+b - a*\tan\left[\frac{c+dx}{2}\right]^2 + b*\tan\left[\frac{c+dx}{2}\right]^2}{a+b}} + (a+b)*(A*b + 4*a*B)*\text{EllipticE}\left[\text{ArcSin}\left[\tan\left[\frac{c+dx}{2}\right]\right], \frac{a-b}{a+b}\right]*\sqrt{1 - \tan\left[\frac{c+dx}{2}\right]^2}*(1 + \tan\left[\frac{c+dx}{2}\right]^2)*\sqrt{\frac{a+b - a*\tan\left[\frac{c+dx}{2}\right]^2 + b*\tan\left[\frac{c+dx}{2}\right]^2}{a+b}} - 2*a*(2*a*A - A*b + 4*b*B)*\text{EllipticF}\left[\text{ArcSin}\left[\tan\left[\frac{c+dx}{2}\right]\right], \frac{a-b}{a+b}\right]*\sqrt{1 - \tan\left[\frac{c+dx}{2}\right]^2}*(1 + \tan\left[\frac{c+dx}{2}\right]^2)*\sqrt{\frac{a+b - a*\tan\left[\frac{c+dx}{2}\right]^2 + b*\tan\left[\frac{c+dx}{2}\right]^2}{a+b}})/(4*a*d*\sqrt{b + a*\cos[c + dx]}*\sqrt{\sec[c + dx]}*(1 + \tan\left[\frac{c+dx}{2}\right]^2)^{(3/2)}*\sqrt{\frac{a+b - a*\tan\left[\frac{c+dx}{2}\right]^2 + b*\tan\left[\frac{c+dx}{2}\right]^2}{1 + \tan\left[\frac{c+dx}{2}\right]^2}})
\end{aligned}$$

fricas [F] time = 1.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(B \cos(dx + c)^2 \sec(dx + c) + A \cos(dx + c)^2\right) \sqrt{b \sec(dx + c) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(dx+c)^2*(A+B*sec(dx+c))*(a+b*sec(dx+c))^(1/2),x, algorithm="fricas")

[Out] integral((B*cos(dx + c)^2*sec(dx + c) + A*cos(dx + c)^2)*sqrt(b*sec(dx + c) + a), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A) \sqrt{b \sec(dx + c) + a} \cos(dx + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(dx+c)^2*(A+B*sec(dx+c))*(a+b*sec(dx+c))^(1/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^2, x)
maple [B] time = 1.96, size = 2065, normalized size = 4.81

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^2*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x)

[Out]
$$-1/4/d*(-1+\cos(d*x+c))^2*(-2*A*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*EllipticPi((-1+\cos(d*x+c))/\sin(d*x+c),-1,((a-b)/(a+b))^{1/2})*b^2*\sin(d*x+c)-4*B*\cos(d*x+c)^2*a^2+A*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*EllipticE((-1+\cos(d*x+c))/\sin(d*x+c),((a-b)/(a+b))^{1/2})*b^2*\sin(d*x+c)+4*B*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*EllipticE((-1+\cos(d*x+c))/\sin(d*x+c),((a-b)/(a+b))^{1/2})*a^2*\sin(d*x+c)+8*A*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*EllipticPi((-1+\cos(d*x+c))/\sin(d*x+c),-1,((a-b)/(a+b))^{1/2})*a^2*\sin(d*x+c)+4*B*\cos(d*x+c)*\sin(d*x+c)*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*EllipticE((-1+\cos(d*x+c))/\sin(d*x+c),((a-b)/(a+b))^{1/2})*a*b-8*B*\cos(d*x+c)*\sin(d*x+c)*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*EllipticF((-1+\cos(d*x+c))/\sin(d*x+c),((a-b)/(a+b))^{1/2})*a*b+A*\cos(d*x+c)*\sin(d*x+c)*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*EllipticE((-1+\cos(d*x+c))/\sin(d*x+c),((a-b)/(a+b))^{1/2})*a*b+2*A*\cos(d*x+c)*\sin(d*x+c)*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*EllipticF((-1+\cos(d*x+c))/\sin(d*x+c),((a-b)/(a+b))^{1/2})*a*b+A*\cos(d*x+c)^2*b^2-A*\cos(d*x+c)*b^2+8*B*\cos(d*x+c)*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*EllipticPi((-1+\cos(d*x+c))/\sin(d*x+c),-1,((a-b)/(a+b))^{1/2})*\sin(d*x+c)*a*b-A*\cos(d*x+c)^2*a*b-2*A*\cos(d*x+c)*a*b+4*B*\cos(d*x+c)^2*a*b-4*B*\cos(d*x+c)*a*b-4*A*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*EllipticF((-1+\cos(d*x+c))/\sin(d*x+c),((a-b)/(a+b))^{1/2})*a^2*\sin(d*x+c)+3*A*\cos(d*x+c)^3*a*b+4*B*\cos(d*x+c)^3*a^2+8*A*\cos(d*x+c)*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*EllipticPi((-1+\cos(d*x+c))/\sin(d*x+c),-1,((a-b)/(a+b))^{1/2})*\sin(d*x+c)*a^2-2*A*\cos(d*x+c)*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*EllipticPi((-1+\cos(d*x+c))/\sin(d*x+c),-1,((a-b)/(a+b))^{1/2})*\sin(d*x+c)*b^2-4*A*\cos(d*x+c)*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*EllipticF((-1+\cos(d*x+c))/\sin(d*x+c),((a-b)/(a+b))^{1/2})*\sin(d*x+c)*a^2+2*A*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*EllipticF((-1+\cos(d*x+c))/\sin(d*x+c),((a-b)/(a+b))^{1/2})*a*b*\sin(d*x+c)+A*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*EllipticE((-1+\cos(d*x+c))/\sin(d*x+c),((a-b)/(a+b))^{1/2})*a*b*\sin(d*x+c)+8*B*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*$$

$(b+a\cos(dx+c))/(1+\cos(dx+c))/(a+b)^{(1/2)} * \text{EllipticPi}((-1+\cos(dx+c))/\sin(dx+c), -1, ((a-b)/(a+b))^{(1/2)} * a * b * \sin(dx+c) - 8 * B * (\cos(dx+c)/(1+\cos(dx+c)))^{(1/2)} * ((b+a\cos(dx+c))/(1+\cos(dx+c))/(a+b))^{(1/2)} * \text{EllipticF}((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{(1/2)} * a * b * \sin(dx+c) + 4 * B * (\cos(dx+c)/(1+\cos(dx+c)))^{(1/2)} * ((b+a\cos(dx+c))/(1+\cos(dx+c))/(a+b))^{(1/2)} * \text{EllipticE}((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{(1/2)} * a * b * \sin(dx+c) - 2 * A * \cos(dx+c)^2 * a^2 + 2 * A * \cos(dx+c)^4 * a^2 + A * \cos(dx+c) * \sin(dx+c) * (\cos(dx+c)/(1+\cos(dx+c)))^{(1/2)} * ((b+a\cos(dx+c))/(1+\cos(dx+c))/(a+b))^{(1/2)} * \text{EllipticE}((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{(1/2)} * b^2 + 4 * B * \cos(dx+c) * \sin(dx+c) * (\cos(dx+c)/(1+\cos(dx+c)))^{(1/2)} * ((b+a\cos(dx+c))/(1+\cos(dx+c))/(a+b))^{(1/2)} * \text{EllipticE}((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{(1/2)} * a^2 * (1+\cos(dx+c)))^2 * ((b+a\cos(dx+c))/\cos(dx+c))^{(1/2)} / (b+a\cos(dx+c))/\sin(dx+c)^5 / a$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A) \sqrt{b \sec(dx + c) + a} \cos(dx + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(dx+c)^2*(A+B*sec(dx+c))*(a+b*sec(dx+c))^(1/2),x, algorithm="maxima")

[Out] integrate((B*sec(dx + c) + A)*sqrt(b*sec(dx + c) + a)*cos(dx + c)^2, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c + dx)^2 \left(A + \frac{B}{\cos(c + dx)} \right) \sqrt{a + \frac{b}{\cos(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^2*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(1/2),x)

[Out] int(cos(c + d*x)^2*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \sec(c + dx)) \sqrt{a + b \sec(c + dx)} \cos^2(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(dx+c)**2*(A+B*sec(dx+c))*(a+b*sec(dx+c))**(1/2),x)

[Out] Integral((A + B*sec(c + d*x))*sqrt(a + b*sec(c + d*x))*cos(c + d*x)**2, x)

$$3.355 \quad \int \cos^3(c+dx) \sqrt{a + b \sec(c + dx)} (A+B \sec(c+dx)) dx$$

Optimal. Leaf size=509

$$\frac{(16a^2A + 6abB - 3Ab^2) \sin(c + dx) \sqrt{a + b \sec(c + dx)}}{24a^2d} + \frac{(a - b) \sqrt{a + b} (16a^2A + 6abB - 3Ab^2) \cot(c + dx) \sqrt{\frac{b(1 - \sec(c + dx))}{a + b}}}{24a^2d}$$

[Out] 1/24*(a-b)*(16*A*a^2-3*A*b^2+6*B*a*b)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/b/d+1/24*(2*a+b)*(8*A*a-3*A*b+6*B*a)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d-1/8*(4*A*a^2*b+A*b^3+8*B*a^3-2*B*a*b^2)*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/d+1/24*(16*A*a^2-3*A*b^2+6*B*a*b)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/a^2/d+1/12*(A*b+6*B*a)*cos(d*x+c)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/a/d+1/3*A*cos(d*x+c)^2*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d

Rubi [A] time = 1.13, antiderivative size = 509, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {4032, 4104, 4058, 3921, 3784, 3832, 4004}

$$\frac{(16a^2A + 6abB - 3Ab^2) \sin(c + dx) \sqrt{a + b \sec(c + dx)}}{24a^2d} + \frac{(a - b) \sqrt{a + b} (16a^2A + 6abB - 3Ab^2) \cot(c + dx) \sqrt{\frac{b(1 - \sec(c + dx))}{a + b}}}{24a^2d}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^3*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]

[Out] ((a - b)*Sqrt[a + b]*(16*a^2*A - 3*A*b^2 + 6*a*b*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))]/(24*a^2*b*d) + (Sqrt[a + b]*(2*a + b)*(8*a*A - 3*A*b + 6*a*B)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))]/(24*a^2*d) - (Sqrt[a + b]*(4*a^2*A*b + A*b^3 + 8*a^3*B - 2*a*b^2*B)*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))]/(8*a^3*d) + ((16*a^2*A - 3*A*b^2 + 6*a*b*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(24*a^2*d) + ((A*b + 6*a*B)*Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x]])/(24*a^2*d)

$c + d*x]]*\sin[c + d*x]/(12*a*d) + (A*\cos[c + d*x]^2*\sqrt{a + b*\sec[c + d*x]})*\sin[c + d*x]/(3*d)$

Rule 3784

$\text{Int}[1/\sqrt{\csc[(c_.) + (d_.)*(x_.)]*(b_.) + (a_.)}, x_Symbol] \rightarrow \text{Simp}[(2*\text{Rt}[a + b, 2]*\sqrt{(b*(1 - \csc[c + d*x]))/(a + b)}*\sqrt{-((b*(1 + \csc[c + d*x]))/(a - b))})*\text{EllipticPi}[(a + b)/a, \text{ArcSin}[\sqrt{a + b*\csc[c + d*x]}/\text{Rt}[a + b, 2]], (a + b)/(a - b)]/(a*d*\text{Cot}[c + d*x]), x] /; \text{FreeQ}\{a, b, c, d\}, x\} \&\& \text{NeQ}[a^2 - b^2, 0]$

Rule 3832

$\text{Int}[\csc[(e_.) + (f_.)*(x_.)]/\sqrt{\csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)}, x_Symbol] \rightarrow \text{Simp}[(-2*\text{Rt}[a + b, 2]*\sqrt{(b*(1 - \csc[e + f*x]))/(a + b)}*\sqrt{-((b*(1 + \csc[e + f*x]))/(a - b))})*\text{EllipticF}[\text{ArcSin}[\sqrt{a + b*\csc[e + f*x]}/\text{Rt}[a + b, 2]], (a + b)/(a - b)]/(b*f*\text{Cot}[e + f*x]), x] /; \text{FreeQ}\{a, b, e, f\}, x\} \&\& \text{NeQ}[a^2 - b^2, 0]$

Rule 3921

$\text{Int}[(\csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))/\sqrt{\csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)}, x_Symbol] \rightarrow \text{Dist}[c, \text{Int}[1/\sqrt{a + b*\csc[e + f*x]}, x], x] + \text{Dist}[d, \text{Int}[\csc[e + f*x]/\sqrt{a + b*\csc[e + f*x]}, x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x\} \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 - b^2, 0]$

Rule 4004

$\text{Int}[(\csc[(e_.) + (f_.)*(x_.)]*(\csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)))/\sqrt{\csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)}, x_Symbol] \rightarrow \text{Simp}[(-2*(A*b - a*B)*\text{Rt}[a + (b*B)/A, 2]*\sqrt{(b*(1 - \csc[e + f*x]))/(a + b)}*\sqrt{-((b*(1 + \csc[e + f*x]))/(a - b))})*\text{EllipticE}[\text{ArcSin}[\sqrt{a + b*\csc[e + f*x]}/\text{Rt}[a + (b*B)/A, 2]], (a*A + b*B)/(a*A - b*B)]/(b^2*f*\text{Cot}[e + f*x]), x] /; \text{FreeQ}\{a, b, e, f, A, B\}, x\} \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{EqQ}[A^2 - B^2, 0]$

Rule 4032

$\text{Int}[(\csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(\csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m*(\csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] \rightarrow \text{Simp}[(A*\text{Cot}[e + f*x]*(a + b*\csc[e + f*x])^m*(d*\csc[e + f*x])^n)/(f*n), x] - \text{Dist}[1/(d*n), \text{Int}[(a + b*\csc[e + f*x])^{m-1}*(d*\csc[e + f*x])^{n+1}*\text{Simp}[A*b*m - a*B*n - (b*B*n + a*A*(n+1))*\csc[e + f*x] - A*b*(m+n+1)*\csc[e + f*x]^2, x], x], x] /; \text{FreeQ}\{a, b, d, e, f, A, B\}, x\} \&\& \text{NeQ}[A*b - a*B, 0] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{LtQ}[0, m, 1] \&\& \text{LeQ}[n, -1]$

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] := Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
+ Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.
))^m, x_Symbol] := Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x] /; FreeQ[{a, b, d,
e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \cos^3(c + dx) \sqrt{a + b \sec(c + dx)} (A + B \sec(c + dx)) dx &= \frac{A \cos^2(c + dx) \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{3d} + \frac{1}{3} \int \\
&= \frac{(Ab + 6aB) \cos(c + dx) \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{12ad} \\
&= \frac{(16a^2A - 3Ab^2 + 6abB) \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{24a^2d} \\
&= \frac{(16a^2A - 3Ab^2 + 6abB) \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{24a^2d} \\
&= \frac{(a - b) \sqrt{a + b} (16a^2A - 3Ab^2 + 6abB) \cot(c + dx) E}{2} \\
&= \frac{(a - b) \sqrt{a + b} (16a^2A - 3Ab^2 + 6abB) \cot(c + dx) E}{2}
\end{aligned}$$

Mathematica [B] time = 20.04, size = 1548, normalized size = 3.04

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^3*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
[Out] (Sqrt[a + b*Sec[c + d*x]]*((A*Sin[c + d*x])/12 + ((A*b + 6*a*B)*Sin[2*(c +
d*x)]/(24*a) + (A*Sin[3*(c + d*x)]/12))/d - (Sqrt[a + b*Sec[c + d*x]]*Sqr
t[(1 - Tan[(c + d*x)/2]^2)^(-1)]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan
[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]*(16*a^3*A*Tan[(c + d*x)/2] + 16*
a^2*A*b*Tan[(c + d*x)/2] - 3*a*A*b^2*Tan[(c + d*x)/2] - 3*A*b^3*Tan[(c + d*
x)/2] + 6*a^2*b*B*Tan[(c + d*x)/2] + 6*a*b^2*B*Tan[(c + d*x)/2] - 32*a^3*A*
Tan[(c + d*x)/2]^3 + 6*a*A*b^2*Tan[(c + d*x)/2]^3 - 12*a^2*b*B*Tan[(c + d*x
)/2]^3 + 16*a^3*A*Tan[(c + d*x)/2]^5 - 16*a^2*A*b*Tan[(c + d*x)/2]^5 - 3*a*
A*b^2*Tan[(c + d*x)/2]^5 + 3*A*b^3*Tan[(c + d*x)/2]^5 + 6*a^2*b*B*Tan[(c +
d*x)/2]^5 - 6*a*b^2*B*Tan[(c + d*x)/2]^5 + 24*a^2*A*b*EllipticPi[-1, ArcSin
[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a +
b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 6*A*b^3*Ellipt
icPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/
2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] +
48*a^3*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1
- Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/
2]^2)/(a + b)] - 12*a*b^2*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b
)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2
+ b*Tan[(c + d*x)/2]^2)/(a + b)] + 24*a^2*A*b*EllipticPi[-1, ArcSin[Tan[(c
+ d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2
]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 6*A
*b^3*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x
)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*
Tan[(c + d*x)/2]^2)/(a + b)] + 48*a^3*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)
/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt
[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 12*a*b^2*
B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2
]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan
[(c + d*x)/2]^2)/(a + b)] + (a + b)*(16*a^2*A - 3*A*b^2 + 6*a*b*B)*Elliptic
E[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(
1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x
)/2]^2)/(a + b)] - 2*a*(-(A*b^2) + 2*a*b*(7*A - 3*B) + 12*a^2*B)*EllipticF[
ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1
+ Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/
2]^2)/(a + b)))/(24*a^2*d*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sqrt
[1 + Tan[(c + d*x)/2]^2]*(a*(-1 + Tan[(c + d*x)/2]^2) - b*(1 + Tan[(c + d*x
)/2]^2)))
```

fricas [F] time = 59.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(B \cos(dx + c)^3 \sec(dx + c) + A \cos(dx + c)^3\right)\sqrt{b \sec(dx + c) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*cos(d*x + c)^3*sec(d*x + c) + A*cos(d*x + c)^3)*sqrt(b*sec(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A) \sqrt{b \sec(dx + c) + a} \cos(dx + c)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^3, x)
```

maple [B] time = 2.49, size = 2954, normalized size = 5.80

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/24/d*(-1+cos(d*x+c))^2*(16*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)+8*A*cos(d*x+c)^3*a^3-16*A*cos(d*x+c)^2*a^3-3*A*cos(d*x+c)^2*b^3-12*B*cos(d*x+c)^2*a^3-A*cos(d*x+c)^3*a*b^2+6*A*cos(d*x+c)^2*a^2*b+3*A*cos(d*x+c)^2*a*b^2-16*A*cos(d*x+c)*a^2*b-2*A*cos(d*x+c)*a*b^2+18*B*cos(d*x+c)^3*a^2*b-6*B*cos(d*x+c)^2*a^2*b+6*B*cos(d*x+c)^2*a*b^2-12*B*cos(d*x+c)*a^2*b-6*B*cos(d*x+c)*a*b^2+3*A*cos(d*x+c)*b^3+10*A*cos(d*x+c)^4*a^2*b-3*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)+12*B*cos(d*x+c)^4*a^3-24*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^3+24*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)-28*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)+2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+16*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))
```

$$\begin{aligned}
& (1+\cos(dx+c))/(a+b)^{1/2} \text{EllipticE}((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{1/2}) * a^2 b \sin(dx+c) - 3A * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * ((b+a \cos(dx+c))/(1+\cos(dx+c)))/(a+b)^{1/2} \text{EllipticE}((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{1/2}) * a * b^2 \sin(dx+c) - 12B * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * ((b+a \cos(dx+c))/(1+\cos(dx+c)))/(a+b)^{1/2} \text{EllipticPi}((-1+\cos(dx+c))/\sin(dx+c), -1, ((a-b)/(a+b))^{1/2}) * a * b^2 \sin(dx+c) + 12B \text{EllipticF}((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{1/2}) * a^2 * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * ((b+a \cos(dx+c))/(1+\cos(dx+c)))/(a+b)^{1/2} \sin(dx+c) * b + 6B * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * ((b+a \cos(dx+c))/(1+\cos(dx+c)))/(a+b)^{1/2} \text{EllipticE}((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{1/2}) * a^2 b \sin(dx+c) + 6B * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * ((b+a \cos(dx+c))/(1+\cos(dx+c)))/(a+b)^{1/2} \text{EllipticE}((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{1/2}) * a * b^2 \sin(dx+c) + 6A * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * ((b+a \cos(dx+c))/(1+\cos(dx+c)))/(a+b)^{1/2} \text{EllipticPi}((-1+\cos(dx+c))/\sin(dx+c), -1, ((a-b)/(a+b))^{1/2}) * \sin(dx+c) * \cos(dx+c) * b^3 + 16A * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * ((b+a \cos(dx+c))/(1+\cos(dx+c)))/(a+b)^{1/2} \text{EllipticE}((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{1/2}) * \sin(dx+c) * \cos(dx+c) * a^3 - 3A * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * ((b+a \cos(dx+c))/(1+\cos(dx+c)))/(a+b)^{1/2} \text{EllipticE}((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{1/2}) * \sin(dx+c) * \cos(dx+c) * b^3 + 48B * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * ((b+a \cos(dx+c))/(1+\cos(dx+c)))/(a+b)^{1/2} \text{EllipticPi}((-1+\cos(dx+c))/\sin(dx+c), -1, ((a-b)/(a+b))^{1/2}) * \sin(dx+c) * \cos(dx+c) * a^3 + 48B * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * ((b+a \cos(dx+c))/(1+\cos(dx+c)))/(a+b)^{1/2} \text{EllipticPi}((-1+\cos(dx+c))/\sin(dx+c), -1, ((a-b)/(a+b))^{1/2}) * a^3 \sin(dx+c) + 6A * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * ((b+a \cos(dx+c))/(1+\cos(dx+c)))/(a+b)^{1/2} \text{EllipticPi}((-1+\cos(dx+c))/\sin(dx+c), -1, ((a-b)/(a+b))^{1/2}) * b^3 \sin(dx+c) - 24B * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * ((b+a \cos(dx+c))/(1+\cos(dx+c)))/(a+b)^{1/2} \text{EllipticF}((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{1/2}) * a^3 \sin(dx+c) + 8A * \cos(dx+c)^5 * a^3 + 6B * \text{EllipticE}((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{1/2}) * \cos(dx+c) * b^2 * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * ((b+a \cos(dx+c))/(1+\cos(dx+c)))/(a+b)^{1/2} \sin(dx+c) * a + 24A * \text{EllipticPi}((-1+\cos(dx+c))/\sin(dx+c), -1, ((a-b)/(a+b))^{1/2}) * \cos(dx+c) * a^2 * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * ((b+a \cos(dx+c))/(1+\cos(dx+c)))/(a+b)^{1/2} \sin(dx+c) * b - 28A * \text{EllipticF}((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{1/2}) * \cos(dx+c) * a^2 * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * ((b+a \cos(dx+c))/(1+\cos(dx+c)))/(a+b)^{1/2} \sin(dx+c) * b + 2A * \text{EllipticF}((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{1/2}) * \cos(dx+c) * b^2 * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * ((b+a \cos(dx+c))/(1+\cos(dx+c)))/(a+b)^{1/2} \sin(dx+c) * a + 16A * \text{EllipticE}((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{1/2}) * \cos(dx+c) * a^2 * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * ((b+a \cos(dx+c))/(1+\cos(dx+c)))/(a+b)^{1/2} \sin(dx+c) * b - 3A * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * ((b+a \cos(dx+c))/(1+\cos(dx+c)))/(a+b)^{1/2} \text{EllipticE}((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{1/2}) * \sin(dx+c) * \cos(dx+c) * a * b^2 - 12B * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * ((b+a \cos(dx+c))/(1+\cos(dx+c)))/(a+b)^{1/2} \text{EllipticPi}((-1+\cos(dx+c))/\sin(dx+c), -1, ((a-b)/(a+b))^{1/2}) * \sin(dx+c) * \cos(dx+c) * a * b^2 + 12B * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * ((b+a \cos(dx+c))/(1+\cos(dx+c)))/(a+b)^{1/2} *
\end{aligned}$$

EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*b+6*B*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*sin(d*x+c)*b*(1+cos(d*x+c))^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)^5/a^2

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A) \sqrt{b \sec(dx + c) + a} \cos(dx + c)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^3, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c + dx)^3 \left(A + \frac{B}{\cos(c + dx)} \right) \sqrt{a + \frac{b}{\cos(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^3*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(1/2),x)

[Out] int(cos(c + d*x)^3*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \sec(c + dx)) \sqrt{a + b \sec(c + dx)} \cos^3(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**3*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))**(1/2),x)

[Out] Integral((A + B*sec(c + d*x))*sqrt(a + b*sec(c + d*x))*cos(c + d*x)**3, x)

$$3.356 \quad \int \sec^3(c+dx)(a+b \sec(c+dx))^{3/2}(A+B \sec(c+dx)) dx$$

Optimal. Leaf size=475

$$\frac{2(-8a^2B + 18aAb - 49b^2B) \tan(c+dx)(a+b \sec(c+dx))^{3/2}}{315b^2d} - \frac{2(-8a^3B + 18a^2Ab - 39ab^2B - 75Ab^3) \tan(c+dx)}{315b^2d}$$

[Out] $2/315*(a-b)*(18*A*a^3*b-246*A*a*b^3-8*B*a^4-33*B*a^2*b^2-147*B*b^4)*\cot(d*x+c)*\text{EllipticE}((a+b*\sec(d*x+c))^{1/2}/(a+b)^{1/2},((a+b)/(a-b))^{1/2})*(a+b)^{1/2}*(b*(1-\sec(d*x+c))/(a+b))^{1/2}*(-b*(1+\sec(d*x+c))/(a-b))^{1/2}/b^4/d-2/315*(a-b)*(3*b^3*(25*A-49*B)-3*a*b^2*(57*A-13*B)-6*a^2*b*(3*A-B)+8*a^3*B)*\cot(d*x+c)*\text{EllipticF}((a+b*\sec(d*x+c))^{1/2}/(a+b)^{1/2},((a+b)/(a-b))^{1/2})*(a+b)^{1/2}*(b*(1-\sec(d*x+c))/(a+b))^{1/2}*(-b*(1+\sec(d*x+c))/(a-b))^{1/2}/b^3/d-2/315*(18*A*a*b-8*B*a^2-49*B*b^2)*(a+b*\sec(d*x+c))^{3/2}*\tan(d*x+c)/b^2/d+2/63*(9*A*b-4*B*a)*(a+b*\sec(d*x+c))^{5/2}*\tan(d*x+c)/b^2/d+2/9*B*\sec(d*x+c)*(a+b*\sec(d*x+c))^{5/2}*\tan(d*x+c)/b/d-2/315*(18*A*a^2*b-75*A*b^3-8*B*a^3-39*B*a*b^2)*(a+b*\sec(d*x+c))^{1/2}*\tan(d*x+c)/b^2/d$

Rubi [A] time = 1.22, antiderivative size = 475, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4033, 4082, 4002, 4005, 3832, 4004}

$$\frac{2(-8a^2B + 18aAb - 49b^2B) \tan(c+dx)(a+b \sec(c+dx))^{3/2}}{315b^2d} - \frac{2(18a^2Ab - 8a^3B - 39ab^2B - 75Ab^3) \tan(c+dx)}{315b^2d}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]

[Out] $(2*(a-b)*\text{Sqrt}[a+b]*(18*a^3*A*b-246*a*A*b^3-8*a^4*B-33*a^2*b^2*B-147*b^4*B)*\text{Cot}[c+d*x]*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[a+b*\text{Sec}[c+d*x]]]/\text{Sqrt}[a+b]],(a+b)/(a-b)]*\text{Sqrt}[(b*(1-\text{Sec}[c+d*x]))/(a+b)]*\text{Sqrt}[-((b*(1+\text{Sec}[c+d*x]))/(a-b))]/(315*b^4*d)-(2*(a-b)*\text{Sqrt}[a+b]*(3*b^3*(25*A-49*B)-3*a*b^2*(57*A-13*B)-6*a^2*b*(3*A-B)+8*a^3*B)*\text{Cot}[c+d*x]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[a+b*\text{Sec}[c+d*x]]]/\text{Sqrt}[a+b]],(a+b)/(a-b)]*\text{Sqrt}[(b*(1-\text{Sec}[c+d*x]))/(a+b)]*\text{Sqrt}[-((b*(1+\text{Sec}[c+d*x]))/(a-b))]/(315*b^3*d)-(2*(18*a^2*A*b-75*A*b^3-8*a^3*B-39*a*b^2*B)*\text{Sqrt}[a+b*\text{Sec}[c+d*x]]*\text{Tan}[c+d*x])/(315*b^2*d)-(2*(18*a*A*b-8*a^2*B-49*b^2*B)*(a+b*\text{Sec}[c+d*x])^{3/2}*\text{Tan}[c+d*x])/(315*b^2*d)+(2*(9*A*b-4*a*B)*(a+b*\text{Sec}[c+d*x])^{5/2}*\text{Tan}[c+d*x])/(63*b^2*d)+(2*B*\text{Sec}[c+d*x]*(a+b*\text{Sec}[c+d*x])^{5/2}*\text{Tan}[c+d*x])/(9*b*d)$

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol]
:> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)]/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol]
:> -Simp[(B*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*Csc[e + f*x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol]
:> Simp[(-2*(A*b - a*B)*Rt[a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A, 2]], (a*A + b*B)/(a*A - b*B)]/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol]
:> Dist[A - B, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 4033

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol]
:> -Simp[(B*d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2))/(b*f*(m + n)), x] + Dist[d^2/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 2)*Simp[a*B*(n - 2) + B*b*(m + n - 1)*Csc[e + f*x] + (A*b*(m + n) - a*B*(n - 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && NeQ[m + n, 0] && !IGtQ[m, 1]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)^(m_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \sec^3(c + dx)(a + b \sec(c + dx))^{3/2}(A + B \sec(c + dx)) dx &= \frac{2B \sec(c + dx)(a + b \sec(c + dx))^{5/2} \tan(c + dx)}{9bd} + \\ &= \frac{2(9Ab - 4aB)(a + b \sec(c + dx))^{5/2} \tan(c + dx)}{63b^2d} + \\ &= -\frac{2(18aAb - 8a^2B - 49b^2B)(a + b \sec(c + dx))^{3/2}}{315b^2d} \\ &= -\frac{2(18a^2Ab - 75Ab^3 - 8a^3B - 39ab^2B)\sqrt{a + b \sec(c + dx)}}{315b^2d} \\ &= -\frac{2(18a^2Ab - 75Ab^3 - 8a^3B - 39ab^2B)\sqrt{a + b \sec(c + dx)}}{315b^2d} \\ &= \frac{2(a - b)\sqrt{a + b}(18a^3Ab - 246aAb^3 - 8a^4B - 33a^2b^2B)}{315b^2d} \end{aligned}$$

Mathematica [B] time = 26.44, size = 3766, normalized size = 7.93

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]), x]
[Out] (Cos[c + d*x]*(a + b*Sec[c + d*x])^(3/2)*((2*(-18*a^3*A*b + 246*a*A*b^3 + 8*a^4*B + 33*a^2*b^2*B + 147*b^4*B)*Sin[c + d*x])/(315*b^3) + (2*Sec[c + d*x]^3*(9*A*b*Ssin[c + d*x] + 10*a*B*Ssin[c + d*x]))/63 + (2*Sec[c + d*x]^2*(72*a*A*b*Ssin[c + d*x] + 3*a^2*B*Ssin[c + d*x] + 49*b^2*B*Ssin[c + d*x]))/(315*b) + (2*Sec[c + d*x]*(9*a^2*A*b*Ssin[c + d*x] + 75*A*b^3*Ssin[c + d*x] - 4*a^3*B*Ssin[c + d*x] + 88*a*b^2*B*Ssin[c + d*x]))/(315*b^2) + (2*b*B*Sec[c + d*x]^3*Tan[c + d*x])/9))/(d*(b + a*Cos[c + d*x])) - (2*((2*a^3*A)/(35*b*Sqrt[b +
```

$$\begin{aligned}
& a \cos[c + dx] \sqrt{\sec[c + dx]} - (82a^2b) / (105 \sqrt{b + a \cos[c + dx]} \sqrt{\sec[c + dx]}) - (11a^2B) / (105 \sqrt{b + a \cos[c + dx]} \sqrt{\sec[c + dx]}) - (8a^4B) / (315b^2 \sqrt{b + a \cos[c + dx]} \sqrt{\sec[c + dx]}) - (7b^2B) / (15 \sqrt{b + a \cos[c + dx]} \sqrt{\sec[c + dx]}) - (31a^2A \sqrt{\sec[c + dx]}) / (105 \sqrt{b + a \cos[c + dx]}) + (2a^4A \sqrt{\sec[c + dx]}) / (35b^2 \sqrt{b + a \cos[c + dx]}) + (5Ab^2 \sqrt{\sec[c + dx]}) / (21 \sqrt{b + a \cos[c + dx]}) - (8a^5B \sqrt{\sec[c + dx]}) / (315b^3 \sqrt{b + a \cos[c + dx]}) - (31a^3B \sqrt{\sec[c + dx]}) / (315b \sqrt{b + a \cos[c + dx]}) + (13abB \sqrt{\sec[c + dx]}) / (105 \sqrt{b + a \cos[c + dx]}) - (82a^2A \cos[2(c + dx)] \sqrt{\sec[c + dx]}) / (105 \sqrt{b + a \cos[c + dx]}) + (2a^4A \cos[2(c + dx)] \sqrt{\sec[c + dx]}) / (35b^2 \sqrt{b + a \cos[c + dx]}) - (8a^5B \cos[2(c + dx)] \sqrt{\sec[c + dx]}) / (315b^3 \sqrt{b + a \cos[c + dx]}) - (11a^3B \cos[2(c + dx)] \sqrt{\sec[c + dx]}) / (105b \sqrt{b + a \cos[c + dx]}) - (7abB \cos[2(c + dx)] \sqrt{\sec[c + dx]}) / (15 \sqrt{b + a \cos[c + dx]}) \sqrt{\cos[(c + dx)/2]^2 \sec[c + dx]} (a + b \sec[c + dx])^{3/2} (2(a + b)(-18a^3Ab + 246aAb^3 + 8a^4B + 33a^2b^2B + 147b^4B) \sqrt{\cos[c + dx]/(1 + \cos[c + dx])} \sqrt{(b + a \cos[c + dx]) / ((a + b)(1 + \cos[c + dx]))} \text{EllipticE}[\text{ArcSin}[\text{Tan}[(c + dx)/2]], (a - b)/(a + b)] - 2b(a + b)(8a^3B - 6a^2b(3A + B) + 3ab^2(57A + 13B) + 3b^3(25A + 49B)) \sqrt{\cos[c + dx]/(1 + \cos[c + dx])} \sqrt{(b + a \cos[c + dx]) / ((a + b)(1 + \cos[c + dx]))} \text{EllipticF}[\text{ArcSin}[\text{Tan}[(c + dx)/2]], (a - b)/(a + b)] + (-18a^3Ab + 246aAb^3 + 8a^4B + 33a^2b^2B + 147b^4B) \cos[c + dx] (b + a \cos[c + dx]) \sec[(c + dx)/2]^2 \tan[(c + dx)/2]) / (315b^3 d (b + a \cos[c + dx])^2 \sqrt{\sec[(c + dx)/2]^2} \sec[c + dx]^{3/2} (-1/315 (a \sqrt{\cos[(c + dx)/2]^2 \sec[c + dx]} \sin[c + dx] (2(a + b)(-18a^3Ab + 246aAb^3 + 8a^4B + 33a^2b^2B + 147b^4B) \sqrt{\cos[c + dx]/(1 + \cos[c + dx])} \sqrt{(b + a \cos[c + dx]) / ((a + b)(1 + \cos[c + dx]))} \text{EllipticE}[\text{ArcSin}[\text{Tan}[(c + dx)/2]], (a - b)/(a + b)] - 2b(a + b)(8a^3B - 6a^2b(3A + B) + 3ab^2(57A + 13B) + 3b^3(25A + 49B)) \sqrt{\cos[c + dx]/(1 + \cos[c + dx])} \sqrt{(b + a \cos[c + dx]) / ((a + b)(1 + \cos[c + dx]))} \text{EllipticF}[\text{ArcSin}[\text{Tan}[(c + dx)/2]], (a - b)/(a + b)] + (-18a^3Ab + 246aAb^3 + 8a^4B + 33a^2b^2B + 147b^4B) \cos[c + dx] (b + a \cos[c + dx]) \sec[(c + dx)/2]^2 \tan[(c + dx)/2])) / (b^3 (b + a \cos[c + dx])^{3/2} \sqrt{\sec[(c + dx)/2]^2} + (\sqrt{\cos[(c + dx)/2]^2 \sec[c + dx]} \tan[(c + dx)/2] (2(a + b)(-18a^3Ab + 246aAb^3 + 8a^4B + 33a^2b^2B + 147b^4B) \sqrt{\cos[c + dx]/(1 + \cos[c + dx])} \sqrt{(b + a \cos[c + dx]) / ((a + b)(1 + \cos[c + dx]))} \text{EllipticE}[\text{ArcSin}[\text{Tan}[(c + dx)/2]], (a - b)/(a + b)] - 2b(a + b)(8a^3B - 6a^2b(3A + B) + 3ab^2(57A + 13B) + 3b^3(25A + 49B)) \sqrt{\cos[c + dx]/(1 + \cos[c + dx])} \sqrt{(b + a \cos[c + dx]) / ((a + b)(1 + \cos[c + dx]))} \text{EllipticF}[\text{ArcSin}[\text{Tan}[(c + dx)/2]], (a - b)/(a + b)] + (-18a^3Ab + 246aAb^3 + 8a^4B + 33a^2b^2B + 147b^4B) \cos[c + dx] (b + a \cos[c + dx]) \sec[(c + dx)/2]^2 \tan[(c + dx)/2])) / (315b^3 \sqrt{b + a \cos[c + dx]} \sqrt{\sec[(c + dx)/2]^2} - (2 \sqrt{\cos[(c + dx)/2]^2 \sec[c + dx]} ((-18a^3Ab + 246aAb^3 + 8a^4B + 33a^2b^2B + 147b^4B) \cos[c +
\end{aligned}$$

$$\begin{aligned}
& d*x)*(b + a*\text{Cos}[c + d*x])*\text{Sec}[(c + d*x)/2]^4/2 + ((a + b)*(-18*a^3*A*b + 246*a^4*B + 8*a^4*B + 33*a^2*b^2*B + 147*b^4*B)*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/((a + b)*(1 + \text{Cos}[c + d*x]))]*\text{EllipticE}[\text{ArcSin}[\text{Tan}[(c + d*x)/2]], (a - b)/(a + b)]*((\text{Cos}[c + d*x]*\text{Sin}[c + d*x])/(1 + \text{Cos}[c + d*x])^2 - \text{Sin}[c + d*x]/(1 + \text{Cos}[c + d*x])))/\text{Sqrt}[\text{Cos}[c + d*x]/(1 + \text{Cos}[c + d*x])] - (b*(a + b)*(8*a^3*B - 6*a^2*b*(3*A + B) + 3*a*b^2*(57*A + 13*B) + 3*b^3*(25*A + 49*B))*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/((a + b)*(1 + \text{Cos}[c + d*x]))]*\text{EllipticF}[\text{ArcSin}[\text{Tan}[(c + d*x)/2]], (a - b)/(a + b)]*((\text{Cos}[c + d*x]*\text{Sin}[c + d*x])/(1 + \text{Cos}[c + d*x])^2 - \text{Sin}[c + d*x]/(1 + \text{Cos}[c + d*x])))/\text{Sqrt}[\text{Cos}[c + d*x]/(1 + \text{Cos}[c + d*x])]) + ((a + b)*(-18*a^3*A*b + 246*a^4*B + 8*a^4*B + 33*a^2*b^2*B + 147*b^4*B)*\text{Sqrt}[\text{Cos}[c + d*x]/(1 + \text{Cos}[c + d*x])]*\text{EllipticE}[\text{ArcSin}[\text{Tan}[(c + d*x)/2]], (a - b)/(a + b)]*(-((a*\text{Sin}[c + d*x])/(a + b)*(1 + \text{Cos}[c + d*x])))) + ((b + a*\text{Cos}[c + d*x])*\text{Sin}[c + d*x])/((a + b)*(1 + \text{Cos}[c + d*x])^2))/\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/((a + b)*(1 + \text{Cos}[c + d*x]))] - (b*(a + b)*(8*a^3*B - 6*a^2*b*(3*A + B) + 3*a*b^2*(57*A + 13*B) + 3*b^3*(25*A + 49*B))*\text{Sqrt}[\text{Cos}[c + d*x]/(1 + \text{Cos}[c + d*x])]*\text{EllipticF}[\text{ArcSin}[\text{Tan}[(c + d*x)/2]], (a - b)/(a + b)]*(-((a*\text{Sin}[c + d*x])/(a + b)*(1 + \text{Cos}[c + d*x])))) + ((b + a*\text{Cos}[c + d*x])*\text{Sin}[c + d*x])/((a + b)*(1 + \text{Cos}[c + d*x])^2))/\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/((a + b)*(1 + \text{Cos}[c + d*x]))] - a*(-18*a^3*A*b + 246*a^4*B + 8*a^4*B + 33*a^2*b^2*B + 147*b^4*B)*\text{Cos}[c + d*x]*\text{Sec}[(c + d*x)/2]^2*\text{Sin}[c + d*x]*\text{Tan}[(c + d*x)/2] - (-18*a^3*A*b + 246*a^4*B + 8*a^4*B + 33*a^2*b^2*B + 147*b^4*B)*(b + a*\text{Cos}[c + d*x])*\text{Sec}[(c + d*x)/2]^2*\text{Sin}[c + d*x]*\text{Tan}[(c + d*x)/2] + (-18*a^3*A*b + 246*a^4*B + 8*a^4*B + 33*a^2*b^2*B + 147*b^4*B)*\text{Cos}[c + d*x]*(b + a*\text{Cos}[c + d*x])*\text{Sec}[(c + d*x)/2]^2*\text{Tan}[(c + d*x)/2]^2 - (b*(a + b)*(8*a^3*B - 6*a^2*b*(3*A + B) + 3*a*b^2*(57*A + 13*B) + 3*b^3*(25*A + 49*B))*\text{Sqrt}[\text{Cos}[c + d*x]/(1 + \text{Cos}[c + d*x])]*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/((a + b)*(1 + \text{Cos}[c + d*x]))]*\text{Sec}[(c + d*x)/2]^2)/(\text{Sqrt}[1 - \text{Tan}[(c + d*x)/2]^2]*\text{Sqrt}[1 - ((a - b)*\text{Tan}[(c + d*x)/2]^2)/(a + b)]) + ((a + b)*(-18*a^3*A*b + 246*a^4*B + 8*a^4*B + 33*a^2*b^2*B + 147*b^4*B)*\text{Sqrt}[\text{Cos}[c + d*x]/(1 + \text{Cos}[c + d*x])]*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/((a + b)*(1 + \text{Cos}[c + d*x]))]*\text{Sec}[(c + d*x)/2]^2*\text{Sqrt}[1 - ((a - b)*\text{Tan}[(c + d*x)/2]^2)/(a + b)])/\text{Sqrt}[1 - \text{Tan}[(c + d*x)/2]^2])/(315*b^3*\text{Sqrt}[b + a*\text{Cos}[c + d*x]]*\text{Sqrt}[\text{Sec}[(c + d*x)/2]^2]) - ((2*(a + b)*(-18*a^3*A*b + 246*a^4*B + 8*a^4*B + 33*a^2*b^2*B + 147*b^4*B)*\text{Sqrt}[\text{Cos}[c + d*x]/(1 + \text{Cos}[c + d*x])]*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/((a + b)*(1 + \text{Cos}[c + d*x]))]*\text{EllipticE}[\text{ArcSin}[\text{Tan}[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(a + b)*(8*a^3*B - 6*a^2*b*(3*A + B) + 3*a*b^2*(57*A + 13*B) + 3*b^3*(25*A + 49*B))*\text{Sqrt}[\text{Cos}[c + d*x]/(1 + \text{Cos}[c + d*x])]*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/((a + b)*(1 + \text{Cos}[c + d*x]))]*\text{EllipticF}[\text{ArcSin}[\text{Tan}[(c + d*x)/2]], (a - b)/(a + b)] + (-18*a^3*A*b + 246*a^4*B + 8*a^4*B + 33*a^2*b^2*B + 147*b^4*B)*\text{Cos}[c + d*x]*(b + a*\text{Cos}[c + d*x])*\text{Sec}[(c + d*x)/2]^2*\text{Tan}[(c + d*x)/2])*(-(\text{Cos}[(c + d*x)/2]*\text{Sec}[c + d*x]*\text{Sin}[(c + d*x)/2]) + \text{Cos}[(c + d*x)/2]^2*\text{Sec}[c + d*x]*\text{Tan}[c + d*x]))/(315*b^3*\text{Sqrt}[b + a*\text{Cos}[c + d*x]]*\text{Sqrt}[\text{Sec}[(c + d*x)/2]^2]*\text{Sqrt}[\text{Cos}[(c + d*x)/2]^2*\text{Sec}[c + d*x]]))
\end{aligned}$$

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb \sec(dx+c)^5 + Aa \sec(dx+c)^3 + (Ba + Ab) \sec(dx+c)^4\right) \sqrt{b \sec(dx+c) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] integral((B*b*sec(d*x + c)^5 + A*a*sec(d*x + c)^3 + (B*a + A*b)*sec(d*x + c)^4)*sqrt(b*sec(d*x + c) + a), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx+c) + A)(b \sec(dx+c) + a)^{\frac{3}{2}} \sec(dx+c)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^3, x)

maple [B] time = 4.00, size = 4395, normalized size = 9.25

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^3*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x)

[Out] 2/315/d*(1+cos(d*x+c))^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))^2*(117*A*cos(d*x+c)^2*a*b^4+53*B*cos(d*x+c)^2*a^2*b^3+85*B*cos(d*x+c)*a*b^4+52*B*cos(d*x+c)^3*a*b^4+30*A*cos(d*x+c)^3*b^5-18*A*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^4*b^4*B*cos(d*x+c)^6*a^4*b-33*B*cos(d*x+c)^6*a^3*b^2-88*B*cos(d*x+c)^6*a^2*b^3-147*B*cos(d*x+c)^6*a*b^4-8*B*cos(d*x+c)^5*a^4*b+34*B*cos(d*x+c)^5*a^3*b^2-33*B*cos(d*x+c)^5*a^2*b^3+10*B*cos(d*x+c)^5*a*b^4+4*B*cos(d*x+c)^4*a^4*b+35*B*b^5+18*A*cos(d*x+c)^6*a^4*b-9*A*cos(d*x+c)^6*a^3*b^2-246*A*cos(d*x+c)^6*a^2*b^3-75*A*cos(d*x+c)^6*a*b^4-18*A*cos(d*x+c)^5*a^4*b+18*A*cos(d*x+c)^5*a^3*b^2+165*A*cos(d*x+c)^5*a^2*b^3-246*A*cos(d*x+c)^5*a*b^4-9*A*cos(d*x+c)^4*a^3*b^2+204*A*cos(d*x+c)^4*a*b^4+81*A*cos(d*x+c)^3*a^2*b^3+68*B*cos(d*x+c)^4*a^2*b^3-B*cos(d*x+c)^3*a^3*b^2-75*A*cos(d*x+c)^5*b^5+8*B*cos(d*x+c)^5*a^5-8*B*cos(d*x+c)^6*a^5+98*B*cos(d*x+c)^4*b^5+45*A*cos(d*x+c)*b^5-147*B*cos(d*x+c)

$$\begin{aligned}
& b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{(1/2)}*EllipticF((-1+\cos(d*x+c))/\sin(d \\
& *x+c), ((a-b)/(a+b))^{(1/2)}*\sin(d*x+c)*a^2*b^3-246*A*\cos(d*x+c)^5*(\cos(d*x+c) \\
&)/(1+\cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{(1/2)}*Ellip \\
& ticF((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{(1/2)}*\sin(d*x+c)*a*b^4+8*B*c \\
& os(d*x+c)^5*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+ \\
& c))/(a+b))^{(1/2)}*EllipticE((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{(1/2)}* \\
& \sin(d*x+c)*a^4*b+33*B*\cos(d*x+c)^5*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((b+a* \\
& \cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{(1/2)}*EllipticE((-1+\cos(d*x+c))/\sin(d*x+c \\
&), ((a-b)/(a+b))^{(1/2)}*\sin(d*x+c)*a^3*b^2+33*B*\cos(d*x+c)^5*(\cos(d*x+c)/(1+ \\
& \cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{(1/2)}*EllipticE(\\
& (-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{(1/2)}*\sin(d*x+c)*a^2*b^3+147*B*co \\
& s(d*x+c)^5*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c \\
&))/(a+b))^{(1/2)}*EllipticE((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{(1/2)}*s \\
& in(d*x+c)*a*b^4-8*B*\cos(d*x+c)^5*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((b+a*co \\
& s(d*x+c))/(1+\cos(d*x+c))/(a+b))^{(1/2)}*EllipticF((-1+\cos(d*x+c))/\sin(d*x+c), \\
& ((a-b)/(a+b))^{(1/2)}*\sin(d*x+c)*a^4*b-2*B*\cos(d*x+c)^5*(\cos(d*x+c)/(1+\cos(d \\
& *x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{(1/2)}*EllipticF((-1+c \\
& os(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{(1/2)}*\sin(d*x+c)*a^3*b^2-33*B*\cos(d*x+ \\
& c)^5*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+ \\
& b))^{(1/2)}*EllipticF((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{(1/2)}*\sin(d*x \\
& +c)*a^2*b^3-186*B*\cos(d*x+c)^5*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((b+a*\cos(\\
& d*x+c))/(1+\cos(d*x+c))/(a+b))^{(1/2)}*EllipticF((-1+\cos(d*x+c))/\sin(d*x+c), ((\\
& a-b)/(a+b))^{(1/2)}*\sin(d*x+c)*a*b^4-18*A*\cos(d*x+c)^4*(\cos(d*x+c)/(1+\cos(d* \\
& x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{(1/2)}*EllipticE((-1+co \\
& s(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{(1/2)}*\sin(d*x+c)*a^4*b-18*A*\cos(d*x+c)^ \\
& 4*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b)) \\
& ^{(1/2)}*EllipticE((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{(1/2)}*\sin(d*x+c) \\
& *a^3*b^2+246*A*\cos(d*x+c)^4*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x \\
& +c))/(1+\cos(d*x+c))/(a+b))^{(1/2)}*EllipticE((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b \\
&)/(a+b))^{(1/2)}*\sin(d*x+c)*a^2*b^3+246*A*\cos(d*x+c)^4*(\cos(d*x+c)/(1+\cos(d* \\
& x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{(1/2)}*EllipticE((-1+co \\
& s(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{(1/2)}*\sin(d*x+c)*a*b^4+18*A*\cos(d*x+c)^ \\
& 4*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b)) \\
& ^{(1/2)}*EllipticF((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{(1/2)}*\sin(d*x+c) \\
& *a^3*b^2-153*A*\cos(d*x+c)^4*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x \\
& +c))/(1+\cos(d*x+c))/(a+b))^{(1/2)}*EllipticF((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b \\
&)/(a+b))^{(1/2)}*\sin(d*x+c)*a^2*b^3-246*A*\cos(d*x+c)^4*(\cos(d*x+c)/(1+\cos(d* \\
& x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{(1/2)}*EllipticF((-1+co \\
& s(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{(1/2)}*\sin(d*x+c)*a*b^4+8*B*\cos(d*x+c)^4 \\
& *(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{(\\
& 1/2)}*EllipticE((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{(1/2)}*\sin(d*x+c)* \\
& a^4*b)/(b+a*\cos(d*x+c))/\cos(d*x+c)^4/\sin(d*x+c)^5/b^3
\end{aligned}$$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(a + \frac{b}{\cos(c+dx)}\right)^{3/2}}{\cos(c+dx)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(3/2))/cos(c + d*x)^3,x)

[Out] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(3/2))/cos(c + d*x)^3, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \sec(c + dx)) (a + b \sec(c + dx))^{\frac{3}{2}} \sec^3(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**3*(a+b*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)),x)

[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**(3/2)*sec(c + d*x)**3, x)

$$3.357 \quad \int \sec^2(c+dx)(a+b \sec(c+dx))^{3/2}(A+B \sec(c+dx)) dx$$

Optimal. Leaf size=388

$$\frac{2(-6a^2B + 21aAb + 25b^2B) \tan(c+dx) \sqrt{a+b \sec(c+dx)}}{105bd} + \frac{2(a-b) \sqrt{a+b} (6a^2B - a(21Ab - 57bB) + b^2(63A - 25B))}{105bd}$$

[Out] $-2/105*(a-b)*(21*A*a^2*b+63*A*b^3-6*B*a^3+82*B*a*b^2)*\cot(d*x+c)*\text{EllipticE}((a+b*\sec(d*x+c))^{1/2}/(a+b)^{1/2},((a+b)/(a-b))^{1/2})*(a+b)^{1/2}*(b*(1-\sec(d*x+c))/(a+b))^{1/2}*(-b*(1+\sec(d*x+c))/(a-b))^{1/2}/b^3/d+2/105*(a-b)*(b^2*(63*A-25*B)+6*a^2*B-a*(21*A*b-57*B*b))*\cot(d*x+c)*\text{EllipticF}((a+b*\sec(d*x+c))^{1/2}/(a+b)^{1/2},((a+b)/(a-b))^{1/2})*(a+b)^{1/2}*(b*(1-\sec(d*x+c))/(a+b))^{1/2}*(-b*(1+\sec(d*x+c))/(a-b))^{1/2}/b^2/d+2/35*(7*A*b-2*B*a)*(a+b*\sec(d*x+c))^{3/2}*\tan(d*x+c)/b/d+2/7*B*(a+b*\sec(d*x+c))^{5/2}*\tan(d*x+c)/b/d+2/105*(21*A*a*b-6*B*a^2+25*B*b^2)*(a+b*\sec(d*x+c))^{1/2}*\tan(d*x+c)/b/d$

Rubi [A] time = 0.83, antiderivative size = 388, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {4010, 4002, 4005, 3832, 4004}

$$\frac{2(-6a^2B + 21aAb + 25b^2B) \tan(c+dx) \sqrt{a+b \sec(c+dx)}}{105bd} + \frac{2(a-b) \sqrt{a+b} (6a^2B - a(21Ab - 57bB) + b^2(63A - 25B))}{105bd}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]

[Out] $(-2*(a-b)*\text{Sqrt}[a+b]*(21*a^2*A*b+63*A*b^3-6*a^3*B+82*a*b^2*B)*\text{Cot}[c+d*x]*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[a+b*\text{Sec}[c+d*x]]/\text{Sqrt}[a+b]],(a+b)/(a-b)]*\text{Sqrt}[(b*(1-\text{Sec}[c+d*x]))/(a+b)]*\text{Sqrt}[-((b*(1+\text{Sec}[c+d*x]))/(a-b))]/(105*b^3*d)+(2*(a-b)*\text{Sqrt}[a+b]*(b^2*(63*A-25*B)+6*a^2*B-a*(21*A*b-57*b*B))*\text{Cot}[c+d*x]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[a+b*\text{Sec}[c+d*x]]/\text{Sqrt}[a+b]],(a+b)/(a-b)]*\text{Sqrt}[(b*(1-\text{Sec}[c+d*x]))/(a+b)]*\text{Sqrt}[-((b*(1+\text{Sec}[c+d*x]))/(a-b))]/(105*b^2*d)+(2*(21*a*A*b-6*a^2*B+25*b^2*B)*\text{Sqrt}[a+b*\text{Sec}[c+d*x]]*\text{Tan}[c+d*x]/(105*b*d)+(2*(7*A*b-2*a*B)*(a+b*\text{Sec}[c+d*x])^{3/2}*\text{Tan}[c+d*x]/(35*b*d)+(2*B*(a+b*\text{Sec}[c+d*x])^{5/2}*\text{Tan}[c+d*x])/(7*b*d)$

Rule 3832

Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]

/Rt[a + b, 2]], (a + b)/(a - b)]/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 4002

Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]

Rule 4004

Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]]/Rt[a + (b*B)/A, 2]], (a*A + b*B)/(a*A - b*B)]/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]

Rule 4005

Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] :> Dist[A - B, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]

Rule 4010

Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && !LtQ[m, -1]

Rubi steps

$$\begin{aligned}
\int \sec^2(c + dx)(a + b \sec(c + dx))^{3/2}(A + B \sec(c + dx)) dx &= \frac{2B(a + b \sec(c + dx))^{5/2} \tan(c + dx)}{7bd} + \frac{2 \int \sec(c + dx) dx}{7bd} \\
&= \frac{2(7Ab - 2aB)(a + b \sec(c + dx))^{3/2} \tan(c + dx)}{35bd} + \frac{2 \int \sec(c + dx) dx}{35bd} \\
&= \frac{2(21aAb - 6a^2B + 25b^2B) \sqrt{a + b \sec(c + dx)} \tan(c + dx)}{105bd} + \frac{2 \int \sec(c + dx) dx}{105bd} \\
&= \frac{2(21aAb - 6a^2B + 25b^2B) \sqrt{a + b \sec(c + dx)} \tan(c + dx)}{105bd} + \frac{2(a - b) \sqrt{a + b} (21a^2Ab + 63Ab^3 - 6a^3B + 82ab^2)}{105bd}
\end{aligned}$$

Mathematica [B] time = 25.26, size = 3342, normalized size = 8.61

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]

[Out] (Cos[c + d*x]*(a + b*Sec[c + d*x])^(3/2)*((-2*(-21*a^2*A*b - 63*A*b^3 + 6*a^3*B - 82*a*b^2*B)*Sin[c + d*x])/(105*b^2) + (2*Sec[c + d*x]^2*(7*A*b*SIN[c + d*x] + 8*a*B*SIN[c + d*x]))/35 + (2*Sec[c + d*x]*(42*a*A*b*SIN[c + d*x] + 3*a^2*B*SIN[c + d*x] + 25*b^2*B*SIN[c + d*x]))/(105*b) + (2*b*B*Sec[c + d*x]^2*Tan[c + d*x])/7)/(d*(b + a*cos[c + d*x])) + (2*(-1/5*(a^2*A)/(Sqrt[b + a*cos[c + d*x]])*Sqrt[Sec[c + d*x]]) - (3*A*b^2)/(5*Sqrt[b + a*cos[c + d*x]])*Sqrt[Sec[c + d*x]]) + (2*a^3*B)/(35*b*Sqrt[b + a*cos[c + d*x]])*Sqrt[Sec[c + d*x]]) - (82*a*b*B)/(105*Sqrt[b + a*cos[c + d*x]])*Sqrt[Sec[c + d*x]]) - (a^3*A*Sqrt[Sec[c + d*x]])/(5*b*Sqrt[b + a*cos[c + d*x]]) + (a*A*b*Sqrt[Sec[c + d*x]])/(5*Sqrt[b + a*cos[c + d*x]]) - (31*a^2*B*Sqrt[Sec[c + d*x]])/(105*Sqrt[b + a*cos[c + d*x]]) + (2*a^4*B*Sqrt[Sec[c + d*x]])/(35*b^2*Sqrt[b + a*cos[c + d*x]]) + (5*b^2*B*Sqrt[Sec[c + d*x]])/(21*Sqrt[b + a*cos[c + d*x]]) - (a^3*A*cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(5*b*Sqrt[b + a*cos[c + d*x]]) - (3*a*A*b*cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(5*Sqrt[b + a*cos[c + d*x]]) - (82*a^2*B*cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(105*Sqrt[b + a*cos[c + d*x]]) + (2*a^4*B*cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(35*b^2*Sqrt[b + a*cos[c + d*x]])*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)*(2*(a + b)*(-21*a^2*A*b - 63*A*b^3 + 6*a^3*B - 82*a*b^2*B)*Sqrt[Sec[c + d*x]])

$$\begin{aligned}
& \text{rt}[\text{Cos}[c + d*x]/(1 + \text{Cos}[c + d*x])] * \text{Sqrt}[(b + a*\text{Cos}[c + d*x])/((a + b)*(1 + \\
& \text{Cos}[c + d*x]))] * \text{EllipticE}[\text{ArcSin}[\text{Tan}[(c + d*x)/2]], (a - b)/(a + b)] + 2*b \\
& *(a + b)*(-6*a^2*B + 3*a*b*(7*A + 19*B) + b^2*(63*A + 25*B)) * \text{Sqrt}[\text{Cos}[c + d \\
& *x]/(1 + \text{Cos}[c + d*x])] * \text{Sqrt}[(b + a*\text{Cos}[c + d*x])/((a + b)*(1 + \text{Cos}[c + d*x] \\
&))] * \text{EllipticF}[\text{ArcSin}[\text{Tan}[(c + d*x)/2]], (a - b)/(a + b)] + (-21*a^2*A*b - \\
& 63*A*b^3 + 6*a^3*B - 82*a*b^2*B) * \text{Cos}[c + d*x] * (b + a*\text{Cos}[c + d*x]) * \text{Sec}[(c + \\
& d*x)/2]^2 * \text{Tan}[(c + d*x)/2]) / (105*b^2*d*(b + a*\text{Cos}[c + d*x])^2 * \text{Sqrt}[\text{Sec}[(c \\
& + d*x)/2]^2] * \text{Sec}[c + d*x]^(3/2) * ((a*\text{Sqrt}[\text{Cos}[(c + d*x)/2]^2 * \text{Sec}[c + d*x]] * \\
& \text{Sin}[c + d*x] * (2*(a + b)*(-21*a^2*A*b - 63*A*b^3 + 6*a^3*B - 82*a*b^2*B)) * \text{Sqr} \\
& \text{t}[\text{Cos}[c + d*x]/(1 + \text{Cos}[c + d*x])] * \text{Sqrt}[(b + a*\text{Cos}[c + d*x])/((a + b)*(1 + \\
& \text{Cos}[c + d*x]))] * \text{EllipticE}[\text{ArcSin}[\text{Tan}[(c + d*x)/2]], (a - b)/(a + b)] + 2*b* \\
& (a + b)*(-6*a^2*B + 3*a*b*(7*A + 19*B) + b^2*(63*A + 25*B)) * \text{Sqrt}[\text{Cos}[c + d* \\
& x]/(1 + \text{Cos}[c + d*x])] * \text{Sqrt}[(b + a*\text{Cos}[c + d*x])/((a + b)*(1 + \text{Cos}[c + d*x] \\
&))] * \text{EllipticF}[\text{ArcSin}[\text{Tan}[(c + d*x)/2]], (a - b)/(a + b)] + (-21*a^2*A*b - 6 \\
& 3*A*b^3 + 6*a^3*B - 82*a*b^2*B) * \text{Cos}[c + d*x] * (b + a*\text{Cos}[c + d*x]) * \text{Sec}[(c + \\
& d*x)/2]^2 * \text{Tan}[(c + d*x)/2]) / (105*b^2*(b + a*\text{Cos}[c + d*x])^(3/2) * \text{Sqrt}[\text{Sec}[(c \\
& + d*x)/2]^2]) - (\text{Sqrt}[\text{Cos}[(c + d*x)/2]^2 * \text{Sec}[c + d*x]] * \text{Tan}[(c + d*x)/2] * (\\
& 2*(a + b)*(-21*a^2*A*b - 63*A*b^3 + 6*a^3*B - 82*a*b^2*B) * \text{Sqrt}[\text{Cos}[c + d*x] \\
& / (1 + \text{Cos}[c + d*x])] * \text{Sqrt}[(b + a*\text{Cos}[c + d*x])/((a + b)*(1 + \text{Cos}[c + d*x]))] \\
&] * \text{EllipticE}[\text{ArcSin}[\text{Tan}[(c + d*x)/2]], (a - b)/(a + b)] + 2*b*(a + b)*(-6*a^ \\
& 2*B + 3*a*b*(7*A + 19*B) + b^2*(63*A + 25*B)) * \text{Sqrt}[\text{Cos}[c + d*x]/(1 + \text{Cos}[c \\
& + d*x])] * \text{Sqrt}[(b + a*\text{Cos}[c + d*x])/((a + b)*(1 + \text{Cos}[c + d*x]))] * \text{EllipticF} \\
& [\text{ArcSin}[\text{Tan}[(c + d*x)/2]], (a - b)/(a + b)] + (-21*a^2*A*b - 63*A*b^3 + 6*a^ \\
& 3*B - 82*a*b^2*B) * \text{Cos}[c + d*x] * (b + a*\text{Cos}[c + d*x]) * \text{Sec}[(c + d*x)/2]^2 * \text{Tan} \\
& [(c + d*x)/2]) / (105*b^2 * \text{Sqrt}[b + a*\text{Cos}[c + d*x]] * \text{Sqrt}[\text{Sec}[(c + d*x)/2]^2]) \\
& + (2*\text{Sqrt}[\text{Cos}[(c + d*x)/2]^2 * \text{Sec}[c + d*x]] * (((-21*a^2*A*b - 63*A*b^3 + 6*a^ \\
& 3*B - 82*a*b^2*B) * \text{Cos}[c + d*x] * (b + a*\text{Cos}[c + d*x]) * \text{Sec}[(c + d*x)/2]^4) / 2 + \\
& ((a + b)*(-21*a^2*A*b - 63*A*b^3 + 6*a^3*B - 82*a*b^2*B) * \text{Sqrt}[(b + a*\text{Cos}[c \\
& + d*x])/((a + b)*(1 + \text{Cos}[c + d*x]))] * \text{EllipticE}[\text{ArcSin}[\text{Tan}[(c + d*x)/2]], \\
& (a - b)/(a + b)] * ((\text{Cos}[c + d*x] * \text{Sin}[c + d*x]) / (1 + \text{Cos}[c + d*x])^2 - \text{Sin}[c \\
& + d*x] / (1 + \text{Cos}[c + d*x]))) / \text{Sqrt}[\text{Cos}[c + d*x] / (1 + \text{Cos}[c + d*x])] + (b*(a + \\
& b)*(-6*a^2*B + 3*a*b*(7*A + 19*B) + b^2*(63*A + 25*B)) * \text{Sqrt}[(b + a*\text{Cos}[c + \\
& d*x])/((a + b)*(1 + \text{Cos}[c + d*x]))] * \text{EllipticF}[\text{ArcSin}[\text{Tan}[(c + d*x)/2]], (a \\
& - b)/(a + b)] * ((\text{Cos}[c + d*x] * \text{Sin}[c + d*x]) / (1 + \text{Cos}[c + d*x])^2 - \text{Sin}[c + \\
& d*x] / (1 + \text{Cos}[c + d*x]))) / \text{Sqrt}[\text{Cos}[c + d*x] / (1 + \text{Cos}[c + d*x])] + ((a + b)* \\
& (-21*a^2*A*b - 63*A*b^3 + 6*a^3*B - 82*a*b^2*B) * \text{Sqrt}[\text{Cos}[c + d*x] / (1 + \text{Cos}[\\
& c + d*x]]) * \text{EllipticE}[\text{ArcSin}[\text{Tan}[(c + d*x)/2]], (a - b)/(a + b)] * (-((a*\text{Sin}[c \\
& + d*x]) / ((a + b)*(1 + \text{Cos}[c + d*x]))) + ((b + a*\text{Cos}[c + d*x]) * \text{Sin}[c + d*x] \\
&) / ((a + b)*(1 + \text{Cos}[c + d*x])^2))) / \text{Sqrt}[(b + a*\text{Cos}[c + d*x]) / ((a + b)*(1 + \\
& \text{Cos}[c + d*x]))] + (b*(a + b)*(-6*a^2*B + 3*a*b*(7*A + 19*B) + b^2*(63*A + 2 \\
& 5*B)) * \text{Sqrt}[\text{Cos}[c + d*x] / (1 + \text{Cos}[c + d*x])] * \text{EllipticF}[\text{ArcSin}[\text{Tan}[(c + d*x)/ \\
& 2]], (a - b)/(a + b)] * (-((a*\text{Sin}[c + d*x]) / ((a + b)*(1 + \text{Cos}[c + d*x]))) + (\\
& (b + a*\text{Cos}[c + d*x]) * \text{Sin}[c + d*x]) / ((a + b)*(1 + \text{Cos}[c + d*x])^2))) / \text{Sqrt}[(b \\
& + a*\text{Cos}[c + d*x]) / ((a + b)*(1 + \text{Cos}[c + d*x]))] - a*(-21*a^2*A*b - 63*A*b^ \\
& 3 + 6*a^3*B - 82*a*b^2*B) * \text{Cos}[c + d*x] * \text{Sec}[(c + d*x)/2]^2 * \text{Sin}[c + d*x] * \text{Tan}
\end{aligned}$$

$$\begin{aligned} & (c + dx)/2] - (-21a^2Ab - 63A^2b^3 + 6a^3B - 82ab^2B)(b + a\cos[c + dx])\sec[(c + dx)/2]^2\sin[c + dx]\tan[(c + dx)/2] + (-21a^2Ab - 63A^2b^3 + 6a^3B - 82ab^2B)\cos[c + dx](b + a\cos[c + dx])\sec[(c + dx)/2]^2\tan[(c + dx)/2]^2 + (b(a + b)(-6a^2B + 3ab(7A + 19B) + b^2(63A + 25B))\sqrt{\cos[c + dx]/(1 + \cos[c + dx])}\sqrt{(b + a\cos[c + dx])/(a + b)(1 + \cos[c + dx])})\sec[(c + dx)/2]^2)/(\sqrt{1 - \tan[(c + dx)/2]^2}\sqrt{1 - ((a - b)\tan[(c + dx)/2]^2)/(a + b)}) + ((a + b)(-21a^2Ab - 63A^2b^3 + 6a^3B - 82ab^2B)\sqrt{\cos[c + dx]/(1 + \cos[c + dx])}\sqrt{(b + a\cos[c + dx])/(a + b)(1 + \cos[c + dx])})\sec[(c + dx)/2]^2\sqrt{1 - ((a - b)\tan[(c + dx)/2]^2)/(a + b)})/\sqrt{1 - \tan[(c + dx)/2]^2})/(105b^2\sqrt{b + a\cos[c + dx]}\sqrt{\sec[(c + dx)/2]^2}) + ((2(a + b)(-21a^2Ab - 63A^2b^3 + 6a^3B - 82ab^2B)\sqrt{\cos[c + dx]/(1 + \cos[c + dx])}\sqrt{(b + a\cos[c + dx])/(a + b)(1 + \cos[c + dx])})\operatorname{EllipticE}[\operatorname{ArcSin}[\tan[(c + dx)/2]], (a - b)/(a + b)] + 2b(a + b)(-6a^2B + 3ab(7A + 19B) + b^2(63A + 25B))\sqrt{\cos[c + dx]/(1 + \cos[c + dx])}\sqrt{(b + a\cos[c + dx])/(a + b)(1 + \cos[c + dx])})\operatorname{EllipticF}[\operatorname{ArcSin}[\tan[(c + dx)/2]], (a - b)/(a + b)] + (-21a^2Ab - 63A^2b^3 + 6a^3B - 82ab^2B)\cos[c + dx](b + a\cos[c + dx])\sec[(c + dx)/2]^2\tan[(c + dx)/2])(-\cos[(c + dx)/2]\sec[c + dx]\sin[(c + dx)/2] + \cos[(c + dx)/2]^2\sec[c + dx]\tan[c + dx])/(105b^2\sqrt{b + a\cos[c + dx]}\sqrt{\sec[(c + dx)/2]^2}\sqrt{\cos[(c + dx)/2]^2\sec[c + dx]}) \end{aligned}$$

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\left(Bb \sec(dx + c)^4 + Aa \sec(dx + c)^2 + (Ba + Ab) \sec(dx + c)^3\right)\sqrt{b \sec(dx + c) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(dx+c)^2*(a+b*sec(dx+c))^(3/2)*(A+B*sec(dx+c)),x, algorithm="fricas")

[Out] integral((B*b*sec(dx + c)^4 + A*a*sec(dx + c)^2 + (B*a + A*b)*sec(dx + c)^3)*sqrt(b*sec(dx + c) + a), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(b \sec(dx + c) + a)^{\frac{3}{2}} \sec(dx + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(dx+c)^2*(a+b*sec(dx+c))^(3/2)*(A+B*sec(dx+c)),x, algorithm="giac")

[Out] integrate((B*sec(dx + c) + A)*(b*sec(dx + c) + a)^(3/2)*sec(dx + c)^2, x)

maple [B] time = 2.92, size = 3424, normalized size = 8.82

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\int (\sec(dx+c)^2(a+b\sec(dx+c))^{3/2}(A+B\sec(dx+c)), x)$

[Out]
$$-2/105/d*(1+\cos(dx+c))^{2*((b+a*\cos(dx+c))/\cos(dx+c))^{1/2}}*(-1+\cos(dx+c))^{2*(21*A*\cos(dx+c)^5*a^3*b+42*A*\cos(dx+c)^5*a^2*b^2+63*A*\cos(dx+c)^5*a*b^3+3*B*\cos(dx+c)^5*a^3*b+82*B*\cos(dx+c)^5*a^2*b^2-21*A*\cos(dx+c)^4*a^3*b+21*A*\cos(dx+c)^4*a^2*b^2-6*B*\cos(dx+c)^4*a^3*b-63*A*\cos(dx+c)^3*a^2*b^2+3*B*\cos(dx+c)^3*a^3*b-68*B*\cos(dx+c)^3*a*b^3-27*B*\cos(dx+c)^2*a^2*b^2-39*B*\cos(dx+c)*a*b^3-63*A*\cos(dx+c)^2*a*b^3-42*A*\cos(dx+c)^3*b^4+25*B*\cos(dx+c)^4*b^4-10*B*\cos(dx+c)^2*b^4+82*B*\sin(dx+c)*\cos(dx+c)^4*((\cos(dx+c)/(1+\cos(dx+c)))^{1/2}*((b+a*\cos(dx+c))/(1+\cos(dx+c)))/(a+b))^{1/2}}*EllipticF((-1+\cos(dx+c))/\sin(dx+c),((a-b)/(a+b))^{1/2})*a*b^3+25*B*\cos(dx+c)^5*a*b^3-55*B*\cos(dx+c)^4*a^2*b^2-15*B*b^4+6*B*\sin(dx+c)*\cos(dx+c)^3*(\cos(dx+c)/(1+\cos(dx+c)))^{1/2}*((b+a*\cos(dx+c))/(1+\cos(dx+c)))/(a+b))^{1/2}}*EllipticE((-1+\cos(dx+c))/\sin(dx+c),((a-b)/(a+b))^{1/2})*a^3*b-82*B*\sin(dx+c)*\cos(dx+c)^3*(\cos(dx+c)/(1+\cos(dx+c)))^{1/2}*((b+a*\cos(dx+c))/(1+\cos(dx+c)))/(a+b))^{1/2}}*EllipticE((-1+\cos(dx+c))/\sin(dx+c),((a-b)/(a+b))^{1/2})*a^2*b^2-82*B*\sin(dx+c)*\cos(dx+c)^3*(\cos(dx+c)/(1+\cos(dx+c)))^{1/2}*((b+a*\cos(dx+c))/(1+\cos(dx+c)))/(a+b))^{1/2}}*EllipticE((-1+\cos(dx+c))/\sin(dx+c),((a-b)/(a+b))^{1/2})*a*b^3-6*B*\sin(dx+c)*\cos(dx+c)^3*(\cos(dx+c)/(1+\cos(dx+c)))^{1/2}*((b+a*\cos(dx+c))/(1+\cos(dx+c)))/(a+b))^{1/2}}*EllipticF((-1+\cos(dx+c))/\sin(dx+c),((a-b)/(a+b))^{1/2})*a^3*b+51*B*\sin(dx+c)*\cos(dx+c)^3*(\cos(dx+c)/(1+\cos(dx+c)))^{1/2}*((b+a*\cos(dx+c))/(1+\cos(dx+c)))/(a+b))^{1/2}}*EllipticF((-1+\cos(dx+c))/\sin(dx+c),((a-b)/(a+b))^{1/2})*a^2*b^2+82*B*\sin(dx+c)*\cos(dx+c)^3*(\cos(dx+c)/(1+\cos(dx+c)))^{1/2}*((b+a*\cos(dx+c))/(1+\cos(dx+c)))/(a+b))^{1/2}}*EllipticF((-1+\cos(dx+c))/\sin(dx+c),((a-b)/(a+b))^{1/2})*a*b^3-21*A*\sin(dx+c)*\cos(dx+c)^4*(\cos(dx+c)/(1+\cos(dx+c)))^{1/2}*((b+a*\cos(dx+c))/(1+\cos(dx+c)))/(a+b))^{1/2}}*EllipticE((-1+\cos(dx+c))/\sin(dx+c),((a-b)/(a+b))^{1/2})*a^3*b-21*A*\sin(dx+c)*\cos(dx+c)^4*(\cos(dx+c)/(1+\cos(dx+c)))^{1/2}*((b+a*\cos(dx+c))/(1+\cos(dx+c)))/(a+b))^{1/2}}*EllipticE((-1+\cos(dx+c))/\sin(dx+c),((a-b)/(a+b))^{1/2})*a^2*b^2-63*A*\sin(dx+c)*\cos(dx+c)^4*(\cos(dx+c)/(1+\cos(dx+c)))^{1/2}*((b+a*\cos(dx+c))/(1+\cos(dx+c)))/(a+b))^{1/2}}*EllipticE((-1+\cos(dx+c))/\sin(dx+c),((a-b)/(a+b))^{1/2})*a*b^3+21*A*\sin(dx+c)*\cos(dx+c)^4*(\cos(dx+c)/(1+\cos(dx+c)))^{1/2}*((b+a*\cos(dx+c))/(1+\cos(dx+c)))/(a+b))^{1/2}}*EllipticF((-1+\cos(dx+c))/\sin(dx+c),((a-b)/(a+b))^{1/2})*a^2*b^2+84*A*\sin(dx+c)*\cos(dx+c)^4*(\cos(dx+c)/(1+\cos(dx+c)))^{1/2}*((b+a*\cos(dx+c))/(1+\cos(dx+c)))/(a+b))^{1/2}}*EllipticF((-1+\cos(dx+c))/\sin(dx+c),((a-b)/(a+b))^{1/2})*a*b^3+6*B*\sin(dx+c)*\cos(dx+c)^4*(\cos(dx+c)/(1+\cos(dx+c)))^{1/2}*((b+a*\cos(dx+c))/(1+\cos(dx+c)))/(a+b))^{1/2}}*EllipticE((-1+\cos(dx+c))/\sin(dx+c),(($$

$$\begin{aligned}
& (a-b)/(a+b))^{(1/2)} * a^3 * b - 82 * B * \sin(d*x+c) * \cos(d*x+c)^4 * (\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)} * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)} * \text{EllipticE}((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{(1/2)} * a^2 * b^2 - 82 * B * \sin(d*x+c) * \cos(d*x+c)^4 * (\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)} * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)} * \text{EllipticE}((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{(1/2)} * a * b^3 - 6 * B * \sin(d*x+c) * \cos(d*x+c)^4 * (\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)} * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)} * \text{EllipticF}((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{(1/2)} * a^3 * b + 51 * B * \sin(d*x+c) * \cos(d*x+c)^4 * (\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)} * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)} * \text{EllipticF}((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{(1/2)} * a^2 * b^2 - 6 * B * \cos(d*x+c)^5 * a^4 - 21 * A * \sin(d*x+c) * \cos(d*x+c)^3 * (\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)} * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)} * \text{EllipticE}((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{(1/2)} * a^3 * b - 21 * A * \sin(d*x+c) * \cos(d*x+c)^3 * (\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)} * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)} * \text{EllipticE}((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{(1/2)} * a^2 * b^2 - 63 * A * \sin(d*x+c) * \cos(d*x+c)^3 * (\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)} * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)} * \text{EllipticE}((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{(1/2)} * a * b^3 + 21 * A * \sin(d*x+c) * \cos(d*x+c)^3 * (\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)} * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)} * \text{EllipticF}((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{(1/2)} * a^2 * b^2 + 84 * A * \sin(d*x+c) * \cos(d*x+c)^3 * (\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)} * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)} * \text{EllipticF}((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{(1/2)} * a * b^3 + 63 * A * \cos(d*x+c)^4 * b^4 + 6 * B * \cos(d*x+c)^4 * a^4 - 21 * A * \cos(d*x+c) * b^4 - 63 * A * \sin(d*x+c) * \cos(d*x+c)^4 * (\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)} * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)} * \text{EllipticE}((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{(1/2)} * b^4 + 63 * A * \sin(d*x+c) * \cos(d*x+c)^4 * (\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)} * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)} * \text{EllipticF}((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{(1/2)} * b^4 + 6 * B * \sin(d*x+c) * \cos(d*x+c)^4 * (\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)} * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)} * \text{EllipticE}((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{(1/2)} * a^4 + 25 * B * \sin(d*x+c) * \cos(d*x+c)^4 * (\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)} * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)} * \text{EllipticF}((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{(1/2)} * b^4 - 63 * A * \sin(d*x+c) * \cos(d*x+c)^3 * (\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)} * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)} * \text{EllipticE}((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{(1/2)} * b^4 + 63 * A * \sin(d*x+c) * \cos(d*x+c)^3 * (\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)} * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)} * \text{EllipticF}((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{(1/2)} * b^4 + 6 * B * \sin(d*x+c) * \cos(d*x+c)^3 * (\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)} * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)} * \text{EllipticE}((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{(1/2)} * a^4 + 25 * B * \sin(d*x+c) * \cos(d*x+c)^3 * (\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)} * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)} * \text{EllipticF}((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{(1/2)} * b^4 + 82 * B * \cos(d*x+c)^4 * a * b^3 / (b+a*\cos(d*x+c)) / \cos(d*x+c)^3 / \sin(d*x+c)^5 / b^2
\end{aligned}$$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(a + \frac{b}{\cos(c+dx)}\right)^{3/2}}{\cos(c+dx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(3/2))/cos(c + d*x)^2,x)

[Out] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(3/2))/cos(c + d*x)^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \sec(c + dx)) (a + b \sec(c + dx))^{\frac{3}{2}} \sec^2(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**2*(a+b*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)),x)

[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**(3/2)*sec(c + d*x)**2, x)

3.358 $\int \sec(c+dx)(a+b \sec(c+dx))^{3/2}(A+B \sec(c+dx)) dx$

Optimal. Leaf size=312

$$\frac{2(a-b)\sqrt{a+b}(3a^2B+20aAb+9b^2B)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b}\sec(c+dx)}{\sqrt{a+b}}\right)\right)\frac{a+b}{a-b}}{15b^2d}$$

[Out] $-2/15*(a-b)*(20*A*a*b+3*B*a^2+9*B*b^2)*\cot(d*x+c)*\text{EllipticE}((a+b*\sec(d*x+c))^{1/2}/(a+b)^{1/2},((a+b)/(a-b))^{1/2})*(a+b)^{1/2}*(b*(1-\sec(d*x+c)))/(a+b))^{1/2}*(-b*(1+\sec(d*x+c)))/(a-b))^{1/2}/b^2/d+2/15*(a-b)*(15*A*a-5*A*b-3*B*a+9*B*b)*\cot(d*x+c)*\text{EllipticF}((a+b*\sec(d*x+c))^{1/2}/(a+b)^{1/2},((a+b)/(a-b))^{1/2})*(a+b)^{1/2}*(b*(1-\sec(d*x+c)))/(a+b))^{1/2}*(-b*(1+\sec(d*x+c)))/(a-b))^{1/2}/b/d+2/5*B*(a+b*\sec(d*x+c))^{3/2}*\tan(d*x+c)/d+2/15*(5*A*b+3*B*a)*(a+b*\sec(d*x+c))^{1/2}*\tan(d*x+c)/d$

Rubi [A] time = 0.57, antiderivative size = 312, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {4002, 4005, 3832, 4004}

$$\frac{2(a-b)\sqrt{a+b}(3a^2B+20aAb+9b^2B)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b}\sec(c+dx)}{\sqrt{a+b}}\right)\right)\frac{a+b}{a-b}}{15b^2d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sec}[c+d*x]*(a+b*\text{Sec}[c+d*x])^{3/2}*(A+B*\text{Sec}[c+d*x]),x]$

[Out] $(-2*(a-b)*\text{Sqrt}[a+b]*(20*a*A*b+3*a^2*B+9*b^2*B)*\text{Cot}[c+d*x]*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[a+b*\text{Sec}[c+d*x]]/\text{Sqrt}[a+b]],(a+b)/(a-b)]*\text{Sqrt}[(b*(1-\text{Sec}[c+d*x]))/(a+b)]*\text{Sqrt}[-((b*(1+\text{Sec}[c+d*x]))/(a-b))])/(15*b^2*d)+(2*(a-b)*\text{Sqrt}[a+b]*(15*a*A-5*A*b-3*a*B+9*b*B)*\text{Cot}[c+d*x]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[a+b*\text{Sec}[c+d*x]]/\text{Sqrt}[a+b]],(a+b)/(a-b)]*\text{Sqrt}[(b*(1-\text{Sec}[c+d*x]))/(a+b)]*\text{Sqrt}[-((b*(1+\text{Sec}[c+d*x]))/(a-b))])/(15*b*d)+(2*(5*A*b+3*a*B)*\text{Sqrt}[a+b*\text{Sec}[c+d*x]]*\text{Tan}[c+d*x])/(15*d)+(2*B*(a+b*\text{Sec}[c+d*x])^{3/2}*\text{Tan}[c+d*x])/(5*d)$

Rule 3832

$\text{Int}[\text{csc}[(e_.)+(f_.)*(x_)]/\text{Sqrt}[\text{csc}[(e_.)+(f_.)*(x_)]*(b_.)+(a_.)],x_Symbol] \rightarrow \text{Simp}[(-2*\text{Rt}[a+b,2]*\text{Sqrt}[(b*(1-\text{Csc}[e+f*x]))/(a+b)]*\text{Sqrt}[-((b*(1+\text{Csc}[e+f*x]))/(a-b))]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[a+b*\text{Csc}[e+f*x]]/\text{Rt}[a+b,2]],(a+b)/(a-b))]/(b*f*\text{Cot}[e+f*x]),x] /; \text{FreeQ}\{a,b,e,f\},x \&\& \text{NeQ}[a^2-b^2,0]$

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_.)]*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := -Simp[(B*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*Csc[e + f*x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)))/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] := Simp[(-2*(A*b - a*B)*Rt[a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]]/Rt[a + (b*B)/A, 2]], (a*A + b*B)/(a*A - b*B)]/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)))/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] := Dist[A - B, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rubi steps

$$\begin{aligned} \int \sec(c + dx)(a + b \sec(c + dx))^{3/2}(A + B \sec(c + dx)) dx &= \frac{2B(a + b \sec(c + dx))^{3/2} \tan(c + dx)}{5d} + \frac{2}{5} \int \sec(c + dx) (a + b \sec(c + dx))^{3/2} dx \\ &= \frac{2(5Ab + 3aB)\sqrt{a + b \sec(c + dx)} \tan(c + dx)}{15d} + \frac{2}{5} \int \sec(c + dx) (a + b \sec(c + dx))^{3/2} dx \\ &= \frac{2(5Ab + 3aB)\sqrt{a + b \sec(c + dx)} \tan(c + dx)}{15d} + \frac{2}{5} \int \sec(c + dx) (a + b \sec(c + dx))^{3/2} dx \\ &= -\frac{2(a - b)\sqrt{a + b} (20aAb + 3a^2B + 9b^2B) \cot(c + dx)}{15d} \end{aligned}$$

Mathematica [A] time = 19.26, size = 502, normalized size = 1.61

$$\frac{\cos^2(c + dx)(a + b \sec(c + dx))^{3/2}(A + B \sec(c + dx)) \left(\frac{2(3a^2B + 20aAb + 9b^2B) \sin(c + dx)}{15b} + \frac{2}{15} \sec(c + dx)(6aB \sin(c + dx)) \right)}{d(a \cos(c + dx) + b)(A \cos(c + dx) + B)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sec[c + d*x]*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]

[Out] (-2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x])*(2*(a + b)*(20*a*A*b + 3*a^2*B + 9*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] * Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] * EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(a + b)*(3*a*(5*A + B) + b*(5*A + 9*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] * Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] * EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (20*a*A*b + 3*a^2*B + 9*b^2*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(15*b*d*(b + a*Cos[c + d*x])^2*(B + A*Cos[c + d*x])*Sqrt[Sec[(c + d*x)/2]^2*Sec[c + d*x]^(5/2) + (Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x])*((2*(20*a*A*b + 3*a^2*B + 9*b^2*B)*Sin[c + d*x])/(15*b) + (2*Sec[c + d*x]*(5*A*b*Sin[c + d*x] + 6*a*B*Sin[c + d*x]))/15 + (2*b*B*Sec[c + d*x]*Tan[c + d*x])/5)]/(d*(b + a*Cos[c + d*x])*(B + A*Cos[c + d*x]))

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral} \left((Bb \sec(dx + c)^3 + Aa \sec(dx + c) + (Ba + Ab) \sec(dx + c)^2) \sqrt{b \sec(dx + c) + a}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] integral((B*b*sec(d*x + c)^3 + A*a*sec(d*x + c) + (B*a + A*b)*sec(d*x + c)^2)*sqrt(b*sec(d*x + c) + a), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(b \sec(dx + c) + a)^{\frac{3}{2}} \sec(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)*sec(d*x + c), x)

maple [B] time = 2.18, size = 2683, normalized size = 8.60

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)), x)

[Out]
$$-2/15/d*(1+\cos(d*x+c))^{2*((b+a*\cos(d*x+c))/\cos(d*x+c))^{1/2}}*(-1+\cos(d*x+c))^{2*(-9*B*\sin(d*x+c)*\cos(d*x+c)^2*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))^{1/2}}*EllipticE((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{1/2})*a*b^2+5*A*\cos(d*x+c)^3*b^3-3*B*\cos(d*x+c)^3*a^3-3*b^3*B+9*B*\cos(d*x+c)^3*b^3-6*B*\cos(d*x+c)^2*b^3+20*A*\cos(d*x+c)^3*a*b^2-25*A*\cos(d*x+c)^2*a*b^2+9*B*\cos(d*x+c)^4*a*b^2+3*B*\cos(d*x+c)^3*a^2*b-9*B*\cos(d*x+c)^2*a^2*b-9*B*\cos(d*x+c)*a*b^2-5*A*\cos(d*x+c)*b^3+20*A*\cos(d*x+c)^4*a^2*b+5*A*\cos(d*x+c)^4*a*b^2-20*A*\cos(d*x+c)^3*a^2*b+6*B*\cos(d*x+c)^4*a^2*b+3*B*\cos(d*x+c)^4*a^3+3*B*\sin(d*x+c)*\cos(d*x+c)^3*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))^{1/2}}*EllipticF((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{1/2})*a^2*b+12*B*\sin(d*x+c)*\cos(d*x+c)^3*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))^{1/2}}*EllipticF((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{1/2})*a*b^2-3*B*\sin(d*x+c)*\cos(d*x+c)^3*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))^{1/2}}*EllipticE((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{1/2})*a^2*b-9*B*\sin(d*x+c)*\cos(d*x+c)^3*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))^{1/2}}*EllipticE((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{1/2})*a*b^2+20*A*\sin(d*x+c)*\cos(d*x+c)^2*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))^{1/2}}*EllipticF((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{1/2})*a*b^2-20*A*\sin(d*x+c)*\cos(d*x+c)^2*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))^{1/2}}*EllipticE((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{1/2})*a^2*b-20*A*\sin(d*x+c)*\cos(d*x+c)^2*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))^{1/2}}*EllipticE((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{1/2})*a^2*b-20*A*\sin(d*x+c)*\cos(d*x+c)^2*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))^{1/2}}*EllipticF((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{1/2})*a^2*b+12*B*\sin(d*x+c)*\cos(d*x+c)^2*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))^{1/2}}*EllipticF((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{1/2})*a*b^2-3*B*\sin(d*x+c)*\cos(d*x+c)^2*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))^{1/2}}*EllipticE((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{1/2})*a^2*b+20*A*\sin(d*x+c)*\cos(d*x+c)^3*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))^{1/2}}*EllipticF((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{1/2})*a*b^2-20*A*\sin(d*x+c)*\cos(d*x+c)^3*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))^{1/2}}*EllipticE((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{1/2})*a^2*b-20*A*\sin(d*x$$

```

+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1
/2))*a*b^2+15*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
(b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),((a-b)/(a+b))^(1/2))*a^2*b+15*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipti
cF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b-3*B*sin(d*x+c)*cos
(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^
3-9*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a
-b)/(a+b))^(1/2))*b^3+5*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3+9*B*sin(d*x+c)*cos(d*x+c)^2*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3-3*B*sin(d*x+c
)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2)
))*a^3-9*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),((a-b)/(a+b))^(1/2))*b^3+5*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3+9*B*sin(d*x+c)*cos(d*x+c)^3*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3)/(b+a*co
s(d*x+c))/cos(d*x+c)^2/sin(d*x+c)^5/b

```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(b \sec(dx + c) + a)^{\frac{3}{2}} \sec(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)*sec(d*x + c), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(a + \frac{b}{\cos(c+dx)}\right)^{3/2}}{\cos(c+dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(3/2))/cos(c + d*x),x)`

[Out] `int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(3/2))/cos(c + d*x), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \sec(c + dx))(a + b \sec(c + dx))^{\frac{3}{2}} \sec(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)*(a+b*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)),x)`

[Out] `Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**(3/2)*sec(c + d*x), x)`

3.359 $\int (a + b \sec(c + dx))^{3/2} (A + B \sec(c + dx)) dx$

Optimal. Leaf size=381

$$\frac{2\sqrt{a+b} \left(-3a^2B - a(6Ab - 4bB) + b^2(3A - B) \right) \cot(c + dx) \sqrt{\frac{b(1-\sec(c+dx))}{a+b}} \sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}} F\left(\sin^{-1}\left(\frac{\sqrt{a+b}\sec(c+dx)}{\sqrt{a+b}}\right)\right)}{3bd}$$

[Out] $-2/3*(a-b)*(3*A*b+4*B*a)*\cot(d*x+c)*\text{EllipticE}((a+b*\sec(d*x+c))^{1/2}/(a+b)^{(1/2}), ((a+b)/(a-b))^{1/2})*(a+b)^{(1/2)}*(b*(1-\sec(d*x+c))/(a+b))^{1/2}*(-b*(1+\sec(d*x+c))/(a-b))^{1/2}/b/d-2/3*(b^2*(3*A-B)-3*a^2*B-a*(6*A*b-4*B*b))*\cot(d*x+c)*\text{EllipticF}((a+b*\sec(d*x+c))^{1/2}/(a+b)^{(1/2}), ((a+b)/(a-b))^{1/2})*(a+b)^{(1/2)}*(b*(1-\sec(d*x+c))/(a+b))^{1/2}*(-b*(1+\sec(d*x+c))/(a-b))^{1/2}/b/d-2*a*A*\cot(d*x+c)*\text{EllipticPi}((a+b*\sec(d*x+c))^{1/2}/(a+b)^{(1/2}), (a+b)/a, ((a+b)/(a-b))^{1/2})*(a+b)^{(1/2)}*(b*(1-\sec(d*x+c))/(a+b))^{1/2}*(-b*(1+\sec(d*x+c))/(a-b))^{1/2}/d+2/3*b*B*(a+b*\sec(d*x+c))^{1/2}*\tan(d*x+c)/d$

Rubi [A] time = 0.46, antiderivative size = 381, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3918, 4058, 3921, 3784, 3832, 4004}

$$\frac{2\sqrt{a+b} \left(-3a^2B - a(6Ab - 4bB) + b^2(3A - B) \right) \cot(c + dx) \sqrt{\frac{b(1-\sec(c+dx))}{a+b}} \sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}} F\left(\sin^{-1}\left(\frac{\sqrt{a+b}\sec(c+dx)}{\sqrt{a+b}}\right)\right)}{3bd}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]

[Out] $(-2*(a-b)*\text{Sqrt}[a+b]*(3*A*b+4*a*B)*\text{Cot}[c+d*x]*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[a+b*\text{Sec}[c+d*x]]/\text{Sqrt}[a+b]], (a+b)/(a-b)]*\text{Sqrt}[(b*(1-\text{Sec}[c+d*x]))/(a+b)]*\text{Sqrt}[-((b*(1+\text{Sec}[c+d*x]))/(a-b))]/(3*b*d) - (2*\text{Sqrt}[a+b]*(b^2*(3*A-B)-3*a^2*B-a*(6*A*b-4*b*B))*\text{Cot}[c+d*x]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[a+b*\text{Sec}[c+d*x]]/\text{Sqrt}[a+b]], (a+b)/(a-b)]*\text{Sqrt}[(b*(1-\text{Sec}[c+d*x]))/(a+b)]*\text{Sqrt}[-((b*(1+\text{Sec}[c+d*x]))/(a-b))]/(3*b*d) - (2*a*A*\text{Sqrt}[a+b]*\text{Cot}[c+d*x]*\text{EllipticPi}[(a+b)/a, \text{ArcSin}[\text{Sqrt}[a+b*\text{Sec}[c+d*x]]/\text{Sqrt}[a+b]], (a+b)/(a-b)]*\text{Sqrt}[(b*(1-\text{Sec}[c+d*x]))/(a+b)]*\text{Sqrt}[-((b*(1+\text{Sec}[c+d*x]))/(a-b))]/d + (2*b*B*\text{Sqrt}[a+b*\text{Sec}[c+d*x]])*\text{Tan}[c+d*x]/(3*d)$

Rule 3784

Int[1/Sqrt[csc[(c_.) + (d_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] :> Simp[(2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)]/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&

NeQ[a^2 - b^2, 0]

Rule 3832

Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)]/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 3918

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1))/(f*m), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m - 2)*Simp[a^2*c*m + (b^2*d*(m - 1) + 2*a*b*c*m + a^2*d*m)*Csc[e + f*x] + b*(b*c*m + a*d*(2*m - 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && GtQ[m, 1] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m]

Rule 3921

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]

Rule 4004

Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A, 2]], (a*A + b*B)/(a*A - b*B)]/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]

Rule 4058

Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned}
\int (a + b \sec(c + dx))^{3/2} (A + B \sec(c + dx)) dx &= \frac{2bB\sqrt{a + b \sec(c + dx)} \tan(c + dx)}{3d} + \frac{2}{3} \int \frac{\frac{3a^2A}{2} + \frac{1}{2}(6aAb + 3a)}{\dots} \\
&= \frac{2bB\sqrt{a + b \sec(c + dx)} \tan(c + dx)}{3d} + \frac{2}{3} \int \frac{\frac{3a^2A}{2} + \left(-\frac{1}{2}b(3Ab + \dots)\right)}{\dots} \\
&= -\frac{2(a - b)\sqrt{a + b} (3Ab + 4aB) \cot(c + dx) E\left(\sin^{-1}\left(\frac{\sqrt{a+b \sec(c+dx)}}{\sqrt{a+b}}\right)\right)}{3bd} \\
&= -\frac{2(a - b)\sqrt{a + b} (3Ab + 4aB) \cot(c + dx) E\left(\sin^{-1}\left(\frac{\sqrt{a+b \sec(c+dx)}}{\sqrt{a+b}}\right)\right)}{3bd}
\end{aligned}$$

Mathematica [B] time = 24.62, size = 6063, normalized size = 15.91

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]), x]

[Out] Result too large to show

fricas [F] time = 23.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb \sec(dx + c)^2 + Aa + (Ba + Ab) \sec(dx + c)\right)\sqrt{b \sec(dx + c) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)), x, algorithm="fricas")

[Out] integral((B*b*sec(d*x + c)^2 + A*a + (B*a + A*b)*sec(d*x + c))*sqrt(b*sec(d*x + c) + a), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(b \sec(dx + c) + a)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)), x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2), x)

maple [B] time = 2.05, size = 2337, normalized size = 6.13

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\int (a+b\sec(dx+c))^{3/2} (A+B\sec(dx+c)), x$

[Out]
$$\begin{aligned} & -2/3/d*(-1+\cos(dx+c))^{2*}(-4*B*\cos(dx+c)^2*a^2+4*B*\cos(dx+c)^2*\sin(dx+c) \\ & *(\cos(dx+c)/(1+\cos(dx+c)))^{1/2}*((b+a*\cos(dx+c))/(1+\cos(dx+c))/(a+b))^{1/2} \\ & *EllipticF((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{1/2})*a*b-4*B*\cos \\ & (dx+c)*\sin(dx+c)*(\cos(dx+c)/(1+\cos(dx+c)))^{1/2}*((b+a*\cos(dx+c))/(1+c \\ & \cos(dx+c))/(a+b))^{1/2}*EllipticE((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{1/2} \\ &)^{1/2})*a*b+4*B*\cos(dx+c)*\sin(dx+c)*(\cos(dx+c)/(1+\cos(dx+c)))^{1/2}*((b+ \\ & a*\cos(dx+c))/(1+\cos(dx+c))/(a+b))^{1/2}*EllipticF((-1+\cos(dx+c))/\sin(dx \\ & +c), ((a-b)/(a+b))^{1/2})*a*b-3*A*\cos(dx+c)^2*\sin(dx+c)*(\cos(dx+c)/(1+\cos \\ & (dx+c)))^{1/2}*((b+a*\cos(dx+c))/(1+\cos(dx+c))/(a+b))^{1/2}*EllipticE((-1 \\ & +\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{1/2})*a*b+6*A*\cos(dx+c)^2*\sin(dx+c) \\ &)*(\cos(dx+c)/(1+\cos(dx+c)))^{1/2}*((b+a*\cos(dx+c))/(1+\cos(dx+c))/(a+b)) \\ & ^{1/2}*EllipticF((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{1/2})*a*b-4*B*co \\ & s(dx+c)^2*\sin(dx+c)*(\cos(dx+c)/(1+\cos(dx+c)))^{1/2}*((b+a*\cos(dx+c))/(\\ & 1+\cos(dx+c))/(a+b))^{1/2}*EllipticE((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b) \\ &))^{1/2})*a*b-3*A*\cos(dx+c)*\sin(dx+c)*(\cos(dx+c)/(1+\cos(dx+c)))^{1/2}*(\\ & (b+a*\cos(dx+c))/(1+\cos(dx+c))/(a+b))^{1/2}*EllipticE((-1+\cos(dx+c))/\sin(\\ & dx+c), ((a-b)/(a+b))^{1/2})*a*b+6*A*\cos(dx+c)*\sin(dx+c)*(\cos(dx+c)/(1+co \\ & s(dx+c)))^{1/2}*((b+a*\cos(dx+c))/(1+\cos(dx+c))/(a+b))^{1/2}*EllipticF((- \\ & 1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{1/2})*a*b+B*\cos(dx+c)^2*b^2-b^2*B+ \\ & 3*A*\cos(dx+c)^2*b^2-3*A*\cos(dx+c)*b^2-3*A*\cos(dx+c)^2*a*b+B*\cos(dx+c)^3 \\ & *a*b+4*B*\cos(dx+c)^2*a*b-5*B*\cos(dx+c)*a*b+3*A*\cos(dx+c)^3*a*b+4*B*\cos(d \\ & *x+c)^3*a^2+6*A*\cos(dx+c)*(\cos(dx+c)/(1+\cos(dx+c)))^{1/2}*((b+a*\cos(dx+x \\ & c))/(1+\cos(dx+c))/(a+b))^{1/2}*EllipticPi((-1+\cos(dx+c))/\sin(dx+c), -1, ((\\ & a-b)/(a+b))^{1/2})*\sin(dx+c)*a^2-3*A*\cos(dx+c)*(\cos(dx+c)/(1+\cos(dx+c)) \\ &)^{1/2}*((b+a*\cos(dx+c))/(1+\cos(dx+c))/(a+b))^{1/2}*EllipticF((-1+\cos(dx \\ & +c))/\sin(dx+c), ((a-b)/(a+b))^{1/2})*\sin(dx+c)*a^2-3*A*\cos(dx+c)^2*\sin(dx \\ & x+c)*(\cos(dx+c)/(1+\cos(dx+c)))^{1/2}*((b+a*\cos(dx+c))/(1+\cos(dx+c))/(a+ \\ & b))^{1/2}*EllipticE((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{1/2})*b^2+3*A \\ & *cos(dx+c)^2*\sin(dx+c)*(\cos(dx+c)/(1+\cos(dx+c)))^{1/2}*((b+a*\cos(dx+c) \\ &)/(1+\cos(dx+c))/(a+b))^{1/2}*EllipticF((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(\\ & a+b))^{1/2})*b^2-4*B*\cos(dx+c)^2*\sin(dx+c)*(\cos(dx+c)/(1+\cos(dx+c)))^{1 \\ & /2}*((b+a*\cos(dx+c))/(1+\cos(dx+c))/(a+b))^{1/2}*EllipticE((-1+\cos(dx+c)) \\ & /sin(dx+c), ((a-b)/(a+b))^{1/2})*a^2+B*\cos(dx+c)^2*\sin(dx+c)*(\cos(dx+c)/ \\ & (1+\cos(dx+c)))^{1/2}*((b+a*\cos(dx+c))/(1+\cos(dx+c))/(a+b))^{1/2}*Ellipti \\ & cF((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{1/2})*b^2-3*A*\cos(dx+c)*\sin(d \\ & *x+c)*(\cos(dx+c)/(1+\cos(dx+c)))^{1/2}*((b+a*\cos(dx+c))/(1+\cos(dx+c))/(a \\ & +b))^{1/2}*EllipticE((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{1/2})*b^2+3* \end{aligned}$$

$$\begin{aligned}
& A \cos(dx+c) \sin(dx+c) (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} ((b+a \cos(dx+c)) / (1+\cos(dx+c)) / (a+b))^{1/2} \text{EllipticF}((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{1/2}) \\
& * b^2 - 4B \cos(dx+c) \sin(dx+c) (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * ((b+a \cos(dx+c)) / (1+\cos(dx+c)) / (a+b))^{1/2} \text{EllipticE}((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{1/2}) \\
& * a^2 + B \cos(dx+c) \sin(dx+c) (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * ((b+a \cos(dx+c)) / (1+\cos(dx+c)) / (a+b))^{1/2} \text{EllipticF}((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{1/2}) \\
& * b^2 - 3A \cos(dx+c)^2 (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * ((b+a \cos(dx+c)) / (1+\cos(dx+c)) / (a+b))^{1/2} \text{EllipticF}((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{1/2}) \\
& * \sin(dx+c) * a^2 + 6A \cos(dx+c)^2 (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * ((b+a \cos(dx+c)) / (1+\cos(dx+c)) / (a+b))^{1/2} \text{EllipticPi}((-1+\cos(dx+c))/\sin(dx+c), -1, ((a-b)/(a+b))^{1/2}) \\
& * \sin(dx+c) * a^2 + 3B \cos(dx+c)^2 (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * ((b+a \cos(dx+c)) / (1+\cos(dx+c)) / (a+b))^{1/2} \text{EllipticF}((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{1/2}) \\
& * \sin(dx+c) * a^2 + 3B \cos(dx+c) (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * ((b+a \cos(dx+c)) / (1+\cos(dx+c)) / (a+b))^{1/2} \text{EllipticF}((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{1/2}) \\
& * \sin(dx+c) * a^2 * ((b+a \cos(dx+c)) / \cos(dx+c))^{1/2} * (1+\cos(dx+c))^2 / (b+a \cos(dx+c)) / \cos(dx+c) / \sin(dx+c)^5
\end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx+c) + A)(b \sec(dx+c) + a)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(dx+c))^(3/2)*(A+B*sec(dx+c)),x, algorithm="maxima")

[Out] integrate((B*sec(dx+c) + A)*(b*sec(dx+c) + a)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(A + \frac{B}{\cos(c+dx)} \right) \left(a + \frac{b}{\cos(c+dx)} \right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + dx))*(a + b/cos(c + dx))^(3/2), x)

[Out] int((A + B/cos(c + dx))*(a + b/cos(c + dx))^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \sec(c + dx))(a + b \sec(c + dx))^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**(3/2), x)
```

$$3.360 \quad \int \cos(c+dx)(a+b \sec(c+dx))^{3/2}(A+B \sec(c+dx)) dx$$

Optimal. Leaf size=361

$$\frac{\sqrt{a+b}(a(A+4B)+2b(A-B)) \cot(c+dx) \sqrt{\frac{b(1-\sec(c+dx))}{a+b}} \sqrt{\frac{b(\sec(c+dx)+1)}{a-b}} F\left(\sin^{-1}\left(\frac{\sqrt{a+b} \sec(c+dx)}{\sqrt{a+b}}\right) \middle| \frac{a+b}{a-b}\right)}{d} (a-b)$$

[Out] (a-b)*(A*a-2*B*b)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2), ((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c)))/(a+b)^(1/2)*(-b*(1+sec(d*x+c)))/(a-b)^(1/2)/b/d+(2*b*(A-B)+a*(A+4*B))*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2), ((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c)))/(a+b)^(1/2)*(-b*(1+sec(d*x+c)))/(a-b)^(1/2)/d-(3*A*b+2*B*a)*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2), (a+b)/a, ((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c)))/(a+b)^(1/2)*(-b*(1+sec(d*x+c)))/(a-b)^(1/2)/d+a*A*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d

Rubi [A] time = 0.45, antiderivative size = 361, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4025, 4058, 3921, 3784, 3832, 4004}

$$\frac{\sqrt{a+b}(a(A+4B)+2b(A-B)) \cot(c+dx) \sqrt{\frac{b(1-\sec(c+dx))}{a+b}} \sqrt{\frac{b(\sec(c+dx)+1)}{a-b}} F\left(\sin^{-1}\left(\frac{\sqrt{a+b} \sec(c+dx)}{\sqrt{a+b}}\right) \middle| \frac{a+b}{a-b}\right)}{d} (a-b)$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]

[Out] ((a - b)*Sqrt[a + b]*(a*A - 2*b*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))]/(b*d) + (Sqrt[a + b]*(2*b*(A - B) + a*(A + 4*B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))]/d - (Sqrt[a + b]*(3*A*b + 2*a*B)*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))]/d + (a*A*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/d

Rule 3784

Int[1/Sqrt[csc[(c_.) + (d_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] :> Simp[(2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)]/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&

NeQ[a^2 - b^2, 0]

Rule 3832

Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)]/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 3921

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]

Rule 4004

Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A, 2]], (a*A + b*B)/(a*A - b*B)]/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]

Rule 4025

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^m*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e + f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LeQ[n, -1]

Rule 4058

Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned}
\int \cos(c + dx)(a + b \sec(c + dx))^{3/2}(A + B \sec(c + dx)) dx &= \frac{aA\sqrt{a + b \sec(c + dx)} \sin(c + dx)}{d} - \int \frac{-\frac{1}{2}a(3Ab + 2b^2)}{d} dx \\
&= \frac{aA\sqrt{a + b \sec(c + dx)} \sin(c + dx)}{d} - \frac{1}{2}(b(aA - 2bB)) \\
&= \frac{(a - b)\sqrt{a + b}(aA - 2bB) \cot(c + dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b}}{\sqrt{a+b \sec(c + dx)}}\right)\right)}{bd} \\
&= \frac{(a - b)\sqrt{a + b}(aA - 2bB) \cot(c + dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b}}{\sqrt{a+b \sec(c + dx)}}\right)\right)}{bd}
\end{aligned}$$

Mathematica [B] time = 16.72, size = 971, normalized size = 2.69

$$\frac{2bB \cos(c + dx) \sin(c + dx)(a + b \sec(c + dx))^{3/2}}{d(b + a \cos(c + dx))} + \frac{\sqrt{\frac{1}{1 - \tan^2\left(\frac{1}{2}(c + dx)\right)}} \left(a^2 A \tan^5\left(\frac{1}{2}(c + dx)\right) - aAb \tan^5\left(\frac{1}{2}(c + dx)\right) \right)}{d}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Cos[c + d*x]*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]

[Out] (2*b*B*Cos[c + d*x]*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(d*(b + a*Cos[c + d*x])) + ((a + b*Sec[c + d*x])^(3/2)*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(a^2*A*Tan[(c + d*x)/2] + a*A*b*Tan[(c + d*x)/2] - 2*a*b*B*Tan[(c + d*x)/2] - 2*b^2*B*Tan[(c + d*x)/2] - 2*a^2*A*Tan[(c + d*x)/2]^3 + 4*a*b*B*Tan[(c + d*x)/2]^3 + a^2*A*Tan[(c + d*x)/2]^5 - a*A*b*Tan[(c + d*x)/2]^5 - 2*a*b*B*Tan[(c + d*x)/2]^5 + 2*b^2*B*Tan[(c + d*x)/2]^5 + 6*a*A*b*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 4*a^2*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 6*a*A*b*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 4*a^2*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2])

$$\begin{aligned} &]^2 * \text{Sqrt}[(a + b - a * \text{Tan}[(c + d * x) / 2]^2 + b * \text{Tan}[(c + d * x) / 2]^2) / (a + b)] + \\ & (a + b) * (a * A - 2 * b * B) * \text{EllipticE}[\text{ArcSin}[\text{Tan}[(c + d * x) / 2]], (a - b) / (a + b)] * \\ & \text{Sqrt}[1 - \text{Tan}[(c + d * x) / 2]^2 * (1 + \text{Tan}[(c + d * x) / 2]^2) * \text{Sqrt}[(a + b - a * \text{Tan}[(c + d * x) / 2]^2 + b * \text{Tan}[(c + d * x) / 2]^2) / (a + b)] - 2 * (2 * a * b * (A - B) + a^2 * B - b^2 * (A + B)) * \text{EllipticF}[\text{ArcSin}[\text{Tan}[(c + d * x) / 2]], (a - b) / (a + b)] * \text{Sqrt}[1 - \text{Tan}[(c + d * x) / 2]^2 * (1 + \text{Tan}[(c + d * x) / 2]^2) * \text{Sqrt}[(a + b - a * \text{Tan}[(c + d * x) / 2]^2 + b * \text{Tan}[(c + d * x) / 2]^2) / (a + b)]] / (d * (b + a * \text{Cos}[c + d * x])^{3/2} * \text{Sec}[c + d * x]^{3/2} * (1 + \text{Tan}[(c + d * x) / 2]^2)^{3/2} * \text{Sqrt}[(a + b - a * \text{Tan}[(c + d * x) / 2]^2 + b * \text{Tan}[(c + d * x) / 2]^2) / (1 + \text{Tan}[(c + d * x) / 2]^2))] \end{aligned}$$

fricas [F] time = 48.20, size = 0, normalized size = 0.00

integral((Bb cos(dx + c) sec(dx + c)² + Aa cos(dx + c) + (Ba + Ab) cos(dx + c) sec(dx + c)) $\sqrt{b \sec(dx + c)}$)

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] integral((B*b*cos(d*x + c)*sec(d*x + c)² + A*a*cos(d*x + c) + (B*a + A*b)*cos(d*x + c)*sec(d*x + c))*sqrt(b*sec(d*x + c) + a), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(b \sec(dx + c) + a)^{\frac{3}{2}} \cos(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)*cos(d*x + c), x)

maple [B] time = 2.04, size = 2196, normalized size = 6.08

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x)

[Out]
$$\begin{aligned} & -1/d * (-1 + \cos(d * x + c))^{3/2} * (2 * B * \text{EllipticF}((-1 + \cos(d * x + c)) / \sin(d * x + c), ((a - b) / (a + b))^{1/2}) * b^2 * (\cos(d * x + c) / (1 + \cos(d * x + c)))^{1/2} * ((b + a * \cos(d * x + c)) / (1 + \cos(d * x + c))) / (a + b)^{1/2} * \sin(d * x + c) - 2 * B * \cos(d * x + c) * \sin(d * x + c) * (\cos(d * x + c) / (1 + \cos(d * x + c)))^{1/2} * ((b + a * \cos(d * x + c)) / (1 + \cos(d * x + c))) / (a + b)^{1/2} * \text{EllipticE}((-1 + \cos(d * x + c)) / \sin(d * x + c), ((a - b) / (a + b))^{1/2}) * a * b + 4 * B * \cos(d * x + c) * \sin(d * x + c) * \end{aligned}$$

$$\begin{aligned}
& (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * ((b+a*\cos(dx+c))/(1+\cos(dx+c)) / (a+b))^{1/2} * \\
& \text{EllipticF}((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{1/2}) * a*b + A*\cos(dx+c)*\sin(dx+c)* \\
& (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * ((b+a*\cos(dx+c))/(1+\cos(dx+c)) / (a+b))^{1/2} * \\
& \text{EllipticE}((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{1/2}) * a*b - 4*A*\cos(dx+c)*\sin(dx+c)* \\
& (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * ((b+a*\cos(dx+c))/(1+\cos(dx+c)) / (a+b))^{1/2} * \\
& \text{EllipticF}((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{1/2}) * a*b - 2*b^2*B + 6*A*\cos(dx+c)*\sin(dx+c)* \\
& (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * ((b+a*\cos(dx+c))/(1+\cos(dx+c)) / (a+b))^{1/2} * \\
& \text{EllipticPi}((-1+\cos(dx+c))/\sin(dx+c), -1, ((a-b)/(a+b))^{1/2}) * a*b - 2*B * \\
& \text{EllipticE}((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{1/2}) * b^2 * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * \\
& ((b+a*\cos(dx+c))/(1+\cos(dx+c)) / (a+b))^{1/2} * \sin(dx+c) + A*\cos(dx+c)^2 * a*b - A*\cos(dx+c)*a*b + 2*B*\cos(dx+c)^2 * a*b - 2*B*\cos(dx+c)*a*b + 4*B*(\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * ((b+a*\cos(dx+c))/(1+\cos(dx+c)) / (a+b))^{1/2} * \\
& \text{EllipticPi}((-1+\cos(dx+c))/\sin(dx+c), -1, ((a-b)/(a+b))^{1/2}) * a^2 * \sin(dx+c) + 2*A * \\
& \text{EllipticF}((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{1/2}) * b^2 * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * ((b+a*\cos(dx+c))/(1+\cos(dx+c)) / (a+b))^{1/2} * \sin(dx+c) + A * \\
& \text{EllipticE}((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{1/2}) * a^2 * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * ((b+a*\cos(dx+c))/(1+\cos(dx+c)) / (a+b))^{1/2} * \sin(dx+c) - 2*B * \\
& \text{EllipticF}((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{1/2}) * a^2 * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * ((b+a*\cos(dx+c))/(1+\cos(dx+c)) / (a+b))^{1/2} * \sin(dx+c) + 2*b^2*B*\cos(dx+c) - 4*A*(\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * ((b+a*\cos(dx+c))/(1+\cos(dx+c)) / (a+b))^{1/2} * \\
& \text{EllipticF}((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{1/2}) * a*b*\sin(dx+c) + A*(\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * ((b+a*\cos(dx+c))/(1+\cos(dx+c)) / (a+b))^{1/2} * \\
& \text{EllipticE}((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{1/2}) * a*b*\sin(dx+c) + 4*B*(\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * ((b+a*\cos(dx+c))/(1+\cos(dx+c)) / (a+b))^{1/2} * \\
& \text{EllipticF}((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{1/2}) * a*b*\sin(dx+c) - 2*B*(\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * ((b+a*\cos(dx+c))/(1+\cos(dx+c)) / (a+b))^{1/2} * \\
& \text{EllipticE}((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{1/2}) * a*b*\sin(dx+c) - A*\cos(dx+c)^2 * a^2 + A*\cos(dx+c)^3 * a^2 + 2*A*\cos(dx+c)*\sin(dx+c)*(\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * ((b+a*\cos(dx+c))/(1+\cos(dx+c)) / (a+b))^{1/2} * \\
& \text{EllipticF}((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{1/2}) * b^2 + 2*B*\cos(dx+c)*\sin(dx+c)*(\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * ((b+a*\cos(dx+c))/(1+\cos(dx+c)) / (a+b))^{1/2} * \\
& \text{EllipticF}((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{1/2}) * b^2 + A*\cos(dx+c)*\sin(dx+c)*(\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * ((b+a*\cos(dx+c))/(1+\cos(dx+c)) / (a+b))^{1/2} * \\
& \text{EllipticE}((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{1/2}) * a^2 - 2*B*\cos(dx+c)*\sin(dx+c)*(\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * ((b+a*\cos(dx+c))/(1+\cos(dx+c)) / (a+b))^{1/2} * \\
& \text{EllipticE}((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{1/2}) * b^2 + 4*B*\cos(dx+c)*\sin(dx+c)*(\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * ((b+a*\cos(dx+c))/(1+\cos(dx+c)) / (a+b))^{1/2} * \\
& \text{EllipticPi}((-1+\cos(dx+c))/\sin(dx+c), -1, ((a-b)/(a+b))^{1/2}) * a^2 + 6*A * \\
& \text{EllipticPi}((-1+\cos(dx+c))/\sin(dx+c), -1, ((a-b)/(a+b))^{1/2}) * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * ((b+a*\cos(dx+c))/(1+\cos(dx+c)) / (a+b))^{1/2} * \sin(dx+c) * a*b - 2*B*\cos(dx+c)*(\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * ((b+a*\cos(dx+c))/(1+\cos(dx+c)) / (a+b))^{1/2} * \\
& \text{EllipticF}((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{1/2}) * \sin(dx+c) * a^2 * (
\end{aligned}$$

$1 + \cos(dx+c)^2 * ((b+a*\cos(dx+c))/\cos(dx+c))^{(1/2)} / (b+a*\cos(dx+c))/\sin(dx+c)^5$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx+c) + A)(b \sec(dx+c) + a)^{\frac{3}{2}} \cos(dx+c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(dx+c)*(a+b*sec(dx+c))^(3/2)*(A+B*sec(dx+c)),x, algorithm="maxima")

[Out] integrate((B*sec(dx+c) + A)*(b*sec(dx+c) + a)^(3/2)*cos(dx+c), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c+dx) \left(A + \frac{B}{\cos(c+dx)} \right) \left(a + \frac{b}{\cos(c+dx)} \right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c+d*x)*(A+B/cos(c+d*x))*(a+b/cos(c+d*x))^(3/2),x)

[Out] int(cos(c+d*x)*(A+B/cos(c+d*x))*(a+b/cos(c+d*x))^(3/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(dx+c)*(a+b*sec(dx+c))**(3/2)*(A+B*sec(dx+c)),x)

[Out] Timed out

3.361 $\int \cos^2(c+dx)(a+b \sec(c+dx))^{3/2}(A+B \sec(c+dx)) dx$

Optimal. Leaf size=428

$$\frac{\sqrt{a+b} (4a^2A + 12abB + 3Ab^2) \cot(c+dx) \sqrt{\frac{b(1-\sec(c+dx))}{a+b}} \sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}} \Pi\left(\frac{a+b}{a}; \sin^{-1}\left(\frac{\sqrt{a+b \sec(c+dx)}}{\sqrt{a+b}}\right) \middle| \frac{a+b}{a-b}\right)}{4ad}$$

[Out] $\frac{1}{4}*(a-b)*(5*A*b+4*B*a)*\cot(d*x+c)*\text{EllipticE}((a+b*\sec(d*x+c))^{1/2}/(a+b)^{(1/2)}, ((a+b)/(a-b))^{1/2})*(a+b)^{(1/2)}*(b*(1-\sec(d*x+c))/(a+b))^{1/2}*(-b*(1+\sec(d*x+c))/(a-b))^{1/2}/b/d+1/4*(2*A*a+5*A*b+4*B*a+8*B*b)*\cot(d*x+c)*\text{EllipticF}((a+b*\sec(d*x+c))^{1/2}/(a+b)^{(1/2)}, ((a+b)/(a-b))^{1/2})*(a+b)^{(1/2)}*(b*(1-\sec(d*x+c))/(a+b))^{1/2}*(-b*(1+\sec(d*x+c))/(a-b))^{1/2}/d-1/4*(4*A*a^2+3*A*b^2+12*B*a*b)*\cot(d*x+c)*\text{EllipticPi}((a+b*\sec(d*x+c))^{1/2}/(a+b)^{(1/2)}, (a+b)/a, ((a+b)/(a-b))^{1/2})*(a+b)^{(1/2)}*(b*(1-\sec(d*x+c))/(a+b))^{1/2}*(-b*(1+\sec(d*x+c))/(a-b))^{1/2}/a/d+1/4*(5*A*b+4*B*a)*\sin(d*x+c)*(a+b*\sec(d*x+c))^{1/2}/d+1/2*a*A*\cos(d*x+c)*\sin(d*x+c)*(a+b*\sec(d*x+c))^{1/2}/d$

Rubi [A] time = 0.79, antiderivative size = 428, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {4025, 4104, 4058, 3921, 3784, 3832, 4004}

$$\frac{\sqrt{a+b} (4a^2A + 12abB + 3Ab^2) \cot(c+dx) \sqrt{\frac{b(1-\sec(c+dx))}{a+b}} \sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}} \Pi\left(\frac{a+b}{a}; \sin^{-1}\left(\frac{\sqrt{a+b \sec(c+dx)}}{\sqrt{a+b}}\right) \middle| \frac{a+b}{a-b}\right)}{4ad}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Cos}[c + d*x]^2*(a + b*\text{Sec}[c + d*x])^{3/2}*(A + B*\text{Sec}[c + d*x]), x]$

[Out] $((a - b)*\text{Sqrt}[a + b]*(5*A*b + 4*a*B)*\text{Cot}[c + d*x]*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[a + b*\text{Sec}[c + d*x]]/\text{Sqrt}[a + b]], (a + b)/(a - b)]*\text{Sqrt}[(b*(1 - \text{Sec}[c + d*x]))/(a + b)]*\text{Sqrt}[-((b*(1 + \text{Sec}[c + d*x]))/(a - b))]/(4*b*d) + (\text{Sqrt}[a + b]*(2*a*A + 5*A*b + 4*a*B + 8*b*B)*\text{Cot}[c + d*x]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[a + b*\text{Sec}[c + d*x]]/\text{Sqrt}[a + b]], (a + b)/(a - b)]*\text{Sqrt}[(b*(1 - \text{Sec}[c + d*x]))/(a + b)]*\text{Sqrt}[-((b*(1 + \text{Sec}[c + d*x]))/(a - b))]/(4*d) - (\text{Sqrt}[a + b]*(4*a^2*A + 3*A*b^2 + 12*a*b*B)*\text{Cot}[c + d*x]*\text{EllipticPi}[(a + b)/a, \text{ArcSin}[\text{Sqrt}[a + b*\text{Sec}[c + d*x]]/\text{Sqrt}[a + b]], (a + b)/(a - b)]*\text{Sqrt}[(b*(1 - \text{Sec}[c + d*x]))/(a + b)]*\text{Sqrt}[-((b*(1 + \text{Sec}[c + d*x]))/(a - b))]/(4*a*d) + ((5*A*b + 4*a*B)*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(4*d) + (a*A*\text{Cos}[c + d*x]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(2*d)$

Rule 3784

$\text{Int}[1/\text{Sqrt}[\text{csc}[(c_.) + (d_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] \rightarrow \text{Simp}[(2*\text{Rt}[a + b, 2]*\text{Sqrt}[(b*(1 - \text{Csc}[c + d*x]))/(a + b)]*\text{Sqrt}[-((b*(1 + \text{Csc}[c + d*x]))$

$$\frac{1}{(a-b)} \text{EllipticPi}\left[\frac{a+b}{a}, \text{ArcSin}\left[\frac{\sqrt{a+b\text{Csc}[c+dx]}}{\text{Rt}[a+b, 2]}\right], \frac{a+b}{a-b}\right] / (a*d\text{Cot}[c+dx]), x \text{ ; FreeQ}\{a, b, c, d\}, x \ \&\& \ \text{NeQ}[a^2 - b^2, 0]$$

Rule 3832

$$\text{Int}\left[\frac{\text{csc}[e] + (f)(x)}{\sqrt{\text{csc}[e] + (f)(x)(b) + (a)}}, x_{\text{Symbol}}\right] \rightarrow \text{Simp}\left[\frac{-2\text{Rt}[a+b, 2]\sqrt{(b(1-\text{Csc}[e+fx]))/(a+b)}\sqrt{-((b(1+\text{Csc}[e+fx]))/(a-b))}\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b\text{Csc}[e+fx]}}{\text{Rt}[a+b, 2]}\right], \frac{a+b}{a-b}\right]}{(b*f\text{Cot}[e+fx])}, x \text{ ; FreeQ}\{a, b, e, f\}, x \ \&\& \ \text{NeQ}[a^2 - b^2, 0]\right]$$

Rule 3921

$$\text{Int}\left[\frac{(\text{csc}[e] + (f)(x))(d) + (c)}{\sqrt{\text{csc}[e] + (f)(x)(b) + (a)}}, x_{\text{Symbol}}\right] \rightarrow \text{Dist}[c, \text{Int}\left[\frac{1}{\sqrt{a+b\text{Csc}[e+fx]}}\right], x] + \text{Dist}[d, \text{Int}\left[\frac{\text{Csc}[e+fx]}{\sqrt{a+b\text{Csc}[e+fx]}}\right], x] \text{ ; FreeQ}\{a, b, c, d, e, f\}, x \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{NeQ}[a^2 - b^2, 0]$$

Rule 4004

$$\text{Int}\left[\frac{(\text{csc}[e] + (f)(x))(c\text{Csc}[e] + (f)(x)(B) + (A))}{\sqrt{\text{csc}[e] + (f)(x)(b) + (a)}}, x_{\text{Symbol}}\right] \rightarrow \text{Simp}\left[\frac{-2(A*b - a*B)\text{Rt}[a + (b*B)/A, 2]\sqrt{(b(1-\text{Csc}[e+fx]))/(a+b)}\sqrt{-((b(1+\text{Csc}[e+fx]))/(a-b))}\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b\text{Csc}[e+fx]}}{\text{Rt}[a + (b*B)/A, 2]}\right], \frac{(a*A + b*B)/(a*A - b*B)}{(b^2*f\text{Cot}[e+fx])}, x \text{ ; FreeQ}\{a, b, e, f, A, B\}, x \ \&\& \ \text{NeQ}[a^2 - b^2, 0] \ \&\& \ \text{EqQ}[A^2 - B^2, 0]\right]$$

Rule 4025

$$\text{Int}\left[\frac{(\text{csc}[e] + (f)(x))(d)^n(\text{csc}[e] + (f)(x)(b) + (a))^m(\text{csc}[e] + (f)(x)(B) + (A))}{(a+b\text{Csc}[e+fx])^{m-1}(d\text{Csc}[e+fx])^n}, x \text{ ; FreeQ}\{a, b, d, e, f, A, B\}, x \ \&\& \ \text{NeQ}[A*b - a*B, 0] \ \&\& \ \text{NeQ}[a^2 - b^2, 0] \ \&\& \ \text{GtQ}[m, 1] \ \&\& \ \text{LeQ}[n, -1]$$

Rule 4058

$$\text{Int}\left[\frac{(A) + \text{csc}[e] + (f)(x)(B) + \text{csc}[e] + (f)(x)^2(C)}{\sqrt{\text{csc}[e] + (f)(x)(b) + (a)}}, x_{\text{Symbol}}\right] \rightarrow \text{Int}\left[\frac{(A + (B - C)\text{Csc}[e+fx])}{\sqrt{a+b\text{Csc}[e+fx]}}\right], x + \text{Dist}[C, \text{Int}\left[\frac{\text{Csc}[e+fx](1 + \text{Csc}[e+fx])}{\sqrt{a+b\text{Csc}[e+fx]}}\right], x] \text{ ; FreeQ}\{a, b, e, f, A,$$

B, C}, x] && NeQ[a^2 - b^2, 0]

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.
))*csc[(e_.) + (f_.)*(x_.)]*(d_.)^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a
_)^m), x_Symbol] := Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\begin{aligned} \int \cos^2(c + dx)(a + b \sec(c + dx))^{3/2}(A + B \sec(c + dx)) dx &= \frac{aA \cos(c + dx) \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{2d} - \frac{1}{2} \\ &= \frac{(5Ab + 4aB) \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{4d} + \frac{aA \cos(c + dx)}{2d} \\ &= \frac{(5Ab + 4aB) \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{4d} + \frac{aA \cos(c + dx)}{2d} \\ &= \frac{(a - b) \sqrt{a + b} (5Ab + 4aB) \cot(c + dx) E\left(\sin^{-1}\left(\frac{\sqrt{a}}{\sqrt{a + b}}\right)\right)}{4bc} \\ &= \frac{(a - b) \sqrt{a + b} (5Ab + 4aB) \cot(c + dx) E\left(\sin^{-1}\left(\frac{\sqrt{a}}{\sqrt{a + b}}\right)\right)}{4bc} \end{aligned}$$

Mathematica [C] time = 19.57, size = 1598, normalized size = 3.73

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a*A*Cos[c + d*x]*(a + b*Sec[c + d*x])^(3/2)*Sin[2*(c + d*x)])/(4*d*(b + a*
Cos[c + d*x])) - ((a + b*Sec[c + d*x])^(3/2)*(5*a*A*b*Sqrt[(-a + b)/(a + b)]
)*Tan[(c + d*x)/2] + 5*A*b^2*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2] + 4*a^
```

```

2*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)/2] + 4*a*b*Sqrt[(-a + b)/(a + b)]*
B*Tan[(c + d*x)/2] - 10*a*A*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^3 - 8
*a^2*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)/2]^3 + 5*a*A*b*Sqrt[(-a + b)/(a
+ b)]*Tan[(c + d*x)/2]^5 - 5*A*b^2*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]
^5 + 4*a^2*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)/2]^5 - 4*a*b*Sqrt[(-a + b
)/(a + b)]*B*Tan[(c + d*x)/2]^5 - (8*I)*a^2*A*EllipticPi[-((a + b)/(a - b))
, I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt
[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*
x)/2]^2)/(a + b)] - (6*I)*A*b^2*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sq
rt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c +
d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a +
b)] - (24*I)*a*b*B*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(
a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sq
rt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (8*I)*a
^2*A*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c
+ d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^
2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (6
*I)*A*b^2*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*T
an[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x
)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]
- (24*I)*a*b*B*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a +
b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c
+ d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a
+ b)] - I*(a - b)*(5*A*b + 4*a*B)*EllipticE[I*ArcSinh[Sqrt[(-a + b)/(a + b
)]]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Ta
n[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2
)/(a + b)] + (2*I)*(a - b)*(2*a*A + b*(A + 4*B))*EllipticF[I*ArcSinh[Sqrt[(
-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)
/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[
(c + d*x)/2]^2)/(a + b)]))/(4*Sqrt[(-a + b)/(a + b)]*d*(b + a*Cos[c + d*x])
^(3/2)*Sec[c + d*x]^(3/2)*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(-1 + Tan[(c
+ d*x)/2]^2)*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b - a*Tan[(c + d*x)/2
]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2))]

```

fricas [F] time = 1.60, size = 0, normalized size = 0.00

integral((Bb cos(dx + c)² sec(dx + c)² + Aa cos(dx + c)² + (Ba + Ab) cos(dx + c)² sec(dx + c))sqrt(b sec(dx + c) + a), x)

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] integral((B*b*cos(d*x + c)^2*sec(d*x + c)^2 + A*a*cos(d*x + c)^2 + (B*a + A*b)*cos(d*x + c)^2*sec(d*x + c))*sqrt(b*sec(d*x + c) + a), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(b \sec(dx + c) + a)^{\frac{3}{2}} \cos(dx + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^2, x)

maple [B] time = 1.91, size = 2439, normalized size = 5.70

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^2*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x)

[Out]
$$\begin{aligned} & -1/4/d*(-1+\cos(d*x+c))^2*(6*A*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*\text{EllipticPi}((-1+\cos(d*x+c))/\sin(d*x+c),-1, \\ & ((a-b)/(a+b))^{1/2})*b^2*\sin(d*x+c)+8*B*\text{EllipticF}((-1+\cos(d*x+c))/\sin(d*x+c), \\ & ((a-b)/(a+b))^{1/2})*b^2*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*\text{E} \\ & \text{llipticE}((-1+\cos(d*x+c))/\sin(d*x+c),((a-b)/(a+b))^{1/2})*b^2*\sin(d*x+c)+4*B \\ & *(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*\text{EllipticE}((-1+\cos(d*x+c))/\sin(d*x+c), \\ & ((a-b)/(a+b))^{1/2})*a^2*\sin(d*x+c)+8*A*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/ \\ & (a+b))^{1/2}*\text{EllipticPi}((-1+\cos(d*x+c))/\sin(d*x+c),-1,((a-b)/(a+b))^{1/2})* \\ & a^2*\sin(d*x+c)+4*B*\cos(d*x+c)*\sin(d*x+c)*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2} \\ & *((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*\text{EllipticE}((-1+\cos(d*x+c))/\sin \\ & (d*x+c),((a-b)/(a+b))^{1/2})*a*b-16*B*\cos(d*x+c)*\sin(d*x+c)*(\cos(d*x+c)/(1+ \\ & \cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*\text{EllipticF} \\ & (-1+\cos(d*x+c))/\sin(d*x+c),((a-b)/(a+b))^{1/2})*a*b+5*A*\cos(d*x+c)*\sin(d*x+c) \\ & *(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b) \\ &)^{1/2}*\text{EllipticE}((-1+\cos(d*x+c))/\sin(d*x+c),((a-b)/(a+b))^{1/2})*a*b+2*A*\cos \\ & (d*x+c)*\sin(d*x+c)*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1 \\ & +\cos(d*x+c))/(a+b))^{1/2}*\text{EllipticF}((-1+\cos(d*x+c))/\sin(d*x+c),((a-b)/(a+b) \\ &)^{1/2})*a*b+5*A*\cos(d*x+c)^2*b^2-5*A*\cos(d*x+c)*b^2+24*B*\cos(d*x+c)*(\cos(d \\ & *x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*\text{E} \\ & \text{llipticPi}((-1+\cos(d*x+c))/\sin(d*x+c),-1,((a-b)/(a+b))^{1/2})*\sin(d*x+c)*a*b \\ & -5*A*\cos(d*x+c)^2*a*b-2*A*\cos(d*x+c)*a*b+4*B*\cos(d*x+c)^2*a*b-4*B*\cos(d*x+c) \\ &)*a*b-4*A*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c) \end{aligned}$$


```

)/(a+b)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^
2*sin(d*x+c)-8*A*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*
b^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*sin(d*x+c)+7*A*cos(d*x+c)^3*a*b+4*B*cos(d*x+c)^3*a^2+8*A*cos(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*sin(d*
x+c)*a^2+6*A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b
)/(a+b))^(1/2))*sin(d*x+c)*b^2-4*A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2+2*A*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b*sin(d*x+c)+5*A*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b*sin(d*x+c)+24*B*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*El
lipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a*b*sin(d*x+c)-
16*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b*sin
(d*x+c)+4*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*
a*b*sin(d*x+c)-2*A*cos(d*x+c)^2*a^2+2*A*cos(d*x+c)^4*a^2+5*A*cos(d*x+c)*sin
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2-
8*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/
(a+b))^(1/2))*b^2+4*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2+8*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2*(1+cos(d*x+c))^2*(b
+a*cos(d*x+c))/cos(d*x+c)^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)^5

```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(b \sec(dx + c) + a)^{\frac{3}{2}} \cos(dx + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^2, x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c + dx)^2 \left(A + \frac{B}{\cos(c + dx)} \right) \left(a + \frac{b}{\cos(c + dx)} \right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^2*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(3/2), x)

[Out] int(cos(c + d*x)^2*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(3/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**2*(a+b*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)), x)

[Out] Timed out

$$3.362 \quad \int \cos^3(c+dx)(a+b \sec(c+dx))^{3/2}(A+B \sec(c+dx)) dx$$

Optimal. Leaf size=520

$$\frac{(16a^2A + 30abB + 3Ab^2) \sin(c + dx) \sqrt{a + b \sec(c + dx)}}{24ad} + \frac{\sqrt{a + b} (16a^2A + 12a^2B + 14aAb + 30abB + 3Ab^2)}{24ad}$$

[Out] 1/24*(a-b)*(16*A*a^2+3*A*b^2+30*B*a*b)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a/b/d+1/24*(16*A*a^2+14*A*a*b+3*A*b^2+12*B*a^2+30*B*a*b)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a/d-1/8*(12*A*a^2*b-A*b^3+8*B*a^3+6*B*a*b^2)*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2), (a+b)/a, ((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d+1/24*(16*A*a^2+3*A*b^2+30*B*a*b)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/a/d+1/12*(7*A*b+6*B*a)*cos(d*x+c)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d+1/3*a*A*cos(d*x+c)^2*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d

Rubi [A] time = 1.28, antiderivative size = 520, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {4025, 4104, 4058, 3921, 3784, 3832, 4004}

$$\frac{(16a^2A + 30abB + 3Ab^2) \sin(c + dx) \sqrt{a + b \sec(c + dx)}}{24ad} + \frac{\sqrt{a + b} (16a^2A + 12a^2B + 14aAb + 30abB + 3Ab^2)}{24ad}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]

[Out] ((a - b)*Sqrt[a + b]*(16*a^2*A + 3*A*b^2 + 30*a*b*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))]/(24*a*b*d) + (Sqrt[a + b]*(16*a^2*A + 14*a*A*b + 3*A*b^2 + 12*a^2*B + 30*a*b*B)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))]/(24*a*d) - (Sqrt[a + b]*(12*a^2*A*b - A*b^3 + 8*a^3*B + 6*a*b^2*B)*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))]/(8*a^2*d) + ((16*a^2*A + 3*A*b^2 + 30*a*b*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(24*a*d) + ((7*A*b + 6*a*B)*Cos[c + d

*x]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(12*d) + (a*A*Cos[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*d)

Rule 3784

Int[1/Sqrt[csc[(c_.) + (d_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] := Simp[(2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)]/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 3832

Int[csc[(e_.) + (f_.)*(x_.)]/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] := Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)]/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 3921

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] := Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]

Rule 4004

Int[(csc[(e_.) + (f_.)*(x_.)]*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)))/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] := Simp[(-2*(A*b - a*B)*Rt[a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A, 2]], (a*A + b*B)/(a*A - b*B)]/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]

Rule 4025

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := Simp[(a*A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n))]*Csc[e + f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LeQ[n, -1]

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)])*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] := Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
+ Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)])*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^m, x_Symbol] := Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x] /; FreeQ[{a, b, d,
e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \cos^3(c + dx)(a + b \sec(c + dx))^{3/2}(A + B \sec(c + dx)) dx &= \frac{aA \cos^2(c + dx) \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{3d} - \\
&= \frac{(7Ab + 6aB) \cos(c + dx) \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{12d} \\
&= \frac{(16a^2A + 3Ab^2 + 30abB) \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{24ad} \\
&= \frac{(16a^2A + 3Ab^2 + 30abB) \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{24ad} \\
&= \frac{(a - b) \sqrt{a + b} (16a^2A + 3Ab^2 + 30abB) \cot(c + dx)}{24ad} \\
&= \frac{(a - b) \sqrt{a + b} (16a^2A + 3Ab^2 + 30abB) \cot(c + dx)}{24ad}
\end{aligned}$$

Mathematica [B] time = 19.46, size = 1535, normalized size = 2.95

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]

[Out] (Cos[c + d*x]*(a + b*Sec[c + d*x])^(3/2)*((a*A*Sin[c + d*x])/12 + ((7*A*b + 6*a*B)*Sin[2*(c + d*x)]/24 + (a*A*Sin[3*(c + d*x)]/12)))/(d*(b + a*Cos[c + d*x])) + ((a + b*Sec[c + d*x])^(3/2)*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(16*a^3*A*Tan[(c + d*x)/2] + 16*a^2*A*b*Tan[(c + d*x)/2] + 3*a*A*b^2*Tan[(c + d*x)/2] + 3*A*b^3*Tan[(c + d*x)/2] + 30*a^2*b*B*Tan[(c + d*x)/2] + 30*a*b^2*B*Tan[(c + d*x)/2] - 32*a^3*A*Tan[(c + d*x)/2]^3 - 6*a*A*b^2*Tan[(c + d*x)/2]^3 - 60*a^2*b*B*Tan[(c + d*x)/2]^3 + 16*a^3*A*Tan[(c + d*x)/2]^5 - 16*a^2*A*b*Tan[(c + d*x)/2]^5 + 3*a*A*b^2*Tan[(c + d*x)/2]^5 - 3*A*b^3*Tan[(c + d*x)/2]^5 + 30*a^2*b*B*Tan[(c + d*x)/2]^5 - 30*a*b^2*B*Tan[(c + d*x)/2]^5 + 72*a^2*A*b*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 6*A*b^3*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 48*a^3*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 36*a*b^2*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 72*a^2*A*b*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 6*A*b^3*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 48*a^3*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 36*a*b^2*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (a + b)*(16*a^2*A + 3*A*b^2 + 30*a*b*B)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 2*a*(12*a^2*B + b^2*(-7*A + 24*B) + a*(26*A*b - 6*b*B))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)))/(24*a*d*(b + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2))]

fricas [F] time = 2.10, size = 0, normalized size = 0.00

integral(((Bb cos(dx + c))^3 sec(dx + c)^2 + Aa cos(dx + c)^3 + (Ba + Ab) cos(dx + c)^3 sec(dx + c))sqrt(b sec(dx + c)))

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^3*sec(d*x + c)^2 + A*a*cos(d*x + c)^3 + (B*a + A*b)*cos(d*x + c)^3*sec(d*x + c))*sqrt(b*sec(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(b \sec(dx + c) + a)^{\frac{3}{2}} \cos(dx + c)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^3, x)
```

maple [B] time = 2.15, size = 3142, normalized size = 6.04

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] -1/24/d*(-1+cos(d*x+c))^2*(16*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)+8*A*cos(d*x+c)^3*a^3-16*A*cos(d*x+c)^2*a^3+3*A*cos(d*x+c)^2*b^3-12*B*cos(d*x+c)^2*a^3+17*A*cos(d*x+c)^3*a*b^2-6*A*cos(d*x+c)^2*a^2*b-3*A*cos(d*x+c)^2*a*b^2-16*A*cos(d*x+c)*a^2*b-14*A*cos(d*x+c)*a*b^2+42*B*cos(d*x+c)^3*a^2*b-30*B*cos(d*x+c)^2*a^2*b+30*B*cos(d*x+c)^2*a*b^2-12*B*cos(d*x+c)*a^2*b-30*B*cos(d*x+c)*a*b^2-3*A*cos(d*x+c)*b^3+22*A*cos(d*x+c)^4*a^2*b+3*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)+12*B*cos(d*x+c)^4*a^3-24*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^3+72*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)-52*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)+14*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))
```



```
*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*sin(d
*x+c)*cos(d*x+c)*a*b^2+12*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b
)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*b+30*B*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*b-48*B*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c))
*(1+cos(d*x+c))^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sin(
d*x+c)^5/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(b \sec(dx + c) + a)^{\frac{3}{2}} \cos(dx + c)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^3, x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c + dx)^3 \left(A + \frac{B}{\cos(c + dx)} \right) \left(a + \frac{b}{\cos(c + dx)} \right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^3*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

$$3.363 \quad \int \sec^3(c+dx)(a+b \sec(c+dx))^{5/2}(A+B \sec(c+dx)) dx$$

Optimal. Leaf size=566

$$\frac{2(-8a^2B + 22aAb - 81b^2B) \tan(c+dx)(a+b \sec(c+dx))^{5/2}}{693b^2d} - \frac{2(-40a^3B + 110a^2Ab - 335ab^2B - 539Ab^3) \tan(c+dx)}{3465b^2d}$$

[Out] 2/3465*(a-b)*(110*A*a^4*b-3069*A*a^2*b^3-1617*A*b^5-40*B*a^5-255*B*a^3*b^2-3705*B*a*b^4)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c)))/(a-b))^(1/2)/b^4/d-2/3465*(a-b)*(6*a*b^3*(209*A-505*B)-3*b^4*(539*A-225*B)-15*a^2*b^2*(121*A-19*B)+40*a^4*B-a^3*(110*A*b-30*B*b))*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c)))/(a-b))^(1/2)/b^3/d-2/3465*(110*A*a^2*b-539*A*b^3-40*B*a^3-335*B*a*b^2)*(a+b*sec(d*x+c))^(3/2)*tan(d*x+c)/b^2/d-2/693*(22*A*a*b-8*B*a^2-81*B*b^2)*(a+b*sec(d*x+c))^(5/2)*tan(d*x+c)/b^2/d+2/99*(11*A*b-4*B*a)*(a+b*sec(d*x+c))^(7/2)*tan(d*x+c)/b^2/d+2/11*B*sec(d*x+c)*(a+b*sec(d*x+c))^(7/2)*tan(d*x+c)/b/d-2/3465*(110*A*a^3*b-1254*A*a*b^3-40*B*a^4-285*B*a^2*b^2-675*B*b^4)*(a+b*sec(d*x+c))^(1/2)*tan(d*x+c)/b^2/d

Rubi [A] time = 1.78, antiderivative size = 566, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4033, 4082, 4002, 4005, 3832, 4004}

$$\frac{2(-8a^2B + 22aAb - 81b^2B) \tan(c+dx)(a+b \sec(c+dx))^{5/2}}{693b^2d} - \frac{2(110a^2Ab - 40a^3B - 335ab^2B - 539Ab^3) \tan(c+dx)}{3465b^2d}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]

[Out] (2*(a - b)*Sqrt[a + b]*(110*a^4*A*b - 3069*a^2*A*b^3 - 1617*A*b^5 - 40*a^5*B - 255*a^3*b^2*B - 3705*a*b^4*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))]/(3465*b^4*d) - (2*(a - b)*Sqrt[a + b]*(6*a*b^3*(209*A - 505*B) - 3*b^4*(539*A - 225*B) - 15*a^2*b^2*(121*A - 19*B) + 40*a^4*B - a^3*(110*A*b - 30*B*b))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))]/(3465*b^3*d) - (2*(110*a^3*A*b - 1254*a*A*b^3 - 40*a^4*B - 285*a^2*b^2*B - 675*b^4*B)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(3465*b^2*d) - (2*(110*a^2*A*b - 539

$$\frac{A^3 b^3 - 40 A^3 B - 335 A^2 b^2 B}{3465 b^2 d} (a + b \sec[c + dx])^{3/2} \tan[c + dx] + \frac{(2(22 A^2 A b - 8 a^2 B - 81 b^2 B) (a + b \sec[c + dx])^{5/2} \tan[c + dx])}{693 b^2 d} + \frac{(2(11 A^2 b - 4 a^2 B) (a + b \sec[c + dx])^{7/2} \tan[c + dx])}{99 b^2 d} + \frac{(2 B \sec[c + dx] (a + b \sec[c + dx])^{7/2} \tan[c + dx])}{11 b d}$$

Rule 3832

$$\text{Int}[\text{csc}[(e_.) + (f_.) (x_)] / \text{Sqrt}[\text{csc}[(e_.) + (f_.) (x_)] (b_.) + (a_)], x_Symbol] \rightarrow \text{Simp}[(-2 \text{Rt}[a + b, 2] \text{Sqrt}[(b(1 - \text{Csc}[e + f x])) / (a + b)] \text{Sqrt}[-((b(1 + \text{Csc}[e + f x])) / (a - b))] \text{EllipticF}[\text{ArcSin}[\text{Sqrt}[a + b \text{Csc}[e + f x]]] / \text{Rt}[a + b, 2]], (a + b) / (a - b))] / (b f \text{Cot}[e + f x]), x] /; \text{FreeQ}[\{a, b, e, f\}, x] \&\& \text{NeQ}[a^2 - b^2, 0]$$

Rule 4002

$$\text{Int}[\text{csc}[(e_.) + (f_.) (x_)] (b_.) + (a_)]^{(m)} (\text{csc}[(e_.) + (f_.) (x_)] (B_.) + (A_)), x_Symbol] \rightarrow -\text{Simp}[(B \text{Cot}[e + f x] (a + b \text{Csc}[e + f x])^m) / (f(m + 1)), x] + \text{Dist}[1 / (m + 1), \text{Int}[\text{Csc}[e + f x] (a + b \text{Csc}[e + f x])^{(m - 1)} \text{Simp}[b B^m + a A (m + 1) + (a B^m + A b (m + 1)) \text{Csc}[e + f x], x], x] /; \text{FreeQ}[\{a, b, A, B, e, f\}, x] \&\& \text{NeQ}[A b - a B, 0] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{GtQ}[m, 0]$$

Rule 4004

$$\text{Int}[(\text{csc}[(e_.) + (f_.) (x_)] (b_.) + (a_)) / \text{Sqrt}[\text{csc}[(e_.) + (f_.) (x_)] (b_.) + (a_)], x_Symbol] \rightarrow \text{Simp}[(-2(A b - a B) \text{Rt}[a + (b B) / A, 2] \text{Sqrt}[(b(1 - \text{Csc}[e + f x])) / (a + b)] \text{Sqrt}[-((b(1 + \text{Csc}[e + f x])) / (a - b))] \text{EllipticE}[\text{ArcSin}[\text{Sqrt}[a + b \text{Csc}[e + f x]]] / \text{Rt}[a + (b B) / A, 2]], (a A + b B) / (a A - b B))] / (b^2 f \text{Cot}[e + f x]), x] /; \text{FreeQ}[\{a, b, e, f, A, B\}, x] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{EqQ}[A^2 - B^2, 0]$$

Rule 4005

$$\text{Int}[(\text{csc}[(e_.) + (f_.) (x_)] (b_.) + (a_)) / \text{Sqrt}[\text{csc}[(e_.) + (f_.) (x_)] (b_.) + (a_)], x_Symbol] \rightarrow \text{Dist}[A - B, \text{Int}[\text{Csc}[e + f x] / \text{Sqrt}[a + b \text{Csc}[e + f x]], x], x] + \text{Dist}[B, \text{Int}[(\text{Csc}[e + f x] (1 + \text{Csc}[e + f x])) / \text{Sqrt}[a + b \text{Csc}[e + f x]], x], x] /; \text{FreeQ}[\{a, b, e, f, A, B\}, x] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{NeQ}[A^2 - B^2, 0]$$

Rule 4033

$$\text{Int}[(\text{csc}[(e_.) + (f_.) (x_)] (d_.)^{(n_)} (\text{csc}[(e_.) + (f_.) (x_)] (b_.) + (a_))^{(m_)} (\text{csc}[(e_.) + (f_.) (x_)] (B_.) + (A_)), x_Symbol] \rightarrow -\text{Simp}[(B d^2 \text{Cot}[e + f x] (a + b \text{Csc}[e + f x])^{(m + 1)} (d \text{Csc}[e + f x])^{(n - 2)}) / (b f (m + n)), x] + \text{Dist}[d^2 / (b(m + n)), \text{Int}[(a + b \text{Csc}[e + f x])^m (d \text{Csc}[e + f x])^{(n - 2)}], x]$$

```

*x])^(n - 2)*Simp[a*B*(n - 2) + B*b*(m + n - 1)*Csc[e + f*x] + (A*b*(m + n)
- a*B*(n - 1))*Csc[e + f*x]^2, x], x] /; FreeQ[{a, b, d, e, f, A, B, m
}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && NeQ[m + n,
0] && !IGtQ[m, 1]

```

Rule 4082

```

Int[csc[(e_.) + (f_.)*(x_.)]*((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e
_.) + (f_.)*(x_.)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

```

Rubi steps

$$\begin{aligned}
\int \sec^3(c + dx)(a + b \sec(c + dx))^{5/2}(A + B \sec(c + dx)) dx &= \frac{2B \sec(c + dx)(a + b \sec(c + dx))^{7/2} \tan(c + dx)}{11bd} + \frac{2(11Ab - 4aB)(a + b \sec(c + dx))^{7/2} \tan(c + dx)}{99b^2d} \\
&= -\frac{2(22aAb - 8a^2B - 81b^2B)(a + b \sec(c + dx))^{5/2} \tan(c + dx)}{693b^2d} \\
&= -\frac{2(110a^2Ab - 539Ab^3 - 40a^3B - 335ab^2B)(a + b \sec(c + dx))^{3/2} \tan(c + dx)}{3465b^2d} \\
&= -\frac{2(110a^3Ab - 1254aAb^3 - 40a^4B - 285a^2b^2B - 612ab^3B)(a + b \sec(c + dx))^{1/2} \tan(c + dx)}{3465b^2d} \\
&= -\frac{2(110a^3Ab - 1254aAb^3 - 40a^4B - 285a^2b^2B - 612ab^3B)(a + b \sec(c + dx))^{1/2} \tan(c + dx)}{3465b^2d} \\
&= \frac{2(a - b)\sqrt{a + b} (110a^4Ab - 3069a^2Ab^3 - 1617Ab^5)}{3465b^2d}
\end{aligned}$$

Mathematica [B] time = 27.34, size = 4227, normalized size = 7.47

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
[Out] (Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2)*((2*(-110*a^4*A*b + 3069*a^2*A*b^3 + 1617*A*b^5 + 40*a^5*B + 255*a^3*b^2*B + 3705*a*b^4*B)*Sin[c + d*x]))/(3465*b^3) + (2*Sec[c + d*x]^4*(11*A*b^2*Sin[c + d*x] + 23*a*b*B*Sin[c + d*x]))/99 + (2*Sec[c + d*x]^3*(209*a*A*b*Sin[c + d*x] + 113*a^2*B*Sin[c + d*x] + 81*b^2*B*Sin[c + d*x]))/693 + (2*Sec[c + d*x]^2*(825*a^2*A*b*Sin[c + d*x] + 539*A*b^3*Sin[c + d*x] + 15*a^3*B*Sin[c + d*x] + 1145*a*b^2*B*Sin[c + d*x]))/(3465*b) + (2*Sec[c + d*x]*(55*a^3*A*b*Sin[c + d*x] + 1793*a*A*b^3*Sin[c + d*x] - 20*a^4*B*Sin[c + d*x] + 1025*a^2*b^2*B*Sin[c + d*x] + 675*b^4*B*Sin[c + d*x]))/(3465*b^2) + (2*b^2*B*Sec[c + d*x]^4*Tan[c + d*x])/11)/(d*(b + a*Cos[c + d*x])^2) - (2*((2*a^4*A)/(63*b*Sqrt[b + a*Cos[c + d*x]])*Sqrt[Sec[c + d*x]] - (31*a^2*A*b)/(35*Sqrt[b + a*Cos[c + d*x]])*Sqrt[Sec[c + d*x]]) - (7*A*b^3)/(15*Sqrt[b + a*Cos[c + d*x]])*Sqrt[Sec[c + d*x]] - (17*a^3*B)/(231*Sqrt[b + a*Cos[c + d*x]])*Sqrt[Sec[c + d*x]] - (8*a^5*B)/(693*b^2*Sqrt[b + a*Cos[c + d*x]])*Sqrt[Sec[c + d*x]] - (247*a*b^2*B)/(231*Sqrt[b + a*Cos[c + d*x]])*Sqrt[Sec[c + d*x]] - (124*a^3*A*Sqrt[Sec[c + d*x]])/(315*Sqrt[b + a*Cos[c + d*x]]) + (2*a^5*A*Sqrt[Sec[c + d*x]])/(63*b^2*Sqrt[b + a*Cos[c + d*x]]) + (38*a*A*b^2*Sqrt[Sec[c + d*x]])/(105*Sqrt[b + a*Cos[c + d*x]]) - (8*a^6*B*Sqrt[Sec[c + d*x]])/(693*b^3*Sqrt[b + a*Cos[c + d*x]]) - (7*a^4*B*Sqrt[Sec[c + d*x]])/(99*b*Sqrt[b + a*Cos[c + d*x]]) - (26*a^2*b*B*Sqrt[Sec[c + d*x]])/(231*Sqrt[b + a*Cos[c + d*x]]) + (15*b^3*B*Sqrt[Sec[c + d*x]])/(77*Sqrt[b + a*Cos[c + d*x]]) - (31*a^3*A*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(35*Sqrt[b + a*Cos[c + d*x]]) + (2*a^5*A*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(63*b^2*Sqrt[b + a*Cos[c + d*x]]) - (7*a*A*b^2*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(15*Sqrt[b + a*Cos[c + d*x]]) - (8*a^6*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(693*b^3*Sqrt[b + a*Cos[c + d*x]]) - (17*a^4*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(231*b*Sqrt[b + a*Cos[c + d*x]]) - (247*a^2*b*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(231*Sqrt[b + a*Cos[c + d*x]]))*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(a + b*Sec[c + d*x])^(5/2)*(2*(a + b)*(-110*a^4*A*b + 3069*a^2*A*b^3 + 1617*A*b^5 + 40*a^5*B + 255*a^3*b^2*B + 3705*a*b^4*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x]])*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(a + b)*(40*a^4*B - 10*a^3*b*(11*A + 3*B) + 15*a^2*b^2*(121*A + 19*B) + 3*b^4*(539*A + 225*B) + 6*a*b^3*(209*A + 505*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x]])*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-110*a^4*A*b + 3069*a^2*A*b^3 + 1617*A*b^5 + 40*a^5*B + 255*a^3*b^2*B + 3705*a*b^4*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2))/(3465*b^3*d*(b + a*Cos[c + d*x])^3*Sqrt[Sec[(c + d*x)/2]^2]*Sec[c + d*x]^(5/2)*(-1/3465*(a*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*x]*(2*(a + b)*(-110*a^4*A*b + 3069*a^2*A*b^3 + 1617*A*b^5 + 40*a^5*B + 255*a^3*b^2*B + 3705*a*b^4*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x]])*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(a + b)*(40*a^4*B - 10*a^3*b*(11*A + 3*B) + 15*a^2*b^2*(121*A
```

$$\begin{aligned}
& + 19*B) + 3*b^4*(539*A + 225*B) + 6*a*b^3*(209*A + 505*B))*\text{Sqrt}[\text{Cos}[c + d*x]/(1 + \text{Cos}[c + d*x])]*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/((a + b)*(1 + \text{Cos}[c + d*x]))]*\text{EllipticF}[\text{ArcSin}[\text{Tan}[(c + d*x)/2]], (a - b)/(a + b)] + (-110*a^4*A*b + 3069*a^2*A*b^3 + 1617*A*b^5 + 40*a^5*B + 255*a^3*b^2*B + 3705*a*b^4*B)*\text{Cos}[c + d*x]*(b + a*\text{Cos}[c + d*x])* \text{Sec}[(c + d*x)/2]^2*\text{Tan}[(c + d*x)/2])]/(b^3*(b + a*\text{Cos}[c + d*x])^(3/2)*\text{Sqrt}[\text{Sec}[(c + d*x)/2]^2]) + (\text{Sqrt}[\text{Cos}[(c + d*x)/2]^2*\text{Sec}[c + d*x]]*\text{Tan}[(c + d*x)/2]*(2*(a + b)*(-110*a^4*A*b + 3069*a^2*A*b^3 + 1617*A*b^5 + 40*a^5*B + 255*a^3*b^2*B + 3705*a*b^4*B)*\text{Sqrt}[\text{Cos}[c + d*x]/(1 + \text{Cos}[c + d*x])]*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/((a + b)*(1 + \text{Cos}[c + d*x]))]*\text{EllipticE}[\text{ArcSin}[\text{Tan}[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(a + b)*(40*a^4*B - 10*a^3*b*(11*A + 3*B) + 15*a^2*b^2*(121*A + 19*B) + 3*b^4*(539*A + 225*B) + 6*a*b^3*(209*A + 505*B))*\text{Sqrt}[\text{Cos}[c + d*x]/(1 + \text{Cos}[c + d*x])]*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/((a + b)*(1 + \text{Cos}[c + d*x]))]*\text{EllipticF}[\text{ArcSin}[\text{Tan}[(c + d*x)/2]], (a - b)/(a + b)] + (-110*a^4*A*b + 3069*a^2*A*b^3 + 1617*A*b^5 + 40*a^5*B + 255*a^3*b^2*B + 3705*a*b^4*B)*\text{Cos}[c + d*x]*(b + a*\text{Cos}[c + d*x])* \text{Sec}[(c + d*x)/2]^2*\text{Tan}[(c + d*x)/2])]/(3465*b^3*\text{Sqrt}[b + a*\text{Cos}[c + d*x]]*\text{Sqrt}[\text{Sec}[(c + d*x)/2]^2]) - (2*\text{Sqrt}[\text{Cos}[(c + d*x)/2]^2*\text{Sec}[c + d*x]]*(((-110*a^4*A*b + 3069*a^2*A*b^3 + 1617*A*b^5 + 40*a^5*B + 255*a^3*b^2*B + 3705*a*b^4*B)*\text{Cos}[c + d*x]*(b + a*\text{Cos}[c + d*x])* \text{Sec}[(c + d*x)/2]^4)/2 + ((a + b)*(-110*a^4*A*b + 3069*a^2*A*b^3 + 1617*A*b^5 + 40*a^5*B + 255*a^3*b^2*B + 3705*a*b^4*B))*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/((a + b)*(1 + \text{Cos}[c + d*x]))]*\text{EllipticE}[\text{ArcSin}[\text{Tan}[(c + d*x)/2]], (a - b)/(a + b)]*((\text{Cos}[c + d*x]*\text{Sin}[c + d*x])/(1 + \text{Cos}[c + d*x])^2 - \text{Sin}[c + d*x]/(1 + \text{Cos}[c + d*x])))/\text{Sqrt}[\text{Cos}[c + d*x]/(1 + \text{Cos}[c + d*x])] - (b*(a + b)*(40*a^4*B - 10*a^3*b*(11*A + 3*B) + 15*a^2*b^2*(121*A + 19*B) + 3*b^4*(539*A + 225*B) + 6*a*b^3*(209*A + 505*B))*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/((a + b)*(1 + \text{Cos}[c + d*x]))]*\text{EllipticF}[\text{ArcSin}[\text{Tan}[(c + d*x)/2]], (a - b)/(a + b)]*((\text{Cos}[c + d*x]*\text{Sin}[c + d*x])/(1 + \text{Cos}[c + d*x])^2 - \text{Sin}[c + d*x]/(1 + \text{Cos}[c + d*x])))/\text{Sqrt}[\text{Cos}[c + d*x]/(1 + \text{Cos}[c + d*x])]) + ((a + b)*(-110*a^4*A*b + 3069*a^2*A*b^3 + 1617*A*b^5 + 40*a^5*B + 255*a^3*b^2*B + 3705*a*b^4*B)*\text{Sqrt}[\text{Cos}[c + d*x]/(1 + \text{Cos}[c + d*x])]*\text{EllipticE}[\text{ArcSin}[\text{Tan}[(c + d*x)/2]], (a - b)/(a + b)]*(-((a*\text{Sin}[c + d*x])/((a + b)*(1 + \text{Cos}[c + d*x])))) + ((b + a*\text{Cos}[c + d*x])* \text{Sin}[c + d*x])/((a + b)*(1 + \text{Cos}[c + d*x])^2)))/\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/((a + b)*(1 + \text{Cos}[c + d*x]))] - (b*(a + b)*(40*a^4*B - 10*a^3*b*(11*A + 3*B) + 15*a^2*b^2*(121*A + 19*B) + 3*b^4*(539*A + 225*B) + 6*a*b^3*(209*A + 505*B))*\text{Sqrt}[\text{Cos}[c + d*x]/(1 + \text{Cos}[c + d*x])]*\text{EllipticF}[\text{ArcSin}[\text{Tan}[(c + d*x)/2]], (a - b)/(a + b)]*(-((a*\text{Sin}[c + d*x])/((a + b)*(1 + \text{Cos}[c + d*x])))) + ((b + a*\text{Cos}[c + d*x])* \text{Sin}[c + d*x])/((a + b)*(1 + \text{Cos}[c + d*x])^2)))/\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/((a + b)*(1 + \text{Cos}[c + d*x]))] - a*(-110*a^4*A*b + 3069*a^2*A*b^3 + 1617*A*b^5 + 40*a^5*B + 255*a^3*b^2*B + 3705*a*b^4*B)*\text{Cos}[c + d*x]* \text{Sec}[(c + d*x)/2]^2*\text{Sin}[c + d*x]*\text{Tan}[(c + d*x)/2] - (-110*a^4*A*b + 3069*a^2*A*b^3 + 1617*A*b^5 + 40*a^5*B + 255*a^3*b^2*B + 3705*a*b^4*B)*(b + a*\text{Cos}[c + d*x])* \text{Sec}[(c + d*x)/2]^2*\text{Sin}[c + d*x]*\text{Tan}[(c + d*x)/2] + (-110*a^4*A*b + 3069*a^2*A*b^3 + 1617*A*b^5 + 40*a^5*B + 255*a^3*b^2*B + 3705*a*b^4*B)*\text{Cos}[c + d*x]*(b + a*\text{Cos}[c + d*x])* \text{Sec}[(c + d*x)/2]^2*\text{Tan}[(c + d*x)/2]^2 - (b*(a + b)*(40*a^4*B - 10*a^3*b*(11*A
\end{aligned}$$

+ 3*B) + 15*a^2*b^2*(121*A + 19*B) + 3*b^4*(539*A + 225*B) + 6*a*b^3*(209*A + 505*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2/(Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[1 - ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]) + ((a + b)*(-110*a^4*A*b + 3069*a^2*A*b^3 + 1617*A*b^5 + 40*a^5*B + 255*a^3*b^2*B + 3705*a*b^4*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2*Sqrt[1 - ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]/Sqrt[1 - Tan[(c + d*x)/2]^2]))/(3465*b^3*Sqrt[b + a*Cos[c + d*x]])*Sqrt[Sec[(c + d*x)/2]^2) - ((2*(a + b)*(-110*a^4*A*b + 3069*a^2*A*b^3 + 1617*A*b^5 + 40*a^5*B + 255*a^3*b^2*B + 3705*a*b^4*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(a + b)*(40*a^4*B - 10*a^3*b*(11*A + 3*B) + 15*a^2*b^2*(121*A + 19*B) + 3*b^4*(539*A + 225*B) + 6*a*b^3*(209*A + 505*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-110*a^4*A*b + 3069*a^2*A*b^3 + 1617*A*b^5 + 40*a^5*B + 255*a^3*b^2*B + 3705*a*b^4*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2))*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2) + Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(3465*b^3*Sqrt[b + a*Cos[c + d*x]])*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]))

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$\int (Bb^2 \sec(dx + c)^6 + Aa^2 \sec(dx + c)^3 + (2Bab + Ab^2) \sec(dx + c)^5 + (Ba^2 + 2Aab) \sec(dx + c)^4) \sqrt{\sec(dx + c)} dx$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] integral((B*b^2*sec(d*x + c)^6 + A*a^2*sec(d*x + c)^3 + (2*B*a*b + A*b^2)*sec(d*x + c)^5 + (B*a^2 + 2*A*a*b)*sec(d*x + c)^4)*sqrt(b*sec(d*x + c) + a), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(b \sec(dx + c) + a)^{\frac{5}{2}} \sec(dx + c)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^3, x)

maple [B] time = 4.07, size = 5368, normalized size = 9.48

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sec(d*x+c)^3*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x)`

[Out] result too large to display

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="maxima")`

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(a + \frac{b}{\cos(c+dx)}\right)^{5/2}}{\cos(c+dx)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(5/2))/cos(c + d*x)^3,x)`

[Out] `int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(5/2))/cos(c + d*x)^3, x)`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)**3*(a+b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)),x)`

[Out] Timed out

$$3.364 \quad \int \sec^2(c+dx)(a+b \sec(c+dx))^{5/2}(A+B \sec(c+dx)) dx$$

Optimal. Leaf size=469

$$\frac{2(-10a^2B + 45aAb + 49b^2B) \tan(c+dx)(a+b \sec(c+dx))^{3/2}}{315bd} + \frac{2(-10a^3B + 45a^2Ab + 114ab^2B + 75Ab^3) \tan(c+dx)}{315bd}$$

[Out] $-2/315*(a-b)*(45*A*a^3*b+435*A*a*b^3-10*B*a^4+279*B*a^2*b^2+147*B*b^4)*\cot(d*x+c)*\text{EllipticE}((a+b*\sec(d*x+c))^{1/2}/(a+b)^{1/2},((a+b)/(a-b))^{1/2})*(a+b)^{1/2}*(b*(1-\sec(d*x+c))/(a+b))^{1/2}*(-b*(1+\sec(d*x+c))/(a-b))^{1/2}/b^3/d-2/315*(a-b)*(3*b^3*(25*A-49*B)-6*a*b^2*(60*A-19*B)+15*a^2*b*(3*A-11*B)-10*a^3*B)*\cot(d*x+c)*\text{EllipticF}((a+b*\sec(d*x+c))^{1/2}/(a+b)^{1/2},((a+b)/(a-b))^{1/2})*(a+b)^{1/2}*(b*(1-\sec(d*x+c))/(a+b))^{1/2}*(-b*(1+\sec(d*x+c))/(a-b))^{1/2}/b^2/d+2/315*(45*A*a*b-10*B*a^2+49*B*b^2)*(a+b*\sec(d*x+c))^{3/2}*\tan(d*x+c)/b/d+2/63*(9*A*b-2*B*a)*(a+b*\sec(d*x+c))^{5/2}*\tan(d*x+c)/b/d+2/9*B*(a+b*\sec(d*x+c))^{7/2}*\tan(d*x+c)/b/d+2/315*(45*A*a^2*b+75*A*b^3-10*B*a^3+114*B*a*b^2)*(a+b*\sec(d*x+c))^{1/2}*\tan(d*x+c)/b/d$

Rubi [A] time = 1.18, antiderivative size = 469, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {4010, 4002, 4005, 3832, 4004}

$$\frac{2(-10a^2B + 45aAb + 49b^2B) \tan(c+dx)(a+b \sec(c+dx))^{3/2}}{315bd} + \frac{2(45a^2Ab - 10a^3B + 114ab^2B + 75Ab^3) \tan(c+dx)}{315bd}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]

[Out] $(-2*(a-b)*\text{Sqrt}[a+b]*(45*a^3*A*b+435*a*A*b^3-10*a^4*B+279*a^2*b^2*B+147*b^4*B)*\text{Cot}[c+d*x]*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[a+b*\text{Sec}[c+d*x]]/\text{Sqrt}[a+b]],(a+b)/(a-b)]*\text{Sqrt}[(b*(1-\text{Sec}[c+d*x]))/(a+b)]*\text{Sqrt}[-((b*(1+\text{Sec}[c+d*x]))/(a-b)))]/(315*b^3*d)-(2*(a-b)*\text{Sqrt}[a+b]*(3*b^3*(25*A-49*B)-6*a*b^2*(60*A-19*B)+15*a^2*b*(3*A-11*B)-10*a^3*B)*\text{Cot}[c+d*x]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[a+b*\text{Sec}[c+d*x]]/\text{Sqrt}[a+b]],(a+b)/(a-b)]*\text{Sqrt}[(b*(1-\text{Sec}[c+d*x]))/(a+b)]*\text{Sqrt}[-((b*(1+\text{Sec}[c+d*x]))/(a-b)))]/(315*b^2*d)+(2*(45*a^2*A*b+75*A*b^3-10*a^3*B+114*a*b^2*B)*\text{Sqrt}[a+b*\text{Sec}[c+d*x]]*\text{Tan}[c+d*x])/(315*b*d)+(2*(45*a*A*b-10*a^2*B+49*b^2*B)*(a+b*\text{Sec}[c+d*x])^{3/2}*\text{Tan}[c+d*x])/(315*b*d)+(2*(9*A*b-2*a*B)*(a+b*\text{Sec}[c+d*x])^{5/2}*\text{Tan}[c+d*x])/(63*b*d)+(2*B*(a+b*\text{Sec}[c+d*x])^{7/2}*\text{Tan}[c+d*x])/(9*b*d)$

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol]
:> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)]/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol]
:> -Simp[(B*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*Csc[e + f*x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol]
:> Simp[(-2*(A*b - a*B)*Rt[a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A, 2]], (a*A + b*B)/(a*A - b*B)]/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol]
:> Dist[A - B, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol]
:> -Simp[(B*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \sec^2(c + dx)(a + b \sec(c + dx))^{5/2}(A + B \sec(c + dx)) dx &= \frac{2B(a + b \sec(c + dx))^{7/2} \tan(c + dx)}{9bd} + \frac{2 \int \sec(c + dx) (a + b \sec(c + dx))^{5/2} dx}{9bd} \\
&= \frac{2(9Ab - 2aB)(a + b \sec(c + dx))^{5/2} \tan(c + dx)}{63bd} + \frac{2 \int \sec(c + dx) (a + b \sec(c + dx))^{3/2} dx}{63bd} \\
&= \frac{2(45aAb - 10a^2B + 49b^2B)(a + b \sec(c + dx))^{3/2}}{315bd} + \frac{2 \int \sec(c + dx) \sqrt{a + b \sec(c + dx)} dx}{315bd} \\
&= \frac{2(45a^2Ab + 75Ab^3 - 10a^3B + 114ab^2B) \sqrt{a + b \sec(c + dx)}}{315bd} + \frac{2 \int \sec(c + dx) \sqrt{a + b \sec(c + dx)} dx}{315bd} \\
&= \frac{2(45a^2Ab + 75Ab^3 - 10a^3B + 114ab^2B) \sqrt{a + b \sec(c + dx)}}{315bd} + \frac{2(a - b) \sqrt{a + b} (45a^3Ab + 435aAb^3 - 10a^4B + 10a^3bB - 10a^2b^2B + 10ab^3B - 10b^4B)}{63bd^2}
\end{aligned}$$

Mathematica [B] time = 26.63, size = 3781, normalized size = 8.06

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]

[Out] (Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2)*((2*(45*a^3*A*b + 435*a*A*b^3 - 10*a^4*B + 279*a^2*b^2*B + 147*b^4*B)*Sin[c + d*x])/(315*b^2) + (2*Sec[c + d*x]^3*(9*A*b^2*Sin[c + d*x] + 19*a*b*B*Sin[c + d*x]))/63 + (2*Sec[c + d*x]^2*(135*a*A*b*Sin[c + d*x] + 75*a^2*B*Sin[c + d*x] + 49*b^2*B*Sin[c + d*x]))/315 + (2*Sec[c + d*x]*(135*a^2*A*b*Sin[c + d*x] + 75*A*b^3*Sin[c + d*x] + 5*a^3*B*Sin[c + d*x] + 163*a*b^2*B*Sin[c + d*x]))/(315*b) + (2*b^2*B*Sec[c + d*x]^3*Tan[c + d*x])/9)/(d*(b + a*Cos[c + d*x])^2) + (2*(-1/7*(a^3*A)/(Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (29*a*A*b^2)/(21*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (2*a^4*B)/(63*b*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (31*a^2*b*B)/(35*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (7*b^3*B)/(15*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (a^4*A*Sqrt[Sec[c + d*x]])/(7*b*Sqrt[b + a*Cos[c + d*x]]) - (2*a^2*A*b*Sqrt[Sec[c + d*x]])/(21*Sqrt[b + a*Cos[c + d*x]]) + (5*A*b^3*Sqrt[Sec[c + d*x]])/(21*Sqrt[b + a*Cos[c + d*x]]) - (124*a^3*B*Sqrt[Sec[c + d*x]])/(315*Sqrt[b + a*Cos[c + d*x]]) + (2*a^5*B*Sqrt[Sec[c + d*x]])/(63*b^2*Sqrt[b + a*Cos[c + d*x]]))

$$\begin{aligned}
& + d*x]]) + (38*a*b^2*B*Sqrt[Sec[c + d*x]])/(105*Sqrt[b + a*Cos[c + d*x]]) \\
& - (a^4*A*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(7*b*Sqrt[b + a*Cos[c + d*x]]) \\
& - (29*a^2*A*b*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(21*Sqrt[b + a*Cos[c + d*x]]) \\
& - (31*a^3*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(35*Sqrt[b + a*Cos[c + d*x]]) \\
& + (2*a^5*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(63*b^2*Sqrt[b + a*Cos[c + d*x]]) \\
& - (7*a*b^2*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(15*Sqrt[b + a*Cos[c + d*x]]) \\
& *Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(a + b*Sec[c + d*x])^(5/2)*(2*(a + b)*(-45*a^3*A*b - 435*a*A*b^3 + 10*a^4*B - 279*a^2*b^2*B - 147*b^4*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x]])*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*b*(a + b)*(-10*a^3*B + 15*a^2*b*(3*A + 11*B) + 6*a*b^2*(60*A + 19*B) + 3*b^3*(25*A + 49*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x]])*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-45*a^3*A*b - 435*a*A*b^3 + 10*a^4*B - 279*a^2*b^2*B - 147*b^4*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/((315*b^2*d*(b + a*Cos[c + d*x])^3*Sqrt[Sec[(c + d*x)/2]^2]*Sec[c + d*x]^(5/2)*((a*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*x]*(2*(a + b)*(-45*a^3*A*b - 435*a*A*b^3 + 10*a^4*B - 279*a^2*b^2*B - 147*b^4*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x]])*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*b*(a + b)*(-10*a^3*B + 15*a^2*b*(3*A + 11*B) + 6*a*b^2*(60*A + 19*B) + 3*b^3*(25*A + 49*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x]])*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-45*a^3*A*b - 435*a*A*b^3 + 10*a^4*B - 279*a^2*b^2*B - 147*b^4*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/((315*b^2*(b + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[(c + d*x)/2]^2]) - (Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*(2*(a + b)*(-45*a^3*A*b - 435*a*A*b^3 + 10*a^4*B - 279*a^2*b^2*B - 147*b^4*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x]])*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*b*(a + b)*(-10*a^3*B + 15*a^2*b*(3*A + 11*B) + 6*a*b^2*(60*A + 19*B) + 3*b^3*(25*A + 49*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x]])*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-45*a^3*A*b - 435*a*A*b^3 + 10*a^4*B - 279*a^2*b^2*B - 147*b^4*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/((315*b^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(((-45*a^3*A*b - 435*a*A*b^3 + 10*a^4*B - 279*a^2*b^2*B - 147*b^4*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^4)/2 + ((a + b)*(-45*a^3*A*b - 435*a*A*b^3 + 10*a^4*B - 279*a^2*b^2*B - 147*b^4*B)*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x]]) + (b*(a + b)*(-10*a^3*B + 15*a^2*b*(3*A + 11*B) + 6*a*b^2*(60*A + 19*B) + 3*b^3*(25*A + 49*B))*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*((Cos[c + d*x]
\end{aligned}$$

```
x]*Sin[c + d*x))/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x]))/S
qrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + ((a + b)*(-45*a^3*A*b - 435*a*A*b^3
+ 10*a^4*B - 279*a^2*b^2*B - 147*b^4*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x]
)]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*(-((a*SIN[c + d*x])
)/((a + b)*(1 + Cos[c + d*x]))) + ((b + a*cos[c + d*x])*Sin[c + d*x])/((a +
b)*(1 + Cos[c + d*x])^2))/Sqrt[(b + a*cos[c + d*x])/((a + b)*(1 + Cos[c +
d*x]))] + (b*(a + b)*(-10*a^3*B + 15*a^2*b*(3*A + 11*B) + 6*a*b^2*(60*A + 1
9*B) + 3*b^3*(25*A + 49*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] *EllipticF
[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*(-((a*SIN[c + d*x])/((a + b)*(1
+ Cos[c + d*x]))) + ((b + a*cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[
c + d*x])^2))/Sqrt[(b + a*cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] - a*
(-45*a^3*A*b - 435*a*A*b^3 + 10*a^4*B - 279*a^2*b^2*B - 147*b^4*B)*Cos[c +
d*x]*Sec[(c + d*x)/2]^2*SIN[c + d*x]*Tan[(c + d*x)/2] - (-45*a^3*A*b - 435*
a*A*b^3 + 10*a^4*B - 279*a^2*b^2*B - 147*b^4*B)*(b + a*cos[c + d*x])*Sec[(c
+ d*x)/2]^2*SIN[c + d*x]*Tan[(c + d*x)/2] + (-45*a^3*A*b - 435*a*A*b^3 + 1
0*a^4*B - 279*a^2*b^2*B - 147*b^4*B)*Cos[c + d*x]*(b + a*cos[c + d*x])*Sec[
(c + d*x)/2]^2*Tan[(c + d*x)/2]^2 + (b*(a + b)*(-10*a^3*B + 15*a^2*b*(3*A +
11*B) + 6*a*b^2*(60*A + 19*B) + 3*b^3*(25*A + 49*B))*Sqrt[Cos[c + d*x]/(1
+ Cos[c + d*x])] *Sqrt[(b + a*cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] *Se
c[(c + d*x)/2]^2)/(Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[1 - ((a - b)*Tan[(c +
d*x)/2]^2)/(a + b)]) + ((a + b)*(-45*a^3*A*b - 435*a*A*b^3 + 10*a^4*B - 279
*a^2*b^2*B - 147*b^4*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] *Sqrt[(b + a*C
os[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] *Sec[(c + d*x)/2]^2*Sqrt[1 - ((a
- b)*Tan[(c + d*x)/2]^2)/(a + b)]/Sqrt[1 - Tan[(c + d*x)/2]^2]))/(315*b^2*
Sqrt[b + a*cos[c + d*x]] *Sqrt[Sec[(c + d*x)/2]^2] + ((2*(a + b)*(-45*a^3*A
*b - 435*a*A*b^3 + 10*a^4*B - 279*a^2*b^2*B - 147*b^4*B)*Sqrt[Cos[c + d*x]/
(1 + Cos[c + d*x])] *Sqrt[(b + a*cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]
 *EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*b*(a + b)*(-10*a^
3*B + 15*a^2*b*(3*A + 11*B) + 6*a*b^2*(60*A + 19*B) + 3*b^3*(25*A + 49*B))*
Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] *Sqrt[(b + a*cos[c + d*x])/((a + b)*(1
+ Cos[c + d*x]))] *EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (
-45*a^3*A*b - 435*a*A*b^3 + 10*a^4*B - 279*a^2*b^2*B - 147*b^4*B)*Cos[c + d
*x]*(b + a*cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]) *(-Cos[(c + d
*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan
[c + d*x]))/(315*b^2*Sqrt[b + a*cos[c + d*x]] *Sqrt[Sec[(c + d*x)/2]^2]*Sqrt
[Cos[(c + d*x)/2]^2*Sec[c + d*x]]))
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$\text{integral}\left(\left(Bb^2 \sec(dx + c)^5 + Aa^2 \sec(dx + c)^2 + (2 Bab + Ab^2) \sec(dx + c)^4 + (Ba^2 + 2 Aab) \sec(dx + c)^3\right)\sqrt{\right.$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] integral((B*b^2*sec(d*x + c)^5 + A*a^2*sec(d*x + c)^2 + (2*B*a*b + A*b^2)*sec(d*x + c)^4 + (B*a^2 + 2*A*a*b)*sec(d*x + c)^3)*sqrt(b*sec(d*x + c) + a), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(b \sec(dx + c) + a)^{\frac{5}{2}} \sec(dx + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^2, x)

maple [B] time = 3.13, size = 4395, normalized size = 9.37

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^2*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x)

[Out]
$$\begin{aligned} & -2/315/d*(1+\cos(d*x+c))^2*((b+a*\cos(d*x+c))/\cos(d*x+c))^{(1/2)}*(-1+\cos(d*x+c)) \\ &)^2*(-180*A*\cos(d*x+c)^2*a*b^4-170*B*\cos(d*x+c)^2*a^2*b^3-130*B*\cos(d*x+c) \\ & *a*b^4-82*B*\cos(d*x+c)^3*a*b^4-30*A*\cos(d*x+c)^3*b^5-45*A*\cos(d*x+c)^5*(\cos \\ & (d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{(1/2)} \\ & *EllipticE((-1+\cos(d*x+c))/\sin(d*x+c),((a-b)/(a+b))^{(1/2)}*\sin(d*x+c)*a^4*b \\ & +5*B*\cos(d*x+c)^6*a^4*b+279*B*\cos(d*x+c)^6*a^3*b^2+163*B*\cos(d*x+c)^6*a^2*b \\ & ^3+147*B*\cos(d*x+c)^6*a*b^4-10*B*\cos(d*x+c)^5*a^4*b-199*B*\cos(d*x+c)^5*a^3* \\ & b^2+279*B*\cos(d*x+c)^5*a^2*b^3+65*B*\cos(d*x+c)^5*a*b^4+5*B*\cos(d*x+c)^4*a^4 \\ & *b-35*B*b^5+45*A*\cos(d*x+c)^6*a^4*b+135*A*\cos(d*x+c)^6*a^3*b^2+435*A*\cos(d* \\ & x+c)^6*a^2*b^3+75*A*\cos(d*x+c)^6*a*b^4-45*A*\cos(d*x+c)^5*a^4*b+45*A*\cos(d*x \\ & +c)^5*a^3*b^2-165*A*\cos(d*x+c)^5*a^2*b^3+435*A*\cos(d*x+c)^5*a*b^4-180*A*\cos \\ & (d*x+c)^4*a^3*b^2-330*A*\cos(d*x+c)^4*a*b^4-270*A*\cos(d*x+c)^3*a^2*b^3-272*B \\ & *\cos(d*x+c)^4*a^2*b^3-80*B*\cos(d*x+c)^3*a^3*b^2+75*A*\cos(d*x+c)^5*b^5+10*B* \\ & \cos(d*x+c)^5*a^5-10*B*\cos(d*x+c)^6*a^5-98*B*\cos(d*x+c)^4*b^5-45*A*\cos(d*x+c) \\ &)*b^5+147*B*\cos(d*x+c)^5*b^5-14*B*\cos(d*x+c)^2*b^5-279*B*\cos(d*x+c)^4*(\cos \\ & (d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{(1/2)}* \\ & EllipticE((-1+\cos(d*x+c))/\sin(d*x+c),((a-b)/(a+b))^{(1/2)}*\sin(d*x+c)*a^3*b^ \\ & 2-279*B*\cos(d*x+c)^4*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1 \\ & +\cos(d*x+c))/(a+b))^{(1/2)}*EllipticE((-1+\cos(d*x+c))/\sin(d*x+c),((a-b)/(a+b) \\ &)^{(1/2)}*\sin(d*x+c)*a^2*b^3-147*B*\cos(d*x+c)^4*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(\\ & 1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{(1/2)}*EllipticE((-1+\cos(d*x+c) \\ &)/\sin(d*x+c),((a-b)/(a+b))^{(1/2)}*\sin(d*x+c)*a*b^4-10*B*\cos(d*x+c)^4*(\cos(\end{aligned}$$

$$\begin{aligned}
& d*x+c)/(1+\cos(d*x+c))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}* \\
& \text{EllipticF}((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{1/2})*\sin(d*x+c)*a^4*b+ \\
& 155*B*\cos(d*x+c)^4*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+c \\
& \cos(d*x+c))/(a+b))^{1/2}*\text{EllipticF}((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{1/2}) \\
& * \sin(d*x+c)*a^3*b^2+279*B*\cos(d*x+c)^4*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2} \\
& *((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*\text{EllipticF}((-1+\cos(d*x+c)) \\
& / \sin(d*x+c), ((a-b)/(a+b))^{1/2})*\sin(d*x+c)*a^2*b^3+261*B*\cos(d*x+c)^4*(\cos \\
& (d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2} \\
& *\text{EllipticF}((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{1/2})*\sin(d*x+c)*a*b^4 \\
& +75*A*\cos(d*x+c)^5*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+c \\
& \cos(d*x+c))/(a+b))^{1/2}*\text{EllipticF}((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{1/2}) \\
& * \sin(d*x+c)*b^5+10*B*\cos(d*x+c)^5*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2} \\
& *\text{EllipticE}((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{1/2})*\sin(d*x+c)*a^5-147*B*\cos(d*x+c)^5*(\cos(d*x+c)/(\\
& 1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*\text{Elliptic} \\
& \text{E}((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{1/2})*\sin(d*x+c)*b^5+147*B*\cos(\\
& d*x+c)^5*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)) \\
& / (a+b))^{1/2}*\text{EllipticF}((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{1/2})*\sin \\
& (d*x+c)*b^5+75*A*\cos(d*x+c)^4*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d \\
& *x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*\text{EllipticF}((-1+\cos(d*x+c))/\sin(d*x+c), ((a \\
& -b)/(a+b))^{1/2})*\sin(d*x+c)*b^5+10*B*\cos(d*x+c)^4*(\cos(d*x+c)/(1+\cos(d*x+c \\
&)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*\text{EllipticE}((-1+\cos(d \\
& *x+c))/\sin(d*x+c), ((a-b)/(a+b))^{1/2})*\sin(d*x+c)*a^5-147*B*\cos(d*x+c)^4*(c \\
& \cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2} \\
& *\text{EllipticE}((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{1/2})*\sin(d*x+c)*b^5 \\
& +147*B*\cos(d*x+c)^4*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+ \\
& \cos(d*x+c))/(a+b))^{1/2}*\text{EllipticF}((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b)) \\
& ^{1/2})*\sin(d*x+c)*b^5-45*A*\cos(d*x+c)^5*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2} \\
& *\text{EllipticE}((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{1/2})*\sin(d*x+c), ((a-b)/(a+b))^{1/2})*\sin(d*x+c)*a^3*b^2-435*A*\cos(d*x+c)^5*(\cos(d*x \\
& +c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*\text{EllipticE}((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{1/2})*\sin(d*x+c)*a^2*b^3-4 \\
& 35*A*\cos(d*x+c)^5*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+c \\
& \cos(d*x+c))/(a+b))^{1/2}*\text{EllipticE}((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{1/2})*\sin(d*x+c)*a*b^4+45*A*\cos(d*x+c)^5*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*\text{EllipticF}((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{1/2})*\sin(d*x+c)*a^3*b^2+405*A*\cos(d*x+c)^5*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*\text{EllipticF}((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{1/2})*\sin(d*x+c)*a^2*b^3+4 \\
& 35*A*\cos(d*x+c)^5*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+c \\
& \cos(d*x+c))/(a+b))^{1/2}*\text{EllipticF}((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{1/2})*\sin(d*x+c)*a*b^4+10*B*\cos(d*x+c)^5*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*\text{EllipticE}((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{1/2})*\sin(d*x+c)*a^4*b-279*B*\cos(d*x+c)^5*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*\text{Ellip}
\end{aligned}$$

```

ticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3*b^2-279
*B*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(
d*x+c)))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/
2))*sin(d*x+c)*a^2*b^3-147*B*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/si
n(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^4-10*B*cos(d*x+c)^5*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^4*b+155*B
*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*
x+c)))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2)
)*sin(d*x+c)*a^3*b^2+279*B*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((
b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b^3+261*B*cos(d*x+c)^5*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*Elli
pticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^4-45*A
*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*
x+c)))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2)
)*sin(d*x+c)*a^4*b-45*A*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+
a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3*b^2-435*A*cos(d*x+c)^4*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b^3-435*A
*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*
x+c)))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2)
)*sin(d*x+c)*a*b^4+45*A*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+
a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3*b^2+405*A*cos(d*x+c)^4*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*Ellipti
cF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b^3+435*A
*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*
x+c)))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2)
)*sin(d*x+c)*a*b^4+10*B*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+
a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^4*b)/(b+a*cos(d*x+c))/cos(d*x+c)^4/si
n(d*x+c)^5/b^2

```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm
="maxima")
```

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(a + \frac{b}{\cos(c+dx)}\right)^{5/2}}{\cos(c+dx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(5/2))/cos(c + d*x)^2,x)

[Out] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(5/2))/cos(c + d*x)^2, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**2*(a+b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)),x)

[Out] Timed out

$$3.365 \quad \int \sec(c+dx)(a+b \sec(c+dx))^{5/2}(A+B \sec(c+dx)) dx$$

Optimal. Leaf size=384

$$\frac{2(15a^2B + 56aAb + 25b^2B) \tan(c+dx) \sqrt{a+b \sec(c+dx)}}{105d} + \frac{2(a-b) \sqrt{a+b} (15a^2(7A-B) - 8ab(7A-15B) + b^2(7A-15B))}{105d}$$

[Out] $-2/105*(a-b)*(161*A*a^2*b+63*A*b^3+15*B*a^3+145*B*a*b^2)*\cot(d*x+c)*\text{EllipticE}((a+b*\sec(d*x+c))^{1/2}/(a+b)^{1/2},((a+b)/(a-b))^{1/2})*(a+b)^{1/2}*(b*(1-\sec(d*x+c))/(a+b))^{1/2}*(-b*(1+\sec(d*x+c))/(a-b))^{1/2}/b^2/d+2/105*(a-b)*(b^2*(63*A-25*B)-8*a*b*(7*A-15*B)+15*a^2*(7*A-B))*\cot(d*x+c)*\text{EllipticF}((a+b*\sec(d*x+c))^{1/2}/(a+b)^{1/2},((a+b)/(a-b))^{1/2})*(a+b)^{1/2}*(b*(1-\sec(d*x+c))/(a+b))^{1/2}*(-b*(1+\sec(d*x+c))/(a-b))^{1/2}/b/d+2/35*(7*A*b+5*B*a)*(a+b*\sec(d*x+c))^{3/2}*\tan(d*x+c)/d+2/7*B*(a+b*\sec(d*x+c))^{5/2}*\tan(d*x+c)/d+2/105*(56*A*a*b+15*B*a^2+25*B*b^2)*(a+b*\sec(d*x+c))^{1/2}*\tan(d*x+c)/d$

Rubi [A] time = 0.81, antiderivative size = 384, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {4002, 4005, 3832, 4004}

$$\frac{2(15a^2B + 56aAb + 25b^2B) \tan(c+dx) \sqrt{a+b \sec(c+dx)}}{105d} + \frac{2(a-b) \sqrt{a+b} (15a^2(7A-B) - 8ab(7A-15B) + b^2(7A-15B))}{105d}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d*x]*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]

[Out] $(-2*(a-b)*\text{Sqrt}[a+b]*(161*a^2*A*b+63*A*b^3+15*a^3*B+145*a*b^2*B)*\cot[c+d*x]*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[a+b*\text{Sec}[c+d*x]]/\text{Sqrt}[a+b]],(a+b)/(a-b)]*\text{Sqrt}[(b*(1-\text{Sec}[c+d*x]))/(a+b)]*\text{Sqrt}[-((b*(1+\text{Sec}[c+d*x]))/(a-b))]/(105*b^2*d)+(2*(a-b)*\text{Sqrt}[a+b]*(b^2*(63*A-25*B)-8*a*b*(7*A-15*B)+15*a^2*(7*A-B))*\text{Cot}[c+d*x]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[a+b*\text{Sec}[c+d*x]]/\text{Sqrt}[a+b]],(a+b)/(a-b)]*\text{Sqrt}[(b*(1-\text{Sec}[c+d*x]))/(a+b)]*\text{Sqrt}[-((b*(1+\text{Sec}[c+d*x]))/(a-b))]/(105*b*d)+(2*(56*a*A*b+15*a^2*B+25*b^2*B)*\text{Sqrt}[a+b*\text{Sec}[c+d*x]]*\text{Tan}[c+d*x]/(105*d)+(2*(7*A*b+5*a*B)*(a+b*\text{Sec}[c+d*x])^{3/2}*\text{Tan}[c+d*x]/(35*d)+(2*B*(a+b*\text{Sec}[c+d*x])^{5/2}*\text{Tan}[c+d*x])/(7*d)$

Rule 3832

Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]

/Rt[a + b, 2]], (a + b)/(a - b)]/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 4002

Int[csc[(e_.) + (f_.)*(x_.)]*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*Csc[e + f*x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]

Rule 4004

Int[(csc[(e_.) + (f_.)*(x_.)]*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)))/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]]/Rt[a + (b*B)/A, 2]], (a*A + b*B)/(a*A - b*B)]/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]

Rule 4005

Int[(csc[(e_.) + (f_.)*(x_.)]*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)))/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] :> Dist[A - B, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]

Rubi steps

$$\begin{aligned}
\int \sec(c + dx)(a + b \sec(c + dx))^{5/2}(A + B \sec(c + dx)) dx &= \frac{2B(a + b \sec(c + dx))^{5/2} \tan(c + dx)}{7d} + \frac{2}{7} \int \sec(c + dx) (a + b \sec(c + dx))^{5/2} dx \\
&= \frac{2(7Ab + 5aB)(a + b \sec(c + dx))^{3/2} \tan(c + dx)}{35d} + \frac{2}{7} \int \sec(c + dx) (a + b \sec(c + dx))^{5/2} dx \\
&= \frac{2(56aAb + 15a^2B + 25b^2B) \sqrt{a + b \sec(c + dx)} \tan(c + dx)}{105d} + \frac{2}{7} \int \sec(c + dx) (a + b \sec(c + dx))^{5/2} dx \\
&= \frac{2(56aAb + 15a^2B + 25b^2B) \sqrt{a + b \sec(c + dx)} \tan(c + dx)}{105d} + \frac{2(a - b) \sqrt{a + b} (161a^2Ab + 63Ab^3 + 15a^3B + 145a^2b^2B)}{105d}
\end{aligned}$$

Mathematica [B] time = 23.17, size = 2957, normalized size = 7.70

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[Sec[c + d*x]*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]

[Out] (Cos[c + d*x]^3*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x])*((2*(161*a^2*A*b + 63*A*b^3 + 15*a^3*B + 145*a*b^2*B)*Sin[c + d*x])/(105*b) + (2*Sec[c + d*x]^2*(7*A*b^2*Sin[c + d*x] + 15*a*b*B*Sin[c + d*x]))/35 + (2*Sec[c + d*x]*(77*a*A*b*Sin[c + d*x] + 45*a^2*B*Sin[c + d*x] + 25*b^2*B*Sin[c + d*x]))/105 + (2*b^2*B*Sec[c + d*x]^2*Tan[c + d*x])/7))/(d*(b + a*Cos[c + d*x])^2*(B + A*Cos[c + d*x])) + (2*((-23*a^2*A*b)/(15*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (3*A*b^3)/(5*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (a^3*B)/(7*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (29*a*b^2*B)/(21*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (8*a^3*A*Sqrt[Sec[c + d*x]])/(15*Sqrt[b + a*Cos[c + d*x]]) + (8*a*A*b^2*Sqrt[Sec[c + d*x]])/(15*Sqrt[b + a*Cos[c + d*x]]) - (a^4*B*Sqrt[Sec[c + d*x]])/(7*b*Sqrt[b + a*Cos[c + d*x]]) - (2*a^2*b*B*Sqrt[Sec[c + d*x]])/(21*Sqrt[b + a*Cos[c + d*x]]) + (5*b^3*B*Sqrt[Sec[c + d*x]])/(21*Sqrt[b + a*Cos[c + d*x]]) - (23*a^3*A*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(15*Sqrt[b + a*Cos[c + d*x]]) - (3*a*A*b^2*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(5*Sqrt[b + a*Cos[c + d*x]]) - (a^4*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(7*b*Sqrt[b + a*Cos[c + d*x]]) - (29*a^2*b*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(21*Sqrt[b + a*Cos[c + d*x]]))*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x])*((-2*(Cos[c + d*x]/(1 + Cos[c + d*x]))^(3/2))*((161*a^2*A*b + 63*A*b^3 + 15*a^3*B + 145*a*b^2*B)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/

$$\begin{aligned}
& (a + b)] - b*(15*a^2*(7*A + B) + 8*a*b*(7*A + 15*B) + b^2*(63*A + 25*B))*E \\
& \text{llipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sec[c + d*x])/Sqrt[(b +} \\
& \text{a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + (161*a^2*A*b + 63*A*b^3 +} \\
& \text{15*a^3*B + 145*a*b^2*B)*Tan[(c + d*x)/2]*(-1 + Tan[(c + d*x)/2]^2))/(105*b} \\
& \text{*d*(b + a*Cos[c + d*x])^2*(B + A*Cos[c + d*x])*Sqrt[Sec[(c + d*x)/2]^2]*Sec} \\
& \text{[c + d*x]^(7/2)*(-1/105*(a*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x])*Sin[c + d*} \\
& \text{x]*((-2*(Cos[c + d*x]/(1 + Cos[c + d*x]))^(3/2)*((161*a^2*A*b + 63*A*b^3 +} \\
& \text{15*a^3*B + 145*a*b^2*B)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)} \\
& \text{] - b*(15*a^2*(7*A + B) + 8*a*b*(7*A + 15*B) + b^2*(63*A + 25*B))*EllipticF} \\
& \text{[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sec[c + d*x])/Sqrt[(b + a*Cos[} \\
& \text{c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + (161*a^2*A*b + 63*A*b^3 + 15*a^3*} \\
& \text{B + 145*a*b^2*B)*Tan[(c + d*x)/2]*(-1 + Tan[(c + d*x)/2]^2))/(b*Sqrt[b + a} \\
& \text{*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) - (Sqrt[b + a*Cos[c + d*x]]*Sqrt[C} \\
& \text{os[(c + d*x)/2]^2*Sec[c + d*x])*Tan[(c + d*x)/2]*((-2*(Cos[c + d*x]/(1 + Co} \\
& \text{s[c + d*x]))^(3/2)*((161*a^2*A*b + 63*A*b^3 + 15*a^3*B + 145*a*b^2*B)*Ellip} \\
& \text{ticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - b*(15*a^2*(7*A + B) + 8*a} \\
& \text{*b*(7*A + 15*B) + b^2*(63*A + 25*B))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a} \\
& \text{- b)/(a + b)]*Sec[c + d*x])/Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c} \\
& \text{+ d*x]))] + (161*a^2*A*b + 63*A*b^3 + 15*a^3*B + 145*a*b^2*B)*Tan[(c + d*x} \\
& \text{)/2]*(-1 + Tan[(c + d*x)/2]^2))/(105*b*Sqrt[Sec[(c + d*x)/2]^2]) + (Sqrt[b} \\
& \text{+ a*Cos[c + d*x]]*((-2*(Cos[c + d*x]/(1 + Cos[c + d*x]))^(3/2)*((161*a^2*A} \\
& \text{*b + 63*A*b^3 + 15*a^3*B + 145*a*b^2*B)*EllipticE[ArcSin[Tan[(c + d*x)/2]],} \\
& \text{(a - b)/(a + b)] - b*(15*a^2*(7*A + B) + 8*a*b*(7*A + 15*B) + b^2*(63*A +} \\
& \text{25*B))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sec[c + d*x])/} \\
& \text{Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + (161*a^2*A*b + 63} \\
& \text{*A*b^3 + 15*a^3*B + 145*a*b^2*B)*Tan[(c + d*x)/2]*(-1 + Tan[(c + d*x)/2]^2} \\
& \text{)*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*S} \\
& \text{ec[c + d*x]*Tan[c + d*x]))/(105*b*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*} \\
& \text{x)/2]^2*Sec[c + d*x])) + (2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Cos[(c + d*x)/2]^} \\
& \text{2*Sec[c + d*x]]*((-3*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x]))*((161*a^2*A*b +} \\
& \text{63*A*b^3 + 15*a^3*B + 145*a*b^2*B)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a -} \\
& \text{b)/(a + b)] - b*(15*a^2*(7*A + B) + 8*a*b*(7*A + 15*B) + b^2*(63*A + 25*B)} \\
& \text{)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sec[c + d*x]*((Cos[} \\
& \text{c + d*x])*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x} \\
& \text{])))/Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + ((Cos[c + d*} \\
& \text{x]/(1 + Cos[c + d*x]))^(3/2)*((161*a^2*A*b + 63*A*b^3 + 15*a^3*B + 145*a*b^} \\
& \text{2*B)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - b*(15*a^2*(7*A} \\
& \text{+ B) + 8*a*b*(7*A + 15*B) + b^2*(63*A + 25*B))*EllipticF[ArcSin[Tan[(c + d*} \\
& \text{x)/2]], (a - b)/(a + b)]*Sec[c + d*x]*(-(a*Sin[c + d*x])/((a + b)*(1 + Co} \\
& \text{s[c + d*x])) + ((b + a*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d} \\
& \text{*x])^2)))/((b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))^(3/2) + (161*} \\
& \text{a^2*A*b + 63*A*b^3 + 15*a^3*B + 145*a*b^2*B)*Sec[(c + d*x)/2]^2*Tan[(c + d*} \\
& \text{x)/2]^2 + ((161*a^2*A*b + 63*A*b^3 + 15*a^3*B + 145*a*b^2*B)*Sec[(c + d*x)/} \\
& \text{2]^2*(-1 + Tan[(c + d*x)/2]^2))/2 - (2*(Cos[c + d*x]/(1 + Cos[c + d*x]))^(3} \\
& \text{/2)*Sec[c + d*x]*(-1/2*(b*(15*a^2*(7*A + B) + 8*a*b*(7*A + 15*B) + b^2*(63*}
\end{aligned}$$

$$\begin{aligned}
 & (A + 25*B)) * \text{Sec}[(c + d*x)/2]^2 / (\text{Sqrt}[1 - \text{Tan}[(c + d*x)/2]^2] * \text{Sqrt}[1 - ((a - b) * \text{Tan}[(c + d*x)/2]^2) / (a + b)]) + ((161*a^2*A*b + 63*A*b^3 + 15*a^3*B + 145*a*b^2*B) * \text{Sec}[(c + d*x)/2]^2 * \text{Sqrt}[1 - ((a - b) * \text{Tan}[(c + d*x)/2]^2) / (a + b)]) / (2 * \text{Sqrt}[1 - \text{Tan}[(c + d*x)/2]^2]) / \text{Sqrt}[(b + a * \text{Cos}[c + d*x]) / ((a + b) * (1 + \text{Cos}[c + d*x]))] - (2 * (\text{Cos}[c + d*x] / (1 + \text{Cos}[c + d*x]))^{3/2} * ((161*a^2*A*b + 63*A*b^3 + 15*a^3*B + 145*a*b^2*B) * \text{EllipticE}[\text{ArcSin}[\text{Tan}[(c + d*x)/2]], (a - b) / (a + b)] - b * (15*a^2*(7*A + B) + 8*a*b*(7*A + 15*B) + b^2*(63*A + 25*B)) * \text{EllipticF}[\text{ArcSin}[\text{Tan}[(c + d*x)/2]], (a - b) / (a + b)]) * \text{Sec}[c + d*x] * \text{Tan}[c + d*x] / \text{Sqrt}[(b + a * \text{Cos}[c + d*x]) / ((a + b) * (1 + \text{Cos}[c + d*x]))]) / (105 * b * \text{Sqrt}[\text{Sec}[(c + d*x)/2]^2]))
 \end{aligned}$$

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}((Bb^2 \sec(dx + c)^4 + Aa^2 \sec(dx + c) + (2Bab + Ab^2) \sec(dx + c)^3 + (Ba^2 + 2Aab) \sec(dx + c)^2) \sqrt{b \sec(dx + c) + a})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] integral((B*b^2*sec(d*x + c)^4 + A*a^2*sec(d*x + c) + (2*B*a*b + A*b^2)*sec(d*x + c)^3 + (B*a^2 + 2*A*a*b)*sec(d*x + c)^2)*sqrt(b*sec(d*x + c) + a), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(b \sec(dx + c) + a)^{\frac{5}{2}} \sec(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)*sec(d*x + c), x)

maple [B] time = 2.69, size = 3637, normalized size = 9.47

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x)

[Out] 2/105/d*(1+cos(d*x+c))^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))^2*(-161*A*cos(d*x+c)^5*a^3*b-77*A*cos(d*x+c)^5*a^2*b^2-63*A*cos(d*x+c)^5*a*b^3-45*B*cos(d*x+c)^5*a^3*b-145*B*cos(d*x+c)^5*a^2*b^2+161*A*cos(d*x+c)^4

$$\begin{aligned}
& +\cos(d*x+c))^{\frac{1}{2}}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{\frac{1}{2}}*EllipticF \\
& ((-1+\cos(d*x+c))/\sin(d*x+c),((a-b)/(a+b))^{\frac{1}{2}})*\sin(d*x+c)*a^3*b-105*A*\cos \\
& (d*x+c)^4*(\cos(d*x+c)/(1+\cos(d*x+c)))^{\frac{1}{2}}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)) \\
&)/(a+b)^{\frac{1}{2}}*EllipticF((-1+\cos(d*x+c))/\sin(d*x+c),((a-b)/(a+b))^{\frac{1}{2}})*\sin \\
& (d*x+c)*a^3*b+161*A*\sin(d*x+c)*\cos(d*x+c)^3*(\cos(d*x+c)/(1+\cos(d*x+c)))^{\frac{1}{2}} \\
&)*(b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b)^{\frac{1}{2}}*EllipticE((-1+\cos(d*x+c)) \\
&)/\sin(d*x+c),((a-b)/(a+b))^{\frac{1}{2}})*a^3*b+161*A*\sin(d*x+c)*\cos(d*x+c)^3*(\cos(d \\
& *x+c)/(1+\cos(d*x+c)))^{\frac{1}{2}}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b)^{\frac{1}{2}}*E \\
& llipticE((-1+\cos(d*x+c))/\sin(d*x+c),((a-b)/(a+b))^{\frac{1}{2}})*a^2*b^2+63*A*\sin(d \\
& *x+c)*\cos(d*x+c)^3*(\cos(d*x+c)/(1+\cos(d*x+c)))^{\frac{1}{2}}*((b+a*\cos(d*x+c))/(1+c \\
& os(d*x+c))/(a+b)^{\frac{1}{2}}*EllipticE((-1+\cos(d*x+c))/\sin(d*x+c),((a-b)/(a+b))^{\frac{1}{2}}) \\
&)*a*b^3-161*A*\sin(d*x+c)*\cos(d*x+c)^3*(\cos(d*x+c)/(1+\cos(d*x+c)))^{\frac{1}{2}} \\
&)*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b)^{\frac{1}{2}}*EllipticF((-1+\cos(d*x+c))/s \\
& in(d*x+c),((a-b)/(a+b))^{\frac{1}{2}})*a^2*b^2-119*A*\sin(d*x+c)*\cos(d*x+c)^3*(\cos(d \\
& *x+c)/(1+\cos(d*x+c)))^{\frac{1}{2}}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b)^{\frac{1}{2}}*E \\
& llipticF((-1+\cos(d*x+c))/\sin(d*x+c),((a-b)/(a+b))^{\frac{1}{2}})*a*b^3-63*A*\cos(d*x \\
& +c)^4*b^4+15*B*\cos(d*x+c)^4*a^4+21*A*\cos(d*x+c)*b^4+63*A*\sin(d*x+c)*\cos(d*x \\
& +c)^4*(\cos(d*x+c)/(1+\cos(d*x+c)))^{\frac{1}{2}}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a \\
& +b))^{\frac{1}{2}}*EllipticE((-1+\cos(d*x+c))/\sin(d*x+c),((a-b)/(a+b))^{\frac{1}{2}})*b^4-63 \\
& *A*\sin(d*x+c)*\cos(d*x+c)^4*(\cos(d*x+c)/(1+\cos(d*x+c)))^{\frac{1}{2}}*((b+a*\cos(d*x+ \\
& c))/(1+\cos(d*x+c))/(a+b)^{\frac{1}{2}}*EllipticF((-1+\cos(d*x+c))/\sin(d*x+c),((a-b) \\
&)/(a+b))^{\frac{1}{2}})*b^4+15*B*\sin(d*x+c)*\cos(d*x+c)^4*(\cos(d*x+c)/(1+\cos(d*x+c))) \\
&)^{\frac{1}{2}}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b)^{\frac{1}{2}}*EllipticE((-1+\cos(d*x+ \\
& c))/\sin(d*x+c),((a-b)/(a+b))^{\frac{1}{2}})*a^4-25*B*\sin(d*x+c)*\cos(d*x+c)^4*(\cos(d \\
& *x+c)/(1+\cos(d*x+c)))^{\frac{1}{2}}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b)^{\frac{1}{2}}*E \\
& llipticF((-1+\cos(d*x+c))/\sin(d*x+c),((a-b)/(a+b))^{\frac{1}{2}})*b^4+63*A*\sin(d*x+c \\
&)*\cos(d*x+c)^3*(\cos(d*x+c)/(1+\cos(d*x+c)))^{\frac{1}{2}}*((b+a*\cos(d*x+c))/(1+\cos(d \\
& *x+c))/(a+b)^{\frac{1}{2}}*EllipticE((-1+\cos(d*x+c))/\sin(d*x+c),((a-b)/(a+b))^{\frac{1}{2}}) \\
&)*b^4-63*A*\sin(d*x+c)*\cos(d*x+c)^3*(\cos(d*x+c)/(1+\cos(d*x+c)))^{\frac{1}{2}}*((b+a \\
& *cos(d*x+c))/(1+\cos(d*x+c))/(a+b)^{\frac{1}{2}}*EllipticF((-1+\cos(d*x+c))/\sin(d*x+ \\
& c),((a-b)/(a+b))^{\frac{1}{2}})*b^4+15*B*\sin(d*x+c)*\cos(d*x+c)^3*(\cos(d*x+c)/(1+\cos \\
& (d*x+c)))^{\frac{1}{2}}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b)^{\frac{1}{2}}*EllipticE((-1 \\
& +\cos(d*x+c))/\sin(d*x+c),((a-b)/(a+b))^{\frac{1}{2}})*a^4-25*B*\sin(d*x+c)*\cos(d*x+c) \\
&)^3*(\cos(d*x+c)/(1+\cos(d*x+c)))^{\frac{1}{2}}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b) \\
&)^{\frac{1}{2}}*EllipticF((-1+\cos(d*x+c))/\sin(d*x+c),((a-b)/(a+b))^{\frac{1}{2}})*b^4-145*B \\
& *\cos(d*x+c)^4*a*b^3)/(b+a*\cos(d*x+c))/\cos(d*x+c)^3/\sin(d*x+c)^5/b
\end{aligned}$$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(a + \frac{b}{\cos(c+dx)}\right)^{5/2}}{\cos(c+dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(5/2))/cos(c + d*x), x)

[Out] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(5/2))/cos(c + d*x), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \sec(c + dx)) (a + b \sec(c + dx))^{\frac{5}{2}} \sec(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)), x)

[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**(5/2)*sec(c + d*x), x)

3.366 $\int (a + b \sec(c + dx))^{5/2} (A + B \sec(c + dx)) dx$

Optimal. Leaf size=442

$$\frac{2(a-b)\sqrt{a+b} (23a^2B + 35aAb + 9b^2B) \cot(c + dx) \sqrt{\frac{b(1-\sec(c+dx))}{a+b}} \sqrt{\frac{b(\sec(c+dx)+1)}{a-b}} E\left(\sin^{-1}\left(\frac{\sqrt{a+b}\sec(c+dx)}{\sqrt{a+b}}\right)\right) \Big|_{\frac{a+b}{a-b}}}{15bd}$$

[Out] $-2/15*(a-b)*(35*A*a*b+23*B*a^2+9*B*b^2)*\cot(d*x+c)*\text{EllipticE}((a+b*\sec(d*x+c))^{1/2}/(a+b)^{1/2}, ((a+b)/(a-b))^{1/2})*(a+b)^{1/2}*(b*(1-\sec(d*x+c)))/(a+b))^{1/2}*(-b*(1+\sec(d*x+c)))/(a-b))^{1/2}/b/d+2/15*(a^2*b*(45*A-23*B)-a*b^2*(35*A-17*B)+b^3*(5*A-9*B)+15*a^3*B)*\cot(d*x+c)*\text{EllipticF}((a+b*\sec(d*x+c))^{1/2}/(a+b)^{1/2}, ((a+b)/(a-b))^{1/2})*(a+b)^{1/2}*(b*(1-\sec(d*x+c)))/(a+b))^{1/2}*(-b*(1+\sec(d*x+c)))/(a-b))^{1/2}/b/d-2*a^2*A*\cot(d*x+c)*\text{EllipticPi}((a+b*\sec(d*x+c))^{1/2}/(a+b)^{1/2}, (a+b)/a, ((a+b)/(a-b))^{1/2})*(a+b)^{1/2}*(b*(1-\sec(d*x+c)))/(a+b))^{1/2}*(-b*(1+\sec(d*x+c)))/(a-b))^{1/2}/d+2/5*b*B*(a+b*\sec(d*x+c))^{3/2}*\tan(d*x+c)/d+2/15*b*(5*A*b+8*B*a)*(a+b*\sec(d*x+c))^{1/2})*\tan(d*x+c)/d$

Rubi [A] time = 0.66, antiderivative size = 442, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3918, 4056, 4058, 3921, 3784, 3832, 4004}

$$\frac{2\sqrt{a+b} (a^2b(45A - 23B) + 15a^3B - ab^2(35A - 17B) + b^3(5A - 9B)) \cot(c + dx) \sqrt{\frac{b(1-\sec(c+dx))}{a+b}} \sqrt{\frac{b(\sec(c+dx)+1)}{a-b}}}{15bd}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + b*\text{Sec}[c + d*x])^{5/2}*(A + B*\text{Sec}[c + d*x]), x]$

[Out] $(-2*(a-b)*\text{Sqrt}[a+b]*(35*a*A*b + 23*a^2*B + 9*b^2*B)*\text{Cot}[c + d*x]*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[a + b*\text{Sec}[c + d*x]]/\text{Sqrt}[a + b]], (a + b)/(a - b)]*\text{Sqrt}[(b*(1 - \text{Sec}[c + d*x]))/(a + b)]*\text{Sqrt}[-((b*(1 + \text{Sec}[c + d*x]))/(a - b))]/(15*b*d) + (2*\text{Sqrt}[a + b]*(a^2*b*(45*A - 23*B) - a*b^2*(35*A - 17*B) + b^3*(5*A - 9*B) + 15*a^3*B)*\text{Cot}[c + d*x]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[a + b*\text{Sec}[c + d*x]]/\text{Sqrt}[a + b]], (a + b)/(a - b)]*\text{Sqrt}[(b*(1 - \text{Sec}[c + d*x]))/(a + b)]*\text{Sqrt}[-((b*(1 + \text{Sec}[c + d*x]))/(a - b))]/(15*b*d) - (2*a^2*A*\text{Sqrt}[a + b]*\text{Cot}[c + d*x]*\text{EllipticPi}[(a + b)/a, \text{ArcSin}[\text{Sqrt}[a + b*\text{Sec}[c + d*x]]/\text{Sqrt}[a + b]], (a + b)/(a - b)]*\text{Sqrt}[(b*(1 - \text{Sec}[c + d*x]))/(a + b)]*\text{Sqrt}[-((b*(1 + \text{Sec}[c + d*x]))/(a - b))]/d + (2*b*(5*A*b + 8*a*B)*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]*\text{Tan}[c + d*x])/d + (2*b*B*(a + b*\text{Sec}[c + d*x])^{3/2}*\text{Tan}[c + d*x])/d)$

Rule 3784

$\text{Int}[1/\text{Sqrt}[\text{csc}[(c_.) + (d_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] \rightarrow \text{Simp}[(2*\text{Rt}[a + b, 2]*\text{Sqrt}[(b*(1 - \text{Csc}[c + d*x]))/(a + b)]*\text{Sqrt}[-((b*(1 + \text{Csc}[c + d*x]))$

$$\frac{1}{(a-b)} \int \frac{\text{EllipticPi}[(a+b)/a, \text{ArcSin}[\sqrt{a+b\text{Csc}[c+dx]}]/\text{Rt}[a+b, 2]], (a+b)/(a-b)]}{(a^2-d^2x^2)} dx$$
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 3832

$$\int \frac{\text{csc}[e+fx] + (f_1)(x_1)}{\sqrt{\text{csc}[e+fx] + (f_1)(x_1)(b_1) + (a_1)}} dx$$
 Symbol \rightarrow
$$\text{Simp}[(-2\text{Rt}[a+b, 2]\sqrt{(b(1-\text{Csc}[e+fx]))/(a+b)})\sqrt{-((b(1+\text{Csc}[e+fx]))/(a-b))}\text{EllipticF}[\text{ArcSin}[\sqrt{a+b\text{Csc}[e+fx]}]/\text{Rt}[a+b, 2]], (a+b)/(a-b)]/(b\text{f}\text{Cot}[e+fx]), x]$$
; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 3918

$$\int (\text{csc}[e+fx] + (f_1)(x_1)(b_1) + (a_1))^m (\text{csc}[e+fx] + (f_1)(x_1)(d_1) + (c_1)) dx$$
 Symbol \rightarrow
$$-\text{Simp}[(b\text{d}\text{Cot}[e+fx](a+b\text{Csc}[e+fx])^{m-1})/(f^m), x] + \text{Dist}[1/m, \text{Int}[(a+b\text{Csc}[e+fx])^{m-2}\text{Simp}[a^{2cm} + (b^2d(m-1) + 2abcm + a^2dm)\text{Csc}[e+fx] + b(bcm + a(2m-1))\text{Csc}[e+fx]^2, x], x], x]$$
; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2c - a^2d, 0] && GtQ[m, 1] && NeQ[a^2 - b^2, 0] && IntegerQ[2m]

Rule 3921

$$\int (\text{csc}[e+fx] + (f_1)(x_1)(d_1) + (c_1))/\sqrt{\text{csc}[e+fx] + (f_1)(x_1)(b_1) + (a_1)} dx$$
 Symbol \rightarrow
$$\text{Dist}[c, \text{Int}[1/\sqrt{a+b\text{Csc}[e+fx]}, x], x] + \text{Dist}[d, \text{Int}[\text{Csc}[e+fx]/\sqrt{a+b\text{Csc}[e+fx]}, x], x]$$
; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2c - a^2d, 0] && NeQ[a^2 - b^2, 0]

Rule 4004

$$\int (\text{csc}[e+fx] + (f_1)(x_1)(c_1) + (A_1))/\sqrt{\text{csc}[e+fx] + (f_1)(x_1)(b_1) + (a_1)} dx$$
 Symbol \rightarrow
$$\text{Simp}[(-2(Ab - aB)\text{Rt}[a + (bB)/A, 2]\sqrt{(b(1-\text{Csc}[e+fx]))/(a+b)})\sqrt{-((b(1+\text{Csc}[e+fx]))/(a-b))}\text{EllipticE}[\text{ArcSin}[\sqrt{a+b\text{Csc}[e+fx]}]/\text{Rt}[a + (bB)/A, 2]], (aA + bB)/(aA - bB)]/(b^2\text{f}\text{Cot}[e+fx]), x]$$
; FreeQ[{a, b, e, f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]

Rule 4056

$$\int ((A_1) + \text{csc}[e+fx] + (f_1)(x_1)(B_1) + \text{csc}[e+fx] + (f_1)(x_1))^2 (C_1) (\text{csc}[e+fx] + (f_1)(x_1)(b_1) + (a_1))^m dx$$
 Symbol \rightarrow
$$-\text{Simp}[(C\text{Cot}[e+fx](a+b\text{Csc}[e+fx])^m)/(f(m+1)), x] + \text{Dist}[1/(m+1), \text{Int}[(a+b\text{Csc}[e+fx])^{m-1}\text{Simp}[aA(m+1) + ((Ab + aB)(m+1) + bCm)\text{Csc}[e+fx] + (bB(m+1) + aCm)\text{Csc}[e+fx]^2, x], x], x]$$
; FreeQ[{a,

b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && IGtQ[2*m, 0]

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] := Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
+ Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
 \int (a + b \sec(c + dx))^{5/2} (A + B \sec(c + dx)) dx &= \frac{2bB(a + b \sec(c + dx))^{3/2} \tan(c + dx)}{5d} + \frac{2}{5} \int \sqrt{a + b \sec(c + dx)} \\
 &= \frac{2b(5Ab + 8aB)\sqrt{a + b \sec(c + dx)} \tan(c + dx)}{15d} + \frac{2bB(a + b \sec(c + dx))^{3/2} \tan(c + dx)}{5d} \\
 &= \frac{2b(5Ab + 8aB)\sqrt{a + b \sec(c + dx)} \tan(c + dx)}{15d} + \frac{2bB(a + b \sec(c + dx))^{3/2} \tan(c + dx)}{5d} \\
 &= -\frac{2(a - b)\sqrt{a + b} (35aAb + 23a^2B + 9b^2B) \cot(c + dx) E\left(\sin^{-1}\left(\frac{\sqrt{a + b \sec(c + dx)}}{\sqrt{a + b}}\right)\right)}{15bd} \\
 &= -\frac{2(a - b)\sqrt{a + b} (35aAb + 23a^2B + 9b^2B) \cot(c + dx) E\left(\sin^{-1}\left(\frac{\sqrt{a + b \sec(c + dx)}}{\sqrt{a + b}}\right)\right)}{15bd}
 \end{aligned}$$

Mathematica [B] time = 25.56, size = 7138, normalized size = 16.15

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]), x]
```

```
[Out] Result too large to show
```

fricas [F] time = 22.71, size = 0, normalized size = 0.00

integral $\left((Bb^2 \sec(dx + c))^3 + Aa^2 + (2 Bab + Ab^2) \sec(dx + c)^2 + (Ba^2 + 2 Aab) \sec(dx + c) \right) \sqrt{b \sec(dx + c)} +$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] integral((B*b^2*sec(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*sec(d*x + c)^2 + (B*a^2 + 2*A*a*b)*sec(d*x + c))*sqrt(b*sec(d*x + c) + a), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(b \sec(dx + c) + a)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2), x)

maple [B] time = 2.42, size = 3285, normalized size = 7.43

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x)

[Out]
$$\begin{aligned} & -2/15/d*(1+\cos(d*x+c))^2*((b+a*\cos(d*x+c))/\cos(d*x+c))^{1/2}*(-1+\cos(d*x+c)) \\ &)^2*(-9*B*\sin(d*x+c)*\cos(d*x+c)^2*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/ \\ & (1+\cos(d*x+c))/(a+b))^{1/2}*EllipticE((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{1/2})* \\ & a*b^2+5*A*\cos(d*x+c)^3*b^3-23*B*\cos(d*x+c)^3*a^3-3*b^3*B+9*B*\cos(d*x+c)^3*b^3-6*B*\cos(d*x+c)^2*b^3+ \\ & 35*A*\cos(d*x+c)^3*a*b^2-40*A*\cos(d*x+c)^2*a*b^2+9*B*\cos(d*x+c)^4*a*b^2+23*B*\cos(d*x+c)^3*a^2*b-34*B*\cos(d*x+c)^2*a^2*b- \\ & 14*B*\cos(d*x+c)*a*b^2-5*A*\cos(d*x+c)*b^3+5*B*\cos(d*x+c)^3*a*b^2+35*A*\cos(d*x+c)^4*a^2*b+ \\ & 5*A*\cos(d*x+c)^4*a*b^2-35*A*\cos(d*x+c)^3*a^2*b+11*B*\cos(d*x+c)^4*a^2*b+23*B*\cos(d*x+c)^4*a^3+23*B*\sin(d*x+c)*\cos(d*x+c)^3* \\ & (\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*EllipticF((-1+\cos(d*x+c))/\sin(d*x+c), \\ & ((a-b)/(a+b))^{1/2})*a^2*b+17*B*\sin(d*x+c)*\cos(d*x+c)^3*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/ \\ & (1+\cos(d*x+c))/(a+b))^{1/2}*EllipticF((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{1/2})*a*b^2-23*B*\sin(d*x+c)*\cos(d*x+c)^3* \\ & (\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*EllipticE((-1+\cos(d*x+c))/\sin(d*x+c), \\ & ((a-b)/(a+b))^{1/2})*a^2*b-9*B*\sin(d*x+c)*\cos(d*x+c)^3*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/ \\ & (1+\cos(d*x+c))/(a+b))^{1/2}*EllipticE((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{1/2})*a*b^2+35*A*\sin(d*x+c)*\cos(d*x+c)^2* \\ & (\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*EllipticF((-1+\cos(d*x+c))/\sin(d*x+c), \\ & ((a-b)/(a+b))^{1/2})*a*b^2-35*A*\sin(d*x+c)*\cos(d*x+c)^2*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/ \\ & (1+\cos(d*x+c))/(a+b))^{1/2}*EllipticE((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{1/2})*a^2*b-35*A*\sin(d*x+c)*\cos(d*x+c)^2* \\ & (\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2} \\ &) \end{aligned}$$

, $((a-b)/(a+b))^{1/2} * a^3 - 9 * B * \sin(dx+c) * \cos(dx+c)^2 * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * ((b+a*\cos(dx+c))/(1+\cos(dx+c)))/(a+b)^{1/2} * \text{EllipticE}((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{1/2}) * b^3 + 5 * A * \sin(dx+c) * \cos(dx+c)^3 * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * ((b+a*\cos(dx+c))/(1+\cos(dx+c)))/(a+b)^{1/2} * \text{EllipticF}((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{1/2}) * b^3 + 9 * B * \sin(dx+c) * \cos(dx+c)^3 * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * ((b+a*\cos(dx+c))/(1+\cos(dx+c)))/(a+b)^{1/2} * \text{EllipticF}((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{1/2}) * b^3 / ((b+a*\cos(dx+c))/\cos(dx+c))^2 / \sin(dx+c)^5$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx+c) + A)(b \sec(dx+c) + a)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(dx+c))^(5/2)*(A+B*sec(dx+c)),x, algorithm="maxima")

[Out] integrate((B*sec(dx+c) + A)*(b*sec(dx+c) + a)^(5/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(A + \frac{B}{\cos(c+dx)} \right) \left(a + \frac{b}{\cos(c+dx)} \right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + dx))*(a + b/cos(c + dx))^(5/2), x)

[Out] int((A + B/cos(c + dx))*(a + b/cos(c + dx))^(5/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \sec(c + dx))(a + b \sec(c + dx))^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(dx+c))**(5/2)*(A+B*sec(dx+c)),x)

[Out] Integral((A + B*sec(c + dx))*(a + b*sec(c + dx))**(5/2), x)

$$3.367 \quad \int \cos(c+dx)(a+b \sec(c+dx))^{5/2}(A+B \sec(c+dx)) dx$$

Optimal. Leaf size=433

$$\frac{\sqrt{a+b} \left(3a^2(A+6B) + 2ab(9A-7B) - 2b^2(3A-B)\right) \cot(c+dx) \sqrt{\frac{b(1-\sec(c+dx))}{a+b}} \sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}} F\left(\sin^{-1}\left(\frac{\sqrt{a+b}\sec(c+dx)}{\sqrt{a}}\right)\right)}{3d}$$

[Out] a*A*(a+b*sec(d*x+c))^(3/2)*sin(d*x+c)/d+1/3*(a-b)*(3*A*a^2-6*A*b^2-14*B*a*b)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c)))/(a+b)^(1/2)*(-b*(1+sec(d*x+c)))/(a-b)^(1/2)/b/d+1/3*(2*a*b*(9*A-7*B)-2*b^2*(3*A-B)+3*a^2*(A+6*B))*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c)))/(a+b)^(1/2)*(-b*(1+sec(d*x+c)))/(a-b)^(1/2)/d-a*(5*A*b+2*B*a)*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2), (a+b)/a, ((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c)))/(a+b)^(1/2)*(-b*(1+sec(d*x+c)))/(a-b)^(1/2)/d-1/3*b*(3*A*a-2*B*b)*(a+b*sec(d*x+c))^(1/2)*tan(d*x+c)/d

Rubi [A] time = 0.70, antiderivative size = 433, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {4025, 4056, 4058, 3921, 3784, 3832, 4004}

$$\frac{\sqrt{a+b} \left(3a^2(A+6B) + 2ab(9A-7B) - 2b^2(3A-B)\right) \cot(c+dx) \sqrt{\frac{b(1-\sec(c+dx))}{a+b}} \sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}} F\left(\sin^{-1}\left(\frac{\sqrt{a+b}\sec(c+dx)}{\sqrt{a}}\right)\right)}{3d}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]

[Out] ((a - b)*Sqrt[a + b]*(3*a^2*A - 6*A*b^2 - 14*a*b*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))]/(3*b*d) + (Sqrt[a + b]*(2*a*b*(9*A - 7*B) - 2*b^2*(3*A - B) + 3*a^2*(A + 6*B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))]/(3*d) - (a*Sqrt[a + b]*(5*A*b + 2*a*B)*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))]/d + (a*A*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/d - (b*(3*a*A - 2*b*B)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/d)

Rule 3784

Int[1/Sqrt[csc[(c_.) + (d_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] :> Simp[(2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))

$$\frac{1}{(a-b)} \text{EllipticPi}\left[\frac{a+b}{a}, \text{ArcSin}\left[\frac{\sqrt{a+b\text{Csc}[c+dx]}}{\text{Rt}[a+b, 2]}\right], \frac{a+b}{a-b}\right] / (a*d\text{Cot}[c+dx]), x] /;$$

$$\text{FreeQ}\{a, b, c, d\}, x \} \&\& \text{NeQ}[a^2 - b^2, 0]$$

Rule 3832

$$\text{Int}\left[\frac{\text{csc}[e] + (f)(x)}{\sqrt{\text{csc}[e] + (f)(x)(b) + (a)}}, x_{\text{Symbol}}\right] \rightarrow \text{Simp}\left[\frac{-2\text{Rt}[a+b, 2]\sqrt{(b(1-\text{Csc}[e+fx]))/(a+b)}\sqrt{-((b(1+\text{Csc}[e+fx]))/(a-b))}\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b\text{Csc}[e+fx]}}{\text{Rt}[a+b, 2]}\right], \frac{a+b}{a-b}\right]}{b*f*\text{Cot}[e+fx]}, x] /;$$

$$\text{FreeQ}\{a, b, e, f\}, x \} \&\& \text{NeQ}[a^2 - b^2, 0]$$

Rule 3921

$$\text{Int}\left[\frac{(\text{csc}[e] + (f)(x))(d) + (c)}{\sqrt{\text{csc}[e] + (f)(x)(b) + (a)}}, x_{\text{Symbol}}\right] \rightarrow \text{Dist}[c, \text{Int}\left[\frac{1}{\sqrt{a+b\text{Csc}[e+fx]}}, x\right], x] + \text{Dist}[d, \text{Int}\left[\frac{\text{Csc}[e+fx]}{\sqrt{a+b\text{Csc}[e+fx]}}, x\right], x] /;$$

$$\text{FreeQ}\{a, b, c, d, e, f\}, x \} \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 - b^2, 0]$$

Rule 4004

$$\text{Int}\left[\frac{(\text{csc}[e] + (f)(x))(csc[e] + (f)(x)(B) + (A))}{\sqrt{\text{csc}[e] + (f)(x)(b) + (a)}}, x_{\text{Symbol}}\right] \rightarrow \text{Simp}\left[\frac{-2(A*b - a*B)\text{Rt}[a + (b*B)/A, 2]\sqrt{(b(1-\text{Csc}[e+fx]))/(a+b)}\sqrt{-((b(1+\text{Csc}[e+fx]))/(a-b))}\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b\text{Csc}[e+fx]}}{\text{Rt}[a + (b*B)/A, 2]}\right], \frac{(a*A + b*B)/(a*A - b*B)}{b^2*f*\text{Cot}[e+fx]}, x] /;$$

$$\text{FreeQ}\{a, b, e, f, A, B\}, x \} \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{EqQ}[A^2 - B^2, 0]$$

Rule 4025

$$\text{Int}\left[\frac{(\text{csc}[e] + (f)(x))(d)^n(\text{csc}[e] + (f)(x)(b) + (a))^m(\text{csc}[e] + (f)(x)(B) + (A))}{(f)^n}, x_{\text{Symbol}}\right] \rightarrow \text{Simp}\left[\frac{a*A*\text{Cot}[e+fx](a+b\text{Csc}[e+fx])^{m-1}(d\text{Csc}[e+fx])^n}{(f)^n}, x\right] + \text{Dist}\left[\frac{1}{(d)^n}, \text{Int}\left[\frac{(a+b\text{Csc}[e+fx])^{m-2}(d\text{Csc}[e+fx])^{n+1}\text{Simp}[a*(a*B*n - A*b*(m-n-1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1+n))]*\text{Csc}[e+fx] + b*(b*B*n + a*A*(m+n))*\text{Csc}[e+fx]^2, x], x], x] /;$$

$$\text{FreeQ}\{a, b, d, e, f, A, B\}, x \} \&\& \text{NeQ}[A*b - a*B, 0] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{GtQ}[m, 1] \&\& \text{LeQ}[n, -1]$$

Rule 4056

$$\text{Int}\left[\frac{(A) + \text{csc}[e] + (f)(x)(B) + \text{csc}[e] + (f)(x)^2(C)}{(\text{csc}[e] + (f)(x)(b) + (a))^m}, x_{\text{Symbol}}\right] \rightarrow -\text{Simp}\left[\frac{C*\text{Cot}[e+fx](a+b\text{Csc}[e+fx])^m}{(f)(m+1)}, x\right] + \text{Dist}\left[\frac{1}{(m+1)}, \text{Int}\left[\frac{(a+b\text{Csc}[e+fx])^{m-1}\text{Simp}[a*A*(m+1) + ((A*b + a*B)*(m+1) + b*C*m)*C}{(f)(m+1)}, x\right], x\right] /;$$

```
sc[e + f*x] + (b*B*(m + 1) + a*C*m)*Csc[e + f*x]^2, x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && IGtQ[2*m, 0]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
+ Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
 \int \cos(c + dx)(a + b \sec(c + dx))^{5/2}(A + B \sec(c + dx)) dx &= \frac{aA(a + b \sec(c + dx))^{3/2} \sin(c + dx)}{d} - \int \sqrt{a + b \sec(c + dx)} dx \\
 &= \frac{aA(a + b \sec(c + dx))^{3/2} \sin(c + dx)}{d} - \frac{b(3aA - 2bB)}{d} \int \sqrt{a + b \sec(c + dx)} dx \\
 &= \frac{aA(a + b \sec(c + dx))^{3/2} \sin(c + dx)}{d} - \frac{b(3aA - 2bB)}{d} \int \sqrt{a + b \sec(c + dx)} dx \\
 &= \frac{(a - b)\sqrt{a + b} (3a^2A - 6Ab^2 - 14abB) \cot(c + dx)E}{d} \\
 &= \frac{(a - b)\sqrt{a + b} (3a^2A - 6Ab^2 - 14abB) \cot(c + dx)E}{d}
 \end{aligned}$$

Mathematica [B] time = 26.08, size = 7745, normalized size = 17.89

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] Result too large to show
```

fricas [F] time = 51.70, size = 0, normalized size = 0.00

```
integral((Bb^2 cos(dx + c) sec(dx + c)^3 + Aa^2 cos(dx + c) + (2 Bab + Ab^2) cos(dx + c) sec(dx + c)^2 + (Ba^2 + 2
```

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] integral((B*b^2*cos(d*x + c)*sec(d*x + c)^3 + A*a^2*cos(d*x + c) + (2*B*a*b + A*b^2)*cos(d*x + c)*sec(d*x + c)^2 + (B*a^2 + 2*A*a*b)*cos(d*x + c)*sec(d*x + c))*sqrt(b*sec(d*x + c) + a), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(b \sec(dx + c) + a)^{\frac{5}{2}} \cos(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)*cos(d*x + c), x)

maple [B] time = 2.28, size = 3215, normalized size = 7.42

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x)

[Out]
$$\begin{aligned} & -1/3/d*(1+\cos(d*x+c))^2*((b+a*\cos(d*x+c))/\cos(d*x+c))^{1/2}*(-1+\cos(d*x+c)) \\ & ^2*(-14*B*\sin(d*x+c)*\cos(d*x+c)^2*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/ \\ & (1+\cos(d*x+c)))/(a+b))^{1/2}*EllipticE((-1+\cos(d*x+c))/\sin(d*x+c) \\ & ,((a-b)/(a+b))^{1/2})*a*b^2-3*A*\cos(d*x+c)^3*a^3+3*A*\sin(d*x+c)*\cos(d*x+c)^ \\ & 2*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/\cos(d*x+c))^{1/2}*((b+a*\cos(d*x+c))/ \\ & (1+\cos(d*x+c)))/(a+b))^{1/2}*EllipticE((-1+\cos(d*x+c))/\sin(d*x+c),((a-b)/(a+b))^{1/2})*a^3-6*A*\sin \\ & (d*x+c)*\cos(d*x+c)^2*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/ \\ & (1+\cos(d*x+c)))/(a+b))^{1/2}*EllipticE((-1+\cos(d*x+c))/\sin(d*x+c),((a-b)/(a+b) \\ &))^{1/2})*b^3+12*B*\sin(d*x+c)*\cos(d*x+c)^2*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2} \\ & *((b+a*\cos(d*x+c))/\cos(d*x+c))^{1/2}*((b+a*\cos(d*x+c))/\cos(d*x+c))^{1/2}*((b+a*\cos(d*x+c))/ \\ & (1+\cos(d*x+c)))/(a+b))^{1/2}*EllipticPi((-1+\cos(d*x+c))/\sin(d*x+c),-1,((a-b)/(a+b))^{1/2})*a^3-2*b^3*B+6*A*\cos(d*x+c)^2*b^3+2*B*\cos \\ & (d*x+c)^2*b^3+6*A*\cos(d*x+c)^3*a*b^2-3*A*\cos(d*x+c)^2*a^2*b-6*A*\cos(d*x+c)^ \\ & 2*a*b^2+14*B*\cos(d*x+c)^3*a^2*b-14*B*\cos(d*x+c)^2*a^2*b+14*B*\cos(d*x+c)^2*a \\ & *b^2-16*B*\cos(d*x+c)*a*b^2+3*A*\cos(d*x+c)^4*a^3+30*A*\sin(d*x+c)*\cos(d*x+c)^ \\ & 2*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/\cos(d*x+c))^{1/2}*((b+a*\cos(d*x+c))/ \\ & (1+\cos(d*x+c)))/(a+b))^{1/2}*EllipticPi((-1+\cos(d*x+c))/\sin(d*x+c),-1,((a-b)/(a+b))^{1/2})*a^2*b- \\ & 6*A*\cos(d*x+c)*b^3+2*B*\cos(d*x+c)^3*a*b^2+3*A*\cos(d*x+c)^3*a^2*b-6*B*(\cos(d \\ & *x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/\cos(d*x+c))^{1/2}*((b+a*\cos(d*x+c))/ \\ & (1+\cos(d*x+c)))/(a+b))^{1/2}*EllipticF((-1+\cos(d*x+c))/\sin(d*x+c),((a-b)/(a+b))^{1/2})*\sin(d*x+c)*\cos(d*x \\ & +c)*a^3+3*A*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/\cos(d*x+c))^{1/2}*((b+a*\cos(d*x+c))/ \\ & (1+\cos(d*x+c)))/(a+b))^{1/2} \end{aligned}$$

$$\begin{aligned}
& c)) / (a+b)^{(1/2)} * \text{EllipticE}((-1+\cos(d*x+c)) / \sin(d*x+c), ((a-b)/(a+b))^{(1/2)}) * \\
& \sin(d*x+c) * \cos(d*x+c) * a^3 - 6*A * (\cos(d*x+c) / (1+\cos(d*x+c)))^{(1/2)} * ((b+a*\cos(d \\
& *x+c)) / (1+\cos(d*x+c))) / (a+b)^{(1/2)} * \text{EllipticE}((-1+\cos(d*x+c)) / \sin(d*x+c), ((a \\
& -b)/(a+b))^{(1/2)}) * \sin(d*x+c) * \cos(d*x+c) * b^3 + 12*B * (\cos(d*x+c) / (1+\cos(d*x+c)) \\
&)^{(1/2)} * ((b+a*\cos(d*x+c)) / (1+\cos(d*x+c))) / (a+b)^{(1/2)} * \text{EllipticPi}((-1+\cos(d* \\
& x+c)) / \sin(d*x+c), -1, ((a-b)/(a+b))^{(1/2)}) * \sin(d*x+c) * \cos(d*x+c) * a^3 + 18*A * \sin \\
& (d*x+c) * \cos(d*x+c)^2 * (\cos(d*x+c) / (1+\cos(d*x+c)))^{(1/2)} * ((b+a*\cos(d*x+c)) / (1 \\
& +\cos(d*x+c))) / (a+b)^{(1/2)} * \text{EllipticF}((-1+\cos(d*x+c)) / \sin(d*x+c), ((a-b)/(a+b) \\
&)^{(1/2)}) * a*b^2 + 3*A * \sin(d*x+c) * \cos(d*x+c)^2 * (\cos(d*x+c) / (1+\cos(d*x+c)))^{(1/2)} \\
&) * ((b+a*\cos(d*x+c)) / (1+\cos(d*x+c))) / (a+b)^{(1/2)} * \text{EllipticE}((-1+\cos(d*x+c)) / s \\
& in(d*x+c), ((a-b)/(a+b))^{(1/2)}) * a^2 * b - 6*A * \sin(d*x+c) * \cos(d*x+c)^2 * (\cos(d*x+c \\
&) / (1+\cos(d*x+c)))^{(1/2)} * ((b+a*\cos(d*x+c)) / (1+\cos(d*x+c))) / (a+b)^{(1/2)} * \text{Ellip \\
& ticE}((-1+\cos(d*x+c)) / \sin(d*x+c), ((a-b)/(a+b))^{(1/2)}) * a*b^2 + 18*B * \sin(d*x+c) * \\
& \cos(d*x+c)^2 * (\cos(d*x+c) / (1+\cos(d*x+c)))^{(1/2)} * ((b+a*\cos(d*x+c)) / (1+\cos(d*x \\
& +c))) / (a+b)^{(1/2)} * \text{EllipticF}((-1+\cos(d*x+c)) / \sin(d*x+c), ((a-b)/(a+b))^{(1/2)}) \\
& * a^2 * b + 14*B * \sin(d*x+c) * \cos(d*x+c)^2 * (\cos(d*x+c) / (1+\cos(d*x+c)))^{(1/2)} * ((b+a \\
& * \cos(d*x+c)) / (1+\cos(d*x+c))) / (a+b)^{(1/2)} * \text{EllipticF}((-1+\cos(d*x+c)) / \sin(d*x+ \\
& c), ((a-b)/(a+b))^{(1/2)}) * a*b^2 - 14*B * \sin(d*x+c) * \cos(d*x+c)^2 * (\cos(d*x+c) / (1+c \\
& os(d*x+c)))^{(1/2)} * ((b+a*\cos(d*x+c)) / (1+\cos(d*x+c))) / (a+b)^{(1/2)} * \text{EllipticE}((\\
& -1+\cos(d*x+c)) / \sin(d*x+c), ((a-b)/(a+b))^{(1/2)}) * a^2 * b + 14*B * \cos(d*x+c) * \sin(d* \\
& x+c) * (\cos(d*x+c) / (1+\cos(d*x+c)))^{(1/2)} * ((b+a*\cos(d*x+c)) / (1+\cos(d*x+c))) / (a+ \\
& b))^{(1/2)} * \text{EllipticF}((-1+\cos(d*x+c)) / \sin(d*x+c), ((a-b)/(a+b))^{(1/2)}) * a*b^2 - 1 \\
& 4*B * \text{EllipticE}((-1+\cos(d*x+c)) / \sin(d*x+c), ((a-b)/(a+b))^{(1/2)}) * \cos(d*x+c) * b^ \\
& 2 * (\cos(d*x+c) / (1+\cos(d*x+c)))^{(1/2)} * ((b+a*\cos(d*x+c)) / (1+\cos(d*x+c))) / (a+b) \\
&)^{(1/2)} * \sin(d*x+c) * a - 18*A * \sin(d*x+c) * \cos(d*x+c)^2 * (\cos(d*x+c) / (1+\cos(d*x+c)) \\
&)^{(1/2)} * ((b+a*\cos(d*x+c)) / (1+\cos(d*x+c))) / (a+b)^{(1/2)} * \text{EllipticF}((-1+\cos(d*x \\
& +c)) / \sin(d*x+c), ((a-b)/(a+b))^{(1/2)}) * a^2 * b + 30*A * \text{EllipticPi}((-1+\cos(d*x+c)) / \\
& \sin(d*x+c), -1, ((a-b)/(a+b))^{(1/2)}) * \cos(d*x+c) * a^2 * (\cos(d*x+c) / (1+\cos(d*x+c) \\
&))^{(1/2)} * ((b+a*\cos(d*x+c)) / (1+\cos(d*x+c))) / (a+b)^{(1/2)} * \sin(d*x+c) * b - 18*A * \text{El \\
& lipticF}((-1+\cos(d*x+c)) / \sin(d*x+c), ((a-b)/(a+b))^{(1/2)}) * \cos(d*x+c) * a^2 * (\cos \\
& (d*x+c) / (1+\cos(d*x+c)))^{(1/2)} * ((b+a*\cos(d*x+c)) / (1+\cos(d*x+c))) / (a+b)^{(1/2)} \\
& * \sin(d*x+c) * b + 18*A * \text{EllipticF}((-1+\cos(d*x+c)) / \sin(d*x+c), ((a-b)/(a+b))^{(1/2)} \\
&) * \cos(d*x+c) * b^2 * (\cos(d*x+c) / (1+\cos(d*x+c)))^{(1/2)} * ((b+a*\cos(d*x+c)) / (1+\cos \\
& (d*x+c))) / (a+b)^{(1/2)} * \sin(d*x+c) * a + 3*A * \text{EllipticE}((-1+\cos(d*x+c)) / \sin(d*x+c) \\
& , ((a-b)/(a+b))^{(1/2)}) * \cos(d*x+c) * a^2 * (\cos(d*x+c) / (1+\cos(d*x+c)))^{(1/2)} * ((b+ \\
& a*\cos(d*x+c)) / (1+\cos(d*x+c))) / (a+b)^{(1/2)} * \sin(d*x+c) * b - 6*A * (\cos(d*x+c) / (1+c \\
& os(d*x+c)))^{(1/2)} * ((b+a*\cos(d*x+c)) / (1+\cos(d*x+c))) / (a+b)^{(1/2)} * \text{EllipticE}((\\
& -1+\cos(d*x+c)) / \sin(d*x+c), ((a-b)/(a+b))^{(1/2)}) * \sin(d*x+c) * \cos(d*x+c) * a*b^2 + \\
& 18*B * (\cos(d*x+c) / (1+\cos(d*x+c)))^{(1/2)} * ((b+a*\cos(d*x+c)) / (1+\cos(d*x+c))) / (a+ \\
& b))^{(1/2)} * \text{EllipticF}((-1+\cos(d*x+c)) / \sin(d*x+c), ((a-b)/(a+b))^{(1/2)}) * \sin(d*x \\
& +c) * \cos(d*x+c) * a^2 * b - 14*B * \text{EllipticE}((-1+\cos(d*x+c)) / \sin(d*x+c), ((a-b)/(a+b) \\
&)^{(1/2)}) * \cos(d*x+c) * a^2 * (\cos(d*x+c) / (1+\cos(d*x+c)))^{(1/2)} * ((b+a*\cos(d*x+c)) \\
& / (1+\cos(d*x+c))) / (a+b)^{(1/2)} * \sin(d*x+c) * b - 6*B * \cos(d*x+c)^2 * (\cos(d*x+c) / (1+c \\
& os(d*x+c)))^{(1/2)} * ((b+a*\cos(d*x+c)) / (1+\cos(d*x+c))) / (a+b)^{(1/2)} * \text{EllipticF}((\\
& -1+\cos(d*x+c)) / \sin(d*x+c), ((a-b)/(a+b))^{(1/2)}) * \sin(d*x+c) * a^3 + 6*A * \sin(d*x+c)
\end{aligned}$$

)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3+2*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3+6*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3+2*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3/sin(d*x+c)^5/(b+a*cos(d*x+c))/cos(d*x+c)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(b \sec(dx + c) + a)^{\frac{5}{2}} \cos(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)*cos(d*x + c), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c + dx) \left(A + \frac{B}{\cos(c + dx)} \right) \left(a + \frac{b}{\cos(c + dx)} \right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(5/2),x)

[Out] int(cos(c + d*x)*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(5/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)),x)

[Out] Timed out

$$3.368 \quad \int \cos^2(c+dx)(a+b \sec(c+dx))^{5/2}(A+B \sec(c+dx)) dx$$

Optimal. Leaf size=450

$$\frac{\sqrt{a+b} \left(2a^2(A+2B) + 3ab(3A+8B) + 8b^2(A-B) \right) \cot(c+dx) \sqrt{\frac{b(1-\sec(c+dx))}{a+b}} \sqrt{\frac{b(\sec(c+dx)+1)}{a-b}} F\left(\sin^{-1}\left(\frac{\sqrt{a+b}\sec(c+dx)}{\sqrt{a+b}}\right)\right)}{4d}$$

[Out] $1/2*a*A*\cos(d*x+c)*(a+b*\sec(d*x+c))^{(3/2)}*\sin(d*x+c)/d+1/4*(a-b)*(9*A*a*b+4*B*a^2-8*B*b^2)*\cot(d*x+c)*\text{EllipticE}((a+b*\sec(d*x+c))^{(1/2)}/(a+b)^{(1/2)},((a+b)/(a-b))^{(1/2)}*(a+b)^{(1/2)}*(b*(1-\sec(d*x+c)))/(a+b)^{(1/2)}*(-b*(1+\sec(d*x+c)))/(a-b))^{(1/2)}/b/d+1/4*(8*b^2*(A-B)+2*a^2*(A+2*B)+3*a*b*(3*A+8*B))*\cot(d*x+c)*\text{EllipticF}((a+b*\sec(d*x+c))^{(1/2)}/(a+b)^{(1/2)},((a+b)/(a-b))^{(1/2)}*(a+b)^{(1/2)}*(b*(1-\sec(d*x+c)))/(a+b)^{(1/2)}*(-b*(1+\sec(d*x+c)))/(a-b))^{(1/2)}/d-1/4*(4*A*a^2+15*A*b^2+20*B*a*b)*\cot(d*x+c)*\text{EllipticPi}((a+b*\sec(d*x+c))^{(1/2)}/(a+b)^{(1/2)},(a+b)/a,((a+b)/(a-b))^{(1/2)}*(a+b)^{(1/2)}*(b*(1-\sec(d*x+c)))/(a+b)^{(1/2)}*(-b*(1+\sec(d*x+c)))/(a-b))^{(1/2)}/d+1/4*a*(7*A*b+4*B*a)*\sin(d*x+c)*(a+b*\sec(d*x+c))^{(1/2)}/d$

Rubi [A] time = 0.83, antiderivative size = 450, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {4025, 4094, 4058, 3921, 3784, 3832, 4004}

$$\frac{\sqrt{a+b} \left(2a^2(A+2B) + 3ab(3A+8B) + 8b^2(A-B) \right) \cot(c+dx) \sqrt{\frac{b(1-\sec(c+dx))}{a+b}} \sqrt{\frac{b(\sec(c+dx)+1)}{a-b}} F\left(\sin^{-1}\left(\frac{\sqrt{a+b}\sec(c+dx)}{\sqrt{a+b}}\right)\right)}{4d}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]

[Out] $((a-b)*\text{Sqrt}[a+b]*(9*a*A*b+4*a^2*B-8*b^2*B)*\text{Cot}[c+d*x]*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[a+b*\text{Sec}[c+d*x]]/\text{Sqrt}[a+b]],(a+b)/(a-b)]*\text{Sqrt}[(b*(1-\text{Sec}[c+d*x]))/(a+b)]*\text{Sqrt}[-((b*(1+\text{Sec}[c+d*x]))/(a-b))]/(4*b*d)+(\text{Sqrt}[a+b]*(8*b^2*(A-B)+2*a^2*(A+2*B)+3*a*b*(3*A+8*B))*\text{Cot}[c+d*x]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[a+b*\text{Sec}[c+d*x]]/\text{Sqrt}[a+b]],(a+b)/(a-b)]*\text{Sqrt}[(b*(1-\text{Sec}[c+d*x]))/(a+b)]*\text{Sqrt}[-((b*(1+\text{Sec}[c+d*x]))/(a-b))]/(4*d)-(\text{Sqrt}[a+b]*(4*a^2*A+15*A*b^2+20*a*b*B)*\text{Cot}[c+d*x]*\text{EllipticPi}[(a+b)/a,\text{ArcSin}[\text{Sqrt}[a+b*\text{Sec}[c+d*x]]/\text{Sqrt}[a+b]],(a+b)/(a-b)]*\text{Sqrt}[(b*(1-\text{Sec}[c+d*x]))/(a+b)]*\text{Sqrt}[-((b*(1+\text{Sec}[c+d*x]))/(a-b))]/(4*d)+(a*(7*A*b+4*a*B)*\text{Sqrt}[a+b*\text{Sec}[c+d*x]]*\text{Sin}[c+d*x]/(4*d)+(a*A*\text{Cos}[c+d*x]*(a+b*\text{Sec}[c+d*x])^{(3/2)}*\text{Sin}[c+d*x])/(2*d)$

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] := Simp[(2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)]/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] := Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)]/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] := Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] := Simp[(-2*(A*b - a*B)*Rt[a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A, 2]], (a*A + b*B)/(a*A - b*B)]/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^m*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] := Simp[(a*A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e + f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LeQ[n, -1]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] := Int[(A + (B - C
```

) * Csc[e + f*x]) / Sqrt[a + b * Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x] * (1 + Csc[e + f*x])) / Sqrt[a + b * Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0]

Rule 4094

Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.)) * (csc[(e_.) + (f_.)*(x_.)]*(d_.))^n * (csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m, x_Symbol] :> Simp[(A * Cot[e + f*x] * (a + b * Csc[e + f*x])^m * (d * Csc[e + f*x])^n) / (f * n), x] - Dist[1 / (d * n), Int[(a + b * Csc[e + f*x])^(m - 1) * (d * Csc[e + f*x])^(n + 1) * Simp[A * b * m - a * B * n - (b * B * n + a * (C * n + A * (n + 1))) * Csc[e + f*x] - b * (C * n + A * (m + n + 1)) * Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]

Rubi steps

$$\begin{aligned} \int \cos^2(c + dx)(a + b \sec(c + dx))^{5/2}(A + B \sec(c + dx)) dx &= \frac{aA \cos(c + dx)(a + b \sec(c + dx))^{3/2} \sin(c + dx)}{2d} - \frac{aA \sin(c + dx)}{2d} \\ &= \frac{a(7Ab + 4aB)\sqrt{a + b \sec(c + dx)} \sin(c + dx)}{4d} + \frac{aA \sin(c + dx)}{2d} \\ &= \frac{a(7Ab + 4aB)\sqrt{a + b \sec(c + dx)} \sin(c + dx)}{4d} + \frac{aA \sin(c + dx)}{2d} \\ &= \frac{(a - b)\sqrt{a + b} (9aAb + 4a^2B - 8b^2B) \cot(c + dx)E}{4d} \\ &= \frac{(a - b)\sqrt{a + b} (9aAb + 4a^2B - 8b^2B) \cot(c + dx)E}{4d} \end{aligned}$$

Mathematica [B] time = 19.79, size = 1326, normalized size = 2.95

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]

[Out] (Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2)*(2*b^2*B*Sin[c + d*x] + (a^2*A*Sin[2*(c + d*x)]/4))/(d*(b + a*Cos[c + d*x])^2) + ((a + b*Sec[c + d*x])^(5/2)*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(9*a^2*A*b*Tan[(c + d*x)/2] + 9*a*A*


```

b^2*Tan[(c + d*x)/2] + 4*a^3*B*Tan[(c + d*x)/2] + 4*a^2*b*B*Tan[(c + d*x)/2]
] - 8*a*b^2*B*Tan[(c + d*x)/2] - 8*b^3*B*Tan[(c + d*x)/2] - 18*a^2*A*b*Tan[
(c + d*x)/2]^3 - 8*a^3*B*Tan[(c + d*x)/2]^3 + 16*a*b^2*B*Tan[(c + d*x)/2]^3
+ 9*a^2*A*b*Tan[(c + d*x)/2]^5 - 9*a*A*b^2*Tan[(c + d*x)/2]^5 + 4*a^3*B*Ta
n[(c + d*x)/2]^5 - 4*a^2*b*B*Tan[(c + d*x)/2]^5 - 8*a*b^2*B*Tan[(c + d*x)/2
]^5 + 8*b^3*B*Tan[(c + d*x)/2]^5 + 8*a^3*A*EllipticPi[-1, ArcSin[Tan[(c + d
*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[
(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 30*a*A*b^2*EllipticPi[-1,
ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sq
rt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 40*a^2*
b*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[
(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/
(a + b)] + 8*a^3*A*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b
)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c +
d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 30*a*A*b^2*EllipticPi[-1, ArcS
in[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c +
d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a +
b)] + 40*a^2*b*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]
*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d
*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (a + b)*(9*a*A*b + 4*a^2*B - 8*
b^2*B)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c
+ d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 +
b*Tan[(c + d*x)/2]^2)/(a + b)] - 2*(2*a^3*A - a^2*b*(A - 12*B) + 12*a*b^2*
(A - B) - 4*b^3*(A + B))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b
)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Ta
n[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)))/(4*d*(b + a*Cos[c + d*x
])^(5/2)*Sec[c + d*x]^(5/2)*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b - a*
Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2))]

```

fricas [F] time = 56.18, size = 0, normalized size = 0.00

integral((Bb^2 cos(dx + c)^2 sec(dx + c)^3 + Aa^2 cos(dx + c)^2 + (2 Bab + Ab^2) cos(dx + c)^2 sec(dx + c)^2 + (Ba

Verification of antiderivative is not currently implemented for this CAS.

```

[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm
="fricas")

```

```

[Out] integral((B*b^2*cos(d*x + c)^2*sec(d*x + c)^3 + A*a^2*cos(d*x + c)^2 + (2*B
*a*b + A*b^2)*cos(d*x + c)^2*sec(d*x + c)^2 + (B*a^2 + 2*A*a*b)*cos(d*x + c
)^2*sec(d*x + c))*sqrt(b*sec(d*x + c) + a), x)

```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(b \sec(dx + c) + a)^{\frac{5}{2}} \cos(dx + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^2, x
)
```

maple [B] time = 2.42, size = 3271, normalized size = 7.27

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] 1/4/d*(-1+cos(d*x+c))^2*(-8*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a
-b)/(a+b))^(1/2))*b^3*sin(d*x+c)+2*A*cos(d*x+c)^2*a^3-4*B*cos(d*x+c)^3*a^3+
8*b^3*B+4*B*cos(d*x+c)^2*a^3+9*A*cos(d*x+c)^2*a^2*b-9*A*cos(d*x+c)^2*a*b^2+
2*A*cos(d*x+c)*a^2*b+9*A*cos(d*x+c)*a*b^2-4*B*cos(d*x+c)^2*a^2*b-8*B*cos(d*
x+c)^2*a*b^2+4*B*cos(d*x+c)*a^2*b+8*B*cos(d*x+c)*a*b^2-2*A*cos(d*x+c)^4*a^3
-11*A*cos(d*x+c)^3*a^2*b-4*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-
b)/(a+b))^(1/2))*a^3*sin(d*x+c)+8*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)+4*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),((a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)-8*A*(cos(d*x+c)/(1+cos(d*x+c)
))^^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)-8*A*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-
1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)-2*A*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)+24
*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2*sin
(d*x+c)-9*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*
a^2*b*sin(d*x+c)-9*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b)
)^(1/2))*a*b^2*sin(d*x+c)+24*B*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/
(a+b))^(1/2))*a^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*b-4*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((
b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),((a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)+8*B*(cos(d*x+c)/(1+cos(d*x+c))
```


)²*(b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)⁵

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(b \sec(dx + c) + a)^{\frac{5}{2}} \cos(dx + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)²*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)², x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c + dx)^2 \left(A + \frac{B}{\cos(c + dx)} \right) \left(a + \frac{b}{\cos(c + dx)} \right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)²*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(5/2),x)

[Out] int(cos(c + d*x)²*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(5/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**2*(a+b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)),x)

[Out] Timed out

$$3.369 \quad \int \cos^3(c+dx)(a+b \sec(c+dx))^{5/2}(A+B \sec(c+dx)) dx$$

Optimal. Leaf size=518

$$\frac{(16a^2A + 54abB + 33Ab^2) \sin(c+dx) \sqrt{a+b \sec(c+dx)}}{24d} + \frac{\sqrt{a+b} (16a^2A + 12a^2B + 26aAb + 54abB + 33Ab^2)}{24d}$$

[Out] $\frac{1}{3} a A \cos(dx+c)^2 (a+b \sec(dx+c))^{3/2} \sin(dx+c) / d + \frac{1}{24} (a-b) (16 A a^2 + 33 A b^2 + 54 B a b) \cot(dx+c) \operatorname{EllipticE}((a+b \sec(dx+c))^{1/2} / (a+b)^{1/2}, ((a+b)/(a-b))^{1/2}) (a+b)^{1/2} (b(1-\sec(dx+c)) / (a+b))^{1/2} (-b(1+\sec(dx+c)) / (a-b))^{1/2} / b + \frac{1}{24} (16 A a^2 + 26 A a b + 33 A b^2 + 12 B a^2 + 54 B a b + 48 B b^2) \cot(dx+c) \operatorname{EllipticF}((a+b \sec(dx+c))^{1/2} / (a+b)^{1/2}, ((a+b)/(a-b))^{1/2}) (a+b)^{1/2} (b(1-\sec(dx+c)) / (a+b))^{1/2} (-b(1+\sec(dx+c)) / (a-b))^{1/2} / d - \frac{1}{8} (20 A a^2 b + 5 A a b^3 + 8 B a^3 + 30 B a b^2) \cot(dx+c) \operatorname{EllipticPi}((a+b \sec(dx+c))^{1/2} / (a+b)^{1/2}, (a+b)/a, ((a+b)/(a-b))^{1/2}) (a+b)^{1/2} (b(1-\sec(dx+c)) / (a+b))^{1/2} (-b(1+\sec(dx+c)) / (a-b))^{1/2} / a + \frac{1}{24} (16 A a^2 + 33 A b^2 + 54 B a b) \sin(dx+c) (a+b \sec(dx+c))^{1/2} / d + \frac{1}{4} a (3 A b + 2 B a) \cos(dx+c) \sin(dx+c) (a+b \sec(dx+c))^{1/2} / d$

Rubi [A] time = 1.37, antiderivative size = 518, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {4025, 4094, 4104, 4058, 3921, 3784, 3832, 4004}

$$\frac{(16a^2A + 54abB + 33Ab^2) \sin(c+dx) \sqrt{a+b \sec(c+dx)}}{24d} + \frac{\sqrt{a+b} (16a^2A + 12a^2B + 26aAb + 54abB + 33Ab^2)}{24d}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]

[Out] $((a-b) \operatorname{Sqrt}[a+b] (16 a^2 A + 33 A b^2 + 54 a b B) \operatorname{Cot}[c+d*x] \operatorname{EllipticE}[\operatorname{ArcSin}[\operatorname{Sqrt}[a+b \operatorname{Sec}[c+d*x]]] / \operatorname{Sqrt}[a+b], (a+b)/(a-b)] \operatorname{Sqrt}[(b(1-\operatorname{Sec}[c+d*x])) / (a+b)] \operatorname{Sqrt}[-((b(1+\operatorname{Sec}[c+d*x])) / (a-b))]) / (24 b d) + (\operatorname{Sqrt}[a+b] (16 a^2 A + 26 a A b + 33 A b^2 + 12 a^2 B + 54 a b B + 48 b^2 B) \operatorname{Cot}[c+d*x] \operatorname{EllipticF}[\operatorname{ArcSin}[\operatorname{Sqrt}[a+b \operatorname{Sec}[c+d*x]]] / \operatorname{Sqrt}[a+b], (a+b)/(a-b)] \operatorname{Sqrt}[(b(1-\operatorname{Sec}[c+d*x])) / (a+b)] \operatorname{Sqrt}[-((b(1+\operatorname{Sec}[c+d*x])) / (a-b))]) / (24 d) - (\operatorname{Sqrt}[a+b] (20 a^2 A b + 5 A b^3 + 8 a^3 B + 30 a b^2 B) \operatorname{Cot}[c+d*x] \operatorname{EllipticPi}[(a+b)/a, \operatorname{ArcSin}[\operatorname{Sqrt}[a+b \operatorname{Sec}[c+d*x]]] / \operatorname{Sqrt}[a+b], (a+b)/(a-b)] \operatorname{Sqrt}[(b(1-\operatorname{Sec}[c+d*x])) / (a+b)] \operatorname{Sqrt}[-((b(1+\operatorname{Sec}[c+d*x])) / (a-b))]) / (8 a d) + ((16 a^2 A + 33 A b^2 + 54 a b B) \operatorname{Sqrt}[a+b \operatorname{Sec}[c+d*x]] \operatorname{Sin}[c+d*x]) / (24 d) + (a (3 A b + 2 a B$

) * Cos[c + d*x] * Sqrt[a + b*Sec[c + d*x]] * Sin[c + d*x] / (4*d) + (a*A*Cos[c + d*x]^2 * (a + b*Sec[c + d*x])^(3/2) * Sin[c + d*x]) / (3*d)

Rule 3784

Int[1/Sqrt[csc[(c_.) + (d_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] := Simp[(2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)]/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 3832

Int[csc[(e_.) + (f_.)*(x_.)]/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] := Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)]/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 3921

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] := Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]

Rule 4004

Int[(csc[(e_.) + (f_.)*(x_.)]*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)))/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] := Simp[(-2*(A*b - a*B)*Rt[a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A, 2]], (a*A + b*B)/(a*A - b*B)]/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]

Rule 4025

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := Simp[(a*A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n))]*Csc[e + f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LeQ[n, -1]

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)])*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] := Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
+ Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)])*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^m, x_Symbol] := Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
+ f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)])*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^m, x_Symbol] := Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \cos^3(c + dx)(a + b \sec(c + dx))^{5/2}(A + B \sec(c + dx)) dx &= \frac{aA \cos^2(c + dx)(a + b \sec(c + dx))^{3/2} \sin(c + dx)}{3d} \\
&= \frac{a(3Ab + 2aB) \cos(c + dx) \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{4d} \\
&= \frac{(16a^2A + 33Ab^2 + 54abB) \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{24d} \\
&= \frac{(16a^2A + 33Ab^2 + 54abB) \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{24d} \\
&= \frac{(a - b) \sqrt{a + b} (16a^2A + 33Ab^2 + 54abB) \cot(c + dx)}{24d} \\
&= \frac{(a - b) \sqrt{a + b} (16a^2A + 33Ab^2 + 54abB) \cot(c + dx)}{24d}
\end{aligned}$$

Mathematica [B] time = 19.53, size = 1551, normalized size = 2.99

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]

[Out] (Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2)*((a^2*A*Sin[c + d*x])/12 + (a*(13*A*b + 6*a*B)*Sin[2*(c + d*x)]/24 + (a^2*A*Sin[3*(c + d*x)]/12)))/(d*(b + a*Cos[c + d*x])^2) + ((a + b*Sec[c + d*x])^(5/2)*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(16*a^3*A*Tan[(c + d*x)/2] + 16*a^2*A*b*Tan[(c + d*x)/2] + 33*a*A*b^2*Tan[(c + d*x)/2] + 33*A*b^3*Tan[(c + d*x)/2] + 54*a^2*b*B*Tan[(c + d*x)/2] + 54*a*b^2*B*Tan[(c + d*x)/2] - 32*a^3*A*Tan[(c + d*x)/2]^3 - 66*a*A*b^2*Tan[(c + d*x)/2]^3 - 108*a^2*b*B*Tan[(c + d*x)/2]^3 + 16*a^3*A*Tan[(c + d*x)/2]^5 - 16*a^2*A*b*Tan[(c + d*x)/2]^5 + 33*a*A*b^2*Tan[(c + d*x)/2]^5 - 33*A*b^3*Tan[(c + d*x)/2]^5 + 54*a^2*b*B*Tan[(c + d*x)/2]^5 - 54*a*b^2*B*Tan[(c + d*x)/2]^5 + 120*a^2*A*b*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 30*A*b^3*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 48*a^3*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2])


```

]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 180
*a*b^2*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 -
Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2
]^2)/(a + b)] + 120*a^2*A*b*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b
)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*
Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 30*A*b^3*EllipticPi[-
1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - T
an[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^
2)/(a + b)] + 48*a^3*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a
+ b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(
c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 180*a*b^2*B*EllipticPi[-1,
ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan
[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)
/(a + b)] + (a + b)*(16*a^2*A + 33*A*b^2 + 54*a*b*B)*EllipticE[ArcSin[Tan[(
c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d
*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b
)] - 2*(24*b^3*(A - B) + 12*a^3*B + a*b^2*(-13*A + 72*B) + a^2*(38*A*b - 6*
b*B))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c
+ d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 +
b*Tan[(c + d*x)/2]^2)/(a + b)))/(24*d*(b + a*Cos[c + d*x])^(5/2)*Sec[c + d
*x]^(5/2)*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2
+ b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)])

```

fricas [F] time = 63.48, size = 0, normalized size = 0.00

integral((Bb² cos(dx + c)³ sec(dx + c)³ + Aa² cos(dx + c)³ + (2 Bab + Ab²) cos(dx + c)³ sec(dx + c)² + (Ba

Verification of antiderivative is not currently implemented for this CAS.

```

[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm
="fricas")

```

```

[Out] integral((B*b^2*cos(d*x + c)^3*sec(d*x + c)^3 + A*a^2*cos(d*x + c)^3 + (2*B
*a*b + A*b^2)*cos(d*x + c)^3*sec(d*x + c)^2 + (B*a^2 + 2*A*a*b)*cos(d*x + c
)^3*sec(d*x + c))*sqrt(b*sec(d*x + c) + a), x)

```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(b \sec(dx + c) + a)^{\frac{5}{2}} \cos(dx + c)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```

[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm
="giac")

```

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^3, x)

maple [B] time = 2.24, size = 3511, normalized size = 6.78

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^3*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x)

[Out] $\frac{1}{24}d \cdot (-1 + \cos(dx+c))^2 \cdot (-16A \cdot (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} \cdot ((b+a \cdot \cos(dx+c))/(1+\cos(dx+c))/(a+b))^{1/2} \cdot \text{EllipticE}((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{1/2}) \cdot a^3 \cdot \sin(dx+c) - 48B \cdot (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} \cdot ((b+a \cdot \cos(dx+c))/(1+\cos(dx+c))/(a+b))^{1/2} \cdot \text{EllipticF}((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{1/2}) \cdot b^3 \cdot \sin(dx+c) - 8A \cdot \cos(dx+c)^3 \cdot a^3 + 16A \cdot \cos(dx+c)^2 \cdot a^3 - 33A \cdot \cos(dx+c)^2 \cdot b^3 + 12B \cdot \cos(dx+c)^2 \cdot a^3 - 59A \cdot \cos(dx+c)^3 \cdot a \cdot b^2 + 18A \cdot \cos(dx+c)^2 \cdot a^2 \cdot b + 33A \cdot \cos(dx+c)^2 \cdot a \cdot b^2 + 16A \cdot \cos(dx+c) \cdot a^2 \cdot b + 26A \cdot \cos(dx+c) \cdot a \cdot b^2 - 66B \cdot \cos(dx+c)^3 \cdot a^2 \cdot b + 54B \cdot \cos(dx+c)^2 \cdot a^2 \cdot b - 54B \cdot \cos(dx+c)^2 \cdot a \cdot b^2 + 12B \cdot \cos(dx+c) \cdot a^2 \cdot b + 54B \cdot \cos(dx+c) \cdot a \cdot b^2 + 33A \cdot \cos(dx+c) \cdot b^3 - 34A \cdot \cos(dx+c)^4 \cdot a^2 \cdot b - 33A \cdot (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} \cdot ((b+a \cdot \cos(dx+c))/(1+\cos(dx+c))/(a+b))^{1/2} \cdot \text{EllipticE}((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{1/2}) \cdot b^3 \cdot \sin(dx+c) - 12B \cdot \cos(dx+c)^4 \cdot a^3 + 48A \cdot (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} \cdot ((b+a \cdot \cos(dx+c))/(1+\cos(dx+c))/(a+b))^{1/2} \cdot \text{EllipticF}((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{1/2}) \cdot b^3 \cdot \sin(dx+c) + 24B \cdot (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} \cdot ((b+a \cdot \cos(dx+c))/(1+\cos(dx+c))/(a+b))^{1/2} \cdot \text{EllipticF}((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{1/2}) \cdot \sin(dx+c) \cdot \cos(dx+c) \cdot a^3 - 120A \cdot (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} \cdot ((b+a \cdot \cos(dx+c))/(1+\cos(dx+c))/(a+b))^{1/2} \cdot \text{EllipticPi}((-1+\cos(dx+c))/\sin(dx+c), -1, ((a-b)/(a+b))^{1/2}) \cdot a^2 \cdot b \cdot \sin(dx+c) + 76A \cdot (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} \cdot ((b+a \cdot \cos(dx+c))/(1+\cos(dx+c))/(a+b))^{1/2} \cdot \text{EllipticF}((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{1/2}) \cdot a^2 \cdot b \cdot \sin(dx+c) - 26A \cdot (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} \cdot ((b+a \cdot \cos(dx+c))/(1+\cos(dx+c))/(a+b))^{1/2} \cdot \text{EllipticF}((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{1/2}) \cdot a \cdot b^2 \cdot \sin(dx+c) - 16A \cdot (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} \cdot ((b+a \cdot \cos(dx+c))/(1+\cos(dx+c))/(a+b))^{1/2} \cdot \text{EllipticE}((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{1/2}) \cdot a^2 \cdot b \cdot \sin(dx+c) - 33A \cdot (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} \cdot ((b+a \cdot \cos(dx+c))/(1+\cos(dx+c))/(a+b))^{1/2} \cdot \text{EllipticE}((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{1/2}) \cdot a \cdot b^2 \cdot \sin(dx+c) - 180B \cdot (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} \cdot ((b+a \cdot \cos(dx+c))/(1+\cos(dx+c))/(a+b))^{1/2} \cdot \text{EllipticPi}((-1+\cos(dx+c))/\sin(dx+c), -1, ((a-b)/(a+b))^{1/2}) \cdot a \cdot b^2 \cdot \sin(dx+c) - 12B \cdot \text{EllipticF}((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{1/2}) \cdot a^2 \cdot (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} \cdot ((b+a \cdot \cos(dx+c))/(1+\cos(dx+c))/(a+b))^{1/2} \cdot \sin(dx+c) \cdot b - 54B \cdot (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} \cdot ((b+a \cdot \cos(dx+c))/(1+\cos(dx+c))/(a+b))^{1/2} \cdot \text{EllipticE}((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{1/2}) \cdot a^2 \cdot b \cdot \sin(dx+c) - 54B \cdot (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} \cdot ((b+a \cdot \cos(dx+c))/(1+\cos(dx+c))/(a+b))^{1/2} \cdot \text{EllipticE}((-1+\cos(dx+c))/\sin(dx+c),$

)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3*(1+cos(d*x+c))^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)^5

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(b \sec(dx + c) + a)^{\frac{5}{2}} \cos(dx + c)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^3, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c + dx)^3 \left(A + \frac{B}{\cos(c + dx)} \right) \left(a + \frac{b}{\cos(c + dx)} \right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^3*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(5/2),x)

[Out] int(cos(c + d*x)^3*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(5/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**3*(a+b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)),x)

[Out] Timed out

$$3.370 \quad \int \cos^4(c+dx)(a+b \sec(c+dx))^{5/2}(A+B \sec(c+dx)) dx$$

Optimal. Leaf size=617

$$\frac{(36a^2A + 104abB + 59Ab^2) \sin(c + dx) \cos(c + dx) \sqrt{a + b \sec(c + dx)}}{96d} + \frac{(128a^3B + 284a^2Ab + 264ab^2B + 15Aa^2b^3 + 128Ba^3 + 264Bab^2 + 15Aa^2b^3 + 128Ba^3 + 264Bab^2 + 15Aa^2b^3) \sin(c + dx) \cos(c + dx) \sqrt{a + b \sec(c + dx)}}{192ad}$$

[Out] $\frac{1}{4}aA \cos(dx+c)^3 (a+b \sec(dx+c))^{3/2} \sin(dx+c)/d + \frac{1}{192}(a-b) (284Aa^2b + 15Aa^2b^3 + 128Ba^3 + 264Bab^2) \cot(dx+c) \text{EllipticE}((a+b \sec(dx+c))^{1/2}/(a+b)^{1/2}, ((a+b)/(a-b))^{1/2}) (a+b)^{1/2} (b(1-\sec(dx+c)))/(a+b)^{1/2} (-b(1+\sec(dx+c)))/(a-b))^{1/2} / a/b/d + \frac{1}{192} (15Aa^2b^3 + 8a^3(9A+16B) + 4a^2b(71A+52B) + 2a^2b^2(59A+132B)) \cot(dx+c) \text{EllipticF}((a+b \sec(dx+c))^{1/2}/(a+b)^{1/2}, ((a+b)/(a-b))^{1/2}) (a+b)^{1/2} (b(1-\sec(dx+c)))/(a+b)^{1/2} (-b(1+\sec(dx+c)))/(a-b))^{1/2} / a/d - \frac{1}{64} (48Aa^4 + 120Aa^2b^2 - 5Aa^2b^4 + 160Ba^3b + 40Ba^2b^3) \cot(dx+c) \text{EllipticPi}((a+b \sec(dx+c))^{1/2}/(a+b)^{1/2}, (a+b)/a, ((a+b)/(a-b))^{1/2}) (a+b)^{1/2} (b(1-\sec(dx+c)))/(a+b)^{1/2} (-b(1+\sec(dx+c)))/(a-b))^{1/2} / a^2/d + \frac{1}{192} (284Aa^2b + 15Aa^2b^3 + 128Ba^3 + 264Bab^2) \sin(dx+c) (a+b \sec(dx+c))^{1/2} / a/d + \frac{1}{96} (36Aa^2 + 59Aa^2b + 104Bab) \cos(dx+c) \sin(dx+c) (a+b \sec(dx+c))^{1/2} / d + \frac{1}{24} a (11Aa^2b + 8Ba^2) \cos(dx+c)^2 \sin(dx+c) (a+b \sec(dx+c))^{1/2} / d$

Rubi [A] time = 1.83, antiderivative size = 617, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {4025, 4094, 4104, 4058, 3921, 3784, 3832, 4004}

$$\frac{(284a^2Ab + 128a^3B + 264ab^2B + 15Ab^3) \sin(c + dx) \sqrt{a + b \sec(c + dx)}}{192ad} + \frac{(36a^2A + 104abB + 59Ab^2) \sin(c + dx) \cos(c + dx) \sqrt{a + b \sec(c + dx)}}{96d}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]

[Out] $((a-b) \sqrt{a+b} (284a^2A + 15Aa^2b^3 + 128a^3B + 264a^2b^2B) \text{Cot}[c+dx] \text{EllipticE}[\text{ArcSin}[\sqrt{a+b \sec[c+dx]}/\sqrt{a+b}], (a+b)/(a-b)] \sqrt{(b(1-\sec[c+dx]))/(a+b)} \sqrt{-((b(1+\sec[c+dx]))/(a-b))}) / (192a^2bd) + (\sqrt{a+b} (15Aa^2b^3 + 8a^3(9A+16B) + 4a^2b(71A+52B) + 2a^2b^2(59A+132B)) \text{Cot}[c+dx] \text{EllipticF}[\text{ArcSin}[\sqrt{a+b \sec[c+dx]}/\sqrt{a+b}], (a+b)/(a-b)] \sqrt{(b(1-\sec[c+dx]))/(a+b)} \sqrt{-((b(1+\sec[c+dx]))/(a-b))}) / (192a^2d) - (\sqrt{a+b} (48a^4A + 120a^2Aa^2b^2 - 5Aa^2b^4 + 160a^3b^2B + 40a^2b^3B) \text{Cot}[c+dx] \text{EllipticPi}[(a+b)/a, \text{ArcSin}[\sqrt{a+b \sec[c+dx]}/\sqrt{a+b}], (a+b)/(a-b)] \sqrt{(b(1-\sec[c+dx]))/(a+b)} \sqrt{-((b(1+\sec[c+dx]))/(a+b))}) / (192a^2d)$

$$\frac{[c + d*x])]/(a - b))]/(64*a^2*d) + ((284*a^2*A*b + 15*A*b^3 + 128*a^3*B + 264*a*b^2*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(192*a*d) + ((36*a^2*A + 59*A*b^2 + 104*a*b*B)*Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(96*d) + (a*(11*A*b + 8*a*B)*Cos[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(24*d) + (a*A*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(4*d)$$

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_.)], x_Symbol] := Simp[(2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)]/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] := Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)]/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] := Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] := Simp[(-2*(A*b - a*B)*Rt[a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A, 2]], (a*A + b*B)/(a*A - b*B)]/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^m*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] := Simp[(a*A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n))]*Csc[e +
```

```
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
LeQ[n, -1]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
+ Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a
_))^m, x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
+ f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a
_))^m, x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \cos^4(c + dx)(a + b \sec(c + dx))^{5/2}(A + B \sec(c + dx)) dx &= \frac{aA \cos^3(c + dx)(a + b \sec(c + dx))^{3/2} \sin(c + dx)}{4d} \\
&= \frac{a(11Ab + 8aB) \cos^2(c + dx) \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{24d} \\
&= \frac{(36a^2A + 59Ab^2 + 104abB) \cos(c + dx) \sqrt{a + b \sec(c + dx)}}{96d} \\
&= \frac{(284a^2Ab + 15Ab^3 + 128a^3B + 264ab^2B) \sqrt{a + b \sec(c + dx)}}{192ad} \\
&= \frac{(284a^2Ab + 15Ab^3 + 128a^3B + 264ab^2B) \sqrt{a + b \sec(c + dx)}}{192ad} \\
&= \frac{(a - b) \sqrt{a + b} (284a^2Ab + 15Ab^3 + 128a^3B + 264ab^2B)}{192ad} \\
&= \frac{(a - b) \sqrt{a + b} (284a^2Ab + 15Ab^3 + 128a^3B + 264ab^2B)}{192ad}
\end{aligned}$$

Mathematica [B] time = 24.27, size = 5172, normalized size = 8.38

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]), x]

[Out] Result too large to show

fricas [F] time = 65.09, size = 0, normalized size = 0.00

integral((Bb^2 cos(dx + c)^4 sec(dx + c)^3 + Aa^2 cos(dx + c)^4 + (2 Bab + Ab^2) cos(dx + c)^4 sec(dx + c)^2 + (Ba^2

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)), x, algorithm="fricas")

[Out] integral((B*b^2*cos(d*x + c)^4*sec(d*x + c)^3 + A*a^2*cos(d*x + c)^4 + (2*B*a*b + A*b^2)*cos(d*x + c)^4*sec(d*x + c)^2 + (B*a^2 + 2*A*a*b)*cos(d*x + c)^4*sec(d*x + c))*sqrt(b*sec(d*x + c) + a), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(b \sec(dx + c) + a)^{\frac{5}{2}} \cos(dx + c)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^4, x)

maple [B] time = 2.28, size = 4231, normalized size = 6.86

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^4*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x)

[Out]
$$\begin{aligned} & -1/192/d*(-1+\cos(d*x+c))^2*(15*A*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*EllipticE((-1+\cos(d*x+c))/\sin(d*x+c), \\ & ((a-b)/(a+b))^{1/2})*b^4*\sin(d*x+c)+184*A*\cos(d*x+c)^5*a^3*b+254*A*\cos(d*x+c)^4*a^2*b^2+272*B*\cos(d*x+c)^4*a^3*b-264*B*\cos(d*x+c)^2*a^2*b^2+264*B*\cos(d*x+c)^2*a*b^3-128*B*\cos(d*x+c)*a^3*b-208*B*\cos(d*x+c)*a^2*b^2-264*B*\cos(d*x+c)*a*b^3-284*A*\cos(d*x+c)^2*a^3*b-15*A*\cos(d*x+c)^2*a*b^3-284*A*\cos(d*x+c)*a^2*b^2-118*A*\cos(d*x+c)*a*b^3+15*A*\cos(d*x+c)^2*b^4-128*B*\cos(d*x+c)^2*a^4+64*B*\cos(d*x+c)^3*a^4-72*A*\cos(d*x+c)^2*a^4-144*B*\cos(d*x+c)^2*a^3*b+172*A*\cos(d*x+c)^3*a^3*b+133*A*\cos(d*x+c)^3*a*b^3+30*A*\cos(d*x+c)^2*a^2*b^2-72*A*\cos(d*x+c)*a^3*b+472*B*\cos(d*x+c)^3*a^2*b^2-30*A*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*EllipticPi((-1+\cos(d*x+c))/\sin(d*x+c), -1, ((a-b)/(a+b))^{1/2})*b^4*\sin(d*x+c)+48*A*\cos(d*x+c)^6*a^4-30*A*\sin(d*x+c)*\cos(d*x+c)*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*EllipticPi((-1+\cos(d*x+c))/\sin(d*x+c), -1, ((a-b)/(a+b))^{1/2})*b^4+128*B*\sin(d*x+c)*\cos(d*x+c)*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*EllipticE((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{1/2})*a^4+284*A*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*EllipticE((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{1/2})*a^3*b*\sin(d*x+c)+284*A*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*EllipticE((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{1/2})*a^2*b^2*\sin(d*x+c)+15*A*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*EllipticE((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{1/2})*a*b^3*\sin(d*x+c)+72*A*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*EllipticF((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{1/2}) \end{aligned}$$

$$\begin{aligned}
& *a^3*b*\sin(d*x+c)-644*A*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x+c)) \\
& / (1+\cos(d*x+c))/(a+b))^{(1/2)}*EllipticF((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a \\
& +b))^{(1/2)})*a^2*b^2*\sin(d*x+c)+118*A*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((b+ \\
& a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{(1/2)}*EllipticF((-1+\cos(d*x+c))/\sin(d*x \\
& +c), ((a-b)/(a+b))^{(1/2)})*a*b^3*\sin(d*x+c)+720*A*(\cos(d*x+c)/(1+\cos(d*x+c))) \\
& ^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{(1/2)}*EllipticPi((-1+\cos(d*x \\
& +c))/\sin(d*x+c), -1, ((a-b)/(a+b))^{(1/2)})*a^2*b^2*\sin(d*x+c)+128*B*(\cos(d*x+c) \\
&)/(1+\cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{(1/2)}*Ellip \\
& ticE((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{(1/2)})*a^3*b*\sin(d*x+c)+264*B \\
& *(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{(\\
& 1/2)}*EllipticE((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{(1/2)})*a^2*b^2*\sin \\
& (d*x+c)+264*B*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d* \\
& x+c))/(a+b))^{(1/2)}*EllipticE((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{(1/2) \\
&)}*a*b^3*\sin(d*x+c)-608*B*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x+c) \\
&)/(1+\cos(d*x+c))/(a+b))^{(1/2)}*EllipticF((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(\\
& a+b))^{(1/2)})*a^3*b*\sin(d*x+c)+208*B*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((b+a \\
& *cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{(1/2)}*EllipticF((-1+\cos(d*x+c))/\sin(d*x+ \\
& c), ((a-b)/(a+b))^{(1/2)})*a^2*b^2*\sin(d*x+c)-384*B*(\cos(d*x+c)/(1+\cos(d*x+c)) \\
&)^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{(1/2)}*EllipticF((-1+\cos(d*x \\
& +c))/\sin(d*x+c), ((a-b)/(a+b))^{(1/2)})*a*b^3*\sin(d*x+c)+960*B*(\cos(d*x+c)/(1+ \\
& \cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{(1/2)}*EllipticPi \\
& ((-1+\cos(d*x+c))/\sin(d*x+c), -1, ((a-b)/(a+b))^{(1/2)})*a^3*b*\sin(d*x+c)+240*B* \\
& (\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{(\\
& 1/2)}*EllipticPi((-1+\cos(d*x+c))/\sin(d*x+c), -1, ((a-b)/(a+b))^{(1/2)})*a*b^3*si \\
& n(d*x+c)+64*B*\cos(d*x+c)^5*a^4+128*B*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((b+ \\
& a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{(1/2)}*EllipticE((-1+\cos(d*x+c))/\sin(d*x \\
& +c), ((a-b)/(a+b))^{(1/2)})*a^4*\sin(d*x+c)-144*A*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(\\
& 1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{(1/2)}*EllipticF((-1+\cos(d*x+c) \\
&)/\sin(d*x+c), ((a-b)/(a+b))^{(1/2)})*a^4*\sin(d*x+c)+288*A*(\cos(d*x+c)/(1+\cos(d \\
& *x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{(1/2)}*EllipticPi((-1+ \\
& \cos(d*x+c))/\sin(d*x+c), -1, ((a-b)/(a+b))^{(1/2)})*a^4*\sin(d*x+c)+24*A*\cos(d*x+ \\
& c)^4*a^4-15*A*\cos(d*x+c)*b^4+720*A*a^2*b^2*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2) \\
& }*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{(1/2)}*\sin(d*x+c)*\cos(d*x+c)*Ellip \\
& ticPi((-1+\cos(d*x+c))/\sin(d*x+c), -1, ((a-b)/(a+b))^{(1/2)})+128*B*\sin(d*x+c)*c \\
& os(d*x+c)*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c) \\
&)/(a+b))^{(1/2)}*EllipticE((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{(1/2)})*a^ \\
& 3*b+264*B*a^2*b^2*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+co \\
& s(d*x+c))/(a+b))^{(1/2)}*\sin(d*x+c)*\cos(d*x+c)*EllipticE((-1+\cos(d*x+c))/\sin(\\
& d*x+c), ((a-b)/(a+b))^{(1/2)})+264*B*b^3*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((b \\
& +a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{(1/2)}*\sin(d*x+c)*\cos(d*x+c)*EllipticE(\\
& (-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{(1/2)})*a-608*B*\sin(d*x+c)*\cos(d*x+ \\
& c)*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b) \\
&)^{(1/2)}*EllipticF((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{(1/2)})*a^3*b+208 \\
& *B*\sin(d*x+c)*\cos(d*x+c)*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x+c) \\
&)/(1+\cos(d*x+c))/(a+b))^{(1/2)}*EllipticF((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(
\end{aligned}$$

$(a+b)^{1/2} \cdot a^2 \cdot b^2 - 384 \cdot B \cdot \sin(dx+c) \cdot \cos(dx+c) \cdot (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} \cdot ((b+a \cdot \cos(dx+c))/(1+\cos(dx+c)))/(a+b)^{1/2} \cdot \text{EllipticF}((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{1/2}) \cdot a \cdot b^3 + 960 \cdot B \cdot \sin(dx+c) \cdot \cos(dx+c) \cdot (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} \cdot ((b+a \cdot \cos(dx+c))/(1+\cos(dx+c)))/(a+b)^{1/2} \cdot \text{EllipticPi}((-1+\cos(dx+c))/\sin(dx+c), -1, ((a-b)/(a+b))^{1/2}) \cdot a^3 \cdot b + 240 \cdot B \cdot \sin(dx+c) \cdot \cos(dx+c) \cdot (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} \cdot ((b+a \cdot \cos(dx+c))/(1+\cos(dx+c)))/(a+b)^{1/2} \cdot \text{EllipticPi}((-1+\cos(dx+c))/\sin(dx+c), -1, ((a-b)/(a+b))^{1/2}) \cdot a \cdot b^3 + 284 \cdot A \cdot a^3 \cdot (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} \cdot ((b+a \cdot \cos(dx+c))/(1+\cos(dx+c)))/(a+b)^{1/2} \cdot \sin(dx+c) \cdot \cos(dx+c) \cdot \text{EllipticE}((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{1/2}) \cdot b + 284 \cdot A \cdot \sin(dx+c) \cdot \cos(dx+c) \cdot (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} \cdot ((b+a \cdot \cos(dx+c))/(1+\cos(dx+c)))/(a+b)^{1/2} \cdot \text{EllipticE}((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{1/2}) \cdot a^2 \cdot b^2 + 15 \cdot A \cdot \sin(dx+c) \cdot \cos(dx+c) \cdot (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} \cdot ((b+a \cdot \cos(dx+c))/(1+\cos(dx+c)))/(a+b)^{1/2} \cdot \text{EllipticE}((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{1/2}) \cdot a \cdot b^3 + 72 \cdot A \cdot a^3 \cdot (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} \cdot ((b+a \cdot \cos(dx+c))/(1+\cos(dx+c)))/(a+b)^{1/2} \cdot \sin(dx+c) \cdot \cos(dx+c) \cdot \text{EllipticF}((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{1/2}) \cdot b - 644 \cdot A \cdot a^2 \cdot b^2 \cdot (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} \cdot ((b+a \cdot \cos(dx+c))/(1+\cos(dx+c)))/(a+b)^{1/2} \cdot \sin(dx+c) \cdot \cos(dx+c) \cdot \text{EllipticF}((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{1/2}) + 118 \cdot A \cdot b^3 \cdot (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} \cdot ((b+a \cdot \cos(dx+c))/(1+\cos(dx+c)))/(a+b)^{1/2} \cdot \sin(dx+c) \cdot \cos(dx+c) \cdot \text{EllipticF}((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{1/2}) \cdot a + 15 \cdot A \cdot \sin(dx+c) \cdot \cos(dx+c) \cdot (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} \cdot ((b+a \cdot \cos(dx+c))/(1+\cos(dx+c)))/(a+b)^{1/2} \cdot \text{EllipticE}((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{1/2}) \cdot b^4 - 144 \cdot A \cdot \sin(dx+c) \cdot \cos(dx+c) \cdot (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} \cdot ((b+a \cdot \cos(dx+c))/(1+\cos(dx+c)))/(a+b)^{1/2} \cdot \text{EllipticF}((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{1/2}) \cdot a^4 + 288 \cdot A \cdot \sin(dx+c) \cdot \cos(dx+c) \cdot (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} \cdot ((b+a \cdot \cos(dx+c))/(1+\cos(dx+c)))/(a+b)^{1/2} \cdot \text{EllipticPi}((-1+\cos(dx+c))/\sin(dx+c), -1, ((a-b)/(a+b))^{1/2}) \cdot a^4 \cdot (1+\cos(dx+c))^2 \cdot ((b+a \cdot \cos(dx+c))/\cos(dx+c))^{1/2} / (b+a \cdot \cos(dx+c)) / \sin(dx+c)^5 / a$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx+c) + A)(b \sec(dx+c) + a)^{5/2} \cos(dx+c)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(dx+c)^4*(a+b*sec(dx+c))^(5/2)*(A+B*sec(dx+c)),x, algorithm="maxima")

[Out] integrate((B*sec(dx+c) + A)*(b*sec(dx+c) + a)^(5/2)*cos(dx+c)^4, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c + dx)^4 \left(A + \frac{B}{\cos(c + dx)} \right) \left(a + \frac{b}{\cos(c + dx)} \right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(c + d*x)^4*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(5/2), x)`

[Out] `int(cos(c + d*x)^4*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(5/2), x)`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)**4*(a+b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)), x)`

[Out] Timed out

$$3.371 \quad \int \frac{\sec^3(c+dx)(A+B \sec(c+dx))}{\sqrt{a+b \sec(c+dx)}} dx$$

Optimal. Leaf size=329

$$\frac{2(a-b)\sqrt{a+b}(-8a^2B+10aAb-9b^2B)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b}\sec(c+dx)}{\sqrt{a+b}}\right)\right)}{15b^4d}$$

[Out] 2/15*(a-b)*(10*A*a*b-8*B*a^2-9*B*b^2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c)))/(a+b)^(1/2)*(-b*(1+sec(d*x+c)))/(a-b))^(1/2)/b^4/d+2/15*(b^2*(5*A-9*B)-8*a^2*B+2*a*b*(5*A+B))*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c)))/(a+b)^(1/2)*(-b*(1+sec(d*x+c)))/(a-b))^(1/2)/b^3/d+2/15*(5*A*b-4*B*a)*(a+b*sec(d*x+c))^(1/2)*tan(d*x+c)/b^2/d+2/5*B*sec(d*x+c)*(a+b*sec(d*x+c))^(1/2)*tan(d*x+c)/b/d

Rubi [A] time = 0.62, antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {4033, 4082, 4005, 3832, 4004}

$$\frac{2\sqrt{a+b}(-8a^2B+2ab(5A+B)+b^2(5A-9B))\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b}\sec(c+dx)}{\sqrt{a+b}}\right)\right)}{15b^3d}$$

Antiderivative was successfully verified.

[In] Int[(Sec[c + d*x]^3*(A + B*Sec[c + d*x]))/Sqrt[a + b*Sec[c + d*x]],x]

[Out] (2*(a-b)*Sqrt[a+b]*(10*a*A*b-8*a^2*B-9*b^2*B)*Cot[c+d*x]*EllipticE[ArcSin[Sqrt[a+b*Sec[c+d*x]]/Sqrt[a+b]],(a+b)/(a-b)]*Sqrt[(b*(1-Sec[c+d*x]))/(a+b)]*Sqrt[-((b*(1+Sec[c+d*x]))/(a-b))]/(15*b^4*d)+(2*Sqrt[a+b]*(b^2*(5*A-9*B)-8*a^2*B+2*a*b*(5*A+B))*Cot[c+d*x]*EllipticF[ArcSin[Sqrt[a+b*Sec[c+d*x]]/Sqrt[a+b]],(a+b)/(a-b)]*Sqrt[(b*(1-Sec[c+d*x]))/(a+b)]*Sqrt[-((b*(1+Sec[c+d*x]))/(a-b))]/(15*b^3*d)+(2*(5*A*b-4*a*B)*Sqrt[a+b*Sec[c+d*x]]*Tan[c+d*x])/((15*b^2*d)+(2*B*Sec[c+d*x]*Sqrt[a+b*Sec[c+d*x]]*Tan[c+d*x]))/(5*b*d)

Rule 3832

Int[csc[(e_.)+(f_.)*(x_)]/Sqrt[csc[(e_.)+(f_.)*(x_)]*(b_.)+(a_)],x_Symbol] :> Simp[(-2*Rt[a+b,2]*Sqrt[(b*(1-Csc[e+f*x]))/(a+b)]*Sqrt[-((b*(1+Csc[e+f*x]))/(a-b))]*EllipticF[ArcSin[Sqrt[a+b*Csc[e+f*x]]/Rt[a+b,2]],(a+b)/(a-b))]/(b*f*Cot[e+f*x]),x] /; FreeQ[{a,b,e,f},x] && NeQ[a^2-b^2,0]

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)))/Sqrt[
sc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
2]], (a*A + b*B)/(a*A - b*B)]/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)))/Sqrt[
sc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x])/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
&& NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 4033

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (
a_.))^(m_)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> -Simp[(B*d^2
*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2))/(b*f*(
m + n)), x] + Dist[d^2/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f
*x])^(n - 2)*Simp[a*B*(n - 2) + B*b*(m + n - 1)*Csc[e + f*x] + (A*b*(m + n)
- a*B*(n - 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m
}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && NeQ[m + n,
0] && !IGtQ[m, 1]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_.)]*((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e
_.) + (f_.)*(x_.)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^3(c+dx)(A+B\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}} dx &= \frac{2B\sec(c+dx)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{5bd} + \frac{2\int \frac{\sec(c+dx)(aB+\frac{3}{2}bB\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}} dx}{5bd} \\
&= \frac{2(5Ab-4aB)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{15b^2d} + \frac{2B\sec(c+dx)\sqrt{a+b\sec(c+dx)}}{5bd} \\
&= \frac{2(5Ab-4aB)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{15b^2d} + \frac{2B\sec(c+dx)\sqrt{a+b\sec(c+dx)}}{5bd} \\
&= \frac{2(a-b)\sqrt{a+b}\left(10aAb-8a^2B-9b^2B\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)\right)}{15b^4d}
\end{aligned}$$

Mathematica [B] time = 22.58, size = 3000, normalized size = 9.12

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^3*(A + B*Sec[c + d*x]))/Sqrt[a + b*Sec[c + d*x]],x]
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]*((2*(-10*a*A*b + 8*a^2*B + 9*b^2*B)*Sin[
c + d*x])/(15*b^3) + (2*Sec[c + d*x]*(5*A*b*Sin[c + d*x] - 4*a*B*Sin[c + d*
x]))/(15*b^2) + (2*B*Sec[c + d*x]*Tan[c + d*x])/(5*b)))/(d*Sqrt[a + b*Sec[c
+ d*x]]) - (2*((2*a*A)/(3*b*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) -
(3*B)/(5*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (8*a^2*B)/(15*b^2*
Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (A*Sqrt[Sec[c + d*x]])/(3*Sq
rt[b + a*Cos[c + d*x]]) + (2*a^2*A*Sqrt[Sec[c + d*x]])/(3*b^2*Sqrt[b + a*Co
s[c + d*x]]) - (8*a^3*B*Sqrt[Sec[c + d*x]])/(15*b^3*Sqrt[b + a*Cos[c + d*x]
]) - (7*a*B*Sqrt[Sec[c + d*x]])/(15*b*Sqrt[b + a*Cos[c + d*x]]) + (2*a^2*A*
Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*b^2*Sqrt[b + a*Cos[c + d*x]]) - (8*
a^3*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(15*b^3*Sqrt[b + a*Cos[c + d*x]
]) - (3*a*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(5*b*Sqrt[b + a*Cos[c + d*x
]]))*Sqrt[Sec[c + d*x]]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(2*(a + b)*(-
10*a*A*b + 8*a^2*B + 9*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x]])*Sqrt[(b
+ a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c +
d*x)/2]], (a - b)/(a + b)] - 2*b*(8*a^2*B + 2*a*b*(-5*A + B) + b^2*(5*A + 9
*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x]])*Sqrt[(b + a*Cos[c + d*x])/((a +
b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)
] + (-10*a*A*b + 8*a^2*B + 9*b^2*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(
c + d*x)/2]^2*Tan[(c + d*x)/2))/(15*b^3*d*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[a
+ b*Sec[c + d*x]]*(-1/15*(a*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sin[c + d
```

$$\begin{aligned}
& *x)*(2*(a + b)*(-10*a*A*b + 8*a^2*B + 9*b^2*B)*\text{Sqrt}[\text{Cos}[c + d*x]/(1 + \text{Cos}[c \\
& + d*x])]*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/((a + b)*(1 + \text{Cos}[c + d*x]))]*\text{EllipticE} \\
& [\text{ArcSin}[\text{Tan}[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(8*a^2*B + 2*a*b*(-5*A + \\
& B) + b^2*(5*A + 9*B))*\text{Sqrt}[\text{Cos}[c + d*x]/(1 + \text{Cos}[c + d*x])]*\text{Sqrt}[(b + a*\text{Cos} \\
& [c + d*x])/((a + b)*(1 + \text{Cos}[c + d*x]))]*\text{EllipticF}[\text{ArcSin}[\text{Tan}[(c + d*x)/2]] \\
& , (a - b)/(a + b)] + (-10*a*A*b + 8*a^2*B + 9*b^2*B)*\text{Cos}[c + d*x]*(b + a*\text{Co} \\
& s[c + d*x])* \text{Sec}[(c + d*x)/2]^2*\text{Tan}[(c + d*x)/2])/ (b^3*(b + a*\text{Cos}[c + d*x]) \\
& ^{(3/2)}*\text{Sqrt}[\text{Sec}[(c + d*x)/2]^2]) + (\text{Sqrt}[\text{Cos}[(c + d*x)/2]^2*\text{Sec}[c + d*x]]* \text{T} \\
& \text{an}[(c + d*x)/2]*(2*(a + b)*(-10*a*A*b + 8*a^2*B + 9*b^2*B)*\text{Sqrt}[\text{Cos}[c + d*x] \\
&]/(1 + \text{Cos}[c + d*x]))*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/((a + b)*(1 + \text{Cos}[c + d*x]) \\
&)]*\text{EllipticE}[\text{ArcSin}[\text{Tan}[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(8*a^2*B + 2* \\
& a*b*(-5*A + B) + b^2*(5*A + 9*B))*\text{Sqrt}[\text{Cos}[c + d*x]/(1 + \text{Cos}[c + d*x])]*\text{Sqr} \\
& \text{t}[(b + a*\text{Cos}[c + d*x])/((a + b)*(1 + \text{Cos}[c + d*x]))]*\text{EllipticF}[\text{ArcSin}[\text{Tan}[(c \\
& + d*x)/2]], (a - b)/(a + b)] + (-10*a*A*b + 8*a^2*B + 9*b^2*B)*\text{Cos}[c + d* \\
& x]*(b + a*\text{Cos}[c + d*x])* \text{Sec}[(c + d*x)/2]^2*\text{Tan}[(c + d*x)/2])/ (15*b^3*\text{Sqrt}[\\
& b + a*\text{Cos}[c + d*x]]*\text{Sqrt}[\text{Sec}[(c + d*x)/2]^2]) - (2*\text{Sqrt}[\text{Cos}[(c + d*x)/2]^2* \\
& \text{Sec}[c + d*x]]*((-10*a*A*b + 8*a^2*B + 9*b^2*B)*\text{Cos}[c + d*x]*(b + a*\text{Cos}[c + \\
& d*x])* \text{Sec}[(c + d*x)/2]^4)/2 + ((a + b)*(-10*a*A*b + 8*a^2*B + 9*b^2*B)*\text{Sqr} \\
& \text{t}[(b + a*\text{Cos}[c + d*x])/((a + b)*(1 + \text{Cos}[c + d*x]))]*\text{EllipticE}[\text{ArcSin}[\text{Tan}[(c \\
& + d*x)/2]], (a - b)/(a + b)]*((\text{Cos}[c + d*x]*\text{Sin}[c + d*x])/(1 + \text{Cos}[c + d* \\
& x])^2 - \text{Sin}[c + d*x]/(1 + \text{Cos}[c + d*x])))/\text{Sqrt}[\text{Cos}[c + d*x]/(1 + \text{Cos}[c + d* \\
& x])] - (b*(8*a^2*B + 2*a*b*(-5*A + B) + b^2*(5*A + 9*B))*\text{Sqrt}[(b + a*\text{Cos}[c \\
& + d*x])/((a + b)*(1 + \text{Cos}[c + d*x]))]*\text{EllipticF}[\text{ArcSin}[\text{Tan}[(c + d*x)/2]], (\\
& a - b)/(a + b)]*((\text{Cos}[c + d*x]*\text{Sin}[c + d*x])/(1 + \text{Cos}[c + d*x])^2 - \text{Sin}[c + \\
& d*x]/(1 + \text{Cos}[c + d*x])))/\text{Sqrt}[\text{Cos}[c + d*x]/(1 + \text{Cos}[c + d*x])] + ((a + b) \\
& *(-10*a*A*b + 8*a^2*B + 9*b^2*B)*\text{Sqrt}[\text{Cos}[c + d*x]/(1 + \text{Cos}[c + d*x])]*\text{Elli} \\
& \text{pticE}[\text{ArcSin}[\text{Tan}[(c + d*x)/2]], (a - b)/(a + b)]*(-((a*\text{Sin}[c + d*x])/(a + \\
& b)*(1 + \text{Cos}[c + d*x]))) + ((b + a*\text{Cos}[c + d*x])* \text{Sin}[c + d*x])/((a + b)*(1 + \\
& \text{Cos}[c + d*x])^2))/\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/((a + b)*(1 + \text{Cos}[c + d*x]))] \\
& - (b*(8*a^2*B + 2*a*b*(-5*A + B) + b^2*(5*A + 9*B))*\text{Sqrt}[\text{Cos}[c + d*x]/(1 + \\
& \text{Cos}[c + d*x])]*\text{EllipticF}[\text{ArcSin}[\text{Tan}[(c + d*x)/2]], (a - b)/(a + b)]*(-((a* \\
& \text{Sin}[c + d*x])/(a + b)*(1 + \text{Cos}[c + d*x]))) + ((b + a*\text{Cos}[c + d*x])* \text{Sin}[c + \\
& d*x])/((a + b)*(1 + \text{Cos}[c + d*x])^2))/\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/((a + b)* \\
& (1 + \text{Cos}[c + d*x]))] - a*(-10*a*A*b + 8*a^2*B + 9*b^2*B)*\text{Cos}[c + d*x]* \text{Sec}[(c \\
& + d*x)/2]^2*\text{Sin}[c + d*x]*\text{Tan}[(c + d*x)/2] - (-10*a*A*b + 8*a^2*B + 9*b^2*B) \\
& *(b + a*\text{Cos}[c + d*x])* \text{Sec}[(c + d*x)/2]^2*\text{Sin}[c + d*x]*\text{Tan}[(c + d*x)/2] + \\
& (-10*a*A*b + 8*a^2*B + 9*b^2*B)*\text{Cos}[c + d*x]*(b + a*\text{Cos}[c + d*x])* \text{Sec}[(c + \\
& d*x)/2]^2*\text{Tan}[(c + d*x)/2]^2 - (b*(8*a^2*B + 2*a*b*(-5*A + B) + b^2*(5*A + \\
& 9*B))*\text{Sqrt}[\text{Cos}[c + d*x]/(1 + \text{Cos}[c + d*x])]*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/((a + \\
& b)*(1 + \text{Cos}[c + d*x]))]* \text{Sec}[(c + d*x)/2]^2)/(\text{Sqrt}[1 - \text{Tan}[(c + d*x)/2]^2]* \\
& \text{Sqrt}[1 - ((a - b)*\text{Tan}[(c + d*x)/2]^2)/(a + b)]) + ((a + b)*(-10*a*A*b + 8*a \\
& ^2*B + 9*b^2*B)*\text{Sqrt}[\text{Cos}[c + d*x]/(1 + \text{Cos}[c + d*x])]*\text{Sqrt}[(b + a*\text{Cos}[c + d \\
& *x])/((a + b)*(1 + \text{Cos}[c + d*x]))]* \text{Sec}[(c + d*x)/2]^2*\text{Sqrt}[1 - ((a - b)*\text{Tan} \\
& [(c + d*x)/2]^2)/(a + b)])/\text{Sqrt}[1 - \text{Tan}[(c + d*x)/2]^2])/ (15*b^3*\text{Sqrt}[b + \\
& a*\text{Cos}[c + d*x]]*\text{Sqrt}[\text{Sec}[(c + d*x)/2]^2]) - ((2*(a + b)*(-10*a*A*b + 8*a^2*B
\end{aligned}$$

$$\begin{aligned}
 & (B + 9*b^2*B)*\text{Sqrt}[\text{Cos}[c + d*x]/(1 + \text{Cos}[c + d*x])]*\text{Sqrt}[(b + a*\text{Cos}[c + d*x]) \\
 &)/((a + b)*(1 + \text{Cos}[c + d*x]))]*\text{EllipticE}[\text{ArcSin}[\text{Tan}[(c + d*x)/2]], (a - b) \\
 & / (a + b)] - 2*b*(8*a^2*B + 2*a*b*(-5*A + B) + b^2*(5*A + 9*B))*\text{Sqrt}[\text{Cos}[c + \\
 & d*x]/(1 + \text{Cos}[c + d*x])]*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/(a + b)*(1 + \text{Cos}[c + d \\
 & *x]))]*\text{EllipticF}[\text{ArcSin}[\text{Tan}[(c + d*x)/2]], (a - b)/(a + b)] + (-10*a*A*b + \\
 & 8*a^2*B + 9*b^2*B)*\text{Cos}[c + d*x]*(b + a*\text{Cos}[c + d*x])* \text{Sec}[(c + d*x)/2]^2*\text{Tan} \\
 & [(c + d*x)/2]*(-(\text{Cos}[(c + d*x)/2]*\text{Sec}[c + d*x]*\text{Sin}[(c + d*x)/2]) + \text{Cos}[(c \\
 & + d*x)/2]^2*\text{Sec}[c + d*x]*\text{Tan}[c + d*x]))/(15*b^3*\text{Sqrt}[b + a*\text{Cos}[c + d*x]]*\text{Sq} \\
 & \text{rt}[\text{Sec}[(c + d*x)/2]^2*\text{Sqrt}[\text{Cos}[(c + d*x)/2]^2*\text{Sec}[c + d*x]]))
 \end{aligned}$$

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{B \sec(dx + c)^4 + A \sec(dx + c)^3}{\sqrt{b \sec(dx + c) + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")

[Out] integral((B*sec(d*x + c)^4 + A*sec(d*x + c)^3)/sqrt(b*sec(d*x + c) + a), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sec(dx + c)^3}{\sqrt{b \sec(dx + c) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^3/sqrt(b*sec(d*x + c) + a), x)

maple [B] time = 2.57, size = 2499, normalized size = 7.60

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x)

[Out]
$$\begin{aligned}
 & -2/15/d*(1+\text{cos}(d*x+c))^2*((b+a*\text{cos}(d*x+c))/\text{cos}(d*x+c))^(1/2)*(-1+\text{cos}(d*x+c)) \\
 &)^2*(-9*B*\text{sin}(d*x+c)*\text{cos}(d*x+c)^2*(\text{cos}(d*x+c)/(1+\text{cos}(d*x+c)))^(1/2)*((b+a*c \\
 & \text{os}(d*x+c))/(1+\text{cos}(d*x+c))/(a+b))^(1/2)*\text{EllipticE}((-1+\text{cos}(d*x+c))/\text{sin}(d*x+c) \\
 & , ((a-b)/(a+b))^(1/2))*a*b^2+5*A*\text{cos}(d*x+c)^3*b^3-8*B*\text{cos}(d*x+c)^3*a^3-3*b^3 \\
 & *B+9*B*\text{cos}(d*x+c)^3*b^3-6*B*\text{cos}(d*x+c)^2*b^3-10*A*\text{cos}(d*x+c)^3*a*b^2+5*A*co
 \end{aligned}$$

$$\begin{aligned}
& s(d*x+c)^2*a*b^2+9*B*cos(d*x+c)^4*a*b^2+8*B*cos(d*x+c)^3*a^2*b-4*B*cos(d*x+c)^2*a^2*b+B*cos(d*x+c)*a*b^2-5*A*cos(d*x+c)*b^3-10*B*cos(d*x+c)^3*a*b^2-10 \\
& *A*cos(d*x+c)^4*a^2*b+5*A*cos(d*x+c)^4*a*b^2+10*A*cos(d*x+c)^3*a^2*b-4*B*cos \\
& s(d*x+c)^4*a^2*b+8*B*cos(d*x+c)^4*a^3+8*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c) \\
& / (1+cos(d*x+c)))^{(1/2)}*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^{(1/2)}*EllipticF((-1+cos(d*x+c))/sin(d*x+c), ((a-b)/(a+b))^{(1/2)})*a^2*b+2*B*sin(d*x+c)* \\
& cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^{(1/2)}*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^{(1/2)}*EllipticF((-1+cos(d*x+c))/sin(d*x+c), ((a-b)/(a+b))^{(1/2)}) \\
& *a*b^2-8*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^{(1/2)}*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^{(1/2)}*EllipticE((-1+cos(d*x+c))/sin(d*x+c), ((a-b)/(a+b))^{(1/2)})*a^2*b-9*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^{(1/2)}*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^{(1/2)}*EllipticE((-1+cos(d*x+c))/sin(d*x+c), ((a-b)/(a+b))^{(1/2)})*a*b^2-10*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^{(1/2)}*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^{(1/2)}*EllipticF((-1+cos(d*x+c))/sin(d*x+c), ((a-b)/(a+b))^{(1/2)})*a*b^2+10*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^{(1/2)}*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^{(1/2)}*EllipticE((-1+cos(d*x+c))/sin(d*x+c), ((a-b)/(a+b))^{(1/2)})*a^2*b+10*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^{(1/2)}*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^{(1/2)}*EllipticE((-1+cos(d*x+c))/sin(d*x+c), ((a-b)/(a+b))^{(1/2)})*a*b^2+8*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^{(1/2)}*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^{(1/2)}*EllipticF((-1+cos(d*x+c))/sin(d*x+c), ((a-b)/(a+b))^{(1/2)})*a^2*b+2*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^{(1/2)}*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^{(1/2)}*EllipticF((-1+cos(d*x+c))/sin(d*x+c), ((a-b)/(a+b))^{(1/2)})*a*b^2-8*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^{(1/2)}*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^{(1/2)}*EllipticE((-1+cos(d*x+c))/sin(d*x+c), ((a-b)/(a+b))^{(1/2)})*a^2*b-10*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^{(1/2)}*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^{(1/2)}*EllipticF((-1+cos(d*x+c))/sin(d*x+c), ((a-b)/(a+b))^{(1/2)})*a*b^2+10*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^{(1/2)}*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^{(1/2)}*EllipticE((-1+cos(d*x+c))/sin(d*x+c), ((a-b)/(a+b))^{(1/2)})*a^2*b+10*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^{(1/2)}*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^{(1/2)}*EllipticE((-1+cos(d*x+c))/sin(d*x+c), ((a-b)/(a+b))^{(1/2)})*a*b^2-8*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^{(1/2)}*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^{(1/2)}*EllipticE((-1+cos(d*x+c))/sin(d*x+c), ((a-b)/(a+b))^{(1/2)})*a^3-9*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^{(1/2)}*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^{(1/2)}*EllipticE((-1+cos(d*x+c))/sin(d*x+c), ((a-b)/(a+b))^{(1/2)})*b^3+5*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^{(1/2)}*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^{(1/2)}*EllipticF((-1+cos(d*x+c))/sin(d*x+c), ((a-b)/(a+b))^{(1/2)})*b^3+9*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^{(1/2)}*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^{(1/2)}*EllipticF((-1+cos(d*x+c))/sin(d*x+c), ((a-b)/(a+b))^{(1/2)})*b^3-8*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^{(1/2)}*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^{(1/2)}*EllipticE((-1+cos(d*x+c))/sin(d*x+c), ((a-b)/(a+b))^{(1/2)})*a^3-9*B*sin(d*x+c)*cos(d
\end{aligned}$$

$$\begin{aligned} & *x+c)^2*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/ \\ & (a+b))^{(1/2)}*EllipticE((-1+\cos(d*x+c))/\sin(d*x+c),((a-b)/(a+b))^{(1/2)})*b^3+ \\ & 5*A*\sin(d*x+c)*\cos(d*x+c)^3*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x \\ & +c))/(1+\cos(d*x+c))/(a+b))^{(1/2)}*EllipticF((-1+\cos(d*x+c))/\sin(d*x+c),((a-b \\ &)/(a+b))^{(1/2)})*b^3+9*B*\sin(d*x+c)*\cos(d*x+c)^3*(\cos(d*x+c)/(1+\cos(d*x+c))) \\ & ^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{(1/2)}*EllipticF((-1+\cos(d*x+ \\ & c))/\sin(d*x+c),((a-b)/(a+b))^{(1/2)})*b^3)/(b+a*\cos(d*x+c))/\cos(d*x+c)^2/\sin(\\ & d*x+c)^5/b^3 \end{aligned}$$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\cos(c+dx)^3 \sqrt{a + \frac{b}{\cos(c+dx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^3*(a + b/cos(c + d*x))^(1/2)),x)

[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^3*(a + b/cos(c + d*x))^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \sec^3(c + dx)}{\sqrt{a + b \sec(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(1/2),x)

[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**3/sqrt(a + b*sec(c + d*x)), x)

$$3.372 \quad \int \frac{\sec^2(c+dx)(A+B \sec(c+dx))}{\sqrt{a+b \sec(c+dx)}} dx$$

Optimal. Leaf size=261

$$\frac{2(a-b)\sqrt{a+b}(3Ab-2aB) \cot(c+dx) \sqrt{\frac{b(1-\sec(c+dx))}{a+b}} \sqrt{\frac{-b(\sec(c+dx)+1)}{a-b}} E\left(\sin^{-1}\left(\frac{\sqrt{a+b \sec(c+dx)}}{\sqrt{a+b}}\right) \middle| \frac{a+b}{a-b}\right) + 2\sqrt{a+b}}{3b^3d}$$

[Out] $-2/3*(a-b)*(3*A*b-2*B*a)*\cot(d*x+c)*\text{EllipticE}((a+b*\sec(d*x+c))^{(1/2)}/(a+b)^{(1/2)},((a+b)/(a-b))^{(1/2)})*(a+b)^{(1/2)}*(b*(1-\sec(d*x+c)))/(a+b)^{(1/2)}*(-b*(1+\sec(d*x+c)))/(a-b)^{(1/2)}/b^3/d-2/3*(3*A*b-(2*a+b)*B)*\cot(d*x+c)*\text{EllipticF}((a+b*\sec(d*x+c))^{(1/2)}/(a+b)^{(1/2)},((a+b)/(a-b))^{(1/2)})*(a+b)^{(1/2)}*(b*(1-\sec(d*x+c)))/(a+b)^{(1/2)}*(-b*(1+\sec(d*x+c)))/(a-b)^{(1/2)}/b^2/d+2/3*B*(a+b*\sec(d*x+c))^{(1/2)}*\tan(d*x+c)/b/d$

Rubi [A] time = 0.40, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {4010, 4005, 3832, 4004}

$$\frac{2\sqrt{a+b}(3Ab-B(2a+b)) \cot(c+dx) \sqrt{\frac{b(1-\sec(c+dx))}{a+b}} \sqrt{\frac{-b(\sec(c+dx)+1)}{a-b}} F\left(\sin^{-1}\left(\frac{\sqrt{a+b \sec(c+dx)}}{\sqrt{a+b}}\right) \middle| \frac{a+b}{a-b}\right) + 2(a-b)\sqrt{a+b}}{3b^2d}$$

Antiderivative was successfully verified.

[In] Int[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/Sqrt[a + b*Sec[c + d*x]],x]

[Out] $(-2*(a-b)*\text{Sqrt}[a+b]*(3*A*b-2*a*B)*\text{Cot}[c+d*x]*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[a+b*\text{Sec}[c+d*x]]/\text{Sqrt}[a+b]],(a+b)/(a-b)]*\text{Sqrt}[(b*(1-\text{Sec}[c+d*x]))/(a+b)]*\text{Sqrt}[-((b*(1+\text{Sec}[c+d*x]))/(a-b))]/(3*b^3*d)-(2*\text{Sqrt}[a+b]*(3*A*b-(2*a+b)*B)*\text{Cot}[c+d*x]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[a+b*\text{Sec}[c+d*x]]/\text{Sqrt}[a+b]],(a+b)/(a-b)]*\text{Sqrt}[(b*(1-\text{Sec}[c+d*x]))/(a+b)]*\text{Sqrt}[-((b*(1+\text{Sec}[c+d*x]))/(a-b))]/(3*b^2*d)+(2*B*\text{Sqrt}[a+b*\text{Sec}[c+d*x]]*\text{Tan}[c+d*x])/(3*b*d)$

Rule 3832

Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)]/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
2]], (a*A + b*B)/(a*A - b*B)]/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x])/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
&& NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Cs
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
0] && !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{\sec^2(c+dx)(A+B\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}} dx &= \frac{2B\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{3bd} + \frac{2\int \frac{\sec(c+dx)\left(\frac{bB}{2} + \frac{1}{2}(3Ab-2aB)\sec(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}}{3b} \\ &= \frac{2B\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{3bd} + \frac{(3Ab-2aB)\int \frac{\sec(c+dx)(1+\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}}{3b} \\ &= -\frac{2(a-b)\sqrt{a+b}(3Ab-2aB)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)\middle|\frac{a+b}{a-b}\right)}{3b^3d} \end{aligned}$$

Mathematica [A] time = 15.84, size = 372, normalized size = 1.43

$$\frac{\sec(c+dx)(a\cos(c+dx)+b)\left(\frac{2(3Ab-2aB)\sin(c+dx)}{3b^2} + \frac{2B\tan(c+dx)}{3b}\right)}{d\sqrt{a+b\sec(c+dx)}} + \frac{2\sqrt{\sec(c+dx)}\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/Sqrt[a + b*Sec[c + d*x]],x]

[Out] (2*Sqrt[Sec[c + d*x]]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(2*(a + b)*(-3*A*b + 2*a*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*b*(3*A*b + (-2*a + b)*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - (3*A*b - 2*a*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2))/(3*b^2*d*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[a + b*Sec[c + d*x]]) + ((b + a*Cos[c + d*x])*Sec[c + d*x]*((2*(3*A*b - 2*a*B)*Sin[c + d*x])/(3*b^2) + (2*B*Tan[c + d*x])/(3*b)))/(d*Sqrt[a + b*Sec[c + d*x]])

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{B \sec(dx + c)^3 + A \sec(dx + c)^2}{\sqrt{b \sec(dx + c) + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")

[Out] integral((B*sec(d*x + c)^3 + A*sec(d*x + c)^2)/sqrt(b*sec(d*x + c) + a), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sec(dx + c)^2}{\sqrt{b \sec(dx + c) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^2/sqrt(b*sec(d*x + c) + a), x)

maple [B] time = 2.30, size = 1563, normalized size = 5.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x)

```
[Out] -2/3/d*(-1+cos(d*x+c))^2*(2*B*cos(d*x+c)^2*a^2-2*B*cos(d*x+c)^2*sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(
1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b+2*B*cos(
d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+co
s(d*x+c)))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(
1/2))*a*b-2*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a
*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*a*b-3*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*EllipticE((-1+
cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b+2*B*cos(d*x+c)^2*sin(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(
1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b-3*A*cos
(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+c
os(d*x+c)))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(
1/2))*a*b+B*cos(d*x+c)^2*b^2-b^2*B+3*A*cos(d*x+c)^2*b^2-3*A*cos(d*x+c)*b^2
-3*A*cos(d*x+c)^2*a*b+B*cos(d*x+c)^3*a*b-2*B*cos(d*x+c)^2*a*b+B*cos(d*x+c)*
a*b+3*A*cos(d*x+c)^3*a*b-2*B*cos(d*x+c)^3*a^2-3*A*cos(d*x+c)^2*sin(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1
/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2+3*A*cos(d
*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+c
os(d*x+c)))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(
1/2))*b^2+2*B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((
b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),((a-b)/(a+b))^(1/2))*a^2+B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2-3*A*cos(d*x+c)*sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(
1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2+3*A*cos(
d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+co
s(d*x+c)))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(
1/2))*b^2+2*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a
*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*a^2+B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2)*((b+a*cos(d*x+c))/cos(d*x+c))^(
1/2)*(1+cos(d*x+c))^2/(b+a*cos(d*x+c))/cos(d*x+c)/sin(d*x+c)^5/b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sec(dx + c)^2}{\sqrt{b \sec(dx + c) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algorithm
="maxima")
```

[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^2/sqrt(b*sec(d*x + c) + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\cos(c+dx)^2 \sqrt{a + \frac{b}{\cos(c+dx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^2*(a + b/cos(c + d*x))^(1/2)), x)

[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^2*(a + b/cos(c + d*x))^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \sec^2(c + dx)}{\sqrt{a + b \sec(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(1/2), x)

[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**2/sqrt(a + b*sec(c + d*x)), x)

$$3.373 \quad \int \frac{\sec(c+dx)(A+B \sec(c+dx))}{\sqrt{a+b \sec(c+dx)}} dx$$

Optimal. Leaf size=210

$$\frac{2\sqrt{a+b}(A-B) \cot(c+dx) \sqrt{\frac{b(1-\sec(c+dx))}{a+b}} \sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}} F\left(\sin^{-1}\left(\frac{\sqrt{a+b \sec(c+dx)}}{\sqrt{a+b}}\right) \middle| \frac{a+b}{a-b}\right) + 2B(a-b)\sqrt{a+b} \cot(c+dx)}{bd}$$

[Out] $-2*(a-b)*B*\cot(d*x+c)*\text{EllipticE}((a+b*\sec(d*x+c))^{1/2}/(a+b)^{1/2}, ((a+b)/(a-b))^{1/2})*(a+b)^{1/2}*(b*(1-\sec(d*x+c)))/(a+b)^{1/2}*(-b*(1+\sec(d*x+c)))/(a-b)^{1/2}/b^2/d+2*(A-B)*\cot(d*x+c)*\text{EllipticF}((a+b*\sec(d*x+c))^{1/2}/(a+b)^{1/2}, ((a+b)/(a-b))^{1/2})*(a+b)^{1/2}*(b*(1-\sec(d*x+c)))/(a+b)^{1/2}*(-b*(1+\sec(d*x+c)))/(a-b)^{1/2}/b/d$

Rubi [A] time = 0.21, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {4005, 3832, 4004}

$$\frac{2\sqrt{a+b}(A-B) \cot(c+dx) \sqrt{\frac{b(1-\sec(c+dx))}{a+b}} \sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}} F\left(\sin^{-1}\left(\frac{\sqrt{a+b \sec(c+dx)}}{\sqrt{a+b}}\right) \middle| \frac{a+b}{a-b}\right) + 2B(a-b)\sqrt{a+b} \cot(c+dx)}{bd}$$

Antiderivative was successfully verified.

[In] Int[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/Sqrt[a + b*Sec[c + d*x]], x]

[Out] $(-2*(a-b)*\text{Sqrt}[a+b]*B*\text{Cot}[c+d*x]*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[a+b*\text{Sec}[c+d*x]]/\text{Sqrt}[a+b]], (a+b)/(a-b)]*\text{Sqrt}[(b*(1-\text{Sec}[c+d*x]))/(a+b)]*\text{Sqrt}[-((b*(1+\text{Sec}[c+d*x]))/(a-b))]/(b^2*d) + (2*\text{Sqrt}[a+b]*(A-B)*\text{Cot}[c+d*x]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[a+b*\text{Sec}[c+d*x]]/\text{Sqrt}[a+b]], (a+b)/(a-b)]*\text{Sqrt}[(b*(1-\text{Sec}[c+d*x]))/(a+b)]*\text{Sqrt}[-((b*(1+\text{Sec}[c+d*x]))/(a-b))])/(b*d)$

Rule 3832

Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)])/ (b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 4004

Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +

$f*x)))/(a - b)) * \text{EllipticE}[\text{ArcSin}[\text{Sqrt}[a + b*\text{Csc}[e + f*x]]/\text{Rt}[a + (b*B)/A, 2]], (a*A + b*B)/(a*A - b*B)]/(b^2*f*\text{Cot}[e + f*x]), x] /; \text{FreeQ}\{a, b, e, f, A, B\}, x] \ \&\& \ \text{NeQ}[a^2 - b^2, 0] \ \&\& \ \text{EqQ}[A^2 - B^2, 0]$

Rule 4005

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(\text{csc}[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)))/\text{Sqrt}[\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] :> \text{Dist}[A - B, \text{Int}[\text{Csc}[e + f*x]/\text{Sqrt}[a + b*\text{Csc}[e + f*x]], x], x] + \text{Dist}[B, \text{Int}[(\text{Csc}[e + f*x]*(1 + \text{Csc}[e + f*x]))/\text{Sqrt}[a + b*\text{Csc}[e + f*x]], x], x] /; \text{FreeQ}\{a, b, e, f, A, B\}, x] \ \&\& \ \text{NeQ}[a^2 - b^2, 0] \ \&\& \ \text{NeQ}[A^2 - B^2, 0]$

Rubi steps

$$\int \frac{\sec(c + dx)(A + B \sec(c + dx))}{\sqrt{a + b \sec(c + dx)}} dx = (A - B) \int \frac{\sec(c + dx)}{\sqrt{a + b \sec(c + dx)}} dx + B \int \frac{\sec(c + dx)(1 + \sec(c + dx))}{\sqrt{a + b \sec(c + dx)}} dx$$

$$= -\frac{2(a - b)\sqrt{a + b} B \cot(c + dx) E\left(\sin^{-1}\left(\frac{\sqrt{a + b \sec(c + dx)}}{\sqrt{a + b}}\right) \middle| \frac{a + b}{a - b}\right) \sqrt{\frac{b(1 - \sec(c + dx))}{a + b}}}{b^2 d}$$

Mathematica [A] time = 14.40, size = 356, normalized size = 1.70

$$\frac{2B \sin(c + dx)(a \cos(c + dx) + b)(A + B \sec(c + dx))}{bd\sqrt{a + b \sec(c + dx)}(A \cos(c + dx) + B)} - \frac{2\sqrt{\cos^2\left(\frac{1}{2}(c + dx)\right) \sec(c + dx)(A + B \sec(c + dx))}{b^2 d} \left(-2b\right)$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/Sqrt[a + b*Sec[c + d*x]],x]

[Out] (2*B*(b + a*Cos[c + d*x])*(A + B*Sec[c + d*x])*Sin[c + d*x])/(b*d*(B + A*Cos[c + d*x])*Sqrt[a + b*Sec[c + d*x]]) - (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x])*(A + B*Sec[c + d*x])*(2*(a + b)*B*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])])*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(A + B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + B*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(b*d*(B + A*Cos[c + d*x])*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]])

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{B \sec(dx+c)^2 + A \sec(dx+c)}{\sqrt{b \sec(dx+c) + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")

[Out] integral((B*sec(d*x + c)^2 + A*sec(d*x + c))/sqrt(b*sec(d*x + c) + a), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx+c) + A) \sec(dx+c)}{\sqrt{b \sec(dx+c) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)/sqrt(b*sec(d*x + c) + a), x)

maple [B] time = 2.14, size = 829, normalized size = 3.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2), x)

[Out]
$$\begin{aligned} & -2/d*(b+a*\cos(d*x+c))/\cos(d*x+c))^(1/2)*(1+\cos(d*x+c))^2*(-1+\cos(d*x+c))^2 \\ & *(A*\cos(d*x+c)*(\cos(d*x+c)/(1+\cos(d*x+c)))^(1/2)*((b+a*\cos(d*x+c))/(1+\cos(d \\ & *x+c)))/(a+b))^(1/2)*\sin(d*x+c)*\text{EllipticF}((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/ \\ & (a+b))^(1/2))*b+B*\cos(d*x+c)*(\cos(d*x+c)/(1+\cos(d*x+c)))^(1/2)*((b+a*\cos(d* \\ & x+c))/(1+\cos(d*x+c)))/(a+b))^(1/2)*\sin(d*x+c)*\text{EllipticF}((-1+\cos(d*x+c))/\sin(\\ & d*x+c), ((a-b)/(a+b))^(1/2))*b-B*\cos(d*x+c)*(\cos(d*x+c)/(1+\cos(d*x+c)))^(1/2) \\ &)*(b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^(1/2)*\sin(d*x+c)*\text{EllipticE}((-1+co \\ & s(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^(1/2))*a-B*\cos(d*x+c)*(\cos(d*x+c)/(1+\cos \\ & (d*x+c)))^(1/2)*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^(1/2)*\sin(d*x+c)*\text{El \\ & lipticE}((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^(1/2))*b+A*(\cos(d*x+c)/(1+ \\ & \cos(d*x+c)))^(1/2)*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^(1/2)*\sin(d*x+c) \\ & *\text{EllipticF}((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^(1/2))*b+B*(\cos(d*x+c)/ \\ & (1+\cos(d*x+c)))^(1/2)*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^(1/2)*\sin(d*x \\ & +c)*\text{EllipticF}((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^(1/2))*b-B*(\cos(d*x+ \end{aligned}$$

$c/(1+\cos(dx+c))^{1/2} * ((b+a\cos(dx+c))/(1+\cos(dx+c)) / (a+b))^{1/2} * \sin(dx+c) * \text{EllipticE}((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{1/2}) * a - B * (\cos(dx+c)/(1+\cos(dx+c))^{1/2} * ((b+a\cos(dx+c))/(1+\cos(dx+c)) / (a+b))^{1/2} * \sin(dx+c) * \text{EllipticE}((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{1/2}) * b + B * \cos(dx+c)^2 * a - B * \cos(dx+c) * a + b * B * \cos(dx+c) - B * b) / \sin(dx+c)^5 / (b+a\cos(dx+c)) / b$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx+c) + A) \sec(dx+c)}{\sqrt{b \sec(dx+c) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(dx+c)*(A+B*sec(dx+c))/(a+b*sec(dx+c))^(1/2),x, algorithm="maxima")

[Out] integrate((B*sec(dx+c) + A)*sec(dx+c)/sqrt(b*sec(dx+c) + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\cos(c+dx) \sqrt{a + \frac{b}{\cos(c+dx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + dx))/(cos(c + dx)*(a + b/cos(c + dx))^(1/2)),x)

[Out] int((A + B/cos(c + dx))/(cos(c + dx)*(a + b/cos(c + dx))^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \sec(c + dx)}{\sqrt{a + b \sec(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(dx+c)*(A+B*sec(dx+c))/(a+b*sec(dx+c))**(1/2),x)

[Out] Integral((A + B*sec(c + dx))*sec(c + dx)/sqrt(a + b*sec(c + dx)), x)

$$3.374 \quad \int \frac{A+B \sec(c+dx)}{\sqrt{a+b \sec(c+dx)}} dx$$

Optimal. Leaf size=208

$$\frac{2B\sqrt{a+b} \cot(c+dx) \sqrt{\frac{b(1-\sec(c+dx))}{a+b}} \sqrt{\frac{b(\sec(c+dx)+1)}{a-b}} F\left(\sin^{-1}\left(\frac{\sqrt{a+b \sec(c+dx)}}{\sqrt{a+b}}\right) \middle| \frac{a+b}{a-b}\right) + 2A\sqrt{a+b} \cot(c+dx) \sqrt{\frac{b(1-\sec(c+dx))}{a+b}}}{bd}$$

[Out] $2*B*\cot(d*x+c)*\text{EllipticF}((a+b*\sec(d*x+c))^{1/2}/(a+b)^{1/2}, ((a+b)/(a-b))^{1/2})*(a+b)^{1/2}*(b*(1-\sec(d*x+c))/(a+b))^{1/2}*(-b*(1+\sec(d*x+c))/(a-b))^{1/2}/b/d - 2*A*\cot(d*x+c)*\text{EllipticPi}((a+b*\sec(d*x+c))^{1/2}/(a+b)^{1/2}, (a+b)/a, ((a+b)/(a-b))^{1/2})*(a+b)^{1/2}*(b*(1-\sec(d*x+c))/(a+b))^{1/2}*(-b*(1+\sec(d*x+c))/(a-b))^{1/2}/a/d$

Rubi [A] time = 0.12, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3921, 3784, 3832}

$$\frac{2B\sqrt{a+b} \cot(c+dx) \sqrt{\frac{b(1-\sec(c+dx))}{a+b}} \sqrt{\frac{b(\sec(c+dx)+1)}{a-b}} F\left(\sin^{-1}\left(\frac{\sqrt{a+b \sec(c+dx)}}{\sqrt{a+b}}\right) \middle| \frac{a+b}{a-b}\right) + 2A\sqrt{a+b} \cot(c+dx) \sqrt{\frac{b(1-\sec(c+dx))}{a+b}}}{bd}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sec[c + d*x])/Sqrt[a + b*Sec[c + d*x]], x]

[Out] $(2*\text{Sqrt}[a + b]*B*\text{Cot}[c + d*x]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[a + b*\text{Sec}[c + d*x]]/\text{Sqrt}[a + b]], (a + b)/(a - b)*\text{Sqrt}[(b*(1 - \text{Sec}[c + d*x]))/(a + b)]*\text{Sqrt}[-((b*(1 + \text{Sec}[c + d*x]))/(a - b))])/(b*d) - (2*A*\text{Sqrt}[a + b]*\text{Cot}[c + d*x]*\text{EllipticPi}[(a + b)/a, \text{ArcSin}[\text{Sqrt}[a + b*\text{Sec}[c + d*x]]/\text{Sqrt}[a + b]], (a + b)/(a - b)]*\text{Sqrt}[(b*(1 - \text{Sec}[c + d*x]))/(a + b)]*\text{Sqrt}[-((b*(1 + \text{Sec}[c + d*x]))/(a - b))])/(a*d)$

Rule 3784

Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_.)], x_Symbol] :> Simp[(2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b))]/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 3832

Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]]

/Rt[a + b, 2]], (a + b)/(a - b)]/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 3921

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]

Rubi steps

$$\int \frac{A + B \sec(c + dx)}{\sqrt{a + b \sec(c + dx)}} dx = A \int \frac{1}{\sqrt{a + b \sec(c + dx)}} dx + B \int \frac{\sec(c + dx)}{\sqrt{a + b \sec(c + dx)}} dx$$

$$= \frac{2\sqrt{a+b} B \cot(c + dx) F\left(\sin^{-1}\left(\frac{\sqrt{a+b \sec(c+dx)}}{\sqrt{a+b}}\right) \middle| \frac{a+b}{a-b}\right) \sqrt{\frac{b(1-\sec(c+dx))}{a+b}} \sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{bd}$$

Mathematica [A] time = 2.34, size = 145, normalized size = 0.70

$$\frac{4 \cos^2\left(\frac{1}{2}(c + dx)\right) \sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}} \sec(c + dx) \sqrt{\frac{a \cos(c+dx)+b}{(a+b)(\cos(c+dx)+1)}} \left((B - A) F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c + dx)\right)\right) \middle| \frac{a-b}{a+b}\right) + 2A \Pi\left(\frac{a-b}{a+b}\right) \right)}{d \sqrt{a + b \sec(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sec[c + d*x])/Sqrt[a + b*Sec[c + d*x]],x]

[Out] (4*Cos[(c + d*x)/2]^2*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*((-A + B)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*A*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)])*Sec[c + d*x])/(d*Sqrt[a + b*Sec[c + d*x]])

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{B \sec(dx + c) + A}{\sqrt{b \sec(dx + c) + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")

[Out] integral((B*sec(d*x + c) + A)/sqrt(b*sec(d*x + c) + a), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{\sqrt{b \sec(dx + c) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)/sqrt(b*sec(d*x + c) + a), x)

maple [A] time = 2.01, size = 215, normalized size = 1.03

$$\frac{2\sqrt{\frac{b+a\cos(dx+c)}{\cos(dx+c)}}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{\frac{b+a\cos(dx+c)}{(1+\cos(dx+c))(a+b)}}(1+\cos(dx+c))^2(-1+\cos(dx+c))\left(A\operatorname{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\right)}{d(b+a\cos(dx+c))\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x)

[Out] -2/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1+cos(d*x+c))^2*(-1+cos(d*x+c))* (A*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))-2*A*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))-B*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2)))/(b+a*cos(d*x+c))/sin(d*x+c)^2

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{\sqrt{b \sec(dx + c) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)/sqrt(b*sec(d*x + c) + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\sqrt{a + \frac{b}{\cos(c+dx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(a + b/cos(c + d*x))^(1/2), x)
```

```
[Out] int((A + B/cos(c + d*x))/(a + b/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \sec(c + dx)}{\sqrt{a + b \sec(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(1/2), x)
```

```
[Out] Integral((A + B*sec(c + d*x))/sqrt(a + b*sec(c + d*x)), x)
```


$$3.375 \quad \int \frac{\cos(c+dx)(A+B \sec(c+dx))}{\sqrt{a+b \sec(c+dx)}} dx$$

Optimal. Leaf size=348

$$\frac{\sqrt{a+b}(Ab-2aB) \cot(c+dx) \sqrt{\frac{b(1-\sec(c+dx))}{a+b}} \sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}} \Pi\left(\frac{a+b}{a}; \sin^{-1}\left(\frac{\sqrt{a+b \sec(c+dx)}}{\sqrt{a+b}}\right) \middle| \frac{a+b}{a-b}\right) + A \sin(c+dx)}{a^2 d}$$

[Out] A*(a-b)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2), ((a+b)/(a-b))^(1/2))* (a+b)^(1/2)*(b*(1-sec(d*x+c)))/(a+b)^(1/2)*(-b*(1+sec(d*x+c)))/(a-b)^(1/2)/a/b/d+A*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2), ((a+b)/(a-b))^(1/2))* (a+b)^(1/2)*(b*(1-sec(d*x+c)))/(a+b)^(1/2)*(-b*(1+sec(d*x+c)))/(a-b)^(1/2)/a/d+(A*b-2*B*a)*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2), (a+b)/a, ((a+b)/(a-b))^(1/2))* (a+b)^(1/2)*(b*(1-sec(d*x+c)))/(a+b)^(1/2)*(-b*(1+sec(d*x+c)))/(a-b)^(1/2)/a^2/d+A*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/a/d

Rubi [A] time = 0.40, antiderivative size = 348, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4034, 4059, 3921, 3784, 3832, 4004}

$$\frac{\sqrt{a+b}(Ab-2aB) \cot(c+dx) \sqrt{\frac{b(1-\sec(c+dx))}{a+b}} \sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}} \Pi\left(\frac{a+b}{a}; \sin^{-1}\left(\frac{\sqrt{a+b \sec(c+dx)}}{\sqrt{a+b}}\right) \middle| \frac{a+b}{a-b}\right) + A \sin(c+dx)}{a^2 d}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]*(A + B*Sec[c + d*x]))/Sqrt[a + b*Sec[c + d*x]], x]

[Out] (A*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))]/(a*b*d) + (A*Sqrt[a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a*d) + (Sqrt[a + b]*(A*b - 2*a*B)*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a^2*d) + (A*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(a*d)

Rule 3784

Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_.)], x_Symbol] :> Simp[(2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b))]/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&

NeQ[a^2 - b^2, 0]

Rule 3832

Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)]/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 3921

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]

Rule 4004

Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A, 2]], (a*A + b*B)/(a*A - b*B)]/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]

Rule 4034

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^m*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + A*a*(n + 1)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]

Rule 4059

Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A - C*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned}
\int \frac{\cos(c+dx)(A+B\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}} dx &= \frac{A\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{ad} - \frac{\int \frac{\frac{1}{2}(Ab-2aB)+\frac{1}{2}Ab\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}} dx}{a} \\
&= \frac{A\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{ad} - \frac{\int \frac{\frac{1}{2}(Ab-2aB)-\frac{1}{2}Ab\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}} dx}{a} - \frac{(Ab)}{a} \\
&= \frac{A(a-b)\sqrt{a+b}\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)\middle|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}}{abd} \\
&= \frac{A(a-b)\sqrt{a+b}\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)\middle|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}}{abd}
\end{aligned}$$

Mathematica [C] time = 17.04, size = 1027, normalized size = 2.95

$$\sqrt{b+a\cos(c+dx)}\sqrt{\sec(c+dx)}\sqrt{\frac{1}{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}}\sqrt{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}\left(-aA\sqrt{\frac{b-a}{a+b}}\sqrt{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}\tan\right)$$

Warning: Unable to verify antiderivative.

```

[In] Integrate[(Cos[c + d*x]*(A + B*Sec[c + d*x]))/Sqrt[a + b*Sec[c + d*x]],x]
[Out] (Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(a*A*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]*Sqrt[1 - Tan[(c + d*x)/2]^2] + A*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]*Sqrt[1 - Tan[(c + d*x)/2]^2] - a*A*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^3*Sqrt[1 - Tan[(c + d*x)/2]^2] + A*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^3*Sqrt[1 - Tan[(c + d*x)/2]^2] + (2*I)*A*b*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (4*I)*a*B*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (2*I)*A*b*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (4*I)*a*B*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]

```

+ d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - I*A*(a - b)*EllipticE[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (2*I)*(A*b - a*B)*EllipticF[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)))/(a*Sqrt[(-a + b)/(a + b)]*d*Sqrt[a + b*Sec[c + d*x]]*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2))]

fricas [F] time = 49.79, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{B \cos(dx + c) \sec(dx + c) + A \cos(dx + c)}{\sqrt{b \sec(dx + c) + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")

[Out] integral((B*cos(d*x + c)*sec(d*x + c) + A*cos(d*x + c))/sqrt(b*sec(d*x + c) + a), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \cos(dx + c)}{\sqrt{b \sec(dx + c) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)/sqrt(b*sec(d*x + c) + a), x)

maple [B] time = 2.17, size = 1025, normalized size = 2.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x)

[Out] -1/d*(-1+cos(d*x+c))^2*(A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a+A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b-2*A*cos(d*x+c)*sin(d*x+c)

$c) * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * ((b+a*\cos(dx+c))/(1+\cos(dx+c)) / (a+b))^{1/2} * \text{EllipticPi}((-1+\cos(dx+c))/\sin(dx+c), -1, ((a-b)/(a+b))^{1/2}) * b + 4 * B * \cos(dx+c) * \sin(dx+c) * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * ((b+a*\cos(dx+c))/(1+\cos(dx+c)) / (a+b))^{1/2} * \text{EllipticPi}((-1+\cos(dx+c))/\sin(dx+c), -1, ((a-b)/(a+b))^{1/2}) * a - 2 * B * \cos(dx+c) * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * ((b+a*\cos(dx+c))/(1+\cos(dx+c)) / (a+b))^{1/2} * \sin(dx+c) * \text{EllipticF}((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{1/2}) * a + A * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * ((b+a*\cos(dx+c))/(1+\cos(dx+c)) / (a+b))^{1/2} * \text{EllipticE}((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{1/2}) * a * \sin(dx+c) + A * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * ((b+a*\cos(dx+c))/(1+\cos(dx+c)) / (a+b))^{1/2} * \text{EllipticE}((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{1/2}) * b * \sin(dx+c) - 2 * A * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * ((b+a*\cos(dx+c))/(1+\cos(dx+c)) / (a+b))^{1/2} * \text{EllipticPi}((-1+\cos(dx+c))/\sin(dx+c), -1, ((a-b)/(a+b))^{1/2}) * b * \sin(dx+c) + 4 * B * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * ((b+a*\cos(dx+c))/(1+\cos(dx+c)) / (a+b))^{1/2} * \text{EllipticPi}((-1+\cos(dx+c))/\sin(dx+c), -1, ((a-b)/(a+b))^{1/2}) * a * \sin(dx+c) - 2 * B * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * ((b+a*\cos(dx+c))/(1+\cos(dx+c)) / (a+b))^{1/2} * \sin(dx+c) * \text{EllipticF}((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{1/2}) * a + A * \cos(dx+c)^3 * a - A * \cos(dx+c)^2 * a + A * \cos(dx+c)^2 * b - A * \cos(dx+c) * b * (1+\cos(dx+c))^2 * ((b+a*\cos(dx+c))/\cos(dx+c))^{1/2} / (b+a*\cos(dx+c)) / \sin(dx+c)^5 / a$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx+c) + A) \cos(dx+c)}{\sqrt{b \sec(dx+c) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(dx+c)*(A+B*sec(dx+c))/(a+b*sec(dx+c))^(1/2),x, algorithm="maxima")

[Out] integrate((B*sec(dx+c) + A)*cos(dx+c)/sqrt(b*sec(dx+c) + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+dx) \left(A + \frac{B}{\cos(c+dx)} \right)}{\sqrt{a + \frac{b}{\cos(c+dx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c+dx)*(A+B/cos(c+dx)))/(a+b/cos(c+dx))^(1/2),x)

[Out] int((cos(c+dx)*(A+B/cos(c+dx)))/(a+b/cos(c+dx))^(1/2),x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c+dx)) \cos(c+dx)}{\sqrt{a + b \sec(c+dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*cos(c + d*x)/sqrt(a + b*sec(c + d*x)), x)
```

$$3.376 \quad \int \frac{\cos^2(c+dx)(A+B \sec(c+dx))}{\sqrt{a+b \sec(c+dx)}} dx$$

Optimal. Leaf size=435

$$\frac{(3Ab - 4aB) \sin(c + dx) \sqrt{a + b \sec(c + dx)}}{4a^2 d} - \frac{\sqrt{a + b} (3Ab - 2a(A + 2B)) \cot(c + dx) \sqrt{\frac{b(1 - \sec(c + dx))}{a + b}} \sqrt{-\frac{b(\sec(c + dx) + 1)}{a - b}}}{4a^2 d}$$

[Out] $-1/4*(a-b)*(3*A*b-4*B*a)*\cot(d*x+c)*\text{EllipticE}((a+b*\sec(d*x+c))^{(1/2)}/(a+b)^{(1/2)}, ((a+b)/(a-b))^{(1/2)}*(a+b)^{(1/2)}*(b*(1-\sec(d*x+c))/(a+b))^{(1/2)}*(-b*(1+\sec(d*x+c))/(a-b))^{(1/2)}/a^2/b/d-1/4*(3*A*b-2*a*(A+2*B))*\cot(d*x+c)*\text{EllipticF}((a+b*\sec(d*x+c))^{(1/2)}/(a+b)^{(1/2)}, ((a+b)/(a-b))^{(1/2)}*(a+b)^{(1/2)}*(b*(1-\sec(d*x+c))/(a+b))^{(1/2)}*(-b*(1+\sec(d*x+c))/(a-b))^{(1/2)}/a^2/d-1/4*(4*A*a^2+3*A*b^2-4*B*a*b)*\cot(d*x+c)*\text{EllipticPi}((a+b*\sec(d*x+c))^{(1/2)}/(a+b)^{(1/2)}, (a+b)/a, ((a+b)/(a-b))^{(1/2)}*(a+b)^{(1/2)}*(b*(1-\sec(d*x+c))/(a+b))^{(1/2)}*(-b*(1+\sec(d*x+c))/(a-b))^{(1/2)}/a^3/d-1/4*(3*A*b-4*B*a)*\sin(d*x+c)*(a+b*\sec(d*x+c))^{(1/2)}/a^2/d+1/2*A*\cos(d*x+c)*\sin(d*x+c)*(a+b*\sec(d*x+c))^{(1/2)}/a/d$

Rubi [A] time = 0.72, antiderivative size = 435, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {4034, 4104, 4058, 3921, 3784, 3832, 4004}

$$\frac{\sqrt{a + b} (4a^2 A - 4abB + 3Ab^2) \cot(c + dx) \sqrt{\frac{b(1 - \sec(c + dx))}{a + b}} \sqrt{-\frac{b(\sec(c + dx) + 1)}{a - b}} \Pi\left(\frac{a + b}{a}; \sin^{-1}\left(\frac{\sqrt{a + b \sec(c + dx)}}{\sqrt{a + b}}\right) \middle| \frac{a + b}{a - b}\right)}{4a^3 d}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^2*(A + B*Sec[c + d*x]))/Sqrt[a + b*Sec[c + d*x]],x]

[Out] $-((a - b)*\text{Sqrt}[a + b]*(3*A*b - 4*a*B)*\text{Cot}[c + d*x]*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[a + b*\text{Sec}[c + d*x]]/\text{Sqrt}[a + b]], (a + b)/(a - b)]*\text{Sqrt}[(b*(1 - \text{Sec}[c + d*x]))/(a + b)]*\text{Sqrt}[-((b*(1 + \text{Sec}[c + d*x]))/(a - b))]/(4*a^2*b*d) - (\text{Sqrt}[a + b]*(3*A*b - 2*a*(A + 2*B))*\text{Cot}[c + d*x]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[a + b*\text{Sec}[c + d*x]]/\text{Sqrt}[a + b]], (a + b)/(a - b)]*\text{Sqrt}[(b*(1 - \text{Sec}[c + d*x]))/(a + b)]*\text{Sqrt}[-((b*(1 + \text{Sec}[c + d*x]))/(a - b))]/(4*a^2*d) - (\text{Sqrt}[a + b]*(4*a^2*A + 3*A*b^2 - 4*a*b*B)*\text{Cot}[c + d*x]*\text{EllipticPi}[(a + b)/a, \text{ArcSin}[\text{Sqrt}[a + b*\text{Sec}[c + d*x]]/\text{Sqrt}[a + b]], (a + b)/(a - b)]*\text{Sqrt}[(b*(1 - \text{Sec}[c + d*x]))/(a + b)]*\text{Sqrt}[-((b*(1 + \text{Sec}[c + d*x]))/(a - b))]/(4*a^3*d) - ((3*A*b - 4*a*B)*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(4*a^2*d) + (A*\text{Cos}[c + d*x]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(2*a*d)$

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] := Simp[(2*Rt[a
+ b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)]/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_.)]/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_S
ymbol] := Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)]/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_
.) + (a_.)], x_Symbol] := Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)))/Sqrt[c
sc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] := Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
2]], (a*A + b*B)/(a*A - b*B)]/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4034

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (
a_.))^m*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*n), x] + Dis
t[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n
- A*b*(m + n + 1) + A*a*(n + 1)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x
]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0]
&& NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_
.))/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] := Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
```


+ Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0]

Rule 4104

Int[((A_.) + csc[(e_.) + (f_.)*(x_)])*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^m, x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]

Rubi steps

$$\begin{aligned} \int \frac{\cos^2(c + dx)(A + B \sec(c + dx))}{\sqrt{a + b \sec(c + dx)}} dx &= \frac{A \cos(c + dx) \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{2ad} - \int \frac{\cos(c + dx) \left(\frac{1}{2}(3Ab - 4aB) - a \right)}{\sqrt{a + b \sec(c + dx)}} dx \\ &= -\frac{(3Ab - 4aB) \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{4a^2d} + \frac{A \cos(c + dx) \sqrt{a + b \sec(c + dx)}}{2a} \\ &= -\frac{(3Ab - 4aB) \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{4a^2d} + \frac{A \cos(c + dx) \sqrt{a + b \sec(c + dx)}}{2a} \\ &= -\frac{(a - b) \sqrt{a + b} (3Ab - 4aB) \cot(c + dx) E \left(\sin^{-1} \left(\frac{\sqrt{a + b \sec(c + dx)}}{\sqrt{a + b}} \right) \middle| \frac{a + b}{a - b} \right)}{4a^2bd} \\ &= -\frac{(a - b) \sqrt{a + b} (3Ab - 4aB) \cot(c + dx) E \left(\sin^{-1} \left(\frac{\sqrt{a + b \sec(c + dx)}}{\sqrt{a + b}} \right) \middle| \frac{a + b}{a - b} \right)}{4a^2bd} \end{aligned}$$

Mathematica [C] time = 16.24, size = 1639, normalized size = 3.77

result too large to display

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]^2*(A + B*Sec[c + d*x]))/Sqrt[a + b*Sec[c + d*x]], x]

[Out] (A*(b + a*cos[c + d*x])*Sec[c + d*x]*Sin[2*(c + d*x)]/(4*a*d*Sqrt[a + b*Sec[c + d*x]]) + (Sqrt[b + a*cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sqrt[(a + b - a

```

*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2))*(-3*a
*A*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2] - 3*A*b^2*Sqrt[(-a + b)/(a + b
)]*Tan[(c + d*x)/2] + 4*a^2*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)/2] + 4*a
*b*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)/2] + 6*a*A*b*Sqrt[(-a + b)/(a + b
)]*Tan[(c + d*x)/2]^3 - 8*a^2*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)/2]^3 -
3*a*A*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 + 3*A*b^2*Sqrt[(-a + b)/
(a + b)]*Tan[(c + d*x)/2]^5 + 4*a^2*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)/
2]^5 - 4*a*b*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)/2]^5 - (8*I)*a^2*A*Elli
pticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2
]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c +
d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (6*I)*A*b^2*EllipticPi[-((a +
b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a
- b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*
Tan[(c + d*x)/2]^2)/(a + b)] + (8*I)*a*b*B*EllipticPi[-((a + b)/(a - b)), I
*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1
- Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/
2]^2)/(a + b)] - (8*I)*a^2*A*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[
(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sq
rt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c +
d*x)/2]^2)/(a + b)] - (6*I)*A*b^2*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[
Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]
^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[
(c + d*x)/2]^2)/(a + b)] + (8*I)*a*b*B*EllipticPi[-((a + b)/(a - b)), I*Arc
Sinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*
x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b
*Tan[(c + d*x)/2]^2)/(a + b)] - I*(a - b)*(-3*A*b + 4*a*B)*EllipticE[I*ArcS
inh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan
[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^
2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (2*I)*(2*a^2*A + 3*A*b^2 - a*b*(A + 4*
B))*EllipticF[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(
a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b -
a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)))/(4*a^2*Sqrt[(-a +
b)/(a + b)]*d*Sqrt[a + b*Sec[c + d*x]]*(-1 + Tan[(c + d*x)/2]^2)*Sqrt[(1 +
Tan[(c + d*x)/2]^2)/(1 - Tan[(c + d*x)/2]^2)]*(a*(-1 + Tan[(c + d*x)/2]^2)
- b*(1 + Tan[(c + d*x)/2]^2)))

```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \cos(dx + c)^2}{\sqrt{b \sec(dx + c) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^2/sqrt(b*sec(d*x + c) + a), x)

maple [B] time = 2.16, size = 1886, normalized size = 4.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x)

[Out]
$$\begin{aligned} & -1/4/d*(-1+\cos(d*x+c))^2*(6*A*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*EllipticPi((-1+\cos(d*x+c))/\sin(d*x+c),-1,((a-b)/(a+b))^{1/2})*b^2*\sin(d*x+c)-4*B*\cos(d*x+c)^2*a^2-3*A*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*EllipticE((-1+\cos(d*x+c))/\sin(d*x+c),((a-b)/(a+b))^{1/2})*b^2*\sin(d*x+c)+4*B*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*EllipticE((-1+\cos(d*x+c))/\sin(d*x+c),((a-b)/(a+b))^{1/2})*a^2*\sin(d*x+c)+8*A*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*EllipticPi((-1+\cos(d*x+c))/\sin(d*x+c),-1,((a-b)/(a+b))^{1/2})*a^2*\sin(d*x+c)+4*B*\cos(d*x+c)*\sin(d*x+c)*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*EllipticE((-1+\cos(d*x+c))/\sin(d*x+c),((a-b)/(a+b))^{1/2})*a*b-3*A*\cos(d*x+c)*\sin(d*x+c)*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*EllipticE((-1+\cos(d*x+c))/\sin(d*x+c),((a-b)/(a+b))^{1/2})*a*b+2*A*\cos(d*x+c)*\sin(d*x+c)*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*EllipticF((-1+\cos(d*x+c))/\sin(d*x+c),((a-b)/(a+b))^{1/2})*a*b-3*A*\cos(d*x+c)^2*b^2+3*A*\cos(d*x+c)*b^2-8*B*\cos(d*x+c)*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*EllipticPi((-1+\cos(d*x+c))/\sin(d*x+c),-1,((a-b)/(a+b))^{1/2})*\sin(d*x+c)*a*b+3*A*\cos(d*x+c)^2*a*b-2*A*\cos(d*x+c)*a*b+4*B*\cos(d*x+c)^2*a*b-4*B*\cos(d*x+c)*a*b-4*A*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*EllipticF((-1+\cos(d*x+c))/\sin(d*x+c),((a-b)/(a+b))^{1/2})*a^2*\sin(d*x+c)-A*\cos(d*x+c)^3*a*b+4*B*\cos(d*x+c)^3*a^2+8*A*\cos(d*x+c)*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*EllipticPi((-1+\cos(d*x+c))/\sin(d*x+c),-1,((a-b)/(a+b))^{1/2})*\sin(d*x+c)*a^2+6*A*\cos(d*x+c)*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*Ellipti \end{aligned}$$

$$\begin{aligned} & c\text{Pi}((-1+\cos(dx+c))/\sin(dx+c), -1, ((a-b)/(a+b))^{1/2}) * \sin(dx+c) * b^2 - 4 * A * \cos(dx+c) * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * ((b+a*\cos(dx+c))/(1+\cos(dx+c))) / (a+b)^{1/2} * \text{EllipticF}((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{1/2}) * \sin(dx+c) * a^2 + 2 * A * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * ((b+a*\cos(dx+c))/(1+\cos(dx+c))) / (a+b)^{1/2} * \text{EllipticF}((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{1/2}) * a * b * \sin(dx+c) - 3 * A * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * ((b+a*\cos(dx+c))/(1+\cos(dx+c))) / (a+b)^{1/2} * \text{EllipticE}((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{1/2}) * a * b * \sin(dx+c) - 8 * B * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * ((b+a*\cos(dx+c))/(1+\cos(dx+c))) / (a+b)^{1/2} * \text{EllipticPi}((-1+\cos(dx+c))/\sin(dx+c), -1, ((a-b)/(a+b))^{1/2}) * a * b * \sin(dx+c) + 4 * B * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * ((b+a*\cos(dx+c))/(1+\cos(dx+c))) / (a+b)^{1/2} * \text{EllipticE}((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{1/2}) * a * b * \sin(dx+c) - 2 * A * \cos(dx+c)^2 * a^2 + 2 * A * \cos(dx+c)^4 * a^2 - 3 * A * \cos(dx+c) * \sin(dx+c) * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * ((b+a*\cos(dx+c))/(1+\cos(dx+c))) / (a+b)^{1/2} * \text{EllipticE}((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{1/2}) * b^2 + 4 * B * \cos(dx+c) * \sin(dx+c) * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * ((b+a*\cos(dx+c))/(1+\cos(dx+c))) / (a+b)^{1/2} * \text{EllipticE}((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{1/2}) * a^2 * (1+\cos(dx+c))^2 * ((b+a*\cos(dx+c))/\cos(dx+c))^{1/2} / (b+a*\cos(dx+c))/\sin(dx+c)^5 / a^2 \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx+c) + A) \cos(dx+c)^2}{\sqrt{b \sec(dx+c) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(dx+c)^2*(A+B*sec(dx+c))/(a+b*sec(dx+c))^(1/2),x, algorithm="maxima")

[Out] integrate((B*sec(dx+c) + A)*cos(dx+c)^2/sqrt(b*sec(dx+c) + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+dx)^2 \left(A + \frac{B}{\cos(c+dx)} \right)}{\sqrt{a + \frac{b}{\cos(c+dx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c+dx)^2*(A+B/cos(c+dx)))/(a+b/cos(c+dx))^(1/2),x)

[Out] int((cos(c+dx)^2*(A+B/cos(c+dx)))/(a+b/cos(c+dx))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c+dx)) \cos^2(c+dx)}{\sqrt{a + b \sec(c+dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*cos(c + d*x)**2/sqrt(a + b*sec(c + d*x)), x)
```

$$3.377 \quad \int \frac{\cos^3(c+dx)(A+B \sec(c+dx))}{\sqrt{a+b \sec(c+dx)}} dx$$

Optimal. Leaf size=525

$$\frac{(5Ab - 6aB) \sin(c + dx) \cos(c + dx) \sqrt{a + b \sec(c + dx)}}{12a^2d} + \frac{(16a^2A - 18abB + 15Ab^2) \sin(c + dx) \sqrt{a + b \sec(c + dx)}}{24a^3d}$$

[Out] $1/24*(a-b)*(16*A*a^2+15*A*b^2-18*B*a*b)*\cot(d*x+c)*\text{EllipticE}((a+b*\sec(d*x+c))^{1/2}/(a+b)^{1/2},((a+b)/(a-b))^{1/2})*(a+b)^{1/2}*(b*(1-\sec(d*x+c)))/(a+b))^{1/2}*(-b*(1+\sec(d*x+c)))/(a-b))^{1/2}/a^3/b/d+1/24*(16*A*a^2-10*A*a*b+15*A*b^2+12*B*a^2-18*B*a*b)*\cot(d*x+c)*\text{EllipticF}((a+b*\sec(d*x+c))^{1/2}/(a+b)^{1/2},((a+b)/(a-b))^{1/2})*(a+b)^{1/2}*(b*(1-\sec(d*x+c)))/(a+b))^{1/2}*(-b*(1+\sec(d*x+c)))/(a-b))^{1/2}/a^3/d+1/8*(4*A*a^2*b+5*A*b^3-8*B*a^3-6*B*a*b^2)*\cot(d*x+c)*\text{EllipticPi}((a+b*\sec(d*x+c))^{1/2}/(a+b)^{1/2},(a+b)/a,((a+b)/(a-b))^{1/2})*(a+b)^{1/2}*(b*(1-\sec(d*x+c)))/(a+b))^{1/2}*(-b*(1+\sec(d*x+c)))/(a-b))^{1/2}/a^4/d+1/24*(16*A*a^2+15*A*b^2-18*B*a*b)*\sin(d*x+c)*(a+b*\sec(d*x+c))^{1/2}/a^3/d-1/12*(5*A*b-6*B*a)*\cos(d*x+c)*\sin(d*x+c)*(a+b*\sec(d*x+c))^{1/2}/a^2/d+1/3*A*\cos(d*x+c)^2*\sin(d*x+c)*(a+b*\sec(d*x+c))^{1/2}/a/d$

Rubi [A] time = 1.17, antiderivative size = 525, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {4034, 4104, 4058, 3921, 3784, 3832, 4004}

$$\frac{(16a^2A - 18abB + 15Ab^2) \sin(c + dx) \sqrt{a + b \sec(c + dx)}}{24a^3d} + \frac{\sqrt{a + b} (16a^2A + 12a^2B - 10aAb - 18abB + 15Ab^2)}{24a^3d}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^3*(A + B*Sec[c + d*x]))/Sqrt[a + b*Sec[c + d*x]],x]

[Out] $((a - b)*\text{Sqrt}[a + b]*(16*a^2*A + 15*A*b^2 - 18*a*b*B)*\text{Cot}[c + d*x]*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[a + b*\text{Sec}[c + d*x]]/\text{Sqrt}[a + b]],(a + b)/(a - b)]*\text{Sqrt}[(b*(1 - \text{Sec}[c + d*x]))/(a + b)]*\text{Sqrt}[-((b*(1 + \text{Sec}[c + d*x]))/(a - b))])/(24*a^3*b*d) + (\text{Sqrt}[a + b]*(16*a^2*A - 10*a*A*b + 15*A*b^2 + 12*a^2*B - 18*a*b*B)*\text{Cot}[c + d*x]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[a + b*\text{Sec}[c + d*x]]/\text{Sqrt}[a + b]],(a + b)/(a - b)]*\text{Sqrt}[(b*(1 - \text{Sec}[c + d*x]))/(a + b)]*\text{Sqrt}[-((b*(1 + \text{Sec}[c + d*x]))/(a - b))])/(24*a^3*d) + (\text{Sqrt}[a + b]*(4*a^2*A*b + 5*A*b^3 - 8*a^3*B - 6*a*b^2*B)*\text{Cot}[c + d*x]*\text{EllipticPi}[(a + b)/a, \text{ArcSin}[\text{Sqrt}[a + b*\text{Sec}[c + d*x]]/\text{Sqrt}[a + b]],(a + b)/(a - b)]*\text{Sqrt}[(b*(1 - \text{Sec}[c + d*x]))/(a + b)]*\text{Sqrt}[-((b*(1 + \text{Sec}[c + d*x]))/(a - b))])/(8*a^4*d) + ((16*a^2*A + 15*A*b^2 - 18*a*b*B)*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(24*a^3*d) - ((5*A*b - 6*a*B)*\text{Cos}[c + d*x]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(12*a^2*d) + (A*\text{Cos}[c + d*x]^2*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(3*a*d)$

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_.)], x_Symbol] := Simp[(2*Rt[a
+ b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)]/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_S
ymbol] := Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)]/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_.)], x_Symbol] := Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] := Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
2]], (a*A + b*B)/(a*A - b*B)]/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4034

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_.))^m*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] := Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f^n), x] + Dis
t[1/(a*d^n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n
- A*b*(m + n + 1) + A*a*(n + 1)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x
]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0]
&& NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
```

```

))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] := Int[(A + (B - C)
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
+ Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]

```

Rule 4104

```

Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*csc[(e_.) + (f_.)*(x_)]*(d_.)^n*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] := Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x] /; FreeQ[{a, b, d,
e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]

```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^3(c+dx)(A+B \sec(c+dx))}{\sqrt{a+b \sec(c+dx)}} dx &= \frac{A \cos^2(c+dx) \sqrt{a+b \sec(c+dx)} \sin(c+dx)}{3ad} - \int \frac{\cos^2(c+dx) \left(\frac{1}{2}(5Ab-6aB)-\right)}{\sqrt{a+b \sec(c+dx)}} dx \\
&= -\frac{(5Ab-6aB) \cos(c+dx) \sqrt{a+b \sec(c+dx)} \sin(c+dx)}{12a^2d} + \frac{A \cos^2(c+dx)}{3ad} \\
&= \frac{(16a^2A+15Ab^2-18abB) \sqrt{a+b \sec(c+dx)} \sin(c+dx)}{24a^3d} - \frac{(5Ab-6aB) \cos(c+dx) \sqrt{a+b \sec(c+dx)} \sin(c+dx)}{12a^2d} \\
&= \frac{(16a^2A+15Ab^2-18abB) \sqrt{a+b \sec(c+dx)} \sin(c+dx)}{24a^3d} - \frac{(5Ab-6aB) \cos(c+dx) \sqrt{a+b \sec(c+dx)} \sin(c+dx)}{12a^2d} \\
&= \frac{(a-b) \sqrt{a+b} (16a^2A+15Ab^2-18abB) \cot(c+dx) E\left(\sin^{-1}\left(\frac{\sqrt{a+b \sec(c+dx)}}{\sqrt{a+b}}\right)\right)}{24a^3bd} \\
&= \frac{(a-b) \sqrt{a+b} (16a^2A+15Ab^2-18abB) \cot(c+dx) E\left(\sin^{-1}\left(\frac{\sqrt{a+b \sec(c+dx)}}{\sqrt{a+b}}\right)\right)}{24a^3bd}
\end{aligned}$$

Mathematica [B] time = 19.79, size = 1569, normalized size = 2.99

result too large to display

Warning: Unable to verify antiderivative.


```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Sec[c + d*x]))/Sqrt[a + b*Sec[c + d*x]],x]
[Out] ((b + a*cos[c + d*x])*Sec[c + d*x]*((A*sin[c + d*x])/(12*a) + ((-5*A*b + 6*
a*B)*Sin[2*(c + d*x)]/(24*a^2) + (A*sin[3*(c + d*x)]/(12*a)))/(d*Sqrt[a +
b*Sec[c + d*x]]) - (Sqrt[b + a*cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sqrt[(1 -
Tan[(c + d*x)/2]^2)^(-1)]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d
*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]*(16*a^3*A*Tan[(c + d*x)/2] + 16*a^2*A*b
*Tan[(c + d*x)/2] + 15*a*A*b^2*Tan[(c + d*x)/2] + 15*A*b^3*Tan[(c + d*x)/2]
- 18*a^2*b*B*Tan[(c + d*x)/2] - 18*a*b^2*B*Tan[(c + d*x)/2] - 32*a^3*A*Tan
[(c + d*x)/2]^3 - 30*a*A*b^2*Tan[(c + d*x)/2]^3 + 36*a^2*b*B*Tan[(c + d*x)/
2]^3 + 16*a^3*A*Tan[(c + d*x)/2]^5 - 16*a^2*A*b*Tan[(c + d*x)/2]^5 + 15*a*A
*b^2*Tan[(c + d*x)/2]^5 - 15*A*b^3*Tan[(c + d*x)/2]^5 - 18*a^2*b*B*Tan[(c +
d*x)/2]^5 + 18*a*b^2*B*Tan[(c + d*x)/2]^5 - 24*a^2*A*b*EllipticPi[-1, ArcS
in[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a
+ b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 30*A*b^3*Ell
ipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*
x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)
] + 48*a^3*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt
[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*
x)/2]^2)/(a + b)] + 36*a*b^2*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a
- b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]
^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 24*a^2*A*b*EllipticPi[-1, ArcSin[Tan[
(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]
^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] -
30*A*b^3*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c +
d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2
+ b*Tan[(c + d*x)/2]^2)/(a + b)] + 48*a^3*B*EllipticPi[-1, ArcSin[Tan[(c +
d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*
Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 36*a*
b^2*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*
x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b
*Tan[(c + d*x)/2]^2)/(a + b)] + (a + b)*(16*a^2*A + 15*A*b^2 - 18*a*b*B)*El
lipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]
^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c
+ d*x)/2]^2)/(a + b)] - 2*a*(5*A*b^2 + 2*a*b*(A - 3*B) + 12*a^2*B)*Ellipti
cF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*
(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*
x)/2]^2)/(a + b)))/(24*a^3*d*Sqrt[a + b*Sec[c + d*x]]*Sqrt[1 + Tan[(c + d*
x)/2]^2]*(a*(-1 + Tan[(c + d*x)/2]^2) - b*(1 + Tan[(c + d*x)/2]^2)))
```

fricas [F] time = 4.12, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{B \cos(dx + c)^3 \sec(dx + c) + A \cos(dx + c)^3}{\sqrt{b \sec(dx + c) + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")

[Out] integral((B*cos(d*x + c)^3*sec(d*x + c) + A*cos(d*x + c)^3)/sqrt(b*sec(d*x + c) + a), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \cos(dx + c)^3}{\sqrt{b \sec(dx + c) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^3/sqrt(b*sec(d*x + c) + a), x)

maple [B] time = 2.31, size = 2954, normalized size = 5.63

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x)

[Out] 1/24/d*(-1+cos(d*x+c))^2*(-16*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)-8*A*cos(d*x+c)^3*a^3+16*A*cos(d*x+c)^2*a^3-15*A*cos(d*x+c)^2*b^3+12*B*cos(d*x+c)^2*a^3-5*A*cos(d*x+c)^3*a*b^2-18*A*cos(d*x+c)^2*a^2*b+15*A*cos(d*x+c)^2*a*b^2+16*A*cos(d*x+c)*a^2*b-10*A*cos(d*x+c)*a*b^2+6*B*cos(d*x+c)^3*a^2*b-18*B*cos(d*x+c)^2*a^2*b+18*B*cos(d*x+c)^2*a*b^2+12*B*cos(d*x+c)*a^2*b-18*B*cos(d*x+c)*a*b^2+15*A*cos(d*x+c)*b^3+2*A*cos(d*x+c)^4*a^2*b-15*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)-12*B*cos(d*x+c)^4*a^3+24*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^3+24*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)+4*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)+10*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)-16*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)-15*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)

$$\begin{aligned}
& *((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{(1/2)}*EllipticE((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{(1/2)}*a*b^2*\sin(d*x+c)-36*B*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{(1/2)}*EllipticPi((-1+\cos(d*x+c))/\sin(d*x+c), -1, ((a-b)/(a+b))^{(1/2)}*a*b^2*\sin(d*x+c)-12*B*EllipticF((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{(1/2)}*a^2*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{(1/2)}*\sin(d*x+c)*b+18*B*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{(1/2)}*EllipticE((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{(1/2)}*a^2*b*\sin(d*x+c)+18*B*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{(1/2)}*EllipticE((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{(1/2)}*a*b^2*\sin(d*x+c)+30*A*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{(1/2)}*EllipticPi((-1+\cos(d*x+c))/\sin(d*x+c), -1, ((a-b)/(a+b))^{(1/2)}*\sin(d*x+c)*\cos(d*x+c)*b^3-16*A*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{(1/2)}*EllipticE((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{(1/2)}*\sin(d*x+c)*\cos(d*x+c)*a^3-15*A*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{(1/2)}*EllipticE((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{(1/2)}*\sin(d*x+c)*\cos(d*x+c)*b^3-48*B*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{(1/2)}*EllipticPi((-1+\cos(d*x+c))/\sin(d*x+c), -1, ((a-b)/(a+b))^{(1/2)}*\sin(d*x+c)*\cos(d*x+c)*a^3-48*B*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{(1/2)}*EllipticPi((-1+\cos(d*x+c))/\sin(d*x+c), -1, ((a-b)/(a+b))^{(1/2)}*a^3*\sin(d*x+c)+30*A*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{(1/2)}*EllipticPi((-1+\cos(d*x+c))/\sin(d*x+c), -1, ((a-b)/(a+b))^{(1/2)}*b^3*\sin(d*x+c)+24*B*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{(1/2)}*EllipticF((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{(1/2)}*a^3*\sin(d*x+c)-8*A*\cos(d*x+c)^5*a^3+18*B*EllipticE((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{(1/2)}*\cos(d*x+c)*b^2*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{(1/2)}*\sin(d*x+c)*a+24*A*EllipticPi((-1+\cos(d*x+c))/\sin(d*x+c), -1, ((a-b)/(a+b))^{(1/2)}*\cos(d*x+c)*a^2*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{(1/2)}*\sin(d*x+c)*b+4*A*EllipticF((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{(1/2)}*\cos(d*x+c)*a^2*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{(1/2)}*\sin(d*x+c)*b+10*A*EllipticF((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{(1/2)}*\cos(d*x+c)*b^2*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{(1/2)}*\sin(d*x+c)*a-16*A*EllipticE((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{(1/2)}*\cos(d*x+c)*a^2*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{(1/2)}*\sin(d*x+c)*b-15*A*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{(1/2)}*EllipticE((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{(1/2)}*\sin(d*x+c)*\cos(d*x+c)*a*b^2-36*B*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{(1/2)}*EllipticPi((-1+\cos(d*x+c))/\sin(d*x+c), -1, ((a-b)/(a+b))^{(1/2)}*\sin(d*x+c)*\cos(d*x+c)*a*b^2-12*B*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{(1/2)}*EllipticF((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{(1/2)}*\sin(d*x+c)*\cos(d*x+c)*a^2*b+18*B*EllipticE((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)
\end{aligned}$$

$$\frac{\cos(dx+c) \cdot a^2 \cdot (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} \cdot ((b+a \cdot \cos(dx+c))/(1+\cos(dx+c)))^{1/2} \cdot \sin(dx+c) \cdot b \cdot (1+\cos(dx+c))^2 \cdot ((b+a \cdot \cos(dx+c))/\cos(dx+c))^{1/2} / (b+a \cdot \cos(dx+c)) / \sin(dx+c)^5 / a^3}{(a+b)^{1/2}}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx+c) + A) \cos(dx+c)^3}{\sqrt{b \sec(dx+c) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(dx+c)^3*(A+B*sec(dx+c))/(a+b*sec(dx+c))^(1/2),x, algorithm="maxima")

[Out] integrate((B*sec(dx+c) + A)*cos(dx+c)^3/sqrt(b*sec(dx+c) + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+dx)^3 \left(A + \frac{B}{\cos(c+dx)} \right)}{\sqrt{a + \frac{b}{\cos(c+dx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c+dx)^3*(A+B/cos(c+dx)))/(a+b/cos(c+dx))^(1/2),x)

[Out] int((cos(c+dx)^3*(A+B/cos(c+dx)))/(a+b/cos(c+dx))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c+dx)) \cos^3(c+dx)}{\sqrt{a + b \sec(c+dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(dx+c)**3*(A+B*sec(dx+c))/(a+b*sec(dx+c))**(1/2),x)

[Out] Integral((A + B*sec(c+dx))*cos(c+dx)**3/sqrt(a + b*sec(c+dx)), x)

$$3.378 \quad \int \frac{\sec^3(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^{3/2}} dx$$

Optimal. Leaf size=329

$$\frac{2a^2(Ab - aB) \tan(c + dx)}{b^2d(a^2 - b^2)\sqrt{a + b \sec(c + dx)}} - \frac{2(-8a^3B + 6a^2Ab + 5ab^2B - 3Ab^3) \cot(c + dx) \sqrt{\frac{b(1-\sec(c+dx))}{a+b}} \sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}}{3b^4d\sqrt{a + b}}$$

[Out] $-2/3*(6*A*a^2*b-3*A*b^3-8*B*a^3+5*B*a*b^2)*\cot(d*x+c)*\text{EllipticE}((a+b*\sec(d*x+c))^{(1/2)}/(a+b)^{(1/2)},((a+b)/(a-b))^{(1/2)})*(b*(1-\sec(d*x+c))/(a+b))^{(1/2)}*(-b*(1+\sec(d*x+c))/(a-b))^{(1/2)}/b^4/d/(a+b)^{(1/2)}-2/3*(2*a+b)*(3*A*b-(4*a+b)*B)*\cot(d*x+c)*\text{EllipticF}((a+b*\sec(d*x+c))^{(1/2)}/(a+b)^{(1/2)},((a+b)/(a-b))^{(1/2)})*(b*(1-\sec(d*x+c))/(a+b))^{(1/2)}*(-b*(1+\sec(d*x+c))/(a-b))^{(1/2)}/b^3/d/(a+b)^{(1/2)}-2*a^2*(A*b-B*a)*\tan(d*x+c)/b^2/(a^2-b^2)/d/(a+b*\sec(d*x+c))^{(1/2)}+2/3*B*(a+b*\sec(d*x+c))^{(1/2)}*\tan(d*x+c)/b^2/d$

Rubi [A] time = 0.72, antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {4028, 4082, 4005, 3832, 4004}

$$\frac{2a^2(Ab - aB) \tan(c + dx)}{b^2d(a^2 - b^2)\sqrt{a + b \sec(c + dx)}} - \frac{2(6a^2Ab - 8a^3B + 5ab^2B - 3Ab^3) \cot(c + dx) \sqrt{\frac{b(1-\sec(c+dx))}{a+b}} \sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}}{3b^4d\sqrt{a + b}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(\text{Sec}[c + d*x])^3*(A + B*\text{Sec}[c + d*x])]/(a + b*\text{Sec}[c + d*x])^{(3/2)}, x]$

[Out] $(-2*(6*a^2*A*b - 3*A*b^3 - 8*a^3*B + 5*a*b^2*B)*\text{Cot}[c + d*x]*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[a + b*\text{Sec}[c + d*x]]/\text{Sqrt}[a + b]], (a + b)/(a - b)]*\text{Sqrt}[(b*(1 - \text{Sec}[c + d*x]))/(a + b)]*\text{Sqrt}[-((b*(1 + \text{Sec}[c + d*x]))/(a - b))]/(3*b^4*\text{Sqrt}[a + b]*d) - (2*(2*a + b)*(3*A*b - (4*a + b)*B)*\text{Cot}[c + d*x]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[a + b*\text{Sec}[c + d*x]]/\text{Sqrt}[a + b]], (a + b)/(a - b)]*\text{Sqrt}[(b*(1 - \text{Sec}[c + d*x]))/(a + b)]*\text{Sqrt}[-((b*(1 + \text{Sec}[c + d*x]))/(a - b))]/(3*b^3*\text{Sqrt}[a + b]*d) - (2*a^2*(A*b - a*B)*\text{Tan}[c + d*x]/(b^2*(a^2 - b^2)*d*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]) + (2*B*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]*\text{Tan}[c + d*x])/(3*b^2*d)$

Rule 3832

$\text{Int}[\text{csc}[(e_.) + (f_.)*(x_.)]/\text{Sqrt}[\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] \rightarrow \text{Simp}[(-2*\text{Rt}[a + b, 2]*\text{Sqrt}[(b*(1 - \text{Csc}[e + f*x]))/(a + b)]*\text{Sqrt}[-((b*(1 + \text{Csc}[e + f*x]))/(a - b))]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[a + b*\text{Csc}[e + f*x]]/\text{Rt}[a + b, 2]], (a + b)/(a - b))]/(b*f*\text{Cot}[e + f*x]), x] /; \text{FreeQ}\{a, b, e, f\}, x \&\& \text{NeQ}[a^2 - b^2, 0]$

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)))/Sqrt[
csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
2]], (a*A + b*B)/(a*A - b*B)]/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)))/Sqrt[
csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x])/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
&& NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 4028

```
Int[csc[(e_.) + (f_.)*(x_.)]^3*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_)*(
csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> -Simp[(a^2*(A*b - a*B)*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x]
+ Dist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*Simp[a*b*(A*b - a*B)*(m + 1) - (A*b - a*B)*(a^2 + b^2*(m + 1))*Csc[e
+ f*x] + b*B*(m + 1)*(a^2 - b^2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_.)]*((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e
_.) + (f_.)*(x_.)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^3(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{3/2}} dx &= -\frac{2a^2(Ab-aB)\tan(c+dx)}{b^2(a^2-b^2)d\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{\sec(c+dx)\left(-\frac{1}{2}ab(Ab-aB)-\frac{1}{2}(2a^2-b^2)\right)}{\sqrt{a+b\sec(c+dx)}} dx}{b^2(a^2-b^2)d} \\
&= -\frac{2a^2(Ab-aB)\tan(c+dx)}{b^2(a^2-b^2)d\sqrt{a+b\sec(c+dx)}} + \frac{2B\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{3b^2d} \\
&= -\frac{2a^2(Ab-aB)\tan(c+dx)}{b^2(a^2-b^2)d\sqrt{a+b\sec(c+dx)}} + \frac{2B\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{3b^2d} \\
&= -\frac{2(6a^2Ab-3Ab^3-8a^3B+5ab^2B)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)\right)}{3b^4\sqrt{a+bd}}
\end{aligned}$$

Mathematica [B] time = 24.64, size = 3460, normalized size = 10.52

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(Sec[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(3/2), x]

[Out] ((b + a*cos[c + d*x])^2*Sec[c + d*x]^2*((2*(-6*a^2*A*b + 3*A*b^3 + 8*a^3*B - 5*a*b^2*B)*Sin[c + d*x])/(3*b^3*(-a^2 + b^2)) + (2*(a^2*A*b*SIN[c + d*x] - a^3*B*SIN[c + d*x]))/(b^2*(-a^2 + b^2)*(b + a*cos[c + d*x])) + (2*B*TAN[c + d*x])/(3*b^2)))/(d*(a + b*Sec[c + d*x])^(3/2)) - (2*(b + a*cos[c + d*x])*((2*a^2*A)/(b*(-a^2 + b^2)*Sqrt[b + a*cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (A*b)/((-a^2 + b^2)*Sqrt[b + a*cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (5*a*B)/(3*(-a^2 + b^2)*Sqrt[b + a*cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (8*a^3*B)/(3*b^2*(-a^2 + b^2)*Sqrt[b + a*cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (2*a*A*Sqrt[Sec[c + d*x]])/((-a^2 + b^2)*Sqrt[b + a*cos[c + d*x]]) + (2*a^3*A*Sqrt[Sec[c + d*x]])/(b^2*(-a^2 + b^2)*Sqrt[b + a*cos[c + d*x]]) - (8*a^4*B*Sqrt[Sec[c + d*x]])/(3*b^3*(-a^2 + b^2)*Sqrt[b + a*cos[c + d*x]]) + (7*a^2*B*Sqrt[Sec[c + d*x]])/(3*b*(-a^2 + b^2)*Sqrt[b + a*cos[c + d*x]]) + (b*B*Sqrt[Sec[c + d*x]])/(3*(-a^2 + b^2)*Sqrt[b + a*cos[c + d*x]]) - (a*A*cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/((-a^2 + b^2)*Sqrt[b + a*cos[c + d*x]]) + (2*a^3*A*cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(b^2*(-a^2 + b^2)*Sqrt[b + a*cos[c + d*x]]) - (8*a^4*B*cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*b^3*(-a^2 + b^2)*Sqrt[b + a*cos[c + d*x]]) + (5*a^2*B*cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*b*(-a^2 + b^2)*Sqrt[b + a*cos[c + d*x]])*Sec[c + d*x]^(3/2)*Sqrt[Cos[(c + d*x)]]

$$\begin{aligned}
& *x)/2]^2 * \text{Sec}[c + d*x]] * (2*(a + b)*(-6*a^2*A*b + 3*A*b^3 + 8*a^3*B - 5*a*b^2 \\
& *B)*\text{Sqrt}[\text{Cos}[c + d*x]/(1 + \text{Cos}[c + d*x])] * \text{Sqrt}[(b + a*\text{Cos}[c + d*x])/((a + b) \\
&)*(1 + \text{Cos}[c + d*x]))] * \text{EllipticE}[\text{ArcSin}[\text{Tan}[(c + d*x)/2]], (a - b)/(a + b)] \\
& - 2*b*(-2*a^2 - a*b + b^2)*(3*A*b + (-4*a + b)*B)*\text{Sqrt}[\text{Cos}[c + d*x]/(1 + \text{C} \\
& \text{os}[c + d*x])] * \text{Sqrt}[(b + a*\text{Cos}[c + d*x])/((a + b)*(1 + \text{Cos}[c + d*x]))] * \text{Ellip} \\
& \text{ticF}[\text{ArcSin}[\text{Tan}[(c + d*x)/2]], (a - b)/(a + b)] + (-6*a^2*A*b + 3*A*b^3 + 8 \\
& *a^3*B - 5*a*b^2*B)*\text{Cos}[c + d*x]*(b + a*\text{Cos}[c + d*x])* \text{Sec}[(c + d*x)/2]^2 * \text{Ta} \\
& \text{n}[(c + d*x)/2]) / (3*b^3*(-a^2 + b^2)*d*\text{Sqrt}[\text{Sec}[(c + d*x)/2]^2]*(a + b*\text{Sec}[\\
& c + d*x])^(3/2)*(-1/3*(a*\text{Sqrt}[\text{Cos}[(c + d*x)/2]^2*\text{Sec}[c + d*x]]*\text{Sin}[c + d*x] \\
& *(2*(a + b)*(-6*a^2*A*b + 3*A*b^3 + 8*a^3*B - 5*a*b^2*B)*\text{Sqrt}[\text{Cos}[c + d*x]/ \\
& (1 + \text{Cos}[c + d*x])] * \text{Sqrt}[(b + a*\text{Cos}[c + d*x])/((a + b)*(1 + \text{Cos}[c + d*x]))] \\
& * \text{EllipticE}[\text{ArcSin}[\text{Tan}[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(-2*a^2 - a*b + \\
& b^2)*(3*A*b + (-4*a + b)*B)*\text{Sqrt}[\text{Cos}[c + d*x]/(1 + \text{Cos}[c + d*x])] * \text{Sqrt}[(b \\
& + a*\text{Cos}[c + d*x])/((a + b)*(1 + \text{Cos}[c + d*x]))] * \text{EllipticF}[\text{ArcSin}[\text{Tan}[(c + d \\
& *x)/2]], (a - b)/(a + b)] + (-6*a^2*A*b + 3*A*b^3 + 8*a^3*B - 5*a*b^2*B)*\text{Co} \\
& \text{s}[c + d*x]*(b + a*\text{Cos}[c + d*x])* \text{Sec}[(c + d*x)/2]^2 * \text{Tan}[(c + d*x)/2]) / (b^3* \\
& (-a^2 + b^2)*(b + a*\text{Cos}[c + d*x])^(3/2)*\text{Sqrt}[\text{Sec}[(c + d*x)/2]^2]) + (\text{Sqrt}[\text{C} \\
& \text{os}[(c + d*x)/2]^2*\text{Sec}[c + d*x]]*\text{Tan}[(c + d*x)/2]*(2*(a + b)*(-6*a^2*A*b + 3 \\
& *A*b^3 + 8*a^3*B - 5*a*b^2*B)*\text{Sqrt}[\text{Cos}[c + d*x]/(1 + \text{Cos}[c + d*x])] * \text{Sqrt}[(b \\
& + a*\text{Cos}[c + d*x])/((a + b)*(1 + \text{Cos}[c + d*x]))] * \text{EllipticE}[\text{ArcSin}[\text{Tan}[(c + \\
& d*x)/2]], (a - b)/(a + b)] - 2*b*(-2*a^2 - a*b + b^2)*(3*A*b + (-4*a + b)*B) \\
&)*\text{Sqrt}[\text{Cos}[c + d*x]/(1 + \text{Cos}[c + d*x])] * \text{Sqrt}[(b + a*\text{Cos}[c + d*x])/((a + b)* \\
& (1 + \text{Cos}[c + d*x]))] * \text{EllipticF}[\text{ArcSin}[\text{Tan}[(c + d*x)/2]], (a - b)/(a + b)] + \\
& (-6*a^2*A*b + 3*A*b^3 + 8*a^3*B - 5*a*b^2*B)*\text{Cos}[c + d*x]*(b + a*\text{Cos}[c + d \\
& *x])* \text{Sec}[(c + d*x)/2]^2 * \text{Tan}[(c + d*x)/2]) / (3*b^3*(-a^2 + b^2)*\text{Sqrt}[b + a*\text{C} \\
& \text{os}[c + d*x]] * \text{Sqrt}[\text{Sec}[(c + d*x)/2]^2]) - (2*\text{Sqrt}[\text{Cos}[(c + d*x)/2]^2*\text{Sec}[c + \\
& d*x]]*(((-6*a^2*A*b + 3*A*b^3 + 8*a^3*B - 5*a*b^2*B)*\text{Cos}[c + d*x]*(b + a*\text{C} \\
& \text{os}[c + d*x])* \text{Sec}[(c + d*x)/2]^4)/2 + ((a + b)*(-6*a^2*A*b + 3*A*b^3 + 8*a^3 \\
& *B - 5*a*b^2*B)*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/((a + b)*(1 + \text{Cos}[c + d*x]))] * \text{Ell} \\
& \text{ipticE}[\text{ArcSin}[\text{Tan}[(c + d*x)/2]], (a - b)/(a + b)]*((\text{Cos}[c + d*x]*\text{Sin}[c + d* \\
& x])/(1 + \text{Cos}[c + d*x])^2 - \text{Sin}[c + d*x]/(1 + \text{Cos}[c + d*x])))/\text{Sqrt}[\text{Cos}[c + d \\
& *x]/(1 + \text{Cos}[c + d*x])] - (b*(-2*a^2 - a*b + b^2)*(3*A*b + (-4*a + b)*B)*\text{Sq} \\
& \text{rt}[(b + a*\text{Cos}[c + d*x])/((a + b)*(1 + \text{Cos}[c + d*x]))] * \text{EllipticF}[\text{ArcSin}[\text{Tan} \\
& (c + d*x)/2]], (a - b)/(a + b)]*((\text{Cos}[c + d*x]*\text{Sin}[c + d*x])/(1 + \text{Cos}[c + d \\
& *x])^2 - \text{Sin}[c + d*x]/(1 + \text{Cos}[c + d*x])))/\text{Sqrt}[\text{Cos}[c + d*x]/(1 + \text{Cos}[c + d \\
& *x])] + ((a + b)*(-6*a^2*A*b + 3*A*b^3 + 8*a^3*B - 5*a*b^2*B)*\text{Sqrt}[\text{Cos}[c + \\
& d*x]/(1 + \text{Cos}[c + d*x])] * \text{EllipticE}[\text{ArcSin}[\text{Tan}[(c + d*x)/2]], (a - b)/(a + b) \\
&]*((-(a*\text{Sin}[c + d*x])/((a + b)*(1 + \text{Cos}[c + d*x]))) + ((b + a*\text{Cos}[c + d*x] \\
&)*\text{Sin}[c + d*x])/((a + b)*(1 + \text{Cos}[c + d*x])^2)))/\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/ \\
& ((a + b)*(1 + \text{Cos}[c + d*x]))] - (b*(-2*a^2 - a*b + b^2)*(3*A*b + (-4*a + b) \\
& *B)*\text{Sqrt}[\text{Cos}[c + d*x]/(1 + \text{Cos}[c + d*x])] * \text{EllipticF}[\text{ArcSin}[\text{Tan}[(c + d*x)/2] \\
&], (a - b)/(a + b)]*((-(a*\text{Sin}[c + d*x])/((a + b)*(1 + \text{Cos}[c + d*x])) + ((b \\
& + a*\text{Cos}[c + d*x])* \text{Sin}[c + d*x])/((a + b)*(1 + \text{Cos}[c + d*x])^2)))/\text{Sqrt}[(b + \\
& a*\text{Cos}[c + d*x])/((a + b)*(1 + \text{Cos}[c + d*x]))] - a*(-6*a^2*A*b + 3*A*b^3 + \\
& 8*a^3*B - 5*a*b^2*B)*\text{Cos}[c + d*x]* \text{Sec}[(c + d*x)/2]^2 * \text{Sin}[c + d*x]* \text{Tan}[(c +
\end{aligned}$$

$d*x)/2] - (-6*a^2*A*b + 3*A*b^3 + 8*a^3*B - 5*a*b^2*B)*(b + a*\text{Cos}[c + d*x])$
 $*\text{Sec}[(c + d*x)/2]^2*\text{Sin}[c + d*x]*\text{Tan}[(c + d*x)/2] + (-6*a^2*A*b + 3*A*b^3 +$
 $8*a^3*B - 5*a*b^2*B)*\text{Cos}[c + d*x]*(b + a*\text{Cos}[c + d*x])* \text{Sec}[(c + d*x)/2]^2*$
 $\text{Tan}[(c + d*x)/2]^2 - (b*(-2*a^2 - a*b + b^2)*(3*A*b + (-4*a + b)*B)*\text{Sqrt}[\text{Co}$
 $s[c + d*x]/(1 + \text{Cos}[c + d*x])]*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/((a + b)*(1 + \text{Cos}[$
 $c + d*x]))]*\text{Sec}[(c + d*x)/2]^2)/(\text{Sqrt}[1 - \text{Tan}[(c + d*x)/2]^2]*\text{Sqrt}[1 - ((a$
 $- b)*\text{Tan}[(c + d*x)/2]^2)/(a + b)]) + ((a + b)*(-6*a^2*A*b + 3*A*b^3 + 8*a^3$
 $*B - 5*a*b^2*B)*\text{Sqrt}[\text{Cos}[c + d*x]/(1 + \text{Cos}[c + d*x])]*\text{Sqrt}[(b + a*\text{Cos}[c + d$
 $*x])/((a + b)*(1 + \text{Cos}[c + d*x]))]*\text{Sec}[(c + d*x)/2]^2*\text{Sqrt}[1 - ((a - b)*\text{Tan}$
 $[(c + d*x)/2]^2)/(a + b)]/\text{Sqrt}[1 - \text{Tan}[(c + d*x)/2]^2]))/(3*b^3*(-a^2 + b^$
 $2)*\text{Sqrt}[b + a*\text{Cos}[c + d*x]]*\text{Sqrt}[\text{Sec}[(c + d*x)/2]^2]) - ((2*(a + b)*(-6*a^2$
 $*A*b + 3*A*b^3 + 8*a^3*B - 5*a*b^2*B)*\text{Sqrt}[\text{Cos}[c + d*x]/(1 + \text{Cos}[c + d*x])]$
 $*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/((a + b)*(1 + \text{Cos}[c + d*x]))]*\text{EllipticE}[\text{ArcSin}[\text{T}$
 $\text{an}[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(-2*a^2 - a*b + b^2)*(3*A*b + (-4*$
 $a + b)*B)*\text{Sqrt}[\text{Cos}[c + d*x]/(1 + \text{Cos}[c + d*x])]*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/$
 $((a + b)*(1 + \text{Cos}[c + d*x]))]*\text{EllipticF}[\text{ArcSin}[\text{Tan}[(c + d*x)/2]], (a - b)/(a$
 $+ b)] + (-6*a^2*A*b + 3*A*b^3 + 8*a^3*B - 5*a*b^2*B)*\text{Cos}[c + d*x]*(b + a*\text{C}$
 $\text{os}[c + d*x])* \text{Sec}[(c + d*x)/2]^2*\text{Tan}[(c + d*x)/2]*(-(\text{Cos}[(c + d*x)/2]*\text{Sec}[c$
 $+ d*x]*\text{Sin}[(c + d*x)/2]) + \text{Cos}[(c + d*x)/2]^2*\text{Sec}[c + d*x]*\text{Tan}[c + d*x]))/$
 $(3*b^3*(-a^2 + b^2)*\text{Sqrt}[b + a*\text{Cos}[c + d*x]]*\text{Sqrt}[\text{Sec}[(c + d*x)/2]^2]*\text{Sqrt}[\text{C}$
 $\text{os}[(c + d*x)/2]^2*\text{Sec}[c + d*x]))))$

fricas [F] time = 1.08, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B \sec(dx+c)^4 + A \sec(dx+c)^3)\sqrt{b \sec(dx+c) + a}}{b^2 \sec(dx+c)^2 + 2ab \sec(dx+c) + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")

[Out] integral((B*sec(d*x + c)^4 + A*sec(d*x + c)^3)*sqrt(b*sec(d*x + c) + a)/(b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx+c) + A) \sec(dx+c)^3}{(b \sec(dx+c) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^3/(b*sec(d*x + c) + a)^(3/2), x)

maple [B] time = 2.55, size = 3333, normalized size = 10.13

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2), x)

[Out]
$$-1/3/d^4^{1/2} * ((b+a*\cos(d*x+c))/\cos(d*x+c))^{1/2} * (-3*A*\cos(d*x+c)^3*a^2*b^2+4*B*\cos(d*x+c)^3*a^3*b-B*\cos(d*x+c)^3*a*b^3-4*B*\cos(d*x+c)^2*a^2*b^2+5*B*\cos(d*x+c)^2*a*b^3+4*B*\cos(d*x+c)*a^3*b-4*B*\cos(d*x+c)*a*b^3-6*A*\cos(d*x+c)^2*a^3*b+3*A*\cos(d*x+c)^2*a*b^3-3*A*\cos(d*x+c)*a^2*b^2+6*A*\sin(d*x+c)*\text{EllipticF}((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{1/2})*\cos(d*x+c)^2*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*a^2*b^2-3*A*\cos(d*x+c)^2*b^4-B*\cos(d*x+c)^2*b^4+8*B*\cos(d*x+c)^2*a^4-8*B*\cos(d*x+c)^3*a^4-8*B*\cos(d*x+c)^2*a^3*b+6*A*\cos(d*x+c)^3*a^3*b-3*A*\cos(d*x+c)^3*a*b^3+6*A*\cos(d*x+c)^2*a^2*b^2+5*B*\cos(d*x+c)^3*a^2*b^2+B*b^4+8*B*\sin(d*x+c)*\cos(d*x+c)*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*\text{EllipticE}((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{1/2})*a^4-a^2*b^2*B-3*A*\sin(d*x+c)*\text{EllipticF}((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{1/2})*\cos(d*x+c)^2*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*b^4+3*A*\sin(d*x+c)*\text{EllipticE}((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{1/2})*\cos(d*x+c)^2*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*b^4-6*A*\sin(d*x+c)*\text{EllipticE}((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{1/2})*\cos(d*x+c)^2*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*a^2*b^2+3*A*\sin(d*x+c)*\text{EllipticF}((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{1/2})*\cos(d*x+c)^2*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*a*b^3-6*A*\sin(d*x+c)*\text{EllipticE}((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{1/2})*\cos(d*x+c)^2*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*a^3*b+3*A*\sin(d*x+c)*\text{EllipticE}((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{1/2})*\cos(d*x+c)^2*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*a^2*b^2+5*B*\sin(d*x+c)*\text{EllipticF}((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{1/2})*\cos(d*x+c)^2*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*a^3*b+8*B*\sin(d*x+c)*\text{EllipticE}((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{1/2})*\cos(d*x+c)^2*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*a^3*b-5*B*\sin(d*x+c)*\text{EllipticE}((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{1/2})*\cos(d*x+c)^2*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\cos(c+dx)^3 \left(a + \frac{b}{\cos(c+dx)}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^3*(a + b/cos(c + d*x))^(3/2)),x)

[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^3*(a + b/cos(c + d*x))^(3/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \sec^3(c + dx)}{(a + b \sec(c + dx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(3/2),x)

[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**3/(a + b*sec(c + d*x))**(3/2), x)

$$3.379 \quad \int \frac{\sec^2(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^{3/2}} dx$$

Optimal. Leaf size=275

$$\frac{2a(Ab - aB) \tan(c + dx)}{bd(a^2 - b^2) \sqrt{a + b \sec(c + dx)}} + \frac{2(-2a^2B + aAb + b^2B) \cot(c + dx) \sqrt{\frac{b(1 - \sec(c+dx))}{a+b}} \sqrt{\frac{b(\sec(c+dx)+1)}{a-b}} E\left(\sin^{-1}\left(\frac{\sqrt{a+b \sec(c+dx)}}{\sqrt{a+b}}\right)\right)}{b^3 d \sqrt{a+b}}$$

[Out] 2*(A*a*b-2*B*a^2+B*b^2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b)^(1/2)*(-b*(1+sec(d*x+c)))/(a-b)^(1/2)/b^3/d/(a+b)^(1/2)+2*(A*b-(2*a+b)*B)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b)^(1/2)*(-b*(1+sec(d*x+c)))/(a-b)^(1/2)/b^2/d/(a+b)^(1/2)+2*a*(A*b-B*a)*tan(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)

Rubi [A] time = 0.46, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {4009, 4005, 3832, 4004}

$$\frac{2a(Ab - aB) \tan(c + dx)}{bd(a^2 - b^2) \sqrt{a + b \sec(c + dx)}} + \frac{2(-2a^2B + aAb + b^2B) \cot(c + dx) \sqrt{\frac{b(1 - \sec(c+dx))}{a+b}} \sqrt{\frac{b(\sec(c+dx)+1)}{a-b}} E\left(\sin^{-1}\left(\frac{\sqrt{a+b \sec(c+dx)}}{\sqrt{a+b}}\right)\right)}{b^3 d \sqrt{a+b}}$$

Antiderivative was successfully verified.

[In] Int[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(3/2),x]

[Out] (2*(a*A*b - 2*a^2*B + b^2*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))]/(b^3*Sqrt[a + b]*d) + (2*(A*b - (2*a + b)*B)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))]/(b^2*Sqrt[a + b]*d) + (2*a*(A*b - a*B)*Tan[c + d*x])/((b*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]])

Rule 3832

Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] := Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)])/((b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)))/Sqrt[
csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
2]], (a*A + b*B)/(a*A - b*B)]/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)))/Sqrt[
csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
&& NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 4009

```
Int[csc[(e_.) + (f_.)*(x_.)]^2*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_)*(
csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> Simp[(a*(A*b - a*B)*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b*(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
Simp[b*(A*b - a*B)*(m + 1) - (a*A*b*(m + 2) - B*(a^2 + b^2*(m + 1)))*Csc[e
+ f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&
NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^{3/2}} dx = \frac{2a(Ab-aB) \tan(c+dx)}{b(a^2-b^2) d \sqrt{a+b \sec(c+dx)}} + \frac{2 \int \frac{\sec(c+dx) \left(-\frac{1}{2}b(Ab-aB) - \frac{1}{2}(aAb-2a^2B+b^2B)\right)}{\sqrt{a+b \sec(c+dx)}}}{b(a^2-b^2)}$$

$$= \frac{2a(Ab-aB) \tan(c+dx)}{b(a^2-b^2) d \sqrt{a+b \sec(c+dx)}} - \frac{(aAb-2a^2B+b^2B) \int \frac{\sec(c+dx)(1+\sec(c+dx))}{\sqrt{a+b \sec(c+dx)}}}{b(a^2-b^2)}$$

$$= \frac{2(aAb-2a^2B+b^2B) \cot(c+dx) E\left(\sin^{-1}\left(\frac{\sqrt{a+b \sec(c+dx)}}{\sqrt{a+b}}\right) \middle| \frac{a+b}{a-b}\right) \sqrt{\frac{b(1-\sec(c+dx))}{a-b}}}{b^3 \sqrt{a+bd}}$$

Mathematica [A] time = 18.08, size = 467, normalized size = 1.70

$$\frac{\sec^2(c + dx)(a \cos(c + dx) + b)^2 \left(\frac{2(-2a^2B + aAb + b^2B) \sin(c + dx)}{b^2(b^2 - a^2)} - \frac{2(aAb \sin(c + dx) - a^2B \sin(c + dx))}{b(b^2 - a^2)(a \cos(c + dx) + b)} \right)}{d(a + b \sec(c + dx))^{3/2}} + \frac{2 \sec^{\frac{3}{2}}(c + dx) \sqrt{\cos^2(c + dx)}}{d(a + b \sec(c + dx))^{3/2}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(3/2), x]

[Out] ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*((2*(a*A*b - 2*a^2*B + b^2*B)*Sin[c + d*x])/(b^2*(-a^2 + b^2)) - (2*(a*A*b*SIN[c + d*x] - a^2*B*SIN[c + d*x]))/(b*(-a^2 + b^2)*(b + a*Cos[c + d*x])))/(d*(a + b*Sec[c + d*x])^(3/2)) + (2*(b + a*Cos[c + d*x])*Sec[c + d*x]^(3/2)*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]])*(2*(a + b)*(-a*A*b) + 2*a^2*B - b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*b*(a + b)*(A*b + (-2*a + b)*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-a*A*b) + 2*a^2*B - b^2*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/((b^2*(-a^2 + b^2)*d*Sqrt[Sec[(c + d*x)/2]^2]*(a + b*Sec[c + d*x])^(3/2))

fricas [F] time = 1.25, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{(B \sec(dx + c)^3 + A \sec(dx + c)^2) \sqrt{b \sec(dx + c) + a}}{b^2 \sec(dx + c)^2 + 2ab \sec(dx + c) + a^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")

[Out] integral((B*sec(d*x + c)^3 + A*sec(d*x + c)^2)*sqrt(b*sec(d*x + c) + a)/(b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sec(dx + c)^2}{(b \sec(dx + c) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^2/(b*sec(d*x + c) + a)^(3/2), x)

maple [B] time = 2.31, size = 2276, normalized size = 8.28

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x)

[Out] $\frac{1}{d^4} \sqrt{\frac{b+a \cos(dx+c)}{\cos(dx+c)}} \left(\frac{B \cos(dx+c)}{1+\cos(dx+c)} \right)^{\frac{1}{2}} \left(\frac{b+a \cos(dx+c)}{1+\cos(dx+c)} \right)^{\frac{1}{2}} \left(\frac{b+a \cos(dx+c)}{a+b} \right)^{\frac{1}{2}} \text{EllipticF} \left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}, \left(\frac{a-b}{a+b} \right)^{\frac{1}{2}} \right) b^3 \sin(dx+c) - b^3 B + a^2 b B - 2 B \cos(dx+c)^2 a^3 + 2 B \cos(dx+c) a^3 + A \cos(dx+c)^2 a^2 b - A \cos(dx+c)^2 a b^2 - A \cos(dx+c) a^2 b + A \cos(dx+c) a b^2 + B \cos(dx+c)^2 a^2 b + B \cos(dx+c)^2 a b^2 - 2 B \cos(dx+c) a^2 b - B \cos(dx+c) a b^2 + A \left(\frac{\cos(dx+c)}{1+\cos(dx+c)} \right)^{\frac{1}{2}} \left(\frac{b+a \cos(dx+c)}{1+\cos(dx+c)} \right)^{\frac{1}{2}} \left(\frac{b+a \cos(dx+c)}{a+b} \right)^{\frac{1}{2}} \text{EllipticF} \left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}, \left(\frac{a-b}{a+b} \right)^{\frac{1}{2}} \right) b^3 \sin(dx+c) + A \left(\frac{\cos(dx+c)}{1+\cos(dx+c)} \right)^{\frac{1}{2}} \left(\frac{b+a \cos(dx+c)}{1+\cos(dx+c)} \right)^{\frac{1}{2}} \left(\frac{b+a \cos(dx+c)}{a+b} \right)^{\frac{1}{2}} \text{EllipticF} \left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}, \left(\frac{a-b}{a+b} \right)^{\frac{1}{2}} \right) a b^2 \sin(dx+c) - A \left(\frac{\cos(dx+c)}{1+\cos(dx+c)} \right)^{\frac{1}{2}} \left(\frac{b+a \cos(dx+c)}{1+\cos(dx+c)} \right)^{\frac{1}{2}} \left(\frac{b+a \cos(dx+c)}{a+b} \right)^{\frac{1}{2}} \text{EllipticE} \left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}, \left(\frac{a-b}{a+b} \right)^{\frac{1}{2}} \right) a^2 b \sin(dx+c) - A \left(\frac{\cos(dx+c)}{1+\cos(dx+c)} \right)^{\frac{1}{2}} \left(\frac{b+a \cos(dx+c)}{1+\cos(dx+c)} \right)^{\frac{1}{2}} \left(\frac{b+a \cos(dx+c)}{a+b} \right)^{\frac{1}{2}} \text{EllipticE} \left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}, \left(\frac{a-b}{a+b} \right)^{\frac{1}{2}} \right) a b^2 \sin(dx+c) - 2 B \text{EllipticF} \left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}, \left(\frac{a-b}{a+b} \right)^{\frac{1}{2}} \right) a^2 \left(\frac{\cos(dx+c)}{1+\cos(dx+c)} \right)^{\frac{1}{2}} \left(\frac{b+a \cos(dx+c)}{1+\cos(dx+c)} \right)^{\frac{1}{2}} \left(\frac{b+a \cos(dx+c)}{a+b} \right)^{\frac{1}{2}} \sin(dx+c) b + 2 B \left(\frac{\cos(dx+c)}{1+\cos(dx+c)} \right)^{\frac{1}{2}} \left(\frac{b+a \cos(dx+c)}{1+\cos(dx+c)} \right)^{\frac{1}{2}} \left(\frac{b+a \cos(dx+c)}{a+b} \right)^{\frac{1}{2}} \text{EllipticE} \left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}, \left(\frac{a-b}{a+b} \right)^{\frac{1}{2}} \right) a^2 b \sin(dx+c) - B \left(\frac{\cos(dx+c)}{1+\cos(dx+c)} \right)^{\frac{1}{2}} \left(\frac{b+a \cos(dx+c)}{1+\cos(dx+c)} \right)^{\frac{1}{2}} \left(\frac{b+a \cos(dx+c)}{a+b} \right)^{\frac{1}{2}} \text{EllipticE} \left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}, \left(\frac{a-b}{a+b} \right)^{\frac{1}{2}} \right) a^2 b \sin(dx+c) - B \left(\frac{\cos(dx+c)}{1+\cos(dx+c)} \right)^{\frac{1}{2}} \left(\frac{b+a \cos(dx+c)}{1+\cos(dx+c)} \right)^{\frac{1}{2}} \left(\frac{b+a \cos(dx+c)}{a+b} \right)^{\frac{1}{2}} \text{EllipticE} \left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}, \left(\frac{a-b}{a+b} \right)^{\frac{1}{2}} \right) a b^2 \sin(dx+c) + 2 B \left(\frac{\cos(dx+c)}{1+\cos(dx+c)} \right)^{\frac{1}{2}} \left(\frac{b+a \cos(dx+c)}{1+\cos(dx+c)} \right)^{\frac{1}{2}} \left(\frac{b+a \cos(dx+c)}{a+b} \right)^{\frac{1}{2}} \text{EllipticE} \left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}, \left(\frac{a-b}{a+b} \right)^{\frac{1}{2}} \right) a^3 - B \left(\frac{\cos(dx+c)}{1+\cos(dx+c)} \right)^{\frac{1}{2}} \left(\frac{b+a \cos(dx+c)}{1+\cos(dx+c)} \right)^{\frac{1}{2}} \left(\frac{b+a \cos(dx+c)}{a+b} \right)^{\frac{1}{2}} \text{EllipticE} \left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}, \left(\frac{a-b}{a+b} \right)^{\frac{1}{2}} \right) \sin(dx+c) b^3 + 2 B \left(\frac{\cos(dx+c)}{1+\cos(dx+c)} \right)^{\frac{1}{2}} \left(\frac{b+a \cos(dx+c)}{1+\cos(dx+c)} \right)^{\frac{1}{2}} \left(\frac{b+a \cos(dx+c)}{a+b} \right)^{\frac{1}{2}} \text{EllipticE} \left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}, \left(\frac{a-b}{a+b} \right)^{\frac{1}{2}} \right) \cos(dx+c) \sin(dx+c) a^3 - B \left(\frac{\cos(dx+c)}{1+\cos(dx+c)} \right)^{\frac{1}{2}} \left(\frac{b+a \cos(dx+c)}{1+\cos(dx+c)} \right)^{\frac{1}{2}} \left(\frac{b+a \cos(dx+c)}{a+b} \right)^{\frac{1}{2}} \text{EllipticE} \left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}, \left(\frac{a-b}{a+b} \right)^{\frac{1}{2}} \right) \cos(dx+c) \sin(dx+c) b^3 + B \cos(dx+c) b^3 - B \cos(dx+c) \sin(dx+c) \left(\frac{\cos(dx+c)}{1+\cos(dx+c)} \right)^{\frac{1}{2}} \left(\frac{b+a \cos(dx+c)}{1+\cos(dx+c)} \right)^{\frac{1}{2}} \left(\frac{b+a \cos(dx+c)}{a+b} \right)^{\frac{1}{2}} \text{EllipticF} \left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}, \left(\frac{a-b}{a+b} \right)^{\frac{1}{2}} \right) a b^2 - B \text{EllipticE} \left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}, \left(\frac{a-b}{a+b} \right)^{\frac{1}{2}} \right) a$

$$\begin{aligned}
 & +b)^{(1/2)} \cdot \cos(dx+c) \cdot b^2 \cdot (\cos(dx+c)/(1+\cos(dx+c)))^{(1/2)} \cdot ((b+a \cdot \cos(dx+c))/ \\
 & (1+\cos(dx+c)))/(a+b)^{(1/2)} \cdot \sin(dx+c) \cdot a + A \cdot \text{EllipticF}((-1+\cos(dx+c))/\sin(dx+c), \\
 & ((a-b)/(a+b))^{(1/2)}) \cdot \cos(dx+c) \cdot b^2 \cdot (\cos(dx+c)/(1+\cos(dx+c)))^{(1/2)} \cdot ((b+a \cdot \cos(dx+c))/ \\
 & (1+\cos(dx+c)))/(a+b)^{(1/2)} \cdot \sin(dx+c) \cdot a - A \cdot \text{EllipticE}((-1+\cos(dx+c))/\sin(dx+c), \\
 & ((a-b)/(a+b))^{(1/2)}) \cdot \cos(dx+c) \cdot a^2 \cdot (\cos(dx+c)/(1+\cos(dx+c)))^{(1/2)} \cdot ((b+a \cdot \cos(dx+c))/ \\
 & (1+\cos(dx+c)))/(a+b)^{(1/2)} \cdot \sin(dx+c) \cdot b - A \cdot (\cos(dx+c)/(1+\cos(dx+c)))^{(1/2)} \cdot ((b+a \cdot \cos(dx+c))/ \\
 & (1+\cos(dx+c)))/(a+b)^{(1/2)} \cdot \text{EllipticE}((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{(1/2)}) \cdot \sin(dx+c) \\
 & \cdot \cos(dx+c) \cdot a \cdot b^2 - 2 \cdot B \cdot (\cos(dx+c)/(1+\cos(dx+c)))^{(1/2)} \cdot ((b+a \cdot \cos(dx+c))/ \\
 & (1+\cos(dx+c)))/(a+b)^{(1/2)} \cdot \text{EllipticF}((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{(1/2)}) \cdot \sin(dx+c) \\
 & \cdot \cos(dx+c) \cdot a^2 \cdot b + 2 \cdot B \cdot \text{EllipticE}((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{(1/2)}) \cdot \cos(dx+c) \\
 & \cdot a^2 \cdot (\cos(dx+c)/(1+\cos(dx+c)))^{(1/2)} \cdot ((b+a \cdot \cos(dx+c))/ \\
 & (1+\cos(dx+c)))/(a+b)^{(1/2)} \cdot \sin(dx+c) \cdot b - B \cdot (\cos(dx+c)/(1+\cos(dx+c)))^{(1/2)} \cdot ((b+a \cdot \cos(dx+c))/ \\
 & (1+\cos(dx+c)))/(a+b)^{(1/2)} \cdot \text{EllipticF}((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{(1/2)}) \cdot a \cdot b^2 \cdot \sin(dx+c) + A \cdot \sin(dx+c) \\
 & \cdot \cos(dx+c) \cdot (\cos(dx+c)/(1+\cos(dx+c)))^{(1/2)} \cdot ((b+a \cdot \cos(dx+c))/ \\
 & (1+\cos(dx+c)))/(a+b)^{(1/2)} \cdot \text{EllipticF}((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{(1/2)}) \cdot b^3 + B \cdot \sin(dx+c) \\
 & \cdot \cos(dx+c) \cdot (\cos(dx+c)/(1+\cos(dx+c)))^{(1/2)} \cdot ((b+a \cdot \cos(dx+c))/ \\
 & (1+\cos(dx+c)))/(a+b)^{(1/2)} \cdot \text{EllipticF}((-1+\cos(dx+c))/\sin(dx+c), ((a-b)/(a+b))^{(1/2)}) \cdot b^3 / (b+a \cdot \cos(dx+c)) / \sin(dx+c) / b^2 / (a+b) / (a-b)
 \end{aligned}$$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(dx+c)^2*(A+B*sec(dx+c))/(a+b*sec(dx+c))^(3/2),x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\cos(c+dx)^2 \left(a + \frac{b}{\cos(c+dx)}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + dx))/(cos(c + dx)^2*(a + b/cos(c + dx))^(3/2)),x)

[Out] int((A + B/cos(c + dx))/(cos(c + dx)^2*(a + b/cos(c + dx))^(3/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \sec^2(c + dx)}{(a + b \sec(c + dx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**2/(a + b*sec(c + d*x))**(3/2),  
x)
```

$$3.380 \quad \int \frac{\sec(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^{3/2}} dx$$

Optimal. Leaf size=254

$$\frac{2(Ab - aB) \tan(c + dx)}{d(a^2 - b^2) \sqrt{a + b \sec(c + dx)}} - \frac{2(Ab - aB) \cot(c + dx) \sqrt{\frac{b(1 - \sec(c+dx))}{a+b}} \sqrt{\frac{b(\sec(c+dx)+1)}{a-b}} E\left(\sin^{-1}\left(\frac{\sqrt{a+b \sec(c+dx)}}{\sqrt{a+b}}\right)\right)}{b^2 d \sqrt{a + b}}$$

[Out] $-2*(A*b-B*a)*\cot(d*x+c)*\text{EllipticE}((a+b*\sec(d*x+c))^{1/2}/(a+b)^{1/2}, ((a+b)/(a-b))^{1/2})*(b*(1-\sec(d*x+c))/(a+b))^{1/2}*(-b*(1+\sec(d*x+c))/(a-b))^{1/2}/b^2/d/(a+b)^{1/2}+2*(A+B)*\cot(d*x+c)*\text{EllipticF}((a+b*\sec(d*x+c))^{1/2}/(a+b)^{1/2}, ((a+b)/(a-b))^{1/2})*(b*(1-\sec(d*x+c))/(a+b))^{1/2}*(-b*(1+\sec(d*x+c))/(a-b))^{1/2}/b/d/(a+b)^{1/2}-2*(A*b-B*a)*\tan(d*x+c)/(a^2-b^2)/d/(a+b*\sec(d*x+c))^{1/2}$

Rubi [A] time = 0.35, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {4003, 4005, 3832, 4004}

$$\frac{2(Ab - aB) \tan(c + dx)}{d(a^2 - b^2) \sqrt{a + b \sec(c + dx)}} - \frac{2(Ab - aB) \cot(c + dx) \sqrt{\frac{b(1 - \sec(c+dx))}{a+b}} \sqrt{\frac{b(\sec(c+dx)+1)}{a-b}} E\left(\sin^{-1}\left(\frac{\sqrt{a+b \sec(c+dx)}}{\sqrt{a+b}}\right)\right)}{b^2 d \sqrt{a + b}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(\text{Sec}[c + d*x]*(A + B*\text{Sec}[c + d*x]))/(a + b*\text{Sec}[c + d*x])^{3/2}, x]$

[Out] $(-2*(A*b - a*B)*\text{Cot}[c + d*x]*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[a + b*\text{Sec}[c + d*x]]]/\text{Sqrt}[a + b]], (a + b)/(a - b)]*\text{Sqrt}[(b*(1 - \text{Sec}[c + d*x]))/(a + b)]*\text{Sqrt}[-((b*(1 + \text{Sec}[c + d*x]))/(a - b))]/(b^2*\text{Sqrt}[a + b]*d) + (2*(A + B)*\text{Cot}[c + d*x]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[a + b*\text{Sec}[c + d*x]]]/\text{Sqrt}[a + b]], (a + b)/(a - b)]*\text{Sqrt}[(b*(1 - \text{Sec}[c + d*x]))/(a + b)]*\text{Sqrt}[-((b*(1 + \text{Sec}[c + d*x]))/(a - b))]/(b*\text{Sqrt}[a + b]*d) - (2*(A*b - a*B)*\text{Tan}[c + d*x])/((a^2 - b^2)*d*\text{Sqrt}[a + b*\text{Sec}[c + d*x]])$

Rule 3832

$\text{Int}[\text{csc}[(e_.) + (f_.)*(x_.)]/\text{Sqrt}[\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] \rightarrow \text{Simp}[(-2*\text{Rt}[a + b, 2]*\text{Sqrt}[(b*(1 - \text{Csc}[e + f*x]))/(a + b)]*\text{Sqrt}[-((b*(1 + \text{Csc}[e + f*x]))/(a - b))]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[a + b*\text{Csc}[e + f*x]]/\text{Rt}[a + b, 2]], (a + b)/(a - b))]/(b*f*\text{Cot}[e + f*x]), x] /; \text{FreeQ}\{a, b, e, f\}, x] \&\& \text{NeQ}[a^2 - b^2, 0]$

Rule 4003

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] := -Simp[((A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(m + 1)*(a^2 - b^2), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[(a*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] := Simp[(-2*(A*b - a*B)*Rt[a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-(b*(1 + Csc[e + f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A, 2]], (a*A + b*B)/(a*A - b*B)]/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] := Dist[A - B, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{3/2}} dx = -\frac{2(Ab-aB)\tan(c+dx)}{(a^2-b^2)d\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{\sec(c+dx)\left(\frac{1}{2}(-aA+bB)-\frac{1}{2}(Ab-aB)\sec(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}} dx}{a^2-b^2}$$

$$= -\frac{2(Ab-aB)\tan(c+dx)}{(a^2-b^2)d\sqrt{a+b\sec(c+dx)}} + \frac{(A+B)\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}} dx}{a+b} + \frac{(Ab-aB)\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}} dx}{a+b}$$

$$= -\frac{2(Ab-aB)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)\middle|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}}{b^2\sqrt{a+bd}}$$

Mathematica [A] time = 15.25, size = 468, normalized size = 1.84

$$\frac{\sec(c + dx)(a \cos(c + dx) + b)^2(A + B \sec(c + dx)) \left(\frac{2(Ab \sin(c+dx) - aB \sin(c+dx))}{(b^2 - a^2)(a \cos(c+dx) + b)} - \frac{2(Ab - aB) \sin(c+dx)}{b(b^2 - a^2)} \right)}{d(a + b \sec(c + dx))^{3/2}(A \cos(c + dx) + B)} \cdot 2\sqrt{\sec(c + dx)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(3/2), x]

[Out] ((b + a*Cos[c + d*x])^2*Sec[c + d*x]*(A + B*Sec[c + d*x])*((-2*(A*b - a*B)*Sin[c + d*x])/(b*(-a^2 + b^2)) + (2*(A*b*Sin[c + d*x] - a*B*Sin[c + d*x]))/((-a^2 + b^2)*(b + a*Cos[c + d*x])))/(d*(B + A*Cos[c + d*x])*(a + b*Sec[c + d*x])^(3/2)) - (2*(b + a*Cos[c + d*x])*Sqrt[Sec[c + d*x]]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(A + B*Sec[c + d*x])*(2*(a + b)*(-(A*b) + a*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*b*(a + b)*(A - B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - (A*b - a*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/((-a^2*b) + b^3)*d*(B + A*Cos[c + d*x])*Sqrt[Sec[(c + d*x)/2]^2*(a + b*Sec[c + d*x])^(3/2))

fricas [F] time = 1.00, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{(B \sec(dx + c)^2 + A \sec(dx + c)) \sqrt{b \sec(dx + c) + a}}{b^2 \sec(dx + c)^2 + 2ab \sec(dx + c) + a^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2), x, algorithm="fricas")

[Out] integral((B*sec(d*x + c)^2 + A*sec(d*x + c))*sqrt(b*sec(d*x + c) + a)/(b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sec(dx + c)}{(b \sec(dx + c) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2), x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)/(b*sec(d*x + c) + a)^(3/2), x)

maple [B] time = 2.11, size = 1633, normalized size = 6.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2), x)

[Out] $\frac{1}{d} \frac{1}{4} \sqrt{\frac{b+a \cos(dx+c)}{\cos(dx+c)}} \sqrt{\frac{A \cos(dx+c) \sin(dx+c) (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} ((b+a \cos(dx+c))/(1+\cos(dx+c)) / (a+b))^{1/2} \text{EllipticE}(-1+\cos(dx+c)/\sin(dx+c), ((a-b)/(a+b))^{1/2}) * a * b + A \cos(dx+c) \sin(dx+c) (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} ((b+a \cos(dx+c))/(1+\cos(dx+c)) / (a+b))^{1/2} \text{EllipticE}(-1+\cos(dx+c)/\sin(dx+c), ((a-b)/(a+b))^{1/2}) * b^2 - A \cos(dx+c) \sin(dx+c) (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} ((b+a \cos(dx+c))/(1+\cos(dx+c)) / (a+b))^{1/2} \text{EllipticF}(-1+\cos(dx+c)/\sin(dx+c), ((a-b)/(a+b))^{1/2}) * a * b - A \cos(dx+c) \sin(dx+c) (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} ((b+a \cos(dx+c))/(1+\cos(dx+c)) / (a+b))^{1/2} \text{EllipticF}(-1+\cos(dx+c)/\sin(dx+c), ((a-b)/(a+b))^{1/2}) * b^2 - B \cos(dx+c) \sin(dx+c) (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} ((b+a \cos(dx+c))/(1+\cos(dx+c)) / (a+b))^{1/2} \text{EllipticE}(-1+\cos(dx+c)/\sin(dx+c), ((a-b)/(a+b))^{1/2}) * a^2 - B \cos(dx+c) \sin(dx+c) (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} ((b+a \cos(dx+c))/(1+\cos(dx+c)) / (a+b))^{1/2} \text{EllipticE}(-1+\cos(dx+c)/\sin(dx+c), ((a-b)/(a+b))^{1/2}) * a * b + B \cos(dx+c) \sin(dx+c) (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} ((b+a \cos(dx+c))/(1+\cos(dx+c)) / (a+b))^{1/2} \text{EllipticF}(-1+\cos(dx+c)/\sin(dx+c), ((a-b)/(a+b))^{1/2}) * a * b + B \cos(dx+c) \sin(dx+c) (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} ((b+a \cos(dx+c))/(1+\cos(dx+c)) / (a+b))^{1/2} \text{EllipticF}(-1+\cos(dx+c)/\sin(dx+c), ((a-b)/(a+b))^{1/2}) * b^2 + A (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} ((b+a \cos(dx+c))/(1+\cos(dx+c)) / (a+b))^{1/2} \text{EllipticE}(-1+\cos(dx+c)/\sin(dx+c), ((a-b)/(a+b))^{1/2}) * a * b * \sin(dx+c) + A (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} ((b+a \cos(dx+c))/(1+\cos(dx+c)) / (a+b))^{1/2} \text{EllipticE}(-1+\cos(dx+c)/\sin(dx+c), ((a-b)/(a+b))^{1/2}) * b^2 * \sin(dx+c) - A (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} ((b+a \cos(dx+c))/(1+\cos(dx+c)) / (a+b))^{1/2} \text{EllipticF}(-1+\cos(dx+c)/\sin(dx+c), ((a-b)/(a+b))^{1/2}) * a * b * \sin(dx+c) - A \text{EllipticF}(-1+\cos(dx+c)/\sin(dx+c), ((a-b)/(a+b))^{1/2}) * b^2 * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} ((b+a \cos(dx+c))/(1+\cos(dx+c)) / (a+b))^{1/2} \sin(dx+c) - B (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} ((b+a \cos(dx+c))/(1+\cos(dx+c)) / (a+b))^{1/2} \text{EllipticE}(-1+\cos(dx+c)/\sin(dx+c), ((a-b)/(a+b))^{1/2}) * a^2 * \sin(dx+c) - B (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} ((b+a \cos(dx+c))/(1+\cos(dx+c)) / (a+b))^{1/2} \text{EllipticE}(-1+\cos(dx+c)/\sin(dx+c), ((a-b)/(a+b))^{1/2}) * a * b * \sin(dx+c) + B (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} ((b+a \cos(dx+c))/(1+\cos(dx+c)) / (a+b))^{1/2} \text{EllipticF}(-1+\cos(dx+c)/\sin(dx+c), ((a-b)/(a+b))^{1/2}) * a * b * \sin(dx+c) + B \text{EllipticF}(-1+\cos(dx+c)/\sin(dx+c), ((a-b)/(a+b))^{1/2}) * b^2 * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} ((b+a \cos(dx+c))/(1+\cos(dx+c)) / (a+b))^{1/2} \sin(dx+c) - A \cos(dx+c)^2 * a * b + A \cos(dx+c)^2 * b^2 + B \cos(dx+c)^2 * a^2 - B \cos(dx+c)^2 * a * b + A \cos(dx+c)^2 * b^2$

$*x+c)*a*b-A*\cos(d*x+c)*b^2-B*\cos(d*x+c)*a^2+B*\cos(d*x+c)*a*b)/(b+a*\cos(d*x+c))/\sin(d*x+c)/b/(a+b)/(a-b)$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\cos(c+dx) \left(a + \frac{b}{\cos(c+dx)}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/(cos(c + d*x)*(a + b/cos(c + d*x))^(3/2)),x)

[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)*(a + b/cos(c + d*x))^(3/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \sec(c + dx)}{(a + b \sec(c + dx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(3/2),x)

[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)/(a + b*sec(c + d*x))**(3/2), x)

$$3.381 \quad \int \frac{A+B \sec(c+dx)}{(a+b \sec(c+dx))^{3/2}} dx$$

Optimal. Leaf size=376

$$\frac{2b(Ab - aB) \tan(c + dx)}{ad(a^2 - b^2) \sqrt{a + b \sec(c + dx)}} - \frac{2A\sqrt{a + b} \cot(c + dx) \sqrt{\frac{b(1 - \sec(c+dx))}{a+b}} \sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}} \Pi\left(\frac{a+b}{a}; \sin^{-1}\left(\frac{\sqrt{a+b \sec(c+dx)}}{\sqrt{a+b}}\right)\right)}{a^2 d}$$

[Out] 2*(A*b-B*a)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a/b/d/(a+b)^(1/2)-2*(A*b-B*a)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a/b/d/(a+b)^(1/2)-2*A*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2), (a+b)/a, ((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d+2*b*(A*b-B*a)*tan(d*x+c)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)

Rubi [A] time = 0.43, antiderivative size = 376, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3923, 4058, 3921, 3784, 3832, 4004}

$$\frac{2b(Ab - aB) \tan(c + dx)}{ad(a^2 - b^2) \sqrt{a + b \sec(c + dx)}} - \frac{2A\sqrt{a + b} \cot(c + dx) \sqrt{\frac{b(1 - \sec(c+dx))}{a+b}} \sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}} \Pi\left(\frac{a+b}{a}; \sin^{-1}\left(\frac{\sqrt{a+b \sec(c+dx)}}{\sqrt{a+b}}\right)\right)}{a^2 d}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sec[c + d*x])/(a + b*Sec[c + d*x])^(3/2), x]

[Out] (2*(A*b - a*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))]/(a*b*Sqrt[a + b]*d) - (2*(A*b - a*B)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))]/(a*b*Sqrt[a + b]*d) - (2*A*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))]/(a^2*d) + (2*b*(A*b - a*B)*Tan[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]])

Rule 3784

Int[1/Sqrt[csc[(c_.) + (d_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] :> Simp[(2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]]/Rt[a + b,

2]], (a + b)/(a - b)]/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 3832

Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] := Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]]/Rt[a + b, 2]], (a + b)/(a - b)]/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 3921

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] := Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]

Rule 3923

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] := Simp[(b*(b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[c*(a^2 - b^2)*(m + 1) - (a*(b*c - a*d)*(m + 1))*Csc[e + f*x] + b*(b*c - a*d)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m]

Rule 4004

Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] := Simp[(-2*(A*b - a*B)*Rt[a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]]/Rt[a + (b*B)/A, 2]], (a*A + b*B)/(a*A - b*B)]/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]

Rule 4058

Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] := Int[(A + (B - C)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned}
\int \frac{A + B \sec(c + dx)}{(a + b \sec(c + dx))^{3/2}} dx &= \frac{2b(Ab - aB) \tan(c + dx)}{a(a^2 - b^2) d \sqrt{a + b \sec(c + dx)}} - \frac{2 \int \frac{-\frac{1}{2}A(a^2 - b^2) + \frac{1}{2}a(Ab - aB) \sec(c + dx) + \frac{1}{2}b(Ab - aB) \sec^2(c + dx)}{\sqrt{a + b \sec(c + dx)}}}{a(a^2 - b^2)} \\
&= \frac{2b(Ab - aB) \tan(c + dx)}{a(a^2 - b^2) d \sqrt{a + b \sec(c + dx)}} - \frac{2 \int \frac{-\frac{1}{2}A(a^2 - b^2) + (\frac{1}{2}a(Ab - aB) - \frac{1}{2}b(Ab - aB)) \sec(c + dx)}{\sqrt{a + b \sec(c + dx)}} dx}{a(a^2 - b^2)} \\
&= \frac{2(Ab - aB) \cot(c + dx) E\left(\sin^{-1}\left(\frac{\sqrt{a + b \sec(c + dx)}}{\sqrt{a + b}}\right) \middle| \frac{a + b}{a - b}\right) \sqrt{\frac{b(1 - \sec(c + dx))}{a + b}} \sqrt{-\frac{b(1 + \sec(c + dx))}{a - b}}}{ab \sqrt{a + b} d} \\
&= \frac{2(Ab - aB) \cot(c + dx) E\left(\sin^{-1}\left(\frac{\sqrt{a + b \sec(c + dx)}}{\sqrt{a + b}}\right) \middle| \frac{a + b}{a - b}\right) \sqrt{\frac{b(1 - \sec(c + dx))}{a + b}} \sqrt{-\frac{b(1 + \sec(c + dx))}{a - b}}}{ab \sqrt{a + b} d}
\end{aligned}$$

Mathematica [C] time = 14.58, size = 1491, normalized size = 3.97

result too large to display

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sec[c + d*x])/(a + b*Sec[c + d*x])^(3/2), x]

[Out] ((b + a*Cos[c + d*x])^2*Sec[c + d*x]*(A + B*Sec[c + d*x])*((2*(-(A*b) + a*B)*Sin[c + d*x])/(a*(a^2 - b^2)) - (2*(-(A*b^2*Sin[c + d*x]) + a*b*B*Sin[c + d*x]))/(a*(a^2 - b^2)*(b + a*Cos[c + d*x])))/(d*(B + A*Cos[c + d*x])*(a + b*Sec[c + d*x])^(3/2)) + (2*(b + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x])*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]*(a*A*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2] + A*b^2*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2] - a^2*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)/2] - a*b*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)/2] - 2*a*A*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^3 + 2*a^2*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)/2]^3 + a*A*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 - A*b^2*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 - a^2*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)/2]^5 + a*b*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)/2]^5 - (2*I)*a^2*A*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (2*I)*A*b^2*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[

$$\begin{aligned} & \left(\frac{c + dx}{2} \right)^2 + b \tan\left[\frac{c + dx}{2}\right]^2 / (a + b) - (2I) a^2 A \operatorname{EllipticPi}\left[-\left(\frac{a + b}{a - b}\right), I \operatorname{ArcSinh}\left[\sqrt{\frac{-a + b}{a + b}} \tan\left[\frac{c + dx}{2}\right]\right], \left(\frac{a + b}{a - b}\right) \tan\left[\frac{c + dx}{2}\right]^2 \sqrt{1 - \tan\left[\frac{c + dx}{2}\right]^2} \sqrt{a + b - a \tan\left[\frac{c + dx}{2}\right]^2 + b \tan\left[\frac{c + dx}{2}\right]^2} / (a + b) + (2I) A b^2 \operatorname{EllipticPi}\left[-\left(\frac{a + b}{a - b}\right), I \operatorname{ArcSinh}\left[\sqrt{\frac{-a + b}{a + b}} \tan\left[\frac{c + dx}{2}\right]\right], \left(\frac{a + b}{a - b}\right) \tan\left[\frac{c + dx}{2}\right]^2 \sqrt{1 - \tan\left[\frac{c + dx}{2}\right]^2} \sqrt{a + b - a \tan\left[\frac{c + dx}{2}\right]^2 + b \tan\left[\frac{c + dx}{2}\right]^2} / (a + b) + I(a - b)(-A * b + a * B) \operatorname{EllipticE}\left[I \operatorname{ArcSinh}\left[\sqrt{\frac{-a + b}{a + b}} \tan\left[\frac{c + dx}{2}\right]\right], \left(\frac{a + b}{a - b}\right) \sqrt{1 - \tan\left[\frac{c + dx}{2}\right]^2} (1 + \tan\left[\frac{c + dx}{2}\right]^2) \sqrt{a + b - a \tan\left[\frac{c + dx}{2}\right]^2 + b \tan\left[\frac{c + dx}{2}\right]^2} / (a + b) + I(a - b)(2 * A * b + a * (A - B)) \operatorname{EllipticF}\left[I \operatorname{ArcSinh}\left[\sqrt{\frac{-a + b}{a + b}} \tan\left[\frac{c + dx}{2}\right]\right], \left(\frac{a + b}{a - b}\right) \sqrt{1 - \tan\left[\frac{c + dx}{2}\right]^2} (1 + \tan\left[\frac{c + dx}{2}\right]^2) \sqrt{a + b - a \tan\left[\frac{c + dx}{2}\right]^2 + b \tan\left[\frac{c + dx}{2}\right]^2} / (a + b) \right) \right) / (a \sqrt{\frac{-a + b}{a + b}} (a^2 - b^2) d * (B + A \cos[c + dx]) * (a + b \sec[c + dx]))^{3/2} * (-1 + \tan\left[\frac{c + dx}{2}\right]^2) \sqrt{(1 + \tan\left[\frac{c + dx}{2}\right]^2) / (1 - \tan\left[\frac{c + dx}{2}\right]^2)} * (a * (-1 + \tan\left[\frac{c + dx}{2}\right]^2) - b * (1 + \tan\left[\frac{c + dx}{2}\right]^2)) \right) \end{aligned}$$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(dx+c))/(a+b*sec(dx+c))^(3/2),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(b \sec(dx + c) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(dx+c))/(a+b*sec(dx+c))^(3/2),x, algorithm="giac")

[Out] integrate((B*sec(dx + c) + A)/(b*sec(dx + c) + a)^(3/2), x)

maple [B] time = 2.10, size = 2010, normalized size = 5.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(dx+c))/(a+b*sec(dx+c))^(3/2),x)

[Out] $-1/d * 4^{1/2} * ((b+a*\cos(dx+c))/\cos(dx+c))^{1/2} * (-2*A*(\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * ((b+a*\cos(dx+c))/(1+\cos(dx+c)))/(a+b))^{1/2} * \operatorname{EllipticPi}((-1+$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(b \sec(dx + c) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)/(b*sec(d*x + c) + a)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\left(a + \frac{b}{\cos(c+dx)}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/(a + b/cos(c + d*x))^(3/2),x)

[Out] int((A + B/cos(c + d*x))/(a + b/cos(c + d*x))^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \sec(c + dx)}{(a + b \sec(c + dx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(3/2),x)

[Out] Integral((A + B*sec(c + d*x))/(a + b*sec(c + d*x))**(3/2), x)

$$3.382 \quad \int \frac{\cos(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^{3/2}} dx$$

Optimal. Leaf size=427

$$\frac{\sqrt{a+b}(3Ab-2aB) \cot(c+dx) \sqrt{\frac{b(1-\sec(c+dx))}{a+b}} \sqrt{\frac{b(\sec(c+dx)+1)}{a-b}} \Pi\left(\frac{a+b}{a}; \sin^{-1}\left(\frac{\sqrt{a+b \sec(c+dx)}}{\sqrt{a+b}}\right) \middle| \frac{a+b}{a-b}\right) + b(a^2A+2abB)}{a^3d(a^2-b^2)}$$

[Out] $(A*a^2-3*A*b^2+2*B*a*b)*\cot(d*x+c)*\text{EllipticE}((a+b*\sec(d*x+c))^{1/2}/(a+b)^{(1/2)}, ((a+b)/(a-b))^{1/2})*(b*(1-\sec(d*x+c))/(a+b))^{1/2}*(-b*(1+\sec(d*x+c))/(a-b))^{1/2}/a^2/b/d/(a+b)^{(1/2)}+(3*A*b+a*(A-2*B))*\cot(d*x+c)*\text{EllipticF}((a+b*\sec(d*x+c))^{1/2}/(a+b)^{(1/2)}, ((a+b)/(a-b))^{1/2})*(b*(1-\sec(d*x+c))/(a+b))^{1/2}*(-b*(1+\sec(d*x+c))/(a-b))^{1/2}/a^2/d/(a+b)^{(1/2)}+(3*A*b-2*B*a)*\cot(d*x+c)*\text{EllipticPi}((a+b*\sec(d*x+c))^{1/2}/(a+b)^{(1/2)}, (a+b)/a, ((a+b)/(a-b))^{1/2})*(a+b)^{(1/2})*(b*(1-\sec(d*x+c))/(a+b))^{1/2}*(-b*(1+\sec(d*x+c))/(a-b))^{1/2}/a^3/d+A*\sin(d*x+c)/a/d/(a+b*\sec(d*x+c))^{1/2}+b*(A*a^2-3*A*b^2+2*B*a*b)*\tan(d*x+c)/a^2/(a^2-b^2)/d/(a+b*\sec(d*x+c))^{1/2}$

Rubi [A] time = 0.70, antiderivative size = 427, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {4034, 4061, 4058, 3921, 3784, 3832, 4004}

$$\frac{b(a^2A+2abB-3Ab^2) \tan(c+dx) (a^2A+2abB-3Ab^2) \cot(c+dx) \sqrt{\frac{b(1-\sec(c+dx))}{a+b}} \sqrt{\frac{b(\sec(c+dx)+1)}{a-b}} E\left(\sin^{-1}\left(\frac{\sqrt{a+b \sec(c+dx)}}{\sqrt{a+b}}\right) \middle| \frac{a+b}{a-b}\right)}{a^2d(a^2-b^2) \sqrt{a+b \sec(c+dx)} + a^2bd \sqrt{a+b}}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(3/2), x]

[Out] $((a^2*A - 3*A*b^2 + 2*a*b*B)*\text{Cot}[c + d*x]*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[a + b*\text{Sec}[c + d*x]]/\text{Sqrt}[a + b]], (a + b)/(a - b)]*\text{Sqrt}[(b*(1 - \text{Sec}[c + d*x]))/(a + b)]*\text{Sqrt}[-((b*(1 + \text{Sec}[c + d*x]))/(a - b))]/(a^2*b*\text{Sqrt}[a + b]*d) + ((3*A*b + a*(A - 2*B))*\text{Cot}[c + d*x]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[a + b*\text{Sec}[c + d*x]]/\text{Sqrt}[a + b]], (a + b)/(a - b)]*\text{Sqrt}[(b*(1 - \text{Sec}[c + d*x]))/(a + b)]*\text{Sqrt}[-((b*(1 + \text{Sec}[c + d*x]))/(a - b))]/(a^2*\text{Sqrt}[a + b]*d) + (\text{Sqrt}[a + b]*(3*A*b - 2*a*B)*\text{Cot}[c + d*x]*\text{EllipticPi}[(a + b)/a, \text{ArcSin}[\text{Sqrt}[a + b*\text{Sec}[c + d*x]]/\text{Sqrt}[a + b]], (a + b)/(a - b)]*\text{Sqrt}[(b*(1 - \text{Sec}[c + d*x]))/(a + b)]*\text{Sqrt}[-((b*(1 + \text{Sec}[c + d*x]))/(a - b))]/(a^3*d) + (A*\text{Sin}[c + d*x])/(a*d*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]) + (b*(a^2*A - 3*A*b^2 + 2*a*b*B)*\text{Tan}[c + d*x])/(a^2*(a^2 - b^2)*d*\text{Sqrt}[a + b*\text{Sec}[c + d*x]])$

Rule 3784

Int[1/Sqrt[csc[(c_.) + (d_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] := Simp[(2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))

$$\frac{1}{(a-b)} \text{EllipticPi}\left[\frac{a+b}{a}, \text{ArcSin}\left[\frac{\sqrt{a+b\text{Csc}[c+dx]}}{\text{Rt}[a+b, 2]}\right], \frac{a+b}{a-b}\right] / (a*d\text{Cot}[c+dx]), x \text{ ; FreeQ}\{a, b, c, d\}, x \ \&\& \ \text{NeQ}[a^2 - b^2, 0]$$

Rule 3832

$$\text{Int}[\text{csc}[e + f*x] / \sqrt{\text{csc}[e + f*x] * (b + a)}, x_Symbol] \text{ :> } \text{Simp}\left[\frac{-2\text{Rt}[a+b, 2] * \sqrt{(b(1 - \text{Csc}[e + f*x])) / (a+b)}}{\text{csc}[e + f*x] * \sqrt{-(b(1 + \text{Csc}[e + f*x])) / (a-b)}} * \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b\text{Csc}[e + f*x]}}{\text{Rt}[a+b, 2]}\right], \frac{a+b}{a-b}\right] / (b*f\text{Cot}[e + f*x]), x \text{ ; FreeQ}\{a, b, e, f\}, x \ \&\& \ \text{NeQ}[a^2 - b^2, 0]\right]$$

Rule 3921

$$\text{Int}[(\text{csc}[e + f*x] * (d + c)) / \sqrt{\text{csc}[e + f*x] * (b + a)}, x_Symbol] \text{ :> } \text{Dist}[c, \text{Int}[1 / \sqrt{a + b\text{Csc}[e + f*x]}, x], x] + \text{Dist}[d, \text{Int}[\text{Csc}[e + f*x] / \sqrt{a + b\text{Csc}[e + f*x]}, x], x] \text{ ; FreeQ}\{a, b, c, d, e, f\}, x \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{NeQ}[a^2 - b^2, 0]$$

Rule 4004

$$\text{Int}[(\text{csc}[e + f*x] * (c\text{Csc}[e + f*x] * (B + A))) / \sqrt{\text{csc}[e + f*x] * (b + a)}, x_Symbol] \text{ :> } \text{Simp}\left[\frac{-2(A*b - a*B)\text{Rt}[a + (b*B)/A, 2] * \sqrt{(b(1 - \text{Csc}[e + f*x])) / (a+b)}}{\text{csc}[e + f*x] * \sqrt{-(b(1 + \text{Csc}[e + f*x])) / (a-b)}} * \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b\text{Csc}[e + f*x]}}{\text{Rt}[a + (b*B)/A, 2]}\right], \frac{(a*A + b*B) / (a*A - b*B)}{b^2 * f\text{Cot}[e + f*x]}\right], x \text{ ; FreeQ}\{a, b, e, f, A, B\}, x \ \&\& \ \text{NeQ}[a^2 - b^2, 0] \ \&\& \ \text{EqQ}[A^2 - B^2, 0]\right]$$

Rule 4034

$$\text{Int}[(\text{csc}[e + f*x] * (d))^{n_1} * (\text{csc}[e + f*x] * (b + a))^{m_1} * (\text{csc}[e + f*x] * (B + A)), x_Symbol] \text{ :> } \text{Simp}[(A*\text{Cot}[e + f*x] * (a + b\text{Csc}[e + f*x])^{m+1} * (d*\text{Csc}[e + f*x])^n) / (a*f^n), x] + \text{Dist}[1/(a*d^n), \text{Int}[(a + b\text{Csc}[e + f*x])^m * (d*\text{Csc}[e + f*x])^{n+1} * \text{Simp}[a*B^n - A*b*(m+n+1) + A*a*(n+1)*\text{Csc}[e + f*x] + A*b*(m+n+2)*\text{Csc}[e + f*x]^2, x], x], x] \text{ ; FreeQ}\{a, b, d, e, f, A, B, m\}, x \ \&\& \ \text{NeQ}[A*b - a*B, 0] \ \&\& \ \text{NeQ}[a^2 - b^2, 0] \ \&\& \ \text{LeQ}[n, -1]$$

Rule 4058

$$\text{Int}[(A + \text{csc}[e + f*x] * (B + c\text{Csc}[e + f*x] * (C + \text{Csc}[e + f*x]))) / \sqrt{\text{csc}[e + f*x] * (b + a)}, x_Symbol] \text{ :> } \text{Int}[(A + (B - C) * \text{Csc}[e + f*x]) / \sqrt{a + b\text{Csc}[e + f*x]}, x] + \text{Dist}[C, \text{Int}[(\text{Csc}[e + f*x] * (1 + \text{Csc}[e + f*x])) / \sqrt{a + b\text{Csc}[e + f*x]}, x], x] \text{ ; FreeQ}\{a, b, e, f, A,$$

B, C}, x] && NeQ[a^2 - b^2, 0]

Rule 4061

Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_), x_Symbol] :> Simp[((A*b^2 + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*b*(A + C)*(m + 1)*Csc[e + f*x] + (A*b^2 + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m] && LtQ[m, -1]

Rubi steps

$$\begin{aligned} \int \frac{\cos(c + dx)(A + B \sec(c + dx))}{(a + b \sec(c + dx))^{3/2}} dx &= \frac{A \sin(c + dx)}{ad\sqrt{a + b \sec(c + dx)}} - \frac{\int \frac{\frac{1}{2}(3Ab - 2aB) - \frac{1}{2}Ab \sec^2(c + dx)}{(a + b \sec(c + dx))^{3/2}} dx}{a} \\ &= \frac{A \sin(c + dx)}{ad\sqrt{a + b \sec(c + dx)}} + \frac{b(a^2A - 3Ab^2 + 2abB) \tan(c + dx)}{a^2(a^2 - b^2)d\sqrt{a + b \sec(c + dx)}} + \frac{2 \int \frac{-\frac{1}{4}(a^2)}{a + b \sec(c + dx)} dx}{a^2(a^2 - b^2)} \\ &= \frac{A \sin(c + dx)}{ad\sqrt{a + b \sec(c + dx)}} + \frac{b(a^2A - 3Ab^2 + 2abB) \tan(c + dx)}{a^2(a^2 - b^2)d\sqrt{a + b \sec(c + dx)}} + \frac{2 \int \frac{-\frac{1}{4}(a^2)}{a + b \sec(c + dx)} dx}{a^2(a^2 - b^2)} \\ &= \frac{(a^2A - 3Ab^2 + 2abB) \cot(c + dx) E\left(\sin^{-1}\left(\frac{\sqrt{a+b \sec(c+dx)}}{\sqrt{a+b}}\right) \middle| \frac{a+b}{a-b}\right) \sqrt{\frac{b(1-\sec(c+dx))}{a+b}}}{a^2b\sqrt{a+bd}} \\ &= \frac{(a^2A - 3Ab^2 + 2abB) \cot(c + dx) E\left(\sin^{-1}\left(\frac{\sqrt{a+b \sec(c+dx)}}{\sqrt{a+b}}\right) \middle| \frac{a+b}{a-b}\right) \sqrt{\frac{b(1-\sec(c+dx))}{a+b}}}{a^2b\sqrt{a+bd}} \end{aligned}$$

Mathematica [B] time = 19.61, size = 1597, normalized size = 3.74

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[(Cos[c + d*x]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(3/2), x]

[Out] ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*((-2*b*(A*b - a*B)*Sin[c + d*x]))/(a^2*(-a^2 + b^2)) + (2*(-(A*b^3*Sin[c + d*x]) + a*b^2*B*Sin[c + d*x]))/(a^2*(

$$\frac{a^2 - b^2}{(b + a \cos(c + dx))} \left(\frac{((b + a \cos(c + dx))^{3/2} \sec(c + dx)^{3/2} \sqrt{(1 - \tan((c + dx)/2)^2})^{-1}) \sqrt{(a + b - a \tan((c + dx)/2)^2 + b \tan((c + dx)/2)^2)} / (1 + \tan((c + dx)/2)^2)}{(a^3 A \tan((c + dx)/2) + a^2 A b \tan((c + dx)/2) - 3 a A b^2 \tan((c + dx)/2) - 3 A b^3 \tan((c + dx)/2) + 2 a^2 b B \tan((c + dx)/2) + 2 a b^2 B \tan((c + dx)/2) - 2 a^3 A \tan((c + dx)/2)^3 + 6 a A b^2 \tan((c + dx)/2)^3 - 4 a^2 b B \tan((c + dx)/2)^3 + a^3 A \tan((c + dx)/2)^5 - a^2 A b \tan((c + dx)/2)^5 - 3 a A b^2 \tan((c + dx)/2)^5 + 3 A b^3 \tan((c + dx)/2)^5 + 2 a^2 b B \tan((c + dx)/2)^5 - 2 a b^2 B \tan((c + dx)/2)^5 - 6 a^2 A b \operatorname{EllipticPi}[-1, \operatorname{ArcSin}[\tan((c + dx)/2)], (a - b)/(a + b)] \sqrt{1 - \tan((c + dx)/2)^2} \sqrt{(a + b - a \tan((c + dx)/2)^2 + b \tan((c + dx)/2)^2)} / (a + b) + 6 A b^3 \operatorname{EllipticPi}[-1, \operatorname{ArcSin}[\tan((c + dx)/2)], (a - b)/(a + b)] \sqrt{1 - \tan((c + dx)/2)^2} \sqrt{(a + b - a \tan((c + dx)/2)^2 + b \tan((c + dx)/2)^2)} / (a + b) + 4 a^3 B \operatorname{EllipticPi}[-1, \operatorname{ArcSin}[\tan((c + dx)/2)], (a - b)/(a + b)] \sqrt{1 - \tan((c + dx)/2)^2} \sqrt{(a + b - a \tan((c + dx)/2)^2 + b \tan((c + dx)/2)^2)} / (a + b) - 4 a b^2 B \operatorname{EllipticPi}[-1, \operatorname{ArcSin}[\tan((c + dx)/2)], (a - b)/(a + b)] \sqrt{1 - \tan((c + dx)/2)^2} \sqrt{(a + b - a \tan((c + dx)/2)^2 + b \tan((c + dx)/2)^2)} / (a + b) - 6 a^2 A b \operatorname{EllipticPi}[-1, \operatorname{ArcSin}[\tan((c + dx)/2)], (a - b)/(a + b)] \tan((c + dx)/2)^2 \sqrt{1 - \tan((c + dx)/2)^2} \sqrt{(a + b - a \tan((c + dx)/2)^2 + b \tan((c + dx)/2)^2)} / (a + b) + 6 A b^3 \operatorname{EllipticPi}[-1, \operatorname{ArcSin}[\tan((c + dx)/2)], (a - b)/(a + b)] \tan((c + dx)/2)^2 \sqrt{1 - \tan((c + dx)/2)^2} \sqrt{(a + b - a \tan((c + dx)/2)^2 + b \tan((c + dx)/2)^2)} / (a + b) + 4 a^3 B \operatorname{EllipticPi}[-1, \operatorname{ArcSin}[\tan((c + dx)/2)], (a - b)/(a + b)] \tan((c + dx)/2)^2 \sqrt{1 - \tan((c + dx)/2)^2} \sqrt{(a + b - a \tan((c + dx)/2)^2 + b \tan((c + dx)/2)^2)} / (a + b) - 4 a b^2 B \operatorname{EllipticPi}[-1, \operatorname{ArcSin}[\tan((c + dx)/2)], (a - b)/(a + b)] \tan((c + dx)/2)^2 \sqrt{1 - \tan((c + dx)/2)^2} \sqrt{(a + b - a \tan((c + dx)/2)^2 + b \tan((c + dx)/2)^2)} / (a + b) + (a + b) (a^2 A - 3 A b^2 + 2 a b B) \operatorname{EllipticE}[\operatorname{ArcSin}[\tan((c + dx)/2)], (a - b)/(a + b)] \sqrt{1 - \tan((c + dx)/2)^2} (1 + \tan((c + dx)/2)^2) \sqrt{(a + b - a \tan((c + dx)/2)^2 + b \tan((c + dx)/2)^2)} / (a + b) - 2 a (a + b) (-A b + a B) \operatorname{EllipticF}[\operatorname{ArcSin}[\tan((c + dx)/2)], (a - b)/(a + b)] \sqrt{1 - \tan((c + dx)/2)^2} (1 + \tan((c + dx)/2)^2) \sqrt{(a + b - a \tan((c + dx)/2)^2 + b \tan((c + dx)/2)^2)} / (a + b) \Big) / (a^2 (a^2 - b^2) d (a + b \sec(c + dx))^{3/2} \sqrt{1 + \tan((c + dx)/2)^2} (a(-1 + \tan((c + dx)/2)^2) - b(1 + \tan((c + dx)/2)^2)))$$

fricas [F] time = 2.06, size = 0, normalized size = 0.00

$$\operatorname{integral} \left(\frac{(B \cos(dx + c) \sec(dx + c) + A \cos(dx + c)) \sqrt{b \sec(dx + c) + a}}{b^2 \sec(dx + c)^2 + 2 ab \sec(dx + c) + a^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(dx+c)*(A+B*sec(dx+c))/(a+b*sec(dx+c))^(3/2),x, algorithm="fricas")

[Out] $\int (B \cos(dx + c) \sec(dx + c) + A \cos(dx + c)) \sqrt{b \sec(dx + c) + a} / (b^2 \sec(dx + c)^2 + 2ab \sec(dx + c) + a^2) dx$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \cos(dx + c)}{(b \sec(dx + c) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(dx+c)*(A+B*sec(dx+c))/(a+b*sec(dx+c))^(3/2),x, algorithm="giac")`

[Out] `integrate((B*sec(dx + c) + A)*cos(dx + c)/(b*sec(dx + c) + a)^(3/2), x)`

maple [B] time = 2.12, size = 2871, normalized size = 6.72

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(dx+c)*(A+B*sec(dx+c))/(a+b*sec(dx+c))^(3/2),x)`

[Out]
$$-1/2/d*4^{(1/2)}*((b+a*\cos(dx+c))/\cos(dx+c))^{(1/2)}*(A*(\cos(dx+c)/(1+\cos(dx+c)))^{(1/2)}*((b+a*\cos(dx+c))/(1+\cos(dx+c)))/(a+b))^{(1/2)}*EllipticE((-1+\cos(dx+c))/\sin(dx+c),((a-b)/(a+b))^{(1/2)})*a^3*\sin(dx+c)+A*\cos(dx+c)^3*a^3-A*\cos(dx+c)^2*a^3-3*A*\cos(dx+c)^2*b^3-A*\cos(dx+c)^3*a*b^2+A*\cos(dx+c)^2*a^2*b+3*A*\cos(dx+c)^2*a*b^2-A*\cos(dx+c)*a^2*b-2*A*\cos(dx+c)*a*b^2-2*B*\cos(dx+c)^2*a^2*b+2*B*\cos(dx+c)^2*a*b^2+2*B*\cos(dx+c)*a^2*b-2*B*\cos(dx+c)*a*b^2+3*A*\cos(dx+c)*b^3-3*A*(\cos(dx+c)/(1+\cos(dx+c)))^{(1/2)}*((b+a*\cos(dx+c))/(1+\cos(dx+c)))/(a+b))^{(1/2)}*EllipticE((-1+\cos(dx+c))/\sin(dx+c),((a-b)/(a+b))^{(1/2)})*b^3*\sin(dx+c)-2*B*(\cos(dx+c)/(1+\cos(dx+c)))^{(1/2)}*((b+a*\cos(dx+c))/(1+\cos(dx+c)))/(a+b))^{(1/2)}*EllipticF((-1+\cos(dx+c))/\sin(dx+c),((a-b)/(a+b))^{(1/2)})*\sin(dx+c)*\cos(dx+c)*a^3-6*A*(\cos(dx+c)/(1+\cos(dx+c)))^{(1/2)}*((b+a*\cos(dx+c))/(1+\cos(dx+c)))/(a+b))^{(1/2)}*EllipticPi((-1+\cos(dx+c))/\sin(dx+c),-1,((a-b)/(a+b))^{(1/2)})*a^2*b*\sin(dx+c)+2*A*(\cos(dx+c)/(1+\cos(dx+c)))^{(1/2)}*((b+a*\cos(dx+c))/(1+\cos(dx+c)))/(a+b))^{(1/2)}*EllipticF((-1+\cos(dx+c))/\sin(dx+c),((a-b)/(a+b))^{(1/2)})*a^2*b*\sin(dx+c)+2*A*(\cos(dx+c)/(1+\cos(dx+c)))^{(1/2)}*((b+a*\cos(dx+c))/(1+\cos(dx+c)))/(a+b))^{(1/2)}*EllipticE((-1+\cos(dx+c))/\sin(dx+c),((a-b)/(a+b))^{(1/2)})*a^2*b*\sin(dx+c)-3*A*(\cos(dx+c)/(1+\cos(dx+c)))^{(1/2)}*((b+a*\cos(dx+c))/(1+\cos(dx+c)))/(a+b))^{(1/2)}*EllipticE((-1+\cos(dx+c))/\sin(dx+c),((a-b)/(a+b))^{(1/2)})*a*b^2*\sin(dx+c)-4*B*(\cos(dx+c)/(1+\cos(dx+c)))^{(1/2)}*((b+a*\cos(dx+c))/(1+\cos(dx+c)))/(a+b))^{(1/2)}*EllipticPi((-1+\cos(dx+c))/\sin(dx+c),-1,$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \cos(dx + c)}{(b \sec(dx + c) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)/(b*sec(d*x + c) + a)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c + dx) \left(A + \frac{B}{\cos(c+dx)} \right)}{\left(a + \frac{b}{\cos(c+dx)} \right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x))^(3/2),x)

[Out] int((cos(c + d*x)*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x))^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \cos(c + dx)}{(a + b \sec(c + dx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(3/2),x)

[Out] Integral((A + B*sec(c + d*x))*cos(c + d*x)/(a + b*sec(c + d*x))**(3/2), x)

$$3.383 \quad \int \frac{\cos^2(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^{3/2}} dx$$

Optimal. Leaf size=531

$$\frac{(5Ab - 4aB) \sin(c + dx) \sqrt{a + b} (4a^2 A - 12abB + 15Ab^2) \cot(c + dx) \sqrt{\frac{b(1 - \sec(c+dx))}{a+b}} \sqrt{\frac{b(\sec(c+dx)+1)}{a-b}} \Pi\left(\frac{a+b}{a}\right)}{4a^2 d \sqrt{a + b \sec(c + dx)} 4a^4 d}$$

[Out] $-1/4*(7*A*a^2*b-15*A*b^3-4*B*a^3+12*B*a*b^2)*\cot(d*x+c)*\text{EllipticE}((a+b*\sec(d*x+c))^{1/2}/(a+b)^{1/2}, ((a+b)/(a-b))^{1/2})*(b*(1-\sec(d*x+c))/(a+b))^{1/2}*(-b*(1+\sec(d*x+c))/(a-b))^{1/2}/a^3/b/d/(a+b)^{1/2}-1/4*(15*A*b^2+a*b*(5*A-12*B)-2*a^2*(A+2*B))*\cot(d*x+c)*\text{EllipticF}((a+b*\sec(d*x+c))^{1/2}/(a+b)^{1/2}, ((a+b)/(a-b))^{1/2})*(b*(1-\sec(d*x+c))/(a+b))^{1/2}*(-b*(1+\sec(d*x+c))/(a-b))^{1/2}/a^3/d/(a+b)^{1/2}-1/4*(4*A*a^2+15*A*b^2-12*B*a*b)*\cot(d*x+c)*\text{EllipticPi}((a+b*\sec(d*x+c))^{1/2}/(a+b)^{1/2}, (a+b)/a, ((a+b)/(a-b))^{1/2})*(a+b)^{1/2}*(b*(1-\sec(d*x+c))/(a+b))^{1/2}*(-b*(1+\sec(d*x+c))/(a-b))^{1/2}/a^4/d-1/4*(5*A*b-4*B*a)*\sin(d*x+c)/a^2/d/(a+b*\sec(d*x+c))^{1/2}+1/2*A*\cos(d*x+c)*\sin(d*x+c)/a/d/(a+b*\sec(d*x+c))^{1/2}-1/4*b*(7*A*a^2*b-15*A*b^3-4*B*a^3+12*B*a*b^2)*\tan(d*x+c)/a^3/(a^2-b^2)/d/(a+b*\sec(d*x+c))^{1/2}$

Rubi [A] time = 1.14, antiderivative size = 531, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {4034, 4104, 4060, 4058, 3921, 3784, 3832, 4004}

$$\frac{b(7a^2Ab - 4a^3B + 12ab^2B - 15Ab^3) \tan(c + dx) (-2a^2(A + 2B) + ab(5A - 12B) + 15Ab^2) \cot(c + dx) \sqrt{\frac{b(1 - \sec(c+dx))}{a+b}}}{4a^3d(a^2 - b^2) \sqrt{a + b \sec(c + dx)} 4a^3d \sqrt{a}}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(3/2), x]

[Out] $-((7*a^2*A*b - 15*A*b^3 - 4*a^3*B + 12*a*b^2*B)*\text{Cot}[c + d*x]*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[a + b*\text{Sec}[c + d*x]]/\text{Sqrt}[a + b]], (a + b)/(a - b)]*\text{Sqrt}[(b*(1 - \text{Sec}[c + d*x]))/(a + b)]*\text{Sqrt}[-((b*(1 + \text{Sec}[c + d*x]))/(a - b))]/(4*a^3*b*\text{Sqrt}[a + b]*d) - ((15*A*b^2 + a*b*(5*A - 12*B) - 2*a^2*(A + 2*B))*\text{Cot}[c + d*x]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[a + b*\text{Sec}[c + d*x]]/\text{Sqrt}[a + b]], (a + b)/(a - b)]*\text{Sqrt}[(b*(1 - \text{Sec}[c + d*x]))/(a + b)]*\text{Sqrt}[-((b*(1 + \text{Sec}[c + d*x]))/(a - b))]/(4*a^3*\text{Sqrt}[a + b]*d) - (\text{Sqrt}[a + b]*(4*a^2*A + 15*A*b^2 - 12*a*b*B)*\text{Cot}[c + d*x]*\text{EllipticPi}[(a + b)/a, \text{ArcSin}[\text{Sqrt}[a + b*\text{Sec}[c + d*x]]/\text{Sqrt}[a + b]], (a + b)/(a - b)]*\text{Sqrt}[(b*(1 - \text{Sec}[c + d*x]))/(a + b)]*\text{Sqrt}[-((b*(1 + \text{Sec}[c + d*x]))/(a - b))]/(4*a^4*d) - ((5*A*b - 4*a*B)*\text{Sin}[c + d*x])/(4*a^2*d*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]) + (A*\text{Cos}[c + d*x]*\text{Sin}[c + d*x])/(2*a*d*\text{Sqrt}[a + b*S$

$\text{ec}[c + d*x]] - (b*(7*a^2*A*b - 15*A*b^3 - 4*a^3*B + 12*a*b^2*B)*\text{Tan}[c + d*x])/(4*a^3*(a^2 - b^2)*d*\text{Sqrt}[a + b*\text{Sec}[c + d*x]])$

Rule 3784

$\text{Int}[1/\text{Sqrt}[\text{csc}[(c_.) + (d_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] \rightarrow \text{Simp}[(2*\text{Rt}[a + b, 2]*\text{Sqrt}[(b*(1 - \text{Csc}[c + d*x]))/(a + b)]*\text{Sqrt}[-((b*(1 + \text{Csc}[c + d*x]))/(a - b))]*\text{EllipticPi}[(a + b)/a, \text{ArcSin}[\text{Sqrt}[a + b*\text{Csc}[c + d*x]]/\text{Rt}[a + b, 2]], (a + b)/(a - b)]/(a*d*\text{Cot}[c + d*x]), x] /; \text{FreeQ}\{a, b, c, d, x\} \&\& \text{NeQ}[a^2 - b^2, 0]$

Rule 3832

$\text{Int}[\text{csc}[(e_.) + (f_.)*(x_.)]/\text{Sqrt}[\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] \rightarrow \text{Simp}[(-2*\text{Rt}[a + b, 2]*\text{Sqrt}[(b*(1 - \text{Csc}[e + f*x]))/(a + b)]*\text{Sqrt}[-((b*(1 + \text{Csc}[e + f*x]))/(a - b))]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[a + b*\text{Csc}[e + f*x]]/\text{Rt}[a + b, 2]], (a + b)/(a - b)]/(b*f*\text{Cot}[e + f*x]), x] /; \text{FreeQ}\{a, b, e, f, x\} \&\& \text{NeQ}[a^2 - b^2, 0]$

Rule 3921

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))/\text{Sqrt}[\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] \rightarrow \text{Dist}[c, \text{Int}[1/\text{Sqrt}[a + b*\text{Csc}[e + f*x]], x], x] + \text{Dist}[d, \text{Int}[\text{Csc}[e + f*x]/\text{Sqrt}[a + b*\text{Csc}[e + f*x]], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, x\} \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 - b^2, 0]$

Rule 4004

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(\text{csc}[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)))/\text{Sqrt}[\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] \rightarrow \text{Simp}[(-2*(A*b - a*B)*\text{Rt}[a + (b*B)/A, 2]*\text{Sqrt}[(b*(1 - \text{Csc}[e + f*x]))/(a + b)]*\text{Sqrt}[-((b*(1 + \text{Csc}[e + f*x]))/(a - b))]*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[a + b*\text{Csc}[e + f*x]]/\text{Rt}[a + (b*B)/A, 2]], (a*A + b*B)/(a*A - b*B)]/(b^2*f*\text{Cot}[e + f*x]), x] /; \text{FreeQ}\{a, b, e, f, A, B, x\} \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{EqQ}[A^2 - B^2, 0]$

Rule 4034

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(d_.))^n*(\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m*(\text{csc}[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] \rightarrow \text{Simp}[(A*\text{Cot}[e + f*x]*(a + b*\text{Csc}[e + f*x])^{m+1}*(d*\text{Csc}[e + f*x])^n)/(a*f^n), x] + \text{Dist}[1/(a*d^n), \text{Int}[(a + b*\text{Csc}[e + f*x])^m*(d*\text{Csc}[e + f*x])^{n+1}*\text{Simp}[a*B^n - A*b*(m + n + 1) + A*a*(n + 1)*\text{Csc}[e + f*x] + A*b*(m + n + 2)*\text{Csc}[e + f*x]^2, x], x], x] /; \text{FreeQ}\{a, b, d, e, f, A, B, m, x\} \&\& \text{NeQ}[A*b - a*B, 0] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{LeQ}[n, -1]$

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)])*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] := Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
+ Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4060

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)])*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^m, x_Symbol] := Simp[((A*b^2 -
a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^
2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
+ 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
+ (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)])*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_.))^m, x_Symbol] := Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{3/2}} dx &= \frac{A\cos(c+dx)\sin(c+dx)}{2ad\sqrt{a+b\sec(c+dx)}} - \int \frac{\cos(c+dx)\left(\frac{1}{2}(5Ab-4aB)-aA\sec(c+dx)-\frac{3}{2}Ab\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{3/2}} \frac{1}{2a} \\
&= -\frac{(5Ab-4aB)\sin(c+dx)}{4a^2d\sqrt{a+b\sec(c+dx)}} + \frac{A\cos(c+dx)\sin(c+dx)}{2ad\sqrt{a+b\sec(c+dx)}} + \int \frac{\frac{1}{4}(4a^2A+15Ab)}{(a+b\sec(c+dx))^{3/2}} \\
&= -\frac{(5Ab-4aB)\sin(c+dx)}{4a^2d\sqrt{a+b\sec(c+dx)}} + \frac{A\cos(c+dx)\sin(c+dx)}{2ad\sqrt{a+b\sec(c+dx)}} - \frac{b(7a^2Ab-15Ab^3)}{4a^3(a+b\sec(c+dx))^{3/2}} \\
&= -\frac{(5Ab-4aB)\sin(c+dx)}{4a^2d\sqrt{a+b\sec(c+dx)}} + \frac{A\cos(c+dx)\sin(c+dx)}{2ad\sqrt{a+b\sec(c+dx)}} - \frac{b(7a^2Ab-15Ab^3)}{4a^3(a+b\sec(c+dx))^{3/2}} \\
&= -\frac{(7a^2Ab-15Ab^3-4a^3B+12ab^2B)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)\right)}{4a^3b\sqrt{a+bd}} \\
&= -\frac{(7a^2Ab-15Ab^3-4a^3B+12ab^2B)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)\right)}{4a^3b\sqrt{a+bd}}
\end{aligned}$$

Mathematica [C] time = 18.30, size = 2667, normalized size = 5.02

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(Cos[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(3/2), x]

[Out] ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*((2*b^2*(A*b - a*B)*Sin[c + d*x]))/(a^3*(-a^2 + b^2)) + (2*(A*b^4*Sin[c + d*x] - a*b^3*B*Sin[c + d*x]))/(a^3*(a^2 - b^2)*(b + a*Cos[c + d*x])) + (A*Sin[2*(c + d*x)]/(4*a^2)))/(d*(a + b*Sec[c + d*x])^(3/2)) + ((b + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]*(-7*a^3*A*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2] - 7*a^2*A*b^2*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2] + 15*a*A*b^3*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2] + 15*A*b^4*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2] + 4*a^4*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)/2] + 4*a^3*b*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)/2] - 12*a^2*b^2*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)/2] - 12*a*b^3*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)/2] + 14*a^3*A*b*Sqrt[(-a + b)

$$\begin{aligned}
&/(a + b)] * \text{Tan}[(c + d*x)/2]^3 - 30*a*A*b^3*\text{Sqrt}[(-a + b)/(a + b)] * \text{Tan}[(c + d \\
&*x)/2]^3 - 8*a^4*\text{Sqrt}[(-a + b)/(a + b)] * B * \text{Tan}[(c + d*x)/2]^3 + 24*a^2*b^2*S \\
&\text{qrt}[(-a + b)/(a + b)] * B * \text{Tan}[(c + d*x)/2]^3 - 7*a^3*A*b*\text{Sqrt}[(-a + b)/(a + b \\
&)] * \text{Tan}[(c + d*x)/2]^5 + 7*a^2*A*b^2*\text{Sqrt}[(-a + b)/(a + b)] * \text{Tan}[(c + d*x)/2] \\
&^5 + 15*a*A*b^3*\text{Sqrt}[(-a + b)/(a + b)] * \text{Tan}[(c + d*x)/2]^5 - 15*A*b^4*\text{Sqrt}[(- \\
&-a + b)/(a + b)] * \text{Tan}[(c + d*x)/2]^5 + 4*a^4*\text{Sqrt}[(-a + b)/(a + b)] * B * \text{Tan}[(c \\
&+ d*x)/2]^5 - 4*a^3*b*\text{Sqrt}[(-a + b)/(a + b)] * B * \text{Tan}[(c + d*x)/2]^5 - 12*a^2 \\
&*b^2*\text{Sqrt}[(-a + b)/(a + b)] * B * \text{Tan}[(c + d*x)/2]^5 + 12*a*b^3*\text{Sqrt}[(-a + b)/(\\
&a + b)] * B * \text{Tan}[(c + d*x)/2]^5 - (8*I)*a^4*A*\text{EllipticPi}[-(a + b)/(a - b)], I \\
&* \text{ArcSinh}[\text{Sqrt}[(-a + b)/(a + b)] * \text{Tan}[(c + d*x)/2]], (a + b)/(a - b)] * \text{Sqrt}[1 \\
&- \text{Tan}[(c + d*x)/2]^2] * \text{Sqrt}[(a + b - a*\text{Tan}[(c + d*x)/2]^2 + b*\text{Tan}[(c + d*x)/ \\
&2]^2)/(a + b)] - (22*I)*a^2*A*b^2*\text{EllipticPi}[-(a + b)/(a - b)], I * \text{ArcSinh} [\\
&\text{Sqrt}[(-a + b)/(a + b)] * \text{Tan}[(c + d*x)/2]], (a + b)/(a - b)] * \text{Sqrt}[1 - \text{Tan}[(c \\
&+ d*x)/2]^2] * \text{Sqrt}[(a + b - a*\text{Tan}[(c + d*x)/2]^2 + b*\text{Tan}[(c + d*x)/2]^2)/(a \\
&+ b)] + (30*I)*A*b^4*\text{EllipticPi}[-(a + b)/(a - b)], I * \text{ArcSinh}[\text{Sqrt}[(-a + b) \\
&/ (a + b)] * \text{Tan}[(c + d*x)/2]], (a + b)/(a - b)] * \text{Sqrt}[1 - \text{Tan}[(c + d*x)/2]^2] * \\
&\text{Sqrt}[(a + b - a*\text{Tan}[(c + d*x)/2]^2 + b*\text{Tan}[(c + d*x)/2]^2)/(a + b)] + (24*I \\
&)*a^3*b*B*\text{EllipticPi}[-(a + b)/(a - b)], I * \text{ArcSinh}[\text{Sqrt}[(-a + b)/(a + b)] * T \\
&\text{an}[(c + d*x)/2]], (a + b)/(a - b)] * \text{Sqrt}[1 - \text{Tan}[(c + d*x)/2]^2] * \text{Sqrt}[(a + b \\
&- a*\text{Tan}[(c + d*x)/2]^2 + b*\text{Tan}[(c + d*x)/2]^2)/(a + b)] - (24*I)*a*b^3*B*E \\
&\text{llipticPi}[-(a + b)/(a - b)], I * \text{ArcSinh}[\text{Sqrt}[(-a + b)/(a + b)] * \text{Tan}[(c + d*x \\
&)/2]], (a + b)/(a - b)] * \text{Sqrt}[1 - \text{Tan}[(c + d*x)/2]^2] * \text{Sqrt}[(a + b - a*\text{Tan}[(c \\
&+ d*x)/2]^2 + b*\text{Tan}[(c + d*x)/2]^2)/(a + b)] - (8*I)*a^4*A*\text{EllipticPi}[-(a \\
&+ b)/(a - b)], I * \text{ArcSinh}[\text{Sqrt}[(-a + b)/(a + b)] * \text{Tan}[(c + d*x)/2]], (a + b) \\
&/ (a - b)] * \text{Tan}[(c + d*x)/2]^2 * \text{Sqrt}[1 - \text{Tan}[(c + d*x)/2]^2] * \text{Sqrt}[(a + b - a*T \\
&\text{an}[(c + d*x)/2]^2 + b*\text{Tan}[(c + d*x)/2]^2)/(a + b)] - (22*I)*a^2*A*b^2*\text{Ellip \\
&\text{ticPi}[-(a + b)/(a - b)], I * \text{ArcSinh}[\text{Sqrt}[(-a + b)/(a + b)] * \text{Tan}[(c + d*x)/2] \\
&], (a + b)/(a - b)] * \text{Tan}[(c + d*x)/2]^2 * \text{Sqrt}[1 - \text{Tan}[(c + d*x)/2]^2] * \text{Sqrt}[(a \\
&+ b - a*\text{Tan}[(c + d*x)/2]^2 + b*\text{Tan}[(c + d*x)/2]^2)/(a + b)] + (30*I)*A*b^4 \\
&* \text{EllipticPi}[-(a + b)/(a - b)], I * \text{ArcSinh}[\text{Sqrt}[(-a + b)/(a + b)] * \text{Tan}[(c + d \\
&*x)/2]], (a + b)/(a - b)] * \text{Tan}[(c + d*x)/2]^2 * \text{Sqrt}[1 - \text{Tan}[(c + d*x)/2]^2] * S \\
&\text{qrt}[(a + b - a*\text{Tan}[(c + d*x)/2]^2 + b*\text{Tan}[(c + d*x)/2]^2)/(a + b)] + (24*I) \\
&*a^3*b*B*\text{EllipticPi}[-(a + b)/(a - b)], I * \text{ArcSinh}[\text{Sqrt}[(-a + b)/(a + b)] * T \\
&\text{an}[(c + d*x)/2]], (a + b)/(a - b)] * \text{Tan}[(c + d*x)/2]^2 * \text{Sqrt}[1 - \text{Tan}[(c + d*x) \\
&/2]^2] * \text{Sqrt}[(a + b - a*\text{Tan}[(c + d*x)/2]^2 + b*\text{Tan}[(c + d*x)/2]^2)/(a + b)] \\
&- (24*I)*a*b^3*B*\text{EllipticPi}[-(a + b)/(a - b)], I * \text{ArcSinh}[\text{Sqrt}[(-a + b)/(a \\
&+ b)] * \text{Tan}[(c + d*x)/2]], (a + b)/(a - b)] * \text{Tan}[(c + d*x)/2]^2 * \text{Sqrt}[1 - \text{Tan}[(c \\
&+ d*x)/2]^2] * \text{Sqrt}[(a + b - a*\text{Tan}[(c + d*x)/2]^2 + b*\text{Tan}[(c + d*x)/2]^2)/(\\
&a + b)] - I*(a - b)*(-7*a^2*A*b + 15*A*b^3 + 4*a^3*B - 12*a*b^2*B)*\text{Elliptic} \\
&\text{E}[I * \text{ArcSinh}[\text{Sqrt}[(-a + b)/(a + b)] * \text{Tan}[(c + d*x)/2]], (a + b)/(a - b)] * \text{Sqrt} \\
&[1 - \text{Tan}[(c + d*x)/2]^2] * (1 + \text{Tan}[(c + d*x)/2]^2) * \text{Sqrt}[(a + b - a*\text{Tan}[(c + \\
&d*x)/2]^2 + b*\text{Tan}[(c + d*x)/2]^2)/(a + b)] + (2*I)*(a - b)*(2*a^3*A + 15*A* \\
&b^3 + a^2*b*(A - 8*B) + 2*a*b^2*(5*A - 6*B))*\text{EllipticF}[I * \text{ArcSinh}[\text{Sqrt}[(-a + \\
&b)/(a + b)] * \text{Tan}[(c + d*x)/2]], (a + b)/(a - b)] * \text{Sqrt}[1 - \text{Tan}[(c + d*x)/2]^ \\
&2] * (1 + \text{Tan}[(c + d*x)/2]^2) * \text{Sqrt}[(a + b - a*\text{Tan}[(c + d*x)/2]^2 + b*\text{Tan}[(c +
\end{aligned}$$

$d*x)/2]^2)/(a + b)])))/(4*a^3*sqrt[(-a + b)/(a + b)]*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)*(-1 + Tan[(c + d*x)/2]^2)*sqrt[(1 + Tan[(c + d*x)/2]^2)/(1 - Tan[(c + d*x)/2]^2)]*(a*(-1 + Tan[(c + d*x)/2]^2) - b*(1 + Tan[(c + d*x)/2]^2)))$

fricas [F] time = 2.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B \cos(dx + c)^2 \sec(dx + c) + A \cos(dx + c)^2) \sqrt{b \sec(dx + c) + a}}{b^2 \sec(dx + c)^2 + 2ab \sec(dx + c) + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")

[Out] integral((B*cos(d*x + c)^2*sec(d*x + c) + A*cos(d*x + c)^2)*sqrt(b*sec(d*x + c) + a)/(b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \cos(dx + c)^2}{(b \sec(dx + c) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^2/(b*sec(d*x + c) + a)^(3/2), x)

maple [B] time = 2.87, size = 3980, normalized size = 7.50

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x)

[Out] $-1/8/d*4^{(1/2)}*((b+a*\cos(d*x+c))/\cos(d*x+c))^{(1/2)}*(15*A*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*EllipticE((-1+\cos(d*x+c))/\sin(d*x+c),((a-b)/(a+b))^{(1/2)})*b^4*\sin(d*x+c)-2*A*\cos(d*x+c)^4*a^2*b^2+12*B*\cos(d*x+c)^2*a^2*b^2-12*B*\cos(d*x+c)^2*a*b^3-4*B*\cos(d*x+c)*a^3*b-8*B*\cos(d*x+c)*a^2*b^2+12*B*\cos(d*x+c)*a*b^3+7*A*\cos(d*x+c)^2*a^3*b-15*A*\cos(d*x+c)^2*a*b^3+7*A*\cos(d*x+c)*a^2*b^2+10*A*\cos(d*x+c)*a*b^3+15*A*\cos(d*x+c)^2*b^4-4*B*\cos(d*x+c)^2*a^4+4*B*\cos(d*x+c)^3*a^4-2*A*\cos(d*x+c)^2*a$

$$\begin{aligned}
&^4+4*B*\cos(d*x+c)^2*a^3*b-5*A*\cos(d*x+c)^3*a^3*b+5*A*\cos(d*x+c)^3*a*b^3-5*A \\
&* \cos(d*x+c)^2*a^2*b^2-2*A*\cos(d*x+c)*a^3*b-4*B*\cos(d*x+c)^3*a^2*b^2-30*A*(c \\
& \cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{(1/ \\
& 2)}* \text{EllipticPi}((-1+\cos(d*x+c))/\sin(d*x+c), -1, ((a-b)/(a+b))^{(1/2)})*b^4*\sin(d* \\
& x+c)-30*A*\sin(d*x+c)*\cos(d*x+c)*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((b+a*\cos \\
& (d*x+c))/(1+\cos(d*x+c))/(a+b))^{(1/2)}* \text{EllipticPi}((-1+\cos(d*x+c))/\sin(d*x+c), \\
& -1, ((a-b)/(a+b))^{(1/2)})*b^4+4*B*\sin(d*x+c)*\cos(d*x+c)*(\cos(d*x+c)/(1+\cos(d* \\
& x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{(1/2)}* \text{EllipticE}((-1+co \\
& s(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{(1/2)})*a^4-7*A*(\cos(d*x+c)/(1+\cos(d*x+c) \\
&))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{(1/2)}* \text{EllipticE}((-1+\cos(d* \\
& x+c))/\sin(d*x+c), ((a-b)/(a+b))^{(1/2)})*a^3*b*\sin(d*x+c)-7*A*(\cos(d*x+c)/(1+c \\
& os(d*x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{(1/2)}* \text{EllipticE}((\\
& -1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{(1/2)})*a^2*b^2*\sin(d*x+c)+15*A*(\cos \\
& (d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{(1/2)} \\
& * \text{EllipticE}((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{(1/2)})*a*b^3*\sin(d*x+c) \\
& +2*A*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+ \\
& b))^{(1/2)}* \text{EllipticF}((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{(1/2)})*a^3*b*s \\
& in(d*x+c)-4*A*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d* \\
& x+c))/(a+b))^{(1/2)}* \text{EllipticF}((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{(1/2)} \\
&)*a^2*b^2*\sin(d*x+c)-10*A*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x+c) \\
&))/(1+\cos(d*x+c))/(a+b))^{(1/2)}* \text{EllipticF}((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/ \\
& (a+b))^{(1/2)})*a*b^3*\sin(d*x+c)+22*A*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((b+a \\
& * \cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{(1/2)}* \text{EllipticPi}((-1+\cos(d*x+c))/\sin(d*x \\
& +c), -1, ((a-b)/(a+b))^{(1/2)})*a^2*b^2*\sin(d*x+c)+4*B*(\cos(d*x+c)/(1+\cos(d*x+c) \\
&)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{(1/2)}* \text{EllipticE}((-1+\cos(d \\
& *x+c))/\sin(d*x+c), ((a-b)/(a+b))^{(1/2)})*a^3*b*\sin(d*x+c)-12*B*(\cos(d*x+c)/(1 \\
& +\cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{(1/2)}* \text{EllipticE} \\
& ((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{(1/2)})*a^2*b^2*\sin(d*x+c)-12*B*(c \\
& os(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{(1/ \\
& 2)}* \text{EllipticE}((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{(1/2)})*a*b^3*\sin(d*x+ \\
& c)+8*B*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(\\
& a+b))^{(1/2)}* \text{EllipticF}((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{(1/2)})*a^3*b \\
& * \sin(d*x+c)+8*B*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(\\
& d*x+c))/(a+b))^{(1/2)}* \text{EllipticF}((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{(1/ \\
& 2)})*a^2*b^2*\sin(d*x+c)-24*B*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x \\
& +c))/(1+\cos(d*x+c))/(a+b))^{(1/2)}* \text{EllipticPi}((-1+\cos(d*x+c))/\sin(d*x+c), -1, (\\
& (a-b)/(a+b))^{(1/2)})*a^3*b*\sin(d*x+c)+24*B*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)} \\
& *((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{(1/2)}* \text{EllipticPi}((-1+\cos(d*x+c))/s \\
& in(d*x+c), -1, ((a-b)/(a+b))^{(1/2)})*a*b^3*\sin(d*x+c)+4*B*(\cos(d*x+c)/(1+\cos(d \\
& *x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{(1/2)}* \text{EllipticE}((-1+c \\
& os(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{(1/2)})*a^4*\sin(d*x+c)-4*A*(\cos(d*x+c)/(\\
& 1+\cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{(1/2)}* \text{Elliptic} \\
& F((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{(1/2)})*a^4*\sin(d*x+c)+8*A*(\cos(d \\
& *x+c)/(1+\cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{(1/2)}*E \\
& llipticPi((-1+\cos(d*x+c))/\sin(d*x+c), -1, ((a-b)/(a+b))^{(1/2)})*a^4*\sin(d*x+c)
\end{aligned}$$

$$\begin{aligned}
& +2*A*\cos(d*x+c)^4*a^4-15*A*\cos(d*x+c)*b^4+22*A*a^2*b^2*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{(1/2)}*\sin(d*x+c)*\cos(d*x+c)*\text{EllipticPi}((-1+\cos(d*x+c))/\sin(d*x+c), -1, ((a-b)/(a+b))^{(1/2)})+4*B*\sin(d*x+c)*\cos(d*x+c)*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{(1/2)}*\text{EllipticE}((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{(1/2)})*a^3*b-12*B*a^2*b^2*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{(1/2)}*\sin(d*x+c)*\cos(d*x+c)*\text{EllipticE}((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{(1/2)})-12*B*b^3*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{(1/2)}*\sin(d*x+c)*\cos(d*x+c)*\text{EllipticE}((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{(1/2)})*a+8*B*\sin(d*x+c)*\cos(d*x+c)*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{(1/2)}*\text{EllipticF}((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{(1/2)})*a^3*b+8*B*\sin(d*x+c)*\cos(d*x+c)*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{(1/2)}*\text{EllipticF}((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{(1/2)})*a^2*b^2-24*B*\sin(d*x+c)*\cos(d*x+c)*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{(1/2)}*\text{EllipticPi}((-1+\cos(d*x+c))/\sin(d*x+c), -1, ((a-b)/(a+b))^{(1/2)})*a^3*b+24*B*\sin(d*x+c)*\cos(d*x+c)*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{(1/2)}*\text{EllipticPi}((-1+\cos(d*x+c))/\sin(d*x+c), -1, ((a-b)/(a+b))^{(1/2)})*a*b^3-7*A*a^3*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{(1/2)}*\sin(d*x+c)*\cos(d*x+c)*\text{EllipticE}((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{(1/2)})*b-7*A*\sin(d*x+c)*\cos(d*x+c)*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{(1/2)}*\text{EllipticE}((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{(1/2)})*a^2*b^2+15*A*\sin(d*x+c)*\cos(d*x+c)*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{(1/2)}*\text{EllipticE}((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{(1/2)})*a*b^3+2*A*a^3*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{(1/2)}*\sin(d*x+c)*\cos(d*x+c)*\text{EllipticF}((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{(1/2)})*b-4*A*a^2*b^2*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{(1/2)}*\sin(d*x+c)*\cos(d*x+c)*\text{EllipticF}((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{(1/2)})-10*A*b^3*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{(1/2)}*\sin(d*x+c)*\cos(d*x+c)*\text{EllipticF}((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{(1/2)})*a+15*A*\sin(d*x+c)*\cos(d*x+c)*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{(1/2)}*\text{EllipticE}((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{(1/2)})*b^4-4*A*\sin(d*x+c)*\cos(d*x+c)*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{(1/2)}*\text{EllipticF}((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{(1/2)})*a^4+8*A*\sin(d*x+c)*\cos(d*x+c)*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{(1/2)}*\text{EllipticPi}((-1+\cos(d*x+c))/\sin(d*x+c), -1, ((a-b)/(a+b))^{(1/2)})*a^4/(b+a*\cos(d*x+c))/\sin(d*x+c)/a^3/(a+b)/(a-b)
\end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \cos(dx + c)^2}{(b \sec(dx + c) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^2/(b*sec(d*x + c) + a)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c + dx)^2 \left(A + \frac{B}{\cos(c+dx)} \right)}{\left(a + \frac{b}{\cos(c+dx)} \right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)^2*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x))^(3/2),x)

[Out] int((cos(c + d*x)^2*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x))^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \cos^2(c + dx)}{(a + b \sec(c + dx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(3/2),x)

[Out] Integral((A + B*sec(c + d*x))*cos(c + d*x)**2/(a + b*sec(c + d*x))**(3/2), x)

$$3.384 \quad \int \frac{\cos^3(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^{3/2}} dx$$

Optimal. Leaf size=630

$$\frac{(7Ab - 6aB) \sin(c + dx) \cos(c + dx)}{12a^2 d \sqrt{a + b \sec(c + dx)}} + \frac{(16a^2 A - 30abB + 35Ab^2) \sin(c + dx)}{24a^3 d \sqrt{a + b \sec(c + dx)}} + \frac{\sqrt{a + b} (-8a^3 B + 12a^2 Ab - 30a^2 B^2)}{24a^3 d \sqrt{a + b \sec(c + dx)}}$$

[Out] 1/24*(16*A*a^4+41*A*a^2*b^2-105*A*b^4-42*B*a^3*b+90*B*a*b^3)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a^4/b/d/(a+b)^(1/2)+1/24*(105*A*b^3+5*a*b^2*(7*A-18*B)+4*a^3*(4*A+3*B)-6*a^2*b*(A+5*B))*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a^4/d/(a+b)^(1/2)+1/8*(12*A*a^2*b+35*A*b^3-8*B*a^3-30*B*a*b^2)*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2), (a+b)/a, ((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a^5/d+1/24*(16*A*a^2+35*A*b^2-30*B*a*b)*sin(d*x+c)/a^3/d/(a+b*sec(d*x+c))^(1/2)-1/12*(7*A*b-6*B*a)*cos(d*x+c)*sin(d*x+c)/a^2/d/(a+b*sec(d*x+c))^(1/2)+1/3*A*cos(d*x+c)^2*sin(d*x+c)/a/d/(a+b*sec(d*x+c))^(1/2)+1/24*b*(16*A*a^4+41*A*a^2*b^2-105*A*b^4-42*B*a^3*b+90*B*a*b^3)*tan(d*x+c)/a^4/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)

Rubi [A] time = 1.67, antiderivative size = 630, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {4034, 4104, 4060, 4058, 3921, 3784, 3832, 4004}

$$\frac{(16a^2 A - 30abB + 35Ab^2) \sin(c + dx)}{24a^3 d \sqrt{a + b \sec(c + dx)}} + \frac{b(41a^2 Ab^2 + 16a^4 A - 42a^3 bB + 90ab^3 B - 105Ab^4) \tan(c + dx)}{24a^4 d (a^2 - b^2) \sqrt{a + b \sec(c + dx)}} + \frac{(-6a^2 B^2)}{24a^4 d (a^2 - b^2) \sqrt{a + b \sec(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(3/2),x]

[Out] ((16*a^4*A + 41*a^2*A*b^2 - 105*A*b^4 - 42*a^3*b*B + 90*a*b^3*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(24*a^4*b*Sqrt[a + b]*d) + ((105*A*b^3 + 5*a*b^2*(7*A - 18*B) + 4*a^3*(4*A + 3*B) - 6*a^2*b*(A + 5*B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(24*a^4*Sqrt[a + b]*d) + (Sqrt[a + b]*(12*a^2*A*b + 35*A*b^3 - 8*a^3*B - 30*a*b^2*B)*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(24*a^4*d*(a^2 - b^2)*Sqrt[a + b*Sec[c + d*x]])

$$\frac{b))}{(8a^5d) + ((16a^2A + 35Ab^2 - 30abB)\sin[c + dx]) / (24a^3d\sqrt{a + b\sec[c + dx]}) - ((7Ab - 6aB)\cos[c + dx]\sin[c + dx]) / (12a^2d\sqrt{a + b\sec[c + dx]}) + (A\cos^2[c + dx]\sin[c + dx]) / (3ad\sqrt{a + b\sec[c + dx]}) + (b(16a^4A + 41a^2Ab^2 - 105Ab^4 - 42a^3bB + 90ab^3B)\tan[c + dx]) / (24a^4(a^2 - b^2)d\sqrt{a + b\sec[c + dx]})}$$

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_.)], x_Symbol] := Simp[(2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]]/Rt[a + b, 2]], (a + b)/(a - b)]/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] := Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]]/Rt[a + b, 2]], (a + b)/(a - b)]/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] := Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] := Simp[(-2*(A*b - a*B)*Rt[a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]]/Rt[a + (b*B)/A, 2]], (a*A + b*B)/(a*A - b*B)]/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4034

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^m*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] := Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f^n), x] + Dist[1/(a*d^n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*B^n - A*b*(m + n + 1) + A*a*(n + 1)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]]
```

$]^2, x], x], x] /; \text{FreeQ}[\{a, b, d, e, f, A, B, m\}, x] \ \&\& \ \text{NeQ}[A*b - a*B, 0]$
 $\&\& \ \text{NeQ}[a^2 - b^2, 0] \ \&\& \ \text{LeQ}[n, -1]$

Rule 4058

$\text{Int}[\left((A_{\cdot}) + \csc[(e_{\cdot}) + (f_{\cdot})(x_{\cdot})]*(B_{\cdot}) + \csc[(e_{\cdot}) + (f_{\cdot})(x_{\cdot})]^2*(C_{\cdot})\right) / \sqrt{\csc[(e_{\cdot}) + (f_{\cdot})(x_{\cdot})]*(b_{\cdot}) + (a_{\cdot})}, x_{\text{Symbol}}] \text{:>} \text{Int}[(A + (B - C) * \csc[e + f*x]) / \sqrt{a + b * \csc[e + f*x]}, x] + \text{Dist}[C, \text{Int}[(\csc[e + f*x] * (1 + \csc[e + f*x])) / \sqrt{a + b * \csc[e + f*x]}, x], x] /; \text{FreeQ}[\{a, b, e, f, A, B, C\}, x] \ \&\& \ \text{NeQ}[a^2 - b^2, 0]$

Rule 4060

$\text{Int}[\left((A_{\cdot}) + \csc[(e_{\cdot}) + (f_{\cdot})(x_{\cdot})]*(B_{\cdot}) + \csc[(e_{\cdot}) + (f_{\cdot})(x_{\cdot})]^2*(C_{\cdot})\right) * (\csc[(e_{\cdot}) + (f_{\cdot})(x_{\cdot})]*(b_{\cdot}) + (a_{\cdot}))^{(m_{\cdot})}, x_{\text{Symbol}}] \text{:>} \text{Simp}[\left((A*b^2 - a*b*B + a^2*C) * \cot[e + f*x] * (a + b * \csc[e + f*x])^{(m + 1)}\right) / (a*f*(m + 1)*(a^2 - b^2)), x] + \text{Dist}[1/(a*(m + 1)*(a^2 - b^2)), \text{Int}[(a + b * \csc[e + f*x])^{(m + 1)} * \text{Simp}[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*\csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m + 2)*\csc[e + f*x]^2, x], x], x] /; \text{FreeQ}[\{a, b, e, f, A, B, C\}, x] \ \&\& \ \text{NeQ}[a^2 - b^2, 0] \ \&\& \ \text{LtQ}[m, -1]$

Rule 4104

$\text{Int}[\left((A_{\cdot}) + \csc[(e_{\cdot}) + (f_{\cdot})(x_{\cdot})]*(B_{\cdot}) + \csc[(e_{\cdot}) + (f_{\cdot})(x_{\cdot})]^2*(C_{\cdot})\right) * (\csc[(e_{\cdot}) + (f_{\cdot})(x_{\cdot})]*(d_{\cdot}))^{(n_{\cdot})} * (\csc[(e_{\cdot}) + (f_{\cdot})(x_{\cdot})]*(b_{\cdot}) + (a_{\cdot}))^{(m_{\cdot})}, x_{\text{Symbol}}] \text{:>} \text{Simp}[(A * \cot[e + f*x] * (a + b * \csc[e + f*x])^{(m + 1)} * (d * \csc[e + f*x])^n) / (a*f*n), x] + \text{Dist}[1/(a*d*n), \text{Int}[(a + b * \csc[e + f*x])^m * (d * \csc[e + f*x])^{(n + 1)} * \text{Simp}[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*\csc[e + f*x] + A*b*(m + n + 2)*\csc[e + f*x]^2, x], x], x] /; \text{FreeQ}[\{a, b, d, e, f, A, B, C, m\}, x] \ \&\& \ \text{NeQ}[a^2 - b^2, 0] \ \&\& \ \text{LeQ}[n, -1]$

Rubi steps

$$\begin{aligned}
\int \frac{\cos^3(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{3/2}} dx &= \frac{A\cos^2(c+dx)\sin(c+dx)}{3ad\sqrt{a+b\sec(c+dx)}} - \int \frac{\cos^2(c+dx)\left(\frac{1}{2}(7Ab-6aB)-2aA\sec(c+dx)-\frac{5}{2}Ab\sec(c+dx)\right)}{(a+b\sec(c+dx))^{3/2}} dx \\
&= -\frac{(7Ab-6aB)\cos(c+dx)\sin(c+dx)}{12a^2d\sqrt{a+b\sec(c+dx)}} + \frac{A\cos^2(c+dx)\sin(c+dx)}{3ad\sqrt{a+b\sec(c+dx)}} + \\
&= \frac{(16a^2A+35Ab^2-30abB)\sin(c+dx)}{24a^3d\sqrt{a+b\sec(c+dx)}} - \frac{(7Ab-6aB)\cos(c+dx)\sin(c+dx)}{12a^2d\sqrt{a+b\sec(c+dx)}} \\
&= \frac{(16a^2A+35Ab^2-30abB)\sin(c+dx)}{24a^3d\sqrt{a+b\sec(c+dx)}} - \frac{(7Ab-6aB)\cos(c+dx)\sin(c+dx)}{12a^2d\sqrt{a+b\sec(c+dx)}} \\
&= \frac{(16a^2A+35Ab^2-30abB)\sin(c+dx)}{24a^3d\sqrt{a+b\sec(c+dx)}} - \frac{(7Ab-6aB)\cos(c+dx)\sin(c+dx)}{12a^2d\sqrt{a+b\sec(c+dx)}} \\
&= \frac{(16a^4A+41a^2Ab^2-105Ab^4-42a^3bB+90ab^3B)\cot(c+dx)E\left(\sin^{-1}\left(\frac{a+b\sec(c+dx)}{a+b}\right)\right)}{24a^4b\sqrt{a+bd}} \\
&= \frac{(16a^4A+41a^2Ab^2-105Ab^4-42a^3bB+90ab^3B)\cot(c+dx)E\left(\sin^{-1}\left(\frac{a+b\sec(c+dx)}{a+b}\right)\right)}{24a^4b\sqrt{a+bd}}
\end{aligned}$$

Mathematica [B] time = 22.29, size = 2319, normalized size = 3.68

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(Cos[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(3/2), x]

[Out] ((b + a*cos[c + d*x])^2*Sec[c + d*x]^2*(-1/12*((a^4*A - a^2*A*b^2 + 24*A*b^4 - 24*a*b^3*B)*Sin[c + d*x]))/(a^4*(-a^2 + b^2)) - (2*(A*b^5*Sin[c + d*x] - a*b^4*B*Sin[c + d*x]))/(a^4*(a^2 - b^2)*(b + a*cos[c + d*x])) + ((-11*A*b + 6*a*B)*Sin[2*(c + d*x)]/(24*a^3) + (A*Sin[3*(c + d*x)]/(12*a^2)))/(d*(a + b*Sec[c + d*x])^(3/2)) - ((b + a*cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]*(16*a^5*A*Tan[(c + d*x)/2] +

$$\begin{aligned}
& 16a^4Ab \operatorname{Tan}[(c+dx)/2] + 41a^3A^2b^2 \operatorname{Tan}[(c+dx)/2] + 41a^2A^3b^3 \operatorname{Tan}[(c+dx)/2] \\
& - 105a^4Ab^4 \operatorname{Tan}[(c+dx)/2] - 105A^5b^5 \operatorname{Tan}[(c+dx)/2] - 42a^4b^2B \operatorname{Tan}[(c+dx)/2] \\
& - 42a^3b^2B^2 \operatorname{Tan}[(c+dx)/2] + 90a^2b^3B^3 \operatorname{Tan}[(c+dx)/2] + 90a^2b^4B^4 \operatorname{Tan}[(c+dx)/2] \\
& - 32a^5A \operatorname{Tan}[(c+dx)/2]^3 - 82a^3A^2b^2 \operatorname{Tan}[(c+dx)/2]^3 + 210a^4A^2b^4 \operatorname{Tan}[(c+dx)/2]^3 \\
& + 84a^4b^2B \operatorname{Tan}[(c+dx)/2]^3 - 180a^2b^3B^3 \operatorname{Tan}[(c+dx)/2]^3 + 16a^5A^5 \operatorname{Tan}[(c+dx)/2]^5 \\
& - 16a^4A^4b \operatorname{Tan}[(c+dx)/2]^5 + 41a^3A^3b^2 \operatorname{Tan}[(c+dx)/2]^5 - 41a^2A^2b^3 \operatorname{Tan}[(c+dx)/2]^5 \\
& - 105a^4Ab^4 \operatorname{Tan}[(c+dx)/2]^5 + 105A^5b^5 \operatorname{Tan}[(c+dx)/2]^5 - 42a^4b^2B \operatorname{Tan}[(c+dx)/2]^5 \\
& + 42a^3b^2B^2 \operatorname{Tan}[(c+dx)/2]^5 + 90a^2b^3B^3 \operatorname{Tan}[(c+dx)/2]^5 - 90a^2b^4B^4 \operatorname{Tan}[(c+dx)/2]^5 \\
& - 72a^4A^4b \operatorname{EllipticPi}[-1, \operatorname{ArcSin}[\operatorname{Tan}[(c+dx)/2]], (a-b)/(a+b)] \operatorname{Sqrt}[1 - \operatorname{Tan}[(c+dx)/2]^2] \\
& \operatorname{Sqrt}[(a+b - a \operatorname{Tan}[(c+dx)/2]^2 + b \operatorname{Tan}[(c+dx)/2]^2)/(a+b)] - 138a^2A^2b^3 \operatorname{EllipticPi}[-1, \operatorname{ArcSin}[\operatorname{Tan}[(c+dx)/2]], \\
& (a-b)/(a+b)] \operatorname{Sqrt}[1 - \operatorname{Tan}[(c+dx)/2]^2] \operatorname{Sqrt}[(a+b - a \operatorname{Tan}[(c+dx)/2]^2 + b \operatorname{Tan}[(c+dx)/2]^2)/(a+b)] \\
& + 210A^5b^5 \operatorname{EllipticPi}[-1, \operatorname{ArcSin}[\operatorname{Tan}[(c+dx)/2]], (a-b)/(a+b)] \operatorname{Sqrt}[1 - \operatorname{Tan}[(c+dx)/2]^2] \\
& \operatorname{Sqrt}[(a+b - a \operatorname{Tan}[(c+dx)/2]^2 + b \operatorname{Tan}[(c+dx)/2]^2)/(a+b)] + 48a^5B^5 \operatorname{EllipticPi}[-1, \operatorname{ArcSin}[\operatorname{Tan}[(c+dx)/2]], \\
& (a-b)/(a+b)] \operatorname{Sqrt}[1 - \operatorname{Tan}[(c+dx)/2]^2] \operatorname{Sqrt}[(a+b - a \operatorname{Tan}[(c+dx)/2]^2 + b \operatorname{Tan}[(c+dx)/2]^2)/(a+b)] \\
& + 132a^3b^2B^2 \operatorname{EllipticPi}[-1, \operatorname{ArcSin}[\operatorname{Tan}[(c+dx)/2]], (a-b)/(a+b)] \operatorname{Sqrt}[1 - \operatorname{Tan}[(c+dx)/2]^2] \\
& \operatorname{Sqrt}[(a+b - a \operatorname{Tan}[(c+dx)/2]^2 + b \operatorname{Tan}[(c+dx)/2]^2)/(a+b)] - 180a^2b^4B^4 \operatorname{EllipticPi}[-1, \operatorname{ArcSin}[\operatorname{Tan}[(c+dx)/2]], \\
& (a-b)/(a+b)] \operatorname{Sqrt}[1 - \operatorname{Tan}[(c+dx)/2]^2] \operatorname{Sqrt}[(a+b - a \operatorname{Tan}[(c+dx)/2]^2 + b \operatorname{Tan}[(c+dx)/2]^2)/(a+b)] \\
& - 72a^4A^4b \operatorname{EllipticPi}[-1, \operatorname{ArcSin}[\operatorname{Tan}[(c+dx)/2]], (a-b)/(a+b)] \operatorname{Tan}[(c+dx)/2]^2 \operatorname{Sqrt}[1 - \operatorname{Tan}[(c+dx)/2]^2] \\
& \operatorname{Sqrt}[(a+b - a \operatorname{Tan}[(c+dx)/2]^2 + b \operatorname{Tan}[(c+dx)/2]^2)/(a+b)] - 138a^2A^2b^3 \operatorname{EllipticPi}[-1, \operatorname{ArcSin}[\operatorname{Tan}[(c+dx)/2]], \\
& (a-b)/(a+b)] \operatorname{Tan}[(c+dx)/2]^2 \operatorname{Sqrt}[1 - \operatorname{Tan}[(c+dx)/2]^2] \operatorname{Sqrt}[(a+b - a \operatorname{Tan}[(c+dx)/2]^2 + b \operatorname{Tan}[(c+dx)/2]^2)/(a+b)] \\
& + 210A^5b^5 \operatorname{EllipticPi}[-1, \operatorname{ArcSin}[\operatorname{Tan}[(c+dx)/2]], (a-b)/(a+b)] \operatorname{Tan}[(c+dx)/2]^2 \operatorname{Sqrt}[1 - \operatorname{Tan}[(c+dx)/2]^2] \\
& \operatorname{Sqrt}[(a+b - a \operatorname{Tan}[(c+dx)/2]^2 + b \operatorname{Tan}[(c+dx)/2]^2)/(a+b)] + 48a^5B^5 \operatorname{EllipticPi}[-1, \operatorname{ArcSin}[\operatorname{Tan}[(c+dx)/2]], \\
& (a-b)/(a+b)] \operatorname{Tan}[(c+dx)/2]^2 \operatorname{Sqrt}[1 - \operatorname{Tan}[(c+dx)/2]^2] \operatorname{Sqrt}[(a+b - a \operatorname{Tan}[(c+dx)/2]^2 + b \operatorname{Tan}[(c+dx)/2]^2)/(a+b)] \\
& + 132a^3b^2B^2 \operatorname{EllipticPi}[-1, \operatorname{ArcSin}[\operatorname{Tan}[(c+dx)/2]], (a-b)/(a+b)] \operatorname{Tan}[(c+dx)/2]^2 \operatorname{Sqrt}[1 - \operatorname{Tan}[(c+dx)/2]^2] \\
& \operatorname{Sqrt}[(a+b - a \operatorname{Tan}[(c+dx)/2]^2 + b \operatorname{Tan}[(c+dx)/2]^2)/(a+b)] - 180a^2b^4B^4 \operatorname{EllipticPi}[-1, \operatorname{ArcSin}[\operatorname{Tan}[(c+dx)/2]], \\
& (a-b)/(a+b)] \operatorname{Tan}[(c+dx)/2]^2 \operatorname{Sqrt}[1 - \operatorname{Tan}[(c+dx)/2]^2] \operatorname{Sqrt}[(a+b - a \operatorname{Tan}[(c+dx)/2]^2 + b \operatorname{Tan}[(c+dx)/2]^2)/(a+b)] \\
& + (a+b)(16a^4A + 41a^2A^2b^2 - 105A^5b^4 - 42a^3b^2B + 90a^2b^3B^3) \operatorname{EllipticE}[\operatorname{ArcSin}[\operatorname{Tan}[(c+dx)/2]], \\
& (a-b)/(a+b)] \operatorname{Sqrt}[1 - \operatorname{Tan}[(c+dx)/2]^2] (1 + \operatorname{Tan}[(c+dx)/2]^2) \operatorname{Sqrt}[(a+b - a \operatorname{Tan}[(c+dx)/2]^2 + b \operatorname{Tan}[(c+dx)/2]^2)/(a+b)] \\
& - 2a(a+b)(-35A^3b^3 + 12a^3B^3 - 2a^2b(5A + 9B) + 3a^2b^2(7A + 10B)) \operatorname{EllipticF}[\operatorname{ArcSin}[\operatorname{Tan}[(c+dx)/2]], \\
& (a-b)/(a+b)] \operatorname{Sqrt}[1 - \operatorname{Tan}[(c+dx)/2]^2] (1 + \operatorname{Tan}[(c+dx)/2]^2) \operatorname{Sqrt}[(a+b - a \operatorname{Tan}[(c+dx)/2]^2)
\end{aligned}$$

$2 + b \cdot \tan\left[\frac{c + dx}{2}\right]^2 / (a + b) \Big) \Big) / (24 \cdot a^4 \cdot (a^2 - b^2) \cdot d \cdot (a + b \cdot \sec[c + dx])^{3/2} \cdot \sqrt{1 + \tan\left[\frac{c + dx}{2}\right]^2} \cdot (a \cdot (-1 + \tan\left[\frac{c + dx}{2}\right]^2) - b \cdot (1 + \tan\left[\frac{c + dx}{2}\right]^2)))$

fricas [F] time = 62.09, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{(B \cos(dx + c)^3 \sec(dx + c) + A \cos(dx + c)^3) \sqrt{b \sec(dx + c) + a}}{b^2 \sec(dx + c)^2 + 2ab \sec(dx + c) + a^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(dx+c)^3*(A+B*sec(dx+c))/(a+b*sec(dx+c))^(3/2),x, algorithm="fricas")

[Out] integral((B*cos(dx + c)^3*sec(dx + c) + A*cos(dx + c)^3)*sqrt(b*sec(dx + c) + a)/(b^2*sec(dx + c)^2 + 2*a*b*sec(dx + c) + a^2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \cos(dx + c)^3}{(b \sec(dx + c) + a)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(dx+c)^3*(A+B*sec(dx+c))/(a+b*sec(dx+c))^(3/2),x, algorithm="giac")

[Out] integrate((B*sec(dx + c) + A)*cos(dx + c)^3/(b*sec(dx + c) + a)^(3/2), x)

maple [B] time = 2.92, size = 5086, normalized size = 8.07

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(dx+c)^3*(A+B*sec(dx+c))/(a+b*sec(dx+c))^(3/2),x)

[Out] result too large to display

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \cos(dx + c)^3}{(b \sec(dx + c) + a)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^3/(b*sec(d*x + c) + a)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c + dx)^3 \left(A + \frac{B}{\cos(c+dx)} \right)}{\left(a + \frac{b}{\cos(c+dx)} \right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)^3*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x))^(3/2),x)

[Out] int((cos(c + d*x)^3*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x))^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \cos^3(c + dx)}{(a + b \sec(c + dx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(3/2),x)

[Out] Integral((A + B*sec(c + d*x))*cos(c + d*x)**3/(a + b*sec(c + d*x))**(3/2), x)

$$3.385 \quad \int \frac{\sec^4(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^{5/2}} dx$$

Optimal. Leaf size=510

$$\frac{2a(Ab - aB) \tan(c + dx) \sec^2(c + dx)}{3bd(a^2 - b^2)(a + b \sec(c + dx))^{3/2}} - \frac{2(-2a^2B + aAb + b^2B) \tan(c + dx) \sqrt{a + b \sec(c + dx)}}{3b^3d(a^2 - b^2)} - \frac{2a^2(-6a^3B + 3a^2A + 3ab^2B)}{3b^3d(a^2 - b^2)}$$

[Out] $-2/3*(8*A*a^4*b-15*A*a^2*b^3+3*A*b^5-16*B*a^5+28*B*a^3*b^2-8*B*a*b^4)*\cot(d*x+c)*\text{EllipticE}((a+b*\sec(d*x+c))^{(1/2)}/(a+b)^{(1/2)},((a+b)/(a-b))^{(1/2)})*(b*(1-\sec(d*x+c))/(a+b))^{(1/2)}*(-b*(1+\sec(d*x+c))/(a-b))^{(1/2)}/(a-b)/b^5/(a+b)^{(3/2)}/d+2/3*(9*a*b^3*(A-B)+b^4*(3*A-B)+16*a^4*B-2*a^2*b^2*(3*A+8*B)-a^3*(8*A*b-12*B*b))*\cot(d*x+c)*\text{EllipticF}((a+b*\sec(d*x+c))^{(1/2)}/(a+b)^{(1/2)},((a+b)/(a-b))^{(1/2)})*(b*(1-\sec(d*x+c))/(a+b))^{(1/2)}*(-b*(1+\sec(d*x+c))/(a-b))^{(1/2)}/b^4/(a^2-b^2)/d/(a+b)^{(1/2)}+2/3*a*(A*b-B*a)*\sec(d*x+c)^2*\tan(d*x+c)/b/(a^2-b^2)/d/(a+b*\sec(d*x+c))^{(3/2)}-2/3*a^2*(3*A*a^2*b-7*A*b^3-6*B*a^3+10*B*a*b^2)*\tan(d*x+c)/b^3/(a^2-b^2)^2/d/(a+b*\sec(d*x+c))^{(1/2)}-2/3*(A*a*b-2*B*a^2+B*b^2)*(a+b*\sec(d*x+c))^{(1/2)}*\tan(d*x+c)/b^3/(a^2-b^2)/d$

Rubi [A] time = 1.59, antiderivative size = 510, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4029, 4090, 4082, 4005, 3832, 4004}

$$\frac{2a(Ab - aB) \tan(c + dx) \sec^2(c + dx)}{3bd(a^2 - b^2)(a + b \sec(c + dx))^{3/2}} - \frac{2a^2(3a^2Ab - 6a^3B + 10ab^2B - 7Ab^3) \tan(c + dx)}{3b^3d(a^2 - b^2)^2 \sqrt{a + b \sec(c + dx)}} - \frac{2(-2a^2B + aAb + b^2B)}{3b^3d(a^2 - b^2)}$$

Antiderivative was successfully verified.

[In] Int[(Sec[c + d*x]^4*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(5/2), x]

[Out] $(-2*(8*a^4*A*b - 15*a^2*A*b^3 + 3*A*b^5 - 16*a^5*B + 28*a^3*b^2*B - 8*a*b^4*B)*\text{Cot}[c + d*x]*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[a + b*\text{Sec}[c + d*x]]/\text{Sqrt}[a + b]], (a + b)/(a - b)]*\text{Sqrt}[(b*(1 - \text{Sec}[c + d*x]))/(a + b)]*\text{Sqrt}[-((b*(1 + \text{Sec}[c + d*x]))/(a - b))]/(3*(a - b)*b^5*(a + b)^{(3/2)*d} + (2*(9*a*b^3*(A - B) + b^4*(3*A - B) + 16*a^4*B - 2*a^2*b^2*(3*A + 8*B) - a^3*(8*A*b - 12*b*B))*\text{Cot}[c + d*x]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[a + b*\text{Sec}[c + d*x]]/\text{Sqrt}[a + b]], (a + b)/(a - b)]*\text{Sqrt}[(b*(1 - \text{Sec}[c + d*x]))/(a + b)]*\text{Sqrt}[-((b*(1 + \text{Sec}[c + d*x]))/(a - b))]/(3*b^4*\text{Sqrt}[a + b]*(a^2 - b^2)*d) + (2*a*(A*b - a*B)*\text{Sec}[c + d*x]^2*\text{Tan}[c + d*x]/(3*b*(a^2 - b^2)*d*(a + b*\text{Sec}[c + d*x])^{(3/2)}) - (2*a^2*(3*a^2*A*b - 7*A*b^3 - 6*a^3*B + 10*a*b^2*B)*\text{Tan}[c + d*x]/(3*b^3*(a^2 - b^2)^2*d*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]) - (2*(a*A*b - 2*a^2*B + b^2*B)*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]*\text{Tan}[c + d*x])/(3*b^3*(a^2 - b^2)*d)$

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol]
:> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)]/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol]
:> Simp[(-2*(A*b - a*B)*Rt[a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A, 2]], (a*A + b*B)/(a*A - b*B)]/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol]
:> Dist[A - B, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 4029

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^m*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol]
:> Simp[(a*d^2*(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[d/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(n - 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2*(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 1]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^m, x_Symbol]
:> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]
```

Rule 4090

```
Int[csc[(e_.) + (f_.)*(x_.)]^2*((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[
(e_.) + (f_.)*(x_.)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_), x
_Symbol] :> Simp[(a*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dist[1/(b^2*(m + 1)*(a^2 - b^
2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(m + 1)*(-(a*(b*B
- a*C)) + A*b^2) + (b*B*(a^2 + b^2*(m + 1)) - a*(A*b^2*(m + 2) + C*(a^2 +
b^2*(m + 1)))*Csc[e + f*x] - b*C*(m + 1)*(a^2 - b^2)*Csc[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{5/2}} dx = \frac{2a(Ab-aB)\sec^2(c+dx)\tan(c+dx)}{3b(a^2-b^2)d(a+b\sec(c+dx))^{3/2}} + \frac{2\int \frac{\sec^2(c+dx)\left(2a(Ab-aB)-\frac{3}{2}b(Ab-a)\right)}{(a+b\sec(c+dx))^{5/2}} dx}{3b(a^2-b^2)d(a+b\sec(c+dx))^{3/2}}$$

$$= \frac{2a(Ab-aB)\sec^2(c+dx)\tan(c+dx)}{3b(a^2-b^2)d(a+b\sec(c+dx))^{3/2}} - \frac{2a^2(3a^2Ab-7Ab^3-6a^3B+3a^2b^2)}{3b^3(a^2-b^2)^2 d\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{2a(Ab-aB)\sec^2(c+dx)\tan(c+dx)}{3b(a^2-b^2)d(a+b\sec(c+dx))^{3/2}} - \frac{2a^2(3a^2Ab-7Ab^3-6a^3B+3a^2b^2)}{3b^3(a^2-b^2)^2 d\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{2a(Ab-aB)\sec^2(c+dx)\tan(c+dx)}{3b(a^2-b^2)d(a+b\sec(c+dx))^{3/2}} - \frac{2a^2(3a^2Ab-7Ab^3-6a^3B+3a^2b^2)}{3b^3(a^2-b^2)^2 d\sqrt{a+b\sec(c+dx)}}$$

$$= -\frac{2(8a^4Ab-15a^2Ab^3+3Ab^5-16a^5B+28a^3b^2B-8ab^4B)\cot(c+dx)}{3(a-b)b^5(a+b\sec(c+dx))^{3/2}}$$

Mathematica [B] time = 27.42, size = 4342, normalized size = 8.51

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^4*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(5/2),
x]
```

```
[Out] ((b + a*Cos[c + d*x])^3*Sec[c + d*x]^3*((2*(8*a^4*A*b - 15*a^2*A*b^3 + 3*A*
b^5 - 16*a^5*B + 28*a^3*b^2*B - 8*a*b^4*B)*Sin[c + d*x])/(3*b^4*(-a^2 + b^2
)^2) + (2*(a^2*A*b*Ssin[c + d*x] - a^3*B*Ssin[c + d*x]))/(3*b^2*(-a^2 + b^2)*
(b + a*Cos[c + d*x])^2) + (2*(-4*a^4*A*b*Ssin[c + d*x] + 8*a^2*A*b^3*Ssin[c +
d*x] + 7*a^5*B*Ssin[c + d*x] - 11*a^3*b^2*B*Ssin[c + d*x]))/(3*b^3*(-a^2 + b
^2)^2*(b + a*Cos[c + d*x])) + (2*B*Tan[c + d*x])/(3*b^3)))/(d*(a + b*Sec[c
+ d*x])^(5/2)) + (2*(b + a*Cos[c + d*x])^2*((5*a^2*A)/((-a^2 + b^2)^2*Sqrt[
b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (8*a^4*A)/(3*b^2*(-a^2 + b^2)^2*S
qrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (A*b^2)/((-a^2 + b^2)^2*Sqrt[
b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (16*a^5*B)/(3*b^3*(-a^2 + b^2)^2*
Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (28*a^3*B)/(3*b*(-a^2 + b^2)
^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (8*a*b*B)/(3*(-a^2 + b^2)
^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (8*a^5*A*Sqrt[Sec[c + d*x
]])/(3*b^3*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) + (17*a^3*A*Sqrt[Sec[c
+ d*x]])/(3*b*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) - (3*a*A*b*Sqrt[Sec[
c + d*x]])/((-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) + (5*a^2*B*Sqrt[Sec[c
+ d*x]])/((-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) + (16*a^6*B*Sqrt[Sec[c +
d*x]])/(3*b^4*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) - (32*a^4*B*Sqrt[Se
c[c + d*x]])/(3*b^2*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) + (b^2*B*Sqrt[
Sec[c + d*x]])/(3*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) - (8*a^5*A*Cos[2
*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*b^3*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d
*x]]) + (5*a^3*A*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(b*(-a^2 + b^2)^2*Sqrt
[b + a*Cos[c + d*x]]) - (a*A*b*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/((-a^2
+ b^2)^2*Sqrt[b + a*Cos[c + d*x]]) + (8*a^2*B*Cos[2*(c + d*x)]*Sqrt[Sec[c +
d*x]])/(3*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) + (16*a^6*B*Cos[2*(c +
d*x)]*Sqrt[Sec[c + d*x]])/(3*b^4*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) -
(28*a^4*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*b^2*(-a^2 + b^2)^2*Sqrt[
b + a*Cos[c + d*x]])*Sec[c + d*x]^(5/2)*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*
x]]*(2*(a + b)*(-8*a^4*A*b + 15*a^2*A*b^3 - 3*A*b^5 + 16*a^5*B - 28*a^3*b^2
*B + 8*a*b^4*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d
*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a -
b)/(a + b)] + 2*b*(a + b)*(-16*a^4*B - 9*a*b^3*(A + B) + b^4*(3*A + B) + 4
*a^3*b*(2*A + 3*B) + 2*a^2*b^2*(-3*A + 8*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c +
d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[A
rcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-8*a^4*A*b + 15*a^2*A*b^3 - 3*
A*b^5 + 16*a^5*B - 28*a^3*b^2*B + 8*a*b^4*B)*Cos[c + d*x]*(b + a*Cos[c + d*
x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*b^4*(a^2 - b^2)^2*d*Sqrt[Sec[(
c + d*x)/2]^2*(a + b*Sec[c + d*x])^(5/2)*((a*Sqrt[Cos[(c + d*x)/2]^2*Sec[c
+ d*x]]*Sin[c + d*x]*(2*(a + b)*(-8*a^4*A*b + 15*a^2*A*b^3 - 3*A*b^5 + 16*
a^5*B - 28*a^3*b^2*B + 8*a*b^4*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqr
t[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(
c + d*x)/2]], (a - b)/(a + b)] + 2*b*(a + b)*(-16*a^4*B - 9*a*b^3*(A + B) +
b^4*(3*A + B) + 4*a^3*b*(2*A + 3*B) + 2*a^2*b^2*(-3*A + 8*B))*Sqrt[Cos[c +
d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d
*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-8*a^4*A*b +
```


$$\begin{aligned}
& 15a^2Ab^3 - 3A^2b^5 + 16a^5B - 28a^3b^2B + 8ab^4B) \cos[c + dx] \\
& * (b + a \cos[c + dx]) \sec[(c + dx)/2]^2 \tan[(c + dx)/2]) / (3b^4(a^2 - b \\
& ^2)^2 * (b + a \cos[c + dx])^{3/2} \sqrt{\sec[(c + dx)/2]^2}) - (\sqrt{\cos[(c + \\
& dx)/2]^2 \sec[c + dx]} \tan[(c + dx)/2] * (2(a + b)(-8a^4Ab + 15a^2A \\
& * b^3 - 3A^2b^5 + 16a^5B - 28a^3b^2B + 8ab^4B) \sqrt{\cos[c + dx]} / (1 \\
& + \cos[c + dx])) \sqrt{(b + a \cos[c + dx]) / ((a + b)(1 + \cos[c + dx]))} * \text{E} \\
& \text{llipticE}[\text{ArcSin}[\tan[(c + dx)/2]], (a - b)/(a + b)] + 2b(a + b)(-16a^4B \\
& - 9a^2b^3(A + B) + b^4(3A + B) + 4a^3b(2A + 3B) + 2a^2b^2(-3A \\
& + 8B)) \sqrt{\cos[c + dx]} / (1 + \cos[c + dx]) \sqrt{(b + a \cos[c + dx]) / ((a \\
& + b)(1 + \cos[c + dx]))} * \text{E} \\
& \text{llipticF}[\text{ArcSin}[\tan[(c + dx)/2]], (a - b)/(a + b)] + (-8a^4Ab + 15a^2A \\
& * b^3 - 3A^2b^5 + 16a^5B - 28a^3b^2B + 8ab^4B) \cos[c + dx] * (b + a \cos[c + \\
& dx]) \sec[(c + dx)/2]^2 \tan[(c + dx)/ \\
& 2]) / (3b^4(a^2 - b^2)^2 \sqrt{b + a \cos[c + dx]} \sqrt{\sec[(c + dx)/2]^2} \\
&) + (2 \sqrt{\cos[(c + dx)/2]^2 \sec[c + dx]} * (((-8a^4Ab + 15a^2A * b^3 - \\
& 3A^2b^5 + 16a^5B - 28a^3b^2B + 8ab^4B) \cos[c + dx] * (b + a \cos[c + \\
& dx]) \sec[(c + dx)/2]^4) / 2 + ((a + b)(-8a^4Ab + 15a^2A * b^3 - 3A^2b^ \\
& 5 + 16a^5B - 28a^3b^2B + 8ab^4B) \sqrt{(b + a \cos[c + dx]) / ((a + b) \\
& * (1 + \cos[c + dx]))} * \text{E} \\
& \text{llipticE}[\text{ArcSin}[\tan[(c + dx)/2]], (a - b)/(a + b)] * \\
& ((\cos[c + dx] \sin[c + dx]) / (1 + \cos[c + dx])^2 - \sin[c + dx] / (1 + \cos[c \\
& + dx]))) / \sqrt{\cos[c + dx]} / (1 + \cos[c + dx]) + (b(a + b)(-16a^4B - \\
& 9a^2b^3(A + B) + b^4(3A + B) + 4a^3b(2A + 3B) + 2a^2b^2(-3A + 8 \\
& * B)) \sqrt{(b + a \cos[c + dx]) / ((a + b)(1 + \cos[c + dx]))} * \text{E} \\
& \text{llipticF}[\text{ArcS} \\
& \text{in}[\tan[(c + dx)/2]], (a - b)/(a + b)] * ((\cos[c + dx] \sin[c + dx]) / (1 + \cos \\
& [c + dx])^2 - \sin[c + dx] / (1 + \cos[c + dx])) / \sqrt{\cos[c + dx]} / (1 + \cos \\
& [c + dx]) + ((a + b)(-8a^4Ab + 15a^2A * b^3 - 3A^2b^5 + 16a^5B - 2 \\
& 8a^3b^2B + 8ab^4B) \sqrt{\cos[c + dx]} / (1 + \cos[c + dx]) \sqrt{\text{E} \\
& \text{llipticE}[\text{Ar} \\
& \text{cSin}[\tan[(c + dx)/2]], (a - b)/(a + b)] * (-((a \sin[c + dx]) / ((a + b)(1 + \\
& \cos[c + dx]))) + ((b + a \cos[c + dx]) \sin[c + dx]) / ((a + b)(1 + \cos[c + \\
& dx])^2)) / \sqrt{(b + a \cos[c + dx]) / ((a + b)(1 + \cos[c + dx]))} + (b(a \\
& + b)(-16a^4B - 9a^2b^3(A + B) + b^4(3A + B) + 4a^3b(2A + 3B) + \\
& 2a^2b^2(-3A + 8B)) \sqrt{\cos[c + dx]} / (1 + \cos[c + dx]) \sqrt{\text{E} \\
& \text{llipticF}[\text{Arc} \\
& \text{Sin}[\tan[(c + dx)/2]], (a - b)/(a + b)] * (-((a \sin[c + dx]) / ((a + b)(1 + \cos \\
& [c + dx]))) + ((b + a \cos[c + dx]) \sin[c + dx]) / ((a + b)(1 + \cos[c + \\
& dx])^2)) / \sqrt{(b + a \cos[c + dx]) / ((a + b)(1 + \cos[c + dx]))} - a(-8 \\
& a^4Ab + 15a^2A * b^3 - 3A^2b^5 + 16a^5B - 28a^3b^2B + 8ab^4B) \cos \\
& [c + dx] \sec[(c + dx)/2]^2 \sin[c + dx] \tan[(c + dx)/2] - (-8a^4Ab + \\
& 15a^2A * b^3 - 3A^2b^5 + 16a^5B - 28a^3b^2B + 8ab^4B) * (b + a \cos[c \\
& + dx]) \sec[(c + dx)/2]^2 \sin[c + dx] \tan[(c + dx)/2] + (-8a^4Ab + 15 \\
& a^2A * b^3 - 3A^2b^5 + 16a^5B - 28a^3b^2B + 8ab^4B) \cos[c + dx] * (b \\
& + a \cos[c + dx]) \sec[(c + dx)/2]^2 \tan[(c + dx)/2]^2 + (b(a + b)(-16a \\
& ^4B - 9a^2b^3(A + B) + b^4(3A + B) + 4a^3b(2A + 3B) + 2a^2b^2(- \\
& -3A + 8B)) \sqrt{\cos[c + dx]} / (1 + \cos[c + dx]) \sqrt{(b + a \cos[c + dx]) \\
&) / ((a + b)(1 + \cos[c + dx]))} * \sec[(c + dx)/2]^2) / (\sqrt{1 - \tan[(c + dx) \\
& /2]^2} \sqrt{1 - ((a - b) \tan[(c + dx)/2]^2) / (a + b)}) + ((a + b)(-8a^4A \\
& * b + 15a^2A * b^3 - 3A^2b^5 + 16a^5B - 28a^3b^2B + 8ab^4B) \sqrt{\cos
\end{aligned}$$

```
[c + d*x]/(1 + Cos[c + d*x]))*Sqrt[(b + a*cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2*Sqrt[1 - ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]/Sqrt[1 - Tan[(c + d*x)/2]^2])/((3*b^4*(a^2 - b^2)^2*Sqrt[b + a*cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + ((2*(a + b)*(-8*a^4*A*b + 15*a^2*A*b^3 - 3*A*b^5 + 16*a^5*B - 28*a^3*b^2*B + 8*a*b^4*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x]))*Sqrt[(b + a*cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*b*(a + b)*(-16*a^4*B - 9*a*b^3*(A + B) + b^4*(3*A + B) + 4*a^3*b*(2*A + 3*B) + 2*a^2*b^2*(-3*A + 8*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x]))*Sqrt[(b + a*cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-8*a^4*A*b + 15*a^2*A*b^3 - 3*A*b^5 + 16*a^5*B - 28*a^3*b^2*B + 8*a*b^4*B)*Cos[c + d*x]*(b + a*cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])*(-Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(3*b^4*(a^2 - b^2)^2*Sqrt[b + a*cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]]))
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{(B \sec(dx + c)^5 + A \sec(dx + c)^4) \sqrt{b \sec(dx + c) + a}}{b^3 \sec(dx + c)^3 + 3ab^2 \sec(dx + c)^2 + 3a^2b \sec(dx + c) + a^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((B*sec(d*x + c)^5 + A*sec(d*x + c)^4)*sqrt(b*sec(d*x + c) + a)/(b^3*sec(d*x + c)^3 + 3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sec(dx + c)^4}{(b \sec(dx + c) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^4/(b*sec(d*x + c) + a)^(5/2), x)
```

maple [B] time = 3.43, size = 8044, normalized size = 15.77

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x)`

[Out] result too large to display

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")`

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\cos(c+dx)^4 \left(a + \frac{b}{\cos(c+dx)}\right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B/cos(c + d*x))/(cos(c + d*x)^4*(a + b/cos(c + d*x))^(5/2)),x)`

[Out] `int((A + B/cos(c + d*x))/(cos(c + d*x)^4*(a + b/cos(c + d*x))^(5/2)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \sec^4(c + dx)}{(a + b \sec(c + dx))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)**4*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(5/2),x)`

[Out] `Integral((A + B*sec(c + d*x))*sec(c + d*x)**4/(a + b*sec(c + d*x))**(5/2), x)`

$$3.386 \quad \int \frac{\sec^3(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^{5/2}} dx$$

Optimal. Leaf size=417

$$\frac{2a^2(Ab - aB) \tan(c + dx)}{3b^2d(a^2 - b^2)(a + b \sec(c + dx))^{3/2}} + \frac{2a(-5a^3B + 2a^2Ab + 9ab^2B - 6Ab^3) \tan(c + dx)}{3b^2d(a^2 - b^2)^2 \sqrt{a + b \sec(c + dx)}} + \frac{2(-8a^3B + 2a^2b(A - B)) \tan(c + dx)}{3b^2d(a^2 - b^2)^2 \sqrt{a + b \sec(c + dx)}}$$

[Out] $2/3*(2*A*a^3*b-6*A*a*b^3-8*B*a^4+15*B*a^2*b^2-3*B*b^4)*\cot(d*x+c)*\text{EllipticE}((a+b*\sec(d*x+c))^{1/2}/(a+b)^{1/2},((a+b)/(a-b))^{1/2})*(b*(1-\sec(d*x+c)))/(a+b))^{1/2}*(-b*(1+\sec(d*x+c)))/(a-b)^{1/2}/(a-b)/b^4/(a+b)^{3/2}/d+2/3*(2*a^2*b*(A-3*B)-3*b^3*(A-B)-8*a^3*B+3*a*b^2*(A+3*B))*\cot(d*x+c)*\text{EllipticF}((a+b*\sec(d*x+c))^{1/2}/(a+b)^{1/2},((a+b)/(a-b))^{1/2})*(b*(1-\sec(d*x+c)))/(a+b))^{1/2}*(-b*(1+\sec(d*x+c)))/(a-b)^{1/2}/b^3/(a^2-b^2)/d/(a+b)^{1/2}-2/3*a^2*(A*b-B*a)*\tan(d*x+c)/b^2/(a^2-b^2)/d/(a+b*\sec(d*x+c))^{3/2}+2/3*a*(2*A*a^2*b-6*A*b^3-5*B*a^3+9*B*a*b^2)*\tan(d*x+c)/b^2/(a^2-b^2)^2/d/(a+b*\sec(d*x+c))^{1/2}$

Rubi [A] time = 1.02, antiderivative size = 417, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {4028, 4080, 4005, 3832, 4004}

$$\frac{2a^2(Ab - aB) \tan(c + dx)}{3b^2d(a^2 - b^2)(a + b \sec(c + dx))^{3/2}} + \frac{2a(2a^2Ab - 5a^3B + 9ab^2B - 6Ab^3) \tan(c + dx)}{3b^2d(a^2 - b^2)^2 \sqrt{a + b \sec(c + dx)}} + \frac{2(2a^2b(A - 3B) - 8a^3B) \tan(c + dx)}{3b^2d(a^2 - b^2)^2 \sqrt{a + b \sec(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[(Sec[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(5/2),x]

[Out] $(2*(2*a^3*A*b - 6*a*A*b^3 - 8*a^4*B + 15*a^2*b^2*B - 3*b^4*B)*\text{Cot}[c + d*x]*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[a + b*\text{Sec}[c + d*x]]/\text{Sqrt}[a + b]],(a + b)/(a - b)]*\text{Sqrt}[(b*(1 - \text{Sec}[c + d*x]))/(a + b)]*\text{Sqrt}[-((b*(1 + \text{Sec}[c + d*x]))/(a - b))])/(3*(a - b)*b^4*(a + b)^{3/2}*d) + (2*(2*a^2*b*(A - 3*B) - 3*b^3*(A - B) - 8*a^3*B + 3*a*b^2*(A + 3*B))*\text{Cot}[c + d*x]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[a + b*\text{Sec}[c + d*x]]/\text{Sqrt}[a + b]],(a + b)/(a - b)]*\text{Sqrt}[(b*(1 - \text{Sec}[c + d*x]))/(a + b)]*\text{Sqrt}[-((b*(1 + \text{Sec}[c + d*x]))/(a - b))])/(3*b^3*\text{Sqrt}[a + b]*(a^2 - b^2)*d) - (2*a^2*(A*b - a*B)*\text{Tan}[c + d*x])/(3*b^2*(a^2 - b^2)*d*(a + b*\text{Sec}[c + d*x])^{3/2}) + (2*a*(2*a^2*A*b - 6*A*b^3 - 5*a^3*B + 9*a*b^2*B)*\text{Tan}[c + d*x])/(3*b^2*(a^2 - b^2)^2*d*\text{Sqrt}[a + b*\text{Sec}[c + d*x]])$

Rule 3832

Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-

```
((b*(1 + Csc[e + f*x]))/(a - b))*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)]/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e + f*x]))/(a - b))])*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A, 2]], (a*A + b*B)/(a*A - b*B)]/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 4028

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(a^2*(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[a*b*(A*b - a*B)*(m + 1) - (A*b - a*B)*(a^2 + b^2*(m + 1))*Csc[e + f*x] + b*B*(m + 1)*(a^2 - b^2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4080

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C))*(m + 1))*Csc[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^3(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{5/2}} dx &= -\frac{2a^2(Ab-aB)\tan(c+dx)}{3b^2(a^2-b^2)d(a+b\sec(c+dx))^{3/2}} - \frac{2\int \frac{\sec(c+dx)\left(-\frac{3}{2}ab(Ab-aB)-\frac{1}{2}(2a^2-3b^2)\right)}{(a+b\sec(c+dx))^{5/2}} dx}{3b^2(a^2-b^2)^2 d\sqrt{a+b\sec(c+dx)}} \\
&= -\frac{2a^2(Ab-aB)\tan(c+dx)}{3b^2(a^2-b^2)d(a+b\sec(c+dx))^{3/2}} + \frac{2a(2a^2Ab-6Ab^3-5a^3B+9a^2B)}{3b^2(a^2-b^2)^2 d\sqrt{a+b\sec(c+dx)}} \\
&= -\frac{2a^2(Ab-aB)\tan(c+dx)}{3b^2(a^2-b^2)d(a+b\sec(c+dx))^{3/2}} + \frac{2a(2a^2Ab-6Ab^3-5a^3B+9a^2B)}{3b^2(a^2-b^2)^2 d\sqrt{a+b\sec(c+dx)}} \\
&= \frac{2(2a^3Ab-6aAb^3-8a^4B+15a^2b^2B-3b^4B)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)\right)}{3(a-b)b^4(a+b)^{3/2}d}
\end{aligned}$$

Mathematica [B] time = 26.77, size = 3920, normalized size = 9.40

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(Sec[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(5/2), x]

[Out] ((b + a*Cos[c + d*x])^3*Sec[c + d*x]^3*((2*(-2*a^3*A*b + 6*a*A*b^3 + 8*a^4*B - 15*a^2*b^2*B + 3*b^4*B)*Sin[c + d*x])/(3*b^3*(-a^2 + b^2)^2) - (2*(a*A*b*SIN[c + d*x] - a^2*B*SIN[c + d*x]))/(3*b*(-a^2 + b^2)*(b + a*Cos[c + d*x])^2) - (2*(-(a^3*A*b*SIN[c + d*x]) + 5*a*A*b^3*SIN[c + d*x] + 4*a^4*B*SIN[c + d*x] - 8*a^2*b^2*B*SIN[c + d*x]))/(3*b^2*(-a^2 + b^2)^2*(b + a*Cos[c + d*x])))/(d*(a + b*Sec[c + d*x])^(5/2)) - (2*(b + a*Cos[c + d*x])^2*((2*a^3*A)/(3*b*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (2*a*A*b)/((-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (5*a^2*B)/((-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (8*a^4*B)/(3*b^2*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (b^2*B)/((-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (5*a^2*A*Sqrt[Sec[c + d*x]])/(3*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) + (2*a^4*A*Sqrt[Sec[c + d*x]])/(3*b^2*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) + (A*b^2*Sqrt[Sec[c + d*x]])/((-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) - (8*a^5*B*Sqrt[Sec[c + d*x]])/(3*b^3*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) + (17*a^3*B*Sqrt[Sec[c + d*x]])/(3*b*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) - (3*a*b*B*Sqrt[Sec[c + d*x]])/((-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) - (2*a

$$\begin{aligned}
& ^2*A*\cos[2*(c + d*x)]*sqrt[sec[c + d*x]]/((-a^2 + b^2)^2*sqrt[b + a*\cos[c + d*x]]) + (2*a^4*A*\cos[2*(c + d*x)]*sqrt[sec[c + d*x]]/(3*b^2*(-a^2 + b^2)^2*sqrt[b + a*\cos[c + d*x]]) - (8*a^5*B*\cos[2*(c + d*x)]*sqrt[sec[c + d*x]])/(3*b^3*(-a^2 + b^2)^2*sqrt[b + a*\cos[c + d*x]]) + (5*a^3*B*\cos[2*(c + d*x)]*sqrt[sec[c + d*x]]/(b*(-a^2 + b^2)^2*sqrt[b + a*\cos[c + d*x]]) - (a*b*B*\cos[2*(c + d*x)]*sqrt[sec[c + d*x]])/((-a^2 + b^2)^2*sqrt[b + a*\cos[c + d*x]])*sec[c + d*x]^(5/2)*sqrt[cos[(c + d*x)/2]^2*sec[c + d*x]]*(2*(a + b)*(-2*a^3*A*b + 6*a*A*b^3 + 8*a^4*B - 15*a^2*b^2*B + 3*b^4*B)*sqrt[cos[c + d*x]/(1 + cos[c + d*x])]*sqrt[(b + a*cos[c + d*x])/((a + b)*(1 + cos[c + d*x]))])*ellipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(a + b)*(3*a*b^2*(A - 3*B) + 8*a^3*B + 3*b^3*(A + B) - 2*a^2*b*(A + 3*B))*sqrt[cos[c + d*x]/(1 + cos[c + d*x])]*sqrt[(b + a*cos[c + d*x])/((a + b)*(1 + cos[c + d*x]))])*ellipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-2*a^3*A*b + 6*a*A*b^3 + 8*a^4*B - 15*a^2*b^2*B + 3*b^4*B)*cos[c + d*x]*(b + a*cos[c + d*x])*sec[(c + d*x)/2]^2*tan[(c + d*x)/2])/((3*b^3*(a^2 - b^2)^2*d*sqrt[sec[(c + d*x)/2]^2]*(a + b*sec[c + d*x])^(5/2)*(-1/3*(a*sqrt[cos[(c + d*x)/2]^2*sec[c + d*x]]*sin[c + d*x]*(2*(a + b)*(-2*a^3*A*b + 6*a*A*b^3 + 8*a^4*B - 15*a^2*b^2*B + 3*b^4*B)*sqrt[cos[c + d*x]/(1 + cos[c + d*x])]*sqrt[(b + a*cos[c + d*x])/((a + b)*(1 + cos[c + d*x]))])*ellipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(a + b)*(3*a*b^2*(A - 3*B) + 8*a^3*B + 3*b^3*(A + B) - 2*a^2*b*(A + 3*B))*sqrt[cos[c + d*x]/(1 + cos[c + d*x])]*sqrt[(b + a*cos[c + d*x])/((a + b)*(1 + cos[c + d*x]))])*ellipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-2*a^3*A*b + 6*a*A*b^3 + 8*a^4*B - 15*a^2*b^2*B + 3*b^4*B)*cos[c + d*x]*(b + a*cos[c + d*x])*sec[(c + d*x)/2]^2*tan[(c + d*x)/2])/((b^3*(a^2 - b^2)^2*(b + a*cos[c + d*x])^(3/2)*sqrt[sec[(c + d*x)/2]^2]) + (sqrt[cos[(c + d*x)/2]^2*sec[c + d*x]]*tan[(c + d*x)/2]*(2*(a + b)*(-2*a^3*A*b + 6*a*A*b^3 + 8*a^4*B - 15*a^2*b^2*B + 3*b^4*B)*sqrt[cos[c + d*x]/(1 + cos[c + d*x])]*sqrt[(b + a*cos[c + d*x])/((a + b)*(1 + cos[c + d*x]))])*ellipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(a + b)*(3*a*b^2*(A - 3*B) + 8*a^3*B + 3*b^3*(A + B) - 2*a^2*b*(A + 3*B))*sqrt[cos[c + d*x]/(1 + cos[c + d*x])]*sqrt[(b + a*cos[c + d*x])/((a + b)*(1 + cos[c + d*x]))])*ellipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-2*a^3*A*b + 6*a*A*b^3 + 8*a^4*B - 15*a^2*b^2*B + 3*b^4*B)*cos[c + d*x]*(b + a*cos[c + d*x])*sec[(c + d*x)/2]^2*tan[(c + d*x)/2])/((3*b^3*(a^2 - b^2)^2*sqrt[b + a*cos[c + d*x]]*sqrt[sec[(c + d*x)/2]^2]) - (2*sqrt[cos[(c + d*x)/2]^2*sec[c + d*x]]*(((-2*a^3*A*b + 6*a*A*b^3 + 8*a^4*B - 15*a^2*b^2*B + 3*b^4*B)*cos[c + d*x]*(b + a*cos[c + d*x])*sec[(c + d*x)/2]^4)/2 + ((a + b)*(-2*a^3*A*b + 6*a*A*b^3 + 8*a^4*B - 15*a^2*b^2*B + 3*b^4*B)*sqrt[(b + a*cos[c + d*x])/((a + b)*(1 + cos[c + d*x]))])*ellipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*(cos[c + d*x]*sin[c + d*x])/(1 + cos[c + d*x])^2 - sin[c + d*x]/(1 + cos[c + d*x])))/sqrt[cos[c + d*x]/(1 + cos[c + d*x])] - (b*(a + b)*(3*a*b^2*(A - 3*B) + 8*a^3*B + 3*b^3*(A + B) - 2*a^2*b*(A + 3*B))*sqrt[(b + a*cos[c + d*x])/((a + b)*(1 + cos[c + d*x]))])*ellipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*(cos[c + d*x]*sin[c + d*x])/(1 + cos[c + d*x])^2 - sin[c + d*x]/(1 + cos[c + d*x])))/sqrt[cos[c + d*x]/(1 + cos[c + d*x])] + ((a + b)
\end{aligned}$$

```

*(-2*a^3*A*b + 6*a*A*b^3 + 8*a^4*B - 15*a^2*b^2*B + 3*b^4*B)*Sqrt[Cos[c + d
*x]/(1 + Cos[c + d*x])] * EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)
] * (-((a*SIN[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((b + a*Cos[c + d*x])
*SIN[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(b + a*Cos[c + d*x])/
(a + b)*(1 + Cos[c + d*x])] - (b*(a + b)*(3*a*b^2*(A - 3*B) + 8*a^3*B + 3*
b^3*(A + B) - 2*a^2*b*(A + 3*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] * Elli
pticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] * (-((a*SIN[c + d*x])/((a +
b)*(1 + Cos[c + d*x]))) + ((b + a*Cos[c + d*x])*SIN[c + d*x])/((a + b)*(1 +
Cos[c + d*x])^2)))/Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]
- a*(-2*a^3*A*b + 6*a*A*b^3 + 8*a^4*B - 15*a^2*b^2*B + 3*b^4*B)*Cos[c + d*
x]*Sec[(c + d*x)/2]^2*SIN[c + d*x]*Tan[(c + d*x)/2] - (-2*a^3*A*b + 6*a*A*b
^3 + 8*a^4*B - 15*a^2*b^2*B + 3*b^4*B)*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2
]^2*SIN[c + d*x]*Tan[(c + d*x)/2] + (-2*a^3*A*b + 6*a*A*b^3 + 8*a^4*B - 15*
a^2*b^2*B + 3*b^4*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*T
an[(c + d*x)/2]^2 - (b*(a + b)*(3*a*b^2*(A - 3*B) + 8*a^3*B + 3*b^3*(A + B)
- 2*a^2*b*(A + 3*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] * Sqrt[(b + a*Cos
[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] * Sec[(c + d*x)/2]^2/(Sqrt[1 - Tan[
(c + d*x)/2]^2] * Sqrt[1 - ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]) + ((a + b)*
(-2*a^3*A*b + 6*a*A*b^3 + 8*a^4*B - 15*a^2*b^2*B + 3*b^4*B)*Sqrt[Cos[c + d*
x]/(1 + Cos[c + d*x])] * Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]
))] * Sec[(c + d*x)/2]^2 * Sqrt[1 - ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]) / Sqrt
[1 - Tan[(c + d*x)/2]^2]) / (3*b^3*(a^2 - b^2)^2 * Sqrt[b + a*Cos[c + d*x]] * Sqr
t[Sec[(c + d*x)/2]^2]) - ((2*(a + b)*(-2*a^3*A*b + 6*a*A*b^3 + 8*a^4*B - 1
5*a^2*b^2*B + 3*b^4*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] * Sqrt[(b + a*Co
s[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] * EllipticE[ArcSin[Tan[(c + d*x)/2]
], (a - b)/(a + b)] - 2*b*(a + b)*(3*a*b^2*(A - 3*B) + 8*a^3*B + 3*b^3*(A +
B) - 2*a^2*b*(A + 3*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] * Sqrt[(b + a*
Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] * EllipticF[ArcSin[Tan[(c + d*x)/
2]], (a - b)/(a + b)] + (-2*a^3*A*b + 6*a*A*b^3 + 8*a^4*B - 15*a^2*b^2*B +
3*b^4*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2 * Tan[(c + d*x)
/2]) * (-Cos[(c + d*x)/2] * Sec[c + d*x] * SIN[(c + d*x)/2]) + Cos[(c + d*x)/2]^
2 * Sec[c + d*x] * Tan[c + d*x]) / (3*b^3*(a^2 - b^2)^2 * Sqrt[b + a*Cos[c + d*x]]
 * Sqrt[Sec[(c + d*x)/2]^2] * Sqrt[Cos[(c + d*x)/2]^2 * Sec[c + d*x]))))

```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{(B \sec(dx + c)^4 + A \sec(dx + c)^3) \sqrt{b \sec(dx + c) + a}}{b^3 \sec(dx + c)^3 + 3ab^2 \sec(dx + c)^2 + 3a^2b \sec(dx + c) + a^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algorithm
="fricas")
```

```
[Out] integral((B*sec(d*x + c)^4 + A*sec(d*x + c)^3)*sqrt(b*sec(d*x + c) + a)/(b^
3*sec(d*x + c)^3 + 3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```


giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sec(dx + c)^3}{(b \sec(dx + c) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^3/(b*sec(d*x + c) + a)^(5/2), x)

maple [B] time = 2.68, size = 6455, normalized size = 15.48

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x)

[Out] result too large to display

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\cos(c+dx)^3 \left(a + \frac{b}{\cos(c+dx)}\right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^3*(a + b/cos(c + d*x))^(5/2)),x)

[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^3*(a + b/cos(c + d*x))^(5/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \sec^3(c + dx)}{(a + b \sec(c + dx))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(5/2), x)

[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**3/(a + b*sec(c + d*x))**(5/2), x)

$$3.387 \quad \int \frac{\sec^2(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^{5/2}} dx$$

Optimal. Leaf size=387

$$\frac{2a(Ab - aB) \tan(c + dx)}{3bd(a^2 - b^2)(a + b \sec(c + dx))^{3/2}} + \frac{2(2a^2B + ab(A + 3B) - 3b^2(A + B)) \cot(c + dx) \sqrt{\frac{b(1 - \sec(c+dx))}{a+b}} \sqrt{-\frac{b(\sec(c+dx))}{a-b}}}{3b^2d\sqrt{a+b}(a^2 - b^2)}$$

[Out] $2/3*(A*a^2*b+3*A*b^3+2*B*a^3-6*B*a*b^2)*\cot(d*x+c)*\text{EllipticE}((a+b*\sec(d*x+c))^{1/2}/(a+b)^{(1/2)},((a+b)/(a-b))^{1/2})*(b*(1-\sec(d*x+c))/(a+b))^{1/2}*(-b*(1+\sec(d*x+c))/(a-b))^{1/2}/(a-b)/b^3/(a+b)^{(3/2)}/d+2/3*(2*a^2*B-3*b^2*(A+B)+a*b*(A+3*B))*\cot(d*x+c)*\text{EllipticF}((a+b*\sec(d*x+c))^{1/2}/(a+b)^{(1/2)},((a+b)/(a-b))^{1/2})*(b*(1-\sec(d*x+c))/(a+b))^{1/2}*(-b*(1+\sec(d*x+c))/(a-b))^{1/2}/b^2/(a^2-b^2)/d/(a+b)^{(1/2)}+2/3*a*(A*b-B*a)*\tan(d*x+c)/b/(a^2-b^2)/d/(a+b*\sec(d*x+c))^{3/2}+2/3*(A*a^2*b+3*A*b^3+2*B*a^3-6*B*a*b^2)*\tan(d*x+c)/b/(a^2-b^2)^2/d/(a+b*\sec(d*x+c))^{1/2}$

Rubi [A] time = 0.69, antiderivative size = 387, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {4009, 4003, 4005, 3832, 4004}

$$\frac{2(a^2Ab + 2a^3B - 6ab^2B + 3Ab^3) \tan(c + dx)}{3bd(a^2 - b^2)^2 \sqrt{a + b \sec(c + dx)}} + \frac{2a(Ab - aB) \tan(c + dx)}{3bd(a^2 - b^2)(a + b \sec(c + dx))^{3/2}} + \frac{2(2a^2B + ab(A + 3B) - 3b^2(A + B)) \cot(c + dx) \sqrt{\frac{b(1 - \sec(c+dx))}{a+b}} \sqrt{-\frac{b(\sec(c+dx))}{a-b}}}{3b^2d\sqrt{a+b}(a^2 - b^2)}$$

Antiderivative was successfully verified.

[In] Int[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(5/2),x]

[Out] $(2*(a^2*A*b + 3*A*b^3 + 2*a^3*B - 6*a*b^2*B)*\text{Cot}[c + d*x]*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[a + b*\text{Sec}[c + d*x]]/\text{Sqrt}[a + b]],(a + b)/(a - b)]*\text{Sqrt}[(b*(1 - \text{Sec}[c + d*x]))/(a + b)]*\text{Sqrt}[-((b*(1 + \text{Sec}[c + d*x]))/(a - b))]/(3*(a - b)*b^3*(a + b)^{(3/2)*d} + (2*(2*a^2*B - 3*b^2*(A + B) + a*b*(A + 3*B))*\text{Cot}[c + d*x]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[a + b*\text{Sec}[c + d*x]]/\text{Sqrt}[a + b]],(a + b)/(a - b)]*\text{Sqrt}[(b*(1 - \text{Sec}[c + d*x]))/(a + b)]*\text{Sqrt}[-((b*(1 + \text{Sec}[c + d*x]))/(a - b))]/(3*b^2*\text{Sqrt}[a + b]*(a^2 - b^2)*d) + (2*a*(A*b - a*B)*\text{Tan}[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*\text{Sec}[c + d*x])^{3/2}) + (2*(a^2*A*b + 3*A*b^3 + 2*a^3*B - 6*a*b^2*B)*\text{Tan}[c + d*x])/(3*b*(a^2 - b^2)^2*d*\text{Sqrt}[a + b*\text{Sec}[c + d*x]])$

Rule 3832

Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] := Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]

/Rt[a + b, 2]], (a + b)/(a - b)]/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 4003

Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] := -Simp[((A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(m + 1)*(a^2 - b^2), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[(a*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]

Rule 4004

Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] := Simp[(-2*(A*b - a*B)*Rt[a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A, 2]], (a*A + b*B)/(a*A - b*B)]/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]

Rule 4005

Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] := Dist[A - B, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]

Rule 4009

Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] := Simp[(a*(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(A*b - a*B)*(m + 1) - (a*A*b*(m + 2) - B*(a^2 + b^2*(m + 1)))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]

Rubi steps

$$\begin{aligned}
\int \frac{\sec^2(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{5/2}} dx &= \frac{2a(Ab-aB)\tan(c+dx)}{3b(a^2-b^2)d(a+b\sec(c+dx))^{3/2}} + \frac{2\int \frac{\sec(c+dx)\left(-\frac{3}{2}b(Ab-aB)+\frac{1}{2}(aAb+2a^2\right)}{(a+b\sec(c+dx))^{3/2}}}{3b(a^2-b^2)}}{3b(a^2-b^2)} \\
&= \frac{2a(Ab-aB)\tan(c+dx)}{3b(a^2-b^2)d(a+b\sec(c+dx))^{3/2}} + \frac{2(a^2Ab+3Ab^3+2a^3B-6ab^2B)}{3b(a^2-b^2)^2d\sqrt{a+b\sec(c+dx)}} \\
&= \frac{2a(Ab-aB)\tan(c+dx)}{3b(a^2-b^2)d(a+b\sec(c+dx))^{3/2}} + \frac{2(a^2Ab+3Ab^3+2a^3B-6ab^2B)}{3b(a^2-b^2)^2d\sqrt{a+b\sec(c+dx)}} \\
&= \frac{2(a^2Ab+3Ab^3+2a^3B-6ab^2B)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)\right)}{3(a-b)b^3(a+b)^{3/2}d}
\end{aligned}$$

Mathematica [B] time = 24.46, size = 3514, normalized size = 9.08

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(5/2), x]

[Out] ((b + a*Cos[c + d*x])^3*Sec[c + d*x]^3*((-2*(a^2*A*b + 3*A*b^3 + 2*a^3*B - 6*a*b^2*B)*Sin[c + d*x]))/(3*b^2*(-a^2 + b^2)^2) + (2*(A*b*Sin[c + d*x] - a*B*Sin[c + d*x]))/(3*(-a^2 + b^2)*(b + a*Cos[c + d*x])^2) + (2*(2*a^2*A*b*Sin[c + d*x] + 2*A*b^3*Sin[c + d*x] + a^3*B*Sin[c + d*x] - 5*a*b^2*B*Sin[c + d*x]))/(3*b*(-a^2 + b^2)^2*(b + a*Cos[c + d*x])))/(d*(a + b*Sec[c + d*x])^(5/2)) + (2*(b + a*Cos[c + d*x])^2*((a^2*A)/(3*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (A*b^2)/((-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (2*a^3*B)/(3*b*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (2*a*b*B)/((-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (a^3*A*Sqrt[Sec[c + d*x]])/(3*b*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) - (a*A*b*Sqrt[Sec[c + d*x]])/(3*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) - (5*a^2*B*Sqrt[Sec[c + d*x]])/(3*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) + (2*a^4*B*Sqrt[Sec[c + d*x]])/(3*b^2*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) + (b^2*B*Sqrt[Sec[c + d*x]])/((-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) + (a^3*A*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*b*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) + (a*A*b*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/((-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) - (2*a^2*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/((-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]])

$$\begin{aligned}
& + d*x)]*Sqrt[Sec[c + d*x]]/((-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) + (2 \\
& *a^4*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]]/(3*b^2*(-a^2 + b^2)^2*Sqrt[b + \\
& a*Cos[c + d*x]])*Sec[c + d*x]^(5/2)*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]* \\
& (2*(a + b)*(a^2*A*b + 3*A*b^3 + 2*a^3*B - 6*a*b^2*B)*Sqrt[Cos[c + d*x]/(1 + \\
& Cos[c + d*x]])*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Ell \\
& ipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(a + b)*(a*b*(A - 3 \\
& *B) + 3*b^2*(A - B) + 2*a^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x]])*Sqrt[(\\
& b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + \\
& d*x)/2]], (a - b)/(a + b)] + (a^2*A*b + 3*A*b^3 + 2*a^3*B - 6*a*b^2*B)*Cos \\
& [c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*(-(\\
& a^2*b) + b^3)^2*d*Sqrt[Sec[(c + d*x)/2]^2]*(a + b*Sec[c + d*x])^(5/2)*((a*S \\
& qrt[Cos[(c + d*x)/2]^2*Sec[c + d*x])*Sin[c + d*x]*(2*(a + b)*(a^2*A*b + 3*A \\
& *b^3 + 2*a^3*B - 6*a*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x]])*Sqrt[(b + \\
& a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d* \\
& x)/2]], (a - b)/(a + b)] - 2*b*(a + b)*(a*b*(A - 3*B) + 3*b^2*(A - B) + 2*a \\
& ^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x]])*Sqrt[(b + a*Cos[c + d*x])/((a + \\
& b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b \\
&)] + (a^2*A*b + 3*A*b^3 + 2*a^3*B - 6*a*b^2*B)*Cos[c + d*x]*(b + a*Cos[c + \\
& d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*(-(a^2*b) + b^3)^2*(b + a*Co \\
& s[c + d*x])^(3/2)*Sqrt[Sec[(c + d*x)/2]^2]) - (Sqrt[Cos[(c + d*x)/2]^2*Sec[\\
& c + d*x]]*Tan[(c + d*x)/2]*(2*(a + b)*(a^2*A*b + 3*A*b^3 + 2*a^3*B - 6*a*b^ \\
& 2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x]])*Sqrt[(b + a*Cos[c + d*x])/((a + \\
& b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b \\
&)] - 2*b*(a + b)*(a*b*(A - 3*B) + 3*b^2*(A - B) + 2*a^2*B)*Sqrt[Cos[c + d*x] \\
& /((1 + Cos[c + d*x]))]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x])) \\
&]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (a^2*A*b + 3*A*b^3 \\
& + 2*a^3*B - 6*a*b^2*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^ \\
& 2*Tan[(c + d*x)/2]))/(3*(-(a^2*b) + b^3)^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Se \\
& c[(c + d*x)/2]^2]) + (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(((a^2*A*b + \\
& 3*A*b^3 + 2*a^3*B - 6*a*b^2*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d \\
& *x)/2]^4)/2 + ((a + b)*(a^2*A*b + 3*A*b^3 + 2*a^3*B - 6*a*b^2*B)*Sqrt[(b + \\
& a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x \\
&)/2]], (a - b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/((1 + Cos[c + d*x])^2 - \\
& Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x]]) - \\
& (b*(a + b)*(a*b*(A - 3*B) + 3*b^2*(A - B) + 2*a^2*B)*Sqrt[(b + a*Cos[c + d* \\
& x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - \\
& b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/((1 + Cos[c + d*x])^2 - Sin[c + d*x \\
&]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x]]) + ((a + b)*(a^ \\
& 2*A*b + 3*A*b^3 + 2*a^3*B - 6*a*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x]) \\
&]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*(-((a*Sin[c + d*x])/ \\
& ((a + b)*(1 + Cos[c + d*x])))) + ((b + a*Cos[c + d*x])*Sin[c + d*x])/((a + b \\
&)*(1 + Cos[c + d*x])^2))/Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d \\
& *x]))] - (b*(a + b)*(a*b*(A - 3*B) + 3*b^2*(A - B) + 2*a^2*B)*Sqrt[Cos[c + \\
& d*x]/(1 + Cos[c + d*x]])*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b \\
&)]*(-((a*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])))) + ((b + a*Cos[c + d*x]
\end{aligned}$$

)*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2))/Sqrt[(b + a*cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] - a*(a^2*A*b + 3*A*b^3 + 2*a^3*B - 6*a*b^2*B)*Cos[c + d*x]*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] - (a^2*A*b + 3*A*b^3 + 2*a^3*B - 6*a*b^2*B)*(b + a*cos[c + d*x])*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] + (a^2*A*b + 3*A*b^3 + 2*a^3*B - 6*a*b^2*B)*Cos[c + d*x]*(b + a*cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2 - (b*(a + b)*(a*b*(A - 3*B) + 3*b^2*(A - B) + 2*a^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2)/(Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[1 - ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]) + ((a + b)*(a^2*A*b + 3*A*b^3 + 2*a^3*B - 6*a*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2*Sqrt[1 - ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]))/Sqrt[1 - Tan[(c + d*x)/2]^2]))/(3*(-(a^2*b) + b^3)^2*Sqrt[b + a*cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + ((2*(a + b)*(a^2*A*b + 3*A*b^3 + 2*a^3*B - 6*a*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(a + b)*(a*b*(A - 3*B) + 3*b^2*(A - B) + 2*a^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (a^2*A*b + 3*A*b^3 + 2*a^3*B - 6*a*b^2*B)*Cos[c + d*x]*(b + a*cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(3*(-(a^2*b) + b^3)^2*Sqrt[b + a*cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]))

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{(B \sec(dx + c)^3 + A \sec(dx + c)^2) \sqrt{b \sec(dx + c) + a}}{b^3 \sec(dx + c)^3 + 3ab^2 \sec(dx + c)^2 + 3a^2b \sec(dx + c) + a^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")

[Out] integral((B*sec(d*x + c)^3 + A*sec(d*x + c)^2)*sqrt(b*sec(d*x + c) + a)/(b^3*sec(d*x + c)^3 + 3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sec(dx + c)^2}{(b \sec(dx + c) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^2/(b*sec(d*x + c) + a)^(5/2), x)

maple [B] time = 2.14, size = 5170, normalized size = 13.36

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x)

[Out] result too large to display

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\cos(c+dx)^2 \left(a + \frac{b}{\cos(c+dx)}\right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^2*(a + b/cos(c + d*x))^(5/2)),x)

[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^2*(a + b/cos(c + d*x))^(5/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \sec^2(c + dx)}{(a + b \sec(c + dx))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(5/2),x)

[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**2/(a + b*sec(c + d*x))**(5/2), x)

$$3.388 \quad \int \frac{\sec(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^{5/2}} dx$$

Optimal. Leaf size=353

$$\frac{2(a^2(-B) + 4aAb - 3b^2B) \tan(c + dx)}{3d(a^2 - b^2)^2 \sqrt{a + b \sec(c + dx)}} - \frac{2(Ab - aB) \tan(c + dx)}{3d(a^2 - b^2)(a + b \sec(c + dx))^{3/2}} - \frac{2(a^2(-B) + 4aAb - 3b^2B) \cot(c + dx)}{3d(a^2 - b^2)(a + b \sec(c + dx))^{3/2}}$$

[Out] $-2/3*(4*A*a*b-B*a^2-3*B*b^2)*\cot(d*x+c)*\text{EllipticE}((a+b*\sec(d*x+c))^{1/2}/(a+b)^{1/2}, ((a+b)/(a-b))^{1/2})*(b*(1-\sec(d*x+c))/(a+b))^{1/2}*(-b*(1+\sec(d*x+c))/(a-b))^{1/2}/(a-b)/b^2/(a+b)^{3/2}/d+2/3*(3*A*a-A*b+B*a-3*B*b)*\cot(d*x+c)*\text{EllipticF}((a+b*\sec(d*x+c))^{1/2}/(a+b)^{1/2}, ((a+b)/(a-b))^{1/2})*(b*(1-\sec(d*x+c))/(a+b))^{1/2}*(-b*(1+\sec(d*x+c))/(a-b))^{1/2}/(a-b)/b/(a+b)^{3/2}/d-2/3*(A*b-B*a)*\tan(d*x+c)/(a^2-b^2)/d/(a+b*\sec(d*x+c))^{3/2}-2/3*(4*A*a*b-B*a^2-3*B*b^2)*\tan(d*x+c)/(a^2-b^2)^2/d/(a+b*\sec(d*x+c))^{1/2}$

Rubi [A] time = 0.60, antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {4003, 4005, 3832, 4004}

$$\frac{2(a^2(-B) + 4aAb - 3b^2B) \tan(c + dx)}{3d(a^2 - b^2)^2 \sqrt{a + b \sec(c + dx)}} - \frac{2(Ab - aB) \tan(c + dx)}{3d(a^2 - b^2)(a + b \sec(c + dx))^{3/2}} - \frac{2(a^2(-B) + 4aAb - 3b^2B) \cot(c + dx)}{3d(a^2 - b^2)(a + b \sec(c + dx))^{3/2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(\text{Sec}[c + d*x]*(A + B*\text{Sec}[c + d*x]))/(a + b*\text{Sec}[c + d*x])^{5/2}, x]$

[Out] $(-2*(4*a*A*b - a^2*B - 3*b^2*B)*\text{Cot}[c + d*x]*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[a + b*\text{Sec}[c + d*x]]/\text{Sqrt}[a + b]], (a + b)/(a - b)]*\text{Sqrt}[(b*(1 - \text{Sec}[c + d*x]))/(a + b)]*\text{Sqrt}[-((b*(1 + \text{Sec}[c + d*x]))/(a - b))]/(3*(a - b)*b^2*(a + b)^{3/2}*d) + (2*(3*a*A - A*b + a*B - 3*b*B)*\text{Cot}[c + d*x]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[a + b*\text{Sec}[c + d*x]]/\text{Sqrt}[a + b]], (a + b)/(a - b)]*\text{Sqrt}[(b*(1 - \text{Sec}[c + d*x]))/(a + b)]*\text{Sqrt}[-((b*(1 + \text{Sec}[c + d*x]))/(a - b))]/(3*(a - b)*b*(a + b)^{3/2}*d) - (2*(A*b - a*B)*\text{Tan}[c + d*x]/(3*(a^2 - b^2)*d*(a + b*\text{Sec}[c + d*x])^{3/2}) - (2*(4*a*A*b - a^2*B - 3*b^2*B)*\text{Tan}[c + d*x]/(3*(a^2 - b^2)^2*d*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]))$

Rule 3832

$\text{Int}[\text{csc}[(e_.) + (f_.)*(x_)]/\text{Sqrt}[\text{csc}[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> \text{Simp}[(-2*\text{Rt}[a + b, 2]*\text{Sqrt}[(b*(1 - \text{Csc}[e + f*x]))/(a + b)]*\text{Sqrt}[-((b*(1 + \text{Csc}[e + f*x]))/(a - b))]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[a + b*\text{Csc}[e + f*x]]/\text{Rt}[a + b, 2]], (a + b)/(a - b))]/(b*f*\text{Cot}[e + f*x]), x] /;$ FreeQ[{a, b, e,

f}, x] && NeQ[a^2 - b^2, 0]

Rule 4003

```
Int[csc[(e_.) + (f_.)*(x_.)]*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := -Simp[((A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(m + 1)*(a^2 - b^2), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[(a*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)))/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] := Simp[(-2*(A*b - a*B)*Rt[a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A, 2]], (a*A + b*B)/(a*A - b*B)]/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)))/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] := Dist[A - B, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{5/2}} dx &= -\frac{2(Ab-aB)\tan(c+dx)}{3(a^2-b^2)d(a+b\sec(c+dx))^{3/2}} - \frac{2\int \frac{\sec(c+dx)\left(-\frac{3}{2}(aA-bB)+\frac{1}{2}(Ab-aB)\sec(c+dx)\right)}{(a+b\sec(c+dx))^{3/2}} dx}{3(a^2-b^2)} \\
&= -\frac{2(Ab-aB)\tan(c+dx)}{3(a^2-b^2)d(a+b\sec(c+dx))^{3/2}} - \frac{2(4aAb-a^2B-3b^2B)\tan(c+dx)}{3(a^2-b^2)^2 d\sqrt{a+b\sec(c+dx)}} \\
&= -\frac{2(Ab-aB)\tan(c+dx)}{3(a^2-b^2)d(a+b\sec(c+dx))^{3/2}} - \frac{2(4aAb-a^2B-3b^2B)\tan(c+dx)}{3(a^2-b^2)^2 d\sqrt{a+b\sec(c+dx)}} \\
&= -\frac{2(4aAb-a^2B-3b^2B)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)\middle|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sqrt{a+b\sec(c+dx)}}{a+b}}}{3(a-b)b^2(a+b)^{3/2}d}
\end{aligned}$$

Mathematica [A] time = 18.87, size = 603, normalized size = 1.71

$$\frac{2\sec^2(c+dx)\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)(a\cos(c+dx)+b)^2(A+B\sec(c+dx))\left((a^2B-4aAb+3b^2B)\cos\right)}}{3(a-b)b^2(a+b)^{3/2}d}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(5/2), x]

[Out] ((b + a*cos[c + d*x])^3*Sec[c + d*x]^2*(A + B*Sec[c + d*x])*((-2*(-4*a*A*b + a^2*B + 3*b^2*B)*Sin[c + d*x]))/(3*b*(-a^2 + b^2)^2) + (2*(A*b^2*Sin[c + d*x] - a*b*B*Sin[c + d*x]))/(3*a*(a^2 - b^2)*(b + a*cos[c + d*x])^2) + (2*(-5*a^2*A*b*Sin[c + d*x] + A*b^3*Sin[c + d*x] + 2*a^3*B*Sin[c + d*x] + 2*a*b^2*B*Sin[c + d*x]))/(3*a*(a^2 - b^2)^2*(b + a*cos[c + d*x])))/(d*(B + A*cos[c + d*x])*(a + b*Sec[c + d*x])^(5/2)) + (2*(b + a*cos[c + d*x])^2*Sec[c + d*x]^(3/2)*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(A + B*Sec[c + d*x])*(2*(a + b)*(-4*a*A*b + a^2*B + 3*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*b*(a + b)*(3*a*A + A*b - a*B - 3*b*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-4*a*A*b + a^2*B + 3*b^2*B)*Cos[c + d*x]*(b + a*cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*b*(a^2 - b^2)^2*d*(B + A*cos[c + d*x])*Sqrt[Sec[(c + d*x)/2]^2*(a + b*Sec[c + d*x])^(5/2))

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{(B \sec(dx+c)^2 + A \sec(dx+c)) \sqrt{b \sec(dx+c) + a}}{b^3 \sec(dx+c)^3 + 3ab^2 \sec(dx+c)^2 + 3a^2b \sec(dx+c) + a^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")

[Out] integral((B*sec(d*x + c)^2 + A*sec(d*x + c))*sqrt(b*sec(d*x + c) + a)/(b^3*sec(d*x + c)^3 + 3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx+c) + A) \sec(dx+c)}{(b \sec(dx+c) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)/(b*sec(d*x + c) + a)^(5/2), x)

maple [B] time = 1.98, size = 4213, normalized size = 11.93

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x)

[Out]
$$-1/3/d*4^{(1/2)}*(A*EllipticF((-1+\cos(d*x+c))/\sin(d*x+c),((a-b)/(a+b))^{(1/2)})*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{(1/2)}*\sin(d*x+c)*b^4+3*A*\cos(d*x+c)^2*EllipticF((-1+\cos(d*x+c))/\sin(d*x+c),((a-b)/(a+b))^{(1/2)})*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{(1/2)}*\sin(d*x+c)*a^3*b-5*A*\cos(d*x+c)^3*a^2*b^2+2*B*\cos(d*x+c)^3*a^3*b+2*B*\cos(d*x+c)^3*a*b^3+4*B*\cos(d*x+c)^2*a^2*b^2-6*B*\cos(d*x+c)^2*a*b^3-B*\cos(d*x+c)*a^2*b^2+4*B*\cos(d*x+c)*a*b^3-4*A*\cos(d*x+c)^2*a^3*b-4*A*\cos(d*x+c)^2*a*b^3-3*A*\cos(d*x+c)*a^2*b^2+4*A*\cos(d*x+c)*a*b^3+4*A*\sin(d*x+c)*EllipticF((-1+\cos(d*x+c))/\sin(d*x+c),((a-b)/(a+b))^{(1/2)})*\cos(d*x+c)^2*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{(1/2)}*a^2*b^2+A*\cos(d*x+c)^3*b^4+3*B*\cos(d*x+c)^2*b^4+B*\cos(d*x+c)^2*a^4-B*\cos(d*x+c)^3*a^4-2*B*\cos(d*x+c)^2*a^3*b+4*A*\cos(d*x+c)^3*a^3*b+8*A*\cos(d*x+c)^2*a^2*b^2-3*B*\cos(d*x+c)^3*a^2*b^2-3*B*EllipticF((-1+\cos(d*x+c))/\sin$$

$$\begin{aligned}
& *x+c)))^{(1/2)} * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)) / (a+b))^{(1/2)} * a^2*b^2+3*B*\sin \\
& (d*x+c)*\text{EllipticE}((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{(1/2)} * \cos(d*x+c) \\
&)^2 * (\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)} * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)) / (a+b) \\
&))^{(1/2)} * a*b^3-A*\cos(d*x+c)*b^4+2*B*\sin(d*x+c)*\cos(d*x+c)*(\cos(d*x+c)/(1+\cos \\
& (d*x+c)))^{(1/2)} * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)) / (a+b))^{(1/2)} * \text{EllipticE}((- \\
& 1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{(1/2)}) * a^3*b+4*B*a^2*b^2*(\cos(d*x+c) \\
& / (1+\cos(d*x+c)))^{(1/2)} * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)) / (a+b))^{(1/2)} * \sin(d* \\
& x+c)*\cos(d*x+c)*\text{EllipticE}((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{(1/2)})+6 \\
& *B*b^3*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)} * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)) / (\\
& a+b))^{(1/2)} * \sin(d*x+c)*\cos(d*x+c)*\text{EllipticE}((-1+\cos(d*x+c))/\sin(d*x+c), ((a- \\
& b)/(a+b))^{(1/2)}) * a-B*\sin(d*x+c)*\cos(d*x+c)*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2) \\
&) * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)) / (a+b))^{(1/2)} * \text{EllipticF}((-1+\cos(d*x+c))/s \\
& in(d*x+c), ((a-b)/(a+b))^{(1/2)}) * a^3*b-5*B*\sin(d*x+c)*\cos(d*x+c)*(\cos(d*x+c)/ \\
& (1+\cos(d*x+c)))^{(1/2)} * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)) / (a+b))^{(1/2)} * \text{Ellipti \\
& cF}((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{(1/2)}) * a^2*b^2-7*B*\sin(d*x+c)*c \\
& os(d*x+c)*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)} * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c) \\
&) / (a+b))^{(1/2)} * \text{EllipticF}((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{(1/2)}) * a \\
& b^3-4*A*a^3*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)} * ((b+a*\cos(d*x+c))/(1+\cos(d*x+ \\
& c)) / (a+b))^{(1/2)} * \sin(d*x+c)*\cos(d*x+c)*\text{EllipticE}((-1+\cos(d*x+c))/\sin(d*x+c) \\
& , ((a-b)/(a+b))^{(1/2)}) * b-8*A*\sin(d*x+c)*\cos(d*x+c)*(\cos(d*x+c)/(1+\cos(d*x+c) \\
&))^{(1/2)} * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)) / (a+b))^{(1/2)} * \text{EllipticE}((-1+\cos(d* \\
& x+c))/\sin(d*x+c), ((a-b)/(a+b))^{(1/2)}) * a^2*b^2-4*A*\sin(d*x+c)*\cos(d*x+c)*(\cos \\
& (d*x+c)/(1+\cos(d*x+c)))^{(1/2)} * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)) / (a+b))^{(1/2) \\
&) * \text{EllipticE}((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{(1/2)}) * a*b^3+3*A*a^3*(\\
& \cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)} * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)) / (a+b))^{(1 \\
& /2)} * \sin(d*x+c)*\cos(d*x+c)*\text{EllipticF}((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b) \\
&))^{(1/2)}) * b+7*A*a^2*b^2*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)} * ((b+a*\cos(d*x+c)) / \\
& (1+\cos(d*x+c)) / (a+b))^{(1/2)} * \sin(d*x+c)*\cos(d*x+c)*\text{EllipticF}((-1+\cos(d*x+c) \\
&) / \sin(d*x+c), ((a-b)/(a+b))^{(1/2)})+5*A*b^3*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)} * \\
& ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)) / (a+b))^{(1/2)} * \sin(d*x+c)*\cos(d*x+c)*\text{Ellipti \\
& cF}((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{(1/2)}) * a+B*\sin(d*x+c)*\text{EllipticE} \\
& ((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{(1/2)}) * \cos(d*x+c)^2 * (\cos(d*x+c) / (\\
& 1+\cos(d*x+c)))^{(1/2)} * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)) / (a+b))^{(1/2)} * a^4+A*si \\
& n(d*x+c)*\text{EllipticF}((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b))^{(1/2)}) * \cos(d*x+ \\
& c)*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)} * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)) / (a+b) \\
&))^{(1/2)} * b^4-3*B*\sin(d*x+c)*\text{EllipticF}((-1+\cos(d*x+c))/\sin(d*x+c), ((a-b)/(a+b) \\
&))^{(1/2)}) * \cos(d*x+c)*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)} * ((b+a*\cos(d*x+c)) / (1 \\
& +\cos(d*x+c)) / (a+b))^{(1/2)} * b^4 * ((b+a*\cos(d*x+c)) / \cos(d*x+c))^{(1/2)} / \sin(d*x+ \\
& c) / (b+a*\cos(d*x+c))^2 / (a-b)^2 / (a+b)^2 / b
\end{aligned}$$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\cos(c+dx) \left(a + \frac{b}{\cos(c+dx)}\right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/(cos(c + d*x)*(a + b/cos(c + d*x))^(5/2)),x)

[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)*(a + b/cos(c + d*x))^(5/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \sec(c + dx)}{(a + b \sec(c + dx))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(5/2),x)

[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)/(a + b*sec(c + d*x))**(5/2), x)

$$3.389 \quad \int \frac{A+B \sec(c+dx)}{(a+b \sec(c+dx))^{5/2}} dx$$

Optimal. Leaf size=495

$$\frac{2A\sqrt{a+b} \cot(c+dx) \sqrt{\frac{b(1-\sec(c+dx))}{a+b}} \sqrt{\frac{b(\sec(c+dx)+1)}{a-b}} \Pi\left(\frac{a+b}{a}; \sin^{-1}\left(\frac{\sqrt{a+b \sec(c+dx)}}{\sqrt{a+b}}\right) \middle| \frac{a+b}{a-b}\right)}{a^3 d} + \frac{2b(Ab - aB) \tan(c+dx)}{3ad(a^2 - b^2)(a + b \sec(c+dx))^{3/2}}$$

[Out] $\frac{2}{3} * (7 * A * a^2 * b - 3 * A * b^3 - 4 * B * a^3) * \cot(d * x + c) * \text{EllipticE}((a + b * \sec(d * x + c))^{1/2} / (a + b)^{1/2}, ((a + b) / (a - b))^{1/2}) * (b * (1 - \sec(d * x + c)) / (a + b))^{1/2} * (-b * (1 + \sec(d * x + c)) / (a - b))^{1/2} / a^2 / (a - b) / b / (a + b)^{3/2} / d - \frac{2}{3} * (6 * A * a^2 * b - A * a * b^2 - 3 * A * b^3 - 3 * B * a^3 + B * a^2 * b) * \cot(d * x + c) * \text{EllipticF}((a + b * \sec(d * x + c))^{1/2} / (a + b)^{1/2}, ((a + b) / (a - b))^{1/2}) * (b * (1 - \sec(d * x + c)) / (a + b))^{1/2} * (-b * (1 + \sec(d * x + c)) / (a - b))^{1/2} / a^2 / (a - b) / b / (a + b)^{3/2} / d - 2 * A * \cot(d * x + c) * \text{EllipticPi}((a + b * \sec(d * x + c))^{1/2} / (a + b)^{1/2}, (a + b) / a, ((a + b) / (a - b))^{1/2}) * (a + b)^{1/2} * (b * (1 - \sec(d * x + c)) / (a + b))^{1/2} * (-b * (1 + \sec(d * x + c)) / (a - b))^{1/2} / a^3 / d + \frac{2}{3} * b * (A * b - B * a) * \tan(d * x + c) / a / (a^2 - b^2) / d / (a + b * \sec(d * x + c))^{3/2} + \frac{2}{3} * b * (7 * A * a^2 * b - 3 * A * b^3 - 4 * B * a^3) * \tan(d * x + c) / a^2 / (a^2 - b^2)^2 / d / (a + b * \sec(d * x + c))^{1/2}$

Rubi [A] time = 0.77, antiderivative size = 495, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3923, 4060, 4058, 3921, 3784, 3832, 4004}

$$\frac{2b(7a^2Ab - 4a^3B - 3Ab^3) \tan(c+dx)}{3a^2d(a^2 - b^2)^2 \sqrt{a+b \sec(c+dx)}} + \frac{2b(Ab - aB) \tan(c+dx)}{3ad(a^2 - b^2)(a + b \sec(c+dx))^{3/2}} - \frac{2(6a^2Ab + a^2bB - 3a^3B - aAb^2 - 3Ab^3)}{3ad(a^2 - b^2)(a + b \sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sec[c + d*x])/(a + b*Sec[c + d*x])^(5/2), x]

[Out] $(2 * (7 * a^2 * A * b - 3 * A * b^3 - 4 * a^3 * B) * \text{Cot}[c + d * x] * \text{EllipticE}[\text{ArcSin}[\text{Sqrt}[a + b * \text{Sec}[c + d * x]] / \text{Sqrt}[a + b]], (a + b) / (a - b)] * \text{Sqrt}[(b * (1 - \text{Sec}[c + d * x])) / (a + b)] * \text{Sqrt}[-((b * (1 + \text{Sec}[c + d * x])) / (a - b))]) / (3 * a^2 * (a - b) * b * (a + b)^{3/2} * d) - (2 * (6 * a^2 * A * b - a * A * b^2 - 3 * A * b^3 - 3 * a^3 * B + a^2 * b * B) * \text{Cot}[c + d * x] * \text{EllipticF}[\text{ArcSin}[\text{Sqrt}[a + b * \text{Sec}[c + d * x]] / \text{Sqrt}[a + b]], (a + b) / (a - b)] * \text{Sqrt}[(b * (1 - \text{Sec}[c + d * x])) / (a + b)] * \text{Sqrt}[-((b * (1 + \text{Sec}[c + d * x])) / (a - b))]) / (3 * a^2 * (a - b) * b * (a + b)^{3/2} * d) - (2 * A * \text{Sqrt}[a + b] * \text{Cot}[c + d * x] * \text{EllipticPi}[(a + b) / a, \text{ArcSin}[\text{Sqrt}[a + b * \text{Sec}[c + d * x]] / \text{Sqrt}[a + b]], (a + b) / (a - b)] * \text{Sqrt}[(b * (1 - \text{Sec}[c + d * x])) / (a + b)] * \text{Sqrt}[-((b * (1 + \text{Sec}[c + d * x])) / (a - b))]) / (a^3 * d) + (2 * b * (A * b - a * B) * \text{Tan}[c + d * x]) / (3 * a * (a^2 - b^2) * d * (a + b * \text{Sec}[c + d * x])^{3/2}) + (2 * b * (7 * a^2 * A * b - 3 * A * b^3 - 4 * a^3 * B) * \text{Tan}[c + d * x]) / (3 * a^2 * (a^2 - b^2)^2 * d * \text{Sqrt}[a + b * \text{Sec}[c + d * x]])$

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_.)], x_Symbol] := Simp[(2*Rt[a
+ b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)]/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_S
ymbol] := Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)]/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_.)], x_Symbol] := Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3923

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] := Simp[(b*(b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f
*x])^(m + 1))/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)
), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[c*(a^2 - b^2)*(m + 1) - (a*(b*c -
a*d)*(m + 1))*Csc[e + f*x] + b*(b*c - a*d)*(m + 2)*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && Ne
Q[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] := Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
2]], (a*A + b*B)/(a*A - b*B)]/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
```

```

))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] := Int[(A + (B - C)
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
+ Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]

```

Rule 4060

```

Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)^(m_), x_Symbol] := Simp[((A*b^2 -
a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^
2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
+ 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
+ (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]

```

Rubi steps

$$\begin{aligned}
\int \frac{A + B \sec(c + dx)}{(a + b \sec(c + dx))^{5/2}} dx &= \frac{2b(Ab - aB) \tan(c + dx)}{3a(a^2 - b^2) d(a + b \sec(c + dx))^{3/2}} - \frac{2 \int \frac{-\frac{3}{2}A(a^2 - b^2) + \frac{3}{2}a(Ab - aB) \sec(c + dx) - \frac{1}{2}b(Ab - aB) \sec^2(c + dx)}{(a + b \sec(c + dx))^{3/2}} dx}{3a(a^2 - b^2)} \\
&= \frac{2b(Ab - aB) \tan(c + dx)}{3a(a^2 - b^2) d(a + b \sec(c + dx))^{3/2}} + \frac{2b(7a^2Ab - 3Ab^3 - 4a^3B) \tan(c + dx)}{3a^2(a^2 - b^2)^2 d \sqrt{a + b \sec(c + dx)}} + \frac{4 \int \frac{A + B \sec(c + dx)}{(a + b \sec(c + dx))^{5/2}} dx}{3a(a^2 - b^2)} \\
&= \frac{2b(Ab - aB) \tan(c + dx)}{3a(a^2 - b^2) d(a + b \sec(c + dx))^{3/2}} + \frac{2b(7a^2Ab - 3Ab^3 - 4a^3B) \tan(c + dx)}{3a^2(a^2 - b^2)^2 d \sqrt{a + b \sec(c + dx)}} + \frac{4 \int \frac{A + B \sec(c + dx)}{(a + b \sec(c + dx))^{5/2}} dx}{3a(a^2 - b^2)} \\
&= \frac{2(7a^2Ab - 3Ab^3 - 4a^3B) \cot(c + dx) E\left(\sin^{-1}\left(\frac{\sqrt{a + b \sec(c + dx)}}{\sqrt{a + b}}\right) \middle| \frac{a + b}{a - b}\right) \sqrt{\frac{b(1 - \sec(c + dx))}{a + b}}}{3a^2(a - b)b(a + b)^{3/2}d} \\
&= \frac{2(7a^2Ab - 3Ab^3 - 4a^3B) \cot(c + dx) E\left(\sin^{-1}\left(\frac{\sqrt{a + b \sec(c + dx)}}{\sqrt{a + b}}\right) \middle| \frac{a + b}{a - b}\right) \sqrt{\frac{b(1 - \sec(c + dx))}{a + b}}}{3a^2(a - b)b(a + b)^{3/2}d}
\end{aligned}$$

Mathematica [C] time = 16.81, size = 2083, normalized size = 4.21

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(A + B*Sec[c + d*x])/(a + b*Sec[c + d*x])^(5/2),x]

[Out]
$$\frac{\left((b + a \cos[c + dx])^3 \sec[c + dx]^2 (A + B \sec[c + dx]) \left((2(-7a^2Ab + 3A^2b^3 + 4a^3B) \sin[c + dx]) / (3a^2(a^2 - b^2)^2) - (2(A^2b^3 \sin[c + dx] - ab^2B \sin[c + dx])) / (3a^2(a^2 - b^2)(b + a \cos[c + dx])^2) - (2(-8a^2Ab^2 \sin[c + dx] + 4A^2b^4 \sin[c + dx] + 5a^3bB \sin[c + dx] - ab^3B \sin[c + dx])) / (3a^2(a^2 - b^2)^2(b + a \cos[c + dx])) \right) \right)}{(d(B + A \cos[c + dx])(a + b \sec[c + dx])^{5/2}) + (2(b + a \cos[c + dx])^{5/2} \sec[c + dx]^{3/2} (A + B \sec[c + dx]) \sqrt{(a + b - a \tan[(c + dx)/2]^2 + b \tan[(c + dx)/2]^2) / (1 + \tan[(c + dx)/2]^2)} + (7a^3Ab \sqrt{(-a + b)/(a + b)} \tan[(c + dx)/2] + 7a^2A^2b^2 \sqrt{(-a + b)/(a + b)} \tan[(c + dx)/2] - 3a^2Ab^3 \sqrt{(-a + b)/(a + b)} \tan[(c + dx)/2] - 3A^2b^4 \sqrt{(-a + b)/(a + b)} \tan[(c + dx)/2] - 4a^4 \sqrt{(-a + b)/(a + b)} B \tan[(c + dx)/2] - 4a^3bB \sqrt{(-a + b)/(a + b)} B \tan[(c + dx)/2] - 14a^3Ab \sqrt{(-a + b)/(a + b)} \tan[(c + dx)/2]^3 + 6a^2A^2b^3 \sqrt{(-a + b)/(a + b)} \tan[(c + dx)/2]^3 + 8a^4 \sqrt{(-a + b)/(a + b)} B \tan[(c + dx)/2]^3 + 7a^3Ab \sqrt{(-a + b)/(a + b)} \tan[(c + dx)/2]^5 - 7a^2A^2b^2 \sqrt{(-a + b)/(a + b)} \tan[(c + dx)/2]^5 - 3a^2A^2b^3 \sqrt{(-a + b)/(a + b)} \tan[(c + dx)/2]^5 + 3A^2b^4 \sqrt{(-a + b)/(a + b)} \tan[(c + dx)/2]^5 - 4a^4 \sqrt{(-a + b)/(a + b)} B \tan[(c + dx)/2]^5 + 4a^3bB \sqrt{(-a + b)/(a + b)} B \tan[(c + dx)/2]^5 - (6I)a^4A \operatorname{EllipticPi}[-(a + b)/(a - b)], I \operatorname{ArcSinh}[\sqrt{(-a + b)/(a + b)} \tan[(c + dx)/2]], (a + b)/(a - b) \sqrt{1 - \tan[(c + dx)/2]^2} \sqrt{(a + b - a \tan[(c + dx)/2]^2 + b \tan[(c + dx)/2]^2) / (a + b)} + (12I)a^2Ab^2 \operatorname{EllipticPi}[-(a + b)/(a - b)], I \operatorname{ArcSinh}[\sqrt{(-a + b)/(a + b)} \tan[(c + dx)/2]], (a + b)/(a - b) \sqrt{1 - \tan[(c + dx)/2]^2} \sqrt{(a + b - a \tan[(c + dx)/2]^2 + b \tan[(c + dx)/2]^2) / (a + b)} - (6I)A^2b^4 \operatorname{EllipticPi}[-(a + b)/(a - b)], I \operatorname{ArcSinh}[\sqrt{(-a + b)/(a + b)} \tan[(c + dx)/2]], (a + b)/(a - b) \sqrt{1 - \tan[(c + dx)/2]^2} \sqrt{(a + b - a \tan[(c + dx)/2]^2 + b \tan[(c + dx)/2]^2) / (a + b)} - (6I)a^4A \operatorname{EllipticPi}[-(a + b)/(a - b)], I \operatorname{ArcSinh}[\sqrt{(-a + b)/(a + b)} \tan[(c + dx)/2]], (a + b)/(a - b) \tan[(c + dx)/2]^2 \sqrt{1 - \tan[(c + dx)/2]^2} \sqrt{(a + b - a \tan[(c + dx)/2]^2 + b \tan[(c + dx)/2]^2) / (a + b)} + (12I)a^2Ab^2 \operatorname{EllipticPi}[-(a + b)/(a - b)], I \operatorname{ArcSinh}[\sqrt{(-a + b)/(a + b)} \tan[(c + dx)/2]], (a + b)/(a - b) \tan[(c + dx)/2]^2 \sqrt{1 - \tan[(c + dx)/2]^2} \sqrt{(a + b - a \tan[(c + dx)/2]^2 + b \tan[(c + dx)/2]^2) / (a + b)} - (6I)A^2b^4 \operatorname{EllipticPi}[-(a + b)/(a - b)], I \operatorname{ArcSinh}[\sqrt{(-a + b)/(a + b)} \tan[(c + dx)/2]], (a + b)/(a - b) \tan[(c + dx)/2]^2 \sqrt{1 - \tan[(c + dx)/2]^2} \sqrt{(a + b - a \tan[(c + dx)/2]^2 + b \tan[(c + dx)/2]^2) / (a + b)} + I(a - b)(-7a^2Ab + 3A^2b^3 + 4a^3B) \operatorname{EllipticE}[I \operatorname{ArcSinh}[\sqrt{(-a + b)/(a + b)} \tan[(c + dx)/2]], (a + b)/(a - b) \sqrt{1 - \tan[(c + dx)/2]^2} (1 + \tan[(c + dx)/2]^2) \sqrt{(a + b - a \tan[(c + dx)/2]^2 + b \tan[(c + dx)/2]^2) / (a + b)} + I(a - b)(-4a^2Ab^2 - 6A^2b^3 + 3a^3(A - B) + a^2b(9A + B)) \operatorname{EllipticF}[I \operatorname{ArcSinh}[\sqrt{(-a + b)/(a + b)} \tan[(c + dx)/2]], (a + b)/(a - b) \sqrt{1 - \tan[(c + dx)/2]^2} (1 + \tan[(c + dx)/2]^2) \sqrt{(a + b - a \tan[(c + dx)/2]^2 + b \tan[(c + dx)/2]^2) / (a + b)}$$

$$\left. \right) / (3a^2 \sqrt{(-a + b)/(a + b)} * (a^2 - b^2)^2 * d * (B + A \cos[c + dx]) * (a + b \sec[c + dx])^{5/2} * (-1 + \tan[(c + dx)/2])^2 * \sqrt{(1 + \tan[(c + dx)/2])^2 / (1 - \tan[(c + dx)/2])^2} * (a * (-1 + \tan[(c + dx)/2])^2 - b * (1 + \tan[(c + dx)/2])^2))$$

fricas [F] time = 22.37, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{(B \sec(dx + c) + A) \sqrt{b \sec(dx + c) + a}}{b^3 \sec(dx + c)^3 + 3ab^2 \sec(dx + c)^2 + 3a^2b \sec(dx + c) + a^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")

[Out] integral((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)/(b^3*sec(d*x + c)^3 + 3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(b \sec(dx + c) + a)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)/(b*sec(d*x + c) + a)^(5/2), x)

maple [B] time = 2.03, size = 5712, normalized size = 11.54

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x)

[Out] result too large to display

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(b \sec(dx + c) + a)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)/(b*sec(d*x + c) + a)^(5/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\left(a + \frac{b}{\cos(c+dx)}\right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/(a + b/cos(c + d*x))^(5/2), x)

[Out] int((A + B/cos(c + d*x))/(a + b/cos(c + d*x))^(5/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \sec(c + dx)}{(a + b \sec(c + dx))^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(5/2), x)

[Out] Integral((A + B*sec(c + d*x))/(a + b*sec(c + d*x))**(5/2), x)

$$3.390 \quad \int \frac{\cos(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^{5/2}} dx$$

Optimal. Leaf size=582

$$\frac{\sqrt{a+b}(5Ab-2aB) \cot(c+dx) \sqrt{\frac{b(1-\sec(c+dx))}{a+b}} \sqrt{\frac{b(\sec(c+dx)+1)}{a-b}} \Pi\left(\frac{a+b}{a}; \sin^{-1}\left(\frac{\sqrt{a+b \sec(c+dx)}}{\sqrt{a+b}}\right) \middle| \frac{a+b}{a-b}\right)}{a^4 d} + \frac{b(3a^2 A + 2abB)}{3a^2 d(a^2 - b^2)}$$

[Out] A*sin(d*x+c)/a/d/(a+b*sec(d*x+c))^(3/2)+1/3*(3*A*a^4-26*A*a^2*b^2+15*A*b^4+14*B*a^3*b-6*B*a*b^3)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/(a-b)/b/(a+b)^(3/2)/d-1/3*(15*A*b^3+a*b^2*(5*A-6*B)-3*a^3*(A-4*B)-a^2*b*(21*A+2*B))*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/(a^2-b^2)/d/(a+b)^(1/2)+(5*A*b-2*B*a)*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2), (a+b)/a, ((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a^4/d+1/3*b*(3*A*a^2-5*A*b^2+2*B*a*b)*tan(d*x+c)/a^2/(a^2-b^2)/d/(a+b*sec(d*x+c))^(3/2)+1/3*b*(3*A*a^4-26*A*a^2*b^2+15*A*b^4+14*B*a^3*b-6*B*a*b^3)*tan(d*x+c)/a^3/(a^2-b^2)^2/d/(a+b*sec(d*x+c))^(1/2)

Rubi [A] time = 1.21, antiderivative size = 582, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {4034, 4061, 4060, 4058, 3921, 3784, 3832, 4004}

$$\frac{b(-26a^2Ab^2 + 3a^4A + 14a^3bB - 6ab^3B + 15Ab^4) \tan(c+dx)}{3a^3d(a^2 - b^2)^2 \sqrt{a+b \sec(c+dx)}} + \frac{b(3a^2A + 2abB - 5Ab^2) \tan(c+dx)}{3a^2d(a^2 - b^2)(a+b \sec(c+dx))^{3/2}} - \frac{(-a^2b(21A + 2abB)) \tan(c+dx)}{3a^2d(a^2 - b^2)(a+b \sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(5/2), x]

[Out] ((3*a^4*A - 26*a^2*A*b^2 + 15*A*b^4 + 14*a^3*b*B - 6*a*b^3*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*a^3*(a - b)*b*(a + b)^(3/2)*d - ((15*A*b^3 + a*b^2*(5*A - 6*B) - 3*a^3*(A - 4*B) - a^2*b*(21*A + 2*B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*a^3*Sqrt[a + b]*(a^2 - b^2)*d) + (Sqrt[a + b]*(5*A*b - 2*a*B)*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a^4*d) + (A

$$\frac{\sin[c + dx]}{a d (a + b \sec[c + dx])^{3/2}} + \frac{b(3a^2 A - 5A b^2 + 2 a b B) \tan[c + dx]}{(3a^2 (a^2 - b^2) d (a + b \sec[c + dx])^{3/2}} + \frac{b(3a^4 A - 26a^2 A b^2 + 15A b^4 + 14a^3 b B - 6a b^3 B) \tan[c + dx]}{(3a^3 (a^2 - b^2)^2 d \sqrt{a + b \sec[c + dx]}}$$

Rule 3784

$$\text{Int}[1/\sqrt{\csc[c] + (d)(x)}(b) + (a)], x_Symbol] \rightarrow \text{Simp}[(2\text{Rt}[a + b, 2] \sqrt{(b(1 - \csc[c + dx]))/(a + b)} \sqrt{-((b(1 + \csc[c + dx]))/(a - b))}] \text{EllipticPi}[(a + b)/a, \text{ArcSin}[\sqrt{a + b \csc[c + dx]}/\text{Rt}[a + b, 2]], (a + b)/(a - b)]/(a d \cot[c + dx]), x] /; \text{FreeQ}\{a, b, c, d\}, x \&\& \text{NeQ}[a^2 - b^2, 0]$$

Rule 3832

$$\text{Int}[\csc[e] + (f)(x)]/\sqrt{\csc[e] + (f)(x)}(b) + (a)], x_Symbol] \rightarrow \text{Simp}[(-2\text{Rt}[a + b, 2] \sqrt{(b(1 - \csc[e + fx]))/(a + b)} \sqrt{-((b(1 + \csc[e + fx]))/(a - b))}] \text{EllipticF}[\text{ArcSin}[\sqrt{a + b \csc[e + fx]}/\text{Rt}[a + b, 2]], (a + b)/(a - b)]/(b f \cot[e + fx]), x] /; \text{FreeQ}\{a, b, e, f\}, x \&\& \text{NeQ}[a^2 - b^2, 0]$$

Rule 3921

$$\text{Int}[(\csc[e] + (f)(x))(d) + (c)]/\sqrt{\csc[e] + (f)(x)}(b) + (a)], x_Symbol] \rightarrow \text{Dist}[c, \text{Int}[1/\sqrt{a + b \csc[e + fx]}, x], x] + \text{Dist}[d, \text{Int}[\csc[e + fx]/\sqrt{a + b \csc[e + fx]}, x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x \&\& \text{NeQ}[b c - a d, 0] \&\& \text{NeQ}[a^2 - b^2, 0]$$

Rule 4004

$$\text{Int}[(\csc[e] + (f)(x))(c \csc[e] + (f)(x)(B) + (A))]/\sqrt{\csc[e] + (f)(x)}(b) + (a)], x_Symbol] \rightarrow \text{Simp}[(-2(A b - a B) \text{Rt}[a + (b B)/A, 2] \sqrt{(b(1 - \csc[e + fx]))/(a + b)} \sqrt{-((b(1 + \csc[e + fx]))/(a - b))}] \text{EllipticE}[\text{ArcSin}[\sqrt{a + b \csc[e + fx]}/\text{Rt}[a + (b B)/A, 2]], (a A + b B)/(a A - b B)]/(b^2 f \cot[e + fx]), x] /; \text{FreeQ}\{a, b, e, f, A, B\}, x \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{EqQ}[A^2 - B^2, 0]$$

Rule 4034

$$\text{Int}[(\csc[e] + (f)(x))(d)^n (\csc[e] + (f)(x))(b) + (a))^m (\csc[e] + (f)(x)(B) + (A)), x_Symbol] \rightarrow \text{Simp}[(A \cot[e + fx] (a + b \csc[e + fx])^{m+1} (d \csc[e + fx])^n / (a f^n), x] + \text{Dist}[1/(a d^n), \text{Int}[(a + b \csc[e + fx])^m (d \csc[e + fx])^{n+1} \text{Simp}[a B^n - A b (m + n + 1) + A a (n + 1) \csc[e + fx] + A b (m + n + 2) \csc[e + fx]^2, x], x], x] /; \text{FreeQ}\{a, b, d, e, f, A, B, m\}, x \&\& \text{NeQ}[A b - a B, 0]$$

&& NeQ[a^2 - b^2, 0] && LeQ[n, -1]

Rule 4058

Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.)))/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] :> Int[(A + (B - C)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0]

Rule 4060

Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m, x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]

Rule 4061

Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m, x_Symbol] :> Simp[((A*b^2 + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*b*(A + C)*(m + 1)*Csc[e + f*x] + (A*b^2 + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m] && LtQ[m, -1]

Rubi steps

$$\begin{aligned}
\int \frac{\cos(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{5/2}} dx &= \frac{A\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}} - \frac{\int \frac{\frac{1}{2}(5Ab-2aB)-\frac{3}{2}Ab\sec^2(c+dx)}{(a+b\sec(c+dx))^{5/2}} dx}{a} \\
&= \frac{A\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}} + \frac{b(3a^2A-5Ab^2+2abB)\tan(c+dx)}{3a^2(a^2-b^2)d(a+b\sec(c+dx))^{3/2}} + \frac{2\int}{3a^2(a^2-b^2)d(a+b\sec(c+dx))^{3/2}} \\
&= \frac{A\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}} + \frac{b(3a^2A-5Ab^2+2abB)\tan(c+dx)}{3a^2(a^2-b^2)d(a+b\sec(c+dx))^{3/2}} + \frac{b(3\int)}{3a^2(a^2-b^2)d(a+b\sec(c+dx))^{3/2}} \\
&= \frac{A\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}} + \frac{b(3a^2A-5Ab^2+2abB)\tan(c+dx)}{3a^2(a^2-b^2)d(a+b\sec(c+dx))^{3/2}} + \frac{b(3\int)}{3a^2(a^2-b^2)d(a+b\sec(c+dx))^{3/2}} \\
&= \frac{(3a^4A-26a^2Ab^2+15Ab^4+14a^3bB-6ab^3B)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a}}{\sqrt{a+b\sec(c+dx)}}\right)\right)}{3a^3(a-b)b(a+b)^{3/2}d} \\
&= \frac{(3a^4A-26a^2Ab^2+15Ab^4+14a^3bB-6ab^3B)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a}}{\sqrt{a+b\sec(c+dx)}}\right)\right)}{3a^3(a-b)b(a+b)^{3/2}d}
\end{aligned}$$

Mathematica [B] time = 21.99, size = 2366, normalized size = 4.07

Result too large to show

Warning: Unable to verify antiderivative.

```

[In] Integrate[(Cos[c + d*x]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(5/2), x]
[Out] ((b + a*cos[c + d*x])^3*sec[c + d*x]^3*((-2*b*(-10*a^2*A*b + 6*A*b^3 + 7*a^3*B - 3*a*b^2*B)*sin[c + d*x]))/(3*a^3*(-a^2 + b^2)^2) + (2*(A*b^4*sin[c + d*x] - a*b^3*B*sin[c + d*x]))/(3*a^3*(a^2 - b^2)*(b + a*cos[c + d*x])^2) + (2*(-11*a^2*A*b^3*sin[c + d*x] + 7*A*b^5*sin[c + d*x] + 8*a^3*b^2*B*sin[c + d*x] - 4*a*b^4*B*sin[c + d*x]))/(3*a^3*(a^2 - b^2)^2*(b + a*cos[c + d*x]))) / (d*(a + b*Sec[c + d*x])^(5/2)) - ((b + a*cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2)*sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]*(3*a^5*A*Tan[(c + d*x)/2] + 3*a^4*A*b*Tan[(c + d*x)/2] - 26*a^3*A*b^2*Tan[(c + d*x)/2] - 26*a^2*A*b^3*Tan[(c + d*x)/2] + 15*a*A*b^4*Tan[(c + d*x)/2] + 15*A*b^5*Tan[(c + d*x)/2] + 14*a^4*b*B*Tan[(c + d*x)/2] + 14*a^3*b^2*B*Tan[(c + d*x)/2] - 6*a^2*

```


fricas [F] time = 1.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B \cos(dx + c) \sec(dx + c) + A \cos(dx + c))\sqrt{b \sec(dx + c) + a}}{b^3 \sec(dx + c)^3 + 3ab^2 \sec(dx + c)^2 + 3a^2b \sec(dx + c) + a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")

[Out] integral((B*cos(d*x + c)*sec(d*x + c) + A*cos(d*x + c))*sqrt(b*sec(d*x + c) + a)/(b^3*sec(d*x + c)^3 + 3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \cos(dx + c)}{(b \sec(dx + c) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)/(b*sec(d*x + c) + a)^(5/2), x)

maple [B] time = 2.01, size = 8545, normalized size = 14.68

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x)

[Out] result too large to display

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \cos(dx + c)}{(b \sec(dx + c) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)/(b*sec(d*x + c) + a)^(5/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c + dx) \left(A + \frac{B}{\cos(c+dx)} \right)}{\left(a + \frac{b}{\cos(c+dx)} \right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x))^(5/2), x)

[Out] int((cos(c + d*x)*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x))^(5/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(5/2), x)

[Out] Timed out

$$3.391 \quad \int \frac{\cos^2(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^{5/2}} dx$$

Optimal. Leaf size=686

$$\frac{(7Ab - 4aB) \sin(c + dx) \sqrt{a + b} (4a^2A - 20abB + 35Ab^2) \cot(c + dx) \sqrt{\frac{b(1-\sec(c+dx))}{a+b}} \sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}} \Pi\left(\frac{a+b \sec(c+dx)}{a}\right)}{4a^2d(a + b \sec(c + dx))^{3/2} 4a^5d}$$

[Out] $-1/4*(7*A*b-4*B*a)*\sin(d*x+c)/a^2/d/(a+b*\sec(d*x+c))^{(3/2)}+1/2*A*\cos(d*x+c)*\sin(d*x+c)/a/d/(a+b*\sec(d*x+c))^{(3/2)}-1/12*(33*A*a^4*b-170*A*a^2*b^3+105*A*b^5-12*B*a^5+104*B*a^3*b^2-60*B*a*b^4)*\cot(d*x+c)*\text{EllipticE}((a+b*\sec(d*x+c))^{(1/2)}/(a+b)^{(1/2)},((a+b)/(a-b))^{(1/2)})*(b*(1-\sec(d*x+c))/(a+b))^{(1/2)}*(-b*(1+\sec(d*x+c))/(a-b))^{(1/2)}/a^4/(a-b)/b/(a+b)^{(3/2)}/d+1/12*(105*A*b^4+5*a*b^3*(7*A-12*B)+6*a^4*(A+2*B)-5*a^2*b^2*(27*A+4*B)-a^3*(27*A*b-84*B*b))*\cot(d*x+c)*\text{EllipticF}((a+b*\sec(d*x+c))^{(1/2)}/(a+b)^{(1/2)},((a+b)/(a-b))^{(1/2)})*(b*(1-\sec(d*x+c))/(a+b))^{(1/2)}*(-b*(1+\sec(d*x+c))/(a-b))^{(1/2)}/a^4/(a^2-b^2)/d/(a+b)^{(1/2)}-1/4*(4*A*a^2+35*A*b^2-20*B*a*b)*\cot(d*x+c)*\text{EllipticPi}((a+b*\sec(d*x+c))^{(1/2)}/(a+b)^{(1/2)},(a+b)/a,((a+b)/(a-b))^{(1/2)}*(a+b)^{(1/2)}*(b*(1-\sec(d*x+c))/(a+b))^{(1/2)}*(-b*(1+\sec(d*x+c))/(a-b))^{(1/2)}/a^5/d-1/12*b*(27*A*a^2*b-35*A*b^3-12*B*a^3+20*B*a*b^2)*\tan(d*x+c)/a^3/(a^2-b^2)/d/(a+b*\sec(d*x+c))^{(3/2)}-1/12*b*(33*A*a^4*b-170*A*a^2*b^3+105*A*b^5-12*B*a^5+104*B*a^3*b^2-60*B*a*b^4)*\tan(d*x+c)/a^4/(a^2-b^2)^2/d/(a+b*\sec(d*x+c))^{(1/2)}$

Rubi [A] time = 2.05, antiderivative size = 686, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {4034, 4104, 4060, 4058, 3921, 3784, 3832, 4004}

$$\frac{b(-170a^2Ab^3 + 33a^4Ab + 104a^3b^2B - 12a^5B - 60ab^4B + 105Ab^5) \tan(c + dx)}{12a^4d(a^2 - b^2)^2 \sqrt{a + b \sec(c + dx)}} - \frac{b(27a^2Ab - 12a^3B + 20ab^2B)}{12a^3d(a^2 - b^2)(a + b)}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(5/2), x]

[Out] $-((33*a^4*A*b - 170*a^2*A*b^3 + 105*A*b^5 - 12*a^5*B + 104*a^3*b^2*B - 60*a*b^4*B)*\text{Cot}[c + d*x]*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[a + b*\text{Sec}[c + d*x]]/\text{Sqrt}[a + b]],(a + b)/(a - b)]*\text{Sqrt}[(b*(1 - \text{Sec}[c + d*x]))/(a + b)]*\text{Sqrt}[-((b*(1 + \text{Sec}[c + d*x]))/(a - b))]/(12*a^4*(a - b)*b*(a + b)^{(3/2)*d} + ((105*A*b^4 + 5*a*b^3*(7*A - 12*B) + 6*a^4*(A + 2*B) - 5*a^2*b^2*(27*A + 4*B) - a^3*(27*A*b - 84*b*B))*\text{Cot}[c + d*x]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[a + b*\text{Sec}[c + d*x]]/\text{Sqrt}[a + b]],(a + b)/(a - b)]*\text{Sqrt}[(b*(1 - \text{Sec}[c + d*x]))/(a + b)]*\text{Sqrt}[-((b*(1 + \text{Sec}[c + d*x]))/(a - b))]/(12*a^4*\text{Sqrt}[a + b]*(a^2 - b^2)*d) - (\text{Sqrt}[a + b]$

$$\begin{aligned} & * (4*a^2*A + 35*A*b^2 - 20*a*b*B) * \text{Cot}[c + d*x] * \text{EllipticPi}[(a + b)/a, \text{ArcSin}[\text{Sqrt}[a + b*\text{Sec}[c + d*x]]/\text{Sqrt}[a + b]], (a + b)/(a - b)] * \text{Sqrt}[(b*(1 - \text{Sec}[c + d*x]))/(a + b)] * \text{Sqrt}[-((b*(1 + \text{Sec}[c + d*x]))/(a - b))]/(4*a^5*d) - ((7*A*b - 4*a*B) * \text{Sin}[c + d*x]) / (4*a^2*d*(a + b*\text{Sec}[c + d*x])^{3/2}) + (A*\text{Cos}[c + d*x] * \text{Sin}[c + d*x]) / (2*a*d*(a + b*\text{Sec}[c + d*x])^{3/2}) - (b*(27*a^2*A*b - 35*A*b^3 - 12*a^3*B + 20*a*b^2*B) * \text{Tan}[c + d*x]) / (12*a^3*(a^2 - b^2)*d*(a + b*\text{Sec}[c + d*x])^{3/2}) - (b*(33*a^4*A*b - 170*a^2*A*b^3 + 105*A*b^5 - 12*a^5*B + 104*a^3*b^2*B - 60*a*b^4*B) * \text{Tan}[c + d*x]) / (12*a^4*(a^2 - b^2)^2*d*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]) \end{aligned}$$

Rule 3784

$$\text{Int}[1/\text{Sqrt}[\text{csc}[(c_.) + (d_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] \rightarrow \text{Simp}[(2*\text{Rt}[a + b, 2] * \text{Sqrt}[(b*(1 - \text{Csc}[c + d*x]))/(a + b)] * \text{Sqrt}[-((b*(1 + \text{Csc}[c + d*x]))/(a - b))] * \text{EllipticPi}[(a + b)/a, \text{ArcSin}[\text{Sqrt}[a + b*\text{Csc}[c + d*x]]/\text{Rt}[a + b, 2]], (a + b)/(a - b)] / (a*d*\text{Cot}[c + d*x]), x] /; \text{FreeQ}\{a, b, c, d\}, x \&\& \text{NeQ}[a^2 - b^2, 0]$$

Rule 3832

$$\text{Int}[\text{csc}[(e_.) + (f_.)*(x_.)]/\text{Sqrt}[\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] \rightarrow \text{Simp}[(-2*\text{Rt}[a + b, 2] * \text{Sqrt}[(b*(1 - \text{Csc}[e + f*x]))/(a + b)] * \text{Sqrt}[-((b*(1 + \text{Csc}[e + f*x]))/(a - b))] * \text{EllipticF}[\text{ArcSin}[\text{Sqrt}[a + b*\text{Csc}[e + f*x]]/\text{Rt}[a + b, 2]], (a + b)/(a - b)] / (b*f*\text{Cot}[e + f*x]), x] /; \text{FreeQ}\{a, b, e, f\}, x \&\& \text{NeQ}[a^2 - b^2, 0]$$

Rule 3921

$$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))/\text{Sqrt}[\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] \rightarrow \text{Dist}[c, \text{Int}[1/\text{Sqrt}[a + b*\text{Csc}[e + f*x]], x], x] + \text{Dist}[d, \text{Int}[\text{Csc}[e + f*x]/\text{Sqrt}[a + b*\text{Csc}[e + f*x]], x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 - b^2, 0]$$

Rule 4004

$$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(\text{csc}[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)))/\text{Sqrt}[\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] \rightarrow \text{Simp}[(-2*(A*b - a*B) * \text{Rt}[a + (b*B)/A, 2] * \text{Sqrt}[(b*(1 - \text{Csc}[e + f*x]))/(a + b)] * \text{Sqrt}[-((b*(1 + \text{Csc}[e + f*x]))/(a - b))] * \text{EllipticE}[\text{ArcSin}[\text{Sqrt}[a + b*\text{Csc}[e + f*x]]/\text{Rt}[a + (b*B)/A, 2]], (a*A + b*B)/(a*A - b*B)] / (b^2*f*\text{Cot}[e + f*x]), x] /; \text{FreeQ}\{a, b, e, f, A, B\}, x \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{EqQ}[A^2 - B^2, 0]$$

Rule 4034

$$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(d_.))^{(n_.)} * (\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^{(m_.)} * (\text{csc}[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] \rightarrow \text{Simp}[(A*\text{Cot}[$$

```
e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*n), x] + Dis
t[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n
- A*b*(m + n + 1) + A*a*(n + 1)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x
]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0]
&& NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)])*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] := Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
+ Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4060

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)])*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_), x_Symbol] := Simp[((A*b^2 -
a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^
2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
+ 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
+ (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)])*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_.))^(m_), x_Symbol] := Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{5/2}} dx &= \frac{A\cos(c+dx)\sin(c+dx)}{2ad(a+b\sec(c+dx))^{3/2}} - \frac{\int \frac{\cos(c+dx)\left(\frac{1}{2}(7Ab-4aB)-aA\sec(c+dx)-\frac{5}{2}Ab\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{5/2}} dx}{2a} \\
&= -\frac{(7Ab-4aB)\sin(c+dx)}{4a^2d(a+b\sec(c+dx))^{3/2}} + \frac{A\cos(c+dx)\sin(c+dx)}{2ad(a+b\sec(c+dx))^{3/2}} + \frac{\int \frac{\frac{1}{4}(4a^2A+35Ab^2)}{(a+b\sec(c+dx))^{5/2}} dx}{2a} \\
&= -\frac{(7Ab-4aB)\sin(c+dx)}{4a^2d(a+b\sec(c+dx))^{3/2}} + \frac{A\cos(c+dx)\sin(c+dx)}{2ad(a+b\sec(c+dx))^{3/2}} - \frac{b(27a^2Ab-12a^3B)}{12a^3} \\
&= -\frac{(7Ab-4aB)\sin(c+dx)}{4a^2d(a+b\sec(c+dx))^{3/2}} + \frac{A\cos(c+dx)\sin(c+dx)}{2ad(a+b\sec(c+dx))^{3/2}} - \frac{b(27a^2Ab-12a^3B)}{12a^3} \\
&= -\frac{(7Ab-4aB)\sin(c+dx)}{4a^2d(a+b\sec(c+dx))^{3/2}} + \frac{A\cos(c+dx)\sin(c+dx)}{2ad(a+b\sec(c+dx))^{3/2}} - \frac{b(27a^2Ab-12a^3B)}{12a^3} \\
&= -\frac{(7Ab-4aB)\sin(c+dx)}{4a^2d(a+b\sec(c+dx))^{3/2}} + \frac{A\cos(c+dx)\sin(c+dx)}{2ad(a+b\sec(c+dx))^{3/2}} - \frac{b(27a^2Ab-12a^3B)}{12a^3} \\
&= -\frac{(33a^4Ab-170a^2Ab^3+105Ab^5-12a^5B+104a^3b^2B-60ab^4B)\cot(c)}{12a^4(a-b)b} \\
&= -\frac{(33a^4Ab-170a^2Ab^3+105Ab^5-12a^5B+104a^3b^2B-60ab^4B)\cot(c)}{12a^4(a-b)b}
\end{aligned}$$

Mathematica [A] time = 15.27, size = 821, normalized size = 1.20

$$\frac{(b+a\cos(c+dx))^3\sec^3(c+dx)\left(\frac{2(10Ba^3-13Aba^2-6b^2Ba+9Ab^3)\sin(c+dx)b^2}{3a^4(b^2-a^2)^2} - \frac{2(Ab^5\sin(c+dx)-ab^4B\sin(c+dx))}{3a^4(a^2-b^2)(b+a\cos(c+dx))^2} - \frac{2(10A\sin(c+dx))}{3a^4(a^2-b^2)(b+a\cos(c+dx))^2}\right)}{d(a+b\sec(c+dx))^{5/2}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(Cos[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(5/2), x]

[Out] ((b + a*Cos[c + d*x])^3*Sec[c + d*x]^3*((2*b^2*(-13*a^2*A*b + 9*A*b^3 + 10*a^3*B - 6*a*b^2*B)*Sin[c + d*x])/(3*a^4*(-a^2 + b^2)^2) - (2*(A*b^5*Sin[c +

$$d*x] - a*b^4*B*\sin[c + d*x]))/(3*a^4*(a^2 - b^2)*(b + a*\cos[c + d*x])^2) -$$

$$(2*(-14*a^2*A*b^4*\sin[c + d*x] + 10*A*b^6*\sin[c + d*x] + 11*a^3*b^3*B*\sin[$$

$$c + d*x] - 7*a*b^5*B*\sin[c + d*x]))/(3*a^4*(a^2 - b^2)^2*(b + a*\cos[c + d*x$$

$$])) + (A*\sin[2*(c + d*x)]/(4*a^3)))/(d*(a + b*\sec[c + d*x])^(5/2)) - ((b +$$

$$a*\cos[c + d*x])^2*\sec[c + d*x]*(-(a*(a + b)*(-33*a^4*A*b + 170*a^2*A*b^3 -$$

$$105*A*b^5 + 12*a^5*B - 104*a^3*b^2*B + 60*a*b^4*B)*\text{EllipticE}[\text{ArcSin}[\text{Tan}[(c$$

$$+ d*x)/2]], (a - b)/(a + b)]*\sec[(c + d*x)/2]^2*\sqrt{((b + a*\cos[c + d*x])$$

$$*\sec[(c + d*x)/2]^2)/(a + b)) + b*(a + b)*(105*A*b^5 + 6*a^5*(A + 2*B) - 3$$

$$0*a*b^4*(7*A + 2*B) + 4*a^3*b^2*(57*A + 10*B) - 3*a^4*b*(13*A + 48*B) + 2*a$$

$$^2*b^3*(-29*A + 60*B))*\text{EllipticF}[\text{ArcSin}[\text{Tan}[(c + d*x)/2]], (a - b)/(a + b)]$$

$$*\sec[(c + d*x)/2]^2*\sqrt{((b + a*\cos[c + d*x])*\sec[(c + d*x)/2]^2)/(a + b)}$$

$$+ 3*(a - b)^2*(a + b)^2*(4*a^2*A + 35*A*b^2 - 20*a*b*B)*((a - b)*\text{EllipticF}$$

$$[\text{ArcSin}[\text{Tan}[(c + d*x)/2]], (a - b)/(a + b)] - 2*a*\text{EllipticPi}[-1, \text{ArcSin}[\text{Tan}$$

$$[(c + d*x)/2]], (a - b)/(a + b)]*\sec[(c + d*x)/2]^2*\sqrt{((b + a*\cos[c + d$$

$$*x])*\sec[(c + d*x)/2]^2)/(a + b)} - a*(-33*a^4*A*b + 170*a^2*A*b^3 - 105*A*$$

$$b^5 + 12*a^5*B - 104*a^3*b^2*B + 60*a*b^4*B)*(b + a*\cos[c + d*x])*(\cos[c +$$

$$d*x]*\sec[(c + d*x)/2]^2)^(3/2)*\sec[c + d*x]*\tan[(c + d*x)/2]))/(12*a^5*(a^2$$

$$- b^2)^2*d*(\cos[c + d*x]*\sec[(c + d*x)/2]^2)^(3/2)*(a + b*\sec[c + d*x])^(5$$

$$/2))$$

fricas [F] time = 57.04, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(B \cos(dx + c)^2 \sec(dx + c) + A \cos(dx + c)^2\right) \sqrt{b \sec(dx + c) + a}}{b^3 \sec(dx + c)^3 + 3ab^2 \sec(dx + c)^2 + 3a^2b \sec(dx + c) + a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")

[Out] integral((B*cos(d*x + c)^2*sec(d*x + c) + A*cos(d*x + c)^2)*sqrt(b*sec(d*x + c) + a)/(b^3*sec(d*x + c)^3 + 3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \cos(dx + c)^2}{(b \sec(dx + c) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^2/(b*sec(d*x + c) + a)^(5/2), x)

maple [B] time = 2.40, size = 10322, normalized size = 15.05

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x)`

[Out] result too large to display

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \cos(dx + c)^2}{(b \sec(dx + c) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")`

[Out] `integrate((B*sec(d*x + c) + A)*cos(d*x + c)^2/(b*sec(d*x + c) + a)^(5/2), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c + dx)^2 \left(A + \frac{B}{\cos(c+dx)} \right)}{\left(a + \frac{b}{\cos(c+dx)} \right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((cos(c + d*x)^2*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x))^(5/2),x)`

[Out] `int((cos(c + d*x)^2*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x))^(5/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \cos^2(c + dx)}{(a + b \sec(c + dx))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)**2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(5/2),x)`

[Out] `Integral((A + B*sec(c + d*x))*cos(c + d*x)**2/(a + b*sec(c + d*x))**(5/2), x)`

$$3.392 \quad \int \frac{\sec(e+fx)(A+A \sec(e+fx))}{\sqrt{a+b \sec(e+fx)}} dx$$

Optimal. Leaf size=105

$$\frac{2A(a-b)\sqrt{a+b} \cot(e+fx) \sqrt{\frac{b(1-\sec(e+fx))}{a+b}} \sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}} E\left(\sin^{-1}\left(\frac{\sqrt{a+b \sec(e+fx)}}{\sqrt{a+b}}\right) \middle| \frac{a+b}{a-b}\right)}{b^2 f}$$

[Out] $-2*A*(a-b)*\cot(f*x+e)*\text{EllipticE}((a+b*\sec(f*x+e))^{(1/2)}/(a+b)^{(1/2)},((a+b)/(a-b))^{(1/2)}*(a+b)^{(1/2)}*(b*(1-\sec(f*x+e)))/(a+b))^{(1/2)}*(-b*(1+\sec(f*x+e)))/(a-b))^{(1/2)}/b^2/f$

Rubi [A] time = 0.08, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.032$, Rules used = {4004}

$$\frac{2A(a-b)\sqrt{a+b} \cot(e+fx) \sqrt{\frac{b(1-\sec(e+fx))}{a+b}} \sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}} E\left(\sin^{-1}\left(\frac{\sqrt{a+b \sec(e+fx)}}{\sqrt{a+b}}\right) \middle| \frac{a+b}{a-b}\right)}{b^2 f}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(\text{Sec}[e + f*x]*(A + A*\text{Sec}[e + f*x]))/\text{Sqrt}[a + b*\text{Sec}[e + f*x]],x]$

[Out] $(-2*A*(a-b)*\text{Sqrt}[a+b]*\text{Cot}[e+f*x]*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[a+b*\text{Sec}[e+f*x]]/\text{Sqrt}[a+b]],(a+b)/(a-b)]*\text{Sqrt}[(b*(1-\text{Sec}[e+f*x]))/(a+b)]*\text{Sqrt}[-(b*(1+\text{Sec}[e+f*x]))/(a-b))]/(b^2*f)$

Rule 4004

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(\text{csc}[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)))/\text{Sqrt}[\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] \rightarrow \text{Simp}[(-2*(A*b - a*B)*\text{Rt}[a + (b*B)/A, 2]*\text{Sqrt}[(b*(1 - \text{Csc}[e + f*x]))/(a + b)]*\text{Sqrt}[-(b*(1 + \text{Csc}[e + f*x]))/(a - b))]*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[a + b*\text{Csc}[e + f*x]]/\text{Rt}[a + (b*B)/A, 2]], (a*A + b*B)/(a*A - b*B)]/(b^2*f*\text{Cot}[e + f*x]), x] /; \text{FreeQ}\{a, b, e, f, A, B\}, x] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{EqQ}[A^2 - B^2, 0]$

Rubi steps

$$\int \frac{\sec(e+fx)(A+A \sec(e+fx))}{\sqrt{a+b \sec(e+fx)}} dx = -\frac{2A(a-b)\sqrt{a+b} \cot(e+fx) E\left(\sin^{-1}\left(\frac{\sqrt{a+b \sec(e+fx)}}{\sqrt{a+b}}\right) \middle| \frac{a+b}{a-b}\right) \sqrt{\frac{b(1-\sec(e+fx))}{a+b}}}{b^2 f}$$

Mathematica [B] time = 10.69, size = 248, normalized size = 2.36

$$A(\sec(e + fx) + 1) \left(2 \tan\left(\frac{1}{2}(e + fx)\right) (a \cos(e + fx) + b) + \frac{\left(\tan^2\left(\frac{1}{2}(e + fx)\right) - 1\right) \sqrt{\sec^2\left(\frac{1}{2}(e + fx)\right)} \sqrt{\cos^2\left(\frac{1}{2}(e + fx)\right)} \sec(e + fx) \left(\tan\left(\frac{1}{2}(e + fx)\right) + \frac{1}{\tan\left(\frac{1}{2}(e + fx)\right)}\right)}{bf\sqrt{a + b\sec(e + fx)}} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(Sec[e + f*x]*(A + A*Sec[e + f*x]))/Sqrt[a + b*Sec[e + f*x]],x]

[Out] (A*(1 + Sec[e + f*x])*(2*(b + a*Cos[e + f*x])*Tan[(e + f*x)/2] + (Sqrt[Sec[(e + f*x)/2]^2]*Sqrt[Cos[(e + f*x)/2]^2*Sec[e + f*x]]*((Sqrt[(a - b)/(a + b)]*(a + b)*Sqrt[(b + a*Cos[e + f*x])/((a + b)*(1 + Cos[e + f*x])])]*EllipticE[ArcSin[Sqrt[(a - b)/(a + b)]*Tan[(e + f*x)/2]], (a + b)/(a - b)]/Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])] + (b + a*Cos[e + f*x])*Tan[(e + f*x)/2])*(-1 + Tan[(e + f*x)/2]^2))/Sqrt[Sec[e + f*x]])/(b*f*Sqrt[a + b*Sec[e + f*x]])

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{A \sec(fx + e)^2 + A \sec(fx + e)}{\sqrt{b \sec(fx + e) + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(f*x+e)*(A+A*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm="fricas")

[Out] integral((A*sec(f*x + e)^2 + A*sec(f*x + e))/sqrt(b*sec(f*x + e) + a), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A \sec(fx + e) + A) \sec(fx + e)}{\sqrt{b \sec(fx + e) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(f*x+e)*(A+A*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm="giac")

[Out] integrate((A*sec(f*x + e) + A)*sec(f*x + e)/sqrt(b*sec(f*x + e) + a), x)

maple [B] time = 2.01, size = 642, normalized size = 6.11

$$2A \sqrt{\frac{b+a \cos(fx+e)}{\cos(fx+e)}} (1 + \cos(fx+e))^2 (-1 + \cos(fx+e))^2 \left(2 \cos(fx+e) \operatorname{EllipticF}\left(\frac{-1+\cos(fx+e)}{\sin(fx+e)}, \sqrt{\frac{a-b}{a+b}}\right) \sqrt{\frac{a-b}{a+b}} \right) \sqrt{\frac{a-b}{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(f*x+e)*(A+A*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x)

[Out] $-2*A/f*((b+a*\cos(f*x+e))/\cos(f*x+e))^{(1/2)}*(1+\cos(f*x+e))^{2*(-1+\cos(f*x+e))^{2*(2*\cos(f*x+e)*\operatorname{EllipticF}((-1+\cos(f*x+e))/\sin(f*x+e),((a-b)/(a+b))^{(1/2))*(\cos(f*x+e)/(1+\cos(f*x+e)))^{(1/2)}*((b+a*\cos(f*x+e))/(1+\cos(f*x+e)))/(a+b))^{(1/2)}*\sin(f*x+e)*b-\cos(f*x+e)*\operatorname{EllipticE}((-1+\cos(f*x+e))/\sin(f*x+e),((a-b)/(a+b))^{(1/2)})*(\cos(f*x+e)/(1+\cos(f*x+e)))^{(1/2)}*((b+a*\cos(f*x+e))/(1+\cos(f*x+e)))/(a+b))^{(1/2)}*\sin(f*x+e)*a-\cos(f*x+e)*\operatorname{EllipticE}((-1+\cos(f*x+e))/\sin(f*x+e),((a-b)/(a+b))^{(1/2)})*(\cos(f*x+e)/(1+\cos(f*x+e)))^{(1/2)}*((b+a*\cos(f*x+e))/(1+\cos(f*x+e)))/(a+b))^{(1/2)}*\sin(f*x+e)*b+2*(\cos(f*x+e)/(1+\cos(f*x+e)))^{(1/2)}*((b+a*\cos(f*x+e))/(1+\cos(f*x+e)))/(a+b))^{(1/2)}*\operatorname{EllipticF}((-1+\cos(f*x+e))/\sin(f*x+e),((a-b)/(a+b))^{(1/2)})*b*\sin(f*x+e)-\operatorname{EllipticE}((-1+\cos(f*x+e))/\sin(f*x+e),((a-b)/(a+b))^{(1/2)})*(\cos(f*x+e)/(1+\cos(f*x+e)))^{(1/2)}*((b+a*\cos(f*x+e))/(1+\cos(f*x+e)))/(a+b))^{(1/2)}*\sin(f*x+e)*a-\operatorname{EllipticE}((-1+\cos(f*x+e))/\sin(f*x+e),((a-b)/(a+b))^{(1/2)})*(\cos(f*x+e)/(1+\cos(f*x+e)))^{(1/2)}*((b+a*\cos(f*x+e))/(1+\cos(f*x+e)))/(a+b))^{(1/2)}*\sin(f*x+e)*b+a*\cos(f*x+e)^2-a*\cos(f*x+e)+b*\cos(f*x+e)-b)/\sin(f*x+e)^5/(b+a*\cos(f*x+e))/b$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A \sec(fx+e) + A) \sec(fx+e)}{\sqrt{b \sec(fx+e) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(f*x+e)*(A+A*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm="maxima")

[Out] integrate((A*sec(f*x + e) + A)*sec(f*x + e)/sqrt(b*sec(f*x + e) + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + \frac{A}{\cos(e+fx)}}{\cos(e+fx) \sqrt{a + \frac{b}{\cos(e+fx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + A/cos(e + f*x))/(cos(e + f*x)*(a + b/cos(e + f*x))^(1/2)),x)
```

```
[Out] int((A + A/cos(e + f*x))/(cos(e + f*x)*(a + b/cos(e + f*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$A \left(\int \frac{\sec(e + fx)}{\sqrt{a + b \sec(e + fx)}} dx + \int \frac{\sec^2(e + fx)}{\sqrt{a + b \sec(e + fx)}} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(A+A*sec(f*x+e))/(a+b*sec(f*x+e))**(1/2),x)
```

```
[Out] A*(Integral(sec(e + f*x)/sqrt(a + b*sec(e + f*x)), x) + Integral(sec(e + f*x)**2/sqrt(a + b*sec(e + f*x)), x))
```

$$3.393 \quad \int \frac{\sec(e+fx)(A-A\sec(e+fx))}{\sqrt{a+b\sec(e+fx)}} dx$$

Optimal. Leaf size=107

$$\frac{2A\sqrt{a-b}(a+b)\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{\frac{b(\sec(e+fx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a-b}}\right)\middle|\frac{a-b}{a+b}\right)}{b^2f}$$

[Out] $2*A*(a+b)*\cot(f*x+e)*\text{EllipticE}((a+b*\sec(f*x+e))^{(1/2)}/(a-b)^{(1/2)},((a-b)/(a+b))^{(1/2)})*(a-b)^{(1/2)}*(b*(1-\sec(f*x+e)))/(a+b)^{(1/2)}*(-b*(1+\sec(f*x+e)))/(a-b)^{(1/2)}/b^2/f$

Rubi [A] time = 0.08, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.031$, Rules used = {4004}

$$\frac{2A\sqrt{a-b}(a+b)\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{\frac{b(\sec(e+fx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a-b}}\right)\middle|\frac{a-b}{a+b}\right)}{b^2f}$$

Antiderivative was successfully verified.

[In] `Int[(Sec[e + f*x]*(A - A*Sec[e + f*x]))/Sqrt[a + b*Sec[e + f*x]],x]`

[Out] `(2*A*Sqrt[a - b]*(a + b)*Cot[e + f*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a - b]], (a - b)/(a + b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b))]/(b^2*f)`

Rule 4004

`Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A, 2]], (a*A + b*B)/(a*A - b*B)]/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]`

Rubi steps

$$\int \frac{\sec(e+fx)(A-A\sec(e+fx))}{\sqrt{a+b\sec(e+fx)}} dx = \frac{2A\sqrt{a-b}(a+b)\cot(e+fx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a-b}}\right)\middle|\frac{a-b}{a+b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}}{b^2f}$$

Mathematica [A] time = 7.94, size = 211, normalized size = 1.97

$$\frac{A(a+b)\sec^2\left(\frac{1}{2}(e+fx)\right)\sqrt{\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{(a+b)(\cos(e+fx)+1)}}\left(\sqrt{\frac{\cos(e+fx)}{\cos(e+fx)+1}}\sqrt{\sec(e+fx)+1}E\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)\right)\right)}{bf\left(\frac{1}{\cos(e+fx)+1}\right)^{3/2}\sqrt{a+b\sec(e+fx)}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(Sec[e + f*x]*(A - A*Sec[e + f*x]))/Sqrt[a + b*Sec[e + f*x]],x]

[Out] (A*(a + b)*Sqrt[(b + a*Cos[e + f*x])/((a + b)*(1 + Cos[e + f*x]))]*Sec[(e + f*x)/2]^2*Sqrt[Sec[e + f*x]]*(Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]*EllipticE[ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)]*Sqrt[1 + Sec[e + f*x]] - Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[(b + a*Cos[e + f*x])/((a + b)*(1 + Cos[e + f*x]))]*Sqrt[Sec[e + f*x]]*Sin[e + f*x]))/(b*f*((1 + Cos[e + f*x])^(-1))^((3/2)*Sqrt[a + b*Sec[e + f*x]))

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{A\sec(fx+e)^2 - A\sec(fx+e)}{\sqrt{b\sec(fx+e)+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(f*x+e)*(A-A*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm="fricas")

[Out] integral(-(A*sec(f*x + e)^2 - A*sec(f*x + e))/sqrt(b*sec(f*x + e) + a), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{(A\sec(fx+e) - A)\sec(fx+e)}{\sqrt{b\sec(fx+e)+a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(f*x+e)*(A-A*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm="giac")

[Out] integrate(-(A*sec(f*x + e) - A)*sec(f*x + e)/sqrt(b*sec(f*x + e) + a), x)

maple [B] time = 1.92, size = 457, normalized size = 4.27

$$2A \sqrt{\frac{b+a \cos(fx+e)}{\cos(fx+e)}} (1 + \cos(fx+e))^2 (-1 + \cos(fx+e))^2 \left(\cos(fx+e) \operatorname{EllipticE}\left(\frac{-1+\cos(fx+e)}{\sin(fx+e)}, \sqrt{\frac{a-b}{a+b}}\right) \sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sec(f*x+e)*(A-A*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x)`

[Out] $-2A/f*((b+a*\cos(f*x+e))/\cos(f*x+e))^{(1/2)}*(1+\cos(f*x+e))^2*(-1+\cos(f*x+e))^{(1/2)}*(\cos(f*x+e)*\operatorname{EllipticE}((-1+\cos(f*x+e))/\sin(f*x+e),((a-b)/(a+b))^{(1/2)})*(\cos(f*x+e)/(1+\cos(f*x+e)))^{(1/2)}*((b+a*\cos(f*x+e))/(1+\cos(f*x+e))^{(1/2)}*\sin(f*x+e)*a+\cos(f*x+e)*\operatorname{EllipticE}((-1+\cos(f*x+e))/\sin(f*x+e),((a-b)/(a+b))^{(1/2)})*(\cos(f*x+e)/(1+\cos(f*x+e)))^{(1/2)}*((b+a*\cos(f*x+e))/(1+\cos(f*x+e))^{(1/2)}*\sin(f*x+e)*b+\operatorname{EllipticE}((-1+\cos(f*x+e))/\sin(f*x+e),((a-b)/(a+b))^{(1/2)})*(\cos(f*x+e)/(1+\cos(f*x+e)))^{(1/2)}*((b+a*\cos(f*x+e))/(1+\cos(f*x+e))^{(1/2)}*\sin(f*x+e)*a+\operatorname{EllipticE}((-1+\cos(f*x+e))/\sin(f*x+e),((a-b)/(a+b))^{(1/2)})*(\cos(f*x+e)/(1+\cos(f*x+e)))^{(1/2)}*((b+a*\cos(f*x+e))/(1+\cos(f*x+e))^{(1/2)}*\sin(f*x+e)*b-a*\cos(f*x+e)^2+a*\cos(f*x+e)-b*\cos(f*x+e)+b)/\sin(f*x+e)^5/(b+a*\cos(f*x+e))/b$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(A \sec(fx+e) - A) \sec(fx+e)}{\sqrt{b \sec(fx+e) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(f*x+e)*(A-A*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm="maxima")`

[Out] `-integrate((A*sec(f*x + e) - A)*sec(f*x + e)/sqrt(b*sec(f*x + e) + a), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A - \frac{A}{\cos(e+fx)}}{\cos(e+fx) \sqrt{a + \frac{b}{\cos(e+fx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A - A/cos(e + f*x))/(cos(e + f*x)*(a + b/cos(e + f*x))^(1/2)),x)`

[Out] `int((A - A/cos(e + f*x))/(cos(e + f*x)*(a + b/cos(e + f*x))^(1/2)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-A \left(\int \left(-\frac{\sec(e + fx)}{\sqrt{a + b \sec(e + fx)}} \right) dx + \int \frac{\sec^2(e + fx)}{\sqrt{a + b \sec(e + fx)}} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(f*x+e)*(A-A*sec(f*x+e))/(a+b*sec(f*x+e))**(1/2), x)`

[Out] `-A*(Integral(-sec(e + f*x)/sqrt(a + b*sec(e + f*x)), x) + Integral(sec(e + f*x)**2/sqrt(a + b*sec(e + f*x)), x))`

$$3.394 \quad \int \sec^2(c + dx)(a + b \sec(c + dx))(A + B \sec(c + dx)) dx$$

Optimal. Leaf size=180

$$\frac{2(aB + Ab) \sin(c + dx) \sec^3(c + dx)}{3d} + \frac{2(5aA + 3bB) \sin(c + dx) \sqrt{\sec(c + dx)}}{5d} + \frac{2(aB + Ab) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)}}{3d}$$

[Out] $2/3*(A*b+B*a)*\sec(d*x+c)^(3/2)*\sin(d*x+c)/d+2/5*b*B*\sec(d*x+c)^(5/2)*\sin(d*x+c)/d+2/5*(5*A*a+3*B*b)*\sin(d*x+c)*\sec(d*x+c)^(1/2)/d-2/5*(5*A*a+3*B*b)*(c+\cos(1/2*d*x+1/2*c)^2)^(1/2)/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^(1/2))*\cos(d*x+c)^(1/2)*\sec(d*x+c)^(1/2)/d+2/3*(A*b+B*a)*(c+\cos(1/2*d*x+1/2*c)^2)^(1/2)/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^(1/2))*\cos(d*x+c)^(1/2)*\sec(d*x+c)^(1/2)/d$

Rubi [A] time = 0.18, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {3997, 3787, 3768, 3771, 2639, 2641}

$$\frac{2(aB + Ab) \sin(c + dx) \sec^3(c + dx)}{3d} + \frac{2(5aA + 3bB) \sin(c + dx) \sqrt{\sec(c + dx)}}{5d} + \frac{2(aB + Ab) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)}}{3d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sec}[c + d*x]^(3/2)*(a + b*\text{Sec}[c + d*x])*(A + B*\text{Sec}[c + d*x]), x]$

[Out] $(-2*(5*a*A + 3*b*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/(5*d) + (2*(A*b + a*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticF}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/(3*d) + (2*(5*a*A + 3*b*B)*\text{Sqrt}[\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(5*d) + (2*(A*b + a*B)*\text{Sec}[c + d*x]^(3/2)*\text{Sin}[c + d*x])/(3*d) + (2*b*B*\text{Sec}[c + d*x]^(5/2)*\text{Sin}[c + d*x])/(5*d)$

Rule 2639

$\text{Int}[\text{Sqrt}[\sin[(c_.) + (d_.)*(x_)]]], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticE}[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 2641

$\text{Int}[1/\text{Sqrt}[\sin[(c_.) + (d_.)*(x_)]]], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticF}[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 3768

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x] *(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3771

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 3997

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] := -Simp[(b*B*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && !LeQ[n, -1]

Rubi steps

$$\begin{aligned}
 \int \sec^{\frac{3}{2}}(c + dx)(a + b \sec(c + dx))(A + B \sec(c + dx)) dx &= \frac{2bB \sec^{\frac{5}{2}}(c + dx) \sin(c + dx)}{5d} + \frac{2}{5} \int \sec^{\frac{3}{2}}(c + dx) \left(\frac{1}{2} \right. \\
 &= \frac{2bB \sec^{\frac{5}{2}}(c + dx) \sin(c + dx)}{5d} + (Ab + aB) \int \sec^{\frac{5}{2}}(c + dx) \\
 &= \frac{2(5aA + 3bB) \sqrt{\sec(c + dx)} \sin(c + dx)}{5d} + \frac{2(Ab + aB)}{5d} \\
 &= \frac{2(5aA + 3bB) \sqrt{\sec(c + dx)} \sin(c + dx)}{5d} + \frac{2(Ab + aB)}{5d} \\
 &= -\frac{2(5aA + 3bB) \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{\sec(c + dx)}}{5d}
 \end{aligned}$$

Mathematica [A] time = 1.86, size = 132, normalized size = 0.73

$$\frac{\sec^{\frac{5}{2}}(c + dx) \left(20(aB + Ab) \cos^{\frac{5}{2}}(c + dx) F\left(\frac{1}{2}(c + dx) \middle| 2\right) - 12(5aA + 3bB) \cos^{\frac{5}{2}}(c + dx) E\left(\frac{1}{2}(c + dx) \middle| 2\right) + 2 \sin\left(\frac{1}{2}(c + dx)\right) \right)}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
[Out] (Sec[c + d*x]^(5/2)*(-12*(5*a*A + 3*b*B)*Cos[c + d*x]^(5/2)*EllipticE[(c + d*x)/2, 2] + 20*(A*b + a*B)*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] + 2*(15*(a*A + b*B) + 10*(A*b + a*B)*Cos[c + d*x] + 3*(5*a*A + 3*b*B)*Cos[2*(c + d*x)])*Sin[c + d*x])/(30*d)
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb \sec(dx + c)^3 + Aa \sec(dx + c) + (Ba + Ab) \sec(dx + c)^2\right) \sqrt{\sec(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="fricas")
[Out] integral((B*b*sec(d*x + c)^3 + A*a*sec(d*x + c) + (B*a + A*b)*sec(d*x + c)^2)*sqrt(sec(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(b \sec(dx + c) + a) \sec(dx + c)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="giac")
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)*sec(d*x + c)^(3/2), x)
```

maple [B] time = 12.44, size = 663, normalized size = 3.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x)
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a*A*(-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
```

$(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)})+2*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^2/\sin(1/2*d*x+1/2*c)^2/(2*\sin(1/2*d*x+1/2*c)^2-1)+2*(A*b+B*a)*(-1/6*\cos(1/2*d*x+1/2*c)*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/(-1/2+\cos(1/2*d*x+1/2*c)^2)^2+1/3*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)})-2/5*B*b/(8*\sin(1/2*d*x+1/2*c)^6-12*\sin(1/2*d*x+1/2*c)^4+6*\sin(1/2*d*x+1/2*c)^2-1)/\sin(1/2*d*x+1/2*c)^2*(12*EllipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)})*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*\sin(1/2*d*x+1/2*c)^4-24*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^6-12*EllipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)})*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*\sin(1/2*d*x+1/2*c)^2+24*\sin(1/2*d*x+1/2*c)^4*\cos(1/2*d*x+1/2*c)+3*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)})-8*\sin(1/2*d*x+1/2*c)^2*\cos(1/2*d*x+1/2*c))*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(b \sec(dx + c) + a) \sec(dx + c)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)*sec(d*x + c)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A + \frac{B}{\cos(c + dx)} \right) \left(a + \frac{b}{\cos(c + dx)} \right) \left(\frac{1}{\cos(c + dx)} \right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))*(a + b/cos(c + d*x))*(1/cos(c + d*x))^(3/2),x)

[Out] int((A + B/cos(c + d*x))*(a + b/cos(c + d*x))*(1/cos(c + d*x))^(3/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**(3/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x)

[Out] Timed out

$$3.395 \quad \int \sqrt{\sec(c+dx)} (a+b \sec(c+dx))(A+B \sec(c+dx)) dx$$

Optimal. Leaf size=143

$$\frac{2(aB + Ab) \sin(c + dx) \sqrt{\sec(c + dx)}}{d} + \frac{2(3aA + bB) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3d} - \frac{2(aB + Ab) \sqrt{\cos(c + dx)}}{3d}$$

[Out] $2/3*b*B*\sec(d*x+c)^{(3/2)}*\sin(d*x+c)/d+2*(A*b+B*a)*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/d-2*(A*b+B*a)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d+2/3*(3*A*a+B*b)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d$

Rubi [A] time = 0.15, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {3997, 3787, 3771, 2641, 3768, 2639}

$$\frac{2(aB + Ab) \sin(c + dx) \sqrt{\sec(c + dx)}}{d} + \frac{2(3aA + bB) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3d} - \frac{2(aB + Ab) \sqrt{\cos(c + dx)}}{3d}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]

[Out] $(-2*(A*b + a*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/d + (2*(3*a*A + b*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticF}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/(3*d) + (2*(A*b + a*B)*\text{Sqrt}[\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/d + (2*b*B*\text{Sec}[c + d*x]^{(3/2)}*\text{Sin}[c + d*x])/(3*d)$

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 3768

Int[(csc[(c_.) + (d_.)*(x_)])*(b_.)^{(n_.)}, x_Symbol] := -Simp[(b*Cos[c + d*x])* (b*Csc[c + d*x])^{(n-1)}/(d*(n-1)), x] + Dist[(b^2*(n-2))/(n-1), I

Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3771

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 3997

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] := -Simp[(b*B*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && !LeQ[n, -1]

Rubi steps

$$\begin{aligned}
 \int \sqrt{\sec(c + dx)} (a + b \sec(c + dx))(A + B \sec(c + dx)) dx &= \frac{2bB \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{3d} + \frac{2}{3} \int \sqrt{\sec(c + dx)} \left(\right. \\
 &= \frac{2bB \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{3d} + (Ab + aB) \int \sec^{\frac{3}{2}}(c + dx) dx \\
 &= \frac{2(Ab + aB) \sqrt{\sec(c + dx)} \sin(c + dx)}{d} + \frac{2bB \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{3d} \\
 &= \frac{2(3aA + bB) \sqrt{\cos(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{\sec(c + dx)}}{3d} \\
 &= -\frac{2(Ab + aB) \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{\sec(c + dx)}}{d}
 \end{aligned}$$

Mathematica [A] time = 0.89, size = 104, normalized size = 0.73

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left((3aA+bB)F\left(\frac{1}{2}(c+dx)\middle|2\right)-3(aB+Ab)E\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{\sin(c+dx)(3(aB+Ab)\cos(c+dx))}{\cos^2(c+dx)}\right)}{3d}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]

[Out] (2*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-3*(A*b + a*B)*EllipticE[(c + d*x)/2, 2] + (3*a*A + b*B)*EllipticF[(c + d*x)/2, 2] + ((b*B + 3*(A*b + a*B)*Cos[c + d*x])*Sin[c + d*x])/Cos[c + d*x]^(3/2)))/(3*d)

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb \sec(dx+c)^2 + Aa + (Ba + Ab) \sec(dx+c)\right)\sqrt{\sec(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] integral((B*b*sec(d*x + c)^2 + A*a + (B*a + A*b)*sec(d*x + c))*sqrt(sec(d*x + c)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx+c) + A)(b \sec(dx+c) + a)\sqrt{\sec(dx+c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)*sqrt(sec(d*x + c)), x)

maple [B] time = 9.67, size = 428, normalized size = 2.99

$$\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}{\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}} \left(\frac{2aA\sqrt{\frac{1-\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}} \right) + \frac{2(Ab+a^2)}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x)`

[Out]
$$-(-(-2*\cos(1/2*d*x+1/2*c)^2+1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*a*A*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)})+2*(A*b+B*a)*(-(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)})+2*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^2)/\sin(1/2*d*x+1/2*c)^2/(2*\sin(1/2*d*x+1/2*c)^2-1)+2*B*b*(-1/6*\cos(1/2*d*x+1/2*c)*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/(-1/2+\cos(1/2*d*x+1/2*c)^2)^2+1/3*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)})))/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(b \sec(dx + c) + a)\sqrt{\sec(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="maxima")`

[Out] `integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)*sqrt(sec(d*x + c)), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A + \frac{B}{\cos(c + dx)} \right) \left(a + \frac{b}{\cos(c + dx)} \right) \sqrt{\frac{1}{\cos(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B/cos(c + d*x))*(a + b/cos(c + d*x))*(1/cos(c + d*x))^(1/2),x)`

[Out] `int((A + B/cos(c + d*x))*(a + b/cos(c + d*x))*(1/cos(c + d*x))^(1/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \sec(c + dx))(a + b \sec(c + dx))\sqrt{\sec(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)**(1/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x)`

[Out] `Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))*sqrt(sec(c + d*x)), x)`

$$3.396 \quad \int \frac{(a+b \sec(c+dx))(A+B \sec(c+dx))}{\sqrt{\sec(c+dx)}} dx$$

Optimal. Leaf size=111

$$\frac{2(aB + Ab)\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{d} + \frac{2(aA - bB)\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{d}$$

[Out] $2*b*B*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/d+2*(A*a-B*b)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c),2^{(1/2)})*\cos(d*x+c)^{(1/2)}* \sec(d*x+c)^{(1/2)}/d+2*(A*b+B*a)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c),2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d$

Rubi [A] time = 0.14, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {3997, 3787, 3771, 2639, 2641}

$$\frac{2(aB + Ab)\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{d} + \frac{2(aA - bB)\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{d}$$

Antiderivative was successfully verified.

[In] Int[((a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]))/Sqrt[Sec[c + d*x]],x]

[Out] $(2*(a*A - b*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/d + (2*(A*b + a*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticF}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/d + (2*b*B*\text{Sqrt}[\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/d$

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 3771

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) +
(a_.)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) +
(a_.))*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> -Simp[(b*B*Cot[e
+ f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && !LeQ[n,
-1]
```

Rubi steps

$$\begin{aligned} \int \frac{(a + b \sec(c + dx))(A + B \sec(c + dx))}{\sqrt{\sec(c + dx)}} dx &= \frac{2bB\sqrt{\sec(c + dx)} \sin(c + dx)}{d} + 2 \int \frac{\frac{1}{2}(aA - bB) + \frac{1}{2}(Ab + aB) \sec(c + dx)}{\sqrt{\sec(c + dx)}} dx \\ &= \frac{2bB\sqrt{\sec(c + dx)} \sin(c + dx)}{d} + (Ab + aB) \int \sqrt{\sec(c + dx)} dx + \frac{1}{2} \int \frac{(aA - bB) + (Ab + aB) \sec(c + dx)}{\sqrt{\sec(c + dx)}} dx \\ &= \frac{2bB\sqrt{\sec(c + dx)} \sin(c + dx)}{d} + ((Ab + aB)\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} + (aA - bB) \sqrt{\sec(c + dx)}) \int \frac{1}{\sqrt{\sec(c + dx)}} dx \\ &= \frac{2(aA - bB)\sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{\sec(c + dx)} + 2(Ab + aB)\sqrt{\cos(c + dx)}}{d} \end{aligned}$$

Mathematica [A] time = 0.29, size = 84, normalized size = 0.76

$$\frac{2\sqrt{\sec(c + dx)} \left((aB + Ab)\sqrt{\cos(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right) + (aA - bB)\sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) + bB \sin(c + dx) \right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]))/Sqrt[Sec[c + d*x]],x]
[Out] (2*Sqrt[Sec[c + d*x]]*((a*A - b*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + (A*b + a*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + b*B*Sin[c + d*x]))/d
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{Bb \sec(dx+c)^2 + Aa + (Ba + Ab) \sec(dx+c)}{\sqrt{\sec(dx+c)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="fricas")

[Out] integral((B*b*sec(d*x + c)^2 + A*a + (B*a + A*b)*sec(d*x + c))/sqrt(sec(d*x + c)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx+c) + A)(b \sec(dx+c) + a)}{\sqrt{\sec(dx+c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)/sqrt(sec(d*x + c)), x)

maple [A] time = 4.59, size = 244, normalized size = 2.20

$$2 \left(A \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \text{EllipticF}\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{2 \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1} b - A \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \text{EllipticE}\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x)

[Out] -2*(A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*b-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a+B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a+B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*b-2*B*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx+c) + A)(b \sec(dx+c) + a)}{\sqrt{\sec(dx+c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)/sqrt(sec(d*x + c)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(a + \frac{b}{\cos(c+dx)}\right)}{\sqrt{\frac{1}{\cos(c+dx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x)))/(1/cos(c + d*x))^(1/2),x)

[Out] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x)))/(1/cos(c + d*x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx))(a + b \sec(c + dx))}{\sqrt{\sec(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)**(1/2),x)

[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))/sqrt(sec(c + d*x)), x)

$$3.397 \quad \int \frac{(a+b \sec(c+dx))(A+B \sec(c+dx))}{\sec^3(c+dx)} dx$$

Optimal. Leaf size=115

$$\frac{2(aA + 3bB)\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3d} + \frac{2(aB + Ab)\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{d}$$

[Out] $2/3*a*A*\sin(d*x+c)/d/\sec(d*x+c)^{(1/2)}+2*(A*b+B*a)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c),2^{(1/2)})*\cos(d*x+c)^{(1/2)}* \sec(d*x+c)^{(1/2)}/d+2/3*(A*a+3*B*b)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c),2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d$

Rubi [A] time = 0.15, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {3996, 3787, 3771, 2639, 2641}

$$\frac{2(aA + 3bB)\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3d} + \frac{2(aB + Ab)\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\frac{(a + b*\text{Sec}[c + d*x])*(A + B*\text{Sec}[c + d*x])}{\text{Sec}[c + d*x]^{(3/2)}}, x]$

[Out] $(2*(A*b + a*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/d + (2*(a*A + 3*b*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticF}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/(3*d) + (2*a*A*\text{Sin}[c + d*x])/(3*d*\text{Sqrt}[\text{Sec}[c + d*x]])$

Rule 2639

$\text{Int}[\text{Sqrt}[\sin[(c_.) + (d_.)*(x_)]]], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticE}[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 2641

$\text{Int}[1/\text{Sqrt}[\sin[(c_.) + (d_.)*(x_)]]], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticF}[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 3771

$\text{Int}[(\text{csc}[(c_.) + (d_.)*(x_)]*(b_.))^{(n_.)}, x_Symbol] \rightarrow \text{Dist}[(b*\text{Csc}[c + d*x])^{(n)}*\text{Sin}[c + d*x]^n, \text{Int}[1/\text{Sin}[c + d*x]^n, x], x] /; \text{FreeQ}\{b, c, d\}, x] \&\&$

EqQ[n^2, 1/4]

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) +
(a_.)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) +
(a_.))*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := Simp[(A*a*Cot[e +
f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
+ 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{(a + b \sec(c + dx))(A + B \sec(c + dx))}{\sec^{\frac{3}{2}}(c + dx)} dx = \frac{2aA \sin(c + dx)}{3d\sqrt{\sec(c + dx)}} - \frac{2}{3} \int \frac{-\frac{3}{2}(Ab + aB) - \frac{1}{2}(aA + 3bB) \sec(c + dx)}{\sqrt{\sec(c + dx)}} dx$$

$$= \frac{2aA \sin(c + dx)}{3d\sqrt{\sec(c + dx)}} - (-Ab - aB) \int \frac{1}{\sqrt{\sec(c + dx)}} dx - \frac{1}{3}(-aA - 3bB) \int \frac{\sec(c + dx)}{\sqrt{\sec(c + dx)}} dx$$

$$= \frac{2aA \sin(c + dx)}{3d\sqrt{\sec(c + dx)}} - ((-Ab - aB)\sqrt{\cos(c + dx)}\sqrt{\sec(c + dx)}) \int \frac{1}{\sqrt{\sec(c + dx)}} dx - \frac{1}{3}(-aA - 3bB) \int \frac{\sec(c + dx)}{\sqrt{\sec(c + dx)}} dx$$

$$= \frac{2(Ab + aB)\sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{\sec(c + dx)}}{d} + \frac{2(aA + 3bB)\sqrt{\sec(c + dx)}}{3d}$$

Mathematica [A] time = 0.26, size = 90, normalized size = 0.78

$$\frac{\sqrt{\sec(c + dx)} \left(2(aA + 3bB)\sqrt{\cos(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right) + 6(aB + Ab)\sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) + aA \sin(2(c + dx)) \right)}{3d}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(3/2),x]

```
[Out] (Sqrt[Sec[c + d*x]]*(6*(A*b + a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2,
2] + 2*(a*A + 3*b*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + a*A*S
in[2*(c + d*x)]))/(3*d)
```


fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{Bb \sec(dx+c)^2 + Aa + (Ba + Ab) \sec(dx+c)}{\sec(dx+c)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="fricas")

[Out] integral((B*b*sec(d*x + c)^2 + A*a + (B*a + A*b)*sec(d*x + c))/sec(d*x + c)^(3/2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx+c) + A)(b \sec(dx+c) + a)}{\sec(dx+c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)/sec(d*x + c)^(3/2), x)

maple [B] time = 4.17, size = 326, normalized size = 2.83

$$2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \left(4Aa \cos\left(\frac{dx}{2} + \frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + aA\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x)

[Out]
$$-2/3*((2*\cos(1/2*d*x+1/2*c)^2-1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(4*A*a*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^4+a*A*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})-3*A*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)})*b-2*A*a*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^2+3*B*b*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})-3*B*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)})*a)/(-2*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}$$

$$4 + \sin(1/2 * d * x + 1/2 * c)^2)^{1/2} / \sin(1/2 * d * x + 1/2 * c) / (2 * \cos(1/2 * d * x + 1/2 * c)^{2-1})^{1/2} / d$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(b \sec(dx + c) + a)}{\sec(dx + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)/sec(d*x + c)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(a + \frac{b}{\cos(c+dx)}\right)}{\left(\frac{1}{\cos(c+dx)}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x)))/(1/cos(c + d*x))^(3/2),x)

[Out] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x)))/(1/cos(c + d*x))^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx))(a + b \sec(c + dx))}{\sec^{\frac{3}{2}}(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)**(3/2),x)

[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))/sec(c + d*x)**(3/2), x)

$$3.398 \quad \int \frac{(a+b \sec(c+dx))(A+B \sec(c+dx))}{\sec^2(c+dx)} dx$$

Optimal. Leaf size=148

$$\frac{2(aB + Ab) \sin(c + dx)}{3d\sqrt{\sec(c + dx)}} + \frac{2(aB + Ab)\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3d} + \frac{2(3aA + 5bB)\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)}}{3d}$$

[Out] $2/5*a*A*\sin(d*x+c)/d/\sec(d*x+c)^{(3/2)}+2/3*(A*b+B*a)*\sin(d*x+c)/d/\sec(d*x+c)^{(1/2)}+2/5*(3*A*a+5*B*b)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d+2/3*(A*b+B*a)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d$

Rubi [A] time = 0.16, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {3996, 3787, 3769, 3771, 2641, 2639}

$$\frac{2(aB + Ab) \sin(c + dx)}{3d\sqrt{\sec(c + dx)}} + \frac{2(aB + Ab)\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3d} + \frac{2(3aA + 5bB)\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)}}{3d}$$

Antiderivative was successfully verified.

[In] Int[((a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(5/2), x]

[Out] $(2*(3*a*A + 5*b*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/(5*d) + (2*(A*b + a*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticF}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/(3*d) + (2*a*A*\text{Sin}[c + d*x])/(5*d*\text{Sec}[c + d*x]^{(3/2)}) + (2*(A*b + a*B)*\text{Sin}[c + d*x])/(3*d*\text{Sqrt}[\text{Sec}[c + d*x]])$

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 3769

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(b*Csc[c + d*x])^(n + 1))/(b*d^n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c + d*x])^(n + 1), x]]

$d*x])^{(n + 2), x], x] /; FreeQ[\{b, c, d\}, x] \&\& LtQ[n, -1] \&\& IntegerQ[2*n]$
 $]$

Rule 3771

$Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.))^{(n_.)}, x_Symbol] :> Dist[(b*Csc[c + d*x]$
 $)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[\{b, c, d\}, x] \&\&$
 $EqQ[n^2, 1/4]$

Rule 3787

$Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^{(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) +$
 $(a_.)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[$
 $(d*Csc[e + f*x])^{(n + 1)}, x], x] /; FreeQ[\{a, b, d, e, f, n\}, x]$

Rule 3996

$Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^{(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) +$
 $(a_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> Simp[(A*a*Cot[e +$
 $f*x]*(d*Csc[e + f*x])^n/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^{(n$
 $+ 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x] /$
 $; FreeQ[\{a, b, d, e, f, A, B\}, x] \&\& NeQ[A*b - a*B, 0] \&\& LeQ[n, -1]$

Rubi steps

$$\int \frac{(a + b \sec(c + dx))(A + B \sec(c + dx))}{\sec^{\frac{5}{2}}(c + dx)} dx = \frac{2aA \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} - \frac{2}{5} \int \frac{-\frac{5}{2}(Ab + aB) - \frac{1}{2}(3aA + 5bB) \sec(c + dx)}{\sec^{\frac{3}{2}}(c + dx)}$$

$$= \frac{2aA \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} - (-Ab - aB) \int \frac{1}{\sec^{\frac{3}{2}}(c + dx)} dx - \frac{1}{5}(-3aA - 5bB) \int \frac{\sec(c + dx)}{\sec^{\frac{3}{2}}(c + dx)}$$

$$= \frac{2aA \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} + \frac{2(Ab + aB) \sin(c + dx)}{3d \sqrt{\sec(c + dx)}} - \frac{1}{3}(-Ab - aB) \int \sqrt{\sec(c + dx)}$$

$$= \frac{2(3aA + 5bB) \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{\sec(c + dx)}}{5d} + \frac{2aA}{5d}$$

$$= \frac{2(3aA + 5bB) \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{\sec(c + dx)}}{5d} + \frac{2(Ab + aB) \sin(c + dx)}{3d \sqrt{\sec(c + dx)}} + \frac{2aA \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)}$$

Mathematica [A] time = 0.69, size = 108, normalized size = 0.73

$$\frac{\sqrt{\sec(c+dx)} \left(\sin(2(c+dx))(3aA \cos(c+dx) + 5aB + 5Ab) + 10(aB + Ab)\sqrt{\cos(c+dx)} F\left(\frac{1}{2}(c+dx)\middle|2\right) + 6\right)}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(5/2), x]
[Out] (Sqrt[Sec[c + d*x]]*(6*(3*a*A + 5*b*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 10*(A*b + a*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (5*A*b + 5*a*B + 3*a*A*Cos[c + d*x])*Sin[2*(c + d*x)]))/(15*d)
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{Bb \sec(dx+c)^2 + Aa + (Ba + Ab) \sec(dx+c)}{\sec(dx+c)^{\frac{5}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2), x, algorithm="fricas")
```

```
[Out] integral((B*b*sec(d*x + c)^2 + A*a + (B*a + A*b)*sec(d*x + c))/sec(d*x + c)^(5/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx+c) + A)(b \sec(dx+c) + a)}{\sec(dx+c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2), x, algorithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)/sec(d*x + c)^(5/2), x)
```

maple [B] time = 4.34, size = 371, normalized size = 2.51

$$2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \left(-24Aa \cos\left(\frac{dx}{2} + \frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + (24aA + 20Ab + 20aB)\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + (24aA + 20Ab + 20aB)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 6\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x)`

[Out]
$$-2/15*((2*\cos(1/2*d*x+1/2*c)^2-1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-24*A*a*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^6+(24*A*a+20*A*b+20*B*a)*\sin(1/2*d*x+1/2*c)^4*\cos(1/2*d*x+1/2*c)+(-6*A*a-10*A*b-10*B*a)*\sin(1/2*d*x+1/2*c)^2*\cos(1/2*d*x+1/2*c)+5*A*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*b-9*A*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)})*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*a+5*B*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*a-15*B*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)})*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*b)/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(b \sec(dx + c) + a)}{\sec(dx + c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="maxima")`

[Out] `integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)/sec(d*x + c)^(5/2), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(a + \frac{b}{\cos(c+dx)}\right)}{\left(\frac{1}{\cos(c+dx)}\right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x)))/(1/cos(c + d*x))^(5/2),x)`

[Out] `int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x)))/(1/cos(c + d*x))^(5/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx))(a + b \sec(c + dx))}{\sec^{\frac{5}{2}}(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)**(5/2),x)

[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))/sec(c + d*x)**(5/2), x)

$$3.399 \quad \int \frac{(a+b \sec(c+dx))(A+B \sec(c+dx))}{7 \sec^2(c+dx)} dx$$

Optimal. Leaf size=180

$$\frac{2(aB + Ab) \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} + \frac{2(5aA + 7bB) \sin(c + dx)}{21d \sqrt{\sec(c + dx)}} + \frac{2(5aA + 7bB) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{21d}$$

[Out] $2/7*a*A*\sin(d*x+c)/d/\sec(d*x+c)^{(5/2)}+2/5*(A*b+B*a)*\sin(d*x+c)/d/\sec(d*x+c)^{(3/2)}+2/21*(5*A*a+7*B*b)*\sin(d*x+c)/d/\sec(d*x+c)^{(1/2)}+6/5*(A*b+B*a)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d+2/21*(5*A*a+7*B*b)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d$

Rubi [A] time = 0.18, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {3996, 3787, 3769, 3771, 2639, 2641}

$$\frac{2(aB + Ab) \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} + \frac{2(5aA + 7bB) \sin(c + dx)}{21d \sqrt{\sec(c + dx)}} + \frac{2(5aA + 7bB) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{21d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + b*\text{Sec}[c + d*x])*(A + B*\text{Sec}[c + d*x])]/\text{Sec}[c + d*x]^{(7/2)}, x]$

[Out] $(6*(A*b + a*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/(5*d) + (2*(5*a*A + 7*b*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticF}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/(21*d) + (2*a*A*\text{Sin}[c + d*x])/(7*d*\text{Sec}[c + d*x]^{(5/2)}) + (2*(A*b + a*B)*\text{Sin}[c + d*x])/(5*d*\text{Sec}[c + d*x]^{(3/2)}) + (2*(5*a*A + 7*b*B)*\text{Sin}[c + d*x])/(21*d*\text{Sqrt}[\text{Sec}[c + d*x]])$

Rule 2639

$\text{Int}[\text{Sqrt}[\sin[(c_.) + (d_.)*(x_.)]], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticE}[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 2641

$\text{Int}[1/\text{Sqrt}[\sin[(c_.) + (d_.)*(x_.)]], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticF}[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 3769


```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x]
)^(n)*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] := Simp[(A*a*Cot[e +
f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
+ 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + b \sec(c + dx))(A + B \sec(c + dx))}{\sec^{\frac{7}{2}}(c + dx)} dx &= \frac{2aA \sin(c + dx)}{7d \sec^{\frac{5}{2}}(c + dx)} - \frac{2}{7} \int \frac{-\frac{7}{2}(Ab + aB) - \frac{1}{2}(5aA + 7bB) \sec(c + dx)}{\sec^{\frac{5}{2}}(c + dx)} dx \\
&= \frac{2aA \sin(c + dx)}{7d \sec^{\frac{5}{2}}(c + dx)} - (-Ab - aB) \int \frac{1}{\sec^{\frac{5}{2}}(c + dx)} dx - \frac{1}{7}(-5aA - 7bB) \int \frac{\sec(c + dx)}{\sec^{\frac{5}{2}}(c + dx)} dx \\
&= \frac{2aA \sin(c + dx)}{7d \sec^{\frac{5}{2}}(c + dx)} + \frac{2(Ab + aB) \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} + \frac{2(5aA + 7bB) \sin(c + dx)}{21d \sqrt{\sec(c + dx)}} \\
&= \frac{2aA \sin(c + dx)}{7d \sec^{\frac{5}{2}}(c + dx)} + \frac{2(Ab + aB) \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} + \frac{2(5aA + 7bB) \sin(c + dx)}{21d \sqrt{\sec(c + dx)}} \\
&= \frac{6(Ab + aB) \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{\sec(c + dx)}}{5d} + \frac{2(5aA + 7bB) \sin(c + dx)}{21d}
\end{aligned}$$

Mathematica [A] time = 1.08, size = 125, normalized size = 0.69

$$\frac{\sqrt{\sec(c + dx)} \left(\sin(2(c + dx))(42(aB + Ab) \cos(c + dx) + 15aA \cos(2(c + dx)) + 65aA + 70bB) + 20(5aA + 7bB) \right)}{210d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(7/2),x]
[Out] (Sqrt[Sec[c + d*x]]*(252*(A*b + a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 20*(5*a*A + 7*b*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (65*a*A + 70*b*B + 42*(A*b + a*B)*Cos[c + d*x] + 15*a*A*Cos[2*(c + d*x)])*Sin[2*(c + d*x)])/(210*d)
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{Bb \sec(dx + c)^2 + Aa + (Ba + Ab) \sec(dx + c)}{\sec(dx + c)^{\frac{7}{2}}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((B*b*sec(d*x + c)^2 + A*a + (B*a + A*b)*sec(d*x + c))/sec(d*x + c)^(7/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(b \sec(dx + c) + a)}{\sec(dx + c)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)/sec(d*x + c)^(7/2), x)
```

maple [A] time = 4.54, size = 413, normalized size = 2.29

$$2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \left(240Aa \cos\left(\frac{dx}{2} + \frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + (-360aA - 168Ab - 168aB)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x)`

[Out]
$$-2/105*((2*\cos(1/2*d*x+1/2*c)^2-1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(240*A*a*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^8+(-360*A*a-168*A*b-168*B*a)*\sin(1/2*d*x+1/2*c)^6*\cos(1/2*d*x+1/2*c)+(280*A*a+168*A*b+168*B*a+140*B*b)*\sin(1/2*d*x+1/2*c)^4*\cos(1/2*d*x+1/2*c)+(-80*A*a-42*A*b-42*B*a-70*B*b)*\sin(1/2*d*x+1/2*c)^2*\cos(1/2*d*x+1/2*c)+25*a*A*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})-63*A*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)})*b+35*B*b*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})-63*B*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)})*a)/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(b \sec(dx + c) + a)}{\sec(dx + c)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm="maxima")`

[Out] `integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)/sec(d*x + c)^(7/2), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(a + \frac{b}{\cos(c+dx)}\right)}{\left(\frac{1}{\cos(c+dx)}\right)^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x)))/(1/cos(c + d*x))^(7/2),x)`

[Out] `int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x)))/(1/cos(c + d*x))^(7/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx))(a + b \sec(c + dx))}{\sec^{\frac{7}{2}}(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)**(7/2),x)`

[Out] `Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))/sec(c + d*x)**(7/2), x)`

$$3.400 \quad \int \sec^2(c+dx)(a+b \sec(c+dx))^2(A+B \sec(c+dx)) dx$$

Optimal. Leaf size=263

$$\frac{2(5a^2A + 6abB + 3Ab^2) \sin(c+dx) \sqrt{\sec(c+dx)}}{5d} - \frac{2(5a^2A + 6abB + 3Ab^2) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} E\left(\frac{1}{2}(c+dx)\right)}{5d}$$

[Out] $2/21*(5*b^2*B+7*a*(2*A*b+B*a))*\sec(d*x+c)^{(3/2)}*\sin(d*x+c)/d+2/35*b*(7*A*b+9*B*a)*\sec(d*x+c)^{(5/2)}*\sin(d*x+c)/d+2/7*b*B*\sec(d*x+c)^{(5/2)}*(a+b*\sec(d*x+c))*\sin(d*x+c)/d+2/5*(5*A*a^2+3*A*b^2+6*B*a*b)*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/d-2/5*(5*A*a^2+3*A*b^2+6*B*a*b)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d+2/21*(5*b^2*B+7*a*(2*A*b+B*a))*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d$

Rubi [A] time = 0.37, antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {4026, 4047, 3768, 3771, 2641, 4046, 2639}

$$\frac{2(5a^2A + 6abB + 3Ab^2) \sin(c+dx) \sqrt{\sec(c+dx)}}{5d} - \frac{2(5a^2A + 6abB + 3Ab^2) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} E\left(\frac{1}{2}(c+dx)\right)}{5d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sec}[c + d*x]^{(3/2)}*(a + b*\text{Sec}[c + d*x])^2*(A + B*\text{Sec}[c + d*x]), x]$

[Out] $(-2*(5*a^2*A + 3*A*b^2 + 6*a*b*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/(5*d) + (2*(5*b^2*B + 7*a*(2*A*b + a*B))*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticF}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/(21*d) + (2*(5*a^2*A + 3*A*b^2 + 6*a*b*B)*\text{Sqrt}[\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(5*d) + (2*(5*b^2*B + 7*a*(2*A*b + a*B))*\text{Sec}[c + d*x]^{(3/2)}*\text{Sin}[c + d*x])/(21*d) + (2*b*(7*A*b + 9*a*B)*\text{Sec}[c + d*x]^{(5/2)}*\text{Sin}[c + d*x])/(35*d) + (2*b*B*\text{Sec}[c + d*x]^{(5/2)}*(a + b*\text{Sec}[c + d*x])*\text{Sin}[c + d*x])/(7*d)$

Rule 2639

$\text{Int}[\text{Sqrt}[\sin[(c_.) + (d_.)*(x_.)]], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticE}[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_.)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 3768

Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.))^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3771

Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 4026

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := -Simp[(b*B*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x] + Dist[1/(m + n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*Simp[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B)*(m + n) + b^2*B*(m + n - 1))*Csc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && !(IGtQ[n, 1] && !IntegerQ[m])

Rule 4046

Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]^2*(C_.) + (A_.)), x_Symbol] := -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]

Rule 4047

Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.)), x_Symbol] := Dist[B/b, Int[(b*Csc[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2), x] /; FreeQ[{b, e, f, A, B, C, m}, x]

Rubi steps

$$\begin{aligned}
\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2(A+B\sec(c+dx))dx &= \frac{2bB\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{7d} + \\
&= \frac{2bB\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{7d} + \\
&= \frac{2(5b^2B+7a(2Ab+aB))\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{21d} \\
&= \frac{2(5a^2A+3Ab^2+6abB)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d} \\
&= \frac{2(5b^2B+7a(2Ab+aB))\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\right)}{21d} \\
&= -\frac{2(5a^2A+3Ab^2+6abB)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\right)}{5d}
\end{aligned}$$

Mathematica [A] time = 4.49, size = 221, normalized size = 0.84

$$\frac{\sec^{\frac{7}{2}}(c+dx)\left(40(7a^2B+14aAb+5b^2B)\cos^{\frac{7}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\right)\Big|_2 - 168(5a^2A+6abB+3Ab^2)\cos^{\frac{7}{2}}(c+dx)\right)}{420d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]), x]
[Out] (Sec[c + d*x]^(7/2)*(-168*(5*a^2*A + 3*A*b^2 + 6*a*b*B)*Cos[c + d*x]^(7/2)*
EllipticE[(c + d*x)/2, 2] + 40*(14*a*A*b + 7*a^2*B + 5*b^2*B)*Cos[c + d*x]^(
7/2)*EllipticF[(c + d*x)/2, 2] + 2*(140*a*A*b + 70*a^2*B + 110*b^2*B + 21*
(15*a^2*A + 13*A*b^2 + 26*a*b*B)*Cos[c + d*x] + 10*(14*a*A*b + 7*a^2*B + 5*
b^2*B)*Cos[2*(c + d*x)] + 105*a^2*A*Cos[3*(c + d*x)] + 63*A*b^2*Cos[3*(c +
d*x)] + 126*a*b*B*Cos[3*(c + d*x)])*Sin[c + d*x])/ (420*d)
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb^2\sec(dx+c)^4 + Aa^2\sec(dx+c) + (2Bab + Ab^2)\sec(dx+c)^3 + (Ba^2 + 2Aab)\sec(dx+c)^2\right)\sqrt{\sec(dx+c)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)), x, algorithm
="fricas")
```

[Out] integral((B*b^2*sec(d*x + c)^4 + A*a^2*sec(d*x + c) + (2*B*a*b + A*b^2)*sec(d*x + c)^3 + (B*a^2 + 2*A*a*b)*sec(d*x + c)^2)*sqrt(sec(d*x + c)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(b \sec(dx + c) + a)^2 \sec(dx + c)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^2*sec(d*x + c)^(3/2), x)

maple [B] time = 16.16, size = 859, normalized size = 3.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x)

[Out]
$$\begin{aligned} & -(-(-2*\cos(1/2*d*x+1/2*c)^2+1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*b^2*B*(-1/56* \\ & \cos(1/2*d*x+1/2*c)*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/(-1 \\ & /2+\cos(1/2*d*x+1/2*c)^2)^4-5/42*\cos(1/2*d*x+1/2*c)*(-2*\sin(1/2*d*x+1/2*c)^4 \\ & +\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/(-1/2+\cos(1/2*d*x+1/2*c)^2)^2+5/21*(\sin(1/2*d* \\ & x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4 \\ & +\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)}))+2*a^2* \\ & A*(-(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(\sin(1/2*d*x+1/2*c) \\ &)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)}) \\ &)+2*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*\cos(1/2*d*x+1 \\ & /2*c)*\sin(1/2*d*x+1/2*c)^2/\sin(1/2*d*x+1/2*c)^2/(2*\sin(1/2*d*x+1/2*c)^2-1) \\ & +2*a*(2*A*b+B*a)*(-1/6*\cos(1/2*d*x+1/2*c)*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2* \\ & d*x+1/2*c)^2)^{(1/2)}/(-1/2+\cos(1/2*d*x+1/2*c)^2)^2+1/3*(\sin(1/2*d*x+1/2*c)^2 \\ &)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2* \\ & d*x+1/2*c)^2)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)}))-2/5*b*(A*b+2*B*a) \\ &)/(8*\sin(1/2*d*x+1/2*c)^6-12*\sin(1/2*d*x+1/2*c)^4+6*\sin(1/2*d*x+1/2*c)^2-1) \\ & /(\sin(1/2*d*x+1/2*c)^2*(12*EllipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)})*(2*\sin(1/2* \\ & d*x+1/2*c)^2-1)^{(1/2)}*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*\sin(1/2*d*x+1/2*c)^4-24* \\ & \cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^6-12*EllipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)}) \\ &)*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*\sin(1/2 \\ & *d*x+1/2*c)^2+24*\sin(1/2*d*x+1/2*c)^4*\cos(1/2*d*x+1/2*c)+3*(\sin(1/2*d*x+1/2 \\ & *c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c), \\ & 2^{(1/2)}))-8*\sin(1/2*d*x+1/2*c)^2*\cos(1/2*d*x+1/2*c))*(-2*\sin(1/2*d*x+1/2*c)^ \end{aligned}$$

$4 + \sin(1/2 * d * x + 1/2 * c)^2)^{1/2} / \sin(1/2 * d * x + 1/2 * c) / (2 * \cos(1/2 * d * x + 1/2 * c)^{2-1})^{1/2} / d$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(A + \frac{B}{\cos(c + dx)} \right) \left(a + \frac{b}{\cos(c + dx)} \right)^2 \left(\frac{1}{\cos(c + dx)} \right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^2*(1/cos(c + d*x))^(3/2),x)

[Out] int((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^2*(1/cos(c + d*x))^(3/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**(3/2)*(a+b*sec(d*x+c))**2*(A+B*sec(d*x+c)),x)

[Out] Timed out

$$3.401 \quad \int \sqrt{\sec(c + dx)} (a + b \sec(c + dx))^2 (A + B \sec(c + dx)) dx$$

Optimal. Leaf size=221

$$\frac{2(3a^2A + 2abB + Ab^2) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3d} + \frac{2(5a(aB + 2Ab) + 3b^2B) \sin(c + dx) \sqrt{\sec(c + dx)}}{5d}$$

[Out] $\frac{2}{15} b (5A^*b + 7B^*a) \sec(d*x+c)^{(3/2)} \sin(d*x+c) / d + \frac{2}{5} b^*B \sec(d*x+c)^{(3/2)} (a + b \sec(d*x+c)) \sin(d*x+c) / d - \frac{2}{5} (3b^2B + 5a(2A^*b + B^*a)) \sin(d*x+c) \sec(d*x+c)^{(1/2)} / d - \frac{2}{5} (3b^2B + 5a(2A^*b + B^*a)) (\cos(1/2*d*x + 1/2*c)^2)^{(1/2)} / \cos(1/2*d*x + 1/2*c) * \text{EllipticE}(\sin(1/2*d*x + 1/2*c), 2^{(1/2)}) * \cos(d*x+c)^{(1/2)} * \sec(d*x+c)^{(1/2)} / d + \frac{2}{3} (3A^*a^2 + A^*b^2 + 2B^*a*b) (\cos(1/2*d*x + 1/2*c)^2)^{(1/2)} / \cos(1/2*d*x + 1/2*c) * \text{EllipticF}(\sin(1/2*d*x + 1/2*c), 2^{(1/2)}) * \cos(d*x+c)^{(1/2)} * \sec(d*x+c)^{(1/2)} / d$

Rubi [A] time = 0.31, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {4026, 4047, 3768, 3771, 2639, 4046, 2641}

$$\frac{2(3a^2A + 2abB + Ab^2) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3d} + \frac{2(5a(aB + 2Ab) + 3b^2B) \sin(c + dx) \sqrt{\sec(c + dx)}}{5d}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]

[Out] $(-2*(3b^2B + 5a(2A^*b + aB))*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/(5*d) + (2*(3a^2A + A^*b^2 + 2a^*b*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticF}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/(3*d) + (2*(3b^2B + 5a(2A^*b + aB))*\text{Sqrt}[\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(5*d) + (2*b*(5A^*b + 7a^*B)*\text{Sec}[c + d*x]^{(3/2)}*\text{Sin}[c + d*x])/(15*d) + (2*b*B*\text{Sec}[c + d*x]^{(3/2)}*(a + b*\text{Sec}[c + d*x])*\text{Sin}[c + d*x])/(5*d)$

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x]
)*(b*Csc[c + d*x])^(n - 1)/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^(n)*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4026

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(m + n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*Sim
p[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B)*(m + n) + b^2*B*(m + n - 1))*C
sc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b
^2, 0] && GtQ[m, 1] && !(IGtQ[n, 1] && !IntegerQ[m])
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_.)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{\sec(c+dx)} (a+b \sec(c+dx))^2 (A+B \sec(c+dx)) dx &= \frac{2bB \sec^{\frac{3}{2}}(c+dx)(a+b \sec(c+dx)) \sin(c+dx)}{5d} + \frac{2}{5} \\
&= \frac{2bB \sec^{\frac{3}{2}}(c+dx)(a+b \sec(c+dx)) \sin(c+dx)}{5d} + \frac{2}{5} \\
&= \frac{2(3b^2B+5a(2Ab+aB)) \sqrt{\sec(c+dx)} \sin(c+dx)}{5d} \\
&= \frac{2(3b^2B+5a(2Ab+aB)) \sqrt{\sec(c+dx)} \sin(c+dx)}{5d} \\
&= -\frac{2(3b^2B+5a(2Ab+aB)) \sqrt{\cos(c+dx)} E\left(\frac{1}{2}(c+dx)\right)}{5d}
\end{aligned}$$

Mathematica [A] time = 2.72, size = 171, normalized size = 0.77

$$\frac{\sec^{\frac{5}{2}}(c+dx) \left(20(3a^2A+2abB+Ab^2) \cos^{\frac{5}{2}}(c+dx) F\left(\frac{1}{2}(c+dx) \middle| 2\right) - 12(5a^2B+10aAb+3b^2B) \cos^{\frac{5}{2}}(c+dx) E\left(\frac{1}{2}(c+dx)\right) \right)}{30d}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]

[Out] (Sec[c + d*x]^(5/2)*(-12*(10*a*A*b + 5*a^2*B + 3*b^2*B)*Cos[c + d*x]^(5/2)*EllipticE[(c + d*x)/2, 2] + 20*(3*a^2*A + A*b^2 + 2*a*b*B)*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] + 2*(15*(2*a*A*b + a^2*B + b^2*B) + 10*b*(A*b + 2*a*B)*Cos[c + d*x] + 3*(10*a*A*b + 5*a^2*B + 3*b^2*B)*Cos[2*(c + d*x)])*Sin[c + d*x])/(30*d)

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb^2 \sec(dx+c)^3 + Aa^2 + (2Bab + Ab^2) \sec(dx+c)^2 + (Ba^2 + 2Aab) \sec(dx+c)\right) \sqrt{\sec(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] integral((B*b^2*sec(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*sec(d*x + c)^2 + (B*a^2 + 2*A*a*b)*sec(d*x + c))*sqrt(sec(d*x + c)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(b \sec(dx + c) + a)^2 \sqrt{\sec(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^2*sqrt(sec(d*x + c)), x)

maple [B] time = 13.53, size = 750, normalized size = 3.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x)

[Out]
$$\begin{aligned} & -(-(-2*\cos(1/2*d*x+1/2*c)^2+1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*a^2*A*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)})+2*a*(2*A*b+B*a)*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)})+2*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^2)/\sin(1/2*d*x+1/2*c)^2/(2*\sin(1/2*d*x+1/2*c)^2-1)-2/5*b^2*B/(8*\sin(1/2*d*x+1/2*c)^6-12*\sin(1/2*d*x+1/2*c)^4+6*\sin(1/2*d*x+1/2*c)^2-1)/\sin(1/2*d*x+1/2*c)^2*(12*EllipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)})*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*\sin(1/2*d*x+1/2*c)^4-24*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^6-12*EllipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)})*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*\sin(1/2*d*x+1/2*c)^2+24*\sin(1/2*d*x+1/2*c)^4*\cos(1/2*d*x+1/2*c)+3*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)})-8*\sin(1/2*d*x+1/2*c)^2*\cos(1/2*d*x+1/2*c))*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}+2*b*(A*b+2*B*a)*(-1/6*\cos(1/2*d*x+1/2*c)*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/(-1/2+\cos(1/2*d*x+1/2*c)^2)^2+1/3*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)})))/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(b \sec(dx + c) + a)^2 \sqrt{\sec(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^2*sqrt(sec(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(A + \frac{B}{\cos(c + dx)} \right) \left(a + \frac{b}{\cos(c + dx)} \right)^2 \sqrt{\frac{1}{\cos(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^2*(1/cos(c + d*x))^(1/2),x)
```

```
[Out] int((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^2*(1/cos(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)*(a+b*sec(d*x+c))**2*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

$$3.402 \quad \int \frac{(a+b \sec(c+dx))^2(A+B \sec(c+dx))}{\sqrt{\sec(c+dx)}} dx$$

Optimal. Leaf size=177

$$\frac{2(3a^2B + 6aAb + b^2B) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} F\left(\frac{1}{2}(c+dx) \middle| 2\right)}{3d} + \frac{2(a^2A - 2abB - Ab^2) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{d}$$

[Out] $2/3*b*(3*A*b+5*B*a)*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/d+2/3*b*B*(a+b*\sec(d*x+c))*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/d+2*(A*a^2-A*b^2-2*B*a*b)*(\cos(1/2*d*x+1/2*c))^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d+2/3*(6*A*a*b+3*B*a^2+B*b^2)*(\cos(1/2*d*x+1/2*c))^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d$

Rubi [A] time = 0.27, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4026, 4047, 3771, 2641, 4046, 2639}

$$\frac{2(3a^2B + 6aAb + b^2B) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} F\left(\frac{1}{2}(c+dx) \middle| 2\right)}{3d} + \frac{2(a^2A - 2abB - Ab^2) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{d}$$

Antiderivative was successfully verified.

[In] Int[((a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]))/Sqrt[Sec[c + d*x]], x]

[Out] $(2*(a^2*A - A*b^2 - 2*a*b*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/d + (2*(6*a*A*b + 3*a^2*B + b^2*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticF}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/(3*d) + (2*b*(3*A*b + 5*a*B)*\text{Sqrt}[\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(3*d) + (2*b*B*\text{Sqrt}[\text{Sec}[c + d*x]]*(a + b*\text{Sec}[c + d*x])*\text{Sin}[c + d*x])/(3*d)$

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4026

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] := -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(m + n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*Simp
[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B)*(m + n) + b^2*B*(m + n - 1))*C
sc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b
^2, 0] && GtQ[m, 1] && !(IGtQ[n, 1] && !IntegerQ[m])
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] := -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] := Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + b \sec(c + dx))^2 (A + B \sec(c + dx))}{\sqrt{\sec(c + dx)}} dx &= \frac{2bB\sqrt{\sec(c + dx)}(a + b \sec(c + dx)) \sin(c + dx)}{3d} + \frac{2}{3} \int \frac{\frac{1}{2}a(3a^2 + 3a^2B + 3Ab^2)}{\sqrt{\sec(c + dx)}} dx \\
&= \frac{2bB\sqrt{\sec(c + dx)}(a + b \sec(c + dx)) \sin(c + dx)}{3d} + \frac{2}{3} \int \frac{\frac{1}{2}a(3a^2 + 3a^2B + 3Ab^2)}{\sqrt{\sec(c + dx)}} dx \\
&= \frac{2b(3Ab + 5aB)\sqrt{\sec(c + dx)} \sin(c + dx)}{3d} + \frac{2bB\sqrt{\sec(c + dx)}(a + b \sec(c + dx)) \sin(c + dx)}{3d} \\
&= \frac{2(6aAb + 3a^2B + b^2B) \sqrt{\cos(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{\sec(c + dx)}}{3d} \\
&= \frac{2(a^2A - Ab^2 - 2abB) \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{\sec(c + dx)}}{d}
\end{aligned}$$

Mathematica [A] time = 1.28, size = 125, normalized size = 0.71

$$\frac{2\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \left((3a^2B + 6aAb + b^2B) F\left(\frac{1}{2}(c + dx) \middle| 2\right) + 3(a^2A - 2abB - Ab^2) E\left(\frac{1}{2}(c + dx) \middle| 2\right) \right)}{3d}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]))/Sqrt[Sec[c + d*x]], x]

[Out] (2*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(3*(a^2*A - A*b^2 - 2*a*b*B)*EllipticE[(c + d*x)/2, 2] + (6*a*A*b + 3*a^2*B + b^2*B)*EllipticF[(c + d*x)/2, 2] + (b*(b*B + 3*(A*b + 2*a*B))*Cos[c + d*x])*Sin[c + d*x])/Cos[c + d*x]^(3/2)))/(3*d)

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{Bb^2 \sec(dx + c)^3 + Aa^2 + (2Bab + Ab^2) \sec(dx + c)^2 + (Ba^2 + 2Aab) \sec(dx + c)}{\sqrt{\sec(dx + c)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="fricas")

[Out] integral((B*b^2*sec(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*sec(d*x + c)^2 + (B*a^2 + 2*A*a*b)*sec(d*x + c))/sqrt(sec(d*x + c)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(b \sec(dx + c) + a)^2}{\sqrt{\sec(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^2/sqrt(sec(d*x + c)), x)

maple [B] time = 10.25, size = 677, normalized size = 3.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x)

[Out]
$$\begin{aligned} & -(-(-2*\cos(1/2*d*x+1/2*c)^2+1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*a^2*A*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})-\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)}))-2*a^2*A*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})+4*A*a*b*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})+2*a^2*B*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})+2*b*(A*b+2*B*a)*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)})+2*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^2/\sin(1/2*d*x+1/2*c)^2/(2*\sin(1/2*d*x+1/2*c)^2-1)+2*b^2*B*(-1/6*\cos(1/2*d*x+1/2*c)*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/(-1/2+\cos(1/2*d*x+1/2*c)^2)^2+1/3*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})))/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(b \sec(dx + c) + a)^2}{\sqrt{\sec(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^2/sqrt(sec(d*x + c)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(a + \frac{b}{\cos(c+dx)}\right)^2}{\sqrt{\frac{1}{\cos(c+dx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^2)/(1/cos(c + d*x))^(1/2),x)

[Out] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^2)/(1/cos(c + d*x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx))(a + b \sec(c + dx))^2}{\sqrt{\sec(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))**2*(A+B*sec(d*x+c))/sec(d*x+c)**(1/2),x)

[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**2/sqrt(sec(c + d*x)), x)

$$3.403 \quad \int \frac{(a+b \sec(c+dx))^2 (A+B \sec(c+dx))}{3 \sec^2(c+dx)} dx$$

Optimal. Leaf size=161

$$\frac{2(a^2A + 6abB + 3Ab^2) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} F\left(\frac{1}{2}(c+dx) \middle| 2\right)}{3d} + \frac{2a^2A \sin(c+dx)}{3d\sqrt{\sec(c+dx)}} - \frac{2(b^2B - a(ab + 2Ab)) \sqrt{\cos(c+dx)}}{3d}$$

[Out] $2/3*a^2*A*\sin(d*x+c)/d/\sec(d*x+c)^{(1/2)}+2*b^2*B*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/d-2*(b^2*B-a*(2*A*b+B*a))*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d+2/3*(A*a^2+3*A*b^2+6*B*a*b)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d$

Rubi [A] time = 0.25, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4024, 4047, 3771, 2641, 4046, 2639}

$$\frac{2(a^2A + 6abB + 3Ab^2) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} F\left(\frac{1}{2}(c+dx) \middle| 2\right)}{3d} + \frac{2a^2A \sin(c+dx)}{3d\sqrt{\sec(c+dx)}} - \frac{2(b^2B - a(ab + 2Ab)) \sqrt{\cos(c+dx)}}{3d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + b*\text{Sec}[c + d*x])^2*(A + B*\text{Sec}[c + d*x])]/\text{Sec}[c + d*x]^{(3/2)}, x]$

[Out] $(-2*(b^2*B - a*(2*A*b + a*B))*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/d + (2*(a^2*A + 3*A*b^2 + 6*a*b*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticF}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/(3*d) + (2*a^2*A*\text{Sin}[c + d*x])/(3*d*\text{Sqrt}[\text{Sec}[c + d*x]]) + (2*b^2*B*\text{Sqrt}[\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/d$

Rule 2639

$\text{Int}[\text{Sqrt}[\sin[(c_.) + (d_.)*(x_.)]], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticE}[(1*(c - \text{Pi}/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 2641

$\text{Int}[1/\text{Sqrt}[\sin[(c_.) + (d_.)*(x_.)]], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticF}[(1*(c - \text{Pi}/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 3771

$\text{Int}[(\text{csc}[(c_.) + (d_.)*(x_.)]*(b_.))^{(n_.)}, x_Symbol] \rightarrow \text{Dist}[(b*\text{Csc}[c + d*x])^{(n)}*\text{Sin}[c + d*x]^n, \text{Int}[1/\text{Sin}[c + d*x]^n, x], x] /; \text{FreeQ}\{b, c, d\}, x] \&\&$

EqQ[n^2, 1/4]

Rule 4024

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (
a_.))^2*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := Simp[(a^2*A*Cos
[e + f*x]*(d*Csc[e + f*x])^(n + 1))/(d*f*n), x] + Dist[1/(d*n), Int[(d*Csc[
e + f*x])^(n + 1)*(a*(2*A*b + a*B)*n + (2*a*b*B*n + A*(b^2*n + a^2*(n + 1))
)*Csc[e + f*x] + b^2*B*n*Csc[e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]^2*(C_.)
+ (A_.)), x_Symbol] := -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_.)]*
(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.)), x_Symbol] := Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + b \sec(c + dx))^2 (A + B \sec(c + dx))}{\sec^{\frac{3}{2}}(c + dx)} dx &= \frac{2a^2 A \sin(c + dx)}{3d \sqrt{\sec(c + dx)}} - \frac{2}{3} \int \frac{-\frac{3}{2}a(2Ab + aB) + \left(A \left(-\frac{a^2}{2} - \frac{3b^2}{2}\right) - 3aB\right)}{\sqrt{\sec(c + dx)}} dx \\
&= \frac{2a^2 A \sin(c + dx)}{3d \sqrt{\sec(c + dx)}} - \frac{2}{3} \int \frac{-\frac{3}{2}a(2Ab + aB) - \frac{3}{2}b^2 B \sec^2(c + dx)}{\sqrt{\sec(c + dx)}} dx \\
&= \frac{2a^2 A \sin(c + dx)}{3d \sqrt{\sec(c + dx)}} + \frac{2b^2 B \sqrt{\sec(c + dx)} \sin(c + dx)}{d} - (b^2 B - a(2Ab + aB)) \int \frac{1}{\sqrt{\sec(c + dx)}} dx \\
&= \frac{2(a^2 A + 3Ab^2 + 6abB) \sqrt{\cos(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{\sec(c + dx)}}{3d} \\
&= -\frac{2(b^2 B - a(2Ab + aB)) \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{\sec(c + dx)}}{d}
\end{aligned}$$

Mathematica [A] time = 0.76, size = 124, normalized size = 0.77

$$\frac{\sqrt{\sec(c+dx)} \left(2 \sin(c+dx) (a^2 A \cos(c+dx) + 3b^2 B) + 2 (a^2 A + 6abB + 3Ab^2) \sqrt{\cos(c+dx)} F\left(\frac{1}{2}(c+dx) \middle| 2\right) \right)}{3d}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(3/2), x]

[Out] (Sqrt[Sec[c + d*x]]*(6*(2*a*A*b + a^2*B - b^2*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 2*(a^2*A + 3*A*b^2 + 6*a*b*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 2*(3*b^2*B + a^2*A*Cos[c + d*x])*Sin[c + d*x]))/(3*d)

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{Bb^2 \sec(dx+c)^3 + Aa^2 + (2Bab + Ab^2) \sec(dx+c)^2 + (Ba^2 + 2Aab) \sec(dx+c)}{\sec(dx+c)^{\frac{3}{2}}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2), x, algorithm="fricas")

[Out] integral((B*b^2*sec(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*sec(d*x + c)^2 + (B*a^2 + 2*A*a*b)*sec(d*x + c))/sec(d*x + c)^(3/2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx+c) + A)(b \sec(dx+c) + a)^2}{\sec(dx+c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2), x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^2/sec(d*x + c)^(3/2), x)

maple [B] time = 4.63, size = 404, normalized size = 2.51

$$2 \left(4A a^2 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) \left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right) \right) + a^2 A \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{2 \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right) \right) - 1} \text{EllipticF}\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x)`

[Out]
$$-2/3*(4*A*a^2*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^4+a^2*A*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})+3*A*b^2*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})-6*A*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)})*a*b-2*A*a^2*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^2+6*B*a*b*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})-3*B*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)})*a^2+3*B*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)})*b^2-6*B*b^2*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^2/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(b \sec(dx + c) + a)^2}{\sec(dx + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="maxima")`

[Out] `integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^2/sec(d*x + c)^(3/2), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(a + \frac{b}{\cos(c+dx)}\right)^2}{\left(\frac{1}{\cos(c+dx)}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^2)/(1/cos(c + d*x))^(3/2),x)`

[Out] `int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^2)/(1/cos(c + d*x))^(3/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx))(a + b \sec(c + dx))^2}{\sec^{\frac{3}{2}}(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2*(A+B*sec(d*x+c))/sec(d*x+c)**(3/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**2/sec(c + d*x)**(3/2),  
x)
```


$$3.404 \quad \int \frac{(a+b \sec(c+dx))^2(A+B \sec(c+dx))}{\sec^2(c+dx)} dx$$

Optimal. Leaf size=171

$$\frac{2(a^2B + 2aAb + 3b^2B) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} F\left(\frac{1}{2}(c+dx) \middle| 2\right)}{3d} + \frac{2(3a^2A + 10abB + 5Ab^2) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{5d}$$

[Out] $2/5*a^2*A*\sin(d*x+c)/d/\sec(d*x+c)^(3/2)+2/3*a*(2*A*b+B*a)*\sin(d*x+c)/d/\sec(d*x+c)^(1/2)+2/5*(3*A*a^2+5*A*b^2+10*B*a*b)*(\cos(1/2*d*x+1/2*c)^2)^(1/2)/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c),2^(1/2))*\cos(d*x+c)^(1/2)*\sec(d*x+c)^(1/2)/d+2/3*(2*A*a*b+B*a^2+3*B*b^2)*(\cos(1/2*d*x+1/2*c)^2)^(1/2)/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c),2^(1/2))*\cos(d*x+c)^(1/2)*\sec(d*x+c)^(1/2)/d$

Rubi [A] time = 0.26, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4024, 4047, 3771, 2639, 4045, 2641}

$$\frac{2(a^2B + 2aAb + 3b^2B) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} F\left(\frac{1}{2}(c+dx) \middle| 2\right)}{3d} + \frac{2(3a^2A + 10abB + 5Ab^2) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{5d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\frac{(a + b*\text{Sec}[c + d*x])^2*(A + B*\text{Sec}[c + d*x])}{\text{Sec}[c + d*x]^(5/2)}, x]$

[Out] $(2*(3*a^2*A + 5*A*b^2 + 10*a*b*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/(5*d) + (2*(2*a*A*b + a^2*B + 3*b^2*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticF}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/(3*d) + (2*a^2*A*\text{Sin}[c + d*x])/(5*d*\text{Sec}[c + d*x]^(3/2)) + (2*a*(2*A*b + a*B)*\text{Sin}[c + d*x])/(3*d*\text{Sqrt}[\text{Sec}[c + d*x]])$

Rule 2639

$\text{Int}[\text{Sqrt}[\sin[(c_.) + (d_.)*(x_)]]], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticE}[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 2641

$\text{Int}[1/\text{Sqrt}[\sin[(c_.) + (d_.)*(x_)]]], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticF}[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 3771

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^n], x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 4024

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^2*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] := Simp[(a^2*A*Cos[e + f*x]*(d*Csc[e + f*x])^(n + 1))/(d*f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*(a*(2*A*b + a*B)*n + (2*a*b*B*n + A*(b^2*n + a^2*(n + 1)))*Csc[e + f*x] + b^2*B*n*Csc[e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]

Rule 4045

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^m*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_.)), x_Symbol] := Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] + Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]

Rule 4047

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^m*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] := Dist[B/b, Int[(b*Csc[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2), x] /; FreeQ[{b, e, f, A, B, C, m}, x]

Rubi steps

$$\begin{aligned}
\int \frac{(a + b \sec(c + dx))^2 (A + B \sec(c + dx))}{\sec^{\frac{5}{2}}(c + dx)} dx &= \frac{2a^2 A \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} - \frac{2}{5} \int \frac{-\frac{5}{2}a(2Ab + aB) + \left(A \left(-\frac{3a^2}{2} - \frac{5b^2}{2}\right) - 5\right)}{\sec^{\frac{3}{2}}(c + dx)} dx \\
&= \frac{2a^2 A \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} - \frac{2}{5} \int \frac{-\frac{5}{2}a(2Ab + aB) - \frac{5}{2}b^2 B \sec^2(c + dx)}{\sec^{\frac{3}{2}}(c + dx)} dx \\
&= \frac{2a^2 A \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} + \frac{2a(2Ab + aB) \sin(c + dx)}{3d \sqrt{\sec(c + dx)}} - \frac{1}{3} (-2aAb - a^2) \sqrt{\sec(c + dx)} \\
&= \frac{2(3a^2 A + 5Ab^2 + 10abB) \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{\sec(c + dx)}}{5d} \\
&= \frac{2(3a^2 A + 5Ab^2 + 10abB) \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{\sec(c + dx)}}{5d}
\end{aligned}$$

Mathematica [A] time = 0.96, size = 128, normalized size = 0.75

$$\frac{\sqrt{\sec(c + dx)} \left(10(a^2 B + 2aAb + 3b^2 B) \sqrt{\cos(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right) + 6(3a^2 A + 10abB + 5Ab^2) \sqrt{\cos(c + dx)} \right)}{15d}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(5/2), x]

[Out] (Sqrt[Sec[c + d*x]]*(6*(3*a^2*A + 5*A*b^2 + 10*a*b*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 10*(2*a*A*b + a^2*B + 3*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + a*(10*A*b + 5*a*B + 3*a*A*Cos[c + d*x])*Sin[2*(c + d*x)]))/(15*d)

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{Bb^2 \sec(dx + c)^3 + Aa^2 + (2Bab + Ab^2) \sec(dx + c)^2 + (Ba^2 + 2Aab) \sec(dx + c)}{\sec(dx + c)^{\frac{5}{2}}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="fricas")

[Out] integral((B*b^2*sec(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*sec(d*x + c)^2 + (B*a^2 + 2*A*a*b)*sec(d*x + c))/sec(d*x + c)^(5/2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(b \sec(dx + c) + a)^2}{\sec(dx + c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^2/sec(d*x + c)^(5/2), x)

maple [B] time = 4.78, size = 487, normalized size = 2.85

$$2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \left(-24Aa^2 \cos\left(\frac{dx}{2} + \frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + (24a^2A + 40Aab + 20a^2B)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2), x)

[Out]
$$\begin{aligned} & -2/15 * ((2 * \cos(1/2 * d * x + 1/2 * c) ^ 2 - 1) * \sin(1/2 * d * x + 1/2 * c) ^ 2) ^ (1/2) * (-24 * A * a ^ 2 * \cos(1/2 * d * x + 1/2 * c) * \sin(1/2 * d * x + 1/2 * c) ^ 6 + (24 * A * a ^ 2 + 40 * A * a * b + 20 * B * a ^ 2) * \sin(1/2 * d * x + 1/2 * c) ^ 4 * \cos(1/2 * d * x + 1/2 * c) + (-6 * A * a ^ 2 - 20 * A * a * b - 10 * B * a ^ 2) * \sin(1/2 * d * x + 1/2 * c) ^ 2 * \cos(1/2 * d * x + 1/2 * c) + 10 * A * a * b * (\sin(1/2 * d * x + 1/2 * c) ^ 2) ^ (1/2) * (2 * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 1) ^ (1/2) * \text{EllipticF}(\cos(1/2 * d * x + 1/2 * c), 2 ^ (1/2)) - 9 * A * (\sin(1/2 * d * x + 1/2 * c) ^ 2) ^ (1/2) * (2 * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 1) ^ (1/2) * \text{EllipticE}(\cos(1/2 * d * x + 1/2 * c), 2 ^ (1/2)) * a ^ 2 - 15 * A * (\sin(1/2 * d * x + 1/2 * c) ^ 2) ^ (1/2) * (2 * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 1) ^ (1/2) * \text{EllipticE}(\cos(1/2 * d * x + 1/2 * c), 2 ^ (1/2)) * b ^ 2 + 5 * a ^ 2 * B * (\sin(1/2 * d * x + 1/2 * c) ^ 2) ^ (1/2) * (2 * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 1) ^ (1/2) * \text{EllipticF}(\cos(1/2 * d * x + 1/2 * c), 2 ^ (1/2)) + 15 * b ^ 2 * B * (\sin(1/2 * d * x + 1/2 * c) ^ 2) ^ (1/2) * (2 * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 1) ^ (1/2) * \text{EllipticF}(\cos(1/2 * d * x + 1/2 * c), 2 ^ (1/2)) - 30 * B * (\sin(1/2 * d * x + 1/2 * c) ^ 2) ^ (1/2) * (2 * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 1) ^ (1/2) * \text{EllipticE}(\cos(1/2 * d * x + 1/2 * c), 2 ^ (1/2)) * a * b) / (-2 * \sin(1/2 * d * x + 1/2 * c) ^ 4 + \sin(1/2 * d * x + 1/2 * c) ^ 2) ^ (1/2) / \sin(1/2 * d * x + 1/2 * c) / (2 * \cos(1/2 * d * x + 1/2 * c) ^ 2 - 1) ^ (1/2) / d \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(b \sec(dx + c) + a)^2}{\sec(dx + c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^2/sec(d*x + c)^(5/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(a + \frac{b}{\cos(c+dx)}\right)^2}{\left(\frac{1}{\cos(c+dx)}\right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^2)/(1/cos(c + d*x))^(5/2),x)

[Out] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^2)/(1/cos(c + d*x))^(5/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) (a + b \sec(c + dx))^2}{\sec^{\frac{5}{2}}(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))**2*(A+B*sec(d*x+c))/sec(d*x+c)**(5/2),x)

[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**2/sec(c + d*x)**(5/2), x)

$$3.405 \quad \int \frac{(a+b \sec(c+dx))^2(A+B \sec(c+dx))}{7 \sec^2(c+dx)} dx$$

Optimal. Leaf size=213

$$\frac{2(5a^2A + 14abB + 7Ab^2) \sin(c + dx)}{21d\sqrt{\sec(c + dx)}} + \frac{2(5a^2A + 14abB + 7Ab^2) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{21d}$$

[Out] 2/7*a^2*A*sin(d*x+c)/d/sec(d*x+c)^(5/2)+2/5*a*(2*A*b+B*a)*sin(d*x+c)/d/sec(d*x+c)^(3/2)+2/21*(5*A*a^2+7*A*b^2+14*B*a*b)*sin(d*x+c)/d/sec(d*x+c)^(1/2)+2/5*(6*A*a*b+3*B*a^2+5*B*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c), 2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/21*(5*A*a^2+7*A*b^2+14*B*a*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c), 2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d

Rubi [A] time = 0.29, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {4024, 4047, 3769, 3771, 2641, 4045, 2639}

$$\frac{2(5a^2A + 14abB + 7Ab^2) \sin(c + dx)}{21d\sqrt{\sec(c + dx)}} + \frac{2(5a^2A + 14abB + 7Ab^2) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{21d}$$

Antiderivative was successfully verified.

[In] Int[((a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(7/2), x]

[Out] (2*(6*a*A*b + 3*a^2*B + 5*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(5*a^2*A + 7*A*b^2 + 14*a*b*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*a^2*A*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (2*a*(2*A*b + a*B)*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (2*(5*a^2*A + 7*A*b^2 + 14*a*b*B)*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x]])

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4024

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_.))^2*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] := Simp[(a^2*A*Cos
[e + f*x]*(d*Csc[e + f*x])^(n + 1))/(d*f*n), x] + Dist[1/(d*n), Int[(d*Csc[
e + f*x])^(n + 1)*(a*(2*A*b + a*B)*n + (2*a*b*B*n + A*(b^2*n + a^2*(n + 1))
)*Csc[e + f*x] + b^2*B*n*Csc[e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_.)), x_Symbol] := Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] := Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + b \sec(c + dx))^2 (A + B \sec(c + dx))}{\sec^{\frac{7}{2}}(c + dx)} dx &= \frac{2a^2 A \sin(c + dx)}{7d \sec^{\frac{5}{2}}(c + dx)} - \frac{2}{7} \int \frac{-\frac{7}{2}a(2Ab + aB) + \left(A\left(-\frac{5a^2}{2} - \frac{7b^2}{2}\right) - 7a\right)}{\sec^{\frac{5}{2}}(c + dx)} dx \\
&= \frac{2a^2 A \sin(c + dx)}{7d \sec^{\frac{5}{2}}(c + dx)} - \frac{2}{7} \int \frac{-\frac{7}{2}a(2Ab + aB) - \frac{7}{2}b^2 B \sec^2(c + dx)}{\sec^{\frac{5}{2}}(c + dx)} dx \\
&= \frac{2a^2 A \sin(c + dx)}{7d \sec^{\frac{5}{2}}(c + dx)} + \frac{2a(2Ab + aB) \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} + \frac{2(5a^2 A + 7Ab^2)}{21d \sqrt{\sec(c + dx)}} \\
&= \frac{2a^2 A \sin(c + dx)}{7d \sec^{\frac{5}{2}}(c + dx)} + \frac{2a(2Ab + aB) \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} + \frac{2(5a^2 A + 7Ab^2)}{21d \sqrt{\sec(c + dx)}} \\
&= \frac{2(6aAb + 3a^2 B + 5b^2 B) \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{\sec(c + dx)}}{5d}
\end{aligned}$$

Mathematica [A] time = 1.58, size = 161, normalized size = 0.76

$$\frac{\sqrt{\sec(c + dx)} \left(\sin(2(c + dx)) \left(5(3a^2 A \cos(2(c + dx)) + 13a^2 A + 28abB + 14Ab^2) + 42a(aB + 2Ab) \cos(c + dx) \right) \right)}{210d}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(7/2), x]

[Out] (Sqrt[Sec[c + d*x]]*(84*(6*a*A*b + 3*a^2*B + 5*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 20*(5*a^2*A + 7*A*b^2 + 14*a*b*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (42*a*(2*A*b + a*B)*Cos[c + d*x] + 5*(13*a^2*A + 14*A*b^2 + 28*a*b*B + 3*a^2*A*Cos[2*(c + d*x)]))*Sin[2*(c + d*x)]))/(210*d)

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{Bb^2 \sec(dx + c)^3 + Aa^2 + (2Bab + Ab^2) \sec(dx + c)^2 + (Ba^2 + 2Aab) \sec(dx + c)}{\sec(dx + c)^{\frac{7}{2}}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm="fricas")

[Out] integral((B*b^2*sec(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*sec(d*x + c)^2 + (B*a^2 + 2*A*a*b)*sec(d*x + c))/sec(d*x + c)^(7/2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(b \sec(dx + c) + a)^2}{\sec(dx + c)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^2/sec(d*x + c)^(7/2), x)

maple [B] time = 4.63, size = 548, normalized size = 2.57

$$2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \left(240A a^2 \cos\left(\frac{dx}{2} + \frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + (-360a^2A - 336Aab - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x)

[Out]
$$\begin{aligned} & -2/105*((2*\cos(1/2*d*x+1/2*c)^2-1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(240*A*a^2*c \\ & \cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^8+(-360*A*a^2-336*A*a*b-168*B*a^2)*\sin \\ & (1/2*d*x+1/2*c)^6*\cos(1/2*d*x+1/2*c)+(280*A*a^2+336*A*a*b+140*A*b^2+168*B*a \\ & ^2+280*B*a*b)*\sin(1/2*d*x+1/2*c)^4*\cos(1/2*d*x+1/2*c)+(-80*A*a^2-84*A*a*b-7 \\ & 0*A*b^2-42*B*a^2-140*B*a*b)*\sin(1/2*d*x+1/2*c)^2*\cos(1/2*d*x+1/2*c)+25*a^2* \\ & A*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*EllipticF(c \\ & \cos(1/2*d*x+1/2*c),2^{(1/2)})+35*A*b^2*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2 \\ & *d*x+1/2*c)^2-1)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)})-126*A*(\sin(1/2 \\ & *d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*EllipticE(\cos(1/2*d*x \\ & +1/2*c),2^{(1/2)})*a*b+70*B*a*b*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1 \\ & /2*c)^2-1)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)})-63*B*(\sin(1/2*d*x+1/ \\ & 2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c) \\ & ,2^{(1/2)})*a^2-105*B*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1) \\ & ^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)})*b^2)/(-2*\sin(1/2*d*x+1/2*c)^4+ \\ & \sin(1/2*d*x+1/2*c)^2)^{(1/2)}/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(b \sec(dx + c) + a)^2}{\sec(dx + c)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^2/sec(d*x + c)^(7/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(a + \frac{b}{\cos(c+dx)}\right)^2}{\left(\frac{1}{\cos(c+dx)}\right)^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^2)/(1/cos(c + d*x))^(7/2),x)

[Out] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^2)/(1/cos(c + d*x))^(7/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx))(a + b \sec(c + dx))^2}{\sec^{\frac{7}{2}}(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))**2*(A+B*sec(d*x+c))/sec(d*x+c)**(7/2),x)

[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**2/sec(c + d*x)**(7/2), x)

$$3.406 \quad \int \frac{(a+b \sec(c+dx))^2 (A+B \sec(c+dx))}{9 \sec^2(c+dx)} dx$$

Optimal. Leaf size=254

$$\frac{2(7a^2A + 18abB + 9Ab^2) \sin(c + dx)}{45d \sec^3(c + dx)} + \frac{2(7a^2A + 18abB + 9Ab^2) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{15d}$$

[Out] 2/9*a^2*A*sin(d*x+c)/d/sec(d*x+c)^(7/2)+2/7*a*(2*A*b+B*a)*sin(d*x+c)/d/sec(d*x+c)^(5/2)+2/45*(7*A*a^2+9*A*b^2+18*B*a*b)*sin(d*x+c)/d/sec(d*x+c)^(3/2)+2/21*(7*b^2*B+5*a*(2*A*b+B*a))*sin(d*x+c)/d/sec(d*x+c)^(1/2)+2/15*(7*A*a^2+9*A*b^2+18*B*a*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/21*(7*b^2*B+5*a*(2*A*b+B*a))*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d

Rubi [A] time = 0.34, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {4024, 4047, 3769, 3771, 2639, 4045, 2641}

$$\frac{2(7a^2A + 18abB + 9Ab^2) \sin(c + dx)}{45d \sec^3(c + dx)} + \frac{2(7a^2A + 18abB + 9Ab^2) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{15d}$$

Antiderivative was successfully verified.

[In] Int[((a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(9/2),x]

[Out] (2*(7*a^2*A + 9*A*b^2 + 18*a*b*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(15*d) + (2*(7*b^2*B + 5*a*(2*A*b + a*B))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*a^2*A*S in[c + d*x])/(9*d*Sec[c + d*x]^(7/2)) + (2*a*(2*A*b + a*B)*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (2*(7*a^2*A + 9*A*b^2 + 18*a*b*B)*Sin[c + d*x])/(45*d*Sec[c + d*x]^(3/2)) + (2*(7*b^2*B + 5*a*(2*A*b + a*B))*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x]])

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_.)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 3769

Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.))^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 3771

Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 4024

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_))^(2*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_))), x_Symbol] := Simp[(a^2*A*Cos[e + f*x]*(d*Csc[e + f*x])^(n + 1))/(d*f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*(a*(2*A*b + a*B)*n + (2*a*b*B*n + A*(b^2*n + a^2*(n + 1)))*Csc[e + f*x] + b^2*B*n*Csc[e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]

Rule 4045

Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.))^(m_)*(csc[(e_.) + (f_.)*(x_.)]^2*(C_.) + (A_)), x_Symbol] := Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] + Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]

Rule 4047

Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.))^(m_)*((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.)), x_Symbol] := Dist[B/b, Int[(b*Csc[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2), x] /; FreeQ[{b, e, f, A, B, C, m}, x]

Rubi steps

$$\begin{aligned}
\int \frac{(a + b \sec(c + dx))^2 (A + B \sec(c + dx))}{\sec^{\frac{9}{2}}(c + dx)} dx &= \frac{2a^2 A \sin(c + dx)}{9d \sec^{\frac{7}{2}}(c + dx)} - \frac{2}{9} \int \frac{-\frac{9}{2}a(2Ab + aB) + \left(A \left(-\frac{7a^2}{2} - \frac{9b^2}{2}\right) - 9\right)}{\sec^{\frac{7}{2}}(c + dx)} dx \\
&= \frac{2a^2 A \sin(c + dx)}{9d \sec^{\frac{7}{2}}(c + dx)} - \frac{2}{9} \int \frac{-\frac{9}{2}a(2Ab + aB) - \frac{9}{2}b^2 B \sec^2(c + dx)}{\sec^{\frac{7}{2}}(c + dx)} dx \\
&= \frac{2a^2 A \sin(c + dx)}{9d \sec^{\frac{7}{2}}(c + dx)} + \frac{2a(2Ab + aB) \sin(c + dx)}{7d \sec^{\frac{5}{2}}(c + dx)} + \frac{2(7a^2 A + 9Ab^2 B)}{45d \sec^{\frac{3}{2}}(c + dx)} \\
&= \frac{2a^2 A \sin(c + dx)}{9d \sec^{\frac{7}{2}}(c + dx)} + \frac{2a(2Ab + aB) \sin(c + dx)}{7d \sec^{\frac{5}{2}}(c + dx)} + \frac{2(7a^2 A + 9Ab^2 B)}{45d \sec^{\frac{3}{2}}(c + dx)} \\
&= \frac{2(7a^2 A + 9Ab^2 B + 18abB) \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{\sec(c + dx)}}{15d} \\
&= \frac{2(7a^2 A + 9Ab^2 B + 18abB) \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{\sec(c + dx)}}{15d}
\end{aligned}$$

Mathematica [A] time = 1.87, size = 189, normalized size = 0.74

$$\sqrt{\sec(c + dx)} \left(\sin(2(c + dx)) \left(7(43a^2 A + 72abB + 36Ab^2) \cos(c + dx) + 5(7a^2 A \cos(3(c + dx)) + 78a^2 B + 18abB) \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(9/2), x]

[Out] (Sqrt[Sec[c + d*x]]*(168*(7*a^2*A + 9*A*b^2 + 18*a*b*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 120*(10*a*A*b + 5*a^2*B + 7*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (7*(43*a^2*A + 36*A*b^2 + 72*a*b*B)*Cos[c + d*x] + 5*(156*a*A*b + 78*a^2*B + 84*b^2*B + 18*a*(2*A*b + a*B))*Cos[2*(c + d*x)] + 7*a^2*A*Cos[3*(c + d*x)]))*Sin[2*(c + d*x)]/(1260*d)

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{Bb^2 \sec(dx + c)^3 + Aa^2 + (2Bab + Ab^2) \sec(dx + c)^2 + (Ba^2 + 2Aab) \sec(dx + c)}{\sec(dx + c)^{\frac{9}{2}}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x, algorithm="fricas")

[Out] integral((B*b^2*sec(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*sec(d*x + c)^2 + (B*a^2 + 2*A*a*b)*sec(d*x + c))/sec(d*x + c)^(9/2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(b \sec(dx + c) + a)^2}{\sec(dx + c)^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^2/sec(d*x + c)^(9/2), x)

maple [B] time = 4.66, size = 610, normalized size = 2.40

$$2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \left(-1120A a^2 \cos\left(\frac{dx}{2} + \frac{c}{2}\right)\left(\sin^{10}\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + (2240a^2A + 1440Aab)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x)

[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*A*a^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(2240*A*a^2+1440*A*a*b+720*B*a^2)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-2072*A*a^2-2160*A*a*b-504*A*b^2-1080*B*a^2-1008*B*a*b)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(952*A*a^2+1680*A*a*b+504*A*b^2+840*B*a^2+1008*B*a*b+420*B*b^2)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-168*A*a^2-480*A*a*b-126*A*b^2-240*B*a^2-252*B*a*b-210*B*b^2)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-147*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-189*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^2+150*A*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-378*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b+75*a^2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+105*b^2

$2*B*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c), 2^{(1/2)})/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(a + \frac{b}{\cos(c+dx)}\right)^2}{\left(\frac{1}{\cos(c+dx)}\right)^{9/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^2)/(1/cos(c + d*x))^(9/2),x)

[Out] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^2)/(1/cos(c + d*x))^(9/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)**(9/2),x)

[Out] Timed out

$$3.407 \quad \int \sec^{\frac{3}{2}}(c+dx)(a+b \sec(c+dx))^3(A+B \sec(c+dx)) dx$$

Optimal. Leaf size=345

$$\frac{2b(22a^2B + 27aAb + 7b^2B) \sin(c+dx) \sec^{\frac{5}{2}}(c+dx)}{45d} + \frac{2(7a^3B + 21a^2Ab + 15ab^2B + 5Ab^3) \sin(c+dx) \sec^{\frac{3}{2}}(c+dx)}{21d}$$

[Out] $2/21*(21*A*a^2*b+5*A*b^3+7*B*a^3+15*B*a*b^2)*\sec(d*x+c)^{(3/2)}*\sin(d*x+c)/d+2/45*b*(27*A*a*b+22*B*a^2+7*B*b^2)*\sec(d*x+c)^{(5/2)}*\sin(d*x+c)/d+2/63*b^2*(9*A*b+13*B*a)*\sec(d*x+c)^{(7/2)}*\sin(d*x+c)/d+2/9*b*B*\sec(d*x+c)^{(5/2)}*(a+b*\sec(d*x+c))^2*\sin(d*x+c)/d+2/15*(15*A*a^3+27*A*a*b^2+27*B*a^2*b+7*B*b^3)*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/d-2/15*(15*A*a^3+27*A*a*b^2+27*B*a^2*b+7*B*b^3)*(c*\cos(1/2*d*x+1/2*c))^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d+2/21*(21*A*a^2*b+5*A*b^3+7*B*a^3+15*B*a*b^2)*(c*\cos(1/2*d*x+1/2*c))^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d$

Rubi [A] time = 0.57, antiderivative size = 345, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {4026, 4076, 4047, 3768, 3771, 2641, 4046, 2639}

$$\frac{2b(22a^2B + 27aAb + 7b^2B) \sin(c+dx) \sec^{\frac{5}{2}}(c+dx)}{45d} + \frac{2(21a^2Ab + 7a^3B + 15ab^2B + 5Ab^3) \sin(c+dx) \sec^{\frac{3}{2}}(c+dx)}{21d}$$

Antiderivative was successfully verified.

[In] `Int[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]`

[Out] $(-2*(15*a^3*A + 27*a*A*b^2 + 27*a^2*b*B + 7*b^3*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/(15*d) + (2*(21*a^2*A*b + 5*A*b^3 + 7*a^3*B + 15*a*b^2*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticF}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/(21*d) + (2*(15*a^3*A + 27*a*A*b^2 + 27*a^2*b*B + 7*b^3*B)*\text{Sqrt}[\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(15*d) + (2*(21*a^2*A*b + 5*A*b^3 + 7*a^3*B + 15*a*b^2*B)*\text{Sec}[c + d*x]^{(3/2)}*\text{Sin}[c + d*x])/(21*d) + (2*b*(27*a*A*b + 22*a^2*B + 7*b^2*B)*\text{Sec}[c + d*x]^{(5/2)}*\text{Sin}[c + d*x])/(45*d) + (2*b^2*(9*A*b + 13*a*B)*\text{Sec}[c + d*x]^{(7/2)}*\text{Sin}[c + d*x])/(63*d) + (2*b*B*\text{Sec}[c + d*x]^{(5/2)}*(a + b*\text{Sec}[c + d*x])^2*\text{Sin}[c + d*x])/(9*d)$

Rule 2639

`Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]`

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 3768

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3771

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 4026

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] := -Simp[(b*B*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x] + Dist[1/(m + n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*Simp[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B))*(m + n) + b^2*B*(m + n - 1))*Csc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && !(IGtQ[n, 1] && !IntegerQ[m])

Rule 4046

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_.)), x_Symbol] := -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]

Rule 4047

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_)*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] := Dist[B/b, Int[(b*Csc[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2), x] /; FreeQ[{b, e, f, A, B, C, m}, x]

Rule 4076

```

Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (
a_)), x_Symbol] := -Simp[(b*C*Csc[e + f*x]*Cot[e + f*x]*(d*Csc[e + f*x])^n
)/(f*(n + 2)), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2)
+ (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n +
2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &&
!LtQ[n, -1]

```

Rubi steps

$$\begin{aligned}
\int \sec^{\frac{3}{2}}(c + dx)(a + b \sec(c + dx))^3(A + B \sec(c + dx)) dx &= \frac{2bB \sec^{\frac{5}{2}}(c + dx)(a + b \sec(c + dx))^2 \sin(c + dx)}{9d} + \frac{2b^2(9Ab + 13aB) \sec^{\frac{7}{2}}(c + dx) \sin(c + dx)}{63d} + \frac{2bB \sec^{\frac{9}{2}}(c + dx) \sin(c + dx)}{63d} \\
&= \frac{2b^2(9Ab + 13aB) \sec^{\frac{7}{2}}(c + dx) \sin(c + dx)}{63d} + \frac{2bB \sec^{\frac{9}{2}}(c + dx) \sin(c + dx)}{63d} \\
&= \frac{2(21a^2Ab + 5Ab^3 + 7a^3B + 15ab^2B) \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{21d} \\
&= \frac{2(15a^3A + 27aAb^2 + 27a^2bB + 7b^3B) \sqrt{\sec(c + dx)} \sin(c + dx)}{15d} \\
&= \frac{2(21a^2Ab + 5Ab^3 + 7a^3B + 15ab^2B) \sqrt{\cos(c + dx)} \sin(c + dx)}{21d} \\
&= -\frac{2(15a^3A + 27aAb^2 + 27a^2bB + 7b^3B) \sqrt{\cos(c + dx)} \cos(c + dx)}{15d}
\end{aligned}$$

Mathematica [A] time = 6.57, size = 452, normalized size = 1.31

$$\frac{\cos^4(c + dx)(a + b \sec(c + dx))^3(A + B \sec(c + dx)) \left(2(35a^3B + 105a^2Ab + 75ab^2B + 25Ab^3) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right)}{105d(a \cos(c + dx) + b)^3(A \cos(c + dx) + B)}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]

```
[Out] (Cos[c + d*x]^4*((2*(-105*a^3*A - 189*a*A*b^2 - 189*a^2*b*B - 49*b^3*B)*EllipticE[(c + d*x)/2, 2])/(Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + 2*(105*a^2*A*b + 25*A*b^3 + 35*a^3*B + 75*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x])/(105*d*(b + a*Cos[c + d*x])^3*(B + A*Cos[c + d*x])) + ((a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]))*((2*(15*a^3*A + 27*a*A*b^2 + 27*a^2*b*B + 7*b^3*B)*Sin[c + d*x])/15 + (2*Sec[c + d*x]^3*(A*b^3*Sin[c + d*x] + 3*a*b^2*B*Sin[c + d*x]))/7 + (2*Sec[c + d*x]*(21*a^2*A*b*Sin[c + d*x] + 5*A*b^3*Sin[c + d*x] + 7*a^3*B*Sin[c + d*x] + 15*a*b^2*B*Sin[c + d*x]))/21 + (2*Sec[c + d*x]^2*(27*a*A*b^2*Sin[c + d*x] + 27*a^2*b*B*Sin[c + d*x] + 7*b^3*B*Sin[c + d*x]))/45 + (2*b^3*B*Sec[c + d*x]^3*Tan[c + d*x])/9))/(d*(b + a*Cos[c + d*x])^3*(B + A*Cos[c + d*x])*Sec[c + d*x]^(7/2))
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb^3 \sec(dx + c)^5 + Aa^3 \sec(dx + c) + (3Bab^2 + Ab^3) \sec(dx + c)^4 + 3(Ba^2b + Aab^2) \sec(dx + c)^3\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((B*b^3*sec(d*x + c)^5 + A*a^3*sec(d*x + c) + (3*B*a*b^2 + A*b^3)*sec(d*x + c)^4 + 3*(B*a^2*b + A*a*b^2)*sec(d*x + c)^3 + (B*a^3 + 3*A*a^2*b)*sec(d*x + c)^2)*sqrt(sec(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(b \sec(dx + c) + a)^3 \sec(dx + c)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^3*sec(d*x + c)^(3/2), x)
```

maple [B] time = 20.42, size = 1193, normalized size = 3.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*b^2*(A*b+3*B*
a)*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+5/21*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
)))+2*b^3*B*(-1/144*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^5-7/180*cos(1/2*d*x+1/2*c)*(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^
2)^3-14/15*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)
^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)+7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos
(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))))+2*a^2*(3*A*b+B*a)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
)))+2*A*a^3*(-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/
2*c)^2-1)-6/5*a*b*(A*b+B*a)/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4
+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2
))*sin(1/2*d*x+1/2*c)^4-24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x
+1/2*c)+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c
))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)
/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm
="maxima")
```

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(A + \frac{B}{\cos(c + dx)} \right) \left(a + \frac{b}{\cos(c + dx)} \right)^3 \left(\frac{1}{\cos(c + dx)} \right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^3*(1/cos(c + d*x))^(3/2), x)`

[Out] `int((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^3*(1/cos(c + d*x))^(3/2), x)`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)**(3/2)*(a+b*sec(d*x+c))**3*(A+B*sec(d*x+c)), x)`

[Out] Timed out

$$3.408 \quad \int \sqrt{\sec(c + dx)} (a + b \sec(c + dx))^3 (A + B \sec(c + dx)) dx$$

Optimal. Leaf size=295

$$\frac{2b(18a^2B + 21aAb + 5b^2B) \sin(c + dx) \sec^{\frac{3}{2}}(c + dx)}{21d} + \frac{2(5a^3B + 15a^2Ab + 9ab^2B + 3Ab^3) \sin(c + dx) \sqrt{\sec(c + dx)}}{5d}$$

[Out] $\frac{2}{21} * b * (21 * A * a * b + 18 * B * a^2 + 5 * B * b^2) * \sec(d * x + c)^{\frac{3}{2}} * \sin(d * x + c) / d + \frac{2}{35} * b^2 * (7 * A * b + 11 * B * a) * \sec(d * x + c)^{\frac{5}{2}} * \sin(d * x + c) / d + \frac{2}{7} * b * B * \sec(d * x + c)^{\frac{3}{2}} * (a + b * \sec(d * x + c))^2 * \sin(d * x + c) / d + \frac{2}{5} * (15 * A * a^2 * b + 3 * A * b^3 + 5 * B * a^3 + 9 * B * a * b^2) * \sin(d * x + c) * \sec(d * x + c)^{\frac{1}{2}} / d - \frac{2}{5} * (15 * A * a^2 * b + 3 * A * b^3 + 5 * B * a^3 + 9 * B * a * b^2) * (\cos(1/2 * d * x + 1/2 * c))^{\frac{1}{2}} / \cos(1/2 * d * x + 1/2 * c) * \text{EllipticE}(\sin(1/2 * d * x + 1/2 * c), 2^{\frac{1}{2}}) * \cos(d * x + c)^{\frac{1}{2}} * \sec(d * x + c)^{\frac{1}{2}} / d + \frac{2}{21} * (21 * A * a^3 + 21 * A * a * b^2 + 21 * B * a^2 * b + 5 * B * b^3) * (\cos(1/2 * d * x + 1/2 * c))^{\frac{1}{2}} / \cos(1/2 * d * x + 1/2 * c) * \text{EllipticF}(\sin(1/2 * d * x + 1/2 * c), 2^{\frac{1}{2}}) * \cos(d * x + c)^{\frac{1}{2}} * \sec(d * x + c)^{\frac{1}{2}} / d$

Rubi [A] time = 0.50, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {4026, 4076, 4047, 3768, 3771, 2639, 4046, 2641}

$$\frac{2b(18a^2B + 21aAb + 5b^2B) \sin(c + dx) \sec^{\frac{3}{2}}(c + dx)}{21d} + \frac{2(15a^2Ab + 5a^3B + 9ab^2B + 3Ab^3) \sin(c + dx) \sqrt{\sec(c + dx)}}{5d}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]

[Out] $(-2 * (15 * a^2 * A * b + 3 * A * b^3 + 5 * a^3 * B + 9 * a * b^2 * B) * \text{Sqrt}[\text{Cos}[c + d * x]] * \text{EllipticE}[(c + d * x) / 2, 2] * \text{Sqrt}[\text{Sec}[c + d * x]]) / (5 * d) + (2 * (21 * a^3 * A + 21 * a * A * b^2 + 21 * a^2 * b * B + 5 * b^3 * B) * \text{Sqrt}[\text{Cos}[c + d * x]] * \text{EllipticF}[(c + d * x) / 2, 2] * \text{Sqrt}[\text{Sec}[c + d * x]]) / (21 * d) + (2 * (15 * a^2 * A * b + 3 * A * b^3 + 5 * a^3 * B + 9 * a * b^2 * B) * \text{Sqrt}[\text{Sec}[c + d * x]] * \text{Sin}[c + d * x]) / (5 * d) + (2 * b * (21 * a * A * b + 18 * a^2 * B + 5 * b^2 * B) * \text{Sec}[c + d * x]^{\frac{3}{2}} * \text{Sin}[c + d * x]) / (21 * d) + (2 * b^2 * (7 * A * b + 11 * a * B) * \text{Sec}[c + d * x]^{\frac{5}{2}} * \text{Sin}[c + d * x]) / (35 * d) + (2 * b * B * \text{Sec}[c + d * x]^{\frac{3}{2}} * (a + b * \text{Sec}[c + d * x])^2 * \text{Sin}[c + d * x]) / (7 * d)$

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 3768

Int[(csc[(c_.) + (d_.)*(x_)])*(b_.))^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x] *(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3771

Int[(csc[(c_.) + (d_.)*(x_)])*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 4026

Int[(csc[(e_.) + (f_.)*(x_)])*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)])*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_)])*(B_.) + (A_.)), x_Symbol] := -Simp[(b*B*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x] + Dist[1/(m + n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*Simp[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B)*(m + n) + b^2*B*(m + n - 1))*Csc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && !(IGtQ[n, 1] && !IntegerQ[m])

Rule 4046

Int[(csc[(e_.) + (f_.)*(x_)])*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)])^2*(C_.) + (A_.)), x_Symbol] := -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]

Rule 4047

Int[(csc[(e_.) + (f_.)*(x_)])*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)])*(B_.) + csc[(e_.) + (f_.)*(x_)])^2*(C_.)), x_Symbol] := Dist[B/b, Int[(b*Csc[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2), x] /; FreeQ[{b, e, f, A, B, C, m}, x]

Rule 4076

Int[((A_.) + csc[(e_.) + (f_.)*(x_)])*(B_.) + csc[(e_.) + (f_.)*(x_)])^2*(C_.) *(csc[(e_.) + (f_.)*(x_)])*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)])*(b_.) + (

```
a_)), x_Symbol] := -Simp[(b*C*Csc[e + f*x]*Cot[e + f*x]*(d*Csc[e + f*x])^n)
/(f*(n + 2)), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2)
+ (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n +
2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &&
!LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
 \int \sqrt{\sec(c + dx)} (a + b \sec(c + dx))^3 (A + B \sec(c + dx)) dx &= \frac{2bB \sec^{\frac{3}{2}}(c + dx) (a + b \sec(c + dx))^2 \sin(c + dx)}{7d} + \\
 &= \frac{2b^2(7Ab + 11aB) \sec^{\frac{5}{2}}(c + dx) \sin(c + dx)}{35d} + \frac{2bB \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{7d} + \\
 &= \frac{2b^2(7Ab + 11aB) \sec^{\frac{5}{2}}(c + dx) \sin(c + dx)}{35d} + \frac{2bB \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{7d} + \\
 &= \frac{2(15a^2Ab + 3Ab^3 + 5a^3B + 9ab^2B) \sqrt{\sec(c + dx)} \sin(c + dx)}{5d} + \\
 &= \frac{2(15a^2Ab + 3Ab^3 + 5a^3B + 9ab^2B) \sqrt{\sec(c + dx)} \sin(c + dx)}{5d} + \\
 &= -\frac{2(15a^2Ab + 3Ab^3 + 5a^3B + 9ab^2B) \sqrt{\cos(c + dx)} \sin(c + dx)}{5d}
 \end{aligned}$$

Mathematica [A] time = 3.88, size = 225, normalized size = 0.76

$$\frac{2\sqrt{\sec(c + dx)} \left(5b(21a^2B + 21aAb + 5b^2B) \tan(c + dx) + 21(5a^3B + 15a^2Ab + 9ab^2B + 3Ab^3) \sin(c + dx) + 5(15a^2Ab + 3Ab^3 + 5a^3B + 9ab^2B) \sqrt{\sec(c + dx)} \sin(c + dx) \right)}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*Sqrt[Sec[c + d*x]]*(-21*(15*a^2*A*b + 3*A*b^3 + 5*a^3*B + 9*a*b^2*B)*Sqr
t[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 5*(21*a^3*A + 21*a*A*b^2 + 21*a
^2*b*B + 5*b^3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 21*(15*a^2
*A*b + 3*A*b^3 + 5*a^3*B + 9*a*b^2*B)*Sin[c + d*x] + 5*b*(21*a*A*b + 21*a^2
*B + 5*b^2*B)*Tan[c + d*x] + 21*b^2*(A*b + 3*a*B)*Sec[c + d*x]*Tan[c + d*x]
+ 15*b^3*B*Sec[c + d*x]^2*Tan[c + d*x]))/(105*d)
```


fricas [F] time = 0.46, size = 0, normalized size = 0.00

integral(((Bb³ sec(dx + c)⁴ + Aa³ + (3 Bab² + Ab³) sec(dx + c)³ + 3(Ba²b + Aab²) sec(dx + c)² + (Ba³ + 3A

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] integral((B*b^3*sec(d*x + c)^4 + A*a^3 + (3*B*a*b^2 + A*b^3)*sec(d*x + c)^3 + 3*(B*a^2*b + A*a*b^2)*sec(d*x + c)^2 + (B*a^3 + 3*A*a^2*b)*sec(d*x + c))*sqrt(sec(d*x + c)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(b \sec(dx + c) + a)^3 \sqrt{\sec(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^3*sqrt(sec(d*x + c)), x)

maple [B] time = 16.36, size = 944, normalized size = 3.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x)

[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*b^3*B*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+6*a*b*(A*b+B*a)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*a^2*(3*A*b+B*a)*(-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+

```

1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)
^2-1)-2/5*b^2*(A*b+3*B*a)/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6
*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*
*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
sin(1/2*d*x+1/2*c)^4-24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1
/2*c)+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))
*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2))/sin(1/2*d*x+1/2*c)/(
2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm
="maxima")
```

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(A + \frac{B}{\cos(c+dx)} \right) \left(a + \frac{b}{\cos(c+dx)} \right)^3 \sqrt{\frac{1}{\cos(c+dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^3*(1/cos(c + d*x))^(1/2),x)
```

```
[Out] int((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^3*(1/cos(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)*(a+b*sec(d*x+c))**3*(A+B*sec(d*x+c)),x)
```

[Out] Timed out

$$3.409 \quad \int \frac{(a+b \sec(c+dx))^3 (A+B \sec(c+dx))}{\sqrt{\sec(c+dx)}} dx$$

Optimal. Leaf size=244

$$\frac{2b(14a^2B + 15aAb + 3b^2B) \sin(c + dx) \sqrt{\sec(c + dx)}}{5d} + \frac{2(3a^3B + 9a^2Ab + 3ab^2B + Ab^3) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)}}{3d}$$

[Out] $\frac{2}{15} b^2 (5A^3 + 9B^3) \sec(dx+c)^{3/2} \sin(dx+c) / d + \frac{2}{5} b (15A^2 + 14B^2 + 3B^2) \sin(dx+c) \sec(dx+c)^{1/2} / d + \frac{2}{5} b B (a+b \sec(dx+c))^2 \sin(dx+c) \sec(dx+c)^{1/2} / d + \frac{2}{5} (5A^3 - 15A^2 b - 15B^2 a^2 - 3B^3) (\cos(1/2 dx + 1/2 c))^2)^{1/2} / \cos(1/2 dx + 1/2 c) \text{EllipticE}(\sin(1/2 dx + 1/2 c), 2^{1/2}) \cos(dx+c)^{1/2} \sec(dx+c)^{1/2} / d + \frac{2}{3} (9A^2 b + A b^3 + 3B^2 a^2 + 3B^3 a b^2) (\cos(1/2 dx + 1/2 c))^2)^{1/2} / \cos(1/2 dx + 1/2 c) \text{EllipticF}(\sin(1/2 dx + 1/2 c), 2^{1/2}) \cos(dx+c)^{1/2} \sec(dx+c)^{1/2} / d$

Rubi [A] time = 0.48, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {4026, 4076, 4047, 3771, 2641, 4046, 2639}

$$\frac{2b(14a^2B + 15aAb + 3b^2B) \sin(c + dx) \sqrt{\sec(c + dx)}}{5d} + \frac{2(9a^2Ab + 3a^3B + 3ab^2B + Ab^3) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)}}{3d}$$

Antiderivative was successfully verified.

[In] Int[((a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]))/Sqrt[Sec[c + d*x]],x]

[Out] $(2*(5a^3A - 15a^2Ab - 15a^2bB - 3b^3B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/(5*d) + (2*(9a^2Ab + A b^3 + 3a^3B + 3a^2bB)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticF}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/(3*d) + (2*b*(15a^2A + 14a^2B + 3b^2B)*\text{Sqrt}[\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(5*d) + (2*b^2*(5A^3 + 9a^2B)*\text{Sec}[c + d*x]^{3/2}*\text{Sin}[c + d*x])/(15*d) + (2*b*B*\text{Sqrt}[\text{Sec}[c + d*x]]*(a + b*\text{Sec}[c + d*x])^2*\text{Sin}[c + d*x])/(5*d)$

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.))^(n_), x_Symbol] :=> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4026

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_)), x_Symbol] :=> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(m + n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*Simp
p[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B)*(m + n) + b^2*B*(m + n - 1))*C
sc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b
^2, 0] && GtQ[m, 1] && !IGtQ[n, 1] && !IntegerQ[m]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.))^(m_)*(csc[(e_.) + (f_.)*(x_.)]^2*(C_.)
+ (A_)), x_Symbol] :=> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.))^(m_)*((A_.) + csc[(e_.) + (f_.)*(x_.)]*
(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.)), x_Symbol] :=> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 4076

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.)
)*(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (
a_)), x_Symbol] :=> -Simp[(b*C*Csc[e + f*x]*Cot[e + f*x]*(d*Csc[e + f*x])^n)
/(f*(n + 2)), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2)
+ (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n +
2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &&
!LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + b \sec(c + dx))^3 (A + B \sec(c + dx))}{\sqrt{\sec(c + dx)}} dx &= \frac{2bB\sqrt{\sec(c + dx)} (a + b \sec(c + dx))^2 \sin(c + dx)}{5d} + \frac{2}{5} \int \frac{(a + b \sec(c + dx))^3 (A + B \sec(c + dx))}{\sqrt{\sec(c + dx)}} dx \\
&= \frac{2b^2(5Ab + 9aB) \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{15d} + \frac{2bB\sqrt{\sec(c + dx)} (a + b \sec(c + dx))^2 \sin(c + dx)}{5d} \\
&= \frac{2b^2(5Ab + 9aB) \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{15d} + \frac{2bB\sqrt{\sec(c + dx)} (a + b \sec(c + dx))^2 \sin(c + dx)}{5d} \\
&= \frac{2b(15aAb + 14a^2B + 3b^2B) \sqrt{\sec(c + dx)} \sin(c + dx)}{5d} + \frac{2b^2(5Ab + 9aB) \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{15d} \\
&= \frac{2(9a^2Ab + Ab^3 + 3a^3B + 3ab^2B) \sqrt{\cos(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3d} \\
&= \frac{2(5a^3A - 15aAb^2 - 15a^2bB - 3b^3B) \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{5d}
\end{aligned}$$

Mathematica [A] time = 2.55, size = 190, normalized size = 0.78

$$\frac{\sec^{\frac{5}{2}}(c + dx) \left(2b \sin(c + dx) \left(9(5a^2B + 5aAb + b^2B) \cos(2(c + dx)) + 15(3a^2B + 3aAb + b^2B) + 10b(3aB + Ab) \right) \right)}{5d}$$

Antiderivative was successfully verified.

[In] Integrate[(((a + b*Sec[c + d*x]))^3*(A + B*Sec[c + d*x]))/Sqrt[Sec[c + d*x]], x]

[Out] (Sec[c + d*x]^(5/2)*(12*(5*a^3*A - 15*a*A*b^2 - 15*a^2*b*B - 3*b^3*B)*Cos[c + d*x]^(5/2)*EllipticE[(c + d*x)/2, 2] + 20*(9*a^2*A*b + A*b^3 + 3*a^3*B + 3*a*b^2*B)*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] + 2*b*(15*(3*a*A*b + 3*a^2*B + b^2*B) + 10*b*(A*b + 3*a*B)*Cos[c + d*x] + 9*(5*a*A*b + 5*a^2*B + b^2*B)*Cos[2*(c + d*x)])*Sin[c + d*x])/(30*d)

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{Bb^3 \sec(dx + c)^4 + Aa^3 + (3Bab^2 + Ab^3) \sec(dx + c)^3 + 3(Ba^2b + Aab^2) \sec(dx + c)^2 + (Ba^3 + 3Aab) \sec(dx + c)}{\sqrt{\sec(dx + c)}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="fricas")

[Out] integral((B*b^3*sec(d*x + c)^4 + A*a^3 + (3*B*a*b^2 + A*b^3)*sec(d*x + c)^3 + 3*(B*a^2*b + A*a*b^2)*sec(d*x + c)^2 + (B*a^3 + 3*A*a^2*b)*sec(d*x + c))/sqrt(sec(d*x + c)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(b \sec(dx + c) + a)^3}{\sqrt{\sec(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^3/sqrt(sec(d*x + c)), x)

maple [B] time = 13.16, size = 997, normalized size = 4.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x)

[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-2*A*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+6*A*a^2*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*a^3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+6*a*b*(A*b+B*a)*(-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)+2*b^2*(A*b+3*B*a)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-2/5*b^3*B/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2)

$\frac{\sin^2(x+c)-1}{\sin(1/2*d*x+1/2*c)^2} * (12*EllipticE(\cos(1/2*d*x+1/2*c), 2^{(1/2)}) * (2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)} * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * \sin(1/2*d*x+1/2*c)^4 - 24*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^6 - 12*EllipticE(\cos(1/2*d*x+1/2*c), 2^{(1/2)}) * (2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)} * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * \sin(1/2*d*x+1/2*c)^2 + 24*\sin(1/2*d*x+1/2*c)^4*\cos(1/2*d*x+1/2*c) + 3*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)} * EllipticE(\cos(1/2*d*x+1/2*c), 2^{(1/2)}) - 8*\sin(1/2*d*x+1/2*c)^2*\cos(1/2*d*x+1/2*c)) * (-2*\sin(1/2*d*x+1/2*c)^4 + \sin(1/2*d*x+1/2*c)^2)^{(1/2)} / \sin(1/2*d*x+1/2*c) / (2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)} / d$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(b \sec(dx + c) + a)^3}{\sqrt{\sec(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^3/sqrt(sec(d*x + c)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(a + \frac{b}{\cos(c+dx)}\right)^3}{\sqrt{\frac{1}{\cos(c+dx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^3)/(1/cos(c + d*x))^(1/2),x)

[Out] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^3)/(1/cos(c + d*x))^(1/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)**(1/2),x)

[Out] Timed out

$$3.410 \quad \int \frac{(a+b \sec(c+dx))^3 (A+B \sec(c+dx))}{\sec^2(c+dx)} dx$$

Optimal. Leaf size=239

$$\frac{2b(2a^2A - 9abB - 3Ab^2) \sin(c+dx) \sqrt{\sec(c+dx)}}{3d} + \frac{2(a^3A + 9a^2bB + 9aAb^2 + b^3B) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{3d}$$

[Out] $-2/3*b^2*(A*a-B*b)*\sec(d*x+c)^{(3/2)}*\sin(d*x+c)/d+2/3*a*A*(a+b*\sec(d*x+c))^{2}* \sin(d*x+c)/d/\sec(d*x+c)^{(1/2)}-2/3*b*(2*A*a^2-3*A*b^2-9*B*a*b)*\sin(d*x+c)* \sec(d*x+c)^{(1/2)}/d+2*(3*A*a^2*b-A*b^3+B*a^3-3*B*a*b^2)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c),2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d+2/3*(A*a^3+9*A*a*b^2+9*B*a^2*b+B*b^3)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c),2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d$

Rubi [A] time = 0.51, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {4025, 4076, 4047, 3771, 2641, 4046, 2639}

$$\frac{2b(2a^2A - 9abB - 3Ab^2) \sin(c+dx) \sqrt{\sec(c+dx)}}{3d} + \frac{2(a^3A + 9a^2bB + 9aAb^2 + b^3B) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{3d}$$

Antiderivative was successfully verified.

[In] Int[((a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(3/2), x]

[Out] $(2*(3*a^2*A*b - A*b^3 + a^3*B - 3*a*b^2*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/d + (2*(a^3*A + 9*a*A*b^2 + 9*a^2*b*B + b^3*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticF}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/(3*d) - (2*b*(2*a^2*A - 3*A*b^2 - 9*a*b*B)*\text{Sqrt}[\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(3*d) - (2*b^2*(a*A - b*B)*\text{Sec}[c + d*x]^{(3/2)}*\text{Sin}[c + d*x])/(3*d) + (2*a*A*(a + b*\text{Sec}[c + d*x])^2*\text{Sin}[c + d*x])/(3*d*\text{Sqrt}[\text{Sec}[c + d*x]])$

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 3771

Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 4025

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> Simp[(a*A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e + f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LeQ[n, -1]

Rule 4046

Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.))^(m_)*(csc[(e_.) + (f_.)*(x_.)]^2*(C_.) + (A_.)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]

Rule 4047

Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.))^(m_)*((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2), x] /; FreeQ[{b, e, f, A, B, C, m}, x]

Rule 4076

Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] :> -Simp[(b*C*Csc[e + f*x]*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 2)), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2) + (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && !LtQ[n, -1]

Rubi steps

$$\begin{aligned}
\int \frac{(a + b \sec(c + dx))^3 (A + B \sec(c + dx))}{\sec^{\frac{3}{2}}(c + dx)} dx &= \frac{2aA(a + b \sec(c + dx))^2 \sin(c + dx)}{3d\sqrt{\sec(c + dx)}} - \frac{2}{3} \int \frac{(a + b \sec(c + dx))}{\sqrt{\sec(c + dx)}} dx \\
&= -\frac{2b^2(aA - bB) \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{3d} + \frac{2aA(a + b \sec(c + dx))}{3d\sqrt{\sec(c + dx)}} \\
&= -\frac{2b^2(aA - bB) \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{3d} + \frac{2aA(a + b \sec(c + dx))}{3d\sqrt{\sec(c + dx)}} \\
&= -\frac{2b(2a^2A - 3Ab^2 - 9abB) \sqrt{\sec(c + dx)} \sin(c + dx)}{3d} - \frac{2b^2(aA - bB)}{3d} \\
&= \frac{2(a^3A + 9aAb^2 + 9a^2bB + b^3B) \sqrt{\cos(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{\sec(c + dx)}}{3d} \\
&= \frac{2(3a^2Ab - Ab^3 + a^3B - 3ab^2B) \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{\sec(c + dx)}}{d}
\end{aligned}$$

Mathematica [A] time = 1.95, size = 166, normalized size = 0.69

$$\frac{\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \left(\frac{\sin(c + dx)(a^3A \cos(2(c + dx)) + a^3A + 6b^2(3aB + Ab) \cos(c + dx) + 2b^3B)}{\cos^{\frac{3}{2}}(c + dx)} + 2(a^3A + 9a^2bB + 9aAb^2 + b^3B) \right)}{3d}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(3/2), x]

[Out] (Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(6*(3*a^2*A*b - A*b^3 + a^3*B - 3*a*b^2*B)*EllipticE[(c + d*x)/2, 2] + 2*(a^3*A + 9*a*A*b^2 + 9*a^2*b*B + b^3*B)*EllipticF[(c + d*x)/2, 2] + ((a^3*A + 2*b^3*B + 6*b^2*(A*b + 3*a*B))*Cos[c + d*x] + a^3*A*Cos[2*(c + d*x)])*Sin[c + d*x])/Cos[c + d*x]^(3/2)))/(3*d)

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{Bb^3 \sec(dx + c)^4 + Aa^3 + (3Bab^2 + Ab^3) \sec(dx + c)^3 + 3(Ba^2b + Aab^2) \sec(dx + c)^2 + (Ba^3 + 3Aab)}{\sec(dx + c)^{\frac{3}{2}}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="fricas")

[Out] integral((B*b^3*sec(d*x + c)^4 + A*a^3 + (3*B*a*b^2 + A*b^3)*sec(d*x + c)^3 + 3*(B*a^2*b + A*a*b^2)*sec(d*x + c)^2 + (B*a^3 + 3*A*a^2*b)*sec(d*x + c))/sec(d*x + c)^(3/2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(b \sec(dx + c) + a)^3}{\sec(dx + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^3/sec(d*x + c)^(3/2), x)

maple [B] time = 13.40, size = 1212, normalized size = 5.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x)

[Out] $\frac{2}{3} * (-(-2 * \cos(1/2 * d * x + 1/2 * c) ^ 2 + 1) * \sin(1/2 * d * x + 1/2 * c) ^ 2) ^ (1/2) / (4 * \sin(1/2 * d * x + 1/2 * c) ^ 4 - 4 * \sin(1/2 * d * x + 1/2 * c) ^ 2 + 1) / \sin(1/2 * d * x + 1/2 * c) ^ 3 * (8 * A * a ^ 3 * \cos(1/2 * d * x + 1/2 * c) * \sin(1/2 * d * x + 1/2 * c) ^ 6 + 2 * A * (\sin(1/2 * d * x + 1/2 * c) ^ 2) ^ (1/2) * \text{EllipticF}(\cos(1/2 * d * x + 1/2 * c), 2 ^ (1/2)) * (2 * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 1) ^ (1/2) * a ^ 3 * \sin(1/2 * d * x + 1/2 * c) ^ 2 + 18 * A * (\sin(1/2 * d * x + 1/2 * c) ^ 2) ^ (1/2) * \text{EllipticF}(\cos(1/2 * d * x + 1/2 * c), 2 ^ (1/2)) * (2 * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 1) ^ (1/2) * a * b ^ 2 * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 18 * A * (\sin(1/2 * d * x + 1/2 * c) ^ 2) ^ (1/2) * \text{EllipticE}(\cos(1/2 * d * x + 1/2 * c), 2 ^ (1/2)) * (2 * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 1) ^ (1/2) * a ^ 2 * b * \sin(1/2 * d * x + 1/2 * c) ^ 2 + 6 * A * (\sin(1/2 * d * x + 1/2 * c) ^ 2) ^ (1/2) * \text{EllipticE}(\cos(1/2 * d * x + 1/2 * c), 2 ^ (1/2)) * (2 * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 1) ^ (1/2) * b ^ 3 * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 8 * A * a ^ 3 * \cos(1/2 * d * x + 1/2 * c) * \sin(1/2 * d * x + 1/2 * c) ^ 4 - 12 * A * b ^ 3 * \cos(1/2 * d * x + 1/2 * c) * \sin(1/2 * d * x + 1/2 * c) ^ 4 + 18 * B * (\sin(1/2 * d * x + 1/2 * c) ^ 2) ^ (1/2) * \text{EllipticF}(\cos(1/2 * d * x + 1/2 * c), 2 ^ (1/2)) * (2 * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 1) ^ (1/2) * a ^ 2 * b * \sin(1/2 * d * x + 1/2 * c) ^ 2 + 2 * B * (\sin(1/2 * d * x + 1/2 * c) ^ 2) ^ (1/2) * \text{EllipticF}(\cos(1/2 * d * x + 1/2 * c), 2 ^ (1/2)) * (2 * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 1) ^ (1/2) * b ^ 3 * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 6 * B * (\sin(1/2 * d * x + 1/2 * c) ^ 2) ^ (1/2) * \text{EllipticE}(\cos(1/2 * d * x + 1/2 * c), 2 ^ (1/2)) * (2 * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 1) ^ (1/2) * a ^ 3 * \sin(1/2 * d * x + 1/2 * c) ^ 2 + 18 * B * (\sin(1/2 * d * x + 1/2 * c) ^ 2) ^ (1/2) * \text{EllipticE}(\cos(1/2 * d * x + 1/2 * c), 2 ^ (1/2)) * (2 * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 1) ^ (1/2) * a * b ^ 2 * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 36 * B * a * b ^ 2 * \cos(1/2 * d * x + 1/2 * c) * \sin(1/2 * d * x + 1/2 * c) ^ 4$

$c) \sin(1/2 dx + 1/2 c)^4 - A a^3 (\sin(1/2 dx + 1/2 c)^2)^{1/2} (2 \sin(1/2 dx + 1/2 c)^2 - 1)^{1/2} \text{EllipticF}(\cos(1/2 dx + 1/2 c), 2^{1/2}) - 9 A a b^2 (\sin(1/2 dx + 1/2 c)^2)^{1/2} (2 \sin(1/2 dx + 1/2 c)^2 - 1)^{1/2} \text{EllipticF}(\cos(1/2 dx + 1/2 c), 2^{1/2}) + 9 A (\sin(1/2 dx + 1/2 c)^2)^{1/2} (2 \sin(1/2 dx + 1/2 c)^2 - 1)^{1/2} \text{EllipticE}(\cos(1/2 dx + 1/2 c), 2^{1/2}) a^2 b - 3 A (\sin(1/2 dx + 1/2 c)^2)^{1/2} (2 \sin(1/2 dx + 1/2 c)^2 - 1)^{1/2} \text{EllipticE}(\cos(1/2 dx + 1/2 c), 2^{1/2}) b^3 + 2 A a^3 \cos(1/2 dx + 1/2 c) \sin(1/2 dx + 1/2 c)^2 + 6 A b^3 \cos(1/2 dx + 1/2 c) \sin(1/2 dx + 1/2 c)^2 - 9 a^2 b B (\sin(1/2 dx + 1/2 c)^2)^{1/2} (2 \sin(1/2 dx + 1/2 c)^2 - 1)^{1/2} \text{EllipticF}(\cos(1/2 dx + 1/2 c), 2^{1/2}) - b^3 B (\sin(1/2 dx + 1/2 c)^2)^{1/2} (2 \sin(1/2 dx + 1/2 c)^2 - 1)^{1/2} \text{EllipticF}(\cos(1/2 dx + 1/2 c), 2^{1/2}) + 3 B (\sin(1/2 dx + 1/2 c)^2)^{1/2} (2 \sin(1/2 dx + 1/2 c)^2 - 1)^{1/2} \text{EllipticE}(\cos(1/2 dx + 1/2 c), 2^{1/2}) a^3 - 9 B (\sin(1/2 dx + 1/2 c)^2)^{1/2} (2 \sin(1/2 dx + 1/2 c)^2 - 1)^{1/2} \text{EllipticE}(\cos(1/2 dx + 1/2 c), 2^{1/2}) a b^2 + 18 B a b^2 \cos(1/2 dx + 1/2 c) \sin(1/2 dx + 1/2 c)^2 + 2 B b^3 \cos(1/2 dx + 1/2 c) \sin(1/2 dx + 1/2 c)^2 (-2 \sin(1/2 dx + 1/2 c)^4 + \sin(1/2 dx + 1/2 c)^2)^{1/2} / (2 \cos(1/2 dx + 1/2 c)^2 - 1)^{1/2} / d$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(b \sec(dx + c) + a)^3}{\sec(dx + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^3/sec(d*x + c)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(a + \frac{b}{\cos(c+dx)}\right)^3}{\left(\frac{1}{\cos(c+dx)}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^3)/(1/cos(c + d*x))^(3/2),x)

[Out] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^3)/(1/cos(c + d*x))^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx))(a + b \sec(c + dx))^3}{\sec^{\frac{3}{2}}(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**3*(A+B*sec(d*x+c))/sec(d*x+c)**(3/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**3/sec(c + d*x)**(3/2),  
x)
```

$$3.411 \quad \int \frac{(a+b \sec(c+dx))^3 (A+B \sec(c+dx))}{5 \sec^2(c+dx)} dx$$

Optimal. Leaf size=236

$$\frac{2a^2(5aB + 9Ab) \sin(c + dx)}{15d\sqrt{\sec(c + dx)}} + \frac{2(a^3B + 3a^2Ab + 9ab^2B + 3Ab^3) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3d} + \dots$$

[Out] $2/5*a*A*(a+b*\sec(d*x+c))^2*\sin(d*x+c)/d/\sec(d*x+c)^(3/2)+2/15*a^2*(9*A*b+5*B*a)*\sin(d*x+c)/d/\sec(d*x+c)^(1/2)-2/5*b^2*(A*a-5*B*b)*\sin(d*x+c)*\sec(d*x+c)^(1/2)/d+2/5*(3*A*a^3+15*A*a*b^2+15*B*a^2*b-5*B*b^3)*(\cos(1/2*d*x+1/2*c)^(2)^(1/2)/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c),2^(1/2))*\cos(d*x+c)^(1/2)*\sec(d*x+c)^(1/2)/d+2/3*(3*A*a^2*b+3*A*a*b^3+B*a^3+9*B*a*b^2)*(\cos(1/2*d*x+1/2*c)^(2)^(1/2)/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c),2^(1/2))*\cos(d*x+c)^(1/2)*\sec(d*x+c)^(1/2)/d$

Rubi [A] time = 0.46, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {4025, 4074, 4047, 3771, 2641, 4046, 2639}

$$\frac{2(3a^2Ab + a^3B + 9ab^2B + 3Ab^3) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3d} + \frac{2(3a^3A + 15a^2bB + 15aAb^2 - 5b^3B) \sin(c + dx)}{15d\sqrt{\sec(c + dx)}} + \dots$$

Antiderivative was successfully verified.

[In] Int[((a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(5/2),x]

[Out] $(2*(3*a^3*A + 15*a*A*b^2 + 15*a^2*b*B - 5*b^3*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/(5*d) + (2*(3*a^2*A*b + 3*A*b^3 + a^3*B + 9*a*b^2*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticF}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/(3*d) + (2*a^2*(9*A*b + 5*a*B)*\text{Sin}[c + d*x])/(15*d*\text{Sqrt}[\text{Sec}[c + d*x]]) - (2*b^2*(a*A - 5*b*B)*\text{Sqrt}[\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(5*d) + (2*a*A*(a + b*\text{Sec}[c + d*x])^2*\text{Sin}[c + d*x])/(5*d*\text{Sec}[c + d*x]^(3/2))$

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 3771

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 4025

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] := Simp[(a*A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e + f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LeQ[n, -1]

Rule 4046

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_.)), x_Symbol] := -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]

Rule 4047

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_)*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] := Dist[B/b, Int[(b*Csc[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2), x] /; FreeQ[{b, e, f, A, B, C, m}, x]

Rule 4074

Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]

Rubi steps

$$\begin{aligned}
\int \frac{(a + b \sec(c + dx))^3 (A + B \sec(c + dx))}{\sec^{\frac{5}{2}}(c + dx)} dx &= \frac{2aA(a + b \sec(c + dx))^2 \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} - \frac{2}{5} \int \frac{(a + b \sec(c + dx))}{\sec^{\frac{3}{2}}(c + dx)} dx \\
&= \frac{2a^2(9Ab + 5aB) \sin(c + dx)}{15d \sqrt{\sec(c + dx)}} + \frac{2aA(a + b \sec(c + dx))^2 \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} \\
&= \frac{2a^2(9Ab + 5aB) \sin(c + dx)}{15d \sqrt{\sec(c + dx)}} + \frac{2aA(a + b \sec(c + dx))^2 \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} \\
&= \frac{2a^2(9Ab + 5aB) \sin(c + dx)}{15d \sqrt{\sec(c + dx)}} - \frac{2b^2(aA - 5bB) \sqrt{\sec(c + dx)} \sin(c + dx)}{5d} \\
&= \frac{2(3a^2Ab + 3Ab^3 + a^3B + 9ab^2B) \sqrt{\cos(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3d} \\
&= \frac{2(3a^3A + 15aAb^2 + 15a^2bB - 5b^3B) \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{5d}
\end{aligned}$$

Mathematica [A] time = 1.69, size = 172, normalized size = 0.73

$$\frac{\sqrt{\sec(c + dx)} \left(2 \sin(c + dx) \left(3(a^3 A \cos(2(c + dx)) + a^3 A + 10b^3 B) + 10a^2(aB + 3Ab) \cos(c + dx) \right) + 20(a^3 B + \dots \right)}{30d}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(5/2), x]

[Out] (Sqrt[Sec[c + d*x]]*(12*(3*a^3*A + 15*a*A*b^2 + 15*a^2*b*B - 5*b^3*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 20*(3*a^2*A*b + 3*A*b^3 + a^3*B + 9*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 2*(10*a^2*(3*A*b + a*B)*Cos[c + d*x] + 3*(a^3*A + 10*b^3*B + a^3*A*Cos[2*(c + d*x)]))*Sin[c + d*x]))/(30*d)

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{Bb^3 \sec(dx + c)^4 + Aa^3 + (3Bab^2 + Ab^3) \sec(dx + c)^3 + 3(Ba^2b + Aab^2) \sec(dx + c)^2 + (Ba^3 + 3Aa^2b) \sec(dx + c) + A^2 + B^2}{\sec(dx + c)^{\frac{5}{2}}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="fricas")

[Out] integral((B*b^3*sec(d*x + c)^4 + A*a^3 + (3*B*a*b^2 + A*b^3)*sec(d*x + c)^3 + 3*(B*a^2*b + A*a*b^2)*sec(d*x + c)^2 + (B*a^3 + 3*A*a^2*b)*sec(d*x + c))/sec(d*x + c)^(5/2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(b \sec(dx + c) + a)^3}{\sec(dx + c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^3/sec(d*x + c)^(5/2), x)

maple [B] time = 5.21, size = 867, normalized size = 3.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2), x)

[Out]
$$\begin{aligned} & -2/15*(-24*A*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*a^3*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^6+4*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*a^2*(6*A*a+15*A*b+5*B*a)*\sin(1/2*d*x+1/2*c)^4*\cos(1/2*d*x+1/2*c)-2*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(3*A*a^3+15*A*a^2*b+5*B*a^3+15*B*b^3)*\sin(1/2*d*x+1/2*c)^2*\cos(1/2*d*x+1/2*c)+15*A*a^2*b*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c), 2^{(1/2)})*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}+15*A*b^3*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c), 2^{(1/2)})*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}-9*A*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c), 2^{(1/2)})*a^3-45*A*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c), 2^{(1/2)})*a*b^2+5*a^3*B*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c), 2^{(1/2)})*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}+45*B*a*b^2*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c), 2^{(1/2)})*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}-45*B*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c), 2^{(1/2)}) \end{aligned}$$

$n(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}$
 $*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)})*a^2$
 $*b+15*B*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}$
 $*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)})$
 $*b^3)/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/\sin(1/2*d*x+1/2*c)$
 $/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(a + \frac{b}{\cos(c+dx)}\right)^3}{\left(\frac{1}{\cos(c+dx)}\right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^3)/(1/cos(c + d*x))^(5/2),x)

[Out] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^3)/(1/cos(c + d*x))^(5/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) (a + b \sec(c + dx))^3}{\sec^{\frac{5}{2}}(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))*3*(A+B*sec(d*x+c))/sec(d*x+c)**(5/2),x)

[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))*3/sec(c + d*x)**(5/2), x)

$$3.412 \quad \int \frac{(a+b \sec(c+dx))^3 (A+B \sec(c+dx))}{\sec^{\frac{7}{2}}(c+dx)} dx$$

Optimal. Leaf size=245

$$\frac{2a(5a^2A + 21abB + 18Ab^2) \sin(c + dx)}{21d\sqrt{\sec(c + dx)}} + \frac{2a^2(7aB + 11Ab) \sin(c + dx)}{35d \sec^{\frac{3}{2}}(c + dx)} + \frac{2(5a^3A + 21a^2bB + 21aAb^2 + 21b^3B)}{21d}$$

[Out] $2/35*a^2*(11*A*b+7*B*a)*\sin(d*x+c)/d/\sec(d*x+c)^{(3/2)}+2/7*a*A*(a+b*\sec(d*x+c))^{2*\sin(d*x+c)/d/\sec(d*x+c)^{(5/2)}+2/21*a*(5*A*a^2+18*A*b^2+21*B*a*b)*\sin(d*x+c)/d/\sec(d*x+c)^{(1/2)}+2/5*(9*A*a^2*b+5*A*b^3+3*B*a^3+15*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^{(1/2)}/cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c),2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d+2/21*(5*A*a^3+21*A*a*b^2+21*B*a^2*b+21*B*b^3)*(cos(1/2*d*x+1/2*c)^2)^{(1/2)}/cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c),2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d$

Rubi [A] time = 0.47, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {4025, 4074, 4047, 3771, 2639, 4045, 2641}

$$\frac{2a(5a^2A + 21abB + 18Ab^2) \sin(c + dx)}{21d\sqrt{\sec(c + dx)}} + \frac{2(5a^3A + 21a^2bB + 21aAb^2 + 21b^3B) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} F}{21d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\frac{(a + b*\text{Sec}[c + d*x])^3*(A + B*\text{Sec}[c + d*x])}{\text{Sec}[c + d*x]^{(7/2)}}, x]$

[Out] $(2*(9*a^2*A*b + 5*A*b^3 + 3*a^3*B + 15*a*b^2*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/(5*d) + (2*(5*a^3*A + 21*a*A*b^2 + 21*a^2*b*B + 21*b^3*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticF}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/(21*d) + (2*a^2*(11*A*b + 7*a*B)*\text{Sin}[c + d*x])/(35*d*\text{Sec}[c + d*x]^{(3/2)}) + (2*a*(5*a^2*A + 18*A*b^2 + 21*a*b*B)*\text{Sin}[c + d*x])/(21*d*\text{Sqrt}[\text{Sec}[c + d*x]]) + (2*a*A*(a + b*\text{Sec}[c + d*x])^2*\text{Sin}[c + d*x])/(7*d*\text{Sec}[c + d*x]^{(5/2)})$

Rule 2639

$\text{Int}[\text{Sqrt}[\sin[(c_.) + (d_.)*(x_.)]], x_Symbol] := \text{Simp}[(2*\text{EllipticE}[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}[\{c, d\}, x]$

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_.)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 3771

Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 4025

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := Simp[(a*A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e + f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LeQ[n, -1]

Rule 4045

Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]^2*(C_.) + (A_.)), x_Symbol] := Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] + Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]

Rule 4047

Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.)), x_Symbol] := Dist[B/b, Int[(b*Csc[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2), x] /; FreeQ[{b, e, f, A, B, C, m}, x]

Rule 4074

Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.))^(n_)*(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] := Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]

Rubi steps

$$\begin{aligned}
\int \frac{(a + b \sec(c + dx))^3 (A + B \sec(c + dx))}{\sec^{\frac{7}{2}}(c + dx)} dx &= \frac{2aA(a + b \sec(c + dx))^2 \sin(c + dx)}{7d \sec^{\frac{5}{2}}(c + dx)} - \frac{2}{7} \int \frac{(a + b \sec(c + dx))}{\sec^{\frac{7}{2}}(c + dx)} dx \\
&= \frac{2a^2(11Ab + 7aB) \sin(c + dx)}{35d \sec^{\frac{3}{2}}(c + dx)} + \frac{2aA(a + b \sec(c + dx))^2 \sin(c + dx)}{7d \sec^{\frac{5}{2}}(c + dx)} \\
&= \frac{2a^2(11Ab + 7aB) \sin(c + dx)}{35d \sec^{\frac{3}{2}}(c + dx)} + \frac{2aA(a + b \sec(c + dx))^2 \sin(c + dx)}{7d \sec^{\frac{5}{2}}(c + dx)} \\
&= \frac{2a^2(11Ab + 7aB) \sin(c + dx)}{35d \sec^{\frac{3}{2}}(c + dx)} + \frac{2a(5a^2A + 18Ab^2 + 21abB) \sin(c + dx)}{21d \sqrt{\sec(c + dx)}} \\
&= \frac{2(9a^2Ab + 5Ab^3 + 3a^3B + 15ab^2B) \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx)\right)}{5d} \\
&= \frac{2(9a^2Ab + 5Ab^3 + 3a^3B + 15ab^2B) \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx)\right)}{5d}
\end{aligned}$$

Mathematica [A] time = 1.36, size = 180, normalized size = 0.73

$$\frac{\sqrt{\sec(c + dx)} \left(a \sin(2(c + dx)) \left(5(3a^2A \cos(2(c + dx)) + 13a^2A + 42abB + 42Ab^2) + 42a(ab + 3Ab) \cos(c + dx) \right) \right)}{5d}$$

Antiderivative was successfully verified.

[In] Integrate[(((a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x])))/Sec[c + d*x]^(7/2), x]

[Out] (Sqrt[Sec[c + d*x]]*(84*(9*a^2*A*b + 5*A*b^3 + 3*a^3*B + 15*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 20*(5*a^3*A + 21*a*A*b^2 + 21*a^2*b*B + 21*b^3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + a*(42*a*(3*A*b + a*B)*Cos[c + d*x] + 5*(13*a^2*A + 42*A*b^2 + 42*a*b*B + 3*a^2*A*Cos[2*(c + d*x)]))*Sin[2*(c + d*x)]))/(210*d)

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{Bb^3 \sec(dx + c)^4 + Aa^3 + (3Bab^2 + Ab^3) \sec(dx + c)^3 + 3(Ba^2b + Aab^2) \sec(dx + c)^2 + (Ba^3 + 3Aab) \sec(dx + c)}{\sec(dx + c)^{\frac{7}{2}}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm="fricas")

[Out] integral((B*b^3*sec(d*x + c)^4 + A*a^3 + (3*B*a*b^2 + A*b^3)*sec(d*x + c)^3 + 3*(B*a^2*b + A*a*b^2)*sec(d*x + c)^2 + (B*a^3 + 3*A*a^2*b)*sec(d*x + c)) /sec(d*x + c)^(7/2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(b \sec(dx + c) + a)^3}{\sec(dx + c)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^3/sec(d*x + c)^(7/2), x)

maple [B] time = 4.50, size = 664, normalized size = 2.71

$$2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \left(240Aa^3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + (-360Aa^3 - 504Aa^2b - 168Ba^3) \sin\left(\frac{dx}{2} + \frac{c}{2}\right) + (280Aa^3 + 504Aa^2b + 420Aa*b^2 + 168Ba^3 + 420B*a^2*b) \sin^3\left(\frac{dx}{2} + \frac{c}{2}\right) + (-80Aa^3 - 126Aa^2*b - 210Aa*b^2 - 42Ba^3 - 210B*a^2*b) \sin^5\left(\frac{dx}{2} + \frac{c}{2}\right) - 189A\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^2 \sqrt{\left(2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \right) \operatorname{EllipticE}\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), 2^{\frac{1}{2}}\right) + 105Aa^2b \sqrt{\left(2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \operatorname{EllipticE}\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), 2^{\frac{1}{2}}\right) + b^3 + 25Aa^3 \sqrt{\left(2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \operatorname{EllipticF}\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), 2^{\frac{1}{2}}\right) + 105Aa^2b \sqrt{\left(2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \operatorname{EllipticF}\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), 2^{\frac{1}{2}}\right) - 63B \sqrt{\left(2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \operatorname{EllipticE}\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), 2^{\frac{1}{2}}\right) + a^3 - 315B \sqrt{\left(2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \operatorname{EllipticE}\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), 2^{\frac{1}{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x)

[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*A*a^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-360*A*a^3-504*A*a^2*b-168*B*a^3)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(280*A*a^3+504*A*a^2*b+420*A*a*b^2+168*B*a^3+420*B*a^2*b)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-80*A*a^3-126*A*a^2*b-210*A*a*b^2-42*B*a^3-210*B*a^2*b)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-189*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b-105*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^3+25*A*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+105*A*a^2*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-63*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3-315*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))

```
in(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2+10
5*a^2*b*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))+105*b^3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(
1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(a + \frac{b}{\cos(c+dx)}\right)^3}{\left(\frac{1}{\cos(c+dx)}\right)^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^3)/(1/cos(c + d*x))^(7/2),x)

[Out] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^3)/(1/cos(c + d*x))^(7/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx))(a + b \sec(c + dx))^3}{\sec^{\frac{7}{2}}(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))*3*(A+B*sec(d*x+c))/sec(d*x+c)**(7/2),x)

[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))*3/sec(c + d*x)**(7/2), x)

$$3.413 \quad \int \frac{(a+b \sec(c+dx))^3 (A+B \sec(c+dx))}{9 \sec^2(c+dx)} dx$$

Optimal. Leaf size=295

$$\frac{2a(7a^2A + 27abB + 22Ab^2) \sin(c+dx)}{45d \sec^3(c+dx)} + \frac{2a^2(9aB + 13Ab) \sin(c+dx)}{63d \sec^5(c+dx)} + \frac{2(5a^3B + 15a^2Ab + 21ab^2B + 7Ab^3) \sin(c+dx)}{21d \sqrt{\sec(c+dx)}}$$

[Out] $2/63*a^2*(13*A*b+9*B*a)*\sin(d*x+c)/d/\sec(d*x+c)^{(5/2)}+2/45*a*(7*A*a^2+22*A*b^2+27*B*a*b)*\sin(d*x+c)/d/\sec(d*x+c)^{(3/2)}+2/9*a*A*(a+b*\sec(d*x+c))^2*\sin(d*x+c)/d/\sec(d*x+c)^{(7/2)}+2/21*(15*A*a^2*b+7*A*b^3+5*B*a^3+21*B*a*b^2)*\sin(d*x+c)/d/\sec(d*x+c)^{(1/2)}+2/15*(7*A*a^3+27*A*a*b^2+27*B*a^2*b+15*B*b^3)*(cos(1/2*d*x+1/2*c))^2/(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d+2/21*(15*A*a^2*b+7*A*b^3+5*B*a^3+21*B*a*b^2)*(cos(1/2*d*x+1/2*c))^2/(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d$

Rubi [A] time = 0.54, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {4025, 4074, 4047, 3769, 3771, 2641, 4045, 2639}

$$\frac{2a(7a^2A + 27abB + 22Ab^2) \sin(c+dx)}{45d \sec^3(c+dx)} + \frac{2(15a^2Ab + 5a^3B + 21ab^2B + 7Ab^3) \sin(c+dx)}{21d \sqrt{\sec(c+dx)}} + \frac{2(15a^2Ab + 5a^3B + 21ab^2B + 7Ab^3) \sin(c+dx)}{21d \sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[((a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(9/2), x]

[Out] $(2*(7*a^3*A + 27*a*A*b^2 + 27*a^2*b*B + 15*b^3*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/(15*d) + (2*(15*a^2*A*b + 7*A*b^3 + 5*a^3*B + 21*a*b^2*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticF}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/(21*d) + (2*a^2*(13*A*b + 9*a*B)*\text{Sin}[c + d*x])/(63*d*\text{Sec}[c + d*x]^{(5/2)}) + (2*a*(7*a^2*A + 22*A*b^2 + 27*a*b*B)*\text{Sin}[c + d*x])/(45*d*\text{Sec}[c + d*x]^{(3/2)}) + (2*(15*a^2*A*b + 7*A*b^3 + 5*a^3*B + 21*a*b^2*B)*\text{Sin}[c + d*x])/(21*d*\text{Sqrt}[\text{Sec}[c + d*x]]) + (2*a*A*(a + b*\text{Sec}[c + d*x])^2*\text{Sin}[c + d*x])/(9*d*\text{Sec}[c + d*x]^{(7/2)})$

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 3769

Int[(csc[(c_.) + (d_.)*(x_)])*(b_.))^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 3771

Int[(csc[(c_.) + (d_.)*(x_)])*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 4025

Int[(csc[(e_.) + (f_.)*(x_)])*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)])*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_)])*(B_.) + (A_.)), x_Symbol] := Simp[(a*A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e + f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LeQ[n, -1]

Rule 4045

Int[(csc[(e_.) + (f_.)*(x_)])*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)])^2*(C_.) + (A_.)), x_Symbol] := Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] + Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]

Rule 4047

Int[(csc[(e_.) + (f_.)*(x_)])*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)])*(B_.) + csc[(e_.) + (f_.)*(x_)])^2*(C_.)), x_Symbol] := Dist[B/b, Int[(b*Csc[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2), x] /; FreeQ[{b, e, f, A, B, C, m}, x]

Rule 4074

Int[((A_.) + csc[(e_.) + (f_.)*(x_)])*(B_.) + csc[(e_.) + (f_.)*(x_)])^2*(C_.) + (a_.)*(csc[(e_.) + (f_.)*(x_)])*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)])*(b_.) + (a_.))^(m_)), x_Symbol] := Simp[(a*(A + C*Csc[e + f*x]^2)*(b*Csc[e + f*x])^m)/(f*m), x] + Dist[(a*(d*Csc[e + f*x])^n*(a + b*Csc[e + f*x])^(m + 1))/(b*d*n), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a*(d*Csc[e + f*x])^n*(a + b*Csc[e + f*x])^(m + 1), 0] && GtQ[m, 1] && LeQ[n, -1]

_), x_Symbol] := Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]

Rubi steps

$$\begin{aligned}
 \int \frac{(a + b \sec(c + dx))^3 (A + B \sec(c + dx))}{\sec^{\frac{9}{2}}(c + dx)} dx &= \frac{2aA(a + b \sec(c + dx))^2 \sin(c + dx)}{9d \sec^{\frac{7}{2}}(c + dx)} - \frac{2}{9} \int \frac{(a + b \sec(c + dx))}{\sec^{\frac{7}{2}}(c + dx)} dx \\
 &= \frac{2a^2(13Ab + 9aB) \sin(c + dx)}{63d \sec^{\frac{5}{2}}(c + dx)} + \frac{2aA(a + b \sec(c + dx))^2 \sin(c + dx)}{9d \sec^{\frac{7}{2}}(c + dx)} \\
 &= \frac{2a^2(13Ab + 9aB) \sin(c + dx)}{63d \sec^{\frac{5}{2}}(c + dx)} + \frac{2aA(a + b \sec(c + dx))^2 \sin(c + dx)}{9d \sec^{\frac{7}{2}}(c + dx)} \\
 &= \frac{2a^2(13Ab + 9aB) \sin(c + dx)}{63d \sec^{\frac{5}{2}}(c + dx)} + \frac{2a(7a^2A + 22Ab^2 + 27abB) \sin(c + dx)}{45d \sec^{\frac{3}{2}}(c + dx)} \\
 &= \frac{2a^2(13Ab + 9aB) \sin(c + dx)}{63d \sec^{\frac{5}{2}}(c + dx)} + \frac{2a(7a^2A + 22Ab^2 + 27abB) \sin(c + dx)}{45d \sec^{\frac{3}{2}}(c + dx)} \\
 &= \frac{2(7a^3A + 27aAb^2 + 27a^2bB + 15b^3B) \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx)\right)}{15d}
 \end{aligned}$$

Mathematica [A] time = 2.03, size = 219, normalized size = 0.74

$$\sqrt{\sec(c + dx)} \left(\sin(2(c + dx)) \left(7a(43a^2A + 108abB + 108Ab^2) \cos(c + dx) + 5(7a^3A \cos(3(c + dx)) + 78a^3B + 15b^3B) \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(9/2), x]

[Out] (Sqrt[Sec[c + d*x]]*(168*(7*a^3*A + 27*a*A*b^2 + 27*a^2*b*B + 15*b^3*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 120*(15*a^2*A*b + 7*A*b^3 + 5*a^3*B + 21*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (7*a*(43*a^2*A + 108*A*b^2 + 108*a*b*B)*Cos[c + d*x] + 5*(234*a^2*A*b + 84*A*b^3 + 7

$8a^3B + 252ab^2B + 18a^2(3Ab + aB)\cos[2(c + dx)] + 7a^3A\cos[3(c + dx)]\sin[2(c + dx)]/(1260d)$

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{Bb^3 \sec(dx + c)^4 + Aa^3 + (3Bab^2 + Ab^3) \sec(dx + c)^3 + 3(Ba^2b + Aab^2) \sec(dx + c)^2 + (Ba^3 + 3Aa^2b) \sec(dx + c)}{\sec(dx + c)^{\frac{9}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x, algorithm="fricas")

[Out] integral((B*b^3*sec(d*x + c)^4 + A*a^3 + (3*B*a*b^2 + A*b^3)*sec(d*x + c)^3 + 3*(B*a^2*b + A*a*b^2)*sec(d*x + c)^2 + (B*a^3 + 3*A*a^2*b)*sec(d*x + c))/sec(d*x + c)^(9/2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(b \sec(dx + c) + a)^3}{\sec(dx + c)^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^3/sec(d*x + c)^(9/2), x)

maple [B] time = 4.76, size = 745, normalized size = 2.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x)

[Out] $-2/315*((2*\cos(1/2*d*x+1/2*c)^2-1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-1120*A*a^3*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^{10}+(2240*A*a^3+2160*A*a^2*b+720*B*a^3)*\sin(1/2*d*x+1/2*c)^8*\cos(1/2*d*x+1/2*c)+(-2072*A*a^3-3240*A*a^2*b-1512*A*a*b^2-1080*B*a^3-1512*B*a^2*b)*\sin(1/2*d*x+1/2*c)^6*\cos(1/2*d*x+1/2*c)+(952*A*a^3+2520*A*a^2*b+1512*A*a*b^2+420*A*b^3+840*B*a^3+1512*B*a^2*b+1260*B*a*b^2)*\sin(1/2*d*x+1/2*c)^4*\cos(1/2*d*x+1/2*c)+(-168*A*a^3-720*A*a^2*b-378*A*a*b^2-210*A*b^3-240*B*a^3-378*B*a^2*b-630*B*a*b^2)*\sin(1/2*d*x+1/2*c)^2*\cos(1/2*d*x+1/2*c)+225*A*a^2*b*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}$

$$2*c)^{2-1})^{1/2} * \text{EllipticF}(\cos(1/2*d*x+1/2*c), 2^{1/2}) + 105*A*b^3 * (\sin(1/2*d*x+1/2*c)^2)^{1/2} * (2*\sin(1/2*d*x+1/2*c)^{2-1})^{1/2} * \text{EllipticF}(\cos(1/2*d*x+1/2*c), 2^{1/2}) - 147*A * (\sin(1/2*d*x+1/2*c)^2)^{1/2} * (2*\sin(1/2*d*x+1/2*c)^{2-1})^{1/2} * \text{EllipticE}(\cos(1/2*d*x+1/2*c), 2^{1/2}) * a^3 - 567*A * (\sin(1/2*d*x+1/2*c)^2)^{1/2} * (2*\sin(1/2*d*x+1/2*c)^{2-1})^{1/2} * \text{EllipticE}(\cos(1/2*d*x+1/2*c), 2^{1/2}) * a*b^2 + 75*a^3*B * (\sin(1/2*d*x+1/2*c)^2)^{1/2} * (2*\sin(1/2*d*x+1/2*c)^{2-1})^{1/2} * \text{EllipticF}(\cos(1/2*d*x+1/2*c), 2^{1/2}) + 315*B*a*b^2 * (\sin(1/2*d*x+1/2*c)^2)^{1/2} * (2*\sin(1/2*d*x+1/2*c)^{2-1})^{1/2} * \text{EllipticF}(\cos(1/2*d*x+1/2*c), 2^{1/2}) - 567*B * (\sin(1/2*d*x+1/2*c)^2)^{1/2} * (2*\sin(1/2*d*x+1/2*c)^{2-1})^{1/2} * \text{EllipticE}(\cos(1/2*d*x+1/2*c), 2^{1/2}) * a^2*b - 315*B * (\sin(1/2*d*x+1/2*c)^2)^{1/2} * (2*\sin(1/2*d*x+1/2*c)^{2-1})^{1/2} * \text{EllipticE}(\cos(1/2*d*x+1/2*c), 2^{1/2}) * b^3 / (-2*\sin(1/2*d*x+1/2*c)^4 + \sin(1/2*d*x+1/2*c)^2)^{1/2} / \sin(1/2*d*x+1/2*c) / (2*\cos(1/2*d*x+1/2*c)^{2-1})^{1/2} / d$$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(a + \frac{b}{\cos(c+dx)}\right)^3}{\left(\frac{1}{\cos(c+dx)}\right)^{9/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^3)/(1/cos(c + d*x))^(9/2),x)

[Out] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^3)/(1/cos(c + d*x))^(9/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))*3*(A+B*sec(d*x+c))/sec(d*x+c)**(9/2),x)

[Out] Timed out

$$3.414 \quad \int \frac{(a+b \sec(c+dx))^3 (A+B \sec(c+dx))}{\frac{11}{\sec^2(c+dx)}} dx$$

Optimal. Leaf size=345

$$\frac{2a(9a^2A + 33abB + 26Ab^2) \sin(c+dx)}{77d \sec^{\frac{5}{2}}(c+dx)} + \frac{2a^2(11aB + 15Ab) \sin(c+dx)}{99d \sec^{\frac{7}{2}}(c+dx)} + \frac{2(7a^3B + 21a^2Ab + 27ab^2B + 9Ab^3) \sin(c+dx)}{45d \sec^{\frac{3}{2}}(c+dx)}$$

[Out] $2/99*a^2*(15*A*b+11*B*a)*\sin(d*x+c)/d/\sec(d*x+c)^{(7/2)}+2/77*a*(9*A*a^2+26*A*b^2+33*B*a*b)*\sin(d*x+c)/d/\sec(d*x+c)^{(5/2)}+2/45*(21*A*a^2*b+9*A*b^3+7*B*a^3+27*B*a*b^2)*\sin(d*x+c)/d/\sec(d*x+c)^{(3/2)}+2/11*a*A*(a+b*\sec(d*x+c))^2*\sin(d*x+c)/d/\sec(d*x+c)^{(9/2)}+2/231*(45*A*a^3+165*A*a*b^2+165*B*a^2*b+77*B*b^3)*\sin(d*x+c)/d/\sec(d*x+c)^{(1/2)}+2/15*(21*A*a^2*b+9*A*b^3+7*B*a^3+27*B*a*b^2)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d+2/231*(45*A*a^3+165*A*a*b^2+165*B*a^2*b+77*B*b^3)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d$

Rubi [A] time = 0.57, antiderivative size = 345, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {4025, 4074, 4047, 3769, 3771, 2639, 4045, 2641}

$$\frac{2a(9a^2A + 33abB + 26Ab^2) \sin(c+dx)}{77d \sec^{\frac{5}{2}}(c+dx)} + \frac{2(21a^2Ab + 7a^3B + 27ab^2B + 9Ab^3) \sin(c+dx)}{45d \sec^{\frac{3}{2}}(c+dx)} + \frac{2(45a^3A + 165a^2bB + 165a^2b^2B + 77ab^3B) \sin(c+dx)}{45d \sec^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\frac{(a + b*\text{Sec}[c + d*x])^3*(A + B*\text{Sec}[c + d*x])}{\text{Sec}[c + d*x]^{(11/2)}}, x]$

[Out] $(2*(21*a^2*A*b + 9*A*b^3 + 7*a^3*B + 27*a*b^2*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/(15*d) + (2*(45*a^3*A + 165*a^2*b*B + 165*a^2*b^2*B + 77*b^3*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticF}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/(231*d) + (2*a^2*(15*A*b + 11*a*B)*\text{Sin}[c + d*x])/(99*d*\text{Sec}[c + d*x]^{(7/2)}) + (2*a*(9*a^2*A + 26*A*b^2 + 33*a*b*B)*\text{Sin}[c + d*x])/(77*d*\text{Sec}[c + d*x]^{(5/2)}) + (2*(21*a^2*A*b + 9*A*b^3 + 7*a^3*B + 27*a*b^2*B)*\text{Sin}[c + d*x])/(45*d*\text{Sec}[c + d*x]^{(3/2)}) + (2*(45*a^3*A + 165*a^2*b*B + 165*a^2*b^2*B + 77*b^3*B)*\text{Sin}[c + d*x])/(231*d*\text{Sqrt}[\text{Sec}[c + d*x]]) + (2*a*A*(a + b*\text{Sec}[c + d*x])^2*\text{Sin}[c + d*x])/(11*d*\text{Sec}[c + d*x]^{(9/2)})$

Rule 2639

$\text{Int}[\text{Sqrt}[\sin[(c_.) + (d_.)*(x_.)]], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticE}[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}[\{c, d\}, x]$

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_.)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 3769

Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.))^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 3771

Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 4025

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := Simp[(a*A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e + f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LeQ[n, -1]

Rule 4045

Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.))^(m_)*(csc[(e_.) + (f_.)*(x_.)]^2*(C_.) + (A_.)), x_Symbol] := Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] + Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]

Rule 4047

Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.))^(m_)*((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.)), x_Symbol] := Dist[B/b, Int[(b*Csc[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2), x] /; FreeQ[{b, e, f, A, B, C, m}, x]

Rule 4074

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)])*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*csc[(e_.) + (f_.)*(x_)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + b \sec(c + dx))^3 (A + B \sec(c + dx))}{\sec^{\frac{11}{2}}(c + dx)} dx &= \frac{2aA(a + b \sec(c + dx))^2 \sin(c + dx)}{11d \sec^{\frac{9}{2}}(c + dx)} - \frac{2}{11} \int \frac{(a + b \sec(c + dx))}{\sec^{\frac{9}{2}}(c + dx)} dx \\
&= \frac{2a^2(15Ab + 11aB) \sin(c + dx)}{99d \sec^{\frac{7}{2}}(c + dx)} + \frac{2aA(a + b \sec(c + dx))^2 \sin(c + dx)}{11d \sec^{\frac{9}{2}}(c + dx)} \\
&= \frac{2a^2(15Ab + 11aB) \sin(c + dx)}{99d \sec^{\frac{7}{2}}(c + dx)} + \frac{2aA(a + b \sec(c + dx))^2 \sin(c + dx)}{11d \sec^{\frac{9}{2}}(c + dx)} \\
&= \frac{2a^2(15Ab + 11aB) \sin(c + dx)}{99d \sec^{\frac{7}{2}}(c + dx)} + \frac{2a(9a^2A + 26Ab^2 + 33abB) \sin(c + dx)}{77d \sec^{\frac{5}{2}}(c + dx)} \\
&= \frac{2a^2(15Ab + 11aB) \sin(c + dx)}{99d \sec^{\frac{7}{2}}(c + dx)} + \frac{2a(9a^2A + 26Ab^2 + 33abB) \sin(c + dx)}{77d \sec^{\frac{5}{2}}(c + dx)} \\
&= \frac{2(21a^2Ab + 9Ab^3 + 7a^3B + 27ab^2B) \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx)\right)}{15d} \\
&= \frac{2(21a^2Ab + 9Ab^3 + 7a^3B + 27ab^2B) \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx)\right)}{15d}
\end{aligned}$$

Mathematica [A] time = 3.23, size = 256, normalized size = 0.74

$$\frac{\sqrt{\sec(c + dx)} \left(\sin(2(c + dx)) (180a (16a^2A + 33abB + 33Ab^2) \cos(2(c + dx)) + 770a^2(aB + 3Ab) \cos(3(c + dx))) \right)}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(11/2), x]
```

[Out] (Sqrt[Sec[c + d*x]]*(3696*(21*a^2*A*b + 9*A*b^3 + 7*a^3*B + 27*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 240*(45*a^3*A + 165*a*A*b^2 + 165*a^2*b*B + 77*b^3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (154*(129*a^2*A*b + 36*A*b^3 + 43*a^3*B + 108*a*b^2*B)*Cos[c + d*x] + 180*a*(16*a^2*A + 33*A*b^2 + 33*a*b*B)*Cos[2*(c + d*x)] + 770*a^2*(3*A*b + a*B)*Cos[3*(c + d*x)] + 15*(531*a^3*A + 1716*a*A*b^2 + 1716*a^2*b*B + 616*b^3*B + 21*a^3*A*Cos[4*(c + d*x)]))*Sin[2*(c + d*x)])/(27720*d)

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{Bb^3 \sec(dx + c)^4 + Aa^3 + (3Bab^2 + Ab^3) \sec(dx + c)^3 + 3(Ba^2b + Aab^2) \sec(dx + c)^2 + (Ba^3 + 3Aa^2b) \sec(dx + c)}{\sec(dx + c)^{\frac{11}{2}}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(11/2),x, algorithm="fricas")

[Out] integral((B*b^3*sec(d*x + c)^4 + A*a^3 + (3*B*a*b^2 + A*b^3)*sec(d*x + c)^3 + 3*(B*a^2*b + A*a*b^2)*sec(d*x + c)^2 + (B*a^3 + 3*A*a^2*b)*sec(d*x + c))/sec(d*x + c)^(11/2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(b \sec(dx + c) + a)^3}{\sec(dx + c)^{\frac{11}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(11/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^3/sec(d*x + c)^(11/2), x)

maple [B] time = 4.81, size = 825, normalized size = 2.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(11/2),x)

[Out] -2/3465*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(20160*A*a^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12+(-50400*A*a^3-36960*A*a^2*b-12320*B*a^3)*sin(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+(56880*A*a^3+73920*A*a^2*

$b+23760Aab^2+24640Bba^3+23760Bba^2b) \sin(1/2dx+1/2c)^8 \cos(1/2dx+1/2c) + (-34920Aa^3-68376Aa^2b-35640Aab^2-5544Ab^3-22792Bba^3-35640Bba^2b-16632Bba^2b) \sin(1/2dx+1/2c)^6 \cos(1/2dx+1/2c) + (13860Aa^3+31416Aa^2b+27720Aab^2+5544Ab^3+10472Bba^3+27720Bba^2b+16632Bba^2b+4620Bb^3) \sin(1/2dx+1/2c)^4 \cos(1/2dx+1/2c) + (-2790Aa^3-5544Aa^2b-7920Aab^2-1386Ab^3-1848Bba^3-7920Bba^2b-4158Bba^2b-2310Bb^3) \sin(1/2dx+1/2c)^2 \cos(1/2dx+1/2c) + 675Aa^3 (\sin(1/2dx+1/2c)^2)^{(1/2)} (2 \sin(1/2dx+1/2c)^2 - 1)^{(1/2)} \text{EllipticF}(\cos(1/2dx+1/2c), 2^{(1/2)}) + 2475Aa^2b (\sin(1/2dx+1/2c)^2)^{(1/2)} (2 \sin(1/2dx+1/2c)^2 - 1)^{(1/2)} \text{EllipticF}(\cos(1/2dx+1/2c), 2^{(1/2)}) - 4851A (\sin(1/2dx+1/2c)^2)^{(1/2)} (2 \sin(1/2dx+1/2c)^2 - 1)^{(1/2)} \text{EllipticE}(\cos(1/2dx+1/2c), 2^{(1/2)}) + a^2b - 2079A (\sin(1/2dx+1/2c)^2)^{(1/2)} (2 \sin(1/2dx+1/2c)^2 - 1)^{(1/2)} \text{EllipticE}(\cos(1/2dx+1/2c), 2^{(1/2)}) + b^3 + 2475a^2bB (\sin(1/2dx+1/2c)^2)^{(1/2)} (2 \sin(1/2dx+1/2c)^2 - 1)^{(1/2)} \text{EllipticF}(\cos(1/2dx+1/2c), 2^{(1/2)}) + 1155b^3B (\sin(1/2dx+1/2c)^2)^{(1/2)} (2 \sin(1/2dx+1/2c)^2 - 1)^{(1/2)} \text{EllipticF}(\cos(1/2dx+1/2c), 2^{(1/2)}) - 1617B (\sin(1/2dx+1/2c)^2)^{(1/2)} (2 \sin(1/2dx+1/2c)^2 - 1)^{(1/2)} \text{EllipticE}(\cos(1/2dx+1/2c), 2^{(1/2)}) + a^3 - 6237B (\sin(1/2dx+1/2c)^2)^{(1/2)} (2 \sin(1/2dx+1/2c)^2 - 1)^{(1/2)} \text{EllipticE}(\cos(1/2dx+1/2c), 2^{(1/2)}) + a^2b^2 / (-2 \sin(1/2dx+1/2c)^4 + \sin(1/2dx+1/2c)^2)^{(1/2)} / \sin(1/2dx+1/2c) / (2 \cos(1/2dx+1/2c)^2 - 1)^{(1/2)}$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(11/2),x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(a + \frac{b}{\cos(c+dx)}\right)^3}{\left(\frac{1}{\cos(c+dx)}\right)^{11/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^3)/(1/cos(c + d*x))^(11/2),x)

[Out] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^3)/(1/cos(c + d*x))^(11/2),x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))**3*(A+B*sec(d*x+c))/sec(d*x+c)**(11/2),x)

[Out] Timed out

$$3.415 \quad \int \frac{\sec^2(c+dx)(A+B \sec(c+dx))}{a+b \sec(c+dx)} dx$$

Optimal. Leaf size=277

$$\frac{2a^2(Ab - aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Pi\left(\frac{2a}{a+b}; \frac{1}{2}(c+dx)\right)}{b^3d(a+b)} - \frac{2(-5a^2B + 5aAb - 3b^2B)\sin(c+dx)\sqrt{\sec(c+dx)}}{5b^3d}$$

[Out] $2/3*(A*b-B*a)*\sec(d*x+c)^{(3/2)}*\sin(d*x+c)/b^2/d+2/5*B*\sec(d*x+c)^{(5/2)}*\sin(d*x+c)/b/d-2/5*(5*A*a*b-5*B*a^2-3*B*b^2)*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/b^3/d+2/5*(5*A*a*b-5*B*a^2-3*B*b^2)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/b^3/d+2/3*(A*b-B*a)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/b^2/d+2*a^2*(A*b-B*a)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticPi}(\sin(1/2*d*x+1/2*c), 2*a/(a+b), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/b^3/(a+b)/d$

Rubi [A] time = 1.01, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {4033, 4102, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$-\frac{2(-5a^2B + 5aAb - 3b^2B)\sin(c+dx)\sqrt{\sec(c+dx)}}{5b^3d} + \frac{2(-5a^2B + 5aAb - 3b^2B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{2a}{a+b}; \frac{1}{2}(c+dx)\right)}{5b^3d}$$

Antiderivative was successfully verified.

[In] Int[(Sec[c + d*x]^(7/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x]),x]

[Out] $(2*(5*a*A*b - 5*a^2*B - 3*b^2*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/(5*b^3*d) + (2*(A*b - a*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticF}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/(3*b^2*d) + (2*a^2*(A*b - a*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticPi}[(2*a)/(a + b), (c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/(b^3*(a + b)*d) - (2*(5*a*A*b - 5*a^2*B - 3*b^2*B)*\text{Sqrt}[\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(5*b^3*d) + (2*(A*b - a*B)*\text{Sec}[c + d*x]^{(3/2)}*\text{Sin}[c + d*x])/(3*b^2*d) + (2*B*\text{Sec}[c + d*x]^{(5/2)}*\text{Sin}[c + d*x])/(5*b*d)$

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2805

Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi/2 + f*x))/2, (2*d)/(c + d)]/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[c + d, 0]

Rule 3771

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^n, x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 3849

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 4033

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^m*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] := -Simp[(B*d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2))/(b*f*(m + n)), x] + Dist[d^2/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 2)*Simp[a*B*(n - 2) + B*b*(m + n - 1)*Csc[e + f*x] + (A*b*(m + n) - a*B*(n - 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && NeQ[m + n, 0] && !IGtQ[m, 1]

Rule 4102

Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a

`_)^(m_), x_Symbol] := -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1) * (d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 0]`

Rule 4106

`Int[((A_.) + csc[(e_.) + (f_.)*(x_)])*(B_.) + csc[(e_.) + (f_.)*(x_)])^2*(C_.)/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))), x_Symbol] := Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0]`

Rubi steps

$$\begin{aligned}
 \int \frac{\sec^{\frac{7}{2}}(c + dx)(A + B \sec(c + dx))}{a + b \sec(c + dx)} dx &= \frac{2B \sec^{\frac{5}{2}}(c + dx) \sin(c + dx)}{5bd} + \frac{2 \int \frac{\sec^{\frac{3}{2}}(c + dx) \left(\frac{3aB}{2} + \frac{3}{2}bB \sec(c + dx) + \frac{5}{2}(Ab - aB) \sec^2(c + dx) \right)}{a + b \sec(c + dx)} dx}{5b} \\
 &= \frac{2(Ab - aB) \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{3b^2d} + \frac{2B \sec^{\frac{5}{2}}(c + dx) \sin(c + dx)}{5bd} + \frac{2 \int \frac{\sec^{\frac{1}{2}}(c + dx) \left(\frac{3aB}{2} + \frac{3}{2}bB \sec(c + dx) + \frac{5}{2}(Ab - aB) \sec^2(c + dx) \right)}{a + b \sec(c + dx)} dx}{5b} \\
 &= -\frac{2(5aAb - 5a^2B - 3b^2B) \sqrt{\sec(c + dx)} \sin(c + dx)}{5b^3d} + \frac{2(Ab - aB) \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{3b^2d} + \frac{2B \sec^{\frac{5}{2}}(c + dx) \sin(c + dx)}{5bd} \\
 &= -\frac{2(5aAb - 5a^2B - 3b^2B) \sqrt{\sec(c + dx)} \sin(c + dx)}{5b^3d} + \frac{2(Ab - aB) \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{3b^2d} + \frac{2 \int \frac{\sec^{\frac{1}{2}}(c + dx) \left(\frac{3aB}{2} + \frac{3}{2}bB \sec(c + dx) + \frac{5}{2}(Ab - aB) \sec^2(c + dx) \right)}{a + b \sec(c + dx)} dx}{5b} \\
 &= -\frac{2(5aAb - 5a^2B - 3b^2B) \sqrt{\sec(c + dx)} \sin(c + dx)}{5b^3d} + \frac{2(Ab - aB) \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{3b^2d} + \frac{2 \int \frac{\sec^{\frac{1}{2}}(c + dx) \left(\frac{3aB}{2} + \frac{3}{2}bB \sec(c + dx) + \frac{5}{2}(Ab - aB) \sec^2(c + dx) \right)}{a + b \sec(c + dx)} dx}{5b} \\
 &= \frac{2a^2(Ab - aB) \sqrt{\cos(c + dx)} \Pi\left(\frac{2a}{a+b}; \frac{1}{2}(c + dx) \middle| 2\right) \sqrt{\sec(c + dx)}}{b^3(a + b)d} - \frac{2(5aAb - 5a^2B - 3b^2B) \sqrt{\sec(c + dx)} \sin(c + dx)}{5b^3d} \\
 &= \frac{2(5aAb - 5a^2B - 3b^2B) \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{\sec(c + dx)}}{5b^3d} - \frac{2(5aAb - 5a^2B - 3b^2B) \sqrt{\sec(c + dx)} \sin(c + dx)}{5b^3d}
 \end{aligned}$$

Mathematica [B] time = 6.94, size = 664, normalized size = 2.40

$$\frac{\sqrt{\sec(c+dx)} \left(\frac{2(5a^2B-5aAb+3b^2B)\sin(c+dx)}{5b^3} + \frac{2\sec(c+dx)(Ab\sin(c+dx)-aB\sin(c+dx))}{3b^2} + \frac{2B\tan(c+dx)\sec(c+dx)}{5b} \right)}{d} - \frac{2(40a^2bB-40aAb^2+}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(Sec[c + d*x]^(7/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x]),x]

[Out] -1/30*((2*(-45*a^2*A*b - 10*A*b^3 + 45*a^3*B + 19*a*b^2*B)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] - EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1])*(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(-40*a*A*b^2 + 40*a^2*b*B + 18*b^3*B)*Cos[c + d*x]^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + ((-15*a^2*A*b + 15*a^3*B + 9*a*b^2*B)*Cos[2*(c + d*x)]*(a + b*Sec[c + d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 2*a*(a - 2*b)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*a^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 4*b^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a^2*b*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2))/(b^3*d) + (Sqrt[Sec[c + d*x]]*((2*(-5*a*A*b + 5*a^2*B + 3*b^2*B)*Sin[c + d*x])/(5*b^3) + (2*Sec[c + d*x]*(A*b*Sin[c + d*x] - a*B*Sin[c + d*x]))/(3*b^2) + (2*B*Sec[c + d*x]*Tan[c + d*x])/(5*b)))/d

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx+c) + A) \sec(dx+c)^{\frac{7}{2}}}{b \sec(dx+c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(7/2)/(b*sec(d*x + c) + a), x)

maple [B] time = 15.57, size = 785, normalized size = 2.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x)

[Out]
$$\begin{aligned} & -(-(-2*\cos(1/2*d*x+1/2*c)^2+1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*(A*b-B*a)*a^3/b^3/(a^2-a*b)*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)})/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticPi(\cos(1/2*d*x+1/2*c),2*a/(a-b),2^{(1/2)})-2/5*B/b/(8*\sin(1/2*d*x+1/2*c)^6-12*\sin(1/2*d*x+1/2*c)^4+6*\sin(1/2*d*x+1/2*c)^2-1)/\sin(1/2*d*x+1/2*c)^2*(12*EllipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)})*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*\sin(1/2*d*x+1/2*c)^4-24*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^6-12*EllipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)})*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*\sin(1/2*d*x+1/2*c)^2+24*\sin(1/2*d*x+1/2*c)^4*\cos(1/2*d*x+1/2*c)+3*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)})-8*\sin(1/2*d*x+1/2*c)^2*\cos(1/2*d*x+1/2*c))*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}-2*(A*b-B*a)/b^3*a*(-(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)})+2*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^2)/\sin(1/2*d*x+1/2*c)^2/(2*\sin(1/2*d*x+1/2*c)^2-1)+2*(A*b-B*a)/b^2*(-1/6*\cos(1/2*d*x+1/2*c)*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)})/(-1/2+\cos(1/2*d*x+1/2*c)^2)^2+1/3*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)})/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)}))/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sec(dx + c)^{\frac{7}{2}}}{b \sec(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(7/2)/(b*sec(d*x + c) + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(\frac{1}{\cos(c+dx)}\right)^{7/2}}{a + \frac{b}{\cos(c+dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(7/2))/(a + b/cos(c + d*x)), x)

[Out] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(7/2))/(a + b/cos(c + d*x)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**(7/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)), x)

[Out] Timed out

$$3.416 \quad \int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B \sec(c+dx))}{a+b \sec(c+dx)} dx$$

Optimal. Leaf size=210

$$\frac{2(Ab - aB) \sin(c + dx) \sqrt{\sec(c + dx)}}{b^2 d} - \frac{2(Ab - aB) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{b^2 d} - \frac{2a(Ab - aB) \sqrt{\cos(c + dx)}}{b^2 d}$$

[Out] $\frac{2}{3} B \sec(d*x+c)^{(3/2)} * \sin(d*x+c) / b / d + 2 * (A*b - B*a) * \sin(d*x+c) * \sec(d*x+c)^{(1/2)} / b^2 / d - 2 * (A*b - B*a) * (\cos(1/2*d*x+1/2*c)^2)^{(1/2)} / \cos(1/2*d*x+1/2*c) * \text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)}) * \cos(d*x+c)^{(1/2)} * \sec(d*x+c)^{(1/2)} / b^2 / d + 2/3 * B * (\cos(1/2*d*x+1/2*c)^2)^{(1/2)} / \cos(1/2*d*x+1/2*c) * \text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)}) * \cos(d*x+c)^{(1/2)} * \sec(d*x+c)^{(1/2)} / b / d - 2 * a * (A*b - B*a) * (\cos(1/2*d*x+1/2*c)^2)^{(1/2)} / \cos(1/2*d*x+1/2*c) * \text{EllipticPi}(\sin(1/2*d*x+1/2*c), 2*a / (a+b), 2^{(1/2)}) * \cos(d*x+c)^{(1/2)} * \sec(d*x+c)^{(1/2)} / b^2 / (a+b) / d$

Rubi [A] time = 0.71, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {4033, 4102, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{2(Ab - aB) \sin(c + dx) \sqrt{\sec(c + dx)}}{b^2 d} - \frac{2(Ab - aB) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{b^2 d} - \frac{2a(Ab - aB) \sqrt{\cos(c + dx)}}{b^2 d}$$

Antiderivative was successfully verified.

[In] Int[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x]),x]

[Out] $(-2*(A*b - a*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/(b^2*d) + (2*B*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticF}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/(3*b*d) - (2*a*(A*b - a*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticPi}[(2*a)/(a + b), (c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/(b^2*(a + b)*d) + (2*(A*b - a*B)*\text{Sqrt}[\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(b^2*d) + (2*B*\text{Sec}[c + d*x]^{(3/2)}*\text{Sin}[c + d*x])/(3*b*d)$

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c -
Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])), x_Symbol] := Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)]/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^n, x_Symbol] := Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_.)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_.)), x_Symbol] := Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4033

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_.))^m*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] := -Simp[(B*d^2
*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2))/(b*f*(
m + n)), x] + Dist[d^2/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f
*x])^(n - 2)*Simp[a*B*(n - 2) + B*b*(m + n - 1)*Csc[e + f*x] + (A*b*(m + n)
- a*B*(n - 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m
}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && NeQ[m + n,
0] && !IGtQ[m, 1]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
.))^m, x_Symbol] := -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
```

$*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e + f*x]^2, x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] \&\& NeQ[a^2 - b^2, 0] \&\& GtQ[n, 0]$

Rule 4106

$Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.)))/(Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))), x_Symbol] := Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] \&\& NeQ[a^2 - b^2, 0]$

Rubi steps

$$\begin{aligned} \int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B \sec(c+dx))}{a+b \sec(c+dx)} dx &= \frac{2B \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{3bd} + \frac{2 \int \frac{\sqrt{\sec(c+dx)} \left(\frac{aB}{2} + \frac{1}{2}bB \sec(c+dx) + \frac{3}{2}(Ab-aB) \sec^2(c+dx) \right)}{a+b \sec(c+dx)} dx}{3b} \\ &= \frac{2(Ab-aB)\sqrt{\sec(c+dx)} \sin(c+dx)}{b^2d} + \frac{2B \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{3bd} + \dots \\ &= \frac{2(Ab-aB)\sqrt{\sec(c+dx)} \sin(c+dx)}{b^2d} + \frac{2B \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{3bd} + \dots \\ &= \frac{2(Ab-aB)\sqrt{\sec(c+dx)} \sin(c+dx)}{b^2d} + \frac{2B \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{3bd} + \dots \\ &= -\frac{2a(Ab-aB)\sqrt{\cos(c+dx)} \Pi\left(\frac{2a}{a+b}; \frac{1}{2}(c+dx) \middle| 2\right) \sqrt{\sec(c+dx)}}{b^2(a+b)d} + \frac{2(Ab-aB)\sqrt{\cos(c+dx)} E\left(\frac{1}{2}(c+dx) \middle| 2\right) \sqrt{\sec(c+dx)}}{b^2d} + \frac{2B\sqrt{\cos(c+dx)}}{3bd} \end{aligned}$$

Mathematica [A] time = 3.66, size = 225, normalized size = 1.07

$$\frac{\cot(c+dx) \left(-2(3a^2B + 3ab(B-A) + b^2(B-3A)) \sqrt{-\tan^2(c+dx)} F\left(\sin^{-1}\left(\sqrt{\sec(c+dx)}\right) \middle| -1\right) + 6a^2B\sqrt{-\tan^2(c+dx)} \right)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x]),x]
[Out] -1/3*(Cot[c + d*x]*(-(b^2*B*Sec[c + d*x]^(5/2)) + b^2*B*Cos[2*(c + d*x)]*Sec[c + d*x]^(5/2) - 6*b*(A*b - a*B)*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] - 2*(3*a^2*B + b^2*(-3*A + B) + 3*a*b*(-A + B))*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] - 6*a*A*b*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] + 6*a^2*B*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2]))/(b^3*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sec(dx + c)^{\frac{5}{2}}}{b \sec(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="giac")
```

[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(5/2)/(b*sec(d*x + c) + a), x)

maple [A] time = 11.19, size = 466, normalized size = 2.22

$$\sqrt{-2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 1} \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right) \right) \frac{2(Ab - aB)a^2 \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{-2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 1} \operatorname{EllipticPi}\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \frac{2a}{a-b}, \sqrt{2}\right)}{b^2(a^2 - ab) \sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x)
```

```
[Out] -((-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*(A*b-B*a)*a^2/b^2/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+2*(A*b-B*a)/b^2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)+2*B/b*(-1/6*cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sec(dx + c)^{\frac{5}{2}}}{b \sec(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(5/2)/(b*sec(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(\frac{1}{\cos(c+dx)}\right)^{5/2}}{a + \frac{b}{\cos(c+dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + b/cos(c + d*x)),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + b/cos(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x)
```

```
[Out] Timed out
```

$$3.417 \quad \int \frac{\sec^3(c+dx)(A+B \sec(c+dx))}{a+b \sec(c+dx)} dx$$

Optimal. Leaf size=126

$$\frac{2(Ab - aB)\sqrt{\cos(c + dx)}\sqrt{\sec(c + dx)}\Pi\left(\frac{2a}{a+b}; \frac{1}{2}(c + dx) \middle| 2\right)}{bd(a + b)} + \frac{2B \sin(c + dx)\sqrt{\sec(c + dx)}}{bd} - \frac{2B\sqrt{\cos(c + dx)}\sqrt{\sec(c + dx)}}{bd}$$

[Out] 2*B*sin(d*x+c)*sec(d*x+c)^(1/2)/b/d-2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c), 2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b/d+2*(A*b-B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c), 2*a/(a+b), 2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b/(a+b)/d

Rubi [A] time = 0.40, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {4033, 4106, 3849, 2805, 12, 3771, 2639}

$$\frac{2(Ab - aB)\sqrt{\cos(c + dx)}\sqrt{\sec(c + dx)}\Pi\left(\frac{2a}{a+b}; \frac{1}{2}(c + dx) \middle| 2\right)}{bd(a + b)} + \frac{2B \sin(c + dx)\sqrt{\sec(c + dx)}}{bd} - \frac{2B\sqrt{\cos(c + dx)}\sqrt{\sec(c + dx)}}{bd}$$

Antiderivative was successfully verified.

[In] Int[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x]),x]

[Out] (-2*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b*d) + (2*(A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b*(a + b)*d) + (2*B*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/b

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]]], x_Symbol] := Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2805

Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi/2 + f*x))/2, 2])/d, x] /; FreeQ[{a, b, c, d, e, f}, x]

$/2 + f*x))/2, (2*d)/(c + d)]/(f*(a + b)*\text{Sqrt}[c + d]), x] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{GtQ}[c + d, 0]$

Rule 3771

$\text{Int}[(\text{csc}[c_.] + (d_.)*(x_)]*(b_.)^n, x_Symbol] \rightarrow \text{Dist}[(b*\text{Csc}[c + d*x])^n*\text{Sin}[c + d*x]^n, \text{Int}[1/\text{Sin}[c + d*x]^n, x], x] /; \text{FreeQ}[\{b, c, d\}, x] \&\& \text{EqQ}[n^2, 1/4]$

Rule 3849

$\text{Int}[(\text{csc}[e_.] + (f_.)*(x_)]*(d_.)^{3/2}/(\text{csc}[e_.] + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] \rightarrow \text{Dist}[d*\text{Sqrt}[d*\text{Sin}[e + f*x]]*\text{Sqrt}[d*\text{Csc}[e + f*x]], \text{Int}[1/(\text{Sqrt}[d*\text{Sin}[e + f*x]]*(b + a*\text{Sin}[e + f*x])), x], x] /; \text{FreeQ}[\{a, b, d, e, f\}, x] \&\& \text{NeQ}[a^2 - b^2, 0]$

Rule 4033

$\text{Int}[(\text{csc}[e_.] + (f_.)*(x_)]*(d_.)^n*(\text{csc}[e_.] + (f_.)*(x_)]*(b_.) + (a_))^{m_}, x_Symbol] \rightarrow -\text{Simp}[(B*d^2*\text{Cot}[e + f*x]*(a + b*\text{Csc}[e + f*x])^{m+1}*(d*\text{Csc}[e + f*x])^{n-2})/(b*f*(m+n)), x] + \text{Dist}[d^2/(b*(m+n)), \text{Int}[(a + b*\text{Csc}[e + f*x])^m*(d*\text{Csc}[e + f*x])^{n-2}*\text{Simp}[a*B*(n-2) + B*b*(m+n-1)*\text{Csc}[e + f*x] + (A*b*(m+n) - a*B*(n-1))*\text{Csc}[e + f*x]^2, x], x], x] /; \text{FreeQ}[\{a, b, d, e, f, A, B, m\}, x] \&\& \text{NeQ}[A*b - a*B, 0] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{GtQ}[n, 1] \&\& \text{NeQ}[m+n, 0] \&\& !\text{IGtQ}[m, 1]$

Rule 4106

$\text{Int}[(A_.) + \text{csc}[e_.] + (f_.)*(x_)]*(B_.) + \text{csc}[e_.] + (f_.)*(x_)]^2*(C_.) / (\text{Sqrt}[\text{csc}[e_.] + (f_.)*(x_)]*(d_.)*(\text{csc}[e_.] + (f_.)*(x_)]*(b_.) + (a_))), x_Symbol] \rightarrow \text{Dist}[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), \text{Int}[(d*\text{Csc}[e + f*x])^{3/2}/(a + b*\text{Csc}[e + f*x]), x], x] + \text{Dist}[1/a^2, \text{Int}[(a*A - (A*b - a*B)*\text{Csc}[e + f*x])/ \text{Sqrt}[d*\text{Csc}[e + f*x]], x], x] /; \text{FreeQ}[\{a, b, d, e, f, A, B, C\}, x] \&\& \text{NeQ}[a^2 - b^2, 0]$

Rubi steps

$$\begin{aligned}
\int \frac{\sec^3(c+dx)(A+B\sec(c+dx))}{a+b\sec(c+dx)} dx &= \frac{2B\sqrt{\sec(c+dx)} \sin(c+dx)}{bd} + \frac{2 \int \frac{-\frac{aB}{2} - \frac{1}{2}bB\sec(c+dx) + \frac{1}{2}(Ab-aB)\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))} dx}{b} \\
&= \frac{2B\sqrt{\sec(c+dx)} \sin(c+dx)}{bd} + \frac{2 \int -\frac{a^2B}{2\sqrt{\sec(c+dx)}} dx}{a^2b} + \frac{(Ab-aB) \int \frac{\sec^3}{a+b\sec}}{b} \\
&= \frac{2B\sqrt{\sec(c+dx)} \sin(c+dx)}{bd} - \frac{B \int \frac{1}{\sqrt{\sec(c+dx)}} dx}{b} + \frac{((Ab-aB)\sqrt{\cos(c+dx)})}{b} \\
&= \frac{2(Ab-aB)\sqrt{\cos(c+dx)} \Pi\left(\frac{2a}{a+b}; \frac{1}{2}(c+dx) \middle| 2\right) \sqrt{\sec(c+dx)}}{b(a+b)d} + \frac{2B\sqrt{\sec(c+dx)}}{b} \\
&= -\frac{2B\sqrt{\cos(c+dx)} E\left(\frac{1}{2}(c+dx) \middle| 2\right) \sqrt{\sec(c+dx)}}{bd} + \frac{2(Ab-aB)\sqrt{\cos(c+dx)}}{b}
\end{aligned}$$

Mathematica [A] time = 1.37, size = 123, normalized size = 0.98

$$\frac{2 \cos(2(c+dx)) \sqrt{-\tan^2(c+dx)} \csc(c+dx) \sec(c+dx) \left((Ab - B(a+b)) F\left(\sin^{-1}\left(\sqrt{\sec(c+dx)}\right) \middle| -1\right) + (aB - B(a+b)) F\left(\sin^{-1}\left(\sqrt{\sec(c+dx)}\right) \middle| -1\right) \right)}{b^2 d (\sec^2(c+dx) - 2)}$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x]),x]

[Out] (-2*Cos[2*(c + d*x)]*Csc[c + d*x]*(b*B*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1] + (A*b - (a + b)*B)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] + -(A*b) + a*B)*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1])*Sec[c + d*x]*Sqrt[-Tan[c + d*x]^2])/(b^2*d*(-2 + Sec[c + d*x]^2))

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sec(dx + c)^{\frac{3}{2}}}{b \sec(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(3/2)/(b*sec(d*x + c) + a), x)

maple [A] time = 8.39, size = 325, normalized size = 2.58

$$\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \left(-\frac{2(Ab-ab)a\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)+1}\operatorname{EllipticPi}\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \frac{2a}{a-b}, \sqrt{2}\right)}{b(a^2-ab)\sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)}} \right) \sin\left(\frac{dx}{2} + \frac{c}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x)

[Out]
$$\begin{aligned} & -(-(-2*\cos(1/2*d*x+1/2*c)^2+1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*(A*b-B*a)/b/ \\ & (a^2-a*b)*a*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/ \\ & (-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*\operatorname{EllipticPi}(\cos(1/2*d*x \\ & +1/2*c), 2*a/(a-b), 2^{(1/2)})+2*B/b*(-(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2 \\ & *c)^2)^{(1/2)}*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}* \\ & \operatorname{EllipticE}(\cos(1/2*d*x+1/2*c), 2^{(1/2)})+2*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d* \\ & x+1/2*c)^2)^{(1/2)}*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^2/\sin(1/2*d*x+1/2* \\ & c)^2/(2*\sin(1/2*d*x+1/2*c)^2-1))/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2 \\ & -1)^{(1/2)}/d \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sec(dx + c)^{\frac{3}{2}}}{b \sec(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(3/2)/(b*sec(d*x + c) + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(\frac{1}{\cos(c+dx)}\right)^{3/2}}{a + \frac{b}{\cos(c+dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + b/cos(c + d*x)), x)

[Out] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + b/cos(c + d*x)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \sec^{\frac{3}{2}}(c + dx)}{a + b \sec(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)), x)

[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**(3/2)/(a + b*sec(c + d*x)), x)

$$3.418 \quad \int \frac{\sqrt{\sec(c+dx)} (A+B \sec(c+dx))}{a+b \sec(c+dx)} dx$$

Optimal. Leaf size=101

$$\frac{2A\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} - \frac{2(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{ad(a+b)}$$

[Out] 2*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d-2*(A*b-B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/(a+b)/d

Rubi [A] time = 0.20, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {4038, 3771, 2641, 3849, 2805}

$$\frac{2A\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} - \frac{2(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{ad(a+b)}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x]),x]

[Out] (2*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d) - (2*(A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*(a + b)*d)

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2805

Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi/2 + f*x))/2, (2*d)/(c + d)])/((f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[c + d, 0]

Rule 3771

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^n, x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&

EqQ[n^2, 1/4]

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_.)]*(b_.) +
(a_.)), x_Symbol] := Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4038

```
Int[((csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) +
(A_.))/(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] := Dist[A/a, Int[(
d*Csc[e + f*x])^n, x], x] - Dist[(A*b - a*B)/(a*d), Int[(d*Csc[e + f*x])^(n
+ 1)/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] &&
NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}(A+B\sec(c+dx))}{a+b\sec(c+dx)} dx = \frac{A \int \sqrt{\sec(c+dx)} dx}{a} - \frac{(Ab-aB) \int \frac{\sec^3(c+dx)}{a+b\sec(c+dx)} dx}{a}$$

$$= \frac{(A\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}) \int \frac{1}{\sqrt{\cos(c+dx)}} dx}{a} - \frac{((Ab-aB)\sqrt{\cos(c+dx)}) \int \frac{1}{\sqrt{\cos(c+dx)}} dx}{a}$$

$$= \frac{2A\sqrt{\cos(c+dx)} F\left(\frac{1}{2}(c+dx) \middle| 2\right) \sqrt{\sec(c+dx)}}{ad} - \frac{2(Ab-aB)\sqrt{\cos(c+dx)} F\left(\frac{1}{2}(c+dx) \middle| 2\right) \sqrt{\sec(c+dx)}}{ad}$$

Mathematica [A] time = 0.58, size = 76, normalized size = 0.75

$$\frac{2\sqrt{-\tan^2(c+dx)} \cot(c+dx) \left((Ab-aB) \Pi\left(-\frac{b}{a}; \sin^{-1}\left(\sqrt{\sec(c+dx)}\right) \middle| -1\right) + aBF\left(\sin^{-1}\left(\sqrt{\sec(c+dx)}\right) \middle| -1\right) \right)}{abd}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x]),x]

```
[Out] (2*Cot[c + d*x]*(a*B*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] + (A*b - a*B)
)*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1])*Sqrt[-Tan[c + d*x]^2]
)/(a*b*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(1/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sqrt{\sec(dx + c)}}{b \sec(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(1/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(b*sec(d*x + c) + a), x)

maple [A] time = 4.44, size = 217, normalized size = 2.15

$$\frac{2 \sqrt{\left(2 \left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right) \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{-2 \left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 1} \left(A \operatorname{EllipticF}\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), 2\right) + B \operatorname{EllipticE}\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), 2\right)\right)}{a(a-b) \sqrt{-2 \left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^(1/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x)

[Out] -2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a-A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b+A*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))*b-B*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))*a)/a/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sqrt{\sec(dx + c)}}{b \sec(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(1/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(b*sec(d*x + c) + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \sqrt{\frac{1}{\cos(c+dx)}}}{a + \frac{b}{\cos(c+dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + b/cos(c + d*x)),x)

[Out] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + b/cos(c + d*x)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \sqrt{\sec(c + dx)}}{a + b \sec(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**(1/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x)

[Out] Integral((A + B*sec(c + d*x))*sqrt(sec(c + d*x))/(a + b*sec(c + d*x)), x)

$$3.419 \quad \int \frac{A+B \sec(c+dx)}{\sqrt{\sec(c+dx)} (a+b \sec(c+dx))} dx$$

Optimal. Leaf size=149

$$\frac{2(Ab - aB)\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{a^2 d} + \frac{2b(Ab - aB)\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \Pi\left(\frac{2a}{a+b}; \frac{1}{2}(c + dx)\right)}{a^2 d(a + b)}$$

[Out] $2*A*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/a/d-2*(A*b-B*a)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/a^2/d+2*b*(A*b-B*a)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticPi}(\sin(1/2*d*x+1/2*c), 2*a/(a+b), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/a^2/(a+b)/d$

Rubi [A] time = 0.26, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {4038, 3771, 2639, 3848, 2803, 2641, 2805}

$$\frac{2(Ab - aB)\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{a^2 d} + \frac{2b(Ab - aB)\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \Pi\left(\frac{2a}{a+b}; \frac{1}{2}(c + dx)\right)}{a^2 d(a + b)}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sec[c + d*x])/(Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])),x]

[Out] $(2*A*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/(a*d) - (2*(A*b - a*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticF}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/(a^2*d) + (2*b*(A*b - a*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticPi}[(2*a)/(a + b), (c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/(a^2*(a + b)*d)$

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2803

Int[Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[d/b, Int[1/Sqrt[c + d*Sin[e + f*x]], x], x]

+ Dist[(b*c - a*d)/b, Int[1/((a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x])], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 2805

Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi/2 + f*x))/2, (2*d)/(c + d)]/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[c + d, 0]

Rule 3771

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^n, x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 3848

Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := Dist[(Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]])/d, Int[Sqrt[d*Sin[e + f*x]]/(b + a*Sin[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 4038

Int[(((csc[(e_.) + (f_.)*(x_)]*(d_.))^n)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)))/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := Dist[A/a, Int[(d*Csc[e + f*x])^n, x], x] - Dist[(A*b - a*B)/(a*d), Int[(d*Csc[e + f*x])^(n + 1)/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned}
\int \frac{A + B \sec(c + dx)}{\sqrt{\sec(c + dx)} (a + b \sec(c + dx))} dx &= \frac{A \int \frac{1}{\sqrt{\sec(c+dx)}} dx}{a} - \frac{(Ab - aB) \int \frac{\sqrt{\sec(c+dx)}}{a+b \sec(c+dx)} dx}{a} \\
&= \frac{(A\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)}) \int \sqrt{\cos(c + dx)} dx}{a} - \frac{((Ab - aB)\sqrt{\cos(c + dx)}) \int \sqrt{\sec(c + dx)} dx}{a} \\
&= \frac{2A\sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{\sec(c + dx)}}{ad} - \frac{((Ab - aB)\sqrt{\cos(c + dx)}) \int \sqrt{\sec(c + dx)} dx}{ad} \\
&= \frac{2A\sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{\sec(c + dx)}}{ad} - \frac{2(Ab - aB)\sqrt{\cos(c + dx)} \int \sqrt{\sec(c + dx)} dx}{ad}
\end{aligned}$$

Mathematica [A] time = 7.02, size = 220, normalized size = 1.48

$$\cot(c + dx) \left(-2Ab\sqrt{-\tan^2(c + dx)} \Pi\left(-\frac{b}{a}; \sin^{-1}\left(\sqrt{\sec(c + dx)}\right) \middle| -1\right) + aA \sec^{\frac{7}{2}}(c + dx) - aA \sec^{\frac{3}{2}}(c + dx) + a \right)$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sec[c + d*x])/(Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])), x]

[Out] (Cot[c + d*x]*(-(a*A*Sec[c + d*x]^(3/2)) - a*A*Cos[2*(c + d*x)]*Sec[c + d*x]^(3/2) + a*A*Sec[c + d*x]^(7/2) + a*A*Cos[2*(c + d*x)]*Sec[c + d*x]^(7/2) - 2*a*A*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] + 2*a*A*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] - 2*A*b*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] + 2*a*B*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2]))/(a^2*d)

fricas [F] time = 144.23, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B \sec(dx + c) + A)\sqrt{\sec(dx + c)}}{b \sec(dx + c)^2 + a \sec(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")

[Out] integral((B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(b*sec(d*x + c)^2 + a*sec(d*x + c)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(b \sec(dx + c) + a) \sqrt{\sec(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c)),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)*sqrt(sec(d*x + c))), x)

maple [A] time = 4.85, size = 295, normalized size = 1.98

$$2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{-2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 1} \left(A \operatorname{EllipticF}\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c)),x)

[Out] 2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b-A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2+A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b+A*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))*b^2-B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2+B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b-B*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))*a*b)/a^2/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(b \sec(dx + c) + a) \sqrt{\sec(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)*sqrt(sec(d*x + c))), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\left(a + \frac{b}{\cos(c+dx)}\right) \sqrt{\frac{1}{\cos(c+dx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/((a + b/cos(c + d*x))*(1/cos(c + d*x))^(1/2)),x)

[Out] int((A + B/cos(c + d*x))/((a + b/cos(c + d*x))*(1/cos(c + d*x))^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \sec(c + dx)}{(a + b \sec(c + dx)) \sqrt{\sec(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)**(1/2)/(a+b*sec(d*x+c)),x)

[Out] Integral((A + B*sec(c + d*x))/((a + b*sec(c + d*x))*sqrt(sec(c + d*x))), x)

$$3.420 \quad \int \frac{A+B \sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+b \sec(c+dx))} dx$$

Optimal. Leaf size=196

$$\frac{2b^2(Ab - aB)\sqrt{\cos(c + dx)}\sqrt{\sec(c + dx)}\Pi\left(\frac{2a}{a+b}; \frac{1}{2}(c + dx)\middle| 2\right)}{a^3d(a + b)} - \frac{2(Ab - aB)\sqrt{\cos(c + dx)}\sqrt{\sec(c + dx)}E\left(\frac{1}{2}(c + dx)\middle| 2\right)}{a^2d}$$

[Out] 2/3*A*sin(d*x+c)/a/d/sec(d*x+c)^(1/2)-2*(A*b-B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/d+2/3*(A*a^2+3*A*b^2-3*B*a*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d-2*b^2*(A*b-B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/(a+b)/d

Rubi [A] time = 0.47, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {4034, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{2(a^2A - 3abB + 3Ab^2)\sqrt{\cos(c + dx)}\sqrt{\sec(c + dx)}F\left(\frac{1}{2}(c + dx)\middle| 2\right)}{3a^3d} - \frac{2b^2(Ab - aB)\sqrt{\cos(c + dx)}\sqrt{\sec(c + dx)}\Pi\left(\frac{1}{2}(c + dx)\middle| 2\right)}{a^3d(a + b)}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])),x]

[Out] (-2*(A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*d) + (2*(a^2*A + 3*A*b^2 - 3*a*b*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a^3*d) - (2*b^2*(A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^3*(a + b)*d) + (2*A*Sin[c + d*x])/(3*a*d*Sqrt[Sec[c + d*x]])

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])], x_Symbol] := Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)]/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_.)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_.)), x_Symbol] := Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4034

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] := Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*n), x] + Dis
t[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n
- A*b*(m + n + 1) + A*a*(n + 1)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x
]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0]
&& NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_.))), x_Symbol] := Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
```

C}, x] && NeQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{A + B \sec(c + dx)}{\sec^{\frac{3}{2}}(c + dx)(a + b \sec(c + dx))} dx &= \frac{2A \sin(c + dx)}{3ad\sqrt{\sec(c + dx)}} - \frac{2 \int \frac{\frac{3}{2}(Ab - aB) - \frac{1}{2}aA \sec(c + dx) - \frac{1}{2}Ab \sec^2(c + dx)}{\sqrt{\sec(c + dx)}(a + b \sec(c + dx))} dx}{3a} \\
 &= \frac{2A \sin(c + dx)}{3ad\sqrt{\sec(c + dx)}} - \frac{2 \int \frac{\frac{3}{2}a(Ab - aB) - \left(\frac{a^2A}{2} + \frac{3}{2}b(Ab - aB)\right) \sec(c + dx)}{\sqrt{\sec(c + dx)}} dx}{3a^3} - \frac{(b^2(Ab - aB))}{3a^3} \\
 &= \frac{2A \sin(c + dx)}{3ad\sqrt{\sec(c + dx)}} - \frac{(Ab - aB) \int \frac{1}{\sqrt{\sec(c + dx)}} dx}{a^2} + \frac{(a^2A + 3Ab^2 - 3abB)}{3a^3} \\
 &= -\frac{2b^2(Ab - aB)\sqrt{\cos(c + dx)} \Pi\left(\frac{2a}{a+b}; \frac{1}{2}(c + dx) \middle| 2\right) \sqrt{\sec(c + dx)}}{a^3(a + b)d} + \frac{2A \sin(c + dx)}{3ad\sqrt{\sec(c + dx)}} \\
 &= -\frac{2(Ab - aB)\sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{\sec(c + dx)}}{a^2d} + \frac{2(a^2A + 3Ab^2 - 3abB)}{3a^3}
 \end{aligned}$$

Mathematica [B] time = 6.74, size = 540, normalized size = 2.76

$$\frac{A \sin(2(c + dx))\sqrt{\sec(c + dx)}}{3ad} - \frac{(3Ab - 3aB) \sin(c + dx) \cos(2(c + dx))(a + b \sec(c + dx)) \left(2a^2 \sqrt{\sec(c + dx)} \sqrt{1 - \sec^2(c + dx)} \Pi\left(-\frac{b}{a}; \sin^{-1}\left(\sqrt{\sec(c + dx)}\right)\right)\right)}{3a^3}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])),x]

[Out] -1/6*((2*(A*b - 3*a*B)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] - EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1])*(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) - (4*A*Cos[c + d*x]^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2) + ((3*A*b - 3*a*B)*Cos[2*(c + d*x)]*(a + b*Sec[c + d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 2*a*(a - 2*b)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*a^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1])*(a + b*Sec[c + d*x])*(1 - Cos[c + d*x]^2))

```
*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 4*b^2*EllipticPi[-
(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c +
d*x]^2])*Sin[c + d*x]]/(a^2*b*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqr
t[Sec[c + d*x]]*(2 - Sec[c + d*x]^2))/(a*d) + (A*Sqrt[Sec[c + d*x]]*Sin[2*
(c + d*x)])/(3*a*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(b \sec(dx + c) + a) \sec(dx + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)*sec(d*x + c)^(3/2)), x
)
```

maple [B] time = 4.87, size = 786, normalized size = 4.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((4*A*a^3-4*A*
a^2*b)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+(-2*A*a^3+2*A*a^2*b)*sin(1/2
*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+A*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-A*a^2*b*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))+3*A*a*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*A*b^3*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
```

$(1/2)*\text{EllipticE}(\cos(1/2*d*x+1/2*c), 2^{(1/2)})*a^2*b-3*A*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticE}(\cos(1/2*d*x+1/2*c), 2^{(1/2)})$
 $*a*b^2+3*A*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}$
 $*\text{EllipticPi}(\cos(1/2*d*x+1/2*c), 2*a/(a-b), 2^{(1/2)})*b^3-3*a^2*b*B*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}$
 $(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c), 2^{(1/2)})+3*B*a*b^2*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}$
 $(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c), 2^{(1/2)})-3*B*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}$
 $(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticE}(\cos(1/2*d*x+1/2*c), 2^{(1/2)})*a^3+3*B*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}$
 $(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticE}(\cos(1/2*d*x+1/2*c), 2^{(1/2)})*a^2*b-3*B*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}$
 $(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticPi}(\cos(1/2*d*x+1/2*c), 2*a/(a-b), 2^{(1/2)})*a*b^2/a^3/(a-b)/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}$
 $/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(b \sec(dx + c) + a) \sec(dx + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)*sec(d*x + c)^(3/2)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\left(a + \frac{b}{\cos(c+dx)}\right) \left(\frac{1}{\cos(c+dx)}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/((a + b/cos(c + d*x))*(1/cos(c + d*x))^(3/2)),x)

[Out] int((A + B/cos(c + d*x))/((a + b/cos(c + d*x))*(1/cos(c + d*x))^(3/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \sec(c + dx)}{(a + b \sec(c + dx)) \sec^{\frac{3}{2}}(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)**(3/2)/(a+b*sec(d*x+c)),x)

[Out] Integral((A + B*sec(c + d*x))/((a + b*sec(c + d*x))*sec(c + d*x)**(3/2)), x
)

$$3.421 \quad \int \frac{A+B \sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+b \sec(c+dx))} dx$$

Optimal. Leaf size=242

$$\frac{2b^3(Ab - aB)\sqrt{\cos(c + dx)}\sqrt{\sec(c + dx)}\Pi\left(\frac{2a}{a+b}; \frac{1}{2}(c + dx) \middle| 2\right)}{a^4d(a + b)} - \frac{2(Ab - aB)\sin(c + dx)}{3a^2d\sqrt{\sec(c + dx)}} - \frac{2(a^2 + 3b^2)(Ab - aB)\sqrt{\cos(c + dx)}}{5a^3d}$$

[Out] $2/5*A*\sin(d*x+c)/a/d/\sec(d*x+c)^{(3/2)}-2/3*(A*b-B*a)*\sin(d*x+c)/a^2/d/\sec(d*x+c)^{(1/2)}+2/5*(3*A*a^2+5*A*b^2-5*B*a*b)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/a^3/d-2/3*(a^2+3*b^2)*(A*b-B*a)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/a^4/d+2*b^3*(A*b-B*a)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticPi}(\sin(1/2*d*x+1/2*c), 2*a/(a+b), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/a^4/(a+b)/d$

Rubi [A] time = 0.76, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {4034, 4104, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{2(a^2 + 3b^2)(Ab - aB)\sqrt{\cos(c + dx)}\sqrt{\sec(c + dx)}F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3a^4d} + \frac{2(3a^2A - 5abB + 5Ab^2)\sqrt{\cos(c + dx)}\sqrt{\sec(c + dx)}}{5a^3d}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(5/2)*(a + b*Sec[c + d*x])),x]

[Out] $(2*(3*a^2*A + 5*A*b^2 - 5*a*b*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/(5*a^3*d) - (2*(a^2 + 3*b^2)*(A*b - a*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticF}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/(3*a^4*d) + (2*b^3*(A*b - a*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticPi}[(2*a)/(a + b), (c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/(a^4*(a + b)*d) + (2*A*\text{Sin}[c + d*x])/(5*a*d*\text{Sec}[c + d*x]^{(3/2)}) - (2*(A*b - a*B)*\text{Sin}[c + d*x])/(3*a^2*d*\text{Sqrt}[\text{Sec}[c + d*x]])$

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2805

Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi/2 + f*x))/2, (2*d)/(c + d)]/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[c + d, 0]

Rule 3771

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^n, x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 3849

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 4034

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^m*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] := Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + A*a*(n + 1)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]

Rule 4104

Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*csc[(e_.) + (f_.)*(x_)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^m, x_Symbol] := Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d

*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]

Rule 4106

Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.))/(Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{A + B \sec(c + dx)}{\sec^{\frac{5}{2}}(c + dx)(a + b \sec(c + dx))} dx &= \frac{2A \sin(c + dx)}{5ad \sec^{\frac{3}{2}}(c + dx)} - \frac{2 \int \frac{\frac{5}{2}(Ab - aB) - \frac{3}{2}aA \sec(c + dx) - \frac{3}{2}Ab \sec^2(c + dx)}{\sec^{\frac{3}{2}}(c + dx)(a + b \sec(c + dx))} dx}{5a} \\
 &= \frac{2A \sin(c + dx)}{5ad \sec^{\frac{3}{2}}(c + dx)} - \frac{2(Ab - aB) \sin(c + dx)}{3a^2 d \sqrt{\sec(c + dx)}} + \frac{4 \int \frac{\frac{3}{4}(3a^2A + 5Ab^2 - 5abB) + \frac{1}{4}a(4a^2 - b^2)}{\sqrt{\sec(c + dx)}} dx}{3a^2} \\
 &= \frac{2A \sin(c + dx)}{5ad \sec^{\frac{3}{2}}(c + dx)} - \frac{2(Ab - aB) \sin(c + dx)}{3a^2 d \sqrt{\sec(c + dx)}} + \frac{4 \int \frac{\frac{3}{4}a(3a^2A + 5Ab^2 - 5abB) - \left(-\frac{1}{4}a(4a^2 - b^2)\right)}{\sqrt{\sec(c + dx)}} dx}{3a^2} \\
 &= \frac{2A \sin(c + dx)}{5ad \sec^{\frac{3}{2}}(c + dx)} - \frac{2(Ab - aB) \sin(c + dx)}{3a^2 d \sqrt{\sec(c + dx)}} - \frac{\left((a^2 + 3b^2)(Ab - aB)\right) \int \frac{1}{\sqrt{\sec(c + dx)}} dx}{3a^4} \\
 &= \frac{2b^3(Ab - aB) \sqrt{\cos(c + dx)} \Pi\left(\frac{2a}{a+b}; \frac{1}{2}(c + dx) \middle| 2\right) \sqrt{\sec(c + dx)}}{a^4(a + b)d} + \frac{2A \sin(c + dx)}{5ad \sec^{\frac{3}{2}}(c + dx)} \\
 &= \frac{2(3a^2A + 5Ab^2 - 5abB) \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{\sec(c + dx)}}{5a^3d} - \frac{2A \sin(c + dx)}{5ad \sec^{\frac{3}{2}}(c + dx)}
 \end{aligned}$$

Mathematica [B] time = 6.91, size = 612, normalized size = 2.53

$$\frac{2(9a^2A-5abB+5Ab^2)\sin(c+dx)\cos^2(c+dx)\sqrt{1-\sec^2(c+dx)}(a+b\sec(c+dx))\left(F\left(\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle| -1\right)-\Pi\left(-\frac{b}{a};\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle| -1\right)\right)}{b(1-\cos^2(c+dx))(a\cos(c+dx)+b)} + \frac{(9a^2A-5abB+5Ab^2)\sin(c+dx)\cos^2(c+dx)\sqrt{1-\sec^2(c+dx)}(a+b\sec(c+dx))\left(F\left(\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle| -1\right)-\Pi\left(-\frac{b}{a};\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle| -1\right)\right)}{b(1-\cos^2(c+dx))(a\cos(c+dx)+b)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(5/2)*(a + b*Sec[c + d*x])),x]
[Out] ((2*(9*a^2*A + 5*A*b^2 - 5*a*b*B)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] - EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1])*(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(8*a*A*b + 10*a^2*B)*Cos[c + d*x]^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + ((9*a^2*A + 15*A*b^2 - 15*a*b*B)*Cos[2*(c + d*x)]*(a + b*Sec[c + d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 2*a*(a - 2*b)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*a^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 4*b^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a^2*b*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2)))/(30*a^2*d) + (Sqrt[Sec[c + d*x]]*(A*Sin[c + d*x])/(10*a) + ((- (A*b) + a*B)*Sin[2*(c + d*x)])/(3*a^2) + (A*Sin[3*(c + d*x)])/(10*a))/d
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(b \sec(dx + c) + a) \sec(dx + c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

[Out] integrate((B*sec(dx + c) + A)/((b*sec(dx + c) + a)*sec(dx + c)^(5/2)), x)

maple [B] time = 5.68, size = 1074, normalized size = 4.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(dx+c))/sec(dx+c)^(5/2)/(a+b*sec(dx+c)), x)

[Out]
$$-2/15 * ((2 * \cos(1/2 * dx + 1/2 * c) - 1) * \sin(1/2 * dx + 1/2 * c)^2)^{(1/2)} * ((-24 * A * a^4 + 24 * A * a^3 * b) * \cos(1/2 * dx + 1/2 * c) * \sin(1/2 * dx + 1/2 * c)^6 + (24 * A * a^4 - 44 * A * a^3 * b + 20 * A * a^2 * b^2 + 20 * B * a^4 - 20 * B * a^3 * b) * \sin(1/2 * dx + 1/2 * c)^4 * \cos(1/2 * dx + 1/2 * c) + (-6 * A * a^4 + 16 * A * a^3 * b - 10 * A * a^2 * b^2 - 10 * B * a^4 + 10 * B * a^3 * b) * \sin(1/2 * dx + 1/2 * c)^2 * \cos(1/2 * dx + 1/2 * c) - 9 * A * (\sin(1/2 * dx + 1/2 * c)^2)^{(1/2)} * (2 * \sin(1/2 * dx + 1/2 * c)^2 - 1)^{(1/2)} * \text{EllipticE}(\cos(1/2 * dx + 1/2 * c), 2^{(1/2)}) * a^4 + 9 * A * (\sin(1/2 * dx + 1/2 * c)^2)^{(1/2)} * (2 * \sin(1/2 * dx + 1/2 * c)^2 - 1)^{(1/2)} * \text{EllipticE}(\cos(1/2 * dx + 1/2 * c), 2^{(1/2)}) * a^3 * b - 15 * A * (\sin(1/2 * dx + 1/2 * c)^2)^{(1/2)} * (2 * \sin(1/2 * dx + 1/2 * c)^2 - 1)^{(1/2)} * \text{EllipticE}(\cos(1/2 * dx + 1/2 * c), 2^{(1/2)}) * a^2 * b^2 + 15 * A * (\sin(1/2 * dx + 1/2 * c)^2)^{(1/2)} * (2 * \sin(1/2 * dx + 1/2 * c)^2 - 1)^{(1/2)} * \text{EllipticE}(\cos(1/2 * dx + 1/2 * c), 2^{(1/2)}) * a * b^3 - 15 * A * (\sin(1/2 * dx + 1/2 * c)^2)^{(1/2)} * (2 * \sin(1/2 * dx + 1/2 * c)^2 - 1)^{(1/2)} * \text{EllipticPi}(\cos(1/2 * dx + 1/2 * c), 2 * a / (a - b), 2^{(1/2)}) * b^4 - 5 * A * (\sin(1/2 * dx + 1/2 * c)^2)^{(1/2)} * (2 * \sin(1/2 * dx + 1/2 * c)^2 - 1)^{(1/2)} * \text{EllipticF}(\cos(1/2 * dx + 1/2 * c), 2^{(1/2)}) * a^3 * b + 5 * A * (\sin(1/2 * dx + 1/2 * c)^2)^{(1/2)} * (2 * \sin(1/2 * dx + 1/2 * c)^2 - 1)^{(1/2)} * \text{EllipticF}(\cos(1/2 * dx + 1/2 * c), 2^{(1/2)}) * a^2 * b^2 - 15 * A * (\sin(1/2 * dx + 1/2 * c)^2)^{(1/2)} * (2 * \sin(1/2 * dx + 1/2 * c)^2 - 1)^{(1/2)} * \text{EllipticF}(\cos(1/2 * dx + 1/2 * c), 2^{(1/2)}) * a * b^3 + 15 * A * (\sin(1/2 * dx + 1/2 * c)^2)^{(1/2)} * (2 * \sin(1/2 * dx + 1/2 * c)^2 - 1)^{(1/2)} * \text{EllipticF}(\cos(1/2 * dx + 1/2 * c), 2^{(1/2)}) * b^4 + 15 * B * (\sin(1/2 * dx + 1/2 * c)^2)^{(1/2)} * (2 * \sin(1/2 * dx + 1/2 * c)^2 - 1)^{(1/2)} * \text{EllipticE}(\cos(1/2 * dx + 1/2 * c), 2^{(1/2)}) * a^3 * b - 15 * B * (\sin(1/2 * dx + 1/2 * c)^2)^{(1/2)} * (2 * \sin(1/2 * dx + 1/2 * c)^2 - 1)^{(1/2)} * \text{EllipticE}(\cos(1/2 * dx + 1/2 * c), 2^{(1/2)}) * a^2 * b^2 + 15 * B * (\sin(1/2 * dx + 1/2 * c)^2)^{(1/2)} * (2 * \sin(1/2 * dx + 1/2 * c)^2 - 1)^{(1/2)} * \text{EllipticPi}(\cos(1/2 * dx + 1/2 * c), 2 * a / (a - b), 2^{(1/2)}) * a * b^3 + 5 * B * (\sin(1/2 * dx + 1/2 * c)^2)^{(1/2)} * (2 * \sin(1/2 * dx + 1/2 * c)^2 - 1)^{(1/2)} * \text{EllipticF}(\cos(1/2 * dx + 1/2 * c), 2^{(1/2)}) * a^4 - 5 * B * (\sin(1/2 * dx + 1/2 * c)^2)^{(1/2)} * (2 * \sin(1/2 * dx + 1/2 * c)^2 - 1)^{(1/2)} * \text{EllipticF}(\cos(1/2 * dx + 1/2 * c), 2^{(1/2)}) * a^3 * b + 15 * B * (\sin(1/2 * dx + 1/2 * c)^2)^{(1/2)} * (2 * \sin(1/2 * dx + 1/2 * c)^2 - 1)^{(1/2)} * \text{EllipticF}(\cos(1/2 * dx + 1/2 * c), 2^{(1/2)}) * a^2 * b^2 - 15 * B * (\sin(1/2 * dx + 1/2 * c)^2)^{(1/2)} * (2 * \sin(1/2 * dx + 1/2 * c)^2 - 1)^{(1/2)} * \text{EllipticF}(\cos(1/2 * dx + 1/2 * c), 2^{(1/2)}) * a * b^3) / a^4 / (a - b) / (-2 * \sin(1/2 * dx + 1/2 * c)^4 + \sin(1/2 * dx + 1/2 * c)^2)^{(1/2)} / \sin(1/2 * dx + 1/2 * c) / (2 * \cos(1/2 * dx + 1/2 * c)^2 - 1)^{(1/2)} / d$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(b \sec(dx + c) + a) \sec(dx + c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)*sec(d*x + c)^(5/2)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\left(a + \frac{b}{\cos(c+dx)}\right) \left(\frac{1}{\cos(c+dx)}\right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/((a + b/cos(c + d*x))*(1/cos(c + d*x))^(5/2)),x)

[Out] int((A + B/cos(c + d*x))/((a + b/cos(c + d*x))*(1/cos(c + d*x))^(5/2)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)**(5/2)/(a+b*sec(d*x+c)),x)

[Out] Timed out

$$3.422 \quad \int \frac{\sec^2(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^2} dx$$

Optimal. Leaf size=406

$$\frac{a(Ab - aB) \sin(c + dx) \sec^{\frac{5}{2}}(c + dx)}{bd(a^2 - b^2)(a + b \sec(c + dx))} - \frac{(-5a^2B + 3aAb + 2b^2B) \sin(c + dx) \sec^{\frac{3}{2}}(c + dx)}{3b^2d(a^2 - b^2)} - \frac{(-5a^2B + 3aAb + 2b^2B) \sin(c + dx) \sec^{\frac{3}{2}}(c + dx)}{3b^2d(a^2 - b^2)}$$

[Out] $-1/3*(3*A*a*b-5*B*a^2+2*B*b^2)*\sec(d*x+c)^{(3/2)}*\sin(d*x+c)/b^2/(a^2-b^2)/d+a*(A*b-B*a)*\sec(d*x+c)^{(5/2)}*\sin(d*x+c)/b/(a^2-b^2)/d/(a+b*\sec(d*x+c))+3*A*a^2*b-2*A*b^3-5*B*a^3+4*B*a*b^2)*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/b^3/(a^2-b^2)/d-(3*A*a^2*b-2*A*b^3-5*B*a^3+4*B*a*b^2)*(\cos(1/2*d*x+1/2*c))^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c),2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/b^3/(a^2-b^2)/d-1/3*(3*A*a*b-5*B*a^2+2*B*b^2)*(\cos(1/2*d*x+1/2*c))^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c),2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/b^2/(a^2-b^2)/d-a*(3*A*a^2*b-5*A*b^3-5*B*a^3+7*B*a*b^2)*(\cos(1/2*d*x+1/2*c))^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticPi}(\sin(1/2*d*x+1/2*c),2*a/(a+b),2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/(a-b)/b^3/(a+b)^2/d$

Rubi [A] time = 1.16, antiderivative size = 406, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {4029, 4102, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{a(Ab - aB) \sin(c + dx) \sec^{\frac{5}{2}}(c + dx)}{bd(a^2 - b^2)(a + b \sec(c + dx))} - \frac{(-5a^2B + 3aAb + 2b^2B) \sin(c + dx) \sec^{\frac{3}{2}}(c + dx)}{3b^2d(a^2 - b^2)} + \frac{(3a^2Ab - 5a^3B + 4ab^2B) \sin(c + dx) \sec^{\frac{3}{2}}(c + dx)}{3b^2d(a^2 - b^2)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(\text{Sec}[c + d*x])^{(7/2)}*(A + B*\text{Sec}[c + d*x])]/(a + b*\text{Sec}[c + d*x])^2,x]$

[Out] $-(((3*a^2*A*b - 2*A*b^3 - 5*a^3*B + 4*a*b^2*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/(b^3*(a^2 - b^2)*d) - ((3*a*A*b - 5*a^2*B + 2*b^2*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticF}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/(3*b^2*(a^2 - b^2)*d) - (a*(3*a^2*A*b - 5*A*b^3 - 5*a^3*B + 7*a*b^2*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticPi}[(2*a)/(a + b), (c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/((a - b)*b^3*(a + b)^2*d) + ((3*a^2*A*b - 2*A*b^3 - 5*a^3*B + 4*a*b^2*B)*\text{Sqrt}[\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(b^3*(a^2 - b^2)*d) - ((3*a*A*b - 5*a^2*B + 2*b^2*B)*\text{Sec}[c + d*x]^{(3/2)}*\text{Sin}[c + d*x])/(3*b^2*(a^2 - b^2)*d) + (a*(A*b - a*B)*\text{Sec}[c + d*x]^{(5/2)}*\text{Sin}[c + d*x])/(b*(a^2 - b^2)*d*(a + b*\text{Sec}[c + d*x]))$

Rule 2639

Int[Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2805

Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)])], x_Symbol] := Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi/2 + f*x))/2, (2*d)/(c + d)]/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[c + d, 0]

Rule 3771

Int[(csc[(c_) + (d_)*(x_)]*(b_))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 3787

Int[(csc[(e_) + (f_)*(x_)]*(d_))^(n_)*(csc[(e_) + (f_)*(x_)]*(b_) + (a_)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 3849

Int[(csc[(e_) + (f_)*(x_)]*(d_))^(3/2)/(csc[(e_) + (f_)*(x_)]*(b_) + (a_)), x_Symbol] := Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 4029

Int[(csc[(e_) + (f_)*(x_)]*(d_))^(n_)*(csc[(e_) + (f_)*(x_)]*(b_) + (a_))^(m_)*(csc[(e_) + (f_)*(x_)]*(B_) + (A_)), x_Symbol] := Simp[(a*d^2*(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[d/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(n - 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2*(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n

, 1]

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a
_))^(m_), x_Symbol] := -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a
_))), x_Symbol] := Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^{\frac{7}{2}}(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^2} dx &= \frac{a(Ab-aB)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{b(a^2-b^2)d(a+b\sec(c+dx))} + \frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(\frac{3}{2}a(Ab-aB)-b(Ab-aB)\right)}{a+b} dx}{b} \\
&= -\frac{(3aAb-5a^2B+2b^2B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b^2(a^2-b^2)d} + \frac{a(Ab-aB)\sec^{\frac{5}{2}}(c+dx)}{b(a^2-b^2)d} \\
&= \frac{(3a^2Ab-2Ab^3-5a^3B+4ab^2B)\sqrt{\sec(c+dx)}\sin(c+dx)}{b^3(a^2-b^2)d} - \frac{(3aAb-5a^2B+2b^2B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b^2(a^2-b^2)d} \\
&= \frac{(3a^2Ab-2Ab^3-5a^3B+4ab^2B)\sqrt{\sec(c+dx)}\sin(c+dx)}{b^3(a^2-b^2)d} - \frac{(3aAb-5a^2B+2b^2B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b^2(a^2-b^2)d} \\
&= \frac{(3a^2Ab-2Ab^3-5a^3B+4ab^2B)\sqrt{\sec(c+dx)}\sin(c+dx)}{b^3(a^2-b^2)d} - \frac{(3aAb-5a^2B+2b^2B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b^2(a^2-b^2)d} \\
&= -\frac{a(3a^2Ab-5Ab^3-5a^3B+7ab^2B)\sqrt{\cos(c+dx)}\Pi\left(\frac{2a}{a+b}; \frac{1}{2}(c+dx)\right)}{(a-b)b^3(a+b)^2d} \\
&= -\frac{(3a^2Ab-2Ab^3-5a^3B+4ab^2B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\right)}{b^3(a^2-b^2)d}
\end{aligned}$$

Mathematica [A] time = 7.32, size = 733, normalized size = 1.81

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{a^2Ab\sin(c+dx)-a^3B\sin(c+dx)}{b^2(b^2-a^2)(a\cos(c+dx)+b)} + \frac{(5a^3B-3a^2Ab-4ab^2B+2Ab^3)\sin(c+dx)}{b^3(b^2-a^2)} + \frac{2B\tan(c+dx)}{3b^2}\right)}{d} + \frac{2(40a^3bB-24a^2Ab^2-28ab^3B+2b^4B)}{b^3(a^2-b^2)d}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(Sec[c + d*x]^(7/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^2, x]

[Out] ((2*(-27*a^3*A*b + 30*a*A*b^3 + 45*a^4*B - 44*a^2*b^2*B - 4*b^4*B)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] - EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1])*(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(-24*a^2*A

```
*b^2 + 12*A*b^4 + 40*a^3*b*B - 28*a*b^3*B)*Cos[c + d*x]^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + ((-9*a^3*A*b + 6*a*A*b^3 + 15*a^4*B - 12*a^2*b^2*B)*Cos[2*(c + d*x)]*(a + b*Sec[c + d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1])*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 2*a*(a - 2*b)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1])*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*a^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1])*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 4*b^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1])*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a^2*b*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2))/(12*(a - b)*b^3*(a + b)*d) + (Sqrt[Sec[c + d*x]]*((( -3*a^2*A*b + 2*A*b^3 + 5*a^3*B - 4*a*b^2*B)*Sin[c + d*x])/(b^3*(-a^2 + b^2)) + (a^2*A*b*Ssin[c + d*x] - a^3*B*Ssin[c + d*x])/(b^2*(-a^2 + b^2)*(b + a*Cos[c + d*x])) + (2*B*Tan[c + d*x])/(3*b^2))))/d
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(dx+c)^(7/2)*(A+B*sec(dx+c))/(a+b*sec(dx+c))^2,x, algorithm="fricas")
```

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx+c) + A) \sec(dx+c)^{\frac{7}{2}}}{(b \sec(dx+c) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(dx+c)^(7/2)*(A+B*sec(dx+c))/(a+b*sec(dx+c))^2,x, algorithm="giac")
```

```
[Out] integrate((B*sec(dx + c) + A)*sec(dx + c)^(7/2)/(b*sec(dx + c) + a)^2, x)
```

maple [B] time = 19.50, size = 1024, normalized size = 2.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(dx+c)^(7/2)*(A+B*sec(dx+c))/(a+b*sec(dx+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a^2*(A*b-2*B*
a)/b^3/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/
2*d*x+1/2*c),2*a/(a-b),2^(1/2))+2*(A*b-2*B*a)/b^3*(-(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)
/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)+2*B/b^2*(-1/6*cos(1/2*d*x+
1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d
*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2)))-2*(A*b-B*a)*a/b^2*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*
c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos(1/2*d*x+1/
2*c)^2-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))+1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2
/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b)
,2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm
="maxima")
```

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(\frac{1}{\cos(c+dx)}\right)^{7/2}}{\left(a + \frac{b}{\cos(c+dx)}\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(7/2))/(a + b/cos(c + d*x))^2,x)
```

```
[Out] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(7/2))/(a + b/cos(c + d*x))^2, x
)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

$$3.423 \quad \int \frac{\sec^2(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^2} dx$$

Optimal. Leaf size=315

$$\frac{a(Ab - aB) \sin(c + dx) \sec^3(c + dx)}{bd(a^2 - b^2)(a + b \sec(c + dx))} - \frac{(-3a^2B + aAb + 2b^2B) \sin(c + dx) \sqrt{\sec(c + dx)}}{b^2d(a^2 - b^2)} + \frac{(Ab - aB) \sqrt{\cos(c + dx)}}{bd}$$

[Out] a*(A*b-B*a)*sec(d*x+c)^(3/2)*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))- (A*a*b-3*B*a^2+2*B*b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/b^2/(a^2-b^2)/d+(A*a*b-3*B*a^2+2*B*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b^2/(a^2-b^2)/d+(A*b-B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b/(a^2-b^2)/d+(A*a^2*b-3*A*b^3-3*B*a^3+5*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/(a-b)/b^2/(a+b)^2/d

Rubi [A] time = 0.84, antiderivative size = 315, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {4029, 4102, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{a(Ab - aB) \sin(c + dx) \sec^3(c + dx)}{bd(a^2 - b^2)(a + b \sec(c + dx))} - \frac{(-3a^2B + aAb + 2b^2B) \sin(c + dx) \sqrt{\sec(c + dx)}}{b^2d(a^2 - b^2)} + \frac{(Ab - aB) \sqrt{\cos(c + dx)}}{bd}$$

Antiderivative was successfully verified.

[In] Int[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^2,x]

[Out] ((a*A*b - 3*a^2*B + 2*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b^2*(a^2 - b^2)*d) + ((A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b*(a^2 - b^2)*d) + ((a^2*A*b - 3*A*b^3 - 3*a^3*B + 5*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/((a - b)*b^2*(a + b)^2*d) - ((a*A*b - 3*a^2*B + 2*b^2*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(b^2*(a^2 - b^2)*d) + (a*(A*b - a*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2805

Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi/2 + f*x))/2, (2*d)/(c + d)]/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[c + d, 0]

Rule 3771

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^n, x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 3849

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 4029

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^m*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] := Simp[(a*d^2*(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[d/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(n - 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2*(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 1]

Rule 4102


```

Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a
_))^(m_), x_Symbol] := -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]

```

Rule 4106

```

Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a
_)), x_Symbol] := Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
C}, x] && NeQ[a^2 - b^2, 0]

```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^2} dx &= \frac{a(Ab-aB) \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{b(a^2-b^2)d(a+b \sec(c+dx))} + \int \frac{\sqrt{\sec(c+dx)} \left(\frac{1}{2}a(Ab-aB) - b(Ab-aB) \right)}{a+b \sec(c+dx)} dx \\
&= -\frac{(aAb-3a^2B+2b^2B) \sqrt{\sec(c+dx)} \sin(c+dx)}{b^2(a^2-b^2)d} + \frac{a(Ab-aB) \sec^{\frac{3}{2}}(c+dx)}{b(a^2-b^2)d(a+b \sec(c+dx))} \\
&= -\frac{(aAb-3a^2B+2b^2B) \sqrt{\sec(c+dx)} \sin(c+dx)}{b^2(a^2-b^2)d} + \frac{a(Ab-aB) \sec^{\frac{3}{2}}(c+dx)}{b(a^2-b^2)d(a+b \sec(c+dx))} \\
&= -\frac{(aAb-3a^2B+2b^2B) \sqrt{\sec(c+dx)} \sin(c+dx)}{b^2(a^2-b^2)d} + \frac{a(Ab-aB) \sec^{\frac{3}{2}}(c+dx)}{b(a^2-b^2)d(a+b \sec(c+dx))} \\
&= \frac{(a^2Ab-3Ab^3-3a^3B+5ab^2B) \sqrt{\cos(c+dx)} \Pi\left(\frac{2a}{a+b}; \frac{1}{2}(c+dx) \middle| 2\right) \sqrt{\sec(c+dx)}}{(a-b)b^2(a+b)^2d} \\
&= \frac{(aAb-3a^2B+2b^2B) \sqrt{\cos(c+dx)} E\left(\frac{1}{2}(c+dx) \middle| 2\right) \sqrt{\sec(c+dx)}}{b^2(a^2-b^2)d} + \dots
\end{aligned}$$

Mathematica [B] time = 7.00, size = 680, normalized size = 2.16

$$\frac{\sqrt{\sec(c+dx)} \left(\frac{(-3a^2B+aAb+2b^2B)\sin(c+dx)}{b^2(b^2-a^2)} + \frac{a^2B\sin(c+dx)-aAb\sin(c+dx)}{b(b^2-a^2)(a\cos(c+dx)+b)} \right)}{d} - \frac{2(8a^2bB-4aAb^2-4b^3B)\sin(c+dx)\cos^2(c+dx)\sqrt{1-\sec^2(c+dx)}}{a(1-\cos^2(c+dx))(a\cos(c+dx)+b)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^2, x]

[Out] -1/4*((2*(-3*a^2*A*b + 4*A*b^3 + 9*a^3*B - 10*a*b^2*B)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] - EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1])*(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(-4*a*A*b^2 + 8*a^2*b*B - 4*b^3*B)*Cos[c + d*x]^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + ((-a^2*A*b) + 3*a^3*B - 2*a*b^2*B)*Cos[2*(c + d*x)]*(a + b*Sec[c + d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 2*a*(a - 2*b)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*a^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 4*b^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a^2*b*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2)))/((a - b)*b^2*(a + b)*d + (Sqrt[Sec[c + d*x]]*(((a*A*b - 3*a^2*B + 2*b^2*B)*Sin[c + d*x])/(b^2*(-a^2 + b^2)) + (-a*A*b*Sin[c + d*x]) + a^2*B*Sin[c + d*x])/(b*(-a^2 + b^2)*(b + a*Cos[c + d*x]))))/d

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx+c) + A) \sec(dx+c)^{\frac{5}{2}}}{(b \sec(dx+c) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(5/2)/(b*sec(d*x + c) + a)^2, x
)
```

maple [B] time = 12.40, size = 877, normalized size = 2.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a^2*B/b^2/(a^
2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*
c),2*a/(a-b),2^(1/2))+2*B/b^2*(-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2/sin(1/2*d*x+1/2*c)^
2/(2*sin(1/2*d*x+1/2*c)^2-1)+2*(A*b-B*a)/b*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2
*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos(1/2*d*x+1
/2*c)^2-a*b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/
2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b
),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm
="maxima")
```

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(\frac{1}{\cos(c+dx)}\right)^{5/2}}{\left(a + \frac{b}{\cos(c+dx)}\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + b/cos(c + d*x))^2,x)

[Out] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + b/cos(c + d*x))^2, x
)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**2,x)

[Out] Timed out

$$3.424 \quad \int \frac{\sec^2(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^2} dx$$

Optimal. Leaf size=257

$$\frac{a(Ab - aB) \sin(c + dx) \sqrt{\sec(c + dx)}}{bd(a^2 - b^2)(a + b \sec(c + dx))} - \frac{(Ab - aB) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{ad(a^2 - b^2)} - \frac{(Ab - aB) \sqrt{\cos(c + dx)}}{ad(a^2 - b^2)}$$

[Out] a*(A*b-B*a)*sin(d*x+c)*sec(d*x+c)^(1/2)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))- (A*b-B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b/(a^2-b^2)/d-(A*b-B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/(a^2-b^2)/d+(A*a^2*b+A*b^3+B*a^3-3*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/(a-b)/b/(a+b)^2/d

Rubi [A] time = 0.53, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {4029, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{a(Ab - aB) \sin(c + dx) \sqrt{\sec(c + dx)}}{bd(a^2 - b^2)(a + b \sec(c + dx))} - \frac{(Ab - aB) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{ad(a^2 - b^2)} - \frac{(Ab - aB) \sqrt{\cos(c + dx)}}{ad(a^2 - b^2)}$$

Antiderivative was successfully verified.

[In] Int[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^2,x]

[Out] -(((A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b*(a^2 - b^2)*d) - ((A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*(a^2 - b^2)*d) + ((a^2*A*b + A*b^3 + a^3*B - 3*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*(a - b)*b*(a + b)^2*d) + (a*(A*b - a*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2805

Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] := Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi/2 + f*x))/2, (2*d)/(c + d)]/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[c + d, 0]

Rule 3771

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^n, x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 3849

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 4029

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^m*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] := Simp[(a*d^2*(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[d/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(n - 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2*(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 1]

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.
))/ (Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^2} dx &= \frac{a(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{b(a^2-b^2)d(a+b\sec(c+dx))} + \frac{\int \frac{-\frac{1}{2}a(Ab-aB)-b(Ab-aB)\sec(c+dx)+}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))} dx}{b(a^2-b^2)} \\
&= \frac{a(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{b(a^2-b^2)d(a+b\sec(c+dx))} + \frac{\int \frac{-\frac{1}{2}a^2(Ab-aB)-\frac{1}{2}ab(Ab-aB)\sec(c+dx)}{\sqrt{\sec(c+dx)}} dx}{a^2b(a^2-b^2)} \\
&= \frac{a(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{b(a^2-b^2)d(a+b\sec(c+dx))} - \frac{(Ab-aB)\int\sqrt{\sec(c+dx)}dx}{2a(a^2-b^2)} \\
&= \frac{(a^2Ab+Ab^3+a^3B-3ab^2B)\sqrt{\cos(c+dx)}\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a(a-b)b(a+b)^2d} \\
&= -\frac{(Ab-aB)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{b(a^2-b^2)d} - \frac{(Ab-aB)\sqrt{\sec(c+dx)}}{2a(a^2-b^2)}
\end{aligned}$$

Mathematica [B] time = 7.00, size = 638, normalized size = 2.48

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{Ab\sin(c+dx)-aB\sin(c+dx)}{(b^2-a^2)(a\cos(c+dx)+b)} - \frac{(Ab-aB)\sin(c+dx)}{b(b^2-a^2)}\right)}{d} + \frac{2(-3a^2B-aAb+4b^2B)\sin(c+dx)\cos^2(c+dx)\sqrt{1-\sec^2(c+dx)}(a+b\sec(c+dx))}{b(1-\cos^2(c+dx))(a+b\sec(c+dx))}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^2,
x]
```

```
[Out] ((2*(-(a*A*b) - 3*a^2*B + 4*b^2*B)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Se
c[c + d*x]]], -1] - EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]))*(a
+ b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x]/(b*(b + a*Cos[c +
```

```

d*x))*(1 - Cos[c + d*x]^2)) + (2*(4*A*b^2 - 4*a*b*B)*Cos[c + d*x]^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + ((a*A*b - a^2*B)*Cos[2*(c + d*x)]*(a + b*Sec[c + d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 2*a*(a - 2*b)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*a^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 4*b^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a^2*b*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2)))/(4*b*(-a + b)*(a + b)*d + (Sqrt[Sec[c + d*x]]*(-((A*b - a*B)*Sin[c + d*x])/(b*(-a^2 + b^2))) + (A*b*Sin[c + d*x] - a*B*Sin[c + d*x])/((-a^2 + b^2)*(b + a*Cos[c + d*x]))))/d

```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sec(dx + c)^{\frac{3}{2}}}{(b \sec(dx + c) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^2, x)
```


maple [B] time = 10.08, size = 715, normalized size = 2.78

$$\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \left(\frac{2A\sqrt{\frac{1-\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)+1}\operatorname{EllipticPi}\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \frac{2a}{a-b}, \sqrt{2}\right)}{(a^2-ab)\sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)}} + \dots \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x)`

[Out]
$$\begin{aligned} & -\left(-2\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2+1\right)\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2\right)^{1/2}\left(-2A/\left(a^2-a*b\right)*\right. \\ & \left.\left(\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2\right)^{1/2}\left(-2\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2+1\right)^{1/2}/\left(-2\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^4+\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2\right)^{1/2}\right. \\ & \left.*\operatorname{EllipticPi}\left(\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right), 2*a/\left(a-b\right), 2^{1/2}\right)+2*\left(-A*b+B*a\right)/a*\left(a^2/b/\left(a^2-b^2\right)*\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right)*\left(-2\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^4+\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2\right)^{1/2}/\left(2*a*\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2-a+b\right)-\right. \right. \\ & \left. \left. 1/2/\left(a+b\right)/b*\left(\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2\right)^{1/2}\left(-2\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2+1\right)^{1/2}/\left(-2\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^4+\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2\right)^{1/2}\right. \right. \\ & \left. \left.*\operatorname{EllipticF}\left(\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right), 2^{1/2}\right)+1/2*a/b/\left(a^2-b^2\right)*\left(\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2\right)^{1/2}\left(-2\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2+1\right)^{1/2}/\left(-2\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^4+\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2\right)^{1/2}\right. \right. \\ & \left. \left.*\operatorname{EllipticF}\left(\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right), 2^{1/2}\right)-1/2*a/b/\left(a^2-b^2\right)*\left(\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2\right)^{1/2}\left(-2\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2+1\right)^{1/2}/\left(-2\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^4+\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2\right)^{1/2}\right. \right. \\ & \left. \left.*\operatorname{EllipticE}\left(\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right), 2^{1/2}\right)-1/2/b/\left(a^2-b^2\right)/\left(a^2-a*b\right)*a^3*\left(\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2\right)^{1/2}\left(-2\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2+1\right)^{1/2}/\left(-2\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^4+\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2\right)^{1/2}\right. \right. \\ & \left. \left.*\operatorname{EllipticPi}\left(\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right), 2*a/\left(a-b\right), 2^{1/2}\right)+3/2*b/\left(a^2-b^2\right)/\left(a^2-a*b\right)*a*\left(\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2\right)^{1/2}\left(-2\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2+1\right)^{1/2}/\left(-2\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^4+\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2\right)^{1/2}\right. \right. \\ & \left. \left.*\operatorname{EllipticPi}\left(\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right), 2*a/\left(a-b\right), 2^{1/2}\right)\right)\right)/\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)/\left(2*\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2-1\right)^{1/2}/d \end{aligned}$$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm="maxima")`

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(\frac{1}{\cos(c+dx)}\right)^{3/2}}{\left(a + \frac{b}{\cos(c+dx)}\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + b/cos(c + d*x))^2,x)

[Out] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + b/cos(c + d*x))^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \sec^{\frac{3}{2}}(c + dx)}{(a + b \sec(c + dx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**2,x)

[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**(3/2)/(a + b*sec(c + d*x))**2, x)

$$3.425 \quad \int \frac{\sqrt{\sec(c+dx)} (A+B \sec(c+dx))}{(a+b \sec(c+dx))^2} dx$$

Optimal. Leaf size=263

$$\frac{(Ab - aB) \sin(c + dx) \sqrt{\sec(c + dx)}}{d(a^2 - b^2)(a + b \sec(c + dx))} + \frac{(2a^2A - abB - Ab^2) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{a^2d(a^2 - b^2)} + \frac{(Ab - aB) \sin(c + dx) \sqrt{\sec(c + dx)}}{d(a^2 - b^2)(a + b \sec(c + dx))}$$

[Out] $-(A*b-B*a)*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/(a^2-b^2)/d/(a+b*\sec(d*x+c))+(A*b-B*a)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/a/(a^2-b^2)/d+(2*A*a^2-A*b^2-B*a*b)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/a^2/(a^2-b^2)/d-(3*A*a^2*b-A*b^3-B*a^3-B*a*b^2)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticPi}(\sin(1/2*d*x+1/2*c), 2*a/(a+b), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/a^2/(a-b)/(a+b)^2/d$

Rubi [A] time = 0.51, antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {4027, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{(Ab - aB) \sin(c + dx) \sqrt{\sec(c + dx)}}{d(a^2 - b^2)(a + b \sec(c + dx))} + \frac{(2a^2A - abB - Ab^2) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{a^2d(a^2 - b^2)} + \frac{(Ab - aB) \sin(c + dx) \sqrt{\sec(c + dx)}}{d(a^2 - b^2)(a + b \sec(c + dx))}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(\text{Sqrt}[\text{Sec}[c + d*x]]*(A + B*\text{Sec}[c + d*x]))/(a + b*\text{Sec}[c + d*x])^2, x]$

[Out] $((A*b - a*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/(a*(a^2 - b^2)*d) + ((2*a^2*A - A*b^2 - a*b*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticF}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/(a^2*(a^2 - b^2)*d) - ((3*a^2*A*b - A*b^3 - a^3*B - a*b^2*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticPi}[(2*a)/(a + b), (c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/(a^2*(a - b)*(a + b)^2*d) - ((A*b - a*B)*\text{Sqrt}[\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/((a^2 - b^2)*d*(a + b*\text{Sec}[c + d*x]))$

Rule 2639

$\text{Int}[\text{Sqrt}[\sin[(c_.) + (d_.)*(x_.)]], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticE}[(1*(c - \text{Pi}/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 2641

$\text{Int}[1/\text{Sqrt}[\sin[(c_.) + (d_.)*(x_.)]], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticF}[(1*(c - \text{Pi}/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])], x_Symbol] := Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)]/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^n, x_Symbol] := Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] := Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4027

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^m*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] := -Simp[(d*(A*
b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)
)/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*Simp[d*(n - 1)*(A*b - a*B) + d
*(a*A - b*B)*(m + 1)*Csc[e + f*x] - d*(A*b - a*B)*(m + n + 1)*Csc[e + f*x]^
2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && Ne
Q[a^2 - b^2, 0] && LtQ[m, -1] && LtQ[0, n, 1]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] := Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B)
)*Csc[e + f*x]/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
```

$C\}, x] \&\& \text{NeQ}[a^2 - b^2, 0]$

Rubi steps

$$\begin{aligned}
 \int \frac{\sqrt{\sec(c+dx)}(A+B\sec(c+dx))}{(a+b\sec(c+dx))^2} dx &= -\frac{(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{(a^2-b^2)d(a+b\sec(c+dx))} + \frac{\int \frac{\frac{1}{2}(-Ab+aB)-(aA-bB)\sec(c+dx)+\frac{1}{2}a^2}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))} dx}{-a^2+b^2} \\
 &= -\frac{(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{(a^2-b^2)d(a+b\sec(c+dx))} - \frac{\int \frac{\frac{1}{2}a(-Ab+aB)-\left(\frac{1}{2}b(-Ab+aB)-a(-aA+bB)\right)\sec(c+dx)+\frac{1}{2}a^2}{\sqrt{\sec(c+dx)}} dx}{a^2(a^2-b^2)} \\
 &= -\frac{(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{(a^2-b^2)d(a+b\sec(c+dx))} + \frac{(Ab-aB)\int \frac{1}{\sqrt{\sec(c+dx)}} dx}{2a(a^2-b^2)} + \\
 &= -\frac{(3a^2Ab-Ab^3-a^3B-ab^2B)\sqrt{\cos(c+dx)}\Pi\left(\frac{2a}{a+b}; \frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2(a-b)(a+b)^2d} \\
 &= \frac{(Ab-aB)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a(a^2-b^2)d} + \frac{(2a^2A-Ab^2B)}{2a(a^2-b^2)}
 \end{aligned}$$

Mathematica [B] time = 6.96, size = 722, normalized size = 2.75

$$\frac{\sec^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+b)^2(A+B\sec(c+dx))\left(\frac{(aB-Ab)\sin(c+dx)}{a(a^2-b^2)} + \frac{Ab^2\sin(c+dx)-abB\sin(c+dx)}{a(a^2-b^2)(a\cos(c+dx)+b)}\right)}{d(a+b\sec(c+dx))^2(A\cos(c+dx)+B)} + \frac{\sec(c+dx)(a\cos(c+dx)+b)}{2a(a^2-b^2)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^2, x]

[Out] ((b + a*Cos[c + d*x])^2*Sec[c + d*x]*(A + B*Sec[c + d*x]))*((2*(-(A*b) + a*B)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] - EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1])*(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(4*a*A - 4*b*B)*Cos[c + d*x]^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + ((A*b - a*B)*Cos[2*(c + d*x)]*(a + b*Sec[c + d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqr

```
t[Sec[c + d*x]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 2*a*(a
- 2*b)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1
- Sec[c + d*x]^2] + 2*a^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1
]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 4*b^2*EllipticPi[-(b/a), Ar
cSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*
Sin[c + d*x])/(a^2*b*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c +
d*x]]*(2 - Sec[c + d*x]^2)))/(4*(a - b)*(a + b)*d*(B + A*Cos[c + d*x])*(a
+ b*Sec[c + d*x])^2) + ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^(3/2)*(A + B*S
ec[c + d*x])*((( - (A*b) + a*B)*Sin[c + d*x])/(a*(a^2 - b^2)) + (A*b^2*Sin[c
+ d*x] - a*b*B*Sin[c + d*x])/(a*(a^2 - b^2)*(b + a*Cos[c + d*x])))))/(d*(B +
A*Cos[c + d*x])*(a + b*Sec[c + d*x])^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm
="fricas")
```

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sqrt{\sec(dx + c)}}{(b \sec(dx + c) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(b*sec(d*x + c) + a)^2, x
)
```

maple [B] time = 10.88, size = 802, normalized size = 3.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A/a^2*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2*(
```

$$\begin{aligned} & -2A^*b+B^*a)/a/(a^2-a^*b)*(sin(1/2*d^*x+1/2*c)^2)^{(1/2)}*(-2*cos(1/2*d^*x+1/2*c) \\ & ^2+1)^{(1/2)/(-2*sin(1/2*d^*x+1/2*c)^4+sin(1/2*d^*x+1/2*c)^2)^{(1/2)}*EllipticPi \\ & (cos(1/2*d^*x+1/2*c),2*a/(a-b),2^{(1/2)})+2*(A^*b-B^*a)*b/a^2*(a^2/b/(a^2-b^2)*c \\ & os(1/2*d^*x+1/2*c)*(-2*sin(1/2*d^*x+1/2*c)^4+sin(1/2*d^*x+1/2*c)^2)^{(1/2)/(2*a \\ & *cos(1/2*d^*x+1/2*c)^2-a+b)-1/2/(a+b)/b*(sin(1/2*d^*x+1/2*c)^2)^{(1/2)}*(-2*cos \\ & (1/2*d^*x+1/2*c)^2+1)^{(1/2)/(-2*sin(1/2*d^*x+1/2*c)^4+sin(1/2*d^*x+1/2*c)^2)^{(1/2)} \\ & *EllipticF(cos(1/2*d^*x+1/2*c),2^{(1/2)})+1/2*a/b/(a^2-b^2)*(sin(1/2*d^*x+1 \\ & /2*c)^2)^{(1/2)}*(-2*cos(1/2*d^*x+1/2*c)^2+1)^{(1/2)/(-2*sin(1/2*d^*x+1/2*c)^4+s \\ & in(1/2*d^*x+1/2*c)^2)^{(1/2)}*EllipticF(cos(1/2*d^*x+1/2*c),2^{(1/2)})-1/2*a/b/(a \\ & ^2-b^2)*(sin(1/2*d^*x+1/2*c)^2)^{(1/2)}*(-2*cos(1/2*d^*x+1/2*c)^2+1)^{(1/2)/(-2* \\ & sin(1/2*d^*x+1/2*c)^4+sin(1/2*d^*x+1/2*c)^2)^{(1/2)}*EllipticE(cos(1/2*d^*x+1/2* \\ & c),2^{(1/2)})-1/2/b/(a^2-b^2)/(a^2-a^*b)*a^3*(sin(1/2*d^*x+1/2*c)^2)^{(1/2)}*(-2* \\ & cos(1/2*d^*x+1/2*c)^2+1)^{(1/2)/(-2*sin(1/2*d^*x+1/2*c)^4+sin(1/2*d^*x+1/2*c)^2 \\ &)^{(1/2)}*EllipticPi(cos(1/2*d^*x+1/2*c),2*a/(a-b),2^{(1/2)})+3/2*b/(a^2-b^2)/(a \\ & ^2-a^*b)*a*(sin(1/2*d^*x+1/2*c)^2)^{(1/2)}*(-2*cos(1/2*d^*x+1/2*c)^2+1)^{(1/2)/(- \\ & 2*sin(1/2*d^*x+1/2*c)^4+sin(1/2*d^*x+1/2*c)^2)^{(1/2)}*EllipticPi(cos(1/2*d^*x+1 \\ & /2*c),2*a/(a-b),2^{(1/2)})))/sin(1/2*d^*x+1/2*c)/(2*cos(1/2*d^*x+1/2*c)^2-1)^{(1 \\ & /2)/d \end{aligned}$$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(1/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \sqrt{\frac{1}{\cos(c+dx)}}}{\left(a + \frac{b}{\cos(c+dx)}\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + b/cos(c + d*x))^2,x)

[Out] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + b/cos(c + d*x))^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \sqrt{\sec(c + dx)}}{(a + b \sec(c + dx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sqrt(sec(c + d*x))/(a + b*sec(c + d*x))**2, x
)
```


$$3.426 \quad \int \frac{A+B \sec(c+dx)}{\sqrt{\sec(c+dx)} (a+b \sec(c+dx))^2} dx$$

Optimal. Leaf size=283

$$\frac{b(Ab - aB) \sin(c + dx) \sqrt{\sec(c + dx)}}{ad(a^2 - b^2)(a + b \sec(c + dx))} + \frac{(2a^2A + abB - 3Ab^2) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{a^2d(a^2 - b^2)} - \frac{(-2a^2A - 3a^2b^2 + B^2) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{a^2d(a^2 - b^2)}$$

[Out] b*(A*b-B*a)*sin(d*x+c)*sec(d*x+c)^(1/2)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))+(2*A*a^2-3*A*b^2+B*a*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/(a^2-b^2)/d-(4*A*a^2*b-3*A*b^3-2*B*a^3+B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/(a^2-b^2)/d+b*(5*A*a^2*b-3*A*b^3-3*B*a^3+B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/(a-b)/(a+b)^2/d

Rubi [A] time = 0.57, antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {4030, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{b(Ab - aB) \sin(c + dx) \sqrt{\sec(c + dx)}}{ad(a^2 - b^2)(a + b \sec(c + dx))} - \frac{(4a^2Ab - 2a^3B + ab^2B - 3Ab^3) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{a^3d(a^2 - b^2)}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sec[c + d*x])/(Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^2), x]

[Out] ((2*a^2*A - 3*A*b^2 + a*b*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*(a^2 - b^2)*d) - ((4*a^2*A*b - 3*A*b^3 - 2*a^3*B + a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^3*(a^2 - b^2)*d) + (b*(5*a^2*A*b - 3*A*b^3 - 3*a^3*B + a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^3*(a - b)*(a + b)^2*d) + (b*(A*b - a*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_.)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2805

Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] := Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi/2 + f*x))/2, (2*d)/(c + d)]/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[c + d, 0]

Rule 3771

Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.))^n, x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 3849

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] := Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 4030

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := Simp[(b*(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[A*(a^2*(m + 1) - b^2*(m + n + 1)) + a*b*B*n - a*(A*b - a*B)*(m + 1)*Csc[e + f*x] + b*(A*b - a*B)*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a
_))), x_Symbol] :=> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{A + B \sec(c + dx)}{\sqrt{\sec(c + dx)} (a + b \sec(c + dx))^2} dx &= \frac{b(Ab - aB) \sqrt{\sec(c + dx)} \sin(c + dx)}{a(a^2 - b^2) d(a + b \sec(c + dx))} - \frac{\int \frac{\frac{1}{2}(-2a^2A + 3Ab^2 - abB) + a(Ab - aB) \sec(c + dx)}{\sqrt{\sec(c + dx)} (a + b \sec(c + dx))^2} dx}{a(a^2 - b^2)} \\ &= \frac{b(Ab - aB) \sqrt{\sec(c + dx)} \sin(c + dx)}{a(a^2 - b^2) d(a + b \sec(c + dx))} - \frac{\int \frac{\frac{1}{2}a(-2a^2A + 3Ab^2 - abB) - (-a^2(Ab - aB) \sec(c + dx))}{\sqrt{\sec(c + dx)} (a + b \sec(c + dx))^2} dx}{a^3(a^2 - b^2)} \\ &= \frac{b(Ab - aB) \sqrt{\sec(c + dx)} \sin(c + dx)}{a(a^2 - b^2) d(a + b \sec(c + dx))} + \frac{(2a^2A - 3Ab^2 + abB) \int \frac{1}{\sqrt{\sec(c + dx)}} dx}{2a^2(a^2 - b^2)} \\ &= \frac{b(5a^2Ab - 3Ab^3 - 3a^3B + ab^2B) \sqrt{\cos(c + dx)} \Pi\left(\frac{2a}{a+b}; \frac{1}{2}(c + dx) \middle| 2\right)}{a^3(a - b)(a + b)^2 d} \\ &= \frac{(2a^2A - 3Ab^2 + abB) \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{\sec(c + dx)}}{a^2(a^2 - b^2) d} \end{aligned}$$

Mathematica [B] time = 6.93, size = 652, normalized size = 2.30

$$\frac{2(-2a^2A + abB + Ab^2) \sin(c + dx) \cos^2(c + dx) \sqrt{1 - \sec^2(c + dx)} (a + b \sec(c + dx)) \left(F(\sin^{-1}(\sqrt{\sec(c + dx)}) \middle| -1) - \Pi\left(-\frac{b}{a}; \sin^{-1}(\sqrt{\sec(c + dx)}) \middle| -1\right) \right)}{b(1 - \cos^2(c + dx))(a \cos(c + dx) + b)} + \frac{(-2a^2A + abB + Ab^2) \sin(c + dx) \cos^2(c + dx) \sqrt{1 - \sec^2(c + dx)} (a + b \sec(c + dx))}{b(1 - \cos^2(c + dx))(a \cos(c + dx) + b)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^2),
x]
```

```
[Out] ((2*(-2*a^2*A + A*b^2 + a*b*B)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c
+ d*x]]], -1] - EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1])*(a + b*
Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(b + a*Cos[c + d*x]
```

```

)*(1 - Cos[c + d*x]^2)) + (2*(4*a*A*b - 4*a^2*B)*Cos[c + d*x]^2*EllipticPi[
-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c
+ d*x]^2]*Sin[c + d*x])/(a*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + ((
-2*a^2*A + 3*A*b^2 - a*b*B)*Cos[2*(c + d*x)]*(a + b*Sec[c + d*x])*(-4*a*b +
4*a*b*Sec[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqr
t[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 2*a*(a - 2*b)*EllipticF[ArcSin[S
qrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*a^2
*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt
[1 - Sec[c + d*x]^2] - 4*b^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]],
-1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a^2*b*(b +
a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^
2)))/(4*a*(-a + b)*(a + b)*d) + (Sqrt[Sec[c + d*x]]*(-((b*(A*b - a*B)*Sin[c
+ d*x])/(a^2*(-a^2 + b^2)))) + (- (A*b^3*Sin[c + d*x]) + a*b^2*B*Sin[c + d*x
])/(a^2*(a^2 - b^2)*(b + a*Cos[c + d*x])))/d

```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^2,x, algorithm
="fricas")
```

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(b \sec(dx + c) + a)^2 \sqrt{\sec(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^2*sqrt(sec(d*x + c))),
x)
```

maple [B] time = 13.45, size = 843, normalized size = 2.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/a^3/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*
b+A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a-B*EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2))*a)-2/a^2*b*(3*A*b-2*B*a)/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))-2*b^2*(A*b-B*a)/
a^3*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)/(2*a*cos(1/2*d*x+1/2*c)^2-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*a/b/(a
^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/
2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*
x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(
1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(b \sec(dx + c) + a)^2 \sqrt{\sec(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^2*sqrt(sec(d*x + c))),
x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\left(a + \frac{b}{\cos(c+dx)}\right)^2 \sqrt{\frac{1}{\cos(c+dx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/((a + b/cos(c + d*x))^2*(1/cos(c + d*x))^(1/2)),x)
```

```
[Out] int((A + B/cos(c + d*x))/((a + b/cos(c + d*x))^2*(1/cos(c + d*x))^(1/2)), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)**(1/2)/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

$$3.427 \quad \int \frac{A+B \sec(c+dx)}{\sec^2(c+dx)(a+b \sec(c+dx))^2} dx$$

Optimal. Leaf size=365

$$\frac{b(Ab - aB) \sin(c + dx)}{ad(a^2 - b^2) \sqrt{\sec(c + dx)} (a + b \sec(c + dx))} + \frac{(2a^2A + 3abB - 5Ab^2) \sin(c + dx)}{3a^2d(a^2 - b^2) \sqrt{\sec(c + dx)}} - \frac{(-2a^3B + 4a^2Ab + 3ab^2B - \dots)}{\dots}$$

[Out] $1/3*(2*A*a^2-5*A*b^2+3*B*a*b)*\sin(d*x+c)/a^2/(a^2-b^2)/d/\sec(d*x+c)^{(1/2)+b*(A*b-B*a)*\sin(d*x+c)/a/(a^2-b^2)/d/(a+b*\sec(d*x+c))/\sec(d*x+c)^{(1/2)}-(4*A*a^2*b-5*A*b^3-2*B*a^3+3*B*a*b^2)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/a^3/(a^2-b^2)/d+1/3*(2*A*a^4+16*A*a^2*b^2-15*A*b^4-12*B*a^3*b+9*B*a*b^3)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/a^4/(a^2-b^2)/d-b^2*(7*A*a^2*b-5*A*b^3-5*B*a^3+3*B*a*b^2)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticPi}(\sin(1/2*d*x+1/2*c), 2*a/(a+b), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/a^4/(a-b)/(a+b)^2/d$

Rubi [A] time = 0.86, antiderivative size = 365, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {4030, 4104, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{b(Ab - aB) \sin(c + dx)}{ad(a^2 - b^2) \sqrt{\sec(c + dx)} (a + b \sec(c + dx))} + \frac{(2a^2A + 3abB - 5Ab^2) \sin(c + dx)}{3a^2d(a^2 - b^2) \sqrt{\sec(c + dx)}} + \frac{(16a^2Ab^2 + 2a^4A - 12a^3bB - \dots)}{\dots}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(A + B*\text{Sec}[c + d*x])/(\text{Sec}[c + d*x]^{(3/2)}*(a + b*\text{Sec}[c + d*x])^2), x]$

[Out] $-(((4*a^2*A*b - 5*A*b^3 - 2*a^3*B + 3*a*b^2*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/(a^3*(a^2 - b^2)*d) + ((2*a^4*A + 16*a^2*A*b^2 - 15*A*b^4 - 12*a^3*b*B + 9*a*b^3*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticF}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/(3*a^4*(a^2 - b^2)*d) - (b^2*(7*a^2*A*b - 5*A*b^3 - 5*a^3*B + 3*a*b^2*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticPi}[(2*a)/(a + b), (c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/(a^4*(a - b)*(a + b)^2*d) + ((2*a^2*A - 5*A*b^2 + 3*a*b*B)*\text{Sin}[c + d*x])/(3*a^2*(a^2 - b^2)*d*\text{Sqrt}[\text{Sec}[c + d*x]]) + (b*(A*b - a*B)*\text{Sin}[c + d*x])/(a*(a^2 - b^2)*d*\text{Sqrt}[\text{Sec}[c + d*x]]*(a + b*\text{Sec}[c + d*x]))$

Rule 2639

$\text{Int}[\text{Sqrt}[\sin[(c_.) + (d_.)*(x_.)]], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticE}[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}\{c, d, x\}$

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2805

Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi/2 + f*x))/2, (2*d)/(c + d)]/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[c + d, 0]

Rule 3771

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^n, x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 3849

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 4030

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^m*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] := Simp[(b*(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[A*(a^2*(m + 1) - b^2*(m + n + 1)) + a*b*B*n - a*(A*b - a*B)*(m + 1)*Csc[e + f*x] + b*(A*b - a*B)*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])

Rule 4104


```

Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a
_))^(m_), x_Symbol] := Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x] /; FreeQ[{a, b, d,
e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]

```

Rule 4106

```

Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a
_))), x_Symbol] := Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
C}, x] && NeQ[a^2 - b^2, 0]

```

Rubi steps

$$\begin{aligned}
\int \frac{A + B \sec(c + dx)}{\sec^{\frac{3}{2}}(c + dx)(a + b \sec(c + dx))^2} dx &= \frac{b(Ab - aB) \sin(c + dx)}{a(a^2 - b^2) d \sqrt{\sec(c + dx)} (a + b \sec(c + dx))} - \int \frac{\frac{1}{2}(-2a^2A + 5Ab^2 - 3abB) + a^3}{\sec^{\frac{3}{2}}} \\
&= \frac{(2a^2A - 5Ab^2 + 3abB) \sin(c + dx)}{3a^2(a^2 - b^2) d \sqrt{\sec(c + dx)}} + \frac{b(Ab - aB) \sin(c + dx)}{a(a^2 - b^2) d \sqrt{\sec(c + dx)} (a + b \sec(c + dx))} \\
&= \frac{(2a^2A - 5Ab^2 + 3abB) \sin(c + dx)}{3a^2(a^2 - b^2) d \sqrt{\sec(c + dx)}} + \frac{b(Ab - aB) \sin(c + dx)}{a(a^2 - b^2) d \sqrt{\sec(c + dx)} (a + b \sec(c + dx))} \\
&= \frac{(2a^2A - 5Ab^2 + 3abB) \sin(c + dx)}{3a^2(a^2 - b^2) d \sqrt{\sec(c + dx)}} + \frac{b(Ab - aB) \sin(c + dx)}{a(a^2 - b^2) d \sqrt{\sec(c + dx)} (a + b \sec(c + dx))} \\
&= -\frac{b^2(7a^2Ab - 5Ab^3 - 5a^3B + 3ab^2B) \sqrt{\cos(c + dx)} \Pi\left(\frac{2a}{a+b}; \frac{1}{2}(c + dx)\right)}{a^4(a - b)(a + b)^2 d} \\
&= -\frac{(4a^2Ab - 5Ab^3 - 2a^3B + 3ab^2B) \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{\sec(c + dx)}}{a^3(a^2 - b^2) d}
\end{aligned}$$

Mathematica [A] time = 7.02, size = 699, normalized size = 1.92

$$\frac{2(4a^3A - 12a^2bB + 8aAb^2) \sin(c+dx) \cos^2(c+dx) \sqrt{1-\sec^2(c+dx)} (a+b \sec(c+dx)) \Pi\left(-\frac{b}{a}; \sin^{-1}(\sqrt{\sec(c+dx)}) \middle| -1\right)}{a(1-\cos^2(c+dx))(a \cos(c+dx)+b)} + \frac{2(6a^3B - 8a^2Ab - 3ab^2B + 5Ab^3) \sin(c+dx) \cos^2(c+dx) \sqrt{1-\sec^2(c+dx)} (a+b \sec(c+dx)) \Pi\left(-\frac{b}{a}; \sin^{-1}(\sqrt{\sec(c+dx)}) \middle| -1\right)}{a(1-\cos^2(c+dx))(a \cos(c+dx)+b)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^2), x]

[Out] ((2*(-8*a^2*A*b + 5*A*b^3 + 6*a^3*B - 3*a*b^2*B)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] - EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1])*(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(b + a*cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(4*a^3*A + 8*a*A*b^2 - 12*a^2*b*B)*Cos[c + d*x]^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(b + a*cos[c + d*x])*(1 - Cos[c + d*x]^2)) + ((-12*a^2*A*b + 15*A*b^3 + 6*a^3*B - 9*a*b^2*B)*Cos[2*(c + d*x)]*(a + b*Sec[c + d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 2*a*(a - 2*b)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*a^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 4*b^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a^2*b*(b + a*cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2))/(12*a^2*(a - b)*(a + b)*d) + (Sqrt[Sec[c + d*x]]*((b^2*(A*b - a*B)*Sin[c + d*x])/(a^3*(-a^2 + b^2)) - ((A*b^4*Sin[c + d*x]) + a*b^3*B*Sin[c + d*x])/(a^3*(a^2 - b^2)*(b + a*cos[c + d*x])) + (A*Sin[2*(c + d*x)])/(3*a^2)))/d

fricas [F] time = 120.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B \sec(dx + c) + A)\sqrt{\sec(dx + c)}}{b^2 \sec(dx + c)^4 + 2ab \sec(dx + c)^3 + a^2 \sec(dx + c)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")

[Out] integral((B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(b^2*sec(d*x + c)^4 + 2*a*b*sec(d*x + c)^3 + a^2*sec(d*x + c)^2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(b \sec(dx + c) + a)^2 \sec(dx + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^2*sec(d*x + c)^(3/2)),
x)
```

maple [B] time = 15.59, size = 1059, normalized size = 2.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/3/a^4*(4*A*a^
2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+a^2*A*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9*
A*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))+6*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b-2*A*a^2*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-6*B*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))*a^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)+2*b^2/a^3*(4*A*b-3*B*a)/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+2*b^3*(A*b-B*a)/a^4
*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)/(2*a*cos(1/2*d*x+1/2*c)^2-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*a/b/(a^2-
b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))
+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2
*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\left(a + \frac{b}{\cos(c+dx)}\right)^2 \left(\frac{1}{\cos(c+dx)}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/((a + b/cos(c + d*x))^2*(1/cos(c + d*x))^(3/2)),x)

[Out] int((A + B/cos(c + d*x))/((a + b/cos(c + d*x))^2*(1/cos(c + d*x))^(3/2)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)**(3/2)/(a+b*sec(d*x+c))**2,x)

[Out] Timed out

$$3.428 \quad \int \frac{\sec^{\frac{9}{2}}(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^3} dx$$

Optimal. Leaf size=583

$$\frac{a(Ab - aB) \sin(c + dx) \sec^{\frac{7}{2}}(c + dx)}{2bd(a^2 - b^2)(a + b \sec(c + dx))^2} + \frac{a(-7a^3B + 3a^2Ab + 13ab^2B - 9Ab^3) \sin(c + dx) \sec^{\frac{5}{2}}(c + dx)}{4b^2d(a^2 - b^2)^2(a + b \sec(c + dx))} - \frac{(-35a^4B + 15a^3Ab - 29a^2A^2b + 8A^3b^2 - 35A^4B + 61A^2b^2 - 8B^2b^4) \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{4b^2d(a^2 - b^2)^2(a + b \sec(c + dx))}$$

[Out] $-1/12*(15*A*a^3*b-33*A*a*b^3-35*B*a^4+61*B*a^2*b^2-8*B*b^4)*\sec(d*x+c)^{(3/2)}$
 $*\sin(d*x+c)/b^3/(a^2-b^2)^2/d+1/2*a*(A*b-B*a)*\sec(d*x+c)^{(7/2)*\sin(d*x+c)/$
 $b/(a^2-b^2)/d/(a+b*\sec(d*x+c))^2+1/4*a*(3*A*a^2*b-9*A*b^3-7*B*a^3+13*B*a*b^2)$
 $*\sec(d*x+c)^{(5/2)*\sin(d*x+c)/b^2/(a^2-b^2)^2/d/(a+b*\sec(d*x+c))+1/4*(15*A$
 $*a^4*b-29*A*a^2*b^3+8*A*b^5-35*B*a^5+65*B*a^3*b^2-24*B*a*b^4)*\sin(d*x+c)*\sec$
 $(d*x+c)^{(1/2)/b^4/(a^2-b^2)^2/d-1/4*(15*A*a^4*b-29*A*a^2*b^3+8*A*b^5-35*B*$
 $a^5+65*B*a^3*b^2-24*B*a*b^4)*(\cos(1/2*d*x+1/2*c))^2)^{(1/2)/\cos(1/2*d*x+1/2*c$
 $)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)*\sec(d*x+c)^{(1/2)/b$
 $^4/(a^2-b^2)^2/d-1/12*(15*A*a^3*b-33*A*a*b^3-35*B*a^4+61*B*a^2*b^2-8*B*b^4)$
 $*(\cos(1/2*d*x+1/2*c))^2)^{(1/2)/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*$
 $c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)*\sec(d*x+c)^{(1/2)/b^3/(a^2-b^2)^2/d-1/4*a*(15*A$
 $*a^4*b-38*A*a^2*b^3+35*A*b^5-35*B*a^5+86*B*a^3*b^2-63*B*a*b^4)*(\cos(1/2*d*x$
 $+1/2*c))^2)^{(1/2)/\cos(1/2*d*x+1/2*c)*\text{EllipticPi}(\sin(1/2*d*x+1/2*c), 2*a/(a+b)$
 $, 2^{(1/2)})*\cos(d*x+c)^{(1/2)*\sec(d*x+c)^{(1/2)/(a-b)^2/b^4/(a+b)^3/d$

Rubi [A] time = 1.78, antiderivative size = 583, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.303$, Rules used = {4029, 4098, 4102, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{a(Ab - aB) \sin(c + dx) \sec^{\frac{7}{2}}(c + dx)}{2bd(a^2 - b^2)(a + b \sec(c + dx))^2} + \frac{a(3a^2Ab - 7a^3B + 13ab^2B - 9Ab^3) \sin(c + dx) \sec^{\frac{5}{2}}(c + dx)}{4b^2d(a^2 - b^2)^2(a + b \sec(c + dx))} - \frac{(15a^3Ab + 15a^2A^2b - 29a^2A^2b^2 + 8A^3b^3 - 35A^4B + 61A^2b^2 - 8B^2b^4) \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{4b^2d(a^2 - b^2)^2(a + b \sec(c + dx))}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(\text{Sec}[c + d*x])^{(9/2)}*(A + B*\text{Sec}[c + d*x])]/(a + b*\text{Sec}[c + d*x])^3, x]$

[Out] $-((15*a^4*A*b - 29*a^2*A*b^3 + 8*A*b^5 - 35*a^5*B + 65*a^3*b^2*B - 24*a*b^4$
 $*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/(4*b^4$
 $*a^2 - b^2)^2*d - ((15*a^3*A*b - 33*a*A*b^3 - 35*a^4*B + 61*a^2*b^2*B - 8$
 $*b^4*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticF}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/(1$
 $2*b^3*(a^2 - b^2)^2*d - (a*(15*a^4*A*b - 38*a^2*A*b^3 + 35*A*b^5 - 35*a^5*$
 $B + 86*a^3*b^2*B - 63*a*b^4*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticPi}[(2*a)/(a + b),$
 $(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/(4*(a - b)^2*b^4*(a + b)^3*d + ((15*a$

$$\begin{aligned} & ^4A*b - 29*a^2*A*b^3 + 8*A*b^5 - 35*a^5*B + 65*a^3*b^2*B - 24*a*b^4*B)*\text{Sqrt}[\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(4*b^4*(a^2 - b^2)^2*d) - ((15*a^3*A*b - 33*a \\ & *A*b^3 - 35*a^4*B + 61*a^2*b^2*B - 8*b^4*B)*\text{Sec}[c + d*x]^{(3/2)}*\text{Sin}[c + d*x] \\ &)/(12*b^3*(a^2 - b^2)^2*d) + (a*(A*b - a*B)*\text{Sec}[c + d*x]^{(7/2)}*\text{Sin}[c + d*x] \\ &)/(2*b*(a^2 - b^2)*d*(a + b*\text{Sec}[c + d*x])^2) + (a*(3*a^2*A*b - 9*A*b^3 - 7* \\ & a^3*B + 13*a*b^2*B)*\text{Sec}[c + d*x]^{(5/2)}*\text{Sin}[c + d*x])/(4*b^2*(a^2 - b^2)^2*d \\ & *(a + b*\text{Sec}[c + d*x])) \end{aligned}$$
Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi/2 + f*x))/2, (2*d)/(c + d)]/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[c + d, 0]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4029

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] := Simp[(a*d^2*
(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n -
2))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[d/(b*(m + 1)*(a^2 - b^2)), Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(n
- 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2*
(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n
, 1]
```

Rule 4098

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] := -Simp[(d*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b
*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1))/(b*f*(a^2 - b^2)*(m + 1)),
x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*C
sc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A -
b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n + b
^2*(m + 1)))*Csc[e + f*x]^2, x], x] /; FreeQ[{a, b, d, e, f, A, B, C},
x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] := -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] := Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B)
)*Csc[e + f*x]/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^{\frac{9}{2}}(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^3} dx &= \frac{a(Ab-aB) \sec^{\frac{7}{2}}(c+dx) \sin(c+dx)}{2b(a^2-b^2)d(a+b \sec(c+dx))^2} + \frac{\int \frac{\sec^{\frac{5}{2}}(c+dx) \left(\frac{5}{2}a(Ab-aB)-2b(Ab-aB) \sec(c+dx)\right)}{(a+b \sec(c+dx))^3} dx}{2b(a^2-b^2)d(a+b \sec(c+dx))^2} \\
&= \frac{a(Ab-aB) \sec^{\frac{7}{2}}(c+dx) \sin(c+dx)}{2b(a^2-b^2)d(a+b \sec(c+dx))^2} + \frac{a(3a^2Ab-9Ab^3-7a^3B+13ab^2B)}{4b^2(a^2-b^2)^2d(a+b \sec(c+dx))} \\
&= -\frac{(15a^3Ab-33aAb^3-35a^4B+61a^2b^2B-8b^4B) \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{12b^3(a^2-b^2)^2d} \\
&= \frac{(15a^4Ab-29a^2Ab^3+8Ab^5-35a^5B+65a^3b^2B-24ab^4B) \sqrt{\sec(c+dx)}}{4b^4(a^2-b^2)^2d} \\
&= \frac{(15a^4Ab-29a^2Ab^3+8Ab^5-35a^5B+65a^3b^2B-24ab^4B) \sqrt{\sec(c+dx)}}{4b^4(a^2-b^2)^2d} \\
&= \frac{(15a^4Ab-29a^2Ab^3+8Ab^5-35a^5B+65a^3b^2B-24ab^4B) \sqrt{\sec(c+dx)}}{4b^4(a^2-b^2)^2d} \\
&= -\frac{a(15a^4Ab-38a^2Ab^3+35Ab^5-35a^5B+86a^3b^2B-63ab^4B) \sqrt{\cos(c+dx)}}{4(a-b)^2b^4(a+b)^3d} \\
&= -\frac{(15a^4Ab-29a^2Ab^3+8Ab^5-35a^5B+65a^3b^2B-24ab^4B) \sqrt{\cos(c+dx)}}{4b^4(a^2-b^2)^2d}
\end{aligned}$$

Mathematica [A] time = 7.50, size = 897, normalized size = 1.54

$$\frac{2(315Ba^6-135Aba^5-641b^2Ba^4+285Ab^3a^3+328b^4Ba^2-168Ab^5a+16b^6B)(F(\sin^{-1}(\sqrt{\sec(c+dx)})|-1)-\Pi(-\frac{b}{a};\sin^{-1}(\sqrt{\sec(c+dx)})|-1))(a+b \sec(c+dx))}{b(b+a \cos(c+dx))(1-\cos^2(c+dx))}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(Sec[c + d*x]^(9/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^3, x]

[Out] ((2*(-135*a^5*A*b + 285*a^3*A*b^3 - 168*a*A*b^5 + 315*a^6*B - 641*a^4*b^2*B + 328*a^2*b^4*B + 16*b^6*B)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] - EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1])*(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(-120*a^4*A*b^2 + 240*a^2*A*b^4 - 48*A*b^6 + 280*a^5*b*B - 512*a^3*b^3*B + 160*a*b^5*B)*Cos[c + d*x]^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + ((-45*a^5*A*b + 87*a^3*A*b^3 - 24*a*A*b^5 + 105*a^6*B - 195*a^4*b^2*B + 72*a^2*b^4*B)*Cos[2*(c + d*x)]*(a + b*Sec[c + d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 2*a*(a - 2*b)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*a^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 4*b^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a^2*b*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2))/(48*(a - b)^2*b^4*(a + b)^2*d) + (Sqrt[Sec[c + d*x]]*(((15*a^4*A*b - 29*a^2*A*b^3 + 8*A*b^5 - 35*a^5*B + 65*a^3*b^2*B - 24*a*b^4*B)*Sin[c + d*x])/(4*b^4*(-a^2 + b^2)^2) + (a^2*A*b*Ssin[c + d*x] - a^3*B*Ssin[c + d*x])/(2*b^2*(-a^2 + b^2)*(b + a*Cos[c + d*x])^2) + (-5*a^4*A*b*Ssin[c + d*x] + 11*a^2*A*b^3*Ssin[c + d*x] + 9*a^5*B*Ssin[c + d*x] - 15*a^3*b^2*B*Ssin[c + d*x])/(4*b^3*(-a^2 + b^2)^2*(b + a*Cos[c + d*x])) + (2*B*Tan[c + d*x])/(3*b^3)))/d

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(9/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sec(dx + c)^{\frac{9}{2}}}{(b \sec(dx + c) + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(9/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(9/2)/(b*sec(d*x + c) + a)^3, x)

maple [B] time = 30.14, size = 2178, normalized size = 3.74

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^(9/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x)

[Out]
$$\begin{aligned} & -(-(-2*\cos(1/2*d*x+1/2*c)^2+1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*a^2*(A*b-3*B* \\ & a)/b^4/(a^2-a*b)*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)} \\ & /(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticPi(\cos(1/ \\ & 2*d*x+1/2*c), 2*a/(a-b), 2^{(1/2)})-2*(A*b-B*a)*a/b^2*(1/2*a^2/b/(a^2-b^2)*\cos(\\ & 1/2*d*x+1/2*c)*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/(2*a*co \\ & s(1/2*d*x+1/2*c)^2-a+b)^2+3/4*a^2*(a^2-3*b^2)/b^2/(a^2-b^2)^2*\cos(1/2*d*x+1 \\ & /2*c)*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/(2*a*cos(1/2*d*x \\ & +1/2*c)^2-a+b)-3/8/(a+b)/(a^2-b^2)/b^2*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*cos \\ & (1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)} \\ & *EllipticF(\cos(1/2*d*x+1/2*c), 2^{(1/2)})*a^2-1/4/(a+b)/(a^2-b^2)/b*(\sin(1 \\ & /2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/ \\ & 2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c), 2^{(1/2)})*a \\ & +7/8/(a+b)/(a^2-b^2)*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1 \\ &)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticF(\cos(\\ & 1/2*d*x+1/2*c), 2^{(1/2)})+3/8*a^3/b^2/(a^2-b^2)^2*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)} \\ &)*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/ \\ & 2*c)^2)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c), 2^{(1/2)})-9/8*a/(a^2-b^2)^2*(\sin(\\ & 1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1 \\ & /2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c), 2^{(1/2)})-3 \\ & /8*a^3/b^2/(a^2-b^2)^2*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^ \\ & 2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticE(c \\ & os(1/2*d*x+1/2*c), 2^{(1/2)})+9/8*a/(a^2-b^2)^2*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(- \\ & 2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c \\ &)^2)^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c), 2^{(1/2)})-3/8/(a-b)/(a+b)/(a^2-b^2)/ \\ & b^2/(a^2-a*b)*a^5*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)} \\ & /(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticPi(\cos(1 \\ & /2*d*x+1/2*c), 2*a/(a-b), 2^{(1/2)})+3/4/(a-b)/(a+b)/(a^2-b^2)/(a^2-a*b)*a^3*(s \\ & in(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d* \\ & x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticPi(\cos(1/2*d*x+1/2*c), 2*a/(a \\ & -b), 2^{(1/2)})-15/8/(a-b)/(a+b)/(a^2-b^2)*b^2/(a^2-a*b)*a*(\sin(1/2*d*x+1/2*c) \\ & ^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/ \\ & 2*d*x+1/2*c)^2)^{(1/2)}*EllipticPi(\cos(1/2*d*x+1/2*c), 2*a/(a-b), 2^{(1/2)})))+2*(\\ & A*b-3*B*a)/b^4*(-(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(\sin(\\ & 1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*EllipticE(\cos(1/2* \\ & d*x+1/2*c), 2^{(1/2)})+2*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}* \end{aligned}$$

$$\frac{\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^2}{\sin(1/2*d*x+1/2*c)^2/(2*\sin(1/2*d*x+1/2*c)^2-1)+2*B/b^3*(-1/6*\cos(1/2*d*x+1/2*c)*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)/(-1/2+\cos(1/2*d*x+1/2*c)^2)^2+1/3*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})}-2*a*(A*b-2*B*a)/b^3*(a^2/b/(a^2-b^2)*\cos(1/2*d*x+1/2*c)*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)/(2*a*\cos(1/2*d*x+1/2*c)^2-a+b)-1/2/(a+b)/b*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})+1/2*a/b/(a^2-b^2)*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})}-1/2*a/b/(a^2-b^2)*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)})}-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)*\text{EllipticPi}(\cos(1/2*d*x+1/2*c),2*a/(a-b),2^{(1/2)})+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)*\text{EllipticPi}(\cos(1/2*d*x+1/2*c),2*a/(a-b),2^{(1/2)})})/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)/d}$$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(9/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(\frac{1}{\cos(c+dx)}\right)^{9/2}}{\left(a + \frac{b}{\cos(c+dx)}\right)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(9/2))/(a + b/cos(c + d*x))^3,x)

[Out] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(9/2))/(a + b/cos(c + d*x))^3, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**(9/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**3,x)

[Out] Timed out

$$3.429 \quad \int \frac{\sec^{\frac{7}{2}}(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^3} dx$$

Optimal. Leaf size=480

$$\frac{a(Ab - aB) \sin(c + dx) \sec^{\frac{5}{2}}(c + dx)}{2bd(a^2 - b^2)(a + b \sec(c + dx))^2} + \frac{a(-5a^3B + a^2Ab + 11ab^2B - 7Ab^3) \sin(c + dx) \sec^{\frac{3}{2}}(c + dx)}{4b^2d(a^2 - b^2)^2(a + b \sec(c + dx))} + \frac{(-5a^3B + a^2Ab + 11ab^2B - 7Ab^3) \sin(c + dx) \sec^{\frac{1}{2}}(c + dx)}{4b^2d(a^2 - b^2)^2(a + b \sec(c + dx))}$$

[Out] $\frac{1}{2}a*(A*b-B*a)*\sec(d*x+c)^{(5/2)}*\sin(d*x+c)/b/(a^2-b^2)/d/(a+b*\sec(d*x+c))^{2+1/4}+ \frac{1}{4}a*(A*a^2*b-7*A*b^3-5*B*a^3+11*B*a*b^2)*\sec(d*x+c)^{(3/2)}*\sin(d*x+c)/b^2/(a^2-b^2)^2/d/(a+b*\sec(d*x+c))- \frac{1}{4}*(3*A*a^3*b-9*A*a*b^3-15*B*a^4+29*B*a^2*b^2-8*B*b^4)*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/b^3/(a^2-b^2)^2/d+ \frac{1}{4}*(3*A*a^3*b-9*A*a*b^3-15*B*a^4+29*B*a^2*b^2-8*B*b^4)*(\cos(1/2*d*x+1/2*c))^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/b^3/(a^2-b^2)^2/d+ \frac{1}{4}*(A*a^2*b-7*A*b^3-5*B*a^3+11*B*a*b^2)*(\cos(1/2*d*x+1/2*c))^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/b^2/(a^2-b^2)^2/d+ \frac{1}{4}*(3*A*a^4*b-6*A*a^2*b^3+15*A*b^5-15*B*a^5+38*B*a^3*b^2-35*B*a*b^4)*(\cos(1/2*d*x+1/2*c))^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticPi}(\sin(1/2*d*x+1/2*c), 2*a/(a+b), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/(a-b)^2/b^3/(a+b)^3/d$

Rubi [A] time = 1.38, antiderivative size = 480, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.303$, Rules used = {4029, 4098, 4102, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{a(Ab - aB) \sin(c + dx) \sec^{\frac{5}{2}}(c + dx)}{2bd(a^2 - b^2)(a + b \sec(c + dx))^2} + \frac{a(a^2Ab - 5a^3B + 11ab^2B - 7Ab^3) \sin(c + dx) \sec^{\frac{3}{2}}(c + dx)}{4b^2d(a^2 - b^2)^2(a + b \sec(c + dx))} - \frac{(3a^3Ab + 29a^2a^2b^2B - 8a^2b^4B) \sin(c + dx) \sec^{\frac{1}{2}}(c + dx)}{4b^2d(a^2 - b^2)^2(a + b \sec(c + dx))}$$

Antiderivative was successfully verified.

[In] Int[(Sec[c + d*x]^(7/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^3,x]

[Out] $((3*a^3*A*b - 9*a*A*b^3 - 15*a^4*B + 29*a^2*b^2*B - 8*b^4*B)*\text{Sqrt}[\text{Cos}[c + d*x]])*\text{EllipticE}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]]/(4*b^3*(a^2 - b^2)^2*d) + ((a^2*A*b - 7*A*b^3 - 5*a^3*B + 11*a*b^2*B)*\text{Sqrt}[\text{Cos}[c + d*x]])*\text{EllipticF}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]]/(4*b^2*(a^2 - b^2)^2*d) + ((3*a^4*A*b - 6*a^2*A*b^3 + 15*A*b^5 - 15*a^5*B + 38*a^3*b^2*B - 35*a*b^4*B)*\text{Sqrt}[\text{Cos}[c + d*x]])*\text{EllipticPi}[(2*a)/(a + b), (c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]]/(4*(a - b)^2*b^3*(a + b)^3*d) - ((3*a^3*A*b - 9*a*A*b^3 - 15*a^4*B + 29*a^2*b^2*B - 8*b^4*B)*\text{Sqrt}[\text{Sec}[c + d*x]])*\text{Sin}[c + d*x]/(4*b^3*(a^2 - b^2)^2*d) + (a*(A*b - a*B)*\text{Sec}[c + d*x]^(5/2)*\text{Sin}[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*\text{Sec}[c + d*x]))$

$(c + dx)^2) + (a(a^2Ab - 7Ab^3 - 5a^3B + 11ab^2B) \operatorname{Sec}[c + dx]^{3/2} \operatorname{Sin}[c + dx]) / (4b^2(a^2 - b^2)^2 d (a + b \operatorname{Sec}[c + dx]))$

Rule 2639

$\operatorname{Int}[\operatorname{Sqrt}[\operatorname{sin}[(c_.) + (d_.)x]], x_Symbol] \rightarrow \operatorname{Simp}[(2 \operatorname{EllipticE}[(1(c - \operatorname{Pi}/2 + dx))/2, 2]) / d, x] /; \operatorname{FreeQ}\{c, d\}, x]$

Rule 2641

$\operatorname{Int}[1/\operatorname{Sqrt}[\operatorname{sin}[(c_.) + (d_.)x]], x_Symbol] \rightarrow \operatorname{Simp}[(2 \operatorname{EllipticF}[(1(c - \operatorname{Pi}/2 + dx))/2, 2]) / d, x] /; \operatorname{FreeQ}\{c, d\}, x]$

Rule 2805

$\operatorname{Int}[1/(((a_.) + (b_.) \operatorname{sin}[(e_.) + (f_.)x]) \operatorname{Sqrt}[(c_.) + (d_.) \operatorname{sin}[(e_.) + (f_.)x]]), x_Symbol] \rightarrow \operatorname{Simp}[(2 \operatorname{EllipticPi}[(2b)/(a + b), (1(e - \operatorname{Pi}/2 + fx))/2, (2d)/(c + d)]) / (f(a + b) \operatorname{Sqrt}[c + d]), x] /; \operatorname{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{NeQ}[a^2 - b^2, 0] \&\& \operatorname{NeQ}[c^2 - d^2, 0] \&\& \operatorname{GtQ}[c + d, 0]$

Rule 3771

$\operatorname{Int}[(\operatorname{csc}[(c_.) + (d_.)x] * (b_.)^n), x_Symbol] \rightarrow \operatorname{Dist}[(b \operatorname{Csc}[c + dx])^n \operatorname{Sin}[c + dx]^n, \operatorname{Int}[1/\operatorname{Sin}[c + dx]^n, x], x] /; \operatorname{FreeQ}\{b, c, d\}, x] \&\& \operatorname{EqQ}[n^2, 1/4]$

Rule 3787

$\operatorname{Int}[(\operatorname{csc}[(e_.) + (f_.)x] * (d_.)^n * (\operatorname{csc}[(e_.) + (f_.)x] * (b_.) + (a_))), x_Symbol] \rightarrow \operatorname{Dist}[a, \operatorname{Int}[(d \operatorname{Csc}[e + fx])^n, x], x] + \operatorname{Dist}[b/d, \operatorname{Int}[(d \operatorname{Csc}[e + fx])^{n+1}, x], x] /; \operatorname{FreeQ}\{a, b, d, e, f, n\}, x]$

Rule 3849

$\operatorname{Int}[(\operatorname{csc}[(e_.) + (f_.)x] * (d_.)^{3/2}) / (\operatorname{csc}[(e_.) + (f_.)x] * (b_.) + (a_)), x_Symbol] \rightarrow \operatorname{Dist}[d \operatorname{Sqrt}[d \operatorname{Sin}[e + fx]] \operatorname{Sqrt}[d \operatorname{Csc}[e + fx]], \operatorname{Int}[1/(\operatorname{Sqrt}[d \operatorname{Sin}[e + fx]] * (b + a \operatorname{Sin}[e + fx])), x], x] /; \operatorname{FreeQ}\{a, b, d, e, f\}, x] \&\& \operatorname{NeQ}[a^2 - b^2, 0]$

Rule 4029

$\operatorname{Int}[(\operatorname{csc}[(e_.) + (f_.)x] * (d_.)^n * (\operatorname{csc}[(e_.) + (f_.)x] * (b_.) + (a_)))^m * (\operatorname{csc}[(e_.) + (f_.)x] * (B_.) + (A_)), x_Symbol] \rightarrow \operatorname{Simp}[(a*d^2 * (A*b - a*B) \operatorname{Cot}[e + fx] * (a + b \operatorname{Csc}[e + fx])^{m+1} * (d \operatorname{Csc}[e + fx])^{n-2}) / (b*f*(m+1)*(a^2 - b^2)), x] - \operatorname{Dist}[d/(b*(m+1)*(a^2 - b^2)), \operatorname{Int}[(a$

```

+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(n
- 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2*
(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n
, 1]

```

Rule 4098

```

Int[((A_.) + csc[(e_.) + (f_.)*(x_)])*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] := -Simp[(d*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b
*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1))/(b*f*(a^2 - b^2)*(m + 1)),
x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*C
sc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A -
b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n + b
^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C},
x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]

```

Rule 4102

```

Int[((A_.) + csc[(e_.) + (f_.)*(x_)])*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] := -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]

```

Rule 4106

```

Int[((A_.) + csc[(e_.) + (f_.)*(x_)])*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] := Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
C}, x] && NeQ[a^2 - b^2, 0]

```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^{\frac{7}{2}}(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^3} dx &= \frac{a(Ab-aB)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2b(a^2-b^2)d(a+b\sec(c+dx))^2} + \int \frac{\sec^{\frac{3}{2}}(c+dx)\left(\frac{3}{2}a(Ab-aB)-2b(Ab-aB)\sec(c+dx)\right)}{(a+b\sec(c+dx))^2} dx \\
&= \frac{a(Ab-aB)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2b(a^2-b^2)d(a+b\sec(c+dx))^2} + \frac{a(a^2Ab-7Ab^3-5a^3B+11ab^2)}{4b^2(a^2-b^2)^2d(a+b\sec(c+dx))} \\
&= -\frac{(3a^3Ab-9aAb^3-15a^4B+29a^2b^2B-8b^4B)\sqrt{\sec(c+dx)}\sin(c+dx)}{4b^3(a^2-b^2)^2d} \\
&= -\frac{(3a^3Ab-9aAb^3-15a^4B+29a^2b^2B-8b^4B)\sqrt{\sec(c+dx)}\sin(c+dx)}{4b^3(a^2-b^2)^2d} \\
&= -\frac{(3a^3Ab-9aAb^3-15a^4B+29a^2b^2B-8b^4B)\sqrt{\sec(c+dx)}\sin(c+dx)}{4b^3(a^2-b^2)^2d} \\
&= \frac{(3a^4Ab-6a^2Ab^3+15Ab^5-15a^5B+38a^3b^2B-35ab^4B)\sqrt{\cos(c+dx)}}{4(a-b)^2b^3(a+b)^3d} \\
&= \frac{(3a^3Ab-9aAb^3-15a^4B+29a^2b^2B-8b^4B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\right)}{4b^3(a^2-b^2)^2d}
\end{aligned}$$

Mathematica [A] time = 7.25, size = 842, normalized size = 1.75

$$\frac{\sqrt{\sec(c+dx)} \left(\frac{(15Ba^4-3Ab^3-29b^2Ba^2+9Ab^3a+8b^4B)\sin(c+dx)}{4b^3(b^2-a^2)^2} + \frac{a^2B\sin(c+dx)-aAb\sin(c+dx)}{2b(b^2-a^2)(b+a\cos(c+dx))^2} + \frac{-5B\sin(c+dx)a^4+Ab\sin(c+dx)a^3+11b^2B}{4b^2(b^2-a^2)^2(b+a\cos(c+dx))} \right)}{d}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(Sec[c + d*x]^(7/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^3, x]

[Out] -1/16*((2*(-9*a^4*A*b + 19*a^2*A*b^3 - 16*A*b^5 + 45*a^5*B - 95*a^3*b^2*B + 56*a*b^4*B)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] - EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1])*(a + b*Sec[c + d*x])*Sqrt


```
[1 - Sec[c + d*x]^2]*Sin[c + d*x))/(b*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(-8*a^3*A*b^2 + 32*a*A*b^4 + 40*a^4*b*B - 80*a^2*b^3*B + 16*b^5*B)*Cos[c + d*x]^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x))/(a*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + ((-3*a^4*A*b + 9*a^2*A*b^3 + 15*a^5*B - 29*a^3*b^2*B + 8*a*b^4*B)*Cos[2*(c + d*x)]*(a + b*Sec[c + d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 2*a*(a - 2*b)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*a^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 4*b^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x))/(a^2*b*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2)))/((a - b)^2*b^3*(a + b)^2*d + (Sqrt[Sec[c + d*x]]*((( -3*a^3*A*b + 9*a*A*b^3 + 15*a^4*B - 29*a^2*b^2*B + 8*b^4*B)*Sin[c + d*x]))/(4*b^3*(-a^2 + b^2)^2) + (- (a*A*b*Ssin[c + d*x]) + a^2*B*Ssin[c + d*x]))/(2*b*(-a^2 + b^2)*(b + a*Cos[c + d*x])^2) + (a^3*A*b*Ssin[c + d*x] - 7*a*A*b^3*Ssin[c + d*x] - 5*a^4*B*Ssin[c + d*x] + 11*a^2*b^2*B*Ssin[c + d*x]))/(4*b^2*(-a^2 + b^2)^2*(b + a*Cos[c + d*x])))/d
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sec(dx + c)^{\frac{7}{2}}}{(b \sec(dx + c) + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(7/2)/(b*sec(d*x + c) + a)^3, x)
```

maple [B] time = 19.72, size = 2024, normalized size = 4.22

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(\sec(dx+c)^{7/2}*(A+B*\sec(dx+c))/(a+b*\sec(dx+c))^3,x)$

[Out]
$$\begin{aligned} & -(-(-2*\cos(1/2*d*x+1/2*c)^2+1)*\sin(1/2*d*x+1/2*c)^2)^{1/2}*(2*a^2*B/b^3/(a^2-a*b) \\ & *(\sin(1/2*d*x+1/2*c)^2)^{1/2}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{1/2}/(-2*\sin(1/2*d*x+1/2*c)^4 \\ & +\sin(1/2*d*x+1/2*c)^2)^{1/2}*EllipticPi(\cos(1/2*d*x+1/2*c), 2*a/(a-b), 2^{1/2})+2*(A*b-B*a)/b \\ & *(1/2*a^2/b/(a^2-b^2)*\cos(1/2*d*x+1/2*c)*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{1/2} \\ & /(2*a*\cos(1/2*d*x+1/2*c)^2-a+b)^{2+3/4} \\ & *a^2*(a^2-3*b^2)/b^2/(a^2-b^2)^2*\cos(1/2*d*x+1/2*c)*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{1/2} \\ & /(2*a*\cos(1/2*d*x+1/2*c)^2-a+b)-3/8/(a+b)/(a^2-b^2)/b^2*(\sin(1/2*d*x+1/2*c)^2)^{1/2} \\ & *(-2*\cos(1/2*d*x+1/2*c)^2+1)^{1/2}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{1/2} \\ & *EllipticF(\cos(1/2*d*x+1/2*c), 2^{1/2})*a^2-1/4/(a+b)/(a^2-b^2)/b*(\sin(1/2*d*x+1/2*c)^2)^{1/2} \\ & *(-2*\cos(1/2*d*x+1/2*c)^2+1)^{1/2}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{1/2} \\ & *EllipticF(\cos(1/2*d*x+1/2*c), 2^{1/2})*a+7/8/(a+b)/(a^2-b^2)*(\sin(1/2*d*x+1/2*c)^2)^{1/2} \\ & *(-2*\cos(1/2*d*x+1/2*c)^2+1)^{1/2}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{1/2} \\ & *EllipticF(\cos(1/2*d*x+1/2*c), 2^{1/2}))+3/8*a^3/b^2/(a^2-b^2)^2*(\sin(1/2*d*x+1/2*c)^2)^{1/2} \\ & *(-2*\cos(1/2*d*x+1/2*c)^2+1)^{1/2}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{1/2} \\ & *EllipticF(\cos(1/2*d*x+1/2*c), 2^{1/2}))-9/8*a/(a^2-b^2)^2*(\sin(1/2*d*x+1/2*c)^2)^{1/2} \\ & *(-2*\cos(1/2*d*x+1/2*c)^2+1)^{1/2}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{1/2} \\ & *EllipticF(\cos(1/2*d*x+1/2*c), 2^{1/2}))-3/8*a^3/b^2/(a^2-b^2)^2*(\sin(1/2*d*x+1/2*c)^2)^{1/2} \\ & *(-2*\cos(1/2*d*x+1/2*c)^2+1)^{1/2}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{1/2} \\ & *EllipticE(\cos(1/2*d*x+1/2*c), 2^{1/2}))+9/8*a/(a^2-b^2)^2*(\sin(1/2*d*x+1/2*c)^2)^{1/2} \\ & *(-2*\cos(1/2*d*x+1/2*c)^2+1)^{1/2}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{1/2} \\ & *EllipticE(\cos(1/2*d*x+1/2*c), 2^{1/2}))-3/8/(a-b)/(a+b)/(a^2-b^2)/b^2/(a^2-a*b) \\ & *a^5*(\sin(1/2*d*x+1/2*c)^2)^{1/2}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{1/2}/(-2*\sin(1/2*d*x+1/2*c)^4 \\ & +\sin(1/2*d*x+1/2*c)^2)^{1/2}*EllipticPi(\cos(1/2*d*x+1/2*c), 2*a/(a-b), 2^{1/2}))+3/4/(a-b) \\ & /(a+b)/(a^2-b^2)/(a^2-a*b)*a^3*(\sin(1/2*d*x+1/2*c)^2)^{1/2}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{1/2} \\ & /(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{1/2} \\ & *EllipticPi(\cos(1/2*d*x+1/2*c), 2*a/(a-b), 2^{1/2}))-15/8/(a-b)/(a+b)/(a^2-b^2)*b^2/(a^2-a*b) \\ & *a*(\sin(1/2*d*x+1/2*c)^2)^{1/2}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{1/2}/(-2*\sin(1/2*d*x+1/2*c)^4 \\ & +\sin(1/2*d*x+1/2*c)^2)^{1/2} \\ & *EllipticPi(\cos(1/2*d*x+1/2*c), 2*a/(a-b), 2^{1/2}))+2*B/b^3*(-(-2*\sin(1/2*d*x+1/2*c)^4 \\ & +\sin(1/2*d*x+1/2*c)^2)^{1/2}*(\sin(1/2*d*x+1/2*c)^2)^{1/2}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{1/2} \\ & *EllipticE(\cos(1/2*d*x+1/2*c), 2^{1/2}))+2*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{1/2} \\ & *\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^2)/\sin(1/2*d*x+1/2*c)^2/(2*\sin(1/2*d*x+1/2*c)^2-1) \\ & -2*a*B/b^2*(a^2/b/(a^2-b^2)*\cos(1/2*d*x+1/2*c)*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{1/2} \\ & /(2*a*\cos(1/2*d*x+1/2*c)^2-a+b)-1/2/(a+b)/b*(\sin(1/2*d*x+1/2*c)^2)^{1/2} \\ & *(-2*\cos(1/2*d*x+1/2*c)^2+1)^{1/2}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{1/2} \\ & *EllipticF(\cos(1/2*d*x+1/2*c), 2^{1/2}))+1/2*a/b/(a^2-b^2)*(\sin(1/2*d*x+1/2*c)^2)^{1/2} \\ & *(-2*\cos(1/2*d*x+1/2*c)^2+1)^{1/2}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{1/2} \end{aligned}$$

$$\frac{1}{2}d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)})-1/2*a/b/(a^2-b^2)*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)})-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticPi(\cos(1/2*d*x+1/2*c),2*a/(a-b),2^{(1/2)})+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticPi(\cos(1/2*d*x+1/2*c),2*a/(a-b),2^{(1/2)})))/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d$$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(\frac{1}{\cos(c+dx)}\right)^{7/2}}{\left(a + \frac{b}{\cos(c+dx)}\right)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(7/2))/(a + b/cos(c + d*x))^3,x)

[Out] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(7/2))/(a + b/cos(c + d*x))^3, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**(7/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**3,x)

[Out] Timed out

$$3.430 \quad \int \frac{\sec^2(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^3} dx$$

Optimal. Leaf size=402

$$\frac{a(Ab - aB) \sin(c + dx) \sec^{\frac{3}{2}}(c + dx)}{2bd(a^2 - b^2)(a + b \sec(c + dx))^2} - \frac{a(3a^3B + a^2Ab - 9ab^2B + 5Ab^3) \sin(c + dx) \sqrt{\sec(c + dx)}}{4b^2d(a^2 - b^2)^2(a + b \sec(c + dx))} + \frac{(a^3B + 3a^2Ab - 3aAb^2 + b^3B) \sin(c + dx) \sqrt{\sec(c + dx)}}{4b^2d(a^2 - b^2)^2(a + b \sec(c + dx))}$$

[Out] $\frac{1}{2} a (A b - B a) \sec(d x + c)^{\frac{3}{2}} \sin(d x + c) / b / (a^2 - b^2) / d / (a + b \sec(d x + c))^{2 - 1/4} a (A a^2 b + 5 A a b^3 + 3 B a^3 - 9 B a a b^2) \sin(d x + c) \sec(d x + c)^{\frac{1}{2}} / b^2 / (a^2 - b^2)^2 / d / (a + b \sec(d x + c)) + 1/4 (A a^2 b + 5 A a b^3 + 3 B a^3 - 9 B a a b^2) (\cos(1/2 d x + 1/2 c))^{\frac{1}{2}} / \cos(1/2 d x + 1/2 c) \text{EllipticE}(\sin(1/2 d x + 1/2 c), 2^{\frac{1}{2}}) \cos(d x + c)^{\frac{1}{2}} \sec(d x + c)^{\frac{1}{2}} / b^2 / (a^2 - b^2)^2 / d + 1/4 (3 A a^2 b + 3 A a b^3 + B a^3 - 7 B a a b^2) (\cos(1/2 d x + 1/2 c))^{\frac{1}{2}} / \cos(1/2 d x + 1/2 c) \text{EllipticF}(\sin(1/2 d x + 1/2 c), 2^{\frac{1}{2}}) \cos(d x + c)^{\frac{1}{2}} \sec(d x + c)^{\frac{1}{2}} / a / b / (a^2 - b^2)^2 / d + 1/4 (A a^4 b - 10 A a^2 b^3 - 3 A a b^5 + 3 B a^5 - 6 B a^3 b^2 + 15 B a a b^4) (\cos(1/2 d x + 1/2 c))^{\frac{1}{2}} / \cos(1/2 d x + 1/2 c) \text{EllipticPi}(\sin(1/2 d x + 1/2 c), 2 a / (a + b), 2^{\frac{1}{2}}) \cos(d x + c)^{\frac{1}{2}} \sec(d x + c)^{\frac{1}{2}} / a / (a - b)^2 / b^2 / (a + b)^3 / d$

Rubi [A] time = 0.91, antiderivative size = 402, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {4029, 4098, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{a(Ab - aB) \sin(c + dx) \sec^{\frac{3}{2}}(c + dx)}{2bd(a^2 - b^2)(a + b \sec(c + dx))^2} - \frac{a(a^2Ab + 3a^3B - 9ab^2B + 5Ab^3) \sin(c + dx) \sqrt{\sec(c + dx)}}{4b^2d(a^2 - b^2)^2(a + b \sec(c + dx))} + \frac{(3a^2Ab + a^3B - 3aAb^2 + b^3B) \sin(c + dx) \sqrt{\sec(c + dx)}}{4b^2d(a^2 - b^2)^2(a + b \sec(c + dx))}$$

Antiderivative was successfully verified.

[In] Int[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^3,x]

[Out] $((a^2 A b + 5 A a b^3 + 3 a^3 B - 9 a a b^2 B) \text{Sqrt}[\text{Cos}[c + d x]] \text{EllipticE}[(c + d x) / 2, 2] \text{Sqrt}[\text{Sec}[c + d x]]) / (4 b^2 (a^2 - b^2)^2 d) + ((3 a^2 A b + 3 A a b^3 + a^3 B - 7 a a b^2 B) \text{Sqrt}[\text{Cos}[c + d x]] \text{EllipticF}[(c + d x) / 2, 2] \text{Sqrt}[\text{Sec}[c + d x]]) / (4 a b (a^2 - b^2)^2 d) + ((a^4 A b - 10 a^2 A a b^3 - 3 A a b^5 + 3 a^5 B - 6 a^3 b^2 B + 15 a a b^4 B) \text{Sqrt}[\text{Cos}[c + d x]] \text{EllipticPi}[(2 a) / (a + b), (c + d x) / 2, 2] \text{Sqrt}[\text{Sec}[c + d x]]) / (4 a (a - b)^2 b^2 (a + b)^3 d) + (a (A b - a B) \text{Sec}[c + d x]^{\frac{3}{2}} \text{Sin}[c + d x]) / (2 b (a^2 - b^2) d (a + b \text{Sec}[c + d x])^2) - (a (a^2 A b + 5 A a b^3 + 3 a^3 B - 9 a a b^2 B) \text{Sqrt}[\text{Sec}[c + d x]] \text{Sin}[c + d x]) / (4 b^2 (a^2 - b^2)^2 d (a + b \text{Sec}[c + d x]))$

Rule 2639

Int[Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2805

Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)])], x_Symbol] := Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi/2 + f*x))/2, (2*d)/(c + d)]/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[c + d, 0]

Rule 3771

Int[(csc[(c_) + (d_)*(x_)]*(b_))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 3787

Int[(csc[(e_) + (f_)*(x_)]*(d_))^(n_)*(csc[(e_) + (f_)*(x_)]*(b_) + (a_)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 3849

Int[(csc[(e_) + (f_)*(x_)]*(d_))^(3/2)/(csc[(e_) + (f_)*(x_)]*(b_) + (a_)), x_Symbol] := Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 4029

Int[(csc[(e_) + (f_)*(x_)]*(d_))^(n_)*(csc[(e_) + (f_)*(x_)]*(b_) + (a_))^(m_)*(csc[(e_) + (f_)*(x_)]*(B_) + (A_)), x_Symbol] := Simp[(a*d^2*(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[d/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(n - 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2*(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n

, 1]

Rule 4098

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a
_)^m), x_Symbol] := -Simp[(d*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b
*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1))/(b*f*(a^2 - b^2)*(m + 1)),
x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*C
sc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A -
b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n + b
^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C},
x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a
_))), x_Symbol] := Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^3} dx &= \frac{a(Ab-aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2b(a^2-b^2)d(a+b\sec(c+dx))^2} + \int \frac{\sqrt{\sec(c+dx)}\left(\frac{1}{2}a(Ab-aB)-2b(Ab-aB)\right)}{(a+b\sec(c+dx))^2} dx \\
&= \frac{a(Ab-aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2b(a^2-b^2)d(a+b\sec(c+dx))^2} - \frac{a(a^2Ab+5Ab^3+3a^3B-9ab^2)}{4b^2(a^2-b^2)^2d(a+b\sec(c+dx))} \\
&= \frac{a(Ab-aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2b(a^2-b^2)d(a+b\sec(c+dx))^2} - \frac{a(a^2Ab+5Ab^3+3a^3B-9ab^2)}{4b^2(a^2-b^2)^2d(a+b\sec(c+dx))} \\
&= \frac{a(Ab-aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2b(a^2-b^2)d(a+b\sec(c+dx))^2} - \frac{a(a^2Ab+5Ab^3+3a^3B-9ab^2)}{4b^2(a^2-b^2)^2d(a+b\sec(c+dx))} \\
&= \frac{(a^4Ab-10a^2Ab^3-3Ab^5+3a^5B-6a^3b^2B+15ab^4B)\sqrt{\cos(c+dx)}\Gamma\left(\frac{1}{2}\right)}{4a(a-b)^2b^2(a+b)^3d} \\
&= \frac{(a^2Ab+5Ab^3+3a^3B-9ab^2B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4b^2(a^2-b^2)^2d}
\end{aligned}$$

Mathematica [A] time = 6.96, size = 795, normalized size = 1.98

$$\frac{2(9Ba^4+3Aba^3-19b^2Ba^2-9Ab^3a+16b^4B)\left(F\left(\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle| -1\right)-\Pi\left(-\frac{b}{a};\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle| -1\right)\right)(a+b\sec(c+dx))\sqrt{1-\sec^2(c+dx)}\sin(c+dx)\cos(c+dx)}{b(b+a\cos(c+dx))(1-\cos^2(c+dx))}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^3, x]

[Out] ((2*(3*a^3*A*b - 9*a*A*b^3 + 9*a^4*B - 19*a^2*b^2*B + 16*b^4*B)*Cos[c + d*x])^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] - EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1])*(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(8*a^2*A*b^2 + 16*A*b^4 + 8*a^3*b*B - 32*a*b^3*B)*Cos[c + d*x]^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + ((a^3*A*b + 5*

```

a*A*b^3 + 3*a^4*B - 9*a^2*b^2*B)*Cos[2*(c + d*x)]*(a + b*Sec[c + d*x])*(-4*
a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1
]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 2*a*(a - 2*b)*EllipticF[Arc
Sin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] +
2*a^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]
*Sqrt[1 - Sec[c + d*x]^2] - 4*b^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*
x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a^2*b
*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c +
d*x]^2)))/(16*(a - b)^2*b^2*(a + b)^2*d) + (Sqrt[Sec[c + d*x]]*(-1/4*((a^2*
A*b + 5*A*b^3 + 3*a^3*B - 9*a*b^2*B)*Sin[c + d*x])/(b^2*(-a^2 + b^2)^2) + (
A*b*Sin[c + d*x] - a*B*Sin[c + d*x])/(2*(-a^2 + b^2)*(b + a*Cos[c + d*x])^2
) + (3*a^2*A*b*Sin[c + d*x] + 3*A*b^3*Sin[c + d*x] + a^3*B*Sin[c + d*x] - 7
*a*b^2*B*Sin[c + d*x])/(4*b*(-a^2 + b^2)^2*(b + a*Cos[c + d*x]))))/d

```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm
="fricas")
```

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sec(dx + c)^{\frac{5}{2}}}{(b \sec(dx + c) + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(5/2)/(b*sec(d*x + c) + a)^3, x
)
```

maple [B] time = 16.34, size = 1768, normalized size = 4.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x)
```



```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*(-A*b+B*a)/a*(1/2*a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos(1/2*d*x+1/2*c)^2-a+b)^2+3/4*a^2*(a^2-3*b^2)/b^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos(1/2*d*x+1/2*c)^2-a+b)-3/8/(a+b)/(a^2-b^2)/b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-1/4/(a+b)/(a^2-b^2)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a+7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8/(a-b)/(a+b)/(a^2-b^2)/b^2/(a^2-a*b)*a^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/4/(a-b)/(a+b)/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))-15/8/(a-b)/(a+b)/(a^2-b^2)*b^2/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))+2*A/a*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos(1/2*d*x+1/2*c)^2-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(\frac{1}{\cos(c+dx)}\right)^{5/2}}{\left(a + \frac{b}{\cos(c+dx)}\right)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + b/cos(c + d*x))^3,x)

[Out] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + b/cos(c + d*x))^3, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**3,x)

[Out] Timed out

$$3.431 \quad \int \frac{\sec^3(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^3} dx$$

Optimal. Leaf size=402

$$\frac{a(Ab - aB) \sin(c + dx) \sqrt{\sec(c + dx)}}{2bd(a^2 - b^2)(a + b \sec(c + dx))^2} + \frac{(a^3B + 3a^2Ab - 7ab^2B + 3Ab^3) \sin(c + dx) \sqrt{\sec(c + dx)}}{4bd(a^2 - b^2)^2(a + b \sec(c + dx))} - \frac{(-3a^3B + 7a^2B)}{4bd(a^2 - b^2)^2(a + b \sec(c + dx))}$$

[Out] $\frac{1}{2} a (A b - B a) \sin(d x + c) \sec(d x + c)^{1/2} / b / (a^2 - b^2) / d / (a + b \sec(d x + c))^{2 + 1/4} + \frac{1}{4} (3 A a^2 b + 3 A a b^3 + B a^3 - 7 B a a b^2) \sin(d x + c) \sec(d x + c)^{1/2} / b / (a^2 - b^2)^2 / d / (a + b \sec(d x + c)) - \frac{1}{4} (5 A a^2 b + A b^3 - B a^3 - 5 B a a b^2) (\cos(1/2 d x + 1/2 c))^2 / \cos(1/2 d x + 1/2 c) \text{EllipticE}(\sin(1/2 d x + 1/2 c), 2^{1/2}) \cos(d x + c)^{1/2} \sec(d x + c)^{1/2} / a / b / (a^2 - b^2)^2 / d - \frac{1}{4} (7 A a^2 b - A b^3 - 3 B a^3 - 3 B a a b^2) (\cos(1/2 d x + 1/2 c))^2 / \cos(1/2 d x + 1/2 c) \text{EllipticF}(\sin(1/2 d x + 1/2 c), 2^{1/2}) \cos(d x + c)^{1/2} \sec(d x + c)^{1/2} / a^2 / (a^2 - b^2)^2 / d + \frac{1}{4} (3 A a^4 b + 10 A a^2 a b^3 - A b^5 + B a^5 - 10 B a^3 b^2 - 3 B a a b^4) (\cos(1/2 d x + 1/2 c))^2 / \cos(1/2 d x + 1/2 c) \text{EllipticPi}(\sin(1/2 d x + 1/2 c), 2 a / (a + b), 2^{1/2}) \cos(d x + c)^{1/2} \sec(d x + c)^{1/2} / a^2 / (a - b)^2 / b / (a + b)^3 / d$

Rubi [A] time = 0.91, antiderivative size = 402, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {4029, 4100, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{(3a^2Ab + a^3B - 7ab^2B + 3Ab^3) \sin(c + dx) \sqrt{\sec(c + dx)}}{4bd(a^2 - b^2)^2(a + b \sec(c + dx))} + \frac{a(Ab - aB) \sin(c + dx) \sqrt{\sec(c + dx)}}{2bd(a^2 - b^2)(a + b \sec(c + dx))^2} - \frac{(7a^2Ab - 3a^3B)}{4bd(a^2 - b^2)^2(a + b \sec(c + dx))}$$

Antiderivative was successfully verified.

[In] Int[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^3,x]

[Out] $-\frac{(5a^2Ab + Ab^3 - a^3B - 5aab^2B) \text{Sqrt}[\text{Cos}[c + dx]] \text{EllipticE}[(c + dx)/2, 2] \text{Sqrt}[\text{Sec}[c + dx]]}{(4ab(a^2 - b^2)^2d) - ((7a^2Ab - Ab^3 - 3a^3B - 3aab^2B) \text{Sqrt}[\text{Cos}[c + dx]] \text{EllipticF}[(c + dx)/2, 2] \text{Sqrt}[\text{Sec}[c + dx]])}{(4a^2(a^2 - b^2)^2d) + ((3a^4Ab + 10a^2Ab^3 - Ab^5 + a^5B - 10a^3b^2B - 3aab^4B) \text{Sqrt}[\text{Cos}[c + dx]] \text{EllipticPi}[(2a)/(a + b), (c + dx)/2, 2] \text{Sqrt}[\text{Sec}[c + dx]])}{(4a^2(a - b)^2b(a + b)^3d) + (a(Ab - aB) \text{Sqrt}[\text{Sec}[c + dx]] \text{Sin}[c + dx])}{(2b(a^2 - b^2)d(a + b \text{Sec}[c + dx])^2) + ((3a^2Ab + 3Ab^3 + a^3B - 7aab^2B) \text{Sqrt}[\text{Sec}[c + dx]] \text{Sin}[c + dx])}{(4b(a^2 - b^2)^2d(a + b \text{Sec}[c + dx]))}$

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2805

Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi/2 + f*x))/2, (2*d)/(c + d)]/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[c + d, 0]

Rule 3771

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^n, x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 3849

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 4029

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^m*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] := Simp[(a*d^2*(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[d/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(n - 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2*(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n

, 1]

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)])*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] := Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
&& NeQ[a^2 - b^2, 0] && LtQ[m, -1] && !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)])*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] := Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^3(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^3} dx &= \frac{a(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{2b(a^2-b^2)d(a+b\sec(c+dx))^2} + \frac{\int \frac{-\frac{1}{2}a(Ab-aB)-2b(Ab-aB)\sec(c+dx)+\frac{1}{2}}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))} dx}{2b(a^2-b^2)d(a+b\sec(c+dx))^2} \\
&= \frac{a(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{2b(a^2-b^2)d(a+b\sec(c+dx))^2} + \frac{(3a^2Ab+3Ab^3+a^3B-7ab^2B)}{4b(a^2-b^2)^2d(a+b\sec(c+dx))} \\
&= \frac{a(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{2b(a^2-b^2)d(a+b\sec(c+dx))^2} + \frac{(3a^2Ab+3Ab^3+a^3B-7ab^2B)}{4b(a^2-b^2)^2d(a+b\sec(c+dx))} \\
&= \frac{a(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{2b(a^2-b^2)d(a+b\sec(c+dx))^2} + \frac{(3a^2Ab+3Ab^3+a^3B-7ab^2B)}{4b(a^2-b^2)^2d(a+b\sec(c+dx))} \\
&= \frac{(3a^4Ab+10a^2Ab^3-Ab^5+a^5B-10a^3b^2B-3ab^4B)\sqrt{\cos(c+dx)}\Pi\left(\frac{1}{2}(c+dx)\right)}{4a^2(a-b)^2b(a+b)^3d} \\
&= \frac{(5a^2Ab+Ab^3-a^3B-5ab^2B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}}{4ab(a^2-b^2)^2d}
\end{aligned}$$

Mathematica [B] time = 6.99, size = 882, normalized size = 2.19

$$\frac{\sec^2(c+dx)(A+B\sec(c+dx))\left(\frac{2(3Ba^3+Ab^2-9b^2Ba+5Ab^3)\left(F(\sin^{-1}(\sqrt{\sec(c+dx)})\right)\right)-1-\Pi\left(-\frac{b}{a};\sin^{-1}(\sqrt{\sec(c+dx)})\right)\right)}{b(b+a\cos(c+dx))(1-\cos^2(c+dx))}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^3, x]

[Out] ((b + a*Cos[c + d*x])^3*Sec[c + d*x]^2*(A + B*Sec[c + d*x])*((2*(a^2*A*b + 5*A*b^3 + 3*a^3*B - 9*a*b^2*B)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] - EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1])*(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(-24*a*A*b^2 + 8*a^2*b*B + 16*b^3*B)*Cos[c + d*x]^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(b + a*Cos[c + d*x])*(1 - Cos[

$c + d*x]^2)) + ((-5*a^2*A*b - A*b^3 + a^3*B + 5*a*b^2*B)*\text{Cos}[2*(c + d*x)]*(a + b*\text{Sec}[c + d*x])*(-4*a*b + 4*a*b*\text{Sec}[c + d*x]^2 - 4*a*b*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[\text{Sec}[c + d*x]]], -1]*\text{Sqrt}[\text{Sec}[c + d*x]]*\text{Sqrt}[1 - \text{Sec}[c + d*x]^2] - 2*a*(a - 2*b)*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[\text{Sec}[c + d*x]]], -1]*\text{Sqrt}[\text{Sec}[c + d*x]]*\text{Sqrt}[1 - \text{Sec}[c + d*x]^2] + 2*a^2*\text{EllipticPi}[-(b/a), \text{ArcSin}[\text{Sqrt}[\text{Sec}[c + d*x]]], -1]*\text{Sqrt}[\text{Sec}[c + d*x]]*\text{Sqrt}[1 - \text{Sec}[c + d*x]^2] - 4*b^2*\text{EllipticPi}[-(b/a), \text{ArcSin}[\text{Sqrt}[\text{Sec}[c + d*x]]], -1]*\text{Sqrt}[\text{Sec}[c + d*x]]*\text{Sqrt}[1 - \text{Sec}[c + d*x]^2])*\text{Sin}[c + d*x])/(a^2*b*(b + a*\text{Cos}[c + d*x])*(1 - \text{Cos}[c + d*x]^2)*\text{Sqrt}[\text{Sec}[c + d*x]]*(2 - \text{Sec}[c + d*x]^2))))/(16*(a - b)^2*b*(a + b)^2*d*(B + A*\text{Cos}[c + d*x])*(a + b*\text{Sec}[c + d*x])^3) + ((b + a*\text{Cos}[c + d*x])^3*\text{Sec}[c + d*x]^(5/2)*(A + B*\text{Sec}[c + d*x])*((5*a^2*A*b + A*b^3 - a^3*B - 5*a*b^2*B)*\text{Sin}[c + d*x])/(4*a*b*(-a^2 + b^2)^2) - ((A*b^2*\text{Sin}[c + d*x]) + a*b*B*\text{Sin}[c + d*x])/(2*a*(a^2 - b^2)*(b + a*\text{Cos}[c + d*x])^2) + (-7*a^2*A*b*\text{Sin}[c + d*x] + A*b^3*\text{Sin}[c + d*x] + 3*a^3*B*\text{Sin}[c + d*x] + 3*a*b^2*B*\text{Sin}[c + d*x])/(4*a*(a^2 - b^2)^2*(b + a*\text{Cos}[c + d*x])))))/(d*(B + A*\text{Cos}[c + d*x])*(a + b*\text{Sec}[c + d*x])^3)$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sec(dx + c)^{\frac{3}{2}}}{(b \sec(dx + c) + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^3, x)

maple [B] time = 16.60, size = 1872, normalized size = 4.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(\sec(dx+c)^{(3/2)}*(A+B*\sec(dx+c))/(a+b*\sec(dx+c))^3,x)$

[Out]
$$\begin{aligned} & -(-(-2*\cos(1/2*d*x+1/2*c)^2+1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*A/a/(a^2-a*b) \\ &)*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/ \\ & 2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticPi(\cos(1/2*d*x+1/2*c),2* \\ & a/(a-b),2^{(1/2)})+2*(A*b-B*a)*b/a^2*(1/2*a^2/b/(a^2-b^2)*\cos(1/2*d*x+1/2*c)* \\ & (-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/(2*a*\cos(1/2*d*x+1/2*c) \\ &)^2-a+b)^2+3/4*a^2*(a^2-3*b^2)/b^2/(a^2-b^2)^2*\cos(1/2*d*x+1/2*c)*(-2*\sin(1 \\ & /2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/(2*a*\cos(1/2*d*x+1/2*c)^2-a+b)- \\ & 3/8/(a+b)/(a^2-b^2)/b^2*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c) \\ & ^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticF(\\ & \cos(1/2*d*x+1/2*c),2^{(1/2)})*a^2-1/4/(a+b)/(a^2-b^2)/b*(\sin(1/2*d*x+1/2*c)^2 \\ &)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2* \\ & d*x+1/2*c)^2)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)})*a+7/8/(a+b)/(a^2- \\ & b^2)*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin \\ & (1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c), \\ & 2^{(1/2)})+3/8*a^3/b^2/(a^2-b^2)^2*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d \\ & *x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*E \\ & llipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)})-9/8*a/(a^2-b^2)^2*(\sin(1/2*d*x+1/2*c)^2 \\ &)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2 \\ & *d*x+1/2*c)^2)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)})-3/8*a^3/b^2/(a^2 \\ & -b^2)^2*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2* \\ & \sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2* \\ & c),2^{(1/2)})+9/8*a/(a^2-b^2)^2*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+ \\ & 1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*Elli \\ & pticE(\cos(1/2*d*x+1/2*c),2^{(1/2)})-3/8/(a-b)/(a+b)/(a^2-b^2)/b^2/(a^2-a*b)*a \\ & ^5*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1 \\ & /2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticPi(\cos(1/2*d*x+1/2*c),2 \\ & *a/(a-b),2^{(1/2)})+3/4/(a-b)/(a+b)/(a^2-b^2)/(a^2-a*b)*a^3*(\sin(1/2*d*x+1/2* \\ & c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(\\ & 1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticPi(\cos(1/2*d*x+1/2*c),2*a/(a-b),2^{(1/2)})-15 \\ & /8/(a-b)/(a+b)/(a^2-b^2)*b^2/(a^2-a*b)*a*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*c \\ & os(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2) \\ & ^{(1/2)}*EllipticPi(\cos(1/2*d*x+1/2*c),2*a/(a-b),2^{(1/2)})+2*(-2*A*b+B*a)/a^2 \\ & *(a^2/b/(a^2-b^2)*\cos(1/2*d*x+1/2*c)*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1 \\ & /2*c)^2)^{(1/2)}/(2*a*\cos(1/2*d*x+1/2*c)^2-a+b)-1/2/(a+b)/b*(\sin(1/2*d*x+1/2* \\ & c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(\\ & 1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)})+1/2*a/b/(a^2- \\ & b^2)*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin \\ & (1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c), \\ & 2^{(1/2)})-1/2*a/b/(a^2-b^2)*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2 \\ & *c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*Elli \\ &pticE(\cos(1/2*d*x+1/2*c),2^{(1/2)})-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(\sin(1/2*d*x+1 \\ & /2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+s \\ & in(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticPi(\cos(1/2*d*x+1/2*c),2*a/(a-b),2^{(1/2)}) \end{aligned}$$

$+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticPi(\cos(1/2*d*x+1/2*c),2*a/(a-b),2^{(1/2)}))/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(\frac{1}{\cos(c+dx)}\right)^{3/2}}{\left(a + \frac{b}{\cos(c+dx)}\right)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + b/cos(c + d*x))^3,x)

[Out] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + b/cos(c + d*x))^3, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**3,x)

[Out] Timed out

$$3.432 \quad \int \frac{\sqrt{\sec(c+dx)} (A+B \sec(c+dx))}{(a+b \sec(c+dx))^3} dx$$

Optimal. Leaf size=402

$$\frac{(Ab - aB) \sin(c + dx) \sqrt{\sec(c + dx)}}{2d(a^2 - b^2)(a + b \sec(c + dx))^2} - \frac{(-3a^3B + 7a^2Ab - 3ab^2B - Ab^3) \sin(c + dx) \sqrt{\sec(c + dx)}}{4ad(a^2 - b^2)^2(a + b \sec(c + dx))} + \frac{(-5a^3B + 9a^2Ab - 3ab^2B - Ab^3) \sin(c + dx) \sqrt{\sec(c + dx)}}{2d(a^2 - b^2)(a + b \sec(c + dx))^2}$$

[Out] $-1/2*(A*b-B*a)*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/(a^2-b^2)/d/(a+b*\sec(d*x+c))^{2-1}$
 $/4*(7*A*a^2*b-A*b^3-3*B*a^3-3*B*a*b^2)*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/a/(a^2-b^2)^{2-1}/d/(a+b*\sec(d*x+c))+1/4*(9*A*a^2*b-3*A*b^3-5*B*a^3-B*a*b^2)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})$
 $*\cos(d*x+c)^{(1/2)*\sec(d*x+c)^{(1/2)}/a^2/(a^2-b^2)^{2-1}/d+1/4*(8*A*a^4-5*A*a^2*b^2+3*A*b^4-7*B*a^3*b+B*a*b^3)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)*\sec(d*x+c)^{(1/2)}/a^3/(a^2-b^2)^{2-1}/d-1/4*(15*A*a^4*b-6*A*a^2*b^3+3*A*b^5-3*B*a^5-10*B*a^3*b^2+B*a*b^4)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticPi}(\sin(1/2*d*x+1/2*c), 2*a/(a+b), 2^{(1/2)})*\cos(d*x+c)^{(1/2)*\sec(d*x+c)^{(1/2)}/a^3/(a-b)^2/(a+b)^3/d$

Rubi [A] time = 0.87, antiderivative size = 402, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {4027, 4100, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{(7a^2Ab - 3a^3B - 3ab^2B - Ab^3) \sin(c + dx) \sqrt{\sec(c + dx)}}{4ad(a^2 - b^2)^2(a + b \sec(c + dx))} - \frac{(Ab - aB) \sin(c + dx) \sqrt{\sec(c + dx)}}{2d(a^2 - b^2)(a + b \sec(c + dx))^2} + \frac{(-5a^2Ab^2 + 8a^2Ab - 3ab^2B - Ab^3) \sin(c + dx) \sqrt{\sec(c + dx)}}{2d(a^2 - b^2)(a + b \sec(c + dx))^2}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^3,x]

[Out] $((9*a^2*A*b - 3*A*b^3 - 5*a^3*B - a*b^2*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/(4*a^2*(a^2 - b^2)^2*d) + ((8*a^4*A - 5*a^2*A*b^2 + 3*A*b^4 - 7*a^3*b*B + a*b^3*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticF}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/(4*a^3*(a^2 - b^2)^2*d) - ((15*a^4*A*b - 6*a^2*A*b^3 + 3*A*b^5 - 3*a^5*B - 10*a^3*b^2*B + a*b^4*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticPi}[(2*a)/(a + b), (c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/(4*a^3*(a - b)^2*(a + b)^3*d) - ((A*b - a*B)*\text{Sqrt}[\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(2*(a^2 - b^2)*d*(a + b*\text{Sec}[c + d*x])^2) - ((7*a^2*A*b - A*b^3 - 3*a^3*B - 3*a*b^2*B)*\text{Sqrt}[\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(4*a*(a^2 - b^2)^2*d*(a + b*\text{Sec}[c + d*x]))$

Rule 2639

Int[Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2805

Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)])], x_Symbol] := Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi/2 + f*x))/2, (2*d)/(c + d)]/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[c + d, 0]

Rule 3771

Int[(csc[(c_) + (d_)*(x_)]*(b_))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 3787

Int[(csc[(e_) + (f_)*(x_)]*(d_))^(n_)*(csc[(e_) + (f_)*(x_)]*(b_) + (a_)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 3849

Int[(csc[(e_) + (f_)*(x_)]*(d_))^(3/2)/(csc[(e_) + (f_)*(x_)]*(b_) + (a_)), x_Symbol] := Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 4027

Int[(csc[(e_) + (f_)*(x_)]*(d_))^(n_)*(csc[(e_) + (f_)*(x_)]*(b_) + (a_))^(m_)*(csc[(e_) + (f_)*(x_)]*(B_) + (A_)), x_Symbol] := -Simp[(d*(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*Simp[d*(n - 1)*(A*b - a*B) + d*(a*A - b*B)*(m + 1)*Csc[e + f*x] - d*(A*b - a*B)*(m + n + 1)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && Ne

$Q[a^2 - b^2, 0]$ && LtQ[m, -1] && LtQ[0, n, 1]

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a
_))^(m_), x_Symbol] := Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
&& NeQ[a^2 - b^2, 0] && LtQ[m, -1] && !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a
_))), x_Symbol] := Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\sec(c+dx)}(A+B\sec(c+dx))}{(a+b\sec(c+dx))^3} dx &= -\frac{(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{2(a^2-b^2)d(a+b\sec(c+dx))^2} - \int \frac{\frac{1}{2}(-Ab+aB)-2(aA-bB)\sec(c+dx)+\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}{2(a^2-b^2)} dx \\
&= -\frac{(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{2(a^2-b^2)d(a+b\sec(c+dx))^2} - \frac{(7a^2Ab-Ab^3-3a^3B-3ab^2)}{4a(a^2-b^2)^2d} \\
&= -\frac{(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{2(a^2-b^2)d(a+b\sec(c+dx))^2} - \frac{(7a^2Ab-Ab^3-3a^3B-3ab^2)}{4a(a^2-b^2)^2d} \\
&= -\frac{(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{2(a^2-b^2)d(a+b\sec(c+dx))^2} - \frac{(7a^2Ab-Ab^3-3a^3B-3ab^2)}{4a(a^2-b^2)^2d} \\
&= -\frac{(15a^4Ab-6a^2Ab^3+3Ab^5-3a^5B-10a^3b^2B+ab^4B)\sqrt{\cos(c+dx)}}{4a^3(a-b)^2(a+b)^3d} \\
&= \frac{(9a^2Ab-3Ab^3-5a^3B-ab^2B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4a^2(a^2-b^2)^2d}
\end{aligned}$$

Mathematica [B] time = 6.98, size = 885, normalized size = 2.20

$$\sec^2(c+dx)(A+B\sec(c+dx)) \left(\frac{2(Ba^3-5Aba^2+5b^2Ba-Ab^3)\left(F\left(\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)-\Pi\left(-\frac{b}{a};\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\right)}{b(b+a\cos(c+dx))(1-\cos^2(c+dx))} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^3, x]

[Out] ((b + a*Cos[c + d*x])^3*Sec[c + d*x]^2*(A + B*Sec[c + d*x])*((2*(-5*a^2*A*b - A*b^3 + a^3*B + 5*a*b^2*B)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] - EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1])*(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(16*a^3*A + 8*a*A*b^2 - 24*a^2*b*B)*Cos[c + d*x]^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(b + a*Cos[c + d*x])*(1 - Cos[c

```

+ d*x)^2)) + ((9*a^2*A*b - 3*A*b^3 - 5*a^3*B - a*b^2*B)*Cos[2*(c + d*x)]*(a
+ b*Sec[c + d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[
Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 2*a*
(a - 2*b)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt
[1 - Sec[c + d*x]^2] + 2*a^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]],
-1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 4*b^2*EllipticPi[-(b/a),
ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2
])*Sin[c + d*x])/(a^2*b*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[
c + d*x]]*(2 - Sec[c + d*x]^2))))/(16*a*(a - b)^2*(a + b)^2*d*(B + A*Cos[c
+ d*x])*(a + b*Sec[c + d*x])^3 + ((b + a*Cos[c + d*x])^3*Sec[c + d*x]^(5/2
))*(A + B*Sec[c + d*x])*(((9*a^2*A*b + 3*A*b^3 + 5*a^3*B + a*b^2*B)*Sin[c +
d*x])/(4*a^2*(-a^2 + b^2)^2) - (A*b^3*Sin[c + d*x] - a*b^2*B*Sin[c + d*x])
/(2*a^2*(a^2 - b^2)*(b + a*Cos[c + d*x])^2) + (11*a^2*A*b^2*Sin[c + d*x] -
5*A*b^4*Sin[c + d*x] - 7*a^3*b*B*Sin[c + d*x] + a*b^3*B*Sin[c + d*x])/(4*a^
2*(a^2 - b^2)^2*(b + a*Cos[c + d*x])))))/(d*(B + A*Cos[c + d*x])*(a + b*Sec[
c + d*x])^3)

```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sqrt{\sec(dx + c)}}{(b \sec(dx + c) + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(b*sec(d*x + c) + a)^3, x
)
```

maple [B] time = 18.14, size = 1959, normalized size = 4.87

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(\sec(dx+c)^{1/2}*(A+B*\sec(dx+c))/(a+b*\sec(dx+c))^3,x)$

[Out]
$$-\left(-(-2*\cos(1/2*d*x+1/2*c)^2+1)*\sin(1/2*d*x+1/2*c)^2\right)^{1/2}*(2*A/a^3*(\sin(1/2*d*x+1/2*c)^2)^{1/2}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{1/2}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{1/2}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{1/2})-2*(-3*A*b+B*a)/a^2/(a^2-a*b)*(\sin(1/2*d*x+1/2*c)^2)^{1/2}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{1/2}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{1/2}*\text{EllipticPi}(\cos(1/2*d*x+1/2*c),2*a/(a-b),2^{1/2})-2*b^2*(A*b-B*a)/a^3*(1/2*a^2/b/(a^2-b^2)*\cos(1/2*d*x+1/2*c)*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{1/2}/(2*a*\cos(1/2*d*x+1/2*c)^2-a+b)^2+3/4*a^2*(a^2-3*b^2)/b^2/(a^2-b^2)^2*\cos(1/2*d*x+1/2*c)*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{1/2}/(2*a*\cos(1/2*d*x+1/2*c)^2-a+b)-3/8/(a+b)/(a^2-b^2)/b^2*(\sin(1/2*d*x+1/2*c)^2)^{1/2}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{1/2}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{1/2}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{1/2})*a^2-1/4/(a+b)/(a^2-b^2)/b*(\sin(1/2*d*x+1/2*c)^2)^{1/2}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{1/2}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{1/2}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{1/2})*a+7/8/(a+b)/(a^2-b^2)*(\sin(1/2*d*x+1/2*c)^2)^{1/2}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{1/2}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{1/2}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{1/2}))+3/8*a^3/b^2/(a^2-b^2)^2*(\sin(1/2*d*x+1/2*c)^2)^{1/2}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{1/2}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{1/2}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{1/2}))-9/8*a/(a^2-b^2)^2*(\sin(1/2*d*x+1/2*c)^2)^{1/2}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{1/2}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{1/2}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{1/2}))-3/8*a^3/b^2/(a^2-b^2)^2*(\sin(1/2*d*x+1/2*c)^2)^{1/2}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{1/2}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{1/2}*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{1/2}))+9/8*a/(a^2-b^2)^2*(\sin(1/2*d*x+1/2*c)^2)^{1/2}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{1/2}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{1/2}*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{1/2}))-3/8/(a-b)/(a+b)/(a^2-b^2)/b^2/(a^2-a*b)*a^5*(\sin(1/2*d*x+1/2*c)^2)^{1/2}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{1/2}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{1/2}*\text{EllipticPi}(\cos(1/2*d*x+1/2*c),2*a/(a-b),2^{1/2}))+3/4/(a-b)/(a+b)/(a^2-b^2)/(a^2-a*b)*a^3*(\sin(1/2*d*x+1/2*c)^2)^{1/2}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{1/2}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{1/2}*\text{EllipticPi}(\cos(1/2*d*x+1/2*c),2*a/(a-b),2^{1/2}))-15/8/(a-b)/(a+b)/(a^2-b^2)*b^2/(a^2-a*b)*a*(\sin(1/2*d*x+1/2*c)^2)^{1/2}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{1/2}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{1/2}*\text{EllipticPi}(\cos(1/2*d*x+1/2*c),2*a/(a-b),2^{1/2}))+2/a^3*b*(3*A*b-2*B*a)*(a^2/b/(a^2-b^2)*\cos(1/2*d*x+1/2*c)*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{1/2}/(2*a*\cos(1/2*d*x+1/2*c)^2-a+b)-1/2/(a+b)/b*(\sin(1/2*d*x+1/2*c)^2)^{1/2}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{1/2}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{1/2}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{1/2}))+1/2*a/b/(a^2-b^2)*(\sin(1/2*d*x+1/2*c)^2)^{1/2}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{1/2}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{1/2}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{1/2}))-1/2*a/b/(a^2-b^2)*(\sin(1/2*d*x+1/2*c)^2)^{1/2}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{1/2}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{1/2}*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{1/2}))-1/2/b/(a^2-b^2)$$

$$\frac{(a^2 - a*b) * a^3 * (\sin(1/2*d*x + 1/2*c)^2)^{(1/2)} * (-2*\cos(1/2*d*x + 1/2*c)^2 + 1)^{(1/2)}}{(-2*\sin(1/2*d*x + 1/2*c)^4 + \sin(1/2*d*x + 1/2*c)^2)^{(1/2)} * \text{EllipticPi}(\cos(1/2*d*x + 1/2*c), 2*a/(a-b), 2^{(1/2)})} + \frac{3/2*b/(a^2 - b^2)}{(a^2 - a*b) * a * (\sin(1/2*d*x + 1/2*c)^2)^{(1/2)} * (-2*\cos(1/2*d*x + 1/2*c)^2 + 1)^{(1/2)}}{(-2*\sin(1/2*d*x + 1/2*c)^4 + \sin(1/2*d*x + 1/2*c)^2)^{(1/2)} * \text{EllipticPi}(\cos(1/2*d*x + 1/2*c), 2*a/(a-b), 2^{(1/2)})} \Big) / \sin(1/2*d*x + 1/2*c) / (2*\cos(1/2*d*x + 1/2*c)^2 - 1)^{(1/2)} / d$$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(1/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \sqrt{\frac{1}{\cos(c+dx)}}}{\left(a + \frac{b}{\cos(c+dx)}\right)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + b/cos(c + d*x))^3,x)

[Out] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + b/cos(c + d*x))^3, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \sqrt{\sec(c + dx)}}{(a + b \sec(c + dx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**(1/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**3,x)

[Out] Integral((A + B*sec(c + d*x))*sqrt(sec(c + d*x))/(a + b*sec(c + d*x))**3, x)

$$3.433 \quad \int \frac{A+B \sec(c+dx)}{\sqrt{\sec(c+dx)} (a+b \sec(c+dx))^3} dx$$

Optimal. Leaf size=427

$$\frac{b(Ab - aB) \sin(c + dx) \sqrt{\sec(c + dx)}}{2ad(a^2 - b^2)(a + b \sec(c + dx))^2} + \frac{b(-7a^3B + 11a^2Ab + ab^2B - 5Ab^3) \sin(c + dx) \sqrt{\sec(c + dx)}}{4a^2d(a^2 - b^2)^2(a + b \sec(c + dx))} + \frac{(8a^4A + \dots)}{\dots}$$

[Out] $\frac{1}{2}b*(A*b-B*a)*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/a/(a^2-b^2)/d/(a+b*\sec(d*x+c))^{2+1/4*b*(11*A*a^2*b-5*A*b^3-7*B*a^3+B*a*b^2)*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/a^2/(a^2-b^2)^2/d/(a+b*\sec(d*x+c))+1/4*(8*A*a^4-29*A*a^2*b^2+15*A*b^4+9*B*a^3*b-3*B*a*b^3)*(\cos(1/2*d*x+1/2*c))^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)*\sec(d*x+c)^{(1/2)}/a^3/(a^2-b^2)^2/d-1/4*(24*A*a^4*b-33*A*a^2*b^3+15*A*b^5-8*B*a^5+5*B*a^3*b^2-3*B*a*b^4)*(\cos(1/2*d*x+1/2*c))^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)*\sec(d*x+c)^{(1/2)}/a^4/(a^2-b^2)^2/d+1/4*b*(35*A*a^4*b-38*A*a^2*b^3+15*A*b^5-15*B*a^5+6*B*a^3*b^2-3*B*a*b^4)*(\cos(1/2*d*x+1/2*c))^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticPi}(\sin(1/2*d*x+1/2*c), 2*a/(a+b), 2^{(1/2)})*\cos(d*x+c)^{(1/2)*\sec(d*x+c)^{(1/2)}/a^4/(a-b)^2/(a+b)^3/d}$

Rubi [A] time = 1.00, antiderivative size = 427, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {4030, 4100, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{b(11a^2Ab - 7a^3B + ab^2B - 5Ab^3) \sin(c + dx) \sqrt{\sec(c + dx)}}{4a^2d(a^2 - b^2)^2(a + b \sec(c + dx))} + \frac{b(Ab - aB) \sin(c + dx) \sqrt{\sec(c + dx)}}{2ad(a^2 - b^2)(a + b \sec(c + dx))^2} - \frac{(-33a^2Ab^3)}{\dots}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sec[c + d*x])/(Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^3), x]

[Out] $((8*a^4*A - 29*a^2*A*b^2 + 15*A*b^4 + 9*a^3*b*B - 3*a*b^3*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/(4*a^3*(a^2 - b^2)^2*d) - ((24*a^4*A*b - 33*a^2*A*b^3 + 15*A*b^5 - 8*a^5*B + 5*a^3*b^2*B - 3*a*b^4*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticF}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/(4*a^4*(a^2 - b^2)^2*d) + (b*(35*a^4*A*b - 38*a^2*A*b^3 + 15*A*b^5 - 15*a^5*B + 6*a^3*b^2*B - 3*a*b^4*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticPi}[(2*a)/(a + b), (c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/(4*a^4*(a - b)^2*(a + b)^3*d) + (b*(A*b - a*B)*\text{Sqrt}[\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(2*a*(a^2 - b^2)*d*(a + b*\text{Sec}[c + d*x])^2) + (b*(11*a^2*A*b - 5*A*b^3 - 7*a^3*B + a*b^2*B)*\text{Sqrt}[\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(4*a^2*(a^2 - b^2)^2*d*(a + b*\text{Sec}[c + d*x]))$

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2805

Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi/2 + f*x))/2, (2*d)/(c + d)]/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[c + d, 0]

Rule 3771

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^n, x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 3849

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 4030

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^m*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] := Simp[(b*(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[A*(a^2*(m + 1) - b^2*(m + n + 1)) + a*b*B*n - a*(A*b - a*B)*(m + 1)*Csc[e + f*x] + b*(A*b - a*B)*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && !(ILtQ[m + 1/2, 0] && ILt

Q[n, 0])

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a
_))^(m_), x_Symbol] := Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
&& NeQ[a^2 - b^2, 0] && LtQ[m, -1] && !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a
_)), x_Symbol] := Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{A + B \sec(c + dx)}{\sqrt{\sec(c + dx)} (a + b \sec(c + dx))^3} dx &= \frac{b(Ab - aB)\sqrt{\sec(c + dx)} \sin(c + dx)}{2a(a^2 - b^2)d(a + b \sec(c + dx))^2} - \int \frac{\frac{1}{2}(-4a^2A + 5Ab^2 - abB) + 2a(Ab - aB) \sec(c + dx)}{\sqrt{\sec(c + dx)} (a + b \sec(c + dx))^3} dx \\
&= \frac{b(Ab - aB)\sqrt{\sec(c + dx)} \sin(c + dx)}{2a(a^2 - b^2)d(a + b \sec(c + dx))^2} + \frac{b(11a^2Ab - 5Ab^3 - 7a^3B + ab^3)}{4a^2(a^2 - b^2)^2 d(a + b \sec(c + dx))} \\
&= \frac{b(Ab - aB)\sqrt{\sec(c + dx)} \sin(c + dx)}{2a(a^2 - b^2)d(a + b \sec(c + dx))^2} + \frac{b(11a^2Ab - 5Ab^3 - 7a^3B + ab^3)}{4a^2(a^2 - b^2)^2 d(a + b \sec(c + dx))} \\
&= \frac{b(Ab - aB)\sqrt{\sec(c + dx)} \sin(c + dx)}{2a(a^2 - b^2)d(a + b \sec(c + dx))^2} + \frac{b(11a^2Ab - 5Ab^3 - 7a^3B + ab^3)}{4a^2(a^2 - b^2)^2 d(a + b \sec(c + dx))} \\
&= \frac{b(35a^4Ab - 38a^2Ab^3 + 15Ab^5 - 15a^5B + 6a^3b^2B - 3ab^4B) \sqrt{\cos(c + dx)}}{4a^4(a - b)^2(a + b)^3d} \\
&= \frac{(8a^4A - 29a^2Ab^2 + 15Ab^4 + 9a^3bB - 3ab^3B) \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx)\right)}{4a^3(a^2 - b^2)^2 d}
\end{aligned}$$

Mathematica [A] time = 7.23, size = 818, normalized size = 1.92

$$\frac{2(8Aa^4 - 5bBa^3 - 7Ab^2a^2 - b^3Ba + 5Ab^4) \left(F\left(\sin^{-1}\left(\sqrt{\sec(c+dx)}\right) \middle| -1\right) - \Pi\left(-\frac{b}{a}; \sin^{-1}\left(\sqrt{\sec(c+dx)}\right) \middle| -1\right) \right) (a + b \sec(c+dx)) \sqrt{1 - \sec^2(c+dx)} \sin(c+dx) \cos^2(c+dx)}{b(b + a \cos(c+dx))(1 - \cos^2(c+dx))}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(A + B*Sec[c + d*x])/(Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^3), x]

[Out] ((2*(8*a^4*A - 7*a^2*A*b^2 + 5*A*b^4 - 5*a^3*b*B - a*b^3*B)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] - EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1])*(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(-32*a^3*A*b + 8*a*A*b^3 + 16*a^4*B + 8*a^2*b^2*B)*Cos[c + d*x]^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + ((8*a^4*A - 29*a^

```

2*A*b^2 + 15*A*b^4 + 9*a^3*b*B - 3*a*b^3*B)*Cos[2*(c + d*x)]*(a + b*Sec[c +
d*x))*(-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c +
d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 2*a*(a - 2*b)*El
lipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c +
d*x]^2] + 2*a^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Se
c[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 4*b^2*EllipticPi[-(b/a), ArcSin[Sqrt
[Sec[c + d*x]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d
*x])/(a^2*b*(b + a*Cos[c + d*x))*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2
- Sec[c + d*x]^2))/(16*a^2*(a - b)^2*(a + b)^2*d) + (Sqrt[Sec[c + d*x]]*(
-1/4*(b*(-13*a^2*A*b + 7*A*b^3 + 9*a^3*B - 3*a*b^2*B)*Sin[c + d*x]))/(a^3*(-
a^2 + b^2)^2) - ((A*b^4*Sin[c + d*x]) + a*b^3*B*Sin[c + d*x]))/(2*a^3*(a^2
- b^2)*(b + a*Cos[c + d*x])^2) + (-15*a^2*A*b^3*Sin[c + d*x] + 9*A*b^5*Sin[
c + d*x] + 11*a^3*b^2*B*Sin[c + d*x] - 5*a*b^4*B*Sin[c + d*x]))/(4*a^3*(a^2
- b^2)^2*(b + a*Cos[c + d*x])))/d

```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(b \sec(dx + c) + a)^3 \sqrt{\sec(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^3*sqrt(sec(d*x + c))),
x)
```

maple [B] time = 20.24, size = 2000, normalized size = 4.68

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/a^4/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(3*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*
b+A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a-B*EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2))*a)-6/a^3*b*(2*A*b-B*a)/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+2*b^3*(A*b-B*a)/a^
4*(1/2*a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)/(2*a*cos(1/2*d*x+1/2*c)^2-a+b)^2+3/4*a^2*(a^2-3*b^2)/b^
2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)/(2*a*cos(1/2*d*x+1/2*c)^2-a+b)-3/8/(a+b)/(a^2-b^2)/b^2*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-1
/4/(a+b)/(a^2-b^2)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+
1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))*a+7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*a^3/b^2/(a^2-b^2)^2*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))-9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))-3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*a/(a^2-b^2)^2*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
-3/8/(a-b)/(a+b)/(a^2-b^2)/b^2/(a^2-a*b)*a^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/4/(a-b)/(a+b)
/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticP
i(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))-15/8/(a-b)/(a+b)/(a^2-b^2)*b^2/(a^2
-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2
*c),2*a/(a-b),2^(1/2))-2*b^2/a^4*(4*A*b-3*B*a)*(a^2/b/(a^2-b^2)*cos(1/2*d*
x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos(1/2*
d*x+1/2*c)^2-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*a/b/(a^2-b^2)*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d
```

```
*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a
/(a-b),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^3,x, algorithm
="maxima")
```

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\left(a + \frac{b}{\cos(c+dx)}\right)^3 \sqrt{\frac{1}{\cos(c+dx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/((a + b/cos(c + d*x))^3*(1/cos(c + d*x))^(1/2)),x)
```

```
[Out] int((A + B/cos(c + d*x))/((a + b/cos(c + d*x))^3*(1/cos(c + d*x))^(1/2)), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)**(1/2)/(a+b*sec(d*x+c))**3,x)
```

[Out] Timed out

$$3.434 \quad \int \frac{A+B \sec(c+dx)}{\sec^2(c+dx)(a+b \sec(c+dx))^3} dx$$

Optimal. Leaf size=521

$$\frac{b(Ab - aB) \sin(c + dx)}{2ad(a^2 - b^2) \sqrt{\sec(c + dx)} (a + b \sec(c + dx))^2} + \frac{b(-9a^3B + 13a^2Ab + 3ab^2B - 7Ab^3) \sin(c + dx)}{4a^2d(a^2 - b^2)^2 \sqrt{\sec(c + dx)} (a + b \sec(c + dx))} + \frac{(8a^4A + 33a^3B) \sin(c + dx)}{4a^2d(a^2 - b^2) \sqrt{\sec(c + dx)} (a + b \sec(c + dx))}$$

[Out] $1/12*(8*A*a^4-61*A*a^2*b^2+35*A*b^4+33*B*a^3*b-15*B*a*b^3)*\sin(d*x+c)/a^3/(a^2-b^2)^2/d/\sec(d*x+c)^{(1/2)}+1/2*b*(A*b-B*a)*\sin(d*x+c)/a/(a^2-b^2)/d/(a+b*\sec(d*x+c))^2/\sec(d*x+c)^{(1/2)}+1/4*b*(13*A*a^2*b-7*A*b^3-9*B*a^3+3*B*a*b^2)*\sin(d*x+c)/a^2/(a^2-b^2)^2/d/(a+b*\sec(d*x+c))/\sec(d*x+c)^{(1/2)}-1/4*(24*A*a^4*b-65*A*a^2*b^3+35*A*b^5-8*B*a^5+29*B*a^3*b^2-15*B*a*b^4)*(\cos(1/2*d*x+1/2*c))^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/a^4/(a^2-b^2)^2/d+1/12*(8*A*a^6+128*A*a^4*b^2-223*A*a^2*b^4+105*A*b^6-72*B*a^5*b+99*B*a^3*b^3-45*B*a*b^5)*(\cos(1/2*d*x+1/2*c))^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/a^5/(a^2-b^2)^2/d-1/4*b^2*(63*A*a^4*b-86*A*a^2*b^3+35*A*b^5-35*B*a^5+38*B*a^3*b^2-15*B*a*b^4)*(\cos(1/2*d*x+1/2*c))^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticPi}(\sin(1/2*d*x+1/2*c), 2*a/(a+b), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/a^5/(a-b)^2/(a+b)^3/d$

Rubi [A] time = 1.44, antiderivative size = 521, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.303$, Rules used = {4030, 4100, 4104, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{b(13a^2Ab - 9a^3B + 3ab^2B - 7Ab^3) \sin(c + dx)}{4a^2d(a^2 - b^2)^2 \sqrt{\sec(c + dx)} (a + b \sec(c + dx))} + \frac{b(Ab - aB) \sin(c + dx)}{2ad(a^2 - b^2) \sqrt{\sec(c + dx)} (a + b \sec(c + dx))^2} + \frac{(-61a^2Ab^2 + 33a^3B) \sin(c + dx)}{4a^2d(a^2 - b^2) \sqrt{\sec(c + dx)} (a + b \sec(c + dx))}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^3), x]

[Out] $-((24*a^4*A*b - 65*a^2*A*b^3 + 35*A*b^5 - 8*a^5*B + 29*a^3*b^2*B - 15*a*b^4*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/(4*a^4*(a^2 - b^2)^2*d) + ((8*a^6*A + 128*a^4*A*b^2 - 223*a^2*A*b^4 + 105*A*b^6 - 72*a^5*b*B + 99*a^3*b^3*B - 45*a*b^5*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticF}[(c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/(12*a^5*(a^2 - b^2)^2*d) - (b^2*(63*a^4*A*b - 86*a^2*A*b^3 + 35*A*b^5 - 35*a^5*B + 38*a^3*b^2*B - 15*a*b^4*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticPi}[(2*a)/(a + b), (c + d*x)/2, 2]*\text{Sqrt}[\text{Sec}[c + d*x]])/(4*a^5*(a - b)^2*(a + b)^3*d) + ((8*a^4*A - 61*a^2*A*b^2 + 35*A*b^4 + 33*a^3*B) \sin(c + dx) / (4*a^2*d*(a^2 - b^2)*\sqrt{\sec(c + dx)}*(a + b*\sec(c + dx))))$

$$\begin{aligned} & *B - 15*a*b^3*B)*\sin[c + d*x])/(12*a^3*(a^2 - b^2)^2*d*\sqrt{\sec[c + d*x]}) \\ & + (b*(A*b - a*B)*\sin[c + d*x])/(2*a*(a^2 - b^2)*d*\sqrt{\sec[c + d*x]}*(a + b \\ & * \sec[c + d*x]^2) + (b*(13*a^2*A*b - 7*A*b^3 - 9*a^3*B + 3*a*b^2*B)*\sin[c + \\ & d*x])/(4*a^2*(a^2 - b^2)^2*d*\sqrt{\sec[c + d*x]}*(a + b*\sec[c + d*x])) \end{aligned}$$
Rule 2639

$$\text{Int}[\sqrt{\sin[(c_.) + (d_.)*(x_)]}], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticE}[(1*(c - \text{Pi}/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}[\{c, d\}, x]$$
Rule 2641

$$\text{Int}[1/\sqrt{\sin[(c_.) + (d_.)*(x_)]}], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticF}[(1*(c - \text{Pi}/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}[\{c, d\}, x]$$
Rule 2805

$$\begin{aligned} & \text{Int}[1/(((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_)])*\sqrt{(c_.) + (d_.)*\sin[(e_.) \\ & + (f_.)*(x_)]}), x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticPi}[(2*b)/(a + b), (1*(e - \text{Pi} \\ & /2 + f*x))/2, (2*d)/(c + d)])/((f*(a + b)*\sqrt{c + d}), x] /; \text{FreeQ}[\{a, b, c \\ & , d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, \\ & 0] \&\& \text{GtQ}[c + d, 0] \end{aligned}$$
Rule 3771

$$\text{Int}[(\csc[(c_.) + (d_.)*(x_)]*(b_.)^{(n_.)}), x_Symbol] \rightarrow \text{Dist}[(b*\csc[c + d*x])^n*\sin[c + d*x]^n, \text{Int}[1/\sin[c + d*x]^n, x], x] /; \text{FreeQ}[\{b, c, d\}, x] \&\& \text{EqQ}[n^2, 1/4]$$
Rule 3787

$$\text{Int}[(\csc[(e_.) + (f_.)*(x_)]*(d_.)^{(n_.)}*(\csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))), x_Symbol] \rightarrow \text{Dist}[a, \text{Int}[(d*\csc[e + f*x])^n, x], x] + \text{Dist}[b/d, \text{Int}[(d*\csc[e + f*x])^{(n + 1)}, x], x] /; \text{FreeQ}[\{a, b, d, e, f, n\}, x]$$
Rule 3849

$$\text{Int}[(\csc[(e_.) + (f_.)*(x_)]*(d_.)^{(3/2)})/(\csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] \rightarrow \text{Dist}[d*\sqrt{d*\sin[e + f*x]}*\sqrt{d*\csc[e + f*x]}, \text{Int}[1/(\sqrt{d*\sin[e + f*x]}*(b + a*\sin[e + f*x])), x], x] /; \text{FreeQ}[\{a, b, d, e, f\}, x] \&\& \text{NeQ}[a^2 - b^2, 0]$$
Rule 4030

$$\text{Int}[(\csc[(e_.) + (f_.)*(x_)]*(d_.)^{(n_.)}*(\csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^{(m_.)}*(\csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))), x_Symbol] \rightarrow \text{Simp}[(b*(A*b$$

```

- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*
(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e
+ f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[A*(a^2*(m + 1) - b^2*(m + n + 1))
+ a*b*B*n - a*(A*b - a*B)*(m + 1)*Csc[e + f*x] + b*(A*b - a*B)*(m + n + 2)*
Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b
- a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && !(ILtQ[m + 1/2, 0] && ILt
Q[n, 0])

```

Rule 4100

```

Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
&& NeQ[a^2 - b^2, 0] && LtQ[m, -1] && !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])

```

Rule 4104

```

Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]

```

Rule 4106

```

Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
C}, x] && NeQ[a^2 - b^2, 0]

```

Rubi steps

$$\begin{aligned}
\int \frac{A + B \sec(c + dx)}{\sec^{\frac{3}{2}}(c + dx)(a + b \sec(c + dx))^3} dx &= \frac{b(Ab - aB) \sin(c + dx)}{2a(a^2 - b^2) d \sqrt{\sec(c + dx)} (a + b \sec(c + dx))^2} - \frac{\int \frac{\frac{1}{2}(-4a^2A + 7Ab^2 - 3abB)}{\sec^{\frac{3}{2}}(c + dx)(a + b \sec(c + dx))^3} dx}{\sec^{\frac{3}{2}}(c + dx)(a + b \sec(c + dx))^3} \\
&= \frac{b(Ab - aB) \sin(c + dx)}{2a(a^2 - b^2) d \sqrt{\sec(c + dx)} (a + b \sec(c + dx))^2} + \frac{b(13a^2Ab - 7Ab^3 - 3ab^2B)}{4a^2(a^2 - b^2)^2 d \sqrt{\sec(c + dx)}} \\
&= \frac{(8a^4A - 61a^2Ab^2 + 35Ab^4 + 33a^3bB - 15ab^3B) \sin(c + dx)}{12a^3(a^2 - b^2)^2 d \sqrt{\sec(c + dx)}} + \frac{b(13a^2Ab - 7Ab^3 - 3ab^2B)}{4a^2(a^2 - b^2)^2 d \sqrt{\sec(c + dx)}} \\
&= \frac{(8a^4A - 61a^2Ab^2 + 35Ab^4 + 33a^3bB - 15ab^3B) \sin(c + dx)}{12a^3(a^2 - b^2)^2 d \sqrt{\sec(c + dx)}} + \frac{b(13a^2Ab - 7Ab^3 - 3ab^2B)}{4a^2(a^2 - b^2)^2 d \sqrt{\sec(c + dx)}} \\
&= \frac{(8a^4A - 61a^2Ab^2 + 35Ab^4 + 33a^3bB - 15ab^3B) \sin(c + dx)}{12a^3(a^2 - b^2)^2 d \sqrt{\sec(c + dx)}} + \frac{b(13a^2Ab - 7Ab^3 - 3ab^2B)}{4a^2(a^2 - b^2)^2 d \sqrt{\sec(c + dx)}} \\
&= -\frac{b^2(63a^4Ab - 86a^2Ab^3 + 35Ab^5 - 35a^5B + 38a^3b^2B - 15ab^4B) \sqrt{\cos(c + dx)}}{4a^5(a - b)^2(a + b)^3d} \\
&= -\frac{(24a^4Ab - 65a^2Ab^3 + 35Ab^5 - 8a^5B + 29a^3b^2B - 15ab^4B) \sqrt{\cos(c + dx)}}{4a^4(a^2 - b^2)^2 d}
\end{aligned}$$

Mathematica [A] time = 7.53, size = 863, normalized size = 1.66

$$\frac{2(24Ba^5 - 56Aba^4 - 21b^2Ba^3 + 73Ab^3a^2 + 15b^4Ba - 35Ab^5) \left(F\left(\sin^{-1}\left(\sqrt{\sec(c+dx)}\right) \middle| -1\right) - \Pi\left(-\frac{b}{a}; \sin^{-1}\left(\sqrt{\sec(c+dx)}\right) \middle| -1\right) \right) (a+b \sec(c+dx)) \sqrt{1-\sec^2(c+dx)}}{b(b+a \cos(c+dx))(1-\cos^2(c+dx))}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^3), x]

[Out] ((2*(-56*a^4*A*b + 73*a^2*A*b^3 - 35*A*b^5 + 24*a^5*B - 21*a^3*b^2*B + 15*a*b^4*B)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] - Elliptic

```

cPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1])*(a + b*Sec[c + d*x])*Sqrt[1 -
Sec[c + d*x]^2]*Sin[c + d*x])/(b*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2))
+ (2*(16*a^5*A + 112*a^3*A*b^2 - 56*a*A*b^4 - 96*a^4*b*B + 24*a^2*b^3*B)*C
os[c + d*x]^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*(a + b*Sec
[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(b + a*Cos[c + d*x])*(
1 - Cos[c + d*x]^2)) + ((-72*a^4*A*b + 195*a^2*A*b^3 - 105*A*b^5 + 24*a^5*B
- 87*a^3*b^2*B + 45*a*b^4*B)*Cos[2*(c + d*x)]*(a + b*Sec[c + d*x])*(-4*a*b
+ 4*a*b*Sec[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*S
qrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 2*a*(a - 2*b)*EllipticF[ArcSin
[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*a
^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sq
rt[1 - Sec[c + d*x]^2] - 4*b^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]
], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a^2*b*(b
+ a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x
]^2)))/(48*a^3*(a - b)^2*(a + b)^2*d + (Sqrt[Sec[c + d*x]]*((b^2*(-17*a^2*
A*b + 11*A*b^3 + 13*a^3*B - 7*a*b^2*B)*Sin[c + d*x])/(4*a^4*(-a^2 + b^2)^2)
- (A*b^5*Sin[c + d*x] - a*b^4*B*Sin[c + d*x])/(2*a^4*(a^2 - b^2)*(b + a*Co
s[c + d*x])^2) + (19*a^2*A*b^4*Sin[c + d*x] - 13*A*b^6*Sin[c + d*x] - 15*a^
3*b^3*B*Sin[c + d*x] + 9*a*b^5*B*Sin[c + d*x])/(4*a^4*(a^2 - b^2)^2*(b + a*
Cos[c + d*x])) + (A*Sin[2*(c + d*x)])/(3*a^3)))/d

```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x, algorithm
="fricas")
```

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(b \sec(dx + c) + a)^3 \sec(dx + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^3*sec(d*x + c)^(3/2)),
x)
```

maple [B] time = 23.62, size = 2216, normalized size = 4.25

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((A+B*\sec(dx+c))/\sec(dx+c)^{(3/2)}/(a+b*\sec(dx+c))^3,x)$

[Out]
$$\begin{aligned} & -(-(-2*\cos(1/2*d*x+1/2*c)^2+1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2/3/a^5*(4*A*a^2*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^4+a^2*A*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)} \\ &)*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)})+18 \\ & *A*b^2*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)})+9*A*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2 \\ & *d*x+1/2*c)^2-1)^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)})*a*b-2*A*a^2*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^2-9*B*a*b*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}* \\ & (2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)})-3*B*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)}) \\ &)*a^2)/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}+4/a^4*b^2*(5*A*b-3*B*a)/(a^2-a*b)*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticPi(\cos(1/2*d*x+1/2*c),2*a/(a-b),2^{(1/2)})-2*b^4*(A*b-B*a)/a^5*(1/2*a^2/b/(a^2-b^2)*\cos(1/2*d*x+1/2*c)*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/(2*a*\cos(1/2*d*x+1/2*c)^2-a+b)^2+3/4*a^2*(a^2-3*b^2)/b^2/(a^2-b^2)^2*\cos(1/2*d*x+1/2*c)*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/(2*a*\cos(1/2*d*x+1/2*c)^2-a+b)-3/8/(a+b)/(a^2-b^2)/b^2*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)})*a^2-1/4/(a+b)/(a^2-b^2)/b*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)})*a+7/8/(a+b)/(a^2-b^2)*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)})+3/8*a^3/b^2/(a^2-b^2)^2*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)})+9/8*a/(a^2-b^2)^2*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)})-3/8/(a-b)/(a+b)/(a^2-b^2)/b^2/(a^2-a*b)*a^5*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticPi(\cos(1/2*d*x+1/2*c),2*a/(a-b),2^{(1/2)})+3/4/(a-b)/(a+b)/(a^2-b^2)/(a^2-a*b)*a^3*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticPi(\cos(1/2*d*x+1/2*c),2*a/(a-b),2^{(1/2)})-15/8/(a-b)/(a+b)/(a^2-b^2)*b^2/(a^2-a*b)*a*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticPi(\cos(1/2*d*x+1/2 \\ & \end{aligned}$$

```

*c),2*a/(a-b),2^(1/2))) + 2/a^5*b^3*(5*A*b-4*B*a)*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos(1/2*d*x+1/2*c)^2-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is undefined.

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\left(a + \frac{b}{\cos(c+dx)}\right)^3 \left(\frac{1}{\cos(c+dx)}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/((a + b/cos(c + d*x))^3*(1/cos(c + d*x))^(3/2)),x)

[Out] int((A + B/cos(c + d*x))/((a + b/cos(c + d*x))^3*(1/cos(c + d*x))^(3/2)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)**(3/2)/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

$$3.435 \quad \int \sec^3(c+dx) \sqrt{a + b \sec(c + dx)} (A+B \sec(c+dx)) dx$$

Optimal. Leaf size=336

$$\frac{(a^2(-B) + 4aAb + 4b^2B) \sqrt{\sec(c + dx)} \sqrt{\frac{a \cos(c+dx)+b}{a+b}} \Pi\left(2; \frac{1}{2}(c + dx) \middle| \frac{2a}{a+b}\right)}{4bd\sqrt{a + b \sec(c + dx)}} + \frac{(aB + 4Ab) \sin(c + dx) \sqrt{\sec(c + dx)}}{4bd}$$

[Out] 1/4*(4*A*b+3*B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c), 2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/d/(a+b*sec(d*x+c))^(1/2)+1/4*(4*A*a*b-B*a^2+4*B*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c), 2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/b/d/(a+b*sec(d*x+c))^(1/2)+1/2*B*sec(d*x+c)^(3/2)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d-1/4*(4*A*b+B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c), 2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/b/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)+1/4*(4*A*b+B*a)*sin(d*x+c)*sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/b/d

Rubi [A] time = 1.11, antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {4031, 4102, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{(a^2(-B) + 4aAb + 4b^2B) \sqrt{\sec(c + dx)} \sqrt{\frac{a \cos(c+dx)+b}{a+b}} \Pi\left(2; \frac{1}{2}(c + dx) \middle| \frac{2a}{a+b}\right)}{4bd\sqrt{a + b \sec(c + dx)}} + \frac{(aB + 4Ab) \sin(c + dx) \sqrt{\sec(c + dx)}}{4bd}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]

[Out] ((4*A*b + 3*a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(4*d*Sqrt[a + b*Sec[c + d*x]]) + ((4*A*a*b - a^2*B + 4*b^2*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(4*b*d*Sqrt[a + b*Sec[c + d*x]]) - ((4*A*b + a*B)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(4*b*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + ((4*A*b + a*B)*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(4*b*d) + (B*Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(2*d)

Rule 2653


```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*Sqrt[a
+ b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)]/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
0] && !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*Ellip
ticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)]/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a
+ b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_)
+ (f_)*(x_)]]), x_Symbol] := Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)]/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_)
+ (f_)*(x_)]]), x_Symbol] := Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
+ f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !GtQ[c + d, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)]/Sqrt[csc[(e_) + (f_)*(x_)]
*(d_)], x_Symbol] := Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
```

b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 3858

Int[Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 3859

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 4031

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> -Simp[(B*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x] + Dist[d/(m + n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n - 1)*Simp[a*B*(n - 1) + (b*B*(m + n - 1) + a*A*(m + n))*Csc[e + f*x] + (a*B*m + A*b*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[0, m, 1] && GtQ[n, 0]

Rule 4035

Int[(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.))/(Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]), x_Symbol] :> Dist[A/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]

Rule 4102

Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 0]

Rule 4108

Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.)))/(Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned}
 \int \sec^{\frac{3}{2}}(c + dx) \sqrt{a + b \sec(c + dx)} (A + B \sec(c + dx)) dx &= \frac{B \sec^{\frac{3}{2}}(c + dx) \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{2d} + \frac{1}{2} \int \sec^{\frac{1}{2}}(c + dx) \sqrt{a + b \sec(c + dx)} (A + B \sec(c + dx)) dx \\
 &= \frac{(4Ab + aB) \sqrt{\sec(c + dx)} \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{4bd} \\
 &= \frac{(4Ab + aB) \sqrt{\sec(c + dx)} \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{4bd} \\
 &= \frac{(4Ab + aB) \sqrt{\sec(c + dx)} \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{4bd} \\
 &= \frac{(4Ab + aB) \sqrt{\sec(c + dx)} \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{4bd} \\
 &= \frac{(4aAb - a^2B + 4b^2B) \sqrt{\frac{b+a \cos(c+dx)}{a+b}} \Pi\left(2; \frac{1}{2}(c + dx) \mid \frac{2a}{a+b}\right)}{4bd \sqrt{a + b \sec(c + dx)}} \\
 &= \frac{(4Ab + 3aB) \sqrt{\frac{b+a \cos(c+dx)}{a+b}} F\left(\frac{1}{2}(c + dx) \mid \frac{2a}{a+b}\right) \sqrt{\sec(c + dx)}}{4d \sqrt{a + b \sec(c + dx)}}
 \end{aligned}$$

Mathematica [C] time = 5.52, size = 422, normalized size = 1.26

$$\sqrt{a + b \sec(c + dx)} \left(\frac{2(-3a^2B + 4aAb + 8b^2B) \Pi\left(2; \frac{1}{2}(c + dx) \mid \frac{2a}{a+b}\right)}{b(a+b) \sqrt{\frac{a \cos(c+dx)+b}{a+b}}} - \frac{2i(aB + 4Ab) \csc(c + dx) \sqrt{-\frac{a(\cos(c+dx)-1)}{a+b}} \sqrt{\frac{a(\cos(c+dx)+1)}{a-b}}}{a} \left(2bF\left(i \sinh^{-1}\left(\sqrt{\frac{a \cos(c+dx)+b}{a+b}}\right)\right) \right) \right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]),
x]
```

```
[Out] (Sqrt[a + b*Sec[c + d*x]]*((8*a*B*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/((
a + b)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*(4*a*A*b - 3*a^2*B + 8*b^2*
B)*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/(b*(a + b)*Sqrt[(b + a*Cos[c
+ d*x])/(a + b)]) - ((2*I)*(4*A*b + a*B)*Sqrt[-((a*(-1 + Cos[c + d*x]))/(a
+ b))]*Sqrt[(a*(1 + Cos[c + d*x]))/(a - b)]*Csc[c + d*x]*(-2*b*(a + b)*Elli
pticE[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a +
b)] + a*(2*b*EllipticF[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x
]]], (-a + b)/(a + b)] + a*EllipticPi[1 - a/b, I*ArcSinh[Sqrt[(a - b)^(-1)]
*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)])))/(a*Sqrt[(a - b)^(-1)]*b^2*
Sqrt[b + a*Cos[c + d*x]]) + (4*(4*A*b + a*B)*Tan[c + d*x])/b + 8*B*Sec[c +
d*x]*Tan[c + d*x]))/(16*d*Sqrt[Sec[c + d*x]])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algo
rithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A) \sqrt{b \sec(dx + c) + a} \sec(dx + c)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algo
rithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(3/2),
x)
```

maple [C] time = 2.24, size = 2521, normalized size = 7.50

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/4/d*(4*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(-(a+b)/(a-b))^(1/2))*cos(d*x+c)^3*a*b-8*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))
)*((a-b)/(a+b))^(1/2)/sin(d*x+c), (a+b)/(a-b), I/((a-b)/(a+b))^(1/2))*cos(d*x+c)^3*a*b-B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(-(a+b)/(a-b))^(1/2))*cos(d*x+c)^3*a*b-2*B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))
)*((a-b)/(a+b))^(1/2)/sin(d*x+c), -(a+b)/(a-b))^(1/2))*cos(d*x+c)^3*a*b+4*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), -(a+b)/(a-b))^(1/2))*cos(d*x+c)^2*a*b-8*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), (a+b)/(a-b), I/((a-b)/(a+b))^(1/2))*cos(d*x+c)^2*a*b-B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), -(a+b)/(a-b))^(1/2))*cos(d*x+c)^2*a*b-2*B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), -(a+b)/(a-b))^(1/2))*cos(d*x+c)^2*a*b-B*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2-4*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*b^2+B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2+4*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^2-2*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*b^2+2*B*((a-b)/(a+b))^(1/2)*b^2-4*A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a*b-2*B*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a*b+4*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b-B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b+3*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b+B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), -(a+b)/(a-b))^(1/2))*cos(d*x+c)^2*a^2+2*B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), (a+b)/(a-b), I/((a-b)/(a+b))^(1/2))*cos(d*x+c)^2*a^2-8*B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), (a+b)/(a-b), I/((a-b)/(a+b))^(1/2))*cos(d*x+c)^2*b^2-2*B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), -(a+b)/(a-b))^(1/2))*cos(d*x+c)^2*a^2+4*B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), -(a+b)/(a-b))^(1/2))*cos(d*x+c)^3*b^2+B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), -(a+b)/(a-b))^(1/2))*cos(d*x+c)^3*a^2+2*B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*E
llipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), (a+b)/(a-b), I/((a-
```

$$\begin{aligned} & b/(a+b))^{(1/2)} * \cos(d*x+c)^3 * a^2 - 8*B*\sin(d*x+c) * ((b+a*\cos(d*x+c))/(1+\cos(d \\ & *x+c)) / (a+b))^{(1/2)} * (1/(1+\cos(d*x+c)))^{(1/2)} * \text{EllipticPi}((-1+\cos(d*x+c)) * ((a \\ & -b)/(a+b))^{(1/2)} / \sin(d*x+c), (a+b)/(a-b), I / ((a-b)/(a+b))^{(1/2)} * \cos(d*x+c)^3 \\ & * b^2 - 2*B*\sin(d*x+c) * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)) / (a+b))^{(1/2)} * (1/(1+\cos \\ & (d*x+c)))^{(1/2)} * \text{EllipticF}((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{(1/2)} / \sin(d*x+c), (- \\ & (a+b)/(a-b))^{(1/2)} * \cos(d*x+c)^3 * a^2 + 4*B*\sin(d*x+c) * ((b+a*\cos(d*x+c))/(1+co \\ & s(d*x+c)) / (a+b))^{(1/2)} * (1/(1+\cos(d*x+c)))^{(1/2)} * \text{EllipticF}((-1+\cos(d*x+c)) * (\\ & (a-b)/(a+b))^{(1/2)} / \sin(d*x+c), -(a+b)/(a-b))^{(1/2)} * \cos(d*x+c)^3 * b^2 - 4*A*si \\ & n(d*x+c) * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)) / (a+b))^{(1/2)} * (1/(1+\cos(d*x+c)))^{(\\ & 1/2)} * \text{EllipticE}((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{(1/2)} / \sin(d*x+c), -(a+b)/(a-b) \\ &)^{(1/2)} * \cos(d*x+c)^2 * b^2 * (1/\cos(d*x+c))^{(3/2)} * ((b+a*\cos(d*x+c))/\cos(d*x+c \\ &))^{(1/2)} / (b+a*\cos(d*x+c)) / \sin(d*x+c) / ((a-b)/(a+b))^{(1/2)} / b \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A) \sqrt{b \sec(dx + c) + a} \sec(dx + c)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(A + \frac{B}{\cos(c + dx)} \right) \sqrt{a + \frac{b}{\cos(c + dx)}} \left(\frac{1}{\cos(c + dx)} \right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(3/2), x)

[Out] int((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(3/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**(3/2)*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))**(1/2),x)

[Out] Timed out

$$3.436 \quad \int \sqrt{\sec(c + dx)} \sqrt{a + b \sec(c + dx)} (A + B \sec(c + dx)) dx$$

Optimal. Leaf size=253

$$\frac{(2aA + bB)\sqrt{\sec(c + dx)} \sqrt{\frac{a \cos(c+dx)+b}{a+b}} F\left(\frac{1}{2}(c + dx) \middle| \frac{2a}{a+b}\right)}{d\sqrt{a + b \sec(c + dx)}} + \frac{(aB + 2Ab)\sqrt{\sec(c + dx)} \sqrt{\frac{a \cos(c+dx)+b}{a+b}} \Pi\left(2; \frac{1}{2}(c + dx)\right)}{d\sqrt{a + b \sec(c + dx)}}$$

[Out] $(2Aa + Bb) \cdot (\cos(1/2 dx + 1/2 c))^2 \cdot \sqrt{\sec(c + dx)} \cdot \sqrt{\frac{a \cos(c + dx) + b}{a + b}} \cdot F\left(\frac{1}{2}(c + dx) \middle| \frac{2a}{a + b}\right) + (aB + 2Ab) \cdot (\cos(1/2 dx + 1/2 c))^2 \cdot \sqrt{\sec(c + dx)} \cdot \sqrt{\frac{a \cos(c + dx) + b}{a + b}} \cdot \Pi\left(2; \frac{1}{2}(c + dx)\right)$
 $\frac{\quad}{d \sqrt{a + b \sec(c + dx)}}$

Rubi [A] time = 0.78, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.343$, Rules used = {4031, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{(2aA + bB)\sqrt{\sec(c + dx)} \sqrt{\frac{a \cos(c+dx)+b}{a+b}} F\left(\frac{1}{2}(c + dx) \middle| \frac{2a}{a+b}\right)}{d\sqrt{a + b \sec(c + dx)}} + \frac{(aB + 2Ab)\sqrt{\sec(c + dx)} \sqrt{\frac{a \cos(c+dx)+b}{a+b}} \Pi\left(2; \frac{1}{2}(c + dx)\right)}{d\sqrt{a + b \sec(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]

[Out] $((2aA + bB) \cdot \sqrt{\sec(c + dx)} \cdot \sqrt{\frac{a \cos(c + dx) + b}{a + b}} \cdot F\left(\frac{1}{2}(c + dx) \middle| \frac{2a}{a + b}\right) + (aB + 2Ab) \cdot \sqrt{\sec(c + dx)} \cdot \sqrt{\frac{a \cos(c + dx) + b}{a + b}} \cdot \Pi\left(2; \frac{1}{2}(c + dx)\right)) / (d \sqrt{a + b \sec(c + dx)})$

Rule 2653

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*Sqrt[a + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,

b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2655

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a + b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2661

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)]/(d*Sqrt[a + b])), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2663

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b) + (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2805

Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi/2 + f*x))/2, (2*d)/(c + d)]/(f*(a + b)*Sqrt[c + d])), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[c + d, 0]

Rule 2807

Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !GtQ[c + d, 0]

Rule 3856

Int[Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)]/Sqrt[csc[(e_) + (f_)*(x_)]*(d_)], x_Symbol] := Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] := Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] := Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]
])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4031

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] := -Simp[(B*d*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x
] + Dist[d/(m + n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n -
1)*Simp[a*B*(n - 1) + (b*B*(m + n - 1) + a*A*(m + n))*Csc[e + f*x] + (a*B*m
+ A*b*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B},
x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[0, m, 1] && GtQ[n, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] := Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)
)/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] := Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
+ b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{\sec(c+dx)} \sqrt{a+b \sec(c+dx)} (A+B \sec(c+dx)) dx &= \frac{B \sqrt{\sec(c+dx)} \sqrt{a+b \sec(c+dx)} \sin(c+dx)}{d} + \int \frac{A \sqrt{\sec(c+dx)} \sqrt{a+b \sec(c+dx)}}{d} dx \\
&= \frac{B \sqrt{\sec(c+dx)} \sqrt{a+b \sec(c+dx)} \sin(c+dx)}{d} + \frac{1}{2} \left(\frac{2A \sqrt{\sec(c+dx)} \sqrt{a+b \sec(c+dx)}}{d} \right) \\
&= \frac{B \sqrt{\sec(c+dx)} \sqrt{a+b \sec(c+dx)} \sin(c+dx)}{d} - \frac{1}{2} \left(\frac{2A \sqrt{\sec(c+dx)} \sqrt{a+b \sec(c+dx)}}{d} \right) \\
&= \frac{B \sqrt{\sec(c+dx)} \sqrt{a+b \sec(c+dx)} \sin(c+dx)}{d} + \frac{1}{2} \left(\frac{2A \sqrt{\sec(c+dx)} \sqrt{a+b \sec(c+dx)}}{d} \right) \\
&= \frac{(2Ab+aB) \sqrt{\frac{b+a \cos(c+dx)}{a+b}} \Pi\left(2; \frac{1}{2}(c+dx) \middle| \frac{2a}{a+b}\right) \sqrt{\sec(c+dx)}}{d \sqrt{a+b \sec(c+dx)}} \\
&= \frac{(2aA+bB) \sqrt{\frac{b+a \cos(c+dx)}{a+b}} F\left(\frac{1}{2}(c+dx) \middle| \frac{2a}{a+b}\right) \sqrt{\sec(c+dx)}}{d \sqrt{a+b \sec(c+dx)}}
\end{aligned}$$

Mathematica [C] time = 6.37, size = 377, normalized size = 1.49

$$\sqrt{a+b \sec(c+dx)} \left(\frac{2(aB+4Ab) \Pi\left(2; \frac{1}{2}(c+dx) \middle| \frac{2a}{a+b}\right)}{(a+b) \sqrt{\frac{a \cos(c+dx)+b}{a+b}}} + \frac{8aAF\left(\frac{1}{2}(c+dx) \middle| \frac{2a}{a+b}\right)}{(a+b) \sqrt{\frac{a \cos(c+dx)+b}{a+b}}} - \frac{2iB \csc(c+dx) \sqrt{-\frac{a(\cos(c+dx)-1)}{a+b}} \sqrt{\frac{a(\cos(c+dx)+1)}{a-b}}}{4d \sqrt{a+b \sec(c+dx)}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]), x]

[Out] (Sqrt[a + b*Sec[c + d*x]]*((8*a*A*EllipticF[(c + d*x)/2, (2*a)/(a + b)]))/((a + b)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*(4*A*b + a*B)*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/((a + b)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) - ((2*I)*B*Sqrt[-((a*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(a*(1 + Cos[c + d*x]))/(a - b)]*Csc[c + d*x]*(-2*b*(a + b)*EllipticE[I*ArcSinh[Sqrt[(a - b)^(-1)]]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*(2*b*EllipticF[I*ArcSinh[Sqrt[(a - b)^(-1)]]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*EllipticPi[1 - a/b, I*ArcSinh[Sqrt[(a - b)^(-1)]]*Sqrt[b + a*Cos[c + d*x]]], (-

$(a + b)/(a + b)))/((a * \sqrt{(a - b)^{-1}} * b * \sqrt{b + a * \cos[c + d * x]}) + 4 * B * \tan[c + d * x]) / (4 * d * \sqrt{\sec[c + d * x]})$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2),x, algorith="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A) \sqrt{b \sec(dx + c) + a} \sqrt{\sec(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2),x, algorith="giac")

[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*sqrt(sec(d*x + c)), x)

maple [C] time = 2.29, size = 1431, normalized size = 5.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2),x)

[Out] $-1/d * (2 * A * ((b + a * \cos(d * x + c)) / (1 + \cos(d * x + c))) / (a + b))^{1/2} * (1 / (1 + \cos(d * x + c)))^{1/2} * \text{EllipticF}((-1 + \cos(d * x + c)) * ((a - b) / (a + b))^{1/2} / \sin(d * x + c), (-a + b) / (a - b))^{1/2} * \sin(d * x + c) * \cos(d * x + c)^{2 * a - 2 * A * ((b + a * \cos(d * x + c)) / (1 + \cos(d * x + c))) / (a + b))^{1/2} * (1 / (1 + \cos(d * x + c)))^{1/2} * \text{EllipticF}((-1 + \cos(d * x + c)) * ((a - b) / (a + b))^{1/2} / \sin(d * x + c), (-a + b) / (a - b))^{1/2} * \sin(d * x + c) * \cos(d * x + c)^{2 * b + 4 * A * ((b + a * \cos(d * x + c)) / (1 + \cos(d * x + c))) / (a + b))^{1/2} * (1 / (1 + \cos(d * x + c)))^{1/2} * \text{EllipticPi}((-1 + \cos(d * x + c)) * ((a - b) / (a + b))^{1/2} / \sin(d * x + c), (a + b) / (a - b), I / ((a - b) / (a + b))^{1/2}) * \sin(d * x + c) * \cos(d * x + c)^{2 * b + 2 * B * ((b + a * \cos(d * x + c)) / (1 + \cos(d * x + c))) / (a + b))^{1/2} * (1 / (1 + \cos(d * x + c)))^{1/2} * \text{EllipticPi}((-1 + \cos(d * x + c)) * ((a - b) / (a + b))^{1/2} / \sin(d * x + c), (a + b) / (a - b), I / ((a - b) / (a + b))^{1/2}) * \sin(d * x + c) * \cos(d * x + c)^{2 * a - B * ((b + a * \cos(d * x + c)) / (1 + \cos(d * x + c))) / (a + b))^{1/2} * (1 / (1 + \cos(d * x + c)))^{1/2} * \text{EllipticE}((-1 + \cos(d * x + c)) * ((a - b) / (a + b))^{1/2} / \sin(d * x + c), (-a + b) / (a - b))^{1/2} * \sin(d * x + c) * \cos(d * x + c)^{2 * a + B * ((b + a * \cos(d * x + c)) / (1 + \cos(d * x + c))) / (a + b))^{1/2}$

```

1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)
/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*b+2*A*((b+a*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+c
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*
cos(d*x+c)*a-2*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*
x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+
b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*b+4*A*((b+a*cos(d*x+c))/(1+cos(d*x+c
)))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/
(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d
*x+c)*b+2*B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c))
)^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-
b),I/((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a-B*((b+a*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*
(a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a
+B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*E
llipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2
))*sin(d*x+c)*cos(d*x+c)*b+B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a-B*cos(d*x+c
)*((a-b)/(a+b))^(1/2)*a+B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b-B*((a-b)/(a+b))
^(1/2)*b*(1/cos(d*x+c))^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(
d*x+c))/sin(d*x+c)/((a-b)/(a+b))^(1/2)

```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A) \sqrt{b \sec(dx + c) + a} \sqrt{\sec(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*sqrt(sec(d*x + c)),
x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(A + \frac{B}{\cos(c + dx)} \right) \sqrt{a + \frac{b}{\cos(c + dx)}} \sqrt{\frac{1}{\cos(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/2),
x)
```

```
[Out] int((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/2),
x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \sec(c + dx)) \sqrt{a + b \sec(c + dx)} \sqrt{\sec(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)**(1/2)*(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sqrt(a + b*sec(c + d*x))*sqrt(sec(c + d*x)),  
x)
```

$$3.437 \quad \int \frac{\sqrt{a+b \sec(c+dx)} (A+B \sec(c+dx))}{\sqrt{\sec(c+dx)}} dx$$

Optimal. Leaf size=208

$$\frac{2A\sqrt{a+b \sec(c+dx)} E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)} \sqrt{\frac{a \cos(c+dx)+b}{a+b}}} + \frac{2aB\sqrt{\sec(c+dx)} \sqrt{\frac{a \cos(c+dx)+b}{a+b}} F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{d\sqrt{a+b \sec(c+dx)}} + \frac{2bB\sqrt{\sec(c+dx)} \sqrt{\frac{a \cos(c+dx)+b}{a+b}}}{d\sqrt{a+b \sec(c+dx)}}$$

[Out] $2*a*B*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)}*(a/(a+b))^{(1/2)})*((b+a*\cos(d*x+c))/(a+b))^{(1/2)}*\sec(d*x+c)^{(1/2)}/d/(a+b*\sec(d*x+c))^{(1/2)}+2*b*B*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticPi}(\sin(1/2*d*x+1/2*c), 2, 2^{(1/2)}*(a/(a+b))^{(1/2)})*((b+a*\cos(d*x+c))/(a+b))^{(1/2)}*\sec(d*x+c)^{(1/2)}/d/(a+b*\sec(d*x+c))^{(1/2)}+2*A*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)}*(a/(a+b))^{(1/2)})*(a+b*\sec(d*x+c))^{(1/2)}/d/((b+a*\cos(d*x+c))/(a+b))^{(1/2)}/\sec(d*x+c)^{(1/2)}$

Rubi [A] time = 0.54, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$, Rules used = {4037, 3854, 3858, 2663, 2661, 3859, 2807, 2805, 3856, 2655, 2653}

$$\frac{2A\sqrt{a+b \sec(c+dx)} E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)} \sqrt{\frac{a \cos(c+dx)+b}{a+b}}} + \frac{2aB\sqrt{\sec(c+dx)} \sqrt{\frac{a \cos(c+dx)+b}{a+b}} F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{d\sqrt{a+b \sec(c+dx)}} + \frac{2bB\sqrt{\sec(c+dx)} \sqrt{\frac{a \cos(c+dx)+b}{a+b}}}{d\sqrt{a+b \sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]))/Sqrt[Sec[c + d*x]], x]

[Out] $(2*a*B*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/(a + b)]*\text{EllipticF}[(c + d*x)/2, (2*a)/(a + b)]*\text{Sqrt}[\text{Sec}[c + d*x]])/(d*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]) + (2*b*B*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/(a + b)]*\text{EllipticPi}[2, (c + d*x)/2, (2*a)/(a + b)]*\text{Sqrt}[\text{Sec}[c + d*x]])/(d*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]) + (2*A*\text{EllipticE}[(c + d*x)/2, (2*a)/(a + b)]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]])/(d*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/(a + b)]*\text{Sqrt}[\text{Sec}[c + d*x]])$

Rule 2653

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2655

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
0] && !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)]/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a
+ b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_)
+ (f_)*(x_)]]), x_Symbol] := Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)]/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_)
+ (f_)*(x_)]]), x_Symbol] := Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
+ f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !GtQ[c + d, 0]
```

Rule 3854

```
Int[Sqrt[csc[(e_) + (f_)*(x_)]*(d_)]*Sqrt[csc[(e_) + (f_)*(x_)]*(b_)
+ (a_)], x_Symbol] := Dist[a, Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f
*x]], x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]
], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)]/Sqrt[csc[(e_) + (f_)*(x_)]
*(d_)], x_Symbol] := Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
```

```

qrt[b + a*Sin[e + f*x]], Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

```

Rule 3858

```

Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_.)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

```

Rule 3859

```

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_.)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]
])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]
]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

```

Rule 4037

```

Int[(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)]*(csc[(e_.) + (f_.)*(x_)]*(B
_.) + (A_.)))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)], x_Symbol] :> Dist[B/d, In
t[Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]], x], x] + Dist[A, Int[Sqrt[
a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]

```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a + b \sec(c + dx)} (A + B \sec(c + dx))}{\sqrt{\sec(c + dx)}} dx &= A \int \frac{\sqrt{a + b \sec(c + dx)}}{\sqrt{\sec(c + dx)}} dx + B \int \sqrt{\sec(c + dx)} \sqrt{a + b \sec(c + dx)} dx \\
&= (aB) \int \frac{\sqrt{\sec(c + dx)}}{\sqrt{a + b \sec(c + dx)}} dx + (bB) \int \frac{\sec^3(c + dx)}{\sqrt{a + b \sec(c + dx)}} dx \\
&= \frac{(aB\sqrt{b + a \cos(c + dx)} \sqrt{\sec(c + dx)}) \int \frac{1}{\sqrt{b + a \cos(c + dx)}} dx}{\sqrt{a + b \sec(c + dx)}} + \frac{(bB) \int \frac{\sec^3(c + dx)}{\sqrt{a + b \sec(c + dx)}} dx}{\sqrt{a + b \sec(c + dx)}} \\
&= \frac{2AE \left(\frac{1}{2}(c + dx) \middle| \frac{2a}{a+b} \right) \sqrt{a + b \sec(c + dx)}}{d \sqrt{\frac{b+a \cos(c+dx)}{a+b}} \sqrt{\sec(c + dx)}} + \frac{(aB \sqrt{\frac{b+a \cos(c+dx)}{a+b}} \sqrt{\sec(c + dx)})}{d \sqrt{\frac{b+a \cos(c+dx)}{a+b}} \sqrt{\sec(c + dx)}} \\
&= \frac{2aB \sqrt{\frac{b+a \cos(c+dx)}{a+b}} F \left(\frac{1}{2}(c + dx) \middle| \frac{2a}{a+b} \right) \sqrt{\sec(c + dx)}}{d \sqrt{a + b \sec(c + dx)}} + \frac{2bB \sqrt{\frac{b+a \cos(c+dx)}{a+b}}}{d \sqrt{a + b \sec(c + dx)}}
\end{aligned}$$

Mathematica [A] time = 2.63, size = 122, normalized size = 0.59

$$\frac{2\sqrt{a + b \sec(c + dx)} \left(A(a + b)E \left(\frac{1}{2}(c + dx) \middle| \frac{2a}{a+b} \right) + B \left(aF \left(\frac{1}{2}(c + dx) \middle| \frac{2a}{a+b} \right) + b\Pi \left(2; \frac{1}{2}(c + dx) \middle| \frac{2a}{a+b} \right) \right) \right)}{d(a + b)\sqrt{\sec(c + dx)} \sqrt{\frac{a \cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]))/Sqrt[Sec[c + d*x]],x]

[Out] (2*(A*(a + b)*EllipticE[(c + d*x)/2, (2*a)/(a + b)] + B*(a*EllipticF[(c + d*x)/2, (2*a)/(a + b)] + b*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]))*Sqrt[a + b*Sec[c + d*x]]/((a + b)*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]])

fricas [F] time = 3.47, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{(B \sec(dx + c) + A)\sqrt{b \sec(dx + c) + a}}{\sqrt{\sec(dx + c)}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")

[Out] integral((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)/sqrt(sec(d*x + c)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)\sqrt{b \sec(dx + c) + a}}{\sqrt{\sec(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2), x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)/sqrt(sec(d*x + c)), x)

maple [C] time = 2.98, size = 1549, normalized size = 7.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2), x)

[Out]
$$\begin{aligned} & -2/d * (-A * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)) / (a+b))^{1/2} * (1/(1+\cos(d*x+c)))^{1/2} * \text{EllipticF}((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{1/2} / \sin(d*x+c), (-a+b)/(a-b))^{1/2}) * \sin(d*x+c) * \cos(d*x+c) * a + A * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)) / (a+b))^{1/2} * (1/(1+\cos(d*x+c)))^{1/2} * \text{EllipticF}((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{1/2} / \sin(d*x+c), (-a+b)/(a-b))^{1/2}) * \sin(d*x+c) * \cos(d*x+c) * b + A * \cos(d*x+c) * \text{EllipticE}((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{1/2} / \sin(d*x+c), (-a+b)/(a-b))^{1/2}) * \sin(d*x+c) * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)) / (a+b))^{1/2} * (1/(1+\cos(d*x+c)))^{1/2} * a - A * \cos(d*x+c) * \text{EllipticE}((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{1/2} / \sin(d*x+c), (-a+b)/(a-b))^{1/2}) * \sin(d*x+c) * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)) / (a+b))^{1/2} * (1/(1+\cos(d*x+c)))^{1/2} * b + B * \cos(d*x+c) * \text{EllipticF}((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{1/2} / \sin(d*x+c), (-a+b)/(a-b))^{1/2}) * \sin(d*x+c) * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)) / (a+b))^{1/2} * (1/(1+\cos(d*x+c)))^{1/2} * a - B * \cos(d*x+c) * \text{EllipticF}((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{1/2} / \sin(d*x+c), (-a+b)/(a-b))^{1/2}) * \sin(d*x+c) * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)) / (a+b))^{1/2} * (1/(1+\cos(d*x+c)))^{1/2} * b + 2 * B * \cos(d*x+c) * \text{EllipticPi}((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{1/2} / \sin(d*x+c), (a+b)/(a-b), I / ((a-b)/(a+b))^{1/2}) * \sin(d*x+c) * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)) / (a+b))^{1/2} * (1/(1+\cos(d*x+c)))^{1/2} * b - A * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)) / (a+b))^{1/2} * (1/(1+\cos(d*x+c)))^{1/2} * \text{EllipticF}((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{1/2} / \sin(d*x+c), (-a+b)/(a-b))^{1/2}) * a * \sin(d*x+c) + A * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)) / (a+b))^{1/2} * (1/(1+\cos(d*x+c)))^{1/2} * \text{EllipticF}((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{1/2} / \sin(d*x+c), (-a+b)/(a-b))^{1/2}) * b * \sin(d*x+c) + A * \text{EllipticE}((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{1/2} / \sin(d*x+c), (-a+b) \end{aligned}$$

$$\frac{1}{(a-b)^{1/2}} * a * \left(\frac{b+a \cos(dx+c)}{1+\cos(dx+c)} \right) / (a+b)^{1/2} * \left(\frac{1}{1+\cos(dx+c)} \right)^{1/2} * \sin(dx+c) - A * \text{EllipticE} \left(\frac{-1+\cos(dx+c)}{(a-b)/(a+b)^{1/2}} / \sin(dx+c), \left(\frac{-1+\cos(dx+c)}{(a-b)/(a+b)^{1/2}} \right)^{1/2} \right) * b * \left(\frac{b+a \cos(dx+c)}{1+\cos(dx+c)} \right) / (a+b)^{1/2} * \left(\frac{1}{1+\cos(dx+c)} \right)^{1/2} * \sin(dx+c) + B * \text{EllipticF} \left(\frac{-1+\cos(dx+c)}{(a-b)/(a+b)^{1/2}} / \sin(dx+c), \left(\frac{-1+\cos(dx+c)}{(a-b)/(a+b)^{1/2}} \right)^{1/2} \right) * a * \left(\frac{b+a \cos(dx+c)}{1+\cos(dx+c)} \right) / (a+b)^{1/2} * \left(\frac{1}{1+\cos(dx+c)} \right)^{1/2} * \sin(dx+c) - B * \text{EllipticF} \left(\frac{-1+\cos(dx+c)}{(a-b)/(a+b)^{1/2}} / \sin(dx+c), \left(\frac{-1+\cos(dx+c)}{(a-b)/(a+b)^{1/2}} \right)^{1/2} \right) * b * \left(\frac{b+a \cos(dx+c)}{1+\cos(dx+c)} \right) / (a+b)^{1/2} * \left(\frac{1}{1+\cos(dx+c)} \right)^{1/2} * \sin(dx+c) + 2 * B * \text{EllipticPi} \left(\frac{-1+\cos(dx+c)}{(a-b)/(a+b)^{1/2}} / \sin(dx+c), \frac{(a+b)/(a-b)}{(a-b)/(a+b)^{1/2}} * b * \left(\frac{b+a \cos(dx+c)}{1+\cos(dx+c)} \right) / (a+b)^{1/2} * \left(\frac{1}{1+\cos(dx+c)} \right)^{1/2} * \sin(dx+c) + A * \cos(dx+c)^2 * \left(\frac{-1+\cos(dx+c)}{(a-b)/(a+b)^{1/2}} \right) * a - A * \cos(dx+c) * \left(\frac{-1+\cos(dx+c)}{(a-b)/(a+b)^{1/2}} \right) * a + A * \cos(dx+c) * \left(\frac{-1+\cos(dx+c)}{(a-b)/(a+b)^{1/2}} \right) * b - A * b * \left(\frac{-1+\cos(dx+c)}{(a-b)/(a+b)^{1/2}} \right) * \left(\frac{b+a \cos(dx+c)}{\cos(dx+c)} \right)^{1/2} / \left(\frac{1}{\cos(dx+c)} \right)^{1/2} / (b+a \cos(dx+c)) / \sin(dx+c) / \left(\frac{-1+\cos(dx+c)}{(a-b)/(a+b)^{1/2}} \right)$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx+c) + A) \sqrt{b \sec(dx+c) + a}}{\sqrt{\sec(dx+c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)/sqrt(sec(d*x + c)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)} \right) \sqrt{a + \frac{b}{\cos(c+dx)}}}{\sqrt{\frac{1}{\cos(c+dx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(1/2))/(1/cos(c + d*x))^(1/2),x)

[Out] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(1/2))/(1/cos(c + d*x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \sqrt{a + b \sec(c + dx)}}{\sqrt{\sec(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+b*sec(d*x+c))**(1/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sqrt(a + b*sec(c + d*x))/sqrt(sec(c + d*x)),  
x)
```

$$3.438 \quad \int \frac{\sqrt{a+b \sec(c+dx)} (A+B \sec(c+dx))}{3 \sec^2(c+dx)} dx$$

Optimal. Leaf size=201

$$\frac{2A(a^2 - b^2) \sqrt{\sec(c+dx)} \sqrt{\frac{a \cos(c+dx)+b}{a+b}} F\left(\frac{1}{2}(c+dx) \middle| \frac{2a}{a+b}\right)}{3ad\sqrt{a+b \sec(c+dx)}} + \frac{2(3aB + Ab)\sqrt{a+b \sec(c+dx)} E\left(\frac{1}{2}(c+dx) \middle| \frac{2a}{a+b}\right)}{3ad\sqrt{\sec(c+dx)} \sqrt{\frac{a \cos(c+dx)+b}{a+b}}}$$

[Out] $2/3*A*(a^2-b^2)*(\cos(1/2*d*x+1/2*c))^2^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)}*(a/(a+b))^{(1/2)})*((b+a*\cos(d*x+c))/(a+b))^{(1/2)}*\sec(d*x+c)^{(1/2)}/a/d/(a+b*\sec(d*x+c))^{(1/2)}+2/3*A*\sin(d*x+c)*(a+b*\sec(d*x+c))^{(1/2)}/d/\sec(d*x+c)^{(1/2)}+2/3*(A*b+3*B*a)*(\cos(1/2*d*x+1/2*c))^2^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)}*(a/(a+b))^{(1/2)})*(a+b*\sec(d*x+c))^{(1/2)}/a/d/((b+a*\cos(d*x+c))/(a+b))^{(1/2)}/\sec(d*x+c)^{(1/2)}$

Rubi [A] time = 0.48, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {4032, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2A(a^2 - b^2) \sqrt{\sec(c+dx)} \sqrt{\frac{a \cos(c+dx)+b}{a+b}} F\left(\frac{1}{2}(c+dx) \middle| \frac{2a}{a+b}\right)}{3ad\sqrt{a+b \sec(c+dx)}} + \frac{2(3aB + Ab)\sqrt{a+b \sec(c+dx)} E\left(\frac{1}{2}(c+dx) \middle| \frac{2a}{a+b}\right)}{3ad\sqrt{\sec(c+dx)} \sqrt{\frac{a \cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(\text{Sqrt}[a + b*\text{Sec}[c + d*x]]*(A + B*\text{Sec}[c + d*x]))/\text{Sec}[c + d*x]^{(3/2)}, x]$

[Out] $(2*A*(a^2 - b^2)*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/(a + b)]*\text{EllipticF}[(c + d*x)/2, (2*a)/(a + b)]*\text{Sqrt}[\text{Sec}[c + d*x]])/(3*a*d*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]) + (2*(A*b + 3*a*B)*\text{EllipticE}[(c + d*x)/2, (2*a)/(a + b)]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]])/(3*a*d*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/(a + b)]*\text{Sqrt}[\text{Sec}[c + d*x]]) + (2*A*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(3*d*\text{Sqrt}[\text{Sec}[c + d*x]])$

Rule 2653

$\text{Int}[\text{Sqrt}[(a_) + (b_)*\sin[(c_) + (d_)*(x_)]], x_Symbol] \rightarrow \text{Simp}[(2*\text{Sqrt}[a + b]*\text{EllipticE}[(1*(c - \text{Pi}/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; \text{FreeQ}\{a, b, c, d\}, x] \ \&\& \ \text{NeQ}[a^2 - b^2, 0] \ \&\& \ \text{GtQ}[a + b, 0]$

Rule 2655

$\text{Int}[\text{Sqrt}[(a_) + (b_)*\sin[(c_) + (d_)*(x_)]], x_Symbol] \rightarrow \text{Dist}[\text{Sqrt}[a + b*\text{Sin}[c + d*x]]/\text{Sqrt}[(a + b*\text{Sin}[c + d*x])/(a + b)], \text{Int}[\text{Sqrt}[a/(a + b) + (b$

```
*Sin[c + d*x]]/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)]/(d*Sqrt[a + b]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a + b*SIN[c + d*x])/(a + b)]/Sqrt[a + b*SIN[c + d*x]], Int[1/Sqrt[a/(a + b) + (b*SIN[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)]/Sqrt[csc[(e_) + (f_)*(x_)]*(d_)], x_Symbol] := Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*SIN[e + f*x]]), Int[Sqrt[b + a*SIN[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_) + (f_)*(x_)]*(d_)]/Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)], x_Symbol] := Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*SIN[e + f*x]])/Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*SIN[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4032

```
Int[(csc[(e_) + (f_)*(x_)]*(d_))^(n_)*(csc[(e_) + (f_)*(x_)]*(b_) + (a_))^(m_)*(csc[(e_) + (f_)*(x_)]*(B_) + (A_)), x_Symbol] := Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*A*(n + 1))*Csc[e + f*x] - A*b*(m + n + 1)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[0, m, 1] && LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_) + (f_)*(x_)]*(B_) + (A_))/(Sqrt[csc[(e_) + (f_)*(x_)]*(d_)]*Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)]), x_Symbol] := Dist[A/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
```

(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{\sqrt{a + b \sec(c + dx)} (A + B \sec(c + dx))}{\sec^3(c + dx)} dx &= \frac{2A\sqrt{a + b \sec(c + dx)} \sin(c + dx)}{3d\sqrt{\sec(c + dx)}} + \frac{2}{3} \int \frac{\frac{1}{2}(Ab + 3aB) + \frac{1}{2}(aA)}{\sqrt{\sec(c + dx)} \sqrt{a}} \\
 &= \frac{2A\sqrt{a + b \sec(c + dx)} \sin(c + dx)}{3d\sqrt{\sec(c + dx)}} + \frac{(A(a^2 - b^2)) \int \frac{\sqrt{\sec(c + dx)}}{\sqrt{a + b \sec(c + dx)}}}{3a} \\
 &= \frac{2A\sqrt{a + b \sec(c + dx)} \sin(c + dx)}{3d\sqrt{\sec(c + dx)}} + \frac{(A(a^2 - b^2) \sqrt{b + a \cos(c + dx)})}{3a\sqrt{a + b \sec(c + dx)}} \\
 &= \frac{2A\sqrt{a + b \sec(c + dx)} \sin(c + dx)}{3d\sqrt{\sec(c + dx)}} + \frac{(A(a^2 - b^2) \sqrt{\frac{b + a \cos(c + dx)}{a + b}})}{3a\sqrt{a + b \sec(c + dx)}} \\
 &= \frac{2A(a^2 - b^2) \sqrt{\frac{b + a \cos(c + dx)}{a + b}} F\left(\frac{1}{2}(c + dx) \middle| \frac{2a}{a + b}\right) \sqrt{\sec(c + dx)}}{3ad\sqrt{a + b \sec(c + dx)}} + \frac{2(A(a^2 - b^2) \sqrt{b + a \cos(c + dx)})}{3ad\sqrt{a + b \sec(c + dx)}}
 \end{aligned}$$

Mathematica [A] time = 0.81, size = 165, normalized size = 0.82

$$\frac{2\sqrt{a + b \sec(c + dx)} \left(A(a^2 - b^2) \sqrt{\frac{a \cos(c + dx) + b}{a + b}} F\left(\frac{1}{2}(c + dx) \middle| \frac{2a}{a + b}\right) + (a + b)(3aB + Ab) \sqrt{\frac{a \cos(c + dx) + b}{a + b}} E\left(\frac{1}{2}(c + dx) \middle| \frac{2a}{a + b}\right) \right)}{3ad\sqrt{\sec(c + dx)} (a \cos(c + dx) + b)}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(3/2), x]

[Out] (2*Sqrt[a + b*Sec[c + d*x]]*((a + b)*(A*b + 3*a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*a)/(a + b)] + A*(a^2 - b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)] + a*A*(b + a*Cos[c + d*x])*Sin[c + d*x))/(3*a*d*(b + a*Cos[c + d*x])*Sqrt[Sec[c + d*x]])

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B \sec(dx+c) + A)\sqrt{b \sec(dx+c) + a}}{\sec(dx+c)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algorithm="fricas")

[Out] integral((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(3/2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx+c) + A)\sqrt{b \sec(dx+c) + a}}{\sec(dx+c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(3/2), x)

maple [B] time = 2.37, size = 1926, normalized size = 9.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x)

[Out]
$$\begin{aligned} & -2/3/d*(A*\text{EllipticF}((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (-a+b)/(a-b))^{1/2}) * a^2 * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)) / (a+b))^{1/2} * (1/(1+\cos(d*x+c)))^{1/2} * \sin(d*x+c) - A*\cos(d*x+c)*\sin(d*x+c)*\text{EllipticF}((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (-a+b)/(a-b))^{1/2}) * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)) / (a+b))^{1/2} * (1/(1+\cos(d*x+c)))^{1/2} * a*b - A*((a-b)/(a+b))^{1/2} * a*b - 3*B*((a-b)/(a+b))^{1/2} * a*b - A*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)) / (a+b))^{1/2} * (1/(1+\cos(d*x+c)))^{1/2} * \text{EllipticE}((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (-a+b)/(a-b))^{1/2}) * b^2 * \sin(d*x+c) + A*\cos(d*x+c)*\sin(d*x+c) * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)) / (a+b))^{1/2} * (1/(1+\cos(d*x+c)))^{1/2} * \text{EllipticE}((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (-a+b)/(a-b))^{1/2}) * a*b + 3*B*\cos(d*x+c)*\sin(d*x+c)*\text{EllipticF}((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (-a+b)/(a-b))^{1/2}) * a*b \end{aligned}$$


```

in(d*x+c), (- (a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*(1/(1+cos(d*x+c)))^(1/2)*a*b-3*B*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x
+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), (- (a+b)/(a-b))^(1/2))*a*b-A*((a-b)/(a+b
))^(1/2)*b^2+3*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2+A*cos(d*x+c)*((a-b)/(
a+b))^(1/2)*b^2+A*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a
+b))^(1/2)/sin(d*x+c), (- (a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c)
))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2-A*cos(d*x+c)*sin(d*x+c)*((b+a*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), (- (a+b)/(a-b))^(1/2))*b^2-3*B
*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*
x+c), (- (a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1
/(1+cos(d*x+c)))^(1/2)*a^2+3*B*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c), (- (a+b)/(a-b))^(1/2))*a^2-A*EllipticF((-1+co
s(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), (- (a+b)/(a-b))^(1/2))*a*b*((b+a*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+A
*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ell
ipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), (- (a+b)/(a-b))^(1/2))
*a*b*sin(d*x+c)+3*B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos
(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), (-
(a+b)/(a-b))^(1/2))*a*b*sin(d*x+c)-3*B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(
1/2)/sin(d*x+c), (- (a+b)/(a-b))^(1/2))*a*b*sin(d*x+c)+2*A*cos(d*x+c)^2*((a-
b)/(a+b))^(1/2)*a*b+3*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b-3*B*cos(d*x+c)*
(a-b)/(a+b))^(1/2)*a^2+A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2-A*a^2*((a-b)/
(a+b))^(1/2)*cos(d*x+c)-A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b-3*B*((b+a*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+
cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), (- (a+b)/(a-b))^(1/2))*a^2*sin(d*
x+c)+3*B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(
1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), (- (a+b)/(a-b)
))^(1/2))*a^2*sin(d*x+c)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^2*(
1/cos(d*x+c))^(3/2)/sin(d*x+c)/(b+a*cos(d*x+c))/((a-b)/(a+b))^(1/2)/a

```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sqrt{b \sec(dx + c) + a}}{\sec(dx + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \sqrt{a + \frac{b}{\cos(c+dx)}}}{\left(\frac{1}{\cos(c+dx)}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(1/2))/(1/cos(c + d*x))^(3/2), x)

[Out] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(1/2))/(1/cos(c + d*x))^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \sqrt{a + b \sec(c + dx)}}{\sec^{\frac{3}{2}}(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*(a+b*sec(d*x+c))**(1/2)/sec(d*x+c)**(3/2), x)

[Out] Integral((A + B*sec(c + d*x))*sqrt(a + b*sec(c + d*x))/sec(c + d*x)**(3/2), x)

$$3.439 \quad \int \frac{\sqrt{a+b \sec(c+dx)} (A+B \sec(c+dx))}{\sec^2(c+dx)} dx$$

Optimal. Leaf size=267

$$\frac{2(a^2 - b^2)(2Ab - 5aB)\sqrt{\sec(c+dx)}\sqrt{\frac{a \cos(c+dx)+b}{a+b}} F\left(\frac{1}{2}(c+dx) \middle| \frac{2a}{a+b}\right)}{15a^2 d \sqrt{a+b \sec(c+dx)}} + \frac{2(9a^2 A + 5abB - 2Ab^2)\sqrt{a+b \sec(c+dx)}}{15a^2 d \sqrt{\sec(c+dx)}\sqrt{\frac{a \cos(c+dx)+b}{a+b}}}$$

[Out] $-2/15*(a^2-b^2)*(2*A*b-5*B*a)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)}*(a/(a+b))^{(1/2)}*((b+a*\cos(d*x+c))/(a+b))^{(1/2)}*\sec(d*x+c)^{(1/2)}/a^2/d/(a+b*\sec(d*x+c))^{(1/2)}+2/5*A*\sin(d*x+c)*(a+b*\sec(d*x+c))^{(1/2)}/d/\sec(d*x+c)^{(3/2)}+2/15*(A*b+5*B*a)*\sin(d*x+c)*(a+b*\sec(d*x+c))^{(1/2)}/a/d/\sec(d*x+c)^{(1/2)}+2/15*(9*A*a^2-2*A*b^2+5*B*a*b)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)}*(a/(a+b))^{(1/2)}*(a+b*\sec(d*x+c))^{(1/2)}/a^2/d/((b+a*\cos(d*x+c))/(a+b))^{(1/2)}/\sec(d*x+c)^{(1/2)})$

Rubi [A] time = 0.75, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {4032, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2(a^2 - b^2)(2Ab - 5aB)\sqrt{\sec(c+dx)}\sqrt{\frac{a \cos(c+dx)+b}{a+b}} F\left(\frac{1}{2}(c+dx) \middle| \frac{2a}{a+b}\right)}{15a^2 d \sqrt{a+b \sec(c+dx)}} + \frac{2(9a^2 A + 5abB - 2Ab^2)\sqrt{a+b \sec(c+dx)}}{15a^2 d \sqrt{\sec(c+dx)}\sqrt{\frac{a \cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(\text{Sqrt}[a + b*\text{Sec}[c + d*x]]*(A + B*\text{Sec}[c + d*x]))/\text{Sec}[c + d*x]^{(5/2)}, x]$

[Out] $(-2*(a^2 - b^2)*(2*A*b - 5*a*B)*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/(a + b)]*\text{EllipticF}[(c + d*x)/2, (2*a)/(a + b)]*\text{Sqrt}[\text{Sec}[c + d*x]])/(15*a^2*d*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]) + (2*(9*a^2*A - 2*A*b^2 + 5*a*b*B)*\text{EllipticE}[(c + d*x)/2, (2*a)/(a + b)]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]])/(15*a^2*d*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/(a + b)]*\text{Sqrt}[\text{Sec}[c + d*x]]) + (2*A*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(5*d*\text{Sec}[c + d*x]^{(3/2)}) + (2*(A*b + 5*a*B)*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(15*a*d*\text{Sqrt}[\text{Sec}[c + d*x]])$

Rule 2653

$\text{Int}[\text{Sqrt}[(a_) + (b_)*\sin[(c_) + (d_)*(x_)]], x_Symbol] \rightarrow \text{Simp}[(2*\text{Sqrt}[a + b]*\text{EllipticE}[(1*(c - \text{Pi}/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; \text{FreeQ}\{a, b, c, d\}, x] \ \&\& \ \text{NeQ}[a^2 - b^2, 0] \ \&\& \ \text{GtQ}[a + b, 0]$

Rule 2655

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
0] && !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)]/(d*Sqrt[a + b]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a
+ b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && !GtQ[a + b, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)]/Sqrt[csc[(e_) + (f_)*(x_)]
*(d_)], x_Symbol] := Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_) + (f_)*(x_)]*(d_)]/Sqrt[csc[(e_) + (f_)*(x_)]*(b_)
+ (a_)], x_Symbol] := Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4032

```
Int[(csc[(e_) + (f_)*(x_)]*(d_))^(n_)*(csc[(e_) + (f_)*(x_)]*(b_) + (
a_))^(m_)*(csc[(e_) + (f_)*(x_)]*(B_) + (A_)), x_Symbol] := Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n
), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp[A*b*m - a
*B*n - (b*B*n + a*A*(n + 1))*Csc[e + f*x] - A*b*(m + n + 1)*Csc[e + f*x]^2,
x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[
a^2 - b^2, 0] && LtQ[0, m, 1] && LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]), x_Symbol] := Dist[A/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m, x_Symbol] := Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{a + b \sec(c + dx)} (A + B \sec(c + dx))}{\sec^{\frac{5}{2}}(c + dx)} dx &= \frac{2A\sqrt{a + b \sec(c + dx)} \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} + \frac{2}{5} \int \frac{\frac{1}{2}(Ab + 5aB) + \frac{1}{2}(3a)}{\sec^{\frac{3}{2}}(c + dx)} dx \\ &= \frac{2A\sqrt{a + b \sec(c + dx)} \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} + \frac{2(Ab + 5aB)\sqrt{a + b \sec(c + dx)}}{15ad\sqrt{\sec(c + dx)}} \\ &= \frac{2A\sqrt{a + b \sec(c + dx)} \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} + \frac{2(Ab + 5aB)\sqrt{a + b \sec(c + dx)}}{15ad\sqrt{\sec(c + dx)}} \\ &= \frac{2A\sqrt{a + b \sec(c + dx)} \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} + \frac{2(Ab + 5aB)\sqrt{a + b \sec(c + dx)}}{15ad\sqrt{\sec(c + dx)}} \\ &= \frac{2A\sqrt{a + b \sec(c + dx)} \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} + \frac{2(Ab + 5aB)\sqrt{a + b \sec(c + dx)}}{15ad\sqrt{\sec(c + dx)}} \\ &= \frac{2(a^2 - b^2)(2Ab - 5aB)\sqrt{\frac{b+a \cos(c+dx)}{a+b}} F\left(\frac{1}{2}(c + dx) \middle| \frac{2a}{a+b}\right) \sqrt{\sec(c + dx)}}{15a^2d\sqrt{a + b \sec(c + dx)}} \end{aligned}$$

Mathematica [A] time = 1.21, size = 200, normalized size = 0.75

$$\frac{2\sqrt{a+b\sec(c+dx)}\left((a^2-b^2)(5aB-2Ab)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)+(a+b)(9a^2A+5abB-2Ab^2)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\right)}{15a^2d\sqrt{\sec(c+dx)}(a\cos(c+dx)+b)}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(5/2), x]

[Out] (2*Sqrt[a + b*Sec[c + d*x]]*((a + b)*(9*a^2*A - 2*A*b^2 + 5*a*b*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*a)/(a + b)] + (a^2 - b^2)*(-2*A*b + 5*a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)] + a*(b + a*Cos[c + d*x])*(A*b + 5*a*B + 3*a*A*Cos[c + d*x])*Sin[c + d*x]))/(15*a^2*d*(b + a*Cos[c + d*x])*Sqrt[Sec[c + d*x]])

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B\sec(dx+c)+A)\sqrt{b\sec(dx+c)+a}}{\sec(dx+c)^{\frac{5}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2), x, algorithm="fricas")

[Out] integral((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(5/2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\sec(dx+c)+A)\sqrt{b\sec(dx+c)+a}}{\sec(dx+c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2), x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(5/2), x)

maple [B] time = 2.24, size = 2737, normalized size = 10.25

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((A+B*\sec(d*x+c))*(a+b*\sec(d*x+c))^{1/2}/\sec(d*x+c)^{5/2}, x)$

[Out] $2/15/d*(-9*A*\text{EllipticE}((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (-a+b)/(a-b))^{1/2})*a^3*(1/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*\sin(d*x+c)-7*A*\sin(d*x+c)*(1/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*\text{EllipticF}((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (-a+b)/(a-b))^{1/2})*\cos(d*x+c)*a^2*b-2*A*\sin(d*x+c)*(1/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*\text{EllipticF}((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (-a+b)/(a-b))^{1/2})*\cos(d*x+c)*a*b^2-5*A*((a-b)/(a+b))^{1/2}*\cos(d*x+c)*a^2*b-4*A*((a-b)/(a+b))^{1/2}*\cos(d*x+c)^3*a^2*b+A*((a-b)/(a+b))^{1/2}*\cos(d*x+c)^2*a*b^2-10*B*((a-b)/(a+b))^{1/2}*\cos(d*x+c)^2*a^2*b-2*A*((a-b)/(a+b))^{1/2}*\cos(d*x+c)*a*b^2+5*B*((a-b)/(a+b))^{1/2}*\cos(d*x+c)*a^2*b-2*A*\text{EllipticE}((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (-a+b)/(a-b))^{1/2})*b^3*(1/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*\sin(d*x+c)-3*A*((a-b)/(a+b))^{1/2}*\cos(d*x+c)^4*a^3-5*B*((a-b)/(a+b))^{1/2}*\cos(d*x+c)^3*a^3+5*B*a^3*((a-b)/(a+b))^{1/2}*\cos(d*x+c)-6*A*((a-b)/(a+b))^{1/2}*\cos(d*x+c)^2*a^3+9*A*((a-b)/(a+b))^{1/2}*\cos(d*x+c)*a^3+2*A*((a-b)/(a+b))^{1/2}*\cos(d*x+c)*b^3+9*A*\sin(d*x+c)*\text{EllipticE}((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (-a+b)/(a-b))^{1/2})*(1/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*\cos(d*x+c)*a^2*b+2*A*\sin(d*x+c)*\text{EllipticE}((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (-a+b)/(a-b))^{1/2})*(1/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*\cos(d*x+c)*a*b^2+5*B*\sin(d*x+c)*(1/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2})*\text{EllipticF}((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (-a+b)/(a-b))^{1/2})*\cos(d*x+c)*a^2*b-5*B*\sin(d*x+c)*\text{EllipticE}((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (-a+b)/(a-b))^{1/2})*(1/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*\cos(d*x+c)*a^2*b+5*B*\sin(d*x+c)*\text{EllipticE}((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (-a+b)/(a-b))^{1/2})*(1/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*\cos(d*x+c)*a*b^2-5*B*\text{EllipticF}((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (-a+b)/(a-b))^{1/2})*a^3*(1/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*\sin(d*x+c)+9*A*\text{EllipticF}((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (-a+b)/(a-b))^{1/2})*a^3*(1/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*\sin(d*x+c)-2*A*b^3*((a-b)/(a+b))^{1/2}+9*A*a^2*b*((a-b)/(a+b))^{1/2}+A*a*b^2*((a-b)/(a+b))^{1/2}+5*B*a^2*b*((a-b)/(a+b))^{1/2}+5*B*a*b^2*((a-b)/(a+b))^{1/2}-5*B*((a-b)/(a+b))^{1/2}*\cos(d*x+c)*a*b^2+9*A*\sin(d*x+c)*(1/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2})*\text{EllipticF}((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (-a+b)/(a-b))^{1/2})*\cos(d*x+c)*a^3-9*A*\sin(d*x+c)*\text{EllipticE}((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (-a+b)/(a-b))^{1/2})*(1/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*\cos(d*x+c)*a^3-2*A*\sin(d*x+c)*\text{EllipticE}((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (-a+b)$

$$\frac{1}{(a-b)^{1/2}} \cdot \frac{1}{(1+\cos(dx+c))^{1/2}} \cdot \frac{(b+a\cos(dx+c))}{(1+\cos(dx+c))} \cdot \frac{1}{(a+b)^{1/2}} \cdot \cos(dx+c) \cdot b^3 - 5B \sin(dx+c) \cdot \frac{1}{(1+\cos(dx+c))^{1/2}} \cdot \frac{(b+a\cos(dx+c))}{(1+\cos(dx+c))} \cdot \frac{1}{(a+b)^{1/2}} \cdot \text{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}, \frac{-(a+b)}{(a-b)^{1/2}}\right) \cdot \cos(dx+c) \cdot a^3 - 7A \cdot \text{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}, \frac{-(a+b)}{(a-b)^{1/2}}\right) \cdot a^2 \cdot b \cdot \frac{1}{(1+\cos(dx+c))^{1/2}} \cdot \frac{(b+a\cos(dx+c))}{(1+\cos(dx+c))} \cdot \frac{1}{(a+b)^{1/2}} \cdot \sin(dx+c) - 2A \cdot \text{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}, \frac{-(a+b)}{(a-b)^{1/2}}\right) \cdot a \cdot b^2 \cdot \frac{1}{(1+\cos(dx+c))^{1/2}} \cdot \frac{(b+a\cos(dx+c))}{(1+\cos(dx+c))} \cdot \frac{1}{(a+b)^{1/2}} \cdot \sin(dx+c) + 9A \cdot \text{EllipticE}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}, \frac{-(a+b)}{(a-b)^{1/2}}\right) \cdot a^2 \cdot b \cdot \frac{1}{(1+\cos(dx+c))^{1/2}} \cdot \frac{(b+a\cos(dx+c))}{(1+\cos(dx+c))} \cdot \frac{1}{(a+b)^{1/2}} \cdot \sin(dx+c) + 2A \cdot \text{EllipticE}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}, \frac{-(a+b)}{(a-b)^{1/2}}\right) \cdot a \cdot b^2 \cdot \frac{1}{(1+\cos(dx+c))^{1/2}} \cdot \frac{(b+a\cos(dx+c))}{(1+\cos(dx+c))} \cdot \frac{1}{(a+b)^{1/2}} \cdot \sin(dx+c) + 5B \cdot \text{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}, \frac{-(a+b)}{(a-b)^{1/2}}\right) \cdot a^2 \cdot b \cdot \frac{1}{(1+\cos(dx+c))^{1/2}} \cdot \frac{(b+a\cos(dx+c))}{(1+\cos(dx+c))} \cdot \frac{1}{(a+b)^{1/2}} \cdot \sin(dx+c) - 5B \cdot \text{EllipticE}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}, \frac{-(a+b)}{(a-b)^{1/2}}\right) \cdot a^2 \cdot b \cdot \frac{1}{(1+\cos(dx+c))^{1/2}} \cdot \frac{(b+a\cos(dx+c))}{(1+\cos(dx+c))} \cdot \frac{1}{(a+b)^{1/2}} \cdot \sin(dx+c) + 5B \cdot \text{EllipticE}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}, \frac{-(a+b)}{(a-b)^{1/2}}\right) \cdot a \cdot b^2 \cdot \frac{1}{(1+\cos(dx+c))^{1/2}} \cdot \frac{(b+a\cos(dx+c))}{(1+\cos(dx+c))} \cdot \frac{1}{(a+b)^{1/2}} \cdot \sin(dx+c) \cdot \frac{(b+a\cos(dx+c))}{\cos(dx+c)} \cdot \frac{1}{(a+b)^{1/2}} \cdot \cos(dx+c)^3 \cdot \frac{1}{\cos(dx+c)^{5/2}} \cdot \frac{1}{\sin(dx+c)} \cdot \frac{1}{(b+a\cos(dx+c))} \cdot \frac{1}{(a-b)^{1/2}} \cdot \frac{1}{(a+b)^{1/2}} \cdot \frac{1}{a^2}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx+c) + A) \sqrt{b \sec(dx+c) + a}}{\sec(dx+c)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(dx+c))*(a+b*sec(dx+c))^(1/2)/sec(dx+c)^(5/2),x, algorithm="maxima")

[Out] integrate((B*sec(dx+c)+A)*sqrt(b*sec(dx+c)+a)/sec(dx+c)^(5/2),x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \sqrt{a + \frac{b}{\cos(c+dx)}}}{\left(\frac{1}{\cos(c+dx)}\right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(1/2))/(1/cos(c + d*x))^(5/2), x)`

[Out] `int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(1/2))/(1/cos(c + d*x))^(5/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \sqrt{a + b \sec(c + dx)}}{\sec^{\frac{5}{2}}(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*sec(d*x+c))*(a+b*sec(d*x+c))**(1/2)/sec(d*x+c)**(5/2), x)`

[Out] `Integral((A + B*sec(c + d*x))*sqrt(a + b*sec(c + d*x))/sec(c + d*x)**(5/2), x)`

$$3.440 \quad \int \frac{\sqrt{a+b \sec(c+dx)} (A+B \sec(c+dx))}{\sec^{\frac{7}{2}}(c+dx)} dx$$

Optimal. Leaf size=343

$$\frac{2(25a^2A + 7abB - 4Ab^2) \sin(c+dx) \sqrt{a+b \sec(c+dx)}}{105a^2d \sqrt{\sec(c+dx)}} + \frac{2(a^2 - b^2)(25a^2A - 14abB + 8Ab^2) \sqrt{\sec(c+dx)} \sqrt{a+b \sec(c+dx)}}{105a^3d \sqrt{a+b \sec(c+dx)}}$$

[Out] $2/105*(a^2-b^2)*(25*A*a^2+8*A*b^2-14*B*a*b)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)}*(a/(a+b))^{(1/2)})*((b+a*\cos(d*x+c))/(a+b))^{(1/2)}*\sec(d*x+c)^{(1/2)}/a^3/d/(a+b*\sec(d*x+c))^{(1/2)}+2/7*A*\sin(d*x+c)*(a+b*\sec(d*x+c))^{(1/2)}/d/\sec(d*x+c)^{(5/2)}+2/35*(A*b+7*B*a)*\sin(d*x+c)*(a+b*\sec(d*x+c))^{(1/2)}/a/d/\sec(d*x+c)^{(3/2)}+2/105*(25*A*a^2-4*A*b^2+7*B*a*b)*\sin(d*x+c)*(a+b*\sec(d*x+c))^{(1/2)}/a^2/d/\sec(d*x+c)^{(1/2)}+2/105*(19*A*a^2*b+8*A*b^3+63*B*a^3-14*B*a*b^2)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)}*(a/(a+b))^{(1/2)})*(a+b*\sec(d*x+c))^{(1/2)}/a^3/d/((b+a*\cos(d*x+c))/(a+b))^{(1/2)}/\sec(d*x+c)^{(1/2)}$

Rubi [A] time = 1.03, antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {4032, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2(25a^2A + 7abB - 4Ab^2) \sin(c+dx) \sqrt{a+b \sec(c+dx)}}{105a^2d \sqrt{\sec(c+dx)}} + \frac{2(a^2 - b^2)(25a^2A - 14abB + 8Ab^2) \sqrt{\sec(c+dx)} \sqrt{a+b \sec(c+dx)}}{105a^3d \sqrt{a+b \sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(7/2), x]

[Out] $(2*(a^2 - b^2)*(25*a^2*A + 8*A*b^2 - 14*a*b*B)*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])]/(a + b))*\text{EllipticF}[(c + d*x)/2, (2*a)/(a + b)]*\text{Sqrt}[\text{Sec}[c + d*x]]/(105*a^3*d*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]) + (2*(19*a^2*A*b + 8*A*b^3 + 63*a^3*B - 14*a*b^2*B))*\text{EllipticE}[(c + d*x)/2, (2*a)/(a + b)]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]/(105*a^3*d*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])]/(a + b))*\text{Sqrt}[\text{Sec}[c + d*x]] + (2*A*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(7*d*\text{Sec}[c + d*x]^(5/2)) + (2*(A*b + 7*a*B))*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]*\text{Sin}[c + d*x]/(35*a*d*\text{Sec}[c + d*x]^(3/2)) + (2*(25*a^2*A - 4*A*b^2 + 7*a*b*B))*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]*\text{Sin}[c + d*x]/(105*a^2*d*\text{Sqrt}[\text{Sec}[c + d*x]])$

Rule 2653

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*Sqrt[a + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,

b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2655

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a + b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2661

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)]/(d*Sqrt[a + b]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2663

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b) + (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 3856

Int[Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)]/Sqrt[csc[(e_) + (f_)*(x_)]*(d_)], x_Symbol] := Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 3858

Int[Sqrt[csc[(e_) + (f_)*(x_)]*(d_)]/Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)], x_Symbol] := Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 4032

Int[(csc[(e_) + (f_)*(x_)]*(d_))^(n_)*(csc[(e_) + (f_)*(x_)]*(b_) + (a_))^(m_)*(csc[(e_) + (f_)*(x_)]*(B_) + (A_)), x_Symbol] := Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*A*(n + 1))*Csc[e + f*x] - A*b*(m + n + 1)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[

$a^2 - b^2, 0]$ && LtQ[0, m, 1] && LeQ[n, -1]

Rule 4035

Int[(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.))/(Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]), x_Symbol] := Dist[A/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]

Rule 4104

Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m, x_Symbol] := Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a + b \sec(c + dx)} (A + B \sec(c + dx))}{\sec^{\frac{7}{2}}(c + dx)} dx &= \frac{2A\sqrt{a + b \sec(c + dx)} \sin(c + dx)}{7d \sec^{\frac{5}{2}}(c + dx)} + \frac{2}{7} \int \frac{\frac{1}{2}(Ab + 7aB) + \frac{1}{2}(5aB)}{\sec^{\frac{5}{2}}(c + dx)} dx \\
&= \frac{2A\sqrt{a + b \sec(c + dx)} \sin(c + dx)}{7d \sec^{\frac{5}{2}}(c + dx)} + \frac{2(Ab + 7aB)\sqrt{a + b \sec(c + dx)}}{35ad \sec^{\frac{3}{2}}(c + dx)} \\
&= \frac{2A\sqrt{a + b \sec(c + dx)} \sin(c + dx)}{7d \sec^{\frac{5}{2}}(c + dx)} + \frac{2(Ab + 7aB)\sqrt{a + b \sec(c + dx)}}{35ad \sec^{\frac{3}{2}}(c + dx)} \\
&= \frac{2A\sqrt{a + b \sec(c + dx)} \sin(c + dx)}{7d \sec^{\frac{5}{2}}(c + dx)} + \frac{2(Ab + 7aB)\sqrt{a + b \sec(c + dx)}}{35ad \sec^{\frac{3}{2}}(c + dx)} \\
&= \frac{2A\sqrt{a + b \sec(c + dx)} \sin(c + dx)}{7d \sec^{\frac{5}{2}}(c + dx)} + \frac{2(Ab + 7aB)\sqrt{a + b \sec(c + dx)}}{35ad \sec^{\frac{3}{2}}(c + dx)} \\
&= \frac{2A\sqrt{a + b \sec(c + dx)} \sin(c + dx)}{7d \sec^{\frac{5}{2}}(c + dx)} + \frac{2(Ab + 7aB)\sqrt{a + b \sec(c + dx)}}{35ad \sec^{\frac{3}{2}}(c + dx)} \\
&= \frac{2A\sqrt{a + b \sec(c + dx)} \sin(c + dx)}{7d \sec^{\frac{5}{2}}(c + dx)} + \frac{2(Ab + 7aB)\sqrt{a + b \sec(c + dx)}}{35ad \sec^{\frac{3}{2}}(c + dx)} \\
&= \frac{2(a^2 - b^2)(25a^2A + 8Ab^2 - 14abB) \sqrt{\frac{b+a \cos(c+dx)}{a+b}} F\left(\frac{1}{2}(c + dx)\right)}{105a^3d\sqrt{a + b \sec(c + dx)}}
\end{aligned}$$

Mathematica [A] time = 1.36, size = 208, normalized size = 0.61

$$\frac{\sqrt{a + b \sec(c + dx)} \left(a \left((115a^2A + 28abB - 16Ab^2) \sin(c + dx) + 3a(2(7aB + Ab) \sin(2(c + dx))) + 5aA \sin(3(c + dx)) \right) \right)}{210a^3d\sqrt{\sec(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(7/2), x]

[Out] (Sqrt[a + b*Sec[c + d*x]]*((4*((19*a^2*A*b + 8*A*b^3 + 63*a^3*B - 14*a*b^2*B)*EllipticE[(c + d*x)/2, (2*a)/(a + b)] + (a - b)*(25*a^2*A + 8*A*b^2 - 14*a*b*B)*EllipticF[(c + d*x)/2, (2*a)/(a + b)])))/Sqrt[(b + a*Cos[c + d*x]]/(

$a + b)] + a*((115*a^2*A - 16*A*b^2 + 28*a*b*B)*\text{Sin}[c + d*x] + 3*a*(2*(A*b + 7*a*B)*\text{Sin}[2*(c + d*x)] + 5*a*A*\text{Sin}[3*(c + d*x)])))/(210*a^3*d*\text{Sqrt}[\text{Sec}[c + d*x]])$

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B \sec(dx + c) + A)\sqrt{b \sec(dx + c) + a}}{\sec(dx + c)^{\frac{7}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2),x, algorithm="fricas")

[Out] integral((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(7/2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)\sqrt{b \sec(dx + c) + a}}{\sec(dx + c)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(7/2), x)

maple [B] time = 2.58, size = 3778, normalized size = 11.01

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2),x)

[Out] $-2/105/d*(-63*B*\text{EllipticF}((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c),(-(a+b)/(a-b))^{1/2})*a^4*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*(1/(1+\cos(d*x+c)))^{1/2}*\sin(d*x+c)+19*A*\cos(d*x+c)*\text{EllipticE}((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c),(-(a+b)/(a-b))^{1/2})*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*(1/(1+\cos(d*x+c)))^{1/2}*\sin(d*x+c)*a^3*b-8*A*\text{EllipticE}((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c),(-(a+b)/(a-b))^{1/2})*b^4*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*(1/(1+\cos(d*x+c)))^{1/2}*\sin(d*x+c)-25*A*a^3*b*((a-b)/(a+b))^{1/2}-19*A*a^2*b^2*((a-b)/(a+b))^{1/2}+4*A$

$$\begin{aligned}
& *a*b^3*((a-b)/(a+b))^{(1/2)}-63*B*a^3*b*((a-b)/(a+b))^{(1/2)}-7*B*a^2*b^2*((a-b)/(a+b))^{(1/2)}+14*B*a*b^3*((a-b)/(a+b))^{(1/2)}+21*B*\cos(d*x+c)^4*((a-b)/(a+b))^{(1/2)}*a^4+42*B*\cos(d*x+c)^2*((a-b)/(a+b))^{(1/2)}*a^4+8*A*\cos(d*x+c)*((a-b)/(a+b))^{(1/2)}*b^4-63*B*\cos(d*x+c)*((a-b)/(a+b))^{(1/2)}*a^4+15*A*\cos(d*x+c)^5*((a-b)/(a+b))^{(1/2)}*a^4+10*A*\cos(d*x+c)^3*((a-b)/(a+b))^{(1/2)}*a^4-25*A*\cos(d*x+c)*((a-b)/(a+b))^{(1/2)}*a^4-8*A*b^4*((a-b)/(a+b))^{(1/2)}-8*A*\cos(d*x+c)*\text{EllipticE}((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c),(-(a+b)/(a-b))^{(1/2)})*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*(1/(1+\cos(d*x+c)))^{(1/2)}*\sin(d*x+c)*b^4+25*A*\cos(d*x+c)*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*(1/(1+\cos(d*x+c)))^{(1/2)}*\text{EllipticF}((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c),(-(a+b)/(a-b))^{(1/2)})*\sin(d*x+c)*a^4+63*B*\cos(d*x+c)*\text{EllipticE}((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c),(-(a+b)/(a-b))^{(1/2)})*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*(1/(1+\cos(d*x+c)))^{(1/2)}*\sin(d*x+c)*a^4-63*B*\cos(d*x+c)*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*(1/(1+\cos(d*x+c)))^{(1/2)}*\text{EllipticF}((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c),(-(a+b)/(a-b))^{(1/2)})*\sin(d*x+c)*a^4+19*A*\text{EllipticE}((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c),(-(a+b)/(a-b))^{(1/2)})*a^3*b*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*(1/(1+\cos(d*x+c)))^{(1/2)}*\sin(d*x+c)-19*A*\text{EllipticE}((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c),(-(a+b)/(a-b))^{(1/2)})*a^2*b^2*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*(1/(1+\cos(d*x+c)))^{(1/2)}*\sin(d*x+c)+8*A*\text{EllipticE}((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c),(-(a+b)/(a-b))^{(1/2)})*a*b^3*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*(1/(1+\cos(d*x+c)))^{(1/2)}*\sin(d*x+c)-19*A*\text{EllipticF}((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c),(-(a+b)/(a-b))^{(1/2)})*a^3*b*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*(1/(1+\cos(d*x+c)))^{(1/2)}*\sin(d*x+c)+2*A*\text{EllipticF}((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c),(-(a+b)/(a-b))^{(1/2)})*a^2*b^2*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*(1/(1+\cos(d*x+c)))^{(1/2)}*\sin(d*x+c)-8*A*\text{EllipticF}((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c),(-(a+b)/(a-b))^{(1/2)})*a*b^3*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*(1/(1+\cos(d*x+c)))^{(1/2)}*\sin(d*x+c)-63*B*\text{EllipticE}((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c),(-(a+b)/(a-b))^{(1/2)})*a^3*b*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*(1/(1+\cos(d*x+c)))^{(1/2)}*\sin(d*x+c)-14*B*\text{EllipticE}((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c),(-(a+b)/(a-b))^{(1/2)})*a^2*b^2*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*(1/(1+\cos(d*x+c)))^{(1/2)}*\sin(d*x+c)+14*B*\text{EllipticE}((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c),(-(a+b)/(a-b))^{(1/2)})*a*b^3*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*(1/(1+\cos(d*x+c)))^{(1/2)}*\sin(d*x+c)+49*B*\text{EllipticF}((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c),(-(a+b)/(a-b))^{(1/2)})*a^3*b*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*(1/(1+\cos(d*x+c)))^{(1/2)}*\sin(d*x+c)+14*B*\text{EllipticF}((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c),(-(a+b)/(a-b))^{(1/2)})*a^2*b^2*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*(1/(1+\cos(d*x+c)))^{(1/2)}*\sin(d*x+c)-19*A*\cos(d*x+c)*\text{EllipticE}((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c),(-(a+b)/(a-b))^{(1/2)})*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*(1/(1+\cos(d*x+c)))^{(1/2)}*\sin(d*x+c)*a^2*b^2+8*A*\cos(d*x+c)*\text{EllipticE}((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c),(-(a+b)/(a-b))^{(1/2)})*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*
\end{aligned}$$

$(1/2)*(1/(1+\cos(d*x+c)))^{1/2}*\sin(d*x+c)*a*b^3-19*A*\cos(d*x+c)*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*(1/(1+\cos(d*x+c)))^{1/2}*EllipticF((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c),(- (a+b)/(a-b))^{1/2})*\sin(d*x+c)*a^3*b+2*A*\cos(d*x+c)*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*(1/(1+\cos(d*x+c)))^{1/2}*EllipticF((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c),(- (a+b)/(a-b))^{1/2})*\sin(d*x+c)*a^2*b^2-8*A*\cos(d*x+c)*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*(1/(1+\cos(d*x+c)))^{1/2}*EllipticF((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c),(- (a+b)/(a-b))^{1/2})*\sin(d*x+c)*a*b^3-63*B*\cos(d*x+c)*EllipticE((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c),(- (a+b)/(a-b))^{1/2})*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*(1/(1+\cos(d*x+c)))^{1/2}*\sin(d*x+c)*a^3*b-14*B*\cos(d*x+c)*EllipticE((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c),(- (a+b)/(a-b))^{1/2})*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*(1/(1+\cos(d*x+c)))^{1/2}*\sin(d*x+c)*a^2*b^2+14*B*\cos(d*x+c)*EllipticE((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c),(- (a+b)/(a-b))^{1/2})*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*(1/(1+\cos(d*x+c)))^{1/2}*\sin(d*x+c)*a*b^3+49*B*\cos(d*x+c)*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*(1/(1+\cos(d*x+c)))^{1/2}*\sin(d*x+c)*a^3*b+14*B*\cos(d*x+c)*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*(1/(1+\cos(d*x+c)))^{1/2}*\sin(d*x+c)*a^2*b^2+25*A*EllipticF((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c),(- (a+b)/(a-b))^{1/2})*a^4*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*(1/(1+\cos(d*x+c)))^{1/2}*\sin(d*x+c)+63*B*EllipticE((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c),(- (a+b)/(a-b))^{1/2})*a^4*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*(1/(1+\cos(d*x+c)))^{1/2}*\sin(d*x+c)-A*\cos(d*x+c)^3*((a-b)/(a+b))^{1/2})*a^2*b^2+28*B*\cos(d*x+c)^3*((a-b)/(a+b))^{1/2})*a^3*b+26*A*\cos(d*x+c)^2*((a-b)/(a+b))^{1/2})*a^3*b+4*A*\cos(d*x+c)^2*((a-b)/(a+b))^{1/2})*a*b^3-7*B*\cos(d*x+c)^2*((a-b)/(a+b))^{1/2})*a^2*b^2-19*A*\cos(d*x+c)*((a-b)/(a+b))^{1/2})*a^3*b+20*A*\cos(d*x+c)*((a-b)/(a+b))^{1/2})*a^2*b^2-8*A*\cos(d*x+c)*((a-b)/(a+b))^{1/2})*a*b^3+35*B*\cos(d*x+c)*((a-b)/(a+b))^{1/2})*a^3*b+18*A*\cos(d*x+c)^4*((a-b)/(a+b))^{1/2})*a^3*b+14*B*\cos(d*x+c)*((a-b)/(a+b))^{1/2})*a^2*b^2-14*B*\cos(d*x+c)*((a-b)/(a+b))^{1/2})*a*b^3*((b+a*\cos(d*x+c))/\cos(d*x+c))^{1/2})*\cos(d*x+c)^4*(1/\cos(d*x+c))^{7/2}/\sin(d*x+c)/(b+a*\cos(d*x+c))/((a-b)/(a+b))^{1/2}/a^3$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)\sqrt{b \sec(dx + c) + a}}{\sec(dx + c)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(7/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \sqrt{a + \frac{b}{\cos(c+dx)}}}{\left(\frac{1}{\cos(c+dx)}\right)^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(1/2))/(1/cos(c + d*x))^(7/2), x)

[Out] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(1/2))/(1/cos(c + d*x))^(7/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*(a+b*sec(d*x+c))**(1/2)/sec(d*x+c)**(7/2), x)

[Out] Timed out

$$3.441 \quad \int \sec^{\frac{3}{2}}(c+dx)(a+b \sec(c+dx))^{3/2}(A+B \sec(c+dx)) dx$$

Optimal. Leaf size=421

$$\frac{(3a^2B + 30aAb + 16b^2B) \sin(c + dx) \sqrt{\sec(c + dx)} \sqrt{a + b \sec(c + dx)}}{24bd} + \frac{(17a^2B + 42aAb + 16b^2B) \sqrt{\sec(c + dx)}}{24d \sqrt{a + b \sec(c + dx)}}$$

[Out] $1/24*(42*A*a*b+17*B*a^2+16*B*b^2)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)}*(a/(a+b))^{(1/2)})*((b+a*\cos(d*x+c))/(a+b))^{(1/2)}*\sec(d*x+c)^{(1/2)}/d/(a+b*\sec(d*x+c))^{(1/2)}+1/8*(6*A*a^2*b+8*A*b^3-B*a^3+12*B*a*b^2)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticPi}(\sin(1/2*d*x+1/2*c), 2, 2^{(1/2)}*(a/(a+b))^{(1/2)})*((b+a*\cos(d*x+c))/(a+b))^{(1/2)}*\sec(d*x+c)^{(1/2)}/b/d/(a+b*\sec(d*x+c))^{(1/2)}+1/12*(6*A*b+7*B*a)*\sec(d*x+c)^{(3/2)}*\sin(d*x+c)*(a+b*\sec(d*x+c))^{(1/2)}/d+1/3*b*B*\sec(d*x+c)^{(5/2)}*\sin(d*x+c)*(a+b*\sec(d*x+c))^{(1/2)}/d-1/24*(30*A*a*b+3*B*a^2+16*B*b^2)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)}*(a/(a+b))^{(1/2)})*(a+b*\sec(d*x+c))^{(1/2)}/b/d/((b+a*\cos(d*x+c))/(a+b))^{(1/2)}/\sec(d*x+c)^{(1/2)}+1/24*(30*A*a*b+3*B*a^2+16*B*b^2)*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}*(a+b*\sec(d*x+c))^{(1/2)}/b/d$

Rubi [A] time = 1.60, antiderivative size = 421, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 13, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {4026, 4102, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{(3a^2B + 30aAb + 16b^2B) \sin(c + dx) \sqrt{\sec(c + dx)} \sqrt{a + b \sec(c + dx)}}{24bd} + \frac{(17a^2B + 42aAb + 16b^2B) \sqrt{\sec(c + dx)}}{24d \sqrt{a + b \sec(c + dx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sec}[c + d*x]^{(3/2)}*(a + b*\text{Sec}[c + d*x])^{(3/2)}*(A + B*\text{Sec}[c + d*x]), x]$

[Out] $((42*a*A*b + 17*a^2*B + 16*b^2*B)*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/(a + b)]*\text{EllipticF}[(c + d*x)/2, (2*a)/(a + b)]*\text{Sqrt}[\text{Sec}[c + d*x]])/(24*d*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]) + ((6*a^2*A*b + 8*A*b^3 - a^3*B + 12*a*b^2*B)*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/(a + b)]*\text{EllipticPi}[2, (c + d*x)/2, (2*a)/(a + b)]*\text{Sqrt}[\text{Sec}[c + d*x]])/(8*b*d*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]) - ((30*a*A*b + 3*a^2*B + 16*b^2*B)*\text{EllipticE}[(c + d*x)/2, (2*a)/(a + b)]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]])/(24*b*d*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/(a + b)]*\text{Sqrt}[\text{Sec}[c + d*x]]) + ((30*a*A*b + 3*a^2*B + 16*$

$$b^2*B)*\text{Sqrt}[\text{Sec}[c + d*x]]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]*\text{Sin}[c + d*x]/(24*b*d) +$$

$$((6*A*b + 7*a*B)*\text{Sec}[c + d*x]^{(3/2)}*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])$$

$$/(12*d) + (b*B*\text{Sec}[c + d*x]^{(5/2)}*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(3$$

$$*d)$$
Rule 2653

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*Sqrt[a
+ b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
0] && !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)]/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a
+ b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_)
+ (f_)*(x_)])], x_Symbol] := Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)]/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_)
+ (f_)*(x_)])], x_Symbol] := Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
+ f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
```

, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !GtQ[c + d, 0]

Rule 3856

Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 3858

Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 3859

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 4026

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] :> -Simp[(b*B*Cos[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x] + Dist[1/(m + n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*Simp[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B)*(m + n) + b^2*B*(m + n - 1))*Csc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && !(IGtQ[n, 1] && !IntegerQ[m])

Rule 4035

Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)]), x_Symbol] :> Dist[A/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]

Rule 4102

```

Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a
_))^(m_), x_Symbol] := -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]

```

Rule 4108

```

Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.)
+ (a_)]), x_Symbol] := Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
+ b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]

```

Rubi steps

$$\begin{aligned}
\int \sec^{\frac{3}{2}}(c + dx)(a + b \sec(c + dx))^{3/2}(A + B \sec(c + dx)) dx &= \frac{bB \sec^{\frac{5}{2}}(c + dx) \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{3d} + \frac{1}{3} \\
&= \frac{(6Ab + 7aB) \sec^{\frac{3}{2}}(c + dx) \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{12d} \\
&= \frac{(30aAb + 3a^2B + 16b^2B) \sqrt{\sec(c + dx)} \sqrt{a + b \sec(c + dx)}}{24bd} \\
&= \frac{(30aAb + 3a^2B + 16b^2B) \sqrt{\sec(c + dx)} \sqrt{a + b \sec(c + dx)}}{24bd} \\
&= \frac{(30aAb + 3a^2B + 16b^2B) \sqrt{\sec(c + dx)} \sqrt{a + b \sec(c + dx)}}{24bd} \\
&= \frac{(30aAb + 3a^2B + 16b^2B) \sqrt{\sec(c + dx)} \sqrt{a + b \sec(c + dx)}}{24bd} \\
&= \frac{(6a^2Ab + 8Ab^3 - a^3B + 12ab^2B) \sqrt{\frac{b+a \cos(c+dx)}{a+b}} \Pi\left(\frac{1}{2}(c + dx)\right)}{8bd \sqrt{a + b \sec(c + dx)}} \\
&= \frac{(42aAb + 17a^2B + 16b^2B) \sqrt{\frac{b+a \cos(c+dx)}{a+b}} F\left(\frac{1}{2}(c + dx)\right)}{24d \sqrt{a + b \sec(c + dx)}}
\end{aligned}$$

Mathematica [C] time = 6.88, size = 673, normalized size = 1.60

$$\frac{(a + b \sec(c + dx))^{3/2} \left(\frac{\sec(c+dx)(3a^2B \sin(c+dx) + 30aAb \sin(c+dx) + 16b^2B \sin(c+dx))}{24b} + \frac{1}{12} \sec^2(c + dx)(7aB \sin(c + dx) + 6Ab) \right)}{d \sec^{\frac{3}{2}}(c + dx)(a \cos(c + dx) + b)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]), x]

```
[Out] -1/96*((a + b*Sec[c + d*x])^(3/2))*((2*(-24*a*A*b^2 - 28*a^2*b*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]/Sqrt[b + a*Cos[c + d*x]] + (2*(-6*a^2*A*b - 48*A*b^3 + 9*a^3*B - 56*a*b^2*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]/Sqrt[b + a*Cos[c + d*x]] + ((2*I)*(30*a^2*A*b + 3*a^3*B + 16*a*b^2*B)*Sqrt[(a - a*Cos[c + d*x])/(a + b)]*Sqrt[(a + a*Cos[c + d*x])/(a - b)]*Cos[2*(c + d*x)]*(-2*b*(a + b)*EllipticE[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*(2*b*EllipticF[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*EllipticPi[1 - a/b, I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)])))*Sin[c + d*x])/(Sqrt[(a - b)^(-1)]*b*Sqrt[1 - Cos[c + d*x]^2]*Sqrt[(a^2 - a^2*Cos[c + d*x]^2)/a^2]*(-a^2 + 2*b^2 - 4*b*(b + a*Cos[c + d*x]) + 2*(b + a*Cos[c + d*x])^2)))/(b*d*(b + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)) + ((a + b*Sec[c + d*x])^(3/2))*((Sec[c + d*x]^2*(6*A*b*Sin[c + d*x] + 7*a*B*Sin[c + d*x]))/12 + (Sec[c + d*x]*(30*a*A*b*Sin[c + d*x] + 3*a^2*B*Sin[c + d*x] + 16*b^2*B*Sin[c + d*x]))/(24*b) + (b*B*Sec[c + d*x]^2*Tan[c + d*x])/3)/(d*(b + a*Cos[c + d*x])*Sec[c + d*x]^(3/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(b \sec(dx + c) + a)^{\frac{3}{2}} \sec(dx + c)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/2), x)
```

maple [C] time = 2.29, size = 4051, normalized size = 9.62

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(\sec(dx+c)^{(3/2)}*(a+b*\sec(dx+c))^{(3/2)}*(A+B*\sec(dx+c)), x)$

[Out]
$$\begin{aligned} & -1/24/d*(3*B*\cos(dx+c)^4*((a-b)/(a+b))^{(1/2)}*a^3+16*B*\cos(dx+c)^3*((a-b)/(a+b))^{(1/2)}*b^3-8*B*\cos(dx+c)^2*((a-b)/(a+b))^{(1/2)}*b^3+12*A*\cos(dx+c)^3 \\ & *((a-b)/(a+b))^{(1/2)}*b^3-30*A*(1/(1+\cos(dx+c)))^{(1/2)}*((b+a*\cos(dx+c))/(1+\cos(dx+c)))/(a+b)^{(1/2)}* \text{EllipticE}((-1+\cos(dx+c))*((a-b)/(a+b))^{(1/2)}/\sin(dx+c), \\ & (-(a+b)/(a-b))^{(1/2)})*\sin(dx+c)*\cos(dx+c)^4*a^2*b-30*A*((a-b)/(a+b))^{(1/2)}*\cos(dx+c)^3*a^2*b-42*A*((a-b)/(a+b))^{(1/2)}*\cos(dx+c)^2*a*b^2-17 \\ & *B*((a-b)/(a+b))^{(1/2)}*\cos(dx+c)^2*a^2*b-8*B*((a-b)/(a+b))^{(1/2)}*b^3-3*B*((a-b)/(a+b))^{(1/2)}*\cos(dx+c)^3*a^3-12*A*((a-b)/(a+b))^{(1/2)}*\cos(dx+c)*b^3 \\ & -22*B*((a-b)/(a+b))^{(1/2)}*\cos(dx+c)*a*b^2+30*A*(1/(1+\cos(dx+c)))^{(1/2)}*((b+a*\cos(dx+c))/(1+\cos(dx+c)))/(a+b)^{(1/2)}* \text{EllipticE}((-1+\cos(dx+c))*((a-b)/(a+b))^{(1/2)}/\sin(dx+c), \\ & (-(a+b)/(a-b))^{(1/2)})*\sin(dx+c)*\cos(dx+c)^4*a*b^2+36*A*(1/(1+\cos(dx+c)))^{(1/2)}*((b+a*\cos(dx+c))/(1+\cos(dx+c)))/(a+b)^{(1/2)}* \text{EllipticPi}((-1+\cos(dx+c))*((a-b)/(a+b))^{(1/2)}/\sin(dx+c), \\ & (a+b)/(a-b), I/((a-b)/(a+b))^{(1/2)})*\sin(dx+c)*\cos(dx+c)^4*a^2*b+14*B* \text{EllipticF}((-1+\cos(dx+c))*((a-b)/(a+b))^{(1/2)}/\sin(dx+c), (-(a+b)/(a-b))^{(1/2)})*(1/(1+\cos(dx+c)))^{(1/2)}* \\ & ((b+a*\cos(dx+c))/(1+\cos(dx+c)))/(a+b)^{(1/2)}*\sin(dx+c)*\cos(dx+c)^4*a^2*b-20*B* \text{EllipticF}((-1+\cos(dx+c))*((a-b)/(a+b))^{(1/2)}/\sin(dx+c), (-(a+b)/(a-b))^{(1/2)})*(1/(1+\cos(dx+c)))^{(1/2)}* \\ & ((b+a*\cos(dx+c))/(1+\cos(dx+c)))/(a+b)^{(1/2)}* \text{EllipticE}((-1+\cos(dx+c))*((a-b)/(a+b))^{(1/2)}/\sin(dx+c), (-(a+b)/(a-b))^{(1/2)})*\sin(dx+c)*\cos(dx+c)^4* \\ & a^2*b-16*B*(1/(1+\cos(dx+c)))^{(1/2)}*((b+a*\cos(dx+c))/(1+\cos(dx+c)))/(a+b)^{(1/2)}* \text{EllipticE}((-1+\cos(dx+c))*((a-b)/(a+b))^{(1/2)}/\sin(dx+c), (-(a+b)/(a-b))^{(1/2)})*\sin(dx+c)*\cos(dx+c)^4* \\ & a*b^2+72*B*(1/(1+\cos(dx+c)))^{(1/2)}*((b+a*\cos(dx+c))/(1+\cos(dx+c)))/(a+b)^{(1/2)}* \text{EllipticPi}((-1+\cos(dx+c))*((a-b)/(a+b))^{(1/2)}/\sin(dx+c), (a+b)/(a-b), I/((a-b)/(a+b))^{(1/2)})*\sin(dx+c)*\cos(dx+c)^4*a*b^2+12*A* \text{EllipticF}((-1+\cos(dx+c))*((a-b)/(a+b))^{(1/2)}/\sin(dx+c), (-(a+b)/(a-b))^{(1/2)})*(1/(1+\cos(dx+c)))^{(1/2)}* \\ & ((b+a*\cos(dx+c))/(1+\cos(dx+c)))/(a+b)^{(1/2)}*\sin(dx+c)*\cos(dx+c)^3*a^2*b+12*A* \text{EllipticF}((-1+\cos(dx+c))*((a-b)/(a+b))^{(1/2)}/\sin(dx+c), (-(a+b)/(a-b))^{(1/2)})*(1/(1+\cos(dx+c)))^{(1/2)}* \\ & ((b+a*\cos(dx+c))/(1+\cos(dx+c)))/(a+b)^{(1/2)}*\sin(dx+c)*\cos(dx+c)^3*a*b^2-30*A*(1/(1+\cos(dx+c)))^{(1/2)}*((b+a*\cos(dx+c))/(1+\cos(dx+c)))/(a+b)^{(1/2)}* \text{EllipticE}((-1+\cos(dx+c))*((a-b)/(a+b))^{(1/2)}/\sin(dx+c), (-(a+b)/(a-b))^{(1/2)})*\sin(dx+c)*\cos(dx+c)^3*a^2*b+30*A*(1/(1+\cos(dx+c)))^{(1/2)}* \\ & ((b+a*\cos(dx+c))/(1+\cos(dx+c)))/(a+b)^{(1/2)}* \text{EllipticE}((-1+\cos(dx+c))*((a-b)/(a+b))^{(1/2)}/\sin(dx+c), (-(a+b)/(a-b))^{(1/2)})*\sin(dx+c)*\cos(dx+c)^3*a*b^2+36*A*(1/(1+\cos(dx+c)))^{(1/2)}*((b+a*\cos(dx+c))/(1+\cos(dx+c)))/(a+b)^{(1/2)}* \text{EllipticPi}((-1+\cos(dx+c))*((a-b)/(a+b))^{(1/2)}/\sin(dx+c), (a+b)/(a-b), I/((a-b)/(a+b))^{(1/2)})*\sin(dx+c)*\cos(dx+c)^3*a^2*b+14*B* \text{EllipticF}((-1+\cos(dx+c))*((a-b)/(a+b))^{(1/2)}/\sin(dx+c), (-(a+b)/(a-b))^{(1/2)})*(1/(1+\cos(dx+c)))^{(1/2)}* \\ & ((b+a*\cos(dx+c))/(1+\cos(dx+c)))/(a+b)^{(1/2)}*\sin(dx+c)*\cos(dx+c)^3*a^2*b-20*B* \text{EllipticF}((-1+\cos(dx+c))*((a-b)/(a+b))^{(1/2)}/\sin(dx+c), (-(a+b)/(a-b))^{(1/2)})*(1/(1+\cos(dx+c)))^{(1/2)}*((b+a*\cos(dx+c))/(1+\cos(dx+c)))/(a+b)^{(1/2)}* \end{aligned}$$

$*x+c)^4*((a-b)/(a+b))^{(1/2)}*a*b^2+30*A*\cos(d*x+c)^3*((a-b)/(a+b))^{(1/2)}*a*b^2+3*B*\cos(d*x+c)^3*((a-b)/(a+b))^{(1/2)}*a^2*b+6*B*\cos(d*x+c)^3*((a-b)/(a+b))^{(1/2)}*a*b^2)*((b+a*\cos(d*x+c))/\cos(d*x+c))^{(1/2)}*(1/\cos(d*x+c))^{(3/2)}/(b+a*\cos(d*x+c))/\cos(d*x+c)/\sin(d*x+c)/b/((a-b)/(a+b))^{(1/2)}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(b \sec(dx + c) + a)^{\frac{3}{2}} \sec(dx + c)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorith="maxima")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(A + \frac{B}{\cos(c + dx)} \right) \left(a + \frac{b}{\cos(c + dx)} \right)^{3/2} \left(\frac{1}{\cos(c + dx)} \right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(3/2), x)

[Out] int((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(3/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**(3/2)*(a+b*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)),x)

[Out] Timed out

$$3.442 \quad \int \sqrt{\sec(c + dx)} (a + b \sec(c + dx))^{3/2} (A + B \sec(c + dx)) dx$$

Optimal. Leaf size=339

$$\frac{(8a^2A + 7abB + 4Ab^2) \sqrt{\sec(c + dx)} \sqrt{\frac{a \cos(c + dx) + b}{a + b}} F\left(\frac{1}{2}(c + dx) \middle| \frac{2a}{a + b}\right)}{4d\sqrt{a + b \sec(c + dx)}} + \frac{(3a^2B + 12aAb + 4b^2B) \sqrt{\sec(c + dx)} \sqrt{\frac{a \cos(c + dx) + b}{a + b}}}{4d\sqrt{a + b \sec(c + dx)}}$$

[Out] $\frac{1}{4} * (8 * A * a^2 + 4 * A * b^2 + 7 * B * a * b) * (\cos(1/2 * d * x + 1/2 * c))^2 \wedge (1/2) / \cos(1/2 * d * x + 1/2 * c) * \text{EllipticF}(\sin(1/2 * d * x + 1/2 * c), 2 \wedge (1/2) * (a / (a + b)) \wedge (1/2)) * ((b + a * \cos(d * x + c)) / (a + b)) \wedge (1/2) * \sec(d * x + c) \wedge (1/2) / d / (a + b * \sec(d * x + c)) \wedge (1/2) + 1/4 * (12 * A * a * b + 3 * B * a^2 + 4 * B * b^2) * (\cos(1/2 * d * x + 1/2 * c))^2 \wedge (1/2) / \cos(1/2 * d * x + 1/2 * c) * \text{EllipticPi}(\sin(1/2 * d * x + 1/2 * c), 2, 2 \wedge (1/2) * (a / (a + b)) \wedge (1/2)) * ((b + a * \cos(d * x + c)) / (a + b)) \wedge (1/2) * \sec(d * x + c) \wedge (1/2) / d / (a + b * \sec(d * x + c)) \wedge (1/2) + 1/2 * b * B * \sec(d * x + c) \wedge (3/2) * \sin(d * x + c) * (a + b * \sec(d * x + c)) \wedge (1/2) / d - 1/4 * (4 * A * b + 5 * B * a) * (\cos(1/2 * d * x + 1/2 * c))^2 \wedge (1/2) / \cos(1/2 * d * x + 1/2 * c) * \text{EllipticE}(\sin(1/2 * d * x + 1/2 * c), 2 \wedge (1/2) * (a / (a + b)) \wedge (1/2)) * (a + b * \sec(d * x + c)) \wedge (1/2) / d / ((b + a * \cos(d * x + c)) / (a + b)) \wedge (1/2) / \sec(d * x + c) \wedge (1/2) + 1/4 * (4 * A * b + 5 * B * a) * \sin(d * x + c) * \sec(d * x + c) \wedge (1/2) * (a + b * \sec(d * x + c)) \wedge (1/2) / d$

Rubi [A] time = 1.21, antiderivative size = 339, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {4026, 4102, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{(8a^2A + 7abB + 4Ab^2) \sqrt{\sec(c + dx)} \sqrt{\frac{a \cos(c + dx) + b}{a + b}} F\left(\frac{1}{2}(c + dx) \middle| \frac{2a}{a + b}\right)}{4d\sqrt{a + b \sec(c + dx)}} + \frac{(3a^2B + 12aAb + 4b^2B) \sqrt{\sec(c + dx)} \sqrt{\frac{a \cos(c + dx) + b}{a + b}}}{4d\sqrt{a + b \sec(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]

[Out] $((8 * a^2 * A + 4 * A * b^2 + 7 * a * b * B) * \text{Sqrt}[(b + a * \text{Cos}[c + d * x]) / (a + b)] * \text{EllipticF}[(c + d * x) / 2, (2 * a) / (a + b)] * \text{Sqrt}[\text{Sec}[c + d * x]]) / (4 * d * \text{Sqrt}[a + b * \text{Sec}[c + d * x]]) + ((12 * a * A * b + 3 * a^2 * B + 4 * b^2 * B) * \text{Sqrt}[(b + a * \text{Cos}[c + d * x]) / (a + b)] * \text{EllipticPi}[2, (c + d * x) / 2, (2 * a) / (a + b)] * \text{Sqrt}[\text{Sec}[c + d * x]]) / (4 * d * \text{Sqrt}[a + b * \text{Sec}[c + d * x]]) - ((4 * A * b + 5 * a * B) * \text{EllipticE}[(c + d * x) / 2, (2 * a) / (a + b)] * \text{Sqrt}[a + b * \text{Sec}[c + d * x]]) / (4 * d * \text{Sqrt}[(b + a * \text{Cos}[c + d * x]) / (a + b)] * \text{Sqrt}[\text{Sec}[c + d * x]]) + ((4 * A * b + 5 * a * B) * \text{Sqrt}[\text{Sec}[c + d * x]] * \text{Sqrt}[a + b * \text{Sec}[c + d * x]] * \text{Sin}[c + d * x]) / (4 * d) + (b * B * \text{Sec}[c + d * x]^(3/2) * \text{Sqrt}[a + b * \text{Sec}[c + d * x]] * \text{Sin}[c + d * x]) / (2 * d)$

Rule 2653

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*Sqrt[a
+ b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
0] && !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)]/(d*Sqrt[a + b]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a
+ b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_)
+ (f_)*(x_)]]), x_Symbol] := Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)]/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_)
+ (f_)*(x_)]]), x_Symbol] := Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
+ f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !GtQ[c + d, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)]/Sqrt[csc[(e_) + (f_)*(x_)]
*(d_)], x_Symbol] := Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
```

b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 3858

Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 3859

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 4026

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] :> -Simp[(b*B*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x] + Dist[1/(m + n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*Simp[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B)*(m + n) + b^2*B*(m + n - 1))*Csc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && !(IGtQ[n, 1] && !IntegerQ[m])

Rule 4035

Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)]), x_Symbol] :> Dist[A/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]

Rule 4102

Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -

$b^2, 0] \ \&\& \text{GtQ}[n, 0]$

Rule 4108

Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.)) / (Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x]) / (Sqrt[d*Csc[e + f*x]]*Sqrt[a + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned}
 \int \sqrt{\sec(c+dx)} (a+b \sec(c+dx))^{3/2} (A+B \sec(c+dx)) dx &= \frac{bB \sec^2(c+dx) \sqrt{a+b \sec(c+dx)} \sin(c+dx)}{2d} + \\
 &= \frac{(4Ab+5aB) \sqrt{\sec(c+dx)} \sqrt{a+b \sec(c+dx)} \sin(c+dx)}{4d} \\
 &= \frac{(4Ab+5aB) \sqrt{\sec(c+dx)} \sqrt{a+b \sec(c+dx)} \sin(c+dx)}{4d} \\
 &= \frac{(4Ab+5aB) \sqrt{\sec(c+dx)} \sqrt{a+b \sec(c+dx)} \sin(c+dx)}{4d} \\
 &= \frac{(4Ab+5aB) \sqrt{\sec(c+dx)} \sqrt{a+b \sec(c+dx)} \sin(c+dx)}{4d} \\
 &= \frac{(12aAb+3a^2B+4b^2B) \sqrt{\frac{b+a \cos(c+dx)}{a+b}} \Pi\left(2; \frac{1}{2}(c+dx)\right)}{4d \sqrt{a+b \sec(c+dx)}} \\
 &= \frac{(8a^2A+4Ab^2+7abB) \sqrt{\frac{b+a \cos(c+dx)}{a+b}} F\left(\frac{1}{2}(c+dx)\right)}{4d \sqrt{a+b \sec(c+dx)}}
 \end{aligned}$$

Mathematica [C] time = 6.88, size = 595, normalized size = 1.76

$$\frac{(a + b \sec(c + dx))^{3/2} \left(\frac{1}{4} \sec(c + dx)(5aB \sin(c + dx) + 4Ab \sin(c + dx)) + \frac{1}{2} bB \tan(c + dx) \sec(c + dx) \right)}{d \sec^2(c + dx)(a \cos(c + dx) + b)} + \frac{(a + b \sec(c + dx))^{3/2}}{d \sec^2(c + dx)(a \cos(c + dx) + b)}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]

[Out] ((a + b*Sec[c + d*x])^(3/2)*((2*(16*a^2*A + 4*a*b*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]/Sqrt[b + a*Cos[c + d*x]] + (2*(20*a*A*b + a^2*B + 8*b^2*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]/Sqrt[b + a*Cos[c + d*x]] + ((2*I)*(-4*a*A*b - 5*a^2*B)*Sqrt[(a - a*Cos[c + d*x])/(a + b)]*Sqrt[(a + a*Cos[c + d*x])/(a - b)]*Cos[2*(c + d*x)]*(-2*b*(a + b)*EllipticE[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*(2*b*EllipticF[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*EllipticPi[1 - a/b, I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)])))*Sin[c + d*x])/(Sqrt[(a - b)^(-1)]*b*Sqrt[1 - Cos[c + d*x]^2]*Sqrt[(a^2 - a^2*Cos[c + d*x]^2)/a^2]*(-a^2 + 2*b^2 - 4*b*(b + a*Cos[c + d*x]) + 2*(b + a*Cos[c + d*x])^2))))/(16*d*(b + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)) + ((a + b*Sec[c + d*x])^(3/2)*((Sec[c + d*x]*(4*A*b*Sin[c + d*x] + 5*a*B*Sin[c + d*x]))/4 + (b*B*Sec[c + d*x]*Tan[c + d*x])/2))/(d*(b + a*Cos[c + d*x])*Sec[c + d*x]^(3/2))

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(b \sec(dx + c) + a)^{\frac{3}{2}} \sqrt{\sec(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)*sqrt(sec(d*x + c)), x)

maple [C] time = 1.90, size = 2947, normalized size = 8.69

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x)

[Out] $\frac{1}{4}d*(4*A*\sin(d*x+c)*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{1/2}*(1/(1+\cos(d*x+c)))^{1/2}*EllipticE((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (-a+b)/(a-b))^{1/2}*\cos(d*x+c)^3*a*b-24*A*\sin(d*x+c)*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{1/2}*(1/(1+\cos(d*x+c)))^{1/2}*EllipticPi((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (a+b)/(a-b), I/((a-b)/(a+b))^{1/2})*\cos(d*x+c)^3*a*b-5*B*\sin(d*x+c)*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{1/2}*(1/(1+\cos(d*x+c)))^{1/2}*EllipticE((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (-a+b)/(a-b))^{1/2}*\cos(d*x+c)^3*a*b-2*B*\sin(d*x+c)*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{1/2}*(1/(1+\cos(d*x+c)))^{1/2}*EllipticF((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (-a+b)/(a-b))^{1/2}*\cos(d*x+c)^3*a*b+4*A*\sin(d*x+c)*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{1/2}*(1/(1+\cos(d*x+c)))^{1/2}*EllipticE((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (-a+b)/(a-b))^{1/2}*\cos(d*x+c)^2*a*b-24*A*\sin(d*x+c)*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{1/2}*(1/(1+\cos(d*x+c)))^{1/2}*EllipticPi((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (a+b)/(a-b), I/((a-b)/(a+b))^{1/2})*\cos(d*x+c)^2*a*b-5*B*\sin(d*x+c)*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{1/2}*(1/(1+\cos(d*x+c)))^{1/2}*EllipticE((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (-a+b)/(a-b))^{1/2}*\cos(d*x+c)^2*a*b-2*B*\sin(d*x+c)*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{1/2}*(1/(1+\cos(d*x+c)))^{1/2}*EllipticF((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (-a+b)/(a-b))^{1/2}*\cos(d*x+c)^2*a*b-5*B*\cos(d*x+c)^3*((a-b)/(a+b))^{1/2}*a^2-4*A*\cos(d*x+c)^2*((a-b)/(a+b))^{1/2}*b^2+5*B*\cos(d*x+c)^2*((a-b)/(a+b))^{1/2}*a^2+4*A*\cos(d*x+c)*((a-b)/(a+b))^{1/2}*b^2-2*B*\cos(d*x+c)^2*((a-b)/(a+b))^{1/2}*b^2+2*B*((a-b)/(a+b))^{1/2}*b^2-4*A*\cos(d*x+c)^3*((a-b)/(a+b))^{1/2}*a*b-2*B*\cos(d*x+c)^3*((a-b)/(a+b))^{1/2}*a*b+4*A*\cos(d*x+c)^2*((a-b)/(a+b))^{1/2}*a*b-5*B*\cos(d*x+c)^2*((a-b)/(a+b))^{1/2}*a*b+7*B*\cos(d*x+c)*((a-b)/(a+b))^{1/2}*a*b+8*A*\sin(d*x+c)*\cos(d*x+c)^3*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{1/2}*(1/(1+\cos(d*x+c)))^{1/2}*EllipticF((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (-a+b)/(a-b))^{1/2})*a*b+8*A*\sin(d*x+c)*\cos(d*x+c)^2*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{1/2}*(1/(1+\cos(d*x+c)))^{1/2}*EllipticF((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (-a+b)/(a-b))^{1/2})*a*b+5*B*\sin(d*x+c)*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{1/2}*(1/(1+\cos(d*x+c)))^{1/2}*EllipticE((-1+\cos(d*x+c)$


```

))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)^2*a^2-6*
B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*(1/(1+cos(d*x+c)
))^1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), (a+b)/(a
-b), I/((a-b)/(a+b))^(1/2))*cos(d*x+c)^2*a^2-8*B*sin(d*x+c)*((b+a*cos(d*x+c)
))/(1+cos(d*x+c)))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d
*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), (a+b)/(a-b), I/((a-b)/(a+b))^(1/2))*co
s(d*x+c)^2*b^2-2*B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)
*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin
(d*x+c), -(a+b)/(a-b))^(1/2))*cos(d*x+c)^2*a^2+4*B*sin(d*x+c)*((b+a*cos(d*x
+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos
(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), -(a+b)/(a-b))^(1/2))*cos(d*x+c)^2*
b^2-4*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*(1/(1+cos(
d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), -(
a+b)/(a-b))^(1/2))*cos(d*x+c)^3*b^2+5*B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos
(d*x+c)))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((
a-b)/(a+b))^(1/2)/sin(d*x+c), -(a+b)/(a-b))^(1/2))*cos(d*x+c)^3*a^2-6*B*sin
(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1
/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), (a+b)/(a-b), I
/((a-b)/(a+b))^(1/2))*cos(d*x+c)^3*a^2-8*B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+
cos(d*x+c)))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c)
))*((a-b)/(a+b))^(1/2)/sin(d*x+c), (a+b)/(a-b), I/((a-b)/(a+b))^(1/2))*cos(d*x
+c)^3*b^2-2*B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*(1/(
1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+
c), -(a+b)/(a-b))^(1/2))*cos(d*x+c)^3*a^2+4*B*sin(d*x+c)*((b+a*cos(d*x+c))/
(1+cos(d*x+c)))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+
c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), -(a+b)/(a-b))^(1/2))*cos(d*x+c)^3*b^2-4
*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*(1/(1+cos(d*x+c
)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), -(a+b)/
(a-b))^(1/2))*cos(d*x+c)^2*b^2-8*A*sin(d*x+c)*cos(d*x+c)^3*((b+a*cos(d*x+c)
))/(1+cos(d*x+c)))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*
x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), -(a+b)/(a-b))^(1/2))*a^2-8*A*sin(d*x+
c)*cos(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*(1/(1+cos(d*x
+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), -(a+b)
)/(a-b))^(1/2))*a^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(1/
2)/(b+a*cos(d*x+c))/cos(d*x+c)/sin(d*x+c)/((a-b)/(a+b))^(1/2)

```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(b \sec(dx + c) + a)^{\frac{3}{2}} \sqrt{\sec(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2), x, algor
ithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)*sqrt(sec(d*x + c)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(A + \frac{B}{\cos(c + dx)} \right) \left(a + \frac{b}{\cos(c + dx)} \right)^{3/2} \sqrt{\frac{1}{\cos(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(1/2), x)

[Out] int((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(1/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c))*sec(d*x+c)**(1/2), x)

[Out] Timed out

$$3.443 \quad \int \frac{(a+b \sec(c+dx))^{3/2}(A+B \sec(c+dx))}{\sqrt{\sec(c+dx)}} dx$$

Optimal. Leaf size=272

$$\frac{(2a^2B + 2aAb + b^2B) \sqrt{\sec(c+dx)} \sqrt{\frac{a \cos(c+dx)+b}{a+b}} F\left(\frac{1}{2}(c+dx) \middle| \frac{2a}{a+b}\right)}{d\sqrt{a+b \sec(c+dx)}} + \frac{(2aA - bB) \sqrt{a+b \sec(c+dx)} E\left(\frac{1}{2}(c+dx) \middle| \frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)} \sqrt{\frac{a \cos(c+dx)+b}{a+b}}}$$

[Out] $(2Aa^2b + 2Aab + b^2B) \cdot (\cos(1/2dx + 1/2c))^2)^{1/2} / \cos(1/2dx + 1/2c) \cdot \text{EllipticF}(\sin(1/2dx + 1/2c), 2^{1/2} \cdot (a/(a+b))^{1/2}) \cdot ((b+a \cdot \cos(dx+c))/(a+b))^{1/2} \cdot \sec(dx+c)^{1/2} / d / (a+b \cdot \sec(dx+c))^{1/2} + b \cdot (2A^2b + 3A^2a) \cdot (\cos(1/2dx + 1/2c))^2)^{1/2} / \cos(1/2dx + 1/2c) \cdot \text{EllipticPi}(\sin(1/2dx + 1/2c), 2, 2^{1/2} \cdot (a/(a+b))^{1/2}) \cdot ((b+a \cdot \cos(dx+c))/(a+b))^{1/2} \cdot \sec(dx+c)^{1/2} / d / (a+b \cdot \sec(dx+c))^{1/2} + (2A^2a - bB) \cdot (\cos(1/2dx + 1/2c))^2)^{1/2} / \cos(1/2dx + 1/2c) \cdot \text{EllipticE}(\sin(1/2dx + 1/2c), 2^{1/2} \cdot (a/(a+b))^{1/2}) \cdot (a+b \cdot \sec(dx+c))^{1/2} / d / ((b+a \cdot \cos(dx+c))/(a+b))^{1/2} / \sec(dx+c)^{1/2} + b \cdot B \cdot \sin(dx+c) \cdot \sec(dx+c)^{1/2} \cdot (a+b \cdot \sec(dx+c))^{1/2} / d$

Rubi [A] time = 0.87, antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.343$, Rules used = {4026, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{(2a^2B + 2aAb + b^2B) \sqrt{\sec(c+dx)} \sqrt{\frac{a \cos(c+dx)+b}{a+b}} F\left(\frac{1}{2}(c+dx) \middle| \frac{2a}{a+b}\right)}{d\sqrt{a+b \sec(c+dx)}} + \frac{(2aA - bB) \sqrt{a+b \sec(c+dx)} E\left(\frac{1}{2}(c+dx) \middle| \frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)} \sqrt{\frac{a \cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

[In] Int[((a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]))/Sqrt[Sec[c + d*x]],x]

[Out] $((2A^2a^2b + 2A^2a^2B + b^2B) \cdot \text{Sqrt}[(b + a \cdot \cos(c + dx))/(a + b]) \cdot \text{EllipticF}[(c + dx)/2, (2a)/(a + b)] \cdot \text{Sqrt}[\sec(c + dx)]) / (d \cdot \text{Sqrt}[a + b \cdot \sec(c + dx)]) + (b \cdot (2A^2b + 3A^2a) \cdot \text{Sqrt}[(b + a \cdot \cos(c + dx))/(a + b]) \cdot \text{EllipticPi}[2, (c + dx)/2, (2a)/(a + b)] \cdot \text{Sqrt}[\sec(c + dx)]) / (d \cdot \text{Sqrt}[a + b \cdot \sec(c + dx)]) + ((2A^2a - bB) \cdot \text{EllipticE}[(c + dx)/2, (2a)/(a + b)] \cdot \text{Sqrt}[a + b \cdot \sec(c + dx)]) / (d \cdot \text{Sqrt}[(b + a \cdot \cos(c + dx))/(a + b]) \cdot \text{Sqrt}[\sec(c + dx)]) + (b \cdot B \cdot \text{Sqrt}[\sec(c + dx)] \cdot \text{Sqrt}[a + b \cdot \sec(c + dx)] \cdot \sin(c + dx)) / d$

Rule 2653

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*Sqrt[a + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,

b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2655

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a + b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2661

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)]/(d*Sqrt[a + b])), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2663

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b) + (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2805

Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi/2 + f*x))/2, (2*d)/(c + d)]/(f*(a + b)*Sqrt[c + d])), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[c + d, 0]

Rule 2807

Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !GtQ[c + d, 0]

Rule 3856

Int[Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)]/Sqrt[csc[(e_) + (f_)*(x_)]*(d_)], x_Symbol] := Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]
])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x])],
x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4026

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(m + n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*Simp
[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B)*(m + n) + b^2*B*(m + n - 1))*C
sc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b
^2, 0] && GtQ[m, 1] && !(IGtQ[n, 1] && !IntegerQ[m])
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)
)/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
+ b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + b \sec(c + dx))^{3/2} (A + B \sec(c + dx))}{\sqrt{\sec(c + dx)}} dx &= \frac{bB \sqrt{\sec(c + dx)} \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{d} + \int \frac{\frac{1}{2} a (2aA - bB)}{\sqrt{\sec(c + dx)}} dx \\
&= \frac{bB \sqrt{\sec(c + dx)} \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{d} + \frac{1}{2} (b(2Ab + 3aA) - a(2aA - bB)) \int \frac{1}{\sqrt{\sec(c + dx)}} dx \\
&= \frac{bB \sqrt{\sec(c + dx)} \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{d} + \frac{1}{2} (2aA - bB) \int \frac{1}{\sqrt{\sec(c + dx)}} dx \\
&= \frac{bB \sqrt{\sec(c + dx)} \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{d} - \frac{((-2aAb - 2a^2A + b^2B))}{d} \int \frac{1}{\sqrt{\sec(c + dx)}} dx \\
&= \frac{b(2Ab + 3aA) \sqrt{\frac{b+a \cos(c+dx)}{a+b}} \Pi\left(2; \frac{1}{2}(c + dx) \middle| \frac{2a}{a+b}\right) \sqrt{\sec(c + dx)}}{d \sqrt{a + b \sec(c + dx)}} \\
&= \frac{(2aAb + 2a^2A + b^2B) \sqrt{\frac{b+a \cos(c+dx)}{a+b}} F\left(\frac{1}{2}(c + dx) \middle| \frac{2a}{a+b}\right) \sqrt{\sec(c + dx)}}{d \sqrt{a + b \sec(c + dx)}}
\end{aligned}$$

Mathematica [C] time = 6.71, size = 554, normalized size = 2.04

$$\frac{bB \sin(c + dx) (a + b \sec(c + dx))^{3/2}}{d \sqrt{\sec(c + dx)} (a \cos(c + dx) + b)} + \frac{(a + b \sec(c + dx))^{3/2} \left(\frac{2(2a^2A + 5abB + 4Ab^2) \sqrt{\frac{a \cos(c+dx)+b}{a+b}} \Pi\left(2; \frac{1}{2}(c+dx) \middle| \frac{2a}{a+b}\right)}{\sqrt{a \cos(c+dx)+b}} + \frac{2i(2a^2A - b^2B)}{\sqrt{a \cos(c+dx)+b}} \right)}{d \sqrt{\sec(c + dx)} (a \cos(c + dx) + b)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[((a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]))/Sqrt[Sec[c + d*x]]], x]

[Out] (b*B*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(d*(b + a*Cos[c + d*x])*Sqrt[Sec[c + d*x]]) + ((a + b*Sec[c + d*x])^(3/2)*((2*(8*a*A*b + 4*a^2*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]/Sqrt[b + a*Cos[c + d*x]] + (2*(2*a^2*A + 4*A*b^2 + 5*a*b*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]/Sqrt[b + a*Cos[c + d*x]] + ((2*I)*(2*a^2*A - a*b*B)*Sqrt[(a - a*Cos[c + d*x])/(a + b)]*Sqrt[(a + a*Cos[c + d*x])/(a - b)]*Cos[2*(c + d*x)]*(-2*b*(a + b)*EllipticE[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*(2*

```
b*EllipticF[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*EllipticPi[1 - a/b, I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)])*Sin[c + d*x]/(Sqrt[(a - b)^(-1)]*b*Sqrt[1 - Cos[c + d*x]^2]*Sqrt[(a^2 - a^2*Cos[c + d*x]^2)/a^2]*(-a^2 + 2*b^2 - 4*b*(b + a*Cos[c + d*x]) + 2*(b + a*Cos[c + d*x])^2)))/(4*d*(b + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(b \sec(dx + c) + a)^{\frac{3}{2}}}{\sqrt{\sec(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)/sqrt(sec(d*x + c)), x)
```

maple [C] time = 2.31, size = 2595, normalized size = 9.54

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x)
```

```
[Out] -1/d*(4*A*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), (-a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b-2*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), (-a+b)/(a-b))^(1/2))*cos(d*x+c)^2*a*b-B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), (-a+b)/(a-b))^(1/2))*cos(d*x+c)^2*a*b-2*B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b)
```


$\frac{(a+b)^{1/2} \cdot (1/\cos(dx+c))^{1/2} \cdot \text{EllipticF}((-1+\cos(dx+c)) \cdot ((a-b)/(a+b))^{1/2}/\sin(dx+c), (-a+b)/(a-b)^{1/2}) \cdot \cos(dx+c)^2 \cdot a^2 - 2A \cdot \cos(dx+c) \cdot ((a-b)/(a+b))^{1/2} \cdot a \cdot b - 2A \cdot \sin(dx+c) \cdot \cos(dx+c)^2 \cdot ((b+a \cdot \cos(dx+c))/(1+\cos(dx+c)))/(a+b)^{1/2} \cdot (1/\cos(dx+c))^{1/2} \cdot \text{EllipticF}((-1+\cos(dx+c)) \cdot ((a-b)/(a+b))^{1/2}/\sin(dx+c), (-a+b)/(a-b)^{1/2}) \cdot a^2 \cdot ((b+a \cdot \cos(dx+c))/\cos(dx+c))^{1/2} \cdot (1/\cos(dx+c))^{1/2} \cdot \sin(dx+c) \cdot (b+a \cdot \cos(dx+c)) / ((a-b)/(a+b))^{1/2}}{(a+b)^{1/2} \cdot (1/\cos(dx+c))^{1/2} \cdot \text{EllipticF}((-1+\cos(dx+c)) \cdot ((a-b)/(a+b))^{1/2}/\sin(dx+c), (-a+b)/(a-b)^{1/2}) \cdot \cos(dx+c)^2 \cdot a^2 - 2A \cdot \cos(dx+c) \cdot ((a-b)/(a+b))^{1/2} \cdot a \cdot b - 2A \cdot \sin(dx+c) \cdot \cos(dx+c)^2 \cdot ((b+a \cdot \cos(dx+c))/(1+\cos(dx+c)))/(a+b)^{1/2} \cdot (1/\cos(dx+c))^{1/2} \cdot \text{EllipticF}((-1+\cos(dx+c)) \cdot ((a-b)/(a+b))^{1/2}/\sin(dx+c), (-a+b)/(a-b)^{1/2}) \cdot a^2 \cdot ((b+a \cdot \cos(dx+c))/\cos(dx+c))^{1/2} \cdot (1/\cos(dx+c))^{1/2} \cdot \sin(dx+c) \cdot (b+a \cdot \cos(dx+c)) / ((a-b)/(a+b))^{1/2}}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx+c) + A)(b \sec(dx+c) + a)^{3/2}}{\sqrt{\sec(dx+c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)/sqrt(sec(d*x + c)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(a + \frac{b}{\cos(c+dx)}\right)^{3/2}}{\sqrt{\frac{1}{\cos(c+dx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(3/2))/(1/cos(c + d*x))^(1/2),x)

[Out] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(3/2))/(1/cos(c + d*x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx))(a + b \sec(c + dx))^{3/2}}{\sqrt{\sec(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x)

[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))^(3/2)/sqrt(sec(c + d*x)), x)

$$3.444 \quad \int \frac{(a+b \sec(c+dx))^{3/2}(A+B \sec(c+dx))}{\sec^2(c+dx)} dx$$

Optimal. Leaf size=276

$$\frac{2(a^2A + 3abB - Ab^2) \sqrt{\sec(c+dx)} \sqrt{\frac{a \cos(c+dx)+b}{a+b}} F\left(\frac{1}{2}(c+dx) \middle| \frac{2a}{a+b}\right)}{3d\sqrt{a+b \sec(c+dx)}} + \frac{2(3aB + 4Ab) \sqrt{a+b \sec(c+dx)} E\left(\frac{1}{2}(c+dx) \middle| \frac{2a}{a+b}\right)}{3d\sqrt{\sec(c+dx)} \sqrt{\frac{a \cos(c+dx)+b}{a+b}}}$$

[Out] $\frac{2}{3} * (A * a^2 - A * b^2 + 3 * B * a * b) * (\cos(1/2 * d * x + 1/2 * c))^2 \wedge (1/2) / \cos(1/2 * d * x + 1/2 * c) * E$
 $\text{llipticF}(\sin(1/2 * d * x + 1/2 * c), 2 \wedge (1/2) * (a / (a + b)) \wedge (1/2)) * ((b + a * \cos(d * x + c)) / (a + b)) \wedge (1/2) * \sec(d * x + c) \wedge (1/2) / d / (a + b * \sec(d * x + c)) \wedge (1/2) + 2 * b^2 * B * (\cos(1/2 * d * x + 1/2 * c))^2 \wedge (1/2) / \cos(1/2 * d * x + 1/2 * c) * \text{EllipticPi}(\sin(1/2 * d * x + 1/2 * c), 2, 2 \wedge (1/2) * (a / (a + b)) \wedge (1/2)) * ((b + a * \cos(d * x + c)) / (a + b)) \wedge (1/2) * \sec(d * x + c) \wedge (1/2) / d / (a + b * \sec(d * x + c)) \wedge (1/2) + 2 / 3 * (4 * A * b + 3 * B * a) * (\cos(1/2 * d * x + 1/2 * c))^2 \wedge (1/2) / \cos(1/2 * d * x + 1/2 * c) * \text{EllipticE}(\sin(1/2 * d * x + 1/2 * c), 2 \wedge (1/2) * (a / (a + b)) \wedge (1/2)) * (a + b * \sec(d * x + c)) \wedge (1/2) / d / ((b + a * \cos(d * x + c)) / (a + b)) \wedge (1/2) / \sec(d * x + c) \wedge (1/2)$

Rubi [A] time = 0.92, antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.343$, Rules used = {4025, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2(a^2A + 3abB - Ab^2) \sqrt{\sec(c+dx)} \sqrt{\frac{a \cos(c+dx)+b}{a+b}} F\left(\frac{1}{2}(c+dx) \middle| \frac{2a}{a+b}\right)}{3d\sqrt{a+b \sec(c+dx)}} + \frac{2(3aB + 4Ab) \sqrt{a+b \sec(c+dx)} E\left(\frac{1}{2}(c+dx) \middle| \frac{2a}{a+b}\right)}{3d\sqrt{\sec(c+dx)} \sqrt{\frac{a \cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + b * \text{Sec}[c + d * x]) \wedge (3/2) * (A + B * \text{Sec}[c + d * x])] / \text{Sec}[c + d * x] \wedge (3/2), x]$

[Out] $(2 * (a^2 * A - A * b^2 + 3 * a * b * B) * \text{Sqrt}[(b + a * \text{Cos}[c + d * x]) / (a + b)] * \text{EllipticF}[(c + d * x) / 2, (2 * a) / (a + b)] * \text{Sqrt}[\text{Sec}[c + d * x]]) / (3 * d * \text{Sqrt}[a + b * \text{Sec}[c + d * x]]) + (2 * b^2 * B * \text{Sqrt}[(b + a * \text{Cos}[c + d * x]) / (a + b)] * \text{EllipticPi}[2, (c + d * x) / 2, (2 * a) / (a + b)] * \text{Sqrt}[\text{Sec}[c + d * x]]) / (d * \text{Sqrt}[a + b * \text{Sec}[c + d * x]]) + (2 * (4 * A * b + 3 * a * B) * \text{EllipticE}[(c + d * x) / 2, (2 * a) / (a + b)] * \text{Sqrt}[a + b * \text{Sec}[c + d * x]]) / (3 * d * \text{Sqrt}[(b + a * \text{Cos}[c + d * x]) / (a + b)] * \text{Sqrt}[\text{Sec}[c + d * x]]) + (2 * a * A * \text{Sqrt}[a + b * \text{Sec}[c + d * x]] * \text{Sin}[c + d * x]) / (3 * d * \text{Sqrt}[\text{Sec}[c + d * x]])$

Rule 2653

$\text{Int}[\text{Sqrt}[(a_) + (b_) * \sin[(c_) + (d_) * (x_)]], x_Symbol] \rightarrow \text{Simp}[(2 * \text{Sqrt}[a + b] * \text{EllipticE}[(1 * (c - \text{Pi} / 2 + d * x)) / 2, (2 * b) / (a + b)]) / d, x] /;$ FreeQ[{a,

b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2655

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a + b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2661

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)]/(d*Sqrt[a + b]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2663

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b) + (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2805

Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi/2 + f*x))/2, (2*d)/(c + d)]/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[c + d, 0]

Rule 2807

Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !GtQ[c + d, 0]

Rule 3856

Int[Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)]/Sqrt[csc[(e_) + (f_)*(x_)]*(d_)], x_Symbol] := Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.)
+ (a_.)], x_Symbol] := Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.)
+ (a_.)], x_Symbol] := Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]
])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x])],
x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (
a_.))^(m_)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.))/(Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]), x_Symbol] := Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.)
)/(Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.)
+ (a_.)]), x_Symbol] := Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
+ b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + b \sec(c + dx))^{3/2} (A + B \sec(c + dx))}{\sec^2(c + dx)} dx &= \frac{2aA\sqrt{a + b \sec(c + dx)} \sin(c + dx)}{3d\sqrt{\sec(c + dx)}} - \frac{2}{3} \int \frac{-\frac{1}{2}a(4Ab + 3aB)}{\sqrt{\sec(c + dx)}} dx \\
&= \frac{2aA\sqrt{a + b \sec(c + dx)} \sin(c + dx)}{3d\sqrt{\sec(c + dx)}} - \frac{2}{3} \int \frac{-\frac{1}{2}a(4Ab + 3aB)}{\sqrt{\sec(c + dx)}} dx \\
&= \frac{2aA\sqrt{a + b \sec(c + dx)} \sin(c + dx)}{3d\sqrt{\sec(c + dx)}} - \frac{1}{3}(-4Ab - 3aB) \int \frac{\sqrt{a + b \sec(c + dx)}}{\sec(c + dx)} dx \\
&= \frac{2aA\sqrt{a + b \sec(c + dx)} \sin(c + dx)}{3d\sqrt{\sec(c + dx)}} - \frac{((-a^2A + Ab^2 - 3abB))}{3d\sqrt{\sec(c + dx)}} \\
&= \frac{2b^2B\sqrt{\frac{b+a \cos(c+dx)}{a+b}} \Pi\left(2; \frac{1}{2}(c + dx) \middle| \frac{2a}{a+b}\right) \sqrt{\sec(c + dx)}}{d\sqrt{a + b \sec(c + dx)}} + \frac{2aA\sqrt{a + b \sec(c + dx)} \sin(c + dx)}{3d\sqrt{\sec(c + dx)}} \\
&= \frac{2(a^2A - Ab^2 + 3abB)\sqrt{\frac{b+a \cos(c+dx)}{a+b}} F\left(\frac{1}{2}(c + dx) \middle| \frac{2a}{a+b}\right) \sqrt{\sec(c + dx)}}{3d\sqrt{a + b \sec(c + dx)}}
\end{aligned}$$

Mathematica [C] time = 4.53, size = 437, normalized size = 1.58

$$(a + b \sec(c + dx))^{3/2} \left(\frac{4(a^2A + 6abB + 3Ab^2)\sqrt{\frac{a \cos(c+dx)+b}{a+b}} F\left(\frac{1}{2}(c+dx) \middle| \frac{2a}{a+b}\right)}{(a \cos(c+dx)+b)^2} + \frac{2(3a^2B + 4aAb + 6b^2B)\sqrt{\frac{a \cos(c+dx)+b}{a+b}} \Pi\left(2; \frac{1}{2}(c+dx) \middle| \frac{2a}{a+b}\right)}{(a \cos(c+dx)+b)^2} + \dots \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[((a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(3/2), x]

[Out] ((a + b*Sec[c + d*x])^(3/2))*((4*(a^2*A + 3*A*b^2 + 6*a*b*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]/(b + a*Cos[c + d*x])^2 + (2*(4*a*A*b + 3*a^2*B + 6*b^2*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]/(b + a*Cos[c + d*x])^2 + ((2*I)*(4*A*b + 3*a*B)*Sqrt[-((a*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(a*(1 + Cos[c + d*x]))/(a - b)]*Csc[c + d*x]*(-2*b*(a + b)*EllipticE[I*ArcSinh[Sqrt[(a - b)^(-1)]]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*(2*b*EllipticF

```
[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)]
+ a*EllipticPi[1 - a/b, I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x
]]], (-a + b)/(a + b)])))/(a*Sqrt[(a - b)^(-1)]*b*(b + a*Cos[c + d*x])^(3/2
)) + (4*a*A*Sin[c + d*x])/(b + a*Cos[c + d*x])))/(6*d*Sec[c + d*x]^(3/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algor
ithm="fricas")
```

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(b \sec(dx + c) + a)^{\frac{3}{2}}}{\sec(dx + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(3/2
), x)
```

maple [C] time = 2.22, size = 2552, normalized size = 9.25

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x)
```

```
[Out] -2/3/d*(A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), (-a+b)/
(a-b))^(1/2))*a^2*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b)^(1/2)*(1/(1+cos(d
*x+c)))^(1/2)*sin(d*x+c)-4*A*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c)
))*((a-b)/(a+b))^(1/2)/sin(d*x+c), (-a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1
+cos(d*x+c)))/(a+b)^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b-A*((a-b)/(a+b))^(1/2
)*a*b-3*B*((a-b)/(a+b))^(1/2)*a*b+3*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b
))^(1/2)/sin(d*x+c), (-a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/
(a+b)^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)*b^2-3*B*((b+a*cos
os(d*x+c))/(1+cos(d*x+c)))/(a+b)^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((
```

$$\begin{aligned}
& -1+\cos(dx+c)) * ((a-b)/(a+b))^{1/2} / \sin(dx+c), (- (a+b)/(a-b))^{1/2}) * b^2 * \sin \\
& (dx+c) - 4 * A * ((b+a * \cos(dx+c)) / (1+\cos(dx+c))) / (a+b)^{1/2} * (1 / (1+\cos(dx+c))) \\
&)^{1/2} * \text{EllipticE}((-1+\cos(dx+c)) * ((a-b)/(a+b))^{1/2} / \sin(dx+c), (- (a+b)/(a \\
& -b))^{1/2}) * b^2 * \sin(dx+c) + 4 * A * \cos(dx+c) * \sin(dx+c) * ((b+a * \cos(dx+c)) / (1+c \\
& os(dx+c))) / (a+b)^{1/2} * (1 / (1+\cos(dx+c)))^{1/2} * \text{EllipticE}((-1+\cos(dx+c)) * \\
& ((a-b)/(a+b))^{1/2} / \sin(dx+c), (- (a+b)/(a-b))^{1/2}) * a * b + 6 * B * \cos(dx+c) * \sin \\
& (dx+c) * \text{EllipticF}((-1+\cos(dx+c)) * ((a-b)/(a+b))^{1/2} / \sin(dx+c), (- (a+b)/(a \\
& -b))^{1/2}) * ((b+a * \cos(dx+c)) / (1+\cos(dx+c))) / (a+b)^{1/2} * (1 / (1+\cos(dx+c))) \\
&)^{1/2} * a * b - 3 * B * \cos(dx+c) * \sin(dx+c) * ((b+a * \cos(dx+c)) / (1+\cos(dx+c))) / (a+b \\
&))^{1/2} * (1 / (1+\cos(dx+c)))^{1/2} * \text{EllipticE}((-1+\cos(dx+c)) * ((a-b)/(a+b))^{1/2} / \\
& 1/2) / \sin(dx+c), (- (a+b)/(a-b))^{1/2}) * a * b - 4 * A * ((a-b)/(a+b))^{1/2} * b^2 + 6 * B * (\\
& (b+a * \cos(dx+c)) / (1+\cos(dx+c))) / (a+b)^{1/2} * (1 / (1+\cos(dx+c)))^{1/2} * \text{Ellip \\
& ticPi}((-1+\cos(dx+c)) * ((a-b)/(a+b))^{1/2} / \sin(dx+c), (a+b)/(a-b), I / ((a-b)/(\\
& a+b))^{1/2}) * b^2 * \sin(dx+c) + 3 * A * \text{EllipticF}((-1+\cos(dx+c)) * ((a-b)/(a+b))^{1/2} / \\
& 2) / \sin(dx+c), (- (a+b)/(a-b))^{1/2}) * b^2 * ((b+a * \cos(dx+c)) / (1+\cos(dx+c))) / (a \\
& +b))^{1/2} * (1 / (1+\cos(dx+c)))^{1/2} * \sin(dx+c) + 3 * B * \cos(dx+c)^2 * ((a-b)/(a+b \\
&))^{1/2} * a^2 + 4 * A * \cos(dx+c) * ((a-b)/(a+b))^{1/2} * b^2 + A * \cos(dx+c) * \sin(dx+c) \\
& * \text{EllipticF}((-1+\cos(dx+c)) * ((a-b)/(a+b))^{1/2} / \sin(dx+c), (- (a+b)/(a-b))^{1/2}) \\
& / 2) * ((b+a * \cos(dx+c)) / (1+\cos(dx+c))) / (a+b)^{1/2} * (1 / (1+\cos(dx+c)))^{1/2} \\
& * a^2 - 4 * A * \cos(dx+c) * \sin(dx+c) * ((b+a * \cos(dx+c)) / (1+\cos(dx+c))) / (a+b)^{1/2} \\
&) * (1 / (1+\cos(dx+c)))^{1/2} * \text{EllipticE}((-1+\cos(dx+c)) * ((a-b)/(a+b))^{1/2} / \sin \\
& (dx+c), (- (a+b)/(a-b))^{1/2}) * b^2 - 3 * B * \cos(dx+c) * \sin(dx+c) * \text{EllipticF}((-1+ \\
& cos(dx+c)) * ((a-b)/(a+b))^{1/2} / \sin(dx+c), (- (a+b)/(a-b))^{1/2}) * ((b+a * \cos(\\
& dx+c)) / (1+\cos(dx+c))) / (a+b)^{1/2} * (1 / (1+\cos(dx+c)))^{1/2} * a^2 + 3 * B * \cos(dx \\
& x+c) * \sin(dx+c) * ((b+a * \cos(dx+c)) / (1+\cos(dx+c))) / (a+b)^{1/2} * (1 / (1+\cos(dx \\
& +c)))^{1/2} * \text{EllipticE}((-1+\cos(dx+c)) * ((a-b)/(a+b))^{1/2} / \sin(dx+c), (- (a+b \\
&) / (a-b))^{1/2}) * a^2 - 4 * A * \text{EllipticF}((-1+\cos(dx+c)) * ((a-b)/(a+b))^{1/2} / \sin(d \\
& *x+c), (- (a+b)/(a-b))^{1/2}) * a * b * ((b+a * \cos(dx+c)) / (1+\cos(dx+c))) / (a+b)^{1/2} \\
& / 2) * (1 / (1+\cos(dx+c)))^{1/2} * \sin(dx+c) + 4 * A * ((b+a * \cos(dx+c)) / (1+\cos(dx+c))) \\
& / (a+b)^{1/2} * (1 / (1+\cos(dx+c)))^{1/2} * \text{EllipticE}((-1+\cos(dx+c)) * ((a-b)/(a+ \\
& b))^{1/2} / \sin(dx+c), (- (a+b)/(a-b))^{1/2}) * a * b * \sin(dx+c) + 6 * B * ((b+a * \cos(dx \\
& +c)) / (1+\cos(dx+c))) / (a+b)^{1/2} * (1 / (1+\cos(dx+c)))^{1/2} * \text{EllipticF}((-1+\cos \\
& (dx+c)) * ((a-b)/(a+b))^{1/2} / \sin(dx+c), (- (a+b)/(a-b))^{1/2}) * a * b * \sin(dx+c \\
&) - 3 * B * ((b+a * \cos(dx+c)) / (1+\cos(dx+c))) / (a+b)^{1/2} * (1 / (1+\cos(dx+c)))^{1/2} \\
&) * \text{EllipticE}((-1+\cos(dx+c)) * ((a-b)/(a+b))^{1/2} / \sin(dx+c), (- (a+b)/(a-b))^{1/2}) \\
&) * a * b * \sin(dx+c) + 5 * A * \cos(dx+c)^2 * ((a-b)/(a+b))^{1/2} * a * b + 3 * B * \cos(dx+c \\
&) * ((a-b)/(a+b))^{1/2} * a * b - 3 * B * \cos(dx+c) * ((a-b)/(a+b))^{1/2} * a^2 + A * \cos(dx+x \\
& c)^3 * ((a-b)/(a+b))^{1/2} * a^2 - A * a^2 * ((a-b)/(a+b))^{1/2} * \cos(dx+c) - 4 * A * \cos(d \\
& *x+c) * ((a-b)/(a+b))^{1/2} * a * b - 3 * B * ((b+a * \cos(dx+c)) / (1+\cos(dx+c))) / (a+b)^{1/2} \\
& / 2) * (1 / (1+\cos(dx+c)))^{1/2} * \text{EllipticF}((-1+\cos(dx+c)) * ((a-b)/(a+b))^{1/2} / \\
& \sin(dx+c), (- (a+b)/(a-b))^{1/2}) * a^2 * \sin(dx+c) + 3 * B * ((b+a * \cos(dx+c)) / (1+c \\
& os(dx+c))) / (a+b)^{1/2} * (1 / (1+\cos(dx+c)))^{1/2} * \text{EllipticE}((-1+\cos(dx+c)) * \\
& ((a-b)/(a+b))^{1/2} / \sin(dx+c), (- (a+b)/(a-b))^{1/2}) * a^2 * \sin(dx+c) - 3 * B * \cos \\
& (dx+c) * \text{EllipticF}((-1+\cos(dx+c)) * ((a-b)/(a+b))^{1/2} / \sin(dx+c), (- (a+b)/(a \\
& -b))^{1/2}) * ((b+a * \cos(dx+c)) / (1+\cos(dx+c))) / (a+b)^{1/2} * (1 / (1+\cos(dx+c)))
\end{aligned}$$

$$\left)^{(1/2)} \cdot \sin(dx+c) \cdot b^2 + 6B \cos(dx+c) \cdot \left(\frac{b+a \cos(dx+c)}{1+\cos(dx+c)}\right) / (a+b) \right)^{(1/2)} \cdot \left(\frac{1}{1+\cos(dx+c)}\right)^{(1/2)} \cdot \text{EllipticPi}\left(\frac{-1+\cos(dx+c)}{2}, \frac{(a-b)/(a+b)}{\sin(dx+c)}, \frac{(a+b)/(a-b)}{I \cdot \left(\frac{a-b}{a+b}\right)^{(1/2)}}, \sin(dx+c) \cdot b^2 \cdot \left(\frac{b+a \cos(dx+c)}{\cos(dx+c)}\right)^{(1/2)} \cdot \cos(dx+c)^2 \cdot \left(\frac{1}{\cos(dx+c)}\right)^{(3/2)} / \sin(dx+c) / \left(\frac{b+a \cos(dx+c)}{(a-b)/(a+b)}\right)^{(1/2)}\right)$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx+c) + A)(b \sec(dx+c) + a)^{3/2}}{\sec(dx+c)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(dx+c))^(3/2)*(A+B*sec(dx+c))/sec(dx+c)^(3/2),x, algorithm="maxima")

[Out] integrate((B*sec(dx+c) + A)*(b*sec(dx+c) + a)^(3/2)/sec(dx+c)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(a + \frac{b}{\cos(c+dx)}\right)^{3/2}}{\left(\frac{1}{\cos(c+dx)}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + dx))*(a + b/cos(c + dx))^(3/2))/(1/cos(c + dx))^(3/2),x)

[Out] int(((A + B/cos(c + dx))*(a + b/cos(c + dx))^(3/2))/(1/cos(c + dx))^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx))(a + b \sec(c + dx))^{3/2}}{\sec^2(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(dx+c))**(3/2)*(A+B*sec(dx+c))/sec(dx+c)**(3/2),x)

[Out] Integral((A + B*sec(c + dx))*(a + b*sec(c + dx))**(3/2)/sec(c + dx)**(3/2), x)

$$3.445 \quad \int \frac{(a+b \sec(c+dx))^{3/2}(A+B \sec(c+dx))}{5 \sec^2(c+dx)} dx$$

Optimal. Leaf size=266

$$\frac{2(a^2 - b^2)(5aB + 3Ab)\sqrt{\sec(c+dx)}\sqrt{\frac{a \cos(c+dx)+b}{a+b}} F\left(\frac{1}{2}(c+dx) \middle| \frac{2a}{a+b}\right)}{15ad\sqrt{a+b \sec(c+dx)}} + \frac{2(9a^2A + 20abB + 3Ab^2)\sqrt{a+b \sec(c+dx)}}{15ad\sqrt{\sec(c+dx)}\sqrt{\frac{a \cos(c+dx)+b}{a+b}}}$$

[Out] $2/15*(a^2-b^2)*(3*A*b+5*B*a)*(cos(1/2*d*x+1/2*c)^2)^{(1/2)}/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c), 2^{(1/2)}*(a/(a+b))^{(1/2)}*((b+a*cos(d*x+c))/(a+b))^{(1/2)}*sec(d*x+c)^{(1/2)}/a/d/(a+b*sec(d*x+c))^{(1/2)}+2/5*a*A*sin(d*x+c)*(a+b*sec(d*x+c))^{(1/2)}/d/sec(d*x+c)^{(3/2)}+2/15*(6*A*b+5*B*a)*sin(d*x+c)*(a+b*sec(d*x+c))^{(1/2)}/d/sec(d*x+c)^{(1/2)}+2/15*(9*A*a^2+3*A*b^2+20*B*a*b)*(cos(1/2*d*x+1/2*c)^2)^{(1/2)}/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c), 2^{(1/2)}*(a/(a+b))^{(1/2)}*(a+b*sec(d*x+c))^{(1/2)}/a/d/((b+a*cos(d*x+c))/(a+b))^{(1/2)}/sec(d*x+c)^{(1/2)})$

Rubi [A] time = 0.79, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {4025, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2(a^2 - b^2)(5aB + 3Ab)\sqrt{\sec(c+dx)}\sqrt{\frac{a \cos(c+dx)+b}{a+b}} F\left(\frac{1}{2}(c+dx) \middle| \frac{2a}{a+b}\right)}{15ad\sqrt{a+b \sec(c+dx)}} + \frac{2(9a^2A + 20abB + 3Ab^2)\sqrt{a+b \sec(c+dx)}}{15ad\sqrt{\sec(c+dx)}\sqrt{\frac{a \cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

[In] Int[((a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(5/2), x]

[Out] $(2*(a^2 - b^2)*(3*A*b + 5*a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(15*a*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(9*a^2*A + 3*A*b^2 + 20*a*b*B)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(15*a*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*a*A*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (2*(6*A*b + 5*a*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[Sec[c + d*x]])$

Rule 2653

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*Sqrt[a + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2655

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
0] && !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)]/(d*Sqrt[a + b]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a
+ b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && !GtQ[a + b, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)]/Sqrt[csc[(e_) + (f_)*(x_)]
*(d_)], x_Symbol] := Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_) + (f_)*(x_)]*(d_)]/Sqrt[csc[(e_) + (f_)*(x_)]*(b_)
+ (a_)], x_Symbol] := Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4025

```
Int[(csc[(e_) + (f_)*(x_)]*(d_))^(n_)*(csc[(e_) + (f_)*(x_)]*(b_) + (
a_))^(m_)*(csc[(e_) + (f_)*(x_)]*(B_) + (A_)), x_Symbol] := Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.))/(Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]), x_Symbol] := Dist[A/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m, x_Symbol] := Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\begin{aligned}
 \int \frac{(a + b \sec(c + dx))^{3/2} (A + B \sec(c + dx))}{\sec^2(c + dx)} dx &= \frac{2aA\sqrt{a + b \sec(c + dx)} \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} - \frac{2}{5} \int \frac{-\frac{1}{2}a(6Ab + 5aB)}{\sec^2(c + dx)} dx \\
 &= \frac{2aA\sqrt{a + b \sec(c + dx)} \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} + \frac{2(6Ab + 5aB)\sqrt{a + b \sec(c + dx)}}{15d \sqrt{\sec(c + dx)}} \\
 &= \frac{2aA\sqrt{a + b \sec(c + dx)} \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} + \frac{2(6Ab + 5aB)\sqrt{a + b \sec(c + dx)}}{15d \sqrt{\sec(c + dx)}} \\
 &= \frac{2aA\sqrt{a + b \sec(c + dx)} \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} + \frac{2(6Ab + 5aB)\sqrt{a + b \sec(c + dx)}}{15d \sqrt{\sec(c + dx)}} \\
 &= \frac{2aA\sqrt{a + b \sec(c + dx)} \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} + \frac{2(6Ab + 5aB)\sqrt{a + b \sec(c + dx)}}{15d \sqrt{\sec(c + dx)}} \\
 &= \frac{2(a^2 - b^2)(3Ab + 5aB)\sqrt{\frac{b+a \cos(c+dx)}{a+b}} F\left(\frac{1}{2}(c + dx) \middle| \frac{2a}{a+b}\right) \sqrt{\sec(c + dx)}}{15ad\sqrt{a + b \sec(c + dx)}}
 \end{aligned}$$

Mathematica [A] time = 1.84, size = 201, normalized size = 0.76

$$\frac{2(a + b \sec(c + dx))^{3/2} \left((a^2 - b^2) (5aB + 3Ab) \sqrt{\frac{a \cos(c+dx)+b}{a+b}} F\left(\frac{1}{2}(c + dx) \middle| \frac{2a}{a+b}\right) + (a + b) (9a^2A + 20abB + 3Ab^2) \right)}{15ad \sec^{\frac{3}{2}}(c + dx)(a \cos(c$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(5/2), x]

[Out] (2*(a + b*Sec[c + d*x])^(3/2)*((a + b)*(9*a^2*A + 3*A*b^2 + 20*a*b*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*a)/(a + b)] + (a^2 - b^2)*(3*A*b + 5*a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)] + a*(b + a*Cos[c + d*x])*(6*A*b + 5*a*B + 3*a*A*Cos[c + d*x])*Sin[c + d*x))/(15*a*d*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^(3/2))

fricas [F] time = 1.19, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{(Bb \sec(dx + c)^2 + Aa + (Ba + Ab) \sec(dx + c)) \sqrt{b \sec(dx + c) + a}}{\sec(dx + c)^{\frac{5}{2}}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2), x, algorithm="fricas")

[Out] integral((B*b*sec(d*x + c)^2 + A*a + (B*a + A*b)*sec(d*x + c))*sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(5/2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(b \sec(dx + c) + a)^{\frac{3}{2}}}{\sec(dx + c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2), x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(5/2), x)

maple [B] time = 2.19, size = 2915, normalized size = 10.96

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((a+b*\sec(d*x+c))^{3/2}*(A+B*\sec(d*x+c))/\sec(d*x+c)^{5/2},x)$

[Out] $-2/15/d*(9*A*\text{EllipticE}((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c),(-(a+b)/(a-b))^{1/2})*a^3*(1/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*\sin(d*x+c)+12*A*\sin(d*x+c)*(1/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*\text{EllipticF}((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c),(-(a+b)/(a-b))^{1/2})*\cos(d*x+c)*a^2*b+15*B*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*(1/(1+\cos(d*x+c)))^{1/2}*\text{EllipticF}((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c),(-(a+b)/(a-b))^{1/2})*a*b^2*\sin(d*x+c)-3*A*\sin(d*x+c)*(1/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*\text{EllipticF}((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c),(-(a+b)/(a-b))^{1/2})*\cos(d*x+c)*a*b^2+9*A*((a-b)/(a+b))^{1/2}*\cos(d*x+c)^3*a^2*b+9*A*((a-b)/(a+b))^{1/2}*\cos(d*x+c)^2*a*b^2+25*B*((a-b)/(a+b))^{1/2}*\cos(d*x+c)^2*a^2*b-3*A*((a-b)/(a+b))^{1/2}*\cos(d*x+c)*a*b^2-20*B*((a-b)/(a+b))^{1/2}*\cos(d*x+c)*a^2*b-3*A*\text{EllipticE}((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c),(-(a+b)/(a-b))^{1/2})*b^3*(1/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*\sin(d*x+c)+3*A*((a-b)/(a+b))^{1/2}*\cos(d*x+c)^4*a^3+5*B*((a-b)/(a+b))^{1/2}*\cos(d*x+c)^3*a^3-5*B*a^3*((a-b)/(a+b))^{1/2}*\cos(d*x+c)+6*A*((a-b)/(a+b))^{1/2}*\cos(d*x+c)^2*a^3-9*A*((a-b)/(a+b))^{1/2}*\cos(d*x+c)*a^3+3*A*((a-b)/(a+b))^{1/2}*\cos(d*x+c)*b^3+15*B*(1/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*\text{EllipticF}((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c),(-(a+b)/(a-b))^{1/2})*\cos(d*x+c)*\sin(d*x+c)*a*b^2-9*A*\sin(d*x+c)*\text{EllipticE}((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c),(-(a+b)/(a-b))^{1/2})*(1/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*\cos(d*x+c)*a^2*b+3*A*\sin(d*x+c)*\text{EllipticE}((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c),(-(a+b)/(a-b))^{1/2})*(1/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*\cos(d*x+c)*a*b^2-20*B*\sin(d*x+c)*(1/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*\text{EllipticF}((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c),(-(a+b)/(a-b))^{1/2})*\cos(d*x+c)*a^2*b+20*B*\sin(d*x+c)*\text{EllipticE}((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c),(-(a+b)/(a-b))^{1/2})*(1/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*\cos(d*x+c)*a^2*b-20*B*\sin(d*x+c)*\text{EllipticE}((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c),(-(a+b)/(a-b))^{1/2})*(1/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*\cos(d*x+c)*a*b^2+5*B*\text{EllipticF}((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c),(-(a+b)/(a-b))^{1/2})*a^3*(1/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*\sin(d*x+c)-9*A*\text{EllipticF}((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c),(-(a+b)/(a-b))^{1/2})*a^3*(1/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*\sin(d*x+c)-3*A*b^3*((a-b)/(a+b))^{1/2}-9*A*a^2*b*((a-b)/(a+b))^{1/2}-6*A*a*b^2*((a-b)/(a+b))^{1/2}-5*B*a^2*b*((a-b)/(a+b))^{1/2}-20*B*a*b^2*((a-b)/(a+b))^{1/2}+20*B*((a-b)/(a+b))^{1/2}*\cos(d*x+c)*a*b^2-9*A*\sin(d*x+c)*(1/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*\text{EllipticF}((-1+\cos(d*x+c))*(($

$(a-b)/(a+b)^{(1/2)}/\sin(dx+c), (-a+b)/(a-b)^{(1/2)}*\cos(dx+c)*a^3+9*A*\sin(dx+c)*\text{EllipticE}((-1+\cos(dx+c))*((a-b)/(a+b))^{(1/2)}/\sin(dx+c), (-a+b)/(a-b))^{(1/2)}*(1/(1+\cos(dx+c)))^{(1/2)}*((b+a*\cos(dx+c))/(1+\cos(dx+c)))/(a+b))^{(1/2)}*\cos(dx+c)*a^3-3*A*\sin(dx+c)*\text{EllipticE}((-1+\cos(dx+c))*((a-b)/(a+b))^{(1/2)}/\sin(dx+c), (-a+b)/(a-b))^{(1/2)}*(1/(1+\cos(dx+c)))^{(1/2)}*((b+a*\cos(dx+c))/(1+\cos(dx+c)))/(a+b))^{(1/2)}*\cos(dx+c)*b^3+5*B*\sin(dx+c)*(1/(1+\cos(dx+c)))^{(1/2)}*((b+a*\cos(dx+c))/(1+\cos(dx+c)))/(a+b))^{(1/2)}*\text{EllipticF}((-1+\cos(dx+c))*((a-b)/(a+b))^{(1/2)}/\sin(dx+c), (-a+b)/(a-b))^{(1/2)}*\cos(dx+c)*a^3+12*A*\text{EllipticF}((-1+\cos(dx+c))*((a-b)/(a+b))^{(1/2)}/\sin(dx+c), (-a+b)/(a-b))^{(1/2)}*a^2*b*(1/(1+\cos(dx+c)))^{(1/2)}*((b+a*\cos(dx+c))/(1+\cos(dx+c)))/(a+b))^{(1/2)}*\sin(dx+c)-3*A*\text{EllipticF}((-1+\cos(dx+c))*((a-b)/(a+b))^{(1/2)}/\sin(dx+c), (-a+b)/(a-b))^{(1/2)}*a*b^2*(1/(1+\cos(dx+c)))^{(1/2)}*((b+a*\cos(dx+c))/(1+\cos(dx+c)))/(a+b))^{(1/2)}*\sin(dx+c)-9*A*\text{EllipticE}((-1+\cos(dx+c))*((a-b)/(a+b))^{(1/2)}/\sin(dx+c), (-a+b)/(a-b))^{(1/2)}*a^2*b*(1/(1+\cos(dx+c)))^{(1/2)}*((b+a*\cos(dx+c))/(1+\cos(dx+c)))/(a+b))^{(1/2)}*\sin(dx+c)+3*A*\text{EllipticE}((-1+\cos(dx+c))*((a-b)/(a+b))^{(1/2)}/\sin(dx+c), (-a+b)/(a-b))^{(1/2)}*a*b^2*(1/(1+\cos(dx+c)))^{(1/2)}*((b+a*\cos(dx+c))/(1+\cos(dx+c)))/(a+b))^{(1/2)}*\sin(dx+c)-20*B*\text{EllipticF}((-1+\cos(dx+c))*((a-b)/(a+b))^{(1/2)}/\sin(dx+c), (-a+b)/(a-b))^{(1/2)}*a^2*b*(1/(1+\cos(dx+c)))^{(1/2)}*((b+a*\cos(dx+c))/(1+\cos(dx+c)))/(a+b))^{(1/2)}*\sin(dx+c)+20*B*\text{EllipticE}((-1+\cos(dx+c))*((a-b)/(a+b))^{(1/2)}/\sin(dx+c), (-a+b)/(a-b))^{(1/2)}*a^2*b*(1/(1+\cos(dx+c)))^{(1/2)}*((b+a*\cos(dx+c))/(1+\cos(dx+c)))/(a+b))^{(1/2)}*\sin(dx+c)-20*B*\text{EllipticE}((-1+\cos(dx+c))*((a-b)/(a+b))^{(1/2)}/\sin(dx+c), (-a+b)/(a-b))^{(1/2)}*a*b^2*(1/(1+\cos(dx+c)))^{(1/2)}*((b+a*\cos(dx+c))/(1+\cos(dx+c)))/(a+b))^{(1/2)}*\sin(dx+c)*((b+a*\cos(dx+c))/\cos(dx+c))^{(1/2)}*\cos(dx+c)^3*(1/\cos(dx+c))^{(5/2)}/\sin(dx+c)/(b+a*\cos(dx+c))/a/((a-b)/(a+b))^{(1/2)}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx+c) + A)(b \sec(dx+c) + a)^{3/2}}{\sec(dx+c)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(dx+c))^(3/2)*(A+B*sec(dx+c))/sec(dx+c)^(5/2), x, algorithm="maxima")

[Out] integrate((B*sec(dx+c) + A)*(b*sec(dx+c) + a)^(3/2)/sec(dx+c)^(5/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(a + \frac{b}{\cos(c+dx)}\right)^{3/2}}{\left(\frac{1}{\cos(c+dx)}\right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(3/2))/(1/cos(c + d*x))^(5/2),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(3/2))/(1/cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

$$3.446 \quad \int \frac{(a+b \sec(c+dx))^{3/2}(A+B \sec(c+dx))}{\sec^2(c+dx)} dx$$

Optimal. Leaf size=342

$$\frac{2(25a^2A + 42abB + 3Ab^2) \sin(c+dx) \sqrt{a+b \sec(c+dx)}}{105ad \sqrt{\sec(c+dx)}} + \frac{2(a^2 - b^2) (25a^2A + 21abB - 6Ab^2) \sqrt{\sec(c+dx)}}{105a^2d \sqrt{a+b \sec(c+dx)}}$$

[Out] 2/105*(a^2-b^2)*(25*A*a^2-6*A*b^2+21*B*a*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/a^2/d/(a+b*sec(d*x+c))^(1/2)+2/7*a*A*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d/sec(d*x+c)^(5/2)+2/35*(8*A*b+7*B*a)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d/sec(d*x+c)^(3/2)+2/105*(25*A*a^2+3*A*b^2+42*B*a*b)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/a/d/sec(d*x+c)^(1/2)+2/105*(82*A*a^2*b-6*A*b^3+63*B*a^3+21*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/a^2/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)

Rubi [A] time = 1.13, antiderivative size = 342, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {4025, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2(25a^2A + 42abB + 3Ab^2) \sin(c+dx) \sqrt{a+b \sec(c+dx)}}{105ad \sqrt{\sec(c+dx)}} + \frac{2(a^2 - b^2) (25a^2A + 21abB - 6Ab^2) \sqrt{\sec(c+dx)}}{105a^2d \sqrt{a+b \sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[((a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(7/2),x]

[Out] (2*(a^2 - b^2)*(25*a^2*A - 6*A*b^2 + 21*a*b*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]]/(105*a^2*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(82*a^2*A*b - 6*A*b^3 + 63*a^3*B + 21*a*b^2*B)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]]/(105*a^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*a*A*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (2*(8*A*b + 7*A*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(35*d*Sec[c + d*x]^(3/2)) + (2*(25*a^2*A + 3*A*b^2 + 42*a*b*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(105*a*d*Sqrt[Sec[c + d*x]])

Rule 2653

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*Sqrt[a + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,

b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2655

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a + b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2661

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)]/(d*Sqrt[a + b]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2663

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b) + (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 3856

Int[Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)]/Sqrt[csc[(e_) + (f_)*(x_)]*(d_)], x_Symbol] := Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 3858

Int[Sqrt[csc[(e_) + (f_)*(x_)]*(d_)]/Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)], x_Symbol] := Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 4025

Int[(csc[(e_) + (f_)*(x_)]*(d_))^(n_)*(csc[(e_) + (f_)*(x_)]*(b_) + (a_))^(m_)*(csc[(e_) + (f_)*(x_)]*(B_) + (A_)), x_Symbol] := Simp[(a*A*Coth[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e + f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&

LeQ[n, -1]

Rule 4035

Int[(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.))/(Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]), x_Symbol] := Dist[A/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]

Rule 4104

Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m, x_Symbol] := Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]

Rubi steps

$$\begin{aligned}
\int \frac{(a + b \sec(c + dx))^{3/2} (A + B \sec(c + dx))}{\sec^2(c + dx)} dx &= \frac{2aA\sqrt{a + b \sec(c + dx)} \sin(c + dx)}{7d \sec^{5/2}(c + dx)} - \frac{2}{7} \int \frac{-\frac{1}{2}a(8Ab + 7aB)}{\sec^2(c + dx)} dx \\
&= \frac{2aA\sqrt{a + b \sec(c + dx)} \sin(c + dx)}{7d \sec^{5/2}(c + dx)} + \frac{2(8Ab + 7aB)\sqrt{a + b \sec(c + dx)}}{35d \sec^{3/2}(c + dx)} \\
&= \frac{2aA\sqrt{a + b \sec(c + dx)} \sin(c + dx)}{7d \sec^{5/2}(c + dx)} + \frac{2(8Ab + 7aB)\sqrt{a + b \sec(c + dx)}}{35d \sec^{3/2}(c + dx)} \\
&= \frac{2aA\sqrt{a + b \sec(c + dx)} \sin(c + dx)}{7d \sec^{5/2}(c + dx)} + \frac{2(8Ab + 7aB)\sqrt{a + b \sec(c + dx)}}{35d \sec^{3/2}(c + dx)} \\
&= \frac{2aA\sqrt{a + b \sec(c + dx)} \sin(c + dx)}{7d \sec^{5/2}(c + dx)} + \frac{2(8Ab + 7aB)\sqrt{a + b \sec(c + dx)}}{35d \sec^{3/2}(c + dx)} \\
&= \frac{2aA\sqrt{a + b \sec(c + dx)} \sin(c + dx)}{7d \sec^{5/2}(c + dx)} + \frac{2(8Ab + 7aB)\sqrt{a + b \sec(c + dx)}}{35d \sec^{3/2}(c + dx)} \\
&= \frac{2(a^2 - b^2)(25a^2A - 6Ab^2 + 21abB) \sqrt{\frac{b+a \cos(c+dx)}{a+b}} F\left(\frac{1}{2}(c + dx)\right)}{105a^2d\sqrt{a + b \sec(c + dx)}}
\end{aligned}$$

Mathematica [A] time = 2.01, size = 255, normalized size = 0.75

$$(a + b \sec(c + dx))^{3/2} \left(a(a \cos(c + dx) + b) \left((115a^2A + 168abB + 12Ab^2) \sin(c + dx) + 3a(2(7aB + 8Ab) \sin(2(c + dx))) \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(((a + b*Sec[c + d*x]))^(3/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(7/2), x]

[Out] ((a + b*Sec[c + d*x])^(3/2)*(4*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*(a^2*(25*a^2*A + 51*A*b^2 + 84*a*b*B)*EllipticF[(c + d*x)/2, (2*a)/(a + b)] + (82*a^2*A*b - 6*A*b^3 + 63*a^3*B + 21*a*b^2*B)*((a + b)*EllipticE[(c + d*x)/2, (2*a)/(a + b)] - b*EllipticF[(c + d*x)/2, (2*a)/(a + b)])) + a*(b + a*Cos[c +

$d*x]))*((115*a^2*A + 12*A*b^2 + 168*a*b*B)*\text{Sin}[c + d*x] + 3*a*(2*(8*A*b + 7*a*B)*\text{Sin}[2*(c + d*x)] + 5*a*A*\text{Sin}[3*(c + d*x)])))/((210*a^2*d*(b + a*\text{Cos}[c + d*x])^2*\text{Sec}[c + d*x]^{3/2}))$

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(Bb \sec(dx + c)^2 + Aa + (Ba + Ab) \sec(dx + c))\sqrt{b \sec(dx + c) + a}}{\sec(dx + c)^{\frac{7}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm="fricas")

[Out] integral((B*b*sec(d*x + c)^2 + A*a + (B*a + A*b)*sec(d*x + c))*sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(7/2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(b \sec(dx + c) + a)^{\frac{3}{2}}}{\sec(dx + c)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(7/2), x)

maple [B] time = 2.45, size = 3752, normalized size = 10.97

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x)

[Out] $-2/105/d*(-63*B*\text{EllipticF}((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (-a+b)/(a-b))^{1/2})*a^4*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{1/2}*(1/(1+\cos(d*x+c)))^{1/2}*\sin(d*x+c)+82*A*\cos(d*x+c)*\text{EllipticE}((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (-a+b)/(a-b))^{1/2})*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{1/2}*(1/(1+\cos(d*x+c)))^{1/2}*\sin(d*x+c)*a^3*b+6*A*\text{EllipticE}((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (-a+b)/(a-b))^{1/2})*b^4*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{1/2}*(1/(1+\cos(d*x+c)))^{1/2}*\sin$

$$\begin{aligned}
& (d*x+c)^{-25}*A*a^3*b*((a-b)/(a+b))^{(1/2)}-82*A*a^2*b^2*((a-b)/(a+b))^{(1/2)}-3*A \\
& *a*b^3*((a-b)/(a+b))^{(1/2)}-63*B*a^3*b*((a-b)/(a+b))^{(1/2)}-42*B*a^2*b^2*((a- \\
& b)/(a+b))^{(1/2)}-21*B*a*b^3*((a-b)/(a+b))^{(1/2)}+21*B*\cos(d*x+c)^4*((a-b)/(a+ \\
& b))^{(1/2)}*a^4+42*B*\cos(d*x+c)^2*((a-b)/(a+b))^{(1/2)}*a^4-6*A*\cos(d*x+c)*((a- \\
& b)/(a+b))^{(1/2)}*b^4-63*B*\cos(d*x+c)*((a-b)/(a+b))^{(1/2)}*a^4+15*A*\cos(d*x+c) \\
& ^5*((a-b)/(a+b))^{(1/2)}*a^4+10*A*\cos(d*x+c)^3*((a-b)/(a+b))^{(1/2)}*a^4-25*A*c \\
& \cos(d*x+c)*((a-b)/(a+b))^{(1/2)}*a^4+6*A*b^4*((a-b)/(a+b))^{(1/2)}+6*A*\cos(d*x+c) \\
&)*EllipticE((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c),(-(a+b)/(a-b))^{(1/2)}) \\
& *((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*(1/(1+\cos(d*x+c)))^{(1/2)} \\
&)*\sin(d*x+c)*b^4+25*A*\cos(d*x+c)*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)} \\
& *(1/(1+\cos(d*x+c)))^{(1/2)}*EllipticF((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/ \\
& \sin(d*x+c),(-(a+b)/(a-b))^{(1/2)})*\sin(d*x+c)*a^4+63*B*\cos(d*x+c)*EllipticE((- \\
& -1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c),(-(a+b)/(a-b))^{(1/2)})*((b+a*c \\
& \cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*(1/(1+\cos(d*x+c)))^{(1/2)}*\sin(d*x+c)* \\
& a^4-63*B*\cos(d*x+c)*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*(1/(1+\cos \\
& (d*x+c)))^{(1/2)}*EllipticF((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c),(- \\
& (a+b)/(a-b))^{(1/2)})*\sin(d*x+c)*a^4+82*A*EllipticE((-1+\cos(d*x+c))*((a-b)/(a \\
& +b))^{(1/2)}/\sin(d*x+c),(-(a+b)/(a-b))^{(1/2)})*a^3*b*((b+a*\cos(d*x+c))/(1+\cos(\\
& d*x+c)))/(a+b))^{(1/2)}*(1/(1+\cos(d*x+c)))^{(1/2)}*\sin(d*x+c)-82*A*EllipticE((-1 \\
& +\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c),(-(a+b)/(a-b))^{(1/2)})*a^2*b^2*(\\
& (b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*(1/(1+\cos(d*x+c)))^{(1/2)}*\sin(d \\
& *x+c)-6*A*EllipticE((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c),(-(a+b)/ \\
& (a-b))^{(1/2)})*a*b^3*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*(1/(1+\cos \\
& (d*x+c)))^{(1/2)}*\sin(d*x+c)-82*A*EllipticF((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/ \\
& \sin(d*x+c),(-(a+b)/(a-b))^{(1/2)})*a^3*b*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(\\
& (a+b))^{(1/2)}*(1/(1+\cos(d*x+c)))^{(1/2)}*\sin(d*x+c)+51*A*EllipticF((-1+\cos(d*x \\
& +c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c),(-(a+b)/(a-b))^{(1/2)})*a^2*b^2*((b+a*\cos \\
& (d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*(1/(1+\cos(d*x+c)))^{(1/2)}*\sin(d*x+c)+6* \\
& A*EllipticF((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c),(-(a+b)/(a-b))^{(1/2)}) \\
&)*a*b^3*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*(1/(1+\cos(d*x+c))) \\
&)^{(1/2)}*\sin(d*x+c)-63*B*EllipticE((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d \\
& *x+c),(-(a+b)/(a-b))^{(1/2)})*a^3*b*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)} \\
& *(1/(1+\cos(d*x+c)))^{(1/2)}*\sin(d*x+c)+21*B*EllipticE((-1+\cos(d*x+c))*((a- \\
& b)/(a+b))^{(1/2)}/\sin(d*x+c),(-(a+b)/(a-b))^{(1/2)})*a^2*b^2*((b+a*\cos(d*x+c)) \\
& / (1+\cos(d*x+c)))/(a+b))^{(1/2)}*(1/(1+\cos(d*x+c)))^{(1/2)}*\sin(d*x+c)-21*B*Ellip \\
& ticE((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c),(-(a+b)/(a-b))^{(1/2)})*a \\
& *b^3*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*(1/(1+\cos(d*x+c)))^{(1/2)} \\
&)*\sin(d*x+c)+84*B*EllipticF((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c),(- \\
& (a+b)/(a-b))^{(1/2)})*a^3*b*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*(1 \\
& / (1+\cos(d*x+c)))^{(1/2)}*\sin(d*x+c)-21*B*EllipticF((-1+\cos(d*x+c))*((a-b)/(a+ \\
& b))^{(1/2)}/\sin(d*x+c),(-(a+b)/(a-b))^{(1/2)})*a^2*b^2*((b+a*\cos(d*x+c))/(1+\cos \\
& (d*x+c)))/(a+b))^{(1/2)}*(1/(1+\cos(d*x+c)))^{(1/2)}*\sin(d*x+c)-82*A*\cos(d*x+c)*E \\
& llipticE((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c),(-(a+b)/(a-b))^{(1/2)} \\
&))*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*(1/(1+\cos(d*x+c)))^{(1/2)}*s \\
& \sin(d*x+c)*a^2*b^2-6*A*\cos(d*x+c)*EllipticE((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}
\end{aligned}$$

$$\begin{aligned} & /2)/\sin(d*x+c), (- (a+b)/(a-b))^{(1/2)} * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)) / (a+b) \\ &)^{(1/2)} * (1/(1+\cos(d*x+c)))^{(1/2)} * \sin(d*x+c) * a*b^3 - 82*A*\cos(d*x+c) * ((b+a*\cos \\ & (d*x+c))/(1+\cos(d*x+c)) / (a+b))^{(1/2)} * (1/(1+\cos(d*x+c)))^{(1/2)} * \text{EllipticF}((-1 \\ & +\cos(d*x+c)) * ((a-b)/(a+b))^{(1/2)} / \sin(d*x+c), (- (a+b)/(a-b))^{(1/2)} * \sin(d*x+c) \\ &) * a^3*b + 51*A*\cos(d*x+c) * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)) / (a+b))^{(1/2)} * (1/(1 \\ & +\cos(d*x+c)))^{(1/2)} * \text{EllipticF}((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{(1/2)} / \sin(d*x+c) \\ &), (- (a+b)/(a-b))^{(1/2)} * \sin(d*x+c) * a^2*b^2 + 6*A*\cos(d*x+c) * ((b+a*\cos(d*x+c)) \\ & / (1+\cos(d*x+c)) / (a+b))^{(1/2)} * (1/(1+\cos(d*x+c)))^{(1/2)} * \text{EllipticF}((-1+\cos(d*x \\ & +c)) * ((a-b)/(a+b))^{(1/2)} / \sin(d*x+c), (- (a+b)/(a-b))^{(1/2)} * \sin(d*x+c) * a*b^3 - \\ & 63*B*\cos(d*x+c) * \text{EllipticE}((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{(1/2)} / \sin(d*x+c), (- \\ & (a+b)/(a-b))^{(1/2)} * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)) / (a+b))^{(1/2)} * (1/(1+\cos \\ & (d*x+c)))^{(1/2)} * \sin(d*x+c) * a^3*b + 21*B*\cos(d*x+c) * \text{EllipticE}((-1+\cos(d*x+c)) * \\ & ((a-b)/(a+b))^{(1/2)} / \sin(d*x+c), (- (a+b)/(a-b))^{(1/2)} * ((b+a*\cos(d*x+c))/(1+\cos \\ & (d*x+c)) / (a+b))^{(1/2)} * (1/(1+\cos(d*x+c)))^{(1/2)} * \sin(d*x+c) * a^2*b^2 - 21*B*\cos \\ & (d*x+c) * \text{EllipticE}((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{(1/2)} / \sin(d*x+c), (- (a+b)/(\\ & a-b))^{(1/2)} * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)) / (a+b))^{(1/2)} * (1/(1+\cos(d*x+c) \\ &))^{(1/2)} * \sin(d*x+c) * a*b^3 + 84*B*\cos(d*x+c) * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)) / \\ & (a+b))^{(1/2)} * (1/(1+\cos(d*x+c)))^{(1/2)} * \text{EllipticF}((-1+\cos(d*x+c)) * ((a-b)/(a+b) \\ &))^{(1/2)} / \sin(d*x+c), (- (a+b)/(a-b))^{(1/2)} * \sin(d*x+c) * a^3*b - 21*B*\cos(d*x+c) * \\ & ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)) / (a+b))^{(1/2)} * (1/(1+\cos(d*x+c)))^{(1/2)} * \text{EllipticF}((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{(1/2)} / \sin(d*x+c), (- (a+b)/(a-b))^{(1/2)} * \sin(d*x+c) * a^2*b^2 + 25*A * \text{EllipticF}((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{(1/2)} / \sin(d*x+c), (- (a+b)/(a-b))^{(1/2)} * a^4 * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)) / (a+b))^{(1/2)} * (1/(1+\cos(d*x+c)))^{(1/2)} * \sin(d*x+c) + 63*B * \text{EllipticE}((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{(1/2)} / \sin(d*x+c), (- (a+b)/(a-b))^{(1/2)} * a^4 * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)) / (a+b))^{(1/2)} * (1/(1+\cos(d*x+c)))^{(1/2)} * \sin(d*x+c) + 27*A * \cos(d*x+c) ^ 3 * ((a-b)/(a+b))^{(1/2)} * a^2*b^2 + 63*B*\cos(d*x+c) ^ 3 * ((a-b)/(a+b))^{(1/2)} * a^3*b + 68*A*\cos(d*x+c) ^ 2 * ((a-b)/(a+b))^{(1/2)} * a^3*b - 3*A*\cos(d*x+c) ^ 2 * ((a-b)/(a+b))^{(1/2)} * a*b^3 + 63*B*\cos(d*x+c) ^ 2 * ((a-b)/(a+b))^{(1/2)} * a^2*b^2 - 82*A*\cos(d*x+c) * ((a-b)/(a+b))^{(1/2)} * a^3*b + 55*A*\cos(d*x+c) * ((a-b)/(a+b))^{(1/2)} * a^2*b^2 + 6*A*\cos(d*x+c) * ((a-b)/(a+b))^{(1/2)} * a*b^3 + 39*A*\cos(d*x+c) ^ 4 * ((a-b)/(a+b))^{(1/2)} * a^3*b - 21*B*\cos(d*x+c) * ((a-b)/(a+b))^{(1/2)} * a^2*b^2 + 21*B*\cos(d*x+c) * ((a-b)/(a+b))^{(1/2)} * a*b^3 * ((b+a*\cos(d*x+c))/\cos(d*x+c))^{(1/2)} * \cos(d*x+c) ^ 4 * (1/\cos(d*x+c))^{(7/2)} / \sin(d*x+c) / (b+a*\cos(d*x+c)) / a^2 / ((a-b)/(a+b))^{(1/2)} \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx+c) + A)(b \sec(dx+c) + a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorith="maxima")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(7/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(a + \frac{b}{\cos(c+dx)}\right)^{3/2}}{\left(\frac{1}{\cos(c+dx)}\right)^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(3/2))/(1/cos(c + d*x))^(7/2), x)

[Out] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(3/2))/(1/cos(c + d*x))^(7/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)**(7/2), x)

[Out] Timed out

$$3.447 \quad \int \frac{(a+b \sec(c+dx))^{3/2}(A+B \sec(c+dx))}{9 \sec^2(c+dx)} dx$$

Optimal. Leaf size=427

$$\frac{2(49a^2A + 72abB + 3Ab^2) \sin(c + dx) \sqrt{a + b \sec(c + dx)}}{315ad \sec^{\frac{3}{2}}(c + dx)} + \frac{2(75a^3B + 88a^2Ab + 9ab^2B - 4Ab^3) \sin(c + dx) \sqrt{a}}{315a^2d \sqrt{\sec(c + dx)}}$$

[Out] $2/315*(a^2-b^2)*(39*A*a^2*b+8*A*b^3+75*B*a^3-18*B*a*b^2)*(cos(1/2*d*x+1/2*c))^2)^{(1/2)}/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c), 2^{(1/2)}*(a/(a+b))^{(1/2)}*((b+a*cos(d*x+c))/(a+b))^{(1/2)}*sec(d*x+c)^{(1/2)}/a^{3/d}/(a+b*sec(d*x+c))^{(1/2)}+2/9*a*A*sin(d*x+c)*(a+b*sec(d*x+c))^{(1/2)}/d/sec(d*x+c)^{(7/2)}+2/63*(10*A*b+9*B*a)*sin(d*x+c)*(a+b*sec(d*x+c))^{(1/2)}/d/sec(d*x+c)^{(5/2)}+2/315*(49*A*a^2+3*A*b^2+72*B*a*b)*sin(d*x+c)*(a+b*sec(d*x+c))^{(1/2)}/a/d/sec(d*x+c)^{(3/2)}+2/315*(88*A*a^2*b-4*A*b^3+75*B*a^3+9*B*a*b^2)*sin(d*x+c)*(a+b*sec(d*x+c))^{(1/2)}/a^2/d/sec(d*x+c)^{(1/2)}+2/315*(147*A*a^4+33*A*a^2*b^2+8*A*b^4+246*B*a^3*b-18*B*a*b^3)*(cos(1/2*d*x+1/2*c))^2)^{(1/2)}/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c), 2^{(1/2)}*(a/(a+b))^{(1/2)}*(a+b*sec(d*x+c))^{(1/2)}/a^{3/d}/((b+a*cos(d*x+c))/(a+b))^{(1/2)}/sec(d*x+c)^{(1/2)})$

Rubi [A] time = 1.50, antiderivative size = 427, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {4025, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2(49a^2A + 72abB + 3Ab^2) \sin(c + dx) \sqrt{a + b \sec(c + dx)}}{315ad \sec^{\frac{3}{2}}(c + dx)} + \frac{2(88a^2Ab + 75a^3B + 9ab^2B - 4Ab^3) \sin(c + dx) \sqrt{a}}{315a^2d \sqrt{\sec(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[((a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(9/2), x]

[Out] $(2*(a^2 - b^2)*(39*a^2*A*b + 8*A*b^3 + 75*a^3*B - 18*a*b^2*B)*Sqrt[(b + a*cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(315*a^3*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(147*a^4*A + 33*a^2*A*b^2 + 8*A*b^4 + 246*a^3*b*B - 18*a*b^3*B)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(315*a^3*d*Sqrt[(b + a*cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*a*A*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2)) + (2*(10*A*b + 9*a*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(63*d*Sec[c + d*x]^(5/2)) + (2*(49*a^2*A + 3*A*b^2 + 72*a*b*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(315*a*d*Sec[c + d*x]^(3/2)) + (2*(88*a^2*A*b - 4*A*b^3 + 75*a^3*B + 9*a*b^2*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(315*a^2*d*Sqrt[Sec[c + d*x]])$

Rule 2653

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*Sqrt[a
+ b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)]/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
0] && !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)]/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a
+ b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && !GtQ[a + b, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)]/Sqrt[csc[(e_) + (f_)*(x_)]
*(d_)], x_Symbol] := Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_) + (f_)*(x_)]*(d_)]/Sqrt[csc[(e_) + (f_)*(x_)]*(b_)
+ (a_)], x_Symbol] := Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4025

```
Int[(csc[(e_) + (f_)*(x_)]*(d_)^(n_)*(csc[(e_) + (f_)*(x_)]*(b_) + (
a_)^(m_)*(csc[(e_) + (f_)*(x_)]*(B_) + (A_)), x_Symbol] := Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dis
```

```
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.))/(Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a
_.))^(m_.), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + b \sec(c + dx))^{3/2} (A + B \sec(c + dx))}{\sec^2(c + dx)} dx &= \frac{2aA\sqrt{a + b \sec(c + dx)} \sin(c + dx)}{9d \sec^{\frac{7}{2}}(c + dx)} - \frac{2}{9} \int \frac{-\frac{1}{2}a(10Ab + 9aB)}{\sec^{\frac{5}{2}}(c + dx)} dx \\
&= \frac{2aA\sqrt{a + b \sec(c + dx)} \sin(c + dx)}{9d \sec^{\frac{7}{2}}(c + dx)} + \frac{2(10Ab + 9aB)\sqrt{a + b \sec(c + dx)}}{63d \sec^{\frac{5}{2}}(c + dx)} \\
&= \frac{2aA\sqrt{a + b \sec(c + dx)} \sin(c + dx)}{9d \sec^{\frac{7}{2}}(c + dx)} + \frac{2(10Ab + 9aB)\sqrt{a + b \sec(c + dx)}}{63d \sec^{\frac{5}{2}}(c + dx)} \\
&= \frac{2aA\sqrt{a + b \sec(c + dx)} \sin(c + dx)}{9d \sec^{\frac{7}{2}}(c + dx)} + \frac{2(10Ab + 9aB)\sqrt{a + b \sec(c + dx)}}{63d \sec^{\frac{5}{2}}(c + dx)} \\
&= \frac{2aA\sqrt{a + b \sec(c + dx)} \sin(c + dx)}{9d \sec^{\frac{7}{2}}(c + dx)} + \frac{2(10Ab + 9aB)\sqrt{a + b \sec(c + dx)}}{63d \sec^{\frac{5}{2}}(c + dx)} \\
&= \frac{2aA\sqrt{a + b \sec(c + dx)} \sin(c + dx)}{9d \sec^{\frac{7}{2}}(c + dx)} + \frac{2(10Ab + 9aB)\sqrt{a + b \sec(c + dx)}}{63d \sec^{\frac{5}{2}}(c + dx)} \\
&= \frac{2aA\sqrt{a + b \sec(c + dx)} \sin(c + dx)}{9d \sec^{\frac{7}{2}}(c + dx)} + \frac{2(10Ab + 9aB)\sqrt{a + b \sec(c + dx)}}{63d \sec^{\frac{5}{2}}(c + dx)} \\
&= \frac{2aA\sqrt{a + b \sec(c + dx)} \sin(c + dx)}{9d \sec^{\frac{7}{2}}(c + dx)} + \frac{2(10Ab + 9aB)\sqrt{a + b \sec(c + dx)}}{63d \sec^{\frac{5}{2}}(c + dx)} \\
&= \frac{2(a^2 - b^2)(39a^2Ab + 8Ab^3 + 75a^3B - 18ab^2B) \sqrt{\frac{b+a \cos(c+dx)}{a+b}}}{315a^3d\sqrt{a + b \sec(c + dx)}}
\end{aligned}$$

Mathematica [A] time = 2.31, size = 313, normalized size = 0.73

$$\frac{(a + b \sec(c + dx))^{3/2} \left(a(a \cos(c + dx) + b) \left(a \left(2 \left(133a^2A + 144abB + 6Ab^2 \right) \sin(2(c + dx)) + 5a(2(9aB + 10Ab) \right) \right) \right)}{315a^3d\sqrt{a + b \sec(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(9/2),x]

```
[Out] ((a + b*Sec[c + d*x])^(3/2)*(8*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*(a^2*(186
*a^2*A*b + 2*A*b^3 + 75*a^3*B + 153*a*b^2*B)*EllipticF[(c + d*x)/2, (2*a)/(
a + b)] + (147*a^4*A + 33*a^2*A*b^2 + 8*A*b^4 + 246*a^3*b*B - 18*a*b^3*B)*(
(a + b)*EllipticE[(c + d*x)/2, (2*a)/(a + b)] - b*EllipticF[(c + d*x)/2, (2
*a)/(a + b)])) + a*(b + a*Cos[c + d*x])*((804*a^2*A*b - 32*A*b^3 + 690*a^3*
B + 72*a*b^2*B)*Sin[c + d*x] + a*(2*(133*a^2*A + 6*A*b^2 + 144*a*b*B)*Sin[2
*(c + d*x)] + 5*a*(2*(10*A*b + 9*a*B)*Sin[3*(c + d*x)] + 7*a*A*Ssin[4*(c + d
*x)])))))/(1260*a^3*d*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^(3/2))
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{(Bb \sec(dx + c)^2 + Aa + (Ba + Ab) \sec(dx + c)) \sqrt{b \sec(dx + c) + a}}{\sec(dx + c)^{\frac{9}{2}}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*b*sec(d*x + c)^2 + A*a + (B*a + A*b)*sec(d*x + c))*sqrt(b*sec(d
*x + c) + a)/sec(d*x + c)^(9/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(b \sec(dx + c) + a)^{\frac{3}{2}}}{\sec(dx + c)^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(9/2
), x)
```

maple [B] time = 3.00, size = 4846, normalized size = 11.35

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x)
```

```
[Out] 2/315/d*(-186*A*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b
))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(
```

$$\begin{aligned}
& 1/2)/\sin(d*x+c), (- (a+b)/(a-b))^{(1/2)} * a^4*b + 147*A*EllipticF((-1+\cos(d*x+c)) \\
& * ((a-b)/(a+b))^{(1/2)}/\sin(d*x+c), (- (a+b)/(a-b))^{(1/2)} * a^5*((b+a*\cos(d*x+c)) \\
& / (1+\cos(d*x+c)))/(a+b))^{(1/2)} * (1/(1+\cos(d*x+c)))^{(1/2)} * \sin(d*x+c) - 147*A*EllipticE((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{(1/2)}/\sin(d*x+c), (- (a+b)/(a-b))^{(1/2)} * \\
& a^5*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)} * (1/(1+\cos(d*x+c)))^{(1/2)} * \\
& \sin(d*x+c) + 147*A*a^4*b*((a-b)/(a+b))^{(1/2)} + 88*A*a^3*b^2*((a-b)/(a+b))^{(1/2)} \\
& + 33*A*a^2*b^3*((a-b)/(a+b))^{(1/2)} - 4*A*a*b^4*((a-b)/(a+b))^{(1/2)} + 75*B*a^4*b* \\
& ((a-b)/(a+b))^{(1/2)} + 246*B*a^3*b^2*((a-b)/(a+b))^{(1/2)} + 9*B*a^2*b^3*((a-b)/(a \\
& +b))^{(1/2)} - 18*B*a*b^4*((a-b)/(a+b))^{(1/2)} - 45*B*\cos(d*x+c)^5*((a-b)/(a+b))^{(\\
& 1/2)} * a^5 - 8*A*\cos(d*x+c)*((a-b)/(a+b))^{(1/2)} * b^5 - 30*B*\cos(d*x+c)^3*((a-b)/(a \\
& +b))^{(1/2)} * a^5 + 75*B*\cos(d*x+c)*((a-b)/(a+b))^{(1/2)} * a^5 - 35*A*\cos(d*x+c)^6*((\\
& a-b)/(a+b))^{(1/2)} * a^5 - 14*A*\cos(d*x+c)^4*((a-b)/(a+b))^{(1/2)} * a^5 - 98*A*\cos(d* \\
& x+c)^2*((a-b)/(a+b))^{(1/2)} * a^5 + 147*A*\cos(d*x+c)*((a-b)/(a+b))^{(1/2)} * a^5 - 246 \\
& *B*EllipticE((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c), (- (a+b)/(a-b))^{(\\
& 1/2)} * a^4*b*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)} * (1/(1+\cos(d*x+c) \\
&))^{(1/2)} * \sin(d*x+c) + 246*B*EllipticE((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin \\
& (d*x+c), (- (a+b)/(a-b))^{(1/2)} * a^3*b^2*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b \\
&))^{(1/2)} * (1/(1+\cos(d*x+c)))^{(1/2)} * \sin(d*x+c) + 18*B*EllipticE((-1+\cos(d*x+c)) \\
& * ((a-b)/(a+b))^{(1/2)}/\sin(d*x+c), (- (a+b)/(a-b))^{(1/2)} * a^2*b^3*((b+a*\cos(d*x \\
& +c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)} * (1/(1+\cos(d*x+c)))^{(1/2)} * \sin(d*x+c) - 18*B*E \\
& llipticE((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c), (- (a+b)/(a-b))^{(1/2) \\
&)) * a*b^4*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)} * (1/(1+\cos(d*x+c)))^{(\\
& 1/2)} * \sin(d*x+c) + 147*A*\sin(d*x+c)*\cos(d*x+c)*((b+a*\cos(d*x+c))/(1+\cos(d*x+c) \\
&))/(a+b))^{(1/2)} * (1/(1+\cos(d*x+c)))^{(1/2)} * EllipticF((-1+\cos(d*x+c))*((a-b)/(a \\
& +b))^{(1/2)}/\sin(d*x+c), (- (a+b)/(a-b))^{(1/2)} * a^5 - 147*A*\sin(d*x+c)*\cos(d*x+c) \\
& * EllipticE((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c), (- (a+b)/(a-b))^{(1 \\
& /2)} * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)} * (1/(1+\cos(d*x+c)))^{(1/2) \\
&) * a^5 + 8*A*\sin(d*x+c)*\cos(d*x+c)*EllipticE((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2) \\
&)/\sin(d*x+c), (- (a+b)/(a-b))^{(1/2)} * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(\\
& 1/2)} * (1/(1+\cos(d*x+c)))^{(1/2)} * b^5 - 75*B*\sin(d*x+c)*\cos(d*x+c)*((b+a*\cos(d*x \\
& +c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)} * (1/(1+\cos(d*x+c)))^{(1/2)} * EllipticF((-1+\cos \\
& (d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c), (- (a+b)/(a-b))^{(1/2)} * a^5 - 186*A*Ell \\
& ipticF((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c), (- (a+b)/(a-b))^{(1/2) \\
&)) * a^4*b*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)} * (1/(1+\cos(d*x+c)))^{(1/ \\
& 2)} * \sin(d*x+c) + 33*A*EllipticF((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c) \\
& , (- (a+b)/(a-b))^{(1/2)} * a^3*b^2*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2) \\
&) * (1/(1+\cos(d*x+c)))^{(1/2)} * \sin(d*x+c) - 2*A*EllipticF((-1+\cos(d*x+c))*((a-b)/ \\
& (a+b))^{(1/2)}/\sin(d*x+c), (- (a+b)/(a-b))^{(1/2)} * a^2*b^3*((b+a*\cos(d*x+c))/(1+ \\
& \cos(d*x+c)))/(a+b))^{(1/2)} * (1/(1+\cos(d*x+c)))^{(1/2)} * \sin(d*x+c) + 8*A*EllipticF(\\
& (-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c), (- (a+b)/(a-b))^{(1/2)} * a*b^4* \\
& ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)} * (1/(1+\cos(d*x+c)))^{(1/2)} * \sin(\\
& d*x+c) + 147*A*EllipticE((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c), (- (a+ \\
& b)/(a-b))^{(1/2)} * a^4*b*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)} * (1/(1+ \\
& \cos(d*x+c)))^{(1/2)} * \sin(d*x+c) - 33*A*EllipticE((-1+\cos(d*x+c))*((a-b)/(a+b))^{(\\
& 1/2)}/\sin(d*x+c), (- (a+b)/(a-b))^{(1/2)} * a^3*b^2*((b+a*\cos(d*x+c))/(1+\cos(d*x
\end{aligned}$$

$$\begin{aligned}
& +c)/(a+b))^{1/2} * (1/(1+\cos(d*x+c)))^{1/2} * \sin(d*x+c) + 33*A*EllipticE((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{1/2} / \sin(d*x+c), (-a+b)/(a-b))^{1/2}) * a^2*b^3 * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)) / (a+b))^{1/2} * (1/(1+\cos(d*x+c)))^{1/2} * \sin(d*x+c) \\
& - 8*A*EllipticE((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{1/2} / \sin(d*x+c), (-a+b)/(a-b))^{1/2}) * a*b^4 * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)) / (a+b))^{1/2} * (1/(1+\cos(d*x+c)))^{1/2} * \sin(d*x+c) + 246*B*EllipticF((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{1/2} / \sin(d*x+c), (-a+b)/(a-b))^{1/2}) * a^4*b * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)) / (a+b))^{1/2} * (1/(1+\cos(d*x+c)))^{1/2} * \sin(d*x+c) - 153*B*EllipticF((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{1/2} / \sin(d*x+c), (-a+b)/(a-b))^{1/2}) * a^3*b^2 * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)) / (a+b))^{1/2} * (1/(1+\cos(d*x+c)))^{1/2} * \sin(d*x+c) - 18*B*EllipticF((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{1/2} / \sin(d*x+c), (-a+b)/(a-b))^{1/2}) * a^2*b^3 * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)) / (a+b))^{1/2} * (1/(1+\cos(d*x+c)))^{1/2} * \sin(d*x+c) + 8*A*b^5 * ((a-b)/(a+b))^{1/2} - 52*A*\cos(d*x+c)^3 * ((a-b)/(a+b))^{1/2} * a^4*b + A*\cos(d*x+c)^3 * ((a-b)/(a+b))^{1/2} * a^2*b^3 - 81*B*\cos(d*x+c)^3 * ((a-b)/(a+b))^{1/2} * a^3*b^2 - 68*A*\cos(d*x+c)^2 * ((a-b)/(a+b))^{1/2} * a^3*b^2 - 85*A*\cos(d*x+c)^5 * ((a-b)/(a+b))^{1/2} * a^4*b - 53*A*\cos(d*x+c)^4 * ((a-b)/(a+b))^{1/2} * a^3*b^2 - 117*B*\cos(d*x+c)^4 * ((a-b)/(a+b))^{1/2} * a^4*b - 4*A*\cos(d*x+c)^2 * ((a-b)/(a+b))^{1/2} * a*b^4 - 204*B*\cos(d*x+c)^2 * ((a-b)/(a+b))^{1/2} * a^4*b + 9*B*\cos(d*x+c)^2 * ((a-b)/(a+b))^{1/2} * a^2*b^3 + 33*A*\sin(d*x+c)*\cos(d*x+c) * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)) / (a+b))^{1/2} * (1/(1+\cos(d*x+c)))^{1/2} * EllipticF((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{1/2} / \sin(d*x+c), (-a+b)/(a-b))^{1/2}) * a^3*b^2 - 2*A*\sin(d*x+c)*\cos(d*x+c) * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)) / (a+b))^{1/2} * (1/(1+\cos(d*x+c)))^{1/2} * EllipticF((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{1/2} / \sin(d*x+c), (-a+b)/(a-b))^{1/2}) * a^2*b^3 + 8*A*\sin(d*x+c)*\cos(d*x+c) * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)) / (a+b))^{1/2} * (1/(1+\cos(d*x+c)))^{1/2} * EllipticF((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{1/2} / \sin(d*x+c), (-a+b)/(a-b))^{1/2}) * a*b^4 + 147*A*\sin(d*x+c)*\cos(d*x+c) * EllipticE((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{1/2} / \sin(d*x+c), (-a+b)/(a-b))^{1/2}) * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)) / (a+b))^{1/2} * (1/(1+\cos(d*x+c)))^{1/2} * a^4*b - 33*A*\sin(d*x+c)*\cos(d*x+c) * EllipticE((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{1/2} / \sin(d*x+c), (-a+b)/(a-b))^{1/2}) * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)) / (a+b))^{1/2} * (1/(1+\cos(d*x+c)))^{1/2} * a^3*b^2 + 33*A*\sin(d*x+c)*\cos(d*x+c) * EllipticE((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{1/2} / \sin(d*x+c), (-a+b)/(a-b))^{1/2}) * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)) / (a+b))^{1/2} * (1/(1+\cos(d*x+c)))^{1/2} * a^2*b^3 - 8*A*\sin(d*x+c)*\cos(d*x+c) * EllipticE((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{1/2} / \sin(d*x+c), (-a+b)/(a-b))^{1/2}) * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)) / (a+b))^{1/2} * (1/(1+\cos(d*x+c)))^{1/2} * a*b^4 + 246*B*\sin(d*x+c)*\cos(d*x+c) * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)) / (a+b))^{1/2} * (1/(1+\cos(d*x+c)))^{1/2} * EllipticF((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{1/2} / \sin(d*x+c), (-a+b)/(a-b))^{1/2}) * a^4*b - 153*B*\sin(d*x+c)*\cos(d*x+c) * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)) / (a+b))^{1/2} * (1/(1+\cos(d*x+c)))^{1/2} * EllipticF((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{1/2} / \sin(d*x+c), (-a+b)/(a-b))^{1/2}) * a^3*b^2 - 18*B*\sin(d*x+c)*\cos(d*x+c) * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)) / (a+b))^{1/2} * (1/(1+\cos(d*x+c)))^{1/2} * EllipticF((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{1/2} / \sin(d*x+c), (-a+b)/(a-b))^{1/2}) * a^2*b^3 - 246*B*\sin(d*x+c)*\cos(d*x+c) * EllipticE((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{1/2} / \sin(d*x+c), (-a+b)/(a-b))^{1/2}) * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)) / (a+b))^{1/2}
\end{aligned}$$

$$\frac{\int \frac{(B \sec(dx+c) + A)(b \sec(dx+c) + a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{9}{2}}} dx}{\int \frac{(A + \frac{B}{\cos(c+dx)}) \left(a + \frac{b}{\cos(c+dx)}\right)^{\frac{3}{2}}}{\left(\frac{1}{\cos(c+dx)}\right)^{\frac{9}{2}}} dx}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx+c) + A)(b \sec(dx+c) + a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(9/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(a + \frac{b}{\cos(c+dx)}\right)^{\frac{3}{2}}}{\left(\frac{1}{\cos(c+dx)}\right)^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(3/2))/(1/cos(c + d*x))^(9/2),x)

```
[Out] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(3/2))/(1/cos(c + d*x))^(9/2), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```


$$3.448 \quad \int \sec^2(c+dx)(a+b \sec(c+dx))^{5/2}(A+B \sec(c+dx)) dx$$

Optimal. Leaf size=513

$$\frac{(59a^2B + 104aAb + 36b^2B) \sin(c + dx) \sec^2(c + dx) \sqrt{a + b \sec(c + dx)}}{96d} + \frac{(15a^3B + 264a^2Ab + 284ab^2B + 128a^3B)}{96d}$$

```
[Out] 1/4*b*B*sec(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(3/2)*sin(d*x+c)/d+1/192*(472*A*a^2*b+128*A*b^3+133*B*a^3+356*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/d/(a+b*sec(d*x+c))^(1/2)+1/64*(40*A*a^3*b+160*A*a*b^3-5*B*a^4+120*B*a^2*b^2+48*B*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/b/d/(a+b*sec(d*x+c))^(1/2)+1/96*(104*A*a*b+59*B*a^2+36*B*b^2)*sec(d*x+c)^(3/2)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d+1/24*b*(8*A*b+11*B*a)*sec(d*x+c)^(5/2)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d-1/192*(264*A*a^2*b+128*A*b^3+15*B*a^3+284*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/b/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)+1/192*(264*A*a^2*b+128*A*b^3+15*B*a^3+284*B*a*b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/b/d
```

Rubi [A] time = 2.00, antiderivative size = 513, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 14, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4026, 4096, 4102, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{(59a^2B + 104aAb + 36b^2B) \sin(c + dx) \sec^2(c + dx) \sqrt{a + b \sec(c + dx)}}{96d} + \frac{(264a^2Ab + 15a^3B + 284ab^2B + 128a^3B)}{96d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
[Out] ((472*a^2*A*b + 128*A*b^3 + 133*a^3*B + 356*a*b^2*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]]/(192*d*Sqrt[a + b*Sec[c + d*x]]) + ((40*a^3*A*b + 160*a*A*b^3 - 5*a^4*B + 120*a^2*b^2*B + 48*b^4*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]]/(64*b*d*Sqrt[a + b*Sec[c + d*x]])
```

) - ((264*a^2*A*b + 128*A*b^3 + 15*a^3*B + 284*a*b^2*B)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(192*b*d*Sqrt[(b + a*cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + ((264*a^2*A*b + 128*A*b^3 + 15*a^3*B + 284*a*b^2*B)*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(192*b*d) + ((104*a*A*b + 59*a^2*B + 36*b^2*B)*Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(96*d) + (b*(8*A*b + 11*a*B)*Sec[c + d*x]^(5/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(24*d) + (b*B*Sec[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(4*d)

Rule 2653

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*Sqrt[a + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2655

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a + b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2661

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2663

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b) + (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2805

Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[c + d, 0]

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])], x_Symbol] := Dist[Sqrt[(c + d*Sin[e + f*x])]/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
+ f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !GtQ[c + d, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] := Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] := Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] := Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4026

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] := -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(m + n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*Sim
p[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B))*(m + n) + b^2*B*(m + n - 1))*C
sc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b
^2, 0] && GtQ[m, 1] && !(IGtQ[n, 1] && !IntegerQ[m])
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] := Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
```

a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]

Rule 4096

Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.)) * (csc[(e_.) + (f_.)*(x_.)]*(d_.))^n * (csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m, x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(m + n + 1), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n*Simp[a*A*(m + n + 1) + a*C*n + ((A*b + a*B)*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) + a*C*m)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && !LeQ[n, -1]

Rule 4102

Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.)) * (csc[(e_.) + (f_.)*(x_.)]*(d_.))^n * (csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m, x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 0]

Rule 4108

Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.))/(Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned}
\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx)) dx &= \frac{bB \sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2} \sin(c+dx)}{4d} \\
&= \frac{b(8Ab+11aB) \sec^{\frac{5}{2}}(c+dx) \sqrt{a+b\sec(c+dx)} \sin(c+dx)}{24d} \\
&= \frac{(104aAb+59a^2B+36b^2B) \sec^{\frac{3}{2}}(c+dx) \sqrt{a+b\sec(c+dx)} \sin(c+dx)}{96d} \\
&= \frac{(264a^2Ab+128Ab^3+15a^3B+284ab^2B) \sqrt{\sec(c+dx)} \sin(c+dx)}{192bd} \\
&= \frac{(264a^2Ab+128Ab^3+15a^3B+284ab^2B) \sqrt{\sec(c+dx)} \sin(c+dx)}{192bd} \\
&= \frac{(264a^2Ab+128Ab^3+15a^3B+284ab^2B) \sqrt{\sec(c+dx)} \sin(c+dx)}{192bd} \\
&= \frac{(264a^2Ab+128Ab^3+15a^3B+284ab^2B) \sqrt{\sec(c+dx)} \sin(c+dx)}{192bd} \\
&= \frac{(264a^2Ab+128Ab^3+15a^3B+284ab^2B) \sqrt{\sec(c+dx)} \sin(c+dx)}{192bd} \\
&= \frac{(40a^3Ab+160aAb^3-5a^4B+120a^2b^2B+48b^4B) \sqrt{a+b\sec(c+dx)} \sin(c+dx)}{64bd\sqrt{a+b\sec(c+dx)}} \\
&= \frac{(472a^2Ab+128Ab^3+133a^3B+356ab^2B) \sqrt{\frac{b+a}{a+b\sec(c+dx)}} \sin(c+dx)}{192d\sqrt{a+b\sec(c+dx)}}
\end{aligned}$$

Mathematica [C] time = 6.96, size = 768, normalized size = 1.50

$$\frac{(a+b\sec(c+dx))^{5/2} \left(\frac{1}{96} \sec^2(c+dx) (59a^2B \sin(c+dx) + 104aAb \sin(c+dx) + 36b^2B \sin(c+dx)) + \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}} \right)}{192d\sqrt{a+b\sec(c+dx)}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]

[Out]
$$-1/768*((a + b*\text{Sec}[c + d*x])^{5/2}*((2*(-416*a^2*A*b^2 - 236*a^3*b*B - 144*a*b^3*B)*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/(a + b)]*\text{EllipticF}[(c + d*x)/2, (2*a)/(a + b)])/\text{Sqrt}[b + a*\text{Cos}[c + d*x]] + (2*(24*a^3*A*b - 832*a*A*b^3 + 45*a^4*B - 436*a^2*b^2*B - 288*b^4*B)*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/(a + b)]*\text{EllipticPi}[2, (c + d*x)/2, (2*a)/(a + b)])/\text{Sqrt}[b + a*\text{Cos}[c + d*x]] + ((2*I)*(264*a^3*A*b + 128*a*A*b^3 + 15*a^4*B + 284*a^2*b^2*B)*\text{Sqrt}[(a - a*\text{Cos}[c + d*x])/(a + b)]*\text{Sqrt}[(a + a*\text{Cos}[c + d*x])/(a - b)]*\text{Cos}[2*(c + d*x)]*(-2*b*(a + b)*\text{EllipticE}[I*\text{ArcSinh}[\text{Sqrt}[(a - b)^{-1}]]*\text{Sqrt}[b + a*\text{Cos}[c + d*x]]], (-a + b)/(a + b)) + a*(2*b*\text{EllipticF}[I*\text{ArcSinh}[\text{Sqrt}[(a - b)^{-1}]]*\text{Sqrt}[b + a*\text{Cos}[c + d*x]]], (-a + b)/(a + b)) + a*\text{EllipticPi}[1 - a/b, I*\text{ArcSinh}[\text{Sqrt}[(a - b)^{-1}]]*\text{Sqrt}[b + a*\text{Cos}[c + d*x]]], (-a + b)/(a + b)))*\text{Sin}[c + d*x])/(\text{Sqrt}[(a - b)^{-1}]*b*\text{Sqrt}[1 - \text{Cos}[c + d*x]^2]*\text{Sqrt}[(a^2 - a^2*\text{Cos}[c + d*x]^2)/a^2]*(-a^2 + 2*b^2 - 4*b*(b + a*\text{Cos}[c + d*x]) + 2*(b + a*\text{Cos}[c + d*x])^2))))/(b*d*(b + a*\text{Cos}[c + d*x])^{5/2}*\text{Sec}[c + d*x]^{5/2}) + ((a + b*\text{Sec}[c + d*x])^{5/2}*((\text{Sec}[c + d*x]^3*(8*A*b^2*\text{Sin}[c + d*x] + 17*a*b*B*\text{Sin}[c + d*x]))/24 + (\text{Sec}[c + d*x]^2*(104*a*A*b*\text{Sin}[c + d*x] + 59*a^2*B*\text{Sin}[c + d*x] + 36*b^2*B*\text{Sin}[c + d*x]))/96 + (\text{Sec}[c + d*x]*(264*a^2*A*b*\text{Sin}[c + d*x] + 128*A*b^3*\text{Sin}[c + d*x] + 15*a^3*B*\text{Sin}[c + d*x] + 284*a*b^2*B*\text{Sin}[c + d*x]))/(192*b) + (b^2*B*\text{Sec}[c + d*x]^3*\text{Tan}[c + d*x])/4))/d*(b + a*\text{Cos}[c + d*x])^2*\text{Sec}[c + d*x]^{5/2}))$$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(b \sec(dx + c) + a)^{\frac{5}{2}} \sec(dx + c)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(3/2), x)

maple [C] time = 2.63, size = 5392, normalized size = 10.51

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x)`

[Out] result too large to display

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(b \sec(dx + c) + a)^{\frac{5}{2}} \sec(dx + c)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="maxima")`

[Out] `integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(3/2), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(A + \frac{B}{\cos(c + dx)} \right) \left(a + \frac{b}{\cos(c + dx)} \right)^{5/2} \left(\frac{1}{\cos(c + dx)} \right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(3/2), x)`

[Out] `int((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(3/2), x)`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)**(3/2)*(a+b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)),x)`

[Out] Timed out

$$3.449 \quad \int \sqrt{\sec(c + dx)} (a + b \sec(c + dx))^{5/2} (A + B \sec(c + dx)) dx$$

Optimal. Leaf size=422

$$\frac{(33a^2B + 54aAb + 16b^2B) \sin(c + dx) \sqrt{\sec(c + dx)} \sqrt{a + b \sec(c + dx)}}{24d} - \frac{(33a^2B + 54aAb + 16b^2B) \sqrt{a + b \sec(c + dx)}}{24d \sqrt{\sec(c + dx)} \sqrt{a \cos(c + dx)}}$$

[Out] $\frac{1}{3} b B \sec(dx+c)^{3/2} (a+b \sec(dx+c))^{3/2} \sin(dx+c)/d + \frac{1}{24} (48 A a^3 + 66 A a^2 b + 59 B a^2 b + 16 B b^3) (\cos(1/2 dx + 1/2 c))^2 \sqrt{\sec(c+dx)} \sqrt{a+b \sec(c+dx)} \operatorname{EllipticF}(\sin(1/2 dx + 1/2 c), 2 \sqrt{\frac{a}{a+b}}) \sqrt{\frac{b+a \cos(dx+c)}{a+b}} \sec(dx+c)^{1/2} / d + \frac{1}{8} (30 A a^2 b + 8 A b^3 + 5 B a^3 + 20 B a^2 b) (\cos(1/2 dx + 1/2 c))^2 \sqrt{\sec(c+dx)} \sqrt{a+b \sec(c+dx)} \operatorname{EllipticPi}(\sin(1/2 dx + 1/2 c), 2 \sqrt{\frac{a}{a+b}}) \sqrt{\frac{b+a \cos(dx+c)}{a+b}} \sec(dx+c)^{1/2} / d + \frac{1}{4} b (2 A b + 3 B a) \sec(dx+c)^{3/2} \sin(dx+c) (a+b \sec(dx+c))^{1/2} / d - \frac{1}{24} (54 A a^2 b + 33 B a^2 + 16 B b^2) (\cos(1/2 dx + 1/2 c))^2 \sqrt{\sec(c+dx)} \sqrt{a+b \sec(c+dx)} \operatorname{EllipticE}(\sin(1/2 dx + 1/2 c), 2 \sqrt{\frac{a}{a+b}}) \sqrt{a+b \sec(c+dx)} / d + \frac{1}{24} (54 A a^2 b + 33 B a^2 + 16 B b^2) \sin(dx+c) \sec(dx+c)^{1/2} (a+b \sec(dx+c))^{1/2} / d$

Rubi [A] time = 1.59, antiderivative size = 422, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 14, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4026, 4096, 4102, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{(33a^2B + 54aAb + 16b^2B) \sin(c + dx) \sqrt{\sec(c + dx)} \sqrt{a + b \sec(c + dx)}}{24d} + \frac{(48a^3A + 59a^2bB + 66aAb^2 + 16b^3B) \sqrt{a + b \sec(c + dx)}}{24d \sqrt{a + b \sec(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]

[Out] $((48 a^3 A + 66 a^2 b B + 59 a^2 b B + 16 b^3 B) \operatorname{Sqrt}[(b + a \operatorname{Cos}[c + d x])]/(a + b) \operatorname{EllipticF}[(c + d x)/2, (2 a)/(a + b)] \operatorname{Sqrt}[\operatorname{Sec}[c + d x]])/(24 d \operatorname{Sqrt}[a + b \operatorname{Sec}[c + d x]]) + ((30 a^2 A b + 8 A b^3 + 5 a^3 B + 20 a^2 b^2 B) \operatorname{Sqrt}[(b + a \operatorname{Cos}[c + d x])]/(a + b) \operatorname{EllipticPi}[2, (c + d x)/2, (2 a)/(a + b)] \operatorname{Sqrt}[\operatorname{Sec}[c + d x]])/(8 d \operatorname{Sqrt}[a + b \operatorname{Sec}[c + d x]]) - ((54 a^2 A b + 33 a^2 B + 16 b^2 B) \operatorname{EllipticE}[(c + d x)/2, (2 a)/(a + b)] \operatorname{Sqrt}[a + b \operatorname{Sec}[c + d x]])/(24 d \operatorname{Sqrt}[(b + a \operatorname{Cos}[c + d x])]/(a + b) \operatorname{Sqrt}[\operatorname{Sec}[c + d x]]) + ((54 a^2 A b + 33 a^2 B + 16 b^2 B) \operatorname{Sqrt}[\operatorname{Sec}[c + d x]] \operatorname{Sqrt}[a + b \operatorname{Sec}[c + d x]] \operatorname{Sin}[c +$

$$\frac{d*x]}{(24*d) + (b*(2*A*b + 3*a*B)*\text{Sec}[c + d*x]^{(3/2)}*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(4*d) + (b*B*\text{Sec}[c + d*x]^{(3/2)}*(a + b*\text{Sec}[c + d*x])^{(3/2)}*\text{Sin}[c + d*x])/(3*d)}$$

Rule 2653

$$\text{Int}[\text{Sqrt}[(a_) + (b_)*\text{sin}[(c_) + (d_)*(x_)]], x_Symbol] \rightarrow \text{Simp}[(2*\text{Sqrt}[a + b]*\text{EllipticE}[(1*(c - \text{Pi}/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; \text{FreeQ}[\{a, b, c, d\}, x] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{GtQ}[a + b, 0]$$

Rule 2655

$$\text{Int}[\text{Sqrt}[(a_) + (b_)*\text{sin}[(c_) + (d_)*(x_)]], x_Symbol] \rightarrow \text{Dist}[\text{Sqrt}[a + b*\text{Sin}[c + d*x]]/\text{Sqrt}[(a + b*\text{Sin}[c + d*x])/(a + b)], \text{Int}[\text{Sqrt}[a/(a + b) + (b*\text{Sin}[c + d*x])/(a + b)], x], x] /; \text{FreeQ}[\{a, b, c, d\}, x] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& !\text{GtQ}[a + b, 0]$$

Rule 2661

$$\text{Int}[1/\text{Sqrt}[(a_) + (b_)*\text{sin}[(c_) + (d_)*(x_)]], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticF}[(1*(c - \text{Pi}/2 + d*x))/2, (2*b)/(a + b)])/d*\text{Sqrt}[a + b], x] /; \text{FreeQ}[\{a, b, c, d\}, x] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{GtQ}[a + b, 0]$$

Rule 2663

$$\text{Int}[1/\text{Sqrt}[(a_) + (b_)*\text{sin}[(c_) + (d_)*(x_)]], x_Symbol] \rightarrow \text{Dist}[\text{Sqrt}[(a + b*\text{Sin}[c + d*x])/(a + b)]/\text{Sqrt}[a + b*\text{Sin}[c + d*x]], \text{Int}[1/\text{Sqrt}[a/(a + b) + (b*\text{Sin}[c + d*x])/(a + b)], x], x] /; \text{FreeQ}[\{a, b, c, d\}, x] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& !\text{GtQ}[a + b, 0]$$

Rule 2805

$$\text{Int}[1/(((a_) + (b_)*\text{sin}[(e_) + (f_)*(x_)])*\text{Sqrt}[(c_) + (d_)*\text{sin}[(e_) + (f_)*(x_)]]), x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticPi}[(2*b)/(a + b), (1*(e - \text{Pi}/2 + f*x))/2, (2*d)/(c + d)])/f*(a + b)*\text{Sqrt}[c + d], x] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{GtQ}[c + d, 0]$$

Rule 2807

$$\text{Int}[1/(((a_) + (b_)*\text{sin}[(e_) + (f_)*(x_)])*\text{Sqrt}[(c_) + (d_)*\text{sin}[(e_) + (f_)*(x_)]]), x_Symbol] \rightarrow \text{Dist}[\text{Sqrt}[(c + d*\text{Sin}[e + f*x])/(c + d)]/\text{Sqrt}[c + d*\text{Sin}[e + f*x]], \text{Int}[1/((a + b*\text{Sin}[e + f*x])*\text{Sqrt}[c/(c + d) + (d*\text{Sin}[e + f*x])/(c + d)]), x], x] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& !\text{GtQ}[c + d, 0]$$

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)], x_Symbol] := Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] := Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] := Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4026

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := -Simp[(b*B*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x] + Dist[1/(m + n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*Simp[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B)*(m + n) + b^2*B*(m + n - 1))*Csc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && !IGtQ[n, 1] && !IntegerQ[m]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.))/(Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]), x_Symbol] := Dist[A/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4096

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_), x_Symbol] := -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
```

```

e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(m + n + 1), Int[(a + b*Csc[e + f
*x])^(m - 1)*(d*Csc[e + f*x])^n*Simp[a*A*(m + n + 1) + a*C*n + ((A*b + a*B)
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) + a*C*m)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && NeQ[a^2 -
b^2, 0] && GtQ[m, 0] && !LeQ[n, -1]

```

Rule 4102

```

Int[((A_.) + csc[(e_.) + (f_.)*(x_)])*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] := -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]

```

Rule 4108

```

Int[((A_.) + csc[(e_.) + (f_.)*(x_)])*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] := Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
+ b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]

```

Rubi steps

$$\begin{aligned}
\int \sqrt{\sec(c+dx)} (a+b \sec(c+dx))^{5/2} (A+B \sec(c+dx)) dx &= \frac{bB \sec^3(c+dx)(a+b \sec(c+dx))^{3/2} \sin(c+dx)}{3d} \\
&= \frac{b(2Ab+3aB) \sec^3(c+dx) \sqrt{a+b \sec(c+dx)} \sin(c+dx)}{4d} \\
&= \frac{(54aAb+33a^2B+16b^2B) \sqrt{\sec(c+dx)} \sqrt{a+b \sec(c+dx)} \sin(c+dx)}{24d} \\
&= \frac{(54aAb+33a^2B+16b^2B) \sqrt{\sec(c+dx)} \sqrt{a+b \sec(c+dx)} \sin(c+dx)}{24d} \\
&= \frac{(54aAb+33a^2B+16b^2B) \sqrt{\sec(c+dx)} \sqrt{a+b \sec(c+dx)} \sin(c+dx)}{24d} \\
&= \frac{(54aAb+33a^2B+16b^2B) \sqrt{\sec(c+dx)} \sqrt{a+b \sec(c+dx)} \sin(c+dx)}{24d} \\
&= \frac{(30a^2Ab+8Ab^3+5a^3B+20ab^2B) \sqrt{\frac{b+a \cos(c+dx)}{a+b}}}{8d \sqrt{a+b \sec(c+dx)}} \\
&= \frac{(48a^3A+66aAb^2+59a^2bB+16b^3B) \sqrt{\frac{b+a \cos(c+dx)}{a+b}}}{24d \sqrt{a+b \sec(c+dx)}}
\end{aligned}$$

Mathematica [C] time = 7.22, size = 678, normalized size = 1.61

$$\frac{(a+b \sec(c+dx))^{5/2} \left(\frac{1}{24} \sec(c+dx) (33a^2B \sin(c+dx) + 54aAb \sin(c+dx) + 16b^2B \sin(c+dx)) + \frac{1}{12} \sec^2(c+dx) \right)}{d \sec^2(c+dx) (a \cos(c+dx) + b)^2}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]

[Out] ((a + b*Sec[c + d*x])^(5/2)*((2*(96*a^3*A + 24*a*A*b^2 + 52*a^2*b*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/Sqrt[b

```
+ a*cos[c + d*x]] + (2*(126*a^2*A*b + 48*A*b^3 - 3*a^3*B + 104*a*b^2*B)*Sqr
t[(b + a*cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]/
Sqrt[b + a*cos[c + d*x]] + ((2*I)*(-54*a^2*A*b - 33*a^3*B - 16*a*b^2*B)*Sqr
t[(a - a*cos[c + d*x])/(a + b)]*Sqrt[(a + a*cos[c + d*x])/(a - b)]*Cos[2*(c
+ d*x)]*(-2*b*(a + b)*EllipticE[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Co
s[c + d*x]]], (-a + b)/(a + b)] + a*(2*b*EllipticF[I*ArcSinh[Sqrt[(a - b)^(-
-1)]*Sqrt[b + a*cos[c + d*x]]], (-a + b)/(a + b)] + a*EllipticPi[1 - a/b, I
*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*cos[c + d*x]]], (-a + b)/(a + b)))*
Sin[c + d*x])/(Sqrt[(a - b)^(-1)]*b*Sqrt[1 - Cos[c + d*x]^2]*Sqrt[(a^2 - a^
2*cos[c + d*x]^2)/a^2]*(-a^2 + 2*b^2 - 4*b*(b + a*cos[c + d*x]) + 2*(b + a*
Cos[c + d*x]^2))))/(96*d*(b + a*cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2)) +
((a + b*Sec[c + d*x])^(5/2)*((Sec[c + d*x]^2*(6*A*b^2*Sin[c + d*x] + 13*a*b
*B*Sin[c + d*x]))/12 + (Sec[c + d*x]*(54*a*A*b*Sin[c + d*x] + 33*a^2*B*Sin[
c + d*x] + 16*b^2*B*Sin[c + d*x]))/24 + (b^2*B*Sec[c + d*x]^2*Tan[c + d*x])
/3))/(d*(b + a*cos[c + d*x])^2*Sec[c + d*x]^(5/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x, algor
ithm="fricas")
```

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(b \sec(dx + c) + a)^{\frac{5}{2}} \sqrt{\sec(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)*sqrt(sec(d*x + c)
), x)
```

maple [C] time = 2.16, size = 4258, normalized size = 10.09

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2), x)
```


$$\begin{aligned}
&))^{1/2} * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2} * \sin(d*x+c) * \cos(d*x+c)^3 * a^2 * b + 44 * B * \text{EllipticF}((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{1/2} / \sin(d*x+c), (- \\
& (a+b)/(a-b))^{1/2}) * (1/(1+\cos(d*x+c)))^{1/2} * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c) \\
&)) / (a+b))^{1/2} * \sin(d*x+c) * \cos(d*x+c)^3 * a * b^2 - 33 * B * (1/(1+\cos(d*x+c)))^{1/2} \\
& * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2} * \text{EllipticE}((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{1/2} / \sin(d*x+c), (- \\
& (a+b)/(a-b))^{1/2}) * \sin(d*x+c) * \cos(d*x+c)^3 * a^2 * b + 16 * B * (1/(1+\cos(d*x+c)))^{1/2} * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b)) \\
& ^{1/2} * \text{EllipticE}((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{1/2} / \sin(d*x+c), (- \\
& (a+b)/(a-b))^{1/2}) * \sin(d*x+c) * \cos(d*x+c)^3 * a * b^2 - 120 * B * (1/(1+\cos(d*x+c)))^{1/2} * ((b \\
& +a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2} * \text{EllipticPi}((-1+\cos(d*x+c)) * ((a-b) \\
&) / (a+b))^{1/2} / \sin(d*x+c), (a+b)/(a-b), I / ((a-b)/(a+b))^{1/2}) * \sin(d*x+c) * \cos \\
& (d*x+c)^3 * a * b^2 + 36 * A * \text{EllipticF}((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{1/2} / \sin(d*x+c), (- \\
& (a+b)/(a-b))^{1/2}) * (1/(1+\cos(d*x+c)))^{1/2} * ((b+a*\cos(d*x+c))/(1+\cos(d \\
& *x+c))/(a+b))^{1/2} * \sin(d*x+c) * \cos(d*x+c)^4 * a^2 * b - 12 * A * \text{EllipticF}((-1+\cos(d \\
& *x+c)) * ((a-b)/(a+b))^{1/2} / \sin(d*x+c), (- \\
& (a+b)/(a-b))^{1/2}) * (1/(1+\cos(d*x+c)))^{1/2} * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c) \\
&))^{1/2} * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2} * \sin(d*x+c) * \cos(d*x+c) \\
& ^4 * a * b^2 + 24 * A * \text{EllipticF}((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{1/2} / \sin(d*x+c), (- \\
& (a+b)/(a-b))^{1/2}) * (1/(1+\cos(d*x+c)))^{1/2} * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c) \\
&)) / (a+b))^{1/2} * \sin(d*x+c) * \cos(d*x+c)^4 * b^3 - 48 * A * (1/(1+\cos(d*x+c)))^{1/2} * (\\
& (b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2} * \text{EllipticPi}((-1+\cos(d*x+c)) * ((a \\
& -b)/(a+b))^{1/2} / \sin(d*x+c), (a+b)/(a-b), I / ((a-b)/(a+b))^{1/2}) * \sin(d*x+c) * c \\
& os(d*x+c)^4 * b^3 - 18 * B * \text{EllipticF}((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{1/2} / \sin(d*x+c), (- \\
& (a+b)/(a-b))^{1/2}) * (1/(1+\cos(d*x+c)))^{1/2} * ((b+a*\cos(d*x+c))/(1+\cos(d \\
& *x+c))/(a+b))^{1/2} * \sin(d*x+c) * \cos(d*x+c)^4 * a^3 + 33 * B * (1/(1+\cos(d*x+c)))^{1/2} * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2} * \text{EllipticE}((-1+\cos(d*x+c)) \\
& * ((a-b)/(a+b))^{1/2} / \sin(d*x+c), (- \\
& (a+b)/(a-b))^{1/2}) * \sin(d*x+c) * \cos(d*x+c)^4 * a^3 - 16 * B * (1/(1+\cos(d*x+c)))^{1/2} * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b) \\
&)^{1/2} * \text{EllipticE}((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{1/2} / \sin(d*x+c), (- \\
& (a+b)/(a-b))^{1/2}) * \sin(d*x+c) * \cos(d*x+c)^4 * b^3 - 30 * B * (1/(1+\cos(d*x+c)))^{1/2} * ((b+a \\
& * \cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2} * \text{EllipticPi}((-1+\cos(d*x+c)) * ((a-b) / \\
& (a+b))^{1/2} / \sin(d*x+c), (a+b)/(a-b), I / ((a-b)/(a+b))^{1/2}) * \sin(d*x+c) * \cos(d \\
& *x+c)^4 * a^3 + 24 * A * \text{EllipticF}((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{1/2} / \sin(d*x+c), (- \\
& (a+b)/(a-b))^{1/2}) * (1/(1+\cos(d*x+c)))^{1/2} * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c) \\
& c)) / (a+b))^{1/2} * \sin(d*x+c) * \cos(d*x+c)^3 * b^3 - 48 * A * (1/(1+\cos(d*x+c)))^{1/2} * (\\
& (b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2} * \text{EllipticPi}((-1+\cos(d*x+c)) * ((\\
& a-b)/(a+b))^{1/2} / \sin(d*x+c), (a+b)/(a-b), I / ((a-b)/(a+b))^{1/2}) * \sin(d*x+c) * \\
& cos(d*x+c)^3 * b^3 - 18 * B * \text{EllipticF}((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{1/2} / \sin(d*x \\
& +c), (- \\
& (a+b)/(a-b))^{1/2}) * (1/(1+\cos(d*x+c)))^{1/2} * ((b+a*\cos(d*x+c))/(1+\cos \\
& (d*x+c))/(a+b))^{1/2} * \sin(d*x+c) * \cos(d*x+c)^3 * a^3 + 33 * B * (1/(1+\cos(d*x+c)))^{1/2} * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2} * \text{EllipticE}((-1+\cos(d*x+c) \\
&) * ((a-b)/(a+b))^{1/2} / \sin(d*x+c), (- \\
& (a+b)/(a-b))^{1/2}) * \sin(d*x+c) * \cos(d*x+c)^3 * a^3 - 16 * B * (1/(1+\cos(d*x+c)))^{1/2} * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b) \\
&))^{1/2} * \text{EllipticE}((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{1/2} / \sin(d*x+c), (- \\
& (a+b)/(a-b))^{1/2}) * \sin(d*x+c) * \cos(d*x+c)^3 * b^3 - 30 * B * (1/(1+\cos(d*x+c)))^{1/2} * ((b+a \\
& * \cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2} * \text{EllipticPi}((-1+\cos(d*x+c)) * ((a-b)
\end{aligned}$$

$$\frac{1}{(a+b)^{1/2} \sin(dx+c)}, \frac{(a+b)}{(a-b)}, I\left(\frac{(a-b)}{(a+b)}\right)^{1/2} \sin(dx+c) \cos(dx+c)^3 a^3 - 54A \cos(dx+c)^4 \left(\frac{(a-b)}{(a+b)}\right)^{1/2} a^2 b - 12A \cos(dx+c)^4 \left(\frac{(a-b)}{(a+b)}\right)^{1/2} a^2 b - 26B \cos(dx+c)^4 \left(\frac{(a-b)}{(a+b)}\right)^{1/2} a^2 b - 16B \cos(dx+c)^4 \left(\frac{(a-b)}{(a+b)}\right)^{1/2} a^2 b - 54A \cos(dx+c)^3 \left(\frac{(a-b)}{(a+b)}\right)^{1/2} a^2 b - 33B \cos(dx+c)^3 \left(\frac{(a-b)}{(a+b)}\right)^{1/2} a^2 b - 18B \cos(dx+c)^3 \left(\frac{(a-b)}{(a+b)}\right)^{1/2} a^2 b^2 \left(\frac{(b+a \cos(dx+c))}{\cos(dx+c)}\right)^{1/2} \left(\frac{1}{\cos(dx+c)}\right)^{1/2} \frac{1}{(b+a \cos(dx+c)) \cos(dx+c)^2 \sin(dx+c) \left(\frac{(a-b)}{(a+b)}\right)^{1/2}}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx+c) + A)(b \sec(dx+c) + a)^{5/2} \sqrt{\sec(dx+c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(dx+c))^(5/2)*(A+B*sec(dx+c))*sec(dx+c)^(1/2),x, algorithm="maxima")

[Out] integrate((B*sec(dx+c) + A)*(b*sec(dx+c) + a)^(5/2)*sqrt(sec(dx+c)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(A + \frac{B}{\cos(c+dx)}\right) \left(a + \frac{b}{\cos(c+dx)}\right)^{5/2} \sqrt{\frac{1}{\cos(c+dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c+dx))*(a + b/cos(c+dx))^(5/2)*(1/cos(c+dx))^(1/2), x)

[Out] int((A + B/cos(c+dx))*(a + b/cos(c+dx))^(5/2)*(1/cos(c+dx))^(1/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(dx+c))**(5/2)*(A+B*sec(dx+c))*sec(dx+c)**(1/2),x)

[Out] Timed out

$$3.450 \quad \int \frac{(a+b \sec(c+dx))^{5/2}(A+B \sec(c+dx))}{\sqrt{\sec(c+dx)}} dx$$

Optimal. Leaf size=359

$$\frac{(8a^2A - 9abB - 4Ab^2) \sqrt{a + b \sec(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2a}{a+b}\right)}{4d\sqrt{\sec(c + dx)} \sqrt{\frac{a \cos(c+dx)+b}{a+b}}} + \frac{b(15a^2B + 20aAb + 4b^2B) \sqrt{\sec(c + dx)} \sqrt{\frac{a \cos(c+dx)+b}{a+b}}}{4d\sqrt{a + b \sec(c + dx)}}$$

[Out] $\frac{1}{2} b B (a + b \sec(dx + c))^{3/2} \sin(dx + c) \sec(dx + c)^{1/2} / d + \frac{1}{4} (16 A a^2 b + 4 A b^3 + 8 B a^3 + 11 B a b^2) (\cos(1/2 dx + 1/2 c))^2)^{1/2} / \cos(1/2 dx + 1/2 c) * \text{EllipticF}(\sin(1/2 dx + 1/2 c), 2^{1/2} (a / (a + b))^{1/2}) * ((b + a \cos(dx + c)) / (a + b))^{1/2} \sec(dx + c)^{1/2} / d / (a + b \sec(dx + c))^{1/2} + \frac{1}{4} b (20 A a b + 15 B a^2 + 4 B b^2) (\cos(1/2 dx + 1/2 c))^2)^{1/2} / \cos(1/2 dx + 1/2 c) * \text{EllipticPi}(\sin(1/2 dx + 1/2 c), 2, 2^{1/2} (a / (a + b))^{1/2}) * ((b + a \cos(dx + c)) / (a + b))^{1/2} \sec(dx + c)^{1/2} / d / (a + b \sec(dx + c))^{1/2} + \frac{1}{4} (8 A a^2 - 4 A b^2 - 9 B a b) (\cos(1/2 dx + 1/2 c))^2)^{1/2} / \cos(1/2 dx + 1/2 c) * \text{EllipticE}(\sin(1/2 dx + 1/2 c), 2^{1/2} (a / (a + b))^{1/2}) * (a + b \sec(dx + c))^{1/2} / d / ((b + a \cos(dx + c)) / (a + b))^{1/2} / \sec(dx + c)^{1/2} + \frac{1}{4} b (4 A b + 7 B a) \sin(dx + c) \sec(dx + c)^{1/2} (a + b \sec(dx + c))^{1/2} / d$

Rubi [A] time = 1.25, antiderivative size = 359, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {4026, 4096, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{(16a^2Ab + 8a^3B + 11ab^2B + 4Ab^3) \sqrt{\sec(c + dx)} \sqrt{\frac{a \cos(c+dx)+b}{a+b}} F\left(\frac{1}{2}(c + dx) \middle| \frac{2a}{a+b}\right)}{4d\sqrt{a + b \sec(c + dx)}} + \frac{(8a^2A - 9abB - 4Ab^2) \sqrt{a + b \sec(c + dx)}}{4d\sqrt{\sec(c + dx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + b \text{Sec}[c + dx])^{5/2} (A + B \text{Sec}[c + dx]) / \text{Sqrt}[\text{Sec}[c + dx]], x]$

[Out] $((16 a^2 A b + 4 A b^3 + 8 a^3 B + 11 a b^2 B) \text{Sqrt}[(b + a \text{Cos}[c + dx]) / (a + b)] * \text{EllipticF}[(c + dx) / 2, (2 a) / (a + b)] * \text{Sqrt}[\text{Sec}[c + dx]]) / (4 d * \text{Sqrt}[a + b \text{Sec}[c + dx]]) + (b (20 a A b + 15 a^2 B + 4 b^2 B) \text{Sqrt}[(b + a \text{Cos}[c + dx]) / (a + b)] * \text{EllipticPi}[2, (c + dx) / 2, (2 a) / (a + b)] * \text{Sqrt}[\text{Sec}[c + dx]]) / (4 d * \text{Sqrt}[a + b \text{Sec}[c + dx]]) + ((8 a^2 A - 4 A b^2 - 9 a b B) * \text{EllipticE}[(c + dx) / 2, (2 a) / (a + b)] * \text{Sqrt}[a + b \text{Sec}[c + dx]]) / (4 d * \text{Sqrt}[(b + a \text{Cos}[c + dx]) / (a + b)] * \text{Sqrt}[\text{Sec}[c + dx]]) + (b (4 A b + 7 a B) \text{Sqrt}[\text{Sec}[c + dx]] * \text{Sqrt}[a + b \text{Sec}[c + dx]] * \text{Sin}[c + dx]) / (4 d) + (b B * \text{Sqrt}[\text{Sec}[c + dx]]) * (a + b \text{Sec}[c + dx])^{3/2} * \text{Sin}[c + dx]) / (2 d)$

Rule 2653

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*Sqrt[a
+ b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)]/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
0] && !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)]/(d*Sqrt[a + b]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a
+ b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_)
+ (f_)*(x_)]]), x_Symbol] := Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)]/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_)
+ (f_)*(x_)]]), x_Symbol] := Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
+ f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !GtQ[c + d, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)]/Sqrt[csc[(e_) + (f_)*(x_)]
*(d_)], x_Symbol] := Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
```

b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 3858

Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 3859

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 4026

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] :> -Simp[(b*B*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x] + Dist[1/(m + n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*Simp[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B)*(m + n) + b^2*B*(m + n - 1))*Csc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && !(IGtQ[n, 1] && !IntegerQ[m])

Rule 4035

Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)]), x_Symbol] :> Dist[A/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]

Rule 4096

Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(m + n + 1), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n*Simp[a*A*(m + n + 1) + a*C*n + ((A*b + a*B)*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) + a*C*m)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && NeQ[a^2 -

$b^2, 0] \ \&\& \text{GtQ}[m, 0] \ \&\& \ !\text{LeQ}[n, -1]$

Rule 4108

$\text{Int}[(A + \csc[e + f x]) \csc[e + f x] (B + \csc[e + f x])^2 (C + \sqrt{\csc[e + f x] (d + \csc[e + f x])} \sqrt{\csc[e + f x] (b + a)})], x, x] \text{ :> } \text{Dist}[C/d^2, \text{Int}[(d \csc[e + f x])^{3/2} / \sqrt{a + b \csc[e + f x]}], x, x] + \text{Int}[(A + B \csc[e + f x]) / (\sqrt{d \csc[e + f x]} \sqrt{a + b \csc[e + f x]}), x] \ /; \text{FreeQ}\{a, b, d, e, f, A, B, C\}, x] \ \&\& \ \text{NeQ}[a^2 - b^2, 0]$

Rubi steps

$$\begin{aligned}
 \int \frac{(a + b \sec(c + dx))^{5/2} (A + B \sec(c + dx))}{\sqrt{\sec(c + dx)}} dx &= \frac{bB \sqrt{\sec(c + dx)} (a + b \sec(c + dx))^{3/2} \sin(c + dx)}{2d} + \frac{1}{2} \int \frac{\sqrt{a + b \sec(c + dx)}}{\sqrt{\sec(c + dx)}} dx \\
 &= \frac{b(4Ab + 7aB) \sqrt{\sec(c + dx)} \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{4d} + \frac{b}{2} \int \frac{\sqrt{a + b \sec(c + dx)}}{\sqrt{\sec(c + dx)}} dx \\
 &= \frac{b(4Ab + 7aB) \sqrt{\sec(c + dx)} \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{4d} + \frac{b}{2} \int \frac{\sqrt{a + b \sec(c + dx)}}{\sqrt{\sec(c + dx)}} dx \\
 &= \frac{b(4Ab + 7aB) \sqrt{\sec(c + dx)} \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{4d} + \frac{b}{2} \int \frac{\sqrt{a + b \sec(c + dx)}}{\sqrt{\sec(c + dx)}} dx \\
 &= \frac{b(4Ab + 7aB) \sqrt{\sec(c + dx)} \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{4d} + \frac{b}{2} \int \frac{\sqrt{a + b \sec(c + dx)}}{\sqrt{\sec(c + dx)}} dx \\
 &= \frac{b(20aAb + 15a^2B + 4b^2B) \sqrt{\frac{b+a \cos(c+dx)}{a+b}} \Pi\left(2; \frac{1}{2}(c + dx) \middle| \frac{2a}{a+b}\right)}{4d \sqrt{a + b \sec(c + dx)}} \\
 &= \frac{(16a^2Ab + 4Ab^3 + 8a^3B + 11ab^2B) \sqrt{\frac{b+a \cos(c+dx)}{a+b}} F\left(\frac{1}{2}(c + dx) \middle| \frac{2a}{a+b}\right)}{4d \sqrt{a + b \sec(c + dx)}}
 \end{aligned}$$

Mathematica [C] time = 6.87, size = 628, normalized size = 1.75

$$\frac{(a + b \sec(c + dx))^{5/2} \left(\frac{1}{4} \sec(c + dx) (9abB \sin(c + dx) + 4Ab^2 \sin(c + dx)) + \frac{1}{2} b^2 B \tan(c + dx) \sec(c + dx) \right)}{d \sec^2(c + dx) (a \cos(c + dx) + b)^2} + \dots$$

Warning: Unable to verify antiderivative.

[In] Integrate[((a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Sqrt[Sec[c + d*x]], x]

[Out] ((a + b*Sec[c + d*x])^(5/2)*((2*(48*a^2*A*b + 16*a^3*B + 4*a*b^2*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]/Sqrt[b + a*Cos[c + d*x]] + (2*(8*a^3*A + 36*a*A*b^2 + 21*a^2*b*B + 8*b^3*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]/Sqrt[b + a*Cos[c + d*x]] + ((2*I)*(8*a^3*A - 4*a*A*b^2 - 9*a^2*b*B)*Sqrt[(a - a*Cos[c + d*x])/(a + b)]*Sqrt[(a + a*Cos[c + d*x])/(a - b)]*Cos[2*(c + d*x)]*(-2*b*(a + b)*EllipticE[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*(2*b*EllipticF[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*EllipticPi[1 - a/b, I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)]))*Sin[c + d*x])/(Sqrt[(a - b)^(-1)]*b*Sqrt[1 - Cos[c + d*x]^2]*Sqrt[(a^2 - a^2*Cos[c + d*x]^2)/a^2]*(-a^2 + 2*b^2 - 4*b*(b + a*Cos[c + d*x]) + 2*(b + a*Cos[c + d*x])^2))))/(16*d*(b + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2)) + ((a + b*Sec[c + d*x])^(5/2)*((Sec[c + d*x]*(4*A*b^2*Sin[c + d*x] + 9*a*b*B*Sin[c + d*x]))/4 + (b^2*B*Sec[c + d*x]*Tan[c + d*x])/2))/(d*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^(5/2))

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2), x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(b \sec(dx + c) + a)^{5/2}}{\sqrt{\sec(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)/sqrt(sec(d*x + c)), x)

maple [C] time = 2.27, size = 3939, normalized size = 10.97

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x)

[Out]
$$-1/4/d*(2*B*\cos(d*x+c)^2*((a-b)/(a+b))^{1/2}*b^3+8*A*((a-b)/(a+b))^{1/2}*\cos(d*x+c)^3*a^2*b-4*A*((a-b)/(a+b))^{1/2}*\cos(d*x+c)^2*a*b^2-9*B*((a-b)/(a+b))^{1/2}*\cos(d*x+c)^2*a^2*b-8*A*\cos(d*x+c)^3*((a-b)/(a+b))^{1/2}*a^3+4*A*\cos(d*x+c)^2*((a-b)/(a+b))^{1/2}*b^3-2*B*((a-b)/(a+b))^{1/2}*b^3+40*A*\cos(d*x+c)^3*\sin(d*x+c)*(1/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*EllipticPi((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (a+b)/(a-b), I/((a-b)/(a+b))^{1/2})*a*b^2+30*B*\cos(d*x+c)^3*\sin(d*x+c)*(1/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*EllipticPi((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (a+b)/(a-b), I/((a-b)/(a+b))^{1/2}))*a^2*b+24*A*\cos(d*x+c)^2*\sin(d*x+c)*EllipticF((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (-a+b)/(a-b))^{1/2}*(1/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*a^2*b-16*A*\cos(d*x+c)^2*\sin(d*x+c)*EllipticF((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (-a+b)/(a-b))^{1/2}*(1/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*a*b^2+40*A*\cos(d*x+c)^2*\sin(d*x+c)*(1/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*EllipticPi((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (a+b)/(a-b), I/((a-b)/(a+b))^{1/2}))*a*b^2-8*A*\cos(d*x+c)^2*\sin(d*x+c)*(1/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*EllipticE((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (-a+b)/(a-b))^{1/2})*a^2*b+8*A*((a-b)/(a+b))^{1/2}*\cos(d*x+c)^4*a^3-4*A*((a-b)/(a+b))^{1/2}*\cos(d*x+c)*b^3-11*B*((a-b)/(a+b))^{1/2}*\cos(d*x+c)*a*b^2-4*A*\cos(d*x+c)^2*\sin(d*x+c)*(1/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*EllipticE((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (-a+b)/(a-b))^{1/2}))*a*b^2-6*B*\cos(d*x+c)^2*\sin(d*x+c)*EllipticF((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (-a+b)/(a-b))^{1/2}*(1/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*a^2*b+2*B*\cos(d*x+c)^2*\sin(d*x+c)*EllipticF((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (-a+b)/(a-b))^{1/2}*(1/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*a*b^2+30*B*\cos(d*x+c)^2*\sin(d*x+c)*(1/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*EllipticPi((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (a+b)/(a-b), I/((a-b)/(a+b))^{1/2}))*a^2*b-9*B*\cos(d*x+c)^2*\sin(d*x+c)*(1/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a$$

$$\begin{aligned}
& +b)^{(1/2)} * \text{EllipticE}((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{(1/2)} / \sin(d*x+c), (-a+b) \\
& / (a-b))^{(1/2)} * a^2 * b + 9 * B * \cos(d*x+c)^2 * \sin(d*x+c) * (1/(1+\cos(d*x+c)))^{(1/2)} * (\\
& (b+a*\cos(d*x+c)) / (1+\cos(d*x+c)) / (a+b))^{(1/2)} * \text{EllipticE}((-1+\cos(d*x+c)) * ((a- \\
& b)/(a+b))^{(1/2)} / \sin(d*x+c), (-a+b)/(a-b))^{(1/2)} * a * b^2 - 8 * A * \sin(d*x+c) * \text{Ellip} \\
& \text{ticF}((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{(1/2)} / \sin(d*x+c), (-a+b)/(a-b))^{(1/2)} * (\\
& 1/(1+\cos(d*x+c)))^{(1/2)} * ((b+a*\cos(d*x+c)) / (1+\cos(d*x+c)) / (a+b))^{(1/2)} * \cos(d \\
& *x+c)^3 * a^3 - 8 * A * \cos(d*x+c)^2 * ((a-b)/(a+b))^{(1/2)} * a^2 * b + 9 * B * \cos(d*x+c)^2 * (a \\
& -b)/(a+b))^{(1/2)} * a * b^2 + 24 * A * \text{EllipticF}((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{(1/2)} / s \\
& \sin(d*x+c), (-a+b)/(a-b))^{(1/2)} * (1/(1+\cos(d*x+c)))^{(1/2)} * ((b+a*\cos(d*x+c)) / \\
& (1+\cos(d*x+c)) / (a+b))^{(1/2)} * \sin(d*x+c) * \cos(d*x+c)^3 * a^2 * b - 16 * A * \text{EllipticF}((- \\
& 1+\cos(d*x+c)) * ((a-b)/(a+b))^{(1/2)} / \sin(d*x+c), (-a+b)/(a-b))^{(1/2)} * (1/(1+co \\
& s(d*x+c)))^{(1/2)} * ((b+a*\cos(d*x+c)) / (1+\cos(d*x+c)) / (a+b))^{(1/2)} * \sin(d*x+c) * c \\
& \cos(d*x+c)^3 * a * b^2 - 8 * A * (1/(1+\cos(d*x+c)))^{(1/2)} * ((b+a*\cos(d*x+c)) / (1+\cos(d*x \\
& +c)) / (a+b))^{(1/2)} * \text{EllipticE}((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{(1/2)} / \sin(d*x+c), \\
& (-a+b)/(a-b))^{(1/2)} * \sin(d*x+c) * \cos(d*x+c)^3 * a^2 * b - 4 * A * (1/(1+\cos(d*x+c)))^{(\\
& 1/2)} * ((b+a*\cos(d*x+c)) / (1+\cos(d*x+c)) / (a+b))^{(1/2)} * \text{EllipticE}((-1+\cos(d*x+c \\
&)) * ((a-b)/(a+b))^{(1/2)} / \sin(d*x+c), (-a+b)/(a-b))^{(1/2)} * \sin(d*x+c) * \cos(d*x+ \\
& c)^3 * a * b^2 - 6 * B * \text{EllipticF}((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{(1/2)} / \sin(d*x+c), (- \\
& a+b)/(a-b))^{(1/2)} * (1/(1+\cos(d*x+c)))^{(1/2)} * ((b+a*\cos(d*x+c)) / (1+\cos(d*x+c) \\
&)) / (a+b))^{(1/2)} * \sin(d*x+c) * \cos(d*x+c)^3 * a^2 * b + 2 * B * \text{EllipticF}((-1+\cos(d*x+c)) * \\
& ((a-b)/(a+b))^{(1/2)} / \sin(d*x+c), (-a+b)/(a-b))^{(1/2)} * (1/(1+\cos(d*x+c)))^{(1/ \\
& 2)} * ((b+a*\cos(d*x+c)) / (1+\cos(d*x+c)) / (a+b))^{(1/2)} * \sin(d*x+c) * \cos(d*x+c)^3 * a \\
& b^2 - 9 * B * (1/(1+\cos(d*x+c)))^{(1/2)} * ((b+a*\cos(d*x+c)) / (1+\cos(d*x+c)) / (a+b))^{(1 \\
& /2)} * \text{EllipticE}((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{(1/2)} / \sin(d*x+c), (-a+b)/(a-b)) \\
& ^{(1/2)} * \sin(d*x+c) * \cos(d*x+c)^3 * a^2 * b + 9 * B * (1/(1+\cos(d*x+c)))^{(1/2)} * ((b+a*co \\
& s(d*x+c)) / (1+\cos(d*x+c)) / (a+b))^{(1/2)} * \text{EllipticE}((-1+\cos(d*x+c)) * ((a-b)/(a+b \\
&))^{(1/2)} / \sin(d*x+c), (-a+b)/(a-b))^{(1/2)} * \sin(d*x+c) * \cos(d*x+c)^3 * a * b^2 + 8 * B \\
& * \text{EllipticF}((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{(1/2)} / \sin(d*x+c), (-a+b)/(a-b))^{(1 \\
& /2)} * (1/(1+\cos(d*x+c)))^{(1/2)} * ((b+a*\cos(d*x+c)) / (1+\cos(d*x+c)) / (a+b))^{(1/2)} \\
& * \sin(d*x+c) * \cos(d*x+c)^3 * a^3 + 8 * A * \cos(d*x+c)^2 * \sin(d*x+c) * (1/(1+\cos(d*x+c))) \\
& ^{(1/2)} * ((b+a*\cos(d*x+c)) / (1+\cos(d*x+c)) / (a+b))^{(1/2)} * \text{EllipticE}((-1+\cos(d*x+ \\
& c)) * ((a-b)/(a+b))^{(1/2)} / \sin(d*x+c), (-a+b)/(a-b))^{(1/2)} * a^3 + 4 * A * \cos(d*x+c) \\
& ^2 * \sin(d*x+c) * (1/(1+\cos(d*x+c)))^{(1/2)} * ((b+a*\cos(d*x+c)) / (1+\cos(d*x+c)) / (a+ \\
& b))^{(1/2)} * \text{EllipticE}((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{(1/2)} / \sin(d*x+c), (-a+b)/ \\
& (a-b))^{(1/2)} * b^3 + 8 * A * \cos(d*x+c)^3 * \sin(d*x+c) * (1/(1+\cos(d*x+c)))^{(1/2)} * ((b+ \\
& a*\cos(d*x+c)) / (1+\cos(d*x+c)) / (a+b))^{(1/2)} * \text{EllipticE}((-1+\cos(d*x+c)) * ((a-b)/ \\
& (a+b))^{(1/2)} / \sin(d*x+c), (-a+b)/(a-b))^{(1/2)} * a^3 + 4 * A * \cos(d*x+c)^3 * \sin(d*x+ \\
& c) * (1/(1+\cos(d*x+c)))^{(1/2)} * ((b+a*\cos(d*x+c)) / (1+\cos(d*x+c)) / (a+b))^{(1/2)} * \text{E} \\
& \text{llipticE}((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{(1/2)} / \sin(d*x+c), (-a+b)/(a-b))^{(1/2)} \\
&)) * b^3 - 4 * B * \cos(d*x+c)^3 * \sin(d*x+c) * \text{EllipticF}((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{(\\
& 1/2)} / \sin(d*x+c), (-a+b)/(a-b))^{(1/2)} * (1/(1+\cos(d*x+c)))^{(1/2)} * ((b+a*\cos(d \\
& *x+c)) / (1+\cos(d*x+c)) / (a+b))^{(1/2)} * b^3 + 8 * B * \cos(d*x+c)^3 * \sin(d*x+c) * (1/(1+co \\
& s(d*x+c)))^{(1/2)} * ((b+a*\cos(d*x+c)) / (1+\cos(d*x+c)) / (a+b))^{(1/2)} * \text{EllipticPi}((\\
& -1+\cos(d*x+c)) * ((a-b)/(a+b))^{(1/2)} / \sin(d*x+c), (a+b)/(a-b), I / ((a-b)/(a+b))^{(\\
& 1/2)} * b^3 - 8 * A * \cos(d*x+c)^2 * \sin(d*x+c) * \text{EllipticF}((-1+\cos(d*x+c)) * ((a-b)/(a+b)
\end{aligned}$$

$$\left. \right)^{(1/2)}/\sin(dx+c), (-\frac{a+b}{a-b})^{(1/2)} * \left(\frac{1}{1+\cos(dx+c)} \right)^{(1/2)} * \left(\frac{b+a\cos(dx+c)}{1+\cos(dx+c)} \right) / (a+b)^{(1/2)} * a^3 + 8B\cos(dx+c)^2 * \sin(dx+c) * \text{EllipticF}\left(\frac{-1+\cos(dx+c)}{2}, \frac{(a-b)/(a+b)^{(1/2)}}{\sin(dx+c)}, \frac{-(a+b)/(a-b)^{(1/2)}}{2} \right) * \left(\frac{1}{1+\cos(dx+c)} \right)^{(1/2)} * \left(\frac{b+a\cos(dx+c)}{1+\cos(dx+c)} \right) / (a+b)^{(1/2)} * a^3 - 4B\cos(dx+c)^2 * \sin(dx+c) * \text{EllipticF}\left(\frac{-1+\cos(dx+c)}{2}, \frac{(a-b)/(a+b)^{(1/2)}}{\sin(dx+c)}, \frac{-(a+b)/(a-b)^{(1/2)}}{2} \right) * \left(\frac{1}{1+\cos(dx+c)} \right)^{(1/2)} * \left(\frac{b+a\cos(dx+c)}{1+\cos(dx+c)} \right) / (a+b)^{(1/2)} * b^3 + 8B\cos(dx+c)^2 * \sin(dx+c) * \left(\frac{1}{1+\cos(dx+c)} \right)^{(1/2)} * \left(\frac{b+a\cos(dx+c)}{1+\cos(dx+c)} \right) / (a+b)^{(1/2)} * \text{EllipticPi}\left(\frac{-1+\cos(dx+c)}{2}, \frac{(a-b)/(a+b)^{(1/2)}}{\sin(dx+c)}, \frac{(a+b)/(a-b)}{2}, \frac{1}{\left(\frac{a-b}{a+b} \right)^{(1/2)} * b^3 + 4A\cos(dx+c)^3 * \left(\frac{a-b}{a+b} \right)^{(1/2)} * a * b^2 + 9B\cos(dx+c)^3 * \left(\frac{a-b}{a+b} \right)^{(1/2)} * a^2 * b + 2B\cos(dx+c)^3 * \left(\frac{a-b}{a+b} \right)^{(1/2)} * a * b^2} \right) * \left(\frac{b+a\cos(dx+c)}{\cos(dx+c)} \right) / \cos(dx+c)^{(1/2)} * \left(\frac{1}{\cos(dx+c)} \right)^{(1/2)} / \sin(dx+c) / \left(\frac{b+a\cos(dx+c)}{\cos(dx+c)} \right) / \cos(dx+c) / \left(\frac{a-b}{a+b} \right)^{(1/2)}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx+c) + A)(b \sec(dx+c) + a)^{5/2}}{\sqrt{\sec(dx+c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(dx+c))^(5/2)*(A+B*sec(dx+c))/sec(dx+c)^(1/2),x, algorithm="maxima")

[Out] integrate((B*sec(dx+c) + A)*(b*sec(dx+c) + a)^(5/2)/sqrt(sec(dx+c)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)} \right) \left(a + \frac{b}{\cos(c+dx)} \right)^{5/2}}{\sqrt{\frac{1}{\cos(c+dx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + dx))*(a + b/cos(c + dx))^(5/2))/(1/cos(c + dx))^(1/2),x)

[Out] int(((A + B/cos(c + dx))*(a + b/cos(c + dx))^(5/2))/(1/cos(c + dx))^(1/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.


```
[In] integrate((a+b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

$$3.451 \quad \int \frac{(a+b \sec(c+dx))^{5/2} (A+B \sec(c+dx))}{\sec^2(c+dx)} dx$$

Optimal. Leaf size=349

$$\frac{(6a^2B + 14aAb - 3b^2B) \sqrt{a + b \sec(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2a}{a+b}\right)}{3d \sqrt{\sec(c + dx)} \sqrt{\frac{a \cos(c+dx)+b}{a+b}}} + \frac{(2a^3A + 12a^2bB + 4aAb^2 + 3b^3B) \sqrt{\sec(c + dx)}}{3d \sqrt{a + b \sec(c + dx)}}$$

[Out] $\frac{2}{3} a A (a+b \sec(dx+c))^{3/2} \sin(dx+c)/d / \sec(dx+c)^{1/2} + \frac{1}{3} (2A a^3 + 4A a^2 b + 12A a b^2 + 3B b^3) (\cos(1/2 dx + 1/2 c))^{1/2} / \cos(1/2 dx + 1/2 c) \operatorname{EllipticF}(\sin(1/2 dx + 1/2 c), 2^{1/2} (a/(a+b))^{1/2}) ((b+a \cos(dx+c))/(a+b))^{1/2} \sec(dx+c)^{1/2} / d / (a+b \sec(dx+c))^{1/2} + b^2 (2A a b + 5B a^2) (\cos(1/2 dx + 1/2 c))^{1/2} / \cos(1/2 dx + 1/2 c) \operatorname{EllipticPi}(\sin(1/2 dx + 1/2 c), 2, 2^{1/2} (a/(a+b))^{1/2}) ((b+a \cos(dx+c))/(a+b))^{1/2} \sec(dx+c)^{1/2} / d / (a+b \sec(dx+c))^{1/2} + \frac{1}{3} (14A a^2 b + 6A a b^2 - 3B b^3) (\cos(1/2 dx + 1/2 c))^{1/2} / \cos(1/2 dx + 1/2 c) \operatorname{EllipticE}(\sin(1/2 dx + 1/2 c), 2^{1/2} (a/(a+b))^{1/2}) (a+b \sec(dx+c))^{1/2} / d / ((b+a \cos(dx+c))/(a+b))^{1/2} / \sec(dx+c)^{1/2} - \frac{1}{3} b (2A a - 3B b) \sin(dx+c) \sec(dx+c)^{1/2} (a+b \sec(dx+c))^{1/2} / d$

Rubi [A] time = 1.25, antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {4025, 4096, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{(2a^3A + 12a^2bB + 4aAb^2 + 3b^3B) \sqrt{\sec(c + dx)} \sqrt{\frac{a \cos(c+dx)+b}{a+b}} F\left(\frac{1}{2}(c + dx) \middle| \frac{2a}{a+b}\right)}{3d \sqrt{a + b \sec(c + dx)}} + \frac{(6a^2B + 14aAb - 3b^2B) \sqrt{a + b \sec(c + dx)}}{3d \sqrt{\sec(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[((a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(3/2),x]

[Out] $((2a^3A + 4a^2Ab + 12a^2bB + 3b^3B) \operatorname{Sqrt}[(b + a \cos[c + d*x])/(a + b)] \operatorname{EllipticF}[(c + d*x)/2, (2a)/(a + b)] \operatorname{Sqrt}[\operatorname{Sec}[c + d*x]]) / (3d \operatorname{Sqrt}[a + b \operatorname{Sec}[c + d*x]]) + (b^2 (2A a b + 5A^2) \operatorname{Sqrt}[(b + a \cos[c + d*x])/(a + b)] \operatorname{EllipticPi}[2, (c + d*x)/2, (2a)/(a + b)] \operatorname{Sqrt}[\operatorname{Sec}[c + d*x]]) / (d \operatorname{Sqrt}[a + b \operatorname{Sec}[c + d*x]]) + ((14A a^2 b + 6A a b^2 - 3B b^3) \operatorname{EllipticE}[(c + d*x)/2, (2a)/(a + b)] \operatorname{Sqrt}[a + b \operatorname{Sec}[c + d*x]]) / (3d \operatorname{Sqrt}[(b + a \cos[c + d*x])/(a + b)] \operatorname{Sqrt}[\operatorname{Sec}[c + d*x]]) - (b (2A a - 3B b) \operatorname{Sqrt}[\operatorname{Sec}[c + d*x]] \operatorname{Sqrt}[a + b \operatorname{Sec}[c + d*x]] \operatorname{Sin}[c + d*x]) / (3d) + (2a^3A (a + b \operatorname{Sec}[c + d*x])^{3/2} \operatorname{Sin}[c + d*x]) / (3d \operatorname{Sqrt}[\operatorname{Sec}[c + d*x]])$

Rule 2653

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*Sqrt[a
+ b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
0] && !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)]/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a
+ b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_)
+ (f_)*(x_)]]), x_Symbol] := Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)]/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_)
+ (f_)*(x_)]]), x_Symbol] := Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
+ f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !GtQ[c + d, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)]/Sqrt[csc[(e_) + (f_)*(x_)]]
```

$\ast(d_)]$, x_Symbol] \rightarrow Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 3858

Int[Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] \rightarrow Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 3859

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] \rightarrow Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 4025

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] \rightarrow Simp[(a*A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e + f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LeQ[n, -1]

Rule 4035

Int[(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.))/(Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]), x_Symbol] \rightarrow Dist[A/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]

Rule 4096

Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_), x_Symbol] \rightarrow -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(m + n + 1), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n*Simp[a*A*(m + n + 1) + a*C*n + ((A*b + a*B)*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) + a*C*m)*Csc[e

+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && !LeQ[n, -1]

Rule 4108

Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.)) / (Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x]) / (Sqrt[d*Csc[e + f*x]]*Sqrt[a + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{(a + b \sec(c + dx))^{5/2} (A + B \sec(c + dx))}{\sec^3(c + dx)} dx &= \frac{2aA(a + b \sec(c + dx))^{3/2} \sin(c + dx)}{3d\sqrt{\sec(c + dx)}} - \frac{2}{3} \int \frac{\sqrt{a + b \sec(c + dx)}}{\sec^2(c + dx)} dx \\
 &= -\frac{b(2aA - 3bB)\sqrt{\sec(c + dx)} \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{3d} \\
 &= -\frac{b(2aA - 3bB)\sqrt{\sec(c + dx)} \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{3d} \\
 &= -\frac{b(2aA - 3bB)\sqrt{\sec(c + dx)} \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{3d} \\
 &= -\frac{b(2aA - 3bB)\sqrt{\sec(c + dx)} \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{3d} \\
 &= \frac{b^2(2Ab + 5aB)\sqrt{\frac{b+a \cos(c+dx)}{a+b}} \Pi\left(2; \frac{1}{2}(c + dx) \middle| \frac{2a}{a+b}\right) \sqrt{\sec(c + dx)}}{d\sqrt{a + b \sec(c + dx)}} \\
 &= \frac{(2a^3A + 4aAb^2 + 12a^2bB + 3b^3B) \sqrt{\frac{b+a \cos(c+dx)}{a+b}} F\left(\frac{1}{2}(c + dx)\right)}{3d\sqrt{a + b \sec(c + dx)}}
 \end{aligned}$$

Mathematica [C] time = 7.02, size = 599, normalized size = 1.72

$$\frac{(a + b \sec(c + dx))^{5/2} \left(\frac{2}{3} a^2 A \sin(c + dx) + b^2 B \tan(c + dx) \right)}{d \sec^2(c + dx) (a \cos(c + dx) + b)^2} + \frac{(a + b \sec(c + dx))^{5/2} \left(\frac{2(4a^3 A + 36a^2 b B + 36a A b^2) \sqrt{\frac{a \cos(c + dx)}{a + b}}}{\sqrt{a \cos(c + dx) + b}} \right)}{d \sec^2(c + dx) (a \cos(c + dx) + b)^2}$$

Warning: Unable to verify antiderivative.

[In] Integrate[((a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(3/2), x]

[Out] ((a + b*Sec[c + d*x])^(5/2)*((2*(4*a^3*A + 36*a*A*b^2 + 36*a^2*b*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]/Sqrt[b + a*Cos[c + d*x]] + (2*(14*a^2*A*b + 12*A*b^3 + 6*a^3*B + 27*a*b^2*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]/Sqrt[b + a*Cos[c + d*x]] + ((2*I)*(14*a^2*A*b + 6*a^3*B - 3*a*b^2*B)*Sqrt[(a - a*Cos[c + d*x])/(a + b)]*Sqrt[(a + a*Cos[c + d*x])/(a - b)]*Cos[2*(c + d*x)]*(-2*b*(a + b)*EllipticE[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*(2*b*EllipticF[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*EllipticPi[1 - a/b, I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)])))*Sin[c + d*x])/(Sqrt[(a - b)^(-1)]*b*Sqrt[1 - Cos[c + d*x]^2]*Sqrt[(a^2 - a^2*Cos[c + d*x]^2)/a^2]*(-a^2 + 2*b^2 - 4*b*(b + a*Cos[c + d*x]) + 2*(b + a*Cos[c + d*x])^2)))/(12*d*(b + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2)) + ((a + b*Sec[c + d*x])^(5/2)*((2*a^2*A*Sin[c + d*x])/3 + b^2*B*Tan[c + d*x]))/(d*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^(5/2))

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2), x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(b \sec(dx + c) + a)^{5/2}}{\sec(dx + c)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(3/2), x)

maple [C] time = 2.25, size = 3663, normalized size = 10.50

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x)

[Out]
$$-1/3/d*(-14*A*\sin(d*x+c)*(1/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*\text{EllipticF}((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (-a+b)/(a-b))^{1/2})*\cos(d*x+c)*a^2*b+18*A*\sin(d*x+c)*(1/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*\text{EllipticF}((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (-a+b)/(a-b))^{1/2})*\cos(d*x+c)*a*b^2-2*A*((a-b)/(a+b))^{1/2}*\cos(d*x+c)*a^2*b+16*A*((a-b)/(a+b))^{1/2}*\cos(d*x+c)^3*a^2*b+14*A*((a-b)/(a+b))^{1/2}*\cos(d*x+c)^2*a*b^2+6*B*((a-b)/(a+b))^{1/2}*\cos(d*x+c)^2*a^2*b-14*A*((a-b)/(a+b))^{1/2}*\cos(d*x+c)*a*b^2-6*B*((a-b)/(a+b))^{1/2}*\cos(d*x+c)*a^2*b-3*B*((a-b)/(a+b))^{1/2}*b^3-14*A*\cos(d*x+c)^2*\sin(d*x+c)*\text{EllipticF}((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (-a+b)/(a-b))^{1/2})*(1/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*a^2*b+18*A*\cos(d*x+c)^2*\sin(d*x+c)*\text{EllipticF}((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (-a+b)/(a-b))^{1/2})*(1/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*a*b^2+14*A*\cos(d*x+c)^2*\sin(d*x+c)*(1/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*\text{EllipticE}((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (-a+b)/(a-b))^{1/2})*a^2*b-6*B*((a-b)/(a+b))^{1/2}*\cos(d*x+c)^2*a^3+3*B*((a-b)/(a+b))^{1/2}*\cos(d*x+c)*b^3+2*A*((a-b)/(a+b))^{1/2}*\cos(d*x+c)^4*a^3+6*B*((a-b)/(a+b))^{1/2}*\cos(d*x+c)^3*a^3-2*A*((a-b)/(a+b))^{1/2}*\cos(d*x+c)^2*a^3-12*B*(1/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*\text{EllipticF}((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (-a+b)/(a-b))^{1/2})*\cos(d*x+c)*\sin(d*x+c)*a*b^2+14*A*\sin(d*x+c)*\text{EllipticE}((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (-a+b)/(a-b))^{1/2})*(1/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*\cos(d*x+c)*a^2*b-14*A*\sin(d*x+c)*\text{EllipticE}((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (-a+b)/(a-b))^{1/2})*(1/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*\cos(d*x+c)*a*b^2+18*B*\sin(d*x+c)*(1/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*\text{EllipticF}((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (-a+b)/(a-b))^{1/2})*\cos(d*x+c)*a^2*b-6*B*\sin(d*x+c)*\text{EllipticE}((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (-a+b)/(a-b))^{1/2})*(1/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*\cos(d*x+c)*a^2*b-3*B*\sin(d*x+c)*\text{EllipticE}((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c)$$

/2))*cos(d*x+c)^2*b^3+2*A*cos(d*x+c)^2*sin(d*x+c)*EllipticF((-1+cos(d*x+c))
 *((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(1+cos(d*x+c)))^(1
 /2)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*a^3-6*B*cos(d*x+c)^2*sin(
 d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-
 b))^(1/2))*(1/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))
 ^((1/2)*a^3)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)*(1/cos(d*x+c))^(
 3/2)/sin(d*x+c)/(b+a*cos(d*x+c))/((a-b)/(a+b))^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(b \sec(dx + c) + a)^{\frac{5}{2}}}{\sec(dx + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(a + \frac{b}{\cos(c+dx)}\right)^{5/2}}{\left(\frac{1}{\cos(c+dx)}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(5/2))/(1/cos(c + d*x))^(3/2),x)

[Out] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(5/2))/(1/cos(c + d*x))^(3/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x)

[Out] Timed out

$$3.452 \quad \int \frac{(a+b \sec(c+dx))^{5/2} (A+B \sec(c+dx))}{\sec^2(c+dx)} dx$$

Optimal. Leaf size=342

$$\frac{2(9a^2A + 35abB + 23Ab^2) \sqrt{a+b \sec(c+dx)} E\left(\frac{1}{2}(c+dx) \middle| \frac{2a}{a+b}\right)}{15d \sqrt{\sec(c+dx)} \sqrt{\frac{a \cos(c+dx)+b}{a+b}}} + \frac{2(5a^3B + 8a^2Ab + 10ab^2B - 8Ab^3) \sqrt{\sec(c+dx)}}{15d \sqrt{a+b \sec(c+dx)}}$$

[Out] $2/5*a*A*(a+b*\sec(d*x+c))^{3/2}*\sin(d*x+c)/d/\sec(d*x+c)^{(3/2)}+2/15*(8*A*a^2*b-8*A*b^3+5*B*a^3+10*B*a*b^2)*(\cos(1/2*d*x+1/2*c))^{1/2}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{1/2}*(a/(a+b))^{1/2})*((b+a*\cos(d*x+c))/(a+b))^{1/2}*\sec(d*x+c)^{(1/2)}/d/(a+b*\sec(d*x+c))^{1/2}+2*b^3*B*(\cos(1/2*d*x+1/2*c))^{1/2}/\cos(1/2*d*x+1/2*c)*\text{EllipticPi}(\sin(1/2*d*x+1/2*c), 2^{1/2}*(a/(a+b))^{1/2})*((b+a*\cos(d*x+c))/(a+b))^{1/2}*\sec(d*x+c)^{(1/2)}/d/(a+b*\sec(d*x+c))^{1/2}+2/15*a*(8*A*b+5*B*a)*\sin(d*x+c)*(a+b*\sec(d*x+c))^{1/2}/d/\sec(d*x+c)^{(1/2)}+2/15*(9*A*a^2+23*A*b^2+35*B*a*b)*(\cos(1/2*d*x+1/2*c))^{1/2}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{1/2}*(a/(a+b))^{1/2})*((a+b*\sec(d*x+c))^{1/2})/d/((b+a*\cos(d*x+c))/(a+b))^{1/2}/\sec(d*x+c)^{(1/2)}$

Rubi [A] time = 1.22, antiderivative size = 342, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {4025, 4094, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2(8a^2Ab + 5a^3B + 10ab^2B - 8Ab^3) \sqrt{\sec(c+dx)} \sqrt{\frac{a \cos(c+dx)+b}{a+b}} F\left(\frac{1}{2}(c+dx) \middle| \frac{2a}{a+b}\right)}{15d \sqrt{a+b \sec(c+dx)}} + \frac{2(9a^2A + 35abB + 23Ab^2) \sqrt{\sec(c+dx)}}{15d \sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[((a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(5/2), x]

[Out] $(2*(8*a^2*A*b - 8*A*b^3 + 5*a^3*B + 10*a*b^2*B)*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])]/(a + b))*\text{EllipticF}[(c + d*x)/2, (2*a)/(a + b)]*\text{Sqrt}[\text{Sec}[c + d*x]]/(15*d*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]) + (2*b^3*B*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])]/(a + b))*\text{EllipticPi}[2, (c + d*x)/2, (2*a)/(a + b)]*\text{Sqrt}[\text{Sec}[c + d*x]]/(d*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]) + (2*(9*a^2*A + 23*A*b^2 + 35*a*b*B))*\text{EllipticE}[(c + d*x)/2, (2*a)/(a + b)]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]/(15*d*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])]/(a + b))*\text{Sqrt}[\text{Sec}[c + d*x]] + (2*a*(8*A*b + 5*a*B))*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]*\text{Sin}[c + d*x]/(15*d*\text{Sqrt}[\text{Sec}[c + d*x]]) + (2*a*A*(a + b*\text{Sec}[c + d*x])^{3/2}*\text{Sin}[c + d*x])/(5*d*\text{Sec}[c + d*x]^{3/2})$

Rule 2653

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*Sqrt[a
+ b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)]/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
0] && !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*Ellip
ticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)]/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a
+ b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_)
+ (f_)*(x_)]]), x_Symbol] := Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)]/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_)
+ (f_)*(x_)]]), x_Symbol] := Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
+ f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !GtQ[c + d, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)]/Sqrt[csc[(e_) + (f_)*(x_)]
*(d_)], x_Symbol] := Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
```

b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 3858

Int[Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 3859

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 4025

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> Simp[(a*A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e + f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LeQ[n, -1]

Rule 4035

Int[(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.))/(Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]), x_Symbol] :> Dist[A/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]

Rule 4094

Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]

Rule 4108

Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.)))/(Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]), x_Symbol] := Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{(a + b \sec(c + dx))^{5/2} (A + B \sec(c + dx))}{\sec^2(c + dx)} dx &= \frac{2aA(a + b \sec(c + dx))^{3/2} \sin(c + dx)}{5d \sec^2(c + dx)} - \frac{2}{5} \int \frac{\sqrt{a + b \sec(c + dx)}}{\sec^2(c + dx)} dx \\
 &= \frac{2a(8Ab + 5aB)\sqrt{a + b \sec(c + dx)} \sin(c + dx)}{15d\sqrt{\sec(c + dx)}} + \frac{2aA(a + b \sec(c + dx))^{3/2} \sin(c + dx)}{5d \sec^2(c + dx)} \\
 &= \frac{2a(8Ab + 5aB)\sqrt{a + b \sec(c + dx)} \sin(c + dx)}{15d\sqrt{\sec(c + dx)}} + \frac{2aA(a + b \sec(c + dx))^{3/2} \sin(c + dx)}{5d \sec^2(c + dx)} \\
 &= \frac{2a(8Ab + 5aB)\sqrt{a + b \sec(c + dx)} \sin(c + dx)}{15d\sqrt{\sec(c + dx)}} + \frac{2aA(a + b \sec(c + dx))^{3/2} \sin(c + dx)}{5d \sec^2(c + dx)} \\
 &= \frac{2a(8Ab + 5aB)\sqrt{a + b \sec(c + dx)} \sin(c + dx)}{15d\sqrt{\sec(c + dx)}} + \frac{2aA(a + b \sec(c + dx))^{3/2} \sin(c + dx)}{5d \sec^2(c + dx)} \\
 &= \frac{2a(8Ab + 5aB)\sqrt{a + b \sec(c + dx)} \sin(c + dx)}{15d\sqrt{\sec(c + dx)}} + \frac{2aA(a + b \sec(c + dx))^{3/2} \sin(c + dx)}{5d \sec^2(c + dx)} \\
 &= \frac{2b^3B\sqrt{\frac{b+a \cos(c+dx)}{a+b}} \Pi\left(2; \frac{1}{2}(c + dx) \middle| \frac{2a}{a+b}\right) \sqrt{\sec(c + dx)}}{d\sqrt{a + b \sec(c + dx)}} + \frac{2a(8Ab + 5aB)\sqrt{a + b \sec(c + dx)} \sin(c + dx)}{15d\sqrt{\sec(c + dx)}} \\
 &= \frac{2(8a^2Ab - 8Ab^3 + 5a^3B + 10ab^2B)\sqrt{\frac{b+a \cos(c+dx)}{a+b}} F\left(\frac{1}{2}(c + dx) \middle| \frac{2a}{a+b}\right)}{15d\sqrt{a + b \sec(c + dx)}} + \frac{2a(8Ab + 5aB)\sqrt{a + b \sec(c + dx)} \sin(c + dx)}{15d\sqrt{\sec(c + dx)}}
 \end{aligned}$$

Mathematica [C] time = 7.00, size = 616, normalized size = 1.80

$$\frac{(a + b \sec(c + dx))^{5/2} \left(\frac{1}{5} a^2 A \sin(2(c + dx)) + \frac{2}{15} a(5aB + 11Ab) \sin(c + dx) \right)}{d \sec^2(c + dx) (a \cos(c + dx) + b)^2} + \frac{(a + b \sec(c + dx))^{5/2} \left(\frac{2i(9a^3 A + 35a^2 b B)}{\dots} \right)}{\dots}$$

Warning: Unable to verify antiderivative.

[In] Integrate[((a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(5/2),x]

[Out] ((a + b*Sec[c + d*x])^(5/2)*((2*(34*a^2*A*b + 30*A*b^3 + 10*a^3*B + 90*a*b^2*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/Sqrt[b + a*Cos[c + d*x]] + (2*(9*a^3*A + 23*a*A*b^2 + 35*a^2*b*B + 30*b^3*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/Sqrt[b + a*Cos[c + d*x]] + ((2*I)*(9*a^3*A + 23*a*A*b^2 + 35*a^2*b*B)*Sqrt[(a - a*Cos[c + d*x])/(a + b)]*Sqrt[(a + a*Cos[c + d*x])/(a - b)]*Cos[2*(c + d*x)]*(-2*b*(a + b)*EllipticE[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*(2*b*EllipticF[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*EllipticPi[1 - a/b, I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)])))*Sin[c + d*x])/(Sqrt[(a - b)^(-1)]*b*Sqrt[1 - Cos[c + d*x]^2]*Sqrt[(a^2 - a^2*Cos[c + d*x]^2)/a^2]*(-a^2 + 2*b^2 - 4*b*(b + a*Cos[c + d*x]) + 2*(b + a*Cos[c + d*x])^2)))/(30*d*(b + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2)) + ((a + b*Sec[c + d*x])^(5/2)*((2*a*(11*A*b + 5*a*B)*Sin[c + d*x])/15 + (a^2*A*Sin[2*(c + d*x)]/5))/(d*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^(5/2))

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorith="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(b \sec(dx + c) + a)^{\frac{5}{2}}}{\sec(dx + c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorith
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(5/2
), x)
```

maple [C] time = 2.38, size = 3564, normalized size = 10.42

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x)
```

```
[Out] -2/15/d*(9*A*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)
/(a-b))^(1/2))*a^3*(1/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+
c)))/(a+b)^(1/2)*sin(d*x+c)+17*A*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*((b+a*
cos(d*x+c))/(1+cos(d*x+c)))/(a+b)^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a
+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*a^2*b+45*B*((b+a*cos
(d*x+c))/(1+cos(d*x+c)))/(a+b)^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2*sin
(d*x+c)-23*A*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d
*x+c)))/(a+b)^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*a*b^2-5*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)
*a^2*b+14*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*a^2*b+34*A*((a-b)/(a+b))^(1/2)
*cos(d*x+c)^2*a*b^2+40*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^2*b-23*A*((a-b)
/(a+b))^(1/2)*cos(d*x+c)*a*b^2-35*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^2*b-23
*A*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(
1/2))*b^3*(1/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))
^(1/2)*sin(d*x+c)+3*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^4*a^3+5*B*((a-b)/(a+b)
)^(1/2)*cos(d*x+c)^3*a^3-5*B*a^3*((a-b)/(a+b))^(1/2)*cos(d*x+c)+6*A*((a-b)/
(a+b))^(1/2)*cos(d*x+c)^2*a^3-9*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^3+23*A*((
a-b)/(a+b))^(1/2)*cos(d*x+c)*b^3+45*B*(1/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d
*x+c))/(1+cos(d*x+c)))/(a+b)^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(
1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^2-9*A*sin(
d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-
b))^(1/2))*1/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))
^(1/2)*cos(d*x+c)*a^2*b+23*A*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a
+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*1/(1+cos(d*x+c)))^(1/2)*((b+a*
cos(d*x+c))/(1+cos(d*x+c)))/(a+b)^(1/2)*cos(d*x+c)*a*b^2-35*B*sin(d*x+c)*(1
/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b)^(1/2)*Ellipt
icF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*co
s(d*x+c)*a^2*b+35*B*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)
)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*1/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c)
```


$-b)^{(1/2)} * b^3 + 30 * B * \sin(dx+c) * \cos(dx+c) * (1/(1+\cos(dx+c)))^{(1/2)} * ((b+a * \cos(dx+c))/(1+\cos(dx+c)) / (a+b))^{(1/2)} * \text{EllipticPi}((-1+\cos(dx+c)) * ((a-b)/(a+b))^{(1/2)} / \sin(dx+c), (a+b)/(a-b), I / ((a-b)/(a+b))^{(1/2)}) * b^3 + 15 * A * \sin(dx+c) * \text{EllipticF}((-1+\cos(dx+c)) * ((a-b)/(a+b))^{(1/2)} / \sin(dx+c), -(a+b)/(a-b))^{(1/2)} * (1/(1+\cos(dx+c)))^{(1/2)} * ((b+a * \cos(dx+c))/(1+\cos(dx+c)) / (a+b))^{(1/2)} * \cos(dx+c) * b^3 * ((b+a * \cos(dx+c))/\cos(dx+c))^{(1/2)} * \cos(dx+c)^3 * (1/\cos(dx+c))^{(5/2)} / \sin(dx+c) / (b+a * \cos(dx+c)) / ((a-b)/(a+b))^{(1/2)}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx+c) + A)(b \sec(dx+c) + a)^{5/2}}{\sec(dx+c)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(dx+c))^(5/2)*(A+B*sec(dx+c))/sec(dx+c)^(5/2),x, algorithm="maxima")

[Out] integrate((B*sec(dx+c) + A)*(b*sec(dx+c) + a)^(5/2)/sec(dx+c)^(5/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(a + \frac{b}{\cos(c+dx)}\right)^{5/2}}{\left(\frac{1}{\cos(c+dx)}\right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + dx))*(a + b/cos(c + dx))^(5/2))/(1/cos(c + dx))^(5/2),x)

[Out] int(((A + B/cos(c + dx))*(a + b/cos(c + dx))^(5/2))/(1/cos(c + dx))^(5/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(dx+c))**(5/2)*(A+B*sec(dx+c))/sec(dx+c)**(5/2),x)

[Out] Timed out

$$3.453 \quad \int \frac{(a+b \sec(c+dx))^{5/2}(A+B \sec(c+dx))}{\sec^2(c+dx)} dx$$

Optimal. Leaf size=340

$$\frac{2(25a^2A + 77abB + 45Ab^2) \sin(c+dx) \sqrt{a+b \sec(c+dx)}}{105d \sqrt{\sec(c+dx)}} + \frac{2(a^2 - b^2)(25a^2A + 56abB + 15Ab^2) \sqrt{\sec(c+dx)}}{105ad \sqrt{a+b \sec(c+dx)}}$$

[Out] $2/7*a*A*(a+b*\sec(d*x+c))^{3/2}*\sin(d*x+c)/d/\sec(d*x+c)^{(5/2)}+2/105*(a^2-b^2)*(25*A*a^2+15*A*b^2+56*B*a*b)*(\cos(1/2*d*x+1/2*c))^{1/2}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{1/2}*(a/(a+b))^{1/2})*((b+a*\cos(d*x+c))/(a+b))^{1/2}*\sec(d*x+c)^{(1/2)}/a/d/(a+b*\sec(d*x+c))^{1/2}+2/35*a*(10*A*b+7*B*a)*\sin(d*x+c)*(a+b*\sec(d*x+c))^{1/2}/d/\sec(d*x+c)^{(3/2)}+2/105*(25*A*a^2+45*A*b^2+77*B*a*b)*\sin(d*x+c)*(a+b*\sec(d*x+c))^{1/2}/d/\sec(d*x+c)^{(1/2)}+2/105*(145*A*a^2*b+15*A*b^3+63*B*a^3+161*B*a*b^2)*(\cos(1/2*d*x+1/2*c))^{1/2}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{1/2}*(a/(a+b))^{1/2})*(a+b*\sec(d*x+c))^{1/2}/a/d/((b+a*\cos(d*x+c))/(a+b))^{1/2}/\sec(d*x+c)^{(1/2)}$

Rubi [A] time = 1.16, antiderivative size = 340, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {4025, 4094, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2(25a^2A + 77abB + 45Ab^2) \sin(c+dx) \sqrt{a+b \sec(c+dx)}}{105d \sqrt{\sec(c+dx)}} + \frac{2(a^2 - b^2)(25a^2A + 56abB + 15Ab^2) \sqrt{\sec(c+dx)}}{105ad \sqrt{a+b \sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + b*\text{Sec}[c + d*x])^{5/2}*(A + B*\text{Sec}[c + d*x])]/\text{Sec}[c + d*x]^{7/2}, x]$

[Out] $(2*(a^2 - b^2)*(25*a^2*A + 15*A*b^2 + 56*a*b*B)*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])]/(a + b))*\text{EllipticF}[(c + d*x)/2, (2*a)/(a + b)]*\text{Sqrt}[\text{Sec}[c + d*x]]/(105*a*d*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]) + (2*(145*a^2*A*b + 15*A*b^3 + 63*a^3*B + 161*a*b^2*B))*\text{EllipticE}[(c + d*x)/2, (2*a)/(a + b)]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]/(105*a*d*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])]/(a + b))*\text{Sqrt}[\text{Sec}[c + d*x]] + (2*a*(10*A*b + 7*a*B))*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]*\text{Sin}[c + d*x]/(35*d*\text{Sec}[c + d*x]^{3/2}) + (2*(25*a^2*A + 45*A*b^2 + 77*a*b*B))*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]*\text{Sin}[c + d*x]/(105*d*\text{Sqrt}[\text{Sec}[c + d*x]]) + (2*a*A*(a + b*\text{Sec}[c + d*x])^{3/2}*\text{Sin}[c + d*x])/((7*d*\text{Sec}[c + d*x]^{5/2}))$

Rule 2653

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*Sqrt[a
+ b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)]/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
0] && !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)]/(d*Sqrt[a + b]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a
+ b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && !GtQ[a + b, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)]/Sqrt[csc[(e_) + (f_)*(x_)]
*(d_)], x_Symbol] := Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_) + (f_)*(x_)]*(d_)]/Sqrt[csc[(e_) + (f_)*(x_)]*(b_)
+ (a_)], x_Symbol] := Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4025

```
Int[(csc[(e_) + (f_)*(x_)]*(d_))^(n_)*(csc[(e_) + (f_)*(x_)]*(b_) + (
a_))^(m_)*(csc[(e_) + (f_)*(x_)]*(B_) + (A_)), x_Symbol] := Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
```

```
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.))/(Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]), x_Symbol] :> Dist[A/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m, x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m, x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + b \sec(c + dx))^{5/2} (A + B \sec(c + dx))}{\sec^2(c + dx)} dx &= \frac{2aA(a + b \sec(c + dx))^{3/2} \sin(c + dx)}{7d \sec^2(c + dx)} - \frac{2}{7} \int \frac{\sqrt{a + b \sec(c + dx)}}{\sec^2(c + dx)} dx \\
&= \frac{2a(10Ab + 7aB)\sqrt{a + b \sec(c + dx)} \sin(c + dx)}{35d \sec^2(c + dx)} + \frac{2aA(a + b \sec(c + dx))^{3/2} \sin(c + dx)}{7d \sec^2(c + dx)} \\
&= \frac{2a(10Ab + 7aB)\sqrt{a + b \sec(c + dx)} \sin(c + dx)}{35d \sec^2(c + dx)} + \frac{2(25a^2A + 15Ab^2 + 56abB)\sqrt{a + b \sec(c + dx)} \sin(c + dx)}{105ad \sec^2(c + dx)} \\
&= \frac{2a(10Ab + 7aB)\sqrt{a + b \sec(c + dx)} \sin(c + dx)}{35d \sec^2(c + dx)} + \frac{2(25a^2A + 15Ab^2 + 56abB)\sqrt{a + b \sec(c + dx)} \sin(c + dx)}{105ad \sec^2(c + dx)} \\
&= \frac{2a(10Ab + 7aB)\sqrt{a + b \sec(c + dx)} \sin(c + dx)}{35d \sec^2(c + dx)} + \frac{2(25a^2A + 15Ab^2 + 56abB)\sqrt{a + b \sec(c + dx)} \sin(c + dx)}{105ad \sec^2(c + dx)} \\
&= \frac{2a(10Ab + 7aB)\sqrt{a + b \sec(c + dx)} \sin(c + dx)}{35d \sec^2(c + dx)} + \frac{2(25a^2A + 15Ab^2 + 56abB)\sqrt{a + b \sec(c + dx)} \sin(c + dx)}{105ad \sec^2(c + dx)} \\
&= \frac{2(a^2 - b^2)(25a^2A + 15Ab^2 + 56abB)\sqrt{\frac{b+a \cos(c+dx)}{a+b}} F\left(\frac{1}{2}(c + dx), \sqrt{\frac{b+a \cos(c+dx)}{a+b}}\right)}{105ad \sqrt{a + b \sec(c + dx)}}
\end{aligned}$$

Mathematica [A] time = 1.83, size = 257, normalized size = 0.76

$$(a + b \sec(c + dx))^{5/2} \left(a \sin(c + dx)(a \cos(c + dx) + b) (15a^2A \cos(2(c + dx)) + 65a^2A + 6a(7aB + 15Ab) \cos(c + dx)) + \dots \right)$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(7/2), x]

[Out] ((a + b*Sec[c + d*x])^(5/2)*(2*Sqrt[(b + a*Cos[c + d*x])]/(a + b))*(a*(25*a^3*A + 135*a*A*b^2 + 119*a^2*b*B + 105*b^3*B)*EllipticF[(c + d*x)/2, (2*a)/(a + b)] + (145*a^2*A*b + 15*A*b^3 + 63*a^3*B + 161*a*b^2*B)*((a + b)*EllipticE[(c + d*x)/2, (2*a)/(a + b)] - b*EllipticF[(c + d*x)/2, (2*a)/(a + b)]))

+ a*(b + a*cos[c + d*x])*(65*a^2*A + 90*A*b^2 + 154*a*b*B + 6*a*(15*A*b + 7*a*B)*cos[c + d*x] + 15*a^2*A*cos[2*(c + d*x)]*sin[c + d*x]))/(105*a*d*(b + a*cos[c + d*x])^3*sec[c + d*x]^(5/2))

fricas [F] time = 1.06, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{(Bb^2 \sec(dx+c)^3 + Aa^2 + (2Bab + Ab^2) \sec(dx+c)^2 + (Ba^2 + 2Aab) \sec(dx+c)) \sqrt{b \sec(dx+c) + a}}{\sec(dx+c)^{\frac{7}{2}}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm="fricas")

[Out] integral((B*b^2*sec(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*sec(d*x + c)^2 + (B*a^2 + 2*A*a*b)*sec(d*x + c))*sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(7/2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx+c) + A)(b \sec(dx+c) + a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(7/2), x)

maple [B] time = 2.67, size = 3980, normalized size = 11.71

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x)

[Out] -2/105/d*(-63*B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b)^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+145*A*cos(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b)^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*a^3*b-15*A*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b

$$\begin{aligned}
& ^4*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*(1/(1+\cos(d*x+c)))^{(1/2)}*s \\
& \sin(d*x+c)-25*A*a^3*b*((a-b)/(a+b))^{(1/2)}-145*A*a^2*b^2*((a-b)/(a+b))^{(1/2)}- \\
& 45*A*a*b^3*((a-b)/(a+b))^{(1/2)}-63*B*a^3*b*((a-b)/(a+b))^{(1/2)}-77*B*a^2*b^2* \\
& ((a-b)/(a+b))^{(1/2)}-161*B*a*b^3*((a-b)/(a+b))^{(1/2)}+105*B*EllipticF((-1+\cos \\
& (d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c),(-(a+b)/(a-b))^{(1/2)})*((b+a*\cos(d*x \\
& +c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*(1/(1+\cos(d*x+c)))^{(1/2)}*\cos(d*x+c)*\sin(d* \\
& x+c)*a*b^3+21*B*\cos(d*x+c)^4*((a-b)/(a+b))^{(1/2)}*a^4+42*B*\cos(d*x+c)^2*((a- \\
& b)/(a+b))^{(1/2)}*a^4+15*A*\cos(d*x+c)*((a-b)/(a+b))^{(1/2)}*b^4-63*B*\cos(d*x+c) \\
& *((a-b)/(a+b))^{(1/2)}*a^4+15*A*\cos(d*x+c)^5*((a-b)/(a+b))^{(1/2)}*a^4+10*A*\cos \\
& (d*x+c)^3*((a-b)/(a+b))^{(1/2)}*a^4-25*A*\cos(d*x+c)*((a-b)/(a+b))^{(1/2)}*a^4-1 \\
& 5*A*b^4*((a-b)/(a+b))^{(1/2)}-15*A*\cos(d*x+c)*EllipticE((-1+\cos(d*x+c))*((a-b) \\
&)/(a+b))^{(1/2)}/\sin(d*x+c),(-(a+b)/(a-b))^{(1/2)})*((b+a*\cos(d*x+c))/(1+\cos(d* \\
& x+c)))/(a+b))^{(1/2)}*(1/(1+\cos(d*x+c)))^{(1/2)}*\sin(d*x+c)*b^4+25*A*\cos(d*x+c)* \\
& ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*(1/(1+\cos(d*x+c)))^{(1/2)}*Elli \\
& pticF((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c),(-(a+b)/(a-b))^{(1/2)})* \\
& \sin(d*x+c)*a^4+63*B*\cos(d*x+c)*EllipticE((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2) \\
&)/\sin(d*x+c),(-(a+b)/(a-b))^{(1/2)})*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(\\
& 1/2)}*(1/(1+\cos(d*x+c)))^{(1/2)}*\sin(d*x+c)*a^4-63*B*\cos(d*x+c)*((b+a*\cos(d*x \\
& +c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*(1/(1+\cos(d*x+c)))^{(1/2)}*EllipticF((-1+\cos \\
& (d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c),(-(a+b)/(a-b))^{(1/2)})*\sin(d*x+c)*a^ \\
& 4+145*A*EllipticE((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c),(-(a+b)/(a \\
& -b))^{(1/2)})*a^3*b*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*(1/(1+\cos(d \\
& *x+c)))^{(1/2)}*\sin(d*x+c)-145*A*EllipticE((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2) \\
&)/\sin(d*x+c),(-(a+b)/(a-b))^{(1/2)})*a^2*b^2*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)) \\
&)/(a+b))^{(1/2)}*(1/(1+\cos(d*x+c)))^{(1/2)}*\sin(d*x+c)+15*A*EllipticE((-1+\cos(d* \\
& x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c),(-(a+b)/(a-b))^{(1/2)})*a*b^3*((b+a*\cos(\\
& d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*(1/(1+\cos(d*x+c)))^{(1/2)}*\sin(d*x+c)-145 \\
& *A*EllipticF((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c),(-(a+b)/(a-b))^{(\\
& 1/2)})*a^3*b*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*(1/(1+\cos(d*x+c) \\
&))^{(1/2)}*\sin(d*x+c)+135*A*EllipticF((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin \\
& (d*x+c),(-(a+b)/(a-b))^{(1/2)})*a^2*b^2*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b \\
&))^{(1/2)}*(1/(1+\cos(d*x+c)))^{(1/2)}*\sin(d*x+c)-15*A*EllipticF((-1+\cos(d*x+c)) \\
& *((a-b)/(a+b))^{(1/2)}/\sin(d*x+c),(-(a+b)/(a-b))^{(1/2)})*a*b^3*((b+a*\cos(d*x+c) \\
&))/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*(1/(1+\cos(d*x+c)))^{(1/2)}*\sin(d*x+c)-63*B*Elli \\
& pticE((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c),(-(a+b)/(a-b))^{(1/2)}) \\
& *a^3*b*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*(1/(1+\cos(d*x+c)))^{(1/ \\
& 2)}*\sin(d*x+c)+161*B*EllipticE((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c \\
&),(-(a+b)/(a-b))^{(1/2)})*a^2*b^2*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/ \\
& 2)}*(1/(1+\cos(d*x+c)))^{(1/2)}*\sin(d*x+c)-161*B*EllipticE((-1+\cos(d*x+c))*((a- \\
& b)/(a+b))^{(1/2)}/\sin(d*x+c),(-(a+b)/(a-b))^{(1/2)})*a*b^3*((b+a*\cos(d*x+c))/(1 \\
& +\cos(d*x+c)))/(a+b))^{(1/2)}*(1/(1+\cos(d*x+c)))^{(1/2)}*\sin(d*x+c)+119*B*Elli \\
& cF((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c),(-(a+b)/(a-b))^{(1/2)})*a^3 \\
& *b*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*(1/(1+\cos(d*x+c)))^{(1/2)}*s \\
& \sin(d*x+c)-161*B*EllipticF((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c),(- \\
& (a+b)/(a-b))^{(1/2)})*a^2*b^2*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*(
\end{aligned}$$

$$\begin{aligned}
& 1/(1+\cos(d*x+c))^{1/2}*\sin(d*x+c)-145*A*\cos(d*x+c)*\text{EllipticE}((-1+\cos(d*x+c)) \\
&)*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (- (a+b)/(a-b))^{1/2})*((b+a*\cos(d*x+c))/(\\
& 1+\cos(d*x+c))/(a+b))^{1/2}*(1/(1+\cos(d*x+c)))^{1/2}*\sin(d*x+c)*a^2*b^2+15*A \\
& *\cos(d*x+c)*\text{EllipticE}((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (- (a+b) \\
&)/(a-b))^{1/2})*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*(1/(1+\cos(d*x \\
& +c)))^{1/2}*\sin(d*x+c)*a*b^3-145*A*\cos(d*x+c)*((b+a*\cos(d*x+c))/(1+\cos(d*x+ \\
& c))/(a+b))^{1/2}*(1/(1+\cos(d*x+c)))^{1/2}*\text{EllipticF}((-1+\cos(d*x+c))*((a-b)/ \\
& (a+b))^{1/2}/\sin(d*x+c), (- (a+b)/(a-b))^{1/2})*\sin(d*x+c)*a^3*b+135*A*\cos(d* \\
& x+c)*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*(1/(1+\cos(d*x+c)))^{1/2} \\
& *\text{EllipticF}((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (- (a+b)/(a-b))^{1/2} \\
&)*\sin(d*x+c)*a^2*b^2-15*A*\cos(d*x+c)*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a \\
& +b))^{1/2}*(1/(1+\cos(d*x+c)))^{1/2}*\text{EllipticF}((-1+\cos(d*x+c))*((a-b)/(a+b)) \\
& ^{1/2}/\sin(d*x+c), (- (a+b)/(a-b))^{1/2})*\sin(d*x+c)*a*b^3-63*B*\cos(d*x+c)*\text{El \\
& lipticE}((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (- (a+b)/(a-b))^{1/2} \\
&)*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*(1/(1+\cos(d*x+c)))^{1/2}*si \\
& n(d*x+c)*a^3*b+161*B*\cos(d*x+c)*\text{EllipticE}((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2} \\
&)/\sin(d*x+c), (- (a+b)/(a-b))^{1/2})*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b)) \\
& ^{1/2}*(1/(1+\cos(d*x+c)))^{1/2}*\sin(d*x+c)*a^2*b^2-161*B*\cos(d*x+c)*\text{Ellipti \\
& cE}((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (- (a+b)/(a-b))^{1/2})*((b \\
& +a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*(1/(1+\cos(d*x+c)))^{1/2}*\sin(d*x \\
& +c)*a*b^3+119*B*\cos(d*x+c)*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*(1 \\
& /(1+\cos(d*x+c)))^{1/2}*\text{EllipticF}((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d* \\
& x+c), (- (a+b)/(a-b))^{1/2})*\sin(d*x+c)*a^3*b-161*B*\cos(d*x+c)*((b+a*\cos(d*x+ \\
& c))/(1+\cos(d*x+c))/(a+b))^{1/2}*(1/(1+\cos(d*x+c)))^{1/2}*\text{EllipticF}((-1+\cos(\\
& d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (- (a+b)/(a-b))^{1/2})*\sin(d*x+c)*a^2 \\
& *b^2+105*B*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*(1/(1+\cos(d*x+c))) \\
& ^{1/2}*\text{EllipticF}((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (- (a+b)/(a- \\
& b))^{1/2})*a*b^3*\sin(d*x+c)+25*A*\text{EllipticF}((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2} \\
&)/\sin(d*x+c), (- (a+b)/(a-b))^{1/2})*a^4*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(\\
& a+b))^{1/2}*(1/(1+\cos(d*x+c)))^{1/2}*\sin(d*x+c)+63*B*\text{EllipticE}((-1+\cos(d*x+ \\
& c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (- (a+b)/(a-b))^{1/2})*a^4*((b+a*\cos(d*x+ \\
& c))/(1+\cos(d*x+c))/(a+b))^{1/2}*(1/(1+\cos(d*x+c)))^{1/2}*\sin(d*x+c)+90*A*co \\
& s(d*x+c)^3*((a-b)/(a+b))^{1/2})*a^2*b^2+98*B*\cos(d*x+c)^3*((a-b)/(a+b))^{1/2} \\
&)*a^3*b+110*A*\cos(d*x+c)^2*((a-b)/(a+b))^{1/2})*a^3*b+60*A*\cos(d*x+c)^2*((a- \\
& b)/(a+b))^{1/2})*a*b^3+238*B*\cos(d*x+c)^2*((a-b)/(a+b))^{1/2})*a^2*b^2-145*A* \\
& \cos(d*x+c)*((a-b)/(a+b))^{1/2})*a^3*b+55*A*\cos(d*x+c)*((a-b)/(a+b))^{1/2})*a^ \\
& 2*b^2-15*A*\cos(d*x+c)*((a-b)/(a+b))^{1/2})*a*b^3-35*B*\cos(d*x+c)*((a-b)/(a+b) \\
&))^{1/2})*a^3*b+60*A*\cos(d*x+c)^4*((a-b)/(a+b))^{1/2})*a^3*b-161*B*\cos(d*x+c) \\
& *((a-b)/(a+b))^{1/2})*a^2*b^2+161*B*\cos(d*x+c)*((a-b)/(a+b))^{1/2})*a*b^3*((\\
& b+a*\cos(d*x+c))/\cos(d*x+c))^{1/2}*\cos(d*x+c)^4*(1/\cos(d*x+c))^{7/2}/\sin(d*x \\
& +c)/(b+a*\cos(d*x+c))/a/((a-b)/(a+b))^{1/2}
\end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(b \sec(dx + c) + a)^{\frac{5}{2}}}{\sec(dx + c)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(7/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(a + \frac{b}{\cos(c+dx)}\right)^{5/2}}{\left(\frac{1}{\cos(c+dx)}\right)^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(5/2))/(1/cos(c + d*x))^(7/2),x)

[Out] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(5/2))/(1/cos(c + d*x))^(7/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)**(7/2),x)

[Out] Timed out

$$3.454 \quad \int \frac{(a+b \sec(c+dx))^{5/2}(A+B \sec(c+dx))}{\sec^2(c+dx)} dx$$

Optimal. Leaf size=425

$$\frac{2(49a^2A + 135abB + 75Ab^2) \sin(c+dx)\sqrt{a+b \sec(c+dx)}}{315d \sec^2(c+dx)} + \frac{2(75a^3B + 163a^2Ab + 135ab^2B + 5Ab^3) \sin(c+dx)}{315ad\sqrt{\sec(c+dx)}}$$

[Out] $2/9*a*A*(a+b*\sec(d*x+c))^{3/2}*\sin(d*x+c)/d/\sec(d*x+c)^{7/2}+2/315*(a^2-b^2)*(114*A*a^2*b-10*A*b^3+75*B*a^3+45*B*a*b^2)*(\cos(1/2*d*x+1/2*c))^2)^{1/2}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{1/2}*(a/(a+b))^{1/2})*((b+a*\cos(d*x+c))/(a+b))^{1/2}*\sec(d*x+c)^{1/2}/a^2/d/(a+b*\sec(d*x+c))^{1/2}+2/21*a*(4*A*b+3*B*a)*\sin(d*x+c)*(a+b*\sec(d*x+c))^{1/2}/d/\sec(d*x+c)^{5/2}+2/315*(49*A*a^2+75*A*b^2+135*B*a*b)*\sin(d*x+c)*(a+b*\sec(d*x+c))^{1/2}/d/\sec(d*x+c)^{3/2}+2/315*(163*A*a^2*b+5*A*b^3+75*B*a^3+135*B*a*b^2)*\sin(d*x+c)*(a+b*\sec(d*x+c))^{1/2}/a/d/\sec(d*x+c)^{1/2}+2/315*(147*A*a^4+279*A*a^2*b^2-10*A*b^4+435*B*a^3*b+45*B*a*b^3)*(\cos(1/2*d*x+1/2*c))^2)^{1/2}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{1/2}*(a/(a+b))^{1/2})*(a+b*\sec(d*x+c))^{1/2}/a^2/d/((b+a*\cos(d*x+c))/(a+b))^{1/2}/\sec(d*x+c)^{1/2}$

Rubi [A] time = 1.52, antiderivative size = 425, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {4025, 4094, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2(49a^2A + 135abB + 75Ab^2) \sin(c+dx)\sqrt{a+b \sec(c+dx)}}{315d \sec^2(c+dx)} + \frac{2(163a^2Ab + 75a^3B + 135ab^2B + 5Ab^3) \sin(c+dx)}{315ad\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[((a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(9/2), x]

[Out] $(2*(a^2 - b^2)*(114*a^2*A*b - 10*A*b^3 + 75*a^3*B + 45*a*b^2*B)*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/(a + b)]*\text{EllipticF}[(c + d*x)/2, (2*a)/(a + b)]*\text{Sqrt}[\text{Sec}[c + d*x]])/(315*a^2*d*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]) + (2*(147*a^4*A + 279*a^2*A*b^2 - 10*A*b^4 + 435*a^3*b*B + 45*a*b^3*B)*\text{EllipticE}[(c + d*x)/2, (2*a)/(a + b)]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]])/(315*a^2*d*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/(a + b)]*\text{Sqrt}[\text{Sec}[c + d*x]]) + (2*a*(4*A*b + 3*a*B)*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(21*d*\text{Sec}[c + d*x]^{5/2}) + (2*(49*a^2*A + 75*A*b^2 + 135*a*b*B)*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(315*d*\text{Sec}[c + d*x]^{3/2}) + (2*(163*a^2*A*b + 5*A*b^3 + 75*a^3*B + 135*a*b^2*B)*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]*\text{Sin}[c +$

$d*x]/(315*a*d*Sqrt[Sec[c + d*x]]) + (2*a*A*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2))$

Rule 2653

$Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]]], x_Symbol] \rightarrow Simp[(2*Sqrt[a + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a, b, c, d}, x] \&\& NeQ[a^2 - b^2, 0] \&\& GtQ[a + b, 0]$

Rule 2655

$Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]]], x_Symbol] \rightarrow Dist[Sqrt[a + b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] \&\& NeQ[a^2 - b^2, 0] \&\& !GtQ[a + b, 0]$

Rule 2661

$Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]]], x_Symbol] \rightarrow Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d/Sqrt[a + b], x] /; FreeQ[{a, b, c, d}, x] \&\& NeQ[a^2 - b^2, 0] \&\& GtQ[a + b, 0]$

Rule 2663

$Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]]], x_Symbol] \rightarrow Dist[Sqrt[(a + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b) + (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] \&\& NeQ[a^2 - b^2, 0] \&\& !GtQ[a + b, 0]$

Rule 3856

$Int[Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)]/Sqrt[csc[(e_) + (f_)*(x_)]*(d_)], x_Symbol] \rightarrow Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] \&\& NeQ[a^2 - b^2, 0]$

Rule 3858

$Int[Sqrt[csc[(e_) + (f_)*(x_)]*(d_)]/Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)], x_Symbol] \rightarrow Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] \&\& NeQ[a^2 - b^2, 0]$

Rule 4025

```

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> Simp[(a*A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e + f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LeQ[n, -1]

```

Rule 4035

```

Int[(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.))/(Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]), x_Symbol] :> Dist[A/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]

```

Rule 4094

```

Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]

```

Rule 4104

```

Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]

```

Rubi steps

$$\begin{aligned}
\int \frac{(a + b \sec(c + dx))^{5/2} (A + B \sec(c + dx))}{\sec^2(c + dx)} dx &= \frac{2aA(a + b \sec(c + dx))^{3/2} \sin(c + dx)}{9d \sec^2(c + dx)} - \frac{2}{9} \int \frac{\sqrt{a + b \sec(c + dx)}}{\sec^2(c + dx)} dx \\
&= \frac{2a(4Ab + 3aB)\sqrt{a + b \sec(c + dx)} \sin(c + dx)}{21d \sec^2(c + dx)} + \frac{2aA(a + b \sec(c + dx))^{3/2} \sin(c + dx)}{9d \sec^2(c + dx)} \\
&= \frac{2a(4Ab + 3aB)\sqrt{a + b \sec(c + dx)} \sin(c + dx)}{21d \sec^2(c + dx)} + \frac{2(49a^2A + 54abB + 300Ab^2) \sin(2(c + dx))}{21d \sec^2(c + dx)} \\
&= \frac{2a(4Ab + 3aB)\sqrt{a + b \sec(c + dx)} \sin(c + dx)}{21d \sec^2(c + dx)} + \frac{2(49a^2A + 54abB + 300Ab^2) \sin(2(c + dx))}{21d \sec^2(c + dx)} \\
&= \frac{2a(4Ab + 3aB)\sqrt{a + b \sec(c + dx)} \sin(c + dx)}{21d \sec^2(c + dx)} + \frac{2(49a^2A + 54abB + 300Ab^2) \sin(2(c + dx))}{21d \sec^2(c + dx)} \\
&= \frac{2a(4Ab + 3aB)\sqrt{a + b \sec(c + dx)} \sin(c + dx)}{21d \sec^2(c + dx)} + \frac{2(49a^2A + 54abB + 300Ab^2) \sin(2(c + dx))}{21d \sec^2(c + dx)} \\
&= \frac{2a(4Ab + 3aB)\sqrt{a + b \sec(c + dx)} \sin(c + dx)}{21d \sec^2(c + dx)} + \frac{2(49a^2A + 54abB + 300Ab^2) \sin(2(c + dx))}{21d \sec^2(c + dx)} \\
&= \frac{2(a^2 - b^2)(114a^2Ab - 10Ab^3 + 75a^3B + 45ab^2B) \sqrt{\frac{b+a \cos(c+dx)}{a+b}}}{315a^2d \sqrt{a + b \sec(c + dx)}}
\end{aligned}$$

Mathematica [A] time = 2.71, size = 313, normalized size = 0.74

$$(a + b \sec(c + dx))^{5/2} \left(a(a \cos(c + dx) + b) \left(a \left((266a^2A + 540abB + 300Ab^2) \sin(2(c + dx)) \right) + 5a(2(9aB + 19Ab^2) \sin(c + dx)) \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(((a + b*Sec[c + d*x]))^(5/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(9/2), x]

[Out] $((a + b \sec[c + d*x])^{5/2} * (8 * \sqrt{(b + a \cos[c + d*x]) / (a + b)}) * (a^{2*} (261 * a^{2*A*b} + 155 * A*b^3 + 75 * a^3 * B + 405 * a * b^2 * B) * \text{EllipticF}[(c + d*x)/2, (2*a)/(a + b)] + (147 * a^4 * A + 279 * a^2 * A * b^2 - 10 * A * b^4 + 435 * a^3 * b * B + 45 * a * b^3 * B) * ((a + b) * \text{EllipticE}[(c + d*x)/2, (2*a)/(a + b)] - b * \text{EllipticF}[(c + d*x)/2, (2*a)/(a + b)])) + a * (b + a \cos[c + d*x]) * (2 * (747 * a^2 * A * b + 20 * A * b^3 + 345 * a^3 * B + 540 * a * b^2 * B) * \sin[c + d*x] + a * ((266 * a^2 * A + 300 * A * b^2 + 540 * a * b * B) * \sin[2 * (c + d*x)] + 5 * a * (2 * (19 * A * b + 9 * a * B) * \sin[3 * (c + d*x)] + 7 * a * A * \sin[4 * (c + d*x)])))) / (1260 * a^2 * d * (b + a \cos[c + d*x])^3 * \sec[c + d*x]^{5/2})$

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{(Bb^2 \sec(dx + c)^3 + Aa^2 + (2Bab + Ab^2) \sec(dx + c)^2 + (Ba^2 + 2Aab) \sec(dx + c)) \sqrt{b \sec(dx + c) + a}}{\sec(dx + c)^{\frac{9}{2}}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x, algorithm="fricas")`

[Out] `integral((B*b^2*sec(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*sec(d*x + c)^2 + (B*a^2 + 2*A*a*b)*sec(d*x + c))*sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(9/2), x)`

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(b \sec(dx + c) + a)^{\frac{5}{2}}}{\sec(dx + c)^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x, algorithm="giac")`

[Out] `integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(9/2), x)`

maple [B] time = 2.89, size = 4847, normalized size = 11.40

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x)`

[Out] `-2/315/d*(261*A*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(`

$$\begin{aligned}
& 1/2)/\sin(d*x+c), (- (a+b)/(a-b))^{(1/2)} * a^4 * b - 147 * A * \text{EllipticF}((-1 + \cos(d*x+c)) \\
& * ((a-b)/(a+b))^{(1/2)} / \sin(d*x+c), (- (a+b)/(a-b))^{(1/2)} * a^5 * ((b+a*\cos(d*x+c)) \\
& / (1 + \cos(d*x+c)) / (a+b))^{(1/2)} * (1/(1 + \cos(d*x+c)))^{(1/2)} * \sin(d*x+c) + 147 * A * \text{EllipticE}((-1 + \cos(d*x+c)) * ((a-b)/(a+b))^{(1/2)} / \sin(d*x+c), (- (a+b)/(a-b))^{(1/2)} * \\
& a^5 * ((b+a*\cos(d*x+c)) / (1 + \cos(d*x+c)) / (a+b))^{(1/2)} * (1/(1 + \cos(d*x+c)))^{(1/2)} * \\
& \sin(d*x+c) - 147 * A * a^4 * b * ((a-b)/(a+b))^{(1/2)} - 163 * A * a^3 * b^2 * ((a-b)/(a+b))^{(1/2)} \\
& - 279 * A * a^2 * b^3 * ((a-b)/(a+b))^{(1/2)} - 5 * A * a * b^4 * ((a-b)/(a+b))^{(1/2)} - 75 * B * a^4 * \\
& b * ((a-b)/(a+b))^{(1/2)} - 435 * B * a^3 * b^2 * ((a-b)/(a+b))^{(1/2)} - 135 * B * a^2 * b^3 * ((a-b) \\
&) / (a+b))^{(1/2)} - 45 * B * a * b^4 * ((a-b)/(a+b))^{(1/2)} + 45 * B * \cos(d*x+c)^5 * ((a-b)/(a+b) \\
&))^{(1/2)} * a^5 - 10 * A * \cos(d*x+c) * ((a-b)/(a+b))^{(1/2)} * b^5 + 30 * B * \cos(d*x+c)^3 * ((a-b) \\
&) / (a+b))^{(1/2)} * a^5 - 75 * B * \cos(d*x+c) * ((a-b)/(a+b))^{(1/2)} * a^5 + 35 * A * \cos(d*x+c) \\
& ^6 * ((a-b)/(a+b))^{(1/2)} * a^5 + 14 * A * \cos(d*x+c)^4 * ((a-b)/(a+b))^{(1/2)} * a^5 + 98 * A * \cos \\
& (d*x+c)^2 * ((a-b)/(a+b))^{(1/2)} * a^5 - 147 * A * \cos(d*x+c) * ((a-b)/(a+b))^{(1/2)} * a^5 \\
& + 435 * B * \text{EllipticE}((-1 + \cos(d*x+c)) * ((a-b)/(a+b))^{(1/2)} / \sin(d*x+c), (- (a+b)/(a-b) \\
&)^{(1/2)} * a^4 * b * ((b+a*\cos(d*x+c)) / (1 + \cos(d*x+c)) / (a+b))^{(1/2)} * (1/(1 + \cos(d \\
& *x+c)))^{(1/2)} * \sin(d*x+c) - 435 * B * \text{EllipticE}((-1 + \cos(d*x+c)) * ((a-b)/(a+b))^{(1/2)} \\
&) / \sin(d*x+c), (- (a+b)/(a-b))^{(1/2)} * a^3 * b^2 * ((b+a*\cos(d*x+c)) / (1 + \cos(d*x+c)) \\
&) / (a+b))^{(1/2)} * (1/(1 + \cos(d*x+c)))^{(1/2)} * \sin(d*x+c) + 45 * B * \text{EllipticE}((-1 + \cos(d* \\
& x+c)) * ((a-b)/(a+b))^{(1/2)} / \sin(d*x+c), (- (a+b)/(a-b))^{(1/2)} * a^2 * b^3 * ((b+a*co \\
& s(d*x+c)) / (1 + \cos(d*x+c)) / (a+b))^{(1/2)} * (1/(1 + \cos(d*x+c)))^{(1/2)} * \sin(d*x+c) - 4 \\
& 5 * B * \text{EllipticE}((-1 + \cos(d*x+c)) * ((a-b)/(a+b))^{(1/2)} / \sin(d*x+c), (- (a+b)/(a-b) \\
&)^{(1/2)} * a * b^4 * ((b+a*\cos(d*x+c)) / (1 + \cos(d*x+c)) / (a+b))^{(1/2)} * (1/(1 + \cos(d*x+c) \\
&))^{(1/2)} * \sin(d*x+c) - 147 * A * \sin(d*x+c) * \cos(d*x+c) * ((b+a*\cos(d*x+c)) / (1 + \cos(d \\
& *x+c)) / (a+b))^{(1/2)} * (1/(1 + \cos(d*x+c)))^{(1/2)} * \text{EllipticF}((-1 + \cos(d*x+c)) * ((a-b) \\
&) / (a+b))^{(1/2)} / \sin(d*x+c), (- (a+b)/(a-b))^{(1/2)} * a^5 + 147 * A * \sin(d*x+c) * \cos(d \\
& *x+c) * \text{EllipticE}((-1 + \cos(d*x+c)) * ((a-b)/(a+b))^{(1/2)} / \sin(d*x+c), (- (a+b)/(a-b) \\
&))^{(1/2)} * ((b+a*\cos(d*x+c)) / (1 + \cos(d*x+c)) / (a+b))^{(1/2)} * (1/(1 + \cos(d*x+c)))^{(1/2)} * a^5 + 10 * A * \sin(d*x+c) * \cos(d*x+c) * \text{EllipticE}((-1 + \cos(d*x+c)) * ((a-b)/(a+b) \\
&)^{(1/2)} / \sin(d*x+c), (- (a+b)/(a-b))^{(1/2)} * ((b+a*\cos(d*x+c)) / (1 + \cos(d*x+c)) / (\\
& a+b))^{(1/2)} * (1/(1 + \cos(d*x+c)))^{(1/2)} * b^5 + 75 * B * \sin(d*x+c) * \cos(d*x+c) * ((b+a*c \\
& os(d*x+c)) / (1 + \cos(d*x+c)) / (a+b))^{(1/2)} * (1/(1 + \cos(d*x+c)))^{(1/2)} * \text{EllipticF}((\\
& -1 + \cos(d*x+c)) * ((a-b)/(a+b))^{(1/2)} / \sin(d*x+c), (- (a+b)/(a-b))^{(1/2)} * a^5 + 261 \\
& * A * \text{EllipticF}((-1 + \cos(d*x+c)) * ((a-b)/(a+b))^{(1/2)} / \sin(d*x+c), (- (a+b)/(a-b))^{(1/2)} * a^4 * b * ((b+a*\cos(d*x+c)) / (1 + \cos(d*x+c)) / (a+b))^{(1/2)} * (1/(1 + \cos(d*x+c) \\
&))^{(1/2)} * \sin(d*x+c) - 279 * A * \text{EllipticF}((-1 + \cos(d*x+c)) * ((a-b)/(a+b))^{(1/2)} / \sin \\
& (d*x+c), (- (a+b)/(a-b))^{(1/2)} * a^3 * b^2 * ((b+a*\cos(d*x+c)) / (1 + \cos(d*x+c)) / (a+b) \\
&))^{(1/2)} * (1/(1 + \cos(d*x+c)))^{(1/2)} * \sin(d*x+c) + 155 * A * \text{EllipticF}((-1 + \cos(d*x+c) \\
&) * ((a-b)/(a+b))^{(1/2)} / \sin(d*x+c), (- (a+b)/(a-b))^{(1/2)} * a^2 * b^3 * ((b+a*\cos(d* \\
& x+c)) / (1 + \cos(d*x+c)) / (a+b))^{(1/2)} * (1/(1 + \cos(d*x+c)))^{(1/2)} * \sin(d*x+c) + 10 * A * \\
& \text{EllipticF}((-1 + \cos(d*x+c)) * ((a-b)/(a+b))^{(1/2)} / \sin(d*x+c), (- (a+b)/(a-b))^{(1/2)} * a * b^4 * ((b+a*\cos(d*x+c)) / (1 + \cos(d*x+c)) / (a+b))^{(1/2)} * (1/(1 + \cos(d*x+c)) \\
&))^{(1/2)} * \sin(d*x+c) - 147 * A * \text{EllipticE}((-1 + \cos(d*x+c)) * ((a-b)/(a+b))^{(1/2)} / \sin(d* \\
& x+c), (- (a+b)/(a-b))^{(1/2)} * a^4 * b * ((b+a*\cos(d*x+c)) / (1 + \cos(d*x+c)) / (a+b))^{(1 \\
& /2)} * (1/(1 + \cos(d*x+c)))^{(1/2)} * \sin(d*x+c) + 279 * A * \text{EllipticE}((-1 + \cos(d*x+c)) * ((a \\
& -b)/(a+b))^{(1/2)} / \sin(d*x+c), (- (a+b)/(a-b))^{(1/2)} * a^3 * b^2 * ((b+a*\cos(d*x+c))
\end{aligned}$$

$$\begin{aligned}
& / (1 + \cos(dx+c)) / (a+b)^{1/2} * (1 / (1 + \cos(dx+c)))^{1/2} * \sin(dx+c) - 279 * A * \text{EllipticE} \\
& ((-1 + \cos(dx+c)) * ((a-b) / (a+b))^{1/2} / \sin(dx+c), (- (a+b) / (a-b))^{1/2}) * \\
& a^2 * b^3 * ((b+a * \cos(dx+c)) / (1 + \cos(dx+c)) / (a+b))^{1/2} * (1 / (1 + \cos(dx+c)))^{1/2} * \\
& \sin(dx+c) - 10 * A * \text{EllipticE}((-1 + \cos(dx+c)) * ((a-b) / (a+b))^{1/2} / \sin(dx+c) \\
&), (- (a+b) / (a-b))^{1/2}) * a * b^4 * ((b+a * \cos(dx+c)) / (1 + \cos(dx+c)) / (a+b))^{1/2} * \\
& (1 / (1 + \cos(dx+c)))^{1/2} * \sin(dx+c) - 435 * B * \text{EllipticF}((-1 + \cos(dx+c)) * ((a-b) / \\
& (a+b))^{1/2} / \sin(dx+c), (- (a+b) / (a-b))^{1/2}) * a^4 * b * ((b+a * \cos(dx+c)) / (1 + \cos \\
& (dx+c)) / (a+b))^{1/2} * (1 / (1 + \cos(dx+c)))^{1/2} * \sin(dx+c) + 405 * B * \text{EllipticF} \\
& ((-1 + \cos(dx+c)) * ((a-b) / (a+b))^{1/2} / \sin(dx+c), (- (a+b) / (a-b))^{1/2}) * a^3 * b^2 * \\
& ((b+a * \cos(dx+c)) / (1 + \cos(dx+c)) / (a+b))^{1/2} * (1 / (1 + \cos(dx+c)))^{1/2} * \sin \\
& (dx+c) - 45 * B * \text{EllipticF}((-1 + \cos(dx+c)) * ((a-b) / (a+b))^{1/2} / \sin(dx+c), (- (\\
& a+b) / (a-b))^{1/2}) * a^2 * b^3 * ((b+a * \cos(dx+c)) / (1 + \cos(dx+c)) / (a+b))^{1/2} * (1 \\
& / (1 + \cos(dx+c)))^{1/2} * \sin(dx+c) + 10 * A * b^5 * ((a-b) / (a+b))^{1/2} + 82 * A * \cos(dx \\
& +c)^3 * ((a-b) / (a+b))^{1/2} * a^4 * b + 80 * A * \cos(dx+c)^3 * ((a-b) / (a+b))^{1/2} * a^2 * b^3 \\
& + 270 * B * \cos(dx+c)^3 * ((a-b) / (a+b))^{1/2} * a^3 * b^2 + 272 * A * \cos(dx+c)^2 * ((a-b) \\
& / (a+b))^{1/2} * a^3 * b^2 + 130 * A * \cos(dx+c)^5 * ((a-b) / (a+b))^{1/2} * a^4 * b + 170 * A * \cos \\
& (dx+c)^4 * ((a-b) / (a+b))^{1/2} * a^3 * b^2 + 180 * B * \cos(dx+c)^4 * ((a-b) / (a+b))^{1/2} * \\
& a^4 * b - 5 * A * \cos(dx+c)^2 * ((a-b) / (a+b))^{1/2} * a * b^4 + 330 * B * \cos(dx+c)^2 * ((a- \\
& b) / (a+b))^{1/2} * a^4 * b + 180 * B * \cos(dx+c)^2 * ((a-b) / (a+b))^{1/2} * a^2 * b^3 - 279 * A * \\
& \sin(dx+c) * \cos(dx+c) * ((b+a * \cos(dx+c)) / (1 + \cos(dx+c)) / (a+b))^{1/2} * (1 / (1 + \cos \\
& (dx+c)))^{1/2} * \text{EllipticF}((-1 + \cos(dx+c)) * ((a-b) / (a+b))^{1/2} / \sin(dx+c), \\
& (- (a+b) / (a-b))^{1/2}) * a^3 * b^2 + 155 * A * \sin(dx+c) * \cos(dx+c) * ((b+a * \cos(dx+c)) \\
& / (1 + \cos(dx+c)) / (a+b))^{1/2} * (1 / (1 + \cos(dx+c)))^{1/2} * \text{EllipticF}((-1 + \cos(dx \\
& +c)) * ((a-b) / (a+b))^{1/2} / \sin(dx+c), (- (a+b) / (a-b))^{1/2}) * a^2 * b^3 + 10 * A * \sin \\
& (dx+c) * \cos(dx+c) * ((b+a * \cos(dx+c)) / (1 + \cos(dx+c)) / (a+b))^{1/2} * (1 / (1 + \cos(dx \\
& +c)))^{1/2} * \text{EllipticF}((-1 + \cos(dx+c)) * ((a-b) / (a+b))^{1/2} / \sin(dx+c), (- (a \\
& +b) / (a-b))^{1/2}) * a * b^4 - 147 * A * \sin(dx+c) * \cos(dx+c) * \text{EllipticE}((-1 + \cos(dx+c) \\
&)) * ((a-b) / (a+b))^{1/2} / \sin(dx+c), (- (a+b) / (a-b))^{1/2}) * ((b+a * \cos(dx+c)) / (\\
& 1 + \cos(dx+c)) / (a+b))^{1/2} * (1 / (1 + \cos(dx+c)))^{1/2} * a^4 * b + 279 * A * \sin(dx+c) * \\
& \cos(dx+c) * \text{EllipticE}((-1 + \cos(dx+c)) * ((a-b) / (a+b))^{1/2} / \sin(dx+c), (- (a+b) \\
& / (a-b))^{1/2}) * ((b+a * \cos(dx+c)) / (1 + \cos(dx+c)) / (a+b))^{1/2} * (1 / (1 + \cos(dx+ \\
& c)))^{1/2} * a^3 * b^2 - 279 * A * \sin(dx+c) * \cos(dx+c) * \text{EllipticE}((-1 + \cos(dx+c)) * (\\
& a-b) / (a+b))^{1/2} / \sin(dx+c), (- (a+b) / (a-b))^{1/2}) * ((b+a * \cos(dx+c)) / (1 + \cos \\
& (dx+c)) / (a+b))^{1/2} * (1 / (1 + \cos(dx+c)))^{1/2} * a^2 * b^3 - 10 * A * \sin(dx+c) * \cos \\
& (dx+c) * \text{EllipticE}((-1 + \cos(dx+c)) * ((a-b) / (a+b))^{1/2} / \sin(dx+c), (- (a+b) / (a- \\
& b))^{1/2}) * ((b+a * \cos(dx+c)) / (1 + \cos(dx+c)) / (a+b))^{1/2} * (1 / (1 + \cos(dx+c))) \\
& ^{1/2} * a * b^4 - 435 * B * \sin(dx+c) * \cos(dx+c) * ((b+a * \cos(dx+c)) / (1 + \cos(dx+c)) / (\\
& a+b))^{1/2} * (1 / (1 + \cos(dx+c)))^{1/2} * \text{EllipticF}((-1 + \cos(dx+c)) * ((a-b) / (a+b) \\
&))^{1/2} / \sin(dx+c), (- (a+b) / (a-b))^{1/2}) * a^4 * b + 405 * B * \sin(dx+c) * \cos(dx+c) * \\
& ((b+a * \cos(dx+c)) / (1 + \cos(dx+c)) / (a+b))^{1/2} * (1 / (1 + \cos(dx+c)))^{1/2} * \text{EllipticF} \\
& ((-1 + \cos(dx+c)) * ((a-b) / (a+b))^{1/2} / \sin(dx+c), (- (a+b) / (a-b))^{1/2}) * \\
& a^3 * b^2 - 45 * B * \sin(dx+c) * \cos(dx+c) * ((b+a * \cos(dx+c)) / (1 + \cos(dx+c)) / (a+b))^{1/2} * \\
& (1 / (1 + \cos(dx+c)))^{1/2} * \text{EllipticF}((-1 + \cos(dx+c)) * ((a-b) / (a+b))^{1/2} / \sin(dx+c), \\
& (- (a+b) / (a-b))^{1/2}) * a^2 * b^3 + 435 * B * \sin(dx+c) * \cos(dx+c) * \text{EllipticE}((-1 + \cos(dx+c) \\
&)) * ((a-b) / (a+b))^{1/2} / \sin(dx+c), (- (a+b) / (a-b))^{1/2}) *
\end{aligned}$$

$$\left(\frac{(b+a\cos(dx+c))/(1+\cos(dx+c))}{(a+b)}\right)^{1/2} \left(\frac{1}{(1+\cos(dx+c))}\right)^{1/2} a^4 b - 435 B \sin(dx+c) \cos(dx+c) \text{EllipticE}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}, \frac{-(a+b)}{(a-b)}\right)^{1/2} \left(\frac{(b+a\cos(dx+c))/(1+\cos(dx+c))}{(a+b)}\right)^{1/2} \left(\frac{1}{(1+\cos(dx+c))}\right)^{1/2} a^3 b^2 + 45 B \sin(dx+c) \cos(dx+c) \text{EllipticE}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}, \frac{-(a+b)}{(a-b)}\right)^{1/2} \left(\frac{(b+a\cos(dx+c))/(1+\cos(dx+c))}{(a+b)}\right)^{1/2} \left(\frac{1}{(1+\cos(dx+c))}\right)^{1/2} a^2 b^3 - 45 B \sin(dx+c) \cos(dx+c) \text{EllipticE}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}, \frac{-(a+b)}{(a-b)}\right)^{1/2} \left(\frac{(b+a\cos(dx+c))/(1+\cos(dx+c))}{(a+b)}\right)^{1/2} \left(\frac{1}{(1+\cos(dx+c))}\right)^{1/2} a b^4 + 10 A \text{EllipticE}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}, \frac{-(a+b)}{(a-b)}\right)^{1/2} b^5 \left(\frac{(b+a\cos(dx+c))/(1+\cos(dx+c))}{(a+b)}\right)^{1/2} \left(\frac{1}{(1+\cos(dx+c))}\right)^{1/2} \sin(dx+c) + 75 B \text{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}, \frac{-(a+b)}{(a-b)}\right)^{1/2} a^5 \left(\frac{(b+a\cos(dx+c))/(1+\cos(dx+c))}{(a+b)}\right)^{1/2} \left(\frac{1}{(1+\cos(dx+c))}\right)^{1/2} \sin(dx+c) - 65 A \cos(dx+c) \left(\frac{(a-b)}{(a+b)}\right)^{1/2} a^4 b - 279 A \cos(dx+c) \left(\frac{(a-b)}{(a+b)}\right)^{1/2} a^3 b^2 + 199 A \cos(dx+c) \left(\frac{(a-b)}{(a+b)}\right)^{1/2} a^2 b^3 + 10 A \cos(dx+c) \left(\frac{(a-b)}{(a+b)}\right)^{1/2} a b^4 - 435 B \cos(dx+c) \left(\frac{(a-b)}{(a+b)}\right)^{1/2} a^4 b + 165 B \cos(dx+c) \left(\frac{(a-b)}{(a+b)}\right)^{1/2} a^3 b^2 - 45 B \cos(dx+c) \left(\frac{(a-b)}{(a+b)}\right)^{1/2} a^2 b^3 + 45 B \cos(dx+c) \left(\frac{(a-b)}{(a+b)}\right)^{1/2} a b^4 \left(\frac{(b+a\cos(dx+c))}{\cos(dx+c)}\right)^{1/2} \cos(dx+c)^5 \left(\frac{1}{\cos(dx+c)}\right)^{9/2} / \sin(dx+c) / (b+a\cos(dx+c)) / a^2 / \left(\frac{(a-b)}{(a+b)}\right)^{1/2}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx+c) + A)(b \sec(dx+c) + a)^{5/2}}{\sec(dx+c)^{9/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(dx+c))^(5/2)*(A+B*sec(dx+c))/sec(dx+c)^(9/2),x, algorith="maxima")

[Out] integrate((B*sec(dx+c) + A)*(b*sec(dx+c) + a)^(5/2)/sec(dx+c)^(9/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(a + \frac{b}{\cos(c+dx)}\right)^{5/2}}{\left(\frac{1}{\cos(c+dx)}\right)^{9/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(5/2))/(1/cos(c + d*x))^(9/2),x)

```
[Out] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(5/2))/(1/cos(c + d*x))^(9/2), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

$$3.455 \quad \int \frac{(a+b \sec(c+dx))^{5/2}(A+B \sec(c+dx))}{\sec^2(c+dx)} dx$$

Optimal. Leaf size=519

$$\frac{2(81a^2A + 209abB + 113Ab^2) \sin(c+dx) \sqrt{a+b \sec(c+dx)}}{693d \sec^2(c+dx)} + \frac{2(539a^3B + 1145a^2Ab + 825ab^2B + 15Ab^3) \sin(c+dx)}{3465ad \sec^2(c+dx)}$$

[Out] $2/11*a*A*(a+b*\sec(d*x+c))^{3/2}*\sin(d*x+c)/d/\sec(d*x+c)^{9/2}+2/3465*(a^2-b^2)*(675*A*a^4+285*A*a^2*b^2+40*A*b^4+1254*B*a^3*b-110*B*a*b^3)*(\cos(1/2*d*x+1/2*c)^2)^{1/2}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{1/2}*(a/(a+b))^{1/2})*((b+a*\cos(d*x+c))/(a+b))^{1/2}*\sec(d*x+c)^{1/2}/a^3/d/(a+b*\sec(d*x+c))^{1/2}+2/99*a*(14*A*b+11*B*a)*\sin(d*x+c)*(a+b*\sec(d*x+c))^{1/2}/d/\sec(d*x+c)^{7/2}+2/693*(81*A*a^2+113*A*b^2+209*B*a*b)*\sin(d*x+c)*(a+b*\sec(d*x+c))^{1/2}/d/\sec(d*x+c)^{5/2}+2/3465*(1145*A*a^2*b+15*A*b^3+539*B*a^3+825*B*a*b^2)*\sin(d*x+c)*(a+b*\sec(d*x+c))^{1/2}/a/d/\sec(d*x+c)^{3/2}+2/3465*(675*A*a^4+1025*A*a^2*b^2-20*A*b^4+1793*B*a^3*b+55*B*a*b^3)*\sin(d*x+c)*(a+b*\sec(d*x+c))^{1/2}/a^2/d/\sec(d*x+c)^{1/2}+2/3465*(3705*A*a^4*b+255*A*a^2*b^3+40*A*b^5+1617*B*a^5+3069*B*a^3*b^2-110*B*a*b^4)*(\cos(1/2*d*x+1/2*c)^2)^{1/2}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{1/2}*(a/(a+b))^{1/2})*((a+b*\sec(d*x+c))^{1/2}/a^3/d/((b+a*\cos(d*x+c))/(a+b))^{1/2})/\sec(d*x+c)^{1/2}$

Rubi [A] time = 1.96, antiderivative size = 519, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {4025, 4094, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2(1145a^2Ab + 539a^3B + 825ab^2B + 15Ab^3) \sin(c+dx) \sqrt{a+b \sec(c+dx)}}{3465ad \sec^2(c+dx)} + \frac{2(81a^2A + 209abB + 113Ab^2) \sin(c+dx)}{693d \sec^2(c+dx)}$$

Antiderivative was successfully verified.

[In] Int[((a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(11/2), x]

[Out] $(2*(a^2 - b^2)*(675*a^4*A + 285*a^2*A*b^2 + 40*A*b^4 + 1254*a^3*b*B - 110*a*b^3*B)*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/(a + b)]*\text{EllipticF}[(c + d*x)/2, (2*a)/(a + b)]*\text{Sqrt}[\text{Sec}[c + d*x]])/(3465*a^3*d*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]) + (2*(3705*a^4*A*b + 255*a^2*A*b^3 + 40*A*b^5 + 1617*a^5*B + 3069*a^3*b^2*B - 110*a*b^4*B)*\text{EllipticE}[(c + d*x)/2, (2*a)/(a + b)]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]])/(3465*$

$$a^3*d*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/(a + b)]*\text{Sqrt}[\text{Sec}[c + d*x]] + (2*a*(14*A*b + 11*a*B)*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(99*d*\text{Sec}[c + d*x]^{(7/2)}) + (2*(81*a^2*A + 113*A*b^2 + 209*a*b*B)*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(693*d*\text{Sec}[c + d*x]^{(5/2)}) + (2*(1145*a^2*A*b + 15*A*b^3 + 539*a^3*B + 825*a*b^2*B)*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(3465*a*d*\text{Sec}[c + d*x]^{(3/2)}) + (2*(675*a^4*A + 1025*a^2*A*b^2 - 20*A*b^4 + 1793*a^3*b*B + 55*a*b^3*B)*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(3465*a^2*d*\text{Sqrt}[\text{Sec}[c + d*x]]) + (2*a*A*(a + b*\text{Sec}[c + d*x])^{(3/2)}*\text{Sin}[c + d*x])/(11*d*\text{Sec}[c + d*x]^{(9/2)})$$
Rule 2653

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*Sqrt[a + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a + b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b) + (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)]/Sqrt[csc[(e_) + (f_)*(x_)]*(d_)], x_Symbol] := Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_.)], x_Symbol] := Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] := Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)]), x_Symbol] := Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_.))^(m_), x_Symbol] := Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
+ f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_.))^(m_), x_Symbol] := Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + b \sec(c + dx))^{5/2} (A + B \sec(c + dx))}{\sec^{\frac{11}{2}}(c + dx)} dx &= \frac{2aA(a + b \sec(c + dx))^{3/2} \sin(c + dx)}{11d \sec^{\frac{9}{2}}(c + dx)} - \frac{2}{11} \int \frac{\sqrt{a + b \sec(c + dx)}}{\sec^{\frac{7}{2}}(c + dx)} dx \\
&= \frac{2a(14Ab + 11aB)\sqrt{a + b \sec(c + dx)} \sin(c + dx)}{99d \sec^{\frac{7}{2}}(c + dx)} + \frac{2aA(a + b \sec(c + dx))^{3/2} \sin(c + dx)}{11d \sec^{\frac{9}{2}}(c + dx)} \\
&= \frac{2a(14Ab + 11aB)\sqrt{a + b \sec(c + dx)} \sin(c + dx)}{99d \sec^{\frac{7}{2}}(c + dx)} + \frac{2(81a^2A + 11aB^2)\sqrt{a + b \sec(c + dx)} \sin(c + dx)}{99d \sec^{\frac{7}{2}}(c + dx)} \\
&= \frac{2a(14Ab + 11aB)\sqrt{a + b \sec(c + dx)} \sin(c + dx)}{99d \sec^{\frac{7}{2}}(c + dx)} + \frac{2(81a^2A + 11aB^2)\sqrt{a + b \sec(c + dx)} \sin(c + dx)}{99d \sec^{\frac{7}{2}}(c + dx)} \\
&= \frac{2a(14Ab + 11aB)\sqrt{a + b \sec(c + dx)} \sin(c + dx)}{99d \sec^{\frac{7}{2}}(c + dx)} + \frac{2(81a^2A + 11aB^2)\sqrt{a + b \sec(c + dx)} \sin(c + dx)}{99d \sec^{\frac{7}{2}}(c + dx)} \\
&= \frac{2a(14Ab + 11aB)\sqrt{a + b \sec(c + dx)} \sin(c + dx)}{99d \sec^{\frac{7}{2}}(c + dx)} + \frac{2(81a^2A + 11aB^2)\sqrt{a + b \sec(c + dx)} \sin(c + dx)}{99d \sec^{\frac{7}{2}}(c + dx)} \\
&= \frac{2a(14Ab + 11aB)\sqrt{a + b \sec(c + dx)} \sin(c + dx)}{99d \sec^{\frac{7}{2}}(c + dx)} + \frac{2(81a^2A + 11aB^2)\sqrt{a + b \sec(c + dx)} \sin(c + dx)}{99d \sec^{\frac{7}{2}}(c + dx)} \\
&= \frac{2a(14Ab + 11aB)\sqrt{a + b \sec(c + dx)} \sin(c + dx)}{99d \sec^{\frac{7}{2}}(c + dx)} + \frac{2(81a^2A + 11aB^2)\sqrt{a + b \sec(c + dx)} \sin(c + dx)}{99d \sec^{\frac{7}{2}}(c + dx)} \\
&= \frac{2(a^2 - b^2)(675a^4A + 285a^2Ab^2 + 40Ab^4 + 1254a^3bB - 110ab^3)}{3465a^3d\sqrt{a + b \sec(c + dx)}}
\end{aligned}$$

Mathematica [A] time = 3.80, size = 380, normalized size = 0.73

$$\frac{(a + b \sec(c + dx))^{5/2} \left(a(a \cos(c + dx) + b) \left(a \left(5a \left((513a^2A + 836abB + 452Ab^2) \sin(3(c + dx)) + 7a((22aB + 46) \right) \right) \right) \right)}{3465a^3d\sqrt{a + b \sec(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(11/2), x]

[Out] ((a + b*Sec[c + d*x])^(5/2)*(16*sqrt[(b + a*cos[c + d*x])]/(a + b))*(a^2*(675*a^4*A + 3315*a^2*A*b^2 + 10*A*b^4 + 2871*a^3*b*B + 1705*a*b^3*B)*EllipticF[(c + d*x)/2, (2*a)/(a + b)] + (3705*a^4*A*b + 255*a^2*A*b^3 + 40*A*b^5 + 1617*a^5*B + 3069*a^3*b^2*B - 110*a*b^4*B)*((a + b)*EllipticE[(c + d*x)/2, (2*a)/(a + b)] - b*EllipticF[(c + d*x)/2, (2*a)/(a + b)])) + a*(b + a*cos[c + d*x])*(2*(6525*a^4*A + 9330*a^2*A*b^2 - 160*A*b^4 + 16434*a^3*b*B + 440*a*b^3*B)*sin[c + d*x] + a*(4*(3095*a^2*A*b + 30*A*b^3 + 1463*a^3*B + 1650*a*b^2*B)*sin[2*(c + d*x)] + 5*a*((513*a^2*A + 452*A*b^2 + 836*a*b*B)*sin[3*(c + d*x)] + 7*a*((46*A*b + 22*a*B)*sin[4*(c + d*x)] + 9*a*A*sin[5*(c + d*x)])))))))/(27720*a^3*d*(b + a*cos[c + d*x])^3*Sec[c + d*x]^(5/2))

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{(Bb^2 \sec(dx + c))^3 + Aa^2 + (2Bab + Ab^2) \sec(dx + c)^2 + (Ba^2 + 2Aab) \sec(dx + c) \sqrt{b \sec(dx + c)}}{\sec(dx + c)^{\frac{11}{2}}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(11/2), x, algorithm="fricas")

[Out] integral((B*b^2*sec(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*sec(d*x + c)^2 + (B*a^2 + 2*A*a*b)*sec(d*x + c))*sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(11/2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(b \sec(dx + c) + a)^{\frac{5}{2}}}{\sec(dx + c)^{\frac{11}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(11/2), x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(11/2), x)

maple [B] time = 3.03, size = 5946, normalized size = 11.46

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(11/2),x)`

[Out] result too large to display

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(b \sec(dx + c) + a)^{\frac{5}{2}}}{\sec(dx + c)^{\frac{11}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(11/2),x, algorithm="maxima")`

[Out] `integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(11/2), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(a + \frac{b}{\cos(c+dx)}\right)^{5/2}}{\left(\frac{1}{\cos(c+dx)}\right)^{11/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(5/2))/(1/cos(c + d*x))^(11/2),x)`

[Out] `int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(5/2))/(1/cos(c + d*x))^(11/2), x)`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)**(11/2),x)`

[Out] Timed out

$$3.456 \quad \int \frac{\sec^2(c+dx)(A+B \sec(c+dx))}{\sqrt{a+b \sec(c+dx)}} dx$$

Optimal. Leaf size=344

$$\frac{(-3a^2B + 4aAb - 4b^2B) \sqrt{\sec(c+dx)} \sqrt{\frac{a \cos(c+dx)+b}{a+b}} \Pi\left(2; \frac{1}{2}(c+dx) \middle| \frac{2a}{a+b}\right)}{4b^2d\sqrt{a+b \sec(c+dx)}} + \frac{(4Ab - 3aB) \sin(c+dx) \sqrt{\sec(c+dx)}}{4b^2d}$$

[Out] $\frac{1}{4}*(4*A*b-B*a)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)}*(a/(a+b))^{(1/2)})*((b+a*\cos(d*x+c))/(a+b))^{(1/2)}*\sec(d*x+c)^{(1/2)}/b/d/(a+b*\sec(d*x+c))^{(1/2)}-1/4*(4*A*a*b-3*B*a^2-4*B*b^2)*(c+\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticPi}(\sin(1/2*d*x+1/2*c), 2, 2^{(1/2)}*(a/(a+b))^{(1/2)})*((b+a*\cos(d*x+c))/(a+b))^{(1/2)}*\sec(d*x+c)^{(1/2)}/b^2/d/(a+b*\sec(d*x+c))^{(1/2)}+1/2*B*\sec(d*x+c)^{(3/2)}*\sin(d*x+c)*(a+b*\sec(d*x+c))^{(1/2)}/b/d-1/4*(4*A*b-3*B*a)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)}*(a/(a+b))^{(1/2)})*(a+b*\sec(d*x+c))^{(1/2)}/b^2/d/((b+a*\cos(d*x+c))/(a+b))^{(1/2)}/\sec(d*x+c)^{(1/2)}+1/4*(4*A*b-3*B*a)*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}*(a+b*\sec(d*x+c))^{(1/2)}/b^2/d$

Rubi [A] time = 1.11, antiderivative size = 344, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {4033, 4102, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{(-3a^2B + 4aAb - 4b^2B) \sqrt{\sec(c+dx)} \sqrt{\frac{a \cos(c+dx)+b}{a+b}} \Pi\left(2; \frac{1}{2}(c+dx) \middle| \frac{2a}{a+b}\right)}{4b^2d\sqrt{a+b \sec(c+dx)}} + \frac{(4Ab - 3aB) \sin(c+dx) \sqrt{\sec(c+dx)}}{4b^2d}$$

Antiderivative was successfully verified.

[In] Int[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/Sqrt[a + b*Sec[c + d*x]], x]

[Out] $((4*A*b - a*B)*\text{Sqrt}[(b + a*\cos[c + d*x])/(a + b)]*\text{EllipticF}[(c + d*x)/2, (2*a)/(a + b)]*\text{Sqrt}[\sec[c + d*x]])/(4*b*d*\text{Sqrt}[a + b*\sec[c + d*x]]) - ((4*a*A*b - 3*a^2*B - 4*b^2*B)*\text{Sqrt}[(b + a*\cos[c + d*x])/(a + b)]*\text{EllipticPi}[2, (c + d*x)/2, (2*a)/(a + b)]*\text{Sqrt}[\sec[c + d*x]])/(4*b^2*d*\text{Sqrt}[a + b*\sec[c + d*x]]) - ((4*A*b - 3*a*B)*\text{EllipticE}[(c + d*x)/2, (2*a)/(a + b)]*\text{Sqrt}[a + b*\sec[c + d*x]])/(4*b^2*d*\text{Sqrt}[(b + a*\cos[c + d*x])/(a + b)]*\text{Sqrt}[\sec[c + d*x]]) + ((4*A*b - 3*a*B)*\text{Sqrt}[\sec[c + d*x]]*\text{Sqrt}[a + b*\sec[c + d*x]]*\sin[c + d*x])/(4*b^2*d) + (B*\sec[c + d*x]^(3/2)*\text{Sqrt}[a + b*\sec[c + d*x]]*\sin[c + d*x])/(2*b*d)$

Rule 2653

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*Sqrt[a
+ b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)]/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
0] && !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)]/(d*Sqrt[a + b]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a
+ b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_)
+ (f_)*(x_)]]), x_Symbol] := Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)]/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_)
+ (f_)*(x_)]]), x_Symbol] := Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
+ f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !GtQ[c + d, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)]/Sqrt[csc[(e_) + (f_)*(x_)]
*(d_)], x_Symbol] := Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
```

b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 3858

Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 3859

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 4033

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] :> -Simp[(B*d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2))/(b*f*(m + n)), x] + Dist[d^2/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 2)*Simp[a*B*(n - 2) + B*b*(m + n - 1)*Csc[e + f*x] + (A*b*(m + n) - a*B*(n - 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && NeQ[m + n, 0] && !IGtQ[m, 1]

Rule 4035

Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)]), x_Symbol] :> Dist[A/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]

Rule 4102

Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -

$b^2, 0]$ && GtQ[n, 0]

Rule 4108

Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.)))/(Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}} dx &= \frac{B\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2bd} + \int \frac{\sqrt{\sec(c+dx)}\left(\frac{aB}{2}+bB\sec(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}} dx \\
 &= \frac{(4Ab-3aB)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4b^2d} + \frac{B\sec^{\frac{3}{2}}(c+dx)}{4b^2d} \\
 &= \frac{(4Ab-3aB)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4b^2d} + \frac{B\sec^{\frac{3}{2}}(c+dx)}{4b^2d} \\
 &= \frac{(4Ab-3aB)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4b^2d} + \frac{B\sec^{\frac{3}{2}}(c+dx)}{4b^2d} \\
 &= \frac{(4Ab-3aB)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4b^2d} + \frac{B\sec^{\frac{3}{2}}(c+dx)}{4b^2d} \\
 &= -\frac{(4aAb-3a^2B-4b^2B)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{4b^2d\sqrt{a+b\sec(c+dx)}} \\
 &= \frac{(4Ab-aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{4bd\sqrt{a+b\sec(c+dx)}} - \frac{(4aAb-3a^2B-4b^2B)\sqrt{\sec(c+dx)}}{4b^2d\sqrt{a+b\sec(c+dx)}}
 \end{aligned}$$

Mathematica [C] time = 4.15, size = 451, normalized size = 1.31

$$\sqrt{\sec(c+dx)} \left(\frac{2(9a^2B-12aAb+8b^2B)\sqrt{\frac{a\cos(c+dx)+b}{a+b}} \Pi\left(2;\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{b^2} + \frac{2i(3aB-4Ab)\csc(c+dx)\sqrt{-\frac{a(\cos(c+dx)-1)}{a+b}}\sqrt{\frac{a(\cos(c+dx)+1)}{a-b}}\sqrt{a\cos(c+dx)}}{b^2} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/Sqrt[a + b*Sec[c + d*x]], x]

[Out] (Sqrt[Sec[c + d*x]]*((8*a*B*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]/b + (2*(-12*a*A*b + 9*a^2*B + 8*b^2*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]/b^2 + ((2*I)*(-4*A*b + 3*a*B)*Sqrt[-((a*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(a*(1 + Cos[c + d*x]))/(a - b)]*Sqrt[b + a*Cos[c + d*x]]*Csc[c + d*x]*(-2*b*(a + b)*EllipticE[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*(2*b*EllipticF[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*EllipticPi[1 - a/b, I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)])))/(a*Sqrt[(a - b)^(-1)]*b^3) - (4*a*(-4*A*b + 3*a*B)*Sin[c + d*x])/b^2 + (8*a*B*Tan[c + d*x])/b + (4*(4*A*b - 3*a*B)*Tan[c + d*x])/b + 8*B*Sec[c + d*x]*Tan[c + d*x]))/(16*d*Sqrt[a + b*Sec[c + d*x]])

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2), x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sec(dx + c)^{\frac{5}{2}}}{\sqrt{b \sec(dx + c) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2), x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(5/2)/sqrt(b*sec(d*x + c) + a), x)

maple [C] time = 2.33, size = 2738, normalized size = 7.96

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2), x)

[Out] $\frac{1}{4}d*(4*A*\sin(d*x+c)*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{1/2}*(1/(1+\cos(d*x+c)))^{1/2}*EllipticE((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (-a+b)/(a-b))^{1/2}*\cos(d*x+c)^3*a*b+8*A*\sin(d*x+c)*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{1/2}*(1/(1+\cos(d*x+c)))^{1/2}*EllipticPi((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (a+b)/(a-b), I/((a-b)/(a+b))^{1/2})*\cos(d*x+c)^3*a*b+3*B*\sin(d*x+c)*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{1/2}*(1/(1+\cos(d*x+c)))^{1/2}*EllipticE((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (-a+b)/(a-b))^{1/2}*\cos(d*x+c)^3*a*b-2*B*\sin(d*x+c)*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{1/2}*(1/(1+\cos(d*x+c)))^{1/2}*EllipticF((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (-a+b)/(a-b))^{1/2}*\cos(d*x+c)^3*a*b+4*A*\sin(d*x+c)*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{1/2}*(1/(1+\cos(d*x+c)))^{1/2}*EllipticE((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (-a+b)/(a-b))^{1/2}*\cos(d*x+c)^2*a*b+8*A*\sin(d*x+c)*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{1/2}*(1/(1+\cos(d*x+c)))^{1/2}*EllipticPi((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (a+b)/(a-b), I/((a-b)/(a+b))^{1/2})*\cos(d*x+c)^2*a*b+3*B*\sin(d*x+c)*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{1/2}*(1/(1+\cos(d*x+c)))^{1/2}*EllipticE((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (-a+b)/(a-b))^{1/2}*\cos(d*x+c)^2*a*b-2*B*\sin(d*x+c)*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{1/2}*(1/(1+\cos(d*x+c)))^{1/2}*EllipticF((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (-a+b)/(a-b))^{1/2}*\cos(d*x+c)^2*a*b+3*B*\cos(d*x+c)^3*((a-b)/(a+b))^{1/2}*a^2-4*A*\cos(d*x+c)^2*((a-b)/(a+b))^{1/2}*b^2-3*B*\cos(d*x+c)^2*((a-b)/(a+b))^{1/2}*a^2+4*A*\cos(d*x+c)*((a-b)/(a+b))^{1/2}*b^2-2*B*\cos(d*x+c)^2*((a-b)/(a+b))^{1/2}*b^2+2*B*((a-b)/(a+b))^{1/2}*b^2-4*A*\cos(d*x+c)^3*((a-b)/(a+b))^{1/2}*a*b-2*B*\cos(d*x+c)^3*((a-b)/(a+b))^{1/2}*a*b+4*A*\cos(d*x+c)^2*((a-b)/(a+b))^{1/2}*a*b+3*B*\cos(d*x+c)^2*((a-b)/(a+b))^{1/2}*a*b-B*\cos(d*x+c)*((a-b)/(a+b))^{1/2}*a*b-8*A*\sin(d*x+c)*\cos(d*x+c)^3*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{1/2}*(1/(1+\cos(d*x+c)))^{1/2}*EllipticF((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (-a+b)/(a-b))^{1/2})*a*b-8*A*\sin(d*x+c)*\cos(d*x+c)^2*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{1/2}*(1/(1+\cos(d*x+c)))^{1/2}*EllipticE((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (-a+b)/(a-b))^{1/2})*\cos(d*x+c)^2*a^2-6*B*\sin(d*x+c)*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{1/2}*(1/(1+\cos(d*x+c)))^{1/2}*EllipticPi((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (a+b)/(a-b),$

$$\frac{I}{((a-b)/(a+b))^{1/2}} \cos(dx+c)^2 a^2 - 8B \sin(dx+c) \frac{(b+a \cos(dx+c))}{(1+\cos(dx+c))} \frac{1}{(a+b)^{1/2}} \left(\frac{1}{1+\cos(dx+c)} \right)^{1/2} \text{EllipticPi} \left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}, \frac{(a+b)}{(a-b)}, \frac{I}{((a-b)/(a+b))^{1/2}} \cos(dx+c)^2 b^2 + 6B \sin(dx+c) \frac{(b+a \cos(dx+c))}{(1+\cos(dx+c))} \frac{1}{(a+b)^{1/2}} \left(\frac{1}{1+\cos(dx+c)} \right)^{1/2} \text{EllipticF} \left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}, \frac{(a-b)}{(a+b)} \right)^{1/2} \frac{1}{\sin(dx+c)}, \left(\frac{-(a+b)}{(a-b)} \right)^{1/2} \cos(dx+c)^2 a^2 + 4B \sin(dx+c) \frac{(b+a \cos(dx+c))}{(1+\cos(dx+c))} \frac{1}{(a+b)^{1/2}} \left(\frac{1}{1+\cos(dx+c)} \right)^{1/2} \text{EllipticF} \left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}, \frac{(a-b)}{(a+b)} \right)^{1/2} \frac{1}{\sin(dx+c)}, \left(\frac{-(a+b)}{(a-b)} \right)^{1/2} \cos(dx+c)^2 b^2 - 4A \sin(dx+c) \frac{(b+a \cos(dx+c))}{(1+\cos(dx+c))} \frac{1}{(a+b)^{1/2}} \left(\frac{1}{1+\cos(dx+c)} \right)^{1/2} \text{EllipticE} \left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}, \frac{(a-b)}{(a+b)} \right)^{1/2} \frac{1}{\sin(dx+c)}, \left(\frac{-(a+b)}{(a-b)} \right)^{1/2} \cos(dx+c)^3 b^2 - 3B \sin(dx+c) \frac{(b+a \cos(dx+c))}{(1+\cos(dx+c))} \frac{1}{(a+b)^{1/2}} \left(\frac{1}{1+\cos(dx+c)} \right)^{1/2} \text{EllipticE} \left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}, \frac{(a-b)}{(a+b)} \right)^{1/2} \frac{1}{\sin(dx+c)}, \left(\frac{-(a+b)}{(a-b)} \right)^{1/2} \cos(dx+c)^3 a^2 - 6B \sin(dx+c) \frac{(b+a \cos(dx+c))}{(1+\cos(dx+c))} \frac{1}{(a+b)^{1/2}} \left(\frac{1}{1+\cos(dx+c)} \right)^{1/2} \text{EllipticPi} \left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}, \frac{(a+b)}{(a-b)}, \frac{I}{((a-b)/(a+b))^{1/2}} \cos(dx+c)^3 a^2 - 8B \sin(dx+c) \frac{(b+a \cos(dx+c))}{(1+\cos(dx+c))} \frac{1}{(a+b)^{1/2}} \left(\frac{1}{1+\cos(dx+c)} \right)^{1/2} \text{EllipticPi} \left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}, \frac{(a+b)}{(a-b)}, \frac{I}{((a-b)/(a+b))^{1/2}} \cos(dx+c)^3 b^2 + 6B \sin(dx+c) \frac{(b+a \cos(dx+c))}{(1+\cos(dx+c))} \frac{1}{(a+b)^{1/2}} \left(\frac{1}{1+\cos(dx+c)} \right)^{1/2} \text{EllipticF} \left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}, \left(\frac{-(a+b)}{(a-b)} \right)^{1/2} \cos(dx+c)^3 a^2 + 4B \sin(dx+c) \frac{(b+a \cos(dx+c))}{(1+\cos(dx+c))} \frac{1}{(a+b)^{1/2}} \left(\frac{1}{1+\cos(dx+c)} \right)^{1/2} \text{EllipticF} \left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}, \left(\frac{-(a+b)}{(a-b)} \right)^{1/2} \cos(dx+c)^3 b^2 - 4A \sin(dx+c) \frac{(b+a \cos(dx+c))}{(1+\cos(dx+c))} \frac{1}{(a+b)^{1/2}} \left(\frac{1}{1+\cos(dx+c)} \right)^{1/2} \text{EllipticE} \left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}, \left(\frac{-(a+b)}{(a-b)} \right)^{1/2} \cos(dx+c)^2 b^2 \right) \frac{1}{\cos(dx+c)} \right)^{5/2} \frac{(b+a \cos(dx+c))}{\sin(dx+c)} \frac{1}{(a+b)^{1/2}}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx+c) + A) \sec(dx+c)^{5/2}}{\sqrt{b \sec(dx+c) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(dx+c)^(5/2)*(A+B*sec(dx+c))/(a+b*sec(dx+c))^(1/2),x, algorithm="maxima")

[Out] integrate((B*sec(dx + c) + A)*sec(dx + c)^(5/2)/sqrt(b*sec(dx + c) + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(\frac{1}{\cos(c+dx)}\right)^{5/2}}{\sqrt{a + \frac{b}{\cos(c+dx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + b/cos(c + d*x))^(1/2), x)

[Out] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + b/cos(c + d*x))^(1/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(1/2), x)

[Out] Timed out

$$3.457 \quad \int \frac{\sec^3(c+dx)(A+B \sec(c+dx))}{\sqrt{a+b \sec(c+dx)}} dx$$

Optimal. Leaf size=256

$$\frac{(2Ab - aB)\sqrt{\sec(c+dx)}\sqrt{\frac{a \cos(c+dx)+b}{a+b}} \Pi\left(2; \frac{1}{2}(c+dx) \middle| \frac{2a}{a+b}\right)}{bd\sqrt{a+b \sec(c+dx)}} + \frac{B \sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b \sec(c+dx)}}{bd} + \dots$$

[Out] B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c), 2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/d/(a+b*sec(d*x+c))^(1/2)+(2*A*b-B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c), 2, 2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/b/d/(a+b*sec(d*x+c))^(1/2)-B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c), 2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/b/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)+B*sin(d*x+c)*sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/b/d

Rubi [A] time = 0.73, antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.343$, Rules used = {4033, 4109, 3859, 2807, 2805, 3862, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{(2Ab - aB)\sqrt{\sec(c+dx)}\sqrt{\frac{a \cos(c+dx)+b}{a+b}} \Pi\left(2; \frac{1}{2}(c+dx) \middle| \frac{2a}{a+b}\right)}{bd\sqrt{a+b \sec(c+dx)}} + \frac{B \sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b \sec(c+dx)}}{bd} + \dots$$

Antiderivative was successfully verified.

[In] Int[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/Sqrt[a + b*Sec[c + d*x]], x]

[Out] (B*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + ((2*A*b - a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(b*d*Sqrt[a + b*Sec[c + d*x]]) - (B*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(b*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (B*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(b*d)

Rule 2653

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*Sqrt[a + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,

b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2655

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a + b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2661

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)]/(d*Sqrt[a + b])), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2663

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b) + (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2805

Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi/2 + f*x))/2, (2*d)/(c + d)]/(f*(a + b)*Sqrt[c + d])), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[c + d, 0]

Rule 2807

Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !GtQ[c + d, 0]

Rule 3856

Int[Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)]/Sqrt[csc[(e_) + (f_)*(x_)]*(d_)], x_Symbol] := Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] := Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] := Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]
])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3862

```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] := Dist[1/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc
[e + f*x]], x], x] - Dist[b/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[
e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4033

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] := -Simp[(B*d^2
*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2))/(b*f*(
m + n)), x] + Dist[d^2/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f
*x])^(n - 2)*Simp[a*B*(n - 2) + B*b*(m + n - 1)*Csc[e + f*x] + (A*b*(m + n)
- a*B*(n - 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m
}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && NeQ[m + n,
0] && !IGtQ[m, 1]
```

Rule 4109

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/(Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] := Dist[C/d^
2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[A, In
t[1/(Sqrt[d*Csc[e + f*x]]*Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b,
d, e, f, A, C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}} dx &= \frac{B\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{bd} + \frac{\int \frac{-\frac{aB}{2} + \frac{1}{2}(2Ab-aB)\sec^2(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}} dx}{b} \\
&= \frac{B\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{bd} - \frac{(aB)\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}} dx}{2b} \\
&= \frac{B\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{bd} + \frac{1}{2}B\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}} dx \\
&= \frac{B\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{bd} + \frac{(B\sqrt{b+a\cos(c+dx)})}{2\sqrt{a}} \\
&= \frac{(2Ab-aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2; \frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{bd\sqrt{a+b\sec(c+dx)}} + \frac{B\sqrt{\sec(c+dx)}}{bd} \\
&= \frac{B\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}} + \frac{(2Ab-aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}{bd}
\end{aligned}$$

Mathematica [C] time = 7.19, size = 339, normalized size = 1.32

$$\sqrt{\sec(c+dx)} \left(2(4Ab-3aB)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\Pi\left(2; \frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right) + 4B\tan(c+dx)(a\cos(c+dx)+b) - \frac{2iB\csc(c+dx)}{b} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/Sqrt[a + b*Sec[c + d*x]],x]

[Out] (Sqrt[Sec[c + d*x]]*(2*(4*A*b - 3*a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)] - ((2*I)*B*Sqrt[-((a*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(a*(1 + Cos[c + d*x]))/(a - b)]*Sqrt[b + a*Cos[c + d*x]])*Csc[c + d*x]*(-2*b*(a + b)*EllipticE[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*(2*b*EllipticF[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*EllipticPi[1 - a/b, I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a

$$\frac{+ b)])))/(a*\text{Sqrt}[(a - b)^{-1}]*b) + 4*B*(b + a*\text{Cos}[c + d*x])* \text{Tan}[c + d*x])}{(4*b*d*\text{Sqrt}[a + b*\text{Sec}[c + d*x]])}$$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sec(dx + c)^{\frac{3}{2}}}{\sqrt{b \sec(dx + c) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(3/2)/sqrt(b*sec(d*x + c) + a), x)

maple [C] time = 2.57, size = 1440, normalized size = 5.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x)

[Out]
$$-1/d*(4*A*((b+a*\text{cos}(d*x+c))/(1+\text{cos}(d*x+c)))/(a+b))^{1/2}*(1/(1+\text{cos}(d*x+c)))^{1/2}*\text{EllipticPi}((-1+\text{cos}(d*x+c))*((a-b)/(a+b))^{1/2}/\text{sin}(d*x+c), (a+b)/(a-b), I/((a-b)/(a+b))^{1/2})*\text{sin}(d*x+c)*\text{cos}(d*x+c)^{2*b-2*A*((b+a*\text{cos}(d*x+c))/(1+\text{cos}(d*x+c)))/(a+b))^{1/2}*(1/(1+\text{cos}(d*x+c)))^{1/2}*\text{EllipticF}((-1+\text{cos}(d*x+c))*((a-b)/(a+b))^{1/2}/\text{sin}(d*x+c), (-a+b)/(a-b))^{1/2})*\text{sin}(d*x+c)*\text{cos}(d*x+c)^{2*b-2*B*((b+a*\text{cos}(d*x+c))/(1+\text{cos}(d*x+c)))/(a+b))^{1/2}*(1/(1+\text{cos}(d*x+c)))^{1/2}*\text{EllipticPi}((-1+\text{cos}(d*x+c))*((a-b)/(a+b))^{1/2}/\text{sin}(d*x+c), (a+b)/(a-b), I/((a-b)/(a+b))^{1/2})*\text{sin}(d*x+c)*\text{cos}(d*x+c)^{2*a+2*B*\text{cos}(d*x+c)^{2*((b+a*\text{cos}(d*x+c))/(1+\text{cos}(d*x+c)))/(a+b))^{1/2}*(1/(1+\text{cos}(d*x+c)))^{1/2}*\text{EllipticF}((-1+\text{cos}(d*x+c))*((a-b)/(a+b))^{1/2}/\text{sin}(d*x+c), (-a+b)/(a-b))^{1/2})*\text{sin}(d*x+c)*a-B*((b+a*\text{cos}(d*x+c))/(1+\text{cos}(d*x+c)))/(a+b))^{1/2}*(1/(1+\text{cos}(d*x+c)))^{1/2})*\text{EllipticE}((-1+\text{cos}(d*x+c))*((a-b)/(a+b))^{1/2}/\text{sin}(d*x+c), (-a+b)/(a-b))^{1/2}$$

```

1/2))*sin(d*x+c)*cos(d*x+c)^2*a+B*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(
1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)
/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*b+4*A*((b+a*cos(d
*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+
cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2
))*sin(d*x+c)*cos(d*x+c)*b-2*A*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2
)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*b-2*B*((b+a*cos(d*x+c)
)/(1+cos(d*x+c)))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d
*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*si
n(d*x+c)*cos(d*x+c)*a+2*B*cos(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b)
)^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*((b+a*cos(d*x+c))/(1+co
s(d*x+c)))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a-B*((b+a*cos(d*x+c))/(1+co
s(d*x+c)))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*
(a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a
+B*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*E
llipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2
))*sin(d*x+c)*cos(d*x+c)*b+B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a-B*cos(d*x+c
)*((a-b)/(a+b))^(1/2)*a+B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b-B*((a-b)/(a+b))^(
1/2)*b)*cos(d*x+c)*(1/cos(d*x+c))^(3/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2
)/sin(d*x+c)/(b+a*cos(d*x+c))/(a-b)/(a+b))^(1/2)/b

```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sec(dx + c)^{\frac{3}{2}}}{\sqrt{b \sec(dx + c) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algo
rithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(3/2)/sqrt(b*sec(d*x + c) + a),
x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(\frac{1}{\cos(c+dx)}\right)^{3/2}}{\sqrt{a + \frac{b}{\cos(c+dx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + b/cos(c + d*x))^(1/2
),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + b/cos(c + d*x))^(1/2), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

$$3.458 \quad \int \frac{\sqrt{\sec(c+dx)} (A+B \sec(c+dx))}{\sqrt{a+b \sec(c+dx)}} dx$$

Optimal. Leaf size=138

$$\frac{2A\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}} + \frac{2B\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}$$

[Out] 2*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/d/(a+b*sec(d*x+c))^(1/2)+2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/d/(a+b*sec(d*x+c))^(1/2)

Rubi [A] time = 0.39, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4036, 3858, 2663, 2661, 3859, 2807, 2805}

$$\frac{2A\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}} + \frac{2B\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x]))/Sqrt[a + b*Sec[c + d*x]],x]

[Out] (2*A*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]]/(d*Sqrt[a + b*Sec[c + d*x]]) + (2*B*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]]/(d*Sqrt[a + b*Sec[c + d*x]]))

Rule 2661

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2663

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b) + (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2805


```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]]), x_Symbol] := Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)]/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]]), x_Symbol] := Dist[Sqrt[(c + d*Sin[e + f*x])]/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
+ f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !GtQ[c + d, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_.)], x_Symbol] := Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_.)], x_Symbol] := Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4036

```
Int[(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (
A_.)))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] := Dist[A, Int[
Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B/d, Int[(d*Cs
c[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\sec(c+dx)}(A+B\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}} dx &= A \int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}} dx + B \int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}} dx \\
&= \frac{(A\sqrt{b+a\cos(c+dx)}\sqrt{\sec(c+dx)}) \int \frac{1}{\sqrt{b+a\cos(c+dx)}} dx}{\sqrt{a+b\sec(c+dx)}} + \frac{(B\sqrt{b+a\cos(c+dx)}) \int \frac{1}{\sqrt{b+a\cos(c+dx)}} dx}{\sqrt{a+b\sec(c+dx)}} \\
&= \frac{(A\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}) \int \frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}} dx}{\sqrt{a+b\sec(c+dx)}} + \frac{(B\sqrt{\frac{b+a\cos(c+dx)}{a+b}}) \int \frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}} dx}{\sqrt{a+b\sec(c+dx)}} \\
&= \frac{2A\sqrt{\frac{b+a\cos(c+dx)}{a+b}} F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right) \sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}} + \frac{2B\sqrt{\frac{b+a\cos(c+dx)}{a+b}} \Pi\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}
\end{aligned}$$

Mathematica [A] time = 0.27, size = 91, normalized size = 0.66

$$\frac{2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}} \left(AF\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right) + B\Pi\left(2;\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right) \right)}{d\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x]))/Sqrt[a + b*Sec[c + d*x]], x]
```

```
[Out] (2*Sqrt[(b + a*Cos[c + d*x])]/(a + b))*(A*EllipticF[(c + d*x)/2, (2*a)/(a + b)] + B*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2), x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\sec(dx+c) + A)\sqrt{\sec(dx+c)}}{\sqrt{b\sec(dx+c) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algorith="giac")

[Out] integrate((B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/sqrt(b*sec(d*x + c) + a), x)

maple [C] time = 2.10, size = 283, normalized size = 2.05

$$2 \left(A \operatorname{EllipticF} \left(\frac{(-1+\cos(dx+c))\sqrt{\frac{a-b}{a+b}}}{\sin(dx+c)}, \sqrt{\frac{a+b}{a-b}} \right) - B \operatorname{EllipticF} \left(\frac{(-1+\cos(dx+c))\sqrt{\frac{a-b}{a+b}}}{\sin(dx+c)}, \sqrt{\frac{a+b}{a-b}} \right) + 2B \operatorname{EllipticPi} \left(\frac{(-1+\cos(dx+c))\sqrt{\frac{a-b}{a+b}}}{\sin(dx+c)}, \sqrt{\frac{a+b}{a-b}} \right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x)

[Out] 2/d*(A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))-B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))+2*B*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*cos(d*x+c)*(1/(1+cos(d*x+c)))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b)^(1/2)*sin(d*x+c)^2*(1/cos(d*x+c))^(1/2))*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(-1+cos(d*x+c))/(b+a*cos(d*x+c))/(a+b)^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx+c) + A) \sqrt{\sec(dx+c)}}{\sqrt{b \sec(dx+c) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algorith="maxima")

[Out] integrate((B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/sqrt(b*sec(d*x + c) + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)} \right) \sqrt{\frac{1}{\cos(c+dx)}}}{\sqrt{a + \frac{b}{\cos(c+dx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + b/cos(c + d*x))^(1/2), x)
```

```
[Out] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + b/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \sqrt{\sec(c + dx)}}{\sqrt{a + b \sec(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)**(1/2)/(a+b*sec(d*x+c))**(1/2), x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sqrt(sec(c + d*x))/sqrt(a + b*sec(c + d*x)), x)
```

$$3.459 \quad \int \frac{A+B \sec(c+dx)}{\sqrt{\sec(c+dx)} \sqrt{a+b \sec(c+dx)}} dx$$

Optimal. Leaf size=150

$$\frac{2A\sqrt{a+b \sec(c+dx)} E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right) - 2(Ab-aB)\sqrt{\sec(c+dx)} \sqrt{\frac{a \cos(c+dx)+b}{a+b}} F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)} \sqrt{\frac{a \cos(c+dx)+b}{a+b}} - ad\sqrt{a+b \sec(c+dx)}}$$

[Out] $-2*(A*b-B*a)*(cos(1/2*d*x+1/2*c)^2)^{(1/2)}/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c), 2^{(1/2)*(a/(a+b))^{(1/2)})*((b+a*cos(d*x+c))/(a+b))^{(1/2)}*sec(d*x+c)^{(1/2)}/a/d/(a+b*sec(d*x+c))^{(1/2)}+2*A*(cos(1/2*d*x+1/2*c)^2)^{(1/2)}/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c), 2^{(1/2)*(a/(a+b))^{(1/2)})*(a+b*sec(d*x+c))^{(1/2)}/a/d/((b+a*cos(d*x+c))/(a+b))^{(1/2)}/sec(d*x+c)^{(1/2)})$

Rubi [A] time = 0.31, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2A\sqrt{a+b \sec(c+dx)} E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right) - 2(Ab-aB)\sqrt{\sec(c+dx)} \sqrt{\frac{a \cos(c+dx)+b}{a+b}} F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)} \sqrt{\frac{a \cos(c+dx)+b}{a+b}} - ad\sqrt{a+b \sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] `Int[(A + B*Sec[c + d*x])/(Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]),x]`

[Out] $(-2*(A*b - a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]]/(a*d*Sqrt[a + b*Sec[c + d*x]]) + (2*A*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(a*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]])$

Rule 2653

`Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*Sqrt[a + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]`

Rule 2655

`Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a + b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]`

Rule 2661

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)]/(d*Sqrt[a + b]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b) + (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)]/Sqrt[csc[(e_) + (f_)*(x_)]*(d_)], x_Symbol] := Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_) + (f_)*(x_)]*(d_)]/Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)], x_Symbol] := Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4035

```
Int[(csc[(e_) + (f_)*(x_)]*(B_) + (A_))/(Sqrt[csc[(e_) + (f_)*(x_)]*(d_) + (a_)])*(Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)]), x_Symbol] := Dist[A/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{A + B \sec(c + dx)}{\sqrt{\sec(c + dx)} \sqrt{a + b \sec(c + dx)}} dx &= \frac{A \int \frac{\sqrt{a+b \sec(c+dx)}}{\sqrt{\sec(c+dx)}} dx}{a} - \frac{(Ab - aB) \int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b \sec(c+dx)}} dx}{a} \\
&= -\frac{\left((Ab - aB) \sqrt{b + a \cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{1}{\sqrt{b+a \cos(c+dx)}} dx}{a \sqrt{a + b \sec(c + dx)}} + \left(\frac{A \sqrt{a+b \sec(c+dx)}}{a} \right) \\
&= -\frac{\left((Ab - aB) \sqrt{\frac{b+a \cos(c+dx)}{a+b}} \sqrt{\sec(c + dx)} \right) \int \frac{1}{\sqrt{\frac{b}{a+b} + \frac{a \cos(c+dx)}{a+b}}} dx}{a \sqrt{a + b \sec(c + dx)}} + \frac{A \sqrt{a+b \sec(c+dx)}}{a} \\
&= -\frac{2(Ab - aB) \sqrt{\frac{b+a \cos(c+dx)}{a+b}} F\left(\frac{1}{2}(c + dx) \middle| \frac{2a}{a+b}\right) \sqrt{\sec(c + dx)}}{ad \sqrt{a + b \sec(c + dx)}} + \frac{2AE\left(\frac{1}{2}(c + dx) \middle| \frac{2a}{a+b}\right)}{ad}
\end{aligned}$$

Mathematica [A] time = 3.89, size = 103, normalized size = 0.69

$$\frac{2\sqrt{\sec(c + dx)} \sqrt{\frac{a \cos(c+dx)+b}{a+b}} \left((aB - Ab) F\left(\frac{1}{2}(c + dx) \middle| \frac{2a}{a+b}\right) + A(a + b) E\left(\frac{1}{2}(c + dx) \middle| \frac{2a}{a+b}\right) \right)}{ad \sqrt{a + b \sec(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sec[c + d*x])/(Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]), x]

[Out] (2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*(A*(a + b)*EllipticE[(c + d*x)/2, (2*a)/(a + b)] + (-A*b) + a*B)*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]]/(a*d*Sqrt[a + b*Sec[c + d*x]])

fricas [F] time = 1.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B \sec(dx + c) + A) \sqrt{b \sec(dx + c) + a} \sqrt{\sec(dx + c)}}{b \sec(dx + c)^2 + a \sec(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2), x, algorithm="fricas")

[Out] integral((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*sqrt(sec(d*x + c))/(b*sec(d*x + c)^2 + a*sec(d*x + c)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{\sqrt{b \sec(dx + c) + a} \sqrt{\sec(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)/(sqrt(b*sec(d*x + c) + a)*sqrt(sec(d*x + c))), x)

maple [B] time = 2.51, size = 940, normalized size = 6.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2), x)

[Out]
$$\begin{aligned} & -2/d*(-A*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{1/2}*(1/(1+\cos(d*x+c)))^{1/2} \\ & *EllipticF((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (- (a+b)/(a-b))^{1/2}) \\ & * \sin(d*x+c)*\cos(d*x+c)*a+A*\cos(d*x+c)*EllipticE((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), \\ & (- (a+b)/(a-b))^{1/2}) * \sin(d*x+c)*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2} \\ & *(1/(1+\cos(d*x+c)))^{1/2} * a - A*\cos(d*x+c)*EllipticE((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), \\ & (- (a+b)/(a-b))^{1/2}) * \sin(d*x+c)*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2} \\ & *(1/(1+\cos(d*x+c)))^{1/2} * b + B*\cos(d*x+c)*EllipticF((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), \\ & (- (a+b)/(a-b))^{1/2}) * \sin(d*x+c)*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2} \\ & *(1/(1+\cos(d*x+c)))^{1/2} * a - A*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2} \\ & *(1/(1+\cos(d*x+c)))^{1/2} * EllipticF((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), \\ & (- (a+b)/(a-b))^{1/2}) * a * \sin(d*x+c) + A*EllipticE((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), \\ & (- (a+b)/(a-b))^{1/2}) * a * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2} \\ & *(1/(1+\cos(d*x+c)))^{1/2} * \sin(d*x+c) - A*EllipticE((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), \\ & (- (a+b)/(a-b))^{1/2}) * b * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2} \\ & *(1/(1+\cos(d*x+c)))^{1/2} * \sin(d*x+c) + B*EllipticF((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), \\ & (- (a+b)/(a-b))^{1/2}) * a * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2} \\ & *(1/(1+\cos(d*x+c)))^{1/2} * \sin(d*x+c) + A*\cos(d*x+c)^2*((a-b)/(a+b))^{1/2} * a - A*\cos(d*x+c)*((a-b)/(a+b))^{1/2} \\ & * a + A*\cos(d*x+c)*((a-b)/(a+b))^{1/2} * b - A*b*((a-b)/(a+b))^{1/2} \\ & *((b+a*\cos(d*x+c))/\cos(d*x+c))^{1/2}/(1/\cos(d*x+c))^{1/2}/\sin(d*x+c)/(b+a*\cos(d*x+c))/((a-b)/(a+b))^{1/2}/a \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{\sqrt{b \sec(dx + c) + a} \sqrt{\sec(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)/(sqrt(b*sec(d*x + c) + a)*sqrt(sec(d*x + c))), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\sqrt{a + \frac{b}{\cos(c+dx)}} \sqrt{\frac{1}{\cos(c+dx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/2)), x)

[Out] int((A + B/cos(c + d*x))/((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \sec(c + dx)}{\sqrt{a + b \sec(c + dx)} \sqrt{\sec(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)**(1/2)/(a+b*sec(d*x+c))**(1/2),x)

[Out] Integral((A + B*sec(c + d*x))/(sqrt(a + b*sec(c + d*x))*sqrt(sec(c + d*x))), x)

$$3.460 \quad \int \frac{A+B \sec(c+dx)}{\sec^2(c+dx) \sqrt{a+b \sec(c+dx)}} dx$$

Optimal. Leaf size=212

$$\frac{2(a^2A - 3abB + 2Ab^2) \sqrt{\sec(c+dx)} \sqrt{\frac{a \cos(c+dx)+b}{a+b}} F\left(\frac{1}{2}(c+dx) \middle| \frac{2a}{a+b}\right)}{3a^2d \sqrt{a+b \sec(c+dx)}} - \frac{2(2Ab - 3aB) \sqrt{a+b \sec(c+dx)} E\left(\frac{1}{2}(c+dx) \middle| \frac{2a}{a+b}\right)}{3a^2d \sqrt{\sec(c+dx)} \sqrt{\frac{a \cos(c+dx)+b}{a+b}}}$$

[Out] $2/3*(A*a^2+2*A*b^2-3*B*a*b)*(cos(1/2*d*x+1/2*c)^2)^{(1/2)}/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c), 2^{(1/2)}*(a/(a+b))^{(1/2)}*((b+a*cos(d*x+c))/(a+b))^{(1/2)}*sec(d*x+c)^{(1/2)}/a^2/d/(a*b*sec(d*x+c))^{(1/2)}+2/3*A*sin(d*x+c)*(a+b*sec(d*x+c))^{(1/2)}/a/d/sec(d*x+c)^{(1/2)}-2/3*(2*A*b-3*B*a)*(cos(1/2*d*x+1/2*c)^2)^{(1/2)}/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c), 2^{(1/2)}*(a/(a+b))^{(1/2)}*(a+b*sec(d*x+c))^{(1/2)}/a^2/d/((b+a*cos(d*x+c))/(a+b))^{(1/2)}/sec(d*x+c)^{(1/2)})$

Rubi [A] time = 0.48, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {4034, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2(a^2A - 3abB + 2Ab^2) \sqrt{\sec(c+dx)} \sqrt{\frac{a \cos(c+dx)+b}{a+b}} F\left(\frac{1}{2}(c+dx) \middle| \frac{2a}{a+b}\right)}{3a^2d \sqrt{a+b \sec(c+dx)}} - \frac{2(2Ab - 3aB) \sqrt{a+b \sec(c+dx)} E\left(\frac{1}{2}(c+dx) \middle| \frac{2a}{a+b}\right)}{3a^2d \sqrt{\sec(c+dx)} \sqrt{\frac{a \cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]),x]

[Out] $(2*(a^2*A + 2*A*b^2 - 3*a*b*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(3*a^2*d*Sqrt[a + b*Sec[c + d*x]]) - (2*(2*A*b - 3*a*B)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*a^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*A*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*a*d*Sqrt[Sec[c + d*x]])$

Rule 2653

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*Sqrt[a + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2655

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
0] && !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)]/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a
+ b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && !GtQ[a + b, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)]/Sqrt[csc[(e_) + (f_)*(x_)]
*(d_)], x_Symbol] := Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_) + (f_)*(x_)]*(d_)]/Sqrt[csc[(e_) + (f_)*(x_)]*(b_)
+ (a_)], x_Symbol] := Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4034

```
Int[(csc[(e_) + (f_)*(x_)]*(d_))^(n_)*(csc[(e_) + (f_)*(x_)]*(b_) + (
a_))^(m_)*(csc[(e_) + (f_)*(x_)]*(B_) + (A_)), x_Symbol] := Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*n), x] + Dis
t[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n
- A*b*(m + n + 1) + A*a*(n + 1)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x
]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0]
&& NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.))/(Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]), x_Symbol] :> Dist[A/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{A + B \sec(c + dx)}{\sec^2(c + dx) \sqrt{a + b \sec(c + dx)}} dx &= \frac{2A \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{3ad \sqrt{\sec(c + dx)}} - \frac{2 \int \frac{\frac{1}{2}(2Ab - 3aB) - \frac{1}{2}aA \sec(c + dx)}{\sqrt{\sec(c + dx)} \sqrt{a + b \sec(c + dx)}} dx}{3a} \\ &= \frac{2A \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{3ad \sqrt{\sec(c + dx)}} - \frac{(2Ab - 3aB) \int \frac{\sqrt{a + b \sec(c + dx)}}{\sqrt{\sec(c + dx)}} dx}{3a^2} + \dots \\ &= \frac{2A \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{3ad \sqrt{\sec(c + dx)}} + \frac{\left(A + \frac{b(2Ab - 3aB)}{a^2} \right) \sqrt{b + a \cos(c + dx)}}{3 \sqrt{a + b \sec(c + dx)}} \\ &= \frac{2A \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{3ad \sqrt{\sec(c + dx)}} + \frac{\left(A + \frac{b(2Ab - 3aB)}{a^2} \right) \sqrt{\frac{b + a \cos(c + dx)}{a + b}} \sqrt{\sec(c + dx)}}{3 \sqrt{a + b \sec(c + dx)}} \\ &= \frac{2 \left(A + \frac{b(2Ab - 3aB)}{a^2} \right) \sqrt{\frac{b + a \cos(c + dx)}{a + b}} F\left(\frac{1}{2}(c + dx) \middle| \frac{2a}{a + b}\right) \sqrt{\sec(c + dx)}}{3d \sqrt{a + b \sec(c + dx)}} - \frac{2(2Ab - 3aB)}{3a^2} \int \frac{\sqrt{a + b \sec(c + dx)}}{\sqrt{\sec(c + dx)}} dx \end{aligned}$$

Mathematica [A] time = 0.88, size = 161, normalized size = 0.76

$$\frac{2 \sqrt{\sec(c + dx)} \left((a^2 A - 3abB + 2Ab^2) \sqrt{\frac{a \cos(c + dx) + b}{a + b}} F\left(\frac{1}{2}(c + dx) \middle| \frac{2a}{a + b}\right) + (a + b)(3aB - 2Ab) \sqrt{\frac{a \cos(c + dx) + b}{a + b}} E\left(\frac{1}{2}(c + dx) \middle| \frac{2a}{a + b}\right) \right)}{3a^2 d \sqrt{a + b \sec(c + dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]), x]
```

```
[Out] (2*Sqrt[Sec[c + d*x]]*((a + b)*(-2*A*b + 3*a*B)*Sqrt[(b + a*Cos[c + d*x])]/(a + b)]*EllipticE[(c + d*x)/2, (2*a)/(a + b)] + (a^2*A + 2*A*b^2 - 3*a*b*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)] + a*A*(b + a*Cos[c + d*x])*Sin[c + d*x])/(3*a^2*d*Sqrt[a + b*Sec[c + d*x]])
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B \sec(dx+c) + A)\sqrt{b \sec(dx+c) + a} \sqrt{\sec(dx+c)}}{b \sec(dx+c)^3 + a \sec(dx+c)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")

[Out] integral((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*sqrt(sec(d*x + c))/(b*sec(d*x + c)^3 + a*sec(d*x + c)^2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx+c) + A}{\sqrt{b \sec(dx+c) + a} \sec(dx+c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)/(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(3/2)), x)

maple [B] time = 2.33, size = 1731, normalized size = 8.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x)

[Out]
$$\begin{aligned} & -2/3/d*(A*EllipticF((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (-a+b)/(a-b))^{1/2}) * a^2 * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)) / (a+b))^{1/2} * (1/(1+\cos(d*x+c)))^{1/2} * \sin(d*x+c) + 2*A*\cos(d*x+c)*\sin(d*x+c)*EllipticF((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (-a+b)/(a-b))^{1/2}) * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)) / (a+b))^{1/2} * (1/(1+\cos(d*x+c)))^{1/2} * a*b - A*((a-b)/(a+b))^{1/2} * a*b - 3*B*((a-b)/(a+b))^{1/2} * a*b + 2*A*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)) / (a+b))^{1/2} * (1/(1+\cos(d*x+c)))^{1/2} * EllipticE((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (-a+b)/(a-b))^{1/2}) * b^2 * \sin(d*x+c) - 2*A*\cos(d*x+c)*\sin(d*x+c) * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)) / (a+b))^{1/2} * (1/(1+\cos(d*x+c)))^{1/2} * EllipticE((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (-a+b)/(a-b))^{1/2}) * a*b - 3*B*\cos(d*x+c)*\sin(d*x+c) * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)) / (a+b))^{1/2} * (1/(1+\cos(d*x+c)))^{1/2} * EllipticE((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2} \end{aligned}$$

```

/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b+2*A*((a-b)/(a+b))^(1/2)*b^2+3*B*cos(d
*x+c)^2*((a-b)/(a+b))^(1/2)*a^2-2*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^2+A*co
s(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*(1/(1
+cos(d*x+c)))^(1/2)*a^2+2*A*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(
d*x+c)))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a
-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2-3*B*cos(d*x+c)*sin(d*
x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b)
)^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(
1/2)*a^2+3*B*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(
1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)
)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2+2*A*EllipticF((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b*((b+a*cos(d*x+c))/(1+cos
(d*x+c)))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-2*A*((b+a*cos(d*x
+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos
(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b*sin(d*x+c
)-3*B*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2
)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(
1/2))*a*b*sin(d*x+c)-A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b+3*B*cos(d*x+c)*
((a-b)/(a+b))^(1/2)*a*b-3*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2+A*cos(d*x+c)
^3*((a-b)/(a+b))^(1/2)*a^2-A*a^2*((a-b)/(a+b))^(1/2)*cos(d*x+c)+2*A*cos(d*x
+c)*((a-b)/(a+b))^(1/2)*a*b-3*B*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/
2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*sin(d*x+c)+3*B*((b+a*cos(d*x+c))/(1+cos
(d*x+c)))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((
a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*sin(d*x+c)*((b+a*co
s(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^2*(1/cos(d*x+c))^(3/2)/sin(d*x+c)/(b
+a*cos(d*x+c))/a^2/((a-b)/(a+b))^(1/2)

```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{\sqrt{b \sec(dx + c) + a} \sec(dx + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x,algor
ithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)/(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(3/2)
, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\sqrt{a + \frac{b}{\cos(c+dx)} \left(\frac{1}{\cos(c+dx)}\right)^{3/2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(3/2)), x)

[Out] int((A + B/cos(c + d*x))/((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(3/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \sec(c + dx)}{\sqrt{a + b \sec(c + dx)} \sec^{\frac{3}{2}}(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)**(3/2)/(a+b*sec(d*x+c))**(1/2), x)

[Out] Integral((A + B*sec(c + d*x))/(sqrt(a + b*sec(c + d*x))*sec(c + d*x)**(3/2)), x)

$$3.461 \quad \int \frac{A+B \sec(c+dx)}{\sec^2(c+dx) \sqrt{a+b \sec(c+dx)}} dx$$

Optimal. Leaf size=280

$$\frac{2(4Ab - 5aB) \sin(c + dx) \sqrt{a + b \sec(c + dx)}}{15a^2 d \sqrt{\sec(c + dx)}} + \frac{2(9a^2 A - 10abB + 8Ab^2) \sqrt{a + b \sec(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2a}{a+b}\right)}{15a^3 d \sqrt{\sec(c + dx)} \sqrt{\frac{a \cos(c+dx)+b}{a+b}}}$$

[Out] $-2/15*(7*A*a^2*b+8*A*b^3-5*B*a^3-10*B*a*b^2)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)}*(a/(a+b))^{(1/2)})*((b+a*\cos(d*x+c))/(a+b))^{(1/2)}*\sec(d*x+c)^{(1/2)}/a^3/d/(a+b*\sec(d*x+c))^{(1/2)}+2/5*A*\sin(d*x+c)*(a+b*\sec(d*x+c))^{(1/2)}/a/d/\sec(d*x+c)^{(3/2)}-2/15*(4*A*b-5*B*a)*\sin(d*x+c)*(a+b*\sec(d*x+c))^{(1/2)}/a^2/d/\sec(d*x+c)^{(1/2)}+2/15*(9*A*a^2+8*A*b^2-10*B*a*b)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)}*(a/(a+b))^{(1/2)})*(a+b*\sec(d*x+c))^{(1/2)}/a^3/d/((b+a*\cos(d*x+c))/(a+b))^{(1/2)}/\sec(d*x+c)^{(1/2)}$

Rubi [A] time = 0.75, antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {4034, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2(7a^2 Ab - 5a^3 B - 10ab^2 B + 8Ab^3) \sqrt{\sec(c + dx)} \sqrt{\frac{a \cos(c+dx)+b}{a+b}} F\left(\frac{1}{2}(c + dx) \middle| \frac{2a}{a+b}\right)}{15a^3 d \sqrt{a + b \sec(c + dx)}} + \frac{2(9a^2 A - 10abB + 8Ab^2) \sqrt{a + b \sec(c + dx)}}{15a^3 d \sqrt{\sec(c + dx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(A + B*\text{Sec}[c + d*x])]/(\text{Sec}[c + d*x]^{(5/2)}*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]), x]$

[Out] $(-2*(7*a^2*A*b + 8*A*b^3 - 5*a^3*B - 10*a*b^2*B)*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])]/(a + b))*\text{EllipticF}[(c + d*x)/2, (2*a)/(a + b)]*\text{Sqrt}[\text{Sec}[c + d*x]]/(15*a^3*d*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]) + (2*(9*a^2*A + 8*A*b^2 - 10*a*b*B)*\text{EllipticE}[(c + d*x)/2, (2*a)/(a + b)]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]])/(15*a^3*d*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])]/(a + b))*\text{Sqrt}[\text{Sec}[c + d*x]]) + (2*A*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(5*a*d*\text{Sec}[c + d*x]^{(3/2)}) - (2*(4*A*b - 5*a*B)*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(15*a^2*d*\text{Sqrt}[\text{Sec}[c + d*x]])$

Rule 2653

$\text{Int}[\text{Sqrt}[(a_) + (b_)*\sin[(c_) + (d_)*(x_)]], x_Symbol] \rightarrow \text{Simp}[(2*\text{Sqrt}[a + b]*\text{EllipticE}[(1*(c - \text{Pi}/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; \text{FreeQ}\{a, b, c, d\}, x] \ \&\& \ \text{NeQ}[a^2 - b^2, 0] \ \&\& \ \text{GtQ}[a + b, 0]$

Rule 2655

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
0] && !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)]/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a
+ b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && !GtQ[a + b, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)]/Sqrt[csc[(e_) + (f_)*(x_)]
*(d_)], x_Symbol] := Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_) + (f_)*(x_)]*(d_)]/Sqrt[csc[(e_) + (f_)*(x_)]*(b_)
+ (a_)], x_Symbol] := Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4034

```
Int[(csc[(e_) + (f_)*(x_)]*(d_))^(n_)*(csc[(e_) + (f_)*(x_)]*(b_) + (
a_))^(m_)*(csc[(e_) + (f_)*(x_)]*(B_) + (A_)), x_Symbol] := Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f^n), x] + Dis
t[1/(a*d^n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n
- A*b*(m + n + 1) + A*a*(n + 1)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x
]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0]
&& NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.))/(Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]), x_Symbol] := Dist[A/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m, x_Symbol] := Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\begin{aligned}
 \int \frac{A + B \sec(c + dx)}{\sec^{\frac{5}{2}}(c + dx) \sqrt{a + b \sec(c + dx)}} dx &= \frac{2A \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{5ad \sec^{\frac{3}{2}}(c + dx)} - \frac{2 \int \frac{\frac{1}{2}(4Ab - 5aB) - \frac{3}{2}aA \sec(c + dx) - Ab \sec^2(c + dx)}{\sec^{\frac{3}{2}}(c + dx) \sqrt{a + b \sec(c + dx)}} dx}{5a} \\
 &= \frac{2A \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{5ad \sec^{\frac{3}{2}}(c + dx)} - \frac{2(4Ab - 5aB) \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{15a^2 d \sqrt{\sec(c + dx)}} \\
 &= \frac{2A \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{5ad \sec^{\frac{3}{2}}(c + dx)} - \frac{2(4Ab - 5aB) \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{15a^2 d \sqrt{\sec(c + dx)}} \\
 &= \frac{2A \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{5ad \sec^{\frac{3}{2}}(c + dx)} - \frac{2(4Ab - 5aB) \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{15a^2 d \sqrt{\sec(c + dx)}} \\
 &= \frac{2A \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{5ad \sec^{\frac{3}{2}}(c + dx)} - \frac{2(4Ab - 5aB) \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{15a^2 d \sqrt{\sec(c + dx)}} \\
 &= \frac{2(7a^2 Ab + 8Ab^3 - 5a^3 B - 10ab^2 B) \sqrt{\frac{b + a \cos(c + dx)}{a + b}} F\left(\frac{1}{2}(c + dx) \middle| \frac{2a}{a + b}\right) \sqrt{a + b \sec(c + dx)}}{15a^3 d \sqrt{a + b \sec(c + dx)}}
 \end{aligned}$$

Mathematica [A] time = 1.39, size = 198, normalized size = 0.71

$$\frac{2\sqrt{\sec(c+dx)} \left((a+b) (9a^2A - 10abB + 8Ab^2) \sqrt{\frac{a \cos(c+dx)+b}{a+b}} E\left(\frac{1}{2}(c+dx) \middle| \frac{2a}{a+b}\right) + (5a^3B - 7a^2Ab + 10ab^2B - 8a^2b^2) \sqrt{a+b} \right)}{15a^3d\sqrt{a+b}\sec(c+dx)}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(5/2)*Sqrt[a + b*Sec[c + d*x]]), x]

[Out] (2*Sqrt[Sec[c + d*x]]*((a + b)*(9*a^2*A + 8*A*b^2 - 10*a*b*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*a)/(a + b)] + (-7*a^2*A*b - 8*A*b^3 + 5*a^3*B + 10*a*b^2*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)] + a*(b + a*Cos[c + d*x])*(-4*A*b + 5*a*B + 3*a*A*Cos[c + d*x])*Sin[c + d*x])/(15*a^3*d*Sqrt[a + b*Sec[c + d*x]])

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B \sec(dx+c) + A)\sqrt{b \sec(dx+c) + a} \sqrt{\sec(dx+c)}}{b \sec(dx+c)^4 + a \sec(dx+c)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2), x, algorithm="fricas")

[Out] integral((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*sqrt(sec(d*x + c))/(b*sec(d*x + c)^4 + a*sec(d*x + c)^3), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx+c) + A}{\sqrt{b \sec(dx+c) + a} \sec(dx+c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2), x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)/(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(5/2)), x)

maple [B] time = 2.48, size = 2738, normalized size = 9.78

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((A+B*\sec(d*x+c))/\sec(d*x+c)^{(5/2)}/(a+b*\sec(d*x+c))^{(1/2)},x)$

[Out] $2/15/d*(-9*A*\text{EllipticE}((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c),(-a+b)/(a-b))^{(1/2)})*a^3*(1/(1+\cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*\sin(d*x+c)-2*A*\sin(d*x+c)*(1/(1+\cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*\text{EllipticF}((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c),(-a+b)/(a-b))^{(1/2)})*\cos(d*x+c)*a^2*b+8*A*\sin(d*x+c)*(1/(1+\cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*\text{EllipticF}((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c),(-a+b)/(a-b))^{(1/2)})*\cos(d*x+c)*a*b^2-10*A*((a-b)/(a+b))^{(1/2)}*\cos(d*x+c)*a^2*b+A*((a-b)/(a+b))^{(1/2)}*\cos(d*x+c)^3*a^2*b-4*A*((a-b)/(a+b))^{(1/2)}*\cos(d*x+c)^2*a*b^2+5*B*((a-b)/(a+b))^{(1/2)}*\cos(d*x+c)^2*a^2*b+8*A*((a-b)/(a+b))^{(1/2)}*\cos(d*x+c)*a*b^2-10*B*((a-b)/(a+b))^{(1/2)}*\cos(d*x+c)*a^2*b+8*A*\text{EllipticE}((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c),(-a+b)/(a-b))^{(1/2)})*b^3*(1/(1+\cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*\sin(d*x+c)-3*A*((a-b)/(a+b))^{(1/2)}*\cos(d*x+c)^4*a^3-5*B*((a-b)/(a+b))^{(1/2)}*\cos(d*x+c)^3*a^3+5*B*a^3*((a-b)/(a+b))^{(1/2)}*\cos(d*x+c)-6*A*((a-b)/(a+b))^{(1/2)}*\cos(d*x+c)^2*a^3+9*A*((a-b)/(a+b))^{(1/2)}*\cos(d*x+c)*a^3-8*A*((a-b)/(a+b))^{(1/2)}*\cos(d*x+c)*b^3+9*A*\sin(d*x+c)*\text{EllipticE}((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c),(-a+b)/(a-b))^{(1/2)})*(1/(1+\cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*\cos(d*x+c)*a^2*b-8*A*\sin(d*x+c)*\text{EllipticE}((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c),(-a+b)/(a-b))^{(1/2)})*(1/(1+\cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*\cos(d*x+c)*a*b^2-10*B*\sin(d*x+c)*(1/(1+\cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*\text{EllipticF}((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c),(-a+b)/(a-b))^{(1/2)})*\cos(d*x+c)*a^2*b+10*B*\sin(d*x+c)*\text{EllipticE}((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c),(-a+b)/(a-b))^{(1/2)})*(1/(1+\cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*\cos(d*x+c)*a^2*b-10*B*\sin(d*x+c)*\text{EllipticE}((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c),(-a+b)/(a-b))^{(1/2)})*(1/(1+\cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*\cos(d*x+c)*a*b^2-5*B*\text{EllipticF}((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c),(-a+b)/(a-b))^{(1/2)})*a^3*(1/(1+\cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*\sin(d*x+c)+9*A*\text{EllipticF}((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c),(-a+b)/(a-b))^{(1/2)})*a^3*(1/(1+\cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*\sin(d*x+c)+8*A*b^3*((a-b)/(a+b))^{(1/2)}+9*A*a^2*b*((a-b)/(a+b))^{(1/2)}-4*A*a*b^2*((a-b)/(a+b))^{(1/2)}+5*B*a^2*b*((a-b)/(a+b))^{(1/2)}-10*B*a*b^2*((a-b)/(a+b))^{(1/2)}+10*B*((a-b)/(a+b))^{(1/2)})*\cos(d*x+c)*a*b^2+9*A*\sin(d*x+c)*(1/(1+\cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*\text{EllipticF}((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c),(-a+b)/(a-b))^{(1/2)})*\cos(d*x+c)*a^3-9*A*\sin(d*x+c)*\text{EllipticE}((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c),(-a+b)/(a-b))^{(1/2)})*(1/(1+\cos(d*x+c)))^{(1/2)}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*\cos(d*x+c)*a^3+8*A*\sin(d*x+c)*\text{EllipticE}((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c),(-a+b)/(a-b))^{(1/2)}/\sin(d*x+c)$

), $(- (a+b)/(a-b))^{(1/2)} * (1/(1+\cos(dx+c)))^{(1/2)} * ((b+a*\cos(dx+c))/(1+\cos(dx+c)))/(a+b)^{(1/2)} * \cos(dx+c) * b^3 - 5*B*\sin(dx+c) * (1/(1+\cos(dx+c)))^{(1/2)} * ((b+a*\cos(dx+c))/(1+\cos(dx+c)))/(a+b)^{(1/2)} * \text{EllipticF}((-1+\cos(dx+c)) * ((a-b)/(a+b))^{(1/2)}/\sin(dx+c), (- (a+b)/(a-b))^{(1/2)} * \cos(dx+c) * a^3 - 2*A*\text{EllipticF}((-1+\cos(dx+c)) * ((a-b)/(a+b))^{(1/2)}/\sin(dx+c), (- (a+b)/(a-b))^{(1/2)} * a^2 * b * (1/(1+\cos(dx+c)))^{(1/2)} * ((b+a*\cos(dx+c))/(1+\cos(dx+c)))/(a+b)^{(1/2)} * \sin(dx+c) + 8*A*\text{EllipticF}((-1+\cos(dx+c)) * ((a-b)/(a+b))^{(1/2)}/\sin(dx+c), (- (a+b)/(a-b))^{(1/2)} * a * b^2 * (1/(1+\cos(dx+c)))^{(1/2)} * ((b+a*\cos(dx+c))/(1+\cos(dx+c)))/(a+b)^{(1/2)} * \sin(dx+c) + 9*A*\text{EllipticE}((-1+\cos(dx+c)) * ((a-b)/(a+b))^{(1/2)}/\sin(dx+c), (- (a+b)/(a-b))^{(1/2)} * a^2 * b * (1/(1+\cos(dx+c)))^{(1/2)} * ((b+a*\cos(dx+c))/(1+\cos(dx+c)))/(a+b)^{(1/2)} * \sin(dx+c) - 8*A*\text{EllipticE}((-1+\cos(dx+c)) * ((a-b)/(a+b))^{(1/2)}/\sin(dx+c), (- (a+b)/(a-b))^{(1/2)} * a * b^2 * (1/(1+\cos(dx+c)))^{(1/2)} * ((b+a*\cos(dx+c))/(1+\cos(dx+c)))/(a+b)^{(1/2)} * \sin(dx+c) - 10*B*\text{EllipticF}((-1+\cos(dx+c)) * ((a-b)/(a+b))^{(1/2)}/\sin(dx+c), (- (a+b)/(a-b))^{(1/2)} * a^2 * b * (1/(1+\cos(dx+c)))^{(1/2)} * ((b+a*\cos(dx+c))/(1+\cos(dx+c)))/(a+b)^{(1/2)} * \sin(dx+c) + 10*B*\text{EllipticE}((-1+\cos(dx+c)) * ((a-b)/(a+b))^{(1/2)}/\sin(dx+c), (- (a+b)/(a-b))^{(1/2)} * a^2 * b * (1/(1+\cos(dx+c)))^{(1/2)} * ((b+a*\cos(dx+c))/(1+\cos(dx+c)))/(a+b)^{(1/2)} * \sin(dx+c) - 10*B*\text{EllipticE}((-1+\cos(dx+c)) * ((a-b)/(a+b))^{(1/2)}/\sin(dx+c), (- (a+b)/(a-b))^{(1/2)} * a * b^2 * (1/(1+\cos(dx+c)))^{(1/2)} * ((b+a*\cos(dx+c))/(1+\cos(dx+c)))/(a+b)^{(1/2)} * \sin(dx+c)) * ((b+a*\cos(dx+c))/\cos(dx+c))^{(1/2)} * \cos(dx+c)^3 * (1/\cos(dx+c))^{(5/2)}/\sin(dx+c)/(b+a*\cos(dx+c))/a^3/((- (a+b)/(a-b))^{(1/2)}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx+c) + A}{\sqrt{b \sec(dx+c) + a} \sec(dx+c)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(dx+c))/sec(dx+c)^(5/2)/(a+b*sec(dx+c))^(1/2),x, algorithm="maxima")

[Out] integrate((B*sec(dx+c) + A)/(sqrt(b*sec(dx+c) + a)*sec(dx+c)^(5/2)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\sqrt{a + \frac{b}{\cos(c+dx)} \left(\frac{1}{\cos(c+dx)}\right)^{5/2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + dx))/((a + b/cos(c + dx))^(1/2)*(1/cos(c + dx))^(5/2)),x)

```
[Out] int((A + B/cos(c + d*x))/((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(5/2)), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)**(5/2)/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

$$3.462 \quad \int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^{\frac{3}{2}}} dx$$

Optimal. Leaf size=371

$$\frac{2a(Ab - aB) \sin(c + dx) \sec^{\frac{3}{2}}(c + dx)}{bd(a^2 - b^2) \sqrt{a + b \sec(c + dx)}} - \frac{(-3a^2B + 2aAb + b^2B) \sin(c + dx) \sqrt{\sec(c + dx)} \sqrt{a + b \sec(c + dx)}}{b^2d(a^2 - b^2)} + (-$$

[Out] $2*a*(A*b-B*a)*\sec(d*x+c)^{(3/2)}*\sin(d*x+c)/b/(a^2-b^2)/d/(a+b*\sec(d*x+c))^{(1/2)}+B*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)}*(a/(a+b))^{(1/2)})*((b+a*\cos(d*x+c))/(a+b))^{(1/2)}*\sec(d*x+c)^{(1/2)}/b/d/(a+b*\sec(d*x+c))^{(1/2)}+(2*A*b-3*B*a)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticPi}(\sin(1/2*d*x+1/2*c), 2, 2^{(1/2)}*(a/(a+b))^{(1/2)})*((b+a*\cos(d*x+c))/(a+b))^{(1/2)}*\sec(d*x+c)^{(1/2)}/b^2/d/(a+b*\sec(d*x+c))^{(1/2)}+(2*A*a*b-3*B*a^2+B*b^2)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)}*(a/(a+b))^{(1/2)})*(a+b*\sec(d*x+c))^{(1/2)}/b^2/(a^2-b^2)/d/((b+a*\cos(d*x+c))/(a+b))^{(1/2)}/\sec(d*x+c)^{(1/2)}-(2*A*a*b-3*B*a^2+B*b^2)*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}*(a+b*\sec(d*x+c))^{(1/2)}/b^2/(a^2-b^2)/d$

Rubi [A] time = 1.27, antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {4029, 4102, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2a(Ab - aB) \sin(c + dx) \sec^{\frac{3}{2}}(c + dx)}{bd(a^2 - b^2) \sqrt{a + b \sec(c + dx)}} - \frac{(-3a^2B + 2aAb + b^2B) \sin(c + dx) \sqrt{\sec(c + dx)} \sqrt{a + b \sec(c + dx)}}{b^2d(a^2 - b^2)} + (-$$

Antiderivative was successfully verified.

[In] Int[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(3/2), x]

[Out] $(B*\text{Sqrt}[b + a*\text{Cos}[c + d*x]]/(a + b))*\text{EllipticF}[(c + d*x)/2, (2*a)/(a + b)]*\text{Sqrt}[\text{Sec}[c + d*x]]/(b*d*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]) + ((2*A*b - 3*a*B)*\text{Sqrt}[(b + a*\text{Cos}[c + d*x]]/(a + b))*\text{EllipticPi}[2, (c + d*x)/2, (2*a)/(a + b)]*\text{Sqrt}[\text{Sec}[c + d*x]]/(b^2*d*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]) + ((2*a*A*b - 3*a^2*B + b^2*B)*\text{EllipticE}[(c + d*x)/2, (2*a)/(a + b)]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]/(b^2*(a^2 - b^2)*d*\text{Sqrt}[(b + a*\text{Cos}[c + d*x]]/(a + b))*\text{Sqrt}[\text{Sec}[c + d*x]]) + (2*a*(A*b - a*B)*\text{Sec}[c + d*x]^(3/2)*\text{Sin}[c + d*x])/(b*(a^2 - b^2)*d*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]) - ((2*a*A*b - 3*a^2*B + b^2*B)*\text{Sqrt}[\text{Sec}[c + d*x]]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(b^2*(a^2 - b^2)*d)$

Rule 2653

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*Sqrt[a
+ b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
0] && !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a
+ b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_)
+ (f_)*(x_)]]), x_Symbol] := Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_)
+ (f_)*(x_)]]), x_Symbol] := Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
+ f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !GtQ[c + d, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)]/Sqrt[csc[(e_) + (f_)*(x_)]]
```


*(d_.)], x_Symbol] := Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 3858

Int[Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] := Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 3859

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] := Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 4029

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := Simp[(a*d^2*(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[d/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(n - 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2*(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 1]

Rule 4035

Int[(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.))/(Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]), x_Symbol] := Dist[A/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]

Rule 4102

Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_), x_Symbol] := -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b

$(m + n + 1) + bC(m + n) \cdot \text{Csc}[e + f*x] + (bB(m + n + 1) - aCn) \cdot \text{Csc}[e + f*x]^2, x], x] /; \text{FreeQ}\{a, b, d, e, f, A, B, C, m\}, x] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{GtQ}[n, 0]$

Rule 4108

$\text{Int}[(A + \text{csc}[e + f*x]) \cdot (B + \text{csc}[e + f*x])^2 \cdot (C + \sqrt{\text{csc}[e + f*x] \cdot (d + \text{csc}[e + f*x])})] / \text{Sqrt}[a + b \cdot \text{Csc}[e + f*x]], x] \text{Symbol} \text{ :> } \text{Dist}[C/d^2, \text{Int}[(d \cdot \text{Csc}[e + f*x])^{3/2} / \text{Sqrt}[a + b \cdot \text{Csc}[e + f*x]], x], x] + \text{Int}[(A + B \cdot \text{Csc}[e + f*x]) / (\text{Sqrt}[d \cdot \text{Csc}[e + f*x]] \cdot \text{Sqrt}[a + b \cdot \text{Csc}[e + f*x]]), x] /; \text{FreeQ}\{a, b, d, e, f, A, B, C\}, x] \&\& \text{NeQ}[a^2 - b^2, 0]$

Rubi steps

$$\begin{aligned} \int \frac{\sec^{\frac{5}{2}}(c + dx)(A + B \sec(c + dx))}{(a + b \sec(c + dx))^{3/2}} dx &= \frac{2a(Ab - aB) \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{b(a^2 - b^2) d \sqrt{a + b \sec(c + dx)}} + \frac{2 \int \frac{\sqrt{\sec(c + dx)} \left(\frac{1}{2} a(Ab - aB) - \frac{1}{2} b(Ab - aB) \right)}{\sqrt{a + b \sec(c + dx)}} dx}{b} \\ &= \frac{2a(Ab - aB) \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{b(a^2 - b^2) d \sqrt{a + b \sec(c + dx)}} - \frac{(2aAb - 3a^2B + b^2B) \sqrt{\sec(c + dx)}}{b^2 (a^2 - b^2)} \\ &= \frac{2a(Ab - aB) \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{b(a^2 - b^2) d \sqrt{a + b \sec(c + dx)}} - \frac{(2aAb - 3a^2B + b^2B) \sqrt{\sec(c + dx)}}{b^2 (a^2 - b^2)} \\ &= \frac{2a(Ab - aB) \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{b(a^2 - b^2) d \sqrt{a + b \sec(c + dx)}} - \frac{(2aAb - 3a^2B + b^2B) \sqrt{\sec(c + dx)}}{b^2 (a^2 - b^2)} \\ &= \frac{2a(Ab - aB) \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{b(a^2 - b^2) d \sqrt{a + b \sec(c + dx)}} - \frac{(2aAb - 3a^2B + b^2B) \sqrt{\sec(c + dx)}}{b^2 (a^2 - b^2)} \\ &= \frac{(2Ab - 3aB) \sqrt{\frac{b+a \cos(c+dx)}{a+b}} \Pi\left(2; \frac{1}{2}(c + dx) \middle| \frac{2a}{a+b}\right) \sqrt{\sec(c + dx)}}{b^2 d \sqrt{a + b \sec(c + dx)}} + \frac{2a(Ab - aB)}{b(a^2 - b^2)} \\ &= \frac{B \sqrt{\frac{b+a \cos(c+dx)}{a+b}} F\left(\frac{1}{2}(c + dx) \middle| \frac{2a}{a+b}\right) \sqrt{\sec(c + dx)}}{bd \sqrt{a + b \sec(c + dx)}} + \frac{(2Ab - 3aB) \sqrt{\frac{b+a \cos(c+dx)}{a+b}}}{b^2 (a^2 - b^2)} \end{aligned}$$

Mathematica [C] time = 5.97, size = 518, normalized size = 1.40

$$\sec^3(c+dx) \left(\frac{4 \tan(c+dx)(a \cos(c+dx)+b)(a(-3a^2B+2aAb+b^2B) \cos(c+dx)+bB(b^2-a^2))}{b^4-a^2b^2} - \frac{(a \cos(c+dx)+b)^{3/2} \left(\frac{2i(3a^2B-2aAb-b^2B) \csc(c+dx) \sqrt{-a}}{\dots} \right)}{\dots} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(3/2), x]

[Out] (Sec[c + d*x]^(3/2)*(-(((b + a*Cos[c + d*x])^(3/2)*((8*a*b*(-(A*b) + a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/Sqrt[b + a*Cos[c + d*x]] + (2*(-6*a^2*A*b + 4*A*b^3 + 9*a^3*B - 7*a*b^2*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/Sqrt[b + a*Cos[c + d*x]] + ((2*I)*(-2*a*A*b + 3*a^2*B - b^2*B)*Sqrt[-((a*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(a*(1 + Cos[c + d*x]))/(a - b)]*Csc[c + d*x]*(-2*b*(a + b)*EllipticE[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*(2*b*EllipticF[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*EllipticPi[1 - a/b, I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)])))/(a*Sqrt[(a - b)^(-1)]*b)))/((a - b)*b^2*(a + b)) + (4*(b + a*Cos[c + d*x])*(b*(-a^2 + b^2)*B + a*(2*a*A*b - 3*a^2*B + b^2*B)*Cos[c + d*x])*Tan[c + d*x])/(-a^2*b^2 + b^4))/(4*d*(a + b*Sec[c + d*x])^(3/2))

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2), x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx+c) + A) \sec(dx+c)^{\frac{5}{2}}}{(b \sec(dx+c) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x, algorith="giac")

[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(5/2)/(b*sec(d*x + c) + a)^(3/2), x)

maple [C] time = 2.56, size = 2656, normalized size = 7.16

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x)

[Out]
$$\frac{1}{d} \cdot (4A \cos(d*x+c) \sin(d*x+c) \text{EllipticF}((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{1/2}) / \sin(d*x+c), (-a+b)/(a-b)^{1/2}) * ((b+a \cos(d*x+c)) / (1+\cos(d*x+c))) / (a+b)^{1/2} * (1/(1+\cos(d*x+c)))^{1/2} * a * b - 2A \sin(d*x+c) * ((b+a \cos(d*x+c)) / (1+\cos(d*x+c))) / (a+b)^{1/2} * (1/(1+\cos(d*x+c)))^{1/2} * \text{EllipticE}((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{1/2}) / \sin(d*x+c), (-a+b)/(a-b)^{1/2}) * \cos(d*x+c)^2 * a * b - 4A \sin(d*x+c) * ((b+a \cos(d*x+c)) / (1+\cos(d*x+c))) / (a+b)^{1/2} * (1/(1+\cos(d*x+c)))^{1/2} * \text{EllipticPi}((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{1/2}) / \sin(d*x+c), (a+b)/(a-b), I / ((a-b)/(a+b))^{1/2}) * \cos(d*x+c)^2 * a * b - 4B \sin(d*x+c) * ((b+a \cos(d*x+c)) / (1+\cos(d*x+c))) / (a+b)^{1/2} * (1/(1+\cos(d*x+c)))^{1/2} * \text{EllipticF}((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{1/2}) / \sin(d*x+c), (-a+b)/(a-b)^{1/2}) * \cos(d*x+c)^2 * a * b + B * ((a-b)/(a+b))^{1/2} * a * b + 2A \text{EllipticF}((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{1/2}) / \sin(d*x+c), (-a+b)/(a-b)^{1/2}) * ((b+a \cos(d*x+c)) / (1+\cos(d*x+c))) / (a+b)^{1/2} * (1/(1+\cos(d*x+c)))^{1/2} * \sin(d*x+c) * \cos(d*x+c) * b^2 - 2A \cos(d*x+c) * \sin(d*x+c) * ((b+a \cos(d*x+c)) / (1+\cos(d*x+c))) / (a+b)^{1/2} * (1/(1+\cos(d*x+c)))^{1/2} * \text{EllipticE}((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{1/2}) / \sin(d*x+c), (-a+b)/(a-b)^{1/2}) * a * b - 4B \cos(d*x+c) * \sin(d*x+c) * \text{EllipticF}((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{1/2}) / \sin(d*x+c), (-a+b)/(a-b)^{1/2}) * ((b+a \cos(d*x+c)) / (1+\cos(d*x+c))) / (a+b)^{1/2} * (1/(1+\cos(d*x+c)))^{1/2} * \text{EllipticPi}((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{1/2}) / \sin(d*x+c), (a+b)/(a-b), I / ((a-b)/(a+b))^{1/2}) * a * b + 6B \sin(d*x+c) * \cos(d*x+c)^2 * ((b+a \cos(d*x+c)) / (1+\cos(d*x+c))) / (a+b)^{1/2} * (1/(1+\cos(d*x+c)))^{1/2} * \text{EllipticPi}((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{1/2}) / \sin(d*x+c), (a+b)/(a-b), I / ((a-b)/(a+b))^{1/2}) * a * b - 4A \sin(d*x+c) * \cos(d*x+c) * ((b+a \cos(d*x+c)) / (1+\cos(d*x+c))) / (a+b)^{1/2} * (1/(1+\cos(d*x+c)))^{1/2} * \text{EllipticPi}((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{1/2}) / \sin(d*x+c), (a+b)/(a-b), I / ((a-b)/(a+b))^{1/2}) * a * b + 6B \sin(d*x+c) * \cos(d*x+c) * ((b+a \cos(d*x+c)) / (1+\cos(d*x+c))) / (a+b)^{1/2} * (1/(1+\cos(d*x+c)))^{1/2} * \text{EllipticPi}((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{1/2}) / \sin(d*x+c), (a+b)/(a-b), I / ((a-b)/(a+b))^{1/2}) * a * b - 3B \cos(d*x+c)^2 * ((a-b)/(a+b))^{1/2} * a^2 - 6B \cos(d*x+c) * \sin(d*x+c) * \text{EllipticF}((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{1/2}) / \sin(d*x+c), (-a+b)/(a-b)^{1/2}) * ((b+a \cos(d*x+c)) / (1+\cos(d*x+c))) / (a+b)^{1/2} * (1/(1+\cos(d*x+c)))^{1/2} * a^2 + 3B \cos(d*x+c) * \sin(d*x+c) * ((b+a \cos(d*x+c)) / (1+\cos(d*x+c))) / (a+b)^{1/2} * (1/(1+\cos(d*x+c)))^{1/2} * \text{EllipticE}((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{1/2}) / \sin(d*x+c), (-a+b)/(a-b)^{1/2}) * a^2 - B \cos(d*x+c) * ((a-b)/(a+b))^{1/2} * b^2 + B * ((a-b)/(a+b))^{1/2} * b^2 + 2A \cos(d*x+c)^2 * ((a-b)/(a+b))^{1/2} * a * b -$$

$$\begin{aligned}
& B \cos(dx+c)^2 \left(\frac{a-b}{a+b}\right)^{1/2} a^2 b + 3 B \cos(dx+c) \left(\frac{a-b}{a+b}\right)^{1/2} a^2 \\
& + 4 A \sin(dx+c) \cos(dx+c)^2 \left(\frac{b+a \cos(dx+c)}{1+\cos(dx+c)}\right) \left(\frac{a+b}{a+b}\right)^{1/2} \\
& \left(\frac{1}{1+\cos(dx+c)}\right)^{1/2} \text{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}, \left(\frac{a-b}{a+b}\right)^{1/2}\right) \\
& \left(\frac{a-b}{a+b}\right)^{1/2} \sin(dx+c) + 3 B \sin(dx+c) \left(\frac{b+a \cos(dx+c)}{1+\cos(dx+c)}\right) \\
& \left(\frac{a+b}{a+b}\right)^{1/2} \left(\frac{1}{1+\cos(dx+c)}\right)^{1/2} \text{EllipticE}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}, \left(\frac{a-b}{a+b}\right)^{1/2}\right) \\
& \cos(dx+c)^2 a^2 + 6 B \sin(dx+c) \left(\frac{b+a \cos(dx+c)}{1+\cos(dx+c)}\right) \left(\frac{a+b}{a+b}\right)^{1/2} \\
& \left(\frac{1}{1+\cos(dx+c)}\right)^{1/2} \text{EllipticPi}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}, \frac{a+b}{a-b}, I\left(\frac{a-b}{a+b}\right)^{1/2}\right) \\
& \cos(dx+c)^2 a^2 - 6 B \sin(dx+c) \left(\frac{b+a \cos(dx+c)}{1+\cos(dx+c)}\right) \left(\frac{a+b}{a+b}\right)^{1/2} \\
& \left(\frac{1}{1+\cos(dx+c)}\right)^{1/2} \text{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}, \left(\frac{a-b}{a+b}\right)^{1/2}\right) \\
& \cos(dx+c)^2 a^2 + 2 A \sin(dx+c) \cos(dx+c)^2 \text{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}, \left(\frac{a-b}{a+b}\right)^{1/2}\right) \\
& \left(\frac{b+a \cos(dx+c)}{1+\cos(dx+c)}\right) \left(\frac{a+b}{a+b}\right)^{1/2} \left(\frac{1}{1+\cos(dx+c)}\right)^{1/2} b^2 - 4 A \sin(dx+c) \cos(dx+c)^2 \\
& \left(\frac{b+a \cos(dx+c)}{1+\cos(dx+c)}\right) \left(\frac{a+b}{a+b}\right)^{1/2} \left(\frac{1}{1+\cos(dx+c)}\right)^{1/2} \text{EllipticPi}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}, \frac{a+b}{a-b}, I\left(\frac{a-b}{a+b}\right)^{1/2}\right) \\
& b^2 - B \sin(dx+c) \cos(dx+c)^2 \left(\frac{b+a \cos(dx+c)}{1+\cos(dx+c)}\right) \left(\frac{a+b}{a+b}\right)^{1/2} \left(\frac{1}{1+\cos(dx+c)}\right)^{1/2} \\
& \text{EllipticE}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}, \left(\frac{a-b}{a+b}\right)^{1/2}\right) b^2 - 4 A \sin(dx+c) \cos(dx+c) \\
& \left(\frac{b+a \cos(dx+c)}{1+\cos(dx+c)}\right) \left(\frac{a+b}{a+b}\right)^{1/2} \left(\frac{1}{1+\cos(dx+c)}\right)^{1/2} \text{EllipticPi}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}, \frac{a+b}{a-b}, I\left(\frac{a-b}{a+b}\right)^{1/2}\right) \\
& b^2 + 6 B \sin(dx+c) \cos(dx+c) \left(\frac{b+a \cos(dx+c)}{1+\cos(dx+c)}\right) \left(\frac{a+b}{a+b}\right)^{1/2} \left(\frac{1}{1+\cos(dx+c)}\right)^{1/2} \\
& \text{EllipticPi}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}, \frac{a+b}{a-b}, I\left(\frac{a-b}{a+b}\right)^{1/2}\right) a^2 - 2 A \cos(dx+c) \left(\frac{a-b}{a+b}\right)^{1/2} \\
& a^2 b - B \sin(dx+c) \cos(dx+c) \left(\frac{b+a \cos(dx+c)}{1+\cos(dx+c)}\right) \left(\frac{a+b}{a+b}\right)^{1/2} \left(\frac{1}{1+\cos(dx+c)}\right)^{1/2} \\
& \text{EllipticE}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}, \left(\frac{a-b}{a+b}\right)^{1/2}\right) \sin(dx+c), \left(\frac{a-b}{a+b}\right)^{1/2} b^2 * \left(\frac{b+a \cos(dx+c)}{\cos(dx+c)}\right)^{1/2} \cos(dx+c)^2 \\
& \left(\frac{1}{\cos(dx+c)}\right)^{5/2} \left(\frac{b+a \cos(dx+c)}{\sin(dx+c)}\right) \left(\frac{a-b}{a+b}\right)^{1/2} / (a+b) / b^2
\end{aligned}$$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(dx+c)^(5/2)*(A+B*sec(dx+c))/(a+b*sec(dx+c))^(3/2),x, algorith="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(\frac{1}{\cos(c+dx)}\right)^{5/2}}{\left(a + \frac{b}{\cos(c+dx)}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + b/cos(c + d*x))^(3/2), x)
```

```
[Out] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + b/cos(c + d*x))^(3/2), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(3/2), x)
```

```
[Out] Timed out
```

$$3.463 \quad \int \frac{\sec^2(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^{3/2}} dx$$

Optimal. Leaf size=220

$$\frac{2a(Ab - aB) \sin(c + dx) \sqrt{\sec(c + dx)}}{bd(a^2 - b^2) \sqrt{a + b \sec(c + dx)}} - \frac{2(Ab - aB) \sqrt{a + b \sec(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2a}{a+b}\right)}{bd(a^2 - b^2) \sqrt{\sec(c + dx)} \sqrt{\frac{a \cos(c+dx)+b}{a+b}}} + \frac{2B \sqrt{\sec(c + dx)} \sqrt{\frac{a \cos(c+dx)+b}{a+b}}}{bd \sqrt{a + b \sec(c + dx)}}$$

[Out] $2*a*(A*b-B*a)*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/b/(a^2-b^2)/d/(a+b*\sec(d*x+c))^{(1/2)}+2*B*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticPi}(\sin(1/2*d*x+1/2*c), 2, 2^{(1/2)}*(a/(a+b))^{(1/2)})*((b+a*\cos(d*x+c))/(a+b))^{(1/2)}*\sec(d*x+c)^{(1/2)}/b/d/(a+b*\sec(d*x+c))^{(1/2)}-2*(A*b-B*a)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)}*(a/(a+b))^{(1/2)})*(a+b*\sec(d*x+c))^{(1/2)}/b/(a^2-b^2)/d/((b+a*\cos(d*x+c))/(a+b))^{(1/2)}/\sec(d*x+c)^{(1/2)}$

Rubi [A] time = 0.63, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {4029, 4108, 3859, 2807, 2805, 21, 3856, 2655, 2653}

$$\frac{2a(Ab - aB) \sin(c + dx) \sqrt{\sec(c + dx)}}{bd(a^2 - b^2) \sqrt{a + b \sec(c + dx)}} - \frac{2(Ab - aB) \sqrt{a + b \sec(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2a}{a+b}\right)}{bd(a^2 - b^2) \sqrt{\sec(c + dx)} \sqrt{\frac{a \cos(c+dx)+b}{a+b}}} + \frac{2B \sqrt{\sec(c + dx)} \sqrt{\frac{a \cos(c+dx)+b}{a+b}}}{bd \sqrt{a + b \sec(c + dx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(\text{Sec}[c + d*x])^{(3/2)}*(A + B*\text{Sec}[c + d*x])]/(a + b*\text{Sec}[c + d*x])^{(3/2)}, x]$

[Out] $(2*B*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])]/(a + b))*\text{EllipticPi}[2, (c + d*x)/2, (2*a)/(a + b)]*\text{Sqrt}[\text{Sec}[c + d*x]]/(b*d*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]) - (2*(A*b - a*B)*\text{EllipticE}[(c + d*x)/2, (2*a)/(a + b)]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]])/(b*(a^2 - b^2)*d*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])]/(a + b))*\text{Sqrt}[\text{Sec}[c + d*x]]) + (2*a*(A*b - a*B)*\text{Sqrt}[\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(b*(a^2 - b^2)*d*\text{Sqrt}[a + b*\text{Sec}[c + d*x]])$

Rule 21

$\text{Int}[(u_.)*((a_.) + (b_.)*(v_.))^{(m_.)*((c_.) + (d_.)*(v_.))^{(n_.)}, x_Symbol] \rightarrow \text{Dist}[(b/d)^m, \text{Int}[u*(c + d*v)^{(m+n)}, x], x] /; \text{FreeQ}\{a, b, c, d, n\}, x] \&\& \text{EqQ}[b*c - a*d, 0] \&\& \text{IntegerQ}[m] \&\& (!\text{IntegerQ}[n] || \text{SimplerQ}[c + d*x, a + b*x])$

Rule 2653

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*Sqrt[a
+ b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)]/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
0] && !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_)
+ (f_)*(x_)]]), x_Symbol] := Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)]/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_)
+ (f_)*(x_)]]), x_Symbol] := Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
+ f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !GtQ[c + d, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)]/Sqrt[csc[(e_) + (f_)*(x_)]
*(d_)], x_Symbol] := Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3859

```
Int[(csc[(e_) + (f_)*(x_)]*(d_))^(3/2)/Sqrt[csc[(e_) + (f_)*(x_)]*(b_
) + (a_)], x_Symbol] := Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4029

```
Int[(csc[(e_) + (f_)*(x_)]*(d_))^(n_)*(csc[(e_) + (f_)*(x_)]*(b_) + (
a_))^(m_)*(csc[(e_) + (f_)*(x_)]*(B_) + (A_)), x_Symbol] := Simp[(a*d^2*
```


$(A*b - a*B)*\text{Cot}[e + f*x]*(a + b*\text{Csc}[e + f*x])^{(m + 1)}*(d*\text{Csc}[e + f*x])^{(n - 2)}/(b*f*(m + 1)*(a^2 - b^2)), x] - \text{Dist}[d/(b*(m + 1)*(a^2 - b^2)), \text{Int}[(a + b*\text{Csc}[e + f*x])^{(m + 1)}*(d*\text{Csc}[e + f*x])^{(n - 2)}*\text{Simp}[a*d*(A*b - a*B)*(n - 2) + b*d*(A*b - a*B)*(m + 1)*\text{Csc}[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2*(n - 1) + b^2*(m + 1)))*\text{Csc}[e + f*x]^2, x], x], x] /;$ FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 1]

Rule 4108

$\text{Int}[(\text{Csc}[e_.] + \text{csc}[(e_.) + (f_.)*(x_)]*(B_.) + \text{csc}[(e_.) + (f_.)*(x_)]^2*(C_.) + (a_.)]/(\text{Sqrt}[\text{csc}[(e_.) + (f_.)*(x_)]*(d_.)]*\text{Sqrt}[\text{csc}[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)]), x_Symbol] := \text{Dist}[C/d^2, \text{Int}[(d*\text{Csc}[e + f*x])^{(3/2)}/\text{Sqrt}[a + b*\text{Csc}[e + f*x]], x], x] + \text{Int}[(A + B*\text{Csc}[e + f*x])/(\text{Sqrt}[d*\text{Csc}[e + f*x]]*\text{Sqrt}[a + b*\text{Csc}[e + f*x]]), x] /;$ FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned} \int \frac{\sec^{\frac{3}{2}}(c + dx)(A + B \sec(c + dx))}{(a + b \sec(c + dx))^{3/2}} dx &= \frac{2a(Ab - aB)\sqrt{\sec(c + dx)} \sin(c + dx)}{b(a^2 - b^2)d\sqrt{a + b \sec(c + dx)}} + \frac{2 \int \frac{-\frac{1}{2}a(Ab - aB) - \frac{1}{2}b(Ab - aB) \sec(c + dx)}{\sqrt{\sec(c + dx)} \sqrt{a + b \sec(c + dx)}} dx}{b(a^2 - b^2)} \\ &= \frac{2a(Ab - aB)\sqrt{\sec(c + dx)} \sin(c + dx)}{b(a^2 - b^2)d\sqrt{a + b \sec(c + dx)}} + \frac{2 \int \frac{-\frac{1}{2}a(Ab - aB) - \frac{1}{2}b(Ab - aB) \sec(c + dx)}{\sqrt{\sec(c + dx)} \sqrt{a + b \sec(c + dx)}} dx}{b(a^2 - b^2)} \\ &= \frac{2a(Ab - aB)\sqrt{\sec(c + dx)} \sin(c + dx)}{b(a^2 - b^2)d\sqrt{a + b \sec(c + dx)}} - \frac{(Ab - aB) \int \frac{\sqrt{a + b \sec(c + dx)}}{\sqrt{\sec(c + dx)}} dx}{b(a^2 - b^2)} \\ &= \frac{2a(Ab - aB)\sqrt{\sec(c + dx)} \sin(c + dx)}{b(a^2 - b^2)d\sqrt{a + b \sec(c + dx)}} + \frac{\left(B \sqrt{\frac{b + a \cos(c + dx)}{a + b}} \sqrt{\sec(c + dx)}\right)}{b\sqrt{a + b \sec(c + dx)}} \\ &= \frac{2B \sqrt{\frac{b + a \cos(c + dx)}{a + b}} \Pi\left(2; \frac{1}{2}(c + dx) \middle| \frac{2a}{a + b}\right) \sqrt{\sec(c + dx)}}{bd\sqrt{a + b \sec(c + dx)}} + \frac{2a(Ab - aB)\sqrt{\sec(c + dx)} \sin(c + dx)}{b(a^2 - b^2)d} \\ &= \frac{2B \sqrt{\frac{b + a \cos(c + dx)}{a + b}} \Pi\left(2; \frac{1}{2}(c + dx) \middle| \frac{2a}{a + b}\right) \sqrt{\sec(c + dx)}}{bd\sqrt{a + b \sec(c + dx)}} - \frac{2(Ab - aB)E\left(\frac{1}{2}(c + dx) \middle| \frac{2a}{a + b}\right)}{b(a^2 - b^2)} \end{aligned}$$

Mathematica [C] time = 4.89, size = 464, normalized size = 2.11

$$\sec^{\frac{3}{2}}(c + dx) \left(\frac{4a(Ab - aB) \sin(c + dx)(a \cos(c + dx) + b)}{a^2 - b^2} + \frac{(a \cos(c + dx) + b)^{3/2} \left(\frac{2(-3a^2B + aAb + 2b^2B) \sqrt{\frac{a \cos(c + dx) + b}{a + b}} \Pi\left(2; \frac{1}{2}(c + dx) \mid \frac{2a}{a + b}\right)}{\sqrt{a \cos(c + dx) + b}} + \frac{4b(Ab - aB) \sqrt{\frac{a \cos(c + dx) + b}{a + b}}}{\sqrt{a \cos(c + dx) + b}} \right)}{a^2 - b^2} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(3/2), x]

[Out] (Sec[c + d*x]^(3/2)*(((b + a*Cos[c + d*x])^(3/2)*((4*b*(A*b - a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]/Sqrt[b + a*Cos[c + d*x]] + (2*(a*A*b - 3*a^2*B + 2*b^2*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]/Sqrt[b + a*Cos[c + d*x]] - ((2*I)*(-A*b) + a*B)*Sqrt[-((a*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(a*(1 + Cos[c + d*x]))/(a - b)]*Csc[c + d*x]*(-2*b*(a + b)*EllipticE[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*(2*b*EllipticF[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*EllipticPi[1 - a/b, I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)])))/(a*Sqrt[(a - b)^(-1)]*b)))/((-a + b)*(a + b)) + (4*a*(A*b - a*B)*(b + a*Cos[c + d*x])*Sin[c + d*x]/(a^2 - b^2)))/(2*b*d*(a + b*Sec[c + d*x])^(3/2))

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2), x, algorith="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sec(dx + c)^{\frac{3}{2}}}{(b \sec(dx + c) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^(3/2), x)

maple [C] time = 3.12, size = 1585, normalized size = 7.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x)

[Out]
$$\begin{aligned} & 2/d*(A*\cos(d*x+c)*\text{EllipticE}((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), \\ & (-a+b)/(a-b)^{1/2})*\sin(d*x+c)*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2} \\ & *(1/(1+\cos(d*x+c)))^{1/2}*b-A*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2} \\ & *(1/(1+\cos(d*x+c)))^{1/2}*\text{EllipticF}((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/ \\ & \sin(d*x+c), (-a+b)/(a-b)^{1/2})*\sin(d*x+c)*\cos(d*x+c)*b-2*B*((b+a*\cos(d*x+c))/ \\ & (1+\cos(d*x+c)))/(a+b)^{1/2}*(1/(1+\cos(d*x+c)))^{1/2}*\text{EllipticPi}((-1+\cos \\ & (d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (a+b)/(a-b), I/((a-b)/(a+b))^{1/2})* \\ & \sin(d*x+c)*\cos(d*x+c)*a-2*B*\cos(d*x+c)*\text{EllipticPi}((-1+\cos(d*x+c))*((a-b)/(a \\ & +b))^{1/2}/\sin(d*x+c), (a+b)/(a-b), I/((a-b)/(a+b))^{1/2})*\sin(d*x+c)*((b+a*c \\ & \cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*(1/(1+\cos(d*x+c)))^{1/2}*b-B*((b+a*c \\ & \cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*(1/(1+\cos(d*x+c)))^{1/2}*\text{EllipticE}((- \\ & -1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (-a+b)/(a-b)^{1/2})*\sin(d*x \\ & +c)*\cos(d*x+c)*a+2*B*\cos(d*x+c)*\text{EllipticF}((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2} \\ & / \sin(d*x+c), (-a+b)/(a-b)^{1/2})*\sin(d*x+c)*((b+a*\cos(d*x+c))/(1+\cos(d*x \\ & +c)))/(a+b)^{1/2}*(1/(1+\cos(d*x+c)))^{1/2}*a+B*\cos(d*x+c)*\text{EllipticF}((-1+\cos \\ & (d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (-a+b)/(a-b)^{1/2})*\sin(d*x+c)*((\\ & b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*(1/(1+\cos(d*x+c)))^{1/2}*b+A*\text{El \\ & lipticE}((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (-a+b)/(a-b)^{1/2} \\ &)*b*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*(1/(1+\cos(d*x+c)))^{1/2}* \\ & \sin(d*x+c)-A*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*(1/(1+\cos(d*x+c) \\ &))^{1/2}*\text{EllipticF}((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (-a+b)/(\\ & a-b)^{1/2})*b*\sin(d*x+c)-2*B*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2} \\ & *(1/(1+\cos(d*x+c)))^{1/2}*\text{EllipticPi}((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin \\ & (d*x+c), (a+b)/(a-b), I/((a-b)/(a+b))^{1/2})*a*\sin(d*x+c)-2*B*\text{EllipticPi}((-1 \\ & +\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (a+b)/(a-b), I/((a-b)/(a+b))^{1/2} \\ &)*b*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*(1/(1+\cos(d*x+c)))^{1/2} \\ &)*\sin(d*x+c)-B*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*(1/(1+\cos(d*x+c) \\ &))^{1/2}*\text{EllipticE}((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (-a+b) \\ & / (a-b))^{1/2})*a*\sin(d*x+c)+2*B*\text{EllipticF}((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2} \\ & / \sin(d*x+c), (-a+b)/(a-b)^{1/2})*a*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b \\ &))^{1/2}*(1/(1+\cos(d*x+c)))^{1/2}*\sin(d*x+c)+B*\text{EllipticF}((-1+\cos(d*x+c))*((\\ & a-b)/(a+b))^{1/2}/\sin(d*x+c), (-a+b)/(a-b)^{1/2})*b*((b+a*\cos(d*x+c))/(1+c \end{aligned}$$

$\cos(dx+c)/(a+b)^{1/2} * (1/(1+\cos(dx+c)))^{1/2} * \sin(dx+c) - A * \cos(dx+c) * ((a-b)/(a+b))^{1/2} * b + B * \cos(dx+c) * ((a-b)/(a+b))^{1/2} * a + A * b * ((a-b)/(a+b))^{1/2} - B * a * ((a-b)/(a+b))^{1/2} * ((b+a * \cos(dx+c))/\cos(dx+c))^{1/2} * \cos(dx+c)^2 * (1/\cos(dx+c))^{3/2} / (b+a * \cos(dx+c)) / \sin(dx+c) / b / (a+b) / ((a-b)/(a+b))^{1/2}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx+c) + A) \sec(dx+c)^{\frac{3}{2}}}{(b \sec(dx+c) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(dx+c)^(3/2)*(A+B*sec(dx+c))/(a+b*sec(dx+c))^(3/2),x, algorithm="maxima")

[Out] integrate((B*sec(dx+c) + A)*sec(dx+c)^(3/2)/(b*sec(dx+c) + a)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(\frac{1}{\cos(c+dx)}\right)^{3/2}}{\left(a + \frac{b}{\cos(c+dx)}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + dx))*(1/cos(c + dx))^(3/2))/(a + b/cos(c + dx))^(3/2),x)

[Out] int(((A + B/cos(c + dx))*(1/cos(c + dx))^(3/2))/(a + b/cos(c + dx))^(3/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(dx+c)**(3/2)*(A+B*sec(dx+c))/(a+b*sec(dx+c))**(3/2),x)

[Out] Timed out

$$3.464 \quad \int \frac{\sqrt{\sec(c+dx)} (A+B \sec(c+dx))}{(a+b \sec(c+dx))^{3/2}} dx$$

Optimal. Leaf size=215

$$\frac{2(Ab - aB) \sin(c + dx) \sqrt{\sec(c + dx)}}{d(a^2 - b^2) \sqrt{a + b \sec(c + dx)}} + \frac{2(Ab - aB) \sqrt{a + b \sec(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2a}{a+b}\right)}{ad(a^2 - b^2) \sqrt{\sec(c + dx)} \sqrt{\frac{a \cos(c+dx)+b}{a+b}}} + \frac{2A \sqrt{\sec(c + dx)} \sqrt{\frac{a \cos(c+dx)+b}{a+b}}}{ad \sqrt{a + b \sec(c + dx)}}$$

[Out] $-2*(A*b-B*a)*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/(a^2-b^2)/d/(a+b*\sec(d*x+c))^{(1/2)}$
 $+2*A*(\cos(1/2*d*x+1/2*c))^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)}*(a/(a+b))^{(1/2)})*((b+a*\cos(d*x+c))/(a+b))^{(1/2)}*\sec(d*x+c)^{(1/2)}/a/d/(a+b*\sec(d*x+c))^{(1/2)}$
 $+2*(A*b-B*a)*(\cos(1/2*d*x+1/2*c))^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)}*(a/(a+b))^{(1/2)})*(a+b*\sec(d*x+c))^{(1/2)}/a/(a^2-b^2)/d/((b+a*\cos(d*x+c))/(a+b))^{(1/2)}/\sec(d*x+c)^{(1/2)}$

Rubi [A] time = 0.57, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {4027, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2(Ab - aB) \sin(c + dx) \sqrt{\sec(c + dx)}}{d(a^2 - b^2) \sqrt{a + b \sec(c + dx)}} + \frac{2(Ab - aB) \sqrt{a + b \sec(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2a}{a+b}\right)}{ad(a^2 - b^2) \sqrt{\sec(c + dx)} \sqrt{\frac{a \cos(c+dx)+b}{a+b}}} + \frac{2A \sqrt{\sec(c + dx)} \sqrt{\frac{a \cos(c+dx)+b}{a+b}}}{ad \sqrt{a + b \sec(c + dx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(\text{Sqrt}[\text{Sec}[c + d*x]]*(A + B*\text{Sec}[c + d*x]))/(a + b*\text{Sec}[c + d*x])^{(3/2)}, x]$

[Out] $(2*A*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/(a + b)]*\text{EllipticF}[(c + d*x)/2, (2*a)/(a + b)])*\text{Sqrt}[\text{Sec}[c + d*x]]/(a*d*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]) + (2*(A*b - a*B)*\text{EllipticE}[(c + d*x)/2, (2*a)/(a + b)]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]])/(a*(a^2 - b^2)*d*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/(a + b)]*\text{Sqrt}[\text{Sec}[c + d*x]]) - (2*(A*b - a*B)*\text{Sqrt}[\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/((a^2 - b^2)*d*\text{Sqrt}[a + b*\text{Sec}[c + d*x]])$

Rule 2653

$\text{Int}[\text{Sqrt}[(a_) + (b_)*\sin[(c_) + (d_)*(x_)]]], x_Symbol] \rightarrow \text{Simp}[(2*\text{Sqrt}[a + b]*\text{EllipticE}[(1*(c - \text{Pi}/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; \text{FreeQ}\{a, b, c, d\}, x] \ \&\& \ \text{NeQ}[a^2 - b^2, 0] \ \&\& \ \text{GtQ}[a + b, 0]$

Rule 2655

$\text{Int}[\text{Sqrt}[(a_) + (b_)*\sin[(c_) + (d_)*(x_)]]], x_Symbol] \rightarrow \text{Dist}[\text{Sqrt}[a + b*\text{Sin}[c + d*x]]/\text{Sqrt}[(a + b*\text{Sin}[c + d*x])/(a + b)], \text{Int}[\text{Sqrt}[a/(a + b) + (b$

```
*Sin[c + d*x]]/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)]/(d*Sqrt[a + b]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a + b*SIN[c + d*x])/(a + b)]/Sqrt[a + b*SIN[c + d*x]], Int[1/Sqrt[a/(a + b) + (b*SIN[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)]/Sqrt[csc[(e_) + (f_)*(x_)]*(d_)], x_Symbol] := Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*SIN[e + f*x]]), Int[Sqrt[b + a*SIN[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_) + (f_)*(x_)]*(d_)]/Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)], x_Symbol] := Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*SIN[e + f*x]])/Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*SIN[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4027

```
Int[(csc[(e_) + (f_)*(x_)]*(d_))^(n_)*(csc[(e_) + (f_)*(x_)]*(b_) + (a_))^(m_)*(csc[(e_) + (f_)*(x_)]*(B_) + (A_)), x_Symbol] := -Simp[(d*(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*Simp[d*(n - 1)*(A*b - a*B) + d*(a*A - b*B)*(m + 1)*Csc[e + f*x] - d*(A*b - a*B)*(m + n + 1)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && LtQ[0, n, 1]
```

Rule 4035

```
Int[(csc[(e_) + (f_)*(x_)]*(B_) + (A_))/(Sqrt[csc[(e_) + (f_)*(x_)]*(d_)]*Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)]), x_Symbol] := Dist[A/a, In
```

$t[\text{Sqrt}[a + b*\text{Csc}[e + f*x]]/\text{Sqrt}[d*\text{Csc}[e + f*x]], x], x] - \text{Dist}[(A*b - a*B)/(a*d), \text{Int}[\text{Sqrt}[d*\text{Csc}[e + f*x]]/\text{Sqrt}[a + b*\text{Csc}[e + f*x]], x], x] /;$ FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{\sec(c+dx)}(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{3/2}} dx &= -\frac{2(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{(a^2-b^2)d\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{\frac{1}{2}(-Ab+aB)-\frac{1}{2}(aA-bB)\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}} dx}{a^2-b^2} \\ &= -\frac{2(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{(a^2-b^2)d\sqrt{a+b\sec(c+dx)}} + \frac{A\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}} dx}{a} + \frac{(Ab-aB)\sin(c+dx)}{a^2-b^2} \\ &= -\frac{2(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{(a^2-b^2)d\sqrt{a+b\sec(c+dx)}} + \frac{(A\sqrt{b+a\cos(c+dx)}\sqrt{\sec(c+dx)})}{a\sqrt{a+b\sec(c+dx)}} \\ &= -\frac{2(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{(a^2-b^2)d\sqrt{a+b\sec(c+dx)}} + \frac{(A\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)})}{a\sqrt{a+b\sec(c+dx)}} \\ &= \frac{2A\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{ad\sqrt{a+b\sec(c+dx)}} + \frac{2(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{a(a^2-b^2)d} \end{aligned}$$

Mathematica [A] time = 0.82, size = 161, normalized size = 0.75

$$\frac{2\sqrt{\sec(c+dx)}\left(A(a^2-b^2)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)+a(aB-Ab)\sin(c+dx)-((a+b)(aB-Ab))\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\right)}{ad(a-b)(a+b)\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(3/2), x]

[Out] (2*Sqrt[Sec[c + d*x]]*(-((a + b)*(-(A*b) + a*B)*Sqrt[(b + a*Cos[c + d*x])]/(a + b)]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]) + A*(a^2 - b^2)*Sqrt[(b + a*Cos[c + d*x])]/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)] + a*(-(A*b) + a*B)*Sin[c + d*x]))/(a*(a - b)*(a + b)*d*Sqrt[a + b*Sec[c + d*x]])

fricas [F] time = 1.72, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B \sec(dx+c) + A)\sqrt{b \sec(dx+c) + a} \sqrt{\sec(dx+c)}}{b^2 \sec(dx+c)^2 + 2ab \sec(dx+c) + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")

[Out] integral((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*sqrt(sec(d*x + c))/(b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx+c) + A)\sqrt{\sec(dx+c)}}{(b \sec(dx+c) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(b*sec(d*x + c) + a)^(3/2), x)

maple [B] time = 2.45, size = 941, normalized size = 4.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x)

[Out] -2/d*(A*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a+A*cos(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b+B*cos(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a-B*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a+A*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*sin(d*x+c)+A*EllipticE((-1+cos(d*x+c)

$$\left. \right) \cdot \left(\frac{a-b}{a+b} \right)^{1/2} / \sin(dx+c), \left(-\frac{a+b}{a-b} \right)^{1/2} \cdot b \cdot \left(\frac{b+a \cos(dx+c)}{1+\cos(dx+c)} \right) / \left(\frac{a+b}{a+b} \right)^{1/2} \cdot \left(\frac{1}{1+\cos(dx+c)} \right)^{1/2} \cdot \sin(dx+c) + B \cdot \text{EllipticF} \left(\frac{-1+\cos(dx+c)}{2}, \left(\frac{a-b}{a+b} \right)^{1/2} / \sin(dx+c), \left(-\frac{a+b}{a-b} \right)^{1/2} \right) \cdot a \cdot \left(\frac{b+a \cos(dx+c)}{1+\cos(dx+c)} \right) / \left(\frac{a+b}{a+b} \right)^{1/2} \cdot \left(\frac{1}{1+\cos(dx+c)} \right)^{1/2} \cdot \sin(dx+c) - B \cdot \left(\frac{b+a \cos(dx+c)}{1+\cos(dx+c)} \right) / \left(\frac{a+b}{a+b} \right)^{1/2} \cdot \left(\frac{1}{1+\cos(dx+c)} \right)^{1/2} \cdot \text{EllipticE} \left(\frac{-1+\cos(dx+c)}{2}, \left(\frac{a-b}{a+b} \right)^{1/2} / \sin(dx+c), \left(-\frac{a+b}{a-b} \right)^{1/2} \right) \cdot a \cdot \sin(dx+c) - A \cdot \cos(dx+c) \cdot \left(\frac{a-b}{a+b} \right)^{1/2} \cdot b + B \cdot \cos(dx+c) \cdot \left(\frac{a-b}{a+b} \right)^{1/2} \cdot a + A \cdot b \cdot \left(\frac{a-b}{a+b} \right)^{1/2} - B \cdot a \cdot \left(\frac{a-b}{a+b} \right)^{1/2} \cdot \cos(dx+c) \cdot \left(\frac{1}{\cos(dx+c)} \right)^{1/2} \cdot \left(\frac{b+a \cos(dx+c)}{\cos(dx+c)} \right)^{1/2} / \left(\frac{b+a \cos(dx+c)}{\sin(dx+c)} \right) / a / \left(\frac{a+b}{a+b} \right)^{1/2}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx+c) + A) \sqrt{\sec(dx+c)}}{(b \sec(dx+c) + a)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(b*sec(d*x + c) + a)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)} \right) \sqrt{\frac{1}{\cos(c+dx)}}}{\left(a + \frac{b}{\cos(c+dx)} \right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + b/cos(c + d*x))^(3/2),x)

[Out] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + b/cos(c + d*x))^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \sqrt{\sec(c + dx)}}{(a + b \sec(c + dx))^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)**(1/2)/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sqrt(sec(c + d*x))/(a + b*sec(c + d*x))**(3/2), x)
```

$$3.465 \quad \int \frac{A+B \sec(c+dx)}{\sqrt{\sec(c+dx)} (a+b \sec(c+dx))^{3/2}} dx$$

Optimal. Leaf size=235

$$\frac{2b(Ab - aB) \sin(c + dx) \sqrt{\sec(c + dx)}}{ad(a^2 - b^2) \sqrt{a + b \sec(c + dx)}} + \frac{2(a^2A + abB - 2Ab^2) \sqrt{a + b \sec(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2a}{a+b}\right)}{a^2d(a^2 - b^2) \sqrt{\sec(c + dx)} \sqrt{\frac{a \cos(c+dx)+b}{a+b}}} - \frac{2(2Ab - a^2)}{ad(a^2 - b^2)}$$

[Out] $2*b*(A*b-B*a)*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/a/(a^2-b^2)/d/(a+b*\sec(d*x+c))^{(1/2)}-2*(2*A*b-B*a)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c),2^{(1/2)}*(a/(a+b))^{(1/2)})*((b+a*\cos(d*x+c))/(a+b))^{(1/2)}* \sec(d*x+c)^{(1/2)}/a^2/d/(a+b*\sec(d*x+c))^{(1/2)}+2*(A*a^2-2*A*b^2+B*a*b)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c),2^{(1/2)}*(a/(a+b))^{(1/2)})*(a+b*\sec(d*x+c))^{(1/2)}/a^2/(a^2-b^2)/d/((b+a*\cos(d*x+c))/(a+b))^{(1/2)}/\sec(d*x+c)^{(1/2)}$

Rubi [A] time = 0.58, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {4030, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2b(Ab - aB) \sin(c + dx) \sqrt{\sec(c + dx)}}{ad(a^2 - b^2) \sqrt{a + b \sec(c + dx)}} + \frac{2(a^2A + abB - 2Ab^2) \sqrt{a + b \sec(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2a}{a+b}\right)}{a^2d(a^2 - b^2) \sqrt{\sec(c + dx)} \sqrt{\frac{a \cos(c+dx)+b}{a+b}}} - \frac{2(2Ab - a^2)}{ad(a^2 - b^2)}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sec[c + d*x])/(Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)),x]

[Out] $(-2*(2*A*b - a*B)*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/(a + b)]*\text{EllipticF}[(c + d*x)/2, (2*a)/(a + b)]*\text{Sqrt}[\text{Sec}[c + d*x]])/(a^2*d*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]) + (2*(a^2*A - 2*A*b^2 + a*b*B)*\text{EllipticE}[(c + d*x)/2, (2*a)/(a + b)]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]])/(a^2*(a^2 - b^2)*d*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/(a + b)]*\text{Sqrt}[\text{Sec}[c + d*x]]) + (2*b*(A*b - a*B)*\text{Sqrt}[\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(a*(a^2 - b^2)*d*\text{Sqrt}[a + b*\text{Sec}[c + d*x]])$

Rule 2653

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2655

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] :> Dist[Sqrt[a + b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b

```
*Sin[c + d*x]]/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)]/(d*Sqrt[a + b]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a + b*SIN[c + d*x])/(a + b)]/Sqrt[a + b*SIN[c + d*x]], Int[1/Sqrt[a/(a + b) + (b*SIN[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)]/Sqrt[csc[(e_) + (f_)*(x_)]*(d_)], x_Symbol] := Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*SIN[e + f*x]]), Int[Sqrt[b + a*SIN[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_) + (f_)*(x_)]*(d_)]/Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)], x_Symbol] := Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*SIN[e + f*x]])/Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*SIN[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4030

```
Int[(csc[(e_) + (f_)*(x_)]*(d_))^(n_)*(csc[(e_) + (f_)*(x_)]*(b_) + (a_))^(m_)*(csc[(e_) + (f_)*(x_)]*(B_) + (A_)), x_Symbol] := Simp[(b*(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[A*(a^2*(m + 1) - b^2*(m + n + 1)) + a*b*B*n - a*(A*b - a*B)*(m + 1)*Csc[e + f*x] + b*(A*b - a*B)*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.))/(Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]), x_Symbol] := Dist[A/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{A + B \sec(c + dx)}{\sqrt{\sec(c + dx)} (a + b \sec(c + dx))^{3/2}} dx &= \frac{2b(Ab - aB)\sqrt{\sec(c + dx)} \sin(c + dx)}{a(a^2 - b^2)d\sqrt{a + b \sec(c + dx)}} - \frac{2 \int \frac{\frac{1}{2}(-a^2A + 2Ab^2 - abB) + \frac{1}{2}a(AB)}{\sqrt{\sec(c + dx)} \sqrt{a + b \sec(c + dx)}} dx}{a(a^2 - b^2)} \\ &= \frac{2b(Ab - aB)\sqrt{\sec(c + dx)} \sin(c + dx)}{a(a^2 - b^2)d\sqrt{a + b \sec(c + dx)}} - \frac{(2Ab - aB) \int \frac{\sqrt{\sec(c + dx)}}{\sqrt{a + b \sec(c + dx)}} dx}{a^2} \\ &= \frac{2b(Ab - aB)\sqrt{\sec(c + dx)} \sin(c + dx)}{a(a^2 - b^2)d\sqrt{a + b \sec(c + dx)}} - \frac{((2Ab - aB)\sqrt{b + a \cos(c + dx)})}{a^2\sqrt{a + b \sec(c + dx)}} \\ &= \frac{2b(Ab - aB)\sqrt{\sec(c + dx)} \sin(c + dx)}{a(a^2 - b^2)d\sqrt{a + b \sec(c + dx)}} - \frac{((2Ab - aB)\sqrt{\frac{b + a \cos(c + dx)}{a + b}})}{a^2\sqrt{a + b \sec(c + dx)}} \\ &= -\frac{2(2Ab - aB)\sqrt{\frac{b + a \cos(c + dx)}{a + b}} F\left(\frac{1}{2}(c + dx) \middle| \frac{2a}{a + b}\right) \sqrt{\sec(c + dx)}}{a^2d\sqrt{a + b \sec(c + dx)}} + \frac{2(a^2 - b^2)\sqrt{\sec(c + dx)} \sin(c + dx)}{a(a^2 - b^2)d\sqrt{a + b \sec(c + dx)}} \end{aligned}$$

Mathematica [A] time = 1.07, size = 178, normalized size = 0.76

$$\frac{2\sqrt{\sec(c + dx)} \left(-(a^2 - b^2)(aB - 2Ab)\sqrt{\frac{a \cos(c + dx) + b}{a + b}} F\left(\frac{1}{2}(c + dx) \middle| \frac{2a}{a + b}\right) - ((a + b)(a^2A + abB - 2Ab^2))\sqrt{\frac{a \cos(c + dx) + b}{a + b}} \right)}{a^2d(a - b)(a + b)\sqrt{a + b \sec(c + dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)), x]
```

```
[Out] (-2*Sqrt[Sec[c + d*x]]*((a + b)*(a^2*A - 2*A*b^2 + a*b*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]) - (a^2 - b^2)*(-2*A*b + a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a
```

)/(a + b)] + a*b*(-(A*b) + a*B)*Sin[c + d*x]))/(a^2*(a - b)*(a + b)*d*sqrt[a + b*Sec[c + d*x]])

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B \sec(dx + c) + A)\sqrt{b \sec(dx + c) + a} \sqrt{\sec(dx + c)}}{b^2 \sec(dx + c)^3 + 2ab \sec(dx + c)^2 + a^2 \sec(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")

[Out] integral((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*sqrt(sec(d*x + c))/(b^2*sec(d*x + c)^3 + 2*a*b*sec(d*x + c)^2 + a^2*sec(d*x + c)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(b \sec(dx + c) + a)^{\frac{3}{2}} \sqrt{\sec(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^(3/2)*sqrt(sec(d*x + c))), x)

maple [B] time = 2.36, size = 1448, normalized size = 6.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x)

[Out] $-2/d*(A*\sin(d*x+c)*\cos(d*x+c)*\text{EllipticE}((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2})/\sin(d*x+c), (-a+b)/(a-b)^{1/2})*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*(1/(1+\cos(d*x+c)))^{1/2}*a^2-2*A*\cos(d*x+c)*\sin(d*x+c)*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*(1/(1+\cos(d*x+c)))^{1/2}*\text{EllipticE}((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2})/\sin(d*x+c), (-a+b)/(a-b)^{1/2})*b^2-A*\cos(d*x+c)*\sin(d*x+c)*\text{EllipticF}((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2})/\sin(d*x+c), (-a+b)/(a-b)^{1/2})*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*(1/(1+\cos(d*x+c)))^{1/2}*a^2-2*A*\cos(d*x+c)*\sin(d*x+c)*\text{EllipticF}((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2})/\sin(d*x+c), (-a+b)/(a-b)^{1/2})*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*(1/(1+\cos(d*x+c)))^{1/2}$

$$\begin{aligned}
& +c)) / (a+b)^{(1/2)} * (1 / (1 + \cos(dx+c)))^{(1/2)} * a*b + B*\cos(dx+c)*\sin(dx+c) * ((b+ \\
& a*\cos(dx+c)) / (1 + \cos(dx+c)) / (a+b)^{(1/2)} * (1 / (1 + \cos(dx+c)))^{(1/2)} * \text{Elliptic} \\
& \text{E}((-1 + \cos(dx+c)) * ((a-b) / (a+b))^{(1/2)} / \sin(dx+c), (- (a+b) / (a-b))^{(1/2)}) * a*b + \\
& B*\cos(dx+c)*\sin(dx+c) * \text{EllipticF}((-1 + \cos(dx+c)) * ((a-b) / (a+b))^{(1/2)} / \sin(dx+c), \\
& (- (a+b) / (a-b))^{(1/2)}) * ((b+a*\cos(dx+c)) / (1 + \cos(dx+c)) / (a+b)^{(1/2)} * (\\
& 1 / (1 + \cos(dx+c)))^{(1/2)} * a^2 + A * \text{EllipticE}((-1 + \cos(dx+c)) * ((a-b) / (a+b))^{(1/2)} \\
& / \sin(dx+c), (- (a+b) / (a-b))^{(1/2)}) * a^2 * ((b+a*\cos(dx+c)) / (1 + \cos(dx+c)) / (a+b) \\
&)^{(1/2)} * (1 / (1 + \cos(dx+c)))^{(1/2)} * \sin(dx+c) - 2 * A * ((b+a*\cos(dx+c)) / (1 + \cos(dx+c)) / (a+b) \\
&)^{(1/2)} * (1 / (1 + \cos(dx+c)))^{(1/2)} * \text{EllipticE}((-1 + \cos(dx+c)) * ((a-b) / (a+b))^{(1/2)} / \sin(dx+c), \\
& (- (a+b) / (a-b))^{(1/2)}) * b^2 * \sin(dx+c) - A * \text{EllipticF} \\
& ((-1 + \cos(dx+c)) * ((a-b) / (a+b))^{(1/2)} / \sin(dx+c), (- (a+b) / (a-b))^{(1/2)}) * a^2 * (\\
& (b+a*\cos(dx+c)) / (1 + \cos(dx+c)) / (a+b)^{(1/2)} * (1 / (1 + \cos(dx+c)))^{(1/2)} * \sin(dx+c) \\
& - 2 * A * \text{EllipticF}((-1 + \cos(dx+c)) * ((a-b) / (a+b))^{(1/2)} / \sin(dx+c), (- (a+b) / (a-b) \\
&)^{(1/2)}) * a*b * ((b+a*\cos(dx+c)) / (1 + \cos(dx+c)) / (a+b)^{(1/2)} * (1 / (1 + \cos(dx+c)))^{(1/2)} * \sin(dx+c) \\
& + B * ((b+a*\cos(dx+c)) / (1 + \cos(dx+c)) / (a+b)^{(1/2)} * (1 / (1 + \cos(dx+c)))^{(1/2)} * \text{EllipticE}((-1 + \cos(dx+c)) * ((a-b) / (a+b))^{(1/2)} / \sin(dx+c), \\
& (- (a+b) / (a-b))^{(1/2)}) * a*b * \sin(dx+c) + B * ((b+a*\cos(dx+c)) / (1 + \cos(dx+c)) / (a+b) \\
&)^{(1/2)} * (1 / (1 + \cos(dx+c)))^{(1/2)} * \text{EllipticF}((-1 + \cos(dx+c)) * ((a-b) / (a+b))^{(1/2)} / \sin(dx+c), \\
& (- (a+b) / (a-b))^{(1/2)}) * a^2 * \sin(dx+c) + A * \cos(dx+c)^2 * (\\
& (a-b) / (a+b))^{(1/2)} * a^2 + A * \cos(dx+c)^2 * ((a-b) / (a+b))^{(1/2)} * a*b - A * a^2 * ((a-b) / (a+b) \\
&)^{(1/2)} * \cos(dx+c) + 2 * A * \cos(dx+c) * ((a-b) / (a+b))^{(1/2)} * b^2 - B * \cos(dx+c) \\
& * ((a-b) / (a+b))^{(1/2)} * a*b - A * ((a-b) / (a+b))^{(1/2)} * a*b - 2 * A * ((a-b) / (a+b))^{(1/2)} * \\
& b^2 + B * ((a-b) / (a+b))^{(1/2)} * a*b * ((b+a*\cos(dx+c)) / \cos(dx+c))^{(1/2)} / (1 / \cos(dx+c))^{(1/2)} / (b+a*\cos(dx+c)) / \sin(dx+c) / ((a-b) / (a+b))^{(1/2)} / (a+b) / a^2
\end{aligned}$$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(dx+c))/(a+b*sec(dx+c))^(3/2)/sec(dx+c)^(1/2),x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\left(a + \frac{b}{\cos(c+dx)}\right)^{3/2} \sqrt{\frac{1}{\cos(c+dx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + dx))/((a + b/cos(c + dx))^(3/2)*(1/cos(c + dx))^(1/2)),x)

[Out] `int((A + B/cos(c + d*x))/((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(1/2)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \sec(c + dx)}{(a + b \sec(c + dx))^{\frac{3}{2}} \sqrt{\sec(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(3/2)/sec(d*x+c)**(1/2), x)`

[Out] `Integral((A + B*sec(c + d*x))/((a + b*sec(c + d*x))**(3/2)*sqrt(sec(c + d*x))), x)`

$$3.466 \quad \int \frac{A+B \sec(c+dx)}{\sec^2(c+dx)(a+b \sec(c+dx))^{3/2}} dx$$

Optimal. Leaf size=326

$$\frac{2(a^2A + 3abB - 4Ab^2) \sin(c + dx) \sqrt{a + b \sec(c + dx)}}{3a^2d(a^2 - b^2) \sqrt{\sec(c + dx)}} + \frac{2b(Ab - aB) \sin(c + dx)}{ad(a^2 - b^2) \sqrt{\sec(c + dx)} \sqrt{a + b \sec(c + dx)}} + \frac{2(a^2A}{$$

```
[Out] 2*b*(A*b-B*a)*sin(d*x+c)/a/(a^2-b^2)/d/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2/3*(A*a^2+8*A*b^2-6*B*a*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/a^3/d/(a+b*sec(d*x+c))^(1/2)+2/3*(A*a^2-4*A*b^2+3*B*a*b)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/a^2/(a^2-b^2)/d/sec(d*x+c)^(1/2)-2/3*(5*A*a^2*b-8*A*b^3-3*B*a^3+6*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/a^3/(a^2-b^2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)
```

Rubi [A] time = 0.84, antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {4030, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2(a^2A + 3abB - 4Ab^2) \sin(c + dx) \sqrt{a + b \sec(c + dx)}}{3a^2d(a^2 - b^2) \sqrt{\sec(c + dx)}} + \frac{2b(Ab - aB) \sin(c + dx)}{ad(a^2 - b^2) \sqrt{\sec(c + dx)} \sqrt{a + b \sec(c + dx)}} + \frac{2(a^2A}{$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2)),x]
[Out] (2*(a^2*A + 8*A*b^2 - 6*a*b*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]]/(3*a^3*d*Sqrt[a + b*Sec[c + d*x]]) - (2*(5*a^2*A*b - 8*A*b^3 - 3*a^3*B + 6*a*b^2*B)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]]/(3*a^3*(a^2 - b^2)*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*b*(A*b - a*B)*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*(a^2*A - 4*A*b^2 + 3*a*b*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*a^2*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
```

b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2655

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a + b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2661

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)]/(d*Sqrt[a + b]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2663

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b) + (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 3856

Int[Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)]/Sqrt[csc[(e_) + (f_)*(x_)]*(d_)], x_Symbol] := Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 3858

Int[Sqrt[csc[(e_) + (f_)*(x_)]*(d_)]/Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)], x_Symbol] := Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 4030

Int[(csc[(e_) + (f_)*(x_)]*(d_))^(n_)*(csc[(e_) + (f_)*(x_)]*(b_) + (a_))^(m_)*(csc[(e_) + (f_)*(x_)]*(B_) + (A_)), x_Symbol] := Simp[(b*(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[A*(a^2*(m + 1) - b^2*(m + n + 1)) + a*b*B*n - a*(A*b - a*B)*(m + 1)*Csc[e + f*x] + b*(A*b - a*B)*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b

- a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])

Rule 4035

Int[(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.))/(Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]), x_Symbol] :> Dist[A/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]

Rule 4104

Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m, x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]

Rubi steps

$$\begin{aligned}
\int \frac{A + B \sec(c + dx)}{\sec^{\frac{3}{2}}(c + dx)(a + b \sec(c + dx))^{3/2}} dx &= \frac{2b(Ab - aB) \sin(c + dx)}{a(a^2 - b^2) d \sqrt{\sec(c + dx)} \sqrt{a + b \sec(c + dx)}} - \frac{2 \int \frac{\frac{1}{2}(-a^2 A + 4Ab^2 - 3abB)}{\sec} dx}{\sec} \\
&= \frac{2b(Ab - aB) \sin(c + dx)}{a(a^2 - b^2) d \sqrt{\sec(c + dx)} \sqrt{a + b \sec(c + dx)}} + \frac{2(a^2 A - 4Ab^2 + 3abB)}{3a^2(a^2 - b^2)} \\
&= \frac{2b(Ab - aB) \sin(c + dx)}{a(a^2 - b^2) d \sqrt{\sec(c + dx)} \sqrt{a + b \sec(c + dx)}} + \frac{2(a^2 A - 4Ab^2 + 3abB)}{3a^2(a^2 - b^2)} \\
&= \frac{2b(Ab - aB) \sin(c + dx)}{a(a^2 - b^2) d \sqrt{\sec(c + dx)} \sqrt{a + b \sec(c + dx)}} + \frac{2(a^2 A - 4Ab^2 + 3abB)}{3a^2(a^2 - b^2)} \\
&= \frac{2b(Ab - aB) \sin(c + dx)}{a(a^2 - b^2) d \sqrt{\sec(c + dx)} \sqrt{a + b \sec(c + dx)}} + \frac{2(a^2 A - 4Ab^2 + 3abB)}{3a^2(a^2 - b^2)} \\
&= \frac{2(a^2 A + 8Ab^2 - 6abB) \sqrt{\frac{b+a \cos(c+dx)}{a+b}} F\left(\frac{1}{2}(c + dx) \middle| \frac{2a}{a+b}\right) \sqrt{\sec(c + dx)}}{3a^3 d \sqrt{a + b \sec(c + dx)}}
\end{aligned}$$

Mathematica [A] time = 1.61, size = 252, normalized size = 0.77

$$\frac{2 \sec^{\frac{3}{2}}(c + dx)(a \cos(c + dx) + b) \left(a(a - b)(a + b) \sin(c + dx) \left(b(a^2 A + 3abB - 4Ab^2) + aA(a^2 - b^2) \cos(c + dx) \right) \right)}{3a^3 d \sqrt{a + b \sec(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2)),x]

[Out] (2*(b + a*Cos[c + d*x])*Sec[c + d*x]^(3/2)*((a^2 - b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*(a^2*(a^2*A + 2*A*b^2 - 3*a*b*B)*EllipticF[(c + d*x)/2, (2*a)/(a + b)] + (-5*a^2*A*b + 8*A*b^3 + 3*a^3*B - 6*a*b^2*B)*((a + b)*EllipticE[(c + d*x)/2, (2*a)/(a + b)] - b*EllipticF[(c + d*x)/2, (2*a)/(a + b)])) + a*(a - b)*(a + b)*(b*(a^2*A - 4*A*b^2 + 3*a*b*B) + a*A*(a^2 - b^2)*Cos[c + d*x])*Sin[c + d*x])/(3*a^3*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x])^(3/2))

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B \sec(dx+c) + A)\sqrt{b \sec(dx+c) + a} \sqrt{\sec(dx+c)}}{b^2 \sec(dx+c)^4 + 2ab \sec(dx+c)^3 + a^2 \sec(dx+c)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")

[Out] integral((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*sqrt(sec(d*x + c))/(b^2*sec(d*x + c)^4 + 2*a*b*sec(d*x + c)^3 + a^2*sec(d*x + c)^2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx+c) + A}{(b \sec(dx+c) + a)^{\frac{3}{2}} \sec(dx+c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/2)), x)

maple [B] time = 2.31, size = 2285, normalized size = 7.01

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x)

[Out]
$$\begin{aligned} & -2/3/d*(6*A*\sin(d*x+c)*(1/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{1/2}*\text{EllipticF}((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), \\ & (-a+b)/(a-b))^{1/2})*\cos(d*x+c)*a^2*b+8*A*\sin(d*x+c)*(1/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{1/2}*\text{EllipticF}((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), \\ & (-a+b)/(a-b))^{1/2})*\cos(d*x+c)*a*b^2+4*A*((a-b)/(a+b))^{1/2}*\cos(d*x+c)*a^2*b+A*((a-b)/(a+b))^{1/2}*\cos(d*x+c)^3*a^2*b-4*A*((a-b)/(a+b))^{1/2}*\cos(d*x+c)^2*a*b^2+3*B*((a-b)/(a+b))^{1/2}*\cos(d*x+c)^2*a^2*b+8*A*\text{EllipticE}((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), \\ & (-a+b)/(a-b))^{1/2})*b^3*(1/(1+\cos(d*x+c)))^{1/2}*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{1/2}*\sin(d*x+c)+A*\cos(d*x+c)^3*((a-b)/(a+b))^{1/2}*a^3+3*B*((a-b)/(a+b))^{1/2}*\cos(d*x+c)^2*a^3-3*B*a^3*((a-b)/(a+b))^{1/2}*\cos(d*x+c)-A*((a-b)/(a+b))^{1/2}*\cos(d*x+c)*a^3-8*A*((a-b)/(a+b))^{1/2}*\cos(d*x+c)* \end{aligned}$$

$$\begin{aligned}
& b^3 - 5A \sin(dx+c) \operatorname{EllipticE}\left(\frac{-1+\cos(dx+c)}{1+\cos(dx+c)}\right) \left(\frac{a-b}{a+b}\right)^{1/2} / \sin(dx+c) \\
& , \left(\frac{-(a+b)}{a-b}\right)^{1/2} \left(\frac{1}{1+\cos(dx+c)}\right)^{1/2} \left(\frac{b+a\cos(dx+c)}{1+\cos(dx+c)}\right) / (a+b) \\
& \left(\frac{1}{1+\cos(dx+c)}\right)^{1/2} \cos(dx+c) a^2 b - 6B \sin(dx+c) \left(\frac{1}{1+\cos(dx+c)}\right)^{1/2} \\
& \left(\frac{b+a\cos(dx+c)}{1+\cos(dx+c)}\right) / (a+b) \left(\frac{1}{1+\cos(dx+c)}\right)^{1/2} \operatorname{EllipticF}\left(\frac{-1+\cos(dx+c)}{1+\cos(dx+c)}\right) \left(\frac{a-b}{a+b}\right)^{1/2} / \sin(dx+c) \\
& , \left(\frac{-(a+b)}{a-b}\right)^{1/2} \cos(dx+c) a^2 b - 6B \sin(dx+c) \operatorname{EllipticE}\left(\frac{-1+\cos(dx+c)}{1+\cos(dx+c)}\right) \left(\frac{a-b}{a+b}\right)^{1/2} / \sin(dx+c) \\
& , \left(\frac{-(a+b)}{a-b}\right)^{1/2} \left(\frac{1}{1+\cos(dx+c)}\right)^{1/2} \left(\frac{b+a\cos(dx+c)}{1+\cos(dx+c)}\right) / (a+b) \\
& \left(\frac{1}{1+\cos(dx+c)}\right)^{1/2} \cos(dx+c) a^2 b - 3B \operatorname{EllipticF}\left(\frac{-1+\cos(dx+c)}{1+\cos(dx+c)}\right) \left(\frac{a-b}{a+b}\right)^{1/2} / \sin(dx+c) \\
& , \left(\frac{-(a+b)}{a-b}\right)^{1/2} a^3 \left(\frac{1}{1+\cos(dx+c)}\right)^{1/2} \left(\frac{b+a\cos(dx+c)}{1+\cos(dx+c)}\right) / (a+b) \\
& \left(\frac{1}{1+\cos(dx+c)}\right)^{1/2} \sin(dx+c) + A \operatorname{EllipticF}\left(\frac{-1+\cos(dx+c)}{1+\cos(dx+c)}\right) \left(\frac{a-b}{a+b}\right)^{1/2} / \sin(dx+c) \\
& , \left(\frac{-(a+b)}{a-b}\right)^{1/2} a^3 \left(\frac{1}{1+\cos(dx+c)}\right)^{1/2} \left(\frac{b+a\cos(dx+c)}{1+\cos(dx+c)}\right) / (a+b) \\
& \left(\frac{1}{1+\cos(dx+c)}\right)^{1/2} \sin(dx+c) + 8A b^3 \left(\frac{a-b}{a+b}\right)^{1/2} - A a^2 b \left(\frac{a-b}{a+b}\right)^{1/2} + 4A a^2 b^2 \left(\frac{a-b}{a+b}\right)^{1/2} - 3B \\
& a^2 b \left(\frac{a-b}{a+b}\right)^{1/2} - 6B a^2 b^2 \left(\frac{a-b}{a+b}\right)^{1/2} + 3B \left(\frac{b+a\cos(dx+c)}{1+\cos(dx+c)}\right) / (a+b) \\
& \left(\frac{1}{1+\cos(dx+c)}\right)^{1/2} \cos(dx+c) \left(\frac{1}{1+\cos(dx+c)}\right)^{1/2} \operatorname{EllipticE}\left(\frac{-1+\cos(dx+c)}{1+\cos(dx+c)}\right) \left(\frac{a-b}{a+b}\right)^{1/2} / \sin(dx+c) \\
& + 6B \left(\frac{a-b}{a+b}\right)^{1/2} \cos(dx+c) a^2 b - 4A \cos(dx+c)^2 \left(\frac{a-b}{a+b}\right)^{1/2} a^2 b + A \sin(dx+c) \left(\frac{1}{1+\cos(dx+c)}\right)^{1/2} \\
& \left(\frac{b+a\cos(dx+c)}{1+\cos(dx+c)}\right) / (a+b) \left(\frac{1}{1+\cos(dx+c)}\right)^{1/2} \operatorname{EllipticF}\left(\frac{-1+\cos(dx+c)}{1+\cos(dx+c)}\right) \left(\frac{a-b}{a+b}\right)^{1/2} / \sin(dx+c) \\
& , \left(\frac{-(a+b)}{a-b}\right)^{1/2} \cos(dx+c) a^3 + 8A \sin(dx+c) \operatorname{EllipticE}\left(\frac{-1+\cos(dx+c)}{1+\cos(dx+c)}\right) \left(\frac{a-b}{a+b}\right)^{1/2} / \sin(dx+c) \\
& , \left(\frac{-(a+b)}{a-b}\right)^{1/2} \left(\frac{1}{1+\cos(dx+c)}\right)^{1/2} \left(\frac{b+a\cos(dx+c)}{1+\cos(dx+c)}\right) / (a+b) \left(\frac{1}{1+\cos(dx+c)}\right)^{1/2} \cos(dx+c) \\
& b^3 - 3B \sin(dx+c) \left(\frac{1}{1+\cos(dx+c)}\right)^{1/2} \left(\frac{b+a\cos(dx+c)}{1+\cos(dx+c)}\right) / (a+b) \left(\frac{1}{1+\cos(dx+c)}\right)^{1/2} \\
& \operatorname{EllipticF}\left(\frac{-1+\cos(dx+c)}{1+\cos(dx+c)}\right) \left(\frac{a-b}{a+b}\right)^{1/2} / \sin(dx+c) , \left(\frac{-(a+b)}{a-b}\right)^{1/2} \cos(dx+c) a^3 + 6A \\
& \operatorname{EllipticF}\left(\frac{-1+\cos(dx+c)}{1+\cos(dx+c)}\right) \left(\frac{a-b}{a+b}\right)^{1/2} / \sin(dx+c) , \left(\frac{-(a+b)}{a-b}\right)^{1/2} a^2 b \left(\frac{1}{1+\cos(dx+c)}\right)^{1/2} \\
& \left(\frac{b+a\cos(dx+c)}{1+\cos(dx+c)}\right) / (a+b) \left(\frac{1}{1+\cos(dx+c)}\right)^{1/2} \sin(dx+c) + 8A \operatorname{EllipticF}\left(\frac{-1+\cos(dx+c)}{1+\cos(dx+c)}\right) \\
& \left(\frac{a-b}{a+b}\right)^{1/2} / \sin(dx+c) , \left(\frac{-(a+b)}{a-b}\right)^{1/2} a^2 b^2 \left(\frac{1}{1+\cos(dx+c)}\right)^{1/2} \left(\frac{b+a\cos(dx+c)}{1+\cos(dx+c)}\right) / (a+b) \\
& \left(\frac{1}{1+\cos(dx+c)}\right)^{1/2} \sin(dx+c) - 5A \operatorname{EllipticE}\left(\frac{-1+\cos(dx+c)}{1+\cos(dx+c)}\right) \left(\frac{a-b}{a+b}\right)^{1/2} / \sin(dx+c) \\
& , \left(\frac{-(a+b)}{a-b}\right)^{1/2} a^2 b \left(\frac{1}{1+\cos(dx+c)}\right)^{1/2} \left(\frac{b+a\cos(dx+c)}{1+\cos(dx+c)}\right) / (a+b) \left(\frac{1}{1+\cos(dx+c)}\right)^{1/2} \\
& \sin(dx+c) - 6B \operatorname{EllipticF}\left(\frac{-1+\cos(dx+c)}{1+\cos(dx+c)}\right) \left(\frac{a-b}{a+b}\right)^{1/2} / \sin(dx+c) , \left(\frac{-(a+b)}{a-b}\right)^{1/2} a^2 b \\
& \left(\frac{1}{1+\cos(dx+c)}\right)^{1/2} \left(\frac{b+a\cos(dx+c)}{1+\cos(dx+c)}\right) / (a+b) \left(\frac{1}{1+\cos(dx+c)}\right)^{1/2} \sin(dx+c) - 6B \operatorname{EllipticE}\left(\frac{-1+\cos(dx+c)}{1+\cos(dx+c)}\right) \\
& \left(\frac{a-b}{a+b}\right)^{1/2} / \sin(dx+c) , \left(\frac{-(a+b)}{a-b}\right)^{1/2} a^2 b^2 \left(\frac{1}{1+\cos(dx+c)}\right)^{1/2} \left(\frac{b+a\cos(dx+c)}{1+\cos(dx+c)}\right) / (a+b) \\
& \left(\frac{1}{1+\cos(dx+c)}\right)^{1/2} \sin(dx+c) + 3B \sin(dx+c) \left(\frac{1}{1+\cos(dx+c)}\right)^{1/2} \left(\frac{b+a\cos(dx+c)}{1+\cos(dx+c)}\right) / (a+b) \\
& \left(\frac{1}{1+\cos(dx+c)}\right)^{1/2} \operatorname{EllipticE}\left(\frac{-1+\cos(dx+c)}{1+\cos(dx+c)}\right) \left(\frac{a-b}{a+b}\right)^{1/2} / \sin(dx+c) , \left(\frac{-(a+b)}{a-b}\right)^{1/2} \cos(dx+c) \\
& a^3 \left(\frac{b+a\cos(dx+c)}{\cos(dx+c)}\right)^{1/2} \cos(dx+c)^2 \left(\frac{1}{\cos(dx+c)}\right)^{3/2} / \sin(dx+c) / (b+a\cos(dx+c)) / a^3 / (a+b) / \left(\frac{a-b}{a+b}\right)^{1/2}
\end{aligned}$$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\left(a + \frac{b}{\cos(c+dx)}\right)^{3/2} \left(\frac{1}{\cos(c+dx)}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(3/2)),x)

[Out] int((A + B/cos(c + d*x))/((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(3/2)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)**(3/2)/(a+b*sec(d*x+c))**(3/2),x)

[Out] Timed out

$$3.467 \quad \int \frac{A+B \sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+b \sec(c+dx))^{3/2}} dx$$

Optimal. Leaf size=423

$$\frac{2(a^2A + 5abB - 6Ab^2) \sin(c+dx) \sqrt{a+b \sec(c+dx)}}{5a^2d(a^2 - b^2) \sec^{\frac{3}{2}}(c+dx)} + \frac{2b(Ab - aB) \sin(c+dx)}{ad(a^2 - b^2) \sec^{\frac{3}{2}}(c+dx) \sqrt{a+b \sec(c+dx)}} - \frac{2(-5a^3B}{$$

[Out] $2*b*(A*b-B*a)*\sin(d*x+c)/a/(a^2-b^2)/d/\sec(d*x+c)^{(3/2)}/(a+b*\sec(d*x+c))^{(1/2)}-2/15*(12*A*a^2*b+48*A*b^3-5*B*a^3-40*B*a*b^2)*(\cos(1/2*d*x+1/2*c))^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c),2^{(1/2)}*(a/(a+b))^{(1/2)})*((b+a*\cos(d*x+c))/(a+b))^{(1/2)}*\sec(d*x+c)^{(1/2)}/a^4/d/(a+b*\sec(d*x+c))^{(1/2)}+2/5*(A*a^2-6*A*b^2+5*B*a*b)*\sin(d*x+c)*(a+b*\sec(d*x+c))^{(1/2)}/a^2/(a^2-b^2)/d/\sec(d*x+c)^{(3/2)}-2/15*(9*A*a^2*b-24*A*b^3-5*B*a^3+20*B*a*b^2)*\sin(d*x+c)*(a+b*\sec(d*x+c))^{(1/2)}/a^3/(a^2-b^2)/d/\sec(d*x+c)^{(1/2)}+2/15*(9*A*a^4+24*A*a^2*b^2-48*A*b^4-25*B*a^3*b+40*B*a*b^3)*(\cos(1/2*d*x+1/2*c))^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c),2^{(1/2)}*(a/(a+b))^{(1/2)})*(a+b*\sec(d*x+c))^{(1/2)}/a^4/(a^2-b^2)/d/((b+a*\cos(d*x+c))/(a+b))^{(1/2)}/\sec(d*x+c)^{(1/2)}$

Rubi [A] time = 1.22, antiderivative size = 423, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {4030, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2(a^2A + 5abB - 6Ab^2) \sin(c+dx) \sqrt{a+b \sec(c+dx)}}{5a^2d(a^2 - b^2) \sec^{\frac{3}{2}}(c+dx)} + \frac{2b(Ab - aB) \sin(c+dx)}{ad(a^2 - b^2) \sec^{\frac{3}{2}}(c+dx) \sqrt{a+b \sec(c+dx)}} - \frac{2(9a^2Ab}{$$

Antiderivative was successfully verified.

[In] $\text{Int}[(A + B*\text{Sec}[c + d*x])/(\text{Sec}[c + d*x]^{(5/2)}*(a + b*\text{Sec}[c + d*x])^{(3/2)}),x]$

[Out] $(-2*(12*a^2*A*b + 48*A*b^3 - 5*a^3*B - 40*a*b^2*B)*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/(a + b)]*\text{EllipticF}[(c + d*x)/2, (2*a)/(a + b)]*\text{Sqrt}[\text{Sec}[c + d*x]])/(15*a^4*d*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]) + (2*(9*a^4*A + 24*a^2*A*b^2 - 48*A*b^4 - 25*a^3*b*B + 40*a*b^3*B)*\text{EllipticE}[(c + d*x)/2, (2*a)/(a + b)]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]])/(15*a^4*(a^2 - b^2)*d*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/(a + b)]*\text{Sqrt}[\text{Sec}[c + d*x]]) + (2*b*(A*b - a*B)*\text{Sin}[c + d*x])/(a*(a^2 - b^2)*d*\text{Sec}[c + d*x]^{(3/2)}*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]) + (2*(a^2*A - 6*A*b^2 + 5*a*b*B)*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(5*a^2*(a^2 - b^2)*d*\text{Sec}[c + d*x]^{(3/2)}) - (2*(9*a^2*A*b - 24*A*b^3 - 5*a^3*B + 20*a*b^2*B)*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(15*a^3*(a^2 - b^2)*d*\text{Sqrt}[\text{Sec}[c + d*x]])$

Rule 2653

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*Sqrt[a
+ b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)]/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
0] && !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)]/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a
+ b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && !GtQ[a + b, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)]/Sqrt[csc[(e_) + (f_)*(x_)]
*(d_)], x_Symbol] := Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_) + (f_)*(x_)]*(d_)]/Sqrt[csc[(e_) + (f_)*(x_)]*(b_)
+ (a_)], x_Symbol] := Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4030

```
Int[(csc[(e_) + (f_)*(x_)]*(d_))^(n_)*(csc[(e_) + (f_)*(x_)]*(b_) + (
a_))^(m_)*(csc[(e_) + (f_)*(x_)]*(B_) + (A_)), x_Symbol] := Simp[(b*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*
```

```
(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e
+ f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[A*(a^2*(m + 1) - b^2*(m + n + 1))
+ a*b*B*n - a*(A*b - a*B)*(m + 1)*Csc[e + f*x] + b*(A*b - a*B)*(m + n + 2)*
Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b
- a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && !(ILtQ[m + 1/2, 0] && ILt
Q[n, 0])
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.))/(Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a
_.))^m, x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{A + B \sec(c + dx)}{\sec^{\frac{5}{2}}(c + dx)(a + b \sec(c + dx))^{3/2}} dx &= \frac{2b(Ab - aB) \sin(c + dx)}{a(a^2 - b^2) d \sec^{\frac{3}{2}}(c + dx) \sqrt{a + b \sec(c + dx)}} - \frac{2 \int \frac{\frac{1}{2}(-a^2 A + 6Ab^2 - 5abB)}{s}}{s} \\
&= \frac{2b(Ab - aB) \sin(c + dx)}{a(a^2 - b^2) d \sec^{\frac{3}{2}}(c + dx) \sqrt{a + b \sec(c + dx)}} + \frac{2(a^2 A - 6Ab^2 + 5abB)}{5a^2(a^2 - b^2)} \\
&= \frac{2b(Ab - aB) \sin(c + dx)}{a(a^2 - b^2) d \sec^{\frac{3}{2}}(c + dx) \sqrt{a + b \sec(c + dx)}} + \frac{2(a^2 A - 6Ab^2 + 5abB)}{5a^2(a^2 - b^2)} \\
&= \frac{2b(Ab - aB) \sin(c + dx)}{a(a^2 - b^2) d \sec^{\frac{3}{2}}(c + dx) \sqrt{a + b \sec(c + dx)}} + \frac{2(a^2 A - 6Ab^2 + 5abB)}{5a^2(a^2 - b^2)} \\
&= \frac{2b(Ab - aB) \sin(c + dx)}{a(a^2 - b^2) d \sec^{\frac{3}{2}}(c + dx) \sqrt{a + b \sec(c + dx)}} + \frac{2(a^2 A - 6Ab^2 + 5abB)}{5a^2(a^2 - b^2)} \\
&= \frac{2b(Ab - aB) \sin(c + dx)}{a(a^2 - b^2) d \sec^{\frac{3}{2}}(c + dx) \sqrt{a + b \sec(c + dx)}} + \frac{2(a^2 A - 6Ab^2 + 5abB)}{5a^2(a^2 - b^2)} \\
&= \frac{2b(Ab - aB) \sin(c + dx)}{a(a^2 - b^2) d \sec^{\frac{3}{2}}(c + dx) \sqrt{a + b \sec(c + dx)}} + \frac{2(a^2 A - 6Ab^2 + 5abB)}{5a^2(a^2 - b^2)} \\
&= \frac{2(12a^2 Ab + 48Ab^3 - 5a^3 B - 40ab^2 B) \sqrt{\frac{b+a \cos(c+dx)}{a+b}} F\left(\frac{1}{2}(c + dx) \middle| \frac{2}{a}\right)}{15a^4 d \sqrt{a + b \sec(c + dx)}}
\end{aligned}$$

Mathematica [A] time = 2.40, size = 316, normalized size = 0.75

$$\frac{\sec^{\frac{3}{2}}(c + dx)(a \cos(c + dx) + b) \left(a(a - b)(a + b) \left(2(a^2 - b^2) (9Ab - 5a^3 B) \sin(c + dx)(a \cos(c + dx) + b) - 3aA \right) \right)}{15a^4 d \sqrt{a + b \sec(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(3/2)),x]

[Out] -1/15*((b + a*Cos[c + d*x])*Sec[c + d*x]^(3/2)*(2*(a^2 - b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*(a^2*(3*a^2*A*b + 12*A*b^3 - 5*a^3*B - 10*a*b^2*B)*El

```
lipticF[(c + d*x)/2, (2*a)/(a + b)] - (9*a^4*A + 24*a^2*A*b^2 - 48*A*b^4 -
25*a^3*b*B + 40*a*b^3*B)*((a + b)*EllipticE[(c + d*x)/2, (2*a)/(a + b)] - b
*EllipticF[(c + d*x)/2, (2*a)/(a + b)]) + a*(a - b)*(a + b)*(30*b^3*(-(A*b
) + a*B)*Sin[c + d*x] + 2*(a^2 - b^2)*(9*A*b - 5*a*B)*(b + a*Cos[c + d*x])*
Sin[c + d*x] - 3*a*A*(a^2 - b^2)*(b + a*Cos[c + d*x])*Sin[2*(c + d*x)])))/(
a^4*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x])^(3/2))
```

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{(B \sec(dx + c) + A) \sqrt{b \sec(dx + c) + a} \sqrt{\sec(dx + c)}}{b^2 \sec(dx + c)^5 + 2ab \sec(dx + c)^4 + a^2 \sec(dx + c)^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*sqrt(sec(d*x + c))/(
b^2*sec(d*x + c)^5 + 2*a*b*sec(d*x + c)^4 + a^2*sec(d*x + c)^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(b \sec(dx + c) + a)^{\frac{3}{2}} \sec(dx + c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(5/
2)), x)
```

maple [B] time = 2.50, size = 3156, normalized size = 7.46

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] 2/15/d*(-5*B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+
b)/(a-b))^(1/2))*a^4*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*(1/(1+co
s(d*x+c)))^(1/2)*sin(d*x+c)-9*A*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/
2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a
+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+48*A*EllipticE((-1+cos(d*x+c
```

$$\begin{aligned}
&)) * ((a-b)/(a+b))^{1/2} / \sin(dx+c), (- (a+b)/(a-b))^{1/2}) * b^4 * ((b+a \cos(dx+c)) / (1+\cos(dx+c)) / (a+b))^{1/2} * (1/(1+\cos(dx+c)))^{1/2} * \sin(dx+c) + 9 * A * a^3 * \\
&b * ((a-b)/(a+b))^{1/2} + 24 * A * a * b^3 * ((a-b)/(a+b))^{1/2} + 5 * B * a^3 * b * ((a-b)/(a+b))^{1/2} - 20 * B * a^2 * b^2 * ((a-b)/(a+b))^{1/2} - 40 * B * a * b^3 * ((a-b)/(a+b))^{1/2} - 5 * B \\
&* \cos(dx+c)^3 * ((a-b)/(a+b))^{1/2} * a^4 - 3 * A * \cos(dx+c)^4 * ((a-b)/(a+b))^{1/2} * \\
&a^4 - 6 * A * \cos(dx+c)^2 * ((a-b)/(a+b))^{1/2} * a^4 - 9 * A * \cos(dx+c) * ((b+a \cos(dx+c)) / (1+\cos(dx+c)) / (a+b))^{1/2} * (1/(1+\cos(dx+c)))^{1/2} * \text{EllipticE}((-1+\cos(d \\
&*x+c)) * ((a-b)/(a+b))^{1/2} / \sin(dx+c), (- (a+b)/(a-b))^{1/2}) * \sin(dx+c) * a^4 - \\
&48 * A * \cos(dx+c) * ((a-b)/(a+b))^{1/2} * b^4 + 5 * B * \cos(dx+c) * ((a-b)/(a+b))^{1/2} * \\
&a^4 + 9 * A * \cos(dx+c) * ((a-b)/(a+b))^{1/2} * a^4 + 6 * A * \cos(dx+c)^3 * ((a-b)/(a+b))^{1/2} * \\
&a^3 * b + 48 * A * b^4 * ((a-b)/(a+b))^{1/2} - 24 * A * \cos(dx+c)^2 * ((a-b)/(a+b))^{1/2} * \\
&a^2 * b^2 + 20 * B * \cos(dx+c)^2 * ((a-b)/(a+b))^{1/2} * a^3 * b + 48 * A * \cos(dx+c) * \text{EllipticE}((-1+\cos(d \\
&*x+c)) * ((a-b)/(a+b))^{1/2} / \sin(dx+c), (- (a+b)/(a-b))^{1/2}) * \\
&((b+a \cos(dx+c)) / (1+\cos(dx+c)) / (a+b))^{1/2} * (1/(1+\cos(dx+c)))^{1/2} * \sin(dx+c) * b^4 + 9 * A * \cos(dx+c) * ((b+a \cos(dx+c)) / (1+\cos(dx+c)) / (a+b))^{1/2} * (1/ \\
&(1+\cos(dx+c)))^{1/2} * \text{EllipticF}((-1+\cos(dx+c)) * ((a-b)/(a+b))^{1/2} / \sin(dx+c), (- (a+b)/(a-b))^{1/2}) * \sin(dx+c) * a^4 - 5 * B * \cos(dx+c) * ((b+a \cos(dx+c)) / (\\
&1+\cos(dx+c)) / (a+b))^{1/2} * (1/(1+\cos(dx+c)))^{1/2} * \text{EllipticF}((-1+\cos(dx+c)) * ((a-b)/(a+b))^{1/2} / \sin(dx+c), (- (a+b)/(a-b))^{1/2}) * \sin(dx+c) * a^4 - 24 * A \\
&* \text{EllipticE}((-1+\cos(dx+c)) * ((a-b)/(a+b))^{1/2} / \sin(dx+c), (- (a+b)/(a-b))^{1/2}) * \\
&a^2 * b^2 * ((b+a \cos(dx+c)) / (1+\cos(dx+c)) / (a+b))^{1/2} * (1/(1+\cos(dx+c)) \\
&))^{1/2} * \sin(dx+c) + 12 * A * \text{EllipticF}((-1+\cos(dx+c)) * ((a-b)/(a+b))^{1/2} / \sin(dx+c), (- (a+b)/(a-b))^{1/2}) * a^3 * b * ((b+a \cos(dx+c)) / (1+\cos(dx+c)) / (a+b))^{1/2} * (1/(1+\cos(dx+c)) \\
&))^{1/2} * \sin(dx+c) + 36 * A * \text{EllipticF}((-1+\cos(dx+c)) * ((a-b)/(a+b))^{1/2} / \sin(dx+c), (- (a+b)/(a-b))^{1/2}) * a^2 * b^2 * ((b+a \cos(dx+c)) / (1+\cos(dx+c)) / (a+b))^{1/2} * (1/(1+\cos(dx+c)) \\
&))^{1/2} * \sin(dx+c) + 48 * A * \text{EllipticF}((-1+\cos(dx+c)) * ((a-b)/(a+b))^{1/2} / \sin(dx+c), (- (a+b)/(a-b))^{1/2}) * \\
&a * b^3 * ((b+a \cos(dx+c)) / (1+\cos(dx+c)) / (a+b))^{1/2} * (1/(1+\cos(dx+c)) \\
&))^{1/2} * \sin(dx+c) + 25 * B * \text{EllipticE}((-1+\cos(dx+c)) * ((a-b)/(a+b))^{1/2} / \sin(dx+c), \\
&(- (a+b)/(a-b))^{1/2}) * a^3 * b * ((b+a \cos(dx+c)) / (1+\cos(dx+c)) / (a+b))^{1/2} * (\\
&1/(1+\cos(dx+c)) \\
&))^{1/2} * \sin(dx+c) - 40 * B * \text{EllipticE}((-1+\cos(dx+c)) * ((a-b)/(a+b))^{1/2} / \sin(dx+c), (- (a+b)/(a-b))^{1/2}) * a * b^3 * ((b+a \cos(dx+c)) / (1+\cos(dx+c)) / (a+b))^{1/2} * (1/(1+\cos(dx+c)) \\
&))^{1/2} * \sin(dx+c) - 30 * B * \text{EllipticF}((-1+\cos(dx+c)) * ((a-b)/(a+b))^{1/2} / \sin(dx+c), (- (a+b)/(a-b))^{1/2}) * a^3 * b * ((b+a \cos(dx+c)) / (1+\cos(dx+c)) / (a+b))^{1/2} * (1/(1+\cos(dx+c)) \\
&))^{1/2} * \sin(dx+c) - 40 * B * \text{EllipticF}((-1+\cos(dx+c)) * ((a-b)/(a+b))^{1/2} / \sin(dx+c), (- (a+b)/(a-b))^{1/2}) * a^2 * b^2 * ((b+a \cos(dx+c)) / (1+\cos(dx+c)) / (a+b))^{1/2} * (1/(1+\cos(dx+c)) \\
&))^{1/2} * \sin(dx+c) - 24 * A * \cos(dx+c) * \text{EllipticE}((-1+\cos(dx+c)) * ((a-b) \\
&)) / (a+b))^{1/2} / \sin(dx+c), (- (a+b)/(a-b))^{1/2}) * ((b+a \cos(dx+c)) / (1+\cos(dx \\
&*x+c)) / (a+b))^{1/2} * (1/(1+\cos(dx+c)) \\
&))^{1/2} * \sin(dx+c) * a^2 * b^2 + 12 * A * \cos(dx+c) * ((b+a \cos(dx+c)) / (1+\cos(dx+c)) / (a+b))^{1/2} * (1/(1+\cos(dx+c)) \\
&))^{1/2} * \text{EllipticF}((-1+\cos(dx+c)) * ((a-b)/(a+b))^{1/2} / \sin(dx+c), (- (a+b)/(a-b))^{1/2}) * \sin(dx+c) * a^3 * b + 36 * A * \cos(dx+c) * ((b+a \cos(dx+c)) / (1+\cos(dx+c)) / (a+b))^{1/2} * (1/(1+\cos(dx+c)) \\
&))^{1/2} * \text{EllipticF}((-1+\cos(dx+c)) * ((a-b)/(a+b))^{1/2} / \sin(dx+c), (- (a+b)/(a-b))^{1/2}) * \sin(dx+c) * a^2 * b^2 + 48 * A * \cos(dx+c) * ((b
\end{aligned}$$

```

+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a*b^3+25*B*cos(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*a^3*b-40*B*cos(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*a*b^3-30*B*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a^3*b-40*B*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a^2*b^2+9*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+6*A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b^2-5*B*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^3*b-6*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3*b-24*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^3+20*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*b^2-6*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3*b+18*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b^2-20*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3*b-3*A*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^3*b+40*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^3)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^3*(1/cos(d*x+c))^(5/2)/sin(d*x+c)/(b+a*cos(d*x+c))/a^4/(a+b)/((a-b)/(a+b))^(1/2)

```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\left(a + \frac{b}{\cos(c+dx)}\right)^{3/2} \left(\frac{1}{\cos(c+dx)}\right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(5/2)),x)

[Out] int((A + B/cos(c + d*x))/((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(5/2)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)**(5/2)/(a+b*sec(d*x+c))**(3/2),x)

[Out] Timed out

$$3.468 \quad \int \frac{\sec^2(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^{5/2}} dx$$

Optimal. Leaf size=399

$$\frac{2a(Ab - aB) \sin(c + dx) \sec^3(c + dx)}{3bd(a^2 - b^2)(a + b \sec(c + dx))^{3/2}} + \frac{2(Ab - aB)\sqrt{\sec(c + dx)} \sqrt{\frac{a \cos(c+dx)+b}{a+b}} F\left(\frac{1}{2}(c + dx) \middle| \frac{2a}{a+b}\right)}{3bd(a^2 - b^2)\sqrt{a + b \sec(c + dx)}} - \frac{2a(3a^3B - 7ab^2B)}{3b^2d(a^2 - b^2)}$$

[Out] $2/3*a*(A*b-B*a)*\sec(d*x+c)^{(3/2)}*\sin(d*x+c)/b/(a^2-b^2)/d/(a+b*\sec(d*x+c))^{(3/2)}-2/3*a*(4*A*b^3+3*B*a^3-7*B*a*b^2)*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/b^2/(a^2-b^2)^2/d/(a+b*\sec(d*x+c))^{(1/2)}+2/3*(A*b-B*a)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)}*(a/(a+b))^{(1/2)})*((b+a*\cos(d*x+c))/(a+b))^{(1/2)}*\sec(d*x+c)^{(1/2)}/b/(a^2-b^2)/d/(a+b*\sec(d*x+c))^{(1/2)}+2*B*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticPi}(\sin(1/2*d*x+1/2*c), 2^{(1/2)}*(a/(a+b))^{(1/2)})*((b+a*\cos(d*x+c))/(a+b))^{(1/2)}*\sec(d*x+c)^{(1/2)}/b^2/d/(a+b*\sec(d*x+c))^{(1/2)}+2/3*(4*A*b^3+3*B*a^3-7*B*a*b^2)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)}*(a/(a+b))^{(1/2)})*(a+b*\sec(d*x+c))^{(1/2)}/b^2/(a^2-b^2)^2/d/((b+a*\cos(d*x+c))/(a+b))^{(1/2)}/\sec(d*x+c)^{(1/2)}$

Rubi [A] time = 1.37, antiderivative size = 399, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {4029, 4098, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2a(Ab - aB) \sin(c + dx) \sec^3(c + dx)}{3bd(a^2 - b^2)(a + b \sec(c + dx))^{3/2}} - \frac{2a(3a^3B - 7ab^2B + 4Ab^3) \sin(c + dx)\sqrt{\sec(c + dx)}}{3b^2d(a^2 - b^2)^2\sqrt{a + b \sec(c + dx)}} + \frac{2(Ab - aB)\sqrt{\sec(c + dx)}}{3bd(a^2 - b^2)}$$

Antiderivative was successfully verified.

[In] Int[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(5/2), x]

[Out] $(2*(A*b - a*B)*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/(a + b)]*\text{EllipticF}[(c + d*x)/2, (2*a)/(a + b)]*\text{Sqrt}[\text{Sec}[c + d*x]])/(3*b*(a^2 - b^2)*d*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]) + (2*B*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/(a + b)]*\text{EllipticPi}[2, (c + d*x)/2, (2*a)/(a + b)]*\text{Sqrt}[\text{Sec}[c + d*x]])/(b^2*d*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]) + (2*(4*A*b^3 + 3*a^3*B - 7*a*b^2*B)*\text{EllipticE}[(c + d*x)/2, (2*a)/(a + b)]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]])/(3*b^2*(a^2 - b^2)^2*d*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/(a + b)]*\text{Sqrt}[\text{Sec}[c + d*x]]) + (2*a*(A*b - a*B)*\text{Sec}[c + d*x]^{(3/2)}*\text{Sin}[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*\text{Sec}[c + d*x])^{(3/2)}) - (2*a*(4*A*b^3 + 3*a^3*B - 7*a*b^2*B)*\text{Sqrt}[\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(3*b^2*(a^2 - b^2)^2*d*\text{Sqrt}[a + b*\text{Sec}[c + d*x]])$

Rule 2653

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*Sqrt[a
+ b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
0] && !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)]/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a
+ b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_)
+ (f_)*(x_)]]), x_Symbol] := Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)]/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_)
+ (f_)*(x_)]]), x_Symbol] := Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
+ f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !GtQ[c + d, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)]/Sqrt[csc[(e_) + (f_)*(x_)]]
```

$\text{*(d_.)}, x_Symbol] \text{ :> Dist}[\text{Sqrt}[a + b*\text{Csc}[e + f*x]]/(\text{Sqrt}[d*\text{Csc}[e + f*x]]*\text{Sqrt}[b + a*\text{Sin}[e + f*x]]), \text{Int}[\text{Sqrt}[b + a*\text{Sin}[e + f*x]], x], x] \text{ /; FreeQ}[\{a, b, d, e, f\}, x] \ \&\& \ \text{NeQ}[a^2 - b^2, 0]$

Rule 3858

$\text{Int}[\text{Sqrt}[\text{csc}[(e_.) + (f_.)*(x_.)]*(d_.)]/\text{Sqrt}[\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] \text{ :> Dist}[(\text{Sqrt}[d*\text{Csc}[e + f*x]]*\text{Sqrt}[b + a*\text{Sin}[e + f*x]])/\text{Sqrt}[a + b*\text{Csc}[e + f*x]], \text{Int}[1/\text{Sqrt}[b + a*\text{Sin}[e + f*x]], x], x] \text{ /; FreeQ}[\{a, b, d, e, f\}, x] \ \&\& \ \text{NeQ}[a^2 - b^2, 0]$

Rule 3859

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(d_.))^{3/2}/\text{Sqrt}[\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] \text{ :> Dist}[(d*\text{Sqrt}[d*\text{Csc}[e + f*x]]*\text{Sqrt}[b + a*\text{Sin}[e + f*x]])/\text{Sqrt}[a + b*\text{Csc}[e + f*x]], \text{Int}[1/(\text{Sin}[e + f*x]*\text{Sqrt}[b + a*\text{Sin}[e + f*x]]), x], x] \text{ /; FreeQ}[\{a, b, d, e, f\}, x] \ \&\& \ \text{NeQ}[a^2 - b^2, 0]$

Rule 4029

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(d_.))^{(n_)}*(\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^{(m_)}*(\text{csc}[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] \text{ :> Simp}[(a*d^2*(A*b - a*B)*\text{Cot}[e + f*x]*(a + b*\text{Csc}[e + f*x])^{(m+1)}*(d*\text{Csc}[e + f*x])^{(n-2)})/(b*f*(m+1)*(a^2 - b^2)), x] - \text{Dist}[d/(b*(m+1)*(a^2 - b^2)), \text{Int}[(a + b*\text{Csc}[e + f*x])^{(m+1)}*(d*\text{Csc}[e + f*x])^{(n-2)}*\text{Simp}[a*d*(A*b - a*B)*(n-2) + b*d*(A*b - a*B)*(m+1)*\text{Csc}[e + f*x] - (a*A*b*d*(m+n) - d*B*(a^2*(n-1) + b^2*(m+1)))*\text{Csc}[e + f*x]^2, x], x], x] \text{ /; FreeQ}[\{a, b, d, e, f, A, B\}, x] \ \&\& \ \text{NeQ}[A*b - a*B, 0] \ \&\& \ \text{NeQ}[a^2 - b^2, 0] \ \&\& \ \text{LtQ}[m, -1] \ \&\& \ \text{GtQ}[n, 1]$

Rule 4035

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.))/(\text{Sqrt}[\text{csc}[(e_.) + (f_.)*(x_.)]*(d_.)]*\text{Sqrt}[\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]), x_Symbol] \text{ :> Dist}[A/a, \text{Int}[\text{Sqrt}[a + b*\text{Csc}[e + f*x]]/\text{Sqrt}[d*\text{Csc}[e + f*x]], x], x] - \text{Dist}[(A*b - a*B)/(a*d), \text{Int}[\text{Sqrt}[d*\text{Csc}[e + f*x]]/\text{Sqrt}[a + b*\text{Csc}[e + f*x]], x], x] \text{ /; FreeQ}[\{a, b, d, e, f, A, B\}, x] \ \&\& \ \text{NeQ}[A*b - a*B, 0] \ \&\& \ \text{NeQ}[a^2 - b^2, 0]$

Rule 4098

$\text{Int}[(\text{(A_.) + csc}[(e_.) + (f_.)*(x_.)]*(B_.) + \text{csc}[(e_.) + (f_.)*(x_.)]^2*(C_.))*(\text{csc}[(e_.) + (f_.)*(x_.)]*(d_.))^{(n_)}*(\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^{(m_)}, x_Symbol] \text{ :> -Simp}[(d*(A*b^2 - a*b*B + a^2*C)*\text{Cot}[e + f*x]*(a + b*\text{Csc}[e + f*x])^{(m+1)}*(d*\text{Csc}[e + f*x])^{(n-1)})/(b*f*(a^2 - b^2)*(m+1)), x] + \text{Dist}[d/(b*(a^2 - b^2)*(m+1)), \text{Int}[(a + b*\text{Csc}[e + f*x])^{(m+1)}*(d*$

```
sc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A -
b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n + b
^2*(m + 1)))*Csc[e + f*x]^2, x], x] /; FreeQ[{a, b, d, e, f, A, B, C},
x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.)
+ (a_.)]), x_Symbol] := Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
+ b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{5/2}} dx &= \frac{2a(Ab-aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b(a^2-b^2)d(a+b\sec(c+dx))^{3/2}} + \frac{2\int \frac{\sqrt{\sec(c+dx)}\left(\frac{1}{2}a(Ab-aB)-\frac{3}{2}b(Ab-a)\right)}{(a+b\sec(c+dx))^{3/2}} dx}{3b(a^2-b^2)} \\
&= \frac{2a(Ab-aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b(a^2-b^2)d(a+b\sec(c+dx))^{3/2}} - \frac{2a(4Ab^3+3a^3B-7ab^2B)\sqrt{\sec(c+dx)}}{3b^2(a^2-b^2)^2d\sqrt{a+b\sec(c+dx)}} \\
&= \frac{2a(Ab-aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b(a^2-b^2)d(a+b\sec(c+dx))^{3/2}} - \frac{2a(4Ab^3+3a^3B-7ab^2B)\sqrt{\sec(c+dx)}}{3b^2(a^2-b^2)^2d\sqrt{a+b\sec(c+dx)}} \\
&= \frac{2a(Ab-aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b(a^2-b^2)d(a+b\sec(c+dx))^{3/2}} - \frac{2a(4Ab^3+3a^3B-7ab^2B)\sqrt{\sec(c+dx)}}{3b^2(a^2-b^2)^2d\sqrt{a+b\sec(c+dx)}} \\
&= \frac{2a(Ab-aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b(a^2-b^2)d(a+b\sec(c+dx))^{3/2}} - \frac{2a(4Ab^3+3a^3B-7ab^2B)\sqrt{\sec(c+dx)}}{3b^2(a^2-b^2)^2d\sqrt{a+b\sec(c+dx)}} \\
&= \frac{2B\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{b^2d\sqrt{a+b\sec(c+dx)}} + \frac{2a(Ab-aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b(a^2-b^2)d(a+b\sec(c+dx))^{3/2}} \\
&= \frac{2(Ab-aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{3b(a^2-b^2)d\sqrt{a+b\sec(c+dx)}} + \frac{2B\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}}{b^2d\sqrt{a+b\sec(c+dx)}}
\end{aligned}$$

Mathematica [C] time = 6.86, size = 726, normalized size = 1.82

$$\frac{\sec^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+b)^3\left(-\frac{2(aAb\sin(c+dx)-a^2B\sin(c+dx))}{3b(b^2-a^2)(a\cos(c+dx)+b)^2}-\frac{2(3a^4B\sin(c+dx)-7a^2b^2B\sin(c+dx)+4aAb^3\sin(c+dx))}{3b^2(b^2-a^2)^2(a\cos(c+dx)+b)}\right)}{d(a+b\sec(c+dx))^{5/2}} + \dots$$

Warning: Unable to verify antiderivative.

[In] Integrate[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(5/2), x]

[Out] ((b + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2)*((2*(2*a^2*A*b^2 + 6*A*b^4 + 4*a^3*b*B - 12*a*b^3*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c +

$d*x)/2, (2*a)/(a + b)]/Sqrt[b + a*Cos[c + d*x]] + (2*(4*a*A*b^3 + 9*a^4*B - 19*a^2*b^2*B + 6*b^4*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]/Sqrt[b + a*Cos[c + d*x]] + ((2*I)*(4*a*A*b^3 + 3*a^4*B - 7*a^2*b^2*B)*Sqrt[(a - a*Cos[c + d*x])/(a + b)]*Sqrt[(a + a*Cos[c + d*x])/(a - b)]*Cos[2*(c + d*x)]*(-2*b*(a + b)*EllipticE[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*(2*b*EllipticF[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*EllipticPi[1 - a/b, I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)]))*Sin[c + d*x])/(Sqrt[(a - b)^(-1)]*b*Sqrt[1 - Cos[c + d*x]^2]*Sqrt[(a^2 - a^2*Cos[c + d*x]^2)/a^2]*(-a^2 + 2*b^2 - 4*b*(b + a*Cos[c + d*x]) + 2*(b + a*Cos[c + d*x])^2)))/(6*(a - b)^2*b^2*(a + b)^2*d*(a + b*Sec[c + d*x])^(5/2)) + ((b + a*Cos[c + d*x])^3*Sec[c + d*x]^(5/2)*(-2*(a*A*b*Sin[c + d*x] - a^2*B*Sin[c + d*x]))/(3*b*(-a^2 + b^2)*(b + a*Cos[c + d*x])^2) - (2*(4*a*A*b^3*Sin[c + d*x] + 3*a^4*B*Sin[c + d*x] - 7*a^2*b^2*B*Sin[c + d*x]))/(3*b^2*(-a^2 + b^2)^2*(b + a*Cos[c + d*x])))/(d*(a + b*Sec[c + d*x])^(5/2))$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sec(dx + c)^{\frac{5}{2}}}{(b \sec(dx + c) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(5/2)/(b*sec(d*x + c) + a)^(5/2), x)

maple [C] time = 2.36, size = 5195, normalized size = 13.02

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x)`

[Out] result too large to display

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sec(dx + c)^{\frac{5}{2}}}{(b \sec(dx + c) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algorith="maxima")`

[Out] `integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(5/2)/(b*sec(d*x + c) + a)^(5/2), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(\frac{1}{\cos(c+dx)}\right)^{5/2}}{\left(a + \frac{b}{\cos(c+dx)}\right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + b/cos(c + d*x))^(5/2),x)`

[Out] `int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + b/cos(c + d*x))^(5/2), x)`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)**(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(5/2),x)`

[Out] Timed out

$$3.469 \quad \int \frac{\sec^3(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^{5/2}} dx$$

Optimal. Leaf size=329

$$\frac{2a(Ab - aB) \sin(c + dx) \sqrt{\sec(c + dx)}}{3bd(a^2 - b^2)(a + b \sec(c + dx))^{3/2}} - \frac{2(Ab - aB) \sqrt{\sec(c + dx)} \sqrt{\frac{a \cos(c + dx) + b}{a + b}} F\left(\frac{1}{2}(c + dx) \middle| \frac{2a}{a + b}\right)}{3ad(a^2 - b^2) \sqrt{a + b \sec(c + dx)}} - \frac{2(3a^2A - 4aB)}{3ad}$$

[Out] $2/3*a*(A*b-B*a)*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/b/(a^2-b^2)/d/(a+b*\sec(d*x+c))^{(3/2)}+2/3*(2*A*a^2*b+2*A*b^3+B*a^3-5*B*a*b^2)*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/b/(a^2-b^2)^2/d/(a+b*\sec(d*x+c))^{(1/2)}-2/3*(A*b-B*a)*(\cos(1/2*d*x+1/2*c))^2^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c),2^{(1/2)}*(a/(a+b))^{(1/2)})*((b+a*\cos(d*x+c))/(a+b))^{(1/2)}*\sec(d*x+c)^{(1/2)}/a/(a^2-b^2)/d/(a+b*\sec(d*x+c))^{(1/2)}-2/3*(3*A*a^2+A*b^2-4*B*a*b)*(\cos(1/2*d*x+1/2*c))^2^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c),2^{(1/2)}*(a/(a+b))^{(1/2)})*(a+b*\sec(d*x+c))^{(1/2)}/a/(a^2-b^2)^2/d/((b+a*\cos(d*x+c))/(a+b))^{(1/2)}/\sec(d*x+c)^{(1/2)}$

Rubi [A] time = 0.84, antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {4029, 4100, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2(2a^2Ab + a^3B - 5ab^2B + 2Ab^3) \sin(c + dx) \sqrt{\sec(c + dx)}}{3bd(a^2 - b^2)^2 \sqrt{a + b \sec(c + dx)}} + \frac{2a(Ab - aB) \sin(c + dx) \sqrt{\sec(c + dx)}}{3bd(a^2 - b^2)(a + b \sec(c + dx))^{3/2}} - \frac{2(Ab - aB)}{3ad}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(\text{Sec}[c + d*x])^{(3/2)}*(A + B*\text{Sec}[c + d*x])]/(a + b*\text{Sec}[c + d*x])^{(5/2)}, x]$

[Out] $(-2*(A*b - a*B)*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/(a + b)]*\text{EllipticF}[(c + d*x)/2, (2*a)/(a + b)]*\text{Sqrt}[\text{Sec}[c + d*x]])/(3*a*(a^2 - b^2)*d*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]) - (2*(3*a^2*A + A*b^2 - 4*a*b*B)*\text{EllipticE}[(c + d*x)/2, (2*a)/(a + b)]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]])/(3*a*(a^2 - b^2)^2*d*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/(a + b)]*\text{Sqrt}[\text{Sec}[c + d*x]]) + (2*a*(A*b - a*B)*\text{Sqrt}[\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*\text{Sec}[c + d*x])^{(3/2)}) + (2*(2*a^2*A*b + 2*A*b^3 + a^3*B - 5*a*b^2*B)*\text{Sqrt}[\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(3*b*(a^2 - b^2)^2*d*\text{Sqrt}[a + b*\text{Sec}[c + d*x]])$

Rule 2653

$\text{Int}[\text{Sqrt}[(a_) + (b_)*\sin[(c_) + (d_)*(x_)]], x_Symbol] \rightarrow \text{Simp}[(2*\text{Sqrt}[a + b]*\text{EllipticE}[(1*(c - \text{Pi}/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; \text{FreeQ}\{a,$

b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2655

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a + b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2661

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)]/(d*Sqrt[a + b]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2663

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b) + (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 3856

Int[Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)]/Sqrt[csc[(e_) + (f_)*(x_)]*(d_)], x_Symbol] := Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 3858

Int[Sqrt[csc[(e_) + (f_)*(x_)]*(d_)]/Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)], x_Symbol] := Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 4029

Int[(csc[(e_) + (f_)*(x_)]*(d_))^(n_)*(csc[(e_) + (f_)*(x_)]*(b_) + (a_))^(m_)*(csc[(e_) + (f_)*(x_)]*(B_) + (A_)), x_Symbol] := Simp[(a*d^2*(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[d/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(n - 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2*(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f,

$A, B\}, x] \&\& \text{NeQ}[A*b - a*B, 0] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{LtQ}[m, -1] \&\& \text{GtQ}[n, 1]$

Rule 4035

$\text{Int}[(\text{csc}[(e_{_}) + (f_{_})*(x_{_})]*(B_{_}) + (A_{_})) / (\text{Sqrt}[\text{csc}[(e_{_}) + (f_{_})*(x_{_})]*(d_{_})] * \text{Sqrt}[\text{csc}[(e_{_}) + (f_{_})*(x_{_})]*(b_{_}) + (a_{_})]), x_Symbol] \rightarrow \text{Dist}[A/a, \text{Int}[\text{Sqrt}[a + b*\text{Csc}[e + f*x]] / \text{Sqrt}[d*\text{Csc}[e + f*x]], x], x] - \text{Dist}[(A*b - a*B) / (a*d), \text{Int}[\text{Sqrt}[d*\text{Csc}[e + f*x]] / \text{Sqrt}[a + b*\text{Csc}[e + f*x]], x], x] /;$ $\text{FreeQ}\{a, b, d, e, f, A, B\}, x] \&\& \text{NeQ}[A*b - a*B, 0] \&\& \text{NeQ}[a^2 - b^2, 0]$

Rule 4100

$\text{Int}[(A_{_}) + \text{csc}[(e_{_}) + (f_{_})*(x_{_})]*(B_{_}) + \text{csc}[(e_{_}) + (f_{_})*(x_{_})]^2*(C_{_})] * (\text{csc}[(e_{_}) + (f_{_})*(x_{_})]*(d_{_}))^{n_{_}} * (\text{csc}[(e_{_}) + (f_{_})*(x_{_})]*(b_{_}) + (a_{_}))^{m_{_}}, x_Symbol] \rightarrow \text{Simp}[(A*b^2 - a*b*B + a^2*C)*\text{Cot}[e + f*x]*(a + b*\text{Csc}[e + f*x])^{m+1}*(d*\text{Csc}[e + f*x])^n] / (a*f*(m+1)*(a^2 - b^2)), x] + \text{Dist}[1/(a*(m+1)*(a^2 - b^2)), \text{Int}[(a + b*\text{Csc}[e + f*x])^{m+1}*(d*\text{Csc}[e + f*x])^n * \text{Simp}[a*(a*A - b*B + a*C)*(m+1) - (A*b^2 - a*b*B + a^2*C)*(m+n+1) - a*(A*b - a*B + b*C)*(m+1)*\text{Csc}[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m+n+2)*\text{Csc}[e + f*x]^2, x], x], x] /;$ $\text{FreeQ}\{a, b, d, e, f, A, B, C, n\}, x] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{LtQ}[m, -1] \&\& !(\text{ILtQ}[m + 1/2, 0] \&\& \text{ILtQ}[n, 0])$

Rubi steps

$$\begin{aligned}
\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{\frac{5}{2}}} dx &= \frac{2a(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{3b(a^2-b^2)d(a+b\sec(c+dx))^{\frac{3}{2}}} + \frac{2\int \frac{-\frac{1}{2}a(Ab-aB)-\frac{3}{2}b(Ab-aB)\sec(c+dx)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))} dx}{3b(a^2-b^2)d\sqrt{a+b\sec(c+dx)}} \\
&= \frac{2a(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{3b(a^2-b^2)d(a+b\sec(c+dx))^{\frac{3}{2}}} + \frac{2(2a^2Ab+2Ab^3+a^3B-5ab^2)}{3b(a^2-b^2)^2d\sqrt{a+b\sec(c+dx)}} \\
&= \frac{2a(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{3b(a^2-b^2)d(a+b\sec(c+dx))^{\frac{3}{2}}} + \frac{2(2a^2Ab+2Ab^3+a^3B-5ab^2)}{3b(a^2-b^2)^2d\sqrt{a+b\sec(c+dx)}} \\
&= \frac{2a(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{3b(a^2-b^2)d(a+b\sec(c+dx))^{\frac{3}{2}}} + \frac{2(2a^2Ab+2Ab^3+a^3B-5ab^2)}{3b(a^2-b^2)^2d\sqrt{a+b\sec(c+dx)}} \\
&= \frac{2a(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{3b(a^2-b^2)d(a+b\sec(c+dx))^{\frac{3}{2}}} + \frac{2(2a^2Ab+2Ab^3+a^3B-5ab^2)}{3b(a^2-b^2)^2d\sqrt{a+b\sec(c+dx)}} \\
&= \frac{2(Ab-aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{3a(a^2-b^2)d\sqrt{a+b\sec(c+dx)}} - \frac{2(3a^2A+3a^2B-4ab^2)}{3b(a^2-b^2)^2d\sqrt{a+b\sec(c+dx)}}
\end{aligned}$$

Mathematica [A] time = 2.31, size = 217, normalized size = 0.66

$$\frac{\sec^{\frac{5}{2}}(c+dx) \left(\frac{2\sin(c+dx)(a\cos(c+dx)+b)(a^3B+a(3a^2A-4abB+Ab^2))\cos(c+dx)+2a^2Ab-5ab^2B+2Ab^3}{(a^2-b^2)^2} - \frac{2(a+b)\left(\frac{a\cos(c+dx)+b}{a+b}\right)^{\frac{5}{2}}((3a^2A-4abB+Ab^2))}{3d(a+b\sec(c+dx))^{\frac{5}{2}}} \right)}{3d(a+b\sec(c+dx))^{\frac{5}{2}}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(5/2), x]

[Out] (Sec[c + d*x]^(5/2)*((-2*(a + b)*((b + a*Cos[c + d*x]))/(a + b))^(5/2)*((3*a^2*A + A*b^2 - 4*a*b*B)*EllipticE[(c + d*x)/2, (2*a)/(a + b)] - (a - b)*(-(A*b) + a*B)*EllipticF[(c + d*x)/2, (2*a)/(a + b)]))/(a*(a - b)^2) + (2*(b + a*Cos[c + d*x])*(2*a^2*A*b + 2*A*b^3 + a^3*B - 5*a*b^2*B + a*(3*a^2*A + A*b^2 - 4*a*b*B)*Cos[c + d*x])*Sin[c + d*x])/(a^2 - b^2)^2)/(3*d*(a + b*Sec[c + d*x])^(5/2))

fricas [F] time = 1.04, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B \sec(dx+c)^2 + A \sec(dx+c))\sqrt{b \sec(dx+c) + a} \sqrt{\sec(dx+c)}}{b^3 \sec(dx+c)^3 + 3ab^2 \sec(dx+c)^2 + 3a^2b \sec(dx+c) + a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")

[Out] integral((B*sec(d*x + c)^2 + A*sec(d*x + c))*sqrt(b*sec(d*x + c) + a)*sqrt(sec(d*x + c))/(b^3*sec(d*x + c)^3 + 3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx+c) + A) \sec(dx+c)^{\frac{3}{2}}}{(b \sec(dx+c) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^(5/2), x)

maple [B] time = 2.40, size = 3138, normalized size = 9.54

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x)

[Out] -2/3/d*(2*A*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), (-a+b)/(a-b))^(1/2))*cos(d*x+c)*a^2*b-3*B*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), (-a+b)/(a-b))^(1/2))*a*b^2*sin(d*x+c)-A*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), (-a+b)/(a-b))^(1/2))*cos(d*x+c)*a*b^2+3*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^2*b-3*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^2*b-A*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a*b^2+4*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^2*b-A*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2))

$2)/\sin(dx+c), (-\frac{a+b}{a-b})^{1/2}) * b^3 * (1/(1+\cos(dx+c)))^{1/2} * ((b+a*\cos(dx+c))/(1+\cos(dx+c)) / (a+b))^{1/2} * \sin(dx+c) - A * \cos(dx+c)^2 * \sin(dx+c) * \text{EllipticF}((-1+\cos(dx+c)) * ((a-b)/(a+b))^{1/2} / \sin(dx+c), (-\frac{a+b}{a-b})^{1/2}) * (1/(1+\cos(dx+c)))^{1/2} * ((b+a*\cos(dx+c))/(1+\cos(dx+c)) / (a+b))^{1/2} * a^2 * b + B * ((a-b)/(a+b))^{1/2} * \cos(dx+c)^2 * a^3 + 3 * A * ((a-b)/(a+b))^{1/2} * \cos(dx+c)^2 * a^3 - 3 * A * ((a-b)/(a+b))^{1/2} * \cos(dx+c) * a^3 + A * ((a-b)/(a+b))^{1/2} * \cos(dx+c) * b^3 - 3 * B * (1/(1+\cos(dx+c)))^{1/2} * ((b+a*\cos(dx+c))/(1+\cos(dx+c)) / (a+b))^{1/2} * \text{EllipticF}((-1+\cos(dx+c)) * ((a-b)/(a+b))^{1/2} / \sin(dx+c), (-\frac{a+b}{a-b})^{1/2}) * \cos(dx+c) * \sin(dx+c) * a * b^2 - 3 * A * \sin(dx+c) * \text{EllipticE}((-1+\cos(dx+c)) * ((a-b)/(a+b))^{1/2} / \sin(dx+c), (-\frac{a+b}{a-b})^{1/2}) * (1/(1+\cos(dx+c)))^{1/2} * ((b+a*\cos(dx+c))/(1+\cos(dx+c)) / (a+b))^{1/2} * \cos(dx+c) * a^2 * b - A * \sin(dx+c) * \text{EllipticE}((-1+\cos(dx+c)) * ((a-b)/(a+b))^{1/2} / \sin(dx+c), (-\frac{a+b}{a-b})^{1/2}) * (1/(1+\cos(dx+c)))^{1/2} * ((b+a*\cos(dx+c))/(1+\cos(dx+c)) / (a+b))^{1/2} * \cos(dx+c) * a * b^2 - 2 * B * \sin(dx+c) * (1/(1+\cos(dx+c)))^{1/2} * ((b+a*\cos(dx+c))/(1+\cos(dx+c)) / (a+b))^{1/2} * \text{EllipticF}((-1+\cos(dx+c)) * ((a-b)/(a+b))^{1/2} / \sin(dx+c), (-\frac{a+b}{a-b})^{1/2}) * \cos(dx+c) * a^2 * b + 4 * B * \sin(dx+c) * \text{EllipticE}((-1+\cos(dx+c)) * ((a-b)/(a+b))^{1/2} / \sin(dx+c), (-\frac{a+b}{a-b})^{1/2}) * (1/(1+\cos(dx+c)))^{1/2} * ((b+a*\cos(dx+c))/(1+\cos(dx+c)) / (a+b))^{1/2} * \cos(dx+c) * a^2 * b + 4 * B * \sin(dx+c) * \text{EllipticE}((-1+\cos(dx+c)) * ((a-b)/(a+b))^{1/2} / \sin(dx+c), (-\frac{a+b}{a-b})^{1/2}) * (1/(1+\cos(dx+c)))^{1/2} * ((b+a*\cos(dx+c))/(1+\cos(dx+c)) / (a+b))^{1/2} * \cos(dx+c) * a * b^2 - A * b^3 * ((a-b)/(a+b))^{1/2} - 2 * A * a^2 * b * ((a-b)/(a+b))^{1/2} + A * a * b^2 * ((a-b)/(a+b))^{1/2} - B * a^2 * b * ((a-b)/(a+b))^{1/2} + 4 * B * a * b^2 * ((a-b)/(a+b))^{1/2} - B * ((a-b)/(a+b))^{1/2} * a^3 - 4 * B * ((a-b)/(a+b))^{1/2} * \cos(dx+c) * a * b^2 - A * \cos(dx+c)^2 * \sin(dx+c) * (1/(1+\cos(dx+c)))^{1/2} * ((b+a*\cos(dx+c))/(1+\cos(dx+c)) / (a+b))^{1/2} * \text{EllipticE}((-1+\cos(dx+c)) * ((a-b)/(a+b))^{1/2} / \sin(dx+c), (-\frac{a+b}{a-b})^{1/2}) * a * b^2 - 3 * B * \cos(dx+c)^2 * \sin(dx+c) * \text{EllipticF}((-1+\cos(dx+c)) * ((a-b)/(a+b))^{1/2} / \sin(dx+c), (-\frac{a+b}{a-b})^{1/2}) * (1/(1+\cos(dx+c)))^{1/2} * ((b+a*\cos(dx+c))/(1+\cos(dx+c)) / (a+b))^{1/2} * a^2 * b + 4 * B * \cos(dx+c)^2 * \sin(dx+c) * (1/(1+\cos(dx+c)))^{1/2} * ((b+a*\cos(dx+c))/(1+\cos(dx+c)) / (a+b))^{1/2} * \text{EllipticE}((-1+\cos(dx+c)) * ((a-b)/(a+b))^{1/2} / \sin(dx+c), (-\frac{a+b}{a-b})^{1/2}) * a^2 * b - A * \cos(dx+c)^2 * ((a-b)/(a+b))^{1/2} * a^2 * b + 3 * A * \sin(dx+c) * (1/(1+\cos(dx+c)))^{1/2} * ((b+a*\cos(dx+c))/(1+\cos(dx+c)) / (a+b))^{1/2} * \text{EllipticF}((-1+\cos(dx+c)) * ((a-b)/(a+b))^{1/2} / \sin(dx+c), (-\frac{a+b}{a-b})^{1/2}) * \cos(dx+c) * a^3 - 3 * A * \sin(dx+c) * \text{EllipticE}((-1+\cos(dx+c)) * ((a-b)/(a+b))^{1/2} / \sin(dx+c), (-\frac{a+b}{a-b})^{1/2}) * (1/(1+\cos(dx+c)))^{1/2} * ((b+a*\cos(dx+c))/(1+\cos(dx+c)) / (a+b))^{1/2} * \cos(dx+c) * a^3 - A * \sin(dx+c) * \text{EllipticE}((-1+\cos(dx+c)) * ((a-b)/(a+b))^{1/2} / \sin(dx+c), (-\frac{a+b}{a-b})^{1/2}) * (1/(1+\cos(dx+c)))^{1/2} * ((b+a*\cos(dx+c))/(1+\cos(dx+c)) / (a+b))^{1/2} * \cos(dx+c) * b^3 + B * \sin(dx+c) * (1/(1+\cos(dx+c)))^{1/2} * ((b+a*\cos(dx+c))/(1+\cos(dx+c)) / (a+b))^{1/2} * \text{EllipticF}((-1+\cos(dx+c)) * ((a-b)/(a+b))^{1/2} / \sin(dx+c), (-\frac{a+b}{a-b})^{1/2}) * \cos(dx+c) * a^3 + 3 * A * \text{EllipticF}((-1+\cos(dx+c)) * ((a-b)/(a+b))^{1/2} / \sin(dx+c), (-\frac{a+b}{a-b})^{1/2}) * a^2 * b * (1/(1+\cos(dx+c)))^{1/2} * ((b+a*\cos(dx+c))/(1+\cos(dx+c)) / (a+b))^{1/2} * \sin(dx+c) - A * \text{EllipticF}((-1+\cos(dx+c)) * ((a-b)/(a+b))^{1/2} / \sin(dx+c), (-\frac{a+b}{a-b})^{1/2}) * a * b^2 * (1/(1+\cos(dx+c)))^{1/2} * ((b+a*\cos(dx+c))/(1+\cos(dx+c)) / (a+b))^{1/2}$

```

*x+c))/(a+b))^(1/2)*sin(d*x+c)-3*A*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(
(1/2)/sin(d*x+c), (-a+b)/(a-b))^(1/2))*a^2*b*(1/(1+cos(d*x+c)))^(1/2)*((b+a
*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*sin(d*x+c)+B*EllipticF((-1+cos(d*x
+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), (-a+b)/(a-b))^(1/2))*a^2*b*(1/(1+cos(d
*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*sin(d*x+c)+4*B*
EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), (-a+b)/(a-b))^(1/
2))*a*b^2*(1/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(
(1/2)*sin(d*x+c)-3*A*cos(d*x+c)^2*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*((b+a
*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c), (-a+b)/(a-b))^(1/2))*a^3+3*A*cos(d*x+c)^2*sin(d*x+c
)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), (-a+b)/(a-b))^(
1/2))*(1/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2
)*a^3+B*cos(d*x+c)^2*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/
2)/sin(d*x+c), (-a+b)/(a-b))^(1/2))*(1/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+
c))/(1+cos(d*x+c)))/(a+b))^(1/2)*a^3*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*co
s(d*x+c)^2*(1/cos(d*x+c))^(3/2)/sin(d*x+c)/(b+a*cos(d*x+c))^2/(a-b)/(a+b)^2
/a/((a-b)/(a+b))^(1/2)

```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sec(dx + c)^{\frac{3}{2}}}{(b \sec(dx + c) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```

[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algor
ithm="maxima")

```

```

[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^(5/2
), x)

```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(\frac{1}{\cos(c+dx)}\right)^{3/2}}{\left(a + \frac{b}{\cos(c+dx)}\right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```

[In] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + b/cos(c + d*x))^(5/2
),x)

```

```

[Out] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + b/cos(c + d*x))^(5/2
), x)

```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(5/2),x)

[Out] Timed out

$$3.470 \quad \int \frac{\sqrt{\sec(c+dx)} (A+B \sec(c+dx))}{(a+b \sec(c+dx))^{5/2}} dx$$

Optimal. Leaf size=346

$$\frac{2(Ab - aB) \sin(c + dx) \sqrt{\sec(c + dx)}}{3d (a^2 - b^2) (a + b \sec(c + dx))^{3/2}} + \frac{2(3a^2 A - abB - 2Ab^2) \sqrt{\sec(c + dx)} \sqrt{\frac{a \cos(c+dx)+b}{a+b}} F\left(\frac{1}{2}(c + dx) \middle| \frac{2a}{a+b}\right)}{3a^2 d (a^2 - b^2) \sqrt{a + b \sec(c + dx)}}$$

[Out] $-2/3*(A*b-B*a)*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/(a^2-b^2)/d/(a+b*\sec(d*x+c))^{(3/2)}-2/3*(5*A*a^2*b-A*b^3-2*B*a^3-2*B*a*b^2)*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/a/(a^2-b^2)^{2/d}/(a+b*\sec(d*x+c))^{(1/2)}+2/3*(3*A*a^2-2*A*b^2-B*a*b)*(\cos(1/2*d*x+1/2*c))^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)}*(a/(a+b))^{(1/2)}*((b+a*\cos(d*x+c))/(a+b))^{(1/2)}*\sec(d*x+c)^{(1/2)}/a^2/(a^2-b^2))/d/(a+b*\sec(d*x+c))^{(1/2)}+2/3*(6*A*a^2*b-2*A*b^3-3*B*a^3-B*a*b^2)*(\cos(1/2*d*x+1/2*c))^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)}*(a/(a+b))^{(1/2)}*(a+b*\sec(d*x+c))^{(1/2)}/a^2/(a^2-b^2)^{2/d}/((b+a*\cos(d*x+c))/(a+b))^{(1/2)}/\sec(d*x+c)^{(1/2)})$

Rubi [A] time = 0.82, antiderivative size = 346, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {4027, 4100, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2(5a^2 Ab - 2a^3 B - 2ab^2 B - Ab^3) \sin(c + dx) \sqrt{\sec(c + dx)}}{3ad (a^2 - b^2)^2 \sqrt{a + b \sec(c + dx)}} - \frac{2(Ab - aB) \sin(c + dx) \sqrt{\sec(c + dx)}}{3d (a^2 - b^2) (a + b \sec(c + dx))^{3/2}} + \frac{2(3a^2 A - abB - 2Ab^2) \sqrt{\sec(c + dx)} \sqrt{\frac{a \cos(c+dx)+b}{a+b}} F\left(\frac{1}{2}(c + dx) \middle| \frac{2a}{a+b}\right)}{3a^2 d (a^2 - b^2) \sqrt{a + b \sec(c + dx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(\text{Sqrt}[\text{Sec}[c + d*x]]*(A + B*\text{Sec}[c + d*x]))/(a + b*\text{Sec}[c + d*x])^{(5/2)}, x]$

[Out] $(2*(3*a^2*A - 2*A*b^2 - a*b*B)*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/(a + b)]*\text{EllipticF}[(c + d*x)/2, (2*a)/(a + b)]*\text{Sqrt}[\text{Sec}[c + d*x]])/(3*a^2*(a^2 - b^2)*d*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]) + (2*(6*a^2*A*b - 2*A*b^3 - 3*a^3*B - a*b^2*B)*\text{EllipticE}[(c + d*x)/2, (2*a)/(a + b)]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]])/(3*a^2*(a^2 - b^2)^{2*d*\text{Sqrt}}[(b + a*\text{Cos}[c + d*x])/(a + b)]*\text{Sqrt}[\text{Sec}[c + d*x]]) - (2*(A*b - a*B)*\text{Sqrt}[\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(3*(a^2 - b^2)*d*(a + b*\text{Sec}[c + d*x])^{(3/2)}) - (2*(5*a^2*A*b - A*b^3 - 2*a^3*B - 2*a*b^2*B)*\text{Sqrt}[\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(3*a*(a^2 - b^2)^{2*d*\text{Sqrt}}[a + b*\text{Sec}[c + d*x]])$

Rule 2653

$\text{Int}[\text{Sqrt}[(a_) + (b_)*\sin[(c_) + (d_)*(x_)]], x_Symbol] \rightarrow \text{Simp}[(2*\text{Sqrt}[a + b]*\text{EllipticE}[(1*(c - \text{Pi}/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; \text{FreeQ}\{a,$

b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2655

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a + b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2661

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)]/(d*Sqrt[a + b])), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2663

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b) + (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 3856

Int[Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)]/Sqrt[csc[(e_) + (f_)*(x_)]*(d_)], x_Symbol] := Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 3858

Int[Sqrt[csc[(e_) + (f_)*(x_)]*(d_)]/Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)], x_Symbol] := Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 4027

Int[(csc[(e_) + (f_)*(x_)]*(d_))^(n_)*(csc[(e_) + (f_)*(x_)]*(b_) + (a_))^(m_)*(csc[(e_) + (f_)*(x_)]*(B_) + (A_)), x_Symbol] := -Simp[(d*(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*Simp[d*(n - 1)*(A*b - a*B) + d*(a*A - b*B)*(m + 1)*Csc[e + f*x] - d*(A*b - a*B)*(m + n + 1)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && Ne

$Q[a^2 - b^2, 0]$ && LtQ[m, -1] && LtQ[0, n, 1]

Rule 4035

Int[(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.))/(Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]), x_Symbol] :> Dist[A/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]

Rule 4100

Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m, x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\sec(c+dx)}(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{5/2}} dx &= -\frac{2(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{3(a^2-b^2)d(a+b\sec(c+dx))^{3/2}} - \frac{2\int \frac{\frac{1}{2}(-Ab+aB)-\frac{3}{2}(aA-bB)\sec(c+dx)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))} dx}{3(a^2-b^2)} \\
&= -\frac{2(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{3(a^2-b^2)d(a+b\sec(c+dx))^{3/2}} - \frac{2(5a^2Ab-Ab^3-2a^3B-2ab^2)}{3a(a^2-b^2)^2d\sqrt{a+b\sec(c+dx)}} \\
&= -\frac{2(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{3(a^2-b^2)d(a+b\sec(c+dx))^{3/2}} - \frac{2(5a^2Ab-Ab^3-2a^3B-2ab^2)}{3a(a^2-b^2)^2d\sqrt{a+b\sec(c+dx)}} \\
&= -\frac{2(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{3(a^2-b^2)d(a+b\sec(c+dx))^{3/2}} - \frac{2(5a^2Ab-Ab^3-2a^3B-2ab^2)}{3a(a^2-b^2)^2d\sqrt{a+b\sec(c+dx)}} \\
&= -\frac{2(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{3(a^2-b^2)d(a+b\sec(c+dx))^{3/2}} - \frac{2(5a^2Ab-Ab^3-2a^3B-2ab^2)}{3a(a^2-b^2)^2d\sqrt{a+b\sec(c+dx)}} \\
&= \frac{2(3a^2A-2Ab^2-abB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{3a^2(a^2-b^2)d\sqrt{a+b\sec(c+dx)}}
\end{aligned}$$

Mathematica [A] time = 2.21, size = 245, normalized size = 0.71

$$\frac{2\sec^2(c+dx)(a\cos(c+dx)+b)\left(\frac{a\sin(c+dx)(a(3a^3B-6a^2Ab+ab^2B+2Ab^3)\cos(c+dx)+b(2a^3B-5a^2Ab+2ab^2B+Ab^3))}{(a^2-b^2)^2} - \frac{(a\cos(c+dx)+b)^3}{a+b}\right)}{3a^2d(a+b\sec(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(5/2), x]

[Out] (2*(b + a*Cos[c + d*x])*Sec[c + d*x]^(5/2)*(-((((b + a*Cos[c + d*x])/(a + b))^(3/2))*((-6*a^2*A*b + 2*A*b^3 + 3*a^3*B + a*b^2*B)*EllipticE[(c + d*x)/2, (2*a)/(a + b)] - (a - b)*(3*a^2*A - 2*A*b^2 - a*b*B)*EllipticF[(c + d*x)/2, (2*a)/(a + b)])))/(a - b)^2 + (a*(b*(-5*a^2*A*b + A*b^3 + 2*a^3*B + 2*a*b^2*B) + a*(-6*a^2*A*b + 2*A*b^3 + 3*a^3*B + a*b^2*B)*Cos[c + d*x])*Sin[c + d*x])/(a^2 - b^2)^2)/(3*a^2*d*(a + b*Sec[c + d*x])^(5/2))

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B \sec(dx+c) + A)\sqrt{b \sec(dx+c) + a} \sqrt{\sec(dx+c)}}{b^3 \sec(dx+c)^3 + 3ab^2 \sec(dx+c)^2 + 3a^2b \sec(dx+c) + a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")

[Out] integral((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*sqrt(sec(d*x + c))/(b^3*sec(d*x + c)^3 + 3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx+c) + A)\sqrt{\sec(dx+c)}}{(b \sec(dx+c) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(b*sec(d*x + c) + a)^(5/2), x)

maple [B] time = 2.25, size = 3857, normalized size = 11.15

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x)

[Out] 2/3/d*(-6*A*cos(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), (-a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b)^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*a^3*b+2*A*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), (-a+b)/(a-b))^(1/2))*b^4*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b)^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-5*A*a^2*b^2*((a-b)/(a+b))^(1/2)+A*a*b^3*((a-b)/(a+b))^(1/2)+2*B*a^3*b*((a-b)/(a+b))^(1/2)-B*a^2*b^2*((a-b)/(a+b))^(1/2)+B*a*b^3*((a-b)/(a+b))^(1/2)+2*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b)^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), (-a+b)/(a-b))^(1/2))*cos(d*x+c)^2*a^2*b^2+2*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b)^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/


```

EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), (- (a+b)/(a-b))^(1/2))
*cos(d*x+c)^2*a^4-3*B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))
^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)
/sin(d*x+c), (- (a+b)/(a-b))^(1/2))*cos(d*x+c)^2*a^4-6*A*cos(d*x+c)*EllipticE
((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), (- (a+b)/(a-b))^(1/2))*((b
+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x
+c)*a^2*b^2+2*A*cos(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c), (- (a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2
)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*a*b^3+5*A*cos(d*x+c)*((b+a*cos(d*x+c)
)/(1+cos(d*x+c)))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*
x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), (- (a+b)/(a-b))^(1/2))*sin(d*x+c)*a^2*b
^2+2*A*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*(1/(1+cos(d
*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), (- (a
+b)/(a-b))^(1/2))*sin(d*x+c)*a*b^3+3*B*cos(d*x+c)*EllipticE((-1+cos(d*x+c))
*((a-b)/(a+b))^(1/2)/sin(d*x+c), (- (a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+
cos(d*x+c)))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*a^3*b+B*cos(d*
x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), (- (a+b)/(a-b)
))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(
1/2)*sin(d*x+c)*a^2*b^2+B*cos(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b)
))^(1/2)/sin(d*x+c), (- (a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(
a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*a*b^3-2*B*cos(d*x+c)*((b+a*
cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), (- (a+b)/(a-b))^(1/2))*sin(d*
x+c)*a^3*b+B*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*(1/(1
+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
), (- (a+b)/(a-b))^(1/2))*sin(d*x+c)*a^2*b^2+6*A*cos(d*x+c)^2*((a-b)/(a+b))^(
1/2)*a^3*b-3*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^3-6*A*cos(d*x+c)*((a-b)
)/(a+b))^(1/2)*a^3*b+6*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b^2+2*A*cos(d*x+
c)*((a-b)/(a+b))^(1/2)*a*b^3-3*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3*b+B*cos
(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b^2-B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^3)
*cos(d*x+c)*(1/cos(d*x+c))^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*
x+c)/(b+a*cos(d*x+c))^2/(a-b)/(a+b)^2/((a-b)/(a+b))^(1/2)/a^2

```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sqrt{\sec(dx + c)}}{(b \sec(dx + c) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2), x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(b*sec(d*x + c) + a)^(5/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \sqrt{\frac{1}{\cos(c+dx)}}}{\left(a + \frac{b}{\cos(c+dx)}\right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + b/cos(c + d*x))^(5/2), x)

[Out] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + b/cos(c + d*x))^(5/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)**(1/2)/(a+b*sec(d*x+c))**(5/2), x)

[Out] Timed out

$$3.471 \quad \int \frac{A+B \sec(c+dx)}{\sqrt{\sec(c+dx)} (a+b \sec(c+dx))^{5/2}} dx$$

Optimal. Leaf size=368

$$\frac{2b(Ab - aB) \sin(c + dx) \sqrt{\sec(c + dx)}}{3ad(a^2 - b^2)(a + b \sec(c + dx))^{3/2}} + \frac{2b(-5a^3B + 8a^2Ab + ab^2B - 4Ab^3) \sin(c + dx) \sqrt{\sec(c + dx)}}{3a^2d(a^2 - b^2)^2 \sqrt{a + b \sec(c + dx)}} - \frac{2(-3a^3B + 2a^2Ab + ab^2B - 4Ab^3) \sin(c + dx) \sqrt{\sec(c + dx)}}{3ad(a^2 - b^2)(a + b \sec(c + dx))^{3/2}}$$

[Out] $2/3*b*(A*b-B*a)*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/a/(a^2-b^2)/d/(a+b*\sec(d*x+c))^{(3/2)}+2/3*b*(8*A*a^2*b-4*A*b^3-5*B*a^3+B*a*b^2)*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/a^2/(a^2-b^2)^2/d/(a+b*\sec(d*x+c))^{(1/2)}-2/3*(9*A*a^2*b-8*A*b^3-3*B*a^3+2*B*a*b^2)*(\cos(1/2*d*x+1/2*c))^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)}*(a/(a+b))^{(1/2)})*((b+a*\cos(d*x+c))/(a+b))^{(1/2)}*\sec(d*x+c)^{(1/2)}/a^3/(a^2-b^2)/d/(a+b*\sec(d*x+c))^{(1/2)}+2/3*(3*A*a^4-15*A*a^2*b^2+8*A*b^4+6*B*a^3*b-2*B*a*b^3)*(\cos(1/2*d*x+1/2*c))^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)}*(a/(a+b))^{(1/2)})*(a+b*\sec(d*x+c))^{(1/2)}/a^3/(a^2-b^2)^2/d/((b+a*\cos(d*x+c))/(a+b))^{(1/2)}/\sec(d*x+c)^{(1/2)}$

Rubi [A] time = 0.94, antiderivative size = 368, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {4030, 4100, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2b(8a^2Ab - 5a^3B + ab^2B - 4Ab^3) \sin(c + dx) \sqrt{\sec(c + dx)}}{3a^2d(a^2 - b^2)^2 \sqrt{a + b \sec(c + dx)}} + \frac{2b(Ab - aB) \sin(c + dx) \sqrt{\sec(c + dx)}}{3ad(a^2 - b^2)(a + b \sec(c + dx))^{3/2}} - \frac{2(9a^2Ab - 5a^3B + ab^2B - 4Ab^3) \sin(c + dx) \sqrt{\sec(c + dx)}}{3ad(a^2 - b^2)(a + b \sec(c + dx))^{3/2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(A + B*\text{Sec}[c + d*x])]/(\text{Sqrt}[\text{Sec}[c + d*x]]*(a + b*\text{Sec}[c + d*x])^{(5/2)}), x]$

[Out] $(-2*(9*a^2*A*b - 8*A*b^3 - 3*a^3*B + 2*a*b^2*B)*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])]/(a + b))*\text{EllipticF}[(c + d*x)/2, (2*a)/(a + b)]*\text{Sqrt}[\text{Sec}[c + d*x]]/(3*a^3*(a^2 - b^2)*d*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]) + (2*(3*a^4*A - 15*a^2*A*b^2 + 8*A*b^4 + 6*a^3*b*B - 2*a*b^3*B)*\text{EllipticE}[(c + d*x)/2, (2*a)/(a + b)]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]])/(3*a^3*(a^2 - b^2)^2*d*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])]/(a + b))*\text{Sqrt}[\text{Sec}[c + d*x]] + (2*b*(A*b - a*B)*\text{Sqrt}[\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*\text{Sec}[c + d*x])^{(3/2)}) + (2*b*(8*a^2*A*b - 4*A*b^3 - 5*a^3*B + a*b^2*B)*\text{Sqrt}[\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(3*a^2*(a^2 - b^2)^2*d*\text{Sqrt}[a + b*\text{Sec}[c + d*x]])$

Rule 2653

$\text{Int}[\text{Sqrt}[(a_) + (b_)*\sin[(c_) + (d_)*(x_)]], x_Symbol] \rightarrow \text{Simp}[(2*\text{Sqrt}[a + b]*\text{EllipticE}[(1*(c - \text{Pi}/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; \text{FreeQ}\{a,$

b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2655

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a + b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2661

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)]/(d*Sqrt[a + b]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2663

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b) + (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 3856

Int[Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)]/Sqrt[csc[(e_) + (f_)*(x_)]*(d_)], x_Symbol] := Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 3858

Int[Sqrt[csc[(e_) + (f_)*(x_)]*(d_)]/Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)], x_Symbol] := Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 4030

Int[(csc[(e_) + (f_)*(x_)]*(d_))^(n_)*(csc[(e_) + (f_)*(x_)]*(b_) + (a_))^(m_)*(csc[(e_) + (f_)*(x_)]*(B_) + (A_)), x_Symbol] := Simp[(b*(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[A*(a^2*(m + 1) - b^2*(m + n + 1)) + a*b*B*n - a*(A*b - a*B)*(m + 1)*Csc[e + f*x] + b*(A*b - a*B)*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b

- a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])

Rule 4035

Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)]), x_Symbol] := Dist[A/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]

Rule 4100

Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^m, x_Symbol] := Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])

Rubi steps

$$\begin{aligned}
\int \frac{A + B \sec(c + dx)}{\sqrt{\sec(c + dx)} (a + b \sec(c + dx))^{5/2}} dx &= \frac{2b(Ab - aB)\sqrt{\sec(c + dx)} \sin(c + dx)}{3a(a^2 - b^2)d(a + b \sec(c + dx))^{3/2}} - \frac{2 \int \frac{\frac{1}{2}(-3a^2A + 4Ab^2 - abB) + \frac{3}{2}a(Ab - aB)}{\sqrt{\sec(c + dx)}} dx}{3a(a^2 - b^2)d(a + b \sec(c + dx))^{3/2}} \\
&= \frac{2b(Ab - aB)\sqrt{\sec(c + dx)} \sin(c + dx)}{3a(a^2 - b^2)d(a + b \sec(c + dx))^{3/2}} + \frac{2b(8a^2Ab - 4Ab^3 - 5a^3B + 2ab^2B)}{3a^2(a^2 - b^2)^2d\sqrt{a + b \sec(c + dx)}} \\
&= \frac{2b(Ab - aB)\sqrt{\sec(c + dx)} \sin(c + dx)}{3a(a^2 - b^2)d(a + b \sec(c + dx))^{3/2}} + \frac{2b(8a^2Ab - 4Ab^3 - 5a^3B + 2ab^2B)}{3a^2(a^2 - b^2)^2d\sqrt{a + b \sec(c + dx)}} \\
&= \frac{2b(Ab - aB)\sqrt{\sec(c + dx)} \sin(c + dx)}{3a(a^2 - b^2)d(a + b \sec(c + dx))^{3/2}} + \frac{2b(8a^2Ab - 4Ab^3 - 5a^3B + 2ab^2B)}{3a^2(a^2 - b^2)^2d\sqrt{a + b \sec(c + dx)}} \\
&= \frac{2b(Ab - aB)\sqrt{\sec(c + dx)} \sin(c + dx)}{3a(a^2 - b^2)d(a + b \sec(c + dx))^{3/2}} + \frac{2b(8a^2Ab - 4Ab^3 - 5a^3B + 2ab^2B)}{3a^2(a^2 - b^2)^2d\sqrt{a + b \sec(c + dx)}} \\
&= \frac{2(9a^2Ab - 8Ab^3 - 3a^3B + 2ab^2B)\sqrt{\frac{b+a \cos(c+dx)}{a+b}} F\left(\frac{1}{2}(c + dx) \middle| \frac{2a}{a+b}\right)}{3a^3(a^2 - b^2)d\sqrt{a + b \sec(c + dx)}}
\end{aligned}$$

Mathematica [A] time = 2.69, size = 297, normalized size = 0.81

$$2 \sec^{\frac{5}{2}}(c + dx)(a \cos(c + dx) + b) \left(-\frac{ab \sin(c + dx)(a(6a^3B - 9a^2Ab - 2ab^2B + 5Ab^3) \cos(c + dx) + b(5a^3B - 8a^2Ab - ab^2B + 4Ab^3))}{(a^2 - b^2)^2} - \frac{\left(\frac{a \cos(c + dx) + b}{a + b}\right)}{3a^3d(a + b \sec(c + dx))} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sec[c + d*x])/(Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(5/2)), x]

[Out] (2*(b + a*Cos[c + d*x])*Sec[c + d*x]^(5/2)*(-((((b + a*Cos[c + d*x])/(a + b))^(3/2)*(-(a^2*(-6*a^2*A*b + 2*A*b^3 + 3*a^3*B + a*b^2*B)*EllipticF[(c + d*x)/2, (2*a)/(a + b)]) - (3*a^4*A - 15*a^2*A*b^2 + 8*A*b^4 + 6*a^3*b*B - 2*a*b^3*B)*((a + b)*EllipticE[(c + d*x)/2, (2*a)/(a + b)] - b*EllipticF[(c + d*x)/2, (2*a)/(a + b)]))))/(a - b)^2*(a + b))) - (a*b*(b*(-8*a^2*A*b + 4*A*b^3 + 5*a^3*B - a*b^2*B) + a*(-9*a^2*A*b + 5*A*b^3 + 6*a^3*B - 2*a*b^2*B))*

os[c + d*x])*Sin[c + d*x])/(a^2 - b^2)^2)/(3*a^3*d*(a + b*Sec[c + d*x])^(5/2))

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B \sec(dx + c) + A)\sqrt{b \sec(dx + c) + a} \sqrt{\sec(dx + c)}}{b^3 \sec(dx + c)^4 + 3ab^2 \sec(dx + c)^3 + 3a^2b \sec(dx + c)^2 + a^3 \sec(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")

[Out] integral((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*sqrt(sec(d*x + c))/(b^3*sec(d*x + c)^4 + 3*a*b^2*sec(d*x + c)^3 + 3*a^2*b*sec(d*x + c)^2 + a^3*sec(d*x + c)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(b \sec(dx + c) + a)^{\frac{5}{2}} \sqrt{\sec(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^(5/2)*sqrt(sec(d*x + c))), x)

maple [B] time = 2.44, size = 5169, normalized size = 14.05

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x)

[Out] result too large to display

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\left(a + \frac{b}{\cos(c+dx)}\right)^{5/2} \sqrt{\frac{1}{\cos(c+dx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(1/2)),x)

[Out] int((A + B/cos(c + d*x))/((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(1/2)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(5/2)/sec(d*x+c)**(1/2),x)

[Out] Timed out

$$3.472 \quad \int \frac{A+B \sec(c+dx)}{\sec^2(c+dx)(a+b \sec(c+dx))^{5/2}} dx$$

Optimal. Leaf size=472

$$\frac{2b(Ab - aB) \sin(c + dx)}{3ad(a^2 - b^2) \sqrt{\sec(c + dx)} (a + b \sec(c + dx))^{3/2}} + \frac{2b(-7a^3B + 10a^2Ab + 3ab^2B - 6Ab^3) \sin(c + dx)}{3a^2d(a^2 - b^2)^2 \sqrt{\sec(c + dx)} \sqrt{a + b \sec(c + dx)}} + \frac{2(a^4A + b^4B) \sin(c + dx)}{3a^3d(a^2 - b^2) \sqrt{\sec(c + dx)} \sqrt{a + b \sec(c + dx)}}$$

[Out] $2/3*b*(A*b-B*a)*\sin(d*x+c)/a/(a^2-b^2)/d/(a+b*\sec(d*x+c))^(3/2)/\sec(d*x+c)^(1/2)+2/3*b*(10*A*a^2*b-6*A*b^3-7*B*a^3+3*B*a*b^2)*\sin(d*x+c)/a^2/(a^2-b^2)^2/d/\sec(d*x+c)^(1/2)/(a+b*\sec(d*x+c))^(1/2)+2/3*(A*a^4+16*A*a^2*b^2-16*A*b^4-9*B*a^3*b+8*B*a*b^3)*(\cos(1/2*d*x+1/2*c))^2^(1/2)/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^(1/2)*(a/(a+b))^(1/2))*((b+a*\cos(d*x+c))/(a+b))^(1/2)*\sec(d*x+c)^(1/2)/a^4/(a^2-b^2)/d/(a+b*\sec(d*x+c))^(1/2)+2/3*(A*a^4-13*A*a^2*b^2+8*A*b^4+8*B*a^3*b-4*B*a*b^3)*\sin(d*x+c)*(a+b*\sec(d*x+c))^(1/2)/a^3/(a^2-b^2)^2/d/\sec(d*x+c)^(1/2)-2/3*(8*A*a^4*b-28*A*a^2*b^3+16*A*b^5-3*B*a^5+15*B*a^3*b^2-8*B*a*b^4)*(\cos(1/2*d*x+1/2*c))^2^(1/2)/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^(1/2)*(a/(a+b))^(1/2))*(a+b*\sec(d*x+c))^(1/2)/a^4/(a^2-b^2)^2/d/((b+a*\cos(d*x+c))/(a+b))^(1/2)/\sec(d*x+c)^(1/2)$

Rubi [A] time = 1.41, antiderivative size = 472, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {4030, 4100, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2(-13a^2Ab^2 + a^4A + 8a^3bB - 4ab^3B + 8Ab^4) \sin(c + dx) \sqrt{a + b \sec(c + dx)}}{3a^3d(a^2 - b^2)^2 \sqrt{\sec(c + dx)}} + \frac{2b(10a^2Ab - 7a^3B + 3ab^2B - 6Ab^3) \sin(c + dx)}{3a^2d(a^2 - b^2)^2 \sqrt{\sec(c + dx)} \sqrt{a + b \sec(c + dx)}} + \frac{2(a^4A + b^4B) \sin(c + dx)}{3a^3d(a^2 - b^2) \sqrt{\sec(c + dx)} \sqrt{a + b \sec(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(5/2)), x]

[Out] $(2*(a^4*A + 16*a^2*A*b^2 - 16*A*b^4 - 9*a^3*b*B + 8*a*b^3*B)*\text{Sqrt}[(b + a*\cos[c + d*x])/(a + b)]*\text{EllipticF}[(c + d*x)/2, (2*a)/(a + b)]*\text{Sqrt}[\sec[c + d*x]])/(3*a^4*(a^2 - b^2)*d*\text{Sqrt}[a + b*\sec[c + d*x]]) - (2*(8*a^4*A*b - 28*a^2*A*b^3 + 16*A*b^5 - 3*a^5*B + 15*a^3*b^2*B - 8*a*b^4*B)*\text{EllipticE}[(c + d*x)/2, (2*a)/(a + b)]*\text{Sqrt}[a + b*\sec[c + d*x]])/(3*a^4*(a^2 - b^2)^2*d*\text{Sqrt}[(b + a*\cos[c + d*x])/(a + b)]*\text{Sqrt}[\sec[c + d*x]]) + (2*b*(A*b - a*B)*\sin[c + d*x])/(3*a*(a^2 - b^2)*d*\text{Sqrt}[\sec[c + d*x]]*(a + b*\sec[c + d*x])^(3/2)) + (2*b*(10*a^2*A*b - 6*A*b^3 - 7*a^3*B + 3*a*b^2*B)*\sin[c + d*x])/(3*a^2*(a^2 - b^2)^2*d*\text{Sqrt}[\sec[c + d*x]]*\text{Sqrt}[a + b*\sec[c + d*x]]) + (2*(a^4*A - 13*a^2*A*b^2 + a^4A + 8a^3bB - 4ab^3B + 8Ab^4) \sin(c + dx) \sqrt{a + b \sec(c + dx)})/(3a^3d(a^2 - b^2)^2 \sqrt{\sec(c + dx)}) + (2b(10a^2Ab - 7a^3B + 3ab^2B - 6Ab^3) \sin(c + dx))/(3a^2d(a^2 - b^2)^2 \sqrt{\sec(c + dx)} \sqrt{a + b \sec(c + dx)}) + (2(a^4A + b^4B) \sin(c + dx))/(3a^3d(a^2 - b^2) \sqrt{\sec(c + dx)} \sqrt{a + b \sec(c + dx)})$

$$2A*b^2 + 8A*b^4 + 8*a^3*b*B - 4*a*b^3*B)*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]*\text{Sin}[c + d*x]]/(3*a^3*(a^2 - b^2)^2*d*\text{Sqrt}[\text{Sec}[c + d*x]])$$
Rule 2653

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*Sqrt[a + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a + b*SIN[c + d*x]]/Sqrt[(a + b*SIN[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b*SIN[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a + b*SIN[c + d*x])/(a + b)]/Sqrt[a + b*SIN[c + d*x]], Int[1/Sqrt[a/(a + b) + (b*SIN[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)]/Sqrt[csc[(e_) + (f_)*(x_)]*(d_)], x_Symbol] := Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*SIN[e + f*x]]), Int[Sqrt[b + a*SIN[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_) + (f_)*(x_)]*(d_)]/Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)], x_Symbol] := Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*SIN[e + f*x]])/Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*SIN[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4030

```

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (
a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := Simp[(b*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*
(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e
+ f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[A*(a^2*(m + 1) - b^2*(m + n + 1))
+ a*b*B*n - a*(A*b - a*B)*(m + 1)*Csc[e + f*x] + b*(A*b - a*B)*(m + n + 2)*
Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b
- a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && !(ILtQ[m + 1/2, 0] && ILt
Q[n, 0])

```

Rule 4035

```

Int[(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.))/(Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]), x_Symbol] := Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]

```

Rule 4100

```

Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a
_.))^(m_.), x_Symbol] := Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
&& NeQ[a^2 - b^2, 0] && LtQ[m, -1] && !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])

```

Rule 4104

```

Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a
_.))^(m_.), x_Symbol] := Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]

```

Rubi steps

$$\begin{aligned}
\int \frac{A + B \sec(c + dx)}{\sec^{\frac{3}{2}}(c + dx)(a + b \sec(c + dx))^{5/2}} dx &= \frac{2b(Ab - aB) \sin(c + dx)}{3a(a^2 - b^2) d \sqrt{\sec(c + dx)} (a + b \sec(c + dx))^{3/2}} - \frac{2 \int \frac{-\frac{3}{2}(a^2 A - 2Ab^2 + ab)}{\dots}}{\dots} \\
&= \frac{2b(Ab - aB) \sin(c + dx)}{3a(a^2 - b^2) d \sqrt{\sec(c + dx)} (a + b \sec(c + dx))^{3/2}} + \frac{2b(10a^2 Ab - 6Ab)}{3a^2(a^2 - b^2)^2 d \sqrt{s}} \\
&= \frac{2b(Ab - aB) \sin(c + dx)}{3a(a^2 - b^2) d \sqrt{\sec(c + dx)} (a + b \sec(c + dx))^{3/2}} + \frac{2b(10a^2 Ab - 6Ab)}{3a^2(a^2 - b^2)^2 d \sqrt{s}} \\
&= \frac{2b(Ab - aB) \sin(c + dx)}{3a(a^2 - b^2) d \sqrt{\sec(c + dx)} (a + b \sec(c + dx))^{3/2}} + \frac{2b(10a^2 Ab - 6Ab)}{3a^2(a^2 - b^2)^2 d \sqrt{s}} \\
&= \frac{2b(Ab - aB) \sin(c + dx)}{3a(a^2 - b^2) d \sqrt{\sec(c + dx)} (a + b \sec(c + dx))^{3/2}} + \frac{2b(10a^2 Ab - 6Ab)}{3a^2(a^2 - b^2)^2 d \sqrt{s}} \\
&= \frac{2b(Ab - aB) \sin(c + dx)}{3a(a^2 - b^2) d \sqrt{\sec(c + dx)} (a + b \sec(c + dx))^{3/2}} + \frac{2b(10a^2 Ab - 6Ab)}{3a^2(a^2 - b^2)^2 d \sqrt{s}} \\
&= \frac{2(a^4 A + 16a^2 Ab^2 - 16Ab^4 - 9a^3 bB + 8ab^3 B) \sqrt{\frac{b+a \cos(c+dx)}{a+b}} F\left(\frac{1}{2}(c + c)\right)}{3a^4(a^2 - b^2) d \sqrt{a + b \sec(c + dx)}}
\end{aligned}$$

Mathematica [A] time = 3.11, size = 353, normalized size = 0.75

$$2 \sec^{\frac{5}{2}}(c + dx)(a \cos(c + dx) + b) \left(\frac{a \sin(c+dx) \left(a^6 A + A(a^3 - ab^2)^2 \cos(2(c+dx)) + 16a^3 b^3 B - 25a^2 Ab^4 + 2ab(2a^4 A + 9a^3 bB - 16a^2 Ab^2 - 5ab^3 B + 10a^2 b^2) \right)}{2(a^2 - b^2)^2} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(5/2)),x]

[Out] (2*(b + a*Cos[c + d*x])*Sec[c + d*x]^(5/2)*(((b + a*Cos[c + d*x])/(a + b))^(3/2)*(a^2*(a^4*A + 7*a^2*A*b^2 - 4*A*b^4 - 6*a^3*b*B + 2*a*b^3*B)*Ellipti

cF[(c + d*x)/2, (2*a)/(a + b)] + (-8*a^4*A*b + 28*a^2*A*b^3 - 16*A*b^5 + 3*a^5*B - 15*a^3*b^2*B + 8*a*b^4*B)*((a + b)*EllipticE[(c + d*x)/2, (2*a)/(a + b)] - b*EllipticF[(c + d*x)/2, (2*a)/(a + b)]))/((a - b)^2*(a + b)) + (a*(a^6*A - 25*a^2*A*b^4 + 16*A*b^6 + 16*a^3*b^3*B - 8*a*b^5*B + 2*a*b*(2*a^4*A - 16*a^2*A*b^2 + 10*A*b^4 + 9*a^3*b*B - 5*a*b^3*B)*Cos[c + d*x] + A*(a^3 - a*b^2)^2*Cos[2*(c + d*x)]*Sin[c + d*x])/(2*(a^2 - b^2)^2))/((3*a^4*d*(a + b*Sec[c + d*x])^(5/2))

fricas [F] time = 0.94, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B \sec(dx + c) + A)\sqrt{b \sec(dx + c) + a} \sqrt{\sec(dx + c)}}{b^3 \sec(dx + c)^5 + 3ab^2 \sec(dx + c)^4 + 3a^2b \sec(dx + c)^3 + a^3 \sec(dx + c)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")

[Out] integral((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*sqrt(sec(d*x + c))/(b^3*sec(d*x + c)^5 + 3*a*b^2*sec(d*x + c)^4 + 3*a^2*b*sec(d*x + c)^3 + a^3*sec(d*x + c)^2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(b \sec(dx + c) + a)^{\frac{5}{2}} \sec(dx + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(3/2)), x)

maple [B] time = 2.63, size = 6746, normalized size = 14.29

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x)

[Out] result too large to display

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\left(a + \frac{b}{\cos(c+dx)}\right)^{5/2} \left(\frac{1}{\cos(c+dx)}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(3/2)),x)

[Out] int((A + B/cos(c + d*x))/((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(3/2)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)**(3/2)/(a+b*sec(d*x+c))**(5/2),x)

[Out] Timed out

$$3.473 \quad \int \frac{A+B \sec(c+dx)}{\sec^2(c+dx)(a+b \sec(c+dx))^{5/2}} dx$$

Optimal. Leaf size=588

$$\frac{2b(Ab - aB) \sin(c + dx)}{3ad(a^2 - b^2) \sec^3(c + dx)(a + b \sec(c + dx))^{3/2}} + \frac{2b(-9a^3B + 12a^2Ab + 5ab^2B - 8Ab^3) \sin(c + dx)}{3a^2d(a^2 - b^2)^2 \sec^3(c + dx) \sqrt{a + b \sec(c + dx)}} + \frac{2(3a^4A + \dots)}{\dots}$$

[Out] $2/3*b*(A*b-B*a)*\sin(d*x+c)/a/(a^2-b^2)/d/\sec(d*x+c)^{(3/2)}/(a+b*\sec(d*x+c))^{(3/2)+2/3*b*(12*A*a^2*b-8*A*b^3-9*B*a^3+5*B*a*b^2)*\sin(d*x+c)/a^2/(a^2-b^2)^2/d/\sec(d*x+c)^{(3/2)}/(a+b*\sec(d*x+c))^{(1/2)-2/15*(17*A*a^4*b+116*A*a^2*b^3-128*A*b^5-5*B*a^5-80*B*a^3*b^2+80*B*a*b^4)*(\cos(1/2*d*x+1/2*c))^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)}*(a/(a+b))^{(1/2)}*((b+a*\cos(d*x+c))/(a+b))^{(1/2)}*\sec(d*x+c)^{(1/2)}/a^5/(a^2-b^2)/d/(a+b*\sec(d*x+c))^{(1/2)+2/15*(3*A*a^4-71*A*a^2*b^2+48*A*b^4+50*B*a^3*b-30*B*a*b^3)*\sin(d*x+c)*(a+b*\sec(d*x+c))^{(1/2)}/a^3/(a^2-b^2)^2/d/\sec(d*x+c)^{(3/2)-2/15*(14*A*a^4*b-98*A*a^2*b^3+64*A*b^5-5*B*a^5+65*B*a^3*b^2-40*B*a*b^4)*\sin(d*x+c)*(a+b*\sec(d*x+c))^{(1/2)}/a^4/(a^2-b^2)^2/d/\sec(d*x+c)^{(1/2)+2/15*(9*A*a^6+55*A*a^4*b^2-212*A*a^2*b^4+128*A*b^6-40*B*a^5*b+140*B*a^3*b^3-80*B*a*b^5)*(\cos(1/2*d*x+1/2*c))^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)}*(a/(a+b))^{(1/2)}*(a+b*\sec(d*x+c))^{(1/2)}/a^5/(a^2-b^2)^2/d/((b+a*\cos(d*x+c))/(a+b))^{(1/2)}/\sec(d*x+c)^{(1/2)}$

Rubi [A] time = 1.88, antiderivative size = 588, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {4030, 4100, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2(-71a^2Ab^2 + 3a^4A + 50a^3bB - 30ab^3B + 48Ab^4) \sin(c + dx) \sqrt{a + b \sec(c + dx)}}{15a^3d(a^2 - b^2)^2 \sec^3(c + dx)} + \frac{2b(12a^2Ab - 9a^3B + 5ab^2B)}{3a^2d(a^2 - b^2)^2 \sec^3(c + dx)}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2)), x]
 [Out] $(-2*(17*a^4*A*b + 116*a^2*A*b^3 - 128*A*b^5 - 5*a^5*B - 80*a^3*b^2*B + 80*a*b^4*B)*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/(a + b)]*\text{EllipticF}[(c + d*x)/2, (2*a)/(a + b)]*\text{Sqrt}[\text{Sec}[c + d*x]])/(15*a^5*(a^2 - b^2)*d*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]) + (2*(9*a^6*A + 55*a^4*A*b^2 - 212*a^2*A*b^4 + 128*A*b^6 - 40*a^5*b*B + 140*a^3*b^3*B - 80*a*b^5*B)*\text{EllipticE}[(c + d*x)/2, (2*a)/(a + b)]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]])/(15*a^5*(a^2 - b^2)^2*d*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/(a + b)]*\text{Sqr}$

$$\begin{aligned} & t[\text{Sec}[c + d*x]] + (2*b*(A*b - a*B)*\text{Sin}[c + d*x])/(3*a*(a^2 - b^2)*d*\text{Sec}[c \\ & + d*x]^{(3/2)}*(a + b*\text{Sec}[c + d*x])^{(3/2)}) + (2*b*(12*a^2*A*b - 8*A*b^3 - 9*a \\ & ^3*B + 5*a*b^2*B)*\text{Sin}[c + d*x])/(3*a^2*(a^2 - b^2)^2*d*\text{Sec}[c + d*x]^{(3/2)}*S \\ & \text{qrt}[a + b*\text{Sec}[c + d*x]]) + (2*(3*a^4*A - 71*a^2*A*b^2 + 48*A*b^4 + 50*a^3*b \\ & *B - 30*a*b^3*B)*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(15*a^3*(a^2 - b^2) \\ & ^2*d*\text{Sec}[c + d*x]^{(3/2)}) - (2*(14*a^4*A*b - 98*a^2*A*b^3 + 64*A*b^5 - 5*a^5 \\ & *B + 65*a^3*b^2*B - 40*a*b^4*B)*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(15* \\ & a^4*(a^2 - b^2)^2*d*\text{Sqrt}[\text{Sec}[c + d*x]]) \end{aligned}$$

Rule 2653

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*Sqrt[a
+ b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
0] && !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a
+ b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && !GtQ[a + b, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)]/Sqrt[csc[(e_) + (f_)*(x_)]
*(d_)], x_Symbol] := Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_) + (f_)*(x_)]*(d_)]/Sqrt[csc[(e_) + (f_)*(x_)]*(b_)
+ (a_)], x_Symbol] := Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
```

$\text{Sqrt}[a + b\text{Csc}[e + f*x]], \text{Int}[1/\text{Sqrt}[b + a*\text{Sin}[e + f*x]], x], x] /;$ FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 4030

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(d_.))^n*(\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^{m_1}*(\text{csc}[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> \text{Simp}[(b*(A*b - a*B)*\text{Cot}[e + f*x]*(a + b*\text{Csc}[e + f*x])^{m+1}*(d*\text{Csc}[e + f*x])^n)/(a*f*(m+1)*(a^2 - b^2)), x] + \text{Dist}[1/(a*(m+1)*(a^2 - b^2)), \text{Int}[(a + b*\text{Csc}[e + f*x])^{m+1}*(d*\text{Csc}[e + f*x])^n*\text{Simp}[A*(a^2*(m+1) - b^2*(m+n+1)) + a*b*B*n - a*(A*b - a*B)*(m+1)*\text{Csc}[e + f*x] + b*(A*b - a*B)*(m+n+2)*\text{Csc}[e + f*x]^2, x], x], x] /;$ FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])

Rule 4035

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.))/(\text{Sqrt}[\text{csc}[(e_.) + (f_.)*(x_.)]*(d_.)]*\text{Sqrt}[\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]), x_Symbol] :> \text{Dist}[A/a, \text{Int}[\text{Sqrt}[a + b*\text{Csc}[e + f*x]]/\text{Sqrt}[d*\text{Csc}[e + f*x]], x], x] - \text{Dist}[(A*b - a*B)/(a*d), \text{Int}[\text{Sqrt}[d*\text{Csc}[e + f*x]]/\text{Sqrt}[a + b*\text{Csc}[e + f*x]], x], x] /;$ FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]

Rule 4100

$\text{Int}[(A_. + \text{csc}[(e_.) + (f_.)*(x_.)]*(B_.) + \text{csc}[(e_.) + (f_.)*(x_.)]^2*(C_.))*(\text{csc}[(e_.) + (f_.)*(x_.)]*(d_.))^n*(\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^{m_1}, x_Symbol] :> \text{Simp}[(A*b^2 - a*b*B + a^2*C)*\text{Cot}[e + f*x]*(a + b*\text{Csc}[e + f*x])^{m+1}*(d*\text{Csc}[e + f*x])^n)/(a*f*(m+1)*(a^2 - b^2)), x] + \text{Dist}[1/(a*(m+1)*(a^2 - b^2)), \text{Int}[(a + b*\text{Csc}[e + f*x])^{m+1}*(d*\text{Csc}[e + f*x])^n*\text{Simp}[a*(a*A - b*B + a*C)*(m+1) - (A*b^2 - a*b*B + a^2*C)*(m+n+1) - a*(A*b - a*B + b*C)*(m+1)*\text{Csc}[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m+n+2)*\text{Csc}[e + f*x]^2, x], x], x] /;$ FreeQ[{a, b, d, e, f, A, B, C, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])

Rule 4104

$\text{Int}[(A_. + \text{csc}[(e_.) + (f_.)*(x_.)]*(B_.) + \text{csc}[(e_.) + (f_.)*(x_.)]^2*(C_.))*(\text{csc}[(e_.) + (f_.)*(x_.)]*(d_.))^n*(\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^{m_1}, x_Symbol] :> \text{Simp}[(A*\text{Cot}[e + f*x]*(a + b*\text{Csc}[e + f*x])^{m+1}*(d*\text{Csc}[e + f*x])^n)/(a*f*n), x] + \text{Dist}[1/(a*d*n), \text{Int}[(a + b*\text{Csc}[e + f*x])^m*(d*\text{Csc}[e + f*x])^{n+1}*\text{Simp}[a*B*n - A*b*(m+n+1) + a*(A + A*n + C*n)*\text{Csc}[e + f*x] + A*b*(m+n+2)*\text{Csc}[e + f*x]^2, x], x], x] /;$ FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]

Rubi steps

$$\begin{aligned}
\int \frac{A + B \sec(c + dx)}{\sec^{\frac{5}{2}}(c + dx)(a + b \sec(c + dx))^{5/2}} dx &= \frac{2b(Ab - aB) \sin(c + dx)}{3a(a^2 - b^2) d \sec^{\frac{3}{2}}(c + dx)(a + b \sec(c + dx))^{3/2}} - \frac{2 \int \frac{1}{2}(-3a^2 A + 8Ab^2 - 5a^2 B) \sec^{\frac{3}{2}}(c + dx)(a + b \sec(c + dx))^{3/2} dx}{3a(a^2 - b^2) d \sec^{\frac{3}{2}}(c + dx)(a + b \sec(c + dx))^{3/2}} \\
&= \frac{2b(Ab - aB) \sin(c + dx)}{3a(a^2 - b^2) d \sec^{\frac{3}{2}}(c + dx)(a + b \sec(c + dx))^{3/2}} + \frac{2b(12a^2 Ab - 8Ab^3)}{3a^2(a^2 - b^2)^2 d \sec^{\frac{3}{2}}(c + dx)(a + b \sec(c + dx))^{3/2}} \\
&= \frac{2b(Ab - aB) \sin(c + dx)}{3a(a^2 - b^2) d \sec^{\frac{3}{2}}(c + dx)(a + b \sec(c + dx))^{3/2}} + \frac{2b(12a^2 Ab - 8Ab^3)}{3a^2(a^2 - b^2)^2 d \sec^{\frac{3}{2}}(c + dx)(a + b \sec(c + dx))^{3/2}} \\
&= \frac{2b(Ab - aB) \sin(c + dx)}{3a(a^2 - b^2) d \sec^{\frac{3}{2}}(c + dx)(a + b \sec(c + dx))^{3/2}} + \frac{2b(12a^2 Ab - 8Ab^3)}{3a^2(a^2 - b^2)^2 d \sec^{\frac{3}{2}}(c + dx)(a + b \sec(c + dx))^{3/2}} \\
&= \frac{2b(Ab - aB) \sin(c + dx)}{3a(a^2 - b^2) d \sec^{\frac{3}{2}}(c + dx)(a + b \sec(c + dx))^{3/2}} + \frac{2b(12a^2 Ab - 8Ab^3)}{3a^2(a^2 - b^2)^2 d \sec^{\frac{3}{2}}(c + dx)(a + b \sec(c + dx))^{3/2}} \\
&= \frac{2b(Ab - aB) \sin(c + dx)}{3a(a^2 - b^2) d \sec^{\frac{3}{2}}(c + dx)(a + b \sec(c + dx))^{3/2}} + \frac{2b(12a^2 Ab - 8Ab^3)}{3a^2(a^2 - b^2)^2 d \sec^{\frac{3}{2}}(c + dx)(a + b \sec(c + dx))^{3/2}} \\
&= \frac{2b(Ab - aB) \sin(c + dx)}{3a(a^2 - b^2) d \sec^{\frac{3}{2}}(c + dx)(a + b \sec(c + dx))^{3/2}} + \frac{2b(12a^2 Ab - 8Ab^3)}{3a^2(a^2 - b^2)^2 d \sec^{\frac{3}{2}}(c + dx)(a + b \sec(c + dx))^{3/2}} \\
&= \frac{2b(Ab - aB) \sin(c + dx)}{3a(a^2 - b^2) d \sec^{\frac{3}{2}}(c + dx)(a + b \sec(c + dx))^{3/2}} + \frac{2b(12a^2 Ab - 8Ab^3)}{3a^2(a^2 - b^2)^2 d \sec^{\frac{3}{2}}(c + dx)(a + b \sec(c + dx))^{3/2}} \\
&= \frac{2(17a^4 Ab + 116a^2 Ab^3 - 128Ab^5 - 5a^5 B - 80a^3 b^2 B + 80ab^4 B) \sqrt{b + c}}{15a^5(a^2 - b^2) d \sqrt{a + b \sec(c + dx)}}
\end{aligned}$$

Mathematica [A] time = 4.30, size = 392, normalized size = 0.67

$$\sec^{\frac{5}{2}}(c + dx)(a \cos(c + dx) + b) \left(a \left(\frac{10b^4(Ab - aB) \sin(c + dx)}{b^2 - a^2} - \frac{10b^3(12a^3 B - 15a^2 Ab - 8ab^2 B + 11Ab^3) \sin(c + dx)(a \cos(c + dx) + b)}{(a^2 - b^2)^2} - 2(14A
\right. \right.$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2)),x]

[Out] ((b + a*cos[c + d*x])*Sec[c + d*x]^(5/2)*((-2*((b + a*cos[c + d*x]))/(a + b))^(3/2)*(a^2*(8*a^4*A*b + 44*a^2*A*b^3 - 32*A*b^5 - 5*a^5*B - 35*a^3*b^2*B + 20*a*b^4*B)*EllipticF[(c + d*x)/2, (2*a)/(a + b)] - (9*a^6*A + 55*a^4*A*b^2 - 212*a^2*A*b^4 + 128*A*b^6 - 40*a^5*b*B + 140*a^3*b^3*B - 80*a*b^5*B)*(a + b)*EllipticE[(c + d*x)/2, (2*a)/(a + b)] - b*EllipticF[(c + d*x)/2, (2*a)/(a + b)])))/((a - b)^2*(a + b)) + a*((10*b^4*(A*b - a*B)*Sin[c + d*x])/(-a^2 + b^2) - (10*b^3*(-15*a^2*A*b + 11*A*b^3 + 12*a^3*B - 8*a*b^2*B)*(b + a*cos[c + d*x])*Sin[c + d*x])/(a^2 - b^2)^2 - 2*(14*A*b - 5*a*B)*(b + a*cos[c + d*x])^2*sin[c + d*x] + 3*a*A*(b + a*cos[c + d*x])^2*sin[2*(c + d*x)])))/(15*a^5*d*(a + b*Sec[c + d*x])^(5/2))

fricas [F] time = 0.99, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B \sec(dx + c) + A)\sqrt{b \sec(dx + c) + a} \sqrt{\sec(dx + c)}}{b^3 \sec(dx + c)^6 + 3ab^2 \sec(dx + c)^5 + 3a^2b \sec(dx + c)^4 + a^3 \sec(dx + c)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")

[Out] integral((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*sqrt(sec(d*x + c))/(b^3*sec(d*x + c)^6 + 3*a*b^2*sec(d*x + c)^5 + 3*a^2*b*sec(d*x + c)^4 + a^3*sec(d*x + c)^3), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(b \sec(dx + c) + a)^{\frac{5}{2}} \sec(dx + c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(5/2)), x)

maple [B] time = 2.94, size = 8251, normalized size = 14.03

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x)`

[Out] result too large to display

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")`

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\left(a + \frac{b}{\cos(c+dx)}\right)^{5/2} \left(\frac{1}{\cos(c+dx)}\right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B/cos(c + d*x))/((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(5/2)),x)`

[Out] `int((A + B/cos(c + d*x))/((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(5/2)), x)`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*sec(d*x+c))/sec(d*x+c)**(5/2)/(a+b*sec(d*x+c))**(5/2),x)`

[Out] Timed out

3.474 $\int (a + b \sec(c + dx))^{2/3} (A + B \sec(c + dx)) dx$

Optimal. Leaf size=126

$$A \operatorname{Int}\left((a + b \sec(c + dx))^{2/3}, x\right) + \frac{\sqrt{2} B \tan(c + dx) (a + b \sec(c + dx))^{2/3} F_1\left(\frac{1}{2}; \frac{1}{2}, -\frac{2}{3}; \frac{3}{2}; \frac{1}{2} (1 - \sec(c + dx)), \frac{b(1 - \sec(c + dx))}{a + b}\right)}{d \sqrt{\sec(c + dx) + 1} \left(\frac{a + b \sec(c + dx)}{a + b}\right)^{2/3}}$$

[Out] B*AppellF1(1/2, -2/3, 1/2, 3/2, b*(1-sec(d*x+c))/(a+b), 1/2-1/2*sec(d*x+c))*(a+b*sec(d*x+c))^(2/3)*2^(1/2)*tan(d*x+c)/d/((a+b*sec(d*x+c))/(a+b))^(2/3)/(1+sec(d*x+c))^(1/2)+A*Unintegrable((a+b*sec(d*x+c))^(2/3), x)

Rubi [A] time = 0.16, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (a + b \sec(c + dx))^{2/3} (A + B \sec(c + dx)) dx$$

Verification is Not applicable to the result.

[In] Int[(a + b*Sec[c + d*x])^(2/3)*(A + B*Sec[c + d*x]), x]

[Out] (Sqrt[2]*B*AppellF1[1/2, 1/2, -2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x]/(d*Sqrt[1 + Sec[c + d*x]])*((a + b*Sec[c + d*x])/(a + b))^(2/3) + A*Defer[Int][(a + b*Sec[c + d*x])^(2/3), x]

Rubi steps

$$\begin{aligned} \int (a + b \sec(c + dx))^{2/3} (A + B \sec(c + dx)) dx &= A \int (a + b \sec(c + dx))^{2/3} dx + B \int \sec(c + dx) (a + b \sec(c + dx))^{2/3} dx \\ &= A \int (a + b \sec(c + dx))^{2/3} dx - \frac{(B \tan(c + dx)) \operatorname{Subst}\left(\int \frac{(a + bx)}{\sqrt{1-x}} dx, \frac{a + b \sec(c + dx)}{a + b}\right)}{d \sqrt{1 - \sec(c + dx)} \sqrt{1 + \sec(c + dx)}} \\ &= A \int (a + b \sec(c + dx))^{2/3} dx - \frac{(B(a + b \sec(c + dx))^{2/3} \tan(c + dx))}{d \sqrt{1 - \sec(c + dx)}} \\ &= \frac{\sqrt{2} B F_1\left(\frac{1}{2}; \frac{1}{2}, -\frac{2}{3}; \frac{3}{2}; \frac{1}{2} (1 - \sec(c + dx)), \frac{b(1 - \sec(c + dx))}{a + b}\right) (a + b \sec(c + dx))^{2/3}}{d \sqrt{1 + \sec(c + dx)} \left(\frac{a + b \sec(c + dx)}{a + b}\right)^{2/3}} \end{aligned}$$

Mathematica [A] time = 22.71, size = 0, normalized size = 0.00

$$\int (a + b \sec(c + dx))^{2/3} (A + B \sec(c + dx)) dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b*Sec[c + d*x])^(2/3)*(A + B*Sec[c + d*x]),x]

[Out] Integrate[(a + b*Sec[c + d*x])^(2/3)*(A + B*Sec[c + d*x]), x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] Timed out

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(b \sec(dx + c) + a)^{2/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(2/3), x)

maple [A] time = 1.08, size = 0, normalized size = 0.00

$$\int (a + b \sec(dx + c))^{2/3} (A + B \sec(dx + c)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x)

[Out] int((a+b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x)

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(b \sec(dx + c) + a)^{2/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(2/3), x)

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A + \frac{B}{\cos(c + dx)} \right) \left(a + \frac{b}{\cos(c + dx)} \right)^{2/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(2/3), x)

[Out] int((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(2/3), x)

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \sec(c + dx)) (a + b \sec(c + dx))^{2/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))**(2/3)*(A+B*sec(d*x+c)), x)

[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**(2/3), x)

$$3.475 \quad \int \sqrt[3]{a + b \sec(c + dx)} (A + B \sec(c + dx)) dx$$

Optimal. Leaf size=126

$$A \operatorname{Int} \left(\sqrt[3]{a + b \sec(c + dx)}, x \right) + \frac{\sqrt{2} B \tan(c + dx) \sqrt[3]{a + b \sec(c + dx)} F_1 \left(\frac{1}{2}; \frac{1}{2}, -\frac{1}{3}; \frac{3}{2}; \frac{1}{2} (1 - \sec(c + dx)), \frac{b(1 - \sec(c + dx))}{a + b} \right)}{d \sqrt{\sec(c + dx) + 1} \sqrt[3]{\frac{a + b \sec(c + dx)}{a + b}}}$$

[Out] B*AppellF1(1/2, -1/3, 1/2, 3/2, b*(1-sec(d*x+c))/(a+b), 1/2-1/2*sec(d*x+c))*(a+b*sec(d*x+c))^(1/3)*2^(1/2)*tan(d*x+c)/d/((a+b*sec(d*x+c))/(a+b))^(1/3)/(1+sec(d*x+c))^(1/2)+A*Unintegrable((a+b*sec(d*x+c))^(1/3), x)

Rubi [A] time = 0.15, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \sqrt[3]{a + b \sec(c + dx)} (A + B \sec(c + dx)) dx$$

Verification is Not applicable to the result.

[In] Int[(a + b*Sec[c + d*x])^(1/3)*(A + B*Sec[c + d*x]), x]

[Out] (Sqrt[2]*B*AppellF1[1/2, 1/2, -1/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(1/3)*Tan[c + d*x]/(d*Sqrt[1 + Sec[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(1/3)) + A*Defer[Int][(a + b*Sec[c + d*x])^(1/3), x]

Rubi steps

$$\begin{aligned} \int \sqrt[3]{a + b \sec(c + dx)} (A + B \sec(c + dx)) dx &= A \int \sqrt[3]{a + b \sec(c + dx)} dx + B \int \sec(c + dx) \sqrt[3]{a + b \sec(c + dx)} dx \\ &= A \int \sqrt[3]{a + b \sec(c + dx)} dx - \frac{(B \tan(c + dx)) \operatorname{Subst} \left(\int \frac{\sqrt[3]{a + bx}}{\sqrt{1-x} \sqrt{1+x}} dx \right)}{d \sqrt{1 - \sec(c + dx)} \sqrt{1 + \sec(c + dx)}} \\ &= A \int \sqrt[3]{a + b \sec(c + dx)} dx - \frac{(B \sqrt[3]{a + b \sec(c + dx)} \tan(c + dx))}{d \sqrt{1 - \sec(c + dx)} \sqrt{1 + \sec(c + dx)}} \\ &= \frac{\sqrt{2} B F_1 \left(\frac{1}{2}; \frac{1}{2}, -\frac{1}{3}; \frac{3}{2}; \frac{1}{2} (1 - \sec(c + dx)), \frac{b(1 - \sec(c + dx))}{a + b} \right) \sqrt[3]{a + b \sec(c + dx)}}{d \sqrt{1 + \sec(c + dx)} \sqrt[3]{\frac{a + b \sec(c + dx)}{a + b}}} \end{aligned}$$

Mathematica [A] time = 19.32, size = 0, normalized size = 0.00

$$\int \sqrt[3]{a + b \sec(c + dx)} (A + B \sec(c + dx)) dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b*Sec[c + d*x])^(1/3)*(A + B*Sec[c + d*x]), x]

[Out] Integrate[(a + b*Sec[c + d*x])^(1/3)*(A + B*Sec[c + d*x]), x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^(1/3)*(A+B*sec(d*x+c)), x, algorithm="fricas")

[Out] Timed out

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(b \sec(dx + c) + a)^{\frac{1}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^(1/3)*(A+B*sec(d*x+c)), x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(1/3), x)

maple [A] time = 1.08, size = 0, normalized size = 0.00

$$\int (a + b \sec(dx + c))^{\frac{1}{3}} (A + B \sec(dx + c)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(d*x+c))^(1/3)*(A+B*sec(d*x+c)), x)

[Out] int((a+b*sec(d*x+c))^(1/3)*(A+B*sec(d*x+c)), x)

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(b \sec(dx + c) + a)^{\frac{1}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^(1/3)*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(1/3), x)

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A + \frac{B}{\cos(c + dx)} \right) \left(a + \frac{b}{\cos(c + dx)} \right)^{1/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(1/3),x)

[Out] int((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(1/3), x)

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \sec(c + dx)) \sqrt[3]{a + b \sec(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))**(1/3)*(A+B*sec(d*x+c)),x)

[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**(1/3), x)

$$3.476 \quad \int \frac{A+B \sec(c+dx)}{\sqrt[3]{a+b \sec(c+dx)}} dx$$

Optimal. Leaf size=126

$$A \operatorname{Int} \left(\frac{1}{\sqrt[3]{a+b \sec(c+dx)}}, x \right) + \frac{\sqrt{2} B \tan(c+dx) \sqrt[3]{\frac{a+b \sec(c+dx)}{a+b}} F_1 \left(\frac{1}{2}; \frac{1}{2}, \frac{1}{3}; \frac{3}{2}; \frac{1}{2} (1 - \sec(c+dx)), \frac{b(1-\sec(c+dx))}{a+b} \right)}{d \sqrt{\sec(c+dx)+1} \sqrt[3]{a+b \sec(c+dx)}}$$

[Out] B*AppellF1(1/2,1/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*((a+b*sec(d*x+c))/(a+b))^(1/3)*2^(1/2)*tan(d*x+c)/d/(a+b*sec(d*x+c))^(1/3)/(1+sec(d*x+c))^(1/2)+A*Unintegrable(1/(a+b*sec(d*x+c))^(1/3),x)

Rubi [A] time = 0.18, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{A+B \sec(c+dx)}{\sqrt[3]{a+b \sec(c+dx)}} dx$$

Verification is Not applicable to the result.

[In] Int[(A + B*Sec[c + d*x])/(a + b*Sec[c + d*x])^(1/3),x]

[Out] (Sqrt[2]*B*AppellF1[1/2, 1/2, 1/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]*((a + b*Sec[c + d*x])/(a + b))^(1/3)*Tan[c + d*x])/(d*Sqrt[1 + Sec[c + d*x]]*(a + b*Sec[c + d*x])^(1/3)) + A*Defer[Int] [(a + b*Sec[c + d*x])^(-1/3), x]

Rubi steps

$$\begin{aligned} \int \frac{A+B \sec(c+dx)}{\sqrt[3]{a+b \sec(c+dx)}} dx &= A \int \frac{1}{\sqrt[3]{a+b \sec(c+dx)}} dx + B \int \frac{\sec(c+dx)}{\sqrt[3]{a+b \sec(c+dx)}} dx \\ &= A \int \frac{1}{\sqrt[3]{a+b \sec(c+dx)}} dx - \frac{(B \tan(c+dx)) \operatorname{Subst} \left(\int \frac{1}{\sqrt{1-x} \sqrt{1+x} \sqrt[3]{a+bx}} dx, x, \sec(c+dx) \right)}{d \sqrt{1-\sec(c+dx)} \sqrt{1+\sec(c+dx)}} \\ &= A \int \frac{1}{\sqrt[3]{a+b \sec(c+dx)}} dx - \frac{\left(B \sqrt[3]{\frac{a+b \sec(c+dx)}{-a-b}} \tan(c+dx) \right) \operatorname{Subst} \left(\int \frac{1}{\sqrt{1-x} \sqrt{1+x} \sqrt[3]{a+bx}} dx, x, \sec(c+dx) \right)}{d \sqrt{1-\sec(c+dx)} \sqrt{1+\sec(c+dx)} \sqrt[3]{a+b \sec(c+dx)}} \\ &= \frac{\sqrt{2} B F_1 \left(\frac{1}{2}; \frac{1}{2}, \frac{1}{3}; \frac{3}{2}; \frac{1}{2} (1 - \sec(c+dx)), \frac{b(1-\sec(c+dx))}{a+b} \right) \sqrt[3]{\frac{a+b \sec(c+dx)}{a+b}} \tan(c+dx)}{d \sqrt{1+\sec(c+dx)} \sqrt[3]{a+b \sec(c+dx)}} + A \int \frac{1}{\sqrt[3]{a+b \sec(c+dx)}} dx \end{aligned}$$

Mathematica [A] time = 3.53, size = 0, normalized size = 0.00

$$\int \frac{A + B \sec(c + dx)}{\sqrt[3]{a + b \sec(c + dx)}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(A + B*Sec[c + d*x])/(a + b*Sec[c + d*x])^(1/3), x]

[Out] Integrate[(A + B*Sec[c + d*x])/(a + b*Sec[c + d*x])^(1/3), x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/3), x, algorithm="fricas")

[Out] Timed out

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(b \sec(dx + c) + a)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/3), x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)/(b*sec(d*x + c) + a)^(1/3), x)

maple [A] time = 1.12, size = 0, normalized size = 0.00

$$\int \frac{A + B \sec(dx + c)}{(a + b \sec(dx + c))^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/3), x)

[Out] int((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/3), x)

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(b \sec(dx + c) + a)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/3),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)/(b*sec(d*x + c) + a)^(1/3), x)

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\left(a + \frac{b}{\cos(c+dx)}\right)^{1/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/(a + b/cos(c + d*x))^(1/3), x)

[Out] int((A + B/cos(c + d*x))/(a + b/cos(c + d*x))^(1/3), x)

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \sec(c + dx)}{\sqrt[3]{a + b \sec(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(1/3),x)

[Out] Integral((A + B*sec(c + d*x))/(a + b*sec(c + d*x))**(1/3), x)

$$3.477 \quad \int \frac{A+B \sec(c+dx)}{(a+b \sec(c+dx))^{2/3}} dx$$

Optimal. Leaf size=126

$$A \operatorname{Int} \left(\frac{1}{(a+b \sec(c+dx))^{2/3}}, x \right) + \frac{\sqrt{2} B \tan(c+dx) \left(\frac{a+b \sec(c+dx)}{a+b} \right)^{2/3} F_1 \left(\frac{1}{2}; \frac{1}{2}, \frac{2}{3}; \frac{3}{2}; \frac{1}{2} (1 - \sec(c+dx)), \frac{b(1-\sec(c+dx))}{a+b} \right)}{d \sqrt{\sec(c+dx)+1} (a+b \sec(c+dx))^{2/3}}$$

[Out] B*AppellF1(1/2,2/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*((a+b*sec(d*x+c))/(a+b))^(2/3)*2^(1/2)*tan(d*x+c)/d/(a+b*sec(d*x+c))^(2/3)/(1+sec(d*x+c))^(1/2)+A*Unintegrable(1/(a+b*sec(d*x+c))^(2/3),x)

Rubi [A] time = 0.16, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{A + B \sec(c + dx)}{(a + b \sec(c + dx))^{2/3}} dx$$

Verification is Not applicable to the result.

[In] Int[(A + B*Sec[c + d*x])/(a + b*Sec[c + d*x])^(2/3), x]

[Out] (Sqrt[2]*B*AppellF1[1/2, 1/2, 2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]*((a + b*Sec[c + d*x])/(a + b))^(2/3)*Tan[c + d*x]/(d*Sqrt[1 + Sec[c + d*x]]*(a + b*Sec[c + d*x])^(2/3)) + A*Defer[Int][(a + b*Sec[c + d*x])^(-2/3), x]

Rubi steps

$$\begin{aligned} \int \frac{A + B \sec(c + dx)}{(a + b \sec(c + dx))^{2/3}} dx &= A \int \frac{1}{(a + b \sec(c + dx))^{2/3}} dx + B \int \frac{\sec(c + dx)}{(a + b \sec(c + dx))^{2/3}} dx \\ &= A \int \frac{1}{(a + b \sec(c + dx))^{2/3}} dx - \frac{(B \tan(c + dx)) \operatorname{Subst} \left(\int \frac{1}{\sqrt{1-x} \sqrt{1+x} (a+bx)^{2/3}} dx, x, \sec(c+dx) \right)}{d \sqrt{1 - \sec(c + dx)} \sqrt{1 + \sec(c + dx)}} \\ &= A \int \frac{1}{(a + b \sec(c + dx))^{2/3}} dx - \frac{\left(B \left(-\frac{a+b \sec(c+dx)}{-a-b} \right)^{2/3} \tan(c + dx) \right) \operatorname{Subst} \left(\int \frac{1}{\sqrt{1-x} \sqrt{1+x} (a+bx)^{2/3}} dx, x, \sec(c+dx) \right)}{d \sqrt{1 - \sec(c + dx)} \sqrt{1 + \sec(c + dx)}} \\ &= \frac{\sqrt{2} B F_1 \left(\frac{1}{2}; \frac{1}{2}, \frac{2}{3}; \frac{3}{2}; \frac{1}{2} (1 - \sec(c + dx)), \frac{b(1-\sec(c+dx))}{a+b} \right) \left(\frac{a+b \sec(c+dx)}{a+b} \right)^{2/3} \tan(c + dx)}{d \sqrt{1 + \sec(c + dx)} (a + b \sec(c + dx))^{2/3}} + \end{aligned}$$

Mathematica [A] time = 3.46, size = 0, normalized size = 0.00

$$\int \frac{A + B \sec(c + dx)}{(a + b \sec(c + dx))^{2/3}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(A + B*Sec[c + d*x])/(a + b*Sec[c + d*x])^(2/3), x]

[Out] Integrate[(A + B*Sec[c + d*x])/(a + b*Sec[c + d*x])^(2/3), x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(2/3), x, algorithm="fricas")

[Out] Timed out

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(b \sec(dx + c) + a)^{2/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(2/3), x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)/(b*sec(d*x + c) + a)^(2/3), x)

maple [A] time = 1.21, size = 0, normalized size = 0.00

$$\int \frac{A + B \sec(dx + c)}{(a + b \sec(dx + c))^{2/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(2/3), x)

[Out] int((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(2/3), x)

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(b \sec(dx + c) + a)^{2/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(2/3),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)/(b*sec(d*x + c) + a)^(2/3), x)

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\left(a + \frac{b}{\cos(c+dx)}\right)^{2/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/(a + b/cos(c + d*x))^(2/3),x)

[Out] int((A + B/cos(c + d*x))/(a + b/cos(c + d*x))^(2/3), x)

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \sec(c + dx)}{(a + b \sec(c + dx))^{2/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(2/3),x)

[Out] Integral((A + B*sec(c + d*x))/(a + b*sec(c + d*x))**(2/3), x)

$$3.478 \quad \int (c \sec(e+fx))^n (a+b \sec(e+fx))^m (A+B \sec(e+fx)) dx$$

Optimal. Leaf size=36

$$\text{Int}\left((A+B \sec(e+fx))(c \sec(e+fx))^n (a+b \sec(e+fx))^m, x\right)$$

[Out] Unintegrable((c*sec(f*x+e))^n*(a+b*sec(f*x+e))^m*(A+B*sec(f*x+e)), x)

Rubi [A] time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (c \sec(e+fx))^n (a+b \sec(e+fx))^m (A+B \sec(e+fx)) dx$$

Verification is Not applicable to the result.

[In] Int[(c*Sec[e + f*x])^n*(a + b*Sec[e + f*x])^m*(A + B*Sec[e + f*x]), x]

[Out] Defer[Int][(c*Sec[e + f*x])^n*(a + b*Sec[e + f*x])^m*(A + B*Sec[e + f*x]), x]

Rubi steps

$$\int (c \sec(e+fx))^n (a+b \sec(e+fx))^m (A+B \sec(e+fx)) dx = \int (c \sec(e+fx))^n (a+b \sec(e+fx))^m (A+B \sec(e+fx)) dx$$

Mathematica [A] time = 4.78, size = 0, normalized size = 0.00

$$\int (c \sec(e+fx))^n (a+b \sec(e+fx))^m (A+B \sec(e+fx)) dx$$

Verification is Not applicable to the result.

[In] Integrate[(c*Sec[e + f*x])^n*(a + b*Sec[e + f*x])^m*(A + B*Sec[e + f*x]), x]

[Out] Integrate[(c*Sec[e + f*x])^n*(a + b*Sec[e + f*x])^m*(A + B*Sec[e + f*x]), x]

fricas [A] time = 0.92, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(B \sec(fx + e) + A\right)\left(b \sec(fx + e) + a\right)^m \left(c \sec(fx + e)\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*sec(f*x+e))^n*(a+b*sec(f*x+e))^m*(A+B*sec(f*x+e)),x, algorithm="fricas")

[Out] integral((B*sec(f*x + e) + A)*(b*sec(f*x + e) + a)^m*(c*sec(f*x + e))^n, x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(fx + e) + A)(b \sec(fx + e) + a)^m (c \sec(fx + e))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*sec(f*x+e))^n*(a+b*sec(f*x+e))^m*(A+B*sec(f*x+e)),x, algorithm="giac")

[Out] integrate((B*sec(f*x + e) + A)*(b*sec(f*x + e) + a)^m*(c*sec(f*x + e))^n, x)

maple [A] time = 3.33, size = 0, normalized size = 0.00

$$\int (c \sec(fx + e))^n (a + b \sec(fx + e))^m (A + B \sec(fx + e)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*sec(f*x+e))^n*(a+b*sec(f*x+e))^m*(A+B*sec(f*x+e)),x)

[Out] int((c*sec(f*x+e))^n*(a+b*sec(f*x+e))^m*(A+B*sec(f*x+e)),x)

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(fx + e) + A)(b \sec(fx + e) + a)^m (c \sec(fx + e))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*sec(f*x+e))^n*(a+b*sec(f*x+e))^m*(A+B*sec(f*x+e)),x, algorithm="maxima")

[Out] integrate((B*sec(f*x + e) + A)*(b*sec(f*x + e) + a)^m*(c*sec(f*x + e))^n, x)

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \left(A + \frac{B}{\cos(e + fx)} \right) \left(a + \frac{b}{\cos(e + fx)} \right)^m \left(\frac{c}{\cos(e + fx)} \right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B/cos(e + f*x))*(a + b/cos(e + f*x))^m*(c/cos(e + f*x))^n,x)`

[Out] `int((A + B/cos(e + f*x))*(a + b/cos(e + f*x))^m*(c/cos(e + f*x))^n, x)`

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (c \sec(e + fx))^n (A + B \sec(e + fx)) (a + b \sec(e + fx))^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*sec(f*x+e))^n*(a+b*sec(f*x+e))^m*(A+B*sec(f*x+e)),x)`

[Out] `Integral((c*sec(e + f*x))^n*(A + B*sec(e + f*x))*(a + b*sec(e + f*x))^m, x)`

$$3.479 \quad \int \sec^m(c+dx)(a+b \sec(c+dx))^4(A+B \sec(c+dx)) dx$$

Optimal. Leaf size=544

$$\frac{b^2 \sin(c+dx) \left(a^2 B (m^2 + 9m + 26) + 2aAb(m+4)^2 + b^2 B(m+3)^2 \right) \sec^{m+2}(c+dx)}{d(m+2)(m+3)(m+4)} + \frac{b \sin(c+dx) \left(2a^3 B (m^2 + 9m + 26) + 2a^2 Ab(m+4)^2 + b^2 B(m+3)^2 \right) \sec^{m+1}(c+dx)}{d(m+2)(m+3)(m+4)}$$

[Out] $b*(A*b^3*(m^2+6*m+8)+4*a*b^2*B*(m^2+6*m+8)+2*a^3*B*(m^2+8*m+19)+a^2*A*b*(5*m^2+37*m+68))*\sec(d*x+c)^{(1+m)}*\sin(d*x+c)/d/(4+m)/(m^2+4*m+3)+b^2*(b^2*B*(3+m)^2+2*a*A*b*(4+m)^2+a^2*B*(m^2+9*m+26))*\sec(d*x+c)^{(2+m)}*\sin(d*x+c)/d/(4+m)/(m^2+5*m+6)+b*(A*b*(4+m)+a*B*(7+m))*\sec(d*x+c)^{(1+m)}*(a+b*\sec(d*x+c))^2*\sin(d*x+c)/d/(3+m)/(4+m)+b*B*\sec(d*x+c)^{(1+m)}*(a+b*\sec(d*x+c))^3*\sin(d*x+c)/d/(4+m)-(A*b^4*m*(2+m)+4*a*b^3*B*m*(2+m)+6*a^2*A*b^2*m*(3+m)+4*a^3*b*B*m*(3+m)+a^4*A*(m^2+4*m+3))*\text{hypergeom}([1/2, 1/2-1/2*m], [3/2-1/2*m], \cos(d*x+c)^2)*\sec(d*x+c)^{(-1+m)}*\sin(d*x+c)/d/(3+m)/(-m^2+1)/(\sin(d*x+c)^2)^{(1/2)}+(b^4*B*(m^2+4*m+3)+4*a*A*b^3*(m^2+5*m+4)+6*a^2*b^2*B*(m^2+5*m+4)+4*a^3*A*b*(m^2+6*m+8)+a^4*B*(m^2+6*m+8))*\text{hypergeom}([1/2, -1/2*m], [1-1/2*m], \cos(d*x+c)^2)*\sec(d*x+c)^m*\sin(d*x+c)/d/m/(2+m)/(4+m)/(\sin(d*x+c)^2)^{(1/2)}$

Rubi [A] time = 1.63, antiderivative size = 544, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {4026, 4096, 4076, 4047, 3772, 2643, 4046}

$$\frac{\sin(c+dx) \left(6a^2 Ab^2 m(m+3) + a^4 A (m^2 + 4m + 3) + 4a^3 b B m(m+3) + 4ab^3 B m(m+2) + Ab^4 m(m+2) \right) \sec^m(c+dx)}{d(1-m)(m+1)(m+3)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d*x]^m*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x]), x]

[Out] $(b*(A*b^3*(8 + 6*m + m^2) + 4*a*b^2*B*(8 + 6*m + m^2) + 2*a^3*B*(19 + 8*m + m^2) + a^2*A*b*(68 + 37*m + 5*m^2))*\text{Sec}[c + d*x]^{(1+m)}*\text{Sin}[c + d*x])/(d*(1+m)*(3+m)*(4+m)) + (b^2*(b^2*B*(3+m)^2 + 2*a*A*b*(4+m)^2 + a^2*B*(26 + 9*m + m^2))*\text{Sec}[c + d*x]^{(2+m)}*\text{Sin}[c + d*x])/(d*(2+m)*(3+m)*(4+m)) + (b*(A*b*(4+m) + a*B*(7+m))*\text{Sec}[c + d*x]^{(1+m)}*(a + b*\text{Sec}[c + d*x])^2*\text{Sin}[c + d*x])/(d*(3+m)*(4+m)) + (b*B*\text{Sec}[c + d*x]^{(1+m)}*(a + b*\text{Sec}[c + d*x])^3*\text{Sin}[c + d*x])/(d*(4+m)) - ((A*b^4*m*(2+m) + 4*a*b^3*B*m*(2+m) + 6*a^2*A*b^2*m*(3+m) + 4*a^3*b*B*m*(3+m) + a^4*A*(3 + 4*m + m^2))*\text{Hypergeometric2F1}[1/2, (1-m)/2, (3-m)/2, \text{Cos}[c + d*x]^2]*\text{Sec}[c + d*x]^{(-1+m)}*\text{Sin}[c + d*x])/(d*(1-m)*(1+m)*(3+m)*\text{Sqrt}[\text{Sin}[c + d*x]^2]) + ((b^4*B*(3 + 4*m + m^2) + 4*a*A*b^3*(4 + 5*m + m^2) + 6*a^2*b^2*B*(4$

$+ 5*m + m^2) + 4*a^3*A*b*(8 + 6*m + m^2) + a^4*B*(8 + 6*m + m^2))*Hypergeometric2F1[1/2, -m/2, (2 - m)/2, Cos[c + d*x]^2]*Sec[c + d*x]^m*Sin[c + d*x]/(d*m*(2 + m)*(4 + m)*Sqrt[Sin[c + d*x]^2])$

Rule 2643

$Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] \&\& !IntegerQ[2*n]$

Rule 3772

$Int[(csc[(c_.) + (d_.)*(x_)])*(b_.)^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x])^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; FreeQ[{b, c, d, n}, x] \&\& !IntegerQ[n]$

Rule 4026

$Int[(csc[(e_.) + (f_.)*(x_)])*(d_.)^(n_)*(csc[(e_.) + (f_.)*(x_)])*(b_.) + (a_.)^(m_)*(csc[(e_.) + (f_.)*(x_)])*(B_.) + (A_.)), x_Symbol] :> -Simp[(b*B*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x] + Dist[1/(m + n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*Simp[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B)*(m + n) + b^2*B*(m + n - 1))*Csc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] \&\& NeQ[A*b - a*B, 0] \&\& NeQ[a^2 - b^2, 0] \&\& GtQ[m, 1] \&\& !(IGtQ[n, 1] \&\& !IntegerQ[m])$

Rule 4046

$Int[(csc[(e_.) + (f_.)*(x_)])*(b_.)^(m_)*(csc[(e_.) + (f_.)*(x_)])^2*(C_.) + (A_.)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] \&\& NeQ[C*m + A*(m + 1), 0] \&\& !LeQ[m, -1]$

Rule 4047

$Int[(csc[(e_.) + (f_.)*(x_)])*(b_.)^(m_)*((A_.) + csc[(e_.) + (f_.)*(x_)])*(B_.) + csc[(e_.) + (f_.)*(x_)])^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2), x] /; FreeQ[{b, e, f, A, B, C, m}, x]$

Rule 4076

$Int[((A_.) + csc[(e_.) + (f_.)*(x_)])*(B_.) + csc[(e_.) + (f_.)*(x_)])^2*(C_.) * (csc[(e_.) + (f_.)*(x_)])*(d_.)^(n_)*(csc[(e_.) + (f_.)*(x_)])*(b_.) + ($

```

a_)), x_Symbol] := -Simp[(b*C*Csc[e + f*x]*Cot[e + f*x]*(d*Csc[e + f*x])^n
/(f*(n + 2)), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2)
+ (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n +
2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &&
!LtQ[n, -1]

```

Rule 4096

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Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a
_)^(m_)), x_Symbol] := -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(m + n + 1), Int[(a + b*Csc[e + f
*x])^(m - 1)*(d*Csc[e + f*x])^n*Simp[a*A*(m + n + 1) + a*C*n + ((A*b + a*B)
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) + a*C*m)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && NeQ[a^2 -
b^2, 0] && GtQ[m, 0] && !LeQ[n, -1]

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Rubi steps

$$\begin{aligned}
\int \sec^m(c + dx)(a + b \sec(c + dx))^4(A + B \sec(c + dx)) dx &= \frac{bB \sec^{1+m}(c + dx)(a + b \sec(c + dx))^3 \sin(c + dx)}{d(4 + m)} + \\
&= \frac{b(Ab(4 + m) + aB(7 + m)) \sec^{1+m}(c + dx)(a + b \sec(c + dx))^2}{d(3 + m)(4 + m)} \\
&= \frac{b^2 (b^2 B(3 + m)^2 + 2aAb(4 + m)^2 + a^2 B(26 + 9m + 6m^2)) \sec^{1+m}(c + dx)(a + b \sec(c + dx))}{d(2 + m)(12 + 7m + 6m^2)} \\
&= \frac{b^2 (b^2 B(3 + m)^2 + 2aAb(4 + m)^2 + a^2 B(26 + 9m + 6m^2)) \sec^{1+m}(c + dx)}{d(2 + m)(12 + 7m + 6m^2)} \\
&= \frac{b(Ab^3(8 + 6m + m^2) + 4ab^2B(8 + 6m + m^2) + 2a^2B(26 + 9m + 6m^2)) \sec^{1+m}(c + dx)}{d(2 + m)(12 + 7m + 6m^2)} \\
&= \frac{b(Ab^3(8 + 6m + m^2) + 4ab^2B(8 + 6m + m^2) + 2a^2B(26 + 9m + 6m^2)) \sec^{1+m}(c + dx)}{d(2 + m)(12 + 7m + 6m^2)} \\
&= \frac{b(Ab^3(8 + 6m + m^2) + 4ab^2B(8 + 6m + m^2) + 2a^2B(26 + 9m + 6m^2)) \sec^{1+m}(c + dx)}{d(2 + m)(12 + 7m + 6m^2)}
\end{aligned}$$

Mathematica [A] time = 4.92, size = 365, normalized size = 0.67

$$\sqrt{-\tan^2(c+dx)} \csc(c+dx) \sec^{m-1}(c+dx)(a+b \sec(c+dx))^4(A+B \sec(c+dx)) \left(\frac{a^4 A \cos^5(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{m}{2}; \frac{m+2}{2}; \sec^2(c+dx)\right)}{m} \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sec[c + d*x]^m*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]

[Out] (Csc[c + d*x]*((a^4*A*Cos[c + d*x]^5*Hypergeometric2F1[1/2, m/2, (2 + m)/2, Sec[c + d*x]^2])/m + (a^3*(4*A*b + a*B)*Cos[c + d*x]^4*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sec[c + d*x]^2]))/(1 + m) + b*((2*a^2*(3*A*b + 2*a*B)*Cos[c + d*x]^3*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Sec[c + d*x]^2]))/(2 + m) + b*((2*a*(2*A*b + 3*a*B)*Cos[c + d*x]^2*Hypergeometric2F1[1/2, (3 + m)/2, (5 + m)/2, Sec[c + d*x]^2]))/(3 + m) + b*((A*b + 4*a*B)*Cos[c + d*x]*Hypergeometric2F1[1/2, (4 + m)/2, (6 + m)/2, Sec[c + d*x]^2]))/(4 + m) + (b*B*Hypergeometric2F1[1/2, (5 + m)/2, (7 + m)/2, Sec[c + d*x]^2]))/(5 + m))))*Sec[c + d*x]^(-1 + m)*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x])*Sqrt[-Tan[c + d*x]^2]]/(d*(b + a*Cos[c + d*x])^4*(B + A*Cos[c + d*x]))

fricas [F] time = 0.99, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb^4 \sec(dx+c)^5 + Aa^4 + (4Bab^3 + Ab^4) \sec(dx+c)^4 + 2(3Ba^2b^2 + 2Aab^3) \sec(dx+c)^3 + 2(2Bab^2 + 2Aa^2b) \sec(dx+c)^2 + (Bb^2 + 4Aab) \sec(dx+c) + Aa\right) \sec(dx+c)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^m*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] integral((B*b^4*sec(d*x + c)^5 + A*a^4 + (4*B*a*b^3 + A*b^4)*sec(d*x + c)^4 + 2*(3*B*a^2*b^2 + 2*A*a*b^3)*sec(d*x + c)^3 + 2*(2*B*a^3*b + 3*A*a^2*b^2)*sec(d*x + c)^2 + (B*a^4 + 4*A*a^3*b)*sec(d*x + c))*sec(d*x + c)^m, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx+c) + A)(b \sec(dx+c) + a)^4 \sec(dx+c)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^m*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^4*sec(d*x + c)^m, x)

maple [F] time = 2.48, size = 0, normalized size = 0.00

$$\int (\sec^m(dx+c))(a+b\sec(dx+c))^4(A+B\sec(dx+c)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sec(d*x+c)^m*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x)`

[Out] `int(sec(d*x+c)^m*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x)`

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^m*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="maxima")`

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(A + \frac{B}{\cos(c+dx)} \right) \left(a + \frac{b}{\cos(c+dx)} \right)^4 \left(\frac{1}{\cos(c+dx)} \right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^4*(1/cos(c + d*x))^m,x)`

[Out] `int((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^4*(1/cos(c + d*x))^m, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \sec(c + dx))(a + b \sec(c + dx))^4 \sec^m(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)**m*(a+b*sec(d*x+c))**4*(A+B*sec(d*x+c)),x)`

[Out] `Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**4*sec(c + d*x)**m, x)`

$$3.480 \quad \int \sec^m(c+dx)(a+b \sec(c+dx))^3(A+B \sec(c+dx)) dx$$

Optimal. Leaf size=366

$$\frac{b \sin(c+dx) \left(2a^2 B(m+4) + 3aAb(m+3) + b^2 B(m+2) \right) \sec^{m+1}(c+dx) \sin(c+dx) \left(a^3 A(m^2+4m+3) + 3 \right)}{d(m+1)(m+3)}$$

[Out] b*(b^2*B*(2+m)+3*a*A*b*(3+m)+2*a^2*B*(4+m))*sec(d*x+c)^(1+m)*sin(d*x+c)/d/(1+m)/(3+m)+b^2*(A*b*(3+m)+a*B*(5+m))*sec(d*x+c)^(2+m)*sin(d*x+c)/d/(2+m)/(3+m)+b*B*sec(d*x+c)^(1+m)*(a+b*sec(d*x+c))^2*sin(d*x+c)/d/(3+m)-(b^3*B*m*(2+m)+3*a*A*b^2*m*(3+m)+3*a^2*b*B*m*(3+m)+a^3*A*(m^2+4*m+3))*hypergeom([1/2, 1/2-1/2*m], [3/2-1/2*m], cos(d*x+c)^2)*sec(d*x+c)^(-1+m)*sin(d*x+c)/d/(3+m)/(-m^2+1)/(sin(d*x+c)^2)^(1/2)+(A*b^3*(1+m)+3*a*b^2*B*(1+m)+3*a^2*A*b*(2+m)+a^3*B*(2+m))*hypergeom([1/2, -1/2*m], [1-1/2*m], cos(d*x+c)^2)*sec(d*x+c)^m*sin(d*x+c)/d/m/(2+m)/(sin(d*x+c)^2)^(1/2)

Rubi [A] time = 0.79, antiderivative size = 366, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4026, 4076, 4047, 3772, 2643, 4046}

$$\frac{\sin(c+dx) \left(a^3 A(m^2+4m+3) + 3a^2 b B m(m+3) + 3a A b^2 m(m+3) + b^3 B m(m+2) \right) \sec^{m-1}(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{1}{2}-\frac{m}{2}; \frac{3}{2}-\frac{m}{2}; \cos^2(c+dx)\right)}{d(m+3)(1-m^2)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d*x]^m*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]

[Out] (b*(b^2*B*(2+m)+3*a*A*b*(3+m)+2*a^2*B*(4+m))*Sec[c+d*x]^(1+m)*Sin[c+d*x])/(d*(1+m)*(3+m))+(b^2*(A*b*(3+m)+a*B*(5+m))*Sec[c+d*x]^(2+m)*Sin[c+d*x])/(d*(2+m)*(3+m))+(b*B*Sec[c+d*x]^(1+m)*(a+b*Sec[c+d*x])^2*Sin[c+d*x])/(d*(3+m))-((b^3*B*m*(2+m)+3*a*A*b^2*m*(3+m)+3*a^2*b*B*m*(3+m)+a^3*A*(3+4*m+m^2))*Hypergeometric2F1[1/2, (1-m)/2, (3-m)/2, Cos[c+d*x]^2]*Sec[c+d*x]^(-1+m)*Sin[c+d*x])/(d*(3+m)*(1-m^2)*Sqrt[Sin[c+d*x]^2])+((A*b^3*(1+m)+3*a*b^2*B*(1+m)+3*a^2*A*b*(2+m)+a^3*B*(2+m))*Hypergeometric2F1[1/2, -m/2, (2-m)/2, Cos[c+d*x]^2]*Sec[c+d*x]^m*Sin[c+d*x])/(d*m*(2+m)*Sqrt[Sin[c+d*x]^2])

Rule 2643

Int[((b_.)*sin[(c_.)+(d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c+d*x]*(b*Sin[c+d*x])^(n+1)*Hypergeometric2F1[1/2, (n+1)/2, (n+3)/2, Sin[c

+ d*x]^2))/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rule 3772

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^ (n_), x_Symbol] :> Simp[(b*Csc[c + d*x])^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

Rule 4026

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^ (n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^ (m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] :> -Simp[(b*B*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x] + Dist[1/(m + n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*Simp[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B)*(m + n) + b^2*B*(m + n - 1))*Csc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && !(IGtQ[n, 1] && !IntegerQ[m])

Rule 4046

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^ (m_)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_.)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]

Rule 4047

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^ (m_)*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2), x] /; FreeQ[{b, e, f, A, B, C, m}, x]

Rule 4076

Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^ (n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] :> -Simp[(b*C*Csc[e + f*x]*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 2)), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2) + (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && !LtQ[n, -1]

Rubi steps

$$\begin{aligned}
\int \sec^m(c+dx)(a+b\sec(c+dx))^3(A+B\sec(c+dx))dx &= \frac{bB\sec^{1+m}(c+dx)(a+b\sec(c+dx))^2\sin(c+dx)}{d(3+m)} \\
&= \frac{b^2(Ab(3+m)+aB(5+m))\sec^{2+m}(c+dx)\sin(c+dx)}{d(2+m)(3+m)} \\
&= \frac{b^2(Ab(3+m)+aB(5+m))\sec^{2+m}(c+dx)\sin(c+dx)}{d(2+m)(3+m)} \\
&= \frac{b(b^2B(2+m)+3aAb(3+m)+2a^2B(4+m))\sec^{1+m}(c+dx)}{d(1+m)(3+m)} \\
&= \frac{b(b^2B(2+m)+3aAb(3+m)+2a^2B(4+m))\sec^{1+m}(c+dx)}{d(1+m)(3+m)} \\
&= \frac{b(b^2B(2+m)+3aAb(3+m)+2a^2B(4+m))\sec^{1+m}(c+dx)}{d(1+m)(3+m)}
\end{aligned}$$

Mathematica [A] time = 2.56, size = 307, normalized size = 0.84

$$\sqrt{-\tan^2(c+dx)}\csc(c+dx)\sec^{m-1}(c+dx)(a+b\sec(c+dx))^3(A+B\sec(c+dx))\left(\frac{a^3A\cos^4(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{m}{2}, \frac{m+2}{2}; \sec^2(c+dx)\right)}{m}\right)$$

Antiderivative was successfully verified.

```

[In] Integrate[Sec[c + d*x]^m*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]), x]
[Out] (Csc[c + d*x]*((a^3*A*Cos[c + d*x]^4*Hypergeometric2F1[1/2, m/2, (2 + m)/2, Sec[c + d*x]^2])/m + (a^2*(3*A*b + a*B)*Cos[c + d*x]^3*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sec[c + d*x]^2]))/(1 + m) + b*((3*a*(A*b + a*B)*Cos[c + d*x]^2*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Sec[c + d*x]^2]))/(2 + m) + b*(((A*b + 3*a*B)*Cos[c + d*x]*Hypergeometric2F1[1/2, (3 + m)/2, (5 + m)/2, Sec[c + d*x]^2]))/(3 + m) + (b*B*Hypergeometric2F1[1/2, (4 + m)/2, (6 + m)/2, Sec[c + d*x]^2]))/(4 + m)))*Sec[c + d*x]^(-1 + m)*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x])*Sqrt[-Tan[c + d*x]^2]]/(d*(b + a*Cos[c + d*x]))^3*(B + A*Cos[c + d*x]))

```

fricas [F] time = 0.80, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb^3\sec(dx+c)^4 + Aa^3 + (3Bab^2 + Ab^3)\sec(dx+c)^3 + 3(Ba^2b + Aab^2)\sec(dx+c)^2 + (Ba^3 + 3Aa^2b)\sec(dx+c) + A^2\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^m*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] integral((B*b^3*sec(d*x + c)^4 + A*a^3 + (3*B*a*b^2 + A*b^3)*sec(d*x + c)^3 + 3*(B*a^2*b + A*a*b^2)*sec(d*x + c)^2 + (B*a^3 + 3*A*a^2*b)*sec(d*x + c))*sec(d*x + c)^m, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(b \sec(dx + c) + a)^3 \sec(dx + c)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^m*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^3*sec(d*x + c)^m, x)

maple [F] time = 1.99, size = 0, normalized size = 0.00

$$\int (\sec^m(dx + c))(a + b \sec(dx + c))^3 (A + B \sec(dx + c)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^m*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x)

[Out] int(sec(d*x+c)^m*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(b \sec(dx + c) + a)^3 \sec(dx + c)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^m*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^3*sec(d*x + c)^m, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(A + \frac{B}{\cos(c + dx)} \right) \left(a + \frac{b}{\cos(c + dx)} \right)^3 \left(\frac{1}{\cos(c + dx)} \right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^3*(1/cos(c + d*x))^m, x)`

[Out] `int((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^3*(1/cos(c + d*x))^m, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \sec(c + dx)) (a + b \sec(c + dx))^3 \sec^m(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)**m*(a+b*sec(d*x+c))**3*(A+B*sec(d*x+c)), x)`

[Out] `Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**3*sec(c + d*x)**m, x)`

$$3.481 \quad \int \sec^m(c+dx)(a+b \sec(c+dx))^2(A+B \sec(c+dx)) dx$$

Optimal. Leaf size=261

$$\frac{\sin(c+dx)(a^2A(m+1)+2abBm+Ab^2m)\sec^{m-1}(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{1-m}{2}; \frac{3-m}{2}; \cos^2(c+dx)\right) \sin(c+dx)(a(m+1)+bB)}{d(1-m^2)\sqrt{\sin^2(c+dx)}} + \frac{\sin(c+dx)(a(m+1)+bB)}{d(1-m^2)\sqrt{\sin^2(c+dx)}}$$

[Out] b*(A*b*(2+m)+a*B*(3+m))*sec(d*x+c)^(1+m)*sin(d*x+c)/d/(1+m)/(2+m)+b*B*sec(d*x+c)^(1+m)*(a+b*sec(d*x+c))*sin(d*x+c)/d/(2+m)-(A*b^2*m+2*a*b*B*m+a^2*A*(1+m))*hypergeom([1/2, 1/2-1/2*m], [3/2-1/2*m], cos(d*x+c)^2)*sec(d*x+c)^(-1+m)*sin(d*x+c)/d/(-m^2+1)/(sin(d*x+c)^2)^(1/2)+(b^2*B*(1+m)+a*(2*A*b+B*a))*(2+m))*hypergeom([1/2, -1/2*m], [1-1/2*m], cos(d*x+c)^2)*sec(d*x+c)^m*sin(d*x+c)/d/m/(2+m)/(sin(d*x+c)^2)^(1/2)

Rubi [A] time = 0.41, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {4026, 4047, 3772, 2643, 4046}

$$\frac{\sin(c+dx)(a^2A(m+1)+2abBm+Ab^2m)\sec^{m-1}(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{1-m}{2}; \frac{3-m}{2}; \cos^2(c+dx)\right) \sin(c+dx)(a(m+1)+bB)}{d(1-m^2)\sqrt{\sin^2(c+dx)}} + \frac{\sin(c+dx)(a(m+1)+bB)}{d(1-m^2)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d*x]^m*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]

[Out] (b*(A*b*(2 + m) + a*B*(3 + m))*Sec[c + d*x]^(1 + m)*Sin[c + d*x])/(d*(1 + m)*(2 + m)) + (b*B*Sec[c + d*x]^(1 + m)*(a + b*Sec[c + d*x])*Sin[c + d*x])/(d*(2 + m)) - ((A*b^2*m + 2*a*b*B*m + a^2*A*(1 + m))*Hypergeometric2F1[1/2, (1 - m)/2, (3 - m)/2, Cos[c + d*x]^2]*Sec[c + d*x]^(-1 + m)*Sin[c + d*x])/(d*(1 - m^2)*Sqrt[Sin[c + d*x]^2]) + ((b^2*B*(1 + m) + a*(2*A*b + a*B))*(2 + m))*Hypergeometric2F1[1/2, -m/2, (2 - m)/2, Cos[c + d*x]^2]*Sec[c + d*x]^m*Ssin[c + d*x])/(d*m*(2 + m)*Sqrt[Sin[c + d*x]^2])

Rule 2643

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(b*Ssin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 4026

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] := -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(m + n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*Simp
[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B)*(m + n) + b^2*B*(m + n - 1))*C
sc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b
^2, 0] && GtQ[m, 1] && !(IGtQ[n, 1] && !IntegerQ[m])
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] := -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] := Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\begin{aligned}
\int \sec^m(c+dx)(a+b\sec(c+dx))^2(A+B\sec(c+dx))dx &= \frac{bB\sec^{1+m}(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{d(2+m)} + \dots \\
&= \frac{bB\sec^{1+m}(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{d(2+m)} + \dots \\
&= \frac{b(Ab(2+m)+aB(3+m))\sec^{1+m}(c+dx)\sin(c+dx)}{d(1+m)(2+m)} \\
&= \frac{b(Ab(2+m)+aB(3+m))\sec^{1+m}(c+dx)\sin(c+dx)}{d(1+m)(2+m)} \\
&= \frac{b(Ab(2+m)+aB(3+m))\sec^{1+m}(c+dx)\sin(c+dx)}{d(1+m)(2+m)}
\end{aligned}$$

Mathematica [A] time = 1.08, size = 239, normalized size = 0.92

$$\sqrt{-\tan^2(c+dx)} \csc(c+dx) \sec^{m+2}(c+dx) \left(a^2 A (m^3 + 6m^2 + 11m + 6) \cos^3(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{m}{2}; \frac{m+2}{2}; \sec^2(c+dx)\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sec[c + d*x]^m*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]

[Out] (Csc[c + d*x]*(a^2*A*(6 + 11*m + 6*m^2 + m^3)*Cos[c + d*x]^3*Hypergeometric2F1[1/2, m/2, (2 + m)/2, Sec[c + d*x]^2] + a*(2*A*b + a*B)*m*(6 + 5*m + m^2)*Cos[c + d*x]^2*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sec[c + d*x]^2] + b*m*(1 + m)*((A*b + 2*a*B)*(3 + m)*Cos[c + d*x]*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Sec[c + d*x]^2] + b*B*(2 + m)*Hypergeometric2F1[1/2, (3 + m)/2, (5 + m)/2, Sec[c + d*x]^2]))*Sec[c + d*x]^(2 + m)*Sqrt[-Tan[c + d*x]^2])/(d*m*(1 + m)*(2 + m)*(3 + m))

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb^2 \sec(dx+c)^3 + Aa^2 + (2Bab + Ab^2) \sec(dx+c)^2 + (Ba^2 + 2Aab) \sec(dx+c)\right) \sec(dx+c)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^m*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] $\text{integral}((B*b^2*\sec(dx + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*\sec(dx + c)^2 + (B*a^2 + 2*A*a*b)*\sec(dx + c))*\sec(dx + c)^m, x)$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(b \sec(dx + c) + a)^2 \sec(dx + c)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(\sec(dx+c)^m*(a+b*\sec(dx+c))^2*(A+B*\sec(dx+c)),x, \text{algorithm}="giac")$

[Out] $\text{integrate}((B*\sec(dx + c) + A)*(b*\sec(dx + c) + a)^2*\sec(dx + c)^m, x)$

maple [F] time = 3.60, size = 0, normalized size = 0.00

$$\int (\sec^m(dx + c))(a + b \sec(dx + c))^2 (A + B \sec(dx + c)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(\sec(dx+c)^m*(a+b*\sec(dx+c))^2*(A+B*\sec(dx+c)),x)$

[Out] $\text{int}(\sec(dx+c)^m*(a+b*\sec(dx+c))^2*(A+B*\sec(dx+c)),x)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(b \sec(dx + c) + a)^2 \sec(dx + c)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(\sec(dx+c)^m*(a+b*\sec(dx+c))^2*(A+B*\sec(dx+c)),x, \text{algorithm}="maxima")$

[Out] $\text{integrate}((B*\sec(dx + c) + A)*(b*\sec(dx + c) + a)^2*\sec(dx + c)^m, x)$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(A + \frac{B}{\cos(c + dx)} \right) \left(a + \frac{b}{\cos(c + dx)} \right)^2 \left(\frac{1}{\cos(c + dx)} \right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((A + B/\cos(c + dx))*(a + b/\cos(c + dx))^2*(1/\cos(c + dx))^m,x)$

[Out] $\text{int}((A + B/\cos(c + dx))*(a + b/\cos(c + dx))^2*(1/\cos(c + dx))^m, x)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \sec(c + dx))(a + b \sec(c + dx))^2 \sec^m(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**m*(a+b*sec(d*x+c))**2*(A+B*sec(d*x+c)),x)

[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**2*sec(c + d*x)**m, x)

$$3.482 \quad \int \sec^m(c+dx)(a+b\sec(c+dx))(A+B\sec(c+dx)) dx$$

Optimal. Leaf size=177

$$\frac{\sin(c+dx)(aA(m+1)+bBm)\sec^{m-1}(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{1-m}{2}; \frac{3-m}{2}; \cos^2(c+dx)\right)}{d(1-m^2)\sqrt{\sin^2(c+dx)}} + \frac{(aB+Ab)\sin(c+dx)\sec^m(c+dx)}{dm\sqrt{\sin^2(c+dx)}}$$

[Out] b*B*sec(d*x+c)^(1+m)*sin(d*x+c)/d/(1+m)-(b*B*m+a*A*(1+m))*hypergeom([1/2, 1/2-1/2*m], [3/2-1/2*m], cos(d*x+c)^2)*sec(d*x+c)^(-1+m)*sin(d*x+c)/d/(-m^2+1)/(sin(d*x+c)^2)^(1/2)+(A*b+B*a)*hypergeom([1/2, -1/2*m], [1-1/2*m], cos(d*x+c)^2)*sec(d*x+c)^m*sin(d*x+c)/d/m/(sin(d*x+c)^2)^(1/2)

Rubi [A] time = 0.20, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {3997, 3787, 3772, 2643}

$$\frac{\sin(c+dx)(aA(m+1)+bBm)\sec^{m-1}(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{1-m}{2}; \frac{3-m}{2}; \cos^2(c+dx)\right)}{d(1-m^2)\sqrt{\sin^2(c+dx)}} + \frac{(aB+Ab)\sin(c+dx)\sec^m(c+dx)}{dm\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d*x]^m*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]), x]

[Out] (b*B*Sec[c + d*x]^(1 + m)*Sin[c + d*x])/(d*(1 + m)) - ((b*B*m + a*A*(1 + m))*Hypergeometric2F1[1/2, (1 - m)/2, (3 - m)/2, Cos[c + d*x]^2]*Sec[c + d*x]^(-1 + m)*Sin[c + d*x])/(d*(1 - m^2)*Sqrt[Sin[c + d*x]^2]) + ((A*b + a*B)*Hypergeometric2F1[1/2, -m/2, (2 - m)/2, Cos[c + d*x]^2]*Sec[c + d*x]^m*Sin[c + d*x])/(d*m*Sqrt[Sin[c + d*x]^2])

Rule 2643

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rule 3772

Int[(csc[(c_.) + (d_.)*(x_)])*(b_.)^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x])^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/((Sin[c + d*x]/b)^n, x]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 3997

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> -Simp[(b*B*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && !LeQ[n, -1]

Rubi steps

$$\begin{aligned} \int \sec^m(c + dx)(a + b \sec(c + dx))(A + B \sec(c + dx)) dx &= \frac{bB \sec^{1+m}(c + dx) \sin(c + dx)}{d(1 + m)} + \frac{\int \sec^m(c + dx)(bBm + aA + bA) dx}{d(1 + m)} \\ &= \frac{bB \sec^{1+m}(c + dx) \sin(c + dx)}{d(1 + m)} + (Ab + aB) \int \sec^{1+m}(c + dx) dx \\ &= \frac{bB \sec^{1+m}(c + dx) \sin(c + dx)}{d(1 + m)} + ((Ab + aB) \cos^m(c + dx) - \int \sec^{1+m}(c + dx) dx) \\ &= \frac{bB \sec^{1+m}(c + dx) \sin(c + dx)}{d(1 + m)} - \frac{\left(aA + \frac{bBm}{1+m}\right) {}_2F_1\left(\frac{1}{2}, \frac{m+1}{2}; \frac{m+3}{2}; \sec^2(c + dx)\right) + aA}{d(1 + m)} \end{aligned}$$

Mathematica [A] time = 0.46, size = 168, normalized size = 0.95

$$\frac{\sqrt{-\tan^2(c + dx)} \csc(c + dx) \sec^{m+1}(c + dx) \left(m(m + 2)(aB + Ab) \cos(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{m+1}{2}; \frac{m+3}{2}; \sec^2(c + dx)\right) + aA\right)}{dm(m + 1)(n)}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[c + d*x]^m*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]

[Out] (Csc[c + d*x]*(a*A*(2 + 3*m + m^2)*Cos[c + d*x]^2*Hypergeometric2F1[1/2, m/2, (2 + m)/2, Sec[c + d*x]^2] + (A*b + a*B)*m*(2 + m)*Cos[c + d*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sec[c + d*x]^2] + b*B*m*(1 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Sec[c + d*x]^2])*Sec[c + d*x]^(1 + m)*Sqrt[-Tan[c + d*x]^2])/(d*m*(1 + m)*(2 + m))

fricas [F] time = 0.97, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb \sec(dx+c)^2 + Aa + (Ba + Ab) \sec(dx+c)\right) \sec(dx+c)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^m*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] integral((B*b*sec(d*x + c)^2 + A*a + (B*a + A*b)*sec(d*x + c))*sec(d*x + c)^m, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx+c) + A)(b \sec(dx+c) + a) \sec(dx+c)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^m*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)*sec(d*x + c)^m, x)

maple [F] time = 2.18, size = 0, normalized size = 0.00

$$\int (\sec^m(dx+c))(a+b \sec(dx+c))(A+B \sec(dx+c)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^m*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x)

[Out] int(sec(d*x+c)^m*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx+c) + A)(b \sec(dx+c) + a) \sec(dx+c)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^m*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)*sec(d*x + c)^m, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A + \frac{B}{\cos(c+dx)}\right) \left(a + \frac{b}{\cos(c+dx)}\right) \left(\frac{1}{\cos(c+dx)}\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B/cos(c + d*x))*(a + b/cos(c + d*x))*(1/cos(c + d*x))^m, x)`

[Out] `int((A + B/cos(c + d*x))*(a + b/cos(c + d*x))*(1/cos(c + d*x))^m, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \sec(c + dx))(a + b \sec(c + dx)) \sec^m(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)**m*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)), x)`

[Out] `Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))*sec(c + d*x)**m, x)`

$$3.483 \quad \int \cos^{\frac{7}{2}}(c + dx)(a + a \sec(c + dx))(A + B \sec(c + dx)) dx$$

Optimal. Leaf size=132

$$\frac{2a(5A + 7B)F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{21d} + \frac{6a(A + B)E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{5d} + \frac{2a(A + B) \sin(c + dx) \cos^{\frac{3}{2}}(c + dx)}{5d} + \frac{2a(5A + 7B) \sin(c + dx)}{5d}$$

[Out] $6/5*a*(A+B)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/d+2/21*a*(5*A+7*B)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/d+2/5*a*(A+B)*\cos(d*x+c)^{(3/2)}*\sin(d*x+c)/d+2/7*a*A*\cos(d*x+c)^{(5/2)}*\sin(d*x+c)/d+2/21*a*(5*A+7*B)*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/d$

Rubi [A] time = 0.23, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {2954, 2968, 3023, 2748, 2635, 2641, 2639}

$$\frac{2a(5A + 7B)F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{21d} + \frac{6a(A + B)E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{5d} + \frac{2a(A + B) \sin(c + dx) \cos^{\frac{3}{2}}(c + dx)}{5d} + \frac{2a(5A + 7B) \sin(c + dx)}{5d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Cos}[c + d*x]^{(7/2)}*(a + a*\text{Sec}[c + d*x])*(A + B*\text{Sec}[c + d*x]), x]$

[Out] $(6*a*(A + B)*\text{EllipticE}[(c + d*x)/2, 2])/(5*d) + (2*a*(5*A + 7*B)*\text{EllipticF}[(c + d*x)/2, 2])/(21*d) + (2*a*(5*A + 7*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sin}[c + d*x])/(21*d) + (2*a*(A + B)*\text{Cos}[c + d*x]^{(3/2)}*\text{Sin}[c + d*x])/(5*d) + (2*a*A*\text{Cos}[c + d*x]^{(5/2)}*\text{Sin}[c + d*x])/(7*d)$

Rule 2635

$\text{Int}[(b*.)*\sin[(c_.) + (d_.)*(x_.)]^{(n_.)}, x_Symbol] \rightarrow -\text{Simp}[(b*\text{Cos}[c + d*x])*(b*\text{Sin}[c + d*x])^{(n - 1)})/(d*n), x] + \text{Dist}[(b^2*(n - 1))/n, \text{Int}[(b*\text{Sin}[c + d*x])^{(n - 2)}, x], x] /;$ FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2639

$\text{Int}[\text{Sqrt}[\sin[(c_.) + (d_.)*(x_.)]], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticE}[(1*(c - P i/2 + d*x))/2, 2])/d, x] /;$ FreeQ[{c, d}, x]

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_.)]], x_Symbol] := Simp[(2*EllipticF[(1*(c -
  Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_.)]^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_.)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_.)]^(p_.), x_Symbol] := Dist[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)]*(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] := Int[(a + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2), x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)] + (C_.)*sin[(e_.) + (f_.)*(x_.)]^2), x_Symbol] := -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))(A+B\sec(c+dx))dx &= \int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))(B+A\cos(c+dx))dx \\
&= \int \cos^{\frac{3}{2}}(c+dx)(aB+(aA+aB)\cos(c+dx)+aA\cos^2(c+dx))dx \\
&= \frac{2aA\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{2}{7} \int \cos^{\frac{3}{2}}(c+dx)(aB+aA\cos(c+dx))dx \\
&= \frac{2aA\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d} + (a(A+B)) \int \cos^{\frac{5}{2}}(c+dx)dx \\
&= \frac{2a(5A+7B)\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2a(A+B)}{21d} \int \cos^{\frac{5}{2}}(c+dx)dx \\
&= \frac{6a(A+B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2a(5A+7B)F\left(\frac{1}{2}(c+dx)\right)}{21d}
\end{aligned}$$

Mathematica [C] time = 6.29, size = 872, normalized size = 6.61

$$a \left(\sqrt{\cos(c+dx)}(\cos(c+dx)+1) \left(-\frac{3(A+B)\cot(c)}{5d} + \frac{(23A+28B)\cos(dx)\sin(c)}{84d} + \frac{(A+B)\cos(2dx)\sin(2c)}{10d} \right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]

[Out] a*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*((-3*(A + B)*Cot[c])/(5*d) + ((23*A + 28*B)*Cos[d*x]*Sin[c])/(84*d) + ((A + B)*Cos[2*d*x]*Sin[2*c])/(10*d) + (A*Cos[3*d*x]*Sin[3*c])/(28*d) + ((23*A + 28*B)*Cos[c]*Sin[d*x])/(84*d) + ((A + B)*Cos[2*c]*Sin[2*d*x])/(10*d) + (A*Cos[3*c]*Sin[3*d*x])/(28*d) - (5*A*(1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]])*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]])]/(21*d*Sqrt[1 + Cot[c]^2]) - (B*(1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x -

$$\frac{\text{ArcTan}[\text{Cot}[c]] \cdot \sqrt{1 - \sin[d*x - \text{ArcTan}[\text{Cot}[c]]]} \cdot \sqrt{-(\sqrt{1 + \text{Cot}[c]^2} \cdot \sin[c] \cdot \sin[d*x - \text{ArcTan}[\text{Cot}[c]]]) \cdot \sqrt{1 + \sin[d*x - \text{ArcTan}[\text{Cot}[c]]]}}}{(3*d*\sqrt{1 + \text{Cot}[c]^2}) - (3*A*(1 + \cos[c + d*x])*\text{Csc}[c]*\text{Sec}[c/2 + (d*x)/2])^2 * ((\text{HypergeometricPFQ}[-1/2, -1/4, \{3/4\}, \cos[d*x + \text{ArcTan}[\text{Tan}[c]]]^2] * \sin[d*x + \text{ArcTan}[\text{Tan}[c]]] * \text{Tan}[c]) / (\sqrt{1 - \cos[d*x + \text{ArcTan}[\text{Tan}[c]]]} * \sqrt{1 + \cos[d*x + \text{ArcTan}[\text{Tan}[c]]]} * \sqrt{\cos[c] * \cos[d*x + \text{ArcTan}[\text{Tan}[c]]]} * \sqrt{1 + \text{Tan}[c]^2}) * \sqrt{1 + \text{Tan}[c]^2}) - ((\sin[d*x + \text{ArcTan}[\text{Tan}[c]]] * \text{Tan}[c]) / \sqrt{1 + \text{Tan}[c]^2} + (2*\cos[c]^2 * \cos[d*x + \text{ArcTan}[\text{Tan}[c]]] * \sqrt{1 + \text{Tan}[c]^2}) / (\cos[c]^2 + \sin[c]^2)) / \sqrt{\cos[c] * \cos[d*x + \text{ArcTan}[\text{Tan}[c]]]} * \sqrt{1 + \text{Tan}[c]^2})} / (10*d) - (3*B*(1 + \cos[c + d*x]) * \text{Csc}[c] * \text{Sec}[c/2 + (d*x)/2])^2 * ((\text{HypergeometricPFQ}[-1/2, -1/4, \{3/4\}, \cos[d*x + \text{ArcTan}[\text{Tan}[c]]]^2] * \sin[d*x + \text{ArcTan}[\text{Tan}[c]]] * \text{Tan}[c]) / (\sqrt{1 - \cos[d*x + \text{ArcTan}[\text{Tan}[c]]]} * \sqrt{1 + \cos[d*x + \text{ArcTan}[\text{Tan}[c]]]} * \sqrt{\cos[c] * \cos[d*x + \text{ArcTan}[\text{Tan}[c]]]} * \sqrt{1 + \text{Tan}[c]^2}) * \sqrt{1 + \text{Tan}[c]^2}) - ((\sin[d*x + \text{ArcTan}[\text{Tan}[c]]] * \text{Tan}[c]) / \sqrt{1 + \text{Tan}[c]^2} + (2*\cos[c]^2 * \cos[d*x + \text{ArcTan}[\text{Tan}[c]]] * \sqrt{1 + \text{Tan}[c]^2}) / (\cos[c]^2 + \sin[c]^2)) / \sqrt{\cos[c] * \cos[d*x + \text{ArcTan}[\text{Tan}[c]]]} * \sqrt{1 + \text{Tan}[c]^2})} / (10*d))$$

fricas [F] time = 1.07, size = 0, normalized size = 0.00

$$\text{integral} \left((Ba \cos(dx + c)^3 \sec(dx + c)^2 + (A + B)a \cos(dx + c)^3 \sec(dx + c) + Aa \cos(dx + c)^3) \sqrt{\cos(dx + c)}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] integral((B*a*cos(d*x + c)^3*sec(d*x + c)^2 + (A + B)*a*cos(d*x + c)^3*sec(d*x + c) + A*a*cos(d*x + c)^3)*sqrt(cos(d*x + c)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(a \sec(dx + c) + a) \cos(dx + c)^{\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)*cos(d*x + c)^(7/2), x)

maple [B] time = 4.66, size = 383, normalized size = 2.90

$$2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} a \left(240A \cos\left(\frac{dx}{2} + \frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + (-528A - 168B)\left(\sin^6\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \dots\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x)`

[Out]
$$-2/105*((2*\cos(1/2*d*x+1/2*c)^2-1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*a*(240*A*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^8+(-528*A-168*B)*\sin(1/2*d*x+1/2*c)^6*\cos(1/2*d*x+1/2*c)+(448*A+308*B)*\sin(1/2*d*x+1/2*c)^4*\cos(1/2*d*x+1/2*c)+(-122*A-112*B)*\sin(1/2*d*x+1/2*c)^2*\cos(1/2*d*x+1/2*c)+25*A*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})-63*A*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)})+35*B*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})-63*B*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)}))/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(a \sec(dx + c) + a) \cos(dx + c)^{\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="maxima")`

[Out] `integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)*cos(d*x + c)^(7/2), x)`

mupad [B] time = 0.89, size = 166, normalized size = 1.26

$$\frac{2Ba \left(\sqrt{\cos(c+dx)} \sin(c+dx) + F\left(\frac{c}{2} + \frac{dx}{2} \middle| 2\right) \right)}{3d} - \frac{2Aa \cos(c+dx)^{7/2} \sin(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{7}{4}; \frac{11}{4}; \cos(c+dx)\right)}{7d \sqrt{\sin(c+dx)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(c + d*x)^(7/2)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x)),x)`

[Out]
$$(2*B*a*(\cos(c + d*x)^{(1/2)}*\sin(c + d*x) + \text{ellipticF}(c/2 + (d*x)/2, 2)))/(3*d) - (2*A*a*\cos(c + d*x)^{(7/2)}*\sin(c + d*x)*\text{hypergeom}([1/2, 7/4], 11/4, \cos(c + d*x)^2))/(7*d*(\sin(c + d*x)^2)^{(1/2)}) - (2*A*a*\cos(c + d*x)^{(9/2)}*\sin(c + d*x)*\text{hypergeom}([1/2, 9/4], 13/4, \cos(c + d*x)^2))/(9*d*(\sin(c + d*x)^2)^{(1/2)}) - (2*B*a*\cos(c + d*x)^{(7/2)}*\sin(c + d*x)*\text{hypergeom}([1/2, 7/4], 11/4, \cos(c + d*x)^2))/(7*d*(\sin(c + d*x)^2)^{(1/2)})$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```


$$3.484 \quad \int \cos^{\frac{5}{2}}(c + dx)(a + a \sec(c + dx))(A + B \sec(c + dx)) dx$$

Optimal. Leaf size=101

$$\frac{2a(A + B)F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3d} + \frac{2a(3A + 5B)E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{5d} + \frac{2a(A + B) \sin(c + dx)\sqrt{\cos(c + dx)}}{3d} + \frac{2aA \sin(c + dx)}{5d}$$

[Out] $2/5*a*(3*A+5*B)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/d+2/3*a*(A+B)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/d+2/5*a*A*\cos(d*x+c)^{(3/2)}*\sin(d*x+c)/d+2/3*a*(A+B)*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/d$

Rubi [A] time = 0.21, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {2954, 2968, 3023, 2748, 2639, 2635, 2641}

$$\frac{2a(A + B)F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3d} + \frac{2a(3A + 5B)E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{5d} + \frac{2a(A + B) \sin(c + dx)\sqrt{\cos(c + dx)}}{3d} + \frac{2aA \sin(c + dx)}{5d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Cos}[c + d*x]^{(5/2)}*(a + a*\text{Sec}[c + d*x])*(A + B*\text{Sec}[c + d*x]), x]$

[Out] $(2*a*(3*A + 5*B)*\text{EllipticE}[(c + d*x)/2, 2])/(5*d) + (2*a*(A + B)*\text{EllipticF}[(c + d*x)/2, 2])/(3*d) + (2*a*(A + B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sin}[c + d*x])/(3*d) + (2*a*A*\text{Cos}[c + d*x]^{(3/2)}*\text{Sin}[c + d*x])/(5*d)$

Rule 2635

$\text{Int}[(b_*)*\sin[(c_*) + (d_*)(x_)]^{(n_)}, x_Symbol] \rightarrow -\text{Simp}[(b*\text{Cos}[c + d*x])*(b*\text{Sin}[c + d*x])^{(n - 1)})/(d*n), x] + \text{Dist}[(b^2*(n - 1))/n, \text{Int}[(b*\text{Sin}[c + d*x])^{(n - 2)}, x], x] /;$ FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2639

$\text{Int}[\text{Sqrt}[\sin[(c_*) + (d_*)(x_)]], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticE}[(1*(c - P i/2 + d*x))/2, 2])/d, x] /;$ FreeQ[{c, d}, x]

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_.)]], x_Symbol] := Simp[(2*EllipticF[(1*(c -
  Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_.)]^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_.)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_.)]^(p_.), x_Symbol] := Dist[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)]*(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] := Int[(a + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2), x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)] + (C_.)*sin[(e_.) + (f_.)*(x_.)]^2), x_Symbol] := -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))(A+B\sec(c+dx))dx &= \int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))(B+A\cos(c+dx))dx \\
&= \int \sqrt{\cos(c+dx)}(aB+(aA+aB)\cos(c+dx)+aA\cos^2(c+dx))dx \\
&= \frac{2aA\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2}{5} \int \sqrt{\cos(c+dx)}dx \\
&= \frac{2aA\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + (a(A+B)) \int \cos^{\frac{3}{2}}(c+dx)dx \\
&= \frac{2a(3A+5B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2a(A+B)\sqrt{\cos(c+dx)}}{3d} \\
&= \frac{2a(3A+5B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2a(A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}
\end{aligned}$$

Mathematica [C] time = 6.22, size = 830, normalized size = 8.22

$$a \left(\sqrt{\cos(c+dx)}(\cos(c+dx)+1) \left(-\frac{(3A+5B)\cot(c)}{5d} + \frac{(A+B)\cos(dx)\sin(c)}{3d} + \frac{A\cos(2dx)\sin(2c)}{10d} + \frac{(A+B)\cos^2(c)}{3d} \right) \right)$$

Warning: Unable to verify antiderivative.

```

[In] Integrate[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
[Out] a*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*(-1/5*((3*A +
5*B)*Cot[c])/d + ((A + B)*Cos[d*x]*Sin[c])/(3*d) + (A*Cos[2*d*x]*Sin[2*c])
/(10*d) + ((A + B)*Cos[c]*Sin[d*x])/(3*d) + (A*Cos[2*c]*Sin[2*d*x])/(10*d))
- (A*(1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*
x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[
1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - A
rcTan[Cot[c]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqrt[1 + Cot[c]^
2]) - (B*(1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin
[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*Sq
rt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x

```

- ArcTan[Cot[c]])) * Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]] / (3*d*Sqrt[1 + Cot[c]^2]) - (3*A*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*(HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2*Sin[d*x + ArcTan[Tan[c]]]*Tan[c]) / (Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c]) / Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]) / (Cos[c]^2 + Sin[c]^2)) / Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])) / (10*d) - (B*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*(HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2*Sin[d*x + ArcTan[Tan[c]]]*Tan[c]) / (Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c]) / Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]) / (Cos[c]^2 + Sin[c]^2)) / Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])) / (2*d))

fricas [F] time = 0.68, size = 0, normalized size = 0.00

integral((Ba cos(dx + c)^2 sec(dx + c)^2 + (A + B)a cos(dx + c)^2 sec(dx + c) + Aa cos(dx + c)^2) sqrt(cos(dx + c)), x)

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] integral((B*a*cos(d*x + c)^2*sec(d*x + c)^2 + (A + B)*a*cos(d*x + c)^2*sec(d*x + c) + A*a*cos(d*x + c)^2)*sqrt(cos(d*x + c)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(a \sec(dx + c) + a) \cos(dx + c)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)*cos(d*x + c)^(5/2), x)

maple [B] time = 4.07, size = 355, normalized size = 3.51

$$2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} a \left(-24A \cos\left(\frac{dx}{2} + \frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + (44A + 20B)\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x)`

[Out]
$$-2/15*((2*\cos(1/2*d*x+1/2*c)^2-1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*a*(-24*A*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^6+(44*A+20*B)*\sin(1/2*d*x+1/2*c)^4*\cos(1/2*d*x+1/2*c)+(-16*A-10*B)*\sin(1/2*d*x+1/2*c)^2*\cos(1/2*d*x+1/2*c)+5*A*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})-9*A*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)})+5*B*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})-15*B*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)}))/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(a \sec(dx + c) + a) \cos(dx + c)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="maxima")`

[Out] `integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)*cos(d*x + c)^(5/2), x)`

mupad [B] time = 0.36, size = 128, normalized size = 1.27

$$\frac{2 A a \left(\sqrt{\cos(c + dx)} \sin(c + dx) + F\left(\frac{c}{2} + \frac{dx}{2} \middle| 2\right) \right)}{3 d} + \frac{2 B a \left(\sqrt{\cos(c + dx)} \sin(c + dx) + F\left(\frac{c}{2} + \frac{dx}{2} \middle| 2\right) \right)}{3 d} + \frac{2 B a}{3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(c + d*x)^(5/2)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x)),x)`

[Out]
$$(2*A*a*(\cos(c + d*x)^{(1/2)}*\sin(c + d*x) + \text{ellipticF}(c/2 + (d*x)/2, 2)))/(3*d) + (2*B*a*(\cos(c + d*x)^{(1/2)}*\sin(c + d*x) + \text{ellipticF}(c/2 + (d*x)/2, 2)))/(3*d) + (2*B*a*\text{ellipticE}(c/2 + (d*x)/2, 2))/d - (2*A*a*\cos(c + d*x)^{(7/2)}*\sin(c + d*x)*\text{hypergeom}([1/2, 7/4], 11/4, \cos(c + d*x)^2))/(7*d*(\sin(c + d*x)^2)^{(1/2)})$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

$$3.485 \quad \int \cos^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))(A + B \sec(c + dx)) dx$$

Optimal. Leaf size=70

$$\frac{2a(A + 3B)F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3d} + \frac{2a(A + B)E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{d} + \frac{2aA \sin(c + dx)\sqrt{\cos(c + dx)}}{3d}$$

[Out] 2*a*(A+B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*a*(A+3*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*a*A*sin(d*x+c)*cos(d*x+c)^(1/2)/d

Rubi [A] time = 0.19, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {2954, 2968, 3023, 2748, 2641, 2639}

$$\frac{2a(A + 3B)F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3d} + \frac{2a(A + B)E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{d} + \frac{2aA \sin(c + dx)\sqrt{\cos(c + dx)}}{3d}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]

[Out] (2*a*(A + B)*EllipticE[(c + d*x)/2, 2])/d + (2*a*(A + 3*B)*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*a*A*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)

Rule 2639

Int[Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2748

Int[((b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)])*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_.))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] := Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
a*d, 0] && !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Int[(a
+ b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] := -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
!LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \cos^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))(A + B \sec(c + dx)) dx &= \int \frac{(a + a \cos(c + dx))(B + A \cos(c + dx))}{\sqrt{\cos(c + dx)}} dx \\
&= \int \frac{aB + (aA + aB) \cos(c + dx) + aA \cos^2(c + dx)}{\sqrt{\cos(c + dx)}} dx \\
&= \frac{2aA\sqrt{\cos(c + dx)} \sin(c + dx)}{3d} + \frac{2}{3} \int \frac{\frac{1}{2}a(A + 3B) + \dots}{\sqrt{\cos(c + dx)}} dx \\
&= \frac{2aA\sqrt{\cos(c + dx)} \sin(c + dx)}{3d} + (a(A + B)) \int \sqrt{\cos(c + dx)} dx \\
&= \frac{2a(A + B)E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{d} + \frac{2a(A + 3B)F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3d}
\end{aligned}$$

Mathematica [C] time = 6.24, size = 309, normalized size = 4.41

$$a(\cos(c + dx) + 1) \sec^2\left(\frac{1}{2}(c + dx)\right) \left(\sqrt{\sin^2(\tan^{-1}(\tan(c)) + dx)}\right) \left(-4(A + 3B) \sin(c) \sqrt{\csc^2(c)} \sqrt{\sec^2(c)} \cos(c + dx)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
[Out] (a*(1 + Cos[c + d*x])*Sec[(c + d*x)/2]^2*(-6*(A + B)*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sec[c]*Sin[d*x + ArcTan[Tan[c]]] + (9*(A + B)*Cos[c - d*x - ArcTan[Tan[c]]]*Csc[c]*Sec[c] + 3*A*Cos[c + d*x + ArcTan[Tan[c]]]*Csc[c]*Sec[c] + 3*B*Cos[c + d*x + ArcTan[Tan[c]]]*Csc[c]*Sec[c] - 12*A*Cos[c + d*x]*Cot[c]*Sqrt[Sec[c]^2] - 12*B*Cos[c + d*x]*Cot[c]*Sqrt[Sec[c]^2] - 4*(A + 3*B)*Cos[c + d*x]*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Sec[c]^2]*Sec[d*x - ArcTan[Cot[c]]]*Sin[c] + 4*A*Cos[c + d*x]*Sqrt[Sec[c]^2]*Sin[c + d*x])*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2]))/(12*d*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2])
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Ba \cos(dx + c) \sec(dx + c)^2 + (A + B)a \cos(dx + c) \sec(dx + c) + Aa \cos(dx + c)\right) \sqrt{\cos(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((B*a*cos(d*x + c)*sec(d*x + c)^2 + (A + B)*a*cos(d*x + c)*sec(d*x + c) + A*a*cos(d*x + c))*sqrt(cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(a \sec(dx + c) + a) \cos(dx + c)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)*cos(d*x + c)^(3/2), x)
```

maple [B] time = 4.40, size = 321, normalized size = 4.59

$$2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} a \left(4A \cos\left(\frac{dx}{2} + \frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + A\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}\right)\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)), x)

[Out] $-2/3*((2*\cos(1/2*d*x+1/2*c)^2-1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*a*(4*A*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^4+A*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c), 2^{(1/2)})-3*A*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticE}(\cos(1/2*d*x+1/2*c), 2^{(1/2)})-2*A*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^2+3*B*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c), 2^{(1/2)})-3*B*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticE}(\cos(1/2*d*x+1/2*c), 2^{(1/2)}))/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(a \sec(dx + c) + a) \cos(dx + c)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)), x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)*cos(d*x + c)^(3/2), x)

mupad [B] time = 2.66, size = 85, normalized size = 1.21

$$\frac{2 A a \left(\sqrt{\cos(c + dx)} \sin(c + dx) + F\left(\frac{c}{2} + \frac{dx}{2} \middle| 2\right) \right)}{3 d} + \frac{2 A a E\left(\frac{c}{2} + \frac{dx}{2} \middle| 2\right)}{d} + \frac{2 B a E\left(\frac{c}{2} + \frac{dx}{2} \middle| 2\right)}{d} + \frac{2 B a F\left(\frac{c}{2} + \frac{dx}{2} \middle| 2\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^(3/2)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x)), x)

[Out] $(2*A*a*(\cos(c + d*x)^{(1/2)}*\sin(c + d*x) + \text{ellipticF}(c/2 + (d*x)/2, 2)))/(3*d) + (2*A*a*\text{ellipticE}(c/2 + (d*x)/2, 2))/d + (2*B*a*\text{ellipticE}(c/2 + (d*x)/2, 2))/d + (2*B*a*\text{ellipticF}(c/2 + (d*x)/2, 2))/d$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**(3/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x)

[Out] Timed out

$$3.486 \quad \int \sqrt{\cos(c + dx)} (a + a \sec(c + dx))(A + B \sec(c + dx)) dx$$

Optimal. Leaf size=66

$$\frac{2a(A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2aB \sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

[Out] 2*a*(A-B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2*a*(A+B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2*a*B*sin(d*x+c)/d/cos(d*x+c)^(1/2)

Rubi [A] time = 0.19, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {2954, 2968, 3021, 2748, 2641, 2639}

$$\frac{2a(A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2aB \sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]

[Out] (2*a*(A - B)*EllipticE[(c + d*x)/2, 2])/d + (2*a*(A + B)*EllipticF[(c + d*x)/2, 2])/d + (2*a*B*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)])*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_.))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] := Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
a*d, 0] && !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Int[(a
+ b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] := -Simp[((A*b^2
- a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{\cos(c + dx)} (a + a \sec(c + dx))(A + B \sec(c + dx)) dx &= \int \frac{(a + a \cos(c + dx))(B + A \cos(c + dx))}{\cos^{\frac{3}{2}}(c + dx)} dx \\
&= \int \frac{aB + (aA + aB) \cos(c + dx) + aA \cos^2(c + dx)}{\cos^{\frac{3}{2}}(c + dx)} dx \\
&= \frac{2aB \sin(c + dx)}{d\sqrt{\cos(c + dx)}} + 2 \int \frac{\frac{1}{2}a(A + B) + \frac{1}{2}a(A - B) \cos(c + dx)}{\sqrt{\cos(c + dx)}} dx \\
&= \frac{2aB \sin(c + dx)}{d\sqrt{\cos(c + dx)}} + (a(A - B)) \int \sqrt{\cos(c + dx)} dx - \\
&= \frac{2a(A - B)E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{d} + \frac{2a(A + B)F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{d}
\end{aligned}$$

Mathematica [C] time = 6.06, size = 252, normalized size = 3.82

$$a(\cos(c + dx) + 1) \sec^2\left(\frac{1}{2}(c + dx)\right) \left(-\frac{2(A-B) \sec(c) \sin(\tan^{-1}(\tan(c)) + dx) {}_2F_1\left(-\frac{1}{2}, -\frac{1}{4}; \frac{3}{4}; \cos^2(dx + \tan^{-1}(\tan(c)))\right)}{\sqrt{\sec^2(c)} \sqrt{\sin^2(\tan^{-1}(\tan(c)) + dx)}} - 4(A+B) \sin(c) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]

[Out] (a*(1 + Cos[c + d*x])*Sec[(c + d*x)/2]^2*((Csc[c]*(3*(A - B)*Cos[c - d*x - ArcTan[Tan[c]]])*Sec[c] + (A - B)*Cos[c + d*x + ArcTan[Tan[c]]])*Sec[c] - 2*((A - 2*B)*Cos[d*x] + A*Cos[2*c + d*x])*Sqrt[Sec[c]^2])/Sqrt[Sec[c]^2] - 4*(A + B)*Cos[c + d*x]*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[Cot[c]]]*Sin[c] - (2*(A - B)*HypergeometricPFQ[-1/2, -1/4], {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2)*Sec[c]*Sin[d*x + ArcTan[Tan[c]]])/(Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2]))/(4*d*Sqrt[Cos[c + d*x]])

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Ba \sec(dx + c)^2 + (A + B)a \sec(dx + c) + Aa\right)\sqrt{\cos(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))*cos(d*x+c)^(1/2),x, algorithm="fricas")

[Out] integral((B*a*sec(d*x + c)^2 + (A + B)*a*sec(d*x + c) + A*a)*sqrt(cos(d*x + c)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(a \sec(dx + c) + a)\sqrt{\cos(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))*cos(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)*sqrt(cos(d*x + c)), x)

maple [B] time = 4.92, size = 240, normalized size = 3.64

$$2a \left(A \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{2 \left(\sin^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) - 1} \text{EllipticF} \left(\cos \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) - A \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{2 \left(\sin^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+a*sec(d*x+c))*(A+B*sec(d*x+c))*cos(d*x+c)^(1/2),x)`

[Out] $-2*a*(A*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)})-A*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)})+B*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)})+B*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)})-2*B*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^2/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(a \sec(dx + c) + a)\sqrt{\cos(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))*cos(d*x+c)^(1/2),x, algorithm="maxima")`

[Out] `integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)*sqrt(cos(d*x + c)), x)`

mupad [B] time = 3.06, size = 96, normalized size = 1.45

$$\frac{2 A a E\left(\frac{c}{2} + \frac{dx}{2} \middle| 2\right)}{d} + \frac{2 A a F\left(\frac{c}{2} + \frac{dx}{2} \middle| 2\right)}{d} + \frac{2 B a F\left(\frac{c}{2} + \frac{dx}{2} \middle| 2\right)}{d} + \frac{2 B a \sin(c + dx) {}_2F_1\left(-\frac{1}{4}, \frac{1}{2}; \frac{3}{4}; \cos(c + dx)^2\right)}{d \sqrt{\cos(c + dx)} \sqrt{\sin(c + dx)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(c + d*x)^(1/2)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x)),x)`

[Out] $(2*A*a*ellipticE(c/2 + (d*x)/2, 2))/d + (2*A*a*ellipticF(c/2 + (d*x)/2, 2))/d + (2*B*a*ellipticF(c/2 + (d*x)/2, 2))/d + (2*B*a*\sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, \cos(c + d*x)^2))/(d*\cos(c + d*x)^{(1/2)}*(\sin(c + d*x)^2)^{(1/2)})$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a \left(\int A \sqrt{\cos(c + dx)} dx + \int A \sqrt{\cos(c + dx)} \sec(c + dx) dx + \int B \sqrt{\cos(c + dx)} \sec(c + dx) dx + \int B \sqrt{\cos(c + dx)} \sec^2(c + dx) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))*cos(d*x+c)**(1/2),x)`

```
[Out] a*(Integral(A*sqrt(cos(c + d*x)), x) + Integral(A*sqrt(cos(c + d*x))*sec(c + d*x), x) + Integral(B*sqrt(cos(c + d*x))*sec(c + d*x), x) + Integral(B*sqrt(cos(c + d*x))*sec(c + d*x)**2, x))
```


$$3.487 \quad \int \frac{(a+a \sec(c+dx))(A+B \sec(c+dx))}{\sqrt{\cos(c+dx)}} dx$$

Optimal. Leaf size=95

$$\frac{2a(3A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} - \frac{2a(A+B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a(A+B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{2aB\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

[Out] $-2*a*(A+B)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/d+2/3*a*(3*A+B)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/d+2/3*a*B*\sin(d*x+c)/d/\cos(d*x+c)^{(3/2)}+2*a*(A+B)*\sin(d*x+c)/d/\cos(d*x+c)^{(1/2)}$

Rubi [A] time = 0.22, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {2954, 2968, 3021, 2748, 2636, 2639, 2641}

$$\frac{2a(3A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} - \frac{2a(A+B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a(A+B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{2aB\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

[In] Int[((a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]))/Sqrt[Cos[c + d*x]], x]

[Out] $(-2*a*(A+B)*\text{EllipticE}[(c+d*x)/2, 2])/d + (2*a*(3*A+B)*\text{EllipticF}[(c+d*x)/2, 2])/(3*d) + (2*a*B*\sin[c+d*x])/(3*d*\cos[c+d*x]^{(3/2)}) + (2*a*(A+B)*\sin[c+d*x])/(d*\text{Sqrt}[\cos[c+d*x]])$

Rule 2636

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), Int[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])], x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2954

Int[((a_.) + csc[(e_.) + (f_.)*(x_)])*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)])*(d_.) + (c_.))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dist[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]

Rule 2968

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])], x_Symbol] :> Int[(a + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2), x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]

Rule 3021

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C))*(m + 1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned}
\int \frac{(a + a \sec(c + dx))(A + B \sec(c + dx))}{\sqrt{\cos(c + dx)}} dx &= \int \frac{(a + a \cos(c + dx))(B + A \cos(c + dx))}{\cos^{\frac{5}{2}}(c + dx)} dx \\
&= \int \frac{aB + (aA + aB) \cos(c + dx) + aA \cos^2(c + dx)}{\cos^{\frac{5}{2}}(c + dx)} dx \\
&= \frac{2aB \sin(c + dx)}{3d \cos^{\frac{3}{2}}(c + dx)} + \frac{2}{3} \int \frac{\frac{3}{2}a(A + B) + \frac{1}{2}a(3A + B) \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx)} dx \\
&= \frac{2aB \sin(c + dx)}{3d \cos^{\frac{3}{2}}(c + dx)} + (a(A + B)) \int \frac{1}{\cos^{\frac{3}{2}}(c + dx)} dx + \frac{1}{3}(a(3A + B)) \int \frac{\cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx)} dx \\
&= \frac{2a(3A + B)F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3d} + \frac{2aB \sin(c + dx)}{3d \cos^{\frac{3}{2}}(c + dx)} + \frac{2a(A + B) \sin(c + dx)}{d \sqrt{\cos(c + dx)}} \\
&= -\frac{2a(A + B)E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{d} + \frac{2a(3A + B)F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3d} + \frac{2a(A + B) \sin(c + dx)}{d \sqrt{\cos(c + dx)}}
\end{aligned}$$

Mathematica [C] time = 6.37, size = 813, normalized size = 8.56

$$a \left(\sqrt{\cos(c + dx)} (\cos(c + dx) + 1) \left(\frac{B \sec(c) \sin(dx) \sec^2(c + dx)}{3d} + \frac{\sec(c)(B \sin(c) + 3A \sin(dx) + 3B \sin(dx)) \sin(c + dx)}{3d} \right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[((a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]))/Sqrt[Cos[c + d*x]],x]

[Out] a*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*((A + B)*Csc[c]*Sec[c])/d + (B*Sec[c]*Sec[c + d*x]^2*Sin[d*x])/(3*d) + (Sec[c]*Sec[c + d*x]*(B*Sin[c] + 3*A*Sin[d*x] + 3*B*Sin[d*x]))/(3*d) - (A*(1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*Sqrt[1 + Cot[c]^2]) - (B*(1 + Cos[c + d*x])

$x) * \text{Csc}[c] * \text{HypergeometricPFQ}[\{1/4, 1/2\}, \{5/4\}, \text{Sin}[d*x - \text{ArcTan}[\text{Cot}[c]]]^2] * \text{Sec}[c/2 + (d*x)/2]^2 * \text{Sec}[d*x - \text{ArcTan}[\text{Cot}[c]]] * \text{Sqrt}[1 - \text{Sin}[d*x - \text{ArcTan}[\text{Cot}[c]]]] * \text{Sqrt}[-(\text{Sqrt}[1 + \text{Cot}[c]^2] * \text{Sin}[c] * \text{Sin}[d*x - \text{ArcTan}[\text{Cot}[c]]])] * \text{Sqrt}[1 + \text{Sin}[d*x - \text{ArcTan}[\text{Cot}[c]]]] / (3*d*\text{Sqrt}[1 + \text{Cot}[c]^2]) + (A*(1 + \text{Cos}[c + d*x]) * \text{Csc}[c] * \text{Sec}[c/2 + (d*x)/2]^2 * (\text{HypergeometricPFQ}[\{-1/2, -1/4\}, \{3/4\}, \text{Cos}[d*x + \text{ArcTan}[\text{Tan}[c]]]^2] * \text{Sin}[d*x + \text{ArcTan}[\text{Tan}[c]]] * \text{Tan}[c]) / (\text{Sqrt}[1 - \text{Cos}[d*x + \text{ArcTan}[\text{Tan}[c]]]] * \text{Sqrt}[1 + \text{Cos}[d*x + \text{ArcTan}[\text{Tan}[c]]]] * \text{Sqrt}[\text{Cos}[c] * \text{Cos}[d*x + \text{ArcTan}[\text{Tan}[c]]] * \text{Sqrt}[1 + \text{Tan}[c]^2]] * \text{Sqrt}[1 + \text{Tan}[c]^2]) - ((\text{Sin}[d*x + \text{ArcTan}[\text{Tan}[c]]] * \text{Tan}[c]) / \text{Sqrt}[1 + \text{Tan}[c]^2] + (2*\text{Cos}[c]^2 * \text{Cos}[d*x + \text{ArcTan}[\text{Tan}[c]]] * \text{Sqrt}[1 + \text{Tan}[c]^2]) / (\text{Cos}[c]^2 + \text{Sin}[c]^2)) / \text{Sqrt}[\text{Cos}[c] * \text{Cos}[d*x + \text{ArcTan}[\text{Tan}[c]]] * \text{Sqrt}[1 + \text{Tan}[c]^2])]) / (2*d) + (B*(1 + \text{Cos}[c + d*x]) * \text{Csc}[c] * \text{Sec}[c/2 + (d*x)/2]^2 * (\text{HypergeometricPFQ}[\{-1/2, -1/4\}, \{3/4\}, \text{Cos}[d*x + \text{ArcTan}[\text{Tan}[c]]]^2] * \text{Sin}[d*x + \text{ArcTan}[\text{Tan}[c]]] * \text{Tan}[c]) / (\text{Sqrt}[1 - \text{Cos}[d*x + \text{ArcTan}[\text{Tan}[c]]]] * \text{Sqrt}[1 + \text{Cos}[d*x + \text{ArcTan}[\text{Tan}[c]]]] * \text{Sqrt}[\text{Cos}[c] * \text{Cos}[d*x + \text{ArcTan}[\text{Tan}[c]]] * \text{Sqrt}[1 + \text{Tan}[c]^2]] * \text{Sqrt}[1 + \text{Tan}[c]^2]) - ((\text{Sin}[d*x + \text{ArcTan}[\text{Tan}[c]]] * \text{Tan}[c]) / \text{Sqrt}[1 + \text{Tan}[c]^2] + (2*\text{Cos}[c]^2 * \text{Cos}[d*x + \text{ArcTan}[\text{Tan}[c]]] * \text{Sqrt}[1 + \text{Tan}[c]^2]) / (\text{Cos}[c]^2 + \text{Sin}[c]^2)) / \text{Sqrt}[\text{Cos}[c] * \text{Cos}[d*x + \text{ArcTan}[\text{Tan}[c]]] * \text{Sqrt}[1 + \text{Tan}[c]^2])]) / (2*d))$

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{Ba \sec(dx+c)^2 + (A+B)a \sec(dx+c) + Aa}{\sqrt{\cos(dx+c)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="fricas")

[Out] integral((B*a*sec(d*x + c)^2 + (A + B)*a*sec(d*x + c) + A*a)/sqrt(cos(d*x + c)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx+c) + A)(a \sec(dx+c) + a)}{\sqrt{\cos(dx+c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)/sqrt(cos(d*x + c)), x)

maple [B] time = 11.02, size = 426, normalized size = 4.48

$$4\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} a \left(\frac{A\sqrt{\frac{1-\cos(dx+c)}{2}} \sqrt{-2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 1} \operatorname{EllipticF}\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{2\sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)}} + \frac{B\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{\dots} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2), x)`

[Out] `-4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(1/2*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2))+1/2*B*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2)))+(1/2*A+1/2*B)*(-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(a \sec(dx + c) + a)}{\sqrt{\cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2), x, algorithm="maxima")`

[Out] `integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)/sqrt(cos(d*x + c)), x)`

mupad [B] time = 3.29, size = 150, normalized size = 1.58

$$\frac{2 A a F\left(\frac{c}{2} + \frac{dx}{2} \middle| 2\right)}{d} + \frac{2 A a \sin(c + dx) {}_2F_1\left(-\frac{1}{4}, \frac{1}{2}; \frac{3}{4}; \cos(c + dx)^2\right)}{d \sqrt{\cos(c + dx)} \sqrt{\sin(c + dx)^2}} + \frac{2 B a \sin(c + dx) {}_2F_1\left(-\frac{1}{4}, \frac{1}{2}; \frac{3}{4}; \cos(c + dx)^2\right)}{d \sqrt{\cos(c + dx)} \sqrt{\sin(c + dx)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x)))/cos(c + d*x)^(1/2), x)`

[Out] $(2*A*a*\text{ellipticF}(c/2 + (d*x)/2, 2))/d + (2*A*a*\sin(c + d*x)*\text{hypergeom}([-1/4, 1/2], 3/4, \cos(c + d*x)^2))/(d*\cos(c + d*x)^{(1/2)}*(\sin(c + d*x)^2)^{(1/2)}) + (2*B*a*\sin(c + d*x)*\text{hypergeom}([-1/4, 1/2], 3/4, \cos(c + d*x)^2))/(d*\cos(c + d*x)^{(1/2)}*(\sin(c + d*x)^2)^{(1/2)}) + (2*B*a*\sin(c + d*x)*\text{hypergeom}([-3/4, 1/2], 1/4, \cos(c + d*x)^2))/(3*d*\cos(c + d*x)^{(3/2)}*(\sin(c + d*x)^2)^{(1/2)})$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a \left(\int \frac{A}{\sqrt{\cos(c + dx)}} dx + \int \frac{A \sec(c + dx)}{\sqrt{\cos(c + dx)}} dx + \int \frac{B \sec(c + dx)}{\sqrt{\cos(c + dx)}} dx + \int \frac{B \sec^2(c + dx)}{\sqrt{\cos(c + dx)}} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/cos(d*x+c)**(1/2), x)`

[Out] `a*(Integral(A/sqrt(cos(c + d*x)), x) + Integral(A*sec(c + d*x)/sqrt(cos(c + d*x)), x) + Integral(B*sec(c + d*x)/sqrt(cos(c + d*x)), x) + Integral(B*sec(c + d*x)**2/sqrt(cos(c + d*x)), x))`

$$3.488 \quad \int \frac{(a+a \sec(c+dx))(A+B \sec(c+dx))}{3 \cos^2(c+dx)} dx$$

Optimal. Leaf size=132

$$\frac{2a(A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} - \frac{2a(5A+3B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2a(A+B)\sin(c+dx)}{3d \cos^{\frac{3}{2}}(c+dx)} + \frac{2a(5A+3B)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}} + \frac{2}{5}$$

[Out] $-2/5*a*(5*A+3*B)*(cos(1/2*d*x+1/2*c)^2)^{(1/2)}/cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/d+2/3*a*(A+B)*(cos(1/2*d*x+1/2*c)^2)^{(1/2)}/cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/d+2/5*a*B*\sin(d*x+c)/d/cos(d*x+c)^{(5/2)}+2/3*a*(A+B)*\sin(d*x+c)/d/cos(d*x+c)^{(3/2)}+2/5*a*(5*A+3*B)*\sin(d*x+c)/d/cos(d*x+c)^{(1/2)}$

Rubi [A] time = 0.23, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {2954, 2968, 3021, 2748, 2636, 2641, 2639}

$$\frac{2a(A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} - \frac{2a(5A+3B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2a(A+B)\sin(c+dx)}{3d \cos^{\frac{3}{2}}(c+dx)} + \frac{2a(5A+3B)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}} + \frac{2}{5}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + a*\text{Sec}[c + d*x])*(A + B*\text{Sec}[c + d*x])/Cos[c + d*x]^{(3/2)}, x]$

[Out] $(-2*a*(5*A + 3*B)*\text{EllipticE}[(c + d*x)/2, 2])/(5*d) + (2*a*(A + B)*\text{EllipticF}[(c + d*x)/2, 2])/(3*d) + (2*a*B*\text{Sin}[c + d*x])/(5*d*\text{Cos}[c + d*x]^{(5/2)}) + (2*a*(A + B)*\text{Sin}[c + d*x])/(3*d*\text{Cos}[c + d*x]^{(3/2)}) + (2*a*(5*A + 3*B)*\text{Sin}[c + d*x])/(5*d*\text{Sqrt}[\text{Cos}[c + d*x]])$

Rule 2636

$\text{Int}[(b_*.*\sin[(c_*) + (d_*)(x_*)])^{(n)}, x_Symbol] \rightarrow \text{Simp}[(\text{Cos}[c + d*x]*(b*\text{Sin}[c + d*x])^{(n + 1)})/(b*d*(n + 1)), x] + \text{Dist}[(n + 2)/(b^2*(n + 1)), \text{Int}[(b*\text{Sin}[c + d*x])^{(n + 2)}, x], x] /;$ $\text{FreeQ}\{b, c, d, x\}$ && $\text{LtQ}[n, -1]$ && $\text{IntegerQ}[2*n]$

Rule 2639

$\text{Int}[\text{Sqrt}[\sin[(c_*) + (d_*)(x_*)]], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticE}[(1*(c - P i/2 + d*x))/2, 2])/d, x] /;$ $\text{FreeQ}\{c, d, x\}$

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_.)]], x_Symbol] := Simp[(2*EllipticF[(1*(c -
Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_.)]^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_.)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_.)]^(p_.), x_Symbol] := Dist[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)]*(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] := Int[(a + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2), x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)] + (C_.)*sin[(e_.) + (f_.)*(x_.)]^2), x_Symbol] := -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C))*(m + 1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + a \sec(c + dx))(A + B \sec(c + dx))}{\cos^{\frac{3}{2}}(c + dx)} dx &= \int \frac{(a + a \cos(c + dx))(B + A \cos(c + dx))}{\cos^{\frac{7}{2}}(c + dx)} dx \\
&= \int \frac{aB + (aA + aB) \cos(c + dx) + aA \cos^2(c + dx)}{\cos^{\frac{7}{2}}(c + dx)} dx \\
&= \frac{2aB \sin(c + dx)}{5d \cos^{\frac{5}{2}}(c + dx)} + \frac{2}{5} \int \frac{\frac{5}{2}a(A + B) + \frac{1}{2}a(5A + 3B) \cos(c + dx)}{\cos^{\frac{5}{2}}(c + dx)} dx \\
&= \frac{2aB \sin(c + dx)}{5d \cos^{\frac{5}{2}}(c + dx)} + (a(A + B)) \int \frac{1}{\cos^{\frac{5}{2}}(c + dx)} dx + \frac{1}{5}(a(5A + 3B)) \int \frac{\cos(c + dx)}{\cos^{\frac{5}{2}}(c + dx)} dx \\
&= \frac{2aB \sin(c + dx)}{5d \cos^{\frac{5}{2}}(c + dx)} + \frac{2a(A + B) \sin(c + dx)}{3d \cos^{\frac{3}{2}}(c + dx)} + \frac{2a(5A + 3B) \sin(c + dx)}{5d \sqrt{\cos(c + dx)}} \\
&= -\frac{2a(5A + 3B)E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{5d} + \frac{2a(A + B)F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3d} + \frac{2a(5A + 3B)\sqrt{\cos(c + dx)}}{5d}
\end{aligned}$$

Mathematica [C] time = 6.43, size = 865, normalized size = 6.55

$$a \sqrt{\cos(c + dx)} (\cos(c + dx) + 1) \left(\frac{B \sec(c) \sin(dx) \sec^3(c + dx)}{5d} + \frac{\sec(c)(3B \sin(c) + 5A \sin(dx) + 5B \sin(dx))}{15d} \right)$$

Warning: Unable to verify antiderivative.

```

[In] Integrate[((a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]))/Cos[c + d*x]^(3/2),x]
[Out] a*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*(((5*A + 3*B)
*Csc[c]*Sec[c])/(5*d) + (B*Sec[c]*Sec[c + d*x]^3*Sin[d*x])/(5*d) + (Sec[c]*
Sec[c + d*x]^2*(3*B*Sin[c] + 5*A*Sin[d*x] + 5*B*Sin[d*x]))/(15*d) + (Sec[c]
*Sec[c + d*x]*(5*A*Sin[c] + 5*B*Sin[c] + 15*A*Sin[d*x] + 9*B*Sin[d*x]))/(15
*d)) - (A*(1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Si
n[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*S
qrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x
- ArcTan[Cot[c]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqrt[1 + Cot

```

$[c]^2) - (B*(1 + \cos[c + dx])*\text{Csc}[c]*\text{HypergeometricPFQ}[\{1/4, 1/2\}, \{5/4\}, \sin[dx - \text{ArcTan}[\text{Cot}[c]]]^2*\text{Sec}[c/2 + (dx)/2]^2*\text{Sec}[dx - \text{ArcTan}[\text{Cot}[c]]]*\text{Sqrt}[1 - \sin[dx - \text{ArcTan}[\text{Cot}[c]]]]*\text{Sqrt}[-(\text{Sqrt}[1 + \text{Cot}[c]^2]*\sin[c]*\sin[dx - \text{ArcTan}[\text{Cot}[c]]])]*\text{Sqrt}[1 + \sin[dx - \text{ArcTan}[\text{Cot}[c]]]])/(3*d*\text{Sqrt}[1 + \text{Cot}[c]^2]) + (A*(1 + \cos[c + dx])*\text{Csc}[c]*\text{Sec}[c/2 + (dx)/2]^2*(\text{HypergeometricPFQ}[\{-1/2, -1/4\}, \{3/4\}, \cos[dx + \text{ArcTan}[\text{Tan}[c]]]^2*\sin[dx + \text{ArcTan}[\text{Tan}[c]]]*\text{Tan}[c])/(\text{Sqrt}[1 - \cos[dx + \text{ArcTan}[\text{Tan}[c]]]]*\text{Sqrt}[1 + \cos[dx + \text{ArcTan}[\text{Tan}[c]]]]*\text{Sqrt}[\cos[c]*\cos[dx + \text{ArcTan}[\text{Tan}[c]]]*\text{Sqrt}[1 + \text{Tan}[c]^2]]*\text{Sqrt}[1 + \text{Tan}[c]^2]) - ((\sin[dx + \text{ArcTan}[\text{Tan}[c]]]*\text{Tan}[c])/(\text{Sqrt}[1 + \text{Tan}[c]^2] + (2*\cos[c]^2*\cos[dx + \text{ArcTan}[\text{Tan}[c]]]*\text{Sqrt}[1 + \text{Tan}[c]^2])/(\cos[c]^2 + \sin[c]^2))/\text{Sqrt}[\cos[c]*\cos[dx + \text{ArcTan}[\text{Tan}[c]]]*\text{Sqrt}[1 + \text{Tan}[c]^2]])))/(2*d) + (3*B*(1 + \cos[c + dx])*\text{Csc}[c]*\text{Sec}[c/2 + (dx)/2]^2*(\text{HypergeometricPFQ}[\{-1/2, -1/4\}, \{3/4\}, \cos[dx + \text{ArcTan}[\text{Tan}[c]]]^2*\sin[dx + \text{ArcTan}[\text{Tan}[c]]]*\text{Tan}[c])/(\text{Sqrt}[1 - \cos[dx + \text{ArcTan}[\text{Tan}[c]]]]*\text{Sqrt}[1 + \cos[dx + \text{ArcTan}[\text{Tan}[c]]]]*\text{Sqrt}[\cos[c]*\cos[dx + \text{ArcTan}[\text{Tan}[c]]]*\text{Sqrt}[1 + \text{Tan}[c]^2]]*\text{Sqrt}[1 + \text{Tan}[c]^2]) - ((\sin[dx + \text{ArcTan}[\text{Tan}[c]]]*\text{Tan}[c])/(\text{Sqrt}[1 + \text{Tan}[c]^2] + (2*\cos[c]^2*\cos[dx + \text{ArcTan}[\text{Tan}[c]]]*\text{Sqrt}[1 + \text{Tan}[c]^2])/(\cos[c]^2 + \sin[c]^2))/\text{Sqrt}[\cos[c]*\cos[dx + \text{ArcTan}[\text{Tan}[c]]]*\text{Sqrt}[1 + \text{Tan}[c]^2]])))/(10*d))$

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{Ba \sec(dx + c)^2 + (A + B)a \sec(dx + c) + Aa}{\cos(dx + c)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(dx+c))*(A+B*sec(dx+c))/cos(dx+c)^(3/2),x, algorithm="fricas")

[Out] integral((B*a*sec(dx + c)^2 + (A + B)*a*sec(dx + c) + A*a)/cos(dx + c)^(3/2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(a \sec(dx + c) + a)}{\cos(dx + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(dx+c))*(A+B*sec(dx+c))/cos(dx+c)^(3/2),x, algorithm="giac")

[Out] integrate((B*sec(dx + c) + A)*(a*sec(dx + c) + a)/cos(dx + c)^(3/2), x)

maple [B] time = 12.33, size = 661, normalized size = 5.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2),x)`

[Out]
$$-4*(-(-2*\cos(1/2*d*x+1/2*c)^2+1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*a*((1/2*A+1/2*B)*(-1/6*\cos(1/2*d*x+1/2*c)*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)})/(-1/2+\cos(1/2*d*x+1/2*c)^2)^2+1/3*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)})/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)}))+1/2*A*(-(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)}))+2*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^2)/\sin(1/2*d*x+1/2*c)^2/(2*\sin(1/2*d*x+1/2*c)^2-1)-1/10*B/(8*\sin(1/2*d*x+1/2*c)^6-12*\sin(1/2*d*x+1/2*c)^4+6*\sin(1/2*d*x+1/2*c)^2-1)/\sin(1/2*d*x+1/2*c)^2*(12*EllipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)})*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*\sin(1/2*d*x+1/2*c)^4-24*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^6-12*EllipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)})*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*\sin(1/2*d*x+1/2*c)^2+24*\sin(1/2*d*x+1/2*c)^4*\cos(1/2*d*x+1/2*c)+3*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)}))-8*\sin(1/2*d*x+1/2*c)^2*\cos(1/2*d*x+1/2*c))*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)})/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(a \sec(dx + c) + a)}{\cos(dx + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="maxima")`

[Out] `integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)/cos(d*x + c)^(3/2), x)`

mupad [B] time = 3.55, size = 177, normalized size = 1.34

$$\frac{2 A a \sin(c + d x) {}_2F_1\left(-\frac{1}{4}, \frac{1}{2}; \frac{3}{4}; \cos(c + d x)^2\right)}{d \sqrt{\cos(c + d x)} \sqrt{\sin(c + d x)^2}} + \frac{2 A a \sin(c + d x) {}_2F_1\left(-\frac{3}{4}, \frac{1}{2}; \frac{1}{4}; \cos(c + d x)^2\right)}{3 d \cos(c + d x)^{3/2} \sqrt{\sin(c + d x)^2}} + \frac{2 B a \sin(c + d x)}{3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((((A + B/cos(c + d*x))*(a + a/cos(c + d*x)))/cos(c + d*x)^(3/2),x)`

[Out]
$$(2*A*a*\sin(c + d*x)*\text{hypergeom}([-1/4, 1/2], 3/4, \cos(c + d*x)^2))/(d*\cos(c + d*x)^{(1/2)}*(\sin(c + d*x)^2)^{(1/2)}) + (2*A*a*\sin(c + d*x)*\text{hypergeom}([-3/4,$$

$1/2], 1/4, \cos(c + d*x)^2)) / (3*d*\cos(c + d*x)^{(3/2)}*(\sin(c + d*x)^2)^{(1/2)})$
 $+ (2*B*a*\sin(c + d*x)*\text{hypergeom}([-3/4, 1/2], 1/4, \cos(c + d*x)^2)) / (3*d*\cos(c + d*x)^{(3/2)}*(\sin(c + d*x)^2)^{(1/2)})$
 $+ (2*B*a*\sin(c + d*x)*\text{hypergeom}([-5/4, 1/2], -1/4, \cos(c + d*x)^2)) / (5*d*\cos(c + d*x)^{(5/2)}*(\sin(c + d*x)^2)^{(1/2)})$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a \left(\int \frac{A}{\cos^{\frac{3}{2}}(c + dx)} dx + \int \frac{A \sec(c + dx)}{\cos^{\frac{3}{2}}(c + dx)} dx + \int \frac{B \sec(c + dx)}{\cos^{\frac{3}{2}}(c + dx)} dx + \int \frac{B \sec^2(c + dx)}{\cos^{\frac{3}{2}}(c + dx)} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/cos(d*x+c)**(3/2), x)

[Out] a*(Integral(A/cos(c + d*x)**(3/2), x) + Integral(A*sec(c + d*x)/cos(c + d*x)**(3/2), x) + Integral(B*sec(c + d*x)/cos(c + d*x)**(3/2), x) + Integral(B*sec(c + d*x)**2/cos(c + d*x)**(3/2), x))

$$3.489 \quad \int \cos^2(c+dx)(a+a \sec(c+dx))^2(A+B \sec(c+dx)) dx$$

Optimal. Leaf size=194

$$\frac{4a^2(5A+6B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} + \frac{4a^2(8A+9B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{2a^2(11A+9B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{63d} + \frac{4a^2(8A+9B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{63d}$$

[Out] $4/15*a^2*(8*A+9*B)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c),2^{(1/2)})/d+4/21*a^2*(5*A+6*B)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c),2^{(1/2)})/d+4/45*a^2*(8*A+9*B)*\cos(d*x+c)^{(3/2)}*\sin(d*x+c)/d+2/63*a^2*(11*A+9*B)*\cos(d*x+c)^{(5/2)}*\sin(d*x+c)/d+2/9*A*\cos(d*x+c)^{(5/2)}*(a^2+a^2*\cos(d*x+c))*\sin(d*x+c)/d+4/21*a^2*(5*A+6*B)*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/d$

Rubi [A] time = 0.40, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {2954, 2976, 2968, 3023, 2748, 2635, 2641, 2639}

$$\frac{4a^2(5A+6B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} + \frac{4a^2(8A+9B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{2a^2(11A+9B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{63d} + \frac{4a^2(8A+9B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{63d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Cos}[c+d*x]^{(9/2)}*(a+a*\text{Sec}[c+d*x])^2*(A+B*\text{Sec}[c+d*x]),x]$

[Out] $(4*a^2*(8*A+9*B)*\text{EllipticE}[(c+d*x)/2,2])/(15*d)+(4*a^2*(5*A+6*B)*\text{EllipticF}[(c+d*x)/2,2])/(21*d)+(4*a^2*(5*A+6*B)*\text{Sqrt}[\text{Cos}[c+d*x]]*\text{Sin}[c+d*x])/(21*d)+(4*a^2*(8*A+9*B)*\text{Cos}[c+d*x]^{(3/2)}*\text{Sin}[c+d*x])/(45*d)+(2*a^2*(11*A+9*B)*\text{Cos}[c+d*x]^{(5/2)}*\text{Sin}[c+d*x])/(63*d)+(2*A*\text{Cos}[c+d*x]^{(5/2)}*(a^2+a^2*\text{Cos}[c+d*x]))*\text{Sin}[c+d*x])/(9*d)$

Rule 2635

$\text{Int}[(b_*)*\sin[(c_*)+(d_*)(x_)]^{(n_)}, x_Symbol] \rightarrow -\text{Simp}[(b*\text{Cos}[c+d*x])*(b*\text{Sin}[c+d*x])^{(n-1)})/(d*n), x] + \text{Dist}[(b^2*(n-1))/n, \text{Int}[(b*\text{Sin}[c+d*x])^{(n-2)}, x], x] /;$ FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2639

$\text{Int}[\text{Sqrt}[\sin[(c_*)+(d_*)(x_)]], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticE}[(1*(c-Pi/2+d*x))/2,2])/d, x] /;$ FreeQ[{c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_.)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_.)]^(m_.))*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2954

Int[((a_.) + csc[(e_.) + (f_.)*(x_.)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_.)]^(p_.), x_Symbol] := Dist[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]

Rule 2968

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^(m_.))*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)]*(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] := Int[(a + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2), x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]

Rule 2976

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^(m_.))*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)]*(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]^(n_.), x_Symbol] := -Simp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] && !LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])

Rule 3023

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^(m_.))*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)] + (C_.)*sin[(e_.) + (f_.)*(x_.)]^2), x_Symbol] := -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&

!LtQ[m, -1]

Rubi steps

$$\begin{aligned}
\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^2(A+B\sec(c+dx))dx &= \int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2(B+A\cos(c+dx))dx \\
&= \frac{2A\cos^{\frac{5}{2}}(c+dx)(a^2+a^2\cos(c+dx))\sin(c+dx)}{9d} \\
&= \frac{2A\cos^{\frac{5}{2}}(c+dx)(a^2+a^2\cos(c+dx))\sin(c+dx)}{9d} \\
&= \frac{2a^2(11A+9B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{63d} + \frac{2A\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{9d} \\
&= \frac{2a^2(11A+9B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{63d} + \frac{2A\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{9d} \\
&= \frac{4a^2(5A+6B)\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} + \frac{4a^2(8A+9B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{4a^2(5A+6B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}
\end{aligned}$$

Mathematica [C] time = 6.31, size = 1086, normalized size = 5.60

result too large to display

Warning: Unable to verify antiderivative.

```

[In] Integrate[Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
[Out] (Cos[c + d*x]^(7/2)*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x])*(-1/15*((8*A + 9*B)*Cot[c])/d + ((46*A + 51*B)*Cos[d*x]*Sin[c])/(168*d) + ((37*A + 36*B)*Cos[2*d*x]*Sin[2*c])/(360*d) + ((2*A + B)*Cos[3*d*x]*Sin[3*c])/(56*d) + (A*Cos[4*d*x]*Sin[4*c])/(144*d) + ((46*A + 51*B)*Cos[c]*Sin[d*x])/(168*d) + ((37*A + 36*B)*Cos[2*c]*Sin[2*d*x])/(360*d) + ((2*A + B)*Cos[3*c]*Sin[3*d*x])/(56*d) + (A*Cos[4*c]*Sin[4*d*x])/(144*d))/ (B + A*Cos[c + d*x]) - (5*A*Cos[c + d*x]^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2)*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqr

```

```
t[1 + Sin[d*x - ArcTan[Cot[c]]]])/(21*d*(B + A*Cos[c + d*x])*Sqrt[1 + Cot[c]^2]) - (2*B*Cos[c + d*x]^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]])*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(7*d*(B + A*Cos[c + d*x])*Sqrt[1 + Cot[c]^2]) - (4*A*Cos[c + d*x]^3*Csc[c]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x])*((HypergeometricPFQ[-1/2, -1/4], {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2)*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]])*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]])*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]])*Sqrt[1 + Tan[c]^2])*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]])*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]])*Sqrt[1 + Tan[c]^2]))/(15*d*(B + A*Cos[c + d*x])) - (3*B*Cos[c + d*x]^3*Csc[c]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x])*((HypergeometricPFQ[-1/2, -1/4], {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2)*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]])*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]])*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]])*Sqrt[1 + Tan[c]^2])*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]])*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]])*Sqrt[1 + Tan[c]^2]))/(10*d*(B + A*Cos[c + d*x]))
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

integral((B*a^2*cos(dx+c)^4*sec(dx+c)^3 + (A+2*B)*a^2*cos(dx+c)^4*sec(dx+c)^2 + (2*A+B)*a^2*cos(dx+c)^4*sec(dx+c), x)

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((B*a^2*cos(d*x + c)^4*sec(d*x + c)^3 + (A + 2*B)*a^2*cos(d*x + c)^4*sec(d*x + c)^2 + (2*A + B)*a^2*cos(d*x + c)^4*sec(d*x + c) + A*a^2*cos(d*x + c)^4)*sqrt(cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(a \sec(dx + c) + a)^2 \cos(dx + c)^{\frac{9}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="giac")
```


[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^2*cos(d*x + c)^(9/2), x)

maple [A] time = 4.87, size = 413, normalized size = 2.13

$$4\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} a^2 \left(-560A \cos\left(\frac{dx}{2} + \frac{c}{2}\right)\left(\sin^{10}\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + (1840A + 360B)\left(\sin^8\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)), x)

[Out] $-4/315*((2*\cos(1/2*d*x+1/2*c)^2-1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*a^2*(-560*A*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^{10}+(1840*A+360*B)*\sin(1/2*d*x+1/2*c)^8*\cos(1/2*d*x+1/2*c)+(-2368*A-1044*B)*\sin(1/2*d*x+1/2*c)^6*\cos(1/2*d*x+1/2*c)+(1568*A+1134*B)*\sin(1/2*d*x+1/2*c)^4*\cos(1/2*d*x+1/2*c)+(-387*A-351*B)*\sin(1/2*d*x+1/2*c)^2*\cos(1/2*d*x+1/2*c)+75*A*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c), 2^{(1/2)})-168*A*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c), 2^{(1/2)})+90*B*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c), 2^{(1/2)})-189*B*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c), 2^{(1/2)}))/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)), x, algorithm="maxima")

[Out] Timed out

mupad [B] time = 3.19, size = 266, normalized size = 1.37

$$\frac{2 B a^2 \left(\sqrt{\cos(c+d x)} \sin(c+d x)+F\left(\frac{c}{2}+\frac{d x}{2}\left|2\right.\right)\right)}{3 d}-\frac{2 A a^2 \cos(c+d x)^{7 / 2} \sin(c+d x) {}_2 F_1\left(\frac{1}{2}, \frac{7}{4} ; \frac{11}{4} ; \cos(c+d x)\right)}{7 d \sqrt{\sin(c+d x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(9/2)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^2,x)
```

```
[Out] (2*B*a^2*(cos(c + d*x)^(1/2)*sin(c + d*x) + ellipticF(c/2 + (d*x)/2, 2)))/(3*d) - (2*A*a^2*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)) - (4*A*a^2*cos(c + d*x)^(9/2)*sin(c + d*x)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2)^(1/2)) - (2*A*a^2*cos(c + d*x)^(11/2)*sin(c + d*x)*hypergeom([1/2, 11/4], 15/4, cos(c + d*x)^2))/(11*d*(sin(c + d*x)^2)^(1/2)) - (4*B*a^2*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)) - (2*B*a^2*cos(c + d*x)^(9/2)*sin(c + d*x)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2)^(1/2))
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(9/2)*(a+a*sec(d*x+c))**2*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

$$3.490 \quad \int \cos^{\frac{7}{2}}(c+dx)(a+a \sec(c+dx))^2(A+B \sec(c+dx)) dx$$

Optimal. Leaf size=161

$$\frac{4a^2(6A+7B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} + \frac{4a^2(3A+4B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2a^2(9A+7B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{35d} + \frac{4a^2(6A+7B)\cos(c+dx)}{35d}$$

[Out] $4/5*a^2*(3*A+4*B)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/d+4/21*a^2*(6*A+7*B)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/d+2/35*a^2*(9*A+7*B)*\cos(d*x+c)^{(3/2)}*\sin(d*x+c)/d+2/7*A*\cos(d*x+c)^{(3/2)}*(a^2+a^2*\cos(d*x+c))*\sin(d*x+c)/d+4/21*a^2*(6*A+7*B)*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/d$

Rubi [A] time = 0.36, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {2954, 2976, 2968, 3023, 2748, 2639, 2635, 2641}

$$\frac{4a^2(6A+7B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} + \frac{4a^2(3A+4B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2a^2(9A+7B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{35d} + \frac{4a^2(6A+7B)\cos(c+dx)}{35d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Cos}[c+d*x]^{(7/2)}*(a+a*\text{Sec}[c+d*x])^2*(A+B*\text{Sec}[c+d*x]),x]$

[Out] $(4*a^2*(3*A+4*B)*\text{EllipticE}[(c+d*x)/2, 2])/(5*d) + (4*a^2*(6*A+7*B)*\text{EllipticF}[(c+d*x)/2, 2])/(21*d) + (4*a^2*(6*A+7*B)*\text{Sqrt}[\text{Cos}[c+d*x]]*\text{Sin}[c+d*x])/(21*d) + (2*a^2*(9*A+7*B)*\text{Cos}[c+d*x]^{(3/2)}*\text{Sin}[c+d*x])/(35*d) + (2*A*\text{Cos}[c+d*x]^{(3/2)}*(a^2+a^2*\text{Cos}[c+d*x])*\text{Sin}[c+d*x])/(7*d)$

Rule 2635

$\text{Int}[(b_*)*\sin[(c_*) + (d_*)*(x_)]^{(n_)}, x_Symbol] \rightarrow -\text{Simp}[(b*\text{Cos}[c+d*x])*(b*\text{Sin}[c+d*x])^{(n-1)})/(d*n), x] + \text{Dist}[(b^2*(n-1))/n, \text{Int}[(b*\text{Sin}[c+d*x])^{(n-2)}, x], x] /;$ FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2639

$\text{Int}[\text{Sqrt}[\sin[(c_*) + (d_*)*(x_)]], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticE}[(1*(c-Pi/2+d*x))/2, 2])/d, x] /;$ FreeQ[{c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_.)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2954

Int[((a_.) + csc[(e_.) + (f_.)*(x_.)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_.)])^(p_.), x_Symbol] := Dist[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]

Rule 2968

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)]*(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] := Int[(a + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2), x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]

Rule 2976

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)]*(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)])^(n_.), x_Symbol] := -Simp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] && !LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])

Rule 3023

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)] + (C_.)*sin[(e_.) + (f_.)*(x_.)]^2), x_Symbol] := -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rubi steps

$$\begin{aligned}
\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^2(A+B\sec(c+dx))dx &= \int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2(B+A\cos(c+dx))dx \\
&= \frac{2A\cos^{\frac{3}{2}}(c+dx)(a^2+a^2\cos(c+dx))\sin(c+dx)}{7d} \\
&= \frac{2A\cos^{\frac{3}{2}}(c+dx)(a^2+a^2\cos(c+dx))\sin(c+dx)}{7d} \\
&= \frac{2a^2(9A+7B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{35d} + \frac{2A\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{7d} \\
&= \frac{2a^2(9A+7B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{35d} + \frac{2A\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{7d} \\
&= \frac{4a^2(3A+4B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{4a^2(6A+7B)\sqrt{\cos(c+dx)}}{21d} \\
&= \frac{4a^2(3A+4B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{4a^2(6A+7B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}
\end{aligned}$$

Mathematica [C] time = 6.27, size = 1040, normalized size = 6.46

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
[Out] (Cos[c + d*x]^(7/2)*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x])*(-1/5*((3*A + 4*B)*Cot[c])/d + ((51*A + 56*B)*Cos[d*x]*Sin[c])/(168*d) + ((2*A + B)*Cos[2*d*x]*Sin[2*c])/(20*d) + (A*Cos[3*d*x]*Sin[3*c])/(56*d) + ((51*A + 56*B)*Cos[c]*Sin[d*x])/(168*d) + ((2*A + B)*Cos[2*c]*Sin[2*d*x])/(20*d) + (A*Cos[3*c]*Sin[3*d*x])/(56*d))/ (B + A*Cos[c + d*x]) - (2*A*Cos[c + d*x]^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]])]/(7*d*(B + A*Cos[c + d*x])*Sqrt[1 + Cot[c]^2]) - (B*Cos[c + d*x]^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x])*Sec[d*x
```

```

- ArcTan[Cot[c]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[
c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]
]]/(3*d*(B + A*Cos[c + d*x])*Sqrt[1 + Cot[c]^2]) - (3*A*Cos[c + d*x]^3*Csc[
c]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x])*((Hyper
geometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + Ar
cTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x
+ ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Tan[c]^2
]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]
]^2) + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2
+ Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]])))/(1
0*d*(B + A*Cos[c + d*x])) - (2*B*Cos[c + d*x]^3*Csc[c]*Sec[c/2 + (d*x)/2]^4
*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x])*((HypergeometricPFQ[{-1/2, -1/
4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(
Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqr
t[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2])
- ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[
d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c
]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]])))/(5*d*(B + A*Cos[c + d*x]
)

```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

integral((Ba² cos(dx + c)³ sec(dx + c)³ + (A + 2B)a² cos(dx + c)³ sec(dx + c)² + (2A + B)a² cos(dx + c)³ sec(dx + c)) dx

Verification of antiderivative is not currently implemented for this CAS.

```

[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm
="fricas")

```

```

[Out] integral((B*a^2*cos(d*x + c)^3*sec(d*x + c)^3 + (A + 2*B)*a^2*cos(d*x + c)^
3*sec(d*x + c)^2 + (2*A + B)*a^2*cos(d*x + c)^3*sec(d*x + c) + A*a^2*cos(d*
x + c)^3)*sqrt(cos(d*x + c)), x)

```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(a \sec(dx + c) + a)^2 \cos(dx + c)^{\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```

[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm
="giac")

```

```

[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^2*cos(d*x + c)^(7/2), x)

```

maple [A] time = 5.05, size = 385, normalized size = 2.39

$$4\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} a^2 \left(120A \cos\left(\frac{dx}{2} + \frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + (-348A - 84B)\left(\sin^6\left(\frac{dx}{2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x)`

[Out] $-4/105*((2*\cos(1/2*d*x+1/2*c)^2-1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*a^2*(120*A*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^8+(-348*A-84*B)*\sin(1/2*d*x+1/2*c)^6*\cos(1/2*d*x+1/2*c)+(378*A+224*B)*\sin(1/2*d*x+1/2*c)^4*\cos(1/2*d*x+1/2*c)+(-17*A-91*B)*\sin(1/2*d*x+1/2*c)^2*\cos(1/2*d*x+1/2*c)+30*A*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)})-63*A*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)})+35*B*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)})-84*B*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)}))/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(a \sec(dx + c) + a)^2 \cos(dx + c)^{\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="maxima")`

[Out] `integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^2*cos(d*x + c)^(7/2), x)`

mupad [B] time = 3.01, size = 231, normalized size = 1.43

$$\frac{2 B a^2 \left(\frac{2 \sqrt{\cos(c+dx)} \sin(c+dx)}{3} + \frac{{}_2F_1\left(\frac{c}{2} + \frac{dx}{2} \middle| 2\right)}{3} \right)}{d} + \frac{2 A a^2 \left(\sqrt{\cos(c+dx)} \sin(c+dx) + F\left(\frac{c}{2} + \frac{dx}{2} \middle| 2\right) \right)}{3d} + \frac{2 B a^2 E\left(\frac{c}{2} + \frac{dx}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(c + d*x)^(7/2)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^2,x)`

```
[Out] (2*B*a^2*((2*cos(c + d*x)^(1/2)*sin(c + d*x))/3 + (2*ellipticF(c/2 + (d*x)/
2, 2))/3))/d + (2*A*a^2*(cos(c + d*x)^(1/2)*sin(c + d*x) + ellipticF(c/2 +
(d*x)/2, 2)))/(3*d) + (2*B*a^2*ellipticE(c/2 + (d*x)/2, 2))/d - (4*A*a^2*co
s(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/
(7*d*(sin(c + d*x)^2)^(1/2)) - (2*A*a^2*cos(c + d*x)^(9/2)*sin(c + d*x)*hyp
ergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2)^(1/2)) - (2
*B*a^2*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c +
d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2))
```

```
sympy [F(-1)]    time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(a+a*sec(d*x+c))**2*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```


$$3.491 \quad \int \cos^{\frac{5}{2}}(c+dx)(a+a \sec(c+dx))^2(A+B \sec(c+dx)) dx$$

Optimal. Leaf size=126

$$\frac{4a^2(A+2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{4a^2(4A+5B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2a^2(7A+5B)\sin(c+dx)\sqrt{\cos(c+dx)}}{15d} + \frac{2A \sin(c+dx)}{d}$$

[Out] $4/5*a^2*(4*A+5*B)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c),2^{(1/2)})/d+4/3*a^2*(A+2*B)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c),2^{(1/2)})/d+2/15*a^2*(7*A+5*B)*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/d+2/5*A*(a^2+a^2*\cos(d*x+c))*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/d$

Rubi [A] time = 0.35, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {2954, 2976, 2968, 3023, 2748, 2641, 2639}

$$\frac{4a^2(A+2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{4a^2(4A+5B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2a^2(7A+5B)\sin(c+dx)\sqrt{\cos(c+dx)}}{15d} + \frac{2A \sin(c+dx)}{d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Cos}[c + d*x]^{(5/2)}*(a + a*\text{Sec}[c + d*x])^2*(A + B*\text{Sec}[c + d*x]),x]$

[Out] $(4*a^2*(4*A + 5*B)*\text{EllipticE}[(c + d*x)/2, 2])/(5*d) + (4*a^2*(A + 2*B)*\text{EllipticF}[(c + d*x)/2, 2])/(3*d) + (2*a^2*(7*A + 5*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sin}[c + d*x])/(15*d) + (2*A*\text{Sqrt}[\text{Cos}[c + d*x]]*(a^2 + a^2*\text{Cos}[c + d*x])*\text{Sin}[c + d*x])/(5*d)$

Rule 2639

$\text{Int}[\text{Sqrt}[\sin[(c_.) + (d_.)*(x_)]]], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticE}[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 2641

$\text{Int}[1/\text{Sqrt}[\sin[(c_.) + (d_.)*(x_)]]], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticF}[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 2748

$\text{Int}[(b_.)*\sin[(e_.) + (f_.)*(x_)]]^{(m_.)*((c_.) + (d_.)*\sin[(e_.) + (f_.)*(x_)])}, x_Symbol] \rightarrow \text{Dist}[c, \text{Int}[(b*\text{Sin}[e + f*x])^m, x], x] + \text{Dist}[d/b, \text{Int}[(b*\text{Sin}[e + f*x])^m, x], x]$

$b \sin[e + f x]^{m+1}, x], x] /; \text{FreeQ}[\{b, c, d, e, f, m\}, x]$

Rule 2954

$\text{Int}[(a_.) + \text{csc}[(e_.) + (f_.)(x_.)]*(b_.)]^{(m_.)} * (\text{csc}[(e_.) + (f_.)(x_.)] * (d_.) + (c_.))^{(n_.)} * ((g_.) \sin[(e_.) + (f_.)(x_.)])^{(p_.)}, x_Symbol] \rightarrow \text{Dist}[g^{(m+n)}, \text{Int}[(g \sin[e + f x])^{(p-m-n)} * (b + a \sin[e + f x])^m * (d + c \sin[e + f x])^n, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, g, p\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& !\text{IntegerQ}[p] \&\& \text{IntegerQ}[m] \&\& \text{IntegerQ}[n]$

Rule 2968

$\text{Int}[(a_.) + (b_.) \sin[(e_.) + (f_.)(x_.)]]^{(m_.)} * ((A_.) + (B_.) \sin[(e_.) + (f_.)(x_.)]) * ((c_.) + (d_.) \sin[(e_.) + (f_.)(x_.)]), x_Symbol] \rightarrow \text{Int}[(a + b \sin[e + f x])^m * (A*c + (B*c + A*d) \sin[e + f x] + B*d \sin[e + f x]^2), x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B, m\}, x] \&\& \text{NeQ}[b*c - a*d, 0]$

Rule 2976

$\text{Int}[(a_.) + (b_.) \sin[(e_.) + (f_.)(x_.)]]^{(m_.)} * ((A_.) + (B_.) \sin[(e_.) + (f_.)(x_.)]) * ((c_.) + (d_.) \sin[(e_.) + (f_.)(x_.)])^{(n_.)}, x_Symbol] \rightarrow -\text{Simp}[(b*B \cos[e + f x] * (a + b \sin[e + f x])^{(m-1)} * (c + d \sin[e + f x])^{(n+1)}) / (d*f*(m+n+1)), x] + \text{Dist}[1/(d*(m+n+1)), \text{Int}[(a + b \sin[e + f x])^{(m-1)} * (c + d \sin[e + f x])^n * \text{Simp}[a*A*d*(m+n+1) + B*(a*c*(m-1) + b*d*(n+1)) + (A*b*d*(m+n+1) - B*(b*c*m - a*d*(2*m+n))] * \sin[e + f x], x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{GtQ}[m, 1/2] \&\& !\text{LtQ}[n, -1] \&\& \text{IntegerQ}[2*m] \&\& (\text{IntegerQ}[2*n] \mid \mid \text{EqQ}[c, 0])$

Rule 3023

$\text{Int}[(a_.) + (b_.) \sin[(e_.) + (f_.)(x_.)]]^{(m_.)} * ((A_.) + (B_.) \sin[(e_.) + (f_.)(x_.)] + (C_.) \sin[(e_.) + (f_.)(x_.)]^2), x_Symbol] \rightarrow -\text{Simp}[(C \cos[e + f x] * (a + b \sin[e + f x])^{(m+1)}) / (b*f*(m+2)), x] + \text{Dist}[1/(b*(m+2)), \text{Int}[(a + b \sin[e + f x])^m * \text{Simp}[A*b*(m+2) + b*C*(m+1) + (b*B*(m+2) - a*C) \sin[e + f x], x], x], x] /; \text{FreeQ}[\{a, b, e, f, A, B, C, m\}, x] \&\& !\text{LtQ}[m, -1]$

Rubi steps

$$\begin{aligned}
\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2(A+B\sec(c+dx))dx &= \int \frac{(a+a\cos(c+dx))^2(B+A\cos(c+dx))}{\sqrt{\cos(c+dx)}}dx \\
&= \frac{2A\sqrt{\cos(c+dx)}(a^2+a^2\cos(c+dx))\sin(c+dx)}{5d} \\
&= \frac{2A\sqrt{\cos(c+dx)}(a^2+a^2\cos(c+dx))\sin(c+dx)}{5d} \\
&= \frac{2a^2(7A+5B)\sqrt{\cos(c+dx)}\sin(c+dx)}{15d} + \frac{2A\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} \\
&= \frac{2a^2(7A+5B)\sqrt{\cos(c+dx)}\sin(c+dx)}{15d} + \frac{2A\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} \\
&= \frac{4a^2(4A+5B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{4a^2(A+2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}
\end{aligned}$$

Mathematica [C] time = 6.31, size = 994, normalized size = 7.89

$$\frac{\cos^{\frac{7}{2}}(c+dx)(\sec(c+dx)a+a)^2(A+B\sec(c+dx))\left(-\frac{(4A+5B)\cot(c)}{5d} + \frac{(2A+B)\cos(dx)\sin(c)}{6d} + \frac{A\cos(2dx)\sin(2c)}{20d} + \frac{(2A+B)\cos(2dx)\sin(2c)}{20d}\right)}{B+A\cos(c+dx)}$$

Warning: Unable to verify antiderivative.

```

[In] Integrate[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
[Out] (Cos[c + d*x]^(7/2)*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x])*(-1/5*((4*A + 5*B)*Cot[c])/d + ((2*A + B)*Cos[d*x]*Sin[c])/(6*d) + (A*Cos[2*d*x]*Sin[2*c])/(20*d) + ((2*A + B)*Cos[c]*Sin[d*x])/(6*d) + (A*Cos[2*c]*Sin[2*d*x])/(20*d)))/(B + A*Cos[c + d*x]) - (A*Cos[c + d*x]^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]))/(3*d*(B + A*Cos[c + d*x])*Sqrt[1 + Cot[c]^2]) - (2*B*Cos[c + d*x]^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*

```

```
Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*
x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]])]/(3*d*(B + A*Cos[
c + d*x])*Sqrt[1 + Cot[c]^2]) - (2*A*Cos[c + d*x]^3*Csc[c]*Sec[c/2 + (d*x)/
2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x])*((HypergeometricPFQ[{-1/2,
-1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c
])/((Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]
*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])*Sqrt[1 + Tan[c]^
2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*
Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]))/(Cos[c]^2 + Sin[c]^2))/Sqrt[C
os[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]))/(5*d*(B + A*Cos[c + d
*x])) - (B*Cos[c + d*x]^3*Csc[c]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^
2*(A + B*Sec[c + d*x])*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + A
rcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/((Sqrt[1 - Cos[d*x + Arc
Tan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + Arc
Tan[Tan[c]]]*Sqrt[1 + Tan[c]^2])*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[T
an[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*
Sqrt[1 + Tan[c]^2]))/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan
[c]]]*Sqrt[1 + Tan[c]^2])))/(2*d*(B + A*Cos[c + d*x]))
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

integral $\left((Ba^2 \cos(dx + c))^2 \sec(dx + c)^3 + (A + 2B)a^2 \cos(dx + c)^2 \sec(dx + c)^2 + (2A + B)a^2 \cos(dx + c)^2 \sec(dx + c) + Aa^2 \cos(dx + c)^2 \sqrt{\cos(dx + c)} \right), x$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] integral((B*a^2*cos(d*x + c)^2*sec(d*x + c)^3 + (A + 2*B)*a^2*cos(d*x + c)^
2*sec(d*x + c)^2 + (2*A + B)*a^2*cos(d*x + c)^2*sec(d*x + c) + A*a^2*cos(d*
x + c)^2)*sqrt(cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(a \sec(dx + c) + a)^2 \cos(dx + c)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^2*cos(d*x + c)^(5/2), x
)
```

maple [B] time = 4.20, size = 357, normalized size = 2.83

$$4\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} a^2 \left(-12A \cos\left(\frac{dx}{2} + \frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + (32A + 10B)\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x)`

[Out] $-4/15*((2*\cos(1/2*d*x+1/2*c)^2-1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*a^2*(-12*A*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^6+(32*A+10*B)*\sin(1/2*d*x+1/2*c)^4*\cos(1/2*d*x+1/2*c)+(-13*A-5*B)*\sin(1/2*d*x+1/2*c)^2*\cos(1/2*d*x+1/2*c)+5*A*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)})-12*A*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)})+10*B*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)})-15*B*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)})/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(a \sec(dx + c) + a)^2 \cos(dx + c)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="maxima")`

[Out] `integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^2*cos(d*x + c)^(5/2), x)`

mupad [B] time = 2.91, size = 153, normalized size = 1.21

$$\frac{2 A a^2 \left(\frac{2 \sqrt{\cos(c+dx)} \sin(c+dx)}{3} + \frac{2 F\left(\frac{c}{2} + \frac{dx}{2} \middle| 2\right)}{3} \right)}{d} + \frac{2 B a^2 \left(\sqrt{\cos(c+dx)} \sin(c+dx) + 6 E\left(\frac{c}{2} + \frac{dx}{2} \middle| 2\right) + 4 F\left(\frac{c}{2} + \frac{dx}{2} \middle| 2\right) \right)}{3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(c + d*x)^(5/2)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^2,x)`

```
[Out] (2*A*a^2*((2*cos(c + d*x)^(1/2)*sin(c + d*x))/3 + (2*ellipticF(c/2 + (d*x)/
2, 2))/3))/d + (2*B*a^2*(cos(c + d*x)^(1/2)*sin(c + d*x) + 6*ellipticE(c/2
+ (d*x)/2, 2) + 4*ellipticF(c/2 + (d*x)/2, 2)))/(3*d) + (2*A*a^2*ellipticE(
c/2 + (d*x)/2, 2))/d - (2*A*a^2*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([
1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2))
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+a*sec(d*x+c))**2*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

$$3.492 \quad \int \cos^{\frac{3}{2}}(c+dx)(a+a \sec(c+dx))^2(A+B \sec(c+dx)) dx$$

Optimal. Leaf size=116

$$\frac{4a^2(2A+3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2a^2(A-3B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d} + \frac{4a^2AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2B\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

[Out] $4*a^2*A*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c),2^{(1/2)})/d+4/3*a^2*(2*A+3*B)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c),2^{(1/2)})/d+2*B*(a^2+a^2*\cos(d*x+c))*\sin(d*x+c)/d/\cos(d*x+c)^{(1/2)}+2/3*a^2*(A-3*B)*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/d$

Rubi [A] time = 0.34, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {2954, 2975, 2968, 3023, 2748, 2641, 2639}

$$\frac{4a^2(2A+3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2a^2(A-3B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d} + \frac{4a^2AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2B\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Cos}[c+d*x]^{(3/2)}*(a+a*\text{Sec}[c+d*x])^2*(A+B*\text{Sec}[c+d*x]),x]$

[Out] $(4*a^2*A*\text{EllipticE}[(c+d*x)/2,2])/d+(4*a^2*(2*A+3*B)*\text{EllipticF}[(c+d*x)/2,2])/(3*d)+(2*a^2*(A-3*B)*\text{Sqrt}[\text{Cos}[c+d*x]]*\text{Sin}[c+d*x])/(3*d)+(2*B*(a^2+a^2*\text{Cos}[c+d*x])* \text{Sin}[c+d*x])/(d*\text{Sqrt}[\text{Cos}[c+d*x]])$

Rule 2639

$\text{Int}[\text{Sqrt}[\sin[(c_.)+(d_.)*(x_)]]],x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticE}[(1*(c-Pi/2+d*x))/2,2])/d,x] /; \text{FreeQ}\{c,d\},x]$

Rule 2641

$\text{Int}[1/\text{Sqrt}[\sin[(c_.)+(d_.)*(x_)]]],x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticF}[(1*(c-Pi/2+d*x))/2,2])/d,x] /; \text{FreeQ}\{c,d\},x]$

Rule 2748

$\text{Int}[(b_.)*\sin[(e_.)+(f_.)*(x_)]]^{(m_.)*((c_.)+(d_.)*\sin[(e_.)+(f_.)*(x_)])},x_Symbol] \rightarrow \text{Dist}[c,\text{Int}[(b*\text{Sin}[e+f*x])^m,x],x] + \text{Dist}[d/b,\text{Int}[(b*\text{Sin}[e+f*x])^m,x],x]$

$b \sin[e + f x]^{m+1}, x], x] /; \text{FreeQ}\{b, c, d, e, f, m\}, x]$

Rule 2954

$\text{Int}[(a_.) + \text{csc}[e_.) + (f_.)(x_.)](b_.))^{m_.}(\text{csc}[e_.) + (f_.)(x_.)](d_.) + (c_.))^{n_.}((g_.)\sin[e_.) + (f_.)(x_.)]^{p_.}, x_Symbol] \rightarrow \text{Dist}[g^{m+n}, \text{Int}[(g \sin[e + f x])^{p-m-n}(b + a \sin[e + f x])^m(d + c \sin[e + f x])^n, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, p\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& !\text{IntegerQ}[p] \&\& \text{IntegerQ}[m] \&\& \text{IntegerQ}[n]$

Rule 2968

$\text{Int}[(a_.) + (b_.)\sin[e_.) + (f_.)(x_.)]^{m_.}((A_.) + (B_.)\sin[e_.) + (f_.)(x_.)](c_.) + (d_.)\sin[e_.) + (f_.)(x_.)], x_Symbol] \rightarrow \text{Int}[(a + b \sin[e + f x])^m(Ac + (Bc + Ad)\sin[e + f x] + B*d \sin[e + f x]^2), x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B, m\}, x] \&\& \text{NeQ}[b*c - a*d, 0]$

Rule 2975

$\text{Int}[(a_.) + (b_.)\sin[e_.) + (f_.)(x_.)]^{m_.}((A_.) + (B_.)\sin[e_.) + (f_.)(x_.)](c_.) + (d_.)\sin[e_.) + (f_.)(x_.)]^{n_.}, x_Symbol] \rightarrow -\text{Simp}[(b^2(Bc - Ad)\cos[e + f x](a + b \sin[e + f x])^{m-1}(c + d \sin[e + f x])^{n+1})/(d*f*(n+1)*(b*c + a*d)), x] - \text{Dist}[b/(d*(n+1)*(b*c + a*d)), \text{Int}[(a + b \sin[e + f x])^{m-1}(c + d \sin[e + f x])^{n+1} \text{Simp}[a*Ad*(m-n-2) - B*(a*c*(m-1) + b*d*(n+1)) - (A*b*d*(m+n+1) - B*(b*c*m - a*d*(n+1))]*\sin[e + f x], x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{GtQ}[m, 1/2] \&\& \text{LtQ}[n, -1] \&\& \text{IntegerQ}[2*m] \&\& (\text{IntegerQ}[2*n] || \text{EqQ}[c, 0])$

Rule 3023

$\text{Int}[(a_.) + (b_.)\sin[e_.) + (f_.)(x_.)]^{m_.}((A_.) + (B_.)\sin[e_.) + (f_.)(x_.)] + (C_.)\sin[e_.) + (f_.)(x_.)]^2), x_Symbol] \rightarrow -\text{Simp}[(C*\cos[e + f x](a + b \sin[e + f x])^{m+1})/(b*f*(m+2)), x] + \text{Dist}[1/(b*(m+2)), \text{Int}[(a + b \sin[e + f x])^m \text{Simp}[A*b*(m+2) + b*C*(m+1) + (b*B*(m+2) - a*C)*\sin[e + f x], x], x], x] /; \text{FreeQ}\{a, b, e, f, A, B, C, m\}, x] \&\& !\text{LtQ}[m, -1]$

Rubi steps

$$\begin{aligned}
\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2(A+B\sec(c+dx))dx &= \int \frac{(a+a\cos(c+dx))^2(B+A\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}dx \\
&= \frac{2B(a^2+a^2\cos(c+dx))\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + 2\int \frac{(a+a\cos(c+dx))^2}{\cos^{\frac{3}{2}}(c+dx)}dx \\
&= \frac{2B(a^2+a^2\cos(c+dx))\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + 2\int \frac{\frac{1}{2}a^2(A+B\sec(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}dx \\
&= \frac{2a^2(A-3B)\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} + \frac{2B(a^2+a^2\cos(c+dx))\sin(c+dx)}{d\sqrt{\cos(c+dx)}} \\
&= \frac{2a^2(A-3B)\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} + \frac{2B(a^2+a^2\cos(c+dx))\sin(c+dx)}{d\sqrt{\cos(c+dx)}} \\
&= \frac{4a^2AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{4a^2(2A+3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}
\end{aligned}$$

Mathematica [C] time = 6.39, size = 735, normalized size = 6.34

$$A \csc(c) \cos^3(c+dx) \sec^4\left(\frac{c}{2} + \frac{dx}{2}\right) (a \sec(c+dx) + a)^2 (A + B \sec(c+dx)) \left(\frac{\tan(c) \sin(\tan^{-1}(\tan(c)+dx))}{\sqrt{\tan^2(c)+1} \sqrt{1-\cos(\tan^{-1}(\tan(c)+dx))} \sqrt{\cos(c)}} \right)$$

$$2d(A \cos(c+dx) + \dots)$$

Warning: Unable to verify antiderivative.

[In] Integrate[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]

[Out] (Cos[c + d*x]^(7/2)*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x])*(-1/4*((2*A - B + 2*A*Cos[2*c] + B*Cos[2*c])*Csc[c]*Sec[c])/d + (A*Cos[d*x]*Sin[c])/(6*d) + (A*Cos[c]*Sin[d*x])/(6*d) + (B*Sec[c]*Sec[c + d*x]*Sin[d*x])/(2*d)))/(B + A*Cos[c + d*x]) - (2*A*Cos[c + d*x]^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(B + A*Cos[c + d*x])*Sqrt[1 + Cot[c]^2]) - (B*Cos[c + d*x]^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 -

$$\frac{\sin[d*x - \text{ArcTan}[\text{Cot}[c]]] * \sqrt{-(\sqrt{1 + \text{Cot}[c]^2} * \sin[c] * \sin[d*x - \text{ArcTan}[\text{Cot}[c]]]) * \sqrt{1 + \sin[d*x - \text{ArcTan}[\text{Cot}[c]]]}}}{d * (B + A * \cos[c + d*x]) * \sqrt{1 + \text{Cot}[c]^2}} - \frac{(A * \cos[c + d*x]^3 * \text{Csc}[c] * \text{Sec}[c/2 + (d*x)/2]^4 * (a + a * \text{Sec}[c + d*x])^2 * (A + B * \text{Sec}[c + d*x]) * (\text{HypergeometricPFQ}[\{-1/2, -1/4\}, \{3/4\}, \cos[d*x + \text{ArcTan}[\text{Tan}[c]]]^2 * \sin[d*x + \text{ArcTan}[\text{Tan}[c]]] * \text{Tan}[c]) / (\sqrt{1 - \cos[d*x + \text{ArcTan}[\text{Tan}[c]]] * \sqrt{1 + \cos[d*x + \text{ArcTan}[\text{Tan}[c]]] * \sqrt{\cos[c] * \cos[d*x + \text{ArcTan}[\text{Tan}[c]]} * \sqrt{1 + \tan[c]^2} * \sqrt{1 + \tan[c]^2}}) - ((\sin[d*x + \text{ArcTan}[\text{Tan}[c]]] * \text{Tan}[c]) / \sqrt{1 + \tan[c]^2} + (2 * \cos[c]^2 * \cos[d*x + \text{ArcTan}[\text{Tan}[c]]] * \sqrt{1 + \tan[c]^2}) / (\cos[c]^2 + \sin[c]^2) / \sqrt{\cos[c] * \cos[d*x + \text{ArcTan}[\text{Tan}[c]]] * \sqrt{1 + \tan[c]^2}})) / (2 * d * (B + A * \cos[c + d*x]))}{1}$$

fricas [F] time = 0.46, size = 0, normalized size = 0.00

integral((Ba² cos(dx + c) sec(dx + c)³ + (A + 2B)a² cos(dx + c) sec(dx + c)² + (2A + B)a² cos(dx + c) sec(dx + c)) * sqrt(cos(dx + c)), x)

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] integral((B*a²*cos(d*x + c)*sec(d*x + c)³ + (A + 2*B)*a²*cos(d*x + c)*sec(d*x + c)² + (2*A + B)*a²*cos(d*x + c)*sec(d*x + c) + A*a²*cos(d*x + c)) * sqrt(cos(d*x + c)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(a \sec(dx + c) + a)^2 \cos(dx + c)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^2*cos(d*x + c)^(3/2), x)

maple [B] time = 4.59, size = 388, normalized size = 3.34

$$4a^2 \left(2A \sqrt{-2 \left(\sin^4 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) + \sin^2 \left(\frac{dx}{2} + \frac{c}{2} \right)} \cos \left(\frac{dx}{2} + \frac{c}{2} \right) \left(\sin^4 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) - \sqrt{-2 \left(\sin^4 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) + \sin^2 \left(\frac{dx}{2} + \frac{c}{2} \right)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x)

```
[Out] -4/3*a^2*(2*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(A+3*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+2*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(a \sec(dx + c) + a)^2 \cos(dx + c)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^2*cos(d*x + c)^(3/2), x)
```

mupad [B] time = 3.16, size = 134, normalized size = 1.16

$$\frac{2 A a^2 \left(\sqrt{\cos(c + dx)} \sin(c + dx) + 6 E\left(\frac{c}{2} + \frac{dx}{2} \middle| 2\right) + 4 F\left(\frac{c}{2} + \frac{dx}{2} \middle| 2\right) \right)}{3 d} + \frac{2 B a^2 E\left(\frac{c}{2} + \frac{dx}{2} \middle| 2\right)}{d} + \frac{4 B a^2 F\left(\frac{c}{2} + \frac{dx}{2} \middle| 2\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^2,x)
```

```
[Out] (2*A*a^2*(cos(c + d*x)^(1/2)*sin(c + d*x) + 6*ellipticE(c/2 + (d*x)/2, 2) + 4*ellipticF(c/2 + (d*x)/2, 2)))/(3*d) + (2*B*a^2*ellipticE(c/2 + (d*x)/2, 2))/d + (4*B*a^2*ellipticF(c/2 + (d*x)/2, 2))/d + (2*B*a^2*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+a*sec(d*x+c))**2*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

$$3.493 \quad \int \sqrt{\cos(c + dx)} (a + a \sec(c + dx))^2 (A + B \sec(c + dx)) dx$$

Optimal. Leaf size=120

$$\frac{4a^2(3A + 2B)F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3d} + \frac{2a^2(3A + 5B)\sin(c + dx)}{3d\sqrt{\cos(c + dx)}} - \frac{4a^2BE\left(\frac{1}{2}(c + dx) \middle| 2\right)}{d} + \frac{2B\sin(c + dx)(a^2\cos(c + dx))^{3/2}}{3d\cos^2(c + dx)}$$

[Out] $-4*a^2*B*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/d+4/3*a^2*(3*A+2*B)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/d+2/3*B*(a^2+a^2*\cos(d*x+c))*\sin(d*x+c)/d/\cos(d*x+c)^{(3/2)}+2/3*a^2*(3*A+5*B)*\sin(d*x+c)/d/\cos(d*x+c)^{(1/2)}$

Rubi [A] time = 0.35, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {2954, 2975, 2968, 3021, 2748, 2641, 2639}

$$\frac{4a^2(3A + 2B)F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3d} + \frac{2a^2(3A + 5B)\sin(c + dx)}{3d\sqrt{\cos(c + dx)}} - \frac{4a^2BE\left(\frac{1}{2}(c + dx) \middle| 2\right)}{d} + \frac{2B\sin(c + dx)(a^2\cos(c + dx))^{3/2}}{3d\cos^2(c + dx)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sqrt}[\text{Cos}[c + d*x]]*(a + a*\text{Sec}[c + d*x])^2*(A + B*\text{Sec}[c + d*x]), x]$

[Out] $(-4*a^2*B*\text{EllipticE}[(c + d*x)/2, 2])/d + (4*a^2*(3*A + 2*B)*\text{EllipticF}[(c + d*x)/2, 2])/(3*d) + (2*a^2*(3*A + 5*B)*\text{Sin}[c + d*x])/(3*d*\text{Sqrt}[\text{Cos}[c + d*x]]) + (2*B*(a^2 + a^2*\text{Cos}[c + d*x])*\text{Sin}[c + d*x])/(3*d*\text{Cos}[c + d*x]^{(3/2)})$

Rule 2639

$\text{Int}[\text{Sqrt}[\sin[(c_.) + (d_.)*(x_)]]], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticE}[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}[\{c, d\}, x]$

Rule 2641

$\text{Int}[1/\text{Sqrt}[\sin[(c_.) + (d_.)*(x_)]]], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticF}[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}[\{c, d\}, x]$

Rule 2748

$\text{Int}[(b_.)*\sin[(e_.) + (f_.)*(x_)]]^{(m_.)*((c_.) + (d_.)*\sin[(e_.) + (f_.)*(x_)])}, x_Symbol] \rightarrow \text{Dist}[c, \text{Int}[(b*\text{Sin}[e + f*x])^m, x], x] + \text{Dist}[d/b, \text{Int}[($

$b \sin[e + f x]^{m+1}, x], x] /;$ FreeQ[{b, c, d, e, f, m}, x]

Rule 2954

Int[((a_.) + csc[(e_.) + (f_.)*(x_)])*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] := Dist[g^(m+n), Int[(g*SIN[e + f*x])^(p-m-n)*(b + a*SIN[e + f*x])^m*(d + c*SIN[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]

Rule 2968

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Int[(a + b*SIN[e + f*x])^m*(A*c + (B*c + A*d)*SIN[e + f*x] + B*d*SIN[e + f*x]^2), x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]

Rule 2975

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^n, x_Symbol] := -Simp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*SIN[e + f*x])^(m-1)*(c + d*SIN[e + f*x])^(n+1))/(d*f*(n+1)*(b*c + a*d)), x] - Dist[b/(d*(n+1)*(b*c + a*d)), Int[(a + b*SIN[e + f*x])^(m-1)*(c + d*SIN[e + f*x])^(n+1)*Simp[a*A*d*(m-n-2) - B*(a*c*(m-1) + b*d*(n+1)) - (A*b*d*(m+n+1) - B*(b*c*m - a*d*(n+1)))*SIN[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])

Rule 3021

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] := -Simp[(A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*SIN[e + f*x])^(m+1)/(b*f*(m+1)*(a^2 - b^2)), x] + Dist[1/(b*(m+1)*(a^2 - b^2)), Int[(a + b*SIN[e + f*x])^(m+1)*Simp[b*(a*A - b*B + a*C)*(m+1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C))*(m+1)*SIN[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned}
\int \sqrt{\cos(c+dx)} (a+a \sec(c+dx))^2 (A+B \sec(c+dx)) dx &= \int \frac{(a+a \cos(c+dx))^2 (B+A \cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)} dx \\
&= \frac{2B(a^2+a^2 \cos(c+dx)) \sin(c+dx)}{3d \cos^{\frac{3}{2}}(c+dx)} + \frac{2}{3} \int \frac{(a+a \cos(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)} dx \\
&= \frac{2B(a^2+a^2 \cos(c+dx)) \sin(c+dx)}{3d \cos^{\frac{3}{2}}(c+dx)} + \frac{2}{3} \int \frac{\frac{1}{2}a^2(3 \cos^2(c+dx)+1)}{\cos^{\frac{5}{2}}(c+dx)} dx \\
&= \frac{2a^2(3A+5B) \sin(c+dx)}{3d \sqrt{\cos(c+dx)}} + \frac{2B(a^2+a^2 \cos(c+dx))}{3d \cos^{\frac{3}{2}}(c+dx)} \\
&= \frac{2a^2(3A+5B) \sin(c+dx)}{3d \sqrt{\cos(c+dx)}} + \frac{2B(a^2+a^2 \cos(c+dx))}{3d \cos^{\frac{3}{2}}(c+dx)} \\
&= -\frac{4a^2 B E\left(\frac{1}{2}(c+dx) \middle| 2\right)}{d} + \frac{4a^2(3A+2B) F\left(\frac{1}{2}(c+dx) \middle| 2\right)}{3d}
\end{aligned}$$

Mathematica [C] time = 6.48, size = 736, normalized size = 6.13

$$B \csc(c) \cos^3(c+dx) \sec^4\left(\frac{c}{2} + \frac{dx}{2}\right) (a \sec(c+dx) + a)^2 (A+B \sec(c+dx)) \left(\frac{\tan(c) \sin(\tan^{-1}(\tan(c)))}{\sqrt{\tan^2(c)+1} \sqrt{1-\cos(\tan^{-1}(\tan(c))+dx)} \sqrt{\cos(c+dx)}} \right)$$

$$2d(A \cos(c+dx) + B \sec(c+dx))$$

Warning: Unable to verify antiderivative.

```

[In] Integrate[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
[Out] (Cos[c + d*x]^(7/2)*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x])*(-1/4*(-A - 4*B + A*Cos[2*c])*Csc[c]*Sec[c])/d + (B*Sec[c]*Sec[c + d*x]^2*Sin[d*x])/(6*d) + (Sec[c]*Sec[c + d*x]*(B*Sin[c] + 3*A*Sin[d*x] + 6*B*Sin[d*x]))/(6*d))/(B + A*Cos[c + d*x]) - (A*Cos[c + d*x]^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]])]/(d*(B + A*Cos[c + d*x])*Sqrt[1 + Cot[c]^2]) - (2*B*Cos[c + d*x]^3*Csc[c]*Hypergeometric

```

PFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]])]/(3*d*(B + A*Cos[c + d*x])*Sqrt[1 + Cot[c]^2]) + (B*Cos[c + d*x]^3*Csc[c]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x])*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])))/(2*d*(B + A*Cos[c + d*x]))

fricas [F] time = 0.47, size = 0, normalized size = 0.00

integral((Ba^2 sec(dx + c)^3 + (A + 2B)a^2 sec(dx + c)^2 + (2A + B)a^2 sec(dx + c) + Aa^2) sqrt(cos(dx + c)), x)

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))*cos(d*x+c)^(1/2),x, algorithm="fricas")

[Out] integral((B*a^2*sec(d*x + c)^3 + (A + 2*B)*a^2*sec(d*x + c)^2 + (2*A + B)*a^2*sec(d*x + c) + A*a^2)*sqrt(cos(d*x + c)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(a \sec(dx + c) + a)^2 \sqrt{\cos(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))*cos(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^2*sqrt(cos(d*x + c)), x)

maple [B] time = 4.81, size = 513, normalized size = 4.28

$$4 \left(6 \sqrt{-2 \left(\sin^4 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) + \sin^2 \left(\frac{dx}{2} + \frac{c}{2} \right)} (A + 2B) \cos \left(\frac{dx}{2} + \frac{c}{2} \right) \left(\sin^4 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) - \sqrt{-2 \left(\sin^4 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) + \sin^2 \left(\frac{dx}{2} + \frac{c}{2} \right)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))*cos(d*x+c)^(1/2),x)

[Out]
$$-4/3*(6*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(A+2*B)*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^4-(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(3*A+7*B)*\sin(1/2*d*x+1/2*c)^2*\cos(1/2*d*x+1/2*c)-2*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(3*A*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})+2*B*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})+3*B*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)}))*\sin(1/2*d*x+1/2*c)^2+3*A*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})+2*B*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}+3*B*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)}))*a^2/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(3/2)}/\sin(1/2*d*x+1/2*c)/d$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(a \sec(dx + c) + a)^2 \sqrt{\cos(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))*cos(d*x+c)^(1/2),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^2*sqrt(cos(d*x + c)), x)

mupad [B] time = 3.94, size = 196, normalized size = 1.63

$$\frac{2 A a^2 E\left(\frac{c}{2} + \frac{dx}{2} \middle| 2\right)}{d} + \frac{4 A a^2 F\left(\frac{c}{2} + \frac{dx}{2} \middle| 2\right)}{d} + \frac{2 B a^2 F\left(\frac{c}{2} + \frac{dx}{2} \middle| 2\right)}{d} + \frac{2 A a^2 \sin(c + dx) {}_2F_1\left(-\frac{1}{4}, \frac{1}{2}; \frac{3}{4}; \cos(c + dx)\right)}{d \sqrt{\cos(c + dx)} \sqrt{\sin(c + dx)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^(1/2)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^2,x)

[Out]
$$(2*A*a^2*\text{ellipticE}(c/2 + (d*x)/2, 2))/d + (4*A*a^2*\text{ellipticF}(c/2 + (d*x)/2, 2))/d + (2*B*a^2*\text{ellipticF}(c/2 + (d*x)/2, 2))/d + (2*A*a^2*\sin(c + d*x)*\text{hypergeom}([-1/4, 1/2], 3/4, \cos(c + d*x)^2))/(d*\cos(c + d*x)^{(1/2)}*(\sin(c + d*x)^2)^{(1/2)}) + (4*B*a^2*\sin(c + d*x)*\text{hypergeom}([-1/4, 1/2], 3/4, \cos(c + d*x)^2))/(d*\cos(c + d*x)^{(1/2)}*(\sin(c + d*x)^2)^{(1/2)}) + (2*B*a^2*\sin(c + d*x)*\text{hypergeom}([-1/4, 1/2], 3/4, \cos(c + d*x)^2))/(d*\cos(c + d*x)^{(1/2)}*(\sin(c + d*x)^2)^{(1/2)})$$

$x) \cdot \text{hypergeom}([-3/4, 1/2], 1/4, \cos(c + d \cdot x)^2) / (3 \cdot d \cdot \cos(c + d \cdot x)^{3/2} \cdot \sin(c + d \cdot x)^{1/2})$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2 \left(\int A \sqrt{\cos(c + dx)} dx + \int 2A \sqrt{\cos(c + dx)} \sec(c + dx) dx + \int A \sqrt{\cos(c + dx)} \sec^2(c + dx) dx + \int B \sqrt{\cos(c + dx)} \sec^3(c + dx) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))**2*(A+B*sec(d*x+c))*cos(d*x+c)**(1/2),x)

[Out] a**2*(Integral(A*sqrt(cos(c + d*x)), x) + Integral(2*A*sqrt(cos(c + d*x))*sec(c + d*x), x) + Integral(A*sqrt(cos(c + d*x))*sec(c + d*x)**2, x) + Integral(B*sqrt(cos(c + d*x))*sec(c + d*x), x) + Integral(2*B*sqrt(cos(c + d*x))*sec(c + d*x)**2, x) + Integral(B*sqrt(cos(c + d*x))*sec(c + d*x)**3, x))

$$3.494 \quad \int \frac{(a+a \sec(c+dx))^2(A+B \sec(c+dx))}{\sqrt{\cos(c+dx)}} dx$$

Optimal. Leaf size=159

$$\frac{4a^2(2A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} - \frac{4a^2(5A+4B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2a^2(5A+7B)\sin(c+dx)}{15d \cos^{\frac{3}{2}}(c+dx)} + \frac{4a^2(5A+4B)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

[Out] $-4/5*a^2*(5*A+4*B)*(cos(1/2*d*x+1/2*c)^2)^{(1/2)}/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c), 2^{(1/2)})/d+4/3*a^2*(2*A+B)*(cos(1/2*d*x+1/2*c)^2)^{(1/2)}/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c), 2^{(1/2)})/d+2/15*a^2*(5*A+7*B)*sin(d*x+c)/d/cos(d*x+c)^{(3/2)}+2/5*B*(a^2+a^2*cos(d*x+c))*sin(d*x+c)/d/cos(d*x+c)^{(5/2)}+4/5*a^2*(5*A+4*B)*sin(d*x+c)/d/cos(d*x+c)^{(1/2)}$

Rubi [A] time = 0.38, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {2954, 2975, 2968, 3021, 2748, 2636, 2639, 2641}

$$\frac{4a^2(2A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} - \frac{4a^2(5A+4B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2a^2(5A+7B)\sin(c+dx)}{15d \cos^{\frac{3}{2}}(c+dx)} + \frac{4a^2(5A+4B)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + a*\text{Sec}[c + d*x])^2*(A + B*\text{Sec}[c + d*x])/Sqrt[\text{Cos}[c + d*x]], x]$

[Out] $(-4*a^2*(5*A + 4*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^2*(2*A + B)*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*a^2*(5*A + 7*B)*Sin[c + d*x])/(15*d*Cos[c + d*x]^{(3/2)}) + (4*a^2*(5*A + 4*B)*Sin[c + d*x])/(5*d*Sqrt[\text{Cos}[c + d*x]]) + (2*B*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(5*d*Cos[c + d*x]^{(5/2)})$

Rule 2636

$\text{Int}[(b_*)*\sin[(c_*) + (d_*)(x_)]^{(n_)}, x_Symbol] \rightarrow \text{Simp}[(\text{Cos}[c + d*x]*(b*\text{Sin}[c + d*x])^{(n+1)})/(b*d*(n+1)), x] + \text{Dist}[(n+2)/(b^2*(n+1)), \text{Int}[(b*\text{Sin}[c + d*x])^{(n+2)}, x], x] /;$ $\text{FreeQ}\{b, c, d\}, x \&\& \text{LtQ}[n, -1] \&\& \text{IntegerQ}[2*n]$

Rule 2639

$\text{Int}[Sqrt[\sin[(c_*) + (d_*)(x_)]], x_Symbol] \rightarrow \text{Simp}[(2*EllipticE[(1*(c - P i/2 + d*x))/2, 2])/d, x] /;$ $\text{FreeQ}\{c, d\}, x$

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_.)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_.)]^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2954

Int[((a_.) + csc[(e_.) + (f_.)*(x_.)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_.)]^(p_.), x_Symbol] := Dist[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]

Rule 2968

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)]*(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] := Int[(a + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2), x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]

Rule 2975

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)]*(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]^(n_.), x_Symbol] := -Simp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])

Rule 3021

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)] + (C_.)*sin[(e_.) + (f_.)*(x_.)]^2), x_Symbol] := -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C))*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,

C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{(a + a \sec(c + dx))^2 (A + B \sec(c + dx))}{\sqrt{\cos(c + dx)}} dx &= \int \frac{(a + a \cos(c + dx))^2 (B + A \cos(c + dx))}{\cos^{\frac{7}{2}}(c + dx)} dx \\
 &= \frac{2B (a^2 + a^2 \cos(c + dx)) \sin(c + dx)}{5d \cos^{\frac{5}{2}}(c + dx)} + \frac{2}{5} \int \frac{(a + a \cos(c + dx))}{\cos^{\frac{5}{2}}(c + dx)} dx \\
 &= \frac{2B (a^2 + a^2 \cos(c + dx)) \sin(c + dx)}{5d \cos^{\frac{5}{2}}(c + dx)} + \frac{2}{5} \int \frac{\frac{1}{2} a^2 (5A + 7B) + \left(\frac{1}{2}\right)}{\cos^{\frac{5}{2}}(c + dx)} dx \\
 &= \frac{2a^2 (5A + 7B) \sin(c + dx)}{15d \cos^{\frac{3}{2}}(c + dx)} + \frac{2B (a^2 + a^2 \cos(c + dx)) \sin(c + dx)}{5d \cos^{\frac{5}{2}}(c + dx)} \\
 &= \frac{2a^2 (5A + 7B) \sin(c + dx)}{15d \cos^{\frac{3}{2}}(c + dx)} + \frac{2B (a^2 + a^2 \cos(c + dx)) \sin(c + dx)}{5d \cos^{\frac{5}{2}}(c + dx)} \\
 &= \frac{4a^2 (2A + B) F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3d} + \frac{2a^2 (5A + 7B) \sin(c + dx)}{15d \cos^{\frac{3}{2}}(c + dx)} + \frac{4a^2 (2A + B) F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3d} \\
 &= -\frac{4a^2 (5A + 4B) E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{5d} + \frac{4a^2 (2A + B) F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3d}
 \end{aligned}$$

Mathematica [C] time = 6.58, size = 1025, normalized size = 6.45

$$\frac{\cos^{\frac{7}{2}}(c + dx) (\sec(c + dx) a + a)^2 (A + B \sec(c + dx)) \left(\frac{B \sec(c) \sin(dx) \sec^3(c + dx)}{10d} + \frac{\sec(c) (3B \sin(c) + 5A \sin(dx) + 10B \sin(dx)) \sec^2(c + dx)}{30d} \right)}{B + A \cos(c + dx)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[((a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]))/Sqrt[Cos[c + d*x]], x]

```
[Out] (Cos[c + d*x]^(7/2)*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x])*(((5*A + 4*B)*Csc[c]*Sec[c])/(5*d) + (B*Sec[c]*Sec[c + d*x]^3*Sin[d*x])/(10*d) + (Sec[c]*Sec[c + d*x]^2*(3*B*Sin[c] + 5*A*Sin[d*x] + 10*B*Sin[d*x]))/(30*d) + (Sec[c]*Sec[c + d*x]*(5*A*Sin[c] + 10*B*Sin[c] + 30*A*Sin[d*x] + 24*B*Sin[d*x]))/(30*d)))/(B + A*Cos[c + d*x]) - (2*A*Cos[c + d*x]^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(B + A*Cos[c + d*x])*Sqrt[1 + Cot[c]^2]) - (B*Cos[c + d*x]^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(B + A*Cos[c + d*x])*Sqrt[1 + Cot[c]^2]) + (A*Cos[c + d*x]^3*Csc[c]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x])*((HypergeometricPFQ[-1/2, -1/4], {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2)*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]))/(2*d*(B + A*Cos[c + d*x])) + (2*B*Cos[c + d*x]^3*Csc[c]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x])*((HypergeometricPFQ[-1/2, -1/4], {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2)*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]))/(5*d*(B + A*Cos[c + d*x]))
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{Ba^2 \sec(dx+c)^3 + (A+2B)a^2 \sec(dx+c)^2 + (2A+B)a^2 \sec(dx+c) + Aa^2}{\sqrt{\cos(dx+c)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*a^2*sec(d*x + c)^3 + (A + 2*B)*a^2*sec(d*x + c)^2 + (2*A + B)*a^2*sec(d*x + c) + A*a^2)/sqrt(cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(a \sec(dx + c) + a)^2}{\sqrt{\cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^2/sqrt(cos(d*x + c)), x)

maple [B] time = 12.76, size = 741, normalized size = 4.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x)

[Out]
$$\begin{aligned} & -8*(-(-2*\cos(1/2*d*x+1/2*c)^2+1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*a^2*(1/4*A*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)} \\ & + (1/4*A+1/2*B)*(-1/6*\cos(1/2*d*x+1/2*c)*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)} / (-1/2+\cos(1/2*d*x+1/2*c)^2)^2+1/3*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)} \\ & *(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * \text{EllipticF}(\cos(1/2*d*x+1/2*c), 2^{(1/2)}) \\ & - 1/20*B/(8*\sin(1/2*d*x+1/2*c)^6-12*\sin(1/2*d*x+1/2*c)^4+6*\sin(1/2*d*x+1/2*c)^2-1) / \sin(1/2*d*x+1/2*c)^2 \\ & * (12*\text{EllipticE}(\cos(1/2*d*x+1/2*c), 2^{(1/2)}) * (2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)} * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * \sin(1/2*d*x+1/2*c)^4 \\ & - 24*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^6-12*\text{EllipticE}(\cos(1/2*d*x+1/2*c), 2^{(1/2)}) * (2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)} \\ & * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * \sin(1/2*d*x+1/2*c)^2) / \sin(1/2*d*x+1/2*c)^2 \\ & + 24*\sin(1/2*d*x+1/2*c)^4*\cos(1/2*d*x+1/2*c)+3*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)} * \text{EllipticE}(\cos(1/2*d*x+1/2*c), 2^{(1/2)}) \\ & - 8*\sin(1/2*d*x+1/2*c)^2*\cos(1/2*d*x+1/2*c)) * (-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)} \\ & + (1/2*A+1/4*B)*(-(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)} * \text{EllipticE}(\cos(1/2*d*x+1/2*c), 2^{(1/2)}) \\ & + 2*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * \cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^2) / \sin(1/2*d*x+1/2*c)^2 \\ & ^2 / (2*\sin(1/2*d*x+1/2*c)^2-1) / \sin(1/2*d*x+1/2*c) / (2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)} / d \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(a \sec(dx + c) + a)^2}{\sqrt{\cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^2/sqrt(cos(d*x + c)), x)

mupad [B] time = 4.15, size = 229, normalized size = 1.44

$$\frac{6 B a^2 \sin(c + d x) {}_2F_1\left(-\frac{5}{4}, \frac{1}{2}; -\frac{1}{4}; \cos(c + d x)^2\right) + 20 B a^2 \cos(c + d x) \sin(c + d x) {}_2F_1\left(-\frac{3}{4}, \frac{1}{2}; \frac{1}{4}; \cos(c + d x)^2\right)}{15 d \cos(c + d x)^{5/2} \sqrt{1 - \cos(c + d x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^2)/cos(c + d*x)^(1/2),x)

[Out] (6*B*a^2*sin(c + d*x)*hypergeom([-5/4, 1/2], -1/4, cos(c + d*x)^2) + 20*B*a^2*cos(c + d*x)*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2) + 30*B*a^2*cos(c + d*x)^2*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(15*d*cos(c + d*x)^(5/2)*(1 - cos(c + d*x)^2)^(1/2)) + (2*A*a^2*ellipticF(c/2 + (d*x)/2, 2))/d + (4*A*a^2*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2)) + (2*A*a^2*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(3*d*cos(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2 \left(\int \frac{A}{\sqrt{\cos(c + dx)}} dx + \int \frac{2A \sec(c + dx)}{\sqrt{\cos(c + dx)}} dx + \int \frac{A \sec^2(c + dx)}{\sqrt{\cos(c + dx)}} dx + \int \frac{B \sec(c + dx)}{\sqrt{\cos(c + dx)}} dx + \int \frac{2B \sec^2(c + dx)}{\sqrt{\cos(c + dx)}} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))**2*(A+B*sec(d*x+c))/cos(d*x+c)**(1/2),x)

[Out] a**2*(Integral(A/sqrt(cos(c + d*x)), x) + Integral(2*A*sec(c + d*x)/sqrt(cos(c + d*x)), x) + Integral(A*sec(c + d*x)**2/sqrt(cos(c + d*x)), x) + Integral(B*sec(c + d*x)/sqrt(cos(c + d*x)), x) + Integral(2*B*sec(c + d*x)**2/sqrt(cos(c + d*x)), x) + Integral(B*sec(c + d*x)**3/sqrt(cos(c + d*x)), x))

$$3.495 \quad \int \frac{(a+a \sec(c+dx))^2(A+B \sec(c+dx))}{\cos^{\frac{3}{2}}(c+dx)} dx$$

Optimal. Leaf size=194

$$\frac{4a^2(7A+6B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} - \frac{4a^2(4A+3B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{4a^2(7A+6B)\sin(c+dx)}{21d \cos^{\frac{3}{2}}(c+dx)} + \frac{2a^2(7A+9B)\sin(c+dx)}{35d \cos^{\frac{5}{2}}(c+dx)}$$

[Out] $-4/5*a^2*(4*A+3*B)*(cos(1/2*d*x+1/2*c)^2)^{(1/2)}/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c), 2^{(1/2)})/d+4/21*a^2*(7*A+6*B)*(cos(1/2*d*x+1/2*c)^2)^{(1/2)}/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c), 2^{(1/2)})/d+2/35*a^2*(7*A+9*B)*sin(d*x+c)/d/cos(d*x+c)^{(5/2)}+4/21*a^2*(7*A+6*B)*sin(d*x+c)/d/cos(d*x+c)^{(3/2)}+2/7*B*(a^2+a^2*cos(d*x+c))*sin(d*x+c)/d/cos(d*x+c)^{(7/2)}+4/5*a^2*(4*A+3*B)*sin(d*x+c)/d/cos(d*x+c)^{(1/2)}$

Rubi [A] time = 0.42, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {2954, 2975, 2968, 3021, 2748, 2636, 2641, 2639}

$$\frac{4a^2(7A+6B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} - \frac{4a^2(4A+3B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{4a^2(7A+6B)\sin(c+dx)}{21d \cos^{\frac{3}{2}}(c+dx)} + \frac{2a^2(7A+9B)\sin(c+dx)}{35d \cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

[In] Int[((a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]))/Cos[c + d*x]^(3/2), x]

[Out] $(-4*a^2*(4*A+3*B)*EllipticE[(c+d*x)/2, 2])/(5*d) + (4*a^2*(7*A+6*B)*EllipticF[(c+d*x)/2, 2])/(21*d) + (2*a^2*(7*A+9*B)*Sin[c+d*x])/(35*d*Cos[c+d*x]^{(5/2)}) + (4*a^2*(7*A+6*B)*Sin[c+d*x])/(21*d*Cos[c+d*x]^{(3/2)}) + (4*a^2*(4*A+3*B)*Sin[c+d*x])/(5*d*Sqrt[Cos[c+d*x]]) + (2*B*(a^2+a^2*Cos[c+d*x])*Sin[c+d*x])/(7*d*Cos[c+d*x]^{(7/2)})$

Rule 2636

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), Int[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_.)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_.)]^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2954

Int[((a_.) + csc[(e_.) + (f_.)*(x_.)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_.)]^(p_.), x_Symbol] := Dist[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]

Rule 2968

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)]*(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] := Int[(a + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2), x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]

Rule 2975

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)]*(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]^(n_.), x_Symbol] := -Simp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])

Rule 3021

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)] + (C_.)*sin[(e_.) + (f_.)*(x_.)]^2), x_Symbol] := -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b

- a*B + b*C)*(m + 1)*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{(a + a \sec(c + dx))^2 (A + B \sec(c + dx))}{\cos^{\frac{3}{2}}(c + dx)} dx &= \int \frac{(a + a \cos(c + dx))^2 (B + A \cos(c + dx))}{\cos^{\frac{9}{2}}(c + dx)} dx \\
 &= \frac{2B (a^2 + a^2 \cos(c + dx)) \sin(c + dx)}{7d \cos^{\frac{7}{2}}(c + dx)} + \frac{2}{7} \int \frac{(a + a \cos(c + dx))}{\cos^{\frac{7}{2}}(c + dx)} dx \\
 &= \frac{2B (a^2 + a^2 \cos(c + dx)) \sin(c + dx)}{7d \cos^{\frac{7}{2}}(c + dx)} + \frac{2}{7} \int \frac{\frac{1}{2}a^2(7A + 9B) + \left(\frac{1}{2}\right)}{\cos^{\frac{7}{2}}(c + dx)} dx \\
 &= \frac{2a^2(7A + 9B) \sin(c + dx)}{35d \cos^{\frac{5}{2}}(c + dx)} + \frac{2B (a^2 + a^2 \cos(c + dx)) \sin(c + dx)}{7d \cos^{\frac{7}{2}}(c + dx)} \\
 &= \frac{2a^2(7A + 9B) \sin(c + dx)}{35d \cos^{\frac{5}{2}}(c + dx)} + \frac{2B (a^2 + a^2 \cos(c + dx)) \sin(c + dx)}{7d \cos^{\frac{7}{2}}(c + dx)} \\
 &= \frac{2a^2(7A + 9B) \sin(c + dx)}{35d \cos^{\frac{5}{2}}(c + dx)} + \frac{4a^2(7A + 6B) \sin(c + dx)}{21d \cos^{\frac{3}{2}}(c + dx)} + \frac{4a^2(4A + 3B)E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{5d} \\
 &= -\frac{4a^2(4A + 3B)E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{5d} + \frac{4a^2(7A + 6B)F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{21d}
 \end{aligned}$$

Mathematica [C] time = 6.64, size = 1067, normalized size = 5.50

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[(((a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]))/Cos[c + d*x]^(3/2), x]

[Out] (Cos[c + d*x]^(7/2)*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x])*(((4*A + 3*B)*Csc[c]*Sec[c])/(5*d) + (B*Sec[c]*Sec[c + d*x]^4*Sin[d*x])/(14*d) + (Sec[c]*Sec[c + d*x]^3*(5*B*Sin[c] + 7*A*Sin[d*x] + 14*B*Sin[d*x]))/(70*d) + (Sec[c]*Sec[c + d*x]^2*(21*A*Sin[c] + 42*B*Sin[c] + 70*A*Sin[d*x] + 60*B*Sin[d*x]))/(210*d) + (Sec[c]*Sec[c + d*x]*(35*A*Sin[c] + 30

$$\begin{aligned}
 & *B*\sin[c] + 84*A*\sin[d*x] + 63*B*\sin[d*x]))/(105*d)))/(B + A*\cos[c + d*x]) \\
 & - (A*\cos[c + d*x]^3*\csc[c]*\text{HypergeometricPFQ}[\{1/4, 1/2\}, \{5/4\}, \sin[d*x - \text{ArcTan}[\cot[c]]]^2]*\sec[c/2 + (d*x)/2]^4*(a + a*\sec[c + d*x])^2*(A + B*\sec[c + d*x])* \\
 & \sec[d*x - \text{ArcTan}[\cot[c]]]*\sqrt{1 - \sin[d*x - \text{ArcTan}[\cot[c]]]}*\sqrt{-(\sqrt{1 + \cot[c]^2}*\sin[c]*\sin[d*x - \text{ArcTan}[\cot[c]]])}*\sqrt{1 + \sin[d*x - \text{ArcTan}[\cot[c]]]})} \\
 & /((3*d*(B + A*\cos[c + d*x])* \sqrt{1 + \cot[c]^2}) - (2*B*\cos[c + d*x]^3*\csc[c]*\text{HypergeometricPFQ}[\{1/4, 1/2\}, \{5/4\}, \sin[d*x - \text{ArcTan}[\cot[c]]]^2]*\sec[c/2 + (d*x)/2]^4*(a + a*\sec[c + d*x])^2*(A + B*\sec[c + d*x])* \\
 & \sec[d*x - \text{ArcTan}[\cot[c]]]*\sqrt{1 - \sin[d*x - \text{ArcTan}[\cot[c]]]}*\sqrt{-(\sqrt{1 + \cot[c]^2}*\sin[c]*\sin[d*x - \text{ArcTan}[\cot[c]]])}*\sqrt{1 + \sin[d*x - \text{ArcTan}[\cot[c]]]})} \\
 & /((7*d*(B + A*\cos[c + d*x])* \sqrt{1 + \cot[c]^2}) + (2*A*\cos[c + d*x]^3*\csc[c]*\sec[c/2 + (d*x)/2]^4*(a + a*\sec[c + d*x])^2*(A + B*\sec[c + d*x])* \\
 & (\text{HypergeometricPFQ}[\{-1/2, -1/4\}, \{3/4\}, \cos[d*x + \text{ArcTan}[\tan[c]]]^2]*\sin[d*x + \text{ArcTan}[\tan[c]]]*\tan[c])/(\sqrt{1 - \cos[d*x + \text{ArcTan}[\tan[c]]]}*\sqrt{1 + \cos[d*x + \text{ArcTan}[\tan[c]]]}*\sqrt{\cos[c]*\cos[d*x + \text{ArcTan}[\tan[c]]]}*\sqrt{1 + \tan[c]^2})*\sqrt{1 + \tan[c]^2}) - ((\sin[d*x + \text{ArcTan}[\tan[c]]]*\tan[c])/ \sqrt{1 + \tan[c]^2} + (2*\cos[c]^2*\cos[d*x + \text{ArcTan}[\tan[c]]]*\sqrt{1 + \tan[c]^2})/(\cos[c]^2 + \sin[c]^2))/\sqrt{\cos[c]*\cos[d*x + \text{ArcTan}[\tan[c]]]}*\sqrt{1 + \tan[c]^2}))/((5*d*(B + A*\cos[c + d*x])) + (3*B*\cos[c + d*x]^3*\csc[c]*\sec[c/2 + (d*x)/2]^4*(a + a*\sec[c + d*x])^2*(A + B*\sec[c + d*x])* \\
 & (\text{HypergeometricPFQ}[\{-1/2, -1/4\}, \{3/4\}, \cos[d*x + \text{ArcTan}[\tan[c]]]^2]*\sin[d*x + \text{ArcTan}[\tan[c]]]*\tan[c])/(\sqrt{1 - \cos[d*x + \text{ArcTan}[\tan[c]]]}*\sqrt{1 + \cos[d*x + \text{ArcTan}[\tan[c]]]}*\sqrt{\cos[c]*\cos[d*x + \text{ArcTan}[\tan[c]]]}*\sqrt{1 + \tan[c]^2})*\sqrt{1 + \tan[c]^2}) - ((\sin[d*x + \text{ArcTan}[\tan[c]]]*\tan[c])/ \sqrt{1 + \tan[c]^2} + (2*\cos[c]^2*\cos[d*x + \text{ArcTan}[\tan[c]]]*\sqrt{1 + \tan[c]^2})/(\cos[c]^2 + \sin[c]^2))/\sqrt{\cos[c]*\cos[d*x + \text{ArcTan}[\tan[c]]]}*\sqrt{1 + \tan[c]^2}))/((10*d*(B + A*\cos[c + d*x]))))
 \end{aligned}$$

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{Ba^2 \sec(dx + c)^3 + (A + 2B)a^2 \sec(dx + c)^2 + (2A + B)a^2 \sec(dx + c) + Aa^2}{\cos(dx + c)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="fricas")

[Out] integral((B*a^2*sec(d*x + c)^3 + (A + 2*B)*a^2*sec(d*x + c)^2 + (2*A + B)*a^2*sec(d*x + c) + A*a^2)/cos(d*x + c)^(3/2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(a \sec(dx + c) + a)^2}{\cos(dx + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^2/cos(d*x + c)^(3/2), x
)
```

maple [B] time = 15.46, size = 851, normalized size = 4.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2),x)
```

```
[Out] -8*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*((1/2*A+1/
4*B)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-1/5*(1/4*A+1/2*B)/(8*sin(1/2
*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x
+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*cos(1/2*d*x+
1/2*c)*sin(1/2*d*x+1/2*c)^6-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^
2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-8*s
in(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)+1/4*A*(-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*s
in(1/2*d*x+1/2*c)^2-1)+1/4*B*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^4-5/42*cos(1/2
*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(
1/2*d*x+1/2*c)^2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1
)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="maxima")

[Out] Timed out

mupad [B] time = 4.69, size = 235, normalized size = 1.21

$$\frac{6 A a^2 \sin(c+d x) {}_2F_1\left(-\frac{5}{4}, \frac{1}{2}; -\frac{1}{4}; \cos(c+d x)^2\right) + 20 A a^2 \cos(c+d x) \sin(c+d x) {}_2F_1\left(-\frac{3}{4}, \frac{1}{2}; \frac{1}{4}; \cos(c+d x)^2\right)}{15 d \cos(c+d x)^{5/2} \sqrt{1-\cos(c+d x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^2)/cos(c + d*x)^(3/2),x)

[Out] (6*A*a^2*sin(c + d*x)*hypergeom([-5/4, 1/2], -1/4, cos(c + d*x)^2) + 20*A*a^2*cos(c + d*x)*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2) + 30*A*a^2*cos(c + d*x)^2*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(15*d*cos(c + d*x)^(5/2)*(1 - cos(c + d*x)^2)^(1/2)) + (30*B*a^2*sin(c + d*x)*hypergeom([-7/4, 1/2], -3/4, cos(c + d*x)^2) + 84*B*a^2*cos(c + d*x)*sin(c + d*x)*hypergeom([-5/4, 1/2], -1/4, cos(c + d*x)^2) + 70*B*a^2*cos(c + d*x)^2*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(105*d*cos(c + d*x)^(7/2)*(1 - cos(c + d*x)^2)^(1/2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2 \left(\int \frac{A}{\cos^{\frac{3}{2}}(c+dx)} dx + \int \frac{2A \sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)} dx + \int \frac{A \sec^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)} dx + \int \frac{B \sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)} dx + \int \frac{2B \sec^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))**2*(A+B*sec(d*x+c))/cos(d*x+c)**(3/2),x)

[Out] a**2*(Integral(A/cos(c + d*x)**(3/2), x) + Integral(2*A*sec(c + d*x)/cos(c + d*x)**(3/2), x) + Integral(A*sec(c + d*x)**2/cos(c + d*x)**(3/2), x) + Integral(B*sec(c + d*x)/cos(c + d*x)**(3/2), x) + Integral(2*B*sec(c + d*x)**2/cos(c + d*x)**(3/2), x) + Integral(B*sec(c + d*x)**3/cos(c + d*x)**(3/2), x))

$$3.496 \quad \int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B \sec(c+dx))}{a+a \sec(c+dx)} dx$$

Optimal. Leaf size=157

$$-\frac{5(A-B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad} + \frac{3(7A-5B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad} - \frac{(A-B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{d(a\cos(c+dx)+a)} + \frac{(7A-5B)\sin(c+dx)}{5ad}$$

[Out] $3/5*(7*A-5*B)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/a/d-5/3*(A-B)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/a/d+1/5*(7*A-5*B)*\cos(d*x+c)^{(3/2)}*\sin(d*x+c)/a/d-(A-B)*\cos(d*x+c)^{(5/2)}*\sin(d*x+c)/d/(a+a*\cos(d*x+c))-5/3*(A-B)*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/a/d$

Rubi [A] time = 0.26, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2954, 2977, 2748, 2635, 2641, 2639}

$$-\frac{5(A-B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad} + \frac{3(7A-5B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad} - \frac{(A-B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{d(a\cos(c+dx)+a)} + \frac{(7A-5B)\sin(c+dx)}{5ad}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(\text{Cos}[c+d*x]^{(5/2)}*(A+B*\text{Sec}[c+d*x]))/(a+a*\text{Sec}[c+d*x]),x]$

[Out] $(3*(7*A-5*B)*\text{EllipticE}[(c+d*x)/2, 2])/(5*a*d) - (5*(A-B)*\text{EllipticF}[(c+d*x)/2, 2])/(3*a*d) - (5*(A-B)*\text{Sqrt}[\text{Cos}[c+d*x]]*\text{Sin}[c+d*x])/(3*a*d) + ((7*A-5*B)*\text{Cos}[c+d*x]^{(3/2)}*\text{Sin}[c+d*x])/(5*a*d) - ((A-B)*\text{Cos}[c+d*x]^{(5/2)}*\text{Sin}[c+d*x])/(d*(a+a*\text{Cos}[c+d*x]))$

Rule 2635

$\text{Int}[(b_*)*\sin[(c_*)+(d_*)(x_*)]^{(n_*)}, x_Symbol] :> -\text{Simp}[(b*\text{Cos}[c+d*x])*(b*\text{Sin}[c+d*x])^{(n-1)})/(d*n), x] + \text{Dist}[(b^2*(n-1))/n, \text{Int}[(b*\text{Sin}[c+d*x])^{(n-2)}, x], x] /;$ FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2639

$\text{Int}[\text{Sqrt}[\sin[(c_*)+(d_*)(x_*)]], x_Symbol] :> \text{Simp}[(2*\text{EllipticE}[(1*(c-Pi/2+d*x))/2, 2])/d, x] /;$ FreeQ[{c, d}, x]

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_.)]], x_Symbol] := Simp[(2*EllipticF[(1*(c -
  Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_.)]^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_.)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_.)]^(p_.), x_Symbol] := Dist[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 2977

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)]*(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]^(n_.), x_Symbol] := Simp[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m + b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{a+a\sec(c+dx)} dx &= \int \frac{\cos^{\frac{5}{2}}(c+dx)(B+A\cos(c+dx))}{a+a\cos(c+dx)} dx \\
&= -\frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} + \frac{\int \cos^{\frac{3}{2}}(c+dx) \left(-\frac{5}{2}a(A-B) + \frac{1}{2}\right)}{a^2} \\
&= -\frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} + \frac{(7A-5B)\int \cos^{\frac{5}{2}}(c+dx) dx}{2a} - \frac{5(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{3ad} \\
&= -\frac{5(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{3ad} + \frac{(7A-5B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5ad} \\
&= \frac{3(7A-5B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad} - \frac{5(A-B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad} - \frac{5(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{3ad}
\end{aligned}$$

Mathematica [C] time = 6.69, size = 1292, normalized size = 8.23

result too large to display

Warning: Unable to verify antiderivative.

```

[In] Integrate[(Cos[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x]),x]
[Out] (((21*I)/20)*A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*(A + B*Sec[c + d*x])*
((2*E^((2*I)*d*x))*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x))*(Cos[c]
+ I*Sin[c])^2])*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d
*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*S
in[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Si
n[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x))*(Cos[c] + I*Si
n[c])^2])*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*S
in[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c
]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c]))/(
(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])) - (((3*I)/4)*B*Cos[c/2 + (d*x)/2
]^2*Csc[c/2]*Sec[c/2]*(A + B*Sec[c + d*x])*((2*E^((2*I)*d*x))*Hypergeometric
2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x))*(Cos[c] + I*Sin[c])^2])*Sqrt[(2*(1 + E^
(2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 +
E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)
*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/
4, 1/2, 3/4, -(E^((2*I)*d*x))*(Cos[c] + I*Sin[c])^2])*Sqrt[(2*(1 + E^((2*I)
*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2
*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*
Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c]))/(B + A*Cos[c + d*x])*(a + a*Sec[

```

$c + dx)) + (\cos[c/2 + (dx)/2]^2 \sqrt{\cos[c + dx]} (A + B \sec[c + dx]) * ((2 * (-5 * A + 5 * B - 16 * A \cos[c] + 10 * B \cos[c]) * \csc[c]) / (5 * d) + (4 * (-A + B) * \cos[dx] * \sin[c]) / (3 * d) + (2 * A \cos[2 * dx] * \sin[2 * c]) / (5 * d) + (2 * \sec[c/2] * \sec[c/2 + (dx)/2] * (-A * \sin[(dx)/2]) + B * \sin[(dx)/2])) / d + (4 * (-A + B) * \cos[c] * \sin[dx]) / (3 * d) + (2 * A \cos[2 * c] * \sin[2 * dx]) / (5 * d)) / ((B + A \cos[c + dx]) * (a + a * \sec[c + dx])) + (5 * A \cos[c/2 + (dx)/2]^2 * \csc[c/2] * \text{HypergeometricPFQ}[\{1/4, 1/2\}, \{5/4\}, \sin[dx - \text{ArcTan}[\text{Cot}[c]]]^2] * \sec[c/2] * (A + B \sec[c + dx]) * \sec[dx - \text{ArcTan}[\text{Cot}[c]]] * \sqrt{1 - \sin[dx - \text{ArcTan}[\text{Cot}[c]]}] * \sqrt{-(\sqrt{1 + \text{Cot}[c]^2} * \sin[c] * \sin[dx - \text{ArcTan}[\text{Cot}[c]]]) * \sqrt{1 + \sin[dx - \text{ArcTan}[\text{Cot}[c]]])}) / (3 * d * (B + A \cos[c + dx]) * \sqrt{1 + \text{Cot}[c]^2} * (a + a * \sec[c + dx])) - (5 * B \cos[c/2 + (dx)/2]^2 * \csc[c/2] * \text{HypergeometricPFQ}[\{1/4, 1/2\}, \{5/4\}, \sin[dx - \text{ArcTan}[\text{Cot}[c]]]^2] * \sec[c/2] * (A + B \sec[c + dx]) * \sec[dx - \text{ArcTan}[\text{Cot}[c]]] * \sqrt{1 - \sin[dx - \text{ArcTan}[\text{Cot}[c]]}] * \sqrt{-(\sqrt{1 + \text{Cot}[c]^2} * \sin[c] * \sin[dx - \text{ArcTan}[\text{Cot}[c]]]) * \sqrt{1 + \sin[dx - \text{ArcTan}[\text{Cot}[c]]])}) / (3 * d * (B + A \cos[c + dx]) * \sqrt{1 + \text{Cot}[c]^2} * (a + a * \sec[c + dx]))$

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{(B \cos(dx + c)^2 \sec(dx + c) + A \cos(dx + c)^2) \sqrt{\cos(dx + c)}}{a \sec(dx + c) + a}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(dx+c)^(5/2)*(A+B*sec(dx+c))/(a+a*sec(dx+c)),x, algorithm="fricas")

[Out] integral((B*cos(dx + c)^2*sec(dx + c) + A*cos(dx + c)^2)*sqrt(cos(dx + c))/(a*sec(dx + c) + a), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \cos(dx + c)^{\frac{5}{2}}}{a \sec(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(dx+c)^(5/2)*(A+B*sec(dx+c))/(a+a*sec(dx+c)),x, algorithm="giac")

[Out] integrate((B*sec(dx + c) + A)*cos(dx + c)^(5/2)/(a*sec(dx + c) + a), x)

maple [A] time = 5.24, size = 282, normalized size = 1.80

$$\sqrt{2 \left(\cos^2 \left(\frac{dx}{2} + \frac{c}{2} \right) - 1 \right) \left(\sin^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)} \left(-\cos \left(\frac{dx}{2} + \frac{c}{2} \right) \sqrt{2 \left(\sin^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) - 1} \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \right) \quad (25A \text{ Elliptic})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x)`

[Out]
$$-1/15*((2*\cos(1/2*d*x+1/2*c)^2-1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-\cos(1/2*d*x+1/2*c)*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(25*A*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})+63*A*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)})-25*B*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})-45*B*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)}))+48*A*\sin(1/2*d*x+1/2*c)^8+(-56*A-40*B)*\sin(1/2*d*x+1/2*c)^6+(-30*A+90*B)*\sin(1/2*d*x+1/2*c)^4+(23*A-35*B)*\sin(1/2*d*x+1/2*c)^2)/a/\cos(1/2*d*x+1/2*c)/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \cos(dx + c)^{\frac{5}{2}}}{a \sec(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="maxima")`

[Out] `integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(5/2)/(a*sec(d*x + c) + a), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c + dx)^{5/2} \left(A + \frac{B}{\cos(c+dx)} \right)}{a + \frac{a}{\cos(c+dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((cos(c + d*x)^(5/2)*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x)),x)`

[Out] `int((cos(c + d*x)^(5/2)*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x)), x)`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)**(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x)`

[Out] Timed out

$$3.497 \quad \int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B \sec(c+dx))}{a+a \sec(c+dx)} dx$$

Optimal. Leaf size=124

$$\frac{(5A-3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad} - \frac{3(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} - \frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{d(a\cos(c+dx)+a)} + \frac{(5A-3B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3ad}$$

[Out] $-3*(A-B)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/a/d+1/3*(5*A-3*B)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/a/d-(A-B)*\cos(d*x+c)^{(3/2)}*\sin(d*x+c)/d/(a+a*\cos(d*x+c))+1/3*(5*A-3*B)*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/a/d$

Rubi [A] time = 0.24, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2954, 2977, 2748, 2639, 2635, 2641}

$$\frac{(5A-3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad} - \frac{3(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} - \frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{d(a\cos(c+dx)+a)} + \frac{(5A-3B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3ad}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(\text{Cos}[c + d*x]^{(3/2)}*(A + B*\text{Sec}[c + d*x]))/(a + a*\text{Sec}[c + d*x]), x]$

[Out] $(-3*(A - B)*\text{EllipticE}[(c + d*x)/2, 2])/(a*d) + ((5*A - 3*B)*\text{EllipticF}[(c + d*x)/2, 2])/(3*a*d) + ((5*A - 3*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sin}[c + d*x])/(3*a*d) - ((A - B)*\text{Cos}[c + d*x]^{(3/2)}*\text{Sin}[c + d*x])/(d*(a + a*\text{Cos}[c + d*x]))$

Rule 2635

$\text{Int}[(b_*)*\sin[(c_*) + (d_*)*(x_*)]^{(n_*)}, x_Symbol] \rightarrow -\text{Simp}[(b*\text{Cos}[c + d*x])*(b*\text{Sin}[c + d*x])^{(n-1)})/(d*n), x] + \text{Dist}[(b^2*(n-1))/n, \text{Int}[(b*\text{Sin}[c + d*x])^{(n-2)}, x], x] /; \text{FreeQ}\{b, c, d, x\} \&\& \text{GtQ}[n, 1] \&\& \text{IntegerQ}[2*n]$

Rule 2639

$\text{Int}[\text{Sqrt}[\sin[(c_*) + (d_*)*(x_*)]], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticE}[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}\{c, d, x\}$

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c -
Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)])*(b_.)^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_.))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] := Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
a*d, 0] && !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 2977

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{a+a\sec(c+dx)} dx &= \int \frac{\cos^{\frac{3}{2}}(c+dx)(B+A\cos(c+dx))}{a+a\cos(c+dx)} dx \\
&= -\frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} + \frac{\int \sqrt{\cos(c+dx)} \left(-\frac{3}{2}a(A-B) + \frac{1}{2}\right)}{a^2} \\
&= -\frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} + \frac{(5A-3B)\int \cos^{\frac{3}{2}}(c+dx) dx}{2a} - \frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2a} \\
&= -\frac{3(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{(5A-3B)\sqrt{\cos(c+dx)}\sin(c+dx)}{3ad} - \frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2a} \\
&= -\frac{3(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{(5A-3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad} + \frac{(5A-3B)\sqrt{\cos(c+dx)}\sin(c+dx)}{2a}
\end{aligned}$$

Mathematica [C] time = 6.60, size = 1239, normalized size = 9.99

result too large to display

Warning: Unable to verify antiderivative.

```

[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x]),x]
[Out] (((-3*I)/4)*A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*(A + B*Sec[c + d*x])*
(2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] +
I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*
x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Si
n[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin
[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin
[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Si
n[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]
])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c]))/((
B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])) + (((3*I)/4)*B*Cos[c/2 + (d*x)/2]
^2*Csc[c/2]*Sec[c/2]*(A + B*Sec[c + d*x])*((2*E^((2*I)*d*x)*Hypergeometric2
F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((
2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 +
E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*
d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4
, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d
*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*
I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*C
os[c] + d*(-1 + E^((2*I)*d*x))*Sin[c]))/(B + A*Cos[c + d*x])*(a + a*Sec[c

```

+ d*x])) + (Cos[c/2 + (d*x)/2]^2*Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x]))*(
 (-2*(-A + B)*(1 + 2*Cos[c])*Csc[c])/d + (4*A*Cos[d*x]*Sin[c])/(3*d) - (2*Se
 c[c/2]*Sec[c/2 + (d*x)/2]*(-(A*Sin[(d*x)/2]) + B*Sin[(d*x)/2]))/d + (4*A*Co
 s[c]*Sin[d*x])/(3*d)))/((B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])) - (5*A*Cos
 [c/2 + (d*x)/2]^2*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x -
 ArcTan[Cot[c]]]^2]*Sec[c/2]*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]
 *Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d
 *x - ArcTan[Cot[c]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(B + A*Cos
 [c + d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])) + (B*Cos[c/2 + (d*x)/2]
 ^2*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^
 2]*Sec[c/2]*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x
 - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]
]]])*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(B + A*Cos[c + d*x])*Sqrt[1 +
 Cot[c]^2]*(a + a*Sec[c + d*x]))

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B \cos(dx + c) \sec(dx + c) + A \cos(dx + c))\sqrt{\cos(dx + c)}}{a \sec(dx + c) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="fricas")

[Out] integral((B*cos(d*x + c)*sec(d*x + c) + A*cos(d*x + c))*sqrt(cos(d*x + c))/(a*sec(d*x + c) + a), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \cos(dx + c)^{\frac{3}{2}}}{a \sec(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(3/2)/(a*sec(d*x + c) + a), x)

maple [A] time = 4.32, size = 262, normalized size = 2.11

$$\sqrt{\left(2 \left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right) \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right)\right) \sqrt{2 \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1} \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \left(5A \text{ EllipticE}\left(\frac{dx+c}{2}, \sqrt{2 \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x)`

[Out]
$$-1/3*((2*\cos(1/2*d*x+1/2*c)^2-1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(\cos(1/2*d*x+1/2*c))*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(5*A*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})+9*A*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)}))-3*B*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})-9*B*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)}))-8*A*\sin(1/2*d*x+1/2*c)^6+(18*A-6*B)*\sin(1/2*d*x+1/2*c)^4+(-7*A+3*B)*\sin(1/2*d*x+1/2*c)^2)/a/\cos(1/2*d*x+1/2*c)/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \cos(dx + c)^{\frac{3}{2}}}{a \sec(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="maxima")`

[Out] `integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(3/2)/(a*sec(d*x + c) + a), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c + dx)^{3/2} \left(A + \frac{B}{\cos(c+dx)} \right)}{a + \frac{a}{\cos(c+dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((cos(c + d*x)^(3/2)*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x)),x)`

[Out] `int((cos(c + d*x)^(3/2)*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x)), x)`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)**(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x)`

[Out] Timed out

$$3.498 \quad \int \frac{\sqrt{\cos(c+dx)} (A+B \sec(c+dx))}{a+a \sec(c+dx)} dx$$

Optimal. Leaf size=88

$$-\frac{(A-B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{(3A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} - \frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}$$

[Out] (3*A-B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a/d-(A-B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a/d-(A-B)*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))

Rubi [A] time = 0.22, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {2954, 2977, 2748, 2641, 2639}

$$-\frac{(A-B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{(3A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} - \frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x]),x]

[Out] ((3*A - B)*EllipticE[(c + d*x)/2, 2])/(a*d) - ((A - B)*EllipticF[(c + d*x)/2, 2])/(a*d) - ((A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*(a + a*Cos[c + d*x]))

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)])*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_.))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] := Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
a*d, 0] && !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{\cos(c+dx)}(A+B \sec(c+dx))}{a+a \sec(c+dx)} dx &= \int \frac{\sqrt{\cos(c+dx)}(B+A \cos(c+dx))}{a+a \cos(c+dx)} dx \\ &= -\frac{(A-B)\sqrt{\cos(c+dx)} \sin(c+dx)}{d(a+a \cos(c+dx))} + \frac{\int \frac{-\frac{1}{2}a(A-B)+\frac{1}{2}a(3A-B) \cos(c+dx)}{\sqrt{\cos(c+dx)}} dx}{a^2} \\ &= -\frac{(A-B)\sqrt{\cos(c+dx)} \sin(c+dx)}{d(a+a \cos(c+dx))} - \frac{(A-B) \int \frac{1}{\sqrt{\cos(c+dx)}} dx}{2a} + \frac{(3A-B)}{2a} \\ &= \frac{(3A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} - \frac{(A-B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} - \frac{(A-B)\sqrt{\cos(c+dx)}}{d(a+a \cos(c+dx))} \end{aligned}$$

Mathematica [C] time = 6.47, size = 1208, normalized size = 13.73

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x]),x]
```

```
[Out] (((3*I)/4)*A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*(A + B*Sec[c + d*x])*((
2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] +
I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x
))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin
[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[
c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[
c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin
[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]]
)/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c]))/(B
+ A*Cos[c + d*x])*(a + a*Sec[c + d*x])) - ((I/4)*B*Cos[c/2 + (d*x)/2]^2*Cs
c[c/2]*Sec[c/2]*(A + B*Sec[c + d*x])*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/
2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*
d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2
*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))
*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2
, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*
Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*
x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c]
+ d*(-1 + E^((2*I)*d*x))*Sin[c]))/(B + A*Cos[c + d*x])*(a + a*Sec[c + d*
x])) + (Cos[c/2 + (d*x)/2]^2*Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x])*((-2*(
A - B + 2*A*Cos[c])*Csc[c])/d + (2*Sec[c/2]*Sec[c/2 + (d*x)/2]*(-(A*Sin[(d*
x)/2]) + B*Sin[(d*x)/2]))/d))/(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])) +
(A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[
d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot
[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*
Sin[d*x - ArcTan[Cot[c]])])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(B + A*
Cos[c + d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])) - (B*Cos[c/2 + (d*x)
/2]^2*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]
]]^2]*Sec[c/2]*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[
d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Co
t[c]])])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(B + A*Cos[c + d*x])*Sqrt[
1 + Cot[c]^2]*(a + a*Sec[c + d*x]))
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B \sec(dx + c) + A)\sqrt{\cos(dx + c)}}{a \sec(dx + c) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(a*sec(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sqrt{\cos(dx + c)}}{a \sec(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(a*sec(d*x + c) + a), x)

maple [A] time = 4.79, size = 244, normalized size = 2.77

$$\frac{\sqrt{\left(2 \left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right) \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right) \sqrt{2 \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1} \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \left(A \operatorname{EllipticF}\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), 2\right) + 3A \operatorname{EllipticE}\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), 2\right) - B \operatorname{EllipticF}\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), 2\right) - B \operatorname{EllipticE}\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), 2\right)\right) + (2A - 2B) \sin\left(\frac{dx}{2} + \frac{c}{2}\right)^4 + (-A + B) \sin\left(\frac{dx}{2} + \frac{c}{2}\right)^2}{a \cos\left(\frac{dx}{2} + \frac{c}{2}\right) + a} \right) / d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x)

[Out] ((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1/2*c)*((2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+(2*A-2*B)*sin(1/2*d*x+1/2*c)^4+(-A+B)*sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sqrt{\cos(dx + c)}}{a \sec(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(a*sec(d*x + c) + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\cos(c + dx)} \left(A + \frac{B}{\cos(c+dx)}\right)}{a + \frac{a}{\cos(c+dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((cos(c + d*x)^(1/2)*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x)),x)`

[Out] `int((cos(c + d*x)^(1/2)*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A\sqrt{\cos(c+dx)}}{\sec(c+dx)+1} dx + \int \frac{B\sqrt{\cos(c+dx)} \sec(c+dx)}{\sec(c+dx)+1} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*sec(d*x+c))*cos(d*x+c)**(1/2)/(a+a*sec(d*x+c)),x)`

[Out] `(Integral(A*sqrt(cos(c + d*x))/(sec(c + d*x) + 1), x) + Integral(B*sqrt(cos(c + d*x))*sec(c + d*x)/(sec(c + d*x) + 1), x))/a`

$$3.499 \quad \int \frac{A+B \sec(c+dx)}{\sqrt{\cos(c+dx)} (a+a \sec(c+dx))} dx$$

Optimal. Leaf size=83

$$\frac{(A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} - \frac{(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}$$

[Out] $-(A-B)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/a/d+(A+B)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/a/d+(A-B)*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/d/(a+a*\cos(d*x+c))$

Rubi [A] time = 0.22, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {2954, 2978, 2748, 2641, 2639}

$$\frac{(A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} - \frac{(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(A + B*\text{Sec}[c + d*x])/(\text{Sqrt}[\text{Cos}[c + d*x]]*(a + a*\text{Sec}[c + d*x])),x]$

[Out] $-\left(\frac{(A-B)*\text{EllipticE}[(c+d*x)/2, 2]}{(a*d)}\right) + \left(\frac{(A+B)*\text{EllipticF}[(c+d*x)/2, 2]}{(a*d)}\right) + \left(\frac{(A-B)*\text{Sqrt}[\text{Cos}[c+d*x]]*\text{Sin}[c+d*x]}{d*(a+a*\text{Cos}[c+d*x])}\right)$

Rule 2639

$\text{Int}[\text{Sqrt}[\sin[(c_.) + (d_.)*(x_.)]], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticE}[(1*(c - \text{Pi}/2 + d*x))/2, 2])/d, x] \text{ ; FreeQ}\{c, d\}, x]$

Rule 2641

$\text{Int}[1/\text{Sqrt}[\sin[(c_.) + (d_.)*(x_.)]], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticF}[(1*(c - \text{Pi}/2 + d*x))/2, 2])/d, x] \text{ ; FreeQ}\{c, d\}, x]$

Rule 2748

$\text{Int}[(b_.)*\sin[(e_.) + (f_.)*(x_.)]^{(m_.)*((c_.) + (d_.)*\sin[(e_.) + (f_.)*(x_.)])}, x_Symbol] \rightarrow \text{Dist}[c, \text{Int}[(b*\text{Sin}[e + f*x])^m, x], x] + \text{Dist}[d/b, \text{Int}[(b*\text{Sin}[e + f*x])^{(m+1)}, x], x] \text{ ; FreeQ}\{b, c, d, e, f, m\}, x]$

Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)])*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_.))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] := Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
a*d, 0] && !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 2978

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&& !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{A + B \sec(c + dx)}{\sqrt{\cos(c + dx)} (a + a \sec(c + dx))} dx &= \int \frac{B + A \cos(c + dx)}{\sqrt{\cos(c + dx)} (a + a \cos(c + dx))} dx \\ &= \frac{(A - B)\sqrt{\cos(c + dx)} \sin(c + dx)}{d(a + a \cos(c + dx))} + \frac{\int \frac{\frac{1}{2}a(A+B) - \frac{1}{2}a(A-B)\cos(c+dx)}{\sqrt{\cos(c+dx)}} dx}{a^2} \\ &= \frac{(A - B)\sqrt{\cos(c + dx)} \sin(c + dx)}{d(a + a \cos(c + dx))} - \frac{(A - B) \int \sqrt{\cos(c + dx)} dx}{2a} + \frac{(A + B) \int \sqrt{\cos(c + dx)} dx}{2a} \\ &= -\frac{(A - B)E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{ad} + \frac{(A + B)F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{ad} + \frac{(A - B)\sqrt{\cos(c + dx)}}{d(a + a \cos(c + dx))} \end{aligned}$$

Mathematica [C] time = 6.51, size = 1204, normalized size = 14.51

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])), x]
```

```
[Out] ((-1/4*I)*A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*(A + B*Sec[c + d*x])*((2
 *E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I
 *Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x)
 )*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[
 2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c
 ]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c
 ])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[
 c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])
 /((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c]))/(B
 + A*Cos[c + d*x])*(a + a*Sec[c + d*x])) + ((I/4)*B*Cos[c/2 + (d*x)/2]^2*Csc
 [c/2]*Sec[c/2]*(A + B*Sec[c + d*x])*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2
 , 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d
 *x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*
 I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*
 Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2,
 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*C
 os[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x
 )*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c]
 + d*(-1 + E^((2*I)*d*x))*Sin[c]))/(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x
 ])) + (Cos[c/2 + (d*x)/2]^2*Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x])*((-2*(-
 A + B)*Csc[c])/d - (2*Sec[c/2]*Sec[c/2 + (d*x)/2]*(-(A*Sin[(d*x)/2]) + B*Si
 n[(d*x)/2]))/d)/((B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])) - (A*Cos[c/2 +
 (d*x)/2]^2*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[
 Cot[c]]]^2]*Sec[c/2]*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1
 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - Arc
 Tan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(B + A*Cos[c + d*x])
 *Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])) - (B*Cos[c/2 + (d*x)/2]^2*Csc[c/2
 ]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2
 ]*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[
 Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt
 [1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(B + A*Cos[c + d*x])*Sqrt[1 + Cot[c]^2]
 *(a + a*Sec[c + d*x]))
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B \sec(dx + c) + A)\sqrt{\cos(dx + c)}}{a \cos(dx + c) \sec(dx + c) + a \cos(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="
 fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(a*cos(d*x + c)*sec(d*x +
 c) + a*cos(d*x + c)), x)
```


giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(a \sec(dx + c) + a) \sqrt{\cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)*sqrt(cos(d*x + c))), x)

maple [A] time = 4.67, size = 243, normalized size = 2.93

$$\frac{\sqrt{\left(2 \left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right) \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) \left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right) \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{2 \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1} \left(A \operatorname{EllipticF}\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), 2\right) - a \cos\left(\frac{dx}{2}\right)\right)}{\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))/(a+a*sec(d*x+c))/cos(d*x+c)^(1/2),x)

[Out] -((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+(2*A-2*B)*sin(1/2*d*x+1/2*c)^4+(-A+B)*sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(a \sec(dx + c) + a) \sqrt{\cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)*sqrt(cos(d*x + c))), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\sqrt{\cos(c+dx)} \left(a + \frac{a}{\cos(c+dx)} \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))), x)

[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A}{\sqrt{\cos(c+dx)} \sec(c+dx) + \sqrt{\cos(c+dx)}} dx + \int \frac{B \sec(c+dx)}{\sqrt{\cos(c+dx)} \sec(c+dx) + \sqrt{\cos(c+dx)}} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))/cos(d*x+c)**(1/2), x)

[Out] (Integral(A/(sqrt(cos(c + d*x))*sec(c + d*x) + sqrt(cos(c + d*x))), x) + Integral(B*sec(c + d*x)/(sqrt(cos(c + d*x))*sec(c + d*x) + sqrt(cos(c + d*x))), x))/a

$$3.500 \quad \int \frac{A+B \sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a \sec(c+dx))} dx$$

Optimal. Leaf size=113

$$\frac{(A-B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{(A-3B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} - \frac{(A-3B)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}} + \frac{(A-B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)}$$

[Out] (A-3*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a/d+(A-B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a/d-(A-3*B)*sin(d*x+c)/a/d/cos(d*x+c)^(1/2)+(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))/cos(d*x+c)^(1/2)

Rubi [A] time = 0.24, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2954, 2978, 2748, 2636, 2639, 2641}

$$\frac{(A-B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{(A-3B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} - \frac{(A-3B)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}} + \frac{(A-B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sec[c + d*x])/((Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])),x]

[Out] ((A - 3*B)*EllipticE[(c + d*x)/2, 2])/(a*d) + ((A - B)*EllipticF[(c + d*x)/2, 2])/(a*d) - ((A - 3*B)*Sin[c + d*x])/(a*d*Sqrt[Cos[c + d*x]]) + ((A - B)*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x]))

Rule 2636

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(b*Ssin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), Int[(b*Ssin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)])*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)])*(d_.) + (c_.))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dist[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 2978

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{A + B \sec(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))} dx &= \int \frac{B + A \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(a + a \cos(c + dx))} dx \\
&= \frac{(A - B) \sin(c + dx)}{d\sqrt{\cos(c + dx)}(a + a \cos(c + dx))} + \frac{\int \frac{-\frac{1}{2}a(A-3B) + \frac{1}{2}a(A-B)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)} dx}{a^2} \\
&= \frac{(A - B) \sin(c + dx)}{d\sqrt{\cos(c + dx)}(a + a \cos(c + dx))} - \frac{(A - 3B) \int \frac{1}{\cos^{\frac{3}{2}}(c+dx)} dx}{2a} + \frac{(A - B) \sin(c + dx)}{d\sqrt{\cos(c + dx)}(a + a \cos(c + dx))} \\
&= \frac{(A - B)F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{ad} - \frac{(A - 3B) \sin(c + dx)}{ad\sqrt{\cos(c + dx)}} + \frac{(A - B) \sin(c + dx)}{d\sqrt{\cos(c + dx)}(a + a \cos(c + dx))} \\
&= \frac{(A - 3B)E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{ad} + \frac{(A - B)F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{ad} - \frac{(A - 3B) \sin(c + dx)}{ad\sqrt{\cos(c + dx)}}
\end{aligned}$$

Mathematica [C] time = 6.71, size = 1240, normalized size = 10.97

result too large to display

Antiderivative was successfully verified.

```

[In] Integrate[(A + B*Sec[c + d*x])/((Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])),x]
[Out] ((I/4)*A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*(A + B*Sec[c + d*x])*((2*E^
((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Si
n[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*S
in[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c
]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c])
- (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^
2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])
/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((
-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c]))/(B + A
*Cos[c + d*x])*(a + a*Sec[c + d*x])) - (((3*I)/4)*B*Cos[c/2 + (d*x)/2]^2*Cs
c[c/2]*Sec[c/2]*(A + B*Sec[c + d*x])*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/
2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*
d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2
*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))
*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2
, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*
Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*

```

$x) \cdot \cos[2c] + I \cdot E^{((2I) \cdot dx) \cdot \sin[2c]} / ((-I) \cdot d \cdot (1 + E^{((2I) \cdot dx) \cdot \cos[c] + d \cdot (-1 + E^{((2I) \cdot dx) \cdot \sin[c]})}) / ((B + A \cdot \cos[c + dx]) \cdot (a + a \cdot \sec[c + dx])) + (\cos[c/2 + (dx)/2]^2 \cdot \sqrt{\cos[c + dx]} \cdot (A + B \cdot \sec[c + dx]) \cdot ((2B - A \cdot \cos[c] + B \cdot \cos[c]) \cdot \csc[c/2] \cdot \sec[c/2] \cdot \sec[c]) / d + (2 \cdot \sec[c/2] \cdot \sec[c/2 + (dx)/2] \cdot (-A \cdot \sin[(dx)/2] + B \cdot \sin[(dx)/2])) / d + (4 \cdot B \cdot \sec[c] \cdot \sec[c + dx] \cdot \sin[dx]) / d) / ((B + A \cdot \cos[c + dx]) \cdot (a + a \cdot \sec[c + dx])) - (A \cdot \cos[c/2 + (dx)/2]^2 \cdot \csc[c/2] \cdot \text{HypergeometricPFQ}[\{1/4, 1/2\}, \{5/4\}, \sin[dx - \text{ArcTan}[\text{Cot}[c]]]^2] \cdot \sec[c/2] \cdot (A + B \cdot \sec[c + dx]) \cdot \sec[dx - \text{ArcTan}[\text{Cot}[c]]] \cdot \sqrt{1 - \sin[dx - \text{ArcTan}[\text{Cot}[c]]}] \cdot \sqrt{-(\sqrt{1 + \text{Cot}[c]^2} \cdot \sin[c] \cdot \sin[dx - \text{ArcTan}[\text{Cot}[c]])}] \cdot \sqrt{1 + \sin[dx - \text{ArcTan}[\text{Cot}[c]]}]) / (d \cdot (B + A \cdot \cos[c + dx]) \cdot \sqrt{1 + \text{Cot}[c]^2} \cdot (a + a \cdot \sec[c + dx])) + (B \cdot \cos[c/2 + (dx)/2]^2 \cdot \csc[c/2] \cdot \text{HypergeometricPFQ}[\{1/4, 1/2\}, \{5/4\}, \sin[dx - \text{ArcTan}[\text{Cot}[c]]]^2] \cdot \sec[c/2] \cdot (A + B \cdot \sec[c + dx]) \cdot \sec[dx - \text{ArcTan}[\text{Cot}[c]]] \cdot \sqrt{1 - \sin[dx - \text{ArcTan}[\text{Cot}[c]]}] \cdot \sqrt{-(\sqrt{1 + \text{Cot}[c]^2} \cdot \sin[c] \cdot \sin[dx - \text{ArcTan}[\text{Cot}[c]])}] \cdot \sqrt{1 + \sin[dx - \text{ArcTan}[\text{Cot}[c]]}]) / (d \cdot (B + A \cdot \cos[c + dx]) \cdot \sqrt{1 + \text{Cot}[c]^2} \cdot (a + a \cdot \sec[c + dx]))$

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{(B \sec(dx + c) + A) \sqrt{\cos(dx + c)}}{a \cos(dx + c)^2 \sec(dx + c) + a \cos(dx + c)^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")

[Out] integral((B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(a*cos(d*x + c)^2*sec(d*x + c) + a*cos(d*x + c)^2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(a \sec(dx + c) + a) \cos(dx + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)*cos(d*x + c)^(3/2)), x)

maple [A] time = 8.91, size = 318, normalized size = 2.81

$$\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)\left(-\cos\left(\frac{dx}{2} + \frac{c}{2}\right)\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)}\sqrt{2\left(\sin^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x)

[Out] $-\left(-\left(-2\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2+1\right)\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2\right)^{\frac{1}{2}}/a\left(-\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)\left(-2\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^4+\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2\right)^{\frac{1}{2}}\left(2\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2-1\right)^{\frac{1}{2}}\left(\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2\right)^{\frac{1}{2}}\left(A\operatorname{EllipticF}\left(\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right),2^{\frac{1}{2}}\right)-A\operatorname{EllipticE}\left(\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right),2^{\frac{1}{2}}\right)-B\operatorname{EllipticF}\left(\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right),2^{\frac{1}{2}}\right)+3B\operatorname{EllipticE}\left(\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right),2^{\frac{1}{2}}\right)\right)+2\left(-2\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^4+\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2\right)^{\frac{1}{2}}\left(A-3B\right)\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^4-\left(-2\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^4+\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2\right)^{\frac{1}{2}}\left(A-5B\right)\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2\right)/\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^3/\left(2\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2-1\right)/\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right)/\left(2\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2-1\right)^{\frac{1}{2}}/d$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(a \sec(dx + c) + a) \cos(dx + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)*cos(d*x + c)^(3/2)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\cos(c+dx)^{\frac{3}{2}} \left(a + \frac{a}{\cos(c+dx)}\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))),x)

[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A}{\cos^{\frac{3}{2}}(c+dx) \sec(c+dx) + \cos^{\frac{3}{2}}(c+dx)} dx + \int \frac{B \sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx) \sec(c+dx) + \cos^{\frac{3}{2}}(c+dx)} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(3/2)/(a+a*sec(d*x+c)), x)

[Out] (Integral(A/(cos(c + d*x)**(3/2)*sec(c + d*x) + cos(c + d*x)**(3/2)), x) + Integral(B*sec(c + d*x)/(cos(c + d*x)**(3/2)*sec(c + d*x) + cos(c + d*x)**(3/2)), x))/a

$$3.501 \quad \int \frac{A+B \sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a \sec(c+dx))} dx$$

Optimal. Leaf size=152

$$\frac{(3A-5B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad} - \frac{3(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{(A-B)\sin(c+dx)}{d \cos^{\frac{3}{2}}(c+dx)(a \cos(c+dx)+a)} - \frac{(3A-5B)\sin(c+dx)}{3ad \cos^{\frac{3}{2}}(c+dx)}$$

[Out] $-3*(A-B)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/a/d-1/3*(3*A-5*B)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/a/d-1/3*(3*A-5*B)*\sin(d*x+c)/a/d/\cos(d*x+c)^{(3/2)}+(A-B)*\sin(d*x+c)/d/\cos(d*x+c)^{(3/2)}/(a+a*\cos(d*x+c))+3*(A-B)*\sin(d*x+c)/a/d/\cos(d*x+c)^{(1/2)}$

Rubi [A] time = 0.26, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2954, 2978, 2748, 2636, 2641, 2639}

$$\frac{(3A-5B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad} - \frac{3(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{(A-B)\sin(c+dx)}{d \cos^{\frac{3}{2}}(c+dx)(a \cos(c+dx)+a)} - \frac{(3A-5B)\sin(c+dx)}{3ad \cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sec[c + d*x])/((Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])),x]

[Out] $(-3*(A-B)*\text{EllipticE}[(c+d*x)/2, 2])/(a*d) - ((3*A-5*B)*\text{EllipticF}[(c+d*x)/2, 2])/(3*a*d) - ((3*A-5*B)*\text{Sin}[c+d*x])/((3*a*d*\text{Cos}[c+d*x]^{(3/2)}) + (3*(A-B)*\text{Sin}[c+d*x])/(a*d*\text{Sqrt}[\text{Cos}[c+d*x]]) + ((A-B)*\text{Sin}[c+d*x])/((d*\text{Cos}[c+d*x]^{(3/2)}*(a+a*\text{Cos}[c+d*x])))$

Rule 2636

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), Int[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_.)]], x_Symbol] := Simp[(2*EllipticF[(1*(c -
Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_.)]^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_.)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_.)]^(p_.), x_Symbol] := Dist[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 2978

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)]*(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]^(n_.), x_Symbol] := Simp[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{A + B \sec(c + dx)}{\cos^{\frac{5}{2}}(c + dx)(a + a \sec(c + dx))} dx &= \int \frac{B + A \cos(c + dx)}{\cos^{\frac{5}{2}}(c + dx)(a + a \cos(c + dx))} dx \\
&= \frac{(A - B) \sin(c + dx)}{d \cos^{\frac{3}{2}}(c + dx)(a + a \cos(c + dx))} + \frac{\int \frac{-\frac{1}{2}a(3A-5B) + \frac{3}{2}a(A-B) \cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)} dx}{a^2} \\
&= \frac{(A - B) \sin(c + dx)}{d \cos^{\frac{3}{2}}(c + dx)(a + a \cos(c + dx))} - \frac{(3A - 5B) \int \frac{1}{\cos^{\frac{5}{2}}(c+dx)} dx}{2a} + \frac{(3A - 5B) \sin(c + dx)}{2a} \\
&= -\frac{(3A - 5B) \sin(c + dx)}{3ad \cos^{\frac{3}{2}}(c + dx)} + \frac{3(A - B) \sin(c + dx)}{ad \sqrt{\cos(c + dx)}} + \frac{(A - B) \sin(c + dx)}{d \cos^{\frac{3}{2}}(c + dx)(a + a \cos(c + dx))} \\
&= -\frac{3(A - B)E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{ad} - \frac{(3A - 5B)F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3ad} - \frac{(3A - 5B) \sin(c + dx)}{3ad \cos^{\frac{3}{2}}(c + dx)}
\end{aligned}$$

Mathematica [C] time = 7.17, size = 1277, normalized size = 8.40

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])),x]
[Out] (((-3*I)/4)*A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*(A + B*Sec[c + d*x])*(
(2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x))*(Cos[c] +
I*Sin[c])^2])*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*
x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Si
n[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin
[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x))*(Cos[c] + I*Sin
[c])^2])*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Si
n[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]
])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c]))/((
B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])) + (((3*I)/4)*B*Cos[c/2 + (d*x)/2]
^2*Csc[c/2]*Sec[c/2]*(A + B*Sec[c + d*x])*((2*E^((2*I)*d*x)*Hypergeometric2
F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x))*(Cos[c] + I*Sin[c])^2])*Sqrt[(2*(1 + E^((
2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 +
E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*
d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4
, 1/2, 3/4, -(E^((2*I)*d*x))*(Cos[c] + I*Sin[c])^2])*Sqrt[(2*(1 + E^((2*I)*d
```

$$\begin{aligned}
 & *x) * \cos[c] + (2*I) * (-1 + E^{(2*I)*d*x}) * \sin[c] / E^{(I*d*x)} * \sqrt{1 + E^{(2*I)*d*x}} * \cos[2*c] + I * E^{(2*I)*d*x} * \sin[2*c] / ((-I)*d*(1 + E^{(2*I)*d*x}) * \cos[c] + d*(-1 + E^{(2*I)*d*x}) * \sin[c])) / ((B + A*\cos[c + d*x])*(a + a*\sec[c + d*x])) + (\cos[c/2 + (d*x)/2]^2 * \sqrt{\cos[c + d*x]} * (A + B*\sec[c + d*x]) * (-((-A + B)*(2 + \cos[c]) * \csc[c/2] * \sec[c/2] * \sec[c]) / d - (2*\sec[c/2] * \sec[c/2 + (d*x)/2] * (-A*\sin[(d*x)/2]) + B*\sin[(d*x)/2])) / d + (4*B*\sec[c] * \sec[c + d*x]^2 * \sin[d*x]) / (3*d) + (4*\sec[c] * \sec[c + d*x] * (B*\sin[c] + 3*A*\sin[d*x] - 3*B*\sin[d*x])) / (3*d)) / ((B + A*\cos[c + d*x])*(a + a*\sec[c + d*x])) + (A*\cos[c/2 + (d*x)/2]^2 * \csc[c/2] * \text{HypergeometricPFQ}[\{1/4, 1/2\}, \{5/4\}, \sin[d*x - \text{ArcTan}[\text{Cot}[c]]]^2] * \sec[c/2] * (A + B*\sec[c + d*x]) * \sec[d*x - \text{ArcTan}[\text{Cot}[c]]] * \sqrt{1 - \sin[d*x - \text{ArcTan}[\text{Cot}[c]]}] * \sqrt{-(\sqrt{1 + \text{Cot}[c]^2} * \sin[c] * \sin[d*x - \text{ArcTan}[\text{Cot}[c]]])} * \sqrt{1 + \sin[d*x - \text{ArcTan}[\text{Cot}[c]]]}) / (d*(B + A*\cos[c + d*x]) * \sqrt{1 + \text{Cot}[c]^2} * (a + a*\sec[c + d*x])) - (5*B*\cos[c/2 + (d*x)/2]^2 * \csc[c/2] * \text{HypergeometricPFQ}[\{1/4, 1/2\}, \{5/4\}, \sin[d*x - \text{ArcTan}[\text{Cot}[c]]]^2] * \sec[c/2] * (A + B*\sec[c + d*x]) * \sec[d*x - \text{ArcTan}[\text{Cot}[c]]] * \sqrt{1 - \sin[d*x - \text{ArcTan}[\text{Cot}[c]]}] * \sqrt{-(\sqrt{1 + \text{Cot}[c]^2} * \sin[c] * \sin[d*x - \text{ArcTan}[\text{Cot}[c]]])} * \sqrt{1 + \sin[d*x - \text{ArcTan}[\text{Cot}[c]]]}) / (3*d*(B + A*\cos[c + d*x]) * \sqrt{1 + \text{Cot}[c]^2} * (a + a*\sec[c + d*x]))
 \end{aligned}$$

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B \sec(dx + c) + A)\sqrt{\cos(dx + c)}}{a \cos(dx + c)^3 \sec(dx + c) + a \cos(dx + c)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")

[Out] integral((B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(a*cos(d*x + c)^3*sec(d*x + c) + a*cos(d*x + c)^3), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(a \sec(dx + c) + a \cos(dx + c))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)*cos(d*x + c)^(5/2)), x)

maple [B] time = 11.49, size = 493, normalized size = 3.24

$$\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \left(2B \left(-\frac{\cos\left(\frac{dx}{2} + \frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)}}{6\left(-\frac{1}{2} + \cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^2} + \frac{\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}}{3\sqrt{-2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c)), x)

[Out]
$$\begin{aligned} & -(-(-2*\cos(1/2*d*x+1/2*c)^2+1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/a*(2*B*(-1/6*\cos \\ & (1/2*d*x+1/2*c)*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/(-1/2+ \\ & \cos(1/2*d*x+1/2*c)^2)^2+1/3*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)*\text{EllipticF}(\cos(1/2*d*x+1/2*c), 2^{(1/2)})}+(2*A-2*B)*(-(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)*\text{EllipticE}(\cos(1/2*d*x+1/2*c), 2^{(1/2)})}+2*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^2}/\sin(1/2*d*x+1/2*c)^2/(2*\sin(1/2*d*x+1/2*c)^2-1)+(-A+B)*(\cos(1/2*d*x+1/2*c)*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)*(\text{EllipticF}(\cos(1/2*d*x+1/2*c), 2^{(1/2)})-\text{EllipticE}(\cos(1/2*d*x+1/2*c), 2^{(1/2)})}-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)/\cos(1/2*d*x+1/2*c)/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(a \sec(dx + c) + a) \cos(dx + c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c)), x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)*cos(d*x + c)^(5/2)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\cos(c+dx)^{5/2} \left(a + \frac{a}{\cos(c+dx)} \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))),x)
```

```
[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(5/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

$$3.502 \quad \int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B \sec(c+dx))}{(a+a \sec(c+dx))^2} dx$$

Optimal. Leaf size=204

$$\frac{5(3A-2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} + \frac{7(8A-5B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d} - \frac{(3A-2B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{a^2d(\cos(c+dx)+1)} + \frac{7(8A-5B)\sin(c+dx)}{a^2d}$$

[Out] $7/5*(8*A-5*B)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c),2^{(1/2)})/a^2/d-5/3*(3*A-2*B)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c),2^{(1/2)})/a^2/d+7/15*(8*A-5*B)*\cos(d*x+c)^{(3/2)}*\sin(d*x+c)/a^2/d-(3*A-2*B)*\cos(d*x+c)^{(5/2)}*\sin(d*x+c)/a^2/d/(1+\cos(d*x+c))-1/3*(A-B)*\cos(d*x+c)^{(7/2)}*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^2-5/3*(3*A-2*B)*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/a^2/d$

Rubi [A] time = 0.42, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2954, 2977, 2748, 2635, 2641, 2639}

$$\frac{5(3A-2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} + \frac{7(8A-5B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d} - \frac{(3A-2B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{a^2d(\cos(c+dx)+1)} + \frac{7(8A-5B)\sin(c+dx)}{a^2d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(\text{Cos}[c+d*x]^{(5/2)}*(A+B*\text{Sec}[c+d*x]))/(a+a*\text{Sec}[c+d*x])^2,x]$

[Out] $(7*(8*A-5*B)*\text{EllipticE}[(c+d*x)/2,2])/(5*a^2*d)-(5*(3*A-2*B)*\text{EllipticF}[(c+d*x)/2,2])/(3*a^2*d)-(5*(3*A-2*B)*\text{Sqrt}[\text{Cos}[c+d*x]]*\text{Sin}[c+d*x])/(3*a^2*d)+(7*(8*A-5*B)*\text{Cos}[c+d*x]^{(3/2)}*\text{Sin}[c+d*x])/(15*a^2*d)-((3*A-2*B)*\text{Cos}[c+d*x]^{(5/2)}*\text{Sin}[c+d*x])/(a^2*d*(1+\text{Cos}[c+d*x]))-((A-B)*\text{Cos}[c+d*x]^{(7/2)}*\text{Sin}[c+d*x])/(3*d*(a+a*\text{Cos}[c+d*x])^2)$

Rule 2635

$\text{Int}[(b_*)*\sin[(c_*)+(d_*)(x_)]^{(n_)}, x_Symbol] := -\text{Simp}[(b*\text{Cos}[c+d*x])*(b*\text{Sin}[c+d*x])^{(n-1)}]/(d*n), x] + \text{Dist}[(b^2*(n-1))/n, \text{Int}[(b*\text{Sin}[c+d*x])^{(n-2)}, x], x] /; \text{FreeQ}\{b, c, d\}, x \&\& \text{GtQ}[n, 1] \&\& \text{IntegerQ}[2*n]$

Rule 2639

$\text{Int}[\text{Sqrt}[\sin[(c_*)+(d_*)(x_)]], x_Symbol] := \text{Simp}[(2*\text{EllipticE}[(1*(c-Pi/2+d*x))/2,2])/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c -
  Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)])*(b_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_.))^(n_)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_), x_Symbol] := Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
  a*d, 0] && !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
  NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^2} dx &= \int \frac{\cos^{\frac{7}{2}}(c+dx)(B+A\cos(c+dx))}{(a+a\cos(c+dx))^2} dx \\
&= -\frac{(A-B)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \int \frac{\cos^{\frac{5}{2}}(c+dx)\left(-\frac{7}{2}a(A-B)+\frac{1}{2}a(11A-5B)\cos(c+dx)\right)}{3a^2(a+a\cos(c+dx))} dx \\
&= -\frac{(3A-2B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{a^2d(1+\cos(c+dx))} - \frac{(A-B)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} \\
&= -\frac{(3A-2B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{a^2d(1+\cos(c+dx))} - \frac{(A-B)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} \\
&= -\frac{5(3A-2B)\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2d} + \frac{7(8A-5B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15a^2d} \\
&= \frac{7(8A-5B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d} - \frac{5(3A-2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} - \frac{5(3A-2B)\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2d}
\end{aligned}$$

Mathematica [C] time = 7.08, size = 1396, normalized size = 6.84

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^2, x]
```

```
[Out] (((28*I)/5)*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x])*((2*E^((2*I)*d*x))*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x))*(Cos[c] + I*Sin[c])^2])*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x))*(Cos[c] + I*Sin[c])^2])*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c]))/(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^2 - (((7*I)/2)*B*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x])*((2*E^((2*I)*d*x))*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x))*(Cos[c] + I*Sin[c])^2])*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]]))
```

$$\frac{\begin{aligned} & c]] / ((3*I)*d*(1 + E^{((2*I)*d*x)})*\cos[c] - 3*d*(-1 + E^{((2*I)*d*x)})*\sin[c] \\ & - (2*\text{Hypergeometric2F1}[-1/4, 1/2, 3/4, -(E^{((2*I)*d*x)})*(\cos[c] + I*\sin[c]) \\ & ^2])*\sqrt{(2*(1 + E^{((2*I)*d*x)})*\cos[c] + (2*I)*(-1 + E^{((2*I)*d*x)})*\sin[c] \\ &)/E^{(I*d*x)}}*\sqrt{1 + E^{((2*I)*d*x)}*\cos[2*c] + I*E^{((2*I)*d*x)}*\sin[2*c]}) / (\\ & (-I)*d*(1 + E^{((2*I)*d*x)})*\cos[c] + d*(-1 + E^{((2*I)*d*x)})*\sin[c])) / ((B + \\ & A*\cos[c + d*x])*(a + a*\sec[c + d*x])^2) + (10*A*\cos[c/2 + (d*x)/2]^4*\csc[c/ \\ & 2]*\text{HypergeometricPFQ}[\{1/4, 1/2\}, \{5/4\}, \sin[d*x - \text{ArcTan}[\text{Cot}[c]]]^2]*\sec[c/ \\ & 2]*\sec[c + d*x]*(A + B*\sec[c + d*x])* \sec[d*x - \text{ArcTan}[\text{Cot}[c]]]*\sqrt{1 - \sin \\ & [d*x - \text{ArcTan}[\text{Cot}[c]]})*\sqrt{-(\sqrt{1 + \text{Cot}[c]^2}*\sin[c]*\sin[d*x - \text{ArcTan}[\text{C} \\ & \text{ot}[c]])})*\sqrt{1 + \sin[d*x - \text{ArcTan}[\text{Cot}[c]]})} / (d*(B + A*\cos[c + d*x])* \sqrt{ \\ & 1 + \text{Cot}[c]^2}*(a + a*\sec[c + d*x])^2) - (20*B*\cos[c/2 + (d*x)/2]^4*\csc[c/2 \\ &]*\text{HypergeometricPFQ}[\{1/4, 1/2\}, \{5/4\}, \sin[d*x - \text{ArcTan}[\text{Cot}[c]]]^2]*\sec[c/2 \\ &]*\sec[c + d*x]*(A + B*\sec[c + d*x])* \sec[d*x - \text{ArcTan}[\text{Cot}[c]]]*\sqrt{1 - \sin \\ & [d*x - \text{ArcTan}[\text{Cot}[c]]})*\sqrt{-(\sqrt{1 + \text{Cot}[c]^2}*\sin[c]*\sin[d*x - \text{ArcTan}[\text{Co} \\ & t[c]])})*\sqrt{1 + \sin[d*x - \text{ArcTan}[\text{Cot}[c]]})} / (3*d*(B + A*\cos[c + d*x])* \sqrt{ \\ & 1 + \text{Cot}[c]^2}*(a + a*\sec[c + d*x])^2) + (\cos[c/2 + (d*x)/2]^4*(A + B*\sec[\\ & c + d*x])*((4*(-20*A + 15*B - 36*A*\cos[c] + 20*B*\cos[c])* \csc[c]) / (5*d) + (8 \\ & *(-2*A + B)*\cos[d*x]*\sin[c]) / (3*d) + (4*A*\cos[2*d*x]*\sin[2*c]) / (5*d) - (2*S \\ & ec[c/2]*\sec[c/2 + (d*x)/2]^3*(-(A*\sin[(d*x)/2]) + B*\sin[(d*x)/2])) / (3*d) + \\ & (4*\sec[c/2]*\sec[c/2 + (d*x)/2]*(-4*A*\sin[(d*x)/2] + 3*B*\sin[(d*x)/2])) / d + \\ & (8*(-2*A + B)*\cos[c]*\sin[d*x]) / (3*d) + (4*A*\cos[2*c]*\sin[2*d*x]) / (5*d) - (2 \\ & *(-A + B)*\sec[c/2 + (d*x)/2]^2*\tan[c/2]) / (3*d))) / (\sqrt{\cos[c + d*x]}*(B + A \\ & *\cos[c + d*x])*(a + a*\sec[c + d*x])^2) \end{aligned}}$$

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{(B \cos(dx + c)^2 \sec(dx + c) + A \cos(dx + c)^2) \sqrt{\cos(dx + c)}}{a^2 \sec(dx + c)^2 + 2a^2 \sec(dx + c) + a^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="fricas")

[Out] integral((B*cos(d*x + c)^2*sec(d*x + c) + A*cos(d*x + c)^2)*sqrt(cos(d*x + c))/(a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \cos(dx + c)^{\frac{5}{2}}}{(a \sec(dx + c) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(5/2)/(a*sec(d*x + c) + a)^2, x)

maple [A] time = 5.01, size = 465, normalized size = 2.28

$$\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \left(96A\left(\cos^{10}\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 352A\left(\cos^8\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 80B\left(\cos^8\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \dots\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x)

[Out]
$$-1/30 * ((2 * \cos(1/2 * d * x + 1/2 * c) ^ 2 - 1) * \sin(1/2 * d * x + 1/2 * c) ^ 2) ^ (1/2) * (96 * A * \cos(1/2 * d * x + 1/2 * c) ^ 10 - 352 * A * \cos(1/2 * d * x + 1/2 * c) ^ 8 + 80 * B * \cos(1/2 * d * x + 1/2 * c) ^ 8 + 120 * A * \cos(1/2 * d * x + 1/2 * c) ^ 6 - 150 * A * (\sin(1/2 * d * x + 1/2 * c) ^ 2) ^ (1/2) * (-2 * \cos(1/2 * d * x + 1/2 * c) ^ 2 + 1) ^ (1/2) * \text{EllipticF}(\cos(1/2 * d * x + 1/2 * c), 2 ^ (1/2)) * \cos(1/2 * d * x + 1/2 * c) ^ 3 - 336 * A * \cos(1/2 * d * x + 1/2 * c) ^ 3 * (\sin(1/2 * d * x + 1/2 * c) ^ 2) ^ (1/2) * (-2 * \cos(1/2 * d * x + 1/2 * c) ^ 2 + 1) ^ (1/2) * \text{EllipticE}(\cos(1/2 * d * x + 1/2 * c), 2 ^ (1/2)) + 60 * B * \cos(1/2 * d * x + 1/2 * c) ^ 6 + 100 * B * (\sin(1/2 * d * x + 1/2 * c) ^ 2) ^ (1/2) * (-2 * \cos(1/2 * d * x + 1/2 * c) ^ 2 + 1) ^ (1/2) * \text{EllipticF}(\cos(1/2 * d * x + 1/2 * c), 2 ^ (1/2)) * \cos(1/2 * d * x + 1/2 * c) ^ 3 + 210 * B * \cos(1/2 * d * x + 1/2 * c) ^ 3 * (\sin(1/2 * d * x + 1/2 * c) ^ 2) ^ (1/2) * (-2 * \cos(1/2 * d * x + 1/2 * c) ^ 2 + 1) ^ (1/2) * \text{EllipticE}(\cos(1/2 * d * x + 1/2 * c), 2 ^ (1/2)) + 266 * A * \cos(1/2 * d * x + 1/2 * c) ^ 4 - 240 * B * \cos(1/2 * d * x + 1/2 * c) ^ 4 - 135 * A * \cos(1/2 * d * x + 1/2 * c) ^ 2 + 105 * B * \cos(1/2 * d * x + 1/2 * c) ^ 2 + 5 * A - 5 * B) / a ^ 2 / \cos(1/2 * d * x + 1/2 * c) ^ 3 / (-2 * \sin(1/2 * d * x + 1/2 * c) ^ 4 + \sin(1/2 * d * x + 1/2 * c) ^ 2) ^ (1/2) / \sin(1/2 * d * x + 1/2 * c) / (2 * \cos(1/2 * d * x + 1/2 * c) ^ 2 - 1) ^ (1/2) / d$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \cos(dx + c)^{\frac{5}{2}}}{(a \sec(dx + c) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(5/2)/(a*sec(d*x + c) + a)^2, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c + dx)^{5/2} \left(A + \frac{B}{\cos(c + dx)}\right)}{\left(a + \frac{a}{\cos(c + dx)}\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(5/2)*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^2,x)
```

```
[Out] int((cos(c + d*x)^(5/2)*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^2, x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

$$3.503 \quad \int \frac{\cos^3(c+dx)(A+B \sec(c+dx))}{(a+a \sec(c+dx))^2} dx$$

Optimal. Leaf size=171

$$\frac{5(2A - B)F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3a^2d} - \frac{(7A - 4B)E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{a^2d} - \frac{(7A - 4B) \sin(c + dx) \cos^{\frac{3}{2}}(c + dx)}{3a^2d(\cos(c + dx) + 1)} + \frac{5(2A - B) \sin(c + dx)}{3a^2d}$$

[Out] $-(7A-4B)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/a^2/d+5/3*(2A-B)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/a^2/d-1/3*(7A-4B)*\cos(d*x+c)^{(3/2)}*\sin(d*x+c)/a^2/d/(1+\cos(d*x+c))-1/3*(A-B)*\cos(d*x+c)^{(5/2)}*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^2+5/3*(2A-B)*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/a^2/d$

Rubi [A] time = 0.40, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2954, 2977, 2748, 2639, 2635, 2641}

$$\frac{5(2A - B)F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3a^2d} - \frac{(7A - 4B)E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{a^2d} - \frac{(7A - 4B) \sin(c + dx) \cos^{\frac{3}{2}}(c + dx)}{3a^2d(\cos(c + dx) + 1)} + \frac{5(2A - B) \sin(c + dx)}{3a^2d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(\text{Cos}[c + d*x]^{(3/2)}*(A + B*\text{Sec}[c + d*x]))/(a + a*\text{Sec}[c + d*x])^2, x]$

[Out] $-(((7A - 4B)*\text{EllipticE}[(c + d*x)/2, 2])/(a^2*d)) + (5*(2A - B)*\text{EllipticF}[(c + d*x)/2, 2])/(3*a^2*d) + (5*(2A - B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sin}[c + d*x])/(3*a^2*d) - ((7A - 4B)*\text{Cos}[c + d*x]^{(3/2)}*\text{Sin}[c + d*x])/(3*a^2*d*(1 + \text{Cos}[c + d*x])) - ((A - B)*\text{Cos}[c + d*x]^{(5/2)}*\text{Sin}[c + d*x])/(3*d*(a + a*\text{Cos}[c + d*x])^2)$

Rule 2635

$\text{Int}[(b_*)*\sin[(c_*) + (d_*)(x_*)]^{(n_)}, x_Symbol] \rightarrow -\text{Simp}[(b*\text{Cos}[c + d*x])*(b*\text{Sin}[c + d*x])^{(n-1)}]/(d*n), x] + \text{Dist}[(b^2*(n-1))/n, \text{Int}[(b*\text{Sin}[c + d*x])^{(n-2)}, x], x] /;$ FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2639

$\text{Int}[\text{Sqrt}[\sin[(c_*) + (d_*)(x_*)]], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticE}[(1*(c - P i/2 + d*x))/2, 2])/d, x] /;$ FreeQ[{c, d}, x]

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]]], x_Symbol] := Simp[(2*EllipticF[(1*(c -
  Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)])*(b_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_.))^(n_)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_), x_Symbol] := Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
  a*d, 0] && !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
  NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^2} dx &= \int \frac{\cos^{\frac{5}{2}}(c+dx)(B+A\cos(c+dx))}{(a+a\cos(c+dx))^2} dx \\
&= -\frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(-\frac{5}{2}a(A-B)+\frac{3}{2}a(3A-B)\cos(c+dx)\right)}{a+a\cos(c+dx)} dx}{3a^2} \\
&= -\frac{(7A-4B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} \\
&= -\frac{(7A-4B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} \\
&= -\frac{(7A-4B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{5(2A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2d} - \\
&= -\frac{(7A-4B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{5(2A-B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} + \frac{5(2A-B)}{3a^2d}
\end{aligned}$$

Mathematica [C] time = 6.81, size = 1352, normalized size = 7.91

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^2, x]
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[Out] (((-7*I)/2)*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x])*((2*E^((2*I)*d*x))*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x))*(Cos[c] + I*Sin[c])^2])*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x))*(Cos[c] + I*Sin[c])^2])*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c]))/(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^2 + ((2*I)*B*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x])*((2*E^((2*I)*d*x))*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x))*(Cos[c] + I*Sin[c])^2])*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])
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/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (
2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x))*(Cos[c] + I*Sin[c])^2])
*sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^
(I*d*x)]*sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]]/((-I)
*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c]))/((B + A*Cos
[c + d*x])*(a + a*Sec[c + d*x])^2) - (20*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*H
ypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*S
ec[c + d*x]*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*sqrt[1 - Sin[d*x
- ArcTan[Cot[c]]])*sqrt[-(sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]
]])]*sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(B + A*Cos[c + d*x])*sqrt[1
+ Cot[c]^2]*(a + a*Sec[c + d*x])^2) + (10*B*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*
HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*
Sec[c + d*x]*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*sqrt[1 - Sin[d*x
- ArcTan[Cot[c]]])*sqrt[-(sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]
]])]*sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(B + A*Cos[c + d*x])*sqrt[1
+ Cot[c]^2]*(a + a*Sec[c + d*x])^2) + (Cos[c/2 + (d*x)/2]^4*(A + B*Sec[c
+ d*x])*((-4*(-3*A + 2*B - 4*A*Cos[c] + 2*B*Cos[c])*Csc[c])/d + (8*A*Cos[d*x]
*Sin[c])/(3*d) + (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(-A*Sin[(d*x)/2]) + B*
Sin[(d*x)/2]))/(3*d) - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(-3*A*Sin[(d*x)/2] +
2*B*Sin[(d*x)/2]))/d + (8*A*Cos[c]*Sin[d*x])/(3*d) + (2*(-A + B)*Sec[c/2 +
(d*x)/2]^2*Tan[c/2])/(3*d))/((sqrt[Cos[c + d*x]]*(B + A*Cos[c + d*x])*(a +
a*Sec[c + d*x])^2)

```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{(B \cos(dx + c) \sec(dx + c) + A \cos(dx + c)) \sqrt{\cos(dx + c)}}{a^2 \sec(dx + c)^2 + 2a^2 \sec(dx + c) + a^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```

[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm
="fricas")

```

```

[Out] integral((B*cos(d*x + c)*sec(d*x + c) + A*cos(d*x + c))*sqrt(cos(d*x + c))/
(a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2), x)

```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \cos(dx + c)^{\frac{3}{2}}}{(a \sec(dx + c) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```

[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm
="giac")

```


[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(3/2)/(a*sec(d*x + c) + a)^2, x)

maple [B] time = 5.54, size = 435, normalized size = 2.54

$$\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)\left(16A\left(\cos^8\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 12A\left(\cos^6\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 20A\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x)

[Out]
$$-1/6*((2*\cos(1/2*d*x+1/2*c)^2-1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(16*A*\cos(1/2*d*x+1/2*c)^8+12*A*\cos(1/2*d*x+1/2*c)^6+20*A*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})*\cos(1/2*d*x+1/2*c)^3+42*A*\cos(1/2*d*x+1/2*c)^3*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)})-24*B*\cos(1/2*d*x+1/2*c)^6-10*B*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})*\cos(1/2*d*x+1/2*c)^3-24*B*\cos(1/2*d*x+1/2*c)^3*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)})-48*A*\cos(1/2*d*x+1/2*c)^4+38*B*\cos(1/2*d*x+1/2*c)^4+21*A*\cos(1/2*d*x+1/2*c)^2-15*B*\cos(1/2*d*x+1/2*c)^2-A+B)/a^2/\cos(1/2*d*x+1/2*c)^3/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \cos(dx + c)^{\frac{3}{2}}}{(a \sec(dx + c) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(3/2)/(a*sec(d*x + c) + a)^2, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c + dx)^{3/2} \left(A + \frac{B}{\cos(c+dx)} \right)}{\left(a + \frac{a}{\cos(c+dx)} \right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(3/2)*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^2,x)
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```
[Out] int((cos(c + d*x)^(3/2)*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^2, x)
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```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

$$3.504 \quad \int \frac{\sqrt{\cos(c+dx)} (A+B \sec(c+dx))}{(a+a \sec(c+dx))^2} dx$$

Optimal. Leaf size=137

$$-\frac{(5A-2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} + \frac{(4A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} - \frac{(5A-2B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3a^2d(\cos(c+dx)+1)} - \frac{(A-B)\sin(c+dx)}{3d(a\cos(c+dx)+1)}$$

[Out] (4*A-B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c), 2^(1/2))/a^2/d-1/3*(5*A-2*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c), 2^(1/2))/a^2/d-1/3*(A-B)*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^2-1/3*(5*A-2*B)*sin(d*x+c)*cos(d*x+c)^(1/2)/a^2/d/(1+cos(d*x+c))

Rubi [A] time = 0.37, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {2954, 2977, 2748, 2641, 2639}

$$-\frac{(5A-2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} + \frac{(4A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} - \frac{(5A-2B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3a^2d(\cos(c+dx)+1)} - \frac{(A-B)\sin(c+dx)}{3d(a\cos(c+dx)+1)}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^2, x]

[Out] ((4*A - B)*EllipticE[(c + d*x)/2, 2])/(a^2*d) - ((5*A - 2*B)*EllipticF[(c + d*x)/2, 2])/(3*a^2*d) - ((5*A - 2*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*a^2*d*(1 + Cos[c + d*x])) - ((A - B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(3*d*(a + a*cos[c + d*x])^2)

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*SIN[e + f*x])^m, x], x] + Dist[d/b, Int[(b*SIN[e + f*x])^m, x], x]

$b*\sin[e + f*x]^{(m + 1)}, x], x] /; \text{FreeQ}[\{b, c, d, e, f, m\}, x]$

Rule 2954

$\text{Int}[(a_.) + \text{csc}[(e_.) + (f_.)*(x_.)]*(b_.)]^{(m_.)}*(\text{csc}[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))^{(n_.)}*((g_.)*\sin[(e_.) + (f_.)*(x_.)])^{(p_.)}, x_Symbol] :> \text{Dist}[g^{(m + n)}, \text{Int}[(g*\sin[e + f*x])^{(p - m - n)}*(b + a*\sin[e + f*x])^m*(d + c*\sin[e + f*x])^n, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, g, p\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& !\text{IntegerQ}[p] \&\& \text{IntegerQ}[m] \&\& \text{IntegerQ}[n]$

Rule 2977

$\text{Int}[(a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)]^{(m_.)}*((A_.) + (B_.)*\sin[(e_.) + (f_.)*(x_.)]*(c_.) + (d_.)*\sin[(e_.) + (f_.)*(x_.)])^{(n_.)}, x_Symbol] :> \text{Simp}[(A*b - a*B)*\text{Cos}[e + f*x]*(a + b*\sin[e + f*x])^m*(c + d*\sin[e + f*x])^n / (a*f*(2*m + 1)), x] - \text{Dist}[1/(a*b*(2*m + 1)), \text{Int}[(a + b*\sin[e + f*x])^{(m + 1)}*(c + d*\sin[e + f*x])^{(n - 1)}*\text{Simp}[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m + b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*\sin[e + f*x], x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{LtQ}[m, -2^{(-1)}] \&\& \text{GtQ}[n, 0] \&\& \text{IntegerQ}[2*m] \&\& (\text{IntegerQ}[2*n] || \text{EqQ}[c, 0])$

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{\cos(c + dx)}(A + B \sec(c + dx))}{(a + a \sec(c + dx))^2} dx &= \int \frac{\cos^{\frac{3}{2}}(c + dx)(B + A \cos(c + dx))}{(a + a \cos(c + dx))^2} dx \\ &= -\frac{(A - B) \cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{3d(a + a \cos(c + dx))^2} + \int \frac{\sqrt{\cos(c + dx)} \left(-\frac{3}{2}a(A - B) + \frac{1}{2}a(7A - B) \cos(c + dx) \right)}{a + a \cos(c + dx)} dx \\ &= -\frac{(5A - 2B) \sqrt{\cos(c + dx)} \sin(c + dx)}{3a^2d(1 + \cos(c + dx))} - \frac{(A - B) \cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{3d(a + a \cos(c + dx))^2} \\ &= -\frac{(5A - 2B) \sqrt{\cos(c + dx)} \sin(c + dx)}{3a^2d(1 + \cos(c + dx))} - \frac{(A - B) \cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{3d(a + a \cos(c + dx))^2} \\ &= \frac{(4A - B)E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{a^2d} - \frac{(5A - 2B)F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3a^2d} - \frac{(5A - 2B)\sqrt{\cos(c + dx)}}{3a^2d} \end{aligned}$$

Mathematica [C] time = 6.70, size = 1318, normalized size = 9.62

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^2, x]

[Out] ((2*I)*A*cos[c/2 + (d*x)/2]^4*csc[c/2]*sec[c/2]*sec[c + d*x]*(A + B*Sec[c + d*x])*((2*E^((2*I)*d*x))*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x))*(Cos[c] + I*Sin[c])^2])*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x))*(Cos[c] + I*Sin[c])^2])*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c]))/(B + A*cos[c + d*x])*(a + a*Sec[c + d*x])^2 - ((I/2)*B*cos[c/2 + (d*x)/2]^4*csc[c/2]*sec[c/2]*sec[c + d*x]*(A + B*Sec[c + d*x])*((2*E^((2*I)*d*x))*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x))*(Cos[c] + I*Sin[c])^2])*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x))*(Cos[c] + I*Sin[c])^2])*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c]))/(B + A*cos[c + d*x])*(a + a*Sec[c + d*x])^2 + (10*A*cos[c/2 + (d*x)/2]^4*csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]])*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]])])*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(B + A*cos[c + d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^2) - (4*B*cos[c/2 + (d*x)/2]^4*csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]])*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]])])*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(B + A*cos[c + d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^2) + (Cos[c/2 + (d*x)/2]^4*(A + B*Sec[c + d*x])*((-4*(2*A - B + 2*A*cos[c])*Csc[c])/d + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(-2*A*Sin[(d*x)/2] + B*Sin[(d*x)/2]))/d - (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(-(A*Sin[(d*x)/2] + B*Sin[(d*x)/2]))/(3*d) - (2*(-A + B)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(3*d)))/(Sqrt[Cos[c + d*x]]*(B + A*cos[c + d*x])*(a + a*Sec[c + d*x])^2)

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{(B \sec(dx + c) + A) \sqrt{\cos(dx + c)}}{a^2 \sec(dx + c)^2 + 2a^2 \sec(dx + c) + a^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")

[Out] integral((B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)\sqrt{\cos(dx + c)}}{(a \sec(dx + c) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(a*sec(d*x + c) + a)^2, x)

maple [B] time = 5.15, size = 421, normalized size = 3.07

$$\sqrt{\left(2 \left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right) \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) \left(24A \left(\cos^6\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 10A \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{-2 \left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 1}\right)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x)

[Out] 1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(24*A*cos(1/2*d*x+1/2*c)^6+10*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+24*A*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-12*B*cos(1/2*d*x+1/2*c)^6-4*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3-6*B*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-38*A*cos(1/2*d*x+1/2*c)^4+20*B*cos(1/2*d*x+1/2*c)^4+15*A*cos(1/2*d*x+1/2*c)^2-9*B*cos(1/2*d*x+1/2*c)^2-A+B)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sqrt{\cos(dx + c)}}{(a \sec(dx + c) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(a*sec(d*x + c) + a)^2, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\cos(c + dx)} \left(A + \frac{B}{\cos(c+dx)} \right)}{\left(a + \frac{a}{\cos(c+dx)} \right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)^(1/2)*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^2,x)

[Out] int((cos(c + d*x)^(1/2)*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A \sqrt{\cos(c+dx)}}{\sec^2(c+dx)+2 \sec(c+dx)+1} dx + \int \frac{B \sqrt{\cos(c+dx)} \sec(c+dx)}{\sec^2(c+dx)+2 \sec(c+dx)+1} dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)**(1/2)/(a+a*sec(d*x+c))**2,x)

[Out] (Integral(A*sqrt(cos(c + d*x))/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x) + Integral(B*sqrt(cos(c + d*x))*sec(c + d*x)/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x))/a**2

$$3.505 \quad \int \frac{A+B \sec(c+dx)}{\sqrt{\cos(c+dx)} (a+a \sec(c+dx))^2} dx$$

Optimal. Leaf size=121

$$\frac{(2A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} - \frac{AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{A \sin(c+dx)\sqrt{\cos(c+dx)}}{a^2d(\cos(c+dx)+1)} - \frac{(A-B) \sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a \cos(c+dx)+a)^2}$$

[Out] $-A*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/a^2/d+1/3*(2*A+B)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/a^2/d+A*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/a^2/d/(1+\cos(d*x+c))-1/3*(A-B)*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/d/(a+a*\cos(d*x+c))^2$

Rubi [A] time = 0.35, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2954, 2977, 2978, 2748, 2641, 2639}

$$\frac{(2A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} - \frac{AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{A \sin(c+dx)\sqrt{\cos(c+dx)}}{a^2d(\cos(c+dx)+1)} - \frac{(A-B) \sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a \cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(A + B*\text{Sec}[c + d*x])]/(\text{Sqrt}[\text{Cos}[c + d*x]]*(a + a*\text{Sec}[c + d*x])^2), x]$

[Out] $-((A*\text{EllipticE}[(c + d*x)/2, 2])/(a^2*d)) + ((2*A + B)*\text{EllipticF}[(c + d*x)/2, 2])/(3*a^2*d) + (A*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sin}[c + d*x])/(a^2*d*(1 + \text{Cos}[c + d*x])) - ((A - B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sin}[c + d*x])/(3*d*(a + a*\text{Cos}[c + d*x])^2)$

Rule 2639

$\text{Int}[\text{Sqrt}[\sin[(c_.) + (d_.)*(x_.)]], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticE}[(1*(c - \text{Pi}/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 2641

$\text{Int}[1/\text{Sqrt}[\sin[(c_.) + (d_.)*(x_.)]], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticF}[(1*(c - \text{Pi}/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 2748

$\text{Int}(((b_.)*\sin[(e_.) + (f_.)*(x_.)])^{(m_.)*((c_.) + (d_.)*\sin[(e_.) + (f_.)*(x_.)])}), x_Symbol] \rightarrow \text{Dist}[c, \text{Int}[(b*\text{Sin}[e + f*x])^m, x], x] + \text{Dist}[d/b, \text{Int}[($

$b \sin[e + f x]^{m+1}, x], x] /; \text{FreeQ}[\{b, c, d, e, f, m\}, x]$

Rule 2954

$\text{Int}[(a_.) + \text{csc}[(e_.) + (f_.)(x_.)]*(b_.)]^{(m_.)}*(\text{csc}[(e_.) + (f_.)(x_.)]*(d_.) + (c_.))^{(n_.)}*((g_.)\sin[(e_.) + (f_.)(x_.)])^{(p_.)}, x_Symbol] := \text{Dist}[g^{(m+n)}, \text{Int}[(g*\sin[e + f*x])^{(p-m-n)}*(b + a*\sin[e + f*x])^m*(d + c*\sin[e + f*x])^n, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, g, p\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& !\text{IntegerQ}[p] \&\& \text{IntegerQ}[m] \&\& \text{IntegerQ}[n]$

Rule 2977

$\text{Int}[(a_.) + (b_.)\sin[(e_.) + (f_.)(x_.)]]^{(m_.)}*((A_.) + (B_.)\sin[(e_.) + (f_.)(x_.)])*((c_.) + (d_.)\sin[(e_.) + (f_.)(x_.)])^{(n_.)}, x_Symbol] := \text{Simp}[(A*b - a*B)*\text{Cos}[e + f*x]*(a + b*\sin[e + f*x])^m*(c + d*\sin[e + f*x])^n/(a*f*(2*m + 1)), x] - \text{Dist}[1/(a*b*(2*m + 1)), \text{Int}[(a + b*\sin[e + f*x])^{(m+1)}*(c + d*\sin[e + f*x])^{(n-1)}*\text{Simp}[A*(a*d*n - b*c*(m+1)) - B*(a*c*m + b*d*n) - d*(a*B*(m-n) + A*b*(m+n+1))*\sin[e + f*x], x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{LtQ}[m, -2^{(-1)}] \&\& \text{GtQ}[n, 0] \&\& \text{IntegerQ}[2*m] \&\& (\text{IntegerQ}[2*n] || \text{EqQ}[c, 0])$

Rule 2978

$\text{Int}[(a_.) + (b_.)\sin[(e_.) + (f_.)(x_.)]]^{(m_.)}*((A_.) + (B_.)\sin[(e_.) + (f_.)(x_.)])*((c_.) + (d_.)\sin[(e_.) + (f_.)(x_.)])^{(n_.)}, x_Symbol] := \text{Simp}[(b*(A*b - a*B)*\text{Cos}[e + f*x]*(a + b*\sin[e + f*x])^m*(c + d*\sin[e + f*x])^{(n+1)})/(a*f*(2*m + 1)*(b*c - a*d)), x] + \text{Dist}[1/(a*(2*m + 1)*(b*c - a*d)), \text{Int}[(a + b*\sin[e + f*x])^{(m+1)}*(c + d*\sin[e + f*x])^n*\text{Simp}[B*(a*c*m + b*d*(n+1)) + A*(b*c*(m+1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m+n+2)*\sin[e + f*x], x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{LtQ}[m, -2^{(-1)}] \&\& !\text{GtQ}[n, 0] \&\& \text{IntegerQ}[2*m] \&\& (\text{IntegerQ}[2*n] || \text{EqQ}[c, 0])$

Rubi steps

$$\begin{aligned}
\int \frac{A + B \sec(c + dx)}{\sqrt{\cos(c + dx)} (a + a \sec(c + dx))^2} dx &= \int \frac{\sqrt{\cos(c + dx)} (B + A \cos(c + dx))}{(a + a \cos(c + dx))^2} dx \\
&= -\frac{(A - B)\sqrt{\cos(c + dx)} \sin(c + dx)}{3d(a + a \cos(c + dx))^2} + \frac{\int \frac{-\frac{1}{2}a(A-B) + \frac{1}{2}a(5A+B) \cos(c+dx)}{\sqrt{\cos(c+dx)} (a+a \cos(c+dx))} dx}{3a^2} \\
&= \frac{A\sqrt{\cos(c + dx)} \sin(c + dx)}{a^2d(1 + \cos(c + dx))} - \frac{(A - B)\sqrt{\cos(c + dx)} \sin(c + dx)}{3d(a + a \cos(c + dx))^2} + \frac{\int \frac{1}{2} \dots}{\dots} \\
&= \frac{A\sqrt{\cos(c + dx)} \sin(c + dx)}{a^2d(1 + \cos(c + dx))} - \frac{(A - B)\sqrt{\cos(c + dx)} \sin(c + dx)}{3d(a + a \cos(c + dx))^2} - \frac{A \int \dots}{\dots} \\
&= -\frac{AE\left(\frac{1}{2}(c + dx) \middle| 2\right)}{a^2d} + \frac{(2A + B)F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3a^2d} + \frac{A\sqrt{\cos(c + dx)} \sin(c + dx)}{a^2d(1 + \cos(c + dx))}
\end{aligned}$$

Mathematica [C] time = 6.57, size = 921, normalized size = 7.61

$$iA \csc\left(\frac{c}{2}\right) \sec\left(\frac{c}{2}\right) \sec(c + dx)(A + B \sec(c + dx)) \left(\frac{2e^{2idx} {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{7}{4}; -e^{2idx}(\cos(c) + i \sin(c))^2\right) \sqrt{e^{-idx}(2(1 + e^{2idx}) \cos(c) + 2i(-1 + e^{2idx}) \sin(c))}}{3id(1 + e^{2idx}) \cos(c) - 3d(-1 + e^{2idx}) \sin(c)} \right)$$

2(B + A

Warning: Unable to verify antiderivative.

[In] Integrate[(A + B*Sec[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^2), x]

[Out] ((-1/2*I)*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x])*((2*E^((2*I)*d*x))*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x))*(Cos[c] + I*Sin[c])^2])*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x))*(Cos[c] + I*Sin[c])^2])*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c]))/(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^2 - (4*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(B + A*Co

$s[c + d*x]) * \text{Sqrt}[1 + \text{Cot}[c]^2] * (a + a * \text{Sec}[c + d*x])^2) - (2 * B * \text{Cos}[c/2 + (d*x)/2]^4 * \text{Csc}[c/2] * \text{HypergeometricPFQ}[\{1/4, 1/2\}, \{5/4\}, \text{Sin}[d*x - \text{ArcTan}[\text{Cot}[c]]]^2] * \text{Sec}[c/2] * \text{Sec}[c + d*x] * (A + B * \text{Sec}[c + d*x]) * \text{Sec}[d*x - \text{ArcTan}[\text{Cot}[c]]] * \text{Sqrt}[1 - \text{Sin}[d*x - \text{ArcTan}[\text{Cot}[c]]]] * \text{Sqrt}[-(\text{Sqrt}[1 + \text{Cot}[c]^2] * \text{Sin}[c] * \text{Sin}[d*x - \text{ArcTan}[\text{Cot}[c]]]) * \text{Sqrt}[1 + \text{Sin}[d*x - \text{ArcTan}[\text{Cot}[c]]]])] / (3 * d * (B + A * \text{Cos}[c + d*x]) * \text{Sqrt}[1 + \text{Cot}[c]^2] * (a + a * \text{Sec}[c + d*x])^2) + (\text{Cos}[c/2 + (d*x)/2]^4 * (A + B * \text{Sec}[c + d*x]) * ((4 * A * \text{Csc}[c]) / d + (4 * A * \text{Sec}[c/2] * \text{Sec}[c/2 + (d*x)/2] * \text{Sin}[(d*x)/2]) / d + (2 * \text{Sec}[c/2] * \text{Sec}[c/2 + (d*x)/2]^3 * (-A * \text{Sin}[(d*x)/2]) + B * \text{Sin}[(d*x)/2])) / (3 * d) + (2 * (-A + B) * \text{Sec}[c/2 + (d*x)/2]^2 * \text{Tan}[c/2]) / (3 * d)) / (\text{Sqrt}[\text{Cos}[c + d*x]] * (B + A * \text{Cos}[c + d*x]) * (a + a * \text{Sec}[c + d*x])^2)$

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B \sec(dx + c) + A)\sqrt{\cos(dx + c)}}{a^2 \cos(dx + c) \sec(dx + c)^2 + 2a^2 \cos(dx + c) \sec(dx + c) + a^2 \cos(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm="fricas")

[Out] integral((B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(a^2*cos(d*x + c)*sec(d*x + c)^2 + 2*a^2*cos(d*x + c)*sec(d*x + c) + a^2*cos(d*x + c)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(a \sec(dx + c) + a)^2 \sqrt{\cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^2*sqrt(cos(d*x + c))), x)

maple [B] time = 5.23, size = 350, normalized size = 2.89

$$\sqrt{\left(2 \left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right) \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) \left(12A \left(\cos^6\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 4A \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{-2 \left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}\right) +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2/cos(d*x+c)^(1/2),x)`

[Out]
$$-1/6*((2*\cos(1/2*d*x+1/2*c)^2-1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(12*A*\cos(1/2*d*x+1/2*c)^6+4*A*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)})*\cos(1/2*d*x+1/2*c)^3+6*A*\cos(1/2*d*x+1/2*c)^3*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)})+2*B*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)})*\cos(1/2*d*x+1/2*c)^3-20*A*\cos(1/2*d*x+1/2*c)^4+2*B*\cos(1/2*d*x+1/2*c)^4+9*A*\cos(1/2*d*x+1/2*c)^2-3*B*\cos(1/2*d*x+1/2*c)^2-A+B)/a^2/\cos(1/2*d*x+1/2*c)^3/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(a \sec(dx + c) + a)^2 \sqrt{\cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm="maxima")`

[Out] `integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^2*sqrt(cos(d*x + c))), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\sqrt{\cos(c+dx)} \left(a + \frac{a}{\cos(c+dx)}\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B/cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^2),x)`

[Out] `int((A + B/cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A}{\sqrt{\cos(c+dx)} \sec^2(c+dx)+2\sqrt{\cos(c+dx)} \sec(c+dx)+\sqrt{\cos(c+dx)}} dx + \int \frac{B \sec(c+dx)}{\sqrt{\cos(c+dx)} \sec^2(c+dx)+2\sqrt{\cos(c+dx)} \sec(c+dx)+\sqrt{\cos(c+dx)}} dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))**2/cos(d*x+c)**(1/2),x)`

```
[Out] (Integral(A/(sqrt(cos(c + d*x))*sec(c + d*x)**2 + 2*sqrt(cos(c + d*x))*sec(c + d*x) + sqrt(cos(c + d*x))), x) + Integral(B*sec(c + d*x)/(sqrt(cos(c + d*x))*sec(c + d*x)**2 + 2*sqrt(cos(c + d*x))*sec(c + d*x) + sqrt(cos(c + d*x))), x))/a**2
```

$$3.506 \quad \int \frac{A+B \sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a \sec(c+dx))^2} dx$$

Optimal. Leaf size=121

$$\frac{(A+2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} + \frac{BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} - \frac{B \sin(c+dx)\sqrt{\cos(c+dx)}}{a^2d(\cos(c+dx)+1)} + \frac{(A-B) \sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a \cos(c+dx)+a)^2}$$

[Out] B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d+1/3*(A+2*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d-B*sin(d*x+c)*cos(d*x+c)^(1/2)/a^2/d/(1+cos(d*x+c))+1/3*(A-B)*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^2

Rubi [A] time = 0.36, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {2954, 2978, 2748, 2641, 2639}

$$\frac{(A+2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} + \frac{BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} - \frac{B \sin(c+dx)\sqrt{\cos(c+dx)}}{a^2d(\cos(c+dx)+1)} + \frac{(A-B) \sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a \cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2),x]

[Out] (B*EllipticE[(c + d*x)/2, 2])/(a^2*d) + ((A + 2*B)*EllipticF[(c + d*x)/2, 2])/(3*a^2*d) - (B*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(a^2*d*(1 + Cos[c + d*x])) + ((A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2)

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(

$b*\sin[e + f*x]^{(m + 1)}, x], x] /; \text{FreeQ}[\{b, c, d, e, f, m\}, x]$

Rule 2954

$\text{Int}[(a_.) + \text{csc}[(e_.) + (f_.)*(x_.)]*(b_.)]^{(m_.)}*(\text{csc}[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))^{(n_.)}*((g_.)*\sin[(e_.) + (f_.)*(x_.)])^{(p_.)}, x_Symbol] := \text{Dist}[g^{(m + n)}, \text{Int}[(g*\sin[e + f*x])^{(p - m - n)}*(b + a*\sin[e + f*x])^m*(d + c*\sin[e + f*x])^n, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, g, p\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& !\text{IntegerQ}[p] \&\& \text{IntegerQ}[m] \&\& \text{IntegerQ}[n]$

Rule 2978

$\text{Int}[(a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)]]^{(m_.)}*((A_.) + (B_.)*\sin[(e_.) + (f_.)*(x_.)])*((c_.) + (d_.)*\sin[(e_.) + (f_.)*(x_.)])^{(n_.)}, x_Symbol] := \text{Simp}[(b*(A*b - a*B)*\text{Cos}[e + f*x]*(a + b*\sin[e + f*x])^m*(c + d*\sin[e + f*x])^{(n + 1)})/(a*f*(2*m + 1)*(b*c - a*d)), x] + \text{Dist}[1/(a*(2*m + 1)*(b*c - a*d)), \text{Int}[(a + b*\sin[e + f*x])^{(m + 1)}*(c + d*\sin[e + f*x])^n*\text{Simp}[B*(a*c*m + b*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2)*\sin[e + f*x], x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{LtQ}[m, -2^{(-1)}] \&\& !\text{GtQ}[n, 0] \&\& \text{IntegerQ}[2*m] \&\& (\text{IntegerQ}[2*n] || \text{EqQ}[c, 0])$

Rubi steps

$$\begin{aligned} \int \frac{A + B \sec(c + dx)}{\cos^3(c + dx)(a + a \sec(c + dx))^2} dx &= \int \frac{B + A \cos(c + dx)}{\sqrt{\cos(c + dx)}(a + a \cos(c + dx))^2} dx \\ &= \frac{(A - B)\sqrt{\cos(c + dx)} \sin(c + dx)}{3d(a + a \cos(c + dx))^2} + \frac{\int \frac{\frac{1}{2}a(A+5B) + \frac{1}{2}a(A-B)\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))} dx}{3a^2} \\ &= -\frac{B\sqrt{\cos(c + dx)} \sin(c + dx)}{a^2d(1 + \cos(c + dx))} + \frac{(A - B)\sqrt{\cos(c + dx)} \sin(c + dx)}{3d(a + a \cos(c + dx))^2} + \int \frac{B}{3a^2} dx \\ &= -\frac{B\sqrt{\cos(c + dx)} \sin(c + dx)}{a^2d(1 + \cos(c + dx))} + \frac{(A - B)\sqrt{\cos(c + dx)} \sin(c + dx)}{3d(a + a \cos(c + dx))^2} + \frac{Bx}{3a^2} \\ &= \frac{BE \left(\frac{1}{2}(c + dx) \Big| 2 \right)}{a^2d} + \frac{(A + 2B)F \left(\frac{1}{2}(c + dx) \Big| 2 \right)}{3a^2d} - \frac{B\sqrt{\cos(c + dx)} \sin(c + dx)}{a^2d(1 + \cos(c + dx))} \end{aligned}$$

Mathematica [C] time = 6.59, size = 921, normalized size = 7.61

$$iB \csc\left(\frac{c}{2}\right) \sec\left(\frac{c}{2}\right) \sec(c+dx)(A+B \sec(c+dx)) \left(\frac{2e^{2idx} {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{7}{4}; -e^{2idx}(\cos(c)+i \sin(c))^2\right) \sqrt{e^{-idx}(2(1+e^{2idx}) \cos(c)+2i(-1+e^{2idx}) \sin(c))}}{3id(1+e^{2idx}) \cos(c)-3d(-1+e^{2idx}) \sin(c)} \right)$$

$$2(B+A)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2), x]

[Out] ((I/2)*B*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x])*((2*E^((2*I)*d*x))*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x))*(Cos[c] + I*Sin[c])^2])*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x))*(Cos[c] + I*Sin[c])^2])*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c]))/(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^2 - (2*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]])*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]])]/(3*d*(B + A*Cos[c + d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^2 - (4*B*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]])*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]])]/(3*d*(B + A*Cos[c + d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^2 + (Cos[c/2 + (d*x)/2]^4*(A + B*Sec[c + d*x])*(-4*B*Csc[c])/d - (4*B*Sec[c/2]*Sec[c/2 + (d*x)/2]*Sin[(d*x)/2])/d - (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(-(A*Sin[(d*x)/2]) + B*Sin[(d*x)/2]))/(3*d) - (2*(-A + B)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(3*d)))/(Sqrt[Cos[c + d*x]]*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^2)

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{(B \sec(dx+c) + A) \sqrt{\cos(dx+c)}}{a^2 \cos(dx+c)^2 \sec(dx+c)^2 + 2a^2 \cos(dx+c)^2 \sec(dx+c) + a^2 \cos(dx+c)^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")

[Out] integral((B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(a^2*cos(d*x + c)^2*sec(d*x + c)^2 + 2*a^2*cos(d*x + c)^2*sec(d*x + c) + a^2*cos(d*x + c)^2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(a \sec(dx + c) + a)^2 \cos(dx + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^2*cos(d*x + c)^(3/2)), x)

maple [B] time = 4.75, size = 350, normalized size = 2.89

$$\sqrt{\left(2 \cos^2\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right) \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \left(2A \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{-2 \cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)} + 1 \operatorname{EllipticF}\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x)

[Out] $-1/6*((2*\cos(1/2*d*x+1/2*c)^2-1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*A*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}*\operatorname{EllipticF}(\cos(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(1/2*d*x+1/2*c)^3-12*B*\cos(1/2*d*x+1/2*c)^6+4*B*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}*\operatorname{EllipticF}(\cos(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(1/2*d*x+1/2*c)^3-6*B*\cos(1/2*d*x+1/2*c)^3*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}*\operatorname{EllipticE}(\cos(1/2*d*x+1/2*c), 2^{(1/2)})+2*A*\cos(1/2*d*x+1/2*c)^4+16*B*\cos(1/2*d*x+1/2*c)^4-3*A*\cos(1/2*d*x+1/2*c)^2-3*B*\cos(1/2*d*x+1/2*c)^2+A-B)/a^2/\cos(1/2*d*x+1/2*c)^3/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\cos(c+dx)^{3/2} \left(a + \frac{a}{\cos(c+dx)}\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^2),x)

[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(3/2)/(a+a*sec(d*x+c))**2,x)

[Out] Timed out

$$3.507 \quad \int \frac{A+B \sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a \sec(c+dx))^2} dx$$

Optimal. Leaf size=164

$$\frac{(2A-5B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} + \frac{(A-4B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} - \frac{(A-4B)\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}} + \frac{(2A-5B)\sin(c+dx)}{3a^2d\sqrt{\cos(c+dx)}(\cos(c+dx))}$$

[Out] (A-4*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d+1/3*(2*A-5*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d-(A-4*B)*sin(d*x+c)/a^2/d/cos(d*x+c)^(1/2)+1/3*(2*A-5*B)*sin(d*x+c)/a^2/d/(1+cos(d*x+c))/cos(d*x+c)^(1/2)+1/3*(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^2/cos(d*x+c)^(1/2)

Rubi [A] time = 0.40, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2954, 2978, 2748, 2636, 2639, 2641}

$$\frac{(2A-5B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} + \frac{(A-4B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} - \frac{(A-4B)\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}} + \frac{(2A-5B)\sin(c+dx)}{3a^2d\sqrt{\cos(c+dx)}(\cos(c+dx))}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^2),x]

[Out] ((A - 4*B)*EllipticE[(c + d*x)/2, 2])/(a^2*d) + ((2*A - 5*B)*EllipticF[(c + d*x)/2, 2])/(3*a^2*d) - ((A - 4*B)*Sin[c + d*x])/(a^2*d*Sqrt[Cos[c + d*x]]) + ((2*A - 5*B)*Sin[c + d*x])/(3*a^2*d*Sqrt[Cos[c + d*x]])*(1 + Cos[c + d*x]) + ((A - B)*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]])*(a + a*Cos[c + d*x])^2

Rule 2636

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), Int[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c -
  Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)])*(b_.)^(m_.)*(csc[(e_.) + (f_.)*(x_)])*(d_.) + (c_.)^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] := Dist[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 2978

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Simp[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{A + B \sec(c + dx)}{\cos^{\frac{5}{2}}(c + dx)(a + a \sec(c + dx))^2} dx &= \int \frac{B + A \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(a + a \cos(c + dx))^2} dx \\
&= \frac{(A - B) \sin(c + dx)}{3d \sqrt{\cos(c + dx)} (a + a \cos(c + dx))^2} + \int \frac{-\frac{1}{2}a(A-7B) + \frac{3}{2}a(A-B) \cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a \cos(c+dx))} dx \\
&= \frac{(2A - 5B) \sin(c + dx)}{3a^2 d \sqrt{\cos(c + dx)} (1 + \cos(c + dx))} + \frac{(A - B) \sin(c + dx)}{3d \sqrt{\cos(c + dx)} (a + a \cos(c + dx))} \\
&= \frac{(2A - 5B) \sin(c + dx)}{3a^2 d \sqrt{\cos(c + dx)} (1 + \cos(c + dx))} + \frac{(A - B) \sin(c + dx)}{3d \sqrt{\cos(c + dx)} (a + a \cos(c + dx))} \\
&= \frac{(2A - 5B) F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3a^2 d} - \frac{(A - 4B) \sin(c + dx)}{a^2 d \sqrt{\cos(c + dx)}} + \frac{(2A - 5B) \sin(c + dx)}{3a^2 d \sqrt{\cos(c + dx)}} \\
&= \frac{(A - 4B) E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{a^2 d} + \frac{(2A - 5B) F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3a^2 d} - \frac{(A - 4B) \sin(c + dx)}{a^2 d \sqrt{\cos(c + dx)}}
\end{aligned}$$

Mathematica [C] time = 6.90, size = 1351, normalized size = 8.24

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^2), x]

[Out] ((I/2)*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x])*((2*E^((2*I)*d*x))*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x))*(Cos[c] + I*Sin[c])^2])*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x))*(Cos[c] + I*Sin[c])^2])*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c]))/((B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^2) - ((2*I)*B*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x])*((2*E^((2*I)*d*x))*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x))*(Cos[c] + I*Sin[c])^2])

]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]]/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x))*(Cos[c] + I*Sin[c])^2])*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]]/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c]))/(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^2 - (4*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]])*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]])])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]])]/(3*d*(B + A*Cos[c + d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^2) + (10*B*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]])*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]])])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]])]/(3*d*(B + A*Cos[c + d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^2) + (Cos[c/2 + (d*x)/2]^4*(A + B*Sec[c + d*x])*((2*(2*B - A*Cos[c] + 2*B*Cos[c])*Csc[c/2]*Sec[c/2]*Sec[c])/d + (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(-A*Sin[(d*x)/2]) + B*Sin[(d*x)/2]))/(3*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(-A*Sin[(d*x)/2]) + 2*B*Sin[(d*x)/2])/d + (8*B*Sec[c]*Sec[c + d*x]*Sin[d*x])/d + (2*(-A + B)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(3*d)))/(Sqrt[Cos[c + d*x]]*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^2)

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B \sec(dx + c) + A)\sqrt{\cos(dx + c)}}{a^2 \cos(dx + c)^3 \sec(dx + c)^2 + 2a^2 \cos(dx + c)^3 \sec(dx + c) + a^2 \cos(dx + c)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")

[Out] integral((B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(a^2*cos(d*x + c)^3*sec(d*x + c)^2 + 2*a^2*cos(d*x + c)^3*sec(d*x + c) + a^2*cos(d*x + c)^3), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(a \sec(dx + c) + a)^2 \cos(dx + c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^2*cos(d*x + c)^(5/2)), x)

maple [B] time = 5.76, size = 492, normalized size = 3.00

$$2\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{2\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1} \sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)} \left(2A \operatorname{EllipticF}\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) + \sqrt{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x)

[Out] 1/6*(2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-5*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+12*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-5*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+12*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*c)-12*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(A-4*B)*sin(1/2*d*x+1/2*c)^6+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(10*A-43*B)*sin(1/2*d*x+1/2*c)^4-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(7*A-37*B)*sin(1/2*d*x+1/2*c)^2/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\cos(c+dx)^{5/2} \left(a + \frac{a}{\cos(c+dx)}\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))^2), x)
```

```
[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))^2), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(5/2)/(a+a*sec(d*x+c))**2, x)
```

```
[Out] Timed out
```


$$3.508 \quad \int \frac{A+B \sec(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+a \sec(c+dx))^2} dx$$

Optimal. Leaf size=197

$$\frac{5(A-2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} - \frac{(4A-7B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{(4A-7B)\sin(c+dx)}{3a^2d \cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)+1)} - \frac{5(A-2B)\sin(c+dx)}{3a^2d \cos^{\frac{3}{2}}(c+dx)}$$

[Out] $-(4A-7B)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/a^2/d-5/3*(A-2*B)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/a^2/d-5/3*(A-2*B)*\sin(d*x+c)/a^2/d/\cos(d*x+c)^{(3/2)}+1/3*(4*A-7*B)*\sin(d*x+c)/a^2/d/\cos(d*x+c)^{(3/2)}/(1+\cos(d*x+c))+1/3*(A-B)*\sin(d*x+c)/d/\cos(d*x+c)^{(3/2)}/(a+a*\cos(d*x+c))^2+(4*A-7*B)*\sin(d*x+c)/a^2/d/\cos(d*x+c)^{(1/2)}$

Rubi [A] time = 0.43, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2954, 2978, 2748, 2636, 2641, 2639}

$$\frac{5(A-2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} - \frac{(4A-7B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{(4A-7B)\sin(c+dx)}{3a^2d \cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)+1)} - \frac{5(A-2B)\sin(c+dx)}{3a^2d \cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(A + B*\text{Sec}[c + d*x])/(\text{Cos}[c + d*x]^{(7/2)}*(a + a*\text{Sec}[c + d*x])^2), x]$

[Out] $-(((4*A - 7*B)*\text{EllipticE}[(c + d*x)/2, 2])/(a^2*d)) - (5*(A - 2*B)*\text{EllipticF}[(c + d*x)/2, 2])/(3*a^2*d) - (5*(A - 2*B)*\text{Sin}[c + d*x])/(3*a^2*d*\text{Cos}[c + d*x]^{(3/2)}) + ((4*A - 7*B)*\text{Sin}[c + d*x])/(a^2*d*\text{Sqrt}[\text{Cos}[c + d*x]]) + ((4*A - 7*B)*\text{Sin}[c + d*x])/(3*a^2*d*\text{Cos}[c + d*x]^{(3/2)}*(1 + \text{Cos}[c + d*x])) + ((A - B)*\text{Sin}[c + d*x])/(3*d*\text{Cos}[c + d*x]^{(3/2)}*(a + a*\text{Cos}[c + d*x])^2)$

Rule 2636

$\text{Int}[(b_*)*\sin[(c_*) + (d_*)*(x_)]^{(n_)}, x_Symbol] \rightarrow \text{Simp}[(\text{Cos}[c + d*x]*(b*\text{Sin}[c + d*x])^{(n+1)})/(b*d*(n+1)), x] + \text{Dist}[(n+2)/(b^2*(n+1)), \text{Int}[(b*\text{Sin}[c + d*x])^{(n+2)}, x], x] /; \text{FreeQ}\{b, c, d\}, x] \&\& \text{LtQ}[n, -1] \&\& \text{IntegerQ}[2*n]$

Rule 2639

$\text{Int}[\text{Sqrt}[\sin[(c_*) + (d_*)*(x_)]], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticE}[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c -
  Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)])*(b_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_.))^(n_)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_), x_Symbol] := Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
  a*d, 0] && !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 2978

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
  b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&& !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{A + B \sec(c + dx)}{\cos^{\frac{7}{2}}(c + dx)(a + a \sec(c + dx))^2} dx &= \int \frac{B + A \cos(c + dx)}{\cos^{\frac{5}{2}}(c + dx)(a + a \cos(c + dx))^2} dx \\
&= \frac{(A - B) \sin(c + dx)}{3d \cos^{\frac{3}{2}}(c + dx)(a + a \cos(c + dx))^2} + \int \frac{-\frac{3}{2}a(A-3B) + \frac{5}{2}a(A-B) \cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a \cos(c+dx))} dx \\
&= \frac{(4A - 7B) \sin(c + dx)}{3a^2d \cos^{\frac{3}{2}}(c + dx)(1 + \cos(c + dx))} + \frac{(A - B) \sin(c + dx)}{3d \cos^{\frac{3}{2}}(c + dx)(a + a \cos(c + dx))} \\
&= \frac{(4A - 7B) \sin(c + dx)}{3a^2d \cos^{\frac{3}{2}}(c + dx)(1 + \cos(c + dx))} + \frac{(A - B) \sin(c + dx)}{3d \cos^{\frac{3}{2}}(c + dx)(a + a \cos(c + dx))} \\
&= -\frac{5(A - 2B) \sin(c + dx)}{3a^2d \cos^{\frac{3}{2}}(c + dx)} + \frac{(4A - 7B) \sin(c + dx)}{a^2d \sqrt{\cos(c + dx)}} + \frac{(4A - 7B) \sin(c + dx)}{3a^2d \cos^{\frac{3}{2}}(c + dx)} \\
&= -\frac{(4A - 7B)E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{a^2d} - \frac{5(A - 2B)F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3a^2d} - \frac{5(A - 2B) \sin(c + dx)}{3a^2d \cos^{\frac{3}{2}}(c + dx)}
\end{aligned}$$

Mathematica [C] time = 7.50, size = 1392, normalized size = 7.07

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^2), x]

[Out] ((-2*I)*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x])*((2*E^((2*I)*d*x))*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x))*(Cos[c] + I*Sin[c])^2])*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x))*(Cos[c] + I*Sin[c])^2])*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c]))/(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^2 + (((7*I)/2)*B*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x])*((2*E^((2

*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]]/(3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c] - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c]))/(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^2 + (10*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]])*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]])]/(3*d*(B + A*Cos[c + d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^2 - (20*B*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]])*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]])]/(3*d*(B + A*Cos[c + d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^2 + (Cos[c/2 + (d*x)/2]^4*(A + B*Sec[c + d*x])*((-2*(-2*A + 4*B - 2*A*Cos[c] + 3*B*Cos[c]))*Csc[c/2]*Sec[c/2]*Sec[c])/d - (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(-(A*Sin[(d*x)/2]) + B*Sin[(d*x)/2]))/(3*d) - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(-2*A*Sin[(d*x)/2] + 3*B*Sin[(d*x)/2]))/d + (8*B*Sec[c]*Sec[c + d*x]^2*Sin[d*x])/d + (8*Sec[c]*Sec[c + d*x]*(B*Sin[c] + 3*A*Sin[d*x] - 6*B*Sin[d*x]))/d - (2*(-A + B)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/d))/Sqrt[Cos[c + d*x]]*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^2)

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{(B \sec(dx + c) + A) \sqrt{\cos(dx + c)}}{a^2 \cos(dx + c)^4 \sec(dx + c)^2 + 2a^2 \cos(dx + c)^4 \sec(dx + c) + a^2 \cos(dx + c)^4}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")

[Out] integral((B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(a^2*cos(d*x + c)^4*sec(d*x + c)^2 + 2*a^2*cos(d*x + c)^4*sec(d*x + c) + a^2*cos(d*x + c)^4), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(a \sec(dx + c) + a)^2 \cos(dx + c)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^2*cos(d*x + c)^(7/2)), x)

maple [B] time = 15.14, size = 750, normalized size = 3.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^2,x)

[Out]
$$-1/2*(-(-2*\cos(1/2*d*x+1/2*c)^2+1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/a^2*(4*B*(-1/6*\cos(1/2*d*x+1/2*c)*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/(-1/2+\cos(1/2*d*x+1/2*c)^2)^2+1/3*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)}))+1/3*(-A+B)*(2*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*(2*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)}))-3*EllipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)}))*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^2-2*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*(2*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)}))-3*EllipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)}))*\cos(1/2*d*x+1/2*c)-12*\sin(1/2*d*x+1/2*c)^6+20*\sin(1/2*d*x+1/2*c)^4-7*\sin(1/2*d*x+1/2*c)^2)/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)/(-1+\sin(1/2*d*x+1/2*c)^2)+(4*A-8*B)*(-(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)}))+2*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^2)/\sin(1/2*d*x+1/2*c)^2/(2*\sin(1/2*d*x+1/2*c)^2-1)+(-2*A+4*B)*(\cos(1/2*d*x+1/2*c)*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*(EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)})-EllipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)}))-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)/\cos(1/2*d*x+1/2*c)/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d$$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\cos(c+dx)^{7/2} \left(a + \frac{a}{\cos(c+dx)}\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))^2), x)

[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))^2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(7/2)/(a+a*sec(d*x+c))**2, x)

[Out] Timed out

$$3.509 \quad \int \frac{\cos^3(c+dx)(A+B \sec(c+dx))}{(a+a \sec(c+dx))^3} dx$$

Optimal. Leaf size=221

$$\frac{(33A - 13B)F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{6a^3d} - \frac{7(17A - 7B)E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{10a^3d} - \frac{7(17A - 7B) \sin(c + dx) \cos^{\frac{3}{2}}(c + dx)}{30d(a^3 \cos(c + dx) + a^3)} + \frac{(33A - 13B)}{a^3d}$$

[Out] $-7/10*(17*A-7*B)*(cos(1/2*d*x+1/2*c)^2)^{(1/2)}/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^{(1/2)})/a^3/d+1/6*(33*A-13*B)*(cos(1/2*d*x+1/2*c)^2)^{(1/2)}/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^{(1/2)})/a^3/d-1/5*(A-B)*cos(d*x+c)^{(7/2)}*sin(d*x+c)/d/(a+a*cos(d*x+c))^3-1/3*(2*A-B)*cos(d*x+c)^{(5/2)}*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^2-7/30*(17*A-7*B)*cos(d*x+c)^{(3/2)}*sin(d*x+c)/d/(a^3+a^3*cos(d*x+c))+1/6*(33*A-13*B)*sin(d*x+c)*cos(d*x+c)^{(1/2)}/a^3/d$

Rubi [A] time = 0.58, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2954, 2977, 2748, 2639, 2635, 2641}

$$\frac{(33A - 13B)F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{6a^3d} - \frac{7(17A - 7B)E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{10a^3d} - \frac{7(17A - 7B) \sin(c + dx) \cos^{\frac{3}{2}}(c + dx)}{30d(a^3 \cos(c + dx) + a^3)} + \frac{(33A - 13B)}{a^3d}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^3,x]

[Out] $(-7*(17*A - 7*B)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + ((33*A - 13*B)*EllipticF[(c + d*x)/2, 2])/(6*a^3*d) + ((33*A - 13*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(6*a^3*d) - ((A - B)*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(5*d*(a + a*cos[c + d*x])^3) - ((2*A - B)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(3*a*d*(a + a*cos[c + d*x])^2) - (7*(17*A - 7*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(30*d*(a^3 + a^3*cos[c + d*x]))$

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*cos[c + d*x])*(b*sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(m_))*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))^(n_)*((g_.)*sin[(e_.) + (f_.)*(x_)]^(p_.), x_Symbol] := Dist[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(m_))*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]*(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] := Simp[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m + b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^3} dx &= \int \frac{\cos^{\frac{7}{2}}(c+dx)(B+A\cos(c+dx))}{(a+a\cos(c+dx))^3} dx \\
&= -\frac{(A-B)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \int \frac{\cos^{\frac{5}{2}}(c+dx)\left(-\frac{7}{2}a(A-B)+\frac{1}{2}a(13A-3B)\cos(c+dx)\right)}{(a+a\cos(c+dx))^2} dx \\
&= -\frac{(A-B)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(2A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3ad(a+a\cos(c+dx))^2} \\
&= -\frac{(A-B)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(2A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3ad(a+a\cos(c+dx))^2} \\
&= -\frac{(A-B)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(2A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3ad(a+a\cos(c+dx))^2} \\
&= -\frac{7(17A-7B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{(33A-13B)\sqrt{\cos(c+dx)}\sin(c+dx)}{6a^3d} \\
&= -\frac{7(17A-7B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{(33A-13B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d} + \frac{(33A-13B)\sqrt{\cos(c+dx)}\sin(c+dx)}{6a^3d}
\end{aligned}$$

Mathematica [C] time = 7.08, size = 1448, normalized size = 6.55

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^3, x]

[Out] (((-119*I)/10)*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x])*((2*E^((2*I)*d*x))*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x))*(Cos[c] + I*Sin[c])^2])*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x))*(Cos[c] + I*Sin[c])^2])*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])

$$d*x)) * \sin[c])) / ((B + A \cos[c + d*x]) * (a + a \sec[c + d*x])^3) + (((49I)/10) * B \cos[c/2 + (d*x)/2]^6 * \csc[c/2] * \sec[c/2] * \sec[c + d*x]^2 * (A + B \sec[c + d*x]) * ((2 * E^{(2I)d*x}) * \text{Hypergeometric2F1}[1/2, 3/4, 7/4, -(E^{(2I)d*x}) * (\cos[c] + I \sin[c])^2]) * \sqrt{(2 * (1 + E^{(2I)d*x}) * \cos[c] + (2I) * (-1 + E^{(2I)d*x})) * \sin[c])} / E^{I d*x}] * \sqrt{1 + E^{(2I)d*x} * \cos[2*c] + I * E^{(2I)d*x} * \sin[2*c]}) / ((3I) * d * (1 + E^{(2I)d*x}) * \cos[c] - 3 * d * (-1 + E^{(2I)d*x}) * \sin[c]) - (2 * \text{Hypergeometric2F1}[-1/4, 1/2, 3/4, -(E^{(2I)d*x}) * (\cos[c] + I \sin[c])^2]) * \sqrt{(2 * (1 + E^{(2I)d*x}) * \cos[c] + (2I) * (-1 + E^{(2I)d*x})) * \sin[c])} / E^{I d*x}] * \sqrt{1 + E^{(2I)d*x} * \cos[2*c] + I * E^{(2I)d*x} * \sin[2*c]}) / ((-I) * d * (1 + E^{(2I)d*x}) * \cos[c] + d * (-1 + E^{(2I)d*x}) * \sin[c])) / ((B + A \cos[c + d*x]) * (a + a \sec[c + d*x])^3) - (22 * A * \cos[c/2 + (d*x)/2]^6 * \csc[c/2] * \text{HypergeometricPFQ}[\{1/4, 1/2\}, \{5/4\}, \sin[d*x - \text{ArcTan}[\text{Cot}[c]]]^2] * \sec[c/2] * \sec[c + d*x]^2 * (A + B \sec[c + d*x]) * \sec[d*x - \text{ArcTan}[\text{Cot}[c]]] * \text{Sqrt}[1 - \sin[d*x - \text{ArcTan}[\text{Cot}[c]]]] * \sqrt{-(\text{Sqrt}[1 + \text{Cot}[c]^2] * \sin[c] * \sin[d*x - \text{ArcTan}[\text{Cot}[c]]])} * \sqrt{1 + \sin[d*x - \text{ArcTan}[\text{Cot}[c]]]}) / (d * (B + A \cos[c + d*x]) * \sqrt{1 + \text{Cot}[c]^2} * (a + a \sec[c + d*x])^3) + (26 * B * \cos[c/2 + (d*x)/2]^6 * \csc[c/2] * \text{HypergeometricPFQ}[\{1/4, 1/2\}, \{5/4\}, \sin[d*x - \text{ArcTan}[\text{Cot}[c]]]^2] * \sec[c/2] * \sec[c + d*x]^2 * (A + B \sec[c + d*x]) * \sec[d*x - \text{ArcTan}[\text{Cot}[c]]] * \text{Sqrt}[1 - \sin[d*x - \text{ArcTan}[\text{Cot}[c]]]] * \sqrt{-(\text{Sqrt}[1 + \text{Cot}[c]^2] * \sin[c] * \sin[d*x - \text{ArcTan}[\text{Cot}[c]]])} * \sqrt{1 + \sin[d*x - \text{ArcTan}[\text{Cot}[c]]]}) / (3 * d * (B + A \cos[c + d*x]) * \sqrt{1 + \text{Cot}[c]^2} * (a + a \sec[c + d*x])^3) + (\cos[c/2 + (d*x)/2]^6 * (A + B \sec[c + d*x]) * ((-4 * (-59 * A + 29 * B - 60 * A * \cos[c] + 20 * B * \cos[c]) * \csc[c]) / (5 * d) + (16 * A * \cos[d*x] * \sin[c]) / (3 * d) - (2 * \sec[c/2] * \sec[c/2 + (d*x)/2]^5 * (- (A * \sin[(d*x)/2]) + B * \sin[(d*x)/2])) / (5 * d) + (4 * \sec[c/2] * \sec[c/2 + (d*x)/2]^3 * (-19 * A * \sin[(d*x)/2] + 14 * B * \sin[(d*x)/2])) / (15 * d) - (4 * \sec[c/2] * \sec[c/2 + (d*x)/2] * (-59 * A * \sin[(d*x)/2] + 29 * B * \sin[(d*x)/2])) / (5 * d) + (16 * A * \cos[c] * \sin[d*x]) / (3 * d) + (4 * (-19 * A + 14 * B) * \sec[c/2 + (d*x)/2]^2 * \tan[c/2]) / (15 * d) - (2 * (-A + B) * \sec[c/2 + (d*x)/2]^4 * \tan[c/2]) / (5 * d))) / (\cos[c + d*x]^{(3/2)} * (B + A \cos[c + d*x]) * (a + a \sec[c + d*x])^3)$$

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{(B \cos(dx + c) \sec(dx + c) + A \cos(dx + c)) \sqrt{\cos(dx + c)}}{a^3 \sec(dx + c)^3 + 3 a^3 \sec(dx + c)^2 + 3 a^3 \sec(dx + c) + a^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="fricas")

[Out] integral((B*cos(d*x + c)*sec(d*x + c) + A*cos(d*x + c))*sqrt(cos(d*x + c))/(a^3*sec(d*x + c)^3 + 3*a^3*sec(d*x + c)^2 + 3*a^3*sec(d*x + c) + a^3), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \cos(dx + c)^{\frac{3}{2}}}{(a \sec(dx + c) + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(3/2)/(a*sec(d*x + c) + a)^3, x)

maple [A] time = 5.70, size = 465, normalized size = 2.10

$$\sqrt{\left(2 \left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right) \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) \left(160A \left(\cos^{10}\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 468A \left(\cos^8\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 330A \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x)

[Out] -1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(160*A*cos(1/2*d*x+1/2*c)^10+468*A*cos(1/2*d*x+1/2*c)^8+330*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+714*A*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-348*B*cos(1/2*d*x+1/2*c)^8-130*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5-294*B*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1058*A*cos(1/2*d*x+1/2*c)^6+578*B*cos(1/2*d*x+1/2*c)^6+474*A*cos(1/2*d*x+1/2*c)^4-264*B*cos(1/2*d*x+1/2*c)^4-47*A*cos(1/2*d*x+1/2*c)^2+37*B*cos(1/2*d*x+1/2*c)^2+3*A-3*B)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \cos(dx + c)^{\frac{3}{2}}}{(a \sec(dx + c) + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(3/2)/(a*sec(d*x + c) + a)^3, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c + dx)^{3/2} \left(A + \frac{B}{\cos(c+dx)} \right)}{\left(a + \frac{a}{\cos(c+dx)} \right)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)^(3/2)*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^3,x)

[Out] int((cos(c + d*x)^(3/2)*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^3, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**3,x)

[Out] Timed out

$$3.510 \quad \int \frac{\sqrt{\cos(c+dx)} (A+B \sec(c+dx))}{(a+a \sec(c+dx))^3} dx$$

Optimal. Leaf size=188

$$-\frac{(13A-3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d} + \frac{(49A-9B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} - \frac{(13A-3B)\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a^3\cos(c+dx)+a^3)} - \frac{(A-B)\sin(c+dx)}{5d(a^3\cos(c+dx)+a^3)}$$

[Out] 1/10*(49*A-9*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d-1/6*(13*A-3*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d-1/5*(A-B)*cos(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^3-1/15*(8*A-3*B)*cos(d*x+c)^(3/2)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^2-1/6*(13*A-3*B)*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a^3+a^3*cos(d*x+c))

Rubi [A] time = 0.55, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {2954, 2977, 2748, 2641, 2639}

$$-\frac{(13A-3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d} + \frac{(49A-9B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} - \frac{(13A-3B)\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a^3\cos(c+dx)+a^3)} - \frac{(A-B)\sin(c+dx)}{5d(a^3\cos(c+dx)+a^3)}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^3,x]

[Out] ((49*A - 9*B)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) - ((13*A - 3*B)*EllipticF[(c + d*x)/2, 2])/(6*a^3*d) - ((A - B)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(5*d*(a + a*cos[c + d*x])^3) - ((8*A - 3*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(15*a*d*(a + a*cos[c + d*x])^2) - ((13*A - 3*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(6*d*(a^3 + a^3*cos[c + d*x]))

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)])*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)])*(d_.) + (c_.))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dist[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 2977

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m + b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\cos(c+dx)}(A+B\sec(c+dx))}{(a+a\sec(c+dx))^3} dx &= \int \frac{\cos^{\frac{5}{2}}(c+dx)(B+A\cos(c+dx))}{(a+a\cos(c+dx))^3} dx \\
&= -\frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \int \frac{\cos^{\frac{3}{2}}(c+dx)\left(-\frac{5}{2}a(A-B)+\frac{1}{2}a(11A-B)\cos(c+dx)\right)}{(a+a\cos(c+dx))^2} dx \\
&= -\frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(8A-3B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} \\
&= -\frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(8A-3B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} \\
&= -\frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(8A-3B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} \\
&= \frac{(49A-9B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} - \frac{(13A-3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d} - \frac{(A-B)}{5d}
\end{aligned}$$

Mathematica [C] time = 6.93, size = 1415, normalized size = 7.53

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^3, x]

[Out] (((49*I)/10)*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x])*((2*E^((2*I)*d*x))*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x))*(Cos[c] + I*Sin[c])^2])*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x))*(Cos[c] + I*Sin[c])^2])*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c]))/(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^3 - (((9*I)/10)*B*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x])*((2*E^((2*I)*d*x))*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x))*(Cos[c]

+ I*Sin[c])^2))*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]]/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x))*(Cos[c] + I*Sin[c])^2])*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]]/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c]))/((B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^3) + (26*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]])*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]])])*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]])]/(3*d*(B + A*Cos[c + d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^3) - (2*B*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]])*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]])])*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]])]/(d*(B + A*Cos[c + d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^3) + (Cos[c/2 + (d*x)/2]^6*(A + B*Sec[c + d*x])*((-4*(29*A - 9*B + 20*A*Cos[c])*Csc[c])/(5*d) + (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^5*(-(A*Sin[(d*x)/2]) + B*Sin[(d*x)/2]))/(5*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(-29*A*Sin[(d*x)/2] + 9*B*Sin[(d*x)/2]))/(5*d) - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(-14*A*Sin[(d*x)/2] + 9*B*Sin[(d*x)/2]))/(15*d) - (4*(-14*A + 9*B)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(15*d) + (2*(-A + B)*Sec[c/2 + (d*x)/2]^4*Tan[c/2])/(5*d)))/(Cos[c + d*x]^(3/2)*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^3)

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{(B \sec(dx + c) + A) \sqrt{\cos(dx + c)}}{a^3 \sec(dx + c)^3 + 3a^3 \sec(dx + c)^2 + 3a^3 \sec(dx + c) + a^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")

[Out] integral((B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(a^3*sec(d*x + c)^3 + 3*a^3*sec(d*x + c)^2 + 3*a^3*sec(d*x + c) + a^3), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sqrt{\cos(dx + c)}}{(a \sec(dx + c) + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(a*sec(d*x + c) + a)^3, x)

maple [B] time = 5.37, size = 451, normalized size = 2.40

$$\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)\left(348A\left(\cos^8\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 130A\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x)

[Out] 1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(348*A*cos(1/2*d*x+1/2*c)^8+130*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+294*A*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-108*B*cos(1/2*d*x+1/2*c)^8-30*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5-54*B*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-578*A*cos(1/2*d*x+1/2*c)^6+198*B*cos(1/2*d*x+1/2*c)^6+264*A*cos(1/2*d*x+1/2*c)^4-114*B*cos(1/2*d*x+1/2*c)^4-37*A*cos(1/2*d*x+1/2*c)^2+27*B*cos(1/2*d*x+1/2*c)^2+3*A-3*B)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)\sqrt{\cos(dx + c)}}{(a \sec(dx + c) + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(a*sec(d*x + c) + a)^3, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\cos(c+dx)} \left(A + \frac{B}{\cos(c+dx)} \right)}{\left(a + \frac{a}{\cos(c+dx)} \right)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)^(1/2)*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^3,x)

[Out] int((cos(c + d*x)^(1/2)*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^3, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A\sqrt{\cos(c+dx)}}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1} dx + \int \frac{B\sqrt{\cos(c+dx)} \sec(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1} dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)**(1/2)/(a+a*sec(d*x+c))**3,x)

[Out] (Integral(A*sqrt(cos(c + d*x))/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d*x) + 1), x) + Integral(B*sqrt(cos(c + d*x))*sec(c + d*x)/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d*x) + 1), x))/a**3

$$3.511 \quad \int \frac{A+B \sec(c+dx)}{\sqrt{\cos(c+dx)} (a+a \sec(c+dx))^3} dx$$

Optimal. Leaf size=182

$$\frac{(3A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d} - \frac{(9A+B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{(9A+B)\sin(c+dx)\sqrt{\cos(c+dx)}}{10d(a^3\cos(c+dx)+a^3)} - \frac{(A-B)\sin(c+dx)}{5d(a\cos(c+dx)+a)}$$

[Out] $-1/10*(9*A+B)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c),2^{(1/2)})/a^3/d+1/6*(3*A+B)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c),2^{(1/2)})/a^3/d-1/5*(A-B)*\cos(d*x+c)^{(3/2)}*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^3-1/15*(6*A-B)*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/a/d/(a+a*\cos(d*x+c))^2+1/10*(9*A+B)*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/d/(a^3+a^3*\cos(d*x+c))$

Rubi [A] time = 0.54, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2954, 2977, 2978, 2748, 2641, 2639}

$$\frac{(3A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d} - \frac{(9A+B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{(9A+B)\sin(c+dx)\sqrt{\cos(c+dx)}}{10d(a^3\cos(c+dx)+a^3)} - \frac{(A-B)\sin(c+dx)}{5d(a\cos(c+dx)+a)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(A + B*\text{Sec}[c + d*x])]/(\text{Sqrt}[\text{Cos}[c + d*x]]*(a + a*\text{Sec}[c + d*x])^3),x]$

[Out] $-((9*A + B)*\text{EllipticE}[(c + d*x)/2, 2])/(10*a^3*d) + ((3*A + B)*\text{EllipticF}[(c + d*x)/2, 2])/(6*a^3*d) - ((A - B)*\text{Cos}[c + d*x]^{(3/2)}*\text{Sin}[c + d*x])/(5*d*(a + a*\text{Cos}[c + d*x])^3) - ((6*A - B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sin}[c + d*x])/(15*a*d*(a + a*\text{Cos}[c + d*x])^2) + ((9*A + B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sin}[c + d*x])/(10*d*(a^3 + a^3*\text{Cos}[c + d*x]))$

Rule 2639

$\text{Int}[\text{Sqrt}[\sin[(c_.) + (d_.)*(x_.)]], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticE}[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}[\{c, d\}, x]$

Rule 2641

$\text{Int}[1/\text{Sqrt}[\sin[(c_.) + (d_.)*(x_.)]], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticF}[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}[\{c, d\}, x]$

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])], x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)])*(b_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))^(n_)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_), x_Symbol] :> Dist[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m + b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{A + B \sec(c + dx)}{\sqrt{\cos(c + dx)} (a + a \sec(c + dx))^3} dx &= \int \frac{\cos^{\frac{3}{2}}(c + dx)(B + A \cos(c + dx))}{(a + a \cos(c + dx))^3} dx \\
&= -\frac{(A - B) \cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{5d(a + a \cos(c + dx))^3} + \int \frac{\sqrt{\cos(c+dx)} \left(-\frac{3}{2}a(A-B) + \frac{1}{2}a(9A+B) \cos(c+dx)\right)}{(a+a \cos(c+dx))^2} dx \\
&= -\frac{(A - B) \cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{5d(a + a \cos(c + dx))^3} - \frac{(6A - B)\sqrt{\cos(c + dx)} \sin(c + dx)}{15ad(a + a \cos(c + dx))^2} \\
&= -\frac{(A - B) \cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{5d(a + a \cos(c + dx))^3} - \frac{(6A - B)\sqrt{\cos(c + dx)} \sin(c + dx)}{15ad(a + a \cos(c + dx))^2} \\
&= -\frac{(A - B) \cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{5d(a + a \cos(c + dx))^3} - \frac{(6A - B)\sqrt{\cos(c + dx)} \sin(c + dx)}{15ad(a + a \cos(c + dx))^2} \\
&= -\frac{(9A + B)E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{10a^3d} + \frac{(3A + B)F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{6a^3d} - \frac{(A - B) \cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{5d(a + a \cos(c + dx))^3}
\end{aligned}$$

Mathematica [C] time = 6.96, size = 1407, normalized size = 7.73

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[(A + B*Sec[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^3), x]

[Out] (((-9*I)/10)*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x])*((2*E^((2*I)*d*x))*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x))*Cos[c] + I*Sin[c]]^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]]/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x))*Cos[c] + I*Sin[c]]^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]]/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c]))/(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^3 - ((I/10)*B*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x])*((2*E^((2*I)*d*x))*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x))*Cos[c] + I

```

*Sin[c]]^2))*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x)
)*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[
2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c
]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x))*(Cos[c] + I*Sin[c
])]^2))*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[
c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])
/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c]))/((B
+ A*cos[c + d*x])*(a + a*Sec[c + d*x])^3) - (2*A*cos[c/2 + (d*x)/2]^6*Csc[c
/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c
/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 -
Sin[d*x - ArcTan[Cot[c]]])*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTa
n[Cot[c]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]])]/(d*(B + A*cos[c + d*x])*S
qrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^3) - (2*B*cos[c/2 + (d*x)/2]^6*Csc[c
/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c
/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 -
Sin[d*x - ArcTan[Cot[c]]])*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTa
n[Cot[c]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]])]/(3*d*(B + A*cos[c + d*x])
*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^3) + (Cos[c/2 + (d*x)/2]^6*(A + B*
Sec[c + d*x])*((4*(9*A + B)*Csc[c])/(5*d) - (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^
5*(-(A*Sin[(d*x)/2] + B*Sin[(d*x)/2]))/(5*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)
/2]*(9*A*Sin[(d*x)/2] + B*Sin[(d*x)/2]))/(5*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)
/2]^3*(-9*A*Sin[(d*x)/2] + 4*B*Sin[(d*x)/2]))/(15*d) + (4*(-9*A + 4*B)*Sec
[c/2 + (d*x)/2]^2*Tan[c/2])/(15*d) - (2*(-A + B)*Sec[c/2 + (d*x)/2]^4*Tan[c
/2])/(5*d)))/(Cos[c + d*x]^(3/2)*(B + A*cos[c + d*x])*(a + a*Sec[c + d*x])^
3)

```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B \sec(dx + c) + A)\sqrt{\cos(dx + c)}}{a^3 \cos(dx + c) \sec(dx + c)^3 + 3a^3 \cos(dx + c) \sec(dx + c)^2 + 3a^3 \cos(dx + c) \sec(dx + c) + a^3 \cos(dx + c)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(a^3*cos(d*x + c)*sec(d*x
+ c)^3 + 3*a^3*cos(d*x + c)*sec(d*x + c)^2 + 3*a^3*cos(d*x + c)*sec(d*x + c
) + a^3*cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(a \sec(dx + c) + a)^3 \sqrt{\cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^3*sqrt(cos(d*x + c))), x)

maple [B] time = 5.61, size = 451, normalized size = 2.48

$$\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)\left(108A\left(\cos^8\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 30A\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3/cos(d*x+c)^(1/2),x)

[Out]
$$-1/60*((2*\cos(1/2*d*x+1/2*c)^2-1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(108*A*\cos(1/2*d*x+1/2*c)^8+30*A*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)})*\cos(1/2*d*x+1/2*c)^5+54*A*\cos(1/2*d*x+1/2*c)^5*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)})+12*B*\cos(1/2*d*x+1/2*c)^8+10*B*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)})*\cos(1/2*d*x+1/2*c)^5+6*B*\cos(1/2*d*x+1/2*c)^5*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)})-198*A*\cos(1/2*d*x+1/2*c)^6-2*B*\cos(1/2*d*x+1/2*c)^6+114*A*\cos(1/2*d*x+1/2*c)^4-24*B*\cos(1/2*d*x+1/2*c)^4-27*A*\cos(1/2*d*x+1/2*c)^2+17*B*\cos(1/2*d*x+1/2*c)^2+3*A-3*B)/a^3/\cos(1/2*d*x+1/2*c)^5/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d$$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\sqrt{\cos(c+dx)} \left(a + \frac{a}{\cos(c+dx)}\right)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^3), x)
```

```
[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^3), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))**3/cos(d*x+c)**(1/2), x)
```

```
[Out] Timed out
```


$$3.512 \quad \int \frac{A+B \sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a \sec(c+dx))^3} dx$$

Optimal. Leaf size=178

$$\frac{(A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d} - \frac{(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{10d(a^3\cos(c+dx)+a^3)} + \frac{(4A+B)\sin(c+dx)\sqrt{\cos(c+dx)}}{15ad(a\cos(c+dx))}$$

[Out] $-1/10*(A-B)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/a^3/d+1/6*(A+B)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/a^3/d-1/5*(A-B)*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/d/(a+a*\cos(d*x+c))^3+1/15*(4*A+B)*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/a/d/(a+a*\cos(d*x+c))^2+1/10*(A-B)*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/d/(a^3+a^3*\cos(d*x+c))$

Rubi [A] time = 0.52, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2954, 2977, 2978, 2748, 2641, 2639}

$$\frac{(A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d} - \frac{(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{10d(a^3\cos(c+dx)+a^3)} + \frac{(4A+B)\sin(c+dx)\sqrt{\cos(c+dx)}}{15ad(a\cos(c+dx))}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(A+B*\text{Sec}[c+d*x])/(\text{Cos}[c+d*x]^{(3/2)}*(a+a*\text{Sec}[c+d*x])^3),x]$

[Out] $-((A-B)*\text{EllipticE}[(c+d*x)/2, 2])/(10*a^3*d) + ((A+B)*\text{EllipticF}[(c+d*x)/2, 2])/(6*a^3*d) - ((A-B)*\text{Sqrt}[\text{Cos}[c+d*x]]*\text{Sin}[c+d*x])/(5*d*(a+a*\text{Cos}[c+d*x])^3) + ((4*A+B)*\text{Sqrt}[\text{Cos}[c+d*x]]*\text{Sin}[c+d*x])/(15*a*d*(a+a*\text{Cos}[c+d*x])^2) + ((A-B)*\text{Sqrt}[\text{Cos}[c+d*x]]*\text{Sin}[c+d*x])/(10*d*(a^3+a^3*\text{Cos}[c+d*x]))$

Rule 2639

$\text{Int}[\text{Sqrt}[\sin[(c_.) + (d_.)*(x_)]], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticE}[(1*(c - P/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 2641

$\text{Int}[1/\text{Sqrt}[\sin[(c_.) + (d_.)*(x_)]], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticF}[(1*(c - P/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])], x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)])*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)])*(d_.) + (c_.))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dist[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m + b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\frac{\sin[c]^2 \sqrt{(2(1 + E^{(2I)d*x}) \cos[c] + (2I)(-1 + E^{(2I)d*x})) \sin[c]) / E^{(I)d*x}} \sqrt{1 + E^{(2I)d*x} \cos[2c] + I E^{(2I)d*x} \sin[2c]}}{(3I)d(1 + E^{(2I)d*x} \cos[c] - 3d(-1 + E^{(2I)d*x}) \sin[c]) - (2 \text{Hypergeometric2F1}[-1/4, 1/2, 3/4, -(E^{(2I)d*x})(\cos[c] + I \sin[c])^2]) \sqrt{(2(1 + E^{(2I)d*x}) \cos[c] + (2I)(-1 + E^{(2I)d*x})) \sin[c]) / E^{(I)d*x}} \sqrt{1 + E^{(2I)d*x} \cos[2c] + I E^{(2I)d*x} \sin[2c]}}}{(-I)d(1 + E^{(2I)d*x} \cos[c] + d(-1 + E^{(2I)d*x}) \sin[c])} / ((B + A \cos[c + dx]) (a + a \sec[c + dx])^3 - (2A \cos[c/2 + (dx)/2]^6 \text{Csc}[c/2] \text{HypergeometricPFQ}[\{1/4, 1/2\}, \{5/4\}, \sin[dx - \text{ArcTan}[\text{Cot}[c]]]^2] \sec[c/2] \sec[c + dx]^2 (A + B \sec[c + dx]) \sec[dx - \text{ArcTan}[\text{Cot}[c]]] \sqrt{1 - \sin[dx - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{-(\sqrt{1 + \text{Cot}[c]^2} \sin[c] \sin[dx - \text{ArcTan}[\text{Cot}[c]])} \sqrt{1 + \sin[dx - \text{ArcTan}[\text{Cot}[c]]]})} / (3d(B + A \cos[c + dx]) \sqrt{1 + \text{Cot}[c]^2} (a + a \sec[c + dx])^3 - (2B \cos[c/2 + (dx)/2]^6 \text{Csc}[c/2] \text{HypergeometricPFQ}[\{1/4, 1/2\}, \{5/4\}, \sin[dx - \text{ArcTan}[\text{Cot}[c]]]^2] \sec[c/2] \sec[c + dx]^2 (A + B \sec[c + dx]) \sec[dx - \text{ArcTan}[\text{Cot}[c]]] \sqrt{1 - \sin[dx - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{-(\sqrt{1 + \text{Cot}[c]^2} \sin[c] \sin[dx - \text{ArcTan}[\text{Cot}[c]])} \sqrt{1 + \sin[dx - \text{ArcTan}[\text{Cot}[c]]]})} / (3d(B + A \cos[c + dx]) \sqrt{1 + \text{Cot}[c]^2} (a + a \sec[c + dx])^3 + (\cos[c/2 + (dx)/2]^6 (A + B \sec[c + dx]) ((-4(-A + B) \text{Csc}[c]) / (5d) - (4 \sec[c/2] \sec[c/2 + (dx)/2] (-A \sin[(dx)/2] + B \sin[(dx)/2])) / (5d) + (2 \sec[c/2] \sec[c/2 + (dx)/2]^5 (-A \sin[(dx)/2] + B \sin[(dx)/2])) / (5d) + (4 \sec[c/2] \sec[c/2 + (dx)/2]^3 (4A \sin[(dx)/2] + B \sin[(dx)/2])) / (15d) + (4(4A + B) \sec[c/2 + (dx)/2]^2 \tan[c/2]) / (15d) + (2(-A + B) \sec[c/2 + (dx)/2]^4 \tan[c/2]) / (5d))) / (\cos[c + dx]^{3/2} (B + A \cos[c + dx]) (a + a \sec[c + dx])^3)$$

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{(B \sec(dx + c) + A) \sqrt{\cos(dx + c)}}{a^3 \cos(dx + c)^2 \sec(dx + c)^3 + 3a^3 \cos(dx + c)^2 \sec(dx + c)^2 + 3a^3 \cos(dx + c)^2 \sec(dx + c) + a^3 \cos(dx + c)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(dx+c))/cos(dx+c)^(3/2)/(a+a*sec(dx+c))^3,x, algorithm="fricas")

[Out] integral((B*sec(dx + c) + A)*sqrt(cos(dx + c))/(a^3*cos(dx + c)^2*sec(dx + c)^3 + 3*a^3*cos(dx + c)^2*sec(dx + c)^2 + 3*a^3*cos(dx + c)^2*sec(dx + c) + a^3*cos(dx + c)^2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(a \sec(dx + c) + a)^3 \cos(dx + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^3*cos(d*x + c)^(3/2)), x)

maple [B] time = 5.32, size = 451, normalized size = 2.53

$$\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)\left(12A\left(\cos^8\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 10A\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}\right) + \dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x)

[Out]
$$-1/60*((2*\cos(1/2*d*x+1/2*c)^2-1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(12*A*\cos(1/2*d*x+1/2*c)^8+10*A*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)})*\cos(1/2*d*x+1/2*c)^5+6*A*\cos(1/2*d*x+1/2*c)^5*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)})-12*B*\cos(1/2*d*x+1/2*c)^8+10*B*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)})*\cos(1/2*d*x+1/2*c)^5-6*B*\cos(1/2*d*x+1/2*c)^5*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)})-2*A*\cos(1/2*d*x+1/2*c)^6+22*B*\cos(1/2*d*x+1/2*c)^6-24*A*\cos(1/2*d*x+1/2*c)^4-6*B*\cos(1/2*d*x+1/2*c)^4+17*A*\cos(1/2*d*x+1/2*c)^2-7*B*\cos(1/2*d*x+1/2*c)^2-3*A+3*B)/a^3/\cos(1/2*d*x+1/2*c)^5/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d$$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\cos(c+dx)^{3/2} \left(a + \frac{a}{\cos(c+dx)}\right)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^3), x)
```

```
[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^3), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(3/2)/(a+a*sec(d*x+c))**3, x)
```

```
[Out] Timed out
```

$$3.513 \quad \int \frac{A+B \sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a \sec(c+dx))^3} dx$$

Optimal. Leaf size=180

$$\frac{(A+3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d} + \frac{(A+9B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} - \frac{(A+9B)\sin(c+dx)\sqrt{\cos(c+dx)}}{10d(a^3\cos(c+dx)+a^3)} + \frac{(A-6B)\sin(c+dx)}{15ad(a\cos(c+dx)+a)}$$

[Out] 1/10*(A+9*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d+1/6*(A+3*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d+1/5*(A-B)*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^3+1/15*(A-6*B)*sin(d*x+c)*cos(d*x+c)^(1/2)/a/d/(a+a*cos(d*x+c))^2-1/10*(A+9*B)*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a^3+a^3*cos(d*x+c))

Rubi [A] time = 0.53, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {2954, 2978, 2748, 2641, 2639}

$$\frac{(A+3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d} + \frac{(A+9B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} - \frac{(A+9B)\sin(c+dx)\sqrt{\cos(c+dx)}}{10d(a^3\cos(c+dx)+a^3)} + \frac{(A-6B)\sin(c+dx)}{15ad(a\cos(c+dx)+a)}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^3),x]

[Out] ((A + 9*B)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + ((A + 3*B)*EllipticF[(c + d*x)/2, 2])/(6*a^3*d) + ((A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(5*d*(a + a*cos[c + d*x])^3) + ((A - 6*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(15*a*d*(a + a*cos[c + d*x])^2) - ((A + 9*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(10*d*(a^3 + a^3*cos[c + d*x]))

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)])*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dist[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 2978

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{A + B \sec(c + dx)}{\cos^{\frac{5}{2}}(c + dx)(a + a \sec(c + dx))^3} dx &= \int \frac{B + A \cos(c + dx)}{\sqrt{\cos(c + dx)} (a + a \cos(c + dx))^3} dx \\
&= \frac{(A - B)\sqrt{\cos(c + dx)} \sin(c + dx)}{5d(a + a \cos(c + dx))^3} + \frac{\int \frac{\frac{1}{2}a(A+9B) + \frac{3}{2}a(A-B) \cos(c+dx)}{\sqrt{\cos(c+dx)} (a+a \cos(c+dx))^2} dx}{5a^2} \\
&= \frac{(A - B)\sqrt{\cos(c + dx)} \sin(c + dx)}{5d(a + a \cos(c + dx))^3} + \frac{(A - 6B)\sqrt{\cos(c + dx)} \sin(c + dx)}{15ad(a + a \cos(c + dx))^2} \\
&= \frac{(A - B)\sqrt{\cos(c + dx)} \sin(c + dx)}{5d(a + a \cos(c + dx))^3} + \frac{(A - 6B)\sqrt{\cos(c + dx)} \sin(c + dx)}{15ad(a + a \cos(c + dx))^2} \\
&= \frac{(A - B)\sqrt{\cos(c + dx)} \sin(c + dx)}{5d(a + a \cos(c + dx))^3} + \frac{(A - 6B)\sqrt{\cos(c + dx)} \sin(c + dx)}{15ad(a + a \cos(c + dx))^2} \\
&= \frac{(A + 9B)E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{10a^3d} + \frac{(A + 3B)F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{6a^3d} + \frac{(A - B)\sqrt{\cos(c + dx)} \sin(c + dx)}{5d(a + a \cos(c + dx))^2}
\end{aligned}$$

Mathematica [C] time = 6.84, size = 1407, normalized size = 7.82

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^3),
x]
```

```
[Out] ((I/10)*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[
c + d*x])*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)
)*(Cos[c] + I*Sin[c])^2])*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 +
E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2
*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)
)*d*x))*Sin[c] - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x))*(Cos
[c] + I*Sin[c])^2])*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*
I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*
x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*S
in[c]))/(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^3 + (((9*I)/10)*B*Cos[
c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x])*((2*
E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x))*(Cos[c] + I*
Sin[c])^2])*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))
```

```

*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2
*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]
) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c]
)^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c
])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/
((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c]))/(B +
A*cos[c + d*x])*(a + a*Sec[c + d*x])^3) - (2*A*cos[c/2 + (d*x)/2]^6*Csc[c/
2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/
2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - S
in[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan
[Cot[c]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(B + A*cos[c + d*x])*
Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^3) - (2*B*cos[c/2 + (d*x)/2]^6*Csc[
c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[
c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 -
Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcT
an[Cot[c]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(B + A*cos[c + d*x])*
Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^3) + (Cos[c/2 + (d*x)/2]^6*(A + B*S
ec[c + d*x])*((-4*(A + 9*B)*Csc[c])/(5*d) - (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^
5*(-(A*Sin[(d*x)/2] + B*Sin[(d*x)/2]))/(5*d) - (4*Sec[c/2]*Sec[c/2 + (d*x)
/2]^3*(-(A*Sin[(d*x)/2] + 6*B*Sin[(d*x)/2]))/(15*d) - (4*Sec[c/2]*Sec[c/2
+ (d*x)/2]*(A*Sin[(d*x)/2] + 9*B*Sin[(d*x)/2]))/(5*d) - (4*(-A + 6*B)*Sec[c
/2 + (d*x)/2]^2*Tan[c/2])/(15*d) - (2*(-A + B)*Sec[c/2 + (d*x)/2]^4*Tan[c/2
])/(5*d)))/(Cos[c + d*x]^(3/2)*(B + A*cos[c + d*x])*(a + a*Sec[c + d*x])^3)

```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{(B \sec(dx + c) + A) \sqrt{\cos(dx + c)}}{a^3 \cos(dx + c)^3 \sec(dx + c)^3 + 3a^3 \cos(dx + c)^3 \sec(dx + c)^2 + 3a^3 \cos(dx + c)^3 \sec(dx + c) + a^3 \cos(dx + c)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorithm
="fricas")

```

```

[Out] integral((B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(a^3*cos(d*x + c)^3*sec(d*
x + c)^3 + 3*a^3*cos(d*x + c)^3*sec(d*x + c)^2 + 3*a^3*cos(d*x + c)^3*sec(d
*x + c) + a^3*cos(d*x + c)^3), x)

```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(a \sec(dx + c) + a)^3 \cos(dx + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^3*cos(d*x + c)^(5/2)), x)

maple [B] time = 5.57, size = 451, normalized size = 2.51

$$\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)\left(12A\left(\cos^8\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 10A\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 1}\right)} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x)

[Out] 1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*A*cos(1/2*d*x+1/2*c)^8-10*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+6*A*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+108*B*cos(1/2*d*x+1/2*c)^8-30*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+54*B*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-22*A*cos(1/2*d*x+1/2*c)^6-138*B*cos(1/2*d*x+1/2*c)^6+6*A*cos(1/2*d*x+1/2*c)^4+24*B*cos(1/2*d*x+1/2*c)^4+7*A*cos(1/2*d*x+1/2*c)^2+3*B*cos(1/2*d*x+1/2*c)^2-3*A+3*B)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\cos(c+dx)^{5/2} \left(a + \frac{a}{\cos(c+dx)}\right)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))^3),x)
```

```
[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))^3), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(5/2)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

$$3.514 \quad \int \frac{A+B \sec(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+a \sec(c+dx))^3} dx$$

Optimal. Leaf size=221

$$\frac{(3A-13B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d} + \frac{(9A-49B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} - \frac{(9A-49B)\sin(c+dx)}{10a^3d\sqrt{\cos(c+dx)}} + \frac{(3A-13B)\sin(c+dx)}{6d\sqrt{\cos(c+dx)}(a^3\cos(c+dx))}$$

[Out] 1/10*(9*A-49*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d+1/6*(3*A-13*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d-1/10*(9*A-49*B)*sin(d*x+c)/a^3/d/cos(d*x+c)^(1/2)+1/5*(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^3/cos(d*x+c)^(1/2)+1/15*(3*A-8*B)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^2/cos(d*x+c)^(1/2)+1/6*(3*A-13*B)*sin(d*x+c)/d/(a^3+a^3*cos(d*x+c))/cos(d*x+c)^(1/2)

Rubi [A] time = 0.58, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2954, 2978, 2748, 2636, 2639, 2641}

$$\frac{(3A-13B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d} + \frac{(9A-49B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} - \frac{(9A-49B)\sin(c+dx)}{10a^3d\sqrt{\cos(c+dx)}} + \frac{(3A-13B)\sin(c+dx)}{6d\sqrt{\cos(c+dx)}(a^3\cos(c+dx))}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^3),x]

[Out] ((9*A - 49*B)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + ((3*A - 13*B)*EllipticF[(c + d*x)/2, 2])/(6*a^3*d) - ((9*A - 49*B)*Sin[c + d*x])/(10*a^3*d*Sqrt[Cos[c + d*x]]) + ((A - B)*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]]*(a + a*cos[c + d*x])^3) + ((3*A - 8*B)*Sin[c + d*x])/(15*a*d*Sqrt[Cos[c + d*x]]*(a + a*cos[c + d*x])^2) + ((3*A - 13*B)*Sin[c + d*x])/(6*d*Sqrt[Cos[c + d*x]]*(a^3 + a^3*cos[c + d*x]))

Rule 2636

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(b*SIN[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), Int[(b*SIN[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_.)], x_Symbol] := Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_.)], x_Symbol] := Simp[(2*EllipticF[(1*(c -
Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_.)]^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x
_.)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_.)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*
(d_.) + (c_.))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_.)]^(p_.), x_Symbol] := Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
a*d, 0] && !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 2978

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^(m_.)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_.)]*(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]^(n_.), x_Symbol] := Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&& !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{A + B \sec(c + dx)}{\cos^{\frac{7}{2}}(c + dx)(a + a \sec(c + dx))^3} dx &= \int \frac{B + A \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(a + a \cos(c + dx))^3} dx \\
&= \frac{(A - B) \sin(c + dx)}{5d \sqrt{\cos(c + dx)} (a + a \cos(c + dx))^3} + \frac{\int \frac{-\frac{1}{2}a(A-11B) + \frac{5}{2}a(A-B) \cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a \cos(c+dx))^2} dx}{5a^2} \\
&= \frac{(A - B) \sin(c + dx)}{5d \sqrt{\cos(c + dx)} (a + a \cos(c + dx))^3} + \frac{(3A - 8B) \sin(c + dx)}{15ad \sqrt{\cos(c + dx)} (a + a \cos(c + dx))^3} \\
&= \frac{(A - B) \sin(c + dx)}{5d \sqrt{\cos(c + dx)} (a + a \cos(c + dx))^3} + \frac{(3A - 8B) \sin(c + dx)}{15ad \sqrt{\cos(c + dx)} (a + a \cos(c + dx))^3} \\
&= \frac{(A - B) \sin(c + dx)}{5d \sqrt{\cos(c + dx)} (a + a \cos(c + dx))^3} + \frac{(3A - 8B) \sin(c + dx)}{15ad \sqrt{\cos(c + dx)} (a + a \cos(c + dx))^3} \\
&= \frac{(3A - 13B)F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{6a^3d} - \frac{(9A - 49B) \sin(c + dx)}{10a^3d \sqrt{\cos(c + dx)}} + \frac{(A - B) \sin(c + dx)}{5d \sqrt{\cos(c + dx)}} \\
&= \frac{(9A - 49B)E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{10a^3d} + \frac{(3A - 13B)F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{6a^3d} - \frac{(9A - 49B) \sin(c + dx)}{10a^3d \sqrt{\cos(c + dx)}}
\end{aligned}$$

Mathematica [C] time = 7.19, size = 1447, normalized size = 6.55

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^3), x]

[Out] (((9*I)/10)*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x])*((2*E^((2*I)*d*x))*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x))*(Cos[c] + I*Sin[c])^2])*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x))*Cos[c] + I*Sin[c])^2])*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^

$$\frac{((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]]/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c]))/((B + A*Cos[c + d*x))*(a + a*Sec[c + d*x])^3) - (((49*I)/10)*B *Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x]) *((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c]))^2]))*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]]/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c]))^2]))*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]]/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c]))/((B + A*Cos[c + d*x))*(a + a*Sec[c + d*x])^3) - (2*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2)*Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]])*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]])]/(d*(B + A*Cos[c + d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^3) + (26*B*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2)*Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]])*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]])]/(3*d*(B + A*Cos[c + d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^3) + (Cos[c/2 + (d*x)/2]^6*(A + B*Sec[c + d*x])*((2*(20*B - 9*A*Cos[c] + 29*B*Cos[c])*Csc[c/2]*Sec[c/2]*Sec[c])/((5*d) + (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^5*(-(A*Sin[(d*x)/2]) + B*Sin[(d*x)/2]))/((5*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(-6*A*Sin[(d*x)/2] + 11*B*Sin[(d*x)/2]))/((15*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(-9*A*Sin[(d*x)/2] + 29*B*Sin[(d*x)/2]))/((5*d) + (16*B*Sec[c]*Sec[c + d*x]*Sin[d*x])/d + (4*(-6*A + 11*B)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/((15*d) + (2*(-A + B)*Sec[c/2 + (d*x)/2]^4*Tan[c/2])/((5*d))))/(Cos[c + d*x]^(3/2)*(B + A*Cos[c + d*x))*(a + a*Sec[c + d*x])^3)$$

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{(B \sec(dx + c) + A) \sqrt{\cos(dx + c)}}{a^3 \cos(dx + c)^4 \sec(dx + c)^3 + 3 a^3 \cos(dx + c)^4 \sec(dx + c)^2 + 3 a^3 \cos(dx + c)^4 \sec(dx + c) + a^3 \cos(dx + c)^4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")

[Out] integral((B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(a^3*cos(d*x + c)^4*sec(d*x + c)^3 + 3*a^3*cos(d*x + c)^4*sec(d*x + c)^2 + 3*a^3*cos(d*x + c)^4*sec(d*x + c) + a^3*cos(d*x + c)^4), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(a \sec(dx + c) + a)^3 \cos(dx + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^3*cos(d*x + c)^(7/2)), x)

maple [B] time = 6.06, size = 685, normalized size = 3.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^3,x)

[Out] 1/60*(-2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(15*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-27*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-65*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+147*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+4*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(15*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-27*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-65*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+147*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(15*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-27*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-65*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+147*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*c)+12*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(9*A-49*B)*sin(1/2*d*x+1/2*c)^8-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(147*A-817*B)*sin(1/2*d*x+1/2*c)^6+6*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(43*A-248*B)*sin(1/2*d*x+1/2*c)^4-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(69*A-439*B)*sin(1/2*d*x+1/2*c)^2/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\cos(c+dx)^{7/2} \left(a + \frac{a}{\cos(c+dx)}\right)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))^3),x)

[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))^3), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(7/2)/(a+a*sec(d*x+c))**3,x)

[Out] Timed out

$$3.515 \quad \int \cos^2(c+dx) \sqrt{a + a \sec(c + dx)} (A+B \sec(c+dx)) dx$$

Optimal. Leaf size=220

$$\frac{2a(8A + 9B) \sin(c + dx) \cos^{\frac{5}{2}}(c + dx)}{63d\sqrt{a \sec(c + dx) + a}} + \frac{4a(8A + 9B) \sin(c + dx) \cos^{\frac{3}{2}}(c + dx)}{105d\sqrt{a \sec(c + dx) + a}} + \frac{16a(8A + 9B) \sin(c + dx) \sqrt{\cos(c + dx)}}{315d\sqrt{a \sec(c + dx) + a}}$$

[Out] $4/105*a*(8*A+9*B)*\cos(d*x+c)^{(3/2)}*\sin(d*x+c)/d/(a+a*\sec(d*x+c))^{(1/2)}+2/63*a*(8*A+9*B)*\cos(d*x+c)^{(5/2)}*\sin(d*x+c)/d/(a+a*\sec(d*x+c))^{(1/2)}+2/9*a*A*\cos(d*x+c)^{(7/2)}*\sin(d*x+c)/d/(a+a*\sec(d*x+c))^{(1/2)}+32/315*a*(8*A+9*B)*\sin(d*x+c)/d/\cos(d*x+c)^{(1/2)}/(a+a*\sec(d*x+c))^{(1/2)}+16/315*a*(8*A+9*B)*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/d/(a+a*\sec(d*x+c))^{(1/2)}$

Rubi [A] time = 0.48, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {2955, 4015, 3805, 3804}

$$\frac{2a(8A + 9B) \sin(c + dx) \cos^{\frac{5}{2}}(c + dx)}{63d\sqrt{a \sec(c + dx) + a}} + \frac{4a(8A + 9B) \sin(c + dx) \cos^{\frac{3}{2}}(c + dx)}{105d\sqrt{a \sec(c + dx) + a}} + \frac{16a(8A + 9B) \sin(c + dx) \sqrt{\cos(c + dx)}}{315d\sqrt{a \sec(c + dx) + a}}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^(9/2)*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]

[Out] $(32*a*(8*A + 9*B)*\sin[c + d*x])/(315*d*\sqrt{\cos[c + d*x]}*\sqrt{a + a*\sec[c + d*x]}) + (16*a*(8*A + 9*B)*\sqrt{\cos[c + d*x]}*\sin[c + d*x])/(315*d*\sqrt{a + a*\sec[c + d*x]}) + (4*a*(8*A + 9*B)*\cos[c + d*x]^{(3/2)}*\sin[c + d*x])/(105*d*\sqrt{a + a*\sec[c + d*x]}) + (2*a*(8*A + 9*B)*\cos[c + d*x]^{(5/2)}*\sin[c + d*x])/(63*d*\sqrt{a + a*\sec[c + d*x]}) + (2*a*A*\cos[c + d*x]^{(7/2)}*\sin[c + d*x])/(9*d*\sqrt{a + a*\sec[c + d*x]})$

Rule 2955

Int[((a_.) + csc[(e_.) + (f_.)*(x_)])*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] := Dist[(g*Csc[e + f*x])^p*(g*Ssin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && IntegerQ[n])

Rule 3804

Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 3805

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^n*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]

Rule 4015

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^n*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] :> Simp[(A*b^2*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]

Rubi steps

$$\begin{aligned}
 \int \cos^{\frac{9}{2}}(c + dx) \sqrt{a + a \sec(c + dx)} (A + B \sec(c + dx)) dx &= \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{\sqrt{a + a \sec(c + dx)}}{\sec^{\frac{9}{2}}(c + dx)} dx \\
 &= \frac{2aA \cos^{\frac{7}{2}}(c + dx) \sin(c + dx)}{9d\sqrt{a + a \sec(c + dx)}} + \frac{1}{9} \left((8A + 9B) \sqrt{\cos(c + dx)} \right) \int \frac{\sqrt{a + a \sec(c + dx)}}{\sec^{\frac{7}{2}}(c + dx)} dx \\
 &= \frac{2a(8A + 9B) \cos^{\frac{5}{2}}(c + dx) \sin(c + dx)}{63d\sqrt{a + a \sec(c + dx)}} + \frac{2aA \cos^{\frac{7}{2}}(c + dx) \sin(c + dx)}{9d\sqrt{a + a \sec(c + dx)}} \\
 &= \frac{4a(8A + 9B) \cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{105d\sqrt{a + a \sec(c + dx)}} + \frac{2a(8A + 9B) \cos^{\frac{5}{2}}(c + dx) \sin(c + dx)}{63d\sqrt{a + a \sec(c + dx)}} \\
 &= \frac{16a(8A + 9B) \sqrt{\cos(c + dx)} \sin(c + dx)}{315d\sqrt{a + a \sec(c + dx)}} + \frac{4a(8A + 9B) \cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{105d\sqrt{a + a \sec(c + dx)}} \\
 &= \frac{32a(8A + 9B) \sin(c + dx)}{315d\sqrt{\cos(c + dx)} \sqrt{a + a \sec(c + dx)}} + \frac{16a(8A + 9B) \cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{315d\sqrt{a + a \sec(c + dx)}}
 \end{aligned}$$

Mathematica [A] time = 0.55, size = 119, normalized size = 0.54

$$\frac{\sin(c + dx)\sqrt{\cos(c + dx)}\sqrt{a(\sec(c + dx) + 1)}(94(8A + 9B)\cos(c + dx) + 4(83A + 54B)\cos(2(c + dx)) + 80A\cos(3(c + dx)) + 35A\cos(4(c + dx)))}{1260d(\cos(c + dx) + 1)}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]^(9/2)*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]), x]

[Out] (Sqrt[Cos[c + d*x]]*(1321*A + 1368*B + 94*(8*A + 9*B)*Cos[c + d*x] + 4*(83*A + 54*B)*Cos[2*(c + d*x)] + 80*A*Cos[3*(c + d*x)] + 90*B*Cos[3*(c + d*x)] + 35*A*Cos[4*(c + d*x)])*Sqrt[a*(1 + Sec[c + d*x])]*Sin[c + d*x])/(1260*d*(1 + Cos[c + d*x]))

fricas [A] time = 0.43, size = 116, normalized size = 0.53

$$\frac{2(35A\cos(dx + c)^4 + 5(8A + 9B)\cos(dx + c)^3 + 6(8A + 9B)\cos(dx + c)^2 + 8(8A + 9B)\cos(dx + c) + 128A + 144B)\sqrt{a\cos(dx + c) + a}}{315(d\cos(dx + c) + d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(9/2)*(A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2), x, algorithm="fricas")

[Out] 2/315*(35*A*cos(d*x + c)^4 + 5*(8*A + 9*B)*cos(d*x + c)^3 + 6*(8*A + 9*B)*cos(d*x + c)^2 + 8*(8*A + 9*B)*cos(d*x + c) + 128*A + 144*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c) + d)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)\sqrt{a \sec(dx + c) + a} \cos(dx + c)^{\frac{9}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(9/2)*(A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2), x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*sqrt(a*sec(d*x + c) + a)*cos(d*x + c)^(9/2), x)

maple [A] time = 1.96, size = 130, normalized size = 0.59

$$2(-1 + \cos(dx + c))\left(35A\left(\cos^4(dx + c)\right) + 40A\left(\cos^3(dx + c)\right) + 45B\left(\cos^3(dx + c)\right) + 48A\left(\cos^2(dx + c)\right) + 32B\cos(dx + c) + 128A + 144B\right)\sqrt{a\cos(dx + c) + a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(d*x+c)^(9/2)*(A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2),x)`

[Out] $-2/315/d*(-1+\cos(d*x+c))*(35*A*\cos(d*x+c)^4+40*A*\cos(d*x+c)^3+45*B*\cos(d*x+c)^3+48*A*\cos(d*x+c)^2+54*B*\cos(d*x+c)^2+64*A*\cos(d*x+c)+72*B*\cos(d*x+c)+128*A+144*B)*(a*(1+\cos(d*x+c))/\cos(d*x+c))^(1/2)*\cos(d*x+c)^(1/2)/\sin(d*x+c)$

maxima [B] time = 0.76, size = 547, normalized size = 2.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)^(9/2)*(A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")`

[Out] $1/5040*(\sqrt{2}*(1890*\cos(8/9*\arctan2(\sin(9/2*d*x + 9/2*c), \cos(9/2*d*x + 9/2*c))) * \sin(9/2*d*x + 9/2*c) + 420*\cos(2/3*\arctan2(\sin(9/2*d*x + 9/2*c), \cos(9/2*d*x + 9/2*c))) * \sin(9/2*d*x + 9/2*c) + 252*\cos(4/9*\arctan2(\sin(9/2*d*x + 9/2*c), \cos(9/2*d*x + 9/2*c))) * \sin(9/2*d*x + 9/2*c) + 45*\cos(2/9*\arctan2(\sin(9/2*d*x + 9/2*c), \cos(9/2*d*x + 9/2*c))) * \sin(9/2*d*x + 9/2*c) - 1890*\cos(9/2*d*x + 9/2*c) * \sin(8/9*\arctan2(\sin(9/2*d*x + 9/2*c), \cos(9/2*d*x + 9/2*c))) - 420*\cos(9/2*d*x + 9/2*c) * \sin(2/3*\arctan2(\sin(9/2*d*x + 9/2*c), \cos(9/2*d*x + 9/2*c))) - 252*\cos(9/2*d*x + 9/2*c) * \sin(4/9*\arctan2(\sin(9/2*d*x + 9/2*c), \cos(9/2*d*x + 9/2*c))) - 45*\cos(9/2*d*x + 9/2*c) * \sin(2/9*\arctan2(\sin(9/2*d*x + 9/2*c), \cos(9/2*d*x + 9/2*c))) + 70*\sin(9/2*d*x + 9/2*c) + 45*\sin(7/9*\arctan2(\sin(9/2*d*x + 9/2*c), \cos(9/2*d*x + 9/2*c))) + 252*\sin(5/9*\arctan2(\sin(9/2*d*x + 9/2*c), \cos(9/2*d*x + 9/2*c))) + 420*\sin(1/3*\arctan2(\sin(9/2*d*x + 9/2*c), \cos(9/2*d*x + 9/2*c))) + 1890*\sin(1/9*\arctan2(\sin(9/2*d*x + 9/2*c), \cos(9/2*d*x + 9/2*c)))) * A * \sqrt{a} - 18*\sqrt{2}*(7*(15*\sin(3*d*x + 3*c) + 5*\sin(2*d*x + 2*c) + \sin(d*x + c)) * \cos(7/2*\arctan2(\sin(d*x + c), \cos(d*x + c))) - (105*\cos(3*d*x + 3*c) + 35*\cos(2*d*x + 2*c) + 7*\cos(d*x + c) + 10) * \sin(7/2*\arctan2(\sin(d*x + c), \cos(d*x + c))) - 7*\sin(5/2*\arctan2(\sin(d*x + c), \cos(d*x + c))) - 35*\sin(3/2*\arctan2(\sin(d*x + c), \cos(d*x + c))) - 105*\sin(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c)))) * B * \sqrt{a})/d$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c + dx)^{9/2} \left(A + \frac{B}{\cos(c + dx)} \right) \sqrt{a + \frac{a}{\cos(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(c + d*x)^(9/2)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(1/2),x)`

[Out] `int(cos(c + d*x)^(9/2)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(1/2), x)`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**(9/2)*(A+B*sec(d*x+c))*(a+a*sec(d*x+c))**(1/2),x)

[Out] Timed out

$$3.516 \quad \int \cos^{\frac{7}{2}}(c+dx) \sqrt{a + a \sec(c + dx)} (A+B \sec(c+dx)) dx$$

Optimal. Leaf size=175

$$\frac{2a(6A + 7B) \sin(c + dx) \cos^{\frac{3}{2}}(c + dx)}{35d\sqrt{a \sec(c + dx) + a}} + \frac{8a(6A + 7B) \sin(c + dx) \sqrt{\cos(c + dx)}}{105d\sqrt{a \sec(c + dx) + a}} + \frac{16a(6A + 7B) \sin(c + dx)}{105d\sqrt{\cos(c + dx)} \sqrt{a \sec(c + dx)}}$$

[Out] $2/35*a*(6*A+7*B)*\cos(d*x+c)^{(3/2)}*\sin(d*x+c)/d/(a+a*\sec(d*x+c))^{(1/2)}+2/7*a*A*\cos(d*x+c)^{(5/2)}*\sin(d*x+c)/d/(a+a*\sec(d*x+c))^{(1/2)}+16/105*a*(6*A+7*B)*\sin(d*x+c)/d/\cos(d*x+c)^{(1/2)}/(a+a*\sec(d*x+c))^{(1/2)}+8/105*a*(6*A+7*B)*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/d/(a+a*\sec(d*x+c))^{(1/2)}$

Rubi [A] time = 0.40, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {2955, 4015, 3805, 3804}

$$\frac{2a(6A + 7B) \sin(c + dx) \cos^{\frac{3}{2}}(c + dx)}{35d\sqrt{a \sec(c + dx) + a}} + \frac{8a(6A + 7B) \sin(c + dx) \sqrt{\cos(c + dx)}}{105d\sqrt{a \sec(c + dx) + a}} + \frac{16a(6A + 7B) \sin(c + dx)}{105d\sqrt{\cos(c + dx)} \sqrt{a \sec(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]

[Out] $(16*a*(6*A + 7*B)*\sin[c + d*x])/(105*d*\text{Sqrt}[\cos[c + d*x]]*\text{Sqrt}[a + a*\sec[c + d*x]]) + (8*a*(6*A + 7*B)*\text{Sqrt}[\cos[c + d*x]]*\sin[c + d*x])/(105*d*\text{Sqrt}[a + a*\sec[c + d*x]]) + (2*a*(6*A + 7*B)*\cos[c + d*x]^{(3/2)}*\sin[c + d*x])/(35*d*\text{Sqrt}[a + a*\sec[c + d*x]]) + (2*a*A*\cos[c + d*x]^{(5/2)}*\sin[c + d*x])/(7*d*\text{Sqrt}[a + a*\sec[c + d*x]])$

Rule 2955

Int[((a_.) + csc[(e_.) + (f_.)*(x_.)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_.)])^(p_.), x_Symbol] :> Dist[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && IntegerQ[n])

Rule 3804

Int[Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]])*

Sqrt[d*Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 3805

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] := Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]

Rule 4015

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := Simp[(A*b^2*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]

Rubi steps

$$\begin{aligned}
 \int \cos^{\frac{7}{2}}(c + dx) \sqrt{a + a \sec(c + dx)} (A + B \sec(c + dx)) dx &= \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{\sqrt{a + a \sec(c + dx)}}{\sec^{\frac{7}{2}}(c + dx)} dx \\
 &= \frac{2aA \cos^{\frac{5}{2}}(c + dx) \sin(c + dx)}{7d\sqrt{a + a \sec(c + dx)}} + \frac{1}{7} \left((6A + 7B) \sqrt{\cos(c + dx)} \right) \\
 &= \frac{2a(6A + 7B) \cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{35d\sqrt{a + a \sec(c + dx)}} + \frac{2aA \cos^{\frac{5}{2}}(c + dx)}{7d\sqrt{a + a \sec(c + dx)}} \\
 &= \frac{8a(6A + 7B) \sqrt{\cos(c + dx)} \sin(c + dx)}{105d\sqrt{a + a \sec(c + dx)}} + \frac{2a(6A + 7B) \cos^{\frac{5}{2}}(c + dx)}{35d\sqrt{a + a \sec(c + dx)}} \\
 &= \frac{16a(6A + 7B) \sin(c + dx)}{105d\sqrt{\cos(c + dx)} \sqrt{a + a \sec(c + dx)}} + \frac{8a(6A + 7B) \cos^{\frac{5}{2}}(c + dx)}{105d\sqrt{a + a \sec(c + dx)}}
 \end{aligned}$$

Mathematica [A] time = 0.37, size = 96, normalized size = 0.55

$$\frac{\sin(c + dx) \sqrt{\cos(c + dx)} \sqrt{a(\sec(c + dx) + 1)} ((141A + 112B) \cos(c + dx) + 6(6A + 7B) \cos(2(c + dx))) + 15A \cos^{\frac{5}{2}}(c + dx)}{210d(\cos(c + dx) + 1)}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]), x]

[Out] (Sqrt[Cos[c + d*x]]*(228*A + 266*B + (141*A + 112*B)*Cos[c + d*x] + 6*(6*A + 7*B)*Cos[2*(c + d*x)] + 15*A*Cos[3*(c + d*x)])*Sqrt[a*(1 + Sec[c + d*x])]*Sin[c + d*x])/(210*d*(1 + Cos[c + d*x]))

fricas [A] time = 0.44, size = 99, normalized size = 0.57

$$\frac{2 \left(15 A \cos(dx + c)^3 + 3(6A + 7B) \cos(dx + c)^2 + 4(6A + 7B) \cos(dx + c) + 48A + 56B \right) \sqrt{\frac{a \cos(dx + c) + a}{\cos(dx + c)}} \sqrt{\cos(dx + c)}}{105(d \cos(dx + c) + d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(7/2)*(A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2), x, algorithm="fricas")

[Out] 2/105*(15*A*cos(d*x + c)^3 + 3*(6*A + 7*B)*cos(d*x + c)^2 + 4*(6*A + 7*B)*cos(d*x + c) + 48*A + 56*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c) + d)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A) \sqrt{a \sec(dx + c) + a} \cos(dx + c)^{\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(7/2)*(A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2), x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*sqrt(a*sec(d*x + c) + a)*cos(d*x + c)^(7/2), x)

maple [A] time = 1.80, size = 108, normalized size = 0.62

$$\frac{2(-1 + \cos(dx + c)) \left(15A \left(\cos^3(dx + c) \right) + 18A \left(\cos^2(dx + c) \right) + 21B \left(\cos^2(dx + c) \right) + 24A \cos(dx + c) + 28B \right)}{105d \sin(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(7/2)*(A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2), x)

[Out] $-2/105/d*(-1+\cos(d*x+c))*(15*A*\cos(d*x+c)^3+18*A*\cos(d*x+c)^2+21*B*\cos(d*x+c)^2+24*A*\cos(d*x+c)+28*B*\cos(d*x+c)+48*A+56*B)*(a*(1+\cos(d*x+c))/\cos(d*x+c))^{(1/2)*\cos(d*x+c)^{(1/2)}/\sin(d*x+c)}$

maxima [B] time = 0.79, size = 418, normalized size = 2.39

$$3\sqrt{2}\left(105\cos\left(\frac{6}{7}\arctan\left(\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right),\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\right)\right)\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right)+35\cos\left(\frac{4}{7}\arctan\left(\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right),\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\right)\right)\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)^(7/2)*(A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2),x, algorith="maxima")`

[Out] $1/840*(3*\sqrt{2}*(105*\cos(6/7*\arctan2(\sin(7/2*d*x + 7/2*c), \cos(7/2*d*x + 7/2*c))) * \sin(7/2*d*x + 7/2*c) + 35*\cos(4/7*\arctan2(\sin(7/2*d*x + 7/2*c), \cos(7/2*d*x + 7/2*c))) * \sin(7/2*d*x + 7/2*c) + 7*\cos(2/7*\arctan2(\sin(7/2*d*x + 7/2*c), \cos(7/2*d*x + 7/2*c))) * \sin(7/2*d*x + 7/2*c) - 105*\cos(7/2*d*x + 7/2*c) * \sin(6/7*\arctan2(\sin(7/2*d*x + 7/2*c), \cos(7/2*d*x + 7/2*c))) - 35*\cos(7/2*d*x + 7/2*c) * \sin(4/7*\arctan2(\sin(7/2*d*x + 7/2*c), \cos(7/2*d*x + 7/2*c))) - 7*\cos(7/2*d*x + 7/2*c) * \sin(2/7*\arctan2(\sin(7/2*d*x + 7/2*c), \cos(7/2*d*x + 7/2*c))) + 10*\sin(7/2*d*x + 7/2*c) + 7*\sin(5/7*\arctan2(\sin(7/2*d*x + 7/2*c), \cos(7/2*d*x + 7/2*c))) + 35*\sin(3/7*\arctan2(\sin(7/2*d*x + 7/2*c), \cos(7/2*d*x + 7/2*c))) + 105*\sin(1/7*\arctan2(\sin(7/2*d*x + 7/2*c), \cos(7/2*d*x + 7/2*c)))) * A*\sqrt{a} - 14*\sqrt{2}*(5*(6*\sin(2*d*x + 2*c) + \sin(d*x + c)) * \cos(5/2*\arctan2(\sin(d*x + c), \cos(d*x + c))) - (30*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 6) * \sin(5/2*\arctan2(\sin(d*x + c), \cos(d*x + c))) - 5*\sin(3/2*\arctan2(\sin(d*x + c), \cos(d*x + c))) - 30*\sin(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c)))) * B*\sqrt{a})/d$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c + dx)^{7/2} \left(A + \frac{B}{\cos(c + dx)} \right) \sqrt{a + \frac{a}{\cos(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(c + d*x)^(7/2)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(1/2),x)`

[Out] `int(cos(c + d*x)^(7/2)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(1/2), x)`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(A+B*sec(d*x+c))*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

$$3.517 \quad \int \cos^{\frac{5}{2}}(c+dx) \sqrt{a + a \sec(c + dx)} (A+B \sec(c+dx)) dx$$

Optimal. Leaf size=130

$$\frac{2a(4A + 5B) \sin(c + dx) \sqrt{\cos(c + dx)}}{15d\sqrt{a \sec(c + dx) + a}} + \frac{4a(4A + 5B) \sin(c + dx)}{15d\sqrt{\cos(c + dx)} \sqrt{a \sec(c + dx) + a}} + \frac{2aA \sin(c + dx) \cos^{\frac{3}{2}}(c + dx)}{5d\sqrt{a \sec(c + dx) + a}}$$

[Out] $2/5*a*A*\cos(d*x+c)^{(3/2)}*\sin(d*x+c)/d/(a+a*\sec(d*x+c))^{(1/2)}+4/15*a*(4*A+5*B)*\sin(d*x+c)/d/\cos(d*x+c)^{(1/2)}/(a+a*\sec(d*x+c))^{(1/2)}+2/15*a*(4*A+5*B)*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/d/(a+a*\sec(d*x+c))^{(1/2)}$

Rubi [A] time = 0.33, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {2955, 4015, 3805, 3804}

$$\frac{2a(4A + 5B) \sin(c + dx) \sqrt{\cos(c + dx)}}{15d\sqrt{a \sec(c + dx) + a}} + \frac{4a(4A + 5B) \sin(c + dx)}{15d\sqrt{\cos(c + dx)} \sqrt{a \sec(c + dx) + a}} + \frac{2aA \sin(c + dx) \cos^{\frac{3}{2}}(c + dx)}{5d\sqrt{a \sec(c + dx) + a}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Cos}[c + d*x]^{(5/2)}*\text{Sqrt}[a + a*\text{Sec}[c + d*x]]*(A + B*\text{Sec}[c + d*x]),x]$

[Out] $(4*a*(4*A + 5*B)*\text{Sin}[c + d*x])/(15*d*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sqrt}[a + a*\text{Sec}[c + d*x]]) + (2*a*(4*A + 5*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sin}[c + d*x])/(15*d*\text{Sqrt}[a + a*\text{Sec}[c + d*x]]) + (2*a*A*\text{Cos}[c + d*x]^{(3/2)}*\text{Sin}[c + d*x])/(5*d*\text{Sqrt}[a + a*\text{Sec}[c + d*x]])$

Rule 2955

$\text{Int}[(a_. + \text{csc}[(e_.) + (f_.)*(x_.)]*(b_.))^{(m_.)}*(\text{csc}[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))^{(n_.)}*((g_.)*\sin[(e_.) + (f_.)*(x_.)])^{(p_.)}, x_Symbol] :> \text{Dist}[(g*\text{Csc}[e + f*x])^p*(g*\text{Sin}[e + f*x])^p, \text{Int}[(a + b*\text{Csc}[e + f*x])^m*(c + d*\text{Csc}[e + f*x])^n]/(g*\text{Csc}[e + f*x])^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, m, n, p\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& !\text{IntegerQ}[p] \&\& !(\text{IntegerQ}[m] \&\& \text{IntegerQ}[n])$

Rule 3804

$\text{Int}[\text{Sqrt}[\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]/\text{Sqrt}[\text{csc}[(e_.) + (f_.)*(x_.)]*(d_.)], x_Symbol] :> \text{Simp}[(-2*a*\text{Cot}[e + f*x])/(f*\text{Sqrt}[a + b*\text{Csc}[e + f*x]]*\text{Sqrt}[d*\text{Csc}[e + f*x]]), x] /; \text{FreeQ}\{a, b, d, e, f\}, x] \&\& \text{EqQ}[a^2 - b^2, 0]$

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.)
+ (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
+ b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.)
+ (a_)]*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
+ f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rubi steps

$$\begin{aligned} \int \cos^{\frac{5}{2}}(c + dx) \sqrt{a + a \sec(c + dx)} (A + B \sec(c + dx)) dx &= \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{\sqrt{a + a \sec(c + dx)}}{\sec^{\frac{5}{2}}(c + dx)} dx \\ &= \frac{2aA \cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{5d\sqrt{a + a \sec(c + dx)}} + \frac{1}{5} \left((4A + 5B) \sqrt{\cos(c + dx)} \right) \\ &= \frac{2a(4A + 5B) \sqrt{\cos(c + dx)} \sin(c + dx)}{15d\sqrt{a + a \sec(c + dx)}} + \frac{2aA \cos^{\frac{3}{2}}(c + dx)}{5d\sqrt{a + a \sec(c + dx)}} \\ &= \frac{4a(4A + 5B) \sin(c + dx)}{15d\sqrt{\cos(c + dx)} \sqrt{a + a \sec(c + dx)}} + \frac{2a(4A + 5B)}{15d\sqrt{a + a \sec(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.16, size = 79, normalized size = 0.61

$$\frac{2 \sin(c + dx) \sqrt{\cos(c + dx)} \sqrt{a(\sec(c + dx) + 1)} \left((4A + 5B) \cos(c + dx) + 3A \cos^2(c + dx) + 8A + 10B \right)}{15d(\cos(c + dx) + 1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),
x]
```

[Out] $(2\sqrt{\cos[c + dx]})(8A + 10B + (4A + 5B)\cos[c + dx] + 3A\cos[c + dx]^2)\sqrt{a(1 + \sec[c + dx])}\sin[c + dx]/(15d(1 + \cos[c + dx]))$

fricas [A] time = 0.43, size = 81, normalized size = 0.62

$$\frac{2(3A\cos(dx+c)^2 + (4A+5B)\cos(dx+c) + 8A+10B)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{15(d\cos(dx+c)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(dx+c)^(5/2)*(A+B*sec(dx+c))*(a+a*sec(dx+c))^(1/2),x, algorithm="fricas")`

[Out] $2/15*(3A*\cos(dx+c)^2 + (4A+5B)*\cos(dx+c) + 8A+10B)*\sqrt{(a*\cos(dx+c)+a)/\cos(dx+c)}*\sqrt{\cos(dx+c)}*\sin(dx+c)/(d*\cos(dx+c)+d)$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\sec(dx+c) + A)\sqrt{a\sec(dx+c) + a}\cos(dx+c)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(dx+c)^(5/2)*(A+B*sec(dx+c))*(a+a*sec(dx+c))^(1/2),x, algorithm="giac")`

[Out] `integrate((B*sec(dx+c) + A)*sqrt(a*sec(dx+c) + a)*cos(dx+c)^(5/2), x)`

maple [A] time = 1.79, size = 86, normalized size = 0.66

$$\frac{2(-1 + \cos(dx+c))\left(3A\left(\cos^2(dx+c)\right) + 4A\cos(dx+c) + 5B\cos(dx+c) + 8A + 10B\right)\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}}{15d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(dx+c)^(5/2)*(A+B*sec(dx+c))*(a+a*sec(dx+c))^(1/2),x)`

[Out] $-2/15/d*(-1+\cos(dx+c))*(3A*\cos(dx+c)^2+4A*\cos(dx+c)+5B*\cos(dx+c)+8A+10B)*(a*(1+\cos(dx+c))/\cos(dx+c))^(1/2)*\cos(dx+c)^(1/2)/\sin(dx+c)$

maxima [B] time = 0.84, size = 296, normalized size = 2.28

$$\sqrt{2}\left(30\cos\left(\frac{4}{5}\arctan\left(\sin\left(\frac{5}{2}dx + \frac{5}{2}c\right),\cos\left(\frac{5}{2}dx + \frac{5}{2}c\right)\right)\right)\sin\left(\frac{5}{2}dx + \frac{5}{2}c\right) + 5\cos\left(\frac{2}{5}\arctan\left(\sin\left(\frac{5}{2}dx + \frac{5}{2}c\right)\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/60*(sqrt(2)*(30*cos(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))
*sin(5/2*d*x + 5/2*c) + 5*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))
*sin(5/2*d*x + 5/2*c) - 30*cos(5/2*d*x + 5/2*c)*sin(4/5*arctan2(sin(5/2*d*x + 5/2*c),
cos(5/2*d*x + 5/2*c))) - 5*cos(5/2*d*x + 5/2*c)*sin(2/5*arctan2(sin(5/2*d*x + 5/2*c),
cos(5/2*d*x + 5/2*c))) + 6*sin(5/2*d*x + 5/2*c) + 5*sin(3/5*arctan2(sin(5/2*d*x + 5/2*c),
cos(5/2*d*x + 5/2*c))) + 30*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))
)*A*sqrt(a) - 10*(3*sqrt(2)*cos(3/2*arctan2(sin(d*x + c), cos(d*x + c)))
*sin(d*x + c) - (3*sqrt(2)*cos(d*x + c) + 2*sqrt(2))*sin(3/2*arctan2(sin(d*x + c),
cos(d*x + c))) - 3*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))))
)*B*sqrt(a))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c + dx)^{5/2} \left(A + \frac{B}{\cos(c + dx)} \right) \sqrt{a + \frac{a}{\cos(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^(5/2)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+B*sec(d*x+c))*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```


$$3.518 \quad \int \cos^2(c+dx) \sqrt{a + a \sec(c + dx)} (A+B \sec(c+dx)) dx$$

Optimal. Leaf size=82

$$\frac{2a(A+3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}} + \frac{2A\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}{3d}$$

[Out] $2/3*a*(A+3*B)*\sin(d*x+c)/d/\cos(d*x+c)^{(1/2)}/(a+a*\sec(d*x+c))^{(1/2)}+2/3*A*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}*(a+a*\sec(d*x+c))^{(1/2)}/d$

Rubi [A] time = 0.26, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {2955, 4013, 3804}

$$\frac{2a(A+3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}} + \frac{2A\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}{3d}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]

[Out] $(2*a*(A+3*B)*\sin[c+d*x])/(3*d*\text{Sqrt}[\cos[c+d*x]]*\text{Sqrt}[a+a*\text{Sec}[c+d*x]]) + (2*A*\text{Sqrt}[\cos[c+d*x]]*\text{Sqrt}[a+a*\text{Sec}[c+d*x]]*\sin[c+d*x])/(3*d)$

Rule 2955

Int[((a_.) + csc[(e_.) + (f_.)*(x_)])*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] := Dist[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && IntegerQ[n])

Rule 3804

Int[Sqrt[csc[(e_.) + (f_.)*(x_)])*(b_.) + (a_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)], x_Symbol] := Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 4013

Int[(csc[(e_.) + (f_.)*(x_)])*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] := Simp[(A*Cot[

$e + f*x]*(a + b*\text{Csc}[e + f*x])^m*(d*\text{Csc}[e + f*x])^n/(f*n), x] - \text{Dist}[(a*A*m - b*B*n)/(b*d*n), \text{Int}[(a + b*\text{Csc}[e + f*x])^m*(d*\text{Csc}[e + f*x])^{n+1}, x], x] /;$ $\text{FreeQ}\{a, b, d, e, f, A, B, m, n\}, x\} \&\& \text{NeQ}[A*b - a*B, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{EqQ}[m + n + 1, 0] \&\& !\text{LeQ}[m, -1]$

Rubi steps

$$\begin{aligned} \int \cos^3(c + dx) \sqrt{a + a \sec(c + dx)} (A + B \sec(c + dx)) dx &= (\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)}) \int \frac{\sqrt{a + a \sec(c + dx)}}{\sec^2(c + dx)} dx \\ &= \frac{2A \sqrt{\cos(c + dx)} \sqrt{a + a \sec(c + dx)} \sin(c + dx)}{3d} + \frac{1}{3} \int \frac{\sqrt{a + a \sec(c + dx)}}{\sec(c + dx)} dx \\ &= \frac{2a(A + 3B) \sin(c + dx)}{3d \sqrt{\cos(c + dx)} \sqrt{a + a \sec(c + dx)}} + \frac{2A \sqrt{\cos(c + dx)}}{3d} \end{aligned}$$

Mathematica [A] time = 0.20, size = 56, normalized size = 0.68

$$\frac{2\sqrt{\cos(c + dx)} \tan\left(\frac{1}{2}(c + dx)\right) \sqrt{a(\sec(c + dx) + 1)} (A \cos(c + dx) + 2A + 3B)}{3d}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]), x]

[Out] (2*Sqrt[Cos[c + d*x]]*(2*A + 3*B + A*Cos[c + d*x])*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(3*d)

fricas [A] time = 0.44, size = 64, normalized size = 0.78

$$\frac{2(A \cos(dx + c) + 2A + 3B) \sqrt{\frac{a \cos(dx + c) + a}{\cos(dx + c)}} \sqrt{\cos(dx + c)} \sin(dx + c)}{3(d \cos(dx + c) + d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")

[Out] 2/3*(A*cos(d*x + c) + 2*A + 3*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c) + d)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A) \sqrt{a \sec(dx + c) + a} \cos(dx + c)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*sqrt(a*sec(d*x + c) + a)*cos(d*x + c)^(3/2), x)

maple [A] time = 1.74, size = 65, normalized size = 0.79

$$\frac{2(-1 + \cos(dx + c))(A \cos(dx + c) + 2A + 3B) \left(\sqrt{\cos(dx + c)} \right) \sqrt{\frac{a(1 + \cos(dx + c))}{\cos(dx + c)}}}{3d \sin(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2),x)

[Out] -2/3/d*(-1+cos(d*x+c))*(A*cos(d*x+c)+2*A+3*B)*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)

maxima [B] time = 0.66, size = 141, normalized size = 1.72

$$\frac{\sqrt{2} \left(3 \cos\left(\frac{2}{3} \arctan\left(\sin\left(\frac{3}{2} dx + \frac{3}{2} c\right)\right), \cos\left(\frac{3}{2} dx + \frac{3}{2} c\right)\right) \right) \sin\left(\frac{3}{2} dx + \frac{3}{2} c\right) - 3 \cos\left(\frac{3}{2} dx + \frac{3}{2} c\right) \sin\left(\frac{2}{3} \arctan\left(\sin\left(\frac{3}{2} dx + \frac{3}{2} c\right)\right), \cos\left(\frac{3}{2} dx + \frac{3}{2} c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")

[Out] 1/6*(sqrt(2)*(3*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(3/2*d*x + 3/2*c) - 3*cos(3/2*d*x + 3/2*c)*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sin(3/2*d*x + 3/2*c) + 3*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*A*sqrt(a) + 12*sqrt(2)*B*sqrt(a)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))))/d

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c + dx)^{3/2} \left(A + \frac{B}{\cos(c + dx)} \right) \sqrt{a + \frac{a}{\cos(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^(3/2)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(1/2), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*sec(d*x+c))*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

$$3.519 \quad \int \sqrt{\cos(c+dx)} \sqrt{a+a\sec(c+dx)} (A+B\sec(c+dx)) dx$$

Optimal. Leaf size=96

$$\frac{2aA \sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}} + \frac{2\sqrt{a}B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)} \sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

[Out] 2*B*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*a^(1/2)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2*a*A*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)

Rubi [A] time = 0.26, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {2955, 4015, 3801, 215}

$$\frac{2aA \sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}} + \frac{2\sqrt{a}B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)} \sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]

[Out] (2*Sqrt[a]*B*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]])*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]/d + (2*a*A*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]])

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 2955

Int[((a_.) + csc[(e_.) + (f_.)*(x_)])*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dist[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && IntegerQ[n])

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.)
+ (a_.)], x_Symbol] := Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.)
+ (a_.)]*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
+ f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rubi steps

$$\begin{aligned} \int \sqrt{\cos(c+dx)} \sqrt{a+a \sec(c+dx)} (A+B \sec(c+dx)) dx &= \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\sqrt{a+a \sec(c+dx)}}{\sqrt{\sec(c+dx)}} dx \\ &= \frac{2aA \sin(c+dx)}{d\sqrt{\cos(c+dx)} \sqrt{a+a \sec(c+dx)}} + \left(B\sqrt{\cos(c+dx)} \right) \int \frac{1}{\sqrt{\sec(c+dx)}} dx \\ &= \frac{2aA \sin(c+dx)}{d\sqrt{\cos(c+dx)} \sqrt{a+a \sec(c+dx)}} - \frac{(2B\sqrt{\cos(c+dx)}) \operatorname{arcsinh}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a+a \sec(c+dx)}}\right)}{d} \\ &= \frac{2\sqrt{a} B \sinh^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a+a \sec(c+dx)}}\right) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{d} \end{aligned}$$

Mathematica [A] time = 0.32, size = 94, normalized size = 0.98

$$\frac{2\sqrt{\cos(c+dx)} \tan\left(\frac{1}{2}(c+dx)\right) \sqrt{a(\sec(c+dx)+1)} \left(A\sqrt{1-\sec(c+dx)} - B\sqrt{\sec(c+dx)} \sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\right)}{d\sqrt{1-\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),
x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*(A*Sqrt[1 - Sec[c + d*x]] - B*ArcSin[Sqrt[Sec[c + d*x]
]]*Sqrt[Sec[c + d*x]])*Sqrt[a*(1 + Sec[c + d*x]]*Tan[(c + d*x)/2])/(d*Sqr
t[1 - Sec[c + d*x]])
```

fricas [A] time = 0.48, size = 298, normalized size = 3.10

$$\frac{4 A \sqrt{\frac{a \cos(dx+c)+a}{\cos(dx+c)}} \sqrt{\cos(dx+c)} \sin(dx+c) + (B \cos(dx+c) + B) \sqrt{a} \log \left(\frac{a \cos(dx+c)^3 - 4 \sqrt{a} \sqrt{\frac{a \cos(dx+c)+a}{\cos(dx+c)}} (\cos(dx+c) - 2) \sqrt{\cos(dx+c)} \sin(dx+c) - 7 a \cos(dx+c)^2 + 8 a}{\cos(dx+c)^3 - 2 (d \cos(dx+c) + d)} \right)}{2 (d \cos(dx+c) + d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(1/2),x, algorith="fricas")

[Out] [1/2*(4*A*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + (B*cos(d*x + c) + B)*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(d*cos(d*x + c) + d), (2*A*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + (B*cos(d*x + c) + B)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*x + c) + d)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx+c) + A) \sqrt{a \sec(dx+c) + a} \sqrt{\cos(dx+c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(1/2),x, algorith="giac")

[Out] integrate((B*sec(d*x + c) + A)*sqrt(a*sec(d*x + c) + a)*sqrt(cos(d*x + c)), x)

maple [B] time = 1.66, size = 169, normalized size = 1.76

$$\frac{(-1 + \cos(dx+c)) \left(2A \sin(dx+c) \sqrt{-\frac{2}{1+\cos(dx+c)}} + B \arctan \left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}} (\cos(dx+c)+1+\sin(dx+c)) \sqrt{2}}{4} \right) \right) \sqrt{2} - B a}{d \sqrt{-\frac{2}{1+\cos(dx+c)}} \sin(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(1/2),x)

[Out] $-1/d*(-1+\cos(d*x+c))*(2*A*\sin(d*x+c)*(-2/(1+\cos(d*x+c)))^(1/2)+B*\arctan(1/4*(-2/(1+\cos(d*x+c)))^(1/2)*(\cos(d*x+c)+1+\sin(d*x+c))*2^(1/2))*2^(1/2)-B*\arctan(1/4*(-2/(1+\cos(d*x+c)))^(1/2)*(\cos(d*x+c)+1-\sin(d*x+c))*2^(1/2))*2^(1/2))*\cos(d*x+c)^(1/2)*(a*(1+\cos(d*x+c))/\cos(d*x+c))^(1/2)/(-2/(1+\cos(d*x+c)))^(1/2)/\sin(d*x+c)^2$

maxima [B] time = 0.64, size = 262, normalized size = 2.73

$$4\sqrt{2}A\sqrt{a}\sin\left(\frac{1}{2}dx + \frac{1}{2}c\right) + B\sqrt{a}\left(\log\left(2\cos\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + 2\sin\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2\right) + 2\sqrt{2}\cos\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 2\sqrt{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")

[Out] $1/2*(4*\sqrt{2}*A*\sqrt{a}*\sin(1/2*d*x + 1/2*c) + B*\sqrt{a}*(\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + \log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2)))/d$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\cos(c + dx)} \left(A + \frac{B}{\cos(c + dx)} \right) \sqrt{a + \frac{a}{\cos(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^(1/2)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(1/2),x)

[Out] int(cos(c + d*x)^(1/2)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a(\sec(c + dx) + 1)} (A + B \sec(c + dx)) \sqrt{\cos(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)**(1/2)*(a+a*sec(d*x+c))**(1/2),x)

[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*(A + B*sec(c + d*x))*sqrt(cos(c + d*x)), x)

$$3.520 \quad \int \frac{\sqrt{a+a \sec(c+dx)} (A+B \sec(c+dx))}{\sqrt{\cos(c+dx)}} dx$$

Optimal. Leaf size=98

$$\frac{\sqrt{a}(2A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d} + \frac{aB\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}$$

[Out] (2*A+B)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*a^(1/2)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+a*B*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2)

Rubi [A] time = 0.26, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {2955, 4016, 3801, 215}

$$\frac{\sqrt{a}(2A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d} + \frac{aB\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]))/Sqrt[Cos[c + d*x]],x]

[Out] (Sqrt[a]*(2*A + B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (a*B*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]])

Rule 215

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 2955

Int[((a_) + csc[(e_) + (f_)*(x_)])*(b_)^(m_)*(csc[(e_) + (f_)*(x_)])*(d_) + (c_))^(n_)*((g_)*sin[(e_) + (f_)*(x_)])^(p_), x_Symbol] := Dist[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && IntegerQ[n])

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.)
+ (a_.)], x_Symbol] := Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.)
+ (a_.)]*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
+ Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x]
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && !
LtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{a + a \sec(c + dx)} (A + B \sec(c + dx))}{\sqrt{\cos(c + dx)}} dx = \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \sqrt{\sec(c + dx)} \sqrt{a + a \sec(c + dx)} dx$$

$$= \frac{aB \sin(c + dx)}{d \cos^{\frac{3}{2}}(c + dx) \sqrt{a + a \sec(c + dx)}} + \frac{1}{2} \left((2A + B) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right)$$

$$= \frac{aB \sin(c + dx)}{d \cos^{\frac{3}{2}}(c + dx) \sqrt{a + a \sec(c + dx)}} - \frac{\left((2A + B) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right)}{d}$$

$$= \frac{\sqrt{a} (2A + B) \sinh^{-1} \left(\frac{\sqrt{a} \tan(c + dx)}{\sqrt{a + a \sec(c + dx)}} \right) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)}}{d}$$

Mathematica [A] time = 0.40, size = 89, normalized size = 0.91

$$\frac{\sqrt{\cos(c + dx)} \sec\left(\frac{1}{2}(c + dx)\right) \sqrt{a(\sec(c + dx) + 1)} \left(\sqrt{2} (2A + B) \tanh^{-1} \left(\sqrt{2} \sin\left(\frac{1}{2}(c + dx)\right) \right) + 2B \sin\left(\frac{1}{2}(c + dx)\right) \right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]))/Sqrt[Cos[c + d*x]
], x]
```

[Out] (Sqrt[Cos[c + d*x]]*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(Sqrt[2]*(2*A + B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]] + 2*B*Sec[c + d*x]*Sin[(c + d*x)/2]))/(2*d)

fricas [A] time = 0.54, size = 351, normalized size = 3.58

$$\frac{4 B \sqrt{\frac{a \cos(dx+c)+a}{\cos(dx+c)}} \sqrt{\cos(dx+c)} \sin(dx+c) + \left((2A+B) \cos(dx+c)^2 + (2A+B) \cos(dx+c) \right) \sqrt{a} \log \left(\frac{a \cos(dx+c)}{\dots} \right)}{4 \left(d \cos(dx+c)^2 + d \cos(dx+c) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")

[Out] [1/4*(4*B*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + ((2*A + B)*cos(d*x + c)^2 + (2*A + B)*cos(d*x + c))*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(d*cos(d*x + c)^2 + d*cos(d*x + c)), 1/2*(2*B*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + ((2*A + B)*cos(d*x + c)^2 + (2*A + B)*cos(d*x + c))*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*x + c)^2 + d*cos(d*x + c))]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sqrt{a \sec(dx + c) + a}}{\sqrt{\cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*sqrt(a*sec(d*x + c) + a)/sqrt(cos(d*x + c)), x)

maple [B] time = 2.41, size = 275, normalized size = 2.81

$$\frac{(-1 + \cos(dx + c)) \left(2A \arctan \left(\frac{\sqrt{\frac{2}{1+\cos(dx+c)}} (\cos(dx+c)+1-\sin(dx+c)) \sqrt{2}}{4} \right) \cos(dx + c) \sqrt{2} - 2A \arctan \left(\frac{\sqrt{\frac{2}{1+\cos(dx+c)}} (\cos(dx+c)+1-\sin(dx+c)) \sqrt{2}}{4} \right) \right)}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x)

[Out] 1/2/d*(-1+cos(d*x+c))*(2*A*arctan(1/4*(-2/(1+cos(d*x+c))))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))*2^(1/2))*cos(d*x+c)*2^(1/2)-2*A*arctan(1/4*(-2/(1+cos(d*x+c))))^(1/2)*(cos(d*x+c)+1+sin(d*x+c))*2^(1/2))*cos(d*x+c)*2^(1/2)+B*arctan(1/4*(-2/(1+cos(d*x+c))))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))*2^(1/2))*cos(d*x+c)*2^(1/2)-B*arctan(1/4*(-2/(1+cos(d*x+c))))^(1/2)*(cos(d*x+c)+1+sin(d*x+c))*2^(1/2))*cos(d*x+c)*2^(1/2)-2*B*(-2/(1+cos(d*x+c))))^(1/2)*sin(d*x+c))*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)^2/(-2/(1+cos(d*x+c))))^(1/2)/cos(d*x+c)^(1/2)

maxima [B] time = 0.86, size = 905, normalized size = 9.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorith="maxima")

[Out] 1/4*(2*A*sqrt(a)*(log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2)) - (4*sqrt(2)*cos(3/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x + 2*c) - 4*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x + 2*c) - (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2)

))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(3/2*arctan2(sin(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))))*B*sqrt(a)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)) /d

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \sqrt{a + \frac{a}{\cos(c+dx)}}}{\sqrt{\cos(c+dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(1/2))/cos(c + d*x)^(1/2), x)

[Out] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(1/2))/cos(c + d*x)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a(\sec(c+dx)+1)}(A+B\sec(c+dx))}{\sqrt{\cos(c+dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*(a+a*sec(d*x+c))**(1/2)/cos(d*x+c)**(1/2), x)

[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*(A + B*sec(c + d*x))/sqrt(cos(c + d*x)), x)

$$3.521 \quad \int \frac{\sqrt{a+a \sec(c+dx)} (A+B \sec(c+dx))}{\cos^{\frac{3}{2}}(c+dx)} dx$$

Optimal. Leaf size=151

$$\frac{a(4A+3B) \sin(c+dx)}{4d \cos^{\frac{3}{2}}(c+dx) \sqrt{a \sec(c+dx)+a}} + \frac{\sqrt{a}(4A+3B) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \sinh^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}}\right)}{4d} + \frac{\sqrt{a} \tan(c+dx)}{2d \cos^{\frac{5}{2}}(c+dx)}$$

[Out] $1/4*(4*A+3*B)*\operatorname{arcsinh}(a^{1/2}*\tan(d*x+c)/(a+a*\sec(d*x+c))^{1/2})*a^{1/2}*\cos(d*x+c)^{1/2}*\sec(d*x+c)^{1/2}/d+1/2*a*B*\sin(d*x+c)/d/\cos(d*x+c)^{5/2}/(a+a*\sec(d*x+c))^{1/2}+1/4*a*(4*A+3*B)*\sin(d*x+c)/d/\cos(d*x+c)^{3/2}/(a+a*\sec(d*x+c))^{1/2}$

Rubi [A] time = 0.33, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2955, 4016, 3803, 3801, 215}

$$\frac{a(4A+3B) \sin(c+dx)}{4d \cos^{\frac{3}{2}}(c+dx) \sqrt{a \sec(c+dx)+a}} + \frac{\sqrt{a}(4A+3B) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \sinh^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}}\right)}{4d} + \frac{\sqrt{a} \tan(c+dx)}{2d \cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(\operatorname{Sqrt}[a + a*\operatorname{Sec}[c + d*x]]*(A + B*\operatorname{Sec}[c + d*x]))/\operatorname{Cos}[c + d*x]^{3/2}, x]$

[Out] $(\operatorname{Sqrt}[a]*(4*A + 3*B)*\operatorname{ArcSinh}[(\operatorname{Sqrt}[a]*\operatorname{Tan}[c + d*x])/\operatorname{Sqrt}[a + a*\operatorname{Sec}[c + d*x]]]*\operatorname{Sqrt}[\operatorname{Cos}[c + d*x]]*\operatorname{Sqrt}[\operatorname{Sec}[c + d*x]])/(4*d) + (a*B*\operatorname{Sin}[c + d*x])/(2*d*\operatorname{Cos}[c + d*x]^{5/2}*\operatorname{Sqrt}[a + a*\operatorname{Sec}[c + d*x]]) + (a*(4*A + 3*B)*\operatorname{Sin}[c + d*x])/(4*d*\operatorname{Cos}[c + d*x]^{3/2}*\operatorname{Sqrt}[a + a*\operatorname{Sec}[c + d*x]])$

Rule 215

$\operatorname{Int}[1/\operatorname{Sqrt}[(a_) + (b_)*(x_)^2], x_Symbol] := \operatorname{Simp}[\operatorname{ArcSinh}[\operatorname{Rt}[b, 2]*x]/\operatorname{Sqrt}[a]/\operatorname{Rt}[b, 2], x] /;$ $\operatorname{FreeQ}\{a, b\}, x \&\& \operatorname{GtQ}[a, 0] \&\& \operatorname{PosQ}[b]$

Rule 2955

$\operatorname{Int}(((a_) + \operatorname{csc}[(e_) + (f_)*(x_)]*(b_))^{(m_)}*(\operatorname{csc}[(e_) + (f_)*(x_)]*(d_) + (c_))^{(n_)}*((g_)*\operatorname{sin}[(e_) + (f_)*(x_)])^{(p_)}, x_Symbol) := \operatorname{Dist}[(g*\operatorname{Csc}[e + f*x])^p*(g*\operatorname{Sin}[e + f*x])^p, \operatorname{Int}(((a + b*\operatorname{Csc}[e + f*x])^m*(c + d*\operatorname{Csc}[e + f*x])^n)/(g*\operatorname{Csc}[e + f*x])^p, x), x] /;$ $\operatorname{FreeQ}\{a, b, c, d, e, f, g, m, n, p\}, x \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{IntegerQ}[p] \&\& \operatorname{IntegerQ}[m] \&\& \operatorname{IntegerQ}[n]$

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_.)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 3803

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_.)], x_Symbol] :> Simp[(-2*b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/
(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(2*a*d*(n - 1))/(b*(2*n -
1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_.)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] :> Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
+ Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && !
LtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a + a \sec(c + dx)} (A + B \sec(c + dx))}{\cos^{\frac{3}{2}}(c + dx)} dx &= \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \sec^{\frac{3}{2}}(c + dx) \sqrt{a + a \sec(c + dx)} \\
&= \frac{aB \sin(c + dx)}{2d \cos^{\frac{5}{2}}(c + dx) \sqrt{a + a \sec(c + dx)}} + \frac{1}{4} \left((4A + 3B) \sqrt{\cos(c + dx)} \right) \\
&= \frac{aB \sin(c + dx)}{2d \cos^{\frac{5}{2}}(c + dx) \sqrt{a + a \sec(c + dx)}} + \frac{a(4A + 3B) \sin(c + dx)}{4d \cos^{\frac{3}{2}}(c + dx) \sqrt{a + a \sec(c + dx)}} \\
&= \frac{aB \sin(c + dx)}{2d \cos^{\frac{5}{2}}(c + dx) \sqrt{a + a \sec(c + dx)}} + \frac{a(4A + 3B) \sin(c + dx)}{4d \cos^{\frac{3}{2}}(c + dx) \sqrt{a + a \sec(c + dx)}} \\
&= \frac{\sqrt{a} (4A + 3B) \sinh^{-1} \left(\frac{\sqrt{a} \tan(c + dx)}{\sqrt{a + a \sec(c + dx)}} \right) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)}}{4d}
\end{aligned}$$

Mathematica [A] time = 0.66, size = 106, normalized size = 0.70

$$\frac{\sqrt{\cos(c + dx)} \sec\left(\frac{1}{2}(c + dx)\right) \sqrt{a(\sec(c + dx) + 1)} \left(\sqrt{2} (4A + 3B) \tanh^{-1} \left(\sqrt{2} \sin\left(\frac{1}{2}(c + dx)\right) \right) + 2 \sin\left(\frac{1}{2}(c + dx)\right) \right)}{8d}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]))/Cos[c + d*x]^(3/2), x]

[Out] (Sqrt[Cos[c + d*x]]*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(Sqrt[2]*(4*A + 3*B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sec[c + d*x]*(4*A + 3*B + 2*B*Sec[c + d*x])*Sin[(c + d*x)/2]))/(8*d)

fricas [A] time = 0.52, size = 401, normalized size = 2.66

$$\left[\frac{4((4A + 3B) \cos(dx + c) + 2B) \sqrt{\frac{a \cos(dx + c) + a}{\cos(dx + c)}} \sqrt{\cos(dx + c)} \sin(dx + c) + ((4A + 3B) \cos(dx + c))^3 + (4A + 3B) \cos(dx + c)}{16(d \cos(dx + c))^3 + d \cos(dx + c)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="fricas")

[Out] [1/16*(4*((4*A + 3*B)*cos(d*x + c) + 2*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + ((4*A + 3*B)*cos(d*x + c)^3 + (4*A + 3*B)*cos(d*x + c)^2)*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2), 1/8*(2*((4*A + 3*B)*cos(d*x + c) + 2*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + ((4*A + 3*B)*cos(d*x + c)^3 + (4*A + 3*B)*cos(d*x + c)^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sqrt{a \sec(dx + c) + a}}{\cos(dx + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*sqrt(a*sec(d*x + c) + a)/cos(d*x + c)^(3/2), x)

maple [B] time = 2.24, size = 342, normalized size = 2.26

$$\frac{(-1 + \cos(dx + c)) \left(-4A \arctan \left(\frac{\sqrt{-\frac{2}{1 + \cos(dx + c)}} (\cos(dx + c) + 1 - \sin(dx + c)) \sqrt{2}}{4} \right) (\cos^2(dx + c)) \sqrt{2} + 4A \arctan \left(\frac{\sqrt{-\frac{2}{1 + \cos(dx + c)}} (\cos(dx + c) + 1 - \sin(dx + c)) \sqrt{2}}{4} \right) \right)}{\cos(dx + c)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x)

[Out] -1/8/d*(-1+cos(d*x+c))*(-4*A*arctan(1/4*(-2/(1+cos(d*x+c))))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))*2^(1/2))*cos(d*x+c)^2*2^(1/2)+4*A*arctan(1/4*(-2/(1+cos(d*x+c))))^(1/2)*(cos(d*x+c)+1+sin(d*x+c))*2^(1/2))*cos(d*x+c)^2*2^(1/2)-3*B*arctan(1/4*(-2/(1+cos(d*x+c))))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))*2^(1/2))*cos(d*x+c)^2*2^(1/2)+3*B*arctan(1/4*(-2/(1+cos(d*x+c))))^(1/2)*(cos(d*x+c)+1+sin(d*x+c))*2^(1/2))*cos(d*x+c)^2*2^(1/2)+8*A*sin(d*x+c)*cos(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)

$$(d*x+c)))^{(1/2)}+6*B*\sin(d*x+c)*\cos(d*x+c)*(-2/(1+\cos(d*x+c)))^{(1/2)}+4*B*(-2/(1+\cos(d*x+c)))^{(1/2)}*\sin(d*x+c))*(a*(1+\cos(d*x+c))/\cos(d*x+c))^{(1/2)}/(-2/(1+\cos(d*x+c)))^{(1/2)}/\sin(d*x+c)^2/\cos(d*x+c)^{(3/2)}$$

maxima [B] time = 0.83, size = 1927, normalized size = 12.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="maxima")

[Out]
$$\begin{aligned} & -1/16*(4*(4*\sqrt{2}*\cos(3/2*\arctan2(\sin(d*x + c), \cos(d*x + c))))*\sin(2*d*x \\ & + 2*c) - 4*\sqrt{2}*\cos(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c))))*\sin(2*d*x + \\ & 2*c) - (\cos(2*d*x + 2*c)^2 + \sin(2*d*x + 2*c)^2 + 2*\cos(2*d*x + 2*c) + 1)* \\ & \log(2*\cos(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c)))^2 + 2*\sin(1/2*\arctan2(\sin \\ & (d*x + c), \cos(d*x + c)))^2 + 2*\sqrt{2}*\cos(1/2*\arctan2(\sin(d*x + c), \cos \\ & (d*x + c)))) + 2*\sqrt{2}*\sin(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c)))) + 2) + \\ & (\cos(2*d*x + 2*c)^2 + \sin(2*d*x + 2*c)^2 + 2*\cos(2*d*x + 2*c) + 1)*\log(2*\cos \\ & (1/2*\arctan2(\sin(d*x + c), \cos(d*x + c)))^2 + 2*\sin(1/2*\arctan2(\sin(d*x + \\ & c), \cos(d*x + c)))^2 + 2*\sqrt{2}*\cos(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c) \\ &)) - 2*\sqrt{2}*\sin(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c))) + 2) - (\cos(2*d \\ & *x + 2*c)^2 + \sin(2*d*x + 2*c)^2 + 2*\cos(2*d*x + 2*c) + 1)*\log(2*\cos(1/2*\ar \\ & ctan2(\sin(d*x + c), \cos(d*x + c)))^2 + 2*\sin(1/2*\arctan2(\sin(d*x + c), \cos \\ & (d*x + c)))^2 - 2*\sqrt{2}*\cos(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c)))) + 2*s \\ & \sqrt{2}*\sin(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c)))) + 2) + (\cos(2*d*x + 2*c \\ &)^2 + \sin(2*d*x + 2*c)^2 + 2*\cos(2*d*x + 2*c) + 1)*\log(2*\cos(1/2*\arctan2(\sin \\ & (d*x + c), \cos(d*x + c)))^2 + 2*\sin(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c) \\ &))^2 - 2*\sqrt{2}*\cos(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c)))) - 2*\sqrt{2}*\sin \\ & (1/2*\arctan2(\sin(d*x + c), \cos(d*x + c)))) + 2) - 4*(\sqrt{2}*\cos(2*d*x + 2 \\ & *c) + \sqrt{2})*\sin(3/2*\arctan2(\sin(d*x + c), \cos(d*x + c))) + 4*(\sqrt{2}*\cos \\ & (2*d*x + 2*c) + \sqrt{2})*\sin(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c))))*A*\sqrt{a} \\ & /(\cos(2*d*x + 2*c)^2 + \sin(2*d*x + 2*c)^2 + 2*\cos(2*d*x + 2*c) + 1) + \\ & (12*(\sqrt{2}*\sin(4*d*x + 4*c) + 2*\sqrt{2}*\sin(2*d*x + 2*c))*\cos(7/2*\arctan \\ & 2(\sin(d*x + c), \cos(d*x + c))) + 4*(\sqrt{2}*\sin(4*d*x + 4*c) + 2*\sqrt{2}*\sin \\ & (2*d*x + 2*c))*\cos(5/2*\arctan2(\sin(d*x + c), \cos(d*x + c))) - 4*(\sqrt{2}*\sin \\ & (4*d*x + 4*c) + 2*\sqrt{2}*\sin(2*d*x + 2*c))*\cos(3/2*\arctan2(\sin(d*x + c), \\ & \cos(d*x + c))) - 12*(\sqrt{2}*\sin(4*d*x + 4*c) + 2*\sqrt{2}*\sin(2*d*x + 2*c) \\ &)*\cos(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c))) - 3*(2*(2*\cos(2*d*x + 2*c) + \\ & 1)*\cos(4*d*x + 4*c) + \cos(4*d*x + 4*c)^2 + 4*\cos(2*d*x + 2*c)^2 + \sin(4*d*x \\ & + 4*c)^2 + 4*\sin(4*d*x + 4*c)*\sin(2*d*x + 2*c) + 4*\sin(2*d*x + 2*c)^2 + 4 \\ & *\cos(2*d*x + 2*c) + 1)*\log(2*\cos(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c)))^2 \\ & + 2*\sin(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c)))^2 + 2*\sqrt{2}*\cos(1/2*\ar \\ & ctan2(\sin(d*x + c), \cos(d*x + c))) + 2*\sqrt{2}*\sin(1/2*\arctan2(\sin(d*x + c), \\ & \cos(d*x + c)))) + 2) + 3*(2*(2*\cos(2*d*x + 2*c) + 1)*\cos(4*d*x + 4*c) + \cos \end{aligned}$$

$(4dx + 4c)^2 + 4\cos(2dx + 2c)^2 + \sin(4dx + 4c)^2 + 4\sin(4dx + 4c)\sin(2dx + 2c) + 4\sin(2dx + 2c)^2 + 4\cos(2dx + 2c) + 1) \log(2\cos(1/2\arctan2(\sin(dx + c), \cos(dx + c)))^2 + 2\sin(1/2\arctan2(\sin(dx + c), \cos(dx + c)))^2 + 2\sqrt{2}\cos(1/2\arctan2(\sin(dx + c), \cos(dx + c)))) - 2\sqrt{2}\sin(1/2\arctan2(\sin(dx + c), \cos(dx + c)))) + 2) - 3(2(2\cos(2dx + 2c) + 1)\cos(4dx + 4c) + \cos(4dx + 4c)^2 + 4\cos(2dx + 2c)^2 + \sin(4dx + 4c)^2 + 4\sin(4dx + 4c)\sin(2dx + 2c) + 4\sin(2dx + 2c)^2 + 4\cos(2dx + 2c) + 1) \log(2\cos(1/2\arctan2(\sin(dx + c), \cos(dx + c)))^2 + 2\sin(1/2\arctan2(\sin(dx + c), \cos(dx + c)))^2 - 2\sqrt{2}\cos(1/2\arctan2(\sin(dx + c), \cos(dx + c)))) + 2\sqrt{2}\sin(1/2\arctan2(\sin(dx + c), \cos(dx + c)))) + 2) + 3(2(2\cos(2dx + 2c) + 1)\cos(4dx + 4c) + \cos(4dx + 4c)^2 + 4\cos(2dx + 2c)^2 + \sin(4dx + 4c)^2 + 4\sin(4dx + 4c)\sin(2dx + 2c) + 4\sin(2dx + 2c)^2 + 4\cos(2dx + 2c) + 1) \log(2\cos(1/2\arctan2(\sin(dx + c), \cos(dx + c)))^2 + 2\sin(1/2\arctan2(\sin(dx + c), \cos(dx + c)))^2 - 2\sqrt{2}\cos(1/2\arctan2(\sin(dx + c), \cos(dx + c)))) - 2\sqrt{2}\sin(1/2\arctan2(\sin(dx + c), \cos(dx + c)))) + 2) - 12(\sqrt{2}\cos(4dx + 4c) + 2\sqrt{2}\cos(2dx + 2c) + \sqrt{2})\sin(7/2\arctan2(\sin(dx + c), \cos(dx + c))) - 4(\sqrt{2}\cos(4dx + 4c) + 2\sqrt{2}\cos(2dx + 2c) + \sqrt{2})\sin(5/2\arctan2(\sin(dx + c), \cos(dx + c)))) + 4(\sqrt{2}\cos(4dx + 4c) + 2\sqrt{2}\cos(2dx + 2c) + \sqrt{2})\sin(3/2\arctan2(\sin(dx + c), \cos(dx + c)))) + 12(\sqrt{2}\cos(4dx + 4c) + 2\sqrt{2}\cos(2dx + 2c) + \sqrt{2})\sin(1/2\arctan2(\sin(dx + c), \cos(dx + c)))) * B\sqrt{a} / (2(2\cos(2dx + 2c) + 1)\cos(4dx + 4c) + \cos(4dx + 4c)^2 + 4\cos(2dx + 2c)^2 + \sin(4dx + 4c)^2 + 4\sin(4dx + 4c)\sin(2dx + 2c) + 4\sin(2dx + 2c)^2 + 4\cos(2dx + 2c) + 1) / d$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \sqrt{a + \frac{a}{\cos(c+dx)}}}{\cos(c+dx)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(1/2))/cos(c + d*x)^(3/2), x)

[Out] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(1/2))/cos(c + d*x)^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a(\sec(c+dx)+1)}(A+B\sec(c+dx))}{\cos^{\frac{3}{2}}(c+dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+a*sec(d*x+c))**(1/2)/cos(d*x+c)**(3/2),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*(A + B*sec(c + d*x))/cos(c + d*x)**(3/2), x)
```

$$3.522 \quad \int \frac{\sqrt{a+a \sec(c+dx)} (A+B \sec(c+dx))}{\cos^{\frac{5}{2}}(c+dx)} dx$$

Optimal. Leaf size=196

$$\frac{a(6A+5B) \sin(c+dx)}{8d \cos^{\frac{3}{2}}(c+dx) \sqrt{a \sec(c+dx)+a}} + \frac{a(6A+5B) \sin(c+dx)}{12d \cos^{\frac{5}{2}}(c+dx) \sqrt{a \sec(c+dx)+a}} + \frac{\sqrt{a}(6A+5B) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{8d}$$

[Out] 1/8*(6*A+5*B)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*a^(1/2)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+1/3*A*B*sin(d*x+c)/d/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(1/2)+1/12*a*(6*A+5*B)*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2)+1/8*a*(6*A+5*B)*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2)

Rubi [A] time = 0.40, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2955, 4016, 3803, 3801, 215}

$$\frac{a(6A+5B) \sin(c+dx)}{8d \cos^{\frac{3}{2}}(c+dx) \sqrt{a \sec(c+dx)+a}} + \frac{a(6A+5B) \sin(c+dx)}{12d \cos^{\frac{5}{2}}(c+dx) \sqrt{a \sec(c+dx)+a}} + \frac{\sqrt{a}(6A+5B) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{8d}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]))/Cos[c + d*x]^(5/2), x]

[Out] (Sqrt[a]*(6*A + 5*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]])*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]/(8*d) + (a*B*Sin[c + d*x])/(3*d*Cos[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]) + (a*(6*A + 5*B)*Sin[c + d*x])/(12*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) + (a*(6*A + 5*B)*Sin[c + d*x])/(8*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]])

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 2955

Int[((a_.) + csc[(e_.) + (f_.)*(x_)])*(b_.)^(m_.)*(csc[(e_.) + (f_.)*(x_)])*(d_.) + (c_.))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] := Dist[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && In

tegerQ[n])

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.)
+ (a_.)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 3803

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.)
+ (a_.)], x_Symbol] :> Simp[(-2*b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/
(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(2*a*d*(n - 1))/(b*(2*n -
1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.)
+ (a_.)]*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A.)), x_Symbol] :> Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
+ Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && !
LtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a + a \sec(c + dx)} (A + B \sec(c + dx))}{\cos^{\frac{5}{2}}(c + dx)} dx &= \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \sec^{\frac{5}{2}}(c + dx) \sqrt{a + a \sec(c + dx)} dx \\
&= \frac{aB \sin(c + dx)}{3d \cos^{\frac{7}{2}}(c + dx) \sqrt{a + a \sec(c + dx)}} + \frac{1}{6} \left((6A + 5B) \sqrt{\cos(c + dx)} \sqrt{a + a \sec(c + dx)} \right) \\
&= \frac{aB \sin(c + dx)}{3d \cos^{\frac{7}{2}}(c + dx) \sqrt{a + a \sec(c + dx)}} + \frac{a(6A + 5B) \sin(c + dx)}{12d \cos^{\frac{5}{2}}(c + dx) \sqrt{a + a \sec(c + dx)}} \\
&= \frac{aB \sin(c + dx)}{3d \cos^{\frac{7}{2}}(c + dx) \sqrt{a + a \sec(c + dx)}} + \frac{a(6A + 5B) \sin(c + dx)}{12d \cos^{\frac{5}{2}}(c + dx) \sqrt{a + a \sec(c + dx)}} \\
&= \frac{aB \sin(c + dx)}{3d \cos^{\frac{7}{2}}(c + dx) \sqrt{a + a \sec(c + dx)}} + \frac{a(6A + 5B) \sin(c + dx)}{12d \cos^{\frac{5}{2}}(c + dx) \sqrt{a + a \sec(c + dx)}} \\
&= \frac{\sqrt{a} (6A + 5B) \sinh^{-1} \left(\frac{\sqrt{a} \tan(c + dx)}{\sqrt{a + a \sec(c + dx)}} \right) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)}}{8d}
\end{aligned}$$

Mathematica [A] time = 1.22, size = 131, normalized size = 0.67

$$\frac{\sec\left(\frac{1}{2}(c + dx)\right) \sqrt{a(\sec(c + dx) + 1)} \left(\sin\left(\frac{1}{2}(c + dx)\right) (4(6A + 5B) \cos(c + dx) + 3(6A + 5B) \cos(2(c + dx))) + 18 \right)}{48d \cos^{\frac{5}{2}}(c + dx)}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]))/Cos[c + d*x]^(5/2), x]

[Out] (Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(3*Sqrt[2]*(6*A + 5*B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^3 + (18*A + 31*B + 4*(6*A + 5*B)*Cos[c + d*x] + 3*(6*A + 5*B)*Cos[2*(c + d*x)])*Sin[(c + d*x)/2])/ (48*d*Cos[c + d*x]^(5/2))

fricas [A] time = 0.55, size = 439, normalized size = 2.24

$$\frac{4 \left(3 (6 A + 5 B) \cos (d x + c)^2 + 2 (6 A + 5 B) \cos (d x + c) + 8 B \right) \sqrt{\frac{a \cos (d x + c) + a}{\cos (d x + c)}} \sqrt{\cos (d x + c)} \sin (d x + c) + 3 \left((6 A + 5 B) \cos (d x + c)^4 + (6 A + 5 B) \cos (d x + c)^3 \right) \sqrt{a} \log \left(\frac{a \cos (d x + c)^3 - 4 \sqrt{a} \sqrt{\frac{a \cos (d x + c) + a}{\cos (d x + c)}} (\cos (d x + c) - 2) \sqrt{\cos (d x + c)} \sin (d x + c) - 7 a \cos (d x + c)^2 + 8 a}{\cos (d x + c)^3 + \cos (d x + c)^2} \right)}{d \cos (d x + c)^4 + d \cos (d x + c)^3} + \frac{1}{48} \frac{2 \left(3 (6 A + 5 B) \cos (d x + c)^2 + 2 (6 A + 5 B) \cos (d x + c) + 8 B \right) \sqrt{\frac{a \cos (d x + c) + a}{\cos (d x + c)}} \sqrt{\cos (d x + c)} \sin (d x + c) + 3 \left((6 A + 5 B) \cos (d x + c)^4 + (6 A + 5 B) \cos (d x + c)^3 \right) \sqrt{-a} \arctan \left(\frac{2 \sqrt{-a} \sqrt{\frac{a \cos (d x + c) + a}{\cos (d x + c)}} \sqrt{\cos (d x + c)} \sin (d x + c)}{a \cos (d x + c)^2 - a \cos (d x + c) - 2 a} \right)}{d \cos (d x + c)^4 + d \cos (d x + c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algorithm="fricas")

[Out] [1/96*(4*(3*(6*A + 5*B)*cos(d*x + c)^2 + 2*(6*A + 5*B)*cos(d*x + c) + 8*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 3*((6*A + 5*B)*cos(d*x + c)^4 + (6*A + 5*B)*cos(d*x + c)^3)*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3), 1/48*(2*(3*(6*A + 5*B)*cos(d*x + c)^2 + 2*(6*A + 5*B)*cos(d*x + c) + 8*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 3*((6*A + 5*B)*cos(d*x + c)^4 + (6*A + 5*B)*cos(d*x + c)^3)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sqrt{a \sec(dx + c) + a}}{\cos(dx + c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*sqrt(a*sec(d*x + c) + a)/cos(d*x + c)^(5/2), x)

maple [B] time = 2.29, size = 404, normalized size = 2.06

$$(-1 + \cos(dx + c)) \left(18A \arctan \left(\frac{\sqrt{\frac{2}{1 + \cos(dx + c)}} (\cos(dx + c) + 1 + \sin(dx + c)) \sqrt{2}}{4} \right) \sqrt{2} (\cos^3(dx + c)) - 18A \arctan \left(\frac{\sqrt{\frac{2}{1 + \cos(dx + c)}} (\cos(dx + c) + 1 + \sin(dx + c)) \sqrt{2}}{4} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((A+B*\sec(d*x+c))*(a+a*\sec(d*x+c))^{(1/2)}/\cos(d*x+c)^{(5/2)},x)$

[Out]
$$\begin{aligned} & -1/48/d*(-1+\cos(d*x+c))*(18*A*\arctan(1/4*(-2/(1+\cos(d*x+c))))^{(1/2)}*(\cos(d*x+c)+1+\sin(d*x+c))*2^{(1/2)}*2^{(1/2)}*\cos(d*x+c)^3-18*A*\arctan(1/4*(-2/(1+\cos(d*x+c))))^{(1/2)}*(\cos(d*x+c)+1-\sin(d*x+c))*2^{(1/2)}*2^{(1/2)}*\cos(d*x+c)^3+15*B*\arctan(1/4*(-2/(1+\cos(d*x+c))))^{(1/2)}*(\cos(d*x+c)+1+\sin(d*x+c))*2^{(1/2)}*2^{(1/2)}*\cos(d*x+c)^3-15*B*\arctan(1/4*(-2/(1+\cos(d*x+c))))^{(1/2)}*(\cos(d*x+c)+1-\sin(d*x+c))*2^{(1/2)}*2^{(1/2)}*\cos(d*x+c)^3+36*A*\sin(d*x+c)*\cos(d*x+c)^2*(-2/(1+\cos(d*x+c)))^{(1/2)}+30*B*\sin(d*x+c)*\cos(d*x+c)^2*(-2/(1+\cos(d*x+c)))^{(1/2)}+24*A*\sin(d*x+c)*\cos(d*x+c)*(-2/(1+\cos(d*x+c)))^{(1/2)}+20*B*\sin(d*x+c)*\cos(d*x+c)*(-2/(1+\cos(d*x+c)))^{(1/2)}+16*B*(-2/(1+\cos(d*x+c)))^{(1/2)}*\sin(d*x+c)) \\ & *(a*(1+\cos(d*x+c))/\cos(d*x+c))^{(1/2)}/\sin(d*x+c)^2/(-2/(1+\cos(d*x+c)))^{(1/2)}/\cos(d*x+c)^{(5/2)} \end{aligned}$$

maxima [B] time = 1.23, size = 3342, normalized size = 17.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((A+B*\sec(d*x+c))*(a+a*\sec(d*x+c))^{(1/2)}/\cos(d*x+c)^{(5/2)},x, \text{algorithm}="maxima")$

[Out]
$$\begin{aligned} & -1/96*(6*(12*(\sqrt{2}*\sin(4*d*x + 4*c) + 2*\sqrt{2}*\sin(2*d*x + 2*c))*\cos(7/2*\arctan2(\sin(d*x + c), \cos(d*x + c))) + 4*(\sqrt{2}*\sin(4*d*x + 4*c) + 2*\sqrt{2}*\sin(2*d*x + 2*c))*\cos(5/2*\arctan2(\sin(d*x + c), \cos(d*x + c))) - 4*(\sqrt{2}*\sin(4*d*x + 4*c) + 2*\sqrt{2}*\sin(2*d*x + 2*c))*\cos(3/2*\arctan2(\sin(d*x + c), \cos(d*x + c))) - 12*(\sqrt{2}*\sin(4*d*x + 4*c) + 2*\sqrt{2}*\sin(2*d*x + 2*c))*\cos(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c))) - 3*(2*(2*\cos(2*d*x + 2*c) + 1)*\cos(4*d*x + 4*c) + \cos(4*d*x + 4*c)^2 + 4*\cos(2*d*x + 2*c)^2 + \sin(4*d*x + 4*c)^2 + 4*\sin(4*d*x + 4*c)*\sin(2*d*x + 2*c) + 4*\sin(2*d*x + 2*c)^2 + 4*\cos(2*d*x + 2*c) + 1)*\log(2*\cos(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c))))^2 + 2*\sin(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c))))^2 + 2*\sqrt{2}*\cos(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c))) + 2*\sqrt{2}*\sin(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c))) + 2) + 3*(2*(2*\cos(2*d*x + 2*c) + 1)*\cos(4*d*x + 4*c) + \cos(4*d*x + 4*c)^2 + 4*\cos(2*d*x + 2*c)^2 + \sin(4*d*x + 4*c)^2 + 4*\sin(4*d*x + 4*c)*\sin(2*d*x + 2*c) + 4*\sin(2*d*x + 2*c)^2 + 4*\cos(2*d*x + 2*c) + 1)*\log(2*\cos(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c))))^2 + 2*\sin(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c))))^2 + 2*\sqrt{2}*\cos(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c))) - 2*\sqrt{2}*\sin(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c))) + 2) - 3*(2*(2*\cos(2*d*x + 2*c) + 1)*\cos(4*d*x + 4*c) + \cos(4*d*x + 4*c)^2 + 4*\cos(2*d*x + 2*c)^2 + \sin(4*d*x + 4*c)^2 + 4*\sin(4*d*x + 4*c)*\sin(2*d*x + 2*c) + 4*\sin(2*d*x + 2*c)^2 + 4*\cos(2*d*x + 2*c) + 1)*\log(2*\cos(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c))))^2 + 2*\sin(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c))))^2 + 2*\sqrt{2}*\cos(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c))) - 2*\sqrt{2}*\sin(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c))) + 2) \end{aligned}$$

$$\begin{aligned}
& c)))^2 - 2\sqrt{2}\cos(1/2\arctan2(\sin(dx + c), \cos(dx + c))) + 2\sqrt{2} \\
&)\sin(1/2\arctan2(\sin(dx + c), \cos(dx + c))) + 2) + 3*(2*(2\cos(2dx + 2 \\
& *c) + 1)\cos(4dx + 4c) + \cos(4dx + 4c)^2 + 4\cos(2dx + 2c)^2 + \sin \\
& (4dx + 4c)^2 + 4\sin(4dx + 4c)\sin(2dx + 2c) + 4\sin(2dx + 2c)^2 \\
& + 4\cos(2dx + 2c) + 1)\log(2\cos(1/2\arctan2(\sin(dx + c), \cos(dx + c \\
&)))^2 + 2\sin(1/2\arctan2(\sin(dx + c), \cos(dx + c)))^2 - 2\sqrt{2}\cos(1/ \\
& 2\arctan2(\sin(dx + c), \cos(dx + c))) - 2\sqrt{2}\sin(1/2\arctan2(\sin(dx \\
& + c), \cos(dx + c))) + 2) - 12*(\sqrt{2}\cos(4dx + 4c) + 2\sqrt{2}\cos(2 \\
& dx + 2c) + \sqrt{2})\sin(7/2\arctan2(\sin(dx + c), \cos(dx + c))) - 4*(\sqrt{ \\
& t(2)\cos(4dx + 4c) + 2\sqrt{2}\cos(2dx + 2c) + \sqrt{2})\sin(5/2\arcta \\
& n2(\sin(dx + c), \cos(dx + c))) + 4*(\sqrt{2}\cos(4dx + 4c) + 2\sqrt{2)*c \\
& os(2dx + 2c) + \sqrt{2})\sin(3/2\arctan2(\sin(dx + c), \cos(dx + c))) + 1 \\
& 2*(\sqrt{2}\cos(4dx + 4c) + 2\sqrt{2}\cos(2dx + 2c) + \sqrt{2})\sin(1/2 \\
& *arctan2(\sin(dx + c), \cos(dx + c)))\sqrt{a}/(2*(2\cos(2dx + 2c) + 1 \\
&)\cos(4dx + 4c) + \cos(4dx + 4c)^2 + 4\cos(2dx + 2c)^2 + \sin(4dx \\
& + 4c)^2 + 4\sin(4dx + 4c)\sin(2dx + 2c) + 4\sin(2dx + 2c)^2 + 4c \\
& os(2dx + 2c) + 1) + (60*(\sqrt{2}\sin(6dx + 6c) + 3\sqrt{2}\sin(4dx \\
& + 4c) + 3\sqrt{2}\sin(2dx + 2c))\cos(11/2\arctan2(\sin(dx + c), \cos(dx \\
& + c))) + 20*(\sqrt{2}\sin(6dx + 6c) + 3\sqrt{2}\sin(4dx + 4c) + 3\sqrt{ \\
& t(2)\sin(2dx + 2c))\cos(9/2\arctan2(\sin(dx + c), \cos(dx + c))) + 168*(\\
& \sqrt{2}\sin(6dx + 6c) + 3\sqrt{2}\sin(4dx + 4c) + 3\sqrt{2}\sin(2dx \\
& + 2c))\cos(7/2\arctan2(\sin(dx + c), \cos(dx + c))) - 168*(\sqrt{2}\sin(6* \\
& dx + 6c) + 3\sqrt{2}\sin(4dx + 4c) + 3\sqrt{2}\sin(2dx + 2c))\cos(5 \\
& /2\arctan2(\sin(dx + c), \cos(dx + c))) - 20*(\sqrt{2}\sin(6dx + 6c) + 3* \\
& \sqrt{2}\sin(4dx + 4c) + 3\sqrt{2}\sin(2dx + 2c))\cos(3/2\arctan2(\sin(\\
& dx + c), \cos(dx + c))) - 60*(\sqrt{2}\sin(6dx + 6c) + 3\sqrt{2}\sin(4d \\
& *x + 4c) + 3\sqrt{2}\sin(2dx + 2c))\cos(1/2\arctan2(\sin(dx + c), \cos(d \\
& *x + c))) - 15*(2*(3\cos(4dx + 4c) + 3\cos(2dx + 2c) + 1)\cos(6dx + \\
& 6c) + \cos(6dx + 6c)^2 + 6*(3\cos(2dx + 2c) + 1)\cos(4dx + 4c) + \\
& 9\cos(4dx + 4c)^2 + 9\cos(2dx + 2c)^2 + 6*(\sin(4dx + 4c) + \sin(2d \\
& *x + 2c))\sin(6dx + 6c) + \sin(6dx + 6c)^2 + 9\sin(4dx + 4c)^2 + 1 \\
& 8\sin(4dx + 4c)\sin(2dx + 2c) + 9\sin(2dx + 2c)^2 + 6\cos(2dx + \\
& 2c) + 1)\log(2\cos(1/2\arctan2(\sin(dx + c), \cos(dx + c)))^2 + 2\sin(1/2 \\
& arctan2(\sin(dx + c), \cos(dx + c)))^2 + 2\sqrt{2}\cos(1/2\arctan2(\sin(dx \\
& + c), \cos(dx + c))) + 2\sqrt{2}\sin(1/2\arctan2(\sin(dx + c), \cos(dx + c \\
&))) + 2) + 15*(2*(3\cos(4dx + 4c) + 3\cos(2dx + 2c) + 1)\cos(6dx + 6 \\
& *c) + \cos(6dx + 6c)^2 + 6*(3\cos(2dx + 2c) + 1)\cos(4dx + 4c) + 9* \\
& \cos(4dx + 4c)^2 + 9\cos(2dx + 2c)^2 + 6*(\sin(4dx + 4c) + \sin(2dx \\
& + 2c))\sin(6dx + 6c) + \sin(6dx + 6c)^2 + 9\sin(4dx + 4c)^2 + 18* \\
& \sin(4dx + 4c)\sin(2dx + 2c) + 9\sin(2dx + 2c)^2 + 6\cos(2dx + 2 \\
& c) + 1)\log(2\cos(1/2\arctan2(\sin(dx + c), \cos(dx + c)))^2 + 2\sin(1/2 \\
& arctan2(\sin(dx + c), \cos(dx + c)))^2 + 2\sqrt{2}\cos(1/2\arctan2(\sin(dx + \\
& c), \cos(dx + c))) - 2\sqrt{2}\sin(1/2\arctan2(\sin(dx + c), \cos(dx + c))) \\
& + 2) - 15*(2*(3\cos(4dx + 4c) + 3\cos(2dx + 2c) + 1)\cos(6dx + 6c \\
&) + \cos(6dx + 6c)^2 + 6*(3\cos(2dx + 2c) + 1)\cos(4dx + 4c) + 9co
\end{aligned}$$

```

s(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x +
2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*si
n(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c)
+ 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arct
an2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c)
, cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) +
2) + 15*(2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c)
+ cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(
4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2
*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(
4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) +
1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan
2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c),
cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2
) - 60*(sqrt(2)*cos(6*d*x + 6*c) + 3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*c
os(2*d*x + 2*c) + sqrt(2))*sin(11/2*arctan2(sin(d*x + c), cos(d*x + c))) -
20*(sqrt(2)*cos(6*d*x + 6*c) + 3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2
*d*x + 2*c) + sqrt(2))*sin(9/2*arctan2(sin(d*x + c), cos(d*x + c))) - 168*(
sqrt(2)*cos(6*d*x + 6*c) + 3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d*x
+ 2*c) + sqrt(2))*sin(7/2*arctan2(sin(d*x + c), cos(d*x + c))) + 168*(sqrt
(2)*cos(6*d*x + 6*c) + 3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d*x + 2
*c) + sqrt(2))*sin(5/2*arctan2(sin(d*x + c), cos(d*x + c))) + 20*(sqrt(2)*c
os(6*d*x + 6*c) + 3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d*x + 2*c) +
sqrt(2))*sin(3/2*arctan2(sin(d*x + c), cos(d*x + c))) + 60*(sqrt(2)*cos(6*
d*x + 6*c) + 3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d*x + 2*c) + sqrt
(2))*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))))*B*sqrt(a)/(2*(3*cos(4*d*
x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 +
6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*cos(
2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) +
sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d*x
+ 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) + 1))/d

```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \sqrt{a + \frac{a}{\cos(c+dx)}}}{\cos(c+dx)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(1/2))/cos(c + d*x)^(5/2), x)

[Out] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(1/2))/cos(c + d*x)^(5/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*(a+a*sec(d*x+c))**(1/2)/cos(d*x+c)**(5/2),x)

[Out] Timed out

$$3.523 \quad \int \cos^{\frac{11}{2}}(c+dx)(a+a \sec(c+dx))^{3/2}(A+B \sec(c+dx)) dx$$

Optimal. Leaf size=275

$$\frac{2a^2(12A + 11B) \sin(c + dx) \cos^{\frac{7}{2}}(c + dx)}{99d\sqrt{a \sec(c + dx) + a}} + \frac{2a^2(168A + 187B) \sin(c + dx) \cos^{\frac{5}{2}}(c + dx)}{693d\sqrt{a \sec(c + dx) + a}} + \frac{4a^2(168A + 187B) \sin(c + dx)}{1155d\sqrt{a \sec(c + dx) + a}}$$

[Out] $4/1155*a^2*(168*A+187*B)*\cos(d*x+c)^{(3/2)}*\sin(d*x+c)/d/(a+a*\sec(d*x+c))^{(1/2)}+2/693*a^2*(168*A+187*B)*\cos(d*x+c)^{(5/2)}*\sin(d*x+c)/d/(a+a*\sec(d*x+c))^{(1/2)}+2/99*a^2*(12*A+11*B)*\cos(d*x+c)^{(7/2)}*\sin(d*x+c)/d/(a+a*\sec(d*x+c))^{(1/2)}+32/3465*a^2*(168*A+187*B)*\sin(d*x+c)/d/\cos(d*x+c)^{(1/2)}/(a+a*\sec(d*x+c))^{(1/2)}+16/3465*a^2*(168*A+187*B)*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/d/(a+a*\sec(d*x+c))^{(1/2)}+2/11*a*A*\cos(d*x+c)^{(9/2)}*\sin(d*x+c)*(a+a*\sec(d*x+c))^{(1/2)}/d$

Rubi [A] time = 0.71, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2955, 4017, 4015, 3805, 3804}

$$\frac{2a^2(12A + 11B) \sin(c + dx) \cos^{\frac{7}{2}}(c + dx)}{99d\sqrt{a \sec(c + dx) + a}} + \frac{2a^2(168A + 187B) \sin(c + dx) \cos^{\frac{5}{2}}(c + dx)}{693d\sqrt{a \sec(c + dx) + a}} + \frac{4a^2(168A + 187B) \sin(c + dx)}{1155d\sqrt{a \sec(c + dx) + a}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Cos}[c + d*x]^{(11/2)}*(a + a*\text{Sec}[c + d*x])^{(3/2)}*(A + B*\text{Sec}[c + d*x]), x]$

[Out] $(32*a^2*(168*A + 187*B)*\text{Sin}[c + d*x])/(3465*d*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sqrt}[a + a*\text{Sec}[c + d*x]]) + (16*a^2*(168*A + 187*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sin}[c + d*x])/(3465*d*\text{Sqrt}[a + a*\text{Sec}[c + d*x]]) + (4*a^2*(168*A + 187*B)*\text{Cos}[c + d*x]^{(3/2)}*\text{Sin}[c + d*x])/(1155*d*\text{Sqrt}[a + a*\text{Sec}[c + d*x]]) + (2*a^2*(168*A + 187*B)*\text{Cos}[c + d*x]^{(5/2)}*\text{Sin}[c + d*x])/(693*d*\text{Sqrt}[a + a*\text{Sec}[c + d*x]]) + (2*a^2*(12*A + 11*B)*\text{Cos}[c + d*x]^{(7/2)}*\text{Sin}[c + d*x])/(99*d*\text{Sqrt}[a + a*\text{Sec}[c + d*x]]) + (2*a*A*\text{Cos}[c + d*x]^{(9/2)}*\text{Sqrt}[a + a*\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(11*d)$

Rule 2955

$\text{Int}[(a_. + \text{csc}[(e_.) + (f_.)*(x_.)]*(b_.))^{(m_.)}*(\text{csc}[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))^{(n_.)}*((g_.)*\sin[(e_.) + (f_.)*(x_.)])^{(p_.)}, x_Symbol] := \text{Dist}[(g*\text{Csc}[e + f*x])^p*(g*\text{Sin}[e + f*x])^p, \text{Int}[(a + b*\text{Csc}[e + f*x])^m*(c + d*\text{Csc}[e + f*x])^n]/(g*\text{Csc}[e + f*x])^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, m, n, p\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& !\text{IntegerQ}[p] \&\& !(\text{IntegerQ}[m] \&\& \text{IntegerQ}[n])$

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^n*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_.)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
+ b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^n*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_.)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] :> Simp[(A*b^2*Co
t[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[
e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_.))^m*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \cos^{\frac{11}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))dx &= (\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}) \int \frac{(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))}{\cos^{\frac{11}{2}}(c+dx)} dx \\
&= \frac{2aA\cos^{\frac{9}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{11d} \\
&= \frac{2a^2(12A+11B)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{99d\sqrt{a+a\sec(c+dx)}} + \frac{2aA}{99d} \\
&= \frac{2a^2(168A+187B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{693d\sqrt{a+a\sec(c+dx)}} + \frac{2aA}{693d} \\
&= \frac{4a^2(168A+187B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{1155d\sqrt{a+a\sec(c+dx)}} + \frac{2aA}{1155d} \\
&= \frac{16a^2(168A+187B)\sqrt{\cos(c+dx)}\sin(c+dx)}{3465d\sqrt{a+a\sec(c+dx)}} + \frac{2aA}{3465d} \\
&= \frac{32a^2(168A+187B)\sin(c+dx)}{3465d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{16a^2(168A+187B)}{3465d}
\end{aligned}$$

Mathematica [A] time = 0.54, size = 131, normalized size = 0.48

$$\frac{2a\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a(\sec(c+dx)+1)}(35(21A+11B)\cos^4(c+dx)+(840A+935B)\cos^3(c+dx)+3465d(\cos(c+dx)+1))}{3465d}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]^(11/2)*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]), x]

[Out] (2*a*Sqrt[Cos[c + d*x]]*(2688*A + 2992*B + 8*(168*A + 187*B)*Cos[c + d*x] + 6*(168*A + 187*B)*Cos[c + d*x]^2 + (840*A + 935*B)*Cos[c + d*x]^3 + 35*(21*A + 11*B)*Cos[c + d*x]^4 + 315*A*Cos[c + d*x]^5)*Sqrt[a*(1 + Sec[c + d*x])]*Sin[c + d*x])/(3465*d*(1 + Cos[c + d*x]))

fricas [A] time = 0.42, size = 142, normalized size = 0.52

$$\frac{2(315Aa\cos(dx+c)^5 + 35(21A+11B)a\cos(dx+c)^4 + 5(168A+187B)a\cos(dx+c)^3 + 6(168A+187B)a\cos(dx+c)^2 + 2aA\cos(dx+c)\sqrt{a+a\sec(c+dx)}\sin(c+dx) + \frac{16a^2(168A+187B)}{3465d}}{3465(d\cos(c+dx)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(11/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] 2/3465*(315*A*a*cos(d*x + c)^5 + 35*(21*A + 11*B)*a*cos(d*x + c)^4 + 5*(168*A + 187*B)*a*cos(d*x + c)^3 + 6*(168*A + 187*B)*a*cos(d*x + c)^2 + 8*(168*A + 187*B)*a*cos(d*x + c) + 16*(168*A + 187*B)*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c) + d)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(a \sec(dx + c) + a)^{\frac{3}{2}} \cos(dx + c)^{\frac{11}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(11/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(11/2), x)

maple [A] time = 2.13, size = 153, normalized size = 0.56

$$2a(-1 + \cos(dx + c)) \left(315A \left(\cos^5(dx + c) \right) + 735A \left(\cos^4(dx + c) \right) + 385B \left(\cos^4(dx + c) \right) + 840A \left(\cos^3(dx + c) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(11/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x)

[Out] -2/3465/d*a*(-1+cos(d*x+c))*(315*A*cos(d*x+c)^5+735*A*cos(d*x+c)^4+385*B*cos(d*x+c)^4+840*A*cos(d*x+c)^3+935*B*cos(d*x+c)^3+1008*A*cos(d*x+c)^2+1122*B*cos(d*x+c)^2+1344*A*cos(d*x+c)+1496*B*cos(d*x+c)+2688*A+2992*B)*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)

maxima [B] time = 0.76, size = 703, normalized size = 2.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(11/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="maxima")


```
[Out] 1/110880*(21*sqrt(2)*(3630*a*cos(10/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c)))*sin(11/2*d*x + 11/2*c) + 990*a*cos(8/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c)))*sin(11/2*d*x + 11/2*c) + 429*a*cos(6/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c)))*sin(11/2*d*x + 11/2*c) + 165*a*cos(4/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c)))*sin(11/2*d*x + 11/2*c) + 55*a*cos(2/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c)))*sin(11/2*d*x + 11/2*c) - 3630*a*cos(11/2*d*x + 11/2*c)*sin(10/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) - 990*a*cos(11/2*d*x + 11/2*c)*sin(8/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) - 429*a*cos(11/2*d*x + 11/2*c)*sin(6/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) - 165*a*cos(11/2*d*x + 11/2*c)*sin(4/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) - 55*a*cos(11/2*d*x + 11/2*c)*sin(2/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) + 30*a*sin(11/2*d*x + 11/2*c) + 55*a*sin(9/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) + 165*a*sin(7/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) + 429*a*sin(5/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) + 990*a*sin(3/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) + 3630*a*sin(1/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))))*A*sqrt(a) - 44*sqrt(2)*(189*(10*a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*cos(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 7*(270*a*cos(4*d*x + 4*c) + 27*a*cos(2*d*x + 2*c) + 5*a)*sin(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 135*a*sin(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 189*a*sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1050*a*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1890*a*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*B*sqrt(a))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c + dx)^{11/2} \left(A + \frac{B}{\cos(c + dx)} \right) \left(a + \frac{a}{\cos(c + dx)} \right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(11/2)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(3/2), x)
```

```
[Out] int(cos(c + d*x)^(11/2)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(11/2)*(a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)), x)
```

```
[Out] Timed out
```

$$3.524 \quad \int \cos^{\frac{9}{2}}(c+dx)(a+a \sec(c+dx))^{3/2}(A+B \sec(c+dx)) dx$$

Optimal. Leaf size=228

$$\frac{2a^2(10A+9B) \sin(c+dx) \cos^{\frac{5}{2}}(c+dx)}{63d\sqrt{a \sec(c+dx)+a}} + \frac{2a^2(34A+39B) \sin(c+dx) \cos^{\frac{3}{2}}(c+dx)}{105d\sqrt{a \sec(c+dx)+a}} + \frac{8a^2(34A+39B) \sin(c+dx)}{315d\sqrt{a \sec(c+dx)+a}}$$

[Out] $2/105*a^2*(34*A+39*B)*\cos(d*x+c)^{(3/2)}*\sin(d*x+c)/d/(a+a*\sec(d*x+c))^{(1/2)}+2/63*a^2*(10*A+9*B)*\cos(d*x+c)^{(5/2)}*\sin(d*x+c)/d/(a+a*\sec(d*x+c))^{(1/2)}+16/315*a^2*(34*A+39*B)*\sin(d*x+c)/d/\cos(d*x+c)^{(1/2)}/(a+a*\sec(d*x+c))^{(1/2)}+8/315*a^2*(34*A+39*B)*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/d/(a+a*\sec(d*x+c))^{(1/2)}+2/9*a*A*\cos(d*x+c)^{(7/2)}*\sin(d*x+c)*(a+a*\sec(d*x+c))^{(1/2)}/d$

Rubi [A] time = 0.69, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2955, 4017, 4015, 3805, 3804}

$$\frac{2a^2(10A+9B) \sin(c+dx) \cos^{\frac{5}{2}}(c+dx)}{63d\sqrt{a \sec(c+dx)+a}} + \frac{2a^2(34A+39B) \sin(c+dx) \cos^{\frac{3}{2}}(c+dx)}{105d\sqrt{a \sec(c+dx)+a}} + \frac{8a^2(34A+39B) \sin(c+dx)}{315d\sqrt{a \sec(c+dx)+a}}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]

[Out] $(16*a^2*(34*A+39*B)*\sin[c+d*x])/(315*d*\sqrt{\cos[c+d*x]}*\sqrt{a+a*\sec[c+d*x]})+(8*a^2*(34*A+39*B)*\sqrt{\cos[c+d*x]}*\sin[c+d*x])/(315*d*\sqrt{a+a*\sec[c+d*x]})+(2*a^2*(34*A+39*B)*\cos[c+d*x]^{(3/2)}*\sin[c+d*x])/(105*d*\sqrt{a+a*\sec[c+d*x]})+(2*a^2*(10*A+9*B)*\cos[c+d*x]^{(5/2)}*\sin[c+d*x])/(63*d*\sqrt{a+a*\sec[c+d*x]})+(2*a*A*\cos[c+d*x]^{(7/2)}*\sqrt{a+a*\sec[c+d*x]}*\sin[c+d*x])/(9*d)$

Rule 2955

Int[((a_.) + csc[(e_.) + (f_.)*(x_)])*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dist[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && IntegerQ[n])

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^n*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
+ b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^n*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[
e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^m*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))dx &= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \frac{(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))}{\sec(c+dx)}dx \\
&= \frac{2aA\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{9d} + \frac{2a^2(10A+9B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{63d\sqrt{a+a\sec(c+dx)}} + \frac{2aA\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{63d\sqrt{a+a\sec(c+dx)}} \\
&= \frac{2a^2(34A+39B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{105d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2(10A+9B)\cos^{\frac{1}{2}}(c+dx)\sin(c+dx)}{105d\sqrt{a+a\sec(c+dx)}} \\
&= \frac{8a^2(34A+39B)\sqrt{\cos(c+dx)}\sin(c+dx)}{315d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2(10A+9B)\sin(c+dx)}{315d\sqrt{a+a\sec(c+dx)}} \\
&= \frac{16a^2(34A+39B)\sin(c+dx)}{315d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{8a^2(34A+39B)\sin(c+dx)}{315d\sqrt{a+a\sec(c+dx)}}
\end{aligned}$$

Mathematica [A] time = 0.42, size = 118, normalized size = 0.52

$$\frac{2a\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a(\sec(c+dx)+1)}\left(5(17A+9B)\cos^3(c+dx)+3(34A+39B)\cos^2(c+dx)+4(34A+39B)\cos(c+dx)+4(34A+39B)\right)}{315d(\cos(c+dx)+1)}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]

[Out] (2*a*Sqrt[Cos[c + d*x]]*(8*(34*A + 39*B) + 4*(34*A + 39*B)*Cos[c + d*x] + 3*(34*A + 39*B)*Cos[c + d*x]^2 + 5*(17*A + 9*B)*Cos[c + d*x]^3 + 35*A*Cos[c + d*x]^4)*Sqrt[a*(1 + Sec[c + d*x])]*Sin[c + d*x])/(315*d*(1 + Cos[c + d*x]))

fricas [A] time = 0.43, size = 124, normalized size = 0.54

$$\frac{2\left(35Aa\cos(dx+c)^4+5(17A+9B)a\cos(dx+c)^3+3(34A+39B)a\cos(dx+c)^2+4(34A+39B)a\cos(dx+c)\right)}{315(d\cos(dx+c)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] 2/315*(35*A*a*cos(d*x + c)^4 + 5*(17*A + 9*B)*a*cos(d*x + c)^3 + 3*(34*A + 39*B)*a*cos(d*x + c)^2 + 4*(34*A + 39*B)*a*cos(d*x + c) + 8*(34*A + 39*B)*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c) + d)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(a \sec(dx + c) + a)^{\frac{3}{2}} \cos(dx + c)^{\frac{9}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(9/2), x)

maple [A] time = 2.57, size = 131, normalized size = 0.57

$$2a(-1 + \cos(dx + c)) \left(35A \left(\cos^4(dx + c) \right) + 85A \left(\cos^3(dx + c) \right) + 45B \left(\cos^3(dx + c) \right) + 102A \left(\cos^2(dx + c) \right) \right)$$

315d s

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x)

[Out] -2/315/d*a*(-1+cos(d*x+c))*(35*A*cos(d*x+c)^4+85*A*cos(d*x+c)^3+45*B*cos(d*x+c)^2+102*A*cos(d*x+c)^2+117*B*cos(d*x+c)^2+136*A*cos(d*x+c)+156*B*cos(d*x+c)+272*A+312*B)*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)

maxima [B] time = 0.70, size = 558, normalized size = 2.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] 1/5040*(sqrt(2)*(3780*a*cos(8/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 1050*a*cos(2/3*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 378*a*cos(4/9*arctan2(sin(

$$\begin{aligned} & \cos(9/2*d*x + 9/2*c), \cos(9/2*d*x + 9/2*c)))*\sin(9/2*d*x + 9/2*c) + 135*a*\cos(2 \\ & /9*\arctan2(\sin(9/2*d*x + 9/2*c), \cos(9/2*d*x + 9/2*c)))*\sin(9/2*d*x + 9/2*c \\ &) - 3780*a*\cos(9/2*d*x + 9/2*c)*\sin(8/9*\arctan2(\sin(9/2*d*x + 9/2*c), \cos(9 \\ & /2*d*x + 9/2*c)) - 1050*a*\cos(9/2*d*x + 9/2*c)*\sin(2/3*\arctan2(\sin(9/2*d*x \\ & + 9/2*c), \cos(9/2*d*x + 9/2*c)) - 378*a*\cos(9/2*d*x + 9/2*c)*\sin(4/9*\arct \\ & an2(\sin(9/2*d*x + 9/2*c), \cos(9/2*d*x + 9/2*c)) - 135*a*\cos(9/2*d*x + 9/2* \\ & c)*\sin(2/9*\arctan2(\sin(9/2*d*x + 9/2*c), \cos(9/2*d*x + 9/2*c))) + 70*a*\sin(\\ & 9/2*d*x + 9/2*c) + 135*a*\sin(7/9*\arctan2(\sin(9/2*d*x + 9/2*c), \cos(9/2*d*x \\ & + 9/2*c))) + 378*a*\sin(5/9*\arctan2(\sin(9/2*d*x + 9/2*c), \cos(9/2*d*x + 9/2* \\ & c))) + 1050*a*\sin(1/3*\arctan2(\sin(9/2*d*x + 9/2*c), \cos(9/2*d*x + 9/2*c))) \\ & + 3780*a*\sin(1/9*\arctan2(\sin(9/2*d*x + 9/2*c), \cos(9/2*d*x + 9/2*c))) * A * \text{sqrt}(a) \\ & - 6*\text{sqrt}(2)*(175*a*\cos(7/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) \\ &))*\sin(2*d*x + 2*c) - 5*(35*a*\cos(2*d*x + 2*c) + 6*a)*\sin(7/4*\arctan2(\sin(2 \\ & *d*x + 2*c), \cos(2*d*x + 2*c))) - 126*a*\sin(5/4*\arctan2(\sin(2*d*x + 2*c), c \\ & os(2*d*x + 2*c))) - 175*a*\sin(3/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c \\ &))) - 1470*a*\sin(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) * B * \text{sqrt}(a \\ &))/d \end{aligned}$$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c + dx)^{9/2} \left(A + \frac{B}{\cos(c + dx)} \right) \left(a + \frac{a}{\cos(c + dx)} \right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^(9/2)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(3/2), x)

[Out] int(cos(c + d*x)^(9/2)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(3/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**(9/2)*(a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)), x)

[Out] Timed out

$$3.525 \quad \int \cos^2(c+dx)(a+a \sec(c+dx))^{3/2}(A+B \sec(c+dx)) dx$$

Optimal. Leaf size=181

$$\frac{2a^2(8A+7B) \sin(c+dx) \cos^{\frac{3}{2}}(c+dx)}{35d\sqrt{a \sec(c+dx)+a}} + \frac{2a^2(52A+63B) \sin(c+dx) \sqrt{\cos(c+dx)}}{105d\sqrt{a \sec(c+dx)+a}} + \frac{4a^2(52A+63B) \sin(c+dx)}{105d\sqrt{\cos(c+dx)} \sqrt{a \sec(c+dx)}}$$

[Out] 2/35*a^2*(8*A+7*B)*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+4/105*a^2*(52*A+63*B)*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+2/105*a^2*(52*A+63*B)*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^(1/2)+2/7*a*A*cos(d*x+c)^(5/2)*sin(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d

Rubi [A] time = 0.62, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2955, 4017, 4015, 3805, 3804}

$$\frac{2a^2(8A+7B) \sin(c+dx) \cos^{\frac{3}{2}}(c+dx)}{35d\sqrt{a \sec(c+dx)+a}} + \frac{2a^2(52A+63B) \sin(c+dx) \sqrt{\cos(c+dx)}}{105d\sqrt{a \sec(c+dx)+a}} + \frac{4a^2(52A+63B) \sin(c+dx)}{105d\sqrt{\cos(c+dx)} \sqrt{a \sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]

[Out] (4*a^2*(52*A + 63*B)*Sin[c + d*x])/(105*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2*(52*A + 63*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(105*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2*(8*A + 7*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(35*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*A*cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(7*d)

Rule 2955

Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)]^(p_.), x_Symbol] := Dist[(g*Csc[e + f*x])^p*(g*SIN[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && IntegerQ[n])

Rule 3804

Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)], x_Symbol] := Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]])*

$\text{Sqrt}[d*\text{Csc}[e + f*x]], x] /; \text{FreeQ}[\{a, b, d, e, f\}, x] \ \&\& \ \text{EqQ}[a^2 - b^2, 0]$

Rule 3805

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(d_.))^n*\text{Sqrt}[\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] :> \text{Simp}[(a*\text{Cot}[e + f*x]*(d*\text{Csc}[e + f*x])^n)/(f*n*\text{Sqrt}[a + b*\text{Csc}[e + f*x]]), x] + \text{Dist}[(a*(2*n + 1))/(2*b*d*n), \text{Int}[\text{Sqrt}[a + b*\text{Csc}[e + f*x]]*(d*\text{Csc}[e + f*x])^{n + 1}, x], x] /; \text{FreeQ}[\{a, b, d, e, f\}, x] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{LtQ}[n, -2^{(-1)}] \ \&\& \ \text{IntegerQ}[2*n]$

Rule 4015

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(d_.))^n*\text{Sqrt}[\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]*(\text{csc}[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> \text{Simp}[(A*b^2*\text{Cot}[e + f*x]*(d*\text{Csc}[e + f*x])^n)/(a*f*n*\text{Sqrt}[a + b*\text{Csc}[e + f*x]]), x] + \text{Dist}[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), \text{Int}[\text{Sqrt}[a + b*\text{Csc}[e + f*x]]*(d*\text{Csc}[e + f*x])^{n + 1}, x], x] /; \text{FreeQ}[\{a, b, d, e, f, A, B\}, x] \ \&\& \ \text{NeQ}[A*b - a*B, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{NeQ}[A*b*(2*n + 1) + 2*a*B*n, 0] \ \&\& \ \text{LtQ}[n, 0]$

Rule 4017

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(d_.))^n*(\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m*(\text{csc}[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> \text{Simp}[(a*A*\text{Cot}[e + f*x]*(a + b*\text{Csc}[e + f*x])^{m - 1}*(d*\text{Csc}[e + f*x])^n)/(f*n), x] - \text{Dist}[b/(a*d*n), \text{Int}[(a + b*\text{Csc}[e + f*x])^{m - 1}*(d*\text{Csc}[e + f*x])^{n + 1}*\text{Simp}[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*\text{Csc}[e + f*x], x], x], x] /; \text{FreeQ}[\{a, b, d, e, f, A, B\}, x] \ \&\& \ \text{NeQ}[A*b - a*B, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{GtQ}[m, 1/2] \ \&\& \ \text{LtQ}[n, -1]$

Rubi steps

$$\begin{aligned}
\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))dx &= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \frac{(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}dx \\
&= \frac{2aA\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{7d} \\
&= \frac{2a^2(8A+7B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{35d\sqrt{a+a\sec(c+dx)}} + \frac{2aA\cos^{\frac{1}{2}}(c+dx)\sin(c+dx)}{35d} \\
&= \frac{2a^2(52A+63B)\sqrt{\cos(c+dx)}\sin(c+dx)}{105d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2(52A+63B)\sin(c+dx)}{105d} \\
&= \frac{4a^2(52A+63B)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{2a^2(52A+63B)\sin(c+dx)}{105d}
\end{aligned}$$

Mathematica [A] time = 0.32, size = 100, normalized size = 0.55

$$\frac{2a\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a(\sec(c+dx)+1)}\left(3(13A+7B)\cos^2(c+dx)+(52A+63B)\cos(c+dx)+2(52A+63B)\right)}{105d(\cos(c+dx)+1)}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]), x]

[Out] (2*a*Sqrt[Cos[c + d*x]]*(2*(52*A + 63*B) + (52*A + 63*B)*Cos[c + d*x] + 3*(13*A + 7*B)*Cos[c + d*x]^2 + 15*A*Cos[c + d*x]^3)*Sqrt[a*(1 + Sec[c + d*x])]*Sin[c + d*x])/(105*d*(1 + Cos[c + d*x]))

fricas [A] time = 0.47, size = 105, normalized size = 0.58

$$\frac{2\left(15Aa\cos(dx+c)^3+3(13A+7B)a\cos(dx+c)^2+(52A+63B)a\cos(dx+c)+2(52A+63B)a\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}}{105(d\cos(dx+c)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)), x, algorith="fricas")

[Out] 2/105*(15*A*a*cos(d*x + c)^3 + 3*(13*A + 7*B)*a*cos(d*x + c)^2 + (52*A + 63*B)*a*cos(d*x + c) + 2*(52*A + 63*B)*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c) + d)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(a \sec(dx + c) + a)^{\frac{3}{2}} \cos(dx + c)^{\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorith="giac")

[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(7/2), x)

maple [A] time = 1.95, size = 109, normalized size = 0.60

$$\frac{2a(-1 + \cos(dx + c)) \left(15A \left(\cos^3(dx + c) \right) + 39A \left(\cos^2(dx + c) \right) + 21B \left(\cos^2(dx + c) \right) + 52A \cos(dx + c) + 63B \right)}{105d \sin(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x)

[Out] -2/105/d*a*(-1+cos(d*x+c))*(15*A*cos(d*x+c)^3+39*A*cos(d*x+c)^2+21*B*cos(d*x+c)^2+52*A*cos(d*x+c)+63*B*cos(d*x+c)+104*A+126*B)*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)

maxima [B] time = 0.72, size = 451, normalized size = 2.49

$$\frac{\sqrt{2} \left(735 a \cos \left(\frac{6}{7} \arctan \left(\sin \left(\frac{7}{2} dx + \frac{7}{2} c \right), \cos \left(\frac{7}{2} dx + \frac{7}{2} c \right) \right) \right) \sin \left(\frac{7}{2} dx + \frac{7}{2} c \right) + 175 a \cos \left(\frac{4}{7} \arctan \left(\sin \left(\frac{7}{2} dx + \frac{7}{2} c \right), \cos \left(\frac{7}{2} dx + \frac{7}{2} c \right) \right) \right) \right)}{\sin(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorith="maxima")

[Out] 1/840*(sqrt(2)*(735*a*cos(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) + 175*a*cos(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) + 63*a*cos(2/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) - 735*a*cos(7/2*d*x + 7/2*c)*sin(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) - 175*a*cos(7/2*d*x + 7/2*c)*sin(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) - 63*a*cos(7/2*d*x + 7/2*c)*sin(2/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 30*a*sin(7/2*d*x + 7/2*c) + 63*a*sin(5/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 175*a*sin(3/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))))

$(\frac{7}{2}dx + \frac{7}{2}c), \cos(\frac{7}{2}dx + \frac{7}{2}c))) + 735a \sin(\frac{1}{7} \arctan2(\sin(\frac{7}{2}dx + \frac{7}{2}c), \cos(\frac{7}{2}dx + \frac{7}{2}c))) * A \sqrt{a} - 84 * (10 \sqrt{2}) * a * \cos(\frac{5}{4} \arctan2(\sin(2dx + 2c), \cos(2dx + 2c))) * \sin(2dx + 2c) - 5 \sqrt{2} * a * \sin(\frac{3}{4} \arctan2(\sin(2dx + 2c), \cos(2dx + 2c))) - 10 \sqrt{2} * a * \sin(\frac{1}{4} \arctan2(\sin(2dx + 2c), \cos(2dx + 2c))) - (10 \sqrt{2}) * a * \cos(2dx + 2c) + \sqrt{2} * a * \sin(\frac{5}{4} \arctan2(\sin(2dx + 2c), \cos(2dx + 2c))) * B \sqrt{a}) / d$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c + dx)^{7/2} \left(A + \frac{B}{\cos(c + dx)} \right) \left(a + \frac{a}{\cos(c + dx)} \right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(c + d*x)^(7/2)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(3/2),x)`

[Out] `int(cos(c + d*x)^(7/2)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(3/2), x)`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)**(7/2)*(a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)),x)`

[Out] Timed out

$$3.526 \quad \int \cos^{\frac{5}{2}}(c+dx)(a+a \sec(c+dx))^{3/2}(A+B \sec(c+dx)) dx$$

Optimal. Leaf size=131

$$\frac{8a^2(3A+5B)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}} + \frac{2a(3A+5B)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}{15d} + \frac{2A\sin(c+dx)}{15d}$$

[Out] $2/5*A*\cos(d*x+c)^{(3/2)}*(a+a*\sec(d*x+c))^{(3/2)}*\sin(d*x+c)/d+8/15*a^2*(3*A+5*B)*\sin(d*x+c)/d/\cos(d*x+c)^{(1/2)}/(a+a*\sec(d*x+c))^{(1/2)}+2/15*a*(3*A+5*B)*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}*(a+a*\sec(d*x+c))^{(1/2)}/d$

Rubi [A] time = 0.38, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {2955, 4013, 3809, 3804}

$$\frac{8a^2(3A+5B)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}} + \frac{2a(3A+5B)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}{15d} + \frac{2A\sin(c+dx)}{15d}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]

[Out] $(8*a^2*(3*A + 5*B)*\sin[c + d*x])/(15*d*\sqrt{\cos[c + d*x]}*\sqrt{a + a*\sec[c + d*x]}) + (2*a*(3*A + 5*B)*\sqrt{\cos[c + d*x]}*\sqrt{a + a*\sec[c + d*x]}*\sin[c + d*x])/(15*d) + (2*A*\cos[c + d*x]^{(3/2)}*(a + a*\sec[c + d*x])^{(3/2)}*\sin[c + d*x])/(5*d)$

Rule 2955

Int[((a_.) + csc[(e_.) + (f_.)*(x_)])*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dist[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && IntegerQ[n])

Rule 3804

Int[Sqrt[csc[(e_.) + (f_.)*(x_)])*(b_.) + (a_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 3809

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*
(d*Csc[e + f*x])^n)/(f*m), x] + Dist[(b*(2*m - 1))/(d*m), Int[(a + b*Csc[e
+ f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && GtQ[m, 1/2] && IntegerQ[2
*m]
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
- b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] && !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \cos^5(c + dx)(a + a \sec(c + dx))^{3/2}(A + B \sec(c + dx)) dx &= \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)}\right) \int \frac{(a + a \sec(c + dx))^{3/2}(A + B \sec(c + dx))}{\sec(c + dx)} dx \\ &= \frac{2A \cos^3(c + dx)(a + a \sec(c + dx))^{3/2} \sin(c + dx)}{5d} \\ &= \frac{2a(3A + 5B)\sqrt{\cos(c + dx)} \sqrt{a + a \sec(c + dx)} \sin(c + dx)}{15d} \\ &= \frac{8a^2(3A + 5B) \sin(c + dx)}{15d\sqrt{\cos(c + dx)} \sqrt{a + a \sec(c + dx)}} + \frac{2a(3A + 5B)}{15d} \end{aligned}$$

Mathematica [A] time = 0.29, size = 80, normalized size = 0.61

$$\frac{2a \sin(c + dx) \sqrt{\cos(c + dx)} \sqrt{a(\sec(c + dx) + 1)} \left((9A + 5B) \cos(c + dx) + 3A \cos^2(c + dx) + 18A + 25B \right)}{15d(\cos(c + dx) + 1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]
), x]
```

[Out] (2*a*Sqrt[Cos[c + d*x]]*(18*A + 25*B + (9*A + 5*B)*Cos[c + d*x] + 3*A*Cos[c + d*x]^2)*Sqrt[a*(1 + Sec[c + d*x])]*Sin[c + d*x])/(15*d*(1 + Cos[c + d*x]))

fricas [A] time = 0.42, size = 86, normalized size = 0.66

$$\frac{2 \left(3 A a \cos(dx + c)^2 + (9 A + 5 B) a \cos(dx + c) + (18 A + 25 B) a \right) \sqrt{\frac{a \cos(dx + c) + a}{\cos(dx + c)}} \sqrt{\cos(dx + c)} \sin(dx + c)}{15 (d \cos(dx + c) + d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] 2/15*(3*A*a*cos(d*x + c)^2 + (9*A + 5*B)*a*cos(d*x + c) + (18*A + 25*B)*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c) + d)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(a \sec(dx + c) + a)^{\frac{3}{2}} \cos(dx + c)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(5/2), x)

maple [A] time = 1.88, size = 87, normalized size = 0.66

$$\frac{2a(-1 + \cos(dx + c)) \left(3A \left(\cos^2(dx + c) \right) + 9A \cos(dx + c) + 5B \cos(dx + c) + 18A + 25B \right) \left(\sqrt{\cos(dx + c)} \right)}{15d \sin(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x)

[Out] -2/15/d*a*(-1+cos(d*x+c))*(3*A*cos(d*x+c)^2+9*A*cos(d*x+c)+5*B*cos(d*x+c)+18*A+25*B)*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)

maxima [B] time = 0.68, size = 276, normalized size = 2.11

$$3 \sqrt{2} \left(20 a \cos \left(\frac{4}{5} \arctan \left(\sin \left(\frac{5}{2} dx + \frac{5}{2} c \right), \cos \left(\frac{5}{2} dx + \frac{5}{2} c \right) \right) \right) \sin \left(\frac{5}{2} dx + \frac{5}{2} c \right) + 5 a \cos \left(\frac{2}{5} \arctan \left(\sin \left(\frac{5}{2} dx + \frac{5}{2} c \right), \cos \left(\frac{5}{2} dx + \frac{5}{2} c \right) \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] 1/60*(3*sqrt(2)*(20*a*cos(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) * sin(5/2*d*x + 5/2*c) + 5*a*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) * sin(5/2*d*x + 5/2*c) - 20*a*cos(5/2*d*x + 5/2*c)*sin(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) - 5*a*cos(5/2*d*x + 5/2*c)*sin(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 2*a*sin(5/2*d*x + 5/2*c) + 5*a*sin(3/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 20*a*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))))*A*sqrt(a) + 20*(sqrt(2)*a*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 9*sqrt(2)*a*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*B*sqrt(a))/d

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c + dx)^{5/2} \left(A + \frac{B}{\cos(c + dx)} \right) \left(a + \frac{a}{\cos(c + dx)} \right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^(5/2)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(3/2),x)

[Out] int(cos(c + d*x)^(5/2)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(3/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**(5/2)*(a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)),x)

[Out] Timed out

$$3.527 \quad \int \cos^2(c+dx)(a+a \sec(c+dx))^{3/2}(A+B \sec(c+dx)) dx$$

Optimal. Leaf size=145

$$\frac{2a^{3/2}B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d} + \frac{2a^2(4A+3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}} + \frac{2aA\sin(c+dx)}{d}$$

[Out] $2a^{3/2}B\operatorname{arcsinh}(a^{1/2}\tan(dx+c)/(a+a\sec(dx+c))^{1/2})\cos(dx+c)^{1/2}\sec(dx+c)^{1/2}/d+2/3a^2(4A+3B)\sin(dx+c)/d/\cos(dx+c)^{1/2}/(a+a\sec(dx+c))^{1/2}+2/3aA\sin(dx+c)\cos(dx+c)^{1/2}(a+a\sec(dx+c))^{1/2}/d$

Rubi [A] time = 0.44, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2955, 4017, 4015, 3801, 215}

$$\frac{2a^2(4A+3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}} + \frac{2a^{3/2}B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d} + \frac{2aA\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]

[Out] $(2a^{3/2}B\operatorname{ArcSinh}[(\operatorname{Sqrt}[a]\operatorname{Tan}[c+d*x])/(\operatorname{Sqrt}[a+a\operatorname{Sec}[c+d*x]])]*\operatorname{Sqrt}[\operatorname{Cos}[c+d*x]]*\operatorname{Sqrt}[\operatorname{Sec}[c+d*x]])/d + (2a^2(4A+3B)\operatorname{Sin}[c+d*x])/(3d*\operatorname{Sqrt}[\operatorname{Cos}[c+d*x]]*\operatorname{Sqrt}[a+a\operatorname{Sec}[c+d*x]]) + (2aA*\operatorname{Sqrt}[\operatorname{Cos}[c+d*x]]*\operatorname{Sqrt}[a+a\operatorname{Sec}[c+d*x]]*\operatorname{Sin}[c+d*x])/(3d)$

Rule 215

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Simp[ArcSinh[Rt[b, 2]*x]/Sqrt[a]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 2955

Int[((a_) + csc[(e_) + (f_)*(x_)]*(b_))^(m_)*(csc[(e_) + (f_)*(x_)]*(d_) + (c_))^(n_)*((g_)*sin[(e_) + (f_)*(x_)])^(p_), x_Symbol] := Dist[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && IntegerQ[n])

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.)
+ (a_.)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.)
+ (a_.)]*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
+ f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (
a_.))^(m_)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \cos^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))^{3/2}(A + B \sec(c + dx)) dx &= \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{(a + a \sec(c + dx))^{3/2}(A + B \sec(c + dx))}{\sec(c + dx)} dx \\
&= \frac{2aA\sqrt{\cos(c + dx)} \sqrt{a + a \sec(c + dx)} \sin(c + dx)}{3d} \\
&= \frac{2a^2(4A + 3B) \sin(c + dx)}{3d\sqrt{\cos(c + dx)} \sqrt{a + a \sec(c + dx)}} + \frac{2aA\sqrt{\cos(c + dx)}}{3d} \\
&= \frac{2a^2(4A + 3B) \sin(c + dx)}{3d\sqrt{\cos(c + dx)} \sqrt{a + a \sec(c + dx)}} + \frac{2aA\sqrt{\cos(c + dx)}}{3d} \\
&= \frac{2a^{3/2}B \sinh^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a+a \sec(c+dx)}}\right) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)}}{d}
\end{aligned}$$

Mathematica [A] time = 0.44, size = 101, normalized size = 0.70

$$\frac{2a^2 \sin(c + dx) \left(\sqrt{1 - \sec(c + dx)} (A \cos(c + dx) + 5A + 3B) + 3B\sqrt{\sec(c + dx)} \sin^{-1} \left(\sqrt{1 - \sec(c + dx)} \right) \right)}{3d\sqrt{\cos(c + dx)} - 1 \sqrt{a(\sec(c + dx) + 1)}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]

[Out] (2*a^2*((5*A + 3*B + A*Cos[c + d*x])*Sqrt[1 - Sec[c + d*x]] + 3*B*ArcSin[Sqrt[1 - Sec[c + d*x]]]*Sqrt[Sec[c + d*x]])*Sin[c + d*x])/(3*d*Sqrt[-1 + Cos[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])

fricas [A] time = 0.49, size = 343, normalized size = 2.37

$$\left[\frac{4(Aa \cos(dx + c) + (5A + 3B)a) \sqrt{\frac{a \cos(dx+c)+a}{\cos(dx+c)}} \sqrt{\cos(dx + c)} \sin(dx + c) + 3(Ba \cos(dx + c) + Ba) \sqrt{a} \log \left(\frac{a \cos(dx+c)+a}{\cos(dx+c)} \right)}{6(d \cos(dx + c) + d)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] [1/6*(4*(A*a*cos(d*x + c) + (5*A + 3*B)*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 3*(B*a*cos(d*x + c) + B*a)*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(d*cos(d*x + c) + d), 1/3*(2*(A*a*cos(d*x + c) + (5*A + 3*B)*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 3*(B*a*cos(d*x + c) + B*a)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*x + c) + d)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(a \sec(dx + c) + a)^{\frac{3}{2}} \cos(dx + c)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/2), x)

maple [A] time = 1.80, size = 201, normalized size = 1.39

$$a \left(\sqrt{\cos(dx + c)} \right) \sqrt{\frac{a(1 + \cos(dx + c))}{\cos(dx + c)}} \left(3B\sqrt{2} \arctan \left(\frac{\sqrt{-\frac{2}{1 + \cos(dx + c)}} (\cos(dx + c) + 1 + \sin(dx + c))\sqrt{2}}{4} \right) \right) \sqrt{-\frac{2}{1 + \cos(dx + c)}} \sin(dx)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x)

[Out] -1/6/d*a*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(3*B*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d*x+c))*2^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-3*B*2^(1/2)*(-2/(1+cos(d*x+c)))^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))*2^(1/2))*sin(d*x+c)+4*A*cos(d*x+c)^2+16*A*cos(d*x+c)+12*B*cos(d*x+c)-20*A-12*B)/sin(d*x+c)

maxima [B] time = 0.86, size = 583, normalized size = 4.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorith="maxima")

[Out] 1/30*(10*(sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 9*sqrt(2)*a*sin(1/2*d*x + 1/2*c))*A*sqrt(a) + 3*(2*sqrt(2)*a*sin(5/2*d*x + 5/2*c) + 40*sqrt(2)*a*sin(1/2*d*x + 1/2*c) - 2*sqrt(2)*a*sin(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 20*sqrt(2)*a*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 5*a*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 5*a*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 5*a*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 5*a*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2))*B*sqrt(a))/d

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c + dx)^{3/2} \left(A + \frac{B}{\cos(c + dx)} \right) \left(a + \frac{a}{\cos(c + dx)} \right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^(3/2)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(3/2),x)

[Out] int(cos(c + d*x)^(3/2)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(3/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**(3/2)*(a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)),x)

[Out] Timed out

$$3.528 \quad \int \sqrt{\cos(c+dx)} (a+a \sec(c+dx))^{3/2} (A+B \sec(c+dx)) dx$$

Optimal. Leaf size=144

$$\frac{a^{3/2}(2A+3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d} + \frac{a^2(2A-B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}} + \frac{aB\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{aB\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

[Out] a^(3/2)*(2*A+3*B)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+a^2*(2*A-B)*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+a*B*sin(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)

Rubi [A] time = 0.43, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2955, 4018, 4015, 3801, 215}

$$\frac{a^2(2A-B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}} + \frac{a^{3/2}(2A+3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d} + \frac{aB\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]), x]

[Out] (a^(3/2)*(2*A + 3*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]])*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]/d + (a^2*(2*A - B)*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (a*B*Sqrt[a + a*Sec[c + d*x]])*Sin[c + d*x]/(d*Sqrt[Cos[c + d*x]])

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 2955

Int[((a_.) + csc[(e_.) + (f_.)*(x_)])*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)])*(d_.) + (c_.))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] := Dist[(g*Csc[e + f*x])^p*(g*Ssin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && IntegerQ[n])

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.)
+ (a_.)], x_Symbol] := Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.)
+ (a_.)]*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
+ f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (
a_.))^m*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && !LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{\cos(c+dx)} (a+a \sec(c+dx))^{3/2} (A+B \sec(c+dx)) dx &= \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{(a+a \sec(c+dx))^{3/2} (A+B \sec(c+dx))}{\sqrt{\cos(c+dx)}} dx \\
&= \frac{aB \sqrt{a+a \sec(c+dx)} \sin(c+dx)}{d \sqrt{\cos(c+dx)}} + \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{(a+a \sec(c+dx))^{3/2} (A+B \sec(c+dx))}{\sqrt{\cos(c+dx)}} dx \\
&= \frac{a^2(2A-B) \sin(c+dx)}{d \sqrt{\cos(c+dx)} \sqrt{a+a \sec(c+dx)}} + \frac{aB \sqrt{a+a \sec(c+dx)} \sin(c+dx)}{d \sqrt{\cos(c+dx)}} \\
&= \frac{a^2(2A-B) \sin(c+dx)}{d \sqrt{\cos(c+dx)} \sqrt{a+a \sec(c+dx)}} + \frac{aB \sqrt{a+a \sec(c+dx)} \sin(c+dx)}{d \sqrt{\cos(c+dx)}} \\
&= \frac{a^{3/2}(2A+3B) \sinh^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a+a \sec(c+dx)}}\right) \sqrt{\cos(c+dx)}}{d}
\end{aligned}$$

Mathematica [A] time = 0.86, size = 133, normalized size = 0.92

$$\frac{a \sqrt{\cos(c+dx)} \tan\left(\frac{1}{2}(c+dx)\right) \sqrt{a(\sec(c+dx)+1)} \left(\sqrt{1-\sec(c+dx)}(2A+B \sec(c+dx)) + 2A \sqrt{\sec(c+dx)}\right)}{d \sqrt{1-\sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]

[Out] (a*Sqrt[Cos[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])]*(2*A*ArcSin[Sqrt[1 - Sec[c + d*x]]]*Sqrt[Sec[c + d*x]] - 3*B*ArcSin[Sqrt[Sec[c + d*x]]]*Sqrt[Sec[c + d*x]] + Sqrt[1 - Sec[c + d*x]]*(2*A + B*Sec[c + d*x]))*Tan[(c + d*x)/2])/(d*Sqrt[1 - Sec[c + d*x]])

fricas [A] time = 0.51, size = 389, normalized size = 2.70

$$\left[\frac{4(2Aa \cos(dx+c) + Ba) \sqrt{\frac{a \cos(dx+c)+a}{\cos(dx+c)}} \sqrt{\cos(dx+c)} \sin(dx+c) + ((2A+3B)a \cos(dx+c))^2 + (2A+3B)a \cos(dx+c)}{4(d \cos(dx+c))^2 + d \cos(dx+c)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))*cos(d*x+c)^(1/2),x, algorith="fricas")

[Out] [1/4*(4*(2*A*a*cos(d*x + c) + B*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + ((2*A + 3*B)*a*cos(d*x + c)^2 + (2*A + 3*B)*a*cos(d*x + c))*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(d*cos(d*x + c)^2 + d*cos(d*x + c)), 1/2*(2*(2*A*a*cos(d*x + c) + B*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + ((2*A + 3*B)*a*cos(d*x + c)^2 + (2*A + 3*B)*a*cos(d*x + c))*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*x + c)^2 + d*cos(d*x + c))]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(a \sec(dx + c) + a)^{\frac{3}{2}} \sqrt{\cos(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))*cos(d*x+c)^(1/2),x, algorith="giac")

[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c)), x)

maple [B] time = 1.88, size = 306, normalized size = 2.12

$$a(-1 + \cos(dx + c)) \left(4A \sin(dx + c) \cos(dx + c) \sqrt{\frac{2}{1 + \cos(dx + c)}} + 2A \arctan \left(\frac{\sqrt{\frac{2}{1 + \cos(dx + c)}} (\cos(dx + c) + 1 + \sin(dx + c))}{4} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))*cos(d*x+c)^(1/2),x)

[Out] -1/2/d*a*(-1+cos(d*x+c))*(4*A*sin(d*x+c)*cos(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)+2*A*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d*x+c))*2^(1/2))*cos(d*x+c)*2^(1/2)-2*A*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))*2^(1/2))*cos(d*x+c)*2^(1/2)+3*B*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d*x+c))*2^(1/2))*cos(d*x+c)*2^(1/2)-3*B*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))*2^(1/2))*cos(d*x+c)*2^(1/2)+2*B*(-2/(1+cos(d*x+c)))^(1/2)*sin(d*x+c))*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)/sin(d*x+c)^2/(-2/(1+cos(d*x+c)))^(1/2)


```
in(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*
d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*
c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2)
- 4*(sqrt(2)*a*cos(3/2*d*x + 3/2*c) - sqrt(2)*a*cos(1/2*d*x + 1/2*c))*sin(
2*d*x + 2*c))*B*sqrt(a)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*
d*x + 2*c) + 1))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\cos(c + dx)} \left(A + \frac{B}{\cos(c + dx)} \right) \left(a + \frac{a}{\cos(c + dx)} \right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^(1/2)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c))*cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

$$3.529 \quad \int \frac{(a+a \sec(c+dx))^{3/2}(A+B \sec(c+dx))}{\sqrt{\cos(c+dx)}} dx$$

Optimal. Leaf size=153

$$\frac{a^{3/2}(12A+7B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d} + \frac{a^2(4A+5B)\sin(c+dx)}{4d\cos^2(c+dx)\sqrt{a\sec(c+dx)+a}} + \frac{aB\sin(c+dx)}{4d\cos^2(c+dx)\sqrt{a\sec(c+dx)+a}}$$

[Out] $1/4*a^{(3/2)}*(12*A+7*B)*\operatorname{arcsinh}(a^{(1/2)}*\tan(d*x+c)/(a+a*\sec(d*x+c))^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d+1/4*a^2*(4*A+5*B)*\sin(d*x+c)/d/\cos(d*x+c)^{(3/2)}/(a+a*\sec(d*x+c))^{(1/2)}+1/2*a*B*\sin(d*x+c)*(a+a*\sec(d*x+c))^{(1/2)}/d/\cos(d*x+c)^{(3/2)}$

Rubi [A] time = 0.45, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2955, 4018, 4016, 3801, 215}

$$\frac{a^2(4A+5B)\sin(c+dx)}{4d\cos^2(c+dx)\sqrt{a\sec(c+dx)+a}} + \frac{a^{3/2}(12A+7B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d} + \frac{aB\sin(c+dx)}{4d\cos^2(c+dx)\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(a+a*\operatorname{Sec}[c+d*x])^{(3/2)}*(A+B*\operatorname{Sec}[c+d*x])]/\operatorname{Sqrt}[\operatorname{Cos}[c+d*x]],x]$

[Out] $(a^{(3/2)}*(12*A+7*B)*\operatorname{ArcSinh}[(\operatorname{Sqrt}[a]*\operatorname{Tan}[c+d*x])/\operatorname{Sqrt}[a+a*\operatorname{Sec}[c+d*x]])*\operatorname{Sqrt}[\operatorname{Cos}[c+d*x]]*\operatorname{Sqrt}[\operatorname{Sec}[c+d*x]])/(4*d)+(a^2*(4*A+5*B)*\operatorname{Sin}[c+d*x])/(4*d*\operatorname{Cos}[c+d*x]^{(3/2)}*\operatorname{Sqrt}[a+a*\operatorname{Sec}[c+d*x]])+(a*B*\operatorname{Sqrt}[a+a*\operatorname{Sec}[c+d*x]]*\operatorname{Sin}[c+d*x])/(2*d*\operatorname{Cos}[c+d*x]^{(3/2)})$

Rule 215

$\operatorname{Int}[1/\operatorname{Sqrt}[(a_.)+(b_.)*(x_)^2],x_Symbol] :> \operatorname{Simp}[\operatorname{ArcSinh}[(\operatorname{Rt}[b,2]*x)/\operatorname{Sqrt}[a]]/\operatorname{Rt}[b,2],x] /; \operatorname{FreeQ}[\{a,b\},x] \&\& \operatorname{GtQ}[a,0] \&\& \operatorname{PosQ}[b]$

Rule 2955

$\operatorname{Int}[(a_.)+\operatorname{csc}(e_.)+(f_.)*(x_)]*(b_.)^{(m_.)}*(\operatorname{csc}(e_.)+(f_.)*(x_)]*(d_.)+(c_.)^{(n_.)}*((g_.)*\sin[(e_.)+(f_.)*(x_)]^{(p_.)},x_Symbol] :> \operatorname{Dist}[(g*\operatorname{Csc}[e+f*x])^p*(g*\operatorname{Sin}[e+f*x])^p,\operatorname{Int}[(a+b*\operatorname{Csc}[e+f*x])^m*(c+d*\operatorname{Csc}[e+f*x])^n]/(g*\operatorname{Csc}[e+f*x])^p,x],x] /; \operatorname{FreeQ}[\{a,b,c,d,e,f,g,m,n,p\},x] \&\& \operatorname{NeQ}[b*c-a*d,0] \&\& !\operatorname{IntegerQ}[p] \&\& !(\operatorname{IntegerQ}[m] \&\& \operatorname{IntegerQ}[n])$

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.)
+ (a_.)], x_Symbol] := Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.)
+ (a_.)]*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
+ Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x]
]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && !
LtQ[n, 0]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (
a_.))^m*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && !LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + a \sec(c + dx))^{3/2} (A + B \sec(c + dx))}{\sqrt{\cos(c + dx)}} dx &= \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \sqrt{\sec(c + dx)} (a + a \sec(c + dx)) dx \\
&= \frac{aB \sqrt{a + a \sec(c + dx)} \sin(c + dx)}{2d \cos^{3/2}(c + dx)} + \frac{1}{2} \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \sqrt{\sec(c + dx)} dx \\
&= \frac{a^2(4A + 5B) \sin(c + dx)}{4d \cos^{3/2}(c + dx) \sqrt{a + a \sec(c + dx)}} + \frac{aB \sqrt{a + a \sec(c + dx)} \sin(c + dx)}{2d \cos^{3/2}(c + dx)} \\
&= \frac{a^2(4A + 5B) \sin(c + dx)}{4d \cos^{3/2}(c + dx) \sqrt{a + a \sec(c + dx)}} + \frac{aB \sqrt{a + a \sec(c + dx)} \sin(c + dx)}{2d \cos^{3/2}(c + dx)} \\
&= \frac{a^{3/2}(12A + 7B) \sinh^{-1} \left(\frac{\sqrt{a} \tan(c + dx)}{\sqrt{a + a \sec(c + dx)}} \right) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)}}{4d}
\end{aligned}$$

Mathematica [A] time = 0.84, size = 107, normalized size = 0.70

$$\frac{a \sqrt{\cos(c + dx)} \sec\left(\frac{1}{2}(c + dx)\right) \sqrt{a(\sec(c + dx) + 1)} \left(\sqrt{2} (12A + 7B) \tanh^{-1} \left(\sqrt{2} \sin\left(\frac{1}{2}(c + dx)\right) \right) + 2 \sin\left(\frac{1}{2}(c + dx)\right) \right)}{8d}$$

Antiderivative was successfully verified.

[In] Integrate[((a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]))/Sqrt[Cos[c + d*x]], x]

[Out] (a*Sqrt[Cos[c + d*x]]*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(Sqrt[2]*(12*A + 7*B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sec[c + d*x]*(4*A + 7*B + 2*B*Sec[c + d*x])*Sin[(c + d*x)/2]))/(8*d)

fricas [A] time = 0.53, size = 409, normalized size = 2.67

$$\left[\frac{4((4A + 7B)a \cos(dx + c) + 2Ba) \sqrt{\frac{a \cos(dx+c)+a}{\cos(dx+c)}} \sqrt{\cos(dx + c)} \sin(dx + c) + ((12A + 7B)a \cos(dx + c)^3 + (12A + 7B)a \cos(dx + c) + 2Ba)}{16(d \cos(dx + c))^3 + \dots} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="fricas")

[Out] [1/16*(4*((4*A + 7*B)*a*cos(d*x + c) + 2*B*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + ((12*A + 7*B)*a*cos(d*x + c)^3 + (12*A + 7*B)*a*cos(d*x + c)^2)*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2), 1/8*(2*((4*A + 7*B)*a*cos(d*x + c) + 2*B*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + ((12*A + 7*B)*a*cos(d*x + c)^3 + (12*A + 7*B)*a*cos(d*x + c)^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(a \sec(dx + c) + a)^{\frac{3}{2}}}{\sqrt{\cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(3/2)/sqrt(cos(d*x + c)), x)

maple [B] time = 2.07, size = 343, normalized size = 2.24

$$a \sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}} (-1 + \cos(dx + c)) \left(12A \arctan \left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}} (\cos(dx+c)+1+\sin(dx+c))\sqrt{2}}{4} \right) (\cos^2(dx + c)) \sqrt{2} - 12 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x)

[Out] -1/8/d*a*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(12*A*arctan(1/4*(-2/(1+cos(d*x+c))))^(1/2)*(cos(d*x+c)+1+sin(d*x+c))*2^(1/2))*cos(d*x+c)^2*2^(1/2)-12*A*arctan(1/4*(-2/(1+cos(d*x+c))))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))*2^(1/2))*cos(d*x+c)^2*2^(1/2)+7*B*arctan(1/4*(-2/(1+cos(d*x+c))))^(1/2)*(cos(d*x+c)+1+sin(d*x+c))*2^(1/2))*cos(d*x+c)^2*2^(1/2)-7*B*arctan(1/4*(-2/(1+cos(d*x+c))))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))*2^(1/2))*cos(d*x+c)^2*2^(1/2)

$$)+8*A*\sin(d*x+c)*\cos(d*x+c)*(-2/(1+\cos(d*x+c)))^{(1/2)}+14*B*\sin(d*x+c)*\cos(d*x+c)*(-2/(1+\cos(d*x+c)))^{(1/2)}+4*B*(-2/(1+\cos(d*x+c)))^{(1/2)}*\sin(d*x+c))/\sin(d*x+c)^2/(-2/(1+\cos(d*x+c)))^{(1/2)}/\cos(d*x+c)^{(3/2)}$$

maxima [B] time = 0.90, size = 3389, normalized size = 22.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="maxima")

[Out]
$$\begin{aligned} & 1/16*(4*(3*(a*\log(2*\cos(1/2*d*x + 1/2*c))^2 + 2*\sin(1/2*d*x + 1/2*c))^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - a*\log(2*\cos(1/2*d*x + 1/2*c))^2 + 2*\sin(1/2*d*x + 1/2*c))^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + a*\log(2*\cos(1/2*d*x + 1/2*c))^2 + 2*\sin(1/2*d*x + 1/2*c))^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*(2*\sin(1/2*d*x + 1/2*c) + 2) - a*\log(2*\cos(1/2*d*x + 1/2*c))^2 + 2*\sin(1/2*d*x + 1/2*c))^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2))*\cos(2*d*x + 2*c)^2 + 3*(a*\log(2*\cos(1/2*d*x + 1/2*c))^2 + 2*\sin(1/2*d*x + 1/2*c))^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - a*\log(2*\cos(1/2*d*x + 1/2*c))^2 + 2*\sin(1/2*d*x + 1/2*c))^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + a*\log(2*\cos(1/2*d*x + 1/2*c))^2 + 2*\sin(1/2*d*x + 1/2*c))^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - a*\log(2*\cos(1/2*d*x + 1/2*c))^2 + 2*\sin(1/2*d*x + 1/2*c))^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2))*\sin(2*d*x + 2*c)^2 + 4*\sqrt{2}*a*\sin(3/2*d*x + 3/2*c) - 4*\sqrt{2}*a*\sin(1/2*d*x + 1/2*c) + 2*(2*\sqrt{2}*a*\sin(3/2*d*x + 3/2*c) - 2*\sqrt{2}*a*\sin(1/2*d*x + 1/2*c) + 3*a*\log(2*\cos(1/2*d*x + 1/2*c))^2 + 2*\sin(1/2*d*x + 1/2*c))^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 3*a*\log(2*\cos(1/2*d*x + 1/2*c))^2 + 2*\sin(1/2*d*x + 1/2*c))^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + 3*a*\log(2*\cos(1/2*d*x + 1/2*c))^2 + 2*\sin(1/2*d*x + 1/2*c))^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 3*a*\log(2*\cos(1/2*d*x + 1/2*c))^2 + 2*\sin(1/2*d*x + 1/2*c))^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2))*\cos(2*d*x + 2*c) + 3*a*\log(2*\cos(1/2*d*x + 1/2*c))^2 + 2*\sin(1/2*d*x + 1/2*c))^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 3*a*\log(2*\cos(1/2*d*x + 1/2*c))^2 + 2*\sin(1/2*d*x + 1/2*c))^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + 3*a*\log(2*\cos(1/2*d*x + 1/2*c))^2 + 2*\sin(1/2*d*x + 1/2*c))^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 3*a*\log(2*\cos(1/2*d*x + 1/2*c))^2 + 2*\sin(1/2*d*x + 1/2*c))^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 4*(\sqrt{2}*a*\cos(3/2*d*x + 3/2*c) - \sqrt{2}*a*\cos(1/2*d*x + 1/2*c))*\sin(2*d*x + 2*c))*A*\sqrt{a}/(\cos(2*d*x + 2*c))^2 + \sin(2*d*x + 2*c))^2 + 2*\cos(2*$$

)^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 7*(a*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + a)*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 4*(3*sqrt(2)*a*cos(3/2*d*x + 3/2*c) + 7*sqrt(2)*a*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 3*sqrt(2)*a*cos(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 7*sqrt(2)*a*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 28*(2*sqrt(2)*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sqrt(2)*a)*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 12*(2*sqrt(2)*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sqrt(2)*a)*sin(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 8*(3*sqrt(2)*a*cos(3/2*d*x + 3/2*c) - 7*sqrt(2)*a*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1))/d

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(a + \frac{a}{\cos(c+dx)}\right)^{3/2}}{\sqrt{\cos(c+dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(3/2))/cos(c + d*x)^(1/2),x)

```
[Out] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(3/2))/cos(c + d*x)^(1/2), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c))/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

$$3.530 \quad \int \frac{(a+a \sec(c+dx))^{3/2}(A+B \sec(c+dx))}{\cos^{\frac{3}{2}}(c+dx)} dx$$

Optimal. Leaf size=200

$$\frac{a^{3/2}(14A + 11B)\sqrt{\cos(c + dx)}\sqrt{\sec(c + dx)} \sinh^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a \sec(c+dx)+a}}\right)}{8d} + \frac{a^2(14A + 11B) \sin(c + dx)}{8d \cos^{\frac{3}{2}}(c + dx)\sqrt{a \sec(c + dx) + a}} + \frac{a^2(6A + 7B) \sin(c + dx)}{12d \cos^{\frac{5}{2}}(c + dx)\sqrt{a \sec(c + dx) + a}}$$

[Out] $1/8*a^{(3/2)}*(14*A+11*B)*\operatorname{arcsinh}(a^{(1/2)}*\tan(d*x+c)/(a+a*\sec(d*x+c))^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d+1/12*a^2*(6*A+7*B)*\sin(d*x+c)/d/\cos(d*x+c)^{(5/2)}/(a+a*\sec(d*x+c))^{(1/2)}+1/8*a^2*(14*A+11*B)*\sin(d*x+c)/d/\cos(d*x+c)^{(3/2)}/(a+a*\sec(d*x+c))^{(1/2)}+1/3*a*B*\sin(d*x+c)*(a+a*\sec(d*x+c))^{(1/2)}/d/\cos(d*x+c)^{(5/2)}$

Rubi [A] time = 0.54, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {2955, 4018, 4016, 3803, 3801, 215}

$$\frac{a^2(14A + 11B) \sin(c + dx)}{8d \cos^{\frac{3}{2}}(c + dx)\sqrt{a \sec(c + dx) + a}} + \frac{a^2(6A + 7B) \sin(c + dx)}{12d \cos^{\frac{5}{2}}(c + dx)\sqrt{a \sec(c + dx) + a}} + \frac{a^{3/2}(14A + 11B)\sqrt{\cos(c + dx)}\sqrt{\sec(c + dx)}}{8d \cos^{\frac{3}{2}}(c + dx)\sqrt{a \sec(c + dx) + a}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(a + a*\operatorname{Sec}[c + d*x])^{(3/2)}*(A + B*\operatorname{Sec}[c + d*x])/ \operatorname{Cos}[c + d*x]^{(3/2)}, x]$

[Out] $(a^{(3/2)}*(14*A + 11*B)*\operatorname{ArcSinh}[(\operatorname{Sqrt}[a]*\operatorname{Tan}[c + d*x])/ \operatorname{Sqrt}[a + a*\operatorname{Sec}[c + d*x]])*\operatorname{Sqrt}[\operatorname{Cos}[c + d*x]]*\operatorname{Sqrt}[\operatorname{Sec}[c + d*x]]/(8*d) + (a^2*(6*A + 7*B)*\operatorname{Sin}[c + d*x])/(12*d*\operatorname{Cos}[c + d*x]^{(5/2)}*\operatorname{Sqrt}[a + a*\operatorname{Sec}[c + d*x]]) + (a^2*(14*A + 11*B)*\operatorname{Sin}[c + d*x])/(8*d*\operatorname{Cos}[c + d*x]^{(3/2)}*\operatorname{Sqrt}[a + a*\operatorname{Sec}[c + d*x]]) + (a*B*\operatorname{Sqrt}[a + a*\operatorname{Sec}[c + d*x]]*\operatorname{Sin}[c + d*x])/(3*d*\operatorname{Cos}[c + d*x]^{(5/2)})$

Rule 215

$\operatorname{Int}[1/\operatorname{Sqrt}[(a_) + (b_)*(x_)^2], x_Symbol] \rightarrow \operatorname{Simp}[\operatorname{ArcSinh}[(\operatorname{Rt}[b, 2]*x)/\operatorname{Sqrt}[a]]/\operatorname{Rt}[b, 2], x] /; \operatorname{FreeQ}\{a, b, x\} \&\& \operatorname{GtQ}[a, 0] \&\& \operatorname{PosQ}[b]$

Rule 2955

$\operatorname{Int}[(a_) + \operatorname{csc}(e_) + (f_)*(x_)]*(b_)^{(m_)}*(\operatorname{csc}(e_) + (f_)*(x_))* (d_) + (c_)^{(n_)}*((g_)*\operatorname{sin}(e_) + (f_)*(x_))^{(p_)}, x_Symbol] \rightarrow \operatorname{Dist}[(g*\operatorname{Csc}[e + f*x])^p*(g*\operatorname{Sin}[e + f*x])^p, \operatorname{Int}[(a + b*\operatorname{Csc}[e + f*x])^m*(c + d*\operatorname{Csc}[e + f*x])^n]/(g*\operatorname{Csc}[e + f*x])^p, x], x] /; \operatorname{FreeQ}\{a, b, c, d, e, f, g, m, n, p\}, x \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{IntegerQ}[p] \&\& \operatorname{IntegerQ}[m] \&\& \operatorname{IntegerQ}[n]$

tegerQ[n])

Rule 3801

Int[Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 + x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]

Rule 3803

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] :> Simp[(-2*b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(2*a*d*(n - 1))/(b*(2*n - 1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]

Rule 4016

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> Simp[(-2*b*B*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && !LtQ[n, 0]

Rule 4018

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> -Simp[(b*B*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n *Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && !LtQ[n, -1]

Rubi steps

$$\begin{aligned}
\int \frac{(a + a \sec(c + dx))^{3/2} (A + B \sec(c + dx))}{\cos^2(c + dx)} dx &= \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \sec^2(c + dx) (a + a \sec(c + dx)) dx \\
&= \frac{aB \sqrt{a + a \sec(c + dx)} \sin(c + dx)}{3d \cos^2(c + dx)} + \frac{1}{3} \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \sec^2(c + dx) dx \\
&= \frac{a^2(6A + 7B) \sin(c + dx)}{12d \cos^2(c + dx) \sqrt{a + a \sec(c + dx)}} + \frac{aB \sqrt{a + a \sec(c + dx)}}{3d \cos^2(c + dx)} \\
&= \frac{a^2(6A + 7B) \sin(c + dx)}{12d \cos^2(c + dx) \sqrt{a + a \sec(c + dx)}} + \frac{a^2(14A + 11B) \sin(c + dx)}{8d \cos^2(c + dx) \sqrt{a + a \sec(c + dx)}} \\
&= \frac{a^2(6A + 7B) \sin(c + dx)}{12d \cos^2(c + dx) \sqrt{a + a \sec(c + dx)}} + \frac{a^2(14A + 11B) \sin(c + dx)}{8d \cos^2(c + dx) \sqrt{a + a \sec(c + dx)}} \\
&= \frac{a^{3/2}(14A + 11B) \sinh^{-1} \left(\frac{\sqrt{a} \tan(c + dx)}{\sqrt{a + a \sec(c + dx)}} \right) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)}}{8d}
\end{aligned}$$

Mathematica [A] time = 1.28, size = 134, normalized size = 0.67

$$\frac{a \sec\left(\frac{1}{2}(c + dx)\right) \sqrt{a(\sec(c + dx) + 1)} \left(\sin\left(\frac{1}{2}(c + dx)\right) (4(6A + 11B) \cos(c + dx) + (42A + 33B) \cos(2(c + dx))) \right)}{48d \cos^2(c + dx)}$$

Antiderivative was successfully verified.

[In] Integrate[((a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]))/Cos[c + d*x]^(3/2),x]

[Out] (a*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(3*Sqrt[2]*(14*A + 11*B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^3 + (7*(6*A + 7*B) + 4*(6*A + 11*B))*Cos[c + d*x] + (42*A + 33*B)*Cos[2*(c + d*x)]*Sin[(c + d*x)/2])/ (48*d*Cos[c + d*x]^(5/2))

fricas [A] time = 0.58, size = 449, normalized size = 2.24

$$4 \left(3 (14 A + 11 B) a \cos(dx + c)^2 + 2 (6 A + 11 B) a \cos(dx + c) + 8 B a \right) \sqrt{\frac{a \cos(dx+c)+a}{\cos(dx+c)}} \sqrt{\cos(dx+c)} \sin(dx+c)$$

96

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="fricas")

[Out] [1/96*(4*(3*(14*A + 11*B)*a*cos(d*x + c)^2 + 2*(6*A + 11*B)*a*cos(d*x + c) + 8*B*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 3*((14*A + 11*B)*a*cos(d*x + c)^4 + (14*A + 11*B)*a*cos(d*x + c)^3)*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3), 1/48*(2*(3*(14*A + 11*B)*a*cos(d*x + c)^2 + 2*(6*A + 11*B)*a*cos(d*x + c) + 8*B*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 3*((14*A + 11*B)*a*cos(d*x + c)^4 + (14*A + 11*B)*a*cos(d*x + c)^3)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(a \sec(dx + c) + a)^{\frac{3}{2}}}{\cos(dx + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(3/2)/cos(d*x + c)^(3/2), x)

maple [B] time = 1.82, size = 405, normalized size = 2.02

$$a \sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}} (-1 + \cos(dx + c)) \left(42A \arctan \left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}} (\cos(dx+c)+1-\sin(dx+c)) \sqrt{2}}{4} \right) \sqrt{2} (\cos^3(dx + c)) - 42A \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((a+a*\sec(d*x+c))^{(3/2)}*(A+B*\sec(d*x+c))/\cos(d*x+c)^{(3/2)},x)$

[Out] $\frac{1}{48}d*a*(a*(1+\cos(d*x+c))/\cos(d*x+c))^{(1/2)}*(-1+\cos(d*x+c))*(42*A*\arctan(1/4*(-2/(1+\cos(d*x+c))))^{(1/2)}*(\cos(d*x+c)+1-\sin(d*x+c))*2^{(1/2)})*2^{(1/2)}*\cos(d*x+c)^3-42*A*\arctan(1/4*(-2/(1+\cos(d*x+c))))^{(1/2)}*(\cos(d*x+c)+1+\sin(d*x+c))*2^{(1/2)})*2^{(1/2)}*\cos(d*x+c)^3+33*B*\arctan(1/4*(-2/(1+\cos(d*x+c))))^{(1/2)}*(\cos(d*x+c)+1-\sin(d*x+c))*2^{(1/2)})*2^{(1/2)}*\cos(d*x+c)^3-33*B*\arctan(1/4*(-2/(1+\cos(d*x+c))))^{(1/2)}*(\cos(d*x+c)+1+\sin(d*x+c))*2^{(1/2)})*2^{(1/2)}*\cos(d*x+c)^3-84*A*\sin(d*x+c)*\cos(d*x+c)^2*(-2/(1+\cos(d*x+c)))^{(1/2)}-66*B*\sin(d*x+c)*\cos(d*x+c)^2*(-2/(1+\cos(d*x+c)))^{(1/2)}-24*A*\sin(d*x+c)*\cos(d*x+c)*(-2/(1+\cos(d*x+c)))^{(1/2)}-44*B*\sin(d*x+c)*\cos(d*x+c)*(-2/(1+\cos(d*x+c)))^{(1/2)}-16*B*(-2/(1+\cos(d*x+c)))^{(1/2)}*\sin(d*x+c))/\cos(d*x+c)^{(5/2)}/\sin(d*x+c)^2/(-2/(1+\cos(d*x+c)))^{(1/2)}$

maxima [B] time = 1.02, size = 4606, normalized size = 23.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((a+a*\sec(d*x+c))^{(3/2)}*(A+B*\sec(d*x+c))/\cos(d*x+c)^{(3/2)},x, \text{algorithm}="maxima")$

[Out] $-1/96*(6*(56*\sqrt{2})*a*\cos(7/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) * \sin(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) - 24*\sqrt{2} * a*\cos(5/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) * \sin(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) - 12*\sqrt{2} * a*\sin(3/2*d*x + 3/2*c) + 28*\sqrt{2} * a*\sin(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) - 4*(3*\sqrt{2} * a*\sin(3/2*d*x + 3/2*c) + 7*\sqrt{2} * a*\sin(7/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) - 3*\sqrt{2} * a*\sin(5/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) - 7*\sqrt{2} * a*\sin(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))) * \cos(8/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) - 8*(3*\sqrt{2} * a*\sin(3/2*d*x + 3/2*c) - 7*\sqrt{2} * a*\sin(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))) * \cos(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) - 7*(a * \cos(8/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + 4*a*\cos(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + a*\sin(8/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + 4*a*\sin(8/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) * \sin(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 4*a*\sin(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + 2*(2*a*\cos(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + a)*\cos(8/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 4*a*\cos(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + a)*\log(2*\cos(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 +$

$$\begin{aligned}
& 2*\sin(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + 2*\sqrt{2} \\
& * \cos(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 2*\sqrt{2} \\
& * \sin(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 2) + 7*(a*\cos \\
& (8/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + 4*a*\cos(4/3 \\
& *\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + a*\sin(8/3*\arctan2 \\
& (\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + 4*a*\sin(8/3*\arctan2(\sin(3 \\
& /2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) * \sin(4/3*\arctan2(\sin(3/2*d*x + 3/2*c) \\
&), \cos(3/2*d*x + 3/2*c))) + 4*a*\sin(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3 \\
& /2*d*x + 3/2*c)))^2 + 2*(2*a*\cos(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2* \\
& d*x + 3/2*c))) + a)*\cos(8/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2 \\
& *c))) + 4*a*\cos(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + \\
& a)*\log(2*\cos(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + 2 \\
& * \sin(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + 2*\sqrt{2} \\
& * \cos(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) - 2*\sqrt{2} * \sin \\
& (1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 2) - 7*(a*\cos \\
& (8/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + 4*a*\cos(4/3*a \\
& rctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + a*\sin(8/3*\arctan2(s \\
& in(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + 4*a*\sin(8/3*\arctan2(\sin(3/2 \\
& *d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) * \sin(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \\
& \cos(3/2*d*x + 3/2*c))) + 4*a*\sin(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2 \\
& *d*x + 3/2*c)))^2 + 2*(2*a*\cos(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d* \\
& x + 3/2*c))) + a)*\cos(8/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c \\
&))) + 4*a*\cos(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + a \\
& * \log(2*\cos(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + 2*s \\
& in(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 - 2*\sqrt{2} * \cos \\
& (1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 2*\sqrt{2} * \sin \\
& (1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 2) + 7*(a*\cos(8 \\
& /3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + 4*a*\cos(4/3*arc \\
& tan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + a*\sin(8/3*\arctan2(\sin \\
& (3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + 4*a*\sin(8/3*\arctan2(\sin(3/2*d \\
& *x + 3/2*c), \cos(3/2*d*x + 3/2*c))) * \sin(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), c \\
& os(3/2*d*x + 3/2*c))) + 4*a*\sin(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d \\
& *x + 3/2*c)))^2 + 2*(2*a*\cos(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x \\
& + 3/2*c))) + a)*\cos(8/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)) \\
&) + 4*a*\cos(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + a)*\log \\
& (2*\cos(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + 2*\sin \\
& (1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 - 2*\sqrt{2} * \cos \\
& (1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) - 2*\sqrt{2} * \sin(1 \\
& /3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 2) + 4*(3*\sqrt{2} \\
& * a*\cos(3/2*d*x + 3/2*c) + 7*\sqrt{2} * a*\cos(7/3*\arctan2(\sin(3/2*d*x + 3/2*c), \\
& \cos(3/2*d*x + 3/2*c))) - 3*\sqrt{2} * a*\cos(5/3*\arctan2(\sin(3/2*d*x + 3/2*c), \\
& \cos(3/2*d*x + 3/2*c))) - 7*\sqrt{2} * a*\cos(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \\
& \cos(3/2*d*x + 3/2*c)))) * \sin(8/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x \\
& + 3/2*c))) - 28*(2*\sqrt{2} * a*\cos(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2* \\
& d*x + 3/2*c))) + \sqrt{2} * a)*\sin(7/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d
\end{aligned}$$

$$\begin{aligned}
& *x + 3/2*c))) + 12*(2*\sqrt{2}*a*\cos(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + \sqrt{2}*a)*\sin(5/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 8*(3*\sqrt{2}*a*\cos(3/2*d*x + 3/2*c) - 7*\sqrt{2}*a*\cos(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))))*\sin(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + A*\sqrt{a}/(2*(2*\cos(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 1)*\cos(8/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + \cos(8/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))))^2 + 4*\cos(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + \sin(8/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + 4*\sin(8/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))*\sin(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 4*\sin(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + 4*\cos(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 1) + (132*(\sqrt{2}*a*\sin(6*d*x + 6*c) + 3*\sqrt{2}*a*\sin(4*d*x + 4*c) + 3*\sqrt{2}*a*\sin(2*d*x + 2*c))*\cos(11/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 44*(\sqrt{2}*a*\sin(6*d*x + 6*c) + 3*\sqrt{2}*a*\sin(4*d*x + 4*c) + 3*\sqrt{2}*a*\sin(2*d*x + 2*c))*\cos(9/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 216*(\sqrt{2}*a*\sin(6*d*x + 6*c) + 3*\sqrt{2}*a*\sin(4*d*x + 4*c) + 3*\sqrt{2}*a*\sin(2*d*x + 2*c))*\cos(7/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) - 216*(\sqrt{2}*a*\sin(6*d*x + 6*c) + 3*\sqrt{2}*a*\sin(4*d*x + 4*c) + 3*\sqrt{2}*a*\sin(2*d*x + 2*c))*\cos(5/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) - 44*(\sqrt{2}*a*\sin(6*d*x + 6*c) + 3*\sqrt{2}*a*\sin(4*d*x + 4*c) + 3*\sqrt{2}*a*\sin(2*d*x + 2*c))*\cos(3/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) - 132*(\sqrt{2}*a*\sin(6*d*x + 6*c) + 3*\sqrt{2}*a*\sin(4*d*x + 4*c) + 3*\sqrt{2}*a*\sin(2*d*x + 2*c))*\cos(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) - 33*(a*\cos(6*d*x + 6*c)^2 + 9*a*\cos(4*d*x + 4*c)^2 + 9*a*\cos(2*d*x + 2*c)^2 + a*\sin(6*d*x + 6*c)^2 + 9*a*\sin(4*d*x + 4*c)^2 + 18*a*\sin(4*d*x + 4*c)*\sin(2*d*x + 2*c) + 9*a*\sin(2*d*x + 2*c)^2 + 2*(3*a*\cos(4*d*x + 4*c) + 3*a*\cos(2*d*x + 2*c) + a)*\cos(6*d*x + 6*c) + 6*(3*a*\cos(2*d*x + 2*c) + a)*\cos(4*d*x + 4*c) + 6*a*\cos(2*d*x + 2*c) + 6*(a*\sin(4*d*x + 4*c) + a*\sin(2*d*x + 2*c))*\sin(6*d*x + 6*c) + a)*\log(2*\cos(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + 2*\sin(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + 2*\sqrt{2}*\cos(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 2*\sqrt{2}*\sin(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 2) + 33*(a*\cos(6*d*x + 6*c)^2 + 9*a*\cos(4*d*x + 4*c)^2 + 9*a*\cos(2*d*x + 2*c)^2 + a*\sin(6*d*x + 6*c)^2 + 9*a*\sin(4*d*x + 4*c)^2 + 18*a*\sin(4*d*x + 4*c)*\sin(2*d*x + 2*c) + 9*a*\sin(2*d*x + 2*c)^2 + 2*(3*a*\cos(4*d*x + 4*c) + 3*a*\cos(2*d*x + 2*c) + a)*\cos(6*d*x + 6*c) + 6*(3*a*\cos(2*d*x + 2*c) + a)*\cos(4*d*x + 4*c) + 6*a*\cos(2*d*x + 2*c) + 6*(a*\sin(4*d*x + 4*c) + a*\sin(2*d*x + 2*c))*\sin(6*d*x + 6*c) + a)*\log(2*\cos(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + 2*\sin(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + 2*\sqrt{2}*\cos(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) - 2*\sqrt{2}*\sin(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 2) - 33*(a*\cos(6*d*x + 6*c)^2 + 9*a*\cos(4*d*x + 4*c)^2 + 9*a*\cos(2*d*x + 2*c)^2 + a*\sin(6*d*x + 6*c)^2 + 9*a*\sin(4*d*x + 4*c)^2 + 18*a*\sin(4*d*x + 4*c)*\sin(2*d*x + 2*c) + 9*a*\sin(2*d*x + 2*c)^2 + 2*(3*a*\cos(4*d*x + 4*c)
\end{aligned}$$

```

+ 3*a*cos(2*d*x + 2*c) + a*cos(6*d*x + 6*c) + 6*(3*a*cos(2*d*x + 2*c) + a)
*cos(4*d*x + 4*c) + 6*a*cos(2*d*x + 2*c) + 6*(a*sin(4*d*x + 4*c) + a*sin(2*
d*x + 2*c))*sin(6*d*x + 6*c) + a*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
)^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sq
rt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 33*(a*cos
(6*d*x + 6*c)^2 + 9*a*cos(4*d*x + 4*c)^2 + 9*a*cos(2*d*x + 2*c)^2 + a*sin(6
*d*x + 6*c)^2 + 9*a*sin(4*d*x + 4*c)^2 + 18*a*sin(4*d*x + 4*c)*sin(2*d*x +
2*c) + 9*a*sin(2*d*x + 2*c)^2 + 2*(3*a*cos(4*d*x + 4*c) + 3*a*cos(2*d*x + 2
*c) + a)*cos(6*d*x + 6*c) + 6*(3*a*cos(2*d*x + 2*c) + a)*cos(4*d*x + 4*c) +
6*a*cos(2*d*x + 2*c) + 6*(a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(6*d
*x + 6*c) + a*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2
+ 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos
(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 132*(sqrt(2)*a*cos(6*d*x + 6
*c) + 3*sqrt(2)*a*cos(4*d*x + 4*c) + 3*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)
*a)*sin(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 44*(sqrt(2)*a*c
os(6*d*x + 6*c) + 3*sqrt(2)*a*cos(4*d*x + 4*c) + 3*sqrt(2)*a*cos(2*d*x + 2
*c) + sqrt(2)*a)*sin(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 216*
(sqrt(2)*a*cos(6*d*x + 6*c) + 3*sqrt(2)*a*cos(4*d*x + 4*c) + 3*sqrt(2)*a*co
s(2*d*x + 2*c) + sqrt(2)*a)*sin(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c))) + 216*(sqrt(2)*a*cos(6*d*x + 6*c) + 3*sqrt(2)*a*cos(4*d*x + 4*c) + 3*
sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(5/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) + 44*(sqrt(2)*a*cos(6*d*x + 6*c) + 3*sqrt(2)*a*cos(4*d*x
+ 4*c) + 3*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(3/4*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))) + 132*(sqrt(2)*a*cos(6*d*x + 6*c) + 3*sqrt(2)*
a*cos(4*d*x + 4*c) + 3*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(1/4*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) * B*sqrt(a)/(2*(3*cos(4*d*x + 4*c)
+ 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(
2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2
*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*
x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) +
9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) + 1))/d

```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(a + \frac{a}{\cos(c+dx)}\right)^{3/2}}{\cos(c+dx)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(3/2))/cos(c + d*x)^(3/2), x)

[Out] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(3/2))/cos(c + d*x)^(3/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c))/cos(d*x+c)**(3/2),x)

[Out] Timed out

$$3.531 \quad \int \frac{(a+a \sec(c+dx))^{3/2}(A+B \sec(c+dx))}{\cos^2(c+dx)} dx$$

Optimal. Leaf size=247

$$\frac{a^{3/2}(88A+75B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{64d} + \frac{a^2(88A+75B)\sin(c+dx)}{64d\cos^3(c+dx)\sqrt{a\sec(c+dx)+a}} + \frac{a^2(8A+9B)\sin(c+dx)}{96d\cos^7(c+dx)\sqrt{a\sec(c+dx)+a}}$$

[Out] 1/64*a^(3/2)*(88*A+75*B)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+1/24*a^2*(8*A+9*B)*sin(d*x+c)/d/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(1/2)+1/96*a^2*(88*A+75*B)*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2)+1/64*a^2*(88*A+75*B)*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2)+1/4*a*B*sin(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d/cos(d*x+c)^(7/2)

Rubi [A] time = 0.63, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {2955, 4018, 4016, 3803, 3801, 215}

$$\frac{a^2(88A+75B)\sin(c+dx)}{64d\cos^3(c+dx)\sqrt{a\sec(c+dx)+a}} + \frac{a^2(88A+75B)\sin(c+dx)}{96d\cos^5(c+dx)\sqrt{a\sec(c+dx)+a}} + \frac{a^2(8A+9B)\sin(c+dx)}{24d\cos^7(c+dx)\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

[In] Int[((a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]))/Cos[c + d*x]^(5/2), x]

[Out] (a^(3/2)*(88*A + 75*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]])*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]/(64*d) + (a^2*(8*A + 9*B)*Sin[c + d*x])/(24*d*Cos[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]) + (a^2*(88*A + 75*B)*Sin[c + d*x])/(96*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) + (a^2*(88*A + 75*B)*Sin[c + d*x])/(64*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (a*B*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(4*d*Cos[c + d*x]^(7/2))

Rule 215

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 2955

Int[((a_) + csc[(e_) + (f_)*(x_)])*(b_)^(m_)*(csc[(e_) + (f_)*(x_)])*(d_) + (c_)^(n_)*((g_)*sin[(e_) + (f_)*(x_)])^(p_), x_Symbol] := Dis

$t[(g*\text{Csc}[e + f*x])^p*(g*\text{Sin}[e + f*x])^p, \text{Int}[\frac{(a + b*\text{Csc}[e + f*x])^m*(c + d*\text{Csc}[e + f*x])^n}{(g*\text{Csc}[e + f*x])^p}, x], x] /;$ FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && IntegerQ[n])

Rule 3801

$\text{Int}[\text{Sqrt}[\text{csc}[(e_.) + (f_.)*(x_.)]*(d_.)]*\text{Sqrt}[\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] :> \text{Dist}[\frac{-2*a*\text{Sqrt}[(a*d)/b]}{(b*f)}, \text{Subst}[\text{Int}[1/\text{Sqrt}[1 + x^2/a], x], x, (b*\text{Cot}[e + f*x])/\text{Sqrt}[a + b*\text{Csc}[e + f*x]]], x] /;$ FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]

Rule 3803

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(d_.))^{(n_.)}*\text{Sqrt}[\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] :> \text{Simp}[\frac{-2*b*d*\text{Cot}[e + f*x]*(d*\text{Csc}[e + f*x])^{(n-1)}}{(f*(2*n-1)*\text{Sqrt}[a + b*\text{Csc}[e + f*x]])}, x] + \text{Dist}[\frac{2*a*d*(n-1)}{(b*(2*n-1))}, \text{Int}[\text{Sqrt}[a + b*\text{Csc}[e + f*x]]*(d*\text{Csc}[e + f*x])^{(n-1)}, x], x] /;$ FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]

Rule 4016

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(d_.))^{(n_.)}*\text{Sqrt}[\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]*(\text{csc}[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> \text{Simp}[\frac{-2*b*B*\text{Cot}[e + f*x]*(d*\text{Csc}[e + f*x])^n}{(f*(2*n+1)*\text{Sqrt}[a + b*\text{Csc}[e + f*x]])}, x] + \text{Dist}[\frac{A*b*(2*n+1) + 2*a*B*n}{(b*(2*n+1))}, \text{Int}[\text{Sqrt}[a + b*\text{Csc}[e + f*x]]*(d*\text{Csc}[e + f*x])^n, x], x] /;$ FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n+1) + 2*a*B*n, 0] && !LtQ[n, 0]

Rule 4018

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(d_.))^{(n_.)}*(\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^{(m_.)}*(\text{csc}[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> -\text{Simp}[(b*B*\text{Cot}[e + f*x]*(a + b*\text{Csc}[e + f*x])^{(m-1)}*(d*\text{Csc}[e + f*x])^n)/(f*(m+n)), x] + \text{Dist}[1/(d*(m+n)), \text{Int}[(a + b*\text{Csc}[e + f*x])^{(m-1)}*(d*\text{Csc}[e + f*x])^n*\text{Simp}[a*A*d*(m+n) + B*(b*d*n) + (A*b*d*(m+n) + a*B*d*(2*m+n-1))*\text{Csc}[e + f*x], x], x], x] /;$ FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && !LtQ[n, -1]

Rubi steps

$$\int \frac{(a + a \sec(c + dx))^{3/2}(A + B \sec(c + dx))}{\cos^{5/2}(c + dx)} dx = (\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)}) \int \sec^{5/2}(c + dx)(a + a \sec(c + dx))$$

$$= \frac{aB\sqrt{a + a \sec(c + dx)} \sin(c + dx)}{4d \cos^{7/2}(c + dx)} + \frac{1}{4} (\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)})$$

$$= \frac{a^2(8A + 9B) \sin(c + dx)}{24d \cos^{7/2}(c + dx) \sqrt{a + a \sec(c + dx)}} + \frac{aB\sqrt{a + a \sec(c + dx)} \sin(c + dx)}{4d \cos^{7/2}(c + dx)}$$

$$= \frac{a^2(8A + 9B) \sin(c + dx)}{24d \cos^{7/2}(c + dx) \sqrt{a + a \sec(c + dx)}} + \frac{a^2(88A + 75B) \sin(c + dx)}{96d \cos^{5/2}(c + dx) \sqrt{a + a \sec(c + dx)}}$$

$$= \frac{a^2(8A + 9B) \sin(c + dx)}{24d \cos^{7/2}(c + dx) \sqrt{a + a \sec(c + dx)}} + \frac{a^2(88A + 75B) \sin(c + dx)}{96d \cos^{5/2}(c + dx) \sqrt{a + a \sec(c + dx)}}$$

$$= \frac{a^2(8A + 9B) \sin(c + dx)}{24d \cos^{7/2}(c + dx) \sqrt{a + a \sec(c + dx)}} + \frac{a^2(88A + 75B) \sin(c + dx)}{96d \cos^{5/2}(c + dx) \sqrt{a + a \sec(c + dx)}}$$

$$= \frac{a^2(8A + 9B) \sin(c + dx)}{24d \cos^{7/2}(c + dx) \sqrt{a + a \sec(c + dx)}} + \frac{a^2(88A + 75B) \sin(c + dx)}{96d \cos^{5/2}(c + dx) \sqrt{a + a \sec(c + dx)}}$$

$$= \frac{a^{3/2}(88A + 75B) \sinh^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a+a \sec(c+dx)}}\right) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)}}{64d}$$

Mathematica [A] time = 1.96, size = 153, normalized size = 0.62

$$a \sec\left(\frac{1}{2}(c + dx)\right) \sqrt{a(\sec(c + dx) + 1)} \left(\sin\left(\frac{1}{2}(c + dx)\right)\right) ((1048A + 1155B) \cos(c + dx) + 4(88A + 75B) \cos(2(c + dx)))$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]))/Cos[c + d*x]^(5/2), x]
```

```
[Out] (a*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(6*Sqrt[2]*(88*A + 75*B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^4 + (352*A + 492*B + (1048*A + 1155*B)*Cos[c + d*x] + 4*(88*A + 75*B)*Cos[2*(c + d*x)] + 264*A*Cos[3*(c + d*x)] + 225*B*Cos[3*(c + d*x)])*Sin[(c + d*x)/2])/((768*d*Cos[c + d*x]^(7/2)))
```

fricas [A] time = 0.63, size = 485, normalized size = 1.96

$$4 \left(3 (88 A + 75 B) a \cos(dx + c)^3 + 2 (88 A + 75 B) a \cos(dx + c)^2 + 8 (8 A + 15 B) a \cos(dx + c) + 48 B a \right) \sqrt{\frac{a \cos(dx + c) + a}{\cos(dx + c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="fricas")

[Out] [1/768*(4*(3*(88*A + 75*B)*a*cos(d*x + c)^3 + 2*(88*A + 75*B)*a*cos(d*x + c)^2 + 8*(8*A + 15*B)*a*cos(d*x + c) + 48*B*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 3*((88*A + 75*B)*a*cos(d*x + c)^5 + (88*A + 75*B)*a*cos(d*x + c)^4)*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(d*cos(d*x + c)^5 + d*cos(d*x + c)^4), 1/384*(2*(3*(88*A + 75*B)*a*cos(d*x + c)^3 + 2*(88*A + 75*B)*a*cos(d*x + c)^2 + 8*(8*A + 15*B)*a*cos(d*x + c) + 48*B*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 3*((88*A + 75*B)*a*cos(d*x + c)^5 + (88*A + 75*B)*a*cos(d*x + c)^4)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*x + c)^5 + d*cos(d*x + c)^4)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(a \sec(dx + c) + a)^{\frac{3}{2}}}{\cos(dx + c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(3/2)/cos(d*x + c)^(5/2), x)

maple [B] time = 1.82, size = 467, normalized size = 1.89

$$a\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}(-1+\cos(dx+c))\left(264A(\cos^4(dx+c))\arctan\left(\frac{\sqrt{\frac{2}{1+\cos(dx+c)}}(\cos(dx+c)+1+\sin(dx+c))\sqrt{2}}{4}}\right)\sqrt{2}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(5/2),x)

[Out]
$$\begin{aligned} & -1/384/d*a*(a*(1+\cos(d*x+c))/\cos(d*x+c))^{(1/2)}*(-1+\cos(d*x+c))*(264*A*\cos(d \\ & *x+c)^4*\arctan(1/4*(-2/(1+\cos(d*x+c))))^{(1/2)}*(\cos(d*x+c)+1+\sin(d*x+c))*2^{(1 \\ & /2))*2^{(1/2)}-264*A*\cos(d*x+c)^4*\arctan(1/4*(-2/(1+\cos(d*x+c))))^{(1/2)}*(\cos(d \\ & *x+c)+1-\sin(d*x+c))*2^{(1/2))*2^{(1/2)}+225*B*\cos(d*x+c)^4*\arctan(1/4*(-2/(1+c \\ & os(d*x+c))))^{(1/2)}*(\cos(d*x+c)+1+\sin(d*x+c))*2^{(1/2))*2^{(1/2)}-225*B*\cos(d*x+ \\ & c)^4*\arctan(1/4*(-2/(1+\cos(d*x+c))))^{(1/2)}*(\cos(d*x+c)+1-\sin(d*x+c))*2^{(1/2) \\ &)*2^{(1/2)}+528*A*\cos(d*x+c)^3*\sin(d*x+c)*(-2/(1+\cos(d*x+c))))^{(1/2)}+450*B*\cos \\ & (d*x+c)^3*\sin(d*x+c)*(-2/(1+\cos(d*x+c))))^{(1/2)}+352*A*\sin(d*x+c)*\cos(d*x+c)^ \\ & 2*(-2/(1+\cos(d*x+c))))^{(1/2)}+300*B*\sin(d*x+c)*\cos(d*x+c)^2*(-2/(1+\cos(d*x+c) \\ &))^{(1/2)}+128*A*\sin(d*x+c)*\cos(d*x+c)*(-2/(1+\cos(d*x+c))))^{(1/2)}+240*B*\sin(d \\ & x+c)*\cos(d*x+c)*(-2/(1+\cos(d*x+c))))^{(1/2)}+96*B*(-2/(1+\cos(d*x+c))))^{(1/2)}*si \\ & n(d*x+c)/(-2/(1+\cos(d*x+c))))^{(1/2)}/\cos(d*x+c)^{(7/2)}/\sin(d*x+c)^2 \end{aligned}$$

maxima [B] time = 1.46, size = 5879, normalized size = 23.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="maxima")

[Out]
$$\begin{aligned} & -1/768*(8*(132*(\sqrt{2})*a*\sin(6*d*x + 6*c) + 3*\sqrt{2})*a*\sin(4*d*x + 4*c) + \\ & 3*\sqrt{2})*a*\sin(2*d*x + 2*c))*\cos(11/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x \\ & + 2*c))) + 44*(\sqrt{2})*a*\sin(6*d*x + 6*c) + 3*\sqrt{2})*a*\sin(4*d*x + 4*c) + \\ & 3*\sqrt{2})*a*\sin(2*d*x + 2*c))*\cos(9/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x \\ & + 2*c))) + 216*(\sqrt{2})*a*\sin(6*d*x + 6*c) + 3*\sqrt{2})*a*\sin(4*d*x + 4*c) + \\ & 3*\sqrt{2})*a*\sin(2*d*x + 2*c))*\cos(7/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x \\ & + 2*c))) - 216*(\sqrt{2})*a*\sin(6*d*x + 6*c) + 3*\sqrt{2})*a*\sin(4*d*x + 4*c) + \\ & 3*\sqrt{2})*a*\sin(2*d*x + 2*c))*\cos(5/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x \\ & + 2*c))) - 44*(\sqrt{2})*a*\sin(6*d*x + 6*c) + 3*\sqrt{2})*a*\sin(4*d*x + 4*c) + \\ & 3*\sqrt{2})*a*\sin(2*d*x + 2*c))*\cos(3/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + \\ & 2*c))) - 132*(\sqrt{2})*a*\sin(6*d*x + 6*c) + 3*\sqrt{2})*a*\sin(4*d*x + 4*c) + \\ & 3*\sqrt{2})*a*\sin(2*d*x + 2*c))*\cos(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + \end{aligned}$$

$$\begin{aligned}
& 2*c))) - 33*(a*\cos(6*d*x + 6*c)^2 + 9*a*\cos(4*d*x + 4*c)^2 + 9*a*\cos(2*d*x \\
& + 2*c)^2 + a*\sin(6*d*x + 6*c)^2 + 9*a*\sin(4*d*x + 4*c)^2 + 18*a*\sin(4*d*x \\
& + 4*c)*\sin(2*d*x + 2*c) + 9*a*\sin(2*d*x + 2*c)^2 + 2*(3*a*\cos(4*d*x + 4*c) \\
& + 3*a*\cos(2*d*x + 2*c) + a)*\cos(6*d*x + 6*c) + 6*(3*a*\cos(2*d*x + 2*c) + a) \\
& *\cos(4*d*x + 4*c) + 6*a*\cos(2*d*x + 2*c) + 6*(a*\sin(4*d*x + 4*c) + a*\sin(2* \\
& d*x + 2*c))*\sin(6*d*x + 6*c) + a)*\log(2*\cos(1/4*\arctan2(\sin(2*d*x + 2*c), c \\
& os(2*d*x + 2*c)))^2 + 2*\sin(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)) \\
&)^2 + 2*\sqrt{2}*\cos(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 2*\sqrt{2} \\
& *\sin(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 2) + 33*(a*\cos \\
& (6*d*x + 6*c)^2 + 9*a*\cos(4*d*x + 4*c)^2 + 9*a*\cos(2*d*x + 2*c)^2 + a*\sin(6 \\
& *d*x + 6*c)^2 + 9*a*\sin(4*d*x + 4*c)^2 + 18*a*\sin(4*d*x + 4*c)*\sin(2*d*x + \\
& 2*c) + 9*a*\sin(2*d*x + 2*c)^2 + 2*(3*a*\cos(4*d*x + 4*c) + 3*a*\cos(2*d*x + 2 \\
& *c) + a)*\cos(6*d*x + 6*c) + 6*(3*a*\cos(2*d*x + 2*c) + a)*\cos(4*d*x + 4*c) + \\
& 6*a*\cos(2*d*x + 2*c) + 6*(a*\sin(4*d*x + 4*c) + a*\sin(2*d*x + 2*c))*\sin(6*d \\
& *x + 6*c) + a)*\log(2*\cos(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 \\
& + 2*\sin(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + 2*\sqrt{2}*\cos \\
& (1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) - 2*\sqrt{2}*\sin(1/4*\arcta \\
& n2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 2) - 33*(a*\cos(6*d*x + 6*c)^2 + 9 \\
& *a*\cos(4*d*x + 4*c)^2 + 9*a*\cos(2*d*x + 2*c)^2 + a*\sin(6*d*x + 6*c)^2 + 9*a \\
& *\sin(4*d*x + 4*c)^2 + 18*a*\sin(4*d*x + 4*c)*\sin(2*d*x + 2*c) + 9*a*\sin(2*d* \\
& x + 2*c)^2 + 2*(3*a*\cos(4*d*x + 4*c) + 3*a*\cos(2*d*x + 2*c) + a)*\cos(6*d*x \\
& + 6*c) + 6*(3*a*\cos(2*d*x + 2*c) + a)*\cos(4*d*x + 4*c) + 6*a*\cos(2*d*x + 2* \\
& c) + 6*(a*\sin(4*d*x + 4*c) + a*\sin(2*d*x + 2*c))*\sin(6*d*x + 6*c) + a)*\log(\\
& 2*\cos(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + 2*\sin(1/4*\arctan \\
& 2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 - 2*\sqrt{2}*\cos(1/4*\arctan2(\sin(2* \\
& d*x + 2*c), \cos(2*d*x + 2*c))) + 2*\sqrt{2}*\sin(1/4*\arctan2(\sin(2*d*x + 2*c) \\
& , \cos(2*d*x + 2*c))) + 2) + 33*(a*\cos(6*d*x + 6*c)^2 + 9*a*\cos(4*d*x + 4*c) \\
& ^2 + 9*a*\cos(2*d*x + 2*c)^2 + a*\sin(6*d*x + 6*c)^2 + 9*a*\sin(4*d*x + 4*c)^2 \\
& + 18*a*\sin(4*d*x + 4*c)*\sin(2*d*x + 2*c) + 9*a*\sin(2*d*x + 2*c)^2 + 2*(3*a \\
& *\cos(4*d*x + 4*c) + 3*a*\cos(2*d*x + 2*c) + a)*\cos(6*d*x + 6*c) + 6*(3*a*\cos \\
& (2*d*x + 2*c) + a)*\cos(4*d*x + 4*c) + 6*a*\cos(2*d*x + 2*c) + 6*(a*\sin(4*d*x \\
& + 4*c) + a*\sin(2*d*x + 2*c))*\sin(6*d*x + 6*c) + a)*\log(2*\cos(1/4*\arctan2(s \\
& in(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + 2*\sin(1/4*\arctan2(\sin(2*d*x + 2*c), \\
& \cos(2*d*x + 2*c)))^2 - 2*\sqrt{2}*\cos(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d \\
& *x + 2*c))) - 2*\sqrt{2}*\sin(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)) \\
&) + 2) - 132*(\sqrt{2})*a*\cos(6*d*x + 6*c) + 3*\sqrt{2})*a*\cos(4*d*x + 4*c) + 3 \\
& *\sqrt{2})*a*\cos(2*d*x + 2*c) + \sqrt{2})*a*\sin(11/4*\arctan2(\sin(2*d*x + 2*c), \\
& \cos(2*d*x + 2*c))) - 44*(\sqrt{2})*a*\cos(6*d*x + 6*c) + 3*\sqrt{2})*a*\cos(4*d* \\
& x + 4*c) + 3*\sqrt{2})*a*\cos(2*d*x + 2*c) + \sqrt{2})*a*\sin(9/4*\arctan2(\sin(2* \\
& d*x + 2*c), \cos(2*d*x + 2*c))) - 216*(\sqrt{2})*a*\cos(6*d*x + 6*c) + 3*\sqrt{2} \\
&)*a*\cos(4*d*x + 4*c) + 3*\sqrt{2})*a*\cos(2*d*x + 2*c) + \sqrt{2})*a*\sin(7/4*\ar \\
& ctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 216*(\sqrt{2})*a*\cos(6*d*x + 6*c \\
&) + 3*\sqrt{2})*a*\cos(4*d*x + 4*c) + 3*\sqrt{2})*a*\cos(2*d*x + 2*c) + \sqrt{2})*a \\
&)*\sin(5/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 44*(\sqrt{2})*a*\cos(\\
& 6*d*x + 6*c) + 3*\sqrt{2})*a*\cos(4*d*x + 4*c) + 3*\sqrt{2})*a*\cos(2*d*x + 2*c)
\end{aligned}$$


```

*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) + 100*(sqrt(2)*a*cos(8*d*x + 8*c) + 4*sqrt(2)*a*cos(6*d*x + 6*c)
+ 6*sqrt(2)*a*cos(4*d*x + 4*c) + 4*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a
*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 300*(sqrt(2)*a*cos(
8*d*x + 8*c) + 4*sqrt(2)*a*cos(6*d*x + 6*c) + 6*sqrt(2)*a*cos(4*d*x + 4*c)
+ 4*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(1/4*arctan2(sin(2*d*x + 2*c)
), cos(2*d*x + 2*c))) *B*sqrt(a)/(2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c)
) + 4*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*co
s(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x +
6*c)^2 + 12*(4*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)
^2 + 16*cos(2*d*x + 2*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2
*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x
+ 4*c) + 2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*
sin(4*d*x + 4*c)^2 + 48*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x +
2*c)^2 + 8*cos(2*d*x + 2*c) + 1))/d

```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(a + \frac{a}{\cos(c+dx)}\right)^{3/2}}{\cos(c+dx)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(3/2))/cos(c + d*x)^(5/2), x)
```

```
[Out] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(3/2))/cos(c + d*x)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c))/cos(d*x+c)**(5/2), x)
```

```
[Out] Timed out
```

$$3.532 \quad \int \cos^{\frac{11}{2}}(c+dx)(a+a \sec(c+dx))^{5/2}(A+B \sec(c+dx)) dx$$

Optimal. Leaf size=275

$$\frac{2a^3(194A + 209B) \sin(c + dx) \cos^{\frac{5}{2}}(c + dx)}{693d\sqrt{a \sec(c + dx) + a}} + \frac{2a^3(710A + 803B) \sin(c + dx) \cos^{\frac{3}{2}}(c + dx)}{1155d\sqrt{a \sec(c + dx) + a}} + \frac{8a^3(710A + 803B) \sin(c + dx) \cos^{\frac{1}{2}}(c + dx)}{3465d\sqrt{a \sec(c + dx) + a}}$$

[Out] $2/11*a*A*\cos(d*x+c)^{(9/2)}*(a+a*\sec(d*x+c))^{(3/2)}*\sin(d*x+c)/d+2/1155*a^3*(710*A+803*B)*\cos(d*x+c)^{(3/2)}*\sin(d*x+c)/d/(a+a*\sec(d*x+c))^{(1/2)}+2/693*a^3*(194*A+209*B)*\cos(d*x+c)^{(5/2)}*\sin(d*x+c)/d/(a+a*\sec(d*x+c))^{(1/2)}+16/3465*a^3*(710*A+803*B)*\sin(d*x+c)/d/\cos(d*x+c)^{(1/2)}/(a+a*\sec(d*x+c))^{(1/2)}+8/3465*a^3*(710*A+803*B)*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/d/(a+a*\sec(d*x+c))^{(1/2)}+2/99*a^2*(14*A+11*B)*\cos(d*x+c)^{(7/2)}*\sin(d*x+c)*(a+a*\sec(d*x+c))^{(1/2)}/d$

Rubi [A] time = 0.83, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2955, 4017, 4015, 3805, 3804}

$$\frac{2a^2(14A + 11B) \sin(c + dx) \cos^{\frac{7}{2}}(c + dx) \sqrt{a \sec(c + dx) + a}}{99d} + \frac{2a^3(194A + 209B) \sin(c + dx) \cos^{\frac{5}{2}}(c + dx)}{693d\sqrt{a \sec(c + dx) + a}} + \frac{2a^3(710A + 803B) \sin(c + dx) \cos^{\frac{3}{2}}(c + dx)}{1155d\sqrt{a \sec(c + dx) + a}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Cos}[c + d*x]^{(11/2)}*(a + a*\text{Sec}[c + d*x])^{(5/2)}*(A + B*\text{Sec}[c + d*x]), x]$

[Out] $(16*a^3*(710*A + 803*B)*\text{Sin}[c + d*x])/(3465*d*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sqrt}[a + a*\text{Sec}[c + d*x]]) + (8*a^3*(710*A + 803*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sin}[c + d*x])/(3465*d*\text{Sqrt}[a + a*\text{Sec}[c + d*x]]) + (2*a^3*(710*A + 803*B)*\text{Cos}[c + d*x]^{(3/2)}*\text{Sin}[c + d*x])/(1155*d*\text{Sqrt}[a + a*\text{Sec}[c + d*x]]) + (2*a^3*(194*A + 209*B)*\text{Cos}[c + d*x]^{(5/2)}*\text{Sin}[c + d*x])/(693*d*\text{Sqrt}[a + a*\text{Sec}[c + d*x]]) + (2*a^2*(14*A + 11*B)*\text{Cos}[c + d*x]^{(7/2)}*\text{Sqrt}[a + a*\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(99*d) + (2*a*A*\text{Cos}[c + d*x]^{(9/2)}*(a + a*\text{Sec}[c + d*x])^{(3/2)}*\text{Sin}[c + d*x])/(11*d)$

Rule 2955

$\text{Int}[(a_. + \text{csc}[(e_.) + (f_.)*(x_.)]*(b_.))^{(m_.)}*(\text{csc}[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))^{(n_.)}*((g_.)*\sin[(e_.) + (f_.)*(x_.)])^{(p_.)}, x_Symbol] := \text{Dist}[(g*\text{Csc}[e + f*x])^p*(g*\text{Sin}[e + f*x])^p, \text{Int}[(a + b*\text{Csc}[e + f*x])^m*(c + d*\text{Csc}[e + f*x])^n]/(g*\text{Csc}[e + f*x])^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, m, n, p\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& !\text{IntegerQ}[p] \&\& !(\text{IntegerQ}[m] \&\& \text{IntegerQ}[n])$

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^n*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_.)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
+ b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^n*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_.)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] :> Simp[(A*b^2*Co
t[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[
e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_.))^m*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \cos^{\frac{11}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))dx &= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \frac{(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))}{\sec(c+dx)}dx \\
&= \frac{2aA\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{11d} \\
&= \frac{2a^2(14A+11B)\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}{99d} \\
&= \frac{2a^3(194A+209B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{693d\sqrt{a+a\sec(c+dx)}} + \frac{2a^3(710A+803B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{1155d\sqrt{a+a\sec(c+dx)}} \\
&= \frac{8a^3(710A+803B)\sqrt{\cos(c+dx)}\sin(c+dx)}{3465d\sqrt{a+a\sec(c+dx)}} + \frac{16a^3(710A+803B)\sin(c+dx)}{3465d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{8a^3(710A+803B)\sin(c+dx)}{3465d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}
\end{aligned}$$

Mathematica [A] time = 0.60, size = 137, normalized size = 0.50

$$\frac{2a^2\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a(\sec(c+dx)+1)}(35(32A+11B)\cos^4(c+dx)+5(355A+286B)\cos^3(c+dx))}{3465d(\cos(c+dx)+1)}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]^(11/2)*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]), x]

[Out] (2*a^2*Sqrt[Cos[c + d*x]]*(8*(710*A + 803*B) + 4*(710*A + 803*B)*Cos[c + d*x] + 3*(710*A + 803*B)*Cos[c + d*x]^2 + 5*(355*A + 286*B)*Cos[c + d*x]^3 + 35*(32*A + 11*B)*Cos[c + d*x]^4 + 315*A*Cos[c + d*x]^5)*Sqrt[a*(1 + Sec[c + d*x])]*Sin[c + d*x])/(3465*d*(1 + Cos[c + d*x]))

fricas [A] time = 0.45, size = 154, normalized size = 0.56

$$2(315Aa^2\cos(dx+c)^5 + 35(32A+11B)a^2\cos(dx+c)^4 + 5(355A+286B)a^2\cos(dx+c)^3 + 3(710A+803B)a^2\cos(dx+c)^2 + 8a^3\sin(dx+c)\sqrt{\cos(dx+c)}\sqrt{a+a\sec(dx+c)})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(11/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] 2/3465*(315*A*a^2*cos(d*x + c)^5 + 35*(32*A + 11*B)*a^2*cos(d*x + c)^4 + 5*(355*A + 286*B)*a^2*cos(d*x + c)^3 + 3*(710*A + 803*B)*a^2*cos(d*x + c)^2 + 4*(710*A + 803*B)*a^2*cos(d*x + c) + 8*(710*A + 803*B)*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c) + d)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(a \sec(dx + c) + a)^{\frac{5}{2}} \cos(dx + c)^{\frac{11}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(11/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(11/2), x)

maple [A] time = 2.08, size = 155, normalized size = 0.56

$$2a^2(-1 + \cos(dx + c)) \left(315A \left(\cos^5(dx + c) \right) + 1120A \left(\cos^4(dx + c) \right) + 385B \left(\cos^4(dx + c) \right) + 1775A \left(\cos^3(dx + c) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(11/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x)

[Out] -2/3465/d*a^2*(-1+cos(d*x+c))*(315*A*cos(d*x+c)^5+1120*A*cos(d*x+c)^4+385*B*cos(d*x+c)^4+1775*A*cos(d*x+c)^3+1430*B*cos(d*x+c)^3+2130*A*cos(d*x+c)^2+2409*B*cos(d*x+c)^2+2840*A*cos(d*x+c)+3212*B*cos(d*x+c)+5680*A+6424*B)*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)

maxima [B] time = 0.75, size = 754, normalized size = 2.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(11/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="maxima")


```
[Out] 1/110880*(5*sqrt(2)*(31878*a^2*cos(10/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c)))*sin(11/2*d*x + 11/2*c) + 8778*a^2*cos(8/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c)))*sin(11/2*d*x + 11/2*c) + 3465*a^2*cos(6/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c)))*sin(11/2*d*x + 11/2*c) + 1287*a^2*cos(4/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c)))*sin(11/2*d*x + 11/2*c) + 385*a^2*cos(2/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c)))*sin(11/2*d*x + 11/2*c) - 31878*a^2*cos(11/2*d*x + 11/2*c)*sin(10/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) - 8778*a^2*cos(11/2*d*x + 11/2*c)*sin(8/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) - 3465*a^2*cos(11/2*d*x + 11/2*c)*sin(6/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) - 1287*a^2*cos(11/2*d*x + 11/2*c)*sin(4/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) - 385*a^2*cos(11/2*d*x + 11/2*c)*sin(2/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) + 126*a^2*sin(11/2*d*x + 11/2*c) + 385*a^2*sin(9/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) + 1287*a^2*sin(7/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) + 3465*a^2*sin(5/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) + 8778*a^2*sin(3/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) + 31878*a^2*sin(1/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))))*A*sqrt(a) + 44*sqrt(2)*(225*a^2*sin(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 378*a^2*sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2100*a^2*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4095*a^2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 63*(65*a^2*sin(4*d*x + 4*c) + 6*a^2*sin(2*d*x + 2*c))*cos(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 7*(585*a^2*cos(4*d*x + 4*c) + 54*a^2*cos(2*d*x + 2*c) + 5*a^2)*sin(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*B*sqrt(a))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c + dx)^{11/2} \left(A + \frac{B}{\cos(c + dx)} \right) \left(a + \frac{a}{\cos(c + dx)} \right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(11/2)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^(11/2)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(11/2)*(a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)),x)
```

[Out] Timed out

$$3.533 \quad \int \cos^2(c+dx)(a+a \sec(c+dx))^{5/2}(A+B \sec(c+dx)) dx$$

Optimal. Leaf size=228

$$\frac{2a^3(124A + 135B) \sin(c + dx) \cos^3(c + dx)}{315d\sqrt{a \sec(c + dx) + a}} + \frac{2a^3(292A + 345B) \sin(c + dx) \sqrt{\cos(c + dx)}}{315d\sqrt{a \sec(c + dx) + a}} + \frac{4a^3(292A + 345B) \sin(c + dx)}{315d\sqrt{\cos(c + dx)}}$$

[Out] $2/9*a*A*\cos(d*x+c)^{(7/2)}*(a+a*\sec(d*x+c))^{(3/2)}*\sin(d*x+c)/d+2/315*a^3*(124*A+135*B)*\cos(d*x+c)^{(3/2)}*\sin(d*x+c)/d/(a+a*\sec(d*x+c))^{(1/2)}+4/315*a^3*(292*A+345*B)*\sin(d*x+c)/d/\cos(d*x+c)^{(1/2)}/(a+a*\sec(d*x+c))^{(1/2)}+2/315*a^3*(292*A+345*B)*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/d/(a+a*\sec(d*x+c))^{(1/2)}+2/21*a^2*(4*A+3*B)*\cos(d*x+c)^{(5/2)}*\sin(d*x+c)*(a+a*\sec(d*x+c))^{(1/2)}/d$

Rubi [A] time = 0.76, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2955, 4017, 4015, 3805, 3804}

$$\frac{2a^2(4A + 3B) \sin(c + dx) \cos^5(c + dx) \sqrt{a \sec(c + dx) + a}}{21d} + \frac{2a^3(124A + 135B) \sin(c + dx) \cos^3(c + dx)}{315d\sqrt{a \sec(c + dx) + a}} + \frac{2a^3(292A + 345B) \sin(c + dx)}{315d\sqrt{\cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]

[Out] $(4*a^3*(292*A + 345*B)*\sin[c + d*x])/(315*d*\sqrt{\cos[c + d*x]}*\sqrt{a + a*\sec[c + d*x]}) + (2*a^3*(292*A + 345*B)*\sqrt{\cos[c + d*x]}*\sin[c + d*x])/(315*d*\sqrt{a + a*\sec[c + d*x]}) + (2*a^3*(124*A + 135*B)*\cos[c + d*x]^{(3/2)}*\sin[c + d*x])/(315*d*\sqrt{a + a*\sec[c + d*x]}) + (2*a^2*(4*A + 3*B)*\cos[c + d*x]^{(5/2)}*\sqrt{a + a*\sec[c + d*x]}*\sin[c + d*x])/(21*d) + (2*a*A*\cos[c + d*x]^{(7/2)}*(a + a*\sec[c + d*x])^{(3/2)}*\sin[c + d*x])/(9*d)$

Rule 2955

Int[((a_.) + csc[(e_.) + (f_.)*(x_)])*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] := Dist[(g*Csc[e + f*x])^p*(g*Sine[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && IntegerQ[n])

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]/Sqrt[csc[(e_.) + (f_.)*(x_.)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.)
+ (a_.)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
+ b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.)
+ (a_.)]*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> Simp[(A*b^2*Co
t[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[
e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (
a_.))^m*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))dx &= (\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}) \int \frac{(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))}{\cos^2(c+dx)} dx \\
&= \frac{2aA\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{9d} \\
&= \frac{2a^2(4A+3B)\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{21d} \\
&= \frac{2a^3(124A+135B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{315d\sqrt{a+a\sec(c+dx)}} + \frac{2a^3(292A+345B)\sqrt{\cos(c+dx)}\sin(c+dx)}{315d\sqrt{a+a\sec(c+dx)}} \\
&= \frac{4a^3(292A+345B)\sin(c+dx)}{315d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{2a^3(292A+345B)\sqrt{\cos(c+dx)}\sin(c+dx)}{315d\sqrt{a+a\sec(c+dx)}}
\end{aligned}$$

Mathematica [A] time = 0.55, size = 116, normalized size = 0.51

$$\frac{2a^2\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a(\sec(c+dx)+1)}(5(26A+9B)\cos^3(c+dx)+3(73A+60B)\cos^2(c+dx)+2(73A+60B)\cos(c+dx)+2a^2)}{315d(\cos(c+dx)+1)}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]), x]

[Out] (2*a^2*Sqrt[Cos[c + d*x]]*(584*A + 690*B + (292*A + 345*B)*Cos[c + d*x] + 3*(73*A + 60*B)*Cos[c + d*x]^2 + 5*(26*A + 9*B)*Cos[c + d*x]^3 + 35*A*Cos[c + d*x]^4)*Sqrt[a*(1 + Sec[c + d*x])]*Sin[c + d*x])/(315*d*(1 + Cos[c + d*x]))

fricas [A] time = 0.46, size = 133, normalized size = 0.58

$$\frac{2(35Aa^2\cos(dx+c)^4+5(26A+9B)a^2\cos(dx+c)^3+3(73A+60B)a^2\cos(dx+c)^2+(292A+345B)a^2\cos(dx+c)+2a^2)}{315(d\cos(dx+c)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] $2/315*(35*A*a^2*\cos(dx + c)^4 + 5*(26*A + 9*B)*a^2*\cos(dx + c)^3 + 3*(73*A + 60*B)*a^2*\cos(dx + c)^2 + (292*A + 345*B)*a^2*\cos(dx + c) + 2*(292*A + 345*B)*a^2)*\sqrt{(a*\cos(dx + c) + a)/\cos(dx + c)}*\sqrt{\cos(dx + c)}*\sin(dx + c)/(d*\cos(dx + c) + d)$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(a \sec(dx + c) + a)^{\frac{5}{2}} \cos(dx + c)^{\frac{9}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(9/2), x)

maple [A] time = 1.91, size = 133, normalized size = 0.58

$$2a^2(-1 + \cos(dx + c))\left(35A(\cos^4(dx + c)) + 130A(\cos^3(dx + c)) + 45B(\cos^3(dx + c)) + 219A(\cos^2(dx + c)) + 315d\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x)

[Out] $-2/315/d*a^2*(-1+\cos(dx+c))*(35*A*\cos(dx+c)^4+130*A*\cos(dx+c)^3+45*B*\cos(dx+c)^2+219*A*\cos(dx+c)+180*B*\cos(dx+c)+292*A*\cos(dx+c)+345*B*\cos(dx+c)+584*A+690*B)*\cos(dx+c)^{1/2}*(a*(1+\cos(dx+c))/\cos(dx+c))^{1/2}/\sin(dx+c)$

maxima [B] time = 0.92, size = 596, normalized size = 2.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] $1/5040*(\sqrt{2}*(8190*a^2*\cos(8/9*\arctan2(\sin(9/2*d*x + 9/2*c), \cos(9/2*d*x + 9/2*c))))*\sin(9/2*d*x + 9/2*c) + 2100*a^2*\cos(2/3*\arctan2(\sin(9/2*d*x + 9/2*c), \cos(9/2*d*x + 9/2*c))))*\sin(9/2*d*x + 9/2*c) + 756*a^2*\cos(4/9*\arctan$

$2(\sin(9/2*d*x + 9/2*c), \cos(9/2*d*x + 9/2*c)) * \sin(9/2*d*x + 9/2*c) + 225*a^2 * \cos(2/9*\arctan2(\sin(9/2*d*x + 9/2*c), \cos(9/2*d*x + 9/2*c))) * \sin(9/2*d*x + 9/2*c) - 8190*a^2 * \cos(9/2*d*x + 9/2*c) * \sin(8/9*\arctan2(\sin(9/2*d*x + 9/2*c), \cos(9/2*d*x + 9/2*c))) - 2100*a^2 * \cos(9/2*d*x + 9/2*c) * \sin(2/3*\arctan2(\sin(9/2*d*x + 9/2*c), \cos(9/2*d*x + 9/2*c))) - 756*a^2 * \cos(9/2*d*x + 9/2*c) * \sin(4/9*\arctan2(\sin(9/2*d*x + 9/2*c), \cos(9/2*d*x + 9/2*c))) - 225*a^2 * \cos(9/2*d*x + 9/2*c) * \sin(2/9*\arctan2(\sin(9/2*d*x + 9/2*c), \cos(9/2*d*x + 9/2*c))) + 70*a^2 * \sin(9/2*d*x + 9/2*c) + 225*a^2 * \sin(7/9*\arctan2(\sin(9/2*d*x + 9/2*c), \cos(9/2*d*x + 9/2*c))) + 756*a^2 * \sin(5/9*\arctan2(\sin(9/2*d*x + 9/2*c), \cos(9/2*d*x + 9/2*c))) + 2100*a^2 * \sin(1/3*\arctan2(\sin(9/2*d*x + 9/2*c), \cos(9/2*d*x + 9/2*c))) + 8190*a^2 * \sin(1/9*\arctan2(\sin(9/2*d*x + 9/2*c), \cos(9/2*d*x + 9/2*c))) * A * \sqrt{a} - 30 * \sqrt{2} * (77*a^2 * \cos(7/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) * \sin(2*d*x + 2*c) - 42*a^2 * \sin(5/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) - 77*a^2 * \sin(3/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) - 630*a^2 * \sin(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) - (77*a^2 * \cos(2*d*x + 2*c) + 6*a^2) * \sin(7/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))) * B * \sqrt{a}) / d$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c + dx)^{9/2} \left(A + \frac{B}{\cos(c + dx)} \right) \left(a + \frac{a}{\cos(c + dx)} \right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^(9/2)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2), x)

[Out] int(cos(c + d*x)^(9/2)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**(9/2)*(a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)), x)

[Out] Timed out

$$3.534 \quad \int \cos^{\frac{7}{2}}(c+dx)(a+a \sec(c+dx))^{5/2}(A+B \sec(c+dx)) dx$$

Optimal. Leaf size=178

$$\frac{64a^3(5A+7B)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}} + \frac{16a^2(5A+7B)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}{105d} + \frac{2a(5A+7B)\sin(c+dx)}{105d}$$

[Out] $2/35*a*(5*A+7*B)*\cos(d*x+c)^{(3/2)}*(a+a*\sec(d*x+c))^{(3/2)}*\sin(d*x+c)/d+2/7*A*\cos(d*x+c)^{(5/2)}*(a+a*\sec(d*x+c))^{(5/2)}*\sin(d*x+c)/d+64/105*a^3*(5*A+7*B)*\sin(d*x+c)/d/\cos(d*x+c)^{(1/2)}/(a+a*\sec(d*x+c))^{(1/2)}+16/105*a^2*(5*A+7*B)*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}*(a+a*\sec(d*x+c))^{(1/2)}/d$

Rubi [A] time = 0.46, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {2955, 4013, 3809, 3804}

$$\frac{64a^3(5A+7B)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}} + \frac{16a^2(5A+7B)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}{105d} + \frac{2a(5A+7B)\sin(c+dx)}{105d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Cos}[c+d*x]^{(7/2)}*(a+a*\text{Sec}[c+d*x])^{(5/2)}*(A+B*\text{Sec}[c+d*x]),x]$

[Out] $(64*a^3*(5*A+7*B)*\text{Sin}[c+d*x])/(105*d*\text{Sqrt}[\text{Cos}[c+d*x]]*\text{Sqrt}[a+a*\text{Sec}[c+d*x]]) + (16*a^2*(5*A+7*B)*\text{Sqrt}[\text{Cos}[c+d*x]]*\text{Sqrt}[a+a*\text{Sec}[c+d*x]]*\text{Sin}[c+d*x])/(105*d) + (2*a*(5*A+7*B)*\text{Cos}[c+d*x]^{(3/2)}*(a+a*\text{Sec}[c+d*x])^{(3/2)}*\text{Sin}[c+d*x])/(35*d) + (2*A*\text{Cos}[c+d*x]^{(5/2)}*(a+a*\text{Sec}[c+d*x])^{(5/2)}*\text{Sin}[c+d*x])/(7*d)$

Rule 2955

$\text{Int}[(a_.) + \text{csc}[(e_.) + (f_.)*(x_)]*(b_.)]^{(m_.)}*(\text{csc}[(e_.) + (f_.)*(x_)]*(d_.) + (c_.)^{(n_.)}*(g_.)*\sin[(e_.) + (f_.)*(x_)]^{(p_.)}, x_Symbol] :> \text{Dist}[(g*\text{Csc}[e+f*x])^p*(g*\text{Sin}[e+f*x])^p, \text{Int}[(a+b*\text{Csc}[e+f*x])^m*(c+d*\text{Csc}[e+f*x])^n/(g*\text{Csc}[e+f*x])^p, x], x] /;$ FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && IntegerQ[n])

Rule 3804

$\text{Int}[\text{Sqrt}[\text{csc}[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)]/\text{Sqrt}[\text{csc}[(e_.) + (f_.)*(x_)]*(d_.)], x_Symbol] :> \text{Simp}[(-2*a*\text{Cot}[e+f*x])/(f*\text{Sqrt}[a+b*\text{Csc}[e+f*x]])*$

Sqrt[d*Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 3809

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m, x_Symbol] :> -Simp[(a*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*m), x] + Dist[(b*(2*m - 1))/(d*m), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && GtQ[m, 1/2] && IntegerQ[2*m]

Rule 4013

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && !LeQ[m, -1]

Rubi steps

$$\begin{aligned}
 \int \cos^{\frac{7}{2}}(c + dx)(a + a \sec(c + dx))^{5/2}(A + B \sec(c + dx)) dx &= \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)}\right) \int \frac{(a + a \sec(c + dx))^{5/2} (A + B \sec(c + dx))}{\sec(c + dx)} dx \\
 &= \frac{2A \cos^{\frac{5}{2}}(c + dx)(a + a \sec(c + dx))^{5/2} \sin(c + dx)}{7d} \\
 &= \frac{2a(5A + 7B) \cos^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))^{3/2} \sin(c + dx)}{35d} \\
 &= \frac{16a^2(5A + 7B) \sqrt{\cos(c + dx)} \sqrt{a + a \sec(c + dx)} \sin(c + dx)}{105d} \\
 &= \frac{64a^3(5A + 7B) \sin(c + dx)}{105d \sqrt{\cos(c + dx)} \sqrt{a + a \sec(c + dx)}} + \frac{16a^2(5A + 7B) \sin(c + dx)}{105d \sqrt{\cos(c + dx)} \sqrt{a + a \sec(c + dx)}}
 \end{aligned}$$

Mathematica [A] time = 0.41, size = 99, normalized size = 0.56

$$\frac{2a^2 \sin(c + dx) \sqrt{\cos(c + dx)} \sqrt{a(\sec(c + dx) + 1)} (3(20A + 7B) \cos^2(c + dx) + (115A + 98B) \cos(c + dx) + 15A)}{105d(\cos(c + dx) + 1)}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]), x]

[Out] (2*a^2*Sqrt[Cos[c + d*x]]*(230*A + 301*B + (115*A + 98*B)*Cos[c + d*x] + 3*(20*A + 7*B)*Cos[c + d*x]^2 + 15*A*Cos[c + d*x]^3)*Sqrt[a*(1 + Sec[c + d*x])]*Sin[c + d*x])/(105*d*(1 + Cos[c + d*x]))

fricas [A] time = 0.45, size = 112, normalized size = 0.63

$$\frac{2 \left(15 A a^2 \cos(dx + c)^3 + 3(20 A + 7 B) a^2 \cos(dx + c)^2 + (115 A + 98 B) a^2 \cos(dx + c) + (230 A + 301 B) a^2 \right) \sqrt{a}}{105 (d \cos(dx + c) + d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)), x, algorithm="fricas")

[Out] 2/105*(15*A*a^2*cos(d*x + c)^3 + 3*(20*A + 7*B)*a^2*cos(d*x + c)^2 + (115*A + 98*B)*a^2*cos(d*x + c) + (230*A + 301*B)*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c) + d)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(a \sec(dx + c) + a)^{\frac{5}{2}} \cos(dx + c)^{\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)), x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(7/2), x)

maple [A] time = 1.95, size = 111, normalized size = 0.62

$$\frac{2a^2(-1 + \cos(dx + c)) \left(15A(\cos^3(dx + c)) + 60A(\cos^2(dx + c)) + 21B(\cos^2(dx + c)) + 115A \cos(dx + c) \right)}{105d \sin(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)), x)

[Out] $-2/105/d*a^2*(-1+\cos(d*x+c))*(15*A*\cos(d*x+c)^3+60*A*\cos(d*x+c)^2+21*B*\cos(d*x+c)^2+115*A*\cos(d*x+c)+98*B*\cos(d*x+c)+230*A+301*B)*\cos(d*x+c)^{(1/2)}*(a*(1+\cos(d*x+c))/\cos(d*x+c))^{(1/2)}/\sin(d*x+c)$

maxima [B] time = 0.69, size = 482, normalized size = 2.71

$$5\sqrt{2}\left(315a^2\cos\left(\frac{6}{7}\arctan\left(\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right),\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\right)\right)\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right)+77a^2\cos\left(\frac{4}{7}\arctan\left(\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right),\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\right)\right)\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right)+21a^2\cos\left(\frac{2}{7}\arctan\left(\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right),\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\right)\right)\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right)-315a^2\cos\left(\frac{6}{7}\arctan\left(\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right),\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\right)\right)\sin\left(\frac{6}{7}\arctan\left(\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right),\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\right)\right)-77a^2\cos\left(\frac{4}{7}\arctan\left(\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right),\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\right)\right)\sin\left(\frac{4}{7}\arctan\left(\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right),\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\right)\right)-21a^2\cos\left(\frac{2}{7}\arctan\left(\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right),\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\right)\right)\sin\left(\frac{2}{7}\arctan\left(\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right),\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\right)\right)+6a^2\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right)+21a^2\sin\left(\frac{5}{7}\arctan\left(\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right),\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\right)\right)+77a^2\sin\left(\frac{3}{7}\arctan\left(\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right),\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\right)\right)+315a^2\sin\left(\frac{1}{7}\arctan\left(\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right),\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\right)\right)*A*\sqrt{a}-28*(75*\sqrt{2})*a^2*\cos\left(\frac{5}{4}\arctan\left(\sin(2*d*x+2*c),\cos(2*d*x+2*c)\right)\right)*\sin(2*d*x+2*c)-25*\sqrt{2})*a^2*\sin\left(\frac{3}{4}\arctan\left(\sin(2*d*x+2*c),\cos(2*d*x+2*c)\right)\right)-75*\sqrt{2})*a^2*\sin\left(\frac{1}{4}\arctan\left(\sin(2*d*x+2*c),\cos(2*d*x+2*c)\right)\right)-3*(25*\sqrt{2})*a^2*\cos(2*d*x+2*c)+\sqrt{2})*a^2*\sin\left(\frac{5}{4}\arctan\left(\sin(2*d*x+2*c),\cos(2*d*x+2*c)\right)\right))*B*\sqrt{a})/d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorith="maxima")

[Out] $1/840*(5*\sqrt{2}*(315*a^2*\cos(6/7*\arctan2(\sin(7/2*d*x+7/2*c),\cos(7/2*d*x+7/2*c)))*\sin(7/2*d*x+7/2*c)+77*a^2*\cos(4/7*\arctan2(\sin(7/2*d*x+7/2*c),\cos(7/2*d*x+7/2*c)))*\sin(7/2*d*x+7/2*c)+21*a^2*\cos(2/7*\arctan2(\sin(7/2*d*x+7/2*c),\cos(7/2*d*x+7/2*c)))*\sin(7/2*d*x+7/2*c)-315*a^2*\cos(6/7*\arctan2(\sin(7/2*d*x+7/2*c),\cos(7/2*d*x+7/2*c)))*\sin(6/7*\arctan2(\sin(7/2*d*x+7/2*c),\cos(7/2*d*x+7/2*c))) - 77*a^2*\cos(4/7*\arctan2(\sin(7/2*d*x+7/2*c),\cos(7/2*d*x+7/2*c)))*\sin(4/7*\arctan2(\sin(7/2*d*x+7/2*c),\cos(7/2*d*x+7/2*c))) - 21*a^2*\cos(2/7*\arctan2(\sin(7/2*d*x+7/2*c),\cos(7/2*d*x+7/2*c)))*\sin(2/7*\arctan2(\sin(7/2*d*x+7/2*c),\cos(7/2*d*x+7/2*c))) + 6*a^2*\sin(7/2*d*x+7/2*c) + 21*a^2*\sin(5/7*\arctan2(\sin(7/2*d*x+7/2*c),\cos(7/2*d*x+7/2*c))) + 77*a^2*\sin(3/7*\arctan2(\sin(7/2*d*x+7/2*c),\cos(7/2*d*x+7/2*c))) + 315*a^2*\sin(1/7*\arctan2(\sin(7/2*d*x+7/2*c),\cos(7/2*d*x+7/2*c))))*A*\sqrt{a}-28*(75*\sqrt{2})*a^2*\cos(5/4*\arctan2(\sin(2*d*x+2*c),\cos(2*d*x+2*c)))*\sin(2*d*x+2*c)-25*\sqrt{2})*a^2*\sin(3/4*\arctan2(\sin(2*d*x+2*c),\cos(2*d*x+2*c))) - 75*\sqrt{2})*a^2*\sin(1/4*\arctan2(\sin(2*d*x+2*c),\cos(2*d*x+2*c))) - 3*(25*\sqrt{2})*a^2*\cos(2*d*x+2*c)+\sqrt{2})*a^2*\sin(5/4*\arctan2(\sin(2*d*x+2*c),\cos(2*d*x+2*c))))*B*\sqrt{a})/d$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+dx)^{7/2} \left(A + \frac{B}{\cos(c+dx)} \right) \left(a + \frac{a}{\cos(c+dx)} \right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c+d*x)^(7/2)*(A+B/cos(c+d*x))*(a+a/cos(c+d*x))^(5/2),x)

[Out] int(cos(c+d*x)^(7/2)*(A+B/cos(c+d*x))*(a+a/cos(c+d*x))^(5/2),x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

$$3.535 \quad \int \cos^2(c+dx) (a+a \sec(c+dx))^{5/2} (A+B \sec(c+dx)) dx$$

Optimal. Leaf size=192

$$\frac{2a^{5/2}B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d} + \frac{2a^3(32A+35B)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}} + \frac{2a^2(8A+5B)}{15d}$$

[Out] $2/5*a*A*\cos(d*x+c)^{(3/2)}*(a+a*\sec(d*x+c))^{(3/2)}*\sin(d*x+c)/d+2*a^{(5/2)}*B*\arcsinh(a^{(1/2)}*\tan(d*x+c)/(a+a*\sec(d*x+c))^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d+2/15*a^3*(32*A+35*B)*\sin(d*x+c)/d/\cos(d*x+c)^{(1/2)}/(a+a*\sec(d*x+c))^{(1/2)}+2/15*a^2*(8*A+5*B)*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}*(a+a*\sec(d*x+c))^{(1/2)}/d$

Rubi [A] time = 0.62, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2955, 4017, 4015, 3801, 215}

$$\frac{2a^3(32A+35B)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}} + \frac{2a^2(8A+5B)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}{15d} + \frac{2a^{5/2}B\sqrt{\cos(c+dx)}}{15d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Cos}[c+d*x]^{(5/2)}*(a+a*\text{Sec}[c+d*x])^{(5/2)}*(A+B*\text{Sec}[c+d*x]),x]$

[Out] $(2*a^{(5/2)}*B*\text{ArcSinh}[(\text{Sqrt}[a]*\text{Tan}[c+d*x])/(\text{Sqrt}[a+a*\text{Sec}[c+d*x]])]*\text{Sqrt}[\text{Cos}[c+d*x]]*\text{Sqrt}[\text{Sec}[c+d*x]])/d + (2*a^3*(32*A+35*B)*\text{Sin}[c+d*x])/(15*d*\text{Sqrt}[\text{Cos}[c+d*x]]*\text{Sqrt}[a+a*\text{Sec}[c+d*x]]) + (2*a^2*(8*A+5*B)*\text{Sqrt}[\text{Cos}[c+d*x]]*\text{Sqrt}[a+a*\text{Sec}[c+d*x]]*\text{Sin}[c+d*x])/(15*d) + (2*a*A*\text{Cos}[c+d*x]^{(3/2)}*(a+a*\text{Sec}[c+d*x])^{(3/2)}*\text{Sin}[c+d*x])/(5*d)$

Rule 215

$\text{Int}[1/\text{Sqrt}[(a_.) + (b_.)*(x_)^2], x_Symbol] \rightarrow \text{Simp}[\text{ArcSinh}[(\text{Rt}[b, 2]*x)/\text{Sqrt}[a]]/\text{Rt}[b, 2], x] /; \text{FreeQ}\{a, b\}, x \ \&\& \ \text{GtQ}[a, 0] \ \&\& \ \text{PosQ}[b]$

Rule 2955

$\text{Int}[(a_.) + \text{csc}[e_.) + (f_.)*(x_)]*(b_.)^{(m_.)}*(\text{csc}[e_.) + (f_.)*(x_)]*(d_.) + (c_.)^{(n_.)}*((g_.)*\sin[e_.) + (f_.)*(x_)]^{(p_.)}, x_Symbol] \rightarrow \text{Dist}[(g*\text{Csc}[e+f*x])^p*(g*\text{Sin}[e+f*x])^p, \text{Int}[(a+b*\text{Csc}[e+f*x])^m*(c+d*\text{Csc}[e+f*x])^n]/(g*\text{Csc}[e+f*x])^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, m, n, p\}, x \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ !\text{IntegerQ}[p] \ \&\& \ !(\text{IntegerQ}[m] \ \&\& \ \text{In}$

tegerQ[n])

Rule 3801

Int[Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 + x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]

Rule 4015

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> Simp[(A*b^2*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]

Rule 4017

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> Simp[(a*A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && LtQ[n, -1]

Rubi steps

$$\begin{aligned}
\int \cos^{\frac{5}{2}}(c+dx)(a+a \sec(c+dx))^{5/2}(A+B \sec(c+dx)) dx &= \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}\right) \int \frac{(a+a \sec(c+dx))^{5/2}(A+B \sec(c+dx))}{\cos^{\frac{5}{2}}(c+dx)} dx \\
&= \frac{2aA \cos^{\frac{3}{2}}(c+dx)(a+a \sec(c+dx))^{3/2} \sin(c+dx)}{5d} \\
&= \frac{2a^2(8A+5B)\sqrt{\cos(c+dx)} \sqrt{a+a \sec(c+dx)} \sin(c+dx)}{15d} \\
&= \frac{2a^3(32A+35B) \sin(c+dx)}{15d\sqrt{\cos(c+dx)} \sqrt{a+a \sec(c+dx)}} + \frac{2a^2(8A+5B) \sin(c+dx)}{15d\sqrt{\cos(c+dx)} \sqrt{a+a \sec(c+dx)}} \\
&= \frac{2a^3(32A+35B) \sin(c+dx)}{15d\sqrt{\cos(c+dx)} \sqrt{a+a \sec(c+dx)}} + \frac{2a^2(8A+5B) \sin(c+dx)}{15d\sqrt{\cos(c+dx)} \sqrt{a+a \sec(c+dx)}} \\
&= \frac{2a^{5/2}B \sinh^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a+a \sec(c+dx)}}\right) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{d}
\end{aligned}$$

Mathematica [A] time = 0.68, size = 118, normalized size = 0.61

$$\frac{2a^3 \sin(c+dx) \left(\sqrt{1-\sec(c+dx)} \left((14A+5B) \cos(c+dx) + 3A \cos^2(c+dx) + 43A + 40B\right) + 15B\sqrt{\sec(c+dx)}\right)}{15d\sqrt{\cos(c+dx)-1} \sqrt{a(\sec(c+dx)+1)}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]), x]

[Out] (2*a^3*((43*A + 40*B + (14*A + 5*B)*Cos[c + d*x] + 3*A*Cos[c + d*x]^2)*Sqrt[1 - Sec[c + d*x]] + 15*B*ArcSin[Sqrt[1 - Sec[c + d*x]]]*Sqrt[Sec[c + d*x]])*Sin[c + d*x])/(15*d*Sqrt[-1 + Cos[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])

fricas [A] time = 0.49, size = 399, normalized size = 2.08

$$\left[\frac{4 \left(3 A a^2 \cos(dx+c)^2 + (14 A + 5 B) a^2 \cos(dx+c) + (43 A + 40 B) a^2 \right) \sqrt{\frac{a \cos(dx+c)+a}{\cos(dx+c)}} \sqrt{\cos(dx+c)} \sin(dx+c)}{30 (d \cos(dx+c))} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] [1/30*(4*(3*A*a^2*cos(d*x + c)^2 + (14*A + 5*B)*a^2*cos(d*x + c) + (43*A + 40*B)*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 15*(B*a^2*cos(d*x + c) + B*a^2)*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(d*cos(d*x + c) + d), 1/15*(2*(3*A*a^2*cos(d*x + c)^2 + (14*A + 5*B)*a^2*cos(d*x + c) + (43*A + 40*B)*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 15*(B*a^2*cos(d*x + c) + B*a^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*x + c) + d)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(a \sec(dx + c) + a)^{\frac{5}{2}} \cos(dx + c)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(5/2), x)

maple [A] time = 1.94, size = 225, normalized size = 1.17

$$a^2 \left(\sqrt{\cos(dx + c)} \right) \sqrt{\frac{a(1 + \cos(dx + c))}{\cos(dx + c)}} \left(15B\sqrt{2} \arctan \left(\frac{\sqrt{\frac{2}{1 + \cos(dx + c)}} (\cos(dx + c) + 1 + \sin(dx + c)) \sqrt{2}}{4} \right) \right) \sqrt{\frac{2}{1 + \cos(dx + c)}} \sin(dx + c)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x)

[Out] -1/30/d*a^2*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(15*B*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d*x+c))*2^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-15*B*2^(1/2)*(-2/(1+cos(d*x+c)))^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))*2^(1/2))*sin(d*x+c)+12*A*cos(d*x+c)^3+44*A*cos(d*x+c)^2+20*B*cos(d*x+c)^2+116*A*cos(d*x+c)+140*B*cos(d*x+c)-172*A-160*B)/sin(d*x+c)

maxima [B] time = 0.68, size = 352, normalized size = 1.83

$$\left(3\sqrt{2}a^2\sin\left(\frac{5}{2}dx + \frac{5}{2}c\right) + 25\sqrt{2}a^2\sin\left(\frac{3}{2}dx + \frac{3}{2}c\right) + 150\sqrt{2}a^2\sin\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right)A\sqrt{a} + 5\left(2\sqrt{2}a^2\sin\left(\frac{3}{2}dx + \frac{3}{2}c\right) + 150\sqrt{2}a^2\sin\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right)B\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] 1/30*((3*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) + 25*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) + 150*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*A*sqrt(a) + 5*(2*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) + 30*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) + 3*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*B*sqrt(a))/d

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c + dx)^{5/2} \left(A + \frac{B}{\cos(c + dx)} \right) \left(a + \frac{a}{\cos(c + dx)} \right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^(5/2)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2),x)

[Out] int(cos(c + d*x)^(5/2)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**(5/2)*(a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)),x)

[Out] Timed out

$$3.536 \quad \int \cos^2(c+dx)(a+a \sec(c+dx))^{5/2}(A+B \sec(c+dx)) dx$$

Optimal. Leaf size=197

$$\frac{a^{5/2}(2A+5B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d} + \frac{a^3(14A+3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}} - \frac{a^2(2A-3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

[Out] $2/3*a*A*(a+a*\sec(d*x+c))^{3/2}*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/d+a^{5/2}*(2*A+5*B)*\operatorname{arcsinh}(a^{1/2}*\tan(d*x+c)/(a+a*\sec(d*x+c))^{1/2})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d+1/3*a^3*(14*A+3*B)*\sin(d*x+c)/d/\cos(d*x+c)^{(1/2)}/(a+a*\sec(d*x+c))^{1/2}-1/3*a^2*(2*A-3*B)*\sin(d*x+c)*(a+a*\sec(d*x+c))^{1/2}/d/\cos(d*x+c)^{(1/2)}$

Rubi [A] time = 0.63, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {2955, 4017, 4018, 4015, 3801, 215}

$$\frac{a^3(14A+3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}} - \frac{a^2(2A-3B)\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\sqrt{\cos(c+dx)}} + \frac{a^{5/2}(2A+5B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{3d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Cos}[c+d*x]^{3/2}*(a+a*\operatorname{Sec}[c+d*x])^{5/2}*(A+B*\operatorname{Sec}[c+d*x]),x]$

[Out] $(a^{5/2}*(2*A+5*B)*\operatorname{ArcSinh}[(\operatorname{Sqrt}[a]*\operatorname{Tan}[c+d*x])/(\operatorname{Sqrt}[a+a*\operatorname{Sec}[c+d*x]])]*\operatorname{Sqrt}[\operatorname{Cos}[c+d*x]]*\operatorname{Sqrt}[\operatorname{Sec}[c+d*x]])/d+(a^3*(14*A+3*B)*\operatorname{Sin}[c+d*x])/(3*d*\operatorname{Sqrt}[\operatorname{Cos}[c+d*x]]*\operatorname{Sqrt}[a+a*\operatorname{Sec}[c+d*x]])-(a^2*(2*A-3*B)*\operatorname{Sqrt}[a+a*\operatorname{Sec}[c+d*x]]*\operatorname{Sin}[c+d*x])/(3*d*\operatorname{Sqrt}[\operatorname{Cos}[c+d*x]])+(2*a*A*\operatorname{Sqrt}[\operatorname{Cos}[c+d*x]]*(a+a*\operatorname{Sec}[c+d*x])^{3/2}*\operatorname{Sin}[c+d*x])/(3*d)$

Rule 215

$\operatorname{Int}[1/\operatorname{Sqrt}[(a_.)+(b_.)*(x_)^2],x_Symbol] \rightarrow \operatorname{Simp}[\operatorname{ArcSinh}[(\operatorname{Rt}[b,2]*x)/\operatorname{Sqrt}[a]]/\operatorname{Rt}[b,2],x] /; \operatorname{FreeQ}\{a,b\},x \ \&\& \ \operatorname{GtQ}[a,0] \ \&\& \ \operatorname{PosQ}[b]$

Rule 2955

$\operatorname{Int}[(a_.)+\operatorname{csc}[e_.)+(f_.)*(x_)]*(b_.)^{(m_.)}*(\operatorname{csc}[e_.)+(f_.)*(x_)]*(d_.)+(c_.)^{(n_.)}*((g_.)*\operatorname{sin}[e_.)+(f_.)*(x_)]^{(p_.)},x_Symbol] \rightarrow \operatorname{Dist}[(g*\operatorname{Csc}[e+f*x])^p*(g*\operatorname{Sin}[e+f*x])^p,\operatorname{Int}[(a+b*\operatorname{Csc}[e+f*x])^m*(c+d*\operatorname{Csc}[e+f*x])^n]/(g*\operatorname{Csc}[e+f*x])^p,x],x] /; \operatorname{FreeQ}\{a,b,c,d,e,f,g,m,n,p\},x \ \&\& \ \operatorname{NeQ}[b*c-a*d,0] \ \&\& \ !\operatorname{IntegerQ}[p] \ \&\& \ !(\operatorname{IntegerQ}[m] \ \&\& \ \operatorname{In}$

tegerQ[n])

Rule 3801

Int[Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 + x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]

Rule 4015

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> Simp[(A*b^2*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]

Rule 4017

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> Simp[(a*A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && LtQ[n, -1]

Rule 4018

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> -Simp[(b*B*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && !LtQ[n, -1]

Rubi steps

$$\begin{aligned}
\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))dx &= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))}{\sec(c+dx)}dx \\
&= \frac{2aA\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{3d} \\
&= -\frac{a^2(2A-3B)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{3d\sqrt{\cos(c+dx)}} + \frac{2a^3(14A+3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} \\
&= \frac{a^3(14A+3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} - \frac{a^2(2A-3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}} \\
&= \frac{a^3(14A+3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} - \frac{a^2(2A-3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}} \\
&= \frac{a^{5/2}(2A+5B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}}{d}
\end{aligned}$$

Mathematica [A] time = 0.68, size = 117, normalized size = 0.59

$$\frac{a^3 \sin(c+dx) \left(\sqrt{1-\sec(c+dx)} (2A \cos(c+dx) + 16A + 3B \sec(c+dx) + 6B) + 3(2A+5B)\sqrt{\sec(c+dx)} \sin^{-1} \left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a+a\sec(c+dx)}} \right) \right)}{3d\sqrt{\cos(c+dx)} - 1 \sqrt{a(\sec(c+dx)+1)}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]), x]

[Out] (a^3*(3*(2*A + 5*B)*ArcSin[Sqrt[1 - Sec[c + d*x]]]*Sqrt[Sec[c + d*x]] + Sqrt[1 - Sec[c + d*x]]*(16*A + 6*B + 2*A*Cos[c + d*x] + 3*B*Sec[c + d*x]))*Sin[c + d*x]/(3*d*Sqrt[-1 + Cos[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])

fricas [A] time = 0.56, size = 449, normalized size = 2.28

$$\left[\frac{4 \left(2 A a^2 \cos(dx+c)^2 + 2 (8 A + 3 B) a^2 \cos(dx+c) + 3 B a^2 \right) \sqrt{\frac{a \cos(dx+c)+a}{\cos(dx+c)}} \sqrt{\cos(dx+c)} \sin(dx+c) + 3 \left((2 A + 5 B) \sqrt{\sec(dx+c)} \sin^{-1} \left(\frac{\sqrt{a} \tan(dx+c)}{\sqrt{a+a\sec(dx+c)}} \right) \right)}{12 \left(d \cos(dx+c) \right)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/12*(4*(2*A*a^2*cos(d*x + c)^2 + 2*(8*A + 3*B)*a^2*cos(d*x + c) + 3*B*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 3*((2*A + 5*B)*a^2*cos(d*x + c)^2 + (2*A + 5*B)*a^2*cos(d*x + c))*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(d*cos(d*x + c)^2 + d*cos(d*x + c)), 1/6*(2*(2*A*a^2*cos(d*x + c)^2 + 2*(8*A + 3*B)*a^2*cos(d*x + c) + 3*B*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 3*((2*A + 5*B)*a^2*cos(d*x + c)^2 + (2*A + 5*B)*a^2*cos(d*x + c))*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*x + c)^2 + d*cos(d*x + c))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(a \sec(dx + c) + a)^{\frac{5}{2}} \cos(dx + c)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(3/2), x)
```

maple [B] time = 2.05, size = 368, normalized size = 1.87

$$a^2 \left(6A \cos(dx + c) \sin(dx + c) \sqrt{-\frac{2}{1+\cos(dx+c)}} \arctan \left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}} (\cos(dx+c)+1+\sin(dx+c)) \sqrt{2}}{4}} \right) \sqrt{2} - 6A \cos(dx + c) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] -1/12/d*a^2*(6*A*cos(d*x+c)*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d*x+c))*2^(1/2))*2^(1/2)-6*A*cos(d*x+c)*sin(d*x+c)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))*2^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*2^(1/2)+15*B*cos(d*x+c)*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d*x+c))*2^(1/2))*2^(1/2)-6*A*cos(d*x+c)*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))*2^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*2^(1/2)+15*B*cos(d*x+c)*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))*2^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*2^(1/2))
```

$$+c)+1+\sin(dx+c))*2^{(1/2)}*2^{(1/2)}-15*B*\cos(dx+c)*\sin(dx+c)*\arctan(1/4*(-2/(1+\cos(dx+c)))^{(1/2)}*(\cos(dx+c)+1-\sin(dx+c))*2^{(1/2)}*(-2/(1+\cos(dx+c))))^{(1/2)}*2^{(1/2)}+8*A*\cos(dx+c)^3+56*A*\cos(dx+c)^2+24*B*\cos(dx+c)^2-64*A*\cos(dx+c)-12*B*\cos(dx+c)-12*B)*(a*(1+\cos(dx+c))/\cos(dx+c))^{(1/2)}/\sin(dx+c)/\cos(dx+c)^{(1/2)}$$

maxima [B] time = 0.84, size = 2589, normalized size = 13.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(dx+c)^(3/2)*(a+a*sec(dx+c))^(5/2)*(A+B*sec(dx+c)),x, algorithm="maxima")

[Out] 1/12*(sqrt(2)*(30*a^2*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(3/2*d*x + 3/2*c) - 30*a^2*cos(3/2*d*x + 3/2*c)*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 3*sqrt(2)*a^2*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 3*sqrt(2)*a^2*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 3*sqrt(2)*a^2*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 3*sqrt(2)*a^2*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 4*a^2*sin(3/2*d*x + 3/2*c) + 30*a^2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*A*sqrt(a) + 3*(4*sqrt(2)*a^2*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*a^2*sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*a^2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^3 - 4*sqrt(2)*a^2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*a^2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(2*sqrt(2)*a^2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - sqrt(2)*a^2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))

$$\begin{aligned}
&))) + 5*(a^2*\cos(5/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + 2*a^2 \\
&* \cos(5/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) * \cos(1/4*\arctan2(\sin(2 \\
&*d*x + 2*c), \cos(2*d*x + 2*c))) + a^2*\cos(1/4*\arctan2(\sin(2*d*x + 2*c), \cos \\
&(2*d*x + 2*c)))^2 + a^2*\sin(5/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)) \\
&)^2 + 2*a^2*\sin(5/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) * \sin(1/4*\ar \\
&ctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + a^2*\sin(1/4*\arctan2(\sin(2*d*x \\
&+ 2*c), \cos(2*d*x + 2*c)))^2 * \log(2*\cos(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2 \\
&*d*x + 2*c)))^2 + 2*\sin(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 \\
&+ 2*\sqrt{2}*\cos(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 2*\sqrt{2} \\
&)*\sin(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 2) - 5*(a^2*\cos(5/ \\
&4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + 2*a^2*\cos(5/4*\arctan2(\sin \\
&(2*d*x + 2*c), \cos(2*d*x + 2*c))) * \cos(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2* \\
&d*x + 2*c))) + a^2*\cos(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + \\
&a^2*\sin(5/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + 2*a^2*\sin(5/4 \\
&* \arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) * \sin(1/4*\arctan2(\sin(2*d*x + 2 \\
&*c), \cos(2*d*x + 2*c))) + a^2*\sin(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + \\
&2*c)))^2 * \log(2*\cos(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + 2 \\
&* \sin(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + 2*\sqrt{2}*\cos(1/4 \\
&* \arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) - 2*\sqrt{2}*\sin(1/4*\arctan2(\sin \\
&(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 2) + 5*(a^2*\cos(5/4*\arctan2(\sin(2*d*x \\
&+ 2*c), \cos(2*d*x + 2*c)))^2 + 2*a^2*\cos(5/4*\arctan2(\sin(2*d*x + 2*c), \cos \\
&(2*d*x + 2*c))) * \cos(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + a^2* \\
&\cos(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + a^2*\sin(5/4*\arctan \\
&2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + 2*a^2*\sin(5/4*\arctan2(\sin(2*d*x \\
&+ 2*c), \cos(2*d*x + 2*c))) * \sin(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2* \\
&c))) + a^2*\sin(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 * \log(2*co \\
&s(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + 2*\sin(1/4*\arctan2(\sin \\
&(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 - 2*\sqrt{2}*\cos(1/4*\arctan2(\sin(2*d*x \\
&+ 2*c), \cos(2*d*x + 2*c))) + 2*\sqrt{2}*\sin(1/4*\arctan2(\sin(2*d*x + 2*c), co \\
&s(2*d*x + 2*c))) + 2) - 5*(a^2*\cos(5/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x \\
&+ 2*c)))^2 + 2*a^2*\cos(5/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) * \cos \\
&(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + a^2*\cos(1/4*\arctan2(\sin \\
&(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + a^2*\sin(5/4*\arctan2(\sin(2*d*x + 2*c), \\
&\cos(2*d*x + 2*c)))^2 + 2*a^2*\sin(5/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + \\
&2*c))) * \sin(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + a^2*\sin(1/4* \\
&\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 * \log(2*\cos(1/4*\arctan2(\sin(2 \\
&*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + 2*\sin(1/4*\arctan2(\sin(2*d*x + 2*c), \cos \\
&(2*d*x + 2*c)))^2 - 2*\sqrt{2}*\cos(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + \\
&2*c))) - 2*\sqrt{2}*\sin(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + \\
&2) + 4*(2*\sqrt{2})*a^2*\sin(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 \\
&+ \sqrt{2})*a^2*\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + \sqrt{2} \\
&(2)*a^2*\sin(5/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 4*(\sqrt{2})* \\
&a^2*\cos(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + \sqrt{2})*a^2*s \\
&\sin(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) * B*\sqrt{a}/(\cos(5/4*\ar \\
&\tan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + 2*\cos(5/4*\arctan2(\sin(2*d*x +
\end{aligned}$$

$2*c), \cos(2*d*x + 2*c))) * \cos(1/4 * \arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + \cos(1/4 * \arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + \sin(5/4 * \arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + 2 * \sin(5/4 * \arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) * \sin(1/4 * \arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + \sin(1/4 * \arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2) / d$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c + dx)^{3/2} \left(A + \frac{B}{\cos(c + dx)} \right) \left(a + \frac{a}{\cos(c + dx)} \right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^(3/2)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2), x)

[Out] int(cos(c + d*x)^(3/2)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**(3/2)*(a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)), x)

[Out] Timed out

$$3.537 \quad \int \sqrt{\cos(c+dx)} (a+a \sec(c+dx))^{5/2} (A+B \sec(c+dx)) dx$$

Optimal. Leaf size=200

$$\frac{a^{5/2}(20A+19B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d} + \frac{a^3(4A-9B)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}} + \frac{a^2(4A+7B)\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{4d\sqrt{\cos(c+dx)}} + \frac{a^{5/2}(20A+19B)\sqrt{\cos(c+dx)}}{4d}$$

[Out] $1/2*a*B*(a+a*\sec(d*x+c))^{(3/2)}*\sin(d*x+c)/d/\cos(d*x+c)^{(1/2)}+1/4*a^{(5/2)}*(20*A+19*B)*\operatorname{arcsinh}(a^{(1/2)}*\tan(d*x+c)/(a+a*\sec(d*x+c))^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d+1/4*a^3*(4*A-9*B)*\sin(d*x+c)/d/\cos(d*x+c)^{(1/2)}/(a+a*\sec(d*x+c))^{(1/2)}+1/4*a^2*(4*A+7*B)*\sin(d*x+c)*(a+a*\sec(d*x+c))^{(1/2)}/d/\cos(d*x+c)^{(1/2)}$

Rubi [A] time = 0.63, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2955, 4018, 4015, 3801, 215}

$$\frac{a^3(4A-9B)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}} + \frac{a^2(4A+7B)\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{4d\sqrt{\cos(c+dx)}} + \frac{a^{5/2}(20A+19B)\sqrt{\cos(c+dx)}}{4d}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]

[Out] $(a^{(5/2)}*(20*A+19*B)*\operatorname{ArcSinh}[(\operatorname{Sqrt}[a]*\tan[c+d*x])/(\operatorname{Sqrt}[a+a*\sec[c+d*x]])]*\operatorname{Sqrt}[\cos[c+d*x]]*\operatorname{Sqrt}[\sec[c+d*x]])/(4*d) + (a^3*(4*A-9*B)*\sin[c+d*x])/(4*d*\operatorname{Sqrt}[\cos[c+d*x]]*\operatorname{Sqrt}[a+a*\sec[c+d*x]]) + (a^2*(4*A+7*B)*\operatorname{Sqrt}[a+a*\sec[c+d*x]]*\sin[c+d*x])/(4*d*\operatorname{Sqrt}[\cos[c+d*x]]) + (a*B*(a+a*\sec[c+d*x])^{(3/2)}*\sin[c+d*x])/(2*d*\operatorname{Sqrt}[\cos[c+d*x]])$

Rule 215

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 2955

Int[((a_) + csc[(e_) + (f_)*(x_)])*(b_)^(m_)*(csc[(e_) + (f_)*(x_)]*(d_) + (c_))^(n_)*((g_)*sin[(e_) + (f_)*(x_)])^(p_), x_Symbol] := Dist[(g*Csc[e + f*x])^p*(g*Ssin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && In

tegerQ[n])

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.)
+ (a_.)], x_Symbol] := Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.)
+ (a_.)]*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
+ f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (
a_.))^(m_)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && !LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{\cos(c+dx)} (a+a \sec(c+dx))^{5/2} (A+B \sec(c+dx)) dx &= \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{(a+a \sec(c+dx))^{5/2} (A+B \sec(c+dx))}{\sqrt{\cos(c+dx)}} dx \\
&= \frac{aB(a+a \sec(c+dx))^{3/2} \sin(c+dx)}{2d\sqrt{\cos(c+dx)}} + \frac{1}{2} \left(\sqrt{\cos(c+dx)} \right) \int \frac{(a+a \sec(c+dx))^{5/2} (A+B \sec(c+dx))}{\sqrt{\cos(c+dx)}} dx \\
&= \frac{a^2(4A+7B)\sqrt{a+a \sec(c+dx)} \sin(c+dx)}{4d\sqrt{\cos(c+dx)}} + \frac{a^3(4A-9B) \sin(c+dx)}{4d\sqrt{\cos(c+dx)} \sqrt{a+a \sec(c+dx)}} + \frac{a^2(4A+7B) \sin(c+dx)}{4d\sqrt{\cos(c+dx)}} \\
&= \frac{a^3(4A-9B) \sin(c+dx)}{4d\sqrt{\cos(c+dx)} \sqrt{a+a \sec(c+dx)}} + \frac{a^2(4A+7B) \sin(c+dx)}{4d\sqrt{\cos(c+dx)}} \\
&= \frac{a^3(4A-9B) \sin(c+dx)}{4d\sqrt{\cos(c+dx)} \sqrt{a+a \sec(c+dx)}} + \frac{a^2(4A+7B) \sin(c+dx)}{4d\sqrt{\cos(c+dx)}} \\
&= \frac{a^5/2(20A+19B) \sinh^{-1} \left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a+a \sec(c+dx)}} \right) \sqrt{\cos(c+dx)}}{4d}
\end{aligned}$$

Mathematica [A] time = 0.95, size = 173, normalized size = 0.86

$$\frac{a^3 \sin(c+dx) \sqrt{\cos(c+dx)} (A+B \sec(c+dx)) \left(\sqrt{1-\sec(c+dx)} \left((4A+11B) \sec(c+dx) + 8A + 2B \sec^2(c+dx) \right) + \sqrt{a+a \sec(c+dx)} \right)}{4d \sqrt{1-\sec(c+dx)} \sqrt{a+a \sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]

[Out] (a^3*Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x])*(20*A*ArcSin[Sqrt[1 - Sec[c + d*x]]]*Sqrt[Sec[c + d*x]] - 19*B*ArcSin[Sqrt[Sec[c + d*x]]]*Sqrt[Sec[c + d*x]] + Sqrt[1 - Sec[c + d*x]]*(8*A + (4*A + 11*B)*Sec[c + d*x] + 2*B*Sec[c + d*x]^2))*Sin[c + d*x])/(4*d*(B + A*Cos[c + d*x])*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])

fricas [A] time = 0.55, size = 453, normalized size = 2.26

$$\frac{4 \left(8 A a^2 \cos(dx + c)^2 + (4 A + 11 B) a^2 \cos(dx + c) + 2 B a^2 \right) \sqrt{\frac{a \cos(dx + c) + a}{\cos(dx + c)}} \sqrt{\cos(dx + c)} \sin(dx + c) + \left((20 A + 19 B) a^2 \cos(dx + c)^3 + (20 A + 19 B) a^2 \cos(dx + c)^2 \right) \sqrt{a} \log\left(\frac{a \cos(dx + c)^3 - 4 \sqrt{a} \sqrt{\frac{a \cos(dx + c) + a}{\cos(dx + c)}} (\cos(dx + c) - 2) \sqrt{\cos(dx + c)} \sin(dx + c) - 7 a \cos(dx + c)^2 + 8 a}{\cos(dx + c)^3 + \cos(dx + c)^2}\right)}{16 \left(d \cos(dx + c)^3 + d \cos(dx + c)^2 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))*cos(d*x+c)^(1/2),x, algorithm="fricas")

[Out] [1/16*(4*(8*A*a^2*cos(d*x + c)^2 + (4*A + 11*B)*a^2*cos(d*x + c) + 2*B*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + ((20*A + 19*B)*a^2*cos(d*x + c)^3 + (20*A + 19*B)*a^2*cos(d*x + c)^2)*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2), 1/8*(2*(8*A*a^2*cos(d*x + c)^2 + (4*A + 11*B)*a^2*cos(d*x + c) + 2*B*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + ((20*A + 19*B)*a^2*cos(d*x + c)^3 + (20*A + 19*B)*a^2*cos(d*x + c)^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(a \sec(dx + c) + a)^{\frac{5}{2}} \sqrt{\cos(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))*cos(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(5/2)*sqrt(cos(d*x + c)), x)

maple [B] time = 2.25, size = 376, normalized size = 1.88

$$\frac{a^2 (-1 + \cos(dx + c)) \left(16 A \sin(dx + c) (\cos^2(dx + c)) \sqrt{-\frac{2}{1 + \cos(dx + c)}} - 20 A \arctan\left(\frac{\sqrt{-\frac{2}{1 + \cos(dx + c)}} (\cos(dx + c) + 1) - \sin(dx + c)}{4}\right) \right)}{16 \left(d \cos(dx + c)^3 + d \cos(dx + c)^2 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))*cos(d*x+c)^(1/2),x)`

[Out] `-1/8/d*a^2*(-1+cos(d*x+c))*(16*A*sin(d*x+c)*cos(d*x+c)^2*(-2/(1+cos(d*x+c)))^(1/2)-20*A*arctan(1/4*(-2/(1+cos(d*x+c))))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))*2^(1/2))*cos(d*x+c)^2*2^(1/2)+20*A*arctan(1/4*(-2/(1+cos(d*x+c))))^(1/2)*(cos(d*x+c)+1+sin(d*x+c))*2^(1/2))*cos(d*x+c)^2*2^(1/2)-19*B*arctan(1/4*(-2/(1+cos(d*x+c))))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))*2^(1/2))*cos(d*x+c)^2*2^(1/2)+19*B*arctan(1/4*(-2/(1+cos(d*x+c))))^(1/2)*(cos(d*x+c)+1+sin(d*x+c))*2^(1/2))*cos(d*x+c)^2*2^(1/2)+8*A*sin(d*x+c)*cos(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)+22*B*sin(d*x+c)*cos(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)+4*B*(-2/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)^2/cos(d*x+c)^(3/2)/(-2/(1+cos(d*x+c)))^(1/2)`

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))*cos(d*x+c)^(1/2),x, algorithm="maxima")`

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{\cos(c+dx)} \left(A + \frac{B}{\cos(c+dx)} \right) \left(a + \frac{a}{\cos(c+dx)} \right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(c+d*x)^(1/2)*(A+B/cos(c+d*x))*(a+a/cos(c+d*x))^(5/2),x)`

[Out] `int(cos(c+d*x)^(1/2)*(A+B/cos(c+d*x))*(a+a/cos(c+d*x))^(5/2),x)`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c))*cos(d*x+c)**(1/2),x)`

[Out] Timed out

$$3.538 \quad \int \frac{(a+a \sec(c+dx))^{5/2}(A+B \sec(c+dx))}{\sqrt{\cos(c+dx)}} dx$$

Optimal. Leaf size=200

$$\frac{a^{5/2}(38A + 25B)\sqrt{\cos(c + dx)}\sqrt{\sec(c + dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8d} + \frac{a^3(54A + 49B)\sin(c + dx)}{24d\cos^2(c + dx)\sqrt{a\sec(c + dx) + a}} + \frac{a^2(2A + 3B)\sin(c + dx)\sqrt{a\sec(c + dx) + a}}{4d\cos^2(c + dx)}$$

[Out] 1/3*a*B*(a+a*sec(d*x+c))^(3/2)*sin(d*x+c)/d/cos(d*x+c)^(3/2)+1/8*a^(5/2)*(3*8*A+25*B)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+1/24*a^3*(54*A+49*B)*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2)+1/4*a^2*(2*A+3*B)*sin(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)

Rubi [A] time = 0.65, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2955, 4018, 4016, 3801, 215}

$$\frac{a^3(54A + 49B)\sin(c + dx)}{24d\cos^2(c + dx)\sqrt{a\sec(c + dx) + a}} + \frac{a^2(2A + 3B)\sin(c + dx)\sqrt{a\sec(c + dx) + a}}{4d\cos^2(c + dx)} + \frac{a^{5/2}(38A + 25B)\sqrt{\cos(c + dx)}}{8d}$$

Antiderivative was successfully verified.

[In] Int[((a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Sqrt[Cos[c + d*x]],x]

[Out] (a^(5/2)*(38*A + 25*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]])*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]/(8*d) + (a^3*(54*A + 49*B)*Sin[c + d*x])/((24*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (a^2*(2*A + 3*B)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/((4*d*Cos[c + d*x]^(3/2)) + (a*B*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/((3*d*Cos[c + d*x]^(3/2))

Rule 215

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 2955

Int[((a_.) + csc[(e_.) + (f_.)*(x_.)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_.)])^(p_.), x_Symbol] := Dist[(g*Csc[e + f*x])^p*(g*Ssin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && In

tegerQ[n])

Rule 3801

Int[Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 + x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]

Rule 4016

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n)*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> Simp[(-2*b*B*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && !LtQ[n, 0]

Rule 4018

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> -Simp[(b*B*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && !LtQ[n, -1]

Rubi steps

$$\begin{aligned}
\int \frac{(a + a \sec(c + dx))^{5/2} (A + B \sec(c + dx))}{\sqrt{\cos(c + dx)}} dx &= (\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)}) \int \sqrt{\sec(c + dx)} (a + a \sec(c + dx)) \\
&= \frac{aB(a + a \sec(c + dx))^{3/2} \sin(c + dx)}{3d \cos^{3/2}(c + dx)} + \frac{1}{3} (\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)}) \\
&= \frac{a^2(2A + 3B)\sqrt{a + a \sec(c + dx)} \sin(c + dx)}{4d \cos^{3/2}(c + dx)} + \frac{aB(a + a \sec(c + dx))^{3/2} \sin(c + dx)}{3d \cos^{3/2}(c + dx)} \\
&= \frac{a^3(54A + 49B) \sin(c + dx)}{24d \cos^{3/2}(c + dx)\sqrt{a + a \sec(c + dx)}} + \frac{a^2(2A + 3B)\sqrt{a + a \sec(c + dx)} \sin(c + dx)}{4d \cos^{3/2}(c + dx)} \\
&= \frac{a^3(54A + 49B) \sin(c + dx)}{24d \cos^{3/2}(c + dx)\sqrt{a + a \sec(c + dx)}} + \frac{a^2(2A + 3B)\sqrt{a + a \sec(c + dx)} \sin(c + dx)}{4d \cos^{3/2}(c + dx)} \\
&= \frac{a^{5/2}(38A + 25B) \sinh^{-1}\left(\frac{\sqrt{a} \tan(c + dx)}{\sqrt{a + a \sec(c + dx)}}\right) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)}}{8d}
\end{aligned}$$

Mathematica [A] time = 1.34, size = 133, normalized size = 0.66

$$\frac{a^2 \sec\left(\frac{1}{2}(c + dx)\right) \sqrt{a(\sec(c + dx) + 1)} \left(\sin\left(\frac{1}{2}(c + dx)\right) (4(6A + 17B) \cos(c + dx) + (66A + 75B) \cos(2(c + dx))) - \frac{1}{2} \cos(c + dx)\right)}{48d \cos^{5/2}(c + dx)}$$

Antiderivative was successfully verified.

[In] Integrate[((a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Sqrt[Cos[c + d*x]], x]

[Out] (a^2*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(3*Sqrt[2]*(38*A + 25*B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^3 + (66*A + 91*B + 4*(6*A + 17*B))*Cos[c + d*x] + (66*A + 75*B)*Cos[2*(c + d*x)])*Sin[(c + d*x)/2])/(48*d*Cos[c + d*x]^(5/2))

fricas [A] time = 0.57, size = 469, normalized size = 2.34

$$4 \left(3 (22 A + 25 B) a^2 \cos(dx + c)^2 + 2 (6 A + 17 B) a^2 \cos(dx + c) + 8 B a^2 \right) \sqrt{\frac{a \cos(dx+c)+a}{\cos(dx+c)}} \sqrt{\cos(dx+c)} \sin(dx)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorith="fricas")

[Out] [1/96*(4*(3*(22*A + 25*B)*a^2*cos(d*x + c)^2 + 2*(6*A + 17*B)*a^2*cos(d*x + c) + 8*B*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 3*((38*A + 25*B)*a^2*cos(d*x + c)^4 + (38*A + 25*B)*a^2*cos(d*x + c)^3)*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3), 1/48*(2*(3*(22*A + 25*B)*a^2*cos(d*x + c)^2 + 2*(6*A + 17*B)*a^2*cos(d*x + c) + 8*B*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 3*((38*A + 25*B)*a^2*cos(d*x + c)^4 + (38*A + 25*B)*a^2*cos(d*x + c)^3)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(a \sec(dx + c) + a)^{\frac{5}{2}}}{\sqrt{\cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorith="giac")

[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(5/2)/sqrt(cos(d*x + c)), x)

maple [B] time = 2.51, size = 407, normalized size = 2.04

$$a^2 \sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}} (-1 + \cos(dx + c)) \left(114 A \arctan \left(\frac{\sqrt{\frac{2}{1+\cos(dx+c)}} (\cos(dx+c)+1-\sin(dx+c)) \sqrt{2}}{4}} \right) \sqrt{2} (\cos^3(dx + c)) - \dots \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((a+a*\sec(d*x+c))^{5/2}*(A+B*\sec(d*x+c))/\cos(d*x+c)^{1/2},x)$

[Out] $\frac{1}{48}d*a^2*(a*(1+\cos(d*x+c))/\cos(d*x+c))^{1/2}*(-1+\cos(d*x+c))*(114*A*\arctan(1/4*(-2/(1+\cos(d*x+c))))^{1/2}*(\cos(d*x+c)+1-\sin(d*x+c))*2^{1/2})*2^{1/2}*\cos(d*x+c)^3-114*A*\arctan(1/4*(-2/(1+\cos(d*x+c))))^{1/2}*(\cos(d*x+c)+1+\sin(d*x+c))*2^{1/2})*2^{1/2}*\cos(d*x+c)^3+75*B*\arctan(1/4*(-2/(1+\cos(d*x+c))))^{1/2}*(\cos(d*x+c)+1-\sin(d*x+c))*2^{1/2})*2^{1/2}*\cos(d*x+c)^3-75*B*\arctan(1/4*(-2/(1+\cos(d*x+c))))^{1/2}*(\cos(d*x+c)+1+\sin(d*x+c))*2^{1/2})*2^{1/2}*\cos(d*x+c)^3-132*A*\sin(d*x+c)*\cos(d*x+c)^2*(-2/(1+\cos(d*x+c)))^{1/2}-150*B*\sin(d*x+c)*\cos(d*x+c)^2*(-2/(1+\cos(d*x+c)))^{1/2}-24*A*\sin(d*x+c)*\cos(d*x+c)*(-2/(1+\cos(d*x+c)))^{1/2}-68*B*\sin(d*x+c)*\cos(d*x+c)*(-2/(1+\cos(d*x+c)))^{1/2}-16*B*(-2/(1+\cos(d*x+c)))^{1/2}*\sin(d*x+c))/\cos(d*x+c)^{5/2}/\sin(d*x+c)^2/(-2/(1+\cos(d*x+c)))^{1/2}$

maxima [B] time = 4.68, size = 6297, normalized size = 31.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((a+a*\sec(d*x+c))^{5/2}*(A+B*\sec(d*x+c))/\cos(d*x+c)^{1/2},x, \text{algorithm}="maxima")$

[Out] $-1/96*(6*(88*\sqrt{2})*a^2*\cos(7/2*d*x + 7/2*c)*\sin(2*d*x + 2*c) - 56*\sqrt{2})*a^2*\cos(5/2*d*x + 5/2*c)*\sin(2*d*x + 2*c) - 28*\sqrt{2})*a^2*\sin(3/2*d*x + 3/2*c) + 44*\sqrt{2})*a^2*\sin(1/2*d*x + 1/2*c) - 19*(a^2*\log(2*\cos(1/2*d*x + 1/2*c))^2 + 2*\sin(1/2*d*x + 1/2*c))^2 + 2*\sqrt{2})*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2})*\sin(1/2*d*x + 1/2*c) + 2) - a^2*\log(2*\cos(1/2*d*x + 1/2*c))^2 + 2*\sin(1/2*d*x + 1/2*c))^2 + 2*\sqrt{2})*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2})*\sin(1/2*d*x + 1/2*c) + 2) + a^2*\log(2*\cos(1/2*d*x + 1/2*c))^2 + 2*\sin(1/2*d*x + 1/2*c))^2 - 2*\sqrt{2})*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2})*\sin(1/2*d*x + 1/2*c) + 2) - a^2*\log(2*\cos(1/2*d*x + 1/2*c))^2 + 2*\sin(1/2*d*x + 1/2*c))^2 - 2*\sqrt{2})*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2})*\sin(1/2*d*x + 1/2*c) + 2))*\cos(4*d*x + 4*c)^2 - 76*(a^2*\log(2*\cos(1/2*d*x + 1/2*c))^2 + 2*\sin(1/2*d*x + 1/2*c))^2 + 2*\sqrt{2})*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2})*\sin(1/2*d*x + 1/2*c) + 2) - a^2*\log(2*\cos(1/2*d*x + 1/2*c))^2 + 2*\sin(1/2*d*x + 1/2*c))^2 + 2*\sqrt{2})*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2})*\sin(1/2*d*x + 1/2*c) + 2) + a^2*\log(2*\cos(1/2*d*x + 1/2*c))^2 + 2*\sin(1/2*d*x + 1/2*c))^2 - 2*\sqrt{2})*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2})*\sin(1/2*d*x + 1/2*c) + 2) - a^2*\log(2*\cos(1/2*d*x + 1/2*c))^2 + 2*\sin(1/2*d*x + 1/2*c))^2 - 2*\sqrt{2})*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2})*\sin(1/2*d*x + 1/2*c) + 2))*\cos(2*d*x + 2*c)^2 - 19*a^2*\log(2*\cos(1/2*d*x + 1/2*c))^2 + 2*\sin(1/2*d*x + 1/2*c))^2 + 2*\sqrt{2})*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2})*\sin(1/2*d*x + 1/2*c) + 2) + 19*a^2*\log(2*\cos(1/2*d*x + 1/2*c))^2 + 2*\sin(1/2*d*x + 1/2*c))^2 + 2*\sqrt{2})*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2})*\sin(1/2*d*x + 1/2*c) +$

$$\begin{aligned}
&) + 7\sqrt{2}a^2\cos(3/2dx + 3/2c) - 11\sqrt{2}a^2\cos(1/2dx + 1/2c) \\
&) - 19(a^2\log(2\cos(1/2dx + 1/2c)^2 + 2\sin(1/2dx + 1/2c)^2 + 2\sqrt{2}\cos(1/2dx + 1/2c) + 2) - a^2\log(2 \\
& * \cos(1/2dx + 1/2c)^2 + 2\sin(1/2dx + 1/2c)^2 + 2\sqrt{2}\cos(1/2dx \\
& + 1/2c) - 2\sqrt{2}\sin(1/2dx + 1/2c) + 2) + a^2\log(2\cos(1/2dx + 1/ \\
& 2c)^2 + 2\sin(1/2dx + 1/2c)^2 - 2\sqrt{2}\cos(1/2dx + 1/2c) + 2\sqrt{2} \\
& (2)\sin(1/2dx + 1/2c) + 2) - a^2\log(2\cos(1/2dx + 1/2c)^2 + 2\sin(1/ \\
& 2dx + 1/2c)^2 - 2\sqrt{2}\cos(1/2dx + 1/2c) - 2\sqrt{2}\sin(1/2dx + \\
& 1/2c) + 2))\sin(2dx + 2c))\sin(4dx + 4c) - 44(2\sqrt{2}a^2\cos(2* \\
& dx + 2c) + \sqrt{2}a^2)\sin(7/2dx + 7/2c) + 28(2\sqrt{2}a^2\cos(2*d* \\
& x + 2*c) + \sqrt{2}a^2)\sin(5/2dx + 5/2c) + 8(7\sqrt{2}a^2\cos(3/2dx \\
& + 3/2c) - 11\sqrt{2}a^2\cos(1/2dx + 1/2c))\sin(2dx + 2c))A\sqrt{a} \\
&)/(2(2\cos(2dx + 2c) + 1)\cos(4dx + 4c) + \cos(4dx + 4c)^2 + 4\cos \\
& (2dx + 2c)^2 + \sin(4dx + 4c)^2 + 4\sin(4dx + 4c)\sin(2dx + 2c) \\
& + 4\sin(2dx + 2c)^2 + 4\cos(2dx + 2c) + 1) - (300\sqrt{2}a^2\cos(1/3 \\
& * \arctan2(\sin(3/2dx + 3/2c), \cos(3/2dx + 3/2c)))\sin(6dx + 6c) - 28 \\
& * \sqrt{2}a^2\sin(9/2dx + 9/2c) + 28\sqrt{2}a^2\sin(3/2dx + 3/2c) - 2 \\
& 8*(\sqrt{2}a^2\sin(9/2dx + 9/2c) - \sqrt{2}a^2\sin(3/2dx + 3/2c))\cos \\
& (6dx + 6c) - 300*(\sqrt{2}a^2\sin(6dx + 6c) + 3\sqrt{2}a^2\sin(8/3a \\
& rctan2(\sin(3/2dx + 3/2c), \cos(3/2dx + 3/2c))) + 3\sqrt{2}a^2\sin(4/3 \\
& * \arctan2(\sin(3/2dx + 3/2c), \cos(3/2dx + 3/2c))))\cos(11/3\arctan2(\sin \\
& (3/2dx + 3/2c), \cos(3/2dx + 3/2c))) - 12(7\sqrt{2}a^2\sin(9/2dx + \\
& 9/2c) - 7\sqrt{2}a^2\sin(3/2dx + 3/2c) - 114\sqrt{2}a^2\sin(7/3\arct \\
& an2(\sin(3/2dx + 3/2c), \cos(3/2dx + 3/2c))) + 114\sqrt{2}a^2\sin(5/3* \\
& arctan2(\sin(3/2dx + 3/2c), \cos(3/2dx + 3/2c))) + 75\sqrt{2}a^2\sin(1 \\
& /3\arctan2(\sin(3/2dx + 3/2c), \cos(3/2dx + 3/2c)))\cos(8/3\arctan2(\sin \\
& (3/2dx + 3/2c), \cos(3/2dx + 3/2c))) - 456*(\sqrt{2}a^2\sin(6dx + 6 \\
& *c) + 3\sqrt{2}a^2\sin(4/3\arctan2(\sin(3/2dx + 3/2c), \cos(3/2dx + 3/2 \\
& *c))))\cos(7/3\arctan2(\sin(3/2dx + 3/2c), \cos(3/2dx + 3/2c))) + 456*(\\
& \sqrt{2}a^2\sin(6dx + 6c) + 3\sqrt{2}a^2\sin(4/3\arctan2(\sin(3/2dx + \\
& 3/2c), \cos(3/2dx + 3/2c))))\cos(5/3\arctan2(\sin(3/2dx + 3/2c), \cos(3 \\
& /2dx + 3/2c))) - 12(7\sqrt{2}a^2\sin(9/2dx + 9/2c) - 7\sqrt{2}a^2\sin \\
& (3/2dx + 3/2c) + 75\sqrt{2}a^2\sin(1/3\arctan2(\sin(3/2dx + 3/2c), \\
& \cos(3/2dx + 3/2c))))\cos(4/3\arctan2(\sin(3/2dx + 3/2c), \cos(3/2dx \\
& + 3/2c))) + 75*(a^2\cos(6dx + 6c)^2 + 9a^2\cos(8/3\arctan2(\sin(3/2dx \\
& + 3/2c), \cos(3/2dx + 3/2c)))^2 + 9a^2\cos(4/3\arctan2(\sin(3/2dx + 3 \\
& /2c), \cos(3/2dx + 3/2c)))^2 + a^2\sin(6dx + 6c)^2 + 9a^2\sin(8/3\ar \\
& ctan2(\sin(3/2dx + 3/2c), \cos(3/2dx + 3/2c)))^2 + 6a^2\sin(6dx + 6* \\
& c)\sin(4/3\arctan2(\sin(3/2dx + 3/2c), \cos(3/2dx + 3/2c))) + 9a^2\sin \\
& (4/3\arctan2(\sin(3/2dx + 3/2c), \cos(3/2dx + 3/2c)))^2 + 2a^2\cos(6d \\
& *x + 6*c) + a^2 + 6*(a^2\cos(6dx + 6c) + 3a^2\cos(4/3\arctan2(\sin(3/2d \\
& *x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + a^2)\cos(8/3\arctan2(\sin(3/2*d*x + 3/ \\
& 2*c), \cos(3/2*d*x + 3/2*c))) + 6*(a^2\cos(6dx + 6c) + a^2)\cos(4/3\arcta \\
& n2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 6*(a^2\sin(6dx + 6c) + \\
& 3a^2\sin(4/3\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))))\sin(8/
\end{aligned}$$


```

^2*sin(6*d*x + 6*c) + 3*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d
*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
)*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))^2 + 2*
sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*
cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*si
n(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 28*(sqrt(
2)*a^2*cos(9/2*d*x + 9/2*c) - sqrt(2)*a^2*cos(3/2*d*x + 3/2*c))*sin(6*d*x +
6*c) + 300*(sqrt(2)*a^2*cos(6*d*x + 6*c) + 3*sqrt(2)*a^2*cos(8/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 3*sqrt(2)*a^2*cos(4/3*arctan2
(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sqrt(2)*a^2)*sin(11/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 12*(7*sqrt(2)*a^2*cos(9/2
*d*x + 9/2*c) - 7*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c) - 114*sqrt(2)*a^2*cos(7/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 114*sqrt(2)*a^2*co
s(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 75*sqrt(2)*a^2
*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(8/3*arct
an2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 456*(sqrt(2)*a^2*cos(6*d
*x + 6*c) + 3*sqrt(2)*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
+ 3/2*c))) + sqrt(2)*a^2)*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c))) - 456*(sqrt(2)*a^2*cos(6*d*x + 6*c) + 3*sqrt(2)*a^2*cos(4/3*ar
ctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sqrt(2)*a^2)*sin(5/3*ar
ctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 12*(7*sqrt(2)*a^2*cos
(9/2*d*x + 9/2*c) - 7*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c) + 75*sqrt(2)*a^2*cos
(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(4/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 300*(sqrt(2)*a^2*cos(6*d*x +
6*c) + sqrt(2)*a^2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/
2*c))))*B*sqrt(a)/(cos(6*d*x + 6*c)^2 + 6*(cos(6*d*x + 6*c) + 3*cos(4/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))) + 1)*cos(8/3*arctan2(sin(
3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 9*cos(8/3*arctan2(sin(3/2*d*x +
3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 6*(cos(6*d*x + 6*c) + 1)*cos(4/3*arctan2
(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 9*cos(4/3*arctan2(sin(3/2*d
*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(6*d*x + 6*c)^2 + 6*(sin(6*d*x +
6*c) + 3*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin
(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 9*sin(8/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 6*sin(6*d*x + 6*c)*sin(
4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 9*sin(4/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*cos(6*d*x + 6*c) + 1))
/d

```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(a + \frac{a}{\cos(c+dx)}\right)^{5/2}}{\sqrt{\cos(c+dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2))/cos(c + d*x)^(1/2),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2))/cos(c + d*x)^(1/2), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c))/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

$$3.539 \quad \int \frac{(a+a \sec(c+dx))^{5/2}(A+B \sec(c+dx))}{\cos^2(c+dx)} dx$$

Optimal. Leaf size=247

$$\frac{a^{5/2}(200A + 163B)\sqrt{\cos(c + dx)}\sqrt{\sec(c + dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{64d} + \frac{a^3(200A + 163B)\sin(c + dx)}{64d\cos^2(c + dx)\sqrt{a\sec(c + dx) + a}} + \dots$$

[Out] $1/4*a*B*(a+a*\sec(d*x+c))^{3/2}*\sin(d*x+c)/d/\cos(d*x+c)^{(5/2)}+1/64*a^{5/2}*(200*A+163*B)*\operatorname{arcsinh}(a^{1/2}*\tan(d*x+c)/(a+a*\sec(d*x+c))^{1/2})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d+1/96*a^3*(104*A+95*B)*\sin(d*x+c)/d/\cos(d*x+c)^{(5/2)}/(a+a*\sec(d*x+c))^{1/2}+1/64*a^3*(200*A+163*B)*\sin(d*x+c)/d/\cos(d*x+c)^{(3/2)}/(a+a*\sec(d*x+c))^{1/2}+1/24*a^2*(8*A+11*B)*\sin(d*x+c)*(a+a*\sec(d*x+c))^{1/2}/d/\cos(d*x+c)^{(5/2)}$

Rubi [A] time = 0.77, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {2955, 4018, 4016, 3803, 3801, 215}

$$\frac{a^3(200A + 163B)\sin(c + dx)}{64d\cos^2(c + dx)\sqrt{a\sec(c + dx) + a}} + \frac{a^3(104A + 95B)\sin(c + dx)}{96d\cos^2(c + dx)\sqrt{a\sec(c + dx) + a}} + \frac{a^2(8A + 11B)\sin(c + dx)\sqrt{a\sec(c + dx)}}{24d\cos^2(c + dx)}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(a + a*\operatorname{Sec}[c + d*x])^{5/2}*(A + B*\operatorname{Sec}[c + d*x])]/\operatorname{Cos}[c + d*x]^{3/2}, x]$

[Out] $(a^{5/2}*(200*A + 163*B)*\operatorname{ArcSinh}[(\operatorname{Sqrt}[a]*\operatorname{Tan}[c + d*x])/(\operatorname{Sqrt}[a + a*\operatorname{Sec}[c + d*x]])]*\operatorname{Sqrt}[\operatorname{Cos}[c + d*x]]*\operatorname{Sqrt}[\operatorname{Sec}[c + d*x]])/(64*d) + (a^3*(104*A + 95*B)*\operatorname{Sin}[c + d*x])/(96*d*\operatorname{Cos}[c + d*x]^{5/2}*\operatorname{Sqrt}[a + a*\operatorname{Sec}[c + d*x]]) + (a^3*(200*A + 163*B)*\operatorname{Sin}[c + d*x])/(64*d*\operatorname{Cos}[c + d*x]^{3/2}*\operatorname{Sqrt}[a + a*\operatorname{Sec}[c + d*x]]) + (a^2*(8*A + 11*B)*\operatorname{Sqrt}[a + a*\operatorname{Sec}[c + d*x]]*\operatorname{Sin}[c + d*x])/(24*d*\operatorname{Cos}[c + d*x]^{5/2}) + (a*B*(a + a*\operatorname{Sec}[c + d*x])^{3/2}*\operatorname{Sin}[c + d*x])/(4*d*\operatorname{Cos}[c + d*x]^{5/2})$

Rule 215

$\operatorname{Int}[1/\operatorname{Sqrt}[(a_) + (b_)*(x_)^2], x_Symbol] \rightarrow \operatorname{Simp}[\operatorname{ArcSinh}[(\operatorname{Rt}[b, 2]*x)/\operatorname{Sqrt}[a]]/\operatorname{Rt}[b, 2], x] /;$ FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 2955

$\operatorname{Int}[(a_) + \operatorname{csc}[(e_) + (f_)*(x_)]*(b_)]^{(m_)}*(\operatorname{csc}[(e_) + (f_)*(x_)]*(d_) + (c_))^{(n_)}*((g_)*\operatorname{sin}[(e_) + (f_)*(x_)]^{(p_)}, x_Symbol] \rightarrow \operatorname{Dis}$

$t[(g \cdot \csc[e + f \cdot x])^p \cdot (g \cdot \sin[e + f \cdot x])^p, \text{Int}[(a + b \cdot \csc[e + f \cdot x])^m \cdot (c + d \cdot \csc[e + f \cdot x])^n / (g \cdot \csc[e + f \cdot x])^p, x], x] /;$ FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && IntegerQ[n])

Rule 3801

$\text{Int}[\text{Sqrt}[\csc[(e_.) + (f_.)(x_.)]*(d_.)]*\text{Sqrt}[\csc[(e_.) + (f_.)(x_.)]*(b_.) + (a_.)], x_Symbol] :> \text{Dist}[(-2*a*\text{Sqrt}[(a*d)/b])/(b*f), \text{Subst}[\text{Int}[1/\text{Sqrt}[1 + x^2/a], x], x, (b*\text{Cot}[e + f*x])/\text{Sqrt}[a + b*\csc[e + f*x]]], x] /;$ FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]

Rule 3803

$\text{Int}[(\csc[(e_.) + (f_.)(x_.)]*(d_.))^{(n_.)}*\text{Sqrt}[\csc[(e_.) + (f_.)(x_.)]*(b_.) + (a_.)], x_Symbol] :> \text{Simp}[(-2*b*d*\text{Cot}[e + f*x]*(d*\csc[e + f*x])^{(n-1)})/(f*(2*n-1)*\text{Sqrt}[a + b*\csc[e + f*x]]), x] + \text{Dist}[(2*a*d*(n-1))/(b*(2*n-1)), \text{Int}[\text{Sqrt}[a + b*\csc[e + f*x]]*(d*\csc[e + f*x])^{(n-1)}, x], x] /;$ FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]

Rule 4016

$\text{Int}[(\csc[(e_.) + (f_.)(x_.)]*(d_.))^{(n_.)}*\text{Sqrt}[\csc[(e_.) + (f_.)(x_.)]*(b_.) + (a_.)]*(\csc[(e_.) + (f_.)(x_.)]*(B_.) + (A_.)), x_Symbol] :> \text{Simp}[(-2*b*B*\text{Cot}[e + f*x]*(d*\csc[e + f*x])^n)/(f*(2*n+1)*\text{Sqrt}[a + b*\csc[e + f*x]]), x] + \text{Dist}[(A*b*(2*n+1) + 2*a*B*n)/(b*(2*n+1)), \text{Int}[\text{Sqrt}[a + b*\csc[e + f*x]]*(d*\csc[e + f*x])^n, x], x] /;$ FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n+1) + 2*a*B*n, 0] && !LtQ[n, 0]

Rule 4018

$\text{Int}[(\csc[(e_.) + (f_.)(x_.)]*(d_.))^{(n_.)}*(\csc[(e_.) + (f_.)(x_.)]*(b_.) + (a_.))^{(m_.)}*(\csc[(e_.) + (f_.)(x_.)]*(B_.) + (A_.)), x_Symbol] :> -\text{Simp}[(b*B*\text{Cot}[e + f*x]*(a + b*\csc[e + f*x])^{(m-1)}*(d*\csc[e + f*x])^n)/(f*(m+n)), x] + \text{Dist}[1/(d*(m+n)), \text{Int}[(a + b*\csc[e + f*x])^{(m-1)}*(d*\csc[e + f*x])^n*\text{Simp}[a*A*d*(m+n) + B*(b*d*n) + (A*b*d*(m+n) + a*B*d*(2*m+n-1))*\csc[e + f*x], x], x], x] /;$ FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && !LtQ[n, -1]

Rubi steps

$$\begin{aligned}
\int \frac{(a + a \sec(c + dx))^{5/2} (A + B \sec(c + dx))}{\cos^{\frac{3}{2}}(c + dx)} dx &= \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \sec^{\frac{3}{2}}(c + dx) (a + a \sec(c + dx)) \\
&= \frac{aB(a + a \sec(c + dx))^{3/2} \sin(c + dx)}{4d \cos^{\frac{5}{2}}(c + dx)} + \frac{1}{4} \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \\
&= \frac{a^2(8A + 11B) \sqrt{a + a \sec(c + dx)} \sin(c + dx)}{24d \cos^{\frac{5}{2}}(c + dx)} + \frac{aB(a + a \sec(c + dx))^{3/2} \sin(c + dx)}{4d \cos^{\frac{5}{2}}(c + dx)} \\
&= \frac{a^3(104A + 95B) \sin(c + dx)}{96d \cos^{\frac{5}{2}}(c + dx) \sqrt{a + a \sec(c + dx)}} + \frac{a^2(8A + 11B) \sqrt{a + a \sec(c + dx)} \sin(c + dx)}{24d \cos^{\frac{5}{2}}(c + dx)} \\
&= \frac{a^3(104A + 95B) \sin(c + dx)}{96d \cos^{\frac{5}{2}}(c + dx) \sqrt{a + a \sec(c + dx)}} + \frac{a^3(200A + 163B) \sin(c + dx)}{64d \cos^{\frac{3}{2}}(c + dx) \sqrt{a + a \sec(c + dx)}} \\
&= \frac{a^3(104A + 95B) \sin(c + dx)}{96d \cos^{\frac{5}{2}}(c + dx) \sqrt{a + a \sec(c + dx)}} + \frac{a^3(200A + 163B) \sin(c + dx)}{64d \cos^{\frac{3}{2}}(c + dx) \sqrt{a + a \sec(c + dx)}} \\
&= \frac{a^{5/2}(200A + 163B) \sinh^{-1} \left(\frac{\sqrt{a} \tan(c + dx)}{\sqrt{a + a \sec(c + dx)}} \right) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)}}{64d}
\end{aligned}$$

Mathematica [A] time = 1.98, size = 154, normalized size = 0.62

$$\frac{a^2 \sec\left(\frac{1}{2}(c + dx)\right) \sqrt{a(\sec(c + dx) + 1)} \left(\sin\left(\frac{1}{2}(c + dx)\right) ((2056A + 2203B) \cos(c + dx) + (544A + 652B) \cos(2(c + dx))) \right)}{64d}$$

Antiderivative was successfully verified.

[In] Integrate[((a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Cos[c + d*x]^(3/2), x]

[Out] (a^2*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(6*Sqrt[2]*(200*A + 163*B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^4 + (544*A + 844*B + (2056*A + 2203*B)*Cos[c + d*x] + (544*A + 652*B)*Cos[2*(c + d*x)] + 600*A*Cos[3*(c + d*x)] + 489*B*Cos[3*(c + d*x)])*Sin[(c + d*x)/2])/((768*d*Cos[c + d*x]^(7/2)))

fricas [A] time = 0.61, size = 509, normalized size = 2.06

$$4 \left(3 (200 A + 163 B) a^2 \cos(dx + c)^3 + 2 (136 A + 163 B) a^2 \cos(dx + c)^2 + 8 (8 A + 23 B) a^2 \cos(dx + c) + 48 B \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="fricas")

[Out] [1/768*(4*(3*(200*A + 163*B))*a^2*cos(d*x + c)^3 + 2*(136*A + 163*B))*a^2*cos(d*x + c)^2 + 8*(8*A + 23*B))*a^2*cos(d*x + c) + 48*B*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 3*((200*A + 163*B)*a^2*cos(d*x + c)^5 + (200*A + 163*B))*a^2*cos(d*x + c)^4)*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(d*cos(d*x + c)^5 + d*cos(d*x + c)^4), 1/384*(2*(3*(200*A + 163*B))*a^2*cos(d*x + c)^3 + 2*(136*A + 163*B))*a^2*cos(d*x + c)^2 + 8*(8*A + 23*B))*a^2*cos(d*x + c) + 48*B*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 3*((200*A + 163*B))*a^2*cos(d*x + c)^5 + (200*A + 163*B))*a^2*cos(d*x + c)^4)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*x + c)^5 + d*cos(d*x + c)^4)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(a \sec(dx + c) + a)^{\frac{5}{2}}}{\cos(dx + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(5/2)/cos(d*x + c)^(3/2), x)

maple [B] time = 2.17, size = 469, normalized size = 1.90

$$a^2 \sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}} (-1 + \cos(dx+c)) \left(600A (\cos^4(dx+c)) \arctan \left(\frac{\sqrt{\frac{2}{1+\cos(dx+c)}} (\cos(dx+c)+1-\sin(dx+c))\sqrt{2}}{4} \right) \right) \sqrt{2} - 60$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2),x)

[Out] 1/384/d*a^2*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(600*A*cos(d*x+c)^4*arctan(1/4*(-2/(1+cos(d*x+c))))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))*2^(1/2))*2^(1/2)-600*A*cos(d*x+c)^4*arctan(1/4*(-2/(1+cos(d*x+c))))^(1/2)*(cos(d*x+c)+1+sin(d*x+c))*2^(1/2))*2^(1/2)+489*B*cos(d*x+c)^4*arctan(1/4*(-2/(1+cos(d*x+c))))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))*2^(1/2))*2^(1/2)-489*B*cos(d*x+c)^4*arctan(1/4*(-2/(1+cos(d*x+c))))^(1/2)*(cos(d*x+c)+1+sin(d*x+c))*2^(1/2))*2^(1/2)-1200*A*cos(d*x+c)^3*sin(d*x+c)*(-2/(1+cos(d*x+c))))^(1/2)-978*B*cos(d*x+c)^3*sin(d*x+c)*(-2/(1+cos(d*x+c))))^(1/2)-544*A*sin(d*x+c)*cos(d*x+c)^2*(-2/(1+cos(d*x+c))))^(1/2)-652*B*sin(d*x+c)*cos(d*x+c)^2*(-2/(1+cos(d*x+c))))^(1/2)-128*A*sin(d*x+c)*cos(d*x+c)*(-2/(1+cos(d*x+c))))^(1/2)-368*B*sin(d*x+c)*cos(d*x+c)*(-2/(1+cos(d*x+c))))^(1/2)-96*B*(-2/(1+cos(d*x+c))))^(1/2)*sin(d*x+c))/sin(d*x+c)^2/cos(d*x+c)^(7/2)/(-2/(1+cos(d*x+c))))^(1/2)

maxima [B] time = 1.41, size = 7331, normalized size = 29.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="maxima")

[Out] 1/768*(8*(300*sqrt(2)*a^2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) * sin(6*d*x + 6*c) - 28*sqrt(2)*a^2*sin(9/2*d*x + 9/2*c) + 28*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) - 28*(sqrt(2)*a^2*sin(9/2*d*x + 9/2*c) - sqrt(2)*a^2*sin(3/2*d*x + 3/2*c))*cos(6*d*x + 6*c) - 300*(sqrt(2)*a^2*sin(6*d*x + 6*c) + 3*sqrt(2)*a^2*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))) + 3*sqrt(2)*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))) * cos(11/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))) - 12*(7*sqrt(2)*a^2*sin(9/2*d*x + 9/2*c) - 7*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) - 114*sqrt(2)*a^2*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))) + 114*sqrt(2)*a^2*sin(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))) + 75*sqrt(2)*a^2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))) * cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))

$$\begin{aligned}
& (4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + a^2*\cos(8/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 6*(a^2*\cos(6*d*x + 6*c) + a^2)*\cos(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 6*(a^2*\sin(6*d*x + 6*c) + 3*a^2*\sin(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))))*\sin(8/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))*\log(2*\cos(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))))^2 + 2*\sin(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 - 2*\sqrt{2}*\cos(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 2*\sqrt{2}*\sin(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 2) - 75*(a^2*\cos(6*d*x + 6*c)^2 + 9*a^2*\cos(8/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))))^2 + 9*a^2*\cos(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + a^2*\sin(6*d*x + 6*c)^2 + 9*a^2*\sin(8/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + 6*a^2*\sin(6*d*x + 6*c)*\sin(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 9*a^2*\sin(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + 2*a^2*\cos(6*d*x + 6*c) + a^2 + 6*(a^2*\cos(6*d*x + 6*c) + 3*a^2*\cos(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))) + a^2*\cos(8/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 6*(a^2*\cos(6*d*x + 6*c) + a^2)*\cos(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 6*(a^2*\sin(6*d*x + 6*c) + 3*a^2*\sin(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))))*\sin(8/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))*\log(2*\cos(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))))^2 + 2*\sin(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 - 2*\sqrt{2}*\cos(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) - 2*\sqrt{2}*\sin(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 2) + 28*(\sqrt{2}*a^2*\cos(9/2*d*x + 9/2*c) - \sqrt{2}*a^2*\cos(3/2*d*x + 3/2*c))*\sin(6*d*x + 6*c) + 300*(\sqrt{2}*a^2*\cos(6*d*x + 6*c) + 3*\sqrt{2}*a^2*\cos(8/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))) + 3*\sqrt{2}*a^2*\cos(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + \sqrt{2}*a^2*\sin(11/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 12*(7*\sqrt{2}*a^2*\cos(9/2*d*x + 9/2*c) - 7*\sqrt{2}*a^2*\cos(3/2*d*x + 3/2*c) - 114*\sqrt{2}*a^2*\cos(7/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))) + 114*\sqrt{2}*a^2*\cos(5/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 75*\sqrt{2}*a^2*\cos(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))))*\sin(8/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 456*(\sqrt{2}*a^2*\cos(6*d*x + 6*c) + 3*\sqrt{2}*a^2*\cos(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))) + \sqrt{2}*a^2*\sin(7/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) - 456*(\sqrt{2}*a^2*\cos(6*d*x + 6*c) + 3*\sqrt{2}*a^2*\cos(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))) + \sqrt{2}*a^2*\sin(5/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 12*(7*\sqrt{2}*a^2*\cos(9/2*d*x + 9/2*c) - 7*\sqrt{2}*a^2*\cos(3/2*d*x + 3/2*c) + 75*\sqrt{2}*a^2*\cos(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))))*\sin(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) - 300*(\sqrt{2}*a^2*\cos(6*d*x + 6*c) + \sqrt{2}*a^2*\sin(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))))*A*\sqrt{a}/(\cos(6*d*x + 6*c)^2 + 6*(\cos(6*d*x + 6*c) + 3*\cos(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))))
\end{aligned}$$

$$\begin{aligned}
& /2*c))) + 1)*\cos(8/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + \\
& 9*\cos(8/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + 6*(\cos(\\
& 6*d*x + 6*c) + 1)*\cos(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c \\
&))) + 9*\cos(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + \sin \\
& (6*d*x + 6*c)^2 + 6*(\sin(6*d*x + 6*c) + 3*\sin(4/3*\arctan2(\sin(3/2*d*x + 3/ \\
& 2*c), \cos(3/2*d*x + 3/2*c))))*\sin(8/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2 \\
& *d*x + 3/2*c))) + 9*\sin(8/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2 \\
& *c)))^2 + 6*\sin(6*d*x + 6*c)*\sin(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2* \\
& d*x + 3/2*c))) + 9*\sin(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2* \\
& c)))^2 + 2*\cos(6*d*x + 6*c) + 1) - (1956*(\sqrt{2})*a^2*\sin(8*d*x + 8*c) + 4* \\
& \sqrt{2})*a^2*\sin(6*d*x + 6*c) + 6*\sqrt{2})*a^2*\sin(4*d*x + 4*c) + 4*\sqrt{2})*a \\
& ^2*\sin(2*d*x + 2*c))*\cos(15/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) \\
& + 652*(\sqrt{2})*a^2*\sin(8*d*x + 8*c) + 4*\sqrt{2})*a^2*\sin(6*d*x + 6*c) + 6*\sqrt{2} \\
& *a^2*\sin(4*d*x + 4*c) + 4*\sqrt{2})*a^2*\sin(2*d*x + 2*c))*\cos(13/4*\arcta \\
& n2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 6204*(\sqrt{2})*a^2*\sin(8*d*x + 8*c \\
&) + 4*\sqrt{2})*a^2*\sin(6*d*x + 6*c) + 6*\sqrt{2})*a^2*\sin(4*d*x + 4*c) + 4*\sqrt{2} \\
& *a^2*\sin(2*d*x + 2*c))*\cos(11/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2 \\
& *c))) - 2060*(\sqrt{2})*a^2*\sin(8*d*x + 8*c) + 4*\sqrt{2})*a^2*\sin(6*d*x + 6*c) \\
& + 6*\sqrt{2})*a^2*\sin(4*d*x + 4*c) + 4*\sqrt{2})*a^2*\sin(2*d*x + 2*c))*\cos(9/4 \\
& *\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 2060*(\sqrt{2})*a^2*\sin(8*d*x \\
& + 8*c) + 4*\sqrt{2})*a^2*\sin(6*d*x + 6*c) + 6*\sqrt{2})*a^2*\sin(4*d*x + 4*c) + \\
& 4*\sqrt{2})*a^2*\sin(2*d*x + 2*c))*\cos(7/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d* \\
& x + 2*c))) - 6204*(\sqrt{2})*a^2*\sin(8*d*x + 8*c) + 4*\sqrt{2})*a^2*\sin(6*d*x + \\
& 6*c) + 6*\sqrt{2})*a^2*\sin(4*d*x + 4*c) + 4*\sqrt{2})*a^2*\sin(2*d*x + 2*c))*\co \\
& s(5/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) - 652*(\sqrt{2})*a^2*\sin(8 \\
& *d*x + 8*c) + 4*\sqrt{2})*a^2*\sin(6*d*x + 6*c) + 6*\sqrt{2})*a^2*\sin(4*d*x + 4* \\
& c) + 4*\sqrt{2})*a^2*\sin(2*d*x + 2*c))*\cos(3/4*\arctan2(\sin(2*d*x + 2*c), \cos(\\
& 2*d*x + 2*c))) - 1956*(\sqrt{2})*a^2*\sin(8*d*x + 8*c) + 4*\sqrt{2})*a^2*\sin(6*d \\
& *x + 6*c) + 6*\sqrt{2})*a^2*\sin(4*d*x + 4*c) + 4*\sqrt{2})*a^2*\sin(2*d*x + 2*c) \\
&)*\cos(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) - 489*(a^2*\cos(8*d*x \\
& + 8*c)^2 + 16*a^2*\cos(6*d*x + 6*c)^2 + 36*a^2*\cos(4*d*x + 4*c)^2 + 16*a^2* \\
& \cos(2*d*x + 2*c)^2 + a^2*\sin(8*d*x + 8*c)^2 + 16*a^2*\sin(6*d*x + 6*c)^2 + 3 \\
& 6*a^2*\sin(4*d*x + 4*c)^2 + 48*a^2*\sin(4*d*x + 4*c)*\sin(2*d*x + 2*c) + 16*a^ \\
& 2*\sin(2*d*x + 2*c)^2 + 8*a^2*\cos(2*d*x + 2*c) + a^2 + 2*(4*a^2*\cos(6*d*x + \\
& 6*c) + 6*a^2*\cos(4*d*x + 4*c) + 4*a^2*\cos(2*d*x + 2*c) + a^2)*\cos(8*d*x + 8 \\
& *c) + 8*(6*a^2*\cos(4*d*x + 4*c) + 4*a^2*\cos(2*d*x + 2*c) + a^2)*\cos(6*d*x + \\
& 6*c) + 12*(4*a^2*\cos(2*d*x + 2*c) + a^2)*\cos(4*d*x + 4*c) + 4*(2*a^2*\sin(6 \\
& *d*x + 6*c) + 3*a^2*\sin(4*d*x + 4*c) + 2*a^2*\sin(2*d*x + 2*c))*\sin(8*d*x + \\
& 8*c) + 16*(3*a^2*\sin(4*d*x + 4*c) + 2*a^2*\sin(2*d*x + 2*c))*\sin(6*d*x + 6*c \\
&))*\log(2*\cos(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + 2*\sin(1/4 \\
& *\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + 2*\sqrt{2})*\cos(1/4*\arctan2 \\
& (\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 2*\sqrt{2})*\sin(1/4*\arctan2(\sin(2*d*x \\
& + 2*c), \cos(2*d*x + 2*c))) + 2) + 489*(a^2*\cos(8*d*x + 8*c)^2 + 16*a^2*\cos \\
& (6*d*x + 6*c)^2 + 36*a^2*\cos(4*d*x + 4*c)^2 + 16*a^2*\cos(2*d*x + 2*c)^2 + a \\
& ^2*\sin(8*d*x + 8*c)^2 + 16*a^2*\sin(6*d*x + 6*c)^2 + 36*a^2*\sin(4*d*x + 4*c)
\end{aligned}$$

$$\begin{aligned}
&^2 + 48a^2\sin(4dx + 4c)\sin(2dx + 2c) + 16a^2\sin(2dx + 2c)^2 + \\
&8a^2\cos(2dx + 2c) + a^2 + 2(4a^2\cos(6dx + 6c) + 6a^2\cos(4dx \\
&+ 4c) + 4a^2\cos(2dx + 2c) + a^2)\cos(8dx + 8c) + 8(6a^2\cos(4d \\
&x + 4c) + 4a^2\cos(2dx + 2c) + a^2)\cos(6dx + 6c) + 12(4a^2\cos(\\
&2dx + 2c) + a^2)\cos(4dx + 4c) + 4(2a^2\sin(6dx + 6c) + 3a^2\sin \\
&(4dx + 4c) + 2a^2\sin(2dx + 2c))\sin(8dx + 8c) + 16(3a^2\sin(4 \\
&dx + 4c) + 2a^2\sin(2dx + 2c))\sin(6dx + 6c))\log(2\cos(1/4\arctan \\
&2(\sin(2dx + 2c), \cos(2dx + 2c)))^2 + 2\sin(1/4\arctan2(\sin(2dx + 2 \\
&c), \cos(2dx + 2c)))^2 + 2\sqrt{2}\cos(1/4\arctan2(\sin(2dx + 2c), \cos \\
&(2dx + 2c))) - 2\sqrt{2}\sin(1/4\arctan2(\sin(2dx + 2c), \cos(2dx + 2 \\
&c))) + 2) - 489(a^2\cos(8dx + 8c)^2 + 16a^2\cos(6dx + 6c)^2 + 36a \\
&^2\cos(4dx + 4c)^2 + 16a^2\cos(2dx + 2c)^2 + a^2\sin(8dx + 8c)^2 \\
&+ 16a^2\sin(6dx + 6c)^2 + 36a^2\sin(4dx + 4c)^2 + 48a^2\sin(4dx \\
&+ 4c)\sin(2dx + 2c) + 16a^2\sin(2dx + 2c)^2 + 8a^2\cos(2dx + 2c \\
&) + a^2 + 2(4a^2\cos(6dx + 6c) + 6a^2\cos(4dx + 4c) + 4a^2\cos(2 \\
&dx + 2c) + a^2)\cos(8dx + 8c) + 8(6a^2\cos(4dx + 4c) + 4a^2\cos(\\
&2dx + 2c) + a^2)\cos(6dx + 6c) + 12(4a^2\cos(2dx + 2c) + a^2)\co \\
&s(4dx + 4c) + 4(2a^2\sin(6dx + 6c) + 3a^2\sin(4dx + 4c) + 2a^2 \\
&\sin(2dx + 2c))\sin(8dx + 8c) + 16(3a^2\sin(4dx + 4c) + 2a^2\sin \\
&(2dx + 2c))\sin(6dx + 6c))\log(2\cos(1/4\arctan2(\sin(2dx + 2c), c \\
&>os(2dx + 2c)))^2 + 2\sin(1/4\arctan2(\sin(2dx + 2c), \cos(2dx + 2c)) \\
&))^2 - 2\sqrt{2}\cos(1/4\arctan2(\sin(2dx + 2c), \cos(2dx + 2c))) + 2\sqrt{ \\
&2}\sin(1/4\arctan2(\sin(2dx + 2c), \cos(2dx + 2c))) + 2) + 489(a^2 \\
&\cos(8dx + 8c)^2 + 16a^2\cos(6dx + 6c)^2 + 36a^2\cos(4dx + 4c)^2 \\
&+ 16a^2\cos(2dx + 2c)^2 + a^2\sin(8dx + 8c)^2 + 16a^2\sin(6dx + 6 \\
&c)^2 + 36a^2\sin(4dx + 4c)^2 + 48a^2\sin(4dx + 4c)\sin(2dx + 2c \\
&) + 16a^2\sin(2dx + 2c)^2 + 8a^2\cos(2dx + 2c) + a^2 + 2(4a^2\cos \\
&(6dx + 6c) + 6a^2\cos(4dx + 4c) + 4a^2\cos(2dx + 2c) + a^2)\cos(\\
&8dx + 8c) + 8(6a^2\cos(4dx + 4c) + 4a^2\cos(2dx + 2c) + a^2)\co \\
&>s(6dx + 6c) + 12(4a^2\cos(2dx + 2c) + a^2)\cos(4dx + 4c) + 4(2 \\
&>a^2\sin(6dx + 6c) + 3a^2\sin(4dx + 4c) + 2a^2\sin(2dx + 2c))\sin \\
&(8dx + 8c) + 16(3a^2\sin(4dx + 4c) + 2a^2\sin(2dx + 2c))\sin(6 \\
&dx + 6c))\log(2\cos(1/4\arctan2(\sin(2dx + 2c), \cos(2dx + 2c)))^2 + \\
&2\sin(1/4\arctan2(\sin(2dx + 2c), \cos(2dx + 2c)))^2 - 2\sqrt{2}\cos(1/ \\
&4\arctan2(\sin(2dx + 2c), \cos(2dx + 2c))) - 2\sqrt{2}\sin(1/4\arctan2(\\
&\sin(2dx + 2c), \cos(2dx + 2c))) + 2) - 1956(\sqrt{2})a^2\cos(8dx + 8 \\
&c) + 4\sqrt{2})a^2\cos(6dx + 6c) + 6\sqrt{2})a^2\cos(4dx + 4c) + 4\sqrt{ \\
&2})a^2\cos(2dx + 2c) + \sqrt{2})a^2)\sin(15/4\arctan2(\sin(2dx + 2c \\
&), \cos(2dx + 2c))) - 652(\sqrt{2})a^2\cos(8dx + 8c) + 4\sqrt{2})a^2\cos \\
&(6dx + 6c) + 6\sqrt{2})a^2\cos(4dx + 4c) + 4\sqrt{2})a^2\cos(2dx \\
&+ 2c) + \sqrt{2})a^2)\sin(13/4\arctan2(\sin(2dx + 2c), \cos(2dx + 2c))) \\
&- 6204(\sqrt{2})a^2\cos(8dx + 8c) + 4\sqrt{2})a^2\cos(6dx + 6c) + 6 \\
&\sqrt{2})a^2\cos(4dx + 4c) + 4\sqrt{2})a^2\cos(2dx + 2c) + \sqrt{2})a^2 \\
&)\sin(11/4\arctan2(\sin(2dx + 2c), \cos(2dx + 2c))) + 2060(\sqrt{2})a^2 \\
&\cos(8dx + 8c) + 4\sqrt{2})a^2\cos(6dx + 6c) + 6\sqrt{2})a^2\cos(4dx
\end{aligned}$$

$x + 4*c) + 4*\sqrt{2}*a^2*\cos(2*d*x + 2*c) + \sqrt{2}*a^2*\sin(9/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) - 2060*(\sqrt{2}*a^2*\cos(8*d*x + 8*c) + 4*\sqrt{2}*a^2*\cos(6*d*x + 6*c) + 6*\sqrt{2}*a^2*\cos(4*d*x + 4*c) + 4*\sqrt{2}*a^2*\cos(2*d*x + 2*c) + \sqrt{2}*a^2*\sin(7/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 6204*(\sqrt{2}*a^2*\cos(8*d*x + 8*c) + 4*\sqrt{2}*a^2*\cos(6*d*x + 6*c) + 6*\sqrt{2}*a^2*\cos(4*d*x + 4*c) + 4*\sqrt{2}*a^2*\cos(2*d*x + 2*c) + \sqrt{2}*a^2*\sin(5/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 652*(\sqrt{2}*a^2*\cos(8*d*x + 8*c) + 4*\sqrt{2}*a^2*\cos(6*d*x + 6*c) + 6*\sqrt{2}*a^2*\cos(4*d*x + 4*c) + 4*\sqrt{2}*a^2*\cos(2*d*x + 2*c) + \sqrt{2}*a^2*\sin(3/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 1956*(\sqrt{2}*a^2*\cos(8*d*x + 8*c) + 4*\sqrt{2}*a^2*\cos(6*d*x + 6*c) + 6*\sqrt{2}*a^2*\cos(4*d*x + 4*c) + 4*\sqrt{2}*a^2*\cos(2*d*x + 2*c) + \sqrt{2}*a^2*\sin(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))))*B*\sqrt{a}/(2*(4*\cos(6*d*x + 6*c) + 6*\cos(4*d*x + 4*c) + 4*\cos(2*d*x + 2*c) + 1)*\cos(8*d*x + 8*c) + \cos(8*d*x + 8*c)^2 + 8*(6*\cos(4*d*x + 4*c) + 4*\cos(2*d*x + 2*c) + 1)*\cos(6*d*x + 6*c) + 16*\cos(6*d*x + 6*c)^2 + 12*(4*\cos(2*d*x + 2*c) + 1)*\cos(4*d*x + 4*c) + 36*\cos(4*d*x + 4*c)^2 + 16*\cos(2*d*x + 2*c)^2 + 4*(2*\sin(6*d*x + 6*c) + 3*\sin(4*d*x + 4*c) + 2*\sin(2*d*x + 2*c))*\sin(8*d*x + 8*c) + \sin(8*d*x + 8*c)^2 + 16*(3*\sin(4*d*x + 4*c) + 2*\sin(2*d*x + 2*c))*\sin(6*d*x + 6*c) + 16*\sin(6*d*x + 6*c)^2 + 36*\sin(4*d*x + 4*c)^2 + 48*\sin(4*d*x + 4*c)*\sin(2*d*x + 2*c) + 16*\sin(2*d*x + 2*c)^2 + 8*\cos(2*d*x + 2*c) + 1))/d$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(a + \frac{a}{\cos(c+dx)}\right)^{5/2}}{\cos(c+dx)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2))/cos(c + d*x)^(3/2), x)

[Out] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2))/cos(c + d*x)^(3/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c))/cos(d*x+c)**(3/2), x)

[Out] Timed out

$$3.540 \quad \int \frac{(a+a \sec(c+dx))^{5/2}(A+B \sec(c+dx))}{\cos^2(c+dx)} dx$$

Optimal. Leaf size=294

$$\frac{a^{5/2}(326A + 283B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{128d} + \frac{a^3(326A + 283B)\sin(c+dx)}{128d\cos^2(c+dx)\sqrt{a\sec(c+dx)+a}} + \dots$$

[Out] $1/5*a*B*(a+a*\sec(d*x+c))^{3/2}*\sin(d*x+c)/d/\cos(d*x+c)^{7/2}+1/128*a^{5/2}*(326*A+283*B)*\operatorname{arcsinh}(a^{1/2}*\tan(d*x+c)/(a+a*\sec(d*x+c))^{1/2})*\cos(d*x+c)^{1/2}*\sec(d*x+c)^{1/2}/d+1/240*a^3*(170*A+157*B)*\sin(d*x+c)/d/\cos(d*x+c)^{7/2}/(a+a*\sec(d*x+c))^{1/2}+1/192*a^3*(326*A+283*B)*\sin(d*x+c)/d/\cos(d*x+c)^{5/2}/(a+a*\sec(d*x+c))^{1/2}+1/128*a^3*(326*A+283*B)*\sin(d*x+c)/d/\cos(d*x+c)^{3/2}/(a+a*\sec(d*x+c))^{1/2}+1/40*a^2*(10*A+13*B)*\sin(d*x+c)*(a+a*\sec(d*x+c))^{1/2}/d/\cos(d*x+c)^{7/2}$

Rubi [A] time = 0.86, antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {2955, 4018, 4016, 3803, 3801, 215}

$$\frac{a^3(326A + 283B)\sin(c+dx)}{128d\cos^2(c+dx)\sqrt{a\sec(c+dx)+a}} + \frac{a^3(326A + 283B)\sin(c+dx)}{192d\cos^2(c+dx)\sqrt{a\sec(c+dx)+a}} + \frac{a^3(170A + 157B)\sin(c+dx)}{240d\cos^2(c+dx)\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(a + a*\operatorname{Sec}[c + d*x])^{5/2}*(A + B*\operatorname{Sec}[c + d*x])]/\operatorname{Cos}[c + d*x]^{5/2}, x]$

[Out] $(a^{5/2}*(326*A + 283*B)*\operatorname{ArcSinh}[(\operatorname{Sqrt}[a]*\operatorname{Tan}[c + d*x])/(\operatorname{Sqrt}[a + a*\operatorname{Sec}[c + d*x]])]*\operatorname{Sqrt}[\operatorname{Cos}[c + d*x]]*\operatorname{Sqrt}[\operatorname{Sec}[c + d*x]])/(128*d) + (a^3*(170*A + 157*B)*\operatorname{Sin}[c + d*x])/(240*d*\operatorname{Cos}[c + d*x]^{7/2}*\operatorname{Sqrt}[a + a*\operatorname{Sec}[c + d*x]]) + (a^3*(326*A + 283*B)*\operatorname{Sin}[c + d*x])/(192*d*\operatorname{Cos}[c + d*x]^{5/2}*\operatorname{Sqrt}[a + a*\operatorname{Sec}[c + d*x]]) + (a^3*(326*A + 283*B)*\operatorname{Sin}[c + d*x])/(128*d*\operatorname{Cos}[c + d*x]^{3/2}*\operatorname{Sqrt}[a + a*\operatorname{Sec}[c + d*x]]) + (a^2*(10*A + 13*B)*\operatorname{Sqrt}[a + a*\operatorname{Sec}[c + d*x]]*\operatorname{Sin}[c + d*x])/(40*d*\operatorname{Cos}[c + d*x]^{7/2}) + (a*B*(a + a*\operatorname{Sec}[c + d*x])^{3/2}*\operatorname{Sin}[c + d*x])/(5*d*\operatorname{Cos}[c + d*x]^{7/2})$

Rule 215

$\operatorname{Int}[1/\operatorname{Sqrt}[(a_) + (b_)*(x_)^2], x_Symbol] \rightarrow \operatorname{Simp}[\operatorname{ArcSinh}[(\operatorname{Rt}[b, 2]*x)/\operatorname{Sqrt}[a]]/\operatorname{Rt}[b, 2], x] /;$ $\operatorname{FreeQ}\{a, b\}, x \&\& \operatorname{GtQ}[a, 0] \&\& \operatorname{PosQ}[b]$

Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)])*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_.))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)])*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_.)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 3803

```
Int[(csc[(e_.) + (f_.)*(x_)])*(d_.))^(n_.)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_.)], x_Symbol] :> Simp[(-2*b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/
(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(2*a*d*(n - 1))/(b*(2*n -
1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)])*(d_.))^(n_.)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_.)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] :> Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
+ Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && !
LtQ[n, 0]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)])*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && !LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + a \sec(c + dx))^{5/2}(A + B \sec(c + dx))}{\cos^2(c + dx)} dx &= \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)}\right) \int \sec^2(c + dx)(a + a \sec(c + dx)) \\
&= \frac{aB(a + a \sec(c + dx))^{3/2} \sin(c + dx)}{5d \cos^2(c + dx)} + \frac{1}{5} \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)}\right) \\
&= \frac{a^2(10A + 13B)\sqrt{a + a \sec(c + dx)} \sin(c + dx)}{40d \cos^2(c + dx)} + \frac{aB(a + a \sec(c + dx))^{3/2} \sin(c + dx)}{5d \cos^2(c + dx)} \\
&= \frac{a^3(170A + 157B) \sin(c + dx)}{240d \cos^2(c + dx)\sqrt{a + a \sec(c + dx)}} + \frac{a^2(10A + 13B)\sqrt{a + a \sec(c + dx)} \sin(c + dx)}{40d \cos^2(c + dx)} \\
&= \frac{a^3(170A + 157B) \sin(c + dx)}{240d \cos^2(c + dx)\sqrt{a + a \sec(c + dx)}} + \frac{a^3(326A + 283B)}{192d \cos^2(c + dx)\sqrt{a + a \sec(c + dx)}} \\
&= \frac{a^3(170A + 157B) \sin(c + dx)}{240d \cos^2(c + dx)\sqrt{a + a \sec(c + dx)}} + \frac{a^3(326A + 283B)}{192d \cos^2(c + dx)\sqrt{a + a \sec(c + dx)}} \\
&= \frac{a^3(170A + 157B) \sin(c + dx)}{240d \cos^2(c + dx)\sqrt{a + a \sec(c + dx)}} + \frac{a^3(326A + 283B)}{192d \cos^2(c + dx)\sqrt{a + a \sec(c + dx)}} \\
&= \frac{a^3(170A + 157B) \sin(c + dx)}{240d \cos^2(c + dx)\sqrt{a + a \sec(c + dx)}} + \frac{a^3(326A + 283B)}{192d \cos^2(c + dx)\sqrt{a + a \sec(c + dx)}} \\
&= \frac{a^3(170A + 157B) \sin(c + dx)}{240d \cos^2(c + dx)\sqrt{a + a \sec(c + dx)}} + \frac{a^3(326A + 283B)}{192d \cos^2(c + dx)\sqrt{a + a \sec(c + dx)}} \\
&= \frac{a^5/2(326A + 283B) \sinh^{-1}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a+a \sec(c+dx)}}\right) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)}}{128d}
\end{aligned}$$

Mathematica [A] time = 2.88, size = 178, normalized size = 0.61

$$\frac{a^2 \sec\left(\frac{1}{2}(c + dx)\right) \sqrt{a(\sec(c + dx) + 1)} \left(\sin\left(\frac{1}{2}(c + dx)\right) (36(650A + 781B) \cos(c + dx) + 4(6730A + 6509B) \cos(2(c + dx)))\right)}{128d}$$

Antiderivative was successfully verified.

[In] Integrate[((a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Cos[c + d*x]^(5/2), x]

[Out] (a^2*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(60*Sqrt[2]*(326*A + 283*B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^5 + (22030*A + 24863*B + 36*(650*A + 781*B)*Cos[c + d*x] + 4*(6730*A + 6509*B)*Cos[2*(c + d*x)] + 6520*A*Cos[3*(c + d*x)] + 5660*B*Cos[3*(c + d*x)] + 4890*A*Cos[4*(c + d*x)] + 4245*B*Cos[4*(c + d*x)])*Sin[(c + d*x)/2])/((15360*d*Cos[c + d*x]^(9/2)))

fricas [A] time = 0.59, size = 549, normalized size = 1.87

$$4 \left(15 (326 A + 283 B) a^2 \cos(dx + c)^4 + 10 (326 A + 283 B) a^2 \cos(dx + c)^3 + 8 (230 A + 283 B) a^2 \cos(dx + c)^2 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="fricas")

[Out] [1/7680*(4*(15*(326*A + 283*B))*a^2*cos(d*x + c)^4 + 10*(326*A + 283*B))*a^2*cos(d*x + c)^3 + 8*(230*A + 283*B))*a^2*cos(d*x + c)^2 + 48*(10*A + 29*B))*a^2*cos(d*x + c) + 384*B*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 15*((326*A + 283*B))*a^2*cos(d*x + c)^6 + (326*A + 283*B))*a^2*cos(d*x + c)^5)*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(d*cos(d*x + c)^6 + d*cos(d*x + c)^5), 1/3840*(2*(15*(326*A + 283*B))*a^2*cos(d*x + c)^4 + 10*(326*A + 283*B))*a^2*cos(d*x + c)^3 + 8*(230*A + 283*B))*a^2*cos(d*x + c)^2 + 48*(10*A + 29*B))*a^2*cos(d*x + c) + 384*B*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 15*((326*A + 283*B))*a^2*cos(d*x + c)^6 + (326*A + 283*B))*a^2*cos(d*x + c)^5)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*x + c)^6 + d*cos(d*x + c)^5)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(a \sec(dx + c) + a)^{\frac{5}{2}}}{\cos(dx + c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(5/2)/cos(d*x + c)^(5/2), x)

maple [B] time = 2.12, size = 531, normalized size = 1.81

$$a^2 \sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}} (-1 + \cos(dx+c)) \left(-4890A \arctan \left(\frac{\sqrt{\frac{2}{1+\cos(dx+c)}} (\cos(dx+c)+1-\sin(dx+c))\sqrt{2}}{4}} \right) \right) (\cos^5(dx+c)) \sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(5/2),x)

[Out]
$$\begin{aligned} & -1/3840/d*a^2*(a*(1+\cos(d*x+c))/\cos(d*x+c))^{(1/2)}*(-1+\cos(d*x+c))*(-4890*A* \\ & \arctan(1/4*(-2/(1+\cos(d*x+c))))^{(1/2)}*(\cos(d*x+c)+1-\sin(d*x+c))*2^{(1/2)}*\cos \\ & (d*x+c)^5*2^{(1/2)}+4890*A*\arctan(1/4*(-2/(1+\cos(d*x+c))))^{(1/2)}*(\cos(d*x+c)+1 \\ & +\sin(d*x+c))*2^{(1/2)}*\cos(d*x+c)^5*2^{(1/2)}-4245*B*\arctan(1/4*(-2/(1+\cos(d*x \\ & +c))))^{(1/2)}*(\cos(d*x+c)+1-\sin(d*x+c))*2^{(1/2)}*\cos(d*x+c)^5*2^{(1/2)}+4245*B* \\ & \arctan(1/4*(-2/(1+\cos(d*x+c))))^{(1/2)}*(\cos(d*x+c)+1+\sin(d*x+c))*2^{(1/2)}*\cos \\ & (d*x+c)^5*2^{(1/2)}+9780*A*(-2/(1+\cos(d*x+c)))^{(1/2)}*\cos(d*x+c)^4*\sin(d*x+c)+ \\ & 8490*B*(-2/(1+\cos(d*x+c)))^{(1/2)}*\cos(d*x+c)^4*\sin(d*x+c)+6520*A*\cos(d*x+c)^ \\ & 3*\sin(d*x+c)*(-2/(1+\cos(d*x+c)))^{(1/2)}+5660*B*\cos(d*x+c)^3*\sin(d*x+c)*(-2/(\\ & 1+\cos(d*x+c)))^{(1/2)}+3680*A*\sin(d*x+c)*\cos(d*x+c)^2*(-2/(1+\cos(d*x+c)))^{(1/ \\ & 2)}+4528*B*\sin(d*x+c)*\cos(d*x+c)^2*(-2/(1+\cos(d*x+c)))^{(1/2)}+960*A*\sin(d*x+c) \\ &)*\cos(d*x+c)*(-2/(1+\cos(d*x+c)))^{(1/2)}+2784*B*\sin(d*x+c)*\cos(d*x+c)*(-2/(1+ \\ & \cos(d*x+c)))^{(1/2)}+768*B*(-2/(1+\cos(d*x+c)))^{(1/2)}*\sin(d*x+c))/\sin(d*x+c)^2 \\ & / \cos(d*x+c)^{(9/2)}/(-2/(1+\cos(d*x+c)))^{(1/2)} \end{aligned}$$

maxima [B] time = 2.28, size = 9242, normalized size = 31.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="maxima")

[Out]
$$\begin{aligned} & -1/7680*(10*(1956*(\sqrt{2})*a^2*\sin(8*d*x + 8*c) + 4*\sqrt{2})*a^2*\sin(6*d*x + \\ & 6*c) + 6*\sqrt{2})*a^2*\sin(4*d*x + 4*c) + 4*\sqrt{2})*a^2*\sin(2*d*x + 2*c))*\cos \\ & (15/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 652*(\sqrt{2})*a^2*\sin(\\ & 8*d*x + 8*c) + 4*\sqrt{2})*a^2*\sin(6*d*x + 6*c) + 6*\sqrt{2})*a^2*\sin(4*d*x + 4 \\ & *c) + 4*\sqrt{2})*a^2*\sin(2*d*x + 2*c))*\cos(13/4*\arctan2(\sin(2*d*x + 2*c), \cos \\ & (2*d*x + 2*c))) + 6204*(\sqrt{2})*a^2*\sin(8*d*x + 8*c) + 4*\sqrt{2})*a^2*\sin(6 \\ & *d*x + 6*c) + 6*\sqrt{2})*a^2*\sin(4*d*x + 4*c) + 4*\sqrt{2})*a^2*\sin(2*d*x + 2* \\ & c))*\cos(11/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) - 2060*(\sqrt{2})*a \\ & ^2*\sin(8*d*x + 8*c) + 4*\sqrt{2})*a^2*\sin(6*d*x + 6*c) + 6*\sqrt{2})*a^2*\sin(4* \\ & d*x + 4*c) + 4*\sqrt{2})*a^2*\sin(2*d*x + 2*c))*\cos(9/4*\arctan2(\sin(2*d*x + 2* \end{aligned}$$

$$\begin{aligned}
& *x + 6*c)) * \log(2*\cos(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + 2 \\
& * \sin(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 - 2*\sqrt{2}*\cos(1/4 \\
& *\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 2*\sqrt{2}*\sin(1/4*\arctan2(s \\
& \sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 2) + 489*(a^2*\cos(8*d*x + 8*c)^2 + 16 \\
& *a^2*\cos(6*d*x + 6*c)^2 + 36*a^2*\cos(4*d*x + 4*c)^2 + 16*a^2*\cos(2*d*x + 2* \\
& c)^2 + a^2*\sin(8*d*x + 8*c)^2 + 16*a^2*\sin(6*d*x + 6*c)^2 + 36*a^2*\sin(4*d* \\
& x + 4*c)^2 + 48*a^2*\sin(4*d*x + 4*c)*\sin(2*d*x + 2*c) + 16*a^2*\sin(2*d*x + \\
& 2*c)^2 + 8*a^2*\cos(2*d*x + 2*c) + a^2 + 2*(4*a^2*\cos(6*d*x + 6*c) + 6*a^2*c \\
& \cos(4*d*x + 4*c) + 4*a^2*\cos(2*d*x + 2*c) + a^2)*\cos(8*d*x + 8*c) + 8*(6*a^2 \\
& *\cos(4*d*x + 4*c) + 4*a^2*\cos(2*d*x + 2*c) + a^2)*\cos(6*d*x + 6*c) + 12*(4* \\
& a^2*\cos(2*d*x + 2*c) + a^2)*\cos(4*d*x + 4*c) + 4*(2*a^2*\sin(6*d*x + 6*c) + \\
& 3*a^2*\sin(4*d*x + 4*c) + 2*a^2*\sin(2*d*x + 2*c))*\sin(8*d*x + 8*c) + 16*(3*a \\
& ^2*\sin(4*d*x + 4*c) + 2*a^2*\sin(2*d*x + 2*c))*\sin(6*d*x + 6*c))*\log(2*\cos(1 \\
& /4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + 2*\sin(1/4*\arctan2(\sin(2 \\
& *d*x + 2*c), \cos(2*d*x + 2*c)))^2 - 2*\sqrt{2}*\cos(1/4*\arctan2(\sin(2*d*x + 2 \\
& *c), \cos(2*d*x + 2*c))) - 2*\sqrt{2}*\sin(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2 \\
& *d*x + 2*c))) + 2) - 1956*(\sqrt{2}*a^2*\cos(8*d*x + 8*c) + 4*\sqrt{2}*a^2*\cos \\
& (6*d*x + 6*c) + 6*\sqrt{2}*a^2*\cos(4*d*x + 4*c) + 4*\sqrt{2}*a^2*\cos(2*d*x + \\
& 2*c) + \sqrt{2}*a^2)*\sin(15/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) - \\
& 652*(\sqrt{2}*a^2*\cos(8*d*x + 8*c) + 4*\sqrt{2}*a^2*\cos(6*d*x + 6*c) + 6*\sqrt{ \\
& t(2)*a^2*\cos(4*d*x + 4*c) + 4*\sqrt{2}*a^2*\cos(2*d*x + 2*c) + \sqrt{2}*a^2)*s \\
& \sin(13/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) - 6204*(\sqrt{2}*a^2*co \\
& s(8*d*x + 8*c) + 4*\sqrt{2}*a^2*\cos(6*d*x + 6*c) + 6*\sqrt{2}*a^2*\cos(4*d*x + \\
& 4*c) + 4*\sqrt{2}*a^2*\cos(2*d*x + 2*c) + \sqrt{2}*a^2)*\sin(11/4*\arctan2(\sin(\\
& 2*d*x + 2*c), \cos(2*d*x + 2*c))) + 2060*(\sqrt{2}*a^2*\cos(8*d*x + 8*c) + 4*s \\
& \sqrt{2}*a^2*\cos(6*d*x + 6*c) + 6*\sqrt{2}*a^2*\cos(4*d*x + 4*c) + 4*\sqrt{2}*a^ \\
& 2*\cos(2*d*x + 2*c) + \sqrt{2}*a^2)*\sin(9/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d \\
& *x + 2*c))) - 2060*(\sqrt{2}*a^2*\cos(8*d*x + 8*c) + 4*\sqrt{2}*a^2*\cos(6*d*x \\
& + 6*c) + 6*\sqrt{2}*a^2*\cos(4*d*x + 4*c) + 4*\sqrt{2}*a^2*\cos(2*d*x + 2*c) + \\
& \sqrt{2}*a^2)*\sin(7/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 6204*(s \\
& \sqrt{2}*a^2*\cos(8*d*x + 8*c) + 4*\sqrt{2}*a^2*\cos(6*d*x + 6*c) + 6*\sqrt{2}*a^ \\
& 2*\cos(4*d*x + 4*c) + 4*\sqrt{2}*a^2*\cos(2*d*x + 2*c) + \sqrt{2}*a^2)*\sin(5/4* \\
& \arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 652*(\sqrt{2}*a^2*\cos(8*d*x + \\
& 8*c) + 4*\sqrt{2}*a^2*\cos(6*d*x + 6*c) + 6*\sqrt{2}*a^2*\cos(4*d*x + 4*c) + 4 \\
& *\sqrt{2}*a^2*\cos(2*d*x + 2*c) + \sqrt{2}*a^2)*\sin(3/4*\arctan2(\sin(2*d*x + 2* \\
& c), \cos(2*d*x + 2*c))) + 1956*(\sqrt{2}*a^2*\cos(8*d*x + 8*c) + 4*\sqrt{2}*a^2 \\
& *\cos(6*d*x + 6*c) + 6*\sqrt{2}*a^2*\cos(4*d*x + 4*c) + 4*\sqrt{2}*a^2*\cos(2*d* \\
& x + 2*c) + \sqrt{2}*a^2)*\sin(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)) \\
&))*A*\sqrt{a}/(2*(4*\cos(6*d*x + 6*c) + 6*\cos(4*d*x + 4*c) + 4*\cos(2*d*x + 2* \\
& c) + 1)*\cos(8*d*x + 8*c) + \cos(8*d*x + 8*c)^2 + 8*(6*\cos(4*d*x + 4*c) + 4*c \\
& \cos(2*d*x + 2*c) + 1)*\cos(6*d*x + 6*c) + 16*\cos(6*d*x + 6*c)^2 + 12*(4*\cos(2 \\
& *d*x + 2*c) + 1)*\cos(4*d*x + 4*c) + 36*\cos(4*d*x + 4*c)^2 + 16*\cos(2*d*x + \\
& 2*c)^2 + 4*(2*\sin(6*d*x + 6*c) + 3*\sin(4*d*x + 4*c) + 2*\sin(2*d*x + 2*c))*s \\
& \sin(8*d*x + 8*c) + \sin(8*d*x + 8*c)^2 + 16*(3*\sin(4*d*x + 4*c) + 2*\sin(2*d*x \\
& + 2*c))*\sin(6*d*x + 6*c) + 16*\sin(6*d*x + 6*c)^2 + 36*\sin(4*d*x + 4*c)^2 +
\end{aligned}$$

$$\begin{aligned}
& 48\sin(4d*x + 4c)\sin(2d*x + 2c) + 16\sin(2d*x + 2c)^2 + 8\cos(2d*x \\
& + 2c) + 1) + (16980*(\sqrt{2})a^2\sin(10d*x + 10c) + 5\sqrt{2})a^2\sin(8 \\
& *d*x + 8c) + 10\sqrt{2})a^2\sin(6d*x + 6c) + 10\sqrt{2})a^2\sin(4d*x + \\
& 4c) + 5\sqrt{2})a^2\sin(2d*x + 2c))\cos(19/4*\arctan2(\sin(2d*x + 2c), \cos(2d*x + 2c))) + 5660*(\sqrt{2})a^2\sin(10d*x + 10c) + 5\sqrt{2})a^2\sin(8d*x + 8c) + 10\sqrt{2})a^2\sin(6d*x + 6c) + 10\sqrt{2})a^2\sin(4d*x + 4c) + 5\sqrt{2})a^2\sin(2d*x + 2c))\cos(17/4*\arctan2(\sin(2d*x + 2c), \cos(2d*x + 2c))) + 81504*(\sqrt{2})a^2\sin(10d*x + 10c) + 5\sqrt{2})a^2\sin(8d*x + 8c) + 10\sqrt{2})a^2\sin(6d*x + 6c) + 10\sqrt{2})a^2\sin(4d*x + 4c) + 5\sqrt{2})a^2\sin(2d*x + 2c))\cos(15/4*\arctan2(\sin(2d*x + 2c), \cos(2d*x + 2c))) + 8320*(\sqrt{2})a^2\sin(10d*x + 10c) + 5\sqrt{2})a^2\sin(8d*x + 8c) + 10\sqrt{2})a^2\sin(6d*x + 6c) + 10\sqrt{2})a^2\sin(4d*x + 4c) + 5\sqrt{2})a^2\sin(2d*x + 2c))\cos(13/4*\arctan2(\sin(2d*x + 2c), \cos(2d*x + 2c))) + 86440*(\sqrt{2})a^2\sin(10d*x + 10c) + 5\sqrt{2})a^2\sin(8d*x + 8c) + 10\sqrt{2})a^2\sin(6d*x + 6c) + 10\sqrt{2})a^2\sin(4d*x + 4c) + 5\sqrt{2})a^2\sin(2d*x + 2c))\cos(11/4*\arctan2(\sin(2d*x + 2c), \cos(2d*x + 2c))) - 86440*(\sqrt{2})a^2\sin(10d*x + 10c) + 5\sqrt{2})a^2\sin(8d*x + 8c) + 10\sqrt{2})a^2\sin(6d*x + 6c) + 10\sqrt{2})a^2\sin(4d*x + 4c) + 5\sqrt{2})a^2\sin(2d*x + 2c))\cos(9/4*\arctan2(\sin(2d*x + 2c), \cos(2d*x + 2c))) - 8320*(\sqrt{2})a^2\sin(10d*x + 10c) + 5\sqrt{2})a^2\sin(8d*x + 8c) + 10\sqrt{2})a^2\sin(6d*x + 6c) + 10\sqrt{2})a^2\sin(4d*x + 4c) + 5\sqrt{2})a^2\sin(2d*x + 2c))\cos(7/4*\arctan2(\sin(2d*x + 2c), \cos(2d*x + 2c))) - 81504*(\sqrt{2})a^2\sin(10d*x + 10c) + 5\sqrt{2})a^2\sin(8d*x + 8c) + 10\sqrt{2})a^2\sin(6d*x + 6c) + 10\sqrt{2})a^2\sin(4d*x + 4c) + 5\sqrt{2})a^2\sin(2d*x + 2c))\cos(5/4*\arctan2(\sin(2d*x + 2c), \cos(2d*x + 2c))) - 5660*(\sqrt{2})a^2\sin(10d*x + 10c) + 5\sqrt{2})a^2\sin(8d*x + 8c) + 10\sqrt{2})a^2\sin(6d*x + 6c) + 10\sqrt{2})a^2\sin(4d*x + 4c) + 5\sqrt{2})a^2\sin(2d*x + 2c))\cos(3/4*\arctan2(\sin(2d*x + 2c), \cos(2d*x + 2c))) - 16980*(\sqrt{2})a^2\sin(10d*x + 10c) + 5\sqrt{2})a^2\sin(8d*x + 8c) + 10\sqrt{2})a^2\sin(6d*x + 6c) + 10\sqrt{2})a^2\sin(4d*x + 4c) + 5\sqrt{2})a^2\sin(2d*x + 2c))\cos(1/4*\arctan2(\sin(2d*x + 2c), \cos(2d*x + 2c))) - 4245*(a^2\cos(10d*x + 10c)^2 + 25a^2\cos(8d*x + 8c)^2 + 100a^2\cos(6d*x + 6c)^2 + 100a^2\cos(4d*x + 4c)^2 + 25a^2\cos(2d*x + 2c)^2 + a^2\sin(10d*x + 10c)^2 + 25a^2\sin(8d*x + 8c)^2 + 100a^2\sin(6d*x + 6c)^2 + 100a^2\sin(4d*x + 4c)^2 + 100a^2\sin(4d*x + 4c)\sin(2d*x + 2c) + 25a^2\sin(2d*x + 2c)^2 + 10a^2\cos(2d*x + 2c) + a^2 + 2*(5a^2\cos(8d*x + 8c) + 10a^2\cos(6d*x + 6c) + 10a^2\cos(4d*x + 4c) + 5a^2\cos(2d*x + 2c) + a^2)\cos(10d*x + 10c) + 10*(10a^2\cos(6d*x + 6c) + 10a^2\cos(4d*x + 4c) + 5a^2\cos(2d*x + 2c) + a^2)\cos(8d*x + 8c) + 20*(10a^2\cos(4d*x + 4c) + 5a^2\cos(2d*x + 2c) + a^2)\cos(6d*x + 6c) + 20*(5a^2\cos(2d*x + 2c) + a^2)\cos(4d*x + 4c) + 10*(a^2\sin(8d*x + 8c) + 2a^2\sin(6d*x + 6c) + 2a^2\sin(4d*x + 4c) + a^2\sin(2d*x + 2c))\sin(10d*x + 10c) + 50*(2a^2\sin(6d*x + 6c) + 2a^2\sin(4d*x + 4c) + a^2\sin(2d*x + 2c))\sin(8d*x + 8c) + 100*(2a^2\sin(4d*x + 4c) + a^2\sin(2d*x + 2c))\sin(
\end{aligned}$$

$$\begin{aligned}
& 6*d*x + 6*c)) * \log(2*\cos(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 \\
& + 2*\sin(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + 2*\sqrt{2}*\cos(\\
& 1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 2*\sqrt{2}*\sin(1/4*\arctan \\
& 2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 2) + 4245*(a^2*\cos(10*d*x + 10*c)^ \\
& 2 + 25*a^2*\cos(8*d*x + 8*c)^2 + 100*a^2*\cos(6*d*x + 6*c)^2 + 100*a^2*\cos(4* \\
& d*x + 4*c)^2 + 25*a^2*\cos(2*d*x + 2*c)^2 + a^2*\sin(10*d*x + 10*c)^2 + 25*a^ \\
& 2*\sin(8*d*x + 8*c)^2 + 100*a^2*\sin(6*d*x + 6*c)^2 + 100*a^2*\sin(4*d*x + 4*c \\
&)^2 + 100*a^2*\sin(4*d*x + 4*c)*\sin(2*d*x + 2*c) + 25*a^2*\sin(2*d*x + 2*c)^2 \\
& + 10*a^2*\cos(2*d*x + 2*c) + a^2 + 2*(5*a^2*\cos(8*d*x + 8*c) + 10*a^2*\cos(6 \\
& *d*x + 6*c) + 10*a^2*\cos(4*d*x + 4*c) + 5*a^2*\cos(2*d*x + 2*c) + a^2)*\cos(1 \\
& 0*d*x + 10*c) + 10*(10*a^2*\cos(6*d*x + 6*c) + 10*a^2*\cos(4*d*x + 4*c) + 5*a \\
& ^2*\cos(2*d*x + 2*c) + a^2)*\cos(8*d*x + 8*c) + 20*(10*a^2*\cos(4*d*x + 4*c) + \\
& 5*a^2*\cos(2*d*x + 2*c) + a^2)*\cos(6*d*x + 6*c) + 20*(5*a^2*\cos(2*d*x + 2*c \\
&) + a^2)*\cos(4*d*x + 4*c) + 10*(a^2*\sin(8*d*x + 8*c) + 2*a^2*\sin(6*d*x + 6* \\
& c) + 2*a^2*\sin(4*d*x + 4*c) + a^2*\sin(2*d*x + 2*c))*\sin(10*d*x + 10*c) + 50 \\
& *(2*a^2*\sin(6*d*x + 6*c) + 2*a^2*\sin(4*d*x + 4*c) + a^2*\sin(2*d*x + 2*c))*\sin(8 \\
& *d*x + 8*c) + 100*(2*a^2*\sin(4*d*x + 4*c) + a^2*\sin(2*d*x + 2*c))*\sin(6 \\
& *d*x + 6*c)) * \log(2*\cos(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + \\
& 2*\sin(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + 2*\sqrt{2}*\cos(1 \\
& /4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) - 2*\sqrt{2}*\sin(1/4*\arctan2 \\
& (\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 2) - 4245*(a^2*\cos(10*d*x + 10*c)^2 \\
& + 25*a^2*\cos(8*d*x + 8*c)^2 + 100*a^2*\cos(6*d*x + 6*c)^2 + 100*a^2*\cos(4*d \\
& *x + 4*c)^2 + 25*a^2*\cos(2*d*x + 2*c)^2 + a^2*\sin(10*d*x + 10*c)^2 + 25*a^2 \\
& *\sin(8*d*x + 8*c)^2 + 100*a^2*\sin(6*d*x + 6*c)^2 + 100*a^2*\sin(4*d*x + 4*c) \\
& ^2 + 100*a^2*\sin(4*d*x + 4*c)*\sin(2*d*x + 2*c) + 25*a^2*\sin(2*d*x + 2*c)^2 \\
& + 10*a^2*\cos(2*d*x + 2*c) + a^2 + 2*(5*a^2*\cos(8*d*x + 8*c) + 10*a^2*\cos(6 \\
& *d*x + 6*c) + 10*a^2*\cos(4*d*x + 4*c) + 5*a^2*\cos(2*d*x + 2*c) + a^2)*\cos(10 \\
& *d*x + 10*c) + 10*(10*a^2*\cos(6*d*x + 6*c) + 10*a^2*\cos(4*d*x + 4*c) + 5*a^ \\
& 2*\cos(2*d*x + 2*c) + a^2)*\cos(8*d*x + 8*c) + 20*(10*a^2*\cos(4*d*x + 4*c) + \\
& 5*a^2*\cos(2*d*x + 2*c) + a^2)*\cos(6*d*x + 6*c) + 20*(5*a^2*\cos(2*d*x + 2*c \\
&) + a^2)*\cos(4*d*x + 4*c) + 10*(a^2*\sin(8*d*x + 8*c) + 2*a^2*\sin(6*d*x + 6*c \\
&) + 2*a^2*\sin(4*d*x + 4*c) + a^2*\sin(2*d*x + 2*c))*\sin(10*d*x + 10*c) + 50 \\
& *(2*a^2*\sin(6*d*x + 6*c) + 2*a^2*\sin(4*d*x + 4*c) + a^2*\sin(2*d*x + 2*c))*\sin \\
& (8*d*x + 8*c) + 100*(2*a^2*\sin(4*d*x + 4*c) + a^2*\sin(2*d*x + 2*c))*\sin(6 \\
& *d*x + 6*c)) * \log(2*\cos(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + \\
& 2*\sin(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 - 2*\sqrt{2}*\cos(1/ \\
& 4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 2*\sqrt{2}*\sin(1/4*\arctan2(\\
& \sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 2) + 4245*(a^2*\cos(10*d*x + 10*c)^2 \\
& + 25*a^2*\cos(8*d*x + 8*c)^2 + 100*a^2*\cos(6*d*x + 6*c)^2 + 100*a^2*\cos(4*d* \\
& x + 4*c)^2 + 25*a^2*\cos(2*d*x + 2*c)^2 + a^2*\sin(10*d*x + 10*c)^2 + 25*a^2* \\
& \sin(8*d*x + 8*c)^2 + 100*a^2*\sin(6*d*x + 6*c)^2 + 100*a^2*\sin(4*d*x + 4*c)^ \\
& 2 + 100*a^2*\sin(4*d*x + 4*c)*\sin(2*d*x + 2*c) + 25*a^2*\sin(2*d*x + 2*c)^2 + \\
& 10*a^2*\cos(2*d*x + 2*c) + a^2 + 2*(5*a^2*\cos(8*d*x + 8*c) + 10*a^2*\cos(6*d \\
& *x + 6*c) + 10*a^2*\cos(4*d*x + 4*c) + 5*a^2*\cos(2*d*x + 2*c) + a^2)*\cos(10* \\
& d*x + 10*c) + 10*(10*a^2*\cos(6*d*x + 6*c) + 10*a^2*\cos(4*d*x + 4*c) + 5*a^2
\end{aligned}$$

$$\begin{aligned}
& * \cos(2*d*x + 2*c) + a^2 * \cos(8*d*x + 8*c) + 20 * (10*a^2 * \cos(4*d*x + 4*c) + 5 \\
& * a^2 * \cos(2*d*x + 2*c) + a^2 * \cos(6*d*x + 6*c) + 20 * (5*a^2 * \cos(2*d*x + 2*c) \\
& + a^2 * \cos(4*d*x + 4*c) + 10 * (a^2 * \sin(8*d*x + 8*c) + 2*a^2 * \sin(6*d*x + 6*c) \\
& + 2*a^2 * \sin(4*d*x + 4*c) + a^2 * \sin(2*d*x + 2*c)) * \sin(10*d*x + 10*c) + 50 * (\\
& 2*a^2 * \sin(6*d*x + 6*c) + 2*a^2 * \sin(4*d*x + 4*c) + a^2 * \sin(2*d*x + 2*c)) * \sin \\
& (8*d*x + 8*c) + 100 * (2*a^2 * \sin(4*d*x + 4*c) + a^2 * \sin(2*d*x + 2*c)) * \sin(6*d \\
& * x + 6*c)) * \log(2 * \cos(1/4 * \arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + 2 \\
& * \sin(1/4 * \arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 - 2 * \sqrt{2} * \cos(1/4 \\
& * \arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) - 2 * \sqrt{2} * \sin(1/4 * \arctan2(s \\
& \sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 2) - 16980 * (\sqrt{2} * a^2 * \cos(10*d*x + \\
& 10*c) + 5 * \sqrt{2} * a^2 * \cos(8*d*x + 8*c) + 10 * \sqrt{2} * a^2 * \cos(6*d*x + 6*c) + \\
& 10 * \sqrt{2} * a^2 * \cos(4*d*x + 4*c) + 5 * \sqrt{2} * a^2 * \cos(2*d*x + 2*c) + \sqrt{2} * \\
& a^2 * \sin(19/4 * \arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))) - 5660 * (\sqrt{2} * \\
& a^2 * \cos(10*d*x + 10*c) + 5 * \sqrt{2} * a^2 * \cos(8*d*x + 8*c) + 10 * \sqrt{2} * a^2 * \cos \\
& (6*d*x + 6*c) + 10 * \sqrt{2} * a^2 * \cos(4*d*x + 4*c) + 5 * \sqrt{2} * a^2 * \cos(2*d*x \\
& + 2*c) + \sqrt{2} * a^2 * \sin(17/4 * \arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))) \\
& - 81504 * (\sqrt{2} * a^2 * \cos(10*d*x + 10*c) + 5 * \sqrt{2} * a^2 * \cos(8*d*x + 8*c) + \\
& 10 * \sqrt{2} * a^2 * \cos(6*d*x + 6*c) + 10 * \sqrt{2} * a^2 * \cos(4*d*x + 4*c) + 5 * \sqrt{2} \\
& (2) * a^2 * \cos(2*d*x + 2*c) + \sqrt{2} * a^2 * \sin(15/4 * \arctan2(\sin(2*d*x + 2*c), \\
& \cos(2*d*x + 2*c)))) - 8320 * (\sqrt{2} * a^2 * \cos(10*d*x + 10*c) + 5 * \sqrt{2} * a^2 * \cos \\
& (8*d*x + 8*c) + 10 * \sqrt{2} * a^2 * \cos(6*d*x + 6*c) + 10 * \sqrt{2} * a^2 * \cos(4*d*x \\
& + 4*c) + 5 * \sqrt{2} * a^2 * \cos(2*d*x + 2*c) + \sqrt{2} * a^2 * \sin(13/4 * \arctan2(s \\
& \sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))) - 86440 * (\sqrt{2} * a^2 * \cos(10*d*x + 10*c) \\
& + 5 * \sqrt{2} * a^2 * \cos(8*d*x + 8*c) + 10 * \sqrt{2} * a^2 * \cos(6*d*x + 6*c) + 10 * \sqrt{2} \\
& (2) * a^2 * \cos(4*d*x + 4*c) + 5 * \sqrt{2} * a^2 * \cos(2*d*x + 2*c) + \sqrt{2} * a^2 * \\
& \sin(11/4 * \arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))) + 86440 * (\sqrt{2} * a^2 * \\
& \cos(10*d*x + 10*c) + 5 * \sqrt{2} * a^2 * \cos(8*d*x + 8*c) + 10 * \sqrt{2} * a^2 * \cos(6* \\
& d*x + 6*c) + 10 * \sqrt{2} * a^2 * \cos(4*d*x + 4*c) + 5 * \sqrt{2} * a^2 * \cos(2*d*x + 2* \\
& c) + \sqrt{2} * a^2 * \sin(9/4 * \arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))) + 83 \\
& 20 * (\sqrt{2} * a^2 * \cos(10*d*x + 10*c) + 5 * \sqrt{2} * a^2 * \cos(8*d*x + 8*c) + 10 * \sqrt{2} \\
& (2) * a^2 * \cos(6*d*x + 6*c) + 10 * \sqrt{2} * a^2 * \cos(4*d*x + 4*c) + 5 * \sqrt{2} * a^2 \\
& * \cos(2*d*x + 2*c) + \sqrt{2} * a^2 * \sin(7/4 * \arctan2(\sin(2*d*x + 2*c), \cos(2*d \\
& * x + 2*c)))) + 81504 * (\sqrt{2} * a^2 * \cos(10*d*x + 10*c) + 5 * \sqrt{2} * a^2 * \cos(8*d \\
& * x + 8*c) + 10 * \sqrt{2} * a^2 * \cos(6*d*x + 6*c) + 10 * \sqrt{2} * a^2 * \cos(4*d*x + 4* \\
& c) + 5 * \sqrt{2} * a^2 * \cos(2*d*x + 2*c) + \sqrt{2} * a^2 * \sin(5/4 * \arctan2(\sin(2*d* \\
& x + 2*c), \cos(2*d*x + 2*c)))) + 5660 * (\sqrt{2} * a^2 * \cos(10*d*x + 10*c) + 5 * \sqrt{2} \\
& (2) * a^2 * \cos(8*d*x + 8*c) + 10 * \sqrt{2} * a^2 * \cos(6*d*x + 6*c) + 10 * \sqrt{2} * a^2 \\
& * \cos(4*d*x + 4*c) + 5 * \sqrt{2} * a^2 * \cos(2*d*x + 2*c) + \sqrt{2} * a^2 * \sin(3/4 * \\
& \arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))) + 16980 * (\sqrt{2} * a^2 * \cos(10*d* \\
& x + 10*c) + 5 * \sqrt{2} * a^2 * \cos(8*d*x + 8*c) + 10 * \sqrt{2} * a^2 * \cos(6*d*x + 6*c \\
&) + 10 * \sqrt{2} * a^2 * \cos(4*d*x + 4*c) + 5 * \sqrt{2} * a^2 * \cos(2*d*x + 2*c) + \sqrt{2} \\
& (2) * a^2 * \sin(1/4 * \arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))) * B * \sqrt{a} / (2 \\
& * (5 * \cos(8*d*x + 8*c) + 10 * \cos(6*d*x + 6*c) + 10 * \cos(4*d*x + 4*c) + 5 * \cos(2* \\
& d*x + 2*c) + 1) * \cos(10*d*x + 10*c) + \cos(10*d*x + 10*c)^2 + 10 * (10 * \cos(6*d* \\
& x + 6*c) + 10 * \cos(4*d*x + 4*c) + 5 * \cos(2*d*x + 2*c) + 1) * \cos(8*d*x + 8*c) +
\end{aligned}$$

```

25*cos(8*d*x + 8*c)^2 + 20*(10*cos(4*d*x + 4*c) + 5*cos(2*d*x + 2*c) + 1)*
cos(6*d*x + 6*c) + 100*cos(6*d*x + 6*c)^2 + 20*(5*cos(2*d*x + 2*c) + 1)*cos
(4*d*x + 4*c) + 100*cos(4*d*x + 4*c)^2 + 25*cos(2*d*x + 2*c)^2 + 10*(sin(8*
d*x + 8*c) + 2*sin(6*d*x + 6*c) + 2*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*si
n(10*d*x + 10*c) + sin(10*d*x + 10*c)^2 + 50*(2*sin(6*d*x + 6*c) + 2*sin(4*
d*x + 4*c) + sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 25*sin(8*d*x + 8*c)^2 + 1
00*(2*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 100*sin(6*d*x
+ 6*c)^2 + 100*sin(4*d*x + 4*c)^2 + 100*sin(4*d*x + 4*c)*sin(2*d*x + 2*c)
+ 25*sin(2*d*x + 2*c)^2 + 10*cos(2*d*x + 2*c) + 1))/d

```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(a + \frac{a}{\cos(c+dx)}\right)^{5/2}}{\cos(c+dx)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2))/cos(c + d*x)^(5/2), x)
```

```
[Out] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2))/cos(c + d*x)^(5/2), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c))/cos(d*x+c)**(5/2), x)
```

```
[Out] Timed out
```

$$3.541 \quad \int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B \sec(c+dx))}{\sqrt{a+a \sec(c+dx)}} dx$$

Optimal. Leaf size=250

$$\frac{2(A-7B) \sin(c+dx) \cos^{\frac{3}{2}}(c+dx)}{35d\sqrt{a \sec(c+dx)+a}} + \frac{2(31A-7B) \sin(c+dx) \sqrt{\cos(c+dx)}}{105d\sqrt{a \sec(c+dx)+a}} - \frac{2(43A-91B) \sin(c+dx)}{105d\sqrt{\cos(c+dx)} \sqrt{a \sec(c+dx)}}$$

[Out] (A-B)*arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d/a^(1/2)-2/35*(A-7*B)*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+2/7*A*cos(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)-2/105*(43*A-91*B)*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+2/105*(31*A-7*B)*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^(1/2)

Rubi [A] time = 0.84, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2955, 4022, 4013, 3808, 206}

$$\frac{2(A-7B) \sin(c+dx) \cos^{\frac{3}{2}}(c+dx)}{35d\sqrt{a \sec(c+dx)+a}} + \frac{2(31A-7B) \sin(c+dx) \sqrt{\cos(c+dx)}}{105d\sqrt{a \sec(c+dx)+a}} - \frac{2(43A-91B) \sin(c+dx)}{105d\sqrt{\cos(c+dx)} \sqrt{a \sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^(7/2)*(A + B*Sec[c + d*x]))/Sqrt[a + a*Sec[c + d*x]],x]

[Out] (Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) - (2*(43*A - 91*B)*Sin[c + d*x])/(105*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (2*(31*A - 7*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(105*d*Sqrt[a + a*Sec[c + d*x]]) - (2*(A - 7*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(35*d*Sqrt[a + a*Sec[c + d*x]]) + (2*A*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d*Sqrt[a + a*Sec[c + d*x]])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2955

Int[((a_.) + csc[(e_.) + (f_.)*(x_)])*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] := Dis

$t[(g*\text{Csc}[e + f*x])^p*(g*\text{Sin}[e + f*x])^p, \text{Int}[(a + b*\text{Csc}[e + f*x])^m*(c + d*\text{Csc}[e + f*x])^n]/(g*\text{Csc}[e + f*x])^p, x], x] /;$ FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && IntegerQ[n])

Rule 3808

$\text{Int}[\text{Sqrt}[\text{csc}[(e_.) + (f_.)*(x_.)]*(d_.)]/\text{Sqrt}[\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] := \text{Dist}[(-2*b*d)/(a*f), \text{Subst}[\text{Int}[1/(2*b - d*x^2)], x], x, (b*\text{Cot}[e + f*x])/(\text{Sqrt}[a + b*\text{Csc}[e + f*x]]*\text{Sqrt}[d*\text{Csc}[e + f*x]])], x] /;$ FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 4013

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(d_.))^n*(\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m*(\text{csc}[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := \text{Simp}[(A*\text{Cot}[e + f*x]*(a + b*\text{Csc}[e + f*x])^m*(d*\text{Csc}[e + f*x])^n)/(f*n), x] - \text{Dist}[(a*A*m - b*B*n)/(b*d*n), \text{Int}[(a + b*\text{Csc}[e + f*x])^m*(d*\text{Csc}[e + f*x])^{n+1}], x], x] /;$ FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && !LeQ[m, -1]

Rule 4022

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(d_.))^n*(\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m*(\text{csc}[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := \text{Simp}[(A*\text{Cot}[e + f*x]*(a + b*\text{Csc}[e + f*x])^m*(d*\text{Csc}[e + f*x])^n)/(f*n), x] - \text{Dist}[1/(b*d*n), \text{Int}[(a + b*\text{Csc}[e + f*x])^m*(d*\text{Csc}[e + f*x])^{n+1}*\text{Simp}[a*A*m - b*B*n - A*b*(m + n + 1)*\text{Csc}[e + f*x], x], x], x] /;$ FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]

Rubi steps

$$\begin{aligned}
\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\sec(c+dx))}{\sqrt{a+a\sec(c+dx)}} dx &= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \frac{A+B\sec(c+dx)}{\sec^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} dx \\
&= \frac{2A\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d\sqrt{a+a\sec(c+dx)}} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \frac{\frac{1}{2}a\cos^{\frac{1}{2}}(c+dx)}{\sec^{\frac{5}{2}}(c+dx)} dx}{7a} \\
&= -\frac{2(A-7B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{35d\sqrt{a+a\sec(c+dx)}} + \frac{2A\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d\sqrt{a+a\sec(c+dx)}} \\
&= \frac{2(31A-7B)\sqrt{\cos(c+dx)}\sin(c+dx)}{105d\sqrt{a+a\sec(c+dx)}} - \frac{2(A-7B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{35d\sqrt{a+a\sec(c+dx)}} \\
&= -\frac{2(43A-91B)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{2(31A-7B)\sqrt{\cos(c+dx)}}{105d\sqrt{a+a\sec(c+dx)}} \\
&= -\frac{2(43A-91B)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{2(31A-7B)\sqrt{\cos(c+dx)}}{105d\sqrt{a+a\sec(c+dx)}} \\
&= \frac{\sqrt{2}(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}d}
\end{aligned}$$

Mathematica [A] time = 1.38, size = 170, normalized size = 0.68

$$\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\left(2\sqrt{1-\sec(c+dx)}\left((91B-43A)\sec^3(c+dx)+(31A-7B)\sec^2(c+dx)-3(A-7B)\right)\right)}{105d\sqrt{1-\sec(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]^(7/2)*(A + B*Sec[c + d*x]))/Sqrt[a + a*Sec[c + d*x]], x]

[Out] (Cos[c + d*x]^(5/2)*(-105*Sqrt[2]*(A - B)*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]])*Sec[c + d*x]^(7/2) + 2*Sqrt[1 - Sec[c + d*x]]*(15*A - 3*(A - 7*B)*Sec[c + d*x] + (31*A - 7*B)*Sec[c + d*x]^2 + (-43*A + 91*B)*Sec[c + d*x]^3)*Sin[c + d*x])/(105*d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])

fricas [A] time = 0.49, size = 400, normalized size = 1.60

$$\frac{4 \left(15 A \cos(dx + c)^3 - 3(A - 7B) \cos(dx + c)^2 + (31A - 7B) \cos(dx + c) - 43A + 91B \right) \sqrt{\frac{a \cos(dx + c) + a}{\cos(dx + c)}} \sqrt{\cos(dx + c)}}{210(ad \cos(dx + c) + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")

[Out] [1/210*(4*(15*A*cos(d*x + c)^3 - 3*(A - 7*B)*cos(d*x + c)^2 + (31*A - 7*B)*cos(d*x + c) - 43*A + 91*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 105*sqrt(2)*((A - B)*a*cos(d*x + c) + (A - B)*a)*log(-(cos(d*x + c)^2 + 2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/sqrt(a))/(a*d*cos(d*x + c) + a*d), -1/105*(105*sqrt(2)*((A - B)*a*cos(d*x + c) + (A - B)*a)*sqrt(-1/a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*sqrt(cos(d*x + c))/sin(d*x + c)) - 2*(15*A*cos(d*x + c)^3 - 3*(A - 7*B)*cos(d*x + c)^2 + (31*A - 7*B)*cos(d*x + c) - 43*A + 91*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \cos(dx + c)^{\frac{7}{2}}}{\sqrt{a \sec(dx + c) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(7/2)/sqrt(a*sec(d*x + c) + a), x)

maple [A] time = 2.07, size = 217, normalized size = 0.87

$$\left(\sqrt{\cos(dx+c)}\right) \sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}} \left(30A(\cos^4(dx+c)) + 105 \arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)\right) \sqrt{-\frac{2}{1+\cos(dx+c)}} A \sin$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x)

[Out] $-1/105/d*\cos(d*x+c)^{(1/2)}*(a*(1+\cos(d*x+c))/\cos(d*x+c))^{(1/2)}*(30*A*\cos(d*x+c)^4+105*\arctan(1/2*\sin(d*x+c)*(-2/(1+\cos(d*x+c)))^{(1/2)}*(-2/(1+\cos(d*x+c))))^{(1/2)}*A*\sin(d*x+c)-36*A*\cos(d*x+c)^3-105*\arctan(1/2*\sin(d*x+c)*(-2/(1+\cos(d*x+c)))^{(1/2)}*(-2/(1+\cos(d*x+c)))^{(1/2)})*B*\sin(d*x+c)+42*B*\cos(d*x+c)^3+68*A*\cos(d*x+c)^2-56*B*\cos(d*x+c)^2-148*A*\cos(d*x+c)+196*B*\cos(d*x+c)+86*A-182*B)/a/\sin(d*x+c)$

maxima [B] time = 0.81, size = 749, normalized size = 3.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")

[Out] $-1/840*(\sqrt{2}*(525*\cos(6/7*\arctan2(\sin(7/2*d*x + 7/2*c), \cos(7/2*d*x + 7/2*c))) * \sin(7/2*d*x + 7/2*c) - 175*\cos(4/7*\arctan2(\sin(7/2*d*x + 7/2*c), \cos(7/2*d*x + 7/2*c))) * \sin(7/2*d*x + 7/2*c) + 21*\cos(2/7*\arctan2(\sin(7/2*d*x + 7/2*c), \cos(7/2*d*x + 7/2*c))) * \sin(7/2*d*x + 7/2*c) - 525*\cos(7/2*d*x + 7/2*c) * \sin(6/7*\arctan2(\sin(7/2*d*x + 7/2*c), \cos(7/2*d*x + 7/2*c))) + 175*\cos(7/2*d*x + 7/2*c) * \sin(4/7*\arctan2(\sin(7/2*d*x + 7/2*c), \cos(7/2*d*x + 7/2*c)))) - 21*\cos(7/2*d*x + 7/2*c) * \sin(2/7*\arctan2(\sin(7/2*d*x + 7/2*c), \cos(7/2*d*x + 7/2*c))) - 420*\log(\cos(1/7*\arctan2(\sin(7/2*d*x + 7/2*c), \cos(7/2*d*x + 7/2*c)))^2 + \sin(1/7*\arctan2(\sin(7/2*d*x + 7/2*c), \cos(7/2*d*x + 7/2*c)))^2 + 2*\sin(1/7*\arctan2(\sin(7/2*d*x + 7/2*c), \cos(7/2*d*x + 7/2*c))) + 1) + 420*\log(\cos(1/7*\arctan2(\sin(7/2*d*x + 7/2*c), \cos(7/2*d*x + 7/2*c)))^2 + \sin(1/7*\arctan2(\sin(7/2*d*x + 7/2*c), \cos(7/2*d*x + 7/2*c)))^2 - 2*\sin(1/7*\arctan2(\sin(7/2*d*x + 7/2*c), \cos(7/2*d*x + 7/2*c))) + 1) - 30*\sin(7/2*d*x + 7/2*c) + 21*\sin(5/7*\arctan2(\sin(7/2*d*x + 7/2*c), \cos(7/2*d*x + 7/2*c))) - 175*\sin(3/7*\arctan2(\sin(7/2*d*x + 7/2*c), \cos(7/2*d*x + 7/2*c))) + 525*\sin(1/7*\arctan2(\sin(7/2*d*x + 7/2*c), \cos(7/2*d*x + 7/2*c)))) * A/\sqrt{a} + 28*(30*\sqrt{2}*\cos(5/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) * \sin(2*d*x + 2*c) - 3*(10*\sqrt{2}*\cos(2*d*x + 2*c) + \sqrt{2}) * \sin(5/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 15*\sqrt{2}*\log(\cos(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))$

$c), \cos(2dx + 2c))^{7/2} + \sin(1/4 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c)))^2 + 2 \sin(1/4 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c))) + 1) - 15 \sqrt{2} \log(\cos(1/4 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c)))^2 + \sin(1/4 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c)))^2 - 2 \sin(1/4 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c))) + 1) + 5 \sqrt{2} \sin(3/4 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c))) - 30 \sqrt{2} \sin(1/4 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c))) * B / \sqrt{a}) / d$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c + dx)^{7/2} \left(A + \frac{B}{\cos(c + dx)} \right)}{\sqrt{a + \frac{a}{\cos(c + dx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)^(7/2)*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^(1/2), x)

[Out] int((cos(c + d*x)^(7/2)*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^(1/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**(7/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(1/2), x)

[Out] Timed out

$$3.542 \quad \int \frac{\cos^2(c+dx)(A+B \sec(c+dx))}{\sqrt{a+a \sec(c+dx)}} dx$$

Optimal. Leaf size=207

$$\frac{2(A-5B) \sin(c+dx) \sqrt{\cos(c+dx)}}{15d \sqrt{a \sec(c+dx)+a}} + \frac{2(13A-5B) \sin(c+dx)}{15d \sqrt{\cos(c+dx)} \sqrt{a \sec(c+dx)+a}} - \frac{\sqrt{2}(A-B) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{15d \sqrt{a \sec(c+dx)+a}}$$

[Out] $-(A-B) \operatorname{arctanh}\left(\frac{1}{2} \sin(d*x+c) * a^{(1/2)} * \sec(d*x+c)^{(1/2)} * 2^{(1/2)} / (a+a*\sec(d*x+c))^{(1/2)}\right) * 2^{(1/2)} * \cos(d*x+c)^{(1/2)} * \sec(d*x+c)^{(1/2)} / d / a^{(1/2)} + 2/5 * A * \cos(d*x+c)^{(3/2)} * \sin(d*x+c) / d / (a+a*\sec(d*x+c))^{(1/2)} + 2/15 * (13*A-5*B) * \sin(d*x+c) / d / \cos(d*x+c)^{(1/2)} / (a+a*\sec(d*x+c))^{(1/2)} - 2/15 * (A-5*B) * \sin(d*x+c) * \cos(d*x+c)^{(1/2)} / d / (a+a*\sec(d*x+c))^{(1/2)}$

Rubi [A] time = 0.63, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2955, 4022, 4013, 3808, 206}

$$\frac{2(A-5B) \sin(c+dx) \sqrt{\cos(c+dx)}}{15d \sqrt{a \sec(c+dx)+a}} + \frac{2(13A-5B) \sin(c+dx)}{15d \sqrt{\cos(c+dx)} \sqrt{a \sec(c+dx)+a}} - \frac{\sqrt{2}(A-B) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{15d \sqrt{a \sec(c+dx)+a}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(\text{Cos}[c+d*x]^{(5/2)} * (A+B*\text{Sec}[c+d*x])) / \text{Sqrt}[a+a*\text{Sec}[c+d*x]], x]$

[Out] $-\left(\frac{\text{Sqrt}[2] * (A-B) * \text{ArcTanh}\left[\frac{\text{Sqrt}[a] * \text{Sqrt}[\text{Sec}[c+d*x]] * \text{Sin}[c+d*x]}{\text{Sqrt}[2] * \text{Sqrt}[a+a*\text{Sec}[c+d*x]]}\right] * \text{Sqrt}[\text{Cos}[c+d*x]] * \text{Sqrt}[\text{Sec}[c+d*x]]}{\text{Sqrt}[a]*d}\right) + \frac{2*(13*A-5*B)*\text{Sin}[c+d*x]}{(15*d*\text{Sqrt}[\text{Cos}[c+d*x]]*\text{Sqrt}[a+a*\text{Sec}[c+d*x]])} - \frac{2*(A-5*B)*\text{Sqrt}[\text{Cos}[c+d*x]]*\text{Sin}[c+d*x]}{(15*d*\text{Sqrt}[a+a*\text{Sec}[c+d*x]])} + \frac{2*A*\text{Cos}[c+d*x]^{(3/2)}*\text{Sin}[c+d*x]}{(5*d*\text{Sqrt}[a+a*\text{Sec}[c+d*x]])}$

Rule 206

$\text{Int}[(a_+ + (b_+)*(x_+)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}\left[\frac{1 * \text{ArcTanh}\left[\frac{\text{Rt}[-b, 2]*x}{\text{Rt}[a, 2]}\right]}{\text{Rt}[a, 2] * \text{Rt}[-b, 2]}\right], x] /;$ FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2955

$\text{Int}[(a_+ + \text{csc}(e_+ + (f_+)*(x_+)) * (b_+))^{(m_+)} * (\text{csc}(e_+ + (f_+)*(x_+)) * (d_+ + (c_+))^{(n_+)} * ((g_+)*\text{sin}[e_+ + (f_+)*(x_+)])^{(p_+)}, x_Symbol] \rightarrow \text{Dis t}[(g_+*\text{Csc}[e_+ + f_*x])^p * (g_+*\text{Sin}[e_+ + f_*x])^p, \text{Int}[(a_+ + b_+*\text{Csc}[e_+ + f_*x])^m * (c_+ + d_+)]]$

*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && IntegerQ[n])

Rule 3808

Int[Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x])*Sqrt[d*Csc[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 4013

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && !LeQ[m, -1]

Rule 4022

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]

Rubi steps

$$\begin{aligned}
\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{\sqrt{a+a\sec(c+dx)}} dx &= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \frac{A+B\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} dx \\
&= \frac{2A\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d\sqrt{a+a\sec(c+dx)}} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \frac{-\frac{1}{2}a(c+dx)}{\sec^{\frac{3}{2}}(c+dx)} dx}{5a} \\
&= -\frac{2(A-5B)\sqrt{\cos(c+dx)}\sin(c+dx)}{15d\sqrt{a+a\sec(c+dx)}} + \frac{2A\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d\sqrt{a+a\sec(c+dx)}} \\
&= \frac{2(13A-5B)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} - \frac{2(A-5B)\sqrt{\cos(c+dx)}\sin(c+dx)}{15d\sqrt{a+a\sec(c+dx)}} \\
&= \frac{2(13A-5B)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} - \frac{2(A-5B)\sqrt{\cos(c+dx)}\sin(c+dx)}{15d\sqrt{a+a\sec(c+dx)}} \\
&= -\frac{\sqrt{2}(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}d}
\end{aligned}$$

Mathematica [A] time = 0.80, size = 154, normalized size = 0.74

$$\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\left(2\sqrt{1-\sec(c+dx)}\left((13A-5B)\sec^2(c+dx)-(A-5B)\sec(c+dx)+3A\right)+15\sqrt{2}(A-5B)\sqrt{\cos(c+dx)}\sin(c+dx)\right)}{15d\sqrt{1-\sec(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/Sqrt[a + a*Sec[c + d*x]], x]

[Out] (Cos[c + d*x]^(3/2)*(15*Sqrt[2]*(A - B)*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]])*Sec[c + d*x]^(5/2) + 2*Sqrt[1 - Sec[c + d*x]]*(3*A - (A - 5*B)*Sec[c + d*x] + (13*A - 5*B)*Sec[c + d*x]^2))*Sin[c + d*x])/(15*d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])

fricas [A] time = 0.47, size = 368, normalized size = 1.78

$$\frac{4 \left(3 A \cos(dx + c)^2 - (A - 5B) \cos(dx + c) + 13A - 5B \right) \sqrt{\frac{a \cos(dx+c)+a}{\cos(dx+c)}} \sqrt{\cos(dx+c)} \sin(dx+c) - \frac{15 \sqrt{2} ((A-B) a \cos(dx+c) + (A-B) a)}{30 (ad \cos(dx+c) + ad)}}{30 (ad \cos(dx+c) + ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")

[Out] [1/30*(4*(3*A*cos(d*x + c)^2 - (A - 5*B)*cos(d*x + c) + 13*A - 5*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 15*sqrt(2)*((A - B)*a*cos(d*x + c) + (A - B)*a)*log(-(cos(d*x + c)^2 - 2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/sqrt(a))/(a*d*cos(d*x + c) + a*d), 1/15*(15*sqrt(2)*((A - B)*a*cos(d*x + c) + (A - B)*a)*sqrt(-1/a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*sqrt(cos(d*x + c))/sin(d*x + c)) + 2*(3*A*cos(d*x + c)^2 - (A - 5*B)*cos(d*x + c) + 13*A - 5*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \cos(dx + c)^{\frac{5}{2}}}{\sqrt{a \sec(dx + c) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(5/2)/sqrt(a*sec(d*x + c) + a), x)

maple [A] time = 2.35, size = 195, normalized size = 0.94

$$\left(\sqrt{\cos(dx+c)}\right) \sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}} \left(15 \arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right) \sqrt{-\frac{2}{1+\cos(dx+c)}} A \sin(dx+c) - 6A \left(\cos^3(dx+c) - \cos(dx+c)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x)`

[Out] `1/15/d*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(15*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*A*sin(d*x+c)-6*A*cos(d*x+c)^3-15*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*B*sin(d*x+c)+8*A*cos(d*x+c)^2-10*B*cos(d*x+c)^2-28*A*cos(d*x+c)+20*B*cos(d*x+c)+26*A-10*B)/a/sin(d*x+c)`

maxima [B] time = 0.75, size = 581, normalized size = 2.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")`

[Out] `1/60*(sqrt(2)*(60*cos(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) * sin(5/2*d*x + 5/2*c) - 5*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) * sin(5/2*d*x + 5/2*c) - 60*cos(5/2*d*x + 5/2*c) * sin(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 5*cos(5/2*d*x + 5/2*c) * sin(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) - 30*log(cos(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + 2*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 1) + 30*log(cos(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 - 2*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 1) + 6*sin(5/2*d*x + 5/2*c) - 5*sin(3/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 60*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) * A/sqrt(a) + 10*(3*sqrt(2)*log(cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 3*sqrt(2)*log(cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 2*sqrt(2)*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 6*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) * B/sqrt(a))/d`

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c + dx)^{5/2} \left(A + \frac{B}{\cos(c+dx)} \right)}{\sqrt{a + \frac{a}{\cos(c+dx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)^(5/2)*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^(1/2), x)

[Out] int((cos(c + d*x)^(5/2)*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^(1/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(1/2), x)

[Out] Timed out

$$3.543 \quad \int \frac{\cos^3(c+dx)(A+B \sec(c+dx))}{\sqrt{a+a \sec(c+dx)}} dx$$

Optimal. Leaf size=162

$$\frac{2(A-3B) \sin(c+dx)}{3d \sqrt{\cos(c+dx)} \sqrt{a \sec(c+dx)+a}} + \frac{\sqrt{2}(A-B) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \tanh^{-1}\left(\frac{\sqrt{a} \sin(c+dx) \sqrt{\sec(c+dx)}}{\sqrt{2} \sqrt{a \sec(c+dx)+a}}\right)}{\sqrt{a} d} + \dots$$

[Out] (A-B)*arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d/a^(1/2)-2/3*(A-3*B)*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+2/3*A*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^(1/2)

Rubi [A] time = 0.45, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2955, 4022, 4013, 3808, 206}

$$\frac{2(A-3B) \sin(c+dx)}{3d \sqrt{\cos(c+dx)} \sqrt{a \sec(c+dx)+a}} + \frac{\sqrt{2}(A-B) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \tanh^{-1}\left(\frac{\sqrt{a} \sin(c+dx) \sqrt{\sec(c+dx)}}{\sqrt{2} \sqrt{a \sec(c+dx)+a}}\right)}{\sqrt{a} d} + \dots$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/Sqrt[a + a*Sec[c + d*x]],x]

[Out] (Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) - (2*(A - 3*B)*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (2*A*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]])

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2955

Int[((a_) + csc[(e_) + (f_)*(x_)]*(b_.))^(m_.)*(csc[(e_) + (f_)*(x_)]*(d_) + (c_))^(n_.)*((g_)*sin[(e_) + (f_)*(x_)])^(p_), x_Symbol] := Dist[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && In

tegerQ[n])

Rule 3808

Int[Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 4013

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && !LeQ[m, -1]

Rule 4022

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]

Rubi steps

$$\begin{aligned}
\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{\sqrt{a+a\sec(c+dx)}} dx &= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \frac{A+B\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} dx \\
&= \frac{2A\sqrt{\cos(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \frac{-\frac{1}{2}a}{\sqrt{\sec(c+dx)}} dx}{3a} \\
&= -\frac{2(A-3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{2A\sqrt{\cos(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} \\
&= -\frac{2(A-3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{2A\sqrt{\cos(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} \\
&= \frac{\sqrt{2}(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}d}
\end{aligned}$$

Mathematica [A] time = 0.35, size = 124, normalized size = 0.77

$$\frac{\sin(c+dx)\left(2\sqrt{1-\sec(c+dx)}(A\cos(c+dx)-A+3B)-3\sqrt{2}(A-B)\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\right)}{3d\sqrt{\cos(c+dx)}-1\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/Sqrt[a + a*Sec[c + d*x]], x]

[Out] ((2*(-A + 3*B + A*Cos[c + d*x])*Sqrt[1 - Sec[c + d*x]] - 3*Sqrt[2]*(A - B)*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]])*Sqrt[Sec[c + d*x]])*Sin[c + d*x])/(3*d*Sqrt[-1 + Cos[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])

fricas [A] time = 0.47, size = 336, normalized size = 2.07

$$\left[\frac{4(A\cos(dx+c) - A + 3B)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c) - \frac{3\sqrt{2}((A-B)a\cos(dx+c)+(A-B)a)\log\left(\frac{\cos(dx+c)^2 + \dots}{\dots}\right)}{6(ad\cos(dx+c) + ad)}}{6(ad\cos(dx+c) + ad)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")

[Out] [1/6*(4*(A*cos(d*x + c) - A + 3*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 3*sqrt(2)*((A - B)*a*cos(d*x + c) + (A - B)*a)*log(-(cos(d*x + c)^2 + 2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/sqrt(a))/(a*d*cos(d*x + c) + a*d), -1/3*(3*sqrt(2)*((A - B)*a*cos(d*x + c) + (A - B)*a)*sqrt(-1/a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*sqrt(cos(d*x + c))/sin(d*x + c)) - 2*(A*cos(d*x + c) - A + 3*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \cos(dx + c)^{\frac{3}{2}}}{\sqrt{a \sec(dx + c) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(3/2)/sqrt(a*sec(d*x + c) + a), x)

maple [A] time = 2.55, size = 173, normalized size = 1.07

$$\frac{(\sqrt{\cos(dx + c)}) \sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}} \left(3 \arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right) \sqrt{-\frac{2}{1+\cos(dx+c)}} A \sin(dx + c) - 3 \arctan\left(\frac{\sin(dx+c)}{3da \sin}\right) \right)}{3da \sin}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x)

[Out] -1/3/d*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(3*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*A*sin(d*x+c) - 3*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*B*sin(d*x+c) + 2*A*cos(d*x+c)^2 - 4*A*cos(d*x+c) + 6*B*cos(d*x+c) + 2*A - 6*B)/a/sin(d*x+c)

maxima [B] time = 0.70, size = 478, normalized size = 2.95

$$\left(3 \sqrt{2} \cos\left(\frac{2}{3} \arctan\left(\sin\left(\frac{3}{2} dx + \frac{3}{2} c\right), \cos\left(\frac{3}{2} dx + \frac{3}{2} c\right)\right)\right) \sin\left(\frac{3}{2} dx + \frac{3}{2} c\right) - 3 \sqrt{2} \cos\left(\frac{3}{2} dx + \frac{3}{2} c\right) \sin\left(\frac{2}{3} \arctan\left(\sin\left(\frac{3}{2} dx + \frac{3}{2} c\right), \cos\left(\frac{3}{2} dx + \frac{3}{2} c\right)\right)\right) - 3 \sqrt{2} \log\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algorith="maxima")

[Out] -1/6*((3*sqrt(2)*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(3/2*d*x + 3/2*c) - 3*sqrt(2)*cos(3/2*d*x + 3/2*c)*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))) - 3*sqrt(2)*log(cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))^2 + sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) + 3*sqrt(2)*log(cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))^2 + sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))^2 - 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) - 2*sqrt(2)*sin(3/2*d*x + 3/2*c) + 3*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*A/sqrt(a) + 3*(sqrt(2)*log(cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))^2 + sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - sqrt(2)*log(cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))^2 + sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))^2 - 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*B/sqrt(a))/d

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c + dx)^{3/2} \left(A + \frac{B}{\cos(c + dx)} \right)}{\sqrt{a + \frac{a}{\cos(c + dx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)^(3/2)*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^(1/2),x)

[Out] int((cos(c + d*x)^(3/2)*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^(1/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

$$3.544 \quad \int \frac{\sqrt{\cos(c+dx)} (A+B \sec(c+dx))}{\sqrt{a+a \sec(c+dx)}} dx$$

Optimal. Leaf size=119

$$\frac{2A \sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a \sec(c+dx)+a}} - \frac{\sqrt{2}(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)} \tanh^{-1}\left(\frac{\sqrt{a} \sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a \sec(c+dx)+a}}\right)}{\sqrt{a}d}$$

[Out] $-(A-B)*\operatorname{arctanh}(1/2*\sin(d*x+c)*a^{(1/2)}*\sec(d*x+c)^{(1/2)}*2^{(1/2)}/(a+a*\sec(d*x+c))^{(1/2)})*2^{(1/2)}*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d/a^{(1/2)}+2*A*\sin(d*x+c)/d/\cos(d*x+c)^{(1/2)}/(a+a*\sec(d*x+c))^{(1/2)}$

Rubi [A] time = 0.29, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {2955, 4013, 3808, 206}

$$\frac{2A \sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a \sec(c+dx)+a}} - \frac{\sqrt{2}(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)} \tanh^{-1}\left(\frac{\sqrt{a} \sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a \sec(c+dx)+a}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(\operatorname{Sqrt}[\operatorname{Cos}[c+d*x]]*(A+B*\operatorname{Sec}[c+d*x]))/\operatorname{Sqrt}[a+a*\operatorname{Sec}[c+d*x]],x]$

[Out] $-\left(\left(\operatorname{Sqrt}[2]*(A-B)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Sqrt}[\operatorname{Sec}[c+d*x]]*\operatorname{Sin}[c+d*x])]/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a+a*\operatorname{Sec}[c+d*x]])\right)*\operatorname{Sqrt}[\operatorname{Cos}[c+d*x]]*\operatorname{Sqrt}[\operatorname{Sec}[c+d*x]]\right)/(\operatorname{Sqrt}[a]*d)+\left(2*A*\operatorname{Sin}[c+d*x]\right)/(d*\operatorname{Sqrt}[\operatorname{Cos}[c+d*x]]*\operatorname{Sqrt}[a+a*\operatorname{Sec}[c+d*x]])$

Rule 206

$\operatorname{Int}[(a_+ + (b_+)*(x_+)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1*\operatorname{ArcTanh}[(\operatorname{Rt}[-b, 2]*x)/\operatorname{Rt}[a, 2]])/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /; \operatorname{FreeQ}\{a, b\}, x] \ \&\& \operatorname{NegQ}[a/b] \ \&\& (\operatorname{Gt}Q[a, 0] \ || \ \operatorname{Lt}Q[b, 0])$

Rule 2955

$\operatorname{Int}[(a_+ + \operatorname{csc}[e_+ + (f_+)*(x_+)]*(b_+))^{(m_+)}*(\operatorname{csc}[e_+ + (f_+)*(x_+)]*(d_+ + (c_+))^{(n_+)}*((g_+)*\sin[e_+ + (f_+)*(x_+)])^{(p_+)}, x_Symbol] \rightarrow \operatorname{Dis}t[(g*\operatorname{Csc}[e+f*x])^p*(g*\operatorname{Sin}[e+f*x])^m, \operatorname{Int}[(a+b*\operatorname{Csc}[e+f*x])^m*(c+d*\operatorname{Csc}[e+f*x])^n]/(g*\operatorname{Csc}[e+f*x])^p, x], x] /; \operatorname{FreeQ}\{a, b, c, d, e, f, g, m, n, p\}, x] \ \&\& \operatorname{NeQ}[b*c - a*d, 0] \ \&\& \operatorname{IntegerQ}[p] \ \&\& \operatorname{IntegerQ}[m] \ \&\& \operatorname{IntegerQ}[n]$

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.)
+ (a_.)], x_Symbol] := Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (
a_.))^m*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
- b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] && !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{\cos(c+dx)}(A+B\sec(c+dx))}{\sqrt{a+a\sec(c+dx)}} dx &= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \frac{A+B\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}} dx \\ &= \frac{2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{((-A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)})}{d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} \\ &= \frac{2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} - \frac{(2(-A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)})}{d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} \\ &= -\frac{\sqrt{2}(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}d} \end{aligned}$$

Mathematica [A] time = 0.31, size = 140, normalized size = 1.18

$$\frac{\sin(c+dx)\sqrt{\cos(c+dx)}(A+B\sec(c+dx))\left(\sqrt{2}(A-B)\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)+2A\sqrt{1-\sec(c+dx)}\right)}{d\sqrt{1-\sec(c+dx)}\sqrt{a(\sec(c+dx)+1)}(A\cos(c+dx)+B)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x]))/Sqrt[a + a*Sec[c + d*x]
],x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(2*A*Sqrt[1 - Sec[c + d*x]] + Sqrt[2]*(A - B)*ArcTan[(S
qrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sqrt[Sec[c + d*x]]*(A +
```


$B \sec[c + dx] \sin[c + dx] / (d(B + A \cos[c + dx]) \sqrt{1 - \sec[c + dx]}) \sqrt{a(1 + \sec[c + dx])}$

fricas [A] time = 0.46, size = 306, normalized size = 2.57

$$\frac{4A \sqrt{\frac{a \cos(dx+c)+a}{\cos(dx+c)}} \sqrt{\cos(dx+c)} \sin(dx+c) - \frac{\sqrt{2}((A-B)a \cos(dx+c) + (A-B)a) \log\left(\frac{\cos(dx+c)^2 - \frac{2\sqrt{2} \sqrt{\frac{a \cos(dx+c)+a}{\cos(dx+c)}} \sqrt{\cos(dx+c)} \sin(dx+c)}{\sqrt{a}}}{\cos(dx+c)^2 + 2 \cos(dx+c) + 1}\right)}{\sqrt{a}}}{2(ad \cos(dx+c) + ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")

[Out] $\frac{1}{2} \left(4A \sqrt{\frac{a \cos(dx+c)+a}{\cos(dx+c)}} \sqrt{\cos(dx+c)} \sin(dx+c) - \sqrt{2} \left((A-B)a \cos(dx+c) + (A-B)a \right) \log\left(-\frac{\cos(dx+c)^2 - 2\sqrt{2} \sqrt{\frac{a \cos(dx+c)+a}{\cos(dx+c)}} \sqrt{\cos(dx+c)} \sin(dx+c) / \sqrt{a} - 3}{\cos(dx+c)^2 + 2\cos(dx+c) + 1} \right) / \sqrt{a} \right) / (a d \cos(dx+c) + a d), \left(\sqrt{2} \left((A-B)a \cos(dx+c) + (A-B)a \right) \sqrt{-1/a} \arctan\left(\sqrt{2} \sqrt{\frac{a \cos(dx+c)+a}{\cos(dx+c)}} \sqrt{\cos(dx+c)} \sin(dx+c) / \sqrt{a} \right) + 2A \sqrt{\frac{a \cos(dx+c)+a}{\cos(dx+c)}} \sqrt{\cos(dx+c)} \sin(dx+c) \right) / (a d \cos(dx+c) + a d) \right]$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx+c) + A) \sqrt{\cos(dx+c)}}{\sqrt{a \sec(dx+c) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x+c) + A)*sqrt(cos(d*x+c))/sqrt(a*sec(d*x+c) + a), x)

maple [A] time = 2.27, size = 142, normalized size = 1.19

$$\frac{2(-1 + \cos(dx + c)) \left(-A \sin(dx + c) \sqrt{\frac{2}{1 + \cos(dx + c)}} + A \arctan\left(\frac{\sin(dx + c) \sqrt{\frac{2}{1 + \cos(dx + c)}}}{2}\right) - B \arctan\left(\frac{\sin(dx + c) \sqrt{\frac{2}{1 + \cos(dx + c)}}}{2}\right) \right)}{da \sqrt{\frac{2}{1 + \cos(dx + c)}} \sin(dx + c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x)`

[Out] `2/d*(-1+cos(d*x+c))*(-A*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)+A*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))-B*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/a/(-2/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^2`

maxima [A] time = 0.65, size = 195, normalized size = 1.64

$$\frac{\left(\sqrt{2} \log\left(\cos\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + \sin\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + 2 \sin\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right) - \sqrt{2} \log\left(\cos\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + \sin\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - 2 \sin\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right) - 4 \sqrt{2} \sin\left(\frac{1}{2}dx + \frac{1}{2}c\right)}{\sqrt{a}}$$

2d

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algorith="maxima")`

[Out] `-1/2*((sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 4*sqrt(2)*sin(1/2*d*x + 1/2*c))*A/sqrt(a) - (sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*B/sqrt(a))/d`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\cos(c + dx)} \left(A + \frac{B}{\cos(c + dx)} \right)}{\sqrt{a + \frac{a}{\cos(c + dx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((cos(c + d*x)^(1/2)*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^(1/2),x)`

[Out] `int((cos(c + d*x)^(1/2)*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^(1/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \sqrt{\cos(c + dx)}}{\sqrt{a(\sec(c + dx) + 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*sec(d*x+c))*cos(d*x+c)**(1/2)/(a+a*sec(d*x+c))**(1/2),x)`

[Out] `Integral((A + B*sec(c + d*x))*sqrt(cos(c + d*x))/sqrt(a*(sec(c + d*x) + 1)), x)`

$$3.545 \quad \int \frac{A+B \sec(c+dx)}{\sqrt{\cos(c+dx)} \sqrt{a+a \sec(c+dx)}} dx$$

Optimal. Leaf size=140

$$\frac{\sqrt{2}(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d} + \frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}}{\sqrt{a}}\right)}{\sqrt{a}d}$$

[Out] 2*B*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d/a^(1/2)+(A-B)*arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d/a^(1/2)

Rubi [A] time = 0.34, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {2955, 4023, 3808, 206, 3801, 215}

$$\frac{\sqrt{2}(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d} + \frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}}{\sqrt{a}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sec[c + d*x])/(Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]),x]

[Out] (2*B*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]/(Sqrt[a]*d) + (Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 2955

Int[((a_.) + csc[(e_.) + (f_.)*(x_)])*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] := Dis

$\int (g \csc[e + f x])^p (g \sin[e + f x])^p \int ((a + b \csc[e + f x])^m (c + d \csc[e + f x])^n) / (g \csc[e + f x])^p dx$; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && IntegerQ[n])

Rule 3801

$\int \sqrt{\csc[e + f x] + (f x)} (d) \sqrt{\csc[e + f x] + (f x)} (b + a)$, x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 + x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] ; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]

Rule 3808

$\int \sqrt{\csc[e + f x] + (f x)} (d) / \sqrt{\csc[e + f x] + (f x)} (b + a)$, x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] ; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 4023

$\int (\csc[e + f x] + (f x) (d))^n (\csc[e + f x] + (f x) (b + a))^m (\csc[e + f x] + (f x) (B + A))$, x_Symbol] :> Dist[(A*b - a*B)/b, Int[(a + b*Csc[e + f*x])^m (d*Csc[e + f*x])^n, x], x] + Dist[B/b, Int[(a + b*Csc[e + f*x])^(m + 1) (d*Csc[e + f*x])^n, x], x] ; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned} \int \frac{A + B \sec(c + dx)}{\sqrt{\cos(c + dx)} \sqrt{a + a \sec(c + dx)}} dx &= \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{\sqrt{\sec(c + dx)} (A + B \sec(c + dx))}{\sqrt{a + a \sec(c + dx)}} dx \\ &= \left((A - B) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{\sqrt{\sec(c + dx)}}{\sqrt{a + a \sec(c + dx)}} dx + \frac{B}{\sqrt{a + a \sec(c + dx)}} \\ &= \frac{\left(2(A - B) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \text{Subst} \left(\int \frac{1}{2a - x^2} dx, x, -\frac{a \sqrt{\sec(c + dx)}}{\sqrt{a + a \sec(c + dx)}} \right)}{d} \\ &= \frac{2B \sinh^{-1} \left(\frac{\sqrt{a} \tan(c + dx)}{\sqrt{a + a \sec(c + dx)}} \right) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)}}{\sqrt{a} d} + \frac{\sqrt{2} (A - B)}{\sqrt{a + a \sec(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.23, size = 115, normalized size = 0.82

$$\frac{\sin(c + dx)\sqrt{\cos(c + dx)} \sec^{\frac{3}{2}}(c + dx) \left(\sqrt{2} (B - A) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{\sec(c + dx)}}{\sqrt{1 - \sec(c + dx)}} \right) - 2B \sin^{-1} \left(\sqrt{\sec(c + dx)} \right) \right)}{d\sqrt{1 - \sec(c + dx)} \sqrt{a(\sec(c + dx) + 1)}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sec[c + d*x])/(Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]),x]

[Out] ((-2*B*ArcSin[Sqrt[Sec[c + d*x]]] + Sqrt[2]*(-A + B)*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]])*Sqrt[Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])

fricas [A] time = 0.49, size = 357, normalized size = 2.55

$$\frac{\sqrt{2}(A - B)\sqrt{a} \log \left(\frac{\cos(dx+c)^2 + \frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{\sqrt{a}} - 2\cos(dx+c) - 3}{\cos(dx+c)^2 + 2\cos(dx+c) + 1} \right) - B\sqrt{a} \log \left(\frac{a\cos(dx+c)^3 - 4\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}}{\cos(dx+c)^2 + 2\cos(dx+c) + 1} \right)}{2ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")

[Out] [-1/2*(sqrt(2)*(A - B)*sqrt(a)*log(-(cos(d*x + c))^2 + 2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - B*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(a*d), -(sqrt(2)*(A - B)*a*sqrt(-1/a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*sqrt(cos(d*x + c))/sin(d*x + c)) - B*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(a*d)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{\sqrt{a \sec(dx + c) + a} \sqrt{\cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algorith="giac")

[Out] integrate((B*sec(d*x + c) + A)/(sqrt(a*sec(d*x + c) + a)*sqrt(cos(d*x + c))), x)

maple [A] time = 2.40, size = 201, normalized size = 1.44

$$\frac{(-1 + \cos(dx + c)) \left(B \arctan \left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}} (\cos(dx+c)+1+\sin(dx+c))\sqrt{2}}{4} \right) \sqrt{2} - B \arctan \left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}} (\cos(dx+c)+1-\sin(dx+c))\sqrt{2}}{4} \right) \right)}{d \sin(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x)

[Out] -1/d*(-1+cos(d*x+c))*(B*arctan(1/4*(-2/(1+cos(d*x+c))))^(1/2)*(cos(d*x+c)+1+sin(d*x+c))*2^(1/2))*2^(1/2)-B*arctan(1/4*(-2/(1+cos(d*x+c))))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))*2^(1/2))*2^(1/2)+2*A*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c))))^(1/2))-2*B*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c))))^(1/2))*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)^2/a/(-2/(1+cos(d*x+c))))^(1/2)

maxima [B] time = 0.72, size = 699, normalized size = 4.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algorith="maxima")

[Out] 1/2*((sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*A/sqrt(a) - (sqrt(2)*log(cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))) + 1) - sqrt(2)*log(cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))) + 1) - log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))

$$\begin{aligned} & \left(\frac{3}{2}c \right) + 2\sqrt{2} \sin\left(\frac{1}{3} \arctan\left(\frac{\sin\left(\frac{3}{2}dx + \frac{3}{2}c\right)}{\cos\left(\frac{3}{2}dx + \frac{3}{2}c\right)}\right)\right) + 2 + \log\left(2 \cos\left(\frac{1}{3} \arctan\left(\frac{\sin\left(\frac{3}{2}dx + \frac{3}{2}c\right)}{\cos\left(\frac{3}{2}dx + \frac{3}{2}c\right)}\right)\right)\right)^2 \\ & + 2 \sin\left(\frac{1}{3} \arctan\left(\frac{\sin\left(\frac{3}{2}dx + \frac{3}{2}c\right)}{\cos\left(\frac{3}{2}dx + \frac{3}{2}c\right)}\right)\right)^2 + 2\sqrt{2} \cos\left(\frac{1}{3} \arctan\left(\frac{\sin\left(\frac{3}{2}dx + \frac{3}{2}c\right)}{\cos\left(\frac{3}{2}dx + \frac{3}{2}c\right)}\right)\right) - \\ & 2\sqrt{2} \sin\left(\frac{1}{3} \arctan\left(\frac{\sin\left(\frac{3}{2}dx + \frac{3}{2}c\right)}{\cos\left(\frac{3}{2}dx + \frac{3}{2}c\right)}\right)\right) + 2 - \log\left(2 \cos\left(\frac{1}{3} \arctan\left(\frac{\sin\left(\frac{3}{2}dx + \frac{3}{2}c\right)}{\cos\left(\frac{3}{2}dx + \frac{3}{2}c\right)}\right)\right)\right)^2 \\ & + 2 \sin\left(\frac{1}{3} \arctan\left(\frac{\sin\left(\frac{3}{2}dx + \frac{3}{2}c\right)}{\cos\left(\frac{3}{2}dx + \frac{3}{2}c\right)}\right)\right)^2 - 2\sqrt{2} \cos\left(\frac{1}{3} \arctan\left(\frac{\sin\left(\frac{3}{2}dx + \frac{3}{2}c\right)}{\cos\left(\frac{3}{2}dx + \frac{3}{2}c\right)}\right)\right) + 2\sqrt{2} \sin\left(\frac{1}{3} \arctan\left(\frac{\sin\left(\frac{3}{2}dx + \frac{3}{2}c\right)}{\cos\left(\frac{3}{2}dx + \frac{3}{2}c\right)}\right)\right) \\ & + 2 + \log\left(2 \cos\left(\frac{1}{3} \arctan\left(\frac{\sin\left(\frac{3}{2}dx + \frac{3}{2}c\right)}{\cos\left(\frac{3}{2}dx + \frac{3}{2}c\right)}\right)\right)\right)^2 + 2 \sin\left(\frac{1}{3} \arctan\left(\frac{\sin\left(\frac{3}{2}dx + \frac{3}{2}c\right)}{\cos\left(\frac{3}{2}dx + \frac{3}{2}c\right)}\right)\right)^2 - 2\sqrt{2} \cos\left(\frac{1}{3} \arctan\left(\frac{\sin\left(\frac{3}{2}dx + \frac{3}{2}c\right)}{\cos\left(\frac{3}{2}dx + \frac{3}{2}c\right)}\right)\right) \\ & - 2\sqrt{2} \sin\left(\frac{1}{3} \arctan\left(\frac{\sin\left(\frac{3}{2}dx + \frac{3}{2}c\right)}{\cos\left(\frac{3}{2}dx + \frac{3}{2}c\right)}\right)\right) + 2 \Big) \cdot \frac{B}{\sqrt{a}} / d \end{aligned}$$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\sqrt{\cos(c+dx)} \sqrt{a + \frac{a}{\cos(c+dx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^(1/2)),x)

[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \sec(c + dx)}{\sqrt{a(\sec(c + dx) + 1)} \sqrt{\cos(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(1/2)/(a+a*sec(d*x+c))**(1/2),x)

[Out] Integral((A + B*sec(c + d*x))/(sqrt(a*(sec(c + d*x) + 1))*sqrt(cos(c + d*x))), x)

$$3.546 \quad \int \frac{A+B \sec(c+dx)}{\cos^3(c+dx) \sqrt{a+a \sec(c+dx)}} dx$$

Optimal. Leaf size=181

$$\frac{\sqrt{2}(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d} + \frac{(2A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d}$$

[Out] (2*A-B)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d/a^(1/2)-(A-B)*arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d/a^(1/2)+B*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2)

Rubi [A] time = 0.50, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2955, 4021, 4023, 3808, 206, 3801, 215}

$$\frac{\sqrt{2}(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d} + \frac{(2A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]), x]

[Out] ((2*A - B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]/(Sqrt[a]*d) - (Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]/(Sqrt[a]*d) + (B*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_.)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*
(d_.) + (c_.))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_.)])^(p_.), x_Symbol] := Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.)
+ (a_.)], x_Symbol] := Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.)
+ (a_.)], x_Symbol] := Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x])*Sqrt[d*Csc[e + f*x]]], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4021

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (
a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := -Simp[(B*d*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x
] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)
*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x] /; FreeQ
[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &&
GtQ[n, 1]
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (
a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{A + B \sec(c + dx)}{\cos^{\frac{3}{2}}(c + dx) \sqrt{a + a \sec(c + dx)}} dx &= \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{\sec^{\frac{3}{2}}(c + dx)(A + B \sec(c + dx))}{\sqrt{a + a \sec(c + dx)}} dx \\
&= \frac{B \sin(c + dx)}{d \cos^{\frac{3}{2}}(c + dx) \sqrt{a + a \sec(c + dx)}} + \frac{\left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{A \sec^{\frac{3}{2}}(c + dx)}{\sqrt{a + a \sec(c + dx)}} dx}{d \cos^{\frac{3}{2}}(c + dx) \sqrt{a + a \sec(c + dx)}} \\
&= \frac{B \sin(c + dx)}{d \cos^{\frac{3}{2}}(c + dx) \sqrt{a + a \sec(c + dx)}} - \left((A - B) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{A \sec^{\frac{3}{2}}(c + dx)}{\sqrt{a + a \sec(c + dx)}} dx \\
&= \frac{B \sin(c + dx)}{d \cos^{\frac{3}{2}}(c + dx) \sqrt{a + a \sec(c + dx)}} + \frac{\left(2(A - B) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{A \sec^{\frac{3}{2}}(c + dx)}{\sqrt{a + a \sec(c + dx)}} dx}{d \cos^{\frac{3}{2}}(c + dx) \sqrt{a + a \sec(c + dx)}} \\
&= \frac{(2A - B) \sinh^{-1} \left(\frac{\sqrt{a} \tan(c + dx)}{\sqrt{a + a \sec(c + dx)}} \right) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} - \sqrt{2} (A - B) \cos(c + dx) \tanh^{-1} \left(\sqrt{2} \sin \left(\frac{1}{2}(c + dx) \right) \right)}{\sqrt{a} d}
\end{aligned}$$

Mathematica [A] time = 0.67, size = 114, normalized size = 0.63

$$\frac{\cos \left(\frac{1}{2}(c + dx) \right) \left(2(A - B) \cos(c + dx) \tanh^{-1} \left(\sin \left(\frac{1}{2}(c + dx) \right) \right) \right) - \sqrt{2} (2A - B) \cos(c + dx) \tanh^{-1} \left(\sqrt{2} \sin \left(\frac{1}{2}(c + dx) \right) \right)}{d \cos^{\frac{3}{2}}(c + dx) \sqrt{a(\sec(c + dx) + 1)}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]), x]

[Out] -((Cos[(c + d*x)/2]*(2*(A - B)*ArcTanh[Sin[(c + d*x)/2]]*Cos[c + d*x] - Sqrt[2]*(2*A - B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x] - 2*B*Sin[(c + d*x)/2]))/(d*Cos[c + d*x]^(3/2)*Sqrt[a*(1 + Sec[c + d*x]))]

fricas [A] time = 0.56, size = 575, normalized size = 3.18

$$4 B \sqrt{\frac{a \cos(dx+c)+a}{\cos(dx+c)}} \sqrt{\cos(dx+c)} \sin(dx+c) - \left((2A-B) \cos(dx+c)^2 + (2A-B) \cos(dx+c) \right) \sqrt{a} \log \left(\frac{a \cos(dx+c)+a}{\cos(dx+c)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")

[Out] [1/4*(4*B*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - ((2*A - B)*cos(d*x + c)^2 + (2*A - B)*cos(d*x + c))*sqrt(a)*log((a*cos(d*x + c)^3 + 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) - 2*sqrt(2)*((A - B)*a*cos(d*x + c)^2 + (A - B)*a*cos(d*x + c))*log(-(cos(d*x + c)^2 - 2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/sqrt(a))/(a*d*cos(d*x + c)^2 + a*d*cos(d*x + c)), 1/2*(2*sqrt(2)*((A - B)*a*cos(d*x + c)^2 + (A - B)*a*cos(d*x + c))*sqrt(-1/a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))*sqrt(-1/a)*sqrt(cos(d*x + c))/sin(d*x + c)) + 2*B*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + ((2*A - B)*cos(d*x + c)^2 + (2*A - B)*cos(d*x + c))*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(a*d*cos(d*x + c)^2 + a*d*cos(d*x + c))]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx+c) + A}{\sqrt{a \sec(dx+c) + a} \cos(dx+c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)/(sqrt(a*sec(d*x + c) + a)*cos(d*x + c)^(3/2)), x)

maple [B] time = 2.34, size = 342, normalized size = 1.89

$$(-1 + \cos(dx + c)) \left(2A \arctan \left(\frac{\sqrt{\frac{2}{1+\cos(dx+c)}} (\cos(dx+c)+1-\sin(dx+c)) \sqrt{2}}{4} \right) \cos(dx + c) \sqrt{2} - 2A \arctan \left(\frac{\sqrt{\frac{2}{1+\cos(dx+c)}}}{4} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x)

[Out] 1/2/d*(-1+cos(d*x+c))*(2*A*arctan(1/4*(-2/(1+cos(d*x+c))))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))*2^(1/2))*cos(d*x+c)*2^(1/2)-2*A*arctan(1/4*(-2/(1+cos(d*x+c))))^(1/2)*(cos(d*x+c)+1+sin(d*x+c))*2^(1/2))*cos(d*x+c)*2^(1/2)-B*arctan(1/4*(-2/(1+cos(d*x+c))))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))*2^(1/2))*cos(d*x+c)*2^(1/2)+B*arctan(1/4*(-2/(1+cos(d*x+c))))^(1/2)*(cos(d*x+c)+1+sin(d*x+c))*2^(1/2))*cos(d*x+c)*2^(1/2)+4*A*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c))))^(1/2))*cos(d*x+c)-2*B*(-2/(1+cos(d*x+c))))^(1/2)*sin(d*x+c)-4*B*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c))))^(1/2))*cos(d*x+c)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)/(-2/(1+cos(d*x+c))))^(1/2)/sin(d*x+c)^2/a

maxima [B] time = 0.83, size = 1509, normalized size = 8.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")

[Out] -1/4*(2*(sqrt(2)*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))))^2 + sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 1) - sqrt(2)*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))))^2 + sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))))^2 - 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 1) - log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2)

$2) \cdot \cos(1/2 \cdot \arctan2(\sin(dx + c), \cos(dx + c))) + 2 \cdot \sqrt{2} \cdot \sin(1/2 \cdot \arctan2(\sin(dx + c), \cos(dx + c))) + 2) + \log(2 \cdot \cos(1/2 \cdot \arctan2(\sin(dx + c), \cos(dx + c)))^2 + 2 \cdot \sin(1/2 \cdot \arctan2(\sin(dx + c), \cos(dx + c)))^2 - 2 \cdot \sqrt{2} \cdot \cos(1/2 \cdot \arctan2(\sin(dx + c), \cos(dx + c))) - 2 \cdot \sqrt{2} \cdot \sin(1/2 \cdot \arctan2(\sin(dx + c), \cos(dx + c))) + 2)) \cdot A / \sqrt{a} + (4 \cdot \sqrt{2} \cdot \cos(3/4 \cdot \arctan2(\sin(2 \cdot dx + 2 \cdot c), \cos(2 \cdot dx + 2 \cdot c))) \cdot \sin(2 \cdot dx + 2 \cdot c) - 4 \cdot \sqrt{2} \cdot \cos(1/4 \cdot \arctan2(\sin(2 \cdot dx + 2 \cdot c), \cos(2 \cdot dx + 2 \cdot c))) \cdot \sin(2 \cdot dx + 2 \cdot c) + (\cos(2 \cdot dx + 2 \cdot c))^2 + \sin(2 \cdot dx + 2 \cdot c)^2 + 2 \cdot \cos(2 \cdot dx + 2 \cdot c) + 1) \cdot \log(2 \cdot \cos(1/4 \cdot \arctan2(\sin(2 \cdot dx + 2 \cdot c), \cos(2 \cdot dx + 2 \cdot c)))^2 + 2 \cdot \sin(1/4 \cdot \arctan2(\sin(2 \cdot dx + 2 \cdot c), \cos(2 \cdot dx + 2 \cdot c)))^2 + 2 \cdot \sqrt{2} \cdot \cos(1/4 \cdot \arctan2(\sin(2 \cdot dx + 2 \cdot c), \cos(2 \cdot dx + 2 \cdot c))) + 2 \cdot \sqrt{2} \cdot \sin(1/4 \cdot \arctan2(\sin(2 \cdot dx + 2 \cdot c), \cos(2 \cdot dx + 2 \cdot c))) + 2) - (\cos(2 \cdot dx + 2 \cdot c))^2 + \sin(2 \cdot dx + 2 \cdot c)^2 + 2 \cdot \cos(2 \cdot dx + 2 \cdot c) + 1) \cdot \log(2 \cdot \cos(1/4 \cdot \arctan2(\sin(2 \cdot dx + 2 \cdot c), \cos(2 \cdot dx + 2 \cdot c)))^2 + 2 \cdot \sin(1/4 \cdot \arctan2(\sin(2 \cdot dx + 2 \cdot c), \cos(2 \cdot dx + 2 \cdot c)))^2 + 2 \cdot \sqrt{2} \cdot \cos(1/4 \cdot \arctan2(\sin(2 \cdot dx + 2 \cdot c), \cos(2 \cdot dx + 2 \cdot c))) - 2 \cdot \sqrt{2} \cdot \sin(1/4 \cdot \arctan2(\sin(2 \cdot dx + 2 \cdot c), \cos(2 \cdot dx + 2 \cdot c))) + 2) + (\cos(2 \cdot dx + 2 \cdot c))^2 + \sin(2 \cdot dx + 2 \cdot c)^2 + 2 \cdot \cos(2 \cdot dx + 2 \cdot c) + 1) \cdot \log(2 \cdot \cos(1/4 \cdot \arctan2(\sin(2 \cdot dx + 2 \cdot c), \cos(2 \cdot dx + 2 \cdot c)))^2 + 2 \cdot \sin(1/4 \cdot \arctan2(\sin(2 \cdot dx + 2 \cdot c), \cos(2 \cdot dx + 2 \cdot c)))^2 - 2 \cdot \sqrt{2} \cdot \cos(1/4 \cdot \arctan2(\sin(2 \cdot dx + 2 \cdot c), \cos(2 \cdot dx + 2 \cdot c))) + 2 \cdot \sqrt{2} \cdot \sin(1/4 \cdot \arctan2(\sin(2 \cdot dx + 2 \cdot c), \cos(2 \cdot dx + 2 \cdot c))) + 2) - (\cos(2 \cdot dx + 2 \cdot c))^2 + \sin(2 \cdot dx + 2 \cdot c)^2 + 2 \cdot \cos(2 \cdot dx + 2 \cdot c) + 1) \cdot \log(2 \cdot \cos(1/4 \cdot \arctan2(\sin(2 \cdot dx + 2 \cdot c), \cos(2 \cdot dx + 2 \cdot c)))^2 + 2 \cdot \sin(1/4 \cdot \arctan2(\sin(2 \cdot dx + 2 \cdot c), \cos(2 \cdot dx + 2 \cdot c)))^2 - 2 \cdot \sqrt{2} \cdot \cos(1/4 \cdot \arctan2(\sin(2 \cdot dx + 2 \cdot c), \cos(2 \cdot dx + 2 \cdot c))) - 2 \cdot \sqrt{2} \cdot \sin(1/4 \cdot \arctan2(\sin(2 \cdot dx + 2 \cdot c), \cos(2 \cdot dx + 2 \cdot c))) + 2) - 2 \cdot (\sqrt{2} \cdot \cos(2 \cdot dx + 2 \cdot c))^2 + \sqrt{2} \cdot \sin(2 \cdot dx + 2 \cdot c)^2 + 2 \cdot \sqrt{2} \cdot \cos(2 \cdot dx + 2 \cdot c) + \sqrt{2} \cdot \log(\cos(1/4 \cdot \arctan2(\sin(2 \cdot dx + 2 \cdot c), \cos(2 \cdot dx + 2 \cdot c)))^2 + \sin(1/4 \cdot \arctan2(\sin(2 \cdot dx + 2 \cdot c), \cos(2 \cdot dx + 2 \cdot c)))^2 + 2 \cdot \sin(1/4 \cdot \arctan2(\sin(2 \cdot dx + 2 \cdot c), \cos(2 \cdot dx + 2 \cdot c))) + 1) + 2 \cdot (\sqrt{2} \cdot \cos(2 \cdot dx + 2 \cdot c))^2 + \sqrt{2} \cdot \sin(2 \cdot dx + 2 \cdot c)^2 + 2 \cdot \sqrt{2} \cdot \cos(2 \cdot dx + 2 \cdot c) + \sqrt{2} \cdot \log(\cos(1/4 \cdot \arctan2(\sin(2 \cdot dx + 2 \cdot c), \cos(2 \cdot dx + 2 \cdot c)))^2 + \sin(1/4 \cdot \arctan2(\sin(2 \cdot dx + 2 \cdot c), \cos(2 \cdot dx + 2 \cdot c)))^2 - 2 \cdot \sin(1/4 \cdot \arctan2(\sin(2 \cdot dx + 2 \cdot c), \cos(2 \cdot dx + 2 \cdot c))) + 1) - 4 \cdot (\sqrt{2} \cdot \cos(2 \cdot dx + 2 \cdot c) + \sqrt{2} \cdot \sin(3/4 \cdot \arctan2(\sin(2 \cdot dx + 2 \cdot c), \cos(2 \cdot dx + 2 \cdot c))) + 4 \cdot (\sqrt{2} \cdot \cos(2 \cdot dx + 2 \cdot c) + \sqrt{2} \cdot \sin(1/4 \cdot \arctan2(\sin(2 \cdot dx + 2 \cdot c), \cos(2 \cdot dx + 2 \cdot c)))) \cdot B / ((\cos(2 \cdot dx + 2 \cdot c))^2 + \sin(2 \cdot dx + 2 \cdot c)^2 + 2 \cdot \cos(2 \cdot dx + 2 \cdot c) + 1) \cdot \sqrt{a})) / d$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\cos(c+dx)^{3/2} \sqrt{a + \frac{a}{\cos(c+dx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B/cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^(1/2)),x)`

[Out] `int((A + B/cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^(1/2)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \sec(c + dx)}{\sqrt{a(\sec(c + dx) + 1)} \cos^{\frac{3}{2}}(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*sec(d*x+c))/cos(d*x+c)**(3/2)/(a+a*sec(d*x+c))**(1/2),x)`

[Out] `Integral((A + B*sec(c + d*x))/(sqrt(a*(sec(c + d*x) + 1))*cos(c + d*x)**(3/2)), x)`

$$3.547 \quad \int \frac{A+B \sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx) \sqrt{a+a \sec(c+dx)}} dx$$

Optimal. Leaf size=230

$$\frac{(4A - B) \sin(c + dx)}{4d \cos^{\frac{3}{2}}(c + dx) \sqrt{a \sec(c + dx) + a}} + \frac{\sqrt{2} (A - B) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \tanh^{-1} \left(\frac{\sqrt{a} \sin(c + dx) \sqrt{\sec(c + dx)}}{\sqrt{2} \sqrt{a \sec(c + dx) + a}} \right)}{\sqrt{a} d} \quad (4A)$$

[Out] $-1/4*(4*A-7*B)*\operatorname{arcsinh}(a^{(1/2)}*\tan(d*x+c)/(a+a*\sec(d*x+c))^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d/a^{(1/2)}+(A-B)*\operatorname{arctanh}(1/2*\sin(d*x+c)*a^{(1/2)}*\sec(d*x+c)^{(1/2)}*2^{(1/2)}/(a+a*\sec(d*x+c))^{(1/2)})*2^{(1/2)}*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d/a^{(1/2)}+1/2*B*\sin(d*x+c)/d/\cos(d*x+c)^{(5/2)}/(a+a*\sec(d*x+c))^{(1/2)}+1/4*(4*A-B)*\sin(d*x+c)/d/\cos(d*x+c)^{(3/2)}/(a+a*\sec(d*x+c))^{(1/2)}$

Rubi [A] time = 0.70, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2955, 4021, 4023, 3808, 206, 3801, 215}

$$\frac{(4A - B) \sin(c + dx)}{4d \cos^{\frac{3}{2}}(c + dx) \sqrt{a \sec(c + dx) + a}} + \frac{\sqrt{2} (A - B) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \tanh^{-1} \left(\frac{\sqrt{a} \sin(c + dx) \sqrt{\sec(c + dx)}}{\sqrt{2} \sqrt{a \sec(c + dx) + a}} \right)}{\sqrt{a} d} \quad (4A)$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(A + B*\operatorname{Sec}[c + d*x])]/(\operatorname{Cos}[c + d*x]^{(5/2)}*\operatorname{Sqrt}[a + a*\operatorname{Sec}[c + d*x]]), x]$

[Out] $-((4*A - 7*B)*\operatorname{ArcSinh}[(\operatorname{Sqrt}[a]*\operatorname{Tan}[c + d*x])/(\operatorname{Sqrt}[a + a*\operatorname{Sec}[c + d*x]])]*\operatorname{Sqrt}[\operatorname{Cos}[c + d*x]]*\operatorname{Sqrt}[\operatorname{Sec}[c + d*x]])/(4*\operatorname{Sqrt}[a]*d) + (\operatorname{Sqrt}[2]*(A - B)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Sqrt}[\operatorname{Sec}[c + d*x]]*\operatorname{Sin}[c + d*x])/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a + a*\operatorname{Sec}[c + d*x]])]*\operatorname{Sqrt}[\operatorname{Cos}[c + d*x]]*\operatorname{Sqrt}[\operatorname{Sec}[c + d*x]])/(\operatorname{Sqrt}[a]*d) + (B*\operatorname{Sin}[c + d*x])/(2*d*\operatorname{Cos}[c + d*x]^{(5/2)}*\operatorname{Sqrt}[a + a*\operatorname{Sec}[c + d*x]]) + ((4*A - B)*\operatorname{Sin}[c + d*x])/(4*d*\operatorname{Cos}[c + d*x]^{(3/2)}*\operatorname{Sqrt}[a + a*\operatorname{Sec}[c + d*x]])$

Rule 206

$\operatorname{Int}[(a_) + (b_)*(x_)^2]^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1*\operatorname{ArcTanh}[(\operatorname{Rt}[-b, 2]*x)/\operatorname{Rt}[a, 2]])/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /; \operatorname{FreeQ}\{a, b\}, x] \&\& \operatorname{NegQ}[a/b] \&\& (\operatorname{GtQ}[a, 0] \parallel \operatorname{LtQ}[b, 0])$

Rule 215

$\operatorname{Int}[1/\operatorname{Sqrt}[(a_) + (b_)*(x_)^2], x_Symbol] \rightarrow \operatorname{Simp}[\operatorname{ArcSinh}[(\operatorname{Rt}[b, 2]*x)/\operatorname{Sqrt}[a]]/\operatorname{Rt}[b, 2], x] /; \operatorname{FreeQ}\{a, b\}, x] \&\& \operatorname{GtQ}[a, 0] \&\& \operatorname{PosQ}[b]$

Rule 2955

Int[((a_.) + csc[(e_.) + (f_.)*(x_)])*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] := Dist[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && IntegerQ[n])

Rule 3801

Int[Sqrt[csc[(e_.) + (f_.)*(x_)])*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] := Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 + x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]

Rule 3808

Int[Sqrt[csc[(e_.) + (f_.)*(x_)])*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] := Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 4021

Int[(csc[(e_.) + (f_.)*(x_)])*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] := -Simp[(B*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && GtQ[n, 1]

Rule 4023

Int[(csc[(e_.) + (f_.)*(x_)])*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] := Dist[(A*b - a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned}
\int \frac{A + B \sec(c + dx)}{\cos^{\frac{5}{2}}(c + dx) \sqrt{a + a \sec(c + dx)}} dx &= \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{\sec^{\frac{5}{2}}(c + dx) (A + B \sec(c + dx))}{\sqrt{a + a \sec(c + dx)}} dx \\
&= \frac{B \sin(c + dx)}{2d \cos^{\frac{5}{2}}(c + dx) \sqrt{a + a \sec(c + dx)}} + \frac{\left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int -}{2d} \\
&= \frac{B \sin(c + dx)}{2d \cos^{\frac{5}{2}}(c + dx) \sqrt{a + a \sec(c + dx)}} + \frac{(4A - B) \sin(c + dx)}{4d \cos^{\frac{3}{2}}(c + dx) \sqrt{a + a \sec(c + dx)}} \\
&= \frac{B \sin(c + dx)}{2d \cos^{\frac{5}{2}}(c + dx) \sqrt{a + a \sec(c + dx)}} + \frac{(4A - B) \sin(c + dx)}{4d \cos^{\frac{3}{2}}(c + dx) \sqrt{a + a \sec(c + dx)}} \\
&= \frac{B \sin(c + dx)}{2d \cos^{\frac{5}{2}}(c + dx) \sqrt{a + a \sec(c + dx)}} + \frac{(4A - B) \sin(c + dx)}{4d \cos^{\frac{3}{2}}(c + dx) \sqrt{a + a \sec(c + dx)}} \\
&= \frac{(4A - 7B) \sinh^{-1} \left(\frac{\sqrt{a} \tan(c + dx)}{\sqrt{a + a \sec(c + dx)}} \right) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} + \sqrt{2} (A - B)}{4\sqrt{a} d} + \frac{\sqrt{2} (A - B)}{4\sqrt{a} d}
\end{aligned}$$

Mathematica [A] time = 1.05, size = 137, normalized size = 0.60

$$\frac{\cos\left(\frac{1}{2}(c + dx)\right) \left(2 \sin\left(\frac{1}{2}(c + dx)\right) \left((4A - B) \cos(c + dx) + 2B \right) + 8(A - B) \cos^2(c + dx) \tanh^{-1} \left(\sin\left(\frac{1}{2}(c + dx)\right) \right) \right)}{4d \cos^{\frac{5}{2}}(c + dx) \sqrt{a(\sec(c + dx) + 1)}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]),x]

[Out] (Cos[(c + d*x)/2]*(8*(A - B)*ArcTanh[Sin[(c + d*x)/2]]*Cos[c + d*x]^2 - Sqrt[2]*(4*A - 7*B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^2 + 2*(2*B + (4*A - B)*Cos[c + d*x])*Sin[(c + d*x)/2])/ (4*d*Cos[c + d*x]^(5/2)*Sqrt[a*(1 + Sec[c + d*x])])

fricas [A] time = 0.56, size = 621, normalized size = 2.70

$$4((4A - B)\cos(dx + c) + 2B)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c) - ((4A - 7B)\cos(dx+c))^3 + (4A - 7B)\cos(dx+c)^2\sqrt{a}\log\left(\frac{(a\cos(dx+c)+a)\sqrt{\cos(dx+c)}\sin(dx+c) - 7a\cos(dx+c)^2 + 8a}{(a\cos(dx+c)+a)\sqrt{\cos(dx+c)}\sin(dx+c) - 2a}\right) - 8\sqrt{2}\sqrt{(A-B)a\cos(dx+c)^3 + (A-B)a\cos(dx+c)^2}\log\left(\frac{-(\cos(dx+c))^2 + 2\sqrt{2}\sqrt{(a\cos(dx+c)+a)/\cos(dx+c)}\sqrt{\cos(dx+c)}\sin(dx+c)/\sqrt{a} - 2\cos(dx+c) - 3}{(\cos(dx+c))^2 + 2\cos(dx+c) + 1}\right)/\sqrt{a})/(a^2d\cos(dx+c)^3 + a^2d\cos(dx+c)^2), -1/8(8\sqrt{2})\sqrt{(A-B)a\cos(dx+c)^3 + (A-B)a\cos(dx+c)^2}\sqrt{-1/a}\arctan\left(\frac{\sqrt{2}\sqrt{(a\cos(dx+c)+a)/\cos(dx+c)}\sqrt{-1/a}\sqrt{\cos(dx+c)}}{\sin(dx+c)} - 2((4A - B)\cos(dx+c) + 2B)\sqrt{(a\cos(dx+c)+a)/\cos(dx+c)}\sqrt{\cos(dx+c)}\sin(dx+c) + ((4A - 7B)\cos(dx+c))^3 + (4A - 7B)\cos(dx+c)^2}\sqrt{-a}\arctan\left(\frac{2\sqrt{-a}\sqrt{(a\cos(dx+c)+a)/\cos(dx+c)}\sqrt{\cos(dx+c)}\sin(dx+c)}{(a\cos(dx+c))^2 - a\cos(dx+c) - 2a}\right)\right)/(a^2d\cos(dx+c)^3 + a^2d\cos(dx+c)^2)]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")

[Out] [1/16*(4*((4*A - B)*cos(d*x + c) + 2*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - ((4*A - 7*B)*cos(d*x + c)^3 + (4*A - 7*B)*cos(d*x + c)^2)*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) - 8*sqrt(2)*((A - B)*a*cos(d*x + c)^3 + (A - B)*a*cos(d*x + c)^2)*log(-(cos(d*x + c))^2 + 2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/sqrt(a))/(a*d*cos(d*x + c)^3 + a*d*cos(d*x + c)^2), -1/8*(8*sqrt(2))*((A - B)*a*cos(d*x + c)^3 + (A - B)*a*cos(d*x + c)^2)*sqrt(-1/a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*sqrt(cos(d*x + c))/sin(d*x + c)) - 2*((4*A - B)*cos(d*x + c) + 2*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + ((4*A - 7*B)*cos(d*x + c)^3 + (4*A - 7*B)*cos(d*x + c)^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(a*d*cos(d*x + c)^3 + a*d*cos(d*x + c)^2)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{\sqrt{a \sec(dx + c) + a} \cos(dx + c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)/(sqrt(a*sec(d*x + c) + a)*cos(d*x + c)^(5/2)), x)

maple [B] time = 2.31, size = 413, normalized size = 1.80

$$(-1 + \cos(dx + c)) \sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}} \left(4A \arctan \left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}} (\cos(dx+c)+1-\sin(dx+c)) \sqrt{2}}{4}} \right) (\cos^2(dx + c)) \sqrt{2} - 4A \arctan \left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}} (\cos(dx+c)+1-\sin(dx+c)) \sqrt{2}}{4}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2), x)

[Out] -1/8/d*(-1+cos(d*x+c))*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(4*A*arctan(1/4*(-2/(1+cos(d*x+c))))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))*2^(1/2))*cos(d*x+c)^2*2^(1/2)-4*A*arctan(1/4*(-2/(1+cos(d*x+c))))^(1/2)*(cos(d*x+c)+1+sin(d*x+c))*2^(1/2))*cos(d*x+c)^2*2^(1/2)-7*B*arctan(1/4*(-2/(1+cos(d*x+c))))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))*2^(1/2))*cos(d*x+c)^2*2^(1/2)+7*B*arctan(1/4*(-2/(1+cos(d*x+c))))^(1/2)*(cos(d*x+c)+1+sin(d*x+c))*2^(1/2))*cos(d*x+c)^2*2^(1/2)+8*A*sin(d*x+c)*cos(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)+16*A*cos(d*x+c)^2*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c))))^(1/2)-2*B*sin(d*x+c)*cos(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)-16*B*cos(d*x+c)^2*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c))))^(1/2)+4*B*(-2/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/a/(-2/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^2/cos(d*x+c)^(3/2)

maxima [B] time = 0.92, size = 2704, normalized size = 11.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2), x, algorithm="maxima")

[Out] -1/16*(4*(4*sqrt(2)*cos(3/2*arctan2(sin(d*x + c), cos(d*x + c))))*sin(2*d*x + 2*c) - 4*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))))*sin(2*d*x + 2*c) + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + (cos(2*d

$$\begin{aligned}
& *x + 2*c)^2 + \sin(2*d*x + 2*c)^2 + 2*\cos(2*d*x + 2*c) + 1)*\log(2*\cos(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c)))^2 + 2*\sin(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c)))^2 - 2*\sqrt{2}*\cos(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c))) + 2*\sqrt{2}*\sin(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c))) + 2) - (\cos(2*d*x + 2*c))^2 + \sin(2*d*x + 2*c)^2 + 2*\cos(2*d*x + 2*c) + 1)*\log(2*\cos(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c)))^2 + 2*\sin(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c)))^2 - 2*\sqrt{2}*\cos(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c))) - 2*\sqrt{2}*\sin(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c))) + 2) - 2*(\sqrt{2}*\cos(2*d*x + 2*c))^2 + \sqrt{2}*\sin(2*d*x + 2*c)^2 + 2*\sqrt{2}*\cos(2*d*x + 2*c) + \sqrt{2})*\log(\cos(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c)))^2 + \sin(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c)))^2 + 2*\sin(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c)))) + 1) + 2*(\sqrt{2}*\cos(2*d*x + 2*c))^2 + \sqrt{2}*\sin(2*d*x + 2*c)^2 + 2*\sqrt{2}*\cos(2*d*x + 2*c) + \sqrt{2})*\log(\cos(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c)))^2 + \sin(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c)))^2 - 2*\sin(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c)))) + 1) - 4*(\sqrt{2}*\cos(2*d*x + 2*c) + \sqrt{2})*\sin(3/2*\arctan2(\sin(d*x + c), \cos(d*x + c))) + 4*(\sqrt{2}*\cos(2*d*x + 2*c) + \sqrt{2})*\sin(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c))))*A/((\cos(2*d*x + 2*c))^2 + \sin(2*d*x + 2*c)^2 + 2*\cos(2*d*x + 2*c) + 1)*\sqrt{a}) - (4*(\sqrt{2}*\sin(4*d*x + 4*c) + 2*\sqrt{2}*\sin(2*d*x + 2*c))*\cos(7/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) - 20*(\sqrt{2}*\sin(4*d*x + 4*c) + 2*\sqrt{2}*\sin(2*d*x + 2*c))*\cos(5/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 20*(\sqrt{2}*\sin(4*d*x + 4*c) + 2*\sqrt{2}*\sin(2*d*x + 2*c))*\cos(3/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) - 4*(\sqrt{2}*\sin(4*d*x + 4*c) + 2*\sqrt{2}*\sin(2*d*x + 2*c))*\cos(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 7*(2*(2*\cos(2*d*x + 2*c) + 1)*\cos(4*d*x + 4*c) + \cos(4*d*x + 4*c))^2 + 4*\cos(2*d*x + 2*c)^2 + \sin(4*d*x + 4*c)^2 + 4*\sin(4*d*x + 4*c)*\sin(2*d*x + 2*c) + 4*\sin(2*d*x + 2*c)^2 + 4*\cos(2*d*x + 2*c) + 1)*\log(2*\cos(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + 2*\sin(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + 2*\sqrt{2}*\cos(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 2*\sqrt{2}*\sin(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 2) - 7*(2*(2*\cos(2*d*x + 2*c) + 1)*\cos(4*d*x + 4*c) + \cos(4*d*x + 4*c))^2 + 4*\cos(2*d*x + 2*c)^2 + \sin(4*d*x + 4*c)^2 + 4*\sin(4*d*x + 4*c)*\sin(2*d*x + 2*c) + 4*\sin(2*d*x + 2*c)^2 + 4*\cos(2*d*x + 2*c) + 1)*\log(2*\cos(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + 2*\sin(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + 2*\sqrt{2}*\cos(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) - 2*\sqrt{2}*\sin(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 2) + 7*(2*(2*\cos(2*d*x + 2*c) + 1)*\cos(4*d*x + 4*c) + \cos(4*d*x + 4*c))^2 + 4*\cos(2*d*x + 2*c)^2 + \sin(4*d*x + 4*c)^2 + 4*\sin(4*d*x + 4*c)*\sin(2*d*x + 2*c) + 4*\sin(2*d*x + 2*c)^2 + 4*\cos(2*d*x + 2*c) + 1)*\log(2*\cos(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + 2*\sin(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 - 2*\sqrt{2}*\cos(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 2*\sqrt{2}*\sin(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))) + 2) - 7*(2*(2*\cos(2*d*x + 2*c) + 1)*\cos(4*d*x + 4*c) + \cos(4*d*x + 4*c))^2 + 4*\cos(2*d*x + 2*c)^2 + \sin(4*d*x + 4*c)^2 + 4*\sin(4*d*x + 4*c)*\sin(2*d*x + 2*c) + 4*\sin(2*d*x + 2*c)^2 + 4*\cos(2*d*x + 2*c) + 1)*\log(2*\cos(1/4*a
\end{aligned}$$

```

rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + 2) - 8*(sqrt(2)*cos(4*d*x + 4*c)^2 + 4*sqrt(2)*cos(2*d*x + 2*c)
^2 + sqrt(2)*sin(4*d*x + 4*c)^2 + 4*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*
c) + 4*sqrt(2)*sin(2*d*x + 2*c)^2 + 2*(2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2)
)*cos(4*d*x + 4*c) + 4*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(cos(1/4*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/4*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c)))) + 1) + 8*(sqrt(2)*cos(4*d*x + 4*c)^2 + 4*sqrt(2)*cos(2*d*x + 2*c)^2
+ sqrt(2)*sin(4*d*x + 4*c)^2 + 4*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c)
+ 4*sqrt(2)*sin(2*d*x + 2*c)^2 + 2*(2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*c
os(4*d*x + 4*c) + 4*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(cos(1/4*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/4*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c)))^2 - 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
))) + 1) - 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(
2))*sin(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 20*(sqrt(2)*cos(
4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(5/4*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) - 20*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*c
os(2*d*x + 2*c) + sqrt(2))*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) + 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*
sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*B/((2*(2*cos(2*d*x +
2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + si
n(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)
^2 + 4*cos(2*d*x + 2*c) + 1)*sqrt(a))/d

```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\cos(c+dx)^{5/2} \sqrt{a + \frac{a}{\cos(c+dx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))^(1/2)),x)

[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))^(1/2)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(5/2)/(a+a*sec(d*x+c))**(1/2),x)

[Out] Timed out

$$3.548 \quad \int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B \sec(c+dx))}{(a+a \sec(c+dx))^{\frac{3}{2}}} dx$$

Optimal. Leaf size=270

$$\frac{(15A - 11B)\sqrt{\cos(c + dx)}\sqrt{\sec(c + dx)} \tanh^{-1}\left(\frac{\sqrt{a} \sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a \sec(c+dx)+a}}\right)}{2\sqrt{2} a^{3/2}d} + \frac{(9A - 5B) \sin(c + dx) \cos^{\frac{3}{2}}(c + dx)}{10ad\sqrt{a \sec(c + dx) + a}} - \frac{(A - B) \cos^{\frac{3}{2}}(c + dx)}{10ad\sqrt{a \sec(c + dx) + a}}$$

[Out] $-1/2*(A-B)*\cos(d*x+c)^{(3/2)}*\sin(d*x+c)/d/(a+a*\sec(d*x+c))^{(3/2)}-1/4*(15*A-11*B)*\operatorname{arctanh}(1/2*\sin(d*x+c)*a^{(1/2)}*\sec(d*x+c)^{(1/2)}*2^{(1/2)}/(a+a*\sec(d*x+c))^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/a^{(3/2)}/d*2^{(1/2)}+1/10*(9*A-5*B)*\cos(d*x+c)^{(3/2)}*\sin(d*x+c)/a/d/(a+a*\sec(d*x+c))^{(1/2)}+1/30*(147*A-95*B)*\sin(d*x+c)/a/d/\cos(d*x+c)^{(1/2)}/(a+a*\sec(d*x+c))^{(1/2)}-1/30*(39*A-35*B)*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/a/d/(a+a*\sec(d*x+c))^{(1/2)}$

Rubi [A] time = 0.87, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {2955, 4020, 4022, 4013, 3808, 206}

$$\frac{(15A - 11B)\sqrt{\cos(c + dx)}\sqrt{\sec(c + dx)} \tanh^{-1}\left(\frac{\sqrt{a} \sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a \sec(c+dx)+a}}\right)}{2\sqrt{2} a^{3/2}d} + \frac{(9A - 5B) \sin(c + dx) \cos^{\frac{3}{2}}(c + dx)}{10ad\sqrt{a \sec(c + dx) + a}} - \frac{(A - B) \cos^{\frac{3}{2}}(c + dx)}{10ad\sqrt{a \sec(c + dx) + a}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(\operatorname{Cos}[c + d*x]^{(5/2)}*(A + B*\operatorname{Sec}[c + d*x]))/(a + a*\operatorname{Sec}[c + d*x]^{(3/2)}, x]$

[Out] $-((15*A - 11*B)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Sqrt}[\operatorname{Sec}[c + d*x]]*\operatorname{Sin}[c + d*x])/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a + a*\operatorname{Sec}[c + d*x]])]*\operatorname{Sqrt}[\operatorname{Cos}[c + d*x]]*\operatorname{Sqrt}[\operatorname{Sec}[c + d*x]])/(2*\operatorname{Sqrt}[2]*a^{(3/2)}*d) - ((A - B)*\operatorname{Cos}[c + d*x]^{(3/2)}*\operatorname{Sin}[c + d*x])/(2*d*(a + a*\operatorname{Sec}[c + d*x]^{(3/2)})) + ((147*A - 95*B)*\operatorname{Sin}[c + d*x])/(30*a*d*\operatorname{Sqrt}[\operatorname{Cos}[c + d*x]]*\operatorname{Sqrt}[a + a*\operatorname{Sec}[c + d*x]]) - ((39*A - 35*B)*\operatorname{Sqrt}[\operatorname{Cos}[c + d*x]]*\operatorname{Sin}[c + d*x])/(30*a*d*\operatorname{Sqrt}[a + a*\operatorname{Sec}[c + d*x]]) + ((9*A - 5*B)*\operatorname{Cos}[c + d*x]^{(3/2)}*\operatorname{Sin}[c + d*x])/(10*a*d*\operatorname{Sqrt}[a + a*\operatorname{Sec}[c + d*x]])$

Rule 206

$\operatorname{Int}[(a_.) + (b_.)*(x_.)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1*\operatorname{ArcTanh}[(\operatorname{Rt}[-b, 2]*x)/\operatorname{Rt}[a, 2]])/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /; \operatorname{FreeQ}\{a, b\}, x] \&\& \operatorname{NegQ}[a/b] \&\& (\operatorname{GtQ}[a, 0] \parallel \operatorname{LtQ}[b, 0])$

Rule 2955

$\operatorname{Int}[(a_.) + \operatorname{csc}[e_.) + (f_.)*(x_.)]*(b_.)^{(m_.)}*(\operatorname{csc}[e_.) + (f_.)*(x_.)]*(d_.) + (c_.)^{(n_.)}*((g_.)*\operatorname{sin}[e_.) + (f_.)*(x_.)])^{(p_.)}, x_Symbol] \rightarrow \operatorname{Dis}$

$t[(g*\text{Csc}[e + f*x])^p*(g*\text{Sin}[e + f*x])^p, \text{Int}[(a + b*\text{Csc}[e + f*x])^m*(c + d*\text{Csc}[e + f*x])^n]/(g*\text{Csc}[e + f*x])^p, x], x] /;$ FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && IntegerQ[n])

Rule 3808

$\text{Int}[\text{Sqrt}[\text{csc}[(e_.) + (f_.)*(x_.)]*(d_.)]/\text{Sqrt}[\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] :> \text{Dist}[(-2*b*d)/(a*f), \text{Subst}[\text{Int}[1/(2*b - d*x^2), x], x, (b*\text{Cot}[e + f*x])/(\text{Sqrt}[a + b*\text{Csc}[e + f*x]]*\text{Sqrt}[d*\text{Csc}[e + f*x]])], x] /;$ FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 4013

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(d_.))^{(n_.)}*(\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^{(m_.)}*(\text{csc}[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> \text{Simp}[(A*\text{Cot}[e + f*x]*(a + b*\text{Csc}[e + f*x])^m*(d*\text{Csc}[e + f*x])^n)/(f*n), x] - \text{Dist}[(a*A*m - b*B*n)/(b*d*n), \text{Int}[(a + b*\text{Csc}[e + f*x])^m*(d*\text{Csc}[e + f*x])^{(n + 1)}], x], x] /;$ FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && !LeQ[m, -1]

Rule 4020

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(d_.))^{(n_.)}*(\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^{(m_.)}*(\text{csc}[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> -\text{Simp}[(A*b - a*B)*\text{Cot}[e + f*x]*(a + b*\text{Csc}[e + f*x])^m*(d*\text{Csc}[e + f*x])^n)/(b*f*(2*m + 1)), x] - \text{Dist}[1/(a^2*(2*m + 1)), \text{Int}[(a + b*\text{Csc}[e + f*x])^{(m + 1)}*(d*\text{Csc}[e + f*x])^n*\text{Simp}[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*\text{Csc}[e + f*x], x], x], x] /;$ FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0]

Rule 4022

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(d_.))^{(n_.)}*(\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^{(m_.)}*(\text{csc}[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> \text{Simp}[(A*\text{Cot}[e + f*x]*(a + b*\text{Csc}[e + f*x])^m*(d*\text{Csc}[e + f*x])^n)/(f*n), x] - \text{Dist}[1/(b*d*n), \text{Int}[(a + b*\text{Csc}[e + f*x])^m*(d*\text{Csc}[e + f*x])^{(n + 1)}*\text{Simp}[a*A*m - b*B*n - A*b*(m + n + 1)*\text{Csc}[e + f*x], x], x], x] /;$ FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]

Rubi steps

$$\begin{aligned}
\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{3/2}} dx &= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \frac{A+B\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} dx \\
&= -\frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int^{\frac{1}{2}}}{2a^2} \\
&= -\frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(9A-5B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{10ad\sqrt{a+a\sec(c+dx)}} \\
&= -\frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(39A-35B)\sqrt{\cos(c+dx)}\sin(c+dx)}{30ad\sqrt{a+a\sec(c+dx)}} \\
&= -\frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(147A-95B)\sin(c+dx)}{30ad\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} \\
&= -\frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(147A-95B)\sin(c+dx)}{30ad\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} \\
&= -\frac{(15A-11B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{2\sqrt{2}a^{3/2}d}
\end{aligned}$$

Mathematica [A] time = 1.35, size = 178, normalized size = 0.66

$$\frac{2 \tan(c+dx)\sqrt{1-\sec(c+dx)}(3(39A-20B)\cos(c+dx)+(10B-6A)\cos(2(c+dx))+3A\cos(3(c+dx))+141B\cos(4(c+dx)))+60d\sqrt{\cos(c+dx)-1}(a(\sec(c+dx)+1))^{3/2}}{60d\sqrt{\cos(c+dx)-1}(a(\sec(c+dx)+1))^{3/2}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(Cos[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(3/2), x]

[Out] (30*Sqrt[2]*(15*A - 11*B)*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]])*Cos[(c + d*x)/2]^2*Sec[c + d*x]^(3/2)*Sin[c + d*x] + 2*(141*A - 85*B + 3*(39*A - 20*B)*Cos[c + d*x] + (-6*A + 10*B)*Cos[2*(c + d*x)] + 3*A*Cos[3*(c + d*x)])*Sqrt[1 - Sec[c + d*x]]*Tan[c + d*x]/(60*d*Sqrt[-1 + Cos[c + d*x]])*(a*(1 + Sec[c + d*x]))^(3/2)

fricas [A] time = 0.48, size = 480, normalized size = 1.78

$$\left[\frac{15\sqrt{2}((15A - 11B)\cos(dx + c)^2 + 2(15A - 11B)\cos(dx + c) + 15A - 11B)\sqrt{a} \log\left(-\frac{a\cos(dx+c)^2 - 2\sqrt{2}\sqrt{a}\sqrt{\dots}}{\dots}\right)}{\dots} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")

[Out] [-1/120*(15*sqrt(2)*((15*A - 11*B)*cos(d*x + c)^2 + 2*(15*A - 11*B)*cos(d*x + c) + 15*A - 11*B)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*(12*A*cos(d*x + c)^3 - 4*(3*A - 5*B)*cos(d*x + c)^2 + 12*(9*A - 5*B)*cos(d*x + c) + 147*A - 95*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), 1/60*(15*sqrt(2)*((15*A - 11*B)*cos(d*x + c)^2 + 2*(15*A - 11*B)*cos(d*x + c) + 15*A - 11*B)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) + 2*(12*A*cos(d*x + c)^3 - 4*(3*A - 5*B)*cos(d*x + c)^2 + 12*(9*A - 5*B)*cos(d*x + c) + 147*A - 95*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \cos(dx + c)^{\frac{5}{2}}}{(a \sec(dx + c) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(5/2)/(a*sec(d*x + c) + a)^(3/2), x)

maple [A] time = 2.37, size = 329, normalized size = 1.22

$$\left(\sqrt{\cos(dx+c)}\right) \sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}} (-1 + \cos(dx+c)) \left(225A \arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right) \sqrt{-\frac{2}{1+\cos(dx+c)}} \cos(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2), x)

[Out] -1/60/d*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(225*A*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c))))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)-24*A*cos(d*x+c)^4-165*B*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c))))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)+225*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c))))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*A*sin(d*x+c)+48*A*cos(d*x+c)^3-165*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c))))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*B*sin(d*x+c)-40*B*cos(d*x+c)^3-240*A*cos(d*x+c)^2+160*B*cos(d*x+c)^2-78*A*cos(d*x+c)+70*B*cos(d*x+c)+294*A-190*B)/sin(d*x+c)^3/a^2

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2), x, algorith="maxima")

[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is undefined.

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+dx)^{5/2} \left(A + \frac{B}{\cos(c+dx)}\right)}{\left(a + \frac{a}{\cos(c+dx)}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c+d*x)^(5/2)*(A+B/cos(c+d*x)))/(a+a/cos(c+d*x))^(3/2), x)

[Out] int((cos(c+d*x)^(5/2)*(A+B/cos(c+d*x)))/(a+a/cos(c+d*x))^(3/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(3/2),x)

[Out] Timed out

$$3.549 \quad \int \frac{\cos^3(c+dx)(A+B \sec(c+dx))}{(a+a \sec(c+dx))^{3/2}} dx$$

Optimal. Leaf size=223

$$\frac{(11A - 7B)\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \tanh^{-1}\left(\frac{\sqrt{a} \sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a \sec(c+dx)+a}}\right)}{2\sqrt{2} a^{3/2}d} + \frac{(7A - 3B) \sin(c + dx)\sqrt{\cos(c + dx)}}{6ad\sqrt{a \sec(c + dx) + a}} - \frac{6ad}{6ad}$$

[Out] $-1/2*(A-B)*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/d/(a+a*\sec(d*x+c))^{(3/2)}+1/4*(11*A-7*B)*\operatorname{arctanh}(1/2*\sin(d*x+c)*a^{(1/2)}*\sec(d*x+c)^{(1/2)}*2^{(1/2)}/(a+a*\sec(d*x+c))^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/a^{(3/2)}/d*2^{(1/2)}-1/6*(19*A-15*B)*\sin(d*x+c)/a/d/\cos(d*x+c)^{(1/2)}/(a+a*\sec(d*x+c))^{(1/2)}+1/6*(7*A-3*B)*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/a/d/(a+a*\sec(d*x+c))^{(1/2)}$

Rubi [A] time = 0.69, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {2955, 4020, 4022, 4013, 3808, 206}

$$\frac{(11A - 7B)\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \tanh^{-1}\left(\frac{\sqrt{a} \sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a \sec(c+dx)+a}}\right)}{2\sqrt{2} a^{3/2}d} + \frac{(7A - 3B) \sin(c + dx)\sqrt{\cos(c + dx)}}{6ad\sqrt{a \sec(c + dx) + a}} - \frac{6ad}{6ad}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(\operatorname{Cos}[c + d*x]^{(3/2)}*(A + B*\operatorname{Sec}[c + d*x]))/(a + a*\operatorname{Sec}[c + d*x]^{(3/2)}), x]$

[Out] $((11*A - 7*B)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Sqrt}[\operatorname{Sec}[c + d*x]]*\operatorname{Sin}[c + d*x])/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a + a*\operatorname{Sec}[c + d*x]])]*\operatorname{Sqrt}[\operatorname{Cos}[c + d*x]]*\operatorname{Sqrt}[\operatorname{Sec}[c + d*x]])/(2*\operatorname{Sqrt}[2]*a^{(3/2)}*d) - ((A - B)*\operatorname{Sqrt}[\operatorname{Cos}[c + d*x]]*\operatorname{Sin}[c + d*x])/(2*d*(a + a*\operatorname{Sec}[c + d*x]^{(3/2)})) - ((19*A - 15*B)*\operatorname{Sin}[c + d*x])/(6*a*d*\operatorname{Sqrt}[\operatorname{Cos}[c + d*x]]*\operatorname{Sqrt}[a + a*\operatorname{Sec}[c + d*x]]) + ((7*A - 3*B)*\operatorname{Sqrt}[\operatorname{Cos}[c + d*x]]*\operatorname{Sin}[c + d*x])/(6*a*d*\operatorname{Sqrt}[a + a*\operatorname{Sec}[c + d*x]])$

Rule 206

$\operatorname{Int}[(a + (b*x)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1*\operatorname{ArcTanh}[(\operatorname{Rt}[-b, 2]*x)/\operatorname{Rt}[a, 2]])/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /; \operatorname{FreeQ}\{a, b, x\} \ \&\& \operatorname{NegQ}[a/b] \ \&\& (\operatorname{GtQ}[a, 0] \ || \operatorname{LtQ}[b, 0])$

Rule 2955

$\operatorname{Int}[(a + \operatorname{csc}(e + f*x) + (b*x)^m)*(c + \operatorname{csc}(e + f*x) + (d + c*x)^n)*(g*\sin(e + f*x))^p], x_Symbol] \rightarrow \operatorname{Dist}[(g*\operatorname{Csc}[e + f*x])^p*(g*\operatorname{Sin}[e + f*x])^p, \operatorname{Int}[(a + b*\operatorname{Csc}[e + f*x])^m*(c + d$

*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && IntegerQ[n])

Rule 3808

Int[Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 4013

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && !LeQ[m, -1]

Rule 4020

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m + 1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0]

Rule 4022

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a*A*m - b*B*n - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]

Rubi steps

$$\begin{aligned}
\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{3/2}} dx &= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \frac{A+B\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} dx \\
&= -\frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \frac{1}{2}}{2a^2} \\
&= -\frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(7A-3B)\sqrt{\cos(c+dx)}\sin(c+dx)}{6ad\sqrt{a+a\sec(c+dx)}} \\
&= -\frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(19A-15B)\sin(c+dx)}{6ad\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} \\
&= -\frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(19A-15B)\sin(c+dx)}{6ad\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} \\
&= \frac{(11A-7B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{2\sqrt{2}a^{3/2}d}
\end{aligned}$$

Mathematica [A] time = 1.24, size = 155, normalized size = 0.70

$$\frac{\sin(c+dx)\left(\sqrt{1-\sec(c+dx)}(\sec(c+dx)(2A\cos(2(c+dx))-17A+15B)+12(B-A))-3\sqrt{2}(11A-7B)\cos^2\right)}{6d\sqrt{\cos(c+dx)-1}(a(\sec(c+dx)+1))^{3/2}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(3/2), x]

[Out] ((-3*Sqrt[2]*(11*A - 7*B)*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]])*Cos[(c + d*x)/2]^2*Sec[c + d*x]^(3/2) + Sqrt[1 - Sec[c + d*x]]*(12*(-A + B) + (-17*A + 15*B + 2*A*Cos[2*(c + d*x)])*Sec[c + d*x]))*Sin[c + d*x])/(6*d*Sqrt[-1 + Cos[c + d*x]]*(a*(1 + Sec[c + d*x]))^(3/2))

fricas [A] time = 0.51, size = 442, normalized size = 1.98

$$\frac{3\sqrt{2}\left((11A-7B)\cos(dx+c)^2+2(11A-7B)\cos(dx+c)+11A-7B\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^2+2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)}{\cos(dx+c)}}}{\cos(dx+c)}\right)}{24(a^2d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")

[Out] [-1/24*(3*sqrt(2)*((11*A - 7*B)*cos(d*x + c)^2 + 2*(11*A - 7*B)*cos(d*x + c) + 11*A - 7*B)*sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*(4*A*cos(d*x + c)^2 - 12*(A - B)*cos(d*x + c) - 19*A + 15*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), -1/12*(3*sqrt(2)*((11*A - 7*B)*cos(d*x + c)^2 + 2*(11*A - 7*B)*cos(d*x + c) + 11*A - 7*B)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) - 2*(4*A*cos(d*x + c)^2 - 12*(A - B)*cos(d*x + c) - 19*A + 15*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx+c) + A) \cos(dx+c)^{\frac{3}{2}}}{(a \sec(dx+c) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(3/2)/(a*sec(d*x + c) + a)^(3/2), x)

maple [A] time = 2.24, size = 307, normalized size = 1.38

$$\frac{(\sqrt{\cos(dx+c)})\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}(-1+\cos(dx+c))\left(33A\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)\sqrt{\frac{2}{1+\cos(dx+c)}}\cos(dx+c)\right)}{24(a^2d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x)`

[Out] $1/12/d*\cos(d*x+c)^{(1/2)}*(a*(1+\cos(d*x+c))/\cos(d*x+c))^{(1/2)}*(-1+\cos(d*x+c))$
 $* (33*A*\arctan(1/2*\sin(d*x+c)*(-2/(1+\cos(d*x+c))))^{(1/2)}*(-2/(1+\cos(d*x+c)))$
 $^{(1/2)}*\cos(d*x+c)*\sin(d*x+c)-21*B*\arctan(1/2*\sin(d*x+c)*(-2/(1+\cos(d*x+c)))$
 $^{(1/2)}*(-2/(1+\cos(d*x+c)))^{(1/2)}*\cos(d*x+c)*\sin(d*x+c)+33*\arctan(1/2*\sin(d$
 $*x+c)*(-2/(1+\cos(d*x+c)))^{(1/2)}*(-2/(1+\cos(d*x+c)))^{(1/2)}*A*\sin(d*x+c)+8*A$
 $*\cos(d*x+c)^3-21*\arctan(1/2*\sin(d*x+c)*(-2/(1+\cos(d*x+c)))^{(1/2)}*(-2/(1+co$
 $s(d*x+c)))^{(1/2)}*B*\sin(d*x+c)-32*A*\cos(d*x+c)^2+24*B*\cos(d*x+c)^2-14*A*\cos($
 $d*x+c)+6*B*\cos(d*x+c)+38*A-30*B)/a^2/\sin(d*x+c)^3$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")`

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+dx)^{3/2} \left(A + \frac{B}{\cos(c+dx)} \right)}{\left(a + \frac{a}{\cos(c+dx)} \right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((cos(c+d*x)^(3/2)*(A+B/cos(c+d*x)))/(a+a/cos(c+d*x))^(3/2),x)`

[Out] `int((cos(c+d*x)^(3/2)*(A+B/cos(c+d*x)))/(a+a/cos(c+d*x))^(3/2),x)`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)**(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(3/2),x)`

[Out] Timed out

$$3.550 \quad \int \frac{\sqrt{\cos(c+dx)} (A+B \sec(c+dx))}{(a+a \sec(c+dx))^{3/2}} dx$$

Optimal. Leaf size=176

$$\frac{(7A-3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d} + \frac{(5A-B)\sin(c+dx)}{2ad\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

[Out] $-1/2*(A-B)*\sin(d*x+c)/d/(a+a*\sec(d*x+c))^{(3/2)}/\cos(d*x+c)^{(1/2)}-1/4*(7*A-3*B)*\operatorname{arctanh}(1/2*\sin(d*x+c)*a^{(1/2)}*\sec(d*x+c)^{(1/2)}*2^{(1/2)}/(a+a*\sec(d*x+c))^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/a^{(3/2)}/d*2^{(1/2)}+1/2*(5*A-B)*\sin(d*x+c)/a/d/\cos(d*x+c)^{(1/2)}/(a+a*\sec(d*x+c))^{(1/2)}$

Rubi [A] time = 0.49, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2955, 4020, 4013, 3808, 206}

$$\frac{(7A-3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d} + \frac{(5A-B)\sin(c+dx)}{2ad\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(\operatorname{Sqrt}[\operatorname{Cos}[c+d*x]]*(A+B*\operatorname{Sec}[c+d*x]))/(a+a*\operatorname{Sec}[c+d*x])^{(3/2)},x]$

[Out] $-((7*A-3*B)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Sqrt}[\operatorname{Sec}[c+d*x]]*\operatorname{Sin}[c+d*x])/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a+a*\operatorname{Sec}[c+d*x]])]*\operatorname{Sqrt}[\operatorname{Cos}[c+d*x]]*\operatorname{Sqrt}[\operatorname{Sec}[c+d*x]])/(2*\operatorname{Sqrt}[2]*a^{(3/2)}*d)-((A-B)*\operatorname{Sin}[c+d*x])/(2*d*\operatorname{Sqrt}[\operatorname{Cos}[c+d*x]]*(a+a*\operatorname{Sec}[c+d*x])^{(3/2)})+((5*A-B)*\operatorname{Sin}[c+d*x])/(2*a*d*\operatorname{Sqrt}[\operatorname{Cos}[c+d*x]]*\operatorname{Sqrt}[a+a*\operatorname{Sec}[c+d*x]])$

Rule 206

$\operatorname{Int}[(a_+ + (b_+)*(x_+)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1*\operatorname{ArcTanh}[(\operatorname{Rt}[-b, 2]*x)/\operatorname{Rt}[a, 2]])/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /; \operatorname{FreeQ}\{a, b\}, x] \&\& \operatorname{NegQ}[a/b] \&\& (\operatorname{GtQ}[a, 0] \parallel \operatorname{LtQ}[b, 0])$

Rule 2955

$\operatorname{Int}[(a_+ + \operatorname{csc}[e_+ + (f_+)*(x_+)]*(b_+))^{(m_+)}*(\operatorname{csc}[e_+ + (f_+)*(x_+)]*(d_+ + (c_+))^{(n_+)})*((g_+)*\operatorname{sin}[e_+ + (f_+)*(x_+)])^{(p_+)}, x_Symbol] \rightarrow \operatorname{Dist}[(g*\operatorname{Csc}[e+f*x])^p*(g*\operatorname{Sin}[e+f*x])^p, \operatorname{Int}[(a+b*\operatorname{Csc}[e+f*x])^m*(c+d*\operatorname{Csc}[e+f*x])^n]/(g*\operatorname{Csc}[e+f*x])^p, x], x] /; \operatorname{FreeQ}\{a, b, c, d, e, f, g, m, n, p\}, x] \&\& \operatorname{NeQ}[b*c-a*d, 0] \&\& !\operatorname{IntegerQ}[p] \&\& !(\operatorname{IntegerQ}[m] \&\& \operatorname{IntegerQ}[n])$

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.)
+ (a_.)], x_Symbol] := Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x])*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (
a_))^m*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_)), x_Symbol] := Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
- b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] && !LeQ[m, -1]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (
a_))^m*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_)), x_Symbol] := -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
+ f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\cos(c+dx)}(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{3/2}} dx &= (\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}) \int \frac{A+B\sec(c+dx)}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}} dx \\
&= -\frac{(A-B)\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{3/2}} + \frac{(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)})}{2ad\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} \\
&= -\frac{(A-B)\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{3/2}} + \frac{(5A-B)\sin(c+dx)}{2ad\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} \\
&= -\frac{(A-B)\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{3/2}} + \frac{(5A-B)\sin(c+dx)}{2ad\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} \\
&= -\frac{(7A-3B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{2\sqrt{2}a^{3/2}d}
\end{aligned}$$

Mathematica [A] time = 1.94, size = 198, normalized size = 1.12

$$\frac{2 \tan(c+dx)\sqrt{1-\sec(c+dx)}(2A^2 \cos(2(c+dx)) + 2A^2 + A(5A+3B)\cos(c+dx) + 5AB - B^2) + 4\sqrt{2}(7A - 3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{4d\sqrt{\cos(c+dx)-1}(a(\sec(c+dx)+1))^{3/2}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(3/2), x]

[Out] (4*Sqrt[2]*(7*A - 3*B)*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]])*Cos[(c + d*x)/2]^3*(B + A*Cos[c + d*x])*Sec[c + d*x]^(3/2)*Sin[(c + d*x)/2] + 2*(2*A^2 + 5*A*B - B^2 + A*(5*A + 3*B)*Cos[c + d*x] + 2*A^2*Cos[2*(c + d*x)])*Sqrt[1 - Sec[c + d*x]]*Tan[c + d*x]/(4*d*Sqrt[-1 + Cos[c + d*x]])*(B + A*Cos[c + d*x])*(a*(1 + Sec[c + d*x]))^(3/2)

fricas [A] time = 0.45, size = 410, normalized size = 2.33

$$\left[\frac{\sqrt{2}((7A-3B)\cos(dx+c)^2 + 2(7A-3B)\cos(dx+c) + 7A-3B)\sqrt{a} \log\left(-\frac{a\cos(dx+c)^2 - 2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}}{\cos(dx+c)^2}\right)}{8(a^2d\cos(dx+c)^2 + 2a)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")

[Out] [-1/8*(sqrt(2)*((7*A - 3*B)*cos(d*x + c)^2 + 2*(7*A - 3*B)*cos(d*x + c) + 7*A - 3*B)*sqrt(a)*log(-(a*cos(d*x + c))^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*(4*A*cos(d*x + c) + 5*A - B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), 1/4*(sqrt(2)*((7*A - 3*B)*cos(d*x + c)^2 + 2*(7*A - 3*B)*cos(d*x + c) + 7*A - 3*B)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) + 2*(4*A*cos(d*x + c) + 5*A - B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)\sqrt{\cos(dx + c)}}{(a \sec(dx + c) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(a*sec(d*x + c) + a)^(3/2), x)

maple [A] time = 2.32, size = 235, normalized size = 1.34

$$(-1 + \cos(dx + c)) \left(4A \sqrt{-\frac{2}{1 + \cos(dx + c)}} (\cos^2(dx + c)) + 7A \arctan \left(\frac{\sin(dx + c) \sqrt{-\frac{2}{1 + \cos(dx + c)}}}{2} \right) \right) \sin(dx + c) + A \sqrt{-\frac{2}{1 + \cos(dx + c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(3/2),x)

[Out] 1/2/d*(-1+cos(d*x+c))*(4*A*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2+7*A*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*sin(d*x+c)+A*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)-3*B*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*sin(d*x+c)-B*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)-5*A*(-2/(1+cos(d*x+c)))^(1/2)

$$)+B*(-2/(1+\cos(dx+c)))^{1/2})*\cos(dx+c)^{1/2}*(a*(1+\cos(dx+c))/\cos(dx+c))^{1/2}/a^2/(-2/(1+\cos(dx+c)))^{1/2}/\sin(dx+c)^3$$

maxima [B] time = 0.94, size = 8208, normalized size = 46.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(dx+c))*cos(dx+c)^(1/2)/(a+a*sec(dx+c))^(3/2),x, algorith="maxima")

[Out]
$$-1/4*((4*(7*\log(\cos(1/2*d*x + 1/2*c))^2 + \sin(1/2*d*x + 1/2*c))^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 7*\log(\cos(1/2*d*x + 1/2*c))^2 + \sin(1/2*d*x + 1/2*c))^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sin(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c)^4 + 63*(\log(\cos(1/2*d*x + 1/2*c))^2 + \sin(1/2*d*x + 1/2*c))^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c))^2 + \sin(1/2*d*x + 1/2*c))^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^4 + 4*(7*\log(\cos(1/2*d*x + 1/2*c))^2 + \sin(1/2*d*x + 1/2*c))^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 7*\log(\cos(1/2*d*x + 1/2*c))^2 + \sin(1/2*d*x + 1/2*c))^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c)^4 + 70*(\log(\cos(1/2*d*x + 1/2*c))^2 + \sin(1/2*d*x + 1/2*c))^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c))^2 + \sin(1/2*d*x + 1/2*c))^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2*\sin(1/2*d*x + 1/2*c)^2 + 7*(\log(\cos(1/2*d*x + 1/2*c))^2 + \sin(1/2*d*x + 1/2*c))^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c))^2 + \sin(1/2*d*x + 1/2*c))^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^4 - 8*\sin(1/2*d*x + 1/2*c)^5 + 28*(7*(\log(\cos(1/2*d*x + 1/2*c))^2 + \sin(1/2*d*x + 1/2*c))^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c))^2 + \sin(1/2*d*x + 1/2*c))^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c) - 8*\cos(1/2*d*x + 1/2*c))*\sin(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c)^3 + 4*(21*(\log(\cos(1/2*d*x + 1/2*c))^2 + \sin(1/2*d*x + 1/2*c))^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c))^2 + \sin(1/2*d*x + 1/2*c))^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) - 24*\sin(1/2*d*x + 1/2*c)^2 - 20)*\sin(3/2*d*x + 3/2*c)^3 - 8*(10*\cos(1/2*d*x + 1/2*c))^2 + 3)*\sin(1/2*d*x + 1/2*c)^3 + ((7*\log(\cos(1/2*d*x + 1/2*c))^2 + \sin(1/2*d*x + 1/2*c))^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 7*\log(\cos(1/2*d*x + 1/2*c))^2 + \sin(1/2*d*x + 1/2*c))^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sin(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c)^2 + 63*(\log(\cos(1/2*d*x + 1/2*c))^2 + \sin(1/2*d*x + 1/2*c))^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c))^2 + \sin(1/2*d*x + 1/2*c))^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 + (7*\log(\cos(1/2*d*x + 1/2*c))^2 + \sin(1/2*d*x + 1/2*c))^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 7*\log(\cos(1/2*d*x + 1/2*c))^2 + \sin(1/2*d*x + 1/2*c))^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c)^2 + 7*(\log(\cos(1/2*d*x + 1/2*c))^2 + \sin(1/2*d*x + 1/2*c))^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c))^2 + \sin(1/2*d*x + 1/2*c))^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 - 8*\sin(1/2*d*x + 1/2*c)^3 + 6*(7*(\log(\cos(1$$

$$\begin{aligned}
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) \\
&) + 1))*\cos(1/2*d*x + 1/2*c) - 8*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c)) \\
& *\cos(3/2*d*x + 3/2*c) + 2*(7*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/ \\
& 2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2 \\
& *d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) - 8*\sin \\
& (1/2*d*x + 1/2*c)^2 - 8)*\sin(3/2*d*x + 3/2*c) - 8*(9*\cos(1/2*d*x + 1/2*c)^2 \\
& + 2)*\sin(1/2*d*x + 1/2*c))*\cos(5/2*d*x + 5/2*c)^2 + (427*(\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) \\
& *\cos(1/2*d*x + 1/2*c)^2 + 35*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/ \\
& 2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2 \\
& *d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 - 40* \\
& \sin(1/2*d*x + 1/2*c)^3 - 8*(61*\cos(1/2*d*x + 1/2*c)^2 + 9)*\sin(1/2*d*x + 1/ \\
& 2*c))*\cos(3/2*d*x + 3/2*c)^2 + ((7*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 7*\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sin(1/2*d*x + 1/2* \\
& c))*\cos(3/2*d*x + 3/2*c)^2 + 63*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(\\
& 1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 + \\
& (7*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/ \\
& 2*c) + 1) - 7*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1 \\
& /2*d*x + 1/2*c) + 1) - 8*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c)^2 + 7*(\\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
&) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d* \\
& x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 - 8*\sin(1/2*d*x + 1/2*c)^3 + 6*(7*(\\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
&) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d* \\
& x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c) - 8*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x \\
& + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(7*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2 \\
& *d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) \\
&) - 8*\sin(1/2*d*x + 1/2*c)^2 - 8)*\sin(3/2*d*x + 3/2*c) - 8*(9*\cos(1/2*d*x + \\
& 1/2*c)^2 + 2)*\sin(1/2*d*x + 1/2*c))*\sin(5/2*d*x + 5/2*c)^2 + (8*(7*\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) \\
& - 7*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1 \\
& /2*c) + 1) - 8*\sin(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c)^2 + 259*(\log(\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2* \\
& c) + 1))*\cos(1/2*d*x + 1/2*c)^2 + 91*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2* \\
& d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) \\
& ^2 - 104*\sin(1/2*d*x + 1/2*c)^3 + 28*(7*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1 \\
& /2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^ \\
& 2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2
\end{aligned}$$

$$\begin{aligned}
& *c) - 8*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) - 8 \\
& *(37*\cos(1/2*d*x + 1/2*c)^2 + 21)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c \\
&)^2 + 2*(2*(7*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1 \\
& /2*d*x + 1/2*c) + 1) - 7*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^ \\
& 2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sin(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2 \\
& *c)^3 + 63*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2 \\
& *d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - \\
& 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^3 + 7*(\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*c \\
& \cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c)^2 - 8*\cos(1/2*d*x + 1/2*c)*\sin(1/2 \\
& *d*x + 1/2*c)^3 + 13*(7*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^ \\
& 2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c) - 8*\cos(1/2* \\
& d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c)^2 + (2*(7*\log(\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& 7*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1) - 8*\sin(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 7*(\log(\cos(1/2*d* \\
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(c \\
& \cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1 \\
&))*\cos(1/2*d*x + 1/2*c) - 8*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c))*\sin(\\
& 3/2*d*x + 3/2*c)^2 + 2*(84*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2* \\
& c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d \\
& *x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 + 7*(lo \\
& g(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 - 8*\sin(1/2*d*x + 1/2*c)^3 - 16*(6*co \\
& s(1/2*d*x + 1/2*c)^2 + 1)*\sin(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(7 \\
& *(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2* \\
& d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 8*\cos(1/2*d* \\
& x + 1/2*c)*\sin(1/2*d*x + 1/2*c)^2 + 2*(7*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(\\
& 1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/ \\
& 2*c) - 8*\sin(1/2*d*x + 1/2*c)^2 - 8)*\cos(3/2*d*x + 3/2*c) - 8*\cos(1/2*d*x + \\
& 1/2*c))*\sin(3/2*d*x + 3/2*c) - 8*(9*\cos(1/2*d*x + 1/2*c)^3 + 2*\cos(1/2*d*x \\
& + 1/2*c))*\sin(1/2*d*x + 1/2*c))*\cos(5/2*d*x + 5/2*c) + 2*(147*(\log(\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - lo \\
& g(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) \\
& + 1))*\cos(1/2*d*x + 1/2*c)^3 + 35*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + si \\
& n(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)*si \\
& n(1/2*d*x + 1/2*c)^2 - 40*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c)^3 - 56* \\
& (3*\cos(1/2*d*x + 1/2*c)^3 + \cos(1/2*d*x + 1/2*c))*\sin(1/2*d*x + 1/2*c))*\cos \\
& (3/2*d*x + 3/2*c) + 2*(2*(7*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*
\end{aligned}$$

$$\begin{aligned}
& c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 7*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2 \\
& *d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sin(1/2*d*x + 1/2*c))*\sin \\
& (3/2*d*x + 3/2*c)^3 + 63*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
& ^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2*\sin(1/2*d \\
& *x + 1/2*c) + 7*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin \\
& (1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
& ^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^3 - 8*\sin(1/2*d*x + \\
& 1/2*c)^4 + (7*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(\\
& 1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) - 8*\sin(1/2*d*x + 1/2* \\
& c)^2 - 4)*\cos(3/2*d*x + 3/2*c)^2 + (35*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2* \\
& c) - 40*\sin(1/2*d*x + 1/2*c)^2 - 36)*\sin(3/2*d*x + 3/2*c)^2 - 4*(18*\cos(1/2 \\
& *d*x + 1/2*c)^2 + 5)*\sin(1/2*d*x + 1/2*c)^2 + 6*(7*(\log(\cos(1/2*d*x + 1/2*c) \\
&)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d* \\
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/ \\
& 2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 8*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + \\
& 1/2*c)^2 - 4*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) - 36*\cos(1/2*d*x + \\
& 1/2*c)^2 + 2*((7*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin \\
& (1/2*d*x + 1/2*c) + 1) - 7*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2* \\
& c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sin(1/2*d*x + 1/2*c))*\cos(3/2*d*x + \\
& 3/2*c)^2 + 63*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(\\
& 1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 + 14*(\log(\cos(1/2*d* \\
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(c \\
& os(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1 \\
&))*\sin(1/2*d*x + 1/2*c)^2 - 16*\sin(1/2*d*x + 1/2*c)^3 + 6*(7*(\log(\cos(1/2*d \\
& *x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\\
& \cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + \\
& 1))*\cos(1/2*d*x + 1/2*c) - 8*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c))*\cos \\
& (3/2*d*x + 3/2*c) - 4*(18*\cos(1/2*d*x + 1/2*c)^2 + 7)*\sin(1/2*d*x + 1/2*c) \\
&)*\sin(3/2*d*x + 3/2*c))*\sin(5/2*d*x + 5/2*c) + 2*(133*(\log(\cos(1/2*d*x + 1/2 \\
& *c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2* \\
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(\\
& 1/2*d*x + 1/2*c)^2*\sin(1/2*d*x + 1/2*c) + 21*(\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x \\
& + 1/2*c)^3 - 24*\sin(1/2*d*x + 1/2*c)^4 + 2*(21*(\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d* \\
& x + 1/2*c) - 24*\sin(1/2*d*x + 1/2*c)^2 - 20)*\cos(3/2*d*x + 3/2*c)^2 - 8*(19 \\
& *\cos(1/2*d*x + 1/2*c)^2 + 7)*\sin(1/2*d*x + 1/2*c)^2 + 16*(7*(\log(\cos(1/2*d* \\
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(c
\end{aligned}$$

$$\begin{aligned}
& \cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1 \\
&))*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 8*\cos(1/2*d*x + 1/2*c)*\sin(1/ \\
& /2*d*x + 1/2*c)^2 - 5*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) - 80*\cos(1 \\
& /2*d*x + 1/2*c)^2)*\sin(3/2*d*x + 3/2*c) - 8*(9*\cos(1/2*d*x + 1/2*c)^4 + 11* \\
& \cos(1/2*d*x + 1/2*c)^2)*\sin(1/2*d*x + 1/2*c))*A*\sqrt{a}/(4*\sqrt{2}*a^2*\cos(\\
& 3/2*d*x + 3/2*c)^4 + 28*\sqrt{2}*a^2*\cos(3/2*d*x + 3/2*c)^3*\cos(1/2*d*x + 1/ \\
& 2*c) + 9*\sqrt{2}*a^2*\cos(1/2*d*x + 1/2*c)^4 + 4*\sqrt{2}*a^2*\sin(3/2*d*x + 3 \\
& /2*c)^4 + 12*\sqrt{2}*a^2*\sin(3/2*d*x + 3/2*c)^3*\sin(1/2*d*x + 1/2*c) + 10*s \\
& \sqrt{2}*a^2*\cos(1/2*d*x + 1/2*c)^2*\sin(1/2*d*x + 1/2*c)^2 + \sqrt{2}*a^2*\sin(\\
& 1/2*d*x + 1/2*c)^4 + (\sqrt{2}*a^2*\cos(3/2*d*x + 3/2*c)^2 + 6*\sqrt{2}*a^2*co \\
& s(3/2*d*x + 3/2*c)*\cos(1/2*d*x + 1/2*c) + 9*\sqrt{2}*a^2*\cos(1/2*d*x + 1/2*c \\
&)^2 + \sqrt{2}*a^2*\sin(3/2*d*x + 3/2*c)^2 + 2*\sqrt{2}*a^2*\sin(3/2*d*x + 3/2* \\
& c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*a^2*\sin(1/2*d*x + 1/2*c)^2)*\cos(5/2*d*x + \\
& 5/2*c)^2 + (61*\sqrt{2}*a^2*\cos(1/2*d*x + 1/2*c)^2 + 5*\sqrt{2}*a^2*\sin(1/2* \\
& d*x + 1/2*c)^2)*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*a^2*\cos(3/2*d*x + 3/2*c)^ \\
& 2 + 6*\sqrt{2}*a^2*\cos(3/2*d*x + 3/2*c)*\cos(1/2*d*x + 1/2*c) + 9*\sqrt{2}*a^2 \\
& *\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*a^2*\sin(3/2*d*x + 3/2*c)^2 + 2*\sqrt{2}*a^ \\
& 2*\sin(3/2*d*x + 3/2*c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*a^2*\sin(1/2*d*x + 1/2 \\
& *c)^2)*\sin(5/2*d*x + 5/2*c)^2 + (8*\sqrt{2}*a^2*\cos(3/2*d*x + 3/2*c)^2 + 28* \\
& \sqrt{2}*a^2*\cos(3/2*d*x + 3/2*c)*\cos(1/2*d*x + 1/2*c) + 37*\sqrt{2}*a^2*\cos(\\
& 1/2*d*x + 1/2*c)^2 + 13*\sqrt{2}*a^2*\sin(1/2*d*x + 1/2*c)^2)*\sin(3/2*d*x + 3 \\
& /2*c)^2 + 2*(2*\sqrt{2}*a^2*\cos(3/2*d*x + 3/2*c)^3 + 13*\sqrt{2}*a^2*\cos(3/2* \\
& d*x + 3/2*c)^2*\cos(1/2*d*x + 1/2*c) + 9*\sqrt{2}*a^2*\cos(1/2*d*x + 1/2*c)^3 \\
& + \sqrt{2}*a^2*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c)^2 + (2*\sqrt{2}*a^2* \\
& \cos(3/2*d*x + 3/2*c) + \sqrt{2}*a^2*\cos(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2* \\
& c)^2 + 2*(12*\sqrt{2}*a^2*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*a^2*\sin(1/2*d*x + \\
& 1/2*c)^2)*\cos(3/2*d*x + 3/2*c) + 2*(2*\sqrt{2}*a^2*\cos(3/2*d*x + 3/2*c)*\sin \\
& (1/2*d*x + 1/2*c) + \sqrt{2}*a^2*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c))* \\
& \sin(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c) + 2*(21*\sqrt{2}*a^2*\cos(1/2*d*x \\
& + 1/2*c)^3 + 5*\sqrt{2}*a^2*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c)^2)*\cos \\
& (3/2*d*x + 3/2*c) + 2*(2*\sqrt{2}*a^2*\sin(3/2*d*x + 3/2*c)^3 + \sqrt{2}*a^2*c \\
& os(3/2*d*x + 3/2*c)^2*\sin(1/2*d*x + 1/2*c) + 6*\sqrt{2}*a^2*\cos(3/2*d*x + 3/ \\
& 2*c)*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) + 9*\sqrt{2}*a^2*\cos(1/2*d*x \\
& + 1/2*c)^2*\sin(1/2*d*x + 1/2*c) + 5*\sqrt{2}*a^2*\sin(3/2*d*x + 3/2*c)^2*\sin(\\
& 1/2*d*x + 1/2*c) + \sqrt{2}*a^2*\sin(1/2*d*x + 1/2*c)^3 + 2*(\sqrt{2}*a^2*\cos(\\
& 3/2*d*x + 3/2*c)^2 + 6*\sqrt{2}*a^2*\cos(3/2*d*x + 3/2*c)*\cos(1/2*d*x + 1/2*c \\
&) + 9*\sqrt{2}*a^2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*a^2*\sin(1/2*d*x + 1/2* \\
& c)^2)*\sin(3/2*d*x + 3/2*c))*\sin(5/2*d*x + 5/2*c) + 2*(6*\sqrt{2}*a^2*\cos(3/2 \\
& *d*x + 3/2*c)^2*\sin(1/2*d*x + 1/2*c) + 16*\sqrt{2}*a^2*\cos(3/2*d*x + 3/2*c)* \\
& \cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) + 19*\sqrt{2}*a^2*\cos(1/2*d*x + 1/ \\
& 2*c)^2*\sin(1/2*d*x + 1/2*c) + 3*\sqrt{2}*a^2*\sin(1/2*d*x + 1/2*c)^3)*\sin(3/2 \\
& *d*x + 3/2*c)) - (3*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/ \\
& 2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(2*d*x + 2*c)^2 + 12*(\log(\cos(1/2* \\
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log
\end{aligned}$$

```
(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) +
  1))*cos(d*x + c)^2 + 3*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^
  2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x
  + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(2*d*x + 2*c)^2 + 12*(log(cos(
  1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) -
  log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*
  c) + 1))*sin(d*x + c)^2 + 2*(6*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x +
  1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(
  1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(d*x + c) + 3*log(cos(
  1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) -
  3*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2
  *c) + 1) - 2*sin(3/2*d*x + 3/2*c) + 2*sin(1/2*d*x + 1/2*c))*cos(2*d*x + 2*c
  ) + 4*(3*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*
  x + 1/2*c) + 1) - 3*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2
  *sin(1/2*d*x + 1/2*c) + 1) + 2*sin(1/2*d*x + 1/2*c))*cos(d*x + c) + 4*(3*(l
  og(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)
  + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x
  + 1/2*c) + 1))*sin(d*x + c) + cos(3/2*d*x + 3/2*c) - cos(1/2*d*x + 1/2*c))
  *sin(2*d*x + 2*c) - 4*(2*cos(d*x + c) + 1)*sin(3/2*d*x + 3/2*c) + 8*cos(3/2
  *d*x + 3/2*c)*sin(d*x + c) - 8*cos(1/2*d*x + 1/2*c)*sin(d*x + c) + 3*log(co
  s(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1)
  - 3*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x +
  1/2*c) + 1) + 4*sin(1/2*d*x + 1/2*c))*B/((sqrt(2)*a*cos(2*d*x + 2*c)^2 + 4*
  sqrt(2)*a*cos(d*x + c)^2 + sqrt(2)*a*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*a*sin(2
  *d*x + 2*c)*sin(d*x + c) + 4*sqrt(2)*a*sin(d*x + c)^2 + 4*sqrt(2)*a*cos(d*x
  + c) + 2*(2*sqrt(2)*a*cos(d*x + c) + sqrt(2)*a)*cos(2*d*x + 2*c) + sqrt(2)
  *a)*sqrt(a))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\cos(c+dx)} \left(A + \frac{B}{\cos(c+dx)} \right)}{\left(a + \frac{a}{\cos(c+dx)} \right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)^(1/2)*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^(3/2), x)

[Out] int((cos(c + d*x)^(1/2)*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \sqrt{\cos(c + dx)}}{(a (\sec(c + dx) + 1))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)**(1/2)/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sqrt(cos(c + d*x))/(a*(sec(c + d*x) + 1))**(3/2), x)
```

$$3.551 \quad \int \frac{A+B \sec(c+dx)}{\sqrt{\cos(c+dx)} (a+a \sec(c+dx))^{3/2}} dx$$

Optimal. Leaf size=127

$$\frac{(3A + B)\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \tanh^{-1}\left(\frac{\sqrt{a} \sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2} \sqrt{a \sec(c+dx)+a}}\right)}{2\sqrt{2} a^{3/2} d} - \frac{(A - B) \sin(c + dx)}{2d \cos^{\frac{3}{2}}(c + dx)(a \sec(c + dx) + a)^{3/2}}$$

[Out] $-1/2*(A-B)*\sin(d*x+c)/d/\cos(d*x+c)^{(3/2)}/(a+a*\sec(d*x+c))^{(3/2)+1/4*(3*A+B)*\operatorname{arctanh}(1/2*\sin(d*x+c)*a^{(1/2)*\sec(d*x+c)^{(1/2)*2^{(1/2)}}/(a+a*\sec(d*x+c))^{(1/2)})*\cos(d*x+c)^{(1/2)*\sec(d*x+c)^{(1/2)}/a^{(3/2)}/d*2^{(1/2)}$

Rubi [A] time = 0.32, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {2955, 4012, 3808, 206}

$$\frac{(3A + B)\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \tanh^{-1}\left(\frac{\sqrt{a} \sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2} \sqrt{a \sec(c+dx)+a}}\right)}{2\sqrt{2} a^{3/2} d} - \frac{(A - B) \sin(c + dx)}{2d \cos^{\frac{3}{2}}(c + dx)(a \sec(c + dx) + a)^{3/2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(A + B*\text{Sec}[c + d*x])/(\text{Sqrt}[\text{Cos}[c + d*x]]*(a + a*\text{Sec}[c + d*x])^{(3/2)}), x]$

[Out] $((3*A + B)*\text{ArcTanh}[(\text{Sqrt}[a]*\text{Sqrt}[\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(\text{Sqrt}[2]*\text{Sqrt}[a + a*\text{Sec}[c + d*x]])]*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sqrt}[\text{Sec}[c + d*x]])/(2*\text{Sqrt}[2]*a^{(3/2)*d} - ((A - B)*\text{Sin}[c + d*x])/(2*d*\text{Cos}[c + d*x]^{(3/2)}*(a + a*\text{Sec}[c + d*x])^{(3/2)})$

Rule 206

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1*\text{ArcTanh}[\text{Rt}[-b, 2]*x]/\text{Rt}[a, 2])]/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b\}, x \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rule 2955

$\text{Int}[(a_ + \text{csc}[e_ + (f_)*(x_)]*(b_))^{(m_)}*(\text{csc}[e_ + (f_)*(x_)]*(d_ + (c_))^{(n_)}*((g_)*\sin[e_ + (f_)*(x_)])^{(p_)}], x_Symbol] \rightarrow \text{Dist}[(g*\text{Csc}[e + f*x])^p*(g*\text{Sin}[e + f*x])^p, \text{Int}[(a + b*\text{Csc}[e + f*x])^m*(c + d*\text{Csc}[e + f*x])^n]/(g*\text{Csc}[e + f*x])^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, m, n, p\}, x \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ !\text{IntegerQ}[p] \ \&\& \ !(\text{IntegerQ}[m] \ \&\& \ \text{IntegerQ}[n])$

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_.)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4012

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] + Dist[(a*A*m + b*B*(m + 1))/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*
x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x
] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && LeQ[m,
-1]
```

Rubi steps

$$\int \frac{A + B \sec(c + dx)}{\sqrt{\cos(c + dx)} (a + a \sec(c + dx))^{3/2}} dx = \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{\sqrt{\sec(c + dx)} (A + B \sec(c + dx))}{(a + a \sec(c + dx))^{3/2}} dx$$

$$= -\frac{(A - B) \sin(c + dx)}{2d \cos^{\frac{3}{2}}(c + dx) (a + a \sec(c + dx))^{3/2}} + \frac{((3A + B) \sqrt{\cos(c + dx)}) \sqrt{\sec(c + dx)}}{(a + a \sec(c + dx))^{3/2}}$$

$$= -\frac{(A - B) \sin(c + dx)}{2d \cos^{\frac{3}{2}}(c + dx) (a + a \sec(c + dx))^{3/2}} - \frac{((3A + B) \sqrt{\cos(c + dx)}) \sqrt{\sec(c + dx)}}{(a + a \sec(c + dx))^{3/2}}$$

$$= \frac{(3A + B) \tanh^{-1} \left(\frac{\sqrt{a} \sqrt{\sec(c + dx)} \sin(c + dx)}{\sqrt{2} \sqrt{a + a \sec(c + dx)}} \right) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)}}{2\sqrt{2} a^{3/2} d}$$

Mathematica [A] time = 0.52, size = 86, normalized size = 0.68

$$\frac{\frac{1}{2}(B - A) \sin(c + dx) + (3A + B) \cos^3 \left(\frac{1}{2}(c + dx) \right) \tanh^{-1} \left(\sin \left(\frac{1}{2}(c + dx) \right) \right)}{ad \sqrt{\cos(c + dx)} (\cos(c + dx) + 1) \sqrt{a(\sec(c + dx) + 1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(3/2)), x]
```

[Out] $((3A + B) \operatorname{ArcTanh}[\sin[(c + dx)/2]] \cos[(c + dx)/2]^3 + ((-A + B) \sin[c + dx])/2) / (a d \sqrt{\cos[c + dx]} (1 + \cos[c + dx]) \sqrt{a(1 + \sec[c + dx])})$

fricas [A] time = 0.47, size = 376, normalized size = 2.96

$$\frac{\sqrt{2} \left((3A + B) \cos(dx + c)^2 + 2(3A + B) \cos(dx + c) + 3A + B \right) \sqrt{a} \log \left(-\frac{a \cos(dx + c)^2 - 2\sqrt{2} \sqrt{a} \sqrt{\frac{a \cos(dx + c) + a}{\cos(dx + c)}} \sqrt{\cos(dx + c)}}{\cos(dx + c)^2 + 2 \cos(dx + c)} \right)}{8 \left(a^2 d \cos(dx + c)^2 + 2 a^2 d \cos(dx + c) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algorith="fricas")`

[Out] $[1/8 * (\sqrt{2} * ((3A + B) \cos(dx + c)^2 + 2 * (3A + B) \cos(dx + c) + 3A + B) \sqrt{a} * \log(- (a \cos(dx + c)^2 - 2 * \sqrt{2} * \sqrt{a} * \sqrt{(a \cos(dx + c) + a) / \cos(dx + c)}) * \sqrt{\cos(dx + c)} * \sin(dx + c) - 2 * a * \cos(dx + c) - 3 * a) / (\cos(dx + c)^2 + 2 * \cos(dx + c) + 1)) - 4 * (A - B) * \sqrt{(a \cos(dx + c) + a) / \cos(dx + c)} * \sqrt{\cos(dx + c)} * \sin(dx + c)) / (a^2 * d * \cos(dx + c)^2 + 2 * a^2 * d * \cos(dx + c) + a^2 * d), -1/4 * (\sqrt{2} * ((3A + B) \cos(dx + c)^2 + 2 * (3A + B) \cos(dx + c) + 3A + B) \sqrt{-a} * \arctan(\sqrt{2} * \sqrt{-a} * \sqrt{(a \cos(dx + c) + a) / \cos(dx + c)}) * \sqrt{\cos(dx + c)}) / (a * \sin(dx + c))) + 2 * (A - B) * \sqrt{(a \cos(dx + c) + a) / \cos(dx + c)} * \sqrt{\cos(dx + c)} * \sin(dx + c)) / (a^2 * d * \cos(dx + c)^2 + 2 * a^2 * d * \cos(dx + c) + a^2 * d)]$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(a \sec(dx + c) + a)^{\frac{3}{2}} \sqrt{\cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algorith="giac")`

[Out] `integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c))), x)`

maple [A] time = 2.11, size = 209, normalized size = 1.65

$$(-1 + \cos(dx + c)) \left(3A \arctan \left(\frac{\sin(dx+c) \sqrt{-\frac{2}{1+\cos(dx+c)}}}{2} \right) \sin(dx + c) + A \sqrt{-\frac{2}{1+\cos(dx+c)}} \cos(dx + c) + B \arctan \left(\frac{\sin(dx+c) \sqrt{-\frac{2}{1+\cos(dx+c)}}}{2} \right) \right)$$

2d a

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x)

[Out] -1/2/d*(-1+cos(d*x+c))*(3*A*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c))))^(1/2)*sin(d*x+c)+A*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)+B*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c))))^(1/2)*sin(d*x+c)-B*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)-A*(-2/(1+cos(d*x+c)))^(1/2)+B*(-2/(1+cos(d*x+c)))^(1/2))*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/a^2/sin(d*x+c)^3/(-2/(1+cos(d*x+c)))^(1/2)

maxima [B] time = 0.83, size = 2166, normalized size = 17.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")

[Out] 1/4*((3*(log(cos(1/2*d*x + 1/2*c))^2 + sin(1/2*d*x + 1/2*c))^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c))^2 + sin(1/2*d*x + 1/2*c))^2 - 2*sin(1/2*d*x + 1/2*c) + 1)*cos(2*d*x + 2*c)^2 + 12*(log(cos(1/2*d*x + 1/2*c))^2 + sin(1/2*d*x + 1/2*c))^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c))^2 + sin(1/2*d*x + 1/2*c))^2 - 2*sin(1/2*d*x + 1/2*c) + 1)*cos(d*x + c)^2 + 3*(log(cos(1/2*d*x + 1/2*c))^2 + sin(1/2*d*x + 1/2*c))^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c))^2 + sin(1/2*d*x + 1/2*c))^2 - 2*sin(1/2*d*x + 1/2*c) + 1)*sin(2*d*x + 2*c)^2 + 12*(log(cos(1/2*d*x + 1/2*c))^2 + sin(1/2*d*x + 1/2*c))^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c))^2 + sin(1/2*d*x + 1/2*c))^2 - 2*sin(1/2*d*x + 1/2*c) + 1)*sin(d*x + c)^2 + 2*(6*(log(cos(1/2*d*x + 1/2*c))^2 + sin(1/2*d*x + 1/2*c))^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c))^2 + sin(1/2*d*x + 1/2*c))^2 - 2*sin(1/2*d*x + 1/2*c) + 1)*cos(d*x + c) + 3*log(cos(1/2*d*x + 1/2*c))^2 + sin(1/2*d*x + 1/2*c))^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 3*log(cos(1/2*d*x + 1/2*c))^2 + sin(1/2*d*x + 1/2*c))^2 + sin(1/2*d*x + 1/2*c))^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 2*sin(3/2*d*x + 3/2*c) + 2*sin(1/2*d*x + 1/2*c))*cos(2*d*x + 2*c) + 4*(3*log(cos(1/2*d*x + 1/2*c))^2 + sin(1/2*d*x + 1/2*c))^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 3*log(cos(1/2*d*x + 1/2*c))^2 + sin(1/2*d*x + 1/2*c))^2 - 2*sin(1/2*d*x

$$\begin{aligned}
& + 1/2*c) + 1) + 2*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) + 4*(3*(\log(\cos(1/2*d \\
& *x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\\
& \cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + \\
& 1))*\sin(d*x + c) + \cos(3/2*d*x + 3/2*c) - \cos(1/2*d*x + 1/2*c))*\sin(2*d*x + \\
& 2*c) - 4*(2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c) + 8*\cos(3/2*d*x + 3/2*c \\
&)*\sin(d*x + c) - 8*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + 3*\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) \\
& + 4*\sin(1/2*d*x + 1/2*c))*A/((\sqrt{2})*a*\cos(2*d*x + 2*c)^2 + 4*\sqrt{2})*a*\cos \\
& (d*x + c)^2 + \sqrt{2})*a*\sin(2*d*x + 2*c)^2 + 4*\sqrt{2})*a*\sin(2*d*x + 2*c)* \\
& \sin(d*x + c) + 4*\sqrt{2})*a*\sin(d*x + c)^2 + 4*\sqrt{2})*a*\cos(d*x + c) + 2*(2 \\
& *\sqrt{2})*a*\cos(d*x + c) + \sqrt{2})*a*\cos(2*d*x + 2*c) + \sqrt{2})*a*\sqrt{a}) \\
& + (4*(\sin(3/2*d*x + 3/2*c) - \sin(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2 \\
& *d*x + 3/2*c))))*\cos(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c) \\
&)) + 8*(\sin(3/2*d*x + 3/2*c) - \sin(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/ \\
& 2*d*x + 3/2*c))))*\cos(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c) \\
&)) + (2*(2*\cos(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + \\
& 1)*\cos(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + \cos(4/3*a \\
& rctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + 4*\cos(2/3*\arctan2(s \\
& in(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + \sin(4/3*\arctan2(\sin(3/2*d*x \\
& + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + 4*\sin(4/3*\arctan2(\sin(3/2*d*x + 3/2*c) \\
&), \cos(3/2*d*x + 3/2*c))*\sin(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x \\
& + 3/2*c))) + 4*\sin(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c) \\
&))^2 + 4*\cos(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 1)*\log \\
& (\cos(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + \sin(1/3 \\
& *\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + 2*\sin(1/3*\arctan2 \\
& (\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 1) - (2*(2*\cos(2/3*\arctan2(\\
& \sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 1)*\cos(4/3*\arctan2(\sin(3/2*d \\
& *x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + \cos(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \\
& \cos(3/2*d*x + 3/2*c)))^2 + 4*\cos(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2 \\
& *d*x + 3/2*c)))^2 + \sin(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2 \\
& *c)))^2 + 4*\sin(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))*\sin \\
& (2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 4*\sin(2/3*\arct \\
& an2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + 4*\cos(2/3*\arctan2(\sin(\\
& 3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 1)*\log(\cos(1/3*\arctan2(\sin(3/2*d \\
& *x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + \sin(1/3*\arctan2(\sin(3/2*d*x + 3/2*c) \\
&), \cos(3/2*d*x + 3/2*c)))^2 - 2*\sin(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3 \\
& /2*d*x + 3/2*c))) + 1) - 4*(\cos(3/2*d*x + 3/2*c) - \cos(1/3*\arctan2(\sin(3/2* \\
& d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))))*\sin(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \\
& \cos(3/2*d*x + 3/2*c))) - 8*(\cos(3/2*d*x + 3/2*c) - \cos(1/3*\arctan2(\sin(3/2 \\
& *d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))))*\sin(2/3*\arctan2(\sin(3/2*d*x + 3/2*c) \\
& , \cos(3/2*d*x + 3/2*c))) + 4*\sin(3/2*d*x + 3/2*c) - 4*\sin(1/3*\arctan2(\sin(3 \\
& /2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))))*B/((\sqrt{2})*a*\cos(4/3*\arctan2(\sin(\\
& 3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + 4*\sqrt{2})*a*\cos(2/3*\arctan2(si \\
& n(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + \sqrt{2})*a*\sin(4/3*\arctan2(si
\end{aligned}$$

$n(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))^{2} + 4*\sqrt{2}*a*\sin(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) * \sin(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 4*\sqrt{2}*a*\sin(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^{2} + 4*\sqrt{2}*a*\cos(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 2*(2*\sqrt{2}*a*\cos(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))) + \sqrt{2}*a*\cos(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + \sqrt{2}*a*\sqrt{a}))/d$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\sqrt{\cos(c+dx)} \left(a + \frac{a}{\cos(c+dx)}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^(3/2)), x)
 [Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^(3/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \sec(c + dx)}{(a(\sec(c + dx) + 1))^{3/2} \sqrt{\cos(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(3/2)/cos(d*x+c)**(1/2), x)
 [Out] Integral((A + B*sec(c + d*x))/((a*(sec(c + d*x) + 1))**(3/2)*sqrt(cos(c + d*x))), x)

$$3.552 \quad \int \frac{A+B \sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a \sec(c+dx))^{\frac{3}{2}}} dx$$

Optimal. Leaf size=185

$$\frac{(A-5B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d} + \frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}$$

[Out] 1/2*(A-B)*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2)+2*B*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(3/2)/d+1/4*(A-5*B)*arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(3/2)/d*2^(1/2)

Rubi [A] time = 0.53, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2955, 4019, 4023, 3808, 206, 3801, 215}

$$\frac{(A-5B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d} + \frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)),x]

[Out] (2*B*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]/(a^(3/2)*d) + ((A - 5*B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d) + ((A - B)*Sin[c + d*x])/(2*d*Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)])*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_.))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)])*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_.)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)])*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_.)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x])*Sqrt[d*Csc[e + f*x]]], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)])*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] :> Simp[(d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)])*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{A + B \sec(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))^{3/2}} dx &= \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{\sec^{\frac{3}{2}}(c + dx)(A + B \sec(c + dx))}{(a + a \sec(c + dx))^{3/2}} dx \\
&= \frac{(A - B) \sin(c + dx)}{2d \cos^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))^{3/2}} + \frac{\left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right)}{4a} \\
&= \frac{(A - B) \sin(c + dx)}{2d \cos^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))^{3/2}} + \frac{\left((A - 5B) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right)}{4a} \\
&= \frac{(A - B) \sin(c + dx)}{2d \cos^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))^{3/2}} - \frac{\left((A - 5B) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right)}{4a} \\
&= \frac{2B \sinh^{-1} \left(\frac{\sqrt{a} \tan(c + dx)}{\sqrt{a + a \sec(c + dx)}} \right) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} + (A - 5B) \tan(c + dx)}{a^{3/2} d}
\end{aligned}$$

Mathematica [A] time = 1.14, size = 113, normalized size = 0.61

$$\frac{(A - B) \tan\left(\frac{1}{2}(c + dx)\right) + (A - 5B) \cos\left(\frac{1}{2}(c + dx)\right) \tanh^{-1}\left(\sin\left(\frac{1}{2}(c + dx)\right)\right) + 4\sqrt{2} B \cos\left(\frac{1}{2}(c + dx)\right) \tanh^{-1}\left(\sqrt{2} \sin\left(\frac{1}{2}(c + dx)\right)\right)}{2ad\sqrt{\cos(c + dx)}\sqrt{a(\sec(c + dx) + 1)}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)), x]

[Out] ((A - 5*B)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2] + 4*Sqrt[2]*B*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[(c + d*x)/2] + (A - B)*Tan[(c + d*x)/2])/(2*a*d*Sqrt[Cos[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])

fricas [A] time = 0.53, size = 592, normalized size = 3.20

$$\left[\frac{\sqrt{2} \left((A - 5B) \cos(dx + c)^2 + 2(A - 5B) \cos(dx + c) + A - 5B \right) \sqrt{a} \log \left(\frac{a \cos(dx + c)^2 + 2\sqrt{2}\sqrt{a} \sqrt{\frac{a \cos(dx + c) + a}{\cos(dx + c)}} \sqrt{\cos(dx + c)}}{\cos(dx + c)^2 + 2 \cos(dx + c)} \right)}{\dots} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")

[Out] [-1/8*(sqrt(2)*((A - 5*B)*cos(d*x + c)^2 + 2*(A - 5*B)*cos(d*x + c) + A - 5*B)*sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*(A - B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 4*(B*cos(d*x + c)^2 + 2*B*cos(d*x + c) + B)*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), -1/4*(sqrt(2)*((A - 5*B)*cos(d*x + c)^2 + 2*(A - 5*B)*cos(d*x + c) + A - 5*B)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c)))/(a*sin(d*x + c))) - 2*(A - B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 4*(B*cos(d*x + c)^2 + 2*B*cos(d*x + c) + B)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(a \sec(dx + c) + a)^{\frac{3}{2}} \cos(dx + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/2)), x)

maple [A] time = 2.22, size = 303, normalized size = 1.64

$$\left(\sqrt{\cos(dx + c)}\right) (-1 + \cos(dx + c)) \left(2B\sqrt{2} \arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}} (\cos(dx+c)+1+\sin(dx+c))\sqrt{2}}{4}}\right) \sin(dx + c) - 2B\sqrt{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x)

```
[Out] -1/2/d*cos(d*x+c)^(1/2)*(-1+cos(d*x+c))*(2*B*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d*x+c))*2^(1/2))*sin(d*x+c)-2*B*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))*2^(1/2))*sin(d*x+c)+A*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*sin(d*x+c)-A*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)-5*B*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*sin(d*x+c)+B*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)+A*(-2/(1+cos(d*x+c)))^(1/2)-B*(-2/(1+cos(d*x+c)))^(1/2))*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/(-2/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^3/a^2
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\cos(c+dx)^{3/2} \left(a + \frac{a}{\cos(c+dx)}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^(3/2)),x)
```

```
[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(3/2)/(a+a*sec(d*x+c))**(3/2),x)
```

[Out] Timed out

$$3.553 \quad \int \frac{A+B \sec(c+dx)}{\cos^2(c+dx)(a+a \sec(c+dx))^{3/2}} dx$$

Optimal. Leaf size=237

$$\frac{(5A-9B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d} + \frac{(2A-3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{si}}{a^{3/2}d}$$

[Out] $1/2*(A-B)*\sin(d*x+c)/d/\cos(d*x+c)^{(5/2)}/(a+a*\sec(d*x+c))^{(3/2)}+(2*A-3*B)*\operatorname{arcsinh}(a^{(1/2)}*\tan(d*x+c)/(a+a*\sec(d*x+c))^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/a^{(3/2)}/d-1/4*(5*A-9*B)*\operatorname{arctanh}(1/2*\sin(d*x+c)*a^{(1/2)}*\sec(d*x+c)^{(1/2)}*2^{(1/2)}/(a+a*\sec(d*x+c))^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/a^{(3/2)}/d*2^{(1/2)}-1/2*(A-3*B)*\sin(d*x+c)/a/d/\cos(d*x+c)^{(3/2)}/(a+a*\sec(d*x+c))^{(1/2)}$

Rubi [A] time = 0.74, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {2955, 4019, 4021, 4023, 3808, 206, 3801, 215}

$$\frac{(5A-9B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d} + \frac{(2A-3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{si}}{a^{3/2}d}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(A+B*\operatorname{Sec}[c+d*x])/(\operatorname{Cos}[c+d*x]^{(5/2)}*(a+a*\operatorname{Sec}[c+d*x])^{(3/2)}),x]$

[Out] $((2*A-3*B)*\operatorname{ArcSinh}[(\operatorname{Sqrt}[a]*\operatorname{Tan}[c+d*x])/(\operatorname{Sqrt}[a+a*\operatorname{Sec}[c+d*x]])]*\operatorname{Sqrt}[\operatorname{Cos}[c+d*x]]*\operatorname{Sqrt}[\operatorname{Sec}[c+d*x]])/(a^{(3/2)}*d)-((5*A-9*B)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Sqrt}[\operatorname{Sec}[c+d*x]]*\operatorname{Sin}[c+d*x])/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a+a*\operatorname{Sec}[c+d*x]])]*\operatorname{Sqrt}[\operatorname{Cos}[c+d*x]]*\operatorname{Sqrt}[\operatorname{Sec}[c+d*x]])/(2*\operatorname{Sqrt}[2]*a^{(3/2)}*d)+((A-B)*\operatorname{Sin}[c+d*x])/((2*d*\operatorname{Cos}[c+d*x]^{(5/2)}*(a+a*\operatorname{Sec}[c+d*x])^{(3/2)})-((A-3*B)*\operatorname{Sin}[c+d*x])/((2*a*d*\operatorname{Cos}[c+d*x]^{(3/2)}*\operatorname{Sqrt}[a+a*\operatorname{Sec}[c+d*x]]))$

Rule 206

$\operatorname{Int}[(a_+ + (b_-)*(x_-)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1*\operatorname{ArcTanh}[(\operatorname{Rt}[-b, 2]*x)/\operatorname{Rt}[a, 2]])/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /;$ FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 215

$\operatorname{Int}[1/\operatorname{Sqrt}[(a_+ + (b_-)*(x_-)^2)], x_Symbol] \rightarrow \operatorname{Simp}[\operatorname{ArcSinh}[(\operatorname{Rt}[b, 2]*x)/\operatorname{Sqrt}[a]]/\operatorname{Rt}[b, 2], x] /;$ FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 2955

Int[((a_.) + csc[(e_.) + (f_.)*(x_)])*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)])*(d_.) + (c_.))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dist[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && IntegerQ[n])

Rule 3801

Int[Sqrt[csc[(e_.) + (f_.)*(x_)])*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)])*(b_.) + (a_.)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 + x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]

Rule 3808

Int[Sqrt[csc[(e_.) + (f_.)*(x_)])*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)])*(b_.) + (a_.)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x])*Sqrt[d*Csc[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 4019

Int[(csc[(e_.) + (f_.)*(x_)])*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)])*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)])*(B_.) + (A_.)), x_Symbol] :> Simp[(d*(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0]

Rule 4021

Int[(csc[(e_.) + (f_.)*(x_)])*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)])*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)])*(B_.) + (A_.)), x_Symbol] :> -Simp[(B*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && GtQ[n, 1]

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (
a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{A + B \sec(c + dx)}{\cos^{\frac{5}{2}}(c + dx)(a + a \sec(c + dx))^{3/2}} dx &= \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{\sec^{\frac{5}{2}}(c + dx)(A + B \sec(c + dx))}{(a + a \sec(c + dx))^{3/2}} dx \\
&= \frac{(A - B) \sin(c + dx)}{2d \cos^{\frac{5}{2}}(c + dx)(a + a \sec(c + dx))^{3/2}} + \frac{\left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right)}{2ad \cos^{\frac{3}{2}}(c + dx) \sqrt{a + a \sec(c + dx)}} \\
&= \frac{(A - B) \sin(c + dx)}{2d \cos^{\frac{5}{2}}(c + dx)(a + a \sec(c + dx))^{3/2}} - \frac{(A - 3B) \sin(c + dx)}{2ad \cos^{\frac{3}{2}}(c + dx) \sqrt{a + a \sec(c + dx)}} \\
&= \frac{(A - B) \sin(c + dx)}{2d \cos^{\frac{5}{2}}(c + dx)(a + a \sec(c + dx))^{3/2}} - \frac{(A - 3B) \sin(c + dx)}{2ad \cos^{\frac{3}{2}}(c + dx) \sqrt{a + a \sec(c + dx)}} \\
&= \frac{(A - B) \sin(c + dx)}{2d \cos^{\frac{5}{2}}(c + dx)(a + a \sec(c + dx))^{3/2}} - \frac{(A - 3B) \sin(c + dx)}{2ad \cos^{\frac{3}{2}}(c + dx) \sqrt{a + a \sec(c + dx)}} \\
&= \frac{(2A - 3B) \sinh^{-1} \left(\frac{\sqrt{a} \tan(c + dx)}{\sqrt{a + a \sec(c + dx)}} \right) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)}}{a^{3/2} d} - \frac{(5A - 9B) \sin(c + dx)}{2ad \cos^{\frac{3}{2}}(c + dx) \sqrt{a + a \sec(c + dx)}}
\end{aligned}$$

Mathematica [A] time = 2.28, size = 288, normalized size = 1.22

$$\frac{\sin(c + dx) \sqrt{\cos(c + dx)} \sec^{\frac{5}{2}}(c + dx) \left(4(A - 3B) \cos^2 \left(\frac{1}{2}(c + dx) \right) \sin^{-1} \left(\sqrt{1 - \sec(c + dx)} \right) - 2\sqrt{2} (5A - 9B) \right)}{a^{3/2} d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x])/((Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/2)), x]
```

```
[Out] -1/4*(Sqrt[Cos[c + d*x]]*Sec[c + d*x]^(5/2)*(4*(A - 3*B)*ArcSin[Sqrt[1 - Sec[c + d*x]]]*Cos[(c + d*x)/2]^2 + 20*A*ArcSin[Sqrt[Sec[c + d*x]]]*Cos[(c + d*x)/2]^2 - 36*B*ArcSin[Sqrt[Sec[c + d*x]]]*Cos[(c + d*x)/2]^2 - 2*Sqrt[2]*(5*A - 9*B)*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Cos[(c + d*x)/2]^2 - 4*B*Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])] + 2*A*Cos[c + d*x]*Sqrt[(-1 + Cos[c + d*x])*Sec[c + d*x]^2] - 6*B*Cos[c + d*x]*Sqrt[(-1 + Cos[c + d*x])*Sec[c + d*x]^2])*Sin[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(3/2))
```

fricas [A] time = 0.58, size = 716, normalized size = 3.02

$$\frac{\sqrt{2} \left((5A - 9B) \cos(dx + c)^3 + 2(5A - 9B) \cos(dx + c)^2 + (5A - 9B) \cos(dx + c) \right) \sqrt{a} \log \left(-\frac{a \cos(dx + c)^2 - 2\sqrt{2}}{\dots} \right)}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x, algorith="fricas")
```

```
[Out] [-1/8*(sqrt(2)*((5*A - 9*B)*cos(d*x + c)^3 + 2*(5*A - 9*B)*cos(d*x + c)^2 + (5*A - 9*B)*cos(d*x + c))*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*((A - 3*B)*cos(d*x + c) - 2*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 2*((2*A - 3*B)*cos(d*x + c)^3 + 2*(2*A - 3*B)*cos(d*x + c)^2 + (2*A - 3*B)*cos(d*x + c))*sqrt(a)*log((a*cos(d*x + c)^3 + 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(a^2*d*cos(d*x + c)^3 + 2*a^2*d*cos(d*x + c)^2 + a^2*d*cos(d*x + c)), 1/4*(sqrt(2)*((5*A - 9*B)*cos(d*x + c)^3 + 2*(5*A - 9*B)*cos(d*x + c)^2 + (5*A - 9*B)*cos(d*x + c))*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) - 2*((A - 3*B)*cos(d*x + c) - 2*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 2*((2*A - 3*B)*cos(d*x + c)^3 + 2*(2*A - 3*B)*cos(d*x + c)^2 + (2*A - 3*B)*cos(d*x + c))*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(a^2*d*cos(d*x + c)^3 + 2*a^2*d*cos(d*x + c)^2 + a^2*d*cos(d*x + c))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(a \sec(dx + c) + a)^{\frac{3}{2}} \cos(dx + c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(5/2)), x)

maple [B] time = 2.24, size = 468, normalized size = 1.97

$$(-1 + \cos(dx + c)) \left(2A \cos(dx + c) \sin(dx + c) \sqrt{2} \arctan \left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}} (\cos(dx+c)+1-\sin(dx+c)) \sqrt{2}}{4} \right) \right) - 2A \cos(dx + c)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x)

[Out] 1/2/d*(-1+cos(d*x+c))*(2*A*cos(d*x+c)*sin(d*x+c)*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))*2^(1/2))-2*A*cos(d*x+c)*sin(d*x+c)*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d*x+c))*2^(1/2))-3*B*cos(d*x+c)*sin(d*x+c)*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))*2^(1/2))+3*B*cos(d*x+c)*sin(d*x+c)*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d*x+c))*2^(1/2))+5*A*cos(d*x+c)*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))-A*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2-9*B*cos(d*x+c)*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))+3*B*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2+A*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)-B*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)-2*B*(-2/(1+cos(d*x+c)))^(1/2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)/sin(d*x+c)^3/(-2/(1+cos(d*x+c)))^(1/2)/a^2

maxima [B] time = 1.43, size = 7057, normalized size = 29.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")

```
[Out] 1/4*((4*(sin(2*d*x + 2*c) + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))))*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*(sqrt(2)*
cos(2*d*x + 2*c)^2 + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c)))^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*sin(2*d*x + 2*c)*sin(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*sin(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt
(2))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*cos(2
*d*x + 2*c) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt
(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1
/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 2*(sqrt(2)*cos(2*d*x
+ 2*c)^2 + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^
2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*sin(2*d*x + 2*c)*sin(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c)))^2 + 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*cos(2*d*x + 2*
c) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2
+ 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(
1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 2*(sqrt(2)*cos(2*d*x + 2*c)^2
+ 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(
2)*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c)))^2 + 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt
(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1
/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) + 2) - 2*(sqrt(2)*cos(2*d*x + 2*c)^2 + 4*sqrt
(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*sin(2*
d*x + 2*c)^2 + 4*sqrt(2)*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c)))^2 + 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(
2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) + 2) - 5*(cos(2*d*x + 2*c)^2 + 4*(cos(2*d*x + 2*c) + 1
)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*cos(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(2*d*x + 2*c)^2 + 4*sin(2*d*x +
2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sin(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*cos(2*d*x + 2*c) + 1)*log(co
s(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/4*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 1) + 5*(cos(2*d*x + 2*c)^2 + 4*(cos(2*d*x + 2*c) + 1)*co
```

$$\begin{aligned}
& s(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 4*\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + \sin(2*d*x + 2*c)^2 + 4*\sin(2*d*x + 2*c) \\
&)*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 4*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + 2*\cos(2*d*x + 2*c) + 1)*\log(\cos(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + \sin(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 - 2*\sin(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 1) - 4*\cos(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) * \sin(2*d*x + 2*c) - 4*(\cos(2*d*x + 2*c) + 2*\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 1)*\sin(3/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))) - 8*\cos(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) * \sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 4*(\cos(2*d*x + 2*c) + 1)*\sin(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))) + 8*\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) * \sin(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))) * A / ((\sqrt{2}*a*\cos(2*d*x + 2*c)^2 + 4*\sqrt{2}*a*\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + \sqrt{2}*a*\sin(2*d*x + 2*c)^2 + 4*\sqrt{2}*a*\sin(2*d*x + 2*c)*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))) + 4*\sqrt{2}*a*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + 2*\sqrt{2}*a*\cos(2*d*x + 2*c) + 4*(\sqrt{2}*a*\cos(2*d*x + 2*c) + \sqrt{2}*a)*\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + \sqrt{2}*a)*\sqrt{a}) - (12*(\sin(4*d*x + 4*c) + 2*\sin(2*d*x + 2*c) + 2*\sin(3/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))) + 2*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))) * \cos(7/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) - 8*(\sin(5/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))) - \sin(3/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))) - 3*\sin(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))) * \cos(3/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 4*(\sin(4*d*x + 4*c) + 2*\sin(2*d*x + 2*c) + 2*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))) * \cos(5/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) - 4*(\sin(4*d*x + 4*c) + 2*\sin(2*d*x + 2*c) + 2*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))) * \cos(3/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) - 12*(\sin(4*d*x + 4*c) + 2*\sin(2*d*x + 2*c)) * \cos(1/4*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 3*(\sqrt{2}*\cos(4*d*x + 4*c)^2 + 4*\sqrt{2}*\cos(2*d*x + 2*c)^2 + 4*\sqrt{2}*\cos(3/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + 4*\sqrt{2}*\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + \sqrt{2}*\sin(4*d*x + 4*c)^2 + 4*\sqrt{2}*\sin(4*d*x + 4*c)*\sin(2*d*x + 2*c) + 4*\sqrt{2}*\sin(2*d*x + 2*c)^2 + 4*\sqrt{2}*\sin(3/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + 4*\sqrt{2}*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + 2*(2*\sqrt{2}*\cos(2*d*x + 2*c) + \sqrt{2})*\cos(4*d*x + 4*c) + 4*(\sqrt{2}*\cos(4*d*x + 4*c) + 2*\sqrt{2}*\cos(2*d*x + 2*c) + 2*\sqrt{2}*\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + \sqrt{2})*\cos(3/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 4*(\sqrt{2}*\cos(4*d*x + 4*c) + 2*\sqrt{2}*\cos(2*d*x + 2*c) + \sqrt{2})*\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 4*(\sqrt{2}*\sin(4*d*x + 4*c) + 2*\sqrt{2}*\sin(2*d*x + 2*c) + 2*\sqrt{2}*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))) * \sin(3/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 4*(\sqrt{2}*\sin(4*d*x + 4*c) + 2*\sqrt{2}*\sin(2*d*x + 2*c)) * \sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 4*\sqrt{2}*\cos(
\end{aligned}$$


```

2*c) + 2*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*sin(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) - 3*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 24*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) * sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 12*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 24*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) * sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*B/((sqrt(2)*a*cos(4*d*x + 4*c)^2 + 4*sqrt(2)*a*cos(2*d*x + 2*c)^2 + 4*sqrt(2)*a*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*sqrt(2)*a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*a*sin(4*d*x + 4*c)^2 + 4*sqrt(2)*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sqrt(2)*a*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*a*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*sqrt(2)*a*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*sqrt(2)*a*cos(2*d*x + 2*c) + 2*(2*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a*cos(4*d*x + 4*c) + 4*(sqrt(2)*a*cos(4*d*x + 4*c) + 2*sqrt(2)*a*cos(2*d*x + 2*c) + 2*sqrt(2)*a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + sqrt(2)*a*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(sqrt(2)*a*cos(4*d*x + 4*c) + 2*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 4*(sqrt(2)*a*sin(4*d*x + 4*c) + 2*sqrt(2)*a*sin(2*d*x + 2*c) + 2*sqrt(2)*a*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(sqrt(2)*a*sin(4*d*x + 4*c) + 2*sqrt(2)*a*sin(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2)*a)*sqrt(a))/d

```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\cos(c+dx)^{5/2} \left(a + \frac{a}{\cos(c+dx)}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))^(3/2)),x)

[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))^(3/2)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(5/2)/(a+a*sec(d*x+c))**(3/2),x)

[Out] Timed out

$$3.554 \quad \int \frac{A+B \sec(c+dx)}{\cos^2(c+dx)(a+a \sec(c+dx))^{3/2}} dx$$

Optimal. Leaf size=287

$$\frac{(9A - 13B)\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \tanh^{-1}\left(\frac{\sqrt{a} \sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2} \sqrt{a \sec(c+dx)+a}}\right)}{2\sqrt{2} a^{3/2} d} - \frac{(12A - 19B)\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \operatorname{arcsinh}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a + a \sec(c+dx)}}\right)}{4a^{3/2} d}$$

[Out] 1/2*(A-B)*sin(d*x+c)/d/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(3/2)-1/4*(12*A-19*B)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(3/2)/d+1/4*(9*A-13*B)*arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(3/2)/d*2^(1/2)-1/2*(A-2*B)*sin(d*x+c)/a/d/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2)+1/4*(6*A-7*B)*sin(d*x+c)/a/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2)

Rubi [A] time = 0.95, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {2955, 4019, 4021, 4023, 3808, 206, 3801, 215}

$$\frac{(9A - 13B)\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \tanh^{-1}\left(\frac{\sqrt{a} \sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2} \sqrt{a \sec(c+dx)+a}}\right)}{2\sqrt{2} a^{3/2} d} - \frac{(12A - 19B)\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \operatorname{arcsinh}\left(\frac{\sqrt{a} \tan(c+dx)}{\sqrt{a + a \sec(c+dx)}}\right)}{4a^{3/2} d}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(3/2)),x]

[Out] -((12*A - 19*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*a^(3/2)*d) + ((9*A - 13*B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d) + ((A - B)*Sin[c + d*x])/(2*d*Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(3/2)) - ((A - 2*B)*Sin[c + d*x])/(2*a*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) + ((6*A - 7*B)*Sin[c + d*x])/(4*a*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 215

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 2955

Int[((a_) + csc[(e_) + (f_)*(x_)])*(b_)^(m_)*(csc[(e_) + (f_)*(x_)]*(d_) + (c_))^(n_)*((g_)*sin[(e_) + (f_)*(x_)])^(p_), x_Symbol] := Dist[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && IntegerQ[n])

Rule 3801

Int[Sqrt[csc[(e_) + (f_)*(x_)]*(d_)]*Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)], x_Symbol] := Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 + x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]

Rule 3808

Int[Sqrt[csc[(e_) + (f_)*(x_)]*(d_)]/Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)], x_Symbol] := Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 4019

Int[(csc[(e_) + (f_)*(x_)]*(d_))^(n_)*(csc[(e_) + (f_)*(x_)]*(b_) + (a_))^(m_)*(csc[(e_) + (f_)*(x_)]*(B_) + (A_)), x_Symbol] := Simp[(d*(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0]

Rule 4021

Int[(csc[(e_) + (f_)*(x_)]*(d_))^(n_)*(csc[(e_) + (f_)*(x_)]*(b_) + (a_))^(m_)*(csc[(e_) + (f_)*(x_)]*(B_) + (A_)), x_Symbol] := -Simp[(B*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &&

GtQ[n, 1]

Rule 4023

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> Dist[(A*b - a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{A + B \sec(c + dx)}{\cos^{\frac{7}{2}}(c + dx)(a + a \sec(c + dx))^{3/2}} dx &= \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{\sec^{\frac{7}{2}}(c + dx)(A + B \sec(c + dx))}{(a + a \sec(c + dx))^{3/2}} dx \\
 &= \frac{(A - B) \sin(c + dx)}{2d \cos^{\frac{7}{2}}(c + dx)(a + a \sec(c + dx))^{3/2}} + \frac{\left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right)}{\dots} \\
 &= \frac{(A - B) \sin(c + dx)}{2d \cos^{\frac{7}{2}}(c + dx)(a + a \sec(c + dx))^{3/2}} - \frac{(A - 2B) \sin(c + dx)}{2ad \cos^{\frac{5}{2}}(c + dx) \sqrt{a + a \sec(c + dx)}} \\
 &= \frac{(A - B) \sin(c + dx)}{2d \cos^{\frac{7}{2}}(c + dx)(a + a \sec(c + dx))^{3/2}} - \frac{(A - 2B) \sin(c + dx)}{2ad \cos^{\frac{5}{2}}(c + dx) \sqrt{a + a \sec(c + dx)}} \\
 &= \frac{(A - B) \sin(c + dx)}{2d \cos^{\frac{7}{2}}(c + dx)(a + a \sec(c + dx))^{3/2}} - \frac{(A - 2B) \sin(c + dx)}{2ad \cos^{\frac{5}{2}}(c + dx) \sqrt{a + a \sec(c + dx)}} \\
 &= \frac{(A - B) \sin(c + dx)}{2d \cos^{\frac{7}{2}}(c + dx)(a + a \sec(c + dx))^{3/2}} - \frac{(A - 2B) \sin(c + dx)}{2ad \cos^{\frac{5}{2}}(c + dx) \sqrt{a + a \sec(c + dx)}} \\
 &= \frac{(A - B) \sin(c + dx)}{2d \cos^{\frac{7}{2}}(c + dx)(a + a \sec(c + dx))^{3/2}} - \frac{(A - 2B) \sin(c + dx)}{2ad \cos^{\frac{5}{2}}(c + dx) \sqrt{a + a \sec(c + dx)}} \\
 &= -\frac{(12A - 19B) \sinh^{-1} \left(\frac{\sqrt{a} \tan(c + dx)}{\sqrt{a + a \sec(c + dx)}} \right) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)}}{4a^{3/2}d} + \dots
 \end{aligned}$$

Mathematica [A] time = 3.52, size = 328, normalized size = 1.14

$$\sin(c + dx) \sec^2(c + dx) \left(2(6A - 7B) \cos^2\left(\frac{1}{2}(c + dx)\right) \sin^{-1}\left(\sqrt{1 - \sec(c + dx)}\right) - 2\sqrt{2}(9A - 13B) \cos^2\left(\frac{1}{2}(c + dx)\right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(3/2)), x]

[Out] (Sec[c + d*x]^(3/2)*(2*(6*A - 7*B)*ArcSin[Sqrt[1 - Sec[c + d*x]]])*Cos[(c + d*x)/2]^2 + 36*A*ArcSin[Sqrt[Sec[c + d*x]]]*Cos[(c + d*x)/2]^2 - 52*B*ArcSin[Sqrt[Sec[c + d*x]]]*Cos[(c + d*x)/2]^2 - 2*Sqrt[2]*(9*A - 13*B)*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Cos[(c + d*x)/2]^2 + 2*B*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(3/2) + 4*A*Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])] - 3*B*Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])] + 6*A*Cos[c + d*x]*Sqrt[(-1 + Cos[c + d*x])*Sec[c + d*x]^2] - 7*B*Cos[c + d*x]*Sqrt[(-1 + Cos[c + d*x])*Sec[c + d*x]^2])*Sin[c + d*x])/(4*d*Sqrt[-1 + Cos[c + d*x]])*(a*(1 + Sec[c + d*x]))^(3/2))

fricas [A] time = 0.60, size = 764, normalized size = 2.66

$$\frac{2\sqrt{2}\left((9A - 13B)\cos(dx + c)^4 + 2(9A - 13B)\cos(dx + c)^3 + (9A - 13B)\cos(dx + c)^2\right)\sqrt{a}\log\left(-\frac{a\cos(dx + c)}{\dots}\right)}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(3/2), x, algorithm="fricas")

[Out] [-1/16*(2*sqrt(2))*((9*A - 13*B)*cos(d*x + c)^4 + 2*(9*A - 13*B)*cos(d*x + c)^3 + (9*A - 13*B)*cos(d*x + c)^2)*sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*((6*A - 7*B)*cos(d*x + c)^2 + (4*A - 3*B)*cos(d*x + c) + 2*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + ((12*A - 19*B)*cos(d*x + c)^4 + 2*(12*A - 19*B)*cos(d*x + c)^3 + (12*A - 19*B)*cos(d*x + c)^2)*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(a^2*d*cos(d*x + c)^4 + 2*a^2*d*cos(d*x + c)^3 + a^2*d*cos(d*x + c)^2), -1/8*(2*sqrt(2))*((9*A -

$13*B*\cos(dx + c)^4 + 2*(9*A - 13*B)*\cos(dx + c)^3 + (9*A - 13*B)*\cos(dx + c)^2*\sqrt{-a}*\arctan(\sqrt{2}*\sqrt{-a}*\sqrt{(a*\cos(dx + c) + a)/\cos(dx + c)})*\sqrt{\cos(dx + c)/(a*\sin(dx + c))} - 2*((6*A - 7*B)*\cos(dx + c)^2 + (4*A - 3*B)*\cos(dx + c) + 2*B)*\sqrt{(a*\cos(dx + c) + a)/\cos(dx + c)}*\sqrt{\cos(dx + c)}*\sin(dx + c) + ((12*A - 19*B)*\cos(dx + c)^4 + 2*(12*A - 19*B)*\cos(dx + c)^3 + (12*A - 19*B)*\cos(dx + c)^2*\sqrt{-a}*\arctan(2*\sqrt{-a}*\sqrt{(a*\cos(dx + c) + a)/\cos(dx + c)})*\sqrt{\cos(dx + c)}*\sin(dx + c)/(a*\cos(dx + c)^2 - a*\cos(dx + c) - 2*a)))/(a^2*d*\cos(dx + c)^4 + 2*a^2*d*\cos(dx + c)^3 + a^2*d*\cos(dx + c)^2)]$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(a \sec(dx + c) + a)^{\frac{3}{2}} \cos(dx + c)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(dx+c))/cos(dx+c)^(7/2)/(a+a*sec(dx+c))^(3/2),x, algorithm="giac")

[Out] integrate((B*sec(dx + c) + A)/((a*sec(dx + c) + a)^(3/2)*cos(dx + c)^(7/2)), x)

maple [B] time = 2.04, size = 531, normalized size = 1.85

$$(-1 + \cos(dx + c)) \sqrt{\frac{a(1 + \cos(dx + c))}{\cos(dx + c)}} \left(-12A\sqrt{2} \arctan\left(\frac{\sqrt{-\frac{2}{1 + \cos(dx + c)}} (\cos(dx + c) + 1 - \sin(dx + c)) \sqrt{2}}{4}}\right) \sin(dx + c) (\cos^2(dx + c) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(dx+c))/cos(dx+c)^(7/2)/(a+a*sec(dx+c))^(3/2),x)

[Out] 1/8/d*(-1+cos(dx+c))*(a*(1+cos(dx+c))/cos(dx+c))^(1/2)*(-12*A*2^(1/2)*arctan(1/4*(-2/(1+cos(dx+c)))^(1/2)*(cos(dx+c)+1-sin(dx+c))*2^(1/2))*sin(dx+c)*cos(dx+c)^2+12*A*2^(1/2)*arctan(1/4*(-2/(1+cos(dx+c)))^(1/2)*(cos(dx+c)+1+sin(dx+c))*2^(1/2))*sin(dx+c)*cos(dx+c)^2+19*B*2^(1/2)*arctan(1/4*(-2/(1+cos(dx+c)))^(1/2)*(cos(dx+c)+1-sin(dx+c))*2^(1/2))*sin(dx+c)*cos(dx+c)^2+19*B*2^(1/2)*arctan(1/4*(-2/(1+cos(dx+c)))^(1/2)*(cos(dx+c)+1+sin(dx+c))*2^(1/2))*sin(dx+c)*cos(dx+c)^2+12*A*(-2/(1+cos(dx+c)))^(1/2)*cos(dx+c)^3-36*A*arctan(1/2*sin(dx+c)*(-2/(1+cos(dx+c)))^(1/2))*sin(dx+c)*cos(dx+c)^2-14*B*(-2/(1+cos(dx+c)))^(1/2)*cos(dx+c)^3+52*B*arctan(1/2*sin(dx+c)*(-2/(1+cos(dx+c)))^(1/2))*sin(dx+c)*cos(dx+c)^2-4*A*(-2/(1+cos(dx+c)))^(1/2)*cos(dx+c)^2+8*B*(-2/(1+cos(dx+c)))^(1/2)*cos(dx+c)^2

$-8*A*(-2/(1+\cos(d*x+c)))^{(1/2)}*\cos(d*x+c)+10*B*(-2/(1+\cos(d*x+c)))^{(1/2)}*\cos(d*x+c)-4*B*(-2/(1+\cos(d*x+c)))^{(1/2)}/a^2/\sin(d*x+c)^3/(-2/(1+\cos(d*x+c)))^{(1/2)}/\cos(d*x+c)^{(3/2)}$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\cos(c+dx)^{7/2} \left(a + \frac{a}{\cos(c+dx)}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))^(3/2)),x)

[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))^(3/2)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(7/2)/(a+a*sec(d*x+c))**(3/2),x)

[Out] Timed out

$$3.555 \quad \int \frac{\cos^2(c+dx)(A+B \sec(c+dx))}{(a+a \sec(c+dx))^{5/2}} dx$$

Optimal. Leaf size=317

$$\frac{(283A - 163B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d} + \frac{(157A - 85B)\sin(c+dx)\cos^3(c+dx)}{80a^2d\sqrt{a\sec(c+dx)+a}}$$

[Out] $-1/4*(A-B)*\cos(d*x+c)^{(3/2)}*\sin(d*x+c)/d/(a+a*\sec(d*x+c))^{(5/2)}-1/16*(21*A-13*B)*\cos(d*x+c)^{(3/2)}*\sin(d*x+c)/a/d/(a+a*\sec(d*x+c))^{(3/2)}-1/32*(283*A-163*B)*\operatorname{arctanh}(1/2*\sin(d*x+c)*a^{(1/2)}*\sec(d*x+c)^{(1/2)}*2^{(1/2)}/(a+a*\sec(d*x+c))^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/a^{(5/2)}/d*2^{(1/2)}+1/80*(157*A-85*B)*\cos(d*x+c)^{(3/2)}*\sin(d*x+c)/a^2/d/(a+a*\sec(d*x+c))^{(1/2)}+1/240*(2671*A-1495*B)*\sin(d*x+c)/a^2/d/\cos(d*x+c)^{(1/2)}/(a+a*\sec(d*x+c))^{(1/2)}-1/240*(787*A-475*B)*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/a^2/d/(a+a*\sec(d*x+c))^{(1/2)}$

Rubi [A] time = 1.12, antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {2955, 4020, 4022, 4013, 3808, 206}

$$\frac{(157A - 85B)\sin(c+dx)\cos^3(c+dx)}{80a^2d\sqrt{a\sec(c+dx)+a}} - \frac{(787A - 475B)\sin(c+dx)\sqrt{\cos(c+dx)}}{240a^2d\sqrt{a\sec(c+dx)+a}} + \frac{(2671A - 1495B)\sin(c+dx)}{240a^2d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(\text{Cos}[c + d*x]^{(5/2)}*(A + B*\text{Sec}[c + d*x]))/(a + a*\text{Sec}[c + d*x]^{(5/2)}), x]$

[Out] $-((283*A - 163*B)*\text{ArcTanh}[(\text{Sqrt}[a]*\text{Sqrt}[\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(\text{Sqrt}[2]*\text{Sqrt}[a + a*\text{Sec}[c + d*x]])]*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sqrt}[\text{Sec}[c + d*x]])/(16*\text{Sqrt}[2]*a^{(5/2)}*d) - ((A - B)*\text{Cos}[c + d*x]^{(3/2)}*\text{Sin}[c + d*x])/(4*d*(a + a*\text{Sec}[c + d*x]^{(5/2)})) - ((21*A - 13*B)*\text{Cos}[c + d*x]^{(3/2)}*\text{Sin}[c + d*x])/(16*a*d*(a + a*\text{Sec}[c + d*x]^{(3/2)})) + ((2671*A - 1495*B)*\text{Sin}[c + d*x])/(240*a^2*d*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sqrt}[a + a*\text{Sec}[c + d*x]]) - ((787*A - 475*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sin}[c + d*x])/(240*a^2*d*\text{Sqrt}[a + a*\text{Sec}[c + d*x]]) + ((157*A - 85*B)*\text{Cos}[c + d*x]^{(3/2)}*\text{Sin}[c + d*x])/(80*a^2*d*\text{Sqrt}[a + a*\text{Sec}[c + d*x]])$

Rule 206

$\text{Int}[(a_+ + (b_+)*(x_+)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1*\text{ArcTanh}[(\text{Rt}[-b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b\}, x] \&\& \text{NegQ}[a/b] \&\& (\text{GtQ}[a, 0] \parallel \text{LtQ}[b, 0])$

Rule 2955

Int[((a_.) + csc[(e_.) + (f_.)*(x_)])*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)])*(d_.) + (c_.))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] := Dist[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && IntegerQ[n])

Rule 3808

Int[Sqrt[csc[(e_.) + (f_.)*(x_)])*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)])*(b_.) + (a_.)], x_Symbol] := Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x])*Sqrt[d*Csc[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 4013

Int[(csc[(e_.) + (f_.)*(x_)])*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)])*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)])*(B_.) + (A_.)], x_Symbol] := Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && !LeQ[m, -1]

Rule 4020

Int[(csc[(e_.) + (f_.)*(x_)])*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)])*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)])*(B_.) + (A_.)], x_Symbol] := -Simp[((A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m + 1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0]

Rule 4022

Int[(csc[(e_.) + (f_.)*(x_)])*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)])*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)])*(B_.) + (A_.)], x_Symbol] := Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]

Rubi steps

$$\begin{aligned}
\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{\frac{5}{2}}} dx &= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \frac{A+B\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{\frac{5}{2}}} dx \\
&= -\frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{\frac{5}{2}}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{\frac{5}{2}}} dx}{4a^2} \\
&= -\frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{\frac{5}{2}}} - \frac{(21A-13B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{\frac{3}{2}}} \\
&= -\frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{\frac{5}{2}}} - \frac{(21A-13B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{\frac{3}{2}}} \\
&= -\frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{\frac{5}{2}}} - \frac{(21A-13B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{\frac{3}{2}}} \\
&= -\frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{\frac{5}{2}}} - \frac{(21A-13B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{\frac{3}{2}}} \\
&= -\frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{\frac{5}{2}}} - \frac{(21A-13B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{\frac{3}{2}}} \\
&= -\frac{(283A-163B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{16\sqrt{2}a^{\frac{5}{2}}d}
\end{aligned}$$

Mathematica [A] time = 2.34, size = 207, normalized size = 0.65

$$2 \tan(c+dx)\sqrt{1-\sec(c+dx)}\sec(c+dx)(5(887A-479B)\cos(c+dx)+16(52A-25B)\cos(2(c+dx)))-40A \cos^2(c+dx)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(Cos[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(5/2), x]

[Out] (60*Sqrt[2]*(283*A - 163*B)*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]])*Cos[(c + d*x)/2]^4*Sec[c + d*x]^(5/2)*Sin[c + d*x] + 2*(3491*A - 1895*B + 5*(887*A - 479*B)*Cos[c + d*x] + 16*(52*A - 25*B)*Cos[2*(c + d*x)])*Sqrt[1 - Sec[c + d*x]]*Sqrt[Sec[c + d*x]]

x)] - 40*A*Cos[3*(c + d*x)] + 40*B*Cos[3*(c + d*x)] + 12*A*Cos[4*(c + d*x)]
)*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]*Tan[c + d*x]/(480*d*Sqrt[-1 + Cos[c
 + d*x]]*(a*(1 + Sec[c + d*x]))^(5/2))

fricas [A] time = 0.53, size = 572, normalized size = 1.80

$$\frac{15\sqrt{2}\left((283A - 163B)\cos(dx + c)^3 + 3(283A - 163B)\cos(dx + c)^2 + 3(283A - 163B)\cos(dx + c) + 283A - 163B\right)\sqrt{a}\log\left(\frac{-a\cos(dx + c)^2 - 2\sqrt{2}\sqrt{a}\sqrt{(a\cos(dx + c) + a)/\cos(dx + c)}}{\sqrt{\cos(dx + c)}\sin(dx + c) - 2a\cos(dx + c) - 3a}\right) + \frac{1}{480}\left(15\sqrt{2}\left((283A - 163B)\cos(dx + c)^3 + 3(283A - 163B)\cos(dx + c)^2 + 3(283A - 163B)\cos(dx + c) + 283A - 163B\right)\sqrt{-a}\arctan\left(\frac{\sqrt{2}\sqrt{-a}\sqrt{(a\cos(dx + c) + a)/\cos(dx + c)}}{a\sin(dx + c)}\right) + 2(96A\cos(dx + c)^4 - 160(A - B)\cos(dx + c)^3 + 32(49A - 25B)\cos(dx + c)^2 + 5(911A - 503B)\cos(dx + c) + 2671A - 1495B)\sqrt{(a\cos(dx + c) + a)/\cos(dx + c)}\sqrt{\cos(dx + c)}\sin(dx + c)}{a^3d\cos(dx + c)^3 + 3a^3d\cos(dx + c)^2 + 3a^3d\cos(dx + c) + a^3d}\right)}{a^3d\cos(dx + c)^3 + 3a^3d\cos(dx + c)^2 + 3a^3d\cos(dx + c) + a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")

[Out] [-1/960*(15*sqrt(2)*((283*A - 163*B)*cos(d*x + c)^3 + 3*(283*A - 163*B)*cos(d*x + c)^2 + 3*(283*A - 163*B)*cos(d*x + c) + 283*A - 163*B)*sqrt(a)*log((-a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*(96*A*cos(d*x + c)^4 - 160*(A - B)*cos(d*x + c)^3 + 32*(49*A - 25*B)*cos(d*x + c)^2 + 5*(911*A - 503*B)*cos(d*x + c) + 2671*A - 1495*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), 1/480*(15*sqrt(2)*((283*A - 163*B)*cos(d*x + c)^3 + 3*(283*A - 163*B)*cos(d*x + c)^2 + 3*(283*A - 163*B)*cos(d*x + c) + 283*A - 163*B)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) + 2*(96*A*cos(d*x + c)^4 - 160*(A - B)*cos(d*x + c)^3 + 32*(49*A - 25*B)*cos(d*x + c)^2 + 5*(911*A - 503*B)*cos(d*x + c) + 2671*A - 1495*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \cos(dx + c)^{\frac{5}{2}}}{(a \sec(dx + c) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(5/2)/(a*sec(d*x + c) + a)^(5/2), x)

maple [A] time = 2.30, size = 461, normalized size = 1.45

$$\left(\sqrt{\cos(dx+c)}\right) \sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}} (-1 + \cos(dx+c))^2 \left(4245A(\cos^2(dx+c)) \arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right) \sqrt{-\frac{2}{1+\cos(dx+c)}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2), x)

[Out] 1/480/d*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))^2*(4245*A*cos(d*x+c)^2*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c))))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-192*A*cos(d*x+c)^5-2445*B*cos(d*x+c)^2*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c))))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+8490*A*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c))))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)+512*A*cos(d*x+c)^4-4890*B*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c))))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)-320*B*cos(d*x+c)^4+4245*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c))))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*A*sin(d*x+c)-3456*A*cos(d*x+c)^3-2445*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c))))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*B*sin(d*x+c)+1920*B*cos(d*x+c)^3-5974*A*cos(d*x+c)^2+3430*B*cos(d*x+c)^2+3768*A*cos(d*x+c)-2040*B*cos(d*x+c)+5342*A-2990*B)/sin(d*x+c)^5/a^3

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2), x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+dx)^{5/2} \left(A + \frac{B}{\cos(c+dx)}\right)}{\left(a + \frac{a}{\cos(c+dx)}\right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(5/2)*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^(5/2),x)
```

```
[Out] int((cos(c + d*x)^(5/2)*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^(5/2), x
)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

$$3.556 \quad \int \frac{\cos^3(c+dx)(A+B \sec(c+dx))}{(a+a \sec(c+dx))^{5/2}} dx$$

Optimal. Leaf size=270

$$\frac{(163A - 75B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)} \tanh^{-1}\left(\frac{\sqrt{a} \sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a \sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d} + \frac{(95A - 39B) \sin(c+dx)\sqrt{\cos(c+dx)}}{48a^2d\sqrt{a \sec(c+dx)+a}}$$

[Out] $-1/4*(A-B)*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/d/(a+a*\sec(d*x+c))^{(5/2)}-1/16*(17*A-9*B)*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/a/d/(a+a*\sec(d*x+c))^{(3/2)}+1/32*(163*A-75*B)*\operatorname{arctanh}(1/2*\sin(d*x+c)*a^{(1/2)}*\sec(d*x+c)^{(1/2)}*2^{(1/2)}/(a+a*\sec(d*x+c))^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/a^{(5/2)}/d*2^{(1/2)}-1/48*(299*A-147*B)*\sin(d*x+c)/a^2/d/\cos(d*x+c)^{(1/2)}/(a+a*\sec(d*x+c))^{(1/2)}+1/48*(95*A-39*B)*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/a^2/d/(a+a*\sec(d*x+c))^{(1/2)}$

Rubi [A] time = 0.91, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {2955, 4020, 4022, 4013, 3808, 206}

$$\frac{(95A - 39B) \sin(c+dx)\sqrt{\cos(c+dx)}}{48a^2d\sqrt{a \sec(c+dx)+a}} - \frac{(299A - 147B) \sin(c+dx)}{48a^2d\sqrt{\cos(c+dx)}\sqrt{a \sec(c+dx)+a}} + \frac{(163A - 75B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{16\sqrt{2}a^{5/2}d}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(\operatorname{Cos}[c+d*x]^{(3/2)}*(A+B*\operatorname{Sec}[c+d*x]))/(a+a*\operatorname{Sec}[c+d*x])^{(5/2)},x]$

[Out] $((163*A - 75*B)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Sqrt}[\operatorname{Sec}[c+d*x]]*\operatorname{Sin}[c+d*x])/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a+a*\operatorname{Sec}[c+d*x]])]*\operatorname{Sqrt}[\operatorname{Cos}[c+d*x]]*\operatorname{Sqrt}[\operatorname{Sec}[c+d*x]])/(16*\operatorname{Sqrt}[2]*a^{(5/2)}*d) - ((A - B)*\operatorname{Sqrt}[\operatorname{Cos}[c+d*x]]*\operatorname{Sin}[c+d*x])/(4*d*(a+a*\operatorname{Sec}[c+d*x])^{(5/2)}) - ((17*A - 9*B)*\operatorname{Sqrt}[\operatorname{Cos}[c+d*x]]*\operatorname{Sin}[c+d*x])/(16*a*d*(a+a*\operatorname{Sec}[c+d*x])^{(3/2)}) - ((299*A - 147*B)*\operatorname{Sin}[c+d*x])/(48*a^2*d*\operatorname{Sqrt}[\operatorname{Cos}[c+d*x]]*\operatorname{Sqrt}[a+a*\operatorname{Sec}[c+d*x]]) + ((95*A - 39*B)*\operatorname{Sqrt}[\operatorname{Cos}[c+d*x]]*\operatorname{Sin}[c+d*x])/(48*a^2*d*\operatorname{Sqrt}[a+a*\operatorname{Sec}[c+d*x]])$

Rule 206

$\operatorname{Int}[(a_.) + (b_.)*(x_.)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1*\operatorname{ArcTanh}[(\operatorname{Rt}[-b, 2]*x)/\operatorname{Rt}[a, 2]])/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /; \operatorname{FreeQ}\{a, b\}, x] \&\& \operatorname{NegQ}[a/b] \&\& (\operatorname{GtQ}[a, 0] \parallel \operatorname{LtQ}[b, 0])$

Rule 2955

$\operatorname{Int}[(a_.) + \operatorname{csc}[e_.) + (f_.)*(x_.)]*(b_.)^{(m_.)}*(\operatorname{csc}[e_.) + (f_.)*(x_.)]*(d_.) + (c_.)^{(n_.)}*((g_.)*\sin[e_.) + (f_.)*(x_.)])^{(p_.)}, x_Symbol] \rightarrow \operatorname{Dis}$

$t[(g \cdot \csc[e + f \cdot x])^p \cdot (g \cdot \sin[e + f \cdot x])^p, \text{Int}[(a + b \cdot \csc[e + f \cdot x])^m \cdot (c + d \cdot \csc[e + f \cdot x])^n / (g \cdot \csc[e + f \cdot x])^p, x], x] /;$ FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && IntegerQ[n])

Rule 3808

$\text{Int}[\text{Sqrt}[\csc[(e_.) + (f_.)(x_.)] \cdot (d_.)] / \text{Sqrt}[\csc[(e_.) + (f_.)(x_.)] \cdot (b_.) + (a_.)], x_Symbol] :> \text{Dist}[(-2 \cdot b \cdot d) / (a \cdot f), \text{Subst}[\text{Int}[1 / (2 \cdot b - d \cdot x^2), x], x, (b \cdot \cot[e + f \cdot x]) / (\text{Sqrt}[a + b \cdot \csc[e + f \cdot x]] \cdot \text{Sqrt}[d \cdot \csc[e + f \cdot x]])], x] /;$ FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 4013

$\text{Int}[(\csc[(e_.) + (f_.)(x_.)] \cdot (d_.))^n \cdot (\csc[(e_.) + (f_.)(x_.)] \cdot (b_.) + (a_.))^m \cdot (\csc[(e_.) + (f_.)(x_.)] \cdot (B_.) + (A_.)), x_Symbol] :> \text{Simp}[(A \cdot \cot[e + f \cdot x] \cdot (a + b \cdot \csc[e + f \cdot x])^m \cdot (d \cdot \csc[e + f \cdot x])^n / (f \cdot n), x] - \text{Dist}[(a \cdot A \cdot m - b \cdot B \cdot n) / (b \cdot d \cdot n), \text{Int}[(a + b \cdot \csc[e + f \cdot x])^m \cdot (d \cdot \csc[e + f \cdot x])^{n+1}, x], x] /;$ FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && !LeQ[m, -1]

Rule 4020

$\text{Int}[(\csc[(e_.) + (f_.)(x_.)] \cdot (d_.))^n \cdot (\csc[(e_.) + (f_.)(x_.)] \cdot (b_.) + (a_.))^m \cdot (\csc[(e_.) + (f_.)(x_.)] \cdot (B_.) + (A_.)), x_Symbol] :> -\text{Simp}[(A \cdot b - a \cdot B) \cdot \cot[e + f \cdot x] \cdot (a + b \cdot \csc[e + f \cdot x])^m \cdot (d \cdot \csc[e + f \cdot x])^n / (b \cdot f \cdot (2 \cdot m + 1)), x] - \text{Dist}[1 / (a^2 \cdot (2 \cdot m + 1)), \text{Int}[(a + b \cdot \csc[e + f \cdot x])^{m+1} \cdot (d \cdot \csc[e + f \cdot x])^n \cdot \text{Simp}[b \cdot B \cdot n - a \cdot A \cdot (2 \cdot m + n + 1) + (A \cdot b - a \cdot B) \cdot (m + n + 1) \cdot \csc[e + f \cdot x], x], x], x] /;$ FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0]

Rule 4022

$\text{Int}[(\csc[(e_.) + (f_.)(x_.)] \cdot (d_.))^n \cdot (\csc[(e_.) + (f_.)(x_.)] \cdot (b_.) + (a_.))^m \cdot (\csc[(e_.) + (f_.)(x_.)] \cdot (B_.) + (A_.)), x_Symbol] :> \text{Simp}[(A \cdot \cot[e + f \cdot x] \cdot (a + b \cdot \csc[e + f \cdot x])^m \cdot (d \cdot \csc[e + f \cdot x])^n / (f \cdot n), x] - \text{Dist}[1 / (b \cdot d \cdot n), \text{Int}[(a + b \cdot \csc[e + f \cdot x])^m \cdot (d \cdot \csc[e + f \cdot x])^{n+1} \cdot \text{Simp}[a \cdot A \cdot m - b \cdot B \cdot n - A \cdot b \cdot (m + n + 1) \cdot \csc[e + f \cdot x], x], x], x] /;$ FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]

Rubi steps

$$\begin{aligned}
\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{5/2}} dx &= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \frac{A+B\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} dx \\
&= -\frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \frac{1}{2}}{4a^2} \\
&= -\frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(17A-9B)\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} \\
&= -\frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(17A-9B)\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} \\
&= -\frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(17A-9B)\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} \\
&= -\frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(17A-9B)\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} \\
&= \frac{(163A-75B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{16\sqrt{2}a^{5/2}d}
\end{aligned}$$

Mathematica [A] time = 1.70, size = 183, normalized size = 0.68

$$\frac{2 \tan(c+dx)\sqrt{1-\sec(c+dx)}\sec(c+dx)((255B-479A)\cos(c+dx)+(48B-80A)\cos(2(c+dx))+8A\cos(3(c+dx)))}{96d\sqrt{\cos(c+dx)-1}(a+a\sec(c+dx))^{5/2}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(5/2), x]

[Out] (-12*Sqrt[2]*(163*A - 75*B)*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]])*Cos[(c + d*x)/2]^4*Sec[c + d*x]^(5/2)*Sin[c + d*x] + 2*(-379*A + 195*B + (-479*A + 255*B)*Cos[c + d*x] + (-80*A + 48*B)*Cos[2*(c + d*x)] + 8*A*Cos[3*(c + d*x)])*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]*Tan[c + d*x]/(96*d*Sqrt[-1 + Cos[c + d*x]]*(a*(1 + Sec[c + d*x]))^(5/2))

fricas [A] time = 0.47, size = 542, normalized size = 2.01

$$3\sqrt{2}\left((163A - 75B)\cos(dx + c)^3 + 3(163A - 75B)\cos(dx + c)^2 + 3(163A - 75B)\cos(dx + c) + 163A - 75B\right)\sqrt{a}\log\left(\frac{-a\cos(dx + c)^2 + 2\sqrt{2}\sqrt{a}\sqrt{(a\cos(dx + c) + a)/\cos(dx + c)}}{\sqrt{\cos(dx + c)}\sin(dx + c) - 2a\cos(dx + c) - 3a}\right) - \frac{4(32A\cos(dx + c)^3 - 32(5A - 3B)\cos(dx + c)^2 - (503A - 255B)\cos(dx + c) - 299A + 147B)\sqrt{(a\cos(dx + c) + a)/\cos(dx + c)}\sqrt{\cos(dx + c)}\sin(dx + c)}{a^3d\cos(dx + c)^3 + 3a^3d\cos(dx + c)^2 + 3a^3d\cos(dx + c) + a^3d} - \frac{1}{96}(3\sqrt{2}\left((163A - 75B)\cos(dx + c)^3 + 3(163A - 75B)\cos(dx + c)^2 + 3(163A - 75B)\cos(dx + c) + 163A - 75B\right)\sqrt{-a}\arctan\left(\frac{\sqrt{2}\sqrt{-a}\sqrt{(a\cos(dx + c) + a)/\cos(dx + c)}}{a\sin(dx + c)}\right) - 2(32A\cos(dx + c)^3 - 32(5A - 3B)\cos(dx + c)^2 - (503A - 255B)\cos(dx + c) - 299A + 147B)\sqrt{(a\cos(dx + c) + a)/\cos(dx + c)}\sqrt{\cos(dx + c)}\sin(dx + c))}{a^3d\cos(dx + c)^3 + 3a^3d\cos(dx + c)^2 + 3a^3d\cos(dx + c) + a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")

[Out] [-1/192*(3*sqrt(2)*((163*A - 75*B)*cos(d*x + c)^3 + 3*(163*A - 75*B)*cos(d*x + c)^2 + 3*(163*A - 75*B)*cos(d*x + c) + 163*A - 75*B)*sqrt(a)*log(-a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*(32*A*cos(d*x + c)^3 - 32*(5*A - 3*B)*cos(d*x + c)^2 - (503*A - 255*B)*cos(d*x + c) - 299*A + 147*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1/96*(3*sqrt(2)*((163*A - 75*B)*cos(d*x + c)^3 + 3*(163*A - 75*B)*cos(d*x + c)^2 + 3*(163*A - 75*B)*cos(d*x + c) + 163*A - 75*B)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) - 2*(32*A*cos(d*x + c)^3 - 32*(5*A - 3*B)*cos(d*x + c)^2 - (503*A - 255*B)*cos(d*x + c) - 299*A + 147*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \cos(dx + c)^{\frac{3}{2}}}{(a \sec(dx + c) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(3/2)/(a*sec(d*x + c) + a)^(5/2), x)

maple [A] time = 2.29, size = 439, normalized size = 1.63

$$\left(\sqrt{\cos(dx+c)}\right) \sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}} (-1+\cos(dx+c))^2 \left(489A(\cos^2(dx+c)) \arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right) \sqrt{-1+\cos(dx+c)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x)

[Out] -1/96/d*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))^2*(489*A*cos(d*x+c)^2*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c))))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-225*B*cos(d*x+c)^2*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c))))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+978*A*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c))))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)+64*A*cos(d*x+c)^4-450*B*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c))))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)+489*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c))))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*A*sin(d*x+c)-384*A*cos(d*x+c)^3-225*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c))))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*B*sin(d*x+c)+192*B*cos(d*x+c)^3-686*A*cos(d*x+c)^2+318*B*cos(d*x+c)^2+408*A*cos(d*x+c)-216*B*cos(d*x+c)+598*A-294*B)/a^3/sin(d*x+c)^5

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+dx)^{3/2} \left(A + \frac{B}{\cos(c+dx)}\right)}{\left(a + \frac{a}{\cos(c+dx)}\right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c+d*x)^(3/2)*(A+B/cos(c+d*x)))/(a+a/cos(c+d*x))^(5/2),x)

```
[Out] int((cos(c + d*x)^(3/2)*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^(5/2), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

$$3.557 \quad \int \frac{\sqrt{\cos(c+dx)} (A+B \sec(c+dx))}{(a+a \sec(c+dx))^{5/2}} dx$$

Optimal. Leaf size=223

$$\frac{(75A - 19B)\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \tanh^{-1}\left(\frac{\sqrt{a} \sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2} \sqrt{a \sec(c+dx)+a}}\right)}{16\sqrt{2} a^{5/2} d} + \frac{(49A - 9B) \sin(c + dx)}{16a^2 d \sqrt{\cos(c + dx)} \sqrt{a \sec(c + dx) + a}}$$

[Out] $-1/4*(A-B)*\sin(d*x+c)/d/(a+a*\sec(d*x+c))^{(5/2)}/\cos(d*x+c)^{(1/2)}-1/16*(13*A-5*B)*\sin(d*x+c)/a/d/(a+a*\sec(d*x+c))^{(3/2)}/\cos(d*x+c)^{(1/2)}-1/32*(75*A-19*B)*\operatorname{arctanh}(1/2*\sin(d*x+c)*a^{(1/2)}*\sec(d*x+c)^{(1/2)}*2^{(1/2)}/(a+a*\sec(d*x+c))^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/a^{(5/2)}/d*2^{(1/2)}+1/16*(49*A-9*B)*\sin(d*x+c)/a^2/d/\cos(d*x+c)^{(1/2)}/(a+a*\sec(d*x+c))^{(1/2)}$

Rubi [A] time = 0.70, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2955, 4020, 4013, 3808, 206}

$$\frac{(49A - 9B) \sin(c + dx)}{16a^2 d \sqrt{\cos(c + dx)} \sqrt{a \sec(c + dx) + a}} - \frac{(75A - 19B)\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \tanh^{-1}\left(\frac{\sqrt{a} \sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2} \sqrt{a \sec(c+dx)+a}}\right)}{16\sqrt{2} a^{5/2} d}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(\operatorname{Sqrt}[\operatorname{Cos}[c + d*x]]*(A + B*\operatorname{Sec}[c + d*x]))/(a + a*\operatorname{Sec}[c + d*x])^{(5/2)}, x]$

[Out] $-((75*A - 19*B)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Sqrt}[\operatorname{Sec}[c + d*x]]*\operatorname{Sin}[c + d*x])/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a + a*\operatorname{Sec}[c + d*x]])]*\operatorname{Sqrt}[\operatorname{Cos}[c + d*x]]*\operatorname{Sqrt}[\operatorname{Sec}[c + d*x]])/(16*\operatorname{Sqrt}[2]*a^{(5/2)}*d) - ((A - B)*\operatorname{Sin}[c + d*x])/(4*d*\operatorname{Sqrt}[\operatorname{Cos}[c + d*x]]*(a + a*\operatorname{Sec}[c + d*x])^{(5/2)}) - ((13*A - 5*B)*\operatorname{Sin}[c + d*x])/(16*a*d*\operatorname{Sqrt}[\operatorname{Cos}[c + d*x]]*(a + a*\operatorname{Sec}[c + d*x])^{(3/2)}) + ((49*A - 9*B)*\operatorname{Sin}[c + d*x])/(16*a^2*d*\operatorname{Sqrt}[\operatorname{Cos}[c + d*x]]*\operatorname{Sqrt}[a + a*\operatorname{Sec}[c + d*x]])$

Rule 206

$\operatorname{Int}[(a_) + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1*\operatorname{ArcTanh}[(\operatorname{Rt}[-b, 2]*x)/\operatorname{Rt}[a, 2]])/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /; \operatorname{FreeQ}\{a, b\}, x \ \&\& \operatorname{NegQ}[a/b] \ \&\& (\operatorname{GtQ}[a, 0] \ || \ \operatorname{LtQ}[b, 0])$

Rule 2955

$\operatorname{Int}[(a_) + \operatorname{csc}[e_] + (f_)*(x_)]*(b_)^{(m_)}*(\operatorname{csc}[e_] + (f_)*(x_)]*(d_) + (c_)]^{(n_)}*((g_)*\operatorname{sin}[e_] + (f_)*(x_)]^{(p_)}, x_Symbol] \rightarrow \operatorname{Dist}[(g*\operatorname{Csc}[e + f*x])^p*(g*\operatorname{Sin}[e + f*x])^p, \operatorname{Int}[(a + b*\operatorname{Csc}[e + f*x])^m*(c + d*\operatorname{Csc}[e + f*x])^n]/(g*\operatorname{Csc}[e + f*x])^p, x], x] /; \operatorname{FreeQ}\{a, b, c, d, e, f, g,$

$m, n, p\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{!IntegerQ}[p] \&\& \text{!(IntegerQ}[m] \&\& \text{IntegerQ}[n])$

Rule 3808

$\text{Int}[\text{Sqrt}[\text{csc}[(e_.) + (f_.)*(x_.)]*(d_.)]/\text{Sqrt}[\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] \text{:>} \text{Dist}[(-2*b*d)/(a*f), \text{Subst}[\text{Int}[1/(2*b - d*x^2)], x], x, (b*\text{Cot}[e + f*x])/(\text{Sqrt}[a + b*\text{Csc}[e + f*x]]*\text{Sqrt}[d*\text{Csc}[e + f*x]])], x] /;$
 $\text{FreeQ}[\{a, b, d, e, f\}, x] \&\& \text{EqQ}[a^2 - b^2, 0]$

Rule 4013

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(d_.))^{(n_.)}*(\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^{(m_.)}*(\text{csc}[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] \text{:>} \text{Simp}[(A*\text{Cot}[e + f*x]*(a + b*\text{Csc}[e + f*x])^m*(d*\text{Csc}[e + f*x])^n)/(f*n), x] - \text{Dist}[(a*A*m - b*B*n)/(b*d*n), \text{Int}[(a + b*\text{Csc}[e + f*x])^m*(d*\text{Csc}[e + f*x])^{(n + 1)}], x], x] /;$
 $\text{FreeQ}[\{a, b, d, e, f, A, B, m, n\}, x] \&\& \text{NeQ}[A*b - a*B, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{EqQ}[m + n + 1, 0] \&\& \text{!LeQ}[m, -1]$

Rule 4020

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(d_.))^{(n_.)}*(\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^{(m_.)}*(\text{csc}[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] \text{:>} -\text{Simp}[(A*b - a*B)*\text{Cot}[e + f*x]*(a + b*\text{Csc}[e + f*x])^m*(d*\text{Csc}[e + f*x])^n)/(b*f*(2*m + 1)), x] - \text{Dist}[1/(a^2*(2*m + 1)), \text{Int}[(a + b*\text{Csc}[e + f*x])^{(m + 1)}*(d*\text{Csc}[e + f*x])^n*\text{Simp}[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*\text{Csc}[e + f*x], x], x], x] /;$
 $\text{FreeQ}[\{a, b, d, e, f, A, B, n\}, x] \&\& \text{NeQ}[A*b - a*B, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{LtQ}[m, -2^{(-1)}] \&\& \text{!GtQ}[n, 0]$

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\cos(c+dx)}(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{5/2}} dx &= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \frac{A+B\sec(c+dx)}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{5/2}} dx \\
&= -\frac{(A-B)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{5/2}} + \frac{(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)})}{16ad\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{5/2}} \\
&= -\frac{(A-B)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{5/2}} - \frac{(13A-5B)\sin(c+dx)}{16ad\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{5/2}} \\
&= -\frac{(A-B)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{5/2}} - \frac{(13A-5B)\sin(c+dx)}{16ad\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{5/2}} \\
&= -\frac{(A-B)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{5/2}} - \frac{(13A-5B)\sin(c+dx)}{16ad\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{5/2}} \\
&= -\frac{(75A-19B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{16\sqrt{2}a^{5/2}d}
\end{aligned}$$

Mathematica [A] time = 2.69, size = 228, normalized size = 1.02

$$\frac{\tan(c+dx)\sqrt{1-\sec(c+dx)}\sec(c+dx)\left(2(73A^2+76AB-13B^2)\cos(c+dx)+16A^2\cos(3(c+dx))+85A^2+32d\sqrt{\cos(c+dx)}\right)}{32d\sqrt{\cos(c+dx)}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(5/2), x]

[Out] (8*Sqrt[2]*(75*A - 19*B)*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]])*Cos[(c + d*x)/2]^5*(B + A*Cos[c + d*x])*Sec[c + d*x]^(5/2)*Sin[(c + d*x)/2] + (85*A^2 + 117*A*B - 18*B^2 + 2*(73*A^2 + 76*A*B - 13*B^2)*Cos[c + d*x] + A*(85*A + 19*B)*Cos[2*(c + d*x)] + 16*A^2*Cos[3*(c + d*x)])*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]*Tan[c + d*x]/(32*d*Sqrt[-1 + Cos[c + d*x]])*(B + A*Cos[c + d*x])*(a*(1 + Sec[c + d*x]))^(5/2)

fricas [A] time = 0.46, size = 504, normalized size = 2.26

$$\sqrt{2} \left((75A - 19B) \cos(dx + c)^3 + 3(75A - 19B) \cos(dx + c)^2 + 3(75A - 19B) \cos(dx + c) + 75A - 19B \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")

[Out] [-1/64*(sqrt(2)*((75*A - 19*B)*cos(d*x + c)^3 + 3*(75*A - 19*B)*cos(d*x + c)^2 + 3*(75*A - 19*B)*cos(d*x + c) + 75*A - 19*B)*sqrt(a)*log(-(a*cos(d*x + c))^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*(32*A*cos(d*x + c)^2 + (85*A - 13*B)*cos(d*x + c) + 49*A - 9*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), 1/32*(sqrt(2)*((75*A - 19*B)*cos(d*x + c)^3 + 3*(75*A - 19*B)*cos(d*x + c)^2 + 3*(75*A - 19*B)*cos(d*x + c) + 75*A - 19*B)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) + 2*(32*A*cos(d*x + c)^2 + (85*A - 13*B)*cos(d*x + c) + 49*A - 9*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sqrt{\cos(dx + c)}}{(a \sec(dx + c) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(a*sec(d*x + c) + a)^(5/2), x)

maple [A] time = 2.09, size = 365, normalized size = 1.64

$$(-1 + \cos(dx + c))^2 \left(32A \sqrt{\frac{2}{1 + \cos(dx + c)}} (\cos^3(dx + c)) + 53A \sqrt{\frac{2}{1 + \cos(dx + c)}} (\cos^2(dx + c)) + 75A \cos(dx + c) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(5/2),x)`

[Out]
$$-1/16/d*(-1+\cos(d*x+c))^2*(32*A*(-2/(1+\cos(d*x+c)))^(1/2)*\cos(d*x+c)^3+53*A*(-2/(1+\cos(d*x+c)))^(1/2)*\cos(d*x+c)^2+75*A*\cos(d*x+c)*\sin(d*x+c)*\arctan(1/2*\sin(d*x+c)*(-2/(1+\cos(d*x+c)))^(1/2))-13*B*(-2/(1+\cos(d*x+c)))^(1/2)*\cos(d*x+c)^2-19*B*\cos(d*x+c)*\sin(d*x+c)*\arctan(1/2*\sin(d*x+c)*(-2/(1+\cos(d*x+c)))^(1/2))-36*A*(-2/(1+\cos(d*x+c)))^(1/2)*\cos(d*x+c)+75*A*\arctan(1/2*\sin(d*x+c)*(-2/(1+\cos(d*x+c)))^(1/2))*\sin(d*x+c)+4*B*(-2/(1+\cos(d*x+c)))^(1/2)*\cos(d*x+c)-19*B*\arctan(1/2*\sin(d*x+c)*(-2/(1+\cos(d*x+c)))^(1/2))*\sin(d*x+c)-49*A*(-2/(1+\cos(d*x+c)))^(1/2)+9*B*(-2/(1+\cos(d*x+c)))^(1/2))*\cos(d*x+c)^(1/2)*(a*(1+\cos(d*x+c))/\cos(d*x+c))^(1/2)/a^3/(-2/(1+\cos(d*x+c)))^(1/2)/\sin(d*x+c)^5$$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(5/2),x, algorith="maxima")`

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\cos(c + dx)} \left(A + \frac{B}{\cos(c + dx)} \right)}{\left(a + \frac{a}{\cos(c + dx)} \right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((cos(c + d*x)^(1/2)*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^(5/2),x)`

[Out] `int((cos(c + d*x)^(1/2)*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^(5/2),x)`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)**(1/2)/(a+a*sec(d*x+c))**(5/2),x)

[Out] Timed out

$$3.558 \quad \int \frac{A+B \sec(c+dx)}{\sqrt{\cos(c+dx)} (a+a \sec(c+dx))^{5/2}} dx$$

Optimal. Leaf size=223

$$\frac{(19A + 5B)\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \tanh^{-1}\left(\frac{\sqrt{a} \sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2} \sqrt{a \sec(c+dx)+a}}\right)}{16\sqrt{2} a^{5/2} d} + \frac{(9A - B) \sin(c + dx)}{16a^2 d \sqrt{\cos(c + dx)} \sqrt{a \sec(c + dx) + a}}$$

[Out] 1/4*(A-B)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2)+1/16*(5*A+3*B)*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2)+1/32*(19*A+5*B)*arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(5/2)/d*2^(1/2)-1/16*(9*A-B)*sin(d*x+c)/a^2/d/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)

Rubi [A] time = 0.71, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {2955, 4019, 4020, 4013, 3808, 206}

$$-\frac{(9A - B) \sin(c + dx)}{16a^2 d \sqrt{\cos(c + dx)} \sqrt{a \sec(c + dx) + a}} + \frac{(19A + 5B)\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \tanh^{-1}\left(\frac{\sqrt{a} \sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2} \sqrt{a \sec(c+dx)+a}}\right)}{16\sqrt{2} a^{5/2} d}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sec[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(5/2)),x]

[Out] ((19*A + 5*B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]*a^(5/2)*d) + ((A - B)*Sin[c + d*x])/(4*d*Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(5/2)) + ((5*A + 3*B)*Sin[c + d*x])/(16*a*d*Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(3/2)) - ((9*A - B)*Sin[c + d*x])/(16*a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]])

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2955

Int[((a_) + csc[(e_) + (f_)*(x_)])*(b_)^(m_)*(csc[(e_) + (f_)*(x_)]*(d_) + (c_))^(n_)*((g_)*sin[(e_) + (f_)*(x_)])^(p_), x_Symbol] := Dist[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,

$m, n, p\}$, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && IntegerQ[n])

Rule 3808

Int[Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 4013

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && !LeQ[m, -1]

Rule 4019

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> Simp[(d*(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0]

Rule 4020

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m + 1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e + f*x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0]

Rubi steps

$$\begin{aligned}
\int \frac{A + B \sec(c + dx)}{\sqrt{\cos(c + dx)} (a + a \sec(c + dx))^{5/2}} dx &= \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{\sqrt{\sec(c + dx)} (A + B \sec(c + dx))}{(a + a \sec(c + dx))^{5/2}} dx \\
&= \frac{(A - B) \sin(c + dx)}{4d \sqrt{\cos(c + dx)} (a + a \sec(c + dx))^{5/2}} + \frac{(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)})}{(a + a \sec(c + dx))^{5/2}} \\
&= \frac{(A - B) \sin(c + dx)}{4d \sqrt{\cos(c + dx)} (a + a \sec(c + dx))^{5/2}} + \frac{(5A + 3B) \sin(c + dx)}{16ad \sqrt{\cos(c + dx)} (a + a \sec(c + dx))^{5/2}} \\
&= \frac{(A - B) \sin(c + dx)}{4d \sqrt{\cos(c + dx)} (a + a \sec(c + dx))^{5/2}} + \frac{(5A + 3B) \sin(c + dx)}{16ad \sqrt{\cos(c + dx)} (a + a \sec(c + dx))^{5/2}} \\
&= \frac{(A - B) \sin(c + dx)}{4d \sqrt{\cos(c + dx)} (a + a \sec(c + dx))^{5/2}} + \frac{(5A + 3B) \sin(c + dx)}{16ad \sqrt{\cos(c + dx)} (a + a \sec(c + dx))^{5/2}} \\
&= \frac{(19A + 5B) \tanh^{-1} \left(\frac{\sqrt{a} \sqrt{\sec(c + dx)} \sin(c + dx)}{\sqrt{2} \sqrt{a + a \sec(c + dx)}} \right) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)}}{16\sqrt{2} a^{5/2} d}
\end{aligned}$$

Mathematica [A] time = 1.02, size = 108, normalized size = 0.48

$$\frac{\sec\left(\frac{1}{2}(c + dx)\right) \left(4 \sin\left(\frac{1}{2}(c + dx)\right) \left((5B - 13A) \cos(c + dx) - 9A + B \right) + 8(19A + 5B) \cos^4\left(\frac{1}{2}(c + dx)\right) \tanh^{-1}\left(\sin\left(\frac{1}{2}(c + dx)\right)\right) \right)}{64ad \cos^{\frac{3}{2}}(c + dx) (a(\sec(c + dx) + 1))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sec[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(5/2)), x]

[Out] (Sec[(c + d*x)/2]*(8*(19*A + 5*B)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^4 + 4*(-9*A + B + (-13*A + 5*B)*Cos[c + d*x])*Sin[(c + d*x)/2])/(64*a*d*Cos[c + d*x]^(3/2)*(a*(1 + Sec[c + d*x]))^(3/2))

fricas [A] time = 0.47, size = 482, normalized size = 2.16

$$\left[\frac{\sqrt{2} \left((19A + 5B) \cos(dx + c)^3 + 3(19A + 5B) \cos(dx + c)^2 + 3(19A + 5B) \cos(dx + c) + 19A + 5B \right) \sqrt{a} \log \left(\frac{\sqrt{a} \sqrt{\sec(c + dx)} \sin(c + dx)}{\sqrt{2} \sqrt{a + a \sec(c + dx)}} \right)}{64 \left(a^3 d \cos(dx + c) \right)^{3/2}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")

[Out] [1/64*(sqrt(2)*((19*A + 5*B)*cos(d*x + c)^3 + 3*(19*A + 5*B)*cos(d*x + c)^2 + 3*(19*A + 5*B)*cos(d*x + c) + 19*A + 5*B)*sqrt(a)*log(-(a*cos(d*x + c))^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*((13*A - 5*B)*cos(d*x + c) + 9*A - B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1/32*(sqrt(2)*((19*A + 5*B)*cos(d*x + c)^3 + 3*(19*A + 5*B)*cos(d*x + c)^2 + 3*(19*A + 5*B)*cos(d*x + c) + 19*A + 5*B)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) + 2*((13*A - 5*B)*cos(d*x + c) + 9*A - B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(a \sec(dx + c) + a)^{\frac{5}{2}} \sqrt{\cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^(5/2)*sqrt(cos(d*x + c))), x)

maple [A] time = 2.24, size = 339, normalized size = 1.52

$$(-1 + \cos(dx + c))^2 \left(13A \sqrt{-\frac{2}{1 + \cos(dx + c)}} (\cos^2(dx + c)) + 19A \cos(dx + c) \sin(dx + c) \arctan \left(\frac{\sin(dx + c) \sqrt{-\frac{2}{1 + \cos(dx + c)}}}{2} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x)

[Out] 1/16/d*(-1+cos(d*x+c))^2*(13*A*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2+19*A*cos(d*x+c)*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))-5*B*

$$\begin{aligned} & (-2/(1+\cos(dx+c)))^{1/2} \cos(dx+c)^2 + 5B \cos(dx+c) \sin(dx+c) \arctan(1/2 \\ & \sin(dx+c) * (-2/(1+\cos(dx+c)))^{1/2} - 4A * (-2/(1+\cos(dx+c)))^{1/2} \cos(dx \\ & x+c) + 19A \arctan(1/2 \sin(dx+c) * (-2/(1+\cos(dx+c)))^{1/2} \sin(dx+c) + 4B * \\ & (-2/(1+\cos(dx+c)))^{1/2} \cos(dx+c) + 5B \arctan(1/2 \sin(dx+c) * (-2/(1+\cos(dx \\ & x+c)))^{1/2} \sin(dx+c) - 9A * (-2/(1+\cos(dx+c)))^{1/2} + B * (-2/(1+\cos(dx+c)) \\ &)^{1/2}) \cos(dx+c)^{1/2} * (a * (1+\cos(dx+c)) / \cos(dx+c))^{1/2} / a^3 / \sin(dx+c) \\ &)^5 / (-2/(1+\cos(dx+c)))^{1/2} \end{aligned}$$

maxima [B] time = 1.55, size = 5924, normalized size = 26.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(dx+c))/(a+a*sec(dx+c))^(5/2)/cos(dx+c)^(1/2),x, algorith
m="maxima")

[Out] 1/32*((19*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(4*d*x + 4*c)^2 + 304*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(3*d*x + 3*c)^2 + 684*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(2*d*x + 2*c)^2 + 304*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(dx + c)^2 + 19*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(4*d*x + 4*c)^2 + 304*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(3*d*x + 3*c)^2 + 684*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(2*d*x + 2*c)^2 + 304*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(dx + c)^2 + 2*(76*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(3*d*x + 3*c) + 114*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(2*d*x + 2*c) + 76*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(dx + c) + 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1

$$\begin{aligned}
& /2*d*x + 1/2*c) + 1) - 19*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
& ^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 26*\sin(7/2*d*x + 7/2*c) - 10*\sin(5/2*d*x \\
& + 5/2*c) + 10*\sin(3/2*d*x + 3/2*c) + 26*\sin(1/2*d*x + 1/2*c))*\cos(4*d*x + \\
& 4*c) + 104*(2*\sin(3*d*x + 3*c) + 3*\sin(2*d*x + 2*c) + 2*\sin(d*x + c))*\cos(7 \\
& /2*d*x + 7/2*c) + 8*(114*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
& ^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(2*d*x + 2*c) + 76*(\log(\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) \\
&) + 1))*\cos(d*x + c) + 19*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
& ^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 19*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2* \\
& d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 10*\sin(5/2*d*x + 5/2*c) + 10 \\
& *\sin(3/2*d*x + 3/2*c) + 26*\sin(1/2*d*x + 1/2*c))*\cos(3*d*x + 3*c) + 40*(3*s \\
& in(2*d*x + 2*c) + 2*\sin(d*x + c))*\cos(5/2*d*x + 5/2*c) + 12*(76*(\log(\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 1 \\
& og(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) \\
& + 1))*\cos(d*x + c) + 19*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^ \\
& 2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 19*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d \\
& *x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) + 10*\sin(3/2*d*x + 3/2*c) + 26* \\
& sin(1/2*d*x + 1/2*c))*\cos(2*d*x + 2*c) + 8*(19*\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 19*\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) + 26*\sin(\\
& 1/2*d*x + 1/2*c))*\cos(d*x + c) + 4*(38*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(3*d*x + 3*c) + \\
& 57*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1 \\
& /2*d*x + 1/2*c) + 1))*\sin(2*d*x + 2*c) + 38*(\log(\cos(1/2*d*x + 1/2*c)^2 + s \\
& in(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2 \\
& *c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c) \\
& + 13*\cos(7/2*d*x + 7/2*c) + 5*\cos(5/2*d*x + 5/2*c) - 5*\cos(3/2*d*x + 3/2*c) \\
& - 13*\cos(1/2*d*x + 1/2*c))*\sin(4*d*x + 4*c) - 52*(4*\cos(3*d*x + 3*c) + 6*c \\
& os(2*d*x + 2*c) + 4*\cos(d*x + c) + 1))*\sin(7/2*d*x + 7/2*c) + 16*(57*(\log(co \\
& s(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) \\
& - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/ \\
& 2*c) + 1))*\sin(2*d*x + 2*c) + 38*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c) + 5*\cos(5/2 \\
& *d*x + 5/2*c) - 5*\cos(3/2*d*x + 3/2*c) - 13*\cos(1/2*d*x + 1/2*c))*\sin(3*d*x \\
& + 3*c) - 20*(6*\cos(2*d*x + 2*c) + 4*\cos(d*x + c) + 1))*\sin(5/2*d*x + 5/2*c) \\
& + 24*(38*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2* \\
& d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 \\
& *\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c) - 5*\cos(3/2*d*x + 3/2*c) - 13*\cos(\\
& 1/2*d*x + 1/2*c))*\sin(2*d*x + 2*c) + 20*(4*\cos(d*x + c) + 1))*\sin(3/2*d*x + \\
& 3/2*c) - 80*\cos(3/2*d*x + 3/2*c))*\sin(d*x + c) - 208*\cos(1/2*d*x + 1/2*c))*\si
\end{aligned}$$

$$\begin{aligned}
& n(dx + c) + 19 \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 + 2 \sin \\
& (1/2 dx + 1/2 c) + 1) - 19 \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c \\
&)^2 - 2 \sin(1/2 dx + 1/2 c) + 1) + 52 \sin(1/2 dx + 1/2 c) * A / ((\sqrt{2}) * a \\
& ^2 \cos(4 dx + 4 c)^2 + 16 \sqrt{2}) * a^2 \cos(3 dx + 3 c)^2 + 36 \sqrt{2}) * a^2 * \\
& \cos(2 dx + 2 c)^2 + 16 \sqrt{2}) * a^2 \cos(dx + c)^2 + \sqrt{2}) * a^2 \sin(4 dx \\
& + 4 c)^2 + 16 \sqrt{2}) * a^2 \sin(3 dx + 3 c)^2 + 36 \sqrt{2}) * a^2 \sin(2 dx + 2 \\
& c)^2 + 48 \sqrt{2}) * a^2 \sin(2 dx + 2 c) * \sin(dx + c) + 16 \sqrt{2}) * a^2 \sin(dx \\
& + c)^2 + 8 \sqrt{2}) * a^2 \cos(dx + c) + \sqrt{2}) * a^2 + 2 * (4 \sqrt{2}) * a^2 \cos \\
& (3 dx + 3 c) + 6 \sqrt{2}) * a^2 \cos(2 dx + 2 c) + 4 \sqrt{2}) * a^2 \cos(dx + c) \\
& + \sqrt{2}) * a^2 * \cos(4 dx + 4 c) + 8 * (6 \sqrt{2}) * a^2 \cos(2 dx + 2 c) + 4 \sqrt{2}) \\
& * a^2 \cos(dx + c) + \sqrt{2}) * a^2 * \cos(3 dx + 3 c) + 12 * (4 \sqrt{2}) * a^2 * \\
& \cos(dx + c) + \sqrt{2}) * a^2 * \cos(2 dx + 2 c) + 4 * (2 \sqrt{2}) * a^2 \sin(3 dx + \\
& 3 c) + 3 \sqrt{2}) * a^2 \sin(2 dx + 2 c) + 2 \sqrt{2}) * a^2 \sin(dx + c) * \sin(4 \\
& dx + 4 c) + 16 * (3 \sqrt{2}) * a^2 \sin(2 dx + 2 c) + 2 \sqrt{2}) * a^2 \sin(dx + c \\
&) * \sin(3 dx + 3 c) * \sqrt{a}) + (4 * (3 \sin(3/2 dx + 3/2 c) + 5 \sin(7/3 \arctan \\
& 2(\sin(3/2 dx + 3/2 c), \cos(3/2 dx + 3/2 c)))) - 3 \sin(5/3 \arctan 2(\sin(3/ \\
& 2 dx + 3/2 c), \cos(3/2 dx + 3/2 c))) - 5 \sin(1/3 \arctan 2(\sin(3/2 dx + 3/ \\
& 2 c), \cos(3/2 dx + 3/2 c)))) * \cos(8/3 \arctan 2(\sin(3/2 dx + 3/2 c), \cos(3/ \\
& 2 dx + 3/2 c))) - 40 * (2 \sin(3 dx + 3 c) + 3 \sin(4/3 \arctan 2(\sin(3/2 dx + \\
& 3/2 c), \cos(3/2 dx + 3/2 c))) + 2 \sin(2/3 \arctan 2(\sin(3/2 dx + 3/2 c), \cos \\
& (3/2 dx + 3/2 c)))) * \cos(7/3 \arctan 2(\sin(3/2 dx + 3/2 c), \cos(3/2 dx + 3/ \\
& 2 c))) + 24 * (2 \sin(3 dx + 3 c) + 3 \sin(4/3 \arctan 2(\sin(3/2 dx + 3/2 c), \\
& \cos(3/2 dx + 3/2 c))) + 2 \sin(2/3 \arctan 2(\sin(3/2 dx + 3/2 c), \cos(3/2 dx \\
& + 3/2 c)))) * \cos(5/3 \arctan 2(\sin(3/2 dx + 3/2 c), \cos(3/2 dx + 3/2 c))) \\
& + 24 * (3 \sin(3/2 dx + 3/2 c) - 5 \sin(1/3 \arctan 2(\sin(3/2 dx + 3/2 c), \cos(\\
& 3/2 dx + 3/2 c)))) * \cos(4/3 \arctan 2(\sin(3/2 dx + 3/2 c), \cos(3/2 dx + 3/2 \\
& c))) + 16 * (3 \sin(3/2 dx + 3/2 c) - 5 \sin(1/3 \arctan 2(\sin(3/2 dx + 3/2 c) \\
& , \cos(3/2 dx + 3/2 c)))) * \cos(2/3 \arctan 2(\sin(3/2 dx + 3/2 c), \cos(3/2 dx \\
& + 3/2 c))) + 5 * (16 \cos(3 dx + 3 c)^2 + 2 * (4 \cos(3 dx + 3 c) + 6 \cos(4/3 \\
& arctan 2(\sin(3/2 dx + 3/2 c), \cos(3/2 dx + 3/2 c)))) + 4 \cos(2/3 \arctan 2(\sin \\
& (3/2 dx + 3/2 c), \cos(3/2 dx + 3/2 c))) + 1) * \cos(8/3 \arctan 2(\sin(3/2 dx \\
& + 3/2 c), \cos(3/2 dx + 3/2 c))) + \cos(8/3 \arctan 2(\sin(3/2 dx + 3/2 c), \cos \\
& (3/2 dx + 3/2 c)))^2 + 12 * (4 \cos(3 dx + 3 c) + 4 \cos(2/3 \arctan 2(\sin(3/ \\
& 2 dx + 3/2 c), \cos(3/2 dx + 3/2 c))) + 1) * \cos(4/3 \arctan 2(\sin(3/2 dx + 3/ \\
& 2 c), \cos(3/2 dx + 3/2 c))) + 36 \cos(4/3 \arctan 2(\sin(3/2 dx + 3/2 c), \cos \\
& (3/2 dx + 3/2 c)))^2 + 8 * (4 \cos(3 dx + 3 c) + 1) * \cos(2/3 \arctan 2(\sin(3/2 \\
& dx + 3/2 c), \cos(3/2 dx + 3/2 c))) + 16 \cos(2/3 \arctan 2(\sin(3/2 dx + 3/ \\
& 2 c), \cos(3/2 dx + 3/2 c)))^2 + 16 \sin(3 dx + 3 c)^2 + 4 * (2 \sin(3 dx + 3 \\
& c) + 3 \sin(4/3 \arctan 2(\sin(3/2 dx + 3/2 c), \cos(3/2 dx + 3/2 c))) + 2 \sin \\
& (2/3 \arctan 2(\sin(3/2 dx + 3/2 c), \cos(3/2 dx + 3/2 c)))) * \sin(8/3 \arctan 2 \\
& (\sin(3/2 dx + 3/2 c), \cos(3/2 dx + 3/2 c))) + \sin(8/3 \arctan 2(\sin(3/2 dx \\
& + 3/2 c), \cos(3/2 dx + 3/2 c)))^2 + 48 * (\sin(3 dx + 3 c) + \sin(2/3 \arctan \\
& 2(\sin(3/2 dx + 3/2 c), \cos(3/2 dx + 3/2 c)))) * \sin(4/3 \arctan 2(\sin(3/2 dx \\
& + 3/2 c), \cos(3/2 dx + 3/2 c))) + 36 \sin(4/3 \arctan 2(\sin(3/2 dx + 3/2 c) \\
& , \cos(3/2 dx + 3/2 c)))^2 + 32 \sin(3 dx + 3 c) * \sin(2/3 \arctan 2(\sin(3/2 dx
\end{aligned}$$

$$\begin{aligned}
& x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 16*\sin(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \\
& \cos(3/2*d*x + 3/2*c)))^2 + 8*\cos(3*d*x + 3*c) + 1)*\log(\cos(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \\
& \cos(3/2*d*x + 3/2*c)))^2 + \sin(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + 2*\sin(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \\
& \cos(3/2*d*x + 3/2*c))) + 1) - 5*(16*\cos(3*d*x + 3*c)^2 + 2*(4*\cos(3*d*x + 3*c) + 6*\cos(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))) + 4 \\
& *\cos(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 1)*\cos(8/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + \cos(8/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + 12*(4*\cos(3*d*x + 3*c) + 4*\cos(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 1)*\cos(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 36*\cos(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + 8*(4*\cos(3*d*x + 3*c) + 1)*\cos(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 16*\cos(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + 16*\sin(3*d*x + 3*c)^2 + 4*(2*\sin(3*d*x + 3*c) + 3*\sin(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))) + 2*\sin(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) * \sin(8/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + \sin(8/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + 48*(\sin(3*d*x + 3*c) + \sin(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))) * \sin(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 36*\sin(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + 32*\sin(3*d*x + 3*c)*\sin(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 16*\sin(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + 8*\cos(3*d*x + 3*c) + 1)*\log(\cos(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + \sin(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 - 2*\sin(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 1) - 48*\cos(3/2*d*x + 3/2*c) * \sin(3*d*x + 3*c) + 80*\cos(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) * \sin(3*d*x + 3*c) + 48*\cos(3*d*x + 3*c) * \sin(3/2*d*x + 3/2*c) - 4*(3 * \cos(3/2*d*x + 3/2*c) + 5*\cos(7/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) - 3*\cos(5/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) - 5*\cos(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))) * \sin(8/3 * \arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 20*(4*\cos(3*d*x + 3 * c) + 6*\cos(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 4*\cos(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 1)*\sin(7/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) - 12*(4*\cos(3*d*x + 3*c) + 6*\cos(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 4*\cos(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 1)*\sin(5/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) - 24*(3*\cos(3/2*d*x + 3/2*c) - 5*\cos(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))) * \sin(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) - 16*(3*\cos(3/2*d*x + 3/2*c) - 5*\cos(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))) * \sin(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) - 20*(4*\cos(3*d*x + 3*c) + 1)*\sin(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 12*\sin(3/2*d*x + 3/2*c)*B/((16*\sqrt{2})*a^2*\cos(3*d*x + 3*c)^2 + \sqrt{2})*a^2*\cos(8/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + 36*\sqrt{2}
\end{aligned}$$

(2)*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 16*sqrt(2)*a^2*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 16*sqrt(2)*a^2*sin(3*d*x + 3*c)^2 + sqrt(2)*a^2*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 36*sqrt(2)*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 32*sqrt(2)*a^2*sin(3*d*x + 3*c)*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 16*sqrt(2)*a^2*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 8*sqrt(2)*a^2*cos(3*d*x + 3*c) + sqrt(2)*a^2 + 2*(4*sqrt(2)*a^2*cos(3*d*x + 3*c) + 6*sqrt(2)*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))) + 4*sqrt(2)*a^2*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sqrt(2)*a^2*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 12*(4*sqrt(2)*a^2*cos(3*d*x + 3*c) + 4*sqrt(2)*a^2*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))) + sqrt(2)*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 8*(4*sqrt(2)*a^2*cos(3*d*x + 3*c) + sqrt(2)*a^2*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))) + 4*(2*sqrt(2)*a^2*sin(3*d*x + 3*c) + 3*sqrt(2)*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))) + 2*sqrt(2)*a^2*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 48*(sqrt(2)*a^2*sin(3*d*x + 3*c) + sqrt(2)*a^2*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sqrt(a))/d

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\sqrt{\cos(c+dx)} \left(a + \frac{a}{\cos(c+dx)}\right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^(5/2)),x)

[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^(5/2)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(5/2)/cos(d*x+c)**(1/2),x)

[Out] Timed out

$$3.559 \quad \int \frac{A+B \sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a \sec(c+dx))^{5/2}} dx$$

Optimal. Leaf size=176

$$\frac{(5A + 3B)\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \tanh^{-1}\left(\frac{\sqrt{a} \sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a \sec(c+dx)+a}}\right)}{16\sqrt{2} a^{5/2}d} + \frac{(5A + 3B) \sin(c + dx)}{16ad \cos^{\frac{3}{2}}(c + dx)(a \sec(c + dx) + a)^{3/2}}$$

[Out] $-1/4*(A-B)*\sin(d*x+c)/d/\cos(d*x+c)^{(5/2)}/(a+a*\sec(d*x+c))^{(5/2)}+1/16*(5*A+3*B)*\sin(d*x+c)/a/d/\cos(d*x+c)^{(3/2)}/(a+a*\sec(d*x+c))^{(3/2)}+1/32*(5*A+3*B)*a$
 $rctanh(1/2*\sin(d*x+c)*a^{(1/2)}*\sec(d*x+c)^{(1/2)}*2^{(1/2)}/(a+a*\sec(d*x+c))^{(1/2)})$
 $*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/a^{(5/2)}/d*2^{(1/2)}$

Rubi [A] time = 0.40, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2955, 4012, 3810, 3808, 206}

$$\frac{(5A + 3B)\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \tanh^{-1}\left(\frac{\sqrt{a} \sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a \sec(c+dx)+a}}\right)}{16\sqrt{2} a^{5/2}d} + \frac{(5A + 3B) \sin(c + dx)}{16ad \cos^{\frac{3}{2}}(c + dx)(a \sec(c + dx) + a)^{3/2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(A + B*\text{Sec}[c + d*x])/(\text{Cos}[c + d*x]^{(3/2)}*(a + a*\text{Sec}[c + d*x])^{(5/2)}), x]$

[Out] $((5*A + 3*B)*\text{ArcTanh}[(\text{Sqrt}[a]*\text{Sqrt}[\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(\text{Sqrt}[2]*\text{Sqrt}[a + a*\text{Sec}[c + d*x]])]*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sqrt}[\text{Sec}[c + d*x]])/(16*\text{Sqrt}[2]*a^{(5/2)}*d) - ((A - B)*\text{Sin}[c + d*x])/((4*d*\text{Cos}[c + d*x]^{(5/2)}*(a + a*\text{Sec}[c + d*x])^{(5/2)}) + ((5*A + 3*B)*\text{Sin}[c + d*x])/(16*a*d*\text{Cos}[c + d*x]^{(3/2)}*(a + a*\text{Sec}[c + d*x])^{(3/2)})$

Rule 206

$\text{Int}[(a_.) + (b_.)*(x_.)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1*\text{ArcTanh}[(\text{Rt}[-b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b\}, x] \&\& \text{NegQ}[a/b] \&\& (\text{GtQ}[a, 0] \parallel \text{LtQ}[b, 0])$

Rule 2955

$\text{Int}[(a_.) + \text{csc}[e_.) + (f_.)*(x_.)]*(b_.)^{(m_.)}*(\text{csc}[e_.) + (f_.)*(x_.)]*(d_.) + (c_.)^{(n_.)}*((g_.)*\sin[e_.) + (f_.)*(x_.)])^{(p_.)}, x_Symbol] \rightarrow \text{Dist}[(g*\text{Csc}[e + f*x])^p*(g*\text{Sin}[e + f*x])^p, \text{Int}[(a + b*\text{Csc}[e + f*x])^m*(c + d*\text{Csc}[e + f*x])^n]/(g*\text{Csc}[e + f*x])^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, m, n, p\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& !\text{IntegerQ}[p] \&\& !(\text{IntegerQ}[m] \&\& \text{In}$

tegerQ[n])

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.)
+ (a_.)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3810

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (
a_.))^(m_), x_Symbol] :> Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc
c[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[(d*(m + 1))/(b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[
{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && LtQ[m, -
2^(-1)] && IntegerQ[2*m]
```

Rule 4012

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (
a_.))^(m_)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] + Dist[(a*A*m + b*B*(m + 1))/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*
x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x
] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && LeQ[m,
-1]
```

Rubi steps

$$\begin{aligned}
\int \frac{A + B \sec(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))^{5/2}} dx &= \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{\sec^{\frac{3}{2}}(c + dx)(A + B \sec(c + dx))}{(a + a \sec(c + dx))^{5/2}} dx \\
&= -\frac{(A - B) \sin(c + dx)}{4d \cos^{\frac{5}{2}}(c + dx)(a + a \sec(c + dx))^{5/2}} + \frac{((5A + 3B)\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)})}{16ad \cos^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))^{5/2}} \\
&= -\frac{(A - B) \sin(c + dx)}{4d \cos^{\frac{5}{2}}(c + dx)(a + a \sec(c + dx))^{5/2}} + \frac{(5A + 3B) \sin(c + dx)}{16ad \cos^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))^{5/2}} \\
&= -\frac{(A - B) \sin(c + dx)}{4d \cos^{\frac{5}{2}}(c + dx)(a + a \sec(c + dx))^{5/2}} + \frac{(5A + 3B) \sin(c + dx)}{16ad \cos^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))^{5/2}} \\
&= \frac{(5A + 3B) \tanh^{-1} \left(\frac{\sqrt{a} \sqrt{\sec(c + dx)} \sin(c + dx)}{\sqrt{2} \sqrt{a + a \sec(c + dx)}} \right) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)}}{16\sqrt{2} a^{5/2} d}
\end{aligned}$$

Mathematica [A] time = 0.83, size = 108, normalized size = 0.61

$$\frac{\cos^2\left(\frac{1}{2}(c + dx)\right) \left(\frac{1}{2} \tan\left(\frac{1}{2}(c + dx)\right)\right) ((5A + 3B) \cos(c + dx) + A + 7B) + (5A + 3B) \cos^3\left(\frac{1}{2}(c + dx)\right) \tanh^{-1}\left(\frac{\sqrt{a} \sqrt{\sec(c + dx)} \sin(c + dx)}{\sqrt{2} \sqrt{a + a \sec(c + dx)}}\right)}{4d \cos^{\frac{5}{2}}(c + dx)(a(\sec(c + dx) + 1))^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(5/2)), x]

[Out] (Cos[(c + d*x)/2]^2*((5*A + 3*B)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^3 + ((A + 7*B + (5*A + 3*B)*Cos[c + d*x])*Tan[(c + d*x)/2])/2)/(4*d*Cos[c + d*x]^(5/2)*(a*(1 + Sec[c + d*x]))^(5/2))

fricas [A] time = 0.49, size = 478, normalized size = 2.72

$$\left[\frac{\sqrt{2} \left((5A + 3B) \cos(dx + c)^3 + 3(5A + 3B) \cos(dx + c)^2 + 3(5A + 3B) \cos(dx + c) + 5A + 3B \right) \sqrt{a} \log \left(\frac{\sqrt{a} \sqrt{\sec(dx + c)} \sin(dx + c)}{\sqrt{2} \sqrt{a + a \sec(dx + c)}} \right)}{64 (a^3 d \cos(dx + c))^{5/2}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")

[Out] [1/64*(sqrt(2)*((5*A + 3*B)*cos(d*x + c)^3 + 3*(5*A + 3*B)*cos(d*x + c)^2 + 3*(5*A + 3*B)*cos(d*x + c) + 5*A + 3*B)*sqrt(a)*log(-(a*cos(d*x + c))^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*((5*A + 3*B)*cos(d*x + c) + A + 7*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1/32*(sqrt(2)*((5*A + 3*B)*cos(d*x + c)^3 + 3*(5*A + 3*B)*cos(d*x + c)^2 + 3*(5*A + 3*B)*cos(d*x + c) + 5*A + 3*B)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) - 2*((5*A + 3*B)*cos(d*x + c) + A + 7*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(a \sec(dx + c) + a)^{\frac{5}{2}} \cos(dx + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(3/2)), x)

maple [B] time = 2.23, size = 340, normalized size = 1.93

$$\frac{(\sqrt{\cos(dx + c)}(-1 + \cos(dx + c)))^2 \left(5A \sqrt{-\frac{2}{1 + \cos(dx + c)}} (\cos^2(dx + c)) - 5A \cos(dx + c) \sin(dx + c) \arctan \left(\frac{\sin(dx + c)}{\cos(dx + c)} \right) \right)}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x)

[Out] -1/16/d*cos(d*x+c)^(1/2)*(-1+cos(d*x+c))^2*(5*A*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2-5*A*cos(d*x+c)*sin(d*x+c)*arctan(1/2*sin(d*x+c))*(-2/(1+cos(d*x+c)))

$$\begin{aligned} & c))^{(1/2)} + 3*B*(-2/(1+\cos(d*x+c)))^{(1/2)}*\cos(d*x+c)^2 - 3*B*\cos(d*x+c)*\sin(d \\ & *x+c)*\arctan(1/2*\sin(d*x+c)*(-2/(1+\cos(d*x+c)))^{(1/2)} - 4*A*(-2/(1+\cos(d*x+c) \\ &))^{(1/2)}*\cos(d*x+c) - 5*A*\arctan(1/2*\sin(d*x+c)*(-2/(1+\cos(d*x+c)))^{(1/2)})*\sin \\ & (d*x+c) + 4*B*(-2/(1+\cos(d*x+c)))^{(1/2)}*\cos(d*x+c) - 3*B*\arctan(1/2*\sin(d*x+c) \\ &)*(-2/(1+\cos(d*x+c)))^{(1/2)})*\sin(d*x+c) - A*(-2/(1+\cos(d*x+c)))^{(1/2)} - 7*B*(-2 \\ & /(1+\cos(d*x+c)))^{(1/2)}*(a*(1+\cos(d*x+c))/\cos(d*x+c))^{(1/2)}/\sin(d*x+c)^5/(- \\ & 2/(1+\cos(d*x+c)))^{(1/2)}/a^3 \end{aligned}$$

maxima [B] time = 1.24, size = 5356, normalized size = 30.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x, algorith="maxima")

[Out]
$$\begin{aligned} & 1/32*((4*(3*\sin(3/2*d*x + 3/2*c) + 5*\sin(7/3*\arctan2(\sin(3/2*d*x + 3/2*c), \\ & \cos(3/2*d*x + 3/2*c)))) - 3*\sin(5/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d* \\ & x + 3/2*c))) - 5*\sin(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c) \\ &))) * \cos(8/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) - 40*(2*\sin \\ & (3*d*x + 3*c) + 3*\sin(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2* \\ & c))) + 2*\sin(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))) * \cos(\\ & 7/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 24*(2*\sin(3*d*x \\ & + 3*c) + 3*\sin(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 2 \\ & * \sin(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))) * \cos(5/3*\arct \\ & an2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 24*(3*\sin(3/2*d*x + 3/2* \\ & c) - 5*\sin(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))) * \cos(4/ \\ & 3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 16*(3*\sin(3/2*d*x \\ & + 3/2*c) - 5*\sin(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))) * \\ & \cos(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 5*(16*\cos(3* \\ & d*x + 3*c)^2 + 2*(4*\cos(3*d*x + 3*c) + 6*\cos(4/3*\arctan2(\sin(3/2*d*x + 3/2* \\ & c), \cos(3/2*d*x + 3/2*c)))) + 4*\cos(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/ \\ & 2*d*x + 3/2*c))) + 1)*\cos(8/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3 \\ & /2*c))) + \cos(8/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + \\ & 12*(4*\cos(3*d*x + 3*c) + 4*\cos(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d* \\ & x + 3/2*c))) + 1)*\cos(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c \\ &))) + 36*\cos(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + 8 \\ & *(4*\cos(3*d*x + 3*c) + 1)*\cos(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x \\ & + 3/2*c))) + 16*\cos(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c) \\ &))^2 + 16*\sin(3*d*x + 3*c)^2 + 4*(2*\sin(3*d*x + 3*c) + 3*\sin(4/3*\arctan2(\sin \\ & (3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 2*\sin(2/3*\arctan2(\sin(3/2*d*x \\ & + 3/2*c), \cos(3/2*d*x + 3/2*c)))) * \sin(8/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos \\ & (3/2*d*x + 3/2*c))) + \sin(8/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3 \\ & /2*c)))^2 + 48*(\sin(3*d*x + 3*c) + \sin(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos \\ & (3/2*d*x + 3/2*c)))) * \sin(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3 \end{aligned}$$

$$\begin{aligned}
& /2*c))) + 36*\sin(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 \\
& + 32*\sin(3*d*x + 3*c)*\sin(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + \\
& 3/2*c))) + 16*\sin(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 \\
& + 8*\cos(3*d*x + 3*c) + 1)*\log(\cos(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 \\
& + \sin(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + 2*\sin(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \\
& \cos(3/2*d*x + 3/2*c))) \\
& + 1) - 5*(16*\cos(3*d*x + 3*c)^2 + 2*(4*\cos(3*d*x + 3*c) + 6*\cos(4/3*\arctan2 \\
& (\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 4*\cos(2/3*\arctan2(\sin(3/2*d* \\
& *x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 1)*\cos(8/3*\arctan2(\sin(3/2*d*x + 3/2* \\
& c), \cos(3/2*d*x + 3/2*c))) + \cos(8/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2* \\
& d*x + 3/2*c)))^2 + 12*(4*\cos(3*d*x + 3*c) + 4*\cos(2/3*\arctan2(\sin(3/2*d*x + \\
& 3/2*c), \cos(3/2*d*x + 3/2*c))) + 1)*\cos(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \\
& \cos(3/2*d*x + 3/2*c))) + 36*\cos(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d* \\
& *x + 3/2*c)))^2 + 8*(4*\cos(3*d*x + 3*c) + 1)*\cos(2/3*\arctan2(\sin(3/2*d*x + \\
& 3/2*c), \cos(3/2*d*x + 3/2*c))) + 16*\cos(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), c \\
& \cos(3/2*d*x + 3/2*c)))^2 + 16*\sin(3*d*x + 3*c)^2 + 4*(2*\sin(3*d*x + 3*c) + 3 \\
& *\sin(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 2*\sin(2/3*a \\
& rctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))))*\sin(8/3*\arctan2(\sin(3/ \\
& 2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + \sin(8/3*\arctan2(\sin(3/2*d*x + 3/2* \\
& c), \cos(3/2*d*x + 3/2*c)))^2 + 48*(\sin(3*d*x + 3*c) + \sin(2/3*\arctan2(\sin(3 \\
& /2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))))*\sin(4/3*\arctan2(\sin(3/2*d*x + 3/2* \\
& c), \cos(3/2*d*x + 3/2*c))) + 36*\sin(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3 \\
& /2*d*x + 3/2*c)))^2 + 32*\sin(3*d*x + 3*c)*\sin(2/3*\arctan2(\sin(3/2*d*x + 3/2 \\
& *c), \cos(3/2*d*x + 3/2*c))) + 16*\sin(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(\\
& 3/2*d*x + 3/2*c)))^2 + 8*\cos(3*d*x + 3*c) + 1)*\log(\cos(1/3*\arctan2(\sin(3/2* \\
& d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + \sin(1/3*\arctan2(\sin(3/2*d*x + 3/2* \\
& c), \cos(3/2*d*x + 3/2*c)))^2 - 2*\sin(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(\\
& 3/2*d*x + 3/2*c))) + 1) - 48*\cos(3/2*d*x + 3/2*c)*\sin(3*d*x + 3*c) + 80*\cos \\
& (1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))*\sin(3*d*x + 3*c) \\
& + 48*\cos(3*d*x + 3*c)*\sin(3/2*d*x + 3/2*c) - 4*(3*\cos(3/2*d*x + 3/2*c) + 5* \\
& \cos(7/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) - 3*\cos(5/3*ar \\
& ctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) - 5*\cos(1/3*\arctan2(\sin(\\
& 3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))*\sin(8/3*\arctan2(\sin(3/2*d*x + 3/2 \\
& *c), \cos(3/2*d*x + 3/2*c))) + 20*(4*\cos(3*d*x + 3*c) + 6*\cos(4/3*\arctan2(si \\
& n(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 4*\cos(2/3*\arctan2(\sin(3/2*d*x \\
& + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 1)*\sin(7/3*\arctan2(\sin(3/2*d*x + 3/2*c), \\
& \cos(3/2*d*x + 3/2*c))) - 12*(4*\cos(3*d*x + 3*c) + 6*\cos(4/3*\arctan2(\sin(3/ \\
& 2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 4*\cos(2/3*\arctan2(\sin(3/2*d*x + 3/ \\
& 2*c), \cos(3/2*d*x + 3/2*c))) + 1)*\sin(5/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos \\
& (3/2*d*x + 3/2*c))) - 24*(3*\cos(3/2*d*x + 3/2*c) - 5*\cos(1/3*\arctan2(\sin(3/ \\
& 2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))*\sin(4/3*\arctan2(\sin(3/2*d*x + 3/2*c \\
&), \cos(3/2*d*x + 3/2*c))) - 16*(3*\cos(3/2*d*x + 3/2*c) - 5*\cos(1/3*\arctan2(\\
& \sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))*\sin(2/3*\arctan2(\sin(3/2*d*x + \\
& 3/2*c), \cos(3/2*d*x + 3/2*c))) - 20*(4*\cos(3*d*x + 3*c) + 1)*\sin(1/3*arcta \\
& n2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 12*\sin(3/2*d*x + 3/2*c))*
\end{aligned}$$

$$\begin{aligned}
& A / \left((16 \sqrt{2} a^2 \cos(3dx + 3c)^2 + \sqrt{2} a^2 \cos(8/3 \arctan2(\sin(3/2 dx + 3/2 c), \cos(3/2 dx + 3/2 c)))^2 + 36 \sqrt{2} a^2 \cos(4/3 \arctan2(\sin(3/2 dx + 3/2 c), \cos(3/2 dx + 3/2 c)))^2 + 16 \sqrt{2} a^2 \cos(2/3 \arctan2(\sin(3/2 dx + 3/2 c), \cos(3/2 dx + 3/2 c)))^2 + 16 \sqrt{2} a^2 \sin(3dx + 3c)^2 + \sqrt{2} a^2 \sin(8/3 \arctan2(\sin(3/2 dx + 3/2 c), \cos(3/2 dx + 3/2 c)))^2 + 36 \sqrt{2} a^2 \sin(4/3 \arctan2(\sin(3/2 dx + 3/2 c), \cos(3/2 dx + 3/2 c)))^2 + 32 \sqrt{2} a^2 \sin(3dx + 3c) \sin(2/3 \arctan2(\sin(3/2 dx + 3/2 c), \cos(3/2 dx + 3/2 c))) + 16 \sqrt{2} a^2 \sin(2/3 \arctan2(\sin(3/2 dx + 3/2 c), \cos(3/2 dx + 3/2 c)))^2 + 8 \sqrt{2} a^2 \cos(3dx + 3c) + \sqrt{2} a^2 + 2(4 \sqrt{2} a^2 \cos(3dx + 3c) + 6 \sqrt{2} a^2 \cos(4/3 \arctan2(\sin(3/2 dx + 3/2 c), \cos(3/2 dx + 3/2 c))) + 4 \sqrt{2} a^2 \cos(2/3 \arctan2(\sin(3/2 dx + 3/2 c), \cos(3/2 dx + 3/2 c))) + \sqrt{2} a^2) \cos(8/3 \arctan2(\sin(3/2 dx + 3/2 c), \cos(3/2 dx + 3/2 c))) + 12(4 \sqrt{2} a^2 \cos(3dx + 3c) + 4 \sqrt{2} a^2 \cos(2/3 \arctan2(\sin(3/2 dx + 3/2 c), \cos(3/2 dx + 3/2 c))) + \sqrt{2} a^2) \cos(4/3 \arctan2(\sin(3/2 dx + 3/2 c), \cos(3/2 dx + 3/2 c))) + 8(4 \sqrt{2} a^2 \cos(3dx + 3c) + \sqrt{2} a^2) \cos(2/3 \arctan2(\sin(3/2 dx + 3/2 c), \cos(3/2 dx + 3/2 c))) + 4(2 \sqrt{2} a^2 \sin(3dx + 3c) + 3 \sqrt{2} a^2 \sin(4/3 \arctan2(\sin(3/2 dx + 3/2 c), \cos(3/2 dx + 3/2 c))) + 2 \sqrt{2} a^2 \sin(2/3 \arctan2(\sin(3/2 dx + 3/2 c), \cos(3/2 dx + 3/2 c)))) \sin(8/3 \arctan2(\sin(3/2 dx + 3/2 c), \cos(3/2 dx + 3/2 c))) + 48(\sqrt{2} a^2 \sin(3dx + 3c) + \sqrt{2} a^2 \sin(2/3 \arctan2(\sin(3/2 dx + 3/2 c), \cos(3/2 dx + 3/2 c)))) \sin(4/3 \arctan2(\sin(3/2 dx + 3/2 c), \cos(3/2 dx + 3/2 c))) \sqrt{a} - (12(\sin(4dx + 4c) + 6 \sin(2dx + 2c) + 4 \sin(3/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c))) + 4 \sin(1/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c)))) \cos(7/4 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c))) - 16(11 \sin(5/4 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c))) \cos(2dx + 2c) - 11 \sin(3/4 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c))) \cos(3/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c))) + 44(\sin(4dx + 4c) + 6 \sin(2dx + 2c) + 4 \sin(1/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c)))) \cos(5/4 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c))) - 44(\sin(4dx + 4c) + 6 \sin(2dx + 2c) + 4 \sin(1/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c)))) \cos(3/4 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c))) - 12(\sin(4dx + 4c) + 6 \sin(2dx + 2c)) \cos(1/4 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c))) - 3(2(6 \cos(2dx + 2c) + 1) \cos(4dx + 4c) + \cos(4dx + 4c)^2 + 36 \cos(2dx + 2c)^2 + 8(\cos(4dx + 4c) + 6 \cos(2dx + 2c) + 4 \cos(1/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c)))) + 1) \cos(3/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c))) + 16 \cos(3/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c)))^2 + 8(\cos(4dx + 4c) + 6 \cos(2dx + 2c) + 1) \cos(1/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c))) + 16 \cos(1/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c)))^2 + \sin(4dx + 4c)^2 + 12 \sin(4dx + 4c) \sin(2dx + 2c) + 36 \sin(2dx + 2c)^2 + 8(\sin(4dx + 4c) + 6 \sin(2dx + 2c) + 4 \sin(1/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c)))) \sin(3/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c))) + 16 \sin(3/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c)))^2 + 8(\sin(4dx + 4c) + 6 \sin(2dx + 2c)) \sin(1/2 \arctan2(
\end{aligned}$$

$$\begin{aligned}
& \sin(2dx + 2c), \cos(2dx + 2c))) + 16\sin(1/2\arctan2(\sin(2dx + 2c), \\
& \cos(2dx + 2c)))^2 + 12\cos(2dx + 2c) + 1)\log(\cos(1/4\arctan2(\sin(2d \\
& dx + 2c), \cos(2dx + 2c)))^2 + \sin(1/4\arctan2(\sin(2dx + 2c), \cos(2 \\
& dx + 2c)))^2 + 2\sin(1/4\arctan2(\sin(2dx + 2c), \cos(2dx + 2c))) + 1 \\
&) + 3*(2*(6\cos(2dx + 2c) + 1)*\cos(4dx + 4c) + \cos(4dx + 4c)^2 + 3 \\
& 6\cos(2dx + 2c)^2 + 8*(\cos(4dx + 4c) + 6\cos(2dx + 2c) + 4\cos(1/2 \\
& *\arctan2(\sin(2dx + 2c), \cos(2dx + 2c))) + 1)*\cos(3/2\arctan2(\sin(2dx \\
& x + 2c), \cos(2dx + 2c))) + 16\cos(3/2\arctan2(\sin(2dx + 2c), \cos(2d \\
& *x + 2c)))^2 + 8*(\cos(4dx + 4c) + 6\cos(2dx + 2c) + 1)*\cos(1/2\arcta \\
& n2(\sin(2dx + 2c), \cos(2dx + 2c))) + 16\cos(1/2\arctan2(\sin(2dx + 2 \\
& c), \cos(2dx + 2c)))^2 + \sin(4dx + 4c)^2 + 12\sin(4dx + 4c)*\sin(2dx \\
& *x + 2c) + 36\sin(2dx + 2c)^2 + 8*(\sin(4dx + 4c) + 6\sin(2dx + 2c \\
&) + 4\sin(1/2\arctan2(\sin(2dx + 2c), \cos(2dx + 2c))))*\sin(3/2\arctan2 \\
& (\sin(2dx + 2c), \cos(2dx + 2c))) + 16\sin(3/2\arctan2(\sin(2dx + 2c) \\
& , \cos(2dx + 2c)))^2 + 8*(\sin(4dx + 4c) + 6\sin(2dx + 2c))*\sin(1/2 \\
& arctan2(\sin(2dx + 2c), \cos(2dx + 2c))) + 16\sin(1/2\arctan2(\sin(2dx \\
& + 2c), \cos(2dx + 2c)))^2 + 12\cos(2dx + 2c) + 1)\log(\cos(1/4\arctan \\
& 2(\sin(2dx + 2c), \cos(2dx + 2c)))^2 + \sin(1/4\arctan2(\sin(2dx + 2c) \\
& , \cos(2dx + 2c)))^2 - 2\sin(1/4\arctan2(\sin(2dx + 2c), \cos(2dx + 2 \\
& c))) + 1) - 12*(\cos(4dx + 4c) + 6\cos(2dx + 2c) + 4\cos(3/2\arctan2(s \\
& in(2dx + 2c), \cos(2dx + 2c))) + 4\cos(1/2\arctan2(\sin(2dx + 2c), c \\
& os(2dx + 2c))) + 1)*\sin(7/4\arctan2(\sin(2dx + 2c), \cos(2dx + 2c))) \\
& + 16*(11\cos(5/4\arctan2(\sin(2dx + 2c), \cos(2dx + 2c))) - 11\cos(3/4 \\
& *\arctan2(\sin(2dx + 2c), \cos(2dx + 2c))) - 3\cos(1/4\arctan2(\sin(2dx \\
& + 2c), \cos(2dx + 2c))))*\sin(3/2\arctan2(\sin(2dx + 2c), \cos(2dx + \\
& 2c))) - 44*(\cos(4dx + 4c) + 6\cos(2dx + 2c) + 4\cos(1/2\arctan2(\sin(\\
& 2dx + 2c), \cos(2dx + 2c))) + 1)*\sin(5/4\arctan2(\sin(2dx + 2c), \cos \\
& (2dx + 2c))) + 44*(\cos(4dx + 4c) + 6\cos(2dx + 2c) + 4\cos(1/2\arc \\
& tan2(\sin(2dx + 2c), \cos(2dx + 2c))) + 1)*\sin(3/4\arctan2(\sin(2dx + \\
& 2c), \cos(2dx + 2c))) - 48\cos(1/4\arctan2(\sin(2dx + 2c), \cos(2dx + \\
& 2c)))*\sin(1/2\arctan2(\sin(2dx + 2c), \cos(2dx + 2c))) + 12*(\cos(4dx \\
& x + 4c) + 6\cos(2dx + 2c) + 1)*\sin(1/4\arctan2(\sin(2dx + 2c), \cos(2 \\
& dx + 2c))) + 48\cos(1/2\arctan2(\sin(2dx + 2c), \cos(2dx + 2c)))*\sin(\\
& 1/4\arctan2(\sin(2dx + 2c), \cos(2dx + 2c)))*B/((\sqrt{2})a^2\cos(4dx \\
& + 4c)^2 + 36\sqrt{2})a^2\cos(2dx + 2c)^2 + 16\sqrt{2})a^2\cos(3/2\arct \\
& an2(\sin(2dx + 2c), \cos(2dx + 2c)))^2 + 16\sqrt{2})a^2\cos(1/2\arctan2 \\
& (\sin(2dx + 2c), \cos(2dx + 2c)))^2 + \sqrt{2})a^2\sin(4dx + 4c)^2 + \\
& 12\sqrt{2})a^2\sin(4dx + 4c)*\sin(2dx + 2c) + 36\sqrt{2})a^2\sin(2dx \\
& + 2c)^2 + 16\sqrt{2})a^2\sin(3/2\arctan2(\sin(2dx + 2c), \cos(2dx + 2 \\
& c)))^2 + 16\sqrt{2})a^2\sin(1/2\arctan2(\sin(2dx + 2c), \cos(2dx + 2c) \\
&))^2 + 12\sqrt{2})a^2\cos(2dx + 2c) + \sqrt{2})a^2 + 2*(6\sqrt{2})a^2\cos(\\
& 2dx + 2c) + \sqrt{2})a^2*\cos(4dx + 4c) + 8*(\sqrt{2})a^2*\cos(4dx + 4 \\
& *c) + 6\sqrt{2})a^2*\cos(2dx + 2c) + 4\sqrt{2})a^2*\cos(1/2\arctan2(\sin(2 \\
& dx + 2c), \cos(2dx + 2c))) + \sqrt{2})a^2*\cos(3/2\arctan2(\sin(2dx + 2 \\
& *c), \cos(2dx + 2c))) + 8*(\sqrt{2})a^2*\cos(4dx + 4c) + 6\sqrt{2})a^2*c
\end{aligned}$$

```
os(2*d*x + 2*c) + sqrt(2)*a^2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + 8*(sqrt(2)*a^2*sin(4*d*x + 4*c) + 6*sqrt(2)*a^2*sin(2*d*x + 2*c)
+ 4*sqrt(2)*a^2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(
3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*a^2*sin(4*d*x
+ 4*c) + 6*sqrt(2)*a^2*sin(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))))*sqrt(a))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\cos(c+dx)^{3/2} \left(a + \frac{a}{\cos(c+dx)}\right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^(5/2)),x)
[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^(5/2)), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(3/2)/(a+a*sec(d*x+c))**(5/2),x)
[Out] Timed out
```

$$3.560 \quad \int \frac{A+B \sec(c+dx)}{\cos^2(c+dx)(a+a \sec(c+dx))^{5/2}} dx$$

Optimal. Leaf size=234

$$\frac{(3A - 43B)\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \tanh^{-1}\left(\frac{\sqrt{a} \sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2} \sqrt{a \sec(c+dx)+a}}\right)}{16\sqrt{2} a^{5/2} d} + \frac{2B\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \sinh^{-1}\left(\frac{\sqrt{a}}{\sqrt{a \sec(c+dx)+a}}\right)}{a^{5/2} d}$$

[Out] 1/4*(A-B)*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(5/2)+1/16*(3*A-11*B)*sin(d*x+c)/a/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2)+2*B*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(5/2)/d+1/32*(3*A-43*B)*arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(5/2)/d*2^(1/2)

Rubi [A] time = 0.71, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2955, 4019, 4023, 3808, 206, 3801, 215}

$$\frac{(3A - 43B)\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \tanh^{-1}\left(\frac{\sqrt{a} \sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2} \sqrt{a \sec(c+dx)+a}}\right)}{16\sqrt{2} a^{5/2} d} + \frac{2B\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \sinh^{-1}\left(\frac{\sqrt{a}}{\sqrt{a \sec(c+dx)+a}}\right)}{a^{5/2} d}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/2)),x]

[Out] (2*B*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(a^(5/2)*d) + ((3*A - 43*B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]*a^(5/2)*d) + ((A - B)*Sin[c + d*x])/(4*d*Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/2)) + ((3*A - 11*B)*Sin[c + d*x])/(16*a*d*Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 2955

Int[((a_.) + csc[(e_.) + (f_.)*(x_)])*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] := Dist[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && IntegerQ[n])

Rule 3801

Int[Sqrt[csc[(e_.) + (f_.)*(x_)])*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] := Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 + x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]

Rule 3808

Int[Sqrt[csc[(e_.) + (f_.)*(x_)])*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] := Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 4019

Int[(csc[(e_.) + (f_.)*(x_)])*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] := Simp[(d*(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0]

Rule 4023

Int[(csc[(e_.) + (f_.)*(x_)])*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] := Dist[(A*b - a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned}
\int \frac{A + B \sec(c + dx)}{\cos^{\frac{5}{2}}(c + dx)(a + a \sec(c + dx))^{5/2}} dx &= \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{\sec^{\frac{5}{2}}(c + dx)(A + B \sec(c + dx))}{(a + a \sec(c + dx))^{5/2}} dx \\
&= \frac{(A - B) \sin(c + dx)}{4d \cos^{\frac{5}{2}}(c + dx)(a + a \sec(c + dx))^{5/2}} + \frac{\left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right)}{\dots} \\
&= \frac{(A - B) \sin(c + dx)}{4d \cos^{\frac{5}{2}}(c + dx)(a + a \sec(c + dx))^{5/2}} + \frac{(3A - 11B) \sin(c + dx)}{16ad \cos^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))^{5/2}} \\
&= \frac{(A - B) \sin(c + dx)}{4d \cos^{\frac{5}{2}}(c + dx)(a + a \sec(c + dx))^{5/2}} + \frac{(3A - 11B) \sin(c + dx)}{16ad \cos^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))^{5/2}} \\
&= \frac{(A - B) \sin(c + dx)}{4d \cos^{\frac{5}{2}}(c + dx)(a + a \sec(c + dx))^{5/2}} + \frac{(3A - 11B) \sin(c + dx)}{16ad \cos^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))^{5/2}} \\
&= \frac{(A - B) \sin(c + dx)}{4d \cos^{\frac{5}{2}}(c + dx)(a + a \sec(c + dx))^{5/2}} + \frac{(3A - 11B) \sin(c + dx)}{16ad \cos^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))^{5/2}} \\
&= \frac{2B \sinh^{-1} \left(\frac{\sqrt{a} \tan(c + dx)}{\sqrt{a + a \sec(c + dx)}} \right) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} (3A - 43B) t}{a^{5/2} d} + \dots
\end{aligned}$$

Mathematica [B] time = 6.17, size = 965, normalized size = 4.12

$$\frac{3A \sin^{-1} \left(\sqrt{1 - \sec(c + dx)} \right) \sqrt{\cos(c + dx)} \sec^{\frac{3}{2}}(c + dx) \sin(c + dx) (\sec(c + dx) + 1)^2}{16d \sqrt{1 - \sec(c + dx)} (a(\sec(c + dx) + 1))^{5/2}} - \frac{11B \sin^{-1} \left(\sqrt{1 - \sec(c + dx)} \right) \sqrt{\cos(c + dx)} \sec^{\frac{3}{2}}(c + dx) \sin(c + dx) (\sec(c + dx) + 1)^2}{16d \sqrt{1 - \sec(c + dx)} (a(\sec(c + dx) + 1))^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/2)), x]

[Out] -1/4*(B*Sin[c + d*x])/(d*Cos[c + d*x]^(9/2)*(a*(1 + Sec[c + d*x]))^(5/2)) - (A*Sin[c + d*x])/(4*d*Cos[c + d*x]^(7/2)*(a*(1 + Sec[c + d*x]))^(5/2)) + (3*B*(1 + Sec[c + d*x])*Sin[c + d*x])/(16*d*Cos[c + d*x]^(9/2)*(a*(1 + Sec[c + d*x]))^(5/2)) - (A*(1 + Sec[c + d*x])*Sin[c + d*x])/(16*d*Cos[c + d*x]^(7/2)*(a*(1 + Sec[c + d*x]))^(5/2)) - (3*B*(1 + Sec[c + d*x])^2*Sin[c + d*x])/(16*d*Cos[c + d*x]^(7/2)*(a*(1 + Sec[c + d*x]))^(5/2)) + (A*(1 + Sec[c + d*x])*Sin[c + d*x])/(16*d*Cos[c + d*x]^(7/2)*(a*(1 + Sec[c + d*x]))^(5/2))

$$\begin{aligned}
& d*x))^{2*Sin[c + d*x]}/(16*d*Cos[c + d*x]^{(5/2)}*(a*(1 + Sec[c + d*x]))^{(5/2)} \\
&) + (7*B*(1 + Sec[c + d*x])^{2*Sin[c + d*x]}/(16*d*Cos[c + d*x]^{(5/2)}*(a*(1 \\
& + Sec[c + d*x]))^{(5/2)}) + (3*A*(1 + Sec[c + d*x])^{2*Sin[c + d*x]}/(16*d*Cos \\
& [c + d*x]^{(3/2)}*(a*(1 + Sec[c + d*x]))^{(5/2)}) - (11*B*(1 + Sec[c + d*x])^{2* \\
& Sin[c + d*x]}/(16*d*Cos[c + d*x]^{(3/2)}*(a*(1 + Sec[c + d*x]))^{(5/2)}) + (3*A \\
& *ArcSin[Sqrt[1 - Sec[c + d*x]]]*Sqrt[Cos[c + d*x]]*Sec[c + d*x]^{(3/2)}*(1 + \\
& Sec[c + d*x])^{2*Sin[c + d*x]}/(16*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + \\
& d*x]))^{(5/2)}) - (11*B*ArcSin[Sqrt[1 - Sec[c + d*x]]]*Sqrt[Cos[c + d*x]]*Sec \\
& [c + d*x]^{(3/2)}*(1 + Sec[c + d*x])^{2*Sin[c + d*x]}/(16*d*Sqrt[1 - Sec[c + d \\
& *x]]*(a*(1 + Sec[c + d*x]))^{(5/2)}) + (3*A*ArcSin[Sqrt[Sec[c + d*x]]]*Sqrt[C \\
& os[c + d*x]]*Sec[c + d*x]^{(3/2)}*(1 + Sec[c + d*x])^{2*Sin[c + d*x]}/(16*d*Sq \\
& rt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^{(5/2)}) - (43*B*ArcSin[Sqrt[Sec[\\
& c + d*x]]]*Sqrt[Cos[c + d*x]]*Sec[c + d*x]^{(3/2)}*(1 + Sec[c + d*x])^{2*Sin[c \\
& + d*x]}/(16*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^{(5/2)}) - (3*A* \\
& ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sqrt[Cos[c + d* \\
& x]]*Sec[c + d*x]^{(3/2)}*(1 + Sec[c + d*x])^{2*Sin[c + d*x]}/(16*Sqrt[2]*d*Sqr \\
& t[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^{(5/2)}) + (43*B*ArcTan[(Sqrt[2]*S \\
& qrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sqrt[Cos[c + d*x]]*Sec[c + d*x]^{ \\
& (3/2)}*(1 + Sec[c + d*x])^{2*Sin[c + d*x]}/(16*Sqrt[2]*d*Sqrt[1 - Sec[c + d*x \\
&]]*(a*(1 + Sec[c + d*x]))^{(5/2)})
\end{aligned}$$

fricas [A] time = 0.57, size = 720, normalized size = 3.08

$$\left[\frac{\sqrt{2} \left((3A - 43B) \cos(dx + c)^3 + 3(3A - 43B) \cos(dx + c)^2 + 3(3A - 43B) \cos(dx + c) + 3A - 43B \right) \sqrt{a}}{\dots} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")

[Out] [-1/64*(sqrt(2))*((3*A - 43*B)*cos(d*x + c)^3 + 3*(3*A - 43*B)*cos(d*x + c)^2 + 3*(3*A - 43*B)*cos(d*x + c) + 3*A - 43*B)*sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*((3*A - 11*B)*cos(d*x + c) + 7*A - 15*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 32*(B*cos(d*x + c)^3 + 3*B*cos(d*x + c)^2 + 3*B*cos(d*x + c) + B)*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*co

$s(dx + c) + a^3d$, $-1/32*(\sqrt{2}*((3*A - 43*B)*\cos(dx + c)^3 + 3*(3*A - 43*B)*\cos(dx + c)^2 + 3*(3*A - 43*B)*\cos(dx + c) + 3*A - 43*B)*\sqrt{-a}*\arctan(\sqrt{2}*\sqrt{-a}*\sqrt{(a*\cos(dx + c) + a)/\cos(dx + c)}*\sqrt{\cos(dx + c)})/(a*\sin(dx + c))) - 2*((3*A - 11*B)*\cos(dx + c) + 7*A - 15*B)*\sqrt{(a*\cos(dx + c) + a)/\cos(dx + c)}*\sqrt{\cos(dx + c)}*\sin(dx + c) - 32*(B*\cos(dx + c)^3 + 3*B*\cos(dx + c)^2 + 3*B*\cos(dx + c) + B)*\sqrt{-a}*\arctan(2*\sqrt{-a}*\sqrt{(a*\cos(dx + c) + a)/\cos(dx + c)}*\sqrt{\cos(dx + c)}*\sin(dx + c)/(a*\cos(dx + c)^2 - a*\cos(dx + c) - 2*a)))/(a^3*d*\cos(dx + c)^3 + 3*a^3*d*\cos(dx + c)^2 + 3*a^3*d*\cos(dx + c) + a^3*d]$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(a \sec(dx + c) + a)^{\frac{5}{2}} \cos(dx + c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(dx+c))/cos(dx+c)^(5/2)/(a+a*sec(dx+c))^(5/2),x, algorith="giac")

[Out] integrate((B*sec(dx + c) + A)/((a*sec(dx + c) + a)^(5/2)*cos(dx + c)^(5/2)), x)

maple [B] time = 2.27, size = 540, normalized size = 2.31

$$(\sqrt{\cos(dx + c)})(-1 + \cos(dx + c))^2 \left(16B \cos(dx + c) \sin(dx + c) \sqrt{2} \arctan \left(\frac{\sqrt{\frac{2}{1 + \cos(dx + c)}} (\cos(dx + c) + 1 + \sin(dx + c))}{4} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(dx+c))/cos(dx+c)^(5/2)/(a+a*sec(dx+c))^(5/2),x)

[Out] $1/16/d*\cos(dx+c)^{(1/2)}*(-1+\cos(dx+c))^{2*}(16*B*\cos(dx+c)*\sin(dx+c)*2^{(1/2)}*\arctan(1/4*(-2/(1+\cos(dx+c)))^{(1/2)}*(\cos(dx+c)+1+\sin(dx+c))*2^{(1/2)})-16*B*\cos(dx+c)*\sin(dx+c)*2^{(1/2)}*\arctan(1/4*(-2/(1+\cos(dx+c)))^{(1/2)}*(\cos(dx+c)+1-\sin(dx+c))*2^{(1/2)})+3*A*\cos(dx+c)*\sin(dx+c)*\arctan(1/2*\sin(dx+c)*(-2/(1+\cos(dx+c)))^{(1/2)})-3*A*(-2/(1+\cos(dx+c)))^{(1/2)}*\cos(dx+c)^2+16*B*2^{(1/2)}*\arctan(1/4*(-2/(1+\cos(dx+c)))^{(1/2)}*(\cos(dx+c)+1+\sin(dx+c))*2^{(1/2)})*\sin(dx+c)-16*B*2^{(1/2)}*\arctan(1/4*(-2/(1+\cos(dx+c)))^{(1/2)}*(\cos(dx+c)+1-\sin(dx+c))*2^{(1/2)})*\sin(dx+c)-43*B*\cos(dx+c)*\sin(dx+c)*\arctan(1/2*\sin(dx+c)*(-2/(1+\cos(dx+c)))^{(1/2)})+11*B*(-2/(1+\cos(dx+c)))^{(1/2)}*\cos(dx+c)^2+3*A*\arctan(1/2*\sin(dx+c)*(-2/(1+\cos(dx+c)))^{(1/2)})*\sin(dx+c)-4*A*(-2/(1+\cos(dx+c)))^{(1/2)}*\cos(dx+c)-43*B*\arctan(1/2*\sin(dx+c)*(-2/(1$

$+ \cos(dx+c)^{1/2} \sin(dx+c) + 4B(-2/(1+\cos(dx+c)))^{1/2} \cos(dx+c) + 7A(-2/(1+\cos(dx+c)))^{1/2} - 15B(-2/(1+\cos(dx+c)))^{1/2} (a(1+\cos(dx+c)))/\cos(dx+c)^{1/2} / (-2/(1+\cos(dx+c)))^{1/2} / \sin(dx+c)^5/a^3$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(dx+c))/cos(dx+c)^(5/2)/(a+a*sec(dx+c))^(5/2),x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\cos(c+dx)^{5/2} \left(a + \frac{a}{\cos(c+dx)}\right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + dx))/(cos(c + dx)^(5/2)*(a + a/cos(c + dx))^(5/2)),x)

[Out] int((A + B/cos(c + dx))/(cos(c + dx)^(5/2)*(a + a/cos(c + dx))^(5/2)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(dx+c))/cos(dx+c)**(5/2)/(a+a*sec(dx+c))**(5/2),x)

[Out] Timed out

$$3.561 \quad \int \frac{A+B \sec(c+dx)}{\cos^2(c+dx)(a+a \sec(c+dx))^{5/2}} dx$$

Optimal. Leaf size=286

$$\frac{(43A - 115B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)} \tanh^{-1}\left(\frac{\sqrt{a} \sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a \sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d} + \frac{(2A - 5B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{a^{5/2}d}$$

[Out] 1/4*(A-B)*sin(d*x+c)/d/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(5/2)+1/16*(7*A-15*B)*sin(d*x+c)/a/d/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2)+(2*A-5*B)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(5/2)/d-1/32*(43*A-115*B)*arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(5/2)/d*2^(1/2)-1/16*(11*A-35*B)*sin(d*x+c)/a^2/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2)

Rubi [A] time = 0.96, antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {2955, 4019, 4021, 4023, 3808, 206, 3801, 215}

$$\frac{(11A - 35B) \sin(c+dx)}{16a^2d \cos^2(c+dx)\sqrt{a \sec(c+dx)+a}} - \frac{(43A - 115B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)} \tanh^{-1}\left(\frac{\sqrt{a} \sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a \sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(5/2)),x]

[Out] ((2*A - 5*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(a^(5/2)*d) - ((43*A - 115*B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]*a^(5/2)*d) + ((A - B)*Sin[c + d*x])/(4*d*Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(5/2)) + ((7*A - 15*B)*Sin[c + d*x])/(16*a*d*Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/2)) - ((11*A - 35*B)*Sin[c + d*x])/(16*a^2*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 215

$\text{Int}[1/\text{Sqrt}[(a_) + (b_)*(x_)^2], x_Symbol] \rightarrow \text{Simp}[\text{ArcSinh}[(\text{Rt}[b, 2]*x)/\text{Sqrt}[a]]/\text{Rt}[b, 2], x] /; \text{FreeQ}\{a, b\}, x\} \ \&\& \ \text{GtQ}[a, 0] \ \&\& \ \text{PosQ}[b]$

Rule 2955

$\text{Int}[(a_) + \text{csc}[(e_) + (f_)*(x_)]*(b_)]^{(m_)} * (\text{csc}[(e_) + (f_)*(x_)] * (d_) + (c_))^{(n_)} * ((g_)*\sin[(e_) + (f_)*(x_)])^{(p_)}, x_Symbol] \rightarrow \text{Dist}[(g*\text{Csc}[e + f*x])^p * (g*\text{Sin}[e + f*x])^p, \text{Int}[(a + b*\text{Csc}[e + f*x])^m * (c + d*\text{Csc}[e + f*x])^n / (g*\text{Csc}[e + f*x])^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, m, n, p\}, x\} \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{!IntegerQ}[p] \ \&\& \ \text{!(IntegerQ}[m] \ \&\& \ \text{IntegerQ}[n])]$

Rule 3801

$\text{Int}[\text{Sqrt}[\text{csc}[(e_) + (f_)*(x_)]*(d_)] * \text{Sqrt}[\text{csc}[(e_) + (f_)*(x_)]*(b_) + (a_)], x_Symbol] \rightarrow \text{Dist}[(-2*a*\text{Sqrt}[(a*d)/b]) / (b*f), \text{Subst}[\text{Int}[1/\text{Sqrt}[1 + x^2/a], x], x, (b*\text{Cot}[e + f*x]) / \text{Sqrt}[a + b*\text{Csc}[e + f*x]]], x] /; \text{FreeQ}\{a, b, d, e, f\}, x\} \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{GtQ}[(a*d)/b, 0]$

Rule 3808

$\text{Int}[\text{Sqrt}[\text{csc}[(e_) + (f_)*(x_)]*(d_)] / \text{Sqrt}[\text{csc}[(e_) + (f_)*(x_)]*(b_) + (a_)], x_Symbol] \rightarrow \text{Dist}[(-2*b*d) / (a*f), \text{Subst}[\text{Int}[1/(2*b - d*x^2), x], x, (b*\text{Cot}[e + f*x]) / (\text{Sqrt}[a + b*\text{Csc}[e + f*x]] * \text{Sqrt}[d*\text{Csc}[e + f*x]])], x] /; \text{FreeQ}\{a, b, d, e, f\}, x\} \ \&\& \ \text{EqQ}[a^2 - b^2, 0]$

Rule 4019

$\text{Int}[(\text{csc}[(e_) + (f_)*(x_)]*(d_))^{(n_)} * (\text{csc}[(e_) + (f_)*(x_)]*(b_) + (a_))^{(m_)} * (\text{csc}[(e_) + (f_)*(x_)]*(B_) + (A_)), x_Symbol] \rightarrow \text{Simp}[(d*(A*b - a*B)*\text{Cot}[e + f*x] * (a + b*\text{Csc}[e + f*x])^m * (d*\text{Csc}[e + f*x])^{(n-1)}) / (a*f*(2*m + 1)), x] - \text{Dist}[1/(a*b*(2*m + 1)), \text{Int}[(a + b*\text{Csc}[e + f*x])^{(m+1)} * (d*\text{Csc}[e + f*x])^{(n-1)} * \text{Simp}[A*(a*d*(n-1)) - B*(b*d*(n-1)) - d*(a*B*(m-n+1) + A*b*(m+n))*\text{Csc}[e + f*x], x], x], x] /; \text{FreeQ}\{a, b, d, e, f, A, B\}, x\} \ \&\& \ \text{NeQ}[A*b - a*B, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{LtQ}[m, -2^{(-1)}] \ \&\& \ \text{GtQ}[n, 0]$

Rule 4021

$\text{Int}[(\text{csc}[(e_) + (f_)*(x_)]*(d_))^{(n_)} * (\text{csc}[(e_) + (f_)*(x_)]*(b_) + (a_))^{(m_)} * (\text{csc}[(e_) + (f_)*(x_)]*(B_) + (A_)), x_Symbol] \rightarrow -\text{Simp}[(B*d*\text{Cot}[e + f*x] * (a + b*\text{Csc}[e + f*x])^m * (d*\text{Csc}[e + f*x])^{(n-1)}) / (f*(m+n)), x] + \text{Dist}[d/(b*(m+n)), \text{Int}[(a + b*\text{Csc}[e + f*x])^m * (d*\text{Csc}[e + f*x])^{(n-1)} * \text{Simp}[b*B*(n-1) + (A*b*(m+n) + a*B*m)*\text{Csc}[e + f*x], x], x], x] /; \text{FreeQ}\{a, b, d, e, f, A, B, m\}, x\} \ \&\& \ \text{NeQ}[A*b - a*B, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\&$

GtQ[n, 1]

Rule 4023

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> Dist[(A*b - a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{A + B \sec(c + dx)}{\cos^{\frac{7}{2}}(c + dx)(a + a \sec(c + dx))^{5/2}} dx &= \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{\sec^{\frac{7}{2}}(c + dx)(A + B \sec(c + dx))}{(a + a \sec(c + dx))^{5/2}} dx \\
 &= \frac{(A - B) \sin(c + dx)}{4d \cos^{\frac{7}{2}}(c + dx)(a + a \sec(c + dx))^{5/2}} + \frac{\left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right)}{16ad \cos^{\frac{5}{2}}(c + dx)(a + a \sec(c + dx))^{5/2}} \\
 &= \frac{(A - B) \sin(c + dx)}{4d \cos^{\frac{7}{2}}(c + dx)(a + a \sec(c + dx))^{5/2}} + \frac{(7A - 15B) \sin(c + dx)}{16ad \cos^{\frac{5}{2}}(c + dx)(a + a \sec(c + dx))^{5/2}} \\
 &= \frac{(A - B) \sin(c + dx)}{4d \cos^{\frac{7}{2}}(c + dx)(a + a \sec(c + dx))^{5/2}} + \frac{(7A - 15B) \sin(c + dx)}{16ad \cos^{\frac{5}{2}}(c + dx)(a + a \sec(c + dx))^{5/2}} \\
 &= \frac{(A - B) \sin(c + dx)}{4d \cos^{\frac{7}{2}}(c + dx)(a + a \sec(c + dx))^{5/2}} + \frac{(7A - 15B) \sin(c + dx)}{16ad \cos^{\frac{5}{2}}(c + dx)(a + a \sec(c + dx))^{5/2}} \\
 &= \frac{(A - B) \sin(c + dx)}{4d \cos^{\frac{7}{2}}(c + dx)(a + a \sec(c + dx))^{5/2}} + \frac{(7A - 15B) \sin(c + dx)}{16ad \cos^{\frac{5}{2}}(c + dx)(a + a \sec(c + dx))^{5/2}} \\
 &= \frac{(2A - 5B) \sinh^{-1} \left(\frac{\sqrt{a} \tan(c + dx)}{\sqrt{a + a \sec(c + dx)}} \right) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)}}{a^{5/2} d} \quad (43A)
 \end{aligned}$$

Mathematica [B] time = 6.20, size = 1061, normalized size = 3.71

$$\frac{11A \sin^{-1}(\sqrt{1 - \sec(c + dx)}) \sqrt{\cos(c + dx)} \sec^3(c + dx) \sin(c + dx) (\sec(c + dx) + 1)^2}{16d \sqrt{1 - \sec(c + dx)} (a(\sec(c + dx) + 1))^{5/2}} + \frac{35B \sin^{-1}(\sqrt{1 - \sec(c + dx)}) \sqrt{\cos(c + dx)} \sec^3(c + dx) \sin(c + dx) (\sec(c + dx) + 1)^2}{16d \sqrt{1 - \sec(c + dx)} (a(\sec(c + dx) + 1))^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(5/2)), x]

[Out]
$$\begin{aligned} & -1/4*(B*\sin[c + d*x])/(d*\cos[c + d*x]^{11/2}*(a*(1 + \sec[c + d*x]))^{5/2}) \\ & - (A*\sin[c + d*x])/(4*d*\cos[c + d*x]^{9/2}*(a*(1 + \sec[c + d*x]))^{5/2}) + \\ & (7*B*(1 + \sec[c + d*x])*\sin[c + d*x])/(16*d*\cos[c + d*x]^{11/2}*(a*(1 + \sec[c + d*x]))^{5/2}) + \\ & (3*A*(1 + \sec[c + d*x])*\sin[c + d*x])/(16*d*\cos[c + d*x]^{9/2}*(a*(1 + \sec[c + d*x]))^{5/2}) - \\ & (7*B*(1 + \sec[c + d*x])^2*\sin[c + d*x])/(16*d*\cos[c + d*x]^{9/2}*(a*(1 + \sec[c + d*x]))^{5/2}) - \\ & (3*A*(1 + \sec[c + d*x])^2*\sin[c + d*x])/(16*d*\cos[c + d*x]^{7/2}*(a*(1 + \sec[c + d*x]))^{5/2}) + \\ & (11*B*(1 + \sec[c + d*x])^2*\sin[c + d*x])/(16*d*\cos[c + d*x]^{7/2}*(a*(1 + \sec[c + d*x]))^{5/2}) + \\ & (7*A*(1 + \sec[c + d*x])^2*\sin[c + d*x])/(16*d*\cos[c + d*x]^{5/2}*(a*(1 + \sec[c + d*x]))^{5/2}) - \\ & (15*B*(1 + \sec[c + d*x])^2*\sin[c + d*x])/(16*d*\cos[c + d*x]^{5/2}*(a*(1 + \sec[c + d*x]))^{5/2}) - \\ & (11*A*(1 + \sec[c + d*x])^2*\sin[c + d*x])/(16*d*\cos[c + d*x]^{3/2}*(a*(1 + \sec[c + d*x]))^{5/2}) + \\ & (35*B*(1 + \sec[c + d*x])^2*\sin[c + d*x])/(16*d*\cos[c + d*x]^{3/2}*(a*(1 + \sec[c + d*x]))^{5/2}) - \\ & (11*A*\arcsin[\sqrt{1 - \sec[c + d*x]}]*\sqrt{\cos[c + d*x]}*\sec[c + d*x]^{3/2}*(1 + \sec[c + d*x])^2*\sin[c + d*x]) / \\ & (16*d*\sqrt{1 - \sec[c + d*x]}*(a*(1 + \sec[c + d*x]))^{5/2}) + (35*B*\arcsin[\sqrt{1 - \sec[c + d*x]}]*\sqrt{\cos[c + d*x]}*\sec[c + d*x]^{3/2}*(1 + \sec[c + d*x])^2*\sin[c + d*x]) / \\ & (16*d*\sqrt{1 - \sec[c + d*x]}*(a*(1 + \sec[c + d*x]))^{5/2}) - (43*A*\arcsin[\sqrt{\sec[c + d*x]}]*\sqrt{\cos[c + d*x]}*\sec[c + d*x]^{3/2}*(1 + \sec[c + d*x])^2*\sin[c + d*x]) / \\ & (16*d*\sqrt{1 - \sec[c + d*x]}*(a*(1 + \sec[c + d*x]))^{5/2}) + (115*B*\arcsin[\sqrt{\sec[c + d*x]}]*\sqrt{\cos[c + d*x]}*\sec[c + d*x]^{3/2}*(1 + \sec[c + d*x])^2*\sin[c + d*x]) / \\ & (16*d*\sqrt{1 - \sec[c + d*x]}*(a*(1 + \sec[c + d*x]))^{5/2}) + (43*A*\arctan[(\sqrt{2}*\sqrt{\sec[c + d*x]})/\sqrt{1 - \sec[c + d*x]}]*\sqrt{\cos[c + d*x]}*\sec[c + d*x]^{3/2}*(1 + \sec[c + d*x])^2*\sin[c + d*x]) / \\ & (16*\sqrt{2}*d*\sqrt{1 - \sec[c + d*x]}*(a*(1 + \sec[c + d*x]))^{5/2}) - (115*B*\arctan[(\sqrt{2}*\sqrt{\sec[c + d*x]})/\sqrt{1 - \sec[c + d*x]}]*\sqrt{\cos[c + d*x]}*\sec[c + d*x]^{3/2}*(1 + \sec[c + d*x])^2*\sin[c + d*x]) / \\ & (16*\sqrt{2}*d*\sqrt{1 - \sec[c + d*x]}*(a*(1 + \sec[c + d*x]))^{5/2}) \end{aligned}$$

fricas [A] time = 0.61, size = 850, normalized size = 2.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")

[Out] [-1/64*(sqrt(2)*((43*A - 115*B)*cos(d*x + c)^4 + 3*(43*A - 115*B)*cos(d*x + c)^3 + 3*(43*A - 115*B)*cos(d*x + c)^2 + (43*A - 115*B)*cos(d*x + c))*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*((11*A - 35*B)*cos(d*x + c)^2 + 5*(3*A - 11*B)*cos(d*x + c) - 16*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 16*((2*A - 5*B)*cos(d*x + c)^4 + 3*(2*A - 5*B)*cos(d*x + c)^3 + 3*(2*A - 5*B)*cos(d*x + c)^2 + (2*A - 5*B)*cos(d*x + c))*sqrt(a)*log((a*cos(d*x + c)^3 + 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + a^3*d*cos(d*x + c)), 1/32*(sqrt(2)*((43*A - 115*B)*cos(d*x + c)^4 + 3*(43*A - 115*B)*cos(d*x + c)^3 + 3*(43*A - 115*B)*cos(d*x + c)^2 + (43*A - 115*B)*cos(d*x + c))*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) - 2*((11*A - 35*B)*cos(d*x + c)^2 + 5*(3*A - 11*B)*cos(d*x + c) - 16*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 16*((2*A - 5*B)*cos(d*x + c)^4 + 3*(2*A - 5*B)*cos(d*x + c)^3 + 3*(2*A - 5*B)*cos(d*x + c)^2 + (2*A - 5*B)*cos(d*x + c))*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + a^3*d*cos(d*x + c))]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(a \sec(dx + c) + a)^{\frac{5}{2}} \cos(dx + c)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(7/2)), x)

maple [B] time = 2.74, size = 821, normalized size = 2.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((A+B*\sec(dx+c))/\cos(dx+c)^{(7/2)}/(a+a*\sec(dx+c))^{(5/2)},x)$

[Out] $\frac{1}{16} \frac{1}{d} (-1 + \cos(dx+c))^2 (-16A^2)^{(1/2)} \arctan\left(\frac{1}{4} \frac{-2}{1 + \cos(dx+c)}\right)^{(1/2)} (\cos(dx+c) + 1 - \sin(dx+c))^2)^{(1/2)} \sin(dx+c) \cos(dx+c)^2 + 16A^2)^{(1/2)} \arctan\left(\frac{1}{4} \frac{-2}{1 + \cos(dx+c)}\right)^{(1/2)} (\cos(dx+c) + 1 + \sin(dx+c))^2)^{(1/2)} \sin(dx+c) \cos(dx+c)^2 + 40B^2)^{(1/2)} \arctan\left(\frac{1}{4} \frac{-2}{1 + \cos(dx+c)}\right)^{(1/2)} (\cos(dx+c) + 1 - \sin(dx+c))^2)^{(1/2)} \sin(dx+c) \cos(dx+c)^2 - 40B^2)^{(1/2)} \arctan\left(\frac{1}{4} \frac{-2}{1 + \cos(dx+c)}\right)^{(1/2)} (\cos(dx+c) + 1 + \sin(dx+c))^2)^{(1/2)} \sin(dx+c) \cos(dx+c)^2 + 11A^3 (-2/(1 + \cos(dx+c)))^{(1/2)} \cos(dx+c)^3 - 43A \arctan\left(\frac{1}{2} \sin(dx+c) \frac{-2}{1 + \cos(dx+c)}\right)^{(1/2)} \sin(dx+c) \cos(dx+c)^2 - 16A \cos(dx+c) \sin(dx+c)^2)^{(1/2)} \arctan\left(\frac{1}{4} \frac{-2}{1 + \cos(dx+c)}\right)^{(1/2)} (\cos(dx+c) + 1 - \sin(dx+c))^2)^{(1/2)} + 16A \cos(dx+c) \sin(dx+c)^2)^{(1/2)} \arctan\left(\frac{1}{4} \frac{-2}{1 + \cos(dx+c)}\right)^{(1/2)} (\cos(dx+c) + 1 + \sin(dx+c))^2)^{(1/2)} - 35B^3 (-2/(1 + \cos(dx+c)))^{(1/2)} \cos(dx+c)^3 + 115B \arctan\left(\frac{1}{2} \sin(dx+c) \frac{-2}{1 + \cos(dx+c)}\right)^{(1/2)} \sin(dx+c) \cos(dx+c)^2 + 40B \cos(dx+c) \sin(dx+c)^2)^{(1/2)} \arctan\left(\frac{1}{4} \frac{-2}{1 + \cos(dx+c)}\right)^{(1/2)} (\cos(dx+c) + 1 - \sin(dx+c))^2)^{(1/2)} - 40B \cos(dx+c) \sin(dx+c)^2)^{(1/2)} \arctan\left(\frac{1}{4} \frac{-2}{1 + \cos(dx+c)}\right)^{(1/2)} (\cos(dx+c) + 1 + \sin(dx+c))^2)^{(1/2)} + 4A^4 (-2/(1 + \cos(dx+c)))^{(1/2)} \cos(dx+c)^2 - 43A \cos(dx+c) \sin(dx+c) \arctan\left(\frac{1}{2} \sin(dx+c) \frac{-2}{1 + \cos(dx+c)}\right)^{(1/2)} - 20B^4 (-2/(1 + \cos(dx+c)))^{(1/2)} \cos(dx+c)^2 + 115B \cos(dx+c) \sin(dx+c) \arctan\left(\frac{1}{2} \sin(dx+c) \frac{-2}{1 + \cos(dx+c)}\right)^{(1/2)} - 15A^4 (-2/(1 + \cos(dx+c)))^{(1/2)} \cos(dx+c) + 39B^4 (-2/(1 + \cos(dx+c)))^{(1/2)} \cos(dx+c) + 16B^4 (-2/(1 + \cos(dx+c)))^{(1/2)}) (a*(1 + \cos(dx+c))/\cos(dx+c))^{(1/2)}/\cos(dx+c)^{(1/2)}/\sin(dx+c)^5/(-2/(1 + \cos(dx+c)))^{(1/2)}/a^3$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((A+B*\sec(dx+c))/\cos(dx+c)^{(7/2)}/(a+a*\sec(dx+c))^{(5/2)},x, \text{algorithm}="maxima")$

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\cos(c+dx)^{7/2} \left(a + \frac{a}{\cos(c+dx)}\right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((A + B/\cos(c + dx))/(\cos(c + dx)^{(7/2)}*(a + a/\cos(c + dx))^{(5/2)}),x)$

```
[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))^(5/2)), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(7/2)/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

$$3.562 \quad \int \cos^2(c + dx)(a + b \sec(c + dx))(A + B \sec(c + dx)) dx$$

Optimal. Leaf size=140

$$\frac{2(5aA + 7bB)F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{21d} + \frac{6(aB + Ab)E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{5d} + \frac{2(aB + Ab) \sin(c + dx) \cos^3(c + dx)}{5d} + \frac{2(5aA + 7bB)}{21d}$$

[Out] $6/5*(A*b+B*a)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/d+2/21*(5*A*a+7*B*b)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/d+2/5*(A*b+B*a)*\cos(d*x+c)^{(3/2)}*\sin(d*x+c)/d+2/7*a*A*\cos(d*x+c)^{(5/2)}*\sin(d*x+c)/d+2/21*(5*A*a+7*B*b)*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/d$

Rubi [A] time = 0.23, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {2954, 2968, 3023, 2748, 2635, 2641, 2639}

$$\frac{2(5aA + 7bB)F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{21d} + \frac{6(aB + Ab)E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{5d} + \frac{2(aB + Ab) \sin(c + dx) \cos^3(c + dx)}{5d} + \frac{2(5aA + 7bB)}{21d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Cos}[c + d*x]^{(7/2)}*(a + b*\text{Sec}[c + d*x])*(A + B*\text{Sec}[c + d*x]), x]$

[Out] $(6*(A*b + a*B)*\text{EllipticE}[(c + d*x)/2, 2])/(5*d) + (2*(5*a*A + 7*b*B)*\text{EllipticF}[(c + d*x)/2, 2])/(21*d) + (2*(5*a*A + 7*b*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sin}[c + d*x])/(21*d) + (2*(A*b + a*B)*\text{Cos}[c + d*x]^{(3/2)}*\text{Sin}[c + d*x])/(5*d) + (2*a*A*\text{Cos}[c + d*x]^{(5/2)}*\text{Sin}[c + d*x])/(7*d)$

Rule 2635

$\text{Int}[(b_*)*\sin[(c_*) + (d_*)(x_)]^{(n_*)}, x_Symbol] \rightarrow -\text{Simp}[(b*\text{Cos}[c + d*x])*(b*\text{Sin}[c + d*x])^{(n-1)})/(d*n), x] + \text{Dist}[(b^2*(n-1))/n, \text{Int}[(b*\text{Sin}[c + d*x])^{(n-2)}, x], x] /;$ FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2639

$\text{Int}[\text{Sqrt}[\sin[(c_*) + (d_*)(x_)]], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticE}[(1*(c - P i/2 + d*x))/2, 2])/d, x] /;$ FreeQ[{c, d}, x]

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_.)]], x_Symbol] := Simp[(2*EllipticF[(1*(c -
  Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_.)]^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_.)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_.)]^(p_.), x_Symbol] := Dist[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)]*(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] := Int[(a + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2), x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)] + (C_.)*sin[(e_.) + (f_.)*(x_.)]^2), x_Symbol] := -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \cos^{\frac{7}{2}}(c+dx)(a+b \sec(c+dx))(A+B \sec(c+dx)) dx &= \int \cos^{\frac{3}{2}}(c+dx)(b+a \cos(c+dx))(B+A \cos(c+dx)) dx \\
&= \int \cos^{\frac{3}{2}}(c+dx) (bB + (Ab + aB) \cos(c+dx) + aA \cos^2(c+dx)) dx \\
&= \frac{2aA \cos^{\frac{5}{2}}(c+dx) \sin(c+dx)}{7d} + \frac{2}{7} \int \cos^{\frac{3}{2}}(c+dx) (bB + (Ab + aB) \cos(c+dx)) dx \\
&= \frac{2aA \cos^{\frac{5}{2}}(c+dx) \sin(c+dx)}{7d} + (Ab + aB) \int \cos^{\frac{3}{2}}(c+dx) dx \\
&= \frac{2(5aA + 7bB) \sqrt{\cos(c+dx)} \sin(c+dx)}{21d} + \frac{2(Ab + aB) \cos(c+dx)}{21d} \\
&= \frac{6(Ab + aB) E\left(\frac{1}{2}(c+dx) \middle| 2\right)}{5d} + \frac{2(5aA + 7bB) F\left(\frac{1}{2}(c+dx) \middle| 2\right)}{21d}
\end{aligned}$$

Mathematica [A] time = 0.92, size = 103, normalized size = 0.74

$$\frac{10(5aA + 7bB) F\left(\frac{1}{2}(c+dx) \middle| 2\right) + 126(aB + Ab) E\left(\frac{1}{2}(c+dx) \middle| 2\right) + \sin(c+dx) \sqrt{\cos(c+dx)} (42(aB + Ab) \cos(c+dx) + 105d)}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
[Out] (126*(A*b + a*B)*EllipticE[(c + d*x)/2, 2] + 10*(5*a*A + 7*b*B)*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(65*a*A + 70*b*B + 42*(A*b + a*B)*Cos[c + d*x] + 15*a*A*Cos[2*(c + d*x)])*Sin[c + d*x])/(105*d)
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb \cos(dx+c)^3 \sec(dx+c)^2 + Aa \cos(dx+c)^3 + (Ba + Ab) \cos(dx+c)^3 \sec(dx+c)\right) \sqrt{\cos(dx+c)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^3*sec(d*x + c)^2 + A*a*cos(d*x + c)^3 + (B*a + A*b)*cos(d*x + c)^3*sec(d*x + c))*sqrt(cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx+c) + A)(b \sec(dx+c) + a) \cos(dx+c)^{\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)*cos(d*x + c)^(7/2), x)

maple [B] time = 5.28, size = 413, normalized size = 2.95

$$2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \left(240Aa \cos\left(\frac{dx}{2} + \frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + (-360aA - 168Ab - 168aB)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x)

[Out]
$$-2/105*((2*\cos(1/2*d*x+1/2*c)^2-1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(240*A*a*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^8+(-360*A*a-168*A*b-168*B*a)*\sin(1/2*d*x+1/2*c)^6*\cos(1/2*d*x+1/2*c)+(280*A*a+168*A*b+168*B*a+140*B*b)*\sin(1/2*d*x+1/2*c)^4*\cos(1/2*d*x+1/2*c)+(-80*A*a-42*A*b-42*B*a-70*B*b)*\sin(1/2*d*x+1/2*c)^2*\cos(1/2*d*x+1/2*c)+25*a*A*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})-63*A*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)})*b+35*B*b*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})-63*B*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)})*a)/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(b \sec(dx + c) + a) \cos(dx + c)^{\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)*cos(d*x + c)^(7/2), x)

mupad [B] time = 0.77, size = 166, normalized size = 1.19

$$\frac{2Bb \left(\sqrt{\cos(c+dx)} \sin(c+dx) + F\left(\frac{c}{2} + \frac{dx}{2} \middle| 2\right) \right)}{3d} - \frac{2Aa \cos(c+dx)^{9/2} \sin(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{9}{4}; \frac{13}{4}; \cos(c+dx)\right)}{9d \sqrt{\sin(c+dx)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(7/2)*(A + B/cos(c + d*x))*(a + b/cos(c + d*x)),x)
```

```
[Out] (2*B*b*(cos(c + d*x)^(1/2)*sin(c + d*x) + ellipticF(c/2 + (d*x)/2, 2)))/(3*d) - (2*A*a*cos(c + d*x)^(9/2)*sin(c + d*x)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2)^(1/2)) - (2*A*b*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)) - (2*B*a*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2))
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

$$3.563 \quad \int \cos^{\frac{5}{2}}(c + dx)(a + b \sec(c + dx))(A + B \sec(c + dx)) dx$$

Optimal. Leaf size=108

$$\frac{2(aB + Ab)F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3d} + \frac{2(3aA + 5bB)E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{5d} + \frac{2(aB + Ab) \sin(c + dx) \sqrt{\cos(c + dx)}}{3d} + \frac{2aA \sin(c + dx)}{3d}$$

[Out] $2/5*(3*A*a+5*B*b)*(cos(1/2*d*x+1/2*c)^2)^{(1/2)}/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^{(1/2)})/d+2/3*(A*b+B*a)*(cos(1/2*d*x+1/2*c)^2)^{(1/2)}/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^{(1/2)})/d+2/5*a*A*cos(d*x+c)^{(3/2)}*sin(d*x+c)/d+2/3*(A*b+B*a)*sin(d*x+c)*cos(d*x+c)^{(1/2)}/d$

Rubi [A] time = 0.21, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {2954, 2968, 3023, 2748, 2639, 2635, 2641}

$$\frac{2(aB + Ab)F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3d} + \frac{2(3aA + 5bB)E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{5d} + \frac{2(aB + Ab) \sin(c + dx) \sqrt{\cos(c + dx)}}{3d} + \frac{2aA \sin(c + dx)}{3d}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]

[Out] $(2*(3*a*A + 5*b*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(A*b + a*B)*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*(A*b + a*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (2*a*A*Cos[c + d*x]^{(3/2)}*Sin[c + d*x])/(5*d)$

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x])*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2641


```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c -
Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)])*(b_.)^(m_.)*(csc[(e_.) + (f_.)*(x_)
*(d_.) + (c_.)]^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] := Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
a*d, 0] && !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Int[(a
+ b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] := -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
!LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \cos^{\frac{5}{2}}(c+dx)(a+b \sec(c+dx))(A+B \sec(c+dx)) dx &= \int \sqrt{\cos(c+dx)}(b+a \cos(c+dx))(B+A \cos(c+dx)) dx \\
&= \int \sqrt{\cos(c+dx)}(bB+(Ab+aB) \cos(c+dx)+aA \cos^2(c+dx)) dx \\
&= \frac{2aA \cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{5d} + \frac{2}{5} \int \sqrt{\cos(c+dx)}(bB+(Ab+aB) \cos(c+dx)) dx \\
&= \frac{2aA \cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{5d} + (Ab+aB) \int \cos^{\frac{3}{2}}(c+dx) dx \\
&= \frac{2(3aA+5bB)E\left(\frac{1}{2}(c+dx) \middle| 2\right)}{5d} + \frac{2(Ab+aB)\sqrt{\cos(c+dx)}(3aA \cos(c+dx)+5a^2)}{3d} \\
&= \frac{2(3aA+5bB)E\left(\frac{1}{2}(c+dx) \middle| 2\right)}{5d} + \frac{2(Ab+aB)F\left(\frac{1}{2}(c+dx) \middle| 2\right)}{3d}
\end{aligned}$$

Mathematica [A] time = 0.44, size = 86, normalized size = 0.80

$$\frac{2\left(5(aB+Ab)F\left(\frac{1}{2}(c+dx) \middle| 2\right)+3(3aA+5bB)E\left(\frac{1}{2}(c+dx) \middle| 2\right)+\sin(c+dx)\sqrt{\cos(c+dx)}(3aA \cos(c+dx)+5a^2)\right)}{15d}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]

[Out] (2*(3*(3*a*A + 5*b*B)*EllipticE[(c + d*x)/2, 2] + 5*(A*b + a*B)*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(5*A*b + 5*a*B + 3*a*A*Cos[c + d*x])*Sin[c + d*x]))/(15*d)

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb \cos(dx+c)^2 \sec(dx+c)^2 + Aa \cos(dx+c)^2 + (Ba+Ab) \cos(dx+c)^2 \sec(dx+c)\right)\sqrt{\cos(dx+c)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] integral((B*b*cos(d*x + c)^2*sec(d*x + c)^2 + A*a*cos(d*x + c)^2 + (B*a + A*b)*cos(d*x + c)^2*sec(d*x + c))*sqrt(cos(d*x + c)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx+c) + A)(b \sec(dx+c) + a) \cos(dx+c)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)*cos(d*x + c)^(5/2), x)

maple [B] time = 4.41, size = 371, normalized size = 3.44

$$2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \left(-24Aa \cos\left(\frac{dx}{2} + \frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + (24aA + 20Ab + 20aB)\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x)

[Out]
$$\begin{aligned} & -2/15 * ((2 * \cos(1/2 * d * x + 1/2 * c) ^ 2 - 1) * \sin(1/2 * d * x + 1/2 * c) ^ 2) ^ (1/2) * (-24 * A * a * \cos(1/2 * d * x + 1/2 * c) * \sin(1/2 * d * x + 1/2 * c) ^ 6 + (24 * A * a + 20 * A * b + 20 * B * a) * \sin(1/2 * d * x + 1/2 * c) ^ 4 * \cos(1/2 * d * x + 1/2 * c) + (-6 * A * a - 10 * A * b - 10 * B * a) * \sin(1/2 * d * x + 1/2 * c) ^ 2 * \cos(1/2 * d * x + 1/2 * c) + 5 * A * b * (\sin(1/2 * d * x + 1/2 * c) ^ 2) ^ (1/2) * (2 * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 1) ^ (1/2) * \text{EllipticF}(\cos(1/2 * d * x + 1/2 * c), 2 ^ (1/2)) - 9 * A * (\sin(1/2 * d * x + 1/2 * c) ^ 2) ^ (1/2) * (2 * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 1) ^ (1/2) * \text{EllipticE}(\cos(1/2 * d * x + 1/2 * c), 2 ^ (1/2)) * a + 5 * a * B * (\sin(1/2 * d * x + 1/2 * c) ^ 2) ^ (1/2) * (2 * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 1) ^ (1/2) * \text{EllipticF}(\cos(1/2 * d * x + 1/2 * c), 2 ^ (1/2)) - 15 * B * (\sin(1/2 * d * x + 1/2 * c) ^ 2) ^ (1/2) * (2 * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 1) ^ (1/2) * \text{EllipticE}(\cos(1/2 * d * x + 1/2 * c), 2 ^ (1/2)) * b) / (-2 * \sin(1/2 * d * x + 1/2 * c) ^ 4 + \sin(1/2 * d * x + 1/2 * c) ^ 2) ^ (1/2) / \sin(1/2 * d * x + 1/2 * c) / (2 * \cos(1/2 * d * x + 1/2 * c) ^ 2 - 1) ^ (1/2) / d \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(b \sec(dx + c) + a) \cos(dx + c)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)*cos(d*x + c)^(5/2), x)

mupad [B] time = 0.62, size = 128, normalized size = 1.19

$$\frac{2Ab \left(\sqrt{\cos(c+dx)} \sin(c+dx) + F\left(\frac{c}{2} + \frac{dx}{2} \middle| 2\right)\right)}{3d} + \frac{2Ba \left(\sqrt{\cos(c+dx)} \sin(c+dx) + F\left(\frac{c}{2} + \frac{dx}{2} \middle| 2\right)\right)}{3d} + \frac{2Bb}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)*(A + B/cos(c + d*x))*(a + b/cos(c + d*x)),x)
```

```
[Out] (2*A*b*(cos(c + d*x)^(1/2)*sin(c + d*x) + ellipticF(c/2 + (d*x)/2, 2)))/(3*d) + (2*B*a*(cos(c + d*x)^(1/2)*sin(c + d*x) + ellipticF(c/2 + (d*x)/2, 2)))/(3*d) + (2*B*b*ellipticE(c/2 + (d*x)/2, 2))/d - (2*A*a*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2))
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

$$3.564 \quad \int \cos^{\frac{3}{2}}(c + dx)(a + b \sec(c + dx))(A + B \sec(c + dx)) dx$$

Optimal. Leaf size=75

$$\frac{2(aA + 3bB)F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3d} + \frac{2(aB + Ab)E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{d} + \frac{2aA \sin(c + dx)\sqrt{\cos(c + dx)}}{3d}$$

[Out] $2*(A*b+B*a)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/d+2/3*(A*a+3*B*b)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/d+2/3*a*A*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/d$

Rubi [A] time = 0.19, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {2954, 2968, 3023, 2748, 2641, 2639}

$$\frac{2(aA + 3bB)F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3d} + \frac{2(aB + Ab)E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{d} + \frac{2aA \sin(c + dx)\sqrt{\cos(c + dx)}}{3d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Cos}[c + d*x]^{(3/2)}*(a + b*\text{Sec}[c + d*x])*(A + B*\text{Sec}[c + d*x]), x]$

[Out] $(2*(A*b + a*B)*\text{EllipticE}[(c + d*x)/2, 2])/d + (2*(a*A + 3*b*B)*\text{EllipticF}[(c + d*x)/2, 2])/(3*d) + (2*a*A*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sin}[c + d*x])/(3*d)$

Rule 2639

$\text{Int}[\text{Sqrt}[\sin[(c_.) + (d_.)*(x_)]]], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticE}[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 2641

$\text{Int}[1/\text{Sqrt}[\sin[(c_.) + (d_.)*(x_)]]], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticF}[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 2748

$\text{Int}[(b_.)*\sin[(e_.) + (f_.)*(x_)]]^{(m_.)*((c_.) + (d_.)*\sin[(e_.) + (f_.)*(x_)])}, x_Symbol] \rightarrow \text{Dist}[c, \text{Int}[(b*\text{Sin}[e + f*x])^m, x], x] + \text{Dist}[d/b, \text{Int}[(b*\text{Sin}[e + f*x])^{(m + 1)}, x], x] /; \text{FreeQ}\{b, c, d, e, f, m\}, x]$

Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)])*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_.))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] := Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
a*d, 0] && !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Int[(a
+ b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] := -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
!LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \cos^{\frac{3}{2}}(c + dx)(a + b \sec(c + dx))(A + B \sec(c + dx)) dx &= \int \frac{(b + a \cos(c + dx))(B + A \cos(c + dx))}{\sqrt{\cos(c + dx)}} dx \\
&= \int \frac{bB + (Ab + aB) \cos(c + dx) + aA \cos^2(c + dx)}{\sqrt{\cos(c + dx)}} dx \\
&= \frac{2aA \sqrt{\cos(c + dx)} \sin(c + dx)}{3d} + \frac{2}{3} \int \frac{\frac{1}{2}(aA + 3bB) + \dots}{\sqrt{\cos(c + dx)}} dx \\
&= \frac{2aA \sqrt{\cos(c + dx)} \sin(c + dx)}{3d} + (Ab + aB) \int \sqrt{\cos(c + dx)} dx \\
&= \frac{2(Ab + aB)E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{d} + \frac{2(aA + 3bB)F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3d}
\end{aligned}$$

Mathematica [A] time = 0.27, size = 67, normalized size = 0.89

$$\frac{2 \left((aA + 3bB) F\left(\frac{1}{2}(c + dx) \middle| 2\right) + 3(aB + Ab) E\left(\frac{1}{2}(c + dx) \middle| 2\right) + aA \sin(c + dx) \sqrt{\cos(c + dx)} \right)}{3d}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]

[Out] (2*(3*(A*b + a*B)*EllipticE[(c + d*x)/2, 2] + (a*A + 3*b*B)*EllipticF[(c + d*x)/2, 2] + a*A*Sqrt[Cos[c + d*x]]*Sin[c + d*x]))/(3*d)

fricas [F] time = 0.46, size = 0, normalized size = 0.00

integral((Bb cos(dx + c) sec(dx + c)^2 + Aa cos(dx + c) + (Ba + Ab) cos(dx + c) sec(dx + c))sqrt(cos(dx + c)), x)

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] integral((B*b*cos(d*x + c)*sec(d*x + c)^2 + A*a*cos(d*x + c) + (B*a + A*b)*cos(d*x + c)*sec(d*x + c))*sqrt(cos(d*x + c)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(b \sec(dx + c) + a) \cos(dx + c)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)*cos(d*x + c)^(3/2), x)

maple [B] time = 4.68, size = 326, normalized size = 4.35

$$\frac{2 \sqrt{\left(2 \left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right) \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) \left(4Aa \cos\left(\frac{dx}{2} + \frac{c}{2}\right) \left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + aA \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{2 \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}\right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x)

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*A*a*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+a*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b-2*A*a*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+3*B*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(b \sec(dx + c) + a) \cos(dx + c)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)*cos(d*x + c)^(3/2), x)
```

mupad [B] time = 0.62, size = 85, normalized size = 1.13

$$\frac{2 A a \left(\sqrt{\cos(c + dx)} \sin(c + dx) + F\left(\frac{c}{2} + \frac{dx}{2} \middle| 2\right) \right)}{3 d} + \frac{2 A b E\left(\frac{c}{2} + \frac{dx}{2} \middle| 2\right)}{d} + \frac{2 B a E\left(\frac{c}{2} + \frac{dx}{2} \middle| 2\right)}{d} + \frac{2 B b F\left(\frac{c}{2} + \frac{dx}{2} \middle| 2\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(A + B/cos(c + d*x))*(a + b/cos(c + d*x)),x)
```

```
[Out] (2*A*a*(cos(c + d*x)^(1/2)*sin(c + d*x) + ellipticF(c/2 + (d*x)/2, 2))/(3*d) + (2*A*b*ellipticE(c/2 + (d*x)/2, 2))/d + (2*B*a*ellipticE(c/2 + (d*x)/2, 2))/d + (2*B*b*ellipticF(c/2 + (d*x)/2, 2))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```


$$3.565 \quad \int \sqrt{\cos(c + dx)} (a + b \sec(c + dx))(A + B \sec(c + dx)) dx$$

Optimal. Leaf size=71

$$\frac{2(aB + Ab)F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{d} + \frac{2(aA - bB)E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{d} + \frac{2bB \sin(c + dx)}{d\sqrt{\cos(c + dx)}}$$

[Out] $2*(A*a-B*b)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/d+2*(A*b+B*a)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/d+2*b*B*\sin(d*x+c)/d/\cos(d*x+c)^{(1/2)}$

Rubi [A] time = 0.19, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {2954, 2968, 3021, 2748, 2641, 2639}

$$\frac{2(aB + Ab)F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{d} + \frac{2(aA - bB)E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{d} + \frac{2bB \sin(c + dx)}{d\sqrt{\cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sqrt}[\text{Cos}[c + d*x]]*(a + b*\text{Sec}[c + d*x])*(A + B*\text{Sec}[c + d*x]), x]$

[Out] $(2*(a*A - b*B)*\text{EllipticE}[(c + d*x)/2, 2])/d + (2*(A*b + a*B)*\text{EllipticF}[(c + d*x)/2, 2])/d + (2*b*B*\text{Sin}[c + d*x])/(d*\text{Sqrt}[\text{Cos}[c + d*x]])$

Rule 2639

$\text{Int}[\text{Sqrt}[\sin[(c_.) + (d_.)*(x_.)]], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticE}[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 2641

$\text{Int}[1/\text{Sqrt}[\sin[(c_.) + (d_.)*(x_.)]], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticF}[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 2748

$\text{Int}[(b_.)*\sin[(e_.) + (f_.)*(x_.)]^{(m_.)*((c_.) + (d_.)*\sin[(e_.) + (f_.)*(x_.)])}, x_Symbol] \rightarrow \text{Dist}[c, \text{Int}[(b*\text{Sin}[e + f*x])^m, x], x] + \text{Dist}[d/b, \text{Int}[(b*\text{Sin}[e + f*x])^{(m + 1)}, x], x] /; \text{FreeQ}\{b, c, d, e, f, m\}, x]$

Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_.)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*
(d_.) + (c_.))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_.)])^(p_.), x_Symbol] := Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
a*d, 0] && !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_.)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] := Int[(a
+ b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_.)] + (C_.)*sin[(e_.) + (f_.)*(x_.)]^2), x_Symbol] := -Simp[((A*b^2
- a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{\cos(c + dx)} (a + b \sec(c + dx))(A + B \sec(c + dx)) dx &= \int \frac{(b + a \cos(c + dx))(B + A \cos(c + dx))}{\cos^{\frac{3}{2}}(c + dx)} dx \\
&= \int \frac{bB + (Ab + aB) \cos(c + dx) + aA \cos^2(c + dx)}{\cos^{\frac{3}{2}}(c + dx)} dx \\
&= \frac{2bB \sin(c + dx)}{d\sqrt{\cos(c + dx)}} + 2 \int \frac{\frac{1}{2}(Ab + aB) + \frac{1}{2}(aA - bB) \cos(c + dx)}{\sqrt{\cos(c + dx)}} dx \\
&= \frac{2bB \sin(c + dx)}{d\sqrt{\cos(c + dx)}} + (Ab + aB) \int \frac{1}{\sqrt{\cos(c + dx)}} dx + \\
&= \frac{2(aA - bB)E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{d} + \frac{2(Ab + aB)F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{d}
\end{aligned}$$

Mathematica [A] time = 0.37, size = 64, normalized size = 0.90

$$\frac{2 \left((aB + Ab) F \left(\frac{1}{2}(c + dx) \middle| 2 \right) + (aA - bB) E \left(\frac{1}{2}(c + dx) \middle| 2 \right) + \frac{bB \sin(c+dx)}{\sqrt{\cos(c+dx)}} \right)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]

[Out] (2*((a*A - b*B)*EllipticE[(c + d*x)/2, 2] + (A*b + a*B)*EllipticF[(c + d*x)/2, 2] + (b*B*Sin[c + d*x])/Sqrt[Cos[c + d*x]]))/d

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral} \left((Bb \sec(dx + c)^2 + Aa + (Ba + Ab) \sec(dx + c)) \sqrt{\cos(dx + c)}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] integral((B*b*sec(d*x + c)^2 + A*a + (B*a + A*b)*sec(d*x + c))*sqrt(cos(d*x + c)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(b \sec(dx + c) + a) \sqrt{\cos(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)*sqrt(cos(d*x + c)), x)

maple [B] time = 5.33, size = 244, normalized size = 3.44

$$\frac{2 \left(A \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \text{EllipticF} \left(\cos \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) \sqrt{2 \left(\sin^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)} - 1 \right) b - A \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \text{EllipticE} \left(\cos \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x)

```
[Out] -2*(A*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a+a*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b-2*B*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(b \sec(dx + c) + a)\sqrt{\cos(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

mupad [B] time = 3.34, size = 96, normalized size = 1.35

$$\frac{2 A a E\left(\frac{c}{2} + \frac{d x}{2} \middle| 2\right)}{d} + \frac{2 A b F\left(\frac{c}{2} + \frac{d x}{2} \middle| 2\right)}{d} + \frac{2 B a F\left(\frac{c}{2} + \frac{d x}{2} \middle| 2\right)}{d} + \frac{2 B b \sin(c + d x) {}_2F_1\left(-\frac{1}{4}, \frac{1}{2}; \frac{3}{4}; \cos(c + d x)^2\right)}{d \sqrt{\cos(c + d x)} \sqrt{\sin(c + d x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(A + B/cos(c + d*x))*(a + b/cos(c + d*x)),x)
```

```
[Out] (2*A*a*ellipticE(c/2 + (d*x)/2, 2))/d + (2*A*b*ellipticF(c/2 + (d*x)/2, 2))/d + (2*B*a*ellipticF(c/2 + (d*x)/2, 2))/d + (2*B*b*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \sec(c + dx))(a + b \sec(c + dx)) \sqrt{\cos(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))*sqrt(cos(c + d*x)), x)
```

$$3.566 \quad \int \frac{(a+b \sec(c+dx))(A+B \sec(c+dx))}{\sqrt{\cos(c+dx)}} dx$$

Optimal. Leaf size=103

$$\frac{2(3aA + bB)F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3d} - \frac{2(aB + Ab)E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{d} + \frac{2(aB + Ab) \sin(c + dx)}{d\sqrt{\cos(c + dx)}} + \frac{2bB \sin(c + dx)}{3d \cos^{\frac{3}{2}}(c + dx)}$$

[Out] $-2*(A*b+B*a)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/d+2/3*(3*A*a+B*b)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/d+2/3*b*B*\sin(d*x+c)/d/\cos(d*x+c)^{(3/2)}+2*(A*b+B*a)*\sin(d*x+c)/d/\cos(d*x+c)^{(1/2)}$

Rubi [A] time = 0.21, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {2954, 2968, 3021, 2748, 2636, 2639, 2641}

$$\frac{2(3aA + bB)F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3d} - \frac{2(aB + Ab)E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{d} + \frac{2(aB + Ab) \sin(c + dx)}{d\sqrt{\cos(c + dx)}} + \frac{2bB \sin(c + dx)}{3d \cos^{\frac{3}{2}}(c + dx)}$$

Antiderivative was successfully verified.

[In] Int[((a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]))/Sqrt[Cos[c + d*x]], x]

[Out] $(-2*(A*b + a*B)*\text{EllipticE}[(c + d*x)/2, 2])/d + (2*(3*a*A + b*B)*\text{EllipticF}[(c + d*x)/2, 2])/(3*d) + (2*b*B*\sin[c + d*x])/(3*d*\cos[c + d*x]^{(3/2)}) + (2*(A*b + a*B)*\sin[c + d*x])/(d*\text{Sqrt}[\cos[c + d*x]])$

Rule 2636

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), Int[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])], x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2954

Int[((a_.) + csc[(e_.) + (f_.)*(x_)])*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)])*(d_.) + (c_.))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] := Dist[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]

Rule 2968

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])], x_Symbol] := Int[(a + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2), x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]

Rule 3021

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2), x_Symbol] := -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C))*(m + 1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned}
\int \frac{(a + b \sec(c + dx))(A + B \sec(c + dx))}{\sqrt{\cos(c + dx)}} dx &= \int \frac{(b + a \cos(c + dx))(B + A \cos(c + dx))}{\cos^{\frac{5}{2}}(c + dx)} dx \\
&= \int \frac{bB + (Ab + aB) \cos(c + dx) + aA \cos^2(c + dx)}{\cos^{\frac{5}{2}}(c + dx)} dx \\
&= \frac{2bB \sin(c + dx)}{3d \cos^{\frac{3}{2}}(c + dx)} + \frac{2}{3} \int \frac{\frac{3}{2}(Ab + aB) + \frac{1}{2}(3aA + bB) \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx)} dx \\
&= \frac{2bB \sin(c + dx)}{3d \cos^{\frac{3}{2}}(c + dx)} + (Ab + aB) \int \frac{1}{\cos^{\frac{3}{2}}(c + dx)} dx + \frac{1}{3}(3aA + bB) \int \frac{\cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx)} dx \\
&= \frac{2(3aA + bB)F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3d} + \frac{2bB \sin(c + dx)}{3d \cos^{\frac{3}{2}}(c + dx)} + \frac{2(Ab + aB) \sin(c + dx)}{d \sqrt{\cos(c + dx)}} \\
&= -\frac{2(Ab + aB)E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{d} + \frac{2(3aA + bB)F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3d} + \frac{2bB \sin(c + dx)}{3d \cos^{\frac{3}{2}}(c + dx)} + \frac{2(Ab + aB) \sin(c + dx)}{d \sqrt{\cos(c + dx)}}
\end{aligned}$$

Mathematica [A] time = 0.60, size = 107, normalized size = 1.04

$$\frac{2 \left((3aA + bB) \sqrt{\cos(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right) - 3(aB + Ab) \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) + 3aB \sin(c + dx) + 3Ab \sin(c + dx) \right)}{3d \sqrt{\cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]))/Sqrt[Cos[c + d*x]], x]

[Out] (2*(-3*(A*b + a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + (3*a*A + b*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 3*A*b*Sin[c + d*x] + 3*a*B*Sin[c + d*x] + b*B*Tan[c + d*x]))/(3*d*Sqrt[Cos[c + d*x]])

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{Bb \sec(dx + c)^2 + Aa + (Ba + Ab) \sec(dx + c)}{\sqrt{\cos(dx + c)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2), x, algorithm="fricas")

[Out] integral((B*b*sec(d*x + c)^2 + A*a + (B*a + A*b)*sec(d*x + c))/sqrt(cos(d*x + c)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(b \sec(dx + c) + a)}{\sqrt{\cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)/sqrt(cos(d*x + c)), x)

maple [B] time = 9.89, size = 428, normalized size = 4.16

$$\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \left(\frac{2aA\sqrt{\frac{1-\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 1}\operatorname{EllipticF}\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)}} + \frac{2(Ab+aB)}{\dots} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x)

[Out]
$$\begin{aligned} & -\left(-\left(-2\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^2+1\right)\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2\right)^{(1/2)}\left(2*a*A*\left(\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^2\right)^{(1/2)}\left(-2\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^2+1\right)^{(1/2)}\left(-2\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^4+\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2\right)^{(1/2)}\operatorname{EllipticF}\left(\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right), 2^{(1/2)}\right)+2*(A*b+B*a)*\left(-\left(-2\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^4+\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2\right)^{(1/2)}\left(\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^2\right)^{(1/2)}\left(2*\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^2-1\right)^{(1/2)}\operatorname{EllipticE}\left(\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right), 2^{(1/2)}\right)+2*\left(-2\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^4+\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2\right)^{(1/2)}\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right)*\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2\right)/\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2\left(2*\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^2-1\right)+2*B*b*\left(-\frac{1}{6}\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)*\left(-2\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^4+\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2\right)^{(1/2)}\left(-\frac{1}{2}+\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^2+1/3*\left(\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^2\right)^{(1/2)}\left(-2\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^2+1\right)^{(1/2)}\left(-2\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^4+\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2\right)^{(1/2)}\operatorname{EllipticF}\left(\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right), 2^{(1/2)}\right)\right)\right)/\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)/\left(2*\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^2-1\right)^{(1/2)}/d \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(b \sec(dx + c) + a)}{\sqrt{\cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)/sqrt(cos(d*x + c)), x)

mupad [B] time = 3.86, size = 150, normalized size = 1.46

$$\frac{2 A a F\left(\frac{c}{2} + \frac{d x}{2} \middle| 2\right)}{d} + \frac{2 A b \sin(c + d x) {}_2F_1\left(-\frac{1}{4}, \frac{1}{2}; \frac{3}{4}; \cos(c + d x)^2\right)}{d \sqrt{\cos(c + d x)} \sqrt{\sin(c + d x)^2}} + \frac{2 B a \sin(c + d x) {}_2F_1\left(-\frac{1}{4}, \frac{1}{2}; \frac{3}{4}; \cos(c + d x)^2\right)}{d \sqrt{\cos(c + d x)} \sqrt{\sin(c + d x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x)))/cos(c + d*x)^(1/2),x)

[Out] (2*A*a*ellipticF(c/2 + (d*x)/2, 2))/d + (2*A*b*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2)) + (2*B*a*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2)) + (2*B*b*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(3*d*cos(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx))(a + b \sec(c + dx))}{\sqrt{\cos(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/cos(d*x+c)**(1/2),x)

[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))/sqrt(cos(c + d*x)), x)

$$3.567 \quad \int \frac{(a+b \sec(c+dx))(A+B \sec(c+dx))}{\cos^{\frac{3}{2}}(c+dx)} dx$$

Optimal. Leaf size=140

$$\frac{2(aB + Ab)F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3d} - \frac{2(5aA + 3bB)E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{5d} + \frac{2(aB + Ab) \sin(c + dx)}{3d \cos^{\frac{3}{2}}(c + dx)} + \frac{2(5aA + 3bB) \sin(c + dx)}{5d \sqrt{\cos(c + dx)}}$$

[Out] $-2/5*(5*A*a+3*B*b)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/d+2/3*(A*b+B*a)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/d+2/5*b*B*\sin(d*x+c)/d/\cos(d*x+c)^{(5/2)}+2/3*(A*b+B*a)*\sin(d*x+c)/d/\cos(d*x+c)^{(3/2)}+2/5*(5*A*a+3*B*b)*\sin(d*x+c)/d/\cos(d*x+c)^{(1/2)}$

Rubi [A] time = 0.23, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {2954, 2968, 3021, 2748, 2636, 2641, 2639}

$$\frac{2(aB + Ab)F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3d} - \frac{2(5aA + 3bB)E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{5d} + \frac{2(aB + Ab) \sin(c + dx)}{3d \cos^{\frac{3}{2}}(c + dx)} + \frac{2(5aA + 3bB) \sin(c + dx)}{5d \sqrt{\cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + b*\text{Sec}[c + d*x])*(A + B*\text{Sec}[c + d*x])]/\text{Cos}[c + d*x]^{(3/2)}, x]$

[Out] $(-2*(5*a*A + 3*b*B)*\text{EllipticE}[(c + d*x)/2, 2])/(5*d) + (2*(A*b + a*B)*\text{EllipticF}[(c + d*x)/2, 2])/(3*d) + (2*b*B*\text{Sin}[c + d*x])/(5*d*\text{Cos}[c + d*x]^{(5/2)}) + (2*(A*b + a*B)*\text{Sin}[c + d*x])/(3*d*\text{Cos}[c + d*x]^{(3/2)}) + (2*(5*a*A + 3*b*B)*\text{Sin}[c + d*x])/(5*d*\text{Sqrt}[\text{Cos}[c + d*x]])$

Rule 2636

$\text{Int}[(b_*)*\sin[(c_*) + (d_*)*(x_*)]^{(n_*)}, x_Symbol] \rightarrow \text{Simp}[(\text{Cos}[c + d*x]*(b*\text{Sin}[c + d*x])^{(n + 1)})/(b*d*(n + 1)), x] + \text{Dist}[(n + 2)/(b^2*(n + 1)), \text{Int}[(b*\text{Sin}[c + d*x])^{(n + 2)}, x], x] /;$ FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2639

$\text{Int}[\text{Sqrt}[\sin[(c_*) + (d_*)*(x_*)]], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticE}[(1*(c - P i/2 + d*x))/2, 2])/d, x] /;$ FreeQ[{c, d}, x]

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c -
Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)])*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_.))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] := Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
a*d, 0] && !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Int[(a
+ b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] := -Simp[((A*b^2
- a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + b \sec(c + dx))(A + B \sec(c + dx))}{\cos^{\frac{3}{2}}(c + dx)} dx &= \int \frac{(b + a \cos(c + dx))(B + A \cos(c + dx))}{\cos^{\frac{7}{2}}(c + dx)} dx \\
&= \int \frac{bB + (Ab + aB) \cos(c + dx) + aA \cos^2(c + dx)}{\cos^{\frac{7}{2}}(c + dx)} dx \\
&= \frac{2bB \sin(c + dx)}{5d \cos^{\frac{5}{2}}(c + dx)} + \frac{2}{5} \int \frac{\frac{5}{2}(Ab + aB) + \frac{1}{2}(5aA + 3bB) \cos(c + dx)}{\cos^{\frac{5}{2}}(c + dx)} dx \\
&= \frac{2bB \sin(c + dx)}{5d \cos^{\frac{5}{2}}(c + dx)} + (Ab + aB) \int \frac{1}{\cos^{\frac{5}{2}}(c + dx)} dx + \frac{1}{5}(5aA + 3bB) \int \frac{\cos(c + dx)}{\cos^{\frac{5}{2}}(c + dx)} dx \\
&= \frac{2bB \sin(c + dx)}{5d \cos^{\frac{5}{2}}(c + dx)} + \frac{2(Ab + aB) \sin(c + dx)}{3d \cos^{\frac{3}{2}}(c + dx)} + \frac{2(5aA + 3bB) \sin(c + dx)}{5d \sqrt{\cos(c + dx)}} \\
&= -\frac{2(5aA + 3bB)E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{5d} + \frac{2(Ab + aB)F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3d} +
\end{aligned}$$

Mathematica [A] time = 1.02, size = 134, normalized size = 0.96

$$\frac{10(aB + Ab) \cos^{\frac{3}{2}}(c + dx) F\left(\frac{1}{2}(c + dx) \middle| 2\right) - 6(5aA + 3bB) \cos^{\frac{3}{2}}(c + dx) E\left(\frac{1}{2}(c + dx) \middle| 2\right) + 15aA \sin(2(c + dx)) + 15bB \sin(2(c + dx))}{15d \cos^{\frac{3}{2}}(c + dx)}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]))/Cos[c + d*x]^(3/2), x]

[Out] (-6*(5*a*A + 3*b*B)*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2] + 10*(A*b + a*B)*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + 10*A*b*Sin[c + d*x] + 10*a*B*Sin[c + d*x] + 15*a*A*Sin[2*(c + d*x)] + 9*b*B*Sin[2*(c + d*x)] + 6*b*B*Tan[c + d*x])/(15*d*Cos[c + d*x]^(3/2))

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{Bb \sec(dx + c)^2 + Aa + (Ba + Ab) \sec(dx + c)}{\cos(dx + c)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2), x, algorithm="fricas")

[Out] $\int \frac{(B \sec(dx + c))^2 + Aa + (Ba + Ab) \sec(dx + c)}{\cos(dx + c)^{3/2}} dx$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(b \sec(dx + c) + a)}{\cos(dx + c)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="giac")`

[Out] `integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)/cos(d*x + c)^(3/2), x)`

maple [B] time = 13.22, size = 663, normalized size = 4.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2),x)`

[Out]
$$\begin{aligned} & -(-(-2 \cos(1/2 dx + 1/2 c)^2 + 1) \sin(1/2 dx + 1/2 c)^2)^{1/2} * (2 * a * A * (-(-2 \sin(1/2 dx + 1/2 c)^4 + \sin(1/2 dx + 1/2 c)^2)^{1/2} * (\sin(1/2 dx + 1/2 c)^2)^{1/2} * \\ & (2 * \sin(1/2 dx + 1/2 c)^2 - 1)^{1/2} * \text{EllipticE}(\cos(1/2 dx + 1/2 c), 2^{1/2})) + 2 * (-2 \sin(1/2 dx + 1/2 c)^4 + \sin(1/2 dx + 1/2 c)^2)^{1/2} * \cos(1/2 dx + 1/2 c) * \sin(1/2 dx + 1/2 c)^2 / \sin(1/2 dx + 1/2 c)^2 / (2 * \sin(1/2 dx + 1/2 c)^2 - 1) + 2 * (A * b + B * a) * \\ & (-1/6 * \cos(1/2 dx + 1/2 c) * (-2 \sin(1/2 dx + 1/2 c)^4 + \sin(1/2 dx + 1/2 c)^2)^{1/2} / (-1/2 + \cos(1/2 dx + 1/2 c)^2)^{2 + 1/3} * (\sin(1/2 dx + 1/2 c)^2)^{1/2} * (-2 \cos(1/2 dx + 1/2 c)^2 + 1)^{1/2} / (-2 \sin(1/2 dx + 1/2 c)^4 + \sin(1/2 dx + 1/2 c)^2)^{1/2} * \text{EllipticF}(\cos(1/2 dx + 1/2 c), 2^{1/2})) - 2/5 * B * b / (8 * \sin(1/2 dx + 1/2 c)^6 - 12 * \sin(1/2 dx + 1/2 c)^4 + 6 * \sin(1/2 dx + 1/2 c)^2 - 1) / \sin(1/2 dx + 1/2 c)^2 * (12 * \text{EllipticE}(\cos(1/2 dx + 1/2 c), 2^{1/2})) * (2 * \sin(1/2 dx + 1/2 c)^2 - 1)^{1/2} * (\sin(1/2 dx + 1/2 c)^2)^{1/2} * \sin(1/2 dx + 1/2 c)^4 - 24 * \cos(1/2 dx + 1/2 c) * \sin(1/2 dx + 1/2 c)^6 - 12 * \text{EllipticE}(\cos(1/2 dx + 1/2 c), 2^{1/2})) * (2 * \sin(1/2 dx + 1/2 c)^2 - 1)^{1/2} * (\sin(1/2 dx + 1/2 c)^2)^{1/2} * \sin(1/2 dx + 1/2 c)^2 + 24 * \sin(1/2 dx + 1/2 c)^4 * \cos(1/2 dx + 1/2 c) + 3 * (\sin(1/2 dx + 1/2 c)^2)^{1/2} * (2 * \sin(1/2 dx + 1/2 c)^2 - 1)^{1/2} * \text{EllipticE}(\cos(1/2 dx + 1/2 c), 2^{1/2})) - 8 * \sin(1/2 dx + 1/2 c)^2 * \cos(1/2 dx + 1/2 c) * (-2 \sin(1/2 dx + 1/2 c)^4 + \sin(1/2 dx + 1/2 c)^2)^{1/2} / \sin(1/2 dx + 1/2 c) / (2 * \cos(1/2 dx + 1/2 c)^2 - 1)^{1/2} / d \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(b \sec(dx + c) + a)}{\cos(dx + c)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)/cos(d*x + c)^(3/2), x)

mupad [B] time = 4.30, size = 177, normalized size = 1.26

$$\frac{2 A a \sin(c+d x) {}_2F_1\left(-\frac{1}{4}, \frac{1}{2}; \frac{3}{4}; \cos(c+d x)^2\right)}{d \sqrt{\cos(c+d x)} \sqrt{\sin(c+d x)^2}} + \frac{2 A b \sin(c+d x) {}_2F_1\left(-\frac{3}{4}, \frac{1}{2}; \frac{1}{4}; \cos(c+d x)^2\right)}{3 d \cos(c+d x)^{3/2} \sqrt{\sin(c+d x)^2}} + \frac{2 B a \sin(c+d x) {}_2F_1\left(-\frac{1}{4}, \frac{1}{2}; \frac{3}{4}; \cos(c+d x)^2\right)}{d \sqrt{\cos(c+d x)} \sqrt{\sin(c+d x)^2}} + \frac{2 B b \sin(c+d x) {}_2F_1\left(-\frac{3}{4}, \frac{1}{2}; \frac{1}{4}; \cos(c+d x)^2\right)}{3 d \cos(c+d x)^{3/2} \sqrt{\sin(c+d x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x)))/cos(c + d*x)^(3/2),x)

[Out] (2*A*a*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2)) + (2*A*b*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(3*d*cos(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2)) + (2*B*a*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(3*d*cos(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2)) + (2*B*b*sin(c + d*x)*hypergeom([-5/4, 1/2], -1/4, cos(c + d*x)^2))/(5*d*cos(c + d*x)^(5/2)*(sin(c + d*x)^2)^(1/2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx))(a + b \sec(c + dx))}{\cos^{\frac{3}{2}}(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/cos(d*x+c)**(3/2),x)

[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))/cos(c + d*x)**(3/2), x)

$$3.568 \quad \int \cos^2(c+dx)(a+b \sec(c+dx))^2(A+B \sec(c+dx)) dx$$

Optimal. Leaf size=182

$$\frac{2(3a^2B + 6aAb + 5b^2B) E\left(\frac{1}{2}(c+dx) \middle| 2\right)}{5d} + \frac{2(5a^2A + 7b(2aB + Ab)) F\left(\frac{1}{2}(c+dx) \middle| 2\right)}{21d} + \frac{2(5a^2A + 7b(2aB + Ab))}{21d}$$

[Out] $2/5*(6*A*a*b+3*B*a^2+5*B*b^2)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/d+2/21*(5*a^2*A+7*b*(A*b+2*B*a))*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/d+2/35*a*(9*A*b+7*B*a)*\cos(d*x+c)^{(3/2)}*\sin(d*x+c)/d+2/7*a*A*\cos(d*x+c)^{(3/2)}*(b+a*\cos(d*x+c))*\sin(d*x+c)/d+2/21*(5*a^2*A+7*b*(A*b+2*B*a))*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/d$

Rubi [A] time = 0.37, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {2954, 2990, 3023, 2748, 2639, 2635, 2641}

$$\frac{2(3a^2B + 6aAb + 5b^2B) E\left(\frac{1}{2}(c+dx) \middle| 2\right)}{5d} + \frac{2(5a^2A + 7b(2aB + Ab)) F\left(\frac{1}{2}(c+dx) \middle| 2\right)}{21d} + \frac{2(5a^2A + 7b(2aB + Ab))}{21d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Cos}[c + d*x]^{(7/2)}*(a + b*\text{Sec}[c + d*x])^2*(A + B*\text{Sec}[c + d*x]), x]$

[Out] $(2*(6*a*A*b + 3*a^2*B + 5*b^2*B)*\text{EllipticE}[(c + d*x)/2, 2])/(5*d) + (2*(5*a^2*A + 7*b*(A*b + 2*a*B))*\text{EllipticF}[(c + d*x)/2, 2])/(21*d) + (2*(5*a^2*A + 7*b*(A*b + 2*a*B))*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sin}[c + d*x])/(21*d) + (2*a*(9*A*b + 7*a*B)*\text{Cos}[c + d*x]^{(3/2)}*\text{Sin}[c + d*x])/(35*d) + (2*a*A*\text{Cos}[c + d*x]^{(3/2)}*(b + a*\text{Cos}[c + d*x])*\text{Sin}[c + d*x])/(7*d)$

Rule 2635

$\text{Int}[(b_*)*\sin[(c_*) + (d_*)(x_)]^{(n_*)}, x_Symbol] \rightarrow -\text{Simp}[(b*\text{Cos}[c + d*x])*(b*\text{Sin}[c + d*x])^{(n-1)}]/(d*n), x] + \text{Dist}[(b^2*(n-1))/n, \text{Int}[(b*\text{Sin}[c + d*x])^{(n-2)}, x], x] /;$ FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2639

$\text{Int}[\text{Sqrt}[\sin[(c_*) + (d_*)(x_)]], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticE}[(1*(c - P i/2 + d*x))/2, 2])/d, x] /;$ FreeQ[{c, d}, x]

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c -
  Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)])*(b_.)^(m_.)*(csc[(e_.) + (f_.)*(x_)])*(d_.) + (c_.)^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] := Dist[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := -Simp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && !(IGtQ[n, 1] && (!IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2), x_Symbol] := -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \cos^{\frac{7}{2}}(c+dx)(a+b \sec(c+dx))^2(A+B \sec(c+dx)) dx &= \int \sqrt{\cos(c+dx)}(b+a \cos(c+dx))^2(B+A \cos(c+dx)) dx \\
&= \frac{2aA \cos^{\frac{3}{2}}(c+dx)(b+a \cos(c+dx)) \sin(c+dx)}{7d} + \frac{2aA \cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{7d} \\
&= \frac{2a(9Ab+7aB) \cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{35d} + \frac{2aA \cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{35d} \\
&= \frac{2a(9Ab+7aB) \cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{35d} + \frac{2aA \cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{35d} \\
&= \frac{2(6aAb+3a^2B+5b^2B) E\left(\frac{1}{2}(c+dx) \middle| 2\right)}{5d} + \frac{2(5a^2A+14abB+7Ab^2) F\left(\frac{1}{2}(c+dx) \middle| 2\right)}{5d} \\
&= \frac{2(6aAb+3a^2B+5b^2B) E\left(\frac{1}{2}(c+dx) \middle| 2\right)}{5d} + \frac{2(5a^2A+14abB+7Ab^2) F\left(\frac{1}{2}(c+dx) \middle| 2\right)}{5d}
\end{aligned}$$

Mathematica [A] time = 1.27, size = 139, normalized size = 0.76

$$\frac{10(5a^2A+14abB+7Ab^2) F\left(\frac{1}{2}(c+dx) \middle| 2\right) + 42(3a^2B+6aAb+5b^2B) E\left(\frac{1}{2}(c+dx) \middle| 2\right) + \sin(c+dx) \sqrt{\cos(c+dx)}}{105d}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]), x]

[Out] (42*(6*a*A*b + 3*a^2*B + 5*b^2*B)*EllipticE[(c + d*x)/2, 2] + 10*(5*a^2*A + 7*A*b^2 + 14*a*b*B)*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(42*a*(2*A*b + a*B)*Cos[c + d*x] + 5*(13*a^2*A + 14*A*b^2 + 28*a*b*B + 3*a^2*A*Cos[2*(c + d*x)]))*Sin[c + d*x])/(105*d)

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral} \left((Bb^2 \cos(dx+c)^3 \sec(dx+c)^3 + Aa^2 \cos(dx+c)^3 + (2Bab + Ab^2) \cos(dx+c)^3 \sec(dx+c)^2 + (Ba^2 + 2Ab^2) \cos(dx+c)^3 \sec(dx+c)) \sqrt{\cos(dx+c)}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)), x, algorithm="fricas")

[Out] integral((B*b^2*cos(d*x + c)^3*sec(d*x + c)^3 + A*a^2*cos(d*x + c)^3 + (2*B*a*b + A*b^2)*cos(d*x + c)^3*sec(d*x + c)^2 + (B*a^2 + 2*A*a*b)*cos(d*x + c)^3*sec(d*x + c))*sqrt(cos(d*x + c)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(b \sec(dx + c) + a)^2 \cos(dx + c)^{\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^2*cos(d*x + c)^(7/2), x)

maple [B] time = 4.83, size = 548, normalized size = 3.01

$$\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \left(240Aa^2 \cos\left(\frac{dx}{2} + \frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + (-360a^2A - 336Aab - 16\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x)

[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*A*a^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-360*A*a^2-336*A*a*b-168*B*a^2)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(280*A*a^2+336*A*a*b+140*A*b^2+168*B*a^2+280*B*a*b)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-80*A*a^2-84*A*a*b-70*A*b^2-42*B*a^2-140*B*a*b)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+25*a^2*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+35*A*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-126*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b+70*B*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-63*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-105*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*b^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(b \sec(dx + c) + a)^2 \cos(dx + c)^{\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^2*cos(d*x + c)^(7/2), x)

mupad [B] time = 3.17, size = 229, normalized size = 1.26

$$\frac{2 A b^2 \left(\sqrt{\cos(c+d x)} \sin(c+d x) + F\left(\frac{c}{2} + \frac{d x}{2} \middle| 2\right) \right)}{3 d} + \frac{2 B b^2 E\left(\frac{c}{2} + \frac{d x}{2} \middle| 2\right)}{d} + \frac{2 B a b \left(\frac{2 \sqrt{\cos(c+d x)} \sin(c+d x)}{3} + \frac{2 F\left(\frac{c}{2} + \frac{d x}{2} \middle| 2\right)}{3} \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^(7/2)*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^2,x)

[Out] (2*A*b^2*(cos(c + d*x)^(1/2)*sin(c + d*x) + ellipticF(c/2 + (d*x)/2, 2)))/(3*d) + (2*B*b^2*ellipticE(c/2 + (d*x)/2, 2))/d + (2*B*a*b*((2*cos(c + d*x)^(1/2)*sin(c + d*x))/3 + (2*ellipticF(c/2 + (d*x)/2, 2))/3))/d - (2*A*a^2*cos(c + d*x)^(9/2)*sin(c + d*x)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2)^(1/2)) - (2*B*a^2*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)) - (4*A*a*b*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**(7/2)*(a+b*sec(d*x+c))**2*(A+B*sec(d*x+c)),x)

[Out] Timed out

$$3.569 \quad \int \cos^{\frac{5}{2}}(c+dx)(a+b \sec(c+dx))^2(A+B \sec(c+dx)) dx$$

Optimal. Leaf size=140

$$\frac{2(a^2B + 2aAb + 3b^2B) F\left(\frac{1}{2}(c+dx) \middle| 2\right)}{3d} + \frac{2(3a^2A + 5b(2aB + Ab)) E\left(\frac{1}{2}(c+dx) \middle| 2\right)}{5d} + \frac{2a(5aB + 7Ab) \sin(c+dx)}{15d}$$

[Out] $2/5*(3*a^2*A+5*b*(A*b+2*B*a))*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/d+2/3*(2*A*a*b+B*a^2+3*B*b^2)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/d+2/15*a*(7*A*b+5*B*a)*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/d+2/5*a*A*(b+a*\cos(d*x+c))*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/d$

Rubi [A] time = 0.33, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2954, 2990, 3023, 2748, 2641, 2639}

$$\frac{2(a^2B + 2aAb + 3b^2B) F\left(\frac{1}{2}(c+dx) \middle| 2\right)}{3d} + \frac{2(3a^2A + 5b(2aB + Ab)) E\left(\frac{1}{2}(c+dx) \middle| 2\right)}{5d} + \frac{2a(5aB + 7Ab) \sin(c+dx)}{15d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Cos}[c + d*x]^{(5/2)}*(a + b*\text{Sec}[c + d*x])^2*(A + B*\text{Sec}[c + d*x]), x]$

[Out] $(2*(3*a^2*A + 5*b*(A*b + 2*a*B))*\text{EllipticE}[(c + d*x)/2, 2])/(5*d) + (2*(2*a*A*b + a^2*B + 3*b^2*B))*\text{EllipticF}[(c + d*x)/2, 2])/(3*d) + (2*a*(7*A*b + 5*a*B))*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sin}[c + d*x])/(15*d) + (2*a*A*\text{Sqrt}[\text{Cos}[c + d*x]]*(b + a*\text{Cos}[c + d*x])*\text{Sin}[c + d*x])/(5*d)$

Rule 2639

$\text{Int}[\text{Sqrt}[\sin[(c_.) + (d_.)*(x_.)]], x_Symbol] := \text{Simp}[(2*\text{EllipticE}[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 2641

$\text{Int}[1/\text{Sqrt}[\sin[(c_.) + (d_.)*(x_.)]], x_Symbol] := \text{Simp}[(2*\text{EllipticF}[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 2748

$\text{Int}[(b_.)*\sin[(e_.) + (f_.)*(x_.)]^{(m_.)*((c_.) + (d_.)*\sin[(e_.) + (f_.)*(x_.)])}, x_Symbol] := \text{Dist}[c, \text{Int}[(b*\text{Sin}[e + f*x])^m, x], x] + \text{Dist}[d/b, \text{Int}[($

$b \sin[e + f x]^{m+1}, x], x] /; \text{FreeQ}[\{b, c, d, e, f, m\}, x]$

Rule 2954

$\text{Int}[(a_.) + \text{csc}[(e_.) + (f_.)(x_.)]*(b_.)]^{(m_.)}*(\text{csc}[(e_.) + (f_.)(x_.)]*(d_.) + (c_.))^{(n_.)}*((g_.)\sin[(e_.) + (f_.)(x_.)])^{(p_.)}, x_Symbol] \text{:> Dist}[g^{(m+n)}, \text{Int}[(g*\sin[e + f*x])^{(p-m-n)}*(b + a*\sin[e + f*x])^m*(d + c*\sin[e + f*x])^n, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, g, p\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{!IntegerQ}[p] \&\& \text{IntegerQ}[m] \&\& \text{IntegerQ}[n]$

Rule 2990

$\text{Int}[(a_.) + (b_.)\sin[(e_.) + (f_.)(x_.)]]^{(m_.)}*((A_.) + (B_.)\sin[(e_.) + (f_.)(x_.)])*((c_.) + (d_.)\sin[(e_.) + (f_.)(x_.)])^{(n_.)}, x_Symbol] \text{:> -Simp}[(b*B*\text{Cos}[e + f*x]*(a + b*\sin[e + f*x])^{(m-1)}*(c + d*\sin[e + f*x])^{(n+1)})/(d*f*(m+n+1)), x] + \text{Dist}[1/(d*(m+n+1)), \text{Int}[(a + b*\sin[e + f*x])^{(m-2)}*(c + d*\sin[e + f*x])^n*\text{Simp}[a^2*A*d*(m+n+1) + b*B*(b*c*(m-1) + a*d*(n+1)) + (a*d*(2*A*b + a*B)*(m+n+1) - b*B*(a*c - b*d*(m+n)))*\sin[e + f*x] + b*(A*b*d*(m+n+1) - B*(b*c*m - a*d*(2*m+n)))*\sin[e + f*x]^2, x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{GtQ}[m, 1] \&\& \text{!(IGtQ}[n, 1] \&\& (\text{!IntegerQ}[m] || (\text{EqQ}[a, 0] \&\& \text{NeQ}[c, 0])))$

Rule 3023

$\text{Int}[(a_.) + (b_.)\sin[(e_.) + (f_.)(x_.)]]^{(m_.)}*((A_.) + (B_.)\sin[(e_.) + (f_.)(x_.)] + (C_.)\sin[(e_.) + (f_.)(x_.)]^2), x_Symbol] \text{:> -Simp}[(C*\text{Cos}[e + f*x]*(a + b*\sin[e + f*x])^{(m+1)})/(b*f*(m+2)), x] + \text{Dist}[1/(b*(m+2)), \text{Int}[(a + b*\sin[e + f*x])^m*\text{Simp}[A*b*(m+2) + b*C*(m+1) + (b*B*(m+2) - a*C)*\sin[e + f*x], x], x], x] /; \text{FreeQ}[\{a, b, e, f, A, B, C, m\}, x] \&\& \text{!LtQ}[m, -1]$

Rubi steps

$$\begin{aligned}
\int \cos^{\frac{5}{2}}(c+dx)(a+b \sec(c+dx))^2(A+B \sec(c+dx)) dx &= \int \frac{(b+a \cos(c+dx))^2(B+A \cos(c+dx))}{\sqrt{\cos(c+dx)}} dx \\
&= \frac{2aA\sqrt{\cos(c+dx)}(b+a \cos(c+dx)) \sin(c+dx)}{5d} + \dots \\
&= \frac{2a(7Ab+5aB)\sqrt{\cos(c+dx)} \sin(c+dx)}{15d} + \frac{2aA\sqrt{\cos(c+dx)}}{15d} \\
&= \frac{2a(7Ab+5aB)\sqrt{\cos(c+dx)} \sin(c+dx)}{15d} + \frac{2aA\sqrt{\cos(c+dx)}}{15d} \\
&= \frac{2(3a^2A+5b(Ab+2aB))E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2(2aA)}{5d}
\end{aligned}$$

Mathematica [A] time = 0.67, size = 106, normalized size = 0.76

$$\frac{2\left(5\left(a^2B+2aAb+3b^2B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)+3\left(3a^2A+10abB+5Ab^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)+a \sin(c+dx)\sqrt{\cos(c+dx)}\right)}{15d}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]

[Out] (2*(3*(3*a^2*A + 5*A*b^2 + 10*a*b*B)*EllipticE[(c + d*x)/2, 2] + 5*(2*a*A*b + a^2*B + 3*b^2*B)*EllipticF[(c + d*x)/2, 2] + a*Sqrt[Cos[c + d*x]]*(10*A*b + 5*a*B + 3*a*A*Cos[c + d*x])*Sin[c + d*x]))/(15*d)

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb^2 \cos(dx+c)^2 \sec(dx+c)^3 + Aa^2 \cos(dx+c)^2 + (2Bab + Ab^2) \cos(dx+c)^2 \sec(dx+c)^2 + (Ba^2\right)
\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] integral((B*b^2*cos(d*x + c)^2*sec(d*x + c)^3 + A*a^2*cos(d*x + c)^2 + (2*B*a*b + A*b^2)*cos(d*x + c)^2*sec(d*x + c)^2 + (B*a^2 + 2*A*a*b)*cos(d*x + c)^2*sec(d*x + c))*sqrt(cos(d*x + c)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx+c) + A)(b \sec(dx+c) + a)^2 \cos(dx+c)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^2*cos(d*x + c)^(5/2), x)

maple [B] time = 5.32, size = 487, normalized size = 3.48

$$2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \left(-24Aa^2 \cos\left(\frac{dx}{2} + \frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + (24a^2A + 40Aab + 20a^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x)

[Out]
$$\begin{aligned} & -2/15*((2*\cos(1/2*d*x+1/2*c)^2-1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-24*A*a^2*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^6+(24*A*a^2+40*A*a*b+20*B*a^2)*\sin(1/2*d*x+1/2*c)^4*\cos(1/2*d*x+1/2*c)+(-6*A*a^2-20*A*a*b-10*B*a^2)*\sin(1/2*d*x+1/2*c)^2*\cos(1/2*d*x+1/2*c)+10*A*a*b*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})-9*A*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)})*a^2-15*A*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)})*b^2+5*a^2*B*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})+15*b^2*B*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})-30*B*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)})*a*b)/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(b \sec(dx + c) + a)^2 \cos(dx + c)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^2*cos(d*x + c)^(5/2), x)

mupad [B] time = 3.04, size = 177, normalized size = 1.26

$$\frac{B a^2 \left(\frac{2 \sqrt{\cos(c+dx)} \sin(c+dx)}{3} + \frac{{}_2F\left(\frac{c}{2} + \frac{dx}{2} \middle| 2\right)}{3} \right)}{d} + \frac{2 A b^2 E\left(\frac{c}{2} + \frac{dx}{2} \middle| 2\right)}{d} + \frac{2 B b^2 F\left(\frac{c}{2} + \frac{dx}{2} \middle| 2\right)}{d} + \frac{2 A a b \left(\frac{2 \sqrt{\cos(c+dx)} \sin(c+dx)}{3} \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(c + d*x)^(5/2)*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^2,x)`

[Out] `(B*a^2*((2*cos(c + d*x)^(1/2)*sin(c + d*x))/3 + (2*ellipticF(c/2 + (d*x)/2, 2))/3))/d + (2*A*b^2*ellipticE(c/2 + (d*x)/2, 2))/d + (2*B*b^2*ellipticF(c/2 + (d*x)/2, 2))/d + (2*A*a*b*((2*cos(c + d*x)^(1/2)*sin(c + d*x))/3 + (2*ellipticF(c/2 + (d*x)/2, 2))/3))/d + (4*B*a*b*ellipticE(c/2 + (d*x)/2, 2))/d - (2*A*a^2*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2))`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)**(5/2)*(a+b*sec(d*x+c))**2*(A+B*sec(d*x+c)),x)`

[Out] Timed out

$$3.570 \quad \int \cos^{\frac{3}{2}}(c+dx)(a+b \sec(c+dx))^2(A+B \sec(c+dx)) dx$$

Optimal. Leaf size=121

$$\frac{2(a^2A + 6abB + 3Ab^2)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2(a^2B + 2aAb - b^2B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a^2A \sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

[Out] 2*(2*A*a*b+B*a^2-B*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*(A*a^2+3*A*b^2+6*B*a*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2*b^2*B*sin(d*x+c)/d/cos(d*x+c)^(1/2)+2/3*a^2*A*sin(d*x+c)*cos(d*x+c)^(1/2)/d

Rubi [A] time = 0.32, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2954, 2988, 3023, 2748, 2641, 2639}

$$\frac{2(a^2A + 6abB + 3Ab^2)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2(a^2B + 2aAb - b^2B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a^2A \sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]

[Out] (2*(2*a*A*b + a^2*B - b^2*B)*EllipticE[(c + d*x)/2, 2])/d + (2*(a^2*A + 3*A*b^2 + 6*a*b*B)*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*b^2*B*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + (2*a^2*A*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(

$b \sin[e + f x]^{m+1}, x], x] /; \text{FreeQ}\{b, c, d, e, f, m\}, x]$

Rule 2954

$\text{Int}[(a + \csc[e + f x])^m (c \csc[e + f x] + d + c \sin[e + f x])^n, x_Symbol] \rightarrow \text{Dist}[g^{m+n}, \text{Int}[g \sin[e + f x]^{p-m-n} (b + a \sin[e + f x])^m (d + c \sin[e + f x])^n, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, p\}, x] \&\& \text{NeQ}[b c - a d, 0] \&\& \text{IntegerQ}[p] \&\& \text{IntegerQ}[m] \&\& \text{IntegerQ}[n]$

Rule 2988

$\text{Int}[(a + b \sin[e + f x])^2 (A + B \sin[e + f x] + C \cos[e + f x])^n, x_Symbol] \rightarrow \text{Simp}[(B c - A d) (b c - a d)^2 \cos[e + f x] (c + d \sin[e + f x])^{n+1} / (f d^2 (n+1) (c^2 - d^2)), x] - \text{Dist}[1 / (d^2 (n+1) (c^2 - d^2)), \text{Int}[(c + d \sin[e + f x])^{n+1} \text{Simp}[d (n+1) (B (b c - a d)^2 - A d (a^2 c + b^2 c - 2 a b d)) - ((B c - A d) (a^2 d^2 (n+2) + b^2 (c^2 + d^2 (n+1))) + 2 a b d (A c d (n+2) - B (c^2 + d^2 (n+1))) \sin[e + f x] - b^2 B d (n+1) (c^2 - d^2) \sin[e + f x]^2, x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B\}, x] \&\& \text{NeQ}[b c - a d, 0] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{LtQ}[n, -1]$

Rule 3023

$\text{Int}[(a + b \sin[e + f x])^m (A + B \sin[e + f x] + C \cos[e + f x])^2, x_Symbol] \rightarrow -\text{Simp}[(C \cos[e + f x] (a + b \sin[e + f x])^{m+1} / (b f (m+2)), x] + \text{Dist}[1 / (b (m+2)), \text{Int}[(a + b \sin[e + f x])^m \text{Simp}[A b (m+2) + b C (m+1) + (b B (m+2) - a C) \sin[e + f x], x], x], x] /; \text{FreeQ}\{a, b, e, f, A, B, C, m\}, x] \&\& \text{!LtQ}[m, -1]$

Rubi steps

$$\begin{aligned}
\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2(A+B\sec(c+dx))dx &= \int \frac{(b+a\cos(c+dx))^2(B+A\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}dx \\
&= \frac{2b^2B\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - 2 \int \frac{-\frac{1}{2}b(Ab+2aB) - \frac{1}{2}(2aAb)}{\cos^{\frac{3}{2}}(c+dx)}dx \\
&= \frac{2b^2B\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{2a^2A\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} \\
&= \frac{2b^2B\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{2a^2A\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} \\
&= \frac{2(2aAb+a^2B-b^2B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2(a^2A+3abB)}{3d}
\end{aligned}$$

Mathematica [A] time = 0.66, size = 102, normalized size = 0.84

$$\frac{2\left((a^2A+6abB+3Ab^2)F\left(\frac{1}{2}(c+dx)\middle|2\right)+3(a^2B+2aAb-b^2B)E\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{\sin(c+dx)(a^2A\cos(c+dx)+3b^2B)}{\sqrt{\cos(c+dx)}}\right)}{3d}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]

[Out] (2*(3*(2*a*A*b + a^2*B - b^2*B)*EllipticE[(c + d*x)/2, 2] + (a^2*A + 3*A*b^2 + 6*a*b*B)*EllipticF[(c + d*x)/2, 2] + ((3*b^2*B + a^2*A*Cos[c + d*x])*Sin[c + d*x])/Sqrt[Cos[c + d*x]]))/(3*d)

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb^2\cos(dx+c)\sec(dx+c)^3+Aa^2\cos(dx+c)+\left(2Bab+Ab^2\right)\cos(dx+c)\sec(dx+c)^2+\left(Ba^2+3abB\right)\cos(dx+c)\sec(dx+c)\right)\sqrt{\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] integral((B*b^2*cos(d*x + c)*sec(d*x + c)^3 + A*a^2*cos(d*x + c) + (2*B*a*b + A*b^2)*cos(d*x + c)*sec(d*x + c)^2 + (B*a^2 + 2*A*a*b)*cos(d*x + c)*sec(d*x + c))*sqrt(cos(d*x + c)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(b \sec(dx + c) + a)^2 \cos(dx + c)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^2*cos(d*x + c)^(3/2), x)

maple [B] time = 5.30, size = 404, normalized size = 3.34

$$\frac{2 \left(4A a^2 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) \left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right) \right) + a^2 A \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{2 \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right) \right) - 1} \operatorname{EllipticF}\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2} \right) \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x)

[Out] -2/3*(4*A*a^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+a^2*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*A*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-6*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b-2*A*a^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+6*B*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2+3*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*b^2-6*B*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(b \sec(dx + c) + a)^2 \cos(dx + c)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^2*cos(d*x + c)^(3/2), x)

mupad [B] time = 3.31, size = 158, normalized size = 1.31

$$\frac{A a^2 \left(\frac{2 \sqrt{\cos(c+d x)} \sin(c+d x)}{3} + \frac{{}_2F\left(\frac{c}{2} + \frac{d x}{2} \middle| 2\right)}{3} \right)}{d} + \frac{2 B a^2 E\left(\frac{c}{2} + \frac{d x}{2} \middle| 2\right)}{d} + \frac{2 A b^2 F\left(\frac{c}{2} + \frac{d x}{2} \middle| 2\right)}{d} + \frac{4 A a b E\left(\frac{c}{2} + \frac{d x}{2} \middle| 2\right)}{d} + \frac{4 B a^2}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^(3/2)*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^2,x)

[Out] (A*a^2*((2*cos(c + d*x)^(1/2)*sin(c + d*x))/3 + (2*ellipticF(c/2 + (d*x)/2, 2))/3))/d + (2*B*a^2*ellipticE(c/2 + (d*x)/2, 2))/d + (2*A*b^2*ellipticF(c/2 + (d*x)/2, 2))/d + (4*A*a*b*ellipticE(c/2 + (d*x)/2, 2))/d + (4*B*a*b*ellipticF(c/2 + (d*x)/2, 2))/d + (2*B*b^2*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**(3/2)*(a+b*sec(d*x+c))**2*(A+B*sec(d*x+c)),x)

[Out] Timed out

$$3.571 \quad \int \sqrt{\cos(c + dx)} (a + b \sec(c + dx))^2 (A + B \sec(c + dx)) dx$$

Optimal. Leaf size=126

$$\frac{2(3a^2B + 6aAb + b^2B)F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3d} + \frac{2(a^2A - 2abB - Ab^2)E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{d} + \frac{2b(2aB + Ab)\sin(c + dx)}{d\sqrt{\cos(c + dx)}} + \frac{2b}{3a}$$

[Out] 2*(A*a^2-A*b^2-2*B*a*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*(6*A*a*b+3*B*a^2+B*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*b^2*B*sin(d*x+c)/d/cos(d*x+c)^(3/2)+2*b*(A*b+2*B*a)*sin(d*x+c)/d/cos(d*x+c)^(1/2)

Rubi [A] time = 0.34, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2954, 2988, 3021, 2748, 2641, 2639}

$$\frac{2(3a^2B + 6aAb + b^2B)F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3d} + \frac{2(a^2A - 2abB - Ab^2)E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{d} + \frac{2b(2aB + Ab)\sin(c + dx)}{d\sqrt{\cos(c + dx)}} + \frac{2b}{3a}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]

[Out] (2*(a^2*A - A*b^2 - 2*a*b*B)*EllipticE[(c + d*x)/2, 2])/d + (2*(6*a*A*b + 3*a^2*B + b^2*B)*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*b^2*B*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*b*(A*b + 2*a*B)*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)])*(b_.)^(m_.)*(csc[(e_.) + (f_.)*(x_)])*(d_.) + (c_.))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] := Dist[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 2988

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[((B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(f*d^2*(n + 1)*(c^2 - d^2)), x] - Dist[1/(d^2*(n + 1)*(c^2 - d^2)), Int[(c + d*Sin[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c - 2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*a*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1)))*Sin[e + f*x] - b^2*B*d*(n + 1)*(c^2 - d^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2), x_Symbol] := -Simp[(A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{\cos(c+dx)} (a+b \sec(c+dx))^2 (A+B \sec(c+dx)) dx &= \int \frac{(b+a \cos(c+dx))^2 (B+A \cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)} dx \\
&= \frac{2b^2 B \sin(c+dx)}{3d \cos^{\frac{3}{2}}(c+dx)} - \frac{2}{3} \int \frac{-\frac{3}{2}b(Ab+2aB) - \frac{1}{2}(6aAb)}{\cos^{\frac{3}{2}}(c+dx)} dx \\
&= \frac{2b^2 B \sin(c+dx)}{3d \cos^{\frac{3}{2}}(c+dx)} + \frac{2b(Ab+2aB) \sin(c+dx)}{d \sqrt{\cos(c+dx)}} - \frac{4}{3} \int \frac{1}{\cos^{\frac{3}{2}}(c+dx)} dx \\
&= \frac{2b^2 B \sin(c+dx)}{3d \cos^{\frac{3}{2}}(c+dx)} + \frac{2b(Ab+2aB) \sin(c+dx)}{d \sqrt{\cos(c+dx)}} - \left(-\frac{2}{d} \right) \\
&= \frac{2(a^2 A - Ab^2 - 2abB) E\left(\frac{1}{2}(c+dx) \middle| 2\right)}{d} + \frac{2(6aAb + \dots)}{d}
\end{aligned}$$

Mathematica [A] time = 1.22, size = 105, normalized size = 0.83

$$\frac{2 \left((3a^2 B + 6aAb + b^2 B) F\left(\frac{1}{2}(c+dx) \middle| 2\right) + 3(a^2 A - 2abB - Ab^2) E\left(\frac{1}{2}(c+dx) \middle| 2\right) + \frac{b \sin(c+dx)(3(2aB+Ab) \cos(c+dx)+b)}{\cos^{\frac{3}{2}}(c+dx)} \right)}{3d}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]

[Out] (2*(3*(a^2*A - A*b^2 - 2*a*b*B)*EllipticE[(c + d*x)/2, 2] + (6*a*A*b + 3*a^2*B + b^2*B)*EllipticF[(c + d*x)/2, 2] + (b*(b*B + 3*(A*b + 2*a*B))*Cos[c + d*x])*Sin[c + d*x])/Cos[c + d*x]^(3/2))/(3*d)

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral} \left((Bb^2 \sec(dx+c)^3 + Aa^2 + (2Bab + Ab^2) \sec(dx+c)^2 + (Ba^2 + 2Aab) \sec(dx+c)) \sqrt{\cos(dx+c)}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] integral((B*b^2*sec(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*sec(d*x + c)^2 + (B*a^2 + 2*A*a*b)*sec(d*x + c))*sqrt(cos(d*x + c)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(b \sec(dx + c) + a)^2 \sqrt{\cos(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^2*sqrt(cos(d*x + c)), x)

maple [B] time = 10.27, size = 677, normalized size = 5.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x)

[Out]
$$\begin{aligned} & -(-(-2*\cos(1/2*d*x+1/2*c)^2+1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*a^2*A*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})-\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)}))-2*a^2*A*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})+4*A*a*b*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})+2*a^2*B*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)})+2*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^2/\sin(1/2*d*x+1/2*c)^2/(2*\sin(1/2*d*x+1/2*c)^2-1)+2*b^2*B*(-1/6*\cos(1/2*d*x+1/2*c)*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/(-1/2+\cos(1/2*d*x+1/2*c)^2)^2+1/3*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})))/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(b \sec(dx + c) + a)^2 \sqrt{\cos(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^2*sqrt(cos(d*x + c)), x)

mupad [B] time = 4.27, size = 194, normalized size = 1.54

$$\frac{2 A a^2 E\left(\frac{c}{2} + \frac{dx}{2} \middle| 2\right)}{d} + \frac{2 B a^2 F\left(\frac{c}{2} + \frac{dx}{2} \middle| 2\right)}{d} + \frac{4 A a b F\left(\frac{c}{2} + \frac{dx}{2} \middle| 2\right)}{d} + \frac{2 A b^2 \sin(c + dx) {}_2F_1\left(-\frac{1}{4}, \frac{1}{2}; \frac{3}{4}; \cos(c + dx)\right)^2}{d \sqrt{\cos(c + dx)} \sqrt{\sin(c + dx)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^(1/2)*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^2,x)

[Out] (2*A*a^2*ellipticE(c/2 + (d*x)/2, 2))/d + (2*B*a^2*ellipticF(c/2 + (d*x)/2, 2))/d + (4*A*a*b*ellipticF(c/2 + (d*x)/2, 2))/d + (2*A*b^2*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2)) + (2*B*b^2*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(3*d*cos(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2)) + (4*B*a*b*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \sec(c + dx)) (a + b \sec(c + dx))^2 \sqrt{\cos(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**(1/2)*(a+b*sec(d*x+c))**2*(A+B*sec(d*x+c)),x)

[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**2*sqrt(cos(c + d*x)), x)

$$3.572 \quad \int \frac{(a+b \sec(c+dx))^2(A+B \sec(c+dx))}{\sqrt{\cos(c+dx)}} dx$$

Optimal. Leaf size=172

$$\frac{2(3a^2A + 2abB + Ab^2)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} - \frac{2(5a^2B + 10aAb + 3b^2B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2(5a^2B + 10aAb + 3b^2B)}{5d\sqrt{\cos(c+dx)}}$$

[Out] $-2/5*(10*A*a*b+5*B*a^2+3*B*b^2)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c),2^{(1/2)})/d+2/3*(3*A*a^2+A*b^2+2*B*a*b)*(c \cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c),2^{(1/2)})/d+2/5*b^2*B*\sin(d*x+c)/d/\cos(d*x+c)^{(5/2)}+2/3*b*(A*b+2*B*a)*\sin(d*x+c)/d/\cos(d*x+c)^{(3/2)}+2/5*(10*A*a*b+5*B*a^2+3*B*b^2)*\sin(d*x+c)/d/\cos(d*x+c)^{(1/2)}$

Rubi [A] time = 0.37, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {2954, 2988, 3021, 2748, 2636, 2639, 2641}

$$\frac{2(3a^2A + 2abB + Ab^2)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} - \frac{2(5a^2B + 10aAb + 3b^2B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2(5a^2B + 10aAb + 3b^2B)}{5d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\frac{(a + b*\text{Sec}[c + d*x])^2*(A + B*\text{Sec}[c + d*x])}{\text{Sqrt}[\text{Cos}[c + d*x]]}, x]$

[Out] $(-2*(10*a*A*b + 5*a^2*B + 3*b^2*B)*\text{EllipticE}[(c + d*x)/2, 2])/(5*d) + (2*(3*a^2*A + A*b^2 + 2*a*b*B)*\text{EllipticF}[(c + d*x)/2, 2])/(3*d) + (2*b^2*B*\text{Sin}[c + d*x])/(5*d*\text{Cos}[c + d*x]^{(5/2)}) + (2*b*(A*b + 2*a*B)*\text{Sin}[c + d*x])/(3*d*\text{Cos}[c + d*x]^{(3/2)}) + (2*(10*a*A*b + 5*a^2*B + 3*b^2*B)*\text{Sin}[c + d*x])/(5*d*\text{Sqrt}[\text{Cos}[c + d*x]])$

Rule 2636

$\text{Int}[(b_*)*\sin[(c_*) + (d_*)(x_)]^{(n_)}, x_Symbol] \rightarrow \text{Simp}[(\text{Cos}[c + d*x]*(b*\text{Sin}[c + d*x])^{(n+1)})/(b*d*(n+1)), x] + \text{Dist}[(n+2)/(b^2*(n+1)), \text{Int}[(b*\text{Sin}[c + d*x])^{(n+2)}, x], x] /;$ $\text{FreeQ}\{b, c, d\}, x \&\& \text{LtQ}[n, -1] \&\& \text{IntegerQ}[2*n]$

Rule 2639

$\text{Int}[\text{Sqrt}[\sin[(c_*) + (d_*)(x_)]], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticE}[(1*(c - P i/2 + d*x))/2, 2])/d, x] /;$ $\text{FreeQ}\{c, d\}, x$

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_.)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_.)]^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2954

Int[((a_.) + csc[(e_.) + (f_.)*(x_.)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_.)]^(p_.), x_Symbol] := Dist[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]

Rule 2988

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^(2*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)]*(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]^(n_)), x_Symbol] := Simp[((B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(f*d^2*(n + 1)*(c^2 - d^2)), x] - Dist[1/(d^2*(n + 1)*(c^2 - d^2)), Int[(c + d*Sin[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c - 2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*a*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1)))*Sin[e + f*x] - b^2*B*d*(n + 1)*(c^2 - d^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]

Rule 3021

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)] + (C_.)*sin[(e_.) + (f_.)*(x_.)]^2), x_Symbol] := -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C))*(m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned}
\int \frac{(a + b \sec(c + dx))^2 (A + B \sec(c + dx))}{\sqrt{\cos(c + dx)}} dx &= \int \frac{(b + a \cos(c + dx))^2 (B + A \cos(c + dx))}{\cos^{\frac{7}{2}}(c + dx)} dx \\
&= \frac{2b^2 B \sin(c + dx)}{5d \cos^{\frac{5}{2}}(c + dx)} - \frac{2}{5} \int \frac{-\frac{5}{2}b(Ab + 2aB) - \frac{1}{2}(10aAb + 5a^2B + \dots)}{\cos^{\frac{5}{2}}(c + dx)} dx \\
&= \frac{2b^2 B \sin(c + dx)}{5d \cos^{\frac{5}{2}}(c + dx)} + \frac{2b(Ab + 2aB) \sin(c + dx)}{3d \cos^{\frac{3}{2}}(c + dx)} - \frac{4}{15} \int \frac{-\frac{3}{4}(10aAb + \dots)}{\cos^{\frac{5}{2}}(c + dx)} dx \\
&= \frac{2b^2 B \sin(c + dx)}{5d \cos^{\frac{5}{2}}(c + dx)} + \frac{2b(Ab + 2aB) \sin(c + dx)}{3d \cos^{\frac{3}{2}}(c + dx)} - \frac{1}{3} (-3a^2 A - A \dots) \\
&= \frac{2(3a^2 A + Ab^2 + 2abB) F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3d} + \frac{2b^2 B \sin(c + dx)}{5d \cos^{\frac{5}{2}}(c + dx)} + \\
&= -\frac{2(10aAb + 5a^2 B + 3b^2 B) E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{5d} + \frac{2(3a^2 A + Ab^2 \dots)}{5d}
\end{aligned}$$

Mathematica [A] time = 1.28, size = 175, normalized size = 1.02

$$\frac{10(3a^2 A + 2abB + Ab^2) \cos^{\frac{3}{2}}(c + dx) F\left(\frac{1}{2}(c + dx) \middle| 2\right) - 6(5a^2 B + 10aAb + 3b^2 B) \cos^{\frac{3}{2}}(c + dx) E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{5d}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]))/Sqrt[Cos[c + d*x]], x]

[Out] (-6*(10*a*A*b + 5*a^2*B + 3*b^2*B)*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2] + 10*(3*a^2*A + A*b^2 + 2*a*b*B)*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + 10*A*b^2*Sin[c + d*x] + 20*a*b*B*Sin[c + d*x] + 30*a*A*b*Sin[2*(c + d*x)] + 15*a^2*B*Sin[2*(c + d*x)] + 9*b^2*B*Sin[2*(c + d*x)] + 6*b^2*B*Tan[c + d*x])/(15*d*Cos[c + d*x]^(3/2))

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{Bb^2 \sec(dx + c)^3 + Aa^2 + (2Bab + Ab^2) \sec(dx + c)^2 + (Ba^2 + 2Aab) \sec(dx + c)}{\sqrt{\cos(dx + c)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="fricas")

[Out] integral((B*b^2*sec(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*sec(d*x + c)^2 + (B*a^2 + 2*A*a*b)*sec(d*x + c))/sqrt(cos(d*x + c)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(b \sec(dx + c) + a)^2}{\sqrt{\cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^2/sqrt(cos(d*x + c)), x)

maple [B] time = 14.56, size = 750, normalized size = 4.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x)

[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a^2*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*a*(2*A*b+B*a)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)-2/5*b^2*B/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c))+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*b*(A*b+2*B*a)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2*cos(1/2*d*x+1/2*c)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic

$\text{icF}(\cos(1/2*d*x+1/2*c), 2^{(1/2)})) / \sin(1/2*d*x+1/2*c) / (2*\cos(1/2*d*x+1/2*c)^{2-1})^{(1/2)}/d$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(b \sec(dx + c) + a)^2}{\sqrt{\cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^2/sqrt(cos(d*x + c)), x)

mupad [B] time = 4.64, size = 227, normalized size = 1.32

$$\frac{6 B b^2 \sin(c + dx) {}_2F_1\left(-\frac{5}{4}, \frac{1}{2}; -\frac{1}{4}; \cos(c + dx)^2\right) + 30 B a^2 \cos(c + dx)^2 \sin(c + dx) {}_2F_1\left(-\frac{1}{4}, \frac{1}{2}; \frac{3}{4}; \cos(c + dx)^2\right)}{15 d \cos(c + dx)^{5/2} \sqrt{1 - \cos(c + dx)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^2)/cos(c + d*x)^(1/2),x)

[Out] (6*B*b^2*sin(c + d*x)*hypergeom([-5/4, 1/2], -1/4, cos(c + d*x)^2) + 30*B*a^2*cos(c + d*x)^2*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2) + 20*B*a*b*cos(c + d*x)*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(15*d*cos(c + d*x)^(5/2)*(1 - cos(c + d*x)^2)^(1/2)) + (2*A*a^2*ellipticF(c/2 + (d*x)/2, 2))/d + (2*A*b^2*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(3*d*cos(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2)) + (4*A*a*b*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx))(a + b \sec(c + dx))^2}{\sqrt{\cos(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c))/cos(d*x+c)**(1/2),x)

[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))^2/sqrt(cos(c + d*x)), x)

$$3.573 \quad \int \frac{(a+b \sec(c+dx))^2(A+B \sec(c+dx))}{\cos^{\frac{3}{2}}(c+dx)} dx$$

Optimal. Leaf size=214

$$\frac{2(7a^2B + 14aAb + 5b^2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} - \frac{2(5a^2A + 6abB + 3Ab^2)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2(7a^2B + 14aAb + 5b^2B)}{21d \cos^{\frac{3}{2}}(c+dx)}$$

[Out] $-2/5*(5*A*a^2+3*A*b^2+6*B*a*b)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c),2^{(1/2)})/d+2/21*(14*A*a*b+7*B*a^2+5*B*b^2)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c),2^{(1/2)})/d+2/7*b^2*B*\sin(d*x+c)/d/\cos(d*x+c)^{(7/2)}+2/5*b*(A*b+2*B*a)*\sin(d*x+c)/d/\cos(d*x+c)^{(5/2)}+2/21*(14*A*a*b+7*B*a^2+5*B*b^2)*\sin(d*x+c)/d/\cos(d*x+c)^{(3/2)}+2/5*(5*A*a^2+3*A*b^2+6*B*a*b)*\sin(d*x+c)/d/\cos(d*x+c)^{(1/2)}$

Rubi [A] time = 0.40, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {2954, 2988, 3021, 2748, 2636, 2641, 2639}

$$\frac{2(7a^2B + 14aAb + 5b^2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} - \frac{2(5a^2A + 6abB + 3Ab^2)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2(7a^2B + 14aAb + 5b^2B)}{21d \cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + b*\text{Sec}[c + d*x])^2*(A + B*\text{Sec}[c + d*x])]/\text{Cos}[c + d*x]^{(3/2)},x]$

[Out] $(-2*(5*a^2*A + 3*A*b^2 + 6*a*b*B)*\text{EllipticE}[(c + d*x)/2, 2])/(5*d) + (2*(14*a*A*b + 7*a^2*B + 5*b^2*B)*\text{EllipticF}[(c + d*x)/2, 2])/(21*d) + (2*b^2*B*\text{Sin}[c + d*x])/(7*d*\text{Cos}[c + d*x]^{(7/2)}) + (2*b*(A*b + 2*a*B)*\text{Sin}[c + d*x])/(5*d*\text{Cos}[c + d*x]^{(5/2)}) + (2*(14*a*A*b + 7*a^2*B + 5*b^2*B)*\text{Sin}[c + d*x])/(21*d*\text{Cos}[c + d*x]^{(3/2)}) + (2*(5*a^2*A + 3*A*b^2 + 6*a*b*B)*\text{Sin}[c + d*x])/(5*d*\text{Sqrt}[\text{Cos}[c + d*x]])$

Rule 2636

$\text{Int}[(b_*)*\sin[(c_*) + (d_*)*(x_*)]^{(n_*)}, x_Symbol] \rightarrow \text{Simp}[(\text{Cos}[c + d*x]*(b*\text{Sin}[c + d*x])^{(n + 1)})/(b*d*(n + 1)), x] + \text{Dist}[(n + 2)/(b^2*(n + 1)), \text{Int}[(b*\text{Sin}[c + d*x])^{(n + 2)}, x], x] /; \text{FreeQ}\{b, c, d, x\} \&\& \text{LtQ}[n, -1] \&\& \text{IntegerQ}[2*n]$

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2954

Int[((a_.) + csc[(e_.) + (f_.)*(x_)])*(b_.)^(m_.)*(csc[(e_.) + (f_.)*(x_)])*(d_. + (c_.))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] := Dist[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]

Rule 2988

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[((B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(f*d^2*(n + 1)*(c^2 - d^2)), x] - Dist[1/(d^2*(n + 1)*(c^2 - d^2)), Int[(c + d*Sin[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c - 2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*a*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1)))*Sin[e + f*x] - b^2*B*d*(n + 1)*(c^2 - d^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]

Rule 3021

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2, x_Symbol] := -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned}
\int \frac{(a + b \sec(c + dx))^2 (A + B \sec(c + dx))}{\cos^{\frac{3}{2}}(c + dx)} dx &= \int \frac{(b + a \cos(c + dx))^2 (B + A \cos(c + dx))}{\cos^{\frac{9}{2}}(c + dx)} dx \\
&= \frac{2b^2 B \sin(c + dx)}{7d \cos^{\frac{7}{2}}(c + dx)} - \frac{2}{7} \int \frac{-\frac{7}{2}b(Ab + 2aB) - \frac{1}{2}(14aAb + 7a^2B + 5b^2B)}{\cos^{\frac{7}{2}}(c + dx)} dx \\
&= \frac{2b^2 B \sin(c + dx)}{7d \cos^{\frac{7}{2}}(c + dx)} + \frac{2b(Ab + 2aB) \sin(c + dx)}{5d \cos^{\frac{5}{2}}(c + dx)} - \frac{4}{35} \int \frac{-\frac{5}{4}(14aAb + 7a^2B + 5b^2B)}{\cos^{\frac{5}{2}}(c + dx)} dx \\
&= \frac{2b^2 B \sin(c + dx)}{7d \cos^{\frac{7}{2}}(c + dx)} + \frac{2b(Ab + 2aB) \sin(c + dx)}{5d \cos^{\frac{5}{2}}(c + dx)} - \frac{1}{5} (-5a^2A - 3Ab^2) E\left(\frac{1}{2}(c + dx) \middle| 2\right) \\
&= \frac{2b^2 B \sin(c + dx)}{7d \cos^{\frac{7}{2}}(c + dx)} + \frac{2b(Ab + 2aB) \sin(c + dx)}{5d \cos^{\frac{5}{2}}(c + dx)} + \frac{2(14aAb + 7a^2B + 5b^2B) \sin(c + dx)}{21d \cos^{\frac{3}{2}}(c + dx)} \\
&= -\frac{2(5a^2A + 3Ab^2 + 6abB) E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{5d} + \frac{2(14aAb + 7a^2B + 5b^2B) \sin(c + dx)}{21d \cos^{\frac{3}{2}}(c + dx)}
\end{aligned}$$

Mathematica [A] time = 5.26, size = 191, normalized size = 0.89

$$\frac{2 \left(5(7a^2B + 14aAb + 5b^2B) F\left(\frac{1}{2}(c + dx) \middle| 2\right) - 21(5a^2A + 6abB + 3Ab^2) E\left(\frac{1}{2}(c + dx) \middle| 2\right) + \frac{5(7a^2B + 14aAb + 5b^2B) \sin(c + dx)}{\cos^{\frac{3}{2}}(c + dx)} \right)}{105d}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]))/Cos[c + d*x]^(3/2), x]

[Out] (2*(-21*(5*a^2*A + 3*A*b^2 + 6*a*b*B)*EllipticE[(c + d*x)/2, 2] + 5*(14*a*A*b + 7*a^2*B + 5*b^2*B)*EllipticF[(c + d*x)/2, 2] + (15*b^2*B*Sin[c + d*x])/Cos[c + d*x]^(7/2) + (21*b*(A*b + 2*a*B)*Sin[c + d*x])/Cos[c + d*x]^(5/2) + (5*(14*a*A*b + 7*a^2*B + 5*b^2*B)*Sin[c + d*x])/Cos[c + d*x]^(3/2) + (21*(5*a^2*A + 3*A*b^2 + 6*a*b*B)*Sin[c + d*x])/Sqrt[Cos[c + d*x]]))/(105*d)

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{Bb^2 \sec(dx + c)^3 + Aa^2 + (2Bab + Ab^2) \sec(dx + c)^2 + (Ba^2 + 2Aab) \sec(dx + c)}{\cos(dx + c)^{\frac{3}{2}}}, x \right)$$

$$\frac{\sin^4(c) + \sin(1/2 dx + 1/2 c)^2 \cos(1/2 dx + 1/2 c) \sin(1/2 dx + 1/2 c)^2}{\sin(1/2 dx + 1/2 c)^2 (2 \sin(1/2 dx + 1/2 c)^2 - 1) + 2 a (2 A b + B a) (-1/6 \cos(1/2 dx + 1/2 c) (-2 \sin(1/2 dx + 1/2 c)^4 + \sin(1/2 dx + 1/2 c)^2)^{1/2} / (-1/2 + \cos(1/2 dx + 1/2 c)^2)^2 + 1/3 (\sin(1/2 dx + 1/2 c)^2)^{1/2} (-2 \cos(1/2 dx + 1/2 c)^2 + 1)^{1/2} / (-2 \sin(1/2 dx + 1/2 c)^4 + \sin(1/2 dx + 1/2 c)^2)^{1/2} \text{EllipticF}(\cos(1/2 dx + 1/2 c), 2^{1/2})} / \sin(1/2 dx + 1/2 c) / (2 \cos(1/2 dx + 1/2 c)^2 - 1)^{1/2} / d$$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="maxima")

[Out] Timed out

mupad [B] time = 4.98, size = 233, normalized size = 1.09

$$\frac{6 A b^2 \sin(c + d x) {}_2F_1\left(-\frac{5}{4}, \frac{1}{2}; -\frac{1}{4}; \cos(c + d x)^2\right) + 30 A a^2 \cos(c + d x)^2 \sin(c + d x) {}_2F_1\left(-\frac{1}{4}, \frac{1}{2}; \frac{3}{4}; \cos(c + d x)^2\right)}{15 d \cos(c + d x)^{5/2} \sqrt{1 - \cos(c + d x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^2)/cos(c + d*x)^(3/2),x)

[Out] (6*A*b^2*sin(c + d*x)*hypergeom([-5/4, 1/2], -1/4, cos(c + d*x)^2) + 30*A*a^2*cos(c + d*x)^2*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2) + 20*A*a*b*cos(c + d*x)*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(15*d*cos(c + d*x)^(5/2)*(1 - cos(c + d*x)^2)^(1/2)) + (30*B*b^2*sin(c + d*x)*hypergeom([-7/4, 1/2], -3/4, cos(c + d*x)^2) + 70*B*a^2*cos(c + d*x)^2*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2) + 84*B*a*b*cos(c + d*x)*sin(c + d*x)*hypergeom([-5/4, 1/2], -1/4, cos(c + d*x)^2))/(105*d*cos(c + d*x)^(7/2)*(1 - cos(c + d*x)^2)^(1/2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) (a + b \sec(c + dx))^2}{\cos^{\frac{3}{2}}(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))*2*(A+B*sec(d*x+c))/cos(d*x+c)**(3/2),x)

[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**2/cos(c + d*x)**(3/2),
x)

$$3.574 \quad \int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B \sec(c+dx))}{a+b \sec(c+dx)} dx$$

Optimal. Leaf size=182

$$\frac{2b^3(Ab - aB)\Pi\left(\frac{2a}{a+b}; \frac{1}{2}(c + dx) \middle| 2\right)}{a^4d(a + b)} - \frac{2(Ab - aB) \sin(c + dx) \sqrt{\cos(c + dx)}}{3a^2d} - \frac{2(a^2 + 3b^2)(Ab - aB)F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3a^4d}$$

[Out] $2/5*(3*A*a^2+5*A*b^2-5*B*a*b)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/a^3/d-2/3*(a^2+3*b^2)*(A*b-B*a)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/a^4/d+2*b^3*(A*b-B*a)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticPi}(\sin(1/2*d*x+1/2*c), 2*a/(a+b), 2^{(1/2)})/a^4/(a+b)/d+2/5*A*\cos(d*x+c)^{(3/2)}*\sin(d*x+c)/a/d-2/3*(A*b-B*a)*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/a^2/d$

Rubi [A] time = 0.86, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {2954, 2990, 3049, 3059, 2639, 3002, 2641, 2805}

$$-\frac{2(a^2 + 3b^2)(Ab - aB)F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3a^4d} + \frac{2(3a^2A - 5abB + 5Ab^2)E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{5a^3d} + \frac{2b^3(Ab - aB)\Pi\left(\frac{2a}{a+b}; \frac{1}{2}(c + dx) \middle| 2\right)}{a^4d(a + b)}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x]),x]

[Out] $(2*(3*a^2*A + 5*A*b^2 - 5*a*b*B)*\text{EllipticE}[(c + d*x)/2, 2])/(5*a^3*d) - (2*(a^2 + 3*b^2)*(A*b - a*B)*\text{EllipticF}[(c + d*x)/2, 2])/(3*a^4*d) + (2*b^3*(A*b - a*B)*\text{EllipticPi}[(2*a)/(a + b), (c + d*x)/2, 2])/(a^4*(a + b)*d) - (2*(A*b - a*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sin}[c + d*x])/(3*a^2*d) + (2*A*\text{Cos}[c + d*x]^(3/2)*\text{Sin}[c + d*x])/(5*a*d)$

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]]], x_Symbol] := Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]]), x_Symbol] := Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)]/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_.))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] := Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
a*d, 0] && !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && !(IGtQ[n
, 1] && (!IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e
_.) + (f_.)*(x_)]^2), x_Symbol] := -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
```

] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && !(IGtQ[n, 0] && (!IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))

Rule 3059

Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)] + (C_.)*sin[(e_.) + (f_.)*(x_.)]^2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x], x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B \sec(c+dx))}{a+b \sec(c+dx)} dx &= \int \frac{\cos^{\frac{5}{2}}(c+dx)(B+A \cos(c+dx))}{b+a \cos(c+dx)} dx \\
 &= \frac{2A \cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{5ad} + \frac{2 \int \frac{\sqrt{\cos(c+dx)} \left(\frac{3Ab}{2} + \frac{3}{2}aA \cos(c+dx) - \frac{5}{2}(Ab-aB) \cos^2(c+dx) \right)}{b+a \cos(c+dx)} dx}{5a} \\
 &= -\frac{2(Ab-aB)\sqrt{\cos(c+dx)} \sin(c+dx)}{3a^2d} + \frac{2A \cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{5ad} \\
 &= -\frac{2(Ab-aB)\sqrt{\cos(c+dx)} \sin(c+dx)}{3a^2d} + \frac{2A \cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{5ad} \\
 &= \frac{2(3a^2A+5Ab^2-5abB) E\left(\frac{1}{2}(c+dx) \middle| 2\right)}{5a^3d} - \frac{2(Ab-aB)\sqrt{\cos(c+dx)} \sin(c+dx)}{3a^2d} \\
 &= \frac{2(3a^2A+5Ab^2-5abB) E\left(\frac{1}{2}(c+dx) \middle| 2\right)}{5a^3d} - \frac{2(a^2+3b^2)(Ab-aB)F\left(\frac{1}{2}(c+dx) \middle| 2\right)}{3a^4d}
 \end{aligned}$$

Mathematica [A] time = 2.80, size = 260, normalized size = 1.43

$$\frac{2a^2(9a^2A-5abB+5Ab^2)\Pi\left(\frac{2a}{a+b}, \frac{1}{2}(c+dx) \middle| 2\right)}{a+b} + \frac{6(3a^2A-5abB+5Ab^2) \sin(c+dx) \left((a^2-2b^2)\Pi\left(-\frac{a}{b}; \sin^{-1}\left(\sqrt{\cos(c+dx)}\right) \middle| -1\right) + 2b(a+b)F\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right) \middle| 2\right) \right)}{b\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.


```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x]),x]
[Out] ((2*a^2*(9*a^2*A + 5*A*b^2 - 5*a*b*B)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) + 2*a^2*(4*A*b + 5*a*B)*(2*EllipticF[(c + d*x)/2, 2] - (2*b*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b)) + 4*a^2*Sqrt[Cos[c + d*x]]*(-5*A*b + 5*a*B + 3*a*A*Cos[c + d*x])*Sin[c + d*x] + (6*(3*a^2*A + 5*A*b^2 - 5*a*b*B)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*b*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (a^2 - 2*b^2)*EllipticPi[-(a/b), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x))/(b*Sqrt[Sin[c + d*x]^2]))/(30*a^4*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \cos(dx + c)^{\frac{5}{2}}}{b \sec(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="giac")
```

[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(5/2)/(b*sec(d*x + c) + a), x)

maple [B] time = 5.94, size = 1074, normalized size = 5.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((-24*A*a^4+24*A*a^3*b)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(24*A*a^4-44*A*a^3*b+20*A*a^2*b^2+20*B*a^4-20*B*a^3*b)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-6*A*a^4+16*A*a^3*b-10*A*a^2*b^2-10*B*a^4+10*B*a^3*b)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-9*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
```

$$\begin{aligned} & \sqrt{\frac{1}{2}} \operatorname{EllipticE}(\cos(\frac{1}{2}dx + \frac{1}{2}c), 2^{\frac{1}{2}}) a^4 + 9A (\sin(\frac{1}{2}dx + \frac{1}{2}c))^2 \\ & \sqrt{\frac{1}{2}} (2 \sin(\frac{1}{2}dx + \frac{1}{2}c)^2 - 1)^{\frac{1}{2}} \operatorname{EllipticE}(\cos(\frac{1}{2}dx + \frac{1}{2}c), 2^{\frac{1}{2}}) \\ & a^3 b - 15A (\sin(\frac{1}{2}dx + \frac{1}{2}c))^2 \sqrt{\frac{1}{2}} (2 \sin(\frac{1}{2}dx + \frac{1}{2}c)^2 - 1)^{\frac{1}{2}} \\ & * \operatorname{EllipticE}(\cos(\frac{1}{2}dx + \frac{1}{2}c), 2^{\frac{1}{2}}) a^2 b^2 + 15A (\sin(\frac{1}{2}dx + \frac{1}{2}c))^2 \\ & \sqrt{\frac{1}{2}} (2 \sin(\frac{1}{2}dx + \frac{1}{2}c)^2 - 1)^{\frac{1}{2}} \operatorname{EllipticE}(\cos(\frac{1}{2}dx + \frac{1}{2}c), 2^{\frac{1}{2}}) \\ & a b^3 - 15A (\sin(\frac{1}{2}dx + \frac{1}{2}c))^2 \sqrt{\frac{1}{2}} (2 \sin(\frac{1}{2}dx + \frac{1}{2}c)^2 - 1)^{\frac{1}{2}} * \\ & \operatorname{EllipticPi}(\cos(\frac{1}{2}dx + \frac{1}{2}c), 2a/(a-b), 2^{\frac{1}{2}}) b^4 - 5A (\sin(\frac{1}{2}dx + \frac{1}{2}c))^2 \\ & \sqrt{\frac{1}{2}} (2 \sin(\frac{1}{2}dx + \frac{1}{2}c)^2 - 1)^{\frac{1}{2}} \operatorname{EllipticF}(\cos(\frac{1}{2}dx + \frac{1}{2}c), 2^{\frac{1}{2}}) \\ & a^3 b + 5A (\sin(\frac{1}{2}dx + \frac{1}{2}c))^2 \sqrt{\frac{1}{2}} (2 \sin(\frac{1}{2}dx + \frac{1}{2}c)^2 - 1)^{\frac{1}{2}} \\ & \operatorname{EllipticF}(\cos(\frac{1}{2}dx + \frac{1}{2}c), 2^{\frac{1}{2}}) a^2 b^2 - 15A (\sin(\frac{1}{2}dx + \frac{1}{2}c))^2 \\ & \sqrt{\frac{1}{2}} (2 \sin(\frac{1}{2}dx + \frac{1}{2}c)^2 - 1)^{\frac{1}{2}} \operatorname{EllipticF}(\cos(\frac{1}{2}dx + \frac{1}{2}c), 2^{\frac{1}{2}}) \\ & a b^3 + 15A (\sin(\frac{1}{2}dx + \frac{1}{2}c))^2 \sqrt{\frac{1}{2}} (2 \sin(\frac{1}{2}dx + \frac{1}{2}c)^2 - 1)^{\frac{1}{2}} \\ & \operatorname{EllipticF}(\cos(\frac{1}{2}dx + \frac{1}{2}c), 2^{\frac{1}{2}}) b^4 + 15B (\sin(\frac{1}{2}dx + \frac{1}{2}c))^2 \sqrt{\frac{1}{2}} \\ & (2 \sin(\frac{1}{2}dx + \frac{1}{2}c)^2 - 1)^{\frac{1}{2}} \operatorname{EllipticE}(\cos(\frac{1}{2}dx + \frac{1}{2}c), 2^{\frac{1}{2}}) * \\ & a^3 b - 15B (\sin(\frac{1}{2}dx + \frac{1}{2}c))^2 \sqrt{\frac{1}{2}} (2 \sin(\frac{1}{2}dx + \frac{1}{2}c)^2 - 1)^{\frac{1}{2}} * \operatorname{El} \\ & \operatorname{lipticE}(\cos(\frac{1}{2}dx + \frac{1}{2}c), 2^{\frac{1}{2}}) a^2 b^2 + 15B (\sin(\frac{1}{2}dx + \frac{1}{2}c))^2 \sqrt{\frac{1}{2}} \\ & (2 \sin(\frac{1}{2}dx + \frac{1}{2}c)^2 - 1)^{\frac{1}{2}} \operatorname{EllipticPi}(\cos(\frac{1}{2}dx + \frac{1}{2}c), 2a/(a-b), \\ & 2^{\frac{1}{2}}) a b^3 + 5B (\sin(\frac{1}{2}dx + \frac{1}{2}c))^2 \sqrt{\frac{1}{2}} (2 \sin(\frac{1}{2}dx + \frac{1}{2}c)^2 - 1)^{\frac{1}{2}} \\ & \operatorname{EllipticF}(\cos(\frac{1}{2}dx + \frac{1}{2}c), 2^{\frac{1}{2}}) a^4 - 5B (\sin(\frac{1}{2}dx + \frac{1}{2}c))^2 \\ & \sqrt{\frac{1}{2}} (2 \sin(\frac{1}{2}dx + \frac{1}{2}c)^2 - 1)^{\frac{1}{2}} \operatorname{EllipticF}(\cos(\frac{1}{2}dx + \frac{1}{2}c), 2^{\frac{1}{2}}) \\ & a^3 b + 15B (\sin(\frac{1}{2}dx + \frac{1}{2}c))^2 \sqrt{\frac{1}{2}} (2 \sin(\frac{1}{2}dx + \frac{1}{2}c)^2 - 1)^{\frac{1}{2}} \\ & * \operatorname{EllipticF}(\cos(\frac{1}{2}dx + \frac{1}{2}c), 2^{\frac{1}{2}}) a^2 b^2 - 15B (\sin(\frac{1}{2}dx + \frac{1}{2}c))^2 \\ & \sqrt{\frac{1}{2}} (2 \sin(\frac{1}{2}dx + \frac{1}{2}c)^2 - 1)^{\frac{1}{2}} \operatorname{EllipticF}(\cos(\frac{1}{2}dx + \frac{1}{2}c), 2^{\frac{1}{2}}) \\ & a b^3) / a^4 / (a-b) / (-2 \sin(\frac{1}{2}dx + \frac{1}{2}c))^4 + \sin(\frac{1}{2}dx + \frac{1}{2}c))^2 \sqrt{\frac{1}{2}} / \sin \\ & (\frac{1}{2}dx + \frac{1}{2}c) / (2 \cos(\frac{1}{2}dx + \frac{1}{2}c)^2 - 1)^{\frac{1}{2}} / d \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \cos(dx + c)^{\frac{5}{2}}}{b \sec(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(5/2)/(b*sec(d*x + c) + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c + dx)^{5/2} \left(A + \frac{B}{\cos(c+dx)} \right)}{a + \frac{b}{\cos(c+dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(5/2)*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x)),x)
```

```
[Out] int((cos(c + d*x)^(5/2)*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x)), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x)
```

```
[Out] Timed out
```

$$3.575 \quad \int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B \sec(c+dx))}{a+b \sec(c+dx)} dx$$

Optimal. Leaf size=136

$$\frac{2b^2(Ab - aB)\Pi\left(\frac{2a}{a+b}; \frac{1}{2}(c+dx)\middle|2\right)}{a^3d(a+b)} - \frac{2(Ab - aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{2(a^2A - 3abB + 3Ab^2)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^3d} + \frac{2A \operatorname{Si}\left(\frac{1}{2}(c+dx)\right)}{a^3d}$$

[Out] $-2*(A*b-B*a)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/a^2/d+2/3*(A*a^2+3*A*b^2-3*B*a*b)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/a^3/d-2*b^2*(A*b-B*a)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticPi}(\sin(1/2*d*x+1/2*c), 2*a/(a+b), 2^{(1/2)})/a^3/(a+b)/d+2/3*A*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/a/d$

Rubi [A] time = 0.59, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {2954, 2990, 3059, 2639, 3002, 2641, 2805}

$$\frac{2(a^2A - 3abB + 3Ab^2)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^3d} - \frac{2b^2(Ab - aB)\Pi\left(\frac{2a}{a+b}; \frac{1}{2}(c+dx)\middle|2\right)}{a^3d(a+b)} - \frac{2(Ab - aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{2A \operatorname{Si}\left(\frac{1}{2}(c+dx)\right)}{a^3d}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(\operatorname{Cos}[c + d*x]^{(3/2)}*(A + B*\operatorname{Sec}[c + d*x]))/(a + b*\operatorname{Sec}[c + d*x]), x]$

[Out] $(-2*(A*b - a*B)*\operatorname{EllipticE}[(c + d*x)/2, 2])/(a^2*d) + (2*(a^2*A + 3*A*b^2 - 3*a*b*B)*\operatorname{EllipticF}[(c + d*x)/2, 2])/(3*a^3*d) - (2*b^2*(A*b - a*B)*\operatorname{EllipticPi}[(2*a)/(a + b), (c + d*x)/2, 2])/(a^3*(a + b)*d) + (2*A*\operatorname{Sqrt}[\operatorname{Cos}[c + d*x]]*\operatorname{Sin}[c + d*x])/(3*a*d)$

Rule 2639

$\operatorname{Int}[\operatorname{Sqrt}[\sin[(c_.) + (d_.)*(x_.)]], x_Symbol] \rightarrow \operatorname{Simp}[(2*\operatorname{EllipticE}[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; \operatorname{FreeQ}\{c, d\}, x]$

Rule 2641

$\operatorname{Int}[1/\operatorname{Sqrt}[\sin[(c_.) + (d_.)*(x_.)]], x_Symbol] \rightarrow \operatorname{Simp}[(2*\operatorname{EllipticF}[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; \operatorname{FreeQ}\{c, d\}, x]$

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]]), x_Symbol] := Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)]/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)])*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_.))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] := Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
a*d, 0] && !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && !(IGtQ[n
, 1] && (!IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] := Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^3(c+dx)(A+B\sec(c+dx))}{a+b\sec(c+dx)} dx &= \int \frac{\cos^3(c+dx)(B+A\cos(c+dx))}{b+a\cos(c+dx)} dx \\
&= \frac{2A\sqrt{\cos(c+dx)}\sin(c+dx)}{3ad} + \frac{2\int \frac{\frac{Ab}{2} + \frac{1}{2}aA\cos(c+dx) - \frac{3}{2}(Ab-aB)\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))} dx}{3a} \\
&= \frac{2A\sqrt{\cos(c+dx)}\sin(c+dx)}{3ad} - \frac{2\int \frac{-\frac{1}{2}aAb - \frac{1}{2}(a^2A+3Ab^2-3abB)\cos(c+dx)}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))} dx}{3a^2} \\
&= -\frac{2(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{2A\sqrt{\cos(c+dx)}\sin(c+dx)}{3ad} - \frac{(b^2(Ab-aB))}{3ad} \\
&= -\frac{2(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{2(a^2A+3Ab^2-3abB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^3d}
\end{aligned}$$

Mathematica [A] time = 1.47, size = 207, normalized size = 1.52

$$\frac{3(aB-Ab)\sin(c+dx)\left((a^2-2b^2)\Pi\left(-\frac{a}{b};\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2b(a+b)F\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)-2abE\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)}{a^2b\sqrt{\sin^2(c+dx)}} + \frac{(3aB-Ab)\Pi\left(\frac{2}{a+}\right)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x]),x]
[Out] (((-(A*b) + 3*a*B)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) + A*(2*EllipticF[(c + d*x)/2, 2] - (2*b*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b)) + 2*A*Sqrt[Cos[c + d*x]]*Sin[c + d*x] + (3*(-(A*b) + a*B)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*b*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (a^2 - 2*b^2)*EllipticPi[-(a/b), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a^2*b*Sqrt[Sin[c + d*x]^2]))/(3*a*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \cos(dx + c)^{\frac{3}{2}}}{b \sec(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(3/2)/(b*sec(d*x + c) + a), x)

maple [B] time = 5.60, size = 786, normalized size = 5.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x)

[Out]
$$-2/3*((2*\cos(1/2*d*x+1/2*c)^2-1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*((4*A*a^3-4*A*a^2*b)*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^4+(-2*A*a^3+2*A*a^2*b)*\sin(1/2*d*x+1/2*c)^2*\cos(1/2*d*x+1/2*c)+A*a^3*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})-A*a^2*b*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})+3*A*a*b^2*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})-3*A*b^3*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})+3*A*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)})*a^2*b-3*A*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)})*a*b^2+3*A*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticPi}(\cos(1/2*d*x+1/2*c),2*a/(a-b),2^{(1/2)})*b^3-3*a^2*b*B*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})+3*B*a*b^2*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})-3*B*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)})*a^3+3*B*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)})*a^2*b-3*B*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticPi}(\cos(1/2*d*x+1/2*c),2*a/(a-b),2^{(1/2)})*a*b^2)/a^3/(a-b)/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \cos(dx + c)^{\frac{3}{2}}}{b \sec(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(3/2)/(b*sec(d*x + c) + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c + dx)^{3/2} \left(A + \frac{B}{\cos(c+dx)} \right)}{a + \frac{b}{\cos(c+dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)^(3/2)*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x)),x)

[Out] int((cos(c + d*x)^(3/2)*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x)

[Out] Timed out

$$3.576 \quad \int \frac{\sqrt{\cos(c+dx)} (A+B \sec(c+dx))}{a+b \sec(c+dx)} dx$$

Optimal. Leaf size=89

$$-\frac{2(Ab - aB)F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{a^2d} + \frac{2b(Ab - aB)\Pi\left(\frac{2a}{a+b}; \frac{1}{2}(c + dx) \middle| 2\right)}{a^2d(a + b)} + \frac{2AE\left(\frac{1}{2}(c + dx) \middle| 2\right)}{ad}$$

[Out] $2*A*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/a/d - 2*(A*b-B*a)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/a^2/d + 2*b*(A*b-B*a)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticPi}(\sin(1/2*d*x+1/2*c), 2*a/(a+b), 2^{(1/2)})/a^2/(a+b)/d$

Rubi [A] time = 0.28, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2954, 3002, 2639, 2803, 2641, 2805}

$$-\frac{2(Ab - aB)F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{a^2d} + \frac{2b(Ab - aB)\Pi\left(\frac{2a}{a+b}; \frac{1}{2}(c + dx) \middle| 2\right)}{a^2d(a + b)} + \frac{2AE\left(\frac{1}{2}(c + dx) \middle| 2\right)}{ad}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(\text{Sqrt}[\text{Cos}[c + d*x]]*(A + B*\text{Sec}[c + d*x]))/(a + b*\text{Sec}[c + d*x]), x]$

[Out] $(2*A*\text{EllipticE}[(c + d*x)/2, 2])/(a*d) - (2*(A*b - a*B)*\text{EllipticF}[(c + d*x)/2, 2])/(a^2*d) + (2*b*(A*b - a*B)*\text{EllipticPi}[(2*a)/(a + b), (c + d*x)/2, 2])/(a^2*(a + b)*d)$

Rule 2639

$\text{Int}[\text{Sqrt}[\sin[(c_.) + (d_.)*(x_.)]], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticE}[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 2641

$\text{Int}[1/\text{Sqrt}[\sin[(c_.) + (d_.)*(x_.)]], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticF}[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 2803

$\text{Int}[\text{Sqrt}[(c_.) + (d_.)*\sin[(e_.) + (f_.)*(x_.)]], x_Symbol] \rightarrow \text{Dist}[d/b, \text{Int}[1/\text{Sqrt}[c + d*\sin[e + f*x]], x], x] + \text{Dist}[(b*c - a*d)/b, \text{Int}[1/((a + b*\sin[e + f*x])*Sqrt[c + d*\sin[e + f*x]])]$

), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 2805

Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi/2 + f*x))/2, (2*d)/(c + d)]/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[c + d, 0]

Rule 2954

Int[((a_.) + csc[(e_.) + (f_.)*(x_)])*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] := Dist[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]

Rule 3002

Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{\sqrt{\cos(c+dx)}(A+B \sec(c+dx))}{a+b \sec(c+dx)} dx &= \int \frac{\sqrt{\cos(c+dx)}(B+A \cos(c+dx))}{b+a \cos(c+dx)} dx \\
 &= \frac{A \int \sqrt{\cos(c+dx)} dx}{a} - \frac{(Ab-aB) \int \frac{\sqrt{\cos(c+dx)}}{b+a \cos(c+dx)} dx}{a} \\
 &= \frac{2AE \left(\frac{1}{2}(c+dx) \middle| 2 \right)}{ad} - \frac{(Ab-aB) \int \frac{1}{\sqrt{\cos(c+dx)}} dx}{a^2} + \frac{(b(Ab-aB)) \int \frac{1}{\sqrt{\cos(c+dx)}} dx}{a^2} \\
 &= \frac{2AE \left(\frac{1}{2}(c+dx) \middle| 2 \right)}{ad} - \frac{2(Ab-aB)F \left(\frac{1}{2}(c+dx) \middle| 2 \right)}{a^2d} + \frac{2b(Ab-aB)\Pi \left(\frac{2}{a} \right)}{a^2(a+)}
 \end{aligned}$$

Mathematica [A] time = 0.99, size = 128, normalized size = 1.44

$$\frac{aB \left(2F \left(\frac{1}{2}(c+dx) \middle| 2 \right) - \frac{2b\pi \left(\frac{2a}{a+b}; \frac{1}{2}(c+dx) \middle| 2 \right)}{a+b} \right) - \frac{2A \sin(c+dx) \left(-(a+b)F \left(\sin^{-1}(\sqrt{\cos(c+dx)}) \middle| -1 \right) + b\pi \left(-\frac{a}{b}; \sin^{-1}(\sqrt{\cos(c+dx)}) \middle| -1 \right) + aE \left(\sin^{-1}(\sqrt{\cos(c+dx)}) \right) \right)}{\sqrt{\sin^2(c+dx)}}}{a^2 d}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x]),x]

[Out] (a*B*(2*EllipticF[(c + d*x)/2, 2] - (2*b*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]))/(a + b) - (2*A*(a*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] - (a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + b*EllipticPi[-(a/b), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/Sqrt[Sin[c + d*x]^2])/(a^2*d)

fricas [F] time = 146.31, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{(B \sec(dx + c) + A) \sqrt{\cos(dx + c)}}{b \sec(dx + c) + a}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(1/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="fricas")

[Out] integral((B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(b*sec(d*x + c) + a), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sqrt{\cos(dx + c)}}{b \sec(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(1/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(b*sec(d*x + c) + a), x)

maple [A] time = 5.16, size = 295, normalized size = 3.31

$$2 \sqrt{\left(2 \left(\cos^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) - 1 \right) \left(\sin^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)} \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{-2 \left(\cos^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) + 1} \left(A \text{EllipticF} \left(\cos \left(\frac{dx}{2} + \frac{c}{2} \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(d*x+c)^(1/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x)`

[Out] $2*((2*\cos(1/2*d*x+1/2*c)^2-1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}*(A*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})*a*b-A*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})*b^2+A*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)})*a^2-A*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)})*a*b+A*\text{EllipticPi}(\cos(1/2*d*x+1/2*c),2*a/(a-b),2^{(1/2)})*b^2-B*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})*a^2+B*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})*a*b-B*\text{EllipticPi}(\cos(1/2*d*x+1/2*c),2*a/(a-b),2^{(1/2)})*a*b)/a^2/(a-b)/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sqrt{\cos(dx + c)}}{b \sec(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)^(1/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="maxima")`

[Out] `integrate((B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(b*sec(d*x + c) + a), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\cos(c + dx)} \left(A + \frac{B}{\cos(c + dx)} \right)}{a + \frac{b}{\cos(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((cos(c + d*x)^(1/2)*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x)),x)`

[Out] `int((cos(c + d*x)^(1/2)*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \sqrt{\cos(c + dx)}}{a + b \sec(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)**(1/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x)`

[Out] `Integral((A + B*sec(c + d*x))*sqrt(cos(c + d*x))/(a + b*sec(c + d*x)), x)`

$$3.577 \quad \int \frac{A+B \sec(c+dx)}{\sqrt{\cos(c+dx)}(a+b \sec(c+dx))} dx$$

Optimal. Leaf size=61

$$\frac{2AF\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} - \frac{2(Ab-aB)\Pi\left(\frac{2a}{a+b}; \frac{1}{2}(c+dx)\middle|2\right)}{ad(a+b)}$$

[Out] $2*A*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/a/d - 2*(A*b-B*a)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticPi}(\sin(1/2*d*x+1/2*c), 2*a/(a+b), 2^{(1/2)})/a/(a+b)/d$

Rubi [A] time = 0.22, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {2954, 3002, 2641, 2805}

$$\frac{2AF\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} - \frac{2(Ab-aB)\Pi\left(\frac{2a}{a+b}; \frac{1}{2}(c+dx)\middle|2\right)}{ad(a+b)}$$

Antiderivative was successfully verified.

[In] `Int[(A + B*Sec[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])),x]`

[Out] $(2*A*\text{EllipticF}[(c+d*x)/2, 2])/(a*d) - (2*(A*b - a*B)*\text{EllipticPi}[(2*a)/(a+b), (c+d*x)/2, 2])/(a*(a+b)*d)$

Rule 2641

`Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]`

Rule 2805

`Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := Simp[(2*EllipticPi[(2*b)/(a+b), (1*(e - Pi/2 + f*x))/2, (2*d)/(c+d)]/(f*(a+b)*Sqrt[c+d]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[c+d, 0]`

Rule 2954

`Int[((a_.) + csc[(e_.) + (f_.)*(x_)])*(b_.)^{(m_.)}*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))^{(n_.)}*((g_.)*sin[(e_.) + (f_.)*(x_)])^{(p_.)}, x_Symbol] := Dist[g^{(m+n)}, Int[(g*Sin[e+f*x])^{(p-m-n)}*(b+a*Sin[e+f*x])^{(m)}*(d+c*Sin[e+f*x])^{(n)}, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -`

$a*d, 0] \&\& !\text{IntegerQ}[p] \&\& \text{IntegerQ}[m] \&\& \text{IntegerQ}[n]$

Rule 3002

$\text{Int}[(((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)])^{(m_.)}*((A_.) + (B_.)*\sin[(e_.) + (f_.)*(x_.)]))/((c_.) + (d_.)*\sin[(e_.) + (f_.)*(x_.)]), x_Symbol] \rightarrow \text{Dist}[B/d, \text{Int}[(a + b*\sin[e + f*x])^m, x], x] - \text{Dist}[(B*c - A*d)/d, \text{Int}[(a + b*\sin[e + f*x])^m/(c + d*\sin[e + f*x]), x], x] /;$ $\text{FreeQ}\{a, b, c, d, e, f, A, B, m\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0]$

Rubi steps

$$\begin{aligned} \int \frac{A + B \sec(c + dx)}{\sqrt{\cos(c + dx)}(a + b \sec(c + dx))} dx &= \int \frac{B + A \cos(c + dx)}{\sqrt{\cos(c + dx)}(b + a \cos(c + dx))} dx \\ &= \frac{A \int \frac{1}{\sqrt{\cos(c+dx)}} dx}{a} - \frac{(Ab - aB) \int \frac{1}{\sqrt{\cos(c+dx)}(b+a \cos(c+dx))} dx}{a} \\ &= \frac{2AF\left(\frac{1}{2}(c + dx) \middle| 2\right)}{ad} - \frac{2(Ab - aB)\Pi\left(\frac{2a}{a+b}; \frac{1}{2}(c + dx) \middle| 2\right)}{a(a + b)d} \end{aligned}$$

Mathematica [A] time = 0.23, size = 58, normalized size = 0.95

$$\frac{2\left((aB - Ab)\Pi\left(\frac{2a}{a+b}; \frac{1}{2}(c + dx) \middle| 2\right) + A(a + b)F\left(\frac{1}{2}(c + dx) \middle| 2\right)\right)}{ad(a + b)}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sec[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])),x]

[Out] (2*(A*(a + b)*EllipticF[(c + d*x)/2, 2] + (-(A*b) + a*B)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]))/(a*(a + b)*d)

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(b \sec(dx + c) + a) \sqrt{\cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c)),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)*sqrt(cos(d*x + c))), x)

maple [A] time = 5.05, size = 217, normalized size = 3.56

$$\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{-2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 1} \left(A \operatorname{EllipticF}\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), 2\right) + \frac{B}{a(a-b)} \sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 1}\right)}{a(a-b) \sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c)),x)

[Out] -2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a-A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b+A*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))*b-B*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))*a)/a/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(b \sec(dx + c) + a) \sqrt{\cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)*sqrt(cos(d*x + c))), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\sqrt{\cos(c+dx)} \left(a + \frac{b}{\cos(c+dx)}\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B/cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))), x)`

[Out] `int((A + B/cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \sec(c + dx)}{(a + b \sec(c + dx)) \sqrt{\cos(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*sec(d*x+c))/cos(d*x+c)**(1/2)/(a+b*sec(d*x+c)), x)`

[Out] `Integral((A + B*sec(c + d*x))/((a + b*sec(c + d*x))*sqrt(cos(c + d*x))), x)`

$$3.578 \quad \int \frac{A+B \sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b \sec(c+dx))} dx$$

Optimal. Leaf size=86

$$\frac{2(Ab - aB)\Pi\left(\frac{2a}{a+b}; \frac{1}{2}(c + dx) \middle| 2\right)}{bd(a + b)} - \frac{2BE\left(\frac{1}{2}(c + dx) \middle| 2\right)}{bd} + \frac{2B \sin(c + dx)}{bd\sqrt{\cos(c + dx)}}$$

[Out] $-2*B*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/b/d+2*(A*b-B*a)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticPi}(\sin(1/2*d*x+1/2*c), 2*a/(a+b), 2^{(1/2)})/b/(a+b)/d+2*B*\sin(d*x+c)/b/d/\cos(d*x+c)^{(1/2)}$

Rubi [A] time = 0.39, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2954, 3000, 3059, 2639, 12, 2805}

$$\frac{2(Ab - aB)\Pi\left(\frac{2a}{a+b}; \frac{1}{2}(c + dx) \middle| 2\right)}{bd(a + b)} - \frac{2BE\left(\frac{1}{2}(c + dx) \middle| 2\right)}{bd} + \frac{2B \sin(c + dx)}{bd\sqrt{\cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])),x]

[Out] $(-2*B*\text{EllipticE}[(c + d*x)/2, 2])/(b*d) + (2*(A*b - a*B)*\text{EllipticPi}[(2*a)/(a + b), (c + d*x)/2, 2])/(b*(a + b)*d) + (2*B*\sin[c + d*x])/(b*d*\text{Sqrt}[\cos[c + d*x]])$

Rule 12

Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2805

Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi /2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,

0] && GtQ[c + d, 0]

Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)])*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_.))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
a*d, 0] && !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
+ n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{A + B \sec(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(a + b \sec(c + dx))} dx &= \int \frac{B + A \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(b + a \cos(c + dx))} dx \\
&= \frac{2B \sin(c + dx)}{bd\sqrt{\cos(c + dx)}} + \frac{2 \int \frac{\frac{1}{2}(Ab - aB) - \frac{1}{2}bB \cos(c + dx) - \frac{1}{2}aB \cos^2(c + dx)}{\sqrt{\cos(c + dx)}(b + a \cos(c + dx))} dx}{b} \\
&= \frac{2B \sin(c + dx)}{bd\sqrt{\cos(c + dx)}} - \frac{2 \int -\frac{a(Ab - aB)}{2\sqrt{\cos(c + dx)}(b + a \cos(c + dx))} dx}{ab} - \frac{B \int \sqrt{\cos(c + dx)}}{b} \\
&= -\frac{2BE\left(\frac{1}{2}(c + dx) \middle| 2\right)}{bd} + \frac{2B \sin(c + dx)}{bd\sqrt{\cos(c + dx)}} + \frac{(Ab - aB) \int \frac{1}{\sqrt{\cos(c + dx)}(b + a \cos(c + dx))} dx}{b} \\
&= -\frac{2BE\left(\frac{1}{2}(c + dx) \middle| 2\right)}{bd} + \frac{2(Ab - aB)\Pi\left(\frac{2a}{a+b}; \frac{1}{2}(c + dx) \middle| 2\right)}{b(a + b)d} + \frac{2B \sin(c + dx)}{bd\sqrt{\cos(c + dx)}}
\end{aligned}$$

Mathematica [B] time = 2.80, size = 206, normalized size = 2.40

$$\frac{-\frac{2B \sin(c+dx)\left((a^2-2b^2)\Pi\left(-\frac{a}{b}; \sin^{-1}\left(\sqrt{\cos(c+dx)}\right) \middle| -1\right)+2b(a+b)F\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right) \middle| -1\right)-2aB\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right) \middle| -1\right)\right)}{ab\sqrt{\sin^2(c+dx)}}}{2bd} + \frac{2(2Ab-3aB)\Pi\left(\frac{2a}{a+b}; \frac{1}{2}(c+dx) \middle| 2\right)}{a+b}$$

Antiderivative was successfully verified.

```

[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])),x]
[Out] ((2*(2*A*b - 3*a*B)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) - (2
*b*B*(2*EllipticF[(c + d*x)/2, 2] - (2*b*EllipticPi[(2*a)/(a + b), (c + d*x
)/2, 2])/(a + b)))/a + (4*B*Sin[c + d*x])/Sqrt[Cos[c + d*x]] - (2*B*(-2*a*b
*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*b*(a + b)*EllipticF[ArcSin[S
qrt[Cos[c + d*x]]], -1] + (a^2 - 2*b^2)*EllipticPi[-(a/b), ArcSin[Sqrt[Cos[
c + d*x]]], -1])*Sin[c + d*x])/(a*b*Sqrt[Sin[c + d*x]^2]))/(2*b*d)

```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x, algorithm="
fricas")

```

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(b \sec(dx + c) + a) \cos(dx + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)*cos(d*x + c)^(3/2)), x)

maple [B] time = 8.68, size = 325, normalized size = 3.78

$$\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \left(-\frac{2(Ab - aB)a\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{-2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 1} \operatorname{EllipticPi}\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \frac{2a}{a-b}, \sqrt{2}\right)}{b(a^2 - ab)\sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)}} \right)}{\sin\left(\frac{dx}{2} + \frac{c}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x)

[Out]
$$\begin{aligned} & -(-(-2*\cos(1/2*d*x+1/2*c)^2+1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*(A*b-B*a)/b/ \\ & (a^2-a*b)*a*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/ \\ & (-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}* \operatorname{EllipticPi}(\cos(1/2*d*x \\ & +1/2*c), 2*a/(a-b), 2^{(1/2)})+2*B/b*(-(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2 \\ & *c)^2)^{(1/2)}*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}* \\ & \operatorname{EllipticE}(\cos(1/2*d*x+1/2*c), 2^{(1/2)})+2*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d* \\ & x+1/2*c)^2)^{(1/2)}*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^2/\sin(1/2*d*x+1/2* \\ & c)^2/(2*\sin(1/2*d*x+1/2*c)^2-1)/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2 \\ & -1)^{(1/2)}/d \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(b \sec(dx + c) + a) \cos(dx + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)*cos(d*x + c)^(3/2)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\cos(c+dx)^{3/2} \left(a + \frac{b}{\cos(c+dx)}\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))),x)

[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \sec(c + dx)}{(a + b \sec(c + dx)) \cos^{\frac{3}{2}}(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(3/2)/(a+b*sec(d*x+c)),x)

[Out] Integral((A + B*sec(c + d*x))/((a + b*sec(c + d*x))*cos(c + d*x)**(3/2)), x)

$$3.579 \quad \int \frac{A+B \sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b \sec(c+dx))} dx$$

Optimal. Leaf size=150

$$\frac{2(Ab - aB)E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{b^2d} - \frac{2a(Ab - aB)\Pi\left(\frac{2a}{a+b}; \frac{1}{2}(c + dx) \middle| 2\right)}{b^2d(a + b)} + \frac{2(Ab - aB) \sin(c + dx)}{b^2d\sqrt{\cos(c + dx)}} + \frac{2BF\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3bd} + \dots$$

[Out] $-2*(A*b-B*a)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/b^2/d+2/3*B*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/b/d-2*a*(A*b-B*a)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticPi}(\sin(1/2*d*x+1/2*c), 2*a/(a+b), 2^{(1/2)})/b^2/(a+b)/d+2/3*B*\sin(d*x+c)/b/d/\cos(d*x+c)^{(3/2)}+2*(A*b-B*a)*\sin(d*x+c)/b^2/d/\cos(d*x+c)^{(1/2)}$

Rubi [A] time = 0.84, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {2954, 3000, 3055, 3059, 2639, 3002, 2641, 2805}

$$\frac{2(Ab - aB)E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{b^2d} - \frac{2a(Ab - aB)\Pi\left(\frac{2a}{a+b}; \frac{1}{2}(c + dx) \middle| 2\right)}{b^2d(a + b)} + \frac{2(Ab - aB) \sin(c + dx)}{b^2d\sqrt{\cos(c + dx)}} + \frac{2BF\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3bd} + \dots$$

Antiderivative was successfully verified.

[In] $\text{Int}[(A + B*\text{Sec}[c + d*x])/(\text{Cos}[c + d*x]^{(5/2)}*(a + b*\text{Sec}[c + d*x])), x]$

[Out] $(-2*(A*b - a*B)*\text{EllipticE}[(c + d*x)/2, 2])/(b^2*d) + (2*B*\text{EllipticF}[(c + d*x)/2, 2])/(3*b*d) - (2*a*(A*b - a*B)*\text{EllipticPi}[(2*a)/(a + b), (c + d*x)/2, 2])/(b^2*(a + b)*d) + (2*B*\text{Sin}[c + d*x])/((3*b*d*\text{Cos}[c + d*x]^{(3/2)})) + (2*(A*b - a*B)*\text{Sin}[c + d*x])/(b^2*d*\text{Sqrt}[\text{Cos}[c + d*x]])$

Rule 2639

$\text{Int}[\text{Sqrt}[\sin[(c_.) + (d_.)*(x_.)]], x_Symbol] := \text{Simp}[(2*\text{EllipticE}[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 2641

$\text{Int}[1/\text{Sqrt}[\sin[(c_.) + (d_.)*(x_.)]], x_Symbol] := \text{Simp}[(2*\text{EllipticF}[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]]), x_Symbol] := Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)]/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)])*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_.))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] := Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
a*d, 0] && !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
+ n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
+ (f_.)*(x_)])^2, x_Symbol] := -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
- a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
+ b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
```

*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))

Rule 3059

Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)] + (C_.)*sin[(e_.) + (f_.)*(x_.)]^2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x], x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{A + B \sec(c + dx)}{\cos^{\frac{5}{2}}(c + dx)(a + b \sec(c + dx))} dx &= \int \frac{B + A \cos(c + dx)}{\cos^{\frac{5}{2}}(c + dx)(b + a \cos(c + dx))} dx \\
 &= \frac{2B \sin(c + dx)}{3bd \cos^{\frac{3}{2}}(c + dx)} + \frac{2 \int \frac{\frac{3}{2}(Ab - aB) + \frac{1}{2}bB \cos(c + dx) + \frac{1}{2}aB \cos^2(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(b + a \cos(c + dx))} dx}{3b} \\
 &= \frac{2B \sin(c + dx)}{3bd \cos^{\frac{3}{2}}(c + dx)} + \frac{2(Ab - aB) \sin(c + dx)}{b^2 d \sqrt{\cos(c + dx)}} + \frac{4 \int \frac{\frac{1}{4}(-3aAb + 3a^2B + b^2B) - \frac{1}{4}b(3a^2 - b^2)}{\sqrt{\cos(c + dx)}(b + a \cos(c + dx))} dx}{3ab^2} \\
 &= \frac{2B \sin(c + dx)}{3bd \cos^{\frac{3}{2}}(c + dx)} + \frac{2(Ab - aB) \sin(c + dx)}{b^2 d \sqrt{\cos(c + dx)}} - \frac{4 \int \frac{\frac{1}{4}a(3aAb - 3a^2B - b^2B) - \frac{1}{4}a^2b(3a^2 - b^2)}{\sqrt{\cos(c + dx)}(b + a \cos(c + dx))} dx}{3ab^2} \\
 &= -\frac{2(Ab - aB)E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{b^2 d} + \frac{2B \sin(c + dx)}{3bd \cos^{\frac{3}{2}}(c + dx)} + \frac{2(Ab - aB) \sin(c + dx)}{b^2 d \sqrt{\cos(c + dx)}} \\
 &= -\frac{2(Ab - aB)E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{b^2 d} + \frac{2BF\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3bd} - \frac{2a(Ab - aB)\Pi\left(\frac{2a}{a + b}, \frac{1}{2}(c + dx) \middle| 2\right)}{b^2(a + b)}
 \end{aligned}$$

Mathematica [A] time = 2.31, size = 260, normalized size = 1.73

$$\frac{2b(9a^2B-9aAb+2b^2B)\Pi\left(\frac{2a}{a+b}; \frac{1}{2}(c+dx)\right)}{a+b} + \frac{6(aB-Ab)\sin(c+dx)\left(\left(a^2-2b^2\right)\Pi\left(-\frac{a}{b}; \sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\right)-1\right)+2b(a+b)F\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\right)-2a}{a\sqrt{\sin^2(c+dx)}}$$

$6b^3d$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])),x]
[Out] ((2*b*(-9*a*A*b + 9*a^2*B + 2*b^2*B)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) + (b*(-6*A*b^2 + 8*a*b*B)*(2*EllipticF[(c + d*x)/2, 2] - (2*b*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b)))/a + (4*b^2*B*Sin[c + d*x])/Cos[c + d*x]^(3/2) + (12*b*(A*b - a*B)*Sin[c + d*x])/Sqrt[Cos[c + d*x]] + (6*(-(A*b) + a*B)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*b*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (a^2 - 2*b^2)*EllipticPi[-(a/b), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*Sqrt[Sin[c + d*x]^2]))/(6*b^3*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(b \sec(dx + c) + a) \cos(dx + c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)*cos(d*x + c)^(5/2)), x)
```

maple [B] time = 11.58, size = 466, normalized size = 3.11

$$\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \left(\frac{2(Ab-ab)a^2 \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{-2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 1} \operatorname{EllipticPi}\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \frac{2a}{a-b}, \sqrt{2}\right)}{b^2(a^2-ab) \sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c)), x)`

[Out]
$$\begin{aligned} & -(-(-2*\cos(1/2*d*x+1/2*c)^2+1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*(A*b-B*a)*a^2 \\ & /b^2/(a^2-a*b)*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)} \\ & /(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*\operatorname{EllipticPi}(\cos(1/2*d*x+1/2*c), \\ & 2*a/(a-b), 2^{(1/2)})+2*(A*b-B*a)/b^2*(-(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)} \\ & *(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\operatorname{EllipticE}(\cos(1/2*d*x+1/2*c), 2^{(1/2)})+2*(-2*\sin(1/2*d*x+1/2*c)^4 \\ & +\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^2/\sin(1/2*d*x+1/2*c)^2 \\ & /((2*\sin(1/2*d*x+1/2*c)^2-1)+2*B/b*(-1/6*\cos(1/2*d*x+1/2*c)^2+1/3*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)} \\ & *(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)} \\ & *\operatorname{EllipticF}(\cos(1/2*d*x+1/2*c), 2^{(1/2)})))/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(b \sec(dx + c) + a) \cos(dx + c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c)), x, algorithm="maxima")`

[Out] `integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)*cos(d*x + c)^(5/2)), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\cos(c+dx)^{5/2} \left(a + \frac{b}{\cos(c+dx)} \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))),x)
```

```
[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(5/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] Timed out
```

$$3.580 \quad \int \frac{A+B \sec(c+dx)}{\cos^2(c+dx)(a+b \sec(c+dx))} dx$$

Optimal. Leaf size=217

$$\frac{2a^2(Ab - aB)\Pi\left(\frac{2a}{a+b}; \frac{1}{2}(c + dx) \middle| 2\right)}{b^3d(a+b)} + \frac{2(-5a^2B + 5aAb - 3b^2B)E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{5b^3d} - \frac{2(-5a^2B + 5aAb - 3b^2B)\sin(c)}{5b^3d\sqrt{\cos(c + dx)}}$$

[Out] $2/5*(5*A*a*b-5*B*a^2-3*B*b^2)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/b^3/d+2/3*(A*b-B*a)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/b^2/d+2*a^2*(A*b-B*a)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticPi}(\sin(1/2*d*x+1/2*c), 2*a/(a+b), 2^{(1/2)})/b^3/(a+b)/d+2/5*B*\sin(d*x+c)/b/d/\cos(d*x+c)^{(5/2)}+2/3*(A*b-B*a)*\sin(d*x+c)/b^2/d/\cos(d*x+c)^{(3/2)}-2/5*(5*A*a*b-5*B*a^2-3*B*b^2)*\sin(d*x+c)/b^3/d/\cos(d*x+c)^{(1/2)}$

Rubi [A] time = 1.19, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {2954, 3000, 3055, 3059, 2639, 3002, 2641, 2805}

$$\frac{2(-5a^2B + 5aAb - 3b^2B)E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{5b^3d} + \frac{2a^2(Ab - aB)\Pi\left(\frac{2a}{a+b}; \frac{1}{2}(c + dx) \middle| 2\right)}{b^3d(a+b)} - \frac{2(-5a^2B + 5aAb - 3b^2B)\sin(c)}{5b^3d\sqrt{\cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(A + B*\text{Sec}[c + d*x])/(\text{Cos}[c + d*x]^{(7/2)}*(a + b*\text{Sec}[c + d*x])), x]$

[Out] $(2*(5*a*A*b - 5*a^2*B - 3*b^2*B)*\text{EllipticE}[(c + d*x)/2, 2])/(5*b^3*d) + (2*(A*b - a*B)*\text{EllipticF}[(c + d*x)/2, 2])/(3*b^2*d) + (2*a^2*(A*b - a*B)*\text{EllipticPi}[(2*a)/(a+b), (c + d*x)/2, 2])/(b^3*(a+b)*d) + (2*B*\text{Sin}[c + d*x])/(5*b*d*\text{Cos}[c + d*x]^{(5/2)}) + (2*(A*b - a*B)*\text{Sin}[c + d*x])/(3*b^2*d*\text{Cos}[c + d*x]^{(3/2)}) - (2*(5*a*A*b - 5*a^2*B - 3*b^2*B)*\text{Sin}[c + d*x])/(5*b^3*d*\text{Sqrt}[\text{Cos}[c + d*x]])$

Rule 2639

$\text{Int}[\text{Sqrt}[\sin[(c_.) + (d_.)*(x_.)]], x_Symbol] := \text{Simp}[(2*\text{EllipticE}[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 2641

$\text{Int}[1/\text{Sqrt}[\sin[(c_.) + (d_.)*(x_.)]], x_Symbol] := \text{Simp}[(2*\text{EllipticF}[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]]), x_Symbol] := Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)]/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_.))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] := Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
a*d, 0] && !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
+ n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] := -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
- a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
```

```

+ b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]
) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || E
qQ[a, 0])))

```

Rule 3059

```

Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)] + (C_.)*sin[(e_.) + (f_.)*(x_.)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]*(c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_.)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]

```

Rubi steps

$$\begin{aligned}
\int \frac{A + B \sec(c + dx)}{\cos^{\frac{7}{2}}(c + dx)(a + b \sec(c + dx))} dx &= \int \frac{B + A \cos(c + dx)}{\cos^{\frac{7}{2}}(c + dx)(b + a \cos(c + dx))} dx \\
&= \frac{2B \sin(c + dx)}{5bd \cos^{\frac{5}{2}}(c + dx)} + \frac{2 \int \frac{\frac{5}{2}(Ab - aB) + \frac{3}{2}bB \cos(c + dx) + \frac{3}{2}aB \cos^2(c + dx)}{\cos^{\frac{5}{2}}(c + dx)(b + a \cos(c + dx))} dx}{5b} \\
&= \frac{2B \sin(c + dx)}{5bd \cos^{\frac{5}{2}}(c + dx)} + \frac{2(Ab - aB) \sin(c + dx)}{3b^2d \cos^{\frac{3}{2}}(c + dx)} + \frac{4 \int \frac{-\frac{3}{4}(5aAb - 5a^2B - 3b^2B) + \frac{1}{4}}{\cos^{\frac{3}{2}}(c + dx)} dx}{3b^2d} \\
&= \frac{2B \sin(c + dx)}{5bd \cos^{\frac{5}{2}}(c + dx)} + \frac{2(Ab - aB) \sin(c + dx)}{3b^2d \cos^{\frac{3}{2}}(c + dx)} - \frac{2(5aAb - 5a^2B - 3b^2B)}{5b^3d \sqrt{\cos(c + dx)}} \\
&= \frac{2B \sin(c + dx)}{5bd \cos^{\frac{5}{2}}(c + dx)} + \frac{2(Ab - aB) \sin(c + dx)}{3b^2d \cos^{\frac{3}{2}}(c + dx)} - \frac{2(5aAb - 5a^2B - 3b^2B)}{5b^3d \sqrt{\cos(c + dx)}} \\
&= \frac{2(5aAb - 5a^2B - 3b^2B) E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{5b^3d} + \frac{2B \sin(c + dx)}{5bd \cos^{\frac{5}{2}}(c + dx)} + \frac{2(Ab - aB) \sin(c + dx)}{3b^2d \cos^{\frac{3}{2}}(c + dx)} \\
&= \frac{2(5aAb - 5a^2B - 3b^2B) E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{5b^3d} + \frac{2(Ab - aB) F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3b^2d}
\end{aligned}$$

Mathematica [A] time = 4.86, size = 326, normalized size = 1.50

$$\frac{6b(5a^2B - 5aAb + 3b^2B) \sin(c + dx)}{\sqrt{\cos(c + dx)}} - \frac{b^2(20a^2B - 20aAb + 9b^2B) \left(2F\left(\frac{1}{2}(c + dx) \middle| 2\right) - \frac{2b\pi\left(\frac{2a}{a+b}; \frac{1}{2}(c + dx) \middle| 2\right)}{a+b} \right)}{a} - \frac{3(5a^2B - 5aAb + 3b^2B) \sin(c + dx) ((a^2 - 2b^2)\pi)}{5b^3d}$$

Antiderivative was successfully verified.

```

[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])),x]
[Out] ((b*(45*a^2*A*b + 10*A*b^3 - 45*a^3*B - 19*a*b^2*B)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) - (b^2*(-20*a*A*b + 20*a^2*B + 9*b^2*B)*(2*EllipticF[(c + d*x)/2, 2] - (2*b*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]))/(a + b)))/a + (6*b^3*B*Sin[c + d*x])/Cos[c + d*x]^(5/2) + (10*b^2*(A*b - a*B)*

```

$\text{Sin}[c + d*x])/\text{Cos}[c + d*x]^{(3/2)} + (6*b*(-5*a*A*b + 5*a^2*B + 3*b^2*B)*\text{Sin}[c + d*x])/\text{Sqrt}[\text{Cos}[c + d*x]] - (3*(-5*a*A*b + 5*a^2*B + 3*b^2*B)*(-2*a*b*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[\text{Cos}[c + d*x]]], -1] + 2*b*(a + b)*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[\text{Cos}[c + d*x]]], -1] + (a^2 - 2*b^2)*\text{EllipticPi}[-(a/b), \text{ArcSin}[\text{Sqrt}[\text{Cos}[c + d*x]]], -1])*\text{Sin}[c + d*x])/(a*\text{Sqrt}[\text{Sin}[c + d*x]^2])/(15*b^4*d)$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(b \sec(dx + c) + a) \cos(dx + c)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c)),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)*cos(d*x + c)^(7/2)), x)

maple [B] time = 15.60, size = 785, normalized size = 3.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c)),x)

[Out] $-(-(-2*\cos(1/2*d*x+1/2*c)^2+1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*(A*b-B*a)*a^3/b^3/(a^2-a*b)*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*\text{EllipticPi}(\cos(1/2*d*x+1/2*c), 2*a/(a-b), 2^{(1/2)})-2/5*B/b/(8*\sin(1/2*d*x+1/2*c)^6-12*\sin(1/2*d*x+1/2*c)^4+6*\sin(1/2*d*x+1/2*c)^2-1)/\sin(1/2*d*x+1/2*c)^2*(12*\text{EllipticE}(\cos(1/2*d*x+1/2*c), 2^{(1/2)})*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*\sin(1/2*d*x+1/2*c)^4-24*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^6-12*\text{EllipticE}(\cos(1/2*d*x+1/2*c), 2^{(1/2)})*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*\sin(1/2*d*x+1/2*c)^2+24*\sin(1/2*d*x+1/2*c)^4*$

$\cos(1/2*d*x+1/2*c)+3*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^{2-1})^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)})-8*\sin(1/2*d*x+1/2*c)^2*\cos(1/2*d*x+1/2*c))*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}-2*(A*b-B*a)/b^3*a*(-(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^{2-1})^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)})+2*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^2)/\sin(1/2*d*x+1/2*c)^2/(2*\sin(1/2*d*x+1/2*c)^{2-1})+2*(A*b-B*a)/b^2*(-1/6*\cos(1/2*d*x+1/2*c)*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/(-1/2+\cos(1/2*d*x+1/2*c)^2)^2+1/3*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)})))/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^{2-1})^{(1/2)}/d$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(b \sec(dx + c) + a) \cos(dx + c)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)*cos(d*x + c)^(7/2)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\cos(c+dx)^{7/2} \left(a + \frac{b}{\cos(c+dx)}\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))),x)

[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(7/2)/(a+b*sec(d*x+c)),x)

[Out] Timed out

$$3.581 \quad \int \frac{\cos^3(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^2} dx$$

Optimal. Leaf size=305

$$\frac{b(Ab - aB) \sin(c + dx) \cos^3(c + dx)}{ad(a^2 - b^2)(a \cos(c + dx) + b)} + \frac{(2a^2A + 3abB - 5Ab^2) \sin(c + dx) \sqrt{\cos(c + dx)}}{3a^2d(a^2 - b^2)} - \frac{(-2a^3B + 4a^2Ab + 3ab^2)}{a^3d}$$

[Out] $-(4Aa^2b - 5Ab^3 - 2Ba^3 + 3Bab^2) * (\cos(1/2dx + 1/2c))^2)^{1/2} / \cos(1/2dx + 1/2c) * \text{EllipticE}(\sin(1/2dx + 1/2c), 2^{1/2}) / a^3 / (a^2 - b^2) / d + 1/3 * (2Aa^4 + 16Aa^2b^2 - 15Ab^4 - 12Ba^3b + 9Bab^3) * (\cos(1/2dx + 1/2c))^2)^{1/2} / \cos(1/2dx + 1/2c) * \text{EllipticF}(\sin(1/2dx + 1/2c), 2^{1/2}) / a^4 / (a^2 - b^2) / d - b^2 * (7Aa^2b - 5Ab^3 - 5Ba^3 + 3Bab^2) * (\cos(1/2dx + 1/2c))^2)^{1/2} / \cos(1/2dx + 1/2c) * \text{EllipticPi}(\sin(1/2dx + 1/2c), 2a/(a+b), 2^{1/2}) / a^4 / (a-b) / (a+b)^2 / d + b * (Ab - Ba) * \cos(dx + c)^{3/2} * \sin(dx + c) / a / (a^2 - b^2) / d / (b + a * \cos(dx + c)) + 1/3 * (2Aa^2 - 5Ab^2 + 3Bab) * \sin(dx + c) * \cos(dx + c)^{1/2} / a^2 / (a^2 - b^2) / d$

Rubi [A] time = 1.02, antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {2954, 2989, 3049, 3059, 2639, 3002, 2641, 2805}

$$\frac{(16a^2Ab^2 + 2a^4A - 12a^3bB + 9ab^3B - 15Ab^4) F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3a^4d(a^2 - b^2)} - \frac{(4a^2Ab - 2a^3B + 3ab^2B - 5Ab^3) E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{a^3d(a^2 - b^2)}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^2,x]

[Out] $-(4a^2Ab - 5Ab^3 - 2a^3B + 3ab^2B) * \text{EllipticE}[(c + dx)/2, 2] / (a^3 * (a^2 - b^2) * d) + ((2a^4A + 16a^2Ab^2 - 15Ab^4 - 12a^3bB + 9a^2b^3B) * \text{EllipticF}[(c + dx)/2, 2]) / (3a^4 * (a^2 - b^2) * d) - (b^2 * (7a^2Ab - 5Ab^3 - 5a^3B + 3ab^2B) * \text{EllipticPi}[(2a)/(a + b), (c + dx)/2, 2]) / (a^4 * (a - b) * (a + b)^2 * d) + ((2a^2A - 5Ab^2 + 3abB) * \text{Sqrt}[\cos[c + dx]] * \sin[c + dx]) / (3a^2 * (a^2 - b^2) * d) + (b * (Ab - aB) * \cos[c + dx]^{3/2} * \sin[c + dx]) / (a * (a^2 - b^2) * d * (b + a * \cos[c + dx]))$

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_.)], x_Symbol] := Simp[(2*EllipticF[(1*(c -
Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_.)]), x_Symbol] := Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)]/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_.)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*
(d_.) + (c_.))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_.)])^(p_.), x_Symbol] := Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
a*d, 0] && !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_.)]*(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)])^(n_.), x_Symbol] := -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1)
]*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_.)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] := Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_.)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)] + (C_.)*sin[(e_
```

```
.) + (f_.)*(x_)^2), x_Symbol] := -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && !IGtQ[n, 0] && (!IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0]))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]*(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] := Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x], x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}
 \int \frac{\cos^{\frac{3}{2}}(c + dx)(A + B \sec(c + dx))}{(a + b \sec(c + dx))^2} dx &= \int \frac{\cos^{\frac{5}{2}}(c + dx)(B + A \cos(c + dx))}{(b + a \cos(c + dx))^2} dx \\
 &= \frac{b(Ab - aB) \cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{a(a^2 - b^2)d(b + a \cos(c + dx))} + \int \frac{\sqrt{\cos(c + dx)} \left(\frac{3}{2} b(Ab - aB) - a(Ab - aB) \cos(c + dx) \right)}{b + a \cos(c + dx)} dx \\
 &= \frac{(2a^2A - 5Ab^2 + 3abB) \sqrt{\cos(c + dx)} \sin(c + dx)}{3a^2(a^2 - b^2)d} + \frac{b(Ab - aB) \cos^{\frac{3}{2}}(c + dx)}{a(a^2 - b^2)d(b + a \cos(c + dx))} \\
 &= \frac{(2a^2A - 5Ab^2 + 3abB) \sqrt{\cos(c + dx)} \sin(c + dx)}{3a^2(a^2 - b^2)d} + \frac{b(Ab - aB) \cos^{\frac{3}{2}}(c + dx)}{a(a^2 - b^2)d(b + a \cos(c + dx))} \\
 &= -\frac{(4a^2Ab - 5Ab^3 - 2a^3B + 3ab^2B) E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{a^3(a^2 - b^2)d} + \frac{(2a^2A - 5Ab^2 + 3abB) \sqrt{\cos(c + dx)} \sin(c + dx)}{a^3(a^2 - b^2)d} \\
 &= -\frac{(4a^2Ab - 5Ab^3 - 2a^3B + 3ab^2B) E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{a^3(a^2 - b^2)d} + \frac{(2a^4A + 16a^2Ab - 5a^2B) \sqrt{\cos(c + dx)} \sin(c + dx)}{a^3(a^2 - b^2)d}
 \end{aligned}$$

Mathematica [A] time = 3.78, size = 318, normalized size = 1.04

$$4 \sin(c + dx) \sqrt{\cos(c + dx)} \left(\frac{3b^2(Ab - aB)}{(b^2 - a^2)(a \cos(c + dx) + b)} + 2A \right) - \frac{8(a^2A - 3abB + 2Ab^2) \left((a+b)F\left(\frac{1}{2}(c+dx) \middle| 2\right) - b\Pi\left(\frac{2a}{a+b}; \frac{1}{2}(c+dx) \middle| 2\right) \right)}{a+b} + \frac{2(6a^3B - 8a^2Ab - 3a^2B^2)}{a+b}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^2, x]

[Out] (4*Sqrt[Cos[c + d*x]]*(2*A + (3*b^2*(A*b - a*B))/((-a^2 + b^2)*(b + a*Cos[c + d*x]))) * Sin[c + d*x] - ((2*(-8*a^2*A*b + 5*A*b^3 + 6*a^3*B - 3*a*b^2*B)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) + (8*(a^2*A + 2*A*b^2 - 3*a*b*B)*((a + b)*EllipticF[(c + d*x)/2, 2] - b*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]))/(a + b) + (6*(-4*a^2*A*b + 5*A*b^3 + 2*a^3*B - 3*a*b^2*B)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*b*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (a^2 - 2*b^2)*EllipticPi[-(a/b), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a^2*b*Sqrt[Sin[c + d*x]^2]))/((-a + b)*(a + b))/(12*a^2*d)

fricas [F] time = 170.43, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{(B \cos(dx + c) \sec(dx + c) + A \cos(dx + c)) \sqrt{\cos(dx + c)}}{b^2 \sec(dx + c)^2 + 2ab \sec(dx + c) + a^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm="fricas")

[Out] integral((B*cos(d*x + c)*sec(d*x + c) + A*cos(d*x + c))*sqrt(cos(d*x + c))/(b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \cos(dx + c)^{\frac{3}{2}}}{(b \sec(dx + c) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^2, x)

maple [B] time = 16.00, size = 1059, normalized size = 3.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x)

[Out]
$$\begin{aligned} & -(-(-2*\cos(1/2*d*x+1/2*c)^2+1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2/3/a^4*(4*A*a^2*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^4+a^2*A*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)} \\ &)*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)})+9*A*b^2*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)}) \\ &)+6*A*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)}) \\ &)*a*b-2*A*a^2*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^2-6*B*a*b*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)} \\ &)*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)})-3*B*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)} \\ &)*EllipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)})*a^2)/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)} \\ & +2*b^2/a^3*(4*A*b-3*B*a)/(a^2-a*b)*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)} \\ &)/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticPi(\cos(1/2*d*x+1/2*c),2*a/(a-b),2^{(1/2)}) \\ & +2*b^3*(A*b-B*a)/a^4*(a^2/b/(a^2-b^2)*\cos(1/2*d*x+1/2*c)*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)} \\ &)/(2*a*\cos(1/2*d*x+1/2*c)^2-a+b)-1/2/(a+b)/b*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)} \\ &)/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)}) \\ & +1/2*a/b/(a^2-b^2)*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)} \\ &)/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)}) \\ & -1/2*a/b/(a^2-b^2)*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)} \\ &)/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)}) \\ & -1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)} \\ &)/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticPi(\cos(1/2*d*x+1/2*c),2*a/(a-b),2^{(1/2)}) \\ & +3/2*b/(a^2-b^2)/(a^2-a*b)*a*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)} \\ &)/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticPi(\cos(1/2*d*x+1/2*c),2*a/(a-b),2^{(1/2)})) \\ &)/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \cos(dx + c)^{\frac{3}{2}}}{(b \sec(dx + c) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^2, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c + dx)^{3/2} \left(A + \frac{B}{\cos(c+dx)} \right)}{\left(a + \frac{b}{\cos(c+dx)} \right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)^(3/2)*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x))^2,x)

[Out] int((cos(c + d*x)^(3/2)*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x))^2, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**2,x)

[Out] Timed out

$$3.582 \quad \int \frac{\sqrt{\cos(c+dx)} (A+B \sec(c+dx))}{(a+b \sec(c+dx))^2} dx$$

Optimal. Leaf size=223

$$\frac{(2a^2A + abB - 3Ab^2) E\left(\frac{1}{2}(c+dx) \middle| 2\right)}{a^2d(a^2 - b^2)} + \frac{b(Ab - aB) \sin(c+dx) \sqrt{\cos(c+dx)}}{ad(a^2 - b^2)(a \cos(c+dx) + b)} - \frac{(-2a^3B + 4a^2Ab + ab^2B - 3Ab^3)}{a^3d(a^2 - b^2)}$$

[Out] (2*A*a^2-3*A*b^2+B*a*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c), 2^(1/2))/a^2/(a^2-b^2)/d-(4*A*a^2*b-3*A*b^3-2*B*a^3+B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c), 2^(1/2))/a^3/(a^2-b^2)/d+b*(5*A*a^2*b-3*A*b^3-3*B*a^3+B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c), 2*a/(a+b), 2^(1/2))/a^3/(a-b)/(a+b)^2/d+b*(A*b-B*a)*sin(d*x+c)*cos(d*x+c)^(1/2)/a/(a^2-b^2)/d/(b+a*cos(d*x+c))

Rubi [A] time = 0.70, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {2954, 2989, 3059, 2639, 3002, 2641, 2805}

$$\frac{(4a^2Ab - 2a^3B + ab^2B - 3Ab^3) F\left(\frac{1}{2}(c+dx) \middle| 2\right)}{a^3d(a^2 - b^2)} + \frac{(2a^2A + abB - 3Ab^2) E\left(\frac{1}{2}(c+dx) \middle| 2\right)}{a^2d(a^2 - b^2)} + \frac{b(5a^2Ab - 3a^3B + ab^2B - 3Ab^3)}{a^3d(a^2 - b^2)}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^2,x]

[Out] ((2*a^2*A - 3*A*b^2 + a*b*B)*EllipticE[(c + d*x)/2, 2])/(a^2*(a^2 - b^2)*d) - ((4*a^2*A*b - 3*A*b^3 - 2*a^3*B + a*b^2*B)*EllipticF[(c + d*x)/2, 2])/(a^3*(a^2 - b^2)*d) + (b*(5*a^2*A*b - 3*A*b^3 - 3*a^3*B + a*b^2*B)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a^3*(a - b)*(a + b)^2*d) + (b*(A*b - a*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(a*(a^2 - b^2)*d*(b + a*cos[c + d*x]))

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]]], x_Symbol] := Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2805


```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]]), x_Symbol] := Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)])*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_.))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] := Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^(m*(d + c
*Ssin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
a*d, 0] && !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Ssin[e + f*x])^(m - 1)*(c +
d*Ssin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Ssin[e + f*x])^(m - 2)*(c + d*Ssin[e + f*x])^(n + 1)
]*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[
B/d, Int[(a + b*Ssin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Ssin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] := Dist[C/(b*d), Int[Sqrt[a + b*Ssin[e + f*x]], x],
x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Ssin[e + f*x]]*(c + d*Ssin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

&& NeQ[c^2 - d^2, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{\sqrt{\cos(c+dx)}(A+B \sec(c+dx))}{(a+b \sec(c+dx))^2} dx &= \int \frac{\cos^{\frac{3}{2}}(c+dx)(B+A \cos(c+dx))}{(b+a \cos(c+dx))^2} dx \\
 &= \frac{b(Ab-aB)\sqrt{\cos(c+dx)} \sin(c+dx)}{a(a^2-b^2)d(b+a \cos(c+dx))} + \frac{\int \frac{\frac{1}{2}b(Ab-aB)-a(Ab-aB) \cos(c+dx)+\frac{1}{2}}{\sqrt{\cos(c+dx)}(b+a \cos(c+dx))} dx}{a(a^2-b^2)d} \\
 &= \frac{b(Ab-aB)\sqrt{\cos(c+dx)} \sin(c+dx)}{a(a^2-b^2)d(b+a \cos(c+dx))} - \frac{\int \frac{-\frac{1}{2}ab(Ab-aB)+\frac{1}{2}(4a^2Ab-3Ab^3-2a^3B)}{\sqrt{\cos(c+dx)}(b+a \cos(c+dx))} dx}{a^2(a^2-b^2)d} \\
 &= \frac{(2a^2A-3Ab^2+abB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2(a^2-b^2)d} + \frac{b(Ab-aB)\sqrt{\cos(c+dx)} \sin(c+dx)}{a(a^2-b^2)d(b+a \cos(c+dx))} \\
 &= \frac{(2a^2A-3Ab^2+abB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2(a^2-b^2)d} - \frac{(4a^2Ab-3Ab^3-2a^3B+ab^2)}{a^3(a^2-b^2)d}
 \end{aligned}$$

Mathematica [A] time = 2.85, size = 281, normalized size = 1.26

$$\frac{4b(Ab-aB) \sin(c+dx) \sqrt{\cos(c+dx)}}{(a^2-b^2)(a \cos(c+dx)+b)} + \frac{\frac{2(2a^2A-abB-Ab^2)\Pi\left(\frac{2a}{a+b}; \frac{1}{2}(c+dx)\middle|2\right)}{a+b} + \frac{2(2a^2A+abB-3Ab^2) \sin(c+dx) \left((a^2-2b^2)\Pi\left(-\frac{a}{b}; \sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2b(a+b)F\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|2\right)\right)}{a^2b \sqrt{\sin^2(c+dx)}}}{(a-b)(a+b)}$$

$4ad$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^2, x]

[Out] ((4*b*(A*b - a*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*(b + a*Cos[c + d*x])) + ((2*(2*a^2*A - A*b^2 - a*b*B)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) + (8*(-(A*b) + a*B)*((a + b)*EllipticF[(c + d*x)/2, 2] - b*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]))/(a + b) + (2*(2*a^2*A - 3*A*b^2 + a*b*B)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*b*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (a^2 - 2*b^2)*EllipticPi[-(a/b), ArcSin[Sqrt[Cos[c + d*x]]], -1]*Sin[c + d*x]))/(a^2*b*Sqrt[Sin[c + d*x]^2]))/(a - b)*(a + b))/(4*a*d)

fricas [F] time = 123.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B \sec(dx+c) + A)\sqrt{\cos(dx+c)}}{b^2 \sec(dx+c)^2 + 2ab \sec(dx+c) + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")

[Out] integral((B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx+c) + A)\sqrt{\cos(dx+c)}}{(b \sec(dx+c) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^2,x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(b*sec(d*x + c) + a)^2, x)

maple [B] time = 13.83, size = 843, normalized size = 3.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^2,x)

[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/a^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(2*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b+A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a-B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a)-2/a^2*b*(3*A*b-2*B*a)/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))-2*b^2*(A*b-B*a)/a^3*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos(1/2*d*x+1/2*c)^2-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*

$$\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c), 2^{(1/2)})-1/2*a/b/(a^2-b^2)*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c), 2^{(1/2)})-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticPi(\cos(1/2*d*x+1/2*c), 2*a/(a-b), 2^{(1/2)})+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticPi(\cos(1/2*d*x+1/2*c), 2*a/(a-b), 2^{(1/2)})))/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sqrt{\cos(dx + c)}}{(b \sec(dx + c) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(b*sec(d*x + c) + a)^2, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\cos(c + dx)} \left(A + \frac{B}{\cos(c+dx)} \right)}{\left(a + \frac{b}{\cos(c+dx)} \right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)^(1/2)*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x))^2,x)

[Out] int((cos(c + d*x)^(1/2)*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x))^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \sqrt{\cos(c + dx)}}{(a + b \sec(c + dx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)**(1/2)/(a+b*sec(d*x+c))**2,x)

[Out] Integral((A + B*sec(c + d*x))*sqrt(cos(c + d*x))/(a + b*sec(c + d*x))**2, x)

$$3.583 \quad \int \frac{A+B \sec(c+dx)}{\sqrt{\cos(c+dx)} (a+b \sec(c+dx))^2} dx$$

Optimal. Leaf size=203

$$\frac{(2a^2A - abB - Ab^2) F\left(\frac{1}{2}(c+dx) \middle| 2\right)}{a^2d(a^2 - b^2)} + \frac{(Ab - aB) E\left(\frac{1}{2}(c+dx) \middle| 2\right)}{ad(a^2 - b^2)} - \frac{(Ab - aB) \sin(c+dx) \sqrt{\cos(c+dx)}}{d(a^2 - b^2)(a \cos(c+dx) + b)} - \frac{a^3(-B)}{d(a^2 - b^2)}$$

[Out] (A*b-B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c), 2^(1/2))/a/(a^2-b^2)/d+(2*A*a^2-A*b^2-B*a*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c), 2^(1/2))/a^2/(a^2-b^2)/d-(3*A*a^2*b-A*b^3-B*a^3-B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c), 2*a/(a+b), 2^(1/2))/a^2/(a-b)/(a+b)^2/d-(A*b-B*a)*sin(d*x+c)*cos(d*x+c)^(1/2)/(a^2-b^2)/d/(b+a*cos(d*x+c))

Rubi [A] time = 0.61, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {2954, 2999, 3059, 2639, 3002, 2641, 2805}

$$\frac{(2a^2A - abB - Ab^2) F\left(\frac{1}{2}(c+dx) \middle| 2\right)}{a^2d(a^2 - b^2)} + \frac{(Ab - aB) E\left(\frac{1}{2}(c+dx) \middle| 2\right)}{ad(a^2 - b^2)} - \frac{(3a^2Ab + a^3(-B) - ab^2B - Ab^3) \Pi\left(\frac{2a}{a+b}; \frac{1}{2}(c+dx) \middle| 2\right)}{a^2d(a-b)(a+b)^2}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sec[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^2), x]

[Out] ((A*b - a*B)*EllipticE[(c + d*x)/2, 2])/(a*(a^2 - b^2)*d) + ((2*a^2*A - A*b^2 - a*b*B)*EllipticF[(c + d*x)/2, 2])/(a^2*(a^2 - b^2)*d) - ((3*a^2*A*b - A*b^3 - a^3*B - a*b^2*B)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a^2*(a - b)*(a + b)^2*d) - ((A*b - a*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*d*(b + a*cos[c + d*x]))

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]]), x_Symbol] := Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)]/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^ (m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_.))^ (n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^ (p_.), x_Symbol] := Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
a*d, 0] && !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 2999

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^ (m_.)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^ (n_.), x_Symbol] := Si
mp[(B*a - A*b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*
x])^n)/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a +
b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[c*(a*A - b*B)*(m
+ 1) + d*n*(A*b - a*B) + (d*(a*A - b*B)*(m + 1) - c*(A*b - a*B)*(m + 2))*S
in[e + f*x] - d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 3002

```
Int((((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^ (m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3059

```
Int(((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] := Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x])*(c + d*Sin[e + f*x]), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{A + B \sec(c + dx)}{\sqrt{\cos(c + dx)} (a + b \sec(c + dx))^2} dx &= \int \frac{\sqrt{\cos(c + dx)} (B + A \cos(c + dx))}{(b + a \cos(c + dx))^2} dx \\
&= -\frac{(Ab - aB)\sqrt{\cos(c + dx)} \sin(c + dx)}{(a^2 - b^2) d(b + a \cos(c + dx))} + \int \frac{\frac{1}{2}(-Ab + aB) + (aA - bB) \cos(c + dx)}{\sqrt{\cos(c + dx)} (b + a \cos(c + dx))} dx \\
&= -\frac{(Ab - aB)\sqrt{\cos(c + dx)} \sin(c + dx)}{(a^2 - b^2) d(b + a \cos(c + dx))} - \frac{\int \frac{\frac{1}{2}a(Ab - aB) - \frac{1}{2}(2a^2A - Ab^2 - abB) \cos(c + dx)}{\sqrt{\cos(c + dx)} (b + a \cos(c + dx))} dx}{a(a^2 - b^2)} \\
&= \frac{(Ab - aB)E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{a(a^2 - b^2) d} - \frac{(Ab - aB)\sqrt{\cos(c + dx)} \sin(c + dx)}{(a^2 - b^2) d(b + a \cos(c + dx))} + \frac{\int \frac{\frac{1}{2}a(Ab - aB) - \frac{1}{2}(2a^2A - Ab^2 - abB) \cos(c + dx)}{\sqrt{\cos(c + dx)} (b + a \cos(c + dx))} dx}{a(a^2 - b^2)} \\
&= \frac{(Ab - aB)E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{a(a^2 - b^2) d} + \frac{(2a^2A - Ab^2 - abB)F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{a^2(a^2 - b^2) d}
\end{aligned}$$

Mathematica [A] time = 2.71, size = 260, normalized size = 1.28

$$\frac{4(aB - Ab) \sin(c + dx) \sqrt{\cos(c + dx)}}{(a^2 - b^2)(a \cos(c + dx) + b)} - \frac{2(Ab - aB) \sin(c + dx) \left((a^2 - 2b^2) \Pi\left(-\frac{a}{b}; \sin^{-1}(\sqrt{\cos(c + dx)}) \middle| -1\right) + 2b(a + b) F\left(\sin^{-1}(\sqrt{\cos(c + dx)}) \middle| -1\right) - 2abE\left(\sin^{-1}(\sqrt{\cos(c + dx)}) \middle| -1\right) \right)}{a^2 b \sqrt{\sin^2(c + dx)}}}{(b - a)(a + b)} \cdot 4d$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sec[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^2), x]

[Out] ((4*(-(A*b) + a*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*(b + a*Cos[c + d*x])) - ((2*(-(A*b) + a*B)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) + ((4*a*A - 4*b*B)*(2*EllipticF[(c + d*x)/2, 2] - (2*b*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b)))/a + (2*(A*b - a*B)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*b*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (a^2 - 2*b^2)*EllipticPi[-(a/b), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a^2*b*Sqrt[Sin[c + d*x]^2]))/((-a + b)*(a + b)))/(4*d)

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

$/2*c), 2*a/(a-b), 2^{(1/2)})))/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(b \sec(dx + c) + a)^2 \sqrt{\cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^2*sqrt(cos(d*x + c))), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\sqrt{\cos(c+dx)} \left(a + \frac{b}{\cos(c+dx)}\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))^2), x)

[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))^2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \sec(c + dx)}{(a + b \sec(c + dx))^2 \sqrt{\cos(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))**2/cos(d*x+c)**(1/2),x)

[Out] Integral((A + B*sec(c + d*x))/((a + b*sec(c + d*x))**2*sqrt(cos(c + d*x))), x)

$$3.584 \quad \int \frac{A+B \sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b \sec(c+dx))^2} dx$$

Optimal. Leaf size=197

$$\frac{(Ab - aB)F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{ad(a^2 - b^2)} - \frac{(Ab - aB)E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{bd(a^2 - b^2)} + \frac{a(Ab - aB) \sin(c + dx) \sqrt{\cos(c + dx)}}{bd(a^2 - b^2)(a \cos(c + dx) + b)} + \frac{(a^3B + a^2Ab - 3a^2bB - a^2b^2B)}{bd(a^2 - b^2)}$$

[Out] $-(A*b-B*a)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/b/(a^2-b^2)/d-(A*b-B*a)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/a/(a^2-b^2)/d+(A*a^2*b+A*b^3+B*a^3-3*B*a*b^2)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticPi}(\sin(1/2*d*x+1/2*c), 2*a/(a+b), 2^{(1/2)})/a/(a-b)/b/(a+b)^2/d+a*(A*b-B*a)*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/b/(a^2-b^2)/d/(b+a*\cos(d*x+c))$

Rubi [A] time = 0.67, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {2954, 3000, 3059, 2639, 3002, 2641, 2805}

$$\frac{(Ab - aB)F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{ad(a^2 - b^2)} - \frac{(Ab - aB)E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{bd(a^2 - b^2)} + \frac{(a^2Ab + a^3B - 3ab^2B + Ab^3) \Pi\left(\frac{2a}{a+b}; \frac{1}{2}(c + dx) \middle| 2\right)}{abd(a-b)(a+b)^2} + \frac{a(a^3B + a^2Ab - 3a^2bB - a^2b^2B)}{bd(a^2 - b^2)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(A + B*\text{Sec}[c + d*x])]/(\text{Cos}[c + d*x]^{(3/2)}*(a + b*\text{Sec}[c + d*x])^2), x]$

[Out] $-(((A*b - a*B)*\text{EllipticE}[(c + d*x)/2, 2])/(b*(a^2 - b^2)*d)) - ((A*b - a*B)*\text{EllipticF}[(c + d*x)/2, 2])/(a*(a^2 - b^2)*d) + ((a^2*A*b + A*b^3 + a^3*B - 3*a*b^2*B)*\text{EllipticPi}[(2*a)/(a + b), (c + d*x)/2, 2])/(a*(a - b)*b*(a + b)^2*d) + (a*(A*b - a*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sin}[c + d*x])/(b*(a^2 - b^2)*d*(b + a*\text{Cos}[c + d*x]))$

Rule 2639

$\text{Int}[\text{Sqrt}[\sin[(c_.) + (d_.)*(x_)]]], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticE}[(1*(c - \text{Pi}/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 2641

$\text{Int}[1/\text{Sqrt}[\sin[(c_.) + (d_.)*(x_)]]], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticF}[(1*(c - \text{Pi}/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])], x_Symbol] := Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)])*(b_.)^(m_.)*(csc[(e_.) + (f_.)*(x_)])*(
(d_.) + (c_.))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] := Dis
t[g^(m + n), Int[(g*SIN[e + f*x])^(p - m - n)*(b + a*SIN[e + f*x])^(m*(d + c
*SIN[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
a*d, 0] && !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*SIN[e + f*x])^(m + 1)*(c + d*SIN[e
+ f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*SIN[e + f*x])^(m + 1)*(c + d*SIN[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*SIN[e + f*x] - b*d*(A*b - a*B)*(m
+ n + 3)*SIN[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])], x_Symbol] := Dist[
B/d, Int[(a + b*SIN[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*SIN[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)])^
2/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])], x_Symbol] := Dist[C/(b*d), Int[Sqrt[a + b*SIN[e + f*x]], x],
x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*SIN[e
+ f*x], x]/(Sqrt[a + b*SIN[e + f*x])*(c + d*SIN[e + f*x]), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

&& NeQ[c^2 - d^2, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{A + B \sec(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(a + b \sec(c + dx))^2} dx &= \int \frac{B + A \cos(c + dx)}{\sqrt{\cos(c + dx)} (b + a \cos(c + dx))^2} dx \\
 &= \frac{a(Ab - aB)\sqrt{\cos(c + dx)} \sin(c + dx)}{b(a^2 - b^2)d(b + a \cos(c + dx))} - \int \frac{\frac{1}{2}(-aAb - a^2B + 2b^2B) + b(Ab - aB) \cos(c + dx)}{\sqrt{\cos(c + dx)} (b + a \cos(c + dx))} dx \\
 &= \frac{a(Ab - aB)\sqrt{\cos(c + dx)} \sin(c + dx)}{b(a^2 - b^2)d(b + a \cos(c + dx))} + \int \frac{\frac{1}{2}a(aAb + a^2B - 2b^2B) - \frac{1}{2}ab(Ab - aB) \cos(c + dx)}{\sqrt{\cos(c + dx)} (b + a \cos(c + dx))} dx \\
 &= -\frac{(Ab - aB)E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{b(a^2 - b^2)d} + \frac{a(Ab - aB)\sqrt{\cos(c + dx)} \sin(c + dx)}{b(a^2 - b^2)d(b + a \cos(c + dx))} \\
 &= -\frac{(Ab - aB)E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{b(a^2 - b^2)d} - \frac{(Ab - aB)F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{a(a^2 - b^2)d} + \frac{(a^2Ab + A^2)}{4bd}
 \end{aligned}$$

Mathematica [A] time = 2.80, size = 273, normalized size = 1.39

$$\frac{2(3a^2B + aAb - 4b^2B)\Pi\left(\frac{2a}{a+b}; \frac{1}{2}(c+dx) \middle| 2\right)}{a+b} + \frac{2(aB - Ab)\sin(c+dx)\left((a^2 - 2b^2)\Pi\left(-\frac{a}{b}; \sin^{-1}(\sqrt{\cos(c+dx)}) \middle| -1\right) + 2b(a+b)F\left(\sin^{-1}(\sqrt{\cos(c+dx)}) \middle| -1\right) - 2abE\left(\sin^{-1}(\sqrt{\cos(c+dx)}) \middle| -1\right)\right)}{ab\sqrt{\sin^2(c+dx)}} + \frac{(a-b)(a+b)}{4bd}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^2), x]

[Out] ((-4*a*(-(A*b) + a*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*(b + a*Cos[c + d*x])) + ((2*(a*A*b + 3*a^2*B - 4*b^2*B)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) + (4*b*(-(A*b) + a*B)*(2*EllipticF[(c + d*x)/2, 2] - (2*b*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b)))/a + (2*(-(A*b) + a*B)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*b*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (a^2 - 2*b^2)*EllipticPi[-(a/b), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*b*Sqrt[Sin[c + d*x]^2]))/(a - b)*(a + b))/(4*b*d)

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(b \sec(dx + c) + a)^2 \cos(dx + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^2*cos(d*x + c)^(3/2)), x)

maple [B] time = 11.37, size = 715, normalized size = 3.63

$$\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}}{\left(\frac{2A\sqrt{\frac{1-\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)+1}\operatorname{EllipticPi}\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \frac{2a}{a-b}, \sqrt{2}\right)}{(a^2-ab)\sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)}} + \dots\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x)

[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*A/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c), 2*a/(a-b), 2^(1/2))+2*(-A*b+B*a)/a*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos(1/2*d*x+1/2*c)^2-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2))

$$\frac{1}{2}c), 2^{(1/2)}) + 1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticF(cos(1/2*d*x+1/2*c), 2^{(1/2)}) - 1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticE(cos(1/2*d*x+1/2*c), 2^{(1/2)}) - 1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticPi(cos(1/2*d*x+1/2*c), 2*a/(a-b), 2^{(1/2)}) + 3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticPi(cos(1/2*d*x+1/2*c), 2*a/(a-b), 2^{(1/2)})))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d$$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\cos(c+dx)^{3/2} \left(a + \frac{b}{\cos(c+dx)}\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^2), x)

[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(3/2)/(a+b*sec(d*x+c))**2,x)

[Out] Timed out

$$3.585 \quad \int \frac{A+B \sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b \sec(c+dx))^2} dx$$

Optimal. Leaf size=255

$$\frac{(Ab - aB)F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{bd(a^2 - b^2)} + \frac{(-3a^2B + aAb + 2b^2B)E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{b^2d(a^2 - b^2)} - \frac{(-3a^2B + aAb + 2b^2B)\sin(c + dx)}{b^2d(a^2 - b^2)\sqrt{\cos(c + dx)}} + \frac{1}{bd(a^2 - b^2)}$$

[Out] (A*a*b-3*B*a^2+2*B*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/b^2/(a^2-b^2)/d+(A*b-B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/b/(a^2-b^2)/d+(A*a^2*b-3*A*b^3-3*B*a^3+5*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))/(a-b)/b^2/(a+b)^2/d-(A*a*b-3*B*a^2+2*B*b^2)*sin(d*x+c)/b^2/(a^2-b^2)/d/cos(d*x+c)^(1/2)+a*(A*b-B*a)*sin(d*x+c)/b/(a^2-b^2)/d/(b+a*cos(d*x+c))/cos(d*x+c)^(1/2)

Rubi [A] time = 0.94, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {2954, 3000, 3055, 3059, 2639, 3002, 2641, 2805}

$$\frac{(Ab - aB)F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{bd(a^2 - b^2)} + \frac{(-3a^2B + aAb + 2b^2B)E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{b^2d(a^2 - b^2)} + \frac{(a^2Ab - 3a^3B + 5ab^2B - 3Ab^3)\Pi\left(\frac{2a}{a+b}, \frac{1}{2}\right)}{b^2d(a-b)(a+b)^2}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sec[c + d*x])/((Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^2), x]

[Out] ((a*A*b - 3*a^2*B + 2*b^2*B)*EllipticE[(c + d*x)/2, 2])/(b^2*(a^2 - b^2)*d) + ((A*b - a*B)*EllipticF[(c + d*x)/2, 2])/(b*(a^2 - b^2)*d) + ((a^2*A*b - 3*A*b^3 - 3*a^3*B + 5*a*b^2*B)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a - b)*b^2*(a + b)^2*d - ((a*A*b - 3*a^2*B + 2*b^2*B)*Sin[c + d*x])/(b^2*(a^2 - b^2)*d*sqrt[Cos[c + d*x]]) + (a*(A*b - a*B)*Sin[c + d*x])/(b*(a^2 - b^2)*d*sqrt[Cos[c + d*x]]*(b + a*cos[c + d*x]))

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c -
Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]]), x_Symbol] := Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)]/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^ (m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_.))^ (n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^ (p_.), x_Symbol] := Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
a*d, 0] && !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^ (m_.)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^ (n_.), x_Symbol] := -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
+ n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^ (m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^ (m_.)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^ (n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
```



```

+ (f_.)*(x_)^2), x_Symbol] := -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
- a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
+ b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]
) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || E
qQ[a, 0])))

```

Rule 3059

```

Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] := Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]

```

Rubi steps

$$\begin{aligned}
\int \frac{A + B \sec(c + dx)}{\cos^{\frac{5}{2}}(c + dx)(a + b \sec(c + dx))^2} dx &= \int \frac{B + A \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(b + a \cos(c + dx))^2} dx \\
&= \frac{a(Ab - aB) \sin(c + dx)}{b(a^2 - b^2) d \sqrt{\cos(c + dx)} (b + a \cos(c + dx))} - \int \frac{\frac{1}{2}(aAb - 3a^2B + 2b^2B) + b(Ab - aB)}{\cos^{\frac{3}{2}}(c + dx)} dx \\
&= -\frac{(aAb - 3a^2B + 2b^2B) \sin(c + dx)}{b^2(a^2 - b^2) d \sqrt{\cos(c + dx)}} + \frac{a(Ab - aB) \sin(c + dx)}{b(a^2 - b^2) d \sqrt{\cos(c + dx)} (b + a \cos(c + dx))} \\
&= -\frac{(aAb - 3a^2B + 2b^2B) \sin(c + dx)}{b^2(a^2 - b^2) d \sqrt{\cos(c + dx)}} + \frac{a(Ab - aB) \sin(c + dx)}{b(a^2 - b^2) d \sqrt{\cos(c + dx)} (b + a \cos(c + dx))} \\
&= \frac{(aAb - 3a^2B + 2b^2B) E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{b^2(a^2 - b^2) d} - \frac{(aAb - 3a^2B + 2b^2B) \sin(c + dx)}{b^2(a^2 - b^2) d \sqrt{\cos(c + dx)}} \\
&= \frac{(aAb - 3a^2B + 2b^2B) E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{b^2(a^2 - b^2) d} + \frac{(Ab - aB) F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{b(a^2 - b^2) d} + \frac{a(Ab - aB) \sin(c + dx)}{b(a^2 - b^2) d \sqrt{\cos(c + dx)} (b + a \cos(c + dx))}
\end{aligned}$$

Mathematica [A] time = 4.73, size = 317, normalized size = 1.24

$$4\sqrt{\cos(c + dx)} \left(\frac{a^2(aB - Ab) \sin(c + dx)}{(a^2 - b^2)(a \cos(c + dx) + b)} + 2B \tan(c + dx) \right) - \frac{8b(-2a^2B + aAb + b^2B) \left((a+b)F\left(\frac{1}{2}(c+dx) \middle| 2\right) - b\Pi\left(\frac{2a}{a+b}; \frac{1}{2}(c+dx) \middle| 2\right) \right)}{a(a+b)} + \frac{2(3a^2B - aAb - 2b^2B) \sin(c + dx)}{b^2(a^2 - b^2) d \sqrt{\cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^2), x]

[Out] (-(((2*(-3*a^2*A*b + 4*A*b^3 + 9*a^3*B - 10*a*b^2*B)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) - (8*b*(a*A*b - 2*a^2*B + b^2*B)*((a + b)*EllipticF[(c + d*x)/2, 2] - b*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]))/(a*(a + b)) + (2*(-(a*A*b) + 3*a^2*B - 2*b^2*B)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*b*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (a^2 - 2*b^2)*EllipticPi[-(a/b), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*b*Sqrt[Sin[c + d*x]^2]))/((a - b)*(a + b))) + 4*Sqrt[Cos[c + d*x]]*((a^2*(-(A*b) + a*B)*Sin[c + d*x])/((a^2 - b^2)*(b + a*Cos[c + d*x])) + 2*B*Tan[c + d*x]))/(4*b^2*d)

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(b \sec(dx + c) + a)^2 \cos(dx + c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^2,x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^2*cos(d*x + c)^(5/2)), x)

maple [B] time = 14.91, size = 877, normalized size = 3.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^2,x)

[Out]
$$\begin{aligned} & -(-(-2*\cos(1/2*d*x+1/2*c)^2+1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*a^2*B/b^2/(a^2-a*b)*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticPi(\cos(1/2*d*x+1/2*c), 2*a/(a-b), 2^{(1/2)})+2*B/b^2*(-(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c), 2^{(1/2)})+2*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^2)/\sin(1/2*d*x+1/2*c)^2/(2*\sin(1/2*d*x+1/2*c)^2-1)+2*(A*b-B*a)/b*(a^2/b/(a^2-b^2)*\cos(1/2*d*x+1/2*c)*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/(2*a*\cos(1/2*d*x+1/2*c)^2-a+b)-1/2/(a+b)/b*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c), 2^{(1/2)})+1/2*a/b/(a^2-b^2)*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c), 2^{(1/2)})-1/2*a/b/(a^2-b^2)*(\sin(1 \end{aligned}$$

$$\frac{1}{2}d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c), 2^{(1/2)})-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticPi(\cos(1/2*d*x+1/2*c), 2*a/(a-b), 2^{(1/2)})+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticPi(\cos(1/2*d*x+1/2*c), 2*a/(a-b), 2^{(1/2)})))/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d$$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\cos(c+dx)^{5/2} \left(a + \frac{b}{\cos(c+dx)}\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))^2), x)

[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))^2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(5/2)/(a+b*sec(d*x+c))**2,x)

[Out] Timed out

$$3.586 \quad \int \frac{A+B \sec(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+b \sec(c+dx))^2} dx$$

Optimal. Leaf size=346

$$\frac{(-5a^2B + 3aAb + 2b^2B) F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3b^2d(a^2 - b^2)} + \frac{a(Ab - aB) \sin(c + dx)}{bd(a^2 - b^2) \cos^{\frac{3}{2}}(c + dx)(a \cos(c + dx) + b)} - \frac{(-5a^2B + 3aAb + 2b^2B) F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3b^2d(a^2 - b^2) \cos^{\frac{3}{2}}(c + dx)(a \cos(c + dx) + b)}$$

[Out] $-(3Aa^2b - 2Ab^3 - 5B^3a + 4B^2ab^2) \cdot (\cos(1/2dx + 1/2c))^2 \cdot \sqrt{\cos(1/2dx + 1/2c)} \cdot \text{EllipticE}(\sin(1/2dx + 1/2c), 2) / b^3 / (a^2 - b^2) / d - 1/3 \cdot (3Aa^2b - 5Ab^3 + 2B^2ab^2) \cdot (\cos(1/2dx + 1/2c))^2 \cdot \sqrt{\cos(1/2dx + 1/2c)} \cdot \text{EllipticF}(\sin(1/2dx + 1/2c), 2) / b^2 / (a^2 - b^2) / d - a \cdot (3Aa^2b - 5Ab^3 - 5B^3a + 7B^2ab^2) \cdot (\cos(1/2dx + 1/2c))^2 \cdot \sqrt{\cos(1/2dx + 1/2c)} \cdot \text{EllipticPi}(\sin(1/2dx + 1/2c), 2a/(a+b), 2) / (a-b) / b^3 / (a+b)^2 / d - 1/3 \cdot (3Aa^2b - 5Ab^3 + 2B^2ab^2) \cdot \sin(dx+c) / b^2 / (a^2 - b^2) / d / \cos(dx+c)^{3/2} + a \cdot (Ab - Ba) \cdot \sin(dx+c) / b / (a^2 - b^2) / d / \cos(dx+c)^{3/2} / (b + a \cos(dx+c)) + (3Aa^2b - 2Ab^3 - 5B^3a + 4B^2ab^2) \cdot \sin(dx+c) / b^3 / (a^2 - b^2) / d / \cos(dx+c)$

Rubi [A] time = 1.31, antiderivative size = 346, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {2954, 3000, 3055, 3059, 2639, 3002, 2641, 2805}

$$\frac{(-5a^2B + 3aAb + 2b^2B) F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3b^2d(a^2 - b^2)} - \frac{(3a^2Ab - 5a^3B + 4ab^2B - 2Ab^3) E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{b^3d(a^2 - b^2)} - \frac{a(3a^2Ab - 5a^3B + 4ab^2B - 2Ab^3) F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{b^3d(a^2 - b^2)}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^2), x]

[Out] $-(((3a^2Ab - 2Ab^3 - 5a^3B + 4ab^2B) \cdot \text{EllipticE}[(c + dx)/2, 2]) / (b^3 \cdot (a^2 - b^2) \cdot d)) - ((3a^2Ab - 5a^3B + 2b^2B) \cdot \text{EllipticF}[(c + dx)/2, 2]) / (3b^2 \cdot (a^2 - b^2) \cdot d) - (a \cdot (3a^2Ab - 5Ab^3 - 5a^3B + 7a^2b^2B) \cdot \text{EllipticPi}[(2a)/(a + b), (c + dx)/2, 2]) / ((a - b) \cdot b^3 \cdot (a + b)^2 \cdot d) - ((3a^2Ab - 5a^2B + 2b^2B) \cdot \text{Sin}[c + d*x]) / (3b^2 \cdot (a^2 - b^2) \cdot d \cdot \text{Cos}[c + d*x]^{3/2}) + ((3a^2Ab - 2Ab^3 - 5a^3B + 4ab^2B) \cdot \text{Sin}[c + d*x]) / (b^3 \cdot (a^2 - b^2) \cdot d \cdot \text{Sqrt}[\text{Cos}[c + d*x]]) + (a \cdot (Ab - aB) \cdot \text{Sin}[c + d*x]) / (b \cdot (a^2 - b^2) \cdot d \cdot \text{Cos}[c + d*x]^{3/2} \cdot (b + a \cdot \text{Cos}[c + d*x]))$

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_.)]], x_Symbol] := Simp[(2*EllipticF[(1*(c -
  Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
  + (f_.)*(x_.)]), x_Symbol] := Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
  /2 + f*x))/2, (2*d)/(c + d)]/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
  , d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
  0] && GtQ[c + d, 0]
```

Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_.)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*
  (d_.) + (c_.))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_.)]^(p_.), x_Symbol] := Dis
  t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
  *Sin[e + f*x])^n, x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
  a*d, 0] && !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^(m_.)*((A_.) + (B_.)*sin[(e_.) +
  (f_.)*(x_.)]*(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]^(n_.), x_Symbol] := -S
  imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
  + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
  *(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
  f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
  + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
  + n + 3)*Sin[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
  x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
  alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(Inte
  gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^(m_.)*((A_.) + (B_.)*sin[(e_.)
  + (f_.)*(x_.)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] := Dist[
  B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
  n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
  , m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.)
+ (f_.)*(x_)^2], x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
- a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
+ b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]
) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || E
qQ[a, 0])))

```

Rule 3059

```

Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)])^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])], x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]

```

Rubi steps

$$\begin{aligned}
\int \frac{A + B \sec(c + dx)}{\cos^{\frac{7}{2}}(c + dx)(a + b \sec(c + dx))^2} dx &= \int \frac{B + A \cos(c + dx)}{\cos^{\frac{5}{2}}(c + dx)(b + a \cos(c + dx))^2} dx \\
&= \frac{a(Ab - aB) \sin(c + dx)}{b(a^2 - b^2) d \cos^{\frac{3}{2}}(c + dx)(b + a \cos(c + dx))} - \int \frac{\frac{1}{2}(3aAb - 5a^2B + 2b^2B) + b(Ab - aB)}{\cos^{\frac{5}{2}}(c + dx)} dx \\
&= -\frac{(3aAb - 5a^2B + 2b^2B) \sin(c + dx)}{3b^2(a^2 - b^2) d \cos^{\frac{3}{2}}(c + dx)} + \frac{a(Ab - aB) \sin(c + dx)}{b(a^2 - b^2) d \cos^{\frac{3}{2}}(c + dx)(b + a \cos(c + dx))} \\
&= -\frac{(3aAb - 5a^2B + 2b^2B) \sin(c + dx)}{3b^2(a^2 - b^2) d \cos^{\frac{3}{2}}(c + dx)} + \frac{(3a^2Ab - 2Ab^3 - 5a^3B + 4ab^2B) E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{b^3(a^2 - b^2) d \sqrt{\cos(c + dx)}} \\
&= -\frac{(3aAb - 5a^2B + 2b^2B) \sin(c + dx)}{3b^2(a^2 - b^2) d \cos^{\frac{3}{2}}(c + dx)} + \frac{(3a^2Ab - 2Ab^3 - 5a^3B + 4ab^2B) E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{b^3(a^2 - b^2) d \sqrt{\cos(c + dx)}} \\
&= -\frac{(3a^2Ab - 2Ab^3 - 5a^3B + 4ab^2B) E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{b^3(a^2 - b^2) d} - \frac{(3aAb - 5a^2B + 2b^2B) \sin(c + dx)}{3b^2(a^2 - b^2) d \cos^{\frac{3}{2}}(c + dx)} \\
&= -\frac{(3a^2Ab - 2Ab^3 - 5a^3B + 4ab^2B) E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{b^3(a^2 - b^2) d} - \frac{(3aAb - 5a^2B + 2b^2B) \sin(c + dx)}{3b^2(a^2 - b^2) d \cos^{\frac{3}{2}}(c + dx)}
\end{aligned}$$

Mathematica [A] time = 6.94, size = 427, normalized size = 1.23

$$\frac{\sqrt{\cos(c + dx)} \left(\frac{a^4 B \sin(c + dx) - a^3 A b \sin(c + dx)}{b^3 (b^2 - a^2) (a \cos(c + dx) + b)} + \frac{2 \sec(c + dx) (A b \sin(c + dx) - 2 a B \sin(c + dx))}{b^3} + \frac{2 B \tan(c + dx) \sec(c + dx)}{3 b^2} \right)}{d} + \frac{(40 a^3 b B - 24 a^2 A b^2 - 24 a^2 B^2 + 24 a b^2 B)}{3 b^2 (a^2 - b^2) d \cos^{\frac{3}{2}}(c + dx)}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^2), x]

[Out] ((2*(-27*a^3*A*b + 30*a*A*b^3 + 45*a^4*B - 44*a^2*b^2*B - 4*b^4*B)*Elliptic Pi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) + ((-24*a^2*A*b^2 + 12*A*b^4 + 4

$$0*a^3*b*B - 28*a*b^3*B)*(2*EllipticF[(c + d*x)/2, 2] - (2*b*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b))/a + (2*(-9*a^3*A*b + 6*a*A*b^3 + 15*a^4*B - 12*a^2*b^2*B)*Cos[2*(c + d*x)]*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*b*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (a^2 - 2*b^2)*EllipticPi[-(a/b), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a^2*b*Sqrt[1 - Cos[c + d*x]^2]*(-1 + 2*Cos[c + d*x]^2))/(12*(a - b)*b^3*(a + b)*d) + (Sqrt[Cos[c + d*x]]*((2*Sec[c + d*x]*(A*b*Sin[c + d*x] - 2*a*B*Sin[c + d*x]))/b^3 + (-a^3*A*b*Sin[c + d*x]) + a^4*B*Sin[c + d*x])/(b^3*(-a^2 + b^2)*(b + a*Cos[c + d*x])) + (2*B*Sec[c + d*x]*Tan[c + d*x])/(3*b^2)))/d$$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(b \sec(dx + c) + a)^2 \cos(dx + c)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^2,x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^2*cos(d*x + c)^(7/2)), x)

maple [B] time = 20.45, size = 1024, normalized size = 2.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^2,x)

[Out]
$$-(-(-2*\cos(1/2*d*x+1/2*c)^2+1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*a^2*(A*b-2*B*a)/b^3/(a^2-a*b)*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticPi(\cos(1/2*d*x+1/2*c), 2*a/(a-b), 2^{(1/2)})+2*(A*b-2*B*a)/b^3*(-(-2*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}))$$

```

4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2
/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)+2*B/b^2*(-1/6*cos(1/2*d*x+
1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d
*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))-2*(A*b-B*a)*a/b^2*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*
c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos(1/2*d*x+1/
2*c)^2-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))+1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2
/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b)
,2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\cos(c+dx)^{7/2} \left(a + \frac{b}{\cos(c+dx)}\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))^2),x)

[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))^2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(7/2)/(a+b*sec(d*x+c))**2,x)

[Out] Timed out

$$3.587 \quad \int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^3} dx$$

Optimal. Leaf size=461

$$\frac{b(Ab - aB) \sin(c + dx) \cos^{\frac{5}{2}}(c + dx)}{2ad(a^2 - b^2)(a \cos(c + dx) + b)^2} + \frac{b(-9a^3B + 13a^2Ab + 3ab^2B - 7Ab^3) \sin(c + dx) \cos^{\frac{3}{2}}(c + dx)}{4a^2d(a^2 - b^2)^2(a \cos(c + dx) + b)} + \frac{(8a^4A + 3a^3B) \sin(c + dx) \cos^{\frac{1}{2}}(c + dx)}{4a^2d(a^2 - b^2)(a \cos(c + dx) + b)}$$

[Out] $-1/4*(24*A*a^4*b-65*A*a^2*b^3+35*A*b^5-8*B*a^5+29*B*a^3*b^2-15*B*a*b^4)*(cos(1/2*d*x+1/2*c)^2)^{(1/2)}/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^{(1/2)})/a^4/(a^2-b^2)^2/d+1/12*(8*A*a^6+128*A*a^4*b^2-223*A*a^2*b^4+105*A*b^6-72*B*a^5*b+99*B*a^3*b^3-45*B*a*b^5)*(cos(1/2*d*x+1/2*c)^2)^{(1/2)}/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^{(1/2)})/a^5/(a^2-b^2)^2/d-1/4*b^2*(63*A*a^4*b-86*A*a^2*b^3+35*A*b^5-35*B*a^5+38*B*a^3*b^2-15*B*a*b^4)*(cos(1/2*d*x+1/2*c)^2)^{(1/2)}/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^{(1/2)})/a^5/(a-b)^2/(a+b)^3/d+1/2*b*(A*b-B*a)*cos(d*x+c)^{(5/2)*sin(d*x+c)/a/(a^2-b^2)/d/(b+a*cos(d*x+c))^2+1/4*b*(13*A*a^2*b-7*A*b^3-9*B*a^3+3*B*a*b^2)*cos(d*x+c)^{(3/2)*sin(d*x+c)/a^2/(a^2-b^2)^2/d/(b+a*cos(d*x+c))+1/12*(8*A*a^4-61*A*a^2*b^2+35*A*b^4+33*B*a^3*b-15*B*a*b^3)*sin(d*x+c)*cos(d*x+c)^{(1/2)}/a^3/(a^2-b^2)^2/d$

Rubi [A] time = 1.58, antiderivative size = 461, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2954, 2989, 3047, 3049, 3059, 2639, 3002, 2641, 2805}

$$\frac{(128a^4Ab^2 - 223a^2Ab^4 + 8a^6A + 99a^3b^3B - 72a^5bB - 45ab^5B + 105Ab^6)F\left(\frac{1}{2}(c + dx) \middle| 2\right) - (65a^2Ab^3 + 24a^4A)}{12a^5d(a^2 - b^2)^2}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^3,x]

[Out] $-((24*a^4*A*b - 65*a^2*A*b^3 + 35*A*b^5 - 8*a^5*B + 29*a^3*b^2*B - 15*a*b^4*B)*EllipticE[(c + d*x)/2, 2])/(4*a^4*(a^2 - b^2)^2*d) + ((8*a^6*A + 128*a^4*A*b^2 - 223*a^2*A*b^4 + 105*A*b^6 - 72*a^5*b*B + 99*a^3*b^3*B - 45*a*b^5*B)*EllipticF[(c + d*x)/2, 2])/(12*a^5*(a^2 - b^2)^2*d) - (b^2*(63*a^4*A*b - 86*a^2*A*b^3 + 35*A*b^5 - 35*a^5*B + 38*a^3*b^2*B - 15*a*b^4*B)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(4*a^5*(a - b)^2*(a + b)^3*d) + ((8*a^4*A - 61*a^2*A*b^2 + 35*A*b^4 + 33*a^3*b*B - 15*a*b^3*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(12*a^3*(a^2 - b^2)^2*d) + (b*(A*b - a*B)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(b + a*cos[c + d*x])^2) + (b*(13*a^2*A*b - 7*$

$$A*b^3 - 9*a^3*B + 3*a*b^2*B)*\text{Cos}[c + d*x]^{(3/2)}*\text{Sin}[c + d*x]/(4*a^2*(a^2 - b^2)^{2*d}*(b + a*\text{Cos}[c + d*x]))$$
Rule 2639

$$\text{Int}[\text{Sqrt}[\text{sin}[(c_.) + (d_.)*(x_.)]], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticE}[(1*(c - \text{Pi}/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}\{c, d\}, x]$$
Rule 2641

$$\text{Int}[1/\text{Sqrt}[\text{sin}[(c_.) + (d_.)*(x_.)]], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticF}[(1*(c - \text{Pi}/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}\{c, d\}, x]$$
Rule 2805

$$\text{Int}[1/(((a_.) + (b_.)*\text{sin}[(e_.) + (f_.)*(x_.)])*\text{Sqrt}[(c_.) + (d_.)*\text{sin}[(e_.) + (f_.)*(x_.)]), x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticPi}[(2*b)/(a + b), (1*(e - \text{Pi}/2 + f*x))/2, (2*d)/(c + d)]/(f*(a + b)*\text{Sqrt}[c + d]), x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{GtQ}[c + d, 0]$$
Rule 2954

$$\text{Int}[(a_.) + \text{csc}[(e_.) + (f_.)*(x_.)]*(b_.)]^{(m_.)}*(\text{csc}[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))^{(n_.)}*((g_.)*\text{sin}[(e_.) + (f_.)*(x_.)]^{(p_.)}, x_Symbol] \rightarrow \text{Dist}[g^{(m + n)}, \text{Int}[(g*\text{Sin}[e + f*x])^{(p - m - n)}*(b + a*\text{Sin}[e + f*x])^{(m)}*(d + c*\text{Sin}[e + f*x])^{(n)}, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, p\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& !\text{IntegerQ}[p] \&\& \text{IntegerQ}[m] \&\& \text{IntegerQ}[n]$$
Rule 2989

$$\text{Int}[(a_.) + (b_.)*\text{sin}[(e_.) + (f_.)*(x_.)]^{(m_.)}*((A_.) + (B_.)*\text{sin}[(e_.) + (f_.)*(x_.)]*(c_.) + (d_.)*\text{sin}[(e_.) + (f_.)*(x_.)]^{(n_.)}, x_Symbol] \rightarrow -\text{Simp}[(b*c - a*d)*(B*c - A*d)*\text{Cos}[e + f*x]*(a + b*\text{Sin}[e + f*x])^{(m - 1)}*(c + d*\text{Sin}[e + f*x])^{(n + 1)})/(d*f*(n + 1)*(c^2 - d^2)), x] + \text{Dist}[1/(d*(n + 1)*(c^2 - d^2)), \text{Int}[(a + b*\text{Sin}[e + f*x])^{(m - 2)}*(c + d*\text{Sin}[e + f*x])^{(n + 1)})*\text{Simp}[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) - a*(b*c - a*d)*(B*c - A*d)*(n + 2))*\text{Sin}[e + f*x] + b*(d*(A*b*c + a*B*c - a*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*\text{Sin}[e + f*x]^2, x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{GtQ}[m, 1] \&\& \text{LtQ}[n, -1]$$
Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3047

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2), x_Symbol] := -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1) - a*c*(n + 2)))] - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1)))]*Sin[e + f*x] + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))]*Sin[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2), x_Symbol] := -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))]*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n + 2))*Sin[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && !(IGtQ[n, 0] && (!IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[(((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] := Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x], x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^3} dx &= \int \frac{\cos^{\frac{7}{2}}(c+dx)(B+A \cos(c+dx))}{(b+a \cos(c+dx))^3} dx \\
&= \frac{b(Ab-aB) \cos^{\frac{5}{2}}(c+dx) \sin(c+dx)}{2a(a^2-b^2)d(b+a \cos(c+dx))^2} + \frac{\int \frac{\cos^{\frac{3}{2}}(c+dx) \left(\frac{5}{2}b(Ab-aB)-2a(Ab-aB)\right)}{(b+a \cos(c+dx))^3} dx}{2a(a^2-b^2)d} \\
&= \frac{b(Ab-aB) \cos^{\frac{5}{2}}(c+dx) \sin(c+dx)}{2a(a^2-b^2)d(b+a \cos(c+dx))^2} + \frac{b(13a^2Ab-7Ab^3-9a^3B+3a^4B)}{4a^2(a^2-b^2)^2d} \sqrt{\cos(c+dx)} \sin(c+dx) \\
&= \frac{(8a^4A-61a^2Ab^2+35Ab^4+33a^3bB-15ab^3B) \sqrt{\cos(c+dx)} \sin(c+dx)}{12a^3(a^2-b^2)^2d} \\
&= \frac{(8a^4A-61a^2Ab^2+35Ab^4+33a^3bB-15ab^3B) \sqrt{\cos(c+dx)} \sin(c+dx)}{12a^3(a^2-b^2)^2d} \\
&= -\frac{(24a^4Ab-65a^2Ab^3+35Ab^5-8a^5B+29a^3b^2B-15ab^4B) E\left(\frac{1}{2}(c+dx)\right)}{4a^4(a^2-b^2)^2d} \\
&= -\frac{(24a^4Ab-65a^2Ab^3+35Ab^5-8a^5B+29a^3b^2B-15ab^4B) E\left(\frac{1}{2}(c+dx)\right)}{4a^4(a^2-b^2)^2d}
\end{aligned}$$

Mathematica [A] time = 6.14, size = 461, normalized size = 1.00

$$\frac{4 \sin(c+dx) \sqrt{\cos(c+dx)} \left(4a^6A+4A(a^3-ab^2)^2 \cos(2(c+dx))+33a^3b^3B-57a^2Ab^4+ab(16a^4A+39a^3bB-83a^2Ab^2-21ab^3B+49Ab^4) \cos(c+dx)-15ab^5B+3a^4B\right)}{(a^2-b^2)^2(a \cos(c+dx)+b)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^3, x]

[Out] ((4*sqrt[Cos[c + d*x]]*(4*a^6*A - 57*a^2*A*b^4 + 35*A*b^6 + 33*a^3*b^3*B - 15*a*b^5*B + a*b*(16*a^4*A - 83*a^2*A*b^2 + 49*A*b^4 + 39*a^3*b*B - 21*a*b^3*B)*Cos[c + d*x] + 4*A*(a^3 - a*b^2)^2*Cos[2*(c + d*x)]*Sin[c + d*x]))/(a

$$\begin{aligned} &^2 - b^2)^2*(b + a*\cos[c + d*x])^2) + ((2*(-56*a^4*A*b + 73*a^2*A*b^3 - 35* \\ &A*b^5 + 24*a^5*B - 21*a^3*b^2*B + 15*a*b^4*B)*\text{EllipticPi}[(2*a)/(a + b), (c \\ &+ d*x)/2, 2])/(a + b) + (16*(2*a^4*A + 14*a^2*A*b^2 - 7*A*b^4 - 12*a^3*b*B \\ &+ 3*a*b^3*B)*((a + b)*\text{EllipticF}[(c + d*x)/2, 2] - b*\text{EllipticPi}[(2*a)/(a + b \\ &), (c + d*x)/2, 2]))/(a + b) + (6*(-24*a^4*A*b + 65*a^2*A*b^3 - 35*A*b^5 + \\ &8*a^5*B - 29*a^3*b^2*B + 15*a*b^4*B)*(-2*a*b*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[\cos[c + \\ &d*x]]], -1] + 2*b*(a + b)*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[\cos[c + d*x]]], -1] + (a^2 \\ &- 2*b^2)*\text{EllipticPi}[-(a/b), \text{ArcSin}[\text{Sqrt}[\cos[c + d*x]]], -1])* \sin[c + d*x])/ \\ &(a^2*b*\text{Sqrt}[\sin[c + d*x]^2]))/((a - b)^2*(a + b)^2)/(48*a^3*d) \end{aligned}$$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \cos(dx + c)^{\frac{3}{2}}}{(b \sec(dx + c) + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^3, x)

maple [B] time = 22.70, size = 2216, normalized size = 4.81

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x)

[Out]
$$\begin{aligned} &-(-(-2*\cos(1/2*d*x+1/2*c)^2+1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2/3/a^5*(4*A*a^ \\ &2*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^4+a^2*A*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)} \\ &)*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c), 2^{(1/2)})+18 \\ &*A*b^2*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{Elliptic} \\ &\text{icF}(\cos(1/2*d*x+1/2*c), 2^{(1/2)})+9*A*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2 \end{aligned}$$

$$\begin{aligned}
& *d*x+1/2*c)^{-2-1}^{(1/2)} * \text{EllipticE}(\cos(1/2*d*x+1/2*c), 2^{(1/2)}) * a*b-2*A*a^2*\cos(1/2*d*x+1/2*c) * \sin(1/2*d*x+1/2*c)^{-2-9*B*a*b} * (\sin(1/2*d*x+1/2*c)^{-2})^{(1/2)} * \\
& (2*\sin(1/2*d*x+1/2*c)^{-2-1})^{(1/2)} * \text{EllipticF}(\cos(1/2*d*x+1/2*c), 2^{(1/2)}) - 3*B * \\
& (\sin(1/2*d*x+1/2*c)^{-2})^{(1/2)} * (2*\sin(1/2*d*x+1/2*c)^{-2-1})^{(1/2)} * \text{EllipticE}(\cos(1/2*d*x+1/2*c), 2^{(1/2)}) * a^2 / (-2*\sin(1/2*d*x+1/2*c)^{-4} + \sin(1/2*d*x+1/2*c)^{-2})^{(1/2)} + 4/a^4*b^2*(5*A*b-3*B*a) / (a^2-a*b) * (\sin(1/2*d*x+1/2*c)^{-2})^{(1/2)} * (-2*\cos(1/2*d*x+1/2*c)^{-2+1})^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^{-4} + \sin(1/2*d*x+1/2*c)^{-2})^{(1/2)} * \text{EllipticPi}(\cos(1/2*d*x+1/2*c), 2*a/(a-b), 2^{(1/2)}) - 2*b^4*(A*b-B*a) / a^5*(1/2*a^2/b / (a^2-b^2) * \cos(1/2*d*x+1/2*c) * (-2*\sin(1/2*d*x+1/2*c)^{-4} + \sin(1/2*d*x+1/2*c)^{-2})^{(1/2)} / (2*a*\cos(1/2*d*x+1/2*c)^{-2-a+b})^{-2+3/4*a^2*(a^2-3*b^2) / b^2 / (a^2-b^2)^2 * \cos(1/2*d*x+1/2*c) * (-2*\sin(1/2*d*x+1/2*c)^{-4} + \sin(1/2*d*x+1/2*c)^{-2})^{(1/2)} / (2*a*\cos(1/2*d*x+1/2*c)^{-2-a+b}) - 3/8 / (a+b) / (a^2-b^2) / b^2 * (\sin(1/2*d*x+1/2*c)^{-2})^{(1/2)} * (-2*\cos(1/2*d*x+1/2*c)^{-2+1})^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^{-4} + \sin(1/2*d*x+1/2*c)^{-2})^{(1/2)} * \text{EllipticF}(\cos(1/2*d*x+1/2*c), 2^{(1/2)}) * a^2-1/4 / (a+b) / (a^2-b^2) / b * (\sin(1/2*d*x+1/2*c)^{-2})^{(1/2)} * (-2*\cos(1/2*d*x+1/2*c)^{-2+1})^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^{-4} + \sin(1/2*d*x+1/2*c)^{-2})^{(1/2)} * \text{EllipticF}(\cos(1/2*d*x+1/2*c), 2^{(1/2)}) * a+7/8 / (a+b) / (a^2-b^2) * (\sin(1/2*d*x+1/2*c)^{-2})^{(1/2)} * (-2*\cos(1/2*d*x+1/2*c)^{-2+1})^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^{-4} + \sin(1/2*d*x+1/2*c)^{-2})^{(1/2)} * \text{EllipticF}(\cos(1/2*d*x+1/2*c), 2^{(1/2)}) + 3/8*a^3/b^2 / (a^2-b^2)^2 * (\sin(1/2*d*x+1/2*c)^{-2})^{(1/2)} * (-2*\cos(1/2*d*x+1/2*c)^{-2+1})^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^{-4} + \sin(1/2*d*x+1/2*c)^{-2})^{(1/2)} * \text{EllipticF}(\cos(1/2*d*x+1/2*c), 2^{(1/2)}) - 9/8*a / (a^2-b^2)^2 * (\sin(1/2*d*x+1/2*c)^{-2})^{(1/2)} * (-2*\cos(1/2*d*x+1/2*c)^{-2+1})^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^{-4} + \sin(1/2*d*x+1/2*c)^{-2})^{(1/2)} * \text{EllipticF}(\cos(1/2*d*x+1/2*c), 2^{(1/2)}) - 3/8*a^3/b^2 / (a^2-b^2)^2 * (\sin(1/2*d*x+1/2*c)^{-2})^{(1/2)} * (-2*\cos(1/2*d*x+1/2*c)^{-2+1})^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^{-4} + \sin(1/2*d*x+1/2*c)^{-2})^{(1/2)} * \text{EllipticE}(\cos(1/2*d*x+1/2*c), 2^{(1/2)}) + 9/8*a / (a^2-b^2)^2 * (\sin(1/2*d*x+1/2*c)^{-2})^{(1/2)} * (-2*\cos(1/2*d*x+1/2*c)^{-2+1})^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^{-4} + \sin(1/2*d*x+1/2*c)^{-2})^{(1/2)} * \text{EllipticE}(\cos(1/2*d*x+1/2*c), 2^{(1/2)}) - 3/8 / (a-b) / (a+b) / (a^2-b^2) / b^2 / (a^2-a*b) * a^5 * (\sin(1/2*d*x+1/2*c)^{-2})^{(1/2)} * (-2*\cos(1/2*d*x+1/2*c)^{-2+1})^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^{-4} + \sin(1/2*d*x+1/2*c)^{-2})^{(1/2)} * \text{EllipticPi}(\cos(1/2*d*x+1/2*c), 2*a/(a-b), 2^{(1/2)}) + 3/4 / (a-b) / (a+b) / (a^2-b^2) / (a^2-a*b) * a^3 * (\sin(1/2*d*x+1/2*c)^{-2})^{(1/2)} * (-2*\cos(1/2*d*x+1/2*c)^{-2+1})^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^{-4} + \sin(1/2*d*x+1/2*c)^{-2})^{(1/2)} * \text{EllipticPi}(\cos(1/2*d*x+1/2*c), 2*a/(a-b), 2^{(1/2)}) - 15/8 / (a-b) / (a+b) / (a^2-b^2) * b^2 / (a^2-a*b) * a * (\sin(1/2*d*x+1/2*c)^{-2})^{(1/2)} * (-2*\cos(1/2*d*x+1/2*c)^{-2+1})^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^{-4} + \sin(1/2*d*x+1/2*c)^{-2})^{(1/2)} * \text{EllipticPi}(\cos(1/2*d*x+1/2*c), 2*a/(a-b), 2^{(1/2)}) + 2/a^5*b^3*(5*A*b-4*B*a) * (a^2/b / (a^2-b^2) * \cos(1/2*d*x+1/2*c) * (-2*\sin(1/2*d*x+1/2*c)^{-4} + \sin(1/2*d*x+1/2*c)^{-2})^{(1/2)} / (2*a*\cos(1/2*d*x+1/2*c)^{-2-a+b}) - 1/2 / (a+b) / b * (\sin(1/2*d*x+1/2*c)^{-2})^{(1/2)} * (-2*\cos(1/2*d*x+1/2*c)^{-2+1})^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^{-4} + \sin(1/2*d*x+1/2*c)^{-2})^{(1/2)} * \text{EllipticF}(\cos(1/2*d*x+1/2*c), 2^{(1/2)}) + 1/2*a/b / (a^2-b^2) * (\sin(1/2*d*x+1/2*c)^{-2})^{(1/2)} * (-2*\cos(1/2*d*x+1/2*c)^{-2+1})^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^{-4} + \sin(1/2*d*x+1/2*c)^{-2})^{(1/2)} * \text{EllipticF}(\cos(1/2*d*x+1/2*c), 2^{(1/2)}) - 1/2*a/b / (a^2-b^2) * (\sin(1/2*d*x+1/2*c)^{-2})^{(1/2)} * (-2*\cos(1/2*d*x+1/2*c)^{-2+1})^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^{-4} + \sin(1/2*d*x+1/2*c)^{-2})^{(1/2)} * \text{EllipticE}(\cos(1/2*d*x+1/2*c), 2^{(1/2)})
\end{aligned}$$

))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+dx)^{3/2} \left(A + \frac{B}{\cos(c+dx)} \right)}{\left(a + \frac{b}{\cos(c+dx)} \right)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)^(3/2)*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x))^3,x)

[Out] int((cos(c + d*x)^(3/2)*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x))^3, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**3,x)

[Out] Timed out

$$3.588 \quad \int \frac{\sqrt{\cos(c+dx)} (A+B \sec(c+dx))}{(a+b \sec(c+dx))^3} dx$$

Optimal. Leaf size=367

$$\frac{b(Ab - aB) \sin(c + dx) \cos^3(c + dx)}{2ad(a^2 - b^2)(a \cos(c + dx) + b)^2} + \frac{b(-7a^3B + 11a^2Ab + ab^2B - 5Ab^3) \sin(c + dx) \sqrt{\cos(c + dx)}}{4a^2d(a^2 - b^2)^2(a \cos(c + dx) + b)} + \frac{(8a^4A + 9a^3bB - 3a^2b^2B - 3ab^3B)}{4a^3d(a^2 - b^2)^2(a \cos(c + dx) + b)}$$

[Out] $\frac{1}{4} * (8 * A * a^4 - 29 * A * a^2 * b^2 + 15 * A * b^4 + 9 * B * a^3 * b - 3 * B * a * b^3) * (\cos(1/2 * d * x + 1/2 * c))^2)^{(1/2)} / \cos(1/2 * d * x + 1/2 * c) * \text{EllipticE}(\sin(1/2 * d * x + 1/2 * c), 2^{(1/2)}) / a^3 / (a^2 - b^2)^2 / d - 1/4 * (24 * A * a^4 * b - 33 * A * a^2 * b^3 + 15 * A * b^5 - 8 * B * a^5 + 5 * B * a^3 * b^2 - 3 * B * a * b^4) * (\cos(1/2 * d * x + 1/2 * c))^2)^{(1/2)} / \cos(1/2 * d * x + 1/2 * c) * \text{EllipticF}(\sin(1/2 * d * x + 1/2 * c), 2^{(1/2)}) / a^4 / (a^2 - b^2)^2 / d + 1/4 * b * (35 * A * a^4 * b - 38 * A * a^2 * b^3 + 15 * A * b^5 - 15 * B * a^5 + 6 * B * a^3 * b^2 - 3 * B * a * b^4) * (\cos(1/2 * d * x + 1/2 * c))^2)^{(1/2)} / \cos(1/2 * d * x + 1/2 * c) * \text{EllipticPi}(\sin(1/2 * d * x + 1/2 * c), 2 * a / (a + b), 2^{(1/2)}) / a^4 / (a - b)^2 / (a + b)^3 / d + 1/2 * b * (A * b - B * a) * \cos(d * x + c)^{(3/2)} * \sin(d * x + c) / a / (a^2 - b^2) / d / (b + a * \cos(d * x + c))^2 + 1/4 * b * (11 * A * a^2 * b - 5 * A * b^3 - 7 * B * a^3 + B * a * b^2) * \sin(d * x + c) * \cos(d * x + c)^{(1/2)} / a^2 / (a^2 - b^2)^2 / d / (b + a * \cos(d * x + c))$

Rubi [A] time = 1.11, antiderivative size = 367, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {2954, 2989, 3047, 3059, 2639, 3002, 2641, 2805}

$$\frac{(-33a^2Ab^3 + 24a^4Ab + 5a^3b^2B - 8a^5B - 3ab^4B + 15Ab^5) F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{4a^4d(a^2 - b^2)^2} + \frac{(-29a^2Ab^2 + 8a^4A + 9a^3bB - 3a^2b^2B - 3ab^3B)}{4a^3d(a^2 - b^2)^2}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^3,x]

[Out] $((8 * a^4 * A - 29 * a^2 * A * b^2 + 15 * A * b^4 + 9 * a^3 * b * B - 3 * a * b^3 * B) * \text{EllipticE}[(c + d * x) / 2, 2]) / (4 * a^3 * (a^2 - b^2)^2 * d) - ((24 * a^4 * A * b - 33 * a^2 * A * b^3 + 15 * A * b^5 - 8 * a^5 * B + 5 * a^3 * b^2 * B - 3 * a * b^4 * B) * \text{EllipticF}[(c + d * x) / 2, 2]) / (4 * a^4 * (a^2 - b^2)^2 * d) + (b * (35 * a^4 * A * b - 38 * a^2 * A * b^3 + 15 * A * b^5 - 15 * a^5 * B + 6 * a^3 * b^2 * B - 3 * a * b^4 * B) * \text{EllipticPi}[(2 * a) / (a + b), (c + d * x) / 2, 2]) / (4 * a^4 * (a - b)^2 * (a + b)^3 * d) + (b * (A * b - a * B) * \text{Cos}[c + d * x]^{(3/2)} * \text{Sin}[c + d * x]) / (2 * a * (a^2 - b^2) * d * (b + a * \text{Cos}[c + d * x])^2) + (b * (11 * a^2 * A * b - 5 * A * b^3 - 7 * a^3 * B + a * b^2 * B) * \text{Sqrt}[\text{Cos}[c + d * x]] * \text{Sin}[c + d * x]) / (4 * a^2 * (a^2 - b^2)^2 * d * (b + a * \text{Cos}[c + d * x]))$

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_.)], x_Symbol] := Simp[(2*EllipticF[(1*(c -
  Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
  + (f_.)*(x_.)]), x_Symbol] := Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
  /2 + f*x))/2, (2*d)/(c + d)]/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
  , d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
  0] && GtQ[c + d, 0]
```

Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_.)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*
  (d_.) + (c_.))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_.)]^(p_.), x_Symbol] := Dis
  t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
  *Sin[e + f*x])^n, x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
  a*d, 0] && !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
  (f_.)*(x_.)]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)])^(n_.), x_Symbol] := -S
  imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
  d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
  *(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1)
  ]*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
  *d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
  a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
  *d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x] /;
  FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
  ] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3002

```
Int((((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
  + (f_.)*(x_.)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] := Dist[
  B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
  n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
  , m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3047

```

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1)))]*Sin[e + f*x] +
b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]

```

Rule 3059

```

Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]

```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\cos(c+dx)}(A+B\sec(c+dx))}{(a+b\sec(c+dx))^3} dx &= \int \frac{\cos^{\frac{5}{2}}(c+dx)(B+A\cos(c+dx))}{(b+a\cos(c+dx))^3} dx \\
&= \frac{b(Ab-aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2a(a^2-b^2)d(b+a\cos(c+dx))^2} + \frac{\int \frac{\sqrt{\cos(c+dx)}\left(\frac{3}{2}b(Ab-aB)-2a(Ab-aB)\right)}{(b+a\cos(c+dx))^3} dx}{2a} \\
&= \frac{b(Ab-aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2a(a^2-b^2)d(b+a\cos(c+dx))^2} + \frac{b(11a^2Ab-5Ab^3-7a^3B+ab^3)}{4a^2(a^2-b^2)^2d(b+a\cos(c+dx))} \\
&= \frac{b(Ab-aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2a(a^2-b^2)d(b+a\cos(c+dx))^2} + \frac{b(11a^2Ab-5Ab^3-7a^3B+ab^3)}{4a^2(a^2-b^2)^2d(b+a\cos(c+dx))} \\
&= \frac{(8a^4A-29a^2Ab^2+15Ab^4+9a^3bB-3ab^3B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^3(a^2-b^2)^2d} + \frac{b(Ab-aB)}{2a} \\
&= \frac{(8a^4A-29a^2Ab^2+15Ab^4+9a^3bB-3ab^3B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^3(a^2-b^2)^2d} - \frac{(24a^3b^2-11a^2b^3-7a^3B+ab^3)}{4a^3(a^2-b^2)^2d}
\end{aligned}$$

Mathematica [A] time = 5.13, size = 390, normalized size = 1.06

$$\frac{8(2a^3B-4a^2Ab+ab^2B+Ab^3)\left(\frac{1}{2}(c+dx)\middle|2\right)-b\pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b} + \frac{(8a^4A-5a^3bB-7a^2Ab^2-ab^3B+5Ab^4)\pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b} + \frac{(8a^4A+9a^3bB-29a^2Ab^2-3ab^3B+15Ab^4)}{(a-b)^2(a+b)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^3, x]

[Out] ((-2*b*Sqrt[Cos[c + d*x]]*(b*(-11*a^2*A*b + 5*A*b^3 + 7*a^3*B - a*b^2*B) + a*(-13*a^2*A*b + 7*A*b^3 + 9*a^3*B - 3*a*b^2*B))*Cos[c + d*x])*Sin[c + d*x])/((a^2 - b^2)^2*(b + a*cos[c + d*x])^2) + (((8*a^4*A - 7*a^2*A*b^2 + 5*A*b^4 - 5*a^3*b*B - a*b^3*B)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) + (8*(-4*a^2*A*b + A*b^3 + 2*a^3*B + a*b^2*B))*((a + b)*EllipticF[(c + d*x)/2, 2] - b*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]))/(a + b) + ((8*a^4*A - 29*a^2*A*b^2 + 15*A*b^4 + 9*a^3*b*B - 3*a*b^3*B)*(-2*a*b*EllipticE[ArcSin

```
[Sqrt[Cos[c + d*x]]], -1] + 2*b*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (a^2 - 2*b^2)*EllipticPi[-(a/b), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a^2*b*Sqrt[Sin[c + d*x]^2])/((a - b)^2*(a + b)^2)/(8*a^2*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sqrt{\cos(dx + c)}}{(b \sec(dx + c) + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(b*sec(d*x + c) + a)^3, x)
```

maple [B] time = 21.89, size = 2000, normalized size = 5.45

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/a^4/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(3*A*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2))*b+A*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2))*a-B*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2))*a)-6/a^3*b*(2*A*b-B*a)/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c), 2*a/(a-b), 2^(1/2))+2*b^3*(A*b-B*a)/a^4*(1/2*a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos(1/2*d*x+1/2*c)^2-a+b)^2+3/4*a^2*(a^2-3*b^2)/b^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
```

$$\begin{aligned}
&)^2)^{(1/2)} / (2*a*\cos(1/2*d*x+1/2*c)^2-a+b) - 3/8 / (a+b) / (a^2-b^2) / b^2 * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * \text{EllipticF}(\cos(1/2*d*x+1/2*c), 2^{(1/2)}) * a^{2-1/4} / (a+b) / (a^2-b^2) / b * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * \text{EllipticF}(\cos(1/2*d*x+1/2*c), 2^{(1/2)}) * a + 7/8 / (a+b) / (a^2-b^2) * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * \text{EllipticF}(\cos(1/2*d*x+1/2*c), 2^{(1/2)}) + 3/8 * a^3 / b^2 / (a^2-b^2)^2 * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * \text{EllipticF}(\cos(1/2*d*x+1/2*c), 2^{(1/2)}) - 9/8 * a / (a^2-b^2)^2 * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * \text{EllipticF}(\cos(1/2*d*x+1/2*c), 2^{(1/2)}) - 3/8 * a^3 / b^2 / (a^2-b^2)^2 * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * \text{EllipticE}(\cos(1/2*d*x+1/2*c), 2^{(1/2)}) + 9/8 * a / (a^2-b^2)^2 * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * \text{EllipticE}(\cos(1/2*d*x+1/2*c), 2^{(1/2)}) - 3/8 / (a-b) / (a+b) / (a^2-b^2) / b^2 / (a^2-a*b) * a^5 * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * \text{EllipticPi}(\cos(1/2*d*x+1/2*c), 2*a / (a-b), 2^{(1/2)}) + 3/4 / (a-b) / (a+b) / (a^2-b^2) / (a^2-a*b) * a^3 * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * \text{EllipticPi}(\cos(1/2*d*x+1/2*c), 2*a / (a-b), 2^{(1/2)}) - 15/8 / (a-b) / (a+b) / (a^2-b^2) * b^2 / (a^2-a*b) * a * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * \text{EllipticPi}(\cos(1/2*d*x+1/2*c), 2*a / (a-b), 2^{(1/2)}) - 2*b^2 / a^4 * (4*A*b-3*B*a) * (a^2/b / (a^2-b^2) * \cos(1/2*d*x+1/2*c) * (-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)} / (2*a*\cos(1/2*d*x+1/2*c)^2-a+b) - 1/2 / (a+b) / b * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * \text{EllipticF}(\cos(1/2*d*x+1/2*c), 2^{(1/2)}) + 1/2 * a / b / (a^2-b^2) * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * \text{EllipticF}(\cos(1/2*d*x+1/2*c), 2^{(1/2)}) - 1/2 * a / b / (a^2-b^2) * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * \text{EllipticE}(\cos(1/2*d*x+1/2*c), 2^{(1/2)}) - 1/2 / b / (a^2-b^2) / (a^2-a*b) * a^3 * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * \text{EllipticPi}(\cos(1/2*d*x+1/2*c), 2*a / (a-b), 2^{(1/2)}) + 3/2 * b / (a^2-b^2) / (a^2-a*b) * a * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * \text{EllipticPi}(\cos(1/2*d*x+1/2*c), 2*a / (a-b), 2^{(1/2)})) / \sin(1/2*d*x+1/2*c) / (2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)} / d
\end{aligned}$$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^3,x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\cos(c+dx)} \left(A + \frac{B}{\cos(c+dx)} \right)}{\left(a + \frac{b}{\cos(c+dx)} \right)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c+d*x)^(1/2)*(A+B/cos(c+d*x)))/(a+b/cos(c+d*x))^3,x)

[Out] int((cos(c+d*x)^(1/2)*(A+B/cos(c+d*x)))/(a+b/cos(c+d*x))^3, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+B \sec(c+dx)) \sqrt{\cos(c+dx)}}{(a+b \sec(c+dx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)**(1/2)/(a+b*sec(d*x+c))**3,x)

[Out] Integral((A+B*sec(c+d*x))*sqrt(cos(c+d*x))/(a+b*sec(c+d*x))**3, x)

$$3.589 \quad \int \frac{A+B \sec(c+dx)}{\sqrt{\cos(c+dx)} (a+b \sec(c+dx))^3} dx$$

Optimal. Leaf size=346

$$\frac{b(Ab - aB) \sin(c + dx) \sqrt{\cos(c + dx)}}{2ad(a^2 - b^2)(a \cos(c + dx) + b)^2} + \frac{(-5a^3B + 9a^2Ab - ab^2B - 3Ab^3) E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{4a^2d(a^2 - b^2)^2} - \frac{(-5a^3B + 9a^2Ab - ab^2B - 3Ab^3) E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{4ad(a^2 - b^2)^2}$$

[Out] $\frac{1}{4} * (9 * A * a^2 * b - 3 * A * b^3 - 5 * B * a^3 * B - B * a * b^2) * (\cos(1/2 * d * x + 1/2 * c))^2^{(1/2)} / \cos(1/2 * d * x + 1/2 * c) * \text{EllipticE}(\sin(1/2 * d * x + 1/2 * c), 2^{(1/2)}) / a^2 / (a^2 - b^2)^2 / d + 1/4 * (8 * A * a^4 - 5 * A * a^2 * b^2 + 3 * A * b^4 - 7 * B * a^3 * b + B * a * b^3) * (\cos(1/2 * d * x + 1/2 * c))^2^{(1/2)} / \cos(1/2 * d * x + 1/2 * c) * \text{EllipticF}(\sin(1/2 * d * x + 1/2 * c), 2^{(1/2)}) / a^3 / (a^2 - b^2)^2 / d - 1/4 * (15 * A * a^4 * b - 6 * A * a^2 * b^3 + 3 * A * b^5 - 3 * B * a^5 - 10 * B * a^3 * b^2 + B * a * b^4) * (\cos(1/2 * d * x + 1/2 * c))^2^{(1/2)} / \cos(1/2 * d * x + 1/2 * c) * \text{EllipticPi}(\sin(1/2 * d * x + 1/2 * c), 2 * a / (a + b), 2^{(1/2)}) / a^3 / (a - b)^2 / (a + b)^3 / d + 1/2 * b * (A * b - B * a) * \sin(d * x + c) * \cos(d * x + c)^{(1/2)} / a / (a^2 - b^2) / d / (b + a * \cos(d * x + c))^2 - 1/4 * (9 * A * a^2 * b - 3 * A * b^3 - 5 * B * a^3 * B - B * a * b^2) * \sin(d * x + c) * \cos(d * x + c)^{(1/2)} / a / (a^2 - b^2)^2 / d / (b + a * \cos(d * x + c))$

Rubi [A] time = 1.10, antiderivative size = 346, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {2954, 2989, 3055, 3059, 2639, 3002, 2641, 2805}

$$\frac{(-5a^2Ab^2 + 8a^4A - 7a^3bB + ab^3B + 3Ab^4) F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{4a^3d(a^2 - b^2)^2} + \frac{(9a^2Ab - 5a^3B - ab^2B - 3Ab^3) E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{4a^2d(a^2 - b^2)^2} - \frac{(-5a^2Ab^2 + 8a^4A - 7a^3bB + ab^3B + 3Ab^4) F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{4a^3d(a^2 - b^2)^2} + \frac{(9a^2Ab - 5a^3B - ab^2B - 3Ab^3) E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{4a^2d(a^2 - b^2)^2}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sec[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^3),x]

[Out] $((9 * a^2 * A * b - 3 * A * b^3 - 5 * a^3 * B - a * b^2 * B) * \text{EllipticE}[(c + d * x) / 2, 2]) / (4 * a^2 * (a^2 - b^2)^2 * d) + ((8 * a^4 * A - 5 * a^2 * A * b^2 + 3 * A * b^4 - 7 * a^3 * b * B + a * b^3 * B) * \text{EllipticF}[(c + d * x) / 2, 2]) / (4 * a^3 * (a^2 - b^2)^2 * d) - ((15 * a^4 * A * b - 6 * a^2 * A * b^3 + 3 * A * b^5 - 3 * a^5 * B - 10 * a^3 * b^2 * B + a * b^4 * B) * \text{EllipticPi}[(2 * a) / (a + b), (c + d * x) / 2, 2]) / (4 * a^3 * (a - b)^2 * (a + b)^3 * d) + (b * (A * b - a * B) * \text{Sqrt}[\text{Cos}[c + d * x]] * \text{Sin}[c + d * x]) / (2 * a * (a^2 - b^2) * d * (b + a * \text{Cos}[c + d * x])^2) - ((9 * a^2 * A * b - 3 * A * b^3 - 5 * a^3 * B - a * b^2 * B) * \text{Sqrt}[\text{Cos}[c + d * x]] * \text{Sin}[c + d * x]) / (4 * a * (a^2 - b^2)^2 * d * (b + a * \text{Cos}[c + d * x]))$

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2805

Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])], x_Symbol] := Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi/2 + f*x))/2, (2*d)/(c + d)]/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[c + d, 0]

Rule 2954

Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^m_*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))^n_*(g_.)*sin[(e_.) + (f_.)*(x_)]^p_, x_Symbol] := Dist[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]

Rule 2989

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^m_*(A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^n_, x_Symbol] := -Simp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) - a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]

Rule 3002

Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^m_*(A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3055

```

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] := -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
- a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
+ b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]
) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || E
qQ[a, 0])))

```

Rule 3059

```

Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] := Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]

```

Rubi steps

$$\begin{aligned}
\int \frac{A + B \sec(c + dx)}{\sqrt{\cos(c + dx)} (a + b \sec(c + dx))^3} dx &= \int \frac{\cos^{\frac{3}{2}}(c + dx)(B + A \cos(c + dx))}{(b + a \cos(c + dx))^3} dx \\
&= \frac{b(Ab - aB)\sqrt{\cos(c + dx)} \sin(c + dx)}{2a(a^2 - b^2)d(b + a \cos(c + dx))^2} + \int \frac{\frac{1}{2}b(Ab - aB) - 2a(Ab - aB) \cos(c + dx)}{\sqrt{\cos(c + dx)}(b + a \cos(c + dx))} dx \\
&= \frac{b(Ab - aB)\sqrt{\cos(c + dx)} \sin(c + dx)}{2a(a^2 - b^2)d(b + a \cos(c + dx))^2} - \frac{(9a^2Ab - 3Ab^3 - 5a^3B - ab^2)}{4a(a^2 - b^2)^2 d(b + a \cos(c + dx))} \\
&= \frac{b(Ab - aB)\sqrt{\cos(c + dx)} \sin(c + dx)}{2a(a^2 - b^2)d(b + a \cos(c + dx))^2} - \frac{(9a^2Ab - 3Ab^3 - 5a^3B - ab^2)}{4a(a^2 - b^2)^2 d(b + a \cos(c + dx))} \\
&= \frac{(9a^2Ab - 3Ab^3 - 5a^3B - ab^2B) E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{4a^2(a^2 - b^2)^2 d} + \frac{b(Ab - aB)\sqrt{\cos(c + dx)} \sin(c + dx)}{2a(a^2 - b^2)d(b + a \cos(c + dx))} \\
&= \frac{(9a^2Ab - 3Ab^3 - 5a^3B - ab^2B) E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{4a^2(a^2 - b^2)^2 d} + \frac{(8a^4A - 5a^2Ab^2 - ab^3)}{4a(a^2 - b^2)d(b + a \cos(c + dx))}
\end{aligned}$$

Mathematica [A] time = 4.85, size = 361, normalized size = 1.04

$$\frac{4 \sin(c+dx) \sqrt{\cos(c+dx)} (a(5a^3B - 9a^2Ab + ab^2B + 3Ab^3) \cos(c+dx) + b(3a^3B - 7a^2Ab + 3ab^2B + Ab^3))}{(a^2 - b^2)^2 (a \cos(c+dx) + b)^2} + \frac{16(2a^2A - 3abB + Ab^2) \left((a+b) F\left(\frac{1}{2}(c+dx) \middle| 2\right) - b \Pi\left(\frac{2a}{a+b}, \frac{1}{2}(c+dx) \middle| 2\right) \right)}{a+b}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sec[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^3), x]

[Out] ((4*Sqrt[Cos[c + d*x]]*(b*(-7*a^2*A*b + A*b^3 + 3*a^3*B + 3*a*b^2*B) + a*(-9*a^2*A*b + 3*A*b^3 + 5*a^3*B + a*b^2*B)*Cos[c + d*x])*Sin[c + d*x])/((a^2 - b^2)^2*(b + a*Cos[c + d*x])^2) + ((2*(-5*a^2*A*b - A*b^3 + a^3*B + 5*a*b^2*B)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) + (16*(2*a^2*A + A*b^2 - 3*a*b*B)*((a + b)*EllipticF[(c + d*x)/2, 2] - b*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]))/(a + b) - (2*(-9*a^2*A*b + 3*A*b^3 + 5*a^3*B + a*b^2*B)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*b*(a + b)*Ellipti

```
cF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (a^2 - 2*b^2)*EllipticPi[-(a/b), ArcSin[Sqrt[Cos[c + d*x]]], -1]*Sin[c + d*x]/(a^2*b*Sqrt[Sin[c + d*x]^2])/((a - b)^2*(a + b)^2)/(16*a*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(b \sec(dx + c) + a)^3 \sqrt{\cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^3*sqrt(cos(d*x + c))), x)
```

maple [B] time = 19.95, size = 1959, normalized size = 5.66

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3/cos(d*x+c)^(1/2),x)
```

```
[Out] -((-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A/a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2*(-3*A*b+B*a)/a^2/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))-2*b^2*(A*b-B*a)/a^3*(1/2*a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos(1/2*d*x+1/2*c)^2-a+b)^2+3/4*a^2*(a^2-3*b^2)/b^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos(1/2*d*x+1/2*c)^2-a+b)-3/8/(a+b)/(a^2-b^2)/b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
```

$$\begin{aligned} & \frac{1}{2}c)^2)^{(1/2)} * \text{EllipticF}(\cos(1/2*d*x+1/2*c), 2^{(1/2)}) * a^{-2-1/4}/(a+b)/(a^2-b^2)/b * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^4 + \sin(1/2*d*x+1/2*c)^2)^{(1/2)} * \text{EllipticF}(\cos(1/2*d*x+1/2*c), 2^{(1/2)}) * a + 7/8/(a+b)/(a^2-b^2) * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^4 + \sin(1/2*d*x+1/2*c)^2)^{(1/2)} * \text{EllipticF}(\cos(1/2*d*x+1/2*c), 2^{(1/2)}) + 3/8*a^3/b^2/(a^2-b^2)^2 * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^4 + \sin(1/2*d*x+1/2*c)^2)^{(1/2)} * \text{EllipticF}(\cos(1/2*d*x+1/2*c), 2^{(1/2)}) - 9/8*a/(a^2-b^2)^2 * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^4 + \sin(1/2*d*x+1/2*c)^2)^{(1/2)} * \text{EllipticF}(\cos(1/2*d*x+1/2*c), 2^{(1/2)}) - 3/8*a^3/b^2/(a^2-b^2)^2 * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^4 + \sin(1/2*d*x+1/2*c)^2)^{(1/2)} * \text{EllipticE}(\cos(1/2*d*x+1/2*c), 2^{(1/2)}) + 9/8*a/(a^2-b^2)^2 * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^4 + \sin(1/2*d*x+1/2*c)^2)^{(1/2)} * \text{EllipticE}(\cos(1/2*d*x+1/2*c), 2^{(1/2)}) - 3/8/(a-b)/(a+b)/(a^2-b^2)/b^2/(a^2-a*b) * a^5 * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^4 + \sin(1/2*d*x+1/2*c)^2)^{(1/2)} * \text{EllipticPi}(\cos(1/2*d*x+1/2*c), 2*a/(a-b), 2^{(1/2)}) + 3/4/(a-b)/(a+b)/(a^2-b^2)/(a^2-a*b) * a^3 * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^4 + \sin(1/2*d*x+1/2*c)^2)^{(1/2)} * \text{EllipticPi}(\cos(1/2*d*x+1/2*c), 2*a/(a-b), 2^{(1/2)}) - 15/8/(a-b)/(a+b)/(a^2-b^2) * b^2/(a^2-a*b) * a * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^4 + \sin(1/2*d*x+1/2*c)^2)^{(1/2)} * \text{EllipticPi}(\cos(1/2*d*x+1/2*c), 2*a/(a-b), 2^{(1/2)})) + 2/a^3 * b * (3*A*b - 2*B*a) * (a^2/b/(a^2-b^2) * \cos(1/2*d*x+1/2*c) * (-2*\sin(1/2*d*x+1/2*c)^4 + \sin(1/2*d*x+1/2*c)^2)^{(1/2)} / (2*a*\cos(1/2*d*x+1/2*c)^2 - a + b) - 1/2/(a+b)/b * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^4 + \sin(1/2*d*x+1/2*c)^2)^{(1/2)} * \text{EllipticF}(\cos(1/2*d*x+1/2*c), 2^{(1/2)}) + 1/2*a/b/(a^2-b^2) * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^4 + \sin(1/2*d*x+1/2*c)^2)^{(1/2)} * \text{EllipticF}(\cos(1/2*d*x+1/2*c), 2^{(1/2)}) - 1/2*a/b/(a^2-b^2) * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^4 + \sin(1/2*d*x+1/2*c)^2)^{(1/2)} * \text{EllipticE}(\cos(1/2*d*x+1/2*c), 2^{(1/2)}) - 1/2/b/(a^2-b^2)/(a^2-a*b) * a^3 * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^4 + \sin(1/2*d*x+1/2*c)^2)^{(1/2)} * \text{EllipticPi}(\cos(1/2*d*x+1/2*c), 2*a/(a-b), 2^{(1/2)}) + 3/2*b/(a^2-b^2)/(a^2-a*b) * a * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^4 + \sin(1/2*d*x+1/2*c)^2)^{(1/2)} * \text{EllipticPi}(\cos(1/2*d*x+1/2*c), 2*a/(a-b), 2^{(1/2)})) / \sin(1/2*d*x+1/2*c) / (2*\cos(1/2*d*x+1/2*c)^2 - 1)^{(1/2)} / d \end{aligned}$$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm

="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\sqrt{\cos(c+dx)} \left(a + \frac{b}{\cos(c+dx)}\right)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))^3), x)

[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))^3), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))**3/cos(d*x+c)**(1/2), x)

[Out] Timed out

$$3.590 \quad \int \frac{A+B \sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b \sec(c+dx))^3} dx$$

Optimal. Leaf size=338

$$\frac{(Ab - aB) \sin(c + dx) \sqrt{\cos(c + dx)}}{2d(a^2 - b^2)(a \cos(c + dx) + b)^2} - \frac{(-3a^3B + 7a^2Ab - 3ab^2B - Ab^3) F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{4a^2d(a^2 - b^2)^2} - \frac{(a^3(-B) + 5a^2Ab - 5ab^2A + Ab^3) E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{4abd(a^2 - b^2)^2} + \frac{(10a^2A - 10a^2B + 10a^2b^2) \operatorname{EllipticPi}\left(\frac{2a}{a+b}, \frac{1}{2}(c + dx) \middle| 2\right)}{4abd(a^2 - b^2)^2}$$

[Out] $-1/4*(5*A*a^2*b+A*b^3-B*a^3-5*B*a*b^2)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/a/b/(a^2-b^2)^2/d-1/4*(7*A*a^2*b-A*b^3-3*B*a^3-3*B*a*b^2)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/a^2/(a^2-b^2)^2/d+1/4*(3*A*a^4*b+10*A*a^2*b^3-A*b^5+B*a^5-10*B*a^3*b^2-3*B*a*b^4)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticPi}(\sin(1/2*d*x+1/2*c), 2*a/(a+b), 2^{(1/2)})/a^2/(a-b)^2/b/(a+b)^3/d-1/2*(A*b-B*a)*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/(a^2-b^2)/d/(b+a*\cos(d*x+c))^2+1/4*(5*A*a^2*b+A*b^3-B*a^3-5*B*a*b^2)*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/b/(a^2-b^2)^2/d/(b+a*\cos(d*x+c))$

Rubi [A] time = 1.01, antiderivative size = 338, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {2954, 2999, 3055, 3059, 2639, 3002, 2641, 2805}

$$\frac{(7a^2Ab - 3a^3B - 3ab^2B - Ab^3) F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{4a^2d(a^2 - b^2)^2} - \frac{(5a^2Ab + a^3(-B) - 5ab^2B + Ab^3) E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{4abd(a^2 - b^2)^2} + \frac{(10a^2A - 10a^2B + 10a^2b^2) \operatorname{EllipticPi}\left(\frac{2a}{a+b}, \frac{1}{2}(c + dx) \middle| 2\right)}{4abd(a^2 - b^2)^2}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(A + B*\operatorname{Sec}[c + d*x])/(\operatorname{Cos}[c + d*x]^{(3/2)}*(a + b*\operatorname{Sec}[c + d*x])^3), x]$

[Out] $-((5*a^2*A*b + A*b^3 - a^3*B - 5*a*b^2*B)*\operatorname{EllipticE}[(c + d*x)/2, 2])/(4*a*b*(a^2 - b^2)^2*d) - ((7*a^2*A*b - A*b^3 - 3*a^3*B - 3*a*b^2*B)*\operatorname{EllipticF}[(c + d*x)/2, 2])/(4*a^2*(a^2 - b^2)^2*d) + ((3*a^4*A*b + 10*a^2*A*b^3 - A*b^5 + a^5*B - 10*a^3*b^2*B - 3*a*b^4*B)*\operatorname{EllipticPi}[(2*a)/(a + b), (c + d*x)/2, 2])/(4*a^2*(a - b)^2*b*(a + b)^3*d) - ((A*b - a*B)*\operatorname{Sqrt}[\operatorname{Cos}[c + d*x]]*\operatorname{Sin}[c + d*x])/(2*(a^2 - b^2)*d*(b + a*\operatorname{Cos}[c + d*x])^2) + ((5*a^2*A*b + A*b^3 - a^3*B - 5*a*b^2*B)*\operatorname{Sqrt}[\operatorname{Cos}[c + d*x]]*\operatorname{Sin}[c + d*x])/(4*b*(a^2 - b^2)^2*d*(b + a*\operatorname{Cos}[c + d*x]))$

Rule 2639

$\operatorname{Int}[\operatorname{Sqrt}[\sin[(c_.) + (d_.)*(x_.)]], x_Symbol] \rightarrow \operatorname{Simp}[(2*\operatorname{EllipticE}[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; \operatorname{FreeQ}\{c, d, x\}$

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_.)], x_Symbol] := Simp[(2*EllipticF[(1*(c -
  Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
  + (f_.)*(x_.)]), x_Symbol] := Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
  /2 + f*x))/2, (2*d)/(c + d)]/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
  , d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
  0] && GtQ[c + d, 0]
```

Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_.)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*
  (d_.) + (c_.))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_.)]^(p_.), x_Symbol] := Dis
  t[g^(m + n), Int[(g*SIN[e + f*x])^(p - m - n)*(b + a*SIN[e + f*x])^m*(d + c
  *SIN[e + f*x])^n, x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
  a*d, 0] && !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 2999

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^(m_.)*((A_.) + (B_.)*sin[(e_.) +
  (f_.)*(x_.)]*(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]^(n_.), x_Symbol] := Si
  mp[((B*a - A*b)*Cos[e + f*x]*(a + b*SIN[e + f*x])^(m + 1)*(c + d*SIN[e + f*
  x])^n)/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a +
  b*SIN[e + f*x])^(m + 1)*(c + d*SIN[e + f*x])^(n - 1)*Simp[c*(a*A - b*B)*(m
  + 1) + d*n*(A*b - a*B) + (d*(a*A - b*B)*(m + 1) - c*(A*b - a*B)*(m + 2))*S
  in[e + f*x] - d*(A*b - a*B)*(m + n + 2)*SIN[e + f*x]^2, x], x] /; FreeQ
  [{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
  NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 3002

```
Int((((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^(m_.)*((A_.) + (B_.)*sin[(e_.)
  + (f_.)*(x_.)]))/(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)], x_Symbol] := Dist[
  B/d, Int[(a + b*SIN[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
  n[e + f*x])^m/(c + d*SIN[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
  , m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int((((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^(m_.)*((c_.) + (d_.)*sin[(e_.) +
  (f_.)*(x_.)]^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)] + (C_.)*sin[(e_.)
```

```

+ (f_.)*(x_)^2), x_Symbol] := -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
- a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
+ b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]
) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || E
qQ[a, 0])))

```

Rule 3059

```

Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] := Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]

```

Rubi steps

$$\begin{aligned}
\int \frac{A + B \sec(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(a + b \sec(c + dx))^3} dx &= \int \frac{\sqrt{\cos(c + dx)}(B + A \cos(c + dx))}{(b + a \cos(c + dx))^3} dx \\
&= -\frac{(Ab - aB)\sqrt{\cos(c + dx)} \sin(c + dx)}{2(a^2 - b^2)d(b + a \cos(c + dx))^2} + \frac{\int \frac{\frac{1}{2}(-Ab + aB) + 2(aA - bB) \cos(c + dx) - \frac{1}{2}}{\sqrt{\cos(c + dx)}(b + a \cos(c + dx))} dx}{2(a^2 - b^2)} \\
&= -\frac{(Ab - aB)\sqrt{\cos(c + dx)} \sin(c + dx)}{2(a^2 - b^2)d(b + a \cos(c + dx))^2} + \frac{(5a^2Ab + Ab^3 - a^3B - 5ab^2B)}{4b(a^2 - b^2)^2 d(b + a \cos(c + dx))} \\
&= -\frac{(Ab - aB)\sqrt{\cos(c + dx)} \sin(c + dx)}{2(a^2 - b^2)d(b + a \cos(c + dx))^2} + \frac{(5a^2Ab + Ab^3 - a^3B - 5ab^2B)}{4b(a^2 - b^2)^2 d(b + a \cos(c + dx))} \\
&= -\frac{(5a^2Ab + Ab^3 - a^3B - 5ab^2B) E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{4ab(a^2 - b^2)^2 d} - \frac{(Ab - aB)\sqrt{\cos(c + dx)} \sin(c + dx)}{2(a^2 - b^2)d(b + a \cos(c + dx))} \\
&= -\frac{(5a^2Ab + Ab^3 - a^3B - 5ab^2B) E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{4ab(a^2 - b^2)^2 d} - \frac{(7a^2Ab - Ab^3 - 3a^3B)}{4a^2}
\end{aligned}$$

Mathematica [A] time = 4.93, size = 364, normalized size = 1.08

$$\frac{2 \sin(c+dx) \sqrt{\cos(c+dx)} (b(a^3B+3a^2Ab-7ab^2B+3Ab^3)-a(a^3B-5a^2Ab+5ab^2B-Ab^3) \cos(c+dx))}{(a^2-b^2)^2 (a \cos(c+dx)+b)^2} + \frac{8b(a^2B-3aAb+2b^2B) \left((a+b)F\left(\frac{1}{2}(c+dx) \middle| 2\right) - b\Pi\left(\frac{2a}{a+b}; \frac{1}{2}(c+dx) \right) \right)}{a(a+b)}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^3), x]

[Out] ((2*Sqrt[Cos[c + d*x]]*(b*(3*a^2*A*b + 3*A*b^3 + a^3*B - 7*a*b^2*B) - a*(-5*a^2*A*b - A*b^3 + a^3*B + 5*a*b^2*B)*Cos[c + d*x])*Sin[c + d*x])/((a^2 - b^2)^2*(b + a*Cos[c + d*x])^2) + (((a^2*A*b + 5*A*b^3 + 3*a^3*B - 9*a*b^2*B)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) + (8*b*(-3*a*A*b + a^2*B + 2*b^2*B)*((a + b)*EllipticF[(c + d*x)/2, 2] - b*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]))/(a*(a + b)) + ((-5*a^2*A*b - A*b^3 + a^3*B + 5*a*b^2*B)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*b*(a + b)*EllipticF

$[\text{ArcSin}[\text{Sqrt}[\text{Cos}[c + d*x]]], -1] + (a^2 - 2*b^2)*\text{EllipticPi}[-(a/b), \text{ArcSin}[\text{Sqrt}[\text{Cos}[c + d*x]]], -1]*\text{Sin}[c + d*x]/(a^2*b*\text{Sqrt}[\text{Sin}[c + d*x]^2])/((a - b)^2*(a + b)^2)/(8*b*d)$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(b \sec(dx + c) + a)^3 \cos(dx + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^3*cos(d*x + c)^(3/2)), x)

maple [B] time = 20.36, size = 1872, normalized size = 5.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x)

[Out] $-\left(-\left(-2*\cos\left(\frac{1}{2}*d*x+\frac{1}{2}*c\right)^2+1\right)*\sin\left(\frac{1}{2}*d*x+\frac{1}{2}*c\right)^2\right)^{\frac{1}{2}}*(-2*A/a/(a^2-a*b)*\left(\sin\left(\frac{1}{2}*d*x+\frac{1}{2}*c\right)^2\right)^{\frac{1}{2}}*(-2*\cos\left(\frac{1}{2}*d*x+\frac{1}{2}*c\right)^2+1)^{\frac{1}{2}}/(-2*\sin\left(\frac{1}{2}*d*x+\frac{1}{2}*c\right)^4+\sin\left(\frac{1}{2}*d*x+\frac{1}{2}*c\right)^2)^{\frac{1}{2}}*\text{EllipticPi}\left(\cos\left(\frac{1}{2}*d*x+\frac{1}{2}*c\right), 2*a/(a-b), 2^{\frac{1}{2}}\right)+2*(A*b-B*a)*b/a^2*(1/2*a^2/b/(a^2-b^2)*\cos\left(\frac{1}{2}*d*x+\frac{1}{2}*c\right)*\left(-2*\sin\left(\frac{1}{2}*d*x+\frac{1}{2}*c\right)^4+\sin\left(\frac{1}{2}*d*x+\frac{1}{2}*c\right)^2\right)^{\frac{1}{2}}/(2*a*\cos\left(\frac{1}{2}*d*x+\frac{1}{2}*c\right)^2-a+b)^2+3/4*a^2*(a^2-3*b^2)/b^2/(a^2-b^2)^2*\cos\left(\frac{1}{2}*d*x+\frac{1}{2}*c\right)*\left(-2*\sin\left(\frac{1}{2}*d*x+\frac{1}{2}*c\right)^4+\sin\left(\frac{1}{2}*d*x+\frac{1}{2}*c\right)^2\right)^{\frac{1}{2}}/(2*a*\cos\left(\frac{1}{2}*d*x+\frac{1}{2}*c\right)^2-a+b)-3/8/(a+b)/(a^2-b^2)/b^2*\left(\sin\left(\frac{1}{2}*d*x+\frac{1}{2}*c\right)^2\right)^{\frac{1}{2}}*(-2*\cos\left(\frac{1}{2}*d*x+\frac{1}{2}*c\right)^2+1)^{\frac{1}{2}}/(-2*\sin\left(\frac{1}{2}*d*x+\frac{1}{2}*c\right)^4+\sin\left(\frac{1}{2}*d*x+\frac{1}{2}*c\right)^2)^{\frac{1}{2}}*\text{EllipticF}\left(\cos\left(\frac{1}{2}*d*x+\frac{1}{2}*c\right), 2^{\frac{1}{2}}\right)*a^2-1/4/(a+b)/(a^2-b^2)/b*\left(\sin\left(\frac{1}{2}*d*x+\frac{1}{2}*c\right)^2\right)^{\frac{1}{2}}*(-2*\cos\left(\frac{1}{2}*d*x+\frac{1}{2}*c\right)^2+1)^{\frac{1}{2}}/(-2*\sin\left(\frac{1}{2}*d*x+\frac{1}{2}*c\right)^4+\sin\left(\frac{1}{2}*d*x+\frac{1}{2}*c\right)^2)^{\frac{1}{2}}*\text{EllipticPi}\left(\cos\left(\frac{1}{2}*d*x+\frac{1}{2}*c\right), 2*a/(a-b), 2^{\frac{1}{2}}\right)$

$$\begin{aligned}
& d*x+1/2*c)^2)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c), 2^{(1/2)})*a+7/8/(a+b)/(a^2- \\
& b^2)*(sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin \\
& (1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c), \\
& 2^{(1/2)})+3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d \\
& *x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^{(1/2)}*E \\
& llipticF(\cos(1/2*d*x+1/2*c), 2^{(1/2)})-9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^ \\
& 2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+sin(1/2 \\
& *d*x+1/2*c)^2)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c), 2^{(1/2)})-3/8*a^3/b^2/(a^2 \\
& -b^2)^2*(sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2* \\
& sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2* \\
& c), 2^{(1/2)})+9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+ \\
& 1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^{(1/2)}*Elli \\
& pticE(\cos(1/2*d*x+1/2*c), 2^{(1/2)})-3/8/(a-b)/(a+b)/(a^2-b^2)/b^2/(a^2-a*b)*a \\
& ^5*(sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1 \\
& /2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticPi(\cos(1/2*d*x+1/2*c), 2 \\
& *a/(a-b), 2^{(1/2)})+3/4/(a-b)/(a+b)/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2* \\
& c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+sin(\\
& 1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticPi(\cos(1/2*d*x+1/2*c), 2*a/(a-b), 2^{(1/2)})-15 \\
& /8/(a-b)/(a+b)/(a^2-b^2)*b^2/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*c \\
& os(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2) \\
& ^{(1/2)}*EllipticPi(\cos(1/2*d*x+1/2*c), 2*a/(a-b), 2^{(1/2)}))+2*(-2*A*b+B*a)/a^2 \\
& *(a^2/b/(a^2-b^2)*\cos(1/2*d*x+1/2*c)*(-2*\sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1 \\
& /2*c)^2)^{(1/2)}/(2*a*\cos(1/2*d*x+1/2*c)^2-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2* \\
& c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+sin(\\
& 1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c), 2^{(1/2)})+1/2*a/b/(a^2- \\
& b^2)*(sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin \\
& (1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c), \\
& 2^{(1/2)})-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2 \\
& *c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^{(1/2)}*Ellipti \\
& cE(\cos(1/2*d*x+1/2*c), 2^{(1/2)})-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1 \\
& /2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+s \\
& in(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticPi(\cos(1/2*d*x+1/2*c), 2*a/(a-b), 2^{(1/2)}) \\
& +3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1 \\
& /2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^{(1/2)}*Ellip \\
& ticPi(\cos(1/2*d*x+1/2*c), 2*a/(a-b), 2^{(1/2)})))/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2 \\
& *d*x+1/2*c)^2-1)^{(1/2)}/d
\end{aligned}$$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x, algorithm
="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\cos(c+dx)^{3/2} \left(a + \frac{b}{\cos(c+dx)}\right)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^3), x)

[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^3), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(3/2)/(a+b*sec(d*x+c))**3, x)

[Out] Timed out

$$3.591 \quad \int \frac{A+B \sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b \sec(c+dx))^3} dx$$

Optimal. Leaf size=342

$$\frac{a(Ab - aB) \sin(c + dx) \sqrt{\cos(c + dx)}}{2bd(a^2 - b^2)(a \cos(c + dx) + b)^2} + \frac{(a^3B + 3a^2Ab - 7ab^2B + 3Ab^3) F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{4abd(a^2 - b^2)^2} + \frac{(3a^3B + a^2Ab - 9ab^2B - 3Ab^3) E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{4b^2d(a^2 - b^2)^2}$$

[Out] $\frac{1}{4} * (A * a^2 * b + 5 * A * b^3 + 3 * B * a^3 - 9 * B * a * b^2) * (\cos(1/2 * d * x + 1/2 * c))^2)^{(1/2)} / \cos(1/2 * d * x + 1/2 * c) * \text{EllipticE}(\sin(1/2 * d * x + 1/2 * c), 2^{(1/2)}) / b^2 / (a^2 - b^2)^2 / d + 1/4 * (3 * A * a^2 * b + 3 * A * b^3 + B * a^3 - 7 * B * a * b^2) * (\cos(1/2 * d * x + 1/2 * c))^2)^{(1/2)} / \cos(1/2 * d * x + 1/2 * c) * \text{EllipticF}(\sin(1/2 * d * x + 1/2 * c), 2^{(1/2)}) / a / b / (a^2 - b^2)^2 / d + 1/4 * (A * a^4 * b - 10 * A * a^2 * b^3 - 3 * A * b^5 + 3 * B * a^5 - 6 * B * a^3 * b^2 + 15 * B * a * b^4) * (\cos(1/2 * d * x + 1/2 * c))^2)^{(1/2)} / \cos(1/2 * d * x + 1/2 * c) * \text{EllipticPi}(\sin(1/2 * d * x + 1/2 * c), 2 * a / (a + b), 2^{(1/2)}) / a / (a - b)^2 / b^2 / (a + b)^3 / d + 1/2 * a * (A * b - B * a) * \sin(d * x + c) * \cos(d * x + c)^{(1/2)} / b / (a^2 - b^2) / d / (b + a * \cos(d * x + c))^2 - 1/4 * a * (A * a^2 * b + 5 * A * b^3 + 3 * B * a^3 - 9 * B * a * b^2) * \sin(d * x + c) * \cos(d * x + c)^{(1/2)} / b^2 / (a^2 - b^2)^2 / d / (b + a * \cos(d * x + c))$

Rubi [A] time = 1.10, antiderivative size = 342, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {2954, 3000, 3055, 3059, 2639, 3002, 2641, 2805}

$$\frac{(3a^2Ab + a^3B - 7ab^2B + 3Ab^3) F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{4abd(a^2 - b^2)^2} + \frac{(a^2Ab + 3a^3B - 9ab^2B + 5Ab^3) E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{4b^2d(a^2 - b^2)^2} + \frac{(-10a^2Ab^3 - 3a^3B + 7ab^2B - 3Ab^3) F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{4b^2d(a^2 - b^2)^2}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^3),x]

[Out] $((a^2 * A * b + 5 * A * b^3 + 3 * a^3 * B - 9 * a * b^2 * B) * \text{EllipticE}[(c + d * x) / 2, 2]) / (4 * b^2 * (a^2 - b^2)^2 * d) + ((3 * a^2 * A * b + 3 * A * b^3 + a^3 * B - 7 * a * b^2 * B) * \text{EllipticF}[(c + d * x) / 2, 2]) / (4 * a * b * (a^2 - b^2)^2 * d) + ((a^4 * A * b - 10 * a^2 * A * b^3 - 3 * A * b^5 + 3 * a^5 * B - 6 * a^3 * b^2 * B + 15 * a * b^4 * B) * \text{EllipticPi}[(2 * a) / (a + b), (c + d * x) / 2, 2]) / (4 * a * (a - b)^2 * b^2 * (a + b)^3 * d) + (a * (A * b - a * B) * \text{Sqrt}[\text{Cos}[c + d * x]] * \text{Sin}[c + d * x]) / (2 * b * (a^2 - b^2) * d * (b + a * \text{Cos}[c + d * x])^2) - (a * (a^2 * A * b + 5 * A * b^3 + 3 * a^3 * B - 9 * a * b^2 * B) * \text{Sqrt}[\text{Cos}[c + d * x]] * \text{Sin}[c + d * x]) / (4 * b^2 * (a^2 - b^2)^2 * d * (b + a * \text{Cos}[c + d * x]))$

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c -
Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])], x_Symbol] := Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)]/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_.))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)]^(p_.), x_Symbol] := Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
a*d, 0] && !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
+ n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])], x_Symbol] := Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] := -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
- a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
+ b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]
) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || E
qQ[a, 0])))

```

Rule 3059

```

Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] := Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]

```

Rubi steps

$$\begin{aligned}
\int \frac{A + B \sec(c + dx)}{\cos^{\frac{5}{2}}(c + dx)(a + b \sec(c + dx))^3} dx &= \int \frac{B + A \cos(c + dx)}{\sqrt{\cos(c + dx)} (b + a \cos(c + dx))^3} dx \\
&= \frac{a(Ab - aB)\sqrt{\cos(c + dx)} \sin(c + dx)}{2b(a^2 - b^2)d(b + a \cos(c + dx))^2} - \frac{\int \frac{\frac{1}{2}(-aAb - 3a^2B + 4b^2B) + 2b(Ab - aB)c}{\sqrt{\cos(c + dx)}(b + a \cos(c + dx))} dx}{2b(a^2 - b^2)d} \\
&= \frac{a(Ab - aB)\sqrt{\cos(c + dx)} \sin(c + dx)}{2b(a^2 - b^2)d(b + a \cos(c + dx))^2} - \frac{a(a^2Ab + 5Ab^3 + 3a^3B - 9ab^2B)}{4b^2(a^2 - b^2)^2d} \\
&= \frac{a(Ab - aB)\sqrt{\cos(c + dx)} \sin(c + dx)}{2b(a^2 - b^2)d(b + a \cos(c + dx))^2} - \frac{a(a^2Ab + 5Ab^3 + 3a^3B - 9ab^2B)}{4b^2(a^2 - b^2)^2d} \\
&= \frac{(a^2Ab + 5Ab^3 + 3a^3B - 9ab^2B) E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{4b^2(a^2 - b^2)^2d} + \frac{a(Ab - aB)\sqrt{\cos(c + dx)} \sin(c + dx)}{2b(a^2 - b^2)d(b + a \cos(c + dx))^2} \\
&= \frac{(a^2Ab + 5Ab^3 + 3a^3B - 9ab^2B) E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{4b^2(a^2 - b^2)^2d} + \frac{(3a^2Ab + 3Ab^3 + 3a^3B - 9ab^2B) F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{(a - b)^2(a + b)^2}
\end{aligned}$$

Mathematica [A] time = 5.14, size = 383, normalized size = 1.12

$$\frac{8b(a^3B + a^2Ab - 4ab^2B + 2Ab^3) \left((a+b)F\left(\frac{1}{2}(c+dx) \middle| 2\right) - b\Pi\left(\frac{2a}{a+b}; \frac{1}{2}(c+dx) \middle| 2\right) \right) + (3a^3B + a^2Ab - 9ab^2B + 5Ab^3) \sin(c+dx) \left((a^2 - 2b^2)\Pi\left(-\frac{a}{b}; \sin^{-1}\left(\frac{\sqrt{\cos(c+dx)}}{b}\right) \middle| -1\right) + 2b(a+b)F\left(\sin^{-1}\left(\frac{\sqrt{\cos(c+dx)}}{b}\right) \middle| 2\right) \right)}{a(a+b) \sqrt{\sin^2(c+dx)} (a-b)^2(a+b)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^3), x]

[Out] ((-2*a*Sqrt[Cos[c + d*x]])*(b*(-(a^2*A*b) + 7*A*b^3 + 5*a^3*B - 11*a*b^2*B) + a*(a^2*A*b + 5*A*b^3 + 3*a^3*B - 9*a*b^2*B)*Cos[c + d*x])*Sin[c + d*x])/((a^2 - b^2)^2*(b + a*Cos[c + d*x])^2) + (((3*a^3*A*b - 9*a*A*b^3 + 9*a^4*B - 19*a^2*b^2*B + 16*b^4*B)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) + (8*b*(a^2*A*b + 2*A*b^3 + a^3*B - 4*a*b^2*B))*((a + b)*EllipticF[(c + d*x)/2, 2] - b*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]))/(a*(a + b)) + ((a^2*A*b + 5*A*b^3 + 3*a^3*B - 9*a*b^2*B)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]]])/(a - b)^2(a + b)^2)

+ d*x]]], -1] + 2*b*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (a^2 - 2*b^2)*EllipticPi[-(a/b), ArcSin[Sqrt[Cos[c + d*x]]], -1]*Sin[c + d*x])/((a*b*Sqrt[Sin[c + d*x]^2]))/((a - b)^2*(a + b)^2)/(8*b^2*d)

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^3,x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(b \sec(dx + c) + a)^3 \cos(dx + c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^3,x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^3*cos(d*x + c)^(5/2)), x)

maple [B] time = 19.44, size = 1768, normalized size = 5.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^3,x)

[Out] -((-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*(-A*b+B*a)/a*(1/2*a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos(1/2*d*x+1/2*c)^2-a+b)^2+3/4*a^2*(a^2-3*b^2)/b^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos(1/2*d*x+1/2*c)^2-a+b)-3/8/(a+b)/(a^2-b^2)/b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-1/4/(a+b)/(a^2-b^2)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a+7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)

$$\begin{aligned}
&)^2)^{(1/2)} * \text{EllipticF}(\cos(1/2*d*x+1/2*c), 2^{(1/2)}) + 3/8*a^3/b^2/(a^2-b^2)^2 * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^4 + \sin(1/2*d*x+1/2*c)^2)^{(1/2)} * \text{EllipticF}(\cos(1/2*d*x+1/2*c), 2^{(1/2)}) \\
&) - 9/8*a/(a^2-b^2)^2 * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^4 + \sin(1/2*d*x+1/2*c)^2)^{(1/2)} * \text{EllipticF}(\cos(1/2*d*x+1/2*c), 2^{(1/2)}) - 3/8*a^3/b^2/(a^2-b^2)^2 * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} \\
&) * (-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^4 + \sin(1/2*d*x+1/2*c)^2)^{(1/2)} * \text{EllipticE}(\cos(1/2*d*x+1/2*c), 2^{(1/2)}) + 9/8*a/(a^2-b^2)^2 * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^4 + \sin(1/2*d*x+1/2*c)^2)^{(1/2)} * \text{EllipticE}(\cos(1/2*d*x+1/2*c), 2^{(1/2)}) - 3/8/(a-b)/(a+b)/(a^2-b^2)/b^2/(a^2-a*b)*a^5 * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^4 + \sin(1/2*d*x+1/2*c)^2)^{(1/2)} * \text{EllipticPi}(\cos(1/2*d*x+1/2*c), 2*a/(a-b), 2^{(1/2)}) + 3/4/(a-b)/(a+b)/(a^2-b^2)/(a^2-a*b)*a^3 * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^4 + \sin(1/2*d*x+1/2*c)^2)^{(1/2)} * \text{EllipticPi}(\cos(1/2*d*x+1/2*c), 2*a/(a-b), 2^{(1/2)}) - 15/8/(a-b)/(a+b)/(a^2-b^2)*b^2/(a^2-a*b)*a * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^4 + \sin(1/2*d*x+1/2*c)^2)^{(1/2)} * \text{EllipticPi}(\cos(1/2*d*x+1/2*c), 2*a/(a-b), 2^{(1/2)})) + 2*A/a*(a^2/b/(a^2-b^2)*\cos(1/2*d*x+1/2*c)*(-2*\sin(1/2*d*x+1/2*c)^4 + \sin(1/2*d*x+1/2*c)^2)^{(1/2)} / (2*a*\cos(1/2*d*x+1/2*c)^2-a+b) - 1/2/(a+b)/b * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^4 + \sin(1/2*d*x+1/2*c)^2)^{(1/2)} * \text{EllipticF}(\cos(1/2*d*x+1/2*c), 2^{(1/2)}) + 1/2*a/b/(a^2-b^2) * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^4 + \sin(1/2*d*x+1/2*c)^2)^{(1/2)} * \text{EllipticF}(\cos(1/2*d*x+1/2*c), 2^{(1/2)}) - 1/2*a/b/(a^2-b^2) * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^4 + \sin(1/2*d*x+1/2*c)^2)^{(1/2)} * \text{EllipticE}(\cos(1/2*d*x+1/2*c), 2^{(1/2)}) - 1/2/b/(a^2-b^2)/(a^2-a*b)*a^3 * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^4 + \sin(1/2*d*x+1/2*c)^2)^{(1/2)} * \text{EllipticPi}(\cos(1/2*d*x+1/2*c), 2*a/(a-b), 2^{(1/2)}) + 3/2*b/(a^2-b^2)/(a^2-a*b)*a * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^4 + \sin(1/2*d*x+1/2*c)^2)^{(1/2)} * \text{EllipticPi}(\cos(1/2*d*x+1/2*c), 2*a/(a-b), 2^{(1/2)})))/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d
\end{aligned}$$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^3,x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\cos(c+dx)^{5/2} \left(a + \frac{b}{\cos(c+dx)}\right)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))^3), x)

[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))^3), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(5/2)/(a+b*sec(d*x+c))**3, x)

[Out] Timed out

$$3.592 \quad \int \frac{A+B \sec(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+b \sec(c+dx))^3} dx$$

Optimal. Leaf size=420

$$\frac{a(Ab - aB) \sin(c + dx)}{2bd(a^2 - b^2) \sqrt{\cos(c + dx)} (a \cos(c + dx) + b)^2} + \frac{(-5a^3B + a^2Ab + 11ab^2B - 7Ab^3) F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{4b^2d(a^2 - b^2)^2} + \frac{a(-5a^3B + a^2Ab + 11ab^2B - 7Ab^3) E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{4b^2d(a^2 - b^2)^2}$$

[Out] $1/4*(3*A*a^3*b-9*A*a*b^3-15*B*a^4+29*B*a^2*b^2-8*B*b^4)*(cos(1/2*d*x+1/2*c)^2)^{(1/2)}/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^{(1/2)})/b^3/(a^2-b^2)^{2/d}+1/4*(A*a^2*b-7*A*b^3-5*B*a^3+11*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^{(1/2)}/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^{(1/2)})/b^2/(a^2-b^2)^{2/d}+1/4*(3*A*a^4*b-6*A*a^2*b^3+15*A*b^5-15*B*a^5+38*B*a^3*b^2-35*B*a*b^4)*(cos(1/2*d*x+1/2*c)^2)^{(1/2)}/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^{(1/2)})/(a-b)^2/b^3/(a+b)^3/d-1/4*(3*A*a^3*b-9*A*a*b^3-15*B*a^4+29*B*a^2*b^2-8*B*b^4)*sin(d*x+c)/b^3/(a^2-b^2)^{2/d}/cos(d*x+c)^{(1/2)}+1/2*a*(A*b-B*a)*sin(d*x+c)/b/(a^2-b^2)/d/(b+a*cos(d*x+c))^2/cos(d*x+c)^{(1/2)}+1/4*a*(A*a^2*b-7*A*b^3-5*B*a^3+11*B*a*b^2)*sin(d*x+c)/b^2/(a^2-b^2)^{2/d}/(b+a*cos(d*x+c))/cos(d*x+c)^{(1/2)}$

Rubi [A] time = 1.48, antiderivative size = 420, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {2954, 3000, 3055, 3059, 2639, 3002, 2641, 2805}

$$\frac{(a^2Ab - 5a^3B + 11ab^2B - 7Ab^3) F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{4b^2d(a^2 - b^2)^2} + \frac{(3a^3Ab + 29a^2b^2B - 15a^4B - 9aAb^3 - 8b^4B) E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{4b^3d(a^2 - b^2)^2}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^3),x]

[Out] $((3*a^3*A*b - 9*a*A*b^3 - 15*a^4*B + 29*a^2*b^2*B - 8*b^4*B)*EllipticE[(c + d*x)/2, 2])/(4*b^3*(a^2 - b^2)^2*d) + ((a^2*A*b - 7*A*b^3 - 5*a^3*B + 11*a*b^2*B)*EllipticF[(c + d*x)/2, 2])/(4*b^2*(a^2 - b^2)^2*d) + ((3*a^4*A*b - 6*a^2*A*b^3 + 15*A*b^5 - 15*a^5*B + 38*a^3*b^2*B - 35*a*b^4*B)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(4*(a - b)^2*b^3*(a + b)^3*d) - ((3*a^3*A*b - 9*a*A*b^3 - 15*a^4*B + 29*a^2*b^2*B - 8*b^4*B)*Sin[c + d*x])/(4*b^3*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]) + (a*(A*b - a*B)*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*(b + a*cos[c + d*x])^2) + (a*(a^2*A*b - 7*A*b^3 - 5*a^3*B + 11*a*b^2*B)*Sin[c + d*x])/(4*b^2*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]])*(b + a*cos[c + d*x]))$

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_.)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_.)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2805

Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] := Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi/2 + f*x))/2, (2*d)/(c + d)]/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[c + d, 0]

Rule 2954

Int[((a_.) + csc[(e_.) + (f_.)*(x_.)]*(b_.))^m_*(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))^n_*(g_.)*sin[(e_.) + (f_.)*(x_.)]^p_, x_Symbol] := Dist[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]

Rule 3000

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^m_*(A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)]*(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]^n_, x_Symbol] := -Simp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && RationalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))

Rule 3002

Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^m_*(A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)])/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] := Dist[B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B

, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.)
+ (f_.)*(x_)^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
- a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
+ b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]
) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)])^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])], x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{A + B \sec(c + dx)}{\cos^{\frac{7}{2}}(c + dx)(a + b \sec(c + dx))^3} dx &= \int \frac{B + A \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(b + a \cos(c + dx))^3} dx \\
&= \frac{a(Ab - aB) \sin(c + dx)}{2b(a^2 - b^2) d \sqrt{\cos(c + dx)} (b + a \cos(c + dx))^2} - \int \frac{\frac{1}{2}(aAb - 5a^2B + 4b^2B) + 2b^3}{\cos^{\frac{3}{2}}(c + dx)} dx \\
&= \frac{a(Ab - aB) \sin(c + dx)}{2b(a^2 - b^2) d \sqrt{\cos(c + dx)} (b + a \cos(c + dx))^2} + \frac{a(a^2Ab - 7Ab^3 - 5b^3A)}{4b^2(a^2 - b^2)^2 d \sqrt{\cos(c + dx)}} \\
&= -\frac{(3a^3Ab - 9aAb^3 - 15a^4B + 29a^2b^2B - 8b^4B) \sin(c + dx)}{4b^3(a^2 - b^2)^2 d \sqrt{\cos(c + dx)}} + \frac{a^2Ab - 7Ab^3 - 5b^3A}{2b(a^2 - b^2)^2 d \sqrt{\cos(c + dx)}} \\
&= -\frac{(3a^3Ab - 9aAb^3 - 15a^4B + 29a^2b^2B - 8b^4B) \sin(c + dx)}{4b^3(a^2 - b^2)^2 d \sqrt{\cos(c + dx)}} + \frac{a^2Ab - 7Ab^3 - 5b^3A}{2b(a^2 - b^2)^2 d \sqrt{\cos(c + dx)}} \\
&= \frac{(3a^3Ab - 9aAb^3 - 15a^4B + 29a^2b^2B - 8b^4B) E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{4b^3(a^2 - b^2)^2 d} - \frac{(3a^3Ab - 9aAb^3 - 15a^4B + 29a^2b^2B - 8b^4B) \sin(c + dx)}{4b^3(a^2 - b^2)^2 d} \\
&= \frac{(3a^3Ab - 9aAb^3 - 15a^4B + 29a^2b^2B - 8b^4B) E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{4b^3(a^2 - b^2)^2 d} + \frac{(a^2Ab - 7Ab^3 - 5b^3A) \sin(c + dx)}{4b^3(a^2 - b^2)^2 d}
\end{aligned}$$

Mathematica [A] time = 6.18, size = 458, normalized size = 1.09

$$\frac{\sqrt{\cos(c+dx)} \left(16B(b^3 - a^2b)^2 \tan(c+dx) + a^2(15a^4B - 3a^3Ab - 29a^2b^2B + 9aAb^3 + 8b^4B) \sin(2(c+dx)) + 2ab(25a^4B - 5a^3Ab - 47a^2b^2B + 11aAb^3 + 16b^4B) \sin(c+dx) \right)}{(a^2 - b^2)^2 (a \cos(c+dx) + b)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^3), x]

[Out] (-(((((-9*a^4*A*b + 19*a^2*A*b^3 - 16*A*b^5 + 45*a^5*B - 95*a^3*b^2*B + 56*a*b^4*B)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) + (8*b*(-(a^3*A*b) + 4*a*A*b^3 + 5*a^4*B - 10*a^2*b^2*B + 2*b^4*B)*((a + b)*EllipticF[(c +

```
d*x)/2, 2] - b*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]]/(a*(a + b)) + ((
-3*a^3*A*b + 9*a*A*b^3 + 15*a^4*B - 29*a^2*b^2*B + 8*b^4*B)*(-2*a*b*Ellipti
cE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*b*(a + b)*EllipticF[ArcSin[Sqrt[Cos[
c + d*x]]], -1] + (a^2 - 2*b^2)*EllipticPi[-(a/b), ArcSin[Sqrt[Cos[c + d*x]
]], -1])*Sin[c + d*x])/(a*b*Sqrt[Sin[c + d*x]^2]))/((a - b)^2*(a + b)^2)) +
(Sqrt[Cos[c + d*x]]*(2*a*b*(-5*a^3*A*b + 11*a*A*b^3 + 25*a^4*B - 47*a^2*b^
2*B + 16*b^4*B)*Sin[c + d*x] + a^2*(-3*a^3*A*b + 9*a*A*b^3 + 15*a^4*B - 29*
a^2*b^2*B + 8*b^4*B)*Sin[2*(c + d*x)] + 16*(-(a^2*b) + b^3)^2*B*Tan[c + d*x
]))/((a^2 - b^2)^2*(b + a*Cos[c + d*x])^2))/(8*b^3*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(b \sec(dx + c) + a)^3 \cos(dx + c)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^3*cos(d*x + c)^(7/2)),
x)
```

maple [B] time = 23.81, size = 2024, normalized size = 4.82

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a^2*B/b^3/(a^
2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*
c),2*a/(a-b),2^(1/2))+2*(A*b-B*a)/b*(1/2*a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)
*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos(1/2*d*x+1/2*
```

$$\begin{aligned}
& c)^{-2-a+b})^2 + 3/4 * a^2 * (a^2 - 3 * b^2) / b^2 / (a^2 - b^2)^2 * \cos(1/2 * d * x + 1/2 * c) * (-2 * \sin(\\
& 1/2 * d * x + 1/2 * c)^4 + \sin(1/2 * d * x + 1/2 * c)^2)^{(1/2)} / (2 * a * \cos(1/2 * d * x + 1/2 * c)^2 - a + b) \\
& - 3/8 / (a + b) / (a^2 - b^2) / b^2 * (\sin(1/2 * d * x + 1/2 * c)^2)^{(1/2)} * (-2 * \cos(1/2 * d * x + 1/2 * c) \\
&)^2 + 1)^{(1/2)} / (-2 * \sin(1/2 * d * x + 1/2 * c)^4 + \sin(1/2 * d * x + 1/2 * c)^2)^{(1/2)} * \text{EllipticF} \\
& (\cos(1/2 * d * x + 1/2 * c), 2^{(1/2)}) * a^{-2-1/4} / (a + b) / (a^2 - b^2) / b * (\sin(1/2 * d * x + 1/2 * c) \\
&)^2)^{(1/2)} * (-2 * \cos(1/2 * d * x + 1/2 * c)^2 + 1)^{(1/2)} / (-2 * \sin(1/2 * d * x + 1/2 * c)^4 + \sin(1/2 \\
& * d * x + 1/2 * c)^2)^{(1/2)} * \text{EllipticF}(\cos(1/2 * d * x + 1/2 * c), 2^{(1/2)}) * a + 7/8 / (a + b) / (a^2 \\
& - b^2) * (\sin(1/2 * d * x + 1/2 * c)^2)^{(1/2)} * (-2 * \cos(1/2 * d * x + 1/2 * c)^2 + 1)^{(1/2)} / (-2 * \sin \\
& (1/2 * d * x + 1/2 * c)^4 + \sin(1/2 * d * x + 1/2 * c)^2)^{(1/2)} * \text{EllipticF}(\cos(1/2 * d * x + 1/2 * c) \\
& , 2^{(1/2)}) + 3/8 * a^3 / b^2 / (a^2 - b^2)^2 * (\sin(1/2 * d * x + 1/2 * c)^2)^{(1/2)} * (-2 * \cos(1/2 * \\
& d * x + 1/2 * c)^2 + 1)^{(1/2)} / (-2 * \sin(1/2 * d * x + 1/2 * c)^4 + \sin(1/2 * d * x + 1/2 * c)^2)^{(1/2)} * \\
& \text{EllipticF}(\cos(1/2 * d * x + 1/2 * c), 2^{(1/2)}) - 9/8 * a / (a^2 - b^2)^2 * (\sin(1/2 * d * x + 1/2 * c) \\
&)^2)^{(1/2)} * (-2 * \cos(1/2 * d * x + 1/2 * c)^2 + 1)^{(1/2)} / (-2 * \sin(1/2 * d * x + 1/2 * c)^4 + \sin(1/ \\
& 2 * d * x + 1/2 * c)^2)^{(1/2)} * \text{EllipticF}(\cos(1/2 * d * x + 1/2 * c), 2^{(1/2)}) - 3/8 * a^3 / b^2 / (a^ \\
& 2 - b^2)^2 * (\sin(1/2 * d * x + 1/2 * c)^2)^{(1/2)} * (-2 * \cos(1/2 * d * x + 1/2 * c)^2 + 1)^{(1/2)} / (-2 \\
& * \sin(1/2 * d * x + 1/2 * c)^4 + \sin(1/2 * d * x + 1/2 * c)^2)^{(1/2)} * \text{EllipticE}(\cos(1/2 * d * x + 1/2 \\
& * c), 2^{(1/2)}) + 9/8 * a / (a^2 - b^2)^2 * (\sin(1/2 * d * x + 1/2 * c)^2)^{(1/2)} * (-2 * \cos(1/2 * d * x \\
& + 1/2 * c)^2 + 1)^{(1/2)} / (-2 * \sin(1/2 * d * x + 1/2 * c)^4 + \sin(1/2 * d * x + 1/2 * c)^2)^{(1/2)} * \text{Ell} \\
& ipticE(\cos(1/2 * d * x + 1/2 * c), 2^{(1/2)}) - 3/8 / (a - b) / (a + b) / (a^2 - b^2) / b^2 / (a^2 - a * b) * \\
& a^5 * (\sin(1/2 * d * x + 1/2 * c)^2)^{(1/2)} * (-2 * \cos(1/2 * d * x + 1/2 * c)^2 + 1)^{(1/2)} / (-2 * \sin(\\
& 1/2 * d * x + 1/2 * c)^4 + \sin(1/2 * d * x + 1/2 * c)^2)^{(1/2)} * \text{EllipticPi}(\cos(1/2 * d * x + 1/2 * c), \\
& 2 * a / (a - b), 2^{(1/2)}) + 3/4 / (a - b) / (a + b) / (a^2 - b^2) / (a^2 - a * b) * a^3 * (\sin(1/2 * d * x + 1/2 \\
& * c)^2)^{(1/2)} * (-2 * \cos(1/2 * d * x + 1/2 * c)^2 + 1)^{(1/2)} / (-2 * \sin(1/2 * d * x + 1/2 * c)^4 + \sin \\
& (1/2 * d * x + 1/2 * c)^2)^{(1/2)} * \text{EllipticPi}(\cos(1/2 * d * x + 1/2 * c), 2 * a / (a - b), 2^{(1/2)}) - 1 \\
& 5/8 / (a - b) / (a + b) / (a^2 - b^2) * b^2 / (a^2 - a * b) * a * (\sin(1/2 * d * x + 1/2 * c)^2)^{(1/2)} * (-2 * \\
& \cos(1/2 * d * x + 1/2 * c)^2 + 1)^{(1/2)} / (-2 * \sin(1/2 * d * x + 1/2 * c)^4 + \sin(1/2 * d * x + 1/2 * c)^2 \\
&)^2)^{(1/2)} * \text{EllipticPi}(\cos(1/2 * d * x + 1/2 * c), 2 * a / (a - b), 2^{(1/2)}) + 2 / b^3 * B * (-(-2 * \sin \\
& (1/2 * d * x + 1/2 * c)^4 + \sin(1/2 * d * x + 1/2 * c)^2)^{(1/2)} * (\sin(1/2 * d * x + 1/2 * c)^2)^{(1/2)} * \\
& (2 * \sin(1/2 * d * x + 1/2 * c)^2 - 1)^{(1/2)} * \text{EllipticE}(\cos(1/2 * d * x + 1/2 * c), 2^{(1/2)}) + 2 * (- \\
& 2 * \sin(1/2 * d * x + 1/2 * c)^4 + \sin(1/2 * d * x + 1/2 * c)^2)^{(1/2)} * \cos(1/2 * d * x + 1/2 * c) * \sin(1 \\
& /2 * d * x + 1/2 * c)^2 / \sin(1/2 * d * x + 1/2 * c)^2 / (2 * \sin(1/2 * d * x + 1/2 * c)^2 - 1) - 2 * a * B / b^2 * \\
& (a^2 / b / (a^2 - b^2) * \cos(1/2 * d * x + 1/2 * c) * (-2 * \sin(1/2 * d * x + 1/2 * c)^4 + \sin(1/2 * d * x + 1/ \\
& 2 * c)^2)^{(1/2)} / (2 * a * \cos(1/2 * d * x + 1/2 * c)^2 - a + b) - 1/2 / (a + b) / b * (\sin(1/2 * d * x + 1/2 * c) \\
&)^2)^{(1/2)} * (-2 * \cos(1/2 * d * x + 1/2 * c)^2 + 1)^{(1/2)} / (-2 * \sin(1/2 * d * x + 1/2 * c)^4 + \sin(1 \\
& /2 * d * x + 1/2 * c)^2)^{(1/2)} * \text{EllipticF}(\cos(1/2 * d * x + 1/2 * c), 2^{(1/2)}) + 1/2 * a / b / (a^2 - b \\
& ^2) * (\sin(1/2 * d * x + 1/2 * c)^2)^{(1/2)} * (-2 * \cos(1/2 * d * x + 1/2 * c)^2 + 1)^{(1/2)} / (-2 * \sin(\\
& 1/2 * d * x + 1/2 * c)^4 + \sin(1/2 * d * x + 1/2 * c)^2)^{(1/2)} * \text{EllipticF}(\cos(1/2 * d * x + 1/2 * c), 2 \\
& ^{(1/2)}) - 1/2 * a / b / (a^2 - b^2) * (\sin(1/2 * d * x + 1/2 * c)^2)^{(1/2)} * (-2 * \cos(1/2 * d * x + 1/2 * \\
& c)^2 + 1)^{(1/2)} / (-2 * \sin(1/2 * d * x + 1/2 * c)^4 + \sin(1/2 * d * x + 1/2 * c)^2)^{(1/2)} * \text{Elliptic} \\
& E(\cos(1/2 * d * x + 1/2 * c), 2^{(1/2)}) - 1/2 / b / (a^2 - b^2) / (a^2 - a * b) * a^3 * (\sin(1/2 * d * x + 1/ \\
& 2 * c)^2)^{(1/2)} * (-2 * \cos(1/2 * d * x + 1/2 * c)^2 + 1)^{(1/2)} / (-2 * \sin(1/2 * d * x + 1/2 * c)^4 + \sin \\
& (1/2 * d * x + 1/2 * c)^2)^{(1/2)} * \text{EllipticPi}(\cos(1/2 * d * x + 1/2 * c), 2 * a / (a - b), 2^{(1/2)}) + \\
& 3/2 * b / (a^2 - b^2) / (a^2 - a * b) * a * (\sin(1/2 * d * x + 1/2 * c)^2)^{(1/2)} * (-2 * \cos(1/2 * d * x + 1/ \\
& 2 * c)^2 + 1)^{(1/2)} / (-2 * \sin(1/2 * d * x + 1/2 * c)^4 + \sin(1/2 * d * x + 1/2 * c)^2)^{(1/2)} * \text{Ellipt} \\
& icPi(\cos(1/2 * d * x + 1/2 * c), 2 * a / (a - b), 2^{(1/2)})) / \sin(1/2 * d * x + 1/2 * c) / (2 * \cos(1/2 *
\end{aligned}$$

$d*x+1/2*c)^{2-1}^{(1/2)}/d$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^3,x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\cos(c+dx)^{7/2} \left(a + \frac{b}{\cos(c+dx)}\right)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))^3), x)

[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))^3), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(7/2)/(a+b*sec(d*x+c))**3,x)

[Out] Timed out

$a*B*\sin[c + d*x])/(2*b*(a^2 - b^2)*d*\cos[c + d*x]^{(3/2)}*(b + a*\cos[c + d*x])^2) + (a*(3*a^2*A*b - 9*A*b^3 - 7*a^3*B + 13*a*b^2*B)*\sin[c + d*x])/(4*b^2*(a^2 - b^2)^2*d*\cos[c + d*x]^{(3/2)}*(b + a*\cos[c + d*x]))$

Rule 2639

$\text{Int}[\text{Sqrt}[\sin[(c_.) + (d_.)*(x_)]], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticE}[(1*(c - \text{Pi}/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 2641

$\text{Int}[1/\text{Sqrt}[\sin[(c_.) + (d_.)*(x_)]], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticF}[(1*(c - \text{Pi}/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 2805

$\text{Int}[1/(((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_)])*\text{Sqrt}[(c_.) + (d_.)*\sin[(e_.) + (f_.)*(x_)]]), x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticPi}[(2*b)/(a + b), (1*(e - \text{Pi}/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*\text{Sqrt}[c + d]), x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{GtQ}[c + d, 0]$

Rule 2954

$\text{Int}[(a_.) + \text{csc}[(e_.) + (f_.)*(x_)]*(b_.)^{(m_.)}*(\text{csc}[(e_.) + (f_.)*(x_)]*(d_.) + (c_.)^{(n_.)}*(g_.)*\sin[(e_.) + (f_.)*(x_)]^{(p_.)}, x_Symbol] \rightarrow \text{Dist}[g^{(m + n)}, \text{Int}[(g*\sin[e + f*x])^{(p - m - n)}*(b + a*\sin[e + f*x])^{(m + 1)}*(c + d*\sin[e + f*x])^n, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, p\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{IntegerQ}[p] \&\& \text{IntegerQ}[m] \&\& \text{IntegerQ}[n]$

Rule 3000

$\text{Int}[(a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_)]^{(m_.)}*((A_.) + (B_.)*\sin[(e_.) + (f_.)*(x_)]*(c_.) + (d_.)*\sin[(e_.) + (f_.)*(x_)]^{(n_.)}, x_Symbol] \rightarrow -\text{Simp}[(A*b^2 - a*b*B)*\cos[e + f*x]*(a + b*\sin[e + f*x])^{(m + 1)}*(c + d*\sin[e + f*x])^{(1 + n)})/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + \text{Dist}[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), \text{Int}[(a + b*\sin[e + f*x])^{(m + 1)}*(c + d*\sin[e + f*x])^n*\text{Simp}[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*\sin[e + f*x] - b*d*(A*b - a*B)*(m + n + 3)*\sin[e + f*x]^2, x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{RationalQ}[m] \&\& m < -1 \&\& ((\text{EqQ}[a, 0] \&\& \text{IntegerQ}[m] \&\& \text{IntegerQ}[n]) || !(\text{IntegerQ}[2*n] \&\& \text{LtQ}[n, -1] \&\& ((\text{IntegerQ}[n] \&\& \text{IntegerQ}[m]) || \text{EqQ}[a, 0])))$

Rule 3002

```

Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

```

Rule 3055

```

Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_) + (C_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] := -Simp[(((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
- a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
+ b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]
) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || E
qQ[a, 0])))

```

Rule 3059

```

Int[(((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] := Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]

```

Rubi steps

$$\begin{aligned}
\int \frac{A + B \sec(c + dx)}{\cos^{\frac{9}{2}}(c + dx)(a + b \sec(c + dx))^3} dx &= \int \frac{B + A \cos(c + dx)}{\cos^{\frac{5}{2}}(c + dx)(b + a \cos(c + dx))^3} dx \\
&= \frac{a(Ab - aB) \sin(c + dx)}{2b(a^2 - b^2) d \cos^{\frac{3}{2}}(c + dx)(b + a \cos(c + dx))^2} - \int \frac{\frac{1}{2}(3aAb - 7a^2B + 4b^2B) + \dots}{\cos} \\
&= \frac{a(Ab - aB) \sin(c + dx)}{2b(a^2 - b^2) d \cos^{\frac{3}{2}}(c + dx)(b + a \cos(c + dx))^2} + \frac{a(3a^2Ab - 9Ab^3 - \dots)}{4b^2(a^2 - b^2)^2 d \cos} \\
&= -\frac{(15a^3Ab - 33aAb^3 - 35a^4B + 61a^2b^2B - 8b^4B) \sin(c + dx)}{12b^3(a^2 - b^2)^2 d \cos^{\frac{3}{2}}(c + dx)} + \frac{\dots}{2b(a^2 - b^2)} \\
&= -\frac{(15a^3Ab - 33aAb^3 - 35a^4B + 61a^2b^2B - 8b^4B) \sin(c + dx)}{12b^3(a^2 - b^2)^2 d \cos^{\frac{3}{2}}(c + dx)} + \frac{(15a^4 - \dots)}{4b^2(a^2 - b^2)} \\
&= -\frac{(15a^3Ab - 33aAb^3 - 35a^4B + 61a^2b^2B - 8b^4B) \sin(c + dx)}{12b^3(a^2 - b^2)^2 d \cos^{\frac{3}{2}}(c + dx)} + \frac{(15a^4 - \dots)}{4b^2(a^2 - b^2)} \\
&= -\frac{(15a^4Ab - 29a^2Ab^3 + 8Ab^5 - 35a^5B + 65a^3b^2B - 24ab^4B) E\left(\frac{1}{2}(c + dx)\right)}{4b^4(a^2 - b^2)^2 d} \\
&= -\frac{(15a^4Ab - 29a^2Ab^3 + 8Ab^5 - 35a^5B + 65a^3b^2B - 24ab^4B) E\left(\frac{1}{2}(c + dx)\right)}{4b^4(a^2 - b^2)^2 d}
\end{aligned}$$

Mathematica [A] time = 7.36, size = 570, normalized size = 1.09

$$\frac{\sqrt{\cos(c + dx)} \left(\frac{a^4B \sin(c+dx) - a^3Ab \sin(c+dx)}{2b^3(b^2 - a^2)(a \cos(c+dx) + b)^2} + \frac{-11a^6B \sin(c+dx) + 7a^5Ab \sin(c+dx) + 17a^4b^2B \sin(c+dx) - 13a^3Ab^3 \sin(c+dx)}{4b^4(b^2 - a^2)^2(a \cos(c+dx) + b)} + \frac{2 \sec(c+dx)}{2b^2(a^2 - b^2)} \right)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(9/2)*(a + b*Sec[c + d*x])^3), x]

[Out] ((2*(-135*a^5*A*b + 285*a^3*A*b^3 - 168*a*A*b^5 + 315*a^6*B - 641*a^4*b^2*B + 328*a^2*b^4*B + 16*b^6*B)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) + ((-120*a^4*A*b^2 + 240*a^2*A*b^4 - 48*A*b^6 + 280*a^5*b*B - 512*a^3*b^3*B + 160*a*b^5*B)*(2*EllipticF[(c + d*x)/2, 2] - (2*b*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b)))/a + (2*(-45*a^5*A*b + 87*a^3*A*b^3 - 24*a*A*b^5 + 105*a^6*B - 195*a^4*b^2*B + 72*a^2*b^4*B)*Cos[2*(c + d*x)]*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*b*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (a^2 - 2*b^2)*EllipticPi[-(a/b), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a^2*b*Sqrt[1 - Cos[c + d*x]^2]*(-1 + 2*Cos[c + d*x]^2)))/(48*(a - b)^2*b^4*(a + b)^2*d) + (Sqrt[Cos[c + d*x]]*((2*Sec[c + d*x]*(A*b*Sin[c + d*x] - 3*a*B*Sin[c + d*x]))/b^4 + (-(a^3*A*b*Sin[c + d*x]) + a^4*B*Sin[c + d*x])/(2*b^3*(-a^2 + b^2)*(b + a*Cos[c + d*x])^2) + (7*a^5*A*b*Sin[c + d*x] - 13*a^3*A*b^3*Sin[c + d*x] - 11*a^6*B*Sin[c + d*x] + 17*a^4*b^2*B*Sin[c + d*x])/(4*b^4*(-a^2 + b^2)^2*(b + a*Cos[c + d*x])) + (2*B*Sec[c + d*x]*Tan[c + d*x])/(3*b^3)))/d

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(9/2)/(a+b*sec(d*x+c))^3,x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(b \sec(dx + c) + a)^3 \cos(dx + c)^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(9/2)/(a+b*sec(d*x+c))^3,x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^3*cos(d*x + c)^(9/2)), x)

maple [B] time = 35.26, size = 2178, normalized size = 4.16

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(9/2)/(a+b*sec(d*x+c))^3,x)

[Out]
$$\begin{aligned} & -(-(-2*\cos(1/2*d*x+1/2*c)^2+1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*a^2*(A*b-3*B*a)/b^4/(a^2-a*b)*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}* \text{EllipticPi}(\cos(1/2*d*x+1/2*c),2*a/(a-b),2^{(1/2)})-2*a*(A*b-B*a)/b^2*(1/2*a^2/b/(a^2-b^2)*\cos(1/2*d*x+1/2*c)*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/(2*a*\cos(1/2*d*x+1/2*c)^2-a+b)^2+3/4*a^2*(a^2-3*b^2)/b^2/(a^2-b^2)^2*\cos(1/2*d*x+1/2*c)*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/(2*a*\cos(1/2*d*x+1/2*c)^2-a+b)-3/8/(a+b)/(a^2-b^2)/b^2*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}* \text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})*a^2-1/4/(a+b)/(a^2-b^2)/b*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}* \text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})*a+7/8/(a+b)/(a^2-b^2)*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}* \text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})+3/8*a^3/b^2/(a^2-b^2)^2*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}* \text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})-9/8*a/(a^2-b^2)^2*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}* \text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})-3/8*a^3/b^2/(a^2-b^2)^2*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}* \text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)})+9/8*a/(a^2-b^2)^2*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}* \text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)})-3/8/(a-b)/(a+b)/(a^2-b^2)/b^2/(a^2-a*b)*a^5*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}* \text{EllipticPi}(\cos(1/2*d*x+1/2*c),2*a/(a-b),2^{(1/2)})+3/4/(a-b)/(a+b)/(a^2-b^2)/(a^2-a*b)*a^3*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}* \text{EllipticPi}(\cos(1/2*d*x+1/2*c),2*a/(a-b),2^{(1/2)})-15/8/(a-b)/(a+b)/(a^2-b^2)*b^2/(a^2-a*b)*a*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}* \text{EllipticPi}(\cos(1/2*d*x+1/2*c),2*a/(a-b),2^{(1/2)})+2*B/b^3*(-1/6*\cos(1/2*d*x+1/2*c)*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/(-1/2+\cos(1/2*d*x+1/2*c)^2)^2+1/3*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}* \text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})+2*(A*b-3*B*a)/b^4*(-(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}* \text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)})+2*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^2)/\sin(1/2*d*x+1/2*c)^2/(2*\sin(1/2*d*x+1/2*c)^2-1)-2*a*(A*b-2*B*a)/b^3*(a^2/b/(a^2-b^2)*\cos(1/2*d*x+1/2*c)*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/(2*a*\cos(1/2*d*x+1/2*c)^2-a+b)-1/2/(a+b)/b*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)} \end{aligned}$$

$$\begin{aligned} &)^4 + \sin(1/2*d*x+1/2*c)^2)^{1/2} * \text{EllipticF}(\cos(1/2*d*x+1/2*c), 2^{1/2}) + 1/2*a \\ &/b/(a^2-b^2) * (\sin(1/2*d*x+1/2*c)^2)^{1/2} * (-2*\cos(1/2*d*x+1/2*c)^2+1)^{1/2} \\ &/(-2*\sin(1/2*d*x+1/2*c)^4 + \sin(1/2*d*x+1/2*c)^2)^{1/2} * \text{EllipticF}(\cos(1/2*d*x \\ &+1/2*c), 2^{1/2}) - 1/2*a/b/(a^2-b^2) * (\sin(1/2*d*x+1/2*c)^2)^{1/2} * (-2*\cos(1/2 \\ &*d*x+1/2*c)^2+1)^{1/2} / (-2*\sin(1/2*d*x+1/2*c)^4 + \sin(1/2*d*x+1/2*c)^2)^{1/2} \\ &* \text{EllipticE}(\cos(1/2*d*x+1/2*c), 2^{1/2}) - 1/2/b/(a^2-b^2)/(a^2-a*b) * a^3 * (\sin(1 \\ &/2*d*x+1/2*c)^2)^{1/2} * (-2*\cos(1/2*d*x+1/2*c)^2+1)^{1/2} / (-2*\sin(1/2*d*x+1/ \\ &2*c)^4 + \sin(1/2*d*x+1/2*c)^2)^{1/2} * \text{EllipticPi}(\cos(1/2*d*x+1/2*c), 2*a/(a-b), \\ &2^{1/2}) + 3/2*b/(a^2-b^2)/(a^2-a*b) * a * (\sin(1/2*d*x+1/2*c)^2)^{1/2} * (-2*\cos(1 \\ &/2*d*x+1/2*c)^2+1)^{1/2} / (-2*\sin(1/2*d*x+1/2*c)^4 + \sin(1/2*d*x+1/2*c)^2)^{1/2} \\ &* \text{EllipticPi}(\cos(1/2*d*x+1/2*c), 2*a/(a-b), 2^{1/2})) / \sin(1/2*d*x+1/2*c) / (2 \\ &* \cos(1/2*d*x+1/2*c)^2-1)^{1/2} / d \end{aligned}$$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(9/2)/(a+b*sec(d*x+c))^3,x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\cos(c+dx)^{9/2} \left(a + \frac{b}{\cos(c+dx)}\right)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^(9/2)*(a + b/cos(c + d*x))^3),x)

[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^(9/2)*(a + b/cos(c + d*x))^3), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(9/2)/(a+b*sec(d*x+c))**3,x)

[Out] Timed out

$$3.594 \quad \int \cos^2(c+dx) \sqrt{a + b \sec(c + dx)} (A + B \sec(c + dx)) dx$$

Optimal. Leaf size=343

$$\frac{2(25a^2A + 7abB - 4Ab^2) \sin(c + dx) \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}}{105a^2d} + \frac{2(a^2 - b^2)(25a^2A - 14abB + 8Ab^2) \sqrt{\cos(c + dx)} \sqrt{a}}{105a^3d \sqrt{\cos(c + dx)} \sqrt{a}}$$

[Out] $2/105*(a^2-b^2)*(25*A*a^2+8*A*b^2-14*B*a*b)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)}*(a/(a+b))^{(1/2)})*((b+a*\cos(d*x+c))/(a+b))^{(1/2)}/a^3/d/\cos(d*x+c)^{(1/2)}/(a+b*\sec(d*x+c))^{(1/2)}+2/35*(A*b+7*B*a)*\cos(d*x+c)^{(3/2)}*\sin(d*x+c)*(a+b*\sec(d*x+c))^{(1/2)}/a/d+2/7*A*\cos(d*x+c)^{(5/2)}*\sin(d*x+c)*(a+b*\sec(d*x+c))^{(1/2)}/d+2/105*(25*A*a^2-4*A*b^2+7*B*a*b)*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}*(a+b*\sec(d*x+c))^{(1/2)}/a^2/d+2/105*(19*A*a^2*b+8*A*b^3+63*B*a^3-14*B*a*b^2)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)}*(a/(a+b))^{(1/2)})*\cos(d*x+c)^{(1/2)}*(a+b*\sec(d*x+c))^{(1/2)}/a^3/d/((b+a*\cos(d*x+c))/(a+b))^{(1/2)}$

Rubi [A] time = 1.22, antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2955, 4032, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2(25a^2A + 7abB - 4Ab^2) \sin(c + dx) \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}}{105a^2d} + \frac{2(a^2 - b^2)(25a^2A - 14abB + 8Ab^2) \sqrt{\cos(c + dx)} \sqrt{a}}{105a^3d \sqrt{\cos(c + dx)} \sqrt{a}}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^(7/2)*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]

[Out] $(2*(a^2 - b^2)*(25*a^2*A + 8*A*b^2 - 14*a*b*B)*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])]/(a + b))*\text{EllipticF}[(c + d*x)/2, (2*a)/(a + b)]/(105*a^3*d*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]) + (2*(19*a^2*A*b + 8*A*b^3 + 63*a^3*B - 14*a*b^2*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, (2*a)/(a + b)]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]])/(105*a^3*d*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])]/(a + b)) + (2*(25*a^2*A - 4*A*b^2 + 7*a*b*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(105*a^2*d) + (2*(A*b + 7*a*B)*\text{Cos}[c + d*x]^{(3/2)}*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(35*a*d) + (2*A*\text{Cos}[c + d*x]^{(5/2)}*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(7*d)$

Rule 2653

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*Sqrt[a
+ b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
0] && !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)]/(d*Sqrt[a + b]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a
+ b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && !GtQ[a + b, 0]
```

Rule 2955

```
Int[((a_) + csc[(e_) + (f_)*(x_)]*(b_))^(m_)*(csc[(e_) + (f_)*(x_)]*
(d_) + (c_))^(n_)*((g_)*sin[(e_) + (f_)*(x_)])^(p_), x_Symbol] := Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3856

```
Int[Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)]/Sqrt[csc[(e_) + (f_)*(x_)]
*(d_)], x_Symbol] := Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_) + (f_)*(x_)]*(d_)]/Sqrt[csc[(e_) + (f_)*(x_)]*(b_)
+ (a_)], x_Symbol] := Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
```

a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 4032

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*A*(n + 1))*Csc[e + f*x] - A*b*(m + n + 1)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[0, m, 1] && LeQ[n, -1]

Rule 4035

Int[(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.))/(Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]), x_Symbol] :> Dist[A/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]

Rule 4104

Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]

Rubi steps

$$\begin{aligned}
\int \cos^{\frac{7}{2}}(c+dx) \sqrt{a+b \sec(c+dx)} (A+B \sec(c+dx)) dx &= \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\sqrt{a+b \sec(c+dx)}}{\sec^{\frac{7}{2}}(c+dx)} dx \\
&= \frac{2A \cos^{\frac{5}{2}}(c+dx) \sqrt{a+b \sec(c+dx)} \sin(c+dx)}{7d} + \frac{1}{7} \int \frac{\sqrt{a+b \sec(c+dx)}}{\sec^{\frac{1}{2}}(c+dx)} dx \\
&= \frac{2(Ab+7aB) \cos^{\frac{3}{2}}(c+dx) \sqrt{a+b \sec(c+dx)} \sin(c+dx)}{35ad} \\
&= \frac{2(25a^2A-4Ab^2+7abB) \sqrt{\cos(c+dx)} \sqrt{a+b \sec(c+dx)}}{105a^2d} \\
&= \frac{2(25a^2A-4Ab^2+7abB) \sqrt{\cos(c+dx)} \sqrt{a+b \sec(c+dx)}}{105a^2d} \\
&= \frac{2(25a^2A-4Ab^2+7abB) \sqrt{\cos(c+dx)} \sqrt{a+b \sec(c+dx)}}{105a^2d} \\
&= \frac{2(25a^2A-4Ab^2+7abB) \sqrt{\cos(c+dx)} \sqrt{a+b \sec(c+dx)}}{105a^2d} \\
&= \frac{2(a^2-b^2)(25a^2A+8Ab^2-14abB) \sqrt{\frac{b+a \cos(c+dx)}{a+b}}}{105a^3d \sqrt{\cos(c+dx)} \sqrt{a+b \sec(c+dx)}}
\end{aligned}$$

Mathematica [C] time = 18.08, size = 455, normalized size = 1.33

$$\frac{\sqrt{\cos(c+dx)} \sqrt{a+b \sec(c+dx)} \left(\frac{(115a^2A+28abB-16Ab^2) \sin(c+dx)}{210a^2} + \frac{(7aB+Ab) \sin(2(c+dx))}{35a} + \frac{1}{14} A \sin(3(c+dx)) \right)}{d}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Cos[c + d*x]^(7/2)*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]), x]

[Out] (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*(((115*a^2*A - 16*A*b^2 + 28*a*b*B)*Sin[c + d*x])/(210*a^2) + ((A*b + 7*a*B)*Sin[2*(c + d*x)])/(35*a) + (

$$\frac{A \sin[3(c + dx)]}{14} / d - (2 \cos[c + dx]^{3/2} (\cos[(c + dx)/2]^{2 \sec[c + dx]^{3/2}} \sqrt{a + b \sec[c + dx]} ((-1)(a + b)(19a^2Ab + 8Ab^3 + 63a^3B - 14ab^2B) \text{EllipticE}[I \text{ArcSinh}[\tan[(c + dx)/2]], (-a + b)/(a + b)] \sec[(c + dx)/2]^{2 \sqrt{((b + a \cos[c + dx]) \sec[(c + dx)/2]^2)/(a + b)}} + I a (a + b) (8Ab^2 - 2ab(3A + 7B) + a^2(25A + 63B)) \text{EllipticF}[I \text{ArcSinh}[\tan[(c + dx)/2]], (-a + b)/(a + b)] \sec[(c + dx)/2]^{2 \sqrt{((b + a \cos[c + dx]) \sec[(c + dx)/2]^2)/(a + b)}} - (19a^2Ab + 8Ab^3 + 63a^3B - 14ab^2B) (b + a \cos[c + dx]) (\sec[(c + dx)/2]^{2(3/2)} \tan[(c + dx)/2]) / (105a^3d(b + a \cos[c + dx]) \sqrt{\sec[c + dx]})$$

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral} \left((B \cos(dx + c)^3 \sec(dx + c) + A \cos(dx + c)^3) \sqrt{b \sec(dx + c) + a} \sqrt{\cos(dx + c)}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(dx+c)^(7/2)*(A+B*sec(dx+c))*(a+b*sec(dx+c))^(1/2),x, algorithm="fricas")

[Out] integral((B*cos(dx + c)^3*sec(dx + c) + A*cos(dx + c)^3)*sqrt(b*sec(dx + c) + a)*sqrt(cos(dx + c)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A) \sqrt{b \sec(dx + c) + a} \cos(dx + c)^{7/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(dx+c)^(7/2)*(A+B*sec(dx+c))*(a+b*sec(dx+c))^(1/2),x, algorithm="giac")

[Out] integrate((B*sec(dx + c) + A)*sqrt(b*sec(dx + c) + a)*cos(dx + c)^(7/2), x)

maple [B] time = 2.87, size = 2364, normalized size = 6.89

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(dx+c)^(7/2)*(A+B*sec(dx+c))*(a+b*sec(dx+c))^(1/2),x)

[Out]
$$\frac{2}{105} d \left(\frac{(b + a \cos(dx + c))}{\cos(dx + c)} \right)^{1/2} \cos(dx + c)^{1/2} (-1 + \cos(dx + c)) (1 + \cos(dx + c)) (18A \left(\frac{a - b}{a + b} \right)^{1/2} \cos(dx + c)^4 a^3 b \left(\frac{1}{1 + \cos(dx + c)} \right)^{1/2} - 14B \left(\frac{b + a \cos(dx + c)}{1 + \cos(dx + c)} \right) \left(\frac{1}{a + b} \right)^{1/2} \text{EllipticE} \left(\frac{-1 + \cos(dx + c)}{\cos(dx + c)} \right) \left(\frac{a - b}{a + b} \right)^{1/2} / \sin(dx + c), \left(\frac{-a + b}{a - b} \right)^{1/2} \sin(dx + c)$$

$s(dx+c) * ((a-b)/(a+b))^{1/2} / \sin(dx+c), (-a+b)/(a-b)^{1/2} * \sin(dx+c) * a^4 - 63 * B * \sin(dx+c) * ((b+a * \cos(dx+c)) / (1 + \cos(dx+c))) / (a+b)^{1/2} * \text{EllipticF}(-1 + \cos(dx+c)) * ((a-b)/(a+b))^{1/2} / \sin(dx+c), (-a+b)/(a-b)^{1/2} * a^4 + 15 * A * ((a-b)/(a+b))^{1/2} * \cos(dx+c)^5 * a^4 * (1 / (1 + \cos(dx+c)))^{1/2} / a^3 / ((a-b)/(a+b))^{1/2} / (b+a * \cos(dx+c)) / (1 / (1 + \cos(dx+c)))^{1/2} / \sin(dx+c)^3$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A) \sqrt{b \sec(dx + c) + a} \cos(dx + c)^{7/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(dx+c)^(7/2)*(A+B*sec(dx+c))*(a+b*sec(dx+c))^(1/2),x, algorithm="maxima")

[Out] integrate((B*sec(dx + c) + A)*sqrt(b*sec(dx + c) + a)*cos(dx + c)^(7/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c + dx)^{7/2} \left(A + \frac{B}{\cos(c + dx)} \right) \sqrt{a + \frac{b}{\cos(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^(7/2)*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(1/2),x)

[Out] int(cos(c + d*x)^(7/2)*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(1/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(dx+c)**(7/2)*(A+B*sec(dx+c))*(a+b*sec(dx+c))**(1/2),x)

[Out] Timed out

$$3.595 \quad \int \cos^2(c+dx) \sqrt{a+b \sec(c+dx)} (A+B \sec(c+dx)) dx$$

Optimal. Leaf size=267

$$\frac{2(a^2 - b^2)(2Ab - 5aB) \sqrt{\frac{a \cos(c+dx)+b}{a+b}} F\left(\frac{1}{2}(c+dx) \middle| \frac{2a}{a+b}\right)}{15a^2 d \sqrt{\cos(c+dx)} \sqrt{a+b \sec(c+dx)}} + \frac{2(9a^2 A + 5abB - 2Ab^2) \sqrt{\cos(c+dx)} \sqrt{a+b \sec(c+dx)}}{15a^2 d \sqrt{\frac{a \cos(c+dx)+b}{a+b}}}$$

[Out] $-2/15*(a^2-b^2)*(2*A*b-5*B*a)*(cos(1/2*d*x+1/2*c)^2)^{(1/2)}/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c), 2^{(1/2)}*(a/(a+b))^{(1/2)})*((b+a*cos(d*x+c))/(a+b))^{(1/2)}/a^2/d/cos(d*x+c)^{(1/2)}/(a+b*sec(d*x+c))^{(1/2)}+2/5*A*cos(d*x+c)^{(3/2)}*sin(d*x+c)*(a+b*sec(d*x+c))^{(1/2)}/d+2/15*(A*b+5*B*a)*sin(d*x+c)*cos(d*x+c)^{(1/2)}*(a+b*sec(d*x+c))^{(1/2)}/a/d+2/15*(9*A*a^2-2*A*b^2+5*B*a*b)*(cos(1/2*d*x+1/2*c)^2)^{(1/2)}/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c), 2^{(1/2)}*(a/(a+b))^{(1/2)})*cos(d*x+c)^{(1/2)}*(a+b*sec(d*x+c))^{(1/2)}/a^2/d/((b+a*cos(d*x+c))/(a+b))^{(1/2)}$

Rubi [A] time = 0.91, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2955, 4032, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2(a^2 - b^2)(2Ab - 5aB) \sqrt{\frac{a \cos(c+dx)+b}{a+b}} F\left(\frac{1}{2}(c+dx) \middle| \frac{2a}{a+b}\right)}{15a^2 d \sqrt{\cos(c+dx)} \sqrt{a+b \sec(c+dx)}} + \frac{2(9a^2 A + 5abB - 2Ab^2) \sqrt{\cos(c+dx)} \sqrt{a+b \sec(c+dx)}}{15a^2 d \sqrt{\frac{a \cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^(5/2)*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]

[Out] $(-2*(a^2 - b^2)*(2*A*b - 5*a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]/(15*a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*(9*a^2*A - 2*A*b^2 + 5*a*b*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(15*a^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*(A*b + 5*a*B)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(15*a*d) + (2*A*Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(5*d)$

Rule 2653

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,

b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2655

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] :> Dist[Sqrt[a + b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2661

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)]/(d*Sqrt[a + b]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2663

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b) + (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2955

Int[((a_) + csc[(e_) + (f_)*(x_)])*(b_)^(m_)*(csc[(e_) + (f_)*(x_)]*(d_) + (c_))^(n_)*((g_)*sin[(e_) + (f_)*(x_)])^(p_), x_Symbol] :> Dist[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && IntegerQ[n])

Rule 3856

Int[Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)]/Sqrt[csc[(e_) + (f_)*(x_)]*(d_)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 3858

Int[Sqrt[csc[(e_) + (f_)*(x_)]*(d_)]/Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 4032

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*A*(n + 1))*Csc[e + f*x] - A*b*(m + n + 1)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[0, m, 1] && LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.))/(Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]), x_Symbol] := Dist[A/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.), x_Symbol] := Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \cos^{\frac{5}{2}}(c + dx) \sqrt{a + b \sec(c + dx)} (A + B \sec(c + dx)) dx &= \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{\sqrt{a + b \sec(c + dx)}}{\sec^{\frac{5}{2}}(c + dx)} dx \\
&= \frac{2A \cos^{\frac{3}{2}}(c + dx) \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{5d} + \frac{2B \cos^{\frac{3}{2}}(c + dx) \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{5d} \\
&= \frac{2(Ab + 5aB) \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{15ad} \\
&= \frac{2(Ab + 5aB) \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{15ad} \\
&= \frac{2(Ab + 5aB) \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{15ad} \\
&= \frac{2(Ab + 5aB) \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{15ad} \\
&= \frac{2(Ab + 5aB) \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{15ad} \\
&= -\frac{2(a^2 - b^2)(2Ab - 5aB) \sqrt{\frac{b+a \cos(c+dx)}{a+b}} F\left(\frac{1}{2}(c + dx)\right)}{15a^2d \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}}
\end{aligned}$$

Mathematica [C] time = 14.92, size = 353, normalized size = 1.32

$$2\sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)} \left(a \sin(c + dx)(3aA \cos(c + dx) + 5aB + Ab) - \frac{\left(\cos^2\left(\frac{1}{2}(c+dx)\right) \sec(c+dx)\right)^{3/2} \left(-9a^2A + \dots\right)}{\dots} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[Cos[c + d*x]^(5/2)*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]), x]

[Out] (2*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*(a*(A*b + 5*a*B + 3*a*A*Cos[c + d*x])*Sin[c + d*x] - ((Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*((-I)*(a + b)*(9*a^2*A - 2*A*b^2 + 5*a*b*B)*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)


```

x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(
1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b+5*B*sin(d*x+c)*((b+a*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)
/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2-7*A*sin(d*x+c)*EllipticF((-1+cos(d*
x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b-2*A*sin(d*x+c)*EllipticF((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*a*b^2+9*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(-(a+b)/(a-b))^(1/2))*a^2*b+2*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a
+b)/(a-b))^(1/2))*a*b^2-4*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*a^2*b*(1/(1+co
s(d*x+c)))^(1/2)+A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a*b^2*(1/(1+cos(d*x+c)
))^(1/2)-2*A*((a-b)/(a+b))^(1/2)*b^3*(1/(1+cos(d*x+c)))^(1/2)-5*B*(1/(1+cos(
d*x+c)))^(1/2))*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*a^3-5*B*sin(d*x+c)*Elliptic
F((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+
a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^3+5*B*((a-b)/(a+b))^(1/2)*cos(d
*x+c)*a^3*(1/(1+cos(d*x+c)))^(1/2)-3*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^4*a^3
*(1/(1+cos(d*x+c)))^(1/2)-6*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^3*(1/(1+co
s(d*x+c)))^(1/2)+9*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^3*(1/(1+cos(d*x+c)))^
(1/2)+2*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)*b^3*(1/(1+cos(d*x+c)))^(1/2)+9*A*(
(a-b)/(a+b))^(1/2)*a^2*b*(1/(1+cos(d*x+c)))^(1/2)+A*((a-b)/(a+b))^(1/2)*a*b
^2*(1/(1+cos(d*x+c)))^(1/2)+5*B*((a-b)/(a+b))^(1/2)*a^2*b*(1/(1+cos(d*x+c)
))^(1/2)+5*B*((a-b)/(a+b))^(1/2)*a*b^2*(1/(1+cos(d*x+c)))^(1/2)+9*A*sin(d*x+
c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^3-9*A*sin(d*x+c)*((b
+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3-2*A*sin(d*x+c)*((b+a*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b)
)^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3/a^2/((a-b)/(a+b))^(1/2)/(b+a*
cos(d*x+c))/sin(d*x+c)^3/(1/(1+cos(d*x+c)))^(1/2)

```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A) \sqrt{b \sec(dx + c) + a} \cos(dx + c)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^(5/2),
x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c + dx)^{5/2} \left(A + \frac{B}{\cos(c + dx)} \right) \sqrt{a + \frac{b}{\cos(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(c + d*x)^(5/2)*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(1/2), x)`

[Out] `int(cos(c + d*x)^(5/2)*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(1/2), x)`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)**(5/2)*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))**(1/2), x)`

[Out] Timed out

$$3.596 \quad \int \cos^2(c+dx) \sqrt{a + b \sec(c + dx)} (A + B \sec(c + dx)) dx$$

Optimal. Leaf size=201

$$\frac{2A(a^2 - b^2) \sqrt{\frac{a \cos(c+dx)+b}{a+b}} F\left(\frac{1}{2}(c+dx) \middle| \frac{2a}{a+b}\right) + 2(3aB + Ab) \sqrt{\cos(c+dx)} \sqrt{a + b \sec(c+dx)} E\left(\frac{1}{2}(c+dx) \middle| \frac{2a}{a+b}\right)}{3ad \sqrt{\cos(c+dx)} \sqrt{a + b \sec(c+dx)}} + \frac{2(3aB + Ab) \sqrt{\cos(c+dx)} \sqrt{a + b \sec(c+dx)} E\left(\frac{1}{2}(c+dx) \middle| \frac{2a}{a+b}\right)}{3ad \sqrt{\frac{a \cos(c+dx)+b}{a+b}}}$$

[Out] $2/3 * A * (a^2 - b^2) * (\cos(1/2 * d * x + 1/2 * c))^2 \wedge (1/2) / \cos(1/2 * d * x + 1/2 * c) * \text{EllipticF}(\sin(1/2 * d * x + 1/2 * c), 2 \wedge (1/2) * (a / (a + b)) \wedge (1/2)) * ((b + a * \cos(d * x + c)) / (a + b)) \wedge (1/2) / a / d / \cos(d * x + c) \wedge (1/2) / (a + b * \sec(d * x + c)) \wedge (1/2) + 2/3 * A * \sin(d * x + c) * \cos(d * x + c) \wedge (1/2) * (a + b * \sec(d * x + c)) \wedge (1/2) / d + 2/3 * (A * b + 3 * B * a) * (\cos(1/2 * d * x + 1/2 * c))^2 \wedge (1/2) / \cos(1/2 * d * x + 1/2 * c) * \text{EllipticE}(\sin(1/2 * d * x + 1/2 * c), 2 \wedge (1/2) * (a / (a + b)) \wedge (1/2)) * \cos(d * x + c) \wedge (1/2) * (a + b * \sec(d * x + c)) \wedge (1/2) / a / d / ((b + a * \cos(d * x + c)) / (a + b)) \wedge (1/2)$

Rubi [A] time = 0.62, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {2955, 4032, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2A(a^2 - b^2) \sqrt{\frac{a \cos(c+dx)+b}{a+b}} F\left(\frac{1}{2}(c+dx) \middle| \frac{2a}{a+b}\right) + 2(3aB + Ab) \sqrt{\cos(c+dx)} \sqrt{a + b \sec(c+dx)} E\left(\frac{1}{2}(c+dx) \middle| \frac{2a}{a+b}\right)}{3ad \sqrt{\cos(c+dx)} \sqrt{a + b \sec(c+dx)}} + \frac{2(3aB + Ab) \sqrt{\cos(c+dx)} \sqrt{a + b \sec(c+dx)} E\left(\frac{1}{2}(c+dx) \middle| \frac{2a}{a+b}\right)}{3ad \sqrt{\frac{a \cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]

[Out] $(2 * A * (a^2 - b^2) * \text{Sqrt}[(b + a * \text{Cos}[c + d * x]) / (a + b)] * \text{EllipticF}[(c + d * x) / 2, (2 * a) / (a + b)]) / (3 * a * d * \text{Sqrt}[\text{Cos}[c + d * x]] * \text{Sqrt}[a + b * \text{Sec}[c + d * x]]) + (2 * (A * b + 3 * a * B) * \text{Sqrt}[\text{Cos}[c + d * x]] * \text{EllipticE}[(c + d * x) / 2, (2 * a) / (a + b)] * \text{Sqrt}[a + b * \text{Sec}[c + d * x]]) / (3 * a * d * \text{Sqrt}[(b + a * \text{Cos}[c + d * x]) / (a + b)]) + (2 * A * \text{Sqrt}[\text{Cos}[c + d * x]] * \text{Sqrt}[a + b * \text{Sec}[c + d * x]] * \text{Sin}[c + d * x]) / (3 * d)$

Rule 2653

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*Sqrt[a + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)]/d, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2655

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a + b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b

```
*Sin[c + d*x]]/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)]/(d*Sqrt[a + b]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a + b*SIN[c + d*x])/(a + b)]/Sqrt[a + b*SIN[c + d*x]], Int[1/Sqrt[a/(a + b) + (b*SIN[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]
```

Rule 2955

```
Int[((a_) + csc[(e_) + (f_)*(x_)]*(b_))^(m_)*(csc[(e_) + (f_)*(x_)]*(d_) + (c_))^(n_)*((g_)*sin[(e_) + (f_)*(x_)])^(p_), x_Symbol] := Dist[(g*Csc[e + f*x])^p*(g*SIN[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && IntegerQ[n])
```

Rule 3856

```
Int[Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)]/Sqrt[csc[(e_) + (f_)*(x_)]*(d_)], x_Symbol] := Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*SIN[e + f*x]]), Int[Sqrt[b + a*SIN[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_) + (f_)*(x_)]*(d_)]/Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)], x_Symbol] := Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*SIN[e + f*x]])/Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*SIN[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4032

```
Int[(csc[(e_) + (f_)*(x_)]*(d_))^(n_)*(csc[(e_) + (f_)*(x_)]*(b_) + (a_))^(m_)*(csc[(e_) + (f_)*(x_)]*(B_) + (A_)), x_Symbol] := Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp[A*b*m - a
```

*B*n - (b*B*n + a*A*(n + 1))*Csc[e + f*x] - A*b*(m + n + 1)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[0, m, 1] && LeQ[n, -1]

Rule 4035

Int[(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.))/(Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]), x_Symbol] :> Dist[A/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned}
 \int \cos^{\frac{3}{2}}(c + dx) \sqrt{a + b \sec(c + dx)} (A + B \sec(c + dx)) dx &= \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{\sqrt{a + b \sec(c + dx)}}{\sec^{\frac{3}{2}}(c + dx)} dx \\
 &= \frac{2A \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{3d} + \dots \\
 &= \frac{2A \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{3d} + \dots \\
 &= \frac{2A \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{3d} + \dots \\
 &= \frac{2A \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{3d} + \dots \\
 &= \frac{2A (a^2 - b^2) \sqrt{\frac{b+a \cos(c+dx)}{a+b}} F\left(\frac{1}{2}(c + dx) \middle| \frac{2a}{a+b}\right)}{3ad \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}} + \dots
 \end{aligned}$$

Mathematica [C] time = 8.95, size = 305, normalized size = 1.52

$$2\sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)} \left(A \sin(c + dx) + \frac{\left(\cos^2\left(\frac{1}{2}(c+dx)\right) \sec(c+dx) \right)^{3/2} \left((3aB + Ab) \tan\left(\frac{1}{2}(c+dx)\right) \sec^2\left(\frac{1}{2}(c+dx)\right) \right)^{3/2} (a \cos(c + dx))}{3ad \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}} \right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]),
x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*(A*Sin[c + d*x] + ((Cos[(c +
d*x)/2]^2*Sec[c + d*x])^(3/2)*(I*(a + b)*(A*b + 3*a*B)*EllipticE[I*ArcSinh
[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c
+ d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - I*a*(a + b)*(A + 3*B)*EllipticF[I*A
rcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a
*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + (A*b + 3*a*B)*(b + a*Cos[c +
d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/(a*(b + a*Cos[c + d*x])
*Sec[c + d*x]^(3/2))))/(3*d)
```

fricas [F] time = 0.94, size = 0, normalized size = 0.00

$$\text{integral}((B \cos(dx + c) \sec(dx + c) + A \cos(dx + c)) \sqrt{b \sec(dx + c) + a} \sqrt{\cos(dx + c)}, x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c)*sec(d*x + c) + A*cos(d*x + c))*sqrt(b*sec(d*x + c)
+ a)*sqrt(cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A) \sqrt{b \sec(dx + c) + a} \cos(dx + c)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^(3/2),
x)
```

maple [B] time = 2.60, size = 1162, normalized size = 5.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] 2/3/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)*(-1+cos(d*x+c))*
(1+cos(d*x+c))*(A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*(1/(1+cos(d*x+c))))^(
```

$1/2)+2*A*\cos(d*x+c)^2*((a-b)/(a+b))^{1/2}*a*b*(1/(1+\cos(d*x+c)))^{1/2}+3*B*\cos(d*x+c)^2*((a-b)/(a+b))^{1/2}*a^2*(1/(1+\cos(d*x+c)))^{1/2}-A*\cos(d*x+c)*((a-b)/(a+b))^{1/2}*a^2*(1/(1+\cos(d*x+c)))^{1/2}-A*\cos(d*x+c)*((a-b)/(a+b))^{1/2}*a*b*(1/(1+\cos(d*x+c)))^{1/2}+A*\cos(d*x+c)*((a-b)/(a+b))^{1/2}*b^2*(1/(1+\cos(d*x+c)))^{1/2}+A*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*EllipticE((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (-a+b)/(a-b))^{1/2})*\sin(d*x+c)*a*b-A*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*EllipticE((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (-a+b)/(a-b))^{1/2})*\sin(d*x+c)*b^2+A*EllipticF((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (-a+b)/(a-b))^{1/2})*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*\sin(d*x+c)*a^2-A*EllipticF((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (-a+b)/(a-b))^{1/2})*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*\sin(d*x+c)*a*b-3*B*\cos(d*x+c)*((a-b)/(a+b))^{1/2}*a^2*(1/(1+\cos(d*x+c)))^{1/2}+3*B*\cos(d*x+c)*((a-b)/(a+b))^{1/2}*a*b*(1/(1+\cos(d*x+c)))^{1/2}+3*B*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*EllipticE((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (-a+b)/(a-b))^{1/2})*\sin(d*x+c)*a^2-3*B*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*EllipticE((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (-a+b)/(a-b))^{1/2})*\sin(d*x+c)*a*b-3*B*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*EllipticF((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (-a+b)/(a-b))^{1/2})*\sin(d*x+c)*a^2+3*B*EllipticF((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (-a+b)/(a-b))^{1/2})*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*\sin(d*x+c)*a*b-A*((a-b)/(a+b))^{1/2}*a*b*(1/(1+\cos(d*x+c)))^{1/2}-A*((a-b)/(a+b))^{1/2}*b^2*(1/(1+\cos(d*x+c)))^{1/2}-3*B*((a-b)/(a+b))^{1/2}*a*b*(1/(1+\cos(d*x+c)))^{1/2})/a/((a-b)/(a+b))^{1/2}/(b+a*\cos(d*x+c))/\sin(d*x+c)^3/(1/(1+\cos(d*x+c)))^{1/2}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A) \sqrt{b \sec(dx + c) + a} \cos(dx + c)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c + dx)^{3/2} \left(A + \frac{B}{\cos(c + dx)} \right) \sqrt{a + \frac{b}{\cos(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^(3/2)*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(1/2), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```


$$3.597 \quad \int \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)} (A + B \sec(c + dx)) dx$$

Optimal. Leaf size=208

$$\frac{2A\sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2a}{a+b}\right)}{d\sqrt{\frac{a \cos(c+dx)+b}{a+b}}} + \frac{2aB\sqrt{\frac{a \cos(c+dx)+b}{a+b}} F\left(\frac{1}{2}(c + dx) \middle| \frac{2a}{a+b}\right)}{d\sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}} + \frac{2bB\sqrt{\frac{a \cos(c+dx)+b}{a+b}}}{d\sqrt{\cos(c + dx)}}$$

[Out] $2*a*B*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)}*(a/(a+b))^{(1/2)})*((b+a*\cos(d*x+c))/(a+b))^{(1/2)}/d/\cos(d*x+c)^{(1/2)}/(a+b*\sec(d*x+c))^{(1/2)}+2*b*B*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticPi}(\sin(1/2*d*x+1/2*c), 2^{(1/2)}*(a/(a+b))^{(1/2)})*((b+a*\cos(d*x+c))/(a+b))^{(1/2)}/d/\cos(d*x+c)^{(1/2)}/(a+b*\sec(d*x+c))^{(1/2)}+2*A*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)}*(a/(a+b))^{(1/2)})*\cos(d*x+c)^{(1/2)}*(a+b*\sec(d*x+c))^{(1/2)}/d/((b+a*\cos(d*x+c))/(a+b))^{(1/2)}$

Rubi [A] time = 0.68, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.343$, Rules used = {2955, 4037, 3854, 3858, 2663, 2661, 3859, 2807, 2805, 3856, 2655, 2653}

$$\frac{2A\sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2a}{a+b}\right)}{d\sqrt{\frac{a \cos(c+dx)+b}{a+b}}} + \frac{2aB\sqrt{\frac{a \cos(c+dx)+b}{a+b}} F\left(\frac{1}{2}(c + dx) \middle| \frac{2a}{a+b}\right)}{d\sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}} + \frac{2bB\sqrt{\frac{a \cos(c+dx)+b}{a+b}}}{d\sqrt{\cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]), x]

[Out] $(2*a*B*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/(a + b)]*\text{EllipticF}[(c + d*x)/2, (2*a)/(a + b)])/(d*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]) + (2*b*B*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/(a + b)]*\text{EllipticPi}[2, (c + d*x)/2, (2*a)/(a + b)])/(d*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]) + (2*A*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, (2*a)/(a + b)]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]])/(d*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/(a + b)])$

Rule 2653

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*Sqrt[a + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)]/d, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2655

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
0] && !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)]/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a
+ b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_)
+ (f_)*(x_)]]), x_Symbol] := Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)]/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_)
+ (f_)*(x_)]]), x_Symbol] := Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
+ f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !GtQ[c + d, 0]
```

Rule 2955

```
Int[((a_) + csc[(e_) + (f_)*(x_)]*(b_))^(m_)*(csc[(e_) + (f_)*(x_)]*
(d_) + (c_))^(n_)*((g_)*sin[(e_) + (f_)*(x_)])^(p_), x_Symbol] := Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3854

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[a, Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f
*x]], x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]
], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4037

```
Int[(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B
_.) + (A_)))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)], x_Symbol] :> Dist[B/d, In
t[Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]], x], x] + Dist[A, Int[Sqrt[
a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{\cos(c+dx)} \sqrt{a+b \sec(c+dx)} (A+B \sec(c+dx)) dx &= (\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}) \int \frac{\sqrt{a+b \sec(c+dx)}}{\sqrt{\sec(c+dx)}} dx \\
&= (A\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}) \int \frac{\sqrt{a+b \sec(c+dx)}}{\sqrt{\sec(c+dx)}} dx \\
&= (aB\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}) \int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b \sec(c+dx)}} dx \\
&= \frac{(aB\sqrt{b+a \cos(c+dx)}) \int \frac{1}{\sqrt{b+a \cos(c+dx)}} dx}{\sqrt{\cos(c+dx)} \sqrt{a+b \sec(c+dx)}} + \frac{(bB\sqrt{\cos(c+dx)}) \int \frac{1}{\sqrt{\sec(c+dx)}} dx}{\sqrt{\cos(c+dx)} \sqrt{a+b \sec(c+dx)}} \\
&= \frac{2A\sqrt{\cos(c+dx)} E\left(\frac{1}{2}(c+dx) \middle| \frac{2a}{a+b}\right) \sqrt{a+b \sec(c+dx)}}{d\sqrt{\frac{b+a \cos(c+dx)}{a+b}}} \\
&= \frac{2aB\sqrt{\frac{b+a \cos(c+dx)}{a+b}} F\left(\frac{1}{2}(c+dx) \middle| \frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)} \sqrt{a+b \sec(c+dx)}} + \frac{2bB\sqrt{\frac{b+a \cos(c+dx)}{a+b}}}{d\sqrt{\cos(c+dx)}}
\end{aligned}$$

Mathematica [C] time = 29.93, size = 25347, normalized size = 121.86

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]), x]

[Out] Result too large to show

fricas [F] time = 2.30, size = 0, normalized size = 0.00

integral((B sec(dx + c) + A) sqrt(b sec(dx + c) + a) sqrt(cos(dx + c)), x)

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2), x, algorithm="fricas")

[Out] integral((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A) \sqrt{b \sec(dx + c) + a} \sqrt{\cos(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c)), x)

maple [C] time = 2.23, size = 822, normalized size = 3.95

$$2(-1 + \cos(dx + c))(1 + \cos(dx + c)) \left(A \sqrt{\frac{a-b}{a+b}} (\cos^2(dx + c)) \sqrt{\frac{1}{1+\cos(dx+c)}} a - A \sqrt{\frac{a-b}{a+b}} \cos(dx + c) \sqrt{\frac{1}{1+\cos(dx+c)}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2),x)

[Out] 2/d*(-1+cos(d*x+c))*(1+cos(d*x+c))*(A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*(1/(1+cos(d*x+c)))^(1/2)*a-A*((a-b)/(a+b))^(1/2)*cos(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*a+A*((a-b)/(a+b))^(1/2)*cos(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*b-A*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b)^(1/2)*a+A*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b)^(1/2)*b+A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b)^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a-A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b)^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b+B*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b)^(1/2)*a-B*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b)^(1/2)*b+2*B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b)^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b-A*((a-b)/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b*cos(d*x+c)^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/((a-b)/(a+b))^(1/2)/(b+a*cos(d*x+c))/(1/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^3

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A) \sqrt{b \sec(dx + c) + a} \sqrt{\cos(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{\cos(c + dx)} \left(A + \frac{B}{\cos(c + dx)} \right) \sqrt{a + \frac{b}{\cos(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^(1/2)*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(1/2),x)

[Out] int(cos(c + d*x)^(1/2)*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \sec(c + dx)) \sqrt{a + b \sec(c + dx)} \sqrt{\cos(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)**(1/2)*(a+b*sec(d*x+c))**(1/2),x)

[Out] Integral((A + B*sec(c + d*x))*sqrt(a + b*sec(c + d*x))*sqrt(cos(c + d*x)), x)

$$3.598 \quad \int \frac{\sqrt{a+b \sec(c+dx)} (A+B \sec(c+dx))}{\sqrt{\cos(c+dx)}} dx$$

Optimal. Leaf size=253

$$\frac{(2aA + bB)\sqrt{\frac{a \cos(c+dx)+b}{a+b}} F\left(\frac{1}{2}(c+dx) \middle| \frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)} \sqrt{a+b \sec(c+dx)}} + \frac{(aB + 2Ab)\sqrt{\frac{a \cos(c+dx)+b}{a+b}} \Pi\left(2; \frac{1}{2}(c+dx) \middle| \frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)} \sqrt{a+b \sec(c+dx)}} + \frac{B \sin(c+dx)\sqrt{a+b}}{d\sqrt{\cos(c+dx)}}$$

[Out] (2*A*a+B*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c), 2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+(2*A*b+B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c), 2, 2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+B*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)-B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c), 2^(1/2)*(a/(a+b))^(1/2))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2)

Rubi [A] time = 0.94, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {2955, 4031, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{(2aA + bB)\sqrt{\frac{a \cos(c+dx)+b}{a+b}} F\left(\frac{1}{2}(c+dx) \middle| \frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)} \sqrt{a+b \sec(c+dx)}} + \frac{(aB + 2Ab)\sqrt{\frac{a \cos(c+dx)+b}{a+b}} \Pi\left(2; \frac{1}{2}(c+dx) \middle| \frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)} \sqrt{a+b \sec(c+dx)}} + \frac{B \sin(c+dx)\sqrt{a+b}}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]))/Sqrt[Cos[c + d*x]], x]

[Out] ((2*a*A + b*B)*Sqrt[(b + a*cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + ((2*A*b + a*B)*Sqrt[(b + a*cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - (B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*cos[c + d*x])/(a + b)]) + (B*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])

Rule 2653

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*Sqrt[a + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,

b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2655

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a + b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2661

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)]/(d*Sqrt[a + b]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2663

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b) + (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2805

Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi/2 + f*x))/2, (2*d)/(c + d)]/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[c + d, 0]

Rule 2807

Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !GtQ[c + d, 0]

Rule 2955

Int[((a_) + csc[(e_) + (f_)*(x_)]*(b_))^(m_)*(csc[(e_) + (f_)*(x_)]*(d_) + (c_))^(n_)*((g_)*sin[(e_) + (f_)*(x_)])^(p_), x_Symbol] := Dist[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && In

tegerQ[n])

Rule 3856

Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 3858

Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 3859

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 4031

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] :> -Simp[(B*d*Cos[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x] + Dist[d/(m + n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n - 1)*Simp[a*B*(n - 1) + (b*B*(m + n - 1) + a*A*(m + n))*Csc[e + f*x] + (a*B*m + A*b*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[0, m, 1] && GtQ[n, 0]

Rule 4035

Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)]), x_Symbol] :> Dist[A/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]

Rule 4108

Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)

```

+ (a_)]), x_Symbol] := Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
+ b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]

```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a + b \sec(c + dx)} (A + B \sec(c + dx))}{\sqrt{\cos(c + dx)}} dx &= \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \sqrt{\sec(c + dx)} \sqrt{a + b \sec(c + dx)} \\
&= \frac{B \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{d \sqrt{\cos(c + dx)}} + \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \\
&= \frac{B \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{d \sqrt{\cos(c + dx)}} + \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \\
&= \frac{B \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{d \sqrt{\cos(c + dx)}} - \frac{1}{2} \left(B \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \\
&= \frac{B \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{d \sqrt{\cos(c + dx)}} + \frac{\left((2aA + bB) \sqrt{b + a \cos(c + dx)} \right)}{2 \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}} \\
&= \frac{(2Ab + aB) \sqrt{\frac{b+a \cos(c+dx)}{a+b}} \Pi \left(2; \frac{1}{2}(c + dx) \middle| \frac{2a}{a+b} \right)}{d \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}} + \frac{B \sqrt{a + b \sec(c + dx)}}{d \sqrt{\cos(c + dx)}} \\
&= \frac{(2aA + bB) \sqrt{\frac{b+a \cos(c+dx)}{a+b}} F \left(\frac{1}{2}(c + dx) \middle| \frac{2a}{a+b} \right)}{d \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}} + \frac{(2Ab + aB) \sqrt{\frac{b+a \cos(c+dx)}{a+b}}}{d \sqrt{\cos(c + dx)}}
\end{aligned}$$

Mathematica [C] time = 32.45, size = 52603, normalized size = 207.92

Result too large to show

Warning: Unable to verify antiderivative.

```

[In] Integrate[(Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]))/Sqrt[Cos[c + d*x]
],x]

```

```

[Out] Result too large to show

```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorith="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sqrt{b \sec(dx + c) + a}}{\sqrt{\cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorith="giac")

[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)/sqrt(cos(d*x + c)), x)

maple [C] time = 2.86, size = 789, normalized size = 3.12

$$\frac{(-1 + \cos(dx + c))(1 + \cos(dx + c)) \left(2A \cos(dx + c) \sin(dx + c) \sqrt{\frac{b + a \cos(dx + c)}{(1 + \cos(dx + c))(a + b)}} \operatorname{EllipticF}\left(\frac{-1 + \cos(dx + c)}{\sin(dx + c)} \sqrt{\frac{a}{a + b}}\right) \right)}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x)

[Out] 1/d*(-1+cos(d*x+c))*(1+cos(d*x+c))*(2*A*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a-2*A*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b+4*A*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), (a+b)/(a-b), I/((a-b)/(a+b))^(1/2))*b+B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*(1/(1+cos(d*x+c)))^(1/2)+2*B*cos(d*x+c)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), (a+b)/(a-b), I/((a-b)/(a+b))^(1/2))*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*a-B*cos(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))

/2))*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*a+B*cos(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*b-B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*(1/(1+cos(d*x+c)))^(1/2)+B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b*(1/(1+cos(d*x+c)))^(1/2)-B*((a-b)/(a+b))^(1/2)*b*(1/(1+cos(d*x+c)))^(1/2))*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/((a-b)/(a+b))^(1/2)/(b+a*cos(d*x+c))/(1/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^3/cos(d*x+c)^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sqrt{b \sec(dx + c) + a}}{\sqrt{\cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)/sqrt(cos(d*x + c)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \sqrt{a + \frac{b}{\cos(c+dx)}}}{\sqrt{\cos(c+dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(1/2))/cos(c + d*x)^(1/2),x)

[Out] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(1/2))/cos(c + d*x)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \sqrt{a + b \sec(c + dx)}}{\sqrt{\cos(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x)

[Out] Integral((A + B*sec(c + d*x))*sqrt(a + b*sec(c + d*x))/sqrt(cos(c + d*x)), x)

$$3.599 \quad \int \frac{\sqrt{a+b \sec(c+dx)} (A+B \sec(c+dx))}{\cos^{\frac{3}{2}}(c+dx)} dx$$

Optimal. Leaf size=336

$$\frac{(a^2(-B) + 4aAb + 4b^2B) \sqrt{\frac{a \cos(c+dx)+b}{a+b}} \Pi\left(2; \frac{1}{2}(c+dx) \middle| \frac{2a}{a+b}\right)}{4bd\sqrt{\cos(c+dx)} \sqrt{a+b \sec(c+dx)}} + \frac{(aB + 4Ab) \sin(c+dx) \sqrt{a+b \sec(c+dx)}}{4bd\sqrt{\cos(c+dx)}} + \frac{(3aB}{4}$$

[Out] $1/4*(4*A*b+3*B*a)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)}*(a/(a+b))^{(1/2)})*((b+a*\cos(d*x+c))/(a+b))^{(1/2)}/d/\cos(d*x+c)^{(1/2)}/(a+b*\sec(d*x+c))^{(1/2)}+1/4*(4*A*a*b-B*a^2+4*B*b^2)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticPi}(\sin(1/2*d*x+1/2*c), 2^{(1/2)}*(a/(a+b))^{(1/2)})*((b+a*\cos(d*x+c))/(a+b))^{(1/2)}/b/d/\cos(d*x+c)^{(1/2)}/(a+b*\sec(d*x+c))^{(1/2)}+1/2*B*\sin(d*x+c)*(a+b*\sec(d*x+c))^{(1/2)}/d/\cos(d*x+c)^{(3/2)}+1/4*(4*A*b+B*a)*\sin(d*x+c)*(a+b*\sec(d*x+c))^{(1/2)}/b/d/\cos(d*x+c)^{(1/2)}-1/4*(4*A*b+B*a)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)}*(a/(a+b))^{(1/2)})*\cos(d*x+c)^{(1/2)}*(a+b*\sec(d*x+c))^{(1/2)}/b/d/((b+a*\cos(d*x+c))/(a+b))^{(1/2)}$

Rubi [A] time = 1.26, antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 14, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2955, 4031, 4102, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{(a^2(-B) + 4aAb + 4b^2B) \sqrt{\frac{a \cos(c+dx)+b}{a+b}} \Pi\left(2; \frac{1}{2}(c+dx) \middle| \frac{2a}{a+b}\right)}{4bd\sqrt{\cos(c+dx)} \sqrt{a+b \sec(c+dx)}} + \frac{(aB + 4Ab) \sin(c+dx) \sqrt{a+b \sec(c+dx)}}{4bd\sqrt{\cos(c+dx)}} + \frac{(3aB}{4}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]))/Cos[c + d*x]^(3/2),x]

[Out] $((4*A*b + 3*a*B)*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/(a + b)]*\text{EllipticF}[(c + d*x)/2, (2*a)/(a + b)])/(4*d*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]) + ((4*a*A*b - a^2*B + 4*b^2*B)*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/(a + b)]*\text{EllipticPi}[2, (c + d*x)/2, (2*a)/(a + b)])/(4*b*d*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]) - ((4*A*b + a*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, (2*a)/(a + b)]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]])/(4*b*d*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/(a + b)]) + (B*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(2*d*\text{Cos}[c + d*x]^(3/2)) + ((4*A*b + a*B)*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(4*b*d*\text{Sqrt}[\text{Cos}[c + d*x]])$

Rule 2653

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*Sqrt[a
+ b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)]/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
0] && !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)]/(d*Sqrt[a + b]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a
+ b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_)
+ (f_)*(x_)]]), x_Symbol] := Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)]/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_)
+ (f_)*(x_)]]), x_Symbol] := Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
+ f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !GtQ[c + d, 0]
```

Rule 2955

```
Int[((a_) + csc[(e_) + (f_)*(x_)]*(b_))^(m_)*(csc[(e_) + (f_)*(x_)]*
(d_) + (c_))^(n_)*((g_)*sin[(e_) + (f_)*(x_)])^(p_), x_Symbol] := Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
```

*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && IntegerQ[n])

Rule 3856

Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 3858

Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 3859

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 4031

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] :> -Simp[(B*d*Cos[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x] + Dist[d/(m + n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n - 1)*Simp[a*B*(n - 1) + (b*B*(m + n - 1) + a*A*(m + n))*Csc[e + f*x] + (a*B*m + A*b*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[0, m, 1] && GtQ[n, 0]

Rule 4035

Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)]), x_Symbol] :> Dist[A/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]

Rule 4102

```

Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a
_))^(m_), x_Symbol] := -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]

```

Rule 4108

```

Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.)
+ (a_)])], x_Symbol] := Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
+ b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]

```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a + b \sec(c + dx)} (A + B \sec(c + dx))}{\cos^{\frac{3}{2}}(c + dx)} dx &= \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \sec^{\frac{3}{2}}(c + dx) \sqrt{a + b \sec(c + dx)} dx \\
&= \frac{B \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{2d \cos^{\frac{3}{2}}(c + dx)} + \frac{1}{2} \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \sec^{\frac{3}{2}}(c + dx) \sqrt{a + b \sec(c + dx)} dx \\
&= \frac{B \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{2d \cos^{\frac{3}{2}}(c + dx)} + \frac{(4Ab + aB) \sqrt{a + b \sec(c + dx)}}{4bd \sqrt{\cos(c + dx)}} \\
&= \frac{B \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{2d \cos^{\frac{3}{2}}(c + dx)} + \frac{(4Ab + aB) \sqrt{a + b \sec(c + dx)}}{4bd \sqrt{\cos(c + dx)}} \\
&= \frac{B \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{2d \cos^{\frac{3}{2}}(c + dx)} + \frac{(4Ab + aB) \sqrt{a + b \sec(c + dx)}}{4bd \sqrt{\cos(c + dx)}} \\
&= \frac{B \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{2d \cos^{\frac{3}{2}}(c + dx)} + \frac{(4Ab + aB) \sqrt{a + b \sec(c + dx)}}{4bd \sqrt{\cos(c + dx)}} \\
&= \frac{B \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{2d \cos^{\frac{3}{2}}(c + dx)} + \frac{(4Ab + aB) \sqrt{a + b \sec(c + dx)}}{4bd \sqrt{\cos(c + dx)}} \\
&= \frac{(4aAb - a^2B + 4b^2B) \sqrt{\frac{b+a \cos(c+dx)}{a+b}} \Pi\left(2; \frac{1}{2}(c + dx) \middle| \frac{2a}{a+b}\right) + B \sqrt{a + b \sec(c + dx)}}{4bd \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}} \\
&= \frac{(4Ab + 3aB) \sqrt{\frac{b+a \cos(c+dx)}{a+b}} F\left(\frac{1}{2}(c + dx) \middle| \frac{2a}{a+b}\right) + (4aAb - a^2B + 4b^2B) \sqrt{a + b \sec(c + dx)}}{4d \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}} + \frac{(4aAb - a^2B + 4b^2B) \sqrt{a + b \sec(c + dx)}}{4bd \sqrt{\cos(c + dx)}}
\end{aligned}$$

Mathematica [C] time = 33.02, size = 77879, normalized size = 231.78

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]))/Cos[c + d*x]^(3/2), x]

[Out] Result too large to show

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algo-
rithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sqrt{b \sec(dx + c) + a}}{\cos(dx + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algo-
rithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)/cos(d*x + c)^(3/2),
x)
```

maple [C] time = 1.97, size = 1475, normalized size = 4.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x)
```

```
[Out] -1/4/d*(-1+cos(d*x+c))*(1+cos(d*x+c))*(4*A*EllipticE((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c), (-a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x
+c)))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*a*b-4*A*EllipticE((-1+cos(d*x+c))
*((a-b)/(a+b))^(1/2)/sin(d*x+c), (-a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+
cos(d*x+c)))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*b^2-8*A*((b+a*cos(d*x+c))/
(1+cos(d*x+c)))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/
sin(d*x+c), (a+b)/(a-b), I/((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a*b-4
*A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a*b*(1/(1+cos(d*x+c)))^(1/2)-2*B*Ellipt
icF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), (-a+b)/(a-b))^(1/2))*si
n(d*x+c)*cos(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*a^2-2*B
*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((
a-b)/(a+b))^(1/2)/sin(d*x+c), (-a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*
a*b+4*B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), (-a+b)/(a
-b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))
^(1/2)*b^2+B*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), (-a+
b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*sin(d*x+c)*c
os(d*x+c)^2*a^2-B*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(-a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*sin(d*x
```

+c)*cos(d*x+c)^2*a*b+2*B*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), (a+b)/(a-b), I/((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^2-8*B*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), (a+b)/(a-b), I/((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*b^2-B*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*(1/(1+cos(d*x+c)))^(1/2)-2*B*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a*b*(1/(1+cos(d*x+c)))^(1/2)+4*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b*(1/(1+cos(d*x+c)))^(1/2)-4*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*b^2*(1/(1+cos(d*x+c)))^(1/2)+B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*(1/(1+cos(d*x+c)))^(1/2)-B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b*(1/(1+cos(d*x+c)))^(1/2)-2*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*b^2*(1/(1+cos(d*x+c)))^(1/2)+4*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^2*(1/(1+cos(d*x+c)))^(1/2)+3*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b*(1/(1+cos(d*x+c)))^(1/2)+2*B*((a-b)/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b^2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/b/((a-b)/(a+b))^(1/2)/(b+a*cos(d*x+c))/(1/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^3/cos(d*x+c)^(3/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sqrt{b \sec(dx + c) + a}}{\cos(dx + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)/cos(d*x + c)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \sqrt{a + \frac{b}{\cos(c+dx)}}}{\cos(c+dx)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(1/2))/cos(c + d*x)^(3/2),x)

[Out] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(1/2))/cos(c + d*x)^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \sqrt{a + b \sec(c + dx)}}{\cos^{\frac{3}{2}}(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+b*sec(d*x+c))**(1/2)/cos(d*x+c)**(3/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sqrt(a + b*sec(c + d*x))/cos(c + d*x)**(3/2),  
x)
```

$$3.600 \quad \int \cos^{\frac{9}{2}}(c+dx)(a+b \sec(c+dx))^{3/2}(A+B \sec(c+dx)) dx$$

Optimal. Leaf size=427

$$\frac{2(49a^2A + 72abB + 3Ab^2) \sin(c+dx) \cos^{\frac{3}{2}}(c+dx) \sqrt{a+b \sec(c+dx)}}{315ad} + \frac{2(75a^3B + 88a^2Ab + 9ab^2B - 4Ab^3)}{315ad}$$

[Out] $2/315*(a^2-b^2)*(39*A*a^2*b+8*A*b^3+75*B*a^3-18*B*a*b^2)*(\cos(1/2*d*x+1/2*c))^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)}*(a/(a+b))^{(1/2)})*((b+a*\cos(d*x+c))/(a+b))^{(1/2)}/a^3/d/\cos(d*x+c)^{(1/2)}/(a+b*\sec(d*x+c))^{(1/2)}+2/315*(49*A*a^2+3*A*b^2+72*B*a*b)*\cos(d*x+c)^{(3/2)}*\sin(d*x+c)*(a+b*\sec(d*x+c))^{(1/2)}/a/d+2/63*(10*A*b+9*B*a)*\cos(d*x+c)^{(5/2)}*\sin(d*x+c)*(a+b*\sec(d*x+c))^{(1/2)}/d+2/9*a*A*\cos(d*x+c)^{(7/2)}*\sin(d*x+c)*(a+b*\sec(d*x+c))^{(1/2)}/d+2/315*(88*A*a^2*b-4*A*b^3+75*B*a^3+9*B*a*b^2)*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}*(a+b*\sec(d*x+c))^{(1/2)}/a^2/d+2/315*(147*A*a^4+33*A*a^2*b^2+8*A*b^4+246*B*a^3*b-18*B*a*b^3)*(\cos(1/2*d*x+1/2*c))^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)}*(a/(a+b))^{(1/2)})*\cos(d*x+c)^{(1/2)}*(a+b*\sec(d*x+c))^{(1/2)}/a^3/d/((b+a*\cos(d*x+c))/(a+b))^{(1/2)}$

Rubi [A] time = 1.71, antiderivative size = 427, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2955, 4025, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2(49a^2A + 72abB + 3Ab^2) \sin(c+dx) \cos^{\frac{3}{2}}(c+dx) \sqrt{a+b \sec(c+dx)}}{315ad} + \frac{2(88a^2Ab + 75a^3B + 9ab^2B - 4Ab^3)}{315ad}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^(9/2)*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]

[Out] $(2*(a^2 - b^2)*(39*a^2*A*b + 8*A*b^3 + 75*a^3*B - 18*a*b^2*B)*\text{Sqrt}[(b + a*\cos[c + d*x])/(a + b)]*\text{EllipticF}[(c + d*x)/2, (2*a)/(a + b)])/(315*a^3*d*\text{Sqrt}[\cos[c + d*x]]*\text{Sqrt}[a + b*\sec[c + d*x]]) + (2*(147*a^4*A + 33*a^2*A*b^2 + 8*A*b^4 + 246*a^3*b*B - 18*a*b^3*B)*\text{Sqrt}[\cos[c + d*x]]*\text{EllipticE}[(c + d*x)/2, (2*a)/(a + b)]*\text{Sqrt}[a + b*\sec[c + d*x]])/(315*a^3*d*\text{Sqrt}[(b + a*\cos[c + d*x])/(a + b)]) + (2*(88*a^2*A*b - 4*A*b^3 + 75*a^3*B + 9*a*b^2*B)*\text{Sqrt}[\cos[c + d*x]]*\text{Sqrt}[a + b*\sec[c + d*x]]*\sin[c + d*x])/(315*a^2*d) + (2*(49*a^2*A + 3*A*b^2 + 72*a*b*B)*\cos[c + d*x]^(3/2)*\text{Sqrt}[a + b*\sec[c + d*x]]*\sin[c +$

$$d*x]]/(315*a*d) + (2*(10*A*b + 9*a*B)*\text{Cos}[c + d*x]^{(5/2)}*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]*\text{Sin}[c + d*x]]/(63*d) + (2*a*A*\text{Cos}[c + d*x]^{(7/2)}*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]*\text{Sin}[c + d*x]]/(9*d)$$
Rule 2653

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*Sqrt[a + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a + b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b) + (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]
```

Rule 2955

```
Int[((a_) + csc[(e_) + (f_)*(x_)]*(b_))^(m_)*(csc[(e_) + (f_)*(x_)]*(d_) + (c_))^(n_)*((g_)*sin[(e_) + (f_)*(x_)])^(p_), x_Symbol] := Dist[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && IntegerQ[n])
```

Rule 3856

```
Int[Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)]/Sqrt[csc[(e_) + (f_)*(x_)]*(d_)], x_Symbol] := Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \cos^{\frac{9}{2}}(c+dx)(a+b \sec(c+dx))^{3/2}(A+B \sec(c+dx)) dx &= \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}\right) \int \frac{(a+b \sec(c+dx))^{3/2}(A+B \sec(c+dx))}{\sec(c+dx)} dx \\
&= \frac{2aA \cos^{\frac{7}{2}}(c+dx) \sqrt{a+b \sec(c+dx)} \sin(c+dx)}{9d} \\
&= \frac{2(10Ab+9aB) \cos^{\frac{5}{2}}(c+dx) \sqrt{a+b \sec(c+dx)} \sin(c+dx)}{63d} \\
&= \frac{2(49a^2A+3Ab^2+72abB) \cos^{\frac{3}{2}}(c+dx) \sqrt{a+b \sec(c+dx)} \sin(c+dx)}{315ad} \\
&= \frac{2(88a^2Ab-4Ab^3+75a^3B+9ab^2B) \sqrt{\cos(c+dx)} \sin(c+dx)}{315a^2d} \\
&= \frac{2(88a^2Ab-4Ab^3+75a^3B+9ab^2B) \sqrt{\cos(c+dx)} \sin(c+dx)}{315a^2d} \\
&= \frac{2(88a^2Ab-4Ab^3+75a^3B+9ab^2B) \sqrt{\cos(c+dx)} \sin(c+dx)}{315a^2d} \\
&= \frac{2(88a^2Ab-4Ab^3+75a^3B+9ab^2B) \sqrt{\cos(c+dx)} \sin(c+dx)}{315a^2d} \\
&= \frac{2(a^2-b^2)(39a^2Ab+8Ab^3+75a^3B-18ab^2B) \sqrt{\cos(c+dx)} \sin(c+dx)}{315a^3d \sqrt{\cos(c+dx)} \sqrt{a+b \sec(c+dx)}}
\end{aligned}$$

Mathematica [C] time = 18.65, size = 540, normalized size = 1.26

$$\frac{\cos^{\frac{3}{2}}(c+dx)(a+b \sec(c+dx))^{3/2} \left(\frac{(133a^2A+144abB+6Ab^2) \sin(2(c+dx))}{630a} + \frac{(345a^3B+402a^2Ab+36ab^2B-16Ab^3) \sin(c+dx)}{630a^2} + \frac{1}{126}(9a^2B-4ab^2) \right)}{d(a \cos(c+dx) + b)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Cos[c + d*x]^(9/2)*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]), x]

[Out] (Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2)*(((402*a^2*A*b - 16*A*b^3 + 345*a^3*B + 36*a*b^2*B)*Sin[c + d*x])/(630*a^2) + ((133*a^2*A + 6*A*b^2 + 144*a*b*B)*Sin[2*(c + d*x)])/(630*a) + ((10*A*b + 9*a*B)*Sin[3*(c + d*x)])/126 + (a*A*Ssin[4*(c + d*x)]/36))/(d*(b + a*Cos[c + d*x])) - (2*Cos[c + d*x]^(3/2)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*(a + b*Sec[c + d*x])^(3/2)*((-1)*(a + b)*(147*a^4*A + 33*a^2*A*b^2 + 8*A*b^4 + 246*a^3*b*B - 18*a*b^3*B)*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + I*a*(a + b)*(8*A*b^3 - 6*a*b^2*(A + 3*B) + 3*a^3*(49*A + 25*B) + 3*a^2*b*(13*A + 57*B))*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - (147*a^4*A + 33*a^2*A*b^2 + 8*A*b^4 + 246*a^3*b*B - 18*a*b^3*B)*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/(315*a^3*d*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^(3/2))

fricas [F] time = 1.68, size = 0, normalized size = 0.00

integral(((Bb cos(dx + c)^4 sec(dx + c)^2 + Aa cos(dx + c)^4 + (Ba + Ab) cos(dx + c)^4 sec(dx + c))sqrt(b sec(dx + c) + a) + Aa cos(dx + c)^4 sec(dx + c)^2 + Bb cos(dx + c)^4 sec(dx + c)), x)

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)), x, algorithm="fricas")

[Out] integral((B*b*cos(d*x + c)^4*sec(d*x + c)^2 + A*a*cos(d*x + c)^4 + (B*a + A*b)*cos(d*x + c)^4*sec(d*x + c))*sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(b \sec(dx + c) + a)^{\frac{3}{2}} \cos(dx + c)^{\frac{9}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)), x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(9/2), x)

maple [B] time = 3.38, size = 3069, normalized size = 7.19

output too large to display

$$\begin{aligned} & d*x+c)) / (a+b))^{(1/2)} * \text{EllipticF}((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{(1/2)} / \sin(d*x+c), \\ & (-a+b)/(a-b))^{(1/2)} * a^3 * b^2 + 2 * A * \sin(d*x+c) * ((b+a*\cos(d*x+c)) / (1+\cos(d*x+c))) / (a+b))^{(1/2)} * \text{EllipticF}((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{(1/2)} / \sin(d*x+c), \\ & (-a+b)/(a-b))^{(1/2)} * a^2 * b^3 - 8 * A * \sin(d*x+c) * ((b+a*\cos(d*x+c)) / (1+\cos(d*x+c))) / (a+b))^{(1/2)} * \text{EllipticF}((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{(1/2)} / \sin(d*x+c), \\ & (-a+b)/(a-b))^{(1/2)} * a * b^4 + 30 * B * ((a-b)/(a+b))^{(1/2)} * \cos(d*x+c)^3 * a^5 * (1 / (1 + \cos(d*x+c)))^{(1/2)} - 75 * B * ((a-b)/(a+b))^{(1/2)} * \cos(d*x+c) * a^5 * (1 / (1 + \cos(d*x+c)))^{(1/2)} + 75 * B * \sin(d*x+c) * \text{EllipticF}((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{(1/2)} / \sin(d*x+c), \\ & (-a+b)/(a-b))^{(1/2)} * ((b+a*\cos(d*x+c)) / (1+\cos(d*x+c))) / (a+b))^{(1/2)} * a^5 - 147 * A * ((a-b)/(a+b))^{(1/2)} * \cos(d*x+c) * a^5 * (1 / (1 + \cos(d*x+c)))^{(1/2)} + 8 * A * ((a-b)/(a+b))^{(1/2)} * \cos(d*x+c) * b^5 * (1 / (1 + \cos(d*x+c)))^{(1/2)} - 147 * A * ((a-b)/(a+b))^{(1/2)} * a^4 * b * (1 / (1 + \cos(d*x+c)))^{(1/2)} - 88 * A * ((a-b)/(a+b))^{(1/2)} * a^3 * b^2 * (1 / (1 + \cos(d*x+c)))^{(1/2)} - 33 * A * ((a-b)/(a+b))^{(1/2)} * a^2 * b^3 * (1 / (1 + \cos(d*x+c)))^{(1/2)} + 4 * A * ((a-b)/(a+b))^{(1/2)} * a * b^4 * (1 / (1 + \cos(d*x+c)))^{(1/2)} - 75 * B * ((a-b)/(a+b))^{(1/2)} * a^4 * b * (1 / (1 + \cos(d*x+c)))^{(1/2)} - 246 * B * ((a-b)/(a+b))^{(1/2)} * a^3 * b^2 * (1 / (1 + \cos(d*x+c)))^{(1/2)} - 9 * B * ((a-b)/(a+b))^{(1/2)} * a^2 * b^3 * (1 / (1 + \cos(d*x+c)))^{(1/2)} + 18 * B * ((a-b)/(a+b))^{(1/2)} * a * b^4 * (1 / (1 + \cos(d*x+c)))^{(1/2)} + 45 * B * ((a-b)/(a+b))^{(1/2)} * \cos(d*x+c)^5 * a^5 * (1 / (1 + \cos(d*x+c)))^{(1/2)} + 35 * A * ((a-b)/(a+b))^{(1/2)} * \cos(d*x+c)^6 * a^5 * (1 / (1 + \cos(d*x+c)))^{(1/2)} + 14 * A * ((a-b)/(a+b))^{(1/2)} * \cos(d*x+c)^4 * a^5 * (1 / (1 + \cos(d*x+c)))^{(1/2)} + 98 * A * ((a-b)/(a+b))^{(1/2)} * \cos(d*x+c)^2 * a^5 * (1 / (1 + \cos(d*x+c)))^{(1/2)} + 147 * A * \sin(d*x+c) * \text{EllipticE}((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{(1/2)} / \sin(d*x+c), (-a+b)/(a-b))^{(1/2)} * ((b+a*\cos(d*x+c)) / (1+\cos(d*x+c))) / (a+b))^{(1/2)} * a^5 - 8 * A * \sin(d*x+c) * \text{EllipticE}((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{(1/2)} / \sin(d*x+c), (-a+b)/(a-b))^{(1/2)} * ((b+a*\cos(d*x+c)) / (1+\cos(d*x+c))) / (a+b))^{(1/2)} * b^5 - 147 * A * \sin(d*x+c) * ((b+a*\cos(d*x+c)) / (1+\cos(d*x+c))) / (a+b))^{(1/2)} * \text{EllipticF}((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{(1/2)} / \sin(d*x+c), (-a+b)/(a-b))^{(1/2)} * a^5 + 18 * B * ((a-b)/(a+b))^{(1/2)} * \cos(d*x+c) * a^2 * b^3 * (1 / (1 + \cos(d*x+c)))^{(1/2)} - 18 * B * ((a-b)/(a+b))^{(1/2)} * \cos(d*x+c) * a * b^4 * (1 / (1 + \cos(d*x+c)))^{(1/2)} / a^3 / ((a-b)/(a+b))^{(1/2)} / (b+a*\cos(d*x+c)) / (1 / (1 + \cos(d*x+c)))^{(1/2)} / \sin(d*x+c)^3 \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(b \sec(dx + c) + a)^{\frac{3}{2}} \cos(dx + c)^{\frac{9}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(9/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c + dx)^{9/2} \left(A + \frac{B}{\cos(c + dx)} \right) \left(a + \frac{b}{\cos(c + dx)} \right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(c + d*x)^(9/2)*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(3/2), x)`

[Out] `int(cos(c + d*x)^(9/2)*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(3/2), x)`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)**(9/2)*(a+b*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)), x)`

[Out] Timed out

$$3.601 \quad \int \cos^2(c+dx)(a+b \sec(c+dx))^{3/2}(A+B \sec(c+dx)) dx$$

Optimal. Leaf size=342

$$\frac{2(25a^2A + 42abB + 3Ab^2) \sin(c+dx) \sqrt{\cos(c+dx)} \sqrt{a+b \sec(c+dx)}}{105ad} + \frac{2(a^2 - b^2)(25a^2A + 21abB - 6Ab^2)}{105a^2d \sqrt{\cos(c+dx)} \sqrt{a+b \sec(c+dx)}}$$

[Out] $2/105*(a^2-b^2)*(25*A*a^2-6*A*b^2+21*B*a*b)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)}*(a/(a+b))^{(1/2)})*((b+a*\cos(d*x+c))/(a+b))^{(1/2)}/a^2/d/\cos(d*x+c)^{(1/2)}/(a+b*\sec(d*x+c))^{(1/2)}+2/35*(8*A*b+7*B*a)*\cos(d*x+c)^{(3/2)}*\sin(d*x+c)*(a+b*\sec(d*x+c))^{(1/2)}/d+2/7*a*A*\cos(d*x+c)^{(5/2)}*\sin(d*x+c)*(a+b*\sec(d*x+c))^{(1/2)}/d+2/105*(25*A*a^2+3*A*b^2+42*B*a*b)*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}*(a+b*\sec(d*x+c))^{(1/2)}/a/d+2/105*(82*A*a^2*b-6*A*b^3+63*B*a^3+21*B*a*b^2)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)}*(a/(a+b))^{(1/2)})*\cos(d*x+c)^{(1/2)}*(a+b*\sec(d*x+c))^{(1/2)}/a^2/d/((b+a*\cos(d*x+c))/(a+b))^{(1/2)}$

Rubi [A] time = 1.30, antiderivative size = 342, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2955, 4025, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2(25a^2A + 42abB + 3Ab^2) \sin(c+dx) \sqrt{\cos(c+dx)} \sqrt{a+b \sec(c+dx)}}{105ad} + \frac{2(a^2 - b^2)(25a^2A + 21abB - 6Ab^2)}{105a^2d \sqrt{\cos(c+dx)} \sqrt{a+b \sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]

[Out] $(2*(a^2 - b^2)*(25*a^2*A - 6*A*b^2 + 21*a*b*B)*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])]/(a + b))*\text{EllipticF}[(c + d*x)/2, (2*a)/(a + b)]/(105*a^2*d*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]) + (2*(82*a^2*A*b - 6*A*b^3 + 63*a^3*B + 21*a*b^2*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, (2*a)/(a + b)]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]])/(105*a^2*d*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])]/(a + b)) + (2*(25*a^2*A + 3*A*b^2 + 42*a*b*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(105*a*d) + (2*(8*A*b + 7*a*B)*\text{Cos}[c + d*x]^{(3/2)}*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(35*d) + (2*a*A*\text{Cos}[c + d*x]^{(5/2)}*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(7*d)$

Rule 2653

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*Sqrt[a
+ b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
0] && !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a
+ b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && !GtQ[a + b, 0]
```

Rule 2955

```
Int[((a_) + csc[(e_) + (f_)*(x_)]*(b_))^(m_)*(csc[(e_) + (f_)*(x_)]*
(d_) + (c_))^(n_)*((g_)*sin[(e_) + (f_)*(x_)])^(p_), x_Symbol] := Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3856

```
Int[Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)]/Sqrt[csc[(e_) + (f_)*(x_)]
*(d_)], x_Symbol] := Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_) + (f_)*(x_)]*(d_)]/Sqrt[csc[(e_) + (f_)*(x_)]*(b_)
+ (a_)], x_Symbol] := Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
```

a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_)])], x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.
))*csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \cos^{\frac{7}{2}}(c+dx)(a+b \sec(c+dx))^{3/2}(A+B \sec(c+dx)) dx &= \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}\right) \int \frac{(a+b \sec(c+dx))^{3/2}(A+B \sec(c+dx))}{\sec(c+dx)} dx \\
&= \frac{2aA \cos^{\frac{5}{2}}(c+dx) \sqrt{a+b \sec(c+dx)} \sin(c+dx)}{7d} \\
&= \frac{2(8Ab+7aB) \cos^{\frac{3}{2}}(c+dx) \sqrt{a+b \sec(c+dx)} \sin(c+dx)}{35d} \\
&= \frac{2(25a^2A+3Ab^2+42abB) \sqrt{\cos(c+dx)} \sqrt{a+b \sec(c+dx)} \sin(c+dx)}{105ad} \\
&= \frac{2(25a^2A+3Ab^2+42abB) \sqrt{\cos(c+dx)} \sqrt{a+b \sec(c+dx)} \sin(c+dx)}{105ad} \\
&= \frac{2(25a^2A+3Ab^2+42abB) \sqrt{\cos(c+dx)} \sqrt{a+b \sec(c+dx)} \sin(c+dx)}{105ad} \\
&= \frac{2(25a^2A+3Ab^2+42abB) \sqrt{\cos(c+dx)} \sqrt{a+b \sec(c+dx)} \sin(c+dx)}{105ad} \\
&= \frac{2(a^2-b^2)(25a^2A-6Ab^2+21abB) \sqrt{\frac{b+a \cos(c+dx)}{a+b}}}{105a^2d \sqrt{\cos(c+dx)} \sqrt{a+b \sec(c+dx)}}
\end{aligned}$$

Mathematica [C] time = 17.25, size = 466, normalized size = 1.36

$$\frac{\cos^{\frac{3}{2}}(c+dx)(a+b \sec(c+dx))^{3/2} \left(\frac{(115a^2A+168abB+12Ab^2) \sin(c+dx)}{210a} + \frac{1}{35}(7aB+8Ab) \sin(2(c+dx)) + \frac{1}{14}aA \sin(3(c+dx)) \right)}{d(a \cos(c+dx)+b)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]

[Out] (Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2)*(((115*a^2*A + 12*A*b^2 + 16*8*a*b*B)*Sin[c + d*x])/(210*a) + ((8*A*b + 7*a*B)*Sin[2*(c + d*x)])/35 + (a

$$\frac{A \sin[3(c + dx)]}{14} / (d(b + a \cos[c + dx])) - (2 \cos[c + dx]^{3/2} (\cos[(c + dx)/2]^2 \sec[c + dx]^{3/2} (a + b \sec[c + dx])^{3/2} (-I)(a + b) (82a^2Ab - 6A^2b^3 + 63a^3B + 21ab^2B) \text{EllipticE}[I \text{ArcSinh}[\tan[(c + dx)/2]], (-a + b)/(a + b)] \sec[(c + dx)/2]^2 \sqrt{(b + a \cos[c + dx]) \sec[(c + dx)/2]^2} / (a + b)) + I a (a + b) (-6A^2b^2 + 3ab(19A + 7B) + a^2(25A + 63B)) \text{EllipticF}[I \text{ArcSinh}[\tan[(c + dx)/2]], (-a + b)/(a + b)] \sec[(c + dx)/2]^2 \sqrt{(b + a \cos[c + dx]) \sec[(c + dx)/2]^2} / (a + b) - (82a^2Ab - 6A^2b^3 + 63a^3B + 21ab^2B) (b + a \cos[c + dx]) (\sec[(c + dx)/2]^2)^{3/2} \tan[(c + dx)/2]) / (105a^2d(b + a \cos[c + dx])^2 \sec[c + dx]^{3/2})$$

fricas [F] time = 0.63, size = 0, normalized size = 0.00

integral $\left((Bb \cos(dx + c)^3 \sec(dx + c)^2 + Aa \cos(dx + c)^3 + (Ba + Ab) \cos(dx + c)^3 \sec(dx + c)) \sqrt{b \sec(dx + c)} \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(dx+c)^(7/2)*(a+b*sec(dx+c))^(3/2)*(A+B*sec(dx+c)),x, algorithm="fricas")

[Out] integral((B*b*cos(dx + c)^3*sec(dx + c)^2 + A*a*cos(dx + c)^3 + (B*a + A*b)*cos(dx + c)^3*sec(dx + c))*sqrt(b*sec(dx + c) + a)*sqrt(cos(dx + c)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(b \sec(dx + c) + a)^{\frac{3}{2}} \cos(dx + c)^{\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(dx+c)^(7/2)*(a+b*sec(dx+c))^(3/2)*(A+B*sec(dx+c)),x, algorithm="giac")

[Out] integrate((B*sec(dx + c) + A)*(b*sec(dx + c) + a)^(3/2)*cos(dx + c)^(7/2), x)

maple [B] time = 2.15, size = 2326, normalized size = 6.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(dx+c)^(7/2)*(a+b*sec(dx+c))^(3/2)*(A+B*sec(dx+c)),x)

[Out] $\frac{2}{105d} \left(\frac{(b+a \cos(dx+c))}{\cos(dx+c)} \right)^{1/2} \cos(dx+c)^{1/2} (-1+\cos(dx+c)) * (1+\cos(dx+c)) * (39A * ((a-b)/(a+b))^{1/2} \cos(dx+c)^4 a^3 b * (1/(1+\cos(dx+c)))$

$$\begin{aligned}
&+c))^{1/2}+21*B*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*EllipticE((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c),(-a+b)/(a-b))^{1/2})*\sin(d*x+c)*a^2*b^2+6*A*((a-b)/(a+b))^{1/2}*b^4*(1/(1+\cos(d*x+c)))^{1/2}-21*B*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*EllipticE((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c),(-a+b)/(a-b))^{1/2})*\sin(d*x+c)*a*b^3+84*B*EllipticF((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c),(-a+b)/(a-b))^{1/2})*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*\sin(d*x+c)*a^3*b-21*B*EllipticF((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c),(-a+b)/(a-b))^{1/2})*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*\sin(d*x+c)*a^2*b^2+27*A*((a-b)/(a+b))^{1/2}*\cos(d*x+c)^3*a^2*b^2*(1/(1+\cos(d*x+c)))^{1/2}-82*A*EllipticF((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c),(-a+b)/(a-b))^{1/2})*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*\sin(d*x+c)*a^3*b+51*A*EllipticF((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c),(-a+b)/(a-b))^{1/2})*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*\sin(d*x+c)*a^2*b^2+6*A*EllipticF((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c),(-a+b)/(a-b))^{1/2})*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*\sin(d*x+c)*a*b^3+82*A*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*EllipticE((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c),(-a+b)/(a-b))^{1/2})*\sin(d*x+c)*a^3*b-82*A*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*EllipticE((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c),(-a+b)/(a-b))^{1/2})*\sin(d*x+c)*a^2*b^2-6*A*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*EllipticE((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c),(-a+b)/(a-b))^{1/2})*\sin(d*x+c)*a^2*b^2-6*A*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*EllipticE((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c),(-a+b)/(a-b))^{1/2})*\sin(d*x+c)*a*b^3+63*B*((a-b)/(a+b))^{1/2}*\cos(d*x+c)^3*a^3*b*(1/(1+\cos(d*x+c)))^{1/2}+68*A*((a-b)/(a+b))^{1/2}*\cos(d*x+c)^2*a^3*b*(1/(1+\cos(d*x+c)))^{1/2}-3*A*((a-b)/(a+b))^{1/2}*\cos(d*x+c)^2*a*b^3*(1/(1+\cos(d*x+c)))^{1/2}+63*B*((a-b)/(a+b))^{1/2}*\cos(d*x+c)^2*a^2*b^2*(1/(1+\cos(d*x+c)))^{1/2}-82*A*((a-b)/(a+b))^{1/2}*\cos(d*x+c)*a^3*b*(1/(1+\cos(d*x+c)))^{1/2}+55*A*((a-b)/(a+b))^{1/2}*\cos(d*x+c)*a^2*b^2*(1/(1+\cos(d*x+c)))^{1/2}+6*A*((a-b)/(a+b))^{1/2}*\cos(d*x+c)*a*b^3*(1/(1+\cos(d*x+c)))^{1/2}-21*B*((a-b)/(a+b))^{1/2}*\cos(d*x+c)*a^2*b^2*(1/(1+\cos(d*x+c)))^{1/2}+21*B*((a-b)/(a+b))^{1/2}*\cos(d*x+c)*a*b^3*(1/(1+\cos(d*x+c)))^{1/2}-63*B*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*EllipticE((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c),(-a+b)/(a-b))^{1/2})*\sin(d*x+c)*a^3*b+25*A*EllipticF((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c),(-a+b)/(a-b))^{1/2})*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*\sin(d*x+c)*a^4+6*A*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*EllipticE((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c),(-a+b)/(a-b))^{1/2})*\sin(d*x+c)*b^4+21*B*((a-b)/(a+b))^{1/2}*\cos(d*x+c)^4*a^4*(1/(1+\cos(d*x+c)))^{1/2}+42*B*((a-b)/(a+b))^{1/2}*\cos(d*x+c)^2*a^4*(1/(1+\cos(d*x+c)))^{1/2}-6*A*((a-b)/(a+b))^{1/2}*\cos(d*x+c)*b^4*(1/(1+\cos(d*x+c)))^{1/2}-63*B*((a-b)/(a+b))^{1/2}*\cos(d*x+c)*a^4*(1/(1+\cos(d*x+c)))^{1/2}-25*A*((a-b)/(a+b))^{1/2}*a^3*b*(1/(1+\cos(d*x+c)))^{1/2}-82*A*((a-b)/(a+b))^{1/2}*a^2*b^2*(1/(1+\cos(d*x+c)))^{1/2}-3*A*((a-b)/(a+b))^{1/2}*a*b^3*(1/(1+\cos(d*x+c)))^{1/2}-63*B*((a-b)/(a+b))^{1/2}*a^3*b*(1/(1+\cos(d*x+c)))^{1/2}-42*B*((a-b)/(a+b))^{1/2}*a^2*b^2*(1/(1+\cos(d*x+c)))^{1/2}-21*B*((a-b)/(a+b))^{1/2}*a*b^3*(1/(1+\cos(d*x+c)))^{1/2}+10*A*((a-b)/(a+b))^{1/2}*\cos(d*x+c)^3*a^4*(1/(1+\cos(d*x+c)))^{1/2}-25*A*((a-b)/(a+b))^{1/2}*\cos(d*x+c)*a^4*(1/
\end{aligned}$$

$(1+\cos(dx+c))^{1/2}+63*B*((b+a*\cos(dx+c))/(1+\cos(dx+c)))/(a+b)^{1/2}*EllipticE((-1+\cos(dx+c))*((a-b)/(a+b))^{1/2}/\sin(dx+c),(-(a+b)/(a-b))^{1/2})*\sin(dx+c)*a^4-63*B*\sin(dx+c)*((b+a*\cos(dx+c))/(1+\cos(dx+c)))/(a+b)^{1/2}*EllipticF((-1+\cos(dx+c))*((a-b)/(a+b))^{1/2}/\sin(dx+c),(-(a+b)/(a-b))^{1/2}))*a^4+15*A*((a-b)/(a+b))^{1/2}*\cos(dx+c)^5*a^4*(1/(1+\cos(dx+c)))^{1/2})/a^2/((a-b)/(a+b))^{1/2}/(b+a*\cos(dx+c))/(1/(1+\cos(dx+c)))^{1/2}/\sin(dx+c)^3$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx+c) + A)(b \sec(dx+c) + a)^{\frac{3}{2}} \cos(dx+c)^{\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(dx+c)^(7/2)*(a+b*sec(dx+c))^(3/2)*(A+B*sec(dx+c)),x, algorith="maxima")

[Out] integrate((B*sec(dx+c) + A)*(b*sec(dx+c) + a)^(3/2)*cos(dx+c)^(7/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c+dx)^{7/2} \left(A + \frac{B}{\cos(c+dx)} \right) \left(a + \frac{b}{\cos(c+dx)} \right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c+d*x)^(7/2)*(A+B/cos(c+d*x))*(a+b/cos(c+d*x))^(3/2),x)

[Out] int(cos(c+d*x)^(7/2)*(A+B/cos(c+d*x))*(a+b/cos(c+d*x))^(3/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(dx+c)**(7/2)*(a+b*sec(dx+c))**(3/2)*(A+B*sec(dx+c)),x)

[Out] Timed out

$$3.602 \quad \int \cos^{\frac{5}{2}}(c+dx)(a+b \sec(c+dx))^{3/2}(A+B \sec(c+dx)) dx$$

Optimal. Leaf size=266

$$\frac{2(a^2 - b^2)(5aB + 3Ab)\sqrt{\frac{a \cos(c+dx)+b}{a+b}} F\left(\frac{1}{2}(c+dx) \middle| \frac{2a}{a+b}\right)}{15ad\sqrt{\cos(c+dx)}\sqrt{a+b \sec(c+dx)}} + \frac{2(9a^2A + 20abB + 3Ab^2)\sqrt{\cos(c+dx)}\sqrt{a+b \sec(c+dx)}}{15ad\sqrt{\frac{a \cos(c+dx)+b}{a+b}}}$$

[Out] $2/15*(a^2-b^2)*(3*A*b+5*B*a)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)}*(a/(a+b))^{(1/2)})*((b+a*\cos(d*x+c))/(a+b))^{(1/2)}/a/d/\cos(d*x+c)^{(1/2)}/(a+b*\sec(d*x+c))^{(1/2)}+2/5*a*A*\cos(d*x+c)^{(3/2)}*\sin(d*x+c)*(a+b*\sec(d*x+c))^{(1/2)}/d+2/15*(6*A*b+5*B*a)*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}*(a+b*\sec(d*x+c))^{(1/2)}/d+2/15*(9*A*a^2+3*A*b^2+20*B*a*b)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)}*(a/(a+b))^{(1/2)})*\cos(d*x+c)^{(1/2)}*(a+b*\sec(d*x+c))^{(1/2)}/a/d/((b+a*\cos(d*x+c))/(a+b))^{(1/2)}$

Rubi [A] time = 0.97, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2955, 4025, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2(a^2 - b^2)(5aB + 3Ab)\sqrt{\frac{a \cos(c+dx)+b}{a+b}} F\left(\frac{1}{2}(c+dx) \middle| \frac{2a}{a+b}\right)}{15ad\sqrt{\cos(c+dx)}\sqrt{a+b \sec(c+dx)}} + \frac{2(9a^2A + 20abB + 3Ab^2)\sqrt{\cos(c+dx)}\sqrt{a+b \sec(c+dx)}}{15ad\sqrt{\frac{a \cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]

[Out] $(2*(a^2 - b^2)*(3*A*b + 5*a*B)*\text{Sqrt}[(b + a*\cos[c + d*x])/(a + b)]*\text{EllipticF}[(c + d*x)/2, (2*a)/(a + b)]/(15*a*d*\text{Sqrt}[\cos[c + d*x]]*\text{Sqrt}[a + b*\sec[c + d*x]]) + (2*(9*a^2*A + 3*A*b^2 + 20*a*b*B)*\text{Sqrt}[\cos[c + d*x]]*\text{EllipticE}[(c + d*x)/2, (2*a)/(a + b)]*\text{Sqrt}[a + b*\sec[c + d*x]])/(15*a*d*\text{Sqrt}[(b + a*\cos[c + d*x])/(a + b)]) + (2*(6*A*b + 5*a*B)*\text{Sqrt}[\cos[c + d*x]]*\text{Sqrt}[a + b*\sec[c + d*x]]*\sin[c + d*x])/(15*d) + (2*a*A*\cos[c + d*x]^{(3/2)}*\text{Sqrt}[a + b*\sec[c + d*x]]*\sin[c + d*x])/(5*d)$

Rule 2653

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,

b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2655

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] :> Dist[Sqrt[a + b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2661

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)]/(d*Sqrt[a + b]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2663

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b) + (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2955

Int[((a_) + csc[(e_) + (f_)*(x_)])*(b_)^(m_)*(csc[(e_) + (f_)*(x_)]*(d_) + (c_))^(n_)*((g_)*sin[(e_) + (f_)*(x_)])^(p_), x_Symbol] :> Dist[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && IntegerQ[n])

Rule 3856

Int[Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)]/Sqrt[csc[(e_) + (f_)*(x_)]*(d_)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 3858

Int[Sqrt[csc[(e_) + (f_)*(x_)]*(d_)]/Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> Simp[(a*A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e + f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.))/(Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]), x_Symbol] :> Dist[A/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \cos^{\frac{5}{2}}(c+dx)(a+b \sec(c+dx))^{3/2}(A+B \sec(c+dx)) dx &= \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}\right) \int \frac{(a+b \sec(c+dx))^{3/2}(A+B \sec(c+dx))}{\cos^{\frac{5}{2}}(c+dx)} dx \\
&= \frac{2aA \cos^{\frac{3}{2}}(c+dx) \sqrt{a+b \sec(c+dx)} \sin(c+dx)}{5d} \\
&= \frac{2(6Ab+5aB) \sqrt{\cos(c+dx)} \sqrt{a+b \sec(c+dx)} \sin(c+dx)}{15d} \\
&= \frac{2(6Ab+5aB) \sqrt{\cos(c+dx)} \sqrt{a+b \sec(c+dx)} \sin(c+dx)}{15d} \\
&= \frac{2(6Ab+5aB) \sqrt{\cos(c+dx)} \sqrt{a+b \sec(c+dx)} \sin(c+dx)}{15d} \\
&= \frac{2(6Ab+5aB) \sqrt{\cos(c+dx)} \sqrt{a+b \sec(c+dx)} \sin(c+dx)}{15d} \\
&= \frac{2(a^2-b^2)(3Ab+5aB) \sqrt{\frac{b+a \cos(c+dx)}{a+b}} F\left(\frac{1}{2}(c+dx)\right)}{15ad \sqrt{\cos(c+dx)} \sqrt{a+b \sec(c+dx)}}
\end{aligned}$$

Mathematica [C] time = 14.54, size = 369, normalized size = 1.39

$$\cos^{\frac{3}{2}}(c+dx)(a+b \sec(c+dx))^{3/2} \left(2 \sin(c+dx)(a \cos(c+dx)+b)(3aA \cos(c+dx)+5aB+6Ab) - \frac{2(\cos^2(\frac{1}{2}(c+dx)))}{\dots} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]

[Out] (Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2)*(2*(b + a*Cos[c + d*x])*(6*A*b + 5*a*B + 3*a*A*Cos[c + d*x])*Sin[c + d*x] - (2*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*((-I)*(a + b)*(9*a^2*A + 3*A*b^2 + 20*a*b*B))*EllipticE[I*Ar

cSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + I*a*(a + b)*(3*b*(A + 5*B) + a*(9*A + 5*B))*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - (9*a^2*A + 3*A*b^2 + 20*a*b*B)*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2))/(a*Sec[c + d*x]^(3/2)))/(15*d*(b + a*Cos[c + d*x])^2)

fricas [F] time = 0.81, size = 0, normalized size = 0.00

integral((Bb cos(dx + c)^2 sec(dx + c)^2 + Aa cos(dx + c)^2 + (Ba + Ab) cos(dx + c)^2 sec(dx + c))sqrt(b sec(dx + c) + a)sqrt(cos(dx + c)), x)

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] integral((B*b*cos(d*x + c)^2*sec(d*x + c)^2 + A*a*cos(d*x + c)^2 + (B*a + A*b)*cos(d*x + c)^2*sec(d*x + c))*sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(b \sec(dx + c) + a)^{\frac{3}{2}} \cos(dx + c)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(5/2), x)

maple [B] time = 2.79, size = 1749, normalized size = 6.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x)

[Out] 2/15/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)*(-1+cos(d*x+c))*(1+cos(d*x+c))*(25*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^2*b*(1/(1+cos(d*x+c)))^(1/2)-3*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a*b^2*(1/(1+cos(d*x+c)))^(1/2)-20*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^2*b*(1/(1+cos(d*x+c)))^(1/2)+20*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a*b^2*(1/(1+cos(d*x+c)))^(1/2)-20*B*sin(d*x+c))*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))

$$\frac{1}{2}) * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{1/2} * a^2 * b + 20 * B * \sin(d*x+c) * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{1/2} * \text{EllipticE}((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{1/2} / \sin(d*x+c), (-a+b)/(a-b))^{1/2}) * a^2 * b - 20 * B * \sin(d*x+c) * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{1/2} * \text{EllipticE}((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{1/2} / \sin(d*x+c), (-a+b)/(a-b))^{1/2}) * a * b^2 + 12 * A * \sin(d*x+c) * \text{EllipticF}((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{1/2} / \sin(d*x+c), (-a+b)/(a-b))^{1/2}) * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{1/2} * a^2 * b - 3 * A * \sin(d*x+c) * \text{EllipticF}((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{1/2} / \sin(d*x+c), (-a+b)/(a-b))^{1/2}) * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{1/2} * a * b^2 - 9 * A * \sin(d*x+c) * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{1/2} * \text{EllipticE}((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{1/2} / \sin(d*x+c), (-a+b)/(a-b))^{1/2}) * a^2 * b + 3 * A * \sin(d*x+c) * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{1/2} * \text{EllipticE}((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{1/2} / \sin(d*x+c), (-a+b)/(a-b))^{1/2}) * a * b^2 + 9 * A * ((a-b)/(a+b))^{1/2} * \cos(d*x+c)^3 * a^2 * b * (1/(1+\cos(d*x+c)))^{1/2} + 9 * A * ((a-b)/(a+b))^{1/2} * \cos(d*x+c)^2 * a * b^2 * (1/(1+\cos(d*x+c)))^{1/2} - 3 * A * ((a-b)/(a+b))^{1/2} * b^3 * (1/(1+\cos(d*x+c)))^{1/2} + 5 * B * (1/(1+\cos(d*x+c)))^{1/2} * ((a-b)/(a+b))^{1/2} * \cos(d*x+c)^3 * a^3 + 5 * B * \sin(d*x+c) * \text{EllipticF}((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{1/2} / \sin(d*x+c), (-a+b)/(a-b))^{1/2}) * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{1/2} * a^3 - 5 * B * ((a-b)/(a+b))^{1/2} * \cos(d*x+c) * a^3 * (1/(1+\cos(d*x+c)))^{1/2} + 3 * A * ((a-b)/(a+b))^{1/2} * \cos(d*x+c)^4 * a^3 * (1/(1+\cos(d*x+c)))^{1/2} + 6 * A * ((a-b)/(a+b))^{1/2} * \cos(d*x+c)^2 * a^3 * (1/(1+\cos(d*x+c)))^{1/2} - 9 * A * ((a-b)/(a+b))^{1/2} * \cos(d*x+c) * a^3 * (1/(1+\cos(d*x+c)))^{1/2} + 3 * A * ((a-b)/(a+b))^{1/2} * \cos(d*x+c) * b^3 * (1/(1+\cos(d*x+c)))^{1/2} - 9 * A * ((a-b)/(a+b))^{1/2} * a^2 * b * (1/(1+\cos(d*x+c)))^{1/2} - 6 * A * ((a-b)/(a+b))^{1/2} * a * b^2 * (1/(1+\cos(d*x+c)))^{1/2} - 5 * B * ((a-b)/(a+b))^{1/2} * a^2 * b * (1/(1+\cos(d*x+c)))^{1/2} - 20 * B * ((a-b)/(a+b))^{1/2} * a * b^2 * (1/(1+\cos(d*x+c)))^{1/2} - 9 * A * \sin(d*x+c) * \text{EllipticF}((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{1/2} / \sin(d*x+c), (-a+b)/(a-b))^{1/2}) * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{1/2} * a^3 + 9 * A * \sin(d*x+c) * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{1/2} * \text{EllipticE}((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{1/2} / \sin(d*x+c), (-a+b)/(a-b))^{1/2}) * a^3 - 3 * A * \sin(d*x+c) * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{1/2} * \text{EllipticE}((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{1/2} / \sin(d*x+c), (-a+b)/(a-b))^{1/2}) * b^3 + 15 * B * \sin(d*x+c) * \text{EllipticF}((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{1/2} / \sin(d*x+c), (-a+b)/(a-b))^{1/2}) * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{1/2} * a * b^2 / a / ((a-b)/(a+b))^{1/2} / (b+a*\cos(d*x+c)) / (1/(1+\cos(d*x+c)))^{1/2} / \sin(d*x+c)^3$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(b \sec(dx + c) + a)^{\frac{3}{2}} \cos(dx + c)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(5/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c + dx)^{5/2} \left(A + \frac{B}{\cos(c + dx)} \right) \left(a + \frac{b}{\cos(c + dx)} \right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(c + d*x)^(5/2)*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(3/2), x)`

[Out] `int(cos(c + d*x)^(5/2)*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(3/2), x)`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)**(5/2)*(a+b*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)), x)`

[Out] Timed out

$$3.603 \quad \int \cos^2(c+dx)(a+b \sec(c+dx))^{3/2}(A+B \sec(c+dx)) dx$$

Optimal. Leaf size=276

$$\frac{2(a^2A + 3abB - Ab^2) \sqrt{\frac{a \cos(c+dx)+b}{a+b}} F\left(\frac{1}{2}(c+dx) \middle| \frac{2a}{a+b}\right)}{3d \sqrt{\cos(c+dx)} \sqrt{a+b \sec(c+dx)}} + \frac{2(3aB + 4Ab) \sqrt{\cos(c+dx)} \sqrt{a+b \sec(c+dx)} E\left(\frac{1}{2}(c+dx) \middle| \frac{2a}{a+b}\right)}{3d \sqrt{\frac{a \cos(c+dx)+b}{a+b}}}$$

[Out] $2/3*(A*a^2-A*b^2+3*B*a*b)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*E$
 $llipticF(\sin(1/2*d*x+1/2*c), 2^{(1/2)}*(a/(a+b))^{(1/2)})*((b+a*\cos(d*x+c))/(a+b$
 $)^{(1/2)}/d/\cos(d*x+c)^{(1/2)}/(a+b*\sec(d*x+c))^{(1/2)+2*b^2*B*(\cos(1/2*d*x+1/2$
 $*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*EllipticPi(\sin(1/2*d*x+1/2*c), 2, 2^{(1/2)}*(a/$
 $(a+b))^{(1/2)})*((b+a*\cos(d*x+c))/(a+b))^{(1/2)}/d/\cos(d*x+c)^{(1/2)}/(a+b*\sec(d*$
 $x+c))^{(1/2)+2/3*a*A*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}*(a+b*\sec(d*x+c))^{(1/2)}/d+2/$
 $3*(4*A*b+3*B*a)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*EllipticE(s$
 $\sin(1/2*d*x+1/2*c), 2^{(1/2)}*(a/(a+b))^{(1/2)})*\cos(d*x+c)^{(1/2)}*(a+b*\sec(d*x+c)$
 $)^{(1/2)}/d/((b+a*\cos(d*x+c))/(a+b))^{(1/2)}$

Rubi [A] time = 1.09, antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {2955, 4025, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2(a^2A + 3abB - Ab^2) \sqrt{\frac{a \cos(c+dx)+b}{a+b}} F\left(\frac{1}{2}(c+dx) \middle| \frac{2a}{a+b}\right)}{3d \sqrt{\cos(c+dx)} \sqrt{a+b \sec(c+dx)}} + \frac{2(3aB + 4Ab) \sqrt{\cos(c+dx)} \sqrt{a+b \sec(c+dx)} E\left(\frac{1}{2}(c+dx) \middle| \frac{2a}{a+b}\right)}{3d \sqrt{\frac{a \cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Cos}[c + d*x]^{(3/2)}*(a + b*\text{Sec}[c + d*x])^{(3/2)}*(A + B*\text{Sec}[c + d*x]), x]$

[Out] $(2*(a^2*A - A*b^2 + 3*a*b*B)*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/(a + b)]*\text{EllipticF}[(c + d*x)/2, (2*a)/(a + b)])/(3*d*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]) + (2*b^2*B*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/(a + b)]*\text{EllipticPi}[2, (c + d*x)/2, (2*a)/(a + b)])/(d*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]) + (2*(4*A*b + 3*a*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, (2*a)/(a + b)]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]])/(3*d*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/(a + b)]) + (2*a*A*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(3*d)$

Rule 2653

$\text{Int}[\text{Sqrt}[(a_) + (b_)*\sin[(c_) + (d_)*(x_)]], x_Symbol] \rightarrow \text{Simp}[(2*\text{Sqrt}[a + b]*\text{EllipticE}[(1*(c - \text{Pi}/2 + d*x))/2, (2*b)/(a + b)]/d, x] /; \text{FreeQ}\{a,$

b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2655

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a + b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2661

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)]/(d*Sqrt[a + b]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2663

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b) + (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2805

Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi/2 + f*x))/2, (2*d)/(c + d)]/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[c + d, 0]

Rule 2807

Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !GtQ[c + d, 0]

Rule 2955

Int[((a_) + csc[(e_) + (f_)*(x_)]*(b_))^(m_)*(csc[(e_) + (f_)*(x_)]*(d_) + (c_))^(n_)*((g_)*sin[(e_) + (f_)*(x_)])^(p_), x_Symbol] := Dist[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && In

tegerQ[n])

Rule 3856

Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 3858

Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 3859

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 4025

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] :> Simp[(a*A*Co t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dis t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a *(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e + f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LeQ[n, -1]

Rule 4035

Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)]), x_Symbol] :> Dist[A/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.
))/ (Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.
+ (a_.))], x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/ (Sqrt[d*Csc[e + f*x]]*Sqrt[a
+ b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \cos^{\frac{3}{2}}(c + dx)(a + b \sec(c + dx))^{3/2}(A + B \sec(c + dx)) dx &= \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{(a + b \sec(c + dx))^{3/2}}{\sec(c + dx)} dx \\
&= \frac{2aA\sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{3d} \\
&= \frac{2aA\sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{3d} \\
&= \frac{2aA\sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{3d} \\
&= \frac{2aA\sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{3d} \\
&= \frac{2b^2B\sqrt{\frac{b+a\cos(c+dx)}{a+b}} \Pi\left(2; \frac{1}{2}(c + dx) \middle| \frac{2a}{a+b}\right) + 2aA\sqrt{\cos(c + dx)}}{d\sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}} \\
&= \frac{2(a^2A - Ab^2 + 3abB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}} F\left(\frac{1}{2}(c + dx) \middle| \frac{2a}{a+b}\right)}{3d\sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}}
\end{aligned}$$

Mathematica [C] time = 34.22, size = 45958, normalized size = 166.51

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]), x]
```

[Out] Result too large to show

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(b \sec(dx + c) + a)^{\frac{3}{2}} \cos(dx + c)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/2), x)

maple [C] time = 2.20, size = 1429, normalized size = 5.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x)

[Out]
$$\frac{2}{3}d \cdot \left(\frac{b+a \cos(dx+c)}{\cos(dx+c)} \right)^{\frac{1}{2}} \cos(dx+c)^{\frac{1}{2}} (-1+\cos(dx+c)) \cdot (1+\cos(dx+c)) \cdot (A \cos(dx+c)^3 \left(\frac{a-b}{a+b} \right)^{\frac{1}{2}} a^2 \left(\frac{1}{1+\cos(dx+c)} \right)^{\frac{1}{2}} + 5A \cos(dx+c)^2 \left(\frac{a-b}{a+b} \right)^{\frac{1}{2}} a \cdot b \left(\frac{1}{1+\cos(dx+c)} \right)^{\frac{1}{2}} + 3B \cos(dx+c)^2 \left(\frac{a-b}{a+b} \right)^{\frac{1}{2}} a^2 \left(\frac{1}{1+\cos(dx+c)} \right)^{\frac{1}{2}} + 4A \left(\frac{b+a \cos(dx+c)}{1+\cos(dx+c)} \right)^{\frac{1}{2}} \operatorname{EllipticE} \left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}, \left(\frac{a-b}{a+b} \right)^{\frac{1}{2}} \right) \cdot \sin(dx+c) \cdot a \cdot b - 4A \left(\frac{b+a \cos(dx+c)}{1+\cos(dx+c)} \right)^{\frac{1}{2}} \operatorname{EllipticE} \left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}, \left(\frac{a-b}{a+b} \right)^{\frac{1}{2}} \right) \cdot \sin(dx+c) \cdot b^2 + A \operatorname{EllipticF} \left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}, \left(\frac{a-b}{a+b} \right)^{\frac{1}{2}} \right) \cdot \sin(dx+c), \left(\frac{a-b}{a+b} \right)^{\frac{1}{2}} \left(\frac{b+a \cos(dx+c)}{1+\cos(dx+c)} \right)^{\frac{1}{2}} \cdot \sin(dx+c) \cdot a^2 - 4A \operatorname{EllipticF} \left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}, \left(\frac{a-b}{a+b} \right)^{\frac{1}{2}} \right) \cdot \sin(dx+c), \left(\frac{a-b}{a+b} \right)^{\frac{1}{2}} \left(\frac{b+a \cos(dx+c)}{1+\cos(dx+c)} \right)^{\frac{1}{2}} \cdot \sin(dx+c) \cdot a \cdot b + 3A \sin(dx+c) \operatorname{EllipticF} \left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}, \left(\frac{a-b}{a+b} \right)^{\frac{1}{2}} \right) \cdot \sin(dx+c), \left(\frac{a-b}{a+b} \right)^{\frac{1}{2}} \left(\frac{b+a \cos(dx+c)}{1+\cos(dx+c)} \right)^{\frac{1}{2}} \cdot b^2 - A \cos(dx+c) \left(\frac{a-b}{a+b} \right)^{\frac{1}{2}} a^2 \left(\frac{1}{1+\cos(dx+c)} \right)^{\frac{1}{2}}$$

```

s(d*x+c)))^(1/2)-4*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b*(1/(1+cos(d*x+c)))^(
(1/2)+4*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^2*(1/(1+cos(d*x+c)))^(1/2)+3*B*(
(b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-
b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a^2-3*B*((b+a*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+
b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a*b-3*B*((b+a*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1
/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a^2+6*B*EllipticF((-1+cos(d
*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*a*b-3*B*sin(d*x+c)*((b+a*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/
2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2+6*B*sin(d*x+c)*((b+a*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b^2-3*B*cos(d*x+c)*((a-b)/(a+b)
)^(1/2)*a^2*(1/(1+cos(d*x+c)))^(1/2)+3*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b
*(1/(1+cos(d*x+c)))^(1/2)-A*((a-b)/(a+b))^(1/2)*a*b*(1/(1+cos(d*x+c)))^(1/2
)-4*A*((a-b)/(a+b))^(1/2)*b^2*(1/(1+cos(d*x+c)))^(1/2)-3*B*((a-b)/(a+b))^(1
/2)*a*b*(1/(1+cos(d*x+c)))^(1/2))/((a-b)/(a+b))^(1/2)/(b+a*cos(d*x+c))/(1/(
1+cos(d*x+c)))^(1/2)/sin(d*x+c)^3

```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(b \sec(dx + c) + a)^{\frac{3}{2}} \cos(dx + c)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorith="maxima")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c + dx)^{3/2} \left(A + \frac{B}{\cos(c + dx)} \right) \left(a + \frac{b}{\cos(c + dx)} \right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^(3/2)*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(3/2),x)

[Out] int(cos(c + d*x)^(3/2)*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(3/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+b*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

$$3.604 \quad \int \sqrt{\cos(c+dx)} (a+b \sec(c+dx))^{3/2} (A+B \sec(c+dx)) dx$$

Optimal. Leaf size=272

$$\frac{(2a^2B + 2aAb + b^2B) \sqrt{\frac{a \cos(c+dx)+b}{a+b}} F\left(\frac{1}{2}(c+dx) \middle| \frac{2a}{a+b}\right)}{d \sqrt{\cos(c+dx)} \sqrt{a+b \sec(c+dx)}} + \frac{(2aA - bB) \sqrt{\cos(c+dx)} \sqrt{a+b \sec(c+dx)} E\left(\frac{1}{2}(c+dx)\right)}{d \sqrt{\frac{a \cos(c+dx)+b}{a+b}}}$$

[Out] $(2*A*a*b+2*B*a^2+B*b^2)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)}*(a/(a+b))^{(1/2)})*((b+a*\cos(d*x+c))/(a+b))^{(1/2)}/d/\cos(d*x+c)^{(1/2)}/(a+b*\sec(d*x+c))^{(1/2)}+b*(2*A*b+3*B*a)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticPi}(\sin(1/2*d*x+1/2*c), 2^{(1/2)}*(a/(a+b))^{(1/2)})*((b+a*\cos(d*x+c))/(a+b))^{(1/2)}/d/\cos(d*x+c)^{(1/2)}/(a+b*\sec(d*x+c))^{(1/2)}+b*B*\sin(d*x+c)*(a+b*\sec(d*x+c))^{(1/2)}/d/\cos(d*x+c)^{(1/2)}+(2*A*a-B*b)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)}*(a/(a+b))^{(1/2)})*\cos(d*x+c)^{(1/2)}*(a+b*\sec(d*x+c))^{(1/2)}/d/((b+a*\cos(d*x+c))/(a+b))^{(1/2)}$

Rubi [A] time = 1.02, antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {2955, 4026, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{(2a^2B + 2aAb + b^2B) \sqrt{\frac{a \cos(c+dx)+b}{a+b}} F\left(\frac{1}{2}(c+dx) \middle| \frac{2a}{a+b}\right)}{d \sqrt{\cos(c+dx)} \sqrt{a+b \sec(c+dx)}} + \frac{(2aA - bB) \sqrt{\cos(c+dx)} \sqrt{a+b \sec(c+dx)} E\left(\frac{1}{2}(c+dx)\right)}{d \sqrt{\frac{a \cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sqrt}[\text{Cos}[c + d*x]]*(a + b*\text{Sec}[c + d*x])^{(3/2)}*(A + B*\text{Sec}[c + d*x]), x]$

[Out] $((2*a*A*b + 2*a^2*B + b^2*B)*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/(a + b)]*\text{EllipticF}[(c + d*x)/2, (2*a)/(a + b)]/(d*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]) + (b*(2*A*b + 3*a*B)*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/(a + b)]*\text{EllipticPi}[2, (c + d*x)/2, (2*a)/(a + b)]/(d*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]) + ((2*a*A - b*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, (2*a)/(a + b)]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]])/(d*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/(a + b)]) + (b*B*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(d*\text{Sqrt}[\text{Cos}[c + d*x]])$

Rule 2653

$\text{Int}[\text{Sqrt}[(a_) + (b_)*\sin[(c_) + (d_)*(x_)]], x_Symbol] \rightarrow \text{Simp}[(2*\text{Sqrt}[a + b]*\text{EllipticE}[(1*(c - \text{Pi}/2 + d*x))/2, (2*b)/(a + b)])/d, x] /;$ FreeQ[{a,

b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2655

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a + b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2661

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)]/(d*Sqrt[a + b]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2663

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b) + (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2805

Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi/2 + f*x))/2, (2*d)/(c + d)]/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[c + d, 0]

Rule 2807

Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !GtQ[c + d, 0]

Rule 2955

Int[((a_) + csc[(e_) + (f_)*(x_)])*(b_)^(m_)*(csc[(e_) + (f_)*(x_)]*(d_) + (c_))^(n_)*((g_)*sin[(e_) + (f_)*(x_)])^(p_), x_Symbol] := Dist[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && In

tegerQ[n])

Rule 3856

Int[Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 3858

Int[Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 3859

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 4026

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> -Simp[(b*B*Cos[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x] + Dist[1/(m + n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*Simp[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B)*(m + n) + b^2*B*(m + n - 1))*Csc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && !(IGtQ[n, 1] && !IntegerQ[m])

Rule 4035

Int[(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.))/(Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]), x_Symbol] :> Dist[A/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/ (Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
+ (a_))], x_Symbol] := Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
+ b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{\cos(c+dx)} (a+b \sec(c+dx))^{3/2} (A+B \sec(c+dx)) dx &= \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{(a+b \sec(c+dx))^{3/2} (A+B \sec(c+dx))}{\sqrt{\cos(c+dx)}} dx \\
&= \frac{bB \sqrt{a+b \sec(c+dx)} \sin(c+dx)}{d \sqrt{\cos(c+dx)}} + \left(\sqrt{\cos(c+dx)} \right) \int \frac{(a+b \sec(c+dx))^{3/2} (A+B \sec(c+dx))}{\sqrt{\cos(c+dx)}} dx \\
&= \frac{bB \sqrt{a+b \sec(c+dx)} \sin(c+dx)}{d \sqrt{\cos(c+dx)}} + \left(\sqrt{\cos(c+dx)} \right) \int \frac{(a+b \sec(c+dx))^{3/2} (A+B \sec(c+dx))}{\sqrt{\cos(c+dx)}} dx \\
&= \frac{bB \sqrt{a+b \sec(c+dx)} \sin(c+dx)}{d \sqrt{\cos(c+dx)}} + \frac{1}{2} \left((2aA - b^2B) \sqrt{\cos(c+dx)} \right) \\
&= \frac{bB \sqrt{a+b \sec(c+dx)} \sin(c+dx)}{d \sqrt{\cos(c+dx)}} - \frac{\left((-2aAb - b^2B) \sqrt{\cos(c+dx)} \right)}{d \sqrt{\cos(c+dx)}} \\
&= \frac{b(2Ab + 3aB) \sqrt{\frac{b+a \cos(c+dx)}{a+b}} \Pi \left(2; \frac{1}{2}(c+dx) \middle| \frac{2a}{a+b} \right)}{d \sqrt{\cos(c+dx)} \sqrt{a+b \sec(c+dx)}} \\
&= \frac{(2aAb + 2a^2B + b^2B) \sqrt{\frac{b+a \cos(c+dx)}{a+b}} F \left(\frac{1}{2}(c+dx) \right)}{d \sqrt{\cos(c+dx)} \sqrt{a+b \sec(c+dx)}}
\end{aligned}$$

Mathematica [C] time = 33.18, size = 66581, normalized size = 244.78

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]), x]
```

[Out] Result too large to show

fricas [F] time = 6.50, size = 0, normalized size = 0.00

$\text{integral}\left(\left(Bb \sec(dx+c)^2 + Aa + (Ba + Ab) \sec(dx+c)\right) \sqrt{b \sec(dx+c) + a} \sqrt{\cos(dx+c)}, x\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))*cos(d*x+c)^(1/2),x, algorithm="fricas")`

[Out] `integral((B*b*sec(d*x + c)^2 + A*a + (B*a + A*b)*sec(d*x + c))*sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c)), x)`

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx+c) + A)(b \sec(dx+c) + a)^{\frac{3}{2}} \sqrt{\cos(dx+c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))*cos(d*x+c)^(1/2),x, algorithm="giac")`

[Out] `integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c)), x)`

maple [C] time = 2.83, size = 1410, normalized size = 5.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))*cos(d*x+c)^(1/2),x)`

[Out] `-1/d*(-1+cos(d*x+c))*(1+cos(d*x+c))*(-2*A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*(1/(1+cos(d*x+c)))^(1/2)+2*A*sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^2-4*A*sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*b+2*A*sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*b^2-2*A*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2+2*A*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-4*A*sin(d*x+c)*cos(d*`

$x+c) * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)) / (a+b))^{1/2} * \text{EllipticPi}((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{1/2} / \sin(d*x+c), (a+b)/(a-b), I / ((a-b)/(a+b))^{1/2}) * b^2 + 2 * A * ((a-b)/(a+b))^{1/2} * \cos(d*x+c)^2 * (1/(1+\cos(d*x+c)))^{1/2} * a^2 - 2 * A * \cos(d*x+c)^2 * ((a-b)/(a+b))^{1/2} * a * b * (1/(1+\cos(d*x+c)))^{1/2} - 2 * B * \sin(d*x+c) * \cos(d*x+c) * \text{EllipticF}((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{1/2} / \sin(d*x+c), (- (a+b)/(a-b))^{1/2}) * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)) / (a+b))^{1/2} * a^2 + 2 * B * \sin(d*x+c) * \cos(d*x+c) * \text{EllipticF}((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{1/2} / \sin(d*x+c), (- (a+b)/(a-b))^{1/2}) * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)) / (a+b))^{1/2} * a * b + B * \sin(d*x+c) * \cos(d*x+c) * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)) / (a+b))^{1/2} * \text{EllipticE}((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{1/2} / \sin(d*x+c), (- (a+b)/(a-b))^{1/2}) * a * b - B * \sin(d*x+c) * \cos(d*x+c) * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)) / (a+b))^{1/2} * \text{EllipticE}((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{1/2} / \sin(d*x+c), (- (a+b)/(a-b))^{1/2}) * b^2 - 6 * B * \sin(d*x+c) * \cos(d*x+c) * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)) / (a+b))^{1/2} * \text{EllipticPi}((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{1/2} / \sin(d*x+c), (a+b)/(a-b), I / ((a-b)/(a+b))^{1/2}) * a * b - B * \cos(d*x+c)^2 * ((a-b)/(a+b))^{1/2} * a * b * (1/(1+\cos(d*x+c)))^{1/2} + 2 * A * \cos(d*x+c) * ((a-b)/(a+b))^{1/2} * a * b * (1/(1+\cos(d*x+c)))^{1/2} + B * \cos(d*x+c) * ((a-b)/(a+b))^{1/2} * a * b * (1/(1+\cos(d*x+c)))^{1/2} - B * ((a-b)/(a+b))^{1/2} * \cos(d*x+c) * (1/(1+\cos(d*x+c)))^{1/2} * b^2 + B * ((a-b)/(a+b))^{1/2} * (1/(1+\cos(d*x+c)))^{1/2} * b^2 * ((b+a*\cos(d*x+c))/\cos(d*x+c))^{1/2} / ((a-b)/(a+b))^{1/2} / (b+a*\cos(d*x+c)) / (1/(1+\cos(d*x+c)))^{1/2} / \cos(d*x+c)^{1/2} / \sin(d*x+c)^3$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(b \sec(dx + c) + a)^{\frac{3}{2}} \sqrt{\cos(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))*cos(d*x+c)^(1/2),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{\cos(c + dx)} \left(A + \frac{B}{\cos(c + dx)} \right) \left(a + \frac{b}{\cos(c + dx)} \right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^(1/2)*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(3/2),x)

[Out] int(cos(c + d*x)^(1/2)*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(3/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c))*cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```


$$3.605 \quad \int \frac{(a+b \sec(c+dx))^{3/2}(A+B \sec(c+dx))}{\sqrt{\cos(c+dx)}} dx$$

Optimal. Leaf size=339

$$\frac{(8a^2A + 7abB + 4Ab^2) \sqrt{\frac{a \cos(c+dx)+b}{a+b}} F\left(\frac{1}{2}(c+dx) \middle| \frac{2a}{a+b}\right)}{4d\sqrt{\cos(c+dx)} \sqrt{a+b \sec(c+dx)}} + \frac{(3a^2B + 12aAb + 4b^2B) \sqrt{\frac{a \cos(c+dx)+b}{a+b}} \Pi\left(2; \frac{1}{2}(c+dx)\right)}{4d\sqrt{\cos(c+dx)} \sqrt{a+b \sec(c+dx)}}$$

[Out] $1/4*(8*A*a^2+4*A*b^2+7*B*a*b)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)}*(a/(a+b))^{(1/2)})*((b+a*\cos(d*x+c))/(a+b))^{(1/2)}/d/\cos(d*x+c)^{(1/2)}/(a+b*\sec(d*x+c))^{(1/2)}+1/4*(12*A*a*b+3*B*a^2+4*B*b^2)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticPi}(\sin(1/2*d*x+1/2*c), 2^{(1/2)}*(a/(a+b))^{(1/2)})*((b+a*\cos(d*x+c))/(a+b))^{(1/2)}/d/\cos(d*x+c)^{(1/2)}/(a+b*\sec(d*x+c))^{(1/2)}+1/2*b*B*\sin(d*x+c)*(a+b*\sec(d*x+c))^{(1/2)}/d/\cos(d*x+c)^{(3/2)}+1/4*(4*A*b+5*B*a)*\sin(d*x+c)*(a+b*\sec(d*x+c))^{(1/2)}/d/\cos(d*x+c)^{(1/2)}-1/4*(4*A*b+5*B*a)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)}*(a/(a+b))^{(1/2)})*\cos(d*x+c)^{(1/2)}*(a+b*\sec(d*x+c))^{(1/2)}/d/((b+a*\cos(d*x+c))/(a+b))^{(1/2)}$

Rubi [A] time = 1.42, antiderivative size = 339, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 14, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2955, 4026, 4102, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{(8a^2A + 7abB + 4Ab^2) \sqrt{\frac{a \cos(c+dx)+b}{a+b}} F\left(\frac{1}{2}(c+dx) \middle| \frac{2a}{a+b}\right)}{4d\sqrt{\cos(c+dx)} \sqrt{a+b \sec(c+dx)}} + \frac{(3a^2B + 12aAb + 4b^2B) \sqrt{\frac{a \cos(c+dx)+b}{a+b}} \Pi\left(2; \frac{1}{2}(c+dx)\right)}{4d\sqrt{\cos(c+dx)} \sqrt{a+b \sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\frac{(a + b*\text{Sec}[c + d*x])^{(3/2)}*(A + B*\text{Sec}[c + d*x])}{\text{Sqrt}[\text{Cos}[c + d*x]]}, x]$

[Out] $((8*a^2*A + 4*A*b^2 + 7*a*b*B)*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/(a + b)]*\text{EllipticF}[(c + d*x)/2, (2*a)/(a + b)]/(4*d*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]) + ((12*a*A*b + 3*a^2*B + 4*b^2*B)*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/(a + b)]*\text{EllipticPi}[2, (c + d*x)/2, (2*a)/(a + b)]/(4*d*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]) - ((4*A*b + 5*a*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, (2*a)/(a + b)]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]])/(4*d*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/(a + b)]) + (b*B*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(2*d*\text{Cos}[c + d*x]^{(3/2)}) + ((4*A*b + 5*a*B)*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(4*d*\text{Sqrt}[\text{Cos}[c + d*x]])$

Rule 2653

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*Sqrt[a
+ b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)]/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
0] && !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)]/(d*Sqrt[a + b]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a
+ b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_)
+ (f_)*(x_)]]), x_Symbol] := Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)]/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_)
+ (f_)*(x_)]]), x_Symbol] := Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
+ f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !GtQ[c + d, 0]
```

Rule 2955

```
Int[((a_) + csc[(e_) + (f_)*(x_)]*(b_))^(m_)*(csc[(e_) + (f_)*(x_)]*
(d_) + (c_))^(n_)*((g_)*sin[(e_) + (f_)*(x_)])^(p_), x_Symbol] := Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
```

*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && IntegerQ[n])

Rule 3856

Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 3858

Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 3859

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 4026

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] :> -Simp[(b*B*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x] + Dist[1/(m + n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*Simp[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B)*(m + n) + b^2*B*(m + n - 1))*Csc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && !(IGtQ[n, 1] && !IntegerQ[m])

Rule 4035

Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)]), x_Symbol] :> Dist[A/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]

Rule 4102

```

Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]

```

Rule 4108

```

Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.)
+ (a_)])], x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
+ b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]

```

Rubi steps

$$\begin{aligned}
\int \frac{(a + b \sec(c + dx))^{3/2} (A + B \sec(c + dx))}{\sqrt{\cos(c + dx)}} dx &= \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \sqrt{\sec(c + dx)} (a + b \sec(c + dx)) dx \\
&= \frac{bB \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{2d \cos^{3/2}(c + dx)} + \frac{1}{2} \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \sqrt{\sec(c + dx)} dx \\
&= \frac{bB \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{2d \cos^{3/2}(c + dx)} + \frac{(4Ab + 5aB) \sqrt{a + b \sec(c + dx)}}{4d \sqrt{\cos(c + dx)}} \\
&= \frac{bB \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{2d \cos^{3/2}(c + dx)} + \frac{(4Ab + 5aB) \sqrt{a + b \sec(c + dx)}}{4d \sqrt{\cos(c + dx)}} \\
&= \frac{bB \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{2d \cos^{3/2}(c + dx)} + \frac{(4Ab + 5aB) \sqrt{a + b \sec(c + dx)}}{4d \sqrt{\cos(c + dx)}} \\
&= \frac{bB \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{2d \cos^{3/2}(c + dx)} + \frac{(4Ab + 5aB) \sqrt{a + b \sec(c + dx)}}{4d \sqrt{\cos(c + dx)}} \\
&= \frac{bB \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{2d \cos^{3/2}(c + dx)} + \frac{(4Ab + 5aB) \sqrt{a + b \sec(c + dx)}}{4d \sqrt{\cos(c + dx)}} \\
&= \frac{(12aAb + 3a^2B + 4b^2B) \sqrt{\frac{b+a \cos(c+dx)}{a+b}} \Pi\left(2; \frac{1}{2}(c + dx) \middle| \frac{2a}{a+b}\right)}{4d \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}} + \frac{(12a^2A + 4Ab^2 + 7abB) \sqrt{\frac{b+a \cos(c+dx)}{a+b}} F\left(\frac{1}{2}(c + dx) \middle| \frac{2a}{a+b}\right)}{4d \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}} + \frac{(12a^2A + 4Ab^2 + 7abB) \sqrt{\frac{b+a \cos(c+dx)}{a+b}} F\left(\frac{1}{2}(c + dx) \middle| \frac{2a}{a+b}\right)}{4d \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}} + \frac{(12a^2A + 4Ab^2 + 7abB) \sqrt{\frac{b+a \cos(c+dx)}{a+b}} F\left(\frac{1}{2}(c + dx) \middle| \frac{2a}{a+b}\right)}{4d \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}}
\end{aligned}$$

Mathematica [C] time = 33.62, size = 79375, normalized size = 234.14

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[((a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]))/Sqrt[Cos[c + d*x]], x]

[Out] Result too large to show

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(b \sec(dx + c) + a)^{\frac{3}{2}}}{\sqrt{\cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)/sqrt(cos(d*x + c)), x)
```

maple [C] time = 2.56, size = 1659, normalized size = 4.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x)
```

```
[Out] 1/4/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(1+cos(d*x+c))*(4
*A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a*b*(1/(1+cos(d*x+c)))^(1/2)+24*A*((b+a
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/
(a+b))^(1/2)/sin(d*x+c), (a+b)/(a-b), I/((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d
*x+c)^2*a*b+8*A*cos(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), (-a+b)/(a-b))^(1/
2))*sin(d*x+c)*a^2-8*A*cos(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), (-a+b)/(a-
b))^(1/2))*sin(d*x+c)*a*b-4*A*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)
/sin(d*x+c), (-a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*sin(d*x+c)*cos(d*x+c)^2*a*b+4*A*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b)
)^(1/2)/sin(d*x+c), (-a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*b^2+5*B*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)
)*a^2*(1/(1+cos(d*x+c)))^(1/2)+2*B*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a*b*(1/
(1+cos(d*x+c)))^(1/2)+6*B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ell
ipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), (a+b)/(a-b), I/((a-b)
/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^2+8*B*((b+a*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c)
```

, (a+b)/(a-b), I/((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*b^2+2*B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), (-a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*a^2+2*B*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), (-a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a*b-4*B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), (-a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*b^2-5*B*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), (-a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*a^2+5*B*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), (-a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*a*b-4*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b*(1/(1+cos(d*x+c)))^(1/2)+4*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*b^2*(1/(1+cos(d*x+c)))^(1/2)-5*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*(1/(1+cos(d*x+c)))^(1/2)+5*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b*(1/(1+cos(d*x+c)))^(1/2)+2*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*b^2*(1/(1+cos(d*x+c)))^(1/2)-4*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^2*(1/(1+cos(d*x+c)))^(1/2)-7*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b*(1/(1+cos(d*x+c)))^(1/2)-2*B*((a-b)/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b^2)/((a-b)/(a+b))^(1/2)/(b+a*cos(d*x+c))/(1/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^3/cos(d*x+c)^(3/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(b \sec(dx + c) + a)^{\frac{3}{2}}}{\sqrt{\cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)/sqrt(cos(d*x + c)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(a + \frac{b}{\cos(c+dx)}\right)^{3/2}}{\sqrt{\cos(c+dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(3/2))/cos(c + d*x)^(1/2),x)

[Out] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(3/2))/cos(c + d*x)^(1/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c))/cos(d*x+c)**(1/2),x)

[Out] Timed out

$$3.606 \quad \int \frac{(a+b \sec(c+dx))^{3/2}(A+B \sec(c+dx))}{\cos^2(c+dx)} dx$$

Optimal. Leaf size=421

$$\frac{(3a^2B + 30aAb + 16b^2B) \sin(c + dx) \sqrt{a + b \sec(c + dx)}}{24bd \sqrt{\cos(c + dx)}} + \frac{(17a^2B + 42aAb + 16b^2B) \sqrt{\frac{a \cos(c+dx)+b}{a+b}} F\left(\frac{1}{2}(c + dx)\right)}{24d \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}}$$

[Out] 1/24*(42*A*a*b+17*B*a^2+16*B*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+1/8*(6*A*a^2*b+8*A*b^3-B*a^3+12*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/b/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+1/3*b*B*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)+1/12*(6*A*b+7*B*a)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+1/24*(30*A*a*b+3*B*a^2+16*B*b^2)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/b/d/cos(d*x+c)^(1/2)-1/24*(30*A*a*b+3*B*a^2+16*B*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/b/d/((b+a*cos(d*x+c))/(a+b))^(1/2)

Rubi [A] time = 1.80, antiderivative size = 421, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 14, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2955, 4026, 4102, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{(3a^2B + 30aAb + 16b^2B) \sin(c + dx) \sqrt{a + b \sec(c + dx)}}{24bd \sqrt{\cos(c + dx)}} + \frac{(17a^2B + 42aAb + 16b^2B) \sqrt{\frac{a \cos(c+dx)+b}{a+b}} F\left(\frac{1}{2}(c + dx)\right)}{24d \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[((a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]))/Cos[c + d*x]^(3/2),x]

[Out] ((42*a*A*b + 17*a^2*B + 16*b^2*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]/(24*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + ((6*a^2*A*b + 8*A*b^3 - a^3*B + 12*a*b^2*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]/(8*b*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - ((30*a*A*b + 3*a^2*B + 16*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(24*b*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (b*B*Sqrt[a + b*Sec[c + d*x]])*Sin[c + d*x]/(3*d*Cos[c + d*x]^(5/2)) + ((6*A*b + 7*a*B)*Sqrt[a + b*Sec[c + d*x]])/d

$$c[c + d*x]]*Sin[c + d*x]]/(12*d*Cos[c + d*x]^(3/2)) + ((30*a*A*b + 3*a^2*B + 16*b^2*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x]]/(24*b*d*Sqrt[Cos[c + d*x]]))$$

Rule 2653

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*Sqrt[a + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a + b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b) + (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !GtQ[c + d, 0]
```

Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)])*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_.))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] := Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)])*(b_.) + (a_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] := Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)])*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_.)], x_Symbol] := Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)])*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_.)], x_Symbol] := Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4026

```
Int[(csc[(e_.) + (f_.)*(x_)])*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] := -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(m + n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*Sim
p[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B))*(m + n) + b^2*B*(m + n - 1))*C
sc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b
^2, 0] && GtQ[m, 1] && !(IGtQ[n, 1] && !IntegerQ[m])
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)]), x_Symbol] := Dist[A/a, In
```

$\int \frac{\sqrt{a + b \csc[e + f x]}}{\sqrt{d \csc[e + f x]}} dx - \text{Dist}[(A b - a B) / (a d), \int \frac{\sqrt{d \csc[e + f x]}}{\sqrt{a + b \csc[e + f x]}} dx, x] /;$
 $\text{FreeQ}\{a, b, d, e, f, A, B\}, x\} \&\& \text{NeQ}[A b - a B, 0] \&\& \text{NeQ}[a^2 - b^2, 0]$

Rule 4102

$\int ((A _) + \csc[(e _) + (f _)(x _)](B _) + \csc[(e _) + (f _)(x _)]^2(C _)) \cdot (\csc[(e _) + (f _)(x _)](d _))^{(n _)} \cdot (\csc[(e _) + (f _)(x _)](b _) + (a _))^{(m _)}, x_Symbol] :> -\text{Simp}[(C d \cot[e + f x] (a + b \csc[e + f x])^{(m + 1)} (d \csc[e + f x])^{(n - 1)}) / (b f (m + n + 1)), x] + \text{Dist}[d / (b (m + n + 1)), \int (a + b \csc[e + f x])^m (d \csc[e + f x])^{(n - 1)} \text{Simp}[a C (n - 1) + (A b (m + n + 1) + b C (m + n)) \csc[e + f x] + (b B (m + n + 1) - a C n) \csc[e + f x]^2, x], x] /;$
 $\text{FreeQ}\{a, b, d, e, f, A, B, C, m\}, x\} \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{GtQ}[n, 0]$

Rule 4108

$\int ((A _) + \csc[(e _) + (f _)(x _)](B _) + \csc[(e _) + (f _)(x _)]^2(C _)) / (\sqrt{\csc[(e _) + (f _)(x _)](d _)} \sqrt{\csc[(e _) + (f _)(x _)](b _) + (a _)}) dx :> \text{Dist}[C / d^2, \int (d \csc[e + f x])^{(3/2)} / \sqrt{a + b \csc[e + f x]} dx, x] + \int (A + B \csc[e + f x]) / (\sqrt{d \csc[e + f x]} \sqrt{a + b \csc[e + f x]}) dx /;$
 $\text{FreeQ}\{a, b, d, e, f, A, B, C\}, x\} \&\& \text{NeQ}[a^2 - b^2, 0]$

Rubi steps

$$\begin{aligned}
\int \frac{(a + b \sec(c + dx))^{3/2} (A + B \sec(c + dx))}{\cos^{\frac{3}{2}}(c + dx)} dx &= \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \sec^{\frac{3}{2}}(c + dx) (a + b \sec(c + dx)) dx \\
&= \frac{bB \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{3d \cos^{\frac{5}{2}}(c + dx)} + \frac{1}{3} \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \sec^{\frac{3}{2}}(c + dx) dx \\
&= \frac{bB \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{3d \cos^{\frac{5}{2}}(c + dx)} + \frac{(6Ab + 7aB) \sqrt{a + b \sec(c + dx)}}{12d \cos^{\frac{3}{2}}(c + dx)} \\
&= \frac{bB \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{3d \cos^{\frac{5}{2}}(c + dx)} + \frac{(6Ab + 7aB) \sqrt{a + b \sec(c + dx)}}{12d \cos^{\frac{3}{2}}(c + dx)} \\
&= \frac{bB \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{3d \cos^{\frac{5}{2}}(c + dx)} + \frac{(6Ab + 7aB) \sqrt{a + b \sec(c + dx)}}{12d \cos^{\frac{3}{2}}(c + dx)} \\
&= \frac{bB \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{3d \cos^{\frac{5}{2}}(c + dx)} + \frac{(6Ab + 7aB) \sqrt{a + b \sec(c + dx)}}{12d \cos^{\frac{3}{2}}(c + dx)} \\
&= \frac{bB \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{3d \cos^{\frac{5}{2}}(c + dx)} + \frac{(6Ab + 7aB) \sqrt{a + b \sec(c + dx)}}{12d \cos^{\frac{3}{2}}(c + dx)} \\
&= \frac{bB \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{3d \cos^{\frac{5}{2}}(c + dx)} + \frac{(6Ab + 7aB) \sqrt{a + b \sec(c + dx)}}{12d \cos^{\frac{3}{2}}(c + dx)} \\
&= \frac{(6a^2 Ab + 8Ab^3 - a^3 B + 12ab^2 B) \sqrt{\frac{b+a \cos(c+dx)}{a+b}} \Pi\left(2; \frac{1}{2}(c + dx) \middle| \frac{2a}{a+b}\right)}{8bd \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}} \\
&= \frac{(42aAb + 17a^2 B + 16b^2 B) \sqrt{\frac{b+a \cos(c+dx)}{a+b}} F\left(\frac{1}{2}(c + dx) \middle| \frac{2a}{a+b}\right)}{24d \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}} + \dots
\end{aligned}$$

Mathematica [C] time = 33.90, size = 104716, normalized size = 248.73

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[((a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]))/Cos[c + d*x]^(3/2),x]

[Out] Result too large to show

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(b \sec(dx + c) + a)^{\frac{3}{2}}}{\cos(dx + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)/cos(d*x + c)^(3/2), x)

maple [C] time = 3.86, size = 2351, normalized size = 5.58

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2),x)

[Out] 1/24/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(1+cos(d*x+c))*(-17*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^2*b*(1/(1+cos(d*x+c)))^(1/2)+48*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*cos(d*x+c)^3*b^3-8*B*((a-b)/(a+b))^(1/2)*b^3*(1/(1+cos(d*x+c)))^(1/2)-22*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a*b^2*(1/(1+cos(d*x+c)))^(1/2)-30*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*a^2*b*(1/(1+cos(d*x+c)))^(1/2)-42*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a*b^2*(1/(1+cos(d*x+c)))^(1/2)+3*B*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^3*(1/(1+cos(d*x+c)))^(1/2)+16*B*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*b^3*(1/(1+cos(d*x+c)))^(1/2)-8*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*b^3*(1/(1+cos(d*x+c)))^(1/2)-3*B*(1/(1+cos(d*x+c)))^(1/2)*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*a^3-12*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)*b^3*(1/(1+cos(d*x+c)))^(1/2)

$$\begin{aligned}
& 1/2)+16*B*\sin(d*x+c)*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*Elliptic \\
& E((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c),(-(a+b)/(a-b))^{1/2})*\cos(\\
& d*x+c)^3*b^3+12*A*\cos(d*x+c)^3*((a-b)/(a+b))^{1/2}*b^3*(1/(1+\cos(d*x+c)))^{1/2} \\
& -24*A*\sin(d*x+c)*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*Elliptic \\
& F((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c),(-(a+b)/(a-b))^{1/2})*\cos(\\
& d*x+c)^3*b^3+30*A*\cos(d*x+c)^3*((a-b)/(a+b))^{1/2}*a*b^2*(1/(1+\cos(d*x+c))) \\
& ^{1/2}+3*B*\cos(d*x+c)^3*((a-b)/(a+b))^{1/2}*a^2*b*(1/(1+\cos(d*x+c)))^{1/2}+ \\
& 6*B*\cos(d*x+c)^3*((a-b)/(a+b))^{1/2}*a*b^2*(1/(1+\cos(d*x+c)))^{1/2}+30*A*\cos(\\
& d*x+c)^4*((a-b)/(a+b))^{1/2}*a^2*b*(1/(1+\cos(d*x+c)))^{1/2}+12*A*\cos(d*x+c) \\
& ^4*((a-b)/(a+b))^{1/2}*a*b^2*(1/(1+\cos(d*x+c)))^{1/2}+14*B*\cos(d*x+c)^4*(\\
& (a-b)/(a+b))^{1/2}*a^2*b*(1/(1+\cos(d*x+c)))^{1/2}+16*B*\cos(d*x+c)^4*((a-b)/ \\
& (a+b))^{1/2}*a*b^2*(1/(1+\cos(d*x+c)))^{1/2}-6*B*\sin(d*x+c)*((b+a*\cos(d*x+c) \\
&)/(1+\cos(d*x+c))/(a+b))^{1/2}*EllipticPi((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2} \\
&)/\sin(d*x+c), (a+b)/(a-b), I/((a-b)/(a+b))^{1/2})*\cos(d*x+c)^3*a^3+6*B*\sin(d* \\
& x+c)*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*EllipticF((-1+\cos(d*x+c) \\
&)*((a-b)/(a+b))^{1/2}/\sin(d*x+c),(-(a+b)/(a-b))^{1/2})*\cos(d*x+c)^3*a^3-3*B \\
& *\sin(d*x+c)*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*EllipticE((-1+\cos \\
& (d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c),(-(a+b)/(a-b))^{1/2})*\cos(d*x+c)^3* \\
& a^3+72*B*\sin(d*x+c)*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*EllipticP \\
& i((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (a+b)/(a-b), I/((a-b)/(a+b) \\
&)^{1/2})*\cos(d*x+c)^3*a*b^2+14*B*\sin(d*x+c)*((b+a*\cos(d*x+c))/(1+\cos(d*x+c) \\
&)/(a+b))^{1/2}*EllipticF((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c),(-(\\
& a+b)/(a-b))^{1/2})*\cos(d*x+c)^3*a^2*b-20*B*\sin(d*x+c)*((b+a*\cos(d*x+c))/(1+ \\
& \cos(d*x+c))/(a+b))^{1/2}*EllipticF((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(\\
& d*x+c),(-(a+b)/(a-b))^{1/2})*\cos(d*x+c)^3*a*b^2+3*B*\sin(d*x+c)*((b+a*\cos(d* \\
& x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*EllipticE((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2} \\
&)/\sin(d*x+c),(-(a+b)/(a-b))^{1/2})*\cos(d*x+c)^3*a^2*b-16*B*\sin(d*x+c)*((\\
& b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*EllipticE((-1+\cos(d*x+c))*((a-b) \\
&)/(a+b))^{1/2}/\sin(d*x+c),(-(a+b)/(a-b))^{1/2})*\cos(d*x+c)^3*a*b^2+36*A*\sin \\
& (d*x+c)*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2}*EllipticPi((-1+\cos(d* \\
& x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (a+b)/(a-b), I/((a-b)/(a+b))^{1/2})*\cos \\
& (d*x+c)^3*a^2*b+12*A*\sin(d*x+c)*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2} \\
& *EllipticF((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c),(-(a+b)/(a-b))^{1/2} \\
&)*\cos(d*x+c)^3*a^2*b+12*A*\sin(d*x+c)*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/ \\
& (a+b))^{1/2}*EllipticF((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c),(-(a+ \\
& b)/(a-b))^{1/2})*\cos(d*x+c)^3*a*b^2-30*A*\sin(d*x+c)*((b+a*\cos(d*x+c))/(1+co \\
& s(d*x+c))/(a+b))^{1/2}*EllipticE((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d* \\
& x+c),(-(a+b)/(a-b))^{1/2})*\cos(d*x+c)^3*a^2*b+30*A*\sin(d*x+c)*((b+a*\cos(d*x \\
& +c))/(1+\cos(d*x+c))/(a+b))^{1/2}*EllipticE((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2} \\
&)/\sin(d*x+c),(-(a+b)/(a-b))^{1/2})*\cos(d*x+c)^3*a*b^2)/b/((a-b)/(a+b))^{1/2} \\
& /((b+a*\cos(d*x+c))/\cos(d*x+c)^{5/2}/(1/(1+\cos(d*x+c)))^{1/2}/\sin(d*x+c)^3
\end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(b \sec(dx + c) + a)^{\frac{3}{2}}}{\cos(dx + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)/cos(d*x + c)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(a + \frac{b}{\cos(c+dx)}\right)^{3/2}}{\cos(c+dx)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(3/2))/cos(c + d*x)^(3/2),x)

[Out] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(3/2))/cos(c + d*x)^(3/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c))/cos(d*x+c)**(3/2),x)

[Out] Timed out

$$3.607 \quad \int \cos^{\frac{11}{2}}(c+dx)(a+b \sec(c+dx))^{5/2}(A+B \sec(c+dx)) dx$$

Optimal. Leaf size=519

$$\frac{2(81a^2A + 209abB + 113Ab^2) \sin(c+dx) \cos^{\frac{5}{2}}(c+dx) \sqrt{a+b \sec(c+dx)}}{693d} + \frac{2(539a^3B + 1145a^2Ab + 825ab^2B)}{693d}$$

```
[Out] 2/11*a*A*cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^(3/2)*sin(d*x+c)/d+2/3465*(a^2-b^2)*(675*A*a^4+285*A*a^2*b^2+40*A*b^4+1254*B*a^3*b-110*B*a*b^3)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/a^3/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2/3465*(1145*A*a^2*b+15*A*b^3+539*B*a^3+825*B*a*b^2)*cos(d*x+c)^(3/2)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/a/d+2/693*(81*A*a^2+113*A*b^2+209*B*a*b)*cos(d*x+c)^(5/2)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d+2/99*a*(14*A*b+11*B*a)*cos(d*x+c)^(7/2)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d+2/3465*(675*A*a^4+1025*A*a^2*b^2-20*A*b^4+1793*B*a^3*b+55*B*a*b^3)*sin(d*x+c)*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/a^2/d+2/3465*(3705*A*a^4*b+255*A*a^2*b^3+40*A*b^5+1617*B*a^5+3069*B*a^3*b^2-110*B*a*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/a^3/d/((b+a*cos(d*x+c))/(a+b))^(1/2)
```

Rubi [A] time = 2.17, antiderivative size = 519, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$, Rules used = {2955, 4025, 4094, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2(81a^2A + 209abB + 113Ab^2) \sin(c+dx) \cos^{\frac{5}{2}}(c+dx) \sqrt{a+b \sec(c+dx)}}{693d} + \frac{2(1145a^2Ab + 539a^3B + 825ab^2B)}{693d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(11/2)*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*(a^2 - b^2)*(675*a^4*A + 285*a^2*A*b^2 + 40*A*b^4 + 1254*a^3*b*B - 110*a*b^3*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]/(3465*a^3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*(3705*a^4*A*b + 255*a^2*A*b^3 + 40*A*b^5 + 1617*a^5*B + 3069*a^3*b^2*B - 110*a*b^4*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Se
```

$$\begin{aligned} & c[c + d*x]]/(3465*a^3*d*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/(a + b)]) + (2*(675*a^4* \\ & A + 1025*a^2*A*b^2 - 20*A*b^4 + 1793*a^3*b*B + 55*a*b^3*B)*\text{Sqrt}[\text{Cos}[c + d*x] \\ &]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(3465*a^2*d) + (2*(1145*a^2*A*b + \\ & 15*A*b^3 + 539*a^3*B + 825*a*b^2*B)*\text{Cos}[c + d*x]^{(3/2)}*\text{Sqrt}[a + b*\text{Sec}[c + \\ & d*x]]*\text{Sin}[c + d*x])/(3465*a*d) + (2*(81*a^2*A + 113*A*b^2 + 209*a*b*B)*\text{Cos}[\\ & c + d*x]^{(5/2)}*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(693*d) + (2*a*(14*A* \\ & b + 11*a*B)*\text{Cos}[c + d*x]^{(7/2)}*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(99*d \\ &) + (2*a*A*\text{Cos}[c + d*x]^{(9/2)}*(a + b*\text{Sec}[c + d*x])^{(3/2)}*\text{Sin}[c + d*x])/(11* \\ & d) \end{aligned}$$

Rule 2653

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*Sqrt[a
+ b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
0] && !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)]/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a
+ b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && !GtQ[a + b, 0]
```

Rule 2955

```
Int[((a_) + csc[(e_) + (f_)*(x_)]*(b_))^(m_)*(csc[(e_) + (f_)*(x_)]*
(d_) + (c_))^(n_)*((g_)*sin[(e_) + (f_)*(x_)])^(p_), x_Symbol] := Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] := Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_.)], x_Symbol] := Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] := Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)]), x_Symbol] := Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_.))^(m_), x_Symbol] := Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
+ f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4104

```

Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.
))*csc[(e_.) + (f_.)*(x_.)]*(d_.)^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]

```

Rubi steps

$$\begin{aligned}
\int \cos^{\frac{11}{2}}(c+dx)(a+b \sec(c+dx))^{5/2}(A+B \sec(c+dx)) dx &= \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}\right) \int \frac{(a+b \sec(c+dx))^{5/2}(A+B \sec(c+dx))}{\cos^{\frac{11}{2}}(c+dx)} dx \\
&= \frac{2aA \cos^{\frac{9}{2}}(c+dx)(a+b \sec(c+dx))^{3/2} \sin(c+dx)}{11d} \\
&= \frac{2a(14Ab+11aB) \cos^{\frac{7}{2}}(c+dx) \sqrt{a+b \sec(c+dx)}}{99d} \\
&= \frac{2(81a^2A+113Ab^2+209abB) \cos^{\frac{5}{2}}(c+dx) \sqrt{a+b \sec(c+dx)}}{693d} \\
&= \frac{2(1145a^2Ab+15Ab^3+539a^3B+825ab^2B) \cos^{\frac{3}{2}}(c+dx) \sqrt{a+b \sec(c+dx)}}{3465ad} \\
&= \frac{2(675a^4A+1025a^2Ab^2-20Ab^4+1793a^3bB+125a^2b^2B) \cos^{\frac{1}{2}}(c+dx) \sqrt{a+b \sec(c+dx)}}{3465ad} \\
&= \frac{2(675a^4A+1025a^2Ab^2-20Ab^4+1793a^3bB+125a^2b^2B) \cos^{\frac{1}{2}}(c+dx) \sqrt{a+b \sec(c+dx)}}{3465ad} \\
&= \frac{2(675a^4A+1025a^2Ab^2-20Ab^4+1793a^3bB+125a^2b^2B) \cos^{\frac{1}{2}}(c+dx) \sqrt{a+b \sec(c+dx)}}{3465ad} \\
&= \frac{2(a^2-b^2)(675a^4A+285a^2Ab^2+40Ab^4+125a^2b^2B) \cos^{\frac{1}{2}}(c+dx) \sqrt{a+b \sec(c+dx)}}{3465a^3d \sqrt{\cos(c+dx)}}
\end{aligned}$$

Mathematica [C] time = 20.64, size = 626, normalized size = 1.21

$$\cos^{\frac{5}{2}}(c+dx)(a+b \sec(c+dx))^{5/2} \left(\frac{(513a^2A+836abB+452Ab^2) \sin(3(c+dx))}{5544} + \frac{1}{88} a^2 A \sin(5(c+dx)) + \frac{(1463a^3B+3095a^2Ab+125a^2b^2B) \cos(5(c+dx))}{3465ad} \right)$$

$d(a \cos(c+dx) + b \sec(c+dx))^{5/2}$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(11/2)*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2)*(((6525*a^4*A + 9330*a^2*A*b^2 - 160*A*b^4 + 16434*a^3*b*B + 440*a*b^3*B)*Sin[c + d*x])/(13860*a^2) + ((3095*a^2*A*b + 30*A*b^3 + 1463*a^3*B + 1650*a*b^2*B)*Sin[2*(c + d*x)])/(6930*a) + ((513*a^2*A + 452*A*b^2 + 836*a*b*B)*Sin[3*(c + d*x)]/5544 + (a*(23*A*b + 11*a*B)*Sin[4*(c + d*x)]/396 + (a^2*A*Ssin[5*(c + d*x)]/88))/(d*(b + a*cos[c + d*x])^2) - (2*cos[c + d*x]^(3/2)*(cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*(a + b*Sec[c + d*x])^(5/2)*((-I)*(a + b)*(3705*a^4*A*b + 255*a^2*A*b^3 + 40*A*b^5 + 1617*a^5*B + 3069*a^3*b^2*B - 110*a*b^4*B)*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + I*a*(a + b)*(40*A*b^4 - 10*a*b^3*(3*A + 11*B) + 15*a^2*b^2*(19*A + 121*B) + 6*a^3*b*(505*A + 209*B) + 3*a^4*(225*A + 539*B))*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - (3705*a^4*A*b + 255*a^2*A*b^3 + 40*A*b^5 + 1617*a^5*B + 3069*a^3*b^2*B - 110*a*b^4*B)*(b + a*cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/(3465*a^3*d*(b + a*cos[c + d*x])^3*Sec[c + d*x]^(5/2))
```

fricas [F] time = 1.33, size = 0, normalized size = 0.00

$$\text{integral} \left((Bb^2 \cos(dx + c))^5 \sec(dx + c)^3 + Aa^2 \cos(dx + c)^5 + (2Bab + Ab^2) \cos(dx + c)^5 \sec(dx + c)^2 + (Ba^2 \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(11/2)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^5*sec(d*x + c)^3 + A*a^2*cos(d*x + c)^5 + (2*B*a*b + A*b^2)*cos(d*x + c)^5*sec(d*x + c)^2 + (B*a^2 + 2*A*a*b)*cos(d*x + c)^5*sec(d*x + c))*sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(b \sec(dx + c) + a)^{\frac{5}{2}} \cos(dx + c)^{\frac{11}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(11/2)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(11/2), x)
```

maple [B] time = 4.77, size = 3816, normalized size = 7.35

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\int (\cos(dx+c)^{(11/2)} * (a+b*\sec(dx+c))^{(5/2)} * (A+B*\sec(dx+c)), x)$

[Out] $\frac{2}{3465}d*(b+a*\cos(dx+c))/\cos(dx+c)^{(1/2)}*\cos(dx+c)^{(1/2)}*(-1+\cos(dx+c))*(1+\cos(dx+c))*(2992*B*\cos(dx+c)^2*((a-b)/(a+b))^{(1/2)}*a^4*b^2*(1/(1+\cos(dx+c)))^{(1/2)}+1535*A*\cos(dx+c)*((a-b)/(a+b))^{(1/2)}*a^4*b^2*(1/(1+\cos(dx+c)))^{(1/2)}-40*A*((a-b)/(a+b))^{(1/2)}*b^6*(1/(1+\cos(dx+c)))^{(1/2)}-40*A*\sin(dx+c)*((b+a*\cos(dx+c))/(1+\cos(dx+c)))/(a+b)^{(1/2)}*EllipticE((-1+\cos(dx+c))*((a-b)/(a+b))^{(1/2)}/\sin(dx+c), (-a+b)/(a-b))^{(1/2)})*b^6-3069*B*((a-b)/(a+b))^{(1/2)}*a^3*b^3*(1/(1+\cos(dx+c)))^{(1/2)}-55*B*((a-b)/(a+b))^{(1/2)}*a^2*b^4*(1/(1+\cos(dx+c)))^{(1/2)}+110*B*((a-b)/(a+b))^{(1/2)}*a*b^5*(1/(1+\cos(dx+c)))^{(1/2)}+315*A*\cos(dx+c)^7*((a-b)/(a+b))^{(1/2)}*a^6*(1/(1+\cos(dx+c)))^{(1/2)}+90*A*\cos(dx+c)^5*((a-b)/(a+b))^{(1/2)}*a^6*(1/(1+\cos(dx+c)))^{(1/2)}+270*A*\cos(dx+c)^3*((a-b)/(a+b))^{(1/2)}*a^6*(1/(1+\cos(dx+c)))^{(1/2)}-675*A*\cos(dx+c)*((a-b)/(a+b))^{(1/2)}*a^6*(1/(1+\cos(dx+c)))^{(1/2)}-1617*B*\sin(dx+c)*EllipticF((-1+\cos(dx+c))*((a-b)/(a+b))^{(1/2)}/\sin(dx+c), (-a+b)/(a-b))^{(1/2)})*((b+a*\cos(dx+c))/(1+\cos(dx+c)))/(a+b)^{(1/2)}*a^6+385*B*\cos(dx+c)^6*((a-b)/(a+b))^{(1/2)}*a^6*(1/(1+\cos(dx+c)))^{(1/2)}+154*B*\cos(dx+c)^4*((a-b)/(a+b))^{(1/2)}*a^6*(1/(1+\cos(dx+c)))^{(1/2)}+1078*B*\cos(dx+c)^2*((a-b)/(a+b))^{(1/2)}*a^6*(1/(1+\cos(dx+c)))^{(1/2)}+40*A*\cos(dx+c)*((a-b)/(a+b))^{(1/2)}*b^6*(1/(1+\cos(dx+c)))^{(1/2)}-1617*B*\cos(dx+c)*((a-b)/(a+b))^{(1/2)}*a^6*(1/(1+\cos(dx+c)))^{(1/2)}-675*A*((a-b)/(a+b))^{(1/2)}*a^5*b*(1/(1+\cos(dx+c)))^{(1/2)}-3705*A*((a-b)/(a+b))^{(1/2)}*a^4*b^2*(1/(1+\cos(dx+c)))^{(1/2)}-1025*A*((a-b)/(a+b))^{(1/2)}*a^3*b^3*(1/(1+\cos(dx+c)))^{(1/2)}-255*A*((a-b)/(a+b))^{(1/2)}*a^2*b^4*(1/(1+\cos(dx+c)))^{(1/2)}+20*A*((a-b)/(a+b))^{(1/2)}*a*b^5*(1/(1+\cos(dx+c)))^{(1/2)}-1617*B*((a-b)/(a+b))^{(1/2)}*a^5*b*(1/(1+\cos(dx+c)))^{(1/2)}-1793*B*((a-b)/(a+b))^{(1/2)}*a^4*b^2*(1/(1+\cos(dx+c)))^{(1/2)}+675*A*\sin(dx+c)*EllipticF((-1+\cos(dx+c))*((a-b)/(a+b))^{(1/2)}/\sin(dx+c), (-a+b)/(a-b))^{(1/2)})*((b+a*\cos(dx+c))/(1+\cos(dx+c)))/(a+b)^{(1/2)}*a^6-255*A*\cos(dx+c)*((a-b)/(a+b))^{(1/2)}*a^3*b^3*(1/(1+\cos(dx+c)))^{(1/2)}+260*A*\cos(dx+c)*((a-b)/(a+b))^{(1/2)}*a^2*b^4*(1/(1+\cos(dx+c)))^{(1/2)}-40*A*\cos(dx+c)*((a-b)/(a+b))^{(1/2)}*a*b^5*(1/(1+\cos(dx+c)))^{(1/2)}-715*B*\cos(dx+c)*((a-b)/(a+b))^{(1/2)}*a^5*b*(1/(1+\cos(dx+c)))^{(1/2)}-3069*B*\cos(dx+c)*((a-b)/(a+b))^{(1/2)}*a^4*b^2*(1/(1+\cos(dx+c)))^{(1/2)}+2189*B*\cos(dx+c)*((a-b)/(a+b))^{(1/2)}*a^3*b^3*(1/(1+\cos(dx+c)))^{(1/2)}+110*B*\cos(dx+c)*((a-b)/(a+b))^{(1/2)}*a^2*b^4*(1/(1+\cos(dx+c)))^{(1/2)}-110*B*\cos(dx+c)*((a-b)/(a+b))^{(1/2)}*a*b^5*(1/(1+\cos(dx+c)))^{(1/2)}+1430*B*\cos(dx+c)^5*((a-b)/(a+b))^{(1/2)}*a^5*b*(1/(1+\cos(dx+c)))^{(1/2)}+430*A*\cos(dx+c)^4*((a-b)/(a+b))^{(1/2)}*a^5*b*(1/(1+\cos(dx+c)))^{(1/2)}+580*A*\cos(dx+c)^4*((a-b)/(a+b))^{(1/2)}*a^3*b^3*(1/(1+\cos(dx+c)))^{(1/2)}+1870*B*\cos(dx+c)^4*((a-b)/(a+b))^{(1/2)}*a^4*b^2*(1/(1+\cos(dx+c)))^{(1/2)}-3705*A*\sin(dx$

+c), $(-(a+b)/(a-b))^{1/2} * a^6 + 2830 * A * \cos(dx+c)^2 * ((a-b)/(a+b))^{1/2} * a^5 * b * (1/(1+\cos(dx+c)))^{1/2} + 700 * A * \cos(dx+c)^2 * ((a-b)/(a+b))^{1/2} * a^3 * b^3 * (1/(1+\cos(dx+c)))^{1/2} + 20 * A * \cos(dx+c)^2 * ((a-b)/(a+b))^{1/2} * a * b^5 * (1/(1+\cos(dx+c)))^{1/2} - 55 * B * \cos(dx+c)^2 * ((a-b)/(a+b))^{1/2} * a^2 * b^4 * (1/(1+\cos(dx+c)))^{1/2} - 3705 * A * \cos(dx+c) * ((a-b)/(a+b))^{1/2} * a^5 * b * (1/(1+\cos(dx+c)))^{1/2} / a^3 / ((a-b)/(a+b))^{1/2} / (b+a * \cos(dx+c)) / (1/(1+\cos(dx+c)))^{1/2} / \sin(dx+c)^3$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx+c) + A)(b \sec(dx+c) + a)^{5/2} \cos(dx+c)^{11/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(dx+c)^(11/2)*(a+b*sec(dx+c))^(5/2)*(A+B*sec(dx+c)),x, algorith="maxima")

[Out] integrate((B*sec(dx+c) + A)*(b*sec(dx+c) + a)^(5/2)*cos(dx+c)^(11/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c+dx)^{11/2} \left(A + \frac{B}{\cos(c+dx)} \right) \left(a + \frac{b}{\cos(c+dx)} \right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c+dx)^(11/2)*(A+B/cos(c+dx))*(a+b/cos(c+dx))^(5/2),x)

[Out] int(cos(c+dx)^(11/2)*(A+B/cos(c+dx))*(a+b/cos(c+dx))^(5/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(dx+c)**(11/2)*(a+b*sec(dx+c))**(5/2)*(A+B*sec(dx+c)),x)

[Out] Timed out

$$3.608 \quad \int \cos^{\frac{9}{2}}(c+dx)(a+b \sec(c+dx))^{5/2}(A+B \sec(c+dx)) dx$$

Optimal. Leaf size=425

$$\frac{2(49a^2A + 135abB + 75Ab^2) \sin(c+dx) \cos^{\frac{3}{2}}(c+dx) \sqrt{a+b \sec(c+dx)}}{315d} + \frac{2(75a^3B + 163a^2Ab + 135ab^2B + 5A^2)}{315d}$$

[Out] $\frac{2}{9} a A \cos(d*x+c)^{(7/2)} (a+b*\sec(d*x+c))^{(3/2)} \sin(d*x+c) / d + \frac{2}{315} (a^2-b^2) * (114*A*a^2*b-10*A*b^3+75*B*a^3+45*B*a*b^2) * (\cos(1/2*d*x+1/2*c)^2)^{(1/2)} / \cos(1/2*d*x+1/2*c) * \text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)} * (a/(a+b))^{(1/2)}) * ((b+a*\cos(d*x+c))/(a+b))^{(1/2)} / a^2/d / \cos(d*x+c)^{(1/2)} / (a+b*\sec(d*x+c))^{(1/2)} + \frac{2}{315} (49*A*a^2+75*A*b^2+135*B*a*b) * \cos(d*x+c)^{(3/2)} * \sin(d*x+c) * (a+b*\sec(d*x+c))^{(1/2)} / d + \frac{2}{21} a * (4*A*b+3*B*a) * \cos(d*x+c)^{(5/2)} * \sin(d*x+c) * (a+b*\sec(d*x+c))^{(1/2)} / d + \frac{2}{315} (163*A*a^2*b+5*A*b^3+75*B*a^3+135*B*a*b^2) * \sin(d*x+c) * \cos(d*x+c)^{(1/2)} * (a+b*\sec(d*x+c))^{(1/2)} / a / d + \frac{2}{315} (147*A*a^4+279*A*a^2*b^2-10*A*b^4+435*B*a^3*b+45*B*a*b^3) * (\cos(1/2*d*x+1/2*c)^2)^{(1/2)} / \cos(1/2*d*x+1/2*c) * \text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)} * (a/(a+b))^{(1/2)}) * \cos(d*x+c)^{(1/2)} * (a+b*\sec(d*x+c))^{(1/2)} / a^2/d / ((b+a*\cos(d*x+c))/(a+b))^{(1/2)}$

Rubi [A] time = 1.72, antiderivative size = 425, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$, Rules used = {2955, 4025, 4094, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2(49a^2A + 135abB + 75Ab^2) \sin(c+dx) \cos^{\frac{3}{2}}(c+dx) \sqrt{a+b \sec(c+dx)}}{315d} + \frac{2(163a^2Ab + 75a^3B + 135ab^2B + 5A^2)}{315d}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^(9/2)*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]

[Out] $(2*(a^2 - b^2) * (114*a^2*A*b - 10*A*b^3 + 75*a^3*B + 45*a*b^2*B) * \text{Sqrt}[(b + a*\cos[c + d*x])/(a + b)] * \text{EllipticF}[(c + d*x)/2, (2*a)/(a + b)]) / (315*a^2*d*\text{Sqrt}[\cos[c + d*x]] * \text{Sqrt}[a + b*\sec[c + d*x]]) + (2*(147*a^4*A + 279*a^2*A*b^2 - 10*A*b^4 + 435*a^3*b*B + 45*a*b^3*B) * \text{Sqrt}[\cos[c + d*x]] * \text{EllipticE}[(c + d*x)/2, (2*a)/(a + b)] * \text{Sqrt}[a + b*\sec[c + d*x]]) / (315*a^2*d*\text{Sqrt}[(b + a*\cos[c + d*x])/(a + b)]) + (2*(163*a^2*A*b + 5*A*b^3 + 75*a^3*B + 135*a*b^2*B) * \text{Sqrt}[\cos[c + d*x]] * \text{Sqrt}[a + b*\sec[c + d*x]] * \sin[c + d*x]) / (315*a*d) + (2*(49*a^2*A + 75*A*b^2 + 135*a*b*B) * \cos[c + d*x]^(3/2) * \text{Sqrt}[a + b*\sec[c + d*x]]) *$

$\frac{\sin[c + dx]}{(315d) + (2a(4A^2b + 3a^2B)\cos[c + dx]^{5/2}\sqrt{a + b}\sec[c + dx])\sin[c + dx]}{(21d) + (2a^2A\cos[c + dx]^{7/2}(a + b\sec[c + dx])^{3/2}\sin[c + dx])}$

Rule 2653

$\text{Int}[\sqrt{(a) + (b)\sin(c) + (d)(x)}], x_Symbol] \rightarrow \text{Simp}[(2\sqrt{a + b}\text{EllipticE}[(1(c - \pi/2 + dx))/2, (2b)/(a + b)]/d, x] /; \text{FreeQ}\{a, b, c, d\}, x] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{GtQ}[a + b, 0]$

Rule 2655

$\text{Int}[\sqrt{(a) + (b)\sin(c) + (d)(x)}], x_Symbol] \rightarrow \text{Dist}[\sqrt{a + b\sin[c + dx]}/\sqrt{(a + b\sin[c + dx])/(a + b)}, \text{Int}[\sqrt{a/(a + b) + (b\sin[c + dx])/(a + b)}, x], x] /; \text{FreeQ}\{a, b, c, d\}, x] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{!GtQ}[a + b, 0]$

Rule 2661

$\text{Int}[1/\sqrt{(a) + (b)\sin(c) + (d)(x)}], x_Symbol] \rightarrow \text{Simp}[(2\text{EllipticF}[(1(c - \pi/2 + dx))/2, (2b)/(a + b)]/(d\sqrt{a + b}), x] /; \text{FreeQ}\{a, b, c, d\}, x] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{GtQ}[a + b, 0]$

Rule 2663

$\text{Int}[1/\sqrt{(a) + (b)\sin(c) + (d)(x)}], x_Symbol] \rightarrow \text{Dist}[\sqrt{(a + b\sin[c + dx])/(a + b)}/\sqrt{a + b\sin[c + dx]}, \text{Int}[1/\sqrt{a/(a + b) + (b\sin[c + dx])/(a + b)}, x], x] /; \text{FreeQ}\{a, b, c, d\}, x] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{!GtQ}[a + b, 0]$

Rule 2955

$\text{Int}[(a + \csc(e) + (f)(x))(b)^{(m)}(\csc(e) + (f)(x))(d + c)^{(n)}((g)\sin(e) + (f)(x))^{(p)}], x_Symbol] \rightarrow \text{Dist}[(g\csc[e + fx])^p(g\sin[e + fx])^p, \text{Int}[(a + b\csc[e + fx])^m(c + d\csc[e + fx])^n]/(g\csc[e + fx])^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, m, n, p\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{!IntegerQ}[p] \&\& \text{!(IntegerQ}[m] \&\& \text{IntegerQ}[n])$

Rule 3856

$\text{Int}[\sqrt{\csc(e) + (f)(x)}(b) + (a)]/\sqrt{\csc(e) + (f)(x)}(d), x_Symbol] \rightarrow \text{Dist}[\sqrt{a + b\csc[e + fx]}/(\sqrt{d\csc[e + fx]}\sqrt{b + a\sin[e + fx]}), \text{Int}[\sqrt{b + a\sin[e + fx]}, x], x] /; \text{FreeQ}\{a, b, d, e, f\}, x] \&\& \text{NeQ}[a^2 - b^2, 0]$

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.)
+ (a_.)], x_Symbol] := Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (
a_.))^(m_)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.))/(Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]), x_Symbol] := Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a
_.))^(m_), x_Symbol] := Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*CSc
[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Cs
c[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a
_.))^(m_), x_Symbol] := Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*Cs
c[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \cos^{\frac{9}{2}}(c+dx)(a+b \sec(c+dx))^{5/2}(A+B \sec(c+dx)) dx &= \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}\right) \int \frac{(a+b \sec(c+dx))^{5/2}(A+B \sec(c+dx))}{\sec(c+dx)} dx \\
&= \frac{2aA \cos^{\frac{7}{2}}(c+dx)(a+b \sec(c+dx))^{3/2} \sin(c+dx)}{9d} \\
&= \frac{2a(4Ab+3aB) \cos^{\frac{5}{2}}(c+dx) \sqrt{a+b \sec(c+dx)} \sin(c+dx)}{21d} \\
&= \frac{2(49a^2A+75Ab^2+135abB) \cos^{\frac{3}{2}}(c+dx) \sqrt{a+b \sec(c+dx)} \sin(c+dx)}{315d} \\
&= \frac{2(163a^2Ab+5Ab^3+75a^3B+135ab^2B) \sqrt{\cos(c+dx)} \sin(c+dx)}{315ad} \\
&= \frac{2(163a^2Ab+5Ab^3+75a^3B+135ab^2B) \sqrt{\cos(c+dx)} \sin(c+dx)}{315ad} \\
&= \frac{2(163a^2Ab+5Ab^3+75a^3B+135ab^2B) \sqrt{\cos(c+dx)} \sin(c+dx)}{315ad} \\
&= \frac{2(163a^2Ab+5Ab^3+75a^3B+135ab^2B) \sqrt{\cos(c+dx)} \sin(c+dx)}{315ad} \\
&= \frac{2(a^2-b^2)(114a^2Ab-10Ab^3+75a^3B+45ab^2B) \sqrt{\cos(c+dx)} \sin(c+dx)}{315a^2d \sqrt{\cos(c+dx)} \sqrt{a+b \sec(c+dx)}}
\end{aligned}$$

Mathematica [C] time = 19.41, size = 542, normalized size = 1.28

$$\frac{\cos^{\frac{5}{2}}(c+dx)(a+b \sec(c+dx))^{5/2} \left(\frac{1}{630} (133a^2A+270abB+150Ab^2) \sin(2(c+dx)) + \frac{1}{36} a^2A \sin(4(c+dx)) \right)}{d(a \cos(c+dx) + b)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(9/2)*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2)*(((747*a^2*A*b + 20*A*b^3 + 345*a^3*B + 540*a*b^2*B)*Sin[c + d*x])/(630*a) + ((133*a^2*A + 150*A*b^2 + 270*a*b*B)*Sin[2*(c + d*x)]/630 + (a*(19*A*b + 9*a*B)*Sin[3*(c + d*x)]/126 + (a^2*A*Ssin[4*(c + d*x)]/36)))/(d*(b + a*Cos[c + d*x])^2) - (2*Cos[c + d*x]^(3/2)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*(a + b*Sec[c + d*x])^(5/2))*((-I)*(a + b)*(147*a^4*A + 279*a^2*A*b^2 - 10*A*b^4 + 435*a^3*b*B + 45*a*b^3*B)*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + I*a*(a + b)*(-10*A*b^3 + 15*a*b^2*(11*A + 3*B) + 3*a^3*(49*A + 25*B) + 6*a^2*b*(19*A + 60*B))*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - (147*a^4*A + 279*a^2*A*b^2 - 10*A*b^4 + 435*a^3*b*B + 45*a*b^3*B)*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/(315*a^2*d*(b + a*Cos[c + d*x])^3*Sec[c + d*x]^(5/2))
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb^2 \cos(dx + c)^4 \sec(dx + c)^3 + Aa^2 \cos(dx + c)^4 + (2Bab + Ab^2) \cos(dx + c)^4 \sec(dx + c)^2 + (Ba^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^4*sec(d*x + c)^3 + A*a^2*cos(d*x + c)^4 + (2*B*a*b + A*b^2)*cos(d*x + c)^4*sec(d*x + c)^2 + (B*a^2 + 2*A*a*b)*cos(d*x + c)^4*sec(d*x + c))*sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(b \sec(dx + c) + a)^{\frac{5}{2}} \cos(dx + c)^{\frac{9}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(9/2), x)
```

maple [B] time = 2.77, size = 3069, normalized size = 7.22

output too large to display


```

s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), (-a+b)/(a-b))^(1/2))*a^3*b^2+155*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), (-a+b)/(a-b))^(1/2))*a^2*b^3+10*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), (-a+b)/(a-b))^(1/2))*a*b^4+30*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*a^5*(1/(1+cos(d*x+c)))^(1/2)-75*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^5*(1/(1+cos(d*x+c)))^(1/2)+75*B*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), (-a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^5-147*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^5*(1/(1+cos(d*x+c)))^(1/2)-10*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)*b^5*(1/(1+cos(d*x+c)))^(1/2)-147*A*((a-b)/(a+b))^(1/2)*a^4*b*(1/(1+cos(d*x+c)))^(1/2)-163*A*((a-b)/(a+b))^(1/2)*a^3*b^2*(1/(1+cos(d*x+c)))^(1/2)-279*A*((a-b)/(a+b))^(1/2)*a^2*b^3*(1/(1+cos(d*x+c)))^(1/2)-5*A*((a-b)/(a+b))^(1/2)*a*b^4*(1/(1+cos(d*x+c)))^(1/2)-75*B*((a-b)/(a+b))^(1/2)*a^4*b*(1/(1+cos(d*x+c)))^(1/2)-435*B*((a-b)/(a+b))^(1/2)*a^3*b^2*(1/(1+cos(d*x+c)))^(1/2)-135*B*((a-b)/(a+b))^(1/2)*a^2*b^3*(1/(1+cos(d*x+c)))^(1/2)-45*B*((a-b)/(a+b))^(1/2)*a*b^4*(1/(1+cos(d*x+c)))^(1/2)+45*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)^5*a^5*(1/(1+cos(d*x+c)))^(1/2)+35*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^6*a^5*(1/(1+cos(d*x+c)))^(1/2)+14*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^4*a^5*(1/(1+cos(d*x+c)))^(1/2)+98*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^5*(1/(1+cos(d*x+c)))^(1/2)+147*A*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), (-a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^5+10*A*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), (-a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*b^5-147*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), (-a+b)/(a-b))^(1/2))*a^5-45*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^2*b^3*(1/(1+cos(d*x+c)))^(1/2)+45*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a*b^4*(1/(1+cos(d*x+c)))^(1/2))/a^2/((a-b)/(a+b))^(1/2)/(b+a*cos(d*x+c))/(1/(1+cos(d*x+c)))^(1/2)/sin(d*x+c))^3

```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(b \sec(dx + c) + a)^{\frac{5}{2}} \cos(dx + c)^{\frac{9}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(9/2), x)
```


mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c + dx)^{9/2} \left(A + \frac{B}{\cos(c + dx)} \right) \left(a + \frac{b}{\cos(c + dx)} \right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(c + d*x)^(9/2)*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(5/2), x)`

[Out] `int(cos(c + d*x)^(9/2)*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(5/2), x)`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)**(9/2)*(a+b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)), x)`

[Out] Timed out

$$3.609 \quad \int \cos^{\frac{7}{2}}(c+dx)(a+b \sec(c+dx))^{5/2}(A+B \sec(c+dx)) dx$$

Optimal. Leaf size=340

$$\frac{2(25a^2A + 77abB + 45Ab^2) \sin(c+dx) \sqrt{\cos(c+dx)} \sqrt{a+b \sec(c+dx)}}{105d} + \frac{2(a^2 - b^2)(25a^2A + 56abB + 15Ab^2)}{105ad \sqrt{\cos(c+dx)} \sqrt{a+b \sec(c+dx)}}$$

[Out] $2/7*a*A*\cos(d*x+c)^{(5/2)}*(a+b*\sec(d*x+c))^{(3/2)}*\sin(d*x+c)/d+2/105*(a^2-b^2)*(25*A*a^2+15*A*b^2+56*B*a*b)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)}*(a/(a+b))^{(1/2)})*((b+a*\cos(d*x+c))/(a+b))^{(1/2)}/a/d/\cos(d*x+c)^{(1/2)}/(a+b*\sec(d*x+c))^{(1/2)}+2/35*a*(10*A*b+7*B*a)*\cos(d*x+c)^{(3/2)}*\sin(d*x+c)*(a+b*\sec(d*x+c))^{(1/2)}/d+2/105*(25*A*a^2+45*A*b^2+77*B*a*b)*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}*(a+b*\sec(d*x+c))^{(1/2)}/d+2/105*(145*A*a^2*b+15*A*b^3+63*B*a^3+161*B*a*b^2)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)}*(a/(a+b))^{(1/2)})*\cos(d*x+c)^{(1/2)}*(a+b*\sec(d*x+c))^{(1/2)}/a/d/((b+a*\cos(d*x+c))/(a+b))^{(1/2)}$

Rubi [A] time = 1.32, antiderivative size = 340, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$, Rules used = {2955, 4025, 4094, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2(25a^2A + 77abB + 45Ab^2) \sin(c+dx) \sqrt{\cos(c+dx)} \sqrt{a+b \sec(c+dx)}}{105d} + \frac{2(a^2 - b^2)(25a^2A + 56abB + 15Ab^2)}{105ad \sqrt{\cos(c+dx)} \sqrt{a+b \sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]

[Out] $(2*(a^2 - b^2)*(25*a^2*A + 15*A*b^2 + 56*a*b*B)*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])]/(a + b))*\text{EllipticF}[(c + d*x)/2, (2*a)/(a + b)]/(105*a*d*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]) + (2*(145*a^2*A*b + 15*A*b^3 + 63*a^3*B + 161*a*b^2*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, (2*a)/(a + b)]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]])/(105*a*d*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])]/(a + b)) + (2*(25*a^2*A + 45*A*b^2 + 77*a*b*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(105*d) + (2*a*(10*A*b + 7*a*B)*\text{Cos}[c + d*x]^{(3/2)}*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(35*d) + (2*a*A*\text{Cos}[c + d*x]^{(5/2)}*(a + b*\text{Sec}[c + d*x])^{(3/2)}*\text{Sin}[c + d*x])/(7*d)$

Rule 2653

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*Sqrt[a
+ b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
0] && !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*Ellip
ticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)]/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a
+ b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && !GtQ[a + b, 0]
```

Rule 2955

```
Int[((a_) + csc[(e_) + (f_)*(x_)])*(b_)^(m_)*(csc[(e_) + (f_)*(x_)]*
(d_) + (c_))^(n_)*((g_)*sin[(e_) + (f_)*(x_)])^(p_), x_Symbol] := Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3856

```
Int[Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)]/Sqrt[csc[(e_) + (f_)*(x_)]
*(d_)], x_Symbol] := Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_) + (f_)*(x_)]*(d_)]/Sqrt[csc[(e_) + (f_)*(x_)]*(b_)
+ (a_)], x_Symbol] := Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
```

a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 4025

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> Simp[(a*A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e + f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LeQ[n, -1]

Rule 4035

Int[(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.))/(Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]), x_Symbol] :> Dist[A/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]

Rule 4094

Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.))^m*(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m, x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]

Rule 4104

Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.))^m*(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m, x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]

Rubi steps

$$\begin{aligned}
\int \cos^{\frac{7}{2}}(c + dx)(a + b \sec(c + dx))^{5/2}(A + B \sec(c + dx)) dx &= \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)}\right) \int \frac{(a + b \sec(c + dx))^{5/2}(A + B \sec(c + dx))}{\sec(c + dx)} dx \\
&= \frac{2aA \cos^{\frac{5}{2}}(c + dx)(a + b \sec(c + dx))^{3/2} \sin(c + dx)}{7d} \\
&= \frac{2a(10Ab + 7aB) \cos^{\frac{3}{2}}(c + dx) \sqrt{a + b \sec(c + dx)}}{35d} \\
&= \frac{2(25a^2A + 45Ab^2 + 77abB) \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}}{105d} \\
&= \frac{2(25a^2A + 45Ab^2 + 77abB) \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}}{105d} \\
&= \frac{2(25a^2A + 45Ab^2 + 77abB) \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}}{105d} \\
&= \frac{2(25a^2A + 45Ab^2 + 77abB) \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}}{105d} \\
&= \frac{2(a^2 - b^2)(25a^2A + 15Ab^2 + 56abB) \sqrt{\frac{b+a \cos(c+dx)}{a+b}}}{105ad \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}}
\end{aligned}$$

Mathematica [C] time = 19.41, size = 470, normalized size = 1.38

$$\frac{\cos^{\frac{5}{2}}(c + dx)(a + b \sec(c + dx))^{5/2} \left(\frac{1}{210} (115a^2A + 308abB + 180Ab^2) \sin(c + dx) + \frac{1}{14} a^2A \sin(3(c + dx)) + \frac{1}{35} a^2A \sin(5(c + dx)) \right) + (a^2 - b^2)(25a^2A + 15Ab^2 + 56abB) \sqrt{\frac{b+a \cos(c+dx)}{a+b}}}{d(a \cos(c + dx) + b)^2}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]

[Out] (Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2)*(((115*a^2*A + 180*A*b^2 + 308*a*b*B)*Sin[c + d*x])/210 + (a*(15*A*b + 7*a*B)*Sin[2*(c + d*x)])/35 + (a

$$\begin{aligned} & \frac{2A \sin[3(c + dx)]}{14} \Big/ (d(b + a \cos[c + dx])^2) - (2 \cos[c + dx])^{3/2} \\ & \cdot (\cos[(c + dx)/2])^2 \sec[c + dx]^{3/2} (a + b \sec[c + dx])^{5/2} \cdot ((-I) \\ & \cdot (a + b) \cdot (145a^2Ab + 15A^2b^3 + 63a^3B + 161ab^2B) \cdot \text{EllipticE}[I \cdot \text{ArcSinh}[\text{Tan}[(c + dx)/2]], \\ & (-a + b)/(a + b)] \cdot \sec[(c + dx)/2]^2 \cdot \text{Sqrt}[(b + a \cos[c + dx]) \cdot \sec[(c + dx)/2]^2] \\ & \Big/ (a + b) + I \cdot a \cdot (a + b) \cdot (15b^2(A + 7B) + 8ab(15A + 7B) + a^2(25A + 63B)) \\ & \cdot \text{EllipticF}[I \cdot \text{ArcSinh}[\text{Tan}[(c + dx)/2]], (-a + b)/(a + b)] \cdot \sec[(c + dx)/2]^2 \\ & \cdot \text{Sqrt}[(b + a \cos[c + dx]) \cdot \sec[(c + dx)/2]^2] \Big/ (a + b) - (145a^2Ab + 15A^2b^3 + 63a^3B + 161ab^2B) \\ & \cdot (b + a \cos[c + dx]) \cdot (\sec[(c + dx)/2]^{3/2} \cdot \tan[(c + dx)/2]) \Big/ (105ad \\ & \cdot (b + a \cos[c + dx])^3 \sec[c + dx]^{5/2}) \end{aligned}$$

fricas [F] time = 1.09, size = 0, normalized size = 0.00

$$\text{integral} \left((B^2 \cos(dx + c)^3 \sec(dx + c)^3 + Aa^2 \cos(dx + c)^3 + (2Bab + Ab^2) \cos(dx + c)^3 \sec(dx + c)^2 + (Ba^2 \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(dx+c)^(7/2)*(a+b*sec(dx+c))^(5/2)*(A+B*sec(dx+c)),x, algorithm="fricas")

[Out] integral((B*b^2*cos(dx + c)^3*sec(dx + c)^3 + A*a^2*cos(dx + c)^3 + (2*B*a*b + A*b^2)*cos(dx + c)^3*sec(dx + c)^2 + (B*a^2 + 2*A*a*b)*cos(dx + c)^3*sec(dx + c))*sqrt(b*sec(dx + c) + a)*sqrt(cos(dx + c)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(b \sec(dx + c) + a)^{\frac{5}{2}} \cos(dx + c)^{\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(dx+c)^(7/2)*(a+b*sec(dx+c))^(5/2)*(A+B*sec(dx+c)),x, algorithm="giac")

[Out] integrate((B*sec(dx + c) + A)*(b*sec(dx + c) + a)^(5/2)*cos(dx + c)^(7/2), x)

maple [B] time = 3.03, size = 2450, normalized size = 7.21

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(dx+c)^(7/2)*(a+b*sec(dx+c))^(5/2)*(A+B*sec(dx+c)),x)

[Out] 2/105/d*((b+a*cos(dx+c))/cos(dx+c))^(1/2)*cos(dx+c)^(1/2)*(-1+cos(dx+c))*(1+cos(dx+c))*(60*A*((a-b)/(a+b))^(1/2)*cos(dx+c)^4*a^3*b*(1/(1+cos(dx+c))))

$$\begin{aligned}
& +c))^{1/2} + 161*B*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2} * \text{EllipticE}((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (- (a+b)/(a-b))^{1/2}) * \sin(d*x+c) * a^2*b^2 - 15*A*((a-b)/(a+b))^{1/2} * b^4 * (1/(1+\cos(d*x+c)))^{1/2} - 161*B*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2} * \text{EllipticE}((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (- (a+b)/(a-b))^{1/2}) * \sin(d*x+c) * a*b^3 + 119*B * \text{EllipticF}((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (- (a+b)/(a-b))^{1/2}) * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2} * \sin(d*x+c) * a^3*b - 161*B * \text{EllipticF}((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (- (a+b)/(a-b))^{1/2}) * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2} * \sin(d*x+c) * a^2*b^2 + 90*A*((a-b)/(a+b))^{1/2} * \cos(d*x+c)^3 * a^2*b^2 * (1/(1+\cos(d*x+c)))^{1/2} - 145*A * \text{EllipticF}((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (- (a+b)/(a-b))^{1/2}) * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2} * \sin(d*x+c) * a^3*b + 135*A * \text{EllipticF}((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (- (a+b)/(a-b))^{1/2}) * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2} * \sin(d*x+c) * a^2*b^2 - 15*A * \text{EllipticF}((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (- (a+b)/(a-b))^{1/2}) * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2} * \sin(d*x+c) * a*b^3 + 145*A * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2} * \text{EllipticE}((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (- (a+b)/(a-b))^{1/2}) * \sin(d*x+c) * a^3*b - 145*A * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2} * \text{EllipticE}((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (- (a+b)/(a-b))^{1/2}) * \sin(d*x+c) * a^2*b^2 + 15*A * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2} * \text{EllipticE}((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (- (a+b)/(a-b))^{1/2}) * \sin(d*x+c) * a*b^3 + 98*B * ((a-b)/(a+b))^{1/2} * \cos(d*x+c)^3 * a^3*b * (1/(1+\cos(d*x+c)))^{1/2} + 110*A * ((a-b)/(a+b))^{1/2} * \cos(d*x+c)^2 * a^3*b * (1/(1+\cos(d*x+c)))^{1/2} + 60*A * ((a-b)/(a+b))^{1/2} * \cos(d*x+c)^2 * a*b^3 * (1/(1+\cos(d*x+c)))^{1/2} + 238*B * ((a-b)/(a+b))^{1/2} * \cos(d*x+c)^2 * a^2*b^2 * (1/(1+\cos(d*x+c)))^{1/2} - 145*A * ((a-b)/(a+b))^{1/2} * \cos(d*x+c) * a^3*b * (1/(1+\cos(d*x+c)))^{1/2} + 55*A * ((a-b)/(a+b))^{1/2} * \cos(d*x+c) * a^2*b^2 * (1/(1+\cos(d*x+c)))^{1/2} - 15*A * ((a-b)/(a+b))^{1/2} * \cos(d*x+c) * a*b^3 * (1/(1+\cos(d*x+c)))^{1/2} - 35*B * ((a-b)/(a+b))^{1/2} * \cos(d*x+c) * a^3*b * (1/(1+\cos(d*x+c)))^{1/2} - 161*B * ((a-b)/(a+b))^{1/2} * \cos(d*x+c) * a^2*b^2 * (1/(1+\cos(d*x+c)))^{1/2} + 161*B * ((a-b)/(a+b))^{1/2} * \cos(d*x+c) * a*b^3 * (1/(1+\cos(d*x+c)))^{1/2} - 63*B * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2} * \text{EllipticE}((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (- (a+b)/(a-b))^{1/2}) * \sin(d*x+c) * a^3*b + 105*B * \sin(d*x+c) * \text{EllipticF}((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (- (a+b)/(a-b))^{1/2}) * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2} * a*b^3 + 25*A * \text{EllipticF}((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (- (a+b)/(a-b))^{1/2}) * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2} * \sin(d*x+c) * a^4 - 15*A * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{1/2} * \text{EllipticE}((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (- (a+b)/(a-b))^{1/2}) * \sin(d*x+c) * b^4 + 21*B * ((a-b)/(a+b))^{1/2} * \cos(d*x+c)^4 * a^4 * (1/(1+\cos(d*x+c)))^{1/2} + 42*B * ((a-b)/(a+b))^{1/2} * \cos(d*x+c)^2 * a^4 * (1/(1+\cos(d*x+c)))^{1/2} + 15*A * ((a-b)/(a+b))^{1/2} * \cos(d*x+c) * b^4 * (1/(1+\cos(d*x+c)))^{1/2} - 63*B * ((a-b)/(a+b))^{1/2} * \cos(d*x+c) * a^4 * (1/(1+\cos(d*x+c)))^{1/2} - 25*A * ((a-b)/(a+b))^{1/2} * a^3*b * (1/(1+\cos(d*x+c)))^{1/2} - 145*A * ((a-b)/(a+b))^{1/2} * a^2*b^2 * (1/(1+\cos(d*x+c)))^{1/2} - 45*A * ((a-b)/(a+b))^{1/2} * a*b^3 * (1/(1+\cos(d*x+c)))^{1/2} - 63*B * ((a-b)/(a+b))^{1/2} * a^3*b * (1/(1+\cos(d*x+c)))^{1/2} - 63*B * ((a-b)/(a+b))^{1/2} * a^3*b * (1/(1+\cos(d*x+c)))^{1/2}
\end{aligned}$$

$$\begin{aligned} & x+c))^{1/2}-77*B*((a-b)/(a+b))^{1/2}*a^2*b^2*(1/(1+\cos(d*x+c)))^{1/2}-161* \\ & B*((a-b)/(a+b))^{1/2}*a*b^3*(1/(1+\cos(d*x+c)))^{1/2}+10*A*((a-b)/(a+b))^{1/2} \\ & * \cos(d*x+c)^3*a^4*(1/(1+\cos(d*x+c)))^{1/2}-25*A*((a-b)/(a+b))^{1/2}*\cos(d \\ & *x+c)*a^4*(1/(1+\cos(d*x+c)))^{1/2}+63*B*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)) \\ & / (a+b))^{1/2}* \text{EllipticE}((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (- (a+b) \\ & / (a-b))^{1/2})*\sin(d*x+c)*a^4-63*B*\sin(d*x+c)*((b+a*\cos(d*x+c))/(1+\cos(d*x+c) \\ &)/(a+b))^{1/2}* \text{EllipticF}((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (\\ & -(a+b)/(a-b))^{1/2})*a^4+15*A*((a-b)/(a+b))^{1/2}*\cos(d*x+c)^5*a^4*(1/(1+\cos \\ & (d*x+c)))^{1/2})/a/((a-b)/(a+b))^{1/2}/(b+a*\cos(d*x+c))/(1/(1+\cos(d*x+c))) \\ & ^{1/2}/\sin(d*x+c)^3 \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(b \sec(dx + c) + a)^{\frac{5}{2}} \cos(dx + c)^{\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(7/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c + dx)^{7/2} \left(A + \frac{B}{\cos(c + dx)} \right) \left(a + \frac{b}{\cos(c + dx)} \right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^(7/2)*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(5/2),x)

[Out] int(cos(c + d*x)^(7/2)*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(5/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**(7/2)*(a+b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)),x)

[Out] Timed out

$$3.610 \quad \int \cos^2(c+dx)(a+b \sec(c+dx))^{5/2}(A+B \sec(c+dx)) dx$$

Optimal. Leaf size=342

$$\frac{2(9a^2A + 35abB + 23Ab^2) \sqrt{\cos(c+dx)} \sqrt{a+b \sec(c+dx)} E\left(\frac{1}{2}(c+dx) \middle| \frac{2a}{a+b}\right)}{15d \sqrt{\frac{a \cos(c+dx)+b}{a+b}}} + \frac{2(5a^3B + 8a^2Ab + 10ab^2B - 8Ab^3)}{15d \sqrt{\cos(c+dx)}}$$

[Out] $2/5*a*A*\cos(d*x+c)^(3/2)*(a+b*\sec(d*x+c))^(3/2)*\sin(d*x+c)/d+2/15*(8*A*a^2*b-8*A*b^3+5*B*a^3+10*B*a*b^2)*(\cos(1/2*d*x+1/2*c)^2)^(1/2)/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^(1/2)*(a/(a+b))^(1/2))*((b+a*\cos(d*x+c))/(a+b))^(1/2)/d/\cos(d*x+c)^(1/2)/(a+b*\sec(d*x+c))^(1/2)+2*b^3*B*(\cos(1/2*d*x+1/2*c)^2)^(1/2)/\cos(1/2*d*x+1/2*c)*\text{EllipticPi}(\sin(1/2*d*x+1/2*c), 2, 2^(1/2)*(a/(a+b))^(1/2))*((b+a*\cos(d*x+c))/(a+b))^(1/2)/d/\cos(d*x+c)^(1/2)/(a+b*\sec(d*x+c))^(1/2)+2/15*a*(8*A*b+5*B*a)*\sin(d*x+c)*\cos(d*x+c)^(1/2)*(a+b*\sec(d*x+c))^(1/2)/d+2/15*(9*A*a^2+23*A*b^2+35*B*a*b)*(\cos(1/2*d*x+1/2*c)^2)^(1/2)/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^(1/2)*(a/(a+b))^(1/2))*\cos(d*x+c)^(1/2)*(a+b*\sec(d*x+c))^(1/2)/d/((b+a*\cos(d*x+c))/(a+b))^(1/2)$

Rubi [A] time = 1.39, antiderivative size = 342, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 14, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2955, 4025, 4094, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2(8a^2Ab + 5a^3B + 10ab^2B - 8Ab^3) \sqrt{\frac{a \cos(c+dx)+b}{a+b}} F\left(\frac{1}{2}(c+dx) \middle| \frac{2a}{a+b}\right)}{15d \sqrt{\cos(c+dx)} \sqrt{a+b \sec(c+dx)}} + \frac{2(9a^2A + 35abB + 23Ab^2) \sqrt{\cos(c+dx)}}{15d \sqrt{\frac{a \cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]), x]

[Out] $(2*(8*a^2*A*b - 8*A*b^3 + 5*a^3*B + 10*a*b^2*B)*\text{Sqrt}[(b + a*\cos[c + d*x])/(a + b)]*\text{EllipticF}[(c + d*x)/2, (2*a)/(a + b)]/(15*d*\text{Sqrt}[\cos[c + d*x]]*\text{Sqrt}[a + b*\sec[c + d*x]]) + (2*b^3*B*\text{Sqrt}[(b + a*\cos[c + d*x])/(a + b)]*\text{EllipticPi}[2, (c + d*x)/2, (2*a)/(a + b)]/(d*\text{Sqrt}[\cos[c + d*x]]*\text{Sqrt}[a + b*\sec[c + d*x]]) + (2*(9*a^2*A + 23*A*b^2 + 35*a*b*B)*\text{Sqrt}[\cos[c + d*x]]*\text{EllipticE}[(c + d*x)/2, (2*a)/(a + b)]*\text{Sqrt}[a + b*\sec[c + d*x]])/(15*d*\text{Sqrt}[(b + a*\cos[c + d*x])/(a + b)]) + (2*a*(8*A*b + 5*a*B)*\text{Sqrt}[\cos[c + d*x]]*\text{Sqrt}[a + b*\sec[c + d*x]]*\sin[c + d*x])/(15*d) + (2*a*A*\cos[c + d*x]^(3/2)*(a + b*\sec[c + d*x])^(3/2)*\sin[c + d*x])/(5*d)$

Rule 2653

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*Sqrt[a
+ b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
0] && !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a
+ b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_)
+ (f_)*(x_)]]), x_Symbol] := Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/d, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_)
+ (f_)*(x_)]]), x_Symbol] := Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
+ f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !GtQ[c + d, 0]
```

Rule 2955

```
Int[((a_) + csc[(e_) + (f_)*(x_)])*(b_)^(m_)*(csc[(e_) + (f_)*(x_)])*
```

$(d_.) + (c_.))^{(n_.)} * ((g_.) * \sin[(e_.) + (f_.) * (x_.)])^{(p_.)}, x_Symbol] := \text{Dist}[(g * \text{Csc}[e + f * x])^p * (g * \sin[e + f * x])^p, \text{Int}[(a + b * \text{Csc}[e + f * x])^m * (c + d * \text{Csc}[e + f * x])^n / (g * \text{Csc}[e + f * x])^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, m, n, p\}, x] \&\& \text{NeQ}[b * c - a * d, 0] \&\& !\text{IntegerQ}[p] \&\& !(\text{IntegerQ}[m] \&\& \text{IntegerQ}[n])$

Rule 3856

$\text{Int}[\text{Sqrt}[\text{csc}[(e_.) + (f_.) * (x_.)] * (b_.) + (a_.)] / \text{Sqrt}[\text{csc}[(e_.) + (f_.) * (x_.)] * (d_.)], x_Symbol] := \text{Dist}[\text{Sqrt}[a + b * \text{Csc}[e + f * x]] / (\text{Sqrt}[d * \text{Csc}[e + f * x]] * \text{Sqrt}[b + a * \sin[e + f * x]]), \text{Int}[\text{Sqrt}[b + a * \sin[e + f * x]], x], x] /; \text{FreeQ}\{a, b, d, e, f\}, x] \&\& \text{NeQ}[a^2 - b^2, 0]$

Rule 3858

$\text{Int}[\text{Sqrt}[\text{csc}[(e_.) + (f_.) * (x_.)] * (d_.)] / \text{Sqrt}[\text{csc}[(e_.) + (f_.) * (x_.)] * (b_.) + (a_.)], x_Symbol] := \text{Dist}[(\text{Sqrt}[d * \text{Csc}[e + f * x]] * \text{Sqrt}[b + a * \sin[e + f * x]]) / \text{Sqrt}[a + b * \text{Csc}[e + f * x]], \text{Int}[1 / \text{Sqrt}[b + a * \sin[e + f * x]], x], x] /; \text{FreeQ}\{a, b, d, e, f\}, x] \&\& \text{NeQ}[a^2 - b^2, 0]$

Rule 3859

$\text{Int}[(\text{csc}[(e_.) + (f_.) * (x_.)] * (d_.))^{(3/2)} / \text{Sqrt}[\text{csc}[(e_.) + (f_.) * (x_.)] * (b_.) + (a_.)], x_Symbol] := \text{Dist}[(d * \text{Sqrt}[d * \text{Csc}[e + f * x]] * \text{Sqrt}[b + a * \sin[e + f * x]]) / \text{Sqrt}[a + b * \text{Csc}[e + f * x]], \text{Int}[1 / (\sin[e + f * x] * \text{Sqrt}[b + a * \sin[e + f * x]]), x], x] /; \text{FreeQ}\{a, b, d, e, f\}, x] \&\& \text{NeQ}[a^2 - b^2, 0]$

Rule 4025

$\text{Int}[(\text{csc}[(e_.) + (f_.) * (x_.)] * (d_.))^{(n_.)} * (\text{csc}[(e_.) + (f_.) * (x_.)] * (b_.) + (a_.))^{(m_.)} * (\text{csc}[(e_.) + (f_.) * (x_.)] * (B_.) + (A_.)), x_Symbol] := \text{Simp}[(a * A * \text{Cot}[e + f * x] * (a + b * \text{Csc}[e + f * x])^{(m - 1)} * (d * \text{Csc}[e + f * x])^n / (f * n), x] + \text{Dist}[1 / (d * n), \text{Int}[(a + b * \text{Csc}[e + f * x])^{(m - 2)} * (d * \text{Csc}[e + f * x])^{(n + 1)} * \text{Simp}[a * (a * B * n - A * b * (m - n - 1)) + (2 * a * b * B * n + A * (b^2 * n + a^2 * (1 + n))) * \text{Csc}[e + f * x] + b * (b * B * n + a * A * (m + n)) * \text{Csc}[e + f * x]^2, x], x], x] /; \text{FreeQ}\{a, b, d, e, f, A, B\}, x] \&\& \text{NeQ}[A * b - a * B, 0] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{GtQ}[m, 1] \&\& \text{LeQ}[n, -1]$

Rule 4035

$\text{Int}[(\text{csc}[(e_.) + (f_.) * (x_.)] * (B_.) + (A_.)) / (\text{Sqrt}[\text{csc}[(e_.) + (f_.) * (x_.)] * (d_.) * \text{Sqrt}[\text{csc}[(e_.) + (f_.) * (x_.)] * (b_.) + (a_.)]), x_Symbol] := \text{Dist}[A / a, \text{Int}[\text{Sqrt}[a + b * \text{Csc}[e + f * x]] / \text{Sqrt}[d * \text{Csc}[e + f * x]], x], x] - \text{Dist}[(A * b - a * B) / (a * d), \text{Int}[\text{Sqrt}[d * \text{Csc}[e + f * x]] / \text{Sqrt}[a + b * \text{Csc}[e + f * x]], x], x] /; \text{FreeQ}\{$

a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]

Rule 4094

Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.)) * (csc[(e_.) + (f_.)*(x_.)]*(d_.))^n * (csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m, x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]

Rule 4108

Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.))/(Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned}
\int \cos^{\frac{5}{2}}(c + dx)(a + b \sec(c + dx))^{5/2}(A + B \sec(c + dx)) dx &= \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)}\right) \int \frac{(a + b \sec(c + dx))^{5/2}(A + B \sec(c + dx))}{\cos(c + dx)} dx \\
&= \frac{2aA \cos^{\frac{3}{2}}(c + dx)(a + b \sec(c + dx))^{3/2} \sin(c + dx)}{5d} \\
&= \frac{2a(8Ab + 5aB)\sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}}{15d} \\
&= \frac{2a(8Ab + 5aB)\sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}}{15d} \\
&= \frac{2a(8Ab + 5aB)\sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}}{15d} \\
&= \frac{2a(8Ab + 5aB)\sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}}{15d} \\
&= \frac{2a(8Ab + 5aB)\sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}}{15d} \\
&= \frac{2b^3 B \sqrt{\frac{b+a \cos(c+dx)}{a+b}} \Pi\left(2; \frac{1}{2}(c + dx) \middle| \frac{2a}{a+b}\right) + 2a(8Ab + 5aB)\sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}}{d \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}} \\
&= \frac{2(8a^2 Ab - 8Ab^3 + 5a^3 B + 10ab^2 B) \sqrt{\frac{b+a \cos(c+dx)}{a+b}}}{15d \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}}
\end{aligned}$$

Mathematica [C] time = 35.49, size = 49609, normalized size = 145.06

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]), x]

[Out] Result too large to show

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

c), $(-(a+b)/(a-b))^{(1/2)} * a * b^2 + 14 * A * ((a-b)/(a+b))^{(1/2)} * \cos(dx+c)^3 * a^2 * b * (1/(1+\cos(dx+c)))^{(1/2)} + 34 * A * ((a-b)/(a+b))^{(1/2)} * \cos(dx+c)^2 * a * b^2 * (1/(1+\cos(dx+c)))^{(1/2)} - 23 * A * ((a-b)/(a+b))^{(1/2)} * b^3 * (1/(1+\cos(dx+c)))^{(1/2)} + 5 * B * (1/(1+\cos(dx+c)))^{(1/2)} * ((a-b)/(a+b))^{(1/2)} * \cos(dx+c)^3 * a^3 + 5 * B * \sin(dx+c) * \text{EllipticF}((-1+\cos(dx+c)) * ((a-b)/(a+b))^{(1/2)} / \sin(dx+c), -(a+b)/(a-b))^{(1/2)} * ((b+a * \cos(dx+c)) / (1+\cos(dx+c))) / (a+b))^{(1/2)} * a^3 - 5 * B * ((a-b)/(a+b))^{(1/2)} * \cos(dx+c) * a^3 * (1/(1+\cos(dx+c)))^{(1/2)} + 3 * A * ((a-b)/(a+b))^{(1/2)} * \cos(dx+c)^4 * a^3 * (1/(1+\cos(dx+c)))^{(1/2)} - 15 * B * \sin(dx+c) * \text{EllipticF}((-1+\cos(dx+c)) * ((a-b)/(a+b))^{(1/2)} / \sin(dx+c), -(a+b)/(a-b))^{(1/2)} * ((b+a * \cos(dx+c)) / (1+\cos(dx+c))) / (a+b))^{(1/2)} * b^3 + 30 * B * \sin(dx+c) * ((b+a * \cos(dx+c)) / (1+\cos(dx+c))) / (a+b))^{(1/2)} * \text{EllipticPi}((-1+\cos(dx+c)) * ((a-b)/(a+b))^{(1/2)} / \sin(dx+c), (a+b)/(a-b), I / ((a-b)/(a+b))^{(1/2)}) * b^3 + 15 * A * \sin(dx+c) * \text{EllipticF}((-1+\cos(dx+c)) * ((a-b)/(a+b))^{(1/2)} / \sin(dx+c), -(a+b)/(a-b))^{(1/2)} * ((b+a * \cos(dx+c)) / (1+\cos(dx+c))) / (a+b))^{(1/2)} * b^3 + 6 * A * ((a-b)/(a+b))^{(1/2)} * \cos(dx+c)^2 * a^3 * (1/(1+\cos(dx+c)))^{(1/2)} - 9 * A * ((a-b)/(a+b))^{(1/2)} * \cos(dx+c) * a^3 * (1/(1+\cos(dx+c)))^{(1/2)} + 23 * A * ((a-b)/(a+b))^{(1/2)} * \cos(dx+c) * b^3 * (1/(1+\cos(dx+c)))^{(1/2)} - 9 * A * ((a-b)/(a+b))^{(1/2)} * a^2 * b * (1/(1+\cos(dx+c)))^{(1/2)} - 11 * A * ((a-b)/(a+b))^{(1/2)} * a * b^2 * (1/(1+\cos(dx+c)))^{(1/2)} - 5 * B * ((a-b)/(a+b))^{(1/2)} * a^2 * b * (1/(1+\cos(dx+c)))^{(1/2)} - 35 * B * ((a-b)/(a+b))^{(1/2)} * a * b^2 * (1/(1+\cos(dx+c)))^{(1/2)} - 9 * A * \sin(dx+c) * \text{EllipticF}((-1+\cos(dx+c)) * ((a-b)/(a+b))^{(1/2)} / \sin(dx+c), -(a+b)/(a-b))^{(1/2)} * ((b+a * \cos(dx+c)) / (1+\cos(dx+c))) / (a+b))^{(1/2)} * a^3 + 9 * A * \sin(dx+c) * ((b+a * \cos(dx+c)) / (1+\cos(dx+c))) / (a+b))^{(1/2)} * \text{EllipticE}((-1+\cos(dx+c)) * ((a-b)/(a+b))^{(1/2)} / \sin(dx+c), -(a+b)/(a-b))^{(1/2)} * a^3 - 23 * A * \sin(dx+c) * ((b+a * \cos(dx+c)) / (1+\cos(dx+c))) / (a+b))^{(1/2)} * \text{EllipticE}((-1+\cos(dx+c)) * ((a-b)/(a+b))^{(1/2)} / \sin(dx+c), -(a+b)/(a-b))^{(1/2)} * b^3 + 45 * B * \sin(dx+c) * \text{EllipticF}((-1+\cos(dx+c)) * ((a-b)/(a+b))^{(1/2)} / \sin(dx+c), -(a+b)/(a-b))^{(1/2)} * ((b+a * \cos(dx+c)) / (1+\cos(dx+c))) / (a+b))^{(1/2)} * a * b^2 / ((a-b)/(a+b))^{(1/2)} / (b+a * \cos(dx+c)) / \sin(dx+c)^3 / (1/(1+\cos(dx+c)))^{(1/2)}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(b \sec(dx + c) + a)^{5/2} \cos(dx + c)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(dx+c)^(5/2)*(a+b*sec(dx+c))^(5/2)*(A+B*sec(dx+c)),x, algorithm="maxima")

[Out] integrate((B*sec(dx + c) + A)*(b*sec(dx + c) + a)^(5/2)*cos(dx + c)^(5/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c + dx)^{5/2} \left(A + \frac{B}{\cos(c + dx)} \right) \left(a + \frac{b}{\cos(c + dx)} \right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^(5/2)*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(5/2), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```


$$3.611 \quad \int \cos^2(c+dx)(a+b \sec(c+dx))^{5/2}(A+B \sec(c+dx)) dx$$

Optimal. Leaf size=349

$$\frac{(6a^2B + 14aAb - 3b^2B) \sqrt{\cos(c+dx)} \sqrt{a+b \sec(c+dx)} E\left(\frac{1}{2}(c+dx) \middle| \frac{2a}{a+b}\right)}{3d \sqrt{\frac{a \cos(c+dx)+b}{a+b}}} + \frac{(2a^3A + 12a^2bB + 4aAb^2 + 3b^3B)}{3d \sqrt{\cos(c+dx)} \sqrt{a+b}}$$

[Out] $2/3*a*A*(a+b*\sec(d*x+c))^(3/2)*\sin(d*x+c)*\cos(d*x+c)^(1/2)/d+1/3*(2*A*a^3+4*A*a*b^2+12*B*a^2*b+3*B*b^3)*(\cos(1/2*d*x+1/2*c)^2)^(1/2)/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^(1/2)*(a/(a+b))^(1/2))*((b+a*\cos(d*x+c))/(a+b))^(1/2)/d/\cos(d*x+c)^(1/2)/(a+b*\sec(d*x+c))^(1/2)+b^2*(2*A*b+5*B*a)*(\cos(1/2*d*x+1/2*c)^2)^(1/2)/\cos(1/2*d*x+1/2*c)*\text{EllipticPi}(\sin(1/2*d*x+1/2*c), 2, 2^(1/2)*(a/(a+b))^(1/2))*((b+a*\cos(d*x+c))/(a+b))^(1/2)/d/\cos(d*x+c)^(1/2)/(a+b*\sec(d*x+c))^(1/2)-1/3*b*(2*A*a-3*B*b)*\sin(d*x+c)*(a+b*\sec(d*x+c))^(1/2)/d/\cos(d*x+c)^(1/2)+1/3*(14*A*a*b+6*B*a^2-3*B*b^2)*(\cos(1/2*d*x+1/2*c)^2)^(1/2)/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^(1/2)*(a/(a+b))^(1/2))*\cos(d*x+c)^(1/2)*(a+b*\sec(d*x+c))^(1/2)/d/((b+a*\cos(d*x+c))/(a+b))^(1/2)$

Rubi [A] time = 1.43, antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 14, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2955, 4025, 4096, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{(2a^3A + 12a^2bB + 4aAb^2 + 3b^3B) \sqrt{\frac{a \cos(c+dx)+b}{a+b}} F\left(\frac{1}{2}(c+dx) \middle| \frac{2a}{a+b}\right)}{3d \sqrt{\cos(c+dx)} \sqrt{a+b \sec(c+dx)}} + \frac{(6a^2B + 14aAb - 3b^2B) \sqrt{\cos(c+dx)} \sqrt{a+b}}{3d \sqrt{\frac{a \cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]

[Out] $((2*a^3*A + 4*a*A*b^2 + 12*a^2*b*B + 3*b^3*B)*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])]/(a + b))*\text{EllipticF}[(c + d*x)/2, (2*a)/(a + b)]/(3*d*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]) + (b^2*(2*A*b + 5*a*B))*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])]/(a + b))*\text{EllipticPi}[2, (c + d*x)/2, (2*a)/(a + b)]/(d*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]) + ((14*a*A*b + 6*a^2*B - 3*b^2*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, (2*a)/(a + b)]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]])/(3*d*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])]/(a + b)) - (b*(2*a*A - 3*b*B))*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]*\text{Sin}[c + d*x]/(3*d*\text{Sqrt}[\text{Cos}[c + d*x]]) + (2*a*A*\text{Sqrt}[\text{Cos}[c + d*x]]*(a + b*\text{Sec}[c + d*x])^(3/2)*\text{Sin}[c + d*x])/(3*d)$

Rule 2653

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*Sqrt[a
+ b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
0] && !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a
+ b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_)
+ (f_)*(x_)]]), x_Symbol] := Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)]/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_)
+ (f_)*(x_)]]), x_Symbol] := Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
+ f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !GtQ[c + d, 0]
```

Rule 2955

```
Int[((a_) + csc[(e_) + (f_)*(x_)])*(b_)^(m_)*(csc[(e_) + (f_)*(x_)])*
```

$(d_.) + (c_.)^{(n_.)} * ((g_.) * \sin[(e_.) + (f_.) * (x_.)])^{(p_.)}$, x_Symbol] := Dist[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && IntegerQ[n])

Rule 3856

Int[Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)], x_Symbol] := Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 3858

Int[Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] := Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 3859

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] := Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 4025

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := Simp[(a*A*Co t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e + f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LeQ[n, -1]

Rule 4035

Int[(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.))/(Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]), x_Symbol] := Dist[A/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{

a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]

Rule 4096

Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.)) * (csc[(e_.) + (f_.)*(x_.)]*(d_.))^n * (csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m, x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(m + n + 1), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n*Simp[a*A*(m + n + 1) + a*C*n + ((A*b + a*B)*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) + a*C*m)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && !LeQ[n, -1]

Rule 4108

Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.))/(Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned}
\int \cos^{\frac{3}{2}}(c+dx)(a+b \sec(c+dx))^{5/2}(A+B \sec(c+dx)) dx &= \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}\right) \int \frac{(a+b \sec(c+dx))^{5/2}(A+B \sec(c+dx))}{\sec(c+dx)} dx \\
&= \frac{2aA\sqrt{\cos(c+dx)}(a+b \sec(c+dx))^{3/2} \sin(c+dx)}{3d} \\
&= -\frac{b(2aA-3bB)\sqrt{a+b \sec(c+dx)} \sin(c+dx)}{3d\sqrt{\cos(c+dx)}} + \\
&= -\frac{b(2aA-3bB)\sqrt{a+b \sec(c+dx)} \sin(c+dx)}{3d\sqrt{\cos(c+dx)}} + \\
&= -\frac{b(2aA-3bB)\sqrt{a+b \sec(c+dx)} \sin(c+dx)}{3d\sqrt{\cos(c+dx)}} + \\
&= -\frac{b(2aA-3bB)\sqrt{a+b \sec(c+dx)} \sin(c+dx)}{3d\sqrt{\cos(c+dx)}} + \\
&= \frac{b^2(2Ab+5aB)\sqrt{\frac{b+a \cos(c+dx)}{a+b}} \Pi\left(2; \frac{1}{2}(c+dx) \middle| \frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)} \sqrt{a+b \sec(c+dx)}} \\
&= \frac{(2a^3A+4aAb^2+12a^2bB+3b^3B) \sqrt{\frac{b+a \cos(c+dx)}{a+b}}}{3d\sqrt{\cos(c+dx)} \sqrt{a+b \sec(c+dx)}}
\end{aligned}$$

Mathematica [C] time = 34.15, size = 73332, normalized size = 210.12

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]), x]

[Out] Result too large to show

fricas [F] time = 6.07, size = 0, normalized size = 0.00

integral(((Bb^2 cos(dx + c) sec(dx + c))^3 + Aa^2 cos(dx + c) + (2Bab + Ab^2) cos(dx + c) sec(dx + c)^2 + (Ba^2 +

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] integral((B*b^2*cos(d*x + c)*sec(d*x + c)^3 + A*a^2*cos(d*x + c) + (2*B*a*b + A*b^2)*cos(d*x + c)*sec(d*x + c)^2 + (B*a^2 + 2*A*a*b)*cos(d*x + c)*sec(d*x + c))*sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(b \sec(dx + c) + a)^{\frac{5}{2}} \cos(dx + c)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(3/2), x)

maple [C] time = 2.85, size = 2073, normalized size = 5.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x)

[Out] 1/3/d*(-1+cos(d*x+c))*(1+cos(d*x+c))*(6*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^2*b*(1/(1+cos(d*x+c)))^(1/2)+2*A*sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^3-3*B*((a-b)/(a+b))^(1/2)*b^3*(1/(1+cos(d*x+c)))^(1/2)-2*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^2*b*(1/(1+cos(d*x+c)))^(1/2)-14*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a*b^2*(1/(1+cos(d*x+c)))^(1/2)-6*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^2*b*(1/(1+cos(d*x+c)))^(1/2)-3*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a*b^2*(1/(1+cos(d*x+c)))^(1/2)+16*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*a^2*b*(1/(1+cos(d*x+c)))^(1/2)+14*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a*b^2*(1/(1+cos(d*x+c)))^(1/2)-6*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3*(1/(1+cos(d*x+c)))^(1/2)+3*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^3*(1/(1+cos(d*x+c)))^(1/2)+6*B*(1/(1+cos(d*x+c)))^(1/2)*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*a^3+2*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^4*a^3*(1/(1+cos(d*x+c)))^(1/2)-2*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^3*(1/(1+cos(d*x+c)))^(1/2)-12*B*sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*b^2-6*B*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b-3*B*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))

$$\begin{aligned} & \sqrt{\frac{1}{2}} \operatorname{EllipticE}\left(\frac{-1+\cos(dx+c)}{\sqrt{\frac{a-b}{a+b}}}, \frac{-\sqrt{\frac{a-b}{a+b}}}{\sqrt{\frac{a-b}{a+b}}}\right) * a^2 b^2 + 30 B \sin(dx+c) \cos(dx+c) * \left(\frac{b+a \cos(dx+c)}{1+\cos(dx+c)}\right) / \sqrt{\frac{a-b}{a+b}} \\ & \operatorname{EllipticPi}\left(\frac{-1+\cos(dx+c)}{\sqrt{\frac{a-b}{a+b}}}, \frac{-\sqrt{\frac{a-b}{a+b}}}{\sqrt{\frac{a-b}{a+b}}}\right) * \sqrt{\frac{a-b}{a+b}} \\ & + I / \left(\frac{a-b}{a+b}\right)^{\frac{1}{2}} * a^2 b^2 + 18 A \sin(dx+c) \cos(dx+c) \operatorname{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sqrt{\frac{a-b}{a+b}}}, \frac{-\sqrt{\frac{a-b}{a+b}}}{\sqrt{\frac{a-b}{a+b}}}\right) * \left(\frac{b+a \cos(dx+c)}{1+\cos(dx+c)}\right) / \sqrt{\frac{a-b}{a+b}} \\ & * a^2 b^2 + 14 A \sin(dx+c) \cos(dx+c) * \left(\frac{b+a \cos(dx+c)}{1+\cos(dx+c)}\right) / \sqrt{\frac{a-b}{a+b}} \\ & \operatorname{EllipticE}\left(\frac{-1+\cos(dx+c)}{\sqrt{\frac{a-b}{a+b}}}, \frac{-\sqrt{\frac{a-b}{a+b}}}{\sqrt{\frac{a-b}{a+b}}}\right) * \left(\frac{a-b}{a+b}\right)^{\frac{1}{2}} / \sin(dx+c), \left(\frac{-\sqrt{\frac{a-b}{a+b}}}{\sqrt{\frac{a-b}{a+b}}}\right)^{\frac{1}{2}} * a^2 b - 14 A \sin(dx+c) \cos(dx+c) * \left(\frac{b+a \cos(dx+c)}{1+\cos(dx+c)}\right) / \sqrt{\frac{a-b}{a+b}} \\ & \operatorname{EllipticE}\left(\frac{-1+\cos(dx+c)}{\sqrt{\frac{a-b}{a+b}}}, \frac{-\sqrt{\frac{a-b}{a+b}}}{\sqrt{\frac{a-b}{a+b}}}\right) * \left(\frac{a-b}{a+b}\right)^{\frac{1}{2}} / \sin(dx+c), \left(\frac{-\sqrt{\frac{a-b}{a+b}}}{\sqrt{\frac{a-b}{a+b}}}\right)^{\frac{1}{2}} * a^2 b - 14 A \sin(dx+c) \cos(dx+c) * \operatorname{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sqrt{\frac{a-b}{a+b}}}, \frac{-\sqrt{\frac{a-b}{a+b}}}{\sqrt{\frac{a-b}{a+b}}}\right) * \left(\frac{b+a \cos(dx+c)}{1+\cos(dx+c)}\right) / \sqrt{\frac{a-b}{a+b}} \\ & * a^2 b + 18 B \sin(dx+c) \cos(dx+c) \operatorname{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sqrt{\frac{a-b}{a+b}}}, \frac{-\sqrt{\frac{a-b}{a+b}}}{\sqrt{\frac{a-b}{a+b}}}\right) / \sin(dx+c), \left(\frac{-\sqrt{\frac{a-b}{a+b}}}{\sqrt{\frac{a-b}{a+b}}}\right)^{\frac{1}{2}} * \left(\frac{b+a \cos(dx+c)}{1+\cos(dx+c)}\right) / \sqrt{\frac{a-b}{a+b}} \\ & * a^2 b - 6 A \sin(dx+c) \cos(dx+c) \operatorname{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sqrt{\frac{a-b}{a+b}}}, \frac{-\sqrt{\frac{a-b}{a+b}}}{\sqrt{\frac{a-b}{a+b}}}\right) / \sin(dx+c), \left(\frac{-\sqrt{\frac{a-b}{a+b}}}{\sqrt{\frac{a-b}{a+b}}}\right)^{\frac{1}{2}} * \left(\frac{b+a \cos(dx+c)}{1+\cos(dx+c)}\right) / \sqrt{\frac{a-b}{a+b}} \\ & * b^3 + 12 A \sin(dx+c) \cos(dx+c) * \left(\frac{b+a \cos(dx+c)}{1+\cos(dx+c)}\right) / \sqrt{\frac{a-b}{a+b}} \\ & \operatorname{EllipticPi}\left(\frac{-1+\cos(dx+c)}{\sqrt{\frac{a-b}{a+b}}}, \frac{-\sqrt{\frac{a-b}{a+b}}}{\sqrt{\frac{a-b}{a+b}}}\right) * \sqrt{\frac{a-b}{a+b}}, I / \left(\frac{a-b}{a+b}\right)^{\frac{1}{2}} * b^3 - 14 A \cos(dx+c)^2 * \left(\frac{a-b}{a+b}\right)^{\frac{1}{2}} * a^2 b * \left(\frac{1}{1+\cos(dx+c)}\right)^{\frac{1}{2}} \\ & + 3 B \cos(dx+c)^2 * \left(\frac{a-b}{a+b}\right)^{\frac{1}{2}} * a^2 b^2 * \left(\frac{1}{1+\cos(dx+c)}\right)^{\frac{1}{2}} - 6 B \sin(dx+c) \cos(dx+c) \operatorname{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sqrt{\frac{a-b}{a+b}}}, \frac{-\sqrt{\frac{a-b}{a+b}}}{\sqrt{\frac{a-b}{a+b}}}\right) * \left(\frac{a-b}{a+b}\right)^{\frac{1}{2}} / \sin(dx+c), \left(\frac{-\sqrt{\frac{a-b}{a+b}}}{\sqrt{\frac{a-b}{a+b}}}\right)^{\frac{1}{2}} * \left(\frac{b+a \cos(dx+c)}{1+\cos(dx+c)}\right) / \sqrt{\frac{a-b}{a+b}} \\ & * a^3 + 6 B \sin(dx+c) \cos(dx+c) * \left(\frac{b+a \cos(dx+c)}{1+\cos(dx+c)}\right) / \sqrt{\frac{a-b}{a+b}} \\ & \operatorname{EllipticE}\left(\frac{-1+\cos(dx+c)}{\sqrt{\frac{a-b}{a+b}}}, \frac{-\sqrt{\frac{a-b}{a+b}}}{\sqrt{\frac{a-b}{a+b}}}\right) * \left(\frac{a-b}{a+b}\right)^{\frac{1}{2}} / \sin(dx+c), \left(\frac{-\sqrt{\frac{a-b}{a+b}}}{\sqrt{\frac{a-b}{a+b}}}\right)^{\frac{1}{2}} * a^3 + 3 B \sin(dx+c) \cos(dx+c) * \left(\frac{b+a \cos(dx+c)}{1+\cos(dx+c)}\right) / \sqrt{\frac{a-b}{a+b}} \\ & \operatorname{EllipticE}\left(\frac{-1+\cos(dx+c)}{\sqrt{\frac{a-b}{a+b}}}, \frac{-\sqrt{\frac{a-b}{a+b}}}{\sqrt{\frac{a-b}{a+b}}}\right) * \left(\frac{a-b}{a+b}\right)^{\frac{1}{2}} / \sin(dx+c), \left(\frac{-\sqrt{\frac{a-b}{a+b}}}{\sqrt{\frac{a-b}{a+b}}}\right)^{\frac{1}{2}} * b^3 * \left(\frac{b+a \cos(dx+c)}{\cos(dx+c)}\right)^{\frac{1}{2}} / \left(\frac{a-b}{a+b}\right)^{\frac{1}{2}} / \left(\frac{b+a \cos(dx+c)}{\sin(dx+c)}\right)^{\frac{3}{2}} / \left(\frac{1}{1+\cos(dx+c)}\right)^{\frac{1}{2}} / \cos(dx+c)^{\frac{1}{2}} \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx+c) + A)(b \sec(dx+c) + a)^{\frac{5}{2}} \cos(dx+c)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(dx+c)^(3/2)*(a+b*sec(dx+c))^(5/2)*(A+B*sec(dx+c)),x, algorith="maxima")

[Out] integrate((B*sec(dx+c) + A)*(b*sec(dx+c) + a)^(5/2)*cos(dx+c)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c+dx)^{\frac{3}{2}} \left(A + \frac{B}{\cos(c+dx)}\right) \left(a + \frac{b}{\cos(c+dx)}\right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^(3/2)*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(5/2), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```


$$3.612 \quad \int \sqrt{\cos(c + dx)} (a + b \sec(c + dx))^{5/2} (A + B \sec(c + dx)) dx$$

Optimal. Leaf size=359

$$\frac{(8a^2A - 9abB - 4Ab^2) \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2a}{a+b}\right) + b(15a^2B + 20aAb + 4b^2B) \sqrt{\frac{a \cos(c + dx)}{a+b}}}{4d \sqrt{\frac{a \cos(c + dx) + b}{a+b}}} + \frac{b(15a^2B + 20aAb + 4b^2B) \sqrt{\frac{a \cos(c + dx)}{a+b}}}{4d \sqrt{\cos(c + dx)} \sqrt{a + b}}$$

[Out] $\frac{1}{2} b B (a + b \sec(dx + c))^{3/2} \sin(dx + c) / d \cos(dx + c)^{1/2} + \frac{1}{4} (16 A a^2 b + 4 A b^3 + 8 B a^3 + 11 B a b^2) (\cos(1/2 dx + 1/2 c))^2)^{1/2} / \cos(1/2 dx + 1/2 c) \operatorname{EllipticF}(\sin(1/2 dx + 1/2 c), 2^{1/2} (a / (a + b))^{1/2}) ((b + a \cos(dx + c)) / (a + b))^{1/2} / d \cos(dx + c)^{1/2} / (a + b \sec(dx + c))^{1/2} + \frac{1}{4} b (20 A a b + 15 B a^2 + 4 B b^2) (\cos(1/2 dx + 1/2 c))^2)^{1/2} / \cos(1/2 dx + 1/2 c) \operatorname{EllipticPi}(\sin(1/2 dx + 1/2 c), 2, 2^{1/2} (a / (a + b))^{1/2}) ((b + a \cos(dx + c)) / (a + b))^{1/2} / d \cos(dx + c)^{1/2} / (a + b \sec(dx + c))^{1/2} + \frac{1}{4} b (4 A b + 7 B a) \sin(dx + c) (a + b \sec(dx + c))^{1/2} / d \cos(dx + c)^{1/2} + \frac{1}{4} (8 A a^2 - 4 A b^2 - 9 B a b) (\cos(1/2 dx + 1/2 c))^2)^{1/2} / \cos(1/2 dx + 1/2 c) \operatorname{EllipticE}(\sin(1/2 dx + 1/2 c), 2^{1/2} (a / (a + b))^{1/2}) \cos(dx + c)^{1/2} (a + b \sec(dx + c))^{1/2} / d / ((b + a \cos(dx + c)) / (a + b))^{1/2}$

Rubi [A] time = 1.43, antiderivative size = 359, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 14, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2955, 4026, 4096, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{(16a^2Ab + 8a^3B + 11ab^2B + 4Ab^3) \sqrt{\frac{a \cos(c + dx) + b}{a+b}} F\left(\frac{1}{2}(c + dx) \middle| \frac{2a}{a+b}\right) + (8a^2A - 9abB - 4Ab^2) \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}}{4d \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}} + \frac{b(15a^2B + 20aAb + 4b^2B) \sqrt{\frac{a \cos(c + dx)}{a+b}}}{4d \sqrt{\frac{a \cos(c + dx)}{a+b}}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]

[Out] $((16 a^2 A b + 4 A b^3 + 8 a^3 B + 11 a b^2 B) \operatorname{Sqrt}[(b + a \operatorname{Cos}[c + d x]) / (a + b)] \operatorname{EllipticF}[(c + d x) / 2, (2 a) / (a + b)]) / (4 d \operatorname{Sqrt}[\operatorname{Cos}[c + d x]] \operatorname{Sqrt}[a + b \operatorname{Sec}[c + d x]]) + (b (20 a A b + 15 a^2 B + 4 b^2 B) \operatorname{Sqrt}[(b + a \operatorname{Cos}[c + d x]) / (a + b)] \operatorname{EllipticPi}[2, (c + d x) / 2, (2 a) / (a + b)]) / (4 d \operatorname{Sqrt}[\operatorname{Cos}[c + d x]] \operatorname{Sqrt}[a + b \operatorname{Sec}[c + d x]]) + ((8 a^2 A - 4 A b^2 - 9 a b B) \operatorname{Sqrt}[\operatorname{Cos}[c + d x]] \operatorname{EllipticE}[(c + d x) / 2, (2 a) / (a + b)] \operatorname{Sqrt}[a + b \operatorname{Sec}[c + d x]]) / (4 d \operatorname{Sqrt}[(b + a \operatorname{Cos}[c + d x]) / (a + b)]) + (b (4 A b + 7 a B) \operatorname{Sqrt}[a + b \operatorname{Sec}[c + d x]] \operatorname{Sin}[c + d x]) / (4 d \operatorname{Sqrt}[\operatorname{Cos}[c + d x]]) + (b B (a + b \operatorname{Sec}[c + d x]))^{3/2} \operatorname{Sin}[c + d x] / (2 d \operatorname{Sqrt}[\operatorname{Cos}[c + d x]])$

Rule 2653

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*Sqrt[a
+ b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
0] && !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a
+ b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_)
+ (f_)*(x_)]]), x_Symbol] := Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/d, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_)
+ (f_)*(x_)]]), x_Symbol] := Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
+ f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !GtQ[c + d, 0]
```

Rule 2955

```
Int[((a_) + csc[(e_) + (f_)*(x_)])*(b_)^(m_)*(csc[(e_) + (f_)*(x_)])*
```

```
(d_.) + (c_.))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_.)]^(p_.), x_Symbol] := Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]/Sqrt[csc[(e_.) + (f_.)*(x_.)]
*(d_.)], x_Symbol] := Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.)
+ (a_.)], x_Symbol] := Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.)
+ (a_.)], x_Symbol] := Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4026

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (
a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(m + n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*Sim
p[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B))*(m + n) + b^2*B*(m + n - 1))*C
sc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b
^2, 0] && GtQ[m, 1] && !(IGtQ[n, 1] && !IntegerQ[m])
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.))/(Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]), x_Symbol] := Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
```

a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]

Rule 4096

Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.)) * (csc[(e_.) + (f_.)*(x_.)]*(d_.))^n * (csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m, x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(m + n + 1), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n*Simp[a*A*(m + n + 1) + a*C*n + ((A*b + a*B)*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) + a*C*m)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && !LeQ[n, -1]

Rule 4108

Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.))/(Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned}
\int \sqrt{\cos(c+dx)} (a+b \sec(c+dx))^{5/2} (A+B \sec(c+dx)) dx &= \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{(a+b \sec(c+dx))^{5/2} (A+B \sec(c+dx))}{\sqrt{\cos(c+dx)}} dx \\
&= \frac{bB(a+b \sec(c+dx))^{3/2} \sin(c+dx)}{2d\sqrt{\cos(c+dx)}} + \frac{1}{2} \left(\sqrt{\cos(c+dx)} \right) \\
&= \frac{b(4Ab+7aB)\sqrt{a+b \sec(c+dx)} \sin(c+dx)}{4d\sqrt{\cos(c+dx)}} + \\
&= \frac{b(4Ab+7aB)\sqrt{a+b \sec(c+dx)} \sin(c+dx)}{4d\sqrt{\cos(c+dx)}} + \\
&= \frac{b(4Ab+7aB)\sqrt{a+b \sec(c+dx)} \sin(c+dx)}{4d\sqrt{\cos(c+dx)}} + \\
&= \frac{b(4Ab+7aB)\sqrt{a+b \sec(c+dx)} \sin(c+dx)}{4d\sqrt{\cos(c+dx)}} + \\
&= \frac{b(4Ab+7aB)\sqrt{a+b \sec(c+dx)} \sin(c+dx)}{4d\sqrt{\cos(c+dx)}} + \\
&= \frac{b(20aAb+15a^2B+4b^2B) \sqrt{\frac{b+a \cos(c+dx)}{a+b}} \Pi\left(2; \frac{1}{2}; \frac{b+a \cos(c+dx)}{a+b}\right)}{4d\sqrt{\cos(c+dx)} \sqrt{a+b \sec(c+dx)}} \\
&= \frac{(16a^2Ab+4Ab^3+8a^3B+11ab^2B) \sqrt{\frac{b+a \cos(c+dx)}{a+b}}}{4d\sqrt{\cos(c+dx)} \sqrt{a+b \sec(c+dx)}}
\end{aligned}$$

Mathematica [C] time = 34.74, size = 97208, normalized size = 270.77

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]

[Out] Result too large to show

fricas [F] time = 9.72, size = 0, normalized size = 0.00

integral((Bb^2 sec(dx + c)^3 + Aa^2 + (2 Bab + Ab^2) sec(dx + c)^2 + (Ba^2 + 2 Aab) sec(dx + c))sqrt(b sec(dx + c))

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))*cos(d*x+c)^(1/2),x, algorithm="fricas")

[Out] integral((B*b^2*sec(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*sec(d*x + c)^2 + (B*a^2 + 2*A*a*b)*sec(d*x + c))*sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(b \sec(dx + c) + a)^{\frac{5}{2}} \sqrt{\cos(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))*cos(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)*sqrt(cos(d*x + c)), x)

maple [C] time = 2.32, size = 2216, normalized size = 6.17

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))*cos(d*x+c)^(1/2),x)

[Out] $\frac{1}{4}d^*(-1+\cos(d*x+c))*(1+\cos(d*x+c))*(-9*B*((a-b)/(a+b))^{(1/2)}*\cos(d*x+c)^2*a^2*b*(1/(1+\cos(d*x+c)))^{(1/2)}-2*B*((a-b)/(a+b))^{(1/2)}*b^3*(1/(1+\cos(d*x+c)))^{(1/2)}-11*B*((a-b)/(a+b))^{(1/2)}*\cos(d*x+c)*a*b^2*(1/(1+\cos(d*x+c)))^{(1/2)}+8*A*((a-b)/(a+b))^{(1/2)}*\cos(d*x+c)^3*a^2*b*(1/(1+\cos(d*x+c)))^{(1/2)}-4*A*((a-b)/(a+b))^{(1/2)}*\cos(d*x+c)^2*a*b^2*(1/(1+\cos(d*x+c)))^{(1/2)}+2*B*\cos(d*x+c)^2*((a-b)/(a+b))^{(1/2)}*b^3*(1/(1+\cos(d*x+c)))^{(1/2)}+8*B*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*\text{EllipticF}((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c),(-(a+b)/(a-b))^{(1/2)}*\cos(d*x+c)^2*\sin(d*x+c)*a^3-4*B*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*\text{EllipticF}((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c),(-(a+b)/(a-b))^{(1/2)}*\cos(d*x+c)^2*\sin(d*x+c)*b^3+8*A*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*\text{EllipticE}((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c),(-(a+b)/(a-b))^{(1/2)}*\cos(d*x+c)^2*\sin(d*x+c)*a^3+4*A*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*\text{EllipticE}((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c),(-(a+b)/(a-b))^{(1/2)}*\cos(d*x+c)^2*\sin(d*x+c)*b^3-8*A*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*\text{EllipticF}((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c),(-(a+b)/(a-b))^{(1/2)}*\cos(d*x+c)^2*\sin(d*x+c)*a^3+8*B*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*\text{EllipticPi}((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^{(1/2)})*\cos(d*x+c)^2*\sin(d*x+c)*b^3+8*A*((a-b)/(a+b))^{(1/2)}*\cos(d*x+c)^4*a^3$

```

*(1/(1+cos(d*x+c)))^(1/2)-4*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)*b^3*(1/(1+cos(
d*x+c)))^(1/2)-8*A*(1/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^3*((a-b)/(a+b))^(1/2
)*a^3+4*A*(1/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*b^3+30*
B*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c), (a+b)/(a-b), I/((a-b)/(a+b))^(1/2))*cos(d*x+c
)^2*sin(d*x+c)*a^2*b-9*B*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*Elli
pticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), -(a+b)/(a-b))^(1/2))*
cos(d*x+c)^2*sin(d*x+c)*a^2*b+9*B*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(
1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), -(a+b)/(a-b)
)^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a*b^2-6*B*((b+a*cos(d*x+c))/(1+cos(d*x+c))
)/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), -(a
+b)/(a-b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^2*b+2*B*((b+a*cos(d*x+c))/(1+co
s(d*x+c)))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*
x+c), -(a+b)/(a-b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a*b^2+40*A*((b+a*cos(d*x
+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(
1/2)/sin(d*x+c), (a+b)/(a-b), I/((a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*
a*b^2-8*A*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*EllipticE((-1+cos(d
*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), -(a+b)/(a-b))^(1/2))*cos(d*x+c)^2*si
n(d*x+c)*a^2*b-4*A*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*EllipticE(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), -(a+b)/(a-b))^(1/2))*cos(d*
x+c)^2*sin(d*x+c)*a*b^2+24*A*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*
EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), -(a+b)/(a-b))^(1/
2))*cos(d*x+c)^2*sin(d*x+c)*a^2*b-16*A*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+
b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), -(a+b)/
(a-b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a*b^2-8*A*cos(d*x+c)^2*((a-b)/(a+b))^(
1/2)*a^2*b*(1/(1+cos(d*x+c)))^(1/2)+9*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a
*b^2*(1/(1+cos(d*x+c)))^(1/2)+4*A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a*b^2*(1
/(1+cos(d*x+c)))^(1/2)+9*B*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b*(1/(1+cos
(d*x+c)))^(1/2)+2*B*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a*b^2*(1/(1+cos(d*x+c)
))^(1/2))*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/((a-b)/(a+b))^(1/2)/(b+a*cos(
d*x+c))/sin(d*x+c)^3/cos(d*x+c)^(3/2)/(1/(1+cos(d*x+c)))^(1/2)

```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sec(dx + c) + A)(b \sec(dx + c) + a)^{\frac{5}{2}} \sqrt{\cos(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))*cos(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)*sqrt(cos(d*x + c)
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{\cos(c + dx)} \left(A + \frac{B}{\cos(c + dx)} \right) \left(a + \frac{b}{\cos(c + dx)} \right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^(1/2)*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(5/2), x)

[Out] int(cos(c + d*x)^(1/2)*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(5/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c))*cos(d*x+c)**(1/2), x)

[Out] Timed out

$$3.613 \quad \int \frac{(a+b \sec(c+dx))^{5/2}(A+B \sec(c+dx))}{\sqrt{\cos(c+dx)}} dx$$

Optimal. Leaf size=422

$$\frac{(33a^2B + 54aAb + 16b^2B) \sin(c + dx) \sqrt{a + b \sec(c + dx)}}{24d \sqrt{\cos(c + dx)}} - \frac{(33a^2B + 54aAb + 16b^2B) \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}}{24d \sqrt{\frac{a \cos(c+dx)+b}{a+b}}}$$

[Out] $1/3*b*B*(a+b*\sec(d*x+c))^(3/2)*\sin(d*x+c)/d/\cos(d*x+c)^(3/2)+1/24*(48*A*a^3+66*A*a*b^2+59*B*a^2*b+16*B*b^3)*(cos(1/2*d*x+1/2*c)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(\sin(1/2*d*x+1/2*c), 2^(1/2)*(a/(a+b))^(1/2))*((b+a*\cos(d*x+c))/(a+b))^(1/2)/d/\cos(d*x+c)^(1/2)/(a+b*\sec(d*x+c))^(1/2)+1/8*(30*A*a^2*b+8*A*b^3+5*B*a^3+20*B*a*b^2)*(cos(1/2*d*x+1/2*c)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(\sin(1/2*d*x+1/2*c), 2, 2^(1/2)*(a/(a+b))^(1/2))*((b+a*\cos(d*x+c))/(a+b))^(1/2)/d/\cos(d*x+c)^(1/2)/(a+b*\sec(d*x+c))^(1/2)+1/4*b*(2*A*b+3*B*a)*\sin(d*x+c)*(a+b*\sec(d*x+c))^(1/2)/d/\cos(d*x+c)^(3/2)+1/24*(54*A*a*b+33*B*a^2+16*B*b^2)*\sin(d*x+c)*(a+b*\sec(d*x+c))^(1/2)/d/\cos(d*x+c)^(1/2)-1/24*(54*A*a*b+33*B*a^2+16*B*b^2)*(cos(1/2*d*x+1/2*c)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(\sin(1/2*d*x+1/2*c), 2^(1/2)*(a/(a+b))^(1/2))*\cos(d*x+c)^(1/2)*(a+b*\sec(d*x+c))^(1/2)/d/((b+a*\cos(d*x+c))/(a+b))^(1/2)$

Rubi [A] time = 1.80, antiderivative size = 422, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 15, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2955, 4026, 4096, 4102, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{(33a^2B + 54aAb + 16b^2B) \sin(c + dx) \sqrt{a + b \sec(c + dx)}}{24d \sqrt{\cos(c + dx)}} + \frac{(48a^3A + 59a^2bB + 66aAb^2 + 16b^3B) \sqrt{\frac{a \cos(c+dx)+b}{a+b}}}{24d \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[((a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Sqrt[Cos[c + d*x]], x]

[Out] $((48*a^3*A + 66*a*A*b^2 + 59*a^2*b*B + 16*b^3*B)*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])]/(a + b))*\text{EllipticF}[(c + d*x)/2, (2*a)/(a + b)]/(24*d*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]) + ((30*a^2*A*b + 8*A*b^3 + 5*a^3*B + 20*a*b^2*B)*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])]/(a + b))*\text{EllipticPi}[2, (c + d*x)/2, (2*a)/(a + b)]/(8*d*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]) - ((54*a*A*b + 33*a^2*B + 16*b^2*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, (2*a)/(a + b)]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]])/(24*d*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])]/(a + b)) + (b*(2*A*b + 3*a*B)*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(4*d*\text{Cos}[c + d*x]^(3/2)) + ((54*a*A*b + 33*a^2*B + 16*b^2*B)*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(24$

$d\sqrt{\cos[c + dx]} + (bB(a + b\sec[c + dx])^{3/2}\sin[c + dx]) / (3d\cos[c + dx]^{3/2})$

Rule 2653

$\text{Int}[\sqrt{(a_.) + (b_.)\sin[(c_.) + (d_.)x]}], x_Symbol] \rightarrow \text{Simp}[(2\sqrt{a + b}\text{EllipticE}[(1*(c - \text{Pi}/2 + dx))/2, (2*b)/(a + b)]) / d, x] /; \text{FreeQ}\{a, b, c, d, x\} \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{GtQ}[a + b, 0]$

Rule 2655

$\text{Int}[\sqrt{(a_.) + (b_.)\sin[(c_.) + (d_.)x]}], x_Symbol] \rightarrow \text{Dist}[\sqrt{a + b\sin[c + dx]} / \sqrt{a + b\sin[c + dx]} / (a + b), \text{Int}[\sqrt{a/(a + b) + (b\sin[c + dx]) / (a + b)}, x], x] /; \text{FreeQ}\{a, b, c, d, x\} \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{!GtQ}[a + b, 0]$

Rule 2661

$\text{Int}[1/\sqrt{(a_.) + (b_.)\sin[(c_.) + (d_.)x]}], x_Symbol] \rightarrow \text{Simp}[(2\text{EllipticF}[(1*(c - \text{Pi}/2 + dx))/2, (2*b)/(a + b)]) / (d\sqrt{a + b}), x] /; \text{FreeQ}\{a, b, c, d, x\} \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{GtQ}[a + b, 0]$

Rule 2663

$\text{Int}[1/\sqrt{(a_.) + (b_.)\sin[(c_.) + (d_.)x]}], x_Symbol] \rightarrow \text{Dist}[\sqrt{(a + b\sin[c + dx]) / (a + b)} / \sqrt{a + b\sin[c + dx]}, \text{Int}[1/\sqrt{a/(a + b) + (b\sin[c + dx]) / (a + b)}, x], x] /; \text{FreeQ}\{a, b, c, d, x\} \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{!GtQ}[a + b, 0]$

Rule 2805

$\text{Int}[1/(((a_.) + (b_.)\sin[(e_.) + (f_.)x])\sqrt{(c_.) + (d_.)\sin[(e_.) + (f_.)x]}), x_Symbol] \rightarrow \text{Simp}[(2\text{EllipticPi}[(2*b)/(a + b), (1*(e - \text{Pi}/2 + fx))/2, (2*d)/(c + d)]) / (f*(a + b)\sqrt{c + d}), x] /; \text{FreeQ}\{a, b, c, d, e, f, x\} \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{GtQ}[c + d, 0]$

Rule 2807

$\text{Int}[1/(((a_.) + (b_.)\sin[(e_.) + (f_.)x])\sqrt{(c_.) + (d_.)\sin[(e_.) + (f_.)x]}), x_Symbol] \rightarrow \text{Dist}[\sqrt{(c + d\sin[e + fx]) / (c + d)} / \sqrt{c + d\sin[e + fx]}, \text{Int}[1/((a + b\sin[e + fx])\sqrt{c/(c + d) + (d\sin[e + fx]) / (c + d)}), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, x\} \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{!GtQ}[c + d, 0]$

Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)])*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_.))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] := Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)])*(b_.) + (a_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] := Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)])*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_.)], x_Symbol] := Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)])*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_.)], x_Symbol] := Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x])],
x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4026

```
Int[(csc[(e_.) + (f_.)*(x_)])*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] := -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(m + n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*Sim
p[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B))*(m + n) + b^2*B*(m + n - 1))*C
sc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b
^2, 0] && GtQ[m, 1] && !(IGtQ[n, 1] && !IntegerQ[m])
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)]), x_Symbol] := Dist[A/a, In
```

$t[\text{Sqrt}[a + b\text{Csc}[e + f*x]]/\text{Sqrt}[d\text{Csc}[e + f*x]], x], x] - \text{Dist}[(A*b - a*B)/(a*d), \text{Int}[\text{Sqrt}[d\text{Csc}[e + f*x]]/\text{Sqrt}[a + b\text{Csc}[e + f*x]], x], x] /;$ FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]

Rule 4096

$\text{Int}[(A + \text{csc}[e] + (f)*(x))*(B) + \text{csc}[e] + (f)*(x)]^2*(C + \text{csc}[e] + (f)*(x))*(d)^n*(\text{csc}[e] + (f)*(x))*(b) + (a)^m, x_Symbol] :> -\text{Simp}[(C*\text{Cot}[e + f*x]*(a + b\text{Csc}[e + f*x])^m*(d\text{Csc}[e + f*x])^n)/(f*(m + n + 1)), x] + \text{Dist}[1/(m + n + 1), \text{Int}[(a + b\text{Csc}[e + f*x])^{m-1}*(d\text{Csc}[e + f*x])^n*\text{Simp}[a*A*(m + n + 1) + a*C*n + ((A*b + a*B)*(m + n + 1) + b*C*(m + n))*\text{Csc}[e + f*x] + (b*B*(m + n + 1) + a*C*m)*\text{Csc}[e + f*x]^2, x], x], x] /;$ FreeQ[{a, b, d, e, f, A, B, C, n}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && !LeQ[n, -1]

Rule 4102

$\text{Int}[(A + \text{csc}[e] + (f)*(x))*(B) + \text{csc}[e] + (f)*(x)]^2*(C + \text{csc}[e] + (f)*(x))*(d)^n*(\text{csc}[e] + (f)*(x))*(b) + (a)^m, x_Symbol] :> -\text{Simp}[(C*d*\text{Cot}[e + f*x]*(a + b\text{Csc}[e + f*x])^{m+1}*(d\text{Csc}[e + f*x])^{n-1})/(b*f*(m + n + 1)), x] + \text{Dist}[d/(b*(m + n + 1)), \text{Int}[(a + b\text{Csc}[e + f*x])^m*(d\text{Csc}[e + f*x])^{n-1}*\text{Simp}[a*C*(n - 1) + (A*b*(m + n + 1) + b*C*(m + n))*\text{Csc}[e + f*x] + (b*B*(m + n + 1) - a*C*n)*\text{Csc}[e + f*x]^2, x], x], x] /;$ FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 0]

Rule 4108

$\text{Int}[(A + \text{csc}[e] + (f)*(x))*(B) + \text{csc}[e] + (f)*(x)]^2*(C + \text{csc}[e] + (f)*(x))/(\text{Sqrt}[\text{csc}[e] + (f)*(x)]*(d)]*\text{Sqrt}[\text{csc}[e] + (f)*(x)]*(b) + (a)], x_Symbol] :> \text{Dist}[C/d^2, \text{Int}[(d\text{Csc}[e + f*x])^{3/2}/\text{Sqrt}[a + b\text{Csc}[e + f*x]], x], x] + \text{Int}[(A + B\text{Csc}[e + f*x])/(\text{Sqrt}[d\text{Csc}[e + f*x]]*\text{Sqrt}[a + b\text{Csc}[e + f*x]]), x] /;$ FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned}
\int \frac{(a + b \sec(c + dx))^{5/2} (A + B \sec(c + dx))}{\sqrt{\cos(c + dx)}} dx &= \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \sqrt{\sec(c + dx)} (a + b \sec(c + dx)) dx \\
&= \frac{bB(a + b \sec(c + dx))^{3/2} \sin(c + dx)}{3d \cos^2(c + dx)} + \frac{1}{3} \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \sqrt{\sec(c + dx)} dx \\
&= \frac{b(2Ab + 3aB) \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{4d \cos^2(c + dx)} + \frac{bB(a + b \sec(c + dx))^{3/2} \sin(c + dx)}{3d \cos^2(c + dx)} \\
&= \frac{b(2Ab + 3aB) \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{4d \cos^2(c + dx)} + \frac{(54aAb + 33a^2B) \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{4d \cos^2(c + dx)} \\
&= \frac{b(2Ab + 3aB) \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{4d \cos^2(c + dx)} + \frac{(54aAb + 33a^2B) \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{4d \cos^2(c + dx)} \\
&= \frac{b(2Ab + 3aB) \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{4d \cos^2(c + dx)} + \frac{(54aAb + 33a^2B) \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{4d \cos^2(c + dx)} \\
&= \frac{b(2Ab + 3aB) \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{4d \cos^2(c + dx)} + \frac{(54aAb + 33a^2B) \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{4d \cos^2(c + dx)} \\
&= \frac{(30a^2Ab + 8Ab^3 + 5a^3B + 20ab^2B) \sqrt{\frac{b+a \cos(c+dx)}{a+b}} \Pi\left(2; \frac{1}{2}(c + dx), \sqrt{\frac{b+a \cos(c+dx)}{a+b}}\right)}{8d \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}} \\
&= \frac{(48a^3A + 66aAb^2 + 59a^2bB + 16b^3B) \sqrt{\frac{b+a \cos(c+dx)}{a+b}} F\left(\frac{1}{2}(c + dx), \sqrt{\frac{b+a \cos(c+dx)}{a+b}}\right)}{24d \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}}
\end{aligned}$$

Mathematica [C] time = 34.62, size = 106199, normalized size = 251.66

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```

```
[Out] Result too large to show
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(b \sec(dx + c) + a)^{\frac{5}{2}}}{\sqrt{\cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)/sqrt(cos(d*x + c)), x)

maple [C] time = 2.87, size = 2441, normalized size = 5.78

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x)

[Out] 1/24/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(1+cos(d*x+c))*(-59*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^2*b*(1/(1+cos(d*x+c)))^(1/2)+48*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), (a+b)/(a-b), I/((a-b)/(a+b))^(1/2))*cos(d*x+c)^3*b^3-8*B*((a-b)/(a+b))^(1/2)*b^3*(1/(1+cos(d*x+c)))^(1/2)-34*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a*b^2*(1/(1+cos(d*x+c)))^(1/2)-54*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*a^2*b*(1/(1+cos(d*x+c)))^(1/2)-66*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a*b^2*(1/(1+cos(d*x+c)))^(1/2)+33*B*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^3*(1/(1+cos(d*x+c)))^(1/2)+16*B*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*b^3*(1/(1+cos(d*x+c)))^(1/2)-8*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*b^3*(1/(1+cos(d*x+c)))^(1/2)-33*B*(1/(1+cos(d*x+c)))^(1/2)*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*a^3-12*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)*b^3*(1/(1+cos(d*x+c)))^(1/2)+16*B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), (-a+b)/(a-b))^(1/2))*co

$$\begin{aligned}
& s(d*x+c)^3*b^3+12*A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*b^3*(1/(1+cos(d*x+c))) \\
& ^{(1/2)}+48*A*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*EllipticF((-1+cos \\
& (d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), (-a+b)/(a-b))^(1/2))*sin(d*x+c)*co \\
& s(d*x+c)^3*a^3-24*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2) \\
&)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), (-a+b)/(a-b))^(\\
& 1/2))*cos(d*x+c)^3*b^3+54*A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a*b^2*(1/(1+co \\
& s(d*x+c)))^(1/2)+33*B*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b*(1/(1+cos(d*x+ \\
& c)))^(1/2)+18*B*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a*b^2*(1/(1+cos(d*x+c)))^(\\
& 1/2)+54*A*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^2*b*(1/(1+cos(d*x+c)))^(1/2)+1 \\
& 2*A*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a*b^2*(1/(1+cos(d*x+c)))^(1/2)+26*B*co \\
& s(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^2*b*(1/(1+cos(d*x+c)))^(1/2)+16*B*cos(d*x+ \\
& c)^4*((a-b)/(a+b))^(1/2)*a*b^2*(1/(1+cos(d*x+c)))^(1/2)+30*B*sin(d*x+c)*((b \\
& +a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b) \\
&)/(a+b))^(1/2)/sin(d*x+c), (a+b)/(a-b), I/((a-b)/(a+b))^(1/2))*cos(d*x+c)^3*a \\
& ^3+18*B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*EllipticF(\\
& (-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), (-a+b)/(a-b))^(1/2))*cos(d* \\
& x+c)^3*a^3-33*B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*El \\
& lipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), (-a+b)/(a-b))^(1/2) \\
&)*cos(d*x+c)^3*a^3+120*B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b)) \\
& ^{(1/2)}*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), (a+b)/(a-b) \\
&), I/((a-b)/(a+b))^(1/2))*cos(d*x+c)^3*a*b^2+26*B*sin(d*x+c)*((b+a*cos(d*x+c) \\
&)/(1+cos(d*x+c)))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2) \\
&)/sin(d*x+c), (-a+b)/(a-b))^(1/2))*cos(d*x+c)^3*a^2*b-44*B*sin(d*x+c)*((b+a \\
& *cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(\\
& a+b))^(1/2)/sin(d*x+c), (-a+b)/(a-b))^(1/2))*cos(d*x+c)^3*a*b^2+33*B*sin(d* \\
& x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c) \\
&)*((a-b)/(a+b))^(1/2)/sin(d*x+c), (-a+b)/(a-b))^(1/2))*cos(d*x+c)^3*a^2*b-1 \\
& 6*B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*EllipticE((-1+ \\
& cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), (-a+b)/(a-b))^(1/2))*cos(d*x+c) \\
& ^3*a*b^2+180*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*Ell \\
& ipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), (a+b)/(a-b), I/((a-b) \\
&)/(a+b))^(1/2))*cos(d*x+c)^3*a^2*b-36*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(\\
& d*x+c)))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+ \\
& c), (-a+b)/(a-b))^(1/2))*cos(d*x+c)^3*a^2*b+12*A*sin(d*x+c)*((b+a*cos(d*x+c) \\
&)/(1+cos(d*x+c)))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2) \\
&)/sin(d*x+c), (-a+b)/(a-b))^(1/2))*cos(d*x+c)^3*a*b^2-54*A*sin(d*x+c)*((b+a \\
& *cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(\\
& a+b))^(1/2)/sin(d*x+c), (-a+b)/(a-b))^(1/2))*cos(d*x+c)^3*a^2*b+54*A*sin(d* \\
& x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c) \\
&)*((a-b)/(a+b))^(1/2)/sin(d*x+c), (-a+b)/(a-b))^(1/2))*cos(d*x+c)^3*a*b^2)/ \\
& ((a-b)/(a+b))^(1/2)/(b+a*cos(d*x+c))/cos(d*x+c)^(5/2)/sin(d*x+c)^3/(1/(1+co \\
& s(d*x+c)))^(1/2)
\end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(b \sec(dx + c) + a)^{\frac{5}{2}}}{\sqrt{\cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)/sqrt(cos(d*x + c)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(a + \frac{b}{\cos(c+dx)}\right)^{5/2}}{\sqrt{\cos(c+dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(5/2))/cos(c + d*x)^(1/2),x)

[Out] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(5/2))/cos(c + d*x)^(1/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c))/cos(d*x+c)**(1/2),x)

[Out] Timed out

$$3.614 \quad \int \frac{(a+b \sec(c+dx))^{5/2}(A+B \sec(c+dx))}{\cos^{\frac{3}{2}}(c+dx)} dx$$

Optimal. Leaf size=513

$$\frac{(59a^2B + 104aAb + 36b^2B) \sin(c + dx) \sqrt{a + b \sec(c + dx)}}{96d \cos^{\frac{3}{2}}(c + dx)} + \frac{(15a^3B + 264a^2Ab + 284ab^2B + 128Ab^3) \sin(c + dx)}{192bd \sqrt{\cos(c + dx)}}$$

[Out] $\frac{1}{4} b B (a + b \sec(dx + c))^{3/2} \sin(dx + c) / d \cos(dx + c)^{5/2} + \frac{1}{192} (472 A a^2 b + 128 A a b^3 + 133 B a^3 + 356 B a b^2) (\cos(1/2 dx + 1/2 c))^2 \sqrt{\cos(1/2 dx + 1/2 c)} \operatorname{EllipticF}(\sin(1/2 dx + 1/2 c), 2^{1/2} (a/(a+b))^{1/2}) ((b + a \cos(dx + c))/(a+b))^{1/2} / d \cos(dx + c)^{1/2} / (a + b \sec(dx + c))^{1/2} + \frac{1}{64} (40 A a^3 b + 160 A a a b^3 - 5 B a^4 + 120 B a^2 b^2 + 48 B b^4) (\cos(1/2 dx + 1/2 c))^2 \sqrt{\cos(1/2 dx + 1/2 c)} \operatorname{EllipticPi}(\sin(1/2 dx + 1/2 c), 2, 2^{1/2} (a/(a+b))^{1/2}) ((b + a \cos(dx + c))/(a+b))^{1/2} / b d \cos(dx + c)^{1/2} / (a + b \sec(dx + c))^{1/2} + \frac{1}{24} b (8 A a b + 11 B a) \sin(dx + c) (a + b \sec(dx + c))^{1/2} / d \cos(dx + c)^{5/2} + \frac{1}{96} (104 A a a b + 59 B a^2 + 36 B b^2) \sin(dx + c) (a + b \sec(dx + c))^{1/2} / d \cos(dx + c)^{3/2} + \frac{1}{192} (264 A a^2 b + 128 A a b^3 + 15 B a^3 + 284 B a a b^2) \sin(dx + c) (a + b \sec(dx + c))^{1/2} / b d \cos(dx + c)^{1/2} - \frac{1}{192} (264 A a^2 b + 128 A a b^3 + 15 B a^3 + 284 B a a b^2) (\cos(1/2 dx + 1/2 c))^2 \sqrt{\cos(1/2 dx + 1/2 c)} \operatorname{EllipticE}(\sin(1/2 dx + 1/2 c), 2^{1/2} (a/(a+b))^{1/2}) \cos(dx + c)^{1/2} (a + b \sec(dx + c))^{1/2} / b d ((b + a \cos(dx + c))/(a+b))^{1/2}$

Rubi [A] time = 2.25, antiderivative size = 513, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 15, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2955, 4026, 4096, 4102, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{(59a^2B + 104aAb + 36b^2B) \sin(c + dx) \sqrt{a + b \sec(c + dx)}}{96d \cos^{\frac{3}{2}}(c + dx)} + \frac{(264a^2Ab + 15a^3B + 284ab^2B + 128Ab^3) \sin(c + dx)}{192bd \sqrt{\cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(a + b \operatorname{Sec}[c + dx])^{5/2} (A + B \operatorname{Sec}[c + dx])] / \operatorname{Cos}[c + dx]^{3/2}, x]$

[Out] $((472 a^2 A b + 128 A a b^3 + 133 a^3 B + 356 a a b^2 B) \operatorname{Sqrt}[(b + a \operatorname{Cos}[c + dx]) / (a + b)] \operatorname{EllipticF}[(c + dx) / 2, (2 a) / (a + b)] / (192 d \operatorname{Sqrt}[\operatorname{Cos}[c + dx]] \operatorname{Sqrt}[a + b \operatorname{Sec}[c + dx]]) + ((40 a^3 A b + 160 a a A b^3 - 5 a^4 B + 120 a^2 b^2 B + 48 b^4 B) \operatorname{Sqrt}[(b + a \operatorname{Cos}[c + dx]) / (a + b)] \operatorname{EllipticPi}[2, (c + dx) / 2, (2 a) / (a + b)] / (64 b d \operatorname{Sqrt}[\operatorname{Cos}[c + dx]] \operatorname{Sqrt}[a + b \operatorname{Sec}[c + dx]]) - ((264 a^2 A b + 128 A a b^3 + 15 a^3 B + 284 a a b^2 B) \operatorname{Sqrt}[\operatorname{Cos}[c + dx]] * \operatorname{Sqrt}[a + b \operatorname{Sec}[c + dx]]) / (192 b d \operatorname{Sqrt}[\operatorname{Cos}[c + dx]])$

EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]]/(192*b*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (b*(8*A*b + 11*a*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(24*d*Cos[c + d*x]^(5/2)) + ((104*a*A*b + 59*a^2*B + 36*b^2*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(96*d*Cos[c + d*x]^(3/2)) + ((264*a^2*A*b + 128*A*b^3 + 15*a^3*B + 284*a*b^2*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(192*b*d*Sqrt[Cos[c + d*x]]) + (b*B*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(4*d*Cos[c + d*x]^(5/2))

Rule 2653

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*Sqrt[a + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2655

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a + b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2661

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2663

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b) + (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2805

Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi/2 + f*x))/2, (2*d)/(c + d)]/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[c + d, 0]

Rule 2807

Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt

$[c + d \sin[e + f x]]$, $\text{Int}[1/((a + b \sin[e + f x]) \sqrt{c/(c + d) + (d \sin[e + f x])/(c + d)})], x], x] /;$ $\text{FreeQ}\{a, b, c, d, e, f, x\}$ && $\text{NeQ}[b c - a d, 0]$ && $\text{NeQ}[a^2 - b^2, 0]$ && $\text{NeQ}[c^2 - d^2, 0]$ && $! \text{GtQ}[c + d, 0]$

Rule 2955

$\text{Int}[(a_.) + \text{csc}[e_.) + (f_.)(x_.)]*(b_.)^{(m_.)}*(\text{csc}[e_.) + (f_.)(x_.)]*(d_.) + (c_.)^{(n_.)}*((g_.)\sin[e_.) + (f_.)(x_.)]^{(p_.)}, x_Symbol] :> \text{Dist}[(g*\text{Csc}[e + f*x])^p*(g*\sin[e + f*x])^p, \text{Int}[(a + b*\text{Csc}[e + f*x])^m*(c + d*\text{Csc}[e + f*x])^n]/(g*\text{Csc}[e + f*x])^p, x], x] /;$ $\text{FreeQ}\{a, b, c, d, e, f, g, m, n, p\}, x]$ && $\text{NeQ}[b*c - a*d, 0]$ && $!\text{IntegerQ}[p]$ && $!(\text{IntegerQ}[m] \&\& \text{IntegerQ}[n])$

Rule 3856

$\text{Int}[\sqrt{\text{csc}[e_.) + (f_.)(x_.)]*(b_.) + (a_.)}/\sqrt{\text{csc}[e_.) + (f_.)(x_.)]*(d_.)}, x_Symbol] :> \text{Dist}[\sqrt{a + b*\text{Csc}[e + f*x]}/(\sqrt{d*\text{Csc}[e + f*x]}*\sqrt{b + a*\sin[e + f*x]}), \text{Int}[\sqrt{b + a*\sin[e + f*x]}, x], x] /;$ $\text{FreeQ}\{a, b, d, e, f, x\}$ && $\text{NeQ}[a^2 - b^2, 0]$

Rule 3858

$\text{Int}[\sqrt{\text{csc}[e_.) + (f_.)(x_.)]*(d_.)}/\sqrt{\text{csc}[e_.) + (f_.)(x_.)]*(b_.) + (a_.)}, x_Symbol] :> \text{Dist}[(\sqrt{d*\text{Csc}[e + f*x]}*\sqrt{b + a*\sin[e + f*x]})/\sqrt{a + b*\text{Csc}[e + f*x]}, \text{Int}[1/\sqrt{b + a*\sin[e + f*x]}, x], x] /;$ $\text{FreeQ}\{a, b, d, e, f, x\}$ && $\text{NeQ}[a^2 - b^2, 0]$

Rule 3859

$\text{Int}[(\text{csc}[e_.) + (f_.)(x_.)]*(d_.))^{(3/2)}/\sqrt{\text{csc}[e_.) + (f_.)(x_.)]*(b_.) + (a_.)}, x_Symbol] :> \text{Dist}[(d*\sqrt{d*\text{Csc}[e + f*x]}*\sqrt{b + a*\sin[e + f*x]})/\sqrt{a + b*\text{Csc}[e + f*x]}, \text{Int}[1/(\sin[e + f*x]*\sqrt{b + a*\sin[e + f*x]}), x], x] /;$ $\text{FreeQ}\{a, b, d, e, f, x\}$ && $\text{NeQ}[a^2 - b^2, 0]$

Rule 4026

$\text{Int}[(\text{csc}[e_.) + (f_.)(x_.)]*(d_.))^{(n_.)}*(\text{csc}[e_.) + (f_.)(x_.)]*(b_.) + (a_.))^{(m_.)}*(\text{csc}[e_.) + (f_.)(x_.)]*(B_.) + (A_.)}, x_Symbol] :> -\text{Simp}[(b*B*\text{Cot}[e + f*x]*(a + b*\text{Csc}[e + f*x])^{(m-1)}*(d*\text{Csc}[e + f*x])^n)/(f*(m+n)), x] + \text{Dist}[1/(m+n), \text{Int}[(a + b*\text{Csc}[e + f*x])^{(m-2)}*(d*\text{Csc}[e + f*x])^n*\text{Simp}[a^2*A*(m+n) + a*b*B*n + (a*(2*A*b + a*B))*(m+n) + b^2*B*(m+n-1))*\text{Csc}[e + f*x] + b*(A*b*(m+n) + a*B*(2*m+n-1))*\text{Csc}[e + f*x]^2, x], x] /;$ $\text{FreeQ}\{a, b, d, e, f, A, B, n\}, x]$ && $\text{NeQ}[A*b - a*B, 0]$ && $\text{NeQ}[a^2 - b^2, 0]$ && $\text{GtQ}[m, 1]$ && $!(\text{IGtQ}[n, 1] \&\& !\text{IntegerQ}[m])$

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.))/(Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]), x_Symbol] := Dist[A/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4096

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m, x_Symbol] := -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(m + n + 1), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n*Simp[a*A*(m + n + 1) + a*C*n + ((A*b + a*B)*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) + a*C*m)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && !LeQ[n, -1]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m, x_Symbol] := -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 0]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.))/(Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]), x_Symbol] := Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + b \sec(c + dx))^{5/2} (A + B \sec(c + dx))}{\cos^{\frac{3}{2}}(c + dx)} dx &= \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \sec^{\frac{3}{2}}(c + dx) (a + b \sec(c + dx)) dx \\
&= \frac{bB(a + b \sec(c + dx))^{3/2} \sin(c + dx)}{4d \cos^{\frac{5}{2}}(c + dx)} + \frac{1}{4} \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \sec^{\frac{3}{2}}(c + dx) dx \\
&= \frac{b(8Ab + 11aB) \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{24d \cos^{\frac{5}{2}}(c + dx)} + \frac{bB(a + b \sec(c + dx))^{3/2} \sin(c + dx)}{4d \cos^{\frac{5}{2}}(c + dx)} \\
&= \frac{b(8Ab + 11aB) \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{24d \cos^{\frac{5}{2}}(c + dx)} + \frac{(104aAb + 5b^2B) \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{24d \cos^{\frac{5}{2}}(c + dx)} \\
&= \frac{b(8Ab + 11aB) \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{24d \cos^{\frac{5}{2}}(c + dx)} + \frac{(104aAb + 5b^2B) \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{24d \cos^{\frac{5}{2}}(c + dx)} \\
&= \frac{b(8Ab + 11aB) \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{24d \cos^{\frac{5}{2}}(c + dx)} + \frac{(104aAb + 5b^2B) \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{24d \cos^{\frac{5}{2}}(c + dx)} \\
&= \frac{b(8Ab + 11aB) \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{24d \cos^{\frac{5}{2}}(c + dx)} + \frac{(104aAb + 5b^2B) \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{24d \cos^{\frac{5}{2}}(c + dx)} \\
&= \frac{b(8Ab + 11aB) \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{24d \cos^{\frac{5}{2}}(c + dx)} + \frac{(104aAb + 5b^2B) \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{24d \cos^{\frac{5}{2}}(c + dx)} \\
&= \frac{(40a^3Ab + 160aAb^3 - 5a^4B + 120a^2b^2B + 48b^4B) \sqrt{\frac{b+a \cos(c+dx)}{a+b}}}{64bd \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}} \\
&= \frac{(472a^2Ab + 128Ab^3 + 133a^3B + 356ab^2B) \sqrt{\frac{b+a \cos(c+dx)}{a+b}} F\left(\frac{1}{2}, \frac{b+a \cos(c+dx)}{a+b}\right)}{192d \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}}
\end{aligned}$$

Mathematica [C] time = 35.40, size = 131553, normalized size = 256.44

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[((a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Cos[c + d*x]^(3/2), x]

[Out] Result too large to show

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2), x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(b \sec(dx + c) + a)^{\frac{5}{2}}}{\cos(dx + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2), x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)/cos(d*x + c)^(3/2), x)

maple [C] time = 2.33, size = 3175, normalized size = 6.19

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2), x)

[Out] 1/192/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(1+cos(d*x+c))*(-264*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^4*a^3*b*(1/(1+cos(d*x+c)))^(1/2)+288*B*cos(d*x+c)^4*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), (a+b)/(a-b), I/((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^4-48*B*((a-b)/(a+b))^(1/2)*b^4*(1/(1+cos(d*x+c)))^(1/2)-472*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*a^2*b^2*(1/(1+cos(d*x+c)))^(1/2)-133*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*a^3*b*(1/(1+cos(d*x+c)))^(1/2)-272*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a*b^3*(1/(1+cos(d*x+c)))^(1/2)-254*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^2*b^2*(1/(1+cos(d*x+c)))^(1/2)-184*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a*b^3*(1/(1+cos(d*x+c)))^(1/2)+264*A*cos(d*x+c)^5*((a-b)

$$\begin{aligned}
&)/(a+b)^{1/2} * a^3 * b * (1/(1+\cos(dx+c)))^{1/2} + 208 * A * \cos(dx+c)^5 * ((a-b)/(a+b))^{1/2} * a^2 * b^2 * (1/(1+\cos(dx+c)))^{1/2} + 128 * A * \cos(dx+c)^5 * ((a-b)/(a+b))^{1/2} * a * b^3 * (1/(1+\cos(dx+c)))^{1/2} + 118 * B * \cos(dx+c)^5 * ((a-b)/(a+b))^{1/2} * a^3 * b * (1/(1+\cos(dx+c)))^{1/2} + 284 * B * \cos(dx+c)^5 * ((a-b)/(a+b))^{1/2} * a^2 * b^2 * (1/(1+\cos(dx+c)))^{1/2} + 72 * B * \cos(dx+c)^5 * ((a-b)/(a+b))^{1/2} * a * b^3 * (1/(1+\cos(dx+c)))^{1/2} + 128 * A * \cos(dx+c)^4 * ((b+a*\cos(dx+c))/(1+\cos(dx+c)))/(a+b)^{1/2} * \text{EllipticE}((-1+\cos(dx+c)) * ((a-b)/(a+b))^{1/2} / \sin(dx+c), (-a+b)/(a-b))^{1/2} * \sin(dx+c) * b^4 + 264 * A * \cos(dx+c)^4 * ((a-b)/(a+b))^{1/2} * a^2 * b^2 * (1/(1+\cos(dx+c)))^{1/2} + 144 * A * \cos(dx+c)^4 * ((a-b)/(a+b))^{1/2} * a * b^3 * (1/(1+\cos(dx+c)))^{1/2} + 15 * B * \cos(dx+c)^4 * ((a-b)/(a+b))^{1/2} * a^3 * b * (1/(1+\cos(dx+c)))^{1/2} - 30 * B * \cos(dx+c)^4 * ((a-b)/(a+b))^{1/2} * a^2 * b^2 * (1/(1+\cos(dx+c)))^{1/2} + 284 * B * \cos(dx+c)^4 * ((a-b)/(a+b))^{1/2} * a * b^3 * (1/(1+\cos(dx+c)))^{1/2} - 172 * B * \cos(dx+c)^3 * ((a-b)/(a+b))^{1/2} * a * b^3 * (1/(1+\cos(dx+c)))^{1/2} + 30 * B * \cos(dx+c)^4 * \text{EllipticF}((-1+\cos(dx+c)) * ((a-b)/(a+b))^{1/2} / \sin(dx+c), (-a+b)/(a-b))^{1/2} * ((b+a*\cos(dx+c))/(1+\cos(dx+c)))/(a+b)^{1/2} * \sin(dx+c) * a^4 - 144 * B * \cos(dx+c)^4 * \text{EllipticF}((-1+\cos(dx+c)) * ((a-b)/(a+b))^{1/2} / \sin(dx+c), (-a+b)/(a-b))^{1/2} * ((b+a*\cos(dx+c))/(1+\cos(dx+c)))/(a+b)^{1/2} * \sin(dx+c) * b^4 - 15 * B * \cos(dx+c)^4 * ((b+a*\cos(dx+c))/(1+\cos(dx+c)))/(a+b)^{1/2} * \text{EllipticE}((-1+\cos(dx+c)) * ((a-b)/(a+b))^{1/2} / \sin(dx+c), (-a+b)/(a-b))^{1/2} * \sin(dx+c) * a^4 - 30 * B * \cos(dx+c)^4 * ((b+a*\cos(dx+c))/(1+\cos(dx+c)))/(a+b)^{1/2} * \text{EllipticPi}((-1+\cos(dx+c)) * ((a-b)/(a+b))^{1/2} / \sin(dx+c), (a+b)/(a-b), I/((a-b)/(a+b))^{1/2} * \sin(dx+c) * a^4 + 15 * B * \cos(dx+c)^5 * ((a-b)/(a+b))^{1/2} * a^4 * (1/(1+\cos(dx+c)))^{1/2} + 128 * A * \cos(dx+c)^4 * ((a-b)/(a+b))^{1/2} * b^4 * (1/(1+\cos(dx+c)))^{1/2} - 64 * A * \cos(dx+c)^3 * ((a-b)/(a+b))^{1/2} * b^4 * (1/(1+\cos(dx+c)))^{1/2} + 72 * B * \cos(dx+c)^4 * ((a-b)/(a+b))^{1/2} * b^4 * (1/(1+\cos(dx+c)))^{1/2} - 24 * B * \cos(dx+c)^2 * ((a-b)/(a+b))^{1/2} * b^4 * (1/(1+\cos(dx+c)))^{1/2} + 960 * A * \cos(dx+c)^4 * ((b+a*\cos(dx+c))/(1+\cos(dx+c)))/(a+b)^{1/2} * \text{EllipticPi}((-1+\cos(dx+c)) * ((a-b)/(a+b))^{1/2} / \sin(dx+c), (a+b)/(a-b), I/((a-b)/(a+b))^{1/2} * \sin(dx+c) * a * b^3 + 118 * B * \cos(dx+c)^4 * \text{EllipticF}((-1+\cos(dx+c)) * ((a-b)/(a+b))^{1/2} / \sin(dx+c), (-a+b)/(a-b))^{1/2} * ((b+a*\cos(dx+c))/(1+\cos(dx+c)))/(a+b)^{1/2} * \sin(dx+c) * a^3 * b - 76 * B * \cos(dx+c)^4 * \text{EllipticF}((-1+\cos(dx+c)) * ((a-b)/(a+b))^{1/2} / \sin(dx+c), (-a+b)/(a-b))^{1/2} * ((b+a*\cos(dx+c))/(1+\cos(dx+c)))/(a+b)^{1/2} * \sin(dx+c) * a^2 * b^2 + 72 * B * \cos(dx+c)^4 * \text{EllipticF}((-1+\cos(dx+c)) * ((a-b)/(a+b))^{1/2} / \sin(dx+c), (-a+b)/(a-b))^{1/2} * ((b+a*\cos(dx+c))/(1+\cos(dx+c)))/(a+b)^{1/2} * \sin(dx+c) * a * b^3 + 15 * B * \cos(dx+c)^4 * ((b+a*\cos(dx+c))/(1+\cos(dx+c)))/(a+b)^{1/2} * \text{EllipticE}((-1+\cos(dx+c)) * ((a-b)/(a+b))^{1/2} / \sin(dx+c), (-a+b)/(a-b))^{1/2} * \sin(dx+c) * a^3 * b - 284 * B * \cos(dx+c)^4 * ((b+a*\cos(dx+c))/(1+\cos(dx+c)))/(a+b)^{1/2} * \text{EllipticE}((-1+\cos(dx+c)) * ((a-b)/(a+b))^{1/2} / \sin(dx+c), (-a+b)/(a-b))^{1/2} * \sin(dx+c) * a^2 * b^2 + 284 * B * \cos(dx+c)^4 * ((b+a*\cos(dx+c))/(1+\cos(dx+c)))/(a+b)^{1/2} * \text{EllipticE}((-1+\cos(dx+c)) * ((a-b)/(a+b))^{1/2} / \sin(dx+c), (-a+b)/(a-b))^{1/2} * \sin(dx+c) * a * b^3 + 720 * B * \cos(dx+c)^4 * ((b+a*\cos(dx+c))/(1+\cos(dx+c)))/(a+b)^{1/2} * \text{EllipticPi}((-1+\cos(dx+c)) * ((a-b)/(a+b))^{1/2} / \sin(dx+c), (a+b)/(a-b), I/((a-b)/(a+b))^{1/2} * \sin(dx+c) * a^2 * b^2 + 144 * A * \cos(dx+c)^4 * \text{EllipticF}((-1+\cos(dx+c)) * ((a-b)/(a+b))^{1/2} / \sin(dx+c), (-a+b)/(a-b))^{1/2}
\end{aligned}$$

/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*a^3*b+208*A*cos(d*x+c)^4*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*a^2*b^2-352*A*cos(d*x+c)^4*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*a*b^3-264*A*cos(d*x+c)^4*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a^3*b+264*A*cos(d*x+c)^4*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a^2*b^2-128*A*cos(d*x+c)^4*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a*b^3+240*A*cos(d*x+c)^4*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), (a+b)/(a-b), I/((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3*b-15*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)^4*a^4*(1/(1+cos(d*x+c)))^(1/2)-64*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)*b^4*(1/(1+cos(d*x+c)))^(1/2)/b/((a-b)/(a+b))^(1/2)/(b+a*cos(d*x+c))/(1/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^3/cos(d*x+c)^(7/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)(b \sec(dx + c) + a)^{\frac{5}{2}}}{\cos(dx + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2),x, algorith="maxima")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)/cos(d*x + c)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + \frac{B}{\cos(c+dx)}\right) \left(a + \frac{b}{\cos(c+dx)}\right)^{5/2}}{\cos(c+dx)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(5/2))/cos(c + d*x)^(3/2),x)

[Out] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(5/2))/cos(c + d*x)^(3/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c))/cos(d*x+c)**(3/2),x)

[Out] Timed out

$$3.615 \quad \int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B \sec(c+dx))}{\sqrt{a+b \sec(c+dx)}} dx$$

Optimal. Leaf size=280

$$\frac{2(4Ab - 5aB) \sin(c + dx) \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}}{15a^2d} + \frac{2(9a^2A - 10abB + 8Ab^2) \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}}{15a^3d \sqrt{\frac{a \cos(c+dx)+b}{a+b}}}$$

[Out] $-2/15*(7*A*a^2*b+8*A*b^3-5*B*a^3-10*B*a*b^2)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)}*(a/(a+b))^{(1/2)})*((b+a*\cos(d*x+c))/(a+b))^{(1/2)}/a^3/d/\cos(d*x+c)^{(1/2)}/(a+b*\sec(d*x+c))^{(1/2)}+2/5*A*\cos(d*x+c)^{(3/2)}*\sin(d*x+c)*(a+b*\sec(d*x+c))^{(1/2)}/a/d-2/15*(4*A*b-5*B*a)*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}*(a+b*\sec(d*x+c))^{(1/2)}/a^2/d+2/15*(9*A*a^2+8*A*b^2-10*B*a*b)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)}*(a/(a+b))^{(1/2)})*\cos(d*x+c)^{(1/2)}*(a+b*\sec(d*x+c))^{(1/2)}/a^3/d/((b+a*\cos(d*x+c))/(a+b))^{(1/2)}$

Rubi [A] time = 0.92, antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2955, 4034, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2(7a^2Ab - 5a^3B - 10ab^2B + 8Ab^3) \sqrt{\frac{a \cos(c+dx)+b}{a+b}} F\left(\frac{1}{2}(c + dx) \middle| \frac{2a}{a+b}\right)}{15a^3d \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}} + \frac{2(9a^2A - 10abB + 8Ab^2) \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}}{15a^3d \sqrt{\frac{a \cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(\text{Cos}[c + d*x])^{(5/2)}*(A + B*\text{Sec}[c + d*x])]/\text{Sqrt}[a + b*\text{Sec}[c + d*x]], x]$

[Out] $(-2*(7*a^2*A*b + 8*A*b^3 - 5*a^3*B - 10*a*b^2*B)*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])]/(a + b))*\text{EllipticF}[(c + d*x)/2, (2*a)/(a + b)]/(15*a^3*d*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]) + (2*(9*a^2*A + 8*A*b^2 - 10*a*b*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, (2*a)/(a + b)]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]])/(15*a^3*d*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/(a + b)]) - (2*(4*A*b - 5*a*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(15*a^2*d) + (2*A*\text{Cos}[c + d*x]^{(3/2)}*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(5*a*d)$

Rule 2653

$\text{Int}[\text{Sqrt}[(a_) + (b_)*\sin[(c_) + (d_)*(x_)]], x_Symbol] \rightarrow \text{Simp}[(2*\text{Sqrt}[a + b]*\text{EllipticE}[(1*(c - \text{Pi}/2 + d*x))/2, (2*b)/(a + b)])/d, x] /;$ $\text{FreeQ}\{a, b, c, d\}, x$ && $\text{NeQ}[a^2 - b^2, 0]$ && $\text{GtQ}[a + b, 0]$

Rule 2655

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
0] && !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)]/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a
+ b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && !GtQ[a + b, 0]
```

Rule 2955

```
Int[((a_) + csc[(e_) + (f_)*(x_)])*(b_)^(m_)*(csc[(e_) + (f_)*(x_)]*
(d_) + (c_))^(n_)*((g_)*sin[(e_) + (f_)*(x_)])^(p_), x_Symbol] := Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3856

```
Int[Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)]/Sqrt[csc[(e_) + (f_)*(x_)]
*(d_)], x_Symbol] := Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_) + (f_)*(x_)]*(d_)]/Sqrt[csc[(e_) + (f_)*(x_)]*(b_)
+ (a_)], x_Symbol] := Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4034

```

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + A*a*(n + 1)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]

```

Rule 4035

```

Int[(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.))/(Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]), x_Symbol] :> Dist[A/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]

```

Rule 4104

```

Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]

```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}} dx &= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \frac{A+B\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}} dx \\
&= \frac{2A\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5ad} - \frac{(2\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)})^3}{15a^2d} \\
&= -\frac{2(4Ab-5aB)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{15a^2d} + \frac{2A\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5ad} \\
&= -\frac{2(4Ab-5aB)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{15a^2d} + \frac{2A\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5ad} \\
&= -\frac{2(4Ab-5aB)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{15a^2d} + \frac{2A\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5ad} \\
&= -\frac{2(4Ab-5aB)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{15a^2d} + \frac{2A\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5ad} \\
&= -\frac{2(4Ab-5aB)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{15a^2d} + \frac{2A\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5ad} \\
&= -\frac{2(7a^2Ab+8Ab^3-5a^3B-10ab^2B)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{15a^3d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}
\end{aligned}$$

Mathematica [C] time = 16.47, size = 363, normalized size = 1.30

$$2a\sin(c+dx)(a\cos(c+dx)+b)(3aA\cos(c+dx)+5aB-4Ab) + \frac{2\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^{3/2}\left((9a^2A-10abB+8Ab^2)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{15a^3d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(Cos[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/Sqrt[a + b*Sec[c + d*x]], x]

[Out] (2*a*(b + a*Cos[c + d*x])*(-4*A*b + 5*a*B + 3*a*A*Cos[c + d*x])*Sin[c + d*x] + (2*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*(I*(a + b)*(9*a^2*A + 8*A*b^2 - 10*a*b*B)*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - I*

$a*(8*A*b^2 + 2*a*b*(A - 5*B) + a^2*(9*A + 5*B))*\text{EllipticF}[I*\text{ArcSinh}[\text{Tan}[(c + d*x)/2]], (-a + b)/(a + b)]*\text{Sec}[(c + d*x)/2]^2*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])*\text{Sec}[(c + d*x)/2]^2/(a + b)] + (9*a^2*A + 8*A*b^2 - 10*a*b*B)*(b + a*\text{Cos}[c + d*x])*(\text{Sec}[(c + d*x)/2]^2)^{(3/2)}*\text{Tan}[(c + d*x)/2])/(\text{Sec}[c + d*x]^{(3/2)})/(15*a^3*d*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]])$

fricas [F] time = 1.02, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{(B \cos(dx + c)^2 \sec(dx + c) + A \cos(dx + c)^2) \sqrt{\cos(dx + c)}}{\sqrt{b \sec(dx + c) + a}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")

[Out] integral((B*cos(d*x + c)^2*sec(d*x + c) + A*cos(d*x + c)^2)*sqrt(cos(d*x + c))/sqrt(b*sec(d*x + c) + a), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \cos(dx + c)^{\frac{5}{2}}}{\sqrt{b \sec(dx + c) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(5/2)/sqrt(b*sec(d*x + c) + a), x)

maple [B] time = 3.36, size = 1700, normalized size = 6.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x)

[Out] $-2/15/d*((b+a*\text{cos}(d*x+c))/\text{cos}(d*x+c))^{(1/2)}*\text{cos}(d*x+c)^{(1/2)}*(-1+\text{cos}(d*x+c))* (1+\text{cos}(d*x+c))* (5*B*((a-b)/(a+b))^{(1/2)}*\text{cos}(d*x+c)^2*a^2*b*(1/(1+\text{cos}(d*x+c)))^{(1/2)}-10*A*((a-b)/(a+b))^{(1/2)}*\text{cos}(d*x+c)*a^2*b*(1/(1+\text{cos}(d*x+c)))^{(1/2)}+8*A*((a-b)/(a+b))^{(1/2)}*\text{cos}(d*x+c)*a*b^2*(1/(1+\text{cos}(d*x+c)))^{(1/2)}-10*B*((a-b)/(a+b))^{(1/2)}*\text{cos}(d*x+c)*a^2*b*(1/(1+\text{cos}(d*x+c)))^{(1/2)}+10*B*((a-b)/(a+b))^{(1/2)}*\text{cos}(d*x+c)*a*b^2*(1/(1+\text{cos}(d*x+c)))^{(1/2)}-10*B*\text{sin}(d*x+c)*\text{Elliptic}$

```

icF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((
b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b+10*B*sin(d*x+c)*((b+a*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b)
)^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b-10*B*sin(d*x+c)*((b+a*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(
1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2-2*A*sin(d*x+c)*EllipticF((-1+co
s(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b+8*A*sin(d*x+c)*EllipticF((-1+cos(d*
x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*a*b^2+9*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x
+c),(-(a+b)/(a-b))^(1/2))*a^2*b-8*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(-(a+b)/(a-b))^(1/2))*a*b^2+A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*a^2*b*(1/(1+
cos(d*x+c)))^(1/2)-4*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a*b^2*(1/(1+cos(d*x
+c)))^(1/2)+8*A*((a-b)/(a+b))^(1/2)*b^3*(1/(1+cos(d*x+c)))^(1/2)-5*B*(1/(1+
cos(d*x+c)))^(1/2))*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*a^3-5*B*sin(d*x+c)*Elli
pticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*
((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^3+5*B*((a-b)/(a+b))^(1/2)*c
os(d*x+c)*a^3*(1/(1+cos(d*x+c)))^(1/2)-3*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^4
*a^3*(1/(1+cos(d*x+c)))^(1/2)-6*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^3*(1/(
1+cos(d*x+c)))^(1/2)+9*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^3*(1/(1+cos(d*x+c
)))^(1/2)-8*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)*b^3*(1/(1+cos(d*x+c)))^(1/2)+9
*A*((a-b)/(a+b))^(1/2)*a^2*b*(1/(1+cos(d*x+c)))^(1/2)-4*A*((a-b)/(a+b))^(1/
2)*a*b^2*(1/(1+cos(d*x+c)))^(1/2)+5*B*((a-b)/(a+b))^(1/2)*a^2*b*(1/(1+cos(d
*x+c)))^(1/2)-10*B*((a-b)/(a+b))^(1/2)*a*b^2*(1/(1+cos(d*x+c)))^(1/2)+9*A*s
in(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/
(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^3-9*A*sin(d*x
+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3+8*A*sin(d*x+c)*((
b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)
)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3/a^3/((a-b)/(a+b))^(1/2
)/(b+a*cos(d*x+c))/(1/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^3

```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \cos(dx + c)^{\frac{5}{2}}}{\sqrt{b \sec(dx + c) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(5/2)/sqrt(b*sec(d*x + c) + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c + dx)^{5/2} \left(A + \frac{B}{\cos(c+dx)} \right)}{\sqrt{a + \frac{b}{\cos(c+dx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)^(5/2)*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x))^(1/2),x)

[Out] int((cos(c + d*x)^(5/2)*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x))^(1/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(1/2),x)

[Out] Timed out

$$3.616 \quad \int \frac{\cos^3(c+dx)(A+B \sec(c+dx))}{\sqrt{a+b \sec(c+dx)}} dx$$

Optimal. Leaf size=212

$$\frac{2(a^2A - 3abB + 2Ab^2) \sqrt{\frac{a \cos(c+dx)+b}{a+b}} F\left(\frac{1}{2}(c+dx) \middle| \frac{2a}{a+b}\right) - 2(2Ab - 3aB) \sqrt{\cos(c+dx)} \sqrt{a+b \sec(c+dx)} E\left(\frac{1}{2}(c+dx) \middle| \frac{2a}{a+b}\right)}{3a^2d \sqrt{\cos(c+dx)} \sqrt{a+b \sec(c+dx)}} - \frac{2(2Ab - 3aB) \sqrt{\cos(c+dx)} \sqrt{a+b \sec(c+dx)} E\left(\frac{1}{2}(c+dx) \middle| \frac{2a}{a+b}\right)}{3a^2d \sqrt{\frac{a \cos(c+dx)+b}{a+b}}}$$

[Out] $2/3*(A*a^2+2*A*b^2-3*B*a*b)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)}*(a/(a+b))^{(1/2)}*((b+a*\cos(d*x+c))/(a+b))^{(1/2)}/a^2/d/\cos(d*x+c)^{(1/2)}/(a+b*\sec(d*x+c))^{(1/2)}+2/3*A*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}*(a+b*\sec(d*x+c))^{(1/2)}/a/d-2/3*(2*A*b-3*B*a)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)}*(a/(a+b))^{(1/2)}*\cos(d*x+c)^{(1/2)}*(a+b*\sec(d*x+c))^{(1/2)}/a^2/d/((b+a*\cos(d*x+c))/(a+b))^{(1/2)})$

Rubi [A] time = 0.63, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {2955, 4034, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2(a^2A - 3abB + 2Ab^2) \sqrt{\frac{a \cos(c+dx)+b}{a+b}} F\left(\frac{1}{2}(c+dx) \middle| \frac{2a}{a+b}\right) - 2(2Ab - 3aB) \sqrt{\cos(c+dx)} \sqrt{a+b \sec(c+dx)} E\left(\frac{1}{2}(c+dx) \middle| \frac{2a}{a+b}\right)}{3a^2d \sqrt{\cos(c+dx)} \sqrt{a+b \sec(c+dx)}} - \frac{2(2Ab - 3aB) \sqrt{\cos(c+dx)} \sqrt{a+b \sec(c+dx)} E\left(\frac{1}{2}(c+dx) \middle| \frac{2a}{a+b}\right)}{3a^2d \sqrt{\frac{a \cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/Sqrt[a + b*Sec[c + d*x]], x]

[Out] $(2*(a^2*A + 2*A*b^2 - 3*a*b*B)*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/(a + b)]*\text{EllipticF}[(c + d*x)/2, (2*a)/(a + b)])/(3*a^2*d*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]) - (2*(2*A*b - 3*a*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, (2*a)/(a + b)]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]])/(3*a^2*d*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/(a + b)]) + (2*A*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(3*a*d)$

Rule 2653

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*Sqrt[a + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)]/d, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2655

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
0] && !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)]/(d*Sqrt[a + b]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a
+ b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && !GtQ[a + b, 0]
```

Rule 2955

```
Int[((a_) + csc[(e_) + (f_)*(x_)]*(b_))^(m_)*(csc[(e_) + (f_)*(x_)]*
(d_) + (c_))^(n_)*((g_)*sin[(e_) + (f_)*(x_)])^(p_), x_Symbol] := Dist
[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && IntegerQ[n])
```

Rule 3856

```
Int[Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)]/Sqrt[csc[(e_) + (f_)*(x_)]
*(d_)], x_Symbol] := Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_) + (f_)*(x_)]*(d_)]/Sqrt[csc[(e_) + (f_)*(x_)]*(b_)
+ (a_)], x_Symbol] := Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4034

```
Int[(csc[(e_) + (f_)*(x_)]*(d_))^(n_)*(csc[(e_) + (f_)*(x_)]*(b_) + (
a_))^(m_)*(csc[(e_) + (f_)*(x_)]*(B_) + (A_)), x_Symbol] := Simp[(A*Cot[
```

$e + f*x](a + b*\text{Csc}[e + f*x])^{m+1}*(d*\text{Csc}[e + f*x])^n/(a*f*n), x] + \text{Dist}[1/(a*d*n), \text{Int}[(a + b*\text{Csc}[e + f*x])^m*(d*\text{Csc}[e + f*x])^{n+1}*\text{Simp}[a*B*n - A*b*(m + n + 1) + A*a*(n + 1)*\text{Csc}[e + f*x] + A*b*(m + n + 2)*\text{Csc}[e + f*x]^2, x], x], x] /;$ $\text{FreeQ}\{a, b, d, e, f, A, B, m\}, x\} \&\& \text{NeQ}[A*b - a*B, 0] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{LeQ}[n, -1]$

Rule 4035

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.))/(\text{Sqrt}[\text{csc}[(e_.) + (f_.)*(x_.)]*(d_.)]*\text{Sqrt}[\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]), x_Symbol] :> \text{Dist}[A/a, \text{Int}[\text{Sqrt}[a + b*\text{Csc}[e + f*x]]/\text{Sqrt}[d*\text{Csc}[e + f*x]], x], x] - \text{Dist}[(A*b - a*B)/(a*d), \text{Int}[\text{Sqrt}[d*\text{Csc}[e + f*x]]/\text{Sqrt}[a + b*\text{Csc}[e + f*x]], x], x] /;$ $\text{FreeQ}\{a, b, d, e, f, A, B\}, x\} \&\& \text{NeQ}[A*b - a*B, 0] \&\& \text{NeQ}[a^2 - b^2, 0]$

Rubi steps

$$\begin{aligned} \int \frac{\cos^2(c+dx)(A+B\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}} dx &= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \frac{A+B\sec(c+dx)}{\sec^2(c+dx)\sqrt{a+b\sec(c+dx)}} dx \\ &= \frac{2A\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3ad} - \frac{(2\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)})}{3ad} \\ &= \frac{2A\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3ad} - \frac{((2Ab-3aB)\sqrt{\cos(c+dx)})}{3ad} \\ &= \frac{2A\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3ad} + \frac{2\left(\frac{a^2A}{2} + \frac{1}{2}b(2Ab-3aB)\right)}{3a^2} \\ &= \frac{2A\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3ad} + \frac{2\left(\frac{a^2A}{2} + \frac{1}{2}b(2Ab-3aB)\right)}{3a^2} \\ &= \frac{2(a^2A+2Ab^2-3abB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{3a^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} - \frac{2(2Ab-3aB)}{3a^2} \end{aligned}$$

Mathematica [C] time = 9.23, size = 311, normalized size = 1.47

$$2 \left(aA \sin(c + dx)(a \cos(c + dx) + b) - \frac{\left(\cos^2\left(\frac{1}{2}(c+dx)\right) \sec(c+dx) \right)^{3/2} \left((2Ab-3aB) \tan\left(\frac{1}{2}(c+dx)\right) \sec^2\left(\frac{1}{2}(c+dx)\right) \right)^{3/2} (a \cos(c+dx)+b)+ia(a(A$$

$3a^2d$

Warning: Unable to verify antiderivative.

[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/Sqrt[a + b*Sec[c + d*x]],x]

[Out] (2*(a*A*(b + a*Cos[c + d*x])*Sin[c + d*x] - ((Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*((-I)*(a + b)*(-2*A*b + 3*a*B)*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + I*a*(-2*A*b + a*(A + 3*B))*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + (2*A*b - 3*a*B)*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/Sec[c + d*x]^(3/2))/(3*a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]])

fricas [F] time = 0.89, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{(B \cos(dx + c) \sec(dx + c) + A \cos(dx + c)) \sqrt{\cos(dx + c)}}{\sqrt{b \sec(dx + c) + a}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")

[Out] integral((B*cos(d*x + c)*sec(d*x + c) + A*cos(d*x + c))*sqrt(cos(d*x + c))/sqrt(b*sec(d*x + c) + a), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \cos(dx + c)^{\frac{3}{2}}}{\sqrt{b \sec(dx + c) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(3/2)/sqrt(b*sec(d*x + c) + a), x)

maple [B] time = 2.41, size = 1080, normalized size = 5.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2), x)

[Out]
$$\frac{2}{3} \frac{d \left((b+a \cos(dx+c)) / \cos(dx+c) \right)^{1/2} \cos(dx+c)^{1/2} (-1+\cos(dx+c)) \left((1+\cos(dx+c)) \left(A \cos(dx+c)^3 \left(\frac{a-b}{a+b} \right)^{1/2} a^2 \left(\frac{1}{1+\cos(dx+c)} \right)^{1/2} - A \cos(dx+c)^2 \left(\frac{a-b}{a+b} \right)^{1/2} a^* b \left(\frac{1}{1+\cos(dx+c)} \right)^{1/2} + 3 B \cos(dx+c)^2 \left(\frac{a-b}{a+b} \right)^{1/2} a^2 \left(\frac{1}{1+\cos(dx+c)} \right)^{1/2} + A \operatorname{EllipticF} \left((-1+\cos(dx+c)) \left(\frac{a-b}{a+b} \right)^{1/2} / \sin(dx+c), \left(-\frac{a+b}{a-b} \right)^{1/2} \right) \left((b+a \cos(dx+c)) / (1+\cos(dx+c)) / (a+b) \right)^{1/2} \sin(dx+c) a^2 + 2 A \operatorname{EllipticF} \left((-1+\cos(dx+c)) \left(\frac{a-b}{a+b} \right)^{1/2} / \sin(dx+c), \left(-\frac{a+b}{a-b} \right)^{1/2} \right) \left((b+a \cos(dx+c)) / (1+\cos(dx+c)) / (a+b) \right)^{1/2} \sin(dx+c) a^* b - 2 A \left((b+a \cos(dx+c)) / (1+\cos(dx+c)) / (a+b) \right)^{1/2} \operatorname{EllipticE} \left((-1+\cos(dx+c)) \left(\frac{a-b}{a+b} \right)^{1/2} / \sin(dx+c), \left(-\frac{a+b}{a-b} \right)^{1/2} \right) \sin(dx+c) a^* b + 2 A \left((b+a \cos(dx+c)) / (1+\cos(dx+c)) / (a+b) \right)^{1/2} \operatorname{EllipticE} \left((-1+\cos(dx+c)) \left(\frac{a-b}{a+b} \right)^{1/2} / \sin(dx+c), \left(-\frac{a+b}{a-b} \right)^{1/2} \right) \sin(dx+c) a^* b - 2 A \cos(dx+c) \left(\frac{a-b}{a+b} \right)^{1/2} a^2 \left(\frac{1}{1+\cos(dx+c)} \right)^{1/2} + 2 A \cos(dx+c) \left(\frac{a-b}{a+b} \right)^{1/2} a^* b \left(\frac{1}{1+\cos(dx+c)} \right)^{1/2} - 2 A \cos(dx+c) \left(\frac{a-b}{a+b} \right)^{1/2} b^2 \left(\frac{1}{1+\cos(dx+c)} \right)^{1/2} - 3 B \left((b+a \cos(dx+c)) / (1+\cos(dx+c)) / (a+b) \right)^{1/2} \operatorname{EllipticF} \left((-1+\cos(dx+c)) \left(\frac{a-b}{a+b} \right)^{1/2} / \sin(dx+c), \left(-\frac{a+b}{a-b} \right)^{1/2} \right) \sin(dx+c) a^2 + 3 B \left((b+a \cos(dx+c)) / (1+\cos(dx+c)) / (a+b) \right)^{1/2} \operatorname{EllipticE} \left((-1+\cos(dx+c)) \left(\frac{a-b}{a+b} \right)^{1/2} / \sin(dx+c), \left(-\frac{a+b}{a-b} \right)^{1/2} \right) \sin(dx+c) a^2 - 3 B \left((b+a \cos(dx+c)) / (1+\cos(dx+c)) / (a+b) \right)^{1/2} \operatorname{EllipticE} \left((-1+\cos(dx+c)) \left(\frac{a-b}{a+b} \right)^{1/2} / \sin(dx+c), \left(-\frac{a+b}{a-b} \right)^{1/2} \right) \sin(dx+c) a^* b - 3 B \cos(dx+c) \left(\frac{a-b}{a+b} \right)^{1/2} a^2 \left(\frac{1}{1+\cos(dx+c)} \right)^{1/2} + 3 B \cos(dx+c) \left(\frac{a-b}{a+b} \right)^{1/2} a^* b \left(\frac{1}{1+\cos(dx+c)} \right)^{1/2} - A \left(\frac{a-b}{a+b} \right)^{1/2} a^* b \left(\frac{1}{1+\cos(dx+c)} \right)^{1/2} + 2 A \left(\frac{a-b}{a+b} \right)^{1/2} b^2 \left(\frac{1}{1+\cos(dx+c)} \right)^{1/2} - 3 B \left(\frac{a-b}{a+b} \right)^{1/2} a^* b \left(\frac{1}{1+\cos(dx+c)} \right)^{1/2} / a^2 \left(\frac{a-b}{a+b} \right)^{1/2} / (b+a \cos(dx+c)) / \left(\frac{1}{1+\cos(dx+c)} \right)^{1/2} / \sin(dx+c) \right)^3$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \cos(dx + c)^{\frac{3}{2}}}{\sqrt{b \sec(dx + c) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2), x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(3/2)/sqrt(b*sec(d*x + c) + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c + dx)^{3/2} \left(A + \frac{B}{\cos(c+dx)} \right)}{\sqrt{a + \frac{b}{\cos(c+dx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)^(3/2)*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x))^(1/2), x)

[Out] int((cos(c + d*x)^(3/2)*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x))^(1/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(1/2), x)

[Out] Timed out

$$3.617 \quad \int \frac{\sqrt{\cos(c+dx)} (A+B \sec(c+dx))}{\sqrt{a+b \sec(c+dx)}} dx$$

Optimal. Leaf size=150

$$\frac{2A\sqrt{\cos(c+dx)}\sqrt{a+b \sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{ad\sqrt{\frac{a \cos(c+dx)+b}{a+b}}}-\frac{2(Ab-aB)\sqrt{\frac{a \cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{ad\sqrt{\cos(c+dx)}\sqrt{a+b \sec(c+dx)}}$$

[Out] $-2*(A*b-B*a)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)}*(a/(a+b))^{(1/2)})*((b+a*\cos(d*x+c))/(a+b))^{(1/2)}/a/d/\cos(d*x+c)^{(1/2)}/(a+b*\sec(d*x+c))^{(1/2)}+2*A*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)}*(a/(a+b))^{(1/2)})*\cos(d*x+c)^{(1/2)}*(a+b*\sec(d*x+c))^{(1/2)}/a/d/((b+a*\cos(d*x+c))/(a+b))^{(1/2)}$

Rubi [A] time = 0.43, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {2955, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2A\sqrt{\cos(c+dx)}\sqrt{a+b \sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{ad\sqrt{\frac{a \cos(c+dx)+b}{a+b}}}-\frac{2(Ab-aB)\sqrt{\frac{a \cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{ad\sqrt{\cos(c+dx)}\sqrt{a+b \sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x]))/Sqrt[a + b*Sec[c + d*x]], x]

[Out] $(-2*(A*b - a*B)*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/(a + b)]*\text{EllipticF}[(c + d*x)/2, (2*a)/(a + b)])/(a*d*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]) + (2*A*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, (2*a)/(a + b)]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]])/(a*d*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/(a + b)])$

Rule 2653

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*Sqrt[a + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2655

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a + b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2661

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)]/(d*Sqrt[a + b]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b) + (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]
```

Rule 2955

```
Int[((a_) + csc[(e_) + (f_)*(x_)]*(b_))^(m_)*(csc[(e_) + (f_)*(x_)]*(d_) + (c_))^(n_)*((g_)*sin[(e_) + (f_)*(x_)])^(p_), x_Symbol] := Dist[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && IntegerQ[n])
```

Rule 3856

```
Int[Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)]/Sqrt[csc[(e_) + (f_)*(x_)]*(d_)], x_Symbol] := Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_) + (f_)*(x_)]*(d_)]/Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)], x_Symbol] := Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4035

```
Int[(csc[(e_) + (f_)*(x_)]*(B_) + (A_))/(Sqrt[csc[(e_) + (f_)*(x_)]*(d_) + (a_)])*(Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)]), x_Symbol] := Dist[A/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```


Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\cos(c+dx)}(A+B\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}} dx &= (\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}) \int \frac{A+B\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}} dx \\
&= \frac{(A\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}) \int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}} dx - ((Ab-aB)\sqrt{\cos(c+dx)}) \int \frac{1}{\sqrt{b+a\cos(c+dx)}} dx}{a} \\
&= -\frac{((Ab-aB)\sqrt{b+a\cos(c+dx)}) \int \frac{1}{\sqrt{b+a\cos(c+dx)}} dx}{a\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{(A\sqrt{\cos(c+dx)}) \int \frac{1}{\sqrt{a+b\sec(c+dx)}} dx}{a\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} \\
&= -\frac{((Ab-aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}) \int \frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}} dx}{a\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{(A\sqrt{\cos(c+dx)}) \int \frac{1}{\sqrt{a+b\sec(c+dx)}} dx}{a\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} \\
&= -\frac{2(Ab-aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}} F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{ad\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2A\sqrt{\cos(c+dx)} E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{ad\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}
\end{aligned}$$

Mathematica [C] time = 7.05, size = 260, normalized size = 1.73

$$2\sqrt{\cos(c+dx)}\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)(A+B\sec(c+dx))} \left(-ia(A+B)\sqrt{\frac{\sec^2\left(\frac{1}{2}(c+dx)\right)(a\cos(c+dx)+b)}{a+b}} F\left(i\sin\left(\frac{1}{2}(c+dx)\right)\middle|\frac{2a}{a+b}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x]))/Sqrt[a + b*Sec[c + d*x]], x]

[Out] (2*Sqrt[Cos[c + d*x]]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(A + B*Sec[c + d*x]))*(I*A*(a + b)*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - I*a*(A + B)*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + A*(b + a*Cos[c + d*x])*Sqrt[Sec[(c + d*x)/2]^2]*Tan[(c + d*x)/2])/(a*d*(B + A*Cos[c + d*x])*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]])

fricas [F] time = 0.96, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B\sec(dx+c)+A)\sqrt{\cos(dx+c)}}{\sqrt{b\sec(dx+c)+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")

[Out] integral((B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/sqrt(b*sec(d*x + c) + a), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sqrt{\cos(dx + c)}}{\sqrt{b \sec(dx + c) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/sqrt(b*sec(d*x + c) + a), x)

maple [B] time = 3.28, size = 564, normalized size = 3.76

$$2(-1 + \cos(dx + c))(1 + \cos(dx + c)) \left(A \sqrt{\frac{a-b}{a+b}} (\cos^2(dx + c)) \sqrt{\frac{1}{1+\cos(dx+c)}} a + A \sin(dx + c) \sqrt{\frac{b+a \cos(dx+c)}{(1+\cos(dx+c))(a+b)}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x)

[Out] 2/d*(-1+cos(d*x+c))*(1+cos(d*x+c))*(A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*(1/(1+cos(d*x+c)))^(1/2)*a+A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a-A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b-A*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*a-A*((a-b)/(a+b))^(1/2)*cos(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*a+A*((a-b)/(a+b))^(1/2)*cos(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*b+B*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*a-A*((a-b)/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b*cos(d*x+c)^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/a/((a-b)/(a+b))^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)^3/(1/(1+cos(d*x+c)))^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sqrt{\cos(dx + c)}}{\sqrt{b \sec(dx + c) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/sqrt(b*sec(d*x + c) + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\cos(c + dx)} \left(A + \frac{B}{\cos(c + dx)} \right)}{\sqrt{a + \frac{b}{\cos(c + dx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)^(1/2)*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x))^(1/2),x)

[Out] int((cos(c + d*x)^(1/2)*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \sqrt{\cos(c + dx)}}{\sqrt{a + b \sec(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)**(1/2)/(a+b*sec(d*x+c))**(1/2),x)

[Out] Integral((A + B*sec(c + d*x))*sqrt(cos(c + d*x))/sqrt(a + b*sec(c + d*x)), x)

$$3.618 \quad \int \frac{A+B \sec(c+dx)}{\sqrt{\cos(c+dx)} \sqrt{a+b \sec(c+dx)}} dx$$

Optimal. Leaf size=138

$$\frac{2A\sqrt{\frac{a\cos(c+dx)+b}{a+b}} F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)} \sqrt{a+b \sec(c+dx)}} + \frac{2B\sqrt{\frac{a\cos(c+dx)+b}{a+b}} \Pi\left(2; \frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)} \sqrt{a+b \sec(c+dx)}}$$

[Out] $2*A*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)}*(a/(a+b))^{(1/2)}*((b+a*\cos(d*x+c))/(a+b))^{(1/2)}/d/\cos(d*x+c)^{(1/2)}/(a+b*\sec(d*x+c))^{(1/2)}+2*B*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticPi}(\sin(1/2*d*x+1/2*c), 2, 2^{(1/2)}*(a/(a+b))^{(1/2)}*((b+a*\cos(d*x+c))/(a+b))^{(1/2)}/d/\cos(d*x+c)^{(1/2)}/(a+b*\sec(d*x+c))^{(1/2)})$

Rubi [A] time = 0.53, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {2955, 4036, 3858, 2663, 2661, 3859, 2807, 2805}

$$\frac{2A\sqrt{\frac{a\cos(c+dx)+b}{a+b}} F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)} \sqrt{a+b \sec(c+dx)}} + \frac{2B\sqrt{\frac{a\cos(c+dx)+b}{a+b}} \Pi\left(2; \frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)} \sqrt{a+b \sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] `Int[(A + B*Sec[c + d*x])/(Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]),x]`

[Out] $(2*A*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/(a + b)]*\text{EllipticF}[(c + d*x)/2, (2*a)/(a + b)])/(d*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]) + (2*B*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/(a + b)]*\text{EllipticPi}[2, (c + d*x)/2, (2*a)/(a + b)])/(d*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]])$

Rule 2661

`Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]`

Rule 2663

`Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b) + (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]`

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]]), x_Symbol] := Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)]/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]]), x_Symbol] := Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
+ f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !GtQ[c + d, 0]
```

Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)])*(b_.)^(m_.)*(csc[(e_.) + (f_.)*(x_)])*
(d_. + (c_.))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] := Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)])*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)])*(b_.)
+ (a_.)], x_Symbol] := Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)])*(d_.)^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)])*(b_.)
+ (a_.)], x_Symbol] := Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4036

```
Int[(Sqrt[csc[(e_.) + (f_.)*(x_)])*(d_.)]*(csc[(e_.) + (f_.)*(x_)])*(B_.) + (
A_.))/Sqrt[csc[(e_.) + (f_.)*(x_)])*(b_.) + (a_.)], x_Symbol] := Dist[A, Int[
Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B/d, Int[(d*Cs
c[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{A + B \sec(c + dx)}{\sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}} dx &= \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{\sqrt{\sec(c + dx)} (A + B \sec(c + dx))}{\sqrt{a + b \sec(c + dx)}} dx \\
&= \left(A \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{\sqrt{\sec(c + dx)}}{\sqrt{a + b \sec(c + dx)}} dx + \left(B \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{\sec(c + dx)}{\sqrt{a + b \sec(c + dx)}} dx \\
&= \frac{\left(A \sqrt{b + a \cos(c + dx)} \right) \int \frac{1}{\sqrt{b + a \cos(c + dx)}} dx}{\sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}} + \frac{\left(B \sqrt{b + a \cos(c + dx)} \right) \int \frac{\sec(c + dx)}{\sqrt{a + b \sec(c + dx)}} dx}{\sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}} \\
&= \frac{\left(A \sqrt{\frac{b + a \cos(c + dx)}{a + b}} \right) \int \frac{1}{\sqrt{\frac{b}{a + b} + \frac{a \cos(c + dx)}{a + b}}} dx}{\sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}} + \frac{\left(B \sqrt{\frac{b + a \cos(c + dx)}{a + b}} \right) \int \frac{\sec(c + dx)}{\sqrt{\frac{b}{a + b} + \frac{a \cos(c + dx)}{a + b}}} dx}{\sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}} \\
&= \frac{2A \sqrt{\frac{b + a \cos(c + dx)}{a + b}} F\left(\frac{1}{2}(c + dx) \middle| \frac{2a}{a + b}\right)}{d \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}} + \frac{2B \sqrt{\frac{b + a \cos(c + dx)}{a + b}} \Pi\left(2; \frac{1}{2}(c + dx) \middle| \frac{2a}{a + b}\right)}{d \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}}
\end{aligned}$$

Mathematica [C] time = 28.81, size = 16611, normalized size = 120.37

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(A + B*Sec[c + d*x])/(Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]), x]

[Out] Result too large to show

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2), x, algorith="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{\sqrt{b \sec(dx + c) + a} \sqrt{\cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)/(sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c))), x)

maple [C] time = 2.37, size = 257, normalized size = 1.86

$$\frac{2 \left(A \operatorname{EllipticF} \left(\frac{(-1+\cos(dx+c))\sqrt{\frac{a-b}{a+b}}}{\sin(dx+c)}, \sqrt{-\frac{a+b}{a-b}} \right) - B \operatorname{EllipticF} \left(\frac{(-1+\cos(dx+c))\sqrt{\frac{a-b}{a+b}}}{\sin(dx+c)}, \sqrt{-\frac{a+b}{a-b}} \right) + 2B \operatorname{EllipticPi} \left(\frac{(-1+\cos(dx+c))\sqrt{\frac{a-b}{a+b}}}{\sin(dx+c)}, \sqrt{-\frac{a+b}{a-b}} \right) \right)}{d\sqrt{\frac{a-b}{a+b}} (b + a \cos(dx + c)) \sqrt{\frac{1}{1+\cos(dx+c)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x)

[Out] -2/d*(A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))-B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))+2*B*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), (a+b)/(a-b), I/((a-b)/(a+b))^(1/2)))*cos(d*x+c)^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b)^(1/2)/((a-b)/(a+b))^(1/2)/(b+a*cos(d*x+c))/(1/(1+cos(d*x+c)))^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{\sqrt{b \sec(dx + c) + a} \sqrt{\cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)/(sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c))), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\sqrt{\cos(c+dx)} \sqrt{a + \frac{b}{\cos(c+dx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B/cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))^(1/2)),x)`

[Out] `int((A + B/cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))^(1/2)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \sec(c + dx)}{\sqrt{a + b \sec(c + dx)} \sqrt{\cos(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*sec(d*x+c))/cos(d*x+c)**(1/2)/(a+b*sec(d*x+c))**(1/2),x)`

[Out] `Integral((A + B*sec(c + d*x))/(sqrt(a + b*sec(c + d*x))*sqrt(cos(c + d*x))), x)`

$$3.619 \quad \int \frac{A+B \sec(c+dx)}{\cos^2(c+dx) \sqrt{a+b \sec(c+dx)}} dx$$

Optimal. Leaf size=256

$$\frac{(2Ab - aB) \sqrt{\frac{a \cos(c+dx)+b}{a+b}} \Pi\left(2; \frac{1}{2}(c+dx) \middle| \frac{2a}{a+b}\right)}{bd \sqrt{\cos(c+dx)} \sqrt{a+b \sec(c+dx)}} + \frac{B \sin(c+dx) \sqrt{a+b \sec(c+dx)}}{bd \sqrt{\cos(c+dx)}} + \frac{B \sqrt{\frac{a \cos(c+dx)+b}{a+b}} F\left(\frac{1}{2}(c+dx) \middle| \frac{2a}{a+b}\right)}{d \sqrt{\cos(c+dx)} \sqrt{a+b \sec(c+dx)}}$$

[Out] $B * (\cos(1/2*d*x+1/2*c)^2)^{(1/2)} / \cos(1/2*d*x+1/2*c) * \text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)} * (a/(a+b))^{(1/2)}) * ((b+a*\cos(d*x+c))/(a+b))^{(1/2)} / d / \cos(d*x+c)^{(1/2)} / (a+b*\sec(d*x+c))^{(1/2)} + (2*A*b-B*a) * (\cos(1/2*d*x+1/2*c)^2)^{(1/2)} / \cos(1/2*d*x+1/2*c) * \text{EllipticPi}(\sin(1/2*d*x+1/2*c), 2^{(1/2)} * (a/(a+b))^{(1/2)}) * ((b+a*\cos(d*x+c))/(a+b))^{(1/2)} / b / d / \cos(d*x+c)^{(1/2)} / (a+b*\sec(d*x+c))^{(1/2)} + B * \sin(d*x+c) * (a+b*\sec(d*x+c))^{(1/2)} / b / d / \cos(d*x+c)^{(1/2)} - B * (\cos(1/2*d*x+1/2*c)^2)^{(1/2)} / \cos(1/2*d*x+1/2*c) * \text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)} * (a/(a+b))^{(1/2)}) * \cos(d*x+c)^{(1/2)} * (a+b*\sec(d*x+c))^{(1/2)} / b / d / ((b+a*\cos(d*x+c))/(a+b))^{(1/2)}$

Rubi [A] time = 0.89, antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {2955, 4033, 4109, 3859, 2807, 2805, 3862, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{(2Ab - aB) \sqrt{\frac{a \cos(c+dx)+b}{a+b}} \Pi\left(2; \frac{1}{2}(c+dx) \middle| \frac{2a}{a+b}\right)}{bd \sqrt{\cos(c+dx)} \sqrt{a+b \sec(c+dx)}} + \frac{B \sin(c+dx) \sqrt{a+b \sec(c+dx)}}{bd \sqrt{\cos(c+dx)}} + \frac{B \sqrt{\frac{a \cos(c+dx)+b}{a+b}} F\left(\frac{1}{2}(c+dx) \middle| \frac{2a}{a+b}\right)}{d \sqrt{\cos(c+dx)} \sqrt{a+b \sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]), x]

[Out] $(B * \text{Sqrt}[(b + a * \text{Cos}[c + d * x]) / (a + b)] * \text{EllipticF}[(c + d * x) / 2, (2 * a) / (a + b)]) / (d * \text{Sqrt}[\text{Cos}[c + d * x]] * \text{Sqrt}[a + b * \text{Sec}[c + d * x]]) + ((2 * A * b - a * B) * \text{Sqrt}[(b + a * \text{Cos}[c + d * x]) / (a + b)] * \text{EllipticPi}[2, (c + d * x) / 2, (2 * a) / (a + b)]) / (b * d * \text{Sqrt}[\text{Cos}[c + d * x]] * \text{Sqrt}[a + b * \text{Sec}[c + d * x]]) - (B * \text{Sqrt}[\text{Cos}[c + d * x]] * \text{EllipticE}[(c + d * x) / 2, (2 * a) / (a + b)] * \text{Sqrt}[a + b * \text{Sec}[c + d * x]]) / (b * d * \text{Sqrt}[(b + a * \text{Cos}[c + d * x]) / (a + b)]) + (B * \text{Sqrt}[a + b * \text{Sec}[c + d * x]] * \text{Sin}[c + d * x]) / (b * d * \text{Sqrt}[\text{Cos}[c + d * x]])$

Rule 2653

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,

b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2655

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a + b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2661

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2663

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b) + (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2805

Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[c + d, 0]

Rule 2807

Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !GtQ[c + d, 0]

Rule 2955

Int[((a_) + csc[(e_) + (f_)*(x_)])*(b_)^(m_)*(csc[(e_) + (f_)*(x_)]*(d_) + (c_))^(n_)*((g_)*sin[(e_) + (f_)*(x_)])^(p_), x_Symbol] := Dist[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && In

tegerQ[n])

Rule 3856

Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 3858

Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 3859

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 3862

Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)]), x_Symbol] :> Dist[1/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[b/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 4033

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] :> -Simp[(B*d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2))/(b*f*(m + n)), x] + Dist[d^2/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 2)*Simp[a*B*(n - 2) + B*b*(m + n - 1)*Csc[e + f*x] + (A*b*(m + n) - a*B*(n - 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && NeQ[m + n, 0] && !IGtQ[m, 1]

Rule 4109

Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)]), x_Symbol] :> Dist[C/d^

2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[A, Int[1/(Sqrt[d*Csc[e + f*x]]*Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, d, e, f, A, C}, x] && NeQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{A + B \sec(c + dx)}{\cos^{\frac{3}{2}}(c + dx) \sqrt{a + b \sec(c + dx)}} dx &= \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{\sec^{\frac{3}{2}}(c + dx) (A + B \sec(c + dx))}{\sqrt{a + b \sec(c + dx)}} dx \\
 &= \frac{B \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{bd \sqrt{\cos(c + dx)}} + \frac{\left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{-\frac{aB}{2}}{\sqrt{\sec(c + dx)}} dx}{b} \\
 &= \frac{B \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{bd \sqrt{\cos(c + dx)}} - \frac{(aB \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)}) \int \frac{1}{\sqrt{\sec(c + dx)}} dx}{2b} \\
 &= \frac{B \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{bd \sqrt{\cos(c + dx)}} + \frac{1}{2} \left(B \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{1}{\sqrt{\sec(c + dx)}} dx \\
 &= \frac{B \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{bd \sqrt{\cos(c + dx)}} + \frac{(B \sqrt{b + a \cos(c + dx)}) \int \frac{1}{\sqrt{b + a \cos(c + dx)}} dx}{2 \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}} \\
 &= \frac{(2Ab - aB) \sqrt{\frac{b+a \cos(c+dx)}{a+b}} \Pi\left(2; \frac{1}{2}(c + dx) \middle| \frac{2a}{a+b}\right)}{bd \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}} + \frac{B \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{bd \sqrt{\cos(c + dx)}} \\
 &= \frac{B \sqrt{\frac{b+a \cos(c+dx)}{a+b}} F\left(\frac{1}{2}(c + dx) \middle| \frac{2a}{a+b}\right)}{d \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}} + \frac{(2Ab - aB) \sqrt{\frac{b+a \cos(c+dx)}{a+b}} \Pi\left(2; \frac{1}{2}(c + dx) \middle| \frac{2a}{a+b}\right)}{bd \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}}
 \end{aligned}$$

Mathematica [C] time = 32.95, size = 51168, normalized size = 199.88

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]), x]

[Out] Result too large to show

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x, algorith="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{\sqrt{b \sec(dx + c) + a} \cos(dx + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x, algorith="giac")

[Out] integrate((B*sec(d*x + c) + A)/(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^(3/2)), x)

maple [C] time = 3.31, size = 776, normalized size = 3.03

$$\sqrt{\frac{b+a \cos(dx+c)}{\cos(dx+c)}} \left(2A \cos(dx+c) \sin(dx+c) \sqrt{\frac{b+a \cos(dx+c)}{(1+\cos(dx+c))(a+b)}} \operatorname{EllipticF} \left(\frac{(-1+\cos(dx+c)) \sqrt{\frac{a-b}{a+b}}}{\sin(dx+c)}, \sqrt{\frac{a+b}{a-b}} \right) b - 4A \cos \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x)

[Out] 1/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(2*A*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b-4*A*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b-B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*(1/(1+cos(d*x+c)))^(1/2)-2*B*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a+2*B*cos(d*x+c)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*a+B*cos(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))

$$\left. \right)^{(1/2)} \sin(dx+c) \left(\frac{b+a \cos(dx+c)}{1+\cos(dx+c)} \right) \left(\frac{1}{a+b} \right)^{(1/2)} a - B \cos(dx+c) \operatorname{EllipticE} \left(\frac{-1+\cos(dx+c)}{2} \left(\frac{a-b}{a+b} \right)^{(1/2)} \frac{1}{\sin(dx+c)}, \frac{-(a+b)}{a-b} \right)^{(1/2)} \sin(dx+c) \left(\frac{b+a \cos(dx+c)}{1+\cos(dx+c)} \right) \left(\frac{1}{a+b} \right)^{(1/2)} b + B \cos(dx+c) \left(\frac{a-b}{a+b} \right)^{(1/2)} a \left(\frac{1}{1+\cos(dx+c)} \right)^{(1/2)} - B \cos(dx+c) \left(\frac{a-b}{a+b} \right)^{(1/2)} b \left(\frac{1}{1+\cos(dx+c)} \right)^{(1/2)} + B \left(\frac{a-b}{a+b} \right)^{(1/2)} b \left(\frac{1}{1+\cos(dx+c)} \right)^{(1/2)} \right) / \left(\frac{a-b}{a+b} \right)^{(1/2)} \left(\frac{b+a \cos(dx+c)}{\sin(dx+c)} \right) \left(\frac{1}{1+\cos(dx+c)} \right)^{(1/2)} \cos(dx+c)^{(1/2)}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx+c) + A}{\sqrt{b \sec(dx+c) + a} \cos(dx+c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)/(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^(3/2)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\cos(c+dx)^{3/2} \sqrt{a + \frac{b}{\cos(c+dx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^(1/2)),x)

[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \sec(c + dx)}{\sqrt{a + b \sec(c + dx)} \cos^{\frac{3}{2}}(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(3/2)/(a+b*sec(d*x+c))**(1/2),x)

[Out] Integral((A + B*sec(c + d*x))/(sqrt(a + b*sec(c + d*x))*cos(c + d*x)**(3/2)), x)

$$3.620 \quad \int \frac{A+B \sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx) \sqrt{a+b \sec(c+dx)}} dx$$

Optimal. Leaf size=344

$$\frac{(-3a^2B + 4aAb - 4b^2B) \sqrt{\frac{a \cos(c+dx)+b}{a+b}} \Pi\left(2; \frac{1}{2}(c+dx) \middle| \frac{2a}{a+b}\right)}{4b^2d \sqrt{\cos(c+dx)} \sqrt{a+b \sec(c+dx)}} + \frac{(4Ab - 3aB) \sin(c+dx) \sqrt{a+b \sec(c+dx)}}{4b^2d \sqrt{\cos(c+dx)}} \quad (4A$$

[Out] $1/4*(4*A*b-B*a)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)}*(a/(a+b))^{(1/2)})*((b+a*\cos(d*x+c))/(a+b))^{(1/2)}/b/d/\cos(d*x+c)^{(1/2)}/(a+b*\sec(d*x+c))^{(1/2)}-1/4*(4*A*a*b-3*B*a^2-4*B*b^2)*(c \cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticPi}(\sin(1/2*d*x+1/2*c), 2, 2^{(1/2)}*(a/(a+b))^{(1/2)})*((b+a*\cos(d*x+c))/(a+b))^{(1/2)}/b^2/d/\cos(d*x+c)^{(1/2)}/(a+b*\sec(d*x+c))^{(1/2)}+1/2*B*\sin(d*x+c)*(a+b*\sec(d*x+c))^{(1/2)}/b/d/\cos(d*x+c)^{(3/2)}+1/4*(4*A*b-3*B*a)*\sin(d*x+c)*(a+b*\sec(d*x+c))^{(1/2)}/b^2/d/\cos(d*x+c)^{(1/2)}-1/4*(4*A*b-3*B*a)*(c \cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)}*(a/(a+b))^{(1/2)})*\cos(d*x+c)^{(1/2)}*(a+b*\sec(d*x+c))^{(1/2)}/b^2/d/((b+a*\cos(d*x+c))/(a+b))^{(1/2)}$

Rubi [A] time = 1.28, antiderivative size = 344, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 14, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2955, 4033, 4102, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{(-3a^2B + 4aAb - 4b^2B) \sqrt{\frac{a \cos(c+dx)+b}{a+b}} \Pi\left(2; \frac{1}{2}(c+dx) \middle| \frac{2a}{a+b}\right)}{4b^2d \sqrt{\cos(c+dx)} \sqrt{a+b \sec(c+dx)}} + \frac{(4Ab - 3aB) \sin(c+dx) \sqrt{a+b \sec(c+dx)}}{4b^2d \sqrt{\cos(c+dx)}} \quad (4A$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(5/2)*Sqrt[a + b*Sec[c + d*x]]),x]

[Out] $((4*A*b - a*B)*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/(a + b)]*\text{EllipticF}[(c + d*x)/2, (2*a)/(a + b)]/(4*b*d*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]) - ((4*a*A*b - 3*a^2*B - 4*b^2*B)*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/(a + b)]*\text{EllipticPi}[2, (c + d*x)/2, (2*a)/(a + b)]/(4*b^2*d*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]) - ((4*A*b - 3*a*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, (2*a)/(a + b)]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]])/(4*b^2*d*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/(a + b)]) + (B*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(2*b*d*\text{Cos}[c + d*x]^(3/2)) + ((4*A*b - 3*a*B)*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(4*b^2*d*\text{Sqrt}[\text{Cos}[c + d*x]])$

Rule 2653

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*Sqrt[a
+ b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
0] && !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)]/(d*Sqrt[a + b]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a
+ b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_)
+ (f_)*(x_)]]), x_Symbol] := Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)]/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_)
+ (f_)*(x_)]]), x_Symbol] := Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
+ f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !GtQ[c + d, 0]
```

Rule 2955

```
Int[((a_) + csc[(e_) + (f_)*(x_)]*(b_))^(m_)*(csc[(e_) + (f_)*(x_)]*
(d_) + (c_))^(n_)*((g_)*sin[(e_) + (f_)*(x_)])^(p_), x_Symbol] := Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
```


*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && IntegerQ[n])

Rule 3856

Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 3858

Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 3859

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 4033

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] :> -Simp[(B*d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2))/(b*f*(m + n)), x] + Dist[d^2/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 2)*Simp[a*B*(n - 2) + B*b*(m + n - 1)*Csc[e + f*x] + (A*b*(m + n) - a*B*(n - 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && NeQ[m + n, 0] && !IGtQ[m, 1]

Rule 4035

Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)]), x_Symbol] :> Dist[A/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]

Rule 4102

```

Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]

```

Rule 4108

```

Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.)
+ (a_)])], x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
+ b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]

```

Rubi steps

$$\begin{aligned}
\int \frac{A + B \sec(c + dx)}{\cos^{\frac{5}{2}}(c + dx) \sqrt{a + b \sec(c + dx)}} dx &= \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{\sec^{\frac{5}{2}}(c + dx)(A + B \sec(c + dx))}{\sqrt{a + b \sec(c + dx)}} dx \\
&= \frac{B \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{2bd \cos^{\frac{3}{2}}(c + dx)} + \frac{\left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{\sqrt{\sec(c + dx)}}{\sqrt{a + b \sec(c + dx)}} dx}{2bd \cos^{\frac{3}{2}}(c + dx)} \\
&= \frac{B \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{2bd \cos^{\frac{3}{2}}(c + dx)} + \frac{(4Ab - 3aB) \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{4b^2 d \sqrt{\cos(c + dx)}} \\
&= \frac{B \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{2bd \cos^{\frac{3}{2}}(c + dx)} + \frac{(4Ab - 3aB) \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{4b^2 d \sqrt{\cos(c + dx)}} \\
&= \frac{B \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{2bd \cos^{\frac{3}{2}}(c + dx)} + \frac{(4Ab - 3aB) \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{4b^2 d \sqrt{\cos(c + dx)}} \\
&= \frac{B \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{2bd \cos^{\frac{3}{2}}(c + dx)} + \frac{(4Ab - 3aB) \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{4b^2 d \sqrt{\cos(c + dx)}} \\
&= \frac{B \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{2bd \cos^{\frac{3}{2}}(c + dx)} + \frac{(4Ab - 3aB) \sqrt{a + b \sec(c + dx)} \sin(c + dx)}{4b^2 d \sqrt{\cos(c + dx)}} \\
&= -\frac{(4aAb - 3a^2B - 4b^2B) \sqrt{\frac{b+a \cos(c+dx)}{a+b}} \Pi\left(2; \frac{1}{2}(c + dx) \middle| \frac{2a}{a+b}\right) + B \sqrt{a + b \sec(c + dx)}}{4b^2 d \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}} \\
&= \frac{(4Ab - aB) \sqrt{\frac{b+a \cos(c+dx)}{a+b}} F\left(\frac{1}{2}(c + dx) \middle| \frac{2a}{a+b}\right) - (4aAb - 3a^2B - 4b^2B) \sqrt{\cos(c + dx)}}{4bd \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}} - \frac{(4aAb - 3a^2B - 4b^2B) \sqrt{\cos(c + dx)}}{4b^2 d \sqrt{\cos(c + dx)}}
\end{aligned}$$

Mathematica [C] time = 32.92, size = 77909, normalized size = 226.48

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(5/2)*Sqrt[a + b*Sec[c + d*x]]), x]

[Out] Result too large to show

$x+c)/(a+b))^{1/2} \sin(dx+c) \cos(dx+c)^2 a^2 b - 6B((b+a \cos(dx+c))/(1+\cos(dx+c)))/(a+b))^{1/2} \text{EllipticPi}((-1+\cos(dx+c))*((a-b)/(a+b))^{1/2}/\sin(dx+c), (a+b)/(a-b), I/((a-b)/(a+b))^{1/2}) \sin(dx+c) \cos(dx+c)^2 a^2 - 8B((b+a \cos(dx+c))/(1+\cos(dx+c)))/(a+b))^{1/2} \text{EllipticPi}((-1+\cos(dx+c))*((a-b)/(a+b))^{1/2}/\sin(dx+c), (a+b)/(a-b), I/((a-b)/(a+b))^{1/2}) \sin(dx+c) \cos(dx+c)^2 b^2 + 6B \text{EllipticF}((-1+\cos(dx+c))*((a-b)/(a+b))^{1/2}/\sin(dx+c), -(a+b)/(a-b))^{1/2} \sin(dx+c) \cos(dx+c)^2 ((b+a \cos(dx+c))/(1+\cos(dx+c)))/(a+b))^{1/2} a^2 - 2B((b+a \cos(dx+c))/(1+\cos(dx+c)))/(a+b))^{1/2} \text{EllipticF}((-1+\cos(dx+c))*((a-b)/(a+b))^{1/2}/\sin(dx+c), -(a+b)/(a-b))^{1/2} \sin(dx+c) \cos(dx+c)^2 a^2 b + 4B \text{EllipticF}((-1+\cos(dx+c))*((a-b)/(a+b))^{1/2}/\sin(dx+c), -(a+b)/(a-b))^{1/2} \sin(dx+c) \cos(dx+c)^2 ((b+a \cos(dx+c))/(1+\cos(dx+c)))/(a+b))^{1/2} b^2 + 3B \cos(dx+c)^3 ((a-b)/(a+b))^{1/2} a^2 * (1/(1+\cos(dx+c)))^{1/2} - 2B \cos(dx+c)^3 ((a-b)/(a+b))^{1/2} a^2 b * (1/(1+\cos(dx+c)))^{1/2} + 4A \cos(dx+c)^2 ((a-b)/(a+b))^{1/2} a^2 b * (1/(1+\cos(dx+c)))^{1/2} - 4A \cos(dx+c)^2 ((a-b)/(a+b))^{1/2} b^2 * (1/(1+\cos(dx+c)))^{1/2} - 3B \cos(dx+c)^2 ((a-b)/(a+b))^{1/2} a^2 b * (1/(1+\cos(dx+c)))^{1/2} - 2B \cos(dx+c)^2 ((a-b)/(a+b))^{1/2} b^2 * (1/(1+\cos(dx+c)))^{1/2} + 4A \cos(dx+c) * ((a-b)/(a+b))^{1/2} a^2 b * (1/(1+\cos(dx+c)))^{1/2} + 2B * ((a-b)/(a+b))^{1/2} * (1/(1+\cos(dx+c)))^{1/2} * b^2) / b^2 / ((a-b)/(a+b))^{1/2} / (b+a \cos(dx+c)) / \sin(dx+c)^3 / (1/(1+\cos(dx+c)))^{1/2} / \cos(dx+c)^{3/2}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx+c) + A}{\sqrt{b \sec(dx+c) + a} \cos(dx+c)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(dx+c))/cos(dx+c)^(5/2)/(a+b*sec(dx+c))^(1/2),x, algorithm="maxima")

[Out] integrate((B*sec(dx+c)+A)/(sqrt(b*sec(dx+c)+a)*cos(dx+c)^(5/2)),x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\cos(c+dx)^{5/2} \sqrt{a + \frac{b}{\cos(c+dx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + dx))/(cos(c + dx)^(5/2)*(a + b/cos(c + dx))^(1/2)),x)

```
[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))^(1/2)), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(5/2)/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

$$3.621 \quad \int \frac{\cos^5(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^{3/2}} dx$$

Optimal. Leaf size=423

$$\frac{2(a^2A + 5abB - 6Ab^2) \sin(c+dx) \cos^3(c+dx) \sqrt{a+b \sec(c+dx)}}{5a^2d(a^2-b^2)} + \frac{2b(Ab - aB) \sin(c+dx) \cos^3(c+dx)}{ad(a^2-b^2) \sqrt{a+b \sec(c+dx)}} - \frac{2}{d}$$

[Out] $2*b*(A*b-B*a)*\cos(d*x+c)^(3/2)*\sin(d*x+c)/a/(a^2-b^2)/d/(a+b*\sec(d*x+c))^(1/2)-2/15*(12*A*a^2*b+48*A*b^3-5*B*a^3-40*B*a*b^2)*(\cos(1/2*d*x+1/2*c))^2/(\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2)))*((b+a*\cos(d*x+c))/(a+b))^(1/2)/a^4/d/\cos(d*x+c)^(1/2)/(a+b*\sec(d*x+c))^(1/2)+2/5*(A*a^2-6*A*b^2+5*B*a*b)*\cos(d*x+c)^(3/2)*\sin(d*x+c)*(a+b*\sec(d*x+c))^(1/2)/a^2/(a^2-b^2)/d-2/15*(9*A*a^2*b-24*A*b^3-5*B*a^3+20*B*a*b^2)*\sin(d*x+c)*\cos(d*x+c)^(1/2)*(a+b*\sec(d*x+c))^(1/2)/a^3/(a^2-b^2)/d+2/15*(9*A*a^4+24*A*a^2*b^2-48*A*b^4-25*B*a^3*b+40*B*a*b^3)*(\cos(1/2*d*x+1/2*c))^2/(\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2)))*\cos(d*x+c)^(1/2)*(a+b*\sec(d*x+c))^(1/2)/a^4/(a^2-b^2)/d/((b+a*\cos(d*x+c))/(a+b))^(1/2)$

Rubi [A] time = 1.40, antiderivative size = 423, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2955, 4030, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2(a^2A + 5abB - 6Ab^2) \sin(c+dx) \cos^3(c+dx) \sqrt{a+b \sec(c+dx)}}{5a^2d(a^2-b^2)} + \frac{2b(Ab - aB) \sin(c+dx) \cos^3(c+dx)}{ad(a^2-b^2) \sqrt{a+b \sec(c+dx)}} - \frac{2}{d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(\text{Cos}[c + d*x])^(5/2)*(A + B*\text{Sec}[c + d*x])]/(a + b*\text{Sec}[c + d*x])^(3/2), x]$

[Out] $(-2*(12*a^2*A*b + 48*A*b^3 - 5*a^3*B - 40*a*b^2*B)*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])]/(a + b))*\text{EllipticF}[(c + d*x)/2, (2*a)/(a + b)]/(15*a^4*d*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]) + (2*(9*a^4*A + 24*a^2*A*b^2 - 48*A*b^4 - 25*a^3*b*B + 40*a*b^3*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, (2*a)/(a + b)]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]])/(15*a^4*(a^2 - b^2)*d*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])]/(a + b)) + (2*b*(A*b - a*B)*\text{Cos}[c + d*x]^(3/2)*\text{Sin}[c + d*x])/(a*(a^2 - b^2)*d*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]) - (2*(9*a^2*A*b - 24*A*b^3 - 5*a^3*B + 20*a*b^2*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(15*a^3*(a^2 - b^2)*d) + (2*(a^2*A - 6*A*b^2 + 5*a*b*B)*\text{Cos}[c + d*x]^(3/2)*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(5*a^2*(a^2 - b^2)*d)$

Rule 2653

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*Sqrt[a
+ b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
0] && !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a
+ b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && !GtQ[a + b, 0]
```

Rule 2955

```
Int[((a_) + csc[(e_) + (f_)*(x_)]*(b_))^(m_)*(csc[(e_) + (f_)*(x_)]*
(d_) + (c_))^(n_)*((g_)*sin[(e_) + (f_)*(x_)])^(p_), x_Symbol] := Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3856

```
Int[Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)]/Sqrt[csc[(e_) + (f_)*(x_)]
*(d_)], x_Symbol] := Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_) + (f_)*(x_)]*(d_)]/Sqrt[csc[(e_) + (f_)*(x_)]*(b_)]
```



```
+ (a_)], x_Symbol] := Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4030

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (
a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := Simp[(b*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*
(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e
+ f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[A*(a^2*(m + 1) - b^2*(m + n + 1))
+ a*b*B*n - a*(A*b - a*B)*(m + 1)*Csc[e + f*x] + b*(A*b - a*B)*(m + n + 2)*
Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b
- a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && !(ILtQ[m + 1/2, 0] && ILt
Q[n, 0])
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.))/(Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]), x_Symbol] := Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a
_.))^(m_.), x_Symbol] := Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{3/2}} dx &= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \frac{A+B\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} dx \\
&= \frac{2b(Ab-aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a(a^2-b^2)d\sqrt{a+b\sec(c+dx)}} - \frac{(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)})}{5a^2} \\
&= \frac{2b(Ab-aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a(a^2-b^2)d\sqrt{a+b\sec(c+dx)}} + \frac{2(a^2A-6Ab^2+5abB)\cos^{\frac{3}{2}}(c+dx)}{5a^2} \\
&= \frac{2b(Ab-aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a(a^2-b^2)d\sqrt{a+b\sec(c+dx)}} - \frac{2(9a^2Ab-24Ab^3-5a^3B+20a^2A)}{15a^4d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} \\
&= \frac{2b(Ab-aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a(a^2-b^2)d\sqrt{a+b\sec(c+dx)}} - \frac{2(9a^2Ab-24Ab^3-5a^3B+20a^2A)}{15a^4d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} \\
&= \frac{2b(Ab-aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a(a^2-b^2)d\sqrt{a+b\sec(c+dx)}} - \frac{2(9a^2Ab-24Ab^3-5a^3B+20a^2A)}{15a^4d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} \\
&= \frac{2b(Ab-aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a(a^2-b^2)d\sqrt{a+b\sec(c+dx)}} - \frac{2(9a^2Ab-24Ab^3-5a^3B+20a^2A)}{15a^4d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} \\
&= \frac{2(12a^2Ab+48Ab^3-5a^3B-40ab^2B)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{15a^4d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}
\end{aligned}$$

Mathematica [C] time = 20.98, size = 533, normalized size = 1.26

$$\frac{(a\cos(c+dx)+b)^2\left(\frac{2(5aB-9Ab)\sin(c+dx)}{15a^3}+\frac{A\sin(2(c+dx))}{5a^2}+\frac{2(Ab^4\sin(c+dx)-ab^3B\sin(c+dx))}{a^3(a^2-b^2)(a\cos(c+dx)+b)}\right)}{d\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(Cos[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(3/2), x]

```
[Out] ((b + a*cos[c + d*x])^2*((2*(-9*A*b + 5*a*B)*sin[c + d*x])/(15*a^3) + (2*(A
*b^4*sin[c + d*x] - a*b^3*B*sin[c + d*x]))/(a^3*(a^2 - b^2)*(b + a*cos[c +
d*x])) + (A*sin[2*(c + d*x)]/(5*a^2)))/(d*cos[c + d*x]^(3/2)*(a + b*sec[c
+ d*x])^(3/2)) - (2*cos[c + d*x]^(3/2)*(b + a*cos[c + d*x])*sec[c + d*x]^(3
/2)*(cos[(c + d*x)/2]^2*sec[c + d*x]^(3/2)*((-1)*(a + b)*(9*a^4*A + 24*a^2
*A*b^2 - 48*A*b^4 - 25*a^3*b*B + 40*a*b^3*B)*EllipticE[I*ArcSinh[Tan[(c + d
*x)/2]], (-a + b)/(a + b)]*sec[(c + d*x)/2]^2*Sqrt[((b + a*cos[c + d*x])*se
c[(c + d*x)/2]^2)/(a + b)] + I*a*(a + b)*(-48*A*b^3 - 6*a^2*b*(2*A + 5*B) +
a^3*(9*A + 5*B) + 4*a*b^2*(9*A + 10*B))*EllipticF[I*ArcSinh[Tan[(c + d*x)/
2]], (-a + b)/(a + b)]*sec[(c + d*x)/2]^2*Sqrt[((b + a*cos[c + d*x])*sec[(c
+ d*x)/2]^2)/(a + b)] - (9*a^4*A + 24*a^2*A*b^2 - 48*A*b^4 - 25*a^3*b*B +
40*a*b^3*B)*(b + a*cos[c + d*x])*(sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2
]))/(15*a^4*(a^2 - b^2)*d*(a + b*sec[c + d*x])^(3/2))
```

fricas [F] time = 1.00, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{(B \cos(dx + c)^2 \sec(dx + c) + A \cos(dx + c)^2) \sqrt{b \sec(dx + c) + a} \sqrt{\cos(dx + c)}}{b^2 \sec(dx + c)^2 + 2ab \sec(dx + c) + a^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c)^2*sec(d*x + c) + A*cos(d*x + c)^2)*sqrt(b*sec(d*x
+ c) + a)*sqrt(cos(d*x + c))/(b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2
), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \cos(dx + c)^{\frac{5}{2}}}{(b \sec(dx + c) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(5/2)/(b*sec(d*x + c) + a)^(3/2
), x)
```

maple [B] time = 3.18, size = 2084, normalized size = 4.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(\cos(d*x+c)^{(5/2)}*(A+B*\sec(d*x+c))/(a+b*\sec(d*x+c))^{(3/2)}, x)$

[Out] $\frac{2}{15}d*((b+a*\cos(d*x+c))/\cos(d*x+c))^{(1/2)}*\cos(d*x+c)^{(1/2)}*(-1+\cos(d*x+c))$
 $* (1+\cos(d*x+c))^{(1/2)}-48*A*((a-b)/(a+b))^{(1/2)}*\cos(d*x+c)^4*a^3*b*(1/(1+\cos(d*x$
 $+c)))^{(1/2)}-48*A*((a-b)/(a+b))^{(1/2)}*b^4*(1/(1+\cos(d*x+c)))^{(1/2)}+40*B*((b+$
 $a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{(1/2)}*\text{EllipticE}((-1+\cos(d*x+c))*((a-b)/$
 $(a+b))^{(1/2)}/\sin(d*x+c), (- (a+b)/(a-b))^{(1/2)})*\sin(d*x+c)*a^3*b+30*B*\text{Ellipti$
 $cF}((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c), (- (a+b)/(a-b))^{(1/2)})*((b$
 $+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{(1/2)}*\sin(d*x+c)*a^3*b+40*B*\text{EllipticF}$
 $(-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c), (- (a+b)/(a-b))^{(1/2)})*((b+a*$
 $\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{(1/2)}*\sin(d*x+c)*a^2*b^2-6*A*((a-b)/(a+b)$
 $)^{(1/2)}*\cos(d*x+c)^3*a^2*b^2*(1/(1+\cos(d*x+c)))^{(1/2)}-12*A*\text{EllipticF}((-1+co$
 $s(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c), (- (a+b)/(a-b))^{(1/2)})*((b+a*\cos(d*$
 $x+c))/(1+\cos(d*x+c))/(a+b))^{(1/2)}*\sin(d*x+c)*a^3*b-36*A*\text{EllipticF}((-1+\cos(d$
 $*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c), (- (a+b)/(a-b))^{(1/2)})*((b+a*\cos(d*x+c)$
 $))/(1+\cos(d*x+c))/(a+b))^{(1/2)}*\sin(d*x+c)*a^2*b^2-48*A*\text{EllipticF}((-1+\cos(d*$
 $x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c), (- (a+b)/(a-b))^{(1/2)})*((b+a*\cos(d*x+c)$
 $))/(1+\cos(d*x+c))/(a+b))^{(1/2)}*\sin(d*x+c)*a*b^3+24*A*((b+a*\cos(d*x+c))/(1+co$
 $s(d*x+c))/(a+b))^{(1/2)}*\text{EllipticE}((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*$
 $x+c), (- (a+b)/(a-b))^{(1/2)})*\sin(d*x+c)*a^2*b^2+5*B*((a-b)/(a+b))^{(1/2)}*\cos(d$
 $*x+c)^3*a^3*b*(1/(1+\cos(d*x+c)))^{(1/2)}+6*A*((a-b)/(a+b))^{(1/2)}*\cos(d*x+c)^2$
 $*a^3*b*(1/(1+\cos(d*x+c)))^{(1/2)}+24*A*((a-b)/(a+b))^{(1/2)}*\cos(d*x+c)^2*a*b^3$
 $*(1/(1+\cos(d*x+c)))^{(1/2)}-20*B*((a-b)/(a+b))^{(1/2)}*\cos(d*x+c)^2*a^2*b^2*(1/$
 $(1+\cos(d*x+c)))^{(1/2)}+6*A*((a-b)/(a+b))^{(1/2)}*\cos(d*x+c)*a^3*b*(1/(1+\cos(d*$
 $x+c)))^{(1/2)}-18*A*((a-b)/(a+b))^{(1/2)}*\cos(d*x+c)*a^2*b^2*(1/(1+\cos(d*x+c)))$
 $^{(1/2)}+20*B*((a-b)/(a+b))^{(1/2)}*\cos(d*x+c)*a^3*b*(1/(1+\cos(d*x+c)))^{(1/2)}-4$
 $0*B*((a-b)/(a+b))^{(1/2)}*\cos(d*x+c)*a*b^3*(1/(1+\cos(d*x+c)))^{(1/2)}-25*B*((b+$
 $a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{(1/2)}*\text{EllipticE}((-1+\cos(d*x+c))*((a-b)/$
 $(a+b))^{(1/2)}/\sin(d*x+c), (- (a+b)/(a-b))^{(1/2)})*\sin(d*x+c)*a^3*b+9*A*((b+a*co$
 $s(d*x+c))/(1+\cos(d*x+c))/(a+b))^{(1/2)}*\text{EllipticE}((-1+\cos(d*x+c))*((a-b)/(a+b)$
 $))^{(1/2)}/\sin(d*x+c), (- (a+b)/(a-b))^{(1/2)})*\sin(d*x+c)*a^4+5*B*\cos(d*x+c)^3*($
 $(a-b)/(a+b))^{(1/2)}*(1/(1+\cos(d*x+c)))^{(1/2)}*a^4+3*A*\cos(d*x+c)^4*((a-b)/(a+$
 $b))^{(1/2)}*a^4*(1/(1+\cos(d*x+c)))^{(1/2)}+6*A*\cos(d*x+c)^2*((a-b)/(a+b))^{(1/2)}$
 $*a^4*(1/(1+\cos(d*x+c)))^{(1/2)}+24*A*\cos(d*x+c)^2*((a-b)/(a+b))^{(1/2)}*a^2*b^2$
 $*(1/(1+\cos(d*x+c)))^{(1/2)}-20*B*\cos(d*x+c)^2*((a-b)/(a+b))^{(1/2)}*a^3*b*(1/(1$
 $+\cos(d*x+c)))^{(1/2)}-6*A*\cos(d*x+c)^3*((a-b)/(a+b))^{(1/2)}*a^3*b*(1/(1+\cos(d*$
 $x+c)))^{(1/2)}-9*A*\text{EllipticF}((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c), ($
 $- (a+b)/(a-b))^{(1/2)})*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{(1/2)}*\sin(d*x+$
 $c)*a^4-48*A*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{(1/2)}*\text{EllipticE}((-1+\cos$
 $(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c), (- (a+b)/(a-b))^{(1/2)})*\sin(d*x+c)*b^$
 $4+48*A*((a-b)/(a+b))^{(1/2)}*\cos(d*x+c)*b^4*(1/(1+\cos(d*x+c)))^{(1/2)}-5*B*((a-$
 $b)/(a+b))^{(1/2)}*\cos(d*x+c)*a^4*(1/(1+\cos(d*x+c)))^{(1/2)}-9*A*((a-b)/(a+b))^{($
 $1/2)}*a^3*b*(1/(1+\cos(d*x+c)))^{(1/2)}-24*A*((a-b)/(a+b))^{(1/2)}*a*b^3*(1/(1+co$
 $s(d*x+c)))^{(1/2)}-5*B*((a-b)/(a+b))^{(1/2)}*a^3*b*(1/(1+\cos(d*x+c)))^{(1/2)}+20*$
 $B*((a-b)/(a+b))^{(1/2)}*a^2*b^2*(1/(1+\cos(d*x+c)))^{(1/2)}+40*B*((a-b)/(a+b))^{($

$\frac{1}{2} * a * b^3 * (1 / (1 + \cos(d * x + c)))^{1/2} - 9 * A * ((a - b) / (a + b))^{1/2} * \cos(d * x + c) * a^4 * (1 / (1 + \cos(d * x + c)))^{1/2} + 5 * B * \sin(d * x + c) * ((b + a * \cos(d * x + c)) / (1 + \cos(d * x + c))) / (a + b)^{1/2} * \text{EllipticF}((-1 + \cos(d * x + c)) * ((a - b) / (a + b))^{1/2} / \sin(d * x + c), (-a + b) / (a - b))^{1/2} * a^4 * ((a - b) / (a + b))^{1/2} * (1 / (1 + \cos(d * x + c)))^{1/2} / a^4 / (b + a * \cos(d * x + c)) / (a - b) / \sin(d * x + c)^3$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c + dx)^{5/2} \left(A + \frac{B}{\cos(c + dx)} \right)}{\left(a + \frac{b}{\cos(c + dx)} \right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)^(5/2)*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x))^(3/2),x)

[Out] int((cos(c + d*x)^(5/2)*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x))^(3/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(3/2),x)

[Out] Timed out

$$3.622 \quad \int \frac{\cos^3(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^{3/2}} dx$$

Optimal. Leaf size=326

$$\frac{2(a^2A + 3abB - 4Ab^2) \sin(c+dx) \sqrt{\cos(c+dx)} \sqrt{a+b \sec(c+dx)}}{3a^2d(a^2 - b^2)} + \frac{2b(Ab - aB) \sin(c+dx) \sqrt{\cos(c+dx)}}{ad(a^2 - b^2) \sqrt{a+b \sec(c+dx)}} + \dots$$

[Out] 2*b*(A*b-B*a)*sin(d*x+c)*cos(d*x+c)^(1/2)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)+2/3*(A*a^2+8*A*b^2-6*B*a*b)*(cos(1/2*d*x+1/2*c))^2^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/a^3/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2/3*(A*a^2-4*A*b^2+3*B*a*b)*sin(d*x+c)*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/a^2/(a^2-b^2)/d-2/3*(5*A*a^2*b-8*A*b^3-3*B*a^3+6*B*a*b^2)*(cos(1/2*d*x+1/2*c))^2^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/a^3/(a^2-b^2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2)

Rubi [A] time = 1.03, antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2955, 4030, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2(a^2A + 3abB - 4Ab^2) \sin(c+dx) \sqrt{\cos(c+dx)} \sqrt{a+b \sec(c+dx)}}{3a^2d(a^2 - b^2)} + \frac{2b(Ab - aB) \sin(c+dx) \sqrt{\cos(c+dx)}}{ad(a^2 - b^2) \sqrt{a+b \sec(c+dx)}} + \dots$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(3/2),x]

[Out] (2*(a^2*A + 8*A*b^2 - 6*a*b*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]/(3*a^3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - (2*(5*a^2*A*b - 8*A*b^3 - 3*a^3*B + 6*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*a^3*(a^2 - b^2)*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*b*(A*b - a*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(a^2*A - 4*A*b^2 + 3*a*b*B)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*a^2*(a^2 - b^2)*d)

Rule 2653

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*Sqrt[a + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,

b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2655

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] :> Dist[Sqrt[a + b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2661

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)]/(d*Sqrt[a + b]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2663

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b) + (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2955

Int[((a_) + csc[(e_) + (f_)*(x_)])*(b_)^(m_)*(csc[(e_) + (f_)*(x_)]*(d_) + (c_))^(n_)*((g_)*sin[(e_) + (f_)*(x_)])^(p_), x_Symbol] :> Dist[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && IntegerQ[n])

Rule 3856

Int[Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)]/Sqrt[csc[(e_) + (f_)*(x_)]*(d_)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 3858

Int[Sqrt[csc[(e_) + (f_)*(x_)]*(d_)]/Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 4030

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> Simp[(b*(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[A*(a^2*(m + 1) - b^2*(m + n + 1)) + a*b*B*n - a*(A*b - a*B)*(m + 1)*Csc[e + f*x] + b*(A*b - a*B)*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.))/(Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]), x_Symbol] :> Dist[A/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{\frac{3}{2}}} dx &= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \frac{A+B\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{\frac{3}{2}}} dx \\
&= \frac{2b(Ab-aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{a(a^2-b^2)d\sqrt{a+b\sec(c+dx)}} - \frac{(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)})}{3} \\
&= \frac{2b(Ab-aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{a(a^2-b^2)d\sqrt{a+b\sec(c+dx)}} + \frac{2(a^2A-4Ab^2+3abB)\sqrt{\cos(c+dx)}}{3} \\
&= \frac{2b(Ab-aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{a(a^2-b^2)d\sqrt{a+b\sec(c+dx)}} + \frac{2(a^2A-4Ab^2+3abB)\sqrt{\cos(c+dx)}}{3} \\
&= \frac{2b(Ab-aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{a(a^2-b^2)d\sqrt{a+b\sec(c+dx)}} + \frac{2(a^2A-4Ab^2+3abB)\sqrt{\cos(c+dx)}}{3} \\
&= \frac{2b(Ab-aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{a(a^2-b^2)d\sqrt{a+b\sec(c+dx)}} + \frac{2(a^2A-4Ab^2+3abB)\sqrt{\cos(c+dx)}}{3} \\
&= \frac{2(a^2A+8Ab^2-6abB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{3a^3d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} - \frac{2(5a^2Ab-)}{3}
\end{aligned}$$

Mathematica [C] time = 18.03, size = 469, normalized size = 1.44

$$\frac{(a\cos(c+dx)+b)^2\left(\frac{2A\sin(c+dx)}{3a^2}-\frac{2(Ab^3\sin(c+dx)-ab^2B\sin(c+dx))}{a^2(a^2-b^2)(a\cos(c+dx)+b)}\right)}{d\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{\frac{3}{2}}}-\frac{2\cos^{\frac{3}{2}}(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)\right)}{3}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(3/2), x]

[Out] ((b + a*Cos[c + d*x])^2*((2*A*Sin[c + d*x])/(3*a^2) - (2*(A*b^3*Sin[c + d*x] - a*b^2*B*Sin[c + d*x]))/(a^2*(a^2 - b^2)*(b + a*Cos[c + d*x])))/(d*Cos[

$$c + d*x]^{(3/2)}*(a + b*\text{Sec}[c + d*x])^{(3/2)}) - (2*\text{Cos}[c + d*x]^{(3/2)}*(b + a*\text{Cos}[c + d*x])*\text{Sec}[c + d*x]^{(3/2)}*(\text{Cos}[(c + d*x)/2]^{(3/2)}*\text{Sec}[c + d*x]^{(3/2)}*((-I)*(a + b)*(-5*a^2*A*b + 8*A*b^3 + 3*a^3*B - 6*a*b^2*B)*\text{EllipticE}[I*\text{ArcSinh}[\text{Tan}[(c + d*x)/2]], (-a + b)/(a + b)]*\text{Sec}[(c + d*x)/2]^{(3/2)}*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])*\text{Sec}[(c + d*x)/2]^{(3/2)}]/(a + b)) + I*a*(a^2 - a*b - 2*b^2)*(-4*A*b + a*(A + 3*B))*\text{EllipticF}[I*\text{ArcSinh}[\text{Tan}[(c + d*x)/2]], (-a + b)/(a + b)]*\text{Sec}[(c + d*x)/2]^{(3/2)}*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])*\text{Sec}[(c + d*x)/2]^{(3/2)}]/(a + b)) - (-5*a^2*A*b + 8*A*b^3 + 3*a^3*B - 6*a*b^2*B)*(b + a*\text{Cos}[c + d*x])*(\text{Sec}[(c + d*x)/2]^{(3/2)})^{(3/2)}*\text{Tan}[(c + d*x)/2])/(3*a^3*(a^2 - b^2)*d*(a + b*\text{Sec}[c + d*x])^{(3/2)})$$

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B \cos(dx + c) \sec(dx + c) + A \cos(dx + c))\sqrt{b \sec(dx + c) + a} \sqrt{\cos(dx + c)}}{b^2 \sec(dx + c)^2 + 2ab \sec(dx + c) + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")

[Out] integral((B*cos(d*x + c)*sec(d*x + c) + A*cos(d*x + c))*sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c))/(b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \cos(dx + c)^{\frac{3}{2}}}{(b \sec(dx + c) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^(3/2), x)

maple [B] time = 2.37, size = 1460, normalized size = 4.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x)

```
[Out] 2/3/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)*(-1+cos(d*x+c))*
(1+cos(d*x+c))^2*(3*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^2*b*(1/(1+cos(d*x+
c)))^(1/2)+4*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^2*b*(1/(1+cos(d*x+c)))^(1/2
)+6*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a*b^2*(1/(1+cos(d*x+c)))^(1/2)-6*B*sin
(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a
-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b-6*B*sin(d*x
+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2+6*A*sin(d*x+c)*
EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/
2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b+8*A*sin(d*x+c)*Elli
pticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*
((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*b^2-5*A*sin(d*x+c)*((b+a*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b
))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b+A*((a-b)/(a+b))^(1/2)*cos(d
*x+c)^3*a^2*b*(1/(1+cos(d*x+c)))^(1/2)-4*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2
*a*b^2*(1/(1+cos(d*x+c)))^(1/2)+8*A*((a-b)/(a+b))^(1/2)*b^3*(1/(1+cos(d*x+c
)))^(1/2)+3*B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elli
pticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*
a^3+3*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3*(1/(1+cos(d*x+c)))^(1/2)-3*B*s
in(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/
(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^3-3*B*((a-b)/
(a+b))^(1/2)*cos(d*x+c)*a^3*(1/(1+cos(d*x+c)))^(1/2)-A*((a-b)/(a+b))^(1/2)*
cos(d*x+c)*a^3*(1/(1+cos(d*x+c)))^(1/2)-8*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)*
b^3*(1/(1+cos(d*x+c)))^(1/2)-A*((a-b)/(a+b))^(1/2)*a^2*b*(1/(1+cos(d*x+c)))
^(1/2)+4*A*((a-b)/(a+b))^(1/2)*a*b^2*(1/(1+cos(d*x+c)))^(1/2)-3*B*((a-b)/(a
+b))^(1/2)*a^2*b*(1/(1+cos(d*x+c)))^(1/2)-6*B*((a-b)/(a+b))^(1/2)*a*b^2*(1/
(1+cos(d*x+c)))^(1/2)+A*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(
1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*a^3+8*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/
2))*b^3+A*(1/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^3-4*A
*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*b*(1/(1+cos(d*x+c)))^(1/2))*((a-b)/(a
+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)/a^3/(b+a*cos(d*x+c))/(a-b)/sin(d*x+c)^3
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x, algo
rithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c + dx)^{3/2} \left(A + \frac{B}{\cos(c+dx)} \right)}{\left(a + \frac{b}{\cos(c+dx)} \right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((cos(c + d*x)^(3/2)*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x))^(3/2), x)`

[Out] `int((cos(c + d*x)^(3/2)*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x))^(3/2), x)`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)**(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(3/2), x)`

[Out] Timed out

$$3.623 \quad \int \frac{\sqrt{\cos(c+dx)} (A+B \sec(c+dx))}{(a+b \sec(c+dx))^{3/2}} dx$$

Optimal. Leaf size=235

$$\frac{2b(Ab - aB) \sin(c + dx)}{ad (a^2 - b^2) \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}} + \frac{2(a^2 A + abB - 2Ab^2) \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)} E\left(\frac{1}{2}(c + dx)\right)}{a^2 d (a^2 - b^2) \sqrt{\frac{a \cos(c + dx) + b}{a + b}}}$$

[Out] 2*b*(A*b-B*a)*sin(d*x+c)/a/(a^2-b^2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)-2*(2*A*b-B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/a^2/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2*(A*a^2-2*A*b^2+B*a*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/a^2/(a^2-b^2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2)

Rubi [A] time = 0.72, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {2955, 4030, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2b(Ab - aB) \sin(c + dx)}{ad (a^2 - b^2) \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}} + \frac{2(a^2 A + abB - 2Ab^2) \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)} E\left(\frac{1}{2}(c + dx)\right)}{a^2 d (a^2 - b^2) \sqrt{\frac{a \cos(c + dx) + b}{a + b}}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(3/2),x]
 [Out] (-2*(2*A*b - a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]/(a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*(a^2*A - 2*A*b^2 + a*b*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(a^2*(a^2 - b^2)*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*b*(A*b - a*B)*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]])

Rule 2653

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2655

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] :> Dist[Sqrt[a + b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b

```
*Sin[c + d*x]]/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)]/(d*Sqrt[a + b]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a + b*SIN[c + d*x])/(a + b)]/Sqrt[a + b*SIN[c + d*x]], Int[1/Sqrt[a/(a + b) + (b*SIN[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]
```

Rule 2955

```
Int[((a_) + csc[(e_) + (f_)*(x_)]*(b_))^(m_)*(csc[(e_) + (f_)*(x_)]*(d_) + (c_))^(n_)*((g_)*sin[(e_) + (f_)*(x_)])^(p_), x_Symbol] := Dist[(g*Csc[e + f*x])^p*(g*SIN[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && IntegerQ[n])
```

Rule 3856

```
Int[Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)]/Sqrt[csc[(e_) + (f_)*(x_)]*(d_)], x_Symbol] := Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*SIN[e + f*x]]), Int[Sqrt[b + a*SIN[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_) + (f_)*(x_)]*(d_)]/Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)], x_Symbol] := Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*SIN[e + f*x]])/Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*SIN[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4030

```
Int[(csc[(e_) + (f_)*(x_)]*(d_))^(n_)*(csc[(e_) + (f_)*(x_)]*(b_) + (a_))^(m_)*(csc[(e_) + (f_)*(x_)]*(B_) + (A_)), x_Symbol] := Simp[(b*(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e
```

$+ f*x])^{(m+1)}*(d*Csc[e+f*x])^n* Simp[A*(a^2*(m+1) - b^2*(m+n+1)) + a*b*B*n - a*(A*b - a*B)*(m+1)*Csc[e+f*x] + b*(A*b - a*B)*(m+n+2)*Csc[e+f*x]^2, x], x] /;$
 $FreeQ[\{a, b, d, e, f, A, B, n\}, x] \&\& NeQ[A*b - a*B, 0] \&\& NeQ[a^2 - b^2, 0] \&\& LtQ[m, -1] \&\& !(ILtQ[m+1/2, 0] \&\& ILtQ[n, 0])$

Rule 4035

$Int[(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.))/(Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]), x_Symbol] :> Dist[A/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /;$
 $FreeQ[\{a, b, d, e, f, A, B\}, x] \&\& NeQ[A*b - a*B, 0] \&\& NeQ[a^2 - b^2, 0]$

Rubi steps

$$\begin{aligned}
 \int \frac{\sqrt{\cos(c+dx)}(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{3/2}} dx &= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \frac{A+B\sec(c+dx)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}} dx \\
 &= \frac{2b(Ab-aB)\sin(c+dx)}{a(a^2-b^2)d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} - \frac{(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)})}{(a+b\sec(c+dx))^{3/2}} \\
 &= \frac{2b(Ab-aB)\sin(c+dx)}{a(a^2-b^2)d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} - \frac{((2Ab-aB)\sqrt{\cos(c+dx)})}{(a+b\sec(c+dx))^{3/2}} \\
 &= \frac{2b(Ab-aB)\sin(c+dx)}{a(a^2-b^2)d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} - \frac{((2Ab-aB)\sqrt{b+a\cos(c+dx)})}{a^2\sqrt{\cos(c+dx)}} \\
 &= \frac{2b(Ab-aB)\sin(c+dx)}{a(a^2-b^2)d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} - \frac{((2Ab-aB)\sqrt{\frac{b+a\cos(c+dx)}{a}})}{a^2\sqrt{\cos(c+dx)}} \\
 &= \frac{2(2Ab-aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{a^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2(a^2A-2Ab^2+ab^2)}{a^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}
 \end{aligned}$$

Mathematica [C] time = 16.52, size = 445, normalized size = 1.89

$$\frac{2(a \cos(c + dx) + b)(A + B \sec(c + dx)) (Ab^2 \sin(c + dx) - abB \sin(c + dx))}{ad(a^2 - b^2) \sqrt{\cos(c + dx)} (a + b \sec(c + dx))^{3/2} (A \cos(c + dx) + B)} \frac{2 \cos^{\frac{3}{2}}(c + dx) \sqrt{\sec(c + dx)} (\cos^2(c + dx) + \sec(c + dx))}{(a + b \sec(c + dx))^{3/2}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(3/2), x]

[Out] (2*(b + a*Cos[c + d*x])*(A + B*Sec[c + d*x])*(A*b^2*Sin[c + d*x] - a*b*B*Sin[c + d*x]))/(a*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*(B + A*Cos[c + d*x])*(a + b*Sec[c + d*x])^(3/2)) - (2*Cos[c + d*x]^(3/2)*(b + a*Cos[c + d*x])*Sqrt[Sec[c + d*x]]*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x])*((-1)*(a + b)*(a^2*A - 2*A*b^2 + a*b*B)*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + I*a*(a + b)*(-2*A*b + a*(A + B))*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - (a^2*A - 2*A*b^2 + a*b*B)*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/(a^2*(a^2 - b^2)*d*(B + A*Cos[c + d*x])*(a + b*Sec[c + d*x])^(3/2))

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{(B \sec(dx + c) + A) \sqrt{b \sec(dx + c) + a} \sqrt{\cos(dx + c)}}{b^2 \sec(dx + c)^2 + 2ab \sec(dx + c) + a^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2), x, algorithm="fricas")

[Out] integral((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c))/(b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \sqrt{\cos(dx + c)}}{(b \sec(dx + c) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2), x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(b*sec(d*x + c) + a)^(3/2), x)

maple [B] time = 3.14, size = 889, normalized size = 3.78

$$2(-1 + \cos(dx + c))(1 + \cos(dx + c))^2 \left(A \sqrt{\frac{a-b}{a+b}} (\cos^2(dx + c)) \sqrt{\frac{1}{1 + \cos(dx + c)}} a^2 + A (\cos^2(dx + c)) \sqrt{\frac{a-b}{a+b}} ab \sqrt{\dots} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2), x)

[Out] 2/d*(-1+cos(d*x+c))*(1+cos(d*x+c))^2*(A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*(1/(1+cos(d*x+c)))^(1/2)*a^2+A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b*(1/(1+cos(d*x+c)))^(1/2)+A*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), (-a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b)^(1/2)*sin(d*x+c)*a^2-2*A*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b)^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), (-a+b)/(a-b))^(1/2))*sin(d*x+c)*b^2-A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), (-a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b)^(1/2)*sin(d*x+c)*a^2-2*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), (-a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b)^(1/2)*sin(d*x+c)*a*b-A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*(1/(1+cos(d*x+c)))^(1/2)+2*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^2*(1/(1+cos(d*x+c)))^(1/2)+B*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b)^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), (-a+b)/(a-b))^(1/2))*sin(d*x+c)*a*b+B*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b)^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), (-a+b)/(a-b))^(1/2))*sin(d*x+c)*a^2-B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b*(1/(1+cos(d*x+c)))^(1/2)-A*((a-b)/(a+b))^(1/2)*a*b*(1/(1+cos(d*x+c)))^(1/2)-2*A*((a-b)/(a+b))^(1/2)*b^2*(1/(1+cos(d*x+c)))^(1/2)+B*((a-b)/(a+b))^(1/2)*a*b*(1/(1+cos(d*x+c)))^(1/2))*cos(d*x+c)^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*((a-b)/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)/a^2/(b+a*cos(d*x+c))/(a-b)/sin(d*x+c))^3

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2), x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\cos(c+dx)} \left(A + \frac{B}{\cos(c+dx)} \right)}{\left(a + \frac{b}{\cos(c+dx)} \right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)^(1/2)*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x))^(3/2), x)

[Out] int((cos(c + d*x)^(1/2)*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x))^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \sqrt{\cos(c + dx)}}{(a + b \sec(c + dx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)**(1/2)/(a+b*sec(d*x+c))**(3/2), x)

[Out] Integral((A + B*sec(c + d*x))*sqrt(cos(c + d*x))/(a + b*sec(c + d*x))**(3/2), x)

$$3.624 \quad \int \frac{A+B \sec(c+dx)}{\sqrt{\cos(c+dx)} (a+b \sec(c+dx))^{3/2}} dx$$

Optimal. Leaf size=215

$$\frac{2(Ab - aB) \sin(c + dx)}{d(a^2 - b^2) \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}} + \frac{2(Ab - aB) \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2a}{a+b}\right)}{ad(a^2 - b^2) \sqrt{\frac{a \cos(c+dx)+b}{a+b}}}$$

[Out] $-2*(A*b-B*a)*\sin(d*x+c)/(a^2-b^2)/d/\cos(d*x+c)^{(1/2)}/(a+b*\sec(d*x+c))^{(1/2)}$
 $+2*A*(\cos(1/2*d*x+1/2*c))^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)}*(a/(a+b))^{(1/2)})*((b+a*\cos(d*x+c))/(a+b))^{(1/2)}/a/d/\cos(d*x+c)^{(1/2)}/(a+b*\sec(d*x+c))^{(1/2)}$
 $+2*(A*b-B*a)*(\cos(1/2*d*x+1/2*c))^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)}*(a/(a+b))^{(1/2)})*\cos(d*x+c)^{(1/2)}*(a+b*\sec(d*x+c))^{(1/2)}/a/(a^2-b^2)/d/((b+a*\cos(d*x+c))/(a+b))^{(1/2)}$

Rubi [A] time = 0.67, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {2955, 4027, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2(Ab - aB) \sin(c + dx)}{d(a^2 - b^2) \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}} + \frac{2(Ab - aB) \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2a}{a+b}\right)}{ad(a^2 - b^2) \sqrt{\frac{a \cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sec[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)), x]

[Out] $(2*A*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/(a + b)]*\text{EllipticF}[(c + d*x)/2, (2*a)/(a + b)])/(a*d*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]) + (2*(A*b - a*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, (2*a)/(a + b)]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]])/(a*(a^2 - b^2)*d*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/(a + b)]) - (2*(A*b - a*B)*\text{Sin}[c + d*x])/((a^2 - b^2)*d*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]])$

Rule 2653

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2655

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] :> Dist[Sqrt[a + b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b

```
*Sin[c + d*x]]/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)]/(d*Sqrt[a + b]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a + b*SIN[c + d*x])/(a + b)]/Sqrt[a + b*SIN[c + d*x]], Int[1/Sqrt[a/(a + b) + (b*SIN[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]
```

Rule 2955

```
Int[((a_) + csc[(e_) + (f_)*(x_)]*(b_))^(m_)*(csc[(e_) + (f_)*(x_)]*(d_) + (c_))^(n_)*((g_)*sin[(e_) + (f_)*(x_)])^(p_), x_Symbol] := Dist[(g*Csc[e + f*x])^p*(g*SIN[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && IntegerQ[n])
```

Rule 3856

```
Int[Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)]/Sqrt[csc[(e_) + (f_)*(x_)]*(d_)], x_Symbol] := Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*SIN[e + f*x]]), Int[Sqrt[b + a*SIN[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_) + (f_)*(x_)]*(d_)]/Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)], x_Symbol] := Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*SIN[e + f*x]])/Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*SIN[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4027

```
Int[(csc[(e_) + (f_)*(x_)]*(d_))^(n_)*(csc[(e_) + (f_)*(x_)]*(b_) + (a_))^(m_)*(csc[(e_) + (f_)*(x_)]*(B_) + (A_)), x_Symbol] := -Simp[(d*(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Csc
```

$c[e + f*x]^{(m + 1)}*(d*Csc[e + f*x])^{(n - 1)}*Simp[d*(n - 1)*(A*b - a*B) + d*(a*A - b*B)*(m + 1)*Csc[e + f*x] - d*(A*b - a*B)*(m + n + 1)*Csc[e + f*x]^2, x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] \&\& NeQ[A*b - a*B, 0] \&\& NeQ[a^2 - b^2, 0] \&\& LtQ[m, -1] \&\& LtQ[0, n, 1]$

Rule 4035

$Int[(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.))/(Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]), x_Symbol] :> Dist[A/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] \&\& NeQ[A*b - a*B, 0] \&\& NeQ[a^2 - b^2, 0]$

Rubi steps

$$\begin{aligned} \int \frac{A + B \sec(c + dx)}{\sqrt{\cos(c + dx)} (a + b \sec(c + dx))^{3/2}} dx &= \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{\sqrt{\sec(c + dx)} (A + B \sec(c + dx))}{(a + b \sec(c + dx))^{3/2}} dx \\ &= -\frac{2(Ab - aB) \sin(c + dx)}{(a^2 - b^2) d \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}} - \frac{(2\sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)})}{(a^2 - b^2) d \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}} \\ &= -\frac{2(Ab - aB) \sin(c + dx)}{(a^2 - b^2) d \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}} + \frac{(A\sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)})}{(a^2 - b^2) d \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}} \\ &= -\frac{2(Ab - aB) \sin(c + dx)}{(a^2 - b^2) d \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}} + \frac{(A\sqrt{b + a \cos(c + dx)})}{a \sqrt{\cos(c + dx)}} \\ &= -\frac{2(Ab - aB) \sin(c + dx)}{(a^2 - b^2) d \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}} + \frac{(A\sqrt{\frac{b+a \cos(c+dx)}{a+b}})}{a \sqrt{\cos(c + dx)}} \\ &= \frac{2A\sqrt{\frac{b+a \cos(c+dx)}{a+b}} F\left(\frac{1}{2}(c + dx) \middle| \frac{2a}{a+b}\right)}{ad \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}} + \frac{2(Ab - aB) \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2a}{a+b}\right)}{a(a^2 - b^2)} \end{aligned}$$

Mathematica [C] time = 10.92, size = 328, normalized size = 1.53

$$2(a \cos(c + dx) + b) \left(\frac{(aB - Ab) \sin(c + dx)}{a^2 - b^2} + \frac{\left(\cos^2\left(\frac{1}{2}(c + dx)\right) \sec(c + dx) \right)^{3/2} \left((Ab - aB) \tan\left(\frac{1}{2}(c + dx)\right) \sec^2\left(\frac{1}{2}(c + dx)\right) \right)^{3/2} (a \cos(c + dx) + b) - ia(a + b)}{d \cos^2\left(\frac{3}{2}(c + dx)\right)} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(A + B*Sec[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)), x]

[Out] (2*(b + a*Cos[c + d*x])*(((A*b) + a*B)*Sin[c + d*x])/(a^2 - b^2) + ((Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*((-I)*(a + b)*(-(A*b) + a*B)*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - I*a*(a + b)*(A - B)*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + (A*b - a*B)*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/((a^3 - a*b^2)*Sec[c + d*x]^(3/2)))/(d*Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2))

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B \sec(dx + c) + A)\sqrt{b \sec(dx + c) + a} \sqrt{\cos(dx + c)}}{b^2 \cos(dx + c) \sec(dx + c)^2 + 2ab \cos(dx + c) \sec(dx + c) + a^2 \cos(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2), x, algorithm="fricas")

[Out] integral((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c))/(b^2*cos(d*x + c)*sec(d*x + c)^2 + 2*a*b*cos(d*x + c)*sec(d*x + c) + a^2*cos(d*x + c)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(b \sec(dx + c) + a)^{\frac{3}{2}} \sqrt{\cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2), x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c))), x)

maple [B] time = 2.44, size = 564, normalized size = 2.62

$$2(-1 + \cos(dx + c))(1 + \cos(dx + c))^2 \left(A \sin(dx + c) \operatorname{EllipticF} \left(\frac{(-1 + \cos(dx + c)) \sqrt{\frac{a-b}{a+b}}}{\sin(dx + c)}, \sqrt{\frac{a+b}{a-b}} \right) \sqrt{\frac{b+a \cos(dx + c)}{(1 + \cos(dx + c))(a+b)}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2), x)

[Out] 2/d*(-1+cos(d*x+c))*(1+cos(d*x+c))^2*(A*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), (-a+b)/(a-b)^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), (-a+b)/(a-b)^(1/2))*b-A*((a-b)/(a+b))^(1/2)*cos(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*b+B*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), (-a+b)/(a-b)^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a-B*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), (-a+b)/(a-b)^(1/2))*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a+B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*(1/(1+cos(d*x+c)))^(1/2)+A*((a-b)/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b-B*((a-b)/(a+b))^(1/2)*a*(1/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*((a-b)/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)/a/(b+a*cos(d*x+c))/(a-b)/sin(d*x+c)^3

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(b \sec(dx + c) + a)^{\frac{3}{2}} \sqrt{\cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2), x, algorith="maxima")

[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c))), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\sqrt{\cos(c+dx)} \left(a + \frac{b}{\cos(c+dx)} \right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B/cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))^(3/2)),x)`

[Out] `int((A + B/cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))^(3/2)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \sec(c + dx)}{(a + b \sec(c + dx))^{\frac{3}{2}} \sqrt{\cos(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(3/2)/cos(d*x+c)**(1/2),x)`

[Out] `Integral((A + B*sec(c + d*x))/((a + b*sec(c + d*x))**(3/2)*sqrt(cos(c + d*x))), x)`

$$3.625 \quad \int \frac{A+B \sec(c+dx)}{\cos^2(c+dx)(a+b \sec(c+dx))^{3/2}} dx$$

Optimal. Leaf size=220

$$\frac{2a(Ab - aB) \sin(c + dx)}{bd(a^2 - b^2) \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}} - \frac{2(Ab - aB) \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2a}{a+b}\right)}{bd(a^2 - b^2) \sqrt{\frac{a \cos(c+dx)+b}{a+b}}}$$

[Out] 2*a*(A*b-B*a)*sin(d*x+c)/b/(a^2-b^2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/b/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)-2*(A*b-B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/b/(a^2-b^2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2)

Rubi [A] time = 0.79, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2955, 4029, 4108, 3859, 2807, 2805, 21, 3856, 2655, 2653}

$$\frac{2a(Ab - aB) \sin(c + dx)}{bd(a^2 - b^2) \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}} - \frac{2(Ab - aB) \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2a}{a+b}\right)}{bd(a^2 - b^2) \sqrt{\frac{a \cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sec[c + d*x])/((Cos[c + d*x])^(3/2)*(a + b*Sec[c + d*x])^(3/2)),x]

[Out] (2*B*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/(b*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - (2*(A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(b*(a^2 - b^2)*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*a*(A*b - a*B)*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]])

Rule 21

Int[(u_)*((a_) + (b_)*(v_))^(m_)*((c_) + (d_)*(v_))^(n_), x_Symbol] := Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplerQ[c + d*x, a + b*x])

Rule 2653

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*Sqrt[a
+ b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)]/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
0] && !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_)
+ (f_)*(x_)]]), x_Symbol] := Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)]/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_)
+ (f_)*(x_)]]), x_Symbol] := Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
+ f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !GtQ[c + d, 0]
```

Rule 2955

```
Int[((a_) + csc[(e_) + (f_)*(x_)]*(b_))^(m_)*(csc[(e_) + (f_)*(x_)]*
(d_) + (c_))^(n_)*((g_)*sin[(e_) + (f_)*(x_)])^(p_), x_Symbol] := Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3856

```
Int[Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)]/Sqrt[csc[(e_) + (f_)*(x_)]
*(d_)], x_Symbol] := Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] := Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4029

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] := Simp[(a*d^2*(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[d/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(n - 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2*(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 1]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] := Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{A + B \sec(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(a + b \sec(c + dx))^{3/2}} dx &= \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{\sec^{\frac{3}{2}}(c + dx)(A + B \sec(c + dx))}{(a + b \sec(c + dx))^{3/2}} dx \\
&= \frac{2a(Ab - aB) \sin(c + dx)}{b(a^2 - b^2) d \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}} + \frac{(2\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)})}{b(a^2 - b^2) d \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}} \\
&= \frac{2a(Ab - aB) \sin(c + dx)}{b(a^2 - b^2) d \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}} + \frac{(2\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)})}{b(a^2 - b^2) d \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}} \\
&= \frac{2a(Ab - aB) \sin(c + dx)}{b(a^2 - b^2) d \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}} + \frac{((-Ab + aB)\sqrt{\cos(c + dx)})}{b(a^2 - b^2) d \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}} \\
&= \frac{2a(Ab - aB) \sin(c + dx)}{b(a^2 - b^2) d \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}} + \frac{\left(B \sqrt{\frac{b+a \cos(c+dx)}{a+b}} \right) \int \frac{1}{\sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}} dx}{b \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}} \\
&= \frac{2B \sqrt{\frac{b+a \cos(c+dx)}{a+b}} \Pi\left(2; \frac{1}{2}(c + dx) \middle| \frac{2a}{a+b}\right)}{bd \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}} + \frac{2a(Ab - aB) \sin(c + dx)}{b(a^2 - b^2) d \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}} \\
&= \frac{2B \sqrt{\frac{b+a \cos(c+dx)}{a+b}} \Pi\left(2; \frac{1}{2}(c + dx) \middle| \frac{2a}{a+b}\right)}{bd \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}} - \frac{2(Ab - aB)\sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2a}{a+b}\right)}{b(a^2 - b^2) d \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}}
\end{aligned}$$

Mathematica [C] time = 32.37, size = 50122, normalized size = 227.83

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2)),x]

[Out] Result too large to show

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(b \sec(dx + c) + a)^{\frac{3}{2}} \cos(dx + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/2)), x)

maple [C] time = 2.95, size = 840, normalized size = 3.82

$$2(-1 + \cos(dx + c)) \sqrt{\frac{b+a \cos(dx+c)}{\cos(dx+c)}} (1 + \cos(dx + c))^2 \left(A \sqrt{\frac{a-b}{a+b}} \cos(dx + c) \sqrt{\frac{1}{1+\cos(dx+c)}} b + A \sin(dx + c) \right) \text{Ell}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x)

[Out] 2/d*(-1+cos(d*x+c))*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(1+cos(d*x+c))^2*(A*((a-b)/(a+b))^(1/2)*cos(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*b+A*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*b-A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b-B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*(1/(1+cos(d*x+c)))^(1/2)-2*B*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*a-B*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*b+B*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*a+2*B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), (a+b)/(a-b), I/((a-b)/(a+b))^(1/2))*a+2*B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), (a+b)/(a-b), I/((a-b)/(a+b))^(1/2))*b-A*((a-b)/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b+B*((a-b)/(a+b))^(1/2)

$/2) * a * (1 / (1 + \cos(dx + c)))^{1/2} * \cos(dx + c)^{1/2} * ((a - b) / (a + b))^{1/2} * (1 / (1 + \cos(dx + c)))^{1/2} / b / (b + a * \cos(dx + c)) / (a - b) / \sin(dx + c)^3$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(b \sec(dx + c) + a)^{3/2} \cos(dx + c)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/2)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\cos(c+dx)^{3/2} \left(a + \frac{b}{\cos(c+dx)}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^(3/2)),x)

[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^(3/2)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(3/2)/(a+b*sec(d*x+c))**(3/2),x)

[Out] Timed out

$$3.626 \quad \int \frac{A+B \sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b \sec(c+dx))^{3/2}} dx$$

Optimal. Leaf size=371

$$\frac{2a(Ab - aB) \sin(c + dx)}{bd(a^2 - b^2) \cos^{\frac{3}{2}}(c + dx) \sqrt{a + b \sec(c + dx)}} - \frac{(-3a^2B + 2aAb + b^2B) \sin(c + dx) \sqrt{a + b \sec(c + dx)}}{b^2d(a^2 - b^2) \sqrt{\cos(c + dx)}} + \frac{(-3a^2B - 2aAb - b^2B) \sin(c + dx) \sqrt{a + b \sec(c + dx)}}{b^2d(a^2 - b^2) \sqrt{\cos(c + dx)}}$$

[Out] $2*a*(A*b-B*a)*\sin(d*x+c)/b/(a^2-b^2)/d/\cos(d*x+c)^{(3/2)}/(a+b*\sec(d*x+c))^{(1/2)}+B*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)}*(a/(a+b))^{(1/2)})*((b+a*\cos(d*x+c))/(a+b))^{(1/2)}/b/d/\cos(d*x+c)^{(1/2)}/(a+b*\sec(d*x+c))^{(1/2)}+(2*A*b-3*B*a)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticPi}(\sin(1/2*d*x+1/2*c), 2, 2^{(1/2)}*(a/(a+b))^{(1/2)})*((b+a*\cos(d*x+c))/(a+b))^{(1/2)}/b^2/d/\cos(d*x+c)^{(1/2)}/(a+b*\sec(d*x+c))^{(1/2)}-(2*A*a*b-3*B*a^2+B*b^2)*\sin(d*x+c)*(a+b*\sec(d*x+c))^{(1/2)}/b^2/(a^2-b^2)/d/\cos(d*x+c)^{(1/2)}+(2*A*a*b-3*B*a^2+B*b^2)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)}*(a/(a+b))^{(1/2)})*\cos(d*x+c)^{(1/2)}*(a+b*\sec(d*x+c))^{(1/2)}/b^2/(a^2-b^2)/d/((b+a*\cos(d*x+c))/(a+b))^{(1/2)}$

Rubi [A] time = 1.45, antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 14, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2955, 4029, 4102, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2a(Ab - aB) \sin(c + dx)}{bd(a^2 - b^2) \cos^{\frac{3}{2}}(c + dx) \sqrt{a + b \sec(c + dx)}} - \frac{(-3a^2B + 2aAb + b^2B) \sin(c + dx) \sqrt{a + b \sec(c + dx)}}{b^2d(a^2 - b^2) \sqrt{\cos(c + dx)}} + \frac{(-3a^2B - 2aAb - b^2B) \sin(c + dx) \sqrt{a + b \sec(c + dx)}}{b^2d(a^2 - b^2) \sqrt{\cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(3/2)), x]

[Out] $(B*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/(a + b)]*\text{EllipticF}[(c + d*x)/2, (2*a)/(a + b)])/(b*d*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]) + ((2*A*b - 3*a*B)*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/(a + b)]*\text{EllipticPi}[2, (c + d*x)/2, (2*a)/(a + b)])/(b^2*d*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]) + ((2*a*A*b - 3*a^2*B + b^2*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, (2*a)/(a + b)]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]])/(b^2*(a^2 - b^2)*d*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/(a + b)]) + (2*a*(A*b - a*B)*\text{Sin}[c + d*x])/(b*(a^2 - b^2)*d*\text{Cos}[c + d*x]^(3/2)*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]) - ((2*a*A*b - 3*a^2*B + b^2*B)*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(b^2*(a^2 - b^2)*d*\text{Sqrt}[\text{Cos}[c + d*x]])$

Rule 2653

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*Sqrt[a
+ b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
0] && !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a
+ b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_)
+ (f_)*(x_)]]), x_Symbol] := Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/d, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_)
+ (f_)*(x_)]]), x_Symbol] := Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
+ f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !GtQ[c + d, 0]
```

Rule 2955

```
Int[((a_) + csc[(e_) + (f_)*(x_)])*(b_)^(m_)*(csc[(e_) + (f_)*(x_)])*
```



```
(d_.) + (c_.))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_.)]^(p_.), x_Symbol] := Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]/Sqrt[csc[(e_.) + (f_.)*(x_.)]
*(d_.)], x_Symbol] := Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.)
+ (a_.)], x_Symbol] := Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.)
+ (a_.)], x_Symbol] := Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4029

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (
a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := Simp[(a*d^2*
(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n -
2))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[d/(b*(m + 1)*(a^2 - b^2)), Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(n
- 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2*
(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n
, 1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.))/(Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]), x_Symbol] := Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
```

a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.)
+ (a_)])], x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
+ b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{A + B \sec(c + dx)}{\cos^{\frac{5}{2}}(c + dx)(a + b \sec(c + dx))^{3/2}} dx &= \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{\sec^{\frac{5}{2}}(c + dx)(A + B \sec(c + dx))}{(a + b \sec(c + dx))^{3/2}} dx \\
&= \frac{2a(Ab - aB) \sin(c + dx)}{b(a^2 - b^2) d \cos^{\frac{3}{2}}(c + dx) \sqrt{a + b \sec(c + dx)}} + \frac{(2\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)})}{b^2(a^2 - b^2) d \cos^{\frac{3}{2}}(c + dx) \sqrt{a + b \sec(c + dx)}} \\
&= \frac{2a(Ab - aB) \sin(c + dx)}{b(a^2 - b^2) d \cos^{\frac{3}{2}}(c + dx) \sqrt{a + b \sec(c + dx)}} - \frac{(2aAb - 3a^2B + b^2B)}{b^2(a^2 - b^2) d \cos^{\frac{3}{2}}(c + dx) \sqrt{a + b \sec(c + dx)}} \\
&= \frac{2a(Ab - aB) \sin(c + dx)}{b(a^2 - b^2) d \cos^{\frac{3}{2}}(c + dx) \sqrt{a + b \sec(c + dx)}} - \frac{(2aAb - 3a^2B + b^2B)}{b^2(a^2 - b^2) d \cos^{\frac{3}{2}}(c + dx) \sqrt{a + b \sec(c + dx)}} \\
&= \frac{2a(Ab - aB) \sin(c + dx)}{b(a^2 - b^2) d \cos^{\frac{3}{2}}(c + dx) \sqrt{a + b \sec(c + dx)}} - \frac{(2aAb - 3a^2B + b^2B)}{b^2(a^2 - b^2) d \cos^{\frac{3}{2}}(c + dx) \sqrt{a + b \sec(c + dx)}} \\
&= \frac{2a(Ab - aB) \sin(c + dx)}{b(a^2 - b^2) d \cos^{\frac{3}{2}}(c + dx) \sqrt{a + b \sec(c + dx)}} - \frac{(2aAb - 3a^2B + b^2B)}{b^2(a^2 - b^2) d \cos^{\frac{3}{2}}(c + dx) \sqrt{a + b \sec(c + dx)}} \\
&= \frac{2a(Ab - aB) \sin(c + dx)}{b(a^2 - b^2) d \cos^{\frac{3}{2}}(c + dx) \sqrt{a + b \sec(c + dx)}} - \frac{(2aAb - 3a^2B + b^2B)}{b^2(a^2 - b^2) d \cos^{\frac{3}{2}}(c + dx) \sqrt{a + b \sec(c + dx)}} \\
&= \frac{(2Ab - 3aB) \sqrt{\frac{b+a \cos(c+dx)}{a+b}} \Pi\left(2; \frac{1}{2}(c + dx) \middle| \frac{2a}{a+b}\right)}{b^2 d \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}} + \frac{2a(Ab - aB) \sin(c + dx)}{b(a^2 - b^2) d \cos^{\frac{3}{2}}(c + dx) \sqrt{a + b \sec(c + dx)}} \\
&= \frac{B \sqrt{\frac{b+a \cos(c+dx)}{a+b}} F\left(\frac{1}{2}(c + dx) \middle| \frac{2a}{a+b}\right)}{bd \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}} + \frac{(2Ab - 3aB) \sqrt{\frac{b+a \cos(c+dx)}{a+b}} \Pi\left(2; \frac{1}{2}(c + dx) \middle| \frac{2a}{a+b}\right)}{b^2 d \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}}
\end{aligned}$$

Mathematica [C] time = 33.84, size = 95694, normalized size = 257.94

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(A + B*Sec[c + d*x])/((Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(3/2)),x]

[Out] Result too large to show

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(b \sec(dx + c) + a)^{\frac{3}{2}} \cos(dx + c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(5/2)), x)

maple [C] time = 2.28, size = 1441, normalized size = 3.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x)

[Out]
$$\begin{aligned} & -1/d*(-1+\cos(d*x+c))*((b+a*\cos(d*x+c))/\cos(d*x+c))^{1/2}*(1+\cos(d*x+c))^{2*} \\ & 2*A*\cos(d*x+c)^2*((a-b)/(a+b))^{1/2}*a*b*(1/(1+\cos(d*x+c)))^{1/2}-2*A*\sin(d \\ & *x+c)*\cos(d*x+c)*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*EllipticE((- \\ & 1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c),(-(a+b)/(a-b))^{1/2})*a*b-4*A* \\ & EllipticPi((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c),(a+b)/(a-b),I/((a \\ & -b)/(a+b))^{1/2})*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*\sin(d*x+c)* \\ & \cos(d*x+c)*a*b-4*A*\sin(d*x+c)*\cos(d*x+c)*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(\\ & a+b)^{1/2}*EllipticPi((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c),(a+b) \\ & /(a-b),I/((a-b)/(a+b))^{1/2})*b^2+4*A*\sin(d*x+c)*\cos(d*x+c)*EllipticF((-1+c \\ & os(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c),(-(a+b)/(a-b))^{1/2})*((b+a*\cos(d \\ & *x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*a*b+2*A*\sin(d*x+c)*\cos(d*x+c)*EllipticF(\\ & (-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c),(-(a+b)/(a-b))^{1/2})*((b+a* \\ & \cos(d*x+c))/(1+\cos(d*x+c)))/(a+b)^{1/2}*b^2-3*B*\cos(d*x+c)^2*((a-b)/(a+b))^{ \\ & (1/2)*a^2*(1/(1+\cos(d*x+c)))^{1/2}-B*\cos(d*x+c)^2*((a-b)/(a+b))^{1/2}*a*b* \end{aligned}$$

```

1/(1+cos(d*x+c))^(1/2)+3*B*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)*a^2-B*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-a+b)/(a-b))^(1/2))*b^2+6*B*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)*a^2+6*B*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a*b-6*B*sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^2-4*B*sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*b-2*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b*(1/(1+cos(d*x+c)))^(1/2)+3*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*(1/(1+cos(d*x+c)))^(1/2)-B*((a-b)/(a+b))^(1/2)*cos(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*b^2+B*((a-b)/(a+b))^(1/2)*a*b*(1/(1+cos(d*x+c)))^(1/2)+B*((a-b)/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b^2)*((a-b)/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)/b^2/(b+a*cos(d*x+c))/cos(d*x+c)^(1/2)/(a-b)/sin(d*x+c)^3

```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\cos(c+dx)^{5/2} \left(a + \frac{b}{\cos(c+dx)}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))^(3/2)),x)

[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))^(3/2)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(5/2)/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```


$$+ d*x))/ (b*(a^2 - b^2)*d*\cos[c + d*x]^{5/2}*\sqrt{a + b*\sec[c + d*x]}) - ((4*a*A*b - 5*a^2*B + b^2*B)*\sqrt{a + b*\sec[c + d*x]}*\sin[c + d*x]) / (2*b^2*(a^2 - b^2)*d*\cos[c + d*x]^{3/2}) + ((12*a^2*A*b - 4*A*b^3 - 15*a^3*B + 7*a*b^2*B)*\sqrt{a + b*\sec[c + d*x]}*\sin[c + d*x]) / (4*b^3*(a^2 - b^2)*d*\sqrt{\cos[c + d*x]})$$

Rule 2653

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*Sqrt[a + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a + b*SIN[c + d*x]]/Sqrt[(a + b*SIN[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b*SIN[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a + b*SIN[c + d*x])/(a + b)]/Sqrt[a + b*SIN[c + d*x]], Int[1/Sqrt[a/(a + b) + (b*SIN[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Dist[Sqrt[(c + d*SIN[e + f*x])/(c + d)]/Sqrt[c + d*SIN[e + f*x]], Int[1/((a + b*SIN[e + f*x])*Sqrt[c/(c + d) + (d*SIN[e + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d,
```


, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !GtQ[c + d, 0]

Rule 2955

Int[((a_.) + csc[(e_.) + (f_.)*(x_)])*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] := Dist[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && IntegerQ[n])

Rule 3856

Int[Sqrt[csc[(e_.) + (f_.)*(x_)])*(b_.) + (a_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)], x_Symbol] := Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 3858

Int[Sqrt[csc[(e_.) + (f_.)*(x_)])*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] := Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 3859

Int[(csc[(e_.) + (f_.)*(x_)])*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] := Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 4029

Int[(csc[(e_.) + (f_.)*(x_)])*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] := Simp[(a*d^2*(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[d/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(n - 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2*(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 1]

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.))/(Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]), x_Symbol] := Dist[A/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m, x_Symbol] := -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 0]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.))/(Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]), x_Symbol] := Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{A + B \sec(c + dx)}{\cos^{\frac{7}{2}}(c + dx)(a + b \sec(c + dx))^{3/2}} dx &= \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{\sec^{\frac{7}{2}}(c + dx)(A + B \sec(c + dx))}{(a + b \sec(c + dx))^{3/2}} dx \\
&= \frac{2a(Ab - aB) \sin(c + dx)}{b(a^2 - b^2) d \cos^{\frac{5}{2}}(c + dx) \sqrt{a + b \sec(c + dx)}} + \frac{(2\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)})}{b(a^2 - b^2) d \cos^{\frac{5}{2}}(c + dx) \sqrt{a + b \sec(c + dx)}} \\
&= \frac{2a(Ab - aB) \sin(c + dx)}{b(a^2 - b^2) d \cos^{\frac{5}{2}}(c + dx) \sqrt{a + b \sec(c + dx)}} - \frac{(4aAb - 5a^2B + b^2B)}{2b^2(a^2 - b^2) d \cos^{\frac{5}{2}}(c + dx) \sqrt{a + b \sec(c + dx)}} \\
&= \frac{2a(Ab - aB) \sin(c + dx)}{b(a^2 - b^2) d \cos^{\frac{5}{2}}(c + dx) \sqrt{a + b \sec(c + dx)}} - \frac{(4aAb - 5a^2B + b^2B)}{2b^2(a^2 - b^2) d \cos^{\frac{5}{2}}(c + dx) \sqrt{a + b \sec(c + dx)}} \\
&= \frac{2a(Ab - aB) \sin(c + dx)}{b(a^2 - b^2) d \cos^{\frac{5}{2}}(c + dx) \sqrt{a + b \sec(c + dx)}} - \frac{(4aAb - 5a^2B + b^2B)}{2b^2(a^2 - b^2) d \cos^{\frac{5}{2}}(c + dx) \sqrt{a + b \sec(c + dx)}} \\
&= \frac{2a(Ab - aB) \sin(c + dx)}{b(a^2 - b^2) d \cos^{\frac{5}{2}}(c + dx) \sqrt{a + b \sec(c + dx)}} - \frac{(4aAb - 5a^2B + b^2B)}{2b^2(a^2 - b^2) d \cos^{\frac{5}{2}}(c + dx) \sqrt{a + b \sec(c + dx)}} \\
&= \frac{2a(Ab - aB) \sin(c + dx)}{b(a^2 - b^2) d \cos^{\frac{5}{2}}(c + dx) \sqrt{a + b \sec(c + dx)}} - \frac{(4aAb - 5a^2B + b^2B)}{2b^2(a^2 - b^2) d \cos^{\frac{5}{2}}(c + dx) \sqrt{a + b \sec(c + dx)}} \\
&= \frac{2a(Ab - aB) \sin(c + dx)}{b(a^2 - b^2) d \cos^{\frac{5}{2}}(c + dx) \sqrt{a + b \sec(c + dx)}} - \frac{(4aAb - 5a^2B + b^2B)}{2b^2(a^2 - b^2) d \cos^{\frac{5}{2}}(c + dx) \sqrt{a + b \sec(c + dx)}} \\
&= \frac{(12aAb - 15a^2B - 4b^2B) \sqrt{\frac{b+a \cos(c+dx)}{a+b}} \Pi\left(2; \frac{1}{2}(c + dx) \middle| \frac{2a}{a+b}\right)}{4b^3 d \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}} + \frac{(4Ab - 5aB) \sqrt{\frac{b+a \cos(c+dx)}{a+b}} F\left(\frac{1}{2}(c + dx) \middle| \frac{2a}{a+b}\right)}{4b^2 d \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}} - \frac{(12aAb - 15a^2B - 4b^2B)}{4b^3 d \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}}
\end{aligned}$$

Mathematica [C] time = 35.28, size = 140027, normalized size = 287.53

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^(3/2)), x]

[Out] Result too large to show

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(3/2), x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(b \sec(dx + c) + a)^{\frac{3}{2}} \cos(dx + c)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(3/2), x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(7/2)), x)

maple [C] time = 2.92, size = 2295, normalized size = 4.71

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(3/2), x)

[Out] $\frac{1}{4}d*(-1+\cos(d*x+c))*((b+a*\cos(d*x+c))/\cos(d*x+c))^{1/2}*(1+\cos(d*x+c))^{2*(-2*B*((a-b)/(a+b))^{1/2}*b^3*(1/(1+\cos(d*x+c)))^{1/2}-4*A*((a-b)/(a+b))^{1/2}*\cos(d*x+c)*a*b^2*(1/(1+\cos(d*x+c)))^{1/2}+5*B*((a-b)/(a+b))^{1/2}*\cos(d*x+c)*a*b^2*(1/(1+\cos(d*x+c)))^{1/2}+5*B*((a-b)/(a+b))^{1/2}*\cos(d*x+c)*a*b^2*(1/(1+\cos(d*x+c)))^{1/2}+12*A*((a-b)/(a+b))^{1/2}*\cos(d*x+c)^3*a^2*b*(1/(1+\cos(d*x+c)))^{1/2}+2*B*\cos(d*x+c)^2*((a-b)/(a+b))^{1/2}*b^3*(1/(1+\cos(d*x+c)))^{1/2}-30*B*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{1/2}*EllipticF((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (-a+b)/(a-b))^{1/2}*\cos(d*x+c)^2*\sin(d*x+c)*a^3-4*B*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{1/2}*EllipticF((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (-a+b)/(a-b))^{1/2})*$

$$\begin{aligned} & \cos(d*x+c)^2*\sin(d*x+c)*b^3+4*A*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)} \\ & *EllipticE((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c), -(a+b)/(a-b))^{(1/2)} \\ & *cos(d*x+c)^2*\sin(d*x+c)*b^3+8*B*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)} \\ & *EllipticPi((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c), (a+b)/(a-b), I/((a-b)/(a+b))^{(1/2)}) \\ & *cos(d*x+c)^2*\sin(d*x+c)*b^3+15*B*\cos(d*x+c)^2*((a-b)/(a+b))^{(1/2)}*a^3*(1/(1+\cos(d*x+c)))^{(1/2)} \\ & -15*B*(1/(1+\cos(d*x+c)))^{(1/2)}*((a-b)/(a+b))^{(1/2)}*\cos(d*x+c)^3*a^3-4*A*((a-b)/(a+b))^{(1/2)} \\ & *\cos(d*x+c)*b^3*(1/(1+\cos(d*x+c)))^{(1/2)}-2*B*((a-b)/(a+b))^{(1/2)}*a*b^2*(1/(1+\cos(d*x+c)))^{(1/2)} \\ & +4*A*(1/(1+\cos(d*x+c)))^{(1/2)}*\cos(d*x+c)^2*((a-b)/(a+b))^{(1/2)}*b^3+30*B*\cos(d*x+c)^2 \\ & *((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*EllipticPi((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c), \\ & (a+b)/(a-b), I/((a-b)/(a+b))^{(1/2)})*\sin(d*x+c)*a^3+15*B*\cos(d*x+c)^2*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)} \\ & *EllipticE((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c), -(a+b)/(a-b))^{(1/2)}*\sin(d*x+c)*a^3-24*A*\cos(d*x+c)^2 \\ & *((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*EllipticPi((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c), \\ & (a+b)/(a-b), I/((a-b)/(a+b))^{(1/2)})*\sin(d*x+c)*a^2*b+8*B*\cos(d*x+c)^2*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)} \\ & *EllipticPi((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c), (a+b)/(a-b), I/((a-b)/(a+b))^{(1/2)})*\sin(d*x+c)*a*b^2 \\ & +30*B*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*EllipticPi((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c), \\ & (a+b)/(a-b), I/((a-b)/(a+b))^{(1/2)})*\cos(d*x+c)^2*\sin(d*x+c)*a^2*b-7*B*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)} \\ & *EllipticE((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c), -(a+b)/(a-b))^{(1/2)}*\cos(d*x+c)^2*\sin(d*x+c)*a*b^2-20*B \\ & *((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*EllipticF((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c), -(a+b)/(a-b))^{(1/2)} \\ & *\cos(d*x+c)^2*\sin(d*x+c)*a^2*b-2*B*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*EllipticF((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c), \\ & -(a+b)/(a-b))^{(1/2)}*\cos(d*x+c)^2*\sin(d*x+c)*a*b^2-24*A*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*EllipticPi((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c), \\ & (a+b)/(a-b), I/((a-b)/(a+b))^{(1/2)})*\cos(d*x+c)^2*\sin(d*x+c)*a*b^2-12*A*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)} \\ & *EllipticE((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c), -(a+b)/(a-b))^{(1/2)}*\cos(d*x+c)^2*\sin(d*x+c)*a^2*b+24*A \\ & *((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*EllipticF((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c), -(a+b)/(a-b))^{(1/2)}*\cos(d*x+c)^2 \\ & *\sin(d*x+c)*a^2*b+16*A*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*EllipticF((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c), \\ & -(a+b)/(a-b))^{(1/2)}*\cos(d*x+c)^2*\sin(d*x+c)*a*b^2-12*A*\cos(d*x+c)^2*((a-b)/(a+b))^{(1/2)}*a^2*b*(1/(1+\cos(d*x+c)))^{(1/2)} \\ & -5*B*\cos(d*x+c)^2*((a-b)/(a+b))^{(1/2)}*a*b^2*(1/(1+\cos(d*x+c)))^{(1/2)}+4*A*\cos(d*x+c)^3*((a-b)/(a+b))^{(1/2)}*a*b^2*(1/(1+\cos(d*x+c)))^{(1/2)} \\ & -5*B*\cos(d*x+c)^3*((a-b)/(a+b))^{(1/2)}*a^2*b*(1/(1+\cos(d*x+c)))^{(1/2)}+2*B*\cos(d*x+c)^3*((a-b)/(a+b))^{(1/2)}*a*b^2*(1/(1+\cos(d*x+c)))^{(1/2)} \\ & *((a-b)/(a+b))^{(1/2)}*(1/(1+\cos(d*x+c)))^{(1/2)}/b^3/(b+a*\cos(d*x+c))/\cos(d*x+c)^{(3/2)}/(a-b)/\sin(d*x+c)^3 \end{aligned}$$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\cos(c+dx)^{7/2} \left(a + \frac{b}{\cos(c+dx)}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))^(3/2)),x)

[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))^(3/2)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(7/2)/(a+b*sec(d*x+c))**(3/2),x)

[Out] Timed out

$$3.628 \quad \int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^{\frac{5}{2}}} dx$$

Optimal. Leaf size=588

$$\frac{2b(Ab - aB) \sin(c + dx) \cos^{\frac{3}{2}}(c + dx)}{3ad(a^2 - b^2)(a + b \sec(c + dx))^{\frac{3}{2}}} + \frac{2b(-9a^3B + 12a^2Ab + 5ab^2B - 8Ab^3) \sin(c + dx) \cos^{\frac{3}{2}}(c + dx)}{3a^2d(a^2 - b^2)^2 \sqrt{a + b \sec(c + dx)}} + \frac{2(3a^4 - 12a^2Ab + 8Ab^3) \sin(c + dx) \cos^{\frac{3}{2}}(c + dx)}{3ad(a^2 - b^2)(a + b \sec(c + dx))^{\frac{3}{2}}}$$

[Out] $2/3*b*(A*b-B*a)*\cos(d*x+c)^{(3/2)}*\sin(d*x+c)/a/(a^2-b^2)/d/(a+b*\sec(d*x+c))^{(3/2)}+2/3*b*(12*A*a^2*b-8*A*b^3-9*B*a^3+5*B*a*b^2)*\cos(d*x+c)^{(3/2)}*\sin(d*x+c)/a^2/(a^2-b^2)^2/d/(a+b*\sec(d*x+c))^{(1/2)}-2/15*(17*A*a^4*b+116*A*a^2*b^3-128*A*b^5-5*B*a^5-80*B*a^3*b^2+80*B*a*b^4)*(\cos(1/2*d*x+1/2*c))^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)}*(a/(a+b))^{(1/2)})*((b+a*\cos(d*x+c))/(a+b))^{(1/2)}/a^5/(a^2-b^2)/d/\cos(d*x+c)^{(1/2)}/(a+b*\sec(d*x+c))^{(1/2)}+2/15*(3*A*a^4-71*A*a^2*b^2+48*A*b^4+50*B*a^3*b-30*B*a*b^3)*\cos(d*x+c)^{(3/2)}*\sin(d*x+c)*(a+b*\sec(d*x+c))^{(1/2)}/a^3/(a^2-b^2)^2/d-2/15*(14*A*a^4*b-98*A*a^2*b^3+64*A*b^5-5*B*a^5+65*B*a^3*b^2-40*B*a*b^4)*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}*(a+b*\sec(d*x+c))^{(1/2)}/a^4/(a^2-b^2)^2/d+2/15*(9*A*a^6+55*A*a^4*b^2-212*A*a^2*b^4+128*A*b^6-40*B*a^5*b+140*B*a^3*b^3-80*B*a*b^5)*(\cos(1/2*d*x+1/2*c))^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)}*(a/(a+b))^{(1/2)})*\cos(d*x+c)^{(1/2)}*(a+b*\sec(d*x+c))^{(1/2)}/a^5/(a^2-b^2)^2/d/((b+a*\cos(d*x+c))/(a+b))^{(1/2)}$

Rubi [A] time = 2.07, antiderivative size = 588, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$, Rules used = {2955, 4030, 4100, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2(-71a^2Ab^2 + 3a^4A + 50a^3bB - 30ab^3B + 48Ab^4) \sin(c + dx) \cos^{\frac{3}{2}}(c + dx) \sqrt{a + b \sec(c + dx)}}{15a^3d(a^2 - b^2)^2} + \frac{2b(12a^2Ab - 8Ab^3) \sin(c + dx) \cos^{\frac{3}{2}}(c + dx)}{3ad(a^2 - b^2)(a + b \sec(c + dx))^{\frac{3}{2}}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(\text{Cos}[c + d*x])^{(5/2)}*(A + B*\text{Sec}[c + d*x])]/(a + b*\text{Sec}[c + d*x])^{(5/2)}, x]$

[Out] $(-2*(17*a^4*A*b + 116*a^2*A*b^3 - 128*A*b^5 - 5*a^5*B - 80*a^3*b^2*B + 80*a*b^4*B)*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/(a + b)]*\text{EllipticF}[(c + d*x)/2, (2*a)/(a + b)]/(15*a^5*(a^2 - b^2)*d*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]) + (2*(9*a^6*A + 55*a^4*A*b^2 - 212*a^2*A*b^4 + 128*A*b^6 - 40*a^5*b*B + 140*a^3*b^3*B - 80*a*b^5*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, (2*a)/(a + b)]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]])/(15*a^5*(a^2 - b^2)^2*d*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/(a + b)]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]])$

$$\begin{aligned} & d*x))/(a + b))] + (2*b*(A*b - a*B)*\text{Cos}[c + d*x]^{(3/2)}*\text{Sin}[c + d*x])/(3*a*(\\ & a^2 - b^2)*d*(a + b*\text{Sec}[c + d*x])^{(3/2)}) + (2*b*(12*a^2*A*b - 8*A*b^3 - 9*a \\ & ^3*B + 5*a*b^2*B)*\text{Cos}[c + d*x]^{(3/2)}*\text{Sin}[c + d*x])/(3*a^2*(a^2 - b^2)^2*d*S \\ & \text{qrt}[a + b*\text{Sec}[c + d*x]]) - (2*(14*a^4*A*b - 98*a^2*A*b^3 + 64*A*b^5 - 5*a^5 \\ & *B + 65*a^3*b^2*B - 40*a*b^4*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]] \\ & *\text{Sin}[c + d*x])/(15*a^4*(a^2 - b^2)^2*d) + (2*(3*a^4*A - 71*a^2*A*b^2 + 48*A \\ & *b^4 + 50*a^3*b*B - 30*a*b^3*B)*\text{Cos}[c + d*x]^{(3/2)}*\text{Sqrt}[a + b*\text{Sec}[c + d*x]] \\ & *\text{Sin}[c + d*x])/(15*a^3*(a^2 - b^2)^2*d) \end{aligned}$$

Rule 2653

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*Sqrt[a
+ b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
0] && !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a
+ b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && !GtQ[a + b, 0]
```

Rule 2955

```
Int[((a_) + csc[(e_) + (f_)*(x_)]*(b_))^(m_)*(csc[(e_) + (f_)*(x_)]*
(d_) + (c_))^(n_)*((g_)*sin[(e_) + (f_)*(x_)])^(p_), x_Symbol] := Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3856


```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] := Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_.)], x_Symbol] := Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4030

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] := Simp[(b*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*
(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e
+ f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[A*(a^2*(m + 1) - b^2*(m + n + 1))
+ a*b*B*n - a*(A*b - a*B)*(m + 1)*Csc[e + f*x] + b*(A*b - a*B)*(m + n + 2)*
Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b
- a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && !(ILtQ[m + 1/2, 0] && ILt
Q[n, 0])
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)]), x_Symbol] := Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_.))^(m_), x_Symbol] := Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1)
- a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
&& NeQ[a^2 - b^2, 0] && LtQ[m, -1] && !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4104

```

Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.
))*csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]

```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{\frac{5}{2}}} dx &= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \frac{A+B\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{\frac{5}{2}}} dx \\
&= \frac{2b(Ab-aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a(a^2-b^2)d(a+b\sec(c+dx))^{\frac{3}{2}}} - \frac{(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)})}{3a^2(a^2-b^2)^2 d} \\
&= \frac{2b(Ab-aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a(a^2-b^2)d(a+b\sec(c+dx))^{\frac{3}{2}}} + \frac{2b(12a^2Ab-8Ab^3-9a^3B+9a^2b^2)}{3a^2(a^2-b^2)^2 d} \\
&= \frac{2b(Ab-aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a(a^2-b^2)d(a+b\sec(c+dx))^{\frac{3}{2}}} + \frac{2b(12a^2Ab-8Ab^3-9a^3B+9a^2b^2)}{3a^2(a^2-b^2)^2 d} \\
&= \frac{2b(Ab-aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a(a^2-b^2)d(a+b\sec(c+dx))^{\frac{3}{2}}} + \frac{2b(12a^2Ab-8Ab^3-9a^3B+9a^2b^2)}{3a^2(a^2-b^2)^2 d} \\
&= \frac{2b(Ab-aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a(a^2-b^2)d(a+b\sec(c+dx))^{\frac{3}{2}}} + \frac{2b(12a^2Ab-8Ab^3-9a^3B+9a^2b^2)}{3a^2(a^2-b^2)^2 d} \\
&= \frac{2b(Ab-aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a(a^2-b^2)d(a+b\sec(c+dx))^{\frac{3}{2}}} + \frac{2b(12a^2Ab-8Ab^3-9a^3B+9a^2b^2)}{3a^2(a^2-b^2)^2 d} \\
&= \frac{2b(Ab-aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a(a^2-b^2)d(a+b\sec(c+dx))^{\frac{3}{2}}} + \frac{2b(12a^2Ab-8Ab^3-9a^3B+9a^2b^2)}{3a^2(a^2-b^2)^2 d} \\
&= \frac{2b(Ab-aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a(a^2-b^2)d(a+b\sec(c+dx))^{\frac{3}{2}}} + \frac{2b(12a^2Ab-8Ab^3-9a^3B+9a^2b^2)}{3a^2(a^2-b^2)^2 d} \\
&= \frac{2(17a^4Ab+116a^2Ab^3-128Ab^5-5a^5B-80a^3b^2B+80ab^4B)\sqrt{\frac{b+}{a+b\sec(c+dx)}}}{15a^5(a^2-b^2)d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}
\end{aligned}$$

Mathematica [C] time = 25.41, size = 4179, normalized size = 7.11

Result too large to show

Warning: Unable to verify antiderivative.

$$\begin{aligned}
& b^3 B - 80 a b^5 B) (b + a \cos[c + dx]) (\sec[(c + dx)/2]^2)^{3/2} \tan[(c + dx)/2] / (a^4 (a^2 - b^2)^2 (b + a \cos[c + dx])^{3/2}) + (\sqrt{\cos[c + dx]}) (\cos[(c + dx)/2]^2 \sec[c + dx])^{3/2} \sin[c + dx] ((-I)(a + b) (9 a^6 A + 55 a^4 A b^2 - 212 a^2 A b^4 + 128 A b^6 - 40 a^5 b B + 140 a^3 b^3 B - 80 a b^5 B) \operatorname{EllipticE}[I \operatorname{ArcSinh}[\tan[(c + dx)/2]], (-a + b)/(a + b)] \sec[(c + dx)/2]^2 \sqrt{((b + a \cos[c + dx]) \sec[(c + dx)/2]^2)/(a + b)} + I a (a + b) (128 A b^5 - 16 a b^4 (6 A + 5 B) + a^5 (9 A + 5 B) + 8 a^3 b^2 (9 A + 10 B) + 4 a^2 b^3 (-29 A + 15 B) - a^4 b (17 A + 45 B)) \operatorname{EllipticF}[I \operatorname{ArcSinh}[\tan[(c + dx)/2]], (-a + b)/(a + b)] \sec[(c + dx)/2]^2 \sqrt{((b + a \cos[c + dx]) \sec[(c + dx)/2]^2)/(a + b)} - (9 a^6 A + 55 a^4 A b^2 - 212 a^2 A b^4 + 128 A b^6 - 40 a^5 b B + 140 a^3 b^3 B - 80 a b^5 B) (b + a \cos[c + dx]) (\sec[(c + dx)/2]^2)^{3/2} \tan[(c + dx)/2] / (5 a^5 (a^2 - b^2)^2 \sqrt{b + a \cos[c + dx]}) - (2 \cos[c + dx])^{3/2} (\cos[(c + dx)/2]^2 \sec[c + dx])^{3/2} (-1/2 ((9 a^6 A + 55 a^4 A b^2 - 212 a^2 A b^4 + 128 A b^6 - 40 a^5 b B + 140 a^3 b^3 B - 80 a b^5 B) (b + a \cos[c + dx]) (\sec[(c + dx)/2]^2)^{5/2}) - I (a + b) (9 a^6 A + 55 a^4 A b^2 - 212 a^2 A b^4 + 128 A b^6 - 40 a^5 b B + 140 a^3 b^3 B - 80 a b^5 B) \operatorname{EllipticE}[I \operatorname{ArcSinh}[\tan[(c + dx)/2]], (-a + b)/(a + b)] \sec[(c + dx)/2]^2 \sqrt{((b + a \cos[c + dx]) \sec[(c + dx)/2]^2)/(a + b)} \tan[(c + dx)/2] + I a (a + b) (128 A b^5 - 16 a b^4 (6 A + 5 B) + a^5 (9 A + 5 B) + 8 a^3 b^2 (9 A + 10 B) + 4 a^2 b^3 (-29 A + 15 B) - a^4 b (17 A + 45 B)) \operatorname{EllipticF}[I \operatorname{ArcSinh}[\tan[(c + dx)/2]], (-a + b)/(a + b)] \sec[(c + dx)/2]^2 \sqrt{((b + a \cos[c + dx]) \sec[(c + dx)/2]^2)/(a + b)} \tan[(c + dx)/2] + a (9 a^6 A + 55 a^4 A b^2 - 212 a^2 A b^4 + 128 A b^6 - 40 a^5 b B + 140 a^3 b^3 B - 80 a b^5 B) (\sec[(c + dx)/2]^2)^{3/2} \sin[c + dx] \tan[(c + dx)/2] - (3 (9 a^6 A + 55 a^4 A b^2 - 212 a^2 A b^4 + 128 A b^6 - 40 a^5 b B + 140 a^3 b^3 B - 80 a b^5 B) (b + a \cos[c + dx]) (\sec[(c + dx)/2]^2)^{3/2} \tan[(c + dx)/2]^2 / 2 - (I/2) (a + b) (9 a^6 A + 55 a^4 A b^2 - 212 a^2 A b^4 + 128 A b^6 - 40 a^5 b B + 140 a^3 b^3 B - 80 a b^5 B) \operatorname{EllipticE}[I \operatorname{ArcSinh}[\tan[(c + dx)/2]], (-a + b)/(a + b)] \sec[(c + dx)/2]^2 ((a \sec[(c + dx)/2]^2 \sin[c + dx]) / (a + b)) + ((b + a \cos[c + dx]) \sec[(c + dx)/2]^2 \tan[(c + dx)/2]) / (a + b)) / \sqrt{((b + a \cos[c + dx]) \sec[(c + dx)/2]^2)/(a + b)} + ((I/2) a (a + b) (128 A b^5 - 16 a b^4 (6 A + 5 B) + a^5 (9 A + 5 B) + 8 a^3 b^2 (9 A + 10 B) + 4 a^2 b^3 (-29 A + 15 B) - a^4 b (17 A + 45 B)) \operatorname{EllipticF}[I \operatorname{ArcSinh}[\tan[(c + dx)/2]], (-a + b)/(a + b)] \sec[(c + dx)/2]^2 ((a \sec[(c + dx)/2]^2 \sin[c + dx]) / (a + b)) + ((b + a \cos[c + dx]) \sec[(c + dx)/2]^2 \tan[(c + dx)/2]) / (a + b)) / \sqrt{((b + a \cos[c + dx]) \sec[(c + dx)/2]^2)/(a + b)} - (a (a + b) (128 A b^5 - 16 a b^4 (6 A + 5 B) + a^5 (9 A + 5 B) + 8 a^3 b^2 (9 A + 10 B) + 4 a^2 b^3 (-29 A + 15 B) - a^4 b (17 A + 45 B)) \sec[(c + dx)/2]^4 \sqrt{((b + a \cos[c + dx]) \sec[(c + dx)/2]^2)/(a + b))} / (2 \sqrt{1 + \tan[(c + dx)/2]^2} \sqrt{1 + ((-a + b) \tan[(c + dx)/2]^2)/(a + b)}) + ((a + b) (9 a^6 A + 55 a^4 A b^2 - 212 a^2 A b^4 + 128 A b^6 - 40 a^5 b B + 140 a^3 b^3 B - 80 a b^5 B) \sec[(c + dx)/2]^4 \sqrt{((b + a \cos[c + dx]) \sec[(c + dx)/2]^2)/(a + b)} \sqrt{1 + ((-a + b) \tan[(c + dx)/2]^2)/(a + b)}) / (2 \sqrt{1 + \tan[(c + dx)/2]^2})) / (15 a^5 (a^2 - b^2)^2 \sqrt{b +
\end{aligned}$$

```
a*cos[c + d*x])) - (Cos[c + d*x]^(3/2)*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]
]*((-I)*(a + b)*(9*a^6*A + 55*a^4*A*b^2 - 212*a^2*A*b^4 + 128*A*b^6 - 40*a^
5*b*B + 140*a^3*b^3*B - 80*a*b^5*B)*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]],
(-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*cos[c + d*x])*Sec[(c + d
*x)/2]^2)/(a + b)] + I*a*(a + b)*(128*A*b^5 - 16*a*b^4*(6*A + 5*B) + a^5*(9*
A + 5*B) + 8*a^3*b^2*(9*A + 10*B) + 4*a^2*b^3*(-29*A + 15*B) - a^4*b*(17*A
+ 45*B))*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c +
d*x)/2]^2*Sqrt[((b + a*cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - (9*a^6*
A + 55*a^4*A*b^2 - 212*a^2*A*b^4 + 128*A*b^6 - 40*a^5*b*B + 140*a^3*b^3*B -
80*a*b^5*B)*(b + a*cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/
2))*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^2
*Sec[c + d*x]*Tan[c + d*x]))/(5*a^5*(a^2 - b^2)^2*Sqrt[b + a*cos[c + d*x]]
))
```

fricas [F] time = 0.98, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{(B \cos(dx + c)^2 \sec(dx + c) + A \cos(dx + c)^2) \sqrt{b \sec(dx + c) + a} \sqrt{\cos(dx + c)}}{b^3 \sec(dx + c)^3 + 3ab^2 \sec(dx + c)^2 + 3a^2b \sec(dx + c) + a^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algo
rithm="fricas")
```

```
[Out] integral((B*cos(d*x + c)^2*sec(d*x + c) + A*cos(d*x + c)^2)*sqrt(b*sec(d*x
+ c) + a)*sqrt(cos(d*x + c))/(b^3*sec(d*x + c)^3 + 3*a*b^2*sec(d*x + c)^2 +
3*a^2*b*sec(d*x + c) + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \cos(dx + c)^{\frac{5}{2}}}{(b \sec(dx + c) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algo
rithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(5/2)/(b*sec(d*x + c) + a)^(5/2
), x)
```

maple [B] time = 2.65, size = 5675, normalized size = 9.65

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x)`

[Out] result too large to display

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")`

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+dx)^{5/2} \left(A + \frac{B}{\cos(c+dx)} \right)}{\left(a + \frac{b}{\cos(c+dx)} \right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((cos(c+d*x)^(5/2)*(A+B/cos(c+d*x)))/(a+b/cos(c+d*x))^(5/2),x)`

[Out] `int((cos(c+d*x)^(5/2)*(A+B/cos(c+d*x)))/(a+b/cos(c+d*x))^(5/2),x)`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)**(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(5/2),x)`

[Out] Timed out

$$3.629 \quad \int \frac{\cos^3(c+dx)(A+B \sec(c+dx))}{(a+b \sec(c+dx))^{5/2}} dx$$

Optimal. Leaf size=472

$$\frac{2b(Ab - aB) \sin(c + dx) \sqrt{\cos(c + dx)}}{3ad(a^2 - b^2)(a + b \sec(c + dx))^{3/2}} + \frac{2b(-7a^3B + 10a^2Ab + 3ab^2B - 6Ab^3) \sin(c + dx) \sqrt{\cos(c + dx)}}{3a^2d(a^2 - b^2)^2 \sqrt{a + b \sec(c + dx)}} + \frac{2(a^4A - 13a^2Ab^2 + a^4A + 8a^3bB - 4ab^3B + 8Ab^4) \sin(c + dx) \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}}{3a^3d(a^2 - b^2)^2} + \frac{2b(10a^2Ab - 7a^3B)}{3a^2d(a^2 - b^2)}$$

[Out] $\frac{2}{3} b (A b - B a) \sin(d x + c) \cos(d x + c)^{(1/2)} / a / (a^2 - b^2) / d / (a + b \sec(d x + c))^{(3/2)} + \frac{2}{3} b (10 A a^2 b - 6 A b^3 - 7 B a^3 + 3 B a b^2) \sin(d x + c) \cos(d x + c)^{(1/2)} / a^2 / (a^2 - b^2)^2 / d / (a + b \sec(d x + c))^{(1/2)} + \frac{2}{3} (A a^4 + 16 A a^2 b^2 - 16 A b^4 - 9 B a^3 b + 8 B a b^3) (\cos(1/2 d x + 1/2 c))^2 / \cos(1/2 d x + 1/2 c) * \text{EllipticF}(\sin(1/2 d x + 1/2 c), 2^{(1/2)} (a / (a + b))^{(1/2)}) * ((b + a \cos(d x + c)) / (a + b))^{(1/2)} / a^4 / (a^2 - b^2) / d / \cos(d x + c)^{(1/2)} / (a + b \sec(d x + c))^{(1/2)} + \frac{2}{3} (A a^4 - 13 A a^2 b^2 + 8 A b^4 + 8 B a^3 b - 4 B a b^3) \sin(d x + c) \cos(d x + c)^{(1/2)} (a + b \sec(d x + c))^{(1/2)} / a^3 / (a^2 - b^2)^2 / d - \frac{2}{3} (8 A a^4 b - 28 A a^2 b^3 + 16 A b^5 - 3 B a^5 + 15 B a^3 b^2 - 8 B a b^4) (\cos(1/2 d x + 1/2 c))^2 / \cos(1/2 d x + 1/2 c) * \text{EllipticE}(\sin(1/2 d x + 1/2 c), 2^{(1/2)} (a / (a + b))^{(1/2)}) * \cos(d x + c)^{(1/2)} (a + b \sec(d x + c))^{(1/2)} / a^4 / (a^2 - b^2)^2 / d / ((b + a \cos(d x + c)) / (a + b))^{(1/2)}$

Rubi [A] time = 1.54, antiderivative size = 472, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$, Rules used = {2955, 4030, 4100, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2(-13a^2Ab^2 + a^4A + 8a^3bB - 4ab^3B + 8Ab^4) \sin(c + dx) \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}}{3a^3d(a^2 - b^2)^2} + \frac{2b(10a^2Ab - 7a^3B)}{3a^2d(a^2 - b^2)}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(5/2),x]

[Out] $(2(a^4A + 16a^2Ab^2 - 16Ab^4 - 9a^3bB + 8ab^3B) \sqrt{(b + a \cos(c + dx)) / (a + b)} * \text{EllipticF}((c + dx) / 2, (2a) / (a + b))) / (3a^4(a^2 - b^2) d \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}) - (2(8a^4Ab - 28a^2Ab^3 + 16Ab^5 - 3a^5B + 15a^3b^2B - 8ab^4B) \sqrt{\cos(c + dx)} * \text{EllipticE}((c + dx) / 2, (2a) / (a + b)) \sqrt{a + b \sec(c + dx)}) / (3a^4(a^2 - b^2)^2 d \sqrt{(b + a \cos(c + dx)) / (a + b)}) + (2b(Ab - aB) \sqrt{\cos(c + dx)} * \sin(c + dx)) / (3a(a^2 - b^2) d (a + b \sec(c + dx))^{(3/2)}) + (2b(10a^2Ab - 6Ab^3 - 7a^3B + 3ab^2B) \sqrt{\cos(c + dx)} * \sin(c + dx)) / (3a^2(a^2 - b^2)^2 d \sqrt{a + b \sec(c + dx)}) + (2(a^4A - 13a^2Ab^2 + a^4A + 8a^3bB - 4ab^3B + 8Ab^4) \sin(c + dx) \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}) / (3a^3d(a^2 - b^2)^2) + \frac{2b(10a^2Ab - 7a^3B)}{3a^2d(a^2 - b^2)}$

$2Ab^2 + 8A^2b^4 + 8a^3b^3B - 4a^2b^3B) \sqrt{\cos[c + dx]} \sqrt{a + b \sin[c + dx]} / (3a^3(a^2 - b^2)^2d)$

Rule 2653

$\text{Int}[\sqrt{(a_.) + (b_.) \sin[(c_.) + (d_.) (x_)]}], x_Symbol] \rightarrow \text{Simp}[(2 \sqrt{a + b} \text{EllipticE}[(1(c - \pi/2 + dx))/2, (2b)/(a + b)]) / d, x] /; \text{FreeQ}\{a, b, c, d\}, x] \ \&\& \ \text{NeQ}[a^2 - b^2, 0] \ \&\& \ \text{GtQ}[a + b, 0]$

Rule 2655

$\text{Int}[\sqrt{(a_.) + (b_.) \sin[(c_.) + (d_.) (x_)]}], x_Symbol] \rightarrow \text{Dist}[\sqrt{a + b \sin[c + dx]} / \sqrt{(a + b \sin[c + dx]) / (a + b)}, \text{Int}[\sqrt{a / (a + b) + (b \sin[c + dx]) / (a + b)}, x], x] /; \text{FreeQ}\{a, b, c, d\}, x] \ \&\& \ \text{NeQ}[a^2 - b^2, 0] \ \&\& \ \text{!GtQ}[a + b, 0]$

Rule 2661

$\text{Int}[1/\sqrt{(a_.) + (b_.) \sin[(c_.) + (d_.) (x_)]}], x_Symbol] \rightarrow \text{Simp}[(2 \text{EllipticF}[(1(c - \pi/2 + dx))/2, (2b)/(a + b)]) / (d \sqrt{a + b}), x] /; \text{FreeQ}\{a, b, c, d\}, x] \ \&\& \ \text{NeQ}[a^2 - b^2, 0] \ \&\& \ \text{GtQ}[a + b, 0]$

Rule 2663

$\text{Int}[1/\sqrt{(a_.) + (b_.) \sin[(c_.) + (d_.) (x_)]}], x_Symbol] \rightarrow \text{Dist}[\sqrt{(a + b \sin[c + dx]) / (a + b)} / \sqrt{a + b \sin[c + dx]}, \text{Int}[1/\sqrt{a / (a + b) + (b \sin[c + dx]) / (a + b)}, x], x] /; \text{FreeQ}\{a, b, c, d\}, x] \ \&\& \ \text{NeQ}[a^2 - b^2, 0] \ \&\& \ \text{!GtQ}[a + b, 0]$

Rule 2955

$\text{Int}[(a_.) + \csc[(e_.) + (f_.) (x_)] (b_.)]^{m_.)} (\csc[(e_.) + (f_.) (x_)] (d_.) + (c_.)^{n_.)} (g_.) \sin[(e_.) + (f_.) (x_)]^{p_.)}, x_Symbol] \rightarrow \text{Dist}[(g \csc[e + fx])^p (g \sin[e + fx])^p, \text{Int}[(a + b \csc[e + fx])^m (c + d \csc[e + fx])^n / (g \csc[e + fx])^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, m, n, p\}, x] \ \&\& \ \text{NeQ}[b \cdot c - a \cdot d, 0] \ \&\& \ \text{!IntegerQ}[p] \ \&\& \ \text{!(IntegerQ}[m] \ \&\& \ \text{IntegerQ}[n])]$

Rule 3856

$\text{Int}[\sqrt{\csc[(e_.) + (f_.) (x_)] (b_.) + (a_.)} / \sqrt{\csc[(e_.) + (f_.) (x_)] (d_.)}], x_Symbol] \rightarrow \text{Dist}[\sqrt{a + b \csc[e + fx]} / (\sqrt{d \csc[e + fx]} \sqrt{b + a \sin[e + fx]}), \text{Int}[\sqrt{b + a \sin[e + fx]}, x], x] /; \text{FreeQ}\{a, b, d, e, f\}, x] \ \&\& \ \text{NeQ}[a^2 - b^2, 0]$

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.)
+ (a_.)], x_Symbol] := Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4030

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (
a_.))^m*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := Simp[(b*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*
(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e
+ f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[A*(a^2*(m + 1) - b^2*(m + n + 1))
+ a*b*B*n - a*(A*b - a*B)*(m + 1)*Csc[e + f*x] + b*(A*b - a*B)*(m + n + 2)*
Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b
- a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && !(ILtQ[m + 1/2, 0] && ILt
Q[n, 0])
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.))/(Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]), x_Symbol] := Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.
))^m*(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a
.))^m, x_Symbol] := Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1)
- a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
&& NeQ[a^2 - b^2, 0] && LtQ[m, -1] && !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.
))^m*(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a
.))^m, x_Symbol] := Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
```

$(d \cdot \text{Csc}[e + f \cdot x])^{(n + 1)} \cdot \text{Simp}[a \cdot B \cdot n - A \cdot b \cdot (m + n + 1) + a \cdot (A + A \cdot n + C \cdot n) \cdot \text{Csc}[e + f \cdot x] + A \cdot b \cdot (m + n + 2) \cdot \text{Csc}[e + f \cdot x]^2, x], x] / ; \text{FreeQ}[\{a, b, d, e, f, A, B, C, m\}, x] \ \&\& \ \text{NeQ}[a^2 - b^2, 0] \ \&\& \ \text{LeQ}[n, -1]$

Rubi steps

$$\begin{aligned}
 \int \frac{\cos^3(c + dx)(A + B \sec(c + dx))}{(a + b \sec(c + dx))^{5/2}} dx &= \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{A + B \sec(c + dx)}{\sec^3(c + dx)(a + b \sec(c + dx))^{5/2}} dx \\
 &= \frac{2b(Ab - aB)\sqrt{\cos(c + dx)} \sin(c + dx)}{3a(a^2 - b^2)d(a + b \sec(c + dx))^{3/2}} - \frac{(2\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)})}{3a^2(a^2 - b^2)^2} \\
 &= \frac{2b(Ab - aB)\sqrt{\cos(c + dx)} \sin(c + dx)}{3a(a^2 - b^2)d(a + b \sec(c + dx))^{3/2}} + \frac{2b(10a^2Ab - 6Ab^3 - 7a^3B)}{3a^2(a^2 - b^2)^2} \\
 &= \frac{2b(Ab - aB)\sqrt{\cos(c + dx)} \sin(c + dx)}{3a(a^2 - b^2)d(a + b \sec(c + dx))^{3/2}} + \frac{2b(10a^2Ab - 6Ab^3 - 7a^3B)}{3a^2(a^2 - b^2)^2} \\
 &= \frac{2b(Ab - aB)\sqrt{\cos(c + dx)} \sin(c + dx)}{3a(a^2 - b^2)d(a + b \sec(c + dx))^{3/2}} + \frac{2b(10a^2Ab - 6Ab^3 - 7a^3B)}{3a^2(a^2 - b^2)^2} \\
 &= \frac{2b(Ab - aB)\sqrt{\cos(c + dx)} \sin(c + dx)}{3a(a^2 - b^2)d(a + b \sec(c + dx))^{3/2}} + \frac{2b(10a^2Ab - 6Ab^3 - 7a^3B)}{3a^2(a^2 - b^2)^2} \\
 &= \frac{2b(Ab - aB)\sqrt{\cos(c + dx)} \sin(c + dx)}{3a(a^2 - b^2)d(a + b \sec(c + dx))^{3/2}} + \frac{2b(10a^2Ab - 6Ab^3 - 7a^3B)}{3a^2(a^2 - b^2)^2} \\
 &= \frac{2(a^4A + 16a^2Ab^2 - 16Ab^4 - 9a^3bB + 8ab^3B) \sqrt{\frac{b+a \cos(c+dx)}{a+b}} F\left(\frac{1}{2}(c + dx)\right)}{3a^4(a^2 - b^2)d\sqrt{\cos(c + dx)}\sqrt{a + b \sec(c + dx)}}
 \end{aligned}$$

Mathematica [C] time = 24.26, size = 3758, normalized size = 7.96

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(5/2), x]

[Out] ((b + a*cos[c + d*x])^3*((2*A*sin[c + d*x])/(3*a^3) + (2*(A*b^4*sin[c + d*x] - a*b^3*B*sin[c + d*x]))/(3*a^3*(a^2 - b^2)*(b + a*cos[c + d*x])^2) + (2*(-12*a^2*A*b^3*sin[c + d*x] + 8*A*b^5*sin[c + d*x] + 9*a^3*b^2*B*sin[c + d*x] - 5*a*b^4*B*sin[c + d*x]))/(3*a^3*(a^2 - b^2)^2*(b + a*cos[c + d*x])))/(d*cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2)) - (2*cos[c + d*x]^(3/2)*(b + a*cos[c + d*x])^2*((-8*a*A*b*Sqrt[Cos[c + d*x]])/(3*(a^2 - b^2)^2*Sqrt[b + a*cos[c + d*x]])*Sqrt[Sec[c + d*x]]) + (28*A*b^3*Sqrt[Cos[c + d*x]])/(3*a*(a^2 - b^2)^2*Sqrt[b + a*cos[c + d*x]])*Sqrt[Sec[c + d*x]]) - (16*A*b^5*Sqrt[Cos[c + d*x]])/(3*a^3*(a^2 - b^2)^2*Sqrt[b + a*cos[c + d*x]])*Sqrt[Sec[c + d*x]]) + (a^2*B*Sqrt[Cos[c + d*x]])/((a^2 - b^2)^2*Sqrt[b + a*cos[c + d*x]])*Sqrt[Sec[c + d*x]]) - (5*b^2*B*Sqrt[Cos[c + d*x]])/((a^2 - b^2)^2*Sqrt[b + a*cos[c + d*x]])*Sqrt[Sec[c + d*x]]) + (8*b^4*B*Sqrt[Cos[c + d*x]])/(3*a^2*(a^2 - b^2)^2*Sqrt[b + a*cos[c + d*x]])*Sqrt[Sec[c + d*x]]) + (a^2*A*Sqrt[Cos[c + d*x]])*Sqrt[Sec[c + d*x]])/(3*(a^2 - b^2)^2*Sqrt[b + a*cos[c + d*x]]) + (7*A*b^2*Sqrt[Cos[c + d*x]])*Sqrt[Sec[c + d*x]])/(3*(a^2 - b^2)^2*Sqrt[b + a*cos[c + d*x]]) - (4*A*b^4*Sqrt[Cos[c + d*x]])*Sqrt[Sec[c + d*x]])/(3*a^2*(a^2 - b^2)^2*Sqrt[b + a*cos[c + d*x]]) - (2*a*b*B*Sqrt[Cos[c + d*x]])*Sqrt[Sec[c + d*x]])/((a^2 - b^2)^2*Sqrt[b + a*cos[c + d*x]]) + (2*b^3*B*Sqrt[Cos[c + d*x]])*Sqrt[Sec[c + d*x]])/(3*a*(a^2 - b^2)^2*Sqrt[b + a*cos[c + d*x]])*Sec[c + d*x]^(5/2)*(Cos[(c + d*x)/2]^2*Sec[c + d*x]^(3/2)*((-I)*(a + b))*(-8*a^4*A*b + 28*a^2*A*b^3 - 16*A*b^5 + 3*a^5*B - 15*a^3*b^2*B + 8*a*b^4*B)*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + I*a*(a + b)*(-16*A*b^4 + 2*a^2*b^2*(8*A - 3*B) - 9*a^3*b*(A + B) + 4*a*b^3*(3*A + 2*B) + a^4*(A + 3*B))*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - (-8*a^4*A*b + 28*a^2*A*b^3 - 16*A*b^5 + 3*a^5*B - 15*a^3*b^2*B + 8*a*b^4*B)*(b + a*cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/(3*a^4*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x])^(5/2)*(-1/3*(Cos[c + d*x]^(3/2)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*Sin[c + d*x]*((-I)*(a + b))*(-8*a^4*A*b + 28*a^2*A*b^3 - 16*A*b^5 + 3*a^5*B - 15*a^3*b^2*B + 8*a*b^4*B)*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + I*a*(a + b)*(-16*A*b^4 + 2*a^2*b^2*(8*A - 3*B) - 9*a^3*b*(A + B) + 4*a*b^3*(3*A + 2*B) + a^4*(A + 3*B))*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - (-8*a^4*A*b + 28*a^2*A*b^3 - 16*A*b^5 + 3*a^5*B - 15*a^3*b^2*B + 8*a*b^4*B)*(b + a*cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/(a^3*(a^2 - b^2)^2*(b + a*cos[c + d*x])^(3/2)) + (Sqrt[Cos[c + d*x]])*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*Sin[c + d*x]*((-I)*(a + b))*(-8*a^4*A*b + 28*a^2*A*b^3 - 16*A*b^5 + 3*a^5*B - 15*a^3*b^2*B + 8*a*b^4*B)*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]

$$\begin{aligned}
& d*x)/2]^2)/(a + b)] + I*a*(a + b)*(-16*A*b^4 + 2*a^2*b^2*(8*A - 3*B) - 9*a^3*b*(A + B) + 4*a*b^3*(3*A + 2*B) + a^4*(A + 3*B))*EllipticF[I*ArcSinh[Tan \\
& [(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d \\
& *x])*Sec[(c + d*x)/2]^2)/(a + b)] - (-8*a^4*A*b + 28*a^2*A*b^3 - 16*A*b^5 + \\
& 3*a^5*B - 15*a^3*b^2*B + 8*a*b^4*B)*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2] \\
& ^2)^(3/2)*Tan[(c + d*x)/2))/(a^4*(a^2 - b^2)^2*Sqrt[b + a*Cos[c + d*x]]) - \\
& (2*Cos[c + d*x]^(3/2)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*(-1/2*((-8*a^4*A*b + 28*a^2*A*b^3 - 16*A*b^5 + 3*a^5*B - 15*a^3*b^2*B + 8*a*b^4*B)*(b + \\
& a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(5/2)) - I*(a + b)*(-8*a^4*A*b + 28*a^2*A*b^3 - 16*A*b^5 + 3*a^5*B - 15*a^3*b^2*B + 8*a*b^4*B))*EllipticE[I*ArcSi \\
& nh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos \\
& [c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Tan[(c + d*x)/2] + I*a*(a + b)*(-16 \\
& *A*b^4 + 2*a^2*b^2*(8*A - 3*B) - 9*a^3*b*(A + B) + 4*a*b^3*(3*A + 2*B) + a^4*(A + 3*B))*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c \\
& + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Tan[(c \\
& + d*x)/2] + a*(-8*a^4*A*b + 28*a^2*A*b^3 - 16*A*b^5 + 3*a^5*B - 15*a^3*b^2*B + 8*a*b^4*B)*(Sec[(c + d*x)/2]^2)^(3/2)*Sin[c + d*x]*Tan[(c + d*x)/2] - \\
& (3*(-8*a^4*A*b + 28*a^2*A*b^3 - 16*A*b^5 + 3*a^5*B - 15*a^3*b^2*B + 8*a*b^4*B)*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]^2)/2 \\
& - ((I/2)*(a + b)*(-8*a^4*A*b + 28*a^2*A*b^3 - 16*A*b^5 + 3*a^5*B - 15*a^3*b^2*B + 8*a*b^4*B))*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]* \\
& Sec[(c + d*x)/2]^2*(-((a*Sec[(c + d*x)/2]^2*Ssin[c + d*x])/(a + b)) + ((b + \\
& a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(a + b))/Sqrt[((b + a \\
& *Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + ((I/2)*a*(a + b)*(-16*A*b^4 + \\
& 2*a^2*b^2*(8*A - 3*B) - 9*a^3*b*(A + B) + 4*a*b^3*(3*A + 2*B) + a^4*(A + 3 \\
& *B))*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x) \\
& /2]^2*(-((a*Sec[(c + d*x)/2]^2*Ssin[c + d*x])/(a + b)) + ((b + a*Cos[c + d*x] \\
&])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(a + b))/Sqrt[((b + a*Cos[c + d*x] \\
&)*Sec[(c + d*x)/2]^2)/(a + b)] - (a*(a + b)*(-16*A*b^4 + 2*a^2*b^2*(8*A - 3 \\
& *B) - 9*a^3*b*(A + B) + 4*a*b^3*(3*A + 2*B) + a^4*(A + 3*B))*Sec[(c + d*x)/ \\
& 2]^4*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)))/(2*Sqrt[1 + T \\
& an[(c + d*x)/2]^2]*Sqrt[1 + ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)]) + ((a + \\
& b)*(-8*a^4*A*b + 28*a^2*A*b^3 - 16*A*b^5 + 3*a^5*B - 15*a^3*b^2*B + 8*a*b^4 \\
& *B))*Sec[(c + d*x)/2]^4*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + \\
& b)]*Sqrt[1 + ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)))/(2*Sqrt[1 + Tan[(c + \\
& d*x)/2]^2]))/(3*a^4*(a^2 - b^2)^2*Sqrt[b + a*Cos[c + d*x]]) - (Cos[c + d*x] \\
&]^(3/2)*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*((-I)*(a + b)*(-8*a^4*A*b + 2 \\
& 8*a^2*A*b^3 - 16*A*b^5 + 3*a^5*B - 15*a^3*b^2*B + 8*a*b^4*B))*EllipticE[I*Ar \\
& cSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a* \\
& Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + I*a*(a + b)*(-16*A*b^4 + 2*a^2 \\
& *b^2*(8*A - 3*B) - 9*a^3*b*(A + B) + 4*a*b^3*(3*A + 2*B) + a^4*(A + 3*B))*E \\
& llipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2 \\
& Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - (-8*a^4*A*b + 28 \\
& a^2*A*b^3 - 16*A*b^5 + 3*a^5*B - 15*a^3*b^2*B + 8*a*b^4*B)*(b + a*Cos[c + d \\
& *x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2])*(-(Cos[(c + d*x)/2]*Sec[c
\end{aligned}$$

+ d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(a^4*(a^2 - b^2)^2*Sqrt[b + a*Cos[c + d*x]]))

fricas [F] time = 1.39, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B \cos(dx + c) \sec(dx + c) + A \cos(dx + c))\sqrt{b \sec(dx + c) + a} \sqrt{\cos(dx + c)}}{b^3 \sec(dx + c)^3 + 3ab^2 \sec(dx + c)^2 + 3a^2b \sec(dx + c) + a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")

[Out] integral((B*cos(d*x + c)*sec(d*x + c) + A*cos(d*x + c))*sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c))/(b^3*sec(d*x + c)^3 + 3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A) \cos(dx + c)^{\frac{3}{2}}}{(b \sec(dx + c) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^(5/2), x)

maple [B] time = 3.24, size = 4480, normalized size = 9.49

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x)

[Out] -2/3/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)*(-1+cos(d*x+c))* (1+cos(d*x+c))^2*(-18*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^4*b^2*(1/(1+cos(d*x+c)))^(1/2)-14*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^4*b^2*(1/(1+cos(d*x+c)))^(1/2)-3*B*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*a^6+3*B*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c)))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^6+A*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^4*b^2*(1/(1+cos(d

$$\begin{aligned}
&)^{1/2} a^3 b^3 (1/(1+\cos(dx+c)))^{1/2} - A \sin(dx+c) \operatorname{EllipticF}((-1+\cos(dx+c)) * ((a-b)/(a+b))^{1/2} / \sin(dx+c), (-a+b)/(a-b))^{1/2}) * ((b+a \cos(dx+c)) / (1+\cos(dx+c))) / (a+b))^{1/2} * a^5 b^9 A \sin(dx+c) \operatorname{EllipticF}((-1+\cos(dx+c)) * ((a-b)/(a+b))^{1/2} / \sin(dx+c), (-a+b)/(a-b))^{1/2}) * ((b+a \cos(dx+c)) / (1+\cos(dx+c))) / (a+b))^{1/2} * a^4 b^2 - 16 A \sin(dx+c) \operatorname{EllipticF}((-1+\cos(dx+c)) * ((a-b)/(a+b))^{1/2} / \sin(dx+c), (-a+b)/(a-b))^{1/2}) * ((b+a \cos(dx+c)) / (1+\cos(dx+c))) / (a+b))^{1/2} * a^3 b^3 + 12 A \sin(dx+c) \operatorname{EllipticF}((-1+\cos(dx+c)) * ((a-b)/(a+b))^{1/2} / \sin(dx+c), (-a+b)/(a-b))^{1/2}) * ((b+a \cos(dx+c)) / (1+\cos(dx+c))) / (a+b))^{1/2} * a^2 b^4 + 16 A \sin(dx+c) \operatorname{EllipticF}((-1+\cos(dx+c)) * ((a-b)/(a+b))^{1/2} / \sin(dx+c), (-a+b)/(a-b))^{1/2}) * ((b+a \cos(dx+c)) / (1+\cos(dx+c))) / (a+b))^{1/2} * a b^5 + 8 A \sin(dx+c) * ((b+a \cos(dx+c)) / (1+\cos(dx+c))) / (a+b))^{1/2} * \operatorname{EllipticE}((-1+\cos(dx+c)) * ((a-b)/(a+b))^{1/2} / \sin(dx+c), (-a+b)/(a-b))^{1/2}) * a^4 b^2 - 28 A \sin(dx+c) * ((b+a \cos(dx+c)) / (1+\cos(dx+c))) / (a+b))^{1/2} * \operatorname{EllipticE}((-1+\cos(dx+c)) * ((a-b)/(a+b))^{1/2} / \sin(dx+c), (-a+b)/(a-b))^{1/2}) * a^2 b^4 + 3 B \sin(dx+c) \operatorname{EllipticF}((-1+\cos(dx+c)) * ((a-b)/(a+b))^{1/2} / \sin(dx+c), (-a+b)/(a-b))^{1/2}) * ((b+a \cos(dx+c)) / (1+\cos(dx+c))) / (a+b))^{1/2} * a^5 b^9 B \sin(dx+c) \operatorname{EllipticF}((-1+\cos(dx+c)) * ((a-b)/(a+b))^{1/2} / \sin(dx+c), (-a+b)/(a-b))^{1/2}) * ((b+a \cos(dx+c)) / (1+\cos(dx+c))) / (a+b))^{1/2} * a^4 b^2 - 6 B \sin(dx+c) \operatorname{EllipticF}((-1+\cos(dx+c)) * ((a-b)/(a+b))^{1/2} / \sin(dx+c), (-a+b)/(a-b))^{1/2}) * ((b+a \cos(dx+c)) / (1+\cos(dx+c))) / (a+b))^{1/2} * a^3 b^3 - 8 B \sin(dx+c) \operatorname{EllipticF}((-1+\cos(dx+c)) * ((a-b)/(a+b))^{1/2} / \sin(dx+c), (-a+b)/(a-b))^{1/2}) * ((b+a \cos(dx+c)) / (1+\cos(dx+c))) / (a+b))^{1/2} * a^2 b^4 - 3 B \sin(dx+c) * ((b+a \cos(dx+c)) / (1+\cos(dx+c))) / (a+b))^{1/2} * \operatorname{EllipticE}((-1+\cos(dx+c)) * ((a-b)/(a+b))^{1/2} / \sin(dx+c), (-a+b)/(a-b))^{1/2}) * a^5 b + 15 B \sin(dx+c) * ((b+a \cos(dx+c)) / (1+\cos(dx+c))) / (a+b))^{1/2} * \operatorname{EllipticE}((-1+\cos(dx+c)) * ((a-b)/(a+b))^{1/2} / \sin(dx+c), (-a+b)/(a-b))^{1/2}) * a^3 b^3 - 8 B \sin(dx+c) * ((b+a \cos(dx+c)) / (1+\cos(dx+c))) / (a+b))^{1/2} * \operatorname{EllipticE}((-1+\cos(dx+c)) * ((a-b)/(a+b))^{1/2} / \sin(dx+c), (-a+b)/(a-b))^{1/2}) * a b^5 + 6 A \cos(dx+c)^3 * ((a-b)/(a+b))^{1/2} * a^4 b^2 * (1/(1+\cos(dx+c)))^{1/2} - 6 A \cos(dx+c)^3 * ((a-b)/(a+b))^{1/2} * a^2 b^4 * (1/(1+\cos(dx+c)))^{1/2} - 3 B \cos(dx+c)^3 * ((a-b)/(a+b))^{1/2} * a^5 b * (1/(1+\cos(dx+c)))^{1/2} + 3 B \cos(dx+c)^3 * ((a-b)/(a+b))^{1/2} * a^3 b^3 * (1/(1+\cos(dx+c)))^{1/2} - 7 A \cos(dx+c)^2 * ((a-b)/(a+b))^{1/2} * a^5 b * (1/(1+\cos(dx+c)))^{1/2} + 34 A \cos(dx+c)^2 * ((a-b)/(a+b))^{1/2} * a^3 b^3 * (1/(1+\cos(dx+c)))^{1/2} - 24 A \cos(dx+c)^2 * ((a-b)/(a+b))^{1/2} * a b^5 * (1/(1+\cos(dx+c)))^{1/2} + 12 B \cos(dx+c)^2 * ((a-b)/(a+b))^{1/2} * a^2 b^4 * (1/(1+\cos(dx+c)))^{1/2} + 2 A \cos(dx+c) * ((a-b)/(a+b))^{1/2} * a^5 b * (1/(1+\cos(dx+c)))^{1/2} - 8 B \cos(dx+c) * \sin(dx+c) * \operatorname{EllipticE}((-1+\cos(dx+c)) * ((a-b)/(a+b))^{1/2} / \sin(dx+c), (-a+b)/(a-b))^{1/2}) * ((b+a \cos(dx+c)) / (1+\cos(dx+c))) / (a+b))^{1/2} * a^2 b^4 + 9 B \cos(dx+c) * \sin(dx+c) * ((b+a \cos(dx+c)) / (1+\cos(dx+c))) / (a+b))^{1/2} * \operatorname{EllipticF}((-1+\cos(dx+c)) * ((a-b)/(a+b))^{1/2} / \sin(dx+c), (-a+b)/(a-b))^{1/2}) * a^5 b - 6 B \cos(dx+c) * \sin(dx+c) * ((b+a \cos(dx+c)) / (1+\cos(dx+c))) / (a+b))^{1/2} * \operatorname{EllipticF}((-1+\cos(dx+c)) * ((a-b)/(a+b))^{1/2} / \sin(dx+c), (-a+b)/(a-b))^{1/2}) * a^4 b^2 - 16 A \cos(dx+c) * \sin(dx+c) * ((b+a \cos(dx+c)) / (1+\cos(dx+c))) / (a+b))^{1/2} * \operatorname{EllipticF}((-1+\cos(dx+c)) * ((a-b)/(a+b))^{1/2} / \sin(dx+c), (-a+b)/(a-b))^{1/2}) * a^4 b^2 + 12 A \cos(dx+c)
\end{aligned}$$

$x+c) \cdot \sin(d*x+c) \cdot ((b+a \cdot \cos(d*x+c)) / (1+\cos(d*x+c))) / (a+b))^{1/2} \cdot \text{EllipticF}((-1+\cos(d*x+c)) \cdot ((a-b)/(a+b))^{1/2} / \sin(d*x+c), (-a+b)/(a-b))^{1/2}) \cdot a^3 \cdot b^3 \cdot ((a-b)/(a+b))^{1/2} \cdot (1/(1+\cos(d*x+c)))^{1/2} / a^4 / (a+b) / (a-b)^2 / (b+a \cdot \cos(d*x+c))^{2/3} / \sin(d*x+c)^3$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+dx)^{3/2} \left(A + \frac{B}{\cos(c+dx)} \right)}{\left(a + \frac{b}{\cos(c+dx)} \right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c+d*x)^(3/2)*(A+B/cos(c+d*x)))/(a+b/cos(c+d*x))^(5/2),x)

[Out] int((cos(c+d*x)^(3/2)*(A+B/cos(c+d*x)))/(a+b/cos(c+d*x))^(5/2),x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(5/2),x)

[Out] Timed out

$$3.630 \quad \int \frac{\sqrt{\cos(c+dx)} (A+B \sec(c+dx))}{(a+b \sec(c+dx))^{5/2}} dx$$

Optimal. Leaf size=368

$$\frac{2b(Ab - aB) \sin(c + dx)}{3ad(a^2 - b^2) \sqrt{\cos(c + dx)} (a + b \sec(c + dx))^{3/2}} + \frac{2b(-5a^3B + 8a^2Ab + ab^2B - 4Ab^3) \sin(c + dx)}{3a^2d(a^2 - b^2)^2 \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}} - \frac{2(-3a^3B + \dots)}{3a^2d(a^2 - b^2)^2 \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}} \quad 3$$

[Out] $2/3*b*(A*b-B*a)*\sin(d*x+c)/a/(a^2-b^2)/d/(a+b*\sec(d*x+c))^(3/2)/\cos(d*x+c)^(1/2)+2/3*b*(8*A*a^2*b-4*A*b^3-5*B*a^3+B*a*b^2)*\sin(d*x+c)/a^2/(a^2-b^2)^2/d/\cos(d*x+c)^(1/2)/(a+b*\sec(d*x+c))^(1/2)-2/3*(9*A*a^2*b-8*A*b^3-3*B*a^3+2*B*a*b^2)*(\cos(1/2*d*x+1/2*c))^2^(1/2)/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^(1/2)*(a/(a+b))^(1/2))*((b+a*\cos(d*x+c))/(a+b))^(1/2)/a^3/(a^2-b^2)/d/\cos(d*x+c)^(1/2)/(a+b*\sec(d*x+c))^(1/2)+2/3*(3*A*a^4-15*A*a^2*b^2+8*A*b^4+6*B*a^3*b-2*B*a*b^3)*(\cos(1/2*d*x+1/2*c))^2^(1/2)/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^(1/2)*(a/(a+b))^(1/2))*\cos(d*x+c)^(1/2)*(a+b*\sec(d*x+c))^(1/2)/a^3/(a^2-b^2)^2/d/((b+a*\cos(d*x+c))/(a+b))^(1/2)$

Rubi [A] time = 1.11, antiderivative size = 368, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2955, 4030, 4100, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2b(8a^2Ab - 5a^3B + ab^2B - 4Ab^3) \sin(c + dx)}{3a^2d(a^2 - b^2)^2 \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}} + \frac{2b(Ab - aB) \sin(c + dx)}{3ad(a^2 - b^2) \sqrt{\cos(c + dx)} (a + b \sec(c + dx))^{3/2}} - \frac{2(9a^2Ab - \dots)}{3a^2d(a^2 - b^2)^2 \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}} \quad 3a$$

Antiderivative was successfully verified.

[In] $\text{Int}[(\text{Sqrt}[\text{Cos}[c + d*x]]*(A + B*\text{Sec}[c + d*x]))/(a + b*\text{Sec}[c + d*x])^(5/2), x]$

[Out] $(-2*(9*a^2*A*b - 8*A*b^3 - 3*a^3*B + 2*a*b^2*B)*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])]/(a + b))*\text{EllipticF}[(c + d*x)/2, (2*a)/(a + b)]/(3*a^3*(a^2 - b^2)*d*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]) + (2*(3*a^4*A - 15*a^2*A*b^2 + 8*A*b^4 + 6*a^3*b*B - 2*a*b^3*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, (2*a)/(a + b)]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]])/(3*a^3*(a^2 - b^2)^2*d*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/(a + b)]) + (2*b*(A*b - a*B)*\text{Sin}[c + d*x])/(3*a*(a^2 - b^2)*d*\text{Sqrt}[\text{Cos}[c + d*x]]*(a + b*\text{Sec}[c + d*x])^(3/2)) + (2*b*(8*a^2*A*b - 4*A*b^3 - 5*a^3*B + a*b^2*B)*\text{Sin}[c + d*x])/(3*a^2*(a^2 - b^2)^2*d*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]])$

Rule 2653

$\text{Int}[\text{Sqrt}[(a_) + (b_)*\sin[(c_) + (d_)*(x_)]]], x_Symbol] \rightarrow \text{Simp}[(2*\text{Sqrt}[a + b]*\text{EllipticE}[(1*(c - \text{Pi}/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; \text{FreeQ}\{a,$

b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2655

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] :> Dist[Sqrt[a + b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2661

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)]/(d*Sqrt[a + b]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2663

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b) + (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2955

Int[((a_) + csc[(e_) + (f_)*(x_)])*(b_)^(m_)*(csc[(e_) + (f_)*(x_)]*(d_) + (c_))^(n_)*((g_)*sin[(e_) + (f_)*(x_)])^(p_), x_Symbol] :> Dist[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && IntegerQ[n])

Rule 3856

Int[Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)]/Sqrt[csc[(e_) + (f_)*(x_)]*(d_)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 3858

Int[Sqrt[csc[(e_) + (f_)*(x_)]*(d_)]/Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 4030

```

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_)), x_Symbol] := Simp[(b*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*
(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e
+ f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[A*(a^2*(m + 1) - b^2*(m + n + 1))
+ a*b*B*n - a*(A*b - a*B)*(m + 1)*Csc[e + f*x] + b*(A*b - a*B)*(m + n + 2)*
Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b
- a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && !(ILtQ[m + 1/2, 0] && ILt
Q[n, 0])

```

Rule 4035

```

Int[(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_)]), x_Symbol] := Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]

```

Rule 4100

```

Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.
))^(m_)*(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a
_))^(m_), x_Symbol] := Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1)
- a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
&& NeQ[a^2 - b^2, 0] && LtQ[m, -1] && !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])

```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\cos(c+dx)}(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{5/2}} dx &= (\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}) \int \frac{A+B\sec(c+dx)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}} dx \\
&= \frac{2b(Ab-aB)\sin(c+dx)}{3a(a^2-b^2)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}} - \frac{(2\sqrt{\cos(c+dx)})}{3a^2(a^2-b^2)^2 d\sqrt{\cos(c+dx)}} \\
&= \frac{2b(Ab-aB)\sin(c+dx)}{3a(a^2-b^2)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}} + \frac{2b(8a^2Ab-4A^2b^2)}{3a^2(a^2-b^2)^2 d\sqrt{\cos(c+dx)}} \\
&= \frac{2b(Ab-aB)\sin(c+dx)}{3a(a^2-b^2)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}} + \frac{2b(8a^2Ab-4A^2b^2)}{3a^2(a^2-b^2)^2 d\sqrt{\cos(c+dx)}} \\
&= \frac{2b(Ab-aB)\sin(c+dx)}{3a(a^2-b^2)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}} + \frac{2b(8a^2Ab-4A^2b^2)}{3a^2(a^2-b^2)^2 d\sqrt{\cos(c+dx)}} \\
&= \frac{2b(Ab-aB)\sin(c+dx)}{3a(a^2-b^2)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}} + \frac{2b(8a^2Ab-4A^2b^2)}{3a^2(a^2-b^2)^2 d\sqrt{\cos(c+dx)}} \\
&= \frac{2(9a^2Ab-8Ab^3-3a^3B+2ab^2B)\sqrt{\frac{b+a\cos(c+dx)}{a+b}} F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{3a^3(a^2-b^2)d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}
\end{aligned}$$

Mathematica [C] time = 19.78, size = 621, normalized size = 1.69

$$\frac{(a\cos(c+dx)+b)^3(A+B\sec(c+dx))\left(-\frac{2(Ab^3\sin(c+dx)-ab^2B\sin(c+dx))}{3a^2(a^2-b^2)(a\cos(c+dx)+b)^2} - \frac{2(6a^3bB\sin(c+dx)-9a^2Ab^2\sin(c+dx)-2ab^3B\sin(c+dx))}{3a^2(a^2-b^2)^2(a\cos(c+dx)+b)}\right)}{d\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}(A\cos(c+dx)+B)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(5/2), x]

[Out] ((b + a*cos[c + d*x])^3*(A + B*Sec[c + d*x])*((-2*(A*b^3*Sin[c + d*x] - a*b^2*B*Sin[c + d*x]))/(3*a^2*(a^2 - b^2)*(b + a*cos[c + d*x])^2) - (2*(-9*a^2*B*Sin[c + d*x] + 5*A*b^4*Sin[c + d*x] + 6*a^3*b*B*Sin[c + d*x] - 2*a*b

$$\begin{aligned} & \left(\frac{(3a^2(a^2 - b^2)^2(b + a\cos[c + dx]))}{(d\cos[c + dx])^{3/2}(B + A\cos[c + dx])(a + b\sec[c + dx])^{5/2}} - (2\cos[c + dx])^{3/2}(b + a\cos[c + dx])^2\sec[c + dx]^{3/2}(\cos[(c + dx)/2])^2\sec[c + dx]^{3/2}(A + B\sec[c + dx])((-I)(a + b)(3a^4A - 15a^2Ab^2 + 8A^2b^4 + 6a^3bB - 2ab^3B))\text{EllipticE}[I\text{ArcSinh}[\text{Tan}[(c + dx)/2]]], \right. \\ & (-a + b)/(a + b)\sec[(c + dx)/2]^2\sqrt{((b + a\cos[c + dx])\sec[(c + dx)/2])^2/(a + b)} + I a(a + b)(8A^2b^3 + 3a^2b(-3A + B) + 3a^3(A + B) - 2ab^2(3A + B))\text{EllipticF}[I\text{ArcSinh}[\text{Tan}[(c + dx)/2]]], (-a + b)/(a + b)\sec[(c + dx)/2]^2\sqrt{((b + a\cos[c + dx])\sec[(c + dx)/2])^2/(a + b)} \\ & \left. - (3a^4A - 15a^2Ab^2 + 8A^2b^4 + 6a^3bB - 2ab^3B)(b + a\cos[c + dx])(\sec[(c + dx)/2])^{3/2}\text{Tan}[(c + dx)/2]) / (3a(a^3 - ab^2)^2d(B + A\cos[c + dx])(a + b\sec[c + dx])^{5/2}) \right) \end{aligned}$$

fricas [F] time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B \sec(dx + c) + A)\sqrt{b \sec(dx + c) + a} \sqrt{\cos(dx + c)}}{b^3 \sec(dx + c)^3 + 3ab^2 \sec(dx + c)^2 + 3a^2b \sec(dx + c) + a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")

[Out] integral((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c))/(b^3*sec(d*x + c)^3 + 3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sec(dx + c) + A)\sqrt{\cos(dx + c)}}{(b \sec(dx + c) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(b*sec(d*x + c) + a)^(5/2), x)

maple [B] time = 2.69, size = 3337, normalized size = 9.07

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

$$\begin{aligned} & \left. \right)^{(1/2)}/\sin(dx+c), (-\frac{a+b}{a-b})^{(1/2)} * a^4 b^4 - 8 * A * \left(\frac{a-b}{a+b} \right)^{(1/2)} * \cos \\ & (dx+c) * b^5 * \left(\frac{1}{1+\cos(dx+c)} \right)^{(1/2)} - 3 * A * \left(\frac{a-b}{a+b} \right)^{(1/2)} * a^3 b^2 * \left(\frac{1}{1+ \right. \\ & \left. \cos(dx+c)} \right)^{(1/2)} - 11 * A * \left(\frac{a-b}{a+b} \right)^{(1/2)} * a^2 b^3 * \left(\frac{1}{1+\cos(dx+c)} \right)^{(1/2)} \\ & + 4 * A * \left(\frac{a-b}{a+b} \right)^{(1/2)} * a b^4 * \left(\frac{1}{1+\cos(dx+c)} \right)^{(1/2)} - 9 * A * \left(\frac{b+a \cos(dx+ \right. \\ & \left. c)}{1+\cos(dx+c)} \right) / (a+b)^{(1/2)} * \text{EllipticF} \left((-1+\cos(dx+c)) * \left(\frac{a-b}{a+b} \right)^{(1/2)} / \sin(dx+c), \right. \\ & \left. \left(\frac{a-b}{a+b} \right)^{(1/2)} \right) * \sin(dx+c) * \cos(dx+c) * a^4 b + 6 * A * \left(\frac{b+a \cos(dx+c)}{1+\cos(dx+c)} \right) / (a+b)^{(1/2)} * \\ & \text{EllipticF} \left((-1+\cos(dx+c)) * \left(\frac{a-b}{a+b} \right)^{(1/2)} / \sin(dx+c), \left(\frac{a-b}{a+b} \right)^{(1/2)} \right) * \sin(dx+c) * \cos(dx+c) * a^3 b^2 + 8 * \\ & A * \left(\frac{b+a \cos(dx+c)}{1+\cos(dx+c)} \right) / (a+b)^{(1/2)} * \text{EllipticF} \left((-1+\cos(dx+c)) * \left(\frac{a-b}{a+b} \right)^{(1/2)} / \sin(dx+c), \right. \\ & \left. \left(\frac{a-b}{a+b} \right)^{(1/2)} \right) * \sin(dx+c) * \cos(dx+c) * a^2 b^3 - 15 * A * \left(\frac{b+a \cos(dx+c)}{1+\cos(dx+c)} \right) / (a+b)^{(1/2)} * \text{EllipticE} \left((-1+\cos(dx+c)) * \left(\frac{a-b}{a+b} \right)^{(1/2)} / \sin(dx+c), \right. \\ & \left. \left(\frac{a-b}{a+b} \right)^{(1/2)} \right) * \sin(dx+c) * \cos(dx+c) * a^3 b^2 + 8 * A * \left(\frac{b+a \cos(dx+c)}{1+\cos(dx+c)} \right) / (a+b)^{(1/2)} * \text{EllipticE} \left((-1+\cos(dx+c)) * \left(\frac{a-b}{a+b} \right)^{(1/2)} / \sin(dx+c), \right. \\ & \left. \left(\frac{a-b}{a+b} \right)^{(1/2)} \right) * \sin(dx+c) * \cos(dx+c) * a b^4 - 3 * B * \left(\frac{b+a \cos(dx+c)}{1+\cos(dx+c)} \right) / (a+b)^{(1/2)} * \text{E} \\ & \text{llipticF} \left((-1+\cos(dx+c)) * \left(\frac{a-b}{a+b} \right)^{(1/2)} / \sin(dx+c), \left(\frac{a-b}{a+b} \right)^{(1/2)} \right) * \sin(dx+c) * \cos(dx+c) * a^4 b - 2 * B * \left(\frac{b+a \cos(dx+c)}{1+\cos(dx+c)} \right) / (a+b)^{(1/2)} * \\ & \text{EllipticF} \left((-1+\cos(dx+c)) * \left(\frac{a-b}{a+b} \right)^{(1/2)} / \sin(dx+c), \left(\frac{a-b}{a+b} \right)^{(1/2)} \right) * \sin(dx+c) * \cos(dx+c) * a^3 b^2 + 6 * B * \left(\frac{b+a \cos(dx+c)}{1+\cos(dx+c)} \right) / (a+b)^{(1/2)} * \\ & \text{EllipticE} \left((-1+\cos(dx+c)) * \left(\frac{a-b}{a+b} \right)^{(1/2)} / \sin(dx+c), \left(\frac{a-b}{a+b} \right)^{(1/2)} \right) * \sin(dx+c) * \cos(dx+c) * a^4 b - 2 * B * \left(\frac{b+a \cos(dx+c)}{1+\cos(dx+c)} \right) / (a+b)^{(1/2)} * \\ & \text{EllipticE} \left((-1+\cos(dx+c)) * \left(\frac{a-b}{a+b} \right)^{(1/2)} / \sin(dx+c), \left(\frac{a-b}{a+b} \right)^{(1/2)} \right) * \sin(dx+c) * \cos(dx+c) * a^2 b^3 + 3 * A * \left(\frac{1}{1+\cos(dx+c)} \right)^{(1/2)} * \cos(dx+c)^3 * \left(\frac{a-b}{a+b} \right)^{(1/2)} * a^5 + 5 * B * \left(\frac{a-b}{a+b} \right)^{(1/2)} * a^3 * \\ & b^2 * \left(\frac{1}{1+\cos(dx+c)} \right)^{(1/2)} - B * \left(\frac{a-b}{a+b} \right)^{(1/2)} * a^2 b^3 * \left(\frac{1}{1+\cos(dx+c)} \right)^{(1/2)} - 2 * B * \left(\frac{a-b}{a+b} \right)^{(1/2)} * a b^4 * \left(\frac{1}{1+\cos(dx+c)} \right)^{(1/2)} - 3 * A * \left(\frac{a-b}{a+b} \right)^{(1/2)} * \cos(dx+c)^2 * a^5 * \left(\frac{1}{1+\cos(dx+c)} \right)^{(1/2)} + 8 * A * \sin(dx+c) * \text{Ellip} \\ & \text{ticE} \left((-1+\cos(dx+c)) * \left(\frac{a-b}{a+b} \right)^{(1/2)} / \sin(dx+c), \left(\frac{a-b}{a+b} \right)^{(1/2)} \right) * \left(\frac{b+a \cos(dx+c)}{1+\cos(dx+c)} \right) / (a+b)^{(1/2)} * b^5 - 2 * B * \left(\frac{a-b}{a+b} \right)^{(1/2)} * \cos(dx+c) * a^2 b^3 * \left(\frac{1}{1+\cos(dx+c)} \right)^{(1/2)} + 2 * B * \left(\frac{a-b}{a+b} \right)^{(1/2)} * \cos(dx+c) * a b^4 * \left(\frac{1}{1+\cos(dx+c)} \right)^{(1/2)} * \cos(dx+c)^{(1/2)} * \left(\frac{b+a \cos(dx+c)}{\cos(dx+c)} \right)^{(1/2)} * \left(\frac{a-b}{a+b} \right)^{(1/2)} * \left(\frac{1}{1+\cos(dx+c)} \right)^{(1/2)} / a^3 / (a+b) / (a-b)^2 / (b+a \cos(dx+c))^2 / \sin(dx+c)^3 \end{aligned}$$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(dx+c))*cos(dx+c)^(1/2)/(a+b*sec(dx+c))^(5/2),x, algorith="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\cos(c+dx)} \left(A + \frac{B}{\cos(c+dx)} \right)}{\left(a + \frac{b}{\cos(c+dx)} \right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)^(1/2)*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x))^(5/2), x)

[Out] int((cos(c + d*x)^(1/2)*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x))^(5/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)**(1/2)/(a+b*sec(d*x+c))**(5/2), x)

[Out] Timed out

$$3.631 \quad \int \frac{A+B \sec(c+dx)}{\sqrt{\cos(c+dx)} (a+b \sec(c+dx))^{5/2}} dx$$

Optimal. Leaf size=346

$$\frac{2(Ab - aB) \sin(c + dx)}{3d(a^2 - b^2) \sqrt{\cos(c + dx)} (a + b \sec(c + dx))^{3/2}} + \frac{2(3a^2A - abB - 2Ab^2) \sqrt{\frac{a \cos(c+dx)+b}{a+b}} F\left(\frac{1}{2}(c + dx) \middle| \frac{2a}{a+b}\right)}{3a^2d(a^2 - b^2) \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}} - \frac{2(-)}{3ad(a^2 - b^2)}$$

[Out] $-2/3*(A*b-B*a)*\sin(d*x+c)/(a^2-b^2)/d/(a+b*\sec(d*x+c))^(3/2)/\cos(d*x+c)^(1/2)-2/3*(5*A*a^2*b-A*b^3-2*B*a^3-2*B*a*b^2)*\sin(d*x+c)/a/(a^2-b^2)^2/d/\cos(d*x+c)^(1/2)/(a+b*\sec(d*x+c))^(1/2)+2/3*(3*A*a^2-2*A*b^2-B*a*b)*(\cos(1/2*d*x+1/2*c))^2^(1/2)/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^(1/2)*(a/(a+b))^(1/2))*((b+a*\cos(d*x+c))/(a+b))^(1/2)/a^2/(a^2-b^2)/d/\cos(d*x+c)^(1/2)/(a+b*\sec(d*x+c))^(1/2)+2/3*(6*A*a^2*b-2*A*b^3-3*B*a^3-B*a*b^2)*(\cos(1/2*d*x+1/2*c))^2^(1/2)/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^(1/2))*(a/(a+b))^(1/2))*\cos(d*x+c)^(1/2)*(a+b*\sec(d*x+c))^(1/2)/a^2/(a^2-b^2)^2/d/((b+a*\cos(d*x+c))/(a+b))^(1/2)$

Rubi [A] time = 1.01, antiderivative size = 346, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2955, 4027, 4100, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2(5a^2Ab - 2a^3B - 2ab^2B - Ab^3) \sin(c + dx)}{3ad(a^2 - b^2)^2 \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}} - \frac{2(Ab - aB) \sin(c + dx)}{3d(a^2 - b^2) \sqrt{\cos(c + dx)} (a + b \sec(c + dx))^{3/2}} + \frac{2(3a^2A - abB - 2Ab^2)}{3a^2d(a^2 - b^2)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(A + B*\text{Sec}[c + d*x])]/(\text{Sqrt}[\text{Cos}[c + d*x]]*(a + b*\text{Sec}[c + d*x])^(5/2)), x]$

[Out] $(2*(3*a^2*A - 2*A*b^2 - a*b*B)*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/(a + b)]*\text{EllipticF}[(c + d*x)/2, (2*a)/(a + b)])/(3*a^2*(a^2 - b^2)*d*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]) + (2*(6*a^2*A*b - 2*A*b^3 - 3*a^3*B - a*b^2*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, (2*a)/(a + b)]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]])/(3*a^2*(a^2 - b^2)^2*d*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/(a + b)]) - (2*(A*b - a*B)*\text{Sin}[c + d*x])/(3*(a^2 - b^2)*d*\text{Sqrt}[\text{Cos}[c + d*x]]*(a + b*\text{Sec}[c + d*x])^(3/2)) - (2*(5*a^2*A*b - A*b^3 - 2*a^3*B - 2*a*b^2*B)*\text{Sin}[c + d*x])/(3*a*(a^2 - b^2)^2*d*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]])$

Rule 2653

$\text{Int}[\text{Sqrt}[(a_) + (b_)*\sin[(c_) + (d_)*(x_)]], x_Symbol] \rightarrow \text{Simp}[(2*\text{Sqrt}[a + b]*\text{EllipticE}[(1*(c - \text{Pi}/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; \text{FreeQ}\{a,$

b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2655

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] :> Dist[Sqrt[a + b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2661

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)]/(d*Sqrt[a + b]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2663

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b) + (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2955

Int[((a_) + csc[(e_) + (f_)*(x_)])*(b_)^(m_)*(csc[(e_) + (f_)*(x_)]*(d_) + (c_))^(n_)*((g_)*sin[(e_) + (f_)*(x_)])^(p_), x_Symbol] :> Dist[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && IntegerQ[n])

Rule 3856

Int[Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)]/Sqrt[csc[(e_) + (f_)*(x_)]*(d_)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 3858

Int[Sqrt[csc[(e_) + (f_)*(x_)]*(d_)]/Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 4027

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := -Simp[(d*(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*Simp[d*(n - 1)*(A*b - a*B) + d*(a*A - b*B)*(m + 1)*Csc[e + f*x] - d*(A*b - a*B)*(m + n + 1)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && LtQ[0, n, 1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.))/(Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]), x_Symbol] := Dist[A/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.), x_Symbol] := Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{A + B \sec(c + dx)}{\sqrt{\cos(c + dx)} (a + b \sec(c + dx))^{5/2}} dx &= \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{\sqrt{\sec(c + dx)} (A + B \sec(c + dx))}{(a + b \sec(c + dx))^{5/2}} dx \\
&= -\frac{2(Ab - aB) \sin(c + dx)}{3(a^2 - b^2) d \sqrt{\cos(c + dx)} (a + b \sec(c + dx))^{3/2}} - \frac{(2\sqrt{\cos(c + dx)})}{3a(a^2 - b^2)^2 d \sqrt{\cos(c + dx)}} \\
&= -\frac{2(Ab - aB) \sin(c + dx)}{3(a^2 - b^2) d \sqrt{\cos(c + dx)} (a + b \sec(c + dx))^{3/2}} - \frac{2(5a^2 Ab - Aa^3)}{3a(a^2 - b^2)^2 d \sqrt{\cos(c + dx)}} \\
&= -\frac{2(Ab - aB) \sin(c + dx)}{3(a^2 - b^2) d \sqrt{\cos(c + dx)} (a + b \sec(c + dx))^{3/2}} - \frac{2(5a^2 Ab - Aa^3)}{3a(a^2 - b^2)^2 d \sqrt{\cos(c + dx)}} \\
&= -\frac{2(Ab - aB) \sin(c + dx)}{3(a^2 - b^2) d \sqrt{\cos(c + dx)} (a + b \sec(c + dx))^{3/2}} - \frac{2(5a^2 Ab - Aa^3)}{3a(a^2 - b^2)^2 d \sqrt{\cos(c + dx)}} \\
&= -\frac{2(Ab - aB) \sin(c + dx)}{3(a^2 - b^2) d \sqrt{\cos(c + dx)} (a + b \sec(c + dx))^{3/2}} - \frac{2(5a^2 Ab - Aa^3)}{3a(a^2 - b^2)^2 d \sqrt{\cos(c + dx)}} \\
&= -\frac{2(Ab - aB) \sin(c + dx)}{3(a^2 - b^2) d \sqrt{\cos(c + dx)} (a + b \sec(c + dx))^{3/2}} - \frac{2(5a^2 Ab - Aa^3)}{3a(a^2 - b^2)^2 d \sqrt{\cos(c + dx)}} \\
&= \frac{2(3a^2 A - 2Ab^2 - abB) \sqrt{\frac{b+a \cos(c+dx)}{a+b}} F\left(\frac{1}{2}(c + dx) \middle| \frac{2a}{a+b}\right)}{3a^2 (a^2 - b^2) d \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}} + \frac{2(6a^2 Ab - Aa^3)}{3a(a^2 - b^2)^2 d \sqrt{\cos(c + dx)}}
\end{aligned}$$

Mathematica [C] time = 18.03, size = 463, normalized size = 1.34

$$(a \cos(c + dx) + b)^2 \left(\frac{2 \sin(c+dx) (a(3a^3 B - 6a^2 Ab + ab^2 B + 2Ab^3) \cos(c+dx) + b(2a^3 B - 5a^2 Ab + 2ab^2 B + Ab^3))}{a(a^2 - b^2)^2 (a \cos(c+dx) + b)} + \frac{2 \left(\cos^2 \left(\frac{1}{2}(c+dx) \right) \sec(c+dx) \right)^{3/2}}{3a(a^2 - b^2)^2 d \sqrt{\cos(c + dx)}} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(A + B*Sec[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(5/2)),x]

[Out] ((b + a*cos[c + d*x])^2*((2*(b*(-5*a^2*A*b + A*b^3 + 2*a^3*B + 2*a*b^2*B) + a*(-6*a^2*A*b + 2*A*b^3 + 3*a^3*B + a*b^2*B))*Cos[c + d*x])*Sin[c + d*x])/(

$$a*(a^2 - b^2)^2*(b + a*\cos[c + d*x]) + (2*(\cos[(c + d*x)/2]^2*\sec[c + d*x])^{3/2}*(-1)*(a + b)*(-6*a^2*A*b + 2*A*b^3 + 3*a^3*B + a*b^2*B)*\text{EllipticE}[I*\text{ArcSinh}[\text{Tan}[(c + d*x)/2]], (-a + b)/(a + b)]*\sec[(c + d*x)/2]^2*\sqrt{((b + a*\cos[c + d*x])* \sec[(c + d*x)/2]^2)/(a + b)} - I*a*(a + b)*(-2*A*b^2 + 3*a^2*(A - B) + a*b*(3*A - B))*\text{EllipticF}[I*\text{ArcSinh}[\text{Tan}[(c + d*x)/2]], (-a + b)/(a + b)]*\sec[(c + d*x)/2]^2*\sqrt{((b + a*\cos[c + d*x])* \sec[(c + d*x)/2]^2)/(a + b)} - (-6*a^2*A*b + 2*A*b^3 + 3*a^3*B + a*b^2*B)*(b + a*\cos[c + d*x])*(\sec[(c + d*x)/2]^2)^{3/2}*\text{Tan}[(c + d*x)/2]))/(a^3 - a*b^2)^2*\sec[c + d*x]^{3/2}))/((3*d*\cos[c + d*x]^{5/2}*(a + b*\sec[c + d*x])^{5/2}))$$

fricas [F] time = 0.97, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B \sec(dx + c) + A)\sqrt{b \sec(dx + c) + a} \sqrt{\cos(dx + c)}}{b^3 \cos(dx + c) \sec(dx + c)^3 + 3ab^2 \cos(dx + c) \sec(dx + c)^2 + 3a^2b \cos(dx + c) \sec(dx + c) + a^3 \cos(dx + c)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algorith="fricas")

[Out] integral((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c))/(b^3*cos(d*x + c)*sec(d*x + c)^3 + 3*a*b^2*cos(d*x + c)*sec(d*x + c)^2 + 3*a^2*b*cos(d*x + c)*sec(d*x + c) + a^3*cos(d*x + c)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(b \sec(dx + c) + a)^2 \sqrt{\cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algorith="giac")

[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^(5/2)*sqrt(cos(d*x + c)))), x)

maple [B] time = 3.17, size = 2416, normalized size = 6.98

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x)

[Out] -2/3/d*(-1+cos(d*x+c))*(1+cos(d*x+c))^2*(-3*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c)))

$$\begin{aligned}
& d*x+c)) / (a+b))^{1/2} * \cos(d*x+c) * \sin(d*x+c) * a^4 + 2*A*((a-b)/(a+b))^{1/2} * b^4 * \\
& (1/(1+\cos(d*x+c)))^{1/2} + B*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)) / (a+b))^{1/2} * \text{El} \\
& \text{lipticE}((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{1/2} / \sin(d*x+c), (- (a+b)/(a-b))^{1/2} \\
&) * \sin(d*x+c) * a * b^3 - 3*B * \text{EllipticF}((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{1/2} / \sin(d* \\
& x+c), (- (a+b)/(a-b))^{1/2}) * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)) / (a+b))^{1/2} * \text{si} \\
& \text{n}(d*x+c) * a^3 * b + B * \text{EllipticF}((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{1/2} / \sin(d*x+c), (\\
& - (a+b)/(a-b))^{1/2}) * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)) / (a+b))^{1/2} * \sin(d*x+ \\
& c) * a^2 * b^2 - 3*A * \text{EllipticF}((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{1/2} / \sin(d*x+c), (- (\\
& a+b)/(a-b))^{1/2}) * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)) / (a+b))^{1/2} * \sin(d*x+c) \\
& * a^3 * b + 3*A * \text{EllipticF}((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{1/2} / \sin(d*x+c), (- (a+b) \\
& / (a-b))^{1/2}) * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)) / (a+b))^{1/2} * \sin(d*x+c) * a^2 \\
& * b^2 + 2*A * \text{EllipticF}((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{1/2} / \sin(d*x+c), (- (a+b)/(\\
& a-b))^{1/2}) * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)) / (a+b))^{1/2} * \sin(d*x+c) * a * b^3 \\
& - 6*A * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)) / (a+b))^{1/2} * \text{EllipticE}((-1+\cos(d*x+c) \\
&) * ((a-b)/(a+b))^{1/2} / \sin(d*x+c), (- (a+b)/(a-b))^{1/2}) * \sin(d*x+c) * a^2 * b^2 + 6 \\
& * A * ((a-b)/(a+b))^{1/2} * \cos(d*x+c)^2 * a^3 * b * (1/(1+\cos(d*x+c)))^{1/2} - 3*A * ((a-b) \\
& / (a+b))^{1/2} * \cos(d*x+c)^2 * a * b^3 * (1/(1+\cos(d*x+c)))^{1/2} - 6*A * ((a-b)/(a+b) \\
&))^{1/2} * \cos(d*x+c) * a^3 * b * (1/(1+\cos(d*x+c)))^{1/2} + 6*A * ((a-b)/(a+b))^{1/2} * \\
& \cos(d*x+c) * a^2 * b^2 * (1/(1+\cos(d*x+c)))^{1/2} + 2*A * ((a-b)/(a+b))^{1/2} * \cos(d*x \\
& +c) * a * b^3 * (1/(1+\cos(d*x+c)))^{1/2} - 3*B * ((a-b)/(a+b))^{1/2} * \cos(d*x+c) * a^3 * b \\
& * (1/(1+\cos(d*x+c)))^{1/2} + B * ((a-b)/(a+b))^{1/2} * \cos(d*x+c) * a^2 * b^2 * (1/(1+co \\
& s(d*x+c)))^{1/2} - B * ((a-b)/(a+b))^{1/2} * \cos(d*x+c) * a * b^3 * (1/(1+\cos(d*x+c)))^{1/2} \\
& (1/2) + 3*B * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)) / (a+b))^{1/2} * \text{EllipticE}((-1+\cos(d \\
& *x+c)) * ((a-b)/(a+b))^{1/2} / \sin(d*x+c), (- (a+b)/(a-b))^{1/2}) * \sin(d*x+c) * a^3 * \\
& b - 3*B * \text{EllipticF}((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{1/2} / \sin(d*x+c), (- (a+b)/(a-b) \\
&))^{1/2}) * \cos(d*x+c) * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)) / (a+b))^{1/2} * \sin(d*x+ \\
& c) * a^4 + 3*B * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)) / (a+b))^{1/2} * \text{EllipticE}((-1+\cos(\\
& d*x+c)) * ((a-b)/(a+b))^{1/2} / \sin(d*x+c), (- (a+b)/(a-b))^{1/2}) * \cos(d*x+c) * \sin \\
& (d*x+c) * a^4 - A * \cos(d*x+c)^2 * ((a-b)/(a+b))^{1/2} * a^2 * b^2 * (1/(1+\cos(d*x+c)))^{1/2} \\
& (1/2) + B * \cos(d*x+c)^2 * ((a-b)/(a+b))^{1/2} * a^3 * b * (1/(1+\cos(d*x+c)))^{1/2} - 6*A * \\
& ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)) / (a+b))^{1/2} * \text{EllipticE}((-1+\cos(d*x+c)) * ((a \\
& -b)/(a+b))^{1/2} / \sin(d*x+c), (- (a+b)/(a-b))^{1/2}) * \cos(d*x+c) * \sin(d*x+c) * a^3 \\
& * b + 2*A * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)) / (a+b))^{1/2} * \text{EllipticE}((-1+\cos(d*x+ \\
& c)) * ((a-b)/(a+b))^{1/2} / \sin(d*x+c), (- (a+b)/(a-b))^{1/2}) * \cos(d*x+c) * \sin(d*x \\
& +c) * a * b^3 + B * \text{EllipticF}((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{1/2} / \sin(d*x+c), (- (a+b) \\
&) / (a-b))^{1/2}) * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)) / (a+b))^{1/2} * \cos(d*x+c) * \text{si} \\
& \text{n}(d*x+c) * a^3 * b + B * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)) / (a+b))^{1/2} * \text{EllipticE}((- \\
& 1+\cos(d*x+c)) * ((a-b)/(a+b))^{1/2} / \sin(d*x+c), (- (a+b)/(a-b))^{1/2}) * \cos(d*x+ \\
& c) * \sin(d*x+c) * a^2 * b^2 + 3*A * \text{EllipticF}((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{1/2} / \sin \\
& (d*x+c), (- (a+b)/(a-b))^{1/2}) * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)) / (a+b))^{1/2} \\
& * \cos(d*x+c) * \sin(d*x+c) * a^3 * b + 2*A * \text{EllipticF}((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{1/2} / \sin \\
& (d*x+c), (- (a+b)/(a-b))^{1/2}) * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)) / (a+b) \\
&)^{1/2} * \cos(d*x+c) * \sin(d*x+c) * a^2 * b^2 + 2*A * ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)) / \\
& (a+b))^{1/2} * \text{EllipticE}((-1+\cos(d*x+c)) * ((a-b)/(a+b))^{1/2} / \sin(d*x+c), (- (a+ \\
& b)/(a-b))^{1/2}) * \sin(d*x+c) * b^4 - 3*B * ((a-b)/(a+b))^{1/2} * \cos(d*x+c)^2 * a^4 * (1
\end{aligned}$$

$$\begin{aligned} & / (1 + \cos(dx + c))^{1/2} - 2A \left(\frac{a-b}{a+b} \right)^{1/2} \cos(dx + c) b^4 \left(\frac{1}{1 + \cos(dx + c)} \right)^{1/2} \\ & + 3B \left(\frac{a-b}{a+b} \right)^{1/2} \cos(dx + c) a^4 \left(\frac{1}{1 + \cos(dx + c)} \right)^{1/2} \\ & - 5A \left(\frac{a-b}{a+b} \right)^{1/2} a^2 b^2 \left(\frac{1}{1 + \cos(dx + c)} \right)^{1/2} + A \left(\frac{a-b}{a+b} \right)^{1/2} \\ & a^3 b^3 \left(\frac{1}{1 + \cos(dx + c)} \right)^{1/2} + 2B \left(\frac{a-b}{a+b} \right)^{1/2} a^3 b^3 \left(\frac{1}{1 + \cos(dx + c)} \right)^{1/2} \\ & - B \left(\frac{a-b}{a+b} \right)^{1/2} a^2 b^2 \left(\frac{1}{1 + \cos(dx + c)} \right)^{1/2} + B \left(\frac{a-b}{a+b} \right)^{1/2} \\ & a^3 b^3 \left(\frac{1}{1 + \cos(dx + c)} \right)^{1/2} \cos(dx + c)^{1/2} \left(\frac{b + a \cos(dx + c)}{\cos(dx + c)} \right)^{1/2} \\ & \left(\frac{a-b}{a+b} \right)^{1/2} \left(\frac{1}{1 + \cos(dx + c)} \right)^{1/2} / a^2 (a+b) / (a-b)^2 / (b + a \cos(dx + c))^2 / \sin(dx + c)^3 \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(b \sec(dx + c) + a)^{5/2} \sqrt{\cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(dx+c))/(a+b*sec(dx+c))^(5/2)/cos(dx+c)^(1/2),x, algorithm="maxima")

[Out] integrate((B*sec(dx + c) + A)/((b*sec(dx + c) + a)^(5/2)*sqrt(cos(dx + c))), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + \frac{B}{\cos(c + dx)}}{\sqrt{\cos(c + dx)} \left(a + \frac{b}{\cos(c + dx)} \right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + dx))/(cos(c + dx)^(1/2)*(a + b/cos(c + dx))^(5/2)),x)

[Out] int((A + B/cos(c + dx))/(cos(c + dx)^(1/2)*(a + b/cos(c + dx))^(5/2)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(dx+c))/(a+b*sec(dx+c))**(5/2)/cos(dx+c)**(1/2),x)

[Out] Timed out

$$3.632 \quad \int \frac{A+B \sec(c+dx)}{\cos^2(c+dx)(a+b \sec(c+dx))^{5/2}} dx$$

Optimal. Leaf size=329

$$\frac{2a(Ab - aB) \sin(c + dx)}{3bd(a^2 - b^2) \sqrt{\cos(c + dx)} (a + b \sec(c + dx))^{3/2}} - \frac{2(Ab - aB) \sqrt{\frac{a \cos(c+dx)+b}{a+b}} F\left(\frac{1}{2}(c + dx) \middle| \frac{2a}{a+b}\right)}{3ad(a^2 - b^2) \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}} - \frac{2(3a^2A - 4a^2B)}{3ad(a^2 - b^2)}$$

[Out] $2/3*a*(A*b-B*a)*\sin(d*x+c)/b/(a^2-b^2)/d/(a+b*\sec(d*x+c))^{3/2}/\cos(d*x+c)^{1/2}+2/3*(2*A*a^2*b+2*A*b^3+B*a^3-5*B*a*b^2)*\sin(d*x+c)/b/(a^2-b^2)^2/d/\cos(d*x+c)^{1/2}/(a+b*\sec(d*x+c))^{1/2}-2/3*(A*b-B*a)*(\cos(1/2*d*x+1/2*c))^{1/2}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{1/2}*(a/(a+b))^{1/2})*((b+a*\cos(d*x+c))/(a+b))^{1/2}/a/(a^2-b^2)/d/\cos(d*x+c)^{1/2}/(a+b*\sec(d*x+c))^{1/2}-2/3*(3*A*a^2+A*b^2-4*B*a*b)*(\cos(1/2*d*x+1/2*c))^{1/2}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{1/2}*(a/(a+b))^{1/2})*\cos(d*x+c)^{1/2}*(a+b*\sec(d*x+c))^{1/2}/a/(a^2-b^2)^2/d/((b+a*\cos(d*x+c))/(a+b))^{1/2}$

Rubi [A] time = 1.04, antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2955, 4029, 4100, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2(2a^2Ab + a^3B - 5ab^2B + 2Ab^3) \sin(c + dx)}{3bd(a^2 - b^2)^2 \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}} + \frac{2a(Ab - aB) \sin(c + dx)}{3bd(a^2 - b^2) \sqrt{\cos(c + dx)} (a + b \sec(c + dx))^{3/2}} - \frac{2(Ab - aB)}{3ad(a^2 - b^2)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(A + B*\text{Sec}[c + d*x])/(\text{Cos}[c + d*x]^{3/2}*(a + b*\text{Sec}[c + d*x])^{5/2}), x]$

[Out] $(-2*(A*b - a*B)*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/(a + b)]*\text{EllipticF}[(c + d*x)/2, (2*a)/(a + b)]/(3*a*(a^2 - b^2)*d*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]) - (2*(3*a^2*A + A*b^2 - 4*a*b*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, (2*a)/(a + b)]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]])/(3*a*(a^2 - b^2)^2*d*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/(a + b)]) + (2*a*(A*b - a*B)*\text{Sin}[c + d*x])/(3*b*(a^2 - b^2)*d*\text{Sqrt}[\text{Cos}[c + d*x]]*(a + b*\text{Sec}[c + d*x])^{3/2}) + (2*(2*a^2*A*b + 2*A*b^3 + a^3*B - 5*a*b^2*B)*\text{Sin}[c + d*x])/(3*b*(a^2 - b^2)^2*d*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]])$

Rule 2653

$\text{Int}[\text{Sqrt}[(a_) + (b_)*\sin[(c_) + (d_)*(x_)]], x_Symbol] \rightarrow \text{Simp}[(2*\text{Sqrt}[a + b]*\text{EllipticE}[(1*(c - \text{Pi}/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; \text{FreeQ}[\{a,$

b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2655

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a + b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2661

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)]/(d*Sqrt[a + b]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2663

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b) + (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2955

Int[((a_) + csc[(e_) + (f_)*(x_)]*(b_))^(m_)*(csc[(e_) + (f_)*(x_)]*(d_) + (c_))^(n_)*((g_)*sin[(e_) + (f_)*(x_)])^(p_), x_Symbol] := Dist[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && IntegerQ[n])

Rule 3856

Int[Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)]/Sqrt[csc[(e_) + (f_)*(x_)]*(d_)], x_Symbol] := Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 3858

Int[Sqrt[csc[(e_) + (f_)*(x_)]*(d_)]/Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)], x_Symbol] := Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 4029

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*d^2*
(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n -
2))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[d/(b*(m + 1)*(a^2 - b^2)), Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(n
- 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2*
(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n
, 1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
&& NeQ[a^2 - b^2, 0] && LtQ[m, -1] && !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{A + B \sec(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(a + b \sec(c + dx))^{5/2}} dx &= \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{\sec^{\frac{3}{2}}(c + dx)(A + B \sec(c + dx))}{(a + b \sec(c + dx))^{5/2}} dx \\
&= \frac{2a(Ab - aB) \sin(c + dx)}{3b(a^2 - b^2) d \sqrt{\cos(c + dx)} (a + b \sec(c + dx))^{3/2}} + \frac{(2\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)})}{(a + b \sec(c + dx))^{5/2}} \\
&= \frac{2a(Ab - aB) \sin(c + dx)}{3b(a^2 - b^2) d \sqrt{\cos(c + dx)} (a + b \sec(c + dx))^{3/2}} + \frac{2(2a^2 Ab + 2Ab^3)}{3b(a^2 - b^2)^2 d \sqrt{\cos(c + dx)}} \\
&= \frac{2a(Ab - aB) \sin(c + dx)}{3b(a^2 - b^2) d \sqrt{\cos(c + dx)} (a + b \sec(c + dx))^{3/2}} + \frac{2(2a^2 Ab + 2Ab^3)}{3b(a^2 - b^2)^2 d \sqrt{\cos(c + dx)}} \\
&= \frac{2a(Ab - aB) \sin(c + dx)}{3b(a^2 - b^2) d \sqrt{\cos(c + dx)} (a + b \sec(c + dx))^{3/2}} + \frac{2(2a^2 Ab + 2Ab^3)}{3b(a^2 - b^2)^2 d \sqrt{\cos(c + dx)}} \\
&= \frac{2a(Ab - aB) \sin(c + dx)}{3b(a^2 - b^2) d \sqrt{\cos(c + dx)} (a + b \sec(c + dx))^{3/2}} + \frac{2(2a^2 Ab + 2Ab^3)}{3b(a^2 - b^2)^2 d \sqrt{\cos(c + dx)}} \\
&= \frac{2a(Ab - aB) \sin(c + dx)}{3b(a^2 - b^2) d \sqrt{\cos(c + dx)} (a + b \sec(c + dx))^{3/2}} + \frac{2(2a^2 Ab + 2Ab^3)}{3b(a^2 - b^2)^2 d \sqrt{\cos(c + dx)}} \\
&= \frac{2(Ab - aB) \sqrt{\frac{b+a \cos(c+dx)}{a+b}} F\left(\frac{1}{2}(c + dx) \middle| \frac{2a}{a+b}\right)}{3a(a^2 - b^2) d \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}} - \frac{2(3a^2 A + Ab^2 - 4Ab^3)}{3b(a^2 - b^2)^2 d \sqrt{\cos(c + dx)}}
\end{aligned}$$

Mathematica [C] time = 16.80, size = 487, normalized size = 1.48

$$\frac{(a \cos(c + dx) + b)^3 \left(\frac{2(Ab \sin(c+dx) - aB \sin(c+dx))}{3(b^2 - a^2)(a \cos(c+dx) + b)^2} + \frac{2(3a^2 A \sin(c+dx) - 4abB \sin(c+dx) + Ab^2 \sin(c+dx))}{3(b^2 - a^2)^2 (a \cos(c+dx) + b)} \right)}{d \cos^{\frac{5}{2}}(c + dx)(a + b \sec(c + dx))^{5/2}} + \frac{2 \cos^{\frac{3}{2}}(c + dx) \sec^{\frac{5}{2}}(c + dx)}{(a + b \sec(c + dx))^{5/2}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(5/2)),x]

[Out] ((b + a*Cos[c + d*x])^3*((2*(A*b*Sin[c + d*x] - a*B*Sin[c + d*x]))/(3*(-a^2 + b^2)*(b + a*Cos[c + d*x])^2) + (2*(3*a^2*A*Sin[c + d*x] + A*b^2*Sin[c +

$$d*x] - 4*a*b*B*\sin[c + d*x]))/(3*(-a^2 + b^2)^2*(b + a*\cos[c + d*x])))/(d*\cos[c + d*x]^{(5/2)}*(a + b*\sec[c + d*x])^{(5/2)} + (2*\cos[c + d*x]^{(3/2)}*(b + a*\cos[c + d*x])^2*\sec[c + d*x]^{(5/2)}*(\cos[(c + d*x)/2]^2*\sec[c + d*x])^{(3/2)}*((-1)*(a + b)*(3*a^2*A + A*b^2 - 4*a*b*B)*\text{EllipticE}[I*\text{ArcSinh}[\text{Tan}[(c + d*x)/2]], (-a + b)/(a + b)]*\sec[(c + d*x)/2]^2*\sqrt{((b + a*\cos[c + d*x])*\sec[(c + d*x)/2]^2)/(a + b)} + I*a*(a + b)*(3*a*A + A*b - a*B - 3*b*B)*\text{EllipticF}[I*\text{ArcSinh}[\text{Tan}[(c + d*x)/2]], (-a + b)/(a + b)]*\sec[(c + d*x)/2]^2*\sqrt{((b + a*\cos[c + d*x])*\sec[(c + d*x)/2]^2)/(a + b)} - (3*a^2*A + A*b^2 - 4*a*b*B)*(b + a*\cos[c + d*x])*(\sec[(c + d*x)/2]^2)^{(3/2)}*\text{Tan}[(c + d*x)/2]))/(3*a*(a^2 - b^2)^2*d*(a + b*\sec[c + d*x])^{(5/2)})$$

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B \sec(dx + c) + A)\sqrt{b \sec(dx + c) + a} \sqrt{\cos(dx + c)}}{b^3 \cos(dx + c)^2 \sec(dx + c)^3 + 3ab^2 \cos(dx + c)^2 \sec(dx + c)^2 + 3a^2b \cos(dx + c)^2 \sec(dx + c) + a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")

[Out] integral((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c))/(b^3*cos(d*x + c)^2*sec(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2*sec(d*x + c)^2 + 3*a^2*b*cos(d*x + c)^2*sec(d*x + c) + a^3*cos(d*x + c)^2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(b \sec(dx + c) + a)^{\frac{5}{2}} \cos(dx + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(3/2)), x)

maple [B] time = 2.29, size = 1925, normalized size = 5.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x)

```
[Out] 2/3/d*(-1+cos(d*x+c))*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(1+cos(d*x+c))^2*
(-3*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^2*b*(1/(1+cos(d*x+c)))^(1/2)+3*A*s
in(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+
c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^3+
3*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^2*b*(1/(1+cos(d*x+c)))^(1/2)-A*((a-b)/
(a+b))^(1/2)*cos(d*x+c)*a*b^2*(1/(1+cos(d*x+c)))^(1/2)+4*B*((a-b)/(a+b))^(1
/2)*cos(d*x+c)*a^2*b*(1/(1+cos(d*x+c)))^(1/2)-4*B*((a-b)/(a+b))^(1/2)*cos(d
*x+c)*a*b^2*(1/(1+cos(d*x+c)))^(1/2)+B*sin(d*x+c)*EllipticF((-1+cos(d*x+c))
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*a^2*b+4*B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-
(a+b)/(a-b))^(1/2))*a*b^2+3*A*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/
(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*a^2*b-A*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))
^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*a*b^2-3*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(
1/2))*a^2*b-B*((a-b)/(a+b))^(1/2)*a^3*(1/(1+cos(d*x+c)))^(1/2)-A*((a-b)/(a
+b))^(1/2)*b^3*(1/(1+cos(d*x+c)))^(1/2)+B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*
a^3*(1/(1+cos(d*x+c)))^(1/2)+3*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^3*(1/(1
+cos(d*x+c)))^(1/2)-3*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^3*(1/(1+cos(d*x+c)
))^(1/2)+A*((a-b)/(a+b))^(1/2)*cos(d*x+c)*b^3*(1/(1+cos(d*x+c)))^(1/2)-2*A*
((a-b)/(a+b))^(1/2)*a^2*b*(1/(1+cos(d*x+c)))^(1/2)+A*((a-b)/(a+b))^(1/2)*a*
b^2*(1/(1+cos(d*x+c)))^(1/2)-B*((a-b)/(a+b))^(1/2)*a^2*b*(1/(1+cos(d*x+c)))
^(1/2)+4*B*((a-b)/(a+b))^(1/2)*a*b^2*(1/(1+cos(d*x+c)))^(1/2)-A*sin(d*x+c)*
((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a
-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3-3*B*sin(d*x+c)*Ellipt
icF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((
b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*b^2-3*B*sin(d*x+c)*cos(d*x+c)
*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1
/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b+4*B*sin(d*x+c)*cos
(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x
+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b-3*A*cos(d*x
+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a^3+B*sin(
d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^3-A*c
os(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*b*(1/(1+cos(d*x+c)))^(1/2)-A*sin(d*x+c)
*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos
(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2-A*sin(d
*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-
(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b)*c
os(d*x+c)^(1/2)*((a-b)/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)/a/(a+b)/(a-b)^
2/(b+a*cos(d*x+c))^2/sin(d*x+c)^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(b \sec(dx + c) + a)^{\frac{5}{2}} \cos(dx + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(3/2)), x)

mpad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\cos(c+dx)^{3/2} \left(a + \frac{b}{\cos(c+dx)}\right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^(5/2)),x)

[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^(5/2)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(3/2)/(a+b*sec(d*x+c))**(5/2),x)

[Out] Timed out

$$3.633 \quad \int \frac{A+B \sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b \sec(c+dx))^{5/2}} dx$$

Optimal. Leaf size=399

$$\frac{2a(Ab - aB) \sin(c + dx)}{3bd(a^2 - b^2) \cos^{\frac{3}{2}}(c + dx)(a + b \sec(c + dx))^{3/2}} + \frac{2(Ab - aB) \sqrt{\frac{a \cos(c+dx)+b}{a+b}} F\left(\frac{1}{2}(c + dx) \middle| \frac{2a}{a+b}\right)}{3bd(a^2 - b^2) \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}} - \frac{2a(3a^3B)}{3b^2d(a^2 - b^2)^2}$$

[Out] $\frac{2}{3}a*(A*b-B*a)*\sin(d*x+c)/b/(a^2-b^2)/d/\cos(d*x+c)^{(3/2)}/(a+b*\sec(d*x+c))^{(3/2)} - \frac{2}{3}a*(4*A*b^3+3*B*a^3-7*B*a*b^2)*\sin(d*x+c)/b^2/(a^2-b^2)^2/d/\cos(d*x+c)^{(1/2)}/(a+b*\sec(d*x+c))^{(1/2)} + \frac{2}{3}*(A*b-B*a)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)}*(a/(a+b))^{(1/2)})*((b+a*\cos(d*x+c))/(a+b))^{(1/2)}/b/(a^2-b^2)/d/\cos(d*x+c)^{(1/2)}/(a+b*\sec(d*x+c))^{(1/2)} + 2*B*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticPi}(\sin(1/2*d*x+1/2*c), 2^{(1/2)}*(a/(a+b))^{(1/2)})*((b+a*\cos(d*x+c))/(a+b))^{(1/2)}/b^2/d/\cos(d*x+c)^{(1/2)}/(a+b*\sec(d*x+c))^{(1/2)} + \frac{2}{3}*(4*A*b^3+3*B*a^3-7*B*a*b^2)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)}*(a/(a+b))^{(1/2)})*\cos(d*x+c)^{(1/2)}*(a+b*\sec(d*x+c))^{(1/2)}/b^2/(a^2-b^2)^2/d/((b+a*\cos(d*x+c))/(a+b))^{(1/2)}$

Rubi [A] time = 1.50, antiderivative size = 399, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 14, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2955, 4029, 4098, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2a(Ab - aB) \sin(c + dx)}{3bd(a^2 - b^2) \cos^{\frac{3}{2}}(c + dx)(a + b \sec(c + dx))^{3/2}} - \frac{2a(3a^3B - 7ab^2B + 4Ab^3) \sin(c + dx)}{3b^2d(a^2 - b^2)^2 \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}} + \frac{2(Ab - aB)}{3bd(a^2 - b^2)}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2)),x]

[Out] $(2*(A*b - a*B)*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/(a + b)]*\text{EllipticF}[(c + d*x)/2, (2*a)/(a + b)])/(3*b*(a^2 - b^2)*d*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]) + (2*B*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/(a + b)]*\text{EllipticPi}[2, (c + d*x)/2, (2*a)/(a + b)])/(b^2*d*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]) + (2*(4*A*b^3 + 3*a^3*B - 7*a*b^2*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, (2*a)/(a + b)]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]])/(3*b^2*(a^2 - b^2)^2*d*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/(a + b)]) + (2*a*(A*b - a*B)*\text{Sin}[c + d*x])/(3*b*(a^2 - b^2)*d*\text{Cos}[c + d*x]^(3/2)*(a + b*\text{Sec}[c + d*x])^(3/2)) - (2*a*(4*A*b^3 + 3*a^3*B - 7*a*b^2*B)*\text{Sin}[c + d*x])/(3*b^2*(a^2 - b^2)^2*d*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]])$

Rule 2653

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*Sqrt[a
+ b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
0] && !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)]/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a
+ b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_)
+ (f_)*(x_)]]), x_Symbol] := Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)]/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_)
+ (f_)*(x_)]]), x_Symbol] := Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
+ f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !GtQ[c + d, 0]
```

Rule 2955

```
Int[((a_) + csc[(e_) + (f_)*(x_)])*(b_)^(m_)*(csc[(e_) + (f_)*(x_)])*
```

```
(d_.) + (c_.))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_.)]^(p_.), x_Symbol] := Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]/Sqrt[csc[(e_.) + (f_.)*(x_.)
*(d_.)], x_Symbol] := Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.)
+ (a_.)], x_Symbol] := Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.)
+ (a_.)], x_Symbol] := Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]]),
x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4029

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (
a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := Simp[(a*d^2*
(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n -
2))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[d/(b*(m + 1)*(a^2 - b^2)), Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(n
- 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2*
(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n,
1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.))/(Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]), x_Symbol] := Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
```

a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]

Rule 4098

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)])*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] := -Simp[(d*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b
*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1))/(b*f*(a^2 - b^2)*(m + 1)),
x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*C
sc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A -
b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n + b
^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C},
x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)])*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] := Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
+ b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{A + B \sec(c + dx)}{\cos^{\frac{5}{2}}(c + dx)(a + b \sec(c + dx))^{5/2}} dx &= \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{\sec^{\frac{5}{2}}(c + dx)(A + B \sec(c + dx))}{(a + b \sec(c + dx))^{5/2}} dx \\
&= \frac{2a(Ab - aB) \sin(c + dx)}{3b(a^2 - b^2) d \cos^{\frac{3}{2}}(c + dx)(a + b \sec(c + dx))^{3/2}} + \frac{(2\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)})}{3b^2(a^2 - b^2)^2 d \sqrt{\cos(c + dx)}} \\
&= \frac{2a(Ab - aB) \sin(c + dx)}{3b(a^2 - b^2) d \cos^{\frac{3}{2}}(c + dx)(a + b \sec(c + dx))^{3/2}} - \frac{2a(4Ab^3 + 3a^2b)}{3b^2(a^2 - b^2)^2 d \sqrt{\cos(c + dx)}} \\
&= \frac{2a(Ab - aB) \sin(c + dx)}{3b(a^2 - b^2) d \cos^{\frac{3}{2}}(c + dx)(a + b \sec(c + dx))^{3/2}} - \frac{2a(4Ab^3 + 3a^2b)}{3b^2(a^2 - b^2)^2 d \sqrt{\cos(c + dx)}} \\
&= \frac{2a(Ab - aB) \sin(c + dx)}{3b(a^2 - b^2) d \cos^{\frac{3}{2}}(c + dx)(a + b \sec(c + dx))^{3/2}} - \frac{2a(4Ab^3 + 3a^2b)}{3b^2(a^2 - b^2)^2 d \sqrt{\cos(c + dx)}} \\
&= \frac{2a(Ab - aB) \sin(c + dx)}{3b(a^2 - b^2) d \cos^{\frac{3}{2}}(c + dx)(a + b \sec(c + dx))^{3/2}} - \frac{2a(4Ab^3 + 3a^2b)}{3b^2(a^2 - b^2)^2 d \sqrt{\cos(c + dx)}} \\
&= \frac{2a(Ab - aB) \sin(c + dx)}{3b(a^2 - b^2) d \cos^{\frac{3}{2}}(c + dx)(a + b \sec(c + dx))^{3/2}} - \frac{2a(4Ab^3 + 3a^2b)}{3b^2(a^2 - b^2)^2 d \sqrt{\cos(c + dx)}} \\
&= \frac{2B \sqrt{\frac{b+a \cos(c+dx)}{a+b}} \Pi\left(2, \frac{1}{2}(c + dx) \middle| \frac{2a}{a+b}\right)}{b^2 d \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}} + \frac{2a(Ab - aB) \sin(c + dx)}{3b(a^2 - b^2) d \cos^{\frac{3}{2}}(c + dx)(a + b \sec(c + dx))^{3/2}} \\
&= \frac{2(Ab - aB) \sqrt{\frac{b+a \cos(c+dx)}{a+b}} F\left(\frac{1}{2}(c + dx) \middle| \frac{2a}{a+b}\right)}{3b(a^2 - b^2) d \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}} + \frac{2B \sqrt{\frac{b+a \cos(c+dx)}{a+b}} \Pi\left(2, \frac{1}{2}(c + dx) \middle| \frac{2a}{a+b}\right)}{b^2 d \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}}
\end{aligned}$$

Mathematica [C] time = 34.31, size = 97528, normalized size = 244.43

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2)), x]

[Out] Result too large to show

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(b \sec(dx + c) + a)^{\frac{5}{2}} \cos(dx + c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(5/2)), x)

maple [C] time = 3.04, size = 3159, normalized size = 7.92

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x)

[Out]
$$\begin{aligned} & 2/3/d*(-1+\cos(d*x+c))*((b+a*\cos(d*x+c))/\cos(d*x+c))^{1/2}*(1+\cos(d*x+c))^{2*} \\ & (-3*A*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{1/2}*EllipticF((-1+\cos(d*x+c) \\ &))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (- (a+b)/(a-b))^{1/2})*\cos(d*x+c)*\sin(d*x+ \\ & c)*a*b^3+6*B*EllipticPi((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (a+b) \\ &)/(a-b), I/((a-b)/(a+b))^{1/2})*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{1/2} \\ & *\cos(d*x+c)*\sin(d*x+c)*a^3*b-6*B*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{1/2} \\ & *EllipticPi((-1+\cos(d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (a+b)/(a-b), \\ & I/((a-b)/(a+b))^{1/2})*\cos(d*x+c)*\sin(d*x+c)*a^2*b^2-6*B*EllipticPi((-1+\cos \\ & (d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (a+b)/(a-b), I/((a-b)/(a+b))^{1/2})* \\ & ((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{1/2}*\cos(d*x+c)*\sin(d*x+c)*a*b^3+9 \\ & *B*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{1/2}*EllipticF((-1+\cos(d*x+c))* \\ & ((a-b)/(a+b))^{1/2}/\sin(d*x+c), (- (a+b)/(a-b))^{1/2})*\cos(d*x+c)*\sin(d*x+c)* \\ & a^2*b^2+3*B*((b+a*\cos(d*x+c))/(1+\cos(d*x+c)))/(a+b))^{1/2}*EllipticF((-1+\cos \\ & (d*x+c))*((a-b)/(a+b))^{1/2}/\sin(d*x+c), (- (a+b)/(a-b))^{1/2})*\cos(d*x+c)*\sin \end{aligned}$$

$$\begin{aligned}
& n(d*x+c)*a*b^3+4*A*((a-b)/(a+b))^{(1/2)}*b^4*(1/(1+\cos(d*x+c)))^{(1/2)}-7*B*((b \\
& +a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{(1/2)}*EllipticE((-1+\cos(d*x+c))*((a-b) \\
& / (a+b))^{(1/2)}/\sin(d*x+c), (- (a+b)/(a-b))^{(1/2)})*\sin(d*x+c)*a*b^3-6*B*Ellipti \\
& cF((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c), (- (a+b)/(a-b))^{(1/2)})*((b \\
& +a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{(1/2)}*\sin(d*x+c)*a^3*b-4*B*EllipticF((\\
& -1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c), (- (a+b)/(a-b))^{(1/2)})*((b+a*c \\
& os(d*x+c))/(1+\cos(d*x+c))/(a+b))^{(1/2)}*\sin(d*x+c)*a^2*b^2+A*EllipticF((-1+c \\
& os(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c), (- (a+b)/(a-b))^{(1/2)})*((b+a*\cos(d \\
& *x+c))/(1+\cos(d*x+c))/(a+b))^{(1/2)}*\sin(d*x+c)*a*b^3-3*A*((a-b)/(a+b))^{(1/2)} \\
& *cos(d*x+c)^2*a*b^3*(1/(1+\cos(d*x+c)))^{(1/2)}+6*B*((a-b)/(a+b))^{(1/2)}*cos(d* \\
& x+c)^2*a^2*b^2*(1/(1+\cos(d*x+c)))^{(1/2)}+4*A*((a-b)/(a+b))^{(1/2)}*cos(d*x+c)* \\
& a*b^3*(1/(1+\cos(d*x+c)))^{(1/2)}-3*B*((a-b)/(a+b))^{(1/2)}*cos(d*x+c)*a^3*b*(1/ \\
& (1+\cos(d*x+c)))^{(1/2)}-7*B*((a-b)/(a+b))^{(1/2)}*cos(d*x+c)*a^2*b^2*(1/(1+\cos(\\
& d*x+c)))^{(1/2)}+7*B*((a-b)/(a+b))^{(1/2)}*cos(d*x+c)*a*b^3*(1/(1+\cos(d*x+c)))^{(\\
& 1/2)}+3*B*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{(1/2)}*EllipticE((-1+\cos(d \\
& *x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c), (- (a+b)/(a-b))^{(1/2)})*\sin(d*x+c)*a^3* \\
& b-6*B*EllipticF((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c), (- (a+b)/(a-b) \\
&))^{(1/2)}*cos(d*x+c)*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{(1/2)}*\sin(d*x+ \\
& c)*a^4+3*B*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{(1/2)}*EllipticE((-1+\cos(\\
& d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c), (- (a+b)/(a-b))^{(1/2)})*cos(d*x+c)*\sin \\
& (d*x+c)*a^4+A*cos(d*x+c)^2*((a-b)/(a+b))^{(1/2)}*a^2*b^2*(1/(1+\cos(d*x+c)))^{(\\
& 1/2)}-B*cos(d*x+c)^2*((a-b)/(a+b))^{(1/2)}*a^3*b*(1/(1+\cos(d*x+c)))^{(1/2)}-3*A* \\
& ((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{(1/2)}*EllipticF((-1+\cos(d*x+c))*((a \\
& -b)/(a+b))^{(1/2)}/\sin(d*x+c), (- (a+b)/(a-b))^{(1/2)})*\sin(d*x+c)*b^4-6*B*Ellipt \\
& icPi((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c), (a+b)/(a-b), I/((a-b)/(a \\
& +b))^{(1/2)})*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{(1/2)}*\sin(d*x+c)*b^4+3* \\
& B*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{(1/2)}*EllipticF((-1+\cos(d*x+c))* \\
& (a-b)/(a+b))^{(1/2)}/\sin(d*x+c), (- (a+b)/(a-b))^{(1/2)})*\sin(d*x+c)*b^4+4*A*((b+ \\
& a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{(1/2)}*EllipticE((-1+\cos(d*x+c))*((a-b)/ \\
& (a+b))^{(1/2)}/\sin(d*x+c), (- (a+b)/(a-b))^{(1/2)})*cos(d*x+c)*\sin(d*x+c)*a*b^3-4 \\
& *B*EllipticF((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c), (- (a+b)/(a-b))^{(\\
& 1/2)})*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{(1/2)}*cos(d*x+c)*\sin(d*x+c)* \\
& a^3*b-7*B*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{(1/2)}*EllipticE((-1+\cos(d \\
& *x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c), (- (a+b)/(a-b))^{(1/2)})*cos(d*x+c)*\sin(\\
& d*x+c)*a^2*b^2+A*EllipticF((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c), (\\
& - (a+b)/(a-b))^{(1/2)})*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a+b))^{(1/2)}*cos(d*x+ \\
& c)*\sin(d*x+c)*a^2*b^2+9*B*\sin(d*x+c)*EllipticF((-1+\cos(d*x+c))*((a-b)/(a+b) \\
&))^{(1/2)}/\sin(d*x+c), (- (a+b)/(a-b))^{(1/2)})*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(\\
& a+b))^{(1/2)}*a*b^3+6*B*EllipticPi((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d* \\
& x+c), (a+b)/(a-b), I/((a-b)/(a+b))^{(1/2)})*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(a \\
& +b))^{(1/2)}*cos(d*x+c)*\sin(d*x+c)*a^4+6*B*((b+a*\cos(d*x+c))/(1+\cos(d*x+c))/(\\
& a+b))^{(1/2)}*EllipticPi((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d*x+c), (a+b) \\
& / (a-b), I/((a-b)/(a+b))^{(1/2)})*\sin(d*x+c)*a^3*b+6*B*((b+a*\cos(d*x+c))/(1+\cos \\
& (d*x+c))/(a+b))^{(1/2)}*EllipticPi((-1+\cos(d*x+c))*((a-b)/(a+b))^{(1/2)}/\sin(d* \\
& x+c), (a+b)/(a-b), I/((a-b)/(a+b))^{(1/2)})*\sin(d*x+c)*a^2*b^2-6*B*EllipticPi((
\end{aligned}$$

$$\begin{aligned}
 & -1 + \cos(dx+c) \cdot \left(\frac{a-b}{a+b}\right)^{1/2} / \sin(dx+c), (a+b)/(a-b), I/\left(\frac{a-b}{a+b}\right)^{1/2} \\
 & \cdot \left(\frac{b+a\cos(dx+c)}{1+\cos(dx+c)}\right) / (a+b)^{1/2} \cdot \sin(dx+c) \cdot a^3 b^4 A \cdot \left(\frac{b+a\cos(dx+c)}{1+\cos(dx+c)}\right) / (a+b)^{1/2} \\
 & \cdot \text{EllipticE}\left(\frac{-1+\cos(dx+c)}{\left(\frac{a-b}{a+b}\right)^{1/2} / \sin(dx+c)}, \left(-\frac{a+b}{a-b}\right)^{1/2}\right) \cdot \sin(dx+c) \cdot b^4 - 3B \cdot \left(\frac{a-b}{a+b}\right)^{1/2} \\
 & \cdot \cos(dx+c)^2 \cdot a^4 \cdot \left(\frac{1}{1+\cos(dx+c)}\right)^{1/2} - 4A \cdot \left(\frac{a-b}{a+b}\right)^{1/2} - \frac{1}{2} \cdot \cos(dx+c) \cdot b^4 \cdot \left(\frac{1}{1+\cos(dx+c)}\right)^{1/2} \\
 & + 3B \cdot \left(\frac{a-b}{a+b}\right)^{1/2} \cdot \cos(dx+c) \cdot a^4 \cdot \left(\frac{1}{1+\cos(dx+c)}\right)^{1/2} - A \cdot \left(\frac{a-b}{a+b}\right)^{1/2} \cdot a^2 \cdot b^2 \cdot \left(\frac{1}{1+\cos(dx+c)}\right)^{1/2} \\
 & - A \cdot \left(\frac{a-b}{a+b}\right)^{1/2} \cdot a \cdot b^3 \cdot \left(\frac{1}{1+\cos(dx+c)}\right)^{1/2} + 4B \cdot \left(\frac{a-b}{a+b}\right)^{1/2} \cdot a^3 \cdot b \cdot \left(\frac{1}{1+\cos(dx+c)}\right)^{1/2} \\
 & + B \cdot \left(\frac{a-b}{a+b}\right)^{1/2} \cdot a^2 \cdot b^2 \cdot \left(\frac{1}{1+\cos(dx+c)}\right)^{1/2} - 7B \cdot \left(\frac{a-b}{a+b}\right)^{1/2} \cdot a \cdot b^3 \cdot \left(\frac{1}{1+\cos(dx+c)}\right)^{1/2} \\
 & \cdot \cos(dx+c)^{1/2} \cdot \left(\frac{a-b}{a+b}\right)^{1/2} \cdot \left(\frac{1}{1+\cos(dx+c)}\right)^{1/2} / b^2 / (a+b) / (a-b)^2 / (b+a\cos(dx+c))^2 / \sin(dx+c)^3
 \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx+c) + A}{(b \sec(dx+c) + a)^{5/2} \cos(dx+c)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(dx+c))/cos(dx+c)^(5/2)/(a+b*sec(dx+c))^(5/2),x, algorithm="maxima")

[Out] integrate((B*sec(dx+c)+A)/((b*sec(dx+c)+a)^(5/2)*cos(dx+c)^(5/2)),x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\cos(c+dx)^{5/2} \left(a + \frac{b}{\cos(c+dx)}\right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B/cos(c+dx))/(cos(c+dx)^(5/2)*(a+b/cos(c+dx))^(5/2)),x)

[Out] int((A+B/cos(c+dx))/(cos(c+dx)^(5/2)*(a+b/cos(c+dx))^(5/2)),x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(dx+c))/cos(dx+c)**(5/2)/(a+b*sec(dx+c))**(5/2),x)

[Out] Timed out

$$3.634 \quad \int \frac{A+B \sec(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+b \sec(c+dx))^{5/2}} dx$$

Optimal. Leaf size=526

$$\frac{2a(Ab - aB) \sin(c + dx)}{3bd(a^2 - b^2) \cos^{\frac{5}{2}}(c + dx)(a + b \sec(c + dx))^{3/2}} - \frac{(-5a^2B + 2aAb + 3b^2B) \sqrt{\frac{a \cos(c+dx)+b}{a+b}} F\left(\frac{1}{2}(c + dx) \middle| \frac{2a}{a+b}\right)}{3b^2d(a^2 - b^2) \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}} + \frac{2a(-}{3b^2d}$$

[Out] $2/3*a*(A*b-B*a)*\sin(d*x+c)/b/(a^2-b^2)/d/\cos(d*x+c)^{(5/2)}/(a+b*\sec(d*x+c))^{(3/2)}+2/3*a*(2*A*a^2*b-6*A*b^3-5*B*a^3+9*B*a*b^2)*\sin(d*x+c)/b^2/(a^2-b^2)^{2/d}/\cos(d*x+c)^{(3/2)}/(a+b*\sec(d*x+c))^{(1/2)}-1/3*(2*A*a*b-5*B*a^2+3*B*b^2)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)}*(a/(a+b))^{(1/2)})*((b+a*\cos(d*x+c))/(a+b))^{(1/2)}/b^2/(a^2-b^2)/d/\cos(d*x+c)^{(1/2)}/(a+b*\sec(d*x+c))^{(1/2)}+(2*A*b-5*B*a)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticPi}(\sin(1/2*d*x+1/2*c), 2^{(1/2)}*(a/(a+b))^{(1/2)})*((b+a*\cos(d*x+c))/(a+b))^{(1/2)}/b^3/d/\cos(d*x+c)^{(1/2)}/(a+b*\sec(d*x+c))^{(1/2)}-1/3*(6*A*a^3*b-14*A*a*b^3-15*B*a^4+26*B*a^2*b^2-3*B*b^4)*\sin(d*x+c)*(a+b*\sec(d*x+c))^{(1/2)}/b^3/(a^2-b^2)^2/d/\cos(d*x+c)^{(1/2)}+1/3*(6*A*a^3*b-14*A*a*b^3-15*B*a^4+26*B*a^2*b^2-3*B*b^4)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)}*(a/(a+b))^{(1/2)})*\cos(d*x+c)^{(1/2)}*(a+b*\sec(d*x+c))^{(1/2)}/b^3/(a^2-b^2)^2/d/((b+a*\cos(d*x+c))/(a+b))^{(1/2)}$

Rubi [A] time = 1.99, antiderivative size = 526, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 15, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2955, 4029, 4098, 4102, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2a(2a^2Ab - 5a^3B + 9ab^2B - 6Ab^3) \sin(c + dx)}{3b^2d(a^2 - b^2)^2 \cos^{\frac{3}{2}}(c + dx) \sqrt{a + b \sec(c + dx)}} + \frac{2a(Ab - aB) \sin(c + dx)}{3bd(a^2 - b^2) \cos^{\frac{5}{2}}(c + dx)(a + b \sec(c + dx))^{3/2}} - \frac{(6a^3Ab + 26a^2b^2B - 15a^4B - 3b^4B) \sin(c + dx)}{3b^2d(a^2 - b^2) \cos^{\frac{5}{2}}(c + dx)(a + b \sec(c + dx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^(5/2)),x]

[Out] $-((2*a*A*b - 5*a^2*B + 3*b^2*B)*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/(a + b)]*\text{EllipticF}[(c + d*x)/2, (2*a)/(a + b)])/(3*b^2*(a^2 - b^2)*d*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]) + ((2*A*b - 5*a*B)*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/(a + b)]*\text{EllipticPi}[2, (c + d*x)/2, (2*a)/(a + b)])/(b^3*d*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]) + ((6*a^3*A*b - 14*a*A*b^3 - 15*a^4*B + 26*a^2*b^2*B - 3*b^4*B)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, (2*a)/(a + b)]*\text{Sqrt}[a +$

$$\begin{aligned} & b*\text{Sec}[c + d*x]]/(3*b^3*(a^2 - b^2)^2*d*\text{Sqrt}[(b + a*\text{Cos}[c + d*x])/(a + b)] \\ &) + (2*a*(A*b - a*B)*\text{Sin}[c + d*x])/(3*b*(a^2 - b^2)*d*\text{Cos}[c + d*x]^{(5/2)}*(a \\ & + b*\text{Sec}[c + d*x])^{(3/2)}) + (2*a*(2*a^2*A*b - 6*A*b^3 - 5*a^3*B + 9*a*b^2*B) \\ &)*\text{Sin}[c + d*x])/(3*b^2*(a^2 - b^2)^2*d*\text{Cos}[c + d*x]^{(3/2)}*\text{Sqrt}[a + b*\text{Sec}[c \\ & + d*x]]) - ((6*a^3*A*b - 14*a*A*b^3 - 15*a^4*B + 26*a^2*b^2*B - 3*b^4*B)*\text{Sqrt}[a + b*\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(3*b^3*(a^2 - b^2)^2*d*\text{Sqrt}[\text{Cos}[c + d*x]]) \end{aligned}$$

Rule 2653

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*Sqrt[a + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a + b*SIN[c + d*x]]/Sqrt[(a + b*SIN[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b*SIN[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)]/(d*Sqrt[a + b]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a + b*SIN[c + d*x])/(a + b)]/Sqrt[a + b*SIN[c + d*x]], Int[1/Sqrt[a/(a + b) + (b*SIN[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi/2 + f*x))/2, (2*d)/(c + d)]/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Dist[Sqrt[(c + d*SIN[e + f*x])/(c + d)]/Sqrt
```

$[c + d \sin[e + f x]]$, $\text{Int}[1/((a + b \sin[e + f x]) \sqrt{c/(c + d) + (d \sin[e + f x])/(c + d)})], x]$, $x]$ /; $\text{FreeQ}\{a, b, c, d, e, f, x\}$ && $\text{NeQ}[b*c - a*d, 0]$ && $\text{NeQ}[a^2 - b^2, 0]$ && $\text{NeQ}[c^2 - d^2, 0]$ && $! \text{GtQ}[c + d, 0]$

Rule 2955

$\text{Int}[(a_.) + \text{csc}[e_.) + (f_.)(x_.)]*(b_.))^m_.)(\text{csc}[e_.) + (f_.)(x_.)]*(d_.) + (c_.))^n_.)(g_.)\sin[e_.) + (f_.)(x_.)]^p_.), x_Symbol] :> \text{Dist}[(g*\text{Csc}[e + f*x])^p*(g*\sin[e + f*x])^p, \text{Int}[(a + b*\text{Csc}[e + f*x])^m*(c + d*\text{Csc}[e + f*x])^n]/(g*\text{Csc}[e + f*x])^p, x], x]$ /; $\text{FreeQ}\{a, b, c, d, e, f, g, m, n, p\}, x]$ && $\text{NeQ}[b*c - a*d, 0]$ && $! \text{IntegerQ}[p]$ && $!(\text{IntegerQ}[m] \&\& \text{IntegerQ}[n])$

Rule 3856

$\text{Int}[\sqrt{\text{csc}[e_.) + (f_.)(x_.)]*(b_.) + (a_.)}/\sqrt{\text{csc}[e_.) + (f_.)(x_.)]*(d_.)}, x_Symbol] :> \text{Dist}[\sqrt{a + b*\text{Csc}[e + f*x]}/(\sqrt{d*\text{Csc}[e + f*x]}*\text{Sqrt}[b + a*\sin[e + f*x]]), \text{Int}[\sqrt{b + a*\sin[e + f*x]], x], x]$ /; $\text{FreeQ}\{a, b, d, e, f, x\}$ && $\text{NeQ}[a^2 - b^2, 0]$

Rule 3858

$\text{Int}[\sqrt{\text{csc}[e_.) + (f_.)(x_.)]*(d_.)}/\sqrt{\text{csc}[e_.) + (f_.)(x_.)]*(b_.) + (a_.)}, x_Symbol] :> \text{Dist}[(\sqrt{d*\text{Csc}[e + f*x]}*\sqrt{b + a*\sin[e + f*x]})/\sqrt{a + b*\text{Csc}[e + f*x]}, \text{Int}[1/\sqrt{b + a*\sin[e + f*x]}, x], x]$ /; $\text{FreeQ}\{a, b, d, e, f, x\}$ && $\text{NeQ}[a^2 - b^2, 0]$

Rule 3859

$\text{Int}[(\text{csc}[e_.) + (f_.)(x_.)]*(d_.))^{3/2}/\sqrt{\text{csc}[e_.) + (f_.)(x_.)]*(b_.) + (a_.)}, x_Symbol] :> \text{Dist}[(d*\sqrt{d*\text{Csc}[e + f*x]}*\sqrt{b + a*\sin[e + f*x]})/\sqrt{a + b*\text{Csc}[e + f*x]}, \text{Int}[1/(\sin[e + f*x]*\sqrt{b + a*\sin[e + f*x]}), x], x]$ /; $\text{FreeQ}\{a, b, d, e, f, x\}$ && $\text{NeQ}[a^2 - b^2, 0]$

Rule 4029

$\text{Int}[(\text{csc}[e_.) + (f_.)(x_.)]*(d_.))^n_.)(\text{csc}[e_.) + (f_.)(x_.)]*(b_.) + (a_.))^m_.)(\text{csc}[e_.) + (f_.)(x_.)]*(B_.) + (A_.)), x_Symbol] :> \text{Simp}[(a*d^2*(A*b - a*B)*\text{Cot}[e + f*x]*(a + b*\text{Csc}[e + f*x])^{m+1}*(d*\text{Csc}[e + f*x])^{n-2})/(b*f*(m+1)*(a^2 - b^2)), x] - \text{Dist}[d/(b*(m+1)*(a^2 - b^2)), \text{Int}[(a + b*\text{Csc}[e + f*x])^{m+1}*(d*\text{Csc}[e + f*x])^{n-2}*\text{Simp}[a*d*(A*b - a*B)*(n-2) + b*d*(A*b - a*B)*(m+1)*\text{Csc}[e + f*x] - (a*A*b*d*(m+n) - d*B*(a^2*(n-1) + b^2*(m+1)))*\text{Csc}[e + f*x]^2, x], x], x]$ /; $\text{FreeQ}\{a, b, d, e, f, A, B\}, x]$ && $\text{NeQ}[A*b - a*B, 0]$ && $\text{NeQ}[a^2 - b^2, 0]$ && $\text{LtQ}[m, -1]$ && $\text{GtQ}[n$

, 1]

Rule 4035

Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)]), x_Symbol] := Dist[A/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]

Rule 4098

Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^m, x_Symbol] := -Simp[(d*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1))/(b*f*(a^2 - b^2)*(m + 1)), x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A - b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]

Rule 4102

Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^m, x_Symbol] := -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 0]

Rule 4108

Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)]), x_Symbol] := Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned}
\int \frac{A + B \sec(c + dx)}{\cos^{\frac{7}{2}}(c + dx)(a + b \sec(c + dx))^{5/2}} dx &= \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{\sec^{\frac{7}{2}}(c + dx)(A + B \sec(c + dx))}{(a + b \sec(c + dx))^{5/2}} dx \\
&= \frac{2a(Ab - aB) \sin(c + dx)}{3b(a^2 - b^2) d \cos^{\frac{5}{2}}(c + dx)(a + b \sec(c + dx))^{3/2}} + \frac{(2\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)})}{3b^2(a^2 - b^2)^2 d \cos^{\frac{5}{2}}(c + dx)(a + b \sec(c + dx))^{3/2}} \\
&= \frac{2a(Ab - aB) \sin(c + dx)}{3b(a^2 - b^2) d \cos^{\frac{5}{2}}(c + dx)(a + b \sec(c + dx))^{3/2}} + \frac{2a(2a^2 Ab - 6Ab^3)}{3b^2(a^2 - b^2)^2 d \cos^{\frac{5}{2}}(c + dx)(a + b \sec(c + dx))^{3/2}} \\
&= \frac{2a(Ab - aB) \sin(c + dx)}{3b(a^2 - b^2) d \cos^{\frac{5}{2}}(c + dx)(a + b \sec(c + dx))^{3/2}} + \frac{2a(2a^2 Ab - 6Ab^3)}{3b^2(a^2 - b^2)^2 d \cos^{\frac{5}{2}}(c + dx)(a + b \sec(c + dx))^{3/2}} \\
&= \frac{2a(Ab - aB) \sin(c + dx)}{3b(a^2 - b^2) d \cos^{\frac{5}{2}}(c + dx)(a + b \sec(c + dx))^{3/2}} + \frac{2a(2a^2 Ab - 6Ab^3)}{3b^2(a^2 - b^2)^2 d \cos^{\frac{5}{2}}(c + dx)(a + b \sec(c + dx))^{3/2}} \\
&= \frac{2a(Ab - aB) \sin(c + dx)}{3b(a^2 - b^2) d \cos^{\frac{5}{2}}(c + dx)(a + b \sec(c + dx))^{3/2}} + \frac{2a(2a^2 Ab - 6Ab^3)}{3b^2(a^2 - b^2)^2 d \cos^{\frac{5}{2}}(c + dx)(a + b \sec(c + dx))^{3/2}} \\
&= \frac{2a(Ab - aB) \sin(c + dx)}{3b(a^2 - b^2) d \cos^{\frac{5}{2}}(c + dx)(a + b \sec(c + dx))^{3/2}} + \frac{2a(2a^2 Ab - 6Ab^3)}{3b^2(a^2 - b^2)^2 d \cos^{\frac{5}{2}}(c + dx)(a + b \sec(c + dx))^{3/2}} \\
&= \frac{2a(Ab - aB) \sin(c + dx)}{3b(a^2 - b^2) d \cos^{\frac{5}{2}}(c + dx)(a + b \sec(c + dx))^{3/2}} + \frac{2a(2a^2 Ab - 6Ab^3)}{3b^2(a^2 - b^2)^2 d \cos^{\frac{5}{2}}(c + dx)(a + b \sec(c + dx))^{3/2}} \\
&= \frac{(2Ab - 5aB) \sqrt{\frac{b+a \cos(c+dx)}{a+b}} \Pi\left(2; \frac{1}{2}(c + dx) \middle| \frac{2a}{a+b}\right)}{b^3 d \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}} + \frac{2a(Ab - aB) \sin(c + dx)}{3b(a^2 - b^2) d \cos^{\frac{5}{2}}(c + dx)(a + b \sec(c + dx))^{3/2}} \\
&= -\frac{(2aAb - 5a^2B + 3b^2B) \sqrt{\frac{b+a \cos(c+dx)}{a+b}} F\left(\frac{1}{2}(c + dx) \middle| \frac{2a}{a+b}\right)}{3b^2(a^2 - b^2) d \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}} + \frac{(2Ab - 5aB) \sin(c + dx)}{b^3 d \sqrt{\cos(c + dx)} \sqrt{a + b \sec(c + dx)}}
\end{aligned}$$

Mathematica [C] time = 37.10, size = 184379, normalized size = 350.53

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^(5/2)),x]

[Out] Result too large to show

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(b \sec(dx + c) + a)^{\frac{5}{2}} \cos(dx + c)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(7/2)), x)

maple [C] time = 2.48, size = 5358, normalized size = 10.19

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(5/2),x)

[Out] result too large to display

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sec(dx + c) + A}{(b \sec(dx + c) + a)^{\frac{5}{2}} \cos(dx + c)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(7/2)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + \frac{B}{\cos(c+dx)}}{\cos(c+dx)^{7/2} \left(a + \frac{b}{\cos(c+dx)}\right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))^(5/2)),x)

[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))^(5/2)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(7/2)/(a+b*sec(d*x+c))**(5/2),x)

[Out] Timed out

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)

(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*      GradeAntiderivative[result,optimal] returns*)
(* "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(* "C" if result involves higher level functions than necessary*)
(* "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(* "A" if result can be considered optimal*)

GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
```

```

If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
  If[LeafCount[result]<=2*LeafCount[optimal],
    "A",
    "B"],
  "C"],
If[FreeQ[result,Integrate] && FreeQ[result,Int],
  "C",
"F"]]

```

```

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)

```

```

ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
        Max[ExpnType[expn[[1]],2]],
      Max[ExpnType[expn[[1]],ExpnType[expn[[2]],3]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],

```



```

If[Head[expn]===RootSum,
  Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
If[Head[expn]===Integrate || Head[expn]===Int,
  Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
9]]]]]]]]]]

ElementaryFunctionQ[func_] :=
MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]

SpecialFunctionQ[func_] :=
MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]

HypergeometricFunctionQ[func_] :=
MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]

AppellFunctionQ[func_] :=
MemberQ[{AppellF1},func]

```

4.0.2 Maple grading function

```

# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017 Use Maple leaf count instead since buildin
#Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017 corrected the check for complex result
#Nasser 10/27/2017 check for leafsize and do not call ExpnType()
#
# if leaf size is "too large". Set at 500,000

```

```

#Nasser 12/22/2019 Added debug flag, added 'dilog' to special functions
# see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

    leaf_count_result:=leafcount(result);
    #do NOT call ExpnType() if leaf size is too large. Recursion problem
    if leaf_count_result > 500000 then
        return "B";
    fi;

    leaf_count_optimal:=leafcount(optimal);

    ExpnType_result:=ExpnType(result);
    ExpnType_optimal:=ExpnType(optimal);

    if debug then
        print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
            ExpnType_optimal);
    fi;

# If result and optimal are mathematical expressions,
# GradeAntiderivative[result,optimal] returns
# "F" if the result fails to integrate an expression that
# is integrable
# "C" if result involves higher level functions than necessary
# "B" if result is more than twice the size of the optimal
# antiderivative
# "A" if result can be considered optimal

#This check below actually is not needed, since I only
#call this grading only for passed integrals. i.e. I check
#for "F" before calling this. But no harm of keeping it here.
#just in case.

if not type(result,freeof('int')) then
    return "F";
end if;

if ExpnType_result<=ExpnType_optimal then
    if debug then
        print("ExpnType_result<=ExpnType_optimal");
    fi;

```

```

if is_contains_complex(result) then
  if is_contains_complex(optimal) then
    if debug then
      print("both result and optimal complex");
    fi;
    #both result and optimal complex
    if leaf_count_result<=2*leaf_count_optimal then
      return "A";
    else
      return "B";
    end if
  else #result contains complex but optimal is not
    if debug then
      print("result contains complex but optimal is not");
    fi;
    return "C";
  end if
else # result do not contain complex
  # this assumes optimal do not as well
  if debug then
    print("result do not contain complex, this assumes optimal do not
as well");
  fi;
  if leaf_count_result<=2*leaf_count_optimal then
    if debug then
      print("leaf_count_result<=2*leaf_count_optimal");
    fi;
    return "A";
  else
    if debug then
      print("leaf_count_result>2*leaf_count_optimal");
    fi;
    return "B";
  end if
end if
else #ExpnType(result) > ExpnType(optimal)
  if debug then
    print("ExpnType(result) > ExpnType(optimal)");
  fi;
  return "C";
end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false

```

```

#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1
    else
      max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'^^') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1
      else
        max(2,ExpnType(op(1,expn)))
      end if
    else
      max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  end if
end proc:

```

```

elif HypergeometricFunctionQ(op(0,expn)) then
  max(5,apply(max,map(ExpnType,[op(expn)])))
elif AppellFunctionQ(op(0,expn)) then
  max(6,apply(max,map(ExpnType,[op(expn)])))
elif op(0,expn)='int' then
  max(8,apply(max,map(ExpnType,[op(expn)]))) else
9
end if
end proc:

ElementaryFunctionQ := proc(func)
  member(func,[
    exp,log,ln,
    sin,cos,tan,cot,sec,csc,
    arcsin,arccos,arctan,arccot,arcsec,arccsc,
    sinh,cosh,tanh,coth,sech,csch,
    arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
    erf,erfc,erfi,
    FresnelS,FresnelC,
    Ei,Ei,Li,Si,Ci,Shi,Chi,
    GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
    EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product. rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
    op(2,u)
  else
    apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

```

```
#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17 Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
    MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#           Port of original Maple grading function by
#           Albert Rich to use with Sympy/Python
#Dec 27, 2019 Nasser. Added `RootSum`. See problem 177, Timofeev file
#           added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
        asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
        asinh,acosh,atanh,acoth,asech,acsch
    ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
        fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
        gamma,loggamma,digamma,zeta,polylog,LambertW,
        elliptic_f,elliptic_e,elliptic_pi,exp_polar
    ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow): #type(expn,'`^`')
        if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')
            return expnType(expn.args[0]) #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)
))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type
(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))

```

```

elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
expn))
    m1 = max(map(expnType, list(expn.args)))
    return max(5,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
elif is_appell_function(expn.func):
    m1 = max(map(expnType, list(expn.args)))
    return max(6,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
elif isinstance(expn,RootSum):
    m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
Apply[List,expn]],7]],
    return max(7,m1)
elif str(expn).find("Integral") != -1:
    m1 = max(map(expnType, list(expn.args)))
    return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"

```


4.0.4 SageMath grading function

```

#Dec 24, 2019. Nasser: Ported original Maple grading function by
#           Albert Rich to use with Sagemath. This is used to
#           grade Fracas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#           'arctan2', 'floor', 'abs', 'log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow: #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
        'sin','cos','tan','cot','sec','csc',
        'arcsin','arccos','arctan','arccot','arcsec','arccsc',
        'sinh','cosh','tanh','coth','sech','csch',
        'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
    ]
    if debug:
        if m:

```

```

        print ("func ", func , " is elementary_function")
    else:
        print ("func ", func , " is NOT elementary_function")

    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
        'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
sinh_integral'
        'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
        'polylog','lambert_w','elliptic_f','elliptic_e',
        'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")

    return m

def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in
sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:

```

```

        return expn.operator() is None
    if expn.parent() in (ZZ, QQ, AA, QQbar):
        return expn in expn.parent() # Should always return True
    if hasattr(expn.parent(), "base_ring") and hasattr(expn.parent(), "gens"):
        return expn in expn.parent().base_ring() or expn in expn.parent().
gens()
    return False

except AttributeError as error:
    return False

def expnType(expn):

    if debug:
        print(">>>>Enter expnType, expn=", expn)
        print(">>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list: #instance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(instance(expn.args[0],
Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
    elif expn.operator() == operator.pow: #instance(expn,Pow)
        if type(expn.operands()[1])==Integer: #instance(expn.args[1],Integer)
            return expnType(expn.operands()[0]) #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational: #instance(expn.args[1],
Rational)
            if type(expn.operands()[0])==Rational: #instance(expn.args[0],
Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
instance(expn,Add) or instance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))

```

```

    return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()): #is_elementary_function(expn.
func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
    return max(4,m1) #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #
is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
    return max(5,m1) #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
    return max(6,m1) #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
#is checked before calling the grading function that is passed.
#but kept it here.
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
    return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

```

```
#main function
```

```
def grade_antiderivative(result,optimal):
```

```

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
leaf_count_optimal=",leaf_count_optimal)

```

```

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

```

```

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
expnType_optimal)

```

```

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex

```

```
        if leaf_count_result <= 2*leaf_count_optimal:
            return "A"
        else:
            return "B"
    else: #result contains complex but optimal is not
        return "C"
else: # result do not contain complex, this assumes optimal do not as
well
    if leaf_count_result <= 2*leaf_count_optimal:
        return "A"
    else:
        return "B"
else:
    return "C"
```